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UMI
NOVEL MULTIMODE FIBRE GRATINGS FOR
MEDICAL RAMAN SPECTROSCOPY

by

VANI PASUPATHY

A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
Graduate Department of Electrical and Computer Engineering
University of Toronto

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0-612-63025-0
NOVEL MULTIMODE FIBRE GRATINGS FOR MEDICAL RAMAN SPECTROSCOPY

A thesis for the degree of Master of Applied Science

Vani Pasupathy

Graduate Department of Electrical and Computer Engineering

University of Toronto

Abstract

We present analysis of a novel multimode fibre tailored to allow the creation of fibre Bragg gratings with narrowband, high reflectivity, response. The proposed fibre consists of concentric cylindrical shells of higher and lower refractive index material. Modal propagation characteristics, and their dependence on fibre structural parameters, are examined using finite element analysis. Grating simulations show a reflection response with a $10^{-4} \lambda$ bandwidth is possible when power is launched into modes with harmonic mode parameter of $p=1$.

An analysis of mode population distributions for excitation by different source geometries is presented. Consequently, a probe geometry is presented for use in Raman Spectroscopy. Simulations of coupling between tissue-scattered light and the proposed probe show that coupling efficiency of the probe is on the order of $10^{-3}$, a value comparable to, or greater than, existing probes. Experimentally determined coupling efficiencies on fabricated fibres are shown to have good agreement with theoretical calculations.
Acknowledgements

I am indebted to many people who have been so free with their help over the course of this project. Each person’s expertise added a vital piece to the puzzle, allowing me to complete the picture. This thesis would not have been possible otherwise.

First, I would like to thank my supervisor, Professor P.W.E. Smith, for his guidance throughout this project. His encouragement of my ideas, and high standards have helped guide this work to completion and have allowed me to grow as a researcher. I would also like to thank my colleagues Thomas, David, Lawrence, Li, Daniel-Steve and Iraklis for all their support and for their willingness to offer a helping hand. Particular thanks goes to Thomas for always making time to answer my questions, and without whom this project would have neither started nor finished.

Special thanks goes to Professor Sipe for providing many fruitful discussions and for being so generous with his time and knowledge.

The invaluable assistance of Robin Tam, and the other members of Photonics Research Ontario with regards to grating fabrication, is greatly appreciated.

I am also grateful to Brian Wilson, Alex Vitkin, Martin Shim and Ralph DaCosta at the Ontario Cancer Institute for their insights about the biological aspects of the project and for providing the required experimental materials.
I am thankful to Photonics Research Ontario for providing funding for this project, and to the Natural Sciences and Engineering Research Council of Canada for their financial support.

Finally, I would like to thank my friends for their encouraging words, my mom for her prayers, my dad for his sage advice, and my fiancé, Suneil, for his unwavering confidence. They have provided the backbone for this endeavour, as they have for all others.
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<tr>
<td>FWHM</td>
<td>full width half maximum</td>
</tr>
<tr>
<td>IVR</td>
<td>In vivo Raman Spectroscopy</td>
</tr>
<tr>
<td>IVRS</td>
<td>In vivo Raman Spectroscopy system</td>
</tr>
<tr>
<td>LAN</td>
<td>local area network</td>
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<tr>
<td>LED</td>
<td>light-emitting diodes</td>
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<tr>
<td>MMF</td>
<td>multimode fibre</td>
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<td>MMFG</td>
<td>multimode fibre Bragg grating</td>
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<td>MPD</td>
<td>mode population distribution</td>
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<td>SMF</td>
<td>single mode fibre</td>
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<td>SMFG</td>
<td>single mode fibre Bragg grating</td>
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<td>WDMA</td>
<td>wavelength division multiple access</td>
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Chapter 1

INTRODUCTION

1.1 MOTIVATION

Fibre grating structures in single mode fibre (SMF) have been studied extensively since the discovery of photosensitivity in germanium doped silica fibre by Hill et al in 1978 [1]. They have been used in numerous commercial applications ranging from wavelength selective filtering in wavelength division multiple access (WDMA) systems, to temperature and strain sensing. To a lesser extent, Bragg gratings in multimode fibres have also received attention [2-5]. Theoretical and experimental evidence shows that the behaviour of Bragg gratings in graded index or step index multimode fibre (MMF) results in a wideband, low reflectivity response. While the wideband, low reflectivity gratings that are possible in standard multimode fibre are useful for certain sensing applications [2] and in the tuning of solid-state lasers [6], a narrowband, high reflectivity response could have a dramatic impact on optical communication systems. MMF’s have the advantage of easy coupling to inexpensive light sources such as light-emitting diodes (LED’s). In particular, graded index MMF’s have relatively low modal dispersion. These two factors in conjunction have led to the predominant use of graded index MMF in local area network (LAN) applications. The possibility of a narrowband fibre Bragg grating in MMF would enable the use of WDM in LAN’s.

Another application that would benefit is the field of in vivo Raman spectroscopy (IVR) in which MMF is used for good light collection efficiency, but
narrowband filtering is required for signal recovery. Raman spectroscopy is used to detect malignant tissue by examining the scattered Raman spectra of the sample. Fiber optic probes are used to deliver the stimulating laser light and to collect the scattered Raman light from the tissue. The primary problem with this approach is the large Raman signal from the silica fibre. Pump light is Rayleigh scattered from the tissue and is intense enough to cause Raman scattering in the collection fibre, which contaminates tissue spectra. Probe heads with built-in filters have been devised in order to filter out the Rayleigh scattered light entering the collection fibre. However, these are generally bulky (2cm) for endoscopic use [7]. The use of a MMF with narrowband grating properties would provide strong filtering of unwanted light while leaving spectral information at nearby wavelengths untouched, while the large diameter of the multimode fiber would ensure efficient collection of the weak Raman signal from the tissue sample. The resulting probe would be compact and inexpensive.

These considerations provide the primary motivating factor behind the work contained in this thesis. In this work, we present and analyze a novel multimode fibre structure that is tailored to facilitate the creation of fiber Bragg gratings with narrowband, high reflectivity, response. Such narrowband grating behaviour in multimode fibre has never previously been demonstrated. We also present a possible probe geometry, which incorporates this novel fibre, for use in an IVR system.
1.2 BACKGROUND

The following two sections provide a brief overview of fibre Bragg grating technology in single mode and multimode fibres, and of in vivo Raman Spectroscopy.

1.2.1 Fibre Bragg Gratings

Photosensitivity, the ability to induce permanent refractive index changes using high intensity light, was first discovered by Hill et al. [1]. It was found that a periodic refractive index perturbation was produced in germania-doped silica fibre by launching argon ion laser light into the fibre at high intensity. It was deduced that the forward propagating input beam was creating a standing wave pattern with the backward propagating Fresnel reflected light from the fibre end. The areas experiencing a high intensity of light underwent increases in refractive index. The resulting fibre Bragg grating had the ability to reflect light at the wavelength of the input laser beam. The value of these gratings in filter fabrication was immediately recognized, although the writing technique did not seem readily applicable at the longer communication wavelengths.

A major advance in grating writing technology occurred in 1989 when Meltz et al. [8] wrote a grating by side exposure of an optical fibre with two interfering ultraviolet (UV) beams with a wavelength of $\lambda=244\text{nm}$. The transverse holographic technique allowed the possibility of writing gratings at longer wavelengths since the wavelength of peak reflectance was dependant on the angle of the two interfering beams. Although still in use today, this technique too
has limitations. Mechanical vibrations and long path lengths in air can cause the interference pattern to change during writing. This limits its application to short writing times. Also, for sources with low spatial coherence, the path lengths between the two interfering beams must be equalized to ensure coherent overlap of spatial profiles.

A second advance in grating writing technology took place in 1993 with the development of the phase mask technique by Hill et al. [9] and Anderson et al. [10]. Phase masks are fabricated in silica plates using e-beam writing and plasma etching techniques. Periodic variations in the optical thickness are etched into the silica plates. The fibre is side exposed through the phase mask and first order diffraction beams produce an intensity pattern, which induces refractive index changes in the fibre. Phase masks simplify manufacturing alignment procedures enabling mass production. This technique also allows the easy manufacturing of useful Bragg spectral responses by allowing flexibility in varying pitch during mask fabrication. However, with this technique, the flexibility to choose the Bragg wavelength is lost since a different phase mask must be purchased for each wavelength.

The physical mechanisms involved in producing refractive index changes in fibre have yet to be completely understood. Although many models have been suggested, the two mechanisms to which photosensitivity in germanium doped fibre are generally attributed are the formation of color centers, and densification and increase in tension [11]. Germanium dopant normally bonds to four silica atoms via bridging oxygen atoms. However, in some instances, defect states
occur in which Ge may be directly bonded to Si. These Ge-Si bonds are responsible for the 240nm absorption peak observed in germanosilicate optical fibres. Upon UV illumination, these bonds break and form colour centers plus one free electron per broken bond. Colour centers consist of a germanium atom bonded to three oxygen atoms and one unsatisfied electron. The free electron produced in this process may recombine with the original colour center or move within the glass matrix and be trapped at another defect site. Through the formation of colour centers, the ultraviolet absorption band is altered. These alterations in the ultraviolet region cause a change in the refractive index at wavelengths through to the infrared as described by the Kramers-Kronig relations. The illumination of ultraviolet light also causes densification of the fibre and stress increase, which contributes to refractive index changes. Ultraviolet light reduces the thickness of glass fibre. This induces stress in the fibre, which affects the bond lengths. These molecular changes cause changes in the absorption spectrum, and therefore the refractive index. Photosensitivity has also been attributed to the formation of other defects such as nonbridging oxygen hole centers, and the peroxy radical. It is generally believed that all of these mechanisms contribute to photosensitivity in germania-doped fibres to a lesser or greater degree [11].

Although photosensitivity was first attributed only to heavily germania-doped fibres, it has since been shown to exist in fibres doped with other elements [12-15]. Also, preparation techniques such as hydrogen loading [16] and flame brushing [17] can enhance photosensitivity.
Although to a much lesser extent, some attention has also been paid to writing gratings in conventional multimode fibre. Wanser et al. [2] calculated the theoretical spectrum of multimode fibre Bragg gratings (MMFG’s) and suggested their use for microbend sensor applications. Figure 1.1 shows the theoretical reflection spectrum of a typical grating in both a graded-index multimode fibre and a singlemode fibre for a length $L=1.5\text{mm}$ and a grating perturbation of $\Delta \varepsilon = 1.5\times 10^{-3}$.

![Reflection response for a single mode fibre and a GRIN multimode fibre for a uniform grating with $\Delta \varepsilon = 1.5\times 10^{-3}$, $L=1.5\text{mm}$. Wavelength scale is normalized to unity.](image)

We can see that the MMFG spectrum has multiple peaks, and comparatively low reflectivity and broad bandwidth. This is due to the Bragg condition being satisfied at a different wavelength for each of the modes in the fibre (refer to Chapter 2 for details). Mizunami et al. [3] experimentally confirmed the spectral properties of MMFG’s. A grating was fabricated in a graded index germano-
silicate fibre that had a reflection spectrum of 15nm width centered at 1560nm, contained multiple peaks, and had a peak reflectivity of 36% with a halogen lamp as a light source. This same group later reported a detailed analysis of MMFG behaviour [4], including temperature and polarization characteristics. Gratings have also been demonstrated in dye-doped polymer optical multimode fibre [5]. The spectral response of this grating was consistent with the behaviour observed by the previously mentioned groups. MMFG’s have also been suggested for use in tuning solid-state lasers in which the high coupling efficiency of multimode fibre is attractive [6]. To the best of our knowledge, in the work performed to date, high reflectivity, narrowband grating reflectivity responses in MMF have yet to be demonstrated.

1.2.2 In vivo Raman Spectroscopy

Professor Chandrasekhara Venkata Raman was the first to observe the Raman effect in 1928. In his Nobel Prize winning work, he found that incident light interacts with molecules in a material in such a way as to cause a small amount of light to be inelastically scattered with a shifted wavelength. In Raman spectroscopy, this shift in wavelength is used to identify characteristic frequencies of the molecules in the sample. In recent years, Raman spectroscopy has been used to study the molecular makeup of a variety of normal and diseased biological tissue such as skin, bone, blood, and brain [18-20]. Such information can be used to identify between healthy and diseased tissue in medical diagnosis. However, much work still remains to be done to optimize performance and cost of IVR systems for a clinical setting.
In this section, we concentrate on describing the structure and light collection capabilities of a new in vivo Raman spectroscopy system (IVRS) being studied at the Ontario Cancer Institute/Princess Margaret Hospital [30, 7] by our collaborators on this project. In future chapters, we will use the capabilities of this probe as a benchmark for comparisons. Shown in figure 1.2 are the components of the biological in vivo Raman spectroscopy system.

![Diagram of in vivo Raman Spectroscopy system](image)

Figure 1.2: In vivo Raman Spectroscopy system. After [30].

The four major components are the excitation source, light delivery/collection optics, spectrograph and detector. Advances in each of these components are responsible for enabling clinical spectroscopy. The system under consideration is an in vivo system. That is, spectra are collected without removing a sample from the organism being studied. This is in contrast to ex
vivo techniques that have been studied most extensively so far, in which a sample must be removed and specially prepared for analysis. In vivo collection is preferable to ex vivo techniques in order to reduce the diagnostic waiting time for the patient.

The system under study at the Ontario Cancer Institute utilizes a laser diode, which provides approximately 500mW of light at a wavelength of 785 nm, as the excitation source [7]. With the use of excitation sources in the near infrared (NIR), the tissue fluorescence, which hinders Raman diagnosis, has been reduced dramatically. Previously, the use of excitation lasers in the visible wavelength range resulted in fluorescence, which was orders of magnitude more intense than the Raman light produced. While it is true that fluorescence spectroscopy has found some success in diagnosing colon and lung cancer, this technique produces spectra without sharp spectral features hindering proper diagnosis. However, with the advent of NIR sources, tissue fluorescence is reduced since the energy of the NIR photons is below the threshold required for fluorescence [22]. Also, NIR photons can penetrate further into tissue samples due to the low absorption coefficient of water at this wavelength. This facilitates in vivo collection of spectra because the water in the tissue does not prohibit accurate spectral readings.

The light delivery/collection fibre optics also plays a vital role in system performance. Calculations of efficiency have been made for a variety of probe designs. The highest light collection efficiency is achieved when one fibre is used for both the delivery of laser light and collection of tissue signal, which provides
complete overlap of illumination and collection “cones” [23]. Unfortunately, this
graphometry is susceptible to spectral artifacts from background signal generated in
the fiber. Fluorescence and Raman signal from the silica fibers contaminate
tissue spectra. Contamination occurs as Raman signal is generated both in the
excitation fibre, by the excitation laser light, and in the collection fibres, by laser
light Rayleigh scattered from the tissue. This causes serious problems since the
intensity of the fibre Raman signal is often greater than that of tissue Raman
signal. Probes incorporating multiple fibres reduce this problem but have low
collection efficiency due to the limited overlap between illumination and collection
cones of light [24]. There have been attempts to improve probe efficiency by
adding GRIN lenses to focus excitation light and collimate collected light [25].

After investigating several probe geometries [26], our collaborators
decided that a 1.5m, multiple fibre probe utilizing beam steering and filtering
(Gaser Light Management System, Enviva Biomedical Raman Probes: Visionex
Inc., Atlanta, GA, USA) would be used in the IVRS. Figure 1.3(a) shows a
diagram of the probe tip. The probe consists of one central delivery fibre
surrounded by six collection fibres. The delivery fibre has a 400μm core diameter
while the collection fibres have a 300μm core diameter. In order to eliminate
silica fluorescence contamination, a 5th order polynomial is fitted to the data to
subtract the broad fluorescence and leave the relatively narrow tissue Raman
spectrum. However, this still leaves the narrow silica Raman spectral
contamination that cannot be eliminated by baseline subtraction.
Internal filtering is incorporated to reduce silica Raman interference. The probe head incorporates dielectric interference filters, which are approximately 2.5cm from the probe tip. The delivery fibre contains a bandpass filter to allow only the excitation wavelength to pass while suppressing silica Raman generated at longer wavelengths. The collection fibres contain long pass filters to reflect Rayleigh scattered pump light while allowing the tissue Raman at longer wavelengths to pass into the fibre. In order to increase the optical efficiency of the system, the collection fibres are beveled and have a reflective coating. This angles the collection "cone" to increase overlap with the delivery fibre illumination cone (see figure 1.3(b) and 1.3(c)).

The collection fibres are aligned along the entrance slit of a spectrograph using holographic optical elements, which deciphers the spectral information.
(Raman intensity versus wavelength). Typically, spectrographs use a prism or grating to disperse the various wavelengths contained in the incident light onto a flat panel detector. Spectrographs using holographic optical components are compact and robust making them an excellent choice for clinical Raman spectroscopy [27, 28]. Many of the experiments performed to date on tissues have been performed using ex vivo Fourier Transform (FT) Raman spectroscopy. Rather than use spectrographs, FT Raman systems use interferometers to obtain the spectral information. A Fourier transform is performed on the resulting interferogram to produce the Raman spectrum. However, the resolution of such a system is adversely affected by many factors, and may have prohibitively long collection times for in vivo clinical applications [26].

A N₂ cooled charge-coupled device (CCD) detector array is used to capture the dispersed light from the spectrograph and display the spectral information. CCDs are particularly optimal for use in NIR Raman spectroscopy since they possess low detector noise and high quantum efficiency at these wavelengths [29].

Overall, the IVRS system performs better than existing clinical systems (mostly FT Raman systems) in several respects. The collection time is reduced from an order of minutes to seconds. This allows clinical diagnostics to take place without the extensive waiting periods associated with lab analysis. It also has an SNR, which is roughly 2.5 times that of a commercial FT Raman system [26]. The IVRS also allows the study of biological tissue as the disease progresses so that new treatments can be developed. With appropriate choice of
excitation power, the tissue would not be damaged by these spectroscopic readings. Also, spectroscopy software can be pre-programmed so that even an inexperienced user can make diagnostics successfully.

However, there are limitations to the in vivo Raman system. The large probe tip (~2cm) makes it awkward for use in endoscopic applications. Also, the current probe is expensive to manufacture which makes it unattractive for routine clinical use. In fact, the current manufacturer of the probe described above has discontinued the making of these probes. Thus, there is currently a need to find a solution that can be used in place of, and improve, the current probe.

1.3 Contributions of This Work

In this work, we develop and analyze a unique multimode fibre tailored to allow the fabrication of fibre Bragg gratings with narrowband, high reflectivity, filter response. A systematic analysis is performed of the various modes in the fibre using a full vectorial, finite element analysis. Fibre parameters are varied and the effects on modal properties are examined. Simulations of grating behaviour show a reflection response with a bandwidth of $10^{-4}\lambda$ is possible when power is launched into modes with a harmonic mode parameter of $p=1$. An analysis of modal distributions for different source excitation conditions is presented. It is shown that only $p=1$ modes are excited by axisymmetric, coherent beams. Also, an analysis is presented of modal distributions excited by tissue-scattered light. Subsequently, a probe geometry for use in an IVRS is presented. Simulations of coupling between tissue-scattered light and the proposed probe show that the power coupling efficiency of the probe is on the
order of $10^{-3}$, a value comparable to, or greater than existing probes. Modal
distributions excited by the tissue-scattered light are shown to be favourable for
enabling the creation of gratings with narrowband response. Thus, theoretical
predictions show that the proposed probe geometry has promising prospects in
an IVRS. Experimentally determined coupling efficiencies on fabricated fibres are
shown to have good agreement with theoretical calculations.

The problem of designing a fibre structure having a set of modes with
almost degenerate effective indices was first studied by T. Szkopek [31]. He
proposed a fibre structure and performed analysis on this structure using a scalar
finite element method. Although a vector finite element analysis program had
been written at that time, it was not until the present work that a vector analysis
of modal behaviour was studied. Sections 2.1 and portions of section 2.2 explain
the fibre design studied in his work, as well as the vector finite element method
used in the present work. The rest of this thesis is a continuation of this
preliminary analysis.

To date, a multimode fibre with these unique properties has yet to be
studied, or even conceptualized. Here, we present an analysis that furthers
understanding of this novel structure with a view to applying it in a clinical in vivo
Raman spectroscopy system. This fibre is shown to have the potential to be a
compact and inexpensive solution in the search for new IVRS probes. In addition,
the potential ramifications of this novel fibre behaviour have yet to be explored for
use in telecom or other photonics applications.
1.4 **Organization of this Thesis**

In Chapter 2 of this thesis, a description of the novel fibre structure and a review of theoretical principles behind modal propagation are given. This is followed by a description of the modes propagating in the proposed structure along with simulations of electric field profiles, and simulations describing modal behaviour dependence on various fibre parameters. Then, after a brief review of grating theory, simulations showing the reflectivity spectrum of a uniform fibre Bragg grating in the novel structure are presented for various mode population distributions. Chapter 3 presents an analysis of modal distributions excited in the fibre by various source geometries. Coherent and incoherent sources are considered and behaviour is examined as a function of axial and radial displacement between the fibre and source. A possible probe geometry is presented and examined. Chapter 4 presents experimental results on a fabricated fibre. This covers attempts at grating writing and experiments to determine coupling efficiency. Finally, Chapter 5 presents a summary of the thesis results, conclusions that can be drawn and a discussion of future work.
Chapter 2

THEORETICAL ANALYSIS OF PROPOSED FIBRE

One of the main aims of the present work is to provide detailed analysis of a fibre configuration that allows a subset of modes to travel with almost degenerate phase velocities. This chapter outlines the proposed structure, its modal behaviour, and its grating response under various launching conditions. The work in this chapter is based on work presented at Bragg Gratings, Photosensitivity and Poling in Glass Waveguides – 2001 [32] and in the Journal of Selected Topics in Quantum Electronics: Specialty Optical Fibers (due for publication in Sept. 2001) [33].

2.1 Fibre structure

The proposed multimode fibre consists of alternating cylindrical shells of higher and lower refractive index. That is, the fibre face resembles a “bulls-eye” pattern with rings of higher refractive index. Figure 2-1 illustrates a four-shell structure with areas of higher refractive index indicated by shading. Also indicated is the refractive index profile as a function of radius where $r_0$ is the inner ring radius, $t$ is the shell thickness, and $\Delta$ is the shell separation. Although cylindrical periodic structures have been proposed for application to distributed feedback lasers [34], and guiding light in air through distributed reflection [35,36], the proposed cylindrical shell structure is meant to function as a dielectric.
waveguide. This structure is intended to minimize the difference in propagation constants of the various guided modes.

![Waveguide diagram](image)

Figure 2.1: (a) Refractive index profile of fibre, and (b) view of fibre face

An intuitive understanding of the operation of the fibre is best understood by considering an analogy between the propagation of an electromagnetic mode within a fibre and a bound quantum mechanical particle in a potential well. The bound particle problem is described by the time independent linear Schrödinger wave equation, which allows determination of the bound states of the particle, and is given by,

$$\left[ \frac{\partial^2}{\partial x^2} - \frac{2m}{\hbar^2} (V(x) - E) \right] \Psi(x) = 0 \quad 2.1$$

where $\Psi(x)$ is the particle wave function, $V(x)$ is the potential well, $E$ is the particle energy, and $m$ is the mass of the particle. In the scalar approximation for the electromagnetic mode field, the radial component of the fibre field can be described by a similar Schrödinger equation as for a bound particle,

$$\left[ \nabla_T^2 + k_0^2 \left( n^2(r) - n_{\text{eff}}^2 \right) \right] \Psi(r) = 0 \quad 2.2$$
where $\nabla_T$ is del operator for transverse spatial co-ordinates, $r$ is the radial position, $k_0$ is the vacuum wavevector, $n(r)$ is the radial index profile, $n_{\text{eff}}$ is the effective index, and $\Psi(r)$ is the scalar field. We see that there is a very strong resemblance in the forms of the equations. This leads to analogous behaviour between the quantum problem, and mode propagation in a fibre. The energy eigenvalues of a bound particle are analogous to the effective indices of the modes in the fibre. Similarly, the quantum well potential is analogous to the radial refractive index profile. In a single quantum well, discrete energy eigenvalues exist. If a quantum well is made sufficiently shallow, it will contain one energy level. This is analogous to a single mode fibre in which only one mode exists. However, when multiple quantum wells are closely spaced, the single energy level splits into several closely spaced energy levels. The behaviour of the different quantum wells is summarized in figure 2-2 below.

Figure 2.2: (a) Single potential well with two energy levels, (b) two wells with two pairs of closely spaced energy levels

That is, if $N$ wells are placed consecutively, and sufficiently close, the single energy level will split into $N$ closely spaced energy levels. As the wells are brought closer together, the splitting between the energy level increases. Once again, by analogy, this implies that a fibre with periodically varying radial index profile as shown in figure 2-1 should have many modes with very closely spaced
effective indices or propagation constants. This is the very property that would enable a narrowband reflection response while still supporting multiple modes. By minimizing the difference in propagation constants, we ensure that all modes interact in roughly the same way with a Bragg grating. This allows a reflection response in this novel multimode fibre that is very similar to that in a single mode fibre (refer to section 2.4 for a more detailed explanation).

To enable this behaviour, the shell properties have to be chosen to allow only a single mode to propagate in each shell. If the shells are made thin compared to the mean radius of the shell, the structure can be approximated by a planar structure. In this case, if we ignore polarization, a planar waveguide analysis can be used to determine the condition which would allow only one mode to propagate in each shell. The restriction on the normalized V parameter to allow only single mode propagation becomes [37],

\[ V = \frac{2\pi t \sqrt{n_{\text{shell}}^2 - n_{\text{cladding}}^2}}{\lambda} < \pi \]  

2-3

where \( n_{\text{shell}} \) and \( n_{\text{cladding}} \) are the shell and cladding refractive indices, t is the shell thickness and \( \lambda \) is the vacuum wavelength. Despite placing this restriction, there are modes associated with different polarizations in each of the shells that cannot be accounted for with scalar analysis. In addition, there are modes with different angular wavevector components. These modes cannot be eliminated and have no analogy in the one-dimensional potential well problem. Thus, an analysis using the full vectorial wave equation is necessary to fully determine modal behaviour.
2.2 Modal Analysis

The guided electromagnetic modes of a fibre can be determined by using Maxwell's equations. When light with angular frequency $\omega$ propagates through a waveguide with a constant refractive index in the direction of propagation, Maxwell's equations can be written [38],

$$\nabla \times \mathbf{E} = -j\omega \mu_0 \mathbf{H}$$
$$\nabla \times \mathbf{H} = -j\omega \varepsilon_0 n^2 \mathbf{E}$$
$$\nabla \cdot \mathbf{H} = 0$$
$$\nabla \cdot \mathbf{E} + \frac{\nabla (n^2)}{n^2} \cdot \mathbf{E} = 0$$

where $\mathbf{E}$ and $\mathbf{H}$ are the complex amplitudes of the electric and magnetic field respectively, and $n$ is the refractive index. Using these equations, the transverse mode profile and propagation constants of each guided mode can be determined.

While analytical solutions are possible for simple structures, a numerical solution is necessary to analyze the more complicated fibre introduced in the previous section. One such numerical method, used in this work, is finite element analysis. For this work, we followed the procedure outlined in [38]. There are numerous types of finite element analysis depending on which electromagnetic field component is used in forming the problem. To some degree, all these methods suffer from the generation of certain nonphysical solutions called spurious solutions. In this work, finite element analysis was performed by formulating the problem using three magnetic field components, which allows for the elimination of spurious solutions through appropriate formalism.
Considering Maxwell's equations for a guided mode within an axially symmetric structure, we assume a magnetic field of the form,

\[ H = \left[ iH_r(r)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}e_r + iH_\theta(r)\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}e_\theta + H_z(r)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}e_z \right]\exp(i(\omega t - \beta z)) \]

2-5

where \( H_r(r), H_\theta(r) \) and \( H_z(r) \) are component radial magnetic field distribution functions, \( e_\theta \) is the unit polar angle vector, \( e_z \) is the unit longitudinal vector, \( p \) is an integer not equal to 0, \( \omega \) is the optical frequency and \( \beta \) is the longitudinal propagation wave vector. In the special case of \( p=0 \), the harmonic function pairs in braces are replaced by a single constant. For a waveguide with both shell and cladding consisting of random anisotropic media, the wave equation for the magnetic field \( H \) is given by,

\[ \nabla \times \left( [\varepsilon_r]^{-1} \nabla \times H \right) - k_0^2 H = 0 \]

2-6

The functional used to solve the above equation is [38],

\[
F = \int_\Omega (\nabla \times H)^* (\varepsilon_r^{-1} \nabla \times H) d\Omega - k_0^2 \int_\Omega H^* \cdot H d\Omega + \int_\Gamma H^* \cdot [e_r \times (\varepsilon_r^{-1} \nabla \times H)] d\Gamma + s \int_\Omega (\nabla \cdot H)^* (\nabla \cdot H) d\Omega \]

2-7

where \( \varepsilon_r \) is the relative electric permittivity, \( \Omega \) is the cross sectional area of the waveguide, \( \Gamma \) is the boundary between the core and the cladding, \( e_r \) is the unit radial vector, \( k_0 \) is the vacuum wavevector and \( s \) is the penalty parameter, as explained below, and is taken to be \( s = 1/n_{\text{cladding}}^2 \). The functional is formulated such that it possesses the property that the value of the functional is stationary for solutions of the wave equation in the dielectric waveguide. Analytic
expressions for the magnetic field in the infinite cladding were used. Solving
Maxwell’s equations for a homogenous cladding yields modified \( p^{th} \) order Bessel
functions of the second kind \( K_p(\cdot) \) and the respective first derivative \( K'_p(\cdot) \). The
analytic expressions used are [39]

\[
\begin{align*}
H_r(r) &= -A \frac{pa}{wr} K_p\left( \frac{wr}{a} \right) + B \frac{\beta a}{w} K'_p\left( \frac{wr}{a} \right) \\
H_\phi(r) &= AK'_p\left( \frac{wr}{a} \right) - B \frac{\beta a}{w^2 r} K_p\left( \frac{wr}{a} \right) \\
H_z(r) &= BK'_p\left( \frac{wr}{a} \right)
\end{align*}
\]

where \( w = \sqrt{\beta^2 - k_0^2 n_{\text{cladding}}} \), the radius at \( \Gamma \) is \( a \), and \( A, B \) are constants which
can be related to the magnetic field of the element adjacent to \( \Gamma \).

If equation 2-7 is solved without the last integral in the equation, spurious
solutions that do not satisfy the condition \( \nabla \cdot \mathbf{H} = 0 \) are generated. The inclusion
of this last integral, known as the penalty method, with the appropriately chosen
penalty parameter as given before, allows the elimination of these spurious
solutions.

The problem was discretized by dividing the cross sectional area of the
waveguide into axisymmetrical annular elements and expanding the magnetic
field in each element in terms of its nodal values. When the discretized form
of the magnetic field is substituted into the equation of the functional, and the
variational principle is applied, the problem of solving for the magnetic fields
reduces to solving the following equation,

\[
[[K] + [Z] + s[L]][\mathbf{H}] - k_0^2 [M][\mathbf{H}] = [0]
\]

\[2-9\]
where the matrices \( [K], [M], [Z], \) and \( [L] \) are functions of the discretized radius within the cross sectional waveguide area \( \Omega \), \( s \) is the penalty parameter and \( \{ H \} \) represents the magnetic field components at each radial annular element. For sake of simplicity, we leave the precise formulation and definitions of these matrices to Appendix A. The important point to note is that this equation is simply the generalized eigenvalue problem with eigenvalue \( k_0^2 \) and eigenvector \( \{ H \} \). The computation requires a first estimate of the longitudinal propagation constant after which iteration is used to refine the value of \( \beta \). Thus, by applying appropriate variational principles to the functional, and solving for stationary solutions, we are able to determine the magnetic field components \( H_r(r) \), \( H_\theta(r) \) and \( H_z(r) \) for radii \( r \) for each guided mode, and their corresponding propagation constants. Once the magnetic field is determined using finite element analysis, the electric field can easily be calculated.

All finite element analysis was implemented using MATLAB. The program was verified by comparing numerical solutions to analytical solutions presented in [38]. It was found that the numerically calculated effective index matched the analytic value of index to the 6th decimal place when a minimum of 25 elements was used. For a more complex structure, the number of elements required was estimated to be on the order of 100 elements. Test simulations were performed on the cylindrical shell structure to determine the minimum number of elements needed for convergence. It was found that the effective indices calculated using 150 elements matched to within \( 10^{-5} \) with the effective indices calculated using 500 elements. Thus, 150 elements were used for most of the simulations below,
as it would allow sufficient accuracy while maintaining a reasonable speed for the calculations.

The finite element program was then used to calculate the modes of a four shell structure with inner radius \( r_0 = 25\lambda \), shell separation \( \Delta = 10\lambda \) and normalized frequency parameter for a shell \( V = 3.0807 \) (refer to section 2.3 for an explanation of the parameter values chosen). The intensity was calculated as the time averaged Poynting vector projected onto the axis of propagation. The results for modes with angular parameter \( p=1 \) and \( p=5 \) are plotted in diagrams 2.3 and 2.4 below. The corresponding dimensionless parameter for each mode, defined as \( b = \frac{(n_{\text{eff}}^2 - n_{\text{cladding}}^2)}{(n_{\text{shell}}^2 - n_{\text{cladding}}^2)} \), is also given. The regions of higher intensity are shaded darker. For clarity, the electric field vectors are plotted in the rings with highest intensity only in each figure.

As expected, the modes are confined in the regions of higher refractive index. The modes with angular parameter \( p=1 \) exhibit mode mixing of the modes in the individual shells and the power is distributed among all four shells. Upon careful observation of the intensity pattern and electric field vectors, one can see the \( \pi \) periodicity in the angular \( \theta \) dependence of the intensity pattern. There are two types of polarizations evident. The almost linear polarization resembles that of a HE\(_{11} \) mode in a step index fibre [40]. The polarization, which exhibits rotation resembles that of the EH\(_{11} \) mode in a step index fibre. The \( 10^{-2} \) variation in \( b \) parameter among the \( p=1 \) modes corresponds to a difference of \( 10^{-4} \) in effective index between the mode with highest \( n_{\text{eff}} \) and lowest \( n_{\text{eff}} \).
Figure 2.3: Contour plots of the optical intensity of modes with \( p=1 \). Increasing levels of darkness indicate greater optical intensity. The coordinates \( x \) and \( y \) are normalized to unit wavelength.
Figure 2.4: Contour plots of the optical intensity of modes with $p=5$. Increasing levels of darkness indicate greater optical intensity. The coordinates $x$ and $y$ are normalized to unit wavelength.
In the case of the modes with angular parameter $p=5$, the modes are localized to individual shells. This trend of increasing localization as $p$ increases was confirmed by observing modes with other $p$ values as well. Upon close inspection, an angular modulation with a periodicity of $\pi/5$ can be seen in the intensity pattern. Once again, the two polarizations present in each shell can be seen to resemble those of a $HE_51$ and $EH_51$ in a step index fibre respectively.

### 2.3 Mode Behaviour Dependence on Fibre Parameters

Once the forms of the guided modes in the fibre were understood, an analysis was performed to determine the changes in modal propagation characteristics with changes in fibre parameters. The first set of simulations was performed for a structure with four cylindrical shells of fixed width but varying positions. The fibre $V$ parameter used was 3.0807 corresponding to $n_{\text{shell}}=1.505$, $n_{\text{cladding}}=1.500$ and $t=4\lambda$. The inner radius of the innermost shell $r_0$ was varied from $10\lambda$ to $30\lambda$, while the shell separation $\Delta$ was independently varied from $3\lambda$ to $15\lambda$. The modes with $p=1$ were selected, since these are the modes to which a linearly polarized, cylindrically symmetric wave would couple (see chapter 3). The plot of figure 2.5 is the resulting maximum spread in the normalized parameter $b = (n_{\text{eff}}^2 - n_{\text{cladding}}^2)/(n_{\text{shell}}^2 - n_{\text{cladding}}^2)$, where $n_{\text{eff}}$ is the effective index of a particular mode. Two trends are apparent. The spread in $b$ parameter ceases to
Figure 2.5: Maximum spread in $b$ parameter as a function of inner shell radius and shell separation for modes with $p=1$. The parameter $r_0$ and $\Delta$ are normalized to unit wavelength.

significantly decrease as the shell separation is increased beyond $\Delta = 10\lambda$. Consequently, the shell separation need not be greater than $10\lambda$. The second trend illustrated is the decrease in $\Delta b$ parameter as the inner shell radius increases. As the inner shell radius increases, the variation in curvature between individual shells decreases. The shells become more similar in geometry and there is thus an expected increase in the degeneracy of the modal effective indices. Significant decrease is not apparent for inner shell radius greater than $r_0=25\lambda$. Hence, all the simulations in the rest of the present work are calculated with these values of inner shell radius and shell separation. Subsequently, the effective index of propagating modes was examined as a function of the harmonic $p$ parameter. The dependence of $b$ upon the $p$ parameter for the proposed MMF structure is shown in diagram 2.6.
These simulations were performed on an MMF with $r_0 = 25\lambda$, $\Delta = 10\lambda$, and $V = 3.0807$. A total of 240 modes (not including the multiplicity associated with $\pi/2$ rotations for HE and EH modes) propagate within the fibre, having a $p$ value ranging from 0 to 43. We see that for lower values of $p$, the $b$ parameters are centered on one number indicating that modes with low $p$ values travel with very similar phase velocities. For values of $p$ below 4, the $b$ parameters are concentrated about the value 0.64. As seen in the previous section, a consequence of degeneracy in the effective indices is that electromagnetic mixing of the mode energy distribution among different shells. However, this desirable quality falls off at higher $p$ values. The modes can be seen to split into four pairs of $b$ values. Each of these pairs corresponds to the two polarizations with field localized in each of the four shells. The innermost shell is unable to
guide modes with p values above roughly p=17. Each shell of subsequently greater mean radius can support modes with greater values of p. The second, third and fourth shells have 'cut-off' p values of 26, 35, and 43 respectively.

The increase in maximum p value with localization radius may be understood by considering the full vector wave equation satisfied by the magnetic field given in equation 2-5. The portion of the equation \( \nabla \times (\varepsilon_{r}^{-1} \nabla \times \mathbf{H}) \) contains terms that are proportional to \( p/r \) and \( p^2/r^2 \). In order to satisfy the equation, a limit is placed on the maximum p by the value of \( k_0^2 \). Thus, in a first order approximation, a shell with radius \( R \) will be able to support localized fibre modes with a cutoff value of p that is proportional to \( R \). By increasing the radius of the shells, one can enable the guiding of modes with higher p values. The trade off, however, is a larger fibre that may be undesirable for certain applications, such as the medical application considered in the next chapter.

2.4 Bragg Grating Behaviour

The grating behaviour in the novel fibre can best be understood by examining the phase matching condition between the propagation constants of the guided modes and the grating wavevectorer. Strong reflection occurs when forward propagating modes are coupled to backward propagating modes. This takes place when the Bragg condition is satisfied. That is,

\[
\beta_i = \frac{2\pi}{\Lambda} + \beta_r
\]

2-10
where $\beta_i$ is the incident longitudinal wavevector, $\beta_r$ is the reflected longitudinal wavevector and $\Lambda$ is the period of the grating. In general, coupling can occur between same modes traveling in opposite directions (self-coupling), or different modes in opposite directions (cross-coupling). However, in the special case where coupling occurs between the same $m^{th}$ mode but traveling in the opposite direction, $\beta_r = -\beta_i$, and the resonant condition for mode $m$ reduces to,

$$\lambda_i = 2n_m\Lambda \quad 2-11$$

where $\lambda_i$ is the incident vacuum wavelength and $n_m$ is the effective index of the $m^{th}$ mode. In a single mode fibre, only one mode exists. Thus, high reflectivity occurs in a narrow band around the wavelength that satisfies the resonant condition. In the case of a multimode step-index or graded index fibre, many modes exist. Therefore, the resonant condition is satisfied at a different wavelength for each of the different modes with different $n_m$. Thus, each mode experiences resonance at a different vacuum wavelength. As a consequence, there is a limit on the bandwidth narrowness that can be achieved by an MMFG in most currently existing multimode fibres. For example, a fibre with a typical refractive index difference of 0.01 between core and cladding, and grating index perturbation strength of $10^{-4}$ has a reflectivity response that is approximately 100 times wider than would be achievable if only one resonance were present. The spread in resonant wavelengths also results in the low peak reflectivity observed in MMFG's written in standard graded-index fibre [4]. Consider a step-index fibre with $N$ guided modes. Since only one mode is in resonance at a particular wavelength, at most only approximately $1/N$ of the total optical power will interact
resonantly with the grating. Thus, both the total bandwidth of the grating can be decreased, and the peak reflectivity increased, by minimizing the difference between the maximum and minimum effective indices of the guided modes. It is this very property that is exploited in the proposed cylindrical shell structured fibre. By minimizing the effective indices of the guided modes, we ensure that a narrowband, high reflectivity can be achieved despite having multiple modes in the fibre.

In this section, we consider the behaviour of a uniform grating in the proposed structure. The grating perturbation is assumed to be cylindrically symmetric and to exist only in the shells of higher refractive index. When a uniform grating is considered, cross coupling between modes having different harmonic p parameter does not occur because the harmonic functions for differing p values are orthogonal. This was also true in the case of the step-index and graded-index MMF. However, coupling does occur between different modes having the same harmonic p parameter. The strength of the self-coupling and cross-coupling can be determined by calculating the coupling strength between all the pairs of modes. The coupling strength between modes $\alpha$ and $\beta$ is given by

$$\kappa_{\alpha\beta} = \frac{-i\omega\varepsilon_0}{4} \int_{\Omega} \Delta n^2(r) \mathbf{E}_\alpha^* \cdot \mathbf{E}_\beta \, d\Omega$$

where $\omega$ is the optical frequency, $\Delta n^2(r)$ is the grating index perturbation squared and $\mathbf{E}_\alpha$ is the electric field distribution of the mode $\alpha$. Calculations of the coupling constant were made for the proposed MMF structure with $V = 3.0807$, $r = 10\lambda$, and $\Delta = 25\lambda$ yielding a maximum ratio of cross-coupling strength to self-coupling
strength of 1.7%. Thus, in our case, the cross coupling between modes of the same \( p \) is weak, especially for modes localized to different shells. With this in mind, an analysis of the spectral response was performed using coupled mode theory while neglecting weak cross-coupling terms. This is the same as assuming that the reflection response is a weighted sum of the individual responses of each of the modes coupling to itself in the counter propagating direction. Assuming only self-coupling terms are relevant, the reflection response can be written [4],

\[
R(L, \lambda) = \sum_\sigma \eta_\sigma \frac{|\kappa_{\sigma,\sigma}|^2 \sinh^2(S_\sigma L)}{\Delta \beta_\sigma^2 \sinh^2(S_\sigma L) + S_\sigma^2 \cosh^2(S_\sigma L)}
\]

where \( \eta_\sigma = P_\sigma / \sum_p P_p \) is the fraction of total optical power in mode \( \sigma \),

\( \Delta \beta_\sigma = 2\pi(n_\sigma / \lambda - 1/2 \lambda) \) is the wavevector detuning and \( S_\sigma = \sqrt{|\kappa_{\sigma,\sigma}|^2 - \Delta \beta_\sigma^2} \) is the detuned coupling strength.

To illustrate the grating behaviour of the novel fibre, simulations were run for a uniform grating with a length of 2 cm and a maximum index depth of \( \Delta n = 1.7 \times 10^{-4} \). In figure 2-7(a), the reflection response is illustrated for the case in which all the optical power is evenly distributed among the \( p=1 \) modes alone. The peak reflectivity is 100% and the FWHM bandwidth is \( 2.5 \times 10^{-4} \lambda \). In plot (b), the optical power is assumed to be distributed evenly among all the modes existing in the fibre (uniform mode population distribution). The peak reflectivity is 31.5% and the FWHM bandwidth is \( \sim 6 \times 10^{-4} \lambda \). For comparison, a graded index fibre was simulated assuming the same core and cladding indices as the
Figure 2.7: (a) Grating response of proposed fibre with \( p=1 \) modes excited. (b) Grating response of proposed fibre with all modes excited. (c) Grating response in graded index multimode fibre. Wavelength scale normalized to unity.

The proposed fibre, and a core radius of 25\( \lambda \), giving \( V = 19.06 \) according to conventional definition of normalized frequency \( V \). In plot 2-7(c), we see the reflection response when assuming uniform mode population distribution yields a peak reflectivity of 7.9\% and a FWHM bandwidth is \( \sim 5 \times 10^{-3} \lambda \). We see that there are multiple individual resonance's contained within the overall response. Each of these responses corresponds to a 'principal mode' of the fibre [4]. That is, each principal mode consists of a set of modes, which have approximately the same propagation constant. For example, the modes \( HE_{12}, HE_{11}, \) and \( HE_{31} \) form a single principal mode in graded index fibres [41]. Thus, the asymmetry in the response is due to the larger number of modes that compose each of the principal modes resonating with the grating on the shorter wavelength side.

The reflectivity response of multimode fibre is a function of the spread in effective indices as well as the mode population distribution assumed. In the case of the proposed shell structure, launching light into modes with low harmonic \( p \) parameter results in greater reflectivity and narrower bandwidth. This is evident from the dependence of \( b \), and thus \( n_{\text{eff}} \), upon \( p \) as illustrated in figure
However, a similar launching of light into low \( p \) valued modes in a conventional fibre would not result in the same behaviour. In a GRIN or step-index fibre, the set of \( p=1 \) modes contains guided modes with effective indices spreading from very near the core refractive index value all the way to the cladding index. In other words, modes with a given \( p \) value in a conventional fibre will be spread among various principal modes and will resonate with the grating differently. Thus, in conventional fibre, selective launching into modes with a given \( p \) value does not result in decreased grating bandwidth and increased reflectivity as it does in the proposed fibre structure.

With the results of this chapter, we now have a thorough understanding of the positive and negative aspects of the novel fibre behaviour. The spread in effective indices of guided modes can be minimized by maximizing inner shell radius, and by maximizing the distance between shells. It was shown that negligible performance improvement could be made through structural modifications beyond a certain choice of inner ring radius and shell separation \((r_0 = 25\lambda, \Delta = 10\lambda)\). Using these optimal parameters, it was shown that the spread in effective indices of modes is dependant on the harmonic \( p \) parameter. For increasing \( p \) values, the spread in effective indices increases. Examination of grating behaviour showed that narrowband reflection response is possible when only modes with low harmonic \( p \) parameter are guided. Even assuming equal mode population distribution (MPD), the reflectivity response of the proposed fibre still has a higher reflectivity and more narrowband response than the case of equal MPD in graded index or step index multimode fibre.
Chapter 3

ANALYSIS OF MODE EXCITATION IN PROPOSED FIBRE

It is evident from the previous section that the narrowband, high reflectivity grating response of the proposed multimode fibre depends heavily on the modal distribution excited in the fibre. In this chapter, we explore this issue in greater detail. We begin by presenting an analysis of the mode population distributions generated by both coherent and incoherent sources. These results suggest a possible probe geometry for an IVR system, which is presented. The collection capability of this probe is examined using a source representing scattered light from biological tissue. Since we are concerned with grating behaviour, we are only concerned with modeling the coupling behaviour of Rayleigh scattered pump light which we want to eliminate by filtering using a Bragg grating. We would expect that to first order, Raman scattered light from the tissue would have roughly the same coupling efficiency. Possible enhancements to the probe design are also discussed.

3.1 MODE LAUNCHING THEORY

To determine the power distribution among modes excited by an arbitrary source, we must begin by specifying the transverse electric or magnetic field at the fibre face [42]. We then represent this field as an expansion over the orthogonal modes of the fibre. From this representation, the power distribution among the modes can easily be determined. Here, we describe the general formalism for illumination by a partially coherent source and then describe the
specific cases of fibre excitation by totally coherent and totally incoherent sources [43].

The geometry considered here is that of an optical fibre aligned with the positive z-axis and surrounded by a uniform medium of lower refractive. Figure 3-1 illustrates the coupling geometry.

![Coupling geometry between source plane and fibre](image)

**Figure 3.1:** Coupling geometry between source plane and fibre

The transverse electric field at the fibre face, $E_t$, can be represented as,

$$E_t(x,y) = \sum_j a_j e_j(x,y) + E_{tr}(x,y)$$  \hspace{1cm} (3-1)

where $a_j$ is the modal amplitude of the $j^{th}$ mode, $e_j$ is the transverse field of mode $j$, $E_{tr}$ is the transverse electric portion of the radiation field at the fibre end face, and the summation is performed over all $j$ modes. The implicit assumption here is that the field has a time dependence of $e^{i\omega t}$ and that the source is monochromatic. The transverse field is taken to be the incident field $E_{\text{inc}}$ in the $z=0$ plane (the fibre end face). Since each mode is orthogonal to all other modes and the radiation fields, by taking the appropriate cross product, it can be shown that,

$$a_j = \left(\frac{c_i}{n_i^2}\right)^{1/2} \int_{A_m} e_j^*(x,y) \cdot E_{\text{inc}}(x,y) dA$$  \hspace{1cm} (3-2)
where \( \mu \) is the permeability (assumed constant) and \( \varepsilon_1 \) is the fibre dielectric constant. It is assumed here that the fibre is weakly guiding due to a small difference in refractive index. We consider only one component of polarization. The other component is treated similarly, and may be added independently to construct the randomly polarized light situation. The power in each mode is then,

\[
P = \sum_{j} P_j = \frac{1}{2} \sum_{j} (a_j \cdot a_j^*) = \frac{1}{2} \int_{A_1} \int_{A_2} e_j^*(x_1, y_1) \Gamma(x_1, y_1, x_2, y_2) e_j(x_2, y_2) dA_1 dA_2
\]

where \( P_j \) is the power in each mode, and \( \Gamma \) is the mutual coherence function defined as,

\[
\Gamma = \langle E_{inc}(x_1, y_1) E_{inc}^*(x_2, y_2) \rangle
\]

where \( \langle \rangle \) denotes ensemble average.

These general equations can be applied to the two extreme cases of coupling from a completely spatially coherent or a completely spatially incoherent source. In the limit of complete coherence, the mutual coherence is reduced to,

\[
\Gamma(x_1, y_1, x_2, y_2) = E(x_1, y_1) E^*(x_2, y_2)
\]

and the power in the \( j \)th mode now becomes,

\[
P_j = \frac{1}{\mu} \int_{A_1} \int_{A_2} |e_j^*(x, y) E(x, y)|^2 dA_1 dA_2
\]

In the case of a totally incoherent quasimonochromatic source, the mutual coherence can be modeled as [44]

\[
\Gamma(x_1, y_1, x_2, y_2) = \kappa \|E(x_1, y_1)\|^2 \delta(x_1 - x_2, y_1 - y_2)
\]

where \( \kappa \) is taken to be \( \kappa = \lambda^2 / \pi \) for an incoherent source [45], in which case the power in the \( j \)th mode becomes,
Note that equation 3-7 implies that coherence does not exist between any two points on a planar source, regardless of the distance between them. This is a mathematical simplification of the physical situation. In reality, coherence exists over a linear dimension of at least a wavelength. However, the model of an incoherent source given in equation 3-7 is often used to simplify cumbersome computations [45].

In later sections, we will see that scattered light from biological tissue acts as a source of incoherent light. As we consider the coupling of such light to the proposed fibre, it becomes important to understand the precise nature of the mutual coherence function as it propagates away from the source. This behaviour is fully described by the Van Cittert-Zernike theorem. This theorem implies that there is a gain of spatial coherence as light propagates away from an incoherent source. As light propagates away from the source, it spreads. Light reaching a plane a distance \( z \) away comes from a set of points, some of which are common. Thus, the light at the output plane is partially correlated though the light at the source plane was incoherent. Diagram 3.2 below illustrates the geometry described by the Van Cittert-Zernike theorem.
Mathematically, for quasimonochromatic light in the Fresnel approximation, the mutual coherence at the output plane is given by the Van Cittert-Zernike theorem,

\[
\Gamma(x_1, y_1; x_2, y_2) = \frac{e^{-i\Psi}}{\pi(z)^2} \int \int I(\xi, \eta) \exp\left[i \frac{\Delta x \xi}{\lambda z} + i \frac{\Delta y \eta}{\lambda z}\right] d\xi d\eta
\]

\[3-9\]

where \(I(\xi, \eta)\) is the intensity of the source, \(\Psi = \frac{\pi}{\lambda z} \left[\left(x_2^2 + y_2^2\right) - \left(x_1^2 + y_1^2\right)\right]\), \(\Delta x = x_2 - x_1\), \(\Delta y = y_2 - y_1\), and the other variables are as shown in figure 3.2. In words, this theorem states that to within some scaling factors, the mutual coherence at a plane some distance \(z\) from the source, can be found by a two-dimensional Fourier transformation of the intensity distribution across the source. The implications of this theorem are profound. If the area of the source is small, the fourier transform has wide spatial extent, so that the mutual intensity in the output plane extends over a wide area and the coherence in the output plane is large. In the limit in which the input plane originates from a point, the radiated field is spatially completely coherent [37]. As we will see, this property plays an important role in the modal distributions excited in the proposed cylindrical shell fibre.
As will be seen in later sections, it is also of interest to know the effect of a single thin lens between the source and output plane. The geometry of interest is illustrated in figure 3.3.

![Figure 3.3: Geometry illustrating propagation of mutual intensity through a thin lens](image)

The mathematical expression at the output plane is given by,

\[
\Gamma_0(u_1,v_1,u_2,v_2) = \frac{1}{(\lambda z_s)^2} \frac{1}{\lambda z_o^2} \exp \left\{ -j \frac{\pi}{\lambda z_o} \left[ (u_2^2 + v_2^2) - (u_1^2 + v_1^2) \right] \right\} \\
\times \int \int \int d\xi_z d\eta_z d\xi_2 d\eta_2 \Gamma_s(\xi_z,\eta_z,\xi_2,\eta_2) \exp \left\{ -j \frac{\pi}{\lambda z_s} \left[ (\xi_z^2 + \eta_z^2) - (\xi_2^2 + \eta_2^2) \right] \right\} \\
\times \int \int \int dx_1 dy_1 dx_2 dy_2 P(x_1,y_1) P^*(x_2,y_2) \\
\times \exp \left\{ -j \frac{\pi}{\lambda} \left[ \frac{1}{z_s} \left( x_2 + y_2 \right) - \frac{1}{f} \left( x_1^2 + y_1^2 \right) \right] \right\} \\
\times \exp \left\{ -j \frac{2\pi}{\lambda} \left[ x_2 \left( \frac{u_2}{z_o} + \frac{\xi_2}{z_s} \right) + y_2 \left( \frac{v_2}{z_o} + \frac{\eta_2}{z_s} \right) - x_1 \left( \frac{u_1}{z_o} + \frac{\xi_1}{z_s} \right) - y_1 \left( \frac{v_1}{z_o} + \frac{\eta_1}{z_s} \right) \right] \right\}
\]

3-10

where \( \Gamma_s \) represents the mutual coherence leaving the object plane, \( f \) is the focal length of the lens, \( P(x,y) \) is the aperture function of the lens which gives the lens amplitude transmittance. The aperture function is taken to have \(|P|=1\) within the area of the lens and \( P=0 \) outside the lens aperture. The phase of \( P \) is used to
account for lens aberrations or to account for apodizations. In the present work, the argument of $P$ is taken to be zero. Paraxial approximations are used in the derivation of equation 3-10.

3.2 MODE EXCITATION BY COHERENT AND INCOHERENT SOURCES

In light of the results in the previous chapter, the most important parameters to quantify have to do with the modal amplitudes launched as a function of the harmonic $p$ parameter. Ideally, we are searching for conditions, which enhance coupling to low $p$ parameter modes where narrowband filtering is possible, while maintaining reasonable power coupling efficiency. The assessment of this behaviour will be the focus of our studies in this section. All scripts were implemented using MATLAB.

3.2.1 Coherent Coupling

We begin by examining modal excitation by a coherent, monochromatic source, emitting a gaussian beam of the form,

$$E_i = \hat{x} \exp \left( -\frac{r^2}{\rho_s^2} \right) \exp \{i k n \text{shell} z \} \ \text{with power} \ P_i = \frac{\pi}{4} \left( \frac{e_0}{\mu_0} \right)^{1/2} n_{\text{shell}} \rho_s^2$$

3-11

where $r$ is the radius, $\rho_s$ is the spot size at the source, corresponding to the radius at which the intensity drops to $1/e^2$ of the peak intensity, and $P_i$ is the power in the input beam. The coupling geometry that was simulated is as illustrated previously in figure 3.1. Simulations were run for beams of spot size $\rho_s$ ranging from $5\lambda$ to $200\lambda$ assuming the beam is aligned with the axis of the fibre.
Figure 3.4: (a) Total coupled power as a function of beam spot size for a coherent source. (b) Power coupled in each mode as a function of harmonic parameter for beam spot size $p_s=63.69\lambda$. Inset shows magnified view of bottom most point on large graph which actually corresponds to six closely spaced values. All powers are normalized to input power and spot size is given in unit wavelength.
and that the fibre to source distance, d, is zero. The results are shown in figure 3.4(a). The maximum coupling occurs at a value of $\rho_{s,\text{max}}=63.69\lambda$. At this value of $\rho_{s,\text{max}}$, 21.17% of the light in the original beam is coupled to the fibre. For spot sizes significantly smaller than $\rho_{s,\text{max}}$, the power is focused in the center of the fibre, allowing little overlap with regions of higher refractive index and resulting in low coupling efficiency. Beams with much larger spot sizes are larger than the fibre face and also result in low coupling efficiency. Figure 3.4(b) shows the modal distribution of power excited by a source with $\rho_s=63.69\lambda$. We see that the on-axis beam only couples to the eight modes with harmonic parameter of $p=1$. This is not surprising because similar behaviour occurs in step-index fibre. Linearly polarized, on-axis gaussian beams only excite $HE_{1m}$ modes in telecom fibre since the overlap integrals are non-zero only for modes with a harmonic dependence of $p=1$ [42]. For the same reasons, coupling in the proposed structure results only in excitation of modes with $p=1$. This also means that as long as axisymmetric illumination is maintained, only $p=1$ modes will be excited regardless of the source to fibre distance.

Among the eight modes having $p=1$, one mode is coupled to with roughly 30 times as much power as the mode with the next highest amount of coupled power. To understand this behaviour, we show in figure 3.5(a) the mode with highest coupling efficiency with electric field vectors indicated in each fibre ring. While the other modes exhibit rotation of the field vectors such as the mode in figure 3.5(b) or exhibit alternating directions of polarization in each ring as in figure 3.5(c), the mode of figure 3.5(a) exhibits quasi-unidirectional linear
polarization across all the rings. Thus, this mode has a structure that is most similar to the incoming linearly polarized light, and therefore is coupled to with the largest efficiency.

Figure 3.5: Electric field distributions for modes with \( p = 1 \). The \( x \) and \( y \) axes are normalized to wavelength

So far, it has been assumed that the source is perfectly aligned with the fibre axis (on-axis). Figure 3.6(a) shows the total power coupled into the fibre as a function of radial displacement \( r_{\text{disp}} \) for a coherent gaussian beam with \( p_s = 63.69\lambda \). Once again it is assumed that the source to fibre distance is zero. The parameter \( r_{\text{disp}} \) was varied from 0 to 95\( \lambda \).
Figure 3.6: (a) Total coupled power as a function of radial displacement (b) Power distribution as a function of harmonic parameter for $r_{\text{disp}}=30\lambda$. All powers are normalized to input power, and radial displacement is given in unit wavelengths.
The power coupling efficiency remains constant for roughly $50\lambda$ with a peak at a displacement of $30\lambda$ and then drops off. Depending on the spot size chosen, the $r_{\text{disp}}$ where the peak occurs may change. The modal distribution at $r_{\text{disp}} = 30\lambda$ is shown in figure 3.6(b). We see that off-axis illumination results in significant coupling to $p=0$ and $p=2$ modes along with $p=1$ modes. Thus, off-axis placement of the incoming beam results in coupling behaviour undesirable for optimum grating performance.

3.2.1 Incoherent Coupling

For comparison, simulations of coupling efficiency from a quasimonochromatic completely spatially incoherent source were also run. The coupling geometry is as illustrated in figure 3.1 with the source to fibre distance $d=0$. The incoherent source was taken to have an intensity distribution $I$ given by,

$$ I = \exp\left(-2\frac{r^2}{\rho_s^2}\right) \text{ with power } P_i = \frac{\pi\rho_s^2}{2} \quad 3.12 $$

where $\rho_s$ is the spot size at the source. Half of the power is taken to be polarized in each of the two directions orthogonal to the direction of propagation due to the random nature of the source. A plot of the total power as a function of spot size is shown in figure 3.7(a).
Figure 3.7: (a) Total coupled power as a function of beam spot size for and incoherent source. (b) Coupled power in each mode as a function of harmonic p parameter for beam spot size \( p_s = 100\lambda \). All powers are normalized to input power, and spot size is normalized to wavelength.
We see that the trend is similar to the coherent case in that the maximum coupling occurs at $\rho_{s,\text{max}}=63.69\lambda$. However, the maximum coupling efficiency has been reduced by a factor of $10^{-2}$. This drop in efficiency is not surprising if one examines the difference between equation 3-6 for modal power excitation in the coherent case, and equation 3-8 for the incoherent case. Equation 3-6 consists of a product of two integrals, each performed over the area of the fibre, whereas 3-8 is a product of one integral over the area of the fibre, and $\kappa$ which is proportional to $\lambda^2$. As a result, we expect the incoherent source to have a small coupling efficiency when compared to coupling from a coherent source.

In figure 3.7(b), a plot of the modal distribution of power in each of the modes as a function of harmonic $p$ parameter is plotted for a source with $\rho_s=100\lambda$. In this case, modal power is distributed almost evenly among the modes. This behaviour is consistent with previously reported work in which it was shown that a uniform incoherent source will excite all fibre modes with equal power [46]. The four horizontal rows correspond to modes localized within each of the four fibre shells. There are two differently polarized modes in each shell that are excited with the same amount of power. Modes localized to the center ring are excited with the highest efficiency because the intensity of the incoming beam is higher in the central regions. Modes localized to shells of subsequently higher mean radius carry less power, with the modes in the outer most ring carrying the least power.
In the above scenario, it is assumed that there is no distance between the fibre and the source. We now examine the coupling efficiency as a function of distance for the incoherent source. As stated previously, in the case of the coherent source, as long as axisymmetric illumination is maintained, only $p=1$ modes will be excited regardless of the distance between source and fibre. However, the modal distribution behaviour of an incoherent source as a function of distance cannot be explained so simply. Figure 3.8 shows four different graphs. Plots 3.8(a) and (b) are simulated assuming a spot size of $p_s=25\lambda$ at the source, while 3.8(c) and (d) are for $p_s=50\lambda$ at the source. Plots 3.8(a) and (c) are for fibre to source spacing of 500\(\lambda\), while plots (b) and (d) are for a spacing of 1000\(\lambda\). We notice two trends from these graphs. As we increase the fibre to source distance, there is a decrease in overall coupled power. This is attributed to the spreading of the beam as it propagates which results in a smaller portion of the light being concentrated on the fibre face where coupling occurs. More interestingly, as distance increases, power is no longer coupled to higher order modes. This is due to the gain in coherence of light from the incoherent source as it propagates away from the source. The farther the distance between the fibre and the source, the more spatially coherent is the light reaching the fibre. Thus, as distance increases, the modal excitation behaviour becomes less like the case of incoherent coupling where all modes carried equal power, and more like the case of completely coherent coupling in which only $p=1$ modes were excited.
Figure 3.8. Coupled power in each mode as a function of harmonic p parameter for (a) $\rho_s=25\lambda$, $d=500\lambda$. (b) $\rho_s=25\lambda$, $d=1000\lambda$. (c) $\rho_s=50\lambda$, $d=500\lambda$. (d) $\rho_s=50\lambda$, $d=1000\lambda$. Mode power is normalized to input power.
Also, as predicted by the Van Cittert-Zernike theorem, the source with smaller spot size gains coherence more rapidly than the source with larger spot size. This is seen by comparing plots 3.8(b) and (d) and noting that (b) has a “cut-off” p value of about 15 whereas the beam with larger spot size in figure (d) has a cut-off value of p=25. This indicates a faster convergence of the smaller beam towards the modal distribution one would expect from a completely coherent source.

In addition, off-axis illumination for the incoherent source was also considered. Figure 3.9(a) shows the drop off in power as a function of radial displacement for an incoherent source assuming a fibre to source distance of zero. The power is constant for roughly 30λ of radial displacement and then drops off. The modal distribution for r_{disp}=70λ is displayed in figure 3.9(b). We see that almost all modes are excited with considerable power as with on-axis illumination, however the third and fourth ring from the center now carry more power than the second or first ring since the peak intensity of the beam is displaced radially to the mean radius of the outer most ring.
Figure 3.9 (a) Modal power as a function of radial displacement for an incoherent source. (b) Modal power as a function of harmonic $p$ parameter for an incoherent source with $r_{disp}=70\lambda$. All powers normalized to input power.
With the results of this section, we now have a general understanding of modal excitation by various source geometries. At this stage, we turn our attention to determining the modal distributions that are excited in the proposed fibre by tissue-scattered light. This will be important in determining the probe geometry that will be required for application in an IVR system. We describe in section 3.3 the tissue model used to calculate the Rayleigh scattered light distributions that will be used as the source for light coupling calculations. This will be followed by a description, in section 3.4, of the probe geometry that will be used in these calculations.

### 3.3 Tissue Model

We begin this section with a description of the scattering properties of a single particle [47] and then extend this to describe a stochastic model which can be used to describe light scattering in a dense volume of scatterers as is the case in biological tissue. It is easiest to describe scattering and absorption properties of a particle by assuming an incident plane wave of the form,

\[
E_i(r) = \hat{e}_i e^{(ik\hat{r} \cdot r)}
\]

where the amplitude $|E_i|$ is chosen to be one (volt/meter), $k = \omega(\mu\varepsilon)^{1/2} = 2\pi/\lambda$ is the wave number, $\lambda$ is the wavelength in the medium, $\hat{r}$ is a unit vector in the wave propagation direction, and $\hat{e}$ is a unit vector in its polarization direction. This wave is incident on the particle which has a relative dielectric constant given by

\[
\varepsilon_r(r) = \frac{\varepsilon(r)}{\varepsilon_0} = \varepsilon_r^e(r) + i\varepsilon_r^m(r)
\]
When a plane wave is incident on a particle, part of the power is scattered and the other part is absorbed. The field at a distance $R$ from a particle in the direction of the unit vector $\hat{o}$ is composed of the incident field $E_i(r)$ and the scattered field $E_s(r)$. In the far field, valid when $R > D^2/\lambda$ where $D$ is the particle diameter, the scattered field can be represented by a spherical wave and is given by,

$$E_s(r) = f(\hat{o}, \hat{i}) e^{i(kR)/R}$$  \hspace{1cm} (3-15)

where $f(\hat{o}, \hat{i})$ represents the amplitude, phase and polarization of the scattered wave in the direction $\hat{o}$ when illuminated by a plane wave propagating in the direction $\hat{i}$. Analytic expressions can be derived for the scattering amplitude as a function of the total field inside the particle. This is given by,

$$f(\hat{o}, \hat{i}) = k^2 \int \{ \hat{o} \times [\hat{o} \times E(r')] \} [\delta(r') - 1] \exp(-ikr' \cdot \hat{o}) dV'$$  \hspace{1cm} (3-16)

where $E(r')$ is the total field inside the particle. If we consider the total observed scattered power at all $4\pi$ steradians surrounding the particle, the cross section that would produce this amount of scattering is called the scattering cross section and is defined as,

$$\sigma_s = \int_4 \|f(\hat{o}, \hat{i})\|^2 d\Omega = \sigma_i \int_4 p(\hat{o}, \hat{i}) d\Omega$$  \hspace{1cm} (3-17)

where $\sigma_i$ is defined below and $p(\hat{o}, \hat{i})$ is a dimensionless quantity called the phase function and is commonly used in radiative transfer theory. Likewise, the cross section corresponding to total power absorbed by the particle is defined as the absorption cross section and is given the symbol $\sigma_a$. For an incident wave with magnitude chosen as unity ($|E_i|=1$), it can be defined as,

$$\sigma_a = \left( \int V |k\epsilon_r' E(r')|^2 dV' \right)$$  \hspace{1cm} (3-18)
The total cross section, $\sigma$, then is just the sum of the scattering cross section and absorption cross section.

For certain simple particle shapes, the scattering amplitude and cross sections can be calculated exactly. For example, an exact solution can be found for a homogeneous dielectric sphere and is referred to as the "Mie" solution. In general, however, the field in the particle is not known exactly, although approximations can be made in certain cases to yield useful results. For instance, if the size of the particle is much smaller than a wavelength, the scattering cross section is found to be inversely proportional to the fourth power of the wavelength and proportional to the square of the particle volume. These characteristics of a small particle are called Rayleigh scattering. It can also be found that a small particle will scatter in a nearly isotropic fashion, meaning light scatters in all $4\pi$ steradian directions with equal intensity. This is in contrast to particles that are much larger than a wavelength, in which scattered light is concentrated in a small forward angular direction. A figure of merit to measure the anisotropy of the scattering is given by the $g$ parameter, which is defined as the average cosine of the scattering angle:

$$g = \frac{1}{4\pi} \int p(\hat{\sigma}, \hat{i})(\hat{\sigma} \cdot \hat{i})d\Omega \quad 3-19$$

A value of 0 indicates isotropic scattering while a value of 1 indicates highly directional scattering in the forward direction.

Although there exist relatively simple expressions to determine the scattering amplitude and cross-sections for a single particle, the problem becomes much more complex when one considers a medium with high particle
density. We must now consider not just one scattering event but many. Two approaches are commonly used to solve the multiple scattering problem in a turbid medium: analytical theory and transport theory. The former begins with the wave equation, introduces particle scattering and absorption characteristics, and obtains integral equations for the statistical quantities of interest. This approach is mathematically rigorous in the sense that all effects can be included, however in practice, it is difficult to obtain such formulations. Hence, this approach is rarely used to model scattering in biological media. The later approach, transport theory, does not begin with the wave equation and concerns itself directly with the transport of energy through a medium. In this heuristic approach, it is assumed that due to multiple scattering effects, there is no correlation between fields, and therefore the addition of powers rather than the addition of fields holds. The most fundamental quantity in transport theory is the specific intensity (also called radiance or brightness). For a given point in a medium, the frequency, phase and amplitude of the wave vary randomly in time, causing the magnitude and direction of the power flux density vector to vary with time. The specific intensity is defined as the average power flux density in a direction $\mathbf{\hat{u}}$, within a unit frequency band centered at $\nu$, within a unit solid angle. This quantity $I(r, \mathbf{\hat{u}})$ is measured in Wm$^{-2}$sr$^{-1}$Hz$^{-1}$ (sr=steradian=unit solid angle). When determining the specific intensity at a point in the medium, various effects need to be taken into account. Let us consider a specific intensity in a direction $\mathbf{\hat{u}}$ incident upon an elementary volume of particles. Each particle absorbs and scatters the incident power, reducing the intensity in the direction $\mathbf{\hat{u}}$. At the same
time, the specific intensity is increased by the intensity incident on this volume from other directions, which are scattered into the direction $\hat{u}$. For turbid media, the diffusion approximation of transport theory is used to calculate the specific intensity at various points in a random media. The diffusion approximation holds when the ratio of the volume occupied by the particles to the total volume of the medium is greater than 1%. This is the case in biological tissue, and hence, it is commonly used to model tissue scattering, and absorption [47]. In the diffusion approximation, it is assumed that the incoming wave is rapidly depolarized by multiple scattering events and hence polarization effects are not included.

The specific intensity can be calculated in the diffusion approximation by solving appropriate integro-differential equations, or by using Monte Carlo methods. In the work presented in this paper, we will use Monte Carlo methods to simulate scattering in tissue. Monte Carlo refers to a technique used to simulate physical processes using probabilistic models. Monte Carlo methods are widely used for predicting light transport in tissue [26,30]. Due to the increased popularity in recent years of this method, software coded in the C programming language by S. Jacques et al has been made publicly available on the Internet [48]. The accuracy of the code has been varied by comparison with solutions of the transport equation published by van de Hulst and Giovanelli [49,50]. This code has been used in several scientific studies on biological media and is also used in the work presented here.

The Monte Carlo simulation is performed by tracing individual photon histories through the tissue as they are scattered and absorbed. As a photon is
propagated by increments of distance, $\Delta s$, the photon is scattered, absorbed, transmitted through the medium, internally reflected, or backscattered from the medium. Each time one of these events occurs, the position is recorded. This continues until the photon finally escapes the medium. As the number of photons launched increases, the results approach an accurate simulation of the physical process. Figure 3.10 illustrates the geometry used in the Monte Carlo simulations.

![Figure 3.10: Movement of photons through tissue as calculated by Monte Carlo simulations](image)

The medium properties are specified by three quantities: the absorption coefficient $\mu_a$ (the probability of photon absorption per unit path length), the scattering coefficient $\mu_s$ (the probability of scatter per unit path length) and the phase function $f(\hat{\Theta}, \hat{I})$ which gives the probability of photon scatter from its initial direction $\hat{I}$ to its final direction $\hat{\Theta}$. The absorption and scattering coefficients are related to the previously defined absorption and scattering cross sections by $\rho \sigma_a$ and $\rho \sigma_s$ respectively where $\rho$ is the particle density. The phase function is
characterized by the Henyey-Greenstein phase function to be described later, and is a function of \(g\), the mean cosine of the scattering angle. For mammalian tissue in the near infrared excitation wavelengths, typical values are \(0.001 \text{mm}^{-1} < \mu_a < 0.5 \text{mm}^{-1}\), \(35 \text{mm}^{-1} < \mu_s < 70 \text{mm}^{-1}\), and \(0.7 < g < 0.99\) [51]. For this work, it was assumed that the index of refraction of tissue was the same as water (\(n=1.33\)) although in practice it may be higher by a few percent.

The photon is initialized to begin propagation at co-ordinates (0,0,0). Once the fluence distributions generated by this infinitely narrow beam input is determined, convolution techniques are then used to determine fluence rates for other input beam shapes. All photons are assumed to initially travel downward, directly into the tissue. Each photon is propagated through the medium by incremental distances \(\Delta s\), which are chosen randomly. The probability distribution describing the distance traveled before an interaction occurs follows Beer's law and is proportional to \(e^{-\mu t \Delta s}\). Thus, \(\Delta s\) is chosen by implementing the function,

\[
\Delta s = \frac{-\ln \xi}{\mu_t}
\]

where \(\Delta s\) is the distance traveled by a photon before it is absorbed or scattered, and \(\xi\) is a uniform random variable generated by the computer. As each photon enters the medium it is given a weight of unity. After each step, a fraction is absorbed and the rest scattered. The fraction absorbed is given by,

\[
\text{absorbed fraction} = \frac{\mu_a}{\mu_a + \mu_s}
\]

This photon weight is reduced by this fraction and recorded as a function of the position at which the photon was partially absorbed. The remaining fraction is
scattered. The direction of scatter is chosen from an appropriate probability density function which is described by a normalized phase function \( f(\theta, i) \). The distribution of the azimuthal angle \( \theta \) is given by Henyey-Greenstein phase function,

\[
\cos \theta = \frac{1}{2g} \left( 1 + g^2 - \left[ \frac{1 - g^2}{1 - g + 2g\xi} \right] \right). \tag{3-22}
\]

In the case of \( g=0 \) (isotropic scattering), the following function is used,

\[
\cos \theta = 2\xi - 1. \tag{3-23}
\]

The angle \( \phi \) is uniformly distributed between 0 and \( 2\pi \) (i.e., \( \phi=2\pi \xi \)). Using the new angles, the old position is updated accordingly, and the new position is determined. If the new position is found to require a crossing from the tissue into air, the probability that the photon will be reflected is determined by the Fresnel reflection coefficient,

\[
R(\theta_i) = \frac{1}{2} \left[ \frac{\sin^2(\theta_i - \theta_i) + \tan^2(\theta_i - \theta_i)}{\sin^2(\theta_i + \theta_i) + \tan^2(\theta_i + \theta_i)} \right] \tag{3-24}
\]

where \( \theta_i \) is the angle of incidence on the boundary and the angle of transmission \( \theta_t \) is given by Snell's law,

\[
n_i \sin \theta_i = n_t \sin \theta_t. \tag{3-25}
\]

The fraction of the photon weight that is reflected continues to propagate as before and the weight of the photon that escapes is recorded as a function of position. To determine intensity distributions in the tissue for various input distributions, convolution techniques are used. The fluence rate that results when a Monte Carlo simulation is performed with photons launched at a single point is the Green's function \( G(x,y,z) \) for the medium. The fluence rate for any arbitrary profile of input beam, \( S(x,y,z) \) can be calculated given the Green's function by,
\[ \Phi(x,y,z) = \int \int G(x',y',z')S(x-x',y-y')dx'dy' \]  

For a gaussian source function with a 1/e^2 radius of R,

\[ S(r) = S_0 e^{-2(r/R)^2}, S_0 = \frac{2P}{\pi R^2} \]  

where \( P \) is the total power in the beam and the fluence equation in cylindrical coordinates becomes,

\[ \Phi(r,z) = S_0 e^{-2(r/R)^2} \int_0^\infty G(r',z)e^{-2(r'/R)^2}I_0(4r'r/R^2)2\pi dr' \]  

where \( I_0(r) \) is the zero order modified Bessel function.

Figure 3.11 illustrates the backscattered intensity as calculated by the Monte Carlo program for a sample simulation.

![Graph](image)

**Figure 3.11: Backscattered intensity versus radius as calculated by the Monte Carlo program**

The input gaussian beam was taken to have a spot size of 0.5mm and to contain 1 J of energy. The medium is assumed to have scattering coefficient, absorption coefficient and g parameter given by \( \mu_s = 50 \text{ mm}^{-1} \), \( \mu_a = 2 \text{ mm}^{-1} \), and \( g = 0.8 \). These medium properties correspond to typical values found in mammalian tissue as stated previously [51], and are used throughout the simulations in the remainder
of the chapter. Because the input beam is axisymmetric, the backscattered light is also axisymmetric and only a function of radius. We note that the intensity profile of the backscattered light as a function of radius is roughly gaussian in shape. The total energy that is backscattered is roughly half the input energy (0.53J) because of the large scattering coefficient of the tissue. Also, because of multiple scattering in the tissue, we know that this backscattered light is completely randomized in terms of polarization and phase.

3.4 Probe Geometry

We are now in a position to understand the design requirements for a biomedical probe. As explained in the previous section, the Rayleigh backscattered light from the tissue most closely resembles an incoherent source due to randomization of the incoming light by multiple scattering events in the tissue. Therefore, we know that if there is no distance between the probe and the tissue, and the excited region covers the area of the fibre end face, all modes will be excited. This will result in poor grating filtering since we know that backscattered light from the tissue must be coupled into lower order \( p \) modes in order to have a narrowband reflectivity response. However, when there is sufficient distance between the probe and the tissue, coupling will be limited to lower order modes. The smaller the source is, the less the axial distance that will be needed to excite modes below a given \( p \) value. The smaller axial distance between source and fibre in turn implies that more total power will be coupled into the fibre because the light from the source does not spread as much and
most of the power stays concentrated on the fibre face. As we saw in section 3.3, the area over which the backscattered light is emitted depends on the size of the excitation beam, which is delivered by the delivery probe. Thus, the behaviour of the proposed fibre as a collection probe in an IVR system relies heavily on delivery fibre geometry as well. Finally, we saw that off-axis illumination results in excitation of higher order modes, so a geometry that allows on-axis illumination is preferred.

One possible probe geometry, that meets these requirements, is illustrated below.

![Figure 3.12: Probe geometry](image)

The proposed fibre is made with a hollow core and the delivery fibre (either step or graded index) is placed through the hollow center. Since the excitation light is delivered centrally, the backscattered light will also be axisymmetric. The cylindrical shells of higher refractive index act as the collection fibre in the probe. Some distance is maintained between probe and tissue to ensure that low order modes are excited. In practice, this can be accomplished by placing a piece of fused quartz at the tip of the fibre with appropriate thickness, so that the overall probe can be placed directly in contact with the tissue. In order to filter unwanted Rayleigh scattered pump light, a grating is written in the collection fibre. The
grating is written at the tip so that mode mixing in the propagation direction is not an issue.

3.5 MODE EXCITATION BY TISSUE SCATTERED LIGHT

Simulations of coupling from tissue scattered light to the probe described in section 3.4 were run. The geometry is illustrated in figure 3.13.

![Figure 3.13: Side view of geometry for coupling of backscattered tissue light to probe](image)

A typical value for the numerical aperture, N.A. = 0.2, was chosen for the delivery fibre. The delivery fibre core was taken to have a diameter of 15λ as this would allow the fibre to be inserted into the central portion of the cylindrical shell geometry described in Chapter 2. From the numerical aperture of the delivery fibre (N.A.=0.2) the spot size illuminating the fibre at a distance d could be calculated. This value was used as the spot size of the illuminating beam in the Monte Carlo program described in section 3.3. The output intensity of the backscattered light distribution was used as the source for the fibre coupling calculations. All simulations were implemented in MATLAB. Due to the numerous integrals in the calculation, scripts took roughly 24 hours to complete. Figure 3.14
shows the modal distribution for a probe to tissue distance of 500\(\lambda\) and 1000\(\lambda\). The total power coupled into the fibre is on the order of \(10^{-5}\). This is comparable to the efficiency experimentally determined by our collaborators of the fibre bundle probe described in Chapter 1 [26]. Thus, with just one fibre, we are able to acquire the same efficiency as a six-fibre probe. However, we see that the modal distribution does not "cut-off" as rapidly as we observed in the previous section for similar fibre to source distances. Also, coupling characteristics for a probe to tissue distance of 1000\(\lambda\) has no more enhanced coupling to lower order modes than a probe to tissue distance of 500\(\lambda\). This is due to the spreading \(\sigma\), the input beam as it propagates from the probe to the tissue. As a consequence of this spreading, the illuminating light from the delivery fibre has a large area. The backscattered light, therefore, is also broad. As we saw in section 3.1, a broad source will not gain spatial coherence as rapidly as a small source. Hence, the light is coupled to almost all modes for any probe to tissue distance. Unfortunately, this is undesirable for Bragg grating behaviour.
Figure 3.14: Power in each mode as a function of harmonic parameter for probe to tissue distance of (a) $500\lambda$ and (b) $1000\lambda$. 
In order to eliminate this behaviour, simulations of a geometry with a lens at the probe tip were performed. The modified geometry is illustrated in figure 3.15.

Figure 3.15: Probe geometry with a thin lens where $f$ is the lens focal length

Simulations were run for $z_s=200\lambda$ and a focal length of $f=100\lambda$. Figure 3.16 shows the results. The lens focuses the light from the delivery fibre to a point at the tissue plane. Thus, the back scattered light has a small spatial extent and gains coherence rapidly, giving the desired effect of exciting lower order modes. In fact, only modes with $p=1$ are excited due to the almost completely coherent nature of the light by the time it reaches the probe. Also, the addition of the lens increases the coupling efficiency of the probe to $10^3$. So, we see that the use of a lens with the probe geometry described earlier results in coupling to only $p=1$ modes which implies a strong reflection will be possible when a grating is written in the collection rings. Also, the results show that coupling efficiency will be higher than currently possible with existing probe geometries.
Figure 3.16: Power coupled as a function of harmonic parameter. Inset shows magnified view of bottom most point on large graph which actually corresponds to four closely spaced values. Power is normalized to input power.

An added benefit is that this probe consists of only one fibre, making it more compact than currently existing probes. However, this geometry is sensitive to misalignment of the lens with the axis of the fibre, or probe tilt, which will cause coupling to higher order modes. Still, if these aspects are kept to a minimum, the excited modes will be among the lower $p$ modes and grating reflectivity will be high. Overall, the results of this chapter suggest that a probe with a lens has promising prospects for use in an IVR system.
Chapter 4

EXPERIMENTAL RESULTS

In this chapter, we outline the various experiments that were tried on cylindrical shell fibres that we fabricated. As we will discuss in more detail in section 4.1, the fabricated fibre did not meet our specifications, and as a result, we were unable to successfully write a grating. Still, for completeness, we include descriptions of the grating experiments that were attempted. In addition, we describe results from coupling experiments that show good agreement with the coupling model described in the previous chapter.

4.1 FIBRE FABRICATION

Three fibres, each consisting of four cylindrical shells of higher refractive index, were fabricated. Each of the three fibres was scaled to operate at a different wavelength. One preform was made and drawn at different rates to create all three fibres. The profile of the preform provided by the manufacturer is shown in figure 4.1 below.

![Index profile of preform of fabricated fibre](image)

Figure 4.1: Index profile of preform of fabricated fibre
The first fibre was scaled for operation at a wavelength of 1.55μm to allow use with standard grating writing equipment. The second was scaled to operate at 785nm, which is the wavelength necessary for the IVR application. The third was scaled to operate in the visible range at 488nm.

The major problem with the fabricated fibre was that the refractive index change between the shell and cladding was half the specified value of Δn=0.005. Because the increased refractive index of the shell was induced by germanium doping, the small index difference indicates low germanium content, and hence lower photosensitivity than desired. In addition, the small index difference results in very poor guiding at the desired wavelengths for which the fibres were made. This was confirmed by performing simulations in the fibre scaled to operate at 785nm using an approximation of the profile in figure 4.1 at a wavelength of 785nm. Simulations showed that guiding was very poor. Light was no longer confined to the regions of higher effective index and the normalized effective index for p=1 modes was centered on a low value of \( b = 0.049 \).

The weak guiding was further confirmed by measuring the output spectrum of the fibre using a halogen source at the input. Figure 4.2 shows the output spectrum for the fibre scaled for 785nm operation. We see that at a wavelength of 785nm, the power guided is 3.5dB lower than the maximum power guided at 613nm. The roll off at the lower wavelength side below 600nm is an artifact due to the limitations of the optical spectrum analyzer, and can be ignored.
This behaviour was observed in all three fibres making them useless at the desired wavelengths. Unfortunately, attempts to have this fibre fabricated a second time to specification have been unsuccessful and further attempts are still in progress at the time of writing of this thesis.

4.2 Grating Experiments

Though the available shell structure fibres did not have adequate guiding at the desired wavelength, simulations of mode profiles at shorter wavelengths indicated reasonable modal guiding. For a wavelength of 980nm in the fibre scaled for 1550nm operation, the normalized effective index took on a value of about \( b=0.47 \) for modes having \( p=1 \). Fortunately, there was one phase mask available at the Photonics Research Ontario grating facility that could be used to write a grating at 980nm. The phase mask used for these experiments was a uniform grating with a length of 3mm, and phase mask period of \( \Lambda_{pm}=675\text{nm} \). The spectrum was measured in transmission during grating writing. Specially crafted bare fibre connectors (made to accommodate the large diameter of the
crafted bare fibre connectors (made to accommodate the large diameter of the novel multimode fibre) were used to connect the fibre to an optical spectrum analyzer at the output and a halogen light source at the input. Before each attempt at grating writing in the novel fibre, the accurate alignment of the phase mask was verified by writing a grating in graded index multimode fibre. The sample spectrum of a grating written at the 980nm wavelength in graded index fibre is shown in figure 4.4. Due to the limited amount of power emitted from the halogen source, it was not possible to couple enough light into a single mode fibre in order to check single mode grating performance. However, according to the analysis of GRIN fibre grating behaviour presented in Chapter 2, a 1.5 dB reflection in a GRIN fibre indicates that a large index perturbation was written.

![Figure 4.3: Transmission spectrum of grin fibre with uniform grating](image)

This suggested that a grating would be possible in the proposed cylindrical shell fibre. Several attempts to write a grating in the shell structure fibre were made using the phase mask method. A range of pulse energies was used on two different grating writing setups. For some attempts, the UV beam was focused tightly at the core of the fibre whereas in other attempts the light was defocused.
in an attempt to write a more uniform grating across the wide fibre diameter (360µm).

Unfortunately, no grating response could be observed. The only observation that could be made was an overall broadband reduction in the power at the output of the fibre, which occurred during grating writing. One possibility is that a non-uniform grating was being written in the core and causing a broadband coupling to cladding modes. However, a rigorous analysis is necessary to verify this. Also, practical factors, such as the short length of the available phase mask, the large diameter of the fibres, and low photosensitivity in the shells of the fibre, may have been contributing factors to our inability to observe a grating response.

In an attempt to determine if it was the thickness of the fibre preventing grating writing, the fibres were etched down to one ring using hydrofluoric acid. The acid was placed in a plastic petri dish and used to etch the fibre. The diameter of the fibre was measured periodically with a digital micrometer scale until the diameter was reduced to 30λ corresponding to a fibre with only one ring. Due to the delicacy of the etched portion of the fibre, only one fibre did not break during removal from the hydrogen loading chambers, though several were etched. The remaining fibre was irradiated through the same 980nm phase mask described previously. Once again, the only change in the transmission characteristic that could be observed was an overall reduction of transmitted power over the 20nm span between 970nm-990nm that was being observed.
4.2 COUPLING EXPERIMENTS

Several experiments were tried to determine the coupling efficiency of the available fibres and to compare them to theoretical calculations. The first experiment was performed to determine overall coupling efficiency to a coherent source. The input source used was a laser at 488nm. This source was used to test coupling efficiency in the fibre originally scaled for operation at 785nm. First, the power of the incoming laser beam at the input of the fibre was measured with a photodetector. The photodetector was then moved to the output of the fibre and the position was optimized for maximum coupling. It was found that the ratio of coupled power to input power was 1.1%. The radius of the laser beam was measured using a Spiricon beam profiler, which fits a gaussian beam to the incoming light. The profiler determined the spot size radius (corresponding to a 1/e^2 fall off in intensity) of the incoming beam to be 1.225mm. This value was then used as the spot size for the simulations. Simulations showed a coupling efficiency of 0.37%, a value that is 0.34 times the experimental value. Thus, we were able to get fairly good agreement between experimental and theoretical values for the case of coupling to a coherent source.

We suspected that experimental values were slightly higher than theoretical calculations due to propagation of cladding modes in the center of the fibre. Despite the use of index matching gel on the outside of the fibre, the cladding modes in the center may not necessarily have been guided out of the fibre. This was confirmed by using a Spiricon beam profiler to capture the intensity profile at the end face of the fibre. Figure 4.4 shows the measurements
of the beam profiler. Figure 4.4 (a) shows the unsaturated intensity. That is, the power was kept low enough not to saturate the camera of the beam profiler. The light levels seem reasonably well confined to the rings of higher refractive index. However, some light can be seen in the central regions. To make this more apparent, the second image in 4.4 (b) is taken after the power was increased just enough for the light in the rings to saturate the camera. Figure 4.4 (b) shows that there is indeed light confined in the central regions of the fibre. Though the light level in the central regions is weak, the area of the fibre in the center region and lower refractive index annular regions is roughly ten times the area of a single ring. A rough estimate made by integrating the image data from the beam profiler indicates that the power in the non-guiding regions is about 23% of the light in the guided regions. Thus, this contribution is responsible, at least in part, for the experimentally measured efficiency being higher than the calculated efficiency. Another factor increasing experimental efficiency could be that the light that is initially coupled to cladding modes in the central region is eventually becoming trapped in guiding regions as
it propagates down the fibre. This light would not have been accounted for by the analysis of Chapter 3, as no theoretical considerations of mode mixing between cladding and guided modes in the propagation direction was modeled.

A second experiment was performed to determine the power efficiency fall off as a function of the radial displacement between the laser source and the fibre. The experimental setup is illustrated in figure 4.5.

Figure 4.5: Experiment for coupling setup; lens focal length $f=25$cm, Aperture diameter=3.5μm

The beam from the Argon ion laser was put through a 4f imaging system to ensure that the beam had uniform cross-section. The beam was then coupled to the fibre and the output light from the fibre was measured using a photodetector. Measurements of output power were made as the fibre was translated radially using a translation stage. Figure 4.6 shows a plot of experimental values of power as a function of radial displacement, indicated with a solid line, and theoretical values indicated by a dashed line.
Once again, the $1/e^2$ spot size radius used for calculations was $\rho_s=1.225\text{mm}$, as indicated by the beam profiler. Plots are normalized to unity power in order to facilitate comparison between experimental and theoretical values. We see relatively good agreement between the two lines indicating that the major features of power fall off are well predicted by the theoretical calculations described in Chapter 3. The slight asymmetry of the experimental results can be attributed to a mismatch in the alignment between the beam center and the fibre axis.

The final experiment was performed to determine the overall coupling efficiency of tissue-scattered light using a tissue simulating phantom medium. Note that in this experiment, coupling takes place between a virtually incoherent source (tissue-scattered light) and the fibre, as opposed to the coherent coupling scenario in the first experiment. The tissue phantom used was 20% stock
Intralipid, which consists of an emulsion of phospholipid micelles and water. This medium has been used extensively as a tissue-simulating medium for scattering and absorption characterization [7]. The experiment was performed at a wavelength of 488nm at which the characteristic properties of 10% stock Intralipid have been characterized [52]. The Intralipid was diluted accordingly and simulations were run using scattering coefficient, absorption coefficient, and anisotropy factor given by $\mu_s = 657\text{cm}^{-1}$, $\mu_a = 0.014\text{ cm}^{-1}$ and $g = 0.86$ [52]. The setup used was as previously shown in figure 4.5 with the addition of a cuvette of 1mm thickness containing Intralipid placed between the incoming beam and the fibre input. The coherent input beam falls off as [47],

$$I'_{\text{coh}} = I_{\text{coh}} \exp(-\mu_s z) \quad 4-1$$

where $\mu_t$ the total scattering coefficient, equal to the sum of the absorption and scattering coefficients. For Intralipid, the intensity falls to $10^{-29}$ value of the input beam after traveling through 1mm of Intralipid meaning that the coherent contribution can be ignored and all light exiting the Intralipid can be taken to be incoherent scattered light. In addition, a 1mm aperture was placed between the beam and the Intralipid to ensure that the area of illumination is well known and all other light is blocked. Using a photodetector, measurements of the input power were made after the Intralipid and before the fibre, and output power was measured at the fibre end face. The ratio of coupled power to input power was found to be $3.6 \times 10^{-5}$. Simulations of modal coupling efficiency were performed and the results are shown in figure 4.7.
We see that all modes are fairly equally excited. The total ratio of coupled power according to simulations came to $1.32 \times 10^{-5}$. The ratio of theoretical coupling efficiency to experimental coupling efficiency was 0.36, a value that is remarkably similar to the ratio of 0.34 found in the coherent coupling experiment. Thus, overall agreement between theoretical and experimental values is high for all three experiments even though very different sources were being studied. This indicates that the model described in Chapter 3 predicts coupling efficiency with a high degree of accuracy for both coherent and incoherent sources.
Chapter 5

CONCLUSIONS AND FUTURE WORK

5.1 Summary and Conclusions

We began by presenting a novel multimode fibre structure consisting of cylindrical shells of higher and lower refractive index. The proposed fibre was designed with the goal of enabling narrowband, high reflectivity gratings to be written in multimode fibre. We showed that the required condition for such narrowband grating response was that of modal effective index degeneracy. Finite element analysis was used to describe the guided modes of the proposed structure and we showed that modes of low harmonic p parameter travel with nearly degenerate effective indices. Effective indices of the modes were shown to depend on fibre parameters such as inner shell radius, and shell separation. We showed that a large inner radius and shell separation minimized the effective index difference between the modes. It was shown that minimal performance improvement could be made through structural modifications beyond a certain choice of inner ring radius and shell separation \((r_0 = 25\lambda, \Delta = 10\lambda)\). Simulations of grating behaviour in the proposed fibre showed that a narrowband reflection response with a FWHM bandwidth of \(10^{-4}\lambda\) is possible only when modes with low harmonic p parameter are guided. However, even assuming equal mode population distribution, the reflectivity response of the proposed fibre still has a substantially higher reflectivity and more narrowband response than the case of equal mode population distribution in graded index multimode fibre.
Subsequently, we presented an analysis of mode distributions in the fibre for excitation by different quasimonochromatic sources. The analysis was used to provide insight into possible probe geometries that could be implemented for an in vivo Raman Spectroscopy system. Simulations showed that excitation by an on-axis, spatially coherent source resulted in excitation of only $p=1$ modes. Spatially incoherent sources excited modes with all harmonic parameters almost uniformly. However, at large distances between fibre and source, the spatially incoherent source was shown to couple only to lower order $p$ modes. A possible probe geometry was presented consisting of a central delivery fibre placed in a hollow collection fibre containing cylindrical shells of higher refractive index. This geometry is more compact than existing multi-fibre probes, and can be made more inexpensively since it requires only one grating to be written in the collection fibre, as oppose to requiring several dielectric filters. Distributions of Rayleigh scattered light from tissue were generated by Monte-Carlo methods and used to determine mode distributions in the suggested probe. We determined that power coupling efficiency of the probe was on the order of $10^{-5}$, a value comparable to existing probes. A lens placed between the probe and tissue was shown to increase the efficiency to $10^{-3}$. In addition, we showed that adding the lens would result in coupling only to $p=1$ modes, which is desirable from the point of view of grating behaviour. These results imply that the suggested probe has promising prospects in the field on in vivo Raman Spectroscopy.

Finally, experiments were performed on fabricated fibres. Attempts to write a grating were unsuccessful due to the poor manufacturing of the fibre. Coupling
experiments were made to determine experimental coupling efficiency for excitation by an on-axis laser source, and for differing radial displacement of this laser source. As well, coupling efficiency of scattered light from a tissue-simulating medium was determined. Good agreement was found between experimental and theoretical results.

With this thesis, we have progressed our understanding of this novel fibre structure, which had never before been studied. We have identified important fibre structural parameters that affect modal behaviour and studied Bragg grating behaviour in this fibre. We have also been able to identify the importance of exciting modes with low harmonic parameter for narrowband grating responses and identify the source characteristics and geometries that allow the desired excitation of modes. Finally, we have illustrated a possible probe geometry that could be used for in vivo Raman Spectroscopy.

5.1 Future Work

Although our theoretical understanding of this novel structure is fairly well developed, there is much that remains to be done in terms of experimental work. Most importantly, a narrowband grating response in a prototype fibre has yet to be demonstrated. This may pose problems on two fronts. First, the fibre has to be made to specification, which is made difficult because of the unusual refractive index profile. Further investigation into this issue remains to be made to eliminate the problem at the manufacturing level. Secondly, a uniform grating must be written which poses difficulties due to the large diameter of the fibre. In
connection with this, non-uniform grating behaviour should be analyzed and the effects determined.

To advance the use of this fibre for the biomedical application presented here, a probe needs to eventually be manufactured. Practical issues such as the effects of source-lens or lens-fibre misalignment need to be examined, as well as the effect of probe handling during clinical trials.

In addition to future work based on further developing the probe, there remains work for the implementation of the proposed multimode fibre in other application areas. One application, mentioned in this thesis, is the possible use of this fibre to implement wavelength division multiplexing in local area networks where multimode fibre is already predominant. However, many gaps in this idea need to be bridged theoretically before a practical implementation can be considered. For example, an efficient method of coupling between the shell structure fibre and the graded index fibre needs to be determined. Also, dispersion issues need to be considered. On a broader scale, the potential of this fibre still remains vastly unexplored in areas such as sensing applications or other biomedical areas. It has yet to be seen what impact can be made with this technology in the photonics industry, and it will be interesting to watch its development in future years.
APPENDIX A

FINITE ELEMENT METHOD

We consider here an axisymmetrical optical fiber with \( n(r) \) in the core (0<\( r < a \)) and \( n_2 \) in the uniform cladding (\( a < r \)). The wave equation of the magnetic field \( H \) is,

\[
\nabla \times \left( \left[ \varepsilon_r \right]^{-1} \nabla \times H \right) - k_0^2 H = 0
\]

where \( H \) is the magnetic field and \( \left[ \varepsilon_r \right] \) is the relative electric permittivity. The general functional used to solve the above equation is [38],

\[
F = \int_{\Omega} (\nabla \times H)^* \cdot \left( \varepsilon_r^{-1} \nabla \times H \right) \, d\Omega - k_0^2 \int_{\Omega} H^* \cdot H \, d\Omega + \int_{\Gamma} H^* \cdot \left( \varepsilon_r^{-1} \nabla \times H \right) \, d\Gamma
\]

where \( \varepsilon_r \) is the relative electric permittivity, \( \Omega \) is the cross sectional area of the waveguide, \( \Gamma \) is the boundary between the core and the cladding, \( \varepsilon_r \) is the unit radial vector, \( k_0 \) is the vacuum wavevector. Expanding the magnetic field in terms of nodal values gives,

\[
H = [N]^T \{H\}_o \exp(-j\theta)
\]

where the nodal magnetic field vector for each element \( e \) is

\[
\{H\}_o = \begin{bmatrix} \{H_1\}_o \\ \{H_2\}_o \end{bmatrix}
\]

and \( [N] \) is given by

\[
[N] = \begin{bmatrix} \{N\} & \{0\} & \{0\} \\ \{0\} & j\{N\} & \{0\} \\ \{0\} & \{0\} & j\{N\} \end{bmatrix}
\]
where \( \{N\} \) is defined, for second order elements by

\[
\{N\} = \left[ \begin{array}{c}
\frac{r_2-r}{r_2-r_1} \\
\frac{r_2-r_1}{r_2-r_1}
\end{array} \right] \left[ \begin{array}{c}
2 \left( \frac{r-r_1}{r_2-r_1} \right) - 1 \\
2 \left( \frac{r-r_1}{r_2-r_1} \right) - 1
\end{array} \right] \left[ \begin{array}{c}
\frac{r_2-r}{r_2-r_1} (r-r_1) \\
\frac{r_2-r_1}{r_2-r_1} (r-r_1)
\end{array} \right]^T \tag{A-6}
\]

The annular elements are shown in the figure below.

![Figure A.1: Annular axisymmetrical elements](image)

Substituting equation A-3 into A-2 and summing over all elements gives,

\[
F = \{H\}^T ([K] - k_0^2 [M]) \{H\} + \int \mathbf{H} \cdot \left[ \mathbf{i}_r \times \left( \frac{1}{n_2^2} \nabla \times \mathbf{H} \right) \right] d\Gamma \tag{A-7}
\]

where

\[
[K] = 2\pi \sum_{\sigma} \int \frac{1}{n^2(r)} \{B\} \{B\}^T \, rdr \tag{A-8}
\]

\[
[M] = 2\pi \sum_{\sigma} \int \{N\} \{N\}^T \, rdr \tag{A-9}
\]

where the matrix \([B]\) is

\[
[B] = \begin{bmatrix}
\{0\} & -j\beta \{N\} & j\beta \{N\}/r \\
-j\beta \{N\} & \{0\} & j\beta \{N\}/r + j\{N\}/r \\
p\{N\}/r & -jd\{N\}/dr & \{0\}
\end{bmatrix} \tag{A-10}
\]

An analytic solution can be determined for the homogeneous cladding and be shown to have the form,
Using this analytic solution, the value of \( e_r \times \left[ \varepsilon_r^{-1} \nabla \times \mathbf{H} \right] \) on the boundary \( \Gamma \) is given by

\[
e_r \times \left[ \varepsilon_r^{-1} \nabla \times \mathbf{H} \right] = j \left[ Z_0 \right]_{r} \{ H \}_r \exp(-j\rho\theta)
\]

where

\[
\left[ Z_0 \right]_{r} = \begin{bmatrix}
0 & 0 & 0 \\
-\omega K_p(\omega) & \rho \beta K_p(\omega) & \frac{\rho \beta}{n_0^2 \omega} K_p'(\omega) \\
0 & \frac{\rho \beta}{n_0^2 \omega} K_p'(\omega) & \frac{\rho \beta}{n_0^2 \omega} K_p'(\omega) + \frac{k_0^2 a}{\omega K_p(\omega)}
\end{bmatrix}
\]

Combining the finite element method with the analytical solution gives a functional of the form,

\[
F = \{ H \}_r^t [A] \{ H \}_r + \{ H \}_r^t [Z]_{r} \{ H \}_r
\]

where

\[
[A] = [K] - k_0^2 [M] \quad \text{A-15}
\]

\[
[Z]_{r} = 2\pi a [Z_0]_{r} \quad \text{A-16}
\]

Application of the variational principle to equation A-14 results in

\[
[\mathcal{A}] \{ H \} = \{ 0 \} \quad \text{A-17}
\]
where

\[
\begin{bmatrix} \Lambda \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} + \begin{bmatrix} Z \end{bmatrix} \tag{A-18}
\]

Thus, the propagation characteristics of the optical fiber can be calculated from

\[
\det(\Lambda) = 0 \tag{A-19}
\]

If we solve this as it is, spurious solutions that do not satisfy \( \nabla \cdot H = 0 \) will still be generated. To prevent these, the penalty method was used. We modify equation A-14 to,

\[
\tilde{F} = F + s \int_{\Omega} (\nabla \cdot H) (\nabla \cdot H \Omega) \tag{A-20}
\]

and use this modified function to calculate the propagation characteristics of the fiber as

\[
\det(\Lambda) + s[L] = 0 \tag{A-21}
\]

where

\[
[L] = 2\pi \sum_{e} \int_{\gamma} \begin{bmatrix} C \end{bmatrix}^\top \begin{bmatrix} C \end{bmatrix} r dr \tag{A-22}
\]

and the vector \([C]\) is

\[
[C] = \begin{bmatrix}
d\{N\}/dr + \{N\}/r \\
p\{N\}/r \\
\beta\{N\}
\end{bmatrix} \tag{A-23}
\]
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