OPTIMIZATION OF A MODEL IV

FLUIDIZED CATALYTIC CRACKING UNIT

by

Ramasubramanian Sundaralingam

A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
Department of Chemical Engineering and Applied Chemistry.
in the University of Toronto

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ABSTRACT

Maximization of a profit function related to a fluidized catalytic cracking unit model combined with a four-lump yield model is carried out by Luus-Jaakola optimization procedure. The 108 nonlinear algebraic equations and 9 ordinary differential equations are grouped into a set of 105 simple equations and a set of 12 difficult algebraic equations.

Despite the low sensitivity and the existence of several local optima, the global optimum was obtained quite easily. When the air blowers were constrained to operate at their maximum capacity, the optimum was 1% lower than the global optimum when the air flow rates were allowed to be free variables. The results show that profit from the unit can be increased by 2.7% by operating the unit closer to the regenerator combustion constraint.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>i</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. PROBLEM FORMULATION</td>
<td>9</td>
</tr>
<tr>
<td>2.1 Problem statement</td>
<td>10</td>
</tr>
<tr>
<td>2.2 Solving the nonlinear algebraic equations and formulating the optimization problem</td>
<td>12</td>
</tr>
<tr>
<td>2.3 Luus Jaakola optimization procedure</td>
<td>39</td>
</tr>
<tr>
<td>3. RESULTS AND DISCUSSIONS</td>
<td>41</td>
</tr>
<tr>
<td>3.1 Effect of the penalty factors</td>
<td>41</td>
</tr>
<tr>
<td>3.2 Effect of the number of random points</td>
<td>45</td>
</tr>
<tr>
<td>3.3 Search for the global optimum</td>
<td>49</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1.1: Schematic of a model IV FCC unit .............................................. 2
Figure 2.1: The four lump yield model .......................................................... 9
Figure 3.1.1: Effect of $\theta$ on the obtained value of the performance index .......... 44
Figure 3.2.1: Effect of number of random points on the obtained value of the performance index .......................................................................................... 48
Figure 3.4.1. Effect of slurry recycle coking factor on the obtained value of the performance index .......................................................................................... 61
Figure 3.4.2. Effect of relaxing the regenerator temperature constraint on the obtained value of the performance index ................................................................. 62
LIST OF TABLES

Table 3.1.1: Variation of performance index and control variables with the penalty factor
........................................................................................................................................ 43

Table 3.2.1: Effect of the number of random points on the obtained value of the performance index and the corresponding control and state variable values
........................................................................................................................................ 46

Table 3.2.2: Effect of the number of random points on the obtained value of the performance index and the corresponding control and state variable values
........................................................................................................................................ 47

Table 3.3.1: Effect of the number of random points on the obtained value of the performance index and the corresponding control and state variable values (case 1)
........................................................................................................................................ 50

Table 3.3.2: Effect of the number of random points on the obtained value of the performance index and the corresponding control and state variable values (case 1)
........................................................................................................................................ 51

Table 3.3.3: Effect of the number of random points on the obtained value of the performance index and the corresponding control and state variable values (case 2)
........................................................................................................................................ 52

Table 3.3.4: Effect of the number of random points on the obtained value of the performance index and the corresponding control and state variable values (case 2)
........................................................................................................................................ 53
Table 3.3.5: Obtained values of performance index, control and state variable values for the global optimum, maximum airflow and for a five dimensional search ................. 56

Table 3.3.6: Obtained values of performance index, control and state variable values at five different local optimum ........................................................................ 57

Table 3.4.1: Effect of variables on the obtained value of the performance index ....... 60
1. INTRODUCTION

In refineries, fluidized catalytic cracking (FCC) is a key process for converting high molecular weight hydrocarbons into lighter more valuable products through contact with catalyst at appropriate process conditions. The FCC unit is a nonlinear process with strong interaction among its various sections and subject to a number of operational constraints that limits the unit production rate. A typical FCC in a refinery receives multiple feeds from different sections of the refinery. Any change in the feed source affects the operation of the FCC unit. The operation of the unit is strongly influenced by factors like ambient temperature, type of catalyst and the operating procedure. Additionally, refinery products are subject to varying demand and the associated price variations (e.g., in Canada, there is a high demand for fuel oil in winter due to heating requirements and lower in summer) dictate the point of operation for the FCC unit. Because of their large throughput, optimization of the operation of FCC units can result in substantial economic benefits. The purpose of this thesis is to investigate the use of direct search optimization to establish the optimum operating point for a Model IV FCC unit, and to study the effect of some of the key variables on the optimum operating point.

The schematic of the Model IV FCC unit is shown in Figure 1.1. Fresh feed preheated in the furnace is mixed with hot slurry recycle from the bottom of the main fractionator and injected into the reactor riser where it mixes with the hot catalyst and is vaporized. In the reactor riser, the high molecular weight feed is partly cracked to the low molecular weight products. The hot catalyst provides the heat for this endothermic cracking reaction. During cracking, coke (C and 5-10% hydrogen) is deposited on the catalyst, reducing the catalyst activity.
Product gas from the reactor goes to the main fractionator for heat recovery and separation into various product streams. Wet gas ($C_4$ and lighter) from the main fractionator is compressed for further separation in downstream fractionators. The entrained catalyst is
separated from the product gas in the cyclones at the top of the reactor and returned to the stripping section of the reactor where steam is injected to strip off the entrained hydrocarbon from the catalyst. The hydrogen in the coke comes from the hydrocarbons entrained in the catalyst since the stripping is never completely successful.

Catalyst from the stripping section of the reactor (called spent catalyst) is transported to the regenerator through the spent catalyst line for burning off the coke using air, to regenerate the catalyst. Air from the lift air blower (LAB) is injected into the regenerator lift pipe to assist in catalyst circulation. Catalyst in the regenerator is fluidized by air supplied from the combustion air blower (CAB), the lift air blower, and the excess air from the lift air blower through the spill air line. The coke in the catalyst is burned off to carbon dioxide, carbon monoxide and water vapor, which are released as flue gases to the atmosphere. The catalyst is separated from the flue gases by cyclones at the top of the regenerator and returned to the reactor through the regenerated catalyst line. The reactions taking place in the regenerator are:

\[
\begin{align*}
4H + O_2 & \rightarrow 2H_2O \\
2C + O_2 & \rightarrow 2CO \\
C + O_2 & \rightarrow CO_2 \\
2CO + O_2 & \rightarrow 2CO_2
\end{align*}
\]

(1.1)  
(1.2)  
(1.3)  
(1.4)

The reactions are exothermic, and the heat generated by burning off the coke in the regenerator provides the major portion of the heat for cracking the feed in the reactor riser. This results in strong interactions between the reactor and regenerator sections of the FCC unit. In the top portion of the regenerator (called the disengaging zone), because of the low density of entrained catalyst, the last reaction becomes significant and can result in temperatures reaching the metallurgical limit of the cyclone. This is called CO afterburn and is a major operational
constraint for FCC units. Hence, the regenerator is operated at conditions of high temperature and excess air with the aim to convert the CO to CO₂.

A model IV FCC unit differs from other FCC units mainly in that the slide valves in the catalyst circulation lines are not used to control the catalyst circulation rate and are available only as emergency shutoff valves. Catalyst circulation in a model IV FCC unit is controlled by the reactor-regenerator pressure difference and by the air supplied to the lift pipe, which changes the catalyst head in the lift pipe by changing the density.

Any control or optimization study of the FCC unit requires a process model, which captures the major effects and interactions taking place in the unit. Three of the most commonly used models in the literature which incorporate the features of earlier models are the one by Lee and Groves (1985), McFarlane et al. (1993) and Arbel et al. (1995).

The Lee-Groves model uses a plug flow reactor (PFR) model for the reactor riser and a continuous stirred tank reactor (CSTR) model for the regenerator with the three lump model of Weekman and Nace (1970) to describe the cracking reactions. The three lumps are gas oil, gasoline, and coke (which includes the wet gas). The three lump model however does not give an accurate representation of the cracking reactions as shown by Ali and Rohani (1996). Also, a PFR model with a dense and dilute phase for the catalyst is more representative of commercial regenerators.

The model by Arbel et al. (1995) uses a PFR model for the reactor with the ten lump model of Jacobs et al. (1976) to describe the cracking kinetics. The regenerator is assumed to be composed of two phases, dense phase and dilute phase. The dense phase is modeled as a CSTR and the dilute phase is assumed to be in plug flow. The advantage of this model is that it
describes the partial CO combustion mode of operation for the regenerator and incorporates a yield model for the cracking reactions.

The main problem in using both of the above models for an optimization study is that they use only material and energy balance equations and do not incorporate mechanistic details like valve characteristics and pressure balances. This forces us to select state variables rather than control variables (with respect to the mechanistic model) as our free variables for optimization. Also, absence of the model equations for pressure balance leads to lower dimensional search during optimization, since pressure cannot be taken as a free variable for optimization.

The Amoco Model IV FCC unit as modeled by McFarlane et al. (1993), uses a CSTR model for the reactor and a distributed parameter model for the regenerator. The fractionator is not modeled to avoid complication. It is a realistic model, which captures the major effects and interactions in the FCC unit and incorporates valve characteristics and pressure balances. The main drawback of the model is the absence of a yield model for the cracking reactions making it unsuitable for optimization studies, as the feed conversion and product distribution cannot be known. In this work, we have used the Amoco model after incorporating the four lump model of Gianetto et al. (1994) to predict product distribution.

There has been a considerable amount of interest in the optimization of FCC units in recent years. Khandalekar and Riggs (1995) combined the Amoco FCC model with the FCC model of Lee and Groves (1985). The optimization was a 3-dimensional search with the feed flow rate, the reactor temperature and the regenerator temperature as the free variables for optimization. The optimization was done in two stages. In the first stage, a search is made for the optimum value of the reactor and regenerator temperatures for a given feed flow rate using
Nelder-Mead optimization. In the second stage, the feed rate is maximized using modified Fibonacci search. This is not the best way to solve the optimization problem. It is well established that a simultaneous search is better than sequential search for obtaining the global optimum. Also, it is a very low dimensional search.

Ellis et al. (1998) used sequential quadratic programming (SQP) to compare on-line and off-line optimization of FCC units. The process is a model IV FCC unit combined with a ten lump yield model and coke deposition kinetics. The SQP algorithm selects the value of the four decision variables, reactor temperature, regenerator temperature, feed flow rate and stack gas oxygen concentration, and uses four nonlinear algebraic equations describing the FCC unit. The remaining process variables are determined using explicit expressions and any process-model mismatch is corrected using parameter estimation. Absence of the model equations for the pressure balances have prevented them from considering the reactor pressure and the reactor-regenerator pressure difference as free variables for optimization. The reactor-regenerator pressure difference allows greater flexibility for varying the catalyst flow rate, a key parameter for control and optimization of FCC units.

Loeblin and Perkins (1999) solved a 2-variable optimization problem with the air flow rate to the regenerator and the catalyst circulation rate as the decision variables, and compared the effect of different control structures on real time optimization of FCC units using the Lee-Groves model (1985). This is a low dimensional search, as it does not incorporate the reactor pressure or the feed flow rate.

Although all the three papers above have solved optimization problems, their main focus has been on online optimization of the FCC unit in the face of external disturbances.
(Loebeln and Perkins, 1999, Ellis et al., 1998) or model based control (Khandalekar and Riggs, 1995).

The present work involves a more general 7-dimensional search for the optimization of the FCC unit. We have used a modified steady state version of the Amoco FCC model as given by McFarlane et al. (1993) combined with the four lump yield model of Gianetto et al. (1994), as the process model. The four lump model has been shown to represent the FCC reactions better than the three lump model (Ali and Rohani, 1996). Also the kinetics given by Gianetto et al. (1994) are more representative of the short residence time encountered in commercial risers. Since the coke and the wet gas are represented by separate lumps, the yield model can be directly incorporated into the Amoco model by replacing the appropriate equations. This model is chosen over the four lump model by Lee et al. (1989) whose four lump model gives data for reactor residence time of 30 sec to 120 sec which are not realistic figures from the industrial standpoint. The 10 lump model of Jacobs et al. (1976), though being more detailed, lumps coke and light gasses together even though the two have widely different properties. This can lead to inaccurate representation of the cracking reactions (Ali and Rohani, 1996). Also, the coke on the spent catalyst has to be estimated by a separate empirical equation. Other models in the literature are omitted because of lack of sufficient data.

Due to the success in the use of the direct search optimization procedure of Luus and Jaakola (1973) (LJ optimization procedure) in solving some difficult fed batch reactor optimization problems (Luus and Hennessey, 1999), we will use this approach for optimization of the FCCU. The LJ optimization procedure has been found by Wang and Luus (1978) to have high reliability in finding the global optimum even for systems exhibiting numerous local optima. For example, for a very difficult catalyst bifunctional problem, for which SQP yielded
26 local optima but not the global optimum (Luus et al., 1992). The use of LJ optimization procedure yielded the global optimum (Bojkov et al., 1993). When used in a multipass manner, LJ optimization yielded the global optimum for a very challenging cancer chemotherapy scheduling problem involving an 84-dimensional problem in optimization and several state constraints (Luus et al., 1995).

Recently it was shown by Luus (1998a) that the efficiency of the LJ optimization procedure can be increased substantially if the method is used in multipass fashion where the region sizes at the beginning of the passes are determined from the extent of variation of the corresponding variables during the previous pass. Such an approach enabled the parameters in the Lotka-Volterra ecological system to be accurately estimated (Luus, 1998b). Since the FCC optimization problem involves state constraints, we use the penalty functions approach as shown by Luus and Wyrwicz (1996) who have used it to handle equality state constraints.

The method is easy to program and involves no auxiliary variables except for the penalty function factors for the state constraint violation in the objective function, keeping the user closer to the problem at hand. Since present day computers are very fast, the extra computational effort required for solution of the large number of model equations and by the LJ optimization procedure is acceptable if we can find the global optimum for the problem.

In this thesis, the effectiveness of the LJ optimization procedure for locating the global optimum for this problem is investigated. Additionally, the study explores the various design possibilities and shows the effect of the model parameters on the location of the global optimum. Finally, the additional benefits that can be gained by operating the unit closer to the regenerator combustion constraint boundaries (McFarlane et al., 1993) are illustrated.
2. PROBLEM FORMULATION

The Model IV FCC unit equations, as provided by McFarlane et al. (1993), are used to model the reactor and the regenerator, and the four lump model of Gianetto et al. (1994) is used to describe the cracking reactions. Gas oil is assumed to crack to gasoline, the most desired product, and to the byproducts, coke and wet gas. The four lumps and their interactions are shown in the Figure 2.1. The rate constants for the cracking reactions are represented by the k's, and are obtained from experimental data of Gianetto et al. (1994). The four lump model uses separate lumps for coke and light gases and is easily incorporated into the Amoco model (McFarlane et al., 1993) by replacing the appropriate equations.

Figure 2.1. The four lump yield model
2.1. Problem statement

We assume that the main products of economic value are gasoline and light gas and that the unconverted feed can be recycled. Therefore, we choose the performance index to be maximized as

\[ P = (F_{\text{ugo}} - F_{\text{oil}})P_{\text{feed}} + F_{\text{gasoline}} P_{\text{gasoline}} + F_{\text{light gas}} P_{\text{light gas}} \]  

where \( F_{\text{ugo}} \) is the unconverted gas oil flowrate. \( F_{\text{oil}} \) is the total feed flowrate. \( F_{\text{gasoline}} \) is the flowrate of gasoline. \( F_{\text{light gas}} \) is the flowrate of light gas and \( P_{\text{feed}} \). \( P_{\text{gasoline}} \) and \( P_{\text{light gas}} \) are the price of feed, gasoline and light gas respectively.

The seven variables that we may use for optimization are: fuel rate to furnace, \( F_5 \), wet gas compressor suction valve position, \( V_{11} \), pressure difference between the regenerator and the reactor, \( \Delta P_{RR} \), combustion air blower suction valve position, \( V_6 \), lift air valve position, \( V_{lift} \), spill air valve position, \( V_9 \), and flow rate of fresh feed to the reactor riser, \( F_3 \). These variables have the following upper and lower bounds:

\[ 0 \leq F_5 \leq 1.1327 \frac{m^3}{s} \]  

\[ 0 \leq V_{11} \leq 0.95 \]  

\[ -3.45 \times 10^4 \leq \Delta P_{RR} \leq 1.38 \times 10^4 \frac{N}{m^2} \]  

\[ 0 \leq V_6 \leq 1 \]  

\[ 0 \leq V_{lift} \leq 0.65 \]  

\[ 0 \leq V_9 \leq 1 \]  

\[ F_3 \geq 0 \frac{kg}{s} \]
It is noted that $\Delta P_{RR}$ and $V_{lift}$ affect the catalyst circulation rate, and $V_6$, $V_{lift}$ and $V_7$ affect the airflow rate. However, for a general search all of the above have to be considered as free variables.

There are also constraints on some of the variables in the model:

\[
P_5 \leq 3.75 \times 10^5 \frac{V}{m^2} \quad (2.1.9)
\]
\[
P_6 \leq 3.75 \times 10^5 \frac{V}{m^2} \quad (2.1.10)
\]
\[
T_{cyc} \leq 710 \, ^\circ C \quad (2.1.11)
\]
\[
T_r \leq 535 \, ^\circ C \quad (2.1.12)
\]
\[
T_3 \leq 926.7 \, ^\circ C \quad (2.1.13)
\]
\[
0 \leq L_{sp} \leq 6.1 \, m \quad (2.1.14)
\]
\[
F_{sucn.lift} > F_{surge.lift} \frac{m^3}{s} \quad (2.1.15)
\]
\[
0 \leq V_{i4} \leq 1 \quad (2.1.16)
\]

Total combustion constraints, set A:

\[
C_{O_2, sr} \geq 1.5 \% \quad (2.1.17)
\]
\[
T_{reg, db} \geq 685 \, ^\circ C \quad (2.1.18)
\]

Total combustion constraints, set B:

\[
T_{diff} = T_{cyc} - T_{reg, db} \leq 11.1 \, ^\circ C \quad (2.1.19)
\]
\[
C_{CO, sr} \leq 350 \, ppm \quad (2.1.20)
\]
Constraints (2.1.9) and (2.1.10) represent design pressure limits on the main fractionator and the regenerator, respectively. Constraint (2.1.11), (2.1.12) and (2.1.13) are the design temperature limits on the regenerator cyclone, reactor riser and furnace firebox respectively. Constraint (2.1.14) is the operational constraint on the level of catalyst in the regenerator standpipe. Constraint (2.1.15) is surge limit on the lift air blower and (2.1.16) is the constraint on regenerator stack gas valve opening. Constraints (2.1.17) to (2.1.20) are operational restrictions on the regenerator.

The model of the FCC unit consists of 112 nonlinear algebraic equations and 9 ordinary differential equations. How to deal with equality constraints in direct search optimization has been actively investigated recently. Originally, Luus (1974) suggested the use of a two-pass method where the equality constraints were replaced by inequalities. A better way of dealing with difficult equality constraints is to use quadratic penalty functions with shifting terms (Luus, 1996). More recently, Luus (2000c) showed that the difficult equality constraints can be solved quite efficiently with Newton’s method at each iteration, thereby removing the need of introducing additional variables. Here we use this latter approach, where we group the equations into two sets, one consisting of a set where the equations can be solved in a seriatim fashion and the second set consisting of difficult equalities which have to be solved numerically with some iterative procedure. Let us consider this aspect of solving algebraic equations.

2.2. Solving the nonlinear algebraic equations and formulating the optimization problem

The difficulty of solving a set of nonlinear algebraic equations has been overestimated by many researchers. Software packages like MAPLE. MATLAB. MATHEMATICA. etc. have difficulty in solving more than 10 non-linear algebraic equations as evident from the paper by
Sacharn et al. (1993). Zain et al. (1995) used a modified nonlinear orthomin method to solve the nonlinear algebraic equations. Gupta (1995) used the bracketing method for the solution. 100% convergence is reported with the BASIC program implemented to pick a new initial guess in case of failure to converge to the solution. All the above methods can be outperformed, both in robustness and computation time using the method shown by Luus (1999, 2000).

The key idea in solving such nonlinear algebraic equations is to arrange the equations into two groups: a set of simple equations which can be solved in a seriatim fashion and a set of difficult equations, to be solved in iterative fashion, for example by Newton's method. Besides computational efficiency, an advantage of grouping the equations is that initial guesses are required only for the difficult equations. When a set of equations is to be solved at each iteration in optimization, good initial guesses are usually available from the previous iteration of the optimization procedure. Since, Newton's method has quadratic convergence near the solution, it has been chosen for the iterative solution of the difficult equations.

If $\mathbf{f}(\mathbf{x})$ is the vector denoting the left hand side of the difficult equations, at each iteration we would like to find a deviation $\delta \mathbf{x}$ that will make $\mathbf{f}(\mathbf{x} + \delta \mathbf{x})$ zero in the next iteration. Using Taylor series and retaining only the 1st derivative term, we get

$$\mathbf{f}(\mathbf{x} + \delta \mathbf{x}) \approx \mathbf{f}(\mathbf{x}) + (\nabla \mathbf{f}^T / \partial \mathbf{x})^T \delta \mathbf{x}$$  \hspace{1cm} (2.2.1)

Since we want $\mathbf{f}(\mathbf{x} + \delta \mathbf{x})$ to be zero, we get

$$(\nabla \mathbf{f}^T / \partial \mathbf{x})^T \delta \mathbf{x} = -\mathbf{f}(\mathbf{x})$$  \hspace{1cm} (2.2.2)

which is a set of linear equations. Using Gaussian elimination, we can solve for the deviation $\delta \mathbf{x}$ and then choose

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \delta \mathbf{x}$$  \hspace{1cm} (2.2.3)

for the next iteration. The details of the method are given by Luus (1999, 2000)
The arrangement of equations into a simple set and a difficult set is not unique. However, we should usually try to minimize the number of equations in the difficult set.

For the FCC unit, we arrange the equations so that we are left with only 12 difficult equations. From those 12 difficult equations, we obtain values for the following variables:

- Temperature of fresh feed to riser $T_2$
- Furnace firebox temperature $T_3$
- Reactor riser temperature $T_r$
- Regenerator dense bed temperature $T_{reg,db}$
- Weight fraction of coke on regenerated catalyst $C_{rge}$
- Wet gas compressor suction pressure $P_w$
- Velocity, regenerated catalyst $v_{rge}$
- Pressure at the bottom of lift pipe $P_{blp}$
- Reactor fractionator pressure $P_5$
- Combustion air blower discharge pressure $P_2$
- Combustion air blower suction pressure $P_1$
- Lift air blower discharge pressure $P_3$

Once values are assigned to these 12 variables, and to the 7 optimization variables $F_j$, $V_{11}$, $\Delta P_{RR}$, $V_6$, $V_{lft}$, $V_9$, and $F_3$, then the rest of the equations are simple in the sense that they can be solved in seriatim fashion. Thus we have the set of simple equations:

$$P_4 = P_5 + \Delta P_{max}$$

(2.2.4)
where we have assumed a constant pressure drop, $\Delta P_{\text{frac.}}$ of 0.65\times10^5\text{N/m}^2 between the reactor and the regenerator.

$$P_6 = P_4 + \Delta P_{\text{AR}}$$  \hfill (2.2.5)

$$M = \frac{k_{\text{avg}} - 1}{k_{\text{avg}} \eta_p}$$  \hfill (2.2.6)

where $k_{\text{avg}}$ is the ratio of specific heats (1.39) and $\eta_p$ is the polytropic efficiency (1.0).

$$F_{\text{rec}} = v_{\text{rec}} A_{\text{ubend}_{\text{rec}}} \rho_c$$  \hfill (2.2.7)

where $A_{\text{ubend}_{\text{rec}}}$ is the cross sectional area of regenerated catalyst U-bend (0.344 m$^2$) and $\rho_c$ is the density of catalyst in U-bend and regenerator standpipe (720.82 kg/m$^3$).

$$v_{\text{rec}} = \frac{F_3 + F_4 + F_{\text{rec}}}{\rho_v + \rho_{\text{part}}}$$  \hfill (2.2.8)

where $\rho_v$ is the vapor density at the reactor riser condition (9.13 kg/m$^3$). $\rho_{\text{part}}$ is the settled density of catalyst (1089.25 kg/m$^3$) and $F_4$ is the flow of slurry to the reactor riser which is taken to be constant (2.38 kg/s).

$$\rho_{\text{rec}} = \frac{F_3 + F_4 + F_{\text{rec}}}{v_{\text{rec}}}$$  \hfill (2.2.9)

where $F_3$ is the flow of slurry to the reactor riser (2.38 kg/s).

$$P_{\text{rh}} = P_4 + \rho_{\text{rec}} h_{\text{rec}} g$$  \hfill (2.2.10)

where $h_{\text{rec}}$ is the height of reactor riser (18.29 m) and $g$ is the acceleration due to gravity (9.81 m/s$^2$).

$$W_{\text{sp}} = A_{\text{v}} \left[ \frac{v_{\text{rec}} f_{\text{ubend}_{\text{rec}}} f_{\text{ubend}_{\text{rec}}}}{A_{\text{ubend}_{\text{rec}}}} - \frac{(P_6 - P_{\text{rh}})}{g} \left( E_{\text{top}} - E_{\text{oil inlet}} \right) \rho_c \right]$$  \hfill (2.2.11)
where $A_{sp}$ is the cross sectional area of the stand pipe (0.65 m$^2$). $L_{ubend_{reg}}$ is the length of regenerated catalyst ubend (17.07 m). $f_{ubend_{reg}}$ is the regenerated catalyst friction factor (83 kg·s/m$^3$). $A_{ubend_{reg}}$ is the cross sectional area of regenerated catalyst U-bend (0.344 m$^2$). $g$ is the acceleration due to gravity (9.81 m/s$^2$). $E_{up}$ is the pressure tap elevation on standpipe (47.24 m). $E_{oil\_inlet}$ is the elevation where oil enters the reactor riser (37.95 m) and $\rho_c$ is the density of catalyst in U-bend (720.82 kg/m$^3$).

\[ v_{sc} = \frac{F_{nx}}{A_{ubend_{reg}} \rho_c} \]  
\hspace{1cm} (2.2.12)

where $A_{ubend_{reg}}$ is the cross sectional area of spent catalyst U-bend (0.483 m$^2$) and $\rho_c$ is the density of catalyst in U-bend and regenerator standpipe (720.82 kg/m$^3$).

\[ W_r = A_{upper} \left[ \frac{v_{sc}}{A_{ubend_{reg}}} L_{ubend_{reg}} f_{ubend_{reg}} \left( \frac{P_4 - P_{UP}}{g} \right) \right] \]  
\hspace{1cm} (2.2.13)

where $A_{upper}$ is the cross sectional area of reactor stripper (5.57 m$^2$). $L_{ubend_{sp}}$ is the length of spent catalyst ubend (17.07 m). $f_{ubend_{sp}}$ is the spent catalyst friction factor (229.47 kg·s/m$^3$). $A_{ubend_{sp}}$ is the cross sectional area of spent catalyst U-bend (0.483 m$^2$). $g$ is the acceleration due to gravity (9.81 m/s$^2$). $E_{upper\_up}$ is the elevation of reactor stripper tap (39.62 m). $E_{lift\_air}$ is the elevation of regenerator lift air injection (40.84 m) and $\rho_c$ is the density of catalyst in U-bend (720.82 kg/m$^3$).

\[ W_{rec} = W_{total} - W_r - W_{up} \]  
\hspace{1cm} (2.2.14)

where $W_{total}$ is the total mass of catalyst in the system (171762 kg).
\begin{equation}
P_{\text{reg}} = P_0 + \frac{W_{\text{reg}} g}{A_{\text{reg}}} \tag{2.2.15}
\end{equation}

where \( g \) is the acceleration due to gravity (9.81 m/s\(^2\)) and \( A_{\text{reg}} \) is the cross sectional area of regenerator (54.81 m\(^2\)).

\( s_a = s_{a}^{\text{min}} + 1100 V_{\text{lif}} \)

\begin{equation}
\text{(2.2.16)}
\end{equation}

where \( s_{a}^{\text{min}} \) is the minimum speed of the lift air blower (5000 rpm).

\begin{equation}
F_\ell = k_{\ell,m} \sqrt{P_3 - P_{\text{reg}}} \tag{2.2.17}
\end{equation}

where \( k_{\ell,m} \) is the combustion air blower discharge pipe flow resistance factor

\begin{equation}
\left( 0.22 \text{kg/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right).
\end{equation}

\begin{equation}
F_{10} = k_{\ell} V_{\ell} \sqrt{P_3 - P_{\text{reg}}} \tag{2.2.18}
\end{equation}

where \( k_{\ell} \) is the lift air blower spill valve flow rating \( \left( 5.46 \times 10^{-2} \text{kg/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \).

\begin{equation}
\rho_x = \frac{1.205 \times 10^{-4} P_0}{T_{\text{reg,db}} + 273.15} \tag{2.2.19}
\end{equation}

\begin{equation}
F_\ell = k_{\ell,m} \sqrt{P_3 - P_{\text{blp}}} \tag{2.2.20}
\end{equation}

where \( k_{\ell,m} \) is the lift air blower discharge pipe flow resistance factor \( (2.73 \times 10^{-2} \text{kg/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}}) \).

\begin{equation}
F_{\text{air}} = \frac{F_\ell + F_\ell + F_{10}}{29} \tag{2.2.21}
\end{equation}

\begin{equation}
v_{\ell} = \frac{F_{\text{air}}}{\rho_x A_{\text{reg}}} \tag{2.2.22}
\end{equation}
where \( A_{reg} \) is the cross sectional area of regenerator (54.81 m\(^2\)).

\[
T_{lm} = \frac{(T_3 - T_1) - (T_1 - T_2)}{\ln \left( \frac{T_1 - T_1}{T_3 - T_2} \right)}
\] (2.2.23)

where \( T_1 \) is the temperature of fresh feed entering furnace (238.3°C).

\[
T_{2,\text{in}} = T_1 + \frac{U_A T_{lm}}{F_c p_{fl}}
\] (2.2.24)

where \( T_1 \) is the temperature of fresh feed entering furnace (238.3°C), \( U_A \) is the furnace overall heat transfer coefficient (47.48 \( \frac{\text{kJ}}{\text{s} \cdot \text{m}^2 \cdot \text{K}} \)) and \( c_{fl} \) is the heat capacity of fresh feed liquid (3.43 \( \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}} \)).

\[
P_{\text{hwc}} = \frac{1.01325 \times 10^5 P_2}{P_1}
\] (2.2.25)

\[
F_{\text{vhc,comb}} = 21.238 + \sqrt{352.15 - 5.855 \times 10^{-9} P_{\text{hwc}}^2}
\] (2.2.26)

\[
F_b = \frac{3.494 \times 10^{-1} P_1 F_{\text{vhc,comb}}}{T_{lm} + 273.15}
\] (2.2.27)

where \( T_{lm} \) is the atmospheric temperature (23.9°C).

\[
f_{pp}(x) = \exp[2/(1 - x) \ln(0.15)], \ x > 0.5
\]

\[
= 0.3x, \quad x \leq 0.5
\] (2.2.28)

where \( x \) is a variable representing valve opening.

\[
F_{V_s} = k_b f_{pp}(V_b) \sqrt{P_{\text{atm}} - P_1}
\] (2.2.29)
where \( k_6 \) is the combustion air blower suction valve flow rating \( 1.37 \text{kg/s} \cdot \sqrt{\frac{N}{m^2}} \) and \( P_{\text{atm}} \) is the atmospheric pressure \((1.01325 \times 10^5 \text{ N/m}^2)\).

\[
F_{i_6} = k_6 f_{rp}(V_{i_6}) \sqrt{P_2 - P_{\text{atm}}}
\] (2.2.30)

where \( k_6 \) is the combustion air blower vent valve flow rating \( 0.08 \text{kg/s} \cdot \sqrt{\frac{N}{m^2}} \) and \( V_{i_6} \) is the combustion air blower vent valve position (0).

\[
C_{\text{w}} = \frac{P_{\text{vr}}}{P_2}
\] (2.2.31)

where \( P_{\text{vr}} \) is the discharge pressure of wet gas compressor to vapor recovery unit \((6.96 \times 10^5 \text{ N/m}^2)\).

\[
H_{\text{wg}} = 12608.56 \times 10^3 \left( C_{\text{w}}^{0.1942} - 1 \right)
\] (2.2.32)

\[
F_{\text{vacn, wg}} = 5.475 + \sqrt{30.426 - 4.955 \times 10^{-16} H_{\text{wg}}^2}
\] (2.2.33)

\[
F_{i_{11}} = 3.675 \times 10^{-7} F_{\text{vacn, wg}} P_2
\] (2.2.34)

\[
F_{i_{ii}} = k_{11} f_{rp}(V_{i_{11}}) \sqrt{P_2 - P_2}
\] (2.2.35)

where \( k_{11} \) is the wet gas compressor suction valve flow rating \( 8.20 \times 10^{-3} \text{kgmol/s} \cdot \sqrt{\frac{N}{m^2}} \).

\[
F_{i_{ii}} = k_{13} V_{i_{13}} P_{\text{vr}}
\] (2.2.36)

where \( k_{13} \) is the wet gas compressor antisurge valve flow rating \( 6.58 \times 10^{-7} \text{kgmol/s} \cdot \sqrt{\frac{N}{m^2}} \).

\( V_{i_{13}} \) is the wet gas compressor vent valve position (0) and \( P_{\text{vr}} \) is the discharge pressure of wet gas compressor to vapor recovery unit \((6.96 \times 10^5 \text{ N/m}^2)\).
\[ F_{v_{12}} = k_{12} V_{12} \sqrt{P_s - P_{\text{atm}}} \]  \hspace{1cm} (2.2.37)

where \( k_{12} \) is the wet gas flare valve flow rating \( \left( 2.73 \times 10^{-3} \text{kgmol/s} \cdot \sqrt{\text{m}^2} \right) \) and \( V_{12} \) is the wet gas flare valve position (0).

\[ W_{m} = \frac{F_{\text{m}} A_{m} h_{m}}{v_{m}} \]  \hspace{1cm} (2.2.38)

where \( A_{m} \) is the cross sectional area of reactor riser (0.89 m\(^2\)) and \( h_{m} \) is the height of reactor riser (18.29 m). \( v_{m} \) is from equation (2.2.8)

\[ \tau_r = \frac{W_{m}}{F_{\text{m}}} \]  \hspace{1cm} (2.2.39)

\[ w_r = (F_3 + F_4) \tau_r \]  \hspace{1cm} (2.2.40)

where \( F_3 \) is the flow of slurry to the reactor riser (2.38 kg/s).

\[ k_{21} = k_{210} \exp \left( \frac{-E_{21}}{RT_r} \right) \]  \hspace{1cm} (2.2.41)

where \( k_{210} \) is the pre exponential factor. gas oil to gasoline \( (0.4272 \times 10^7 \frac{\text{m}^6}{\text{kgmol} \cdot \text{kgecat} \cdot \text{s}}) \). \( E_{21} \) is the activation energy \( (87821.4 \text{ J/kgmol}) \) and \( R \) is the universal gas constant \( (8314.39 \frac{\text{J}}{\text{kgmol} \cdot \text{K}}) \).

\[ J = \] \hspace{1cm} \text{kgmol} \cdot \text{K}.

\[ k_{31} = k_{310} \exp \left( \frac{-E_{31}}{RT_r} \right) \]  \hspace{1cm} (2.2.42)

where \( k_{310} \) is the pre exponential factor. gas oil to wet gas \( (0.1012 \times 10^8 \frac{\text{m}^6}{\text{kgmol} \cdot \text{kgecat} \cdot \text{s}}) \) and \( E_{31} \) is the activation energy \( (97552.4 \text{ J/kgmol}) \).
\[ k_{41} = k_{410} \exp \left( -\frac{E_{41}}{RT_r} \right) \]  

(2.2.43)

where \( k_{410} \) is the pre exponential factor, gas oil to coke (0.5504 \( \times 10^5 \) \( \text{m}^3 \) \( \text{kgcat} \cdot \text{s} \)) and \( E_{41} \) is the activation energy (87504.1 J/kgmol).

\[ k_{42} = k_{420} \exp \left( -\frac{E_{42}}{RT_r} \right) \]  

(2.2.44)

where \( k_{420} \) is the pre exponential factor, gasoline to coke (0.1337 \( \times 10^5 \) \( \text{m}^6 \) \( \text{kgmol} \cdot \text{kgcat} \cdot \text{s} \)) and \( E_{42} \) is the activation energy, gasoline to coke (72988.7 J/kgmol).

\[ \frac{dY_i}{d\tau_r} = -\Phi(k_{21} + k_{31} + k_{41}) \frac{Y_i^2 w_{rs}^i V_T}{M_A V_T^2} Y_i(0) = 1 \]  

(2.2.45)

where \( M_A \) is the molecular weight of fresh feed (333 kg/kgmol) and \( V_T \) is the volume of the reactor riser (16.3 m\(^3\)).

\[ \frac{dY_B}{d\tau_r} = \Phi \left[ \left( \frac{w_T}{M_A V_T} \right) k_{31} Y_i^2 - k_{42} Y_B \right] \frac{w_{rs}^i V_T}{V_T} \]  

(2.2.46)

\[ \frac{dY_C}{d\tau_r} = \Phi \left[ \left( \frac{w_T}{M_A V_T} \right) k_{31} Y_i^2 \right] \frac{w_{rs}^i V_T}{V_T} \]  

(2.2.47)

\[ \frac{dY_D}{d\tau_r} = \Phi \left[ \left( \frac{w_T}{M_A V_T} \right) k_{41} Y_i^2 + k_{42} Y_B \right] \frac{w_{rs}^i V_T}{V_T} \]  

(2.2.48)

where

\[ \Phi = \exp \left( -\alpha Y_D w_T \right) \frac{w_{rs}}{W_{rs}} \]  

(2.2.49)

where \( \alpha \) is the exponential decay function (391).
\[ S_f = \frac{(1 - c_{\text{slurry}} Y_D)}{(1 - Y_D)} \]  \hspace{1cm} (2.2.50)

where \( c_{\text{slurry}} \) is the factor representing coking tendency of slurry recycle relative to fresh feed (3.5 for base case).

\[ F_{\text{light gas}} = F_3 Y_C + F_4 S_f Y_C \]  \hspace{1cm} (2.2.51)

where \( F_4 \) is the flow of slurry to the reactor riser (2.38 kg/s).

\[ F_{wg} = \frac{F_{\text{light gas}}}{M_{wg}} \]  \hspace{1cm} (2.2.52)

where \( M_{wg} \) is the molecular weight of wet gas (40 kg/kmol).

\[ F_{\text{gasoil}} = F_3 Y_A + F_4 S_f Y_A \]  \hspace{1cm} (2.2.53)

\[ F_{\text{gasoline}} = F_3 Y_B + F_4 S_f Y_B \]  \hspace{1cm} (2.2.54)

\[ F_{\text{coker}} = F_3 Y_D + F_4 S_f Y_D \]  \hspace{1cm} (2.2.55)

\[ \Delta H_{\text{crack}} = 401.7 + 12560(T_r - T_{\text{ref}}) \]  \hspace{1cm} (2.2.56)

where \( T_{\text{ref}} \) is the base temperature for reactor riser energy balance (537.8°C).

\[ Q_{\text{cracking}} = [F_3 (1 - Y_A) + F_4 (1 - S_f Y_A)] \Delta H_{\text{crack}} \]  \hspace{1cm} (2.2.57)

where \( F_4 \) is the flow of slurry to the reactor riser (2.38 kg/s).

\[ Q_{ff} = F_3 \left[ c_{\text{pfe}} (T_r - T_{\text{ref}}) + Q_{fr} \right] \]  \hspace{1cm} (2.2.58)

where \( c_{\text{pfe}} \) is the heat capacity of fresh feed vapor (3.39 \( \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} \)), \( T_{\text{ref}} \) is the base temperature for reactor riser energy balance (537.8°C) and \( Q_{fr} \) is the heat required to raise temperature of fresh feed from 371.1°C liquid to 537.8°C vapor (718.73 kJ/kg).
\[ Q_{\text{slurry}} = F_s [c_{pv} (T_r - T_{ref}) + Q_{sr}] \] (2.2.59)

where \( F_s \) is the flow of slurry to the reactor riser (2.38 kg/s), \( c_{pv} \) is the heat capacity of slurry vapor (3.35 \( \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}} \)), \( T_{ref} \) is the base temperature for reactor riser energy balance (537.8°C) and \( Q_{sr} \) is the heat required to raise temperature of slurry from 260°C liquid to 537.8°C vapor (958.31 kJ/kg).

\[ F_{sc} = v_{sc} A_{ubend} \rho_c \] (2.2.60)

where \( A_{ubend} \) is the cross sectional area of spent catalyst U-bend (0.483 m\(^2\)) and \( \rho_c \) is the density of catalyst in U-bend (720.82 kg/m\(^3\)). \( v_{sc} \) is from equation (2.12).

\[ Q_{\text{catout}} = F_{sc} c_{pc} (T_r - T_{base}) \] (2.2.61)

where \( c_{pc} \) is the heat capacity of catalyst (1.30 \( \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}} \)) and \( T_{base} \) is the base temperature for regenerator energy balance (537.8°C).

\[ Q_{\text{out reactor}} = Q_{\text{catout}} + Q_{\text{slurry}} + Q_{\text{cracking}} + Q_{ff} \] (2.2.62)

\[ Q_{rgc} = F_{rgc} c_{pc} (T_{reg,db} - T_{base}) \] (2.2.63)

where \( c_{pc} \) is the heat capacity of catalyst (1.30 \( \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}} \)) and \( T_{base} \) is the base temperature for regenerator energy balance (537.8°C). \( F_{rgc} \) is from equation (2.2.7).

\[ Q_{\text{in reactor}} = Q_{rgc} + F_3 c_{pfl} (T_2 - T_{bbase,f}) \] (2.2.64)

where \( c_{pfl} \) is the heat capacity of fresh feed liquid (3.43 \( \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}} \)) and \( T_{base,f} \) is the base temperature of reactor fresh feed (371.1°C).
\[ P_{\text{base},d} = \left[ \left( P_3^M - P_{\text{atm}}^M \right) \left( \frac{S_d^2}{S_{\text{atm}}^2} + P_{\text{atm}}^M \right) \right]^{\frac{1}{3}} \]  

(2.2.65)

where \( P_{\text{atm}} \) is the atmospheric pressure \( (1.01325 \times 10^5 \text{ N/m}^2) \) and \( s_b \) is the base speed of the lift air blower \( (5950 \text{ rpm}) \). \( s_a \) is from equation (2.2.16) and \( M \) is from equation (2.2.6).

\[ F_{\text{base}} = 4.059 + \sqrt{57.511 - 5.007 \times 10^{-10} P_{\text{base},d}^2} \]  

(2.2.66)

\[ F_{\text{sucn.lift}} = F_{\text{base}} \left( \frac{S_d}{S_b} \right) \]  

(2.2.67)

where \( s_b \) is the base speed of the lift air blower \( (5950 \text{ rpm}) \).

\[ F_8 = \frac{3.494 \times 10^{-3} P_{\text{atm}} F_{\text{sucn.lift}}}{T_{\text{atm}} + 273.15} \]  

(2.2.68)

where \( P_{\text{atm}} \) is the atmospheric pressure \( (1.01325 \times 10^5 \text{ N/m}^2) \) and \( T_{\text{atm}} \) is the atmospheric temperature \( (23.9^\circ \text{C}) \).

\[ F_{\text{surge.lift}} = 2.372 + 7.669 \times 10^{-6} P_3 \]  

(2.2.69)

\[ \rho_{\text{c.dilute}} = -14.064 + 30.586 v_s \]  

(2.2.70)

\( v_s \) is from equation (2.2.22).

\[ M_c = A_{\text{reg}} v_r \rho_{\text{c.dilute}} \]  

(2.2.71)

where \( A_{\text{reg}} \) is the cross sectional area of the regenerator \( (54.81 \text{ m}^2) \). \( v_r \) is from equation (2.2.22).

\[ \varepsilon_f = 0.332 + 0.197 v_r \]  

(2.2.72)

\[ \rho_{\text{c.dense}} = \rho_{\text{part}} \left( 1 - \varepsilon_f \right) \]  

(2.2.73)

where \( \rho_{\text{part}} \) is the settled density of the catalyst \( (1089.25 \text{ kg/m}^3) \).
where \( z_{\text{bed}} \) is the height of cyclone inlet (13.72 m), and \( A_{\text{reg}} \) is the cross sectional area of the regenerator (54.81 m\(^2\)). \( W_{\text{reg}} \) is from equation (2.2.14)

\[
T_{\text{sc}} = T_r - \Delta T_{\text{stripper}}
\]

where a constant temperature drop \( \Delta T_{\text{stripper}} \) (19.44°C) has been assumed across the reactor stripper.

\[
\rho_{\text{arc}} = \frac{29 P_o}{R(T_{\text{sc}} + 273.15)}
\]

where \( R \) is the universal gas constant (8314.39 J kgmol\(^{-1}\)K\(^{-1}\)). \( P_o \) is from equation (2.2.5).

\[
v_{\text{arc, inlet}} = \frac{F_o}{A_{lp \rho_{\text{arc}}}}
\]

where \( A_{lp} \) is the cross sectional area of lift pipe (0.811 m\(^2\)). \( F_o \) is from equation (2.2.20).

\[
v_{\text{cat, inlet}} = \max \left\{ v_{\text{arc, inlet}} - v_{\text{slip}} \frac{F_{sc}}{A_{lp \rho_{\text{cat}}}} \right\}
\]

where \( v_{\text{slip}} \) is the slip velocity (0.671 m/s). \( A_{lp} \) is the cross sectional area of lift pipe (0.811 m\(^2\)) and \( \rho_{\text{cat}} \) is the settled density of the catalyst (1089.25 kg/m\(^3\)).

\[
\rho_{\text{cat}} = \frac{F_{sc}}{v_{\text{cat, inlet}} A_{lp}} + \rho_{\text{arc}}
\]

where \( A_{lp} \) is the cross sectional area of lift pipe (0.811 m\(^2\)). \( F_{sc} \) is from equation (2.260).
\[ F_{i_s} = k_s V_s \sqrt{p_s - p_{am}} \]  
\hspace{10cm} (2.2.80)

where \( k_s \) is the lift air blower vent valve flow rating \( \left( 2.73 \times 10^{-3} \text{ kg/s} \cdot \sqrt{\frac{N}{m^2}} \right) \) and \( V_s \) is the lift air blower vent valve position (0).

\[ F_t = F_r + F_6 + F_{10} \]  
\hspace{10cm} (2.2.81)

\[ C_n = \left( \frac{F_{reg} C_{reg} + F_{coke}}{F_{reg}} \right) \]  
\hspace{10cm} (2.2.82)

\( F_{reg} \) is from equation (2.2.7) and \( F_{coke} \) is from equation (2.2.55).

\[ F_{ii} = F_{sc} \left( C_{sc} - C_{reg} \right) C_{ii} \]  
\hspace{10cm} (2.2.83)

where \( C_{ii} \) is the weight fraction of hydrogen in coke (0.075 kg H\(_2\)/kg coke) and \( F_{sc} \) is from equation (2.2.60).

\[ Q_{ii} = F_{ii} \Delta H_{ii} \]  
\hspace{10cm} (2.2.84)

where \( \Delta H_{ii} \) is the heat of combustion of hydrogen (141.904 kJ/kg).

\[ Q_{air} = F_{air} c_{p air} (T_{air} - T_{base}) \]  
\hspace{10cm} (2.2.85)

where \( c_{p air} \) is the heat capacity of air \( (29.64 \frac{\text{kJ}}{\text{kgmol} \cdot ^\circ \text{C}}) \), \( T_{base} \) is the base temperature for regenerator energy balance (537.8°C) and \( T_{air} \) is the temperature of air entering regenerator (132.2°C). \( F_{air} \) is from equation (2.2.21).

\[ Q_{sc} = F_{sc} c_{p c} (T_{sc} - T_{base}) \]  
\hspace{10cm} (2.2.86)

where \( c_{p c} \) is the heat capacity of catalyst \( (1.30 \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}}) \) and \( T_{base} \) is the base temperature for regenerator energy balance (537.8°C). \( T_{sc} \) is from equation (2.2.75).
\[ \varepsilon_e = \min \left( 1, \max \left( \varepsilon_j, \varepsilon_j + \frac{0.580 + 0.363v_s - 0.157v_s^2}{z_{\text{bed}}} \right) \right) \]  

\[ (2.2.87) \]

\( \varepsilon_e \) is from equation (2.2.72), \( v_s \) is from equation (2.2.22) and \( z_{\text{bed}} \) is from equation (2.2.74).

\[ \rho_{\beta}(z) = 1 - \varepsilon_e \quad 0 \leq z \leq z_{\text{bed}} \]  

\[ (2.2.88) \]

\[ \rho_{\beta}(z) = (1 - \varepsilon_e) \exp \left( \frac{-3280.83F_{\text{air}}(z - z_{\text{bed}})}{A_{\text{reg}}v_s \rho_{\text{c,dilute}}} \right) \quad z_{\text{bed}} < z \leq z_{\text{cyc}} \]  

\[ (2.2.89) \]

where \( A_{\text{reg}} \) is the cross sectional area of the regenerator \((54.81 \text{ m}^2)\) and \( z_{\text{cyc}} \) is the height of the cyclone inlet \((13.72 \text{ m})\). \( F_{\text{air}} \) is from equation (2.2.21), \( z_{\text{bed}} \) is from equation (2.2.74), \( v_s \) is from equation (2.2.22) and \( \rho_{\text{c,dilute}} \) is from equation (2.2.70).

\[ \rho_{\beta}(z) = 0 \quad z > z_{\text{cyc}} \]  

\[ (2.2.90) \]

\[ X_{\text{th}}(z = 0) = \frac{1}{F_{\text{air}}} \left\{ 0.21F_{\text{air}} - 0.25F_{\beta} \right\} \]  

\[ (2.2.91) \]

\( F_{\text{air}} \) is from equation (2.2.21) and \( F_{\beta} \) is from equation (2.2.83).

\[ \frac{dX_{\text{th}}}{dz} = \left[ 100k_1 \rho_{\beta}(z)C_{\text{rec}} - 2k_3 X_{\text{th}}(z) \right] \frac{X_{\text{th}}(z)}{v_s} \quad X_{\text{th}}(z = 0) = 0 \]  

\[ (2.2.92) \]

\( v_s \) is from equation (2.2.22).

\[ \frac{dX_{\text{th}}}{dz} = \left[ 100(-0.5k_1 - k_2) \rho_{\beta}(z)C_{\text{rec}} - k_3 X_{\text{th}}(z) \right] \frac{X_{\text{th}}(z)}{v_s} \]  

\[ (2.2.93) \]

\[ \frac{dX_{\text{th}}}{dz} = \frac{dX_{\text{th}}}{dz} - 0.5 \frac{dX_{\text{th}}}{dz} \quad X_{\text{th}}(z = 0) = 0 \]  

\[ (2.2.94) \]

\[ \delta_z = 1 \quad z < z_{\text{cyc}} \]  

\[ (2.2.95) \]

\[ \delta_z = 0 \quad z \geq z_{\text{cyc}} \]  

\[ (2.2.96) \]
\[ c_p(z) = 0.79 c_{N_2} + X_{CO} c_{CO} + X_{CO_2} c_{CO_2} + X_{O_2} c_{O_2} \]
\[ + \left( 0.5 F_H c_{H_2O} + \delta z c_{pc} M_e \right) \frac{1}{F_{air}} \]

(2.2.97)

where \( c_{N_2} \) (30.23), \( c_{CO} \) (30.48), \( c_{CO_2} \) (46.05), \( c_{O_2} \) (31.90), \( c_{H_2O} \) (36.09) and \( c_{pc} \) (1.30) are the heat capacities of nitrogen, carbon monoxide, carbon dioxide, oxygen, steam and catalyst respectively in \( \frac{kJ}{kg \cdot ^\circ C} \). \( F_{air} \) is from equation (2.2.21). \( F_H \) is from equation (2.2.83) and \( M_e \) is from equation (2.2.71).

\[ \frac{dT_{reg}(z)}{dz} = 0, \quad T_{reg}(z = 0) = T_{reg,db} \]
\[ 0 \leq z \leq z_{ned} \]

(2.2.98)

\[ \frac{dT_{reg}(z)}{dz} = \left( \Delta H_1 \frac{dX_{CO}(z)}{dz} + \Delta H_2 \frac{dX_{CO_2}(z)}{dz} \right) \frac{1}{c_p(z)} z_{ned} < z \leq z_{np} \]

(2.2.99)

where

\[ k_1 = 6.9547 \exp \left\{ 19.88 - \frac{18888.89}{T_{reg}(z) + 273.15} \right\} \]

(2.2.100)

\[ k_2 = 0.69148 \exp \left\{ 15.06 - \frac{13888.89}{T_{reg}(z) + 273.15} \right\} \]

(2.2.101)

\[ k_3 = 0.6412 P_6 \exp \left\{ 25.55 - \frac{25000.00}{T_{reg}(z) + 273.15} \right\} \]

(2.2.102)

and \( \Delta H_1 \) (109.545.1) and \( \Delta H_2 \) (395.837.9) are the heats of formation of CO and CO₂ respectively in (kJ/kgmol of gas produced). \( P_6 \) is from equation (2.2.5).

\[ C_{CO,sg} = \frac{\left( 28 \times 10^5 \right) X_{CO,sg}}{28 X_{CO,sg} + 44 X_{CO_2,sg} + 32 X_{O_2,sg} + 22.12 + 9 F_H} \]

(2.2.103)
$F_H$ is from equation (2.2.83). Gas concentration values are obtained from the integration of the differential equations (2.2.92, 93 and 94).

\[ C_{\text{CO}_2,\text{W}} = 100X_{\text{CO}_2,\text{W}} \quad (2.2.104) \]

\[ Q_i = F_{\text{air}} \left( X_{\text{O}_2,\text{W}} \Delta H_1 + X_{\text{CO},\text{W}} \Delta H_2 \right) \quad (2.2.105) \]

where $\Delta H_1 (109.545.1)$ and $\Delta H_2 (395.837.9)$ are the heats of formation of CO and CO$_2$, respectively in (kJ/kgmol of gas produced).

\[ Q_{fr} = \left[ F_{air} \left( X_{\text{O}_2,\text{W}} c_{\text{pO}_2} + X_{\text{CO},\text{W}} c_{\text{pCO}} + X_{\text{CO}_2,\text{W}} c_{\text{pCO}_2} + 0.79 c_{\text{pN}_2} \right) \right] + 0.5 F_H c_{\text{pH}_2O} \quad (2.2.106) \]

where $c_{\text{pO}_2} (31.90)$. $c_{\text{pCO}} (30.48)$. $c_{\text{pCO}_2} (46.05)$. $c_{\text{pN}_2} (30.23)$, and $c_{\text{pH}_2O} (36.09)$ are the heat capacities of oxygen, carbon monoxide, carbon dioxide, nitrogen and steam in $\frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$ and $T_{\text{base}}$ is the base temperature for regenerator energy balance (537.8°C). Gas concentration values and cyclone temperature $T_{\text{cyc}}$ are obtained from the integration of the differential equations (2.2.92, 93, 94 and 99).

\[ Q_{\text{in regenerator}} = Q_{\text{air}} + Q_H + Q_i + Q_{sc} \quad (2.2.107) \]

$Q_{\text{air}}$ is from equation (2.2.85). $Q_H$ from (2.2.84). $Q_i$ from (2.2.105) and $Q_{sc}$ from (2.2.86).

\[ Q_{\text{out regenerator}} = Q_{fr} + Q_{rgc} + Q_e \quad (2.2.108) \]

where $Q_e$ is the total heat loss from regenerator into environment (586.61 kJ/s). $Q_{fr}$ is from equation (2.2.106) and $Q_{rgc}$ from (2.2.63).

\[ L_p = \frac{W_{sp}}{A_p \rho_c} \quad (2.2.109) \]
where $A_{sp}$ is the cross sectional area stand pipe (0.65 m$^2$) and $\rho_c$ is the density of catalyst in the regenerator standpipe (720.82 kg/m$^3$). $W_{sp}$ is from equation (2.2.11).

$$d = \min(0.9144, h_{sp} - L_{sp})$$  \hspace{1cm} (2.2.110)

where $h_{sp}$ is the height of the regenerator standpipe (6.10 m).

$$F_{sp} = f_{sp} \sqrt{A_{sp} (z_{bed} - z_{sp})} - 2233.95 - 29.76(0.9144 - d)$$  \hspace{1cm} (2.2.111)

where $f_{sp}$ is the overflow factor (2070.14 kg m$^{-2}$ s$^{-1}$), $A_{sp}$ is the cross sectional area of the stand pipe (0.65 m$^2$) and $z_{sp}$ is the standpipe exit height from the bottom of the regenerator (3.96 m).

$z_{bed}$ is from equation (2.2.74).

$$V_{rcx,i} = A_{rcx} z_{cy} - A_{rcx} z_{bed} (1 - \varepsilon_c)$$  \hspace{1cm} (2.2.112)

where $A_{rcx}$ is the cross sectional area of regenerator (54.81 m$^2$) and $z_{cy}$ is the height of cyclone inlet (13.72 m). $\varepsilon_c$ is from equation (2.2.87).

$$V_{t4} = \frac{F_{irp}}{k_{14} \sqrt{P_{0} - P_{atm}}}$$  \hspace{1cm} (2.2.113)

where $k_{14}$ is the regenerator stack gas valve flow rating $\left(6.01 \times 10^{-3} \text{kgmol/s} \cdot \sqrt{\frac{N}{m^2}}\right)$ and $P_{atm}$ is the atmospheric pressure (1.01325 $\times 10^5$ N/m$^2$).

The 12 variables that we needed for the above simple equations are now obtained from the solution of the set of difficult equations:

$$T_{2,ax} - T_2 = 0$$  \hspace{1cm} (2.2.114)

$T_{2,ax}$ is from equation (2.2.24).
\[ F_2 \Delta H_{fu} - UA_f T_{lm} - a_1 F_5 T_5 + a_2 = 0 \] (2.2.115)

where \( \Delta H_{fu} \) is the heat of combustion of furnace fuel (37,258.55 kJ/m\(^3\)). \( UA_f \) is the furnace overall heat transfer coefficient (47.48 \( \text{kJ/s} \cdot \text{o} \text{C} \)), and \( a_1 \left( \frac{1.87 + 32}{T_j} \right) \text{kJ/m}^3 \cdot \text{o} \text{C} \). \( a_2 \) (211.01 kJ/s) are furnace heat loss parameters. \( T_{lm} \) is from equation (2.2.23).

\[ Q_{in\_reactor} - Q_{out\_reactor} = 0 \] (2.2.116)

\( Q_{in\_reactor} \) is from equation (2.2.64) and \( Q_{out\_reactor} \) from equation (2.2.62).

\[ Q_{in\_regenerator} - Q_{out\_regenerator} = 0 \] (2.2.117)

\( Q_{in\_regenerator} \) is from equation (2.2.107) and \( Q_{out\_regenerator} \) from equation (2.2.108).

\[ (F_{sc} C_{sc} - F_H) - (F_{sc} C_{rge} + 12 F_{air} \left( X_{C7O_{.sr}} + X_{C7O_{.sr}} \right)) = 0 \] (2.2.118)

\( F_{air} \) is from equation (2.2.21). \( F_H \) is from equation (2.2.83) and \( F_{sc} \) is from equation (2.2.60).

\[ F_{i\_i} - F_6 = 0 \] (2.2.119)

\( F_{i\_i} \) is from equation (2.2.29) and \( F_6 \) from equation (2.2.27).

\[ F_{sc} - F_{sp} = 0 \] (2.2.120)

\( F_{sc} \) is from equation (2.2.60) and \( F_{sp} \) is from equation (2.2.111).

\[ P_{spr} - P_6 - \rho_{spr} h_{i\_i} g - (z_{bed} - z_{ip}) \rho_{c\_dense} g = 0 \] (2.2.121)

where \( h_{i\_i} \) is the height of lift pipe (10.36 m). \( g \) is the acceleration due to gravity (9.81 m/s\(^2\)) and \( z_{ip} \) is the height of lift pipe discharge (3.35 m). \( P_6 \) is from equation (2.2.5). \( \rho_{spr} \) is from equation (2.2.79). \( z_{bed} \) is from equation (2.2.74) and \( \rho_{c\_dense} \) is from equation (2.2.73).

\[ F_{sw} - F_{i\_i} - F_{i\_i} + F_{i\_i} = 0 \] (2.2.122)
$F_{wg}$ is from equation (2.2.52), $F_{1i}$ is from equation (2.2.35), $F_{1i'}$ is from equation (2.2.37) and $F_{1i''}$ from equation (2.2.36).

\[ F_6 - F_3 - F_s = 0 \quad (2.2.123) \]

$F_6$ is from equation (2.2.27), $F_3$ is from equation (2.2.30) and $F_s$ from equation (2.2.17).

\[ F_{1i} - F_{11} = 0 \quad (2.2.124) \]

$F_{1i}$ is from equation (2.2.35). $F_{11}$ is from equation (2.2.34).

\[ F_s - F_{1i} - F_9 - F_{10} = 0 \quad (2.2.125) \]

$F_s$ is from equation (2.2.68). $F_{1i}$ is from equation (2.2.80). $F_9$ is from equation (2.2.20) and $F_{10}$ from equation (2.2.18).

In the above equations, the equations used for the calculation of wet gas and coke yield in the model given by McFarlane et al. (1993) have been replaced by equations from the four lump model. Equation (2.2.24) has been adjusted to include the fresh feed specific heat term. Equation (2.2.103) has been adjusted to account for the water vapor in the stack gas and $F_{rgc}$ has been corrected to $F_{sc}$ in Equation (2.2.61). The reference temperatures $T_{base}$ and $T_{ref}$ have been changed to 537.8°C. The last two changes do not affect the steady state version of the model equations, but are important in the dynamic behaviour. The values of $\Delta H_f$, $\Delta H_1$ and $\Delta H_2$ have been adjusted to account for the change in the base temperature $T_{base}$. To account for the slurry recycle, we have assumed it to have a coking factor, 3.5 times the feed, for the base case. Also, an average molecular weight of 40 has been assumed for the wet gas based on calculations from industrial data (Sadeighbeigi, 1995, pp. 42-43).
A brief description of some of the model equations is given below:

<table>
<thead>
<tr>
<th>Eq. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.4</td>
<td>Reactor pressure $P_r$ is calculated assuming constant pressure drop between reactor and main fractionator.</td>
</tr>
<tr>
<td>2.2.5</td>
<td>Regenerator pressure $P_d$ is controlled based on reactor pressure and reactor-regenerator pressure difference.</td>
</tr>
<tr>
<td>2.2.7</td>
<td>Flow of regenerated catalyst $F_{rec}$ is calculated based on catalyst velocity in the regenerated catalyst line.</td>
</tr>
<tr>
<td>2.2.10</td>
<td>Pressure at the reactor bottom $P_{rb}$ is the sum of the reactor pressure and pressure due to catalyst head.</td>
</tr>
<tr>
<td>2.2.11</td>
<td>Mass of catalyst in the standpipe.</td>
</tr>
<tr>
<td>2.2.12</td>
<td>Velocity of the spent catalyst is calculated based on regenerated catalyst flow at steady state.</td>
</tr>
<tr>
<td>2.2.13</td>
<td>Mass of the catalyst in the reactor.</td>
</tr>
<tr>
<td>2.2.14</td>
<td>Mass of catalyst in the regenerator is calculated based on material balance for catalyst.</td>
</tr>
<tr>
<td>2.2.15</td>
<td>Pressure at the bottom of the regenerator is the sum of regenerator pressure and pressure due to weight of the catalyst bed.</td>
</tr>
<tr>
<td>2.2.17.18</td>
<td>Combustion, spill and lift air flows calculated based on valve characteristics and pressure difference.</td>
</tr>
</tbody>
</table>
Eq. No. | Description
--- | ---
2.2.21 | Molar flow rate of air is the sum of combustion air blower flow, spill air flow and lift air flow.
2.2.24 | Steady state feed temperature at furnace outlet calculated from energy balance around the furnace.
2.2.27 | Combustion air blower throughput calculated from ideal gas law and compressor head characteristic curve.
2.2.34 | Wet gas flow to vapour recovery unit.
2.2.41 to | Equations for calculating yields from cracking reactions in the riser based on catalyst residence time, $\tau_c$.
2.2.55 | Empirical equation based on riser temperature for calculating the heat of cracking.
2.2.57 | Energy consumed for cracking based on conversion of feed.
2.2.58 and | Enthalpy into the riser due to fresh feed and slurry recycle flow.
2.2.59 | Flow of spent catalyst $F_{sc}$ is calculated based on catalyst velocity in the spent catalyst line.
2.2.61 | Enthalpy of catalyst leaving the reactor.
2.2.63 | Enthalpy of catalyst from the regenerator.
2.2.68 | Lift air blower throughput calculated based on ideal gas law and characteristic curve equation.
2.2.69 | Equation for lift air blower surge line.
<table>
<thead>
<tr>
<th>Eq. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.70 and 2.2.73</td>
<td>Empirical equations for calculating dilute and dense phase catalyst densities respectively.</td>
</tr>
<tr>
<td>2.2.74</td>
<td>Empirical equation for calculating regenerator dense bed height.</td>
</tr>
<tr>
<td>2.2.82</td>
<td>Carbon on the spent catalyst based on carbon material balance around riser.</td>
</tr>
<tr>
<td>2.2.84</td>
<td>Energy released due to combustion of Hydrogen from the hydrocarbon entrained in the spent catalyst.</td>
</tr>
<tr>
<td>2.2.85</td>
<td>Enthalpy of air entering the regenerator.</td>
</tr>
<tr>
<td>2.2.86</td>
<td>Enthalpy of catalyst leaving the reactor.</td>
</tr>
<tr>
<td>2.2.87</td>
<td>Empirical equation for calculating the effective void fraction of regenerator catalyst dense bed.</td>
</tr>
<tr>
<td>2.2.92 to 2.2.102</td>
<td>Equations for calculating temperature and gas concentrations along the vertical height in the regenerator.</td>
</tr>
<tr>
<td>2.2.105</td>
<td>Energy released due to combustion of carbon to CO and CO₂ in the regenerator.</td>
</tr>
<tr>
<td>2.2.106</td>
<td>Enthalpy associated with flue gases exiting the regenerator.</td>
</tr>
<tr>
<td>2.2.111</td>
<td>Empirical equation for calculating the flow of catalyst into the standpipe.</td>
</tr>
<tr>
<td>2.2.113</td>
<td>Regenerator stack gas valve opening calculated based on the assumption that, at steady state, air flow into the regenerator is equal to the stack gas flow.</td>
</tr>
<tr>
<td>2.2.114 and 2.2.115</td>
<td>Energy balance equation around the furnace firebox.</td>
</tr>
<tr>
<td>2.2.116 and 2.2.117</td>
<td>Energy balance equations around reactor and regenerator, respectively.</td>
</tr>
<tr>
<td>Eq. No.</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>2.2.118</td>
<td>Carbon mass balance around regenerator.</td>
</tr>
<tr>
<td>2.2.119</td>
<td>Mass balance around wet gas compressor.</td>
</tr>
<tr>
<td>2.2.120</td>
<td>Catalyst mass balance around regenerator standpipe</td>
</tr>
<tr>
<td>2.2.121</td>
<td>Force balance in the regenerator lift pipe.</td>
</tr>
<tr>
<td>2.2.122</td>
<td>Mass balance around fractionator.</td>
</tr>
<tr>
<td>2.2.123, 124</td>
<td>Mass balance around combustion air blower, wet gas compressor and lift air and 125 blower, respectively.</td>
</tr>
</tbody>
</table>

Since Newton’s method requires 'good' initial guesses, initially the 12 free variables required for the solution are chosen from the physical nature of the problem. 8 of the free variables ($T_2$, $T_3$, $T_r$, $T_{reg,db}$, $C_{rgc}$, $P^*$, $v_{rgc}$, and $P_{blp}$) are chosen randomly in a region based on a reasonable range of values these variables can take. The remaining 4 free variables were calculated with the help of the model equations.

The following equations were used to get good initial guesses for the free variables for the equation solver and to pre-empt infeasible conditions arising from the physical nature of the problem.

1. $T_1 \neq T_2 \neq T_3$ from the Equation (2.2.23).
2. $v_{rgc} > 0$.
3. $P_5 > P_{atm}$ from Equation (2.2.37).
   
   $P_5 > P_7$ from Equation (2.2.35).
1. \( P_1 > 1.036 \times 10^5 \) from Equation (2.2.33).

4. \( v_1 > 1.5086 \) from Equation (2.2.70).

5. \( 0 < \frac{P_2}{P_1} < 2.42 \) from Equation (2.2.26).

\[ P_2 > P_{am} \cdot P_{rgb} \] from Equations (2.2.30) and (2.2.17) respectively.

\[ P_1 < P_{am} \] from Equation (2.2.29).

6. \( P_3 > P_{rph} \cdot P_{hpi} \cdot P_{am} \) from Equations (2.2.18), (2.2.20) and (2.2.80) respectively.

\[ P_3 < \left[ 10.2336 \frac{S_{u}^2}{S_{b}^2} + P_{am}^{M} \right]^{i} \] from Equation (2.2.65).

Subsequently, the free variable values corresponding to the best value of the performance index are taken as the starting conditions for the equation solver. Additionally, we were able to improve the region of convergence for the Newton’s method somewhat by using a stepping parameter of 0.5 for updating these 12 free variables for the first two iterations.

For integration of the differential equations, we used the subroutine DVERK (Hull et al., 1976) with a local error tolerance of \( 10^{-6} \) for the yield equations and \( 10^{-8} \) for the regenerator temperature and concentration equations.

To handle the state inequality constraints, we use the penalty function approach as was successfully used with iterative dynamic programming by Luus (1991). We have assumed that the combustion air blower and the wet gas compressor are operating far from the surge line (McFarlane et al., 1993) and hence their surge flow limits are not considered as state constraints.
To handle the state constraints, we construct the augmented performance index

$$J = (F_{ugo} - F_{oil})P_{feed} + F_{gasoline}P_{gasoline} + F_{lightgas}P_{lightgas} - \sum_{i=1}^{12} \theta_i p_i$$  \hspace{1cm} (2.2.126)

Where $p_i$'s are the penalty functions corresponding to the 12 constrained state variables and are given by

$$p_1 = \begin{cases} 
0 & \text{if } P_s \leq 3.75 \times 10^4 \frac{N}{m^2} \\
10 & \text{if } P_s > 3.75 \times 10^4 \frac{N}{m^2}
\end{cases} \hspace{1cm} (2.2.127)$$

$$p_2 = \begin{cases} 
0 & \text{if } P_o \leq 3.75 \times 10^5 \frac{N}{m^2} \\
10 & \text{if } P_o > 3.75 \times 10^5 \frac{N}{m^2}
\end{cases} \hspace{1cm} (2.2.128)$$

$$p_3 = \begin{cases} 
0 & \text{if } C_{\text{ethane}} \geq 1.5\% \\
1.5 - C_{\text{ethane}} & \text{if } C_{\text{ethane}} < 1.5\%
\end{cases} \hspace{1cm} (2.2.129)$$

$$p_4 = \begin{cases} 
0 & \text{if } T_{\text{ref}} \geq 685^\circ C \\
685 - T_{\text{ref}} & \text{if } T_{\text{ref}} < 685^\circ C
\end{cases} \hspace{1cm} (2.2.130)$$

$$p_5 = \begin{cases} 
0 & \text{if } T_{vce} \leq 710^\circ C \\
10 & \text{if } T_{vce} > 710^\circ C
\end{cases} \hspace{1cm} (2.2.131)$$

$$p_6 = \begin{cases} 
0 & \text{if } T_r \leq 535^\circ C \\
10 & \text{if } T_r > 535^\circ C
\end{cases} \hspace{1cm} (2.2.132)$$

$$p_7 = \begin{cases} 
0 & \text{if } T_3 \leq 926.7^\circ C \\
10 & \text{if } T_3 > 926.7^\circ C
\end{cases} \hspace{1cm} (2.2.133)$$

$$p_8 = \begin{cases} 
10 & \text{if } L_v < 0 \\
0 & \text{if } 0 \leq L_v \leq 6.1 \\
L_v - 6.1 & \text{if } L_v > 6.1
\end{cases} \hspace{1cm} (2.2.134)$$
\[ p_0 = \begin{cases} 0 & \text{if } F_{\text{cwn, left}} > F_{\text{cwn, right}} \\ 10 & \text{if } F_{\text{cwn, left}} \leq F_{\text{cwn, right}} \end{cases} \]  \hspace{1cm} (2.2.135)

\[ p_{10} = \begin{cases} 10 & \text{if } V_{14} < 0 \\ 0 & \text{if } 0 \leq V_{14} \leq 1 \\ 10 & \text{if } V_{14} > 1 \end{cases} \]  \hspace{1cm} (2.2.136)

\[ p_{11} = \begin{cases} 0 & \text{if } C_{\text{v, C, w}} \leq 350 \text{ ppm} \\ 10 & \text{if } C_{\text{v, C, w}} > 350 \text{ ppm} \end{cases} \]  \hspace{1cm} (2.2.137)

\[ p_{12} = \begin{cases} 0 & \text{if } T_{\text{c, c}} - T_{\text{reg}} \leq 11.1^\circ C \\ 10 & \text{if } T_{\text{c, c}} - T_{\text{reg}} > 11.1^\circ C \end{cases} \]  \hspace{1cm} (2.2.138)

and \( \theta \)'s are the positive penalty factors. We have used a relatively large value, namely \( p_i = 10 \) for the penalty functions where the violation of the inequality constraint can lead to difficulties.

2.3 Luus Jaakola optimization procedure

As was shown in a recent review paper (Luus, 2000d), despite its wide range of applicability, the LJ optimization procedure is very simple and basically involves only three steps:

1. Given some initial point \( u^* \) for the variables over which optimization is to be carried out, choose a number of random points \( R \) around it through the equation

\[ u = u^* + Dr \]  \hspace{1cm} (2.3.1)

where \( D \) is a diagonal matrix of randomly chosen elements in the interval \([-1, +1]\), and \( r \) is the region size vector for the variables.
2. Check the feasibility of each chosen point with respect to the inequality constraints. For each feasible point evaluate the augmented performance index $J$ and keep the best $u$ value that gives the largest value to $J$.

3. An iteration is defined by steps 1 and 2. At the end of each iteration, $u^*$ is replaced by the best feasible $u$ obtained in step 2, and the region size vector $r$ is reduced by $\gamma$ through

$$r^{j+1} = \gamma r^j$$

(2.3.2)

where $\gamma$ is a region contraction factor such as 0.95, and $j$ is the iteration index. This procedure is continued for a number of iterations and the results are examined.

The efficiency of the LJ optimization procedure is increased by using the multi-pass procedure, as shown by Luus et al. (1995). In the multi-pass method the three-step procedure is repeated after a given number of iterations, usually with a smaller initial region size than the previous pass. Recently Luus (1998a) showed that the strategy of choosing the region size at the beginning of a pass to be the range over which the variables have changed during the previous pass is an effective way of further increasing the efficiency.

Here we use the LJ optimization procedure in a multi-pass fashion with a region restoration factor of 0.85 of the initial region sizes for the first 5 passes, followed by the extent of variation, for region size determination. A minimum region size of $1.0 \times 10^{-5}$ is used to prevent the region sizes from collapsing to zero. After every iteration the region size is reduced by the factor $\gamma = 0.95$. 

40
3. RESULTS AND DISCUSSIONS:

The primary objective of making several runs is to find the global optimum for the problem. We would also like to know the effect of varying the various parameters in the LJ optimization procedure and how it influences the efficiency of the search in locating the global optimum. A few preliminary runs were conducted using different number of random points per iteration and different values of the penalty factors to find their effects and the approximate location of the global optimum.

3.1. Effect of the penalty function factors

For simplicity, the values of the penalty factors $\theta_1, \theta_2, \theta_3, \ldots, \theta_7$ were taken to be the same, each equal to $\theta$. Table 3.1.1 gives the results obtained from various runs conducted with different values of $\theta$. For these runs the starting conditions for the control values $[F_3, V_{11}, \Delta P_{RR}, V_6, V_{10}, V_9, F_j]$ were taken to be [0.9, 0.9, -2.75×10^4, 0.8, 0.4, 0.5, 63.5] with initial region size [0.08, 0.05, 0.7×10^4, 0.20, 0.20, 0.50, 1.5]. A region restoration factor of 0.85 for the first 5 passes followed by the extent of variation for region size determination as suggested recently by Luus (1998a) for the next 10 passes was used for conducting these runs. A minimum region size of 1.0×10^-5 has been used to prevent region collapse. Using a restoration factor for the region size for the first few passes followed by the extent of deviation helps to make the search less dependent on the starting values of the control variables.

For very low values of $\theta$ (for example, 0.1 in Table 3.1.1) constraints tend to be violated as can be seen from the value of the augmented performance index, which is less than the value of the performance index. However, for a given number of random points, values of $\theta$ between
1 and 250 give almost identical results as seen from Figure 3.1.1 for \( \theta=1 \) and \( \theta=100 \). Thus we have a wide range over which to choose the penalty function factor for which we can get convergence to the vicinity of the global optimum. For subsequent runs we have used a value of 1 for \( \theta \). Figure 3.1.1 gives the convergence profile for different values of \( \theta \). Note that the plots for \( \theta=1 \) and 100 are identical and overlap each other.
<table>
<thead>
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<th></th>
<th></th>
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<td>2.60505</td>
<td>2.60517</td>
<td>2.60508</td>
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<tr>
<td>Aug. Perf. Index</td>
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<td>2.60505</td>
<td>2.60517</td>
<td>2.60508</td>
<td>2.60508</td>
<td></td>
</tr>
<tr>
<td>$F_3$</td>
<td>1.03363</td>
<td>1.04202</td>
<td>1.04203</td>
<td>1.04204</td>
<td>1.04204</td>
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<tr>
<td>$V_{11}$</td>
<td>0.9353</td>
<td>0.8562</td>
<td>0.8562</td>
<td>0.8561</td>
<td>0.8561</td>
<td></td>
</tr>
<tr>
<td>$\Delta P_{RR} \times 10^{-4}$</td>
<td>-2.759</td>
<td>-3.448</td>
<td>-3.449</td>
<td>-3.448</td>
<td>-3.448</td>
<td></td>
</tr>
<tr>
<td>$V_6$</td>
<td>0.8863</td>
<td>0.7136</td>
<td>0.7134</td>
<td>0.7135</td>
<td>0.7135</td>
<td></td>
</tr>
<tr>
<td>$V_{l0}$</td>
<td>0.4731</td>
<td>0.5371</td>
<td>0.5371</td>
<td>0.5371</td>
<td>0.5371</td>
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</tr>
<tr>
<td>$V_0$</td>
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<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td></td>
</tr>
<tr>
<td>$F_3$</td>
<td>64.8515</td>
<td>64.8248</td>
<td>64.8279</td>
<td>64.8257</td>
<td>64.8257</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1.1 Variation of performance index and control variables with the penalty factor
Figure 3.1.1 Effect of $\theta$ on the obtained value of the performance index.
3.2. Effect of the number of random points

To find the effect of the number of random points the starting conditions for the control variables \([F_5, V_{11}, \Delta P_{RR}, V_6, V_{i6}, V_9, F_3]\) was taken to be \([0.9, 0.9, -2.75 \times 10^4, 0.8, 0.4, 0.5, 63.5]\) with region size \([0.08, 0.05, 0.7 \times 10^4, 0.20, 0.20, 0.50, 1.5]\) and the number of random points per iteration was varied. Each run consisted of 15 passes with a restoration factor of 0.85 for the first 5 passes followed by extent of deviation for region size determination. All the runs were conducted with 30 iterations per pass and a region contraction factor of 0.95.

Tables 3.2.1 and 3.2.2 give the result of the runs. The best value for the performance index obtained from the runs is 2.60585 with 85 random points per iteration and 15 passes. However none of the other runs gives us the same result which calls for further investigation for locating the global optimum.

Although there is no specific trend in the obtained value of the performance index for different number of random points, there seems to be a minimum number of random points to be used to get a reasonably good value for the performance index. Using too large a number of random points however, does not necessarily give better results. In all the cases there has been no constraint violation.

Figure 3.2.1 gives the convergence profile for the number of random points on the obtained value of the performance index.
<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td></td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>Perf. Index ($/s)</td>
<td>2.60389</td>
<td>2.60397</td>
<td>2.60505</td>
</tr>
<tr>
<td>$F_5$ (m³/s)</td>
<td>1.05265</td>
<td>1.04790</td>
<td>1.04202</td>
</tr>
<tr>
<td>$V_{II}$</td>
<td>0.94997</td>
<td>0.92583</td>
<td>0.85616</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m²)</td>
<td>$-2.917 \times 10^4$</td>
<td>$-2.668 \times 10^4$</td>
<td>$-3.448 \times 10^4$</td>
</tr>
<tr>
<td>$V_6$</td>
<td>0.71711</td>
<td>0.97307</td>
<td>0.71355</td>
</tr>
<tr>
<td>$V_{lift}$</td>
<td>0.44253</td>
<td>0.64998</td>
<td>0.53706</td>
</tr>
<tr>
<td>$V_9$</td>
<td>0.60295</td>
<td>0.02454</td>
<td>0.99998</td>
</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.79346</td>
<td>64.79627</td>
<td>64.82475</td>
</tr>
<tr>
<td>$P_5$ (N/m²)</td>
<td>$1.65 \times 10^5$</td>
<td>$1.66 \times 10^5$</td>
<td>$1.71 \times 10^5$</td>
</tr>
<tr>
<td>$P_6$ (N/m²)</td>
<td>$2.01 \times 10^5$</td>
<td>$2.05 \times 10^5$</td>
<td>$2.02 \times 10^5$</td>
</tr>
<tr>
<td>$C_{O2,SG}$ (%)</td>
<td>1.53308</td>
<td>1.52981</td>
<td>1.50604</td>
</tr>
<tr>
<td>$T_{nrg,db}$ (°C)</td>
<td>685</td>
<td>685</td>
<td>685</td>
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<tr>
<td>$T_r$ (°C)</td>
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<td>535</td>
<td>535</td>
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<tr>
<td>$T_{cyc}$ (°C)</td>
<td>691.1</td>
<td>691.2</td>
<td>691.1</td>
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<td>$T_f$ (°C)</td>
<td>926.7</td>
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<td>921.1</td>
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<tr>
<td>$L_{sp}$ (m)</td>
<td>4.578</td>
<td>4.241</td>
<td>5.367</td>
</tr>
<tr>
<td>$V_{14}$</td>
<td>0.63493</td>
<td>0.62426</td>
<td>0.63286</td>
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<tr>
<td>$T_{diff}$ (°C)</td>
<td>6.1</td>
<td>6.2</td>
<td>6.1</td>
</tr>
<tr>
<td>$SRGM$ (m³/s)</td>
<td>3.397</td>
<td>1.830</td>
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<td>$C_{CO,SG}$</td>
<td>130</td>
<td>108</td>
<td>133</td>
</tr>
<tr>
<td>$COR$</td>
<td>5.377</td>
<td>5.385</td>
<td>5.396</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>1.20681</td>
<td>1.20854</td>
<td>1.20970</td>
</tr>
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</table>

Table 3.2.1 Effect of the number of random points $R$ on the value of the performance index and the corresponding control and state variable values.
<table>
<thead>
<tr>
<th>$R$</th>
<th>70</th>
<th>85</th>
<th>100</th>
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<tbody>
<tr>
<td>Perf. Index ($$/s)</td>
<td>2.60573</td>
<td>2.60585</td>
<td>2.60205</td>
</tr>
<tr>
<td>$F_3$ (m$^3$/s)</td>
<td>1.04636</td>
<td>1.04797</td>
<td>1.00590</td>
</tr>
<tr>
<td>$V_{11}$</td>
<td>0.83945</td>
<td>0.88891</td>
<td>0.88085</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m$^2$)</td>
<td>$-3.351 \times 10^4$</td>
<td>$-3.015 \times 10^4$</td>
<td>$-3.262 \times 10^4$</td>
</tr>
<tr>
<td>$V_6$</td>
<td>0.98249</td>
<td>0.85299</td>
<td>0.78317</td>
</tr>
<tr>
<td>$V_{lift}$</td>
<td>0.21168</td>
<td>0.32515</td>
<td>0.30999</td>
</tr>
<tr>
<td>$V_9$</td>
<td>0.55474</td>
<td>0.40214</td>
<td>0.71428</td>
</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.84171</td>
<td>64.84436</td>
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</tr>
<tr>
<td>$P_3$ (N/m$^2$)</td>
<td>$1.73 \times 10^5$</td>
<td>$1.69 \times 10^5$</td>
<td>$1.69 \times 10^5$</td>
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<td>$P_6$ (N/m$^2$)</td>
<td>$2.05 \times 10^5$</td>
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<tr>
<td>$C_{02, SG}$ (%)</td>
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<td>$T_{reg, db}$ (°C)</td>
<td>685</td>
<td>685</td>
<td>685</td>
</tr>
<tr>
<td>$T_r$ (°C)</td>
<td>535</td>
<td>535</td>
<td>535</td>
</tr>
<tr>
<td>$T_{cyc}$ (°C)</td>
<td>691.2</td>
<td>691.2</td>
<td>691.0</td>
</tr>
<tr>
<td>$T_3$ (°C)</td>
<td>923.4</td>
<td>924.2</td>
<td>902.2</td>
</tr>
<tr>
<td>$L_{sp}$ (m)</td>
<td>5.221</td>
<td>4.740</td>
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<td>$V_{14}$</td>
<td>0.62388</td>
<td>0.62675</td>
<td>0.64062</td>
</tr>
<tr>
<td>$T_{diff}$ (°C)</td>
<td>6.2</td>
<td>6.2</td>
<td>6.0</td>
</tr>
<tr>
<td>$SRGM$ (m$^3$/s)</td>
<td>2.184</td>
<td>2.453</td>
<td>2.978</td>
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<tr>
<td>$C_{CO2, SG}$</td>
<td>117</td>
<td>123</td>
<td>140</td>
</tr>
<tr>
<td>$COR$</td>
<td>5.389</td>
<td>5.389</td>
<td>5.458</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>1.20810</td>
<td>1.20746</td>
<td>1.22303</td>
</tr>
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</table>

Table 3.2.2 Effect of the number of random points $R$ on the value of the performance index and the corresponding control and state variable values.
Figure 3.2.1 Effect of number of random points on the obtained value of the performance index.
3.3 Search for the global optimum

The global optimum of 2.60601 was obtained by rerunning the case with 40 random points with \( \theta = 1.0 \) using the control variable values obtained from the previous run as the starting condition. Region size for the control variables \([F_5, V_{IL}, \Delta P_{RR}, V_6, V_{th}, V_y, F_3]\) was \([0.0008, 0.0005, 0.7 \times 10^2, 0.002, 0.002, 0.005, 0.015]\) for this run. This method of using information from the previous runs to locate the global optimum for difficult optimization problems was successfully used by Luus et al. (1995) in solving the optimal drug scheduling problem in cancer chemotherapy.

The values of the control and state variables corresponding to the global optimum are given in Table 3.3.5. Note that at the optimum, all the constraints are satisfied. Also, riser temperature, \( T_r \) concentration of \( O_2 \) in the stack gas, \( C_{O_{2SG}} \) and regenerator temperature, \( T_{reg,db} \) are at the constraint boundaries.

To see whether we can get the same value of the global optimum runs were made with different starting conditions for the control variable values \([F_5, V_{IL}, \Delta P_{RR}, V_6, V_{th}, V_y, F_3]\) with different number of random points per iteration. The number of passes and the number of iteration were kept the same at 15 and 30 respectively. The region size for these runs were kept the same at \([0.08, 0.05, 0.7 \times 10^4, 0.20, 0.20, 0.50, 1.5]\). For runs starting from the value (case 1) \([1.049, 0.941, -3.140 \times 10^4, 0.687, 0.396, 0.849, 64.839]\) the best value of performance index obtained was 2.60566 with a run using 30 random points per iteration. Tables 3.3.1 and 3.3.2 give the results obtained from these runs. For runs starting from the value (case 2) \([0.913, 0.942, -3.040 \times 10^4, 0.990, 0.152, 0.413, 64.587]\) with the same parameters for LJ optimization procedure, the best value of performance index obtained was 2.60097 for a run with 60 random points. Tables 3.3.3 and 3.3.4 give the results obtained from these runs.
Table 3.3.1 Effect of the number of random points $R$ on the value of the performance index and the corresponding control and state variable values (case 1).
<table>
<thead>
<tr>
<th>$R$</th>
<th></th>
<th></th>
<th></th>
</tr>
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<tr>
<td>Perf. Index ($/s)$</td>
<td>2.59758</td>
<td>2.60377</td>
<td>2.59987</td>
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<tr>
<td>$F_3$ (m$^3$/s)</td>
<td>0.93938</td>
<td>1.02347</td>
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<tr>
<td>$V_{11}$</td>
<td>0.92290</td>
<td>0.93642</td>
<td>0.91151</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m$^2$)</td>
<td>$-3.132 \times 10^4$</td>
<td>$-3.444 \times 10^4$</td>
<td>$-2.954 \times 10^4$</td>
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<tr>
<td>$V_6$</td>
<td>0.74084</td>
<td>0.66551</td>
<td>0.79479</td>
</tr>
<tr>
<td>$V_{lift}$</td>
<td>0.46725</td>
<td>0.32364</td>
<td>0.55445</td>
</tr>
<tr>
<td>$V_9$</td>
<td>0.75234</td>
<td>0.99998</td>
<td>0.45871</td>
</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.65874</td>
<td>64.79533</td>
<td>64.71113</td>
</tr>
<tr>
<td>$P_5$ (N/m$^2$)</td>
<td>$1.67 \times 10^5$</td>
<td>$1.66 \times 10^5$</td>
<td>$1.67 \times 10^5$</td>
</tr>
<tr>
<td>$P_6$ (N/m$^2$)</td>
<td>$2.01 \times 10^5$</td>
<td>$1.97 \times 10^5$</td>
<td>$2.03 \times 10^5$</td>
</tr>
<tr>
<td>$C_{O_2,SG}$ (%)</td>
<td>1.51324</td>
<td>1.50001</td>
<td>1.50258</td>
</tr>
<tr>
<td>$T_{reg,db}$ (°C)</td>
<td>685.0</td>
<td>685.0</td>
<td>685.0</td>
</tr>
<tr>
<td>$T_r$ (°C)</td>
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<td>535.0</td>
<td>535.0</td>
</tr>
<tr>
<td>$T_{cyc}$ (°C)</td>
<td>690.9</td>
<td>690.9</td>
<td>691.0</td>
</tr>
<tr>
<td>$T_3$ (°C)</td>
<td>866.6</td>
<td>911.4</td>
<td>877.2</td>
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<td>$L_{sp}$ (m)</td>
<td>5.216</td>
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<td>$V_{14}$</td>
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<td>0.64593</td>
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<td>$T_{diff}$ (°C)</td>
<td>5.9</td>
<td>5.9</td>
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<tr>
<td>$SRGM$ (m$^3$/s)</td>
<td>3.686</td>
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<tr>
<td>$C_{CO,SG}$</td>
<td>169</td>
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<td>148</td>
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<tr>
<td>$COR$</td>
<td>5.576</td>
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<td>5.542</td>
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<tr>
<td>$F_{air}$</td>
<td>1.24705</td>
<td>1.21574</td>
<td>1.23962</td>
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</table>

Table 3.3.2 Effect of the number of random points $R$ on the value of the performance index and the corresponding control and state variable values (case 1).
<table>
<thead>
<tr>
<th>$R$</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perf. Index ($$/s$)</td>
<td>2.58619</td>
<td>2.59300</td>
<td>2.60097</td>
</tr>
<tr>
<td>$F_5$ ($m^3/s$)</td>
<td>0.78810</td>
<td>0.89480</td>
<td>0.98994</td>
</tr>
<tr>
<td>$V_{11}$</td>
<td>0.90352</td>
<td>0.86216</td>
<td>0.84708</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m$^2$)</td>
<td>$-3.047\times10^4$</td>
<td>$-3.450\times10^4$</td>
<td>$-3.219\times10^4$</td>
</tr>
<tr>
<td>$V_6$</td>
<td>0.94758</td>
<td>0.86025</td>
<td>0.83028</td>
</tr>
<tr>
<td>$V_{lift}$</td>
<td>0.63667</td>
<td>0.32545</td>
<td>0.59312</td>
</tr>
<tr>
<td>$V_y$</td>
<td>0.41774</td>
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</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.43614</td>
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<tr>
<td>$P_5$ (N/m$^2$)</td>
<td>$1.68\times10^5$</td>
<td>$1.71\times10^5$</td>
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<tr>
<td>$P_6$ (N/m$^2$)</td>
<td>$2.03\times10^5$</td>
<td>$2.02\times10^5$</td>
<td>$2.06\times10^5$</td>
</tr>
<tr>
<td>$C_{O2:SG}$ (%)</td>
<td>1.50695</td>
<td>1.53579</td>
<td>1.52031</td>
</tr>
<tr>
<td>$T_{reg,db}$ (°C)</td>
<td>685</td>
<td>685</td>
<td>685</td>
</tr>
<tr>
<td>$T_r$ (°C)</td>
<td>535</td>
<td>535</td>
<td>535</td>
</tr>
<tr>
<td>$T_{cyc}$ (°C)</td>
<td>690.7</td>
<td>690.8</td>
<td>691.1</td>
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<tr>
<td>$T_3$ (°C)</td>
<td>782.0</td>
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<td>$L_{sp}$ (m)</td>
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<tr>
<td>$V_{t4}$</td>
<td>0.68042</td>
<td>0.66309</td>
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</tr>
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<td>$T_{diff}$ (°C)</td>
<td>5.7</td>
<td>5.8</td>
<td>6.1</td>
</tr>
<tr>
<td>$SRGM$ ($m^3/s$)</td>
<td>3.650</td>
<td>3.082</td>
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<tr>
<td>$C_{CO:SG}$</td>
<td>195</td>
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<td>119</td>
</tr>
<tr>
<td>$COR$</td>
<td>5.856</td>
<td>5.656</td>
<td>5.486</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>1.30426</td>
<td>1.26460</td>
<td>1.22909</td>
</tr>
</tbody>
</table>

Table 3.3.3 Effect of the number of random points $R$ on the value of the performance index and the corresponding control and state variable values (case 2).
<table>
<thead>
<tr>
<th>$R$</th>
<th>70</th>
<th>85</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perf. Index ($$/s$)</td>
<td>2.59443</td>
<td>2.59218</td>
<td>2.58940</td>
</tr>
<tr>
<td>$F_5$ (m$^3$/s)</td>
<td>0.91300</td>
<td>0.85650</td>
<td>0.84060</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>0.94151</td>
<td>0.87590</td>
<td>0.94921</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m$^2$)</td>
<td>-3.040×10$^4$</td>
<td>-3.040×10$^4$</td>
<td>-3.040×10$^4$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>0.98981</td>
<td>0.88996</td>
<td>0.99995</td>
</tr>
<tr>
<td>$V_{th}$</td>
<td>0.15193</td>
<td>0.31768</td>
<td>0.05810</td>
</tr>
<tr>
<td>$V_g$</td>
<td>0.41299</td>
<td>0.68885</td>
<td>0.55492</td>
</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.58739</td>
<td>64.55329</td>
<td>64.48989</td>
</tr>
<tr>
<td>$P_5$ (N/m$^2$)</td>
<td>1.66×10$^5$</td>
<td>1.70×10$^5$</td>
<td>1.65×10$^5$</td>
</tr>
<tr>
<td>$P_6$ (N/m$^2$)</td>
<td>2.01×10$^5$</td>
<td>2.01×10$^5$</td>
<td>1.98×10$^5$</td>
</tr>
<tr>
<td>$\text{O}_{22SG}$ (%)</td>
<td>1.53257</td>
<td>1.50001</td>
<td>1.51575</td>
</tr>
<tr>
<td>$T_{reg,db}$ ($^\circ$C)</td>
<td>685</td>
<td>685</td>
<td>685</td>
</tr>
<tr>
<td>$T_r$ ($^\circ$C)</td>
<td>535</td>
<td>535</td>
<td>535</td>
</tr>
<tr>
<td>$T_{cyc}$ ($^\circ$C)</td>
<td>690.8</td>
<td>690.7</td>
<td>690.6</td>
</tr>
<tr>
<td>$T_3$ ($^\circ$C)</td>
<td>852.2</td>
<td>820.9</td>
<td>812.0</td>
</tr>
<tr>
<td>$L_{sp}$ (m)</td>
<td>5.158</td>
<td>5.850</td>
<td>5.740</td>
</tr>
<tr>
<td>$V_{1,t}$</td>
<td>0.66373</td>
<td>0.67170</td>
<td>0.68743</td>
</tr>
<tr>
<td>$T_{diff}$ ($^\circ$C)</td>
<td>5.8</td>
<td>5.7</td>
<td>5.6</td>
</tr>
<tr>
<td>$\text{SRGM}$ (m$^3$/s)</td>
<td>1.90</td>
<td>3.015</td>
<td>1.851</td>
</tr>
<tr>
<td>$C_{O_{22SG}}$</td>
<td>169</td>
<td>192</td>
<td>225</td>
</tr>
<tr>
<td>$\text{COR}$</td>
<td>5.623</td>
<td>5.728</td>
<td>5.756</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>1.25758</td>
<td>1.27757</td>
<td>1.28393</td>
</tr>
</tbody>
</table>

Table 3.3.4 Effect of the number of random points $R$ on the value of the performance index and the corresponding control and state variable values (case 2).
The computation time for a run consisting of 15 passes, 30 iterations per pass and 60 random points per iteration is 2 hrs. and 45 mins., so we can get close to the global optimum in reasonable amount of computation time. Also, in the industry, we generally move from one steady state to a nearby one, so good starting conditions are available in the practical situation, making the method easy to implement in an actual FCC unit.

As mentioned before, of the seven control variables, \( \Delta P_{RR} \) and \( V_{\theta} \) affect the catalyst circulation rate and \( V_{\theta} \). \( V_{\theta} \) and \( V_{\phi} \) affect the air flowrate. However reducing the dimensionality of the problem from a seven variable to a five variable search and solving results in a performance index of 2.55048. The value of the control state and other important variables are shown in Table 3.3.5. The performance index value is more than 2% away from the global optimum of 2.60601. The control variable \( V_{\theta} \) has hit its upper bound of 1.0. Hence, a seven dimensional search is necessary for this problem.

Making reruns as above using the information from the previous run, however, does not give the global optimum and gets stuck at the local optimum for the other cases. Table 3.3.6 gives the performance index obtained from 5 different runs and the corresponding control values. It can be seen from the table that apart from a lack of trend in the value of the variables, the performance index is very insensitive to the value of the variables, pressure \( P_5 \), air flow rate \( F_{air} \) and catalyst to oil ratio, \( COR \). This low sensitivity makes it difficult to obtain accurately the global optimum for this problem.

An interesting observation evident from table 3.3.6 is that, in general, operating the FCC unit at a lower catalyst to oil flow rate and a lower air flow rate keeping the other constraints satisfied, give a higher value for the performance index. This means that for a higher
value of the performance index, the unit prefers to take a lesser amount of energy from catalyst circulation and a greater amount from the preheater.
<table>
<thead>
<tr>
<th>Control variable</th>
<th>Value at Global optimum</th>
<th>Value at maximum airflow</th>
<th>Value with a 5 dimensional search</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance index</td>
<td>2.60601</td>
<td>2.58028</td>
<td>2.55048</td>
</tr>
<tr>
<td>$F_3$ (m$^3$/s)</td>
<td>1.05261</td>
<td>1.05056</td>
<td>1.05036</td>
</tr>
<tr>
<td>$V_{11}$</td>
<td>0.94980</td>
<td>0.79152</td>
<td>0.95</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m$^2$)</td>
<td>-2.932x10$^4$</td>
<td>-3.450x10$^4$</td>
<td>-2.670x10$^4$</td>
</tr>
<tr>
<td>$V_6$</td>
<td>0.71513</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$V_{lift}$</td>
<td>0.44573</td>
<td>0.64485</td>
<td>0.0</td>
</tr>
<tr>
<td>$V_9$</td>
<td>0.59660</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.84774</td>
<td>64.18982</td>
<td>63.43260</td>
</tr>
<tr>
<td>$P_3$ (N/m$^2$)</td>
<td>1.65x10$^5$</td>
<td>1.78x10$^5$</td>
<td>1.62x10$^5$</td>
</tr>
<tr>
<td>$P_6$ (N/m$^2$)</td>
<td>2.01x10$^5$</td>
<td>2.09x10$^5$</td>
<td>2.01x10$^5$</td>
</tr>
<tr>
<td>$C_{O_{2,SG}}$ (%)</td>
<td>1.5</td>
<td>1.87</td>
<td>1.51</td>
</tr>
<tr>
<td>$T_{rec,db}$ ($^\circ$C)</td>
<td>685</td>
<td>685</td>
<td>685</td>
</tr>
<tr>
<td>$T_r$ ($^\circ$C)</td>
<td>535</td>
<td>535</td>
<td>534.1</td>
</tr>
<tr>
<td>$T_{cvc}$ ($^\circ$C)</td>
<td>691.1</td>
<td>691</td>
<td>691.4</td>
</tr>
<tr>
<td>$T_3$ ($^\circ$C)</td>
<td>926.6</td>
<td>926.1</td>
<td>926.7</td>
</tr>
<tr>
<td>$L_{sp}$ (m)</td>
<td>4.61</td>
<td>5.23</td>
<td>3.80</td>
</tr>
<tr>
<td>$V_{14}$</td>
<td>0.63444</td>
<td>0.61859</td>
<td>0.6089</td>
</tr>
<tr>
<td>$T_{diff}$ ($^\circ$C)</td>
<td>6.1</td>
<td>6.0</td>
<td>6.40</td>
</tr>
<tr>
<td>$SRGM$ (m$^3$/s)</td>
<td>3.40</td>
<td>4.20</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>$C_{CO_{2,SG}}$</td>
<td>141</td>
<td>34</td>
<td>112</td>
</tr>
<tr>
<td>$COR$</td>
<td>5.379</td>
<td>5.360</td>
<td>5.225</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>1.20567</td>
<td>1.21896</td>
<td>1.1539</td>
</tr>
</tbody>
</table>

Table 3.3.5: Obtained values of performance index, control and state variable values for the global optimum, maximum airflow and for a five dimensional search.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Performance Index ($/s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.60583</td>
</tr>
<tr>
<td>$F_S$ (m$^3$/s)</td>
<td>1.04648</td>
</tr>
<tr>
<td>$V_{II}$</td>
<td>0.83890</td>
</tr>
<tr>
<td>$\Delta P_{RR}$ (N/m$^2$)</td>
<td>-3.355x10$^4$</td>
</tr>
<tr>
<td>$V_6$</td>
<td>0.98207</td>
</tr>
<tr>
<td>$V_{lift}$</td>
<td>0.21319</td>
</tr>
<tr>
<td>$V_q$</td>
<td>0.55691</td>
</tr>
<tr>
<td>$F_3$ (kg/s)</td>
<td>64.84407</td>
</tr>
<tr>
<td>$P_S$ (N/m$^2$)</td>
<td>1.73x10$^5$</td>
</tr>
<tr>
<td>$P_6$ (N/m$^2$)</td>
<td>2.05x10$^5$</td>
</tr>
<tr>
<td>$C_{O2,SG}$ (%)</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_{reg,db}$ (°C)</td>
<td>685</td>
</tr>
<tr>
<td>$T_r$ (°C)</td>
<td>535</td>
</tr>
<tr>
<td>$T_{cyc}$ (°C)</td>
<td>691.2</td>
</tr>
<tr>
<td>$T_j$ (°C)</td>
<td>923.4</td>
</tr>
<tr>
<td>$L_{sp}$ (m)</td>
<td>5.23</td>
</tr>
<tr>
<td>$V_{j,4}$</td>
<td>0.62376</td>
</tr>
<tr>
<td>$T_{diff}$ (°C)</td>
<td>6.2</td>
</tr>
<tr>
<td>$SRGM$ (m$^3$/s)</td>
<td>2.19</td>
</tr>
<tr>
<td>$C_{CO,SG}$</td>
<td>117</td>
</tr>
<tr>
<td>COR (kg cat/kg oil)</td>
<td>5.389</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>1.20802</td>
</tr>
</tbody>
</table>

Table 3.3.6. Obtained values of performance index, control and state variable values at five different local optimum.
3.4. Effect of variables

From the control values for the global optimum we can see that the airflow rate is not at its maximum value as indicated by the valve positions \( V_6 \) and \( V_9 \), which are not in full open condition. Operating the unit with both the combustion and the lift air blowers at the maximum capacity gives a performance index of 2.58028 which is 1\% below the global optimum. resulting in a loss of nearly $800,000/year with the given pricing structure. Comparing the values of the variables for the global optimum and maximum air flow rate as given by Table 3.3.5, it is clear that no significant advantage in terms of ease of operation will be gained by operating the unit at maximum air flow rate. In the maximum air flow case, we do not gain any significant margin by operating away from the surge limits, design temperatures or design pressures.

Table 3.4.1 shows the effect of changing some of the variables, on the performance index. The performance index has very low sensitivity to the heat of cracking. This is consistent with earlier observations (Ellis et al., 1998). The performance index is very sensitive and increases with increasing amount of hydrogen (in the form of unstripped hydrocarbon) in the spent catalyst. This is opposite to the opinion (Sadeighbeigi, 1995) that the performance index should be lower due to reduced throughput caused by higher regenerator temperatures due to the higher heat of combustion of hydrogen compared to carbon. The pricing structure has negligible effect on the control variables at the optimum. This might be because of the simplified yield model. A pricing structure based on octane value of the gasoline can be expected to behave differently.

The performance index shows a steady increase with increasing value of the coking factor for the slurry recycle as shown in Figure 3.4.1. It is believed that the combustion
constraint set A is a conservative one, and that there are significant advantages in operating the unit by relaxing constraint set A and operating closer to the more stringent constraint set B (McFarlane et al., 1993). Figure 3.4.2 shows the behavior of the performance index when the regenerator temperature is relaxed. Relaxing the regenerator temperature constraint by 2°C results in nearly 2.7% increase in the performance index, which is considerable.
<table>
<thead>
<tr>
<th>Change in parameter</th>
<th>Perf. Index</th>
<th>$F_3$</th>
<th>$T_r$</th>
<th>$T_{reg,db}$</th>
<th>COR</th>
<th>$O_{2,sg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta H_{crack}$</td>
<td>+5.0%</td>
<td>-0.12%</td>
<td>64.7767</td>
<td>535</td>
<td>685</td>
<td>5.463</td>
</tr>
<tr>
<td></td>
<td>-5.0%</td>
<td>+0.04%</td>
<td>64.8752</td>
<td>535</td>
<td>685</td>
<td>5.345</td>
</tr>
<tr>
<td>$C_{II}$</td>
<td>+5.0%</td>
<td>+2.37%</td>
<td>66.4305</td>
<td>535</td>
<td>685</td>
<td>5.427</td>
</tr>
<tr>
<td></td>
<td>-5.0%</td>
<td>-2.75%</td>
<td>63.0175</td>
<td>535</td>
<td>685</td>
<td>5.318</td>
</tr>
<tr>
<td>$P_{gasoline}$</td>
<td>+10.0%</td>
<td>+19.5%</td>
<td>64.8479</td>
<td>535</td>
<td>685</td>
<td>5.378</td>
</tr>
<tr>
<td></td>
<td>-10.0%</td>
<td>-19.5%</td>
<td>64.8480</td>
<td>535</td>
<td>685</td>
<td>5.378</td>
</tr>
</tbody>
</table>

Table 3.4.1. Effect of variables on the obtained value of the performance index
Figure 3.4.1. Effect of slurry recycle coking factor on the obtained value of the performance index.
Figure 3.4.2. Effect of relaxing the regenerator temperature constraint on the obtained value of the performance index.
4. CONCLUSIONS

A complex chemical reactor system was optimized by LJ optimization procedure. The model consisting of a large number of nonlinear algebraic equations was solved at each iteration efficiently by grouping the equations, so that the LJ optimization procedure with penalty functions to handle the state constraints, could be used for obtaining the global optimum for the FCC unit.

The presence of numerous local optima makes the problem a very challenging one. There is a wide range of values for the penalty factors for which the method works well. The grouping of equations into classes as shown in this thesis provides a robust and efficient method for solving the nonlinear steady state algebraic equations in the FCC model. Therefore, the global optimum could be obtained in reasonable computation time.

At the global optimum, the air flow rate was not at its maximum value, so the unit profitability can be improved by not operating the unit at maximum airflow. The coking tendency of the slurry recycle has a significant effect on the performance index. The unit profitability can be increased by 2.7% if the regenerator temperature constraint could be relaxed by only 2.0°C and operating the unit closer to the combustion constraints. Therefore the LJ optimization procedure provided a convenient way of studying the effects of design parameters.
5. NOTATION

$A_{lp}$  cross sectional area of lift pipe, (0.811 m$^2$)

$A_{reg}$ cross sectional area of regenerator, (64.81 m$^2$)

$A_{rx}$ cross sectional area of reactor riser, (0.89 m$^2$)

$A_{sp}$ cross sectional area stand pipe, (0.65 m$^2$)

$A_{strip}$ cross sectional area of reactor stripper, (5.57 m$^2$)

$A_{ubend_{reg}}$ cross sectional area of regenerated catalyst U-bend, (0.344 m$^2$)

$A_{ubend_{sp}}$ cross sectional area of spent catalyst U-bend, (0.483 m$^2$)

$a_1$ furnace heat loss parameter, (5.59 $\left(\frac{1.871 + 32}{72}\right)$ kJ/m$^3$)$^0$C

$a_2$  furnace heat loss parameter, (211.01 kJ/s)

$C_{CO_{reg}}$ concentration of carbon monoxide in regenerator stack gas, (ppm)

$C_H$ weight fraction of hydrogen in coke, (0.075 kg H$_2$/kg coke)

$COR$ catalyst to oil ratio, (kg catalyst/kg oil)

$C_{O_{reg}}$ concentration of oxygen in regenerator stack gas, (kg mol%)

$c_p(z)$ average heat capacity, (kJ/kgmol $^0$C)

$c_{air}$ heat capacity of air, (29.64 kJ/kgmol $^0$C)

$c_{pc}$ heat capacity of catalyst, (1.30 kJ/kg $^0$C)

$c_{pc}(CO)$ heat capacity of carbon monoxide, (30.48 kJ/kgmol $^0$C)

$c_{pc}(CO_2)$ heat capacity of carbon dioxide, (46.05 kJ/kgmol $^0$C)

$c_{iff}$ heat capacity of fresh feed liquid, (3.43 kJ/kg $^0$C)
heat capacity of fresh feed vapor, \( (3.39 \frac{kJ}{kg \cdot ^0C}) \)

heat capacity of steam. \( (36.09 \frac{kJ}{kg \cdot mol \cdot ^0C}) \)

heat capacity of nitrogen. \( (30.23 \frac{kJ}{kg \cdot mol \cdot ^0C}) \)

heat capacity of carbon dioxide. \( (31.90 \frac{kJ}{kg \cdot mol \cdot ^0C}) \)

heat capacity of slurry vapor. \( (3.35 \frac{kJ}{kg \cdot ^0C}) \)

weight fraction of coke on regenerated catalyst. (kg coke/kg catalyst)

wet gas compressor compression ratio

weight fraction of coke on spent catalyst. (kg coke/kg catalyst)

factor representing coking tendency of slurry recycle relative to fresh feed. (3.5 for base case).

d parameter used in calculation of catalyst flow into the regenerator standpipe

D diagonal matrix

elevation of regenerator lift air injection. (40.84 m)

elevation where oil enters the reactor riser. (37.95 m)

elevation of reactor stripper tap. (39.62 m)

pressure tap elevation on standpipe. (47.24 m)

activation energy. gas oil to gasoline. (87821.4 J/kgmol)

activation energy. gasoline to coke. (72988.7 J/kgmol)

activation energy. gas oil to wet gas. (97552.4 J/kgmol)

activation energy. gas oil to coke. (87504.1 J/kgmol)

air flowrate into regenerator. (kgmol/s)

air lift compressor inlet suction flow at base conditions. (m³/s)

production of coke in reactor riser. (kg/s)
\( F_{\text{gasoline}} \) \text{ flow rate of Gasoline, (kg/s)}

\( F_H \) \text{ burning rate of hydrogen, (kg/s)}

\( F_{\text{light gas}} \) \text{ light gas flow rate, (kg/s)}

\( f_{of} \) \text{ overflow factor. (2070.14 } \frac{\text{kg}}{\text{m}^3 \cdot \text{s}} \text{)}

\( F_{\text{oil}} \) \text{ total feed flow rate, (kg/s)}

\( f_{pp}(x) \) \text{ nonlinear valve flowrate function}

\( F_{\text{rec}} \) \text{ flowrate of regenerated catalyst, (kg/s)}

\( F_{\text{sc}} \) \text{ flowrate of spent catalyst, (kg/s)}

\( F_{\text{sp}} \) \text{ flow into standpipe, (kg/s)}

\( F_{\text{vacm, comb}} \) \text{ combustion air blower inlet suction flow, (m}^3\text{/s)}

\( F_{\text{vacm, lift}} \) \text{ lift air blower inlet suction flow, (m}^3\text{/s)}

\( F_{\text{vacn, wg}} \) \text{ wet gas compressor inlet suction flow, (m}^3\text{/s)}

\( F_{\text{surge lift}} \) \text{ lift air blower surge flow, (m}^3\text{/s)}

\( F_f \) \text{ air flowrate into regenerator, (kg/s)}

\( f_{\text{ubend}_{\text{rec}}} \) \text{ regenerated catalyst friction factor. (83 } \frac{\text{kg} \cdot \text{s}}{\text{m}^3} \text{)}

\( f_{\text{ubend}_{\text{sc}}} \) \text{ spent catalyst friction factor, (229.47 } \frac{\text{kg} \cdot \text{s}}{\text{m}^3} \text{)}

\( F_{\text{unconv}} \) \text{ unconverted gas oil flow rate, (kg/s)}

\( F_{\text{v1}} \) \text{ flow through combustion air blower suction valve } V_a. \text{ (kg/s)}

\( F_{\text{v2}} \) \text{ flow through combustion air blower vent valve } V'. \text{ (kg/s)}

\( F_{\text{v3}} \) \text{ flow through lift air blower vent valve } V_s. \text{ (kg/s)}

\( F_{\text{v11}} \) \text{ flow through wet gas compressor suction valve } V_{11}. \text{ (kgmol/s)}

\( F_{\text{v12}} \) \text{ flow through wet gas flare valve } V_{12}. \text{ (kgmol/s)}

\( F_{\text{v13}} \) \text{ flow through wet gas compressor anti-surge valve } V_{13}. \text{ (kgmol/s)}

\( F_{\text{wg}} \) \text{ wet gas production in the reactor, (kgmol/s)}
\[ F_3 \quad \text{flow of fresh feed to the reactor riser. (kg/s)} \]

\[ F_4 \quad \text{flow of slurry to the reactor riser. 2.38 (kg/s)} \]

\[ F_5 \quad \text{flow of fuel to furnace. (m}^3/\text{s)} \]

\[ F_6 \quad \text{combustion air blower throughput. (kg/s)} \]

\[ F_7 \quad \text{combustion air flow to the regenerator. (kg/s)} \]

\[ F_8 \quad \text{lift air blower throughput. (kg/s)} \]

\[ F_9 \quad \text{lift air flow to the regenerator. (kg/s)} \]

\[ F_{10} \quad \text{spill air flow to the regenerator. (kg/s)} \]

\[ F_{11} \quad \text{wet gas flow to the vapour recovery unit. (kgmol/s)} \]

\[ g \quad \text{acceleration due to gravity. (9.81 m/s}^2) \]

\[ h_{\text{lift}} \quad \text{height of lift pipe. (10.36 m)} \]

\[ h_{\text{rv}} \quad \text{height of reactor riser. (18.29 m)} \]

\[ h_{\text{rp}} \quad \text{height of regenerator standpipe. (6.10 m)} \]

\[ H_{\text{wc}} \quad \text{wet gas compressor head. (N/m}^2) \]

\[ J \quad \text{augmented performance index} \]

\[ k_{\text{unit}} \quad \text{average ratio of specific heats. (1.39)} \]

\[ k_{\text{comb}} \quad \text{combustion air blower discharge pipe flow resistance factor. } \left( 0.22 \text{kg/s} \cdot \frac{\sqrt{N}}{\sqrt{m^2}} \right) \]

\[ k_{\text{lift}} \quad \text{lift air blower discharge pipe flow resistance factor. } \left( 2.73 \times 10^{-2} \text{ kg/s} \cdot \frac{\sqrt{N}}{\sqrt{m^2}} \right) \]

\[ k_1 \quad \text{reaction rate constant. (s}^{-1}) \]

\[ k_2 \quad \text{reaction rate constant. (s}^{-1}) \]

\[ k_3 \quad \text{reaction rate constant. (kgmol air/s.kg mol CO)} \]

\[ k_6 \quad \text{combustion air blower suction valve flow rating. } \left( 1.37 \text{kg/s} \cdot \frac{\sqrt{N}}{\sqrt{m^2}} \right) \]

\[ k_7 \quad \text{combustion air blower vent valve flow rating. } \left( 0.08 \text{ kg/s} \cdot \frac{\sqrt{N}}{\sqrt{m^2}} \right) \]
\( k_s \) lift air blower vent valve flow rating, \( \left( 2.73 \times 10^{-2} \text{kg/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \)

\( k_v \) lift air blower spill valve flow rating, \( \left( 5.46 \times 10^{-2} \text{kg/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \)

\( k_{11} \) wet gas compressor suction valve flow rating, \( \left( 8.20 \times 10^{-3} \text{kgmol/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \)

\( k_{12} \) wet gas flare valve flow rating, \( \left( 2.73 \times 10^{-3} \text{kgmol/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \)

\( k_{13} \) wet gas compressor antisurge valve flow rating, \( \left( 6.58 \times 10^{-7} \text{kgmol/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \)

\( k_{14} \) regenerator stack gas valve flow rating, \( \left( 6.01 \times 10^{-3} \text{kgmol/s} \cdot \sqrt{\frac{\text{N}}{\text{m}^2}} \right) \)

\( k_{21} \) rate constant, gas oil to gasoline, \( \left( \frac{m^b}{\text{kgmol} \cdot \text{kgcat} \cdot \text{s}} \right) \)

\( k_{31} \) rate constant, gas oil to wet gas, \( \left( \frac{m^b}{\text{kgmol} \cdot \text{kgcat} \cdot \text{s}} \right) \)

\( k_{41} \) rate constant, gas oil to coke, \( \left( \frac{m^3}{\text{kgcat} \cdot \text{s}} \right) \)

\( k_{42} \) rate constant, gasoline to coke, \( \left( \frac{m^b}{\text{kgmol} \cdot \text{kgcat} \cdot \text{s}} \right) \)

\( k_{210} \) pre exponential factor, gas oil to gasoline, \( \left( 0.4272 \times 10^7 \frac{m^b}{\text{kgmol} \cdot \text{kgcat} \cdot \text{s}} \right) \)

\( k_{310} \) pre exponential factor, gas oil to wet gas, \( \left( 0.1012 \times 10^8 \frac{m^b}{\text{kgmol} \cdot \text{kgcat} \cdot \text{s}} \right) \)

\( k_{410} \) pre exponential factor, gas oil to coke, \( \left( 0.5504 \times 10^5 \frac{m^3}{\text{kgcat} \cdot \text{s}} \right) \)
$k_{120}$ pre exponential factor, gasoline to coke. \( (0.1337 \times 10^3 \frac{m^b}{kgmol \cdot kgcat \cdot s}) \)

$L_{sp}$ level of catalyst in standpipe, (m)

$L_{ubend_r}$ length of regenerated catalyst ubend, (17.07 m)

$L_{ubend_s}$ length of spent catalyst ubend, (17.07 m)

$M$ polytropic exponent

$M_f$ molecular weight of fresh feed, (333 kg/kgmol)

$M_e$ flowrate of entrained catalyst from dense bed into dilute phase, (kg/s)

$M_{wt}$ molecular weight, wet gas, (40 kg/kgmol)

$P$ performance index to be maximized, ($/s)$

$P_{atm}$ atmospheric pressure, (1.01325 $\times 10^5$ N/m$^2$)

$P_{basc}$ combustion air blower base discharge pressure, (N/m$^2$)

$P_{basc,d}$ lift air blower base discharge pressure, (N/m$^2$)

$P_{bip}$ pressure at the bottom of lift pipe, (N/m$^2$)

$P_{feed}$ feed price, 0.088 $/kg$

$P_{gasoline}$ gasoline price, 0.140 $/kg$

$p_i$ penalty functions

$P_{light\ gas}$ light gas price, 0.132 $/kg$

$P_{rh}$ pressure at the bottom of reactor riser, (N/m$^2$)

$P_{rgh}$ pressure at the bottom of regenerator, (N/m$^2$)

$P_{vra}$ discharge pressure of wet gas compressor to vapor recovery unit, (6.96 $\times 10^5$ N/m$^2$)

$P_1$ combustion air blower suction pressure, (N/m$^2$)

$P_2$ combustion air blower discharge pressure, (N/m$^2$)

$P_3$ lift air blower discharge pressure, (N/m$^2$)

$P_4$ reactor pressure, (N/m$^2$)
$P_s$ reactor fractionator pressure. (N/m$^2$)

$P_r$ regenerator pressure. (N/m$^2$)

$P_t$ wet gas compressor suction pressure. (N/m$^2$)

$Q_{air}$ enthalpy of air to regenerator. (kJ/s)

$Q_c$ total heat of burning carbon. (kJ/s)

$Q_{catalyst}$ enthalpy of catalyst out of reactor riser. (kJ/s)

$Q_{cracking}$ heat generated from cracking. (kJ/s)

$Q_c$ total heat loss from regenerator into environment. (586.61 kJ/s)

$Q_{if}$ heat required to bring fresh feed to reactor riser temperature. (kJ/s)

$Q_{rc}$ enthalpy of outgoing regenerator stack gas. (kJ/s)

$Q_{rr}$ heat required to raise temperature of fresh feed from 371.1°C (liq.) to 537.8°C (vapor). (718.73 kJ/kg)

$Q_{rf}$ enthalpy of hydrogen to regenerator. (kJ/s)

$Q_{in, reactor}$ enthalpy into reactor. (kJ/s)

$Q_{out, reactor}$ enthalpy out of reactor. (kJ/s)

$Q_{in, regenerator}$ enthalpy into regenerator. (kJ/s)

$Q_{out, regenerator}$ enthalpy out of regenerator. (kJ/s)

$Q_{rec}$ enthalpy of regenerated catalyst. (kJ/s)

$Q_{sc}$ enthalpy of spent catalyst. (kJ/s)

$Q_{slurry}$ heat required to bring slurry to reactor riser temperature. (kJ/s)

$Q_{sr}$ heat required to raise temperature of slurry from 260°C (liq.) to 537.8°C vapor. (958.31 kJ/kg)

$r$ region size vector

$R$ number of random points per iteration

$R$ universal gas constant. ($8314.39 \frac{J}{kgmol \cdot K}$)
\( s_a \) actual speed of the lift air blower. (rpm)

\( s_{min} \) minimum speed of the lift air blower. (5000 rpm)

\( s_b \) base speed of the lift air blower. (5950 rpm)

\( S_f \) slurry factor. takes into account the effect of slurry recycle on the system

\( SRGM \) surge margin for lift air blower. \( F_{acr, lift} - F_{surge, lift} \) (m³/s)

\( T_{air} \) temperature of air entering regenerator. (132.2°C)

\( T_{am} \) atmospheric temperature. (23.9°C)

\( T_{base} \) base temperature. (537.8°C)

\( T_{base, f} \) base temperature of reactor fresh feed. (371.1°C)

\( T_{comb, d} \) combustion air blower discharge temperature. (88°C)

\( T_{v,c} \) regenerator stack gas temperature at cyclone. (°C)

\( T_{diff} \) temperature difference between cyclone and regenerator bed. (°C)

\( T_{lift, d} \) lift air blower discharge temperature. (°C)

\( T_{lm} \) furnace log mean temperature difference. (°C)

\( T_r \) temperature of reactor riser. (°C)

\( T_{ref} \) base temperature for reactor riser energy balance. (537.8°C)

\( T_{reg} \) temperature of regenerator. (°C)

\( T_{reg, db} \) temperature of regenerator dense bed. (°C)

\( T_w \) temperature of spent catalyst entering regenerator. (°C)

\( T_1 \) temperature of fresh feed entering furnace. (238.3°C)

\( T_2 \) temperature of fresh feed entering the reactor riser. (°C)

\( T_{2, ss} \) steady state furnace outlet temperature. (°C)

\( T_3 \) furnace firebox temperature. (°C)

\( u \) vector of variables used for optimization (7×1)
optimization variable values corresponding to the maximum value of the augmented performance index, at the end of each iteration (7x1)

\( u^* \)

\( UA_f \)  
furnace overall heat transfer coefficient, \((47.48 \frac{kJ}{s \cdot ^oC})\)

\( v_{air, lift} \)  
velocity of air in lift pipe, (m/s)

\( v_{cat, lift} \)  
velocity of catalyst in lift pipe, (m/s)

\( V_{lift} \)  
manipulated variable for lift air blower steam valve, (0-1.2)

\( v_{rex} \)  
velocity of regenerated catalyst, (m/s)

\( v_{rx} \)  
volumetric flowrate in reactor riser, (m³/s)

\( v_s \)  
superficial velocity in the regenerator, (m/s)

\( v_{w} \)  
velocity of spent catalyst, (m/s)

\( v_{slip} \)  
slip velocity, (0.671 m/s)

\( V_T \)  
volume, reactor riser, (16.3 m³)

\( V_s \)  
combustion air blower suction valve position, (0-1)

\( V_l \)  
combustion air blower vent valve position, (0-1)

\( V_{8} \)  
lift air blower vent valve position, (0-1)

\( V_{9} \)  
spill air blower valve position, (0-1)

\( V_{11} \)  
wet gas compressor suction valve position, (0-1)

\( V_{12} \)  
wet gas flare valve position, (0-1)

\( V_{13} \)  
wet gas compressor vent valve position, (0-1)

\( V_{14} \)  
stack gas valve position, (0-1)

\( W_c \)  
mass of carbon in regenerator, (kg)

\( W_r \)  
mass of catalyst in reactor, (kg)

\( W_{rex} \)  
mass of catalyst in regenerator, (kg)

\( W_{rr} \)  
mass of catalyst in riser, (kg)

\( W_{sp} \)  
mass of catalyst in standpipe, (kg)

\( w_T \)  
mass of hydrocarbon in the riser, (kg)
$W_{total}$  total catalyst in the system, (171762 kg)

$x$  variable representing valve opening

$X_{CO}$  molar ratio of CO to air. (kgmol CO/kgmol air)

$X_{CO,SG}$  molar ratio of CO to air in stack gas. (kgmol CO/kgmol air)

$X_{O_2}$  molar ratio of $O_2$ to air. (kgmol $O_2$/kgmol air)

$X_{CO,SG}$  molar ratio of $CO_2$ to air. (kgmol $CO_2$/kgmol air)

$X_{O_2,SG}$  molar ratio of $O_2$ to air in stack gas. (kgmol $O_2$/kgmol air)

$X_N$  molar ratio of $N_2$ to air. (kgmol $N_2$/kgmol air)

$X_O$  molar ratio of $O_2$ to air. (kgmol $O_2$/kgmol air)

$Y_A$  mass fraction, gas oil feed

$Y_B$  mass fraction, gasoline

$Y_C$  mass fraction, wet gas

$Y_D$  mass fraction, coke

$z$  vertical distance from the bottom of the regenerator. (m)

$z_{Reb}$  regenerator dense bed height. (m)

$z_{Cyc}$  height of cyclone inlet. (13.72 m)

$z_{lp}$  height of lift pipe discharge. (3.35 m)

$z_{xp}$  standpipe exit height from the bottom of the regenerator. (3.96 m)

$z_{top}$  O$_2$, CO measurement point. (15.09 m)

**Greek symbols**

$\alpha$  exponential decay function. (391)

$\gamma$  region contraction factor

$\delta_z$  Dirac delta function

$\Delta H_{crack}$  heat of cracking. (kJ/kg)

$\Delta H_{fu}$  heat of combustion of furnace fuel. (37.259 kJ/m$^3$)
\( \Delta H_H \)  
heat of combustion of hydrogen, (141.904 kJ/kg)

\( \Delta H_1 \)  
heat of formation of CO, (109.545 kJ/kgmol CO produced)

\( \Delta H_2 \)  
heat of formation of CO\(_2\), (395.838 kJ/kgmol CO\(_2\) produced)

\( \Delta P_{\text{frac}} \)  
pressure drop across reactor main fractionator, (0.65\times10^5 N/m\(^2\))

\( \Delta P_{\text{RR}} \)  
differential pressure between regenerator and reactor, (N/m\(^2\))

\( \Delta T_{\text{stripper}} \)  
temperature drop across reactor stripper, (19.44°C)

\( \varepsilon_e \)  
effective void fraction in regenerator dense phase bed

\( \varepsilon_f \)  
apparent void fraction in regenerator dense phase bed

\( \eta_p \)  
polytropic efficiency, (1.0)

\( \theta, \theta_i \)  
penalty function factors

\( \rho_{\text{airg}} \)  
density of air at regenerator conditions, (kg/m\(^3\))

\( \rho_\text{v} \)  
volume fraction of catalyst

\( \rho_\text{c} \)  
density of catalyst in U-bend and regenerator standpipe, (720.82 kg/m\(^3\))

\( \rho_{\text{c, dense}} \)  
density of catalyst in the dense phase, (kg/m\(^3\))

\( \rho_{\text{c, dilute}} \)  
density of catalyst in the dilute phase, (kg/m\(^3\))

\( \rho_\text{g} \)  
density of exit gas, (kgmol/m\(^3\))

\( \rho_{\text{lift}} \)  
density of catalyst in lift pipe, (kg/m\(^3\))

\( \rho_{\text{part}} \)  
settled density of catalyst, (1089.25 kg/m\(^3\))

\( \rho_{\text{rs}} \)  
average density of material in the reactor riser, (kg/m\(^3\))

\( \rho_v \)  
vapor density at the reactor riser condition, (9.13 kg/m\(^3\))

\( \tau_r \)  
catalyst residence time in the riser, (s)

\( \Phi \)  
fraction of active sites in catalyst
6. REFERENCES


