Mesostucture Quantification of Fibre-reinforced Composites

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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0-612-49722-4
Abstract

Computer-aided image processing techniques are developed to determine the position of fibre from a cross-section image. Two-dimensional mesostructure information is obtained by constructing Voronoi Diagrams. Fibre misalignment and fibre waviness are analyzed from the images in successive sections. Three laboratory-made samples, including carbon-epoxy pultrusion, carbon-PEEK laminate, glass-epoxy laminate, are tested. A composite sample from Bombardier Aerospace and a multi-layer filament-wound tube from FRE Composites are also examined.

The paths of fibres in carbon-epoxy pultrusion sample are traced individually and fitted as sinusoidal functions. Misalignment angle distribution is also measured and it is found that it can be used to estimate the fibre waviness. In the analysis of the commercial filament-wound tube, an increasing trend of waviness from the outside layer to the inside layer is found.

Methods for quantifying fibre spatial regularity and fibre movement synchronization are proposed. From the two-dimensional mesostructure of the materials examined, it is found that high volume fraction gives more uniform fibre packing and more synchronized fibre movement.
Acknowledgement

I would like to acknowledge and express my gratitude to Professor M. R. Piggott, for providing the motivations of this interesting study and for his guidance, enthusiasm, patience and thoughtful opinions throughout the course of this work. Special thanks to Jenny Clifford for her friendship, care and encouragement.

I would like to extend my special thanks to Professor Kuhn, for examining my thesis and coordinating my graduate committee. I also thank the other members of my committee, Professor Kirk and Professor Jia, for their time and effort.

I would also like to thank FRE Composites and Bombardier Aerospace for generously supplying composite samples.

I am thankful to all members of Advanced Composites, Physics and Chemistry Group for their support and friendship, especially to Dr. Karen Lui for her great assistance and encouragement. Also, I am thankful to all other members (Alex, Asif, Jim, Lynn, Mimi, Shane, Shu-ren) for providing a fun and enjoyable research environment. Their challenges have undoubtedly improved my computer literacy in a great extent.

I extend my thanks to my friends who helped me without reciprocation in these years. Thanks Max for sharing his insight in image processing and programming, Ricky for his help to keep me awake, and Alex, Erik, Hai, Joseph and Xintong for their friendships.

Finally, I express my deepest gratitude to my parents for their laissez faire economic support, forgiveness and patience.
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1 Introduction

1.1 Composite Materials

In general, a composite consists of fibres embedded in or bonded to a matrix with distinct interfaces between two constituent phases. The fibres usually have high strength and modulus and therefore they are used as the principal load-bearing members. The matrix keeps the fibres in a desired location and orientation, separating fibres from each other to avoid mutual abrasion during periodic straining of the composite. The matrix acts as a load transfer medium between fibres, and in less ideal cases where loads are complex, the matrix may even have to bear loads transverse to the fibre axis. Since the matrix is generally more ductile than the fibres, it is the source of composite toughness. The matrix also serves to protect the fibres from environmental damage before, during, and after composites processing. In a composite, both fibres and matrix largely retain their identities and yet result in many properties that cannot be achieved with either of the constituents acting alone.

A wide variety of fibres are available for use in composites. The most commonly used fibres in polymer matrices are various types of carbon, glass, and aramid e.g. Kevlar fibres. Also still in limited use are alumina, boron, silicon nitride, ceramic fibres and metal wires. The most common materials used as matrices are thermoplastic or thermoset polymers, metals (e.g. aluminiums, titanium, and superalloys), or ceramics. Reinforcing fibres are also available in wide variety of forms, as shown in Figure 1.1. Composites can differ in the amount of fibre, fibre type, fibre length, and fibre orientation.
By nature, continuous-fibre composites are highly anisotropic. Maximum properties can be achieved if all the fibres are aligned in one direction. The properties, such as modulus and strength, decrease rapidly in directions away from the fibre direction. To obtain more orthotropic properties, alternative layers of fibres may vary from 0° to 90°, resulting in less directionality, but at the expense of absolute properties in the fibre direction. A laminate is fabricated by stacking a number of thin layers of fibre and matrix, consolidating them into the desired thickness. A laminate is the most common form of composite for structural applications. The fibre orientation in each layer as well as the stacking sequences of various layers can be manipulated to produce a wide range
of physical and mechanical properties.

For cylindrical structures such as pressure vessels, filament winding is the best method. Pultrusion is particularly suitable for producing long structural elements with constant cross-sections from composites. 3-D weaving, braiding, knitting, etc. are under consideration for structural composites, particularly when interlaminar shear strength, damage tolerance, and thickness-direction properties of composites are important design consideration.

1.2 Elastic Properties of Composites

In order to predict the properties of fibre composites, the integrity of composite structures is treated as simply as possible. For unidirectional fibre composites, fibres are assumed to be packed in hexagonal or rectangular arrays and perfectly straight. The properties of composites are the volumetric weighted average properties of fibre and matrix. Assuming an iso-strain condition, the Young’s Modulus along the fibre direction, $E_1$, of composites is

$$E_1 = E_f V_f + E_m V_m$$  \hspace{1cm} (1.1)

and the transverse modulus, $E_2$, is given by

$$1/E_2 = V_f/E_f + V_m/E_m$$  \hspace{1cm} (1.2)
where \( E_f \) and \( E_m \) are the Young's Modulus of fibre and matrix; \( V_f \) and \( V_m \) are volume fraction of fibre and matrix.

Equation (1.1) is the well-known ‘rule of mixture’ equation. The Poisson’s ratio \( \nu_{12} \), shear modulus \( G_{12} \) for unidirectional composites are developed in a similar fashion:

\[
\nu_{12} = \nu_f V_f + \nu_m V_m
\]

\[
\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}
\]

In the orthotropic state, the stress-strain relationship of unidirectional fibre composite laminates can be expressed as

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_2
\end{bmatrix} =
\begin{bmatrix}
1/E_1 & -\nu_{12}/E_2 & 0 \\
-\nu_{12}/E_2 & 1/E_2 & 0 \\
0 & 0 & 1/G_{12}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

The mechanical properties of angle-ply and cross-ply laminate are quite different because of the strong directional nature of the mechanical properties of the layers and the complex interactions between layers. Classical laminate theory was developed many years ago and it uses anisotropic elasticity theory, which involves extensive matrix algebra. Full descriptions can be found in many textbooks, e.g. Jones [1].

The elastic properties of discontinuous fibre- and whisker-reinforced polymer composites can be also estimated using laminate theory. The material is divided into pseudo laminae with each lamina having an appropriate proportion of fibres in one given direction. Elastic properties are obtained by summing the resolved stiffnesses or compliances of the pseudo laminae.
1.3 The Mesostructure

The simple models for fibre composites are easy to understand and apply yet there are some discrepancies in elastic properties of real composite samples. In any piece of a real composite, the lack of fibre straightness and fibre packing regularity are the main reasons that the simple models have been violated. Fibre waviness and packing non-uniformities are often regarded as part of the 'mesostructure'. The mesostructure is defined as the structures on the scale between about 30μm and 30mm [2]. These imperfections are unintentional and the variations of mesostructures are often governed by the processing conditions. Voids, fibre misalignment, and fibre synchronization are also considered as mesostructural features.

1.3.1 Fibre Misalignment

Short-fibre composites can be manufactured by injection or compression molding. In these processes, the fibres and the resin are transported into the mold cavity. As the resin or molding compound deforms to achieve the desired shape, the orientation of the fibres is changing. Fibre orientation changes stop when the matrix solidifies and the orientation pattern becomes part of the mesostructure of the finished product (Figure 1.2). There have been extensive investigations of the factors affecting the fibre orientation during the molding process. Ranganathan & Advani [3] classified fibre suspensions into three concentration regimes according to the fibre volume fraction ($V_f$) and fibre aspect ratio ($s$) (Figure 1.3), defined as the ratio of the fibre length to fibre diameter. In the dilute region, the spacing between the fibres is greater than
their length. The fibres are free to move and rotate and hydrodynamic interactions between the fibres are rare. In the concentrated region, on the contrary, the fibre-fibre interaction becomes significant. The mold geometry can influence the fibre orientation in various sections of the mold [4]. Simulations were made to predict the interaction between fibre orientations and flow kinematics during filling [4, 5].

![Figure 1.2 Flow-induced fiber misalignment of an injection-moulded composite.](image)

**Fig. 1.2** Flow-induced fiber misalignment of an injection-moulded composite.

![Figure 1.3 Classification of fibers suspensions.](image)

**Fig. 1.3** Classification of fibers suspensions.

For continuous fibre composites, fibre misalignment is the partial or localized dis-orientation of the reinforcing fibres with respect to the nominal fibre axis (Figure 1.4).
Some composite materials are fabricated from pre-impregnated tapes. Fibre misalignment is thought to originate in the manufacture of fibre tows, in the prepreg tape preparation, or in the subsequent composite fabrication. Fibre misalignment is often used more to describe the spatial orientation-imperfections in the fibrous reinforcement, whereas fibre waviness describes the sinusoidal-like deformation induced during processing. The fibre misalignment is sometimes considered as a localized feature of fibre waviness because fibre misalignment is determined from a cross-section of a composite material (Figure 1.5). The misalignment angle is a 'snapshot' of the whole fibre path profile.

Fig. 1.4 Fiber misalignment in a laminate composite.

Fig. 1.5 Fibre misalignment originating from fibre waviness.
For short-fibre composites, image analysis is the most successful method to evaluate the distribution of fibre misalignment. Light microscope equipped with a video camera can be used to capture cross sections of well-polished composite surfaces. Major and minor axes of the sectioning ellipse are used to determine the out-of-plane (inclined) misalignment angles (Figure 1.6). This method was used by Fischer & Eyerer [6], Toll & Andersson [7], and Hine et al [8] to examine short glass-fibre reinforced composite.

![Diagram](image)

\[
\phi = \sin^{-1}(a/d_f)
\]

\(I\) : major axis  
\(d_f\) : minor axis  
\(\phi\) : misalignment angle

**Fig. 1.6 Calculation of misalignment angle.**

For continuous fibre composites, due to the small angular deviation and the limited resolution of the cross-section captured, sectioning perpendicular to the fibre direction yields a poor result because the length difference in elliptical major and minor axes is small. Yurgartis [9] has developed a simple method to overcome this difficulty. His technique requires specimen sectioning at a small angle \(\phi_{sect}\) to the vertical plane (Figure 1.7). It gives a larger axes ratio of elliptical cross section of fibre and the actual misalignment angle \((\phi)\) is given by:

\[
\phi = \phi_{sect} - \sin^{-1}(d_f/a)
\]  

\(1.6\)
Using confocal laser scanning microscopy (CLSM) to measure fibre misalignment has been developed by Clark, Davidson & Archenhold [10]. A parallel beam of laser light passes through a beam splitter, the microscope optics and the sample. The reflected light, and any fluorescence emission from the sample, returns through the microscope optics, the beam splitter and small aperture before reaching a detector. Providing that the sample is sufficiently transparent, the laser beam can penetrate the sample and a series of optical sections at different depths within the sample can be acquired. Fibres in each section are recognized and matched between two adjacent sections. Then fibre misalignment is determined by simple trigonometry. However, the maximum usable depth depends on the fibre-matrix system. It is reported that the usable depth is about 150μm for glass/epoxy composites with less than 30% fibre volume fraction and about 30-40μm for higher volume fraction (~50%).

Any two independent angles in three-dimensions can be used to describe the direction of a fibre. However, the definition of misalignment angle pair is different from
different researchers, as illustrated in Figure 1.8.

Fig. 1.8 Misalignment angles pairs defined (a) by Toll & Andersson [7], (b) by Yurgartis [9], and (c) in this project. The original fibre direction is shown in (d).

1.3.2 Fibre Waviness

Fibre waviness can result from a variety of manufacturing induced phenomena. Fibre wrinkling may occur as a result of laminate/tool interaction resulting from the mismatch in thermal expansion coefficients and instabilities encountered during forming operations [11]. Fibre waviness can be categorized as either in-plane or out-of-plane waviness (Figure 1.9). In-plane waviness involves the deformation of many fibres within the plane of the laminate. Out-of-plane waviness, also known as layer waviness or ply waviness, involves the motion of multiple plies through the thickness of the laminate.
and is prevalent in thick composites. Ply waviness is also common in certain fibre placement processes such as filament winding of composite cylinders. For thermoset composite cylinders, the waviness in the hoop layers results from compressive strains during final consolidation.

**Fig. 1.9** Type of fiber waviness in a composite material.

In the 1960s, Bolotin [12] and Tarnopolskii [13] did some tensile experiments which indicated that for relatively small curvature of the fibres, there was a decrease in the modulus of elasticity:

\[
E = \frac{E_1}{1 + E_1/G_{12}(A/\lambda)^2}
\]

(1.7)

where \(E_1\) and \(G_{12}\) are the Young's and shear modulus of the reinforced material with straight fibre; \(A\) and \(\lambda\) are the amplitude and wavelength of the fibre.

In compression, Rosen [35] distinguished a shear mode and an extensional mode of elastic micro-buckling due to fibre waviness (Figure 1.10). Swift [14], following Rosen approach, concluded that the shear deformations gave larger reductions in moduli than
extensional mode deformation and the reduction was also larger for fibre with longer wavelength. Martinez et al [15] showed that the composite compressive strength increases as the minimum radius of curvature increases. In other words, the compressive strength becomes higher when the fibres are straighter.

Piggott & Mrse [16] observed that fibre waviness decreases the compressive modulus of carbon/PEEK composites approximately as the square of the mean fibre angular deviation but did not affect the tensile modulus significantly. Carbon/PEKK and glass/PEKK composites laminates were tested in compression by Bogetti et al [17] and they suggested the stiffness reduction was due to out-of-plane rotation of wavy plies. The magnitude of the property reduction increases as the amplitude of the undulation increases and the wavelength decreases. Highsmith et al [18] predicted that the compressive strength of the composite is controlled by regions where the waviness is large, whereas the stiffness of the composites is controlled by some average value of waviness. The effect of fibre waviness on compressive strength and stiffness, fatigue endurance, shear strength, and delamination resistance were reviewed and summarized by Piggott [19].

![Image](image.png)

**Fig. 1.10** Two mode of failure mechanisms due to fibre waviness.
A method of tracing the fibre waviness directly on the composite surface was practiced by Highsmith et al [18]. First, a unidirectional specimen was polished, acid etched, and the surface was photographed by optical microscope. The enlarged view of this negative was used and the waviness was traced. The same method was used by Bogetti et al [17] to measure the fibre waviness of filament wound thermoplastic cylinders.

Successive sectioning is the other method to measure the fibre waviness. This method originated from the work of Davis [20]. Boron/epoxy composite was used and the cross sections, perpendicular to the fibre, were polished successively at regular intervals along the fibre direction. Misalignment of individual fibres was characterized by matching the fibre in each section. This method was modified by Paluch [21] to measure the fibre waviness. About 40 to 50 sections with small regular intervals (~2μm) were used. The image of each section was digitized and the fibre centers were located by image analysis. Fibres between adjacent planes were matched using the criterion that the displacement of each fibre between planes could not be larger than a certain value. After the whole path of a fibre was traced, it was fitted as a sinusoidal function.

1.3.3 Fibre Packing

Volume fraction distribution, packing regularity, or fibre spatial distribution, are the names used to describe the degree of fibre dispersion within the matrix. Uneven packing gives resin-rich and fibre-rich regions in a composite (Figure 1.11). Again, this mesostructural morphology is usually influenced by processing conditions. In a pultrusion, fibre tows are pulled through a mould and the tows may be twisted and not spaced
regularly. Fibre tows in prepreg are incorporated in random fashion and this results in a laminate with irregular fibre packing.

Hillig [22] showed that in a fibre-dense region, the fibres grouped as a bundle give resistance to crack propagation and to ultimate failure. The packing mesostructure also affects the apparent shear strength [23]. Intermeshing of fibres at very high fibre volume fraction will increase the composite apparent shear strength by causing deviation of the shear fracture plane. A factor of $2\sqrt{3}$ is estimated when hexagonal packing is assumed:

$$\tau_{13u} = 2\tau_{mu}\sqrt{3} \quad (1.8)$$

If $V_t$ is the fraction of intermeshing regions in the composites, the shear strength can be determined as

$$\tau_{13u} = \tau_{mu}\left(\frac{2}{\sqrt{3}}V_t + (1 + V_t)\right) \quad (1.9)$$
Using the concept of Voronoi diagram to characterize fibre spatial distribution is practiced by many researchers. The Voronoi diagram is also known as Dirichlet tessellation or Thiessen tessellation. Delaunay triangulation can be generated directly from the Voronoi diagram because both diagrams are constructed using information from nodal neighbourhood connections [25]. The Voronoi diagram, Delaunay triangulation and the definition of neighbourhood are shown in Figure 1.12. In the case of composite material, the center of fibre axes are treated as nodes. Everett & Chu [26] simulated the Voronoi diagram by using randomly placed fibres. The distribution of cell volume fraction, inter-fibre distance, and nearest-neighbour distance were suggested as modelling tools. Everett [27] suggested the use of a radial distribution function to represent the nearest-neighbour distance while Pyrz [28] and Ghosh et al [29] used a second-order intensity function and a pair distribution function to describe nearest-neighbour distances. The inclusion angle between nearest neighbours has also been used to describe the fibre spatial relationship.
Fig. 1.12 Definition of neighbourhood: If any 3 nodes can form a circle in which no other nodes are located inside of this circle, these 3 nodes are neighbours. (a) The Delaunay triangulation diagram is constructed by connecting all neighbouring nodes. (b) The Voronoi diagram is constructed by connecting the perpendicular bisectors of neighbouring nodes.

1.3.4 Voids

Empty cavities, or voids, are present in most composites materials. There are two basic types of voids. The first type is the voids along individual fibres. which may be spherical or elongated into ellipsoidal cavities parallel to fibres. The void diameter is related to the fibre spacing and is typically in the range 5-20μm. The second type is the empty cavities present in the resin-rich regions. They are similar in shape and size to the first type (Figure 1.13).
Voids arise from two main causes. Firstly, incomplete wetting out of the fibres by the resin: this results in entrapment of air and is more likely in systems where the dry fibres are closely spaced and the viscosity of the resin is high. Secondly, volatile gases are produced during curing of some thermoset resins. These gases are trapped in the resin and finally become empty spheres in the resin. Some of the inter-laminate voids are due to gross defects in the manufacturing operation.

It is generally agreed that the apparent interlaminar shear strength decreases as the voids content increases [30]. Beaumont & Harris [31] showed that 10% voids reduced the apparent interlaminar shear strength of carbon-fibre reinforced composites by about 20%. Judd & Wright [32] concluded that the apparent interlaminar shear strength decreases by about 7% for each 1% of voids up to total void content of about 4%. Hull [24] showed that the transverse tensile strength of glass/epoxy laminate decreases as void content increases.

The volume fraction of voids \( (V_v) \) is given by:

\[
V_v = 1 - (V_f + V_m) \tag{1.10}
\]
The examination of the presence of some larger voids (e.g. split, crack) and estimation the volume fraction of those voids can be undertaken by ultrasonic or optical methods. Ultrasonic inspection is a non-destructive testing technique [33]. The simplest method is to transmit a short pulse of high frequency (typical 50kHz - 100kHz) energy through the specimen and to measure the transit time or attenuation of the signal. Conventional ultrasonic scan method can detect the size and position of crack by analyzing the acoustic wave behavior in composites.

The 2D-image analysis of voids on a section plane is popular because of its simplicity. Assuming that infinite numbers of two-dimensional sections are taken, the void volume fraction will be equal to the void area fraction:

\[ V_v = \frac{\sum \text{Area}_{\text{void}}}{\sum \text{Area}_{\text{image}}} \]  

(1.11)

However, in practice only one section is taken and a finite number of random images frames are analyzed for voids. So, this procedure must assume that the voids are randomly distributed throughout the sample.

1.3.5 Image Analysis

As mentioned above, some mesostructure quantification methods require the technique of computer-aided image analysis. The advantage of employing digital image analysis is that it can provide direct information about composite composition effectively, and in a reasonable time. Image analysis allows mesostructural measurements to become much
more convenient and therefore large volume of data can be obtained, and hence increases statistical confidence. The technique is also reproducible, reduces human errors, and once developed it can be applied for different composite materials.

Nevertheless, digital image analysis also has difficulties. Although computer pattern recognition has been studied extensively, satisfactory results are case-dependent. Applying image analysis for a particular project requires a specific approach to give successful data. Sophisticated work with digital images is required to achieve good accuracy but most of the analysis algorithms are computationally intensive, and therefore a large equipment investment is required. So, in designing a tailor-made image analysis algorithm, the computational complexity, accuracy and efficiency should be optimized under the constraint of equipment availability.
1.4 Objectives of this Study

The study of influence of mesostructure on composite material is in its infancy and there is still no unified approach to measure and quantify mesostructural parameters. This project was designed to find a better method and algorithm to determine fibre divagations and packing anomalies in composite materials. The following aims are set:

- This method should applicable to different kinds of composite materials;
- This method should yield high measurement accuracy;
- This method should require little computation so that analysis can be done with an ordinary personal computer;
- Results should reproducible and user-independent; i.e. personal judgment is minimized.
2 Experimental Method

2.1 Materials

Composite samples used in this study are listed in Table 2.1.

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>System</th>
<th>Type</th>
<th>Source</th>
<th>Manufacturing Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>carbon-epoxy</td>
<td>unidirectional composite</td>
<td>colleague</td>
<td>pultrusion</td>
</tr>
<tr>
<td>II</td>
<td>carbon-PEEK</td>
<td>unidirectional laminate</td>
<td>colleague</td>
<td>hot-press</td>
</tr>
<tr>
<td>III</td>
<td>carbon-epoxy</td>
<td>unidirectional composite</td>
<td>Bombardier Aerospace</td>
<td>autoclave</td>
</tr>
<tr>
<td>IV</td>
<td>glass-epoxy</td>
<td>unidirectional laminate</td>
<td>colleague</td>
<td>hot-press</td>
</tr>
<tr>
<td>V</td>
<td>hybrid</td>
<td>angle-ply</td>
<td>FRE Composites</td>
<td>filament winding</td>
</tr>
</tbody>
</table>

Types I to IV were composite materials consisting of one type of fibre and one type of polymer matrix. Type V was a multi-layer, multi-system filament wound tube. The detailed structure of this commercial sample is shown in Figure 2.1 and Table 2.2.

2.2 Sample Preparation

Unidirectional composite samples were cut at right angles to the nominal fiber direction. The thickness \( t \) of the samples was ranged from a few millimetres (type II, IV) to two centimetres (type I, III). The width \( w \) of the samples was cut to about 1 cm. The height \( l \) of each sample was cut to 3 cm. The schematic of the sample is shown in Figure 2.2. The four lateral sides \( (s1, s2, s3, s4) \) were wet abraded with successively...
finer silicon carbide paper, down to 1200 grit, and finally polished with 3 μm alumina powder paste. The polishing was done carefully so that the cross sections of the samples were uniform and sharp corners were obtained. The samples were placed, with the cross-section face down, into a rectangular silicone rubber mould. The samples were stuck firmly on the mould so that the sample was upright. Epoxy/curing agent mixture was poured into the mould and the whole system was heated in an oven for curing. After curing, the bottom of the cured sample-embedded epoxy block was polished.

Fig. 2.1 Detail structure of Type V sample - commercial filament wound tube.
### Table 2.2 Details of Type V material.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Fibre Type</th>
<th>Orientation</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>glass</td>
<td>$0^\circ$</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>carbon</td>
<td>$0^\circ$</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>carbon</td>
<td>$60^\circ$</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>carbon</td>
<td>$-60^\circ$</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>carbon</td>
<td>$60^\circ$</td>
<td>1.0</td>
</tr>
<tr>
<td>6</td>
<td>carbon</td>
<td>$-60^\circ$</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>carbon</td>
<td>$0^\circ$</td>
<td>0.7</td>
</tr>
<tr>
<td>8</td>
<td>glass</td>
<td>$0^\circ$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Fig 2.2** Schematic of sample prepared for analysis.

### 2.3 Successive Sectioning

The resin block was mounted in an aluminum holder (Figure 2.3) during polishing. The detail of the polishing scheme was as follows:
About 100μm in thickness was removed under this scheme. The thickness change of the resin block was measured by Mitutoyo Venier Caliper, with an estimated accuracy of 0.01mm. The thickness recorded was the average height measured from four sides of the resin block. The block was then washed with water to remove any residual alumina powder. A final rinse in acetone avoided any water marks.

<table>
<thead>
<tr>
<th>Silicon carbide paper grit #</th>
<th>Grinding time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>5</td>
</tr>
<tr>
<td>800</td>
<td>10</td>
</tr>
<tr>
<td>1200</td>
<td>10</td>
</tr>
<tr>
<td>10μm alumina powder paste</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 2.3 Glass/carbon composite embedded resin block mounted onto the aluminium holder.

2.4 Image Capturing System

The image capturing system is shown in Figure 2.4. It consisted of an Olympus Vanox light microscope, Sony XC-77 CCD video camera, Scion LG-3 frame grabber and Macintosh computer. Freeware IMAGE v1.57 by National Institute of Health was used for acquiring and presenting images from this system. The resolution of the digitized
image was $640 \times 480$ pixels with 8-bit grayscale palette. The magnification was set to 330 times and the dimensions of the image captured were about $320 \times 240\mu m$. In such resolution, about 300 to 800 fibres were found in an image.

Fig. 2.4 Image capturing equipment.

2.5 Image Analysis

Graphic-user interface programs were written in Digital Visual Fortran 5.0 and run in a Pentium II 450MHz PC under Windows 98. All the programs developed were aimed at high accuracy and speed, and ease of use. The operation method and algorithm of all programs are described in Appendix A.

The following steps were used to analyze each image:
• Thresholding of the raw image

• Separating and resolving of each fibre

• Determining inter-fibre relationships by constructing Delaunay and Voronoi diagrams

If the cross-section was not perpendicular to the fibre direction, the fibres appeared as ellipses. Local misalignment angles could be analyzed directly from this image. When more than two images of successive sections for a particular composite specimen were obtained, fibre waviness was calculated.

The analysis performed on each type of specimen was as follows (Table 2.4):

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Fibre Packing</th>
<th>Inter-fibre Relationships</th>
<th>Local Misalignment Angles</th>
<th>Fibre Waviness</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>II</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

2.5.1 Fibre Packing

The analysis of fibre packing was represented by the fibre volume fraction distribution. This employed the concept of Voronoi diagram. The fibre volume fraction is defined as the ratio of the area of Voronoi cell to fibre cross-section area (Figure 2.5b). The fibre diameter was assumed to be constant in the calculation. Fibres located around the image edges were ignored.
Fig 2.5 (a) Illustrations to show the definitions of inter-fibre distance \( (L) \) and inter-fibre angle \( (\theta) \). The first subscript refers to the reference fibre. For inter-fibre distance, the second subscript refers to the target fibre. For inter-fibre angle, the second and third subscript pair indicate the included angle of the neighbouring fibre in clockwise direction. (b) The fibre cell volume fraction for fibre 1: \( v_{f1} = \) fibre cross-section area / shaded area.

2.5.2 Inter-fibre Relationships

Inter-fibre distance and angle were found for each type of composite sample. These two parameters were obtained by constructing the Delaunay diagram (Figure 2.5a). The inter-fibre distance was defined as the distance between two neighbouring fibres from center to center. So, the minimum distance was equal to the fibre diameter. The inter-fibre angle for a reference fibre was defined as the included angle between two neighbouring fibres, in a clockwise direction, with respect to this reference fibre.

The data associated with the fibres located around the edge of the image was ignored, using the rule as follows:

For a fibre \( k \) which has neighbours 1, 2, 3... \( i \),

if \( \theta_{k-i-1,i} > 180^\circ \), then fibre \( k \) is ignored.

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Misleading values were generated without this rule, because the information about the vicinity of this fibre was incomplete.

2.5.3 Fibre Waviness

For fibre waviness measurement, several successive polished sections were prepared and scanned from a composite sample. The digitized image of each polished section was analyzed and the coordinates of each fibre cross-section were found (see Appendix A6 and A7). After linking the fibre-center 3-D coordinates of a fibre through the thickness, the whole path of an individual fibre filament could be recognized (see Appendix A9). For each fibre filament, the associated point set was projected on the mean inertial plane. This projected point set was mapped onto another two-dimensional plane, which was spanned by vectors \( \vec{u} \) and \( \vec{v} \). The mapped point set was then rotated by an angle \( \delta \) to fit the sinusoidal function:

\[
y = A \cos(Bx + C) + y_0,
\]

where

- \( A \) is the amplitude;
- \( B \) is the radial frequency (\( 2\pi/\lambda \));
- \( C \) is the phase shift;
- \( y_0 \) is the vertical offset;
- \( \lambda \) is the wavelength.

The 3-D representation of this fitting scheme is illustrated in Figure 2.6a and the details are in Appendix A10. The amplitude and wavelength were the parameters used to characterize the fibre path while the other parameters were used for fitting only.
2.5.4 Local Fibre Misalignment

Local Fibre misalignment was characterized by a pair of angles: in-plane misalignment angle ($\phi_{in}$) and out-of-plane misalignment angle ($\phi_{out}$), as shown in Figure 2.6b. These
two angles described the three dimensional deviation of a fibre from the main direction.

For Type I material, these misalignment angle pairs were determined by matching the fibre cross-section from two adjacent planes. Local misalignment angles (??) were also determined from the cross-section of an angle-ply composite. The digitized image of the cross-section contained ellipses, instead of circles as obtained from the unidirectional composites. The ratio of the major axis of the ellipse to the fibre diameter (which was equal to the minor axis) was the local slope of the fibre at that section. The method of calculating the local misalignment angles from an ellipse was shown in Figure 2.7.

\[ n = \text{no. of pixel of an ellipse (area)} \]
\[ M_x = \frac{\sum x_i}{n} = X_{\text{centroid}} \]
\[ M_y = \frac{\sum y_i}{n} = Y_{\text{centroid}} \]
\[ M_{xx} = \frac{\sum x_i^2}{n} - M_x^2 \]
\[ M_{yy} = \frac{\sum y_i^2}{n} - M_y^2 \]
\[ M_{xy} = \frac{\sum x_i y_i}{n} - M_x M_y \]
\[ a = \sqrt{2 (M_{xx} + M_{yy}) + \sqrt{2 \left[(M_{xx} - M_{yy})^2 + 4 M_{xy}^2\right]}} \]
\[ b = \sqrt{2 (M_{xx} - M_{yy}) + \sqrt{2 \left[(M_{xx} - M_{yy})^2 + 4 M_{xy}^2\right]}} \]
\[ \Phi = \frac{1}{2} \tan^{-1}\left(\frac{2 M_{xy}}{M_{xx} - M_{yy}}\right) \]
\[ \phi = \sin^{-1}(b/a) \]

**Fig. 2.7** Moment analysis of ellipse.
3 Results

3.1 Two-dimensional Mesostructure Analysis

3.1.1 Single-Fibre, Single-Matrix System

Cross-section images of each type of material were selected at random. Typical Images of Type I to Type IV material shown in Figure 3.1 indicate that the fibre cross-section for each type of specimen is circular and the diameter is fairly constant. The fibre diameters were measured from the images, as given in Table 3.1.

The distributions of inter-fibre angles, inter-fibre distances, number of neighbours and fibre cell volume fractions of Type I to Type IV materials are shown in Figures 3.2 to 3.5. In order to make quantitative comparisons, all the histograms for each type of distribution were constructed using the same class width. The raw relative frequency plot in each graph is smoothed by discrete Fast Fourier Transform. The means and standard deviations for each category of 2-D parameter are listed in Table 3.2.

<table>
<thead>
<tr>
<th>Table 3.1 Information for 2-D analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of images used</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>Type I</td>
</tr>
<tr>
<td>Type II</td>
</tr>
<tr>
<td>Type III</td>
</tr>
<tr>
<td>Type IV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3.2 Results of 2-D analysis of Type I to Type IV material.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-fibre distance(no. of d)</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Type I</td>
</tr>
<tr>
<td>Type II</td>
</tr>
<tr>
<td>Type III</td>
</tr>
<tr>
<td>Type IV</td>
</tr>
</tbody>
</table>
Fig. 3.1 Typical cross-section images of testing materials. These images are only a portion (1/10 in size) of their originals.
Fig. 3.2 Distribution of inter-fibre distance of Type I to Type IV material.

Fig. 3.3 Distribution of inter-fibre angle of Type I to Type IV material.
Fig. 3.4 Distribution of number of neighbouring fibre in Type I to Type IV material.

Fig. 3.5 Distribution of fibre cell volume fraction of Type I to Type IV material.
The change of 2-D parameters through the thickness of a composite material was also investigated. Five successive sections of Type I material were used. The height and thickness differences between sections are given in Tables 3.3 and 3.4. The maximum $\delta z$ value is the largest height measurement difference for a section.

Three images, located at three different corners of the composite sample, were obtained from each section. The changes in the 2-D mesostructure parameters between sections are very small, as indicated in Table 3.5.

**Table 3.3 Heights of Type I sample in different sections.**

<table>
<thead>
<tr>
<th>Sample height ($\mu m$)</th>
<th>Section 1</th>
<th>Section 2</th>
<th>Section 3</th>
<th>Section 4</th>
<th>Section 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner 1</td>
<td>18.07</td>
<td>18.02</td>
<td>17.92</td>
<td>17.88</td>
<td>17.79</td>
</tr>
<tr>
<td>Corner 2</td>
<td>18.09</td>
<td>18.03</td>
<td>17.93</td>
<td>17.90</td>
<td>17.80</td>
</tr>
<tr>
<td>Corner 3</td>
<td>18.17</td>
<td>18.11</td>
<td>18.05</td>
<td>17.98</td>
<td>17.88</td>
</tr>
<tr>
<td>Corner 4</td>
<td>18.10</td>
<td>18.06</td>
<td>17.97</td>
<td>17.92</td>
<td>17.83</td>
</tr>
<tr>
<td>Average</td>
<td>18.11</td>
<td>18.06</td>
<td>17.97</td>
<td>17.92</td>
<td>17.83</td>
</tr>
<tr>
<td>max. $\delta z (\mu m)$</td>
<td>100</td>
<td>90</td>
<td>130</td>
<td>120</td>
<td>90</td>
</tr>
</tbody>
</table>

**Table 3.4 Inter-section distance of Type I material for 2-D analysis.**

<table>
<thead>
<tr>
<th>Section Number</th>
<th>Thickness ($\mu m$)</th>
<th>Thickness (no. of $d_f$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 2</td>
<td>50</td>
<td>6</td>
</tr>
<tr>
<td>2 to 3</td>
<td>90</td>
<td>11</td>
</tr>
<tr>
<td>3 to 4</td>
<td>50</td>
<td>6</td>
</tr>
<tr>
<td>4 to 5</td>
<td>90</td>
<td>11</td>
</tr>
<tr>
<td>Total</td>
<td>28</td>
<td>34</td>
</tr>
</tbody>
</table>

**Table 3.5 2-D mesostructure results for Type I material at different depths.**

<table>
<thead>
<tr>
<th>Section</th>
<th>Inter-fibre angle ($^\circ$)</th>
<th>Inter-fibre distance ($\mu m$)</th>
<th>Number of neighbour</th>
<th>Fibre cell volume fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61.0 ± 23.4</td>
<td>23.9 ± 8.4</td>
<td>5.96 ± 0.95</td>
<td>48.5 ± 12.5</td>
</tr>
<tr>
<td>2</td>
<td>61.0 ± 23.1</td>
<td>23.9 ± 8.3</td>
<td>5.96 ± 0.94</td>
<td>48.3 ± 12.7</td>
</tr>
<tr>
<td>3</td>
<td>61.1 ± 23.4</td>
<td>24.0 ± 8.5</td>
<td>5.96 ± 0.94</td>
<td>48.4 ± 12.5</td>
</tr>
<tr>
<td>4</td>
<td>61.2 ± 23.5</td>
<td>24.0 ± 8.7</td>
<td>5.96 ± 0.93</td>
<td>48.3 ± 12.6</td>
</tr>
<tr>
<td>5</td>
<td>61.1 ± 23.5</td>
<td>24.1 ± 8.8</td>
<td>5.95 ± 0.95</td>
<td>48.3 ± 13.1</td>
</tr>
</tbody>
</table>
3.1.2 Multi-Fibre, Multi-Matrix System

Only the first two layers of Type V material were used in the 2-D mesostructure analysis. Typical cross-section images of the first four layers are shown in Figure 3.6. The distributions of 2-D mesostructure parameters and the numerical result of Layer 1 and Layer 2 are shown in Figure 3.7 to 3.10, and given in Table 3.6.

### Table 3.6 2-D mesostructure result of Type V material at different depth.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Inter-fibre Angle(°)</th>
<th>Inter-fibre distance (µm)</th>
<th>Number of neighbour</th>
<th>Fibre cell volume fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61 ± 22</td>
<td>25 ± 7</td>
<td>6.0 ± 0.9</td>
<td>44 ± 10</td>
</tr>
<tr>
<td>2</td>
<td>61 ± 23</td>
<td>24 ± 7</td>
<td>6.0 ± 1.0</td>
<td>48 ± 12</td>
</tr>
</tbody>
</table>

Fig. 3.6 Typical cross-section images of Type V material. These images are only a portion of its originals.
Fig. 3.7 Distribution of inter-fibre angle of Type V material.

Fig. 3.8 Distribution of inter-fibre distance of Type V material.
Fig. 3.9 Distribution of number of neighbouring fibre of Type V material.

Fig. 3.10 Distribution of fibre cell volume fraction of Type V material.
3.2 Three-dimensional Mesostructure Analysis

3.2.1 Single-Fibre, Single-Matrix System

The 3-D mesostructure of Type I material was found. Five successive sections from three corners of the composite sample, with the thickness differences given in Table 3.4, were used. Only 86% of fibres recognized were used to construct the complete fibre path. This was because some fibres, especially the fibres located at the edges of the image, may enter or leave the observation volume. Some fibres may discontinue through the observation volume thus their paths were not complete. Data on the number of fibres, which could be followed through all five sections, are given in Table 3.7.

<table>
<thead>
<tr>
<th></th>
<th>Fibre observed</th>
<th>Fibre recognized</th>
<th>Percentage Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner A</td>
<td>505</td>
<td>396</td>
<td>22%</td>
</tr>
<tr>
<td>Corner B</td>
<td>696</td>
<td>613</td>
<td>12%</td>
</tr>
<tr>
<td>Corner C</td>
<td>634</td>
<td>572</td>
<td>10%</td>
</tr>
<tr>
<td>Total</td>
<td>1835</td>
<td>1581</td>
<td>14%</td>
</tr>
</tbody>
</table>

Table 3.7 Result of fibre recognition.

Waviness parameters for each fibre were calculated, as outlined in Section 2.5.3. The distribution of wavelength and amplitude are shown in Figures 3.11 and 3.12, respectively. The distribution of amplitude divided by wavelength ($A/\lambda$) is given in Figure 3.13. Some curve fitting examples are shown in Figure 3.14.
Fig. 3.11 Distribution of fibre wavelength of Type I material.

Fig. 3.12 Distribution of amplitude of Type I material.
Fig. 3.13 Distribution of $A/\lambda$ of Type I material.
Fig. 3.14 Examples of fitted fibre path. Waviness parameters shown are the program-optimized values which give the minimum point-to-fitting line distance ($SE2$).
The first two layers of Type I material were used to determine the misalignment angle distributions, as shown in Figures 3.15a to 3.15c, and statistical values can be found in Table 3.8. The fitted curves found in those graphs are normal distribution curves. These curves were determined by estimating of mean and standard deviation that could cover most of the data and manifest the peak shape.

<table>
<thead>
<tr>
<th>Table 3.8 Misalignment angles of Type I material.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misalignment angle (°)</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>In-plane mean</td>
</tr>
<tr>
<td>In-plane standard deviation</td>
</tr>
<tr>
<td>Out-of-plane mean</td>
</tr>
<tr>
<td>Out-of-plane standard deviation</td>
</tr>
</tbody>
</table>

![Graph showing misalignment angles](image)

**Fig. 3.15a Type I / Corner 1**
Fig. 3.15b Type I / Corner 2

Fig. 3.15c Type I / Corner 3

Fig. 3.15 Distributions of in-plane and out-of-plane misalignment angles of Type I materials.
3.2.2 Multi-Fibre, Multi-Matrix System

The misalignment angles between two layers of Type V materials was analyzed. Two successive sections, with 35μm apart, of Layer 1 and Layer 2 were obtained. Fibre centers between two sections were connected and the distributions of in-plane ($\phi_{in}$) and out-of-plane ($\phi_{out}$) misalignment angles were determined ( Table 3.9, Figures 3.16a and 3.16b). The local misalignment angle ($\phi$) of Layer 1 and 2 can be calculated from in-plane and out-of-plane misalignment angles:

$$
\phi = \tan^{-1} \left[ \tan^2 \phi_{in} + \tan^2 \phi_{out} \right]^{1/2}
$$

(3.1)

The fibre cross section in Layer 3 and Layer 4 of Type V material was elliptical in shape so the local misalignment angle of each fibre can be obtained using the geometry of each ellipse, as shown in Figure 2.7. The number of fibres analyzed from each layer is given in Table 3.10.

<table>
<thead>
<tr>
<th>Table 3.9 Misalignment angles of Type V material.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misalignment angle(°)</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>In-plane mean</td>
</tr>
<tr>
<td>In-plane standard deviation</td>
</tr>
<tr>
<td>Out-of-plane mean</td>
</tr>
<tr>
<td>Out-of-plane standard deviation</td>
</tr>
</tbody>
</table>
Fig. 3.16a Type V / Layer 1

Fig. 3.16a Type V / Layer 2

Fig. 3.16 Distributions of in-plane and out-of-plane misalignment angles of Type V material
Fig. 3.17 Local misalignment angles of Layer 1 to 4 of Type V material.

Assuming each fibre has the same wavelength and amplitude and in a form of $y = A \sin \left(\frac{2\pi x}{\lambda}\right)$, the slope at any point of the fibre is the first derivative of $y$:

$$y' = 2\pi \left(\frac{A}{\lambda}\right) \cos \left(\frac{2\pi x}{\lambda}\right)$$  \hspace{1cm} (3.2)

Then, the slope is squared to remove the sign. The average value of the square of slope $\langle y'^2 \rangle$ can be estimated by:

$$\langle y'^2 \rangle = \frac{1}{\lambda/2} \int_0^{\lambda/2} A^2 B^2 \cos^2 (Bx) \, dx = A^2 B^2 / 2 = 2\pi^2 A^2 / \lambda^3$$  \hspace{1cm} (3.3)

where $B = 2\pi / \lambda$. This definite integral is bounded between zero to $\lambda/2$ because the function $y$ is repeated in every interval of $\lambda/2$. 

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The variance of $y^2$ can also be found by:

$$\sigma_{y^2}^2 = \frac{1}{\lambda/2} \int_0^{\lambda/2} \left( A^2 B^2 \cos^2 (Bx) - \frac{A^2 B^2}{2} \right)^2 \, dx = 2\pi^4 A^4 / \lambda^4 \quad (3.4)$$

Since the slope is the tangent of the local misalignment angle $\phi$, the $A/\lambda$ value of each layer can be deduced:

$$A/\lambda = \sqrt{\langle y^2 \rangle / 2\pi^2} \quad (3.5)$$

$$A/\lambda = \sqrt{\sigma_{y^2}^2 / \sqrt{2\pi^2}} \quad (3.6)$$

where

$$\langle y^2 \rangle = \sum_{i=1}^{n} \tan^2 \phi_i \quad (3.7)$$

$$\sigma_{y^2}^2 = \sum_{i=1}^{n} \left( \tan^2 \phi_i - \langle y^2 \rangle \right) / n \quad (3.8)$$

The $A/\lambda$ values, determined by both equations, for each layer is given in Table 3.10 and also plotted in Figure 3.18.

<table>
<thead>
<tr>
<th>Table 3.10 Waviness measurement of Type IV material.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) is calculated from eqn. 3.5; (2) is calculated from eqn. 3.6.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of fibres Analyzed</th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3</th>
<th>Layer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A/\lambda \times 10^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\langle y^2 \rangle$</td>
<td>7.0</td>
<td>10</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td>$A/\lambda \times 10^3$</td>
<td>10</td>
<td>19</td>
<td>55</td>
<td>62</td>
</tr>
</tbody>
</table>

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Fig 3.18 The trend of $A/\lambda$ in different layers of Type IV material.
4 Discussion

4.1 Analysis of Results

4.1.1 Two-dimensional Mesostructures

In this project, four two-dimensional mesostructure characteristics (inter-fibre angle, inter-fibre distance, number of neighbour and fibre cell volume fraction) for five different composite materials were measured. As shown in Figures 3.3 and 3.7, the inter-fibre angles are almost equal to 60° for all type of materials. Moreover, for all type of materials the numbers of neighbour are close to six, as indicated in Figures 3.4 and 3.9. As a result, these two parameters are not useful for further quantitative analysis due to their small differences in value.

The remaining parameters (inter-fibre distance and fibre cell volume fraction) for five types of materials have significant differences. It means that these two parameters can reflect and differentiate the fibre spatial arrangement for each kind of composite material. However, basic models of fibre spatial arrangement should be considered first in order to understand how these two parameters contribute in mesostructure quantification.

In general, fibre spatial arrangement in a composite can be modeled as an array of hexagons, squares or triangles (Figure 4.1). The spatial properties for each model are listed in Table 4.1. The hexagonal array is the most commonly used model to describe the fibre arrangement. Indeed, as mentioned above, all types of composite sample analyzed have about six neighbours and an inter-fiber angle about 60°. In conclusion.
hexagonal model is the preferred fibre arrangement structure.

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Neighbours</th>
<th>Inter-fibre Angle(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>Square</td>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>Triangle</td>
<td>3</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 4.1 Spatial properties of ideal models.

Fig. 4.1 Three type of ideal models for fibre spatial arrangement.

Before further model development, it is necessary to clarify the differences between general fibre volume fraction ($V_f$) and fibre cell volume fraction ($v_f$). General fibre volume fraction is usually determined by the volume ratio of total fibre used to the composite material. It is measured by weight, or length, of fibre used, then it is derived from fibre and matrix densities. Fibre cell volume fraction is the area ratio of the fibre cross-section to the area of a Voronoi cell. It is the property of a single fibre entity and it may be different from one fibre to others. The average value of both fibre volume

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fractions are found as

\[ \bar{V}_f = \frac{nA_f}{nA_f + \Sigma A_{c.i}} \]  \hspace{1cm} (4.1)

\[ \bar{v}_f = \frac{\Sigma v_{f,i}}{n} = \frac{\Sigma (A_f/A_{c,i})}{n} \]  \hspace{1cm} (4.2)

where  \( A_f \) is the fibre cross-section area
\( A_c \) is the Voronoi cell area
\( n \) is the number of fibre

Numerically, Equations 4.1 and 4.2 are equal if and only if \( A_{c.1} = A_{c.2} = A_{c.3} = \ldots = A_{c,n} \). In other words, if all the areas of Voronoi cell are identical, both equations should give the same value. This condition is valid in all three ideal models mentioned above.

Inter-fibre distance and cell volume fraction can be associated to these three models. The inter-fibre distance, which is the measurement of center-to-center distance of fibre cross-section, is not appropriate for modeling because it is not a dimensionless parameter. This parameter is changed to a dimensionless form by dividing it by the fibre diameter, and gives the reduced inter-fibre distance \((L')\). The relationships between reduced inter-fibre distance and fibre cell volume fraction for three different models can be calculated from the symmetric geometry of fibre spatial arrangement (Appendix B) and are summarized in Table 4.2. When \( L' \) is equal to one, it indicates that the inter-fibre distance is equal to the fibre diameter and all fibres are closely packed. In other words, the cell volume fraction reaches to the maximum value when \( L' \) approaches to unity.

52
Table 4.2 Relationships between reduced inter-fibre distance and cell volume fraction.

<table>
<thead>
<tr>
<th>Model</th>
<th>Relationship between $L'$ and $V_f$</th>
<th>Limiting Volume Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagon</td>
<td>$L' = \sqrt{\frac{\pi}{2\sqrt{3}}V_f^{-1/2}}$</td>
<td>90.7%</td>
</tr>
<tr>
<td>Square</td>
<td>$L' = \sqrt{2\frac{\pi}{2}}V_f^{-1/2}$</td>
<td>78.5%</td>
</tr>
<tr>
<td>Triangle</td>
<td>$L' = \sqrt{\frac{\pi}{3\sqrt{3}}}V_f^{-1/2}$</td>
<td>60.5%</td>
</tr>
</tbody>
</table>

A plot of reduced inter-fibre distance versus fibre cell volume fraction for all three models is shown in Figure 4.2. The experimental results from Tables 3.2 and 3.5 are also plotted on the same figure.

![Graph](image)

**Fig. 4.2** The plot of $L'$ vs. $V_f$ with experimental results.

All data points are located above all three curves. Theoretically, this indicates that...
all the composite samples contain a mixture of triangular, square and hexagonal configurations. However, the results of inter-fibre angle and number of neighbour strongly suggest that they should be in hexagonal pattern in general. In order to distinguish the differences in fibre spatial configuration of each sample, the hexagonal model is used as a common reference. Using the plot of $L'$ vs. $v_f$, the regularity of hexagonal pattern of fibre arrangement can be reflected by the distance between the data point and the curve. Thus, the regularity of fibre spatial arrangement of a composite can be defined as:

$$\text{Regularity} = \left(1 - \frac{L' - L'_{\text{ideal}}}{L'_{\text{ideal}}} \right) \times 100\% \quad (4.3)$$

The regularity can also be used to indicate the deviation between the real and ideal structure. The regularity value for Type I to Type V composite materials are computed below:

<table>
<thead>
<tr>
<th>Material</th>
<th>$v_f(%)$</th>
<th>$L'$</th>
<th>$L'_{\text{ideal}}$</th>
<th>Regularity(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>56</td>
<td>1.31</td>
<td>1.21</td>
<td>91.9</td>
</tr>
<tr>
<td>Type II</td>
<td>60</td>
<td>1.23</td>
<td>1.17</td>
<td>94.7</td>
</tr>
<tr>
<td>Type III</td>
<td>63</td>
<td>1.23</td>
<td>1.14</td>
<td>91.7</td>
</tr>
<tr>
<td>Type IV</td>
<td>50</td>
<td>1.49</td>
<td>1.28</td>
<td>83.4</td>
</tr>
<tr>
<td>Type V / Layer1</td>
<td>44</td>
<td>1.53</td>
<td>1.36</td>
<td>87.7</td>
</tr>
<tr>
<td>Type V / Layer2</td>
<td>48</td>
<td>1.49</td>
<td>1.30</td>
<td>85.5</td>
</tr>
</tbody>
</table>

High regularity indicates that the fibre spatial arrangement is very uniform, which can be found in high fibre volume fraction material (Type II). The contrary is also true. Low fibre volume fraction can cause lower regularity, as seen from the trend between $v_f$ and regularity in Table 4.3.
4.1.2 Three-Dimensional Mesostuctures

Misalignment angles (in-plane $\phi_{in}$, out-of-plane $\phi_{out}$ and local $\phi$) are the preliminary quantities to characterize a 3-D fibre path. These values are a direct measurement because they are obtained from the position change of fibres between two successive sections, or from analyzing the elliptical fibre cross-sections from an inclined section. However, fibre waviness is the major origin of fibre misalignment and fibre waviness has stronger influence on mechanical properties. In section 3.22 the relationship between misalignment angle distribution and fibre waviness is derived. Equations 3.5 and 3.6 are based on the assumptions that (1) all fibres have the same wavelength and amplitude; and (2) fibres are not in-phase, i.e. the phase-shifts are random. The effectiveness of this estimation method can be tested by comparing the waviness results between this estimation and the detailed waviness analysis. The in-plane and out-of-plane misalignment angles of Type I material in different corners are shown in Figure 3.15. Using these results and Equation 3.1, one can calculate the local misalignment angles for each corner. The $A/\lambda$ values, based on Equations 3.5 and 3.6, are determined subsequently. The results are listed in Table 4.4.

| Table 4.4 Waviness parameter estimation for Type I material. |
|---|---|---|---|
| $\langle y'^2 \rangle$ | $\sigma_{y'^2}$ | $A/\lambda$ (Eqn. 3.5) | $A/\lambda$ (Eqn. 3.6) |
| Corner A | 0.0074 | 0.010 | 0.019 | 0.032 |
| Corner B | 0.0050 | 0.014 | 0.016 | 0.027 |
| Corner C | 0.0023 | 0.0028 | 0.011 | 0.012 |

Another independent analysis of $A/\lambda$ value of Type I material, determined by using the method of fibre tracking through five successive sections, is shown in Figure 3.13, which indicates that the distribution has a peak lying in the range of $\log(-2)$ to $\log(-1)$.  

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or 0.01 to 0.1. On the other hand, from Table 4.4, the $A/\lambda$ value is from 0.01 to 0.03. By comparing these two sets of results, the estimated $A/\lambda$ values derived from misalignment angle measurement agree within about 30% of the result from detail waviness analysis. Thus, misalignment angle distribution can give a quick and rough estimate in $A/\lambda$ value. However, if one wants to find fibre waviness or wavelength distribution individually, multi-section fibre tracking method is the only solution.

4.1.3 Fibre Movement Synchronization

As stated in the Introduction, the position of a fibre is governed by the process condition and material used. Fibres sometimes group as a bundle and ‘move’ in the same direction in certain areas in a composite. Conversely, they can also act freely, without the influence of neighbouring fibres. So, in order to make meaningful comparison, some quantification of fibre movement synchronization in the volume of composite is needed. Again, by using Voronoi diagram and fibre neighbourhood, the average of the relative angular movement between a fibre and its neighbouring fibres is computed (Figure 4.3):

$$\theta_{\text{syn}} = \frac{\sum |\theta_o - \theta_i|}{n_n}$$

where $n_n$ is the number of neighbouring fibres.

This angular average indicates the degree of synchronization in the area analyzed. The smallest value of this angle is $0^\circ$, which means that all fibres are moved in the same direction. This angle can be as large as $90^\circ$.
Diagrams in Figure 4.4 are generated by connecting the fibre centers in two adjacent sections of Type I material. This is a helpful tool to visualize the general trends of fibre movement. The characteristics of fibre bundles, which is the area with synchronized fibre movement, and also the number of fibre with this bundle, can also be estimated from this diagram.

![Diagram of fibre synchronization angle](image)

**Fig. 4.3** Definition of fibre synchronization angle.

$\sigma_{y'^2}$ values, synchronization angles and fibre cell volume fraction of Type I material at different corners are listed in Table 4.5. By comparing these numbers, one can see that when fibre cell volume fraction increases, both synchronization angles and $\sigma_{y'^2}$ values decrease. It can be explained that when fibres are packed closely, the fibre-fibre interaction is high. Fibres are gathered as a bundle and therefore have more synchronized movement in high volume fraction area.
4.2 Sources of Error and its Effects

4.2.1 Sample Surface Imperfections

Polishing the surface of a sample to obtain high degree of flatness is important in sample preparation. Ideally, the surface should be perfectly flat and at right angles to the four lateral sides. In the experiment, there are two possible situations in which the surface is not perfect. The first situation is the coarse surface resulting from an incomplete grinding process. The second situation is an inclined surface resulted from improper grinding (Figure 4.5).

The fibre centers in a particular section are stored as \((x, y, z)\), a three-dimensional coordination set. For simplicity, the measurement error in the \(y\)-coordinate and \(x\)-coordinate are assumed to be equal. As shown in Figure 4.5, the recorded \(z\)-coordinate is at section B, which is the average position of section A and section C. So, \(\delta z\) is the measuring error in \(z\)-direction. The distance between the recorded and the actual position of a fibre cross-section is \(\delta x\). The \(\delta x\) value depends on the local misalignment angle of a fibre and \(\delta x\) is always zero if a fibre is straight and perpendicular to section plane.
In order to estimate $\delta x$, we consider that all the fibre misalignments are induced from fibre waviness. The misalignment angle of a fibre found at any particular section is a local value, or a 'snapshot' at a certain position of a fibre. According to the coordinate system specified in Figure 4.5, the fibre waviness can be modeled as

$$x = A \sin\left(\frac{2\pi z}{\lambda}\right)$$

(4.5)

The maximum misalignment angle $\phi$ is $\tan^{-1}(2\pi A/\lambda)$. Since this misalignment angle is also equal to $\tan^{-1}(\delta x/\delta z)$, the maximum value of $\delta x$ becomes $2\delta z \pi A/\lambda$. In other words, the $\delta x$ value is proportional to the waviness parameter $(A/\lambda)$ and the measurement error in the $z$-direction.
δz values for Type I material analyzed are listed in Table 3.3. Using the δz value of 130 μm and A/λ of 0.03 obtained from the Results section, the maximum δx value for the carbon-epoxy pultrusion sample is about 25 μm, or three fibre diameters.

We further assume that the measurement errors of the position of a fibre cross-section center in x- or y-coordinate is distributed normally. It can be conceived that the maximum value of δx is about three standard deviations from the mean error (or 99.9% probability that this error is within the range of the maximum value of δx). Therefore, the average value of δx can be represented by one standard deviation of the maximum value of δx, which is $2δzπA/3λ$, or one fibre diameter for Type I material.

4.2.2 Connection of Fibre Cross-sections between Sections

Connecting fibre cross-sections between sections involves two steps. First, fibre centers in a section are grouped as a whole and moved to match the physical position of the adjacent section. The image of the upper section cannot be superimposed directly on the lower image because the positions of the sample holder under the microscope are not identical during image capturing. Second, fibre centers in both sections are linked individually.

Without using any reference position, Paluch [21] combined both steps by linking fibre centers first, and then deduced the positional offset between two sections. Under his approach, the change in position of two associated fibre centers in two adjacent sections is resolved into vectors $\bar{a}$ (actual fibre movement) and $\bar{b}$ (section positional offset). The pictorial representation is shown in Figure 4.6. He assumed that the
summation of all vectors $\vec{a}$ was zero because vector $\vec{a}$ was associated with the real amplitude of the fibre undulation. Then, vector $\vec{b}$ was determined from vector $\vec{d}$ by an iteration scheme. The shortcoming of this method, as he also pointed out, is that the positional offset must not be greater than half of the distance between fibres; otherwise most of the linking of fibre centers could be incorrect. Another shortcoming is that the rotational offset is ignored in his calculation. For an image of $640 \times 480$ pixel, one degree of rotational offset can give an error of about 11 pixels, which is more than half the fibre diameter. So, both translational and rotational offsets should be determined before the linking process.

![Diagram](image)

**Fig. 4.6** Section matching method used by Paluch.

As outlined in the Experimental section, the corner of the sample, which is the intersection point of vertical and horizontal edge of the sample, is used as an absolute reference position for different successive sections. Using this information, section positions are matched by determining the translational and rotational offset between two
adjacent sections. This method gives more accurate result because it does not require any assumption.

The height difference between two sections ($\Delta z$) is vital to the accuracy in linking fibre cross-sections. A possible error is shown in Figure 4.7. If $\Delta z$ is too large, the correct fibre cross-section in the next section will be out of reach in the searching region, which is a circular region with the radius of $nd_f (n$ times fibre diameter).

It is difficult to determine maximum usable $\Delta z$ value because it depends on the fibre waviness. In theory the maximum $\Delta z$ value is $2\pi Ad_f / \lambda$, which is derived from the maximum slope of Equation 4.5 and using the searching radius of one $d_f$. Because the waviness is unknown before experiment, we need some a priori assumptions, or try-and-error, to predict the initial $\Delta z$ value. However, this value can be refined during experiment by observing the fibre position between sections.

![Diagram showing possible error in constructing fibre path.](image)

**Fig. 4.7** Possible error in constructing fibre path.
4.2.3 Waviness Parameters Optimization

Most of the researchers modeled the fibre waviness in a continuous fibre composite as a standard sinusoidal function (Equation 4.5). This is a reasonable assumption because of its simplicity and ease of modeling. However, the undulation of a fibre is far more complex in reality. Two categories of fibre path are classified below.

(1) Two-dimensional fibre path

If the whole fibre path lies on a plane, this path can be mapped onto a two-dimensional coordinate system. In other words, the locus of the fibre path is along a two-dimensional space. This fibre path can be completely straight in an arbitrary direction. The most ideal case is that fibres follow a straight path parallel to one of the surfaces of the composite sample. Otherwise, the straight fibre path is described as a misaligned fibre, characterized by a pair of misalignment angles (in-plane and out-of-plane). The second possibility is that the fibre path is wavy. The simplest case, as mentioned above, is that the fibre is in a shape of a sinusoidal function and the central axis is parallel to one of the surface of the composite sample. Indeed, the central axis can be in any direction with the whole fibre path lying on a plane. Non-straight fibre paths can be described with other functions, e.g. Fourier series, spline function.

(2) Three-dimensional fibre path

This is a fibre path that does not lie on a plane. To analyze this kind of fibre path, the path is projected to its mean inertial plane thus it becomes a two-dimensional path. For example, a helix is a simple three-dimensional fibre path. The parametric equations of a helix are \( x = \sin t, y = \cos t, z = t \). The mean inertial plane is the \( xz \)- or \( yz \)-plane.
and the projected form of this path is \( x = \sin z \) or \( y = \cos z \).

**Fig. 4.8** Types of fibre path. A: 2-dimensional fibre (straight); B: 2-dimensional fibre (wavy); C: 3-dimensional wavy fibre.

These two types of fibre path are illustrated in Figure 4.8. In practice, all fibre paths are assumed to be three-dimensional in waviness analysis. The average distance between the real path and the projected path can be calculated by

\[
\overline{L_p} = \Sigma \sqrt{(x - x_p)^2 + (y - y_p)^2 + (z - z_p)^2} / N_{sec}
\]

where \( x \) and \( y \) are the original coordinates; \( x_p \) and \( y_p \) are the projected coordinates and \( N_{sec} \) is the number of sections used. This parameter indicates the discrepancy between the real and projected fibre path.

In the waviness analysis, the \( \overline{L_p} \) value of Type I material was about \( 1.9 \pm 1.5 \mu m \) and its distribution is shown in Figure 4.19. Comparing this \( \overline{L_p} \) value with the fibre diameter (\( 3 \mu m \)), it shows strong agreement that the fibres of Type I material is indeed a two-dimensional fibre path.
The central axis of a fibre path can be in any direction, not necessarily parallel to the mean inertial plane. So, when the fibre path is fitted to a sinusoidal curve, the orientation of this curve should also be considered. Paluch used the eigenvector associated with the largest eigenvalue of the fibre center coordinates to determine the central axis of the fibre path. It is valid only if the length of a fibre path is longer than several multiples of wavelength. Otherwise, as illustrated in Figure 4.10, incorrect amplitude and wavelength would be determined.

To overcome this problem, the eigenvector associated with the smallest eigenvalue of the fibre center coordinates is used as the normal vector $\vec{n}$ of the mean inertial plane. Then vector $\vec{v}$, which is parallel to $xy$-plane, and vector $\vec{v}'$, which is the cross product of $\vec{n}$ and $\vec{v}$, are calculated accordingly. Fibre center coordinates are mapped to a plane spanned by $\vec{v}$ and $\vec{v}'$. Finally the mapped points are fitted to a rotated sinusoidal function. The advantage of this method is that waviness parameters can be determined from a small portion of the fibre path.

Another error measurement for waviness analysis is the average distance between the projected points and the sine-wave-fitted coordinates, denoted as $\overline{L_f}$ and is defined as follow:

$$\overline{L_f} = \Sigma \sqrt{(x_p - x_f)^2 + (y_p - y_f)^2}/N_{sec}$$

(4.7)

where $x_p$ and $y_p$ are the projected coordinates, and $x_f$ and $y_f$ are the fitted coordinates. This value indicates how well this fibre path is fitted as a sinusoidal wave. The $\overline{L_f}$
value for the waviness analysis of Type I material was found to be $0.62 \pm 0.56 \mu m$ and the distribution is shown in Figure 4.9. This is a relative smaller value (4% of fibre diameter) and it shows that the fibre path is in good sinusoidal shape.

**Fig. 4.9** Distributions of errors in waviness analysis of Type I material.

**Fig. 4.10** Possible errors in Paluch’s method.

$\lambda$ : original wavelength  
$\lambda_a$ : apparent wavelength  
$A$ : original amplitude  
$A_a$ : apparent amplitude
4.2.4 Errors in Image Analysis

The limitations of digitized image and possible errors in optical microscopy have been mentioned by Clark, Davidson & Archenhold [33]. Firstly, the noise in pixilated images affects the measurement of fibre cross-section coordinates and fibre ellipticity (Figure 4.11). Secondly, the fibre ellipticity is difficult to determine for the fibre with non-circular cross-section.

![Image](image.png)

**Fig. 4.11 Influence of noise in ellipticity measurement.**

In the analysis of the fibre cross-section for all type of material in this project, the fibre positions were determined by image analysis, and then they are corrected manually if the position of a fibre center was not matched with the original image. In this case, the amount of personal judgment involved depends on the clarity of the image. Nevertheless, the highest accuracy is still obtained by the judgment of the human vision.

Based on the results from this project, more than twenty thousand fibres have been analyzed and in average about 80% of fibres can be automatically recognized from the images. However, the positions of fibres in an image are definite and thus the fibres positions, either determined by machine or human vision, should be user-independent and reproducible.
4.3 Advantages, Advances and Applications of Present Method

This mesostructure quantification method is based on image analysis. The strength of this technique is that one can analyze a variety of composite materials by using the same set of computational routine. In this project, five different types of material were successfully analyzed.

The equipment required for this method is rather simple: a light microscope with digital image capturing device, a grinder and a personal computer. All the equipment is very common in many research laboratories and therefore this method can be adapted without extensive capital investment.

This quantification method involves the steps of sample preparation, grinding and polishing, image capturing and analysis, and waviness parameters optimization. The flow sheet of a complete waviness analysis is shown in Figure 4.12. About one day of work is enough to find a complete and accurate fibre waviness distribution and fibre spatial characteristics of a composite.

![Flow sheet of waviness analysis.](image)

Fig. 4.12 Flow sheet of waviness analysis.

This method can obtain a higher accuracy of waviness measurement in comparison with the methods developed by Paluch and Highsmith. It is because the three-dimensional path of individual fibre is determined explicitly. No assumption is used in
the calculation and the reference point for each section is an absolute position. Moreover, the algorithm used by Paluch is revised so that the number of cross-sections used for waviness analysis is reduced substantially. Since sample surface polishing is a labour-intensive process, the present method is more cost-effective and less time-consuming.

Fibre spatial regularity, which is derived from basic 2-D mesostructure properties, and synchronization angle, which is a combined property of misalignment angle and fibre neighbourhood, are introduced in this project. The basic and extended parameters are useful for mesostructure comparison, meso-mechanical analysis and realistic composite modeling. Error analysis is also outlined to verify the quality and reliability of data.
5 Conclusions

1. A set of computer programs suitable for running on personal computers was developed for mesostructure quantification. Two-dimensional and three-dimensional mesostructures of five different types of composite material were successfully analyzed.

2. Waviness for individual fibre could be determined by successive sectioning. Estimation of fibre waviness parameter ($A/\lambda$) could be achieved by using misalignment angle distribution.

3. An increasing trend of fibre waviness from outside layer to inside layer of filament-wound tube was found.

4. Quantification of fibre packing regularity and fibre movement synchronization were proposed.

5. It was found that high fibre volume fraction gives more uniform fibre packing and more synchronized fibre movement.

6 Recommendations

1. The relationships of sand paper grit/ grinding time/ composite surface removed should be found in order to determine the eroding rate in sample polishing.

2. More research in analyzing the void fraction is needed.

3. More statistical analysis is needed to verify the significance of each parameter. Sensitivity test is needed to examine the effect of noise on each parameter.

4. In 3-D analysis, a more accurate reference point on a composite sample should be found.

5. Further research, in real sample analysis and computer simulation, is needed to gain a better understanding in the mesostructures-properties-processing links.
7 Nomenclature

Greek Letter

\( \varepsilon \)  Tensile strain
\( \Delta z \)  height of sample removed between two successive sections
\( \Phi \)  Rotational angle of ellipse
\( \phi \)  Local misalignment angle
\( \phi_1, \phi_2 \)  Arbitrary misalignment angles
\( \phi_{out} \)  Out-of-plane misalignment angle
\( \phi_{in} \)  In-plane misalignment angle
\( \theta \)  Inter-fibre angle
\( \delta \)  Rotational angle of fitted fibre
\( \delta_x \)  measurement error of fibre cross-section center in x-direction
\( \delta_y \)  measurement error of fibre cross-section center in y-direction
\( \delta_z \)  measurement error of fibre cross-section center in z-direction
\( \theta_{syn} \)  Synchronization angle
\( \gamma \)  Shear strain
\( \lambda \)  Fibre wavelength
\( \lambda_a \)  Apparent fibre wavelength
\( \nu \)  Poisson ratio
\( \rho \)  Density
\( \sigma \)  Tensile stress
\( \sigma_{y^2} \)  Standard deviation of the square of slope
\( \tau \)  Shear strength
\( \zeta \)  Rotational offset

Latin Symbols

\( A \)  Fibre amplitude
\( A_a \)  Apparent fibre amplitude
\( A_c \)  Voronoi cell area
sl.
s3.
s3.
s4
t
l
z
viaJuL
QXlJ
u1
cllpat;
vector represents the displacement of fibre center between
two successive sections
C
Phase shift of harmonic function
b
Minor axis of ellipse
\overrightarrow{b}
positionional offset of two successive sections
d_f
Fibre diameter
\overrightarrow{d}
Total displacement of fibre center between two successive
layer (\overrightarrow{a} + \overrightarrow{b})
E
Young's modulus
G
Shear modulus
L
Inter-fibre distance
L'
reduced inter-fibre distance
L_e
Ellipse major axis
L_f
Average distance between fitted and projected fibre path
L_p
Average distance between real and projected fibre path
l
Sample length
N
Number of Neighbour
N_{sec}
Number of section
n
Number of fibre
\overrightarrow{n}
Normal vector of mean inertial plane
r_f
Radius of fibre
s
Fibre aspect ratio
s_1, s_2, s_3, s_4
Four lateral sides of sample
t
Sample thickness
w
Sample weight
\underbar{u}
Orthonormal vector in mean inertial plane

73
$V_f$ General fibre volume fraction
$u_f$ Voronoi cell fibre volume fraction
$\vec{v}$ Orthonormal vector in mean inertial plane
$x, y, z$ Coordinate variables
$x_p, y_p$ Projected coordinates
$x_f, y_f$ Fitted coordinates
$y_o$ Vertical offset of harmonic function

Subscripts

1, 2, 3 Composite material direction
$m$ Composite matrix
$f$ Composite fibre
8 References


12. Bolotin, V. V., Polymer Mechanics, 2 1 (1966) 11-19


23. Piggott, M. R., “Mesostructure in Fiber Composites: Shear Strength”, 1A31-1A37, Proceeding of CANCOM’91, Montreal, Quebec (1991)


APPENDIX A

User Manual

of

Fibre-reinforced Composites Mesostructure

Quantification Program Set
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A1. Introduction

The TC (University of Toronto Chemical Engineering and Applied Chemistry) application series is a set of programs designed to measure and quantify the mesostructure of fibre-reinforced composites. The aim of this program set is to unify the approach of mesostructure quantification and standardize the parameters to describe fibre volume fraction, inter-fibre relationships, fibre misalignment and fibre waviness.

Traditional image-processing program provides a large variety of function to satisfy different needs. This means that a job is broken down to a series of simple task and lots of parameters are needed to set before operation. The user always has the difficulties in choosing the correct functions, parameters and task sequence to do a specific task.

The TC application series aggregates the essential image processing functions and numerical algorithms necessary for mesostructural quantification. The number of steps and adjustments involved are minimized so that the operation is straightforward and all the parameters obtained can be user-independent. Thus, meaningful comparisons and conclusions can be drawn. Moreover, these parameters can be used in mesomechanical modeling or finite element analysis.

A2. Functions of Each Program

TCBinary  Conversion of an 8-bit gray-scale image to a black-and-white image; volume fraction distribution calculation and binary image processing operation.

TCLabel   Recognize fibre position from a binary image.

TCTriangle Construct Voronoi and Delaunay diagram. Determine and analyze inter-fibre angles, inter-fibre distances, number of fibre neighbours and cell fibre volume fraction.

TCLink    Match the orientation between layers; link fibre centers from different layers to construct the entire fibre path.

TCWaviness Projecting fibre path into 2D coordinates; analyze fibre waviness.
A3. System Requirement

- PC with an Intel (or 100% compatible) Pentium series processor (Pentium 200 MHz or faster processor recommended)
- Operating system: Microsoft Windows 98 / Windows NT 4.0 with SP3 (or later) / Windows 2000 beta 3
- A minimum of 12 MB of memory, 32 MB recommended
- Video output of 16-bit colour with a resolution of 800x600, 1280x1024 recommended

A4. Installation

Each program is a self-executable file and can be placed in any directory. No additional dynamically linked library is required. These programs do not modify any Windows registry entries.

A5. TC Application Series Project Page

All the programs, source codes and manuals are obtainable from the TC Project Page at ‘http://www.chem-eng.toronto.edu/~hoskc/’. Information about new releases, patches, etc. is also available from this page. Any questions, comments or suggestions can be sent to the author by e-mail to hoskc@chem-eng.toronto.edu.
A6. TCBinary

Input: 8-bit gray scale bitmap  
Output: black and white bitmap

A6.1 Program Operation

A6.1.1 Image Thresholding

1. An 8-bit gray-scale image is selected through an open-file dialog box (Fig. A1). The image is shown in a separated window (Fig. A2).

2. The user is prompted to choose the size of sub-image grid. Standard grid sizes are suggested (1x1, 2x2, 4x4, etc). The user can use customized grid size for an image with uneven illumination. The size of each sub-image is bounded by the red gridlines (Fig. A3).

3. The threshold image is shown in a separated window (Fig. A4). The user can select other grids if the result is unsatisfactory.

4. The threshold image can be saved in either bitmap or text format.

A6.1.2 Image Manipulation

Erosion, accretion and partitioning can be performed on the binary image. The user can input different erosion, dilation or partitioning coefficient to manipulate the image. The resulting image will be shown in a separated window (Fig. A5). The user can save the resulting image as a different file.
Fig. A1. Gray-scale bitmap (CPEEK1.bmp) is loaded by an open file dialog box.

Fig. A2. CPEEK1.bmp is loaded and showed in a separate window.
Fig. A3. A $10 \times 10$ (64 x 48 pixels) grid is used for thresholding.

Fig. A4. The binary image of CPEEK1 is shown in a separate window.
Fig. A5. CPEEK1.bmp is eroded twice with $k_E = 6$
A6.2 Algorithm

A6.2.1 Iterative thresholding

The histogram of a gray image is typically bimodal. The threshold value is the local minimum point between the two peaks. The algorithm described below can move the threshold value from the average value to close to the local minimum by refining the partition point and distinguishing the optimal thresholding point.

The pictorial representation of this process is shown in Fig. A6. A gray-scale image is divided into sub-images with dimensions of m x n pixels. For each sub-image:

(1) The average gray value (T) of the sub-image is determined.
(2) The image is partitioned into two groups, R1 and R2, using the threshold value T.
(3) The mean gray values μ1 and μ2 of the partitions R1 and R2 are found.
(4) A new threshold value T_{new} is calculated by

\[ T_{\text{new}} = \frac{1}{2} (\mu_1 + \mu_2) \]  

(A1)

(5) Steps (2) - (4) are repeated until |T - T_{\text{new}}| < \epsilon

where \epsilon is the absolute change of the threshold value in successive iterations. It is set to 1 because this is the minimum gray level increment.

(6) The sub-image f is transformed into binary sub-image g as follows:

\[ g(i,j) = \begin{cases} 1 & \text{for } f(i,j) \geq T_{\text{new}} \\ 0 & \text{for } f(i,j) < T_{\text{new}} \end{cases} \]  

(A2)

\[ \]  

Fig. A6. The typical gray value histogram is bimodal. The initial threshold value $T$ is the average of the sub-image gray level. This value is shifted to the local minimum until the difference between $T$ and $T_{\text{new}}$ is less than one.

A6.2.2 Binary Image Processing

*Erosion / Accretion*

For the pixel $P_{i,j}$ in a binary image $P$, a $3 \times 3$ mask around this pixel is constructed:

$$
\begin{array}{ccc}
P_{i-1,j-1} & P_{i,j-1} & P_{i-1,j-1} \\
P_{i-1,j} & P_{ij} & P_{i-1,j} \\
P_{i-1,j+1} & P_{i,j+1} & P_{i-1,j+1}
\end{array}
$$

*Fig. A7. A 3 x 3 mask for binary operation*
By letting $S = \sum P_{ij}$ and defining $k_E$ and $k_A$ as the erosion and accretion coefficient, the erosion and dilation operation are as follows:

Erosion: if $P_{ij} = 1$, then $P_{ij} = 0$ if $S < k_E$;
Dilation: if $P_{ij} = 0$, then $P_{ij} = 1$ if $S > k_A$.

$S$ is in the range of 0 to 9, as are $k_E$ and $k_A$. A larger $k_E$ value allows faster image shrinking whereas a smaller $k_A$ value allows faster image expanding. If an image contains holes or incomplete circular area, the process of accretion followed by erosion can fill up and smoothen the incomplete part of the image. Conversely, noise can be removed by erosion followed by accretion.

**Partitioning**

It is a quick process to separate large objects that are connected by small parts. If a chain of black pixels, in a column or in a row, contains less than or equal to the partitioning coefficient $k_p$, all these pixels are turned to white. So, for any row in a binary image $P$, if

$$P_{i-1,j} = 0, \text{ and } \sum_{i=1}^{m} P_{ij} \leq k_S, \text{ and } P_{i-m-1,j} = 0$$

(A3)

then $P_{ij} = 0$ for $i = 1$ to $m$.

For any column in a binary image $P$, if

$$P_{i,j-1} = 0, \text{ and } \sum_{j=1}^{m} P_{ij} \leq k_S, \text{ and } P_{i,j+m-1} = 0$$

(A4)

then $P_{ij} = 0$ for $j = 1$ to $m$.

Examples of erosion, accretion and partitioning processes are shown in Fig. A8.
A: Original image K. 1 = black pixel; empty space = white pixel.

The first pixel next to the edges is assigned to zero.

B: numeric sum of 3 x 3 mask     C: Erosion of K (kE = 5)
D: Accretion of K (kA = 5)      E: Erosion of K (kE = 7)
F: Accretion of K (kA = 3)      G: Partitioning of K (kp = 2)
H: Partitioning of K (kp = 3)

Fig. A8. Examples of binary operations
A7. TCLabet

Input: Black and white bitmap
Output: text file containing fibre coordinates

A7.1 Program Operation

A7.1.1 Fibres Recognition Process

(1) A binary image is loaded via an open-file dialog box.

(2) The user is asked to enter the name of a text file to store the result. The User should estimate the fibre diameter (in pixels) from the original image before using this program. Once a binary image is loaded, fibre centers are recognized by a two-pass operation. Parameters for this operation can be changed to obtain the optimum result. The default parameters are:

Fibre diameter = 18
First-pass erosion coefficient ($k_{E1}$) = 4
First-pass partitioning coefficient ($k_{P1}$) = 2
Second-pass erosion coefficient ($k_{E2}$) = 7
Second-pass partitioning coefficient ($k_{P2}$) = 3

(3) (i) In the first pass, each discrete black area, or "unit", in the image is numbered by sequential labeling. A unit can be an incomplete fibre, a single complete fibre, or several fibres with touching edges (clumped-fibre). The area of each unit is calculated. If the area is less than half of a single fibre cross-section area, this unit will be removed from the image. If the area is larger than ten times that of a single fibre cross-section area, the unit will be processed later.

(ii) The remaining units, which will consist of less than ten fibres, are isolated individually. The expected number of fibres in the unit is determined by the unit area and unit perimeter. If both methods give different numbers, the smaller value is used. Erosion and partitioning are used to operate on a unit until the number of separated fibres is equal to the expected number of fibres. The center coordinates of separated fibres are calculated by moment analysis.
(iii) The remaining units are processed by erosion and partitioning. The effective fibre diameter is reduced by two pixels. Step (i) is repeated until the fibre diameter is equal to four or no more fibre can be recognized.

(iv) The recognized fibre cross-sections that are removed in steps (i) to (iii), are redrawn in white on the original image. So, the resulting image will only contain the fibre units that cannot be recognized in the first-pass.

(v) Step (i) to step (iii) are re-applied on the resulting image, but using the second-pass parameters.

The flow chart of this fibre recognition process is shown in Fig. A9.

### A7.1.2 Fibre Redrawing Process

1. The user is asked to load an image and a text file containing fibre centers corresponding to the image. This image can be an original gray-scale image or a threshold binary image.

2. Fibres are redrawn and superimposed on this image. The user can change the appearance of the redrawn fibres by manipulating the drawing parameters. The user can add and delete any fibre to match the original image. The text file containing fibre centers is updated in every session.

The drawing parameters are:

- **Drawing radius (8*)**
- **Showing original image** (1 = yes*, 2 = no)
- **Fibre color** (blue*, green, red, gray)
- **Filling the fibres** (1 = filled circle, 2 = empty circle*)
- **Showing the fibres** (1 = yes*, 2 = no)
- **Showing the fibre labels** (1 = yes*, 2 = no)
- **Label color** (blue, green, red*, black, white)
* Default value

Fig. A10 and Fig. A11 show a sample result of both processes.
Fig. A9. Flow charts of fibre recognition process
Fig. A10. Fibre recognition of binary image of CPEEK1. The recognized fibres are outlined in red. This result using the default settings.

Fig. A11. Fibre recognition of CPEEK1 after manual addition and correction of the previous result as shown in Fig. A10. 722 recognized fibres are drawn in red circle. Fibres located around the edges are ignored.
A7.2 Algorithm

A7.2.1 Sequential Labeling

This is a method to label separated objects in a binary image. The value of each pixel is changed to the label of a separated object that this pixel belongs to. Using a 2 x 2 mask, this method scans an image from left to right, in a raster from top to bottom.

\[
\begin{array}{cc}
D & C \\
B & A
\end{array}
\]

Fig. A52. 2 x 2 mask for sequential labeling

Sequential labeling pseudo code:

1. Do
2. NewLabel = 1
3. If A = 0, nothing to do and move on;
4. If A = 1 and D > 1, then A = D;
5. If A = 1, B > 1 and C ≤ 1, then A = B;
6. If A = 1, B ≤ 1 and C > 1, then A = C;
7. If A = 1, B ≤ 1 and C ≤ 1, then
   NewLabel = NewLabel + 1; A = NewLabel;
   Special note on the connectivity of B and C
8. If A = 1, B > 1 and C > 1, then A = MIN(B,C);
9. End Do

Lines 4 to 6 indicate that if one of the neighbours of A is labeled, this label is simply copied to A and the scanning moves on. Line 7 checks for the case that neither B nor C is labeled. If so a new label is chosen for A. If both B and C carry labels and both labels are equal, this label is simply copied to A. However, if B and C are different, A is copied as the smaller value between B and C and the relationship between the labels of B and C are recorded. This is because both labels belong to the same object. When this scenario is encountered, all the linking information between different labels are collected and complied. This result is applied in the second scan so that each object can carry a unique label. Finally, all labels are sorted from left to right, top to bottom and the total number of separated objects will equal the number of labels.

---

A8. TCTriangle

Input: Text file containing fibre center coordinates
Output: Voronoi and Delaunay diagrams

Text files containing 2-D mesostructure results

A8.1 Program Operation

A text file containing fibre centers is requested. The user is asked to enter the fibre diameter (in pixels), the maximum distance between neighbours, and the number of fibres used to construct Voronoi and Delaunay diagram, which are shown after the computation (Fig. A13). The inter-fibre distance and angle, fibre volume fraction and number of fibre neighbours are calculated and saved as text files.

A8.2 Algorithm

A8.2.1 Voronoi / Delaunay diagram Construction

The history and definition of Voronoi diagram will not be presented here. This information can be obtained from different sources, e.g. see Boots et al.

The scheme for diagrams constructing presented as follows:

1) Assume the image contains the cross-section of N fibres, with center points $P_i(x_i, y_i)$.
2) Any three points $P_1, P_2, P_3$, are chosen and a circle is constructed. The center $O(x_c, y_c)$ and radius ($R$) of this circle are calculated as follows:

\[
A = x_2 - x_1; \quad B = y_2 - y_1; \\
C = x_1 - x_2; \quad D = y_1 - y_2; \\
E = A (x_1 + x_2) + B (y_1 + y_2); \\
F = C (x_1 + x_2) + D (y_1 + y_2); \\
G = 2 [ A (y_3 - y_2) - B (x_3 - x_2) ]; \\
x_c = (DE - BF) / G \\
y_c = (AF - CE) / G \\
R = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}
\]

If $G = 0$, these three points are collinear and no finite radius circle through them exists.

---

(3) If the distance between \( P_i (i = 4 \text{ to } N) \) and \( O \) is less then \( R \), points \( P_1, P_2, P_3 \) cannot form a Delaunay triangle (because it is not an empty circle). Otherwise, these three points can be linked together and the linkages are: \( P_1P_2, P_1P_3 \) and \( P_2P_3 \). These three fibres are defined as neighbours to each other. Any three fibres in the fibre set are checked for the linkage. The number of iterations for this process is about \( N^2 \).

(4) The Delaunay diagram of this fibre set can be constructed by connecting all the links obtained in step (3). The inter-fibre distance is defined as the length of center-to-center distance between two neighbours. The inter-fibre angle is the angle between two adjacent neighbour fibres.

(5) The Voronoi cell of fibre \( P_i \) is bounded by perpendicular bisectors of the Delaunay triangles constructed between \( P_i \) and its neighbours. The fibre cell volume fraction \( \nu_f \) of fibre \( P_i \) in this cell is defined as

\[
\nu_f = \frac{R_i^2 \pi}{\text{Voronoi cell Area}}
\]

Fig. A13. Voronoi and Delaunay diagram for the image CPEEK01
A9. TCLink

Input: Text files containing fibre coordinates
Output: Text file containing coordinates of 3-D fibre paths

A9.1 Program Operation

A9.1.1 Section Orientation Matching
The images of two adjacent sections are requested. One is the ‘reference image’ and the other one is the ‘matching image’. The aim of this step is to move the matching image to match the reference image. The associated text files containing fibre center coordinates of these two images are also loaded. Then, the user can mark the reference point in each image. The reference point is the corner of composite sample. This is the intersection point of the horizontal and vertical edge (Fig. A14). To obtain this point, the user is asked to keep choosing points at the edge of the composite sample. A straight line along the edge is shown and the position is updated when a new point is chosen. After the reference point is confirmed, the fibre of the matching image is moved and the new coordinates are saved in a user-selected file.

A9.1.2 Inter-section Fibre Linking
Two text files, containing the fibre centers for two adjacent planes, are requested. The user is asked to enter the searching area radius (ndr) in the matching plane (Fig. A15). The result is stored in a user-selected file.
Fig. A14. Two layers are matched by defining the reference positions (edges and corner)

Fig. A15. Diagram to show the searching radius (nRf) in inter-section fibre linking process
**A9.2 Algorithm**

**Section Orientation Matching**

Fibre matching between two consecutive sections involves two steps: first the images of the two sections are superimposed. The points in the upper section is translated and rotated to match the point cluster in the lower section. Second, points between sections are connected individually by the scheme described below.

The center coordinates of fibres of the bottom section is arranged as matrix $P$:

$$ P = \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix} $$

The $x$- and $y$-offset of the reference points between adjacent planes are $k_1$ and $k_2$ respectively. The average rotational offset is $\zeta$. The new center coordinates of fibres in the bottom section becomes:

$$ P' = M_\zeta (P - K)^T $$

where $M_\zeta = \begin{bmatrix} \cos \zeta & -\sin \zeta \\ \sin \zeta & \cos \zeta \end{bmatrix}$ and $K = \begin{bmatrix} k_1 & k_2 \\ \vdots & \vdots \\ k_1 & k_2 \end{bmatrix}$.

**Inter-section Fibre Linking**

In this part of the matching scheme, a point in the upper section is searched to connect the closest point in the lower section. The maximum distance between sections is set in order to eliminate disconnected fibres. An example is given below to show how this scheme works.

A simple fibre-overlaying diagram is shown in Fig.A6. The fibres labeled by letters (A,B,C,D) are from the upper layer and labeled by digits (1,2,3,4) are from the lower layer.

![Fig. A16. Example of fibre overlay diagram](image-url)
(1) Constructing distance table and Ranking table

The distance table contains the distances between each point, with the format of:

<table>
<thead>
<tr>
<th>point</th>
<th>A</th>
<th>B</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3</td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.7</td>
<td>2.2</td>
<td></td>
</tr>
</tbody>
</table>

From this table, each column is sorted in ascending order of distance and this sorting order, with respect to the bottom layer fibre label is stored in the Ranking table. The complete Ranking table for Fig. A17 is:

<table>
<thead>
<tr>
<th>Ranking</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(2) Eliminating repeated linking

Each top fibre should link with a unique bottom fibre. Since both fibres C and D compete for fibre 3 and the distance of D-3 is shorter than C-3, column C is 'pop-pushed': the top element is of column C is popped away and the column of data is pushed up, with the last one assigned to 0. So, the Ranking table becomes:

<table>
<thead>
<tr>
<th>Ranking</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Again, E-2 is shorter than C-2, so column is 'pop-pushed' again. Finally, the result is:

<table>
<thead>
<tr>
<th>Ranking</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Although each fibre has a unique match, the distance of C-4 is too large. So, it is assigned as a discontinued fibre and C-4 becomes 0.
A10. TCWaviness

Input: Text file containing coordinates of 3-D fibre paths  
Output: Text file containing waviness parameters

A10.1 Program Operation

The user is requested to select a text file containing the 3-D fibre center coordinates for all individual fibre in different layers. Once this file is loaded, the calculation progresses as a batch process. The 3-D coordinates for the harmonically fitted path, accompanying the estimated wavelength, amplitude and other fitting parameters are saved in a user-selected file.

A10.2 Algorithm

Fibre Path Reconstructing Algorithm

PART I: 3D to 2D Mapping

A set of m points associated with the fibre center coordinates in 3-D is stored in a matrix

\[
X = \begin{bmatrix}
    x_1 & y_1 & z_1 \\
    x_2 & y_2 & z_2 \\
    \vdots & \vdots & \vdots \\
    x_m & y_m & z_m
\end{bmatrix}
\]

All the points are first shifted by the centroid \((\bar{x}, \bar{y}, \bar{z})\) of the point cluster and the resulting coordinates \(S\) becomes

\[
S = X - DX
\]  \hspace{1cm} (A7)

where \(D\) is a \(m \times m\) matrix with all elements of \(1/m\).

Then, the covariance matrix of the points is

\[
C = S^T S
\]  \hspace{1cm} (A8)

The normalized eigenvector \(\bar{n} = (n_1, n_2, n_3)\) associated with the smallest eigenvalue of \(C\) is the normal of the mean plane of the point cluster. The plane equation (fig. A7) is given by

\[
n_1x + n_2y + n_3z = k
\]  \hspace{1cm} (A9)

where \(k = \bar{x} n_1 + \bar{y} n_2 + \bar{z} n_3.\)
All the points are projected to this plane and

\[ X_p = X + (K - XN^T)N \]  \hspace{1cm} (A10)

where \( K \) is a \( m \times 1 \) matrix with all elements of \( k \).

The 3-D projected points are then mapped to form a set of 2-D points by projecting onto a plane formed by two orthonormal vectors \( \vec{v} \) and \( \vec{u} \):

\[ \vec{v} = (v_1, v_2, 0); \hspace{0.5cm} \vec{u} = \vec{n} \times \vec{v} \]

where \( v_1 = n_1 / \sqrt{n_1^2 + n_2^2}; \hspace{0.5cm} v_2 = n_2 / \sqrt{n_1^2 + n_2^2} \)

So, the mapped points are

\[ X_m = X_p [v \mid u] \]

PART II : Waviness Estimation

The 2-D points obtained in Part I are fitted to a rotated harmonic function:

\[
\begin{bmatrix}
  x_m' \\
  y_m'
\end{bmatrix} = \begin{bmatrix}
  \cos \delta & \sin \delta \\
  -\sin \delta & \cos \delta
\end{bmatrix} \begin{bmatrix}
  x_m \\
  y_m
\end{bmatrix}
\]  \hspace{1cm} (A11)

and

\[ y_m' = A \sin (Bx_m' + C) + y_o \]  \hspace{1cm} (A12)

where \( A \) is the amplitude; \( B \) is the radial frequency ( = \( 2\pi/\lambda \)); \( C \) is the phase shift and \( y_o \) is the vertical offset.

To estimate the waviness parameters, the fibre path is rotated by an angle \( \delta \). Then this rotated path is fitted as a standard sinusoid by random search optimization. The scheme is as follows:

(1) guess \( \delta \) and \( \lambda \).

(2) Eqn.(A12) is expanded to

\[ y_m' = A \cos C \sin Bx_m' + A \sin C \cos Bx_m' + y_o \]

\[ y_m' = \beta_o + \beta_1 \alpha_1 + \beta_2 \alpha_2 \]

where \( \alpha_1 = \sin Bx_m' \); \( \alpha_2 = \sin Bx_m' \).

Then, \( \beta_o, \beta_1 \) and \( \beta_2 \) are determined by linear regression to give

\[ C = \tan^{-1} (\beta_2 / \beta_1); \hspace{0.5cm} A = \beta_1 / \cos (C) \text{ and } y_o = \beta_o. \]

(3) Use the above results to calculate \( y_{m*} \) and \( x_{m*} \):

\[ y_{m*} = A \sin (Bx_{m*} + C) + y_o \]
The sum of error (SE) is defined as the sum of distances between the actual and predicted points:

\[
SE = \sum \sqrt{(x_m^* - x_m)^2 + (y_m^* - y_m)^2}
\]

(5) Repeat steps (1) to (4) to minimize SE.

Fig. A17. Definition of direction vectors for waviness analysis.

Fig. A18. Rotation of projected fibre path to give standard sinusoid.
APPENDIX B

Fibre Spatial Arrangement

The development of three fibre spatial arrangement models is explained. Figure B1 illustrates the physical arrangement of each model and its unit cell. A numerical example is provided to show how the extended parameters are measured.

**Fig. B1.** Fibre spatial arrangement models
Hexagon Model:
Unit cell area = 12 x shaded area
\[ = 12 \times \left[ \frac{L}{2} \times \frac{L(\tan30^\circ)}{2} \times \frac{1}{2} \right] = \frac{\sqrt{3}L^2}{2} \]
\[ \nu_f = \frac{\pi d_f^2}{4} \times \frac{\sqrt{3}L^2}{2} = \frac{\pi d_f^2}{2\sqrt{3}L^2} \]
Substitute \( L' = d_f L \) gives
\[ \nu_f = \frac{\pi}{2\sqrt{3}L'^2} \]

Square Model:
Unit cell area = \( L^2 \)
\[ \nu_f = \frac{\pi d_f^2}{4} / L^2 = \frac{\pi}{4L^2} \]

Triangle Model:
Unit cell area = \( 6 \times \left[ \frac{L}{2} \times \frac{L(\tan60^\circ)}{2} \times \frac{1}{2} \right] = \frac{3\sqrt{3}L^2}{4} \]
\[ \nu_f = \frac{\pi d_f^2}{4} / \frac{3\sqrt{3}L^2}{4} = \frac{\pi}{3\sqrt{3}L'^2} \]

Numerical Examples:
A composite material consists of two blocks with identical size (Fig. B2). Each block contains fibre with uniform and constant spatial configuration and the specifications for different cases are listed in Table B1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Fibre packing</th>
<th>Fibre cell volume fraction (%)</th>
<th>Fibre packing</th>
<th>Fibre cell volume fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Hexagon</td>
<td>90</td>
<td>Hexagonal</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>Hexagon</td>
<td>70</td>
<td>Hexagonal</td>
<td>60</td>
</tr>
<tr>
<td>C</td>
<td>Hexagon</td>
<td>90</td>
<td>Hexagonal</td>
<td>60</td>
</tr>
<tr>
<td>D</td>
<td>Square</td>
<td>90</td>
<td>Hexagonal</td>
<td>60</td>
</tr>
<tr>
<td>E</td>
<td>Square</td>
<td>90</td>
<td>Square</td>
<td>60</td>
</tr>
</tbody>
</table>

Fig. B2. The cross-section of composite material
Case A

General volume fraction \( \overline{V_f} = (90\% + 40\%) / 2 = 65\% \)

Unit cell area = \( \pi r^2 / \nu_f \)

number of fibre (N) = Block area / unit cell area

\( \therefore \frac{N_1}{N_2} = \frac{\nu_f}{\nu_f} = 90\% / 40 \% = 2.25 \)

Average cell volume fraction \( \overline{V_f} = \frac{(2.25 \times 90\%) + 40\%}{3.25} = 74.6\% \)

Average reduced inter-fibre distance \( \overline{L'_{i}} = \)

\[ = \frac{2.25 \times \sqrt{\frac{\pi}{2 \sqrt{3}(0.9)}} + \sqrt{\frac{\pi}{2 \sqrt{3}(0.4)}}}{3.25} = 1.16 \]

\( L'_{\text{ideal}} = \sqrt{\frac{\pi}{2 \sqrt{3}(0.746)}} = 1.10 \)

Regularity = \( (1 - |(\overline{L'} - L'_{\text{ideal}}) / L'_{\text{ideal}}|) \times 100\% = 94.9\% \)

Case B

General volume fraction \( \overline{V_f} = (70\% + 60\%) / 2 = 65\% \)

\( \frac{N_1}{N_2} = \frac{\nu_f}{\nu_f} = 90\% / 60 \% = 1.167 \)

Average cell volume fraction \( \overline{V_f} = \frac{(1.167 \times 70\%) + 60\%}{2.167} = 65.4\% \)

Average reduced inter-fibre distance \( \overline{L'_{i}} = \)

\[ = \frac{1.167 \times \sqrt{\frac{\pi}{2 \sqrt{3}(0.9)}} + \sqrt{\frac{\pi}{2 \sqrt{3}(0.4)}}}{2.167} = 1.180 \]

\( L'_{\text{ideal}} = \sqrt{\frac{\pi}{2 \sqrt{3}(0.654)}} = 1.178 \)

Regularity = \( (1 - |(\overline{L'} - L'_{\text{ideal}}) / L'_{\text{ideal}}|) \times 100\% = 99.8\% \)

Case C

General volume fraction \( \overline{V_f} = (90\% + 60\%) / 2 = 75\% \)

\( \frac{N_1}{N_2} = \frac{\nu_f}{\nu_f} = 70\% / 60 \% = 1.5 \)
Average cell volume fraction $\bar{V}_f = \frac{(2.25 \times 90\%) + 40\%}{3.25} = 78.0\%$

Average reduced inter-fibre distance $\bar{L}' =
\frac{1.5 \times \frac{\pi}{\sqrt{2\sqrt{3}(0.9)}} + \frac{\pi}{\sqrt{2\sqrt{3}(0.4)}}}{2.5} = 1.094$

$L'_{\text{ideal}} = \sqrt{\frac{\pi}{2\sqrt{3}(0.78)}} = 1.078$

Regularity $= (1 - |(\bar{L}' - L'_{\text{ideal}})/L'_{\text{ideal}}|) \times 100\% = 98.5\%$

**Case D**

General volume fraction $\bar{V}_f = (90\% + 60\%) / 2 = 75\%$

Average cell volume fraction $\bar{V}_f = \frac{(1.5 \times 90\%) + 40\%}{2.5} = 78.0\%$

Average reduced inter-fibre distance $\bar{L}'$
\[
= \frac{1.5 \times \frac{\pi}{\sqrt{2\sqrt{3}(0.9)}} + \frac{\pi}{\sqrt{4(0.6)}}}{2.5} = 1.060
\]

$L'_{\text{ideal}} = \sqrt{\frac{\pi}{2\sqrt{3}(0.78)}} = 1.078$ (based on hexagon model)

$L'_{\text{ideal}} = \sqrt{\frac{\pi}{4(0.78)}} = 1.003$ (based on hexagon model)

Regularity = 98.3\% (based on hexagon model)

Regularity = 94.4\% (based on square model)

**Case E**

General volume fraction $\bar{V}_f = (90\% + 60\%) / 2 = 75\%$

Average cell volume fraction $\bar{V}_f = \frac{(1.5 \times 90\%) + 40\%}{2.5} = 78.0\%$

Average reduced inter-fibre distance $\bar{L}'$
\[
= \frac{1.5 \times \frac{\pi}{\sqrt{4(0.9)}} + \frac{\pi}{\sqrt{4(0.6)}}}{2.5} = 1.018
\]

$L'_{\text{ideal}} = \sqrt{\frac{\pi}{4(0.78)}} = 1.003$

Regularity $= 1 - |(\bar{L}' - L'_{\text{ideal}})/L'_{\text{ideal}}| \times 100\% = 98.5\%$
Table B2. Summary of numerical results

<table>
<thead>
<tr>
<th>Case</th>
<th>Packing</th>
<th>Block 1</th>
<th>Packing</th>
<th>Block 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>V_f (%)</td>
<td></td>
<td>V_f (%)</td>
<td>V_f (%)</td>
</tr>
<tr>
<td>A</td>
<td>Hexagon</td>
<td>90</td>
<td></td>
<td>Hexagonal</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
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<td>Hexagonal</td>
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</tr>
<tr>
<td>C</td>
<td>Hexagon</td>
<td>90</td>
<td></td>
<td>Hexagonal</td>
<td>60</td>
</tr>
<tr>
<td>D</td>
<td>Square</td>
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<td></td>
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<td>60</td>
</tr>
<tr>
<td>E</td>
<td>Square</td>
<td>90</td>
<td></td>
<td>Square</td>
<td>60</td>
</tr>
</tbody>
</table>

![Fig. B3. L' vs. v_f plot with numerical results](image-url)