MILATRAS
Microsimulation Learning-based Approach to Transit Assignment

By

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Abstract

Public transit is considered a cost-effective alternative to mitigate the effects of traffic gridlock through the implementation of innovative service designs, and deploying new smart systems for operations control and traveller information. Public transport planners use transit assignment models to predict passenger loads and levels of service.

Existing transit assignment approaches have limitations in evaluating the effects of information technologies, since they are neither sensitive to the types of information that may be provided to travellers nor to the traveller’s response to that information. Moreover, they are not adequate for evaluating the impacts of Intelligent Transportation Systems (ITS) deployments on service reliability, which in turn affect passengers’ behaviour.

This dissertation presents an innovative transit assignment framework, namely the MIcrosimulation Learning-based Approach to TRansit ASsignment – MILATRAS. MILATRAS uses learning and adaptation to represent the dynamic feedback of passengers’ trip choices and their adaptation to service performance. Individual passengers adjust their behaviour (i.e. trip choices) according to their experience with the transit system performance. MILATRAS
introduces the concept of ‘mental model’ to maintain and distinguish between the individual’s experience with service performance and the information provided about system conditions.

A dynamic transit path choice model is developed using concepts of Markovian Decision Process (MDP) and Reinforcement Learning (RL). It addresses the departure time and path choices with and without information provision. A parameter-calibration procedure using a generic optimization technique (Genetic Algorithms) is also proposed. A proof-of-concept prototype has been implemented; it investigates the impact of different traveller information provision scenarios on departure time and path choices, and network performance. A large-scale application, including parameter calibration, is conducted for the Toronto Transit Commission (TTC) network.

MILATRAS implements a microsimulation, stochastic (nonequilibrium-based) approach for modelling within-day and day-to-day variations in the transit assignment process, where aggregate travel patterns can be extracted from individual choices. MILATRAS addresses many limitations of existing transit assignment models by exploiting methodologies already established in the areas of traffic assignment and travel behaviour modeling. Such approaches include the microsimulation of transportation systems, learning-based algorithms for modelling travel behaviour, agent-based representation for travellers, and the adoption of Geographical Information Systems (GIS).

This thesis presents a significant step towards the advancement of the modelling for the transit assignment problem by providing a detailed operational specification for an integrated dynamic modelling framework – MILATRAS.
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GLOSSARY

APTS  Advanced Public Transportation Systems
APC  Automatic Passenger Counter
API  Application Programmer Interface
ATIS  Automated Traveller Information Systems
AVI  Automatic Vehicle Information
AVL  Automatic Vehicle Location
BRT  Bus Rapid Transit
\( d \)  Iteration #
DP  Dynamic Programming
DES  Destination Stop State
DMG  Data Management Group
DNS  Destination State
ETAM  *Equilibrium* Transit Assignment Model
ETAP  *Equilibrium* Transit Assignment Problem
FIFO  First-In-First-Out
GA  Genetic Algorithm
GC  Generalized Cost
GIS  Geographical Information Systems
GIS-T  Geographical Information Systems – Transit
GO  Government of Ontario
GRE  Global Relative Error
GTA  Greater Toronto Area
iDRS  interactive Data Retrieval System
ILUTE  Integrated Land Use, Transportation and Environment
IRL  Inverse Reinforcement Learning
ITS  Intelligent Transportation Systems
LRT  Light Rail Transit
MASS  Multi-Agent Simulation Systems
MaxEnt  Maximum Entropy
MC  Monte Carlo
MDP  Markovian Decision Process
MILATRAS  Microsimulation Learning-based Approach to TRansit ASsignment
OD  Origin-Destination
OFS  Off Stop State
OGN  Origin State
OGS  Origin Stop State
ONS  On Stop State
PFS  Pattern First Search
PMRE  Point Mean Relative Error
\[ \text{Pr}^{ij} \]  Transition probability from state \( i \) to state \( j \)
RL  Reinforcement Learning
RUT  Route State
\( \pi \)  Reinforcement Learning Policy
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\pi^*$</td>
<td>Reinforcement Learning Optimal Policy</td>
</tr>
<tr>
<td>SAT</td>
<td>Scheduled Arrival Time</td>
</tr>
<tr>
<td>SD</td>
<td>Schedule Delay</td>
</tr>
<tr>
<td>SUE</td>
<td>Stochastic User Equilibrium</td>
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<tr>
<td>TAP</td>
<td>Transit Assignment Problem</td>
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<tr>
<td>TAZ</td>
<td>Traffic Analysis Zones</td>
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<tr>
<td>TD</td>
<td>Temporal Difference</td>
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<tr>
<td>TDM</td>
<td>Travel Demand Management</td>
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<td>TOP</td>
<td>Transportation Object Platform</td>
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<td>TPCP</td>
<td>Transit Path Choice Problem</td>
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<td>TSP</td>
<td>Transit Signal Priority</td>
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<td>TTC</td>
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<td>Visual Basic Application</td>
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1 INTRODUCTION

The transportation system is one of the basic components of an urban area’s social, economic and physical structure. Recently, urban planners and politicians have been increasingly alarmed by the deteriorating levels of network gridlock and its impacts on quality of life, road safety, environmental quality and economic viability. In the Greater Toronto Area (GTA), gridlock has an enormous impact on our communities. Moving people and goods around Central Ontario is increasingly difficult. About 70% of our highways are almost at total capacity during rush hour - and in some parts of the central zone, the so-called rush hour now lasts for 13 hours a day. Transit share of passenger travel is declining. In 1986, 22% of all morning peak travel was by transit, while in 1991, it was down to 19%; and in 1996, it further declined to 15%. Congestion costs the economy billions of dollars each year due to delays of goods shipments. Congestion in the GTA and Hamilton is estimated to cost the economy $2 billion per year because it delays the movement of people and goods. Vehicles idling on our roads and highways produce pollution. Carbon dioxide emissions double when speeds drop from 55 to 30 km/hr, and hydrocarbon emissions triple at speeds less than 60 km/hr compared to a constant speed of 80 km/hr. Poor air quality causes 5,000 hospital admissions in the Greater Toronto Area each year.

Transportation planning, a sub-field of urban planning, plays a direct role in shaping our quality of life. The choice between transit, highway and pedestrian investments affects greatly the activity patterns of individuals and the movements of goods. Public transit, as part of a multimodal transportation network, has been widely considered as a cost-effective alternative to mitigate the effects of traffic gridlock. Public transit is envisioned to provide an attractive alternative by implementing innovative service designs and deploying new smart systems and technologies for operations control and customer information. Examples of such emerging transit systems include Bus Rapid Transit (BRT) and Light Rail Transit (LRT) systems. Public transport planners use transit assignment models to predict passenger loads and level of services on a given transit network. These models are widely used as an important planning tool at the

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1 Statistics in this paragraph are extracted from the report by the Central Ontario Smart Growth Panel (August 2002), “Interim Advice on Unlocking Gridlock and Promoting Liveable Communities in Central Ontario”
strategic and operational levels. Also, transportation planners use transit assignment models as a critical component of multimodal network models of urban transportation systems. Important decisions concerning investment in public transport infrastructure or services are normally supported by evaluation methodologies based on transit assignment models. This requires the modelling of the foundational relationships that govern travel behaviour to be able to address ‘what-if’ scenarios based on questions of alternative transportation policies.

Nevertheless, there are small amounts of research on public transportation policy and planning evaluation or management tools by comparison with those of automobiles on roads. Moreover, the existing methodologies for the evaluation for public transport policies and operations cannot represent realistic important features of the emerging public transport smart systems. Emerging information, communication, sensor technologies and innovative transit operations control strategies are becoming critical elements of the viable, competitive public transit system. Advanced Public Transportation Systems (APTS) and Automated Travellers Information Systems (ATIS), through a variety of data collection and communication capabilities, support improved operations planning and real-time transit operations management. Such information systems are designed to provide timely information to transit passengers on the conditions of the network, thus affecting travel choice behaviour. Traditional transportation planning methods have serious limitations in evaluating the effects of information technologies, since they are neither sensitive to the types of information that may be provided to travellers (i.e. lack of dynamic representation of the transport network) nor to the traveller’s response to that information (i.e. lack of behavioural modelling that explicitly treats information provision). BRT and LRT systems require the implementation of a variety of APTS applications; existing transit assignment models are not adequate for representing BRT and LRT characteristics. Meanwhile, traffic assignment procedures have recently implemented a microsimulation approach to describe the detailed behaviour of the transportation system. Geographical Information Systems (GIS) have been widely adapted in urban planning applications. Learning-based algorithms, for modelling travel behaviour with agent-based representation, have been shown to result in different and more realistic assignments. These advances present great opportunities for further advancing the state-of-the-art of transit assignment modelling.
In order to address the limitations of traditional models, an integrated dynamic modeling framework is needed that (Wahba, 2004):

1. is sensitive to time-dependent and stochastic transit service characteristics (supply modeling)
2. models adaptive departure time and path decisions by passengers (demand modeling)
3. captures the interaction between passenger decisions and transit network performance (via an integrated framework).

This integrated dynamic modeling framework needs to address:

1. the time-dependent pattern of flows and their distribution over space, as well as the systematic changes of the passenger decisions within the day and from day to day, and
2. the interaction between the passenger decisions and the system performance. As such, the modeling framework has to deal explicitly with trip timing and path selection, and the mechanism through which passengers adjust these decisions in response to experienced congestion, control measures, and supplied information.

This dissertation represents a significant step towards the advancement of existing transit assignment procedures.

1.1 Objective and Scope

The objective of this dissertation is to develop a new modelling framework for the assignment of transit passenger demand. It also presents a large-scale real-world application of the proposed modelling system for the Toronto Transit Commission (TTC) public transportation system within the context of the Greater Toronto Area (GTA), Ontario, as a case study – see Figure 1.1.
This investigation focuses on the issues concerning the theoretical aspects of transit assignment modelling, in particular path choice under information provision. The study considers multiple dimensions of the transit path choice problem: the departure time choice, the stop choice, and the route (or run) choice. Such dimensions were either simplified (e.g. stop choice) or ignored (e.g. departure time) in existing approaches. The developed assignment procedure is capable of modelling the day-to-day and within-day dynamics of the transit service, as well as passengers’ responses. Furthermore, it presents a coherent behavioural integrated framework, which deals with the issue of congestion (i.e. influence of individual traveller’s options on travel choices of all others) endogenously. Hence, aggregate travel patterns can be properly extracted from individual choices.
The proposed framework, namely the MIicrosimulation Learning-based Approach to TRansit ASsignment – MILATRAS, is an agent-based model. Individual passengers are represented by agents with their learning and planning activities. Learning activities, in this context, are concerned with the values of the generalized cost for the transit trip (e.g. perceived in-vehicle time, perceived waiting time, etc.), while planning activities are concerned with the choice mechanism by which rational passengers decide about their trip choices (e.g. departure time choice, run choice). Without loss of generality, the framework focuses on recurring trips (e.g. work and school trips) in the peak periods, where learning from experience is evident. The proposed procedure considers the access/egress mode choices as part of the transit path choice problem; however, primary mode of travel and destination choices, in this investigation, are treated as an input. It is our understanding that these types of choices require different level of learning and adaptation that is outside the scope of this thesis.

1.2 Motivation

The work in this thesis was originally motivated to explore the following research question:

*What is the impact of Intelligent Transportation Systems (ITS) technologies on the performance of transportation networks, and in particular public transit networks?*

A reasonable approach to answer this question is to compare the performance of the transportation network, under investigation, with and without ITS technology deployments. This brings up two other research questions:

*How can we ‘model’ the deployment of ITS technologies?*

*How can we ‘measure’ the effect of ITS deployments on the performance of the transportation network?*
One of the traditional measures of the performance of transportation networks is, for example, average travel time per traveller. This brings travellers into the equation. The research question then becomes:

*How can we model travellers’ behaviour in response to ITS deployments?*

This implies that the modelling of ITS deployments and travellers’ behaviour in response to these deployments are essential components in the proper modelling of the impact of ITS policies and technologies on the performance of transportation networks, along with the resulting (dynamic) supply-demand interaction, see Figure 1.2.

For consistency of comparison, the modelling system that is to be used to analyse the base-scenario (i.e. *without* ITS deployments) should be the same to analyse the scenario with ITS services and technologies. This means that the analysis should integrate the modelling of dynamic transportation service characteristics with the modelling of dynamic travellers’ responses into one framework. For public transit networks, emerging information and communication technologies, commonly used in ITS and Advanced Public Transportation Systems (APTS) are expected to improve public transit services. Such information systems are designed to provide timely information to transit passengers on the conditions of the network, thus affecting travel choice behaviour. ITS technologies and policies are usually envisioned to improve the ‘efficiency’ and ‘reliability’ of the transportation system. For transit networks, efficiency can be enhanced through service improvement technologies (e.g. Transit-Signal Priority, TSP, and Automatic Vehicle Location, AVL). These technologies address dynamic service characteristics, such as, for example, arrival time at intersection. A reliable transit service is important for transit passengers, who perceive reliability through schedule adherence and/or trustworthy Automated Traveller Information System (ATIS). This requires modelling of passengers’ responses to the provision of ATIS – see Figure 1.3.
Figure 1.2 Components for proper modeling of impacts of ITS deployments

Figure 1.3 ITS connections with Service Characteristics and Travellers’ Responses
In this context, assignment procedures are concerned with modelling travellers’ behaviour (i.e. trip choices), and such procedures distribute a given travel demand on a network. This helps determine travel volumes over roads and transit routes, as well as it reflects the service quality of the transport network, which affects passengers’ behaviour (and hence travel volumes). For public transport networks, transit assignment models are used to predict passenger loads and levels of services in a given transit network; this is called the Transit Assignment Problem (TAP). Transit assignment procedures include a path choice model that describes the behaviour of transit riders with regards to their trip choices to travel between trip origins and destinations. The inputs to such choice models usually include information on the transit service performance. When the within-day performance changes (due to, for example, ITS deployment), it is expected that travellers will respond by adjusting their trip choices.

The motivation has become to enhance the current state-of-the-art of transit assignment modelling to be responsive to the impacts of emerging ITS, and to enhance the current state-of-the-practice by devising a modelling tool that can be used by planners to experiment with different policies and strategies and provide a consistent framework for measuring the impact of ITS deployments.

1.3 The Approach

A key aspect of urban travel is the adaptive behaviour of people in response to a change in their environment. It is well known that there is a mutual dependence between traveller behaviour and system performance; congestion is the result of the execution of departure time and path choices of many individuals over a transit network with a constraint capacity, but individual choices are based on the anticipation of congestion. One approach is to model each passenger as a microscopic entity and model that entity’s reaction to the system directly, while modelling the time-dependent system performance as a response to passengers’ behaviour; this approach is formulated in MILATRAS within a “multi-agent” simulation environment.

Passengers making travel decisions are examples of agents who independently make decisions as a function of their own attributes, experiences, and the state of the system that they find
themselves within. These actions, in turn, change the system state over time (e.g. changing congestion levels). Agents collect information about their environment as they interact with it, and use such information to develop anticipatory models of the environment. The decision process arises from an adaptive learning process driven by the agent’s desire to maximize some payoff through its actions over time. To properly represent the environment, an appropriate network performance model is needed to obtain the experienced travel and waiting times, convenience measures, and congestion and capacity effects, etc. that change within-day and from day to day for each passenger. MILATRAS includes a network performance model that has two modules: a GIS-T module (for static representation) and a network microsimulation module (for dynamic representation).

MILATRAS is based on representing passengers and both their learning and planning activities explicitly. The learning process is concerned with the specification of different trip components (e.g. in-vehicle time, out-of-vehicle time, convenience measures, etc). The planning process considers how experience and information about those components on previous days influence the choice on the current day. The underlying hypothesis is that individual passengers are expected to adjust their behaviour (i.e. trip choices) according to their experience with the transit system performance. Individual passengers base their daily travel decisions on the accumulated experience (stored in the mental model) gathered from repetitively travelling through the transit network on consecutive days. Individual behaviour, therefore, should be modelled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. By repeatedly making a decision, an individual acquires knowledge (i.e. learns) about his environment and thereby forms expectations about attributes of the environment. Individuals may make different choices over time and thus learn which of these choices is more effective in achieving particular goals.

The system state evolves through agent interaction and communication with other agents and objects and their environment(s). The multi-agent system will then evolve to a pattern from which useful macro-level information can be extracted. This is important in modeling space-time dynamics within urban systems since it allows for studying the relationships between micro-level
individual actions (e.g. boarding decision) and the emerging macro-level phenomenon (e.g. overcrowding).

1.4 Thesis Roadmap

The dissertation starts with a general description of the challenges to traditional transit assignment procedures for modelling advanced public transportation systems. It then outlines the motivation behind this research effort. It also gives a high-level description of the proposed approach.

Chapter 2 reviews the literature of transit assignment procedures. It presents a brief history of the developments in the field of transit assignment from the early developments to the recent advancements. Section 2.4 describes the author’s vision for the future developments in the area of transit assignment modelling.

Chapter 3 provides the conceptual development of the proposed approach. It describes, in detail, the structure of the MIcrosimulation Learning-based Approach to TRansit ASsignment, MILATRAS. It discusses the requirements and benefits of the development of the proposed approach. The concept of passenger-agent with mental models and learning-based travel behaviour is proposed. Section 3.3 outlines the connections with urban planning models. It ends with a discussion of applications of intelligent transportation systems in public transport networks.

The theoretical foundation for a dynamic path choice model for transit riders is presented in Chapter 4. The chapter starts with a review of the Markovian Decision Process (MDP) and the concepts related to Reinforcement Learning (RL). It provides a detailed description of the mathematical formulation for the departure time and path choices with and without information provision. A parameter-calibration procedure using a generic optimization technique (Genetic Algorithms) is proposed.
Chapter 5 reports on the implementation of the proposed approach using a prototype network. It investigates the impact of different traveller information provision scenarios on transit riders departure time and path choices, and network performance, using the proposed multi-agent microsimulation learning-based approach. A large-scale application is presented in Chapter 6. Issues regarding data requirements and modelling assumptions are discussed. Section 6.5 presents the model outputs with various reporting features.

Chapter 7 provides the conclusions and the contributions of this thesis with a discussion of a future research agenda.
2 LITERATURE REVIEW

The conventional trip-based approach to transport modelling, exemplified in the four-step model (or Urban Transportation Modelling System, UTMS), aims to determine the flow (auto and passenger) over transportation network facilities (road and transit networks). The first step, *Trip Generation*, defines the intensity of travel demand (i.e. frequency by trip purpose). Trip ends (productions and attractions) are estimated independently as functions of household and zonal activity characteristics. *Trip Distribution*, the second step, is concerned with distributing trip productions in proportion to the estimated attraction distribution and as a function of travel disutility; this results in the traditional trip tables or origin-destination matrices. OD matrices are then factored to reflect relative proportions of trips by alternative modes, *Mode Choice* step. In the fourth step, *Trip Assignment*, mode-specific OD matrices are assigned to mode-specific networks using assignment procedures; this is done through assigning trips to specific paths from origins to destinations, generating estimates of travel flows. Specifically, transit assignment procedures are used to distribute the transit OD matrix over a fixed set of public transportation routes defining the transit service.

The Transit Assignment Problem (TAP) is the problem of predicting passenger loads and levels of services on a given transit network that consists of a set of fixed lines. By modeling passengers’ travel behaviour on their journey from origins to destinations, transit assignment procedures distribute a given travel demand on a network and attempt to model the interaction between the travel demand and the network supply. Not only does this help determine volumes on transit lines, but it also reflects the service quality of the transport network. Transit assignment models are widely used as an important planning tool at the strategic and operational levels. They are, therefore, a critical component of multimodal network models of urban transportation systems. Important decisions concerning investment in public transport infrastructure or services are normally supported by evaluation methodologies based on transit assignment models.

Public transport assignment models, like those for road, include *supply models*, *path choice models*, and *supply/demand interaction models*. The core of an assignment model is the path
choice model, which has to be specified according to user behavioural hypotheses that depend on specific user characteristics (frequent, well-informed user; occasional, ill-informed user, etc.) and service characteristics (frequency, regularity, pre-trip/en-route information availability). While much attention has been given to auto-traffic assignment models, it is well acknowledged in the literature that the transit assignment process is more complicated than auto-traffic assignment (Nielsen, 2000; Friedrich et al., 2001). Wahba and Shalaby (2005) outlined the reasons for such complexity as follows:

- The common lines problem, where multiple lines with the same or different frequencies serve the same road segment. That is; passengers, waiting at a stop, are faced with various travel options between an origin and a destination where each line is characterized by a frequency and performance measures.
- Parallel lines on parallel roads are normal features of public transport networks. This represents an extension to the common lines problem when parallel roads are close in distance.
- Unlike car drivers who are free to depart at anytime and free to choose a route, transit riders are strongly restricted by the network timetable and fixed-route structure. The supply of transit service is discontinuous; it is available at certain times.
- The out-of-vehicle time component is significant for the transit assignment process and it is not straightforward to relate to the in-vehicle time component.
- Transit riders are faced with the transfer connection problem, which encompasses temporal and spatial constraints.
- Trip choices in public transport networks are dependent on each other, the network structure and the service performance.
- The existence of transit sub-modes complicates the transit assignment process and creates the transit mode-chain complexity.
- The complexity of various fare structures; transit riders often deal with mixed fare structures during their journeys. For example, inter-regional services usually apply distance-based fare structure (e.g. GO service in Ontario), while urban transit systems implement flat-fare (e.g. Toronto Transit Commission, TTC, service), modal-fare (e.g. downtown express service offered by TTC), or zonal-based fare (e.g. London Transport, UK).
• The public transport network structure is very complicated, and the assumption that each passenger is aware of all available routes may not be justifiable.

2.1 Early Developments

When the 4SM was introduced in late 1950s, road network assignment procedures were proposed, after small refinements, to solve the transit assignment problem ignoring the significant differences between the auto and passenger assignment problems. As the recognition of the distinctive features of the transit assignment problem increased, many researchers started to develop new transit assignment procedures; examples are Dial (1967), Le Clercq (1972), Andreasson (1976), and Rapp et al. (1976). Those studies dealt explicitly with the out-of-vehicle time component (i.e. waiting time at stops) and the common lines aspect of the transit route choice problem. They basically proposed heuristic approaches for combining out-of-vehicle and in-vehicle time components with the consideration of the common lines factor in computing the shortest path for a given OD pair. For example, Andreasson (1976) suggested a heuristic approach to estimate waiting times at stops and route shares where a route is considered in the attractive set of lines if boarding this route results in a travel time value that is less than the waiting time plus the travel time upon boarding the minimum time route. Later, this heuristic approach was extended for transit networks with multiple lines and sub-modes between one origin and one destination (Jansson and Ridderstolpe, 1992).

At this stage, the most notable contribution was made by Chriqui and Robillard (1975) who argued that passengers consider a set of attractive lines in order to minimize the expected sum of out-of-vehicle and in-vehicle time components. That was revolutionary since the all-or-nothing notion was dominating the transit assignment filed. They also proposed a behaviourally-sound route choice model whereby passengers choose to board the first line of the set of attractive lines that arrives at a stop. The proposed concepts received a tremendous recognition since it was able to explain passenger’s trip choices based on the transit service characteristics at that time; however, the model was only explained for a single corridor served by a set of bus lines.
Parallel to highway large-scale infrastructure expansion projects in 1970s, the need for designing the infrastructure of transit networks emerged as part of the urban modelling and planning framework. The evaluation of the impacts of such investments persuaded researchers to further develop transit assignment procedures to be used in the context of transport network design models (such as Scheele, 1977; Mandle, 1980) and in the context of multimodal network equilibrium (such as Florian, 1977; Florian and Spiess, 1983).

In all the aforementioned studies, the transit assignment problem was treated as a route-choice problem where passengers are assumed to be waiting at stops and transit assignment procedures were only concerned with the boarding decision. Stop and departure time choices were not included in the assignment procedure. The assignment procedures did not account for capacity constraints and congestion effects on the out-of-vehicle and in-vehicle time components; this was reasonably justified since congestion was not widely observed with transit services. Congestion effects refer to the flow-dependent in-vehicle and out-of-vehicle time components, where capacity constraints reflect the physically limited supply represented by service frequency and vehicle capacities. An attempt for modelling congestion effects in transit networks was presented by Last and Leak (1976), TRANSEPT, yet only applicable for very special radial networks which are not common real-world applications.

2.2 The First Wave – The Strategy-based Approach

It was not until the 1980s when mathematical formulations for the transit assignment problem were proposed. While there were various significant contributions, they were all rooted to the concepts of the set of attractive lines (e.g. Chriqui and Robillard, 1975) and the common lines treatment (e.g. Le Clercq, 1972).

Spiess and Florian (1989) presented a linear programming model and a solution algorithm for the transit assignment problem, extending the work done by Chriqui and Robillard (1975) from a simple one origin-destination transit network to general transit services. This procedure is referred to as the headway-based approach, the frequency-based approach, the line-based approach, or more commonly the strategy-based approach. The proposed model assumes that
passengers minimize “generalized travel times” as they adapt “strategies” rather than simple all-or-nothing paths to make their origin-destination trips over a transit network. Nguyen and Pallotino (1988), extending the work by Chriqui and Robillard (1975), proposed the concept of “hyperpath” as an acyclic directed graph, similar to the “strategy” concept. A different treatment was proposed by De Cea and Fernandez (1989) where the concept of common lines is restricted to lines that only share the next stop to be served and route choice is allowed among this newly defined attractive set of lines.

The strategy-based assignment model is based on the concept of the strategy, which is defined as “a set of rules that, when applied, allows the traveller to reach his or her destination” (Speiss and Florian, 1989, p. 84). A strategy can be formulated as the set of ‘attractive links’ in the transit network. Since there may be more than one line passing through a link, and the waiting time for boarding a line is probabilistic, there may be more than one minimum cost path for each OD pair – known as the common lines problem. By assessing the in-vehicle travel time on each link that is part of the strategy, the waiting time at each node, and the probability of leaving that node on each link for a particular strategy, the total expected travel time for the strategy can be calculated. The optimal strategy is, then, determined as the one that minimizes the expected total travel time including waiting time. The waiting time, which depends on the arrival time of the passenger and the arrival times of buses of different lines, is computed by assuming that transit riders wait on the average half of the interarrival interval and that frequencies are combined linearly. The “line probability”, which is based on the probability that a bus will arrive from that attractive line first, is the ratio of its frequency divided by the combined frequency. In strategy-based assignment models, the path choice model assumes that users always choose in an indifferent adaptive way, that is they board the first arriving run belonging to their attractive line set. The underlying hypothesis that allow such results is that transit vehicles and users arrive at stops randomly following a pre-defined probability distribution.

Strategy-based mathematical models are capable of spreading the transit demand where there are common lines. In general, they assume that the passenger demand is constant within the specified period of analysis. They, however, assume flow independent in-vehicle and out-of-vehicle times, and hence do not consider congestion effects. This is a very serious limitation,
especially when transit assignment models are used to study transit networks operating with high congestion levels where passengers may not be able to board the first vehicle to arrive at a stop. The frequency-based transit assignment models simplify the search for minimum path by using average values for the transfer time; they do not explicitly calculate transfer times but assume that they depend on the headway, and hence do not consider timetable coordination.

As congestion was observed in transit networks, the concept of equilibrium was introduced in the modelling of transit assignment, resulting in the development of Equilibrium Transit Assignment Models (ETAM). One of the earliest contributions was proposed by Gendreau (1984) where the congestion effects on the in-vehicle travel costs and waiting time were explicitly considered. The study treated congestion at bus stops based on queuing theory, resulting in a general transit assignment model with congestion. The mathematical model formulation was however restricted to a particular case of a transit system without common lines (stops served by one line) (De Cea and Fernandez, 2002). For the general case, transit systems with common lines, they proposed a practical formulation for the Equilibrium Transit Assignment Problem (ETAP). Gendreau’s contribution was basically the consideration of passenger-flow-dependent waiting times at transit stops due to the capacity constraints of transit vehicles and high demand volumes. De Cea and Fernandez (1993) extended their earlier contribution (De Cea and Fernandez, 1989) by considering the effects of congestion at boarding stops and on-board of transit vehicles. Using heuristics, they developed an asymmetric equilibrium model and a solution algorithm using the Jacobi method. The model employs an augmented graph of the transit service and conditions, thus requiring significant computational resources. Spiess and Florian (1989) gave a general version of their linear model in which in-vehicle travel times (or “generalized travel costs”) are increasing functions of passenger flows (called “discomfort functions”). However, as the same authors acknowledge, the model presents important limitations, the main one being that waiting times at stops are not affected by transit volumes, reducing the congestion phenomenon to a comfort problem. This is similar to the model proposed by Nguyen and Pallottino (1988). Bell et al. (2002) presents another example of static capacity constrained frequency-based modelling approach.
The one finding that was common to all early ETAM studies is that congestion effects do not only impact waiting times at boarding stops, but also the flow distribution over attractive routes. This introduced the differentiation between nominal frequency and effective frequency notions. Nominal frequency is defined as the inverse of the ‘line-headway’. The line’s effective frequency is equivalent to its nominal frequency when the arrival of lines at stops is independent of its travel flow. The line’s effective frequency can be computed as the inverse of the ‘waiting time’ of the route. As congestion increases, the line’s effective frequency becomes less than its nominal frequency values and this is consequently reflected in the flow distribution over attractive routes. When congestion impacts are only incorporated in waiting times at boarding stops, this is called a semi-congested model. When the effect on flow splits over attractive routes is considered, this is called a fully-congested model.

Wu et al. (1994) introduced another mathematical formulation for the ETAP. The formulation uses the concept of hyperpath. The cost of walking, boarding, and alighting links is flow independent, while the cost of waiting and in-vehicle links are flow dependent with asymmetric interactions. The in-vehicle cost has two components (an in-vehicle travel cost and a discomfort cost), and the cost of waiting links is a function of the frequency of the transit lines and congestion effects due to queues at stops. These models define passenger-flow-dependent generalized cost functions. However, this mathematical treatment did not consider the distribution of flows among attractive lines as a function of line-flows; route-loads are assumed to be proportional to nominal frequencies and hence this treatment represents a semi-congested model. This work was extended by Bouzaïene-Ayari et al. (1995) to relate flow distribution to line-loads, representing a fully-congested model. The model combines line flows (or arc flows) as a fixed-point problem and hyperpath flows as a variational inequality problem. They proposed a solution algorithm similar to the method of successive averages. While it captures congestion, the hyperpath problem suffers from the curse of dimensionality (i.e. combinatorial complexity), and this limits the model’s application to only small-size networks.

Despite the incorporation of congestion effects in ETAP models, capacity constraints were not strictly applied, as resulting flows often exceeded the capacity of transit vehicles. The strategy-based approach is usually criticized for the bias towards over-assigning riders to lines with high
combined frequency of transit services and under-assigning riders to those with low combined frequency of services. Moreover, low frequency transit lines may not be assigned as a travel option at all. These limitations are particularly serious when such assignment models are used to study transit networks with crowded vehicles and congested stations/stops due to insufficient capacity of the transit services.

To account for congestion effects, Lam et al. (1999) proposed a mathematical formulation, and a solution algorithm, for the modelling of transit assignment in congested networks, where the passenger delay at bottleneck stops is determined endogenously. At bottleneck nodes, only a proportion of passengers board the first arriving vehicles and residual passengers are served by the next vehicle or by transferring to alternative routes. In this formulation, transit vehicle capacity is explicitly constrained. Kurauchi et al. (2003) proposed a model in which over-capacity is explicitly treated through re-routing passengers who cannot board over spill-links. The proposed procedure uses residual capacities of transit vehicles to compute the boarding probabilities and construct network hyperpaths. Hamdouch et al. (2004) proposed a model where vehicle capacities are explicitly incorporated, as the probability of boarding a line is proportional to its residual capacity and inversely proportional to the number of users wanting to board the same line. They developed a frequency-based equilibrium assignment model where passengers are assigned to strategies that minimize their expected travel cost and accounting for loading priorities among passengers.

Cominetti and Correa (2001) analyzed the congestion effects using the Wardrop equilibrium concept and the dynamic programming approach; they developed a headway-based transit assignment model that considers congestion effects on waiting times and the flow distribution. The model uses dynamic programming to compute the equilibrium flows. Capacity constraints are modelled by realistic waiting time functions that behave asymptotically at vehicle capacities thus affect effective frequency calculations when the flows exceed the line capacity. With this treatment, the model is defined in the space of line flows (or arc flows) only; this is significant as large-scale applications become more practical. For this formulation, the equilibrium conditions were stated and the existence of network equilibrium flows was shown. However, the solution algorithm was not presented. Cepeda et al. (2006) provided an alternative formulation for the
problem stated in Cominetti and Correa (2001), assuming a fully-congested scenario, and proposed a solution algorithm for the alternative formulation. The proposed solution algorithm uses a heuristic optimization technique, the method of successive averages, to compute the network equilibrium flows.

To capture the potential differences between passengers’ preferences, research efforts were directed towards developing Stochastic User Equilibrium (SUE) transit assignment models. In ETAM, transit passengers are assumed to have identical perceptions regarding travel disutility components, commonly interpreted as perfect information available for all passengers. In congested networks with unreliable service performance, passengers are not expected to have perfect information about the network conditions. Lam et al. (1999) considered the differences among passenger’s perception and proposed a stochastic user equilibrium formulation for the full-congested scenario of transit networks. Lam et al. (2002) extended the model to consider the elasticity between transit line frequencies and passenger flows on transit lines. Nielsen (2000) presented a frequency-based framework of transit assignment using the probit-type SUE model. The proposed model included more stochastic components in order to describe differences in passengers’ preferences. Niu and Mao (2007) presented a frequency-based transit assignment model that accounts for congestion effects and passengers’ preferences. They model travel preference through the formulation of passenger user-groups, or market segments. The model however considers constant in-vehicle travel times and flow-independent waiting times.

Yang and Lam (2006) proposed a probit-type reliability-based assignment model for congested transit networks with unreliable services. The trade-off between transit service reliability and travel behaviour is captured in a newly developed disutility function. The reliability measures are related to in-vehicle time on transit links which is dependent on unreliable traffic conditions for bus services on the road network. This also affects passengers’ waiting times at stops and capacity-related delays. They show that the traditional stochastic user equilibrium transit assignment is a special case of the proposed model when there is no variation on the travel times for transit service.
Florian (2003) and Bell (2003) provide a description of frequency-based models and capacity constraints equilibrium assignment models, respectively, while Schmocker and Bell (2007) highlight the reasons and merits of using a frequency-based approach in a dynamic, capacity-constrained setting.

In headway-based transit assignment models, passenger demand is assumed to be constant within the period of analysis. Also, transit routes are assumed to operate a constant headway and the passenger waiting time is a probabilistic function of arrival patterns at stops. The strategy-based approach is called static, as it considers only the average values of demand and supplied services, without taking into account their within-day and day-to-day variability. Recently, the need for a proper modelling of overcrowded high-frequency transit networks using the frequency-based approach has grown. However, the static representation of line-based procedures has limitations in modeling situations where passengers might not be able to board the next arriving vehicle. Other situations include the peak-hour bottlenecks for scheduled-runs, where a transfer-point observes high-demand levels at only certain times during the modelling period (e.g. high-demand for subway lines at Union Station, Toronto, upon the arrival of scheduled GO services). Such static representations, while allowing capacity constraints to be considered, do not allow the prediction of the number of passengers that many not be able to board transit vehicles. Research efforts have been directed towards the development of a dynamic frequency-based approach. The basic idea is to vary the OD demand by large-enough time-intervals to replicate various overcrowding conditions for the transit service, allowing passenger’s choices to be directly related to levels of services in each time interval. This seems to provide a better estimation of route loads since low-frequency routes with low-overcrowding conditions may become more attractive compared to traditional static frequency-based procedures. The joint representation of schedule-based services and frequency-based approaches is introduced to explicitly model vehicle’s capacity. Examples of such attempts are Meschini et al. (2007), Teklu et al. (2008) and Schmocker et al. (2008).
2.3 The Second Wave – The Schedule-based Approach

A second wave of transit assignment models has focused on the distribution of passengers to scheduled runs. That is, a passenger is assigned to a specific vehicle trip that belongs to a route, compared to just a route in headway-based models. The transit service timetable is used to describe the time-dependent movement of scheduled vehicle trips, hence the name schedule-based or timetable-based approach. The schedule-based approach is considered dynamic as it aims to model the clock-dependent evolution of supply and demand to generate transit vehicle loads and network level of service.

Since the timetable-based approach models scheduled transit vehicles, it requires a time-dependent origin-destination demand matrix. On the other hand, the strategy-based approach assumes fixed-demand for the modelling period. The waiting time for boarding a line in headway-based assignment models is assumed to be probabilistic and depends on the arrival profile of passengers at boarding stops. In schedule-based approaches, the passenger waiting time for boarding a scheduled-run is deterministic; it is a function of the arrival time of the passenger at the boarding stop and the arrival time of the scheduled-run at the stop. In strategy-based approaches, passengers are assumed to choose in an indifferent adaptive way as they board the first vehicle to arrive from their attractive list of routes. This is based on the assumption that passengers arrive at stops following a probability distribution while transit vehicles arrive at stops at random based on a distribution of line’s headway. Passengers, in a schedule-based assignment model, choose in an intelligent adaptive way considering actual vehicle arrival times at stops and comparing that with other scheduled-runs.

The application of headway-based assignment models is widely accepted for transit service with high-frequency services, un-congested scenarios, low punctuality and no user information. The dynamic representation of transit service by scheduled vehicle trips was initially introduced for the modelling of low-frequency transit systems (such as inter-city service or in small-size cities), of situations where peak volumes are not properly represented, when passengers cannot board the first vehicle to arrive from an attractive set, and to represent schedule-penalties due to timetable constraints.
For timetable-based assignment, a supply representation of the clock-dependent movements of each transit vehicle is needed. There have been basically two methodologies. One is to expand the network representation in the time-dimension (i.e. individual time-scheduled runs are represented). The minimum path problem can be then solved by conventional algorithms at the expense of computational efficiency and memory requirements. The other alternative is to keep the simplicity of the headway-based supply representation while developing more complex minimum path finding algorithms.

Tong and Richardson (1984) presented one of the first contributions to the modelling of high-frequency transit networks using a schedule-based approach. They proposed a time-dependent optimal path algorithm for transit networks where passenger demand and vehicle movements are time-dependent. The model uses a Dijkstra shortest-path algorithm and a set of heuristic path efficiency rules to find the quickest path from origin to destination assuming four trip disutility components: in-vehicle time, waiting time, walking time, and transfer penalty. A different representation, considering traditional frequency-based network models, is called the mixed line-based/database approach. This approach uses headway-based representation to model the spatial topology of the transit service. It adds a database for links and nodes to define the time-dependent movement of transit vehicles. Florian (1998) presented a deterministic path-finding algorithm for the selection of the least generalized cost path. It is an enhanced version of the mixed line-based/database approach where the time-dependent movements of scheduled runs is expanded in real-time (during execution) for only relevant parts of the network. This results in significant improvement in memory usage and computational efficiency. This representation is implemented in EMME/2 release 9.

For low-frequency services, Nuzzolo and Russo (1996) introduced the space-time diachronic graphs. In the diachronic graph, each scheduled vehicle trip is represented by a sub-graph where temporal-nodes represent arrival and departure times of the transit vehicle at stops. Similar sub-graphs are used to represent times and locations of trip arrivals and departures, where access and egress links connect run-graphs and trip-graphs. A dual graph representation, initially meant to represent turn probabilities and restrictions in road networks (see Anez et al., 1996), was
extended to model public transport networks. In such representation, each scheduled vehicle run is represented by a *node* and connections between nodes are through *links* to model vehicle arrivals and departures. Such representation was used in modelling a large-scale transit system for the Copenhagen-Ringsted railway system (Nielsen and Jovicic, 1999). Nielsen et al. (2000), using mixed line-based/database representation, reported an improved computational time of their earlier work for the Copenhagen-Ringsted application with *dual* graph representation. Nguyen et al. (2001) proposed a graph representation based on the concept of *path available capacity* to model flow priorities at boarding points and to account for the FIFO rule. It uses discrete space-time graphs with space-time nodes and links. Friedrich et al. (2001) proposed a branch and bound path finding algorithm for the modelling of schedule-based transit services; this path finding algorithm implemented in the commercial transportation planning software VISUM.

Since the schedule-based approach was first introduced for modelling intercity and low-frequency services, it has received growing interest to model urban transit systems as it is capable of representing more coherent user behavioural hypothesis in response to service reliability and the introduction of ITS. For within-day dynamics, Hickman and Wilson (1995) developed a framework to evaluate path choices in public transit systems where passengers receive information in real time regarding projected vehicle travel times. They developed a path choice model that not only accounts for time-dependent and stochastic transit service characteristics, but also for *dynamic* path choice behaviour, where passengers update their decisions while waiting at transit stops. This model was further developed and its statistical aspects are analyzed in Hickman and Bernstein (1997). Tong and Wong (1999) presented a dynamic transit assignment model, where transit service was represented by scheduled timetables. Passengers’ perceptions to various travel cost components were explicitly modelled through sensitivity coefficients that vary among passengers. They considered time-varying supply and demand conditions, where a time-dependent transit OD matrix can be estimated (Wong and Tong, 1998). Poon et al. (2004) proposed a user-equilibrium model and a solution algorithm for congested, dynamic and schedule-based transit services. They explicitly modelled individual vehicle’s capacity and FIFO queuing discipline at boarding stops. They, however, did not consider departure time and entry/exit point choices, simplifying the path choice to only a
run-choice problem. Bungartz et al. (2005) presented an extension of the within-day schedule-based assignment procedure proposed in earlier work (Friedrich et al., 2001) to multi-day assignment modelling system to capture demand varying within a week-period, typical for railway systems.

Nuzzolo et al. (2001) presented a dynamic stochastic path choice model which considers variations in transit services and passengers’ learning and adaptation. The model is called “doubly dynamic” as it accounts for within-day and day-to-day variations. Passengers are assumed to make a discrete choice of trip choices based on travel disutility (e.g. in-vehicle time, out-of-vehicle time, transfer penalty) and the scheduled arrival time at destination. Nuzzolo et al. (2003) report on the theoretical foundation of the schedule-based approach for modelling travellers’ behaviour in transit networks. It provides a treatment of the transit path choice problem in low-frequency and high-frequency transit services using the diachronic graph representation of the transit service. In these models, path choice model implements the Logit/Probit formulations to calculate the choice probabilities. The run-choice is varied in both within-day and day-to-day assignment based on learning, which is represented by an exponential updating mechanism of run attributes. This is not considered a learning-driven approach as the learning process does not follow an explicit learning algorithm and it is done on an aggregate level using a common pool of information that is accessed by all travellers.

Tong et al. (2001) provide a summary of the inherent advantages of the schedule-based modelling framework over frequency-based assignment models. Wilson and Nuzzolo (2004, 2008) provide a collection of contributions towards the development of theoretical foundations and applications of the modelling of transit services based on a timetable representation.

As recent efforts are directed towards the dynamic representation of frequency-based approaches, similar attempts suggest the use of strategy-concepts in the schedule-base framework. Hamdoucha and Lawphongpanich (2008) propose a user equilibrium transit assignment algorithm with detailed description of transit service dynamics and modelling travellers’ behaviour using strategies and user preferences sets. They use time-extended graphs to represent the transit service dynamics over pre-defined time-intervals; thus allowing them to
represent the assignment problem as a variation inequality and providing a solution algorithm using the method of successive averages that outputs travel strategies.

Other applications of the schedule-based approach have been proposed in timetable setting (e.g. Russo, 1998; Crisalli and Coppola, 2001) and time-dependent demand estimation (Wong and Tong, 1998; Nuzzolo and Crisalli, 2001).

2.4 The Next Wave – The Microsimulation-, Learning-based Approach

From reviewing the literature, it is clear that the modelling of service dynamics is the driving force for the latest developments. The proper representation of service dynamics is not only important for modelling existing conditions but also for the evaluation of ITS technologies that require detailed representation of service conditions. The change in supply representation necessitates the change in travellers’ behaviour modelling; it is more likely that passengers will consider individual vehicle trips when planning or executing transit trips compared to just line frequencies. The introduction of Automated Travellers Information Systems for both auto-travellers and transit-riders has motivated researchers to explore more behavioural models that deal with the role of information, knowledge levels and decision-styles.

As has been the tradition, the evolution of transit assignment procedures follows the steps of the progress of traffic assignment approaches, yet acknowledging the distinctive features of the public transport networks. When static traffic equilibrium models were applied for evaluation of highway expansion projects (see Florian, 1977), only heuristic approaches were available for the transit assignment problem. By 1990, there were several successful attempts for dynamic modelling of the traffic assignment problem using random utility theory and stochastic user equilibrium methods (Small, 1982; Alfa, 1986; Ben Akiva et al., 1984, 1986; Sheffi, 1985). At that time, the schedule-based approach for dynamic representation of transit services was just born. Recently, traffic assignment procedures implemented a microsimulation approach to account for the non-linear relationship between the transportation system behaviour and changes in supply and/or demand (Mahmassani and Liu, 1999; Balmer et al., 2004). Microsimulation has been particularly attractive for its ability to describe various behavioural hypotheses of individual
decision-makers, from the traditional utility maximization models to models with basis in psychology. In this regard, learning-based models have been shown to result in different and more realistic assignments (Nakayama et al., 1999; Arentze and Timmermans 2003; Ettema et al., 2005). With microsimulation representation, individual travellers are explicitly modelled as cognitive agents. This has led to the shift to an agent-based microsimulation modelling framework that has been successfully and aggressively applied in activity-based models (e.g. Timmermans et al., 2003; Salvini and Miller, 2003; Roorda and Miller, 2006) and in traffic assignment models with information provision (e.g. de Palma and Marchal, 2002; Rossetti and Liu, 2005; Ettema et al., 2005). It should not then come as a surprise that the next generation of dynamic transit assignment algorithms will implement microsimulation, learning-based and multi-agent concepts. This is driven by the increasing popularity and applicability of agent-based, microsimulation activity-based modelling systems which are not naturally linked with existing transit assignment models.

In the traffic assignment field, Schofer et al. (1997) suggest that, while drivers value real-time traffic information, drivers seek to incorporate their own knowledge and perspectives in the development of route plans, and expect these to be superior to those prepared through a navigation computer. The traffic simulation analysis conducted by Nakayama et al. (1999) indicated the possibility that drivers’ beliefs toward travel time formed by initial driving experiences may be deluded (i.e. the travel time of a route is believed to be longer than it actually is due to an excessive experienced travel time resulting in that the driver abandon this route). Furthermore, the simulation analysis has shown that this delusion may result in deluded equilibrium, which is different from user equilibrium and may represent much less efficient traffic conditions. Thus, understanding the formation of anticipated travel time through initial driving experience and information acquisition would be crucial for understanding drivers’ learning in route choice and also for linking microscopic drivers’ route choice behaviour and macroscopic dynamics in network traffic flow.

Although several previous studies have investigated the effect of traffic information on auto driver behaviour (such as route and departure time choices), fewer studies have attempted to explore the impact of transit information on passenger behaviour. For example, Polak and Jones
(1993) examined, based on an in-home pre-trip information system, the relationship between the process of information acquisition and changes in travel behaviour. Hickman and Day (1996) reviewed several transit information systems. Kitamura et al. (1995) conducted in-laboratory interviews to evaluate the benefits of a pre-trip information system, using a PC-based prototype. Khattak et al. (1996) surveyed different Advanced Public Transportation Systems (APTS) and information systems and their potential impacts on travellers’ behaviour and transit operations. Abdel-Aty (2001) investigated the effect of ATIS on transit ridership, and emphasized the potential of ATIS for increasing the acceptance of transit as a commute mode.

Recently Gentile et al. (2005) proposed a general framework for determining the probability of boarding each line available at a stop when online information on bus waiting times is provided to passengers. They derived a mathematical model for route choice behaviour where passengers utilize online information about the waiting times of the available lines in addition to the headway distributions and the expected travel times. They expanded their results to general headway distributions (the Erlang family of distributions), and they showed that the case of common lines with no information provision (e.g. exponential distribution for headways) is a special case of the proposed model. Using a test network, they showed that online information had a significant impact on route loads. One interesting result was that while total expected travel time decreased with information provision at stops, the expected waiting time increased as passengers are more likely to accept longer waiting times to board a faster route. This results was more visible with high service regularity.

Our proposed framework fits within the vision for the next wave of transit assignment models. It is inspired by the early work of Cascetta and Cantarella (1991) and the recent contribution of Ettema et al. (2005) in the modelling of within-day and day-to-day dynamics of auto assignment and the explicit manipulation of auto-travellers’ learning and adaptation, respectively. The developed approach combines the nonequilibrium framework for assignment procedures proposed by Cascetta and Cantarella and the agent-based, learning-driven path choice model presented in Ettema et al. It applies similar concepts to the transit assignment problem and passengers’ behaviour modelling, taking into consideration the distinctive features of public transport networks and transit riders’ travel behaviour.
One of the pioneering work in non-equilibrium based assignment models for auto-demand was presented by Cascetta and Cantarella (1991). They addressed the lack of explicit modelling of day-to-day and within-day variations in supply and demand within static equilibrium procedures where a fixed-point solution is searched. They argued that the equilibrium framework is not structurally compatible with observed phenomena, such as habitual behaviour, random variations in demand and network conditions, and transient states of network conditions following modifications. They then proposed a stochastic process model that generates steady-state arc-flows (comparable to Stochastic User Equilibrium outputs) with the explicit representation of day-to-day adjustments in travellers’ decisions and within-day variations of demand. They defined the states of the stochastic process in terms of path-flows; path-flows are assumed to take different values (i.e. various feasible states) in different days due to the variation in users’ decisions and network conditions. The evolution of the path-flows in successive days is then modelled as a Markovian stochastic process, where the uniqueness and existence of steady-state distributions are guaranteed when the Markovian process is ergodic and irreducible (see Chapter 4 for description of such properties). They also provided a solution algorithm to finding the steady-state distributions. Cantarella and Cascetta (1995) proposed conditions for the existence and uniqueness of stationary probability distributions. The model, however, did not consider en-route replanning due to variations in network conditions or information provision. The model represents learning and adaptation through a pool of experience and information accessible by all passengers (i.e. not agent-based). Examples of research efforts in this direction include Davis and Nihan (1993), Watling (1996), and Hazelton and Watling (2004)

The work by Ettema et al. (2005) represents learning and experience on the individual level for the auto-assignment problem. They promote the use of microsimulation methods to model the system-level phenomenon, e.g. congestion, from individual-level choices. They used a microsimulation platform to account for the travel time uncertainties on departure time choices, thus allowing for the modelling of day-to-day adjustments of travellers’ behaviour. They used formal models of knowledge and acquisition to integrate new experiences with past knowledge of the traffic conditions. They used mental models to represent traveller’s experience of network conditions. Travellers are assumed to be decision-makers who decide on the trip departure time
from origin. The study therefore did not consider other trip choices, e.g. route choice. In addition, the application did not present a full linkage between individual decision-making and simulated experiences; they only modelled a sample of the population and it was not straightforward to estimate the impact of travel time uncertainty on the congestion levels (which is related to 100% sample).

To the author’s knowledge and based on the literature review, the first attempt to model the transit assignment problem using a multi-agent, learning-based approach was presented by Wahba (2004) and published in Wahba and Shalaby (2006a). We presented a conceptual development of a modelling framework for a dynamic transit assignment procedure based on agent- and learning-based concepts. The agent-based representation allows the explicit modelling of the learning and decision-making activities of individual passengers. The framework presents a stochastic process approach (i.e. nonequilibrium-based) for modelling within-day and day-to-day variations in the transit assignment problem. It allows for the handling of learning and adaptation of passengers’ travel behaviour (i.e. trip choices). While it overcomes the structural limitations of equilibrium-based models, it still gives the steady-state run loads (comparable with stochastic user equilibrium loads). The connections between the proposed framework and emerging activity-based models were highlighted. They however did not present the theoretical foundation and detailed representation for modelling travellers’ behaviour (i.e. path choice modelling) in a dynamic setting.

Wahba and Shalaby (2005) present a prototype implementation, originally developed in Wahba (2004), of the conceptual framework for a hypothetical medium-size transit network. The prototype modelled the day-to-day variations and dynamics of travellers’ behaviour using the n-armed bandit algorithm from the Reinforcement Learning (Sutton and Barto, 1998) field to represent the path choice probabilities. It results in a dynamic network manipulation through the microsimulation model, time-dependent trip choices, and a dynamic network loading procedure. This is important for modelling fixed-scheduled transit networks when the system performance is measured by passenger volumes (run-loads) and not route-flows. However, there are some limitations to the previous work of Wahba (2004). The implemented n-armed bandit problem considers departure time, stop and path choices to be joint – i.e. there is no en-route replanning
as passengers are assumed to decide on all trip choices prior to trip execution. The passengers’ adaptation to within-day variations was not treated in the prototype implementation. Due to the medium-size network used, the common-lines problem was not explored in great detail. Therefore, entry/exit points and transfer choices were not explicitly accounted for. More importantly, there was no formal procedure to calibrate the model parameters. The work presented in this thesis extends on the developments of Wahba (2004) to overcome such limitations along with other contributions (see Chapter 7). This document also presents a large-scale application of the proposed modelling system to showcase its applicability and significance.
This chapter discusses the conceptual development of the proposed multi-agent, learning-based approach to the transit assignment problem. It provides a general description of the framework components. It introduces the mental-model concept and its linkage to the modelling of traveller’s choices. It also highlights the relevance of the proposed approach and the requirements for the modelling of ITS deployments. Details on the proposed dynamic path choice model are presented in the next chapter with a prototype implementation in Chapter 5 and a full-scale application in Chapter 6.

There is a growing need for a new modelling framework for the transit assignment problem, particularly to represent behavioural responses under information provision. Emerging information and communication technologies, which are used commonly in Intelligent Transportation Systems (ITS) and Advanced Public Transportation Systems (APTS), are expected to improve transportation system performance, in particular transit services. Such information systems are designed to provide timely information to transit passengers on the conditions of the network, thus affecting travel choice behaviour. Traditional transportation planning methods have serious limitations in evaluating the effects of information technologies, since they are neither sensitive to the *types* of information that may be provided to travellers nor to the *traveller's response* to that information. Existing approaches are not adequate for evaluating the impacts of ITS deployments on service reliability, which in turn affect passengers’ behaviour.

Traditional travel behavioural models for path and departure time choices need to be modified to be applicable in the context of dynamic transport conditions and ITS. This chapter discusses the issues concerning the development and implementation of a new modelling framework for the

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2 Portions of this chapter has been reproduced, with modifications, from the following previously published materials:
- Wahba and Shalaby (2008)
- Wahba and Shalaby (2007)
transit assignment problem, namely the MIcrosimulation Learning-based Approach to TRansit ASSignment – MILATRAS.

The proposed framework is inspired by the early work of Cascetta and Cantarella (1991) and the recent contribution of Ettema et al. (2005) in the modelling of within-day and day-to-day dynamics of auto assignment and the explicit manipulation of travellers’ learning and adaptation, respectively. The developed approach combines the nonequilibrium framework for assignment procedures proposed by Cascetta and Cantarella and the agent-based, learning-driven path choice model presented in Ettema et al (see Section 2.4 for more details). MILATRAS applies similar concepts to the transit assignment problem and passengers’ behaviour modelling.

MILATRAS can be adapted to model systems of large-size, high frequency transit networks without ITS deployment, as many existing static models are capable of. MILATRAS, in addition, is suitable for modelling systems of medium-size networks with medium to low frequencies, where behavioural hypotheses of existing static models are violated (e.g. random arrival of passengers at stops). MILATRAS is also geared to modelling recurring trips such as work and school trips in the AM peak period.

The goal is to create an environment for travel demand modellers to experiment with dynamic microsimulation sub-models of dynamic departure time and transit path choices, passenger’s perception updating and passenger’s within-day and day-to-day travel choice dynamics. In this context, the proposed modelling framework provides an experimental tool that helps transit planners and researchers to analyse a broad range of transportation policies and methods.

3.1 The Transit Assignment Problem – Dimensions of Choice

When a passenger $P$ leaves origin $O$ for destination $D$ at departure time $T$, this passenger’s decision, along with other passengers’ trip-timing decisions, determine the temporal pattern of origin-destination trip demand for the transit network, which is important for capturing the within-day temporal variation of demand. Passenger $P$ may revise the timing of the trip from one day to the next to determine whether it should be changed or not, depending on his experience
with the transit network conditions and information provided to him; this is important for capturing the *day-to-day temporal* variation of demand. Under the provision of real-time information, the trip timing decision may be changed on a *real time* basis, rather than a planned basis, such as delaying departure time from work to avoid congestion. Passenger P also decides on a transit path A (consisting of an origin stop, a run or a sequence of runs, and a destination stop) to take from the origin O to the destination D, when leaving the origin at departure time T. While the departure time choice influences the temporal pattern of trip demand for the transit network, the path choice directly determines the spatial distribution of trips in the network. The modelling of path choice is important for capturing the *within-day* and the *day-to-day spatial* variation of demand (or route loads). Based on provided real-time information, the path choice decision may also be changed on a *real time* basis to reflect the “adaptive” behaviour of transit travellers. Given a path A and a departure time T, determining a passenger’s travel/waiting time along a segment of the transit network is not straightforward; this is because of the existence of nonlinear interactions among individual decisions, which are made non-cooperatively by passengers. In particular, the in-vehicle and waiting times for a particular run is dependent on the congestion levels generated along that path, which depend on the decisions of all other passengers.

Because in-vehicle and waiting times are not constant, the need arises for simultaneous consideration of passenger decisions and the system performance, which adds more complexity to the problem. This yields the three primary time frames of interest in the analysis of temporal and spatial aspects of traveller’s decisions and associated network performance: day-to-day, within-day, and real-time. Also, there are three dimensions in the transit passenger’s decision making process: the departure time choice, the stop choice, and the run(s) choice. Such dimensions were either simplified (e.g. stop choice) or ignored (e.g. departure time) in current approaches. In order to properly model path choices in these contexts, an integrated dynamic modelling framework is needed that is: sensitive to time-dependent and stochastic transit service characteristics (supply modelling), that models adaptive departure time and path decisions by passengers (demand modelling), and that captures the interaction between passenger decisions and transit network performance (*via* an integrated framework). This integrated dynamic modeling framework needs to address: the time-dependent pattern of flows and their distribution...
over space, the systematic changes of the passenger decisions within the day and from day to
day, and the interaction between the passenger decisions and the system performance. As such,
the modeling framework has to deal explicitly with trip timing and path selection, and the
mechanism through which passengers adjust these decisions in response to experienced
congestion, control measures, and supplied information. MILATRAS, the MIcrosimulation
Learning-based Approach to TRansit ASsignment, has been conceived based on the above
premises. MILATRAS has an overall microsimulation framework that consists of a number of
individual modules, and it is structured to be integrated with Urban Transportation Models
(UTMs) – see Figure 3.1.

3.2 MILATRAS – An Integrated Modelling Framework

The impact of any service change, including infrastructure improvements as well as providing
real-time information, depends critically on passenger’s behaviour. This implies that simulating
individual passengers directly to capture such impacts may be useful. Forecasting the impact of
service change, in particular with the existence of ITS technology, is difficult. It has been shown
that learning and adaptation methodology is a powerful tool in modeling the dynamics in
responses over time (Ettema et al., 2005). The transportation system, in particular the transit
system, is complicated, and given the system’s path dependencies and the time-varying factors,
system equilibrium is often not achieved. This represents a great challenge to equilibrium-based
models. Therefore, in the absence of explicit equilibrium conditions, a future state of the
transportation system can only be estimated by explicitly tracing the evolutionary path of the
system over time, beginning with current knowledge conditions (Miller, 2003). The proposed
framework considers the possibility of emergent behaviour to be predicted, which is not
hardwired into the model.

The framework consists of two key components: an overall microsimulation framework and a
number of individual sub-models, as shown in Figure 3.1. The microsimulation environment
means that the supply side is represented by a time-dependent and stochastic model, while
individual transit passengers are modelled as agents who learn and adapt to service variations.
The interaction between supply modelling and demand representation is facilitated by three
assistant-managers: the feeder-manager, the loader-manager, and the feedback-manager. The purpose of the assistant-managers is to build modularity into the framework and separate the supply and demand components via ‘bridges’. These bridges enable all the combinations of different technologies and/or architectures of the sub-modules implementations. Besides, each assistant-manager has another task for the transit assignment process.

The input to MILATRAS is the demand for the transit network. This demand (transit OD matrix or trip table) is converted to passenger-agents, each representing a transit trip. Each passenger has a planner component that is responsible for selecting only one path and a departure time that reflects that passenger’s preferences and is based on the mental model of previous experiences. The feeder-manager connects the GIS-T module and the network microsimulation module and feeds passenger-agents with the constructed mental model. This results in a stochastic process of different choices for individual passengers; therefore, the loader-manager’s task is to communicate dynamically passengers’ choices to the network-microsimulation module. Then, the microsimulation model (through the transit-handling module) handles the dynamics of the transportation network according to the passengers’ choices and provides experienced measurements for individual passengers. Afterwards, the feedback-manager is responsible for updating each passenger’s memory, according to a learning mechanism. The whole process repeats for many days.

MILATRAS provides an environment where independent researchers can build sub-models, such as perception updating mechanism or day-to-day departure time choice models with “plug-and-play” features within the dynamic modelling setup. The framework, complete with its sub-models, is an appropriate tool for experimenting with policy decisions and observing the changes in simulation outcomes. The microsimulation environment and the individual modules are explained next.
Figure 3.1 The MILATRAS Framework
3.2.1 The Microsimulation Environment

One of the most interesting aspects of transportation modelling is the adaptive behaviour of people in response to a change in their environment. It is well known that there is a mutual dependence between traveller behaviour and system performance; congestion is the result of *the execution* of departure time and path choices in a capacity constrained system, but individual choices are based on *the anticipation* of congestion. Learning techniques are important to model passenger’s adaptation to the changing environment; this is because the transportation system constantly fluctuates and learning methods allow us to represent how travellers react to these changes without necessarily converging the system to an equilibrium state (Brenner, 2004).

One approach is to model each passenger as a microscopic entity and model that entity’s reaction to the system directly, while modelling the time-dependent system performance as a response to passengers’ behaviour; this approach is referred to as the “multi-agent” simulation environment. In Multi-Agent Simulation Systems (MASS), it is possible to represent individual agents coupled with their behaviour; the main point is not only the functionality of single agents in the transportation system, but also how they interact. Behavioural rules define how agents behave and how the environment changes as a result of agent’s decisions. Interaction rules describe how agents interact with each other and the environment, see Figure 3.2. The system state evolves through agent interaction and communication with other agents and objects and their environment(s). The multi-agent system then evolves to a pattern from which useful macro-level information can be extracted.

An agent’s success in achieving his goal(s) depends on his ability to anticipate the environment status and, accordingly, the return of his possible actions. An agent representing a person would therefore need: the ability to perceive information about the environment, the functionality to update his experience, a model to anticipate his environment status in the future, and a decision process that selects behaviour based upon the anticipatory model of the environment to achieve some set of goals. This is important in modeling space-time dynamics within urban systems.
since it allows for studying the relationships between micro-level individual actions and the emergent macro-level phenomena.

Agents are objects of particular interest in microsimulation models in that they exhibit autonomous behaviour that is typically the primary focus of the microsimulation. Trip makers making travel decisions are examples of agents who independently make decisions as a function of their own attributes, experiences and the state of the system that they find themselves within. These actions, in turn, change the system state over time (e.g. congestion levels). Agents collect information about their environment as they interact with it, and use it to develop anticipatory models of the environment. The decision process arises from an adaptive learning process driven by the agent’s desire to maximize some payoff through its actions over time.
Figure 3.2 Schematic Relationship between Agents, Objects and Environment

Environment
the topological space where agents and objects are located, move and act, and
where signals, due to agents’ actions, propagate

Objects
the set of all represented passive entities that do not respond to stimuli and have physical characteristics (e.g. Stop)

Interaction Rules
Describe how agents interact with each other and the environment

Agents
like people (or vehicles) who have characteristics, goals and rules of behaviour

Behavioural Rules
Define how agents behave based on the state they find themselves in

Observations are generated through agents interaction/communication with other agents, objects and their environment(s)
Transit assignment is a process of interactions between individual passengers and transit services. Traditional transit assignment procedures assume User Equilibrium (UE). Mahmassani (1997) questioned the wisdom of investing much effort to solve a difficult problem that may not be particularly relevant to reality, given that the existence and nature of a time-dependent equilibrium is itself a tentative proposition at best. This is manifested by the fact of limited behavioural contents of most formulations and algorithmic procedures proposed for the network problem. By moving away from any assumption about equilibrium, a simulation assignment model that tracks individual trips through the network appears more appropriate. It provides a framework for examining the network-level implications of behavioural rules for the various tripmakers decision-making processes in a way that is consistent with reality, rather than imposing a rigid set of assumptions for the sake of mathematical and computational convenience.

If equilibrium has been reached, this means that travel times are identical across all paths as long as equilibrium is maintained. For equilibrium to be maintained, a traveller, who is repeatedly making the same trip (e.g. work trip), has to choose a departure time and path so that equilibrium will not be disturbed. Since it is impossible to know other traveller’s choices before travelling (i.e. non-cooperative game), it is thus more likely to assume that equilibrium will not be sustained than assuming that the traveller’s decision making process is consistent with network equilibrium. Therefore, it is important to trace the reasons for reaching, or not reaching, equilibrium rather than assuming equilibrium apriori (Goodwin, 1998). Some studies found that under reasonable users’ learning and adaptation mechanism, the system may not converge to an equilibrium state (Horowitz, 1984).

The transportation system is path dependent (i.e. equilibrium may be attained through different paths); this is a complex property that cannot be captured by traditional equilibrium analysis. In a study by Mahmassani et al. (1986), traffic flow converged because of the “indifferent band” assumption derived from the notion of “satisficing” (Simon, 1990). In another study by Nakayama and Kitamura (2000), incorrect beliefs and heterogeneity in drivers’ perceptions of travel times seem to be the cause of convergence. It is worthy to note in both studies, where convergence is attained, drivers are not assumed to behave completely rationally with perfect
information. These results imply that perfect information may not be a necessary condition for convergence to user equilibrium. To the contrary, the presence of a large number of drivers who firmly believe in their route choices, but in fact have incorrect travel time information, may lead to network equilibrium (i.e. convergence to a stable state). Nakayama and Kitamura (2000) call such phenomenon *deluded equilibrium*; and they argue that it may result in habitual behaviour and can be dissolved by providing reliable real-time information.

When the time-dependent network flows, the understanding of travel demand forecasting, or the sensitivity analysis of control measures are the main concerns, equilibrium analysis may not be the appropriate method. Equilibrium analysis assumes that the state in which the traveller’s utility has been maximized has been achieved. While this is assumed *apriori*, the process of adjustments or the evolutionary path toward equilibrium are not properly addressed. Equilibrium analysis is thus often viewed as static, rather than dynamic in nature.

A different methodology to approach the original problem is using *learning algorithms*. It is arguable that learning algorithms no longer guarantee a UE solution. One can, however, assume that learning algorithms converge to a fixed point (if everything is deterministic) or go towards a steady-state density (for stochastic systems if they are Markovian) (Raney and Nagel, 2003). Learning is important to model passenger’s adaptation to the changing environment; the transportation systems permanently fluctuate and learning is rather important because it allows us to react to these changes and not because it converges to an equilibrium. The removal of the equilibrium assumption through the framework enables a more dynamic and behavioural approach for the transit assignment modelling process.

### 3.2.2 Supply Modelling

This component includes a transit service model that represents explicitly vehicle travel time characteristics in a stochastic and time-dependent fashion. The model is sufficiently microscopic to allow stochastic vehicle departure and running times, but it also allows a representation of the larger interactions among the transit routes of the network. Previous transit route models, such as Andersson and Scalia-Tomba (1981), Powell and Sheffi (1983) and Marguier (1985), describe
the stochastic movement of vehicles on a single bus route, incorporating passenger boardings and alightings as well as stochastic running times between stops. Because of the microscopic description of vehicle movements and passenger boarding and alighting, these analytic models are too detailed to allow aggregation from a single route to a more complete transit network. This restriction limits the use of these models for trips that may involve several (connecting) routes. An appropriate network performance model is needed to obtain the experienced travel and waiting times, convenience measures, and congestion and capacity effects, etc. that change from day to day for each passenger. MILATRAS includes a network performance model that has two modules: a GIS-T module (for static representation) and a network microsimulation module (for dynamic representation) – see Figure 3.1.

3.2.2.1 The GIS-T Module

The term “GIS-T” refers to a Geographic Information System (GIS) representation of the transportation network. It is acknowledged that the topology of the transit network is very complex. Recently, some transit applications have included a GIS model as an essential component to treat the complex nature of the transit network, with different public transport modes, lines and transfer points (Huang and Peng, 2001a and 2001b). The GIS representation is meant to enhance the path-finding capability of the proposed assignment framework. Huang (2007) proposed a pattern first search (PFS) algorithm that is dependent on the representation of the transit service on a GIS platform. As the need for online transit planning services increases, GIS representations of transit services become essential in order to generate spatio-temporal, schedule-coordinated paths for trips. Examples of online trip planners with GIS platforms are numerous, e.g. Mississauga Transit Planner (www4.mississauga.ca/ClicknRide/)

Nielsen et al. (2001a) discussed the evolution of the transit network models in GIS. In parallel with the development of topological models in GIS, transport-modelling packages usually include data models for the transit network representation and management. Examples include TransCAD (2008) and VISUM (2008). Transportation Object Platform (TOP), another example, is a complex data storage and management tool, developed by the research team at the Center for Traffic and Transport in Technical University of Denmark (Nielsen et al., 2001b). TOP is based
on the integration of relational databases, GIS and methodologies of object-oriented software design, and implemented within the framework of ArcInfo, ESRI (http://www.esri.com/). In TOP, each stop is geocoded, and the transit route network is superimposed on the road map and developed as a sequence of stops. The transit network timetable is encoded and the transfers between stops are represented. This enables the coding of an integrated multi-modal transportation network.

The purpose of the GIS-T module is to store the geocoded data of transit trips (origin and destination of each trip), to determine the available/accessible transit stops for each passenger-agent based on a pre-defined catchment area, and to define for each transit trip access/egress walking times between any trip origin/destination and a particular transit stop. Not only does the access walking time to an origin transit stop affect the route choice, but also the egress walking time and/or accessibility from the destination transit stop. While it has always been overlooked, the stop choice is very critical to the transit assignment process and may affect considerably the loads on all routes; changing a stop most probably results in changing the route (and hence the transfer connection). The GIS-T module is important to test and evaluate land-use policies, especially when spatial analysis is required.

The input to the framework is the demand for the transit service; this can be the traditional OD transit matrix – see Table 3-1 for an example. From Figure 3.1, this can be done through a user interface, or the framework can be integrated with a larger trip-based (or agent, activity-based) urban transportation model. This matrix is then disaggregated into individual trips (i.e. an OD trip list); that is, the GIS-T module generates individual trips such that summing up the trips over zones will result in the given OD matrix – see Table 3-2 as an example. The modelling of intrazonal trips is dependent on whether the input OD transit matrix contains this information or not. While the OD transit matrix would assume traffic analysis zones (TAZs), the framework models transit trips at the stop and link levels. The GIS-T module has a sub-module, the OD-Generator, that seeks to make the trip list geographically anonymous by randomly generating an OD-Geo List with origin and destination geographical locations, where each OD pair in this list corresponds to an OD pair from the OD trip list – see Table 3-3.
The GIS-T module has another sub-module, *the Path-Generator*, to determine for each passenger-agent the initial set of possible/eligible transit paths, where spatial and/or temporal constraints may apply (e.g. no more than one transfer is accepted). In order to implement the *Path-Generator* sub-module, a *data model* of the transit network GIS representation needs to be developed. Figure 3.3 shows the different information stored in the GIS-T data model about the transit demand and the transit network characteristics. The data model description is provided in Section 6.3.1 with an example of the Toronto Transit Commission (TTC) service network.
### Table 3-1 The OD Transit Matrix

<table>
<thead>
<tr>
<th>OD Transit Matrix</th>
<th>O</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>25</td>
</tr>
<tr>
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<td>5</td>
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<tr>
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<td>15</td>
<td>40</td>
<td>15</td>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>

### Table 3-2 The OD Trip List (Disaggregated OD Matrix)

<table>
<thead>
<tr>
<th>OD Transit Trip List</th>
<th>Ozone</th>
<th>Dzone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip #</td>
<td>Ozone</td>
<td>Dzone</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
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<tr>
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<td>3</td>
<td>2</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 3-3 The OD Geo List (Disaggregated OD Matrix)

<table>
<thead>
<tr>
<th>OD Geo List</th>
<th>Ozone-Geocode</th>
<th>Dzone-Geocode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip #</td>
<td>Ozone Geocode</td>
<td>Dzone Geocode</td>
</tr>
<tr>
<td>1</td>
<td>(X_{O1}, Y_{O1})</td>
<td>(X_{D1}, Y_{D1})</td>
</tr>
<tr>
<td>2</td>
<td>(X_{O2}, Y_{O2})</td>
<td>(X_{D2}, Y_{D2})</td>
</tr>
<tr>
<td>...</td>
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</tr>
<tr>
<td>25</td>
<td>(X_{O25}, Y_{O25})</td>
<td>(X_{D25}, Y_{D25})</td>
</tr>
<tr>
<td>26</td>
<td>(X_{O26}, Y_{O26})</td>
<td>(X_{D26}, Y_{D26})</td>
</tr>
<tr>
<td>...</td>
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</tr>
<tr>
<td>40</td>
<td>(X_{O40}, Y_{O40})</td>
<td>(X_{D40}, Y_{D40})</td>
</tr>
<tr>
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<td>(X_{D41}, Y_{D41})</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>63</td>
<td>(X_{O63}, Y_{O63})</td>
<td>(X_{D63}, Y_{D63})</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>100</td>
<td>(X_{O100}, Y_{O100})</td>
<td>(X_{D100}, Y_{D100})</td>
</tr>
</tbody>
</table>
Figure 3.3 A Sample Transit Network and its GIS-T Data Model Representation
3.2.2.2 The Network Microsimulation Module

The network microsimulation module is essential to the framework, as services in a transit network are time-dependent. Although there may be a pre-defined schedule, transit service performance varies by the time of day and day of week; the optimal path, therefore, from an origin to a destination varies accordingly. In order for passenger-agents to experience these variations, a microsimulation representation of the transit network is important. Hickman and Wilson (1995) define the uncertainty and time-dependence in vehicle travel times as the two critical elements of transit service that have not been adequately included in transit service models. Mahmassani (1997), while modelling within-day variation of commuter trips, suggested that models should well recognize the interrelation between user decisions and system performance, and thus treat the system attributes as endogenous – i.e. congestion should be modelled endogenously.

Not only do microsimulation models describe the behaviour of individual decision makers, but they can also incorporate the interaction between the system level and the individual level, due for instance to limitations of the system capacity. Interactions among individuals as well as between individuals and the system affect the assignment process; for example, the trip duration is influenced by the occurrence of congestion that is determined by the interaction between transit supply and decisions of individuals to use the transit network at particular times on particular routes. To capture the dynamics of the transit system, a time-dependent self-updated representation is needed, i.e. a microsimulation model.

Current assignment models do not consider properly the interaction between transit vehicles and other general traffic sharing the same road, although transit vehicles are usually delayed by other general traffic. In principle, to describe these delays, auto and transit assignment models should interact at the link level. The argument that these delays are usually reflected by the timetables does not hold for long-term forecast, where it might be easier to model delays instead of specifying timetable-based in-vehicle time manually for each planned scenario (Nielsen, 2000). In addition, the current practice of using nominal frequencies to determine the set of attractive
lines for a given pair of nodes is no longer correct. Nominal frequencies should be replaced by effective frequencies, which depend on the flows over the transit network. This means that attractive transit links cannot be defined in advance. In other words, the trip assignment process and defining attractive sets cannot be separated (De Cea and Fernandez, 2002). Besides the aforementioned dynamics of the transit network, Wahba and Shalaby (2005) provide more reasons why a network-microsimulation module is essential to the modelling of the transit assignment process.

It is important to mention that, because of the dynamic representation of the transportation network (i.e. a microsimulation model), passengers’ adaptive choice can be modelled not only relative to the transit line, as in static models, but also to the specific run of each line. In other words, the proposed approach considers the path choice as time-dependent. Microsimulation models have recently been considered as an essential component in urban transportation planning models, such as ILUTE (Salvini and Miller, 2003), MIT-SIMLab (2004), DYNASMART (2003) and DYNAMIT (2004).

3.2.3 Demand Modelling

The proposed framework is based on representing passengers and both their learning and planning activities explicitly. Learning activities include updating passenger’s mental model contents such as perception regarding transit network conditions (in order to provide predictions about transit network conditions), and updating perception regarding the accuracy of real-time information provided (i.e. value or reliability of real-time information systems). Planning activities include making trip choices (i.e. departure time choice, stop choice and run, or sequence of runs, choice). The learning process is concerned with the specification of the generalized cost function components, by which passengers consider their choices on day $d$. The planning process considers how experience and information about those components on previous days influence the choice on the current day.

The underlying hypothesis is that individual passengers are expected to adjust their behaviour (i.e. trip choices) according to their experience with the transit system performance. Individual
passengers base their daily travel decisions on the accumulated experience (i.e. mental model) gathered from repetitively traveling through the transit network on consecutive days; this is similar to the concept implemented by Ettema et al. (2005) for auto path choice. Individual behaviour, therefore, should be modelled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. Passengers plan their trips in non-stationary, and therefore uncertain, environments. This means that, in a dynamic sense, choices are contingent upon outcomes of previous choices. By repeatedly making a decision, an individual acquires knowledge (i.e. learns) about his environment and thereby forms expectations about attributes of the environments. Individuals may make different choices over time and thus learn which of these choices is more effective in achieving particular goals.

### 3.2.3.1 Mental Model Representation

Knowledge is a particular kind of data that represents information accumulated over time. Passengers make “experiments” in an attempt to understand the service characteristics of various travel choices under a variety of service conditions. The basic idea is that individuals develop mental models about their surroundings (or at least the part of the real world that is relevant to the individual). The theory of mental models in psychology was propagated by Johnson-Laird (1983), and even before that Kevin Lynch (1960) discussed the concept of a mental map as a representation of a city that is unique to an individual.

Mental models are the internal representation that individuals’ cognitive systems create to interpret the environment. In other words, individual’s mental model is the sum of all beliefs and knowledge that the individual holds about the world, including the results that different actions will bring about, and is updated according to experience and information obtained from different sources. Mental models are used to make predictions about the future and the consequences of actions. This, in turn, is the basis for choosing an adequate action. New information is interpreted and included according to and within existing mental models. It is well known that individuals memorize their experience with certain situations and use this experience to choose a rewarding action. Experimentation and experience collection are two features of any learning process. While learning techniques are based on collecting experiences, they usually differ in the way in
which experience is processed to provide expectations and predictions about the outputs of actions.

Choice set generation is an important modelling aspect in the development of a realistic travel behaviour model. It is formulated based on travelers’ perception regarding available alternatives, which in turn is based on their acquired information. In the literature, choice set generation models are rarely explicitly specified and are calibrated based on indirect information (Cascetta et al., 2002).

A proper mental model representation for the (relevant) transit network and its conditions is important for the learning and planning activities to be carried out. In this section, we propose a mental model structure for transit passengers, while in the following sections we present different mechanisms for updating and using this mental model. The basic principle is that human beings are able to form their expectations of available choices. And, travellers store information related to their trip, and therefore they do not possess perfect knowledge of the whole transportation system. It is not uncommon that a passenger remembers how long he waited for or spent on a run. Passengers will even remember the best (minimum) and worst (maximum) waiting and travel times for a specific run. Passengers are able to construct their expectations for the travel time for different alighting stops if they are on board a specific run.

The proposed mental model structure assumes that a transit route segment may be modelled as a shuttle service. The (relevant) transit network is represented by a set of route segments. A route segment approach is convenient so that a passenger may examine travel times on a route segment using only the initial wait at the origin stop and the travel time from the origin stop to the destination stop upon boarding a vehicle on that route. The path generator sub-module returns for each OD pair of the OD-List a set of transit paths (each transit path includes an origin stop, a run or a sequence of runs, and a destination stop). Figure 3.4 shows a sample output for the path-generator sub-module for a specific transit trip (trip #63 from Table 3-3). This output is based on applying some dominance and heuristic rules in addition to a number of factors. For example, no more than one transfer is accepted, access/egress distances from stops A, B, G and H are within an acceptable access distance buffer zone for transit passengers, and the passenger is aware of
available transit routes and possible transfers. Stops C and D are not within the acceptable walking distance from origin. Similarly, stops E and F are not within the acceptable walking distance to destination. For instance, if access distance to stop G is greater than the maximum acceptable access distance, then five transit paths will not be generated for that trip. If the passenger is not aware of the possible transfers to transit route #6, then three transit paths will not be generated. While stop B may be closer in distance, stop A may be more attractive because it is part of a larger number of transit paths. This representation considers the Common Lines problem: a transit passenger who chooses stop A as the origin boarding stop will be faced with the choice between routes #1 and #2; dynamic path choice models specifically deal with this situation.

The path generator sub-module can be used to represent different awareness levels about the transit network for different passenger-types, e.g. frequent users and occasional users – frequent users usually have a higher level of awareness and their set of transit paths contains more options than the set for occasional users. In this context, different policies that target travellers’ awareness of the transit network can be evaluated, e.g. the development of an on-line trip planner which advises (occasional) transit passengers with all possible transit options.
A transfer connection contains an off stop and an on stop. They could be different, like the transfer between R3 and R6 (T1); they could be the same like the transfer between R1 and R4/R6 (T6).

For each On stop, the passenger stores the access time (walking time), list of attractive routes passing by this stop, stop convenience (e.g., terminal or not, stop queue capacity, etc).

For each segment (i.e., an on stop, a route, and an off stop), the passenger memorizes the experienced waiting time, seat availability, route reliability, running time to the off stop, etc. Each passenger may only memorize the last K experiences about each attribute, allowing the passenger to forget far in the past experiences. Information provided about any attribute is received in the memory as a recent experience with relevant reliability.

Figure 3.4 A Sample Path-Generator Output and Mental Model Structure for Trip (i.e., Passenger) #63 (O=3, D=2)
3.2.3.2 Mental Model Updating

Learning occurs both with the evolution of within-day and the evolution of day-to-day experiences and conditions. The generalized cost function has some fixed components that do not change from day-to-day or within-day, such as the number of transfers for a certain path and walking distances to certain stops; no updating is required for these components. Some components change due to within-day dynamics; these attributes are direct functions of service features, such as waiting time, travel time, transfer time and comfort levels. Passenger-agents update their perceptions about these components through their experience with the transit system, or, in the case of available ITS and ATIS, this information could be supplied by the system.

In the context of within-day and day-to-day dynamics, especially in the presence of information, it is highly likely that passenger’s knowledge (i.e. perceptions) of network performance will vary depending on their past experience, accessibility to ATIS and personal attributes. Modelling the process by which travellers combine their experience and other information received through different sources is important. The role of information integration and learning in the decision making process in transportation applications has not been extensively studied. The proposed approach makes a clear distinction between individual knowledge (i.e. experience), provided information, and the possible experience-information integration. Perceived information may be assigned different reliability (or trust) values based on the type of information (waiting time information is more reliable than running time information) or based on the location when information is provided (at home, at stop, or on board). Ben-Akiva et al. (1991) proposed a convex combination approach for integrating past experience and perceived information, with a parameter that indicates the relative importance of information and historical perception in the integration process.

Not all perceived experience or information are stored in the mental model; some experiences and information may not be representative of the transit conditions (e.g. severe congestion due to an accident, while severe delays due to a normal road congestion are considered representative),
or this experience may be stored with a low reliability value, to reflect that it is not effective in the prediction process. In this regard, biased perceptions can be modelled, which may lead to a deluded equilibrium state (Nakayama and Kitamura, 2000).

The process of updating the mental model (perceiving experience and information and integrating them for the use in the decision making process) is a rich area for experimentation, keeping in mind the trade-off between being behaviourally realistic and computationally efficient. The mental model updating process occurs in real-time (when real-time information is provided at a stop or on board), within-day (either pre-trip at origin or en-route) and day-to-day (post-trip, to reflect upon previous experiences).

Passengers update their mental model and utilize it as the base for their behaviour (i.e. trip choices); different passengers would provide different expectations about their transit trip components according to their mental model contents and prediction mechanisms. This is similar to the concept suggested by Lotan (1998), who proposed a two stage approach that corresponds to a sequential process in which, at the first stage perception and existing knowledge are updated and, at the second stage, a choice is made based on the updated perceptions.

3.2.3.3 Mental Model Utilization

Individual passengers are decision makers, who choose a departure time from an origin, an origin stop, a destination stop and a route between a given origin and a destination each day. While learning methods can represent equilibrium, the reason of using learning techniques is to represent the dynamics of individuals’ adaptations, rather than representing an equilibrium state of the system.

For the proposed framework, the decision making process at the individual level has two stages. In the first stage, each passenger perceives new experiences and information and integrates them in light of previous knowledge (i.e. updating of mental model contents). This process may occur pre-trip (when information is provided at origin), en-route (when real-time information is provided) and post-trip (when experience is gathered). In the second stage, a passenger decides
on a departure time, a stop, and a run (or sequence of runs) for the transit trip. A passenger, in a transit network, is either at the origin (e.g. home for work and school trip), waiting at a stop (origin or a transfer stop), on board of a vehicle (travelling to a transfer stop or a destination stop), or at destination (end of the trip). The second stage may therefore happen pre-trip (a departure time and an origin stop choices) or en-route (a run choice, and accordingly a transfer connection and a destination stop choice). Pre-trip and en-route updating and decision-making processes are considered in the within-day and real-time dynamics context, while post-trip updating process is expected to affect the day-to-day dynamics.

The framework considers the departure time choice, the stop choice and the run (or sequence of runs) choice. The departure time and origin stop choices are assumed to be at-origin choices (i.e. pre-trip), in which passenger-agents consider available information obtained from previous days, in addition to pre-trip information provision (if any). Once a passenger-agent arrives at a stop, a specific run choice is considered an adaptive choice, in which the passenger considers situations that occur during the trip, for example the variation in waiting time. The existence of information, through ITS and APTS, will influence the passenger-agent adaptive choice behaviour during the trip. At the origin (e.g. home for a work or school trip), a passenger develops an initial (tentative) travel plan, based on his updated mental model (historical experience and pre-trip information provided by the system). This travel plan includes a departure time, an origin stop (which are fixed) and a (tentative) run (or sequence of runs). The initial plan reflects the passenger’s preferences and expectations, and would be followed if the reality (en-route dynamics) matches, to a great extent, the passenger’s expectations. The dynamic path choice model for developing such a plan is explained in the next Chapter.

3.3 Connections with Agent-Based Urban Planning Models

The evolution of travel demand modeling is now leading to the new activity-based models, as the core of the next generation of transportation forecasting models. This evolution has been driven by the need of greater sensitivity to policies that affect more than just the broad characteristics of urban form, and target the mechanisms that produce human travel behaviour. Wahba (2004) showed the potential connectivity with the agent-based structure in MILATRAS and emerging
activity-based, agent-based urban planning models. Such emerging models require the transit assignment process to be responsive to dynamic *temporal* variations in travel demand. In addition, the introduction of the GIS-based component allows for appropriate handling of spatial land-use issues that are difficult to be addressed by a transportation microsimulation model alone.

It has been traditionally assumed that highly detailed dynamic transit assignment models are not adequate for strategic planning as they require detailed input data for planning scenarios. This has been due to the fact that existing traditional planning models (e.g., four-stage planning models) do not produce detailed future output to be used by transit assignment models. On the other hand, emerging microsimulation planning models are capable of providing detailed output for future scenarios (e.g., ILUTE); such details are not being optimally utilized with existing transit assignment models. Therefore, there is a mutual benefit by connecting MILATRAS-type transit assignment models with emerging activity-based microsimulation urban transportation models.

### 3.4 Transit ITS Technologies and Opportunities

The cornerstone of public transport assignment is the path choice model, which depends on service attributes and user characteristics. The main service factors affecting user behaviour in path choice are service frequency, service regularity and information available to users. Given the stochastic and time-dependent nature of travel times in transit, real-time information about the transit service may be helpful to passengers in planning their trips. Traditional transportation planning methods have serious limitations in evaluating the effects of information technologies, since they are not adequately sensitive to the types of information that may be provided to travelers. They also lack an adequate representation of the traveler’s response to that information.

Few studies have addressed the impact of transit information provision on passenger behaviour (see Section 2.4 for examples of those studies). Hickman and Wilson (1995) developed an analytical framework to evaluate path choices and travel time benefits resulting from real time
information, based on at-stop en-route information systems. Nuzzolo and Russo (1998) developed stochastic within-day dynamic path choice models for regular and irregular high-frequency services, with or without user information systems. These models evolved to a doubly dynamic stochastic path choice model (Nuzzolo et al. 2001) that considers the within-day and day-to-day variations of services and user learning for high frequency services with information at stops. This model was extended to accommodate different user classes, such as regular and occasional users (Nuzzolo et al. 2000). Gentile et al. (2005) proposed a general framework for determining the probability of boarding each line available at a stop when online information on bus waiting times is provided to passengers. Wahba and Shalaby (2005) outlined a framework to model the day-to-day learning and adaptation processes of passengers using a multi-agent learning-based approach, representing explicitly passengers as agents with their learning and decision making activities. Wahba and Shalaby (2007) investigate the impact of different traveller information provision scenarios on transit riders departure time and path choices, and network performance, using an agent-based microsimulation learning-based approach. Four information provision scenarios were investigated and the impacts on transit rider travel choices were examined. The results show that, for a medium-size transit network with low to medium frequency services, the stop and departure time choices seem to be more important than the run choice, as they significantly affect the trip time. Information provided only at stops did not help passengers reduce their trip time, as run choices are limited and dependent on previous stop and departure time choices in such networks with limited services. Information targeting the transfer stop choice was found to be more effective.

### 3.4.1 Transit ITS Technologies

Emerging information, communication, sensor technologies and innovative transit operations control strategies are becoming critical elements of the viable, competitive public transit system. Intelligent Transportation Systems (ITS) have significantly expanded the range of information available to the traveler through Advanced Traveler Information Systems (ATIS). ITS also improve the performance of transit vehicles, in terms of regularity, through the use of Advanced Public Transportation Systems (APTS).
APTS and ATIS, through a variety of data collection and communication capabilities, support improved operations planning and real-time transit operations management. ATIS are designed to provide timely information to transit passengers on the conditions of the network, thus affecting travel choice behaviour. The information provided to passengers in real time is concerned with current or projected system conditions (such as vehicle locations, expected departure times, and expected travel or running times), in addition to static route and schedule information. Different types of real-time information on service can be available to the user in different places (pre-trip at home and/or en-route at stops). Typical real-time information include waiting times for arriving vehicles, while more advanced information systems could give information on travel times and on-board occupancy.

Intelligent Transportation Systems technology is promoted as a means for improving public transit services. The intended benefits of ITS include better and more regular information, seamless transportation services, and improved productivity. ITS deployment in public transit is seen as a tool to improve efficiency, increasing service quality, and ultimately attracting more choice riders. ITS technologies are also intended to improve fleet management, hence reducing operating costs. Technology such as Automatic Passenger Counters (APC) and Automatic Vehicle Location (AVL) systems may allow transit operators to better balance supply and demand, and improve reliability of the service, through, for example, schedule adherence. Electronic fare cards may reduce dwell time and make fare payment more convenient. Traveler information services can enhance and expedite trip-planning and provide real time schedule information to travellers, hence affecting their decisions. In a demand-responsive setting, AVL and automated dispatching may increase productivity and reduce passenger travel time, affecting the level-of-service of the transit network. Signal priority systems can reduce travel time, affecting (hopefully improving) the transit network performance, which in return affects traveler’s choices. New technology that makes possible coordination between services, perceived as seamless integration of transit from the perspective of the passenger, has an effect on the convenience measure, which can be incorporated into the transit assignment framework. Other applications of ITS technologies and services in public transit include travel demand management (TDM) applications which would combine technologies and strategies to promote the use of existing transportation infrastructure to serve the increased demand for transit.
Forecasting the impact of service change, in particular with the existence of ITS technology, is difficult; real-time information can change at any given point in time, compared to the introduction of new routes which remain there for many years. In many transportation simulation applications with information provision, behavioural responses of individual travelers are important. The impact of any service change, including infrastructure improvements as well as providing real-time information, depends critically on passenger’s behaviour. In the transit assignment field, the dynamic modelling approach is the focus of growing interest, because of the importance of explicit system simulations to know how user flow for each run and the performance of different service networks in terms of times and comfort levels, and to enable user decisions to be evaluated if ITS is used.

On the supply side, APTS, for example, provide valuable input to planning applications that may lead to better transit system design (e.g. improving scheduling and route planning). APTS also enable a variety of real-time operational strategies (e.g. holding and dispatching) that directly affect transit vehicle movements. Traditional approaches treat travel times and level of services as given or input to the system; none of these approaches is well suited for a dynamic framework analysis that incorporates time varying congestion on roadways, or the impact of technologies such as Transit Signal Priority (TSP) on transit travel time. For any of the ITS technologies to be tested or evaluated, the transit assignment process needs not be deterministic, but rather it should allow for within-day and day-to-day variability. A fare collection method that enhances the dwell time cannot be evaluated using traditional approaches, simply because dwell time is given and not calculated.

On the demand side, APTS and ATIS allow the sharing of real-time performance information with travelers to influence demand and improve passenger level-of-service. Information available to travelers is not explicitly modeled in the existing approaches and hence the limitation to assess ATIS. Last, but not least, ITS technologies are directed to attract choice riders from other modes, while mode choice is not a part of traditional transit assignment modeling frameworks and demand for transit is given, such effects cannot be captured.
3.4.2 Transit ITS Opportunities

As innovative technological solutions are integrated with transit services, it is useful to have a tool for evaluating and refining new strategies prior to the deployment. Within the framework of public transit assignment, the usage of dynamic analytical models is the focus of growing interest. This is motivated by the need to explicitly simulate the evolution of the transport system for two reasons:

1. In analyzing and planning transport systems, dynamic models allow better evaluation of link flows and network performance, such as travel times and service levels
2. For the use of ITS (APTS and ATIS), dynamic models are the only ones that enable user decisions to be evaluated with regard to the evolution of the service in real time, and thus provide information on predicted flows.

In order, however, to evaluate Transit-ITS opportunities, the transit assignment model should be based on a detailed representation of service operations. Service operations are subject to a number of incidental and controlling factors, for instance service schedule design and passenger demand. Morgan et al., (2004) listed five elements that should be incorporated in any dynamic transit assignment model to capture these variations:

1. *Transit System Representation*, including detailed representation of the transit network structure, schedules and fleet assignments.
2. *Transit Vehicle Movement and Interactions*, including microscopic vehicle movements, such as lane-changing, as well the behaviours of non-transit vehicles in the presence of transit vehicles.
3. *Demand Representation*, referring to the passengers and their behaviour with regard to the use of the system, including pre-trip and/or en-route mode and route choice, as well as behaviour at transit stops.
4. *APTS Representation*, which involves the representation of surveillance and monitoring systems that generate and distribute real-time information, the application of that data to real-time control strategies, and the provision of information to travelers.
5. *Measures of Effectiveness*, including indicators, level of services and other measures that are used to evaluate the performance of an APTS strategy. The reliability of the measures
of effectiveness generated by a simulation is dependent upon the strength of the former four requirements.

The design of the proposed framework, MILATRAS, complies with the requirements for a dynamic transit assignment model mentioned above.

Balmer et al. (2004) differentiate between simulation-based ITS evaluation and simulation-based ITS application. When the ITS system, as a “black box”, is plugged into the simulation system, this is referred to as the evaluation process. The simulation system generates synthetic sensor output and communicates it to the ITS system. The ITS “black box” receives this data, computes its response, and sends the corresponding measures, such as the estimation of vehicle running time, to the simulation system. Concurrently, the simulation system has the passengers react according to their behavioral rules. When the simulation system is used to generate the ITS system response, this is an application of ITS. An example of ITS application is the generation of the optimal Transit Signal Priority plan, which is usually not known apriori (Ling and Shalaby, 2004). In this context, the proposed framework can be used to generate and evaluate different transit-ITS policies.
4 THE TRANSIT PATH CHOICE PROBLEM (TPCP)

One significant contribution of this thesis is about the modelling of the transit path choice problem as a Markovian Decision Process, making use of the well-established theoretical foundation for analyzing Markov Chains to study the transit path choice behaviour.

The theoretical foundation for a dynamic path choice model for transit riders is presented in this chapter. It starts with a review of the Markovian Decision Process (MDP) and the concepts related to Reinforcement Learning (RL). It provides a detailed description of the mathematical formulation for the departure time and path choices with and without information provision. A parameter-calibration procedure using a generic optimization technique (Genetic Algorithms) is proposed.

4.1 Markovian Decision Process

In this section, an introduction to the Markovian Decision Process (MDP) is presented in a manner sufficient for understanding the theoretical analysis presented in the following section.

4.1.1 Stochastic Processes and The Markovian Property

A stochastic process (Ross, 1983) is an ordered collection of random variables; it is conventional to order the random variables by time. When an observer is counting the number of cars crossing an intersection per hour, the data collected over a period of time represent a stochastic process. In any stochastic process, a random variable is defined. This random variable represents the phenomenon being observed (e.g. number of cars). The defined random variable is expected to take different values at different points in time. For example, the observed number of cars takes different values for different hours. All possible values belong to the ‘state space’ (or sample space) of this variable. For example, the state space of the random variable representing the observed number of cars crossing an intersection includes all positive integer values plus zero. Let X be the random variable observed, a stochastic process can be written as $X(t), t \geq 0$; this is referred to as a ‘continuous-time’ stochastic process. When time is discretised, the random
process is called a ‘discrete-time’ stochastic process – $X(n), n = \{1,2,...\}$– which is a collection of random variables at discrete points in time. The value of the random variable at a given time $t$ represents the ‘state’ of the system at that time.

A stochastic process is specified by properties of the joint distribution that describes the relationship among the sequence of random values for the process state variable. When rolling a die, for instance, the outcome of the rolling process is represented by a random variable. The ‘state space’ (i.e. the possible values) for this random variable has 6 elements: 1, 2, 3, 4, 5 and 6. If this process is repeated $n$ times, then the outcome of the process is $n$ ordered random values, $X(i), i = \{1,2,...,n\}$; this represents a discrete-time stochastic process. $X(3)$ represents the ‘state’ of the system (or the experiment) at time step 3. The relationship among these variables, $X(i)$, is that they are independently and identically distributed (iid). When the relationship among the random variables satisfies the Markov property, the stochastic process is then called a ‘Markov Process’.

The Markov property means that the conditional probability distribution of future states of the process (i.e. values of the random variable at future time instances), given the present state and all past states, depends only upon present state and not on any past state. In other words, the distribution of future values is conditionally independent of the past states (the path of the process) given the present state. This also means that the system’s stationary state is independent of the system’s initial conditions. The present state of the system is sufficient and necessary to predict the future state of the process; the past state(s) of the system are not necessary for the prediction process. This is sometimes referred to as the ‘memoryless property’ of the Markov process.

For a queuing system, the stochastic process can represent the change in the number of customers waiting for a service in a queue over time. This number changes at times when a new customer joins the queue or when a customer leaves the queue. At any given time, the system state is described by the number of customers in the queue. The prediction of the number of customers in a queue at the next time instance will depend only on the present system state (i.e.
the number of current customers in the queue) and is *independent* of the past trajectory of the queue-length.

The concept of the Markov property and Markovian Decision Process (MDP) is illustrated by the following example. Assume that there are three individuals playing Frisbee – see Figure 4.1 – namely individuals P1, P2 and P3. The location of the Frisbee at a specific point in time \( t \) is not known *apriori*; the location of the Frisbee is *random*. If we collect observations about the location of the Frisbee at various points in time, this is considered a stochastic process.

In this stochastic process, the random variable represents the location of the Frisbee, \( l \). The state space (or sample space) for the random variable has three elements: \{P1, P2, P3\}. At \( t = 1 \), let \( l_1 = P1 \); this means that individual P1 has the Frisbee at the beginning. After observing the process for time \( T \), a set of probabilities are calculated that describe the observed transition of the Frisbee location over time – see Figure 4.2. These probabilities are called the ‘state transition’ probabilities. They describe the transition of the system from one state to another state. In other words, they define the probability of the system ‘state’ taking a possible value from the ‘state space’, given a pre-defined present state. In the Frisbee example, these probabilities define the likelihood that the location of the Frisbee (system state) be at one of the three individuals (state space); that is the probability of the Frisbee being at location \( l_{r+1} \) at time \( t+1 \), given that it was at location \( l_r \) at time \( t \), location \( l_{r-1} \) at time \( t-1 \), location \( l_{r-2} \) at time \( t-2 \), ..., and location \( l_1 = P1 \) at time \( t = 1 \).
It is noteworthy that these probabilities are time-independent; the probability of the Frisbee being at individual, say, P2 at time \( t + 1 \) given it is at individual P3 at time \( t \) is \((1-c)\), the transition probability at P3). This is the same as the probability of the Frisbee being at individual P2 at time \( t \) given it is at individual P3 at time \( t - 1 \). In such cases, the state transition probabilities are called the ‘stationary’ transition probabilities. In the discussions to follow, we are only concerned with stationary systems where state transition probabilities are time-independent.

There are two questions that are of interest at time \( t \):

1) Who has the Frisbee at time \( t \)? Or what is the probability that individual P1 has the Frisbee, the probability that individual P2 has the Frisbee and the probability that individual P3 has the Frisbee?
The answer to this question is the stationary distribution for the stochastic process. This distribution defines the probability that the system state takes on a value of the state space.

2) Who will have the Frisbee at time \( t + 1 \)?

The answer to this question is the state transition probabilities mentioned above.

At any time \( t \), the information available about the system state (Frisbee location) are the location of the Frisbee at time \( t \) (defined as \( l_t \)), location at time \( t - 1 \) (defined as \( l_{t-1} \)), location at time \( t - 2 \) (defined as \( l_{t-2} \),..., and location at time \( t = 1 \) (defined as \( l_1 = P1 \)). When the only information needed to define the state of the system at time \( t + 1 \) (\( l_{t+1} \)) is the present state of the system (\( l_t \)), the random process is defined as a Markov Process, or said to have the Markovian Property. In a mathematical notation, the Markovian property (or the memoryless property) is stated as:

\[
Pr(l_{t+1} = j|l_t = i, l_{t-1} = i_{t-1}, l_{t-2} = i_{t-2}, ..., l_1 = i_1) = Pr(l_{t+1} = j|l_t = i_t)
\]

(4.1)
In the Frisbee example, the described stochastic process is a Markovian process; that is the probability distribution of future system states is conditionally independent of the past states (the path of the process) given the present state. For instance:

\[
\Pr(l_{t+1} = P1|l_t = P2, l_{t-1} = x_{t-1}, l_{t-2} = x_{t-2}, \ldots, l_1 = x_1) = \Pr(l_{t+1} = P1|l_t = P2) = d
\]

where \( x_i = \{P1,P2,P3\} \) and \( i = \{t-1,t-2,\ldots,1\} \)

Mistakenly, the memoryless property sometimes is viewed as that the path of the process to the present state is not relevant. This of course is not correct; the evolution of the process defines the present state of the system, which is needed to predict the future state of the system.
In the above example, the system state is represented by one piece of information: who has the Frisbee (or the Frisbee location) at time \( t \). The state space has three values for three possible locations. If it is observed that the state transition probabilities (Figure 4.2) are not only related to who has the Frisbee at time \( t \) but also who had it at time \( t-1 \), the above system state representation would fail to predict the future system state based only on the present system state (location of Frisbee at time \( t \)). In this case the system state needs to be represented by two variables: Frisbee location at time \( t \) and Frisbee location at time \( t-1 \). The state space will then have nine elements (representing all possible combinations for the two state variables)

\[
\{(l_t, l_{t-1}): l_t \in \{P1, P2, P3\}, l_{t-1} \in \{P1, P2, P3\}\}^3.
\]

The point here is that the definitions of the ‘system state’ and ‘state space’ are important for describing a stochastic process and for showing that a stochastic process has the Markovian property. The idea is that the present system state should summarize all relevant information about the past, making the prediction of the future state of the system only conditional on the present state and not conditionally dependent on the knowledge of the past.

An ordered sequence of system state occurrences from a Markovian process is called a ‘Markov Chain’. For the Frisbee example, a finite ordered sequence of Frisbee locations at different points in time constitutes a Markov Chain. The Frisbee example represents a time-homogeneous Markov Chain, as the transition probabilities are time-independent. That is

\[
\Pr(l_{t+1} = j|l_t = i_t) = \Pr(l_t = j|l_{t-1} = i_{t-1}), \forall t.
\]

It is conventional to represent Markov Chains by a directed graph, where the edges are labelled by the probabilities of going from one state to the other states (Figure 4.2).

4.1.2 Markov Chains and The Ergodic Theory

The Ergodic Theory provides the foundation for studying stochastic processes that have the Markovian property. It establishes the conditions under which a Markov Chain can be analysed to determine its transition probabilities and steady state behaviour. The Ergodic Theory states that if a Markov Chain is ergodic, then a unique steady state distribution exists and is

\[\text{Note that the probability of some state space values is zero, based on Figure 4.2 (e.g. } \Pr(l_t = P1, l_{t-1} = P1) = 0\)\]
independent of the initial conditions. The proof of this theory is well established in the literature and will therefore not be provided here – see Petersen (1990).

A Markov Chain is called ‘ergodic’ if, and only if, the chain is irreducible, positive-recurrent and aperiodic. The conditions for a Markov Chain to be ergodic are discussed below.

A Markov Chain is ‘irreducible’ if, and only if, state \( i \) and state \( j \) ‘communicate’ \( \forall i, j \). States \( i \) and \( j \) communicate if, and only if, state \( j \) (\( i \)) is reachable from state \( i \) (\( j \)) in a finite number of steps. Figure 4.3 shows two examples of an irreducible Markov Chain and a reducible chain. In the irreducible figure, all states communicate with each other; state 1, for example, is reachable from all states and all states are reachable from state 1 in a finite number of steps. On the other hand, state 1 is not reachable from all states (e.g. state 4 and state 2) in the reducible chain. A strong state is a state that communicates with all other states in the Markov Chain – e.g. state 1 in the irreducible figure – and the existence of one strong state in the Markov Chain guarantees that the chain is irreducible.

A Markov Chain is ‘positive-recurrent’ if, and only if, all states in the chain are ‘positive-recurrent’. In a Markov Chain, a state is recurrent (transient) if, and only if, the probability (as \( t \to \infty \)) of returning to this state, after leaving it, is 1 (< 1). In Figure 4.3, state 1 is recurrent in the irreducible chain and transient in the reducible chain. In the irreducible chain,

\[
\Pr^{11} = \sum_{n=1}^{\infty} \Pr^{11}(n) = 1, \text{ where } \Pr^{11}(n) \text{ is the probability of returning to state } 1 \text{ in } n \text{ steps, after leaving state } 1, \text{ whereas } \Pr^{11} = 0 \text{ in the reducible chain.}
\]

If the number of states in the chain is finite, and consequently the number of steps to return to a state is also finite, then the recurrent state is called a nonnull-recurrent or positive-recurrent.

When the number of states is infinite, a state \( j \), with \( \Pr^j = \sum_{n=1}^{\infty} \Pr^j(n) = 1 \), is called null-recurrent. If \( \Pr^j(n = 1) = 1 \), then state \( j \) is called an ‘absorbing’ state; a chain with an absorbing state is obviously non-irreducible.
If state $i$ is positive-recurrent and communicates with state $j$, then it can be easily concluded that state $j$ is positive-recurrent. Leaving state $j$, state $i$ is reachable in a finite number of steps (since state $i$ is positive-recurrent), and state $j$ is also reachable from state $i$ in a finite number of steps, hence state $j$ is positive-recurrent. Therefore, an irreducible Markov Chain, with a finite number of states, is a positive-recurrent Markov Chain since all states will be positive-recurrent.

A Markov Chain is ‘aperiodic’ (or in other words ‘acyclic’) if, and only if, all its states are ‘aperiodic’ or ‘acyclic’. A state is periodic or cyclic if, and only if, there is a number of steps to return back to the state, after leaving it, as a multiple of an integer $d$ – i.e. the cycle of returning back to the state has $d, 2d, 3d, ..., cd$ steps, where $d \neq 1$. That is, $\Pr_i^n(0) = 0, \forall n \neq d, 2d, 3d, ..., cd$. Otherwise, the state is called acyclic or aperiodic. Figure 4.4 shows an example of cyclic and aperiodic Markov Chains. In the cyclic chain, state 1 has a cycle of 3 steps and its multiples – e.g. state $1 \rightarrow$ state 4 $\rightarrow$ state 0 $\rightarrow$ state 1 or state $1 \rightarrow$ state 3 $\rightarrow$ state 0 $\rightarrow$ state 1. In the aperiodic chain, state 1 is aperiodic since it can be reached through 2 steps ($s1 \rightarrow s2 \rightarrow s1$), 3 steps, 4 steps ($s1 \rightarrow s4 \rightarrow s0 \rightarrow s2 \rightarrow s1$), 6 steps ($s1 \rightarrow s4 \rightarrow s0 \rightarrow s2 \rightarrow s1 \rightarrow s2 \rightarrow s1$), and so on.

The presence of one aperiodic state in an irreducible, finite-state, Markov Chain guarantees that the whole chain is aperiodic. Also, the self-loop of any state in an irreducible, finite-state, Markov Chain guarantees that the chain is aperiodic. In addition, the existence of a state, with a possible self-loop transition, in an irreducible, finite-state, chain makes the state acyclic.
With reference to the Ergodic Theory, if a Markov Chain is irreducible, positive-recurrent and aperiodic, then a unique steady state distribution exists which is independent of the initial conditions. The positive-recurrent and aperiodic characteristics guarantee that the steady state distribution exists, while irreducibility guarantees that the steady state distribution is unique and independent of initial conditions.

### 4.1.3 Actions, Rewards and Decision Making in Markov Chains

In Markov Processes, the outcome of the stochastic process is totally random, or not controlled but rather observed. Situations with the underlying stochastic process having the Markov property and where outcomes are partly random and partly under the control of a decision maker are studied using Markov Decision Processes (MDPs). MDPs provide a mathematical framework for modelling the decision-making process in such situations. MDPs are useful for studying a wide range of optimization problems solved via Dynamic Programming (or similar approaches).

A Markov Decision Process is a discrete-time stochastic process, where a decision-maker partly controls the transition probabilities through deliberate choices at discrete points in time. In a MDP, in addition to the system state representation, at each state there are several actions from which the decision maker must choose. This decision-making behaviour, consequently, influences the transition probability from one system state to another. Associated with the
decision-making capability is the concept of a reward. Rewards can be earned for each state visited, or for each state-action pair implemented. The decision-maker in a Markov Decision Process has a goal of optimizing some cumulative function of the rewards – i.e. the decision-maker acts rationally. The main differences between Markov Processes and Markov Decision Processes are the addition of actions (allowing choices) and rewards (giving motivation).

In the Frisbee example, the underlying stochastic process has the Markov property. Let individual P1 be a decision-maker (i.e. intelligent-agent or called hereafter agent) while individuals P2 and P3 are objects without any decision-making capability. When the location of the Frisbee is at P1, the transition probabilities $\Pr_{11}$, $\Pr_{12}$, and $\Pr_{13}$ are partly controlled by the decision-making behaviour of agent P1. Assuming that agent P1 always passes the Frisbee and does not keep it, then $\Pr_{11} = 0$. If there is a fixed immediate-reward associated with passing the Frisbee by agent P1 to individual P2, $r_{12}$, and to individual P3, $r_{13} > r_{12}$, then it might be expected that a reward-maximizing, rational, agent will always pass the Frisbee to individual P3, i.e. $\Pr_{13} = 1$ and $\Pr_{12} = 0$. However, the future returns, $V_{1i}$, (rather than only immediate-rewards) should be considered. The future return, $V_{1i}$, is a function of the system-state transitions in response to the action taken by agent P1. While the immediate-reward may have a fixed value, the future return is always a random variable due to the stochastic behaviour of the underlying Markov process. The decision-making process is therefore dependent on the reward and return values or functions. If the location of the Frisbee is at non-intelligent agents P2 or P3, then there are no actions available and the outcome of the transition is random following the underlying Markov process.

Table 4-1 shows the possible state-action pairs for the Frisbee example based on the aforementioned system state representation; this is different from the transition possibilities since there is a possible transition from state P2 to state P3, but there is no action available at state P2 to the decision-maker agent P1. Since the decision-making capability is only available for agent P1, the state representation can be modified to reflect agent P1 behaviour, see Table 4-2. Note that the transition between the states of system (from ‘has Frisbee’ to ‘does not have Frisbee’ and vice versa) is not only partly controlled by agent P1 decision-making behaviour but also by the random transition between individuals P2 and P3 and agent P1. Table 4-3 shows a different way
of representing the state-action pairs. In this representation, the decision type or context is rather modelled. The probability of taking a certain action is a function of the immediate-reward and future returns; this function, in other words, represents the decision-making behaviour.

Table 4-1 State-Action Table for the Frisbee Example modeled as a Markov process

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
<th>Pass to P1</th>
<th>Pass to P2</th>
<th>Pass to P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>At P1</td>
<td></td>
<td></td>
<td>( r^{12}, V^{12} )</td>
<td>( r^{13}, V^{13} )</td>
</tr>
<tr>
<td>At P2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>At P3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-2 State-Action Table for the Frisbee Example modeled as a Markov Decision Process for agent P1

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
<th>Keep Frisbee</th>
<th>Pass to P2</th>
<th>Pass to P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has Frisbee</td>
<td></td>
<td></td>
<td>( r^{12}, V^{12} )</td>
<td>( r^{13}, V^{13} )</td>
</tr>
<tr>
<td>Does not have Frisbee</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-3 Another representation for State-Action pairs for the Markov Decision Process for agent J

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
<th>Keep Frisbee</th>
<th>Pass Frisbee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has Frisbee</td>
<td></td>
<td></td>
<td>( Pr^J = f(r^J, V^J) )</td>
</tr>
<tr>
<td>Does not have Frisbee</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If the reward and return functions are specified, then the Markov Decision Process becomes an (stochastic) optimization problem where the decision variables are the transition probabilities and the objective function is about the maximization of the expected return. When the underlying Markov Process is ergodic, then a unique steady state distribution exists; this also means that a unique state-transition probability matrix \([Pr^J]\) exists, which ensures that the above formulated optimization problem has a solution and it is unique (i.e. global optima). A solution, or a possible \([Pr^J]\), represents a policy \((\pi)\) that the agent can follow; this policy directs the agent to
what actions to choose at any given state *regardless of prior history*. The optimal solution represents the *optimal policy* \( \pi^* \) which, if followed, maximizes the expected return.

For stochastic optimization, dynamics programming has been widely applied to solve general stochastic optimal control problems, such as Markov Decision Processes. For the MDP, if there is a closed-form for the expected return function, optimization techniques and algorithms such as dynamic programming algorithms can be used to find the optimal policy. Dynamic programming however suffers from what is called “the curse of dimensionality”, meaning that its computational requirements grow exponentially with the number of state variables or state-action pairs.

When the agent-environment interaction is complex, and with the existence of dependence between actions and environment-response, a closed-form for the expected return function is hard to formulate. In such cases, simulation-based optimization techniques are applied; that is the expected return is sampled from a simulation model that describes the *stochastic* agent-environment interaction and averaged over time. As \( t \to \infty \), the *estimated* expected return approaches the *true* expected return and then the optimization of returns (of finding the optimal policy) becomes possible. Reinforcement Learning (RL) represents an approach to estimate the policy that optimizes the expected return. RL also has an advantage over dynamic programming for situations that are too large or complicated to explicitly enumerate the next-state transition probabilities in a mathematical form.

RL (Sutton and Barto, 1998) uses a formal framework defining the interaction between a goal-directed agent and its environment in terms of states, actions and rewards. It is a *computational* approach to model goal-directed learning and decision-making; it is does not model the internal decision-making process by agents nor the psychological learning behaviour. It does, however, emphasize learning by the individual from direct interaction with the environment. It uses exploitation of experience and exploration of available actions to converge to the policy that yields the maximum return. It therefore returns a policy \( \pi \) (or more precisely the optimal policy \( \pi^* \)); that is how the agent will choose actions at each state regardless of prior history.
There are four components of any RL model:

1- A policy $\pi$, $\Pr(s,s')$, or $\Pr(a|s)$: the policy describes the agent’s behaviour at any given state, by defining the likelihood of choosing action $a$ (or moving from state $s$ to $s'$).

2- A reward function ($r$): the reward function signals the immediate reward of a specific state (or state-action pair).

3- A value or return function ($V$): the value function calculates the accumulated reward over time for a specific state (or state-action pair). It also ensures that state-action pairs with short-term high reward but long-term low value are not preferred.

4- A model of the environment: this is used to estimate the immediate reward and long-term value for each state (or state-action pair)

Under the RL umbrella, there are different techniques to estimate the optimal policy $\pi'$. These techniques include Dynamic Programming (DP), Monte-Carlo (MC) Methods and Temporal-Difference (TD) learning. TD learning is one of the novel RL techniques which has been proven to converge to the optimal policy $\pi'$ with probability 1 (Sutton and Barto, 1998). A famous TD algorithm, developed in late 1980, is known as Q-learning (Watkins, 1989), expressed as follows:

$$Q(s,a_t) \leftarrow Q(s,a_t) + \alpha \left[ r_{t+1} + \gamma \max_{a'} Q(s',a') - Q(s,a_t) \right]$$

(4.3)

where:
- $Q(s,a_t)$ is the learned state-action value at time $t$,
- $r_t$ is the immediate reward of choosing action $a$ at state $s$,
- $\alpha$ is the weight of new experience relative to old one
- $\gamma$ is a discount factor of future rewards

The learned state-action value, $Q$, directly approximates the true state-action value, $Q^*$. That is, it has been shown that, if all state-action pairs continue to be updated and visited, $Q$ converges with probability 1 to $Q^*$. The proof is based on the assumption that the underlying stochastic process has the Markov property and is ergodic. The existence of a unique $Q^*$ is linked to the existence of a unique steady-state distribution and state-transition probability matrix.
The decision-making behaviour should balance between exploiting the acquired experience and exploring available actions. When exploiting, \( P_s(a) = \begin{cases} 1 & Q(s,a) = \max_{a'} Q(s,a') \\ 0 & \text{otherwise} \end{cases} \), and when exploring \( P_s(a) = \frac{f(Q(s,a))}{\sum_{i \in A} f(Q(s,i))} \). If the probability of exploring is \( \varepsilon \), this is called the \( \varepsilon \)-greedy method. Figure 4.5 shows the procedure for the Q-learning algorithm.
Back to the Frisbee example, the Markov Chain (Figure 4.2 and Table 4-3) is irreducible, positive-recurrent, and aperiodic; the Markov Chain is ergodic. Hence, Q-learning can be used to estimate an optimal policy for agent P1. If followed, agent P1 is expected to maximize the expected return. The output of the Q-learning is a steady-state transition probability matrix, \( \text{Pr}^\gamma \) that describes agent P1’s behaviour (or decisions). For Q-learning to estimate the optimal policy, the immediate-reward, \( r \), and value function, \( V \), need to be known or calculated from the environment model.

Looking at the Frisbee example from a different angle, and holding the assumption that agent P1 is behaving rationally (i.e. goal-directed), the observation of agent P1’s steady decisions over time can be summarized as a steady-state transition probability matrix, \( \text{Pr}^\gamma_{\text{observed}} \). This transition probability matrix represents a policy, or the optimal policy \( \pi^* \) in this case, followed by agent P1 to maximize the expected return. Although not directly observed, agent P1 must have a function for calculating immediate-reward and future value; the existence of this function is, however, manifested in the observed transition probabilities. While one can make (educated) guesses on the form, \( f \), of the unobserved function, the assumed form can be validated if it
generates a similar policy $\pi^*$, with similar $[Pr^\beta]_{observed}$. If the assumed function form has parameters, $f(\beta)$, then these parameters can be calibrated such that the likelihood of regenerating $[Pr^\beta]_{observed}$ is maximized. The problem of estimating the value function is called Inverse Reinforcement Learning (IRL) (Russell, 1998). It is about the determination of the reward function that the agent is optimizing. This analogy will be used to model the transit path choice problem as a Markovian Decision Process.

4.2 The Transit Path Choice Problem as a Markovian Decision Process

The research in dynamic path choice models is motivated by the fact that passengers may further improve their path decision by updating their expected travel times based on information gained while waiting. In traditional static models, information gained while waiting is not relevant to the boarding decision. On the other hand, dynamic path choice models allow passengers to update their expectations while waiting. The need for dynamic path choice models is supported by other facts. The stochastic and time-dependent nature of vehicle movements in the transit network may require more “adaptive” boarding/alighting decisions by passengers. A passenger may receive real-time information on the status of transit vehicles, which may require passengers to adjust their departure time and path decisions in a real-time manner (e.g. Hickman, and Wilson, 1995). Since a passenger can make one of several possible transfers between transit routes, he may wait until arriving at a transfer point before deciding which transfer to take (Hall, 1986). Under the assumption of static knowledge, models lack the recognition that the interaction with the environment generally leads to adaptation of rules of behaviour through learning. Alternatively, models under the assumption of learning assume that travellers learn based on experience. In a dynamic context, passenger’s responses depend on their perceptions of travel and waiting times in the network; it is, therefore, important to model the process by which passengers update their perceptions of travel times.

Travellers are assumed to be goal-directed intelligent-agents. A traveller has a goal of minimizing the travel cost (e.g. time, money, inconveniences, etc.) and, for some trip purposes, maximizing the probability of arriving on a scheduled arrival time. In order to achieve their
goals, travellers follow policies to optimize the return of their trip. These policies manifest themselves through the observed travel behaviour (or trip choices) of individual passengers. Assuming that travellers are rational, it is logical to expect that they follow their optimal policy. Based on the notion that actions receive rewards (or penalties), it is also expected that travellers value their choices according to a value function; although this value function is not directly observed by the modeller, it is known to the individual traveller.

In the transit assignment context, a transit user is faced with various types of choices during the trip, see Figure 4.6. A passenger needs to decide on an origin departure time, an origin stop, a run (or route) to board, and possibly a connection to transfer at. In addition, in a multimodal system, access and egress mode choices to and from the transit service may be included.
For a recurring trip (e.g. home-based work trip), a passenger settles on the trip choices by trying (or judging) different options until certain path choices prove to be the optimal for this particular trip’s objectives. For a home-based school trip, objectives are to arrive on (or close to) a scheduled arrival time, while minimizing travel costs. Alternatively, for a home-based discretionary trip, the objective could be only to minimize travel cost. After settling on one path, a passenger adjusts the origin departure time to optimize his or her objective. This optimization process is based on the passenger’s experience with the transit service performance for different origin departure time values. It is worth to note that switching paths is more common than large adjustments in the origin departure time.

We are interested in the process through which passengers settle on their trip choices. This is important to be able to model the shifts in trip choices when changes to the transit service are introduced. A passenger at origin has a choice set of different combinations of departure time and path choices. Figure 4.7 represents the choice set for the transit trip shown in Figure 4.6, where the trip choices are highlighted by solid circles. There is one path choice with zero transfers, while there is another path with two transfers. Some origin stops are closer than others, and some have more than one attractive route. The same transfer stop can be reached through multiple paths. Similarly, the same destination stop can be reached by different trip choices and some destination stops are closer to the final destination than others. Over time, a passenger chooses the combination of departure time, origin stop, run (or route), and transfer stop choices that minimizes the travel cost. In order to reach this combination, the passenger must have valued (tried or judged) all other possible choices and found that the chosen combination outperforms all other options, with reference to a value function.
At the origin, the passenger has the objective of minimizing travel cost for the trip. At the \textit{star} stop in Figure 4.7, the passenger’s objective is still to minimize the travel cost for the remainder of the trip, regardless of prior choices. This is does not mean that previous choices are not important; it is rather the relative impact of prior choices on future decisions than the actual choices. This impact can be shown by, for example, the arrival time of the passenger at the \textit{star} stop and the fare paid to reach the \textit{star} stop (which is dependent on previous choices). This influences future choices, expressed in the values of $p$ (the choice probability for one alternative) and $1-p$. The values of $p$ and $1-p$ also depend on the transit service performance. The outcome of the passenger’s choice, in turn, affects the transit service performance through, for example, reducing the available capacity on the chosen route by 1.
The change in transit service performance represented by, for example, route loads is a stochastic process. Route loads take different values at different points in time. This stochastic process is partly controlled by the decisions made by individual travellers. If we define the system state as the set of variables representing route loads, the state space (or sample space) will be all possible combinations of route load values. For example, if there are only two routes in the network; route $R^1$ with load capacity of $C^{R^1} = 50$ and route $R^2$ with load capacity of $C^{R^2} = 65$, then the system state is represented by two variables $L^{R^1}$ and $L^{R^2}$ expressing route $R^1$ and $R^2$ load values. The state space, in this case, will have 3366 possible combinations $\{L^{R^1},L^{R^2}\}: (0,0),(1,0),...,(50,0),(1,0),(1,1),...,(50,1),...,(50,65)$. This number grows exponentially with the increase of number of routes and/or route capacities, to the extent that it becomes impractical to study the underlying stochastic process using this system state representation.

Another representation could refer to the path flows, instead of route loads (similar to Cascetta and Cantarella, 1991, for auto-assignment). This means that the state space for the whole transit system is expressed in $k$ path flows, covering all possible paths in the system for all origin-destination demands. In this approach, user’s behaviour is modelled by departure time and path choices. The path choice does not differentiate among origin stop choice, route choice and transfer connection choice. Moreover, within-day dynamics are considered only until the departure time; en-route adaptive behaviour is not explicitly modelled.

The change in the location of the transit rider during the trip represents another stochastic process. Passengers, while travelling, move to different locations in the transit network at different points in time (e.g. at stop, on board). The state variable in this case represents the location of the transit rider, which takes a value out of the state space. The state space represents all possible locations for a transit rider; it consists of possible origin stop choices, route choices, transfer stop choices and destination stop choices. A passenger at the start stop in Figure 4.7, has made three transitions: from origin-location to origin-stop-location (associated with an origin stop choice), from origin-stop-location to onboard-route-location (associated with a route choice), and from onboard-route-location to transfer-stop-location (associated with a transfer
stop choice). Each transition depends on the current state of the passenger (i.e. location of passenger) and on the transit service performance. The passenger would need to make two more transitions to reach the trip destination: from transfer-stop-location to onboard-route-location (associated with a route choice) and from onboard-route-location to destination-stop-location and final destination (no choices are made since there is only one option). Given the present state (i.e. location), the passenger decides on future transitions, regardless of the history of states (i.e. prior locations). This resembles of a stochastic process with the Markov property. It should be stated that the location information only might not be enough to decide on future transitions; information related to previous transitions, such as fare paid, remaining time to scheduled arrival time, or real-time information about the transit system performance, are important. Instead of having to memorize previous transitions, the present state (or the system state) should summarize all information that is assumed to be available to the passenger at any time $t$.

This stochastic process is partly dependent on the transit service performance and partly controlled by the transit rider. This can be analyzed as a Markovian Decision Process. In a MDP, actions are rewarded and hence optimal policies can be estimated. For an origin stop choice, there is an immediate-cost expressed in the value of the access cost (time and/or money). Also, there is a future value of a specific stop choice expressed in the expected travel cost of the subsequent available route and transfer connection choices. For a route choice, there is an immediate-cost expressed as the value of waiting time. Future cost, associated with a route choice, is related to the value of possible transfer connections and the probability of arriving at the scheduled arrival time.

Assuming passengers are rational, it is logical to expect passengers to follow their optimal policy and to optimize their trip return (or cost in this regard). The effect of such an optimal policy is observed through either disaggregate individual choices, $[Pr_i]^{observed}$, and/or aggregate route loads, $L^{observed}$. The value of $p$, Figure 4.7, represents one cell in the state transition probability matrix, $[Pr_i]^{observed}$. If the underlying Markov process is ergodic, then there exists a unique optimal policy $\pi^*$; if followed, passengers will optimize their return. Associated with $\pi^*$ is a steady state transition probability matrix $[Pr_i]$. Passengers devise their optimal policies based
on a value function for the state-action evaluation. While the optimal policy $\pi^*$ is observed (or its effect through route loads), the only unknown (to the modeller) is the value function. By reconstructing the transit path choice problem as a Markovian Decision Process, the value function (or its parameters), used by individual passengers, can be estimated/calibrated. This is similar to the process of Inverse Reinforcement Learning (IRL).

4.2.1 Mental Model Structure

In order to ensure the uniqueness and existence of the optimal policy $\pi^*$ in the reconstructed Markovian Decision Process, the underlying Markov process needs to be ergodic. To show that a Markov Chain is ergodic, its state-action transition diagram has to be irreducible, positive-recurrent, and aperiodic (see Section 4.1.2).

The state-action transition diagram for the transit path choice problem is represented by the ‘mental model’ of the relevant parts of the transit network. For simplicity of analysis, and without losing generality, assume that the system state is only represented by the location of the passenger during the trip. At each state, $s$, there is a set of possible actions, $a_i \in A(s)$, available for each passenger. The value of a state-action pair is measured based on the travel cost association with this choice.

For the transit assignment process, passengers need to make choices when they are at home (pre-trip) and during the trip (en-route). At any point, any passenger agent may be in one of 7 state categories: “at home”, “accessing origin stop”, “waiting at a stop”, “on board of a bus”, “transferring to a stop”, “egressing from destination”, or “at the destination”. At each state, passengers may be faced with a set of possible actions. When a passenger is in the “at home” state, possible actions include a set of home departure time and origin stop choices combinations. A passenger, at home, chooses a specific home departure time and an origin stop; this results in a change in the passenger’s state – from the “at home” state to the “accessing origin stop” state. While being in the “accessing origin stop” state, passengers will continue until they reach the origin stop. A passenger waiting at a stop needs to decide on which bus run to board. Assuming no traveller information provision, passengers make the boarding decision when a run arrives.
from a list of attractive routes. A “board” decision will result in a state change – from “waiting at a stop” to “on board of a bus”. A “do not board” decision yields no state change. This means that the “waiting at stop” state has a possible self-loop transition. A “board” decision is expected to change the environment of the passenger – e.g. increase the dwell time at the stop, reduce on-board seat availability, etc. When a passenger is “on board of a bus” and arriving at a stop, possible actions are “stay on board” or “alight”. When a passenger alights at a stop, his state changes to either “transferring to a stop” or “egressing from destination stop” depending on whether the stop is a destination stop or a transfer point. When a passenger arrives at destination (i.e. in the “at the destination” state), the transit trip ends and no further action is needed. When a passenger is at a state \( s \) and decides to take action \( a \), this is referred to as the state-action pair: \((s,a)\), and action \( a \) moves the passenger from state \( s \) to state \( s' \), expressed as \( a : s \rightarrow s' \). Table 4-4 summarizes the state-action pairs available to each passenger and Table 4-5 shows the state-action table for this Markovian Decision Process.
Table 4-4 Summary of states and actions for the transit path choice problem

<table>
<thead>
<tr>
<th>State, $s$</th>
<th>Possible Actions, $A(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Origin ($O$)</td>
<td>Departure Time and Origin Stop Combination</td>
</tr>
<tr>
<td>ACCESSING Origin Stop ($W$)</td>
<td>$While not at Origin Stop$ {</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} CONTINUE TO ACCESS</td>
</tr>
<tr>
<td>WAITING at a STOP ($S$)</td>
<td>$When a RUN arrives at a STOP$ {</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} either Board \hspace{1cm} KEEP WAITING (i.e. Do Not Board)</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} $}$ \hspace{1cm} otherwise KEEP WAITING</td>
</tr>
<tr>
<td>On-Board of a Transit Vehicle ($V$)</td>
<td>$When Transit Vehicle arrives at a STOP$ {</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} either Alight \hspace{1cm} STAY On-BOARD (i.e. Do Not Alight)</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} $}$ \hspace{1cm} otherwise STAY On-BOARD</td>
</tr>
<tr>
<td>TRANSFERRING to a STOP ($F$)</td>
<td>STOP Choice</td>
</tr>
<tr>
<td>EGRESSING from Destination Stop ($W$)</td>
<td>$While not at Destination$ {</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} CONTINUE TO EGRESS</td>
</tr>
<tr>
<td></td>
<td>\hspace{1cm} $}$</td>
</tr>
<tr>
<td>At the Destination ($D$)</td>
<td>(no action)</td>
</tr>
</tbody>
</table>
It is clear that passengers need to make decisions only at discrete points in time (e.g. when a run arrives at a stop). Therefore, the location of the passenger changes at discrete points in time; and hence the underlying Markovian process is considered a discrete-time stochastic process. From Table 4-5, there are four types of choices: departure time choice, stop choice, route (or run) choice and transfer choice. The transfer choice contains in fact two selections: alighting off-stop choice and transfer on-stop choice.

<table>
<thead>
<tr>
<th></th>
<th>Departure Time Choice</th>
<th>Stop Choice</th>
<th>Transfer Choice</th>
<th>Route (or Run) Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>√</td>
<td>√</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>S</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>√</td>
</tr>
<tr>
<td>V</td>
<td>x</td>
<td>x</td>
<td>√</td>
<td>x</td>
</tr>
<tr>
<td>F</td>
<td>x</td>
<td>√</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

The schematic representation of the state-action pairs is shown in Figure 4.8. This figure represents the state transitions in the underlying stochastic process. In this representation, the system state variable represents the location of the passenger during the transit trip. The state space has 7 possible locations, shown by 7 different colors in Figure 4.8. The underlying stochastic process has the Markov property, since the present location of the passenger is necessary and sufficient for predicting the future location of the passenger, regardless of prior locations.

For any transit trip, the number of states is finite (i.e. possible locations are finite). The state \( OGN \) is a strong state; that is, any state \( j \) (where \( j \neq OGN \)) is reachable from state \( OGN \) in a finite number of steps. With the existence of one strong state, a Markov Chain with a finite number of states is guaranteed to be irreducible. Therefore, the underlying Markov process for the transit path choice problem is irreducible.
The state $OGN$ is a positive-recurrent state, since after leaving $OGN$ the probability of returning back to $OGN$ in any number of steps is 1. In a Markov Chain with a finite number of states, the existence of one positive-recurrent state guarantees that the Markov Chain is positive-recurrent. Therefore, the underlying Markov process for the transit path choice problem is positive-recurrent.

The state $OGN$ has a possible self-loop transition, representing the departure time choice from origin. Similarly, the $OGS$ and $ONS$ states have a possible self-loop transition, representing the no-boarding decision. Therefore, states $OGN$ and $ONS$ are aperiodic. With the irreducibility property and finite number of states, this means that the underlying Markov process for the transit path choice problem is aperiodic.

The Markov Chain representing the transit path choice problem is therefore ergodic, meaning that there exists a unique optimal policy $\pi^*$ that will optimize the return of the transit trip (or minimize the travel cost in this regard) given a value function. Alternatively, given an observed optimal policy $\pi^*$, a value function can be estimated so as to regenerate the observed optimal policy (or observed choices). If the form of the value function is assumed, then its parameters can be calibrated to maximize the likelihood of reproducing the observed choices.
Figure 4.8 Schematic representation of the state-action pairs for the transit path choice problem
When the objective is to find the probability of choosing action \( a \) at state \( s \), \( P_s(a) \), Reinforcement Learning (RL) techniques provide a systematic procedure for estimating these probabilities. RL algorithms attempt to find a policy \( \pi^*: S \rightarrow A \) that the agent follows and it gives the probability \( P_s(a)^* \). In particular, the Q-Learning algorithm shown in Figure 4.5 is adopted in this framework.

In RL terminology, passengers need to follow a policy that defines the passenger’s behaviour (i.e. trip choices) at a given time. Passengers need a reward function which signals the immediate reward of a specific state-action pair. For instance, the immediate reward of boarding a bus \( i \) from a stop \( j \) is represented by the experienced waiting time for bus \( i \) at stop \( j \). A value function calculates the accumulated reward over time of a specific state-action pair. The value function ensures that state-action pairs with short-term high reward but with a long-term low value are not preferred. For example, the value of boarding a bus \( i \) from a stop \( j \) is calculated as the expected travel time (out-of-vehicle and in-vehicle time) to the destination, starting from this stop and boarding this bus. In the transit assignment context, Q-value represents the state-action utility (or called hereafter the Generalized Cost, GC). For a passenger,

\[
GC(s,a) = \sum_{i=1}^{n} \hat{\beta}_i \cdot X_i = \sum_{j=1}^{k} \hat{\beta}_j \cdot X_j + \gamma \min_{\forall a' \in A(s')} GC(s',a'), \text{ and}
\]

\[
GC(s,a) \leftarrow [1 - \alpha] \cdot GC(s,a) + \alpha \cdot \left[ \sum_{j=1}^{k} \hat{\beta}_j \cdot X_j + \gamma \min_{\forall a' \in A(s')} GC(s',a') \right]
\]

(4.4)

(4.5)

Where

- \( X_i \) is a passenger-experienced alternative-specific (state-action) attribute.

- \( \sum_{j=1}^{k} \hat{\beta}_j \cdot X_j \) represents the immediate reward (or cost) for the state-action pair.

- \( \gamma \min_{\forall a' \in A(s')} GC(s',a') \) represents long-term expected reward for choosing this state-action pair.

Note that \( X_r \), (e.g. \( X_{WT} \): waiting time) is a random variable, and its realization at time step \( t \) depends on the transit network conditions and other passengers’ choices. Therefore, the GC
value of a specific state-action pair is a random variable and the GC function acts in replacement of the unobserved value function.

The choice rule follows a mixed \{\epsilon-\text{greedy}, \text{SoftMax}\} action-choice model, with a \((1-\epsilon)\) probability of exploitation, \(\epsilon\) probability of exploration, and a SoftMax model such that:

- When exploiting, \(P_s(a) = \begin{cases} 1 & \text{GC}(s,a) = \min_{a'} \text{GC}(s,a'), \text{ and} \\ 0 & \text{otherwise} \end{cases}\)

- When exploring, \(P_s(a) = \frac{V(s,a)}{\sum_{a' \in \mathcal{A}(s)} V(s,a')} \cdot V(s,a) = g(\text{GC}(s,a))\).

The \(\epsilon-\text{greedy}\) method means that agents behave greedy most of the time (i.e. they exploit current knowledge to maximize immediate reward) and every once in a while, with probability \((\epsilon)\), they select an action at random, uniformly and independently of the action-value estimates. The drawback with this choice rule is that as agents explore they choose equally among all actions. This means that it is likely to choose the worst action as it is to choose the best action. The SoftMax method varies the action-choice probabilities based on the estimated reward of each action. In this method, the greedy action is still given the highest selection probability, and all other actions are weighted according to their estimated values.

### 4.2.2 Learning-Based Transit Assignment Model

The underlying hypothesis is that individual passengers are expected to adjust their behaviour (i.e. trip choices) according to their experience with the transit system performance. Individual passengers base their daily travel decisions on the accumulated experience gathered from repetitively travelling through the transit network on consecutive days. Travellers’ behaviour, therefore, should be modeled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. This decision-making process is based on a ‘mental model’ of the transit network conditions.
The fundamental concept of Q-learning is that an obvious way to estimate the $Q(s,a)$ from experience is simply to average the returns observed after visits to that state $s$ and taking the same action $a$. As more returns are observed, the average should converge to the expected value. In an episodic task, e.g. chess game, more visits are encountered by repeatedly re-starting the task.

The learning and decision-making processes of passengers are assumed in the proposed model to follow Reinforcement Learning (RL) principles for experience updating and choice techniques. In particular, the Temporal-Difference (TD) Q-Learning algorithm is applied. Reinforcement learning is based on the concept that agents learn, by interacting with their environment, what actions to take so as to maximize a reward signal. Agents are not instructed what actions to take, but rather they discover which actions yield the most reward by trying them. Assuming passengers as agents, transit riders need to discover what combination of departure time and transit path choices will maximize their reward (i.e. minimize the expected travel cost).

It has been shown that; based on the proposed mental model structure, the underlying stochastic process is ergodic. In the transportation context, this means that each possible combination of path choices can be tried by iterative dynamics (i.e. all path options can be tried through repeatedly making the same trip over days). Iterative dynamics, in the modelling of transit assignment, are carried out by iteratively *loading* passengers at their origin after they reach their destination; thus passengers are repeatedly making choices and updating perceptions.

The mental model represents the passenger’s *memory* where previous experiences are stored, and it reflects the passenger’s perception (i.e. knowledge) about the transit network conditions. The mental model tree-like structure is a more efficient representation of the state-action table traditionally developed for Q-learning problems – see Figure 4.9. The probability of deciding on action $a$ at state $s$ is based on the accumulated experience, GC-value. The GC($s,a$) is updated every time state $s$ is visited and action $a$ is chosen. The GC($s,a$) has two components: an immediate reward for choosing action $a$ at state $s$, and an estimated accumulated future reward for this specific *state-action* pair. This GC-value represents the passenger’s experience with the transit network conditions. While there are only five types of different choices (Figure 4.9),
passengers sometimes need to make the same type of decision more than once during the trip. For example, the passenger in Figure 4.7 needs to make one departure time choice, one origin stop choice, two run choices, one off-stop choice, and one on-stop choice. The agent-experience and network-conditions, based on which the passenger decides on the run choice, are different for the two run choices. For each run choice, the GC-value is accumulated over time under different network conditions (with different choices for other passengers). The accumulated specific agent-experience for each state-action pair is stored in the passenger’s mental model.

Previous studies refer to the path choice problem as the decision passengers make to board a vehicle departing from a stop, at a specific point in time, which is in a set of attractive paths serving the origin and destination for that passenger – see Hickman and Bernstein (1997) for definitions of the static and dynamic path choice problems. Why a passenger is at this stop (i.e. origin stop choice) at this point in time (i.e. departure time choice) is not addressed adequately in the literature as part of the path choice problem. Often, the travel time on paths including a transfer depends on when the passenger begins the trip: paths which are attractive at one time may be less so later when there is a smaller probability of making a particular connection. The proposed learning-based choice model considers the departure time choice, the stop choice and the run (or sequence of runs) choice. To accommodate the modelling of departure time choice, another state category is introduced, departure-time tempo-location (T) which , explicitly represented in Figure 4.9. Table 4-6 shows the updated state-action pairs table with five state categories and five choice types.

The departure time and origin stop choices are assumed to be at-home choices (i.e. pre-trip), in which passenger-agents consider available information obtained from previous trips, in addition to pre-trip information provision (if any). Once a passenger-agent arrives at a stop, the bus run choice is considered an adaptive choice, in which, besides previous information, the passenger considers developments that occur during the trip. It is important to mention that, because of the dynamic representation of the transportation network (i.e. a microsimulation model), the adaptive choice is relative to the specific run of each line. In other words, the proposed approach considers the path choice as time-dependent.
Figure 4.9 The Mental Model Structure, with different types of choices

Table 4-6 Updated State-Action pairs for the transit path choice problem for passenger-agents at any time $t$

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
<th>Departure Time Choice $t \in A(O)$</th>
<th>Origin-Stop Choice $g \in A(T)$</th>
<th>Off-Stop Choice $f \in A(V)$</th>
<th>On-Stop Choice $n \in A(F)$</th>
<th>Route (or Run) Choice $r \in A(S), s = {g,n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>✓</td>
<td></td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>T</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>S</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>V</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>F</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
At the origin (i.e. home for a home-based work or school trip), a passenger develops an initial (tentative) travel plan, based on his updated mental model (historical experience and pre-trip information, if provided). This travel plan includes a departure time, an origin stop (which are fixed) and a (tentative) run (or sequence of runs). The initial plan reflects the passenger’s preferences and expectations, and he would follow it if the reality (en-route dynamics) matches, to a great extent, his expectations (which may be different from published static schedule information). The process, through which the passenger decides on the initial plan, is modelled by a mixed \{\text{\textit{greedy}}, \text{\textit{SoftMax}}\} action-choice model. Based on the nested-structure outlined in Figure 4.9, the choice of departure time precedes the choice of origin-stop choice. As in nested-logit model structures, the order of choices does not always represent the real sequence of passenger’s decisions; passengers often make their path choices simultaneously and the assumed order is for analysis purposes.

At origin, on iteration \(d\), the departure time choice is modelled according to the choice rule outlined in Figure 4.10 (\textit{a mixed \{\text{\textit{greedy}}, \text{\textit{SoftMax}}\} action choice model}). Being at state \(O\), a specific departure time choice signals an immediate reward and a future return for a passenger-agent \(z\) and on iteration \(d\), which can be written as:

\[
\overrightarrow{d} GC_{Z}(O, t) \leftarrow \left[1 - \alpha \right] \cdot \overrightarrow{d-1} GC_{Z}(O, t) + \alpha \cdot \left[ \overrightarrow{d} \Gamma_{Z} + \gamma \cdot \min_{g \in A_{z}(T)} (GC_{Z}(T, g)) \right]
\]

(4.6)

Where \(t : O \rightarrow T\), and \(\overrightarrow{d} \Gamma_{Z} = \Theta_{Z}'\); \(\Theta_{Z}'\) is a constant (or a variable) representing the utility of leaving at time \(t\) (e.g. auto availability, activity-schedule fitting value). This is an input to the transit path choice model from activity-based models.

After completing the trip with departure time \(t\), the \(\overrightarrow{d} GC_{Z}(O, t)\) is updated to \(\overrightarrow{d} GC_{Z}(O, t)\) reflecting the passenger-agent experience with this state-action pair. It is important to note that the choice at iteration \(d\) is based partly on the most updated experience from iteration \(d - 1\); passengers use their experience to predict the travel cost associated with their trip. Without information
provision, passengers have no means of knowing the \textit{actual} travel cost of their trip on iteration $d$ before starting the trip, however they utilize what they have \textit{learned} to make an estimate. If the estimated value, $\overline{GC}_z(O,t)$, proves to be close to the actual travel cost $\Gamma$ then the associated choice is reinforced in future iterations.

It is also important to note that the update of the GC-value occurs \textit{after} the choice has been made since the Q-value of only visited state-action pairs is updated. This does not contradict with the sequence of mental model \textit{updating} then \textit{utilization} mentioned in section 3.2.3.3; in the absence of information provision, the mental model contents updated at the end of iteration $d-1$ are representative of the mental model contents at the beginning of iteration $d$. Due to exploration and the ergodicity property, all state-action pairs are guaranteed to be visited as $d \to \infty$. Under information provision, an additional mental model updating step, prior to mental model utilization, is required. It becomes possible to estimate $\overline{MIN}_{\forall \tau \in A(T)} \langle GC(T,\tau) \rangle$ before the trip starts.

Note that $\overline{MIN}_{\forall \tau \in A(T)} \langle GC(T,\tau) \rangle$ is not only dependent on information provided about transit service performance for iteration $d$, but it is also dependent on $\overline{MIN}_{\forall \tau \in A(T)} \langle GC(T,\tau) \rangle$. The topic of \textit{experience-information integration} is addressed in the next chapter.

\footnote{Note the difference between this expression and the one used in updating $\overline{GC}_z(O,t)$, equation (4.6). When no information provision is assumed, the value $\overline{MIN}_{\forall \tau \in A(T)} \langle GC(T,\tau) \rangle$ is not known \textit{apriori}, and can only be calculated at the end of iteration $d.$}
- Generate $u = \text{Uniform}(0,1)$

- if $u \leq \varepsilon$ [Exploration]

  \[
  \frac{d}{P_o(t)} = \sum_{\forall i \in A(O)} f\left(\frac{d-1}{GC(O,i)}\right)
  \]

else [if $u > \varepsilon$] [Exploitation]

  \[
  \frac{d}{P_o(t)} = \begin{cases} 
  1 & \text{iff } GC(O,t) = \text{MIN}_{\forall i \in A(O)} GC(O,i) \\
  0 & \text{Otherwise}
  \end{cases}
  \]

Figure 4.10 a mixed $\{\varepsilon - \text{greedy, SoftMax}\}$ action choice model for the departure time choice$^5$

---

$^5$ The passenger-agent subscript $z$ is dropped from the equations in Figure 4.10 and for the rest of the discussion in this chapter, with the understanding that all equations represent the decision-making behaviour of a passenger-agent $z$. 
When a passenger-agent is at state \( t : O \rightarrow T \) (i.e. after deciding on a departure time), the origin stop choice follows a similar procedure to the one outlined in Figure 4.10, where:

\[
\frac{d}{P_T(g)} = \frac{\sum_{i \in A(T)} f \left( \frac{d-1}{GC(T,g)} \right)}{f \left( \frac{d-1}{GC(T,i)} \right)} \quad \text{when exploring experience,}
\]

\[
\text{and } \frac{d}{P_T(g)} = \begin{cases} 
1 & \text{iff } \frac{d-1}{GC(T,g)} = \text{MIN} \left( \frac{d-1}{GC(T,i)} \right) \\
0 & \text{Otherwise}
\end{cases} \quad \text{when exploiting experience.}
\]

The origin stop choice has an immediate cost (e.g. access time) and a future value represented by the travel cost associated with possible attractive route choices from the origin stop. The generalized cost for an origin stop, for a passenger-agent \( z \) on iteration \( d \), is updated as follows:

\[
\frac{d}{GC(T,g)} \leftarrow [1 - \alpha] \cdot \frac{d}{GC(T,g)} + \alpha \cdot \left[ \frac{d}{\sum_{r \in A(S), r \neq g} \beta_j \cdot X_j} + \gamma \cdot \text{MIN} \left( \frac{d-1}{GC(S,r)} \right) \right]
\]

Where \( g : T \rightarrow S \), and \( \frac{d}{\Gamma^g} = \sum_{j=1}^{k} \beta_j \cdot X_j \):

- \( X_j \) represent the immediate travel cost associated with stop \( g \); for example, access time and access cost to stop \( g \). The modelling of access mode choice may be introduced at this level by varying \( X_r \) as a function of the access mode.
- \( \beta_j \) can be an alternative-specific parameter; for example, access time for train stations is perceived differently from bus stops. \( \beta_r \) may also be a general-parameter for all alternatives, such as the fare parameter that reflects the perceived monetary cost associated with a specific \( g \) choice.
It is worth mentioning that the origin state \((O)\) category has only one element; there is only one origin for each trip. The departure time state \((T)\), on the other hand, has more than one element, representing the possible origin departure time options. Similarly, the origin stop state \((S = g)\) category might have more than one element; there could be multiple accessible and attractive origin stops for a trip. Therefore, the estimated travel cost of choosing stop \(g\) is associated with a specific departure time \(t\). This permits the representation of different transit network conditions at different times during the modelling period. For example, the access time for stop \(g\) associated with departure time choice \(t_i\) may be different from the access time for stop \(g\) associated with departure time choice \(t_2\), due to, for example, congestion building or access mode availability. Accordingly, the perceived travel cost for stop \(g\) while leaving at \(t_i\), \(GC_{t_i,g}( )\) will be different from \(GC_{t_2,g}( )\) which is the perceived travel cost for stop \(g\) while leaving at \(t_2\). This is reflected in the departure time choice through the term \(MIN_{\forall t \in A(T)} (GC(T,g))\) in equation (4.6).

When a passenger arrives at a stop \(s \in \{g,n\}\), an initial route choice is made. This route choice follows the mixed \(\varepsilon\text{-greedy, SoftMax}\) action choice model with \(\frac{d}{P_s(r)} = \left[ f \left( \frac{d}{GC(S,r)} \right) \right] \sum_{\forall s \in A(S)} f \left( \frac{d}{GC(S,i)} \right)\) and

\[
\frac{d}{P_s(r)} = \begin{cases} 
1 \quad \text{iff} \quad GC_{S,r} = \min_{\forall s \in A(S)} \left( GC(S,i) \right) \\
0 \quad \text{Otherwise}
\end{cases}
\]

for experience exploration and exploitation, respectively. The value of a run \(r\) (or route) choice at a stop \(s\) is updated based on the following equation:

\[
\frac{d}{GC(S,r)} \leftarrow \left[ 1 - \alpha \right] \cdot \frac{d}{GC(S,r)} + \alpha \cdot \left[ \frac{d}{\Gamma'} + \gamma \cdot \min_{\forall f \in A(V)} \left( GC(V,f) \right) \right]
\]

\[(4.10)\]

Where \(r : S \rightarrow V\), and \(\Gamma' = \sum_{j=1}^{k} \beta_j \cdot X_j\).
- $X_j$ represents immediate travel cost associated with run $r$, for example, waiting time for run $r$ (of route $R$), crowding level of run $r$, etc.
- $\beta_j$ can be an alternative-specific parameter; for example, waiting time for trains is perceived differently from waiting time for buses.

This decision-making and experience updating mechanism, however, reflects previous experiences of the chosen route and does not take into consideration the within-day dynamics of the transit service. Coupled with the aforementioned mechanism, a number of SWITCH routines are introduced. There are four SWITCH routines: ‘SWITCH AND WAIT’, ‘SWITCH AND BOARD’, ‘SWITCH AND STAY’, and ‘SWITCH AND ALIGHT’. The first two routines are concerned with the adaptive behaviour when passengers are at stops, while the latter two routines are used to model the adaptive behaviour when passengers are on-board of a transit vehicle. The outcome of these routines is of a boolean type, returning 0 (NO) or 1 (YES) answers – see Figure 4.11.

For a passenger waiting at stop $s \in \{g,n\}$, there is an initial route choice $r$. When a transit vehicle from route $r$ arrives at stop $s$, the SWITCH AND WAIT routine is called for this passenger. If the expected travel cost, $GC(S,r)$, is in line with (i.e. matches or better than) the transit service performance and the within-day dynamics, $\Gamma^r + \gamma \cdot \min_{V \in A(V)} (GC(V,f))$, then the SWITCH AND WAIT routine returns NO; meaning that the passenger will board run $r$. By examining equation (4.10), the within-day dynamics, for iteration $d$, are represented by the term $\Gamma^r$. Without provision of information, a passenger has no knowledge of the performance of other attractive runs on iteration $d$ except through previous experiences, summarized up to iteration $d-1$. Since the experiences for all attractive runs were considered in the initial route choice, then there is no need to re-evaluate the initial choice as reality matches expectations (for the initial choice).
When within-day dynamics show that the expected performance of run $r$ does not match (i.e. worse than) reality, a reconsideration of the initial route choice is warranted. In this case, the route choice mechanism is applied with \[ \Gamma' + \gamma \cdot \min_{V_f \in A(V)} \{GC(V, f)\} \] as a replacement of $GC(S, r)$ while $GC(S, i), \forall i \neq r$ remain the same. The SWITCH AND WAIT routine will return NO if the initial route choice still outperforms other attractive routes (based on experience summarized up to iteration $d-1$ for other route options). A YES will be returned if the within-day dynamics of run $r$ has resulted in another attractive route $r'$ outperforming route $r^\circ$. The passenger then waits (and hence the name of the routine) for a run from route $r'$ to arrive. Regardless of the output of the SWITCH AND WAIT routine $GC(S, r)$ is calculated based on equation (4.10), since $\Gamma'$ has been already experienced.

For a passenger waiting at stop $s \in \{g, n\}$ with an initial route choice $r \in A(s)$, the SWITCH AND BOARD routine is called when a transit vehicle arrives from route $r' \in A(s)$. The updated travel cost for route $r'$, \[ \Gamma' + \gamma \cdot \min_{V_f \in A(V)} \{GC(V, f)\} \], is calculated. If its updated travel cost matches the expected value, $GC(S, r')$, then there is no need to reconsider the initial route choice $r$ and the SWITCH AND BOARD routine will return NO. If the updated travel cost for route $r'$ is improved, then the initial route choice, $r$, needs reconsideration. In this situation, \[ \Gamma' + \gamma \cdot \min_{V_f \in A(V)} \{GC(V, f)\} \] is compared to $GC(S, r)$, without considering other attractive routes since the passenger’s expectations regarding their performance remain the same, $GC(S, i), \forall i \neq \{r, r'\}$, and they were inferior to the initial route choice, $r$. If the updated travel cost of route $r'$ becomes more attractive then the SWITCH AND BOARD routine returns YES and the

---

6 This outcome is represented by the self-loop transition of stop-state in Figure 4.8, proving that the underlying Markov Chain is aperiodic.
When a passenger is on board of a transit vehicle, \( V \), of route \( r \), there are two possibilities. First, this route \( r \) reaches the trip destination and no more travel choices are required (except egress mode choice, if any). In this case, \( A(V) \) has only one element, \( \{\text{des}\} \), representing the destination stop associated with route \( r \). Based on the mental model structure, there can be only one destination stop associated with each route. The immediate travel cost of deciding on alighting at a destination stop can be thought of as, for example, the in-vehicle time and fare to reach the destination stop. There is also a future travel cost associated with this decision; this cost includes egress time and egress monetary cost. When the passenger arrives at destination, the schedule
delay is calculated. It represents the deviation from the scheduled arrival time by reaching the destination earlier or later than desired.

It is worth mentioning that the destination stop choice, \( \{ \text{des} \} \), is linked to a route \( r \). The route choice is associated with a stop choice \( s \), which is for a departure time \( t \). This has two implications. First, the experience for path choices is relevant to the departure time choice. It is not uncommon that passengers make different trip decisions for different origin departure time choices. The transit service performance is observed to be time-dependent and passengers have time-dependent expectations about path choices, hence the departure time choice. Note that the transit service performance is time-dependent not only due to the passengers’ decisions to leave from origin but also because of other factors such as traffic congestion. Therefore the relationship between passengers’ departure time choice and transit service performance does not follow a vicious logic; the cycle is broken by external factors – see Figure 4.12.

![Figure 4.12 Relationship between passengers’ choices and transit service performance](image)

The second implication is related to the schedule delay (\( SD \)). In this formulation, \( SD \) is dependent on both the departure time and trip choices. This can be noted from equation (4.6); \( SD \) does not directly appear in the travel cost calculation for departure time choice and is implicitly included through the term \( \min_{g \in A_2(T)} \{ GC_g(T, g) \} \), by recursive computations.
Since no more trip decisions are needed at destination, the destination location represents the terminating-state. At this state, the recursive calculations end and are propagated backwards to update perceptions for the departure time choice made at the beginning of the trip. The $GC(V, des)$ is updated based on the following equation:

$$
GC(V, des) \leftarrow [1 - \alpha] \cdot GC(V, des) + \alpha \cdot \left[ \Gamma^{des} + \gamma \cdot \Omega^{des} \right]
$$

(4.11)

Where $des: V \rightarrow D$, $\Gamma^{des} = \sum_{j=1}^{k} \beta_j \cdot X_j$:

- $X_j$ represents immediate travel cost associated with the destination stop, for example, in-vehicle time to reach the destination stop, fare to reach the destination stop.
- $\beta_j$ can be an alternative-specific parameter; for example, in-vehicle time for subways is perceived differently from in-vehicle time for buses.

and $\Omega^{des} = \sum_{a=1}^{k} \beta_a \cdot X_a$:

- $X_a$ represents future travel cost associated with the destination stop, for example, egress time and egress monetary cost to reach the trip destination from the destination stop. It also represents the schedule delay associated with trip choices.
- $\beta_a$ reflects the various perceptions with regard to future travel cost components. For instance, the early/late schedule delay values.

The second possibility for on-board passengers occurs when a transfer connection is needed to reach the trip destination. In such situations, a passenger decides on an initial off-stop choice upon boarding the transit vehicle $V$ of route $r$ following the same action-choice yielding $P_{V}(f)$.
\[
\frac{f \left( \frac{d^{-1}}{GC(V,f)} \right)}{\sum_{i \in A(V)} f \left( \frac{d^{-1}}{GC(V,i)} \right)} \text{ for exploration and } \frac{d}{P_r(f)} = \begin{cases} 
1 & \text{iff } GC(V,f) = \min_{i \in A(V)} \left( \frac{d^{-1}}{GC(V,i)} \right) \\
0 & \text{Otherwise} 
\end{cases}
\]

when exploiting of experience occurs. The GC-value for the off-stop choice is updated as follows:

\[
GC(V,f) \leftarrow [1 - \alpha] \cdot GC(V,f) + \alpha \left[ \Gamma' + \gamma \cdot \min_{i \in A(V)} \left\{ GC(F,n) \right\} \right]
\]

(4.12)

Where \( f : V \rightarrow F \), and \( \Gamma' = \sum_{j=1}^{k} \beta_j \cdot X_j \):

- \( X_j \) represents immediate travel cost associated with off-stop \( f \), for example, in-vehicle time to each off-stop \( f \), number of transfers to reach destination from off-stop \( f \), etc.
- \( \beta_j \) can be an alternative-specific parameter (in-vehicle time for subways is perceived differently from in-vehicle time for buses) or a general-parameter (transfer penalty).

This initial choice is based on previous experiences, \( GC(V,f) \), and does not reflect within-day dynamics of the transit service. The \( SWITCH AND STAY \) and \( SWITCH AND ALIGHT \) routines capture passengers’ adaptive behaviour in response to service dynamics.

When the transit vehicle arrives at the initially chosen off-stop \( f \), the passenger deploys the \( SWITCH AND STAY \) routine. The routine returns \( NO \) to represent the situation when the current experience for the chosen off-stop matches expectations and the passenger decides to alight at the off-stop \( f \). In other situations, passengers may experience an \textit{unexpected} travel cost associated with the initial off-stop choice \( f \) (e.g. an unexpected delay). This warrants the re-evaluation of the initial off-stop choice since it may not be the optimal choice anymore (assuming exploitation is
in effect). The updated travel cost, \( \frac{d}{\Gamma^f} + \gamma \cdot \min_{n \in A(F)} \left( GC(F,n) \right) \), is compared to \( GC(V,f'), \forall f' \in A(V) \). If another off-stop option \( f' \) becomes more attractive then \SWITCH AND STAY \ returns \( YES \) and the passenger decides to stay on board and the initial off-stop choice changes to \( f' \). Otherwise, the passenger alights at stop \( f \). This is also restricted to the availability of subsequent feasible off-stop choices, while on-board of route \( r \). In all cases, the GC-value of off-stop \( f \) is updated according to equation (4.12) since the experience with this choice, \( \Gamma^f \), is encountered.

When the transit vehicle arrives at a feasible off-stop \( f' \in A(V) \), \( f' \neq f \) and \( f \) is the passenger’s initial off-stop choice, the \SWITCH AND ALIGHT \ routine is called. If the travel cost associated with off-stop \( f' \) is observed to be improved, then \( \frac{d}{\Gamma^f} + \gamma \cdot \min_{n \in A(F)} \left( GC(F,n) \right) \) is compared to \( GC(V,f) \). The \SWITCH AND ALIGHT \ routine returns \( YES \) when a passenger changes the initial off-stop choice \( f \) and decides to alight at the current stop \( f' \). A \( NO \)-outcome means that the passenger stays on-board for the arrival at the initially chosen off-stop \( f \). Note that \( f' \) is always encountered prior to \( f \) (even if \( f \) is updated through the \SWITCH AND STAY \ routine).

When a passenger alights at an off-stop \( f' \), a decision concerning the transfer on-stop, \( n \in A(F) \), needs to be made. This decision is based on the expected travel cost associated with each possible transfer on-stop. These expectations reflect previous experiences and choices are made following a similar decision-making behaviour outlined for previous choices. When exploiting, and in the absence of real-time information, a passenger chooses a transfer on-stop \( n \) such that 
\[
P_r(n) = \begin{cases} 
1 & \text{iff } GC(F,n) = \min_{i \in A(F)} GC(F,i) \\
0 & \text{Otherwise}
\end{cases}
\]

The probability of choosing a
transfer on-stop \( n \) when exploring experience is calculated as \( P_r(n) = \frac{\sum_{i \in A(F)} f\left(\frac{d_{i,n}^{d-1}}{GC(F,n)}\right)}{d_{i,n}^{d-1}} \). When a passenger arrives at a transfer on-stop \( n \), an initial route choice needs to be made. The boarding decision is dependent on the within-day dynamics of the transit service as explained before.

There could be an immediate penalty associated with the choice of a transfer on-stop \( n \) such as inconvenience due to walking or crossing multiple intersections to reach the on-stop \( n \). There is also a future travel cost related to the choice expressed in the trip return of attractive routes \( r \in A(S), s = n \). The travel cost associated with a transfer on-stop \( n \) choice is updated as:

\[
\Gamma^n = \left(1 - \alpha\right) \cdot GC(F,n) + \alpha \cdot \sum_{i \in A(S), s = n} \left(\frac{d_{i,n}^{d-1}}{GC(F,n)}\right) + \gamma \cdot MIN_{r \in A(S), s = n} (GC(S,r))
\]

(4.13)

Where \( n : F \rightarrow S \), and \( \Gamma^n = \sum_{r=1}^{d} \beta_j \cdot X_j \):

- \( X_j \) represents the immediate travel cost associated with an on-stop choice \( n \); for example, transfer walk time.
- \( \beta_j \) reflects the passenger’s perception of the transfer penalty associated with on-stop choices

The proposed model assumes that individual passengers are decision makers, who choose a departure time from home, an origin stop, a destination stop and a route between a given pair of an origin and a destination each day. This decision-making behaviour, as explained, consists of a two-step process: making choices and updating perceptions – see Figure 4.13. When real-time information is not provided, the step of making choices precedes perception updating. Perception updating occurs only when the state-action pair is visited. Pre-trip decisions are treated as fixed choices, while en-route choices are adaptive.
Figure 4.13 Decision-making tree for transit riders
4.3 Notes on the Proposed Model

4.3.1 A Note on the Modelling of Information Provision

The benefits of Advanced Traveller Information Systems (ATIS) applications can be assessed by comparing the path choice behaviour and cost-savings of informed passengers with non-informed ones. However, conventional transit assignment models assume that passengers have full information about the network conditions and infinite information processing capabilities; this is referred to as the “perfect knowledge of network” assumption. These models are not appropriate for modelling information provision since information regarding network conditions are assumed to be available, anyway, to all passengers. The emergence and increased deployment of ATIS make it practically important to relax the assumption of perfect information or perfect knowledge in transit assignment studies.

MILATRAS represents a deviation from the traditional perfect information assumption. The system performance on iteration \(d\) is dependent on choices of other passengers; these choices are not known apriori to the subject passenger. In the proposed model and with no provision of real-time information, passengers do not have perfect knowledge about the system performance on iteration \(d\). Instead, passengers form their expectations about the performance of the transit service.

Under information provision, an additional ‘mental model update’ step is required before making choices: \(\Gamma^a\) can now be obtained for all \(a \in A(S)\) and \(P_s(a) = f \left( \frac{d}{GC(s,a)} \right)\), compared to \(P_s(a) = f \left( \frac{d-1}{GC(s,a)} \right)\) when real-time information is not provided. This is reflected in the SWITCHES routines. For example, the SWITCH AND WAIT routine compares
$\left[ d \left( \Gamma' + \gamma \cdot \min_{v \in A(V)} \langle GC(v, f) \rangle \right) \right]^{d-1}$ and $\langle GC(S, i), \forall i \neq r \rangle$. When information is not provided, $\langle GC(S, i), \forall i \neq r \rangle$ represents passengers’ expectations for the unknown performance of other attractive routes, in iteration $d$. With information provision, $\Gamma'$, $\forall i \neq r$ is now obtainable.

Based on the above, information provision will become effective 1) when great variability in service performance exists and average GC-values are not representative of state-action pair values, 2) when non-recurrent congestion occurs – due to, for example, frequent traffic incidents –, where $\langle GC(S, i), \forall i \neq r \rangle$ are not representative of the network dynamics in iteration $d$, and 3) for non-familiar travellers whose expectations are not representative for the network performance. In such cases, SWITCH routines will have more role in the decision-making process. The impact of provided information can then be estimated by comparing passengers’ usage of SWITCH routines in the case of absence of information and the case of information provision.

In normal conditions and for frequent travellers, GC-values will approximate the transit service performance and will be close to provided information, if any. Therefore, providing real-time information is not expected to have an effect on passengers’ travel decisions more than their expectations have. SWITCH routines, in these conditions, have another role. These routines make sure that passengers’ experiences are realistic by adaptively re-evaluating choices, when needed.

4.3.2 A Note on the Number of Reinforcement Learning Agents

Transit assignment is a process of interactions between individual passengers and transit services. These interactions are in both directions: the execution of path choices affects service performance, yet the expectation of performance influences choices. There is a mutual-dependence among passengers’ choices.
While the mental model (or Q-value table) structure is assumed to be the same for all passenger-agents, the specification of the mental model for each passenger might be different. The state space (e.g. possible locations) for passengers is not universal; different origin stops are available (accessible) for different passengers. In addition, all modelled passengers are assumed to be following the mixed $\{\varepsilon-greedy,\text{SoftMax}\}$ action choice model.

In cases where all passengers belong to the same socio-economic class with the same value function specification (variables and parameters), there is in effect one RL-agent. Each modelled passenger represents a cloned version of this RL-agent with a separate Q-value table (or a separate mental model). If there are two passengers who have the exact mental model structure (similar trip origin and destination geo-location with same scheduled arrival time), both passenger-agents will converge to the same optimal policy $\pi^*, P_s(a) : s \rightarrow s'$. While both passengers will experience different transit service performance at different points in time (due to other passengers’ choices and exploration behaviour), both passengers will have the same GC-value (or Q-value) for each state-action pair as $d \rightarrow \infty$. When capacity is constrained, the within-day dynamics (e.g. over-capacity) force some passengers to adapt their choices based on the instantaneous GC-value (for the current iteration, it is $\Gamma_{a \in A(S)}^{d} + \gamma \cdot \min_{a' \in A(S')} \left\langle GC(S', a') \right\rangle$) of state-action pairs, and not the average GC-values over all iterations (i.e. $\overline{GC(S, a)}$).

For two passengers from the same socio-economic class with different mental model specification, each passenger-agent will converge to a unique optimal policy $\pi^*, P_s(a) : s \rightarrow s'$. This is expected since GC-values for state-action pairs are different, even though both passengers utilize the same decision-making behaviour (same action choice model and value function specification in terms of parameters and variables).

In cases where there is market segmentation (e.g. socio-economic classes), the number of RL-agents is the same as the number of modelled socio-economic groups. For example, if home-based work trip makers and home-based school trip makers are considered as two market
segments, then there are two RL-agents. This classification should be reflected in the value function specification (parameters and/or variables). Since the value function is specific to the socio-economic group, then each group’s optimal policy $\pi^* \cdot P_t(a):s \rightarrow s'$ is expected to be different, even for two passengers with the same mental model specification while each belonging to a different group.

### 4.3.3 A Note on the Modelling of Multi-Agents in a Non-Cooperative Environment

The Ergodic Theory proves that, as $t \rightarrow \infty$, there exists a unique steady state distribution for the underlying Markovian Decision Process if the corresponding Markov Chain is irreducible, positive-recurrent, and aperiodic. With multi-agent modelling, the state space for each agent becomes large, particularly when there is a mutual dependency among agents’ choices (i.e. environment performance depends on agents’ choices). In reality, simulation models will be run for a finite number of iterations $n \neq \infty$. For a small number of iterations, effects such as broken ergodicity (Palmer, 1989) can be expected. Broken ergodicity means that the system is trapped in sub-areas of the state space for long periods of time although it is mathematically ergodic. In practice, this rarely occurs as simulation models relax reasonably quickly to recover from the broken ergodicity property; this was observed in dynamic traffic assignment models (Raney and Nagel, 2003).

### 4.3.4 A Note on the Classification of the Proposed Choice Model

The decision rule (or choice rule) in the proposed model is stochastic, allowing exploitation and exploration of individual’s experience. The utility (or generalized travel cost) term is formed by the passenger, and it has no random error component. Therefore, the proposed model is classified as a bounded rational model, with a constant utility term. On the other hand, random utility models are characterised by a deterministic decision rule and a stochastic utility term.

When the decision making process of transit travellers (see Figure 4.13) is modelled as a nested-logit model (or a cross-nested logit model), the parameter estimation process in the decision-making tree is performed from the bottom nest all the way up to the root. At the run choice-nest,
for example, the parameter estimation procedure *forgets* (or does not consider) previous choice-nests (e.g. stop choice-nest) and takes into consideration further nests, assuming that information needed to make the run choice is preserved at the run choice-nest level. This in many ways is analogous to the Memoryless property of the Markovian Decision Process.

### 4.4 Parameter Calibration Procedure

Thus far, we have focused on the theoretical development of the transit path choice model. In this section, we will address the issue of parameter calibration.

By reconstructing the transit path choice problem as a Markovian Decision Process and representing passengers as agents, the *utility* (or *value*) function can be estimated. Assuming that the value function takes a certain form, \( f(\bar{\beta}) \), the steady state transition probabilities become a function of the value function parameter, \( \Pr^{u} = g(\bar{\beta}) \). Therefore, the function parameters, \( \bar{\beta} \), are calibrated such that the calibrated values minimize a misfit function \( D_{\bar{\beta}} \), where

\[
D = f(\text{observed, simulated}_{\bar{\beta}})
\]

Simulated and observed data can be system-wide (e.g. route loads, \( L \)) or passenger-specific (likelihood of choices, \( \Pr^{u} \)). In other words, the parameters are estimated such that the calibrated values maximize the likelihood of regenerating \( \Pr^{u} \) as \( \Pr^{u}_{\text{observed}} \) or maximize the entropy of \( L^* \) (associated with \( \Pr^{u} \)) to reproduce \( L_{\text{observed}}^* \).

In the specified model, there are variables \( (X_r) \), and parameters \( (\bar{\beta}) \). Model variables are calculated values based on the model dynamics; for example, the passenger waiting time at stop is calculated as the difference between the arrival of a passenger at the stop and the arrival of the route-run at the stop. Model variables are input to the model (i.e. exogenous to the model), such as fare values.

The model parameters \( \bar{\beta} \) need to be estimated endogenously to the model. This estimation process should produce parameter values (i.e. calibrated values) such that the travel behaviour
(disaggregate choices) or travel demand (route loads) predicted by the model conform, to the
greatest extent possible, to travel behaviour or travel demands as observed in reality. To judge
the conformity of predicted travel behaviour or demand, a fitness function is used to measure the
goodness of the calibrated values of parameters.

If the value function is expressed in a closed-form equation, then mathematical optimization
techniques (such as dynamic programming) can be used to find \( \hat{\beta}^* \). The value function does
depend on transit service performance, which depends on other passengers’ choices and the
interactions between transit service and passengers’ decisions. It becomes difficult to represent
these interactions (that are related to unknown passengers’ decisions) in a closed-form
mathematical expression, and hence mathematical optimization techniques for the parameter
estimation problem cannot be utilized. Simulation-based optimization techniques for analyzing
Markovian Decision Process can be used instead. The estimation procedure employs Genetic
Algorithms (GAs), which find the set of (optimal) parameters, out of possible parameter
populations, that will minimize the misfit function – see Figure 4.14.

Genetic Algorithms are stochastic optimization techniques. Their optimization methodology is
based on the phenomenon of “survival of the fittest” from natural evolution. GAs search for the
optimal solution from a wide variety of starting points (i.e. initial population). This initial step,
along with the process of mutation, has proven to prevent GAs from getting trapped in local
optima. GAs have lately been used as a generic, system-independent optimization tool
(Goldberg, 1989 and Krishnakumar et al., 1995). This optimization technique has been favoured
for its capability of effectively searching a multidimensional search space and finding a global
(or best local) optimal solution; it is usually referred to as a combinatorial optimization
technique.

Recently, GA-based parameter estimation procedures have been adapted in the transportation
field; Ma and Abdulhai (2002) used a GA-based procedure to calibrate the traffic
microsimulation model of Paramics®. Parveen et al. (2007) developed the G-EMME/2 tool to
calibrate the EMME/2 transit assignment model parameters using Genetic Algorithms. Roorda et

The evolution of the optimal solution in GAs is based on the cyclic processes of generating genes and natural evolution. A chromosome is a collection of genes; a collection of chromosomes represents a generation. Some genes have desirable characteristics that, with time, will prevail. The fitness of the chromosome is defined by the desirability of its genes. The concept of evolution is reflected in the mating process among chromosomes; a new generation evolves as parent chromosomes mate each other. This involves the exchange of genes’ characteristics. Fit chromosomes have higher probability of mating (i.e. survival of the fittest) and hence children, as hybrid chromosomes, are expected to have better fitness than their parents. As this process continues, the overall fitness of generations is improved until all chromosomes have the same fitness and further evolutions do not produce better children. Based on the fitness evaluation, we are interested in the most fit chromosome in the final generation. The process of mutation, where genes’ characteristics are modified randomly, occurs infrequently. This prevents the evolution process from being stopped when further improvements is possible.

For the proposed transit path choice model and in GA terminology, a gene represents a parameter \( \hat{\beta}_i \). A collection of genes is a chromosome, \( \tilde{\beta} \). Each chromosome is a feasible solution to the parameter estimation optimization problem. A generation of chromosomes is a collection of potential set of parameters. The fittest chromosome in the final generation is the global optimal solution, \( \tilde{\beta}^* \). Since the existence and uniqueness of \( \left[ \Pr^{\tilde{\beta}} \right] \) is guaranteed by the Ergodic Theory (section 4.1.2), the existence and uniqueness of \( \tilde{\beta}^* \) is derived. The fitness (or misfit) function, \( D = f(\text{observed}, \text{simulated} | \tilde{\beta}) \), to evaluate chromosomes is the link between the parameter-calibration optimization problem and the generic, system-independent GA stochastic optimization technique. To calculate the fitness of each chromosome \( \tilde{\beta} \), a transit assignment process needs to be completed. For a given \( \tilde{\beta} \), which specifies the value function, the Q-learning algorithm will generate a \( \pi\left(\tilde{\beta}\right) : S \rightarrow A \). Therefore, one complete transit assignment process
means that a convergence in $\left[ Pr^\beta \right]$ (or $P_s(a) = g(\tilde{X} | \tilde{\beta})$) is achieved, yielding $\left[ Pr^\beta \right] | \tilde{\beta}$. This may require a number of iterations of passengers making choices and updating perceptions.
Figure 4.14 Parameter Estimation Procedure using Genetic Algorithms (GA)
5 PROTOTYPE IMPLEMENTATION7

This chapter presents the implementation of a prototype that is used to demonstrate the applicability and feasibility of the proposed approach. This prototype shows the main principles of MILATRAS; it does not, however, represent a full-scale implementation of the approach. It is important to state that the reported results reflect a hypothetical situation presented by the prototype. The application of MILATRAS to a large scale case study is presented in the next chapter.

The study investigates the impact of different traveller information provision scenarios on transit rider departure time and path choices, as well as network performance, using the proposed agent-based microsimulation learning-based approach. A simulation-based modeling tool was developed that examines transit passenger behaviour from the perspective of individual travelers (agents). This prototype presents an analysis of different information strategies for passengers in a medium-sized transit system. Four information provision scenarios are investigated and the impacts on transit rider travel choices are examined. The study starts with a no information baseline, and compares this with modeled passenger behaviour for pre-trip, at-stop, and on-route (i.e., transfer) information scenarios. The innovative structure here is the passenger as agent in the simulation. This is to our knowledge the only simulation model that explicitly incorporates individual transit passenger behaviour in a fairly comprehensive way, including decision-making, learning, and information strategies explicitly.

5.1 The MILATRAS System Prototype8

One of the objectives of MILATRAS is to provide an environment for travel demand modellers to experiment with dynamic microsimulation sub-models of dynamic departure time and transit path choices, passenger’s perception updating and passenger’s within-day and day-to-day travel choice dynamics. It improves the predictive power of such models by employing an integrated

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7 This chapter has been reproduced, with modifications, from the following previously published material:
- Wahba and Shalaby (2007)
8 Some concepts from Chapter 3 and Chapter 4 are reiterated in this section.
framework that properly models and combines the supply side, the demand side and the interaction between both.

MILATRAS is based on representing passengers as agents and their learning and decision-making activities *explicitly*. The underlying hypothesis is that individual passengers are expected to adjust their behaviour (i.e. trip choices) according to their experience over time with the transit system performance. Individual passengers base their daily travel decisions on the accumulated experience (i.e. *mental model*) gathered from repetitively travelling through the transit network on consecutive days. Individual behaviour, therefore, is modelled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. MILATRAS models each passenger as a microscopic entity (i.e. agent) and models that entity’s reaction to the system directly, while modelling the time-dependent system performance as a response to passenger behavioural choices; this approach is referred to as the “multi-agent” simulation environment.

MILATRAS has an overall microsimulation framework that consists of a number of individual modules – see Figure 3.1. The microsimulation environment represents the transit supply side with a time-dependent and stochastic model, while individual passengers, along with their learning and adaptation activities, are explicitly modelled. MILATRAS provides a simulation assignment model that tracks individual trips through the network. It provides a framework for examining the network-level implications of behavioural rules for the decision-making processes of various tripmakers.

The interaction between the supply model and demand representation is facilitated by the following assistant-managers: the feeder-manager, the loader-manager, and the feedback-manager. The purpose of the assistant-managers is to ‘bridge’ the gap between the supply and demand components while maintaining modularity into the framework. Besides, each assistant-manager has another task for the transit assignment process as follows. The input to MILATRAS is the OD trip matrix for the transit mode. The GIS-T module generates for each transit trip (i.e. passenger) a set of possible path choices (called hereafter *action space*) and passes it to the *feeder-manager*, which attaches it to the passenger-agent. Each passenger-agent has a *planner*
component that is responsible for selecting only one path, including a departure time, that reflects that passenger’s preferences and is based on the mental model of previous experiences. This results in a stochastic process of different choices for individual passengers; hence, the loader-manager’s task is to communicate dynamically passengers’ choices to the network-microsimulation module. Then, the microsimulation model (through the transit-handling module) handles the dynamics of the transportation network according to passengers’ choices and provides experienced measurements for individual passenger-agents. Afterwards, the feedback-manager is responsible for updating each passenger’s memory, according to a learning mechanism. The whole process repeats for many ‘days’ (i.e. iterations). The developed simulation-based assignment model well recognizes the interrelation between user decisions and system performance, and thus treats the system attributes as endogenous – i.e. congestion evolves within the model rather than separately modelled.

### 5.1.1 Prototype Description

A transit network has been coded on two platforms: a GIS platform (ArcMap©) and a microsimulation platform (Paramics©). This transit network represents the skeleton of the major transit lines of the City of Brampton, Ontario (downtown Brampton City and surrounding areas) – see Figure 5.1. A zoning system, consisting of 30 zones, was also coded that approximates the traffic analysis zones (TAZ) used by the City of Brampton Planning Department. A total of 22 single-directional routes, representing major routes in the Brampton Transit system, were coded with a total of 284 stops covering all possible transfers and common line stops.

The main input to MILATRAS is the transit demand for the simulated period. For this exercise, a 30x30 OD hypothetical transit OD matrix was compiled with a total of 3000 trips (i.e. transit riders) to model the transit assignment process in the morning peak period for recurring trips (e.g. work/school trips). The Brampton Transit System carries about 32,000 riders every day, with about 8,000 trips in the morning peak period; this represents about 10% modal share. While MILATRAS models transit trips at the stop and link levels, which enables the assignment of intrazonal trips, such data were not included in the input OD matrix, since conventional planning models do not produce such information. The generated transit demand reflects the direction of
travel in the morning peak period, with downtown/employment areas attracting most of the trips and suburb/residential areas producing most of the trips.

The Brampton transit system operates over 300 buses on 40 routes, serving nearly 32,000 daily rides. Service frequencies range from medium (10-minute headway) to low (30-minute headway). Only a few routes run on common segments of the street network, where they serve the same stops. Also, common segments are usually before a bifurcation node. These characteristics affect the choice set generation process for passengers, where only a limited number of transit paths are available for each origin-destination pair. The common lines problem is not a major issue here, as in large transit networks with high-frequency services. Most importantly is the significance of departure time choice in order to minimize the anticipated long waiting times, associated with medium-to-low frequency services. Previous research focused on modelling the run choice within the context of the ‘common lines’ problem with information provision at stops, which is characteristic of transit systems with high frequency services. When considering transit systems of medium-size networks with medium to low frequencies, the departure time and the path choices become equally important to commuters. A few researchers have addressed the departure time and route choice decision problem in the context of transit assignment models (such as Sumi et al., 1990; Nguyen et al. 2001).
Figure 5.1 The Brampton Transit Network Structure in ArcMap© GIS
5.2 Supply Modelling

5.2.1 The GIS-T Model

A GIS model was developed in ArcMap© for the transit network. The inputs for the GIS representation include a street network layer, TAZ layer (and preferably land use layers), transit route layer, and transit stop layer. The GIS component has two sub-modules: the OD-Generator and the Path-Generator.

The OD-Generator’s task is to disaggregate the OD transit matrix into individual trips, generating an OD trip list. Each element in the OD trip list is a pair of an origin-zone and a destination-zone such that summing up the trips in the generated OD list by zone will result in the input OD transit matrix. For the compiled OD matrix, a list of 3000 pairs is generated such that the total number of trips generated or attracted by a zone matches the input OD matrix data. Each element is then geographically coded by randomly generating a point in the origin-zone (representing trip origin, e.g. home) and a point in the destination-zone (representing trip destination, e.g. workplace/school). This yields the OD-Geo List, with origin and destination geographical locations, where each OD pair in this list corresponds to an OD pair from the OD trip list.

While the OD-Geo List is generated randomly for privacy reasons, this process is enhanced by including land use maps, such as a residential area layer and an employment area layer. Geographic origin points are generated only within residential areas (i.e., excluding non-residential areas in each zone) and geographic destination points are generated only within employment areas of each zone. The added layers were obtained from land use maps for the Greater Toronto Area (GTA), where the City of Brampton exists. These maps reflect the fact that most of residential areas are in the suburbs and most of the employment areas are in the downtown area.
The Path-Generator module, a choice set generation algorithm, loops on each pair in the OD-Geo List (i.e. transit trip/ rider) and generates a set of possible/eligible transit paths (i.e. an origin stop, a route or sequence of routes, a destination stop) each of which connects the geographically coded origin-point and destination-point in that pair. The path generator can be used to represent different awareness levels about the transit network for different passenger-types. The path generator module considers different criteria when finding feasible transit options for each trip. In this exercise, two parameters were specified: the maximum walking distance to/from origin/destination stop is 800 meters and the maximum number of possible transfers per trip is 2. The OD-Generator and Path-Generator processes can be run iteratively for validation purposes. In this implementation, the average number of transit paths per passenger was found to be about 7 options, with an average walking/egress time of 6 minutes (equivalent to 450 meters with 4km/h walking speed).

5.2.2 The Network-Microsimulation Model

A replica of the GIS transit network has been developed and coded on a microsimulation platform (Paramics©), which matches the street network, transit route and transit stop layers. A zoning system, similar to the TAZ layer, has also been coded. A total of 22 single-directional routes representing major routes in the Brampton Transit system were coded with a total of 284 stops covering all possible transfers and common line stops. The coded transportation network contains more roads than just the transit service roads – see Figure 5.2. This study focuses on recurring trips in the AM peak period, 6:00am to 9:00am. The transit assignment process includes transit trips starting from 7:00am up to 8:00am. The simulation, however, starts earlier (i.e. 6:00am) allowing a warm-up period. Paramics© takes the scheduled bus-runs for each route as an input; the Brampton Transit published timetable was used for scheduling buses on the start stop of each of the modelled routes. The arrival of buses at downstream stops is not an input to the model (i.e. not scheduled) but rather occurs according to the dynamics of the transit and traffic conditions.

A traffic demand matrix is also developed, based on an 85% modal share for auto. The simulated traffic conditions are moderate such that buses are neither running in free-flow conditions nor the
transportation network is heavily congested, which represents typical traffic conditions in
medium-size cities where auto-usage is high but population density is not. Most of the traffic
demand is released during the period between 7:00am and 9:00am. The traffic demand also
respects the direction of heavy travel in the morning peak period, with most trips originating in
suburban/residential areas and destined to downtown/employment areas. Current transit
assignment models do not consider properly the interaction between transit vehicles and other
general traffic sharing the same road, although transit vehicles are usually delayed by the general
traffic. The problem of transit service reliability is obviously rooted at the uncertainty of traffic
conditions in the whole transport network, including its transit and traffic sub-networks.

Within a microsimulation platform, it is feasible to represent transit routes with various
frequencies (e.g. Route #2 has a 10-minute headway, representing a medium frequency service,
and Route #15 has a 30-minute headway, representing a low frequency service), and transit
routes with different frequencies for different periods (e.g. Route #77 has a headway of 15
minutes from 7:00am to 8:00am and a headway of 20 minutes from 8:00am to 9:00am). The
microsimulation-network model is sufficiently microscopic to allow stochastic vehicle departure
and running times, but it also allows a representation of the larger interactions among the transit
routes of the network.

With most, if not all, existing microsimulation packages having a principal focus on auto traffic
simulation, a problem arises when using such packages to model transit systems in detail.
Paramics®, as no exception, has several limitations which presented a challenge for
operationalizing our prototype. For example, it handles transit demand as random arrivals at
transit stops, given an arrival rate for each transit stop. Also, Paramics® does not support
transfers between transit routes, nor does it support tracking transit passenger identities. And, the
number of alighting passengers at any transit stop is determined as a percentage of the stopping
transit vehicle occupancy. Paramics® is, therefore, unable to provide passenger-specific
measurements, such as passenger waiting time at the origin stop, which obviously differs among
passengers due to different stop-arrival times. To overcome these challenges, the transit-
handling module, developed by Wahba (2004) which can be integrated with Paramics®, was
enhanced. The enhanced module version:
1. represents each passenger as an agent, with attributes;
2. accepts transit demand at the zone level, not the stop level;
3. is capable of tracing every passenger-agent through the transit network;
4. supports transfers between routes;
5. deals with boarding and alighting at the passenger level;
6. provides passenger-specific measurements. It provides the experienced travel and waiting times, convenience measures, and congestion and capacity effects, etc. that change from day to day for each passenger; and
7. models the (adaptive) behaviour of transit riders when faced with a departure time choice, stop choice or run choice.

The transit-handling component was coded using the Programmer, which is the Application Programming Interface (API) of the Paramics© package, and used as a plug-in for the Paramics© microsimulation platform. This module represents a significant enhancement to the current transit-modelling capabilities of Paramics©.
Figure 5.2 The Brampton Transit Network Structure in Paramics©
5.3 Demand Modelling

5.3.1 The Passenger-Agent representation

MILATRAS represents passengers and both their learning and planning activities explicitly. Passenger choices (and adaptation of choices) depend on the expected return (e.g. trip cost) of alternative actions. However, these returns are not known \textit{apriori} and can only be learned from experience. This implies that, without prior experience, passengers have no reason to distinguish between potential actions and conditions. This is referred to as the “explorative” behaviour that individuals follow when finding themselves in new environments. Through accumulated experience, transit riders will then gradually learn to distinguish between various alternative state-decision pairs with respect to their expected return; i.e. they will be able to choose the action that maximizes the expected return, given their current state.

For the prototype implementation, a passenger-agent structure is generated, synthesised and associated with each of the 3,000 trips in the OD-Geo list, yielding a population of 3,000 transit riders. Each passenger-agent structure has a set of attributes; for example, trip-related information includes trip purpose, scheduled arrival time at destination, etc. Other attributes may include socioeconomic characteristics such as income level, employment category, auto-ownership, etc. These attributes can be used for mode choice modelling, when other travel modes are available for passengers. In this application, demand for transit is assumed to be fixed for the simulated period. Transit riders experiment in an attempt to understand the services characteristics of various paths under a variety of conditions. Each passenger, therefore, develops a mental model of the (relevant) transit network and its conditions. A proper mental model representation for the (relevant) transit network and its conditions is important for the learning and planning activities to be carried out.
5.3.2 Mental Model Representation

The output of the Path-Generator module is a set of transit paths for each passenger (i.e. transit trip). These transit paths are associated with each passenger-agent and stored in a mental model. In this implementation, the mental model represents the relevant transit network structure for each passenger and its conditions as each passenger experiences it. The mental model is structured as follows – see Figure 4.9. Each passenger has a set of origin stops (accessible and within the acceptable walking distance). The mental model stores fixed (e.g. access walking time) and dynamic (e.g. familiarity) parameters for each origin stop. Each origin stop has a list of attractive routes for this passenger. Each attractive route has a set of all possible alighting stops, along with other attributes such as experienced waiting time. An alighting stop is either a feasible destination stop or a transfer point to other stops. An egress walking time is associated with a final destination stop. A transfer point has a list of feasible transfer on-stops, whose structure is similar to the structure of an origin stop. In the mental model, the relevant transit network is represented by a set of route segments. For each segment (i.e. a boarding stop, a route and an alighting stop), the passenger stores the experienced waiting time, running time to the alighting stop and measures of seat availability and route reliability for the last $K$ past experiences. $K$ is a passenger-specific parameter, and it is assumed to be 7 for all passengers in this study.

In most path choice models, passengers are assumed to be able to calculate travel times on alternative paths in some manner (e.g. weighted average of experienced travel times), then choose which path to take. It is clear that a passenger generally does not remember travel times as a precise continuum, but rather should be assumed to store waiting and travel times in memory only approximately. Therefore, we propose two time resolutions for different pieces of experiences stored in the mental model. For a transit trip, there are two categories of time that are perceived differently among passengers: out-of-vehicle time (walking times and waiting time) and in-vehicle time. It is known that waiting time is perceived to have a higher disutility than in-vehicle time. The proposed time resolution is inversely related to the amount of disutility; in-vehicle time has a larger time unit (e.g. 5 minutes) than waiting time, which has two minutes as the time unit (weights that are assigned to waiting time and travel time usually reflect a ratio of
2). This means that all waiting times are stored to the nearest two minutes and travel times to the nearest 5 minutes (walking times are fixed and do not change over days and are stored to the nearest minute). Also a fuzziness component is to be added to represent human judgment; for example, a travel time (waiting time) of the value of 23 minutes has, say, a 50% chance of being stored as 20 (22) minutes or 25 (24) minutes. These time resolutions will be used when storing information, updating perceptions, and providing expectations regarding network conditions.

Passengers, with recurring trips, are assumed to have knowledge of the transit network conditions for different intervals of the peak period. In other words, transit riders have different expectations regarding the transit system performance for different departure times from the origin. The mental model structure therefore distinguishes between transit network conditions for different departure time choices from the origin. This mental model representation can be viewed as the spatial-temporal knowledge-base for each passenger.

5.3.3 Learning and Adaptation Representation

The learning and decision making processes of passengers are assumed in this study to follow Reinforcement Learning (RL) principles for experience updating and choice techniques, as explained in Chapter 4. Reinforcement learning is based on the concept that agents learn, by interacting with their environment, what actions to take so as to maximize a reward signal. Agents are not instructed what actions to take, but rather they discover which actions yield the most reward by trying them. Assuming passengers as agents and travel time as the only evaluation criterion, transit riders need to discover what combination of home departure time and transit path choice will maximize their reward (i.e. minimize the expected travel time).

For the transit assignment process, passengers need to make choices when they are at home (pre-trip) and during the trip (en-route). During the transit trip, passenger agents may be in one of seven states: “at home”, “accessing origin stop”, “waiting at a stop”, “on board of a bus”, “transferring to a stop”, “egressing from destination stop”, or “at the destination” – see Table 4-4. As explained in Section 4.2, passengers may be faced at each state with a set of possible actions. When a passenger is in the “at home” state, possible actions include a set of home
departure time and origin stop choices combinations. A passenger, at home, chooses a specific home departure time and an origin stop; this results in a change in the passenger’s state – from the “at home” state to the “accessing origin stop” state. While being in the “accessing origin stop” state, passengers will continue till they reach the origin stop. A passenger waiting at a stop needs to decide on which bus run to board. Assuming no traveller information provision, passengers make the boarding decision when a bus arrives from a list of attractive routes. A “board” decision will result in a state change – from “waiting at a stop” to “on board of a bus”. A “do not board” decision yields no state change. A “board” decision is expected to change the environment of other passengers – e.g. increase the dwell time at the stop, reduce on-board seat availability, etc. When a passenger is “on board of a bus” and arriving at a stop, possible actions are “keep on board” or “alight”. When a passenger alights at a stop, his state changes to either “transferring to a stop” or “egressing from destination stop” depending on whether the stop is a destination stop or a transfer point. When a passenger arrives at destination (i.e. in the “at the destination” state), the transit trip ends and no further action is needed. When a passenger is at a state \( s \) and decides to take action \( a \), this is referred to as the state-action pair: \( (s, a) \).

In this implementation, the choice model considers the departure time choice, the stop choice and the run (or sequence of runs) choice. The departure time and origin stop choices are assumed to be at-home choices (i.e. pre-trip), in which passenger-agents consider available information obtained from previous trips, in addition to pre-trip information provision (if any). Once a passenger-agent arrives at a stop, the bus run choice is considered an adaptive choice, in which, besides previous information, the passenger considers developments that occur during the trip. It is worth mentioning that, because of the dynamic representation of the transportation network (i.e. a microsimulation model), the adaptive choice is relative to the specific run of each line (time-dependent path choice).

### 5.3.3.1 Pre-Trip Planning

At the origin (i.e. home for a work or school trip), a passenger develops an initial (tentative) travel plan, based on his updated mental model (historical experience and pre-trip information provided by the system). This travel plan includes a departure time, an origin stop (which are
fixed) and a (tentative) run (or sequence of runs). The initial plan reflects the passenger’s preferences and expectations, and he would follow it if the reality (en-route dynamics) matches, to a great extent, his expectations (which may be different from published static schedule information). The process through which the passenger decides on the initial plan, given expectations based on his mental model, can be modelled using different techniques. In this implementation, it has been assumed that passengers will choose the travel plan that minimizes the expected travel time of the transit trip. This includes exhaustive search for all possible travel plans simultaneously and in no preferred order.

The simulation macroscopic structure is shown in Error! Reference source not found.. At the origin, passengers “update” their mental model with most recent experiences and information provided (static, such as online schedules, or dynamic, such as real-time information), if any. Passengers then “utilize” their mental model to choose a home departure time and an origin stop combination. Passengers recall from the updated mental model what is the minimum expected travel time (including out-of-vehicle and in-vehicle) to destination for each home departure time, using the “ExptTmToDest (X)” routine, where X represents different home departure times, as outlined in Figure 5.4. For a certain home departure time, the routine recalls the associated experience of the transit network conditions – i.e. experienced waiting times at different stops and in-vehicle times for different runs when leaving at this time from origin. The routine calculates the accumulated travel time from all possible origin stops to the destination and returns the minimum accumulated travel time for this home departure time. Then, the routine loops for all possible home departure times and selects the home departure time with the minimum expected accumulated travel time to destination. Upon deciding on a home departure time, each passenger selects the origin stop that has the minimum expected accumulated travel time to the destination. The home departure time and origin stop choices are fixed, while the run (or sequence of runs) to the destination with the minimum expected travel time to destination represents a tentative travel plan.

The departure time choice can be evaluated based on the trip duration and the schedule delay associated with travelling at the chosen time, (Small, 1982). In order to calculate the schedule delay, a scheduled arrival time at destination (e.g. work start time) needs to be associated with
each trip (i.e. passenger). The schedule delay also affects the departure time adjustments from day-to-day. The passenger will then choose to travel at the time that is expected to minimize the travel time and maximize the probability of arriving at the destination at the scheduled arrival time. In this prototype, the departure time choice is only based on trip duration, since obtaining scheduled arrival times, and most importantly associating its value to the right passenger, is not a straightforward task. At origin, a passenger will choose to leave at the time which is expected to minimize the transit trip time. Transit riders will choose to leave the origin at a time between 7:00am and 8:00am, and they will use an interval of 5 minutes (or its multiples) to adjust (or shift) their departure time.

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9 In the large-scale case study presented in Chapter 6, the scheduled arrival time is considered for the modelling of departure time choice.
Figure 5.3 Simulation Procedure Macroscopic Structure
Expected\( (X) = \sum_{i=1}^{k} W_i \times X_i \), where \( W_i = \frac{1}{i} \), \( K \) is the number of last experiences

Figure 5.4 The “ExptTmeToDest” Routine
5.3.3.2 En-Route Adaptive Behaviour

When a passenger-agent decides on a transit path and a departure time, this passenger-agent is loaded on its origin stop. The arrival time at the origin stop is determined by the departure time from home and the walking distance to the origin stop. Passengers, at any stop, form a queue with a first-come-first-served discipline. When a bus from a route \( r \) arrives at a particular stop \( s \), each passenger, who has stop \( s \) as a destination/off-transfer stop, needs to make a decision, either to alight at this stop or not. A Passenger, with stop \( s \) as the desired alighting stop, will get off if the “SWITCH AND STAY” routine returns “NO”, which means that the existing conditions (experienced travel time) match expectations (expected travel time). If existing conditions do not match expectations (i.e. worse than expectations), the passenger will evaluate the value of other feasible downstream off-stops; the value is calculated as the expected time to destination from stop \( s \) and staying on-board. If the calculated value for all feasible downstream off-stops is not better than stop \( s \)’s value, then the “SWITCH AND STAY” routine returns “NO” and the passenger gets off at stop \( s \). For this passenger, the in-vehicle travel time for this segment is inserted as a recent experience in the passenger’s mental model. If the passenger finds a better choice, the “SWITCH AND STAY” routine will return “YES” and the passenger updates his/her desired alighting stop to the chosen one with the minimum expected travel time to destination from stop \( s \). This means that the passenger chooses to stay on-board, and also updates the in-vehicle travel time experience for this segment.

For other on-board passengers, who do not have stop \( s \) as the desired alighting stop but it is feasible off-stop, each passenger needs to decide whether to alight at stop \( s \) or stay on-board for the desired alighting stop. If the existing conditions are better than the passenger’s expectations, then the passenger might consider changing the desired alighting stop to stop \( s \). If the value of stop \( s \) (i.e. the expected travel time to destination from the current stop) is better than the value of the initially desired alighting downstream stop plus the expected in-vehicle travel time from stop \( s \) to the initially desired alighting downstream stop, then the “SWITCH AND ALIGHT” routine will return “YES”. This means that the passenger gets off at stop \( s \) and updates the mental model contents regarding the in-vehicle travel time for this segment. If the “SWITCH
AND ALIGHT” routine returns “NO”, then the passenger stays on-board and also updates the in-vehicle travel time for stop (s).

Passengers, who get off at stop (s), are either ending their transit trip by walking to their destination, or transferring to another route, being loaded to a connecting on-stop and waiting for the next bus to arrive. Then the bus occupancy is calculated to reflect the updated on-board occupancy.

Each passenger waiting at stop (s) will choose whether to board the bus from route (r) or not as long as the bus is not full. A passenger, who has route (r) as the desired route, will board the bus if the existing conditions (e.g. experienced waiting time) match his/her expectations. That is; the “SWITCH AND WAIT” routine returns “NO” for this passenger and there is no need to reconsider his/her initial choice (which has the minimum expected travel time to destination based on his/her expectations). If current conditions are worse than expected and there are other attractive routes passing by stop (s), the passenger needs to reconsider his initial choice. First, the passenger checks if the existing conditions affect other attractive routes as well (i.e. current waiting time is more than expected for the other attractive route); in this case, the initial choice might still be desirable. Then, the passenger calculates the value of waiting for another attractive route (a) and compares it with the value of boarding the route (r). The value of boarding route (r) is the minimum expected travel time to the destination from stop (s) and boarding route (r), while the value of waiting for attractive route (a) is the expected waiting time for the next arrival of attractive route (a) at stop (s) plus the minimum expected travel time to the destination from stop (s) and boarding attractive route (a). If no attractive routes are found to be more desirable than route (r), the “SWITCH AND WAIT” routine returns “NO” and the passenger boards the bus – if the bus capacity is not reached. For this passenger, the waiting time for this segment is inserted as a recent experience in the passenger’s mental model. If attractive route (a) is found to be more desirable, the “SWITCH AND WAIT” routine returns “YES” and the passenger decides not to board and the passenger is loaded to the end of the queue of passengers waiting for attractive route (a). The passenger registers the experienced waiting time for the stop (s) and route (r) combination in the mental model.
For passengers waiting at stop \((s)\) for buses from other routes, they may consider boarding the current bus from route \((r)\), if the bus is not full. First, a passenger who does not have route \((r)\) among the list of attractive routes at stop \((s)\) will not board the current bus. A passenger with route \((r)\) in his/her attractive set will consider boarding the current bus if the existing conditions associated with route \((r)\) is better than his/her expectations (e.g. experience waiting time for route \((r)\) is less than expected waiting time for route \((r)\) at stop \((s)\)). If the value of \textit{boarding} route \((r)\) from stop \((s)\) now is better than the value of \textit{waiting} for the initially desired route and boarding it from stop \((s)\), then the “SWITCH AND BOARD” routine will return “YES” and the passenger will board the current bus from route \((r)\) and stop \((s)\). For this passenger, the waiting time experience for this segment is updated. Otherwise, the “SWITCH AND BOARD” routine returns “NO”; this means that the passenger will not board the current bus from route \((r)\) and will keep waiting at stop \((s)\), maintaining his position in the queue, for the next bus arrival. Meanwhile, the passenger’s waiting time experience is updated for route \((r)\) and stop \((s)\) combination.

When a passenger decides to board the bus from route \((r)\), the bus occupancy is updated. If bus capacity (assumed 65 passengers) is reached, all passengers at stop \((s)\) continue to wait for the next bus to arrive from any route serving stop \((s)\). This \textit{dynamic} updating of bus occupancy reflects the asymmetric capacity effects between passengers; passengers boarding first affect the occupancy levels for passengers boarding later (or even not able to board because the bus is full) but not vice versa. It also reflects the asymmetric congestion effects between passengers; passengers boarding at upstream stops affect the waiting time experience for passengers waiting at downstream stops but not vice versa. Without an agent-based representation and a microsimulation representation of the transit service, such effects cannot properly be modelled.

When passengers arrive at their destination, the deviation from their scheduled arrival time (SAT) could be calculated. In this implementation, no SAT is assigned to individual passengers and hence schedule delay is not part of the travel cost. Figure 5.5 shows the SWITCH routines.
Figure 5.5 SWTICH routines with ExptTmToDest procedure

The prototype incorporates learning, imperfect knowledge, expectation updates, search and decision heuristics, and information acquisition into a coherent framework of passenger travel behavior. It has the following characteristics:

- It provides a simulation assignment model that tracks individual trips through the network;
- It provides a framework for examining the network-level implications of behavioural rules for the decision-making processes of various trip-makers;
- It well recognizes the interrelation between user decisions and system performance, and thus treats the system attributes as endogenous – i.e. congestion evolves within the model rather than exogenously acquired;
- It represents the transit network using a sufficiently microscopic model to allow stochastic vehicle departure and running times, but it also allows a representation of the larger interactions among the transit routes of the network;
- It incorporates the experienced travel and waiting times, convenience measures, and congestion and capacity effects, etc. that change from day to day for each passenger;
- It models the behaviour of passengers as a dynamic process of repetitively making decisions and updating perceptions, using Q-Learning techniques from the Reinforcement Learning field; and
- It represents, using a microsimulation model, the interaction between transit vehicles and other general traffic sharing the same road.

5.4 Information Provision Policy Analysis

With the rapid growth of Intelligent Transportation Systems (ITS) applications, the need for dynamic models of travel behaviour and network performance has been growing. The development of microsimulation models has been strongly motivated by the necessity to use tools that are able to deal with the dynamic characteristics of ITS applications. In particular, the provision of real-time travel information on transit services is increasingly being recognized as a potential strategy for influencing transit rider behaviour on departure time choice, path choice and hopefully attracting auto-mode users. Understanding and modelling travellers’ responses to this information is therefore critical to the design and implementation of effective intelligent transport systems strategies such as Advanced Traveller Information Systems (ATIS). The benefits of ATIS applications can be assessed by comparing the path choice behaviour and time savings of informed passengers with non-informed ones.

The MILATRAS modelling framework was used to investigate the impact of four different information provision scenarios on passenger home departure time and path choices.

5.4.1 Base-case Scenario – ‘No Information Provision’

The details of the “no information provision” scenario are described in sections 5.1 (for a description of situational factors), 5.2 (for supply modeling) and 5.3 (for demand and path choice modeling).
We assume no previous knowledge about the transit system performance for all passenger-agents. On run #1, transit riders are indifferent to all home departure times and path choices. The expected trip time associated with every home departure is therefore assigned a value of zero (to force all of them to be tried). Similarly, the transit network conditions (i.e. waiting times and vehicle times) are initialized. Passengers are expected to explore their choices to build up their experience, and then exploit the best home departure time and path choices over time.

### 5.4.2 Information Provision Implementation Requirements

All investigated scenarios (explained in Section 5.5) assume that the information provided is related to the expected waiting time (or arrival time) for the next run. This information is updated and displayed on stops every minute, and values shown are approximated by the minute. Passengers still have their expectations of in-vehicle times, which are not supplied by the system. For the modelling of information provision, there need to be a model for travel time prediction and a procedure for information-experience integration.

### 5.4.3 Travel Time Prediction Model

In all information provision scenarios, we employ a travel time prediction model within MILATRAS to produce the “expected” waiting time at each stop for the next run from each route. The model uses information, generated by the simulation, about the previous runs within the last 45 minutes of each route to produce the expected arrival times – see Figure 5.6. The travel time prediction model is activated after 45 minutes from the simulation starting time; which is 6:00am. The 45 minutes period was chosen to ensure enough information of buses on routes with low frequencies. The algorithm shown in Figure 5.6 is repeated every minute of the simulation time. That is, the travel time prediction model is sensitive to the traffic network conditions. However, by using a microsimulation platform, the displayed expected waiting times are not 100% accurate, except at starting stops where the displayed times are 100% accurate since it is not affected by travel time predictions (assuming availability of fleet).
Using the *Travel Time Prediction Model*, information is quantified and provided to passengers regarding the expected waiting time (i.e. arrival time) for the next run at each stop for all bus routes. This information is compared with the passenger’s experience regarding the expected waiting time for the next run at the current stop. This expectation is made available using the passenger’s mental model contents. In this implementation, the expected waiting time for the next run is calculated as the weighted average (with higher weight for most recent experiences) of all previously experienced (and stored) waiting times for this stop-route pair. Note that the mental model distinguishes experiences based on network conditions and recalls experiences that are only relevant to the current conditions for the current situation.
When information is provided, passengers *integrate* this information with their experience using the logic outlined in Figure 5.7. The confidence level is dynamically updated to reflect the reliability of the information provided by the system. This happens only when a bus actually arrives at the stop; this way, passengers update their confidence in the system according to the latest information provided regarding the expected waiting time. The information about the expected waiting time is displayed and updated every minute, and passengers will consider this information only when a boarding decision needs to be made.

When the information provided does not match the expectations of passengers, the integration of information and experience will not yield a useful output. For example, if a passenger has an expectation value of the waiting time for a specific run as 5 minutes, and because of a non-recurrent congestion situation the displayed waiting time for this run is 30 minutes, then averaging both values will result in an incorrect expected value. At this time, passengers need to choose whether to trust the information system or continue to use their expectation.

In this study, $\alpha$ is initialized by 0 (i.e. no confidence) and is updated by adding (subtracting) a value $\beta$ for each passenger every time the information provided matches (deviates) from “reality”. In this study, information is considered matching “reality” if the deviation is less than 50%. And $\beta$ is assumed to be 0.1 for all passengers. The minimum value for $\alpha$ is zero, representing “no confidence” in the information provided and this means that passengers base their travel choices on their expectations, even with information provision. A 100% level of confidence means that passengers will trust the information provided about the expected waiting time and will base their decisions only on this information. Note that, in-vehicle travel time information is not provided by the system and passengers use their expectations (based on experience).
5.5 Information Provision Scenarios

5.5.1 Scenario I – ‘Information at Boarding Stops’

Scenario I represents the situation where information is posted at the stop. This information is accessible at stops and therefore will only have an effect on the run choice. For medium-size transit networks, where the “common lines” problem is not a major issue, it is expected that information on stop will not have a significant effect on passengers’ choices. The average trip time is not expected to improve (i.e. decrease) with regard to the base-case scenario. Since run choice will not significantly be affected, route and bus loads are expected to remain as in the base-case scenario.

5.5.2 Scenario II – ‘Information at Alighting Stops’

In this scenario, it is assumed that information on the “expected” waiting times at each stop is provided as well as at the nearby transferable stops; i.e. for connecting passengers, they use this information before committing themselves to a connecting stop. This is expected to be useful
when passengers are faced with different transfer connection options after they alight at a particular stop, which is the case in a grid network such as the one used in this study.

5.5.3 Scenario III – ‘Information On-Board’

Scenario III is an extension to Scenario II, where information is also provided on-board. When a bus is approaching a stop, the information regarding the expected waiting time of the next run of each route on that stop, as well as possible transferable stops, are displayed on-board (e.g. by using wireless transmission technology). In this situation, passengers can decide whether to alight and make a connection, or stay on-board to the next available alighting stop. This way, passengers can avoid arriving at a connecting stop just after the departure of a run. This information is expected to be most useful in a grid-network with medium to low frequency services, where a significant portion of the trip time is experienced waiting at stops. The average out-of-vehicle time is expected to show significant improvements. Also, bus (and consequently route) loads are expected to be affected.

5.5.4 Scenario IV – ‘Pre-Trip Information’

The last Scenario, IV, investigates the importance of providing information at home (i.e. pre-trip information). Traditionally, static information about the transit network is provided at the origin, including maps, schedules, etc. In this scenario, we assume that the information on the transit system performance of the previous day (i.e. simulation run) is available for passengers at home. In an ideal situation, it would be preferable to provide passengers with predictive information regarding the transit network conditions based on anticipated passengers’ decisions; this is, however, difficult to obtain. In the previous scenarios, passengers will base their departure time and origin stop choices on their expectations with the transit network conditions at different times during the peak period. Under pre-trip information provision, passengers plan their trips, not based on their expectations, but based on the actual network conditions of the previous day. Passengers will choose the departure time and origin stop combination which has the minimum trip time to their destination, based on the information provided. This scenario can be considered as the “perfect information” assumption in traditional assignment models. The provision of
information at home is expected to affect the home departure time choice, and possibly the path choice. Information according to Scenario III is also considered for the en-route adaptive choice.

5.6 Results and Discussion

For the base-case scenario, it takes about 3 minutes on a Pentium M 2.0GHz computer, to execute the proposed model for one morning peak period. After the “exploration” period, the model is tested for the convergence of OD transit trip times, route loads, and bus loads using equation (5.1).

\[
\Delta(X_{D-1}, X_D) = \Delta(X_{D}, X_{D+1})
\]

(5.1)

Where

\[
X^D \text{ represents OD trip times, route loads, or bus loads for day } D, \text{ and }
\]

\[
\Delta(X_{D-1}, X_D) = \frac{\sum_{i=1}^{n} |X_{i}^{D-1} - X_{i}^{D}|}{\sum_{i=1}^{n} X_{i}^{D-1}} = GRE \text{ (Global Relative Error)}
\]

or \[
\Delta(X_{D-1}, X_D) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{X_{i}^{D-1} - X_{i}^{D}}{X_{i}^{D-1}} \right)^2} = PMRE \text{ (Point Mean Relative Error)}
\]

and \(n\) is the number of OD pairs, routes, or buses.

Note that \(\Delta(X_{D-1}, X_D)\) will not converge to zero but to a value representing transit network performance variability between different days (i.e. simulation iterations). From Figure 5.8, route loads (an aggregate output) showed stability after 25 iterations, while bus loads have some variability, as a result of the day-to-day dynamics of the travel behaviour process. OD trip times converged to a stable state, which can be interpreted as “equilibrium”. For the base-case scenario, the average trip time is 33 minutes for all OD pairs, with an average of 14 minutes of out-of-vehicle time (not including walking times) and 19 minutes of in-vehicle time. This is expected since the Brampton Transit Network operates medium to low frequency services and
passengers are expected to spend a significant amount of time waiting (in the absence of information).

Figure 5.9 shows the route load distribution over the transit network, along with the route headway information. Routes 77, 18 and 15 service the centre of the downtown area and are expected to be attractive for most of the passengers at origin stops or at transfer stops. Routes 3, 51 and 30 service the suburban areas, and therefore have lower loads. Figure 5.9 shows also the loads for different runs on Route 77E. As expected, runs get more congested as more transit demand is released. Also, the number of passengers missing “runs” has decreased dramatically over time; that is, passengers learned to avoid the over-loaded runs and seek alternative paths. This is attributed to the fact that missing a run in such a network, with medium to low transit services, will result in experiencing long waiting times.

![Model Convergence](image)

Figure 5.8 Model Convergence – Base-Case Scenario
Figure 5.9 Route Loads – Base-Case Scenario
This study is concerned with the transit assignment process in the morning peak period; in particular, transit trips starting between 7:00am and 8:00am. Figure 5.10 shows the departure time distribution from the origin (i.e. home). The departure time choice can be evaluated based on the trip duration and the schedule delay associated with travelling at the chosen time. In order to calculate the schedule delay, a scheduled arrival time at destination (e.g. work start time) needs to be associated with each trip (i.e. passenger). The schedule delay also affects the departure time adjustments from day-to-day. The passenger will then choose to travel at the time that is expected to minimize the total travel time and to maximize the probability of arriving at the destination at the scheduled arrival time. In this prototype, the departure time choice is only based on trip duration. About 50% leave home before 7:15am, after experiencing modest traffic conditions and an un-congested transit network. Fewer passengers leave home during the middle of the peak period (7:15-7:45) due to the congested transit network and travel time variability. This percentage increases again close to 8:00am where transit network conditions improve.

Figure 5.10 Home Departure Time Distribution – Base-Case Scenario
As expected, the average trip time in Scenario I remained close to 33 minutes (32 minutes) with the same proportion between out-of-vehicle time and in-vehicle time. The model reached a stable state (i.e. converged) after a few iterations (5 iterations); although the model was traced over another 20 iterations to confirm that variations in route and bus loads between simulation iterations have reached the minimum. Since the average trip times were not significantly influenced by the information provided at stops, home departure time choices (and subsequently the home departure time distribution) did not change either. Nonetheless, it was observed that passengers, when they needed to make a run choice, made use of the information. The model is able to output how many passengers switched their choices based on the information provided. On iteration 1, in Scenario I, the number of passengers who switched their initial route choice, based on their expectations, was found to be 200 passengers. This number, however, decreased over time, as passengers built up their expectations. In this study, the run and stop choices are based on the trip time minimization criteria. Therefore, other soft measures such as the convenience of the availability of ATIS will not affect paths choices for passengers.

In Scenario I, it was found that passengers cannot make use of the information after they are committed to a stop; passengers would need more information to make the stop choice. The results of Scenario II show the impact of providing the same information as in Scenario I, but at a different location to target another aspect of the path choice problem: the transfer stop choice. Under Scenario II, the model converged to a stable state after 20 iterations. The average trip decreased from 33 minutes (base-case scenario) to 30 minutes. The average-out-of-vehicle time, mainly the average transfer waiting time, has decreased from an average of 14 minutes (base-case scenario) to 12 minutes. This reduction is mainly felt in the trips going to the downtown/employment area where the transit network coverage is high and many transfer stops become feasible.

Scenario III was more effective in reducing the average trip time from 33 minutes to 25 minutes. About 70% of the reduction is attributed to avoiding missing transfers due to unreliable transit service caused by variable traffic conditions; for example, a passenger would know if the connecting route has just left the transfer stop or the passenger will have enough time to catch the run. It was observed that the number of switches was higher than in Scenarios I and II,
reflecting the fact that passengers made use of the information provided to them. This is also reflected in the variation in route and bus loads compared to the base-case scenario – see Figure 5.11 – as route loads are distributed more evenly. Figure 5.11 shows the bus load distribution of peak period runs from Route 77E for different scenarios. One can notice the changes in bus loads, with less buses being congested. This is also attributed to the changes in home departure time choice. Figure 5.12 shows the home departure time distribution for Scenario II and Scenario III compared to the Base-case scenario. With the improvement of average trip time, particularly the out-of-vehicle time reduction, passengers are tempted to leave home later. If we were to consider in our model the scheduled arrival time at destination as a guiding criterion, it would be expected that the departure time distribution would show more passengers leaving later from origin, as a result of decreased trip time. This would leave more time for passengers at the origin to perform other activities, which is an important issue that activity-based modelling systems attempt to deal with. Without considering departure time choice, benefits from information provision systems can be underestimated.

Different route loads, as shown in Figure 5.11, may represent different “equilibrium” states. The base-case scenario represents a “deluded equilibrium” state (as defined by Nakayama et al., 1999, for traffic assignment) where passengers’ experiences are deluded, and it represents less efficient operating conditions for the transit network. One possible way to break out of deluded equilibrium is to provide passengers with (reliable) actual transit conditions (e.g. expected waiting times). This sheds light on the significance of the location of provided information. It also shows that the run choice is not a major decision for passengers in a medium-size transit network. Decisions about the transfer stops could be more important.

For Scenario IV, and after about 20 iterations, convergence was not reached – see Figure 5.13. Under the pre-trip information provision scenario, passengers base their travel plan choices on the information provided, and not on their expectations (i.e. mental model), regarding the transit network conditions. Passengers, with complete information seeking optimality, will always try to out-perform the system, resulting in changing their trip choices from one day to another. For convergence to be reached under these conditions, a passenger, who is repeatedly making the same trip, has to choose a departure time and a path so that other passengers’ choices are not
disturbed. Since it is impossible to know other passengers’ choices before travelling (i.e. non-cooperative game), it is thus more likely that convergence will not be obtained.

The results from a prototype implementation showed that, for a medium-size transit network with low to medium frequency services, run choice is not a major concern for passengers due to the transit network structure. In such networks, the stop and departure time choices seem to be more important, as they significantly affect the trip time. Information provided only at stops, as in Scenario I, did not help passengers reduce their trip time, as run choices are limited and dependent on previous stop and departure time choices in such networks with limited services. Information targeting the transfer stop choice was found to be more effective, as shown in Scenarios II and III. Not only would such information affect the average trip time, but it could also generate a different departure time distribution which would affect the transit network performance. Interestingly, historical information provided at origin, targeting home departure time, did not enable the system to reach a stable state. While it needs more examination, this raises some concern on the types of information that should be provided to passengers and/or the optimality assumption regarding passenger behaviour. In this study, traffic demand was represented on the same network. Information regarding transit network conditions can be made available as well to auto-drivers and traffic conditions can be provided to transit riders. Therefore, mode choice can be integrated into the overall modelling framework, using the learning-based approach.
Figure 5.11 Route Loads – Scenarios II and III
Figure 5.12 Home Departure Time Distribution – Scenarios II and III
Figure 5.13 Model Convergence – Multiple Scenarios
6 LARGE-SCALE CASE STUDY – THE TORONTO TRANSIT COMMISSION APPLICATION

This chapter documents the effort to operationalize the conceptual model presented in Chapter 3 and the transit path choice model proposed in Chapter 4. The methodology outlined in the previous chapter is applied to develop a large-scale, real-world application to the Toronto Transit Commission (TTC) public transportation system, see Figure 6.1. This application demonstrates the benefits of the proposed approach and builds on the lessons learned from the prototype implementation described in the previous chapter. In addition, this large-scale application confirms the feasibility of the multi-agent, learning-based approach.

Section 6.1 gives a background on the application context and the analysis scope. Then, the modelling of the supply service is presented. The procedure for mental model construction is illustrated in Section 6.4.3. The specification for the generalized travel cost function of the departure time and path choices is presented. At the end of the chapter, the parameter calibration procedure is explained using the case study implementation. The results and discussions are mentioned in Section 6.8.
Figure 6.1 Toronto Transit Commission Public Transportation System
6.1 Application Background

6.1.1 The Transportation Tomorrow Survey (TTS)

The Transportation Tomorrow Survey (TTS) is a comprehensive travel survey, which collects detailed demographic and travel information on all household members for 5% of all households in the Greater Toronto Area (GTA) and surrounding regions. The GTA area is divided into five regions; namely Toronto, Durham, Peel, York and Halton. Each region is identified with a range of zone codes. For example, zone-codes in the City of Toronto range from 1 to 481, see Figure 6.2. The collected data include travel information about the trips made by the household members over an entire weekday. The TTS was first conducted in 1986 and it is carried out every 5 years since then (1991, 1996, 2001, and 2006). The data used in this application refer to the collected TTS records in the year 2001.

Data collected for transit trips include trip start time, trip purpose, origin and destination geo-locations, and the sequence of transit routes for those trips, to name a few. While the survey covers only 5% of the households, expansion factors are used to expand the information to the 100% level. The expansion factors are determined and verified based on Census Sub-Division level data (Joint Program in Transportation, 2003b). In 2001, for example, 85,095 individuals across the GTA reported that they used transit as the mode of travel; this is expanded to 1,469,237 transit trips across the GTA for a weekday.
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Figure 6.2 GTA Regions and Zone-Coding for Year 2001 (source: www.jpint.utoronto.ca/gta01/GTA.html)


6.1.2 Application Context

The TTC system provides service coverage within the City of Toronto. The City of Toronto is located in the middle of the Greater Toronto Area (GTA), with the Lake of Ontario defining its south boundary. It has jurisdictional boundaries from the west with the City of Mississauga and City of Brampton, York Region from the north, and Durham Region from the east.

Toronto is considered the economic heart of the GTA, where most businesses and facilities are located. While there are 5 regions in the GTA, about 45% of the GTA daily work trips are destined to Toronto alone, with a 25% modal split for transit (Joint Program in Transportation, 2003a). According to the TTS 2001 numbers, out of the 1,469,237 transit trips made everyday in the GTA, about 78% of those trips use the TTC service, with over 332,000 trips in the morning peak period. Based on the mentioned statistics, the TTC system was selected for the development of the operational model due to its importance and significance to the GTA transportation network.

6.1.3 The TTC System

In 2001, the TTC operated 294 one-directional routes (or 147 two-directional routes) during the AM peak period (TTC Report, 2001). Some of these routes run on different branches, covering different segments, bringing the total of individual branches modelled to 480 branches. These branches serve just about 10,000 stops during the AM period. The frequency of service over these branches spans from high frequency service (2 minute headway) to low frequency service (60 minute headway), with different values representing medium frequency services. The TTC system operates four different types of service:

1- Traditional bus service, e.g. Route #85 – Sheppard West
2- Express bus service, e.g. Route #141 – Downtown Express Route
3- Light Rail Transit/Streetcar service, e.g. Route #510 – Spadina Streetcar
4- Rapid Rail Transit/Subway service, e.g. Route 601 – Bloor-Danforth Line
6.1.4 Demand for TTC Service

While the TTC operates within the City of Toronto jurisdictions, some interaction with neighbouring transit systems exists along the city boundary. Transfers between Mississauga Transit or York Region Transit systems and the TTC service are common. In 2001, about 45% of all recorded TTS work trips for all modes of travel were destined to Toronto. 31% of those were originating in the 905 belt surrounding the City of Toronto. About 12% (4%) of the trips that use the TTC in the AM peak period originated (destined for) outside the City of Toronto boundaries – see Figure 6.3. In order to treat these trips properly, the modelling boundary of the case study should not be restricted to the physical boundary of the TTC service. Therefore, this application deals with such trips originating (destined for) outside the Toronto boundary at their origin (destination); i.e. trips originating (destined for) outside Toronto are tracked from their origin (to their destination) and not just loaded (or un-loaded) from the TTC service at the city boundary. Section 6.4.3 explains this treatment in more detail.
Figure 6.3 Demand for TTC Service (Origin and Destination Zones according to TTS2001 Data)
Inter-regional rail and bus services also exist in the GTA; this is represented by the GO (Government of Ontario) service. GO operates a number of inter-regional train and bus routes that are heavily used during the peak periods. The major GO hub is located at the downtown Toronto area, at Union Station, a major hub for the TTC service as well. A great deal of interaction between GO users and TTC services is evident at many major TTC/GO connection points, e.g. Finch Station. Moreover, the TTC and GO services operate different fare structures. To capture these interactions at an appropriate level, GO services are also modelled. GO network covers large parts of the GTA, which further expands the geographical boundary of the case study. GO services are not frequent and are largely dependent on the time of day and area of coverage. This presents a temporal-constraint on the possible connections that GO users may consider to transfer to the TTC service, and vice versa. In other words, the usage of the TTC service by GO riders is time-dependent and needs to be modelled dynamically. This, in turn, may affect the trip choices made by other TTC users (non-GO users) due to capacity and congestion effects.

From the TTS records, it was reported that some TTC users, with trip origin (and/or destination) outside the TTC physical boundary, had ‘walking’ as the chosen access (and/or egress) mode. Where there is no walk-accessible TTC/GO service (i.e. stops) for those users, it is shown from the TTS records that passengers accessed the TTC service through local municipality transit services. This would require special treatment when dealing with the loading (and/or unloading) of these trips, and calculating the associated trip cost (e.g. local transit fare).

6.1.5 Analysis Scope and Definitions

The following outlines the scope and definitions of the analysis

1. All data used in the application reflect the year 2001 conditions, unless otherwise stated.
2. All modelling and analysis are carried out for the AM peak period. The AM peak period in this application spans from 6:00am to 9:00am, unless otherwise stated.
3. The ‘TTS2001’ term denotes data collected through the Transportation Tomorrow Survey (TTS) in the year 2001 and refers to TTS (Joint Program in Transportation 2003a, 2003b) sources.
4. Convention of route codes  
   a. Route codes between 140 and 149 represent Express Service  
   b. Route codes between 500 and 599 represent Streetcar Service  
   c. Route codes between 600 and 699 represent Subway Service  
   d. Route codes between 9000 and 9999 represent the GO service.  
   e. All other route codes represent surface road TTC bus service  

5. A trip-end is considered within the TTC service coverage area if:  
   a. It is located within the City of Toronto boundary (GTA-zones 1-481)  
   b. Or, it is located within the maximum walking distance from a TTC service stop (e.g. TTC stops on Steeles Avenue might be accessible to York Region residents if the trip origin/destination location is within the buffer zone)  

6.2 Data, Data and Data  

Three data sources are used in this implementation. Different types of data were collected from each source for use at various stages in the modelling process. These sources are:  

1. The Toronto Transit Commission Service Planning Department  
   The TTC service planning department provided the following, some of which needed processing to be input to the model:  
   - The TTC service summary report for the year 2001. This report contains the following information for each route operating in the year 2001:  
     o Route Name and Number (and description of coverage area if needed)  
     o Number of Branches and coverage area  
     o Service characteristics by time of day; 5 periods are included  
        ▪ Morning peak period (6:00am-9:00am)  
        ▪ Midday period (9:00am-3:00pm)  
        ▪ Afternoon peak period (3:00pm-7:00pm)  
        ▪ Early evening period (7:00pm-10:00pm)  
        ▪ Late evening period (10:00pm-1:00am)

10 Contact persons are Bernard Farrol and Trevor Pitman, Service Planning Analyst, TTC Service Planning Department
For each period, the following information are reported:

- Route/Branch headway
- Route/Branch average travel speed along the whole route/branch
- Route/Branch terminal time
- Route/Branch fleet size
- Route/Branch travel distance
- Route/Branch average travel time along the whole route/branch

- A partial list of route (not branch) boardings over multiple years (2002-2006) for the AM peak period. The list did not contain the boardings for the whole TTC network for one year nor the boardings of one route over multiple years.

- A complete list of route-stop sequences. This text file includes for each route/bound/branch a complete list of stops (with stop number and intersection information) as well as the sequence of stops for each branch.
  - Data in this file include the most up-to-date TTC routes and stop configurations.
  - Data also include the ‘distance from previous stop’ attribute. For the route/branch start stop, this value is obviously zero.

- Geographical Information System (GIS) shape files for:
  - A partial list of all TTC stops, about 90% (8967 stops out of 9969 stops) of all stops included in the route-stop list. This means that some of the stops were not geo-coded.
  - A complete list of most up-to-date TTC routes. Each branch is represented by a single line (GIS polyline shape component) from its start terminal to its end terminal.
  - The routes shape file was not linked to the stops shape file.

2. **The Data Management Group (DMG) at the University of Toronto**

The DMG at the University of Toronto is the custodian of the data sets collected and derived from the Transportation Tomorrow Survey.

2.1. **The Data Retrieval System** (The iDRS, online source, [www.jpint.utoronto.ca/drs/index.html](http://www.jpint.utoronto.ca/drs/index.html), with username and password authentication)
The DMG provides online access to the TTS data sets (derived data, not the raw collected records) through the iDRS using individual accounts that are granted based on request. A set of procedures were applied to the TTS2001 data set to extract the inputs needed for the modelling exercise:

- The following filters were applied first:
  - Only trips that use the TTC (i.e. trips which have TTC flag as ‘yes’)
  - Only trips that have ‘start time of trip’ variable between 6:00am and 9:00am
- These data are tabulated by their origin and destination zone information

The output of this procedure is a table, where each row has three pieces of information: the origin zone code, the destination zone code, and the number of transit trips that use TTC and start between 6:00am and 9:00am. This is conventional to the OD Matrix for transit demand with the above mentioned filters – hereafter referred to as the OD Matrix. The total number of trips in this matrix is 332,073.

- This OD Matrix is further divided by the access mode of the trip to the transit service and the egress mode to the destination from the transit service.
  - Note that the transit service accessed (or egressed from) is not necessarily the TTC, as trips might originate outside the City of Toronto and passengers access a local transit service or GO transit first. Similarly, trips not destined for the City of Toronto may involve egress from another municipal transit service or GO transit.
  - Three modes of access and egress are considered: walk mode, auto-passenger mode and auto-driver mode. The ‘walk’ mode means individuals access (egress from) transit service by walk. The ‘auto-driver’ mode means that passengers access (egress from) transit service using automobiles involving park and ride; while the ‘auto-passenger’ mode means that the traveller uses automobiles, however the trip-maker is not the driver. This results in a total of 9 combinations for possible access/egress mode choices. Table 6-1 shows each combination and the percentage of the total trips representing each combination in the data set extracted from the iDRS. Other modes, representing less than 0.01, are ignored (e.g. taxi passenger or cycle).
Similar information is compiled for each Origin-Destination pair. For example, from Zone 310 to Zone 222, there are 50 transit trips. Out of those 50 trips, 27 trips have walk as the access and egress mode, 13 trips have access mode as passenger and 10 trips have access mode a driver. All 23 trips have walk as the egress mode.

2.2. The Joint Program in Transportation website (www.jpint.utoronto.ca/drs/)

A collection of documents is available on the Joint Program website that report the validation of TTS2001 data. In the TTS2001 validation report, a list of the TTC 2001 route loads (i.e. total boardings per route) is provided for the AM peak period (6:00am-9:00am) in comparison with the TTS2001 expanded TTC route loads for the same period. This list is extracted and is referred to hereafter as the ‘TTC 2001 observed counts’ list.

2.3. The Joint Program in Transportation\(^{11}\)

The iDRS does not provide disaggregate data records through the online access service. The following was obtained upon request:

- The frequency distribution of each OD pair trips with regard to the start time of trip.
- The frequency distribution of each OD pair trips with regard to the trip purpose.
- The previous frequency distributions are not joint; that is, the number of trips, which have a certain trip start time and a certain trip purpose, is not given.
- Disaggregate data were obtained for individual records surveyed. This includes information such as trip origin geo-location, route (not branch) choices, stop choices (only for TTC Subway stops and GO stations), start time of trip, and trip destination geo-location. However, no socio-economic data was given for privacy purposes. This made it difficult to analyse travel behaviour of individual records based on, for example, trip purpose.
- A list of the GTA GO transit service routes was extracted from the 2001 transportation network maintained by the DMG. This includes information on route service characteristics (e.g. average speed, headway).

\(^{11}\) Contact person is Susanna Choy, The Joint Program in Transportation, University of Toronto
3. The University of Toronto Map Library (online source, www.main.library.utoronto.ca)

3.1. The map library at the University of Toronto provides the following GIS shape files for the GO transit system network

- A list of all GO rail and bus routes for the year 2002
- A list of all GO stops for rail and bus routes for the year 2002

3.2. Land use maps for the Province of Ontario. These maps group land usage based on seven categories: Commercial, Governmental and Institutional, Open Area, Parks and Recreational, Residential, Resources and Industrial, Water body.

- Commercial, Resources and Industrial, and Government and Institutional categories are considered as a potential ‘destination’ location for trips made during the modelling period.
- The residential category was used as a potential ‘origin’ location for trips modelled.

<table>
<thead>
<tr>
<th></th>
<th>Access</th>
<th>Egress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td>87.89% (291,861)</td>
<td>0.35% (1,155)</td>
</tr>
<tr>
<td></td>
<td>0.06% (199)</td>
<td>88.3% (293,220)</td>
</tr>
<tr>
<td>Passenger</td>
<td>5.0% (16,603)</td>
<td>0.10% (325)</td>
</tr>
<tr>
<td></td>
<td>0% (0)</td>
<td>5.1% (16,935)</td>
</tr>
<tr>
<td>Driver</td>
<td>6.6% (21,916)</td>
<td>~0% (14)</td>
</tr>
<tr>
<td></td>
<td>0% (0)</td>
<td>6.6% (21,918)</td>
</tr>
<tr>
<td>Total Egress</td>
<td>99.49% (330,380)</td>
<td>0.45% (1,494)</td>
</tr>
<tr>
<td></td>
<td>0.06% (199)</td>
<td>100% (332,073)</td>
</tr>
</tbody>
</table>

6.3 Supply Modelling

In this application, the supply of transit services is provided by:

- The TTC service network (routes/branches and stops)
- The GO service network (routes/branches and stops)
- Other Municipal transit services that provide access/egress to/from TTC and/or GO services.
6.3.1 The GIS-T Model

A Geographic Information System (GIS) static representation of the supply side is developed in ArcMap\textsuperscript{©}. In this implementation, the GIS data for local transit services, other than the TTC, were not available. In addition, the data available for TTC and GO transit service modelling were not ready to be directly imported and used. The following tasks were carried out:

1. From the route-stop sequence file provided by the TTC, a master-list of stops serviced by the TTC AM transit network was compiled. This list includes all stops that are covered by 480 TTC branches. Note that a stop could serve different branches on the same route and/or multiple routes. This list includes entries of 9,969 stops for all different types of services provided by TTC; namely: bus service, streetcar service, and subway service. This will later help investigate the ‘common-lines’ problem.
   a. It is worth mentioning that, after performing some validation checks on the data provided by the TTC in the route-stop sequence file, several inconsistencies were found. These errors were brought to the attention of the TTC Service Planning Department and were clarified. The outcome of this was an updated route-stop sequence file.

2. This master-list is compared to the stops shape file provided by the TTC. As mentioned before, the shape file had only 90\% of these stops. Therefore, a “mini-master-list” was extracted for each route/branch to identify the missing stops from the shape file.

3. Then, the missing stops (total of 1,002 stops) were geo-coded and the stops shape file was updated to have a total of 9,969 stops for TTC AM peak period service. This geo-coding process was automated through a tool, which was developed in the ArcGIS\textsuperscript{©} Visual Basic Application (VBA) environment. The tool was developed by the author of this thesis. The tool, in principle, follows the following steps:
   a. It compares the mini-master list for each route/branch with the stops shape file. If a stop in the list is not found in the shape file, it activates the ‘geo-builder’ algorithm.
   b. The ‘geo-builder’ basically locates the position of the missing stop with regard to the stop sequence (or stop-order). The geo-coded stops, located before and after the missing stop in the sequence file, are used to calculate the geo-location of the missing stop, with the consideration of the route-segment direction. For example, if the route-
segment with the missing stop is directed east-west, then the coordinates of the missing stop are assumed to have the same latitude as the before and after geo-coded stops, but a different longitude value. This latitude value is calculated based on information about ‘distance from previous stop’ attribute provided in the route-stop sequence file.

c. If there is more than one missing consecutive stops, the procedure is applied recursively, up to the route/branch start stop. This was designed after a validation check that all route/branch start stops are not missing.

d. Due to some irregularity in route/branch directions (and sometimes looping structures), a manual check was performed for all the geo-coded stops using the ‘geo-builder’ tool.

4. The GO service shape files (stops and routes) were also imported and overlaid on the TTC service network to establish spatial connections between both service networks.

   a. A similar route-stop sequence file was compiled for the GO service, as well as a master-list for all GO stops, using data collected from the DMG at the University of Toronto and data in the shape files downloaded from the University of Toronto Library. The DMG data represent the year 2001 GO (train and bus routes) and shape files represent the service coverage and stop geo-locations for the year 2002. Only the routes that were reported by the DMG for the year 2001 are used, however the stations associated with them represent the year 2002. There were no maps available for GO transit service for the year 2001. Since the TTS2001 was conducted in the fall of 2001, we assume that differences, if any, of the GO transit service between the year 2001 and the year 2002 are minimal, and consequently their impacts are insignificant.

   b. A mini-stop-list was also compiled for each GO route (rail or bus). There was no need to use the ‘geo-builder’ for the GIS modelling of GO service since all stops were included in the shape files.

   c. This results in a GO network of 15 routes (30 one-directional routes) and 91 GO stations.

5. The data in the route-stop sequence file was not linked to the stops shape file nor the routes shape file. This was true of the TTC as well as GO services. A ‘data model’ was developed to
connect the transit service supply information – see Figure 6.4. This data model links the different elementary units of transit service (routes/branches and stops) by defining the type and direction of the connecting-relationship.

a. Stops are fixed entities, defined by their geo-location.

b. Routes span over a region and cover multiple-geographical areas, denoted by the route-segments. Routes are directionless, or in other words, they represent the possible two-directions.

c. Bounds, on the other hand, are defined by direction and represent one possible direction for the route, either ‘inbound’ or ‘outbound’.

d. Branches are connected to a specific bound, and therefore are restricted by direction. Each branch is geographically defined by the sequence of stops it serves. It is also defined by a set of service characteristics. These characteristics can be defined at the route-level, the bound-level, or the branch-level. A vehicle type and a fleet size are associated with each branch; this data are utilized along with schedule information to generate runs for each branch.

e. A sequence of transit stops forms a branch. A transit stop is not a physical entity, however it is linked to a physical location represented by a stop. A transit stop has a specific order in the branch’s stop list; for instance, the start-stop has 1 as its order attribute. For the same fixed-location stop, different transit stops may exist with different orders for multiple branches. Note that in branches with looping structures, two transit stops may exist with different order that point to the same fixed-location stop.

f. When the same stop services multiple branches (and possibly routes), this stop will appear in multiple branch-transit stop lists. To make this information visible at the physical stop level, each stop has a list of ‘common lines’. This list contains a link to all branches (that are linked with bounds and consequently routes) that service this stop. For all stops, this list has at least one item, since all geo-coded stops are linked to routes/branches from the route-stop sequence file. This list has many entries at hubs where multiple routes are connected, such as Union Station.

g. When a transfer is possible between geo-located stops, a ‘transfer-connection’ group is created. This group has a list of (pointers to) stops that are accessible to a geo-
located stop. For example, at a four-way intersection, the transfer-connection group of
the north-east stop will contain links to the other three stops (north-west, south-east
and south-west stops). The transfer-connection group is linked to a stop, which can be
retrieved from a transit stop structure.

h. Each branch operates a service schedule. Scheduled service is represented by a set of
‘runs’. Each run is defined by its release time from the branch start-stop, which may
depend on the branch headway attribute or take specific-values. A run represents the
interaction between a transit vehicle and the transit stops associated with the branch.
A Run’s level of service is aggregated to represent the level of service of its branch,
which is linked to a bound that is a part of a route.

i. Transit vehicles are the physical entities that move from one transit stop to another,
covering route-segments. A transit vehicle has different attributes, for instance,
capacity, acceleration/deceleration rates, number of doors for boarding/alighting, and
method of payment. In this application, there are five classes of vehicles: bus,
streetcar, subway, GO-bus and GO-train. The information collected when a transit
vehicle arrives/departs from a transit stop is stored in a stop-record.

j. A stop-record contains information needed to monitor and evaluate the transit service
performance. Stop-records include information such as transit vehicle
arriving/departing occupancy, dwell time at stop, arrival/departure times at stop, and
schedule/headway adherence. A set of stop-records is linked to each run to describe
the run performance along its movement at different geo-located stops. The service
performance can also be viewed at specific geo-located stops by a set of stop-records
that are associated with each stop. This set describes the performance of all the
‘common lines’ that pass by this stop.

6. The local transit service that provides access/egress to/from TTC and GO services were not
explicitly represented in the GIS-T model due to the lack of geo-information about these
services. This required a special treatment for trips originating/ending outside the TTC and
GO services buffer areas. This treatment is explained in section 6.4.3, ‘Mental Model
Building’.
The structure outlined for the data model in Figure 6.4 was developed for the TTC and GO services. This data model facilitates the development of several tools for modeling the dynamics of the transit service and generating the ‘mental model’ for individual passengers.

6.3.2 The Network Simulation Model

While the use of a microsimulation model is recommended, it is also recognized that the development of a microsimulation model for the transit service in the proposed case-study is a time-consuming task, especially given that traffic demand calibration is required for the validation of transit vehicle travel times. Therefore, a mesoscopic model is developed to represent the dynamics of the transit service at the network level with a detailed representation of branch/vehicle-level operations. When further level of detail is required, the model can be linked to a microsimulation representation of the part of the network under investigation – see Figure 6.5.
The developed mesoscopic model represents the movement of each transit vehicle between stops as a function of the link speed, without the representation of the general traffic. Meanwhile, it microscopically represents individual passenger alighting and boarding activities at each stop, including the interactions among passenger-agents and between passenger-agents and the transit network. The supply model acknowledges loading priorities at stops and represents congestion through fail-to-board handling. This is modelled as a “discrete-time, event-driven” simulation model, where the simulation model clock advances every second and handles events as they occur at varying increments. There are two types of events: unconditional events and conditional events that occur during the simulation of the transit service dynamics. Unconditional events are the ones that “fuel” the simulation and keep it running; for example, the release of the first transit vehicle from the start stop of a specific route/branch. The arrival of transit vehicles, on the other hand, at subsequent stops is conditional on the release from the first stop and travel time between stops. Similarly, the release of the second transit vehicle from the start stop is conditioned on the headway of this specific route/branch and the availability of transit vehicles for this route/branch. The departure of a transit vehicle from a stop is conditioned on the arrival of this transit vehicle at this stop and the dwell time at this stop.

In this context, the fuel to the “discrete-time event-driven” simulation comes from the release time of the first run for each transit route/branch. This “release” event will generate two events: the release of the next run at a specified time (conditioned on headway and transit vehicle availability) and the “arrival” event of the transit vehicle at the next stop, see Figure 6.6. This arrival event will generate a “departure” event from the current stop, which will generate an “arrival” event at the next stop (based on the stop sequence structure for each route/branch), see Figure 6.7 and Figure 6.8. When the transit vehicle arrives at a stop, and this stop is the end-terminal for this specific route/branch, it does not generate an “arrival” event. In such simulation models, events are handled by an “event list”, which is ordered chronologically based on time of event, and events are added to this list as they are scheduled, see Figure 6.9. For efficient use of memory, events that are scheduled after the simulation end time are not added to the event-list and are simply ignored. It is worth mentioning that such a mesoscopic model allows the proper
modelling of the dynamics of each branch, with regard to run-by-run representation and passenger-network interactions, while accounting for network-level (emerging) effects.

With an object-oriented structure, the developed mesoscopic simulation model can address many issues at the route level, while representing the whole network:
- Fleet size constraints for each route/branch
  - When there is a delay experienced by a run on a specific route/branch, this is propagated to future runs as their release from start stop may be delayed due to the shortage of transit vehicles.
  - Due to the microscopic representation of transit stops and runs, a range of policies that address this issue can be modelled and tested (e.g. short-run, fleet re-assignment from another route). This model presents an advantage of observing the network-level performance (or emerging/unpredicted deficiency) of such policies rather than focusing on single-route operations.
- Route/Branch-specific attributes
  - Transit Vehicle Characteristics
    - Different routes/branches could operate different types of transit vehicles, with different attributes, even with the ‘Bus Class’ of vehicles (e.g. capacity, acceleration/deceleration rates, number of doors for boarding/alighting, accessibility-features)
      - For example, streetcar routes now operate two types of vehicles: single-unit streetcar (e.g. Spadina route, #509) and articulated streetcar (e.g. Queens route, #501).
  - Different headway-values during the modelling period
    - Within the proposed platform, it is feasible to represent transit routes with various frequencies (e.g. Route #6, Bay Street, has a 5-minute headway, representing a high frequency service; Route #85, Sheppard East, has a 12-minute headway, representing a medium frequency service; and Route #192, Airport Rocket, has a 45-minute headway, representing a low frequency service), and transit routes with different frequencies for different periods (e.g. Route #9001, GO Lakeshore West, has a headway
of 15 minutes from 6:00am to 9:00am and a headway of 60 minutes during off-peak periods).

- ‘Method of Payment’
  - The method of payment affects the dwell time at stops, and consequently run’s travel time along route segments. Different methods of payment can be associated with different routes, and possibly capture the impact of method of payments on route level-of-service.

- Automatic Vehicle Location (AVL) and Automatic Vehicle Information (AVI) capabilities
  - The deployment of some control strategies requires the location of the transit vehicle to be known (e.g. holding control strategy, short-run control strategy). In the case where not all the fleet is equipped with AVL, routes (or even runs) can be selectively chosen for the application of such strategies.

- Automated Traveller Information System (ATIS) capabilities
  - If some routes are equipped with ATIS at stop and/or on-board, this can be represented on a route/run-level. This becomes important to model the gradual introduction of ATIS.

The developed mesoscopic simulation model represents a dynamic replica of the GIS-T model for the TTC and GO transit services. A total of 324 one-directional routes (TTC and GO transit services) with 510 branches were coded – or more specifically loaded from the GIS-T model – along with 10,060 stops and stations covering all possible transfers and common line stops.
Figure 6.5 Potential Connection between Mesoscopic and Microscopic Representation for a selected route (Spadina Streetcar, R#510)
**RELEASE_FROM_STARTSTOP**

- **Vehicle to Release?**
  - **Yes**
    - **Generate RUN**
    - **Board Passengers**
    - **Update StopRecords (Stop/Run)** Boardings/alights/dwelltime/occupancy
    - **Schedule Next Release from Branch at (t + headway)**
    - **Schedule Departure From Stop at (t + dwell time)**
    - **Update FleetSize for Branch** *Reduce* number of vehicles to release
    - **Delete ‘Release_from_StartStop’ From event-list**
  - **No**
    - **Re-Schedule This Release at (t + 1 minute)**

---

**DEPARTURE_FROM_STOP**

- **Retrieve ‘Next Stop in Sequence’** For this branch, call it ‘next stop’
- **Schedule Arrival At Next Stop at (t + time to next stop)**
Figure 6.8 ‘Arrival at a Stop’ Event Procedure
Figure 6.9 ‘Run Iteration’ Event Procedure
6.4 Demand Modelling

In this application, the demand for the TTC transit service has the following characteristics:

- Trips started between 6:00am and 9:00am, based on the TTS2001 records. It is important to mention that such trips may end after 9:00am. This will require the extension of the modelling period as discussed later in section 6.6. It is also important to note when the calibration process is carried out, observed counts for TTC service are available for the period between 6:00am and 9:00 and not for trips starting during this period.

- Access and egress modes can be ‘walk’, ‘passenger’, or ‘driver’.

- Four categories for trip purpose are modelled:
  - Home-based Work trips (67%)
  - Home-based School trips (27%)
  - Home-based Other trips (4%)
  - Non-home-based trips (2%)

6.4.1 On the accuracy of the TTS2001 extracted data

From the TTS2001, a transit OD-matrix was compiled based on the criteria listed in section 6.2. Based on the TTS2001 collected data, 67,090 individuals reported using the TTC service as a means to complete their trips. About 19,520 individuals reported using TTC service, and their trips started during the AM peak period (6:00-9:00am). Those 19,520 records are expanded to 332,073 trips, representing the total number of trips that use TTC and start during the AM peak period. From the iDRS, it is possible to extract those trips in an Origin-Destination Matrix format.

A validation step was carried out before using the extracted OD matrix for the TTS2001 TTC trips. It was found that a total of 124 trips (equivalent to 13 records) are not correct. Based on the origin and destination zone codes associated with those trips, it is unrealistic that the individuals who completed those trips used the TTC service due to the geo-location of the origins and destinations of these trips. For one example, the origin and destination geo-locations of the transit trip are in the Waterloo region (west of Toronto area). This anomaly could be due to data-
entering errors during the encoding of the TTS2001 data. These inaccuracies were brought to the attention of the DMG personnel, and the inaccuracies were acknowledged on their part. Those records were also observed in the disaggregate data obtained from the DMG staff. Those 124 trips were excluded from the analysis.

6.4.2 From Aggregate Transit OD-Matrix to Disaggregate Transit Trip Geo-List

The input to MILATRAS is the traditional Origin-Destination (OD) transit matrix (or the agent-based transit demand). The OD matrix, extracted from the TTS2001 for the abovementioned records, has 931 origin-zone codes (Figure 6.10) and 701 destination-zone records (Figure 6.11), with the total number of transit trips being 332,073. While this OD matrix has more than 652,500 cells, only 14,794 cells have non-zero values (representing about 332,000 trips). Therefore, the transit OD matrix was converted to a transit OD-list, with only 14,794 records and three columns. The three columns represent the origin-zone ID, the destination-zone ID, and the number of transit trips for the OD pair. The total number of trips for all OD pairs sums up to the total of the OD matrix (i.e. 332,073). This OD-list is further disaggregated to a list of trips; the total number of records in this list is 332,073. Each record has three attributes: trip id (auto-generated attribute from 1 to 332,073), trip origin-zone id, and trip destination-zone id. This final list is consistent with the original transit OD-matrix such that the aggregation over origin zones and/or destination zones will result in the original OD matrix – refer to Table 3-2 for illustration. For example, if the number of trips originating in Zone 1 is 420 trips, with 54 of those trips destined for Zone 6, then the aggregation of the OD-list, for trips that have Zone 1 as the origin-zone, will result in 420 trips. If the aggregation is for trips that have Zone 1 as the origin-zone and Zone 6 as the destination-zone, the result will be 54 records.
Figure 6.10 Origin Zones for TTC Trips during the AM Peak Period according to TTS2001 Data
Figure 6.11 Destination Zones for the TTC Trips during the AM Peak Period according to TTS2001 Data
Using the *OD-Generator* (section 3.2.2.1), the *OD-Geo List* is generated; this list includes a geo-record for each trip-record in the *OD-list*. The geo-code for each trip origin and destination is generated *randomly* within the corresponding zones. While disaggregate data, including geolocation, are available for the TTS2001, this is only 5% of the whole population and this geolocation information is distorted due to obvious privacy issues. The *OD-Geo List* generation process was enhanced by utilizing the land use data downloaded from the University of Toronto Map Library. In this application, and since we are concerned with AM peak period trips, it was assumed that trip origins will be located in a residential area and trip destinations will be within an employment-type areas. To accomplish this, each TTS zone was attached to the residential area(s) and employment area(s) (‘Commercial’, ‘Resources and Industrial’, and ‘Government and Institutional’) within that TTS zone. It is important to note that some of the residential and employment areas are overlapping, where mixed land-use development exists – see Figure 6.12 and Figure 6.13.
Figure 6.12 Residential Areas in the Greater Toronto Area
Figure 6.13 Employment Areas in The Greater Toronto Area
To randomly generate a trip-end geo-location for each of the 332,073 trips, the following procedure is implemented. The trip origin zone is identified; then, the residential areas in this zone are retrieved. The probability of the trip origin geo-location being generated in a specific residential area is proportional to the area of this residential lot compared to the total area of all residential lots inside the identified zone – see Figure 6.14. A similar approach is carried out for the trip destination; that is, the destination zone is identified and the employment locations are retrieved. The probability of the trip destination geo-location being generated in an employment area is proportional to the weight of this area relative to the total employment areas within the destination zone – see Figure 6.15. The output of this process is the OD-Geo List, where each trip has a geo-coded origin point and a geo-coded destination point; each trip-origin and trip-destination are linked through the trip-id, which is also referred to as agent-id or passenger-id. The reference to each trip’s origin-zone and destination-zone codes (as input from TTS2001) is still maintained for aggregation and validation purposes. A few points are worth mentioning:

1- For each trip, the distance between its origin geo-point and its destination geo-point is set to be greater than 1000 meters. This is to warrant a transit trip between the geo-origin and geo-destination locations.

2- While the probability of generating a geo-point within a land use region is related to the area land-coverage, it is better to relate the probabilities with the use of the land in this area. For example, high-rise office buildings are small in area but represent a potential destination for a larger number of trips in the AM peak period than low-rise retail buildings that are spread over a large area of land with few employment positions. This level of information was not available, but would of course enhance the modelling accuracy.

3- It does not come as a surprise that transit stops are surrounded by residential and employment areas. The opposite, however, is not true; residential and employment areas are not always surrounded by transit stops. It was intentionally assumed that the random generation of each trip’s origin and destination is not targeted at locations close to transit stops. The rationale behind this assumption is that by restricting trips to be generated close to transit stops, the stop-choice will be biased and somehow imposed. Alternatively, trip-ends are generated within residential and employment areas, without any information on the transit stop location.
The input OD matrix has some intra-zonal trips. This poses a challenge to traditional transit assignment models since those models load trips to zone-centroids. On the other hand, the proposed framework naturally deals with these trips since it models transit trips on the stop and link level rather than the zone-centroid level.
Figure 6.14 Trip Origin Geo-Locations (partial view of the GTA)
Figure 6.15 Trip Destination Geo-Locations (partial view of the GTA)
Based on the data obtained on the trip purpose geo-distribution (i.e. trip-purpose distribution by OD pair), a trip-purpose attribute is synthesised and attached to each trip; this trip purpose is: home-based work, home-based school, home-based other, or non-home-based. The synthesis process uses Monte Carlo methods so that the aggregation of the individual synthesised attributes matches the OD distributions.

6.4.3 Mental Model Building Process

6.4.3.1 Path Choice Set Generation

For each trip in the OD-Geo List, the Path Generator (section 3.2.3.1) produces its choice set. The choice set is stored according to the proposed mental model structure as in Figure 4.9. The structure of the mental model represents the relevant transit network for a specific trip. Its contents, accumulated over time, represent the relevant transit network conditions experienced by the passenger. The process of building the mental model is illustrated with an example in the following.

Passenger-agent #316 (called hereafter P316) was reported to have used the TTC service during the morning peak period of a typical weekday in 2001. To be more precise, TTS2001 records show one individual travelling from GTA-zone #1 to GTA-zone #194. This record was given an expansion factor of 13. Therefore, this record is represented by 13 trips in the OD-Geo List with different origin and destination geo-locations within GTA-zone #1 and GTA-zone #194, respectively. One of these trips is trip #316

The purpose of P316’s trip is (synthesised as) work-related (i.e. home-based work). The origin of the trip is located in a residential area in GTA-zone #1 (a suburb of the City of Toronto) and it is destined for an employment location in GTA-zone #194 (downtown of the City of Toronto) – see Figure 6.16. The access and egress modes for this trip are synthesised as walking.

In simple terms, the path generator first identifies the accessible origin stops and destination stops based on some predefined criteria. Second, the path generator finds realistic path options
between the identified accessible origin and destination stops. Two predefined criteria are considered for the mental model building process:

1- When the access or egress mode is walking, the maximum walking distance is 1000 meters – unless otherwise stated. This is based on the study by Alshalalfah and Shalaby (2007). The catchment area is expected to vary by mode – passengers are willing to walk more to a subway station compared to a bus stop. The maximum walking distance is assumed as the upper bound for the rapid rail transit mode in order to mitigate any impact of the randomness in generating geo-locations for trip-ends.

2- In a grid-like network such as the TTC service network, it is possible to connect almost any two stops through a number of transfers. Path options with two or more transfer connections greater than the minimum number of transfers for a given trip are not considered. This means that, if the minimum number of transfers to complete a certain OD trip is 1, then path options with number of transfers greater than or equal to 3 are not generated. Meanwhile, path options with 2 transfers for the same trip are considered.
Figure 6.16 Origin and Destination Locations of P316’s Trip
For P316, the selected origin stops are highlighted in Figure 6.17. While there are many stops within the buffer zone (a circle of 1000 meter radius), the mental model building process shows that only three stops are considered by the passenger-agent as potential origin stops. The list of origin stops is incrementally constructed based on the following:

1- Stops, within the accessible buffer zone, are added to the origin stop list if and only if they are not dominated by stops already inserted in the list. Dominance is measured based on distance and list of common lines passing by a stop.
   a. Stop a dominates stop b if and only if stop a is closer by distance to the trip origin geo-location and the list of common lines passing by stop b is a subset of the list of common lines passing by stop a. Note that at this stage in the mental model building process, attractive routes passing by each stop have not been identified yet.

2- Stops, in the origin stop list, are removed from the list if and only if they are dominated by a newly added stop to the list. This means that if a stop s is to be added to the list (which implies that stop s is not dominated by any stop in the current list), any stop s’ that is dominated by stop s is removed from the list.

3- Stops, in the origin stop list, are removed from the list if and only if they are dominated by a group of stops from the list. A stop s with three common lines (r₁, r₂ and r₃) that is further away from stop s’ with only two common lines (r₁ and r₂) cannot be removed. If there exists another stop s’’ that is closer than stop s and has r₃ in its list of common lines, then stop s is removed from the origin stop list since it is considered to be dominated by the group (s’ and s’’). For P316, for example, route #123 (going north) passes by many stops within the buffer zone; however, the nearby stops dominate the further away stops.

The highlighted stop to the west of the origin in Figure 6.17 is a GO service station. This is important to mention since the choice set should include all available path options, and not only TTC-related choices. Despite the fact that the passenger reported using the TTC services, we are interested in modelling the decision-making behaviour with regard to all available options. GO service represents an alternative: faster service with a higher cost and a limited capacity. Even for trips that could be completed with the TTC service alone, other alternatives should be
considered in the choice set, if possible, to properly describe travel choices. This has implications for the departure time choice as well: for a fixed scheduled arrival time, passengers may choose to leave at a later time from origin and use a faster service (with a higher fare) rather than to leave earlier and use a slower service (while paying less). When considering the activity-scheduling dimension of passengers’ travel behaviour, the significance of including such alternatives becomes evident.
Figure 6.17 Accessible and Selected Origin Stops for P316
The list of potential destination stops is generated following a similar set of rules to the one used for the list of origin stops. Figure 6.18 shows the buffer zone for the destination location. Clearly, the buffer zone covers more stops compared to Figure 6.17 as the trip’s destination is in the downtown area that is heavily serviced by transit. Four potential destination stops are selected (highlighted in Figure 6.18). Interestingly, one of the highlighted stops is located outside the destination zone boundary. Traditional transit assignment models with zone-centroid representation of the transit network fail to capture such alternatives, and hence do not properly model path choices.

By examining Figure 6.16, one can only imagine the number of possible paths for P316’s trip. The path generator initially identifies the minimum number of transfers to complete the trip; for P316, the minimum number of transfers is found to be 1 – i.e. there is no direct connection between the selected origin and destination stops. Then, the path generator constructs the choice set for the passenger, considering all path choices with number of transfers less than or equal to the minimum number of transfers plus one. The path choices for P316 consider all transit options with 1 and 2 transfers. Even with this restriction, the possible number of path choices is large. The path generator algorithm starts by generating all path options with the minimum and maximum number of transfers between all origin and destination stops. In this process, the algorithm considers avoiding loops and flip-flopping between GO and TTC services. Then, the path generator algorithm follows a set of heuristic rules to exclude unrealistic options; previous studies implemented heuristic path finding rules such as Tong and Richardson (1984) and Florian (1998). These heuristic rules are:

- For any route $r$ in the attractive set of an origin stop, if route $r$ is accessible from a transfer stop then delete this route from the attractive set of the transfer stop.
- For any route $r$ in the attractive set of an origin stop, with $t$ minimum number of transfers to the destination:
  - Delete route $r$ from the attractive set of any origin stop or transfer stop, if the minimum number of transfers to the destination is greater than $t$
  - Delete any route in the opposite direction of route $r$ and with a minimum number of transfers to the destination greater than $t$
For any rail transit (i.e. subway or GO Transit) route \( r \) in the attractive set of an origin stop or a transfer stop, with a minimum travel distance to the destination \( \text{minDis} \)
- Delete any route \( r' \) that is in the attractive set of the same origin stop or transfer stop if it is not a rail transit and has a minimum travel distance to the destination greater than \( \text{minDes} \)

- For any light rail transit (i.e. streetcar) route \( r \) in the attractive set of an origin stop or a transfer stop, with a minimum travel distance to the destination \( \text{minDis} \)
  - Delete any route \( r' \) that is in the attractive set of the same origin stop or transfer stop if it operates buses on surface roads and has a minimum travel distance to the destination greater than \( \text{minDes} \)

- For any rail transit or light rail transit route \( r \) with more than one possible alighting stop, keep only alighting stops that maximize the distance travelled using rail or light rail transit while maintaining the minimum travel distance to destination. This is to represent the preference to complete a large part of the trip using rail transit (if possible).

- An origin stop \( s \) is deleted if and only if its set of attractive routes is dominated by a group of origin stops

- Travel time (in-vehicle and out-of-vehicle) is considered last to keep the number of path choices under a predefined maximum value. If the total number of paths still exceeds the pre-defined maximum number, then path options with the largest travel distance (and travel time) are deleted iteratively until the pre-defined maximum number of paths is reached. During this process, path options with rail connections are given priority to remain in the choice set.

For the TTC application, the maximum number of paths is set to 7; this is in line with Cascetta et al. (2002) as they suggested that travellers realistically consider between 4 and 7 alternatives for travel choices. By setting the maximum number of paths to 7, this means that the maximum number of available choices at each stage during the trip is also 7. For instance, the number of potential origin stops is guaranteed to be less than or equal to 7, similarly the number of attractive routes at any stop.
For P316, the constructed mental model is shown in Figure 6.19. All paths require one transfer; options with two transfers were excluded based on the heuristic rules. One possible path includes a transfer between the GO service (route #9001) and the TTC service at Union station (route #602); while this might be the fastest, it requires paying two separate fares. A couple of paths consider transferring at the east-west subway line #601 (Bloor-Danforth line). These two paths require using the TTC surface bus routes and the rail transit subway system. Two other paths suggest using the light rail transit streetcar system (route #501) and transfer to either the subway system (route #602, University-Spadina-Yonge line) or the express service (route # 142).

From the TTS2001 disaggregate data, the trip, with an origin in GTA-zone #1 and a destination in GTA-zone #194, was recorded to have used the GO service (route #9001) and the TTC service (route #602) – see Figure 6.20. The record from TTS2001 is represented by 13 trips. Not all trips have the same origin and destination geo-locations, and hence the mental model could be different for each trip and not necessarily covering the reported path choices (#9001 and #601). This warrants a more realistic assignment procedure compared to generating route loads assuming that the 13 trips (representing the TTS2001 record) will make the exact route choices.
Figure 6.18 Accessible and Selection Destination Stops for P316
Figure 6.19 Constructed Mental Model for P316’s Trip
Figure 6.20 TTS2001 Record for Trip with an origin in GTA-zone #1 and a destination in GTA-zone #194
The output of this process is the choice set of each passenger-agent. This includes a list of origin stops; for each origin stop, there is a list of attractive routes. For each attractive route, there is a list of possible connections (off stops and on stops). The mental model building process is repeated for each trip of the 265,574 (80%) trips in the OD-Geo list that has an origin and destination within the TTC service coverage and has walking as the access and egress mode. For other trips, some special treatment is needed. Those trips have either an origin (or a destination) outside the TTC service coverage area or their trip-end is located within the City of Toronto with auto-driver or auto-passenger as the access or egress mode:

1- With access (or egress) mode as walking:
   (i) Passengers transfer to the TTC service through local transit or GO services.
   (ii) Transfers happen at GO stations or subway stations.
   (iii) Since information about local transit networks in the GTA was not available, the list of accessible origin (or destination) stops includes the nearby GO and subway stations.
   (iv) If the selected GO or subway station is not within the maximum walking distance, then a local-fare cost is considered.
   (v) The stop access (or egress) time needs to be approximated. With no information on the local-transit service performance, a time-distance curve is used to calculate the access (or egress) access time, where walking speed is assumed to be 4km/h and average driving speed as 25km/h – see Figure 6.21.
   (vi) Transfers occurring while using the local-transit network are not considered.

2- With access (or egress) mode as auto-driver or auto-passenger:
   (i) Passengers use auto mode (as a driver or as a passenger) to access (or egress form) GO and subway stations only.
   (ii) Since GO and subway service networks are explicitly modelled at the stop-level of detail, a list of accessible nearby GO and subway stations is generated.
(iii) Travel cost using the auto-passenger or auto-driver mode is not considered in this implementation. The generalized cost accuracy can be enhanced by information regarding gas cost, parking cost and other automobile-driving related expenses.

(iv) It is important to note that, without auto-usage cost, it might seem preferable to passengers to access (or egress from) the farthest stop away from the trip-end and the closest to destination. Such choices are restricted by giving priority to the nearby GO and subway stations.

(v) The stop access (or egress) travel time using the automobile needs to be approximated. With no information on the network conditions of the surrounding cities, the speed-curve in Figure 6.21 is used to approximate the travel time to access (or egress from) a stop using auto-passenger or auto-driver mode.

While the above treatment introduces some assumptions, it only affects 15% of the modelled trips\textsuperscript{12} as shown in Table 6-2. At this point, the mental model of the 332,073 trips is generated. The mental model building process was validated by comparing the disaggregate choices from the TTS records with the choice sets generated for individual trips. For 96% of all TTS records, the reported chosen routes for a particular trip are included in the generated choice set for the same trip attributes. According to Figure 4.9, the only missing choice dimension is the departure time choice; the departure time choice set generation is illustrated next.

\textsuperscript{12} Practically, the affected percentage is less than 15% because some trips, which have an origin or a destination outside the City of Toronto, are still within the coverage area of the TTC service; examples include trips originating or destined in North York region or Etobicoke
Table 6-2 Number of trips that received special treatment during the mental model building process

<table>
<thead>
<tr>
<th>Number of Modelled Trips from TTS2001</th>
<th>Origin Location</th>
<th>Access Mode</th>
<th>Destination Location</th>
<th>Egress Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>265,574 (80%)</td>
<td>Inside Toronto</td>
<td>Walking</td>
<td>Inside Toronto</td>
<td>Walking</td>
</tr>
<tr>
<td>482 (&lt;1%)</td>
<td>Inside Toronto</td>
<td>Walking</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
</tr>
<tr>
<td>120 (~0%)</td>
<td>Inside Toronto</td>
<td>Walking</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
</tr>
<tr>
<td>7,215 (2.1%)</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
<td>Inside Toronto</td>
<td>Walking</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
</tr>
<tr>
<td>14 (~0%)</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
</tr>
<tr>
<td>8,697 (2.6%)</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
<td>Inside Toronto</td>
<td>Walking</td>
</tr>
<tr>
<td>198 (~0%)</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
</tr>
<tr>
<td>9,586 (2.9%)</td>
<td>Inside Toronto</td>
<td>Walking</td>
<td>Outside Toronto</td>
<td>Walking*</td>
</tr>
<tr>
<td>611 (&lt;1%)</td>
<td>Inside Toronto</td>
<td>Walking</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
</tr>
<tr>
<td>55 (~0%)</td>
<td>Inside Toronto</td>
<td>Walking</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
</tr>
<tr>
<td>45 (~0%)</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
<td>Outside Toronto</td>
<td>Walking*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
</tr>
<tr>
<td>79 (~0%)</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
<td>Outside Toronto</td>
<td>Walking*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
</tr>
<tr>
<td>58 (0%)</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
</tr>
<tr>
<td>15,311 (4.6%)</td>
<td>Outside Toronto</td>
<td>Walking*</td>
<td>Inside Toronto</td>
<td>Walking</td>
</tr>
<tr>
<td>53 (~0%)</td>
<td>Outside Toronto</td>
<td>Walking*</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
</tr>
<tr>
<td>40 (~0%)</td>
<td>Outside Toronto</td>
<td>Walking*</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
</tr>
<tr>
<td>14,608 (4.4%)</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
<td>Inside Toronto</td>
<td>Walking</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
<td>Inside Toronto</td>
<td>Auto-Driver*</td>
</tr>
<tr>
<td>7,745 (2.3%)</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
<td>Inside Toronto</td>
<td>Walking</td>
</tr>
<tr>
<td>70 (~0%)</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
<td>Inside Toronto</td>
<td>Auto-Passenger</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
<td>Inside Toronto</td>
<td>Auto-Driver</td>
</tr>
<tr>
<td>1,302 (&lt;1%)</td>
<td>Outside Toronto</td>
<td>Walking*</td>
<td>Outside Toronto</td>
<td>Walking*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Walking*</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Walking*</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
</tr>
<tr>
<td>37 (~0%)</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
<td>Outside Toronto</td>
<td>Walking*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
</tr>
<tr>
<td>102 (~0%)</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
<td>Outside Toronto</td>
<td>Walking*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
</tr>
<tr>
<td>0 (0%)</td>
<td>Outside Toronto</td>
<td>Auto-Passenger*</td>
<td>Outside Toronto</td>
<td>Auto-Driver*</td>
</tr>
</tbody>
</table>

* trip-ends receiving special treatment
Figure 6.21 A distance-time curve used to approximate the auto-travel time for GTA areas outside the City of Toronto
6.4.3.2 Departure Time Choice Set Generation

Almost all research on timing of travel shares a common concept that is founded on the idea that a traveler chooses his time of travel based on the journey duration and the schedule delay associated with travelling at a time other than the originally preferred time of travel. By far, the most popular specification for this problem is that due to Small (1982), which was based on the investigation of McFadden and Talvitie (1977). The underlying assumption is that the only reason why travellers should shift from their preferred time of travel is because the resultant loss associated with schedule disutility is outweighed by the gain from reduced travel time. This usually reflects the case in heavily congested areas, with a reasonable variation in travel times associated with each time option. To measure the schedule disutility, a scheduled arrival time at the destination is needed. In the transit assignment field, studies that explicitly model time-dependent vehicle movements (using service timetables) take into account users’ scheduled arrival times at destinations (e.g. Nielsen and Jovici, 1999; Nuzzolo et al., 2001; Florian, 1998; Nguyen et al., 2001).

From the TTS2001, the scheduled arrival time for each trip is not collected; rather, the start time is surveyed for each trip record. In order to generate the departure time choice set, a scheduled arrival time at the destination is needed. This is also required to compute the schedule delay associated with any departure time and path choices. The following procedure is used to synthesise a scheduled arrival time for each trip record; for datasets with scheduled arrival time information, this step should be skipped.

Based on the output from the path generator, each passenger-agent has a set of paths that connects the trip origin and destination geo-locations. The frequency distribution of trip start time by origin-destination information is then used to synthesise (using Monte Carlo methods) a trip start time for each passenger-agent. For each trip, the minimum and maximum travel time (in-vehicle and out of vehicle) from its origin to its destination are computed. A range of possible scheduled arrival time values is then established: the lower bound is calculated by adding the minimum travel time to the start time of the trip, and the upper bound is obtained by adding the
maximum travel time to the trip’s start time. The lower and upper bound values are approximated to the nearest 5 minutes. This range is discretized to data points representing 5 minutes increments from the lower bound value to the upper bound value. One data point is then selected randomly and assigned to the passenger-agent to represent the Scheduled Arrival Time (SAT) for the trip. This is similar to previous efforts where the path choice set is defined based on the target arrival time at destination by computing the maximum earliness and lateness times and then identifying a time-band within which all path alternatives are considered (Carraesi et al., 1996; Nguyen et al., 2001, Florian, 1998).

For P316, the synthesised start time of the home-based work trip is 8:20am. The minimum and maximum travel time associated with the constructed paths are 32 and 73 minutes, respectively. Possible values for P316’s scheduled arrival time range from 8:45am and 9:45am. After discretizing the range, a scheduled arrival time of 9:25am is randomly selected. This procedure is done only once and the selected schedule arrival time for each trip is fixed for the modelling exercise. For the 13 trips representing the TTS2001 record of the TTC trips from GTA-zone #1 to GTA-zone #194, the randomly selected SAT differs from one trip to another since the constructed paths are not exactly the same even though the 13 trips have the same trip start time.

For a trip with a predefined scheduled arrival time at its destination, a range of potential departure times from origin can be established. The upper bound for this range is calculated by subtracting the minimum travel time for the trip from its scheduled arrival time. The lower bound is computed by subtracting the maximum travel time for the trip from the scheduled arrival time. The range is further expanded from both ends by \( t \) minutes to avoid the risk of missing small shifts in the departure time choice due to schedule disutility. The constructed range, with 5 minute data points, represent the choice set for the departure time choice. The value of \( t \) is dependent on the acceptable deviation from the scheduled arrival time without experiencing schedule disutility. This value will be associated with each trip based on the trip purpose.

The possible departure time choices for P316’s home-based work trip range between 7:50am and 8:50am. The ranges for the other 12 trips (with same origin and destination GTA-zone codes) do not necessarily match, however there exists a great deal of overlapping.
A replica of the constructed path options is attached to each departure time value – see Figure 6.22. The content of each replica is updated when the passenger decides to leave the origin at the corresponding departure time value. To prevent unrealistic departure time and path choices, the path choice set replica is customized for each departure time value. For example, a path is removed from the replica associated with any departure time value if the in-vehicle travel time alone of this path guarantees that the expected arrival time at destination is later than the scheduled arrival time. This is usually the case when there is an alternative GO option. For P316, the path with GO connection is the fastest option. Due to the travel time associated with this path option, departure time values such as 8:45am and 8:40am are included in the departure time choice set. However, when P316 leaves at 8:50am, it is not expected that the passenger will choose the slowest path option (paths with route #501), since it is guaranteed that the passenger will arrive after the scheduled arrival time associated with the trip.

It is worth mentioning that the time-saving benefits at the origin due to the choice of a later departure time are not considered in this exercise. Faster, more costly travel choices might be equivalent to slower, less costly path choices in terms of the generalized travel cost. However, when considered within the activity-based paradigm, activity-schedule adherence could explain the preferential behaviour regarding such choices.

At this stage, a complete mental model is constructed for every passenger-agent of the 332,073 travellers, according to Figure 4.9. For every passenger-agent, the mental model represents its Q-value table. The structure of the mental model stays fixed during the assignment process, while the content of the mental model (i.e. passenger’s experience) is updated over time as passenger-agents make choices and perceive rewards. The dynamic process of repetitively making decisions and updating perceptions is modelled according to a learning process.
Figure 6.22 Sample Output for the Departure Time and Path Generation Procedure for the sample trip shown in Figure 4.7
6.5 The Generalized Cost Specification

6.5.1 GC Components

The available choices are organized in a nested-like structure. The travel cost of any choice-level depends on the immediate reward of making such a decision and the expected return from the choice’s sub-tree of alternatives. The generalized cost components are also defined at each choice-level based on the direct travel disutility and the expected generalized cost of future travel decisions. This means that the travel cost associated with sub-tree alternatives is implicitly included in the parent’s generalized cost. In this implementation, the generalized travel cost has the following components, by choice-level from top-to-bottom according to Figure 4.13:

1- Departure time, $GC(O,t)$
   a. Direct-disutility related components, $\Gamma'$
      i. Activity-schedule adherence cost, $Y_{schedule}^t$
   b. Future-return related components, $GC(T.g), \forall g \in A(T)$

2- Origin-stop, $GC(T,g)$
   a. Direct-disutility related components, $\Gamma^g$
      i. Access travel time; by walking, auto-passenger, or auto-driver, $Y_{AccessT}^g$
      ii. Access fare, by local transit (if any), $Y_{AccessF}^g$
   b. Future-return related components, $GC(S,r), \forall r \in A(S), s = g$

3- Run/Route, $GC(S,r)$
   a. Direct-disutility related components, $\Gamma'$
      i. Waiting time at stop, $X_{waiting}^r \in \{X_{waitingG}^r, X_{waitingN}^r\}$, where $X_{waitingG}^r$ represents the waiting time at an origin stop, $S = g$, and $X_{waitingN}^r$ represents the waiting at a transfer connection, $S = n$.
      b. Future-return related components, $GC(V,f), \forall f \in A(V)$ or $GC(V,des)$

4- Off-stop, $GC(V,f)$
   a. Direct-disutility related components, $\Gamma^f$
i. In-vehicle travel time to the off-stop, $X_{invehicleT}^f$, given a boarding stop $s$ and route $r$.

ii. Fare value to travel to the off-stop, $Y_{invehicleF}^f$, given a boarding stop $s$ and route $r$.

b. Future-return related components, $GC(F,n),\forall n \in A(F)$

5- On-stop, $GC(F,n)$

a. Direct-disutility related components, $\Gamma^n$

i. Transfer access cost (e.g. inconvenience due to transfer), $Y_{transfer}^n$

b. Future-return related components, $GC(S,r),\forall r \in A(S), s = n$

6- Destination-stop, $GC(V,des)$

a. Direct-disutility related components, $\Gamma^{des}$

i. In-vehicle travel time to the destination-stop, $X_{invehicleT}^{des}$, given a boarding stop $s$ and route $r$.

ii. Fare value to travel to the destination-stop, $Y_{invehicleF}^{des}$, given a boarding stop $s$ and route $r$.

iii. Egress travel time; by walking, auto-passerger, or auto-passenger, $Y_{EgressT}^{des}$

iv. Egress fare, by local transit (if any), $Y_{EgressF}^{des}$

v. Schedule delay disutility associated with a trip-purpose, $X_{SD}^{TRP}$. It is important to notice that the schedule delay is associated with departure time and path choices, not only the departure time choice.

b. No future-return related components.

All components denoted by the symbol $Y$ are considered fixed-cost components; since their values are independent of the passenger’s travel decisions and other passengers’ trip choices. These values do not change as a result of within-day or day-to-day dynamics, and hence the name fixed-cost. An example of fixed-cost components is transit fare ($Y_{AccessF}^g$, $Y_{invehicleF}^f$, $Y_{invehicleF}^{des}$, and $Y_{EgressF}^{des}$), which is considered fixed (at least on the short term). In the above specification, $Y_{schedule}^t$ is considered fixed since the current implementation is not linked to an activity-based urban planning modelling system. If such a link exists, $Y_{schedule}^t$ could vary based
on the activity-scheduling scheme. Similarly, \( Y^{des}_{EgressT} \) and \( Y^{g}_{AccessT} \) are considered fixed since there is no representation of local transit networks or road network to be able to accurately measure those components based on passengers’ decisions and the transportation network performance. \( Y^{n}_{transfer} \) is assumed to be fixed, representing inconveniences due to physical transfer structures (e.g. tunnels with long-walk such as the transfer between Yonge-University subway line and Bloor-Danforth subway line at Spadina station).

The waiting and in-vehicle times are considered variable-cost components, denoted by the symbol \( X \). These components (\( X^{waiting}_{\text{waiting}} \) and \( X^{invehicleT}_{\text{invehicleT}} \)) largely depend on passengers’ travel choices and the transportation network performance. Therefore, these values are not exogenous to the assignment process; these values are not deterministic and take different values based on the within-day and day-to-day dynamics of the transit network conditions. \( X^{waiting}_{\text{waiting}} \) does not only represent waiting time at a stop but it also works as a proxy for the effects of capacity constraints; crowding, for instance, results in longer waiting times experienced by passengers. Due to the dynamic representation of the transit service and the departure time choice dimension, such experiences are time-dependent and sensitive to within-day and day-to-day dynamics. Likewise, \( X^{invehicleT}_{\text{invehicleT}} \) is a proxy for congestion effects due to interaction between general traffic and transit vehicles. Schedule delay, \( X^{TRP}_{SD} \), depends not only on the departure time and path choices but also on transit service dynamics. A variable-cost component therefore represents a stochastic process; it is considered a random variable as its realization at time \( t \) depends on the transit network conditions and passenger travel choice. For variable components, a capital letter \( X \) represents the random variable for the underlying stochastic process, where as a small letter \( x \) denotes an instantiation of the corresponding random variable on iteration \( d \).

The generalized cost is then a random variable since it is a function of fixed-cost and variable-cost components. The learning process is concerned with the expected value of the generalized cost of each state-action pair: \( GC(S,a) \).
6.5.2 GC Evaluation – Function form and Parameters

The TTC operates various types of transit services: rail transit, light rail transit, and bus transit. GO services include train and bus operations. Each service has its characteristics and appeals differently to transit users. For instance, passengers may perceive a minute spent in a subway train differently from a minute spent in a surface route vehicle. Transit riders may be willing to walk longer distances to access a subway station. In the same way, passengers might consider a longer egress time from a rail station. The waiting time at the origin stop may be weighted differently from the waiting time at a transfer connection stop. Moreover, travellers with different socio-economic characteristics may value various travel cost components differently. For example, high-income individuals perceive the fare cost disutility differently from low-income individuals. For discretionary trips, passengers are most likely not concerned with early or late arrival (i.e. schedule delay) disutility. For school trips, on the other hand, late arrivals are highly undesirable. In this regard, the cost components for departure time and path choices are calculated based on perceived travel cost. The components (fixed and variable) are weighted to compute the generalized cost value. These weights, $\bar{\beta}$, need to be estimated (or calibrated) following the procedure outlined in section 4.4.

The generalized cost, GC, is assumed to be a function of the parameters $\bar{\beta}$, fixed-cost components $\bar{Y}$, and variable-cost components $\bar{X}$. The following is considered for the modelling exercise:

1- Passengers’ perception of direct-disutility depends on the type of the transit service. In this implementation, four groups of transit services are considered
   a. TTC rail transit service, denoted by $RT$ hereafter.
   b. TTC light rail transit service, denoted by $LRT$ hereafter
   c. TTC surface route service, denoted by $BUS$ hereafter
   d. GO bus and rail services, denoted by $GO$ hereafter

2- Similarly, four stop categories are considered based on the type of service provided at the stop. Due to the geo-coding of the transit network, almost all stops are associated with one type of service. Union station, for instance, has two representations, one for subway service and another for GO service. These stop categories are, where $s \in \{g,D\}$:
a. Transit stop with rail transit service, denoted by \( sRT \) hereafter.
b. Transit stop with light rail transit service, denoted by \( sLRT \) hereafter
c. Transit stop with surface route service, denoted by \( sBUS \) hereafter
d. Transit stop with GO bus and rail services, denoted by \( sGO \) hereafter

3- Passengers perceive schedule delays differently in relation to their trip purpose. Also, travellers usually differentiate between early arrivals (\( ESD \)) and late arrivals (\( LSD \)). For the TTC application, four trip purposes (\( TRP \)) are modelled:

   a. Home-based work trips, denoted by \( HBW \)
   b. Home-based school trips, denoted by \( HBS \)
   c. Home-based others trips, denoted by \( HBO \)
   d. Non-home-based trips, denoted by \( NHB \)

4- Acceptable deviations (\( ACD \)) from the trip’s scheduled arrival time are related to the trip purpose. It is understandable that passengers do not arrive on time at the destination; some deviation from the scheduled arrival time is usually acceptable and it depends on the trip purpose. When a passenger’s arrival time is within the acceptable deviation period, no schedule delay is perceived (i.e. \( SD \) is ignored). For this exercise:

   a. An acceptable deviation of 15 minutes is assumed for \( HBW \) trips. This is to represent some flexibility with regard to work-start times. Passengers arriving at work early or late by 15 minutes do not experience schedule delay.
   b. An acceptable deviation of only 10 minutes is assumed for \( HBS \) trips. School programs have more rigid start times than work.
   c. Passengers making \( HBO \) and \( NHB \) trips are assumed to be more concerned with en-route travel cost. Therefore, an acceptable deviation of \( \infty \) minutes is assumed; in other words, schedule delay is not computed for such trips. In this implementation, the departure time choice set is generated based on the surveyed trip start time from TTS2001. For such trip purposes, the departure time choice is related only to en-route travel cost.
   d. \( ACD^{TRP} \in \{ ACD^{HBW}(=15), ACD^{HBS}(=10), ACD^{HBO}(=\infty), ACD^{NHB}(=\infty) \} \)

5- In the case of early or late arrival (i.e. schedule delay is more than acceptable deviation), schedule delay is calculated as the difference between scheduled arrival time and actual arrival time at destination. The computed value of \( SD \), \( X_{SD}^{TRP} \), is then approximated using
a resolution similar to the departure time resolution of 5-minute increments. This allows passengers to adjust their departure time choice to offset their schedule delay, if possible.

6- Income-levels are not surveyed in the TTS2001; however, a trip-maker occupation attribute is attached to each trip record. This data could be used as a proxy for the income-level information. Travellers’ perception of monetary cost is expected to be related to their income-level or occupation-status. Five trip-maker occupation (TOC) groups were surveyed for the TTS2001 transit data:

a. General Office/Clerical, denoted by $G$

b. Manufacturing/Construction/Trades, denoted by $M$

c. Professional/Management/Technical, denoted by $P$

d. Retail Sales and Service, denoted by $S$

e. Not Employed (e.g. students), denoted by $O$

Some of the above assumptions were made due to lack of information (e.g. 15 minutes acceptable deviation for work-related trips). In the case when sufficient information is provided, these assumptions need to be revisited.

The generalized cost function is specified based on the following parameters $\tilde{\beta}$:

1- $\beta^c_{\text{AccessT}} \in \{\beta^{RT}_{\text{AccessT}}, \beta^{LRT}_{\text{AccessT}}, \beta^{BUS}_{\text{AccessT}}, \beta^{GO}_{\text{AccessT}}\}$

2- $\beta^c_{\text{waitingG}} \in \{\beta^{RT}_{\text{waitingG}}, \beta^{LRT}_{\text{waitingG}}, \beta^{BUS}_{\text{waitingG}}, \beta^{GO}_{\text{waitingG}}\}$

3- $\beta^c_{\text{waitingN}} \in \{\beta^{RT}_{\text{waitingN}}, \beta^{LRT}_{\text{waitingN}}, \beta^{BUS}_{\text{waitingN}}, \beta^{GO}_{\text{waitingN}}\}$

4- $\beta_{\text{Transfer}}$

5- $\beta^c_{\text{invehicleT}} \in \{\beta^{RT}_{\text{invehicleT}}, \beta^{LRT}_{\text{invehicleT}}, \beta^{BUS}_{\text{invehicleT}}, \beta^{GO}_{\text{invehicleT}}\}$

6- $\beta^c_{\text{EgressT}} \in \{\beta^{RT}_{\text{EgressT}}, \beta^{LRT}_{\text{EgressT}}, \beta^{BUS}_{\text{EgressT}}, \beta^{GO}_{\text{EgressT}}\}$

7- $\beta^c_{\text{ESD}} \in \{\beta^{HBW}_{\text{ESD}}, \beta^{HBS}_{\text{ESD}}\}$

8- $\beta^c_{\text{LSD}} \in \{\beta^{HBW}_{\text{LSD}}, \beta^{HBS}_{\text{LSD}}\}$

9- $\beta^c_{\text{Fare}} \in \{\beta^G_{\text{Fare}}, \beta^M_{\text{Fare}}, \beta^P_{\text{Fare}}, \beta^S_{\text{Fare}}, \beta^O_{\text{Fare}}\}$
The GC disutility function for departure time and path choices is outlined in section 4.2.2. With the aforementioned parameter specification, the following GC functions are used in this exercise:

\[ d \quad GC(O,t) \leftarrow [1 - \alpha] \cdot GC(O,t) + \alpha \cdot \left[ d \quad \Gamma^t + \gamma \cdot \min_{\forall g \in A(T)} (GC(T,g)) \right] \]

- \( \Gamma^t = \gamma^t \cdot \text{schedule} \),

- \( d \quad GC(T,g) \leftarrow [1 - \alpha] \cdot GC(T,g) + \alpha \cdot \left[ d \quad \Gamma^g + \gamma \cdot \min_{\forall r \in A(V) \cdot (\forall f)} (GC(S,r)) \right] \)

- \( d \quad \Gamma^g = \beta_{\text{AccessT}}^g \cdot \gamma^g_{\text{AccessT}} + \beta_{\text{TOC}}^{\text{TOC}} \cdot \gamma^g_{\text{AccessF}} \)

- \( d \quad GC(S,r) \leftarrow [1 - \alpha] \cdot GC(S,r) + \alpha \cdot \left[ d \quad \Gamma^r + \gamma \cdot \min_{\forall f \in A(V)} (GC(V,f)) \right] \)

- Where \( S = g \), then \( \Gamma^r = \beta_{\text{waitingG}}^r \cdot x_{\text{waiting}}^r \)

- Where \( S = n \), then \( \Gamma^r = \beta_{\text{waitingN}}^r \cdot x_{\text{waiting}}^r \)

- For \( A(V) = \{\text{des}\} \), then \( d \quad GC(V,\text{des}) \leftarrow [1 - \alpha] \cdot GC(V,\text{des}) + \alpha \cdot \left[ d \quad \Gamma^{\text{des}} + \gamma \cdot \Omega^{\text{des}} \right] \)

- \( d \quad \Gamma^{\text{des}} = \beta_{\text{invehicleT}}^{\text{des}} \cdot x_{\text{invehicleT}}^r + \beta_{\text{TOC}}^{\text{TOC}} \cdot x_{\text{invehicleF}}^r \)

- \( d \quad \Omega^{\text{des}} = \beta_{\text{EgressT}}^{\text{des}} \cdot y_{\text{EgressT}}^r + \beta_{\text{TOC}}^{\text{TOC}} \cdot y_{\text{EgressF}}^r + \beta_{\text{TRP}}^{\text{SD}} \cdot x_{\text{TRP}}^r \)

- \( \beta_{\text{SD}}^{\text{TRP}} = l_{\text{SD}} \cdot \beta_{\text{TRP}}^{\text{SD}} + (1 - l_{\text{SD}}) \cdot \beta_{\text{LSD}}^{\text{TRP}} \)

- \( l_{\text{SD}} = \begin{cases} 0 & x_{\text{TRP}}^r \geq 0 \\ 1 & x_{\text{TRP}}^r < 0 \end{cases} \)

- \( x_{\text{TRP}}^r = \begin{cases} (\text{ArrvTm} - \text{SAT})^{\text{approx}(5)} & (\text{ArrvTm} - \text{SAT})^{\text{approx}(5)} \geq ACD^{\text{TRP}} \\ 0 & (\text{ArrvTm} - \text{SAT})^{\text{approx}(5)} < ACD^{\text{TRP}} \end{cases} \)
\[ \text{For } f \in A(V), \text{ then } GC(V, f) \leftarrow (1 - \alpha) \cdot GC(V, f) + \alpha \cdot \left[ \Gamma_f + \gamma \cdot \min_{n \in A(f)} GC(F, n) \right] \]

- \( \Gamma_f \) = \beta_{\text{invehicle}T}^{r} \cdot x_{\text{invehicle}T}^{r} + \beta_{\text{Fare}}^{r} \cdot Y_{\text{invehicle}F}^{r} + \beta_{\text{Transfer}}^{r} \)

- \( GC(F, n) \leftarrow (1 - \alpha) \cdot GC(F, n) + \alpha \cdot \left[ \Gamma_{n}^{r} + \gamma \cdot \min_{\exists t \in A(S), s \neq n} GC(S, r) \right] \)

- \( \Gamma_{n}^{r} = Y_{\text{transfer}}^{n} \)

- \( GC(S, r) \) as above

The decision-making behaviour follows the mixed \{ \varepsilon - greedy, SoftMax \} action-choice model, with a \( (1 - \varepsilon) \) probability of exploitation, \( \varepsilon \) probability of exploration, and a SoftMax choice mechanism such that (Section 0):

- When exploiting, \( P_{s}(a) = \begin{cases} 1 & \text{GC}(s, a) = \min_{\forall a' \in A(s)} \text{GC}(s, a') \text{, and} \\ 0 & \text{otherwise} \end{cases} \)

- When exploring, \( P_{s}(a) = \frac{V(s, a)}{\sum_{\forall a' \in A(s)} V(s, a')} \) \( V(s, a) = 1 \cdot \text{GC}(s, a) \)

- \( S \in \{ O, T, G, V, F, N, D \} \) and \( a \in \{ t, g, r, f, n \} \) \( \exists a : s \to s' \)

Note that the action-choice model in iteration \( d \) is based on expectations of the generalized cost from previous iterations, summarized as \( GC(s, a) \). The experienced generalized cost of the state-action pair \( (S, a) \) (i.e. \( \Gamma^{a} \)) in iteration \( d \) is integrated with previous expectations, yielding \( GC(s, a) \). Hence passenger learning is modelled by the process of first making choices (i.e. based on expectations from previous experiences) and second updating perceptions.
An analogous classification of the fixed and variable travel cost components is proposed by Nuzzolo et al. (2003) in the context of modelling transit path choices using discrete choice models. However, the classification mentioned above is proposed for the modelling of information provision scenarios. In practical terms, passengers do not learn about the generalized cost as a whole term, but rather they update their perceptions about its variable components \( X \) only. For the fixed components \( Y \), once known, they are not part of the learning process. That is; 

\[
\text{\text{Access}}_{t}^{d} \text{, for instance, is equivalent to } Y_{t}^{d} \text{ and } Y_{t}^{d+1}.
\]

Based on the above specification, passengers can form their expectation about the generalized cost of boarding route/run \( r \) from a boarding stop \( s \): \( GC(S,r) \). The passenger’s expected waiting time, \( X_{\text{waiting}} \), for the same state-action pair cannot be computed; however, an instantiation of the waiting time \( x_{\text{waiting}} \) is experienced by the passenger. Automated Traveller Information Systems (ATIS), on the other hand, provide waiting time for run \( r \) at a boarding stop \( s \); this is not directly comparable with \( GC(S,r) \). To allow passengers to form their expectations with regard to variable components while using the generalized cost as the evaluation criteria, the \( GC \) function is formulated as follows for a state-action pair \( (S,a) \):

\[
\begin{align*}
\overline{d}^{d} \quad GC(S,a) & \leftarrow \text{fixed immediate cost} + \text{expected immediate cost (i.e. variable component)} + \\
& \text{expected future-return} \\
\overline{d}^{d} \quad GC(S,a) & \leftarrow \sum \beta_{i} X_{i} + \sum \beta_{j} X_{j} + \gamma \cdot \min_{a' \in \mathcal{A}(S')} \bigl\langle GC(S',a') \bigr\rangle, \text{ where perception update for } X \text{ follows} \\
\overline{d}^{d} \quad X & \leftarrow (1 - \alpha) \cdot X + \alpha \cdot x
\end{align*}
\]

Therefore, the \( GC \) representation for the transit path choice model becomes:

\[
\overline{d}^{d} \quad - GC(O,t) \leftarrow \Gamma' + \gamma \cdot \min_{g \in \mathcal{A}(T)} \bigl\langle GC(T,g) \bigr\rangle
\]
\[ \Gamma^i = \gamma^i \text{ schedule}, \]

\[ \frac{d}{d} \Gamma^g = \gamma \left( \gamma^g + \text{MIN}_{\gamma \in \Lambda(S), \gamma < \gamma^g} \langle GC(S, r) \rangle \right) \]

- \[ \frac{d}{d} \Gamma^g = \beta^g_{\text{Access}} \cdot \gamma^g_{\text{Access}} + \beta_{\text{TOC}} \cdot \gamma^g_{\text{AccessF}} \]

\[ \frac{d}{d} \Gamma^r = \gamma \left( \gamma^r + \text{MIN}_{\gamma \in \Lambda(V)} \langle GC(V, f) \rangle \right) \]

- Where \( S = g \), then \( \Gamma^r = \beta^r_{\text{waitingG}} \cdot X^r_{\text{waiting}} \)

- Where \( S = n \), then \( \Gamma^r = \beta^r_{\text{waitingN}} \cdot X^r_{\text{waiting}} \)

and \( X^r_{\text{waiting}} \leftarrow (1 - \alpha) \cdot X^r_{\text{waiting}} + \alpha \cdot x^r_{\text{waiting}} \)

- For \( A(V) = \{ \text{des} \} \), then \( \frac{d}{d} \Gamma_{\text{des}} = \gamma \left( \gamma_{\text{des}} + \gamma \cdot \Omega_{\text{des}} \right) \)

\[ \frac{d}{d} \Gamma_{\text{des}} = \beta^k_{\text{TOC}} \cdot \gamma_{\text{des}}^k_{\text{invehicleF}} + \beta^k_{\text{des}} \cdot X^k_{\text{invehicleF}} \]

\[ \frac{d}{d} \Gamma_{\text{des}} = \beta^k_{\text{invehicleT}} \cdot \gamma_{\text{des}}^k_{\text{invehicleT}} \]

\[ \frac{d}{d} \Gamma_{\text{des}} = (1 - \alpha) \cdot X^r_{\text{invehicleT}} + \alpha \cdot x^r_{\text{invehicleT}} \]

\[ \Omega^k_{\text{des}} = \beta^k_{\text{EgressT}} \cdot \gamma^k_{\text{EgressT}}_{\text{des}} + \beta^k_{\text{TOC}} \cdot \gamma^k_{\text{des}}_{\text{EgressF}} + \beta^k_{\text{TRP}} \cdot \left( X^k_{\text{TRP}} \right) \]

\[ \beta^k_{\text{SD}} \cdot \gamma^k_{\text{ESD}}_{\text{des}} + (1 - \beta^k_{\text{SD}}) \cdot \beta^k_{\text{ESD}} \]

\[ I_{\text{SD}} = \begin{cases} 0 & X^d_{\text{SD}} \geq 0 \\ 1 & X^d_{\text{SD}} < 0 \end{cases} \]

where \( X^d_{\text{SD}} \leftarrow (1 - \alpha) \cdot X^d_{\text{SD}} + \alpha \cdot X^d_{\text{SD}} \), and

\[ \frac{d}{d} X^d_{\text{SD}} = \begin{cases} (\text{ArrvTm} - \text{SAT})_{\text{approx}}^{(5)} & (\text{ArrvTm} - \text{SAT})_{\text{approx}}^{(5)} \geq ACD^\text{TRP} \\ 0 & (\text{ArrvTm} - \text{SAT})_{\text{approx}}^{(5)} < ACD^\text{TRP} \end{cases} \]
• For $f \in A(V)$, then
  \[
  \frac{d}{\Gamma f} \left( GC(V, f) \right) \leftarrow \frac{d}{\gamma} \frac{\min}{\forall n \in A(f)} \left( GC(F, n) \right)
  \]

  \[
  \frac{d}{\Gamma f} = \beta_{\text{Fare}} \cdot Y^r_{\text{invehicleT}} + \beta_{\text{Transfer}}^r \cdot x^r_{\text{invehicleT}}
  \]

  and

  \[
  \frac{d}{X}_{\text{invehicleT}} \leftarrow (1 - \alpha) \cdot \frac{d}{X^r_{\text{invehicleT}}} + \alpha \cdot x^r_{\text{invehicleT}}
  \]

  \[
  \frac{d}{\Gamma^n} = Y^n_{\text{transfer}}
  \]

  \[
  GC(S, r) \text{ as above}
  \]

With this representation, passengers learn about the variable components (e.g. waiting and in-vehicle times) and are able to construct the generalized cost of any state-action pair using their knowledge regarding their fixed cost components.

\[\frac{d}{X}\] approximates the expected value of the variable component; as $d \rightarrow \infty$, \[\frac{d}{X}\] converges to the true average value of $X$. Other descriptive measures could be included in this representation for the estimation of random variable $X$; for example, the standard deviation \[\frac{d}{\sigma_X}\] can be updated with subsequent experiences of $X$ and then used to provide an expectation of $X$ in iteration $d$.

Implementing the aforementioned representation enables the consideration of information provision at downstream locations and pre-trip information provision in the decision-making process. Passengers construct the generalized cost of reaching the terminal state (i.e. destination) given the current state-action pair based on their expectations. By GC-construction, passengers’ perceptions of a state-action pair are computed using most recent experiences and real-time information on service conditions.
Without such representation, \( GC(S,a) \) is dependent on expected values of future states based on previous experience,

\[
GC(S,a) \leftarrow (1 - \alpha) \cdot GC(S,a) + \alpha \cdot \left[ \sum_{a' \in A(S')} \gamma \cdot MIN\left( GC(S',a') \right) \right].
\]

Information provided about future states are not utilized in the decision-making process using this representation; however, this information will be used to update the perception regarding future states Q-values.

For the \textit{SWITCH AND WAIT} routine, for instance, \( MIN\left( \sum_{V \in A(V)} GC(V,f) \right) \) could be accessible through online repository of real-time service conditions available to public or through text messaging as a business transaction. Without such information provision capability, real-time downstream service conditions are not available to passengers; however, passengers form their expectations. In the case of traffic accidents, passengers are expected to utilize such information if provided. This representation therefore is suitable for the modelling of transit service within-day dynamics and evaluating the potential of information provision in atypical conditions.

### 6.5.3 Fare Calculations

In 2001, there was no integrated fare system in the Greater Toronto Area (GTA). Passengers transferring between different transit systems are required to pay for the usage of each transit service.

For the TTC service, a flat-fare structure is implemented; the fare is not based on the distance travelled nor the zones crossed. However, extra fare is required for using the downtown express service. The GO system implements a distance-based fare structure. Local transit services provide reduced-fare value for accessing nearby GO and TTC subway stations.

In this application, all possible fare combinations are considered and calculated accordingly. For example, if a passenger uses the GO system, the fare for this segment is calculated based on the
boarding and alighting stops. When the passenger transfers to the TTC system, a TTC fare is added. If the passenger requires a transfer using the (non-express) TTC service to reach destination, no extra monetary cost is added. However, if the passenger transferred to the downtown express service, an extra fare is added. If the trip starts by boarding an express service route, the TTC base fare and the express service extra fare are combined. When using local transit service, local fare values are applied.

6.6 Simulation-, Learning-based Assignment Environment

The use of simulation-based experiments for the analysis of dynamic departure time and path choices is becoming more popular for at least 3 reasons. First, the complexity and (possibly) nonlinearity (i.e. stochasticity and dynamics) of the problem in question necessitate the deployment of simulation-based frameworks. Second, it is difficult to use survey data and, at the same time, control for experimental factors; the simulated environment becomes the only possible way to systematically examine these effects while controlling others to avoid confounding impacts. Third, it is not easy to observe user responses at the desired temporal resolution. Further, the observed evolutionary path (in the real-world) is only one possible sample from a set of possible stochastic realizations.

The simulation environment used to model the assignment process is depicted in Figure 6.23. At the start of iteration $d$, all passengers are at state origin, associated with a zone. Based on previous experiences, and without information provision, passengers decide on a departure time from origin and an origin stop. When it is time to leave the origin, passengers are loaded to the chosen origin stop based on the access travel time value. Passengers waiting at a stop have an initial route choice. When this route arrives, the SWITCH AND WAIT routine is invoked. If another attractive route arrives, then the SWITCH AND BOARD routine is called – see Figure 4.11. Due to the microscopic representation of the transit service, capacity constraints are considered, affecting passenger experience at stops.
Figure 6.23 Macroscopic structure of the simulation environment for the learning-based assignment procedure
For on-board passenger, possible transfer connections are considered using the SWITCH AND ALIGHT and SWITCH AND STAY routines. Upon arrival at destination, schedule delay is computed. During the trip, the content of the mental model is dynamically updated (i.e. perception updating) and utilized in the decision making process.

In this implementation, HBW and HBS trips are considered to be recurrent, while HBO and NHB are modelled as non-recurrent trips. This affects the rate of perception update for each passenger based on the trip purpose. For passengers making HBW or HBS trips, the mental model contents (or passenger’s perceptions) are updated every iteration \(d\). On the other hand, for non-recurrent trips, a trip-frequency, \(frq\), attribute is added. Passengers making HBO and NHB trips are not expected to have the same level of knowledge of the transit service conditions as regular travellers. This is reflected in the contents of their mental models – the mental model contents are not updated regularly (i.e. every iteration) but rather based on the trip-frequency attribute (assumed as \(frq \sim U(5,15)\)). Non-frequent travellers decide on their travel options to the best of their knowledge of the transit service conditions; this knowledge however may not represent the actual transit service conditions. It is worth noting that non-frequent travellers still behave rationally, but their expectations are deluded. Information provision could provide non-frequent travellers with valuable guidance during their trip.

In each iteration, recurrent and non-recurrent trips are modelled. Frequent travellers (94%, representing HBW and HBS trips) make choices and update perceptions every iteration. Non-frequent travellers (6%, representing HBO and NHB trips), on the other hand, decide on their travel choices based on their recently mental model contents, and only update their perception every \(frq\) iterations. It is important to highlight that each iteration represents 100% of the demand; this is crucial for modelling capacity effects.

An arbitrary learning rate of 0.70 is considered in this exercise (\(\alpha = 0.70\)). This means that passengers weight recent experiences more than older ones.\(^\text{13}\) Since the transit trip represents an

\(^{13}\) A sensitivity analysis could be performed later to investigate the impact of the learning speed on the assignment process outputs.
episodic task (i.e. it terminates), the discount factor \( \gamma \) (equation (4.3)) is set to 1; passengers value future travel cost similar to current travel cost. The travel cost, however, is a weighted-average of the future travel disutility based on \( \bar{\beta} \). The rate of exploration \( \epsilon \), as in Figure 4.10, is updated every iteration \( d \) to reflect the fact that passengers \textit{explore less as they learn more}. A learning period is assumed (90 iterations); during this period, passengers update their mental model components while exploring various options. As \( d \) increases, the rate of exploration decreases linearly such that at \( d = 90, \epsilon = 6\% \) and this value is fixed as \( d > 90, \epsilon = 6\% \) is chosen based on the percentage of occasional travellers (non-recurrent trips) to represent the level of exploration taking place in one iteration.

Before the assignment process starts, all fixed-component values are calculated for each passenger. At iteration \( d = 1 \), passengers are assumed to have no information, except average travel times between stops. This information can be obtained through transit service on-line scheduling system. Waiting times are initialized with zero values; waiting times are dependent on headways, other passengers’ choices and capacity constraints. Over time, passengers build their expectations of waiting times and update their expectations of in-vehicle times based on their trip choices and other passengers’ travel behaviour.

The mesoscopic supply model considers the variability in transit vehicle’s speed to reflect various congestion levels and operating conditions. This is represented by a percentage of variability in the average-speed of each route/branch. There were no available data on the travel speed/time distribution for each route/branch. There were only average-speed data available from the TTC service summary report (TTC, 2001) and GO service data. A 3\% variability in average-speeds is assumed for RT routes with dedicated exclusive Right-Of-Way (e.g. subway lines and GO-Rail routes). The variability for surface routes is assumed to be 3 times of ROW services (10\%) representing the high congestion levels in the GTA network during the AM peak period.

While the AM peak period is considered from 6:00am to 9:00am (as in TTS2001), the simulation horizon needs to be expanded beyond 9:00am. AM peak transit trips, as defined by TTS2001, include trips that have a \textit{start time} up to 9:00am; consequently, passengers leaving their origin close to 9:00am may arrive at their destination after 9:00am. The latest scheduled arrival time is
11:50am, for a trip originating in downtown Toronto (zone 390) and destined to Region Simcoe/Orillia (zone 3792) with a TTS2001 trip start time as 8:50am. As the learning process takes place en-route and post-trip, the simulation period per iteration is extended till the arrival of passengers at destinations. The simulation also starts at 5:30am, providing a 30-minute warm-up period.

The learning process is considered to have converged when the passengers’ travel behaviour (i.e. trip choices) remains unchanged as $d$ increases (after the learning period). In an explorative environment, the conventional equilibrium state cannot be reached as exploration of new choices is part of the learning process and hence passengers may unilaterally change their choices. However, a stationary state is achievable where passenger-agents have learned an optimal policy $\pi^*$ that guides their choices based on the state they find themselves in.

In this exercise, convergence is achieved when only a percentage of less than 6% of the modelled passengers change their trip choices from iteration to the next (equivalent to the occasional travellers percentage). This means that passenger-agents have found an optimal policy $\pi^*$ and that by following it, they are maximizing their return. This also means that the Q-value table (or the mental model content) for each passenger-agent has converged. To avoid broken ergodicity (see section 4.3.3), convergence of the learning process is declared only if the previous condition is met for a number of iterations (assumed 7 in this implementation).

Upon convergence of the learning process, macro-level outputs can be extracted based on micro-level behaviour of passenger-agents. For example, route-loads are endogenous to the assignment process; they are constructed based on individual choices. Route loading-profiles (i.e. run-loads) can also be obtained due the time-dependent path choice algorithm. In a purely microsimulation environment (where in-vehicle travel time is determined based on traffic conditions), route’s schedule- and headway-adherence can be computed and the efficiency of the transit service can be measured. Basically, the model has all the micro-level information which can be aggregated, using various factors, to generate macro-level measures of effectiveness and travel patterns. Prior to policy analysis, the model parameters need to be estimated (or calibrated).
6.7 Model Parameter Estimation and Calibration

The model with its parameters unspecified represents a general framework to model the assignment process for passenger demand. When applying the model to a specific transit service, the model parameters need to be calibrated based on the observable conditions of the transportation network and patterns of travel demand for the system under investigation.

At stated in section 4.4, the steady state transition probabilities are a function of the value function parameters, \(\Pr^{ij} = g(\tilde{\beta})\). In other words, the travel choice probabilities \(P_s(a)\) are a function of the generalized cost parameters \(\tilde{\beta}\). The objective of the parameter calibration procedure is to find \(\tilde{\beta}\) that minimizes a misfit function \(D|_{\tilde{\beta}}\), where \(D = f(\text{observed}, \text{simulated} | \tilde{\beta})\) - see Figure 4.14.

There are two types of observed data that can be used in the calibration process: system-wide, aggregate observations (e.g. route-loads) or passenger-specific disaggregate behaviour (e.g. trip choices). In either case, the calibration process should produce parameter values such that the travel behaviour (disaggregate choices) or travel demand (route loads) predicted by the model conform, to the greatest extent possible, to travel behaviour or travel demand as observed in reality. The goodness of the calibrated values is measured by the ability to maximize the likelihood of regenerating \([\Pr^{ij}]^*\) as \([\Pr^{ij}]^{\text{observed}*}\) or maximize the entropy of \(L\) (associated with \([\Pr^{ij}]^*\)) to reproduce \(L^{\text{observed}*}\).

The TTS2001 disaggregate data provides the individual choices to a certain level of detail. The data represents only 5% of the TTC users in the AM peak period (19,650 records), whereas the modelled demand represents a 100% of the TTC users (332,073 passengers). Moreover, stop choice (origin stop, transfer connection, destination stop) is only recorded for trips using the TTC subway system and the GO service; boarding and alighting stop choices are not recorded for trips (or trip segments) using surface transit service. The percentage of TTC trips with no subway or
GO connection is about 65%, meaning that the stop choice is not recorded for about 35% of observed individual travel behaviour. The other 35% of the recorded trips include trip-segments with no subway or GO connections. Due to privacy concerns, individuals’ trip choices were not given with a linkage to the trip-purpose or the individual’s job type (e.g. Professional, Retail service, etc.). As a result, the relationship among travel choices, trip purpose and individual’s income level cannot be established based on the observed choices. For example, a trip record that shows that the passenger chooses a fast, reliable, and expensive service (e.g. GO service) does not provide an explanation of whether this is because of the trip purpose or the passenger’s income level/job occupancy (or both), especially when a cheaper, slower alternative exists (e.g. TTC service).

In order to use disaggregate choices for parameter calibration, at most 3.25% of the estimated individual choices can be compared to observed data (after establishing one-to-one linkage between observed records and passenger-agents using origins and destinations geo-locations provided by the TTS2001 disaggregate data). Therefore, the goodness of the calibrated parameters is not measured by the ability to maximize the likelihood of regenerating individual choices. However, the 5% sample of choices will be used to validate the calibrated parameters.

On the other hand, the TTS2001 validation report (Joint Program in Transportation 2003a) provides aggregate route-loads (more specifically, route boardings) for the TTC service in the year 2001 for the AM peak period (6:00-9:00am). This aggregate data represents a 100% level of demand for TTC users in the AM peak period. Passengers’ transfer choices are also reflected in aggregate route loads. Therefore, the goodness of the calibrated parameters is relative to the likelihood of regenerating aggregate-route loads from individual choices similar to observed route loads.

To judge the conformity of predicted travel behaviour or demands, different misfit functions can be used to measure the goodness of the calibrated parameters with reference to aggregate route-loads. $D|\hat{\beta}$ can be assumed as
- The Global Relative Error (GRE) between the observed route loads and simulated route loads, 
\[ D_{\beta} = \frac{\sum_{i=1}^{n} \left| L_{i}^{\text{obs}} - L_{i}^{\text{simulated}}(\hat{\beta}) \right|}{\sum_{i=1}^{n} L_{i}^{\text{obs}}} \], where \( L_{i}^{\text{obs}} \) represents the observed load, \( L_{i}^{\text{simulated}}(\hat{\beta}) \) is the simulated route load (after convergence) for route \( i \), and \( n \) is the number of routes.

- The Point Mean Relative Error (PMRE) between the observed and simulated route loads, 
\[ D_{\beta} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left( \frac{L_{i}^{\text{obs}} - L_{i}^{\text{simulated}}(\hat{\beta})}{L_{i}^{\text{obs}}} \right)^{2} \], where \( L_{i}^{\text{obs}} \) represents the observed load, \( L_{i}^{\text{simulated}}(\hat{\beta}) \) is the simulated route load (after convergence) for route \( i \), and \( n \) is the number of routes.

Since route loads are for the AM peak period (6:00am-9:00am) while TTS2001 TTC AM peak period demand represents trips starting between 6:00am and 9:00am, both GRE and PMRE misfit functions cannot be used as they compare the simulated value with the “exact” value of the observed data. TTC observed route loads may include trips that start before 6:00am and the simulated route loads may include trips ending after 9:00am. Instead, the parameters are calibrated such that the entropy of the simulated route loads is optimized with reference to the observed route loads.

Route loads are random variables; the observed route loads represent a realization of an underlying distribution. We are interested in reproducing the underlying distribution more than regenerating one instance of that distribution. Meanwhile, we only have partial information with regard to the real distribution of route loads. The concept of Maximum Entropy (MaxEnt) Estimation, or Information Theory, provides a procedure to build probability distributions on the basis of partial knowledge (Jaynes, 1957). The estimated values are the least biased parameter estimates based on the given information (i.e. observed counts) that maximize the entropy of the constructed distribution. By maximizing the entropy of a distribution, the estimated distribution becomes noncommittal with regards to missing information and most resembles some reference distribution.
The terms *entropy* and *uncertainty*, from the perspective of probabilities, are used more or less interchangeably. For event, denoted by $A$ with probability $P_A$, the term $\log \left( \frac{1}{P_A} \right)$ is called the “surprise” or “uncertainty” of observing event $A$ (Tribus, 1961). When $P_A = 1$, there is no uncertainty (or surprise) with the data, as event $A$ will always occur. When $P_A$ takes a very small value, then it comes as a surprise to observe event $A$. When $P_A = 0$, the uncertainty of event $A$ is undefined and its surprise term is set to zero; that is, event $A$ will never surprise us.

The entropy, $E$, or uncertainty of a distribution is defined as a weighted average of surprises, as

$$E = \sum_{i=1}^{k} P_i \cdot \log \left( \frac{1}{P_i} \right).$$

The concept of MaxEnt has been applied in urban and regional models to estimation/calibrate model parameters (Wilson, 1970). The goodness of the calibrated values is assessed on the basis of the comparison of the entropy of the predicted values to that of the observed values. That is, the term $|E_{L(\hat{\beta})} - E_{L_{\text{obs}}}|$ needs to be minimized, where $E_{L_{\text{obs}}} = \sum_{i=1}^{n} P_{L_{\text{obs}}} \cdot \log \left( \frac{1}{P_{L_{\text{obs}}}} \right)$ and

$$P_{L_{\text{obs}}} = \frac{L_{i_{\text{obs}}}}{\sum_{j=1}^{n} L_{j_{\text{obs}}}}.$$ Similarly, $E_{L(\hat{\beta})} = \sum_{i=1}^{n} P_{L_{i}(\hat{\beta})} \cdot \log \left( \frac{1}{P_{L_{i}(\hat{\beta})}} \right)$ and $P_{L_{i}(\hat{\beta})} = \frac{L_{i}(\hat{\beta})}{\sum_{j=1}^{n} L_{j}(\hat{\beta})}$. $E_{L_{\text{obs}}}$ is a fixed value calculated based on the observed TTC route loads for the AM peak period ($E_{L_{\text{obs}}} = 5.17$); it represents the amount of uncertainty of the underlying distribution of observed route loads.

Due to the dynamic representation of the transit service and capacity constraints, a set of $\hat{\beta}$ that favours surface routes over subway lines will result in significant delays for passengers (large schedule delay values). However, the entropy of the simulated loads might not be as bad, since the entropy calculation is based on the distribution of route-loads and not their absolute values. In other words, the entropy formula does not control for the total simulated route loads to be
similar to the total observed loads. For example, it might be that \( \sum_{j=1}^{n} L_{j}^{obs} = 1000 \) and \( \sum_{j=1}^{n} L_{j}(\hat{\beta}) = 100 \), but \( |E_{L_{i}(\hat{\beta})} - E_{L_{i,\text{obs}}}| \) might take a value of zero if \( P_{L_{i}}^{\text{obs}} = P_{L_{i}}(\hat{\beta}) \), even given that about 90% of the passengers experienced significant delay and could not board their chosen routes.

To overcome this drawback, a proxy to the schedule delay is considered for the assessment of the calibrated values. This is represented by the percentage of passengers that arrive at their scheduled arrival time. For the abovementioned situation, the goodness of the calibrated parameter values will be affected by the percentage of passengers not arriving on time. Therefore, the misfit (or goodness) function for evaluating the calibrated set of parameters \( \hat{\beta} \) is:

\[
D_{\hat{\beta}} = \frac{E_{L_{i}(\hat{\beta})} - E_{L_{i,\text{obs}}} \bigg)}{E_{L_{i,\text{obs}}} + \left[1 - \frac{\text{ArrivalsOnTime}}{\text{TotalDemand}} \right]} + \left(1 - \frac{\text{ArrivalsOnTime}}{\text{TotalDemand}} \right)
\]

6.7.1 Estimation Procedure

The estimation procedure employs Genetic Algorithms (GAs), which find the set of (optimal) parameters, out of possible parameter populations, that will minimize the misfit function \( D_{\hat{\beta}} \). The GA process is based on a cyclic iterative process of generating genes and natural evolution. A gene represents a parameter \( \hat{\beta}_i \), and a collection of genes is a chromosome, \( \hat{\beta} \). To calculate the fitness of one chromosome, \( \hat{\beta} \), the misfit function \( D_{\hat{\beta}} \) needs to be computed by running the simulation-\, learning-based model until the convergence of the passenger-agents’ learning process is achieved. A Parallel Genetic Algorithms engine called GenoTrans, developed at the University of Toronto (Mohamed, 2007) is deployed for the parameter calibration process. GenoTrans uses cluster computing over a network of workstations to speed up the genetic algorithm convergence process. It takes 4.5 minutes to run one iteration of the assignment process for the AM peak period on a Pentium IV 2.0GH workstation. Considering the learning
period (90 iterations), it takes about 9 hours to evaluate one chromosome using one workstation. It is conventional that the number of chromosomes in each generation is proportional to the number of genes (or parameters in this case). For a 30-gene chromosome, the generation size (or the number of chromosomes in one generation) should be at least 30. This means that the assessment of one generation will take about 12 days on one workstation or 9 hours on a cluster of 30 computers. Therefore, the parameter calibration process will greatly benefit from the parallel computing feature of GenoTrans. To reach the optimal set of parameters, the iterative evolution process will require the assessment of many generations. For only 10 generations, it takes about 120 days on one computer or 4 days using GenoTrans.

For a set of parameters \( \tilde{\beta} \) with \( n \) parameters, the estimation process is concerned with only \( n-1 \) parameters with the \( n^{th} \) parameter being fixed. In this application, the \( \beta_{BUS}^{imvehicleT} \) is fixed to a value of 1 in all chromosomes.

### 6.7.2 Model Specification

The model outlined in section 6.5 has 30 parameters. The fare parameters, 
\[ \beta_{Fare}^{T{OC}} \in \{ \beta_{Fare}^{G}, \beta_{Fare}^{M}, \beta_{Fare}^{p}, \beta_{Fare}^{S}, \beta_{Fare}^{O} \}, \]
cannot be estimated due to the lack of information on the job occupation. Therefore, this sub-set of parameters is replaced by one parameter, \( \beta_{Fare} \). Because there is no information regarding the arrival times at destination, the early \( (\beta_{ESD}^{TRP} \in \{ \beta_{ESD}^{HW}, \beta_{ESD}^{HS} \}) \) and late \( (\beta_{LSD}^{TRP} \in \{ \beta_{LSD}^{HW}, \beta_{LSD}^{HS} \}) \) parameters are replaced by one schedule delay parameter for each trip purpose \( \beta_{SD}^{TRP} \in \{ \beta_{SD}^{HW}, \beta_{SD}^{HS} \} \). Due to the limited number of workstations connected to the GenoTrans cluster at present, there is a need to reduce the number of genes per chromosome to manage the calibration process within the resources available. This reduction process is based on the following assumptions:

1- The access and egress travel time parameters are assumed to be the same for each transit service.

2- The surface route services are combined (denoted by \( SUR \)). This means that all parameters that represent LRT and BUS services separately are replaced by one parameter to represent surface route.
3- The GO and subway services are combined as well (denoted by \( RT \)).

4- The waiting time parameters are considered equivalent for origin waiting times and transfer waiting times.

This results in 9 parameters, \( \tilde{\beta} \in \{ \beta_{RT}^{RT}, \beta_{SUR}, \beta_{RT}^{waiting}, \beta_{SUR}^{waiting}, \beta_{Transfer}, \beta_{invehicleT}^{RT}, \beta_{SD}^{HBW}, \beta_{SD}^{HBS}, \beta_{Fare} \} \), to be calibrated and \( \beta_{invehicleT}^{SUR} \) is fixed as 1. Currently, there is only one computer connected to the GenoTrans Genetic Algorithm engine. This restricts the number of chromosomes in each generation and the number of iterations to achieve optimality. In this implementation, the generation-size is set to 20 (number of \( \tilde{\beta} \) in each iteration). The evaluation process for one generation takes 180 hours (or 7.5 days).

6.8 Results and Discussion

6.8.1 Parameter Calibration Results

The output of the genetic-based optimization process is a set of parameters that minimizes the pre-specified misfit function. The MILATRAS framework, integrated with the GenoTrans engine, outputs the calibrated parameters of the transit assignment process. The solution is a set of values for the 9 specified parameters (Section 6.7.2).

There are many decisions to be made when designing a problem-specific Genetic Algorithm. Although there are many versions of GAs, all of them have two basic steps: selection and replacement. The selection process determines how the parents are selected for producing offspring. The replacement process determines how offspring will be inserted into the new generation. GenoTrans provides the option to choose from a list of various GA implementations reflecting different mechanisms for selection and replacement procedures. The algorithm used in this application implements a real-variable representation of chromosomes, using “RealBlend” and “RealGaussian” algorithms (Mohamed, 2007) for crossover (with probability of 90%) and mutation (with probability of 5%), respectively.
Figure 6.24 shows the misfit function value for various sets of parameters (or chromosomes $\tilde{\beta}$) over a number of generations. In concept, the misfit value (Section 6.7) is a function of $\tilde{\beta}$; however there is no closed-form equation to represent this relationship. The optimization-by-simulation technique applied in the calibration process enabled such relationship to be studied and optimized. Since it is a nine-dimensional problem and the calibration process searches for the optimal values randomly in the feasible solution space, it is possible to obtain from the final population several chromosomes (or solutions $\tilde{\beta}$) that posses the same or similar fitness value. One can identify the optimal set of parameters from Figure 6.24; it is the set with the least misfit function depicted from the graph.

The genetic-based optimization procedure is based on evolution and survival of the fittest; this is clearly shown in Figure 6.25, where fitness values are plotted in an ascending order per generation. The first generation represents 20 randomly chosen feasible solutions, while the second generation is constructed using methods of crossover and mutation to produce a better generation with regard to average fitness value. This process is repeated for generations 3 and afterwards, shown by a steady decrease in the average misfit value as the number of generations increases. Ultimately, the average fitness of a population will approach the optimal fitness value (e.g. zero) as the number of generations goes to infinity. While it is not practical to run the genetic-optimization procedure indefinitely for obvious reasons, a threshold value for the generation average fitness value is usually specified as a stopping criterion to bring the optimization process to an end. In this application, the optimization process is aborted after generation $n$ if the average fitness (or misfit) of the chromosomes in generation $n$ is below $\varphi = 0.1$. This was achieved after 6 generations (each with 20 chromosomes). Due to technical issues with running GenoTrans on a cluster of computers, MILATRAS used the genetic-based optimization tool on only one workstation. It took a little over 45 days for the genetic algorithm to converge based on the pre-defined stopping criterion ($\varphi$). It is important to note that with 20 workstations, the calibration process would have taken less than 2.5 days – the addition of more workstations will not reduce the calibration time but will improve the search process by increasing the population size to more than 20 chromosomes; the calibration running time is restricted by the computational time needed for one assignment process to be carried out.
The (unconstrained) genetic-based optimization procedure is problem-independent; that is, it does not impose explicit restrictions on the calibrated values of the parameters except the feasibility requirement. This means that the relationships between the parameter values are not defined \textit{apriori}, but rather these relationships are constructed from the optimal solution. Then, these relationships are compared with prior expectations and common knowledge for validation purposes. In this application the feasibility requirement is represented by the non-negativity constraints for all parameters. This is assumed with the understanding that all GC components (defined in Section 6.5.1) will \textit{add} to the disutility or the inconvenience of the transit trip in terms of time or monetary costs.

Using a microsimulation-based assignment procedure that considers explicit congestion effects and capacity constraints (Sections 6.3.2 and 6.6), the relatively poor levels of service and limited capacity of road surface transit services ensure that parameter values that favour road service over rapid transit result in low fitness value (or high misfit value). This can be clearly shown in Figure 6.26, where the parameter values for the best and worst chromosomes in the final generation of the genetic algorithm evolution (or \( \hat{\beta} \)) are plotted. When surface transit service is preferred (e.g. less disutility for access/egress, waiting time and in-vehicle time compared to rail transit disutility), the percentage of passengers arriving on time reaches its lowest value and the simulated route loads show significant discrepancy with reference to the observed loads (based on the entropy calculations). On the other hand, the set of parameter values with the best fitness (or least misfit value) demonstrates expected relationships between parameter values with regard to preference of rapid transit services compared to surface road services. The microsimulation assignment environment, with less structural constraints, has shown that the parameter calibration procedure (outlined in Section 4.4) can integrate supply and demand interactions in the estimation/calibration process, which is critical for congested networks.
Figure 6.24 Misfit function values for various sets of parameters
Figure 6.25 Evolution of the parameter calibration process using Genetic Algorithm
Figure 6.26 Parameter values for Best/Worst Fitness Values
The following represents the author’s expectations regarding the relationships between the values of the calibrated parameters, sometimes with reference to the parameter of in-vehicle travel time for surface road transit services, $\beta_{invehicle}^{SUR}$ (which is set to 1):

1. $\beta_{RT}^{RT} < \beta_{RT}^{SUR}$, $\beta_{waiting}^{RT} < \beta_{waiting}^{SUR}$, $\beta_{invehicle}^{RT} < \beta_{invehicle}^{SUR}$ reflecting the less disutility or extra convenience usually perceived by transit riders when using rapid transit services compared to surface road services due to high levels of uncertainty and unreliability commonly identified with road operations specially in congested traffic situations.

2. $\beta_{invehicle}^{RT} < \beta_{waiting}^{RT}$ based on the well-established observations that travellers value a minute spent waiting more than a minute on-board. In extreme weather conditions and unsafe areas, this relationship might be even more pronounced.

The output from the calibration process is shown in Figure 6.27. The calibrated values suggest that passengers perceive a minute spent in a rail transit system (e.g. subway) less than a minute spent in a surface road service (e.g. bus or streetcar), $\beta_{invehicle}^{RT} (0.8) < \beta_{invehicle}^{SUR} (1.0)$. The waiting time disutility is higher than the in-vehicle time disutility, $\beta_{waiting}^{RT} (1.4) > \beta_{invehicle}^{RT} (0.8)$, $\beta_{waiting}^{SUR} (5.0) > \beta_{invehicle}^{SUR} (1.0)$. The travel time components related to rail transit systems are valued less in comparison to travel time components associated with surface transit service, e.g. $\beta_{waiting}^{RT} < \beta_{waiting}^{SUR}$; this is in line with our expectation and common findings for transit assignment procedures.

The relationship between the perceived value of access/egress times for surface transit services and rail transit services is interesting. The calibrated numbers suggest a factor of about 8 in favour of rail transit. While this is higher than traditionally assumed values, it presents a unique estimate and should not be directly compared with previous research since the stop-choice is explicitly modelled and stop-locations, origins and destinations are geo-coded. Also, this high factor could be attributed to the fact that rail transit systems are sometimes accessed using auto (as a driver or passenger) and there might be an indirect utility for such convenience related to using the car. However, $\beta_{RT}^{RT} < \beta_{invehicle}^{RT}$, which suggests that passengers have a higher disutility for a minute spent travelling in rail transit more than a minute spent to access rail transit. This might seem contradicting with common expectations; it could also be interpreted in light of
future reward related to using rail transit services since the stop-choice is the first decision to access and use rail transit. Similarly, the high disutility of accessing surface road transit compared to accessing rail rapid transit could be attributed to expected future high travel cost associated with surface road transit services.

The ratio of a minute spent waiting for rail transit and a minute spent travelling using rail transit service is 1.47; for surface road transit, the ratio is about 5. This reflects on passengers’ perceptions regarding waiting for rail transit. Since rail transit operates either a high-frequency service (e.g. subway system with 2 minute headway) or a reliable, low-frequency service (e.g. GO service with 30 minute headway), the waiting time in both cases is minimum (as passengers time their arrivals for low-frequency services). The transfer penalty parameter value is estimated to be 6.1; the TTC uses 10.0 as a value for transfer penalty. While the estimated value is lower than the TTC value, the waiting time weight for surface transit is higher than what is used by TTC (between 1 and 3). This could represent a better explanation to the transfer inconvenience (as a combination of high disutility for waiting time and transfer penalty) rather than a high, unjustified disutility for the transfer penalty alone.

The fare weight is estimated as 8.5, and it has a unit of minute per $. This is extrapolated to a money value of time as $7/hour (12c/minute), which is close to the minimum wage in Ontario for the year 2001 ($6.85/hour). The flexibility in activity start times for work trips is higher than for school trips; therefore one might expect that the inconvenience from a late-arrival at destination for school trips is weighted more than for work trips. On the contrary, the value of time for a worker is more than for a student, which could be represented by a high disutility for late arrivals for work trips. In this implementation, about 67% of all modelled trips are work-related. Moreover, the misfit function is proportional to the number of passengers arriving on time. The calibrated values suggest that $\beta_{SD}^{HBW} > \beta_{SD}^{HBS}$, which means that, with everything else equal, workers have a higher disutility for arriving late (or early) than students. Also, when considering this inconvenience in terms of monetary values, the cost of schedule delay is approximately equivalent to the minimum wage value.
Figure 6.27 Values of Calibration Parameters for the TTC Application
6.8.2 Assignment Outputs

The assignment procedure outputs details on the transit service operations (e.g. run/route loads, stop-run records) and details on travellers’ behaviour (e.g. trip choices and experience accumulated). The assignment process is concerned with distributing passenger demand over a fixed set of routes with scheduled runs for each route. The calibrated parameters maximize the likelihood that the modelled loads match, to a great extent, the observed loads. Figure 6.28 shows the frequency distribution of observed loads and modelled loads. Since Subway Lines have much larger capacities (44% of observed demand), they could be considered as outliers and are removed from the figure for better visibility of surface road service distribution (see Figure 6.29).

In general, the modelled loads approximate the distribution underlying the observed loads. There is a discrepancy between the observed and modelled loads for the Queens Streetcar route #501 and the King Streetcar route #504; the model underestimates the demand for King Streetcar route and overestimates the demand for Queens Streetcar route. However, the total modelled demand for both routes is similar to the total observed demand for both routes. In reality, Queens Streetcar route and King Streetcar route run on two parallel routes (Queens Street and King Street) which are separated by a 300m block. King street has more employment high-rise buildings than Queens street, and the land-use data do not provide this information.

The Sheppard East Bus route #85 was observed to be over crowded with regard to total route loads and individual run occupancy. The Sheppard East Bus route was replaced by a rail transit line in 2002, which would have been justified by the model based on the high level of demand and since Sheppard East route had already a combined headway of 3 minutes (for 4 branches). A future study should compare the service improvements implemented in the year 2002 and onwards with the model findings.

By examining Figure 6.29, it is noticed that the model underestimates the demand for streetcar routes and downtown express services. This was somehow expected since streetcar and bus
routes were augmented into surface road service during the calibration process. While there are 134 bus routes (48% of observed demand), there are only 11 streetcar routes (8% of observed demand) and 4 express bus service (0.1% of observed demand); therefore, the calibrated parameters for surface transit services might be biased towards reproducing demand for bus routes more than other sub-modes. Also, subway lines and GO services (carrying about 20% of TTC users) do not share the same characteristics; subway lines provide a high-frequency, high-capacity service, while GO lines have schedule-based operations with limited capacities and high monetary cost.

The mesoscopic representation of the transit service provides sufficient details for the analysis of route and run operations and the testing of control strategies – see Table 6-3 for an example. This detailed output is beneficial in many ways. One can examine the on-time performance of any route in the transit network, by comparing the pre-announced schedule with the actual performance. Possible operation control strategies, such as holding, can be assessed. APTS technologies (e.g. Transit Signal Priority) can be evaluated, through their implementation into the microsimulation model for selected corridors. One can observe congestion areas over the network, propose and test possible relief policies.
Table 6-3 A detailed report for a sample run in one iteration

<table>
<thead>
<tr>
<th>Stop ID (end stop)</th>
<th>arrival time</th>
<th>arriving occupancy</th>
<th>queue length</th>
<th>alights</th>
<th>boardings</th>
<th>departure occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>13682*</td>
<td>8:10:44</td>
<td>47</td>
<td>107</td>
<td>47</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7226*</td>
<td>8:10:08</td>
<td>46</td>
<td>132</td>
<td>6</td>
<td>7</td>
<td>47</td>
</tr>
<tr>
<td>7191*</td>
<td>8:07:52</td>
<td>42</td>
<td>235</td>
<td>1</td>
<td>5</td>
<td>46</td>
</tr>
<tr>
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*stops with common lines.*
Figure 6.28 Observed and Modelled Route Loads for the TTC Application (with Subway Lines)
Figure 6.29: Observed and Modelled Route Loads for the TTC Application (without Subway Lines)
On the demand side, the assignment procedure considers the departure time choice as part of traveller’s behaviour. The TTS2001 records the start time for surveyed trips, this is compared to the departure time distribution generated from the assignment procedure. From Figure 6.30, the frequency distribution of the modelled departure time choices peaks at time-points similar to the frequency distribution of the surveyed departure time choices, with the highest peak at 8:00am. However, the figure shows a significant difference in the peak-frequencies. This could be attributed to two factors. When passengers are asked about their trip start time, they might respond with approximating the trip start time to the nearest clock-time (e.g. :00, :15, :30, :45). Also, the expansion factors used to extrapolate TTS surveyed records to population totals contribute to the observed peak demands at certain clock-times. This might impose unrealistic congestion effects if the TTS2001 synthesised departure time distribution is used as an input for the assignment process. Secondly, it might be convenient for passengers to choose a clock-time when planning their departure time choice and this needs to be considered in modelling departure time choices. When aggregating departure time choices for every 30 minute-period, the modelled frequency distribution resembles more the surveyed trip start time frequency distribution – see Figure 6.31. The differences could also be attributed to the unobserved scheduled arrival time; nonetheless, the assignment framework considers the departure time choice and this choice is responsive to service characteristics and passengers’ choices.

P316 has 7 options in the constructed mental model, and the stationary probabilities suggest that P316 would leave at 8:15am and will walk to the GO station to take the Lakeshore West inbound train towards Union station where he switches to the subway system and gets off close to his destination. The TTS2001 records show that P316 (or equivalent) chose to leave home at 8:20am and made exact trip choices as the modelled P316 – see Figure 6.20. The surveyed choices are compared with the modelled departure time and path choices – see Figure 6.32. About 99% of the surveyed departure time choices were replicated by passenger agents with similar origin, destination, access mode and egress mode attributes. The first route choice was matched for 79% of observed choices, while the predictability of the exact sequence of route transfers was about 60%. The first boarding stop choice was reproduced for about 75% of the surveyed origin stop choices and 75% of the exact sequence of transfer point choices were correctly predicted by the off-stop/on-stop choice mechanism.
Figure 6.30 Departure Time Frequency Distribution for Surveyed and Modeled AM Transit Trips
Figure 6.31 Departure Time Frequency Distribution for Surveyed and Modelled AM Transit Trips, with 30 minute-period aggregation
Figure 6.32 Learning-based Departure Time and Path Choice Model Predictive Power
The mental model contains information on all other options available for P316. This is a unique feature for the proposed framework since not only does it explain why a certain travel option is chosen by a passenger, but also it provides information on why a certain travel option is not chosen by a passenger. This is important for analysing passengers’ preferential treatment of travel options. In equilibrium-based models, passengers do not change their choices unilaterally; the choices of other passengers are required for the decision-making process. In reality, passengers change their choices based on their perceived travel cost of available options; when the service characteristics of these options change (e.g. faster bus service), passengers may change their trip choices without knowing others’ decisions. Through learning and adaptation within the microsimulation environment, a new steady state of passengers’ choices and service performance is reached.

The SWITCH routines reflect the en-route replanning behaviour of transit riders. Passengers utilize the SWITCH routines more when the within-day dynamics do not match their expectations or experiences with the transit service. When there is no experience accumulated, the usage of SWITCH routines is high; this is reduced as passengers accumulate experience and are able to make better decisions based on their anticipatory models of the transit service. This is evident as the number of passengers who utilize the SWITCH routines decreases significantly with time – see Figure 6.33. Other indicators on passengers’ learning is overcrowding which was observed to decline as the number of iterations increase – this is due to passengers’ adaptation to experienced congestion levels and capacity constraints.

The learning and adaptation is clear from tracing the individual-level behaviour (e.g. en-route replanning and day-to-day trip choices) and the network-level statistics over days (or iterations). When passengers have no prior knowledge of the transit service, they are exploring their choices and often not making the best decision. This occurs during the learning period, as show in Figure 6.33, as the percentage of passengers arriving on time is about 55% in iteration #1. The number of passengers experiencing congestion at origin-stop and transfer-stops also decreases significantly as passenger learn about their environment and they experience the congestion effects and capacity constraints. Since transfer is perceived as a disutility, passengers, through
learning and adaptation mechanisms, avoid transfers when possible – the number of passengers making a transfer shows a decline as the number of iterations increases.

At iteration #1, many passengers could not reach their destination on the scheduled arrival time and some did not reach their destination at all due to long delays at bus stops. As passengers build their knowledge-base of the service performance for different departure time options, passengers can plan their trip in order to minimize the travel cost (including schedule delay disutility). At the final iteration, all passengers arrived at their destination; however, some of them arrived earlier or later than their scheduled arrival time. The total schedule delay experienced by all passengers also is observed to decrease with time – see Figure 6.34. The average schedule delay for passengers also decreased significantly. When considering only delayed passengers, it is observed that some passengers still experience significant delay; those passengers are mostly the occasional travellers. The provision of information is expected to help avoid such long delays as anticipatory models of non-frequent travellers are not reproducing representative performance of the transit service and they are more inclined to use the information provided by Automated Traveller Information Systems (ATIS).

The model outputs details on individual passenger-agents’ trip choices, accumulated experiences stored in a mental model structure, departure time frequency distributions, dynamic transit-OD matrices for the modelling period, run-level performance indicators, and tracking of travellers’ behaviour during the trip and over time. This raw data can be aggregated using various factors to generate useful network-level and demand-level outputs and findings. For example, route-loads are not stored but rather they are calculated from the run-loads. Similarly, the number of passengers arriving on time is computed based on individuals’ experiences and trip choices.
Figure 6.33 Network-level Indicators on Passengers’ Learning and Adaptation
Figure 6.34 Schedule-Delay Output for (delayed) Passengers
The assignment process converges to a steady state where passengers continue to make the same choices over days and such trip decisions are reinforced as they minimize the generalized travel cost compared to other available options. The steady state is also observed regarding aggregate route loads that show stability over time – see Figure 6.35. The variation of route loads (as an aggregate network-level output) is less compared to variations of bounds and branches (a more disaggregate output). It is interesting to show the variations of stop-loads (number of passengers boarding from a stop); the stability in route loads alone might be deceiving as route loads can be reproduced through different levels of boardings from all stops along the route. However, the stability in stop-loads shows that the reproduced route loads are a result of a unique stationary distribution of travel choices for individual passengers. It also reflects the convergence of stop-choices for individual passengers.

This steady state is reached without the availability of perfect knowledge to all passenger and without assuming equilibrium \textit{apriori}. The assignment procedure also acknowledges the existence of within-day and day-to-day dynamics in both passengers' trip choices and service performance.
Figure 6.35 Steady State Convergence of Model Outputs
7 Conclusions

This thesis focuses on the issues concerning the theoretical aspects of transit assignment modelling, in particular trip choices under information provision. The study considers multiple dimensions of the transit path choice problem: the departure time choice, the stop choice, and the route (or run) choice. Such dimensions were either simplified (e.g. stop choice) or ignored (e.g. departure time) in existing approaches. The developed assignment procedure is capable of modelling the day-to-day and within-day dynamics of the transit service, as well as passengers’ responses. Furthermore, it represents a coherent behavioural integrated framework, which deals with the issue of congestion (i.e. influence of individual traveller’s options on travel choices of all others) endogenously. Hence, aggregate travel patterns can be properly extracted from individual choices. It addresses many limitations of existing transit assignment models by exploiting emerging methodologies already established in the areas of traffic assignment and travel behaviour modeling. Such emerging approaches include the microsimulation of transportation systems, learning-based algorithms for modelling travel behaviour, agent-based representation of travellers, and the adoption of Geographical Information Systems (GIS). It provides transit planners as well as operators with a platform for experimenting with smart transit systems applications and initiatives.

Transit assignment is a process of interactions between individual passengers and transit services. These interactions are in both directions: the execution of path choices leads to congestion, yet the expectation of congestion influences choices; and such interactions cannot be overlooked. In reality, this cyclical process manifests itself through a feedback mechanism, which could appropriately be represented by a learning process. The proposed framework assumes that individual passengers base their daily travel decisions on the accumulated experience gathered from repetitively travelling through the transit network on consecutive days and the information presented to them. Individual behaviour, therefore, should be modeled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. Transit passengers are assumed to plan their transit trip based on their experience with the transportation network and according to the information presented to them. This is a different perspective to view traveler behaviour compared to equilibrium analysis. An
operational prototype of the proposed modelling framework has been developed and tested. The purpose of this prototype is to demonstrate the feasibility and applicability of the new framework – refer to Chapter 5. A full implementation, using the Toronto Transit Commission (TTC) transit service as a case study, was conducted – refer to Chapter 6.

The proposed approach, with a microsimulation representation, is capable of providing different types of information regarding transit system performance to both transit operators (e.g. schedule adherence) and users (e.g. next arrivals). It is also sensitive to the travellers’ responses to such information through the incorporation of the effect of information provision in the modelling of within-day and day-to-day trip choices.

For modelling Bus Rapid Transit (BRT) or Light Rail Transit (LRT) systems, the proposed approach is sensitive to the unique characteristics of such systems. Using a detailed representation of the transit system, the model adequately represents the effect of stop-spacing alternatives usually considered when proposing new BRT systems. APTS deployments at stops require detailed representation of the physical structure and operations of the transit network. APTS applications, such as Automatic Passenger Counter (APC) and smart card for fare payments, affect for example dwell times as stops and vehicle travel time. Exclusive lanes for bus operations are another major component of BRT deployments; exclusive lanes increase speed, reduce travel time, and improve reliability, making BRT more competitive with car travel. This is expected to affect the transportation network in general; the microsimulation approach naturally combines traffic and transit operations. The evaluation of alternative designs of entrances and exits from exclusive lanes requires detailed representation of the transit and transportation systems. Queue jumping and transit signal priority are other features of BRT systems that require detailed intersection representations. Congestion and capacity constraints can be explicitly modelled, specially when comparing BRT and LRT systems. BRT uses ITS systems to track vehicle locations, control traffic signals, and provide vehicle arrival information. With agent-based, learning-driven representation of passengers and their travel behaviour (choices and adjustments), the influence of information provision can be captured and various scenarios can be investigated. With a GIS engine, the accessibility and area-coverage issues related to large investments such as BRT and LRT systems can be extensively studied.
MILATRAS provides an integrated dynamic modelling framework that: is sensitive to time-dependent and stochastic transit service characteristics (supply modeling), models adaptive departure time and path decisions by passengers (demand modeling), and captures the interaction between passenger decisions and transit network performance. This integrated dynamic modeling framework addresses: the time-dependent pattern of flows and their distribution over space, the systematic changes of the passenger decisions within the day and from day to day, and the interaction between the passenger decisions and the system performance. As such, the modeling framework deals explicitly with trip timing and path selection, and the mechanism through which passengers adjust these decisions in response to experienced congestion, control measures, and supplied information.

7.1 Summary

Chapter 1 presents the motivation behind this research effort and it discusses the challenges to traditional transit assignment procedures for modelling Advanced Public Transportation Systems. After reviewing the literature in Chapter 2, the author presents in section 2.4 his vision for the future developments in the area of transit assignment modelling.

Chapter 3 discusses the issues concerning the development and implementation of a new modelling framework for the transit assignment problem, namely the MIcrosimulation Learning-based Approach for TRansit ASsignment – MILATRAS. The proposed modelling framework is sensitive to time-dependent and stochastic transit service characteristics (supply modelling). It also models adaptive departure time and path choices by passengers (demand modelling), and captures the interaction between passenger decisions and transit network performance (via an integrated framework). MILATRAS is developed for the modeling of day-to-day and within-day dynamics of the transit assignment problem. MILATRAS considers multiple dimensions of the transit path choice problem: the departure time choice, the stop choice, and the route (or run) choice. MILATRAS represents passengers and both their learning and planning activities explicitly. The learning process is concerned with the specification of different trip components (e.g. in-vehicle time, out-of-vehicle time, convenience measures, etc). The planning process
considers how experience and information about those components on previous days influence the choice on the current day. The underlying hypothesis is that individual passengers are expected to adjust their behaviour (i.e. trip choices) according to their experience with the transit system performance as stored in a ‘mental model’. Individual passengers base their daily travel decisions on the accumulated experience gathered from repetitively travelling through the transit network on consecutive days. Individual behaviour, therefore, is modelled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. By repeatedly making a decision, an individual acquires knowledge (i.e. learns) about his environment and thereby forms expectations about attributes of the environment. Individuals may make different choices over time and thus learn which of these choices is more effective in achieving particular goals.

Chapter 4 presents the theoretical development of a departure time and transit path choice model based on the Markovian Decision Process; this model is the core of MILATRAS. Passengers, while travelling, move to different locations in the transit network at different points in time (e.g. at stop, on board), representing a stochastic process. This stochastic process is partly dependent on the transit service performance and partly controlled by the transit rider. This can be analyzed as a Markovian Decision Process. In an MDP, actions are rewarded and hence passengers’ optimal policies can be estimated. The proposed learning-based choice model considers the departure time choice, the stop choice and the run (or sequence of runs) choice. The proposed model is classified as a bounded rational model, with a constant utility term and a stochastic choice rule. The model is appropriate for modelling information provision since it distinguishes between individual’s experience with the service performance and information provided about the system dynamic characteristics. A parameter-calibration procedure using a generic optimization technique (Genetic Algorithms) is also proposed.

Chapter 5 investigates the impact of different traveller information provision scenarios on transit riders departure time and path choices, and network performance, using an agent-based microsimulation learning-based approach presented in Chapter 3 and a dynamic departure time and path choice model proposed in Chapter 4. It is assumed that individual passengers adjust their behaviour (i.e. trip choices) according to their accumulated experience gathered from
repetitively travelling through the transit network on consecutive days. Individual behaviour, therefore, is modelled as a dynamic process of repetitively making decisions and updating perceptions, according to a learning process. Four information provision scenarios were investigated and the impacts on transit rider travel choices were examined. The results show that, for a medium-size transit network with low to medium frequency services, the stop and departure time choices seem to be more important than the run choice, as they significantly affect the trip time. Information provided only at stops did not help passengers reduce their trip time, as run choices are limited and dependent on previous stop and departure time choices in such networks with limited services. Information targeting the transfer stop choice was found to be more effective. Interestingly, pre-trip information targeting home departure time choice did not enable the system to reach a stable state.

Chapter 6 documents the efforts to operationalize the conceptual framework of MILATRAS and its component models of departure time and path choices. It presents a large-scale real-world application, namely the multi-modal transit network of Toronto which is operated by the TTC (Toronto Transit Commission). The transit service is represented by over 500 branches with more than 10,000 stops. About 332,000 passenger agents are modelled to represent the demand for the TTC in the AM peak period. The learning-based departure time and path choice model, proposed in Chapter 4, was adopted using the concept of mental models for the modelling of the transit assignment problem. The choice model parameters were calibrated such that the entropy of the simulated route loads was optimized with reference to the observed route loads. A Parallel Genetic Algorithm engine was used for the parameter calibration process. The modelled route loads, based on the calibrated parameters, greatly approximate the distribution underlying the observed loads. 75% of the exact sequence of transfer point choices were correctly predicted by the off-stop/on-stop choice mechanism. The model predictability of the exact sequence of route transfers was about 60%. In this application, transit passengers were assumed to plan their transit trip based on their experience with the transportation network; with no prior (or perfect) knowledge of service performance.
7.2 Contributions

This thesis presents a significant step towards the advancement of modelling the transit assignment problem by providing a detailed operational specification for an integrated dynamic modelling framework – MILATRAS. The development of an operational model has shown that the proposed approach can simultaneously predict how passengers will choose their routes and estimate the total passenger travel cost in a congested network, as well as transit service performance (e.g. run loads on different transit routes). It results in a microscopic network manipulation, time-dependent trip choices, and a dynamic network loading procedure; significant enhancements that the field of transit assignment modelling is in need for.

MILATRAS uses learning and adaptation to represent the dynamic feedback of passengers’ trip choices and passengers’ adaptation to service performance. The dynamic integration of supply and demand at the proposed disaggregate level is unique to the proposed modelling system. That is, the framework represents the supply network at the stop geolocation and link levels, where it represents individual passengers as agents with their learning and planning activities explicitly modelled. The concept of ‘mental model’ of public transport service for transit riders, introduced in section 3.2.3, is another innovative design aspect of the proposed modelling system. It addresses the ongoing issue of realistic choice set generation by representing only the relative parts of the transit network for each passenger. Moreover, it is structured to maintain and distinguish between individual’s experience with the service performance and information provided about the system dynamic characteristics.

On the behavioural side, the proposed approach presents a different perspective on modelling passenger behaviour compared to equilibrium analysis; similar arguments have been proposed in recently developed dynamic traffic assignment models (or microsimulation tools). In MILATRAS, individual passengers base their daily travel decisions on the accumulated experience gathered from repetitively travelling through the transit network on consecutive days. Moreover, the proposed agent-based approach differentiates between the modelling of recurrent trips (or frequent users, e.g. work trips) and non-recurrent trips (or occasional users, e.g. discretionary trips). Individual behaviour is modeled as a dynamic process of repetitively making
decisions and updating perceptions, according to a learning process. The advantage of such approach is twofold. First, the removal of the equilibrium assumption through the framework enables a more dynamic and behavioural approach for the transit assignment modelling process. Second, while learning algorithms no longer guarantee a user equilibrium solution, they are proven to converge to a fixed point or go towards a steady-state density. This is rather interesting as it provides insights on the path to the steady state of the system. Without imposing an equilibrium condition, the proposed framework models the evolving interaction between system performance and passengers’ choices considering transient conditions and temporal fluctuations.

The transportation system, in particular the transit system, is complicated, and given the systems’ path dependencies and the time-varying factors, system equilibrium is often not achieved. This represents a great challenge to equilibrium-based models. Therefore, in the absence of explicit equilibrium conditions, a future state of the transportation system can only be estimated by explicitly tracing the evolutionary path of the system over time, beginning with current knowledge conditions (Miller, 2003). The microsimulation multi-agent representation increases the possibility of emergent behaviour to be predicted, which is not hardwired into the model.

Prior to the work accomplished in this thesis, behavioural-based microscopic activity-based models would have to resort to aggregate, equilibrium-based transit assignment models for the assignment of transit demand and the performance of the transit service. This results in inconsistencies in the modelling process, in addition to aggregation-disaggregation potential discrepancies. The effort outlined in this dissertation presents a significant step towards a more unified framework for urban and travel choices modelling. The proposed agent-based structure makes it natural and feasible to connect with recently emerging agent-based activity-based models for urban transportation system, such as ILUTE (Salvini and Miller, 2003).

The MILATRAS structure is innovative in combining the departure time choice, stop choice and run (or sequence of runs) choice in one framework along with the representation of day-to-day and within-day dynamics in travellers’ choices as well as transit service. MILATRAS is unique in dealing with the network-level effects of such interactions; existing approaches either deal with less choice dimensions (e.g. ignore stop choice) on the network-level or incorporate
multiple choice dimensions with a microscopic representation of only a subset of the transit network.

Geographical Information Systems (GIS) have been used in transportation planning applications for a long time. The integration of GIS representation and Microsimulation of transport networks, however, has only been recently explored. Based on the cited literature, such integration in the transit assignment analysis was initiated by our work in 2004 (Wahba, 2004; Wahba and Shalaby, 2005). Lately, the integration of GIS spatial-capabilities with the added benefits of microsimulation has received attention in the academic and professional communities. This is evident by the progress in transportation modelling software developments; earlier packages are either GIS-focused with no microsimulation advantage (e.g. MADITUC, 2008, for transit modelling, TransCAD (2008) for transportation planning) or microsimulation-focused with no GIS spatial capabilities (e.g. VISSIM, 2008, and Paramics, 2008, for traffic and transit microsimulation). Recently, packages that integrate GIS and microsimulation features are becoming the state-of-the-practice (e.g. TransModeller, 2008; VISUM, 2008).

MILATRAS can be adapted to model systems of large-size, high frequency transit networks without ITS deployment, as many existing static models are capable of. MILATRAS, in addition, is suitable for modelling systems of medium-size networks with medium to low frequencies, where behavioural hypotheses of existing static models are violated (e.g. random arrival of passengers at stops). Such networks exist in cities that are growing rapidly in population size and where major transit initiatives (e.g. Bus Rapid Transit and Light Rapid Transit) are being considered. Several cities that surround Toronto, Canada fall in this category (e.g. Brampton, Mississauga). When ITS deployment is considered, the proposed approach acknowledges the importance of maintaining explicit representation of information available to passengers as well as dynamic representation of service characteristics. It, therefore, allows for explicit modeling and evaluations of operational impacts of investing in new technologies (e.g. Automated Traveller Information System, ATIS; Automatic Vehicle Location, AVL, Automatic Passenger Counter, APC). It is possible to analyze and evaluate different planning polices at the operational level (such as Transit Signal Priority and control operation strategies that address
reliability issues, for example holding policy) as well as at the strategic level (such as the introduction of a new Bus Rapid Transit line or schedule changes).

On the practical side, for corridor-level studies in large-scale applications when microsimulation is difficult to develop for the whole transit network, MILATRAS provides a resolution to the gap between aggregate, zonal-level transit assignment models and disaggregate, stop/link-level microsimulation models. Aggregate transit assignment models, such as the ones used in EMME/2 (INRO, 2003), generate transit demand loadings at major stops and with a uniform loading profile for the assignment period. Microsimulation models, on the other hand, demand run-based loading profiles at the stop-link level, which usually requires the manipulation of the output of aggregate models. MILATRAS has provided a more direct input to microsimulation models than the output from aggregate assignment models. This is of particular interest for the modelling of Bus Rapid Transit/Light Rapid Transit investments, where microscopic details are relevant (e.g. stop geolocation).

7.3 Future Research

Because of the inherent complexity and the dynamics of trip choices under information provision, research in the area of transit assignment needs to be more open to experimentation with new approaches and less constrained by traditional concepts of travel demand analysis. This section presents some thoughts for future research:

1- Stochastic processes have been applied in traffic assignment procedures. It has been shown that static and stochastic user equilibrium solutions for the traffic assignment problem can be reproduced as particular cases of stochastic processes assignment models (Cantarella and Cascetta, 1995). It is worth investigating these findings in the transit assignment field. The proposed approach provides the framework for using stochastic processes in modelling the transit assignment process. A comparative study of equilibrium and non-equilibrium based models for the transit assignment problem will be a valuable contribution.
2- Transit-ITS polices are not only directed to improve transit service performance and help existing transit passengers make effective path choices, but also these polices have the potential to help marketing transit to those who normally travel by other modes. Large investments in transit services (e.g. the introduction of Bus Rapid Transit and real-time information systems) target auto drivers, and a full-scale assessment of such policies should not be constrained to benefits in travel times and path choices alone. Therefore, a mode choice module needs to be integrated in the framework structure. MILATRAS provides a consistent way of combining traffic and transit in a simultaneous modeling framework; therefore, it is able to represent the impact of roadway congestion on transit service and *vice versa*. In a microsimulation environment, traffic demand can be represented on the same network. Information regarding transit network conditions can be made available as well to auto-drivers and traffic conditions can be provided to transit riders. Therefore, mode choice can be integrated into the overall modelling framework, using the learning-based approach.

3- Information provision is expected to be more significant when non-recurring traffic conditions, like accidents, occur. Also, Advanced Public Transportation Systems (APTS) are expected to enable transit operators to efficiently manage the transit service under such circumstances. Using a microsimulation representation, the evaluation of different ATIS and APTS policies in this regard seems a natural extension.

4- The availability of *before* and *after* passenger trip choices for applications of ATIS will provide a unique source of data for the validation of various travel behaviour models that target the deployment of Transit-ITS policies and technologies.

5- Transit assignment is a key component of land-use and transportation models, which require transit assignment models to be sensitive to dynamic variations in travel demand, and have the ability to provide feedback on average transit travel times in a way that is consistent with traffic congestion and service interruptions. The proposed approach focuses on departure time and path choices. These are clearly not the only choice dimensions available to travellers. Moreover, travel demand management policies usually
target other choice dimensions. In the short-run, departure time and path choices seem to be the only mechanisms available to passengers to respond to congestion. When modelling long-term policies, other choice dimensions need to be considered, such as mode choice, trip chaining and activity scheduling. Therefore, the integration of dynamic transit assignment models (such as the proposed model) and activity-based urban planning models is needed.

6- The full implementation of the proposed framework should include a module to predict the transit network changes (i.e. to evolve the transit network) through the analysis period particularly if that period spans more than a year; this is roughly the timeframe of service reviews, which result in route service changes (e.g. new frequencies, timetables). Also, the proposed framework is suitable for addressing the transit network design problem as part of a larger transportation network design problem. Examples of optimal service design components where the proposed model can be utilized include route design, timetable construction, vehicle routing and crew scheduling.

7- More research is needed in developing theoretical constructs for representing traveller behaviour, especially with regard to capturing day-to-day learning and travel time prediction processes of travellers in response to actual experience and exogenous information. The application in Chapter 6 presents one way of modeling traveller learning and adaptation. The framework however encourages researchers to experiment with various methods and conduct comparative studies.

8- Further developments is needed to build demand-forecasting tools that recognize the temporal evolution of the transportation systems and acknowledge the day-to-day and within day variations, even if these systems are represented by a steady-state in the future.

9- The development of procedures to estimate dynamic transit-OD from observed route flows is another natural extension to the framework. The dynamic representation of transit services and trip choices enables such development.
10- Recently, urban areas across North America have been experiencing increasing urban sprawl characterized by medium to low densities and dispersed land use. In the Greater Toronto Area (GTA), more residents now live in the 905 belt with such land use characteristics, than in the City of Toronto with its compact and dense urban form. With 45% of the GTA work trips destined to Toronto, about 1/3 of those originated in the 905 belt with a 25% modal split for transit (Joint Program in Transportation, 2003a). For cross-regional transit journeys, passengers transfer between different modes and different networks, forming what is called “intermodal trips” which has been receiving considerable attention lately. Therefore, inter-urban multimodal transport network assignment models are an essential component of any region-wide planning exercise. Conventional models deal with multimodal transport networks in a static framework and dynamic frameworks are limited to the single mode (e.g. auto). Future efforts should be directed to the modelling of passengers’ travel choices in a multimodal network, using dynamic departure time and path choice models.

11- The growing trend in using smart cards (e.g. new GTA smart card fare system “Presto”, Chicago Card Plus, and the Charlie Card in Boston) provides a rich source for data on transit path choices in particular and multi-modal trips in general. Such data sources will present great opportunities for calibrating and validating dynamic transit path choice models.

12- In this dissertation, adaptation is assumed to be related to adjustments in trip choices in response to congestion and information provision. Another form of adaptation concerns the long term adjustment of the learning methodology itself, including the preferences towards various travel costs. Research efforts should be directed to study this kind of adaptation, especially when considering long-term travel demand management policies.

13- Different passenger groups can be easily represented due to the agent-based passenger representation of MILATRAS. This does not only represent traditional socio-economic groups, but also refers to different learning and adaptation techniques. Future
developments of travel survey designs should include questions that attempt to identify various groups.

A transit planner’s dream is to have a tool that: is sensitive to policy variables, is easy to use, produces logical and reasonable results, and could be connected to larger, land-use and transportation planning modelling systems. The proposed framework is a step in this direction. MILATRAS, with its innovative structure, departs from the traditional modelling paradigm at this time stage of the development of the transportation modelling process, seeking to arrive at a better understanding of the transit path choice behaviour (hopefully on time!).

In conclusion, while this thesis is meant to answer a few research questions, it triggers more. The level of details captured in this modelling effort opens a new dimension for transit planners as well as researchers to experiment with a wide range of policy scenarios and modelling techniques. This, in turn, will help tackle previously un-answered questions and set the stage for future research programs.
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