FLOW CHARACTERISTICS OF A DIRECT CURRENT STIRRING DEVICE

By

Katherine Bryce

A thesis submitted in conformity with the requirements for the degree of Master of Applied Science in the University of Toronto

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Flow Characteristics of a Direct Current Stirring Device
Master of Applied Science (2001)
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Abstract

Electromagnetic stirring is employed to improve the quality of continuously cast steel product. This thesis proposes a device that may form the basis for the development of a direct current stirring system.

An analytical model describing the magnetic flux density and magnetic body force was developed. A numerical model was constructed to describe the conduction, electromagnetic, and fluid flow characteristics of the system. This model was used to explore the influence of coil separation, coil radius, the magnitude of the current in the system, and the direction of current flow in the coils on the fluid flow profile. An experimental model was built and tested.

The results show that it is feasible to stir liquid metal using the device proposed. The flow pattern created is rotational, though not unidirectional. It is recommended that the coil configuration which would generate a unidirectional flow profile throughout the fluid, and reduce the stirring velocity at the surface to zero, be determined.
I would like to thank Prof. Bendzsak for not only supporting me during this project but also for mentoring me throughout my undergraduate studies. I would also like to thank the other professors in our department; all of you helped make my whole university career a real pleasure. Thanks also to Fanny Strumas, Teresa Miniaci, and Louisa Kung, for keeping this place together. Thanks to Phil Poulos and Bob Manson for keeping the computers up and running.

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I would also like to thank Warren Adolphe, who in no small part paved the way for me to complete this work, and also for helping in the construction linear tracking motor assembly. Thanks also to my classmate and friend Ka-Ming Lin, who always had the right gadget or screwdriver, and most importantly the time to help out.

Most of all I’d like to thank my family: my parents, for keeping the safety net strung up these extra two years; my brothers and sister also for their support, and for actually listening after making the mistake of asking “but what exactly is it that you do?”; and my partner, rdm, for “seeing things through”. You truly are some kind of wonderful.
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CHAPTER 1: Introduction

1.1 Electromagnetic Stirring During Continuous Casting of Steel
Electromagnetic stirring of metal during continuous casting is well established science. It has been shown to result in improved metallurgical properties in the solidified strand [1-4]. These improvements include (but are not limited to) increased equiaxed zone size, as well as decreased porosity, segregation, and surface defects, improving mechanical properties and workability.

The interaction between electrical currents and magnetic fields produces body forces in conducting media. This phenomenon has been exploited to stir the molten part of solidifying strands during continuous casting of steel. Although the force field can be generated by both static (dc) or alternating (ac) electromagnetic fields, at the present time only ac fields are commonly employed for this purpose.

Stirring has also been achieved using direct current devices [5-8]. Because dc devices require that current be injected into the strand, and that stirring be in the direction of casting, these devices have seen limited application.

The principle purpose of this thesis is to explore a basic device for rotary stirring of steel in the casting mould. The concepts explored can be extended in the development of future devices that will create higher velocity flow fields over increased lengths of the strand.

1.2 Electromagnetic Stirring using Alternating Current
D. A. Shtanko proposed the stirring of liquid metal to control solidification processes as early as 1917 [5]. In the same year, Kuerth suggested the use of rotating magnetic fields rather than mechanical rotation of the mould for the same purpose [9]. The first experimentation with electromagnetic stirring of cast product was done by Shtanko, in 1933, when he placed a small steel sand casting in the rotary field of the stator of an induction motor [5]. Real interest in the technique began in 1952, with the advent of continuous casting of steel [4]. The first industrial application of electromagnetic stirring on a continuous caster was designed by Junghans and Schaaber in 1952 [5]. The use of travelling magnetic fields generated by linear induction motors was first explored by Sundberg in the 1960s [9].
The principle of electromagnetic stirring (EMS) is based on the principles governing the induction motor. The stirring coil is energized with alternating current, which generates a travelling magnetic field. This field induces eddy currents in the solidifying strand. These eddy currents are orthogonal to the inducing magnetic field, and the interaction of the induced current with the magnetic field generates a body force, known as the Lorenz force. The Lorenz force acts in the direction orthogonal to both the direction of the induced current flow, and the direction of the magnetic field, to stir the liquid metal. The two main types of induction based EMS techniques are those based on the rotary induction motor and those based on the linear induction motor. EMS systems generating more complex flow patterns (such as helical) have been developed using modifications of these two techniques [10], but in principle they are the same.

Rotary EMS systems are typically used on billets and blooms of square, or approximately square, cross sections. The maximum product dimension is approximately 30cm (~12”). These devices have also been applied to the stirring of round billets of less than 35cm (~14”) in diameter [9].

The linear EMS systems can be applied to any shape of billet, bloom, or slab. Typically, billets and blooms are stirred in the direction of casting (Fig.1.1a). For slabs, the stirring direction can either be longitudinal or transverse, that is, stirring takes place across the broad face of the strand, orthogonal to the casting direction (Fig 1.1b) [11].

![Fig. 1.1 Schematic representation of (a) longitudinal stirring of billets (b) horizontal stirring of slabs (adapted from [11])](image-url)
Fig. 1.1 shows the induction stirrer on one side of the billet, bloom or slab, however a stirring unit may also be present on the opposite face of the strand. In these cases, there are two possibilities for the creation of forces. They can be either in the same or opposite direction, with the choice dependent on the desired flow profile (shown schematically in Fig. 1.2).

![Diagram showing induction stirrer on one side of the billet, bloom or slab.](image)

**Fig. 1.2 Longitudinal stirring in a billet or bloom with linear EMS systems in combination**
(a) symmetrical (b) opposed (adapted from [12])

A summary of the typical EMS systems used for billet and bloom casting is presented in Table 1.1. The perceived metallurgical benefits associated with each type include [5]:

**Surface Quality:**
- reduction of slag entrapment
- reduction of pinholes

**Sub-Surface Quality:**
- reduction of large non-metallic inclusions
- reduction of blowholes

**Centreline Quality:**
- reduction of large non-metallic inclusions
- reduced centreline segregation
- reduced centerline porosity
- increased equiaxed zone size.
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<td>✗</td>
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<td>✓</td>
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† Reported on slab caster

✓ = benefits applicable, ✗ = benefits not applicable
1.2.1 Attenuation of the Magnetic Field in Induction Based EMS Systems
The stirring action of all induction based in-mould EMS systems is limited by the attenuation of the magnetic field by the copper mould. The electromagnetic field generated by the induction coils creates eddy currents in the mould that reduce the magnitude of the field exponentially, hence very small amounts of the magnetic field actually reaches the steel product. This phenomenon is governed by the concept of skin depth, given by

\[ \delta = \frac{\sqrt{2}}{\mu_0 \omega \sigma} \]  

(1.1)

where \( \mu \) is the magnetic permeability of the medium,
\( \omega \) is the angular velocity of the magnetic field, and
\( \sigma \) is the electrical conductivity.

For a copper mould the electrical conductivity is on the order of \( 4 \times 10^7 \, (\Omega \cdot m)^{-1} \), \( \mu = 4\pi \times 10^{-7} \, H/m \), and for line current \( \omega \) is approximately 377Hz. Thus the penetration depth is on the order of 10mm. This means that the mould will reduce the magnitude of a 60Hz electromagnetic field at the surface of the steel to approximately 10%, or less, of the original value. The actual value is dependent on the copper grade, mould wall thickness and the geometry of the mould.

Hence low frequencies (2-10Hz [5,9]) must be used to power induction based in-mould EMS systems in order to get adequate penetration of the bath by the magnetic field. This will increase the strength of the field by \( \sqrt{\frac{60}{2}} \) to \( \sqrt{\frac{60}{10}} \) times.

1.3 Electromagnetic Stirring using Direct Current
The systems developed to utilize direct current for electromagnetic stirring are few. In 1979 Sumitomo Metal Industries Ltd. of Japan developed a below-mould conductive (dc) EMS system. It used stationary permanent magnets as a stator to supply the dc magnetic field, and current was injected through the rollers [12], as illustrated in Fig. 1.3.
In this system, the direction of the magnetic field is reversed by an alternating arrangement of the magnetic poles of the permanent magnets (see Fig. 1.3b). This assembly generates a flow field in the liquid metal consisting of many small vortices [5], as illustrated in Fig. 1.4.
In 1980 Sumitomo Metal Industries developed an alternate below-mould conductive electromagnetic stirrer. It was shown to be effective in improving the internal quality of the steel blooms cast [6]. In this device, direct current flows across the slab between the rollers (as shown in Fig. 1.5), and rectangular coils enclose the strand. The coils are supplied by direct current to generate a static magnetic field in the axial direction, orthogonal to the direction of current flow. The force generated by the interaction of the current supplied by the upper set of rollers with the magnetic field is in the direction that is orthogonal to both (in this case, out of the page). The force generated by the lower set of roller is into the page (Fig. 1.5b). This generates a transverse stirring action. The flow direction is across the broad face of the slab, as illustrated in Fig. 1.5c.

![Diagram of Sumitomo conductive EMS for slab casting](image)

Fig. 1.5 The Sumitomo conductive EMS for slab casting (a) schematic of installation (b) schematic of body force profile (c) schematic of flow direction profile generated (adapted from [6])

In 1984 V. M. Fedotov et al. [7] developed a below-mould dc EMS system for slab casters which employs two sets of the rollers of the continuous caster to supply the electric current, and a third set to act as the cores of electromagnets. The current flow was down the length of the slab, between two of the sets of rollers (illustrated in Fig. 1.6a), and the magnetic flux density was distributed across the slab, between the third set of rollers (located between the first two sets), orthogonal to the current flow direction. The interaction between the current density and the magnetic flux density present generated a stirring force across the width of the slab.

In 1985 I.M. Kirko et al. developed an "electrovortex" technique, which was proposed as a simplification of the system by Fedotov [8]. It also employs the guide rollers of the continuous caster to supply the electric current, which flows down the length of the strand, to a second set of rollers (Fig 1.6b). The difference between the two designs was that rather than using guide
rollers as the core of the electromagnets to supply the magnetic flux density, the system by Kirko et al. uses a U-shaped ferromagnetic core to do so [8]. The ferromagnetic core generates a magnetic flux density transversely across the slab, which, upon interaction with the current density generates a stirring force across the broad face of the slab.

Both designs generate a flow pattern like that illustrated in Fig 1.5c.

A survey of the relevant literature indicates that all conductive electromagnetic stirrers that have been developed to date are below-mould systems used on slab casters.

1.4 Thesis Objective
The purpose of this thesis was to investigate the fundamental characteristics of a new type of dc, rotary electromagnetic stirring system, which had not been examined previously in the literature.

The main objectives of this thesis are:
1. The examination of the variation in the flow fields generated by the stirrer due to changes in system geometry and current, using finite element analysis. This is discussed in Chapter 4.
2. To examine the effect of current flow direction in the stirring coils on the flow fields generated by the stirrer. This is discussed in Chapter 5.
3. To determine the feasibility of the system using finite element analysis and experimental work on a laboratory scale model. The experimental work is discussed in Chapter 6.

4. A simplified model for the electromagnetic portion of the system is described in Chapter 2. It will be used as a "reference" model in the interpretation of the more detailed results of the numerical analysis.

Consequently, the investigation of possible metallurgical benefits arising from the use of this stirrer is beyond the scope of this work.
CHAPTER 2: Theoretical Considerations

2.1 Introduction
Magnetohydrodynamics is the study of the motion of electrically conducting fluids under the influence of combined electric and magnetic fields [14]. When electric current flows in a fluid in the presence of a magnetic field, the interaction of the current in the fluid with the magnetic field generates an electromagnetic body force in the fluid, causing it to flow. The motion of the fluid through the magnetic field induces current in the fluid, and this current generates a magnetic field of its own. This second magnetic field acts in opposition to the first, and therefore the motion of the fluid is affecting the electromagnetic state of the system [15]. These basic considerations show that the problems of magnetohydrodynamics (MHD) are those of inherently coupled fields.

The electromagnetic aspects of magnetohydrodynamic problems are described by Maxwell's equations, and the fluid flow aspects by the Navier-Stokes equations. Under certain conditions these equations may be decoupled, that is, for some problems, the motion of the fluid does not significantly affect the magnetic field. This was the case in this thesis. The equations, and the requirement for decoupling them, are discussed in the following sections.

The equations of MHD are fundamentally nonlinear. Consequently, only a few analytical solutions are available for highly idealized and linearized systems. The solutions of realistic 3-D problems are only accessible via numerical methods.

Computer software packages dedicated to the solution of the multi-physics problems inherent in MHD have been developed. These include Algor®, ANSYS®, and Nastran®. These packages require the user to understand the problem and its formulation, but not expert knowledge in the numerical solution of the diverse equations of mathematical physics. In this thesis, solution of the 3-D MHD problem was attained using ANSYS®.
2.2 The Governing Equations of Magnetohydrodynamics (MHD) for Incompressible Fluids

2.2.1 Maxwell’s Equations

The basic Maxwell’s equations for magnetostatic fields are [16]

\[ \nabla \times \mathbf{H} = \mathbf{J} \quad (2.1) \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (2.2) \]
\[ \mathbf{B} = \mu_0 \mu_r \mathbf{H} \quad (2.3) \]
\[ \nabla \cdot \mathbf{J} = 0 \quad (2.4) \]

where:
- \( \mathbf{B} \) is the magnetic flux density (T, or Wb/m²)
- \( \mathbf{H} \) is the magnetic field intensity (A/m)
- \( \mathbf{J} \) is the current density (A/m²)
- \( \mu_0 \) is the permeability of free space, \( 4\pi \times 10^{-7} \) H/m, and
- \( \mu_r \) is the relative permeability of the medium.

The current density is determined by Ohm's Law which states that the total current density is due to both an electric field and the motion of the conducting fluid in the presence of a magnetic field (equation 2.5) [16].

\[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad (2.5) \]

where:
- \( \mathbf{E} \) is the electric field intensity and
- \( \mathbf{V} \) is the velocity of the fluid.

It can be shown that the magnetic Reynold’s number \( (R_m) \) is a measure of the strength of coupling between the equations of electromagnetics and fluid flow.

\[ R_m = \frac{V L \sigma \mu_0}{\mu_r} \quad (2.6) \]

where:
- \( V \) is the velocity of the conducting fluid (m/s)
- \( L \) is a characteristic length (m)
- \( \sigma \) is the electrical conductivity of the fluid ((\Omega m)^{-1}).
If the quantity $R_m < 1$ the coupling between the fields is negligible, and the equations of fluid mechanics and electromagnetics decouple to form two systems that can be solved independently of each other.

For the present system, $\alpha = 4.35 \times 10^9 (\Omega m)^{-1}$, the velocities are low (less than 0.5 m/s), and the characteristic length is approximately 0.1 m. Thus, $R_m \approx 0.09$, and therefore equation 2.5 becomes

$$J = \sigma E$$

(2.7)

where

$$E = -\nabla V$$

(2.8)

$V$ is the applied voltage, which satisfies Laplace's equation for a potential field

$$\nabla^2 V = 0$$

(2.9)

Therefore the current density in the bath is entirely due to the potential difference applied to it, and currents induced by the magnetic field are not considered in this thesis.

2.2.2 Fluid Flow Equations

The continuity equation for an incompressible fluid in steady state is given by [16]

$$\nabla \cdot V = 0$$

(2.10)

Equation 2.10 is the law of conservation of mass.

The equation of motion for incompressible viscous flows under the MHD approximations is [16]

$$\rho \left( \frac{\partial V}{\partial t} + (V \cdot V) V \right) = -\nabla P + \nabla \cdot \tau + J \times B$$

(2.11)

Term: (i) (ii) (iii) (iv) (v)

where:

- $\rho$ is the density of the fluid
- $P$ is the pressure
- $\tau$ is the viscous stress tensor
- $J \times B$ is the magnetic body force acting on the fluid

Equation 2.11 is the conservation of momentum for Newtonian fluids. The terms of the Navier-Stokes equations include [17]:

Chapter 2: Theoretical Considerations

(i) local acceleration of a fluid particle
(ii) convective acceleration of a fluid particle
(iii) acceleration due to pressure differential
(iv) viscous deceleration due to frictional resistance (dissipation)
(v) electromagnetic body force acting on the fluid (all other forces are considered negligible).

Equations 2.10 and 2.11 together determine the flow field.

2.3 A Simplified Magnetic and Body Force Model
The considerations of Section 2.2 show that the most appropriate solution of the Navier-Stokes equations are numerical. However, the force field \((J \times B)\) can be approximated by a simple analytical model. Since the flow is created by this force field, it is advantageous to explore its nature using an analytical approach.

The approximate magnetic model for the system was constructed using the analytical equations for a single circular filament loop (Fig. 2.1). The underlying assumptions of the equations are that the current flows in a filament with zero cross sectional area and is ideal and circular, and thus the effects of current source leads connecting to the loop are negligible [18].

![Fig. 2.1 Single circular current loop centred at O (adapted from [18])](image-url)
2.3.1 Analytical Equations for a Single Filament Loop

Magnetic field intensity and magnetic flux density in equations 2.1 and 2.2 are related (in a vacuum) by Ampère’s Law

\[ B = \mu_0 H \]  (2.12)

Combining equation 2.12 with equation 2.1 gives

\[ \nabla \times H = \frac{1}{\mu_0} \nabla \times B = J \]  (2.13)

In order to simplify the analysis of the magnetic fields, it is customary to introduce the concept of a vector potential, \( A \). This magnetic vector potential in cylindrical coordinates is defined as

\[ A = a_r A_r + a_\phi A_\phi + a_z A_z \]  (2.14)

where:

- \( A_r \) is the radial component
- \( A_\phi \) is the azimuthal component
- \( A_z \) is the axial component of the magnetic vector potential, and
- \( a_r, a_\phi, \) and \( a_z \) are the unit vectors in these same directions, respectively.

Magnetic flux density is related to magnetic vector potential by

\[ B = \nabla \times A \]  (2.15)

Equation 2.15 can be substituted into equation 2.12 to show that (see Appendix A) [19]

\[ \nabla^2 A = -\mu_0 J \]  (2.16)

Therefore

\[
\begin{align*}
\nabla^2 A_r &= -\mu_0 J_r, \\
\nabla^2 A_\phi &= -\mu_0 J_\phi, \\
\nabla^2 A_z &= -\mu_0 J_z
\end{align*}
\]  (2.17)

Equation 2.17 shows that the magnetic vector potential in any of the three orthogonal directions is a function of the current density in the same direction. For a circular filament loop the current is entirely in the azimuthal direction \( (I = I_\phi) \), \( J_r = J_z = 0 \). Therefore the components of magnetic vector potential \( A \) are also zero in the radial and axial directions. Thus \( A \) is described entirely by its \( \phi \) component, given by equation 2.18 [18],

\[ A_\phi = \frac{\mu_0}{2\pi} I \left[ (r + a)^2 + z^2 \right]^{1/2} \left[ \frac{1}{2} k^2 \right] K(k) - E(k) \]  (2.18)
where:

- $r$ is the radial distance from the centre of the loop
- $z$ is the height for the position of interest $P$ (as shown in Fig. 2.1),
- $I$ is the current in the filament and
- $\mu$ is the magnetic permeability of the medium, in this case free space ($\mu_0 = 4\pi \times 10^{-7}$ H/m)

$K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, with the modulus $k$ defined as [18]:

$$k^2 = \frac{4ra}{(r + a)^2 + z^2}$$  \hspace{1cm} (2.19)

The components of magnetic flux density vectors due to a circular current filament are [18]:

$$B_r = -\frac{\partial A_\phi}{\partial z} = \frac{\mu}{2\pi} I \frac{z}{r} \left[ \frac{(r+a)^2 + z^2}{z} \right]^{1/2} \left[ -K(k) + \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2} E(k) \right]$$  \hspace{1cm} (2.20)

$$B_z = \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) = \frac{\mu}{2\pi} I \left[ \frac{(r+a)^2 + z^2}{z} \right]^{1/2} \left[ +K(k) + \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2} E(k) \right]$$  \hspace{1cm} (2.21)

and $B_\phi = 0$ due to axial symmetry.

### 2.3.2 Two Loop System

When a system of two filament loops of equal radii is considered, the equations are modified to include the separation between the loops, $2h$ (Fig. 2.2).

![Fig. 2.2 Two filament loops (adapted from [18])](image)
The fields are given by

\[
A_{\varphi 1} = \frac{\mu}{2\pi} I_1 \left[ (r + a)^2 + (z - h)^2 \right]^{\frac{1}{2}} \left[ (l - \frac{1}{2} k_1^2) K(k_1) - E(k_1) \right]
\]
\[
A_{\varphi 2} = \frac{\mu}{2\pi} I_2 \left[ (r + a)^2 + (z + h)^2 \right]^{\frac{1}{2}} \left[ (l - \frac{1}{2} k_2^2) K(k_2) - E(k_2) \right]
\]

(2.22)

for the top and bottom filaments (numbered 1 and 2 respectively), and accordingly

\[
k_1^2 = \frac{4ra}{(r + a)^2 + (z - h)^2}
\]
\[
k_2^2 = \frac{4ra}{(r + a)^2 + (z + h)^2}
\]

(2.23)

The vector components of the magnetic flux density due to each current carrying filament are then

\[
B_{r1} = -\frac{\partial A_{\varphi 1}}{\partial z} = \frac{\mu}{2\pi} I_1 \frac{z - h}{r} \left[ (r + a)^2 + (z - h)^2 \right]^{\frac{1}{2}} \left[ -K(k_1) + \frac{a^2 + r^2 + (z - h)^2}{(a - r)^2 + (z - h)^2} E(k_1) \right]
\]

(2.24)

\[
B_{r2} = -\frac{\partial A_{\varphi 2}}{\partial z} = \frac{\mu}{2\pi} I_2 \frac{z + h}{r} \left[ (r + a)^2 + (z + h)^2 \right]^{\frac{1}{2}} \left[ -K(k_2) + \frac{a^2 + r^2 + (z + h)^2}{(a - r)^2 + (z + h)^2} E(k_2) \right]
\]

and

\[
B_{z1} = \frac{1}{r} \frac{\partial (rA_{\varphi 1})}{\partial r} = \frac{\mu}{2\pi} I_1 \left[ (r + a)^2 + (z - h)^2 \right]^{\frac{1}{2}} \left[ +K(k_1) + \frac{a^2 - r^2 - (z - h)^2}{(a - r)^2 + (z - h)^2} E(k_1) \right]
\]

(2.25)

\[
B_{z2} = \frac{1}{r} \frac{\partial (rA_{\varphi 2})}{\partial r} = \frac{\mu}{2\pi} I_2 \left[ (r + a)^2 + (z + h)^2 \right]^{\frac{1}{2}} \left[ +K(k_2) + \frac{a^2 - r^2 - (z + h)^2}{(a - r)^2 + (z + h)^2} E(k_1) \right]
\]
It is useful to normalize equations 2.22 through 2.25 with respect to filament radius $a$ and separation $h$. This gives a set of equations which can be used to generate a generalized case. Factoring the radius $a$ out of the equations results in

$$
A_{\phi 1} = \frac{\mu}{2\pi} I_1 \left( (r*+1)^2 + (z*-h*)^2 \right)^{1/2} \left( 1 - \frac{1}{2} k_1 a^2 \right) K(k_1) - E(k_1)
$$

$$
A_{\phi 2} = \frac{\mu}{2\pi} I_2 \left( (r*-1)^2 + (z*-h*)^2 \right)^{1/2} \left( 1 - \frac{1}{2} k_2 a^2 \right) K(k_2) - E(k_2)
$$

Factor (2.26)

where $r*=r/a$, $z*=z/a$ and $h*=h/a$, and $0 \leq r* \leq +\infty$, $-\infty \leq z* \leq +\infty$ and $0 \leq h* \leq +\infty$. The corresponding modulus is

$$
k_1 = \frac{4r*}{(r*-1)^2 + (z*-h*)^2}
$$

$$
k_2 = \frac{4r*}{(r*-1)^2 + (z*+h*)^2}
$$

(2.27)

The effect on the field streamlines caused by changes in filament loop radius and filament loop separation can be examined by solving equation 2.26 for different values of $h*$.

Figures 2.3 and 2.4 illustrate the shape of the fields for $h*=0.25$ (filament separation equals one half filament radius) and $h*=0.5$ (filament separation equals filament radius), for two filaments with equal current but opposite current flow direction ($I_1=I_2$).

Figures 2.5 and 2.6 illustrate the shape of the fields for $h*=0.25$ and $h*=0.5$ for two filaments with equal current and the same current flow direction ($I_1=I_2$).

A decrease in the ratio $h*$ can be interpreted as either an increase in filament radius, or as a decrease in filament separation, in each case with the other parameter held constant. Throughout this discussion of this analytical model the example of filament separation will be used.
Fig. 2.3 Equipotential surface for $h^*=0.25$, Current flow is in the opposite direction in each filament loop ($I_1=-I_2$).

Fig. 2.4 Equipotential surface for $h^*=0.5$, Current flow is in the opposite direction in each filament loop ($I_1=-I_2$).
Fig. 2.5 Equipotential surface for $h^* = 0.25$, Current flow is in the same direction in each filament loop ($I_1 = I_2$)

Fig. 2.6 Equipotential surface for $h^* = 0.5$, Current flow is in the same direction in each filament loop ($I_1 = I_2$)
The normalized forms of the magnetic flux density equations are of the form

\[ B_{r_1} = \left( \frac{\mu}{2\pi a} \right) \zeta_{r_1} \]
\[ B_{r_2} = \left( \frac{\mu}{2\pi a} \right) \zeta_{r_2} \]  

(2.28)

and

\[ B_{z_1} = \left( \frac{\mu}{2\pi a} \right) \zeta_{z_1} \]
\[ B_{z_2} = \left( \frac{\mu}{2\pi a} \right) \zeta_{z_2} \]  

(2.28b)

where

\[ \zeta_{r_1} = \left( \frac{z^* - h^*}{r^*} \right) \left[ (r^* + 1)^2 + (z^* - h^*)^2 \right]^{1/2} \left[ -K(k_1^*) + \frac{1 + r^* + (z^* - h^*)^2}{(1 - r^*)^2 + (z^* - h^*)^2} E(k_1^*) \right] \]
\[ \zeta_{r_2} = \left( \frac{z^* + h^*}{r^*} \right) \left[ (r^* + 1)^2 + (z^* + h^*)^2 \right]^{1/2} \left[ -K(k_2^*) + \frac{1 + r^* + (z^* + h^*)^2}{(1 - r^*)^2 + (z^* + h^*)^2} E(k_2^*) \right] \]  

(2.29)

and

\[ \zeta_{z_1} = \left[ (r^* + 1)^2 + (z^* - h^*)^2 \right]^{1/2} \left[ +K(k_1^*) + \frac{1 - r^* + (z^* - h^*)^2}{(1 - r^*)^2 + (z^* - h^*)^2} E(k_1^*) \right] \]
\[ \zeta_{z_2} = \left[ (r^* + 1)^2 + (z^* + h^*)^2 \right]^{1/2} \left[ +K(k_2^*) + \frac{1 - r^* - (z^* + h^*)^2}{(1 - r^*)^2 + (z^* + h^*)^2} E(k_2^*) \right] \]  

(2.29b)

Thus for the case in which the current in the filaments is equal, but opposite in direction \((I_1 = -I_2)\), the total flux in the radial and \(z\) directions is

\[ B_{r_{total}} = \left( \frac{\mu}{2\pi a} \right) (\zeta_{r_1} - \zeta_{r_2}) \]  

(2.30)

\[ B_{z_{total}} = \left( \frac{\mu}{2\pi a} \right) (\zeta_{z_1} - \zeta_{z_2}) \]  

(2.31)

and if the current in the filaments is equal, and in the same direction \((I_1 = I_2)\), then the total flux is
2.3.3 The Electromagnetic Body Force

The electromagnetic body force (in N/m²) acting on a current conducting medium in the presence of a magnetic field is given by [16]

\[ f_r = J \times B \]  

(2.32)

where \( J \) is the current density in the conducting medium (A/m²). In general

\[ f_r = \begin{bmatrix} a_r & a_\phi & a_z \end{bmatrix} \begin{bmatrix} J_r & J_\phi & J_z \end{bmatrix} = (J_\phi B_z - J_z B_\phi)a_r - (J_z B_\phi - J_\phi B_z)a_\phi + (J_\phi B_z - J_z B_\phi)a_z \]  

(2.33)

In the electromagnetic stirring system modeled \( J_\phi = J_z = 0 \), and due to the axisymmetry of the filaments the angular component of magnetic flux density due to the current in them is zero \((B_\phi = 0)\). Thus equation 2.31 reduces to

\[ f_r = J_z B_\phi \]  

(2.34)

Therefore it is only the radial component of the magnetic flux density vector which contributes to the force on the conducting fluid, and since at \( z = 0 \) both \( J_z \) and \( B_r \) vectors are in the negative direction, the resulting force in that plane is in the positive \( \phi \) direction.

In the EMS system considered in this thesis the current density in the bath gives rise to self-generated magnetic field resulting in an angular component of \( B \). This creates a force in the radial direction

\[ f_r = -J_z B_\phi \]  

(2.35)

This is called a pinch force. The force acts towards the axial centreline of the bath and it is constant magnitude down almost the entire length of the bath. Hence it cannot influence the nature of the flow. Note: At current entry and exit, however, the electrical field is distorted and end effects do generate secondary flows in localized areas. These are small and hence neglected in this simplified model.
If the value of the magnetic flux density in the radial direction at a point P is determined by

\[ B_r(P) = \left( \frac{\mu}{2\pi a} \right) [\xi_{rTotal}(P)] \tag{2.36} \]

(from equation 2.30) then the body force acting at point P is given by

\[ f_{b}(P) = J_s B_r = \left( J_s \right) \left( \frac{\mu}{2\pi a} \right) [\xi_{rTotal}(P)] \tag{2.37} \]

In the electromagnetic stirring system modeled the coils are comprised of n turns, and in such a case equation 2.37 becomes

\[ f_{b}(P) = \left( \frac{\mu J_s}{2\pi a} nI \right) [\xi_{rTotal}(P)] \tag{2.38} \]

Therefore plotting the multiplication factor \( \xi_{Total} \) illustrates the behaviour of both the radial component of the magnetic flux density and the body force acting on the fluid. Values specific to a geometry and current can then be obtained by multiplying the factor \( \xi_{rTotal}(P) \) by \( \frac{\mu}{2\pi a} nI \) or \( \frac{\mu J_s}{2\pi a} nI \) for radial magnetic flux density or force, respectively. Thus these equations form the basis for a simplified model for the force field in the mould. Once the characteristics of these equations are determined, the approximate behaviour of the force field is obtained. Note: this assumes that the current density distribution is uniform throughout the bath. It is shown in Section 4.2 that this is essentially the case. Thus for the purposes of this approximate model this assumption is valid.

2.3.4 Magnetic Flux Density and Magnetic Body Force Distribution in the Fluid for Two Filaments with Equal Current Flowing in Opposite Directions

The previous section showed that the electromagnetic body force acting on the fluid in this simplified model is due to the radial component of magnetic flux density only. Therefore determining the influence of \( h^* \) on the multiplication factor \( \xi_r \) provides an approximate model of the behaviour of the electromagnetic body force throughout the bath.

Figure 2.7 illustrates the variation of \( \xi_r \) with \( z^* \), at \( r^* = 0.387 \) for a single filament loop. The filament loop is centred at \((0,0,0)\), and at this exact location the radial component of the magnetic flux density is zero (the flux is entirely in the \( z \) direction). Above and below the filament the radial component of the magnetic flux density increases to a maximum of ±0.595 at \( z^* = ±0.43 \).
The position $r^*=0.387$ was chosen to correspond to analysis work performed using the finite element model. This model will be discussed in Chapter 3.

For two filaments with equal current but opposite current flow directions ($I_1=-I_2$), the location of the peak indicates the ideal separation for maximizing the radial component of the magnetic flux density ($B_r$) between the filaments. The greatest value of $B_r$ is obtained when the peaks for the two filaments overlap with their respective maximums aligned. Thus, to maximize $B_r$ for this value of $r^*$, the filaments should be located such that the distance between them is two times $z^*=0.43$, or $z^*=0.86$. This corresponds to $h=0.43^*a$ (and therefore the ideal separation for maximizing the flux between the filaments is 86% of the filament radius).

This is confirmed by plotting the factor $\zeta_{Total}$ against $h^*$, for $z=0$ and $r^*=0.387$. Figure 2.8 shows that the maximum of the graph (note that it is negative) occurs at $h^*=0.43$. The value of $\zeta_{Total}$ at that location is $-1.19$, which is twice the peak value for the individual filament.
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**Fig. 2.8** Multiplication factor $\zeta_{\text{Total}}$ as a function of normalized coil separation. $r^* = 0.387$, Current flow is in the opposite direction in each filament loop ($I_i = -I_j$).

---

**Fig. 2.9** The variation of multiplication factor $\zeta_{\text{Total}}$ with normalized filament radius, for two filament loops at a separation of $0.86a$. Current flow is in the opposite direction in each filament loop ($I_i = -I_j$).
When two filaments are separated by $0.86 \ast a$, superposition of $\zeta_1$ and $\zeta_2$ generates a curve like Figure 2.9. This curve has a maximum of $-1.19$ at $z=0$, the centreline between the two filaments.

Figure 2.10 Variation of multiplication factor $\zeta_{Total}$ as a function of normalized height for different values of $h^*$, with $r^* = 0.387$, $I_1 = -I_2$.

Figure 2.10 shows the effect of changing the value of $h^*$ on the multiplication factor $\zeta_{Total}$. The curves shown are for $h^*$ values of 0.27, 0.42, and 0.58. This corresponds to $h$ values 3.5cm, 5.5cm, and 7.5cm, for a filament radius of 13cm. These values of $h^*$ and $h$ were chosen to correspond with solutions obtained using the finite element model. Table 2.1 summarizes the results of the graph.

<table>
<thead>
<tr>
<th>$h^*$</th>
<th>Position of Filaments ($z^*$)</th>
<th>Peak $\zeta_{Total}$</th>
<th>Percentage of Maximum Possible ($\zeta_{Total}$)</th>
<th>Position at Which $\zeta_{Total}=0$ ($z^*$)</th>
<th>$\Delta z^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>±0.27</td>
<td>-1.0369</td>
<td>87.1%</td>
<td>±0.48</td>
<td>0.19</td>
</tr>
<tr>
<td>0.42</td>
<td>±0.42</td>
<td>-1.1896</td>
<td>99.9%</td>
<td>±0.56</td>
<td>0.14</td>
</tr>
<tr>
<td>0.58</td>
<td>±0.58</td>
<td>-1.1116</td>
<td>93.4%</td>
<td>±0.66</td>
<td>0.08</td>
</tr>
</tbody>
</table>
The key points observed on Figure 2.10 are:

1. The largest peak $\zeta_{\text{Total}}$ occurs for the $h^*=0.42$ case.

2. That the presence of a second filament offsets the height at which $\zeta_{\text{Total}}=0$ from the plane of the first filament. This is indicated by the fact that the heights at which $\zeta_{\text{Total}}=0$ do not correspond to the locations of the filaments ($\Delta z^*\neq 0$).

3. As the filament separation increases the curves widen. As the filaments are separated, the contribution of one filament to the value of $\zeta_{\text{Total}}$ in the immediate vicinity of the other filament decreases, and the position at which the curve crosses the axis ($\zeta_{\text{Total}}=0$) approaches the plane of the filament ($\Delta z^*\rightarrow 0$).

4. As the filament separation is increased, the value of the secondary maxima increases. This trend would be observed up to a maximum value of $\zeta_r$ for a single filament. This is illustrated by equation 2.24, as follows:

   For a point P located above the top filament (filament 1), an increase in $h$ decreases the value of $\zeta_r$. Since the superposition of the fluxes above and below the top and bottom filaments is destructive, as $\zeta_r$ decreases $\zeta_{\text{Total}}$ increases to a maximum of $\zeta_{r1}$.

5. As the filament separation is increased, the distance between the plane of the filament and the position of the secondary maxima approaches $0.43\alpha$, the position of the maximum for a single filament ($z^*=0.43$).
Figure 2.11 illustrates the variation in $\zeta_{\text{Total}}$ along a line extending in the radial direction from the origin at $(0,0,0)$, to a radius of $r=2a$ $(2a,0,0)$. The factor $\zeta_{\text{Total}}$ reaches a maximum of $-4.02$ at below the filament. At $r^*=0.387$, $\zeta_{\text{Total}}$ equals approximately 1.19, which agrees with the result on Fig. 2.9.

Figure 2.12 illustrates the case of two filaments for $h^*=0.27$, 0.42, and 0.58cm. As previously illustrated, for this set of $h^*$, the maximum $\zeta_{\text{Total}}$ (and therefore the maximum flux) between the filaments occurs for $h^*=0.42$. For $h^*=0.27$, the filaments are too close together, and at $h^*=0.58$ they are too far apart to maximize $\zeta_{\text{Total}}$. 
As mentioned in Section 2.3.2 these normalized equations, and therefore these curves, Figures 2.10 and 2.12, are equally valid for \( h^* \) variations due to increasing or decreasing filament radius. Thus the 14.7% increase in \( \zeta_{\text{Total}} \) obtained by increasing \( h \) from 3.5cm to 5.5cm for a filament radius of 13cm (corresponding to an increase in \( h^* \) from 0.27 to 0.42) could also be attributed to a decrease in filament radius from 20cm to 13cm for \( h=5.5 \).

2.3.5 Magnetic Flux Density and Magnetic Body Force Distribution in the Fluid for Two Filaments with Equal Current Flowing in the Same Direction

If the current in each filament is equal and flows in the same direction, the superposition of \( \zeta_1 \) and \( \zeta_2 \) generates a pattern that is much different from filaments with equal current in opposite directions (Fig. 2.13). Between the filaments \( \zeta_1 \) and \( \zeta_2 \) have opposite signs, and as a result the superposition of these is destructive in nature. At \( z=0 \), \( \zeta_1 \) and \( \zeta_2 \) are exactly equal but opposite, and therefore \( \zeta_{\text{Total}}=0 \). The maxima of the graph occur at equal distances above and below the upper and lower filaments.
Figure 2.13 Two filament loops, at a separation of 0.86α, Current flow is in the same direction in each filament loop (I₁=I₂)

Figure 2.14 illustrates the variation in ζ_{Total} with normalized filament separation h^*. The maximum ζ_{Total} occurs at h^*=0. Thus, unlike the I₁=-I₂ case the two filaments cannot be positioned so that their maximum fluxes exactly overlap since this would place the coils within the same space. However, the maximum ζ_{Total} for a given filament geometry can be obtained by placing the filaments so that they are immediately adjacent. Therefore if two filaments with I₁=I₂ have equal diameters d, then the B_r produced by these two filaments can be maximized by locating them such that h=d/2, where h is measured from the axial centreline of the filament.

Figure 2.15 illustrates the variation in ζ_{Total} with normalized height z^*. h=0.001, resulting in a nominal filament separation of 0.002α, and with r^*=0.387. The maximum values of ζ_{Total} occur at z^*=±0.43, with a value of ±1.19.
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Fig. 2.14 Multiplication factor $\zeta_{\text{Total}}$ as a function of normalized filament separation. $r^* = 0.387$
Current flow is in the same direction in each filament loop ($I_1 = I_2$)

Fig. 2.15 Two filament loops, at a separation of $h = 0.001$, Current flow is in the same direction in each filament loop ($I_1 = I_2$)
The decrease in $\zeta_{\text{Total}}$ which occurs with increasing filament separation when $I_1=I_2$ is shown in Fig. 2.16. The values of $h^*$ examined were 0 ($h=0.001$), 0.19, and 0.42. Table 2.2 summarizes the features of the graph.

![Graph showing multiplication factor $\zeta_{\text{Total}}$ as a function of normalized height for different values of $h^*$.

$r^*=0.387$, Current flow is in the same direction in each filament loop ($I_1=I_2$)

<table>
<thead>
<tr>
<th>$h^*$</th>
<th>Position of Filaments ($z^*$)</th>
<th>Peak $\zeta_{\text{Total}}$</th>
<th>Percentage of Maximum Possible ($\zeta_{\text{Total}}$)</th>
<th>Percentage of $\zeta_r$ (Single Coil)</th>
<th>Position of Peak $\zeta_{\text{Total}}$ ($z^*$)</th>
<th>$\Delta z^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\pm 0.001$</td>
<td>-1.1899</td>
<td>100%</td>
<td>200%</td>
<td>$\pm 0.43$</td>
<td>0.43</td>
</tr>
<tr>
<td>0.19</td>
<td>$\pm 0.19$</td>
<td>-1.0462</td>
<td>87.9%</td>
<td>175.8%</td>
<td>$\pm 0.52$</td>
<td>0.33</td>
</tr>
<tr>
<td>0.42</td>
<td>$\pm 0.42$</td>
<td>-0.8040</td>
<td>67.6%</td>
<td>135.2%</td>
<td>$\pm 0.79$</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The key points observed on Fig. 2.16 are:

1. The largest peak $\zeta_{\text{Total}}$ occurs for the $h^*=0$ case.

2. When the filaments are less than 0.43$\alpha$ apart, the height at which $\zeta_{\text{Total}}=0$ is $z=0$. 
Note: Though not shown on the graph, when the filaments are more than 0.43\(a\) apart, secondary maxima occur between the planes of the two filaments. As the separation increases, these secondary maxima approach the value for a single coil.

3. Increasing \(h\) reduces \(\zeta_{\text{Total}}\) to a minimum possible value of \(\zeta_{r1}=\zeta_{r2}\) (equation 2.28b), as the contribution of the second filament approaches zero in the region of the first.

4. For \(0<h^*<0.43\), the location of the maxima are closer to the planes of the filaments than 0.43\(a\). This is due to the contribution to \(\zeta_{\text{Total}}\) of the second filament. As the separation increases, the contribution of the second filament to \(\zeta_{\text{Total}}\) decreases, and the distance from the plane of the filament to the height at which \(\zeta_{\text{Total}}\) is at a maximum increases to 0.43\(a\).

Figure 2.17 illustrates the variation in \(\zeta_{\text{Total}}\) that occurs with changing \(r^*\), along a line extending in the radial direction from the origin at (0,0,0) to a radius of \(r=2a\) (2a,0,0). For this plot, \(z^*=0.43\). The factor \(\zeta_{\text{Total}}\) is 0 at the origin, and reaches a maximum of -4.02 at \(r^*=0.97\) (\(r=0.97a\)). At \(r^*=0.387\) \(\zeta_{\text{Total}}\) equals approximately 1.19, in agreement with Fig. 2.15.

![Graph of Multiplication factor \(\zeta_{\text{Total}}\) as a function of normalized filament radius, \(h^*=0, z^*=0.43\). Current flow is in the same direction in each filament loop (\(I_1=I_2\)).](image-url)
Figure 2.18 illustrates the variation in $\zeta_{\text{Total}}$ with normalized radius, $r^*$. The value of $\zeta_{\text{Total}}$ decreases with increasing filament separation for all points on the curve. Note: the curve is plotted for $z^*=0.43$, which only coincides with the location of the maxima if $h^*=0$ (Fig. 2.16).

Fig. 2.18 Normalized filament radius vs. the multiplication factor $\zeta_{\text{Total}}$, $z^*=0.43$. Current flow is in the same direction in each filament loop ($I_1=I_2$)

As previously noted for the $I_1=I_2$ case, although the example of changes in filament separation has been used to demonstrate the usefulness of the normalized equations, changes in the ratio $h^*$ can also be interpreted as changes in filament radius, with $h$ held at a constant.

2.3.6 Conclusions Based on the Approximate Model
The analytical model shows that:

1. The magnetic flux density (radial component) and magnetic body force distributions which arise from the magnetic field generated by the current filament loops can both be described by a factor, $\zeta_{\text{Total}}$. 
2. The factor $\xi_{Total}$ is a function of a ratio $h^*$, the ratio of half the filament separation to filament radius. Thus, examination of $\xi_{Total}$ as a function of $h^*$ describes the variation of the magnetic flux density (radial component) and magnetic body force which occurs with both
   i. changes in filament separation
   ii. changes in filament radius

3. The $\xi_{Total}$ distribution changes with filament configuration.
   Two filament configurations have been examined:
   i. Current is equal in both filaments, current flow direction is opposite in each filament
      The $\xi_{Total}$ distribution has three distinct regions:
      - above the filaments: secondary maxima ($\xi_{Total}$ is positive)
      - between the filaments: primary maxima ($\xi_{Total}$ is negative)
      - below the filaments: secondary maxima ($\xi_{Total}$ is positive)
   ii. Current is equal in both filaments, current flow direction is the same in each filament
      The $\xi_{Total}$ distribution has two distinct regions:
      - above the filaments: primary maxima ($\xi_{Total}$ is positive)
      - below the filaments: primary maxima ($\xi_{Total}$ is negative)
      This will be reflected in the stirring generated by the body forces.

4. The peak $\xi_{Total}$ possible for a given filament current is higher if the filaments are configured such that the current flow direction is opposite in the two loops.

5. For the case examined in this analytical model (location of interest is $r^*=0.387$), if the filaments are configured such that current flow direction is opposite in the two loops, the factor $\xi_{Total}$ increases with increasing $h^*$, up to $h^*=0.43$, after which it will decrease.

6. For the case examined in this analytical model (location of interest is $r^*=0.387$), if the filaments are configured such that current flow direction is the same in the two loops, the factor $\xi_{Total}$ increases with decreasing $h^*$, to $h^*=0$.

7. As the ratio $h^*$ increases (coil separation increases or filament radius decreases), the relative contribution of the second filament loop to the total magnetic field in the vicinity of the first decreases.
2.4 Current Density in the Bath

The current is injected into the system via an electrode on the top of the mould. Therefore initially current flow is entirely through the mould itself. When the current reaches the position where the tin and mould are in contact, current enters the bath and two parallel paths are formed, as illustrated in Fig. 2.19.

![Diagram showing current paths in a bath](image)

Fig. 2.19 Mould / tin bath: (a) schematic, (b) approximate equivalent circuit

The proportion of the current to flow through each path is determined by approximate resistances. In general

$$R = \frac{\rho \ell}{A} \quad (2.39)$$

Where \(\rho\) is the resistivity of the material,

\(\ell\) is the length over which the current flows, and

\(A\) is the cross sectional area of the material normal to the direction of current flow.

The resistance for each component can be approximated by equation 2.38
\[ R_{m1} = \frac{\rho_{m1} \ell_{m1}}{A_{m1}} \]
\[ R_{m2} = \frac{\rho_{m2} \ell_{m2}}{A_{m2}} \]
\[ R_b = \frac{\rho_b \ell_b}{A_b} \]

where the subscript \( m1 \) indicates the section of mould not in contact with the liquid tin, \( m2 \) indicates the section of mould in contact with the liquid tin, and \( b \) indicates the liquid tin.

The required voltage to supply a total current of \( I \) amps is then
\[ V = I \left[ R_{m1} + \frac{R_{m2} R_b}{R_{m2} + R_b} \right] \] (2.41)

The components \( I_b \) and \( I_{m2} \) are calculated using equation 2.42
\[ I_b = I \left( \frac{R_{m2}}{R_b + R_{m2}} \right) \]
\[ I_{m2} = I \left( \frac{R_b}{R_b + R_{m2}} \right) \] (2.42)

For a copper mould with an inner cross-section of 10.2cm (4"), and mould wall thickness of 1.6cm (5/8") containing tin with a bath height of 50cm, it can be shown that 85% of the current will flow through the mould itself, and only 15% through the tin (see Appendix B). Therefore if 5000A are supplied to the system approximately 750A will flow though the tin bath.

If 750A are flowing through the tin bath, then the current density is
\[ J_z = \frac{I_z}{A} = \frac{750}{(0.102)^2} = 72100 \, A/m^2 \]

If the radial component of the magnetic flux density is approximately 0.035T, then the body force acting on the liquid metal will be
\[ f_r = J_z B_r = (72100)(0.035) \approx 2500 \, N/m^3 \]

These equations show that this method of current injection will generate a current density in the bath of sufficient intensity to create a significant amount of force on the liquid tin.
Chapter 3: Numerical Model Description

3.1 General Features of Modern Finite Element Analysis Packages
The finite element model is constructed by first building the system geometry as a set of
volumes, and then breaking down these volumes into finite elements. Finite elements are basic
shapes such as cubes and pyramids. They are represented by nodes, which are located at the
corners of the elements. The assembly of elements and nodes make up the finite element mesh.
The quality of the mesh is determined by how well it obtains the desired solution. This is
dependent on the nature of the problem, and the structure of the mesh. A finely meshed model
with no sudden changes in element size is preferred. However, increasing the number of
elements used increases the length of time required by the computer to obtain a solution. Thus a
balance is required between an appropriate level of model accuracy, and processing time.

Finite Element Analysis (FEA) software contains large libraries of these elements, each specific
to some analysis type. Each element type has equations appropriate to its analysis type associated
with it. For example, thermal elements have heat transfer equations associated with them, and
not fluid flow equations, and as a result they can be used to solve thermal distributions, but not
flow fields. Some elements are coupled elements. For example, an element may have both
thermal and structural equations associated with it, so that analyses of structures at high
temperature may be performed.

The material properties for each component of the system are input into the computer model. The
software utilizes this information, as well as excitation (voltages, current sources, forces) and
boundary condition information (symmetry, inlet pressure) to solve for the desired quantity
(current density, magnetic flux density, fluid velocity) at each node using matrix algebra.

3.2 Modelling of the EMS System using Finite Elements
The appropriateness of the computer model to represent the problem to be solved is dependent
on the correct representation of the actual system through the model geometry, understanding of
the appropriate assumptions, and application of the correct boundary conditions and excitations
for each analysis.
The main components of the EMS system are the mould, the metal to be stirred, and the coils. For the solution of the magnetics problem, the air region surrounding the coils was also included in the model. Connecting cables, the support frame, coil spacers and other auxiliary systems were not included.

The geometry of the model was constructed using a parameterized code. That is, the features of the geometry such as width, height and thickness of the mould and coils were assigned variables, or scalar parameters. At the beginning of the code these parameters were assigned values, which corresponded to the system to be modelled (i.e. coil size and placement). The code contained commands which constructed the desired geometry, and applied all boundary conditions. The benefit of this method lies in the fact that to change a system dimension, only the value assigned to the appropriate parameter need be changed, not numerous commands throughout the code. An outline of the code given in Table 3.1.

![Diagram of EM system components](image)

Fig. 3.1 Geometry of the computer model of the electromagnetic stirrer
(a) front view (b) top view, showing symmetry section modelled.

The EMS system components required for the computer model are shown in Fig. 3.1. In the horizontal plane the system possesses quarter symmetry (Fig. 3.1b), consequently only one quarter of the geometry need be modelled to represent the whole. Fig. 3.2 is a schematic of the FEA model, indicating some of the parameters used in its construction.
Colour Plate 1 shows the geometry of the finite element model. Colour plates are at the end of each chapter, as appropriate.

<table>
<thead>
<tr>
<th>Table 3.1 Structure of the Parameterized Code</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong></td>
</tr>
<tr>
<td>1. Scalar Parameters</td>
</tr>
<tr>
<td>2. Materials Properties</td>
</tr>
<tr>
<td><strong>Commands For:</strong></td>
</tr>
<tr>
<td>3. Model Geometry Construction</td>
</tr>
<tr>
<td>4. Finite Element Mesh Construction</td>
</tr>
<tr>
<td>5. Application of Excitation and Boundary Conditions for Analyses</td>
</tr>
<tr>
<td>6. Solution of System and Storage of Results</td>
</tr>
</tbody>
</table>

![Diagram showing model geometry]

**Fig. 3.2** The model geometry: a) half of the longitudinal cross-section b) a quarter of the top view

### 3.2.1 Major Assumptions of the Analysis

In order to simplify the analysis, several assumptions were made:

1. The flow, the electric and magnetic fields, and hence the force field, are all steady state.
2. The system is isothermal.

The solidification in the mould represents a small fraction of the material. Hence, the temperature of the bulk of the material being stirred is approximately constant, at the
liquidus. Thus for the purpose of investigating the flow patterns in the bath, an isothermal assumption is justified. This means:
   i. negligible natural convection
   ii. viscosity is constant and independent of temperature
3. There is no free surface deformation.
4. The effects of surface tension are neglected.

3.2.2 Building Blocks of an FEA Model for the EMS System
The magnetohydrodynamic problem presented by the electromagnetic stirring system requires a coupled-field solution. That is, the flow field is determined by the interaction of the electric and magnetic fields, and the properties of the fluid on which the resulting forces act. Thus the model consists of three main parts:

1. The conduction analysis: solution of the voltage distribution within the mould and the metal bath, to determine the current density distribution, $J$.

2. The magnetics analysis:
   *Part A*: determining the magnetic flux density distribution throughout the bath arising from the current flowing in the liquid. The main component, $B_\phi$, gives rise to a pinch force, which may adversely affect the interaction between the current density $J$ in the bath, and the magnetic flux density produced by the coils (Part B)
   *Part B*: determining the magnetic flux density distribution throughout the bath due to the current in the two coils. The main component, $B_n$, gives rise to the stirring forces experienced by the liquid.

3. The fluids analysis: determining the flow profile which develops from the magnetic body force acting upon the tin bath.

Each of these components were run separately, on the same finite element mesh. These three solutions will be discussed separately in the next three sections.
3.2.2.1 The Conduction Analysis (Part 1)
The conduction analysis involved determination of the current density distribution in the tin bath. This solution was obtained by applying a potential difference down the length of the mould. The system is illustrated in Fig. 2.19. The required voltage was determined using the properties of the mould material and tin at 200°C (source: [20]), the desired amperage in the mould / tin bath assembly (i.e. 400A), and equation 2.41. The boundary conditions applied made the problem independent of temperature.

The current density distribution was retained for use in the magnetics analysis, and the determination of the magnetic body force.

3.2.2.2 The Magnetic Analysis (Part 2)

3.2.2.2.1 The Magnetic Analysis, Part A
The first part of the magnetic analysis was the solution of the magnetic flux density distribution arising from the current density in the tin bath. The current density distribution obtained in Part 1 was used as the excitation on the model for this part of the analysis. The Neumann boundary condition was applied the symmetry faces. Thus, at the symmetry faces the flux vectors are entirely normal. This condition defines the model as only one quarter of the whole. Fig. 3.3 illustrates the importance of applying the Neumann boundary condition. The figure shows the shape of the magnetic flux density vectors obtained for the magnetics solution with (a) and without (b) the boundary condition. In case (b) the computer model does not recognize the desired symmetry faces. The boundary condition on the exterior of the air regions was that magnetic vector potential on these faces was zero. This defined the outer boundary of the model.

Fig. 3.3 Top view of the mould / liquid assembly used in the magnetics analysis
(a) with the correct boundary condition (b) no boundary condition
The results of the first part of the magnetics analysis were retained for future use. The boundary conditions and excitations were removed from the model.

3.2.2.2 The Magnetic Analysis, Part B
The second part of the magnetic analysis was the solution of the magnetic flux density distribution due to the current in the coils. For this analysis the coils were excited with the appropriate number of amps per coil turn. This amperage per coil turn was equal to the number of amps running through the mould / tin bath assembly. The symmetry faces did not require a boundary condition, as the Cauchy boundary condition, by which all flux vectors are parallel to the boundaries of the model, is naturally occurring. On the exterior air faces the magnetic potential was set to zero.

The magnetic flux density distribution was retained. The magnetic flux density distributions from the two parts of the magnetic analysis were combined to form an overall distribution. The total magnetic flux density and the current density distribution were combined (as in equation 2.33) to determine the magnetic body force acting at each node throughout the tin bath.

3.2.2.3 The Fluids Analysis (Part 3)
The fluids analysis required some assumptions to be made. These were as follows [21]:

1. The fluid is incompressible
2. The fluid is Newtonian (linear relationship between shear stress and shear strain rate [22])
3. There is only one phase
4. The problem domain does not change (no deformation)

For the fluids analysis the magnetic body force at each node determined in Parts 1 and 2 was applied to the tin bath as the excitation.

The boundary conditions were as follows:

- Symmetry faces: Periodic boundary condition. Thus the velocity profile is unknown but identical on the two symmetry faces.
- **Surface:** $V_z = 0$. The vertical component of velocity was zero. Therefore, there was no surface deformation, which means that no meniscus was modelled.

- **Mould walls and bottom:** $V_r = V_i = V_z = 0$. Where the tin is in contact with the mould, the boundary condition was that of *no slip*.

- **The centreline of the melt:** The nodes located where the two symmetry faces come together are restricted to flow in the longitudinal direction, since $V_r = V_i = 0$ at that location (the fluid is incompressible).

These boundary conditions are shown schematically in Fig. 3.4. (Note: the mould is shown for illustrative clarity, but it was not part of the problem domain.)

\[ Re = \frac{\alpha L V_0}{\eta} \]  

where, for tin, $\rho$, the density, is 6900Kg/m$^3$ at 230°C and $\eta$, the viscosity, is 0.00185Ns/m$^2$ (source:[20]). The characteristic length for the system, $L$, is half the length of the side wall, (0.0485m), and $V_0$ is the velocity (~0.1m/s). Thus the Reynolds Number for the system is approximately 18000, well in excess of the range of $Re$ for laminar flow ($Re < 2000$ [22]).

In turbulent flows, the instantaneous velocity is fluctuating at every point [21]. Thus the velocity in the x-direction at a point may be defined as (from [21])
\[ V_x = \bar{V}_x + V_x' \]  

(3.2)

where \( \bar{V}_x \) is the mean velocity in the x-direction and \( V_x' \) is the fluctuating component of the total velocity.

The substitution of the "total" velocity into the Navier-Stokes equations gives a set of new relationships for the fluctuating components of velocity known as the *Reynolds equations*. These introduce a new component of viscosity, \( \mu_t \), which is a property of the flow. The Reynolds equations require a solution of not only the "steady" part of the flow, but also of the fluctuating terms. This approach yields a large system of coupled, non-linear partial differential equations, whose solution defy the capabilities of the largest computer systems.

In order to simplify the solution of these equations it was proposed that only the turbulent kinetic energy, \( k \), and its dissipation rate, \( \varepsilon \), be studied to determine the parameters of an effective "turbulent" viscosity \( \mu_t \) [22]. This is known as the \( k-\varepsilon \) model for turbulence and was used in this thesis for the calculation of viscosity within the fluid.

In the \( k-\varepsilon \) turbulence model the turbulent viscosity is calculated as a function of the turbulent kinetic energy \( k \) and the turbulent kinetic energy dissipation rate \( \varepsilon \), using (from [21])

\[ \mu_t = C_\mu \rho \frac{k^2}{\varepsilon} \]  

(3.3)

where \( C_\mu \) is a constant. The parameters \( k \) and \( \varepsilon \) are defined by differential equations [21]

\[
\frac{\partial \rho k}{\partial t} + \frac{\partial (\rho V_x k)}{\partial x} + \frac{\partial (\rho V_y k)}{\partial y} + \frac{\partial (\rho V_z k)}{\partial z} = \frac{\partial}{\partial x} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial z} \right) + \mu_t \Phi - \rho \varepsilon + B \]  

(3.4)

and

\[
\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial (\rho V_x \varepsilon)}{\partial x} + \frac{\partial (\rho V_y \varepsilon)}{\partial y} + \frac{\partial (\rho V_z \varepsilon)}{\partial z} = \frac{\partial}{\partial x} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) + C_{t\varepsilon} \frac{\varepsilon}{k} - C_{2\varepsilon} \frac{\varepsilon^2}{k} + B \]  

(3.5)
where $\Phi$ is a viscous dissipation term, and $B$ is a buoyancy term, which is not relevant to this analysis (therefore $B=0$).

The simplifications inherent in the study of turbulence introduce a set of arbitrary parameters. These were obtained experimentally by Launder [22] and are given in Table 3.2 for the "standard" model. Since $\mu_t$ is a function of the flow, the values in Table 3.2 change with the application. The guides for selection of proper values for a given problem are confusing, and it is an accepted practice in numerical solutions of fluid mechanics to use the quoted values. This practice was followed in the analysis of the present problem.

<table>
<thead>
<tr>
<th>Table 3.2 Parameters of the $k-\varepsilon$ turbulence model [21]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$C_i, C_{iz}$</td>
</tr>
<tr>
<td>$C_2$</td>
</tr>
<tr>
<td>$C_{\mu}$</td>
</tr>
<tr>
<td>$\sigma_k$</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
</tr>
</tbody>
</table>

The solution of Equations 3.4 and 3.5 require boundary conditions for $k$ and $\varepsilon$. These can be estimated from experimental data regarding flow conditions near a solid wall [21]. The Log-Law-of-the-Wall has been used in this thesis to evaluate near wall flow conditions.

The flow field is determined by iterative solution of the velocity, pressure, and turbulence equations. For this analysis, approximately 150 global iterations were required to reach convergence. Within each global iteration, the pressure equation underwent 500 iterations, the turbulence equations 10 iterations, and the velocity 1 iteration.

A flow chart illustrating the entire computer model is given in Fig. 3.5.
Fig. 3.5 Structure of the computer model. Dashed arrows indicate flow of data. Solid arrows indicate computer analysis process. Square boxes indicate input excitations. Rounded boxes indicate resultant information.
Colour Plate 1: The finite element model geometry. One quarter of the mould and bath. Arrows indicate direction of current flow in the coils (in this case, opposite).
Chapter 4: Analysis of Stirring Systems by Means of a FEA Model

4.1 Introduction

In order to evaluate the effectiveness of the dc EMS system for practical casting operations, the parameters of the computer model were adjusted to approximate the dimensions of an industrial sized billet mould, given in Table 4.1.

<table>
<thead>
<tr>
<th>Table 4.1 Industrial Billet Mould</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner cross section</td>
</tr>
<tr>
<td>Mould wall thickness</td>
</tr>
<tr>
<td>Mould length</td>
</tr>
<tr>
<td>Bath height</td>
</tr>
<tr>
<td>Coil height</td>
</tr>
<tr>
<td>Coil width</td>
</tr>
<tr>
<td>Coil inner radius</td>
</tr>
<tr>
<td>Turns per coil</td>
</tr>
<tr>
<td>Current per coil turn</td>
</tr>
<tr>
<td>Current in the tin bath</td>
</tr>
<tr>
<td>Current Density in the tin bath</td>
</tr>
</tbody>
</table>

The mould was short by industrial standards. This was necessary due to the element size restriction in ANSYS. The coils were positioned with the centreline between them at 15cm below the surface of the bath (Fig. 4.1). Current flow was in opposite directions in the two coils ($I_1 = -I_2$). The current was applied to the bath via a potential difference between a simulated collar around the top of the copper mould, and a simulated electrode covering the bottom of the mould and bath.

A means of comparing results obtained from different coil geometries was required. Preliminary results showed the presence of three distinct regions in the magnetic, force, and stirring velocity results. The first of these was at the height at which the coils were centred. The second, above the centreline of the top coil, and the third below the centreline of the bottom coil. In order that these regions could be quantitatively compared, a system of paths was developed. These paths were chosen to be straight lines through the geometry over which results such as magnetic flux density, force, and velocity were subsequently mapped.
Eleven paths through the bath were studied. These were classed as follows:

- **Edge Paths** began at the axial centreline of the melt, and extended to the mould orthogonally.
- **Cross Paths** began at the axial centreline of the bath and extended to the corner of the mould.
- **Longitudinal Paths** began at the surface of the bath and ran the length of the bath to the bottom, being orthogonal to both surfaces.

The path locations are summarized in Table 4.2 and illustrated in Fig. 4.2.

The top and bottom paths (Fig. 4.2) were not equidistant from the middle path, rather they were placed at the approximate centre of the region which they represented. Thus the top paths were 8 cm above the middle paths, while the bottom paths were 18 cm below the middle paths. This was so that the paths better represented the field, force, or velocity experienced by the bulk of the fluid in that region.
Table 4.2 The Paths

<table>
<thead>
<tr>
<th>Type</th>
<th>Location</th>
<th>Designation</th>
<th>Start Coordinate (cm)</th>
<th>End Coordinate (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge</td>
<td>Surface</td>
<td>ES</td>
<td>(0,0,15)</td>
<td>(5.08,0,15)</td>
</tr>
<tr>
<td></td>
<td>Top</td>
<td>ET</td>
<td>(0,0,8)</td>
<td>(5.08,0,8)</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>EM</td>
<td>(0,0,0)</td>
<td>(5.08,0,0)</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>EB</td>
<td>(0,0,-18)</td>
<td>(5.08,0,-18)</td>
</tr>
<tr>
<td>Cross</td>
<td>Surface</td>
<td>CS</td>
<td>(0,0,15)</td>
<td>(5.08,5.08,15)</td>
</tr>
<tr>
<td></td>
<td>Top</td>
<td>CT</td>
<td>(0,0,8)</td>
<td>(5.08,5.08,8)</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>CM</td>
<td>(0,0,0)</td>
<td>(5.08,5.08,0)</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>CB</td>
<td>(0,0,-18)</td>
<td>(5.08,5.08,-18)</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>Symmetry face</td>
<td>LF</td>
<td>(5.03,0,15)</td>
<td>(5.03,0,-35)</td>
</tr>
<tr>
<td></td>
<td>22.5° Offset</td>
<td>L22.5</td>
<td>(5.03,-2.015,15)</td>
<td>(5.03,-2.015,-35)</td>
</tr>
<tr>
<td></td>
<td>45° Offset</td>
<td>L45</td>
<td>(5.03,-5.03,15)</td>
<td>(5.03,-5.03,-35)</td>
</tr>
</tbody>
</table>

Fig. 4.2 The system of paths used for quantitative comparison of results
(geometrical not shown to scale)
4.2 Current Density Distribution in the Bath

Table 4.1 lists the current density in the bath as \( \sim 72000 \text{A/m}^2 \). As discussed in Section 2.4 the current density in the bath arises from a voltage applied to the mould itself. Therefore the current density in the bath will not be identical at all points.

Figure 4.3 shows that the current density is essentially constant throughout the bulk of the bath. The data was collected along the path LF, but with the exception of approximately the top 5 cm of the bath the current density was uniform.

![Current Density Distribution](image)

**Fig. 4.3** The current density in the melt due to the applied current.

Colour Plate 2 (at the end of the chapter) shows the distribution of the z component of current density throughout the bath. The mould (not shown) is on the right-hand side of the bath. The top right corner of the image is the location where the current enters the bath from the mould. There is some variation in the z component of the current density at the top of the bath, but throughout the remainder of the bath it is essentially uniform.
4.3 The Magnetic Flux Density Distribution in the Tin Bath

In order to show the effect of each coil, the magnetic flux densities should be examined for each coil individually. The following graphs show the radial component of magnetic flux density plotted against position for longitudinal paths that are located on the symmetry face, at intervals of 20% of the distance from the axial centreline of the tin bath to the mould wall.

![Graph showing magnetic flux density vs. position for the bottom coil.](image)

**Fig. 4.4 Magnetic flux density vs. position for the bottom coil.**

The bottom and top coils were located with their centrelines at 20cm and 10cm below the surface of the bath. The curves cross the axis ($B_r=0$) at these respective locations. The maximum flux occurs at an average distance of 3cm from the edges of the coil. This is in good agreement with the analytical model. The coil radius is 13cm, and 3cm from the edge of the coil places the maximum flux at $h^*=0.42$, since the coil itself is 5cm in height.

For both the top and bottom coil, the absolute value of the maximum flux values is approximately 0.0181T. Dividing by the factor $\frac{\mu}{2\pi a} I$ (with $I=20000$A, and $a=0.13$m) gives a $\zeta_r$ value of 0.5882. This is within 2% of the value calculated using the analytical model ($\zeta_r=0.5950$, Section 2.3.4).
Superposition of the magnetic fields when the two coils are used in tandem creates an overall curve like the one below:

![Graph showing magnetic flux density vs. position for the top coil](image1)

**Fig. 4.5** Magnetic flux density vs. position for the top coil

![Graph showing magnetic flux density with position for two coil system - 2cm coil separation](image2)

**Fig. 4.6** Magnetic flux density with position for two coil system – 2cm coil separation
These curves show the magnetic flux density in the radial direction is greatest along the path closest to the coils. This follows from Fig. 2.11, which shows that as \( r \rightarrow a \ (r^2 \rightarrow 1) \), \( \zeta_{\text{Total}} \), and therefore \( B \), increases in magnitude.

The maximum flux for each individual coil being 3cm from the coil edge indicates that the correct coil separation for maximizing the flux between the coils would be approximately 6cm. The curve for 2cm of separation (Fig. 4.6) shows that the maximum possible \( B_r \) is not reached. The peak is at a value of -0.0307T, while on the curve for 6cm of separation (Fig. 4.7) the maximum between the coils is -0.0362T, which equals the sum of two -0.0181T peaks.

![Graph showing magnetic flux density with position for two coil system - 6cm coil separation](image)

Fig. 4.7 Magnetic flux density with position for two coil system – 6cm coil separation

The analytical model showed that the ideal filament separation for maximizing the radial component of magnetic flux density between the coils (at the location of the path) was \( 2h^* = 0.86 \), and a coil separation of 6cm places the centrelines of the coils of the FEA model 11cm apart. This corresponds to 84.6% of the radius, and so again the analytical result and the FEA model are within 2%. 

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These two curves also illustrate the weakening of the influence of one coil in the vicinity of the other with increasing coil separation. Increasing the separation from 2 to 6 centimetres decreases the difference in height between the location of the coil centreline and the height at which $B_r=0$, as well as increasing the magnitude of the secondary maxima. This agrees with the findings of the analytical model (Section 2.3.4).

4.4 Influence of Coil Separation on the Parameters of the EMS System

The effect of coil separation on the effectiveness of the stirrer was studied next. The coils were adjusted from 0cm to 10cm of separation in 2cm increments. The results are included below. The positive radial direction is outward from the axial centreline of the tin bath.

4.4.1 Superposition of the Radial Component of the Magnetic Flux Density

Fig. 4.8 shows the relative placement of the two coils against the longitudinal path LF ($r^*=0.387$). The coils are centred at 10cm and 20cm below the melt, and are marked schematically on the figure.

![Fig. 4.8 The superposition of the flux along the longitudinal path LF ($r^*=0.387$). $I_t=I_r; h^*=0.385$](image)
The coils are 5cm in height and the distance between the coils is 5cm. This places the centrelines of the coils 10cm apart, corresponding to a value of $h=5$, or $h^*=0.385$ (See Fig. 4.9). Increasing coil separation shifts the curves further apart relative to the x-axis, as marked by the direction of the arrows. The positions of the top and bottom path are marked on the figure.

At the location of the bottom path, the magnitude of the radial component of the magnetic flux density ($B_r$) is almost entirely due to the bottom coil. At the location of the top coil, the fluxes produced by each of the top and bottom coil are greater in magnitude. In both locations superposition of the peaks is destructive.

At the location of the bottom path the flux density from the bottom coil ($B_{r2}$) is relatively large and positive, while the flux density from the top coil ($B_{r1}$) is small and negative. At the location of the top path, $B_{r1}$ is large and positive, and $B_{r2}$ is negative but almost equally as large. Thus the magnitude of $B_{rTotal}$ at the location of the bottom path will be depend almost entirely on $B_{r2}$, while the magnitude of $B_{rTotal}$ at the location of the top path will be much more dependent on the magnitude of each component. This difference is due to the distance which each path is to the centreline of the coils. The bottom path is more than twice the distance away from the centreline than is the top path. Thus the top coil has a much smaller effect on the magnetic field at that location.
Increasing the separation between the coils will shift these curves relative to the x-axis in the directions indicated by the arrows. Thus increasing coil separation from 0cm to 10cm will shift the position of maximum flux for the bottom coil towards the path EB, while decreasing $B_{r1}$ at that location. The increase will shift the position of the maximum flux due to the top coil towards and then past the position of the top path ET, while decreasing the magnitude of $B_{r2}$ at that location.

4.4.2 Magnetic Flux Density Variations with Changes in Coil Separation
Figure 4.10 illustrates that as coil separation increases and the bottom coil approaches the bottom path, the magnetic flux density along that path increases from 0.0033T at 0cm, to 0.0107T at 10cm of separation.

With 10cm of separation the leading edge of the bottom coil is still 8cm away from the path. Thus the magnetic flux density along the path does not reach its maximum within the range of the experiment. An increase could be expected for a further travel of 5cm for this bottom coil (placing the leading edge 3cm from the path, and therefore the maximum flux aligned with the path, as determined in Section 4.3), or a corresponding increase to 24 cm of separation.
The location of the top path is much closer to the coils than the bottom path and therefore along these paths the superposition of the magnetic flux generated by each coil plays a much more significant role. The curves (Fig. 4.11) show an increase and then a decrease in the magnetic flux density. $B_{rTotal}$ increases with coil separation from 0cm to 2cm of separation, as the strong magnetic flux densities above the top coil approach the path. At greater than 2cm of separation $B_{rTotal}$ decreases. This occurs because the magnetic flux density due to the top coil has begun to decrease in magnitude.

The greatest separation for which the magnetic flux density vectors are all in the same direction is 6cm. This corresponds to the leading edge of the top coil aligning with the path. As the centreline of the coil approaches the path the radial component from the top coil is no longer as strong and the opposing magnetic flux densities from the bottom coil become dominant. This is illustrated by the negative values on the curves for 8cm and 10cm of separation, and confirmed by the LF path (Fig. 4.13), along which the flux does not change direction until above the height of the path. If only the top coil was operating, the magnetic flux density vectors would not change direction until the centreline of the coil had passed the path (Fig. 4.5).

![Magnetic flux density variation with coil separation along the top path - ET](Fig. 4.11)
The lines on the two graphs look markedly different - on the ET graph, where superposition plays a large role in the magnitude of the magnetic flux densities, and the top coil is very close to the path the lines are curved, while on the EB graph, where only one coil makes a significant contribution to the magnetic flux field, the lines are very linear. This linearity would not be maintained as the coil approaches and passes the path location. As $B_{r_2}$ approaches its maximum value the magnitude increases more rapidly, as is observed on the curves for the individual coil (Fig. 4.4, Fig. 4.8)

The difference in the degree to which each coil influences the region of the two paths EB and ET is also reflected in the magnetic flux density values on the two graphs. The $B_r$ values on the ET graph are influenced more strongly by the bottom coil than the values on the EB graph are by the top coil. Since the superposition of the curves is destructive above the coil, the magnitudes on the ET graph are much smaller overall than those on the EB graph.

The curves for the path at the centreline of the coils (EM – Fig. 4.12) show that the magnitude of the magnetic flux density increases with coil separation up to 6cm (to the value of 0.0362T). Beyond that separation the magnetic flux densities decrease. This is within 2.5% of the value calculated using the analytical model (Fig. 2.12 illustrates the analytical model for 2cm, 6cm, and 10cm of separation).

The longitudinal path LF shows the superimposed magnetic flux density curves widening with increasing coil separation (Fig. 4.13). The values are in good accordance with the analytical model, within approximately 5%. Most of the points examined (maxima, secondary maxima, and the location at which $B_r=0$) are within approximately 2% of the values predicted by the reference model (Fig. 2.10 illustrates the analytical model for 2cm, 6cm, and 10cm of separation).
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Fig. 4.12 Magnetic flux density variation along the centreline path – EM

Fig. 4.13 Magnetic flux density variation along the longitudinal path LF
4.4.3 Magnetic Body Force Variations with Changes in Coil Separation

Equation 2.37 gave the relationship between force and the current density and magnetic flux density vectors for a two filament system with equal current in opposite directions. Since the components of the magnetic flux density vector and the force vector both depend on the value of the multiplication factor $\zeta_{\text{Total}}$, the curves which describe the variation of $B_t$ and $f_x$ with position have the same shape (Section 2.3.3). Their total magnitude is determined by the system-specific factors $\frac{\mu}{2\pi} n I$ and $\frac{\mu}{2\pi} n I$ respectively. A current density in the negative $z$-direction (as is the case in this study) will result in these factors having opposite signs, and therefore in the flux and force curves occupying opposite sides of the $x$-axis.

Fig. 4.14 illustrates the variation of force with position for the bottom path EB. The shape of the curves is very similar to those for the magnetic flux. The variations are due to the coarseness of the finite element mesh used. A finer mesh would have produced a more uniform result, without the sudden change in slope at approximately 0.7cm along the path.
On the EB path the forces increase steadily as the coil approaches the path, from $-250.6 \, \text{N/m}^3$ at 0cm of separation up to $-782.8 \, \text{N/m}^3$ at 10cm. The negative indicates that the forces are acting in the negative $\phi$-direction.

On the EM path (Fig. 4.15), the forces increase, and then decrease after reaching a maximum force of $2551.1 \, \text{N/m}^3$ at 6cm of separation. As with the magnetic flux density curves these show good agreement with the analytical model. The values for 6cm and 10cm of separation are within 5% of those illustrated on Fig. 2.12. The value for 2cm of separation is within approximately 10% of the analytically predicted result.

The ET curves (Fig. 4.16) show good agreement with the shape and relative magnitude of the corresponding magnetic flux density curves (Fig. 4.11).
The force along the path LF is shown in Fig. 4.17. Three distinct regions of force are present, corresponding to the regions of positive and negative radial component magnetic flux density. The force on the central region is the highest. The upper and lower regions have peak forces of approximately 30% of the peak force on the central region.

The curves match their corresponding flux curves from Fig. 4.13. The forces are within 10% of those predicted by the analytical model. The maximum force does not correspond to the 10cm coils separation, however the total force exerted over the central region is greatest for the 10cm separation, as indicated by the higher average force over the width of the parabola the force curve inscribes (the area under the curve for the 10cm case is almost 25% greater than the area under the 6cm curve).

The slightly greater disparity in the results in terms of force rather than flux being compared to the analytical model is explained by the analytical model assuming a current density of exactly 72200A/m² at all points, when this may not be the case. Their linear relationship causes a 1% difference in current density to similarly change the magnitude of the force by 1%.
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Colour Plates 3 through 6 are vector plots illustrating $f_b$ at bath heights corresponding to ES, ET, EM, and EB for the 6cm separation case.

![Fig. 4.17 Force variation with position for the longitudinal path - LF](image)

4.4.4 Stirring Velocity Variations with Changes in Coil Separation

The magnetic body force acting on the fluid generates a rotational flow. The nature of the body force distribution (with three distinct regions of force) causes three distinct stirring regions to develop:

1. *upper stirring region*: flow is in the negative $\phi$ direction
2. *central stirring region*: flow is in the positive $\phi$ direction
3. *lower stirring region*: flow is in the negative $\phi$ direction

The force on the central region is greater than the force on the upper and lower regions (Fig 4.17), and correspondingly the velocity in this region is highest.

In the corner of the mould the flow is stagnant. Here the flow is not in a clearly defined direction, and velocities are very low.
Colour Plates 7-9 are composite vector plots of the symmetry face, surface, and wall face, for 0cm, 6cm, and 10cm of separation, providing an overview of the 3-D flow field.

Colour Plates 10 through 12 are contour plots of rotational (stirring) velocity on the symmetry face of the bath. The four contours indicate the regions of moderate to fast stirring in the negative and positive directions (greater than 2cm/s) and slow stirring in the negative and positive directions (less than 2cm/s). The contour plots are for the 0cm, 6cm, and 10cm coil separations.

On all three plots the stirring is concentrated at the region near the mould wall. Each indicates viscous effects propagating the stirring down the length of the mould, which is shown by the region of moderate to fast stirring extending towards the centreline of the bath as the distance down the bath increases. Each plot has extensive dead zones (less than 2cm/s in either direction), comprising approximately half the bath in the 0cm separation case, a third of the bath in the 6cm separation case, and a quarter of the bath in the 10cm separation case. The presence of dead zones extending to the mould wall indicated the change in direction of the flow between stirring regions.

The LF path (Fig. 4.18) illustrates the three clearly defined rotational stirring regions which exist in the system, which are generated by the oppositely wound coils. These will be referred to as the upper stirring region, generated for the most part by the positive $B_r$ vectors above the top coil, the central stirring region, generated by the sum of the negative $B_r$ vectors between the two coils, and the lower stirring region, largely due to the positive $B_r$ vectors below the bottom coil. In the central stirring region, stirring is in the positive $\phi$-direction while in the upper and lower stirring regions, it is in the negative $\phi$-direction.

The force curve for the path LF (Fig. 4.17) showed a widening of the central region with increasing coil separation. The width of the central stirring region increases accordingly with increasing coil separation, a difference of about 6cm between the 0cm and 10cm cases. The results are summarized Table 4.3.
Although 6cm of separation between the coils has been shown to maximize the peak values of flux and body force between the coils, that separation does not generate either the highest peak value of velocity or the highest average velocity. As previously discussed, the coil separations of 8cm and 10cm have a greater amount of force acting in the central region due to a higher average force acting over a greater area. These separations give a higher stirring velocity than the 6cm case.

The L22.5 path is closer to the corner of the mould and flow in the vicinity of the path is adversely affected by this fact. The velocities are lower than along the path LF. The curves for this path are shown in Figure 4.21. Table 4.4 summarizes the results.
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Fig. 4.19 Stirring velocity in the $\phi$ direction (rotational) along the longitudinal path - L22.5

<table>
<thead>
<tr>
<th>Separation (cm)</th>
<th>Width of Positive Stirring Zone (cm)</th>
<th>Average Velocity in Zone (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.8</td>
<td>7.2</td>
</tr>
<tr>
<td>2</td>
<td>13.0</td>
<td>9.9</td>
</tr>
<tr>
<td>4</td>
<td>13.5</td>
<td>10.6</td>
</tr>
<tr>
<td>6</td>
<td>14.5</td>
<td>11.7</td>
</tr>
<tr>
<td>8</td>
<td>15.8</td>
<td>11.6</td>
</tr>
<tr>
<td>10</td>
<td>18.0</td>
<td>11.6</td>
</tr>
</tbody>
</table>

The rotational flow established by the body force acting on the fluid is essentially destroyed in the corner of the mould. Figure 4.20 illustrates the velocity along the L45 path. Most of the curves show a lack of the clearly defined stirring regions that are present throughout the bulk of the fluid, and the velocity is low. The fluid in this region is essentially stagnant.
The velocity profiles for EB, ET, EM, and ES are all shown below. For the path EB (Fig 4.21), the velocities increase as coil separation increases and the bottom coil approaches the path, exerting a higher force in the region. A maximum velocity of 10cm/s occurs at the 10cm separation.

The bend in the curves at approximately 0.7cm along the path is the result of a coarse finite element mesh. That is, the element mesh used in the fluids analysis did not contain enough nodes to adequately solve the problem. This was due to a model size restriction within the FEA program used. This also caused "unlikely" bends in other curves as well, for example, at the 3cm mark of the EM path (Fig. 4.21). It is also possible that the fluctuations present in the curves of the longitudinal paths are a result of this. This is indicated by the fact that the fluctuations increase as the velocity increases (Fig. 4.18). The flow in the mould is rotational and turbulent, and a fine mesh is required to adequately represent the local fluid flow profile at all locations.

The problem of insufficient nodes is a serious one. Because there is direct evidence of this on the graphs, there is question as to the accuracy of the shape of the curves.
The boundary condition at the wall is no-slip: fluid velocity is zero. However, the body force acting on the fluid is highest near the wall (see Figs. 4.14, and 4.15 for example). Thus the velocity will rise sharply adjacent to the wall. However, the curves on the edge path graphs (Figs. 4.21, and 4.22 for example) should be slightly more rounded as velocity begins to decrease with increasing distance from the wall. The pointed bump at approximately 4.8cm along the path (Fig. 4.21, for example) should be smoothed off. It is likely that if more nodes were available for the analysis then a (more accurate) smoother, more rounded profile would result.

Along the path EM, the peak velocity is 19.3cm/s. This is approximately double the peak velocities in the EB, ET, and ES paths, as would be expected since the body force acting on the tin in the central stirring region is nearly double that in the upper and lower regions. An unexpected aspect of the EM path graph is the presence of two distinct stirring directions. The small negative velocities from 0 to 2 or 2.5cm along the path for the coil separations below 6cm show that the stirring direction is reversed close to the axial centreline of the melt. Only for those coil separations of 6cm or greater (though the 8cm case shows a slight dip also) is the stirring direction positive and robust throughout.
Fig. 4.22 Stirring velocity in the $\phi$ direction (rotational) along the centreline path – EM

The following curves show the changes in velocity profile for longitudinal paths at 20%, 40%, 60%, and 80%. These curves indicate why there are two stirring directions on some of the edge paths. The longitudinal path at 99% was left off because the magnitude of the velocities along that path would obscure the other results.

The curves showing the changes in velocity profile show that the stirring in the 4cm and 8cm cases stays slow and in the negative direction down the core of the melt. The 8cm case has two small regions of low velocity positive stirring within the 40% path, in both cases the stirring is essentially zero in the region closest to the centreline of the coils, where stirring velocity would be expected to be highest.
Fig. 4.23 Stirring velocity in the $\phi$ direction for several FL paths across the symmetry face a) 4cm Coil Separation
b) 8cm Coil Separation

Examining the graph of forces for the 4cm coil separation, the stirring forces along the 20% path are only 25% of the forces at the 80% path, and less than 500N/m$^3$. The forces are insufficient to
overcome the viscous effects. The stirring is unable to switch directions twice within the short width of the positive stirring zone.

The ET path (Fig. 4.25) shows curves to either side of the axis, indicating that the path is close to the region where the transition from one stirring direction to another occurs for different coils separations. Clearly in the 8cm and 10cm cases the transition has not yet fully occurred (this is also visible on the LF curve, where it can be seen that the curves cross the axis at about 6cm, and the path itself is located at 7cm). The positive stirring direction is still dominating, though for the region closer to the axial centreline the transition to negative stirring direction has already occurred.

The ES path (Fig. 4.26) shows a high peak velocity for the 10cm case. This is because at 10cm of separation, the leading edge of the top coil would be just 5cm from the surface of the bath. Since the maximum flux occurs at about 3cm above the leading edge of the coil, this peak nearly coincides with the free surface path (Fig. 4.19).
Fig. 4.25 Stirring velocity in the $\phi$ direction (rotational) along the top path - ET

Fig. 4.26 Stirring velocity in the $\phi$ direction (rotational) along the surface path - ES
Colour Plates 13 through 16 illustrate $V_y$ at bath heights corresponding to ES, ET, EM, and EB for the 6cm separation case.

It should be noted that the maximum velocities for each coil separation are not necessarily mapped by the paths examined here. The following table summarizes these values.

<table>
<thead>
<tr>
<th>Table 4.5 Maximum Velocity Summary for the Coil Separation Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil Separation (cm)</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

4.5 Influence of Coil Radius on the Parameters of the EMS System
The effect of coil radius ($a$, Fig. 4.27) on the effectiveness of the stirring system was studied next. The radius of each coil was set to the standard 13cm, and increased in 2cm increments to 19cm. The current through the coils and all other parameters, such as coil placement, remained the same (Table 4.1). All the model runs were performed with a coil separation of 5cm.

4.5.1 Magnetic Flux Density Variations with Changes in Coil Radius
The LF path results (Fig. 4.28) show a decrease in the magnitude of the magnetic flux density with increasing coils radius; the source of the flux is further from the path. With a radius of 13cm, the value of $B_r$ reaches 0.0362T, decreased to 0.0131T with a coil radius of 19cm. This is a decrease of approximately 65%. Fig. 2.10 illustrates the change in the $\zeta_{\text{Total}}$ curve with decreasing $h^*$. Since the radius is not constant in this analysis the values of $\zeta_{\text{Total}}$ determined in the analytical model must be adjusted by dividing by the radius $a$ (direct comparison of the $\zeta_{\text{Total}}$ curves for the purposes of comparing flux or force magnitude for different cases assumes a
constant factor of \( \frac{\mu}{2\pi a} \). The analytical model predicts a decrease in \( B_r \) upon increasing the radius from 13cm to 19cm of just 40%. Examination of the magnetic flux density results for each radius for both models shows an increasing departure from the analytically predicted result with increasing coil radius in ANSYS (Table 4.6).

<table>
<thead>
<tr>
<th>Coil Radius (cm)</th>
<th>( B_r ) (Analytical Model)</th>
<th>( B_r ) (ANSYS Model)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.0363</td>
<td>0.0357</td>
<td>1.7</td>
</tr>
<tr>
<td>15</td>
<td>0.0303</td>
<td>0.0247</td>
<td>18.5</td>
</tr>
<tr>
<td>17</td>
<td>0.0255</td>
<td>0.0174</td>
<td>31.8</td>
</tr>
<tr>
<td>19</td>
<td>0.0184</td>
<td>0.0130</td>
<td>39.8</td>
</tr>
</tbody>
</table>

This is the result of an insufficient number of elements (and therefore nodes) available in the ANSYS 5.5.3 Student Edition to properly handle the magnetics problem. As the coil is moved further from the problem domain, the poor mesh of the air region deteriorates the result. The widening of the curves with increasing coil radius is further evidence of this. Fig. 2.10 shows that the analytical model does not predict this effect.
4.5.2 Magnetic Body Force Variations with Changes in Coil Radius

The relationship between magnetic body force and magnetic flux density has been well established in previous sections (2.3.3, 4.4.3), and so no plots showing the variation of body force with coil radius along any paths are included here. However the trend in magnetic body force with changing coil radius is shown below (Fig. 4.29). The data is from the LF path, at z=0. It shows that the body force due to the coils with a 19cm radius would be just 36% of the force due to coils with a 13cm radius. Adjusting the coil radius from 13cm to 15cm is an increase of 15%, and it increases the distance from the coil to the location of the path by 25%. At 15cm the body force would be less than 70% of the value for 13cm of radius. However as noted in the magnetics analysis, this value is greater than that predicted by the analytical model.

![Graph showing magnetic body force density variation with coil radius](image)

**Fig. 4.29** Magnetic body force density variation with coil radius

4.5.2 Stirring Velocity Variations with Changes in Coil Radius

Examining the velocity profiles, again there are the central, upper and lower stirring regions, and they each follow the expected trend of velocity decreases as coil radius increases. LF (Fig. 4.30) shows the widening of the central stirring region with increasing coil radius. Although this follows from Fig. 4.28 as discussed in Section 4.4.1 this effect is erroneous and a widening of the central stirring region would not be this marked with increasing coil radius, since from Fig. 2.10,
the width of the force curves for each would be approximately equal. Table 4.7 summarizes the results.

Table 4.7 Summary of Rotational Stirring Velocity - LF

<table>
<thead>
<tr>
<th>Coil Radius (cm)</th>
<th>Average Velocity in Central Zone (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>13.1</td>
</tr>
<tr>
<td>15</td>
<td>10.2</td>
</tr>
<tr>
<td>17</td>
<td>8.6</td>
</tr>
<tr>
<td>19</td>
<td>8.6</td>
</tr>
</tbody>
</table>

For the L22.5 path the velocities are less (Table 4.8), and the positive stirring zone narrower. This decrease in velocity occurs despite the fact that the forces acting in the theta direction are higher than for the LF path. The reason for this lies in the geometry of the model. Though the L22.5 path is closer to the coils, and therefore experience a higher force (LF is located at \( r^e = 0.387 \), L22.5 is located at \( r^e = 0.437 \), and since \( \zeta_{r_{Total}} \) increases as \( r^e \to 1 \) (\( r \to a \)), the forces are greater along the path L22.5), it is also closer to the corner of the mould, which acts to decrease the velocity.
The EB, EM, ET, and ES graphs all show that velocity decreases as coil radius increases. For EB, and EM (Figs. 4.31 and 4.32) with \( a=19 \text{cm} \) the peak velocity along the path is approximately 75% of the peak velocity for \( a=13 \text{cm} \). Along ES, it is half of the peak velocity.

As with the coil separation study, the EM graph shows two stirring directions are present for both the 17cm and 19cm radii, however these velocities are so slight as to be negligible, and these regions (from about zero to the two centimeter point of the path) could be considered to be a dead zone for each case.

<table>
<thead>
<tr>
<th>Table 4.8 Summary of Rotational Velocity – L22.5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coil Radius (cm)</strong></td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>19</td>
</tr>
</tbody>
</table>

Fig. 4.31 Stirring velocity in the \( \phi \) direction (rotational) along the bottom path – EB
The ET path shows the two distinct stirring directions much more clearly (Fig. 4.33) - they are well defined. The 13cm case is the only one that has undergone the transition from the central (positive) stirring zone to the upper (negative) stirring zone. Note that this includes positive stirring close to the axial centreline. The 15cm case has begun the transition - the region close to the axis has already reversed. At the surface path ES the flow is completely in the negative direction, the maximum velocity is 10cm/s for the 13cm radius case (Fig. 4.34).

Figure 4.35 illustrates the velocity profile throughout the melt for a=15cm. The three different stirring regions are not clearly defined throughout the bath. Closer to the axial centreline of the bath the transition from the lower to central stirring region occurs higher in the bath. This shift is due to the viscous propagation of the lower stirring region. The weaker forces nearer the centreline of the bath are not strong enough to overcome the momentum of the stirring that has developed in the lower stirring region. Thus the transition from positive to negative stirring direction does not occur at the same bath height for each LF path position, and as a result the path curves will show two stirring directions for some cases.
Fig. 4.33 Stirring velocity in the $\phi$ direction (rotational) along the top path - ET

Fig. 4.34 Stirring velocity in the $\phi$ direction (rotational) along the surface path - ES
As with the coil separation study, it should be noted that the maximum velocities for each coil radius are not necessarily mapped by the paths examined here. The following table summarizes these values.

<table>
<thead>
<tr>
<th>Coil Radius (cm)</th>
<th>Maximum Velocity (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>24.4</td>
</tr>
<tr>
<td>15</td>
<td>18.1</td>
</tr>
<tr>
<td>17</td>
<td>14.2</td>
</tr>
<tr>
<td>19</td>
<td>12.4</td>
</tr>
</tbody>
</table>

**4.6 Influence of Current Magnitude on the Parameters of the EMS System**

The effect of current was examined by adjusting the number of amps flowing in the system to 3000A, 4000A, and 6000A, and comparing with the standard 5000A. Table 4.10 summarizes the current in each system component. The coil radius was maintained at 13cm, and the coil separation at 5cm. As previously discussed (Section 2.3), only approximately 15% of the total system current will flow through the bath, the remaining 85% flows through the mould.
### Table 4.10 Summary of the Current in Each System Component

<table>
<thead>
<tr>
<th>System Current (A)</th>
<th>Current in Tin Bath (I_a) (A)</th>
<th>Current Density in Tin Bath (A/m²)</th>
<th>Current Per Coil Turn (A)</th>
<th>Total Current Per Coil (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>~ 450</td>
<td>43300</td>
<td>3000</td>
<td>12000</td>
</tr>
<tr>
<td>4000</td>
<td>~ 600</td>
<td>57800</td>
<td>4000</td>
<td>16000</td>
</tr>
<tr>
<td>5000</td>
<td>~ 750</td>
<td>72200</td>
<td>5000</td>
<td>20000</td>
</tr>
<tr>
<td>6000</td>
<td>~ 900</td>
<td>86700</td>
<td>6000</td>
<td>24000</td>
</tr>
</tbody>
</table>

#### 4.6.1 Magnetic Flux Density Variations with Changes in System Current

The analytical model showed that the radial component of magnetic flux density and the body force acting on the tin bath can be determined by calculation of a non-dimensionized factor \( \zeta_{\text{Total}} \) and multiplication of that factor by either the term \( \frac{\mu}{2na} nI \), in the case of magnetic flux density, or by the term \( \frac{\mu I}{2ma} nI \), in the case of body force. A comparison of the results from the ANSYS model may be made by reference to the analytical model and calculation of these terms for each system current. They are summarized in Table 4.11.

### Table 4.11 Summary of Multiplication Terms

<table>
<thead>
<tr>
<th>System Current (A)</th>
<th>Magnetic Flux Density Term ( \frac{\mu}{2na} nI ) (T)</th>
<th>Magnetic Body Force Term ( \frac{\mu I}{2ma} nI ) (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>0.0185</td>
<td>800</td>
</tr>
<tr>
<td>4000</td>
<td>0.0246</td>
<td>1400</td>
</tr>
<tr>
<td>5000</td>
<td>0.0308</td>
<td>2200</td>
</tr>
<tr>
<td>6000</td>
<td>0.0369</td>
<td>3200</td>
</tr>
</tbody>
</table>

The magnetic flux density term is seen to vary linearly with system current, while the force varies exponentially. A two-fold increase in system current doubles the magnetic flux density at a point, but increases the magnetic body force at that location by four times.

For each of the EB, EM, and ET paths, the maximum radial flux occurred for the 6000A case and the maximum values for the 3000A case were exactly half of those. The EM path is illustrated in Fig. 4.36. The magnetic flux densities were found to be within 3.5% of those calculated by the analytical model.
Fig. 4.36 Magnetic flux density variation along the middle path - EM

Fig. 4.37 illustrates that the variation in current in the coils does not change the shape of the flux field, merely the magnitude of the flux, and so the curves all cross the axis at the same points.

Fig. 4.37 Magnetic flux density variation along the longitudinal path – LF
### 4.6.2 Stirring Velocity Variations with Changes in System Current

The velocity profiles all show velocity increasing with increasing current. The maximum velocity is reached by the 6000A case, at just under 23cm/s. The LF path curves are shown below (Fig. 4.38). Table 4.12 summarizing the average velocities in the central stirring region for each current.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Width of Positive Stirring Zone (cm)</th>
<th>Average Velocity in Zone (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>16.5</td>
<td>6.9</td>
</tr>
<tr>
<td>4000</td>
<td>15.5</td>
<td>10.2</td>
</tr>
<tr>
<td>5000</td>
<td>15.5</td>
<td>13.6</td>
</tr>
<tr>
<td>6000</td>
<td>15.5</td>
<td>15.3</td>
</tr>
</tbody>
</table>

The width of the central stirring region is essentially constant, which is in accordance with the fact that the width of the flux curves is also equal for that region.

![Fig. 4.38 Stirring velocity in the φ direction (rotational) along the longitudinal path - LF](image)

For the most part, the average velocities are lower on the L22.5 curve (Fig. 4.39).
The lack of trend in the width of the central stirring region indicates that the fluid flow effects related to the path's proximity to the corner of the mould are dominating over the width that would be determined by the magnetic body force.

On each of the EB and EM paths (Figs. 4.40, 4.41), as well as the ES path, the peak velocity obtained due to a system current of 6000A is more than double the velocity obtained when the system current is 3000A.
Fig. 4.40 Stirring velocity in the \( \phi \) direction (rotational) along the bottom path - EB

Fig. 4.41 Stirring velocity in the \( \phi \) direction (rotational) along the middle path - EM
The ET graph shows that at that bath height, the 4000A and 6000A cases have already fully undergone the transition from positive to negative stirring regions (Fig. 4.42). The 3000A and 5000A cases still have residual positive regions. Again these regions are explained upon examination of Fig. 4.43, which shows the velocity profile across the symmetry face. The transitions from upper to central to lower stirring regions do not occur at equal heights throughout the bath.

![Fig. 4.42 Stirring velocity in the \( \phi \) direction (rotational) along the top path - ET](image)

These graphs illustrate an almost linear relationship between the increase in the magnitude of the current applied to the system and stirring velocity. It was shown in Section 4.6.1 that doubling the magnitude of the current applied to the system increases the body force on the fluid by four times. Thus the fourfold increase in body force has not caused a fourfold increase in stirring velocity. Possible explanations for this include the coarseness of the finite element grid, and the effect of turbulence on the flow.
Again, the maximum velocities were not necessarily met on any path examined. Table 4.14 summarizes the maximum velocities for the current study.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Maximum Velocity (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>11.0</td>
</tr>
<tr>
<td>4000</td>
<td>18.1</td>
</tr>
<tr>
<td>5000</td>
<td>22.8</td>
</tr>
<tr>
<td>6000</td>
<td>32.9</td>
</tr>
</tbody>
</table>
The contour plot shows that the current density in the z-direction is between 65900A/m² and 79000A/m² throughout the bulk of the bath. Fig. 4.3 showed that it was essentially constant at 72000A/m².

This colour plate illustrates the end effect at the top of the bath, where current enters the fluid. The current density is not uniform in this region. It is approximately the top 5cm of the bath. The current density distribution at the bottom of the bath is not strongly affected by the exit of the current, as the entire bottom of the bath acts as the bottom electrode.

For the following vector plots, and those accompanying Chapter 5, the relative size of the vector arrows indicates, at a glance, the relative magnitude of force or velocity, as the case may be. The colour bar to the right of each vector plot indicates the range within which the vector is located.
Colour Plate 3: Vector plot of the body forces acting on the bath, at the location of path ES (bath surface). Current flow is opposite directions in the coils (\(I_1=-I_2\)). Coil Separation is 6cm.

Colour Plate 4: Vector plot of the body forces acting on the bath, at the location of path ET (7cm below bath surface). Current flow is opposite directions in the coils (\(I_1=-I_2\)). Coil Separation is 6cm.
Colour Plate 5: Vector plot of the body forces acting on the bath, at the location of path EM (15cm below bath surface). Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 6cm.

Colour Plate 6: Vector plot of the body forces acting on the bath, at the location of path EB (32cm below bath surface). Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 6cm.
Colour Plate 7: Composite vector plot of velocities in the bulk, on the symmetry face, wall face, and surface. Current flow is opposite directions in the coils ($I = -$). Coil Separation is 0cm.

0cm Coil Separation - Oppositely Wound Coils
Colour Plate 8: Composite vector plot of velocities in the bath, on the symmetry face, wall face, and surface. Current flow is opposite directions in the coils ($I_1=-I_2$). Coil Separation is 6cm.
Colour Plate 9: Composite vector plot of velocities in the bath, on the symmetry face, wall face, and surface. Current flow is opposite directions in the coils ($I_1=-I_2$). Coil Separation is 10cm.
Contour plot of velocities in the bath on the symmetry face, showing regions of slow, and moderate to fast stirring. Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 0cm.

Contour plot of velocities in the bath on the symmetry face, showing regions of slow, and moderate to fast stirring. Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 6cm.
Colour Plate 12: Contour plot of velocities in the bath on the symmetry face, showing regions of slow, and moderate to fast stirring. Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 10cm.
Colour Plate 13: Vector plot of stirring velocity, at the location of path ES (bath surface). Current flow is opposite directions in the coils ($I_1=-I_2$). Coil Separation is 6 cm.

Colour Plate 14: Vector plot of stirring velocity, at the location of path ET (7 cm below bath surface). Current flow is opposite directions in the coils ($I_1=-I_2$). Coil Separation is 6 cm.
Colour Plate 15: Vector plot of stirring velocity, at the location of path ET (15cm below bath surface). Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 6cm.

Colour Plate 16: Vector plot of stirring velocity, at the location of path EB (32cm below bath surface). Current flow is opposite directions in the coils ($I_1 = -I_2$). Coil Separation is 6cm.
Chapter 5: Influence of Current Flow Direction in the EMS System Stirring Coils

5.1 Introduction
The effect of having both coils wound in the same direction was studied. The current was the same in each coil. In this analysis, rather than the coils adding to give a strong central stirring zone, the fields in between the coils cancel each other out, though the fields elsewhere add constructively. The model geometry was the same as that for the oppositely wound coils. It is reiterated in the table below.

<table>
<thead>
<tr>
<th>Table 5.1 Industrial Billet Mould Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner cross section</td>
</tr>
<tr>
<td>Mould wall thickness</td>
</tr>
<tr>
<td>Mould length</td>
</tr>
<tr>
<td>Bath height</td>
</tr>
<tr>
<td>Coil height</td>
</tr>
<tr>
<td>Coil width</td>
</tr>
<tr>
<td>Coil inner radius</td>
</tr>
<tr>
<td>Turns per coil</td>
</tr>
<tr>
<td>Current per coil turn</td>
</tr>
<tr>
<td>Current in the tin bath</td>
</tr>
<tr>
<td>Current Density in the tin bath</td>
</tr>
</tbody>
</table>

The system of paths used in the examination of the results was the same as in Chapter 4 (Table 4.2, Fig. 4.2).

5.2. The Magnetic Flux Density Distribution in the Tin Bath
The following graph (Fig. 5.1) shows the radial component of magnetic flux density due to one coil plotted against position for longitudinal paths that are located on the symmetry face, at intervals of 20% of the distance from the axial centreline of the tin bath to the mould wall. Since the coils are identical, and therefore the flux curves are identical, only the curve for the top coil is shown.

For this figure, the top coil was located with its centreline at 10cm below the surface of the bath. The curve crosses the axis ($B_r=0$) at this location. The parameters are identical to the $I_1=-I_2$ case, and so again the maximum flux occurs at an average distance of 3cm from the edges of the coil, in good agreement with the analytical model (places the maximum flux at $h^*=0.42$).
5.2.1 Superposition of the Radial Component of the Magnetic Flux Density

Fig. 5.2 shows the relative placement of the two coils against the longitudinal path LF \( (r^*=0.387) \). The coils are centred at 12.5cm and 17.5cm below the melt, and are marked schematically on the figure. This places the coils with no separation between them, however the height of the coils is 5cm each, and so this corresponds to \( h=2.5 \), or \( h^*=0.19 \) (See Fig. 4.9). Increasing coil separation shifts the curves further apart relative to the x-axis, as marked by the direction of the arrows. The positions of the top and bottom path are marked on the figure. At the locations of both the top and bottom paths, the magnitude of the radial component of the magnetic flux density is due to the constructive superposition of the fluxes from both the top and bottom coil.

At the location of the top path, the flux density due to each coil are greater in magnitude than at the location of the bottom path. This difference is due to the distance at which each path is to the centreline of the coils. The bottom path is more than twice the distance away from the centreline than is the top path. Thus each coil has a much smaller radial component of magnetic flux density at that location. Since the current is identical in each coil, and the coils are always
equidistant from the centreline \((z=0)\), \(B_r\) at that location is zero, because the flux densities are identical but opposite between the coils.

![Diagram](image)

**Fig. 5.2** The superposition of the flux along the longitudinal path \(LF (r^*=0.387)\). \(I_1=I_2\), \(h^*=0.19\)

Increasing the separation between the coils will shift these curves relative to the \(x\)-axis in the directions indicated by the arrows. Thus, increasing the coil separation from 0cm to 10cm will shift the position of maximum flux for the bottom coil towards the path \(EB\), while decreasing \(B_{r1}\) at that location. The increase will shift the position of the maximum flux due to the top coil towards and then past the position of the top path \(ET\), while decreasing the magnitude of \(B_{r2}\) at that location.

5.3 Influence of Coil Separation on the Parameters of the EMS System
In order that the effect of the direction of current flow in the coils could be studied, the effectiveness of the EMS system with coil separation was examined for \(I_1=I_2\). The parameters of the model were identical to those in Chapter 4, to allow direct comparison between these results and those for \(I_1=-I_2\). The coils were adjusted to between 0cm and 10cm of separation in 2cm increments. Each coil had a current of 5000A per coil turn, and a radius of 13cm. They were centred at 15cm below the surface of the bath.
5.3.1 Magnetic Flux Density variation with Changing Coil Separation

The EB path shows an increase in $B_{r_{\text{Total}}}$ with increasing coil separation (Fig. 5.3). This increase is due to the coil approaching the path. In changing from 0cm to 10cm of separation the peak radial magnetic flux density along the path increases by almost 40%. Fig. 5.2 shows that the contribution of the bottom coil to the value of $B_{r_{\text{Total}}}$ is much greater than that of the top coil along EB. This small $B_{r1}$ contribution decreases with increasing coil separation, but $B_{r2}$ is increasing, and at a faster rate (as evidenced by the slopes of the two curves). Thus overall $B_{r_{\text{Total}}}$ increases.

Along ET (Fig. 5.4) the radial magnetic flux density decreases with increasing coil separation. Fig. 5.2 shows that when the coils are at 0cm of separation the maximum flux from the top coil is in alignment with the location of ET. As the coils are separated, both of the components $B_{r1}$ and $B_{r2}$ are rapidly decreasing in intensity along the path (again, the slopes of the curves on Fig. 5.2 in the vicinity of ET are steep, and thus there are relatively large changes in $B_{r1}$ and $B_{r2}$ with 2cm incremental change in separation. The effect on $B_{r_{\text{Total}}}$ is compounded by the fact that it is dependent on the superposition of two rapidly decreasing values). The peak $B_{r_{\text{Total}}}$ on the path decreases by over 225% upon increasing the coil separation from 0cm to 10cm.

![Magnetic flux density variation along the bottom path - EB](image)
Examining the LF path it is clear that the top path ET coincides almost exactly with the maximum flux obtained by superposition of the fluxes from the two coils (Fig. 5.5). This maximum occurs at a height of 8cm below the bath surface, the ET path is at 7cm.

The curves on the plot of the LF path show good agreement with the analytical model. Fig. 2.16 shows the same path for three cases: 0cm of separation and 6cm of separation ($h^*=0.19$, $h^*=0.42$), as well as a third case which, in the analytical model, represents 0cm of \textit{filament} separation with a nominal filament radius (so $h^*=0$). Though this is physically impossible for the billet mould EMS system modeled here (as the coils would have to occupy the same space), it can be approximated by directly superimposing two individual curves. This has little value other than allowing further comparison of results given by the two analysis techniques – the analytical model and ANSYS. The peak values of the magnetic flux density for the hypothetical case, and the 0cm and 6cm separation cases are ±0.0362T, ±0.0315T, and ±0.0242T respectively. Division of these results by $\frac{\mu}{2\pi}I$ gives their corresponding $\zeta_{\text{Total}}$ values of ±1.177, ±1.023, and ±0.786, all of which are within 2.5% of the analytical model. The locations of the maxima predicted by the numerical model are within approximately 3.5% of those determined analytically.
Fig. 5.5 Magnetic flux density variation along the longitudinal path - LF

The curves on Figs. 5.5 and 2.16 appear to be reversed, however this is not the case. The start of the path, on the left in Fig. 5.5, corresponds to a positive value of \( z^* \) (the surface of the bath). Fig. 2.16 plots \( \zeta_{\text{Total}} \) against normalized height, from \(-3z^* \) to \(+3z^*\), beginning with \(-z^* \) on the left, by convention. This was not observed on the plots where \( I_1 = -I_2 \), because of the symmetry in the curves.

5.3.2 Magnetic Body Force Variation with Changing Coil Separation

As discussed in Section 4.3.3, the curves describing the variation of \( B_r \) and \( f_\phi \) with position have the same shape, since the magnetic flux density vector and the force vector both depend on the value of the multiplication factor \( \zeta_{\text{Total}} \). Their total magnitude is determined by the system specific factors \( \frac{\mu}{2\pi a} nI \) and \( \frac{\mu I}{2\pi a} nI \) respectively. Since in this model the current density vector is in the negative \( z \)-direction these factors have opposite signs, and therefore the flux and force curves occupy opposite sides of the \( x \)-axis.
Figures 5.6 and 5.7 illustrate the variation of force with position for the paths EB and ET. The shape of the curves is very similar to those for the magnetic flux. As with Figures 4.14 and 4.15 there are variations due to the coarseness of the finite element mesh used.
Examining the distribution of the forces, for the EB path (Fig. 5.6), the maximum force along the path is $1068 \text{N/m}^3$ for the 10cm separation case, an increase of approximately 40% from the 0cm case. The ET graph (Fig. 5.7) shows a maximum of $2240.3 \text{N/m}^3$ of force in the negative direction for the 0cm case, an increase of almost 225% from the 10cm case. These values are in agreement with the changes in magnetic flux density shown on Figs. 5.3 and 5.4.

The LF graph (Fig. 5.8) shows two distinct regions of significant force. The maximum occurs for the 0cm case, with the upper and lower flux regions peaking at $-2241.2 \text{ N/m}^3$ 8cm below the bath surface and $2215.4 \text{ N/m}^3$ at 22cm below the bath surface respectively. These values should be identical considering the symmetry of the system, however they are within 1.5% of each other and each is within 5% of the value predicted by the analytical model ($2325.3 \text{ N/m}^3$). The total force exerted over the path is highest for the 0cm case (the area under the curve is largest).

Colour Plates 17 through 20 illustrate $f_y$ at bath heights corresponding to ES, ET, EM, and EB, for the 0cm separation case. The EM path corresponds to the transition between force in the positive and negative directions.
5.3.3 Stirring Velocity Variations with Changing Coil Separation

The magnetic body force acting on the fluid generates a rotational flow. The nature of the body force distribution (with two distinct regions of force) causes two distinct stirring regions to develop:

1. upper stirring region: flow is in the negative $\phi$ direction
2. lower stirring region: flow is in the positive $\phi$ direction

The force on each region is of relatively equal magnitude (Fig 5.8), and correspondingly the velocities in both regions are very similar.

In the corner of the mould the flow is stagnant. Here the flow is not in a clearly defined direction, and velocities are very low.

Colour Plates 21 through 23 are composite vector plots of the symmetry face, surface, and wall face, for 0cm, 6cm, and 10cm of separation, providing an overview of the 3-D flow field.

Colour Plates 24 through 26 are contour plots of rotational (stirring) velocity on the symmetry face of the bath. The four contours indicate regions of moderate to fast stirring in the negative and positive directions (greater than 2cm/s), and slow stirring in the negative and positive directions (less than 2cm/s). The contour plots are for the 0cm, 6cm, and 10cm coil separations. Each shows stirring at greater than 2cm/s in the majority of the bath. The 0cm and 6cm coil separation contour plots are very similar, the 10cm separation plot shows reduced stirring in the upper (negative) region. Viscous propagation of the flow is evident as the moderate to fast stirring contour encroaches on the axial centreline of the bath towards the bottom of the tin. The dead zones present extend less than two thirds of the distance down the bath, and in general occupy less than half the width.

The LF velocity curves (Figs. 5.9, 5.10) show two distinct stirring directions, in the upper stirring region stirring occurs in the negative direction, in the lower stirring region the stirring is in the positive direction. There is a single dead zone, where the two regions meet.
The average velocities are summarized in the table below.

<table>
<thead>
<tr>
<th>Coil Separation (cm)</th>
<th>Average Velocity Upper Region (cm/s)</th>
<th>Average Velocity Lower Region (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-14.3</td>
<td>10.8</td>
</tr>
<tr>
<td>2</td>
<td>-14.6</td>
<td>10.4</td>
</tr>
<tr>
<td>4</td>
<td>-13.0</td>
<td>10.0</td>
</tr>
<tr>
<td>6</td>
<td>-12.0</td>
<td>9.5</td>
</tr>
<tr>
<td>8</td>
<td>-10.2</td>
<td>9.2</td>
</tr>
<tr>
<td>10</td>
<td>-9.6</td>
<td>8.5</td>
</tr>
</tbody>
</table>

The average velocity for the lower stirring region is lower than for the upper stirring region, due to the lower velocities at the bottom of the mould. In the top half of the lower stirring region, where the body forces acting on the fluid are equal to those acting on the fluid in the upper region, the average velocities are very similar to those of the upper stirring region.

As with the coil separation study for $I_1=-I_2$, the L22.5 path has slightly lower velocities (Fig. 5.10, Table 5.3). This is due to fluid flow effects arising from the path's proximity to the corner. The corner acts to reduce the fluid velocity.
The effectiveness of the corner at reducing the rotational stirring is illustrated in Fig. 5.11. The longitudinal path LF shows a lack of significant stirring or established stirring zones. The fluid is essentially stagnant.
Chapter 5: Analyzing the Effect of Current Flow Direction

The EB path shows little variation between the curves. The average peak velocity is approximately 11 cm/s (Fig. 5.12)

Fig. 5.11 Stirring velocity in the \( \phi \) direction (rotational) along the longitudinal path L45

Fig. 5.12 Stirring velocity in the \( \phi \) direction (rotational) along the bottom path - EB
Note: As was previously mentioned in Section 4.4.4, the sudden changes in slope which occur on some of the curves (for example, at approximately 0.7cm along the path on Fig. 5.12, at approximately 1.2cm along the path on Fig 5.13) are due to a coarse finite element mesh. An insufficient number of nodes was available to mesh the fluid region as finely as required to obtain a more accurate solution. This means that there is question as to the accuracy of the exact shape of the graphs.

Similar to the coil separation study in Chapter 4 (with \( I_f = -I_2 \)), in which the ET path was close to (and in some cases, within) the transition zone from the central stirring region to the upper stirring region, in this study the EM path is in the region of the transition zone from the lower stirring region to the upper stirring region. Due to this the velocities along the EM path are low (Fig. 5.13), and in most cases the curves show distinct positive and negative regions.

![Fig. 5.13 Stirring velocity in the \( \phi \) direction (rotational) along the middle path - EM](image)

The ET path, which is almost coincidental with the height of maximum force for the upper region, shows correspondingly high stirring velocities (Fig. 5.14). The maximum velocity is for the 2cm case, at 17.5cm/s. The 0cm case, which experiences the greater force, peaks at approximately 16cm/s
Chapter 5: Analyzing the Effect of Current Flow Direction

The surface path (Fig. 5.15) lacks a trend that could be attributed to the magnitude of the forces experienced along the path for each coil separation.
The peak velocity occurs on the 4cm separation path, at 16.5 cm/s. The 0cm separation case has a maximum velocity of 13.2cm/s, and the run with the largest force, the 10cm separation case, is very similar at 13.5cm/s.

The telling pattern that does emerge in the graph is the shape of the curves. At the 3cm point along the path, for example, the slope of the curves increases with decreasing coil separation. With the exception of the 6cm case, which deviates from the pattern at approximately the 3.5cm mark, this holds until the velocities begin to rapidly increase for all cases, about 4.5cm along the path. This indicates the propagation of the rotational flow due to viscous effects. Using the 0cm case as the example, the height of maximum force occurs 8cm below the surface of the bath (Fig. 5.8). The force is very high, relatively speaking, at 2241.2N/m$^2$ in the stirring direction. At that height, the force is greater for the 0cm case than for any of the others, and the velocity at that position is correspondingly high. Between the ET and ES paths, the 0cm case makes the transition from being the highest force at a given location, to being the lowest. However, viscous effects have propagated that high velocity in the region of the ET path both towards the axial centreline of the bath and towards the surface. This same phenomenon occurs for each case, though to a lesser degree corresponding to the lesser force experienced.

The similar peak velocities are due to the fact that as the coil separation increases from 0cm to 10cm, the force acting on the surface of the bath is increasing. Thus the 8cm and 10cm cases (for example) have surface velocities similar to the other separations, despite the lower force that they exert overall over the upper stirring region.

Colour plates 29 through 32 illustrate $V_\phi$ at bath heights corresponding to ES, ET, EM, and EB, for the 0cm separation case.

In contrast to the oppositely wound coils, the similarly wound coil experiments exhibit a change in stirring direction throughout the melt. Unlike Fig. 4.23, which shows that the central core of the bath never actually changes stirring direction (it is negative throughout), Fig. 5.16, illustrating the 0cm coil separation, shows that even at 20% of the distance from the axial
centreline to the mould, the transition from positive to negative stirring direction is definite. The velocity is not high at that location, but the transition is clear.

![Graph showing stirring velocity in the φ direction for several FL paths across the symmetry face](image)

Fig. 5.16 Stirring velocity in the φ direction for several FL paths across the symmetry face

Again, the maximum stirring velocities may not have been reached on any of the paths examined, and so the peak velocities for each separation are summarized in the table below.

<table>
<thead>
<tr>
<th>Table 5.4 Maximum Velocity Summary for the I₁-I₂ Coll Separation Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coll Separation (cm)</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>
5.4 Comparison Between Coil Configurations $I_1=I_2$ and $I_1=-I_2$

By comparing the magnetic flux density distributions it can be shown that the oppositely wound coils give the maximum force. It is possible to align the coils such that the maximum fluxes from each overlap, giving the highest possible force for the amperage supplied. However, the $B_r$ curves in Fig. 5.17 show that this location between the centrelines of the coils is the only location where the flux densities from the individual coils add constructively. Elsewhere, to each side of the pair of coils, the flux densities oppose each other, and so the total flux density there is much less that the amount which would be supplied by a single coil.

Conversely, having the coils wound in the same direction means that between the coils the $B_r$ curves from the individual coils oppose each other, but to either side of the centrelines of each coil the flux densities add. While it is not possible to maximize the force for the given amperage by positioning of the coils (due to coil geometry), by placing the coils with no separation between them it is possible to maximize the force for the given coil geometry. This is shown in Fig. 5.18.
Fig. 5.19 shows the percentage change in the magnitude of $B_r$ on transition from an $I_1=-I_2$ system to an $I_1=I_2$ system. Except for the region between the centrelines of the coils, having $I_1=I_2$ is a consistent improvement over the opposite direction system. It maintains a value of $B_r$ at least 38% higher.

The percentage of the bath height over which having current flow in the same direction in the coils generates a greater body force than having current flow in opposite directions in the coils decreases as coil separation increases from 0cm to 10cm.

- At 0cm of separation, having the coils wound such that $I_1=I_2$ generates a higher body force than having them wound such that $I_1=-I_2$, over 90% of the bath height. Over this height the body force generated by the $I_1=I_2$ case is over 195% greater than that generated by the $I_1=-I_2$ case.
- At 10cm of separation, having the coils wound such that $I_1=I_2$ generates a higher body force than having them wound such that $I_1=-I_2$, over 70% of the bath height. Over this height the body force generated by the $I_1=I_2$ case is at least 38% greater than that generated by the $I_1=-I_2$ case.
Chapter 5: Analyzing the Effect of Current Flow Direction

Fig. 5.19 The percentage change in force from $I_1 = -I_2$ to $I_1 = I_2$.

It should be noted that these coil configurations, $I_1 = -I_2$, and $I_1 = I_2$, are not optimized at the same coil separation. That is, the flux patterns generated by the two configurations are very different. Chapter 4 showed that the ideal coil separation for maximizing the peak radial component of magnetic flux density between the coils (at the LF path location) if $I_1 = -I_2$ was 6cm. Chapter 5 showed that the magnetic flux density and the magnetic body force were maximized for $I_1 = I_2$ if there was no separation between the coils. Thus, comparing 0cm data for each case is comparing a reasonable coil separation for $I_1 = -I_2$, with the best coil separation for $I_1 = I_2$.

This is addressed by direct comparison of the 6cm coil separation case which gives the highest values of flux density for when $I_1 = -I_2$, with the 0cm separation case which gives the highest values of flux density for when $I_1 = -I_2$.

Having the current in the same direction in both coils generates a superior flux for all but 7cm (14%) of the total bath height. For the most part, it is at least 38% superior, in that the forces generated are at least 38% higher (Fig. 5.20).
For clarity the peaks have been removed from the graph. The maximums occur where the force is zero for the oppositely wound case, which coincides closely with the maximum force for the similarly wound case. The magnitude of the peaks was such that showing them on the graph obscured the rest of the results.

![Graph showing percentage change in force](image)

Fig. 5.20 The percentage change in force from oppositely wound (6cm) to similarly wound (0cm)

Colour plates 10-12 and 24-26 also offer a good comparison between the two systems. In each case, the contour plots for \( I_1 = I_2 \) show a higher percentage of moderate to high stirring in the bath. The best case in the \( I_1 = I_2 \) study (0cm of separation) is superior to each case for \( I_1 = -I_2 \), in that it develops moderate to high stirring throughout more of the bath.

Table 4.3 lists the average stirring velocities in the central stirring region for different coil separations and the configuration \( I_1 = I_2 \). Table 5.2 lists the average stirring velocities in the upper stirring region for different coil separations and the configuration \( I_1 = I_2 \). The tables show that the average stirring velocity obtained using the optimum coil separations for the \( I_1 = I_2 \) configuration is higher than those for the \( I_1 = -I_2 \) configuration. The average velocity for 0cm of separation, \( I_1 = I_2 \) is 14.3cm/s, while for 6cm of separation and \( I_1 = -I_2 \), it is 12.9cm/s.
In addition, the upper and lower stirring regions for the configuration $I_1=-I_2$ have lower average velocities than the central region, because the body force on the fluid in these regions is much less. However for the $I_1=I_2$ configuration, the average velocity of approximately the upper half of the lower stirring region is comparable to that of the upper region.

From these results it is clear that having the current flow in the same direction in both coils provides:

- a higher radial component of magnetic flux density and therefore a greater body force on average throughout the bath
- correspondingly higher average velocities throughout the bulk of the bath.

In addition, having the coils wound in the same direction results in only one dead zone occurring in the bath, since the stirring direction only changes once.
Colour Plate 17: Vector plot of the body forces acting on the bath, at the location of path ES (bath surface). Current flow is in the same direction in the coils ($I_1 = I_2$). Coil Separation is 0cm.

Colour Plate 18: Vector plot of the body forces acting on the bath, at the location of path ET (7cm below bath surface). Current flow is in the same direction in the coils ($I_1 = I_2$). Coil Separation is 0cm.
Colour Plate 19: Vector plot of the body forces acting on the bath, at the location of path EM (15cm below bath surface). Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.

Colour Plate 20: Vector plot of the body forces acting on the bath, at the location of path ET (32cm below bath surface). Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.
Colour Plate 21: Composite vector plot of velocities in the bath, on the symmetry face, wall face, and surface.

Current flow is in the same direction in the coils ($r = A$). Coil Separation is 0 cm.

0cm Coil Separation - Similarly Wound Coils
Colour Plate 22: Composite vector plot of velocities in the bath, on the symmetry face, wall face, and surface. Current flow is the same direction in the coils ($I_1=I_2$). Coil Separation is 6cm.
Colour Plate 2. Composite vector plot of velocities in the bath, on the symmetry face, wall face, and surface. Current flow is the same direction in the coils $(i=j)$. Coil Separation is 10cm.

10cm Coil Separation - Similarly Wound Coils
Colour Plate 24: Contour plot of velocities in the bath on the symmetry face, showing regions of slow, and moderate to fast stirring. Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.

Colour Plate 25: Contour plot of velocities in the bath on the symmetry face, showing regions of slow, and moderate to fast stirring. Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 6cm.
Colour Plate 26: Contour plot of velocities in the bath on the symmetry face, showing regions of slow, and moderate to fast stirring. Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 10cm.
Colour Plate 27: Vector plot of stirring velocity, at the location of path ES (bath surface). Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.

Colour Plate 28: Vector plot of stirring velocity, at the location of path ET (7cm below bath surface). Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.
Colour Plate 29: Vector plot of stirring velocity, at the location of path EM (15cm below bath surface). Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.

Colour Plate 30: Vector plot of stirring velocity, at the location of path EB (32cm below bath surface). Current flow is in the same direction in the coils ($I_1=I_2$). Coil Separation is 0cm.
Chapter 6: Experimental Results

6.1 Introduction
Experiments were performed on a laboratory model in order to compare the results with the theoretical results obtained using the finite element model. Measurements of the velocity near the surface of the bath were taken and compared with the theoretically calculated velocity at the location of the probe.

6.2 Experimental Apparatus
The laboratory model consisted of a soft iron closed-bottom mould with a square cross-section, and wound copper coils supported on a frame. The frame could be raised and lowered to adjust the height of the coils. Wooden spacers allowed for adjustment of coil separation. The mould was approximately 3/4 full of commercially pure tin. The tin was melted using an industrial hot plate and resistance heaters. The mould was thermally insulated with Fibrafrax insulation, and the coils were insulated electrically and thermally with silicon based high temperature electrical tape. The dc current was supplied by a welding power source. The details of these components are summarized in Table 6.1.

<table>
<thead>
<tr>
<th>Table 6.1 Experimental apparatus components summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
</tr>
<tr>
<td>Linear tracking motor</td>
</tr>
<tr>
<td>Coil support frame</td>
</tr>
<tr>
<td>Coils</td>
</tr>
<tr>
<td>Coil insulation</td>
</tr>
<tr>
<td>Mould</td>
</tr>
<tr>
<td>Mould insulation</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Hot plate</td>
</tr>
<tr>
<td>Resistance heaters</td>
</tr>
<tr>
<td>DC power source</td>
</tr>
<tr>
<td>Multimeter</td>
</tr>
<tr>
<td>Shunt</td>
</tr>
</tbody>
</table>

A 100mv/200A shunt was inserted in series with the coils to measure the magnitude of the direct current. The current supply could be adjusted using the controls on the power source and a multimeter gage inserted in parallel with the shunt. The coils were wound in the same direction so that current flow in each coil would be in the same direction. From the bottom coil a cable carried the current to the top electrode of the mould. The current then flowed through both the
mould wall and the tin. From the bottom electrode of the mould a power cable completed the electric circuit. This is shown schematically in Fig. 6.1 The geometry of the salient features is outlined in Table 6.1. Fig. 6.2 is a photograph of the system.

---

**Fig. 6.1 Schematic of the experimental apparatus**

<table>
<thead>
<tr>
<th>Table 6.2 Geometry of Laboratory Scale Billet Mould Apparatus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner cross section</td>
</tr>
<tr>
<td>Mould wall thickness</td>
</tr>
<tr>
<td>Mould length</td>
</tr>
<tr>
<td>Bath height</td>
</tr>
<tr>
<td>Coil height</td>
</tr>
<tr>
<td>Coil width</td>
</tr>
<tr>
<td>Coil inner radius (Radius)</td>
</tr>
<tr>
<td>Turns per coil</td>
</tr>
<tr>
<td>Current per coil turn</td>
</tr>
<tr>
<td>Current in the tin bath</td>
</tr>
<tr>
<td>Current Density in the tin bath</td>
</tr>
<tr>
<td>Coil centreline height</td>
</tr>
<tr>
<td>Coil separation</td>
</tr>
</tbody>
</table>
6.2.1 Data Acquisition
The data acquisition was performed using a CCD camera recording onto VHS tape.

<table>
<thead>
<tr>
<th>Table 6.3 Data acquisition components summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
</tr>
<tr>
<td>CCD Camera</td>
</tr>
<tr>
<td>CCD Camera lens</td>
</tr>
<tr>
<td>S-VHS VCR</td>
</tr>
<tr>
<td>Television</td>
</tr>
<tr>
<td>Stop Watch</td>
</tr>
</tbody>
</table>
6.2.2 Velocity Measurement

High temperature and the presence of an electromagnetic field make velocity measurements of electromagnetically stirred metal difficult. Due to the complicated construction and extensive calibration at operating temperature required of the probes designed for liquid metal, velocity measurements throughout the fluid were not feasible within the scope of this work.

Therefore only the velocity near the surface of the bath could be measured. This was achieved using an aluminum probe. The probe consisted of a ring, 6cm in outer diameter, 1cm in height and 0.4cm in thickness. On the underside of the ring were mounted 4 fins, each measuring 1.5cm in width and 2cm in height. The fins were made of 0.05cm aluminum sheet. Both the ring and the fins were polished to remove any surface roughness. On the top of the ring was an indicator, so that revolutions could be counted. The probe is illustrated in Fig. 6.3.

![Probe Diagram](image)

Fig. 6.3 Schematic representation of the probe used to measure velocity (a) side view (b) top view

Aluminum was chosen for the probe material because it is non-magnetic, and therefore does not interfere with the magnetic fields generated by the coils, and also because aluminum is not wetted by tin. Therefore the probe remained clean during experiments, and did not develop a build up of tin. In addition, the specific gravity of aluminum is less than tin. This meant that the body of the probe sat on the surface of the bath, it did not sink down into it. The fins penetrated the bath, and the stirring action of the tin spun the probe. The revolutions per second of the probe indicated the approximate velocity of the bath at the location of the fins.
6.3 Experimental Process
Once stirring was established, the probe was placed on the surface of the bath. A CCD camera was used to capture footage of the probe rotating on the surface, which was recorded on the SVHS VCR. Approximately 20 seconds of the probe spinning was filmed for each experiment. In between experimental runs the probe was removed from the tin, and the surface of the tin bath was cleaned to remove the oxide layer.

Rotational velocity was calculated from the revolutions per second of the probe. Analysis in frame by frame mode allowed determination of the elapsed time during 5 complete revolutions of the probe. The frequency, \( f \) (revolutions per second, RPS) of the probe was found using

\[
RPS = f = \frac{5}{\text{time elapsed}}
\]

From the frequency, it was possible to determine the angular velocity (\( \omega \)), and subsequently the velocity of the probe (equations 3.2 and 3.3).

\[
\omega = 2\pi f
\]

\[
V = \omega r
\]

6.4 Experimental Results
The apparatus was tested under 4 conditions. These are outlined in Table 6.4.

<table>
<thead>
<tr>
<th>Experimental Run</th>
<th>Coil Separation (cm)</th>
<th>Coil Position (cm below surface)</th>
<th>System Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>2</td>
<td>9</td>
<td>-400</td>
</tr>
<tr>
<td>4-6</td>
<td>8</td>
<td>9</td>
<td>-400</td>
</tr>
<tr>
<td>7, 8</td>
<td>8</td>
<td>2</td>
<td>-400</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>2</td>
<td>300</td>
</tr>
</tbody>
</table>

The apparatus was tested at 2cm and 8cm of separation, with the coils centred 9cm below the bath. This corresponds to three quarters of the way up the bath. Further experiments were performed with 8cm of separation, with the coils centred just below the surface of the bath.

For each run, several measurements of velocity were made. The average velocity for each run best describing the experimental conditions (i.e. most correct system current) is given in Table 6.5.
The system current was chosen to be approximately 400A and 300A, with the exact values listed in Appendix C. The variation in system current introduced during experimentation permits the comparison of system current with the resultant velocity. This is shown in Fig. 6.4, for the experimental conditions where the coils were centred 9cm below the surface of the bath. Clearly as system current increases from approximately 380A to approximately 400A the velocity of the metal is increasing. As the current increases from 378A to 388A, the average velocity increases by about 17%. A further increase of over 5% occurs on increasing the current from 388A to 398A. The increase in the 8cm coil separation data is approximately 3.5%, upon increasing from 394A to 400A of system current.

Table 6.5 Summary of Average Stirring Velocities

<table>
<thead>
<tr>
<th>Experimental Run</th>
<th>System Current (A)</th>
<th>Velocity (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>398</td>
<td>16.9</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>14.2</td>
</tr>
<tr>
<td>7</td>
<td>400</td>
<td>14.6</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Fig. 6.4 The variation in average velocity with system current, for runs 1 through 6. (Coils centred 9cm below the surface of the bath)

The increase in average velocity (from Table 6.5) on increasing the system current from 300A to 400A for the 8cm separation case centred just below the bath surface is over 70%.
6.5 Comparison of Experimental and Theoretical Results

The same system parameters were modelled using FEA. Some differences exist between this model and the model used to examine the industrial billet mould. The geometry is different, as noted in Tables 4.1 and 6.2, the lab scale model is significantly smaller. However the primary difference is in the mould material. The billet mould model uses copper, which has a relative permeability of 1, and therefore is non-magnetic. The lab model mould is soft iron, which is a magnetic material, and so does not have a relative permeability of 1. The permeability of the lab mould at any point is dependent on the strength of the magnetic field at that point. The computer model of the laboratory scale model accommodates this non-linear material property by calculating the relevant value of permeability for all points using the B-H curve for soft iron.

The results from the FEA of the laboratory scale model are shown in Figures 6.5 and 6.6. Fig. 6.5 is an edge path located 1cm below the surface. Fig. 6.6 is a longitudinal path at r=3.55cm, the distance from the axial centreline of the melt to the centre of the probe fins, the location used in calculation of the probe velocity.
The velocity predicted by ANSYS at 1 cm below the bath surface at the location of the fin was -7.7 cm/s, for 2 cm of separation and -3.6 cm/s, for 8 cm of separation, when the coils are centred at 9 cm below the surface of the bath.

For 8 cm of separation, with the coils centred 2 cm below the surface of the bath, and a system current of 400 A and 300 A, the velocities predicted by ANSYS at 1 cm below the bath surface at the location of the fin were 11.7 cm/s and 8.0 cm/s respectively. These results are within 20% of the FEA model.

An analysis on the effect of the non-linear permeability was performed. This was carried out to determine if the difference in velocity was due to the computer model overestimating the shielding provided by the (magnetic) soft iron mould. Model analyses were performed for 2 cm and 8 cm of separation, with the coils centred 9 cm below the surface of the bath, but with the non-linear B-H data replaced by μ = 1. This simulates the temperature of the mould exceeding the Curie point (at which a magnetic material becomes non-magnetic). The results from this test are shown in Fig. 6.7.
The model with $\mu=1$ predicts a slightly higher velocity at most bath heights for each case. However, the velocities on the curves are still below the velocities measured in the lab. In addition, for each of the experiments performed in the lab, the velocity probe was spun in a counter-clockwise direction. This corresponds to a "positive" stirring direction in ANSYS. However the finite element model predicted that the geometry which placed the coils 9cm below the surface of the bath would produce a flow field with two distinct flow regions, and that the upper stirring region would be in the negative (clockwise) direction.

![Graph](image)

**Fig. 6.7** Comparison of ANSYS predicted result using B-H data and $\mu=1$ to represent the permeability of soft iron

The most likely reason for the disparity in the results is the finite element model size. The ANSYS 5.5.3 University License permits a maximum of 32000 elements in the model. This limit was reached in the modelling of the dc EMS system. The necessary inclusion of the air region to model the magnetics problem adequately allowed for only approximately 5600 of these elements to be used in the modelling of the tin bath. Because the problem was turbulent in nature, the majority of these elements were located in the outer section of the bath, close to the mould walls. The elements greatly increased in size towards the axial centreline of the bath. There were insufficient elements to adequately model the flow field.
Because the magnetohydrodynamic analysis of the dc EMS system is a coupled field problem, the same finite element mesh must be used for all parts of the analysis. Thus the geometry could not be re-meshed following the magnetics portion in order to concentrate more elements on the fluid region. The only way to properly model the fluids portion of this analysis would be to use a version of ANSYS with a higher element allotment.

In light of these findings, it is reasonable to extend this to the previous analyses – those outlined in Chapters 4 and 5 – that there were insufficient elements to properly model the flow in those tests also. The industrial billet mould model used just over 6000 elements in total to represent the fluid region. It is very possible that tests of the EMS system described in this thesis on a billet mould of the geometry studied here would show much higher stirring velocities than those predicted by the computer model.
Chapter 7: Conclusions

1. Regardless of the direction of current flow in the coils, the system causes rotational stirring in the fluid. The stirring is not unidirectional.

   i. When the direction of current flow is opposite in the two coils, the system causes a flow profile consisting of 3 distinct stirring regions. The velocities in the stirring regions are not equal. In the central region, where stirring is in the positive direction, stirring velocity is higher than in the upper and lower regions, where stirring is in the negative direction.

   ii. When the direction of current flow is the same in the two coils, the system causes a flow profile consisting of 2 distinct stirring regions. The average velocity in the upper region was greater than in the lower region.

3. Because the forces created by the coils change direction, it is possible to position the coils so that the velocity at the surface of the bath is zero.

4. For the mould geometry considered, the rotational stirring extends the length of the fluid, well above and below the location of the stirring coils.

5. Flow velocity in the vertical direction is minimal compared to that of the rotational.

6. The fluid flow in the corners of the mould is stagnant, that is, velocities are low, and stirring directions are not clearly defined.

7. Experimental results and predicted flow velocities are within 20% for the configuration which centred the coils 9cm below the surface of the bath. However, there is disagreement between the experimental results and predicted flow velocities for the configuration which centred the coils 2cm below the surface of the bath. The reason for this discrepancy is not known.
Chapter 8: Recommendations for Future Work

1. It was shown that the analytical equations for the distribution of magnetic flux density due to two current filament loops predicts that of the numerical model to within 3%. This indicates that there is little gain in the inclusion of the detailed numerical model in the overall framework of the analysis. The nodes liberated by use of the analytical model could overcome the limitation introduced by the size restriction in the software package.

   It is recommended that the magnetic flux density distribution in the fluid be determined analytically, and that this distribution be applied to the finite element model. The conduction analysis should still be performed using the finite element model, since current density is not entirely uniform throughout the bath.

2. Correct placement and configuration of the two stirring coils should generate a fluid flow field with zero velocity at the free surface. This would mean that no meniscus would be created by the stirring action of the EMS system, which would be beneficial from a metallurgical standpoint. The correct coil configuration and placement to achieve zero stirring velocity at the surface should be determined using a finite element model.

3. Correct coil placement and configuration coupled with the use of a third, or more, stirring coils should be examined to achieve unidirectional stirring throughout the bath.

4. It was shown that the magnitude of the magnetic flux density varies linearly with the magnitude of the current applied to the system, and that the magnitude of the magnetic body force created varies as the square of this current. The velocity did not vary with the square of the current, rather it varied approximately linearly with increasing system current.

   Possible explanations for this include the coarseness of the finite element grid, and the effect of turbulence on fluid flow. It is recommended that the reason for this discrepancy be explored.

5. The metallurgical benefits of this dc EMS system should be examined and quantified.
Appendix A – Dependence of the Vector Components of Magnetic Vector Potential on the Current Density

(Adapted from [19])

Maxwell’s curl \( \mathbf{H} \) equation:
\[ \nabla \times \mathbf{H} = \mathbf{J} \]  
(A.1)

Ampère’s Law:
\[ \mathbf{B} = \mu_0 \mathbf{H} \]  
(A.2)

Magnetic vector potential (A) is defined as
\[ \mathbf{B} = \nabla \times \mathbf{A} \]  
(A.3)

Taking the curl of both sides and substituting for \( \mathbf{B} \) using equation A.2 gives
\[ \nabla \times \mathbf{H} = \frac{1}{\mu_0} \nabla \times (\nabla \times \mathbf{A}) \]  
(A.4)

Therefore (from equation A.1)
\[ \frac{1}{\mu_0} \nabla \times (\nabla \times \mathbf{A}) = \mathbf{J} \]  
(A.5)

Employing the following vector identity for the curl of a curl of a vector:
\[ \nabla \times (\nabla \times Z) = \nabla (\nabla \cdot Z) - \nabla^2 Z \]  
(A.6)

this becomes
\[ \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J} \]  
(A.7)

If \( \nabla \cdot \mathbf{A} = 0 \), then
\[ \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \]  
(A.8)

and therefore
\[ \nabla^2 A_r = -\mu_0 J_r \]
\[ \nabla^2 A_\phi = -\mu_0 J_\phi \]  
(A.9)
\[ \nabla^2 A_z = -\mu_0 J_z \]

The magnetic vector potential in any of the three orthogonal directions is a function of the current density in the same direction.
Appendix B – Distribution of Current Between the Mould and the Bath

| Inner cross section of mould | 10.2cm |
| Mould wall thickness         | 1.6cm  |
| Cross-sectional area of tin bath | 0.01032m² |
| Cross-sectional area of copper mould | 0.00746m² |
| Length of mould             | 58cm   |
| Height of bath ($l_b = l_{m2}$) | 50cm |
| Resistivity of tin ($\rho_b$) | $0.230 \times 10^{-7} (\Omega \text{m})^{-1}$ |
| Resistivity of copper ($\rho_m$) | $0.293 \times 10^{-7} (\Omega \text{m})^{-1}$ |
| Current                     | 5000A  |

The resistances in the equivalent circuit are

$$R_{m1} = \frac{\rho_{m1} l_{m1}}{A_{m1}} = \frac{(0.293 \times 10^{-7})(0.08)}{0.00746} = 3.142 \times 10^{-7} \Omega$$

$$R_{m2} = \frac{\rho_{m2} l_{m2}}{A_{m2}} = \frac{(0.230 \times 10^{-6})(0.5)}{0.00746} = 1.964 \times 10^{-6} \Omega$$

$$R_b = \frac{\rho_b l_b}{A_b} = \frac{(0.230 \times 10^{-6})(0.5)}{0.01032} = 1.14 \times 10^{-5} \Omega$$

Thus the current in each component is

$$I_{m2} = I \left( \frac{R_b}{R_b + R_{m2}} \right) = 5000 \left( \frac{1.14 \times 10^{-5}}{1.14 \times 10^{-5} + 1.964 \times 10^{-6}} \right) = 5000(0.85) = 4250 \text{A}$$

$$I_b = I \left( \frac{R_{m2}}{R_b + R_{m2}} \right) = 5000 \left( \frac{1.964 \times 10^{-6}}{1.14 \times 10^{-5} + 1.964 \times 10^{-6}} \right) = 5000(0.15) = 750 \text{A}$$

Thus 85% of the current supplied will travel through the mould, and 15% through the tin.
### Appendix C: Summary of Experimental Data

<table>
<thead>
<tr>
<th>Run</th>
<th>Separation (cm)</th>
<th>Location (cm)</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Begin (s)</th>
<th>End (s)</th>
<th>Elapsed (s)</th>
<th>Rotations</th>
<th>RPS (ls)</th>
<th>Omega (m/s)</th>
<th>Velocity (m/s)</th>
<th>Average (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
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