A State Variable Approach for the Calculation of Switching Transients on a Power Transmission Line

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Abstract—This paper gives the outline of a new method for the calculation of electromagnetic transients on transmission lines above homogeneous ground. The basic idea is that line transfer functions which normally are calculated in terms of hyperbolic functions of ω are now expressed in terms of $s = \sigma + j\omega$. This is possible due to the existence of analytical expressions of line impedances and admittances in terms of $j\omega$, or s in the new approach. It is then possible to find the poles of the transfer function by the secant method and then the corresponding residues. The partial fraction approximation of the transfer functions is incomplete and a correction may be used to account for the missing poles. The calculations are performed numerically only for samples of s needed for the calculation of poles and residues. The end result is a set of state equations with v and i as inputs/outputs at the switch terminals. Inputs and outputs are interchanged as the switch poles open or close. The integration can be numerical or closed form for sinusoidal inputs (generator voltages) and the state variables carry the past information from one stage of switching to the next. Essential parts of the procedures are illustrated by numerical results.

I. INTRODUCTION

THE SIMULATION of electromagnetic transients in a power system network is of great importance for the design of line and station insulation and selection of equipment. Earlier programs considered the transmission lines lossless and traveling wave methods have been applied [1]. Frequency domain methods [2] have been used in conjunction with the Fourier transform to take into account the frequency dependence of transmission line parameters as quantified originally by Carson [3]. Later an Electro-Magnetic Transients Program (EMTP) has been developed, based on the procedure of [1], and has included frequency dependent modeling of transmission line parameters [4]–[7], while retaining the traveling wave approach as its basic procedure. The main advantage in using traveling waves is that geographically distant nodes are computationally separated at any given instant, due to the travel delay. The computations are at present performed sequentially but the problem is of reduced size and amenable to parallel processing. The disadvantages consist in the difficulty of creating reduced order system equivalents and in the requirement for small integration steps imposed by the discretization needed for traveling wave front representation. A step size of, say, 10^{-5} s leads to lengthy computations,

with possible accumulation of roundoff errors, if the total study time is of, say, 1 s. The transient may have lasted only about 10^{-2} s. Because of this, and for theoretical reasons, there has been some evidence of interest in lumped parameter, rather than traveling wave modeling [8]. Such methods present, however, difficulties because of the need to represent by rational polynomials the transfer functions of transmission lines. Indeed, Carson's corrections [3] require numerical evaluations, and the transfer functions contain square roots and hyperbolic functions of logarithmic expressions and of Carson's corrections, so that a fitting procedure is very difficult.

The basic new idea of this paper consists in using the complex variable $s = \sigma + j\omega$, instead of $j\omega$, for frequency domain modeling. This has become possible due to very good closed form expressions for the line impedance Z in terms of the complex penetration depth

$$p = \frac{1}{\sqrt{j\omega\mu_0\kappa}} \quad (1)$$

where κ is the conductivity of earth. We simply replace $j\omega$ in (1) by s and obtain $Z(s)$. Similarly, we write $Y(s) = G + sC$ for the line admittance. Then all required transfer functions become functions of s .

The complex depth p is used for resistive homogeneous earth return for establishing the position of an image return conductor. This is placed at a (complex) depth deeper by $2p$ than its position with perfectly conducting earth [9]–[11]. The approximation, with Carson's equations as reference, has errors of at most a few percent [12] for all frequencies and conductor positions.

Once transfer functions are expressed in terms of s , they or their combinations can easily be approximated by rational polynomials and realized in the form of state equations with not too many state variables. Their integration can be done using variable step size and, when no nonlinearities are considered in the system, the integration can be performed even in closed form since the inputs are sinusoidal generator voltages.

These ideas and procedures constitute the contributions of this paper to the realization theory of overhead transmission lines. The paper gives the outline of the new method and illustrates its essential features. Many prob-

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Fig. 1. Location of poles. (a) General case; P represents hidden poles. (b) Poles of $H(s) = V_2(s)/E(s)$ of the system of Fig. 4; typically $\sigma \cong -0.03\omega$.

lems of detail are of routine numerical nature and successful procedures will only be indicated without explaining any details of implementation.

II. OUTLINE OF METHOD

In order to minimize analytical calculations, a transfer function $H(s_k)$ has to be calculated for a specific, numerically defined, complex frequency $s_k = \sigma_k + j\omega_k$. For this the following calculations are performed for a given s_k .

a) For all lines Z and Y are calculated; then a modal analysis is performed if the line is not single phase; and, finally, line transfer function matrices (TFM) are obtained.

b) The line transfer functions are assembled in a matrix for the whole system. This could be an admittance matrix for structured calculations in power system networks [8]. The matrix is reduced numerically eliminating nodes of no interest and retaining terminals for inputs, output, and nonlinear attached branches [8]. For simplicity we consider as end result a TFM, $H(s_k)$.

Thus $H(s_k)$ is the result of numerical evaluations. In

Fig. 2. $H(s)$ and $H_R(s)$ evaluated for $s = j\omega$ related to (13) and the circuit of Fig. 4. (a) Magnitude. (b) Real part. (c) Imaginary part. $\circ H(s)$, $\Delta H_R(s)$ } practically superimposed.

simple cases, as the one used for illustration in this paper, $H(s)$ can be obtained in analytical form.

It has been found that the poles of $H(s)$ are close to the ω -axis (Fig. 1). Fig. 2 shows profiles of $H(s)$ taken along the ω -axis and Fig. 3 shows profiles along an axis $s =$

Fig. 3. Profile of $H(s)$ along $s = (-0.03 + j)\omega$. (a) Magnitude. (b) Real part. (c) Imaginary part.

$(-0.03 + j)\omega$ crossing the s -plane more closely to the peaks.

In order to locate the poles, we have developed the following procedure. The ω -axis is first scanned (by computer) and the peaks of the profiles are located. On occasion poles are "hidden" from the ω -axis (see poles P in Fig. 1a): then the profiles of Fig. 3 are more useful. Two values of s are taken, one before and one after the peak, s_1 and s_2 , and $H(s_1)$ and $H(s_2)$ are evaluated. Then the secant method is used to iteratively determine the exact location of the pole. The desired number of poles is thus located.

The residues are calculated by a single evaluation of $H(s)$ very close to the pole.

For the poles at high frequencies, which are not calcu-

lated, a single equivalent pole is evaluated. This yields a correction term for the missing poles.

The end result is a partial fraction approximation for the scalar (or TFM) $H(s)$. It is converted in a straightforward way into state equations

$$\dot{x} = Ax + Bu \quad (2')$$

$$y = Cx + Du. \quad (2'')$$

If care is taken to have $|D| \neq 0$, inputs and outputs in (2), i.e., voltages and current can be interchanged, to follow the switching operations in appropriate sequence. The state variables remain the same. The integration of (2) is performed numerically or in closed form if u is a sine function (vector).

III. CALCULATION OF POLES AND RESIDUES OF TRANSFER FUNCTIONS

A. Approximation to Carson's Formulas

Consider the complex depth of a mirroring surface

$$p = \frac{1}{\sqrt{s\mu_0\kappa}}. \quad (1')$$

Then the mutual impedance between two conductors i and j at height h and h' , respectively, and at horizontal distance D is [11], [12]

$$Z_{ij}(s) = j\omega \frac{\mu_0}{2\pi} \ln \frac{\sqrt{(h + h' + 2p)^2 + D^2}}{\sqrt{(h - h')^2 + D^2}}. \quad (3)$$

As shown in reference [12], expression (3) is remarkably accurate for most practical purposes.

If $i = j$ (3) reduces to the self impedance if the conductor impedance is added. This is also function of s (see discussion to reference [7]).

Thus the line impedance Z is function of s . It is a matrix in the case of a three-phase line. The same applies for Y , except that there $p = 0$.

B. Transfer Functions of Single Phase Lines for $s = s_k$

Consider a line of length l with voltages V_1, V_2 and injected currents I_1, I_2 at the terminals. The following TFM relations are pertinent:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y' & Y'' \\ Y'' & Y' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (4a)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z' & Z'' \\ Z'' & Z' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (4b)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & A \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (A^2 - BC = 1) \quad (4c)$$

$$\begin{bmatrix} V_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} L & M \\ N & L \end{bmatrix} \begin{bmatrix} V_2 \\ I_1 \end{bmatrix} \quad (L^2 - MN = 1). \quad (4d)$$

Denote

$$\gamma = \sqrt{Z(s)Y(s)} \quad (5')$$

and

$$Y_c = \sqrt{\frac{Y(s)}{Z(s)}}. \quad (5'')$$

Then

$$Y' = Y_c \frac{1}{\tanh \gamma l} \quad Y'' = -Y_c \frac{1}{\sinh \gamma l} \quad (6a)$$

$$Z' = Y_c^{-1} \frac{1}{\tanh \gamma l} \quad Z'' = Y_c^{-1} \frac{1}{\sinh \gamma l} \quad (6b)$$

$$A = \cosh \gamma l \quad B = Y_c^{-1} \sinh \gamma l \quad C = Y_c \sinh \gamma l \quad (6c)$$

$$L = \frac{1}{\cosh \gamma l} \quad M = Y_c^{-1} \tanh \gamma l \quad N = -Y_c \tanh \gamma l. \quad (6d)$$

Clearly, the elements of all TFM's are functions of γ and Y_c of (5) and can be easily evaluated for any $s = s_k$. Only two such elements are independent.

We note at this stage that these functions are not rational in s . Hyperbolic functions in x are rational, of infinite order, in x and partial fraction expansion are given for such functions [14] with an infinite number but discrete poles. However, the hyperbolic functions in (6) are functions of γ , not of s . For all frequencies which are not close to zero, γ is almost proportional to s : therefore, for most frequencies of interest, the hyperbolic functions are close to rational. Also, for these frequencies, Y_c is almost constant. Consequently, all functions (6) are close to rational functions in s , with a very large number of poles. This explains why the rational approximations used for these functions are so good.

C. Transfer Functions of Three-Phase Lines for $s = s_k$

For untransposed three-phase lines modal transformation matrices [13], [5] have to be calculated for any $s = s_k$. These are the eigenvector matrices T_V , and T_I of ZY and YZ , respectively, defined by

$$V = T_V V_m \quad (7')$$

$$I = T_I I_m \quad (7'')$$

where V_m and I_m are vectors of modal voltages and currents.

One can define modal impedance and admittance matrices by

$$V_m = Z_m I_m \quad I_m = Y_m V_m \quad (8')$$

or, using (7), by

$$Z_m = T_V^{-1} Z T_I \quad Y_m = T_I^{-1} Y T_V. \quad (8'')$$

The matrices Z_m and Y_m are diagonal and therefore the calculations in (5) and (6) can be performed directly (i.e., independently, for each mode). The result is Y'_m, Y''_m , etc. Then the phase domain TFM results by using the definitions (7). For example, from (4a)

$$I_1 = Y' V_1. \quad (9)$$

Substitution of relations (7) into (9) yields

$$Y' = T_I Y'_m T_V^{-1}. \quad (10')$$

Fig. 4. Energization of open circuited single phase line. T = transformer, B = busbars with capacitance, $l = 150$ km, $R = 6.5 \Omega$, $L = 0.034$ H, $C = 0.005 \mu\text{F}$. (a) General layout. (b) Circuit diagram.

Similarly

$$Y'' = T_I Y''_m T_V^{-1}. \quad (10'')$$

In conclusion, for any $s = s_k$, we first calculate T_V and T_I as the eigenvector-matrices of ZY and YZ , respectively. Then we calculate Z_m and Y_m using (8'') and the desired modal TFM's using (6). Finally we convert these into the phase domain, as shown in (10).

D. Transfer Functions of a Complex System for $s = s_k$

The transfer function of a complex system can be obtained by assembling its components. For a larger system it is perhaps best to use a structured approach by means of a system nodal admittance matrix. Details of such a procedure are described, in another context, in reference [8]. Often, simpler cases are of interest and, then, no general procedure is needed. The example of the energization of the single phase line shown in Fig. 4 will be discussed next.

For the source network we can write, with the switch closed

$$E = (1 + (R + sL)sC)V_1 + (R + sL)I_1 \quad (11)$$

and for the line equations (4c) and (6c) yield

$$V_1 = \cosh \gamma l V_2 \quad (12')$$

$$I_1 = Y_c \sinh \gamma l V_2. \quad (12'')$$

Substitution of (12) into (11) gives the transfer function

$$H(s) = \frac{V_2}{E} = \frac{1}{(1 + (R + sL)sC) \cosh \gamma l + (R + sL) Y_c \sinh \gamma l}. \quad (13)$$

This can easily be evaluated for any $s = s_k$.

E. Calculation of Poles by the Secant method

It is quite clear from the particular cases described that derivatives should not be calculated because of the complexity of the expressions. Therefore, the secant method [15] was used to obtain the poles of the transfer function $H(s)$. For this we define $F(s) = 1/H(s)$ and calculate the roots of $F(s) = 0$. We found that the secant method worked well with complex variables (see also reference [16]). Its

iteration algorithm is

$$s_{k+2} = \frac{F(s_k)s_{k+1} - F(s_{k+1})s_k}{F(s_k) - F(s_{k+1})}. \quad (14)$$

We have used double precision calculation because of the differences in (14) which may produce roundoff errors close to the root. It is interesting to note that poles in the s -plane could be reached with starting points on the ω -axis. But, on occasions, a pole hidden behind other poles closer to the ω -axis (see Fig. 1a) could not be reached even though it could be detected from the ω -axis. As already mentioned, it was then found more reliable to do the searching and starting from a line slightly deviating from the pure imaginary ω -axis (see Fig. 3).

It is also useful to emphasize here the significance of being able to evaluate $H(s)$ for a complex s_k . If this is not possible (as is the case in other approaches) and $H(s)$ can be evaluated only for $s = j\omega_k$, then we have the problem of rational approximation

$$H(s) = C \frac{\pi(s - z_i)}{\pi(s - p_j)} \quad (15)$$

with the zeros z_i and poles p_j to be determined so that (15) is satisfied for a given $H(s)$ at a sufficient number N of imaginary values $s = j\omega_k$. This is equivalent to solving (perhaps in a least squares sense) N nonlinear equations, in z_i and p_j . The problem is difficult and often produces unstable solutions (z_i, p_i in the right half-plane) even if $H(s)$ is stable. With the secant method, using complex s_k , an arbitrary number of poles can be accurately located, usually with very little computer time (of the order of a few seconds).

F. Calculation of Residues

Consider the partial fraction expansion

$$H(s) = \sum_{j=1}^{\infty} \frac{r_j}{s - p_j} \quad (16)$$

equivalent to (15). The evaluation of the poles p_j has been discussed in the previous section.

The residue r_j can be obtained from

$$r_j = \lim_{s \rightarrow p_j} \{H(s)(s - p_j)\}. \quad (17)$$

The usual concern with (17) is that, at the limit, $r_j = \infty \times 0$ so that derivatives are normally used if the denominator of $H(s)$ is not available in factorized form. But since we want to avoid derivatives we evaluate (17) at some value s very close to p_j . It does not matter from which direction we approach p_j but for better accuracy double precision arithmetic is recommended.

The end result of these calculations is a rational approximation $H_R(s)$, in partial fraction expansion, of the original transfer function (or TFM) $H(s)$. Fig. 2 gives an example of $H(s)$ and its rational approximation $H_R(s)$, evaluated for $s = j\omega$, for the transfer function (13) related to the circuit of Fig. 4.

IV. THE RATIONAL TRANSFER FUNCTION

A. Effect of Truncation

The rational transfer function

$$H_R(s) = \sum_{j=1}^n \frac{r_j}{s - p_j} \quad (16')$$

has n poles identical to the first n poles of the original function $H(s)$. Because of the missing poles and also because $H(s)$ is only close to being rational itself, $H_R(j\omega)$ only approximates $H(j\omega)$. The difference is so small that it can not be seen in Fig. 2 where both $H(j\omega)$ and $H_R(j\omega)$ are plotted.

One can note that (16') is strictly proper even if the original function $H(s)$ is proper. The examples of possible transfer functions of equations (6) indicate that they can be of all types: strictly proper, such as $L = 1/\cos \gamma l$; proper, such as $Y' = Y_c / \tanh \gamma l$; nonproper, such as $A = \cosh \gamma l$. Their nature results from their value at $s = \infty$.

If the original function is proper the approximation (16') becomes more and more accurate for smaller values of $s = j\omega$ but, of course, at $s = j\infty$ $H_R \rightarrow 0$ while $H \rightarrow H_\infty \neq 0$.

One will then seek a correction term for (16') to account for the missing terms and to make $H_R(s)$ proper if that was the nature of the original function or to keep $H_R(s)$ strictly proper if that is the case. Nonproper transfer functions have not been considered.

The case of proper transfer functions is very important. If $H(s)$ is proper, then so is $1/H(s)$, the inverse transfer function for which input and output are interchanged [17]. The input may be a voltage, the output a current when the switch pole is closed but, when the switch pole is open the two have to be interchanged. Input/output interchange is only possible if $H(s)$ is proper and, correspondingly, $|D| \neq 0$ in the state equation realization (2). Therefore, if $H(s)$ is proper, it is important to have the approximation $H_R(s)$ of (16') corrected to become also proper.

The aggregate value of the effect of missing poles $p_i = \sigma_i + j\Omega_i$ ($\sigma_i < 0$), evaluated at $s = j\omega$ for $\omega \ll \Omega_i$, is

$$\Delta H(j\omega) = \sum_{i=n+1}^{\infty} \left(\frac{r'_i + jr''_i}{j\omega - (\sigma_i + j\Omega_i)} + \frac{r'_i - jr''_i}{j\omega - (\sigma_i - j\Omega_i)} \right). \quad (18')$$

The linear part of the Taylor series expansion of (18') in terms of $j\omega$ is

$$\Delta H(j\omega) \cong a + jb\omega \quad (18'')$$

where a and b are real constants.

Equation (18'') shows that the error due to the missing terms is approximately constant in the plot of $\text{Re}\{H(j\omega)\}$. This appears clearly in Fig. 5(b) where $H(s) = \tanh \gamma(s)l - 1$. Equation (18'') also shows that the error in the plot of $\text{Im}\{H(j\omega)\}$ is approximately proportional to ω , but this error is small and therefore not perceptible in Fig. 5(c).

The correction term for equation (16') should be adequate if it provides a compensation as shown in equation (18'').

Fig. 5. Profile of $H(s) = \tanh \gamma(s)l - 1$ along $s = j\omega$. (a) Magnitude. (b) Real part. (c) Imaginary part. $\circ H(j\omega)$. $\Delta H_R(j\omega)$.

It is not intended to calculate the constants a and b in (18'') from information about the missing poles. Equation (18'') simply indicates that a correction term should produce a close to linear variation of $\Delta H(j\omega)$. It should, therefore, have two parameters for this purpose, as has (18''), with probably a pole at high frequency.

Fig. 6 gives the result of a correction term being added to the $H_R(j\omega)$ for the example of Fig. 5. This corresponds to a strictly proper transfer function and the details of the calculations are given in the next section.

B. Correction for Missing Poles for Strictly Proper Transfer Functions

We write the correction in the strictly proper form

$$\Delta H(s) = \frac{r_\infty}{s - p_\infty} \quad (19)$$

Fig. 6. Profile of $H(s) = \tanh \gamma(s)l - 1$ along $s = j\omega$ (with correction). (a) Magnitude. (b) Real part. (c) Imaginary part. $\circ H(j\omega)$ }
 $\Delta H_R(j\omega)$ }
 practically superimposed.

with r_∞ and p_∞ assumed to be real.

From (19) and (18'') we obtain for $s = 0$

$$-\frac{r_\infty}{p_\infty} = \Delta H(0) \quad (20')$$

and for $s = j\omega_{\max}$, where $\omega_{\max}$ is the highest frequency of interest, we obtain

$$\frac{r_\infty}{j\omega_{\max} - p_\infty} = \Delta H(j\omega_{\max}). \quad (20'')$$

Fig. 7. Profile of $H(s) = \tanh \gamma(s)l$ along $s = j\omega$. (a) Magnitude. (b) Real part. (c) Imaginary part. $\circ H(j\omega)$, $\Delta H_R(j\omega)$ practically superimposed.

This yields, for $p_\infty \gg \omega_{\max}$ and using equation (20')

$$p_\infty = \omega_{\max} \frac{\Delta H(0)}{\text{Im}\{\Delta H(j\omega_{\max})\}} \quad (21')$$

and

$$r_\infty = -p_\infty \Delta H(0). \quad (21'')$$

Thus the correction (19) is calculated by using only two values of the approximation error $\Delta H(s) = H(s) - H_R(s)$,

$\Delta H(0)$ and $\Delta H(j\omega_{\max})$ evaluated at $s = 0$ and $s = j\omega_{\max}$, respectively.

C. Correction for Missing Poles for Proper Transfer Functions

We write the correction in the form

$$\Delta H(s) = H_\infty \frac{s - z_\infty}{s - p_\infty} \quad (22)$$

so that the corrected transfer function $H_R(s)$ of (16') becomes proper, with the value H_∞ for $s = j\infty$.

For $s = 0$ (22) gives

$$\frac{z_\infty}{p_\infty} = \frac{\Delta H(0)}{H_\infty} \quad (23')$$

and for $s = j\omega_{\max}$ (22) becomes

$$H_\infty \frac{j\omega_{\max} - z_\infty}{j\omega_{\max} - p_\infty} = \Delta H(j\omega_{\max}). \quad (23'')$$

The solution of (23) is

$$p_\infty = \text{Re} \left\{ j\omega_{\max} \frac{\Delta H(j\omega_{\max}) - H_\infty}{\Delta H(j\omega_{\max}) - \Delta H(0)} \right\} \\ \cong \omega_{\max} \frac{\Delta H(0) - H_\infty}{\text{Im}\{\Delta H(j\omega_{\max})\}} \quad (24')$$

$$z_\infty = p_\infty \frac{\Delta H(0)}{H_\infty}. \quad (24'')$$

Fig. 7 shows the fitting for the proper transfer function $H(s) = \tanh \gamma(s)l$. $H_R(s)$ is not corrected and a correction does not appear to be necessary in this particular case.

V. STATE EQUATIONS AND THEIR INTEGRATION

A. Interchanging Input and Output Variables

Assume that in (2) u is the vector of voltages and y the vector of currents. As an example consider the energization of a three-phase line (Fig. 8) by sequentially closing the poles of a breaker. When pole a is closed the inputs are $u_a, y_b = 0, y_c = 0$; when poles a and b are closed the inputs are $u_a, u_b, y_c = 0$; and when finally all three poles are closed the inputs are u_a, u_b, u_c . When, subsequently, all three poles are open the inputs are $y_a = 0, y_b = 0, y_c = 0$. This example shows that some or all of the inputs and outputs have to be interchanged.

The mathematical procedure is shown for a complete interchange (inverse system [17]) in the state equations (2). If D is nonsingular equation (2'') yields

$$u = -D^{-1}Cx + D^{-1}y. \quad (25')$$

Substitution of (25') with (2') yields

$$\dot{x} = (A - BD^{-1}C)x + BD^{-1}y. \quad (25'')$$

Equations (25) can be rewritten as

$$\dot{x} = A'x + B'y \quad (26')$$

$$u = C'x + D'y \quad (26'')$$

Fig. 8. Receiving end voltage transient for the system of Fig. 4.
 ○ Inverse Fourier transform solution } practically superimposed.
 Δ Closed form solution }

where

$$A' = A - BD^{-1}C \quad B' = BD^{-1} \quad C' = -D^{-1}C \quad D' = D^{-1}. \quad (27)$$

It can be seen from (26) that the inversion does not affect the state variables, but all coefficient matrices are changed; the natural modes of the inverse system are changed too because A and A' have different eigenvalues.

B. Initialization for Sinusoidal Variables

A transient will often start from a sinusoidal steady-state with frequency ω_0 . It is then easy to obtain the initial value $x(0)$ of the state variables x of equation (2'). For this purpose we represent the input voltages

$$u_i = U_{i_{\max}} \sin(\omega_0 t + \phi_i) \quad (28')$$

by phasors

$$\bar{U}_i = U_{i_{\max}} e^{j\phi_i}. \quad (28'')$$

In equation (2') we represent x also by phasors. Equation (2') yields

$$\bar{X} = \bar{G} \bar{U} \quad (29')$$

where

$$\bar{G} = (j\omega_0 I - A)^{-1} B. \quad (29'')$$

One element of (29') is

$$\bar{X}_k = \sum_i \bar{G}_{ki} \bar{U}_i \quad (30')$$

with

$$\bar{G}_{ki} = G_{ki} e^{j\theta_{ki}}. \quad (30'')$$

In the time domain (30') gives

$$x_k(t) = \sum_i G_{ki} U_{i_{\max}} \sin(\omega_0 t + \phi_i + \theta_{ki}). \quad (31)$$

Equation (31) gives the initial value of x_k for $t = 0$.

C. Closed-Form Solution for Sinusoidal Input

We consider first the single input case for equation (2')

$$\dot{x} = Ax + bu \quad (32')$$

Fig. 9. Receiving end voltage transient at simultaneous energization of a three phase line, 150 km, 500 kV, transposed, source inductance = 0.034 H. ○ Phase a. Δ Phase b. □ Phase c.

where

$$u = U_{\max} \sin(\omega_0 t + \phi). \quad (32'')$$

In equation (32') the vector $x(0)$ is known from the value of $x(t)$ at the end of the previous stage of the transient or from the sinusoidal steady-state, discussed in the previous section.

We note that A is diagonal since (32') has been obtained from $H_R(s)$ in the form of a series of simple fractions. It has the poles p_j on its diagonal and the vector b consists of the residues r_j of the simple fractions.

Integration of (32) yields for state variable x_j related to a single partial fraction

$$\begin{aligned} x_j(t) = & e^{p_j t} x_j(0) \\ & - U_{\max} \frac{r_j}{\omega_0^2 + p_j^2} \left[p_j \sin(\omega_0 t + \phi) + \omega_0 \cos(\omega_0 t + \phi) \right. \\ & \left. - e^{p_j t} (p_j \sin \phi + \omega_0 \cos \phi) \right]. \end{aligned} \quad (33)$$

We note that, if p_j and r_j are complex, they appear in conjugate pairs and so will $x_j(t)$. Therefore the resultant output will be real, as expected.

Fig. 8 represents the receiving end voltage transient resulting from the frequency domain data of Fig. 2 corresponding to the energization of the line shown in Fig. 4. One plot corresponds to the closed form integration of the state equations derived from $H_R(s)$. The other plot represents the inverse Fourier transform of the original $H(j\omega)$. As can be seen, the agreement is very good.

Fig. 9 gives an example for the closed form calculation of transients on a three-phase transmission line. Because of the simultaneous pole closing, chosen for simplicity, there is no ground mode damping in this case. Fig. 10 illustrates the case of sequential energization.

The total computer CPU time, including the rational fitting, is of the order of a few seconds. The storage requirements are very low.

D. Numerical Integration for Systems with Nonlinear Elements

For nonlinear elements connected to the network we will consider special terminals with inputs u' and outputs y' ,

Fig. 10. Receiving end voltages on phases a, b, c at sequential energization of the same line as in Fig. 9, at $t_a = 0$, $t_b = 2.5$ ms, $t_c = 5$ ms.

nonlinearly interrelated:

$$\dot{x} = Ax + Bu + B'u' \quad (34')$$

$$y = Cx + Du + D'u' \quad (34'')$$

$$y' = Ex + Fu + F'u' \quad (34''')$$

$$u' = f(y') \quad (35)$$

where equation (35) symbolizes a nonlinear feedback. Clearly, the system is now nonlinear and numerical techniques will be needed for the integration. Our main remark here is that small step sizes will be required until the

transients attenuate but later they can be significantly increased to an upper limit determined only by the power frequency ω_0 .

VI. CONCLUSIONS

A new method has been developed for the calculation of transients in power system networks. It is based on expressions of transmission line parameters in terms of the complex frequency $s = \sigma + j\omega$. This permits the approximation of a transfer function $H(s)$ by a rational form for which the poles and residues can easily be calculated by direct search procedures in the s -plane.

The resultant rational transfer functions are converted analytically into state equations. The integration of the state equations can be performed either numerically or in closed form. Computational requirements are low both for computing time and storage. No stability or convergence problems have been experienced.

The advantage of the new method appears to be that it permits the powerful tools of state equations and s -domain formulations to be applied to realistic transmission lines with possible application of important system theory procedures, such as model reduction (network equivalencing for transients), optimization (direct calculation of maximized over-voltages), and control. Long range transients can now be calculated advantageously using closed form solutions. On the other hand, the preparatory work for larger systems appears at this stage to be difficult.

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A Numerical Design Procedure for General Canonic LC One-Ports

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Abstract—A numerical method has been developed and a computer program has been written to design arbitrary, canonic, inductor-capacitor one-ports. The design procedure employs a matrix tridiagonalization method in conjunction with a Newton-Raphson technique to solve for the element values in a canonic LC network to yield a prescribed driving-point impedance function. The numerical method is extremely stable and numerically

well-conditioned; in one of our examples, a 14th degree, nonplanar LC structure was designed (using single precision arithmetic) yielding element values accurate to six significant figures.

I. INTRODUCTION

NCESSARY AND SUFFICIENT topological conditions for a lossless one-port to be canonic have recently been discovered [1]. This result is in the form of an existence theorem, and hence does not provide a means for actually calculating the circuit-element values in an arbitrary

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