This dissertation consists of three independent essays in Macroeconomics. The first essay analyzes monetary coordination between currency areas. It is shown that search frictions can generate the deviations from the law of one price and that each country is tempted to exploit these deviations by inflation. Monetary coordination eliminates the inefficiency caused by inflation. The welfare gains from coordination increase when the two economies become more integrated. In contrast to traditional models, the need for coordination exists even after each country is allowed to directly tax foreign holdings of its currency.

The second essay studies the behavior of exchange rates in an environment with search frictions. In contrast to traditional models, even without any nominal rigidity, the model can generate enough volatility of exchange rates found in the data. The changes in the behavior of exchange rates under different regimes are also examined in this essay. The model shows a sharp increase in the volatility of exchange rates when moving from a pegged to a floating exchange regime,
while there is no such systematic change in fluctuations of output or consumption. Moreover, the co-movements of output and consumption across countries are higher under a fixed rate regime than under a flexible rate regime. These results are consistent with empirical findings.

The final essay focuses on the competition between groups of allied firms. In the essay we propose a model of group fitness and develop an approach to evaluate the fitness of groups and the utility of their member firms. A group has high fitness if member firms have four features: (i) high capacity, (ii) being embedded in dense relationships, (iii) holding complementary resources and (iv) having limited competition and conflict. We illustrate the effectiveness of our model and methodology by applying it to the airline groups between 1997 and 2002. By examining what really happened to the airline groups afterwards, we found that the predictions based on the comparison between the fitness scores of actual groups formed and those of the corresponding population constructed are reasonably accurate, and that the implications based on the ranking of individual firm utility within each group are generally supported.
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Introduction

This thesis consists of three independent essays in Macroeconomics. In the first essay, we investigate the need for monetary coordination between currency areas in an environment with search frictions. The second essay focuses on the dynamics of exchange rates and thus goes beyond the steady-state analysis that explains only the behavior of exchange rates. The final essay studies interfirm networks and evaluates the interfirm group fitness in the global airline industry. Each essay is presented as a separate chapter in the thesis.

Whether countries should coordinate their monetary policy has been an important issue in international economics and in policy debate. Given the increase in trade flows across countries, this subject becomes more important and more interesting. The models presented in literature on monetary coordination usually impose ad hoc assumptions such as cash-in-advance constraints or money in the utility function, which have generated mixed results regarding the need for monetary coordination based on different assumptions. The lack of a strong microfoundation for money renders welfare analyses in these models questionable, subjecting them to the Lucas critique.

In the first essay, chapter 1, we integrate monetary search theory into international economics and present a tractable framework with a strong microfoundation for money. This framework is effective in dealing with the issues of currencies and exchange rates since values for the currencies can be derived endogenously from detailed descriptions of preferences and technologies. Currency areas are modelled in this chapter, and it is shown that the nominal exchange rate and the size of a currency area are determinate in equilibrium. Although prices are perfectly flexible and all goods are tradable between the two countries, search frictions can generate deviations from the
law of one price. Each country is tempted to exploit these deviations by increasing inflation. As a result, both countries set their money growth rates above the Friedman rule in a non-cooperative game. Monetary coordination restores inflation to the social optimal level and improves the welfare for all countries. Moreover, the welfare gains from monetary coordination increase when the two economies become more integrated in the goods market.

Monetary coordination is not a Nash Equilibrium. Thus, the coordination outcome would not be sustainable if there were no punishment on the deviations. Adopting a unified currency is a reliable way to provide a commitment to maintain the efficient outcome. The model in the first chapter suggests that two highly integrated economies are more likely to adopt a common currency.

Inflation can generate the effect of "beggar-thy-neighbour" in traditional models, in which it serves effectively as a tax on foreign holdings of the currency. The role of inflation usually can be eliminated if each country is allowed to directly tax foreign holdings of its currency. In our model, however, the tax is not a perfect substitute for inflation because inflation affects the deviations from the law of one price in more ways than the tax. The need for monetary coordination exists even after each country sets such a tax at an optimal level.

The highly volatile and persistent exchange rates have always been a central puzzle in the theory of international business cycles. The behavior of exchange rates evidenced by many empirical works can hardly be explained by the structural models estimated on data. The second essay, chapter 2, studies the behavior of exchange rates in a stochastic environment with search frictions.

The conventional explanation for fluctuations of exchange rates usually rests on the interaction of nominal rigidities and monetary shocks. While such models are potentially capable of accounting for the dynamics of exchange rates, they typically need a long period of price stickiness
to capture the volatility and persistence of exchange rates shown in the data. Recent empirical works, however, casts serious doubt on the validity of such an assumption.

Do we have to impose the assumption of long-lived nominal rigidities to produce enough volatility and persistence of exchange rates found in the data? To answer this question, we deviate substantially from current theories by integrating monetary search theory into international economics to derive the role of money in a two-country economy. Even though prices are fully flexible and goods are tradable across countries, search frictions in the goods market are capable of generating the price differentials in the goods market. The law of one price is violated and generally purchasing power parity (PPP) does not hold. We show that fluctuations of the nominal exchange rate come entirely from the changes in the differential in the valuations of the two currencies, and the variations of this differential account for a large part of fluctuations in the real exchange rate.

In accordance with empirical studies, monetary shocks explain most fluctuations of exchange rates in our model, mainly due to the deviations from the law of one price which depend critically on the differential between the valuation by each country of the two currencies. Because money growth shocks directly affect this differential, they account for a larger part of the volatility in exchange rates than productivity shocks.

The behavior of exchange rates in different regimes is also examined in this chapter. The model shows a sharp increase in the volatility of the real exchange rate when moving from a pegged to a floating exchange regime, while fluctuations of output and consumption do not change systematically with the regimes. Moreover, the co-movements of output and consumption across countries are higher under a fixed rate regime than under a flexible rate regime. These results are consistent with the empirical findings.
Chapter 2 suggests that even without any nominal rigidity, search frictions in the goods market are capable of generating the high volatility of exchange rates as observed in the data.

The final essay, chapter 3, deals with the issue of interfirm networks. As it is well-known, interfirm networks are collections of firms joined by ties that vary in formality but are stable and significant enough to create reasonably persistent interfirm structures. These structures are reshaping our view of traditional organizational governance mechanisms of market and hierarchy.

Research on interfirm networks tends to focus on examining general motivations for interfirm collaboration and firms’ partner selection decisions. The unit of analysis in these studies focuses mainly on individual firms or firm dyads. Little attention has been paid to interfirm groups. Since competition increasingly takes the form of groups of allied firms against other groups instead of the traditional battle of firm versus firm, it is essential for us to focus the analysis on interfirm groups rather than individual firms. Chapter 3 is motivated by the dearth of reported research on overall group fitness.

In this study, we propose a model of group fitness and develop an approach for calculating the fitness of groups and the utility of their member firms. More specifically, a group has high fitness if member firms have four features: (i) high capacity, (ii) being embedded in dense relationships, (iii) holding complementary resources and (iv) having limited competition and conflict.

We illustrate the effectiveness of our model and methodology by applying it to the airline groups between 1997 and 2002. Given a plausible set of assumptions concerning firm capacity, tie embeddedness, resource complementarity and rivalry, we compute group fitness and individual firm utility within each group. The predictions based on the comparison between the fitness scores of actual groups formed and those of the corresponding population constructed are consistent with the genuine results, and the implications based on the ranking of individual firms’ utilities within each group are generally supported.
By focusing on the fitness of groups as a whole, the study departs substantially from traditional network literature, which tends to focus on individual characteristics to explain issues of network involvement and firm performance. Our work on interfirm groups provides a new perspective of studying interfirm networks and extends theories of organizational networks. Although mainly focused on the competition between groups, our work also contains implications for the study of individual firms’ entry and exit decisions. By examining the ranking of each firm’s utility within each group, the model can be used to study the overall network evolution and dynamics.

Compared with previous work on interfirm groups (Axelrod et al., 1993), we develop a more general method to deal with the social and instrumental antecedents, exploring different fitness specifications, including some that are nonlinear, and allowing asymmetric variables in the fitness function. Practically, the approach in the essay can be used to examine the current status of an interfirm group and evaluate the value-added by potential group member candidates.
CHAPTER 1

Currency Areas and Monetary Coordination

1.1. Introduction

There is convincing evidence that the law of one price (LOP for short) does not hold between countries (see Goldberg and Knetter, 1997, and Crucini et al., 2005). Moreover, deviations from the LOP depend regularly on the nominal exchange rate (Knetter, 1989). Given this dependence, each country may have incentive to use inflation to exploit the deviations. However, because inflation generates a deadweight loss, all countries can gain from coordinating on reducing inflation. We capture this need for monetary coordination in an environment with two currency areas. By integrating a monetary search model into open-economy macro, we derive a role of each currency and allow individuals to choose the frequency of using each currency. Despite perfectly flexible prices, search frictions generate deviations from the law of one price, which create the temptation to inflate and, hence, the need for monetary coordination.

Monetary coordination is not a new issue. At least as early as Mundell (1961), open-economy macroeconomists have examined the gains and losses for countries to coordinate on monetary policy or to form a currency area. The focus has been on whether countries should use short-run inflation to offset other shocks when nominal rigidities give rise to the Phillips curve (e.g., Obstfeld and Rogoff, 2002, and Ching and Devereux, 2003). In similar environments, there are also analyses of whether monetary coordination is beneficial when optimal monetary policy is time inconsistent (e.g., Rogoff, 1985). However, this literature does not emphasize deviations from the law of one price as a reason for monetary coordination.
Another shortcoming of the literature is that it does not derive the role of a currency from basic characteristics such as preferences and technologies. Instead, it assumes cash-in-advance constraints and money in the utility function. These assumptions are at odds with the observation that a country’s exports are often invoiced in more than one currency (see Page, 1981). Even for the U.S., which seems the only country whose imports and exports are both invoiced predominantly in the domestic currency, the invoicing arrangement is a result of choices rather than regulations and it can respond to changes in monetary policy and the trading environment. Fixing the role of a currency by assumptions renders welfare analysis of monetary policy vulnerable to the Lucas critique, especially for policies on currency competition and monetary coordination.

To see how sensitively the results in the literature depend on the assumptions on currencies, consider the model by Helpman (1981) where the goods in two countries are perfectly substitutable to each other but goods must be purchased only with the currency of the seller’s country. In contrast to the endowment economy in Helpman, assume that goods are produced using elastically supplied labor. Then, a unilateral increase in inflation reduces a country’s consumption and welfare by reducing the purchasing power of that country’s currency. There is no incentive to inflate and, hence, no need to coordinate on long-run inflation. The result is reversed if all currencies can be used to purchase all goods. In this case, the nominal exchange rate is constant over time and a version of the so-called Gresham’s Law holds. That is, the currency whose supply grows at a higher rate will asymptotically occupy the entire portfolio of currency holdings (see Kareken and Wallace, 1981). This feature provides a strong incentive for each country to inflate. Coordination reduces inflation and increases welfare for both countries.

The sensitivity is not unique to cash-in-advance models. In models with money in utility functions (e.g., Obstfeld and Rogoff, 1998), the result on coordination depends on the assumption of how substitutable different currencies are in utility functions.
To eliminate this problem, we derive a role of each currency using the search theory of money (see Shi, 1995, and Trejos and Wright, 1995). In contrast to the large literature on open-economy macro, we assume perfectly flexible prices, focus on long-run inflation, and abstract from the issue of time inconsistency. To emphasize the deviations from the LOP, we assume that the two countries have the same production technology and preferences. The countries produce an identical set of goods and all goods are tradeable between the two countries. There are two goods markets and one currency market. Markets are separated and so arbitrage between two markets is not possible. The currency market is centralized and Walrasian. The exchange in goods markets is decentralized and modeled as random matching. In equilibrium, a currency is valued because it alleviates the difficulty of exchange, but it does not yield direct utility or facilitate production.

Each goods market uses a particular currency and is referred to as a currency area. However, households can choose the frequency of using each currency. Because goods sold in the two areas are perfect substitutes, the choice between the currencies implies that there is no cash-in-advance constraint. Nevertheless, the nominal exchange rate and the size of a currency area are determinate in equilibrium. Key to the determinacy is an asymmetry in the matching technology, which we call local congestion. That is, by entering a goods market, a buyer increases congestion to other buyers from the same country by more than to the other country’s buyers. Local congestion is a realistic feature, because foreign buyers typically buy from particular distributional networks or regions. Facing local congestion, a household chooses a unique optimal allocation of buyers and sellers to each currency area, which determines the nominal exchange rate.

The model not only provides a microfoundation of exchange rates, it also uncovers two new features that are important for monetary coordination. One is that consumption depends on

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1The common usage of the term “currency area” refers to a group of countries that issue the same currency. For convenience of the analysis, we use the term to refer to a group of transactions that use the same currency. Our usage captures the fact that a country’s exports are invoiced in more than one currency (see Page, 1981).
the number of trades as well as the quantity of goods traded in each match. This feature is a natural implication of search frictions and non-Walrasian pricing. The other feature is deviations from the LOP. Even after controlling for the buyer’s country and the currency used, sellers from different countries charge different prices. Similarly, even after controlling for the seller’s country and the currency used, buyers from different countries pay different amounts. These deviations exist as long as the two countries have different money growth rates.

The deviations from the LOP arise from asymmetric monetary transfers and local congestion in matching. Because monetary transfers are given to only domestic households, the two countries value a given currency differently if the two countries have different money growth rates. This differential in the valuation cannot be eliminated by arbitrage in the currency market – Such arbitrage only equates the relative valuation of the two currencies between the two countries, not the absolute valuation of a given currency. Moreover, because of local congestion in the goods market, it is not optimal for the households to adjust their allocations of buyers and sellers to the extent of eliminating the cross-country differential in the valuation of a currency. This differential implies that the sellers in different countries are willing to give different quantities of goods for a given currency, and that the buyers in different countries are willing to pay different amounts of a given currency for the same goods.

Moreover, the deviations from the LOP depend on money growth and this dependence motivates each country to inflate. For example, suppose that country 1’s money growth is higher than country 2’s. Then, country 1’s households hold more of both currencies in the steady state than country 2’s households and, consequently, value each currency by less than country 2’s households. A seller of country 2 is willing to sell more goods for a currency than does a seller of country 1, and a buyer of country 2 is willing to pay more for the same goods than does a buyer of country 1. As a result, country 1 gains from its higher money growth. As each country inflates to exploit
the deviations, money growth rates in the two countries are higher than the Friedman rule in the equilibrium. They are bounded above because inflation also reduces the purchasing power of money. Both countries are worse off relative to the Friedman rule. Coordination between the countries reduces inflation and improves welfare for the two countries. Moreover, as the barriers to selling in the foreign goods market fall, the temptation to inflate increases and, hence, the gain from monetary coordination increases.

The focus on the deviations from the LOP contrasts sharply with the traditional mechanism that countries inflate to manipulate the terms of trade (e.g., Cooley and Quadrini, 2003). The terms of trade are not important in our model because all goods are tradable and the two countries’ products are perfectly substitutable. This deliberate setup not only helps us to focus on a different mechanism, but also makes our results more robust in the following sense. In models that rely on the terms of trade, inflation acts effectively as a tax on foreign use of a currency. Once such direct taxes are introduced, there will be no incentive for a country to inflate and, hence, no need for monetary coordination. This is not the case in our model. After allowing each country to set the direct taxes optimally on foreign use of the country’s currency, we show that the temptation to inflate continues to exist in our model. The reason is that inflation affects all relative prices between the two countries, while the taxes affect only those in particular types of matches.2

In comparison with the literature of money search, our model provides a tractable extension of the literature to a two-country environment.3 The closest paper is the one by Head and Shi

---

2Alessandria (2004) uses search frictions to generate deviations from the LOP, albeit differently. Our model is complementary to his in the focus and results. First, his model is a real model, which has nothing to say about currencies or the effects of monetary policy on the deviations from the LOP. Second, deviations from the LOP in his model rely on a differential in the marginal cost of production between domestic and foreign firms, which we abstract from deliberately.

3One of the first papers in search theory of money is Kiyotaki and Wright (1993). Early applications of the theory to multiple currencies and exchange rates include Matsuyama, et al. (1993), Shi (1995), Zhou (1997) and Wright and Trejos (2001). These applications have assumed money or goods, or both, to be indivisible. As a result, they are not suitable for analyzing money growth and inflation. Camera et al. (2004) allow agents to hold two units of money. The current model eliminates the restriction of indivisibility by using the device in Shi (1997), while Lagos
(2003), with which our model shares a number of important ingredients. The main difference lies in the focus. Head and Shi focus on the determination of the nominal exchange rate, and so they do not examine monetary coordination. Another notable difference is that we model the currency market as a Walrasian market, while Head and Shi model currency exchange as random matching. The modeling assumption here is more realistic (see Obstfeld, 1998, for a discussion), and it makes deviations from the LOP more robust.

1.2. The Model

1.2.1. Two Countries and Currency Areas

Consider a world with two countries, indexed by \( i \in \{1, 2\} \). The two countries have the same size, preferences and production technology. All goods are tradeable between the two countries and there is no distinction between the same type of goods produced in the two countries. This description of the world abstracts from a large part of the gains from international trade in reality, but the abstraction is useful for two reasons. First, the abstraction sharpens our focus on competing currencies, which traditional models have sidestepped with assumptions such as cash-in-advance constraints. Second, if monetary coordination is beneficial in this simple environment, the benefits are likely to be higher when the gains from trade are larger.

In each country, there are many types of infinitely-lived households. The households of each type can produce only a particular type of goods, at a cost (disutility) \( \phi(q) = q^\sigma \) for \( q \) units of production, where \( \sigma > 1 \). They consume only another type of good, which is produced by some other households in the two countries. Hence, there is no double coincidence of wants between two households. Goods are nonstorable. Without loss of generality, we let the utility function of consumption be \( u(q) = Aq \), where \( A > 0 \) is a constant.

and Wright (2005) provide an alternative way to do so. Craig and Waller (2004) also construct a model with two currencies, but their model admits only numerical solutions.
A household consists of a continuum of members. The members do not make decisions; instead, they trade according to the instructions given by the household and regard the household’s utility as the common objective. The measure of members in the household is 2, divided into a unit measure of buyers and a unit measure of sellers. Sellers produce and sell goods, while buyers use money to buy consumption goods for all members to share. This construct of large households keeps the analysis tractable: By smoothing the matching risks within a household, the assumption makes the distribution of money holdings across households degenerate.4

Each country issues one currency. A currency can facilitate the exchange in the goods market by alleviating the difficulty of a double coincidence of wants. However, a currency does not generate direct utility or facilitate production. Moreover, there is no requirement that a country’s goods must be purchased with a particular currency; instead, either currency can be used.

There are two markets for goods and one market for currencies. These markets are separated from each other. The currency market is centralized and Walrasian, but the goods markets are decentralized and modeled as random matching. Each goods market uses a specific currency; the one using currency $i$ is referred to as country $i$’s domestic currency area. We take the structure of currency areas as given, although it is possible to describe the environment in more details to endogenously generate the division of the exchange into currency areas.5 This allows us to focus on how often a household will choose to trade in each currency area.

The size of a currency area is endogenous, determined by the households’ allocation of the members to the area. Let $n_i$ be the fraction of buyers and $s_i$ the fraction of sellers whom a household in country $i$ sends to the domestic currency area. The household can choose $n_i$ to be

---

4This modelling device is a proxy for a single agent’s time allocation during a period. See Shi (1997) for a discussion. Lagos and Wright (2005) use a different device to make the distribution of money holdings degenerate.

5For example, imagine that goods must be sold through vending machines and that each machine is programmed to take in only one type of currency (perhaps because there is an additional cost to allow a machine to take in two types of currencies). Without currency areas, mismatches can occur as a machine may not take the particular currency carried by a buyer. Separating the machines into two areas eliminates the mismatches.
any number in \([0, 1]\). For the allocation of sellers, we will fix \(s_i = s \in [1/2, 1]\) until section 1.8. Fixing \(s\) allows us to assume that the buyer in a trade makes a take-it-or-leave-it offer, which simplifies the analysis greatly.\(^6\) In addition, this assumption strengthens the finding of deviations from the LOP, by allowing such deviations to arise even when sellers have the least bargaining power in trade. By doing so, the assumption emphasizes trading frictions in the goods market as a cause of the deviations, as opposed to the emphasis in the literature on sellers’ market power (see Goldberg and Knetter, 1997).

However, neither the fixed \(s\) nor the assumption on the offers is necessary for the main results. In section 1.8, we will allow each household to choose \(s\) under the assumption that sending sellers to the foreign currency area is more costly than to the domestic currency area. With such endogenous \(s\), we will illustrate similar results numerically. Moreover, the equilibrium will generate \(s > 1/2\), as assumed above, and the number \((1 - s)\) will increase in the cost differential between sending the sellers to the two areas. When the cost differential vanishes, \(s\) will approach 1/2. Anticipating these results, we refer to \((1 - s)\) as the degree of the integration between the two areas, and to the limit case \(s \to 1/2\) as a fully integrated economy.

At this point, we would like to clarify two issues regarding the assumptions of currency areas and the fixed \(s\). First, these assumptions do not imply cash-in-advance constraints. Any meaningful link between our model and cash-in-advance constraints is severed by the household’s allocation of buyers and the assumptions on goods. On purchases, a household can completely avoid using a particular currency by not sending any buyer to the area which uses that currency. This choice does not reduce the set of goods the household can consume; to the contrary, the household can continue to enjoy an identical set of goods by buying in the other area. On selling,

\(^6\)Under this assumption on the offers, any fixed \(s\) is consistent with an equilibrium. That is, if a household were allowed to choose \(s\), it would have no incentive to deviate from the particular value at which we fix \(s\), because sellers obtain zero surplus in all trades.
a household sells goods for both currencies, provided \( s < 1 \). Only in the case \( s = 1 \) are goods sold entirely for the seller’s domestic currency.

Second, the assumptions of currency areas and the fixed \( s \) are not sufficient for determining the nominal exchange rate uniquely, as we will discuss in section 1.3.4. This insufficiency illustrates in another way that our model does not have cash-in-advance constraints, because such constraints would lead to a uniquely determined exchange rate (e.g., Helpman, 1981). To determine the exchange rate in our model, some differences between the countries are needed.

We introduce three differences between the two countries. First, there is local congestion in the goods markets, which we will describe in the next subsection. Second, monetary transfers are asymmetric or local, in the sense that the households receive only the transfers of the domestic currency. Third, the two goods markets may not be fully integrated, i.e., \( s \) may not necessarily be equal to \( 1/2 \). The first difference is critical for the determination of the exchange rate, while the last two differences affect how the equilibrium depends on money growth.

**1.2.2. Country Characteristics, Matches and Trading Quantities**

In a currency area, agents are matched randomly and bilaterally. Because there is no double coincidence of wants, trade is possible only in a match of a single coincidence of wants, where the seller can produce the consumption goods for the buyer. We call such a match a trade match.

In each area \( k \), there are four types of trade matches as listed in Table 1.1. The notation \( S_{jk} \) is the number of country \( j \) sellers in area \( k \), where \( S_{jk} = s \) if \( j = k \) and \( S_{jk} = 1 - s \) if \( j \neq k \). \( N_{ik} \) denotes the number of country \( i \) buyers in area \( k \), where \( N_{ik} = N_i \) if \( i = k \) and \( N_{ik} = 1 - N_i \) if \( i \neq k \). \( T_{ij}^k \) denotes the total number of trade matches in area \( k \) between country \( i \) buyers and country \( j \) sellers. We assume that

\[
T_{ij}^k = L(N_{ik})S_{jk},
\]  
\[(1.1)\]
<table>
<thead>
<tr>
<th>buyers</th>
<th>sellers</th>
</tr>
</thead>
<tbody>
<tr>
<td>country 1 ((N_1k))</td>
<td>(T_{11}^k, (x_{11k}, q_{11k}))</td>
</tr>
<tr>
<td>country 2 ((N_2k))</td>
<td>(T_{21}^k, (x_{21k}, q_{21k}))</td>
</tr>
<tr>
<td>country 1 ((N_1k))</td>
<td>(T_{12}^k, (x_{12k}, q_{12k}))</td>
</tr>
<tr>
<td>country 2 ((N_2k))</td>
<td>(T_{22}^k, (x_{22k}, q_{22k}))</td>
</tr>
</tbody>
</table>

Table 1.1. Trade matches and trading quantities in currency area \(k\)

where \(L(N) = \min\{L_0(N)^\psi, N\}\) with \(\psi \in (0, 1)\) and \(0 < L_0 < 2^{\psi-1}\). The upper bound on \(L_0\) ensures that not every buyer is matched when \(N = 1/2\). Because \(S\) is fixed, the matching function above is equivalent to a Cobb-Douglas function up to rescaling \(L_0\).

Denote \(\mu(N) = L(N)/N\). A buyer from country \(i\) has a trade match with a seller from country \(j\) in area \(k\) with probability \(T_{ij}^k/N_{ik} = \mu(N_{ik})S_{jk}\). Similarly, a seller from country \(j\) has a trade match with a buyer from country \(i\) in area \(k\) with probability \(T_{ij}^k/S_{jk}\). Because the function \(L\) is bounded below \(N\), \(\mu(N) \leq 1\) and so the trading probabilities are indeed bounded below one.

The matching function introduces an asymmetry between the two countries, which we call local congestion in the market. That is, if a buyer enters an area, he crowds out other buyers from the same country by more than crowding out the other country’s buyers. We push this asymmetry to the limit in which a buyer only crowds out other buyers from the same country. For example, a country 1 buyer meets a country 2 seller in area \(k\) with probability \(\mu(N_{1k})S_{jk}\), which decreases with the number \(N_{1k}\) but is independent of \(N_{2k}\). Asymmetric matching is necessary for our results, but the extreme asymmetry is not.

Local congestion is a reasonable distinction between countries. To see what the assumption entails in general, consider the following environment. There are two trading posts in each market, indexed by \(a = 1, 2\). The buyers of country \(a\) go to post \(a\) with probability \(p\) and to post \(a'\) with probability \((1-p)\), where \(a' \neq a\). The sellers who come to the area go to each post with probability \(1/2\). At each post, agents are randomly matched and the number of matches is \(L(N)S\), where \(N\) and \(S\) are the numbers of buyers and sellers at the post. If \(p = 1\), this matching process delivers the matching function assumed above. In general, local congestion exists if \(p > 1/2\), that is, if
foreign and domestic buyers are not distributed to the two posts in the same way. There are a number of reasons why \( p > 1/2 \) in reality. For example, foreigners may typically buy goods in regions near the national border, or the networks of distributing goods to foreign buyers are different from those to domestic buyers.

Local congestion is captured by \( \psi < 1 \). Coincidently, the assumption \( \psi < 1 \) also implies that the matching function above is concave in the number of buyers. However, concavity per se is not critical for the determinacy of the nominal exchange rate (see section 1.3.4).\(^7\)

Now, let us turn to the notation \((x, q)\) in Table 1.1. These are the quantities of money and goods exchanged in a trade, instructed by the traders’ households. We assume that the buyer in a match makes a take-it-or-leave-it offer. The buyer offers \( x \) units of money for \( q \) units of goods, and the seller can respond to the offer by saying either “yes” or “no”. The first subscript of \( x \) and \( q \) indicates the buyer’s country, the second subscript indicates the currency used in the trade, and the superscript \( f \) indicates that the trade is carried out between agents from different countries. For example, in a match with a country 2 seller in area 1, a buyer from country 1 proposes \((x_{11}^f, q_{11}^f)\), and a buyer from country 2 proposes \((x_{21}, q_{21})\).

To facilitate the analysis of a stationary equilibrium, we normalize all nominal prices and quantities involving currency \( k \) by the total stock of currency \( k, M_k \). For example, the actual amount of money \( k \) that a buyer of country \( i \) offers to a seller of country \( i \) is \( x_{ik}M_k \). Similarly, we normalize the nominal exchange rate by the ratio of the stocks of the two currencies, and denote it as \( e \). The actual exchange rate is \( eM_2/M_1 \), which is the amount of currency 2 that is needed to exchange for one unit of currency 1. Moreover, whenever the index \( i' \) is used together with \( i \), it should be understood that \( i' \neq i \) and \( i, i' \in \{1, 2\} \).

\(^7\)Concavity helps ruling out the corner solutions \( n_1 = 1 \) and \( n_1 = 0 \) as an equilibrium. If no other buyer goes to an area, the matching function implies that a particular buyer going to that area will obtain a match almost surely provided that there are some sellers in that area. This feature makes the corner choices not optimal.
1.2.3. Timing of Events

The events in an arbitrary period \( t \) unfold as follows. At the beginning of period \( t \), each household receives lump-sum transfers of the domestic currency. Let the amount of transfer of currency \( k \) in period \( t \) be \( r_{k,t-1} M_{k,t-1} \), where

\[
r_{k,t-1} = \gamma_{k,t-1} - 1, \quad i = 1, 2.
\]

Thus, the stock of currency \( k \) grows at the (gross) rate \( \gamma_{k,t} \) between periods \( t \) and \( t + 1 \). After the transfer, the household’s money holdings are measured, where the (normalized) holdings of currency \( k \) by a country \( i \) household is measured as \( m_{ik} \).

The currency market opens before the goods markets. In the currency market, a country \( i \) household sells \( f_{ii} \) units of currency \( i \) for \( f_{i'i'} \) units of currency \( i' \), at the exchange rate \( e \). Thus, \( f_{i'i'} = e^{i'i} f_{ii} \), where \( i' \neq i \). After the currency market closes, a country \( i \) household chooses \( n_i \), the fraction of buyers going to currency area \( i \). All buyers who go to the same area are given the same amount of currency. Then, the goods markets open, the agents obtain matches, and they trade the quantities \((x,q)\). During the exchange, matches are separated from each other. After the exchange, the members bring back the goods and money to the household, which gives all members the same amount to consume. Then, time proceeds to the next period.

1.2.4. The Representative Household’s Decision Problem

We pick an arbitrary household in country \( i \) as the representative household of the country. The time subscript \( t \) is suppressed. If the current period is \( t \), we shorten the time subscript

\cite{footnote:timing}

\footnote{The timing assumed here simplifies the algebra, but it is not critical for the results. Because there is no uncertainty in our model, whether the currency market opens before, after, or at the same time as the goods markets is not important to the analytical results. The important assumption is that these markets are separated so that there is no instantaneous arbitrage between them.}
t \pm t' \text{ to } \pm t', \text{ where } t' > 0. \text{ The representative household makes the following decisions: (i) the allocation of buyers to the currency areas, } (n_i, 1 - n_i); \text{ (ii) the trading quantities, } (x_{ik}, q_{ik}) \text{ and } (x'_{ik}, q'_{ik}), \text{ which a buyer will propose in a trade match in area } k; \text{ (iii) future money holdings of each currency, } m_{ik,+1}; \text{ and (iv) the amounts of currency exchange, } f_{ii} \text{ and } f_{ii'}, \text{ where } f_{ii} \text{ is the amount of currency } i \text{ sold and } f_{ii'} \text{ the amount of currency } i' \text{ purchased in the currency market. As said earlier, nominal quantities } (x_{ik}, x'_{ik}, f_{ik}, m_{ik}) \text{ are ones normalized by the aggregate stock of money } k, M_k. \text{ Denote the household’s decisions in a period as}

\[ h_i = [n_i, m_{i1,+1}, m_{i2,+1}, f_{ii}, (x_{ik}, q_{ik}, x'_{ik}, q'_{ik})_{k=1,2}] . \]

We omit } f_{ii'} \text{ in this list, because } f_{ii'} = e^{t' - t} f_{ii}. \text{ Use the corresponding upper-case letters to denote other households’ decisions or aggregate variables.}

Let } v(m_{i1}, m_{i2}) \text{ be the value function of the representative household in country } i. \text{ Define}

\[ \omega_{ik} = \frac{\beta}{\gamma_k} v_k (m_{i1,+1}, m_{i2,+1}), i = 1, 2; k = 1, 2. \]

Here, the subscript } k \text{ of the function } v \text{ indicates the partial derivative with respect to the } k\text{th argument. The variable } \omega_{ik} \text{ is country } i \text{ representative household’s marginal value of next period’s holdings of currency } k, \text{ discounted to the current period. Discounting includes the money growth rate, as well as timing discounting, because } m \text{ is normalized by the aggregate stock which grows over time. Similarly, the valuation of currency } k \text{ of other households in country } i \text{ is } \Omega_{ik}. \text{ We formulate the maximization problem of the representative household of country 1. (A similar formulation applies to country 2 household’s problem.) To do so, compute the household’s}
consumption in a period as:

\[ C = n_1 \left( \frac{T_{11}^1 q_{11}}{N_1} + \frac{T_{12}^1 q_{11}^f}{N_1} \right) + (1 - n_1) \left( \frac{T_{11}^2}{1 - N_1} q_{12} + \frac{T_{12}^2}{1 - N_1} q_{12}^f \right) \]  

(1.2)

The terms in the first pair of brackets are the expected amount of goods obtained from currency area 1 by one of the household’s buyers, and the terms in the second pair of brackets are the expected amount of goods from area 2. There are two terms inside each pair of brackets because a buyer in an area has positive probability of trading with a seller from either country.

Similarly, the household’s disutility of production in a period is:

\[ P = \left[ T_{11}^1 (Q_{11})^\sigma + T_{21}^1 (Q_{21}^f)^\sigma \right] + \left[ T_{11}^2 (Q_{12})^\sigma + T_{21}^2 (Q_{22}^f)^\sigma \right] \]  

(1.3)

Notice the difference between the lower-case and upper-case letters in (1.2) and (1.3).

The representative household chooses \( h_1 \) to solve the following maximization problem:

\[(PH) \quad v(m_{11}, m_{12}) = \max \{ u(C) - P + \beta v(m_{11,+1}, m_{12,+1}) \}\]

subject to following constraints:

\[ \Omega_{1k} x_{1k} - (q_{1k})^\sigma = 0; \quad k = 1, 2; \]  

(1.4)

\[ \Omega_{2k} x_{1k}^f - (q_{1k}^f)^\sigma = 0; \quad k = 1, 2; \]  

(1.5)

\[ \frac{m_{11} - f_{11}}{n_1} \geq x_{11}; \quad \frac{m_{11} - f_{11}}{n_1} \geq x_{11}^f; \]  

(1.6)

\[ \frac{m_{12} + ef_{11}}{1 - n_1} \geq x_{12}; \quad \frac{m_{12} + ef_{11}}{1 - n_1} \geq x_{12}^f. \]  

(1.7)
In addition, money holdings must obey the following laws of motion:

\[
\gamma_1 m_{11,+1} = (m_{11} - f_{11}) - n_1 \left( \frac{T_{11}^1}{N_1} x_{11} + \frac{T_{12}^1}{N_1} x_{11}^f \right) + \left( T_{11}^1 X_{11} + T_{21}^1 X_{21}^f \right) + (\gamma_1 - 1); \quad (1.8)
\]

\[
\gamma_2 m_{12,+1} = (m_{12} + e f_{11}) - (1 - n_1) \left( \frac{T_{11}^2}{1 - N_1} x_{12} + \frac{T_{12}^2}{1 - N_1} x_{12}^f \right) + \left( T_{11}^2 X_{12} + T_{21}^2 X_{22}^f \right)
\]

(1.9)

The constraints (1.4) and (1.5) are outcomes of the assumption that buyers make take-it-or-leave-it offers in trade. The left-hand side of each constraint is the seller’s surplus from trade. The constraints (1.6) and (1.7) require that a buyer should not spend more money than he carries into the trade. These restrictions are necessary because the matches are separated from each other, which prevents the buyers from using money balances left over in the period by other buyers in the household. Note that the amount of money each buyer carries into the goods markets includes the amount the household received from the currency trade.

Finally, (1.8) and (1.9) are the laws of motion of the household’s holdings of the two currencies. In (1.8), the left-hand side of this constraint is the household’s holdings of currency 1 in the next period, where the money growth rate \( \gamma_1 \) appears because we normalize nominal quantities and prices associated with currency 1 by \( M_1 \). This amount of holdings comes from the holdings after the currency trade in this period (the first group of terms), minus the spending by the buyers (the second group of terms), plus the amount of currency 1 received by the household’s sellers (the third group of terms), and plus the transfers of currency 1 received at the beginning of the next period. Similarly, (1.9) describes the law of motion of holdings of money 2. Its main difference from (1.8) is that a country 1 household does not receive transfers of currency 2.
1.3. Equilibrium

1.3.1. Definition

A monetary equilibrium consists of the representative households’ decisions \((h_1, h_2)\), other households’ decisions \((H_1, H_2)\), and the nominal exchange rate \(e\) such that the following requirements are met for all \(t\): (i) given \((e, H_1, H_2)\) and other aggregate variables, \(h_1\) solves \((PH)\) and \(h_2\) solves a similar problem for a country 2’s household; (ii) the decisions are symmetric within each country: \(h_i = H_i\) for \(i = 1, 2\); (iii) the currency market clears: \(f_{22} = ef_{11}\); (iv) money holdings add up: \(m_{1k} + m_{2k} = 1\), for \(k = 1, 2\); and (v) both currencies have positive and bounded values: \(\omega_{1i}m_{1i} > 0, \omega_{2i}m_{2i} > 0\) and \(0 < \omega_{1i}m_{1i} + \omega_{2i}m_{2i} < \infty\) for \(i = 1, 2\).

Under symmetry within each country, we have \(N_{ii} = n_i, N_{ii'} = 1 - n_i, S_{ii} = s,\) and \(S_{ii'} = 1 - s\) for \(i = 1, 2\) \((i' \neq i)\). Thus, the total number of buyers in currency area \(i\) is \((n_i + 1 - n_{i'})\) and the total number of sellers in area \(i\) is \((s + 1 - s) = 1\). However, we do not impose symmetry between the two countries’ decisions. In general, the symmetry, \(n_1 = n_2\), does not hold.

An important variable in the equilibrium is the relative valuation of a given currency by the two countries. Call this the relative valuation of currency \(k\) and denote it as

\[
\theta_k \equiv \frac{\omega_{1k}}{\omega_{2k}}, \quad k = 1, 2. \tag{1.10}
\]

If \(\theta_1 = \theta_2\), the cross-country relative valuation is the same for the two currencies. If \(\theta_1 = \theta_2 = 1\), the two countries have the same absolute valuation of each currency.

1.3.2. Equilibrium Conditions

To characterize optimal decisions, let \(\lambda_{ik}\) be the shadow price of the money constraint faced by a country \(i\) buyer in a trade with a domestic seller in area \(k\), and \(\lambda_{ik}^f\) be the shadow price in a
trade in area \(k\) with a foreign seller. For example, \(\lambda_{11}\) is the shadow price of the first constraint in (1.6) and \(\lambda_{11}'\) of the second constraint. These shadow prices are the non-pecuniary returns to the currencies generated by relaxing the trade restrictions. It is convenient to rescale each of these multipliers by the number of the corresponding trade matches that the household has. That is, \(\lambda_{ii}\) is multiplied by \(n_i T_{ii}/N_i\), \(\lambda_{ii}'\) by \(n_i T_{ii}'/N_i\), \(\lambda_{ii''}\) by \((1 - n_i) T_{ii''}/(1 - N_i)\), and \(\lambda_{ii''}'\) by \((1 - n_i) T_{ii''}'/(1 - N_i)\). We collected the derivations for (1.11) through (1.22) below in Appendix A.4.

The following conditions are necessary for the decisions to be optimal.

(i) On the trading quantities \((x_{ik}, q_{ik})\) and \((x_{ik}', q_{ik}')\):

\[
A = (\omega_{ik} + \lambda_{ik}) \sigma (q_{ik})^{\sigma - 1} / \Omega_{ik}, \tag{1.11}
\]

\[
A = (\omega_{ik} + \lambda_{ik}') \sigma (q_{ik}')^{\sigma - 1} / \Omega_{ik}', \tag{1.12}
\]

Since the two equations are similar, we explain the first one only. The left-hand side of the equation is the marginal utility of consumption, and hence the marginal benefit of an additional unit of good to a buyer. The right-hand side is the cost to a country \(i\) buyer of asking a country \(i\) seller in area \(k\) to supply an additional unit of good. To obtain the additional unit of good, the buyer must offer \(\sigma(q_{ik})^{\sigma - 1}/\Omega_{ik}\) units of currency \(k\) (see (1.4)). The cost of giving up one unit of currency \(k\) is the sum of the future marginal value of the currency, \(\omega_{ik}\), and the shadow price of the trading restriction on the currency, \(\lambda_{ik}\).

(ii) On the choice of \(n_i\) (the fraction of buyers of country \(i\) sent to area \(i\)):

\[
\frac{T_{ii}}{N_i} A q_{ii} + \frac{T_{ii}'}{N_i} A q_{ii}' = \frac{T_{ii}'}{1 - N_i} A q_{ii'} + \frac{T_{ii}'}{1 - N_i} A q_{ii'}', \quad i' \neq i. \tag{1.13}
\]
By sending one additional buyer to currency $i$, the household obtains more consumption goods from area $i$, which increases the household’s utility by the amount given by the left-hand side. The cost is that the household must reduce the number of buyers sent to area $i'$ by one, which reduces the utility of consumption by the amount given by the right-hand side. These marginal benefits and costs must be equal to each other when an interior value of $n_i$ is optimal.

(iii) On the amounts of currency exchanges, $f_{11}$ and $f_{22}$:

\[
\omega_{11} + \frac{T_{11}}{N_1} \lambda_{11} + \frac{T_{12}}{N_1} \lambda'_{11} = \left( \omega_{12} + \frac{T_{11}}{1 - N_1} \lambda_{12} + \frac{T_{12}}{1 - N_1} \lambda'_{12} \right) e; \tag{1.14}
\]

\[
\omega_{21} + \frac{T_{22}}{1 - N_2} \lambda_{21} + \frac{T_{21}}{1 - N_2} \lambda'_{21} = \left( \omega_{22} + \frac{T_{22}}{N_2} \lambda_{22} + \frac{T_{22}}{N_2} \lambda'_{22} \right) e. \tag{1.15}
\]

Since these two conditions are similar, we explain only the first one by considering a country $1$ household’s choice of $f_{11}$. By increasing $f_{11}$ by one unit, the household reduces the holdings of currency $1$ by one unit and increases the holdings of currency $2$ by $e$ units. The expected values of these two amounts of currencies, given by the two sides of (1.14), must be equal to each other. Note that the value of a currency before matches take place consists of the future value of the currency, $\omega$, and the expected non-pecuniary returns that the currency can generate by relaxing the money constraints in trade matches.

(iv) On money holdings, $m_{ii}$ and $m_{ii'}$ (the envelope conditions):

\[
\omega_{ii,-1} = \frac{\beta}{\gamma_i} \left( \omega_{ii} + \frac{T_{ii}}{N_i} \lambda_{ii} + \frac{T_{ii'}}{N_i} \lambda'_{ii} \right); \tag{1.16}
\]

\[
\omega_{ii',-1} = \frac{\beta}{\gamma_{i'}} \left( \omega_{ii'} + \frac{T_{ii'}}{1 - N_i} \lambda_{ii'} + \frac{T_{ii'}}{1 - N_i} \lambda'_{ii'} \right). \tag{1.17}
\]

We explain the first condition only. The left-hand side of (1.16) is the marginal value of currency $i$ in the current period to a country $i$ household. The right-hand side is the discounted value of currency $i$ in the next period plus the expected non-pecuniary return that the currency can
generate in the current period. The condition (1.16) says that the rate of the non-pecuniary return to currency \( i \) must be equal to the nominal interest rate, which is given by \( \gamma_i / \beta \).

1.3.3. Characterizing a Stationary Equilibrium

Set money growth rates to be constant over time, i.e., \( \gamma_{kt} = \gamma_k \) for all \( t \). This enables us to focus on a stationary equilibrium, where \( h_{i,t+1} = h_i \) and \( \omega_{ik,t+1} = \omega_{ik} \) for all \( i \) and \( k \). Stationarity implies that the exchange rate, \( e \), is constant over time, and so the exchange rate without the normalization is depreciating at the rate \( (\gamma_1 - \gamma_2) \). Moreover, we restrict \( \gamma_1 > \beta \) and \( \gamma_2 > \beta \), which are necessary for the money constraints in trade to be binding (see Lemma 1 later).

The stationary equilibrium has the following properties. First, the two currencies’ relative value is the same for the two countries and is equal to the nominal exchange rate:

\[
\frac{\gamma_1 \omega_{11}}{\gamma_2 \omega_{12}} = e = \frac{\gamma_1 \omega_{21}}{\gamma_2 \omega_{22}} \tag{1.18}
\]

This result comes from the conditions for optimal currency exchanges, (1.14) and (1.15), and the envelope conditions for money holdings. Here, we have used the requirement of stationarity: \( e_{t+1} = e \). The above condition is necessary and sufficient for eliminating arbitrage in the currency market. For example, if \( \gamma_1 \omega_{11}/(\gamma_2 \omega_{12}) < e \), then a household in country 1 could gain by selling additional amounts of currency 1 for currency 2 in the currency market.

Second, the above condition implies that the two countries have the same relative valuation of the two currencies. That is,

\[
\frac{\omega_{12}}{\omega_{22}} = \frac{\omega_{11}}{\omega_{21}} \equiv \theta. \tag{1.19}
\]

However, \( \theta \neq 1 \) in general. This differential in the absolute valuation sustains in equilibrium because the households are unable to arbitrage between the goods markets and the currency
market. Note that a country 1 household either values both currencies more than a country 2 household (if \( \theta > 1 \)) or values both currencies less than a country 2 household (if \( \theta < 1 \)). We will explain this result in section 1.4 and tie it to the differential in money growth.

Third, after controlling for the currency used, the quantity of goods that a buyer obtains from a country 1 seller relative to that from a country 2 seller is the same in the two areas:

\[
\frac{q_{11}}{q'_{11}} = \frac{q_{12}}{q'_{12}} = \frac{q_{21}}{q'_{21}} = \frac{q_{22}}{q'_{22}} = \theta^{1/\sigma}.
\] (1.20)

If \( \theta \neq 1 \), there is a differential between the quantities of goods sold by the two countries’ sellers in a trade, even after controlling for the currency used. For example, if \( \theta > 1 \), a country 1 seller sells more goods for the same type and amount of currency than a country 2 seller does. This result is intuitive because, when \( \theta > 1 \), money is more valuable to a country 1 seller’s household than to a country 2 seller’s household. As we will show later, this quantity differential implies a price differential in each area and failures of the LOP.

Fourth, there is a differential in the quantity of goods that a buyer gets in the two areas. To see this, solve the \( \lambda \)’es from (1.11) and (1.12) and use (1.18) – (1.20) to express the solutions as functions of \((q, \theta)\). Then, (1.16) and (1.17) imply the following equations for \( i \in \{1, 2\} \):

\[
q_{ii} = \left[ \left( \frac{A}{\sigma} \right) \frac{s + (1 - s) \theta^{i-i'}/\sigma}{1 + \left( \frac{\gamma^i}{\beta} - 1 \right)/\mu(n_i)} \right]^{1/(\sigma-1)},
\] (1.21)

\[
q_{ii'} = \left[ \left( \frac{A}{\sigma} \right) \frac{(1 - s) + s\theta^{i-i'}/\sigma}{1 + \left( \frac{\gamma^{i'}}{\beta} - 1 \right)/\mu(1 - n_i)} \right]^{1/(\sigma-1)}.
\] (1.22)

As defined before, \( \mu(n) = L(n)/n \). The ratio \( q_{ii}/q_{ii'} \) is the relative quantity of goods that a country \( i \) buyer gets from a country \( i \) seller in the two currency areas. Note that this relative
quantity is independent of the seller whom the buyer meets. That is, \( q_{11}/q_{12} = q_{11}^f/q_{12}^f \) and
\( q_{21}^f/q_{22}^f = q_{21}/q_{22} \), as implied by (1.20).

Finally, the equilibrium satisfies the following lemma (see Appendix A.1 for a proof):

**Lemma 1.** In the stationary equilibrium, \( n_2 = 1 - n_1 \). If \( \gamma_1 = \gamma_2 = \gamma \), then \( \theta = 1 \),
\( n_1 = n_2 = 1/2 \) and \( q_{ij} = q_{ij}^f = Q(\gamma) \) for all \( i, j = 1, 2 \), where

\[
Q(\gamma) = \left( \frac{A/\sigma}{1 + \left( \frac{2}{\beta} - 1 \right) / \mu \left( \frac{1}{2} \right)} \right)^{\frac{1}{\sigma-1}}. 
\]  
(1.23)

All money constraints bind in this case if and only if \( \gamma > \beta \). If \( \gamma_1 \neq \gamma_2 \), all money constraints
bind only if \( \gamma_1 > \beta \) and \( \gamma_2 > \beta \).

The fraction of buyers allocated to a given currency area is the same for the two countries,
because the two countries have the same relative valuation of the two currencies. For example,
if a household in country 1 chooses to allocate more than a half of the buyers to area 1, the
household must value currency 1 more than currency 2. In this case, a household in country 2
must also value currency 1 more than currency 2, because the currency market ensures that the
two countries have the same relative valuation of the two currencies. Then, it is also optimal for
a country 2 household to allocate more than a half of the buyers to area 1.

The feature \( n_2 = 1 - n_1 \) implies the following result from (1.21) and (1.22):

\[
\frac{q_{21}}{q_{11}} = \frac{q_{22}}{q_{12}} = \theta^{1/[\sigma(\sigma-1)]}. 
\]  
(1.24)

That is, the relative quantity of goods that the two countries’ buyers get from their domestic
sellers is the same in the two currency areas.
Determining the equilibrium amounts to solving for \((n_1, \theta)\). In Appendix A.1, we show that \(n_1\) and \(\theta\) solve the following two equations:

\[
\frac{1 - s + s\theta^{1/\sigma}}{s + (1 - s)\theta^{1/\sigma}} = \left[\frac{\mu(1 - n_1)}{\mu(n_1)}\right]^{1 - \frac{1}{\sigma}} \left[\frac{\left(\frac{\gamma_2}{\beta} - 1\right)}{\mu(n_1) + 1}\right]^{1/\sigma},
\]

\[
\theta_{\pi^\gamma} = \frac{[\gamma_2 - 1 + s\mu(1 - n_1)](1 - n_1)[G(n_1, \theta)]^\sigma + (1 - s)n_1\mu(n_1)}{[\gamma_1 - 1 + s\mu(n_1)]n_1 + (1 - s)(1 - n_1)\mu(n_1)[G(n_1, \theta)]^\sigma},
\]

where

\[
G(n_1, \theta) \equiv \frac{\mu(n_1)}{\mu(1 - n_1)} \left[\frac{1 - s + s\theta^{1/\sigma}}{s + (1 - s)\theta^{1/\sigma}}\right].
\]

Furthermore, the (normalized) nominal exchange rate is given as

\[
e = \frac{\gamma_1}{\gamma_2} \frac{n_1}{1 - n_1} [G(n_1, \theta)]^{-\sigma}.
\]

1.3.4. Determinacy and the Roles of Various Assumptions

The equilibrium is determinate, provided that (1.25) and (1.26) solve for a unique pair \((n_1, \theta)\). In the next section, we will show that the solutions for \(n_1\) and \(\theta\) are indeed unique, at least for a fully integrated economy. The unique allocation of buyers generates a relative valuation of the two currencies for each country’s households. Since this relative valuation must be equal between the two countries in order to prevent profitable arbitrage in the currency market (see the explanation following (1.18)). Only a particular nominal exchange rate can achieve this equilibrium.

The determinacy of the exchange rate contrasts with the well-known indeterminacy in Kareken and Wallace (1981). Like our model, Kareken and Wallace do not assume cash-in-advance constraints or any other intrinsic role of a currency in preferences and the production technology. In contrast to our model, they model the goods markets as Walrasian markets and show that a
continuum of nominal exchange rates is consistent with equilibrium. For an exchange rate to be an equilibrium value there, it only needs to be constant over time, so as to yield the same rate of return to holding the two currencies. Given an arbitrary exchange rate, there is a corresponding relative price of goods that eliminates profitable arbitrage between the two goods markets. By modeling the exchange of goods as random matches, we introduce a new determinant of an equilibrium, i.e., the number of trades in each market. This feature is necessary for the nominal exchange rate to be determinate here.

However, the non-Walrasian feature and the structure of currency areas are not sufficient for the determinacy. To see this, let us change only the parameter $\psi$ in the matching function from $\psi < 1$ to $\psi = 1$. Then, matching in each area is symmetric between the two countries’ buyers. The result $n_2 = 1 - n_1$ in Lemma 1 no longer holds; in fact, the equilibrium yields no relationship between $n_2$ and $n_1$. There is a range of values of $(n_1, n_2)$, each of which is consistent with an equilibrium (see Appendix A.1). Hence, the exchange rate is indeterminate.

To determine the nominal exchange rate, a household must care about the composition of the trades in each market, as well as the total number of trades. With the assumption of identical preferences and production technologies between the two countries, the composition of matches matters to a household only if the matching process is asymmetric between the two countries. We modeled the asymmetry as local congestion by the assumption $\psi < 1$. With local congestion, there is a unique allocation of buyers to the two areas that maximizes a household’s expected gain from trading in the two areas. More specifically, the assumption $\psi < 1$ is needed for the result $n_2 = 1 - n_1$ in Lemma 1. As explained at the beginning of this section, the unique allocation of buyers leads to the determinacy of the exchange rate.

The assumption $\psi < 1$ is important because it implies local congestion, not because it implies concavity in the matching function. To see this, consider alternative matching functions that
preserve concavity but not local congestion. Let \( \mu_{ik} \) be the probability with which a buyer of country \( i \) gets a trade match in area \( k \). Let \( \mu_{ik} \) be strictly decreasing in \( N_{ik} \), in which case the matching function is strictly concave in \( N \). Assume \( \mu_{1k} = \mu_{2k} = \mu_k \) for \( k = 1, 2 \), so that the matching process is symmetric between the two countries’ buyers. One such example is that the total number of trade matches in area \( i \) is given by \( L(N_k)S_k \), where \( N_i = N_{1i} + N_{2i} \) is the total number of buyers in area \( k \), \( S_k = 1 \) is the total number of sellers in area \( k \), and \( L(N) = L_0N^\psi \), with \( \psi < 1 \). With symmetric matching, (1.25) is modified by replacing \( \mu(n_1) \) with \( \mu_1 \) and replacing \( \mu(1 - n_1) \) with \( \mu_2 \). Because the resulting equation is the same for the two countries, the equilibrium does not pin down \( n_1 \) and \( n_2 \) uniquely. Hence, the exchange rate is indeterminate. (For a detailed proof, see Appendix A.4.)

Determinacy of the equilibrium enables us to analyze how the equilibrium depends on the fundamental factors. Lemma 1 states that all differences between the two countries in the equilibrium allocation are driven by the difference between the growth rates of the two currencies. If the growth rates are equal, then each country will allocate exactly a half of the buyers to each area and agents will trade the same quantity of goods in all trade matches. In this case, the two other differences between the two countries, i.e., asymmetric monetary transfers and the possibility of \( s > 1/2 \), are neutralized by the currency trade and the allocation of buyers.

### 1.4. Integrated Economy

In this and the next section, we study the benchmark case, i.e., the case of full integration \( (s = 1/2) \). In section 1.6, we will use numerical examples to illustrate the robustness of the results to changes in \( s \). The proofs for this section are collected in Appendix A.2.
1.4.1. The Equilibrium and the Size of a Currency Area

In the integrated economy, (1.25) and (1.28) are simplified to

\[
\left[ \frac{\mu(n_1)}{\mu(1-n_1)} \right]^{\sigma-1} = \frac{1 + \left( \frac{\gamma_1}{\beta} - 1 \right) / \mu(n_1)}{1 + \left( \frac{\gamma_2}{\beta} - 1 \right) / \mu(1-n_1)}
\]

(1.29)

\[
e = \left( \frac{\gamma_1}{\gamma_2} \right) \frac{n_1}{1 - n_1} \left[ \frac{\mu(1-n_1)}{\mu(n_1)} \right]^\sigma.
\]

(1.30)

The following proposition describes the existence of the equilibrium:

**Proposition 2.** Denote \( D = \{ (\gamma_1, \gamma_2) : \gamma_i < (\beta L_0 + \gamma_i - \beta) / L_0^\gamma \text{ for } i = 1, 2 \} \). A unique equilibrium exists, provided \( \gamma_1, \gamma_2 \in D \). Denote this equilibrium value of \( n_1 \) as \( n_1 = N(\gamma_1, \gamma_2) \).

Then, \( N_1(\gamma_1, \gamma_2) < 0 \) and \( N_2(\gamma_1, \gamma_2) > 0 \). Moreover, \( N(\gamma, \gamma) = 1/2 \) for all \( \gamma > \beta \), and \( N(\gamma_1, \gamma_2) > 1/2 \) iff \( \gamma_1 < \gamma_2 \).

The allocation of buyers to the two areas depends on money growth rates in an intuitive way. When the growth rate of currency 1 is higher than that of currency 2, all households send fewer buyers to area 1 and more to area 2. This re-allocation of buyers arises from the effect of money growth on \( \theta \), the valuation of currencies of a country 1 household relative to that of a country 2 household. To explain, suppose \( \gamma_1 > \gamma_2 \) so that \( \theta < 1 \) (see section 1.4.2). In this case, a buyer obtains more goods from trading with a country 2 seller than with a country 1 seller (see (1.20)). Given this quantity differential, it is beneficial for a country 1 household to reduce the frequency of purchases from domestic sellers and increase the frequency from foreign sellers. The household can achieve this outcome only by sending more buyers to area 2. The same re-allocation of buyers is beneficial for a country 2 household.

By affecting the allocation of buyers, the differential in money growth rates changes the size of a currency area. We measure the size of a currency area by the total number of trades that
take place in the area in each period. The size of currency area $k$ is:

$$T^k_{11} + T^k_{12} + T^k_{21} + T^k_{22} = 2n_k\mu(n_k), \quad k = 1, 2.$$ 

The equality comes from the result $1 - n_2 = n_1$. Because $n\mu(n)$ increases in $n$, then a currency area shrinks with the growth rate of the currency used in the area and expands with the growth rate of the competing currency. Only when the two currencies have the same growth rate do the two areas have the same size.

Similarly, the differential in money growth rates affects the nominal exchange rate, $e$. The rate $e$ decreases in $\gamma_1$ and increases in $\gamma_2$, with $e < 1$ if and only if $\gamma_1 > \gamma_2$ (see the proof of Lemma 3 later). Recall that $e$ is normalized by the ratio of the stocks of the two currencies. Without normalization, the exchange rate is $eM_2/M_1$. Since $e$ is stationary, currency 1 depreciates against currency 2 over time at the rate $(\gamma_1 - \gamma_2)$.

### 1.4.2. Cross-Country Differential in the Valuation of Currencies

The relative valuation of a currency by the two countries, represented by $\theta$, is an important variable in the equilibrium. This differential is generated by the difference between the two currencies’ growth rates. Loosely speaking, a household in country 1 has a higher valuation of both currencies than a household in country 2 if currency 1 grows less quickly in its supply than currency 2. The following lemma formally states the properties of $\theta$.

**Lemma 3.** If $\gamma_1 = \gamma_2$, then $\theta = 1$. If $\gamma_2 \leq 1$, then $\theta > 1$ for all $\beta < \gamma_1 < \gamma_2$. If $\gamma_2 > 1$, then there exists $\gamma_0 \in (1, \gamma_2]$ such that $\theta > 1$ for all $\beta < \gamma_1 < \gamma_0$.

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Another way to measure the size of a market is to use the aggregate transaction value in goods market. This measure is more complicated to compute because all four types of trade in an area result in different quantities of goods traded. However, the results are similar.

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9Another way to measure the size of a market is to use the aggregate transaction value in goods market. This measure is more complicated to compute because all four types of trade in an area result in different quantities of goods traded. However, the results are similar.
To explain, suppose $\gamma_1 > 1$ and $\gamma_2 > 1$ so that a household receives positive transfers from the domestic monetary authority. Because a household does not receive transfers of the foreign currency, there is a potential imbalance in the household’s portfolio of currencies. To eliminate this imbalance, a household sells part of the received transfers for the other currency. If $\gamma_1 = \gamma_2$, the transfers of the two currencies have the same real value, and so a household sells exactly a half of the received transfers for the other currency. As a result, each country holds a half of the stock of each currency, and the two countries have the same absolute valuation of each currency. In contrast, if $\gamma_1 < \gamma_2$ and if $\gamma_2$ is not very high, then the real value of currency 2 transfers exceeds that of currency 1 transfers. In this case, households in country 1 sell more than a half of the received transfers for currency 2. Hence, households in country 1 hold less than a half of the stock of each currency and they value both currencies more than households in country 2.

The currency market cannot eliminate the difference between the two countries’ valuations of a currency; but rather, it equalizes only the relative valuation of a currency between the two countries. If a household in country 1 values one currency more than a household in country 2, it also values the other currency more. To reduce this gap between the two countries’ valuations of currency 1, a household in country 1 would have to buy more of the first currency and sell more of currency 2 in the currency market than it does in the described equilibrium. But this would widen the gap between the two countries’ valuations of currency 2, which cannot be an equilibrium.

To eliminate this gap, the households need to trade goods directly for currencies in the currency market, which is impossible in the environment. Note that the trade of goods for money in the goods markets cannot replicate the missing trade of goods for money in the centralized currency market. The former is fragmented as a result of bilateral matches and cannot ensure uniform prices in different types of matches. We now turn to these price differentials.
1.4.3. Failures of the Law of One Price

The differential $\theta$ generates the differentials in relative quantities, as documented in section 1.3, and failures of the LOP. Let us define prices. In area $k$, there are four types of trades and hence four prices, two for each country’s buyers. Fix the buyer’s country index $i$. The price implied by a trade with a domestic seller, $p_{ik}$, and the price implied by a trade with a foreign seller, $p_{ik}^f$, are

$$p_{ik} = \frac{\hat{m}_{ik}}{n_{ik}q_{ik}}, \quad p_{ik}^f = \frac{\hat{m}_{ik}}{n_{ik}q_{ik}^f},$$

where $\hat{m}_{ik}$ is the amount of currency $k$ that a country $i$ household has after the trade in the currency market. For example, $\hat{m}_{11} = m_{11} - f_{11}$ and $\hat{m}_{12} = m_{12} + e_{11}$. Since $\hat{m}_{ik}$ is normalized by the total stock of currency $k$, prices defined above are also normalized.

We can compute the relative price charged by sellers from different countries when selling to the same buyers for the same currency. There are four such relative prices, but they are all equal to a common value denoted $R_1$:

$$R_1 = \frac{p_{11}}{p_{11}^f} = \frac{p_{12}}{p_{12}^f} = \frac{p_{21}}{p_{21}^f} = \frac{p_{22}}{p_{22}^f}.$$  

The equalities come from (1.20). Similarly, we can compute the relative price paid by the two countries’ buyers when buying from the same sellers with the same currency. There are four such relative prices, which are equal to a common value denoted $R_2$:

$$R_2 = \frac{p_{11}}{p_{21}^f} = \frac{p_{12}}{p_{22}^f} = \frac{p_{11}^f}{p_{21}} = \frac{p_{12}^f}{p_{22}}.$$  

The relative prices above have the finest controls for the trade characteristics. They are different from the trading quantities or real exchange rates, which often aggregate prices over a
large range of trades. If either $R_1$ or $R_2$ differs from unity, the LOP fails. Using (1.20) and (1.24), it is easy to establish the following corollary (the proof is omitted):

**Corollary 4.** For all $s \in [0.5, 1]$, $R_1 = \theta^{-1/\sigma}$ and $R_2 = \theta^{-1}$ in the stationary equilibrium.

The LOP fails if and only if $\theta \neq 1$, and hence if and only if the two currencies have different growth rates. For example, when currency 1 has a higher growth rate than currency 2, the households in country 1 value each currency less than the households in country 2, i.e. $\theta < 1$. As a result, sellers in country 1 charge higher prices than sellers in country 2 when selling to the same buyers in the same currency area, and buyers in country 1 pay higher prices than buyers in country 2 when buying from the same sellers in the same currency area. This dependence of relative prices on the differential in money growth provides an important incentive for a country to inflate, as we will examine in the next section.

Deviations from the LOP are linked to changes in the nominal exchange rate, because both are driven by the differential in money growth. Knetter (1989) finds evidence for this link. Also, the result $R_2 \neq 1$ is an indication of “pricing to market” which has been documented extensively (see Goldberg and Knetter, 1997). It is a strong form of pricing to market, because the relative price $R_2$ is calculated by controlling the invoicing currency.

### 1.5. Optimal Monetary Policy

Now we examine the countries’ choices of long-run money growth rates under two institutional regimes: the noncooperative regime and the cooperative regime. We also refer to the noncooperative regime as *policy competition*. At the end of this section, we will remark on the outcome under a unified currency. This section analyzes only the integrated economy, $s = 1/2$, with all proofs collected in Appendix A.3.
1.5.1. Noncooperative Monetary Policy

In the noncooperative regime, each country chooses its own money growth to maximize the country’s welfare, taking the growth rate of the other currency as given.\(^{10}\) We measure a country’s welfare with the steady state utility, per period, of the country’s representative household. Denote this welfare measure for country \(i\) as \(W_i = (1 - \beta) v(m_{i1}, m_{i2})\). Then,

\[
W_i = A \left( T_i^{ii} q_{i1} + T_i^{i'i} q_{i'i}^{f} \right) + A \left( T_i^{ii'} q_{i'i'} + T_i^{i'i'} q_{i'i'}^{f} \right) - \left[ T_i^{ii} (q_{i1})^{\sigma} + T_i^{i'i} (q_{i'i})^{\sigma} \right] - \left[ T_i^{ii'} (q_{i'i'})^{\sigma} + T_i^{i'i'} (q_{i'i'})^{\sigma} \right].
\]  

(1.31)

The first two groups of terms on the right-hand side are total utility of consumption and the last two groups of terms are disutility of production. After substituting the equilibrium quantities, \(W_i\) is a function of money growth rates, \((\gamma_1, \gamma_2)\), denoted \(W_i(\gamma_1, \gamma_2)\).

We use the so-called Friedman rule, \(\gamma = \beta\), as the reference point because this rule is often optimal in a closed economy.\(^{11}\) Suppose that one country fixes money growth at the Friedman rule. Then, the other country can increase its utility by setting its own money growth rate above the Friedman rule, as stated formally in the following proposition.

**Proposition 5.** Given \(\gamma_{i'} = \beta\), growth of currency \(i\) has the following effects near \(\gamma_i = \beta\):

\[d n_i/d \gamma_i < 0, \ d(\theta^{i'} - i)/d \gamma_i < 0, \ dW_i/d \gamma_i > 0 \text{ and } dW_i/d \gamma_i < 0.\]

Deviations from the LOP are the source of the gain from unilaterally increasing money growth above the Friedman rule. Because these deviations depend on the cross-country relative valuation of a currency, \(\theta\), they provide incentive for each country to use money growth to affect \(\theta\), thereby increasing the country’s purchasing power relative to the other country. To explain, suppose that

\(^{10}\)As is common in the literature, the game is a one-shot game. In particular, we do not examine trigger strategies. As is well known in game theory, allowing for trigger strategies generates a large set of equilibria, since any individually rational outcome can be supported by trigger strategies in an infinitely repeated game.

\(^{11}\)In this paper, we refer to the Friedman rule as the limit \(\gamma \rightarrow \beta\), because we have imposed \(\gamma_1 > \beta\) and \(\gamma_2 > \beta\) to ensure that all money constraints in trade bind.
country 1 sets its money growth rate above country 2’s. Then, country 1 households will end up having more of each currency in the equilibrium than country 2 households (see the explanation for Lemma 3). The higher holdings depress country 1 households’ valuation of the currencies relative to country 2 households. That is, \( \theta \) falls below one and the LOP fails. In particular, because the households in country 2 value the currencies more, they are willing to sell more goods for the currencies. Thus, a country 1 buyer is able to pay a lower price and buy a higher quantity from a foreign seller than from a domestic seller (see (1.20) and Corollary 4). This shift in the purchasing power increases country 1’s utility.

The gain from a unilateral deviation from the Friedman rule is redistributive, because it reduces the other country’s utility. There are three remarks on this redistributive effect. First, the channel of the redistributive effect is the deviations from the LOP, which do not exist in standard models. In this sense, the redistributive effect is significantly different from the classic effect of “beggar-thy-neighbor”. We will explore this distinction further in section 1.7. Second, because \( d\eta_i/d\gamma_i < 0 \), the redistributive gain from increasing growth of a currency comes with a reduction in the size of that currency area. Third, prices tend to respond to mitigate the redistributive gain. For example, when currency 1 has a higher growth rate than currency 2, all sellers charge higher prices to country 1 buyers than to country 2 buyers. However, this price disadvantage does not wipe out the redistributive gain to country 1.

Proposition 5 implies that the Friedman rule cannot be a Nash equilibrium under policy competition. To determine the Nash equilibrium, we need to examine the best response of one country’s money growth to any arbitrarily given growth rate of the other currency, not just the best response to the Friedman rule. When the other country does not follow the Friedman rule, the redistributive effect of inflation described above still exists. But inflation also erodes the purchasing power of currencies. When a country’s money growth is low, the redistributive effect
of inflation dominates, in which case the country’s welfare increases in its money growth. When a country’s money growth is sufficiently high, the negative effect of inflation on the purchasing power dominates, in which case the country’s welfare decreases in its money growth. Thus, there is an interior money growth rate that maximizes the country’s welfare, given the other country’s money growth. Moreover, this optimal money growth rate is higher if the other country’s money growth is higher. These features lead to the following proposition:\footnote{It may be reasonable to conjecture that all Nash equilibria must have $\gamma_1 = \gamma_2$, or that the equilibrium is unique under the restriction $\gamma_1 = \gamma_2$. Although we have not found any numerical example that contradicts either conjecture, we are not able to establish the result analytically, because a country’s best response is characterized by a complicated equation.}

**Proposition 6.** Assuming $\beta > 1 - \mu(1)$, then a Nash equilibrium exists. Equilibrium inflation rates are higher than the Friedman rule, i.e., $\gamma_1^* = \gamma_2^* = \gamma^* > \beta$.

1.5.2. Monetary Policy under Coordination

Under policy coordination or cooperation, the two countries choose growth rates of the two currencies jointly to maximize the world’s aggregate welfare. Since the two countries are symmetric, we measure aggregate welfare by the average of the two countries’ steady state utilities and denote it as $W_c = \frac{1}{2}(W_1 + W_2)$, where $W_1$ and $W_2$ are given by (1.31). The optimal monetary policy under the cooperative regime is a pair $(\gamma_1, \gamma_2)$ that maximizes $W_c$.

Because the two countries are symmetric, the optimal policy must have the same money growth rate of both currencies, say $\gamma$. By Lemma 1, then $\theta = 1$, $n_1 = n_2 = 1/2$, and $q_{ij} = q_{1ij} = Q(\gamma)$ for all $i, j = 1, 2$, where $Q(\gamma)$ is defined by (1.23). Thus, $W_c = W(\gamma)$ where

$$W(\gamma) \equiv 2L \left(\frac{1}{2}\right) \{AQ(\gamma) - [Q(\gamma)]^\sigma\}. \quad (1.32)$$
Since the right-hand side is increasing in $Q$ and $Q$ is decreasing in $\gamma$, $W'(\gamma) < 0$. Thus, the optimal policy under cooperation is $\gamma_1 = \gamma_2 = \beta$; i.e., the Friedman rule for both currencies.

It is not surprising that the Friedman rule is optimal under coordination. When the objective is to maximize aggregate welfare, the redistributive effect of inflation is irrelevant. The only effect to be considered is the negative effect of inflation in reducing the purchasing power of money. The Friedman rule minimizes this negative effect and, hence, maximizes aggregate welfare.

Not only does the cooperative regime generates higher aggregate welfare than the non-cooperative regime, it also increases each country’s welfare. That is, the cooperative outcome Pareto dominates the non-cooperative outcome. To see this, recall that the non-cooperative equilibrium generates $\gamma_1 = \gamma_2 = \gamma^*$. Thus, it has $\theta = 1$, $n_1 = n_2 = 1/2$, and $q_{ij} = q^f_{ij} = Q(\gamma^*)$ for all $i, j = 1, 2$. As a result, each country’s welfare level in the non-cooperative regime is given by the function $W(\gamma^*)$ defined above. Since $\gamma^* > \beta$, then $W(\gamma^*) < W(\beta)$.

The above analysis has established the following proposition.

**Proposition 7.** The optimal policy under coordination is the Friedman rule for both currencies. Welfare is higher for both countries under coordination than under no coordination.

The cooperative outcome is identical to that of currency unification. To see this, suppose that the two countries adopt a unified currency and each country receives a half of the transfers of the unified currency. Then, $n_1 = n_2 = 1/2$ in the equilibrium, and each country holds a half of the money stock. As a result, $q_{ik} = q^f_{ik} = Q(\gamma)$ for all $i, k = 1, 2$, where $\gamma$ is the growth rate of the unified currency. The welfare of each country is equal to $W(\gamma)$, which is maximized at $\gamma = \beta$. The implied allocation are the same as that of policy coordination. Thus, we can view currency unification as a way to coordinate monetary policy between the countries.
1.6. World Economy without Fully Integrated Markets

In this section we extend the main results in the last two sections to the case $s > 1/2$. We interpret $(1 - s)$ as the degree of market integration and examine how the optimal policy depends on market integration. Only the non-cooperative regime will be examined, because the optimal policy under coordination is still the Friedman rule.

With $s > 1/2$, a country’s money growth generates a redistributive effect as in the benchmark case. In addition, money growth generates an extensive effect that affects the number of trades experienced by a household’s sellers. To see this extensive effect, consider a country 1 household. The number of trades experienced by the household’s sellers in a period is

$$T_1 \equiv T_{11}^1 + T_{21}^1 + T_{11}^2 + T_{21}^2 = 2\left[n_1\mu(n_1)s + (1 - n_1)\mu(1 - n_1)(1 - s)\right].$$

Start with $\gamma_1 = \gamma_2$ so that $n_1 = 1/2$ initially. Since $dn_1/d\gamma_1 < 0$ and $\mu(n) + n\mu'(n) > 0$, $T_1$ decreases in $\gamma_1$ for all $s > 1/2$. This reduction reflects the simple fact that, because there are more country 1 sellers in area 1 than in area 2, the shift of the buyers to area 2 reduces matches for country 1 sellers in area 1 by more than increasing matches for country 1 sellers in area 2.

The reduction in $T_1$ increases the utility of country 1 households, as it reduces the disutility of production. That is, growth of currency 1 generates a positive extensive effect on country 1’s utility. This extensive effect is an externality, generated by all households’ allocations of buyers. Note that the total number of trades experienced by the household’s buyers does not change when a country’s money growth increases marginally above the other country’s.\(^{13}\)

\(^{13}\)This can be verified by computing the buyers’ number of trades as $[n_1\mu(n_1) + (1 - n_1)\mu(1 - n_1)]$. Starting from any $\gamma_1 = \gamma_2$ so that $n_1 = 1/2$, this number does not change with $n_1$ and hence not with $\gamma_1$. 
The effect of country 1’s money growth on country 1’s welfare can be decomposed as follows:

\[ q \sigma \frac{dW_1}{d\gamma_1} = \frac{\sigma(2s - 1)}{2\beta(\sigma - 1)} - \mu \left( \frac{1}{2} \right) \left[ \frac{s^2 + (1-s)^2}{\sigma - 1} + 1 \right] \frac{d\theta}{d\gamma_1} + \frac{dT_1}{d\gamma_1} \]  

(1.33)

Here, all the derivatives are evaluated at \( \gamma_1 = \gamma_2 = \beta \). Now, the redistributive effect has two parts: the effect through \( \theta \) and a direct effect of money growth on the quantities of trade. When \( s \) is close to 1/2, the redistributive effect and the extensive effect of inflation are both positive. In this case, the Nash equilibrium under policy competition has \( \gamma_1 = \gamma_2 > \beta \), as in the case \( s = 1/2 \).

When \( s \) is significantly above 1/2, the analytical result is not clear. We resort to numerical examples. Consider the following parameter values:

\[ \beta = 0.995, \quad A = 4, \quad \sigma = 2, \quad \psi = 0.5. \]

The value of \( \beta \) suggests that a period can be interpreted as one month. We start with three values of \( s \): 0.5, 0.75 and 1. Fixing \( \gamma_2 = \beta \), we depict country 1’s welfare as a function of \( \gamma_1 \) in Figure 1.1. Because a country’s welfare depends on its money growth in a hump-shaped pattern, as in the benchmark case, the optimal money growth is above the Friedman rule. Note that the peak in Figure 1.1 occurs at higher levels of \( \gamma_1 \) when \( s \) is smaller, which indicates that the incentive to inflate is higher when the two areas are more integrated with each other.\(^\text{14}\)

To see more clearly how the incentive to inflate changes with \( s \), we now examine the entire range \( s \in [0.5, 1] \). Figure 1.2 depicts how the overall welfare effect of inflation and its decomposition depend on \( s \). In this figure, \( \gamma_2 \) is fixed at \( \beta \) while \( \gamma_1 \) is increased marginally above \( \beta \). Other parameters are given the same values as in Figure 1.1. As the degree of integration falls (i.e., as

\(^\text{14}\)In contrast to the benchmark case, an increase in \( \gamma_1 \) does not necessarily reduce \( \theta \) when \( s > 1/2 \). For example, when \( s = 0.75 \) or 1, an increase in \( \gamma_1 \) increases \( \theta \) (not shown in Figure 1). Despite this different response of \( \theta \), an increase in \( \gamma_1 \) still reduces \( n_1 \) and generates a positive redistributive effect to the country. The redistributive effect is the same because it now has two terms (see (1.33)), the first of which is always positive when \( s > 1/2 \).
$W_1$ as a function of $\gamma_1$

Figure 1.1. Country 1’s welfare $W_1$ as a function of $\gamma_1$

Welfare effects

Figure 1.2. Effects of $\gamma_1$ on country 1’s welfare and its composition

$s$ increases), the redistributive effect falls, the extensive effect first increases and then falls, and the overall welfare gain falls. The overall welfare gain is positive for all $s$.

Figure 1.3 depicts how the common money growth rate in the Nash equilibrium changes with $s$. Equilibrium inflation is always above the Friedman rule and, in fact, positive. Also, it increases with market integration and reaches the maximum at $s = 1/2$. Because inflation is not Pareto
efficient in the economy, the results in Figure 1.3 suggest that the integration of the two areas increases the need for monetary coordination between the two areas.

It is worthwhile mentioning that the incentive to inflate, and hence the need for coordination, exists even in the case $s = 1$. When $s = 1$, all goods are sold for the seller country’s currency, as in the cash-in-advance model described by Helpman (1981). Despite this resemblance, our model’s implication for monetary coordination is opposite to Helpman’s. To facilitate the comparison, we modify Helpman’s model so that output is produced with elastically supplied labor, a feature of our model. With this modification, Helpman’s model implies that a unilateral deviation from the Friedman rule reduces the country’s consumption and welfare (see Appendix A.4). Since the Friedman rule is optimal for each country, there is no need for coordination. In contrast, our model generates a gain from coordination even in the case $s = 1$. The reason is that, even when $s = 1$, our model still possesses the channels through which a country can gain from inflation, namely, the failures of the LOP and the extensive effect of inflation. These channels disappear when the goods markets are Walrasian, as in Helpman’s model.
1.7. Directly Taxing Foreign Use of a Currency

It is well known that inflation can generate the effect of “beggar-thy-neighbor”. For example, this effect can arise in models by Kareken and Wallace (1981), King et al. (1992) and Cooley and Quadrini (2003). Inflation motivated by this effect alone serves effectively as a tax on foreign use of the currency and, as such, its role is eliminated if each country can directly impose such taxes. Can such taxes eliminate the incentive to inflate in our model? The answer is no. To support this answer, we introduce the taxes below.\textsuperscript{15}

Consider a buyer of country $i$ in a trade in area $k$. When the buyer buys from a domestic seller, let the amount of money spent by the buyer be $x_{ik}/(1 - \tau_{ik})$. A fraction $\tau_{ik}$ of this amount is paid as a tax and the remaining amount, $x_{ik}$, goes to the seller. Similarly, when this buyer buys from a foreign seller, the amount spent is $x_{ik}^f/(1 - \tau_{ik}^f)$, of which the tax is a fraction $\tau_{ik}^f$. Without loss of generality, we assume that the tax receipts are burned.

Of this specification of taxes, we first examine the following combination: each country $i$ chooses $\tau_{i'i}^f = \tau_{i'i} = \tau_i$, where $i' \neq i$, and sets all other taxes to zero. That is, each country $i$ can tax foreign buyers on all transactions in which foreign buyers use currency $i$. Note that the tax $\tau_{i'i}$ is imposed on transactions in which both the seller and buyer are foreigners. Thus, the combination of taxes gives a country more power to tax foreign buyers than it has in reality. If inflation in our model were just a tax on the use of a currency by foreign buyers, it would be superseded by the optimal choice of $\tau_i$.

We compute optimal taxes and inflation numerically for the case $s = 1/2$, with the same parameter values as in section 1.6. Fixing $\gamma_2 = \beta$, we express country 1’s optimal choices of $(\tau_1, \gamma_1)$ as functions of $\tau_2$ and depict them in Figures 1.4 and 1.5. Two traditional features appear in these figures. First, a country has incentive to tax foreign use of its currency. For

\textsuperscript{15}We thank Narayana Kocherlakota for suggesting this exercise.
example, the optimal choice of $\tau_1$ is positive for all $\tau_2 \geq 0$. Second, a country’s incentive to inflate is lower in the presence of the tax. For example, country 1’s optimal inflation falls as the country’s tax, $\tau_1$, increases with $\tau_2$. Thus, part of the role of inflation is to tax foreign use of a currency.

However, the tax does not eliminate a country’s incentive to inflate. For all given values of $\tau_2$, country 1’s optimal inflation is above the Friedman rule even after $\tau_1$ is chosen optimally. This implies that optimal inflation is above the Friedman rule in the Nash equilibrium. To see this, note that country 2’s optimal tax, $\tau_2 = \tau_2(\tau_1)$, is the inverse function of the graph in Figure 1.4, because the two countries are symmetric. Putting this function in Figure 1.4, we can see that the tax rates in the Nash equilibrium is $\tau_1 = \tau_2 = 0.24$. Then, inflation in the Nash equilibrium is $\gamma_1 = \gamma_2 = 1.013$. Thus, monetary coordination between the two countries can still improve welfare of the two countries in the presence of taxes.

The tax is not a perfect substitute for inflation, because inflation can affect the deviations from the LOP in more ways than the tax does. There are at least two such differences between

Figure 1.4. Optimal tax $\tau_1$ as a function of $\tau_2$
Figure 1.5. Optimal money growth $\gamma_1$ as a function of $\tau_2$

inflation and the tax. First, a country’s tax redistributes the purchasing power from foreign buyers to domestic buyers only in trades that use the country’s currency. It does not achieve the redistribution in trades that use the other country’s currency. In contrast, a country’s money growth redistributes the purchasing power to the country’s buyers in both types of trades, by depressing $\theta$. Second, a tax on foreign buyers reduces foreign households’ valuation of that country’s currency, and hence increases the relative price that the country’s buyers must pay to foreign sellers versus domestic sellers. This response of the relative price is opposite to what happens under inflation, and it reduces the benefit of the tax to the country’s buyers. Because of these differences, the incentive to inflate remains in the presence of the optimal tax.

The above results are not caused by the restriction $\tau_{fi}^f = \tau_{fi}^v$. After allowing each country $i$ to choose $\tau_{fi}^f$ and $\tau_{fi}^v$ independently, we found the similar results. Moreover, we have investigated other combinations of taxes in which each country can choose two taxes. So far, all the numerical exercises have failed to yield a combination of taxes that can restore the Friedman rule as the optimal policy. To eliminate the incentive to inflate, each country must be able to choose a more
comprehensive combination of taxes. Even if such a combination exists, it goes much beyond the conventional argument that inflation is a tax on foreign use of a country’s currency.

1.8. Endogenous Allocation of Sellers

In this section, we allow each household to choose the allocation of sellers to the two areas, as well as the allocation of buyers. This extension is intended to illustrate three results. First, the main results in previous sections are robust to the extension. Second, the meaning of market integration can be made precise and linked to the number \((1 - s)\). Third, the qualitative results are not sensitive to the assumption on bargaining that we used in previous sections.

Let \(s_i\) be the fraction of sellers whom a household in country \(i\) sends to area \(i\). To make this choice nontrivial, a seller must obtain a positive surplus from trade. Thus, we need to modify the assumption that buyers make a take-it-or-leave-it offer in trade. One way to do so is to adopt the generalized Nash bargaining formula. This approach complicates the analysis significantly, even for numerical exercises.\(^{16}\) Instead, we assume that each seller sells the goods with a fixed markup, \(\nu > 0\). That is, in the transactions involving a country \(i\) buyer in area \(k\), where \(i, k \in \{1, 2\}\), the quantities of trade satisfy the following conditions:

\[
\Omega_{ik} x_{ik} = (1 + \nu) (q_{ik})^\sigma, \quad \Omega_{ik} x_{ik}^f = (1 + \nu) \left( q_{ik}^f \right)^\sigma.
\]

The surplus of trade to the seller is \(\nu (q_{ik})^\sigma\) in the first trade and \(\nu (q_{ik}^f)^\sigma\) in the second trade. The special case \(\nu = 0\) corresponds to the model in previous sections.\(^{17}\)

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\(^{16}\)The complication comes not only from the non-linearity of the equation system, but also from the fact that it is no longer possible to reduce the dimension of the equation system by substituting a subset of quantities as functions of other variables, as we did in section 1.3.

\(^{17}\)We have also allowed sellers to have a different markup in the foreign area from that in the domestic area. The results are similar to the ones presented here and, hence, omitted.
To link market integration to the number \((1 - s_i)\), we assume that it is more costly to send the sellers to the foreign currency area than to the domestic currency area. This additional cost, expressed in utils and denoted as \(c(1 - s_i)\) for country \(i\), has the following form:

\[
c(1 - s_i) = c_0 (1 - s_i)^\alpha, \quad c_0 > 0, \quad \alpha > 1.
\]

We interpret the number \(1/c_0\) as the degree of market integration. The higher the number \(1/c_0\), the lower the additional cost of selling in the foreign currency area, and hence the higher the degree of integration. When \(c_0 = 0\), we say that the two goods markets are fully integrated.\(^{18}\)

We compute the extended model and perform the sensitivity analysis with respect to the parameters. The main findings are as follows. First, the qualitative results remain the same as in the model with a fixed \(s\). In particular, (i) the exchange rate is determinate, (ii) deviations from the LOP exist and depend on the differential in money growth and (iii) money growth is above the Friedman rule in the Nash equilibrium, and so monetary coordination improves welfare for both countries. Second, the extension justifies the results in section 1.6 as comparative statics with respect to market integration. In particular, the solution for \(1 - s\) increases in the new measure of market integration, \(1/c_0\). As \(1/c_0\) increases to \(\infty\), \((1 - s)\) increases to \(1/2\), which we called a fully integrated economy before. Third, an increase in the markup, \(\nu\), increases the fraction of sellers allocated to the foreign market, \((1 - s)\). This is simply because an increase in \(\nu\) reduces the disadvantage of selling in the foreign market compared to the domestic market.

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\(^{18}\)Another purpose of introducing the additional cost is as follows. When the households choose the allocation of both buyers and sellers, these choices respond to changes in the differential between the two money growth rates in a bang-bang fashion, because we have assumed that the goods produced in the two countries are perfect substitutes. The introduction of the cost prevents these drastic responses. Similar costs have been used in models of open economy macro in the adjustment of the capital stock.
1.9. Conclusion

In this chapter we integrate monetary search theory into open-economy macro to construct a microfoundation of exchange rates. Each currency derives a value from alleviating the trading difficulties. By postulating a matching function that exhibits local congestion, we determine the exchange rate between the currencies and the size of each currency area endogenously. Search frictions generate deviations from the law of one price, despite the fact that prices are perfectly flexible. Because the deviations depend on the differential between countries’ money growth rate, each country has temptation to use inflation to exploit these deviations. Thus, when countries set money growth in a non-cooperative game, inflation is higher than the socially optimal level. Monetary coordination reduces inflation and improves welfare for all countries.

In contrast to most models on monetary coordination, the incentive to inflate in our model does not come from the terms of trade, because all goods are tradable and the two countries produce perfectly substitutable goods. Instead, the incentive comes from the deviations from the law of one price, which arise from search frictions and asymmetric monetary transfers. Thus, the incentive to inflate and the gain from monetary coordination remain even after each country sets optimally the direct taxes on foreign use of its currency. Such taxes would eliminate the incentive to inflate if manipulating the terms of trade were the motivation for inflation.

We also show that the temptation to exploit the deviations from the LOP becomes stronger when the barriers to selling in the foreign currency area fall. Higher integration between the markets enables individuals to avoid more easily the country’s inflation by transacting more often in the foreign market. Thus, the gain from monetary coordination increases when goods markets become more integrated between countries. However, because monetary coordination is not a Nash equilibrium in a one-shot game, one way to achieve the coordination outcome is to adopt a
common currency. Our results suggest that the need for currency unification increases when the
two areas are more integrated with each other in trade.

In addition to the policy analysis, our paper provides a tractable framework that generates
values for the currencies from detailed descriptions of preferences and technologies, without re-
sorting to ad hoc assumptions on the use of a particular currency. This framework should be
useful in the future for examining a wide range of issues on currencies and exchange rates. In
particular, the framework generates deviations from the LOP in a natural way and without any
specific assumptions on the market power of individual sellers. In the second Chapter, I explore
the implications of these deviations for the volatility of exchange rates. After calibrating the
model with shocks to productivity and money growth, it is shown that the model is able to
capture most of the volatility of the real exchange rate, without any price stickiness.
CHAPTER 2

The Volatility of Exchange Rates in a Search Model

2.1. Introduction

There are several well-known facts that characterize the behavior of real exchange rates in the business cycle data: i) The real exchange rates is highly volatile relative to other macroeconomic variables such as aggregate output and consumption; ii) Changes in the real exchange rate are very persistent and the real exchange rate follows approximately a random walk; iii) There are substantial and systematic differences in the behavior of real exchange rates under fixed versus floating exchange rate regimes, while fluctuations of output and consumption do not seem to change systematically with the regimes. As an international relative price, the real exchange rate is expected to play an important role in real allocation across countries. Thus, the real exchange rate movement should be strongly correlated with fluctuations in real variables. However, the above mentioned empirical facts suggest only a weak link between the behavior of the real exchange rate and other macroeconomic variables, which poses a non-trivial challenge for international business cycle models.

The conventional explanation for fluctuations of exchange rates usually rests on the interaction of the nominal price rigidities and monetary shocks. The basic idea is simple: Monetary shocks can induce an immediate change in the nominal exchange rate. Since prices are sticky in the short run, fluctuations in the nominal exchange rate translate one for one into the real exchange rate movement. The volatility of exchange rates can be much higher than their underlying fundamentals because of the sticky price and high capital mobility. The earliest version of the sticky price model goes back to the celebrated Mundell-Fleming-Dornbusch framework. With
the ad hoc link between money demand and saving-investment decision, as well as the lack of an explicit theory of price setting, this framework is inadequate for understanding the real exchange rate. The similar idea can also be embedded in the general equilibrium framework. Following Obstfeld and Rogoff’s (1995) pioneering work, a series of papers have applied the general equilibrium model with microfounded price setting to explore the features of exchange rates. Chari, Kehoe and McGrattan (2002) evaluate such models quantitatively and show that sticky price models are potentially capable of accounting for the highly volatile and persistent real exchange rates.\(^1\) To generate enough volatility, however, they have to assume an implausibly high risk aversion.\(^2\) Moreover, their model typically needs 11 quarters or longer of price stickiness to replicate the persistence in the data. Recent evidence in Bils and Klenow (2005), however, seriously calls into question the assumption of such a long-lived price stickiness.\(^3\)

Duarte (2003) applies the new open-macroeconomics models to explain the systematic changes in the behavior of real exchange rates across regimes. Using the assumption that firms set prices one period in advance, her model shows no persistence in real exchange rates and much less volatile exchange rates than in the data. To get better quantitative results, a longer-term price setting in such sticky price models seems necessary.

An alternative approach to explain the exchange rate volatility is to use models with flexible price, an idea that has been implemented firstly in Stockman (1988). Backus, Kehoe and Kydland (1992, 1994), and Stockman and Tesar (1995) advance the so-called "international real business cycle models" in a series of influential papers and try to use such models to match up the international business data. Quantitatively, however, these models fail to explain the facts above

\(^1\)Bergin and Feenstra (2001) obtain similar results.  
\(^2\)Their model sets up a link between real exchange rates and the ratio of the marginal utility of consumption in the two countries. With separable utility function, the volatility of real exchange rates is essentially determined by the risk aversion parameter and the volatility of consumption. Such a model can easily generate highly volatile exchange rates by imposing a high value of risk aversion parameter.  
\(^3\)Bils and Klenow (2005) find that half of all posted prices last less than 4.3 months.
that characterize the behavior of real exchange rates. The law of one price always holds in these models since the goods market is Walrasian and frictionless. It seems unlikely that models without deviations from the LOP can generate highly volatile real exchange rates. Moreover, fluctuations of the real exchange rate in such models are driven entirely by real disturbances, and persistence in real exchange rate is due to persistence in the underlying real shocks. These results are inconsistent with empirical findings which support the view that monetary shocks, rather than real shocks, account for a substantial fraction of the variability of the real exchange rate.

In order to understand the real exchange rate volatility, it seems clear that we need to introduce at least some type of goods market frictions into the flexible-price model to generate the deviations from the LOP. Monetary search theory is a natural framework to capture these frictions. In this chapter, we integrate the monetary search theory into international economics to derive the role of money in a two-country, two-currency economy. All goods are tradable between the two countries and prices are fully flexible. In contrast to the Walrasian market in standard models, the goods market is decentralized and modelled as randomly and bilaterally matching. Search frictions generate the differential in a household’s valuations of the two currencies and create the deviations from the LOP, which contribute to the majority of fluctuations of exchange rates in our model. Even without any nominal rigidity, quantitatively the model can produce enough volatility of exchange rates to be consistent with the data.

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4In an empirical work, Alessandria and Kaboski (2004) demonstrate that the deviations from the law of one price are an important source of violations of absolute PPP across countries.

5Rogers (1999) studies the variation in the real pound-dollar exchange rate and estimates the contribution of various shocks. He finds that monetary shocks account for up to 60 percent of the variations in real exchange rate, while the contribution of fiscal and productivity shocks combined is only 4 to 26 percent. Faust and Rogers (2003) also show that the contribution of monetary shocks can be more than 50 percent of the variance shares.
For simplicity, we assume that the two countries have the same size, identical preferences and production technologies. Both consumption and production are specialized. Because of the lack-of-double-coincidence-of-wants problem as well as the absence of a perfect record-keeping device, money is essential in the economy as a medium of exchange. Both countries can issue their own currencies and inject money into the economy as a lump-sum transfer to their domestic residents. Thus, money injection is asymmetric in the sense that households receive only the transfer of domestic currency. Both currencies are acceptable in the goods market for any transaction, and there is a centralized currency market where households can exchange currencies. Although a currency does not directly generate utility or facilitate production in the model, it obtains positive value in an equilibrium by alleviating the difficulty of exchange. The relative role of each currency in the trade determines the nominal exchange rate in the currency market.

The valuation of the same currency differs for households in different countries because of the presence of search frictions in the goods market. The centralized currency market only equalizes the relative valuations of the two currencies but not the levels of the valuations. The gap between the two countries’ valuations of a currency does not disappear unless households can trade goods directly for currency in the currency market, which is impossible since transactions in the goods market are formed in random and bilateral matches. There is no way to arbitrage between goods market and currency market to eliminate this differential in the valuations of currencies. Therefore, even considering the same goods, buyers from different countries face different prices (pricing-to-market behavior) and buyers with different types of currency pay different prices. Thus, there are deviations from the LOP, despite the fact that prices are fully flexible and all goods are tradable between the two countries. We decompose the fluctuations in real exchange rates and illustrate that fluctuations in the deviations from the LOP are the main component of the real exchange rate movement.
We calibrate the model to U.S. and European aggregate data and evaluate it quantitatively. The model closely replicates the feature of exchange rates listed earlier as stylized facts. The real exchange rate is much more volatile than output and consumption, exchange rates are highly persistent as in the data, and the cross-correlations between most variables are close to those observed in the data.

The deviations from the LOP come from the relative valuation of the two currencies and the relative effective money holdings across countries, both of which depend critically on the differential between the two countries’ money growth rates. Since monetary shocks directly affect this differential, they account for most of the fluctuations in the real exchange rate in our model. This result is consistent with the empirical evidence that suggests monetary shocks, rather than real shocks, cause more real exchange rate fluctuations.

Real shocks, on the other hand, do play a role, albeit a less important one, in explaining the fluctuations of real exchange rates in our model. Real shocks do not change the relative valuation of the two currencies; instead, they do change the quantities exchanged in a trade match and hence the level of valuations of the currencies. However, since the goods are sold for both currencies, real shocks affect the valuation of both currencies in the same way and so the differential in the valuation of the two currencies remains the same.

Search frictions in the goods market are the key feature of our model. The simulated results suggest that the volatility of exchange rates increases with the degree of frictions. Moreover, to illustrate the importance of search frictions in generating volatility and the persistence of exchange rates, we construct a Walrasian model with cash-in-advance constraints in a similar environment.6 We apply the same preference and technology as those applied in the benchmark model. The only difference is that the goods market is now Walrasian and frictionless. The LOP always holds

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6This model is a variant of Helpman’s (1981) model with elastic labor supply as in our model.
in this CIA model. Using the same parameter values and shock processes, the simulated results show that neither monetary shocks nor real shocks are capable of generating much volatility of exchange rates without the presence of search frictions in the goods market. The persistence in exchange rates in this CIA model comes entirely from the persistence in the underlying shocks.

The behavior of exchange rates in different regimes is also examined in this chapter. The model shows a sharp increase in the volatility of the real exchange rate when a country moves from a pegged to a floating regime, while fluctuations of output and consumption do not change systematically with exchange rate regimes. Moreover, the co-movements of output and consumption across countries are higher under a fixed rate regime than under a flexible rate regime. These results are consistent with the empirical findings. These results are intuitive: To maintain the pegged rate, one country has to follow the other country’s monetary policy and keep the money growth at the same rate as the other country. Since the driving force of the deviations of the LOP, the differential in money growth rates across countries, does not exist in this case, the LOP holds and, as a result, there is hardly much fluctuation in the real exchange rate. The movements of other macroeconomic variables, such as output and consumption, do not rely on the two countries’ money growth rate differential. Therefore, the switch of exchange rate regimes does not have a systematic effect on these variables.

Many researchers have investigated the deviations from the LOP as a factor generating the fluctuations in the real exchange rate. The explanations of these persistent deviations from the LOP in previous models either rely on the assumption of pricing-to-market or simply assume sticky prices with local-currency-pricing (see the work of Dornbusch (1987), Krugman (1987), Knetter (1989), Goldberg and Knetter (1997), Betts and Devereux (1996, 2000), Chari, Kehoe and McGrattan (2002).). In our paper, however, we focus on the search frictions as the factor
that produces the deviations from the LOP. This feature of the model yields new insights into the behavior of exchange rates in the business cycle.

How a country’s welfare varies under different exchange rate systems is an interesting issue. Our model is especially suitable to deal with such questions since the role of each currency is derived endogenously from detailed descriptions of preferences and technologies. While exchange-rate stability is often viewed as favorable to trade and therefore welfare improving, whether such a view can be supported with a rigorous model still needs to be examined. Although this examination is interesting, it is outside the scope of this chapter and hence is left for future research.

As mentioned earlier, our model builds on the recent development in monetary search theory. Early applications of this theory to multiple currencies and exchange rates include Matsuyama, et al. (1993), Shi (1995), Trejos and Wright (1996) and Zhou (1997). These applications have assumed money or goods, or both, to be indivisible. As a result, these models are not suitable for analyzing issues related to money growth and inflation. The current model eliminates this restriction by using the construct of large households in Shi (1997).

The papers closest to ours are Head and Shi (2003) and Liu and Shi (2007). Using a similar setup, Head and Shi focus on the determination of the nominal exchange rate while Liu and Shi model currency areas, examining the welfare effects of long-run money growth. Both papers work in a deterministic environment and focus on stationary equilibrium. In this chapter, however, we are interested in the dynamics of exchange rates and hence go beyond the steady-state analysis, introducing monetary shocks and technology shocks into the model. Moreover, in contrast to the

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7The construct of large households keeps the analysis tractable: By smoothing the matching risks within a household, the assumption makes the distribution of money holdings across households degenerate. Lagos and Wright (2005) use a different device to make the distribution of money holdings degenerate.
theoretical analysis in the search literature, we evaluate the model quantitatively and use the search theoretical model to capture the features of exchange rates observed in the data.

2.2. The Model

2.2.1. Countries and households

Consider a world economy consisting of country 1 and country 2. To keep the problem simple, the two countries are assumed to have the same size, the same preference and production technology. Each country is populated by a large number of infinitely-lived households, which belong to different types. All the households are specialized in both consumption and production. Specifically, a type $k$ household produces only good $k$ and consumes only good $k+1 \ (\text{mod} \ J, J \geq 3)$. Therefore, the problem of lack-of-double-coincidence-of-wants exists in this environment, which generates a need for the use of a medium in the goods exchange.

All goods are tradeable between the two countries and households consume both local and foreign produced goods. The same type of goods produced in the two countries, however, are imperfect substitutes in consumption, with constant elasticity of substitution $\eta > 1$. Let $c_{ih,t}$ and $c_{if,t}$ denote date $t$ country $i \ (i = \{1, 2\})$ household’s consumption of local and imported goods, respectively.\(^8\) The consumption index of a household in country $i$ is defined as

$$C_{i,t} = \left[ \rho^{1/\eta} c_{id,t}^{(\eta-1)/\eta} + (1 - \rho)^{1/\eta} c_{if,t}^{(\eta-1)/\eta} \right]^\eta/\eta.$$ \hspace{1cm} (2.1)

where the parameter $\eta$ represents the elasticity of substitution between local and imported goods and the weight $\rho$ determines the agents’s bias for the consumption of local goods, $c_{id}$.

\(^8\)Note that some of the symbols used in this Chapter do not necessarily have the same meanings as the ones used in the first Chapter.
Each household consists of a continuum of infinitely-lived members, who are divided into two groups according to their roles in the exchange. Those who carry money and purchase goods are called buyers and those who produce and sell goods are called sellers. The individual member of a household does not make decisions. Instead, he simply follows the rules prescribed by the household.\footnote{This large household setting is a device that makes the distribution of money holdings degenerate across households and so that allows for a tractable analysis for issues related to money growth, see Shi (1997, 1998, 1999).} For simplicity, the composition of buyers and sellers in each household is set to be fixed and thus all households in the economy have the same number of buyers and sellers. We normalize the number of buyers in a household to one and use $s$ to denote the number of sellers.

Each country issues fiat money which does not generate direct utility or facilitate the production. The monies are indexed as currency 1 and currency 2, corresponding to the issuing countries. In contrast to the cash-in-advance constraints in traditional models, we do not impose any restrictions on the use of currency in a purchase and therefore both currencies are acceptable in the goods market.\footnote{Note that a buyer can buy local goods with foreign currency. In our model, if imposing symmetry, the volume of transactions of local goods by domestic buyers with domestic currency is the same as that by domestic buyers with foreign currency. Looking at the real world, however, most transactions of local goods by domestic buyers involve domestic currency rather than foreign currency. The share of trade is poorly matched with the data because we are capturing only traded goods in the model. By adding non-traded goods that is only sold for domestic currency, the model will then characterize the world economy much better and bring the share of trade closer to the data.} There is a restriction on carrying currency, however, that a buyer can bring only one type of currency into a trade at a time. Thus, although buyers can use either of the two currencies to buy the goods, they cannot use both currencies in one purchase.\footnote{If buyers can carry both currencies into the trade and make arbitrage between trades, there is the problem of indeterminancy in the model. Head and Shi (2003) make a similar argument and provide a detailed discussion.}

The buyers and sellers of the households in the two countries are sent to the goods market, meeting with each other bilaterally and randomly. Meetings in which a successful transaction occurs we call a \textit{trade match}. For simplicity, we exclude the possibility of barter. Therefore, all the trade matches involve money-goods transaction, where a buyer who carries money meets with a seller who produces the type of goods that the buyer’s family prefers. There are in total eight
different types of matches which result in trade: a country $i$ buyer holding domestic currency meets with a country $i$ ($i'$) seller, and a country $i$ buyer holding foreign currency meets with a country $i$ ($i'$) seller.\textsuperscript{12} We use $T_{ij}^k$ to denote the aggregate number of trade matches between a country $i$ buyer who carries currency $k$ and a country $j$ seller, where $i, j, k \in \{1, 2\}$.

The aggregate matching function is assumed to take the form of the commonly used Cobb-Douglas functions:

\[
T_{ii}^k = T_0 N_{ik} s_i \psi_1^{1-\psi_i}, \quad T_{ii'}^k = \alpha T_0 N_{ik} s_{i'}^{1-\psi_i};
\]  

(2.2)

where $\alpha, \psi \in (0, 1)$. $N_{ik}$ is the number of country $i$ buyers who carry currency $k$. $s_i$ is the number of sellers from country $i$ and $s_{i'}$ is the number of sellers from country $i'$. An important feature of this matching function is that the matching frequencies within and across countries are asymmetric. Since $\alpha < 1$, according to the matching function, agents are more likely to meet domestic people than foreign people ($T_{ii'}^k / N_{ik} = \alpha T_{ii}^k / N_{ik}$). This assumption is reasonable since trades across countries are generally much more difficult than within a country. Thus, a country can be interpreted as a group of households that have a higher possibility of meeting with each other.

Buyers are assumed to take all the bargaining power in the transactions.\textsuperscript{13} They propose an offer of $(x_{ik}, q_{ik})$ in the trade matches with domestic sellers and $(x_{i'k}, q_{i'k})$ with foreign sellers. $x$’s denote the amount of money paid by the buyers and $q$’s denote the quantity of goods sold by the sellers. The first subscript of $x$ and $q$ indicates the index of the buyer’s country and the second subscript indicates the type of currency used in the trade. Superscript $f$ indicates

\textsuperscript{12}In this chapter, $i$ and $i'$ denote the index of a country, where $i, i' \in \{1, 2\},$ and $i' \neq i$.

\textsuperscript{13}The assumption of buyers’ taking all the bargaining power is not necessarily needed to obtain the main results. In fact, since the deviations from the LOP are the driven force of the fluctuations in exchange rates, the model can generate even larger volatility of exchange rates if sellers obtain some market power.
Table 2.1. Trading arrangements and trade matches in the goods market

<table>
<thead>
<tr>
<th>Buyer’s country</th>
<th>Seller’s country</th>
<th>Trading arrangement</th>
<th>Number of matches</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>i</td>
<td>$x_{ik} \leftrightarrow q_{ik}$</td>
<td>$T_{ii}$</td>
</tr>
<tr>
<td>i</td>
<td>$i'$</td>
<td>$x_{ik} \leftrightarrow q_{ik}'$</td>
<td>$T_{ip}$</td>
</tr>
<tr>
<td>$i'$</td>
<td>i</td>
<td>$x_{i'k} \leftrightarrow q_{i'k}$</td>
<td>$T_{i'i}$</td>
</tr>
<tr>
<td>$i'$</td>
<td>$i'$</td>
<td>$x_{i'k} \leftrightarrow q_{i'k}$</td>
<td>$T_{i'i'}$</td>
</tr>
</tbody>
</table>

that the purchase is made between agents from different countries. Table 2.1 lists the trading arrangements in all the types of trade matches. We will describe in details about how $x$’s and $q$’s are determined in section 2.2.3.

Besides the decentralized goods market, there is also a currency market where households can exchange currencies with each other. In contrast to the goods market, the currency market is centralized and frictionless and a nominal exchange rate clears the market in equilibrium. Note that the goods market and the currency market are separated from each other, and there is no possibility of arbitrage between the two market.

2.2.2. Aggregate Shocks and Timing of Events

Two aggregate shocks are considered in this economy: One to monetary growth rates and the other to productivity. Money is injected into the economy at the beginning of each period. Let $M_{i,t}$ denote the aggregate stock of currency $i$ per household in period $t$. Each household in country $i$ receives a lump-sum transfer of domestic currency $(\gamma_{i,t+1} - 1) M_{i,t}$ at the beginning of period $t+1$. Therefore, $\gamma_{i,t+1} = M_{i,t+1}/M_{i,t}$ is the gross growth rate of currency $i$ between period $t$ and $t + 1$. The money growth rate $\gamma_{i,t+1}$ follows a stochastic process which we will describe later. Productivity shocks are captured by the disutility function of production. By producing the amount of $q$ of goods, each household suffers a cost of $A_i q^\sigma$, measured in terms of utility. $\sigma$ is
greater than 1 so that the cost function is convex. $A_i$ is a stochastic variable that represents the deviation of innovations to productivity in country $i$.\footnote{Labor supply is assumed to be elastic. The disutility of production can be derived from disutility of labor and the production function. Suppose production function takes the form of $Q = zF(l)$, where $z$ captures technology shocks. Therefore, to produce quantity $Q$ of goods, the labor input required is $l = F^{-1}(Q/z)$. Substituting it into the function of disutility of labor $c(l)$, we obtain the disutility of production. $A$ denotes the part that involves $z$ and hence represents the technology shocks in our model.}

The events in an arbitrary period $t$ unfold as in Figure 2.1, with the subscript $t$ suppressed. At the beginning of period $t$, the two aggregate shocks are realized and each household receives a lump-sum monetary transfer of only domestic currency. Then, aggregate money stock, $M_i$, and each household’s money holdings are measured. Denote the money holdings of currency $k$ in country $i$ household as $m_{ik}$.\footnote{We are interested in the stationary equilibrium. To make the economy stationary, we deflate all the nominal variables ($m_{ik}, x_{ik}, z^f_{ik}, e, f_{ii}$) with the level of corresponding money stock. For example, $m_{ik}$ is normalized by $M_k$ and represents the relative measure of money holdings across countries.}

We assume the currency market opens before the goods market.\footnote{This assumption is just to simplify the algebra. As long as the two markets are separated and agents cannot arbitrage between the markets, the sequence of market opening does not affect the main results.} A country $i$ household sells $f_{ii}$ unit of domestic currency for foreign currency at an exchange rate $e$ in the currency market. After the currency market closes, the household divides its buyers into two groups and allocates domestic and foreign currencies to these two groups, respectively. In particular, in a country $i$ household, there are a number of $n_i$ buyers holding domestic currency and a number of $(1 - n_i)$ buyers holding foreign currency. Meanwhile, the household also determines the trading arrangement, $(q, x)$, for his buyers to offer in the trade matches. After these decisions are made, all the buyers and sellers are sent to the goods market. Matches are formed and exchanges then follow. The agents carry out trades according to the rules described by their households. After matches and exchanges are completed, all buyers and sellers return home and each household pools the receipts from trade, sharing the goods among all the members to consume until the next period arrives.
2.2.3. Trading Arrangement in the Goods Market

Before describing a household’s trading arrangement in the goods market, we first need to define four variables that represent a household’s valuations of the two currencies. Let \( V(m_{i1}, m_{i2}) \) be the value function of a representative household in country \( i \). Define:\(^{17}\)

\[
\omega_{ik} = E \left[ \frac{\beta}{\gamma_{k,+1}} V_k (m_{i1,+1}, m_{i2,+1}) \right], \quad k = 1, 2.
\]

where \( V_k \) denotes the derivative of the value function with respect to the \( k \)th argument. Expectations in these equations are conditional on the information available in the period following the realization of the shocks. By this definition, \( \omega_{ik} \) refers to a country \( i \) household’s marginal value of money holdings of currency \( k \) in the next period, discounted to the current period.\(^{18}\) We can regard \( \omega_{ik} \) as country \( i \) household’s valuation of money holdings of currency \( k \) in current period.

Consider a trade match that involves a country \( i \) buyer holding currency \( k \). As we described in the previous section, the buyer proposes \( (x_{ik}, q_{ik}) \) when meeting with a domestic seller and

---

\(^{17}\)To simplify notation, in this paper, we suppress the time subscript \( t \) and shorten the time subscript \( t \pm t' \) to \( \pm t' \), where \( t' > 0 \).

\(^{18}\)Note that the next period value of money is discounted by the money growth rate, \( \gamma_{k,+1} \), as well as by \( \beta \), because we normalize the money holdings \( m_{ik} \) by the aggregate money stock \( M_k \).
a country $i$ seller receives $x_{ik}$ unit of currency $k$ in the trade match, whose value to the seller’s household is $\Omega_{ik} x_{ik}$. Meanwhile, the seller has to produce $q_{ik}$ unit of goods and suffers a cost of $A_i (q_{ik})^\sigma$. Thus, the surplus to a country $i$ seller in the trade match is captured by the term $[\Omega_{ik} x_{ik} - A_i (q_{ik})^\sigma]$. Similarly, a country $i'$ seller’s surplus in the trade match is denoted by the term $[\Omega_{i'k} x_{i'k} - A_{i'} (q_{i'k})^\sigma]$. Setting these surpluses to zero, we have

$$\Omega_{ik} x_{ik} - A_i (q_{ik})^\sigma = 0;$$

(2.3)

$$\Omega_{i'k} x_{i'k} - A_{i'} (q_{i'k})^\sigma = 0.$$  

(2.4)

In fact, these constraints are the participation constraints for sellers, which prohibit buyers from proposing offers in the trade matches that leave sellers worse off than if they were not to trade. Otherwise, sellers will refuse to produce.

Moreover, all the buyers in the goods market are temporarily separated in the sense that it is impossible for buyers to borrow money across matches. Therefore, the payment in a trade match, $x$, should not exceed the amount of money that a buyer carries into the trade. After currency exchange, a country $i$ household holds $(m_{ii} - f_{ii})$ unit of domestic money and $(m_{ii} + e^\nu_{i} - f_{ii})$ unit of foreign money. As we described in the previous section, the household allocates domestic currency to a number of $n_i$ buyers and foreign currency to the other buyers. Then, in a country $i$ household, each buyer either holds $(m_{ii} - f_{ii}) / n_i$ of domestic money, or holds $(m_{ii} + e^\nu_{i} - f_{ii}) / (1 - n_i)$ foreign money. Therefore, the following money-goods constraints must

---

$^{19}$In this chapter, all the lowercase variables denote individual variables, which can be chosen by a household when making the decisions, and all the uppercase variables denote aggregate variables, which are determined by the representative household and taken as given when the household makes the decisions.
be satisfied in a trade match,

\[
\frac{m_{ii} - f_{ii}}{n_i} \geq x_{ii}; \quad \frac{m_{ii} - f_{ii}}{n_i} \geq x_{i}'i;
\]

(2.5)

\[
\frac{m_{ii'} + e^{i'-i} f_{ii}}{(1 - n_i)} \geq x_{ii'}; \quad \frac{m_{ii'} + e^{i'-i} f_{ii}}{(1 - n_i)} \geq x_{i}'i'.
\]

(2.6)

Sellers in country \(i\) will agree to trade if their trading partners offer \((X_{ik}, Q_{ik})\) or \((X'_{ik}, Q'_{ik})\)
which follows the similar constraints as (2.3) and (2.4).

2.2.4. Household’s Decision Problem

Pick an arbitrary household in country \(i\) as the representative household. In this section, we use dynamic programming to formulate the representative household’s decision problem. In each period, the endogenous state variables for the representative household are current period money holdings \((m_{ik})_{k=1,2}\), and the choice variables are \(h_i = [n_i, f_{ii}, (x_{ik}, q_{ik}, x'_{ik}, q'_{ik}, m_{ik+1})_{k=1,2}]\).

Taking the aggregate variables as given, the representative household in country \(i\) faces the following maximization problem:

\[
(PH) \quad V(m_{i1}, m_{i2}) = \max \{u(C_i) - P_i + \beta E[V(m_{i1+1}, m_{i2+1})]\}.
\]

The constraints are (2.3)-(2.6) and the following conditions:

\[
C_i = \left[ \rho^{1/\eta} c^{(q-1)/\eta}_{id} + (1 - \rho)^{1/\eta} c^{(q-1)/\eta}_{if} \right]^{\eta/(q-1)};
\]

(2.7)

\[
c_{id} = n_i \frac{T_i^i}{N_i} q_{ii} + (1 - n_i) \frac{T_i^{i'}}{(1 - N_i)} q_{ii'};
\]

(2.8)

\[
c_{if} = n_i \frac{T_i^{i'}}{N_i} q_{ii} + (1 - n_i) \frac{T_i^{i'}}{(1 - N_i)} q_{ii'};
\]

(2.9)
\[ \mathcal{P}_i = \left[ T_{ii}^i A_i (Q_{ii})^\sigma + T_{ii}^i A_i (Q_{ii})^\sigma \right] + \left[ T_{ii}^i A_i i (Q_{ii})^\sigma + T_{ii}^i A_i (Q_{ii})^\sigma \right]; \]  
(2.10)

\[ \gamma_{i+1} m_{ii+1} = m_{ii} - f_{ii} - n_i \left( T_{ii}^i x_{ii} + T_{ii}^i x_{ii}^f \right) + \left[ T_{ii}^i X_{ii} + T_{ii}^i X_{ii}^f \right] + (\gamma_{i+1} - 1); \]  
(2.11)

\[ \gamma_{i'+1} m_{ii'+1} = m_{ii'} + f_{ii'} e^{-i} - (1 - n_{ii'}) \left( T_{ii'}^i x_{ii'} + T_{ii'}^i x_{ii'}^f \right) + \left[ T_{ii'}^i X_{ii'} + T_{ii'}^i X_{ii'}^f \right]. \]  
(2.12)

The constraints (2.3)-(2.6) come from the earlier discussion on the household’s trading arrangement. The expected consumption of a country \( i \) household is given by constraints (2.7)-(2.9). As described earlier, \( C_i \) is consumption indices of local goods, \( c_{id} \), and imported goods, \( c_{if} \). Note that both consumption of local goods and imported goods consist of two parts because goods can be purchased with both currencies. The first terms on the right-hand side of (2.8) and (2.9) are the expected amount of goods purchased with domestic currency and the second terms are the expected amount of goods purchased with foreign currency.

The household’s disutility of production is given by constraint (2.10). The terms in the first bracket are the costs incurred in trade matches with domestic buyers and the terms in the second bracket are the costs incurred in trade matches with foreign buyers. There are two terms inside each set of brackets because each seller has the positive possibility of trading with a buyer holding either currency. Note that here the quantities of goods produced are aggregate variables, since prices are posted by buyers and the household takes it as given when making the decisions.

The last two constraints are the laws of motion of the household’s money holdings. We explain the first one here.\(^{20}\) The left-hand side of the constraint (2.11) is the money holdings of currency \( i \) in the next period, where \( \gamma_{i+1} \) appears here because the money holdings are normalized by

\(^{20}\) The explanation for the second one is similar, except that the household does not receive the monetary transfer of foreign currency.
current period money stock $M_i$. The right-hand side describes the changes in the money holdings in the current period, which comes from the selling in the currency market ($f_{ii}$), the buying and selling in the goods market (the two terms with bracket)\(^{21}\) and monetary transfer at the beginning of the next period ($\gamma_{i+1}$). The laws of motion of money holdings describe the changes in the household’s money holdings between two adjacent periods.

### 2.2.5. Equilibrium

Note that all of the money constraints (2.5) and (2.6) hold with equality, provided that $\omega_{ik}$ are positive. Let $\lambda_{ik}$ and $\lambda_{i}^{f}$ be the shadow price of the money constraints that contain $x_{ik}$ and $x_{i}^{f}$.

These shadow prices imply the non-pecuniary returns of the currencies that come from their role in relaxing trading restrictions. The solution to the household’s problem ($PH$) is characterized by the following conditions, in addition to the binding money constrains (2.3)-(2.6), and (2.7)-(2.9):

\[
\rho^{1/\eta} u'(C_i) C_i^\frac{1}{\eta} c_i^{\sigma-1} = (\lambda_{ik} + \omega_{ik}) \frac{A_i \sigma (q_{ik})^{\sigma-1}}{\Omega_{ik}}; \tag{2.13}
\]

\[
(1 - \rho)\frac{1}{\eta} u'(C_i) C_i^\frac{1}{\eta} c_i^{\sigma-1} = (\lambda_{i}^{f} + \omega_{ik}) \frac{A_i \sigma (q_{i}^{f})^{\sigma-1}}{\Omega_{i}^{f}}; \tag{2.14}
\]

\[
\rho^{1/\eta} c_{id} \frac{T_{ii}}{N_i} q_{ii} + (1 - \rho)\frac{1}{\eta} c_{if} \frac{T_{ii}}{N_i} q_{ii} = \rho^{1/\eta} \frac{1}{\eta} c_{id} \frac{T_{ii}}{1 - N_i} q_{ii} + (1 - \rho)^{1/\eta} \frac{1}{\eta} c_{if} \frac{T_{ii}}{1 - N_i} q_{ii}; \tag{2.15}
\]

\[
\omega_{11} + T_{11}^2 N_1 \lambda_{11} + T_{12}^1 N_1 \lambda_{12}^f = e \left[ \omega_{12} + T_{11}^2 (1 - N_1) \lambda_{12} + T_{12}^2 (1 - N_1) \lambda_{12}^f \right]; \tag{2.16}
\]

\[
\omega_{21} + T_{21}^2 (1 - N_2) \lambda_{21} + T_{22}^1 (1 - N_2) \lambda_{21}^f = e \left[ \omega_{22} + T_{22}^2 (1 - N_2) \lambda_{22} + T_{22}^2 (1 - N_2) \lambda_{22}^f \right]; \tag{2.17}
\]

\(^{21}\)Note the difference between the buying (choice variables) and selling (aggregate variables).
\[ \omega_{ii} = E \left[ \frac{\beta}{\gamma_{i+1}} \left( \omega_{ii+1} + \frac{T_{ii+1}^i}{N_{i+1}} \lambda_{ii+1} + \frac{T_{ii+1}^f}{N_{i+1}} \lambda_{ii+1}^f \right) \right] ; \]  
\[ \omega_{i'i'} = E \left[ \frac{\beta}{\gamma_{i'+1}} \left( \omega_{i'i'+1} + \frac{T_{i'i'+1}^{i'}}{N_{i'+1}} \lambda_{i'i'+1} + \frac{T_{i'i'+1}^{f'}}{N_{i'+1}} \lambda_{i'i'+1}^f \right) \right] . \]  

(2.18)

(2.19)

Constraints (2.13) and (2.14) prescribe the household’s optimal choices of trading quantities \((x_{ik}, q_{ik})\) and \((x_{i'k}, q_{i'k})\), respectively. These conditions require that the marginal utility of consumption (the left-hand side of the equations) must be equal to the marginal value of corresponding currencies spent on purchasing the consumption goods (the right-hand side of the equations).\(^{22}\) Note that the cost of giving up one unit of currency \(k\) is not fully measured by a household’s valuation of this amount of currency, \(\omega_{ik}\). By losing one unit of currency \(k\), the buyer also gives up the non-pecuniary return of holding the currency \(k\), which is captured by \(\lambda_{ik}\) and \(\lambda_{ik}^f\).

The optimal allocation of money holders, \(n_i\), has to satisfy the condition (2.15). Suppose the household decides to allocate one additional buyer to carry domestic currency. There is a possibility of \(T_{ii}^i/N_i\) for this buyer to purchase from a domestic seller, thereby increasing the household’s expected utility by \(\rho^{1/n} c_{id}^{-\frac{1}{n}} q_{ii} T_{ii}^i/N_i\). The possibility of trading with a foreign seller for him is \(T_{i'i'}^i/N_i\), which increases the household’s expected utility by \((1 - \rho)^{1/n} c_{if}^{-\frac{1}{n}} q_{i'i'} T_{i'i'}^i/N_i\). Thus, the left-hand side of the equation is the expected gain due to allocating an additional buyer carrying domestic currency. Similarly, the right-hand side can be interpreted as the expected loss due to the removal of one buyer carrying foreign currency. The optimal value of \(n_i\) must equalize the expected marginal benefit and cost.

Conditions (2.16) and (2.17) are the optimal choices of currency exchange of the households in country 1 and 2, \(f_{ii}\), respectively. These two conditions are similar and we explain the first

\(^{22}\)According to (2.3), to get one unit of consumption goods, the amount of \(A_i \sigma (q_{ik})^{\sigma-1} / \Omega_{ik}\) of currency \(k\) is needed for the purchase.
one here. A household in country 1 can sell one unit of domestic currency for \( e \) unit of foreign currency. By reducing one unit of domestic currency, the household not only loses the expected future value of currency, \( \omega_{11} \), but also the expected non-pecuniary returns of the currency in goods exchange, \( (T_{11}^i/N_1) \lambda_{11} + (T_{12}^i/N_1) \lambda_{12}^f \). On the other hand, the increased \( e \) units of foreign currency increase household’s expected utility by its own value, \( \omega_{12} \), as well as the expected non-pecuniary returns of the foreign currency, \[ \left\{ \left[ T_{11}^2/(1 - N_1) \right] \lambda_{12} + \left[ T_{12}^2/(1 - N_1) \right] \lambda_{12}^f \right\} e. \] In equilibrium, the marginal costs of selling currencies should be equal to the marginal benefits the exchanged currencies bring to the household.

The last two conditions, (2.18) and (2.19), are the envelop conditions. Note that we move the time index of the two conditions forward by one period. The expectations are conditional on the information available in the period following the realization of the shocks. Since the two envelop conditions have similar interpretations, we only explain (2.18) here. The left-hand side of (2.18) is the marginal value of currency \( i \) to a country \( i \) household. The right-hand side is the expected value of the currency in the next period, plus the non-pecuniary return of holding the currency in the next period, discounted to the current period. Envelop conditions maintain that the expected return of money should be equal to the nominal interest rate.

Now we define a stationary and symmetric equilibrium for this economy as follows:

**Definition 8.** An equilibrium in this economy consists of each household’s choice variables \( h_i \) and other household’s choices \( H_i \), where \( h_i = \left[ n_i, f_{ii}, (x_{ik}, q_{ik}, x_{ik}^f, q_{ik}^f, m_{ik}, +1)_{k=1,2} \right] \), and a nominal exchange rate \( e \) such that, for any given initial state \((m_{ik0})_{i,k=1,2} \) and the exogenous shock processes, the following requirements are satisfied: (i) optimality: \( h_i \) solves country \( i \) household’s decision problem \((PH)\) given \( H_i \) and \((m_{i1}, m_{i2}) \), \( i = 1,2 \); (ii) symmetry: \( h_i = H_i \); (iii) the currency market clears: \( f_{22} = e_f_{11} \); (iv) money holdings add up: \( m_{1k} + m_{2k} = 1, k = 1,2 \); and (v) the value of \((\omega_{1k} m_{1k} + \omega_{2k} m_{2k})_{k=1,2} \) lies in \((0, \infty)\).
2.3. Linearized Dynamic System

2.3.1. Solution technique

The whole dynamic system of the solution to the model in the previous section consists of 32 equations, five of which involve period $t+1$ variables.\textsuperscript{23} To solve the model, we first compute the non-stochastic symmetric steady state. Then, log-linearize around the steady state and derive a system of linear difference equations. Let lower case letters with a hat represent log deviations from an initial steady state, i.e., $\hat{y} = \ln(Y) - \ln(\bar{Y})$. We choose to keep two predetermined variables ($\hat{m}_{11}, \hat{m}_{12}$) and three jump variables ($\hat{\omega}_{11}, \hat{\omega}_{12}, \hat{\omega}_{22}$), eliminating all the other variables using those 27 equations that contain only current period variables. Then, we obtain a much simpler dynamic system that only consists of those five choice variables ($\hat{m}_{11}, \hat{m}_{12}, \hat{\omega}_{11}, \hat{\omega}_{12}, \hat{\omega}_{22}$) and four exogenous state variables. Now, the model can be easily solved with the method similar to Blanchard and Khan's (1980). The detailed solution method of the model is described in the Appendix B. (Appendix to Chapter 2). Here we limit the discussion to some key equations.

2.3.2. Differentials in the Valuation of the Two Currencies

Combining the equilibrium conditions of currency exchange, (2.16) and (2.17), with envelop conditions, (2.18) and (2.19), we can easily derive the relationship between a household’s valuations of currencies, $\hat{\omega}_{ik}$. The following Lemma describes this relationship:

\textbf{Lemma 9.} In an equilibrium, the changes in a household’s valuations of currencies must satisfy the following conditions for $i = 1, 2$:

\[ \hat{\omega}_{i1} - \hat{\omega}_{i2} = E [\hat{e}_{i+1} - (\hat{\gamma}_{1, i+1} - \hat{\gamma}_{2, i+1})] . \]  

\textsuperscript{23}In fact, there are six equations that involve next period variables: the four envelop conditions as well as the two laws of motion of money holdings. However, one of the envelop conditions is redundant and we can derive a linear relationship between $\hat{\omega}_{ik}$: $\hat{\omega}_{i1} - \hat{\omega}_{i2} = \hat{\omega}_{21} - \hat{\omega}_{22}$.
\( (\hat{\omega}_{i1} - \hat{\omega}_{i2}) \) is the differential in country \( i \) household’s valuations of the two currencies. As we will discuss later, this variable is important to the interpretation of our results. We use \( \theta \) to represent this differential, i.e.,

\[
\theta \equiv \hat{\omega}_{11} - \hat{\omega}_{12} = \hat{\omega}_{21} - \hat{\omega}_{22}.
\]

This lemma shows that although households in different countries value the two currencies at different worths, the relative valuations of the two currencies are the same in the two countries. Moreover, the differential in the valuations depends on the expected change in the next period nominal exchange rate and the differential in the two currencies’ growth rates. In particular, \( \theta \) is increasing in \( \tilde{e}_{+1} \) and decreasing in \( (\tilde{\gamma}_{1,+1} - \tilde{\gamma}_{2,+1}) \). This result is quite intuitive: an increase in the nominal exchange rate means more currency 2 can be exchanged by currency 1 and, hence, the value of currency 1 to a household increases. On the other hand, a higher inflation rate of a currency reduces the purchasing power of that currency. Thus, if a currency has a higher growth rate, its value to a household is lower compared to the value of the other currency.

Note that (2.20) also implies that the cross-country differentials in the valuations of currencies are the same, i.e., \( \hat{\omega}_{11} - \hat{\omega}_{21} = \hat{\omega}_{12} - \hat{\omega}_{22} \). Thus, if a country values a currency higher than the other country does, it values the other currency higher as well. Keeping in mind that money injection is asymmetric and only domestic households receive the increased monetary transfer of domestic currency, suppose \( \gamma_1 > 1 \) and country 1 increases injection of currency 1. There is a potential imbalance in the portfolio of each household’s money holdings after receiving the transfer. The households then sell part of the increased transfer for currency 2. As a result, the asymmetric money injection of currency 1 increases country 1 households’ relative holdings of both currencies. Accordingly, the country 1 households value both currencies less than the country 2 households do.
The differential in the valuations of the two currencies comes from search frictions in the goods market. The centralized currency market only equalizes the relative valuation of currencies in the two countries. Arbitrage in the currency market cannot eliminate the difference in the absolute values of currencies. The gap between valuations of the two currencies does not disappear unless households can trade goods directly for currencies in the currency market. However, transactions in the goods market are formed in bilateral matches and there are different prices for different types of matches. There is no way to arbitrage between the goods market and the currency market to eliminate this differential in valuations of currencies.

2.3.3. Quantity Differentials across Matches

As we have described, there are eight different types of trade matches in the goods market. Since agents from different countries have different valuations of each currency, the quantity traded in each type of trade match is not the same. However, the centralized currency market equalizes the relative valuation of currencies across countries and as a result, there are certain rules the trading quantities in different types of matches follow. The following lemma summarizes the findings:

Lemma 10. In an equilibrium, the quantities of goods exchanged in the goods market satisfy the following conditions for \( i = 1, 2 \) and \( k = 1, 2 \):

\[
\hat{q}_{1k} - \hat{q}_{1k}^f = \hat{q}_{2k} - \hat{q}_{2k}^f = \frac{1}{\sigma} (\hat{\omega}_{11} - \hat{\omega}_{21}) - \frac{1}{\sigma} \hat{\omega}_1 + \frac{1}{\sigma} \hat{\omega}_2; \tag{2.21}
\]

\[
\hat{q}_{i1} - \hat{q}_{i2} = \hat{q}_{i1}^f - \hat{q}_{i2}^f = \frac{1 - \psi}{1 + \sigma (1 - \psi)} \theta. \tag{2.22}
\]

(2.21) is derived from the binding money constraints. It says that a buyer receives different quantities of goods when trading with sellers from different countries, even though the buyer pays the same amount of currency. This is easy to understand since agents from different countries...
have different valuations of the currencies. For example, if a country 1 household values both currencies higher than a country 2 household does, sellers from country 1 will provide more goods than sellers from country 2 when dealing with the same buyer. Moreover, this quantity differential is always the same regardless of which country the buyer is from or which currency is used in the trade. This result is derived from the fact that the quantity differential depends on the differential in the two countries’ valuations of currencies and that the cross-country valuations of the currency are equal.

Now, consider two trade matches that involve the same buyer and seller but different types of currencies. Since households value the two currencies at different values, even in a trade with the same seller, the quantities of goods a buyer obtains when paying with different types of currency are not the same. (2.22) states that the quantity differential between such trade matches is always the same and relies on the relative valuation of the two currencies, $\theta$. The higher the relative value of a currency, the more goods a buyer can purchase in a trade with that currency compared to using the other currency.

Note that search frictions in the goods market generate households’ different valuations of currencies, and are responsible for the quantity differentials across trade matches. The quantity differentials imply price differentials and the deviations from the LOP. We will later discuss how this is significant for explaining the fluctuations in exchange rates.

2.3.4. Division of Money Holders and the Nominal Exchange Rate

The nominal exchange rate is determinate in our model. As we described, in each period, households divide their buyers into two groups and allocate money stocks of the two currencies to the two groups, respectively. The division of money holders generates a relative valuation of the two
currencies to the households in each country, and this relative valuation of currencies determines the nominal exchange rate.

Log-linearizing the condition for the optimal division of money holders, (2.15), and rearranging it, we get

\[ 2(1 + \mu_1)(1 - \psi)\hat{n}_i = \hat{q}_{ii} - \hat{q}_{ii'} + \mu_1 \left( \hat{q}_{ii}^f - \hat{q}_{ii'}^f \right), \quad \mu_1 = \alpha^{1+\frac{1}{\rho}} \left( \frac{1 - \rho}{\rho} \right)^{1/\eta}. \]  

(2.23)

Therefore, the optimal decision on division of money holders relies on the quantity differential that a buyer can purchase with different types of currencies. According to lemma 10, the quantity differentials are proportional to \( \theta \). Therefore, we immediately obtain the following proposition:

**Proposition 11.** The changes in the division of money holders in a country \( i \) household, \( \hat{n}_i \), only depend on the relative valuation of the two currencies, and

\[ \hat{n}_1 = -\hat{n}_2 = \frac{1}{2[1 + \sigma(1 - \psi)]}\theta. \]  

(2.24)

\( \hat{n}_i \) denotes the change in the number of buyers holding domestic currency in country \( i \). Proposition 11 determines that the changes in the number of buyers who carry the same currency are exactly the same in the two countries. This is not a surprising result since the allocation of money holders depends on a household’s relative valuation of the two currencies, and the relative valuations are equalized across countries. Note that \( \hat{n}_1 \) responds positively to the differential in the valuations of the two currencies. The higher a household values a currency, the more buyers are assigned to hold the currency.
The nominal exchange rate is determined by the relative role of the two currencies in the trade. Log-linearizing (2.16) and rearranging it, we obtain,

$$\frac{\pi_e}{\beta} = (\hat{\omega}_{11} - \hat{\omega}_{12}) + \mu_0 \left[ \frac{\bar{h}}{\bar{\Omega}} \left( \hat{\lambda}_{11} - \hat{\lambda}_{12} \right) + \frac{\alpha x'}{\bar{\Omega}} \left( \hat{x}'_{11} - \hat{x}'_{12} \right) \right] - \frac{2\mu_0 \left( 1 - \psi \right)}{\bar{\Omega}} \left( \bar{h} + \alpha x' \right) \hat{n}_1,$$

(2.25)

where $\mu_0$ is the steady-state value of matching rate of a buyer meeting with domestic sellers.

The changes in the nominal exchange rate come from three sources: i) changes in the relative value of the two currencies; ii) changes in relative non-pecuniary returns of the two currencies; and iii) changes in the division of money holders. The first two sources are easy to understand since the nominal exchange rate is the relative price of the two currencies. Thus, the changes in the nominal exchange rate should reflect the changes in the relative valuations, as well as the non-pecuniary returns of holding the two currencies. The effect of $\hat{n}_1$ on $\hat{e}$, however, is less obvious. Changes in the number of money holders affect the matching rate of buyers, as well as the total number of trade matches. Thus, it changes expected returns of currencies in the trades and then has an influence on the nominal exchange rate.

We can show that the differentials in non-pecuniary returns to the two currencies depend on the relative valuation of the two currencies, as well as quantity differentials in trades with different currencies:

$$\hat{\lambda}_{11} - \hat{\lambda}_{12} = (\hat{\omega}_{11} - \hat{\omega}_{12}) - (\sigma - 1) \frac{(X + \Omega)}{X} (\hat{q}_{11} - \hat{q}_{12});$$

$$\hat{x}'_{11} - \hat{x}'_{12} = (\hat{\omega}_{11} - \hat{\omega}_{12}) - (\sigma - 1) \frac{(X' + \Omega)}{X'} (\hat{q}'_{11} - \hat{q}'_{12}).$$

$^{24}$In the paper, a variable with a bar represents the steady state value of this variable.
Since the changes in both the quantity differentials and the division of money holders depend only on the relative valuation of the two currencies, $\theta$, equilibrium condition (2.25) implies that the change in the nominal exchange rate also relies solely on $\theta$.

**Proposition 12.** The changes in the nominal exchange rate come entirely from the differential in a household’s valuations of the two currencies, and

$$
\hat{e} = \frac{\gamma + \beta (1 - \psi) \left[ \sigma - \mu_0 (1 + \alpha) (\sigma - 1) \right]}{\tau + \sigma (1 - \psi) } \theta.
$$

(2.26)

Note that lemma 9 states that $\theta$ depends on $E[\hat{e}_{t+1}]$ and $E[(\hat{\gamma}_{1,t+1} - \hat{\gamma}_{2,t+1})]$. Thus, we can derive the evolution rule of $\theta$ as,

$$
\theta = \mu_2 E[\theta_{t+1}] - E \left[ (\hat{\gamma}_{1,t+1} - \hat{\gamma}_{2,t+1}) \right],
$$

(2.27)

where $\mu_2 \equiv \frac{\gamma + \beta (1 - \psi) \left[ \sigma - \mu_0 (1 + \alpha) (\sigma - 1) \right]}{\tau + \sigma (1 - \psi) }$.

The formation of $\theta$ only involves the differential in the two countries’ money growth rates.\textsuperscript{25} This result is important for the interpretation of the relative role of monetary and real shocks which we will discussed later. Thus, the difference between the two countries’ money growth rates is the only factor that affects the fluctuations of the nominal exchange rate in our model. A higher growth rate of a currency causes a lower valuation of this currency by households, thereby resulting the depreciation of the currency.

\textsuperscript{25}Note that this result depends crucially on the assumption that goods can be purchased by either currency. If assuming domestic buyers only use domestic currency to buy local goods, then real shocks will affect $\theta$ too.
2.3.5. Net Currency Trades and Relative Money Holdings across Countries

The net amount of currency exchange and the relative money holdings per household across countries are two important factors affecting the price levels in the goods market. In this subsection, we examine how these variables respond to shocks in the model.

The following condition for net currency trade, \( \hat{f}_{11} \), can be derived from the binding money constraints:

\[
2\hat{f}_{11} = \frac{1}{(\gamma - 1)}[(2\gamma - 1) \hat{m}_{11} - \hat{m}_{12}] - \hat{e}. \tag{2.28}
\]

Thus, the changes in net currency exchange depend on the changes in the relative money holdings of the two currencies, \( \hat{m}_{11} \) and \( \hat{m}_{12} \), as well as the changes in the nominal exchange rate, \( \hat{e} \). We have shown in the previous subsections that the changes in the nominal exchange rate depend exclusively on the differential in valuations of the two currencies, \( \theta \), which is driven entirely by the difference between the two currencies’ gross growth rates. Therefore, equation (2.28) suggests that the currency trade is affected by the relative money holdings across countries and the money growth rate differential across countries.

Log-linearizing the two laws of motion of money holdings, (2.11) and (2.12), together with the quantity differentials and equations for \( \hat{n}_1 \) (2.24), we can rewrite the two equations as,

\[
\gamma \hat{m}_{11, +1} = (1 - 2\alpha \mu_0) \hat{m}_{11} - (1 - 2\alpha \mu_0) \left( \frac{\gamma - 1}{2\gamma - 1} \right) \hat{f}_{11} + \frac{\gamma}{(2\gamma - 1)} \hat{\gamma}_{1, +1}; \tag{2.29}
\]

\[
\gamma \hat{m}_{12, +1} = (1 - 2\alpha \mu_0) \hat{m}_{12} + (1 - 2\alpha \mu_0) (\gamma - 1) \left( \hat{f}_{11} + \hat{\gamma} \right) - \gamma \hat{\gamma}_{2, +1}. \tag{2.30}
\]

These two equations describe the evolution rules for the relative money holdings of the two currencies. Note that \( \hat{f}_{11} \) is a function of \( \hat{m}_{11}, \hat{m}_{12}, \) and \( \theta \). Thus, the next period money holdings
depend on the current period money holdings, $\hat{m}_{11}$ and $\hat{m}_{12}$, the relative valuation of currencies, $\theta$, and the exogenous money growth shocks, $\hat{\gamma}_{1,+1}$ and $\hat{\gamma}_{2,+1}$. According to (2.27), the formation of $\theta$ only involves the differential in the growth rates of the two currencies. Therefore, the two laws of motion of money holdings imply that the relative money holdings of the two currencies can only be affected by monetary shocks. Real shocks play no role in the evolutionary path of the relative money holdings in our model.

Note that $\hat{m}_{11}$ and $\hat{m}_{12}$ are the measure of money holdings of a country 1 household at the beginning of each period. The actual amount of money brought into the goods market is the amount of currency a household has after the currency trades. We call the amount of money an individual buyer carries into the goods market effective money holdings and denote it as $m^e_{ik}$. For example, $m^e_{11} = (m_{11} - f_{11})/n_1$ and $m^e_{12} = (m_{12} + e f_{11}) / (1 - n_1)$. The effective money holdings of currencies are what really matter for the price levels in the trade matches.

Let $\hat{m}^e_k = \hat{m}^e_{1k} - \hat{m}^e_{2k}$ denote the changes in the relative effective money holdings of currency $k$ across countries. We can show that:

$$
\hat{m}^e_k = \frac{(2\gamma - 1)}{\gamma} \hat{m}_{11} + \frac{1}{\gamma} \hat{m}_{12} + \frac{(\gamma - 1) \mu_2}{\gamma} \theta;
$$

Therefore, the changes in relative effective money holdings across countries are the same for both currencies. If, after the currency trade, a household in one country holds more currency 1 than a household in the other country does, the household in the first country also holds more currency 2.

Combining the two laws of motion of money holdings and using equation (2.20), we obtain the evolutionary rule for the relative effective money holdings across countries:

$$
\gamma \hat{m}^e_{+1} = (1 - 2\alpha \mu_0) \hat{m}^e + (\gamma - 1) \theta + \gamma (\hat{\gamma}_{1,+1} - \hat{\gamma}_{2,+1}).
$$

(2.31)
Since \( \theta \) is driven entirely by the growth rate differential of the two currencies, \((\hat{\gamma}_1 - \hat{\gamma}_2)\), the above equation implies that the changes in relative effective money holdings across countries, \( \hat{m}^e \), can only be affected by the difference between the two countries’ money growth rates.

### 2.3.6. Price Differentials and Deviations from the Law of One Price

Consider a country \( i \) buyer who is carrying currency \( k \). \((X_{ik}, Q_{ik})\) is the trading arrangement in a trade match with a domestic seller, and \((X'_{ik}, Q'_{ik})\) is the trading arrangement in a trade match with a foreign seller. Thus, the prices implied in these trade matches are:

\[
P_{ik} = \frac{X_{ik}}{Q_{ik}}; \quad P'_{ik} = \frac{X'_{ik}}{Q'_{ik}}.
\]

For the goods produced by a country \( i \) household, there are four market prices: \( P_{ik} \) is the price faced by domestic buyers carrying currency \( k \) and \( P'_{ik} \) is the price faced by foreign buyers carrying currency \( k \). The quantity differentials we discussed in section 2.3.3 imply following price differentials.

**Corollary 13.** If \( \gamma_1 \neq \gamma_2 \), the four prices of goods produced by country \( i \) households, \( \hat{p}_{ik} \) and \( \hat{p}'_{ik} \), are not equal to each other. Moreover, the price differentials must satisfy:

\[
\hat{p}_1 - \hat{p}_2 = \hat{p}'_{1i} - \hat{p}'_{2i} = -\frac{2 - \psi}{1 + \sigma (1 - \psi)} \theta;
\]

\[
\hat{p}_1 - \hat{p}'_{1i} = \hat{p}_2 - \hat{p}'_{2i} = \frac{(\sigma - 1)}{\sigma} \hat{m}^e.
\]

This corollary suggests a strong violation of the LOP. Even after converting to the same currency, the same goods are sold at different prices in different trade matches. The differential in valuations of the two currencies, \( \theta \), and the relative effective money holdings across countries, \( \hat{m}^e \), are the two important components of the price differentials. Note that both \( \theta \) and \( \hat{m}^e \) appear
because of the differential in the two countries’ money growth rates. Thus, as long as the money growth rates differ in the two countries, the above price differentials always exist and the LOP is violated.

We can further determine whether the LOP holds at the aggregate price level. Let $P_{id}$ denotes the domestic price of local goods in country $i$ and $P_{if}$ denotes the domestic price of imported goods in country $i$. Since local (imported) goods can be purchased by either domestic currency at the price $P_{ii}$ ($P_{iif}$) or foreign currency at the price $P_{iit}$ ($P_{iif'}$), we define $P_{id}$ and $P_{if}$ as follows:

$$P_{id} = v_{id}P_{ii} + (1 - v_{id}) P_{ii}^{'e^{i-t'}};$$

$$P_{if} = v_{if}P_{iif'} + (1 - v_{if}) P_{iif'}^{'e^{i-t'}},$$

where $v_{id} = T_{ii} Q_{ii}/(T_{ii} Q_{ii} + T_{i0} Q_{i0})$ is the consumption share of local goods purchased by domestic money, and $v_{if} = T_{iif'} Q_{iif'}/(T_{iif'} Q_{iif'} + T_{i0f'} Q_{i0f'})$ is the consumption share of imported goods purchased by domestic money.

The LOP requires the price of country $i$'s goods sold in country $i$ ($P_{id}$) be equal to the price sold in country $i'$ ($P_{iif}$) after converting to the same currency. However, we can show that:

$$\hat{P}_{id} + \hat{e} - \hat{P}_{iif} = \eta_{id}(\hat{p}_{i1} - \hat{p}_{i2} + \hat{e}) + \eta_{if}(\hat{p}_{i1f} - \hat{p}_{i2f} + \hat{e}) + (\hat{p}_{i2} - \hat{p}_{i2f} + \hat{e});$$

where $\eta_{id}$ and $\eta_{if}$ are the steady state values of $v_{id}$ and $v_{if}$, respectively.

The difference between the two countries’ price levels of goods $i$ can be broken down into three 3 parts: i) the price differential faced by domestic buyers holding different currencies; ii) the price differential faced by foreign buyers holding different currencies; iii) the price differential faced by buyers from different countries. Search frictions cause market segment and generate differential in valuations of currencies across countries. Price differentials across trade matches
always exist since the valuations of currencies to households from different countries are different. The LOP is violated in the model as long as two countries have different money growth rates.\footnote{There are empirical evidences that support this result. Cheung and Fujii (2008), for example, use 25 years of monthly data on individual retail prices to study the behavior of the behavior of product-specific LOP deviations. They find that the deviations from the LOP are positively related with the inflation rate differentials.}

### 2.3.7. Real Exchange Rate

The real exchange rate is usually defined as the relative price of the common basket of goods where prices are converted into a common numeraire. According to the CES consumption index, the consumption-based price index \( P_i \) in country \( i \) is derived as

\[
P_i = \left[ \rho P_{id}^{1-\eta} + (1 - \rho) P_{if}^{1-\eta} \right]^{\frac{1}{1-\eta}},
\]

where \( P_{id} \) is the domestic money price of local goods in country \( i \), and \( P_{if} \) is the domestic money price of imported goods in country \( i \).

Thus, the real exchange rate can be defined as

\[
RER = \frac{eP_1}{P_2}.
\]

We can decompose the changes in the real exchange rate as follows:

\[
\Delta RER = \hat{e} + \hat{p}_1 - \hat{p}_2
\]

\[
= \rho \left[ \hat{p}_{1d} + \hat{e} - \hat{p}_{2f} \right] + (1 - \rho) \left[ \hat{p}_{1f} + \hat{e} - \hat{p}_{2d} \right] + (2\rho - 1) \left[ \hat{p}_{2f} - \hat{p}_{2d} \right].
\]

The first two terms on the right-hand side of the equation indicate the deviations from the LOP. The last term appears because of the difference in the consumption bundles of the households in
the two countries, determined by

\[ \hat{p}_{2f} - \hat{p}_{2d} = \frac{1}{\sigma} (\hat{\omega}_{22} - \hat{\omega}_{12}) + \frac{1}{\sigma} \hat{a}_1 - \frac{1}{\sigma} \hat{a}_2. \]  

(2.32)

Thus, this price differential comes from the two countries’ different valuations of the currency, 
\((\hat{\omega}_{22} - \hat{\omega}_{12})/\sigma\), as well as the real disturbances, 
\((\hat{a}_1 - \hat{a}_2)/\sigma\).

The decomposition suggests that a large part of variations in the real exchange rate are generated by the search frictions in the goods market. Without search frictions, households value all the currencies at the same level, goods are sold at the same price and the LOP holds. The primary driven force of price differential in imported and local goods disappears, causing all of the variations in the real exchange rate to come merely from the real disturbance.

2.4. Calibration

This section describes how we choose functional forms and benchmark parameter values. The discount factor, \(\beta\), is set equal to 0.99 to get an annual real interest rate of 4%. The utility of consumption takes the form of CRRA function,

\[ U(C) = \frac{1}{1 - \epsilon} C^{1-\epsilon}, \]

where \(\epsilon\) is the coefficient of risk aversion. We set \(\epsilon = 2\), which is a standard value in the literature.\(^{27}\) For the consumption index \(\mathcal{C}\) defined as (2.1), the elasticity of substitution between local goods and imported goods is set to 1.5, since empirical studies suggest a value between 1 and 2 for U.S. data. The home bias parameter, \(\rho\), is chosen in such a way that the standard

\(^{27}\)In sticky price models such as ones put forth by Chari, Kehoe and McGrattan (2001), a high risk aversion is needed to generate enough volatility for exchange rates. Our model, however, does not need such a requirement.
deviation of consumption relative to the standard deviation of output is consistent with data. For our benchmark economy, $\rho$ is chosen to be 0.89.

Then consider the parameters in the matching functions (2.2). In the steady state, $\alpha = \bar{\pi}_f / \bar{\pi}_d$, which can be interpreted as the import share in a country. We set $\alpha = 0.15$ in the model to match U.S. data. The constant $T_0$ indicates the degree of search frictions in the goods market. The lower the value of $T_0$ is, the lower possibility for an agent to find a trade match. In the computation, we choose the value of $T_0$ so that the standard deviation of output generated from the model matches the data. The buyer’s share in the formation of matches, $\psi$, however, cannot be identified. We choose $\psi = 0.5$ for the benchmark economy and examine how sensitive the results are to this parameter later.

The ratio of sellers to buyers in a household, $s$, can be interpreted as the ratio of working time to shopping time. According to Juster and Stafford (1991), the shopping time of the population is 11.17% of the working time, thus, we set $s = 8.9$.

The disutility of production, $A_iQ^\sigma$, is derived from the production function and disutility of labor. We assume that production takes place according to a decreasing returns to scale production function,

$$F (l) = z l^\varphi, 0 < \varphi < 1,$$

where $z$ captures technology shocks and follows a stochastic process. We set $\varphi = 2/3$. Therefore, the labor input required to produce $Q$ units of goods can be found by inverting the above production function. That is, $l = (Q/z)^{1/\varphi}$. In addition, the disutility of labor takes the form of the function,

$$c (l) = \bar{\phi} l^{1+\bar{\phi}} / (1 + \bar{\phi}),$$
where \( \phi_0 > 0 \) and \( \phi > 0 \). According to the recent work by Rogerson and Wallenius (2007), the intertemporal elasticity of substitution in aggregate labor supply is high and therefore we set \( \phi = 0.33 \). Then, the cost of producing \( Q \) unit of goods, measured in terms of utility, is 
\[
c(Q) = AQ^\sigma,
\]
where \( \sigma = (1 + \phi) / \phi = 1.995 \), and \( A = \phi_0 / (1 + \phi) z^{-\sigma} \). We normalize the steady state value of \( A \) to 1.

\( A \) in our model represents technology shocks and obviously it follows

\[
\log A_i = -\sigma \log z_i,
\]

We assume the stochastic process of \( \log z \) obeys the following vector autoregressive (VAR) process:

\[
\log z = \Gamma \log z_{-1} + \varepsilon_z, \quad \varepsilon_z \sim N(0, \Sigma),
\]

where \( z = (z_1, z_2) \) and \( \varepsilon_z = (\varepsilon_{z1}, \varepsilon_{z2}) \). Following the estimation in most of the international business cycle literature, for example, Backus, Kehoe and Kydland (1992, 1995), the real shocks are very persistent, and the autocorrelation is set equal to 0.95, i.e.,

\[
\hat{\Gamma} = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}.
\]

The standard deviation of innovations is set to 0.007. The shocks are positively cross-correlated and the cross-correlation is set to 0.25.

The details of the monetary rules followed in the U.S. and Europe are extensively debated, and there is no a widely-accepted way existed to capture the monetary shock process. To make a comparison of our results with those from sticky price model by Chari, Kehoe, and McGrattan (2003), we follow their paper and assume the growth rates of money stocks in both countries
follow the same simple rule in the following form:\textsuperscript{28}

\[
\log \gamma = \rho_\gamma \log \gamma_{t-1} + \varepsilon_\gamma, \quad \varepsilon_\gamma \sim N(0, \sigma_\gamma).
\]  \hspace{1cm} (2.33)

where \(\varepsilon_\gamma\) is a normally distributed, mean-zero shock. We follow the estimation of Chari, Kehoe, and McGrattan (2003) and run a regression of the above equation on quarterly U.S. data for M1 from 1973 : 1 to 2007 : 1 and set \(\rho_\gamma = 0.69\) and \(\sigma_\gamma = 0.014\).\textsuperscript{29}

\subsection*{2.5. Results}

In this section, we evaluate the quantitative performance of the model. The central interest is the dynamics of exchange rates which is generated by the model. Note that all variables that we discuss in this section are measured by the percentage deviation from their steady state values. Table 2.2 reports the second moments of key variables generated by the model. Three cases are considered: (i) both money growth shocks and technology shocks are present; (ii) only money growth shocks are considered; (iii) only technology shocks are considered.

\textbf{2.5.1. Volatility, Persistence and Cross-correlation}

Our model reaches some successes in accounting for the main properties of the international business cycle, at least qualitatively. Compared to output, both the nominal and real exchange rate are highly volatile. The volatility of the nominal and real exchange rate are 0.079 and 0.089, respectively, which are more than 4 times that of output. Both values are closed to the data. Moreover, the model also produces substantial persistence for the real exchange rate (0.96),

\textsuperscript{28}Another popular way to estimate the shock processes is using VAR estimation, based on the idea that the monetary authorities may respond to technology shocks. We also estimate the shock processes using VAR estimation. The main results still stand and the model can generate excess volatility of exchange rates.

\textsuperscript{29}M1 data comes from the Board of Governors of the Federal Reserve System.
which is even higher than that in the data. The persistence of nominal exchange rate (0.69) falls below the level in the data. As discussed earlier, the nominal exchange rate relies entirely on the differential in valuations of currencies, which is affected only by monetary disturbances. Persistence in nominal exchange rate inherits roughly the same persistence in the underlying shocks to money growth rates, which is 0.69 in our model. Autocorrelations of output and consumption are all close to the data.

It is well-known that the international business cycle models with complete asset market set up a link between the real exchange rate and the ratio of marginal utility of consumption, which suggests a high correlation between the real exchange rate and relative consumption. For example, Chari, Kehoe and McGrattan (2002) construct a sticky price model with LCP (local currency pricing) and their quantitative exercise shows that such correlation is actually 1. While
this correlations in the data are zero or negative, such anomaly remains an interesting puzzle in international economics.\textsuperscript{30} The asset market is incomplete in our model in the sense that no state-contingent bonds are provided in current environment. Therefore, the link between the real exchange rate and the ratio of marginal utility of consumption is absent in our model. In fact, our model replicates successfully the negative relationship between the real exchange rate and relative consumption and the correlation between these two variables is close to the data.

The relationships between the real exchange rate and output, domestic and foreign outputs, and domestic and foreign consumptions, suggested by the model, are all consistent with those in the data.

Empirical studies show that the nominal and real exchange rate are highly correlated. Our model, however, fails to generate this property of exchange rates. Instead, the model shows a negative correlation between the nominal and real exchange rate. To explain this, consider a positive monetary shock to currency 1. Households value currency 1 less and then the nominal exchange rate of currency 1 depreciates. However, the relative valuation of currencies across countries changes too. As we discussed earlier, country 1 households value currency 1 less than country 2 households do and this improves terms of trade for country 1 households. As the result, the real exchange rate of currency 1 appreciates.

Although the volatility of real and nominal exchange rates generated by the benchmark model is about right, it does less successfully in accounting for the volatility in price ratio across countries. In fact, our model generates too much volatility of price ratio relative to that observed in the data.

\textsuperscript{30}This anomaly was first documented by Backus and Smith (1993).
2.5.2. The Roles of the Two Shocks

Many empirical studies support the view that monetary shocks account for a substantial fraction of the variability of the real exchange rate (e.g., Clarida and Gali (1994), Rogers (1999), etc.). In this section, we investigate the roles of money growth shocks and technology shocks in our model. To illustrate this point, we consider the world economy is affected by both monetary and real shocks and take the model with both shocks as a benchmark case. Then, we examine two highly abstracted cases: i) the real shocks are taken away and only monetary shocks are considered; ii) the monetary shocks are taken away and only real shocks are considered. The results from these two cases give us the clue about how much variations in the real exchange rate are generated by the two shocks, respectively. The results are reported in Table 2.2.

It is clear that monetary shocks in our model account for most of the volatility of exchange rates. With only technology shocks presented, there is no variability at all in the nominal exchange rate and volatility of the real exchange rate drops dramatically from 0.089 to 0.021. The main reason that monetary shocks play a relatively more important role in the model is that the main components of the variability of exchange rates, $\theta$ and $\tilde{m}^e$, only respond to the changes in the two countries’ money growth rate differential, which can only be directly affected by monetary shocks. As discussed in section 2.3.4, the changes in the nominal exchange rate depend exclusively on $\theta$, while the evolution rule of $\theta$, (2.27), shows that the formation of $\theta$ only involves the differential in growth rates of the two currencies. Technology shocks, $\tilde{a}_i$, also affect a household’s valuation of currencies. However, they affect the valuations of both currencies in the same way, and thus leave the differential unchanged. For example, suppose that a positive technology shock occurs in country 1. Sellers from country 1 can produce more because of technology improvement and hence buyers may purchase more goods in a trade match with a country 1 seller. Therefore, households value the currency involved in the transaction with country 1’s goods more. Since
country 1’s goods are sold for both currencies, households value both currencies higher. The differential in the valuations, however, does not change. Without monetary shocks, \( \theta \) does not change. As a result, the nominal exchange rate remains the same all the time.

Our model suggests that monetary shocks contribute significantly to the variations in the real exchange rate.\(^{31}\) As we have shown in subsection 2.3.7, the changes in the real exchange rate can be decomposed into the deviations from the LOP and the price differential caused by the difference in consumption bundles across countries. The deviations from the LOP are caused by the changes in \( \theta \) and the relative money holdings across countries, both of which can only be affected by the differential in the two countries’ money growth rates. In the absence of monetary shocks, the LOP holds. All deviations from PPP arise from the differentials in consumption bundles between the two countries.

Technology shocks, however, help to explain some basic features of output and consumption in the data. Money growth shocks alone cannot generate adequate volatility of output and consumption. The standard deviations of output and consumption are only 65% of that in the data when real shocks are taken away. Moreover, in the absence of real shocks, the persistence of output and consumption is too low compared to that in the data. The correlations between the domestic and the foreign output and consumption match the data more closely if we incorporate the real shocks.

2.5.3. The Role of Search Frictions in the Goods Market

The non-Walrasian feature in the goods market plays an important role in generating high volatility in the exchange rates. Agents meet bilaterally and randomly in the goods market. They cannot

\(^{31}\) The assumption that goods can be purchased by either currency is crucial to generate this result. If assuming domestic buyers only use domestic currency to buy local goods, real shocks will affect \( \theta \) and hence, contribute more to the variations of the real exchange rate. The main conclusion that search frictions generate excess volatility of exchange rates, however, does not change. As long as search frictions exist in the goods market, the shocks can produce the fluctuations in the deviations from the LOP and the excess volatility of real exchange rates.
Figure 2.2. Volatility of the real exchange rate at different value of $T_0$

arbitrage across matches and markets. Households form different valuations of the two currencies and the prices differ in different types of trade matches. Search frictions generate the deviations from the LOP and, hence, the main part of variations in the real exchange rates.

The parameter, $T_0$, can be interpreted as the degree of frictions in the goods market. The lower the value $T_0$ is, the lower the matching rates for the agents in the goods market are, and the harder it is to find a trade partner in the market. Figure 2.2 shows the relationship between $T_0$ and the volatility of the real exchange rate (horizontal axis denotes $T_0$ and vertical axis denotes the volatility of the real exchange rate).

Clearly, the volatility of the real exchange rate increases with the degree of search frictions in the goods market. With more difficulty in finding a successful transaction in the goods market, the differences in the household’s valuations of currencies are larger, which means higher deviations from the LOP and larger variations in the real exchange rate.

To illustrate the importance of the decentralized goods market, we construct a standard Walrasian model with the cash-in-advance constraints.\[^{32}\] We apply the same preference and the

\[^{32}\]The model is a variation of Helpman’s (1981) model with elastic labor supply.
Data Model with Walrasian goods Market

<table>
<thead>
<tr>
<th>Standard deviations</th>
<th>Data</th>
<th>Model with Walrasian goods Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both shocks</td>
<td>Monetary shock</td>
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<tr>
<td>Output</td>
<td>0.0182</td>
<td>0.0154</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0171</td>
<td>0.0152</td>
</tr>
<tr>
<td>Price ratio</td>
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<tr>
<td>Real exchange rate</td>
<td>0.080</td>
<td>0.0096</td>
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<table>
<thead>
<tr>
<th>Autocorrelations</th>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.88</td>
<td>0.845</td>
<td>0.68</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.89</td>
<td>0.845</td>
<td>0.68</td>
</tr>
<tr>
<td>Price ratio</td>
<td>0.87</td>
<td>0.845</td>
<td>0.68</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>0.86</td>
<td>0.831</td>
<td>0.68</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.84</td>
<td>0.831</td>
<td>0.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cross-correlations</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Real and nominal exchange rate</td>
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<td>-1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>RER and output</td>
<td>0.08</td>
<td>0.635</td>
<td>0.72</td>
</tr>
<tr>
<td>RER and relative consumption</td>
<td>-0.35</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>foreign and domestic output</td>
<td>0.6</td>
<td>0.15</td>
<td>-0.03</td>
</tr>
<tr>
<td>foreign and domestic consumption</td>
<td>0.38</td>
<td>0.18</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Table 2.3. Simulated moments in a CIA model

same technology function. Using the same shock processes, we simulate this simple CIA model and report the results in Table 2.3.

Just as we discussed in section 2.3.7, without any friction in the goods market, the model cannot generate much volatility of both the nominal and real exchange rate. All the persistence of exchange rates comes from the underlying disturbances.

2.5.4. Flexible versus Fixed Regime

Empirical works have shown that when countries move from a pegged to a floating exchange rate system, the volatility of the real exchange rate increases dramatically, while the behaviors of other macroeconomic variables do not seem to change systematically.33 Moreover, the co-movement

of output, consumption and investment are usually higher under the fixed rate regime. In this subsection, we examine the model’s performance when the nominal exchange rate is fixed. The simulated results are consistent with those empirical evidences appearing in the switches of exchange rate regimes.

Under the fixed rate regime, country 2’s monetary authority is assumed to unilaterally peg the nominal exchange rate at a constant level. The model is otherwise identical to the benchmark model described in the previous section, where the nominal exchange rate is allowed to change. Therefore, to maintain the fixed exchange rate, monetary policy in country 2 loses independence and responds according to country 1’s monetary policy. More specifically, $\gamma_{1,t}$ is still exogenous and follows the stochastic process (2.33), but $\gamma_{2,t}$ has to be endogenous to maintain the fixed exchange rate.

In our model, to peg the nominal exchange rate at the level $\tau$, country 2 has to follow the same monetary policy as country 1 and maintain $\gamma_{2,t} = \gamma_{1,t}$ in all periods. To make a comparison between fixed and flexible rate regimes, we report the simulated results under different exchange rate systems in table 2.4.

The model does pretty well to replicate the evidence suggested by those empirical studies comparing fixed rate regime with flexible rate regime. The real exchange rate is the variable that is most affected by the change of exchange rate regime: it drops from 0.089 to 0.026 after changing to a fixed regime. Moreover, the real exchange rate is also highly persistent under a fixed rate regime (0.92). The behavior of the other variables, however, does not seem to be sensitive to the exchange rate regime: the standard deviations of output and consumption barely change after the switch of regimes.

34 Sopraseuth (2000) studies the data of countries participating and not participating in the ERM and finds that the EMS seems to favor a greater degree of synchronization among EMS countries.
### Table 2.4. Simulated results in fixed and flexible rate regimes

<table>
<thead>
<tr>
<th></th>
<th>Fixed Rate Regime</th>
<th>Flexible Rate Regime</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>$\theta$</td>
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<td>0.0843</td>
</tr>
<tr>
<td>Output</td>
<td>0.0175</td>
<td>0.0182</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0171</td>
<td>0.0171</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>-</td>
<td>0.0787</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.026</td>
<td>0.089</td>
</tr>
<tr>
<td><strong>Autocorrelations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>-</td>
<td>0.68</td>
</tr>
<tr>
<td>Output</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.79</td>
<td>0.82</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>-</td>
<td>0.68</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.92</td>
<td>0.96</td>
</tr>
<tr>
<td><strong>Cross-correlations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real and nominal exchange rate</td>
<td>-</td>
<td>-0.52</td>
</tr>
<tr>
<td>foreign and domestic output</td>
<td>0.59</td>
<td>0.11</td>
</tr>
<tr>
<td>foreign and domestic consumption</td>
<td>0.64</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The absence of the differential in valuation of currencies, $\theta$, as well as the changes in the relative effective money holdings, $\hat{m}^e$, in the fixed regime is the main reason for the substantial reduction in the variation of the real exchange rate. Under a fixed regime, the two countries have to apply the same monetary policy to maintain the fixed nominal exchange rate. The same money growth rates across countries eliminate the differential in valuations of currencies, as well as the changes in the relative effective money holdings, which removes the two key components of the fluctuations in the real exchange rate. Volatilities of output and consumption do not depend on $\theta$ or $\hat{m}^e$, and hence are not affected by the change of the exchange rate regime.

In accordance with the evidence suggested by the empirical works, the cross-correlations between domestic and foreign output and consumption increase when moving from pegged to flexible exchange rate regime. This is not a surprising result since the two countries have to coordinate their monetary policies to peg the nominal exchange rate. Therefore, the monetary policies of the two countries are correlated under a fixed rate regime instead of being independent.
under a flexible rate regime. As a result, the cross-country correlations of the variables in the model are higher under fixed rate regime than under flexible rate regime.

2.6. Conclusion

It has been a well known fact in the international business cycle that the real exchange rate movements disconnect with other macroeconomic aggregates. The behavior of the real exchange rate poses a non-trivial challenge to international business cycle models in which real exchange rates play an important role in real allocation. Traditional explanation for this feature of exchange rate relies on the interaction of the nominal price rigidities and monetary shocks. However, to generate the volatility and persistence of exchange rates that are high enough to match the data, such models with nominal price rigidities need to assume unrealistically long-lived price stickiness. Given the above-mentioned problem in the sticky price model, in this chapter we deviate substantially from the literature by developing a two-country dynamic search model to examine the behavior of exchange rates. The prices in our model are fully flexible and the most important feature of the model is the presence of search frictions in the goods market.

Our model successfully replicates the behavior of exchange rates shown by empirical studies: both the nominal and real exchange rates are highly volatile and persistent, and the behavior of real exchange rates changes systematically across different exchange rate systems. In sharp contrast to previous models which rely on nominal rigidities, the model in this chapter focuses on the role of search frictions in the goods market and shows that such a model is capable of capturing the main features of exchange rates in the international business cycle.

The deviations from the LOP are the key components of fluctuations of exchange rates in our model. Search frictions in the goods market induce a differential between the two countries’ valuations of a currency, thus creating price differentials across different types of trade matches.
The numerical results suggest that the deviations from the LOP contribute about 60 percent to the fluctuations of the real exchange rate in our model.

Monetary shocks play an important role in explaining the fluctuations of exchange rates in our model. The main reason lies in the fact that the deviations from the LOP is driven exclusively by the differential in the two currencies’ growth rates, which is directly affected by monetary shocks. Since monetary shocks are responsible for the deviations from the LOP, they account for a large part of the volatility of exchange rates. This result is consistent with the empirical findings which show that monetary shocks play an more important role in generating fluctuations in exchange rates.

Search frictions in the goods market is the key feature that generates high volatility of exchange rates. The simulated results show that the volatility of exchange rates increases with the degree of frictions in the goods market. In addition, we construct a CIA model with a Walrasian goods market. To make a comparison, the other features of the model are set up alike except that the goods market is frictionless. Without search frictions, neither monetary shocks nor real shocks can generate much volatility of real exchange rates, and the persistence of real exchange rates comes entirely from the persistence in the underlying shocks.

We are also interested in the behavior of exchange rates under different regimes. Our results show that a country experiences a dramatic increase in the volatility of the real exchange rate when moving from a pegged to a floating exchange rate regime. The change in the exchange rate regime, however, does not affect the behavior of the other variables such as output and consumption. Moreover, a higher co-movement of output and consumption across countries is found under a fixed rate regime than under a flexible rate regime. These results are consistent with the empirical studies.
The main discrepancy between the model and the data is that the model generates a negative correlation between the nominal and real exchange rates, while the data suggests they are highly positively correlated. This result arises from the highly abstracted description of the world economy. To sharpen our focus on the role of search frictions, the model is set in the simplest way that the main difference between the two countries comes from their money growth rates. Such a highly abstracted model prevents us from providing a satisfactory explanation for the correlation between variables. To reconcile this mismatch generated by the model, more differences between the two countries should be incorporated into the model. The next step of our research is to introduce the non-traded good sector. With the non-traded good sector, both the monetary shocks and real shocks play a role in generating the deviations from the LOP, and hence the fluctuations of nominal and real exchange rates. The correlation between the nominal and real exchange rate may then be consistent with the data.
A Dynamic Evaluation of Interfirm Group Fitness in the Global Airline Industry

3.1. Introduction

Interfirm networks are collections of firms joined by ties that vary in formality but are stable and significant enough to create reasonably persistent interfirm structures. These structures are reshaping our view of traditional organizational governance mechanisms of market and hierarchy. In the past 30 years, few phenomena have exceeded interfirm networks in attracting the attention of management scholars and practitioners.

Studies on interfirm networks tend to focus on examining general motivations for interfirm collaboration and firms’ partner selection decisions. These research show, for example, that firms enter interfirm alliances and choose partners selectively in order to reduce uncertainty resulting from resource requirements (Gulati, 1995), to access new knowledge and complementary assets (Kogut, 1988; Teece, 1986, 1992), to contend with market and hierarchy failures (Oxley, 1997; Williamson, 1991), or to fulfill their performance aspirations (Baum, Rowley, Shipilov and Chuang, 2005).

While previous works shed light on the organizational antecedents and outcomes of network formation, the unit of analysis in these studies focuses mainly on individual firms or firm dyads. Little attention has been paid to interfirm groups. A group here refers to a set of firms embedded in the network that are connected by certain relationships and work more closely with each other than they work with other firms outside the group (Gomes-Casseres, 1996; Rowley, Greve, Rao, Baum and Shipilov, 2005; Wasserman and Faust, 1994).
of groups of allied firms against other groups instead of the traditional battle of firm versus firm, it is essential for us to focus the analysis on interfirm group rather than individual firm. In most past studies, however, the collective nature of organizational action and the fitness of interfirm groups that maintain stable collective structures enabling coordination among interdependent parties have been largely ignored.

Recently, a limited number of studies have focused on groups that represent the mesolevel structure between dyads and networks. However, questions thus far examined in these works generally revolve around how the characteristics of group structures affect the performance and exit decisions of individual firms embedded within groups (Lazzarini, 2007; Rowley et al., 2004, 2005). The lack of research at the group level is problematic. Although focus on individual firm performance is clearly appropriate when outcomes can readily be attributed to the activities of individual firms or firm dyads, not all problems can be solved or understood by simply analyzing individual actions. Particularly in groups of firms bonded by multilateral relationships, individual firm outcome inextricably hinges on its interactions with other group members and the group operation as a whole. Moreover, customers’ welfare also depends more on the group-level operation than on an individual firm’s performance. Improving our understanding of group-level operation would provide insights into the preferable structure of firms within groups, the individual firm performance, the stability and changes of groups and, as well, by inference, the evolution and dynamics of overall networks.

In this chapter, we propose a group fitness construct and define it as the overall happiness or satisfaction of group members. Given the uncertain nature of group-based outcomes, a firm cannot determine a priori how profitable a given group will be and what proportion of any profits the firm will garner (Axelrod et al., 1993). Thus, strict profit maximization is not an immediate objective of firms choosing among competing groups. Instead, firms rely on their expectations of
which group is likely to do better than others. Firms, therefore, maximize their utility by ranking their preferences over competing groups. The firm utility will take on specific meaning with empirical data but, for the moment, it is broadly defined as a measure of the relative happiness or satisfaction (gratification) of firms gained by participating in different groups.

Given the originality of the group fitness construct, past research provides little direct guidance on what factors are important in modeling individual firm’s utility and overall group fitness. To identify those factors and their relationships, we first build on the clues suggested by previous academic studies on interfirm networks.

Past literature suggests that four factors affect the individual firm’s utility of being in a certain group and the fitness of the interfirm group as a whole: (i) member firms’ capacity, (ii) member embeddedness in past direct and indirect ties, (iii) resource complementarity and (iv) member firms’ internal competition. The effect of the four factors on firm’s utility and group fitness depends on their intricate interplay. To further our theoretical argument and provide support for our empirical modeling, we employ an inductive approach in verifying these four factors based on an extensive compilation and investigation of all public documents related to interfirm groups in the global airline industry.\(^2\) Industry insights from some top executives involved in the airline group operation are also used to corroborate our theoretical argument and model specification.

Building on these primitives, we propose a method to measure the utility of individual firms within a certain group and the overall fitness of each group. To assess group fitness, we construct the sample groups and compute the distribution of the group fitness for the sample groups. Then, we compare the fitness of actual groups with the distribution of the sample groups and a higher level of group fitness indicates that the group is stable and in a good shape. The distribution of

\(^2\)To study the evolution and dynamics of interfirm networks, we need an industry where a few interfirm groups have been formed for a certain years and a lot of entries and exists of individual firms occur in the groups. The global airline industry is chosen because it satisfies these conditions and provides a rich setting for our analysis.
individual carrier’s utilities within each actual group is also examined to predict the stability of
groups, which carrier is most likely to exit the group, and which is most likely to stay.

We test the firm utility and group fitness model by studying the groups formed in the global
airline industry between 1997 and 2002 based on archival data and verify the validity of the utility
and fitness constructs by comparing the empirical results against the industry evidence. Since
global airline firms are sophisticated actors that simultaneously engage in cooperative relation-
ships and compete for clients and partner carriers both within and across groups, this industry
is a rich setting for analyzing the outcomes of each individual carrier and the aggregate group
fitness. In an airline group, the outcomes of individual carriers are fundamentally intertwined
with those of other carriers that belong to the same group, and the integration and aggregation
of individual outcome leads to the overall group fitness and effectiveness.

The results computed from the data using our model can accurately reflect the real facts in
the airline industry. An interfirm group is indeed highly stable with a relatively higher group
fitness (or satisfaction), and the rank and trend of the utilities of individual firms in the group
can be used to indicate which firms are most likely to leave or stay.

In contrast to previous work on interfirm groups (Axelrod et al., 1993), we have developed a
more general means of operationalizing the social and instrumental antecedents, explored different
fitness specifications including some that are nonlinear and also allowed asymmetric variables
(route overlap) in the fitness function. The two constructs developed in this chapter, individual
firm’s utility and overall group fitness, hold great promise in fostering more group-level studies.

3.2. Groups: Concept and Evidence in the Global Airline Industry

Following deregulation of the world’s air transport industry in the 1980s and the growing
need of travelers for airlines serving a large number of cities, airlines are motivated to establish
extensive global networks. However, it is extremely difficult for a single airline to create a truly global network on its own. A foreign carrier has far more constraints in setting up an efficient service network in a host country than a domestic carrier. There are also legal constraints on international mergers and acquisitions in the airline industry. Even in the absence of any legal constraints, it is still impractical for a single airline to spend a huge amount of financial resources and time in building an individual service network (Oum & Park, 1997). In order to overcome these problems and set up a seamless global network, most airlines have formed strategic alliances with other airlines residing in foreign countries or continents (Oum & Park, 1997). As of June 1998, approximately 502 alliances had been formed between 196 international airlines (Airline Business, 1998).

While the first airline alliances were bilateral, involving agreements between pairs of carriers, broader airline alliances involving multiple carriers emerged in the mid 1990s. These multilateral alliances among multiple autonomous carriers, such as Star Alliance, Oneworld and Qualiflyer, are normally referred to as interfirm groups. Concomitant with the emergence of these airline groups is the shift of rivalry from the firm or dyad level to the group level (See Gomes-Casseres, 1996).

The main benefits derived from membership in these groups stem from the potential externalities generated by traffic sharing and joint resource use among member airlines (Grandori & Soda, 1995; Richardson, 1972; Teece, 1992). Interfirm groups have committed to enhancing closer coordination among members and providing greater connectivity and complementary point-to-point service that best meets passengers’ needs. The size of the aggregate customer

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3There are several benefits for travelers to have an airline which covers a large number of cities: (i) with an airline covering an extensive route network, consumers can minimize their time cost of travel planning and (ii) carriers covering a large number of destinations tend to offer a consistently higher quality of service and more attractive frequent flyer programs (Oum and Park, 1997).
base brought by member airlines within a group is, therefore, a determinant of competitive advantage in group-based competition which hinges on market share and economics of scale and scope (Gomes-Casseres, 1996). Resource complementarity is also essential for groups to compete effectively with each other for both clients and potential partner airlines. For example, by controlling alternative hubs and servicing alternative routes, group members can pool passenger traffic emanating from different regions. In addition, less market overlap could help member airlines to avoid or reduce within-group competition (Axelrod, Mitchell, Thomas, Bennett and Bruderer, 1995; Gomes-Casseres, 1994; Lazzarini, 2007). Carriers operating on the same routes may compete fiercely for passengers. This internal competition can fragment a group as partner carriers’ competing interests pull in different directions and appropriation concerns derail cooperative efforts (Gomes-Casseres, 1996; Rowley et al., 2005). In summary, the utility of a carrier participating in a certain group and the overall group fitness depend on the delicate interplay between collaboration and competition among group members.

3.3. Toward a Theory of Interfirm Group Fitness

3.3.1. The Necessity and Importance of an Interfirm Group Fitness Construct

The theory of the firm typically makes the convenient assumption of rational profit-maximizing behavior on the part of an individual firm. In various cases of the real world, however, firms do not appear to behave consistently in such a manner. One such case appears in interfirm groups where mutual interdependence among group members exists.

Three concerns render the focus on individual firm profit-maximization inappropriate in interfirm groups. Firstly, in interfirm groups, the very existence of interdependence transforms the context in which decisions are made. Neither individual firm strategies nor market equilibriums can be analyzed through the traditional profit-maximization approach. Interactions among group
members must be considered when accounting for the outcomes of individual firms and the group as a whole. Secondly, there is a growing recognition that interfirm groups have an identity apart from the individual members of which it is comprised. The performance of interfirm groups may not be fully captured by a simple amalgamation of individual motives and actions. For instance, in the area of community-based health care and social services for such groups as the homeless, people with severe mental illness, drug and alcohol abusers, and the elderly, a focus on individual firm outcomes is insufficient. Individual outcomes reflect only how well individual providers are performing their particular component of the many services needed by the clients. However, the overall well-being of clients depends on the integrated and coordinated actions of many different agencies within the whole system (Provan and Milward, 1995). The rewards are, therefore, determined at the group level and efforts should be put on encouraging team behavior, not simply individual behavior. Thirdly, different firms usually employ different accounting methods, especially when they operate in different countries and institutional environments. This renders the objective financial profit data barely comparable across firms and the aggregation of the data to the group level meaningless. Furthermore, the group involvement may be only part of much larger businesses of individual members. Therefore, the financial benefits derived directly from group participation may not be reported separately, which makes the calculation of the group-related benefits of individual members impossible (Gomes-Casseres, 1996). All these three concerns justify new constructs of individual firm and group level performance: individual firm utility and interfirm group fitness.
3.3.2. Social and Instrumental Antecedents of Group Fitness

To propose a theory of individual firm utility and interfirm group fitness, we draw upon previous research on interfirm networks, which indicates that a combination of four social and instrumental factors, rather than social or instrumental factors in isolation, explains economic behavior (Granovetter, 1985; Rowley et al., 2005).

3.3.2.1. Social antecedents: embeddedness of ties. Connections among member firms in the groups based on previous direct ties and common third party ties are an important source of differentiation in the fitness of groups. Drawing on Coleman’s (1988) network closure logic, researchers have argued that dense connections among a set of firms increase information sharing among partners, build familiarity and norms, enhance trust and, as a result, foster a normative environment that facilitates cooperation (Anand and Khanna, 2000; Baker, Faulkner and Fisher, 1998; Burt, 1992; Gulati and Gargiulo, 1999). The interfirm links among group members also act as a governance mechanism that controls the potential opportunistic behavior and coordination problems, and lowers information, monitoring and enforcement costs (North, 1990; Zenger, Lazzarini and Poppo, 2002). Norms of exchanges and trust among member firms are more effective in densely embedded groups. Thus, dense groups create more value through cooperative exchanges and smooth coordination than less dense groups (Walker, Kogut and Shan, 1997; Rowley et al., 2005).

In the airline groups, ties may promote norms of flexibility, solidarity, and information exchange (Poppo & Zenger, 2002). Past relationships, both direct and through common third parties, facilitate member airlines’ adaptation to unforeseeable contingencies and make them more flexible. Solidarity ensures member airlines act jointly and adjust collectively when problems and conflicts arise. Socially related partners are also willing to share more information, which
in turn facilitates flexibility and solidarity. The social relationships developed over time will enhance social embeddedness among member airlines, which creates a macroculture in airline groups with widely shared assumptions and values. This macroculture based on social relationships will complement formal contractual governance in guiding and coordinating collective actions among independent carriers (Das and Teng, 2002; Jones, Hesterly, Fladmoe-Lindquist and Borgatti, 1998).

As one CEO of an airline group remarked, “The key to a powerful partnership is the strength of the bonds that you develop between individual members.... Our members are constantly involved in an exchange of experience.... This gives you an idea of the level of trust that exists amongst member airlines.” (Albrecht, 2003)

Thus, we can propose the following assumption:

**Assumption 1.** Individual firm utility and overall group fitness are positively related to tie embeddedness among group members.

### 3.3.2.2. Instrumental antecedents: Firm capacity.

The total capacity of the group, which is the aggregation of member firms’ capacity, is valued because it is an indicator of the likelihood the group will succeed in attracting a larger customer base and in building an extensive global network. A substantial capacity in a group can be an advantage in group-based competition, particularly in a context of network externalities. Also, a large aggregate customer base can make the group more attractive for potential customers when the benefits that individuals attain by consuming the products of the group increase with the expected number of users (see Economides, 1996; Katz and Shapiro, 1985). This effect is particularly prominent when customers face significant switching cost to pursue alternative products (Klemperer, 1987a, 1987b; Lazzarini, 2007). In the airline industry, joint frequent flyer programs (FFPs), which represent a kind of repeat-purchase discount, have been regarded as an example of endogenous switching
costs. Customers will have increasing benefits if they continue using a particular FFP program and be penalized if they switch between different programs offered by competitors. Therefore, a group with a large installed base of passengers who face significant switching costs will generate substantial traffic externalities for member carriers. For example, a CEO stated, “Members carry almost 300 million passengers a year and generate revenue of over 75 billion U.S. dollars. These figures put us in a clear lead... That also explains why we are where we are today.” (Albrecht, 2003)

Groups with a large installed customer base are also able to reduce unit cost and improve service quality under jointly coordinated operations. This is due to the increasing returns to scale and scope based on cost savings when firms in the group increase the production of a given good or service and the variety of goods and services (Besanko, Dranove, Shanley and Schaefer, 2004; Tirole, 1988). When these arguments are applied to the airline industry, the implication is that joint operations on certain routes and joint market activities among group members are likely to reduce unit costs when the joint passenger base is large. The joint coordination on certain routes will also enable member airlines to coordinate their flight schedules to minimize connection times, to be located in the same terminal with neighboring gates to allow for convenient transfers, and to enhance flexibility to protect, monitor and re-accommodate connections on a proactive basis in case delays occur (Marchand and Gomes-Casseres, 2000). Group members also obtain scale economies when they make some joint non-redeployable investments in brand names, common administrative structures and common IT infrastructure. All these help group members achieve synergies and offer consistent and reliable services across routes for passengers, which enhance the effectiveness of the whole group.

Following this argument, we get:
Assumption 2. *Individual firm utility and overall group fitness are positively related with the member firm capacity.*

3.3.2.3. Instrumental antecedents: Resource complementarity. The effect of capacity externality among group members is augmented when firms operate in different niches and specialize in different roles. In that case, firms typically seek out distinct functions and build specific capabilities in which they hold competitive advantages. A CEO explained, “By having moved under one roof at many hubs and locations around the world we have improved customer service towards our overall aim of ‘seamless service’. At the same time we have created synergies for our partners.” (Albrecht, 2003) The mix of firms is critical in groups for combining resources, capabilities and markets, which enables groups to exploit economics of scope inherent in complex businesses and generate synergies among group members. The synergies result from the increasing interfirm externalities, leading to a more beneficial joint use of resources (Lorenzoni and Ornati, 1988; Richardson, 1972). For instance, a carrier within an airline group comprised of members who control alternative hubs and service alternative routes can, on the one hand, pool passenger traffic emanating from different regions and transport them to their destinations, and on the other hand, gather passengers and send them to diverse places by partners’ routes.

Thus, another assumption can be made as follows:

Assumption 3. *Individual firm utility and overall group fitness are positively related to the resource complementarity among member firms.*

3.3.2.4. Instrumental antecedents: Internal competition. Group fitness hinges on the appropriation of the created value among its members as well as the creation of value within each group. The value captured by a group member depends on the within-group competition intensity experienced by that member. Firms specializing in similar roles and markets are close rivals,
competing for similar resources and potential partners (Hannan and Freeman, 1977; Silverman and Baum, 2002). Some degree of conflict is usually inevitable among group members operating in the same industry. To a point, internal competition may even increase flexibility and foster innovation, however, too much conflict reduces the gains from collaboration and fragments the whole group (Gomes-Casseres, 1996; Rowley et al., 2005). If an airline group is composed of many carriers operating along the same routes, there likely is a high level of competition for passengers and potential partner carriers. None of the member carriers in such a group can reach a high level of capacity utilization or earn sufficient return to reinvest in growth, which indicates that the overall fitness of the group is poor. A CEO echoed this internal competition concern, “Very important, we keep each partner’s individual identity. . . . We agreed to work together in our own best interest. Win-win is the short formula to describe this. Most parts of our networks don’t overlap.” (Albrecht, 2003)

The above argument leads us to the final assumption:

**Assumption 4.** *Individual firm utility and overall group fitness are negatively related to the level of within-group competition.*

### 3.3.3. Model Specification

The foregoing discussion suggests that all four factors are equally important concerns in evaluating the value of participating in a particular group. A group that lacks a single factor will be significantly underrated by potential partners. Together with the relationships suggested by the literature, this indicates a multiplicative combination of the four factors in calculating the utility
to firm $i$ of participating in group $A$, $U_i (A)$, as follows:\(^4\)

$$U_i (A) = \left( \sum_{j \in A, i \neq j} S_j PT_{ij} RC_{ij} / CP_{ij} \right) / \left[ N(A) - 1 \right], \quad (3.1)$$

where $j$ is a member firm in the same group as firm $i$; $S_j$ is the capacity of firm $j$; $PT_{ij}$ measures the past direct and indirect ties between firm $i$ and firm $j$; $RC_{ij}$ measures the resource complementarity between firm $i$ and firm $j$; and $CP_{ij}$ indicates the pairwise competitive intensity. The equation is scaled by the number of firms in the group except the focal firm $[N(A) - 1]$ to capture the average utility to firm $i$ in participating in group $A$. This scaling also avoids the possibility that a high firm utility only results from the high number of group members rather than the four factors we focus on.\(^5\)

The next step is to define the fitness ($F$) of a group as a whole as the weighted sum of the utility of each firm in that group, where the weights are the capacity of the firms ($S_i$) and the number of firms in the group ($N$). We scale the group fitness score by the number of firms in the group in order to make it comparable over time and across different groups. This gives the group fitness as

$$F (A) = \sum_i S_i U_i (A) / N (A). \quad (3.2)$$

\(^4\)Note that the multiplicative specification is not the only way that the four factors should be considered to be incorporated into the group fitness function. There are other forms which are consistent with the assumptions we made in previous section. We did check some other forms and will discuss the results later.

\(^5\)Note that the pairwise competitive intensity measure may be asymmetric when firm $i$ and $j$ treat each other as a rival to a different degree. This specification treats a firm as a bounded rational and myopic entity which bases its evaluation of a group only on pairwise relationships between itself and potential group members. This is consistent with the dyad-level partnering literature which does not consider relationships among the partners of the pair within a dyad (Gulati, 1995; Podolny, 1994).
3.3.4. Exploratory Field Study

Given that firm utility and group fitness have not been studied in previous works, we use an inductive, field-based study to verify the four factors we identified and their multiplicative relationships in modeling firm utility and group fitness.

Inductive research lacks a generally accepted model for its central creative process. In the absence of a standard, the following approach is applied in this work: We firstly conducted an exhaustive search and compilation of all public documents related to the interfirm airline groups. These documents included all articles available in the Factiva database that mentioned the “global airline alliances” or “airline groups” from 1990 when the first global airline group was formed through 2006. This search resulted in 1,043 relevant articles with 1,878 pages.

Most of these articles are business releases about the entries and exits of new member carriers to a group and the operation of airline groups. They provide important information on why a certain carrier chose to join a certain group, what benefits the focal carrier can derive from other group members and what benefits the other group members can garner with the entrance of a new member. We then randomly picked 50 articles out of the 1,043 documents. Each author pored over each article independently and manually coded the factors mentioned in each article relevant to carriers’ group choices and the benefits of group membership.

A list of important factors that are supposed to affect firm utility and group fitness was thus generated by each author for each article. Immediately after the coding, the authors cross-checked their lists and discrepancies were solved based on further communication. When the authors could not reach an agreement on certain factors, a third party who holds substantial consulting experience in the airline industry was called upon for an in-depth discussion. A list of 12 factors finally emerged as important in affecting firm utility and group fitness (See Table 3.1).
Based on this list of factors, each author carried on to finish reading the other 993 articles and map these factors to each article. The authors noted any new factors not included in the list. Another round of cross-checking was done on the 993 articles and discrepancies were solved. Then each author independently mapped the 12 factors to the four general factors proposed in our theory. In cases where we could not map certain factors to the four general categories, new categories were added based by argument between authors. Finally, we mapped the 12 factors to seven general categories and counted the occurrences of mentions of each general category in all 1,043 articles and plotted them in Figure 3.1.

This Figure corroborates that firm capacity, tie embeddedness, resource complementarity and internal competition are the most important factors affecting an individual carrier utility within a certain airline group and overall group fitness. And more importantly, these four factors are mentioned side by side in 70 percent of the articles, indicating that carriers simultaneously evaluate the four factors in their group choice. The high frequency of simultaneous mentions of the four considerations provides further support of the multiplicative specification of our model.
3.4. Quantitative Analysis

3.4.1. Sample and Data

We test the validity of the two constructs, individual firm’s utility and group fitness, using archival data in the global airline industry. The original sample included 104 airline firms, 38 of which appeared in at least one airline group during the observation period between 1997 and 2002. To define the risk set for this chapter, we firstly ran a binomial logistic regression of group entry decision based on a list of variables: firm capacity, international positioning, number of destinations, number of ties, average tie age, average hub distance and average route overlap. We then calculated the predicted group entry value for each firm based on the estimation, and the lowest predicted value was identified among the 38 group members. We included in the final sample for analysis all firms that have a larger (or equal) predicted value than the lowest value identified among actual group members. This amounts to a total number of 75 carriers with 38 group members and 37 non-group members. These 37 non-group members are regarded as potential group entry candidates.6 We choose 1997 as the starting year because the most prominent and significant airline groups first emerged in that year. (Tables 3.2-3.7 summarizes the airline groups operating during the 1997 – 2002 period.) The carriers in this final sample represent about 92.6 percent of the total world passenger traffic in 2002 (World Air Transport Statistics, 2002). We created all possible dyads among the 75 firms along with their corresponding variables. These data were used for constructing an event history for each feasible dyad for each year studied. The resulting data structure is best characterized as a cross-sectional time-series

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6The top ranked non-group carriers based on the predicted group entry value, such as Continental Airlines, Japan Airlines and South African Airways, all joined an airline group in subsequent years, which provides additional support for the approach we used to define the risk set.
panel in which the units are unique dyads. Each record included the time-varying and time-
constant covariates characterizing the dyad in that year, which are used to calculate the current
and lagged pairwise variables.

Data on alliances formed and the composition of airline groups came from several issues of
Airline Business magazine. This magazine conducts the annual Airline Alliance Survey, which
provides information on alliances at a particular time, including the alliance partners, the starting
date of the alliance, the status of equity participation and the cooperation areas (e.g. code sharing,
joint purchasing, etc.). This is regarded as a very comprehensive and reliable source for tracking
airline alliance activities. Since the cut-off date for the annual alliance survey is normally June
or July, we assume that an alliance is formed in a given year if its starting date is in the first
half of that year, i.e., between January and June. If an alliance is dissolved in a given year, we
assume that the alliance is in place in that year if the termination occurs in the second half of
that year (Lazzarini, 2007). We excluded reports of planned alliances that never materialized. To
be consistent, we also coded the group membership based on the same cut-off date as shown in
Tables 3.2-3.7. For each carrier, we also collected their operational information, such as traffic and
capacity, from the World Air Transport Statistics compiled by the International Air Transport
Association (IATA) for each year between 1997 and 2002. Finally, we collected detailed flight
schedule data by combining the Official Airline Guide (OAG) database and the Department of
Transportation’s Origin and Destination Survey (O&D) database.

Insert Tables 3.2-3.7 about here
3.4.2. Key Variables

For a better understanding of our method and the results, four variables need to be described in details.

i). Firm Capacity ($RPK$).

The appropriate capacity measure varies in different empirical settings. Current market share is usually regarded as a good indicator of future market power especially when the market shows strong network effects (Axelrod et al., 1993; Katz and Shapiro, 1985). In the airline industry, we judge the scheduled passenger traffic to be the best available empirical estimate of a carrier’s future importance and, thus, current attractiveness when traffic externalities are essential. We measure the scheduled passenger traffic with $\text{revenue passenger kilometers (RPK)}$, which corresponds to the sum of the products obtained by multiplying the number of revenue passengers carried on each flight stage by the flight stage distance.


Two variables were created for measuring resource complementarity: $\text{international positioning (ITP)}$ and $\text{hub distance (HBD)}$. Following Lazzarini & Joaquim (2004), we define the $\text{international positioning (ITP)}$ of a carrier as the ratio of traffic – $\text{revenue passenger kilometers (RPK)}$ – emanating from international passenger flows to the total traffic of that carrier. Then we calculated the ratio of the largest to the smallest international positioning score of member airlines within each dyad for each year between 1997 and 2002. This ratio measures the resource complementarity in the sense that two carriers can generate more externalities if one specializes in domestic markets and the other specializes in international routes.
In addition, using our combined schedule data, we identified the top three hubs for each carrier in each year based on the number of departing flights.\(^7\) We then calculated average *hub distance* \((HBD)\) between the top hubs of carriers according to their latitudes and longitudes adjusted for the earth’s curvature.\(^8\) This hub distance variable captures the fact that distant hubs expand the possibilities for connections, which lead to more diverse routes offered to consumers.

A third dimension of resource complementarity among group members can be measured by *number of nonoverlap destinations* \((DES)\). This variable counts the number of new destinations provided to the focal carrier by its group peers. This variable is, therefore, a group level variable. It captures resource complementarity in the sense that new destinations provided by peer carriers extend the focal carrier’s market coverage and offer air travelers more seamless service.

iii). **Internal competition within groups (CMR).**

Internal competition within groups is measured by the market overlap among member firms. The degree of presence that a carrier manifests in the markets where it overlaps with a focal carrier is determined by two factors: the strategic importance each of the markets the focal carrier shares with the rival carrier and that rival carrier’s market share in those markets. Market in the airline industry is normally defined as a route (Gimeno2004; Karnani and Wernerfelt, 1985). Therefore, we measure the internal competition within airline groups based on the *route overlap* \((CMR)\)

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\(^7\)Hubs are defined as the airports that, for a particular carrier, have the highest number of departing connections to other airports (Lazzarini, 2007).

\(^8\)According to Sorenson and Audia (2000) we calculated distances using spherical geometry, which computes the distance between two points \(A\) and \(B\) as:

\[
d(A, B) = 6370 \arccos [\sin (l_A) \sin (l_B) + \cos (l_A) \cos (l_B) \cos (|g_A - g_B|)]
\]

where \(l_i\) is the latitude of location \(i\), and \(g_i\) is the longitude of location \(i\). Both latitude and longitude are measured in radians. The constant 6370.997 is the earth’s radius in kilometers, and converts the distance into units of one kilometer.
between member carriers: 9

\[ CMR_{ij} = \sum_{k=1}^{n} \left( \frac{NF_{ik}}{NF_i} \right) \left( \frac{NF_{jk}}{NG_k} \right); \]

where \( CMR_{ij} \) denotes route overlap that airline \( j \) has with the focal airline \( i \); \( NF_{ik} \) denotes the number of flights served by \( i \) in route \( k \); \( NF_i \) denotes the total number of flights served by \( i \) across all \( n \) route; \( NG_k \) denotes the number of flights served by all airlines in route \( k \).

A higher value of \( CMR_{ij} \) means the airline \( i \) and \( j \) have more overlaps in their routes. Therefore, the level of competition between those two firms is higher.

iv). Past ties (\( PBT, TRD \)).

To measure the prior relationships between carriers within a dyad, we created two dichotomous variables based on the airline alliance network constructed every year: prior bilateral ties (\( PBT \)) and third-party embeddedness (\( TRD \)). Prior bilateral ties (\( PBT \)) is coded 1 if members in a dyad were involved in some alliances in the past year and 0 otherwise. Third-party embeddedness (\( TRD \)) is coded 1 if both carriers of a dyad were tied to at least one common third carrier in the past year but were not directly linked at the same time and 0 otherwise. In order to obtain unbiased measures of these relationship variables, we based our calculation of these variables on the overall network constructed with the 104 carriers in our original data set, each of which was involved in at least one alliance during 1994 - 2002.

To be comparable, all variables were subsequently standardized by subtracting their mean and then divided by their standard deviation based on the sample of 75 firms and the corresponding 16,069 dyads constructed for the six-year observation period. The standardization allows for the combination of these variables on a comparable scale. To avoid computational complications, we

\[ \text{We also calculated another variable, common departure airport (CDA), as an alternative measure of internal competition. Airlines departing from the same airports are likely to be close competitors for passengers.} \]
then identified the global minimum value among all standardized variables and subtracted it from all standardized results to ensure that all standardized variables are nonnegative.

3.4.3. Customized Fitness Models to the Airline Groups

For the airline industry data, we specify the final form of the general utility model in equation (3.1) and fitness model in equation (3.2) as follows:

\[
U_i(A) = \frac{\sum_{j \in A, i \neq j} RPK_j (PBT_{ij} + TRD_{ij}) (ITP_{ij} + HBD_{ij} + DES_{iA}) / CMR_{ij}}{N(A) - 1}; \quad (3.3)
\]

\[
F(A) = \sum_{i,j} RPK_i U_i(A) / N(A); \quad (3.4)
\]

where the variables are defined in the previous section (see Table 3.1 for a summary of all measures).

To evaluate whether the actual groups formed are fit, the ideal approach will consider in each year all possible number of groups and also consider all possible combinations of firms in each group based on our sample of 75 airline firms. However, this exhaustive simulation, although ideal, is not feasible based on our large number of firms.\(^{10}\) Therefore, we treat the number of groups and the number of firms in each group as exogenous in each observation year. Then we randomly pick carriers from the sample to generate different compositions of each group in each year, taking the number of groups and the number of firms in each group as given. All hypothetical configurations of each group in each year based on random assignments constitute the population configurations of that group that year. Then based on equation (3.3) and (3.4), we can calculate the population distribution of the fitness scores for each group in each year,

\(^{10}\)For example, the possible number of random assignments of the 75 firms to just two groups will be \(2^{75}\), which is way too much computation work on a personal computer. The computation work for three or more groups will be way much more.
and map the fitness score of each actual group composition (hereafter actual group fitness) into its population distribution. The relative position of the actual group fitness in its population distribution is defined by the mean fitness score of the population and the percentile of fitness scores in the population below the actual fitness score. A fit actual group will be characterized by two attributes: (i) the actual fitness score is higher than the population mean and (ii) the higher the percentile, the better fitness of the actual group composition.

According to Tables 3.2-3.7, the number of groups formed during the observation period ranges from three to five, and the number of firms in each group varies from two to 12. Even when we take the number of groups and the number of firms in each group as given in each observation year, the population of group configurations based on the random reassignments of the 75 sample firms is extremely large. It is again infeasible to generate all possible configurations. To approximate the population distribution of the fitness scores of each group, we used a Monte Carlo simulation method. Specifically, we randomly reassigned the 75 firms to groups in each year for 100,000 times, thus generating 100,000 hypothetical configurations for each group each year. To verify whether the results of the 100,000 hypothetical configurations are representative for the population configurations of each group each year, we reran the program 10 times independently, each time with 100,000 random reassignments. Each time we reran the program, we calculated the mean fitness score of the 100,000 hypothetical configurations for each group each year and the percentile of fitness scores below the actual group fitness score. Based on the comparison across the 10 mean fitness scores and percentiles of each group each year, we found the differences are trivial (coefficient of variance under 0.0001), indicating that 100,000 random reassignments are enough to determine the representative results of the population distribution of fitness scores for each group each year.
3.4.4. Main Results

We present the results of the analysis in Figure 3.2 – Figure 3.9. Both the fitness score of each group as a whole and the individual firm’s utility score within each group are plotted in these figures. In order to illustrate the patterns of group fitness and individual utilities over time, we focus on four groups – QualiFlyer, Star Alliance, Atlantic Excellence and SkyTeam – all of which exhibit distinct and interesting evolution patterns. In the figure of group fitness scores, two curves are plotted, one labeled as “actual” and the other labeled as “random”. The “actual” curve represents the fitness scores that are calculated using equation (3.4) based on the composition of the actual groups formed (i.e., actual group fitness). The “random” curve represents the mean fitness score of the population configurations for each group each year. The graph of an individual firm’s utility shows the ranking of individual firm’s utility within each actual group formed, based on calculations using equation (3.3).

By comparing the fitness scores of actual groups and corresponding population means and percentiles, we are able to assess the fitness of actual groups in each year, how it changed over time and the stability of the group in the future. The plot for an individual firm’s utility shows which firms benefited most within a group and which firms had the lowest utilities as a group member in each year, to what extent the utilities were unequal, and how the utilities changed over time for each firm. While the graph of group fitness scores indicates whether a certain group was in a good shape at a certain point of time and how it evolved over time, the graph of an individual firm’s utility offers the underlying reasons for explaining the (in)stability of the group.

Several patterns were observed in Figure 3.2 – Figure 3.9:

(i) QualiFlyer consistently underperformed its population mean as shown in Figure 3.2, which suggests poor fitness of the group as a whole. More than 95 percent of the population fitness scores are greater than the actual group fitness score of QualiFlyer during the observation period (refer
to Table 3.8). The utility of all member firms in the actual group except one started declining after a temporary increase in 1999. This suggests that Qualiflyer was likely to be unstable after 1999 and had a high probability of being dissolved.

Insert Table 3.8 about here

Insert Figures 3.2 and 3.3 about here

Several factors may have led to the poor fit of this group. By looking closely at its membership, we found that most members, such as AOM French Airlines and LOT Polish, are second-tier European carriers with small capacity. The proximity in terms of member carriers’ hubs indicates that they were playing similar roles in close geographic markets, which led to few complementary resources and significant competition. In addition, member carriers were rarely linked directly or through common third parties.

The main reason Qualiflyer was formed was because Swissair, in order to compete with other airline groups, took the lead and pooled a troupe of second-league carriers into a single group generally by purchasing shares in those members. By doing so, Swissair unknowingly began its path to bankruptcy. Firstly, the participation of those smaller carriers was largely involuntary. Rather their participation was a result of Swissair acquiring them. Secondly, the acquisition was costly and many of the Qualiflyer Group’s airlines had been financial losers. For example, Sabena, the Qualiflyer Group’s largest member after Swissair, had only made a profit once in its entire history (www.wiki.com). Since it was a partial subsidiary of Swissair, Swissair was forced to make up for its losses. All the evidence suggests that Qualiflyer was not in good shape from
the very beginning, although it managed to exist for three years before its demise.\textsuperscript{11} As predicted by our model, Qualifyer disbanded in 2001.

(ii) Star Alliance had higher fitness scores than its population mean and the difference increased overtime, suggesting a high and increasing group fitness. Specifically, only two percent of the population fitness scores are greater than the fitness score of the actual Star Alliance in 1997. This percentage decreases to 0.6 percent in 2000, followed by a small increase in 2001 and 2002. The utility of most members also, on average, shows an increasing trend with one notable exception, Thai Airlines International.

The good fitness of Star Alliance and the increasing utility of its member carriers can be explained by the four social and instrumental antecedents. The member airlines of Star Alliance came from five continents and included the biggest and most successful carriers in Asia, Europe, North America, South America and Australia. This composition resulted in significant complementary resources which could be jointly used, creating substantial traffic externalities among group members. The few overlaps in members’ route coverage also reduced within-group competition and attenuated internal conflict. Consistent with these results, Star Alliance is arguably the best performing airline group today.

\begin{center}
\hline
\textbf{Insert Figures 3.4 and 3.5 about here}
\end{center}

\textsuperscript{11}Qualifyer was able to exist for four years largely because of inertia, sunk cost commitment and limited alternative choices of group members. For example, executives may be reluctant to signal the defeat of their essential strategies by undertaking new courses of actions at the first signs of failure (Staw, 1976; Staw, Sandelands and Dutton, 1981).
Three interesting phenomena come up in Figure 3.5. First, the utility of Thai Airlines International follows a decreasing trend, especially when Singapore Airlines partnered with United Airlines and joined the Star Alliance in 2000. Indeed the media noted that the entrance of Singapore Airlines to Star would cause “a great deal of soul searching in Bangkok” (*Airline Business*, 1997: 14). Therefore, our model suggests that there would be a tension between Singapore Airlines and Thai Airlines International which may lead to potential conflict within Star Alliance.

Secondly, Ansett Australia, which joined in 1999, had the highest utility in the group, even higher than Lufthansa and United Airlines, which are regarded as the leaders of Star Alliance. This is because Ansett Australia was a carrier specializing in domestic routes in Australia, with 80 percent of its traffic coming from domestic flights. The membership of Star Alliance helped Ansett Australia absorb great traffic flows from its partners, most of which are international carriers, and at the same time its domestic specialization further attenuated its competition with partners.

Thirdly, Austrian Airlines switched from Qualiflyer and Atlantic Excellence to Star Alliance in 2000. Austrian increased its utility score right after this switch, indicating that carriers seek to become a member of a group where it can derive higher utility.

(iii) Atlantic Excellence appeared to have had a sharp decrease in its fitness level based on our estimation from 1997 to 1999. It finally dissolved in November 1999 when its actual fitness score fell lower than the corresponding population mean.

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Insert Figures 3.6 and 3.7 about here

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Figures 3.6 and 3.7 show that Atlantic Excellence was composed of four members: Delta Airlines from North America, and Austrian, Sabena and Swissair from Europe. Delta’s utility
showed the most significant decrease from 1997 to 1999. Therefore, Delta was likely to be the one that triggered the dissolution of the group. As the only carrier from North America, Delta had long been questioning the benefits of being a member of Atlantic Excellence and finally came to the conclusion that Atlantic Excellence was not fulfilling its aspirations (Airline Business, 1999). Delta then formed an alliance with Air France in 1999, which was considered a close competitor by the three European group members. Undoubtedly, this potential conflict and competition within the group made it unlikely that all five firms could operate as a cooperative group, leading to the sharp decrease in group fitness and thus the dissolution of Atlantic Excellence. This again reinforces the model and verifies its prediction power.

(iv) SkyTeam had good fitness from its inception to 2000. While higher than the population mean, its actual fitness score was declining and the difference between actual score and the population mean was decreasing as well (refer to Table 3.8 and Figures 3.8 and 3.9). This trend may have resulted from the recruitment of two additional carriers, CSA Czech in 2001 and Alitalia in 2002. These two carriers are both second-tier European carriers and did not contribute much to the overall group fitness based on our model. The most critical candidate members for SkyTeam not covered by current group members would be from Asia, Australia, and Africa. The recent developments of SkyTeam corroborated this estimation. For example, in 2006 Aeroflot from Russia joined SkyTeam followed by China Southern Airline’s entrance. SkyTeam is also working on incorporating Kenya Airways into its group network. Finding a partner from Australia, however, may be more difficult, because both Quantas and Air New Zealand have already committed to the competing groups OneWorld and Star Alliance, respectively. More data are needed to trace the long-run evolution of SkyTeam.
3.4.5. Robust Check

In order to check the robustness of the patterns we found, we varied equation (3.3) and (3.4), and examined 24 alternative specifications that combined the social and instrumental antecedents in different ways. Despite different specifications, all models reflect the basic propositions we made on how group fitness depends on the four social and instrumental antecedents.

Firstly, we combined the four factors in a linear specification. Secondly, we included a measure of market growth (total air traffic growth by region) as an extra factor in the model because market growth was shown as a fairly important consideration in group fitness based on the field study as shown in Figure 1. Thirdly, we substituted tie age for prior bilateral ties (PBT) and third-party embeddedness (TRD) as an alternative measure of embedded relationships. Finally, we also tried taking the log of each factor in order to capture the potential declining marginal utility with the increases in each factor. We add a constant 1 to each variable to ensure the log of each factor is positive. There are no significant changes in results in all these alternative specifications.

The results are, therefore, robust with the patterns reported in this chapter generally holding across all specifications. For brevity, we do not present the other specifications in this chapter. More importantly, by showing the robustness of the model presented in this chapter, we sought a model that included only those four factors, which are suggested as the most important in affecting firm utility and group fitness by the literature and verified by the inductive field-based study. The general principle is that for a given level of accuracy, the simplest model that fits
well with the data and industry evidence is the most parsimonious model and is preferable to the more complex one, known as Occam’s Razor.

3.5. Conclusion

Competition increasingly takes the form of groups of allied firms against other groups instead of the traditional battle of firm versus firm. It becomes more and more essential for researchers to choose higher levels of analysis than the individual actor, dyads and even ego networks. Focusing on groups as a whole holds the promise of transcending the legacy of actor-centered theorizing and investigation. The dearth of reported research on overall group fitness was an important motivation for this chapter.

In this chapter, we have pinpointed four important factors in affecting individual firm’s utility of being in a certain group and the overall group fitness based on literature search and field study, also developing a model for measuring these two constructs. We then tested the validity of the model by calculating the two constructs using archival data and comparing the results against industry evidence. This mix of fieldwork and quantitative analysis allowed us to examine how group compositions such as group capacity, tie density, resource complementarity and internal competition affect individual firm’s utility and overall group fitness. The comparison over the observation period between the actual group fitness and the population distribution of fitness scores indicates whether the actual groups are in a good shape. The rank and trend of the utilities of individual firms in the actual groups help us predict which firms are most likely to leave or stay.

We illustrated the effectiveness of our model and methodology by applying it to the airline groups between 1997 and 2002. Given a plausible set of assumptions concerning firm capacity, tie embeddedness, resource complementarity and rivalry, we found a robust estimate of group fitness and individual firm’s utility within each group. By looking at what really happened to the
airline groups afterwards, we found that the predictions based on the comparison between the fitness scores of actual groups formed and those of the corresponding population constructed are reasonably accurate, and the implications based on the ranking of individual firm’s utility within each group are generally supported.

Compared with previous work on interfirm groups (Axelrod et al., 1993), we have developed a more general means of operationalizing the social and instrumental antecedents, explored different fitness specifications including some that are nonlinear and also allowed asymmetric variables (route overlap) in the fitness function. The two constructs that we developed in this chapter, individual firm’s utility and overall group fitness, are reasonably supported by the data.

Our study can be used promisingly in fostering more group-level studies. For instance, researchers can employ the individual firm utility and the group fitness measure as independent variables in future study of other aspects of the group phenomena. Practically, the approach developed in this chapter can be used to examine the current status of their groups and evaluate the value-added by potential group member candidates.

Just as with any other research, this study has limitations of its own. Because the research is exploratory, we chose to base the firm’s utility and group fitness construct on factors identified by past literature and field study in order to determine which aspects, if any, were relevant. We make no presumption, however, that the four factors considered offer the only possible explanations of individual firm’s utility and group fitness. Much more research needs to be done on the implementability of the constructs in other industries. Nothing is known, for example, about the practical difficulties involved in identifying factors affecting group fitness, or about to what extent one, in practice, can combine those factors in the same way as suggested in this chapter. The potential alternative explanations and specifications offer avenues for future research. Furthermore, the number of groups formed and the number of firms within each group are treated
as exogenous in this study. Being able to estimate these variables endogenously would add great
value to the methodology. The ideal simulation will have to consider any possible number of
groups and number of firms in each group in each year and calculate the fitness for each possible
group configuration. While this approach is ideal and theoretically possible, it is empirically
infeasible and intractable given the large number of firms in our sample. By fixing the number
of groups and number of firms within each group, our simulation is easily tractable and at least
provides a reasonable first approximation. This first approximation shows its value by generating
consistent results with observed industry evidence. Finally, future study covering a longer period
of time may uncover more interesting patterns on group fitness as well as on the utility changes
of individual firms. Despite these limitations, this chapter has provided a useful starting point
and illustrated the power and potential value of our approach.

Overall, this chapter has outlined some basic elements of a theory of interfirm group fitness.
With a high degree of robustness, the results of our study indicate that a fit airline group is
composed of firms with large capacity, embedded relationships, highly complementary resources
and a low degree of market overlap. Given the difficulty of assessing group fitness objectively,
this chapter makes a critical step forward in drawing some preliminary conclusions about what
factors may be important considerations. By focusing on the fitness of groups as a whole, the study
departs from traditional network perspectives, particularly resource dependence and transaction
cost economics, which tend to focus on individual characteristics to explain issues of network
involvement and firm performance. By examining the ranking of individual firm's utility within
each group, our study also contains implications for the study of firms’ group entry and exit
decisions and thus the overall network evolution and dynamics. Focusing on groups holds high
promise for extending theories of organizational networks. We encourage further development
of the model presented in this chapter and expect great theoretical and practical rewards from studying groups.
<table>
<thead>
<tr>
<th>Factors</th>
<th>General Categories</th>
<th>Sub-Categories</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group Member</td>
<td>Capacity</td>
<td>Revenue Passenger</td>
</tr>
<tr>
<td></td>
<td>Resource</td>
<td>International Positioning</td>
<td>Positioning Ratio</td>
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<td></td>
<td>Complementarity</td>
<td>Hub Distance</td>
<td>Hub Distance (HBD_{ij})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Market Coverage</td>
<td>No. of Destinations (DES_{iA})</td>
</tr>
<tr>
<td></td>
<td>Tie Embeddedness</td>
<td>Prior Direct Tie</td>
<td>Prior Bilateral Tie (PBT_{ij})</td>
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<td>Prior Third Party Tie</td>
<td>Tie Age (AGE_{ij})</td>
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<td>Internal Competition</td>
<td>Route Overlap</td>
<td>Route Overlap (CMR_{ij})</td>
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<td>Growth Rates of Home Country</td>
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<td>Brand Match</td>
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<td></td>
<td>Other</td>
<td>Customer Service Standards</td>
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<td>Culture Fit</td>
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<tr>
<td></td>
<td></td>
<td>Employee Training etc.</td>
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</tr>
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Table 3.1. Factor Identification Based On Field Study

<table>
<thead>
<tr>
<th>Group</th>
<th>Date Founded</th>
<th>Group Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global Excellence,12</td>
<td>1990</td>
<td>Delta Airlines Singapore Airlines Swissair</td>
</tr>
<tr>
<td>Atlantic Excellence</td>
<td>Feb 1997</td>
<td>Austrian Airlines Delta Airlines Sabena Swissair</td>
</tr>
<tr>
<td>Star Alliance</td>
<td>May 1997</td>
<td>Air Canada Lufthansa SAS International Airlines</td>
</tr>
</tbody>
</table>

Table 3.2. Airline Groups 1997 (Actual groups formed)

<table>
<thead>
<tr>
<th>Group</th>
<th>Date Founded</th>
<th>Group Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic Excellence</td>
<td>Feb 1997</td>
<td>Austrian Airlines Delta Airlines Sabena Swissair</td>
</tr>
<tr>
<td>Qualifyer</td>
<td>Mar 1998</td>
<td>AOM French Airlines Airlines Crossair Laudia Air Sabena Swissair</td>
</tr>
<tr>
<td>Star Alliance</td>
<td>May 1997</td>
<td>Air Canada Lufthansa SAS International Airlines Varig</td>
</tr>
</tbody>
</table>

Table 3.3. Airline Groups 1998 (Actual groups formed)
<table>
<thead>
<tr>
<th>Group</th>
<th>Date Founded</th>
<th>Group Member</th>
<th>Group</th>
<th>Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic</td>
<td>Feb 1997</td>
<td>Austrian Airlines</td>
<td>Delta Airlines</td>
<td>Sabena Swissair</td>
</tr>
<tr>
<td>Excellence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oneworld</td>
<td>Sep 1998</td>
<td>American Airlines</td>
<td>British Airways</td>
<td>Cathay Pacific Qantas Airlines</td>
</tr>
<tr>
<td>Qualiflyer</td>
<td>Mar 1998</td>
<td>AOM Austrian Airlines</td>
<td>French Airlines</td>
<td>Crossair Lauda Air Sabena Swissair</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TAP Air Turkish Airlines</td>
<td>Portugal</td>
<td></td>
</tr>
<tr>
<td>Star Alliance</td>
<td>May 1997</td>
<td>Air Canada Lufthansa SAS International Airlines Varig Thai Airlines United</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>New Zealand Australia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wings</td>
<td>1999</td>
<td>KLM Northwest</td>
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Table 3.4. Airline Groups 1999 (Actual groups formed)

<table>
<thead>
<tr>
<th>Group</th>
<th>Date Founded</th>
<th>Group Member</th>
<th>Group</th>
<th>Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oneworld</td>
<td>Sep 1998</td>
<td>American Airlines</td>
<td>British Airways</td>
<td>Cathay Pacific Qantas Finnair Iberia</td>
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<tr>
<td>Qualiflyer</td>
<td>Mar 1998</td>
<td>AOM Portuguese Airlines</td>
<td>French Airlines</td>
<td>Crossair LOT Polish Sabena Swissair</td>
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<tr>
<td></td>
<td></td>
<td>TAP Air Turkish Airlines</td>
<td>Portugal</td>
<td></td>
</tr>
<tr>
<td>SkyTeam</td>
<td>Sep 1999</td>
<td>Aeromexico Air France Korean Airlines Delta Airlines Thai Airlines United</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Star Alliance</td>
<td>May 1997</td>
<td>Air Canada Lufthansa SAS International Airlines Varig</td>
<td>Thai Airlines United</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>New Zealand Australia Airways Airlines Mexican Airlines Singapore Nippon Austrian</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wings</td>
<td>1999</td>
<td>KLM Northwest</td>
<td></td>
<td></td>
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</table>

Table 3.5. Airline Groups 2000 (Actual groups formed)
<table>
<thead>
<tr>
<th>Group</th>
<th>Date</th>
<th>Group Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oneworld</td>
<td>Sep 1998</td>
<td>American Airline, British Airways, Cathay Pacific, Qantas, Finnair, Iberia</td>
</tr>
<tr>
<td>Qualifyer</td>
<td>Mar 1998</td>
<td>AOM Airlines, British Airways, Cathay Pacific, Qantas, Finnair, Iberia</td>
</tr>
<tr>
<td>SkyTeam</td>
<td>Sep 1999</td>
<td>Aeromexico, Air France, Korean Airlines, Delta Airlines, CSA Czech, Thai Airlines, United</td>
</tr>
<tr>
<td>Star Alliance</td>
<td>May 1997</td>
<td>Air Canada, Lufthansa, SAS, International Airlines, Varig</td>
</tr>
<tr>
<td>Wings</td>
<td>1999</td>
<td>KLM, Northwest</td>
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Table 3.6. Airline Groups 2001 (Actual groups formed)

<table>
<thead>
<tr>
<th>Group</th>
<th>Date</th>
<th>Group Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oneworld</td>
<td>Sep 1998</td>
<td>American Airline, British Airways, Cathay Pacific, Qantas, Finnair, Iberia</td>
</tr>
<tr>
<td>SkyTeam</td>
<td>Sep 1999</td>
<td>Aeromexico, Air France, Korean Airlines, Delta Airlines, CSA Czech, Thai Airlines, United</td>
</tr>
<tr>
<td>Star Alliance</td>
<td>May 1997</td>
<td>Air Canada, Lufthansa, SAS, International Airlines, Varig</td>
</tr>
<tr>
<td>Wings</td>
<td>1999</td>
<td>KLM, Northwest</td>
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Table 3.7. Airline Groups 2002 (Actual groups formed)

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</thead>
<tbody>
<tr>
<td>Qualifyer</td>
<td>-</td>
<td>0.59%</td>
<td>4.49%</td>
<td>2.34%</td>
<td>3.14%</td>
<td>-</td>
</tr>
<tr>
<td>Star Alliance</td>
<td>97.92%</td>
<td>98.14%</td>
<td>99.17%</td>
<td>99.34%</td>
<td>98.62%</td>
<td>97.45%</td>
</tr>
<tr>
<td>Atlantic Excellence</td>
<td>69.67%</td>
<td>65.39%</td>
<td>59.20%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SkyTeam</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>97.35%</td>
<td>89.56%</td>
<td>79.64%</td>
</tr>
</tbody>
</table>

Table 3.8. Percentile of Actual Group Fitness Scores
Figure 3.1. Frequency of Factor Mentions
Figure 3.2. Qualifyer Group Fitness

Figure 3.3. Qualifyer Firm Utility
Figure 3.4. Star Alliance Group Fitness

Figure 3.5. Star Alliance Firm Utility
Figure 3.6. Atlantic Excellence Group Fitness

Figure 3.7. Atlantic Excellence Firm Utility
Figure 3.8. SkyTeam Group Fitness

Figure 3.9. SkyTeam Firm Utility
References


APPENDIX A

Appendix to Chapter 1

A.1. Proof of Lemma 1 and the Derivations for (1.25) and (1.26)

First, we derive (1.25) from (1.13). Substituting (1.20) into (1.13), we have:

\[
\frac{q_{12}}{q_{11}} = G(n_1, \theta), \quad \frac{q_{21}}{q_{22}} = G(n_2, \frac{1}{\theta}),
\]

where the function \(G\) is defined by (1.27). Substituting \((q_{11}, q_{12})\) and \((q_{21}, q_{22})\) from (1.21) and (1.22), the first equation above yields (1.25) and the second equation yields:

\[
\frac{1 - s + s\theta^{1/\sigma}}{s + (1 - s)\theta^{1/\sigma}} = \left[ \frac{\mu(n_2)}{\mu(1 - n_2)} \right]^{1 - \frac{1}{\psi}} \left[ \left( \frac{\gamma_2}{\sigma} - 1 \right) / \mu(1 - n_2) + 1 \right]^{\frac{1}{\psi}}. \tag{A.1}
\]

Second, we show \(n_2 = 1 - n_1\). Denote the right-hand side of (1.25) temporarily as \(RHS(n_1)\). Then, the right-hand side of (A.1) is equal to \(RHS(1 - n_2)\). Since the left-hand sides of (1.25) and (A.1) are identical, then \(RHS(1 - n_2) = RHS(n_1)\). To show \(1 - n_2 = n_1\), it suffices to show that \(RHS(n)\) is increasing in \(n\). Because \(\psi < 1\), the function \(\mu(n)\) is decreasing. Then, \(RHS(n)\) is indeed increasing under the maintained assumption \(\gamma_1, \gamma_2 > \beta\).

Third, we derive (1.28) and (1.26). To do so, first use (1.4) – (1.7) to eliminate the \(x^\prime\)es and express the \(\omega^\prime\)es as functions of \((q, f)\). Substituting the resulting \(\omega^\prime\)es, we rewrite (1.18) as:

\[
e = \frac{\gamma_1 n_1}{\gamma_2 (1 - n_1)} \left( \frac{q_{11}}{q_{12}} \right)^{\sigma} \frac{m_{12} + cf_{11}}{m_{11} - f_{11}}.
\]
Similarly, we rewrite (1.19) as:

\[
\theta = \frac{\omega_{11}}{\omega_{21}} = \left( \frac{q_{11}}{q_{21}} \right)^\sigma \frac{1 - m_{11} + f_{11}}{m_{11} - f_{11}},
\]

\[
\theta = \frac{\omega_{12}}{\omega_{22}} = \left( \frac{q_{12}}{q_{22}} \right)^\sigma \frac{1 - m_{12} - e f_{11}}{m_{12} + e f_{11}}.
\]

Here, we have used the result \( n_2 = 1 - n_1 \) and the market clearing conditions: \( m_{21} = 1 - m_{11}, \)
\( m_{22} = 1 - m_{12}, \) and \( f_{21} = f_{11}. \) From the last two equations above, we can solve:

\[
m_{11} = f_{11} + \left[ 1 + \theta \left( \frac{q_{21}}{q_{11}} \right)^\sigma \right]^{-1}, \quad m_{12} = -e f_{11} + \left[ 1 + \theta \left( \frac{q_{22}}{q_{12}} \right)^\sigma \right]^{-1}.
\]

Substituting these solutions into the above expression for \( e, \) and using (1.21) and (1.22) to compute the ratios of the \( q \)'s, we obtain (1.28). To obtain (1.26), use the market clearing conditions in the laws of motion of money holdings, (1.8) and (1.9). We have:

\[
\gamma_1 f_{11} = (1 - m_{11} + f_{11}) \left[ \gamma_1 - 1 + \mu(n_1) \right] - \mu(n_1)(1 - s),
\]

\[
\gamma_2 e f_{11} = (m_{12} + e f_{11}) \left[ \gamma_2 - 1 + \mu(1 - n_1) \right] - \mu(1 - n_1)(1 - s).
\]

Again, we used the result \( n_2 = 1 - n_1 \). Dividing the second equation by the first, substituting \((m_{11}, m_{12})\) above, and substituting \( e \) from (1.28), we obtain (1.26).

Fourth, we examine the case \( \gamma_1 = \gamma_2 = \gamma \). In this case, it is easy to verify that (1.25) and (1.26) are satisfied by \( (n_1, \theta) = (1/2, 1) \). Then, (1.20), (1.21) and (1.22) imply \( q_{ij} = q_{ij}^f = Q(\gamma) \) for all \( i, j = 1, 2, \) where \( Q(\gamma) \) is defined by (1.23). Form (1.11) and (1.12), we can solve for the \( \lambda \)'s, the Lagrangian multipliers of the money constraints in trade, and verify that all \( \lambda \)'s are positive if and only if \( Q(\gamma) < (A/\sigma)^{1/(\sigma - 1)} \). This condition is equivalent to \( \gamma > \beta \). More generally, if \( \gamma_1 \) and \( \gamma_2 \) are closely to each other, then all the \( \lambda \)'s are positive if \( \gamma_1, \gamma_2 > \beta \).
Fifth, we show that the conditions $\gamma_1 > \beta$ and $\gamma_2 > \beta$ are necessary for the money constraints to bind in the general case. Consider the trades which involve the buyers from country 1. Solve the $\lambda$'es from (1.11) and (1.12), and then use (1.19) and (1.20) to substitute $\omega_{2k}$ and $q_{1k}^f$. It can then be verified that $\lambda_{11} > 0$ and $\lambda_{11}^f > 0$ if and only if

$$q_{11} < \left( \frac{A}{\sigma} \right)^{\frac{1}{\sigma-1}} \max \left\{ 1, \theta^{-\frac{1}{\sigma}} \right\}.$$  

Using (1.21), we rewrite this condition as:

$$1 + \frac{\gamma_1}{\mu(n_1)} > \max \left\{ s + (1 - s)\theta^{-1/\sigma}, 1 - s + s\theta^{1/\sigma} \right\}.$$  

Because the right-hand side of this inequality is at least 1, a necessary condition for the inequality to hold is $\gamma_1 > \beta$. Similarly, the requirements $\lambda_{12} > 0$ and $\lambda_{12}^f > 0$ lead to $\gamma_2 > \beta$.

Finally, note the use of the assumption $\psi < 1$ in the above proof. If $\psi = 1$, then $\mu(n) = L_0$ for all $n$ and so $RHS(n)$ is independent of $n$. In this case, the equation $RHS(1-n_2) = RHS(n_1)$ holds for all $(n_1, n_2)$, and hence it does not lead to the relation $n_2 = 1 - n_1$ or any other restriction on $n_1$ and $n_2$. Equation (1.25) still holds and it determines $\theta$ uniquely. However, (1.26) does not hold, because its derivation required the use of the relation $n_2 = 1 - n_1$. Instead, the equation will involve $n_2$ as well as $n_1$ and $\theta$. In this case, $n_1$ and $n_2$ are not determinate in equilibrium. Neither is the exchange rate, $e$. QED

### A.2. Proofs for Section 1.4

We first prove Proposition 2. To establish existence and uniqueness of the equilibrium, we need to show that there is a unique solution for $n_1$ to (1.29). Once this is done, the $q$'s are given by (1.20), (1.21), (1.22) and (1.24). $\theta$ is given by (1.26) and $e$ by (1.28). Moreover, from the laws
of motion of money holdings, we can solve country 1’s holdings as follows:

\[ m_{11} = \frac{1}{\gamma_1} \left[ \gamma_1 - 1 + s \mu(n_1) + \frac{1 - \mu(n_1)}{1 + \theta \sigma} \right], \]

\[ m_{12} = \frac{1}{\gamma_2} \left[ (1 - s) \mu(1 - n_1) + \frac{1 - \mu(1 - n_1)}{1 + \theta \sigma} \right]. \]

Here, \( s = 1/2 \). The holdings by country 2 are \( m_{2k} = 1 - m_{1k} \) for \( k = 1, 2 \). The amounts of currency exchange are given by \( f_{22} = e f_{11} \) and \( f_{11} = m_{11} - \left( 1 + \theta \sigma \right)^{-1} \). These quantities and prices constitute a unique equilibrium.

To show that (1.29) has a unique solution, denote its left-hand side as \( LHS(n_1) \) and the right-hand side as \( RHS(n_1) \). Note that \( LHS'(n) \leq 0 \) and \( RHS'(n) \geq 0 \) for all \( n \). Moreover, under the maintained assumption \( L_0 < 2^{\psi - 1} \), the region of \( n \) where \( LHS'(n) = 0 \) does not overlap with the region where \( RHS'(n) = 0 \). Thus, the solution to \( LHS(n) = RHS(n) \) is unique if it exists. For existence, it suffices to show that \( LHS(0) > RHS(0) \) and \( LHS(1) < RHS(1) \). These conditions are equivalent to the requirement that the parameter values lie in the set \( D \) specified in the proposition.

For given \( n_1 \), \( RHS(n_1) \) is an increasing function of \( \gamma_1 \) and decreasing function of \( \gamma_2 \), while \( LHS(n_1) \) does not depend on \( (\gamma_1, \gamma_2) \). Thus, it is easy to show the solution for \( n_1 \) to (1.29), denoted \( N(\gamma_1, \gamma_2) \), has the properties \( N_1(\gamma_1, \gamma_2) < 0 \) and \( N_2(\gamma_1, \gamma_2) > 0 \). Moreover, if \( \gamma_1 = \gamma_2 = \gamma \), then \( n_1 = 1/2 \) solves (1.29). This completes the proof of Proposition 2.

To prove Lemma 3, define \( e(\gamma_1, \gamma_2) \) by (1.30). Setting \( s = 1/2 \) in (1.26), we have

\[ \theta^{\sigma \tau} = 1 + \frac{\gamma_2 (1 - n_1) [G(n_1, \theta)]^\sigma}{(\gamma_1 - 1) n_1 + \frac{1}{2} L(n_1) + \frac{1}{2} L(1 - n_1) [G(n_1, \theta)]^\sigma} \Delta(\gamma_1, \gamma_2), \]
where

\[ \Delta(\gamma_1, \gamma_2) = 1 - \frac{1}{\gamma_2} - \left(1 - \frac{1}{\gamma_1}\right) e(\gamma_1, \gamma_2). \]

Then, it is clear that \( \theta > 1 \) if and only if \( \Delta > 0 \).

For \( \Delta \), we examine \( e \) first. Using (1.29) to substitute for \( \mu(n_1)/\mu(1-n_1) \), we rewrite (1.30) as:

\[ e(\gamma_1, \gamma_2) = \frac{n}{1-n} \frac{1 - [1 - \mu(1-n)]}{\gamma_2}, \]

where \( n = N(\gamma_1, \gamma_2) \) is the solution to (1.29) for \( n_1 \). Since \( \mu(n) < 1 \) for all \( n \), the right-hand side of the above expression for \( e \) is an increasing function of \( n \) and \( \gamma_2 \), and a decreasing function of \( \gamma_1 \). Since \( n = N(\gamma_1, \gamma_2) \) decreases in \( \gamma_1 \) and increases in \( \gamma_2 \), then \( e(\gamma_1, \gamma_2) \) decreases in \( \gamma_1 \) and increases in \( \gamma_2 \). Because \( e(\gamma, \gamma) = 1 \) for all \( \gamma > \beta \), then \( e > 1 \) if and only if \( \gamma_1 < \gamma_2 \).

Now, return to \( \Delta(\gamma_1, \gamma_2) \). Clearly, \( \Delta(\gamma, \gamma) = 0 \) for all \( \gamma > \beta \). Suppose \( \gamma_1 < \gamma_2 \). Then, \( e(\gamma_1, \gamma_2) > 1 \). We derive the conditions (in the lemma) for \( \Delta > 0 \), and hence for \( \theta > 1 \). If \( \gamma_2 \leq 1 \), then for all \( \beta < \gamma_1 < \gamma_2 \), we have:

\[ \Delta > \left(1 - \frac{1}{\gamma_2}\right) [1 - e(\gamma_1, \gamma_2)] \geq 0. \]

The first inequality comes from \( \gamma_1 < \gamma_2 \) and the second inequality comes from \( \gamma_2 \leq 1 \) and \( e(\gamma_1, \gamma_2) > 1 \). If \( \gamma_2 > 1 \), then for all \( \beta < \gamma_1 \leq 1 \), we have:

\[ \Delta > \left(1 - \frac{1}{\gamma_1}\right) [1 - e(\gamma_1, \gamma_2)] \geq 0. \]
For $\gamma_2 > 1$, let $\gamma_0$ be the minimum value of $\gamma (> \beta)$ that satisfies $\Delta(\gamma, \gamma_2) = 0$. Because $\Delta(\gamma_2, \gamma_2) = 0$, then $\gamma_0$ exists and $\gamma_0 \leq \gamma_2$. Also, $\gamma_0 > 1$, because $\Delta(1, \gamma_2) > 0$ for $\gamma_2 > 1$. Thus, if $\gamma_2 > 1$, then $\Delta > 0$ for all $\gamma_1 < \gamma_0$. This completes the proof of Lemma 3. QED

A.3. Proofs for Section 1.5

We prove Proposition 5. Fix $\gamma_2 = \beta$ and examine the effects of $\gamma_1$ near $\gamma_1 = \beta$. Rewrite country 1’s welfare level as:

$$W_1 = T_{11}^1 [Aq_{11} - (q_{11})^\sigma] + T_{12}^2 [Aq_{12} - (q_{12})^\sigma] + [T_{12}^1 Aq_{11}^{\sigma} - T_{21}^1 (q_{21}^{f})^\sigma] + [T_{12}^2 Aq_{12}^{\sigma} - T_{21}^2 (q_{22}^{f})^\sigma].$$

The first two terms are net surpluses that the household’s agents obtain in trades with domestic agents and the last two terms are net surpluses in trades with foreign agents. Differentiate the above welfare level with respect to $\gamma_1$ and evaluate the derivative at $\gamma_1 = \gamma_2 = \beta$. Utilizing (1.20), (1.21), (1.22), (1.24) and the result $n_2 = 1 - n_1$, we get:

$$q^{-\sigma} \frac{dW_1}{d\gamma_1} \bigg|_{\gamma_1=\gamma_2=\beta} = -\mu \left( \frac{1}{2} \right) \left[ \frac{1}{2(\sigma - 1)} + 1 \right] \frac{d\theta}{d\gamma_1}$$ (A.2)

Here $\gamma_2$ is taken as given and the derivatives are evaluated at $\gamma_1 = \gamma_2 = \beta$. Differentiating (1.29) with respect to $\gamma_1$ and evaluating the derivative at $\gamma_1 = \gamma_2 = \beta$, we get:

$$\frac{dn_1}{d\gamma_1} = - \left[ \beta (1 - \psi) (\sigma - 1) \mu \left( \frac{1}{2} \right) \right]^{-1} < 0.$$ (A.3)

Using this result and differentiating (1.26), we have:

$$\frac{d\theta}{d\gamma_1} = - \left( 1 - \frac{1}{\sigma} \right) \frac{(1 - \beta) [1 + 2\sigma (1 - \psi)] + 2\beta (1 - \psi) (\sigma - 1) \mu \left( \frac{1}{2} \right)}{\beta \mu \left( \frac{1}{2} \right) (1 - \psi) (\sigma - 1) [\beta - 1 + \mu \left( \frac{1}{2} \right)]} < 0.$$ (A.4)
Then, (A.2) implies \( \frac{dW_1}{d\gamma_1} > 0 \) at \( \gamma_1 = \gamma_2 = \beta \). Similarly, we can show that \( \frac{dW_2}{d\gamma_1} < 0 \) around \( \gamma_1 = \gamma_2 = \beta \). This completes the proof of Proposition 5.

Now we prove Proposition 6. A Nash equilibrium is a pair, \((\gamma_1, \gamma_2)\), which solves the following equations that characterize the two countries’ best responses to each other:

\[
\frac{dW_1}{d\gamma_1} = 0, \quad \frac{dW_2}{d\gamma_2} = 0.
\]

We restrict our search for the solutions to the ones that have \( \gamma_1 = \gamma_2 \). Calculating the derivative \( \frac{dW_1}{d\gamma_1} \) and evaluating it at \( \gamma_1 = \gamma_2 = \gamma \), we have:

\[
\frac{1}{T_1} \frac{dW_1}{d\gamma_1}|_{\gamma_1=\gamma_2=\gamma} = F(\gamma),
\]

where

\[
F(\gamma) = -2 \left[ q^\sigma + \frac{1}{\sigma - 1} Aq \right] \theta' - \frac{2(A - \sigma q^{\sigma-1})}{\beta \mu \left( \frac{1}{2} \right)} q^\sigma,
\]

\[
\theta' = -\left( \frac{\sigma - 1}{\sigma} \right) \frac{4(\gamma - 1) [\sigma (1 - \psi) + 1] n' + 1}{\gamma - 1 + \mu \left( \frac{1}{2} \right)},
\]

\[
n' = -\frac{1}{4(1 - \psi)} \left[ \beta (\sigma - 1) \mu \left( \frac{1}{2} \right) + (\gamma - \beta) \sigma \right]^{-1}.
\]

Thus, \( \frac{dW_1}{d\gamma_1}|_{\gamma_1=\gamma_2=\gamma} = 0 \) if and only if \( F(\gamma) = 0 \). Similarly, \( \frac{dW_2}{d\gamma_2}|_{\gamma_1=\gamma_2=\gamma} = 0 \) if and only if \( F(\gamma) = 0 \). Therefore, a solution to \( F(\gamma) = 0 \) constitutes a Nash equilibrium.

The function \( F(\gamma) \) is continuous for all \( \gamma \geq \beta \), provided that \( \beta > 1 - \mu \left( \frac{1}{2} \right) \). Proposition 5 shows that \( F(\beta) > 0 \). Let \( \gamma_A \) be such that \( \theta' = 0 \). Clearly, \( F(\gamma_A) < 0 \). Also, because \( n' < 0 \), it is easy to verify that \( \gamma_A > 1 > \beta \). Thus, there exists \( \gamma^* \in (\beta, \gamma_A) \) such that \( F(\gamma^*) = 0 \). This level \( \gamma^* \) is the common rate of growth of the two currencies in a Nash equilibrium. QED
A.4. Supplementary Appendix

See http://individual.utoronto.ca/liuqing/mzsappendix1.pdf
APPENDIX B

Appendix to Chapter 2

B.1. Log-linearized System

Here we derive the results that obtained in Section 2.3 of the paper. The equilibrium is characterized by the equations described in section 2.2.5. To solve this system, we take a linear approximation around the initial symmetric steady state, and get the following 32 linear equations:

\[
\hat{C}_i = \phi_1 \hat{c}_{ih} + (1 - \phi_1) \hat{c}_{if}; \quad \phi_1 = \rho^{1/\eta} \left( \frac{\bar{c}_h}{\bar{c}} \right)^{(\eta-1)/\eta};
\]  

(B.1)

\[
2\hat{c}_{ih} = \hat{q}_{ii} + \hat{q}_{iv};
\]  

(B.2)

\[
2\hat{c}_{if} = \hat{q}_{ii}^f + \hat{q}_{iv}^f;
\]  

(B.3)

\[
\phi_2 \hat{\lambda}_{ii} = \left( \frac{1}{\eta} - \epsilon \right) \hat{C}_i - \frac{1}{\eta} \hat{c}_{ih} + \phi_2 \hat{\omega}_{ii} - \hat{a}_i - (\sigma - 1) \hat{q}_{ii}; \quad \phi_2 = \frac{\lambda}{(\lambda + \Omega)};
\]  

(B.4)

\[
\phi_3 \hat{\lambda}_{ii} = \left( \frac{1}{\eta} - \epsilon \right) \hat{C}_i - \frac{1}{\eta} \hat{c}_{if} + \phi_3 \hat{\omega}_{iv} - \hat{a}_i - (\sigma - 1) \hat{q}_{ii}^f; \quad \phi_3 = \frac{\lambda^f}{(\lambda^f + \Omega)};
\]  

(B.5)

\[
\phi_2 \hat{\lambda}_{iv} = \left( \frac{1}{\eta} - \epsilon \right) \hat{C}_i - \frac{1}{\eta} \hat{c}_{ih} + \phi_2 \hat{\omega}_{iv} - \hat{a}_i - (\sigma - 1) \hat{q}_{iv};
\]  

(B.6)

\[
\phi_3 \hat{\lambda}_{iv} = \left( \frac{1}{\eta} - \epsilon \right) \hat{C}_i - \frac{1}{\eta} \hat{c}_{if} + \phi_3 \hat{\omega}_{iv} - \hat{a}_i - (\sigma - 1) \hat{q}_{iv}^f;
\]  

(B.7)

\[
\left( 2 - \frac{1}{\gamma} \right) \hat{m}_{11} - \left( 1 - \frac{1}{\gamma} \right) \hat{f}_{11} = \sigma \hat{q}_{11} + \hat{a}_1 + \hat{n}_1 - \hat{\omega}_{11};
\]  

(B.8)

\[
\left( 2 - \frac{1}{\gamma} \right) \hat{m}_{11} - \left( 1 - \frac{1}{\gamma} \right) \hat{f}_{11} = \sigma \hat{q}_{11}^f + \hat{a}_2 + \hat{n}_1 - \hat{\omega}_{21};
\]  

(B.9)

\[
\frac{1}{\gamma} \hat{m}_{12} + \left( 1 - \frac{1}{\gamma} \right) \left( \hat{f}_{11} + \hat{e} \right) = \sigma \hat{q}_{12} + \hat{a}_1 - \hat{\omega}_{12} - \hat{n}_1;
\]  

(B.10)
\[
\begin{align*}
\frac{1}{\gamma} \hat{m}_{12} + \left(1 - \frac{1}{\gamma}\right) \left(\hat{f}_{11} + \hat{e}\right) &= \sigma \hat{q}_{12}^f + \hat{a}_2 - \hat{\omega}_{22} - \hat{n}_1; & (B.11) \\
-\frac{1}{\gamma} \hat{m}_{12} - \left(1 - \frac{1}{\gamma}\right) \left(\hat{f}_{11} + \hat{e}\right) &= \sigma \hat{q}_{22} + \hat{a}_2 + \hat{n}_2 - \hat{\omega}_{22}; & (B.12) \\
-\frac{1}{\gamma} \hat{m}_{12} - \left(1 - \frac{1}{\gamma}\right) \left(\hat{f}_{11} + \hat{e}\right) &= \sigma \hat{q}_{22}^f + \hat{a}_1 + \hat{n}_2 - \hat{\omega}_{12}; & (B.13) \\
\left(1 - \frac{1}{\gamma}\right) \hat{f}_{11} - \left(1 - \frac{1}{\gamma}\right) \hat{m}_{11} &= \sigma \hat{q}_{21} + \hat{a}_2 - \hat{n}_2 - \hat{\omega}_{21}; & (B.14) \\
\left(1 - \frac{1}{\gamma}\right) \hat{f}_{11} - \left(1 - \frac{1}{\gamma}\right) \hat{m}_{11} &= \sigma \hat{q}_{21}^f + \hat{a}_1 - \hat{n}_2 - \hat{\omega}_{11}; & (B.15) \\
2 (1 + \phi_4) (1 - \psi) \hat{n}_1 &= \hat{q}_{11} - \hat{q}_{12} + \phi_4 \left(\hat{q}_{11}^f - \hat{q}_{12}^f\right); & (B.16) \\
2 (1 + \phi_4) (1 - \psi) \hat{n}_2 &= \hat{q}_{22} - \hat{q}_{21} + \phi_4 \left(\hat{q}_{22}^f - \hat{q}_{21}^f\right); & (B.17) \\
\frac{\tau}{\beta} \hat{e} &= (\hat{\omega}_{11} - \hat{\omega}_{12}) + \mu_0 \frac{\phi_3}{1 - \phi_2} \left(\hat{\lambda}_{11} - \hat{\lambda}_{12}\right) + \alpha \mu_0 \frac{\phi_3}{1 - \phi_3} \left(\hat{\lambda}_{11}^f - \hat{\lambda}_{12}^f\right) \\
&- 2 \left(\frac{\tau}{\beta} - 1\right) (1 - \psi) \hat{n}_1; & (B.18) \\
\frac{\tau}{\beta} \hat{e} &= (\hat{\omega}_{21} - \hat{\omega}_{22}) + \mu_0 \frac{\phi_3}{1 - \phi_2} \left(\hat{\lambda}_{21} - \hat{\lambda}_{22}\right) + \alpha \mu_0 \frac{\phi_3}{1 - \phi_3} \left(\hat{\lambda}_{21}^f - \hat{\lambda}_{22}^f\right) \\
&+ 2 \left(\frac{\tau}{\beta} - 1\right) (1 - \psi) \hat{n}_2; & (B.19) \\
\frac{\tau}{\beta} \hat{\omega}_{11} &= E \left[ -\frac{\tau}{\beta} \hat{\gamma}_{1,1} + \hat{\omega}_{11,11} + \mu_0 \frac{\phi_3}{1 - \phi_2} \hat{\lambda}_{11,11} + \alpha \mu_0 \frac{\phi_3}{1 - \phi_3} \hat{\lambda}_{11,11}^f \right] - \left(\frac{\tau}{\beta} - 1\right) (1 - \psi) \hat{n}_{1,11}; & (B.20) \\
\frac{\tau}{\beta} \hat{\omega}_{12} &= E \left[ -\frac{\tau}{\beta} \hat{\gamma}_{2,1} + \hat{\omega}_{12,11} + \mu_0 \frac{\phi_3}{1 - \phi_2} \hat{\lambda}_{12,11} + \alpha \mu_0 \frac{\phi_3}{1 - \phi_3} \hat{\lambda}_{12,11}^f \right] + \left(\frac{\tau}{\beta} - 1\right) (1 - \psi) \hat{n}_{1,11}; & (B.21) \\
\frac{\tau}{\beta} \hat{\omega}_{22} &= E \left[ -\frac{\tau}{\beta} \hat{\gamma}_{2,2} + \hat{\omega}_{22,11} + \mu_0 \frac{\phi_3}{1 - \phi_2} \hat{\lambda}_{22,11} + \alpha \mu_0 \frac{\phi_3}{1 - \phi_3} \hat{\lambda}_{22,11}^f \right] - \left(\frac{\tau}{\beta} - 1\right) (1 - \psi) \hat{n}_{2,11}; & (B.22)
\end{align*}
\]
\[
\frac{T}{\beta} \hat{\omega}_{21} = E \left[ -\frac{T}{\beta} \hat{\gamma}_{1,1+1} + \hat{\omega}_{21,1+1} + \mu_0 \frac{\phi_2}{1-\phi_2} \hat{\lambda}_{21,1+1} + \alpha \mu_0 \frac{\phi_3}{1-\phi_3} \hat{\lambda}_{21,1+1} \right] + \left( \frac{T}{\beta} - 1 \right) (1 - \psi) \hat{n}_{2,1+1}; \quad (B.23)
\]

\[
(2\gamma - 1) \hat{m}_{11,1+1} = \alpha \mu_0 \left[ \sigma \left( \hat{q}_{21}^f - \hat{q}_{11}^f \right) - \psi (\hat{n}_1 + \hat{n}_2) + \hat{a}_1 + \hat{a}_2 + \hat{\omega}_{21} - \hat{\omega}_{11} \right] + \left( 2 - \frac{1}{\gamma} \right) \hat{m}_{11} - \left( 1 - \frac{1}{\gamma} \right) \hat{f}_{11} + \hat{\gamma}_{1,1+1}; \quad (B.24)
\]

\[
\hat{m}_{12,1+1} = \alpha \mu_0 \left[ \sigma \left( \hat{q}_{22}^f - \hat{q}_{12}^f \right) + \psi (\hat{n}_1 + \hat{n}_2) + \hat{a}_1 - \hat{a}_2 + \hat{\omega}_{22} - \hat{\omega}_{12} \right] + \frac{1}{\gamma} \hat{m}_{12} + \left( 1 - \frac{1}{\gamma} \right) \left( \hat{f}_{11} + \hat{\epsilon} \right) - \hat{\gamma}_{2,1+1}; \quad (B.25)
\]

Combining equation (B.18), (B.20) and (B.21), we get

\[
\hat{\omega}_{11} - \hat{\omega}_{12} = E \left[ \hat{e}_{1+1} - \hat{\gamma}_{1,1+1} + \hat{\gamma}_{2,1+1} \right];
\]

Similarly, \( \hat{\omega}_{21} - \hat{\omega}_{22} = E \left[ \hat{e}_{1+1} - \hat{\gamma}_{1,1+1} + \hat{\gamma}_{2,1+1} \right] \) can be derived from (B.19), (B.22) and (B.23). Thus, we obtain the relationship between valuations of the two currencies (2.20). The quantity differentials (2.21) and (2.22) can be easily derived from equations (B.8)-(B.15). Using these quantity differentials and equation (B.16) and (B.17), we can immediately get proposition 11. Note that variables \( \left( \hat{c}_i, \hat{c}_{ih}, \hat{c}_{if}, \hat{\lambda}_{ii}, \hat{\lambda}_{i}^f \right) \) can be written as functions of \( \left( \hat{q}_{ik}, \hat{q}_{ik}^f, \hat{\omega}_{ik}, \hat{a}_k \right) \) by (B.1)-(B.7). Substituting \( \hat{\lambda}_{ik}, \hat{\lambda}_{ik}^f \) and \( \hat{n}_1 \) into (B.18), we can show that the nominal exchange rate \( \hat{e} \) is proportional to \( \theta \), i.e. (2.26). \( \hat{f}_{11} \) can be solved by (B.8) and (B.10), which leads to (2.28). Since both \( \hat{e} \) and \( \hat{n}_1 \) are the function of \( \theta \), it’s not hard to show \( \hat{f}_{11} \) is a function of \( \theta \), \( \hat{m}_{11} \) and \( \hat{m}_{12} \). Thus, money-goods constraints (B.8)-(B.15) imply that all the \( \hat{q}_{ik} \) and \( \hat{q}_{ik}^f \) can be written as functions of \( \left( \hat{m}_{11}, \hat{m}_{12}, \hat{\omega}_{11}, \hat{\omega}_{12}, \hat{\omega}_{22}, \hat{a}_1, \hat{a}_2 \right) \).
Above results can be used further to simplify the two laws of motion of money holdings, (B.24) and (B.25). Substituting $q_{ik}$ and $\hat{n}_t$ into the two equations, we obtain,

$$\gamma \hat{m}_{11,+1} = (1 - 2\alpha \mu_0) \hat{m}_{11} - (1 - 2\alpha \mu_0) \left( \frac{(\gamma - 1)}{(2\gamma - 1)} \hat{f}_{11} + \frac{\gamma}{(2\gamma - 1)} \hat{\gamma}_{1,+1} \right); \quad (B.26)$$

$$\gamma \hat{m}_{12,+1} = (1 - 2\alpha \mu_0) \hat{m}_{12} + (1 - 2\alpha \mu_0) \left( \hat{f}_{11} + \hat{e} \right) - \gamma \hat{\gamma}_{2,+1}; \quad (B.27)$$

Note that all the variables $\{\hat{C}_i, \hat{c}_{ih}, \hat{C}_f, \hat{\lambda}_{ii}, \hat{\lambda}_{if}, \hat{q}_{ik}, \hat{q}_{ik}, \hat{n}_t, \hat{e}\}$ can be expressed as a function of $(\hat{m}_{11}, \hat{m}_{12}, \hat{\omega}_{11}, \hat{\omega}_{12}, \hat{\omega}_{22})$ and exogenous state variables. Therefore, the above two laws of motion of money holdings and three envelop conditions (B.20)-(B.22) constitute a much simpler dynamic system, which only involves two predetermined variables, $\hat{m}_{11}$ and $\hat{m}_{12}$, three jump variables, $\hat{\omega}_{11}$, $\hat{\omega}_{12}$, and $\hat{\omega}_{22}$, and exogenous state variables. Once we solve this dynamic system, all the other variables can then be derived accordingly.

### B.2. Solution Technique

We can apply Banchard and Khan’s method to solve the dynamic system. Let $x = [\hat{m}_{11}, \hat{m}_{12}, \hat{\omega}_{11}, \hat{\omega}_{12}, \hat{\omega}_{22}]^T$, where $\hat{m}_{11}$ and $\hat{m}_{12}$ are predetermined variables and the other three variables are jump variables. Let $z = [\hat{\gamma}_{1}, \hat{\gamma}_{2}, \hat{a}_1, \hat{a}_2]^T$ denote the vector of exogenous variables. Therefore, the dynamic system can be rewritten as,

$$E_t [x_{t+1}] = A \cdot x_t + R \cdot E_t [z_{t+1}];$$

First, we transform $A$ into the Jordan canonical form,

$$A = C^{-1}JC;$$

Note that some of the symbols used here do not necessarily have the same meanings as the ones used in the text.
where the diagonal elements of $J$, which are the eigenvalues of $A$, are ordered by increasing absolute value.

For the system to have a stable solution, there should be exactly two eigenvalues of $A$ outside the unit circle. The calibrated parameter values in our model indeed generate such eigenvalues.

Decompose $J$ further as,

$$
J = \begin{bmatrix}
J_1 & 0 \\
(2 \times 2) & J_2 \\
0 & (3 \times 3)
\end{bmatrix}
$$

where all eigenvalues of $J_1$ are on or inside the unit circle, all eigenvalues of $J_2$ are outside the unit circle.

Matrix $C$, $C^{-1}$, and $R$ are decomposed accordingly:

$$
C = \begin{bmatrix}
C_{11} & C_{12} \\
(2 \times 2) & (2 \times 3) \\
C_{21} & C_{22} \\
(3 \times 2) & (3 \times 3)
\end{bmatrix};
C^{-1} = \begin{bmatrix}
B_{11} & B_{12} \\
(2 \times 2) & (2 \times 3) \\
B_{21} & B_{22} \\
(3 \times 2) & (3 \times 3)
\end{bmatrix};
R = \begin{bmatrix}
R_1 \\
(2 \times 4) \\
R_2 \\
(3 \times 4)
\end{bmatrix};
$$

For given initial value $x_0$, the solution to above system is given by,

$$
x_t (1 : 2) = B_{11}J_1B_{11}^{-1}x_{t-1} (1 : 2) + R_1 E_{t-1}z_t
- (B_{11}J_1C_{12} + B_{12}J_2C_{22}) C_{22}^{-1}\sum_{i=0}^{\infty} J_2^{-i-1} [(C_{21}R_1 + C_{22}R_2) E_{t-1}z_{t+i}] , \text{ for } t > 0;
$$

$$
x_t (3 : 5) = -C_{22}^{-1}C_{21}x_t (1 : 2) - C_{22}^{-1}\sum_{i=0}^{\infty} J_2^{-i-1} [(C_{21}R_1 + C_{22}R_2) E_t z_{t+i+1}] , \text{ for } t \geq 0;
$$
Given the exogenous process of $z_t$, we can calculate the time path of $x_t$ recursively.