EXPERIMENT AND THEORY OF A NOVEL, MULTIPLE WAVELENGTH, 
ERBIUM-DOPED FIBER LASER

by

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A thesis submitted in conformity with the requirements 
for the degree of Master’s of Applied Science 
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Electrical and Computer Engineering 
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0-612-63033-1
Abstract

Experiment and Theory of a Novel. Multiple Wavelength. Erbium-Doped Fiber Laser

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2001

For the first time, four-wave mixing and intracavity spectral slicing is shown to produce simultaneous multiple wavelength laser oscillation in an erbium doped fiber laser. A key application is multiple optical carrier generation for wavelength encoded, light-wave communication systems. The experimental prototype used active modelocking for greater peak powers and thus increased four-wave mixing within the cavity. We report here the experimental demonstration of simultaneous oscillation of 6 wavelength channels with ±5 dB variation in power and 45 GHz channel spacing.

Numerical simulations based on a split-step Fourier method are presented, agreeing qualitatively with experiment. Simulations indicate we may expect improved temporal pulse profiles with the insertion of an all-fiber, ultrafast saturable absorber.
Pour le Parcier et Maître de l’Église Métropolitaine de l’Art de Jésus Conducteur.
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Chapter 1

Introduction

1.1 Motivation

The first operating fiber optic communication link was installed in a police station in the city of Bournemouth, United Kingdom in 1975 as a lightning resistant replacement for a two-way radio [1]. Out of such humble beginnings, the fiber optics industry has burgeoned into the juggernaut behind the internet and the unprecedented economy of long distance telephony. Data transfer among internet users has on average doubled every year since 1996 [2]. Bit rates supported by fiber optic trunks are pushed to multiples of terabits per second to accommodate the growth in data traffic.

Over a quarter century since the Bournemouth link, the fiber optics industry has matured under the moniker of photonics. Functional, reliable, commercial components are available for the construction of a photonic information infrastructure incorporating long haul trunks, metropolitan area networks (MAN’s) and local area networks (LAN’s). Wavelength division multiplexing (WDM) is the prevailing multiplexing scheme in long haul trunks and MAN’s. Data channels are simply allotted slots in the optical frequency domain in WDM schemes. All optical carriers are intensity modulated with their respective data. A key benefit of WDM is that a large aggregate bit rate can be supported
while the bit rates of each channel are low enough to allow for modulation with electro-optic devices (typically 10 Gbit/s to 40 Gbit/s). Photonic components are now readily available to generate, modulate, multiplex, transport, amplify, route, demultiplex and detect optical signals as required in WDM networks.

At present, semiconductor distributed feedback lasers (DFB's) perform well as optical sources in WDM networks. However, each WDM channel requires an individual temperature stabilized DFB, resulting in highly complex and costly systems [3]. It is desirable to reduce both cost and complexity of optical sources for small-scale (~10 channel) WDM networks, such as those currently used for metropolitan exchange or those proposed for local access [4]. Doing so increases the economic viability of WDM technology, thus augmenting efforts towards proliferation of photonic infrastructure.

A robust multiple wavelength source is ideal as an alternative to an array of DFB’s. The desired properties of such a multiple wavelength source are:

- the capability to simultaneously generate multiple wavelength carriers with suitable channel spacing. A paradigmatic WDM system employs a channel grid with nanometer to sub-nanometer spacing, thus giving a similar requirement for any proposed source.

- the incorporation of wavelength stabilisation for the entire channel grid. Ideally, we wish to maintain control over only two variables, the regular channel spacing and the absolute frequency offset of the channel source with the network channel grid.

- simple fabrication procedures. Although it is difficult to quantify a complexity metric, the multiple wavelength source should be competitive with DFB rack arrays considering resources in its manufacture, installation and online operation.

There are numerous candidate WDM sources that are currently being investigated by the research community. A short list includes solid state lasers, integrated DFB's,
integrated vertical cavity surface emitting lasers, semiconductor optical amplifier lasers and erbium doped fiber lasers (EDFL's).

Not all candidates are equally attractive however. Solid state lasers, recently demonstrated with suitable wavelength spacing in Cr:forsterite [5], are bulky and typically very sensitive to the environment. Although capable of supplying more power than any other scheme, integration with fiber optic networks is not trivial. Research work has been fervent with compact, integrated DFB arrays but wavelength stabilisation must still be performed on a per channel basis [6, 7]. Integrated vertical cavity surface emitting lasers are also being actively pursued, but there is considerable effort required in their fabrication [8, 9]. Semiconductor optical amplifier lasers have been demonstrated with bulk components [10, 11] and with all fiber components [12, 13]. The limitations are again size and sensitivity for bulk optic lasers. All fiber semiconductor optical amplifier lasers are perhaps the most promising multiple wavelength sources, although there is presently limited commercial production of semiconductor optical amplifiers.

EDFL's are the candidates investigated further in this thesis. Constructed from off the shelf fiber optic components, EDFL's are obviously the most compatible technology with fiber optic networks. It is the purpose of this thesis to demonstrate a novel multiple wavelength EDFL which satisfies the above defined criteria. The on-going search for a compact, robust, economic, source of multiple carrier frequencies is the motivation for this thesis. We briefly review E DFA's here to familiarize readers with the technology dealt with throughout this thesis.

1.2 Erbium Doped Fiber Amplifiers

The first fiber laser and amplifier were demonstrated by Koester et al. [14] in 1964. The elegant design consisted of a neodymium doped silica glass fiber wound into a coil about a flash lamp. Further development of fiber lasers was hampered by difficulties in the
reliable doping of optical fibers.

The breakthrough came with the invention of an extended modified chemical vapour deposition (MCVD) technique by Poole et al. [15], which is still in use today for the fabrication of erbium doped silica fiber. The extended MCVD technique opened the way for reliable fabrication of silica doped with rare-earths [16]. The most important result for EDFL's was the new found ability to form low loss fibers with Er$_2$O$_3$ concentrations of up to >1 wt%. achieved with Al$_2$O$_3$ codoping [17]. Rapid development followed in erbium doped fiber amplifier (EDFA) technology. Integrated modules pumped with 980 nm laser diodes, the industry standard today, already began to appear by 1990 [18, 19]. Gains from 30 dB to 46 dB at 1.53 $\mu$m were readily provided [20, 21], as were 3 dB spectral gain bandwidths of 35 nm [22].

We explain here the basic operation of an EDFA 's, and refer the reader to the comprehensive treatment of Desurvire [23] for advanced topics. The prototypical EDFA is illustrated in Fig. 1.1(A). A pump beam, commonly a 980 nm laser diode source, is directed through erbium doped fiber with a wavelength dependent fiber coupler. The pump light produces an inversion in the electronic states of Er$^{3+}$ ions which varies across the length of the amplifier.

The relevant energy structure and transition rates of an Er$^{3+}$ ion within a glass matrix are illustrated in Fig. 1.1(B). The energy structure and transition rates are in fact critical to the success of EDFA technology. An inversion can be readily established in an EDFA first because of the relatively quick decay from the pump manifold $^4I_{11/2}$ to the excited manifold $^4I_{13/2}$ through multiple phonon collisions [24]. Secondly, the lifetime of the $^4I_{13/2}$ manifold is relatively long ($\sim$ 10 ms). Radiative dipole transitions at 1550 nm are in fact highly improbably for erbium ions [25], counter intuitive perhaps for an optical amplifier operating at 1550 nm. Once inversion is established, gain is provided by stimulated emission from the erbium ions over a wavelength range spanning tens of nanometers. There are many possible configurations of pump and signal directions giving different
Figure 1.1: (A) A simplified schematic of an erbium doped fiber amplifier in a counter-propagating pump-signal configuration. (B) Most significant energy levels and rates for trivalent erbium ions in a silicate glass host.

inversion profiles and thus different maximum gains, saturation powers and noise figures [23].

The width of the gain spectrum is due to numerous factors. The greatest contribution to the spectral width of the EDFA gain is the manifold nature of the levels involved in laser transitions. The erbium ions find themselves in a silica glass host which does not possess long range order, but does possess short range order. The local electric field produces a splitting in energy levels referred to as Stark splitting, giving rise to the observed manifold structure. The Stark splitting gives an approximate spectral width of 50 nm [26]. However, further complications arise because the manifolds are in fact statistical distributions of energy levels due to variations in the local environment of each erbium ion within the glass matrix. Prediction of spectral shape from first principles
is an untractable problem. and thus EDFA spectral properties are most successfully characterized with various empirical models [23].

### 1.2.1 EDFL Development

The rapid evolution of EDFA technology resulted in a concomitant development of EDFL's. Both continuous wave (CW) and Q-switching modes of laser operation were investigated for linear cavities in early years [27, 28, 29]. The availability of 30 dB EDFA gain and greater fiber optic component functionality led to a cornucopia of EDFL designs. Most notably, active modelocking and soliton pulse shaping were demonstrated [30, 31]. The high peak powers achievable with short optical pulses (sub 10 ps) resulted in significant self phase modulation due to the Kerr nonlinearity of the silica fiber itself.

Single longitudinal mode EDF ring lasers have been demonstrated with linewidths as narrow as 10 kHz [32]. Such lasers take advantage of the very high quality factors achievable in all-fiber ring resonators. On the other hand, EDF can provide gain over a 35 nm bandwidth, and thus short pulse sources can be developed. Passive modelocking in the form of additive pulse modelocking has been successfully implemented in various ways [33]. The principle behind additive pulse modelocking is nonlinear interferometry providing lower losses to higher energy pulses. The technique has proven successful and has led to the generation of pulses as short as 100 fs [34]. There are many configurations with performance ranging between the extremes mentioned.

### 1.3 Contributions of this Thesis

We pursue here an experimental and theoretical investigation of a novel multiple wavelength EDFL. The main contributions of the work here are,

- a novel concept for generating multiple wavelengths within an EDFL is proposed.

The novel feature of the technique is the combination of intracavity spectral slicing
and four-wave mixing.

- the results of a detailed experimental study of the novel multiple wavelength EDFL is performed. We present a full characterization of the EDFL, noting that pulse energy fluctuation limits the immediate utility of the configuration demonstrated here.

- a refined laser design has been proposed using an ultrafast saturable absorber. Numerical simulations indicate a superior pulse profile may be expected in experiment.

### 1.4 Organization of this Thesis

This thesis is organized into five chapters followed by an extensive appendix. The present chapter, being the first, is the prolegomenon for the work that follows. The second chapter puts the present work into context among contemporaneous research efforts. Discussions of the proposed technique, the principle of operation and physical implementation are found in the second chapter. The third chapter gives a detailed account of the experimental results from the laboratory demonstration, including time average measurements and dynamical measurements. The fourth chapter presents the methods adopted for numerical simulation of the proposed EDFL. Discussions regarding a refined laser cavity and corresponding simulation results are presented as well. Finally, the fifth chapter brings the main body of the thesis to a close with concluding remarks regarding future prospects for the demonstrated multiple wavelength EDFL. Within the appendices, one will find details which the author believes would only detract from the main arguments of the thesis with tedious longueurs.
Chapter 2

Erbium Doped Fiber Lasers

2.1 Introduction

The focus of this chapter is erbium doped fiber lasers (EDFL’s) designed for operation at multiple wavelengths. We proceed to examine a general theory regarding simultaneous laser oscillation of multiple wavelengths, arriving at the required criteria for stable operation. The technical difficulty of achieving stable multiple wavelength operation are made apparent. To put the work here in context, we provide a survey of techniques previously employed to generate multiple wavelengths in EDFL’s. Finally, the technique proposed and demonstrated in this thesis is described.

2.2 Multiple Wavelength Lasing

Development of multiple wavelength EDFL’s has been comparatively slow compared to that of other EDFL categories. The retarded development is due to the inherent difficulty in achieving stable multiple wavelength laser operation. Soon after the discovery of the laser itself, the broadening of spectral gain bandwidths was classified as homogeneous, inhomogeneous or a combination of both. As will be shown, the nature of the spectral broadening for gain media plays a key role in the stability of multiple wavelength lasers.
Homogeneous broadening is caused by each individual atom, ion or molecule having the same finite spectral width for a given optical transition. Contributing mechanisms include dipole interactions with the vacuum field\textsuperscript{1} [35] and phonon stimulated broadening [36]. Such mechanisms are more generally referred to as lifetime broadening. Each radiating entity is spectrally identical, hence the name homogeneous. Also, note that homogeneous broadening is always present although it may not be the dominant contribution to the spectral width.

Inhomogeneous broadening is caused by a redistribution of resonance frequencies among a collection of atoms, ions or molecules. Each entity interacting with the electromagnetic field is spectrally different. In solids, inhomogeneous broadening is caused by the inevitable variations in the host material structure at the local site of each ion, atom or molecule [36]. The importance in the distinction between the two broadening mechanisms is crucial because of the different saturation behaviour that results from each, as discussed further below. The saturation behaviour of an amplifying medium determines the relative ease with which stable multiple wavelength operation can be achieved.

### 2.2.1 Saturation

First, let us consider a purely homogeneously broadened medium represented as a collection of two level systems pumped externally in an unspecified way. We assume a homogeneous linewidth $\Delta \omega$, a resonant frequency $\omega_a$ and effective transition cross-section $\sigma$. The complex susceptibility $\chi(\omega)$ relating the material polarization phasor $P(\omega)$ to the electric field phasor $E(\omega)$ takes the following form\textsuperscript{2}

$$\chi(\omega) = N \chi_h^{\omega_a}(\omega) = N \frac{c}{\omega_a} \sigma \frac{1}{2(\omega - \omega_a)/\Delta \omega + i}$$

where $c$ is the vacuum speed of light and $N = N_{\text{excited}} - N_{\text{ground}}$ is the population inversion of the active medium. The optical gain provided by our prototypical amplifier is simply

\textsuperscript{1}spontaneous emission in the semiclassical theory of photons and atoms
\textsuperscript{2}refer to the Appendix D for a derivation of this result from the Bloch equations
the imaginary part of the familiar complex Lorentzian of Eq. 2.1. Saturation of the gain medium is the result of a reduction of the inversion $N$ according to [37].

$$N = \frac{N^{eq}}{1 + I/I_{\text{sat}}}$$

where $N^{eq}$ is the inversion due to external pumping in the absence of the resonant signal of intensity $I$ and $I_{\text{sat}}$ is the saturation intensity of the transition. Obviously, saturation of a homogeneously broadened medium results in the reduction of gain across the entire spectral width of the transition.

Let us now consider saturation of a medium exhibiting inhomogeneous broadening as well. Note that lifetime broadening will always be non-zero, and thus some homogeneous spectral broadening is always present. The statistical distribution of resonances, denoted $\zeta(\omega_a)$ here, is assumed normalized such that $\int \zeta(\omega_a)d\omega_a = 1$. The susceptibility $\tilde{x}(\omega)$ under saturation by an optical signal of frequency $\omega_1$ and intensity $I_1$ can be written in the form [37].

$$\tilde{x}(\omega) = N^{eq} \int_{-\infty}^{\infty} \zeta(\omega_a)S(\omega_a)\chi^{\omega_a}_{\text{h}}(\omega)d\omega_a$$

where the saturation factor $S(\omega_a)$ is given by,

$$S(\omega_a) = \frac{1}{1 + F(\omega_a, \omega_1, I_1, I_{\text{sat}})}$$

and where the depth of the saturation is determined by.

$$F(\omega_a, \omega_1, I_1, I_{\text{sat}}) = \frac{I_1}{I_{\text{sat}}} \frac{1}{1 + [2(\omega_1 - \omega_a)/\Delta \omega]^2}$$

The depth of saturation increases with saturating intensity and proximity to the saturating signal frequency, as one would expect.

Inhomogeneous broadening leads to a frequency dependent saturation\(^3\). Plots of the normalized gain spectra under varying saturation conditions are illustrated below in Fig. 2.1. Spectral hole-burning is clearly observed, a phenomenon unobserved in purely homogeneously broadened gain media. The greater the ratio of inhomogeneous broadening

\(^3\)refer to Appendix A for an analytic method to calculate the saturated susceptibility $\tilde{x}(\omega)$
to homogeneous broadening, the narrower the spectral hole. In the strongly inhomogeneous limit, the spectral holes are twice the homogeneous linewidth for weak saturation conditions. The spectral hole burning effect is reduced as the medium becomes more homogeneous. The broadening mechanisms of amplifying media must be understood in detail for any prediction of saturation behaviour.

Figure 2.1: The saturated gain, normalized to an unsaturated unit peak, is plotted above for an inhomogeneously broadened medium. The frequency is normalized to the width of the assumed Gaussian inhomogeneous distribution. The homogeneous width is $\Delta \omega = 0.01$ and the saturating frequency is $\omega_1 = -0.1$. The plots are labelled with normalized saturating powers $I_1/I_{sat}$.

It is worth discussing the nature of spectral broadening in EDF. The homogeneous linewidth of typical germano-alumina-silica based EDF at room temperature is approximately $\sim 10$ nm [38]. Although moderately well approximated as a homogeneous medium in optical signal amplification applications [39], deep saturation results in complex spec-
tral hole burning behaviour [40. 41. 42. 43. 44]. Our two-level assumption is a simplification of the case for EDF. As noted earlier, there is a multiplicity of electronic energy levels giving a ground state manifold and excited state manifold. To date, there does not exist a tractable theory allowing one to calculate the contributions to the susceptibility from each transition among the manifolds, even for crystalline hosts. Interaction between electron wavefunction configurations are forbidding due to the sheer size of the many-electron atom problem [45. 46]. An empirical model was developed by Desurvire [47] to overcome the mathematical difficulties of a first principles treatment. The requisite Stark split energy levels have been measured through low temperature spectroscopy [26. 48], but the requisite phenomenological dipole moments of each transition have yet to be measured. Presently, the homogeneously broadened two-level approximation is the best available for EDFL simulations.

2.2.2 Stability Criterion

The conditions under which stable laser oscillation are expected to ensue were investigated early in the development of lasers by Lamb [49]. The criterion for stable dual wavelength, continuous wave oscillation was determined by Lamb through linearization of the gain saturation in the rate equations for a laser cavity with two modes,

\[
\begin{align*}
\frac{dI_1}{dt} &= \frac{g_1 I_1}{1 + \kappa_{11} I_1 + \kappa_{12} I_2} - \alpha_1 I_1 \approx I_1 [g_1 (1 - \kappa_{11} I_1 - \kappa_{12} I_2) - \alpha_1] \\
\frac{dI_2}{dt} &= \frac{g_2 I_2}{1 + \kappa_{21} I_1 + \kappa_{22} I_2} - \alpha_2 I_2 \approx I_2 [g_2 (1 - \kappa_{21} I_1 - \kappa_{22} I_2) - \alpha_2]
\end{align*}
\]  

(2.6)

where \( t \) is time, \( I_1 \) and \( I_2 \) are the optical intensities at two different frequencies, \( g_j \) are the optical gains, \( \alpha_j \) are the optical loss, and \( \kappa_{jk} \) are the gain saturation coefficients. Neglecting wavelength dependent loss (\( \alpha_1 = \alpha_2 \)), the criterion for stable laser oscillation was found to be that of weak beam coupling [49],

\[
\kappa_{21} \kappa_{12} < \kappa_{11} \kappa_{22}
\]  

(2.7)
In other words, the cross saturation of gain must be less than the self saturation of gain in order to achieve stable oscillation. In view of the previous discussion pertaining to homogeneous and inhomogeneous gain broadening, one readily concludes that the spectral hole burning effect results in weak coupling and thus supports stable multiple wavelength oscillation. The width of a spectral hole, proportional to the homogeneous linewidth to first order, determines the order of magnitude of the minimum spacing of stable multiple wavelength oscillations. Once within a homogeneous linewidth, gain cross saturation approaches gain self saturation for the two oscillating frequencies. Instability arises because any wavelength dependent loss can upset the fine balance achieved between the two wavelengths competing for gain through cross saturation. A completely homogeneous gain medium will not support stable multiple wavelength operation. However, any mechanism which reduces net gain cross saturation below net self saturation can induce stability. Thus, although the homogeneous linewidth imposes a limit on the minimum separation for stable oscillation, this can be overcome with external methods to reduce effective gain cross saturation.

2.3 Multiple Wavelength Fiber Lasers

The 10 nm homogeneous linewidth of EDF and a desired channel spacing of less than 10 nm necessitates the use of another physical mechanism for the reduction of net gain cross saturation to allow stable operation. Research in the field is fervid and numerous stratagems have been proposed and demonstrated with varying degrees of success. The techniques previously employed can be categorised roughly as follows 4:

- gain equalization
- cooling

---

4we do not report here methods obtaining simultaneous laser oscillation with 8 nm channel spacing or greater
• spatial multiplexing
• polarization multiplexing
• stimulated Brillouin scattering
• frequency shifting
• extracavity spectral slicing

In the following paragraphs, we briefly describe the principle used in each technique to overcome the limit imposed by a 10 nm room temperature homogeneous linewidth. A summary of experimentally demonstrated results is given.

**Gain Equalization**

Multiple frequencies will compete for gain in the erbium gain medium. Each frequency saturates the gain, with the result that the frequency with the highest unsaturated net gain (gain minus loss) will experience the greatest amplification and eventually saturate the gain below the loss level for all other frequencies. This phenomenon is referred to as gain clamping. Gain equalization is the technique by which one adjusts the loss of each channel such that no single frequency will experience preferential net amplification per round trip. As with all other techniques below, the channel spacing is determined by an intracavity filter unless otherwise specified. Experimental demonstration of this technique showed a minimum 4.8 nm channel spacing could be obtained [50, 51, 52]. The gain equalization technique is sensitive to cavity perturbations because only a slight wavelength dependent loss can upset the balanced amplification of each channel undergoing laser oscillation.

**Cooling**

The homogeneous linewidth of erbium doped fiber exhibits an approximate $T^{1.73}$ temperature dependence down to 20 K [38]. Cooling erbium doped fiber to 77 K with readily
available liquid nitrogen reduces the homogeneous linewidth to approximately 1 nm. Experimental demonstration of the technique [53, 54] has shown its utility in increasing the stability of multiple wavelength EDFL’s. In fact, cooling is often used in conjunction with other techniques listed here to increase output power stability. However, the use of liquid nitrogen limits the practical use of this technique outside research laboratories.

**Spatial Multiplexing**

Spatial multiplexing includes all schemes in which different frequency channels access spatially distinct erbium ions for gain. The most mundane and costly approach is to force each frequency to access separate EDFA’s for individual amplification [55, 56]. An elegant implementation of the technique is the use of dual core erbium doped fiber [57]. The oscillation of optical power between the cores of the fiber is frequency dependent. Thus the overlap of optical signals with disparate frequencies is reduced in the amplifying cores. A similar reduction in overlap can be achieved through the use of a miniature, highly doped fiber\(^5\) as a standing wave resonator [58]. Again, disparate frequencies experience a reduced overlap within the amplifying medium and thus a reduction in gain competition is effected. An 0.4 nm wavelength separation can be achieved. The primary limitation of this technique is the required fabrication of novel erbium doped fiber structures.

**Polarization Multiplexing**

Polarization multiplexing is based upon the phenomenon of polarization dependent gain cross saturation, often referred to as polarization hole burning [59]. Following amplifier saturation by a pulse of definite polarization, a probe pulse will experience more gain when orthogonally polarized. This phenomenon was used to reduce gain competition

\(^5\) the high doping concentration was achieved by co-doping with ytterbium, which did not dramatically change the homogeneous linewidth
between frequencies by wrapping erbium doped fiber around a spool to induce birefringence: a 1.1 nm channel spacing was obtained [60]. Other mechanisms such as spectral hole burning due to gain inhomogeneity were suggested to aid the multiple wavelength operation.

**Stimulated Brillouin Scattering**

Spontaneous Brillouin scattering is the process by which light is back scattered by an acoustic wave within fiber [61]. Stimulated Brillouin scattering (SBS) occurs when sufficient Brillouin scattered light interferes with the incident light, resulting in the generation of more acoustic waves and thus increased Brillouin scattering. SBS results in a frequency shift dependent upon the frequency of the acoustic wave. Typical frequency shifts are on the order of 10 GHz [61]. Employing two EDFA's, 53 channels with a 0.08 nm channel spacing were generated through feedback of successive signals generated by SBS [62]. The problem of gain competition is avoided here because of the amplification provided through the SBS process within undoped fiber. Of course, SBS provides no control over the channel spacing, since it is determined by acoustic wave frequencies within the fiber.

**Frequency Shifting**

Frequency shifting by an externally driven acousto-optic modulator finds application in generating multiple wavelengths from a single EDFL. Experimental demonstrations have utilized a frequency shift below that of the channel spacing by a factor of at least 1000 [63, 64]. With sub-channel frequency shifting, the laser is operating in a dynamic equilibrium. Optical energy is continuously being shifted in one frequency direction among the cavity modes, preventing any single channel from clamping the gain for all other channels. The spectral width of each channel was verified to be less than that of the defining intracavity spectral filter, indicative of laser oscillation. However, a quantitative theoretical explanation of this technique is yet to be given.
Extracavity Spectral Slicing

Extracavity spectral slicing is the technique in which broadband optical power is spectrally filtered external to the laser cavity. A pulsed mode of operation is required to generate the several nanometers of bandwidth required through nonlinear effects within fiber. Pulses from sub-picosecond fiber lasers have been filtered with wavelength demultiplexers for multiple wavelength output [65]. Nonlinear pulse compression and supercontinuum generation have also been used to generate sufficient bandwidth for spectral slicing [66, 67]. The contrast between peak to trough in the output power spectral density is limited by that of the extracavity filter. Furthermore, extracavity spectral slicing rejects a significant fraction of the output power, reducing the final power efficiency of the source.

Summary of Techniques

A comparison of the aforementioned techniques is found in Table 2.1. The minimum channel spacing and corresponding number of channels are indicated.

<table>
<thead>
<tr>
<th>Method</th>
<th>Channel Spacing</th>
<th>Number of Channels</th>
<th>cw / pulsed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Equalization [50]</td>
<td>4.8 nm</td>
<td>6</td>
<td>cw</td>
</tr>
<tr>
<td>Cooling [53]</td>
<td>0.65 nm</td>
<td>11</td>
<td>cw</td>
</tr>
<tr>
<td>Spatial Multiplexing [57]</td>
<td>0.5 nm</td>
<td>8</td>
<td>cw</td>
</tr>
<tr>
<td>Polarization Multiplexing [60]</td>
<td>1.1 nm</td>
<td>7</td>
<td>cw</td>
</tr>
<tr>
<td>SBS [62]</td>
<td>0.08 nm</td>
<td>53</td>
<td>cw</td>
</tr>
<tr>
<td>Frequency Shifting [63]</td>
<td>0.8 nm</td>
<td>14</td>
<td>cw</td>
</tr>
<tr>
<td>Extracavity Spectral Slicing [66]</td>
<td>0.08 nm</td>
<td>150</td>
<td>pulsed</td>
</tr>
</tbody>
</table>

Table 2.1: Comparison of techniques for multiple wavelength generation in EDFL's
2.4 Proposed Technique

We wish to combine the desirable properties of previously mentioned techniques, namely the environmental stability of extracavity spectral slicing and the high peak to trough contrast of the other techniques with intracavity filters. The stratagem proposed here is to use four-wave mixing (FWM) to couple optical power between multiple wavelengths defined by an intracavity spectral filter. A schematic of the pulsed laser demonstrated in this work is illustrated below in Fig. 2.2. We first discuss how FWM, intracavity spectral slicing and active mode locking work together in the demonstrated laser to achieve multiple wavelength oscillation. We finally discuss the propagation of a pulse around the laser cavity to illustrate the significant effects at play.

![Figure 2.2: A schematic of the laser demonstrated in this body of work.](image-url)
2.4.1 Four-Wave Mixing

A classical model of FWM is the mixing of three electromagnetic waves in a medium by virtue of a third order susceptibility tensor $\chi^{(3)}$ according to.

$$P_{NL} = \epsilon_0 \chi^{(3)}:E_EE$$ (2.8)

where $\epsilon_0$ is the vacuum permittivity, $P_{NL}$ is the nonlinear material polarization and $E$ the electric field. The resulting material polarization oscillates at frequencies given by all possible signed sums of the incident frequencies to drive a fourth electromagnetic wave [68]. A special case of degenerate (or nearly degenerate) FWM is the optical Kerr effect, namely an intensity dependent refractive index. The optical Kerr effect results in what is known as self phase modulation for obvious reasons. Third order nonlinearity typically arises from the nonresonant, optical driving of electrons in anharmonic potential wells within the medium of interest.

We illustrate here how FWM acts to reduce effective cross-saturation with a simple example. Let us consider a laser with a homogeneously broadened gain medium operating at a single frequency $\omega_p$ due to gain clamping. We define the respective electric field,

$$E_p = \frac{1}{2}e [E_p \exp(-i\omega_pt) + E_p^* \exp(i\omega_pt)]$$ (2.9)

where $e$ is a unit vector defining the polarization state and $E_p$ is the complex electric field phasor including all spatial dependency. In the absence of any nonlinearity other than gain saturation, the electric field would continue to oscillate at a single frequency.

If now the cavity is assumed to possess a $\chi^{(3)}$ nonlinearity, FWM will occur. Let us call our original single frequency electric field the "pump". A full quantum mechanical treatment is required for us to observe how light at other frequencies will build up. We take the result here that spontaneous emission from the gain medium and vacuum field fluctuations themselves will result in broadband optical noise [35]. Let us consider noise $E_c$ at frequency $\omega_c = \omega_p + \delta$, which we will call the "conjugate" field. The
nonlinearity within the cavity will result in a material polarization with many terms of different frequency dependence. We are interested in polarization terms of the form 

\[ \epsilon_0 \chi^{(3)} E_p E_p E_c^* \exp(-i\omega_s t) \]

where \( \omega_s = \omega_p - \delta \). The oscillating material polarization will thus drive a "signal" electric field \( E_s \) at frequency \( \omega_s \). The signal and conjugate are in fact interchangeable here, and each will lead to the growth of the other. One may think of FWM as the annihilation of two pump photons leading to the creation of a signal photon and a conjugate photon.

The effect of FWM is now clear. A wavelength pump channel will divert power to adjacent wavelength channels via FWM. The gain medium is saturated by the pump so as to provide insufficient gain to other wavelengths for laser oscillation, but gain proportional to pump power is provided through the FWM mechanism. Thus, FWM reduces the effective gain cross saturation and aids us in achieving multiple wavelength laser oscillation. Furthermore, a regular channel spacing enforced with an intracavity filter will facilitate the FWM process. Any channel with sufficient power will serve as a pump beam for adjacent signal and conjugate channels.

The medium that mediates the FWM process within the demonstrated laser cavity is the silica fiber itself. Pulsed operation and propagation over extended lengths can be used to increase the total FWM generated. Fiber nonlinearity \( (\chi^{(3)} = 6 \times 10^{-15} \text{cm}^2/\text{erg} \) [68]) is sufficiently large that FWM can provide gain over a 200 GHz bandwidth with a 1 W continuous wave pump in standard telecommunications fiber, resulting in an effect known as modulation instability [69].

### 2.4.2 Active Modelocking

To obtain the greatest nonlinearity possible, and thus maximize FWM, the demonstrated laser is operated in pulsed mode. The pulse propagation direction is fixed to be clockwise through the use of an isolator within the EDFA. With the same average power provided by the EDFA to the laser cavity, the peak power of a pulse increases in proportion
to $T/\Delta t$, where $T$ is the round trip time and $\Delta t$ the pulse width. To achieve pulsed operation, an active modelocking technique is employed here.

The amplitude modulator responsible for modelocking the EDFL is a LiNbO$_3$ electro-optic (EO) modulator in a Mach-Zehnder interferometer configuration. An applied voltage controls the optical transmission through the modulator by inducing a phase shift in one arm of a Mach-Zehnder interferometer via the electro-optic effect. To obtain a single pulse circulating within the cavity at a time, we apply a voltage pulse to the EO modulator with a repetition rate equal to the fundamental cavity resonance frequency (the frequency with which an optical pulse traverses the laser cavity once). A transmission window is opened at the precise time an optical pulse arrives at the EO modulator each round trip. The process is self-starting because noise which passes through the transmission window will experience less loss than noise which does not. The circulating pulse experiences minimum loss and will thus be the preferred mode of operation. The formation of a pulse train periodic in time within the cavity is equivalent to locking the phase between the longitudinal cavity modes, hence the name modelocking.

### 2.4.3 Intracavity Spectral Slicing

We have discussed how FWM will provide gain through fiber nonlinearity to counter gain cross-saturation. However, an intracavity spectral slicing filter is required to define the wavelength channels. The filter we employ is a Lyot filter composed of highly birefringent fiber, polarization controllers and the electro-optic (EO) modulator previously discussed. We use the modulator here as a polarization state analyzer. The modulator is implemented with planar waveguides which are designed to allow the propagation of a single polarization only.

The Lyot filter operates as follows. Linearly polarized light from the modulator impinges on a polarization controller. This polarization controller, which can perform arbitrary polarization transformations, is adjusted to launch light with linear polarization
\[ \frac{\pi}{4} \text{ radians from the principal axes of the highly birefringent fiber. As light propagates down the birefringent fiber, the polarization state evolves as in Fig. 2.3.} \]

Figure 2.3: Polarization rotation in a birefringent fiber is shown above, after Agrawal. The polarization evolves with a spatial period known as the beat length.

The rate at which polarization state evolves is determined by the accumulation of phase difference \[ \phi = \omega \Delta n L / c, \] where \( \omega \) is the optical radian frequency, \( \Delta n \) is the birefringence of the fiber, \( c \) is the vacuum speed of light and \( L \) is the fiber length. The polarization rotation is frequency dependent, and thus we can perform spectral slicing with a polarization analyzer at the terminal end of the birefringent fiber. A polarization controller at the terminal end is thus adjusted to set the polarization state which will experience minimal loss through our analyzer, the EO modulator. The polarization controllers are used in the laser as a means to achieve the desired alignment between the principle axes of the birefringent fiber and the transmitting axis of the EO modulator.

A detailed analysis of the Lyot filter with arbitrary polarization controller settings is presented in Appendix B. We present some general results here, and leave the reader to the appendix for details. The transmission function for the electric field phasor of an optical signal at carrier frequency \( \omega \) is

\[ H(\omega) = ab + \sqrt{(1-a^2)(1-b^2)} \exp[-i(2\pi\omega/\Delta\omega + \Phi)] \]  \hspace{1cm} (2.10)\]

\footnote{\textsuperscript{6} We denote the transmission function by \( H(\omega) \) rather than \( t \) here to avoid confusion with the time variable}
where \( a, b \) and \( \Phi \) are constants determined by polarization controller setting. The undesirable situation of frequency independent transmission \((a = b = 1\ or\ a = b = 0)\), occurs when light is launched or analyzed along a principle axis of the birefringent fiber. The free spectral range (FSR) of the Lyot filter is given in Hz by.

\[
\Delta \nu = \frac{c}{\Delta n L}
\]  

(2.11)

The time-domain impulse response of the Lyot filter is given by,

\[
h(t) = 2\pi ab \delta(t) + 2\pi \exp(-i\Phi) \sqrt{(1 - a^2)(1 - b^2)} \delta(t - 1/\Delta \nu)
\]

(2.12)

where \( \delta(t) \) is the Dirac delta function. An incident pulse thus produces two output pulses with a delay \( \tau = 1/\Delta \nu \) in between them. The first (second) pulse corresponds to the portion of the pulse which traversed the fast (slow) axis of the birefringent fiber. We discuss the importance of spectral-temporal relationships further below.

There are several attractive features of the Lyot filter as a spectral slicing filter. The FSR is slightly adjustable through fiber birefringence and length, and can be finely tuned by bending the fiber for stress induced birefringence [70]. For a typical 10 m fiber section, the FSR can be tuned by approximately 3 GHz with a birefringence tuning of \(10^{-6}\), as could be induced with a bend radius of about 3 cm for a typical fiber [70]. Another desirable feature is that the FSR of the Lyot filter exhibits negligible frequency dependence over the wavelength span of interest. The FSR depends upon the birefringence, and thus only dispersion of the birefringence itself contributes to a variation in FSR with frequency. The high birefringence of the fiber used in this work, a so called “bow-tie” fiber, is due to a permanent stress induced birefringence within the fiber core [71]. The frequency dependence of silica photoelasticity over a 10 nm bandwidth is negligible. One can readily keep the FSR variation to below 500 MHz over a 10 nm bandwidth, that is, 1 part in 100 for a prototypical WDM channel spacing of 50 GHz.

\[\text{we assume here a permanent fiber birefringence of approximately } 10^{-4}, \text{ a representative value for the work here}\]
Given the uniformity of the Lyot filter response, one would only be required to monitor and control two degrees of freedom for wavelength stabilisation in a system application: channel grid spacing and channel grid bias. The bias point and contrast ratio of the Lyot filter can be adjusted through the polarization controllers. The Lyot filter is thus excellent for intracavity spectral slicing.

2.4.4 Laser Operation

We trace a pulse propagating through the cavity to give further insight into the operation of the demonstrated laser. We begin with a linearly polarized pulse entering the EDFA. We assume that numerous wavelength channels add to give our single pulse.

 Upon propagating through the EDFA, the wavelength channel at the gain peak of the EDFA will be amplified more than adjacent channels. A fraction of the pulse energy is diverted to the output of the laser by an all fiber coupler and the remainder is kept within the cavity. While traversing the fiber ring, FWM occurs within the fiber to produce two effects. First, FWM provides gain to lower power channels as described earlier. Second, pulsed operation implies a finite spectral width for each wavelength channel and thus FWM acts to broaden the spectrum of each channel itself. In the absence of spectral filtering, we would obtain a pulsed broadband output from the EDFL.

The pulse then propagates through the Lyot filter elements. As described, the Lyot filter produces two weighted replicas of the incident pulse with a delay $\tau = 1/\Delta \nu$ between them. The Lyot filter narrows the width of each wavelength channel. The result of repeated spectral filtering of a short pulse, as would be incurred through several round trips, is illustrated in Fig. 2.4. One can readily see that spectral broadening by FWM will be balanced by the spectral filtering of the Lyot filter. The periodic temporal structure enforced by repeated filtering is seen in the experimental and numerical work reported later in this thesis.

Finally, the pulse must pass through the EO modulator. It will be noticed that
we have in fact two round trip times through the ring cavity corresponding to the fast and slow paths through the birefringent fiber. When the modulator driving voltage resonates with the fast axis, a portion of the pulse will always arrive late for the modulator transmission window. The pulse energy will thus accumulate at the falling edge of the transmission window. If resonating with the slow axis, pulse energy will accumulate on the rising edge. The attenuation of the portion of the pulse which takes the "wrong" path through the cavity increases the effective cavity loss. Furthermore, the accumulation of pulse energy at the edge of the modulator window brings about sensitivity to timing jitter and pulse shape in the voltage signal driving the EO modulator. Discussions regarding these issues are presented with the experimental and numerical results of Chapters 3 and 4, respectively. The pulse, once shaped by the modulator transmission window, emerges with linear polarization at the incident side of the EDFA. We have followed a pulse over a round trip.
2.5 Summary

We have given a brief theoretical survey of relevant physical effects in multiple wavelength lasers. In particular, we have reviewed the nature of gain saturation and the stability criterion for multiple wavelength laser oscillation. We have also presented a brief review of techniques demonstrated experimentally by other groups to achieve multiple wavelength operation of EDFL's. No single technique presented itself as an outstanding practical solution for multiple wavelength generation for an WDM optical network.

We have proposed a novel technique to induce stable multiple wavelength oscillation in an EDFL. The role of FWM in reducing gain cross-saturation was explained in detail. A description of the operation of the proposed EDFL structure was given, highlighting the importance of FWM, active modelocking and intracavity spectral slicing. The number of effects at play and the nonlinear nature of EDFA gain saturation and FWM motivates the numerical simulations presented in Chapter 4 for a quantitative theoretical treatment.
Chapter 3

Experimental Demonstration

3.1 Introduction

The novel fiber laser was constructed and characterized experimentally in two configurations. We present the time average spectral and temporal characterization of the laser pulses under different operating conditions. The time average spectra of the pulsed laser output are encouraging. Measurement of the pulse substructure agrees well with that expected from the spectra, although imperfect modelocking was found. Finally, statistical measurements of the output pulse energy are reported to quantify the wave noise inherent to the laser.

3.2 Laser Implementation

Two lasers were constructed according to Fig. 2.2 for the experiments reported here. Both cavities were identical apart from the EDFA used. The details regarding common cavity elements are described first.

All fiber components were connected to each other with angle cleaved, physical contact connectors where possible to avoid back reflections. The exceptions were at the input and output ends of the EDFA’s, where standard physical contact connectors were used.
The output coupler was a commercial (ExB) 10:90 fused-fiber coupler. Both polarization controllers were based on commercial rotation paddles (Thor Labs FPC560) wound with loops of industry standard single-mode fiber (Corning SMF-28). The three paddles of each controller were wound with 2, 4 and 2 loops of fiber to best approximate the 1/4 waveplate, 1/2 waveplate and 1/4 waveplate combination that can perform arbitrary polarization transformations.

The highly birefringent fiber used was a 12.4 m segment of commercial “bow-tie” fiber (Newport F-SPPC-15) fusion spliced to SMF-28 segments with angle cleaved, physical contact connectors. The birefringence was found by constructing a Lyot filter with the fiber segment and fiber coupled polarizers. The Lyot filter transmission was measured with an optical spectrum analyzer and broadband spontaneous emission source. The fiber birefringence corresponding to the Lyot filter FSR of 45.2 ± 0.5 GHz is $5.35 \times 10^{-4} \pm 0.01 \times 10^{-4}$.

The commercial EO modulator used was a LiNbO$_3$ Mach-Zehnder interferometer implemented with planar waveguides (Uniphase S5-150-1-1-C2-P1-AP). The insertion loss at maximum transmission was measured to be approximately 6 dB. The on/off extinction ratio was specified as 24.5 dB with an RF electrode $V_\pi$ of 3.0 V. The 3 dB bandwidth of the RF modulation response was specified as 3.4 GHz. A DC supply was used to bias the EO modulator to null transmission. The RF source used to drive the EO modulator consisted of two parts. A frequency synthesizer (Hewlett-Packard 8656B) was used to trigger a short pulse generator (Avtech AVMN-1-PS-UTI) capable of generating sub-picosecond pulses. The pulse generator was specified with a trigger to output timing jitter of 15 ps RMS, a rise time of less than 100 ps and a fall time of less than 200 ps. For the experimental results reported here, an approximately square pulse of duration 1.01 ns and peak amplitude 4.48 V was used. Direct measurement with a digital sampling oscilloscope (Tektronix TK11805) was used to adjust the pulse source to give the shortest pulse possible with a symmetric square shape.
As mentioned previously, two different EDFA's were used in the laser cavity for the experiments reported here. The first EDFA was used only for the test between laser oscillation and spontaneous emission. This EDFA was constructed within the laboratory according to the diagram in Fig. 3.1. An optical circulator (JDS Fitel CR5500-3P) defined the direction for the signal to traverse the EDFA. The EDF was a 7.5 m segment from Institute National d'Optique with a doping of approximately 790 ppm by weight of Er$_2$O$_3$, a numerical aperature of 0.22 and an effective confinement factor of 0.5. A wavelength dependent coupler (JDS Fitel WD 915-T4-A) directed pump light from a 980.30 nm semiconductor laser diode (SDL 2564-145-BN) into the EDF. A loop mirror was constructed from a third polarization controller and a 50:50 fused fiber coupler (ExB). The loop mirror with polarization controller served to reflect the signal for a second pass through the amplifier. The exceedingly short strand of EDF did not provide sufficient gain with a single pass, and thus the double pass configuration was employed. The unsaturated gain was measured to be 15 dB when pumped with 74 mW from the semiconductor laser diode. Apart from physical contact connectors at the 50:50 coupler, all fiber connections were made with fusion splices.

The EDFA used for the remaining experiments reported here was a commercial unit (Corning FGM-S-045-01). An unsaturated gain of 28.2 dB at a wavelength of 1535 nm was provided. The gain spectrum was limited by a 5 nm FWHM filter within the EDFA.

3.3 Lasers versus Spontaneous Emission Sources

We consider this question to distinguish the behaviour of our source from others [63] where the distinction between laser oscillation and amplification of spontaneous emission was less clear and the subject of debate. To determine experimentally whether a source is a laser or a spontaneous emission source, one need only observe the presence of a threshold optical pump power. Below threshold, the spontaneous emission is so low that
Figure 3.1: The double-pass EDFA constructed for initial experiments with the multiple wavelength laser.

The there is negligible probability of photon feedback and stimulated emission. Above threshold, the spontaneous emission rate is great enough that there is an increase of photon density and stimulated emission eventually becomes significant. The stimulated emission rate will dominate over spontaneous emission and the optical gain will finally saturate to equal the photon loss rate. Detailed treatments of the onset of laser oscillation are many, and Loudon's [35] is but one excellent treatment of this fundamental subject.

A plot of output power versus EDFA pump power from the 980.30 nm semiconductor laser diode is given in Fig. 3.2. The pump power was determined by noting the input pump current and using a supplied calibration curve for the laser diode. Cavity losses were not minimized in the laser, but a threshold is clearly visible. The multiple wavelength source is indeed a laser.
Figure 3.2: The output power versus the pump power for the proposed fiber laser with a low gain (~15 dB) EDFA. Note the threshold indicative of the onset of laser oscillation.

3.4 Power Spectral Density

The time average power spectral density of the pulsed laser output was investigated. The FWM mechanism for spectral broadening is power dependent and thus spectra were observed with different peak intracavity pulse energies. The intracavity power was varied by controlling the EDFA pump power and thus the EDFA saturation power. The pulse energy was determined through measurement of the output power with an Si photodetector (Newport 818-IR). A simple calculation requiring knowledge of the output coupling and pulse repetition rate gives the peak intracavity pulse energy. A summary of some laser parameters is presented in Table 3.1.

Power spectral density measurements of the laser output were performed with an ANDO AQ6317 optical spectrum analyzer. The spectra are illustrated below in Fig. 3.4 for six different intracavity pulse energies. The channel spacing was measured to be 360±5 pm (45.0±0.6 GHz). The change in spectral shape with peak intracavity pulse energy is that expected from fiber nonlinearity. The triangular shape of the spectrum on a logarithmic scale is consistent with a sech² function. The sech function is self Fourier
Table 3.1: Parameters of the fiber laser used to obtain spectral, temporal and statistical measurements. The RF repetition rate was tuned for resonance with the fast axis path. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsaturated EDFA gain</td>
<td>28.2 dB</td>
</tr>
<tr>
<td>Lyot filter FSR</td>
<td>45.2 GHz</td>
</tr>
<tr>
<td>Output coupling loss</td>
<td>0.5 dB</td>
</tr>
<tr>
<td>EO modulator loss</td>
<td>6 dB</td>
</tr>
<tr>
<td>Connection losses</td>
<td>0.5 dB</td>
</tr>
<tr>
<td>RF pulse width</td>
<td>1.01 ns</td>
</tr>
<tr>
<td>RF repetition rate</td>
<td>2.9015 MHz</td>
</tr>
</tbody>
</table>

Transforming [72], implying a hyperbolic secant shape in field envelope. Such a shape is that of pulses experiencing dispersion and self phase modulation, the omnipresent solitons [73].

With 40 pJ of peak intracavity pulse energy, the laser is operating just above threshold and the spectrum is the narrowest observed. With increasing pulse energies, two trends are observable. The envelope of the spectrum broadens and each channel undergoes spectral broadening, in accordance with the discussion of laser operation in Chapter 2. With 1902 pJ peak pulse energies, the channel 3 dB widths were 60±5 pm. Allowing for a 5 dB variation in power, we observe 6 wavelength channels oscillating simultaneously.

3.5 Temporal Characterization

From spectral measurements we may conclude that fine substructure is present in the pulsed output, as expected from our discussion in Chapter 2. A total spectral width exceeding 2 nm corresponds to a sub-picosecond pulse width assuming a Fourier transform-limited time-bandwidth product [74]. In order to resolve pulse substructure on the pi-
Figure 3.3: Output power spectral densities measured with an optical spectrum analyzer with a resolution bandwidth of 10 pm. The abscissae are wavelengths in units of nm and the ordinates are optical powers in units of dBm. The peak intracavity pulse energies are: A. 40 pJ; B. 255 pJ; C. 510 pJ; D. 989 pJ; E. 1268 pJ; and F, 1902 pJ.
cosecond scale, the autocorrelation technique is used. We briefly discuss the technique and we report key features of the autocorrelator constructed. We then discuss the observed autocorrelation traces.

The purpose of the autocorrelation technique is to allow the resolution of temporal features of optical pulses that are beyond the limits of direct photodetection. High-speed photodetectors are typically limited to response times of \( \sim 100 \) ps due to photocarrier transit times [75]. The technique used here is that of background free, non-interferometric second harmonic generation (SHG). We briefly discuss this technique here: a more general account of autocorrelation is given by Ippen et al. [74].

**Autocorrelation by Second Harmonic Generation**

A schematic of the autocorrelator constructed is illustrated in Fig. 3.5. The principle of operation is as follows. An incident pulse train is split into two equal intensity trains. An optical trombone is used to vary the delay between the two pulse trains as they are focussed onto a crystal possessing large second order nonlinearity characterized by a second order susceptibility \( \chi^{(2)} \). Three second harmonic beams are generated with comparable efficiency if the incident beams are aligned with the crystal to match wavevector phase. The details of phase matching are given in Appendix C. Two second harmonic beams are generated by the material polarization excited by each beam individually. The other second harmonic beam is the result of material polarization excited by the interaction of the two incident beams in the \( \chi^{(2)} \) crystal. Using an aperture located appropriately, the SHG resulting from the overlap of both incident beams is selected and detected with a highly sensitive photodetector such as a photomultiplier tube.

There are four notable features of the autocorrelator constructed for fiber laser characterization:

- the scanning length of the optical trombone is approximately 10 cm, which allows for a delay range of approximately \( \pm 300 \) ps. The trombone consists of a retrore-
Figure 3.4: A schematic of the autocorrelator designed and built to characterize the pulse substructure of the experimental fiber laser.
flector mounted upon a translation stage with stepping motor control. A minimum 10 \( \mu \text{m} \) resolution in position gives a minimum 66 fs resolution in the autocorrelation trace. The trombone was chosen with parameters to allow for measurement of sub-picosecond structure on pulses that were measured to be somewhat wider than 100 ps via direct photodetection.

- The crystal used for SHG is commercially available LiIO\(_3\) (Femtochrome) with an anti-reflection coating. The 1mm thick crystal was cut with a 30° angle between crystal axis and normal. Type I [61] phase matching was employed, for which a detailed calculation appears in appendix C. Phase matching is polarization sensitive, and thus a polarization controller was required at the entrance to the autocorrelator to ensure correct linear polarization of the fundamental beams within the LiIO\(_3\) crystal.

- An optical chopper and lock-in amplifier (Stanford Research Systems 830DSP) are present. The total photocurrent shotnoise is proportional to the square root of the electrical measurement bandwidth [76]. The optical chopper and lock-in amplifier were used to reduce the electrical measurement bandwidth to 0.3 Hz, thereby significantly increasing the electrical signal to noise ratio.

- The photomultiplier tube is a Hamamatsu model R928, which has a spectral response extended to the near infra-red. The quantum efficiency at the second harmonic wavelength of interest (765 nm) is 4%, while at the fundamental (1530nm) there is a negligible quantum efficiency.

We now give a quantitative description of the SHG detected, referred to as the autocorrelation trace. The greater the pulse coincidence within the \( \chi^{(2)} \) crystal, the greater the second harmonic. The second harmonic generated in the present scheme will be of a relative intensity \( S(\tau) \) given by [74].

\[
S(\tau) \propto g^{(2)}(\tau) + r(\tau)
\]  

(3.1)
where \( \tau \) is the time delay between the fundamental signals at the \( \chi^{(2)} \) crystal. The first term contributing to the second harmonic is the second order temporal coherence.

\[
g^{(2)}(\tau) = \frac{\int E^\ast(t)E^\ast(t + \tau)E(t + \tau)E(t)dt}{\int E^\ast(t)E^\ast(t)E(t)E(t)dt}
\]

(3.2)

where \( E(t) \) is the complex electric field at time \( t \) and the integration time is determined by that of the measurement system bandwidth. The second term of Eq. 3.1 is a rapidly oscillating function of delay time \( \tau \) due to interference of the two fundamental beams. Only the average of \( r(\tau) \), which is zero, is actually detected because the interference pattern of the two fundamental beams is spatially averaged over the entire volume of non-collinear overlap within the crystal. Lock-in amplification gives a measurement integration time of 300 ms resulting in an averaging of over one million pulses. Thus, the autocorrelation trace will be proportional to the ensemble average of the second order temporal coherence integrated over a single output pulse:

\[
g^{(2)}(\tau) = \frac{\int \langle E^\ast(t)E^\ast(t + \tau)E(t + \tau)E(t) \rangle dt}{\int \langle E^\ast(t)E^\ast(t)E(t)E(t) \rangle dt} = \frac{\int \langle I(t)I(t + \tau) \rangle dt}{\int \langle I(t)I(t) \rangle dt}
\]

(3.3)

The pulse shape cannot be extracted from the autocorrelation trace because all field phase information is lost, although some information about pulse substructure can be determined.

### 3.5.1 Autocorrelation Traces

Having discussed the autocorrelation technique employed, we now present and discuss experimentally observed autocorrelation traces. A commercial EDFA (Calmar Optcom EDFA-02) was used to amplify the EDFL output to several mW for increased SHG within the autocorrelator. The measurements are shown consistent with the observed spectra and the contribution of FWM to laser operation is clearly shown.

With the same laser parameters and powers as used to obtain the results in Fig. 3.4, autocorrelation traces were obtained and are presented below. A number of features
are worthy of note. Each autocorrelation trace consists of a number of peaks situated upon a broad pedestal. The delay between adjacent peaks is $22.1\pm0.1$ ps, indicating the presence of some such periodicity in pulse substructure. The inverse of the peak separation is $45.2\pm0.2$ GHz, agreeing well to the periodicity of the frequency spectra observed.

There is a noticeable narrowing of the autocorrelation peaks with the increase of peak pulse energy from 40 pJ to 255 pJ, with no noticeable change for further increases. The narrowing of the autocorrelation peaks is the result of pulse substructure shortening by the combination of optical Kerr effect and anomalous dispersion within the fiber ring. Sub-pulses of short duration accumulate phase by both the Kerr effect and anomalous dispersion. The sub-pulses may thus take on a soliton-like character. In fact, it is a property of media with anomalous dispersion and Kerr nonlinearity that solitons typically form after the propagation of an arbitrary pulse shape of sufficient peak power [73]. Solitons, as nonlinear phenomena, exhibit the property that their duration is inversely proportional to their energy [69]. We thus expect to see a reduction in the soliton-like sub-pulse duration as the energy of the total pulse increases.

Higher resolution scans (66 fs stepping resolution) were used to probe more accurately the central peak of the autocorrelation traces. Illustrated below are pulse substructure widths inferred from the autocorrelation traces as a function of peak intracavity power. A hyperbolic secant, transform-limited sub-pulse shape was assumed to obtain the pulse FWHM from the autocorrelation trace FWHM. It is emphasized that since the measured SHG is in fact the ensemble average of the second order temporal coherence function, we are in fact probing an average substructure width. The minimum feature width obtained here is 590 fs, which corresponds to a spectral width of 4.3 nm. It has already been noted however, that the power spectral density envelope had an approximate 2 nm FWHM. The apparent discrepancy is easily explained by the fact that only a fraction of the pulse energy is in the form of sub-picosecond structures, and thus contributes to the average
Figure 3.5: Autocorrelation traces observed as a function of peak intracavity pulse energy. The abscissae are delay times $\tau$ in units of ps and the ordinates are second harmonic intensities normalized to peak values. The peak intracavity pulse energies are: A. 40 pJ; B. 255 pJ; C. 510 pJ; D. 989 pJ; E. 1268 pJ; and F. 1902 pJ.
Figure 3.6: Minimum pulse substructure width inferred from the central autocorrelation peak under the assumption of negligible chirp and $sech^2()$ pulse shape.

power spectral density proportionately. The large width pedestal clearly indicates a large fraction of pulse energy is not found in the sub-picosecond structure.

We now consider the ratio of the central peak height to the pedestal background, as is presented in Fig. 3.5.1 for various intracavity pulse energies. The background autocorrelation level was taken to be that midway between the central peak and adjacent peaks. The ratio rapidly increases from approximately 2 at low pulse energies to 4 at high pulse energies. We wish to assign meaning to the observed ratios. Hence, we consider the theoretically expected second order temporal coherence for chaotic light bursts with Gaussian power spectral density, partially modelocked pulses and also perfectly modelocked pulses.

It was shown by Grutter et al. [77] that for chaotic light bursts with Gaussian power spectral density, the second order coherence function satisfies the condition $g^{(2)}(0) = 2$ and $g^{(2)}(\tau) = 1$ for $\tau$ greater than the coherence time of the chaotic light. Thus, we have a ratio of 2 between central peak and pedestal. A perfectly modelocked pulse train is predicted to satisfy $g^{(2)}(0) \approx T/2\Delta t$ where $\Delta t$ is the pulse width and $T$ is the pulse train repetition rate. The factor in an exact relation depends upon pulse shape. Furthermore,
Figure 3.7: Ratios of central peak SHG to pedestal SHG as a function of peak intracavity pulse energy.

There is predicted to be zero pedestal for perfect modelocking. Partial modelocking results in a reduction of $g^{(2)}(0)$, and an increase in background pedestal. Calculation of accurate peak to pedestal ratios is difficult for intermediate degrees of temporal coherence and results vary depending upon the formalism one adopts to model partial modelocking [77, 78].

Looking back upon the observed autocorrelation traces, we can see that the low pulse energy traces correspond well to that expected for Gaussian noise bursts, with a coherence time roughly equal to 6 ps. With increasing pulse energy, the ratio of peak to pedestal reaches approximately 4. There is a reduction in phase noise among the modes of the cavity, meaning the cavity modes have become partially modelocked. However, the peak to pedestal ratio is still far below what is expected for a perfectly modelocked source. From the autocorrelation ratios, we see that partial modelocking was improved by increasing the pulse energies, but there was still significant phase noise present even at the highest energies.
3.6 Statistical Measurements

The power spectral densities indicate multiple wavelength operation was achieved, although perfect modelocking was not achieved. The statistical distribution of output pulse energy was measured in order to determine the intrinsic noise of the laser with the same laser parameters as in Table 3.1. The experimental setup, illustrated below, was used to measure pulse energy. The operating peak pulse energy was chosen to be 1430 pJ, the energy at which we observe close to minimum pulse substructure width. The photocurrent from a 125 MHz bandwidth InGaAs p-i-n photodiode (New Focus 1811) was measured with a digital oscilloscope of 100 MHz bandwidth and transferred to a computer for processing. The oscilloscope window encompassed seven pulses which were each integrated numerically to determine the relative pulse energies. The apparatus was used to collect 10 000 series, each series consisting of seven pulse energies.

3.6.1 Pulse Energy Correlation

We quantified the inherent noise of the laser through analysis of the statistical distributions of the pulse energies. The fluctuation of pulse energy was significant, with an observed standard deviation equal to 0.395 of the mean pulse energy. Scatter plots of normalized pulse energies against each other are illustrated in Fig. 3.8. For instance, graph A records the second pulse energy against the first pulse energy for each series. A distribution concentrated along the line $E_{n+1} = E_n$ indicates a high correlation between first and second pulse energies. For a given pulse, the energy of the following pulse will most often be almost equal to the given pulse energy. As the number of round trips between observed pulses increases, a greater deviation in energy is indicated by expansion of the distribution outwards from the equal energy axis.

The correlation function of pulse energy measurements quantifies the correlation of measurements between pulses some number of round trips apart [79]. A value of zero indi-
Figure 3.8: Scatter plots of normalized pulse energies over 10 000 collected series. The abscissae are first pulse energies $E_n$ and the ordinates are: A, second pulse energies $E_{n+1}$; B, fourth pulse energies $E_{n+3}$; C, sixth pulse energies $E_{n+5}$; and D, seventh pulse energies $E_{n+6}$. 
Figure 3.9: The correlation between energy measurements of pulses separated by the indicated number of round trips.

cates no correlation and statistical independence, while ±1 indicates perfect correlation and anticorrelation, respectively. The correlation function is plotted in Fig. 3.9. Six round trips are sufficient to bring the pulse energies close to statistical independence.

3.7 Conclusions

We have presented here numerous experiments characterizing our fiber laser source. A brief summary of the key experimental results is:

- A threshold in output power versus optical pumping power was observed, indicating the onset of laser oscillation.

- A time average power spectral density with a 45.0±0.6 GHz channel spacing was measured, corresponding well to the Lyot filter FSR. Spectral broadening of up was observed with features characteristic of that due to Kerr nonlinearity within fiber.
Simultaneous lasing at multiple wavelengths was thus successfully demonstrated with 6 adjacent channels exhibiting less than 5 dB power variation.

- Autocorrelation by SHG was used to characterize pulse substructure. Autocorrelation peaks were observed at 22.1 ps intervals as expected from the Lyot filter FSR. A 4:1 peak to pedestal ratio was observed, indicating partial modelocking was achieved.

- Measurement of pulse energies by direct photodetection revealed fluctuations with standard deviation 0.395 of the mean pulse energy.
Chapter 4

Numerical Simulation

4.1 Introduction

We present here numerical simulations which qualitatively predict the experimentally observed laser performance. Numerical simulations give us a means to investigate origins of instability. The theoretical model applied to describe the operation of the fiber laser is described first, with care taken to describe the approximations made. The numerical techniques used to implement the theoretical model of the EDFL are briefly reviewed due to their importance in obtaining meaningful results.

The numerical simulation results are presented for the EDFL configuration investigated in this thesis. Agreement with experimental results are good, however some discrepancies appear and are discussed. Finally, the issue of stability is tackled with the proposed insertion of a fast saturable absorber. The reasoning, modelling and simulation results of this solution are discussed.

4.2 Theory

The theoretical model we employ for the EDFL demonstrated is based upon the propagation of a broadband (several nanometers) pulse through the laser cavity. We describe
the pulse circulating through the ring cavity with a single carrier and a single envelope that can possess fine substructure.

The laser was constructed in single mode fiber and thus the electric field can be described by\textsuperscript{1}.

\[
E(r, t) = \frac{1}{2} (\hat{e} F(x, y) A(z, t) \exp[i(\beta_0 z - \omega_0 t)] + c.c.)
\]  

(4.1)

where \(\hat{e}\) is a polarization unit vector, \(F(x, y)\) is the mode field distribution, \(A(z, t)\) is the pulse envelope, \(\beta_0\) is the carrier propagation constant and \(\omega_0\) is the carrier frequency. The pulse envelope gives both amplitude and phase, thus completely characterizing the pulse. The Fourier transform of the envelope is here defined as,

\[
A(z, \omega) = \int_{-\infty}^{\infty} A(z, t) \exp(i\omega t) dt
\]  

(4.2)

For the propagation of the envelope through fiber, it is required that the envelope be a baseband function. In other words, the spectral components of \(A(z, \omega)\) are possibly nonzero strictly for \(|\omega| \ll \omega_0\). We discuss each non-trivial element within the laser cavity, supplying the equations used to transform the envelope appropriately. A schematic of the cavity element models and the propagation of \(A(z, t)\) is presented below in Fig. 4.1.

**Standard Single-Mode Fiber**

The propagation of a pulse through single mode fiber is described by a nonlinear Schrödinger (NLS) equation [69].

\[
\frac{\partial}{\partial z} A(z, T) = \left(-i\frac{1}{2} \beta_2 \frac{\partial^2}{\partial T^2} + i\gamma |A(z, T)|^2 \right) A(z, T)
\]  

(4.3)

where \(T = t - z/v_g\) is the time in a reference frame moving with the group velocity \(v_g\) of the pulse, \(\beta_2 = \partial^2 \beta/\partial \omega^2\) is the group velocity dispersion and \(\gamma\) is a nonlinearity coefficient given by,

\[
\gamma = \frac{n_2 \omega_0}{c A_{eff}}
\]  

(4.4)

\textsuperscript{1}refer to appendix D for a description of this expression
Figure 4.1: An illustration of the theoretical model used for numerical simulations of the fiber laser experimentally demonstrated.
where $n_2$ is the nonlinear Kerr index of the glass and $A_{\text{eff}}$ is an effective area related to the mode field by:

$$A_{\text{eff}} = \frac{(\iint F^* F dx dy)^2}{\iint F^* F F dx dy} \quad (4.5)$$

In the expressions above, it is assumed that $|A(z,T)|^2$ is normalized to instantaneous power within the pulse. It is of interest to look at the form of the NLS equation in the Fourier domain to see more clearly the effect of propagation through fiber.

$$\frac{\partial}{\partial z} A(z, \omega) = i \frac{1}{2} \beta_2 \omega^2 A(z, \omega) - i \frac{\gamma}{4\pi^2} \iint A^*(z, \omega') A(z, \omega'') A(z, \omega + \omega' - \omega'') d\omega' d\omega'' \quad (4.6)$$

The above equation clearly illustrates the quadratic phase accumulation of the pulse due to group velocity dispersion. Also, the optical Kerr nonlinearity takes the form of an integration over all possible contributions from FWM. We see explicitly the FWM within the envelope that will result in both the coupling between wavelength channels and the spectral broadening of each channel.

**Lyot Filter**

Nonlinear polarization rotation effects are assumed negligible within the Lyot filter. The large temporal walk-off due to group velocity mismatch does not allow nonlinear Kerr induced phase to accumulate between the two polarization components within the birefringent fiber [69]. The Lyot filter thus linearly filters the pulse envelopes according to the amplitude transmission equation derived in Appendix B\(^2\), rewritten in the form.

$$H(\omega) = ab + \sqrt{(1 - a^2)(1 - b^2)} \exp[-i(2\pi\omega/\Delta\omega + \Phi)] \quad (4.7)$$

where $a$, $b$ and $\Phi$ are constants. For the simulations, we assume $a = b = 1/\sqrt{2}$ and $\Phi = 0$. This corresponds to the physical situation in which light is launched and analyzed at $\pi/4$ radians to the principle axes of the birefringent fiber.

\(^2\)In the context here, we use $H(\omega)$ rather than $t$ to avoid confusion with the time variable.
Electro-optic Modulation

The electro-optic modulator is a simple Mach-Zehnder interferometer with a variable refractive index in one arm. Application of a voltage to the single arm effects refractive index changes via the electro-optic effect. Neglecting the linear wavelength dependence of the half-wave voltage $V_\pi$ required to switch the modulator, the temporal transmission function is [80].

$$j(t) = \frac{\sqrt{T_{EOM}}}{2} \left[ 1 + \exp(i\pi V(t)/V_\pi) \right]$$

(4.8)

where $\sqrt{T_{EOM}}$ is the peak modulator transmission. $V(t)$ is the voltage appearing across the interferometer arm. The form of the voltage applied is assumed to be a rather general biased Gaussian.

$$\frac{V(t)}{V_\pi} = 1 - \exp \left[ -4\ln(2) \left( \frac{t}{\sigma_t} \right)^2 \right]$$

(4.9)

where $\sigma_t$ is the FWHM of the voltage function. The bias is present to effect high transmission over the FWHM of the voltage function. The negative sign proceeding the Gaussian could just as well have been chosen to be positive; the only effect being the sign of the induced chirp on the envelope at the edges of the modulator window.

EDFA

The envelope is propagated through the EDFA with a NLS equation including gain and gain dispersion. The modified NLS equation is in fact a Ginzburg-Landau (GL) equation of the form [69].

$$\frac{\partial}{\partial z} A(z, T) = \left( -i \frac{1}{2} (\beta_2 + igT_2^2) \frac{\partial^2}{\partial T^2} + \frac{1}{2} g + i\gamma |A(z, T)|^2 \right) A(z, T)$$

(4.10)

where $g$ is the gain of the amplifier per unit length of fiber and $T_2 = 2/\Delta\omega$ is the homogeneous lifetime inversely proportional to the homogeneous linewidth $\Delta\omega$. The gain term clearly leads to amplification of the envelope. The gain dispersion term is included to incorporate the finite gain bandwidth of the EDFA. The optical Kerr nonlinearity and the fiber dispersion are included in the (GL) equation. The inversion of the erbium ions,
and thus the gain $g$ was assumed constant along the length of the fiber amplifier for each round trip. From the time dependence of the inversion derived in Appendix D, we find the gain $g$ is governed by the equation.

$$\dot{g} - \frac{g - g^{eq}}{T_1} = \frac{g}{T_1} \frac{I}{I_{sat}}$$  \hspace{1cm} (4.11)$$

where $g^{eq}$ is the unsaturated gain. $T_1$ is the population relaxation time. $I$ is the average optical intensity over the fiber area and $I_{sat}$ is the saturation intensity averaged over the fiber mode area. The population relaxation time is 10 ms. while a round trip is approximately 350 ns. We thus assume a constant EDFA gain per round trip.

### 4.3 Numerical Method

The analysis of the propagation of the envelope $A(z, T)$ is trivial for the Lyot filter, output coupler and electro-optic modulator. We treat them here as lumped elements acting upon the envelope in either the time or frequency domain. However, analysis of the propagation of the envelope through the doped and undoped fiber requires integration of Eqs. 4.3 and 4.10. The method chosen here is the split-step Fourier method [69]. We briefly consider the method here in the context of the NLS equation. The form of the NLS equation is that of propagation by the sum of a linear operator $\hat{D}$ and a nonlinear operator $\hat{N}$, where.

$$\hat{D} = -i \frac{1}{2} \beta_2 \frac{\partial^2}{\partial T^2}$$

$$\hat{N} = i \gamma |A(z, T)|^2$$ \hspace{1cm} (4.12)

The NLS equation takes the simple form $\partial A/\partial z = (\hat{D} + \hat{N})A$. The split-step Fourier method consists of taking the step size $\Delta z = \hbar$ for numerical integration sufficiently small that we may assume the operators $\hat{D}$ and $\hat{N}$ commute. Under said assumption, step wise integration may be performed with the formal symmetric recipe,

$$A(z + \hbar, T) = \exp \left( \frac{\hbar}{2} \hat{D} \right) \exp \left( \int_z^{z+h} \hat{N}(z')dz' \right) \exp \left( \frac{\hbar}{2} \hat{D} \right) A(z, T)$$ \hspace{1cm} (4.13)
The linear exponential operators are performed in the frequency domain. We take advantage of the Fourier operator equivalence $\partial / \partial T \Leftrightarrow -i \omega$ in applying $\exp(h \hat{D}/2)$. The linear operator is conveniently diagonal in the frequency domain. The nonlinear operation is performed in the time domain. Where in this case we use a simple first order approximation.

$$\int_{z}^{z+h} \hat{N}(z') dz' \approx h \hat{N}(z)$$  \hspace{1cm} (4.14)

We take advantage of the simplicity of the form of the optical Kerr nonlinearity represented in the time domain. The present computational ease of applying Fourier transforms renders the split-step Fourier method attractive.

The numerical algorithm was implemented in a MATLAB environment. In order to minimize computation time, adaptive step sizing was used. It is important to choose the step size sufficiently small so that the phase of the envelope changes gradually with each step. This is a requirement for accurate simulation with the split-step method. We choose here to keep the phase change below $\pi/2$ for each step. Although the phase change due to dispersion is negligible over a round trip, the Kerr nonlinear phase shift may not be. Estimation of the maximum phase accumulation in the envelope for a single step leads us to the condition.

$$h \gamma \max \{|A(0,T)|^2\} < \eta \pi / 2$$  \hspace{1cm} (4.15)

where the numerical factor $\eta$ was chosen as a compromise between computational time and accuracy. For the simulations presented here, we chose $\eta = 0.1$. The value was based upon the accuracy of simulation results for the propagation of a 1 ps soliton over many soliton periods. The step size was updated once per round trip.

Integration of the GL equation 4.10 was performed also with a split-step Fourier method. The first modification required is the generalization of the linear operator $\hat{D}$ to incorporate the gain and gain dispersion of Eq. 4.10. The second modification is in the choice of step size. Not only may the phase of the envelope change significantly with a
single step, but the amplitude will change as well in the presence of gain. The step size is chosen to be the minimum of that determined by Eq. 4.15 or \( hg < \eta' \), where again \( \eta' \) is a numerical factor chosen as a compromise between computation time and accuracy. For the simulations reported here, we have chosen \( \eta' = 0.2 \). This ensures the envelope does not grow by more than a factor of 1.2 per step. Again, the step size was updated once per round trip.

### 4.4 Simulation Results

Numerical simulation of the demonstrated laser was undertaken according to the scheme described. The pulse envelope was propagated through the laser cavity with Gaussian white noise as the initial conditions. The pulse envelope was discretized into 8192 samples across a total time window of 800 ps. The relevant laser parameters used in the simulations are presented below in Table 4.1, matching those from experiment. We have assumed \( \beta_2 = -20 \) ps\(^2\) km\(^{-1}\) and \( \gamma = 3 \) rad km\(^{-1}\) as for standard SMF-28 fiber. We assume these same values for the highly birefringent fiber and the EDF within the commercial EDFA because these parameters are of approximately the same order of magnitude as in SMF-28. The EDF length was estimated knowing the experimental cavity resonance frequency and direct measurement of all other fiber lengths. The unsaturated gain per unit length was thus estimated. The remaining erbium ion parameters were taken to be those typically found [81]. The modulator pulse width was assumed to be 40\% of that in experiment to reduce the time required for simulations. The modulator window width is less important than the window shape because the pulse energy typically bunches up along a window edge.

The last important parameter still not mentioned is that of the modulator pulse repetition rate, which is not necessarily exactly equal to the fast (slow) cavity round trip time. The repetition rate was chosen to be that which gave the best correspondance
Table 4.1: Laser simulation parameters and their respective values taken to match those in experiment unless otherwise noted in the text.

with the experimentally observed data. A 2 ps incremental (decremental) detuning per round trip from the slow (fast) axis repetition rate was chosen. This detuning resulted in numerically simulated behaviour which qualitatively best matched the time average experimental results reported in Chapter 3. All the experimental results quoted were performed following no less than 600 round trips with Gaussian white noise initial conditions for the pulse envelope. The population response of the EDF was assumed instantaneous during the initial 600 round trips so that convergence towards the average inversion would be accelerated.

The simulated time average output spectrum and autocorrelation trace are illustrated below in Figs. 4.2 and 4.3. The simulated peak intracavity pulse energy was an aver-
Figure 4.2: The simulated time average spectrum of the fiber laser assuming slight detuning from the slow axis. The spectrum is normalized to unit peak.

The energy 1390 pJ, in accordance with experiment where the energy was varied from 40 pJ to 1920 pJ. The theoretical spectrum exhibits the 45 GHz channel spacing as expected and approximately linear roll-off of the spectral envelope. Both features agree well with the experimental data of Fig. 3.4. The average autocorrelation trace corresponds qualitatively with the experimentally observed traces of Fig. 3.5.1. Autocorrelation peaks separated by 22.1 ps are readily observed upon a broad pedestal. The 2:1 peak to pedestal ratio of the numerically simulated trace indicates poorer modelocking than the experimentally observed 4:1 ratio. Also, the simulated pedestal width is a factor of 2 wider than the experimentally observed pedestal. In general we have qualitative agreement, indicating our model may at least be useful for qualitative predictions.

There are two simplifications in the model which may affect the agreement with experimental results. First, the nonidealities of the modulator transmission have been omitted. No timing jitter in the transmission window has been assumed, although a maximum jitter of 100 ps was observed in the electrical pulses driving the EO modulator.
Figure 4.3: The simulated time average autocorrelation trace of the fiber laser assuming slight detuning from the slow axis.

We have also neglected pulse distortion due to the capacitance of the electrodes and slight impedance mismatch with the pulse generator. Both effects make an exact modelling of the transmission window shape extremely difficult at best. As will be discussed further, the modulator window shape may be significant in determining laser dynamics.

The second simplification was done with regards to the EDFA. A gain response based upon a simple, homogeneously broadened two level system was used in the model. We have neglected such difficult topics as spectral hole burning of deeply saturated amplifiers resulting from gain inhomogeneity. We have also ignored the manifold nature of the ground and excited states of erbium ions with silica fiber. A more rigourous treatment of the EDFA requires more detailed knowledge of the EDF than is currently available in the literature. In particular, the transition wavelengths, the inhomogeneous broadening of each transition, and relative strengths of each transition involved are required. The detailed modelling of deeply saturated amplifiers thus rests upon new approaches to spectroscopic studies and theoretical modelling of EDF.
Nonetheless, our numerical model allows us to probe the qualitative nature of the dynamics of the fiber laser, giving insight into the physical mechanisms involved. The instantaneous power of the simulated output pulses is illustrated in Fig. 4.4 as a function of round trip number. A number of interesting features are apparent. We see the pulse energy gather towards the rising edge of the modulator window. With each round trip, the portion of the pulse which travels along the fast axis leads ahead by 22.1 ps. An increase in effective loss is created by the continuous shifting of the pulses traversing the fast path of the Lyot filter. The slope of the edge of the modulator window determines the rate at which energy is lost due to this shifting. A closer examination of the striations in the profile evolution plot reveals that a 2 ps per round trip shift can be observed, as obviously expected due to the detuning from resonance with the slow axis.

Rapid fluctuations in pulse shape are visible, much like pulse break up expected due to Kerr induced modulation instability [82]. Nonlinearly-induced variations in the refractive index lead to instabilities that typically result in sub-picosecond formations. An equivalent interpretation is spectral broadening due to FWM and the resulting beating between the various spectral components gives rise to a rapidly varying temporal pulse profile.

We take advantage of the numerical simulations in another respect by probing the dynamics of the total pulse energy as it propagates through the laser cavity. The pulse energy transmission through the Lyot filter, electro-optic modulator, EDFA and undoped fiber are individually plotted for 800 round trips in Fig. 4.5. From the pulse energy transmission plot we can discern the elements giving rise to the output pulse energy fluctuations observed. Pulse energy transmission through the spectral slicing filter is observed to fluctuate by as much as 25%. The variation in transmission is caused by the nonlinear generation of spectral components by FWM which are removed by the Lyot filter. In particular, broadening of the individual channels through FWM contributes to a nonlinear loss through the Lyot filter. Similarly, variations in EDFA gain are caused
Figure 4.4: Instantaneous pulse power indicated by dark shading versus time referenced to peak modulator transmission on the abscissa. The evolution of pulse profile is shown with corresponding round trip numbers along the ordinate. The modulator window is assumed to be a 400 ps FWHM supergaussian.
by filtering of spectral components lying outside the gain bandwidth [83] determined through the dipole dephasing time (recall $\Delta \omega = 2/T_2$ for a homogeneous transition). The nonlinear generation of spectral components through FWM outside the gain bandwidth varies from round trip to round trip. Transmission variation through the modulator is simply a result of the shifting of energy out of the transmission window. Successive round trips give different pulse energy distributions at the modulator window edge and thus a variation in loss following propagation through the Lyot filter. Transmission through undoped fiber unsurprisingly gives no variation in transmission. The conclusion that can be drawn from the above analysis is that spectral broadening by FWM gives rise to cavity losses which vary significantly from round trip to round trip. Although FWM counteracts gain competition to allow multiple wavelength laser oscillation, undesired spectral broadening of each channel leads to pulse energy instability.

The magnitude of the simulated pulse energy fluctuations is less than that observed in experiment. The standard deviation of the output pulse energy was simulated to be a fraction 0.208 of the mean pulse energy. The corresponding ratio is 0.395 for the experimental results. We thus conjecture that the experimental pulse energy fluctuation would be reduced with a decrease in the RF source timing jitter. The distribution of pulse energy along the EO modulator window edge results in extreme sensitivity to any jitter in the modulator window. A single instance of a 100 ps jitter is sufficient to completely null the transmission of a 100 ps pulse that would normally pass the EO modulator on the edge of the window.

4.5 Pulse Shaping

In the previous section we have shown how the numerical model we have employed predicts similar behaviour to the laser experimentally realized. An undesirable feature of the output pulses inferred from the numerical simulations is the randomly fluctuating
Figure 4.5: Pulse energy transmission through various cavity elements are plotted above as a function of round trip number: $T_1$ Lyot filter, $T_2$ electro-optic modulator. $T_3$ EDFA and $T_4$ undoped fiber.
pulse profile. We propose here a method for pulse shaping within the laser cavity with the intent to create a more uniform train of pulse substructure.

Before introducing the method and experimental results, we consider in a bit more detail the nature of the fluctuations. Short pulse substructures in the simulations are typically sub-picosecond in width and multiple Watts in peak power. The first order soliton power for the fiber used gives a peak power of 20 W assuming 1 ps pulse width. The pulse substructures are thus not far from that of low order solitons. Considering the sub pulses as a collection of low order solitons allows us to use the conclusions drawn from perturbative treatments of interacting solitons [84, 85]. The principal result of interest to us is that regularly spaced solitons are only very weakly bound to each other, and are thus very easily perturbed. We thus can not expect the Lyot filter to produce a pulse substructure with perfect periodicity. Modulational instability inevitably results in the random fluctuations reported here.

We show here that pulse quality is dramatically improved through the introduction of a nonlinear pulse shaping element. In the language of lasers, an additive pulse modelocking technique is being proposed. An illustration of the proposed structure is given in Fig. 4.6. An ultrafast saturable absorber based upon nonlinear polarization rotation has been incorporated into the laser. Demonstrated experimentally [86] and analyzed theoretically [87, 88, 89], nonlinear polarization rotation now often finds application as a saturable absorber for passively modelocked lasers [33, 90].

Nonlinear polarization rotation is simply the result of an intensity dependent index of refraction. A pulse experiences an intensity dependent polarization rotation since the indices of refraction along the fast and slow axes change according to the power launched along each axis. A polarization analyzer at the terminal end of the birefringent section of fiber acts to project the pulses from each axis to a common polarization. We thus

calculation using the formula quoted in [69]
a detailed analysis [87, 89] reveals subtleties that we need not consider here.
Figure 4.6: The modified laser cavity here includes an all fiber saturable absorber consisting of two polarization controllers, a length of low birefringence fiber and a polarization analyzer.

have a nonlinear interferometer of the generic sort used in additive pulse modelocking. Appropriate adjustment of the polarization analyzer will give a transmission increasing with intensity, a saturable absorber in other words.

We reiterate an important point here. The Lyot filter can be approximated as a linear interferometer because of the great disparity in group velocities along each axis. The saturable absorber exhibits its nonlinear behaviour because the birefringence of the fiber segment is sufficiently low that pulses travel together, allowing significant nonlinear interaction.

Insertion of saturable absorption into a fiber laser cavity is a simple and common technique to achieve modelocking. High intensity peaks, experiencing less loss, are more energetically favourable within a laser cavity and hence short pulse operation ensues. Incident pulses are shortened with each pass through the saturable absorber. The ultimate width of the pulse is usually limited not by the ultrafast response times of the optical
Kerr effect (100 fs [61]), but by the finite gain bandwidth of the EDFA within the cavity.

The pulse shaping mechanism introduces greater nonlinearity into the laser cavity. The favourable reduction in loss for high peak power subpulses makes a regular train of pulses energetically more favourable than the fluctuating pulse structures of Fig. 4.4. The Lyot filter acts to introduce temporal coherence with periodicity on the order of 22.1 ps through coupling of pulse components 22.1 ps apart. The combined effect of the Lyot filter and saturable absorber effectively increase the binding energy of the pulses, making the desired pulse substructure more resilient against perturbations due to EDFA spectral filtering and temporal modulation.

The arguments above were tested with a numerical simulation of the new cavity structure in Fig. 4.6. The ultrafast saturable absorber was assumed to have instantaneous response, since we have noted that the EDFA gain bandwidth will be the limiting bandwidth and not the nonlinear response of optical fiber. The absorption was thus assumed to take the form.

$$\alpha = \frac{\alpha_0}{1 + P(t)/P_{\text{sat}}}$$  \hspace{1cm} (4.16)

where $\alpha_0$ is the unsaturated absorption and $P_{\text{sat}}$ is the saturation power. Typically available values of $\alpha_0 = 0.9$ and $P_{\text{sat}} = 5$ W were chosen here [86]. The peak transmission through the modulator was assumed to be an optimistic $T_{\text{EOM}} = 0.9$ for the simulations with saturable absorbers.

The results of numerical simulations are plotted below in Figs. 4.7, 4.8 and 4.9. First note that multiple wavelength oscillation is still occuring, although the contrast between the channel power spectral density and background spectral density has been reduced. The change in pulse structure is more dramatic. No longer a rapidly fluctuating profile, the pulse has taken on a much less random and more periodic structure. The quality of the modelocking is superior because of the reduction in noise among cavity modes as indicated by the vastly improved peak to pedestal ratio. This should not come as a surprise since lasers modelocked with saturable absorbers based upon nonlinear
polarization rotation are known to exhibit exceptional pulse uniformity [90].

Figure 4.7: Simulated power spectral density versus optical frequency for the modified laser cavity including an ultrafast saturable absorber. The spectrum is normalized to unit peak.

4.6 Summary

This chapter has presented the theoretical model used to perform numerical simulations of the proposed fiber laser. The model did not incorporate jitter in the EO modulator transmission window and the EDFA was approximated with homogeneously broadened two level systems. but qualitative agreement between experiment and numerical simulation was found. Variation in loss from round trip to round trip was predicted, although not to the extent observed in experiment. Spectral components generated each round trip by FWM were found to be filtered by the Lyot filter and EDFA in a nonlinear manner. Furthermore, the simulations indicate the intrinsic pulse energy fluctuation from the laser is less than that observed. We conjecture that pulse energy fluctuation can be reduced by
Figure 4.8: The simulated time average autocorrelation trace of the fiber laser including an ultrafast saturable absorber.

the reduction of timing jitter in the RF pulse source driving the EO modulator. Finally, an ultrafast saturable absorber was found to dramatically improve the pulse profile in numerical simulations.
Figure 4.9: Simulated instantaneous pulse power indicated by dark shading versus time. An ultrafast saturable absorber has been assumed.
Chapter 5

Concluding Remarks

We have introduced in this thesis a novel technique for simultaneous multiple wavelength generation in an EDFL, striving to satisfy the criteria outlined in Chapter 1 for practical application in WDM networks. The novelty here lies in the simultaneous use of FWM and intracavity spectral slicing to achieve multiple wavelength laser oscillation. Following an introduction to the challenge in developing stable, multiple wavelength EDFL's, a brief survey of previous techniques demonstrated was given in Chapter 2. The principle and experimental implementation of the laser proposed here was then introduced. A summary of the experimental setup was given in Chapter 3. Temporal, spectral and statistical characterization of the output laser pulses were presented. The time average spectra indicated multiple wavelength oscillation was achieved, but fluctuation in pulse energy was significant. In Chapter 4, we introduced the theoretical model which formed the basis for numerical simulations. Although a number of approximations were made, qualitative agreement was found between experiment and numerical simulation. Finally, we proposed a modified laser structure including an all fiber ultrafast saturable absorber. Numerical simulations indicate the expected pulse shaping yields improved pulse profiles.

The important demonstrations of this thesis include,

- the combination of FWM and intracavity spectral slicing has been shown as a
method for multiple wavelength laser oscillation in EDFL’s

- time average spectra of 6 channels with 5 dB power variation were demonstrated with a 45 GHz channel spacing

- numerical simulations of a laser incorporating an ultrafast saturable absorber indicate improved pulse quality may be expected in experiment

The work performed here provides avenues for fruitful research into multiple wavelength sources.

- experimental demonstration of the EDFL with a tuneable RF source with less timing jitter should be performed. Timing jitter is believed to be a limiting factor in the EDFL performance.

- experimental demonstration of the modified EDFL incorporating a saturable absorber should be performed. The expected improvement in pulse quality could easily be evinced from an autocorrelation measurement.

- the numerical simulations of the laser, although giving qualitative agreement with experiment, require a model that can incorporate inhomogeneous effects that become important in strongly saturated EDFA’s. A comprehensive spectroscopic study of EDF to extract the phenomenological parameters required for the more complete empirical model of Desurvire [26] would enable more accurate, multiple wavelength EDFL modelling.

- the incorporation of a semiconductor optical amplifier (SOA), in lieu of an EDFA, should be demonstrated as well. The success of the proposed technique for EDFL’s indicates it should easily be applied to SOA lasers since FWM occurs within SOA’s themselves and the SOA can be used as a modelocking element itself due to much quicker gain dynamics.
In conclusion, a new technique for multiple wavelength generation within EDFL's has been demonstrated. The results of the proof of principle demonstration are encouraging. Continuing development of the technique proposed, including an experimental investigation of the modified laser structure proposed, should be pursued for ultimate application to WDM lightwave systems.
Appendix A

Inhomogeneously Broadened Media Under Saturation

A.1 Introduction

The susceptibility $\chi(\omega)$ of an inhomogeneously broadened medium is given by an integration over the resonance frequency distribution $\zeta(\omega_a)$ with the homogeneous susceptibility $\chi''_h(\omega)$ and a saturation factor $S(\omega_a)$. In general, we may assume a Gaussian resonance distribution [36].

$$\zeta(\omega_a) = \sqrt{\frac{\ln 2}{\pi}} \frac{2}{\Delta \omega_a} \exp \left[ -4 \ln 2 \left( \frac{\omega_a - \omega_0}{\Delta \omega_a} \right)^2 \right] \quad (A.1)$$

where $\omega_0$ is the mean resonance frequency and $\Delta \omega_a$ is the FWHM of the resonance frequency distribution. Even with this common distribution, the saturated susceptibility does not lend itself to a tractable analytic expression in closed form. In order to avoid numerical integration, we provide here a derivation of an analytic expression for the saturated susceptibility.
A.2 Voigt Profile

First, we consider the unsaturated susceptibility. The susceptibility is given by [37]

$$
\chi_0(\omega) = \Theta \sqrt{\frac{\ln 2}{\pi}} \frac{2}{\Delta \omega_a} \int_{-\infty}^{\infty} \exp \left[ -4 \ln 2 \left( \frac{\omega - \omega_0}{\Delta \omega_a} \right)^2 \right] \frac{1}{2(\omega - \omega_a)/\Delta \omega + i} d\omega_a
$$

(A.2)

where $\Theta = N_{eq}^2 \omega_0 L |\mu|^2 / 3 c \hbar \epsilon_0 \Delta \omega$ denotes the strength of the transition in terms of the dipole moment $\mu$. Lorentz correction factor $L$ and unsaturated inversion $N_{eq}$. The lineshape is the ubiquitous Voigt profile, the convolution of a Gaussian distribution and a Lorentzian lineshape. A compact expression for the above is

$$
\tilde{\chi}_0(\omega) = -i \Theta \sqrt{\pi \ln 2} w(z)
$$

(A.3)

where $w(z)$ is the Faddeeva function defined by

$$
w(z) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{z - t} dt
$$

(A.4)

with $z = x + iy$ and $t$ given by

$$
x = 2 \sqrt{\ln 2} \frac{\omega}{\Delta \omega_a} \\
y = \sqrt{\ln 2} \frac{\Delta \omega}{\Delta \omega_a}
$$

(A.5)

Efficient algorithms exist for calculation of the Faddeeva function [91, 92], and it is thus as accessible as $\exp()$ for example. The unsaturated susceptibility is thus known in terms of a well characterized analytic function.

A.3 Susceptibility Saturation

In order to include saturation, we follow Siegman [37] and consider the change in susceptibility due to saturation by a signal of intensity $I_1$ and frequency $\omega_1$,

$$
\delta \tilde{\chi}(\omega) = \tilde{\chi}(\omega) - \tilde{\chi}_0(\omega)
$$

$$
= \int_{-\infty}^{\infty} \zeta(\omega_a)[1 - S(\omega_a)] \chi_n^{\omega_1}(\omega) d\omega_a
$$

(A.6)
where the saturation factor \( S(\omega_a) \) is given by

\[
S(\omega_a) = \frac{1}{1 + F(\omega_a, \omega_1, I_1, I_{\text{sat}})}
\]  

(A.7)

and where the depth of the saturation is determined by

\[
F(\omega_a, \omega_1, I_1, I_{\text{sat}}) = \frac{I_1}{I_{\text{sat}}} \frac{1}{1 + [2(\omega_1 - \omega_a)/\Delta \omega]^2}
\]  

(A.8)

The reason for considering the change in susceptibility rather than the susceptibility as a whole is that \( 1 - S(\omega_a) \) is a simpler expression than \( S(\omega_a) \) and the unsaturated susceptibility is easily calculated.

The mathematics is simplified when we transform to normalized variables according to the scheme.

\[
p = I_1/I_{\text{sat}}
\]

\[
u = 2(\omega_a - \omega_1)/\Delta \omega
\]

\[
u = 2(\omega - \omega_1)/\Delta \omega
\]

\[
r = 2(\omega_0 - \omega_1)/\Delta \omega
\]

\[
\alpha = \left( \frac{\Delta \omega}{\Delta \omega_a} \right)^2 4 \ln 2
\]

\[
\Theta' = \Theta \sqrt{\frac{\ln 2}{\pi}} \frac{\Delta \omega}{\Delta \omega_a}
\]

(A.9)

Our susceptibility change thus may be written.

\[
\delta \chi(\omega) = \Theta' \int_{-\infty}^{\infty} \exp(-\alpha(u - r)^2) \frac{p}{1 + p + u^2 (v - u) + i} \, du
\]

(A.10)

The integrand does not lend itself to any simple integration technique. An approximation is therefore made. Using \( \exp(\alpha(u - r)^2) \approx 1 + \alpha(u - r)^2 + \alpha(u - r)^4/2 \) for sufficiently small \(|u - r|\), we can approximate the Gaussian distribution with an "extended" Lorentzian,

\[
\exp(-\alpha(u - r)^2) \approx \frac{1}{1 + \alpha(u - r)^2 + \alpha^2(u - r)^4/2}
\]

(A.11)
As $|u - r|$ increases, the approximation does not fail catastrophically because the expansion is found in the denominator. The integral thus becomes,

$$
\delta \tilde{\chi}(\omega) = \Theta' \int_{-\infty}^{\infty} \frac{1}{1 + \alpha(u - r)^2 + \alpha^2(u - r)^4/2} \frac{p}{1 + p + u^2(v - u) + i} du \quad (A.12)
$$

Complex contour integration is readily applicable to the above integral. The poles of the integrand in the complex $u$ plane are all simple and are given by

$$
u_m \in \{ r \pm 1/\sqrt{\alpha} \exp(i3\pi/8), r \pm 1/\sqrt{\alpha} \exp(i5\pi/8), \pm i \sqrt{1 + p}, v + i \} \quad (A.13)
$$

We now consider the clockwise complex contour integral along path $\gamma$ consisting of the real axis and an arc in the lower half plane.

$$
I = \oint_\gamma \prod_m \frac{1}{u - u_m} du \quad (A.14)
$$

We can easily verify that integration along an arc about the centre of the complex plane can be made vanishingly small. Thus, we can apply Cauchy's residue theorem for complex contour integration to obtain.

$$
\delta \tilde{\chi}(\omega) = -2\pi i \Theta' \sum_n \text{residue} \left( \prod_m \frac{1}{u - u_m}, u = u_n \right) \quad (A.15)
$$

Explicitly calculating the residues leaves us with the following analytical formula.

$$
\frac{\delta \tilde{\chi}(\omega)}{\Theta'} = \frac{-i\pi/\sqrt{\alpha}}{(\phi_1 - \phi_1^2)(1 + p + (r - \phi_1/\sqrt{\alpha})^2)(v - r + i + \phi_1/\sqrt{\alpha})} - \frac{i\pi/\sqrt{\alpha}}{(\phi_2 - \phi_2^2)(1 + p + (r - \phi_2/\sqrt{\alpha})^2)(v - r + i + \phi_2/\sqrt{\alpha})} + \frac{\pi}{\sqrt{1 + p} (1 + \alpha(r + i\sqrt{1 + p})^2 + \alpha^2/2(r + i\sqrt{1 + p})^4)} (v + i + i\sqrt{1 + p}) \quad (A.16)
$$

where $\phi_1 = \exp(i3\pi/8)/2^{1/4}$ and $\phi_2 = \exp(i5\pi/8)/2^{1/4}$. Although the analytic formula does not give one insight into the nature of the spectrum saturation, it allows for quick numerical calculations of saturated susceptibilities. The method can also be applied with higher order expansions, albeit at the expense of factoring higher order polynomials.
Appendix B

Theory of the Lyot Filter

B.1 Introduction

The demonstrated multiple wavelength laser employs a Lyot filter for intracavity spectral slicing. Based on linear polarization rotation, the Lyot filter is a simple device. However, the linear theory of Lyot filter operation is described here in detail because it is not as ubiquitous as other frequency comb filters such as dielectric stacks and Fabry-Perot resonators. Nonlinear polarization rotation is neglected because the high birefringence of the fiber used in the Lyot filter produces a large velocity mismatch between the polarization components of a pulse. Thus, the temporal walk-off inhibits nonlinear interaction between each polarization component. A second reason to study the linear theory of a Lyot filter is to ascertain what parameters are critical to the Lyot filter characteristics, such as free spectral range (FSR) and extinction ratio between pass and stop bands.

We begin this appendix chapter with a description of birefringence in optical fibers. We then describe Poincaré sphere formalism, which will be useful in our description of polarization rotation. An analytical expression for the transmission function of a Lyot filter is given and we provide illuminating examples.


B.2 Birefringent Optical Fibers

A single mode optical fiber, with perfect axial symmetry, in fact supports the propagation of two co-propagating modes at a given optical frequency. The doubly degenerate mode comes about due to the polarization degree of freedom, analogous to the double degeneracy associated with co-propagating plane waves in vacuum. Any HE_{11} mode field distribution and a duplicate rotated through an angle $\pi/2$ about the symmetry axis serve as a basis for all possible guided optical excitations. The mode fields approximate two linear polarizations, and the literature often speaks of both modes as such.

The introduction of fiber asymmetry about a plane through the axis destroys the degeneracy of both HE_{11} modes. All guided optical excitations are now described by a unique basis constituted by two modes. Each mode will possess a different longitudinal propagation constant and a different mode field distribution. If we again identify each mode with linear polarization, we have an obvious analogue to the propagation of linearly polarized plane waves in a birefringent crystal [93]. Birefringent fiber designs employ geometric core asymmetry or stress induced index birefringence. As in the case of a birefringent crystal, we identify a fast axis and a slow axis with the fiber, although we are strictly speaking of a fast mode (high phase velocity, low effective index) and a slow mode (low phase velocity, high effective index).

The modes of a birefringent fiber are not linearly polarized, but it is convenient to speak of them in this way for two reasons:

- the difference is immaterial except at the entrance and exit of the birefringent fiber. The exact electromagnetic field distribution does not matter except when we consider the launching of light into and out of the birefringent fiber.

- the modes ARE very close to being linearly polarized, especially for weakly guiding fibers.

For the above reasons, we will use the language of plane wave polarization to elucidate
the operation of the fiber Lyot filter.

**B.3 Poincaré Sphere Formalism**

In order to fully characterize the operation of the Lyot filter, we present a Poincaré sphere formalism. Although it may seem to the reader as though we are cracking the proverbial walnut with a sledgehammer, the Poincaré sphere provides a clear visualization of the Lyot filter operation giving further meaning to the mathematics. The assumptions we make here are that negligible loss and negligible mode coupling are induced over the length of the birefringent fiber. We begin by expressing the electric field within the fiber as a superposition of modes at a point $z_0$ along the fiber axis.

$$E_u = \frac{1}{2} \{E_1 e_1 + E_2 e_2\} + c.c.$$  \hspace{1cm} (B.1)

$$= \frac{1}{2} E_0 [\cos(\phi)e_1 + \sin(\phi)e_2 \exp(-i\theta)] \exp(i(\beta_1 z_0 - \omega t)) + c.c.$$  

where $e_1$ and $e_2$ are the slow and fast mode field distributions respectively. Similarly, $\beta_1$ and $\beta_2$ are the slow and fast longitudinal propagation constants. It is convenient to write the superposition of modes in a normalized vector form as in the calculus of Jones,

$$u = \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \exp(-i\theta) \end{bmatrix}$$  \hspace{1cm} (B.2)

Note that we only require two real variables $\phi$ and $\theta$ to describe the polarization state since absolute magnitude and absolute phase are irrelevant.

The Poincaré formalism makes use of the Stokes parameters [94], defined as follows,

$$S_0 = |E_1|^2 + |E_2|^2$$  

$$S_1 = |E_1|^2 - |E_2|^2$$  \hspace{1cm} (B.3)

$$S_2 = 2 \Re(E_1 E_2^*)$$

$$S_3 = 2 \Im(E_1 E_2^*)$$
It can easily be shown that \( S_0^2 = S_1^2 + S_2^2 + S_3^2 \), the equation of a sphere. The Poincaré sphere \( \Sigma \) is the set of all possible polarization states in the Cartesian coordinate system \( \{ \mathbf{S} \} = \{(S_1, S_2, S_3)\} \in \mathbb{R}^3 \). We assume \( S_0 = 1 \) for convenience because the polarization state of the electromagnetic field is independent of amplitude. The conventional spherical co-ordinate system describing \( \Sigma \) is:

\[
S_1 = \cos(2\psi) \sin(2\chi) \\
S_2 = \sin(2\psi) \sin(2\chi) \\
S_3 = \cos(2\chi)
\]

where the angles \( \psi \) and \( \chi \) are given by:

\[
\psi = \frac{1}{2} \arctan(\Re(E_1 E_2^*)/\Im(E_1 E_2^*)) \\
\chi = \frac{1}{2} \arccos(2\Im(E_1 E_2^*))
\]

The physical interpretation of \( \psi \) and \( \chi \) are worth noting. For a general elliptical polarization state, the electric field vector will describe a locus at fixed \( z \). The major axis of the polarization ellipse is an angle \( \psi \) away from the axis defined by \( \mathbf{e}_1 \). The tangent of the angle \( \chi \) is the ratio of major axis to minor axis of the polarization ellipse. An illustration of the standard coordinate system is given in Fig. B.1 (A). With the angles defined as above, we can easily associate points on \( \Sigma \) with definite polarization states. Linear polarization states are located on the \( S_1 - S_2 \) plane. Linear polarization states on the fast and slow axes are located at \((-1, 0, 0)\) and \((1, 0, 0)\), respectively. Polarization states at \(+\pi/4\) and \(-\pi/4\) with respect to the slow axis are located at \((0, 1, 0)\) and \((0, -1, 0)\). Left- and right-handed polarizations are located at \((0, 0, -1)\) and \((0, 0, 1)\).

For our purposes, however, we use the less conventional angles \( \phi \) and \( \theta \) to describe the polarization state on \( \Sigma \) as illustrated in Fig. B.1 (B). The Stokes parameters can be
Figure B.1: The Poincaré sphere $\Sigma$ illustrated with two co-ordinate systems: A. the conventional $\psi$ and $\chi$; B. the alternative system $\varphi$ and $\theta$.

written in terms of the angles $\theta$ and $\varphi$ as.

$$S_1 = \cos(2\varphi)$$

$$S_2 = \cos(\theta) \sin(2\varphi)$$

$$S_3 = \sin(\theta) \sin(2\varphi)$$

(B.6)

The angle $\theta$ is simply the phase of the ratio $E_1/E_2$. The cotangent of $\varphi$ is the amplitude ratio $|E_1|/|E_2|$. The utility of these angles will soon be apparent.

Having introduced the Poincaré sphere formalism, we now introduce three useful properties of $\Sigma$.

1. For every polarization state $\mathbf{S}$, there is a unique polarization state $\mathbf{\tilde{S}}$ orthogonal to $\mathbf{S}$. The state $\mathbf{\tilde{S}}$ is located diametrically opposite to $\mathbf{S}$ on $\Sigma$.

2. The evolution of a polarization state is described by a unitary transformation $\mathbf{U}$ that corresponds to rotation of $\Sigma$ about the axis described by the orthogonal polarization eigenvectors of the transformation $\mathbf{U}$. 
3. The inner product of two fields described by polarization states on \( \Sigma \) has a magnitude equal to \( \cos(\Psi) \), where \( 2\Psi \) is the minimum angle subtending both polarization states.

Proofs of the above are outlined in [93]. With the above arsenal, we are now able to describe the operation of the Lyot filter mathematically and graphically.

### B.4 Frequency Transmission Function

We now consider the Lyot filter structure. The incident polarization analyzer and polarization controller launch light into the birefringent fiber with a polarization state \( S_u \). Propagation through the birefringent fiber gives a wavelength dependent polarization state \( S_{u'} \). The output polarization analyzer and polarization controller are set to pass a polarization state \( S_v \) without loss.

Assuming we have an initial polarization state \( S_u \) at \( z = 0 \), the Jones vector will be,

\[
\mathbf{u} = \begin{bmatrix} \cos(\phi_u) \\ \sin(\phi_u) \exp(-i\theta_u) \end{bmatrix}
\]  

(B.7)

Propagation by a distance \( z \) through the birefringent fiber is given by a diagonal unitary transformation in the basis of the fast and slow modes. The result is a polarization state \( S_{u'} \) with Jones vector.

\[
\mathbf{u}' = \begin{bmatrix} \exp(i\beta_1 z) & 0 \\ 0 & \exp(i\beta_2 z) \end{bmatrix} \begin{bmatrix} \cos(\phi_u) \\ \sin(\phi_u) \exp(-i\theta_u) \end{bmatrix}
\]  

(B.8)

We factor out an irrelevant global phase of \( \beta_1 z \) radians to obtain,

\[
\mathbf{u}' = \begin{bmatrix} \cos(\phi_u) \\ \sin(\phi_u) \exp(-i(\theta_u + 2\pi \Delta w z/c)) \end{bmatrix}
\]  

(B.9)
where $\Delta n = n_1 - n_2$. We see that the angle $\theta$ increases by $\Delta n \omega z / c$ radians upon propagation through the birefringent medium. On the Poincaré sphere, we have an equivalent rotation about the $S_1$ axis, as given by the two polarization eigenvectors corresponding to linear polarization along the fast and slow axes of the fiber. For a given length $L$ of fiber, we thus have an optical frequency periodicity $\Delta \nu$ in polarization state given by:

$$\Delta \nu = \frac{c}{\Delta n L}$$

We take the inner product of the polarization states $S_v$ and $S_{u'}$ to calculate the amplitude transmission function $t$. We do this explicitly in the Jones calculus.

$$t = v^\dagger u = \begin{bmatrix} \cos(\phi_v) & \sin(\phi_v) \exp(i\theta_v) \end{bmatrix} \begin{bmatrix} \cos(\phi_u) \\ \sin(\phi_u) \exp(-i(\theta_u + 2\pi \nu / \Delta \nu)) \end{bmatrix}$$

$$= \cos(\phi_v) \cos(\phi_u) + \sin(\phi_v) \sin(\phi_u) \exp(-i(2\pi \nu / \Delta \nu + \theta_u - \theta_v))$$

The transmittance $T = t^*t$ is readily obtained.

$$T = \cos^2(\phi_v) \cos^2(\phi_u) + \sin^2(\phi_v) \sin^2(\phi_u) + 2 \cos(\phi_v) \cos(\phi_u) \sin(\phi_v) \sin(\phi_u) \cos(2\pi \nu / \Delta \nu + \theta_u - \theta_v)$$

We immediately see that the angle $\theta_u - \theta_v$ simply represents a bias phase, shifting the entire transmission spectrum. The angles $\phi_u$ and $\phi_v$ determine the maximum and minimum transmission.

The transmission amplitude loci and transmittances are illustrated below in Figs. B.2,B.4 and B.3,B.5 respectively for two cases. In the first case, $\phi_u = \phi_v$ at a number of different values. We observe a peak unity transmission and a minimum dependent on the value of $\phi_u = \phi_v$. Launching optical power along fast or slow axes results in unity transmission for all frequencies, while equal optical power in fast and slow axes gives transmission minima of zero. On the Poincaré sphere, the loci are circles of intersection between $\Sigma$ and planes with the normal $S_1$. Transmission maxima occur with the coincidence of $S_{u'}$ and $S_v$. Transmission minima occur when $S_{u'}$ is maximally distant from...
Figure B.2: The amplitude transmission function with $\phi_u = \phi_v$ as the labelled parameter. The trajectories in the complex plane are clockwise with increasing optical frequency.

$S_v$ on $\Sigma$. Nulls in transmission will only occur if the locus of polarization state includes the state $\bar{S}_v$ opposite $S_v$. Only equal excitation of both fast and slow axes allows the polarization to evolve through orthogonal states, such as opposite circular polarizations or linear polarizations with $\theta = \pm \pi/4$.

The second case considered is that of fixed polarization analysis with $\phi_v = \pi/4$ and variable $\phi_u$. The maximum transmittance is unity only for the case of $\phi_u = \pi/4$, as is made obvious through consideration of polarization state loci on $\Sigma$. Furthermore, by considering the loci for $\phi_u \neq \pi/4$, as illustrated below, we see the transmission nulls occur only for $\phi_u = \pi/4$ also. The present case corresponds to launching elliptical polarizations of light into the birefringent fiber and analyzing with a linear polarizer $\pi/4$ radians to the
Figure B.3: The power transmission function with $\phi_u = \phi_v$. One abscissa is the optical frequency normalized to the Lyot FSR and referenced to a bias frequency. The other abscissa is the angle $\phi_u$ normalized to $\pi/2$. 
Figure B.4: The amplitude transmission function with $\phi_v = \pi/4$ and $\phi_u$ as the labelled parameter. The trajectories in the complex plane are clockwise with increasing optical frequency.

slow axis. Maxima (minima) occur when the output polarization ellipticity is greatest and the major axis is $\pi/4$ ($-\pi/4$) radians to the slow axis.

The above analysis gives us the explicit form of the transmittance of the Lyot filter. The periodicity of the transmittance in the frequency domain is determined by the total length of fiber and size of the index difference between slow and fast axes. We can also see that to obtain unity maxima and null minima, we are required to set the polarization controllers and polarization analyzers such that $\phi_u = \phi_v = \pi/4$. In other words, we must excite equally the fast and slow axes of the birefringent fiber. We must also analyze a polarization corresponding to equal excitation of the fast and slow axes of the optical
Figure B.5: The power transmission function with $\phi_v = \pi/4$. One abscissa is the optical frequency normalized to the Lyot FSR and referenced to a bias frequency. The other abscissa is the angle $\phi_u$ normalized to $\pi/2$. 
fiber. Finally, we note that shifting the transmission spectrum is done readily by varying the phase difference between the fields exciting the fast and slow axes of the fiber (or by varying the analyzed polarization state in a like wise manner).

**B.5 Summary**

In conclusion, we have given a complete theoretical description of the linear operation of a Lyot filter. Mathematical and graphical descriptions were provided to elucidate Lyot filter operation and facilitate Lyot filter design.
Appendix C

Phase-Matching for Second Harmonic Generation

C.1 Introduction

The autocorrelation technique employed here was based upon second harmonic generation (SHG) in birefringent LiIO₃. The efficiency of the SHG process is strongly dependent on satisfaction of the phase matching condition [61]. We present here calculations based on a Type I angle tuning approach to achieve phase matching. Two fundamental, ordinary beams are combined to create a second harmonic, extraordinary beam.

C.2 Phase-Matching Condition

The non-interferometric, background free autocorrelation technique requires the efficient generation of SHG from two, non-collinear fundamental beams. The wavevectors of the fundamental beams are here denoted \( k_{\omega+} \) and \( k_{\omega-} \). We let the wavevector of the desired SHG beam be \( k_{2\omega} \). The phase matching condition is simply,

\[
k_{2\omega} = k_{\omega+} + k_{\omega-}
\]  

(C.1)
The phase-matching condition is illustrated below in Fig. C.1. together with the optic axis $c$ of the crystal. The polarizations of both fundamental beams are out of the page, corresponding to ordinary beams. The second harmonic beam is an extraordinary beam. The polarization arrangement here is according to the Type I phase matching scheme. Substituting vacuum wavelengths and indices of refraction for the wavevectors in Eq. C.1. one readily obtains the relation.

$$\tilde{n}_{2\omega,c}(\varphi) = \cos(\psi)n_{\omega,o}$$ (C.2)

where $\varphi$ is the angle between the second harmonic and the optic axis. $2\psi$ is the angular spread of the fundamental beams. $\tilde{n}_{2\omega,c}(\varphi)$ is the $\varphi$-dependent refractive index of the extraordinary second harmonic beam and $n_{\omega,o}$ is the ordinary refractive index of the fundamental beams.

Figure C.1: The phase matching condition for two ordinary fundamental beams with wavevectors $k_{\omega\pm}$ and an extraordinary second harmonic beam with wavevector $k_{2\omega}$. The optic axis $c$ of the crystal is also illustrated.
C.3 Angle-Tuning

In order to satisfy Eq. C.2, we note the refractive index of the extraordinary beam can be represented as [95].

\[
\frac{1}{n_{2\omega}^2(\varphi)} = \frac{\cos^2(\varphi)}{n_{2\omega,o}^2} + \frac{\sin^2(\varphi)}{n_{2\omega,e}^2}
\]

(C.3)

where \(n_{2\omega,o}^2\) and \(n_{2\omega,e}^2\) are the ordinary and extraordinary refractive indices within the crystal. Manipulation of the above and substitution of Eq. C.2 gives us the relation.

\[
\sin^2(\varphi) = \frac{1}{\cos^2(\varphi) n_{2\omega,o}^2} - \frac{1}{n_{2\omega,e}^2}
\]

(C.4)

The required angle of the second harmonic to the optic axis can thus be determined from the angular spread of the fundamental beams in the crystal. The indices of refraction for LiIO\(_3\) can be determined with the Sellmeir formulae [96].

\[
\begin{align*}
n_o^2 &= 2.083648 + \frac{1.332068\lambda^2}{\lambda^2 - 0.035306} - 0.008525\lambda^2 \\
n_e^2 &= 1.673463 + \frac{1.245229\lambda^2}{\lambda^2 - 0.028224} - 0.003641\lambda^2
\end{align*}
\]

(C.5)

where \(\lambda\) is the vacuum wavelength in micrometers. Here, we use \(\lambda = 1.530\ \mu m\), the approximate centre wavelength of fiber laser pulses we wish to characterize.

C.4 External Angles

We are interested in the external angles of the beams when aligning the crystal within the autocorrelator. The refraction of each beam is illustrated in Fig. C.2 below. The optic axis angle in the crystal used here is \(\theta = 30^\circ\).

Snell's law is applied with the angles as illustrated above to yield the following,

\[
\begin{align*}
\sin(\xi^+) &= n_{\omega,o} \sin(\theta - \varphi - \psi) \\
\sin(\xi^-) &= n_{\omega,o} \sin(\theta - \varphi + \psi) \\
\sin(\zeta) &= n_{2\omega,e}(\varphi) \sin(\theta - \varphi)
\end{align*}
\]

(C.6)
Figure C.2: The refraction at the air-crystal interface for the two fundamental beams and the second harmonic beam are illustrated. The shaded region represents the crystal; the unshaded region represents air.

Putting together Eqs. C.6, C.5 and C.4, the external second harmonic angle $\zeta$ can be plotted versus the external fundamental beam spread $\Delta \xi = \xi^- - \xi^+$. The result is given in Fig. C.3 (A) below.
Figure C.3: (A) The phase matched, second harmonic external angle $\zeta$ as a function of the external fundamental beam spread $\Delta \xi$. (B) The mean fundamental external angle $1/2(\xi^+ + \xi^-)$ versus the external fundamental beam spread $\Delta \xi$. The crystal axis is $30^\circ$ with respect to normal, as in the crystal used.
Appendix D

Derivation of the Ginzburg-Landau Equation

D.1 Introduction

The theoretical treatment we give here is based upon that first given by Agrawal [81, 69]. but we provide here corrections and further details as is seen to be helpful. Notes regarding the validity of various approximations are presented as the approximations are made. We begin with the Bloch equations for the interaction of optical fields with a collection of noninteracting rare-earth ions. We then proceed to apply Maxwell’s equations to a lossless fiber, with the rare-earth ion interaction with the optical field and Kerr nonlinearity treated as first order perturbations. The final equation of motion derived is shown to be of Ginzburg-Landau form.

D.2 Bloch Equations

We begin first by considering the interaction of an optical electromagnetic field with a collection of rare earth ions sitting in bulk silica glass. The relevant energy structure of erbium ions is not a simple three-level system, but is in fact three manifolds due to
Stark splitting and inhomogeneity in ion sites within the glass matrix [25]. However, experiments by Desurvire et al. [39] have shown that EDFA’s behave similarly to strictly homogeneously broadened media at room temperature. Thus, we consider the erbium ions as simple, homogeneously broadened three level systems.

We will now apply the Bloch equations of motion for the ions interacting with an electromagnetic field. The characteristic time $T_1$ for population relaxation from the excited state to the ground state is on the order of 10 ms [97]. The dipole dephasing time $T_2$, or the inverse of the homogeneous linewidth, is on the order of 100 fs [38]. We are concerned with pulses that possess widths greater than $T_2$ but less than $T_1$. We assume a material inversion changing in time on scales of $T_1$, since coherent effects such as Rabi oscillations will last for times comparable to $T_2$ and will generally be unobservable at room temperature. The Bloch equations take the following form [61],

$$\ddot{P} + \frac{2}{T_2} \dot{P} + \omega_a^2 P = -\frac{2\omega_a L |\mu|^2 N}{3\hbar\epsilon_0} E$$

$$\dot{N} + \frac{N - N_{eq}}{T_1} = \frac{2}{\hbar\omega_a} \langle \dot{P} \cdot E \rangle$$

where $P$ is the rare-earth polarization, $E$ the electric field, time derivatives are indicated by raised dots, time averages are given by angled brackets, $\epsilon_0$ is the vacuum permittivity, $\omega_a$ is the resonance frequency of the ions, $|\mu|$ is the dipole moment moment of the transition, $N = N_2 - N_1$ is the population inversion, $N_{eq}$ is the equilibrium population inversion in the absence of the resonant optical field and $L$ is the Lorentz correction factor relating macroscopic fields to microscopic fields. Note that a factor of $1/3$ is present to account for random orientations of the dipoles within the glass host. The polarization of the electric field and rare-earth polarization are assumed constant and aligned. Since there is only residual birefringence within the EDFA, this is an appropriate approximation.

Introducing the Fourier transforms of $P$ and $E$ as follows,

$$P(r, \omega) = \int_{-\infty}^{\infty} P(r, t) \exp(i\omega t) dt$$

$$E(r, \omega) = \int_{-\infty}^{\infty} E(r, t) \exp(i\omega t) dt$$

(D.2)
we can arrive at a susceptibility $\chi(\omega)$ satisfying the relation.

$$P(r, \omega) = \epsilon_0 \chi(\omega) E(r, \omega)$$

(D.3)

Applying Fourier transforms to the Bloch equations of Eq. D.1, we obtain

$$\chi(\omega) = \frac{2\omega_a L|\mu|^2 N}{3\hbar\epsilon_0} \frac{1}{\omega^2 - \omega_a^2 + 2i\omega/T_2}$$

(D.4)

Since we are concerned with optical frequencies $\omega \approx \omega_a$, we can approximate $\omega^2 - \omega_a^2 \approx 2\omega(\omega - \omega_a)$ and thus rewrite $\chi(\omega)$ in the form

$$\chi(\omega) = \frac{c}{\omega} \sigma N \frac{1}{(\omega - \omega_a)T_2 + i}$$

(D.5)

where $c$ is the vacuum speed of light and we have introduced the cross-section $\sigma = T_2 \omega_a L|\mu|^2/3c\hbar\epsilon_0$. The reason for the introduction of $\sigma$ defined in the above manner will become apparent as will its interpretation as a cross-section. The form of $\chi(\omega)$ is the familiar complex Lorentzian as expected for resonant interactions of light with atoms or ions.

Before we continue in deriving an equation of motion for optical pulse propagation in fiber, we further specify $N$ by simplification of the second of Eq. D.1. The final result, whose technical derivation is given by Siegman [37], can be written in the form

$$\dot{N} = -\frac{N - N^{eq}}{T_1} = \frac{N}{T_1} \frac{I}{I_{sat}}$$

(D.6)

where $I$ is the optical intensity averaged over a time much greater than $T_2$ but shorter than $T_1$. The saturation intensity is given by $I_{sat} = \hbar\omega/2\sigma T_1$. The expression above is strictly valid for a narrow optical spectrum with zero detuning from resonance, but the reduction in saturation due to the violation of these restrictions is negligible in cases of interest to us here. It is edifying to consider the form of $N$ at steady state,

$$N = \frac{N^{eq}}{1 + I/I_{sat}}$$

(D.7)

The cause of homogeneous gain saturation is clearly seen to be the reduction of the gain medium inversion.
D.3 Maxwell's Equations

We now turn our attention to Maxwell's equations, beginning by considering an undoped, lossless, linear fiber first. We follow the method of Argawal [69], beginning with Maxwell's vector equation for the electric field.

\[ \nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \]  

(D.8)

For weakly guiding fibers, as is the case of interest, the semi-vectorial approximation applies.

\[ \nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \approx -\nabla^2 \mathbf{E} \]  

(D.9)

Maxwell's wave equation thus takes the familiar form.

\[ \nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}(\mathbf{r}, t)}{\partial t^2} \]  

(D.10)

Taking Fourier transforms in time, we can use the relation \( \mathbf{P}(\mathbf{r}, \omega) = \epsilon_0 \chi_0(\mathbf{r}, \omega) \mathbf{E}(\mathbf{r}, \omega) \) where \( \chi_0(\mathbf{r}, \omega) \) is the susceptibility of the silica constituents of the fiber.

\[ \nabla^2 \mathbf{E}(\mathbf{r}, \omega) + \frac{\omega^2}{c^2} (1 + \chi_0(\mathbf{r}, \omega)) \mathbf{E}(\mathbf{r}, \omega) = 0 \]  

(D.11)

The z-axis is chosen as the fiber axis and we define a relative permittivity dependent solely on transverse coordinates \( \epsilon_r(x, y, \omega) = 1 + \chi_0(\mathbf{r}, \omega) \). In determining the fiber modes, we assume a solution of the form \( \mathbf{E}(\mathbf{r}, \omega) = \hat{e} F(x, y, \omega) \exp(i\beta(\omega)z) \). Simply, \( \hat{e} \) is a unit direction vector. \( F(x, y, \omega) \) is the mode profile for our single mode fiber and \( \beta(\omega) \) is the longitudinal propagation constant. The propagation constant and mode profile are determined through substitution into Eq. D.11.

\[ \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] F(x, y, \omega) + \epsilon_r(x, y, \omega) \frac{\omega^2}{c^2} F(x, y, \omega) = \beta(\omega)^2 F(x, y, \omega) \]  

(D.12)

For a given relative permittivity, the above equation can be solved by numerous methods. Analytic solutions exist only for certain index profiles. More complex profiles require the use of numerical methods such as the finite element method, which is a robust and efficient method widely employed [98]. The mode profile and propagation constant are assumed to be known from here onwards.
D.4 First-Order Perturbation Theory

We are now in a position to consider perturbations to the propagation of light in a linear, lossless fiber. Following Agrawal [69], the relative permittivity is perturbed to give the following to first order.

\[ \epsilon(\omega) = n^2(\omega) + in(\omega) \frac{c}{\omega} \alpha + \chi(\omega) + \Delta n^2(\omega) \]  

(D.13)

where \( n(\omega) = \beta(\omega)/c\omega \) is the effective mode index, \( \alpha \) is the fiber loss, \( \chi(\omega) \) is the susceptibility of the rare-earth ions and \( \Delta n^2(\omega) \) is the change in the square of effective index due to the optical Kerr nonlinearity of the silica glass. Explicitly, the optical Kerr nonlinearity is the intensity dependence of refractive index

\[ n_f = n_0 + n_2 I(x, y) \]  

(D.14)

where \( n_f \) is the silica refractive index, \( n_0 \) is the linear refractive index, \( n_2 \) is the Kerr coefficient and \( I(x, y) \) is the local light intensity.

The perturbations to the relative permittivity may be considered individually, and are taken into account by considering Eq. D.12 to determine the perturbation to the propagation constant \( \beta(\omega) \). Letting \( \tilde{\beta} \) be the perturbed propagation constant, we have

\[ \tilde{\beta}^2 - \beta^2 = \frac{\omega^2}{c^2} \iint F^* \Delta \epsilon_r F \, dx \, dy \]  

(D.15)

Assuming a small perturbation as is required in our first order approach, we can approximate \( \tilde{\beta}^2 - \beta^2 \approx 2\beta (\tilde{\beta} - \beta) \). Thus, the perturbation to the propagation constant is,

\[ \tilde{\beta} - \beta = \frac{\omega}{2n(\omega)c} \iint F^* \Delta \epsilon_r F \, dx \, dy \]  

(D.16)

We now apply the above formula to calculate the propagation constant perturbation from loss, gain and optical Kerr nonlinearity respectively. We assume uniform loss across the transverse plane and obtain the simple result,

\[ (\tilde{\beta} - \beta)_{\text{loss}} = \frac{1}{2} i\alpha \]  

(D.17)
The perturbation due to gain requires more careful treatment since the rare-earth ions will be confined to a volume within the fiber core. We thus assume a transverse density profile of ions $\rho(x, y)$, normalized such that $\iint \rho(x, y) dx dy = 1$. The rare-earth ions will thus contribute.

$$\langle \beta - \beta \rangle_{RE} = \frac{1}{2} \frac{\omega}{n(\omega)c} \Gamma \chi(\omega)$$  \hspace{1cm} (D.18)

where the overlap factor $\Gamma$ is given by

$$\Gamma = \frac{\iint F^* F dx dy}{\iint F^* F dx dy}$$  \hspace{1cm} (D.19)

The Kerr nonlinearity requires a similar treatment. Using the approximation $\Delta n^2 \approx 2n(\omega)n_2 I(x, y)$, we again arrive at a simple expression for the $\beta$ perturbation.

$$\langle \beta - \beta \rangle_{Kerr} = \gamma P_0$$  \hspace{1cm} (D.20)

where $P_0$ is the total optical power and $\gamma = n_2 \omega / c A_{eff}$. The effective area $A_{eff}$ is defined as.

$$A_{eff} = \frac{\left(\iint F^* F dx dy\right)^2}{\iint F^* F F dx dy}$$  \hspace{1cm} (D.21)

Adding the contributions of Eqs. D.17, D.18, D.20, we arrive at the total perturbation.

$$\beta - \beta = i \alpha + \frac{\omega}{2 n(\omega)c} \Gamma \chi(\omega) + \gamma P_0$$  \hspace{1cm} (D.22)

With the above perturbation, we can proceed to derive an equation of motion for optical pulses.

As was mentioned earlier, we assume pulses of width greater than $T_2$, but less than $T_1$. In this regime, an envelope approximation is appropriate.

$$E(r, t) = \frac{1}{2} (\hat{\epsilon} F(x, y) A(z, t) \exp[i(\beta_0 z - \omega_0 t)] + c.c.)$$  \hspace{1cm} (D.23)

We will also make use of the Fourier transform of the envelope.

$$A(z, \omega) = \int_{-\infty}^{\infty} A(z, t) \exp(i \omega t) dt$$  \hspace{1cm} (D.24)
APPENDIX D. DERIVATION OF THE GINZBURG-LANDAU EQUATION

The complex envelope $A(z, t)$ thus carries the information of pulse amplitude and phase at a carrier optical frequency $\omega_0$ with corresponding propagation constant $\beta_0$. Furthermore, under our assumptions about pulse width, we expect $A(z, t)$ to have significant spectral components only for frequencies $|\omega| \ll \omega_0$. An equation of motion for $A(z, t)$ will now be determined. We take the Fourier transform of Eq. D.23 with the intent to substitute the result into Eq. D.11.

$$
\mathbf{E}(r, \omega) = \frac{1}{2} (\hat{e} F(x, y) A(z, \omega - \omega_0) \exp(i\beta_0 z) + \hat{e} F^*(x, y) A^*(z, \omega + \omega_0) \exp(i\beta_0 z)) \\
= \frac{1}{2} \hat{e} F(x, y) A(z, \omega - \omega_0) \exp(i\beta_0 z)
$$

(D.25)

The approximation we have made above, based upon the assumption that $A(z, t)$ has components only for frequencies less than the optical carrier, restricts us to forward (positive $z$) propagating pulses only. To simplify the continuing analysis, we use the shifted frequency $\tilde{\omega} = \omega - \omega_0$. Substitution of the above result for $\mathbf{E}(r, \omega)$ into Eq. D.11 yields the following after some algebraic gymnastics.

$$(\beta^2(\omega) - \beta_0^2) A(z, \tilde{\omega}) + \frac{\partial^2 A(z, \tilde{\omega})}{\partial z^2} + 2i\beta_0 \frac{\partial A(z, \tilde{\omega})}{\partial z} = 0$$

(D.26)

where $\beta(\omega)$ is determined from Eq. D.12. The slowly varying envelope approximation (SVEA) can be used for pulse envelopes which change negligibly over distances on the order of optical wavelengths. We are considering such pulses and thus the SVEA allows us to neglect $\partial^2 A(z, \tilde{\omega})/\partial z^2$ relative to the other terms appearing in Eq. D.26. Using $\beta^2(\omega) - \beta_0^2 \approx 2\beta_0 (\beta(\omega) - \beta_0)$, we arrive at the peculiarly elegant equation of motion for $A(z, \tilde{\omega})$.

$$
\frac{\partial}{\partial z} A(z, \tilde{\omega}) = i(\beta(\omega) - \beta_0) A(z, \tilde{\omega})
$$

(D.27)

The above equation is quite general, and numerous physical effects may be taken into account through appropriate modification of $\beta(\omega)$. We proceed to derive the promised Ginzburg-Landau equation by including the physical effects claimed to be relevant. Prior
to considering the perturbations given in Eq. D.22, we expand $\beta(\omega)$ in a Taylor series.

\[
\begin{aligned}
\beta(\omega) &= \beta_0 + \left( \frac{\partial \beta}{\partial \omega} \right)_{\omega=\omega_0} \tilde{\omega} + \frac{1}{2} \left( \frac{\partial^2 \beta}{\partial \omega^2} \right)_{\omega=\omega_0} \tilde{\omega}^2 + \frac{1}{6} \left( \frac{\partial^3 \beta}{\partial \omega^3} \right)_{\omega=\omega_0} \tilde{\omega}^3 + \ldots \\
&= \beta_0 + \beta_1 \tilde{\omega} + \frac{1}{2} \beta_2 \tilde{\omega}^2 + \frac{1}{6} \beta_3 \tilde{\omega}^3 + \ldots 
\end{aligned}
\] (D.28)

The dependence of $\beta$ upon $\omega$ corresponds to the combination of material and waveguide dispersion within the fiber. In the work considered here, $\tilde{\omega}$ is sufficiently small that we need only consider terms up to second order in $\tilde{\omega}$. Inspection of the terms in the above expansion allows one to make the following physical connections. The phase velocity is trivially given by $v_\phi = \omega_0 / \beta_0$, as expected with our assumption about carrier velocity. The term $\beta_1 = \partial \beta / \partial \omega$ is the inverse of the group velocity $v_g = \partial \omega / \partial \beta$. The second order coefficient $\beta_2$ is referred to as the group velocity dispersion (GVD) parameter. The greater the GVD parameter, the greater the curvature of $\beta$ with $\omega$. The physical result is a difference in group velocities of constituent spectral components of a pulse, resulting in chirping and broadening with propagation. The above expansion includes effects which preserve pulse energy, but result in phase accumulation and possible pulse spreading or compression.

Prior to deriving the specific equation of motion from Eq. D.27, we make one final Taylor expansion. The susceptibility $\chi(\omega)$ of Eq. D.5 gives the detailed frequency dependence of the rare-earth response. As with the material and waveguide dispersion of the optical fiber, we are interested in the material response over a selected frequency range. The work here is concerned with laser oscillations, and we thus expect a carrier frequency for our pulses to correspond to the gain peak in the absence of cavity losses forcing oscillation elsewhere. Thus, we set $\omega_0 = \omega_a$ and expand the susceptibility up to second order. Forseeing that we will be summing the rare-earth perturbation to $\beta(\omega)$.
we give the modified coefficients of expansion of $\beta(\omega)$.

\[
\beta_0^{eff} = \beta_0 - \frac{i}{2} \frac{\Gamma\sigma}{n(\omega_0)} N \\
\beta_1^{eff} = \beta_1 + \frac{1}{2} \frac{\Gamma\sigma}{n(\omega_0)} NT_2 \\
\beta_2^{eff} = \beta_2 + i \frac{\Gamma\sigma}{n(\omega_0)} NT_2^2
\]

(D.29)

The physical meaning behind the modifications of the $\beta(\omega)$ are easily seen. The imaginary contribution to $\beta_0^{eff}$ is the gain provided by the rare-earth ions, proportional to the inversion $N$. The real contribution to $\beta_1^{eff}$ is the result of the group delay at the linecentre of the Lorentzian response. The second order contribution to $\beta_2^{eff}$ is imaginary and is the curvature of the Lorentzian gain profile. The curvature models the limited bandwidth of the gain provided by the erbium ions.

Incorporating the expansion coefficients of $\beta(\omega)$ in Eq. D.29 and including the perturbation terms of Eqs. D.17, D.20, the equation of motion D.27 becomes the following upon inverse Fourier transformation.

\[
\frac{\partial}{\partial z} A(z,t) = \left( -\beta_1^{eff} \frac{\partial}{\partial t} - i \frac{1}{2} (\beta_2 + gT_2^2) \frac{\partial^2}{\partial t^2} + \frac{1}{2} (g - \alpha) + i\gamma |A(z,t)|^2 \right) A(z,t)
\]

(D.30)

where we have defined the gain $g = \Gamma\sigma N/n(\omega_0)$ and we have assumed the pulse envelope $A(z,t)$ is scaled to root power, meaning $P_0 = |A(z,t)|^2$. The transformation to a moving reference frame with $T = t - \beta_1^{eff} z$ gives us our heralded equation of motion.

\[
\frac{\partial}{\partial z} A(z,T) = \left( -i \frac{1}{2} (\beta_2 + gT_2^2) \frac{\partial^2}{\partial T^2} + \frac{1}{2} (g - \alpha) + i\gamma |A(z,T)|^2 \right) A(z,T)
\]

(D.31)

The above equation is in the promised form of a Ginzburg-Landau equation.
Bibliography


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