INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600
AN EXPERIMENTAL INVESTIGATION OF FREE-SURFACE STRUCTURES IN TURBULENT CHANNEL FLOW AT LOW REYNOLDS NUMBERS

by

Peter Mang-Yu Chung

A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Graduate Department of Chemical Engineering and Applied Chemistry
University of Toronto

© Copyright by Peter Mang-Yu Chung (1998)
The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L’auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L’auteur conserve la propriété du droit d’auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-40904-X
AN EXPERIMENTAL INVESTIGATION OF FREE-SURFACE STRUCTURES IN TURBULENT CHANNEL FLOW AT LOW REYNOLDS NUMBERS

Master of Applied Science, 1998

Peter Mang-Yu Chung

Department of Chemical Engineering and Applied Chemistry

University of Toronto

ABSTRACT

Flow visualization using photochromic dye activation (PDA) and image processing was performed in a smooth, open channel to study the turbulence beneath the shearfree and waveless, gas-liquid interface. Experiments were done over a low-Reynolds number range of 800 to 2,400 based on liquid depth and a Froude number range of 0.25 to 0.32. Turbulence quantities obtained from statistical analysis of the instantaneous streamwise velocity profiles include the mean velocity distribution, probability density function, turbulence intensity, skewness and flatness factors and Reynolds stress. On the video recordings, coherent motions of large-scale free-surface structures were qualitatively observed, namely upwellings, downdrafts, spiral eddies and streamwise vortices. Visual counting of wall events showed that the scaling of their temporal frequency with the inner variables of viscous shear stress and viscosity improved with increasing Reynolds number, yielding average ejection and bursting periods of 39 and 119 for the highest Reynolds number, respectively.
ACKNOWLEDGEMENTS

The writer wishes to express his gratitude to Professor M. Kawaji for his invaluable comments and guidance during the course of this research. The assistance of Mr. E.G. Stamatious in conducting the numerous experiments is gratefully acknowledged. In addition, the author would like to thank Dr. G. Karimi for his helpful discussions and many suggestions on this work. Finally, the author is grateful to Mr. C. Jowlabar and Dr. A. Kariyasaki for their help in setting up the experiments.
TABLE OF CONTENTS

ABSTRACT ............................................................................................................................. ii

ACKNOWLEDGEMENTS ......................................................................................................... iii

TABLE OF CONTENTS .............................................................................................................. iv

LIST OF TABLES ....................................................................................................................... viii

LIST OF FIGURES ..................................................................................................................... ix

NOMENCLATURE ..................................................................................................................... xiii

CHAPTER 1 BACKGROUND ..................................................................................................... 1

1.1 Introduction ....................................................................................................................... 1

1.2 Scope .................................................................................................................................. 3

CHAPTER 2 THEORY OF TWO-DIMENSIONAL TURBULENT STRUCTURES .............. 4

2.1 Nomenclature and fundamental concepts ........................................................................... 4

2.2 Mean-Flow Equations ........................................................................................................ 7

2.2.1 Mean-flow energy budget .............................................................................................. 7

2.2.2 The law of the wall ....................................................................................................... 9

2.2.3 The velocity-defect law ............................................................................................... 10

2.3 Reynolds shear stress ........................................................................................................ 11

2.4 Eddy viscosity .................................................................................................................. 11

2.5 Mixing length ................................................................................................................... 11

2.6 Sub-division of the open-channel flow ............................................................................. 12

2.6.1 Wall region ................................................................................................................... 12
CHAPTER 5 EXPERIMENTAL RESULTS AND DISCUSSION ........................................ 48

5.1 Statistical structures of turbulence ................................................................. 48

5.1.1 Instantaneous flow properties .................................................................... 49

5.1.2 Correlation functions .................................................................................. 56

5.1.3 Mean flow properties .................................................................................. 60

5.1.3.1 Friction velocity .................................................................................... 60

5.1.3.2 The law of the wall ................................................................................ 62

5.1.4 Frequency distribution of velocity fluctuations ........................................... 68

5.1.5 Turbulence intensity .................................................................................... 70

5.1.6 Higher-order statistics ................................................................................. 80

5.1.7 Reynolds shear stress .................................................................................. 86

5.1.8 Eddy viscosity distribution .......................................................................... 91

5.1.9 Mixing-length distribution .......................................................................... 94

5.1.10 Three-dimensional flow structures ............................................................ 94

5.2 Coherent structures ....................................................................................... 97

5.2.1 Detection methods ..................................................................................... 97

5.2.1.1 Spectral analysis .................................................................................. 97

5.2.1.2 Conditional sampling .......................................................................... 101

5.2.1.3 Visual count ......................................................................................... 102

5.2.2 Scaling laws .................................................................................................. 102

5.2.2.1 Ejection period .................................................................................... 102

5.2.2.2 Bursting period .................................................................................... 105

5.2.3 Flow visualization ....................................................................................... 109
5.2.3.1 Bursting phenomena ........................................................................................................... 114
5.2.3.2 Large-scale vortical motions ............................................................................................... 131
5.2.3.3 Summary ............................................................................................................................ 135

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS ................................................................. 143

6.1 Conclusions .................................................................................................................................. 143
6.2 Recommendations .......................................................................................................................... 145

REFERENCES ................................................................................................................................... 146

APPENDIX ......................................................................................................................................... A-1
LIST OF TABLES

Table 4.1. Experimental conditions .................................................................................................................................. 47

Table A.1. Comparison of different methods to calculate wall friction velocity ......................................................... A-6

Table A.2. Comparison of different methods to calculate ejection frequency ................................................................. A-7
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Geometrical terminology for coordinate system</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Sub-division of flow field in open channel</td>
<td>13</td>
</tr>
<tr>
<td>3.1</td>
<td>Conceptual model of bursting phenomena</td>
<td>19</td>
</tr>
<tr>
<td>3.2</td>
<td>Generation of hairpin vortex</td>
<td>21</td>
</tr>
<tr>
<td>3.3</td>
<td>Double structure of turbulence</td>
<td>23</td>
</tr>
<tr>
<td>3.4</td>
<td>Conceptual illustration of burst-interface interactions</td>
<td>25</td>
</tr>
<tr>
<td>3.5</td>
<td>Splatting produced by a pair of streamwise vortices</td>
<td>28</td>
</tr>
<tr>
<td>4.1</td>
<td>Experimental flow loop</td>
<td>31</td>
</tr>
<tr>
<td>4.2</td>
<td>Laser arrangement</td>
<td>36</td>
</tr>
<tr>
<td>4.3</td>
<td>Video setup</td>
<td>38</td>
</tr>
<tr>
<td>4.4</td>
<td>Image analysis procedure</td>
<td>40</td>
</tr>
<tr>
<td>4.5</td>
<td>Polynomial fit to digitized data</td>
<td>41</td>
</tr>
<tr>
<td>4.6</td>
<td>Apparent failure to satisfy the no-slip boundary condition due to misalignment of camera</td>
<td>44</td>
</tr>
<tr>
<td>4.7</td>
<td>Uncertainty of interfacial position with distance of trace moving laterally</td>
<td>45</td>
</tr>
<tr>
<td>5.1</td>
<td>Instantaneous and mean velocity distributions</td>
<td>50</td>
</tr>
<tr>
<td>5.2</td>
<td>Eulerian time history of instantaneous streamwise velocity</td>
<td>52</td>
</tr>
<tr>
<td>5.3</td>
<td>Eulerian time history of streamwise velocity fluctuations</td>
<td>53</td>
</tr>
<tr>
<td>5.4</td>
<td>Lagrangian time history of instantaneous streamwise velocity</td>
<td>54</td>
</tr>
<tr>
<td>5.5</td>
<td>Velocity histories from 30&lt;y&lt;122</td>
<td>55</td>
</tr>
<tr>
<td>5.6</td>
<td>Velocity histories from 20&lt;y&lt;30</td>
<td>57</td>
</tr>
<tr>
<td>5.7</td>
<td>Cross-covariance functions of instantaneous streamwise velocities</td>
<td>59</td>
</tr>
<tr>
<td>5.8</td>
<td>Cross-correlation functions of instantaneous streamwise velocities</td>
<td>61</td>
</tr>
<tr>
<td>5.9</td>
<td>Mean velocity distribution in wall coordinates</td>
<td>63</td>
</tr>
</tbody>
</table>
Figure 5.10. Instantaneous and mean velocity distributions in wall coordinates .............................. 64
Figure 5.11. Error bars of mean velocity distribution ........................................................................ 66
Figure 5.12. Mean velocity distributions in wall coordinates for all runs ............................................ 67
Figure 5.13. Histograms of velocity fluctuations from 30<y^+<122 .................................................... 69
Figure 5.14. Histograms of velocity fluctuations from 20<y^+<30 .................................................... 71
Figure 5.15. Probability density functions of velocity fluctuations from 30<y^+<122 ...................... 72
Figure 5.16. Probability density functions of velocity fluctuations from 20<y^+<30 ...................... 73
Figure 5.17. Turbulence intensity .................................................................................................... 74
Figure 5.18. Dimensionless turbulence intensity ............................................................................. 75
Figure 5.19. Turbulence intensity in wall coordinates ........................................................................ 77
Figure 5.20. Turbulence intensities for all runs ................................................................................ 78
Figure 5.21. Maximum turbulence intensity versus Reynolds number ............................................ 79
Figure 5.22. Dimensionless turbulence intensities for all runs ........................................................ 81
Figure 5.23. Turbulence intensities in wall coordinates for all runs .................................................. 82
Figure 5.24. Dimensionless distributions of skewness and flatness factors .................................... 83
Figure 5.25. Distributions of skewness and flatness factors in wall coordinates ............................. 84
Figure 5.26. Dimensionless distributions of skewness and flatness factors for all runs .............. 87
Figure 5.27. Nonlinear regression of mean velocity distribution ....................................................... 89
Figure 5.28. Dimensionless Reynolds shear stress distribution ........................................................ 90
Figure 5.29. Reynolds shear stress distribution in wall coordinates ................................................ 92
Figure 5.30. Eddy viscosity distribution ............................................................................................ 93
Figure 5.31. Mixing length distribution ............................................................................................ 95
Figure 5.32. Schematic flow pattern of cellular secondary currents in narrow open channels ... 96
Figure 5.33. Power density spectra ................................................................................................. 100
Figure 5.34. Typical dye-line patterns of wall turbulence events ...................................................... 103
Figure 5.35. Scaling of ejection frequency ................................................................. 104
Figure 5.36. Ejection frequency versus friction velocity ........................................... 106
Figure 5.37. Scaled ejection periods for all runs ......................................................... 107
Figure 5.38. Scaled bursting period versus shear Reynolds number ......................... 110
Figure 5.39. Scaled bursting period versus flow Reynolds number ......................... 111
Figure 5.40. Inflow and eddy .................................................................................. 113
Figure 5.41. Lift-up ................................................................................................. 115
Figure 5.42. Formation of streamwise vortex .......................................................... 116
Figure 5.43. Formation of secondary streamwise vortex ........................................... 118
Figure 5.44. Streamwise vortex near interface ......................................................... 119
Figure 5.45. Lateral movement of traces .................................................................. 120
Figure 5.46. Evolution of three ejections ................................................................. 122
Figure 5.47. Collision of sweep with shear layer ...................................................... 124
Figure 5.48. Formation of spanwise vortex .............................................................. 125
Figure 5.49. Ascension of spanwise vortex from shear layer .................................... 126
Figure 5.50. Spanwise vortex near interface ............................................................. 127
Figure 5.51. Counter-rotating spanwise vortices from shear layer ......................... 128
Figure 5.52. Formation of hairpin vortex ................................................................. 129
Figure 5.53. Oscillation of hairpin vortex ................................................................. 130
Figure 5.54. Streamwise acceleration by upwelling ............................................... 132
Figure 5.55. Upwelling near interface .................................................................... 133
Figure 5.56. Collision of upwellings ....................................................................... 134
Figure 5.57. Formation of spiral eddy .................................................................... 136
Figure 5.58. Counter-clockwise rotation of spiral eddy near interface ..................... 138
Figure 5.59. Spiral eddy .......................................................................................... 139
Figure 5.60. Descension of downdraft to wall ................................................................. 140
Figure 5.61. Deflection of downdraft off shear layer ......................................................... 141
Figure A.1. Variance of density with temperature for Shell – Sol 715 ............................... A-1
Figure A.2. Variance of dynamic (absolute) viscosity with temperature for Shell – Sol 715 ... A-2
Figure A.3. Wake strength parameter as a function of Reynolds number ............................. A-3
Figure A.4. Scaled ejection frequency versus Reynolds number (from Nasr-Esfahany, 1998) A-4
Figure A.5. Mean bursting period, scaled by inner variables, versus Reynolds number (from Nezu and Nakagawa, 1993) ................................................................................. A-5
NOMENCLATURE

Unless stated otherwise, numbers in parentheses after the description refer to the equation in which the symbol is first used or defined. Dimensions are given according to the International System of Units (SI). Symbols that appear infrequently are not listed.

a Parameter of hyperbolic function (5.13), dimensionless
A Integration constant (2.29), dimensionless
A Cross-sectional area of flow (4.4), m²
b Channel width (4.1), m
b Parameter of hyperbolic function (5.13), dimensionless
B Damping factor (2.27), dimensionless
c Velocity of elementary surface wave (5.4), m/s
C Constant (2.38), dimensionless
C_k Constant (2.37), dimensionless
C_\text{xy}(\tau) Cross-covariance function (5.6)
D_u Constant (2.37), dimensionless
D_h Hydraulic diameter (4.5), m
f_s Actual sampling frequency (5.15), 1/s
f_{s,\text{theoretical}} Theoretical sampling frequency (5.16), 1/s
f_{\text{max}} Maximum response frequency (5.18), 1/s
F(u^{'}) Flatness factor (5.12), dimensionless
g Gravitational acceleration (5.4), m/s²
G Rate of turbulent energy generation (2.13), m²/s³
h Flow depth (2.12), m
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Turbulence intensity (2.4), dimensionless</td>
</tr>
<tr>
<td>$k$</td>
<td>Turbulent kinetic energy (2.19), m$^2$/s$^2$</td>
</tr>
<tr>
<td>$k_{\text{max}}$</td>
<td>Maximum wave number (5.18), 1/m</td>
</tr>
<tr>
<td>$L$</td>
<td>Threshold value (5.21), dimensionless</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Macroscale (5.19), m</td>
</tr>
<tr>
<td>$n$</td>
<td>Order of moving average (5.5), dimensionless</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of observations (5.5), dimensionless</td>
</tr>
<tr>
<td>$N_{u_i}$</td>
<td>Number of times that amplitude of velocity fluctuation occurs (5.10), dimensionless</td>
</tr>
<tr>
<td>$\dot{N}_F$</td>
<td>Frame rate of video camera (5.16), fps</td>
</tr>
<tr>
<td>$\overline{N}_F$</td>
<td>Number of frames required to calculate velocity profile (5.16), dimensionless</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure fluctuation (2.18), N/m$^2$</td>
</tr>
<tr>
<td>$P$</td>
<td>Wetted perimeter (4.4), m</td>
</tr>
<tr>
<td>$P(u')$</td>
<td>Probability of amplitude of velocity fluctuation (5.10), dimensionless</td>
</tr>
<tr>
<td>$P_D$</td>
<td>Pressure energy diffusion (2.13), m$^3$/s$^3$</td>
</tr>
<tr>
<td>$R_h$</td>
<td>Hydraulic radius (4.4), m</td>
</tr>
<tr>
<td>$R_{xy}(\tau)$</td>
<td>Cross-correlation function (5.7)</td>
</tr>
<tr>
<td>$s$</td>
<td>Scaling factor (4.6), dimensionless</td>
</tr>
<tr>
<td>$S(u')$</td>
<td>Skewness factor (5.11), dimensionless</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (2.1), s</td>
</tr>
<tr>
<td>$t_i$</td>
<td>Integration limit (2.2), s</td>
</tr>
<tr>
<td>$T$</td>
<td>Period of record (5.6), s</td>
</tr>
<tr>
<td>$T_D$</td>
<td>Turbulent energy diffusion (2.13), m$^3$/s$^3$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\bar{T}_E$</td>
<td>Average period of ejection (5.22), s</td>
</tr>
<tr>
<td>$\bar{T}_B$</td>
<td>Average period of burst (5.23), s</td>
</tr>
<tr>
<td>$\bar{T}_S$</td>
<td>Average period of sweep (5.23), s</td>
</tr>
<tr>
<td>$\bar{T}_E^*$</td>
<td>Average period of ejection scaled by inner variables (5.22), dimensionless</td>
</tr>
<tr>
<td>$\bar{T}_B^*$</td>
<td>Average period of burst scaled by inner variables (5.24), dimensionless</td>
</tr>
<tr>
<td>$u$</td>
<td>Instantaneous streamwise velocity (2.1), m/s</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>Mean streamwise velocity (2.1), m/s</td>
</tr>
<tr>
<td>$u'$</td>
<td>Fluctuating streamwise velocity (2.1), m/s</td>
</tr>
<tr>
<td>$u^*$</td>
<td>Friction velocity (2.20), m/s</td>
</tr>
<tr>
<td>$u^{*\infty}$</td>
<td>Mean streamwise velocity normalized by inner scales (2.21), dimensionless</td>
</tr>
<tr>
<td>$\bar{u}_{\text{Bulk}}$</td>
<td>Bulk mean velocity (2.4), m/s</td>
</tr>
<tr>
<td>$u_{\text{Max}}$</td>
<td>Maximum streamwise velocity (5.23), m/s</td>
</tr>
<tr>
<td>$u_{\text{RMS}}$</td>
<td>Root-mean-square of fluctuating streamwise velocity (2.3), m/s</td>
</tr>
<tr>
<td>$v'$</td>
<td>Fluctuating vertical velocity (2.7), m/s</td>
</tr>
<tr>
<td>$V_D$</td>
<td>Viscous diffusion (2.13), m²/s³</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Viscous term (2.32), dimensionless</td>
</tr>
<tr>
<td>$w(y/h)$</td>
<td>Coles' wake function (2.32), dimensionless</td>
</tr>
<tr>
<td>$x$</td>
<td>Streamwise distance (2.1), m</td>
</tr>
<tr>
<td>$x(t)$</td>
<td>Time history record (5.6)</td>
</tr>
<tr>
<td>$\bar{x}_{\text{Mov}}$</td>
<td>Moving average (5.5)</td>
</tr>
<tr>
<td>$y$</td>
<td>Vertical distance (2.1), m</td>
</tr>
<tr>
<td>$y(t)$</td>
<td>Time history record (5.6)</td>
</tr>
</tbody>
</table>
\[ y^* \quad \text{Vertical distance normalized by inner scales (2.22), dimensionless} \]

\[ z \quad \text{Spanwise distance (2.1), m} \]

\[ \ell \quad \text{Mixing length (2.10), m} \]

\[ \ell^* \quad \text{Mixing length normalized by inner scales (2.25), dimensionless} \]

\textbf{Overlines}

\[ \~ \quad \text{Time-smoothed (2.1)} \]

\[ \bar{\dots} \quad \text{Time rate (5.16)} \]

\textbf{Superscripts}

\[ \cdot \quad \text{Perturbation from average value (2.1)} \]

\[ \cdot \quad \text{Normalized by inner scales (2.21)} \]

\[ \cdot \quad \text{Normalized by inner scales (2.21)} \]

\textbf{Subscripts}

\[ \text{B} \quad \text{Burst (5.23)} \]

\[ \text{Bulk} \quad \text{Bulk mean velocity (2.4)} \]

\[ c \quad \text{Critical (4.2)} \]

\[ D \quad \text{Diffusion (2.13)} \]

\[ D \quad \text{Based on hydraulic diameter (5.2)} \]

\[ E \quad \text{Ejection (5.22)} \]

\[ F \quad \text{Frame (5.16)} \]

\[ h \quad \text{Hydraulic (4.3)} \]

\[ h \quad \text{Based on flow depth (5.1)} \]

\[ \text{lamin} \quad \text{Laminar shear stress (2.5)} \]

\[ \text{max} \quad \text{Maximum (5.17)} \]
Moving average (5.5)

Root-mean-square (2.3)

Sampling (5.15)

Theoretical sampling (5.16)

Sweep (5.23)

Turbulent (2.8)

Total shear stress (2.5)

Turbulent shear stress (2.5)

Streamwise velocity (2.37)

Fluctuating streamwise velocity (5.10)

Wall (2.5)

Time history record (5.6)

Two different time history records (5.6)

Time history record (5.6)

Aspect ratio of flow (4.1), dimensionless

Critical aspect ratio of flow (4.2), dimensionless

Difference (4.6)

Turbulent dissipation (2.13), m²/s³

van Driest damping function (2.27), dimensionless

von Karman constant (2.26), dimensionless

Dynamic viscosity (2.6), Ns/m²

Mean value of time history record (5.6)
\( \mu_y \)  Mean value of time history record (5.6) \\
\( \nu \)  Kinematic viscosity (2.16), m\(^2\)/s \\
\( \nu_i \)  Eddy viscosity (2.8), m\(^2\)/s \\
\( \pi \)  Constant (2.32), dimensionless \\
\( \Pi \)  Coles' wake strength parameter (2.32), dimensionless \\
\( \rho \)  Weight density (2.7), kg/m\(^3\) \\
\( \tau \)  Time delay (5.6), s \\
\( \tau_{yn(lam)} \)  Laminar shear stress (2.5), N/m\(^2\) \\
\( \tau_{yn(turb)} \)  Turbulent shear stress (2.5), N/m\(^2\) \\
\( \tau_{yn(total)} \)  Total shear stress (2.5), N/m\(^2\) \\
\( \tau_w \)  Wall shear stress (2.6), N/m\(^2\) \\
\( \xi \)  Vertical distance normalized by outer scales (2.24), dimensionless \\

Dimensionless Groups \\
\( Fr \)  Froude number (5.4) \\
\( Re' \)  Reynolds number based on friction velocity (2.28) \\
\( Re_h \)  Reynolds number based on flow depth (5.1) \\
\( Re_D \)  Reynolds number based on hydraulic diameter (5.2)
1.1 Introduction

Extensive research on the nature of boundary-layer turbulence in single-phase flow has lead to the discovery of quasi-periodic patterns of coherent structures. These organized vortex patterns undergo a regular chain of events, commonly termed a turbulent burst. Appearing as a streak, the turbulence structure lifts away from the bottom wall, oscillates and breaks up from the instability. In the process, low-speed fluid is ejected into the outer flow. An insweep of high velocity fluid from the bulk flow to replace the ejected fluid completes the cycle. It is the most important coherent structure associated with the production of Reynolds stress and turbulent energy. Although the general characteristics of these turbulence structures are established, questions regarding their evolution and interactions at the free surface are still unanswered.

New developments indicate that upwellings originate from the impingement on to the free surface of bursts rising from the channel bottom. After forming surface patches, these upsurging spanwise vortices roll back from the interface and mix into the bulk flow in the form of downdrafts. Spiral eddies are generated at the edge of upwellings and will interact with each other and slowly dissipate, unless destroyed by other upwellings. Recently, streamwise vortices are suspected to rise from the bottom wall to the free surface.

A free surface implies that shear is not imposed atop the liquid layer. An example of the application of free-surface turbulence is in the exchange of carbon dioxide and oxygen between the atmosphere and ocean, a concern to long-term climate dynamics. For most examples of scalar transfer across the gas-liquid interface, the liquid flow is turbulent and the transport process is controlled by the transfer coefficient on the liquid side of the surface. Thus, insight into free-surface turbulence will advance the understanding of interfacial heat and mass transfer. The present findings have profound implications to thermal pollution and reaeration of oxidation ponds and rivers, and industrial relevance to contacting equipment like gas absorbers and
evaporators. In addition, the global position of marine vessels is determined by radar reflection from their wake signatures on the ocean surface. Difficulties in experimental measurements and computer simulations have prohibited the understanding of the structures near the free surface until lately.

In the last few decades, a variety of investigative techniques have been employed to explore the mean flow and turbulence structures in a horizontal channel. Bar none, this canonical flow is the simplest situation and best understood. Direct Numerical Simulation (DNS) and Large-Eddy Simulation (LES) are at the forefront of numerical simulations of organized structures in open-channel flow. Hot-wire (HWA) or Hot-film Anemometry (HFA), Laser Doppler Anemometry (LDA) and the hydrogen or oxygen bubble technique are among the most popular experimental methods to study the turbulent flow structure.

The instantaneous information from photochromic dye activation (PDA) betters the conventional probes of hot film anemometry (HFA) where the typical averaging technique of such would mask the intercorrelations between velocity components which connote turbulent coherent motions. Nonetheless feasible, the determination of the tracer position in space and time by flow visualization is difficult.

Insight into the local structure of turbulence underneath a fluid-fluid interface has spawned interest in the chemical industry, and environmental and geophysical sciences on account of the significant role that the turbulence structure plays in the transport of mass, momentum and heat between the phases. With a deeper knowledge of the mechanisms that control turbulence at the free surface, mathematical modelling of interfacial transport phenomena can be developed with greater accuracy.

The objective of the present work is to investigate the evolution in space and time of the free-surface turbulence structures through the determination of the instantaneous and mean velocity profiles and statistics on the velocity fluctuations near the interface. Also, flow
visualization will be used to explore the turbulent flow structure in the vicinity of the liquid surface. Theories and numerical predictions of the organized motions in open-channel flows from the scientific literature will precede the explanation of the current experimental work.

1.2 Scope

This thesis will examine coherent structures of turbulence in open-channel flow, elicited from the statistical analysis of instantaneous streamwise velocities and flow visualization. The statistical theory of turbulence structures in two-dimensional open-channel flow is reviewed in Chapter 2. A brief introduction and classification of large-scale coherent motions near and away from the bed of open-channel flow is given in Chapter 3. Conceptual illustrations are included to clarify the interaction between the bursting motions and large-scale vortical motions observed over a wide range of Reynolds numbers in steady flow. Error analysis and the experimental conditions are detailed in Chapter 4. In Chapter 5, the present experimental results are discussed and compared to past experimental data and numerical simulations. The summary of Chapter 6 recaps the major findings of this research.
The two-dimensional flow structure of uniform and fully-developed flows in open channels has been analyzed from the statistical theory of turbulence. The knowledge gained out of this standpoint has advanced the concepts and numerical models of open-channel turbulence. Johns et al. (1975), Akai et al. (1981), Fabre et al. (1983), and Murata et al. (1991) made measurements of turbulence statistics.

2.1 Nomenclature and fundamental concepts

In an open channel, a no-slip wall at the bottom and a free surface at the top bound the flow. This simple flow configuration is often taken as the basis for numerical and experimental work because a steady state is feasible at which the fluid depth and root-mean-square (RMS) of the fluctuating velocity, \( \sqrt{u''^2} \), are invariant with time, \( t \). The uniform and fully-developed flow is driven by a streamwise pressure gradient. Common names, symbols and terminology used in the Cartesian coordinate system hereafter are shown in Figure 2.1. The longitudinal direction with the origin at the channel entrance is designated by \( x \), the spanwise direction with the origin at the channel center by \( z \), and the wall-normal direction with the origin at the channel bed by \( y \). The instantaneous velocity, \( u \), is decomposed into the mean velocity, \( \bar{u} \), and turbulent fluctuating components, \( u' \):

\[
    u = \bar{u}(x, y, z) + u'(x, y, z, t)
\]  

[2.1]

Hereinafter, an overbar means an average over \( x \) and \( t \) and a prime means perturbation from this average. Now, the foregoing time-averaged velocity or first moment is evaluated from

\[
    \bar{u} = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} u(x, y, z, t) dt,
\]

[2.2]
<table>
<thead>
<tr>
<th>tensor subscript</th>
<th>axis/direction</th>
<th>axis name</th>
<th>view name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>streamwise</td>
<td>end</td>
</tr>
<tr>
<td></td>
<td></td>
<td>axial</td>
<td>cross</td>
</tr>
<tr>
<td></td>
<td></td>
<td>longitudinal</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>y</td>
<td>wall-normal</td>
<td>plan</td>
</tr>
<tr>
<td></td>
<td></td>
<td>vertical</td>
<td>top</td>
</tr>
<tr>
<td></td>
<td></td>
<td>up</td>
<td>over</td>
</tr>
<tr>
<td></td>
<td></td>
<td>out</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>z</td>
<td>spanwise</td>
<td>side</td>
</tr>
<tr>
<td></td>
<td></td>
<td>tranverse</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>lateral</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.1. Geometrical terminology for coordinate system (from Robinson, 1991).
where the integration limit, \( t_i \), is much longer than the period of any fluctuation or time scale of the turbulence. Whereas the turbulent fluctuations have a zero arithmetic mean, i.e., they are as often positive as they are negative on the average, the root-mean-square (RMS) value or second moment of the fluctuations is nonzero. The RMS of the streamwise velocity fluctuation is

\[ u'_{\text{RMS}} = \sqrt{u'^2(x, y, z, t)}. \]  

[2.3]

It is known that the RMS value is a critical quantity upon which the rates of heat and mass transfer depend. The turbulence intensity, \( I \), is defined as

\[ I \equiv \sqrt{\frac{u'^2 + v'^2 + w'^2}{3}}, \]  

[2.4]

with \( \overline{u}_{\text{bulk}} \) as the bulk mean velocity of the incompressible flow. Henceforth, the turbulence intensity will be synonymous with the RMS of velocity fluctuations. The RMS is sometimes dubbed the quadratic mean.

In turbulent flow, the total shear stress, \( \tau_{yx(\text{total})} \), has a laminar, \( \tau_{yx(\text{lam})} \), and turbulent, \( \tau_{yx(\text{turb})} \), component:

\[ \tau_{yx(\text{total})} = \tau_{yx(\text{lam})} + \tau_{yx(\text{turb})}. \]  

[2.5]

Next to the wall in parallel, laminar flow, the shear stress, \( \tau_{yx(\text{lam})} \), is related to the derivative of the velocity:

\[ \tau_{yx(\text{lam})} = \tau_w = \mu \frac{du}{dy}. \]  

[2.6]

The turbulent stress, \( \tau_{yx(\text{turb})} \),

\[ \tau_{yx(\text{turb})} = -\rho \overline{u'v'}, \]  

[2.7]

is called the Reynolds stress and is solely expressed by the fluctuating flow properties which can not be solved for analytically. A simple theoretical model for the turbulent shear stress in two-dimensional turbulent flow is the eddy viscosity analogy by Boussinesq (1877):
The eddy viscosity, denoted by \( \nu \), differs with flow condition and will vary in space for that flow. Alternatively, the turbulent viscosity, \( \nu_t \), is given by

\[
\nu_t = -\frac{\bar{u}' \bar{v}'}{\bar{u}' / \bar{v}'}.
\]  

Another phenomenological theory to relate the turbulent stress to the average velocity is the mixing-length hypothesis proposed by Prandtl (1925):

\[
\tau_{yx(turb)} = \rho \ell^2 \left| \frac{\bar{d}u}{\bar{d}y} \right| \frac{\bar{d}u}{\bar{d}y}.
\]

Be wary that the momentum mixing length, \( \ell \), is also a property of the flow and not of the fluid. This model lacks generality by excluding the effects of flow history and transport on the turbulence. Comparison with the prior equation for the eddy viscosity yields

\[
\nu_t = \ell^2 \left| \frac{\bar{d}u}{\bar{d}y} \right|.
\]

Equipped with the Reynolds stress from a turbulence model, the velocity field can be constructed by solving the two-dimensional forms of the continuity and streamwise momentum equations.

2.2 Mean-Flow Equations

2.2.1 Mean-flow energy budget

For fully-developed, two-dimensional flow in a smooth open channel, the total shear stress, \( \tau_{yx(total)} \), can be written as

\[
\tau_{yx(total)} = \rho \nu \frac{\bar{d}u}{\bar{d}y} - \rho u' \bar{v}' = \tau_w \left( 1 - \frac{y}{h} \right),
\]

which is obtained from the Navier-Stokes equations of motion. Equation [2.12] specifies that the total shear stress, \( \tau_{yx(total)} \), varies linearly from the value, \( \tau_w \), at the wall (\( y=0 \)) to zero at the free
surface \((y=h)\). Recall that the exchange of momentum in the former term, better known as Newton’s law of viscosity, results from random molecular action while that in the latter is a consequence of macroscopic turbulent fluctuations. The large-scale fluctuating properties of the flow are exclusively embodied in the turbulent shear stress, whereas the viscous stress encompasses a fluid property and the mean rate of shear strain.

For a two-dimensional and uniform flow in an open channel, the turbulent energy equation is given by

\[ G = \varepsilon + T_D + P_D + V_D, \]  

where

\[ G = \frac{\partial}{\partial y} \frac{d \tilde{u}}{d y}, \]  

\[ \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \]  

\[ \varepsilon_i = \nu \left( \frac{\partial \tilde{u}_i}{\partial x}^2 + \frac{\partial \tilde{u}_i}{\partial y}^2 + \frac{\partial \tilde{u}_i}{\partial z}^2 \right) > 0 \text{ for } i = 1,2,3, \]  

\[ T_D = \frac{\partial}{\partial y} \left( \frac{1}{2} \left( u^2 + v^2 + w^2 \right) v \right), \]  

\[ P_D = \frac{\partial}{\partial y} \left( \frac{P}{\rho} v \right), \]  

\[ V_D = -\nu \frac{\partial^2 k}{\partial y^2}. \]

The generation rate of turbulent energy, \(G\), is realized at the expense of the mean-flow which works against the wall shear stress. Turbulent fluctuations associated with large-scale eddies are produced and turbulent energy is conveyed from the large-scale eddies to smaller-scale eddies through the energy-cascade process, ultimately dissipating into heat by molecular viscosity. The turbulent production is balanced on the right-hand side of equation [2.13] by the total turbulent
dissipation, $\varepsilon$, turbulent energy diffusion, $T_D$, pressure energy diffusion, $P_D$, and viscous diffusion, $V_D$.

### 2.2.2 The law of the wall

In the wall region of $\xi = y/h < 0.2$, the characteristic scales of length and velocity are the viscous length, $v/u^*$, and friction velocity, $u^*$, respectively. The friction velocity, $u^*$, by which the inner parameters are scaled is not really a velocity of the fluid but merely a quantity with the units of velocity:

$$u^* = \sqrt{\frac{\tau_w}{\rho}}. \quad [2.20]$$

Often, the x-component of mean velocity and y-coordinate are normalized by the friction velocity, $u^*$, and viscous length, $v/u^*$, enabling the velocity profile to be written in dimensionless form:

$$u^* \equiv \frac{u}{u^*} ; \quad [2.21]$$

$$y^* \equiv \frac{yu^*}{v}. \quad [2.22]$$

The substitution of the mixing-length formulation \[2.10\] into the total shear stress relation \[2.12\] yields a velocity gradient,

$$\frac{du^*}{dy^*} = \frac{2(1-\xi)}{1 + \sqrt{1 + 4\ell^2(1-\xi)}}, \quad [2.23]$$

which can be integrated to give the dimensionless velocity distribution insofar as the mixing-length, $\ell^*$, distribution is known. $\xi$ is the dimensionless vertical distance from the channel bed and $\ell^*$ is the nondimensional equivalent of the mixing-length, $\ell$:

$$\xi = \frac{y}{h} ; \quad [2.24]$$

$$\ell^* = \frac{\ell u^*}{v}. \quad [2.25]$$
To account for viscous effects in the near-wall region, the premise of a linear mixing length distribution, $\ell = \kappa y$, is modified with the van Driest (1956) damping function, $\Gamma(y^*)$:

$$\ell^* = \kappa y^* \cdot \Gamma(y^*); \quad [2.26]$$

$$\Gamma(y^*) = 1 - \exp(-y^*/B). \quad [2.27]$$

For open-channel flow, Nezu and Rodi (1986) reported a damping factor, $B = 26$, in the van Driest damping function, $\Gamma(y^*)$. $\kappa$ is the von Karman constant.

Therefore, from the integration of the dimensionless velocity gradient [2.23] using the dimensionless mixing length distribution [2.26], the standard equations pertaining to the law of the wall are the viscous-sublayer formula,

$$u^* = y^* \quad \text{for } y^* \ll B \quad [2.28]$$

and the log-law,

$$u^* = \frac{1}{\kappa} \ln(y^*) + A \quad \text{for } B < y^* < 0.2 \text{Re}^*, \quad [2.29]$$

where the von Karman constant, $\kappa$, and constant of integration, $A$, have been widely accepted through experimental verification as 0.412 and 5.287, respectively. The shear Reynolds number, $\text{Re}^*$, is based on the depth of the liquid stream, friction velocity at the no-slip wall and kinematic viscosity:

$$\text{Re}^* = hu^*/\nu \quad [2.30]$$

For the buffer layer of $5 < y^* < 30$, the van Driest mixing length model [2.26] can be similarly inserted into the dimensionless velocity gradient [2.23] and integrated to get the velocity distribution.

2.2.3 The velocity-defect law

In the outer region, $y/h > 0.2$, predictions of the mean velocity distribution with the log-wake law has been proven reliable. A wake function $w(y/h)$ added to the log-law [2.29] of the
wall region corrects for the systematic departure from the standard distribution at the free surface for high Reynolds numbers:

\[
 u^* = \frac{1}{\kappa} \ln(y^*) + A + w(y/h) \tag{2.31}
\]

The empirical wake function \( w(y/h) \) from Coles (1956) is generally accepted as the most suitable extension of the log-law [2.29]:

\[
 w(y/h) = \frac{2\Pi}{\kappa} \sin^2 \left( \frac{\pi y}{2h} \right), \tag{2.32}
\]

where \( \Pi \) is Coles' wake strength parameter that expresses the deviation from the log profile [2.29] in the outer region.

### 2.3 Reynolds shear stress

By manipulating [2.12] and [2.31], the theoretical Reynolds stress distribution is rendered:

\[
 -u'v'/u'' = (1 - y/h) - V_t(y/h). \tag{2.33}
\]

The viscous term, \( V_t(y/h) \), is negligible at large Reynolds numbers, \( \text{Re}^* \):

\[
 V_t(y/h) = \frac{1}{\kappa \text{Re}^*} \left[ \left( \frac{y}{h} \right)^{-1} + \pi \Pi \sin \left( \frac{\pi y}{h} \right) \right]. \tag{2.34}
\]

### 2.4 Eddy viscosity

Rearranging and rewriting equations [2.9], [2.12] and [2.31], the theoretical curve for the eddy viscosity, \( v_t \), is

\[
 \frac{v_t}{hu^*} = \frac{\kappa (1 - y/h) - \kappa V_t}{(h/y) + \pi \Pi \sin(\pi y/h)}. \tag{2.35}
\]

### 2.5 Mixing length

In an analogous fashion, the theoretical mixing-length is described by
Nezu and Nakagawa (1993) provide clarification of the equations in Sections 2.2 to 2.4 in their monograph. Equation [2.36] was derived by Nezu and Rodi (1986).

2.6 Sub-division of the open-channel flow

Like boundary layer and closed-channel flows, open-channel flows consist of two regions classified according to the relative strengths of the viscous and turbulent contributions to the total shear stress, $\tau_{yx(\text{total})}$. A schematic diagram of the inner region and outer region is sketched in Figure 2.2.

2.6.1 Wall region

The near-wall region is controlled by the inner variables, the kinematic viscosity, $\nu$, and the friction velocity, $u^*$. Stemming from the experimental observation that the total shear stress, $\tau_{yx(\text{total})}$, is almost constant and equal to the viscous shear, $\tau_w$, in the wall region of the open-channel flow field, this region is bounded by $0<y/h<0.2$. In conformity to the 'inner layer' of the turbulent boundary layer, three distinct regions are quite apparent in the wall region or inner region: (i) a laminar sublayer, (ii) a buffer intermediary and (iii) a turbulent layer. The viscous sublayer is the subdivision of the inner region controlled by viscous shear. Adversely, in the fully turbulent constant-stress layer of the wall region, the turbulent part is dominant. A transitional buffer layer of equal viscous and Reynolds stresses lies between the two subregions.

The relevant velocity scale for the wall region is the friction velocity, $u^*$, which has the same order of magnitude as the RMS value of the velocity fluctuations, $u_{\text{RMS}}$, and the appropriate length scale is the inner length scale, $\nu/u^*$. Consequently, if the law of the wall of the inner flow field is satisfied, then the turbulent structure is governed by these inner variables. It is in this region that the rate of turbulent energy production, $G$, exceeds the dissipation rate, $\varepsilon$. 

$$\frac{\ell}{h} = \frac{\kappa \sqrt{1 - \xi \Gamma(\xi)}}{1/\xi + \pi \Gamma \sin(\pi \xi)^{}}. \quad [2.36]$$
Figure 2.2. Sub-division of the flow field in open channel (from Nezu and Nakagawa, 1993).
owing to the occurrence of bursts. These flow structures are of immense importance as will be seen in the succeeding chapter on coherent structures.

2.6.2 Intermediate region

Within $0.2 \leq y/h \leq 0.6$, the intermediate region exists, to which a characteristic length scale of $y$ and velocity scale of $(\tau/\rho)^{1/2}$ are applicable. Neither the wall nor the free surface exercises a strong influence on this region. This is where turbulent energy extracted from the mean flow is transferred to smaller-scale eddies. The turbulent energy budget is also roughly in equilibrium; the production rate, $G$, is balanced by the dissipation rate, $\varepsilon$.

2.6.3 Free-surface region

The free-surface region stretches across $0.6 < y/h \leq 1$. Herein, turbulent structures are dictated by the outer variables of length and velocity scales particularly given by the flow depth, $h$ and the maximum mainstream velocity, $u_{\text{max}}$. The preceding log-wake law [2.31] is applied in this region. Unlike the inner region, the turbulent generation rate, $G$, is smaller than the dissipation rate, $\varepsilon$, and turbulent energy from the wall diffuses toward the surface. The free surface exclusively affects this outer region. Of fascination is the damping of the vertical turbulence intensity, $v_{\text{RMS}}$, and other turbulence characteristics in proximity to the interface seen by Komori et al. (1982), and Celik and Rodi (1984). Solely on this outer part does the shear-free surface exert an influence. Together, the intermediate and free-surface regions combine to assemble the 'outer region' or 'outer layer'.

It is resolved that surface waves affect turbulent structure. Nezu and Rodi (1986) claimed that waves amplify the turbulence intensity near the free surface as demonstrated by the scatter of experimental data. Stamatiou (1998) examined the effect of surface waves on the turbulence structures of the present open-channel flow and his results should be consulted.
2.7 Universal function for turbulence intensity

Lacking a model, Nezu (1977) devised a universal function for the turbulence intensity, $u'_{\text{RMS}}$, distribution from a phenomenological perspective:

$$\frac{u'_{\text{RMS}}}{u'} = D_0 e^{-C_s \xi}. \quad [2.37]$$

Data collected with hot-film anemometry by Nezu (1977) unveil the empirical constants $D_0=2.30$ and $C_s=1.0$ in the semi-theoretical exponential function to be independent of the Reynolds and Froude numbers. This longitudinal turbulence intensity relation is exclusive to the equilibrium region $50<y^*<0.6Re^*$. Near the wall, $y^*<50$, in which the turbulence generation, $G$, outweighs the dissipation, $\varepsilon$, as a result of the bursting phenomena, the empirical formula,

$$\frac{u'_{\text{RMS}}}{u'} = Cy^* \quad [2.38]$$

with $C=0.3$ holds. To span the whole flow depth, Nezu (1977) adopted a weighted average of the cited expressions in [2.37] and [2.38] along with the damping function $\Gamma(y^*)$ of [2.27] to get

$$\frac{u'_{\text{RMS}}}{u'} = D_0 e^{-y^*/Re^*} \Gamma(y^*) + Cy^* (1 - \Gamma(y^*)), \quad [2.39]$$

where the constants, $D_0=2.30$ and $C=0.3$, are fixed as in [2.37] and [2.38]. In this instance, the damping coefficient, $B$, of the damping function, $\Gamma(y^*)$, [2.27] is chosen as 10.
3.1 Vortices

Despite the intuitive reference to eddying motions in real fluid flows as 'vortices', the definition of the term in classical hydrodynamics deserves mentioning. The concept of a vortex is comprehensible in an ideal fluid by virtue of the sharp boundary separating the rotational and irrotational regions of the flow. This boundary becomes less distinct for viscous fluids. However, vorticity being the curl of the velocity field is equally applicable to both real and ideal fluids and may be thought of as the propensity of the fluid to rotate. When the incident stream of velocity, \( u \), contacts the wall, vorticity is generated on the surface in compliance with the 'no-slip' boundary condition. This vortex motion diffuses away from the viscous layer as the fluid progresses downstream.

Akin to a cascade of energy, the turbulent energy of a larger eddy undergoing decay is distributed to smaller eddies. Conversely, the turbulent energy from the coalescence of large-scaled eddies is accumulated by the large eddy.

3.2 Coherent structures

Coherent structures are fluid parcels that have a life cycle of birth, growth, interaction and breakdown. Other labels tagged to these eddies or vortices are organized motion and ordered motion. Robinson (1991) generalized the definition of coherent motion to a three-dimensional region of the flow over which at least one fundamental flow variable (velocity component, density, temperature, etc.) exhibits significant correlation with itself or with another variable over a range of space and or time that is significantly larger than the smallest local scales of the flow. The definition of coherent eddy structures by Blackwelder (1988) runs similar.

Notwithstanding their life cycle, coherent structures are not deterministic in the strictest sense but behave dynamically. Bursting motions are not perfectly periodic in space and time.
Their size, orientation and convection velocity are random. Functionally, their motion is quasi-periodic or quasi-organized. Statistical analysis sans a detecting condition cannot conclusively identify coherent eddies.

Turbulence energy is removed from the mean flow by the interactions of large-scale eddies so coherent eddies contain the bulk of this energy and therein lies their significance to fluid mechanics and hydraulics.

### 3.3 Classification of coherent structures in open-channel flows

According to Nezu and Nakagawa (1993), organized structures in rivers are divided into two categories: (i) bursting phenomena which preserve turbulent shear flows near walls and (ii) large-scale vortical motions that occur periodically.

#### 3.3.1 Bursting phenomena

A burst is a cycle of ejections and sweeps whereby the low-speed streaky structures lying next to the wall mixes with the outer flow region. In the ejection process, a low-speed parcel of fluid goes through a chain of events: (i) lift-up from the wall, (ii) oscillation in three dimensions and (iii) break-up. Some portions of the low-speed fluid are expelled into the outer flow for the last step (iii). The motion of a high-speed parcel of fluid to remove the low-speed fluid that remains at the wall constitutes the sweep or inrush process. These phenomena near the channel floor, $y^+ \leq 100$, are coherent although the bursting eddies emerge sporadically in space and time.

Kline et al. (1967) discovered from pressure gradient data that the bursting rate is intertwined with the production rate of turbulent kinetic energy and Reynolds stress. That is, turbulent bursts govern the generation of turbulence through shear at a solid surface and regulate transport phenomena. Inside the turbulent boundary layer, kinetic energy from the mean flow is transformed into turbulent fluctuations and thereafter converted into internal energy via viscous dissipation. They also conjectured that the violent ejection of fluid during a burst was the primary avenue for the conveyance of energy and other turbulent quantities to the outer parts of
the boundary layer. Within the viscous sublayer devoid of turbulent energy where $G<\varepsilon$, the sweep motion is key to the transfer of energy to the wall.

### 3.3.2 Large-scale vortical motions

Large-scale vortical motions are encountered at a greater distance away from the wall than common to bursting phenomena. Nezu and Nakagawa (1993) sorted large-scale vortical motions into six groups, of which only two types are germane: (i) kolk-boil vortices and (ii) secondary currents of Prandtl’s second kind. A kolk is a strong vortex with an upward motion that turns into a boil at the free surface. Furthermore, three classes of boils are acknowledged from their mechanism of generation near the bed. Boils of the third kind, which are pertinent to this study, are interrelated with the development of bursting motions and appear rather randomly in space and time for that reason. Corner and cellular currents are driven by turbulence in straight open channels as will be made clear in Section 5.1.10. Their irregular manifestation nullifies them as coherent structures. Thereupon, attention will be focussed on kolk-boil vortices affiliated with bursting phenomena. These boils were called surface-renewal eddies by Komori et al. (1988 and 1989).

### 3.4 Physical models of coherent structures

A conceptual model is an idealized account of the physical processes underlying the behaviour of coherent motions. Physical models have been hypothesized to accommodate the findings in laboratory flumes and geophysical flows like rivers and estuaries, covering a range of Reynolds numbers.

### 3.4.1 Bursting phenomena

Most ideas of the bursting process revolve around an inclined horseshoe- or hairpin-shaped vortex. This vortex model in all its forms has been named the $\Pi$-shaped eddy, $\Omega$-shaped eddy, $\Lambda$-shaped vortex and loop vortex. It will be addressed now as a horseshoe or hairpin vortex. Wallace (1985) envisaged the bursting phenomena of the viscous sublayer in Figure 3.1.
Figure 3.1. Conceptual model of bursting phenomena (from Wallace, 1985).
He adapted the horseshoe vortex model of Theodorsen (1955) to include the spatial characteristics of the bursting structure. Three-dimensional disturbances give rise to the movement of a spanwise vortex tube away from the wall. Flow visualization and conditional averages of velocities support the description of bursting motions as hairpin vortices.

The evolution of bursting motions as recounted by Nezu and Nakagawa (1993) is redrawn in Figure 3.2. For simplicity, the model is pictured to be symmetric, coherent and isolated from other events. The model starts with a low-speed streak that grows mainly in the streamwise and wall-normal directions by the ongoing accumulation of low-momentum fluid. A local adverse pressure gradient from the passage of a fluid with a substantial amount of vorticity brings on a local deceleration of the flow near the wall. Subsequently, the streak is unstable to the small local disturbances of the background turbulence in the outer region and oscillation in three dimensions is imminent. Eventually, the oscillation prompts the vorticity sheet that envelops the streak region to roll up over the top and sides of the streak in a shape much like a hairpin vortex. The lateral pressure gradient of the trailing legs is behind the mechanism that ejects low-momentum fluid away from the wall and perpetuates the streak region following ejection. Further, there is an upward motion of the legs and head of the hairpin from the induced velocity that each leg imposes upon the other.

It is worth mentioning the secondary streamwise vortices that are encountered outside of the counter-rotating legs of the hairpin vortices. These secondary vortical structures are generated by the inrush of fluid coming from the rotating action of the legs.

3.4.2 Large-scale vortical motions

To simply the coherent structures of wall turbulence, Nezu and Nakagawa (1993) proposed a physical model of both bursting and large-scale vortical motions titled double structure of turbulence. The bursting motion is the active component that produces turbulence and Reynolds stress while the large-scale vortical motion is the somewhat inactive component of
Figure 3.2. Generation of hairpin vortex (from Nezu and Nakagawa, 1993).
turbulence. In the model by Rashidi (1997), interfacial deformations and free-surface structures are interconnected to wall turbulence activity.

3.4.2.1 Double structure of turbulence

Turbulence in the outer region is said to have a double structure as portrayed in Figure 3.3. Elongated horseshoe or hairpin vortices originate from the wall region and extend into the outer flow at an angle of about 45° to the wall. The head and legs of these hairpin vortices evolve in this direction of maximum strain. On the other hand, large-scale vortical motions consist of random arrays of suchlike hairpin vortices and are inclined to the wall at an angle of 15° to 30°. The many heads of these vortices are aligned at an angle of about 20° to the wall to form a boundary between the high-speed and low-speed zones referred to as a high-speed front. In Figure 3.3, the boundary is represented by a dotted line through the heads of neighbouring hairpin vortices. The large-scale vortical motions are related to the history of the upstream flow and appear at a lower frequency. As indicated in Figure 3.3, the pre-existing large-scale vortical motions form a shear layer that along with sweeps stimulate the creation of hairpin vortices.

It is imaginable from Figure 3.3 that the legs of multiple hairpin vortices will coalesce and agglomerate streamwise vorticity. The legs constitute counter-rotating pairs of streamwise vortices. Concentrations of vorticity from coalescence enable the heads of hairpin vortices, which are transverse vortices, to strengthen and climb to the free surface as weak boils.

For geophysical flows of high Reynolds numbers, there is a marked contrast between small-scale and large-scale bursting motions. The initial hairpin vortex whose scale is smaller than that of the mean-flow is called a small-scale bursting motion. A large-scale bursting motion emerges from the agglomeration of these hairpin vortices. However, the bursts cannot be differentiated as such in laboratory flumes of low Reynolds numbers. An elaborate treatise on the subject is found in the book by Nezu and Nakagawa (1993). They revised the model by Smith (1984) so that large-scale vortical motions incite the formation of low-speed streaks and
Figure 3.3. Double structure of turbulence (from Smith, 1984; and Nezu and Nakagawa, 1993).
initiation of bursts and sweep motions toward the wall trigger the lift-up of the streaky structures. An inrush of high-speed fluid links the large-scale vortical motions to the bursting process. There is no effect of the outer variables in the original model.

3.4.2.2 Free-surface turbulence

As schematized in Figure 3.4, Rashidi (1997) provided a sketch of the burst-free surface interactions to characterize the bursting processes and their engagement with the free surface. This sketch illustrates recent observations of flow structures on the free surface omitted in the previous conceptual model by Rashidi and Banerjee (1990a).

From the figure, a low-speed streak of appreciable length is noticed near the wall. Rashidi and Banerjee (1990b) also saw streaky structures near a sheared interface and ascribed their generation to the shear rate. The differences in the boundary conditions are of lesser significance. On passing, higher momentum fluid lifts up the low-speed streak. Thereafter, a pair of longitudinal vortices straddling the streak is formed from the tendency of the lifted portion to roll. This vortex pair resembles the counter-rotating legs of a horseshoe or hairpin vortex whose head is inclined to the wall at a bigger angle than its legs. Likewise, Kaftori et al. (1994) attributed these helical motions to funnel shaped eddies that look like twisted ribbons. Exceeding a certain distance from the bottom wall, the hairpin vortex flutters in all directions and ejects wall fluid. Several ejections are undergone prior to the vortex loop losing its identity. In the course of transporting low momentum fluid, a spanwise vortex is generated that rolls with the flow. Nearing the free surface, the flow between the upsurging vortex and interface is accelerated in the streamwise direction. A raised surface and surface patches ensue from the splatter. In the downswing, the surface drops as the spanwise eddy is reflected off the surface and heads back to the wall. The reader is referred to Rashidi (1997) for an excellent explanation.

Three distinctive structures become visible at the free surface atop regions of turbulence generation: (i) upwellings or surface-renewal eddies, (ii) downdrafts and (iii) spiral eddies.
Figure 3.4. Conceptual illustration of burst-interface interactions (from Rashidi, 1997).
Bursting motions emanate from the bottom wall to bring about the patches of the interface and roll back from the free surface during the inflow. Controversy surrounds the activation of the quasi-two-dimensional surface vortices. Spiral eddies may be mistaken for attached vortices in view of the weak downflow of the cores. Pan and Banerjee (1995) envisioned a conceptual mechanism for the origin and extinction of a spiral eddy. Horizontal vorticity near the surface results from the impingement of the surface by an ascending upwelling. This coherent structure carries spanwise vorticity to the interface. The reorientation of the surface-parallel components of vorticity into the surface-normal direction at the edge of the upwelling introduces instability and high shear into the flow, encouraging the vertical component of vorticity to reconnect with the free surface and form a vortex. They speculated that the high shear region at the edge of an upwelling was conducive to the creation of a vortex in the same way that a vortex tube is connected to the free surface. Kida et al. (1989) attributed the surface-parallel vorticity to the vertical motion of the burst relative to the ambient fluid. Rashidi (1997) credited the surface deformations (surface rise and fall) and difference in free-surface velocities of the upwelling and downdraft motions. Kumar et al. (1998) contended that surface-normal vorticity could arise from the relative streamwise motion of the low-speed fluid to the surrounding fluid.

Vortex structures are persistent in turbulent flow, decaying slowly once formed and moving with the interface. They engage in several modes of interaction with each other. Two codirectional vortices will merge into a larger vortical structure. Two counter-rotating vortices will work to cancel the rotation of each other and may consolidate into a weaker vortex. Later, the pair may detach as individual vortices.

A spiral eddy is terminated by three mechanisms as stated by Kumar et al. (1998). For flows of small depths, vortex mergers are the prevalent means of destruction while annihilation by another upwelling impinging upon the free surface and viscous dissipation are common to flows at higher depths. The last method is effective when the upwellings are weak.
From Figure 3.5, Nagaosa and Saito (1997) implemented direct numerical simulations (DNS) to deduce a vortex pair aligned in the streamwise direction. They concluded that the rotating motion of these vortices near the free surface causes the upwellings which they named splattings. In the figure, the upward thrust between the two vortices corresponds to the centre of the splatting. Their numerical study indicated that the quasi-streamwise vortices of the wall region could reach the interface. The measurements of Komori et al. (1993) also suggested that these tube-like vortical structures are the remnants of the coherent motions from the bottom wall.

3.5 Numerical simulation

Despite the advent of supercomputer technology, the numerical resolution of turbulent motions at all scales is time consuming and costly. Turbulence modelling is useful for turbulence statistics but does not lead to identification of coherent structures because of their inherent long-time averaging. Coherent motions from numerical simulations are acquired theoretically in the sense that the turbulence was calculated and not modelled. Neither direct numerical simulation (DNS) or large-eddy simulation (LES) are practical for engineering purposes and numerical methods are limited to idealized situations of low Reynolds numbers for which the spectrum of scales is narrow. Anyhow, these computed databases can render turbulent quantities that are vital to turbulence modelling but are difficult or impractical to measure experimentally and aspects of coherent structures not quantifiable in the laboratory. Lam and Banerjee (1988) initially attempted to solve the Navier-Stokes equations by direct numerical simulation (DNS) in which the free surface was replaced by a rigid slip wall on the presumption that high gravitational and surface tension forces prevent surface waves and subdue vertical motion at the interface. Komori et al (1993) treated the free-surface boundary conditions in their entirety but the surface motions continued to be limited to perturbations of small amplitude.

Computational models for free-surface turbulence could incorporate structural features. Moderate success in predicting wall turbulence with structural models based on hairpin vortices
Figure 3.5. Splatting produced by a pair of streamwise vortices (from Nagaosa and Saito, 1997).
is already evident. In addition, the quasi-two-dimensionality nature of free-surface turbulence promises substantial savings and simplicity in the direct simulations.
4.1 Experimental apparatus

As visualized in Figure 4.1, the experiment was performed using a straight flow channel in a kerosene flow loop. The smooth test channel was designed from three sections of a rectangular channel with internal dimensions of 100 mm in width and 50 mm in height. For a particular bed width, b, and flow depth, h, the aspect ratio, $\alpha$, is defined by

$$\alpha = \frac{b}{h}.$$  \[4.1\]

With successive runs, the aspect ratio, $\alpha$, is lowered progressively from 7 to 4. Nakagawa et al. (1983) advocated an aspect ratio $\alpha \geq 6$ for the side-wall effects to disappear and to fulfill their criterion of two-dimensional flow in the central zone,

$$|\alpha|/h \leq (\alpha - \alpha_c)/2,$$  \[4.2\]

where the critical value, $\alpha_c$, is 4. Thus, two-dimensionality of the flow is debatable for the final runs at higher Reynolds numbers.

Three sections, 2.4 m long each, were joined end-to-end by flanges to ensure that the inner channel walls were flush and the channel bed horizontal. A measuring station at 440 cm downstream from the inlet tank was sufficiently far from the liquid inlet to obey the condition for fully-developed flow by Gessner (1981):

$$x/4R_h = x/D_h \geq 60.$$  \[4.3\]

The hydraulic radius, $R_h$, is defined as

$$R_h = \frac{A}{P} = \frac{hb}{2h + b},$$  \[4.4\]

where $A=bh$ is the cross-sectional area and $P=2h+b$ is the channel wetted perimeter which is in contact with the fluid (excludes the gas-liquid interface). Similarly, the hydraulic diameter, $D_h$, is a characteristic length that describes the cross-sectional size for a specific shape and includes a
Figure 4.1. Experimental flow loop.
factor of 4 to its ratio of cross-sectional flow area, \( A \), to wetted perimeter, \( P \), so that the diameter and effective diameter, \( D_h \), for a round pipe are equal:

\[ D_h = 4A/P = 4hb/(2h + b). \]  

For the present experiments, \( 63 < x/D_h < 93 \) and fully-developed flows can be presumed. This suggests that the mean velocity profile and other properties of the primary flow will remain stolid to streamwise location, i.e., \( \partial/\partial x = 0 \).

Kerosene as the working fluid was continuously filtered to remove particles bigger than 5 \( \mu \)m. A centrifugal pump stably recirculated the fluid and the settling chamber at the entrance ensured uniform incoming flow by damping large-scale disturbances. Along the horizontal channel, the liquid level and temperature were measured. As well, a cooling coil of tap water was immersed in the inlet chamber to sustain an isothermal condition. Electric fans strategically placed along the open channel helped maintain a stable temperature and uniform properties in the streamwise and lateral directions. Heat is imparted to the test fluid by the pump, laser and numerous light sources. Although remote in this study, stratified flows with fluid layers of unequal density require appreciable deflections in pressure and temperature to change the density of the fluid and induce local convection motion. In the same facility, Nasr-Esfahany (1998) studied cocurrent and countercurrent shear flows with a wavy interface. He simultaneously conducted flow visualization with photochromic dye activation (PDA) and measurements with hot-film anemometry (HFA).

4.2 Flow visualization

Point or probe measurements, in which a detection instrument is inserted into the fluid flow to measure velocities at one or more points, are established and accurate techniques for turbulence measurement. This way of determining velocity fluctuations is nevertheless not without problems. Hot-film anemometry (HFA) requires velocity and flow directional sensitivity calibration. At a sensor length of 1 mm, the flow pattern around the region of
measurement is modified considerably by the wake effect of the hot-film probe. The degree of
the velocity defect, of course, rests upon the main stream direction and proximity of the probe to
a wall. Temperature changes and impurities in the fluid affect its accuracy. Of greater
importance is that very low or inverse velocity can not be measured. Nalluri and Novak (1977)
used hot-film anemometry (HFA) in turbulent channel flow.

Likewise, laser-Doppler anemometry (LDA) needs intensive digital signal processing.
Quantitative point measurements can be made to elucidate the ordered structure of the regular
vortex, whether by the correlation of anemometer signals or the recognition of patterns in
conditional sampling. Laser Doppler Anemometry (LDA) was used by Nezu and Rodi (1986),
and Fabre et al. (1987) in an open channel, to name a few.

Flow visualization, whereupon a tracer or other indicator is added to the fluid to render
the flow pattern discernable, is especially effective in characterizing coherent structures.
Einstein and Li (1956) first uncovered coherent structures by the bed using dye injection.
Quantitative measurements of velocity fluctuations are obtainable by manually digitizing the
time lines of the tracers, supplementing the qualitative observations of flow phenomenon.
Experiments have been conducted with oxygen bubbles by Rashidi and Banerjee (1988, 1990a
and 1990b) and hydrogen bubbles by Grass (1971).

4.3 Photochromic dye activation (PDA) technique

In the present work, a non-intrusive photochromic tracer technique was used to provide
instantaneous images of the liquid motion under the smooth gas-liquid interface without
disturbing the flow. The selected dye, 1',3',3'-trimethylindolone-6-nitro-benzospiropyran or
TNSB, has a short ultraviolet (UV) absorption spectrum and is soluble only in organic liquids.
This photochromic dye was dissolved in deodorized kerosene (Van - Sol 715) at a dilute
concentration of 80 ppm (0.008% by weight). The dye solution is normally colourless. Density
and viscosity data for the test fluid at different temperatures are plotted in Figures A.1 and A.2 of
the appendix, respectively. Experimental values of density and viscosity for each run are tabulated in those figures, too. This concentration was chosen so as to yield the best trace contrast yet maintain the appropriate penetration depth of the UV laser beam. Changes in the physical properties of kerosene from the dissolution of the dye, such as surface tension and viscosity, have been concluded to be negligible by Ahmad (1993).

Traces are created in the photochromic dye solution as soon as the dye molecules in the path of a laser beam are activated, and a colour change ensues in the previously clear liquid. The absorption of electromagnetic radiation shifts the dye molecules from one structural state to another with a different absorption spectrum just temporarily, for the transition is reversible.

Adjustments made to the optical system ensured that time-lines were formed entirely in the central vertical plane of the channel, z=0. By filming the displacement of the trace in the flow, instantaneous velocity profiles could be measured.

Originally developed by Popovich and Hummel (1967) for studies of single-phase flow, this flow visualization technique has since been extended to two-phase flow experiments (Kawaji et al., 1993; 1997; and Kawaji, 1998). Improvements in the optical elements, video photography and image analysis have enhanced the accuracy of the velocity measurements by producing sharper and thinner traces. Distortion of the free surface by conventional instrumentation like a hot-film anemometer probe has increased the popularity of the photochromic dye activation (PDA) method as a tool for the measurement of turbulent parameters below the interface. Lorencez et al. (1993) investigated open-channel flow with this technique.

4.4 Optical system

Here, two lasers were operated in tandem because the individual laser could not penetrate the whole liquid depth of up to 27 mm. A nitrogen (N₂) laser (Model UV24 from Laser Photonics) discharged a pulsed light in the ultraviolet (UV) spectrum at a wavelength of 337 nm, while a helium-cadmium (He-Cd) laser (Model IK3102R-G from Kimmon Electric) emitted a
continuous UV light of 325 nm in wavelength. Subsequent to the mirror alignment, the output radiation was focused using a lens with a focal length of 250 mm for the former laser beam and a lens of 300 nm in focal length for the latter. The two lasers were arranged so that the beam of the N₂ laser on top would coincide with that of the He-Cd laser from below to give a single ray that penetrated the entire liquid layer from the free surface to the channel floor. An optical chopper (model SR540 from Stanford Research Systems) was utilized to divide the continuous emission out of the He-Cd laser into pulses. A Helium-Neon (He-Ne) laser (Model 05-LHR-111 from Melles Griot) fired a continuous ray through the same chopper to get a light pulse at an identical repetition rate to that of the He-Cd laser. This procedure was required to synchronize the UV pulses from the N₂ and He-Cd lasers. The optoelectronic circuit utilized in conjunction to the He-Ne laser was assembled from a phototransistor, protected by a light shield to prevent false triggering, and an external power supply. Upon the detection of a pulse input from the He-Ne laser, the light sensor transmitted a signal to the trigger generator of the N₂ laser. See Figure 4.2 for a schematic showing how the lasers were synchronized.

To visualize the motion of the traces in the liquid from a direction normal to the liquid flow, i.e. facing the channel, two colour video cameras were installed beside and above the channel to record the side view (Model VK-C370 by Hitachi) and top view (Model GR-S77 by JVC) of the flow, respectively. The side view camera was set up 59 cm from the centreline of the channel to get a 30 mm × 25 mm viewing window, while the top camera was raised 43 cm from the channel bottom to shoot a 8 cm × 10 cm segment of the flow. Their high horizontal resolutions of over 400 lines and frame rates, \( N_f \), of 30 fps permitted the playback in flicker-free slow motion and capturing of specific frames for precise data analysis. The side view camera was tilted slightly upward to enable the interfacial position to be exactly located since the liquid surface acted like a mirror and reflected the traces. The interface was recognized as the sharp discontinuity in the trace curvature. To furnish a reliable source of background
illumination, the white light of a stroboscope (Model PS-240 from Sugawara) was synchronized with the side view camera by means of the sync-signal generator in a monochrome video camera (Model DC-77RR from Sony). A side boroscope (Flolite Industries) mounted to another colour video camera (Model VK-C370 from Hitachi) was placed inside the test section facing the flow to capture the oncoming view. The rigid boroscope of 6 mm in outer diameter and 253 mm in working length offered a 90° direction of view, a 60° field of view, a depth of field from 4 mm to ∞ and complete rotation of the shaft.

The video camera system was arranged to allow flow visualization in both a fixed (Eulerian) and moving (Lagrangian) frame of reference. Since a Lagrangian viewpoint is essential to confirm the existence of the vortex, the speed of the cameras moving along a rail was set constant so that the course of a coherent structure could be followed over a lengthened period of time. In other words, the convecting axis moved with the coherent structure.

To accommodate all three views from different angles on one screen for simultaneous viewing, a colour quad (Model MV85 from Robot) was employed. A combination of four video cassette recorders (VCR's) were used to store the trace images from each camera (Panasonic Model PV-S4580-K for the end view, Panasonic Model AG-7355 for the side view and Mitsubishi Model HS-715UR for the top view) and the quad (Mitsubishi Model HS-U80). Apart from the videotaping of the top view on an ordinary VHS tape deck, all recordings were made in SVHS mode with higher spatial resolution. The layout of the video recording system is shown in Figure 4.3. The same video arrangement and test section was shared with Stamatiou (1998) who looked into the effect of waves on turbulence beneath a shearfree interface.

4.5 Image processing system

By digitally freezing the desired frame on a VCR (Model AG-7355 from Panasonic) and using a videographics card (TARGA+ from Truevision) in conjunction with image analysis software (Mocha V1.20 from Jandel) installed on a personal computer (PC), stills of the trace
images were captured frame by frame onto a PC. Refer to Figure 4.4 for a representation of the digitization operation.

Lorencez et al. (1997) succeeded in measuring the instantaneous streamwise and vertical velocity field of two-phase wavy flow in the same channel through the formation of a photochromic grid pattern, but recent attempts to restart the EXCIMER laser (Series TE-860-4 from Lumonics) used earlier have been unsuccessful. Hence, only the longitudinal velocity profiles were obtained from the displacement of non-intersecting traces, as was done by Lorencez et al. (1991) for a liquid flow with a smooth interface. Approximate values for the axial velocity were computed from a variation of the methodology proposed by Karimi et al. (1998) in the study of turbulent annular flow. This procedure entailed fitting a polynomial equation of modest order to the digitized data points of a trace as illustrated in Figure 4.5. The polynomial order for that trace could change in later video frames if it undergoes deformation due to the turbulent bursts. A large number of grid points spaced equidistant on the polynomial was taken and by assuming that they matched the corresponding nodes on the same trace of the successive frame, the streamwise velocities of these interpolated points were evaluated. Nasr-Esfahany (1998) affirmed the foregoing supposition of uni-directional flow. He declared the vertical velocity to be small compared to the axial velocity.

4.6 Data processing

Only the streamwise component, u, of instantaneous velocity was estimated from the measurement of the x-coordinates of the tracer position over a certain interval of time, $\Delta t$,

$$u(t) = \frac{\Delta x}{s \Delta t},$$  \[4.6\]

where $\Delta x$ is the horizontal displacement between two time-lines on the digitization tablet, and $s$ is the magnification scale for the tablet. Two-dimensional velocity profiles were computed on the basis that the trace stayed in the same vertical plane (no lateral movement) and negligible vertical velocity.
Figure 4.4. Image analysis procedure.
Figure 4.5. Polynomial fit to digitized data.
4.7 Error analysis

By far the largest source of uncertainty in the measurement of the x-displacement, \( \Delta x \), is from image processing. Errors in the measurement of \( \Delta x \) can originate from digitization and reading error, limited resolution of the tablet, optical distortion, and misalignment of the optical equipment. An attempt was made to compensate the digitization error by always selecting the middle of the trace width throughout the liquid layer. The width changes with level of turbulence and flow rate. Uncertainty of the mid-width, as a fraction of the dye-line displacement, was estimated at 4% beneath the interface and 20% near the bottom wall, where the width is thickest and displacement smallest. Error from tracer interpolation at places of discontinuity in the dye-line is incorporated into this error. With later runs of greater flow depth, the failure of the two lasers to penetrate the whole liquid depth becomes noticeable. Imperfect coincidence of the lasers from the top and bottom of the channel contributes an approximate error of 8% to the tracer displacement.

Faulty triggering can produce uncertainty of 10 to 17% in the time interval over which the trace has moved, if the lasers are out of phase by a hypothetical frame field. The alignment of the He-Ne laser with the photodiode that triggers the \( \text{N}_2 \) laser is a meticulous task.

Errors in the velocity profile measurements can be attributed to the limitation in the temporal resolution of the image analysis. Since the elapsed time interval to determine the velocity is finite, velocity fluctuations of shorter time scale than the time over which averaging is done can not be detected. For example, fluctuations of frequencies higher than the frame rate of 30 fps will not be recorded. For a liquid with a turbulence intensity of 0.15, Lorencez (1994) estimated a relative error of 2.5% in the adoption of this procedure.

The jitter of the video tape during digitization introduces an error of up to 3 pixels in the position of the interface, upon which the wall location depends. As realized now, the slight inclination of the side camera misrepresented the exact position of the wall, \( y=0 \). The location of
the wall as alleged from the captured image is elevated above the actual plane. By not originating from the same spot on the wall and so in violation of the no-slip boundary condition, the traces bear witness to this error clearly revealed in Figure 4.6 for a representative run. Consequently, the traces were extrapolated to the true wall position, as resolved from the measured liquid depth and interfacial position.

A frame contains two fields yet only the frame number was recorded, irrespective of which field. The time of trace displacement is therefore uncertain by a field, e.g., the counter will display the advancement of 2 frames or 4 fields when the trace has in reality moved 3 or 5 fields across the monitor.

There is still the problem of representing a flow structure of three-dimensional geometry on a two-dimensional plane, e.g., side view. When the trace moves laterally away from the x-y plane of the laser beam on which the side camera is focussed, the trace image will either be enlarged or reduced depending on its proximity to the camera such that the image of a trace drawn towards the camera will be magnified. This could portray a seemingly wavy surface as in Figure 4.7, if the interface inferred from trace reflection is watched from the side, yet the flow is obviously quiescent when inspected from the fixed vertical plane at the side wall nearest the camera. The velocity profile measurements hereafter must assume one-dimensionality of the flow in the streamwise direction or constant liquid depth (parallel flow) while tracing dye displacement; a stipulation rationalized by the comparatively small vertical velocity. All traces were digitized up to the gas-liquid interface at the centreline of the channel to assure consistency in trace height, unaffected by the appearance of variable interfacial position due to the sideways motion of the time-lines.

The flow stagnation in front of the boroscope can cause serious error if velocity measurements are made too close to it. Although the effect of flow stagnation likely varies with the Reynolds number of the flow, this error was minimized by focussing the camera lenses for all
Figure 4.6. Apparent failure to fulfill the no-slip boundary condition due to upward tilt of camera.
Figure 4.7. Conflicting impressions of interfacial position with distance of sideward moving trace from camera.
three views on the traces at 32 cm upstream of the boroscope and recording the images in that
neighbourhood. Streamwise velocity is affected beyond 20 cm from the location of trace
formation.

4.8 Experimental Conditions

Seven runs were conducted in the Lagrangian reference frame to explore the flow
structure beneath the liquid interface, each differing in the Reynolds number. They were then
repeated to establish reproducibility of the results. For the Eulerian frame, 14 runs of varying
Reynolds number were undertaken to gather quantitative data, but only the first and last runs
were duplicated to check the reproducibility of the results. The experiments were conducted at
near atmospheric pressure and room temperature. Table 4.1 summarizes the experimental
conditions for both types of measurements, where the volumetric flow rate, \( Q \), liquid depth, \( h \),
dimensionless flow depth, \( h^+ \), aspect ratio, \( \alpha \), hydraulic diameter, \( D_h \), flow Reynolds number
based on the liquid depth, \( Re_h \), Reynolds number in terms of the hydraulic diameter, \( Re_D \), friction
Reynolds number, \( Re^* \), bulk mean velocity, \( \bar{u}_{\text{bulk}} \), wall friction velocity, \( u^* \), and Froude number,
\( Fr \), are given. The wall friction velocity, \( u^* \), was determined by comparing the measured free
stream velocity profile to the universal velocity profile for turbulent flow in an open channel
[2.31], in the manner used by Lorenzez et al (1997).
Table 4.1. Experimental conditions.

<table>
<thead>
<tr>
<th>Frame</th>
<th>Run</th>
<th>Q [LPM]</th>
<th>h [mm]</th>
<th>h'</th>
<th>α</th>
<th>D_h [m]</th>
<th>Re_h</th>
<th>Re_o</th>
<th>Re'_o</th>
<th>u_{bulk} [m/s]</th>
<th>u' [m/s]</th>
<th>Fr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eulerian</td>
<td>1</td>
<td>8.8</td>
<td>14.8</td>
<td>87</td>
<td>6.8</td>
<td>0.046</td>
<td>840</td>
<td>2600</td>
<td>87</td>
<td>0.10</td>
<td>0.010</td>
<td>0.26</td>
</tr>
<tr>
<td>Eulerian</td>
<td>2</td>
<td>9.9</td>
<td>15.4</td>
<td>83</td>
<td>6.5</td>
<td>0.047</td>
<td>940</td>
<td>2900</td>
<td>83</td>
<td>0.11</td>
<td>0.009</td>
<td>0.28</td>
</tr>
<tr>
<td>Eulerian</td>
<td>3</td>
<td>11.3</td>
<td>16.4</td>
<td>89</td>
<td>6.1</td>
<td>0.049</td>
<td>1100</td>
<td>3200</td>
<td>89</td>
<td>0.11</td>
<td>0.010</td>
<td>0.29</td>
</tr>
<tr>
<td>Eulerian</td>
<td>4</td>
<td>12.7</td>
<td>17.4</td>
<td>98</td>
<td>5.8</td>
<td>0.052</td>
<td>1200</td>
<td>3600</td>
<td>98</td>
<td>0.12</td>
<td>0.010</td>
<td>0.30</td>
</tr>
<tr>
<td>Eulerian</td>
<td>5</td>
<td>14.1</td>
<td>18.4</td>
<td>108</td>
<td>5.4</td>
<td>0.054</td>
<td>1300</td>
<td>3900</td>
<td>108</td>
<td>0.13</td>
<td>0.010</td>
<td>0.30</td>
</tr>
<tr>
<td>Eulerian</td>
<td>6</td>
<td>15.3</td>
<td>19.1</td>
<td>115</td>
<td>5.2</td>
<td>0.055</td>
<td>1400</td>
<td>4200</td>
<td>115</td>
<td>0.13</td>
<td>0.011</td>
<td>0.31</td>
</tr>
<tr>
<td>Eulerian</td>
<td>7</td>
<td>16.6</td>
<td>20.0</td>
<td>123</td>
<td>5.0</td>
<td>0.057</td>
<td>1600</td>
<td>4500</td>
<td>123</td>
<td>0.14</td>
<td>0.011</td>
<td>0.31</td>
</tr>
<tr>
<td>Eulerian</td>
<td>8</td>
<td>18.0</td>
<td>21.0</td>
<td>132</td>
<td>4.8</td>
<td>0.059</td>
<td>1700</td>
<td>4800</td>
<td>132</td>
<td>0.14</td>
<td>0.011</td>
<td>0.32</td>
</tr>
<tr>
<td>Eulerian</td>
<td>9</td>
<td>19.7</td>
<td>21.7</td>
<td>141</td>
<td>4.6</td>
<td>0.061</td>
<td>1900</td>
<td>5200</td>
<td>141</td>
<td>0.15</td>
<td>0.011</td>
<td>0.33</td>
</tr>
<tr>
<td>Eulerian</td>
<td>10</td>
<td>20.8</td>
<td>22.7</td>
<td>152</td>
<td>4.4</td>
<td>0.062</td>
<td>2000</td>
<td>5400</td>
<td>152</td>
<td>0.15</td>
<td>0.012</td>
<td>0.32</td>
</tr>
<tr>
<td>Eulerian</td>
<td>11</td>
<td>22.2</td>
<td>24.8</td>
<td>165</td>
<td>4.0</td>
<td>0.066</td>
<td>1900</td>
<td>5100</td>
<td>165</td>
<td>0.15</td>
<td>0.013</td>
<td>0.30</td>
</tr>
<tr>
<td>Eulerian</td>
<td>12</td>
<td>23.5</td>
<td>25.5</td>
<td>179</td>
<td>3.9</td>
<td>0.068</td>
<td>2000</td>
<td>5400</td>
<td>179</td>
<td>0.15</td>
<td>0.014</td>
<td>0.31</td>
</tr>
<tr>
<td>Eulerian</td>
<td>13</td>
<td>24.9</td>
<td>26.0</td>
<td>177</td>
<td>3.8</td>
<td>0.068</td>
<td>2100</td>
<td>5700</td>
<td>177</td>
<td>0.16</td>
<td>0.013</td>
<td>0.32</td>
</tr>
<tr>
<td>Eulerian</td>
<td>14</td>
<td>26.3</td>
<td>26.8</td>
<td>192</td>
<td>3.7</td>
<td>0.070</td>
<td>2300</td>
<td>5900</td>
<td>192</td>
<td>0.16</td>
<td>0.014</td>
<td>0.32</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>1</td>
<td>8.5</td>
<td>14.8</td>
<td>87</td>
<td>6.8</td>
<td>0.046</td>
<td>800</td>
<td>2500</td>
<td>-</td>
<td>0.10</td>
<td>-</td>
<td>0.25</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>2</td>
<td>11.2</td>
<td>16.4</td>
<td>89</td>
<td>6.1</td>
<td>0.049</td>
<td>1100</td>
<td>3200</td>
<td>-</td>
<td>0.11</td>
<td>-</td>
<td>0.28</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>3</td>
<td>14.1</td>
<td>18.4</td>
<td>108</td>
<td>5.4</td>
<td>0.054</td>
<td>1300</td>
<td>3900</td>
<td>-</td>
<td>0.13</td>
<td>-</td>
<td>0.30</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>4</td>
<td>16.9</td>
<td>20.0</td>
<td>123</td>
<td>5.0</td>
<td>0.057</td>
<td>1600</td>
<td>4600</td>
<td>-</td>
<td>0.14</td>
<td>-</td>
<td>0.32</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>5</td>
<td>19.4</td>
<td>21.7</td>
<td>141</td>
<td>4.6</td>
<td>0.061</td>
<td>1800</td>
<td>5100</td>
<td>-</td>
<td>0.15</td>
<td>-</td>
<td>0.32</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>6</td>
<td>22.3</td>
<td>24.8</td>
<td>165</td>
<td>4.0</td>
<td>0.066</td>
<td>2100</td>
<td>5600</td>
<td>-</td>
<td>0.15</td>
<td>-</td>
<td>0.30</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>7</td>
<td>24.9</td>
<td>26.0</td>
<td>177</td>
<td>3.8</td>
<td>0.068</td>
<td>2400</td>
<td>6200</td>
<td>-</td>
<td>0.16</td>
<td>-</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Note:

\[ h^* = h \frac{u^*}{\nu} \]
Conventional probabilistic tools of long-term averaging are utilized for the statistical
description of turbulence. Flow visualization allows the realization of the short-lived coherent
structures that are not manifested in the statistical properties of turbulence unless conditional
sampling is performed.

5.1 Statistical structures of turbulence

A statistical analysis was completed on the fourteen Eulerian runs. Run 1 was plagued by
difficulties throughout the digitization stage, largely from inexperience, and will not be reported
due to possible inaccuracy. Of the thirteen runs left, whose results are given collectively, the
turbulence statistics for Run 7 will be explained in detail.

Open-channel flow is described by the Reynolds number involving fluid viscosity and the
Froude number involving gravity. The effective Reynolds number is defined as

\[ \text{Re}_h = \frac{\bar{u}_{\text{Bulk}} h}{\nu} \]  \hspace{1cm} [5.1]

where \( \nu \) is the kinematic viscosity, \( h \) is the flow depth and \( \bar{u}_{\text{Bulk}} \) is the bulk mean velocity. The
latter quantity is averaged over the cross section of the liquid layer by dividing the volumetric
flow rate, \( Q \), by the cross-sectional area, \( A = bh \). Likewise, the Reynolds number, \( \text{Re}_D \), can be
defined using the hydraulic diameter, \( D_h \), and mean velocity:

\[ \text{Re}_D = \frac{\bar{u}_{\text{Bulk}} D_h}{\nu}. \]  \hspace{1cm} [5.2]

This Reynolds number is roughly four times the flow Reynolds number, \( \text{Re}_h \),

\[ \text{Re}_D \approx 4 \text{Re}_h. \]  \hspace{1cm} [5.3]

Munson et al. (1990) stipulated a general guideline for laminar flow in an open channel to
be \( \text{Re}_D < 1,000 \). In the current investigation, every run exceeded this critical Reynolds number as
anticipated. Moreover, they said that the flow is turbulent when \( \text{Re}_D > 25,000 \). Nezu and
Nakagawa (1993) used a criterion for low Reynolds number, Re<500, the satisfaction of which for all runs considered is further suggestive of turbulent flow at low Reynolds number. In addition, interfacial waves are averted for Re<15,000 (Rashidi and Bancrjee, 1988).

In open-channel flow where the gravitational acceleration, g, can influence the shape of the free surface, another meaningful dimensionless parameter is the Froude number, Fr, defined as

\[ Fr = \frac{\bar{u}_{\text{Bulk}}}{\sqrt{gh}} = \frac{\bar{u}_{\text{Bulk}}}{c}. \]  

This ratio of the fluid velocity, \( \bar{u}_{\text{Bulk}} \), to the speed of an elementary surface wave, c, reflects the relative importance of the inertial force on a fluid element to the weight of that element just as the Reynolds number relates the inertial and viscous forces. The flow is subcritical (Fr<1) when the flow speed is less than that for a hypothetical wave of small amplitude on the surface, i.e., \( \bar{u}_{\text{Bulk}} < c \). A low Fr number signifies a stiff free surface, essential to the suppression of vertical eddy movement. When the flow velocity, \( \bar{u}_{\text{Bulk}} \), is equal to the wave speed, c, the flow is critical (Fr=1). If the flow velocity, \( \bar{u}_{\text{Bulk}} \), is greater than the wave speed, c, a supercritical (Fr>1) condition ensues and surface waves emerge.

Surface tension is a property of the fluid, dependent on temperature and any other fluid it is in contact with at the interface. It is assumed to be irrelevant for free-surface flows without capillary waves since inertial, gravitational, and viscous forces are more dominant.

5.1.1 Instantaneous flow properties

Figure 5.1 exhibits the streamwise velocity profiles at consecutive instances in time for Run 7. The extrapolated points are not shown and hence the measured values begin at \( y=0.5 \) mm from the wall. The mean velocity was time-averaged over 100 instantaneous velocity profiles and indicated by a solid curve. From Figure 5.1, the largest departures of the instantaneous
Figure 5.1. Instantaneous and mean velocity distributions.
velocity profiles from the mean velocity profile seem to occur in the bulk flow, \( y > 3 \) mm, implicative of wall turbulence. Forward and backward bulges represent sweep and ejection events, respectively. Fluid of low streamwise velocity moving up to the interface and that of high streamwise velocity moving down to the bottom wall are evident. The inflections in the velocity profiles are suggestive of turbulent organized structures and poor mixing as a uniform profile implies that there is no turbulence generation.

A typical time-history of the instantaneous axial velocity is presented in Figure 5.2. Figure 5.3 gives a time history of fluctuating velocity. Take note that the pointers opposite the flow direction imply a negative velocity slower than the average velocity, not backward flow. Forward arrows depict a positive velocity faster than the reference speed. The deviation from the mean flow is proportional to the vector dimension. From this illustration, two controlling events are disclosed: (i) an outward ejection of fluid with low streamwise momentum from the wall and (ii) an inrush of fluid with high streamwise momentum from the free surface. Low-momentum fluid is discharged from the wall at time \( t = 15.6 \) s. An inflow of high momentum fluid from the interface arrives at time \( t = 16.6 \) s. By then, the upsurge has risen to the interface. At time \( t = 17.6 \) s, a fresh sweep descends to counteract the oncoming ejection. An inflow from the interface toward the wall succeeds each deceleration of the bulk fluid in a cyclic fashion. See how the transition from ejection to sweep motions is abrupt whereas the reverse is progressive.

Figure 5.4 redraws the velocity profiles of Figure 5.2 from a Lagrangian frame moving with the bulk mean velocity. The negative velocity is greatest at the wall because of the no-slip boundary condition for viscous fluids. As a useful indicator of bursting events in the profiles, the crossover point lies furthest to the surface for outflows and closest to the wall for high-speed sweeps.

Next, consider the graph of instantaneous velocity versus time in Figure 5.5. Four arbitrary locations were selected, extending from the outskirts of the buffer region at \( y^+ = 30 \) to the
Figure 5.2. Eulerian time history of instantaneous streamwise velocity.
Figure 5.3. Eulerian time history of streamwise velocity fluctuation.
Figure 5.4. Lagrangian time history of instantaneous streamwise velocity.
Figure 5.5. Velocity histories from 30<y<122.

Moving average

- $y=4.9$ mm, $y^+=30$
- $y=10.0$ mm, $y^+=61$
- $y=15.0$ mm, $y^+=92$
- $y=20.0$ mm, $y^+=122$

Streamwise velocity, u [m/s]

Run 07

$Re_U=4500$

$Re_V=1600$
interface. For comparison, the moving average is also indicated by a dotted curve. Note that only the long-term average of [2.2] was used in the statistical analysis. A moving average, $\bar{x}_{\text{Mov}}$, of order $n$ for a $N$ set of readings, $x_1, x_2, \ldots, x_N$, is a succession of $N-n+1$ arithmetic means,

$$\bar{x}_{\text{Mov}} = \frac{x_1 + \ldots + x_n}{n}, \frac{x_2 + \ldots + x_{n+1}}{n}, \ldots, \frac{x_{N-n+1} + \ldots + x_N}{n},$$

that smoothes the unwanted fluctuations in a time series. To effectively average out the fluctuating component, the period of fluctuations in the time sequence was elected as the order $n$. This period is recognized as the reciprocal of the dominant frequency from spectral analysis.

The steadiness of the underlying trend in the curves shown in Figure 5.5 substantiates the attainment of a steady state or stationary data for which the average value stays constant with time. The origin of time is incidental. Amplified deviations from the moving average in the buffer region, $5<y^+<30$, of Figure 5.5 that diminish in the direction of the interface is convincingly ascribed to wall turbulence activity. Deceleration proceeded by acceleration agrees with the changeover from ejection to sweep motions. Figure 5.6 shows the region, $20<y^+<30$, in more detail. Notice how the oscillatory behaviour of the time series is maintained with the wall-normal distance.

5.1.2 Correlation functions

As a test for linear dependency of different streamwise velocities across the flow depth, the following correlation functions were employed: (i) cross-covariance and (ii) cross-correlation functions. The cross-covariance, $C_{xy}$, and cross-correlation, $R_{xy}$, functions for two time history records, $x(t)$ and $y(t)$, with a time delay, $\tau$, are defined by

$$C_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \{x(t) - \mu_x\} \{y(t + \tau) - \mu_y\} dt = R_{xy}(\tau) - \mu_x \mu_y$$

and

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)y(t + \tau) dt,$$
Figure 5.6. Velocity histories from 20 mm to 30 mm.
where $T$ is the period of the record, $\mu_x$ is the mean value of $x(t)$ and $\mu_y$ is the mean value of $y(t)$. Basically, the cross-covariance function, $C_{xy}$, is the cross-correlation function, $R_{xy}$, with the mean values of the random variables removed. Unrelated or statistically independent time history data will have a cross-covariance function, $C_{xy}$, of zero as opposed to data of linear correlation with a positive average product. It is worthwhile to remember that this function is a measure of linearity and not association. Thus, a zero covariance can still be obtained notwithstanding the existence of a nonlinear relationship for two sequences of data. If large values of the variable $x(t)$ induce large values of the variable $y(t)$ or small values of the variable $x(t)$ prompt small values of the variable $y(t)$, the mean covariation will tend to be positive. On the contrary, if large values of the variable $x(t)$ cause small values in the variable $y(t)$ or vice versa, a negative sign of the covariance is calculated to disclose the inverse variation.

To probe the degree of linear regression between velocities at various heights from the wall, the time history of the streamwise velocity was converted to a series of equally spaced sample values using a cubic spline interpolation of the data with a time-resolution interval of 0.001 s. Thereafter, the raw covariance and correlation were estimated from the resulting interpolant with no normalization.

The cross-covariance function, $C_{xy}$, computed for Run 7 is revealed in Figure 5.7. In the analysis conducted, the $x(t)$ data represented a streamwise velocity in the buffer zone at $y^*$=30 and the $y(t)$ data the velocity at another elevation and measured after a time delay, $\tau$. In order to assess the rapidity in ascension of a low-momentum fluid ejected from the wall, the $y(t)$ sample records for incremental heights to the free surface was entered, i.e., 10 mm, 15 mm and 20 mm. The average vertical velocity of an ejection is approximated by the ratio of the spatial separation between the two signals over the time delay, $\tau$, for the cross-covariance peak. From Figure 5.7, the sharp and distinct peak of the cross-covariance function, $C_{xy}$, is lost beyond $y^*$=61 or $y$=10.0
Figure 5.7. Cross-covariance functions of instantaneous streamwise velocities.
Any inference drawn on the extent of concurrent variation should proceed with prudence bearing in mind the coarse data assemblage.

This shortcoming of the low sampling rate is brought to light in Figure 5.8 where the x(t) and y(t) data of Run 7 are primarily correlated at zero time delay, \( \tau \), for every measurement of y(t) from the wall. The outcome of the preceding correlation analysis on Run 7 is typical of that for the other runs omitted in this discussion.

5.1.3 Mean flow properties

5.1.3.1 Friction velocity

The normalization of mean velocity and turbulence demands a careful evaluation of the wall shear velocity. Firstly, the wall friction velocity, \( u^* \), was derived from the fitting of the measured mean velocity profile to the log-law of [2.31] by the least-square method. In [2.31], the von Karman constant, \( \kappa = 0.41 \), and the integral constant, \( A = 5.3 \), were assumed, regardless of flow properties. These universal constants for open-channel flows are similar to those found by Nikuradse for smooth-pipe flow - \( \kappa = 0.4 \) and \( A = 5.5 \). To plot the theoretical log-wake law of the outer region [2.31], the wake strength parameter, \( \Pi \), was assumed to be zero. The wake strength parameter, \( \Pi \), as a function of the friction Reynolds number, \( Re^* \), is reproduced in Figure A.3 and hints at that value for low-Reynolds number flows, \( Re^* < 500 \).

Secondly, the Blasius formula for flat plates was employed to verify the results of the above method:

\[
    u^* = 0.167 \bar{u}_{bulk} R_{Re}^{-0.125}.
\]

[5.8]

The discrepancy varies from 14% to 31% as seen in Table A.1 of the appendix.

Thirdly, the shear velocity, \( u^* \), was resolved from the calculation of the wall shear stress, \( \tau_w \), distribution [2.6] in the defining equation [2.20]. One hundred instantaneous values were averaged to derive the mean wall shear stress of [2.6]. This alternative was abandoned because
Figure 5.8. Cross-correlation functions of instantaneous streamwise velocities.
of uncertainty in the velocity measurements of the thin viscous sublayer. A disparity of 20% to the friction velocity of the first approach was even attained. Thus, the first method was adopted to evaluate the velocity scale, $u^*$. The wall friction velocity, $u^*$, from all three methods is tabulated in Table A.1.

Disagreement with the scientific literature is introduced by the manner by which the normalizing factor, $u^*$, was acquired. Instead of reading experimental measurements, the wall shear velocity, $u^*$, was indirectly elicited from the fitting of the logarithmic formula to the mean velocity profile.

5.1.3.2 The law of the wall

A sample size of one hundred instantaneous velocity profiles, large enough for statistical analysis, was time-averaged to obtain the mean velocity profile for every experimental run. Figure 5.9 reveals a good fit of the mean velocity data to the law of the wall [2.31] for Run 7. The collapse of the profiles signifies the adequacy of the sample population used for averaging. Discretion is forewarned in the viscous sublayer, $y^+<5$, more so than in the rest of the flow, as the logarithmic scale in Figure 5.9 tends to exaggerate the divergence from the law of the wall [2.31] in the laminar sublayer, $y^+<5$. Still, the universal velocity profile [2.31] overestimates the actual flow in the neighbourhood of $20<y^+<50$. Steffler et al. (1983) faced a contrasting underestimation in the buffer zone, $5<y^+<30$. He erred in determining the wall position, $y=0$. It is not surprising that the analytic expression for pipe flow by Spalding (1961),

$$y^+ = u^* + e^{-\kappa A} \left[ -1 - \kappa u^* - \frac{(\kappa u^*)^2}{2} - \frac{(\kappa u^*)^3}{6} \right]$$

with $\kappa=0.4$ and $A=5.5$ equally holds for open-channel flow as noted in Figure 5.9.

In Figure 5.10, the instantaneous velocity profiles are normalized by the wall variables, i.e., the wall friction velocity derived previously. Consequently, the envelope of the data is
Figure 5.9. Mean velocity distribution in wall coordinates.

Run 07
Re_h = 1600
Re_D = 4500
\( \bar{u}_{bulk} = 0.14 \text{ m/s} \)
h = 20.0 mm
\( u^* = 0.011 \text{ m/s} \)

- Law of the wall [2.31]
- Experimental data
- Spalding formula [5.9]
Figure 5.10. Instantaneous and mean velocity distributions in wall coordinates.
indicative of the spread in the nondimensional streamwise velocity and the fluctuations appear to decrease immediate to the interface.

The constant horizontal error bars and variable vertical error bars of Figure 5.11 display uncertainty in the velocity measurements of Run 7 arising from uncertainty of the exact wall position and possible miscount of frame field, respectively. This error band in data acquisition totally encloses the law of the wall. The instantaneous velocity profiles would have similar error bands.

The nondimensional mean velocity data for all runs are plotted in Figure 5.12 and infers no particular dependency on Reynolds number for open-channel flows. However, a better correspondence of the measured mean velocity data with the predicted log-law curve [2.31] at higher Reynolds numbers is apparent, notably in the buffer region. Perhaps this behaviour typifies turbulent flow at low Reynolds numbers. Altogether, it is fair to say that quantitative data from the photochromic dye activation (PDA) technique brought about rather good mean velocity profiles. Nezu and Rodi (1986) postulated the departure from the log-law profile [2.29] as the free surface is approached at high Reynolds numbers, $Re_h>10,000$ or $Re_D>55,000$, and recommended the appendage of a wake function to the log-law [2.31] to rectify this flaw. On closer scrutiny, their amendment is not warranted for flows at the low Reynolds numbers investigated in this study. At these Reynolds numbers, interfacial waves are of no importance and the velocity fluctuations are solely due to turbulence.

Turbulent activity is a three-dimensional phenomenon at considerable depths from the free surface, yet turbulence within the vicinity of the surface is quasi-two-dimensional. Sarpkaya and Suthon (1991) speculated that the free-surface turbulence, located away from the region of turbulence generation, is two-dimensional such that $w=0$ and there is no deviation of mean properties in the transverse direction. Only the average velocity is two-dimensional; the turbulent fluctuating velocities are nonetheless three-dimensional.
Figure 5.11. Error bars of mean velocity distribution in wall coordinates.
Figure 5.12. Mean velocity distributions in wall coordinates for all runs.
The channel depth ran from 83 to 192 wall units and the free-surface domain can normally be regarded as the layer below the surface of 20 to 30 wall units in thickness. A difference in the distance from the wall to the interface in wall units is expected of dissimilar Reynolds numbers such that the free surface in wall units is further for greater Reynolds numbers.

5.1.4 Frequency distribution of velocity fluctuations

A histogram is a rectangular graph of a frequency distribution that tabulates the number of occurrences of each group of data. To construct the amplitude histogram of fluctuating velocity, intervals of equal width were created to sort the data, enclosing the sample space. In Figure 5.13, the frequency distribution of the streamwise velocity fluctuations for Run 7 is shown at several depths of the liquid stream. The distribution has 56 classes of 2 mm/s in width. It readily illustrates the variability in the data and presence of turbulence. It is safe to remark that the fluctuating velocity is not uniform across the liquid depth as the distribution differs with height – an extensive scattering near the bed bottom \( (y^+ = 30) \) and a narrow, symmetric distribution at the interface \( (y^+ = 122) \). Owing to wall turbulence, the amplitude of velocity fluctuation is much larger close to the wall, \( y^+ \leq 30 \), as demonstrated by the spread of the histogram. A slight skewness from the incorporation of a few more negative measurements than positive ones in the histogram is noticed in the outer region \( (y^+ > 61) \). Intermittent ejections of low momentum fluid from the wall headed for the free surface could explain this asymmetry as will be discussed further. Nearing the interface, the curve of the frequency distribution shifts from a bimodal contour with two maxima to a bell-shaped profile in which observations are distributed somewhat symmetrically about the maximum. Since turbulent components are never independently random but mutually related in space and time, an exact normal or Gaussian distribution is elusive.
Run 07
$Re_h=1600$
$Re_D=4500$
$\bar{u}_{\text{bulk}}=0.14 \text{ m/s}$
$h=20.0 \text{ mm}$

Figure 5.13. Histograms of velocity fluctuations from $30<y^+<122$. 
From Figure 5.14, a tail in the negative direction is already unfolding at $y^+=23$. Not shown here, the dispersion of observations is unaffected by the Reynolds number. The range remains constant at about ±0.06 m/s in the wall region for all the Reynolds numbers tested.

When written in the probabilistic terms, the bar charts of Figures 5.13 and 5.14 are in fact the areas under probability density functions. Recall that the probability of an instantaneous amplitude falling within a defined window at a certain time requires calculating the area under the density function between the two amplitudes. The probability, $P(u')$, of an outcome is expressed by its relative frequency of occurrence,

$$P(u') = \frac{N_u}{N},$$

where $N_u$ is the number of times that the measured velocity fluctuation amplitude occurs and $N$ is the total number of measurements. Probability functions of the foregoing histograms are presented in Figures 5.15 and 5.16. Their low values point to a meager sample population.

5.1.5 Turbulence intensity

Figure 5.17 shows the streamwise turbulence intensity for Run 7. A marked feature is that the fluctuating turbulence rapidly climbs to a maximum at a distance, $y=2$ mm, above which it gradually descends due to diffusion of turbulent energy.

The RMS velocity fluctuation, $u_{\text{RMS}}$, is normalized by the wall shear velocity, $u^*$, and plotted against the dimensionless normal distance, $y/h$, in Figure 5.18. Such scaling implies that turbulence intensity varies with the Reynolds number, regardless of the liquid depth. It becomes apparent that the streamwise turbulence intensity is slightly heightened near the surface, $y/h>0.9$. Consequent to interfacial damping of the vertical motions by surface tension and gravity, there is suspected a redistribution of turbulent kinetic energy from the wall-normal, $y$, to the axial, $x$, and transverse, $z$, directions through the pressure. Indeed, any argument of energy transfer is weak lacking experimental validation of the vertical turbulent intensity and pressure fluctuation. Lam
Strearnwise velocity fluctuation, $u'$ - 4.9 mm, $y^+ = 3$

$y = 4.9 \text{ mm}, y^+ = 30$

$y = 4.2 \text{ mm}, y^+ = 26$

$y = 3.8 \text{ mm}, y^+ = 23$

$y = 3.3 \text{ mm}, y^+ = 20$

Figure 5.14. Histograms of velocity fluctuations from $20 < y^+ < 30$. 
Figure 5.15. Probability density functions of velocity fluctuations from $30 < y^+ < 122$. 

Run 07  
Re$_h$=1600  
Re$_D$=4500  
u$_{Bulk}$=0.14 m/s  
h=20.0 mm
Figure 5.16. Probability density functions of velocity fluctuations from $20 < y^+ < 30$. 

Run 07
Re$_h$=1600
Re$_D$=4500
$ar{u}_{Bull}$=0.14 m/s
h=20.0 mm
Figure 5.17. Turbulence intensity.

Run 07
Re_h=1600
Re_D=4500
u_Bulk=0.14 m/s
h=20.0 mm
Figure 5.18. Dimensionless turbulence intensity.
and Banerjee (1988) clarified this issue with an inspection of the pressure-strain correlation terms in the turbulent kinetic energy transport equations. The Froude number, Fr, ranged from 0.25 to 0.32 for the present runs. However small, surface waves are imminent for Fr>0.2 and the local suppression of velocity fluctuation by the interface is decreased somewhat.

In Figure 5.19, the normal distance, y, is rescaled by the wall variables. Good agreement with previous data as represented by equation [2.39] is evident in the shape of the normalized RMS profile with respect to position and broadness of the peaks. The coincidence of the peak streamwise fluctuation at \( y^* = 12 \) with the area of maximum production, \( G \) [2.14], was initially pointed out by Kim et al. (1987) and viscous effects on the fluctuating velocity seem important only for \( y^* < 12 \). The maximum value of \( \frac{u_{\text{rms}}}{u^*} = 2.6 \) at \( y^* = 13.5 \) compares favourably with that of \( \frac{u_{\text{rms}}}{u^*} = 2.8 \) from Nezu (1986).

Except for the region, \( 30 < y^* < 80 \), equation [2.39] agrees well with the experimental data. This agreement is of no surprise, considering that Nezu (1977) found the empirical parameters, \( D_u = 2.30 \) and \( C_k = 1.0 \), to be independent of the Reynolds and Froude numbers. Furthermore, he chose the damping constant, \( B = 10 \), to confine viscous effects on the velocity fluctuation to \( y^* \leq 10 \). Turbulence measurements showed that the RMS distribution had a maximum over \( y^* = 10-20 \). Rashidi and Banerjee (1988) mentioned the failure of the exponential fit [2.39] to capture the ascent in streamwise intensity, a defect not quite discernable in the present data.

From Figure 5.20, an increase in turbulence intensity with Reynolds number is observed, especially near the wall. The graph of maximum turbulence intensity against increasing Reynolds number in Figure 5.21 clearly illustrates this trend that could be explained by a promotion of bursting events at larger Reynolds number. As expected, the frequency of velocity fluctuation amplitude in Figure 5.13 is distributed less symmetrically close to the wall. The
Figure 5.19. Turbulence intensity in wall coordinates.
Normal distance, y [mm]

Streamwise turbulence intensity, u RMS [m/s]

- Run 02, \( \text{Re}_h=940, \text{Re}_D=2900 \)
- Run 03, \( \text{Re}_h=1100, \text{Re}_D=3200 \)
- Run 04, \( \text{Re}_h=1200, \text{Re}_D=3600 \)
- Run 05, \( \text{Re}_h=1300, \text{Re}_D=3900 \)
- Run 06, \( \text{Re}_h=1400, \text{Re}_D=4200 \)
- Run 07, \( \text{Re}_h=1600, \text{Re}_D=4500 \)
- Run 08, \( \text{Re}_h=1700, \text{Re}_D=4800 \)
- Run 09, \( \text{Re}_h=1900, \text{Re}_D=5200 \)
- Run 10, \( \text{Re}_h=2000, \text{Re}_D=5400 \)
- Run 11, \( \text{Re}_h=1900, \text{Re}_D=5100 \)
- Run 12, \( \text{Re}_h=2000, \text{Re}_D=5400 \)
- Run 13, \( \text{Re}_h=2100, \text{Re}_D=5700 \)
- Run 14, \( \text{Re}_h=2300, \text{Re}_D=5900 \)

Figure 5.20. Turbulence intensities for all runs.
Figure 5.21. Maximum turbulence intensity as a function of Reynolds number
ejection of slow-moving fluid with negative velocity fluctuation leads to the long tail of this histogram. This topic will be revisited when organized structures are examined in greater length.

All cases shared a similar contour when the measurements were renormalized by the friction velocity, $u^*$, and flow depth, $h$, in Figure 5.22. Amid the experimental scatter, adherence of the exponential equation [2.37] to the turbulence intensity data improves for $y/h > 0.4$. Their nonzero value at $y = 0$ is indicative of the tremendous difficulty in making accurate measurements at the wall.

Finally, the streamwise fluctuations are plotted against the wall coordinates in Figure 5.23 and the semi-empirical formula [2.38] of the inner region is replotted to display agreement for the viscous sublayer, $y^* < 5$. In this figure, a Reynolds number dependence of the profiles is irrefutable; the decline after $y^* = 20$ begins to diminish for higher Reynolds numbers and eventually $u_{RMS}/u^*$ hits a plateau at 1. Coles (1978) references channel flow data in support of this Reynolds-number effect on relative turbulence intensity. With a peak of the RMS value at about $y^* = 20$, the crest is seen to broaden at larger Reynolds numbers.

5.1.6 Higher-order statistics

Information on the symmetry of a data set or asymmetry in the probability density function of turbulent fluctuations is given by the standardized skewness measure or third moment about the mean, $S(u')$:

$$S\left(\overline{u'(x, y, z, t)} \right) = \frac{\overline{u'^3(x, y, z, t)}}{\left(\overline{u'^2(x, y, z, t)}\right)^{3/2}}. \quad [5.11]$$

The skewness factor, $S(u')$, will be positive or negative according to the direction of skewness. If the elements in the group are distributed symmetrically about the mean, the moment coefficient of skewness, $S(u')$, will be zero.

Figures 5.24 and 5.25 present the distribution of skewness, $S(u')$, and flatness, $F(u')$, factors across the flow stream for Run 7 against $y/h$ and $y^*$, respectively. With higher-order
Figure 5.22. Dimensionless turbulence intensities for all runs.
Figure 5.23. Turbulence intensities in wall coordinates for all runs.
Figure 5.24. Dimensionless distributions of skewness and flatness factors.
Figure 5.25. Distributions of skewness and flatness factors in wall coordinates.
statistics as such, the small asymmetry and oscillations of the above profiles raise doubt about
the sufficiency of the sample size. As Figure 5.24 plainly depicts at a short normalized distance
from the wall, the skewness factor, $S(u')$, interchanges sign from positive to negative well below
the free surface. This concurs with the frequency curves of Figure 5.13 skewed to the left,
recognizable by the steeper slope to the right of the maximum than that to the left. It infers that
the ejection of low momentum fluid leaves the inner region and reaches the interface.

The enhanced deceleration of streamwise velocity caused by bursting is shown by the
high negative skewness relative to the local RMS velocity fluctuation. Although missing in
Figure 5.24, there is vague agreement between the present experimental results and those of Lam
et al. (1992) and numerical simulation of Kim et al. (1987). Reminiscent of an inrush event, the
skewness factor builds to a value of 0.8 at the wall. Lam and Banerjee (1988) computed a higher
value of 1.15 very close to the wall. To recite Kim et al. (1987) nevertheless, skewness and
flatness factors at the wall should be received with some reservation since the numerator and
denominator of the high-order statistics there assume zero and can intensify any inaccuracy in its
value.

Supposing negative skewness indicates ejection events and positive skewness indicates
sweep events in Figure 5.25, then the crossover point at $y^+=20$ for Run 7 surpasses that reported
by Kim et al. (1987). They said that at $y^+=12$, the contributions from the ejection and sweep
events are equivalent.

Lastly, the standardized kurtosis measure or fourth moment about the mean, $F$, represents
the peakedness of the data set or intermittency of turbulence. Graphs of the kurtosis data are also
provided in Figures 5.24 and 5.25. Also known as the flatness factor, $F(u')$, a magnitude of 3
designates a Gaussian distribution to which it is usually compared.

$$F(u'(x, y, z, t)) = \frac{u'^4(x, y, z, t)}{[u'^2(x, y, z, t)]^2}$$  \[5.12\]
Three kinds of shape to classify the peak of a distribution are acknowledged: (i) leptokurtic, with a definite and high peak and denoted by large values of $F(u') > 3$; (ii) mesokurtic, accompanied by a medium peak native to the normal distribution and denoted by values of $F(u') = 3$; and (iii) platykurtic, along with a nondescriptive and low peak and denoted by small values of $F(u') < 3$.

The flatness factor, $F(u')$, appears to stray from the previous baseline value of 3 for a normal distribution, notably near the interface, which suggests that the turbulent motions there were not purely random but recurrent. Growth of the flatness factor, $F(u')$, in the interface region, $0.75 < y/h < 0.95$, attests to the arrival of recurring episodes of bursts at the surface from the bottom. Strong upflows with low speeds are perceived. There is a perceptible changeover in the fluctuating velocity distribution from a flat outline near the wall to a pointed outline in the free-surface region. So, the assertion expressed in Figure 5.13 of a narrowing of the distribution with proximity to the interface is confirmed. Incidentally, the increase in flatness factor near the wall was absent, unlike the prediction by Lam and Banerjee (1988) of an upsurge from the intermittent burst events of ejection and sweep.

Figure 5.26 shows the skewness $S(u')$ and flatness $F(u')$ factors for Runs 2 to 14. The majority of the cases retained a negative skewness up to the interface. Notice the excursion of the peak flatness from the vicinity of the surface to half of the liquid depth with increasing Reynolds number for which the reasons are not obvious. A likely explanation may be that the influence of bursts in producing Reynolds stress fades near the interface as the region of burst generation, $y^* < 50$, thins with larger Reynolds numbers.

### 5.1.7 Reynolds shear stress

Lacking the vertical velocity fluctuations, the Reynolds stress, $-u'v'$, was assessed from the averaged momentum equation [2.12]. The rate of shearing strain, $\ddot{u}/dy$, therein was
Figure 5.26. Dimensionless distributions of skewness and flatness factors for all runs.
deduced from a best-fit curve through the $\bar{u}$ against $y$ data in lieu of the measured velocity gradient, $d\bar{u}/dy$, whose non-monotonic tendency sends slopes of interchanging signs into the turbulence quantity. A rectangular hyperbolic function was picked for nonlinear regression of the measured mean velocity profile:

$$\bar{u}(y) = \frac{ay}{b + y}. \quad [5.13]$$

This best-fit curve starts from $\bar{u}(y) = 0$ at $y=0$ and passes through $\bar{u}(y) = a/2$ at $y=b$ before stabilizing at $\bar{u}(y) = a$. For Run 7, the parameters, $a=0.204$ and $b=2.225$, were determined to best describe the mean velocity profile. In Figure 5.27, the rendition by the hyperbola is found to be satisfactory. Taking the analytical derivative of the hyperbolic function delivers the rate of strain, $d\bar{u}/dy$,

$$\frac{d\bar{u}(y)}{dy} = \frac{ab}{(b + y)^2}. \quad [5.14]$$

In Figure 5.28, the Reynolds shear stress, $-\bar{u} \bar{v}$, nondimensionalized by the square of the friction velocity, $u^*$, is given. Observe that the wall shear velocity, $u^*$, was evaluated by applying the velocity gradient from the curve fit in the defining relations of [2.6] and [2.20] in place of that from the log law. Aside from the overshoot of the inner region, the theoretical curve [2.33] correlated well with the measured values. Included in Figure 5.28 is the total shear stress, $\tau_{yx(\text{total})}$, that combines the viscous stress, $v d\bar{u}/dy$, and Reynolds stress, $-\bar{u} \bar{v}$, from an overall momentum balance. The equilibrium total shear stress $\tau_{yx(\text{total})}$ is denoted by the straight line going from unity at the wall to zero at the free surface. As a result, the open-channel flow data presented in Figure 5.28 achieved equilibrium. Commencing at $y/h=0.2$, the convergence of the Reynolds stress, $-\bar{u} \bar{v}$, with the total stress, $\tau_{yx}$, suggests the confinement of the viscous stress, $v d\bar{u}/dy$, to the inner layer, $0<y/h<0.2$, as per the prediction of Komori et al. (1993).
Mean streamwise velocity, $\bar{u}$ [m/s]

- Experimental data
- Nonlinear regression $\bar{u} = 0.204y/(2.225+y)$ [5.13]

Figure 5.27. Nonlinear regression of mean velocity distribution.
Figure 5.26: Dimensionless Reynolds shear stress distribution.

- Theoretical curve [2.33]
- Total shear stress [2.12]
- Experimental data

Dimensionless normal distance, \( y/h \)

Dimensionless Reynolds shear stress, \(-\tau'/\nu^2\)
Although not shown, turbulence data for all other runs imply that the Reynolds number does not affect the linear profile of the Reynolds stress, $-\bar{u} \bar{v}$, approaching the interface. The collapse of the Reynolds shear stress, $-\bar{u} \bar{v}$, into one curve for different Reynolds numbers is possible.

As viewed in Figure 5.29, the peak location at $y^*=23$ tends to concur with that predicted by Lam and Banerjee (1988) at $y^*=25$ for open-channel flow rather than that measured by Eckelmann (1974) and computed by Kim et al. (1987) at $y^*=30$ for channel flow. As well, Kim et al. (1987) saw a local maximum at $y^*=12$ which matches the region of greatest Reynolds stress production and streamwise velocity fluctuations from the earlier plots of turbulent intensity. Moin and Kim (1982) and Moser and Moin (1984) postulated this weak rise to come from organized motion in the wall region or more precisely, the ejection and sweep events.

5.1.8 Eddy viscosity distribution

The eddy viscosity, $\nu$, calculated from [2.9] was normalized by the channel depth, $h$, and shear velocity, $u^*$. Featured in Figure 5.30 is the distribution of eddy viscosity, $\nu/hu^*$, together with the theoretical equation [2.35]. Although the experimental results take on the near-parabolic form of the theoretical curve, the turbulence viscosity data are shifted up such that the measured values are overestimated below the mid-depth, $y/h<0.5$, and underestimated above the mid-depth, $y/h>0.5$. Departure from the theory was encountered in the scattered data of Nezu and Rodi (1986), as well. It is believed that the decrease in eddy viscosity near the free surface is brought upon by the enhancement of the energy dissipation rate that reduces the turbulence length scale and restricts the size of the turbulent eddies.

The eddy viscosity $\nu$, [2.9] is isotropic and would not replicate secondary motions driven by turbulence, giving way to a more refined turbulence model. Being a scalar, the eddy viscosity, $\nu$, is identical for the various stress components. Secondary currents will be discussed shortly.
Figure 5.29. Reynolds shear stress distribution in wall coordinates.

- Dimensionless normal distance, $y^+$
- Dimensionless Reynolds shear stress, $-\frac{\tau}{\mu U}$

- Total shear stress [2.12]
- Experimental data

Parameters:
- $u = 0.011 \text{ m/s}$
- $h = 2.0 \text{ mm}$
- $U_{1/4} = 0.14 \text{ m/s}$
- $Re_D = 4500$
- $Re_H = 1600$
- Run 07
Figure 5.30. Eddy viscosity distribution.
5.1.9 Mixing-length distribution

Figure 5.31 shows the mixing length, \( \ell \), obtained experimentally from [2.11] and accompanied by its theoretical relation of [2.36]. The measurements of Figure 5.31 are scaled by the liquid depth, \( h \). It is noteworthy that the mixing length dramatically drops to zero near the surface. Notwithstanding the resemblance of the profiles, the divergence of the data in the outer region, \( y/h > 0.5 \), is illustrative of the sensitivity to the velocity gradient as reported by Nezu and Rodi (1986).

5.1.10 Three-dimensional flow structures

Non-homogeneity and anisotropy of turbulence occasionally give rise to secondary motions while anisotropy at the solid walls and free surface is spurred by their boundary conditions. Anisotropy means that the structure of turbulence in the horizontal plane varies in the streamwise, \( x \), and wall-normal, \( y \), directions. Therefore, secondary currents originate at the free surface and corners of narrow open channels. The flow pattern of cellular secondary currents in Figure 5.32 is an idealized sketch that ignores the side-wall and free-surface effects. Save for wide channels, the free-surface vortex is bigger and stronger than the counter-rotating vortex at the bottom.

Otherwise known as turbulence-driven secondary currents or secondary currents of Prandtl’s second kind, they are recognized by the depression of the maximum velocity below the interface. This velocity-dip phenomenon owes to the free-surface effect of open-channel flows. From Figure 5.32, secondary currents or vortices carry low momentum fluid from the side walls to the channel centre near the free surface and transport high momentum fluid from the interface toward mid-depth. The strong downflow in the middle of the channel induces the velocity dip. Nezu and Rodi (1985) discovered that this dip appears when the aspect ratio, \( \alpha \), is less than a critical value, \( \alpha_c \), of about 5. Nakagawa et al. (1983) suggested a critical value of \( \alpha_c = 4 \) and recommended an aspect ratio, \( \alpha \), of at least 6 (see equation [4.2]) to preserve two-dimensional
Figure 5.31. Mixing length distribution.
Figure 5.32. Schematic flow pattern of cellular secondary currents in narrow open channels.
flow. As stated initially in Section 4.1, the two-dimensionality of the flow was not retained for some runs and anisotropy from the side walls of the rectangular channel is possible.

Situated at the central axis, $z=0$, the mean velocity distribution of the present data were well described by the log-wake law [2.31] even for three-dimensional flows. Approaching the side walls, the velocity should decline and the profile would level off from [2.31].

5.2 Coherent structures

Coherent motions of the turbulent boundary layer were first implied from the statistics of the fluctuations. Visual studies will now complement the quantitative information about turbulent flows and complete the particulars of the underlying structures. It is not obvious a priori what statistics are needed to thoroughly represent the eddies. Nakagawa and Nezu (1981) and Smith and Metzler (1983) gave a statistical description of wall structures.

5.2.1 Detection methods

Three quantitative methods to evaluate the mean ejection period, $\bar{T}_e$, were attempted, none of which are absolute. They are the (i) inversion of the dominant frequency from spectral analysis, (ii) employment of the instantaneous streamwise velocity profiles as the detection signal, according to an appropriate criterion for identifying ejections, and (iii) counting of the frequency with which coherent motions were visually observed. The visual method was considered the most reliable but entailed a degree of discretion. Each of these three methods, in turn, will be discussed at length.

5.2.1.1 Spectral analysis

The power spectral density separates the different frequency components of the velocity signal by assessing the energy at various frequencies. For the conversion to the frequency domain, a fast Fourier transform (FFT) algorithm was applied to the time domain signal. Interestingly, fast Fourier transform (FFT) processing is one way to compute the Doppler frequency of the burst signals in laser-Doppler anemometry (LDA). The data that the fast
Fourier transform (FFT) operates on is presumed to be periodic over the interval being analyzed, or else high frequency components may be introduced. To acquire the finite discrete signal demanded of the Fourier analysis, a one-dimensional spline interpolation was carried out on the velocity history. A cubic spline affords a smoother regression of the data than a linear interpolation.

The constant time interval separating the observations is the inverse of the sampling frequency, \( f_s \),
\[
f_s = N / T ,
\]
where \( N \) is the number of observations and \( T \) is the duration of the signal. For Run 7, the sampling frequency, \( f_s \), was 3 Hz, typical of the runs investigated. The tedious and laborious work of manually converting the analogue signal to a digital format constrained the sampling frequency of data processing to 3-5 Hz, substantially lower than that experienced with hot-film (HFA) and laser-Doppler (LDA) anemometry. In an effort to encompass all ejection events and further ensure that the sampling frequency was higher than any frequencies in the signal, a larger sampling frequency, \( f_{s,\text{theoretical}} \), was instead entered in the spectral analysis:
\[
f_{s,\text{theoretical}} = \frac{N_F}{\overline{N}_F} \]
Now, \( N_F \) is the frame rate of the video camera and \( \overline{N}_F \) is the average number of frames taken to generate the local velocity profile. Run 7 has a theoretical sampling frequency, \( f_{s,\text{theoretical}} \), of 7 Hz, consistent with most runs.

A general criterion for the sampling frequency, \( f_s \), required for data processing of the signals from hot-film (HFA) or laser Doppler (LDA) anemometry is
\[
f_s \geq 2 f_{\text{max}} ,
\]
where \( f_{\text{max}} \) is the maximum response frequency of the device,
\[
f_{\text{max}} = k_{\text{max}} \overline{U}_{\text{bulk}} / 2\pi .
\]
Outside the maximum wave-number, \( k_{\text{max}} \), the scale for turbulence velocity fluctuations, the signal is noise. The satisfaction of [5.17] allows for the elimination of data aliasing.

To analyze the spectral distribution down to the viscous subrange, the following condition on the dimensionless wave-number, \( L_xk_{\text{max}} \), composed of the macroscale, \( L_x \), and the maximum wave-number, \( k_{\text{max}} \), should be met:

\[
L_xk_{\text{max}} \geq 100. \quad [5.19]
\]

Since the macroscale, \( L_x \), has the same order of magnitude as the flow depth, \( h \), in the outer region, the maximum response frequency, \( f_{\text{max}} \), of the turbulence can be estimated by

\[
f_{\text{max}} \geq 100\overline{u}_{\text{Bulk}}/2\pi L_x \approx 50\overline{u}_{\text{Bulk}}/\pi h \quad [5.20]
\]

from [5.18] and [5.19] (Nezu and Nakagawa, 1993).

From [5.20], a hot-film (HFA) or laser-Doppler (LDA) anemometer will respond to a frequency up to 119 Hz in Run 7. Applying [5.17] to flow visualization, the deficiency in sampling frequency for Run 7 is quickly realized. One hundred samples were collected in an irregular fashion for 33 s in Run 7. Again, any interpretation and trend formulated must be made with caution and generalization should be avoided because of the low sampling rate involved. This is not surprising considering that the power spectrum is the Fourier transform of the correlation function which confronted the same problem.

Figure 5.33 shows the power spectral density graphed with respect to frequency. The contour of the plot and major peak at \( f_{\text{Dom}}=0.12 \) Hz are consistent at all distances from \( y^+=20 \) to 30. It is clear that the peak frequency would be too low to implicate coherent structures. Recall that the original data was biased by the selection of certain traces for digitization, prior to polynomial curve fitting.
Figure 5.33. Power density spectra.
5.2.1.2 Conditional sampling

Inferring the ejection frequency from the dominant frequency in a power density spectrum was questionable as the sampling rate used was judged to be inadequate earlier. Counting the number of wall turbulence events over a prescribed time is another option and will be detailed below. The u-level technique of Lu and Willmarth (1973), from which this scheme is heavily borrowed, identifies an event when the instantaneous velocity fluctuation, \( u' \), is less than the negative product of the RMS fluctuating velocity, \( u'_{\text{RMS}} \), and a threshold level, \( L \).

\[
u' < -Lu'_{\text{RMS}}
\]  

[5.21]

Instead, a reduction in streamwise velocity was deemed an ejection if it surpassed a fraction of the largest drop in velocity for that run. An initial threshold value of 30% was used and counting the incidence of trace deformation on video corroborated the ensuing frequency. This threshold level is arbitrary and hence requires validation. The next section goes through the details of the third method of visual observation. To remedy the inconsistency of the two methods, the sampling criterion was relaxed to detect more events. Table A.2 indicates that the ejection frequency from conditional sampling lags that of visual counting, even for a criterion of zero percent which incorporates all reductions in velocity. Conceivably, the poor sampling rate could cause some turbulence structures to go undetected.

Of greater bearing is that the streamwise velocity, \( u \), cannot be treated alone. The quadrant splitting of the \( u'v' \) signal uses both the fluctuating components of the streamwise, \( u' \), and wall-normal, \( v' \), velocities. Wallace et al. (1972) and Willmarth and Lu (1972) defined an ejection as a second-quadrant event (-\( u' \), +\( v' \)) in the instantaneous \( u'v' \)-plane. Lacking the vertical velocity, \( v \), a distinction cannot be made between second- and third-quadrant events. Negative contributions to the Reynolds shear stress are made in the third-quadrant (-\( u' \), -\( v' \)).
5.2.1.3 Visual count

Any pattern recognition necessitates some individual judgement. Visual data have oftentimes been stigmatized as primitive and subjective to interpretation but flow visualization is the only reliable way to embody all the traits of coherent structures. From the video pictures, wall turbulence events were counted in a window of 30 x 25 mm² in area for a certain period of time. The frequencies of occurrence will be presented later with regards to that velocity field fixed in space - not normalized per unit area.

5.2.2 Scaling laws

Whether the average periods of ejection, $\bar{T}_E$, and bursting, $\bar{T}_B$, can be nondimensionalized by the inner or outer variables is an unresolved issue, in spite of the numerous laboratory experiments on various aspects of the bursting process. It is unclear if the turbulence production processes are controlled by near-wall events or triggered by the passage of motions in the outer flow.

5.2.2.1 Ejection period

In Figure 5.34(a), the folding of the parallel dye-lines denotes the occurrence of an ejection away from the wall. The event pierces through the smooth dye-lines to bring on a chaotic pattern. Its trajectory could be retraced through the liquid layer to the solid surface. As stated in Section 5.1.1, an ejection produces an inflection in the velocity profile. The trace pattern of Figure 5.34(b) is associated with a burst for which the inclination angle of the trace to the channel floor is about 45°.

As said initially, the temporal ejection frequency was determined from the visual count data. Figure 5.35 shows the frequency of ejections, $f_E$, scaled by the inner variables, $\nu/u^2$, outer variables, $h/\bar{u}_{bulk}$, and a mixture of the inner and outer variables. It is evident that the ejection frequency scaled by the inner variables, $f_E\nu/u^2$, is approximately constant over the range of
Figure 5.34. Typical dye-line patterns of wall turbulence events in Run 07: (a) ejection; and (b) burst.
Reynolds number, $Re_h$

Dimensionless ejection frequency scaled with inner variables, $f_{E/v^*}^2$

Dimensionless ejection frequency scaled with mixed variables, $f_{E(h/v^*u_{Bulk}^*)}^{2,1/2}$

Dimensionless ejection frequency scaled with outer variables, $f_{Eh/u_{Bulk}}$

Figure 5.35. Dimensionless ejection frequencies.
Reynolds numbers tested. The frequency of occurrence normalized by the mixed variables,
\[ f_E \left( \frac{h v}{\bar{u}_{bulk} u^*} \right)^2, \]
is a strong function of the Reynolds number. In Figure A.4 of the appendix, the dimensionless frequencies from conditional sampling by Nasr-Esfahany (1998) seem to behave similarly. His ejection frequency, \( f_E \), scaled best with the inner variables, \( v/u^* \).

Dimensional frequency of ejection events versus wall friction velocity is plotted in logarithmic scale in Figure 5.36 and implies that ejections occur more frequently when the shear stress is larger at the wall. Linear regression through the data discloses a stronger dependence of the ejection rate on the shear velocity than was found by Nasr-Esfahany (1998), whose values are also presented in the figure. Furthermore, the straight line confirms that the variables, \( f_E \) and \( u^* \), are related by a power function. The slope is the exponent of the power equation and a value of 3 was obtained. Nasr-Esfahany (1998) reported a slope of 2 for a channel flow of similar Reynolds number but different Froude number, because of a higher average velocity and shallower depth.

For low Reynolds numbers, \( Re_h<15,000 \), Rashidi (1997) maintained that the mean period between ejections, \( \bar{T}_E \), is best scaled with the wall variables, \( u^*/v \),
\[ \bar{T}_E^* = \frac{\bar{T}_E u^*/v}{38}. \] [5.22]
In Figure 5.37, the ejection periods for all runs are scaled by the inner variables. Nonlinear regression of the data yields a curve that falls with an increasing Reynolds number to a dimensionless period of \( \bar{T}_E^* = 39 \) at \( Re_h=2300 \), consistent with the assertion by Rashidi (1997).

### 5.2.2.2 Bursting period

The temporal frequency of bursting events was obtained by visually counting the events, and the mean bursting period, \( \bar{T}_B \), was subsequently computed. Adjacent ejection events were grouped together and treated as a single bursting event in line with the comments by Kaftori et al. (1994) and Rashidi (1997) that a burst can be comprised of two or more ejections. The
Figure 5.36. Ejection frequency against friction velocity.
Figure 5.37. Dimensionless ejection periods based on inner variables.
number of bursting events was equated to the number of grouped ejections. Naturally, a certain amount of discretion pertains to the grouping of the ejection events. Moreover, bursting motions were correlated with coherent motions that were sloped at an angle of 45° to the channel floor. Longitudinal vortices lift up chaotically or are ejected at that angle. Those inclined at a smaller angle and observed below \( y^+ = 20 \) were disregarded. Nezu and Nakagawa (1993) characterized the bursting motion as a hairpin vortex inclined to the wall at that angle. Recall that the visual observations take place in a field that spans a streamwise distance of 30 mm. All video recordings were examined in their entirety.

There is a lack of consensus concerning the scaling of the mean bursting period. Nakagawa and Nezu (1978) remarked that the mean periods of ejection, \( \bar{T}_E \), sweep, \( \bar{T}_s \) and bursting, \( \bar{T}_b \), are alike when normalized by such outer variables as the maximum velocity, \( u_{\text{Max}} \), and flow depth, \( h \):

\[
\frac{\bar{T}_E u_{\text{Max}}}{h} \approx \frac{\bar{T}_s u_{\text{Max}}}{h} \approx \frac{\bar{T}_b u_{\text{Max}}}{h} = (1.5 - 3.0). \tag{5.23}
\]

These relations are independent of the Reynolds and Froude numbers, and suggestive of the scaling of the mean bursting period, \( \bar{T}_b \), with the outer variables, \( h/u_{\text{Max}} \). Kim et al. (1971), Rao et al. (1971) and Lu and Willmarth (1973) understood the bursting phenomenon as an inner-wall process governed by the outer region. Implicit in [4.28] is an assumption that a burst, on the average, is made up of one ejection and sweep in space and time. The field data for geophysical flows of high Reynolds numbers, \( Re = 10^6 - 10^7 \), seem to uphold the outer hypothesis.

Blackwelder and Haritonidis (1983) and Luchik and Tiederman (1987) insisted on the scaling of the bursting frequency, \( f_b \), with the inner variables. Blackwelder and Haritonidis (1983) argued that the mean bursting period, \( \bar{T}_b \), is inflated when the length of the probe used to detect bursting is longer than \( 20v/u^* \). Many investigators violated this criterion at high Reynolds
numbers in which this spatial averaging effect is heightened and may have been misled to the outer hypothesis.

Adding to the confusion, Alfredsson and Johansson (1984) accentuated the interaction of the inner and outer regions with a mixture of inner and outer variables to scale the bursting frequency, \( f_B \).

As illustrated in Figure 5.38, the average bursting period ranged in value from \( \bar{T}_b^* = 110 \) to 150. When scaled with the wall variables, it stays fairly constant over the ranges of Reynolds, Re, and Froude, Fr, numbers covered. For comparison, the mean bursting period from Nakagawa and Nezu (1978) is reproduced in Figure A.5 of the appendix. Their bursting periods are shorter and lengthened with Reynolds number.

Nonlinear regression of the present data in Figure 5.39 yields a trend that slowly descends to a scaled bursting period, \( \bar{T}_b^* \), of 119 for the last run. Other people (see Rashidi, 1997; Hetsroni and Mosyak, 1996; Kline et al., 1967; and Kim et al., 1971) have measured the average bursting period,

\[
\bar{T}_b^* = \bar{T}_b \frac{u^*}{v} = 85 \sim 100. \tag{5.24}
\]

Kasagi et al. (1986) claimed a bursting period of 150 based on modelling and experiment. Therefore, the bursting period from this study is proportional to the inner time scale, \( v/u^* \), at least at low Reynolds numbers and the period of bursting motions is regulated by the inner variables.

5.2.3 Flow visualization

To overcome complications brought on by variations in the spatial shape and convection velocity of coherent structures and their three-dimensional trajectories, the integration of flow visualization and velocity measurements was done. Visual pictures permit the qualitative observation of the spatial structure and genesis of spiral eddies and their interaction with the free
Reynolds number, $Re^*$

- Run 02, $Re^* = 83$, $Re_h = 940$, $Re_D = 2900$
- Run 03, $Re^* = 89$, $Re_h = 1100$, $Re_D = 3200$
- Run 04, $Re^* = 98$, $Re_h = 1200$, $Re_D = 3600$
- Run 05, $Re^* = 108$, $Re_h = 1300$, $Re_D = 3900$
- Run 06, $Re^* = 115$, $Re_h = 1400$, $Re_D = 4200$
- Run 07, $Re^* = 123$, $Re_h = 1600$, $Re_D = 4500$
- Run 08, $Re^* = 132$, $Re_h = 1700$, $Re_D = 4800$
- Run 09, $Re^* = 141$, $Re_h = 1900$, $Re_D = 5200$
- Run 10, $Re^* = 152$, $Re_h = 2000$, $Re_D = 5400$
- Run 11, $Re^* = 165$, $Re_h = 1900$, $Re_D = 5100$
- Run 12, $Re^* = 179$, $Re_h = 2000$, $Re_D = 5400$
- Run 13, $Re^* = 177$, $Re_h = 2100$, $Re_D = 5700$
- Run 14, $Re^* = 192$, $Re_h = 2300$, $Re_D = 5900$

Figure 5.38. Dimensionless bursting period based on inner variables against shear Reynolds number.
Figure 5.39. Dimensionless bursting period based on inner variables against flow Reynolds number.
surface, disclosing the direction, location and shape of these vortices. Their qualitative
information can enrich the quantitative data from which an outline of a physical model can be
assembled. Dye traces uncover the local and active motions in the flow. Nonetheless, caution
should be taken when interpreting the visual observations as the dye tends to be cumulative and
exhibits a flow history.

Simultaneous visualization of the flow from the top, side and front was performed but
matching the trajectories of the traces in different planes to the same coherent motion to recreate
the organized structure in space was not fully pursued for lack of time. To facilitate the
surveillance from all three directions, all the recordings from the quad should be analyzed. Some
structures were watched from multiple views to avoid erroneous interpretation of the motion of
the dye traces.

For the Lagrangian flow description, an individual trace was followed so as to tag a
coherent structure during its development. Several other arbitrary markers were tracked in the
preliminary trials to ensure that the speed of the video camera platform equaled the mean liquid
velocity. The properties of the turbulent structures could be observed over a stretch of 1.5 m.
Fastening a video camera to a moving carriage going at the same speed as the coherent motion in
the longitudinal direction is still problematic and far from trivial since the eddy is limited in
magnitude and changes its shape as it moves.

An effort was made to prevent flow separation and the creation of a wake region
upstream of the immersed boroscope, which moved through the liquid, by streamlining its
cylindrical body. A fold of aluminum foil like the bow of a ship served this purpose.

Whenever the method of flow visualization is used, some prudence is imperative and
more so when the Lagrangian view is applied. From the perspective of the viewer in motion, the
inrush (A) and eddy (B) of Figure 5.40 look as though they are flowing backward. On further
contemplation, the structures appear as such by reason of their streamwise velocities being lower
Figure 5.40. Inflow (A) and eddy (B) in Lagrangian frame (Run 02).
than the speed of the moving camera. There is no backward flow. The Lagrangian description of flow reduces the longitudinal velocity to emphasize the lateral and surface-normal components.

To illustrate the flow pattern in an unequivocal manner, the still images from the side view offered the most clarity and information. From the end view, existing traces in the foreground and those newly formed in the background obscured the tracer trajectory. Insufficient lighting and video equipment also contributed to the less than adequate quality of the pictures captured from the top view. Insight into the lifetime and stability of the vortices were inconclusive from the plan view due to the rapid fading of the traces. When the video tape was played in slow motion, the grainy character of still frames could be averted and organized motions became apparent.

5.2.3.1 Bursting phenomena

Figure 5.41 shows the traces from the end view in which the mean streamwise flow goes out of the paper. The free surface is at the top and the no-slip boundary is at the bottom. This view is of a cross-sectional plane in the spanwise direction. The evolution of the flow pattern is given with respect to time units. In Figure 5.41, the breakdown of the low-speed streak with the lift-up of the streaky structure into the bulk flow can be viewed. In the region of interest, the streak as an elongated mass of low axial velocity can be made out. Kline et al. (1967) discovered streaks when their hydrogen microbubbles were separated into an alternating array of high-speed and low-speed regions.

Coupled with the streak breakdown is the advent of a cone-shaped structure as spotted near the solid boundary in Figure 5.42. For the side view, the flow moves from the right to the left. Blackweldier and Eckelmann (1979) extracted details of the counter-rotating streamwise vortices from hot-film sensors and elements affixed to the wall. Rashidi and Banerjee (1990a) used small oxygen bubbles to elicit sequential pictures of low-speed streaks between pairs of
Figure 5.41. Lift-up in Lagrangian frame (Run 02).

(a) t=0

(b) t=0.27 s

(c) t=0.53 s

END VIEW
Figure 5.42. Formation of streamwise vortex in Eulerian frame (Run 03).
longitudinal vortices near the boundary. The vortices beside the wall are primarily quasi-streamwise. Regard the streak behind the streamwise vortex in the video sequences (d) to (g) that is lifted by the higher momentum fluid of the core. As anticipated from the conception of Figure 3.2, an inclined vortex pair arises from the ascension of the vorticity sheet encompassing the wall streak. A complete hairpin vortex with two legs is seldom seen and less so for flows of low Reynolds numbers. Instead, hockeystick vortices composed of a single leg are more prolific.

A secondary streamwise vortex is formed to the periphery of the prevailing vortical structure in Figure 5.43. Fluid drawn to the bottom floor by the rotating motion of the first vortex counter-rotates as stated in Section 3.4.1.

In Figure 5.44, the sequence of frames reveals a clockwise rotation of the vortex near the interface as deduced from the variation in space and time of the curved trace. Perhaps this is the tubular structure of Figure 3.5 that redistributes the normal component of turbulent kinetic energy to the transverse component, leading to anisotropy in turbulence near the free surface. The circular trace is really a cross-section of the legs of a hairpin vortex. Its outset from the wall region could not be observed. Experimental validation of such structures is new and the findings of this work supplement that of Nasr-Esfahany (1997) who gave a plan view of a streamwise vortex below the free surface. It would be instructive in the future to complement the aforementioned qualitative discussion with some quantitative aspects such as the direction of rotation, scale, frequency and temporal longevity of the longitudinal vortices.

The traces of the outer region are prone to flow sideways as learned in Figure 5.45. This action corroborates the presence of free-surface structures and some coherent motion in the spanwise direction. Near the wall, the lateral motion is subdued by the no-slip condition. It should be pointed out that the flow configuration is narrow in contrast to its extent in the flow direction and the spanwise velocity is small relative to that in the direction of flow. In close proximity to the boroscope, there is an exaggeration of the actual movement of the traces and
Figure 5.44. Streamwise vortex near interface in Lagrangian frame (Run 07).
Figure 5.45. Lateral movement of traces from left to right in Lagrangian frame (Run 01).
scale of the flow structure. Additionally, the fact that the extent of the flow across the channel must be finite begs the question about the encouragement of instability of the vortex tubes by side-wall effects. For open channels of low aspect ratios, bursting motions from the side walls are of concern.

Three consecutive ejections are depicted from a sequence of side-view images in Figure 5.46. Rolling spanwise vortices look as if emanating from the sloping shear layer in the first five frames and they nearly reach the free surface. The shear-layer interface is constructed when the high-speed fluid impacts the low-speed fluid. A sweeping event is already descending upon the shear layer as it emerges in Figure 5.47. Ejections renew the surface region with fresh fluid from the wall while sweeps carry surface fluid to the bulk flow. Both phases of bursting are important to interfacial heat and mass transfer across the interface.

Dye visualization in Figure 5.48 renders a picture of a spanwise vortex formed from the roll-up of low-momentum fluid. A coherent vortex as the head of the hairpin vortex, exits from the shear layer of Figure 5.49 and another arrives at the interface in Figure 5.50. Nasr-Esfahany (1997) saw a rolling eddy moving along the interface. It is believed that the spanwise vortex rolling in the direction of flow is responsible for the redistribution of the vertical component of turbulent energy to the streamwise component near the free surface. A pair of spanwise vortices rotating in opposite directions is implied in Figure 5.51 from the movement of the traces.

The likeness of an embryonic hairpin vortex is discernable in the successive frames of Figure 5.52. The transverse vortex from the side view can be thought as the head of the hairpin vortex. As established, the inclination angle between the legs of a hairpin vortex and the bottom wall is more or less 20°. The angle of the head or transverse vortex is much steeper at about 45°. In frames (c) and (d), there may be a concentration of low-moving fluid between the legs. Figure 5.53 sort of shows the onset of hairpin vortex oscillation to be proceeded by one or more ejections of wall fluid. A section of the vortex is suddenly expelled towards the interface.
Figure 5.46. (continued) Evolution of three ejections in Lagrangian frame (Run 02).
Figure 5.47. Collision of sweep with shear layer in Lagrangian frame (Run 02).
Figure 5.48. Formation of spanwise vortex in Eulerian frame (Run 08).
Figure 5.49. Ascension of spanwise vortex from shear layer in Lagrangian frame (Run 03).
Figure 5.50. Spanwise vortex near interface in Lagrangian frame (Run 03).
Figure 5.51. Counter-rotating spanwise vortices from shear layer in Lagrangian frame (Run 03).
Figure 5.52. Formation of hairpin vortex in Eulerian frame (Run 05).
Figure 5.53. Oscillation of hairpin vortex in Lagrangian frame (Run 2).
Reinforced by smoke visualization, Head and Bandyopadhyay (1981) showed a hairpin structure in the log region of the turbulent boundary layer.

5.2.3.2 Large-scale vortical motions

A coherent structure such as a burst travelling upward from the wall region produces an upwelling moving towards the free surface. This upwelling verges upon the surface and accelerates the fluid caught between the interface and itself. In Figure 5.54, a dark lump of fluid is stretched in the axial direction. The streamwise acceleration could result in renewed interfacial patches and a slight elevation of the surface. In the photo sequence of Figure 5.55, the impact of an upwelling on the free surface is akin to a jet hitting a solid surface and yielding regions of strong rotational fluid movement. Watch how the redirected fluid rolls down towards the wall, giving rise to spanwise vortices near the interface. Kumar et al. (1998) gave simultaneous visualization of an upwelling from the side view and on the free surface using dye injection and particle image velocimetry (PIV). Boils of the third kind have been observed for flow at low Reynolds number and implied in conditionally averaged velocity patterns, nevertheless their weak nature and the presence of small waves on the surface hampered their detection.

As the upwelling approaches the surface, the instantaneous streamlines above the upwelling should radiate outwards to create a donut-shaped aggregate of high vorticity in the macroscopic fluid motion, not to be mistaken with a vortex ring. Originally parallel, the streamlines would converge at the edge of an upwelling (Pan and Banerjee, 1995). Rashidi (1997) visualized interfacial patches affiliated with the burst renewal of the interface using oxygen bubble tracers. Upwellings and spiral eddies were detected by Kumar et al. (1998) who applied neutrally buoyant microballoons to flow visualization.

From the footage of Figure 5.56, adjacent upwellings appear to collide. Although not shown, the top view of the free surface would disclose a bundle of streamlines resulting from the
Figure 5.54. Streamwise acceleration by upwelling in Lagrangian frame (Run 02).
Figure 5.55. Upwelling near interface in Lagrangian frame (Run 04).
Figure 5.56. Collision of upwellings in Lagrangian frame (Run 02).
radial motion of the upsurging vortices. Streamlines fan outwards to converge into an intense stagnation line with which downdrafts are identified. Spiral eddies are located at the edges of the upwellings, beside the stagnation lines.

The series of photographs in Figure 5.57 support the notion of a vortex reconnection process indifferent to surface deformation. A spiral eddy was generated over a time interval in excess of 4 s. It is argued that the surface vortex grows normal to the liquid surface by reflection of the wall bursts but a comprehensive explanation of its evolutionary process remains elusive. This persistent structure is oriented surface normal and retains its quasi-two-dimensional identity over a sizable distance until annihilated by a fresh upwelling. Due to the poor resolution of the top view camera, the persistence of the spiral eddy was not explored. Eddies that rotate in the same direction amalgamate while those that spin in opposite directions annul each other. Vertical vorticity is indicated in Figures 5.58 and 5.59. In Figure 5.58, counterclockwise rotation of the two traces is perceived. The trace of Figure 5.59 turns in a clockwise direction like a whirlpool. From the plan view, surface vortices are recognized by the streamlines in a spiral pattern. Their movement in the spanwise direction is owed to local instability. Mainly, upward and downward flows and spiral eddies are observed in the areas close to the free surface. Kumar et al. (1998) illustrated the inception and interaction of spiral eddies through the measurement of the velocity field by particle image velocimetry (PIV).

The motion of a downswinging spanwise vortex almost touches the wall in Figure 5.60. Figure 5.61 portrays a downdraft that is diverted into the streamwise direction, probably by an upwelling.

5.2.3.3 Summary

Coherent structures were examined over a low-Reynolds number range of 800 to 2,400 based on liquid depth, flow conditions at which some organized turbulent motions have not been reported before. Low-speed streaky structures were lifted from the channel bottom by the
Figure 5.57. Formation of spiral eddy in Lagrangian frame (Run 02).
Figure 5.57. (continued) Formation of spiral eddy in Lagrangian frame (Run 02).
Figure 5.58. Counter-clockwise rotation of spiral eddy near interface in Lagrangian frame (Run 04).
Figure 5.59. Spiral eddy in Lagrangian frame (Run 04).
Figure 5.60. Descension of downdraft to wall in Lagrangian frame (Run 03).
Figure 5.61. Deflection of downdraft off shear layer in Lagrangian frame (Run 03).
upward motion of funnel-shaped eddies. Moreover, the inflow of the counter-rotating streamwise vortices brought about secondary streamwise vortices. Traces beneath the gas-liquid interface were observed to reorient their axes of rotation and attach to the free surface to form spiral eddies. For the first time, this experimental finding substantiates the origin of spiral eddies from a realignment of the surface-parallel vorticity to the surface-normal direction as conjectured by Kumar et al. (1988). Spanwise vortices from the roll-up of low-momentum fluid were seen to ascend to the interface, whereupon they reflect downwards toward the wall. Streamwise acceleration of the flow between the upwellings and free surface was also noticed. If unimpeded, the downdrafts descended to the bottom wall. The named structures of turbulence have been observed or predicted in the past, but for open-channel flow at higher Reynolds numbers. Tamburrino (1997) visualized only spanwise and streamwise vortical motions at the lowest flow Reynolds number, $Re_h=2005$, reported in the literature.
6.1 Conclusions

Open-channel flow was investigated under some flow conditions that have not been explored previously. Experiments were performed over a low-Reynolds number, $Re_h$, range of 800 to 2,400 based on flow depth and a Froude number, $Fr$, range of 0.25 to 0.32. Statistical parameters obtained from photochromic dye-trace measurements of instantaneous velocity profiles coincided well with the numerical calculations by other workers for the mean velocity, $\bar{u}$, turbulence intensity, $u'_{\text{RMS}}$, and skewness, $S(u')$, and flatness, $F(u')$, factors. At higher Reynolds numbers, the measured mean velocity agreed with the law of the wall and the equation for pipe flow by Spalding (1961).

For Run 7 at $Re_h=1600$, a slight increase in the turbulence intensity, $u'_{\text{RMS}}$, for $y/h>0.9$ suggested the interfacial damping of vertical movement by surface tension and gravity, and redistribution of turbulent kinetic energy from the vertical to the streamwise and transverse directions. The turbulence intensity had a maximum value of $u'_{\text{RMS}}/u^* = 2.6$ at $y^*=13.5$. The RMS fluctuations in the streamwise velocity obtained agreed with the universal function for turbulence intensity by Nezu (1977). Towards increasing Reynolds numbers, the relative RMS of fluctuations was enhanced by the occurrence of more bursting motions. Above $y^*=20$, the turbulence intensity levelled off to $u'_{\text{RMS}}/u^* \approx 1$.

The negative skewness of the streamwise fluctuating velocity in the bulk flow implied that bursts from the wall travelled the entire liquid depth for Run 7. The skewness factor interchanged signs from positive to negative at $y^*=20$ in that run. At greater Reynolds numbers, the peak in the flatness of velocity fluctuations moved to mid-depth from the interface because bursts generated from the bottom wall have less impact on turbulence near the interface at larger
Reynolds numbers. This was supported by the shortening of the negative tails with increasing height in the probability density functions.

A good fit of the Reynolds stress estimated from the measurements with the theoretical values was seen in Run 7 and the measured Reynolds stress peaked at $y^+=23$.

A novel method was proposed to identify wall turbulence events from the instantaneous velocity profiles obtained in a Lagrangian frame. It was recognized that the switch from negative to positive velocity occurs furthest to the free surface for ejections and nearest to the wall for sweeps.

Quantitative measurements unveiled the impression of vortex-loop structures embedded in the bottom boundary layer. This turbulent boundary layer consists of hairpin vortices and possesses a mean velocity profile that adheres to a log-type distribution.

Both the present experiment and direct numerical simulations by other researchers show that upwellings are the main turbulent structures that occur at the free surface of an unsheared and waveless liquid stream under turbulent flow in an open channel. Driving the flow outward, upwellings were seen to arise from the impingement of large organized structures on the surface. Situated amidst upwellings were downdrafts. The surface-parallel vortices were believed to reconnect to the free surface to form cyclone-like vortices called spiral eddies. Vortical tubes along the interface are believed to emanate from the streamwise vortices of the wall region. These characteristics were verified at low Reynolds numbers through flow visualization, which confirmed qualitatively the velocity field constructed by numerical techniques.

From visual counting of turbulent bursts near the bottom wall, the physical parameters characterizing wall structures were found to be governed by the inner variables. The ejection frequency was proportional to the cube of the wall friction velocity. The inner scaling of the ejection and bursting periods improved at higher Reynolds numbers. When scaled by the friction velocity and viscosity, mean ejection and bursting periods of 39 and 119 were obtained for the
highest Reynolds number of $Re_h=2300$, respectively. These results were consistent with the values reported in other studies for the lowest Reynolds numbers available of $2500<Re_h<3100$ (see Rashidi, 1997; and Hetsoni and Mosyak, 1996).

If direct numerical simulations of free-surface turbulence are conducted at the low-Reynolds number range reported here, numerical predictions could be compared with the results of the present work.

6.2 Recommendations

Unquestionably, the most serious defect of the image processing technique used here is that the vertical velocity cannot be calculated from the continuous lines of dye. The fluctuating surface-normal velocity, $v'$, can be determined by tracking the trajectory of a coherent structure in physical space. Subjected to time-constraints, this step was not taken and a more precise analysis of the two-dimensional flow field awaits further work. Also, grids of dye traces can be used to resolve the vertical velocity flow field.

More numerical simulation and experimentation are necessary to elucidate the surface-normal distance over which two-dimensional behaviour preside and ascertain the mechanism behind the formation of spiral eddies. It would be constructive to conduct future tests in a channel of sufficient width in order to be free of edge effects and sustain two-dimensional flow. Additionally, experiments over an extended range of Reynolds numbers may solve the discrepancy of the scaling laws.
REFERENCES


Prandtl, L., 1925. ZAMM 5, 136.


Figure A.1. Density of test fluid against temperature (from Van Waters & Rogers Ltd.).
Figure A.2. Viscosity of test fluid against temperature (from Van Waters & Rogers Ltd.).
Figure A.3. Wake strength parameter, $\Pi$, against Reynolds numbers, $Re'$ and $Re_D$ (from Nezu and Nakagawa, 1993).

Note:
$R_e = Re'$
$R_h = Re_D$
Figure A.4. Dimensionless ejection frequency (from Nasr-Esfahany, 1998).
Figure A.5. Average bursting period, scaled with the inner variables, against Reynolds numbers, Re_0 and Re* (from Nezu and Nakagawa, 1993).

Note:
\[ \bar{T}_B u'^2 / \nu = 0.65 Re_0^{0.73} \]  [8.26]
\[ R_0 = Re_0 \]
\[ R_* = Re^* \]
### Table A.1. Comparison of wall friction velocity by different methods.

<table>
<thead>
<tr>
<th>Run</th>
<th>(Re_h)</th>
<th>(Re_D)</th>
<th>(Re^*)</th>
<th>Fr</th>
<th>(u_{\text{log-law}} [\text{m/s}])</th>
<th>(u^*_{\text{Blasius}} [\text{m/s}])</th>
<th>((u_{\text{log-law}}-u^<em>_{\text{Blasius}})/u^</em>_{\text{log-law}} [%])</th>
<th>(u_{\text{vis}}[\text{m/s}])</th>
<th>((u_{\text{log-law}}-u_{\text{vis}})/u_{\text{log-law}}[%])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>840</td>
<td>2600</td>
<td>87</td>
<td>0.26</td>
<td>0.0104</td>
<td>0.0072</td>
<td>31</td>
<td>0.0100</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>940</td>
<td>2900</td>
<td>83</td>
<td>0.28</td>
<td>0.0095</td>
<td>0.0076</td>
<td>20</td>
<td>0.0113</td>
<td>-20</td>
</tr>
<tr>
<td>3</td>
<td>1100</td>
<td>3200</td>
<td>89</td>
<td>0.29</td>
<td>0.0096</td>
<td>0.0080</td>
<td>17</td>
<td>0.0110</td>
<td>-14</td>
</tr>
<tr>
<td>4</td>
<td>1200</td>
<td>3600</td>
<td>98</td>
<td>0.30</td>
<td>0.0100</td>
<td>0.0084</td>
<td>16</td>
<td>0.0116</td>
<td>-16</td>
</tr>
<tr>
<td>5</td>
<td>1300</td>
<td>3900</td>
<td>108</td>
<td>0.30</td>
<td>0.0104</td>
<td>0.0087</td>
<td>16</td>
<td>0.0118</td>
<td>-14</td>
</tr>
<tr>
<td>6</td>
<td>1400</td>
<td>4200</td>
<td>115</td>
<td>0.31</td>
<td>0.0106</td>
<td>0.0090</td>
<td>15</td>
<td>0.0124</td>
<td>-17</td>
</tr>
<tr>
<td>7</td>
<td>1600</td>
<td>4500</td>
<td>123</td>
<td>0.31</td>
<td>0.0108</td>
<td>0.0092</td>
<td>15</td>
<td>0.0111</td>
<td>-2</td>
</tr>
<tr>
<td>8</td>
<td>1700</td>
<td>4800</td>
<td>132</td>
<td>0.32</td>
<td>0.0111</td>
<td>0.0095</td>
<td>15</td>
<td>0.0111</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>1900</td>
<td>5200</td>
<td>141</td>
<td>0.33</td>
<td>0.0115</td>
<td>0.0099</td>
<td>14</td>
<td>0.0117</td>
<td>-2</td>
</tr>
<tr>
<td>10</td>
<td>2000</td>
<td>5400</td>
<td>152</td>
<td>0.32</td>
<td>0.0118</td>
<td>0.0099</td>
<td>16</td>
<td>0.0112</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>1900</td>
<td>5100</td>
<td>165</td>
<td>0.30</td>
<td>0.0129</td>
<td>0.0097</td>
<td>24</td>
<td>0.0118</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>2000</td>
<td>5400</td>
<td>179</td>
<td>0.31</td>
<td>0.0135</td>
<td>0.0099</td>
<td>27</td>
<td>0.0124</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>2100</td>
<td>5700</td>
<td>177</td>
<td>0.32</td>
<td>0.0131</td>
<td>0.0102</td>
<td>22</td>
<td>0.0120</td>
<td>9</td>
</tr>
<tr>
<td>14</td>
<td>2300</td>
<td>5900</td>
<td>192</td>
<td>0.32</td>
<td>0.0138</td>
<td>0.0104</td>
<td>25</td>
<td>0.0123</td>
<td>11</td>
</tr>
</tbody>
</table>

Note:
- \(u_{\text{log-law}}\) = wall friction velocity from fitting of mean velocity data to log-law [2.29].
- \(u^*_{\text{Blasius}}\) = wall friction velocity from Blasius formula [4.10].
- \(u_{\text{vis}}\) = wall friction velocity from Newtonian viscosity law [2.6] and friction velocity definition [2.20].
Table A.2. Comparison of ejection frequency by different methods.

<table>
<thead>
<tr>
<th>Run</th>
<th>Re_h</th>
<th>Re_D</th>
<th>Re*</th>
<th>f_{E(visual)} [Hz]</th>
<th>f_{E(u')} [Hz]</th>
<th>f_{E(spectral)} [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>840</td>
<td>2600</td>
<td>87</td>
<td>0.33</td>
<td>1.53</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>940</td>
<td>2900</td>
<td>83</td>
<td>0.80</td>
<td>1.40</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>1100</td>
<td>3200</td>
<td>89</td>
<td>0.85</td>
<td>1.41</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>1200</td>
<td>3600</td>
<td>98</td>
<td>1.20</td>
<td>1.61</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>1300</td>
<td>3900</td>
<td>108</td>
<td>1.38</td>
<td>1.70</td>
<td>0.38</td>
</tr>
<tr>
<td>6</td>
<td>1400</td>
<td>4200</td>
<td>115</td>
<td>1.16</td>
<td>1.50</td>
<td>0.46</td>
</tr>
<tr>
<td>7</td>
<td>1600</td>
<td>4500</td>
<td>123</td>
<td>1.60</td>
<td>1.50</td>
<td>0.12</td>
</tr>
<tr>
<td>8</td>
<td>1700</td>
<td>4800</td>
<td>132</td>
<td>1.69</td>
<td>1.90</td>
<td>0.18</td>
</tr>
<tr>
<td>9</td>
<td>1900</td>
<td>5200</td>
<td>141</td>
<td>1.56</td>
<td>1.90</td>
<td>0.73</td>
</tr>
<tr>
<td>10</td>
<td>2000</td>
<td>5400</td>
<td>152</td>
<td>2.15</td>
<td>1.76</td>
<td>0.26</td>
</tr>
<tr>
<td>11</td>
<td>1900</td>
<td>5100</td>
<td>165</td>
<td>2.03</td>
<td>1.35</td>
<td>0.61</td>
</tr>
<tr>
<td>12</td>
<td>2000</td>
<td>5400</td>
<td>179</td>
<td>2.16</td>
<td>1.91</td>
<td>0.15</td>
</tr>
<tr>
<td>13</td>
<td>2100</td>
<td>5700</td>
<td>177</td>
<td>2.72</td>
<td>2.44</td>
<td>0.05</td>
</tr>
<tr>
<td>14</td>
<td>2300</td>
<td>5900</td>
<td>192</td>
<td>2.56</td>
<td>2.29</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note:
- $f_{E(visual)}$: Ejection frequency from visual counting
- $f_{E(u')}$: Ejection frequency from conditional sampling for threshold value of 0 % at $y^*=20$
- $f_{E(spectral)}$: Ejection frequency from inverse of dominant frequency at $y^*=20$