OPTIMIZATION OF IMAGING PERFORMANCE AND
CONSPICUITY IN DUAL-ENERGY X-RAY RADIOGRAPHY

by

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ABSTRACT

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Dual-energy (DE) x-ray imaging of the chest decomposes two radiographs acquired at low- and high x-ray energies into 'soft-tissue' and 'bone' images, reducing the influence of background anatomical noise and providing increased conspicuity of subtle underlying structures compared to conventional radiography. This thesis derives a quantitative theoretical model of imaging performance in DE x-ray imaging and employs the resulting framework to system optimization in thoracic imaging. Fourier domain metrics of signal and noise performance - including the noise-power spectrum (NPS), modulation transfer function (MTF), detective quantum efficiency (DQE), and noise-equivalent quanta (NEQ) - were computed using cascaded systems analysis extended to DE imaging and combined with a quantitative model of imaging task to yield estimates of detectability across a broad range of DE image acquisition and decomposition techniques. Specifically, the detectability index provided an objective function for optimizing the selection of kVp pair, added filtration, allocation of dose between low- and high-energy views, and choice of decomposition algorithm and parameters therein. Theoretical calculations were validated in comparison to measurements of NPS, MTF, DQE, and NEQ performed on an experimental DE imaging system and through human observer studies for a variety of imaging tasks. Overall, the detectability index was found to provide a reliable predictor of human observer performance. Results identified optimal DE image acquisition and decomposition techniques that boost detectability beyond that achieved by conventional radiography or other DE imaging approaches, in many cases boosting conspicuity of subtle lesions from barely visible to highly conspicuous at fixed dose to the patient. The results are particularly encouraging, as such performance was achieved with the DE imaging dose equivalent to that of a single chest radiograph. The theoretical framework provided a valuable guide to optimization of a clinical prototype for high-performance DE chest imaging and may be extended to other DE imaging approaches, such as DE mammography and DE computed tomography.
An expert is a man who has made all the mistakes which can be made in a very narrow field.

-Niels Bohr
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<td>Anti-Correlated Noise Reduction</td>
</tr>
<tr>
<td>AMLCD</td>
<td>Active Matrix Liquid Crystal Display</td>
</tr>
<tr>
<td>BSF</td>
<td>Backscatter Factor</td>
</tr>
<tr>
<td>CBCT</td>
<td>Cone-beam Computed Tomography</td>
</tr>
<tr>
<td>CE</td>
<td>Contrast Enhancement</td>
</tr>
<tr>
<td>CR</td>
<td>Computed Radiography</td>
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<tr>
<td>CSA</td>
<td>Cascaded Systems Analysis</td>
</tr>
<tr>
<td>CT</td>
<td>Computed Radiography</td>
</tr>
<tr>
<td>DE</td>
<td>Dual Energy</td>
</tr>
<tr>
<td>DQE</td>
<td>Detective Quantum Efficiency</td>
</tr>
<tr>
<td>DR</td>
<td>Digital Radiography</td>
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<td>EAS</td>
<td>Edge-Adaptive Smoothing</td>
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<tr>
<td>ESD</td>
<td>Entrance Surface Dose</td>
</tr>
<tr>
<td>ESE</td>
<td>Entrance Surface Exposure</td>
</tr>
<tr>
<td>ESF</td>
<td>Edge Spread Function</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast-Fourier Transform</td>
</tr>
<tr>
<td>FH</td>
<td>Fisher Hotelling</td>
</tr>
<tr>
<td>FHE</td>
<td>Fisher Hotelling with Eye</td>
</tr>
<tr>
<td>FPD</td>
<td>Flat-Panel Detector</td>
</tr>
<tr>
<td>GDQE</td>
<td>Generalized Detective Quantum Efficiency</td>
</tr>
<tr>
<td>GLNR</td>
<td>Generalized Linear Noise Reduction</td>
</tr>
<tr>
<td>GNEQ</td>
<td>Generalized Noise equivalent Quanta</td>
</tr>
<tr>
<td>HDPE</td>
<td>High Density Polyethylene</td>
</tr>
<tr>
<td>HPF</td>
<td>High-Pass Filter</td>
</tr>
<tr>
<td>HVL</td>
<td>Half-Value Layer</td>
</tr>
<tr>
<td>kVp</td>
<td>Peak Kilovoltage</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>LPF</td>
<td>Low-Pass Filter</td>
</tr>
<tr>
<td>LSF</td>
<td>Line Spread Function</td>
</tr>
<tr>
<td>MAFC</td>
<td>Multiple Alternative Forced Choice</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
</tr>
<tr>
<td>MTF</td>
<td>Modulation Transfer Function</td>
</tr>
<tr>
<td>NC</td>
<td>Noise Clipping</td>
</tr>
<tr>
<td>NEQ</td>
<td>Noise Equivalent Quanta</td>
</tr>
<tr>
<td>NMR</td>
<td>Nuclear Magnetic Resonance</td>
</tr>
<tr>
<td>NNPS</td>
<td>Normalized Noise-Power Spectrum</td>
</tr>
<tr>
<td>NPS</td>
<td>Noise-Power Spectrum</td>
</tr>
<tr>
<td>NPW</td>
<td>Non-Prewhitening</td>
</tr>
<tr>
<td>NPWE</td>
<td>Non-Prewhitening with EYE</td>
</tr>
<tr>
<td>OTF</td>
<td>Optical Transfer Function</td>
</tr>
<tr>
<td>PA</td>
<td>Posterior-Anterior</td>
</tr>
<tr>
<td>PMMA</td>
<td>Poly(Methyl Methacrylate)</td>
</tr>
<tr>
<td>PET</td>
<td>Positron Emission Tomography</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>ROC</td>
<td>Receiver Operating Characteristic</td>
</tr>
<tr>
<td>ROI</td>
<td>Region of Interest</td>
</tr>
<tr>
<td>SDNR</td>
<td>Signal Difference to Noise Ratio</td>
</tr>
<tr>
<td>SLS</td>
<td>Standard Log Subtraction</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SSH</td>
<td>Simple Smoothing High</td>
</tr>
<tr>
<td>TFT</td>
<td>Thin-Film Transistors</td>
</tr>
<tr>
<td>US</td>
<td>Ultrasound</td>
</tr>
<tr>
<td>WSS</td>
<td>Wide-Sense Stationarity</td>
</tr>
</tbody>
</table>
CHAPTER 1: INTRODUCTION

I CONSPICUITY IN MEDICAL IMAGES

The ability to detect subtle lesions in x-ray projection images is an enduring challenge despite
the advances in x-ray sources and detector technologies over the latter half of the 20th century.
Still, the x-ray projection image remains a mainstay of medical imaging, providing the most
common medical imaging modality for screening, diagnosis, and image guidance. Although
recent improvement in detector efficiency permits diagnostic imaging at lower patient dose
without sacrifice in image quality, such has not in itself significantly improved diagnostic
performance. Several studies have shown that little discernable change in reader error and
variability has occurred with the advent of digital flat-panel detector (FPD) technologies
compared to x-ray film and computed radiography (CR) technologies.1-3 These observations
point to other challenges that must be overcome to improve lesion conspicuity in projection
imaging. Engel4 defined visual conspicuity as the “… combination of properties of a visible
object in its background...” and similarly, Revesz and Kundel define conspicuity as the ratio between contrast of the object and complexity of the background. Both interpretations suggest that anatomical background (rather than dose and detector efficiency) is intimately linked to object conspicuity and must be addressed to improve diagnostic performance. The hypothesis that anatomical background structure is a significant factor in the conspicuity of subtle lesions is a central theme of this work and motivates investigation of dual-energy (DE) x-ray imaging as a means of boosting lesion conspicuity. With the benefit of tissue cancellation, as discussed below, DE imaging is shown to reduce anatomical background noise. A theoretical framework for DE imaging performance is developed, applied to the optimization of DE image acquisition and decomposition techniques, and shown to correlate well with the performance of real human observers.

This chapter introduces DE imaging in the context of chest radiography and provides some motivation for the investigation and development of DE imaging systems. Secondly, basic principles of DE imaging and acquisition techniques are discussed. Finally, a wide range of current and potential clinical applications for DE imaging is presented.

I.1 Current Challenges

There are several physical factors that can affect lesion conspicuity in medical imaging. The section below provides an overview of the most prevalent challenges in x-ray imaging that need to be addressed for improved lesion conspicuity.

Contrast, the relative signal intensity with respect to its background, was shown in early work to significantly affect an observer’s ability to distinguish a lesion in a uniform background below a certain threshold. In x-ray imaging, contrast arises from the difference in x-ray attenuation
between the lesion and background and depends on the material, material thickness, and x-ray energy as described by the Beers-Lambert law:

\[ N = N_0 \exp(-\mu(E)t) \] (1.1)

where \( N \) denotes the transmitted number of x-rays, \( N_0 \) denotes the initial number of x-rays, \( \mu \) denotes the linear attenuation coefficient of the material, \( E \) is the x-ray energy, and \( t \) denotes the distance travelled by the x-ray through the material. Contrast can be improved by imaging at energies where the difference in attenuation between the lesion and the background is the greatest, while also accounting for the x-ray transmission \( (N/N_0) \) through the patient, as shown in Fig 1.1 – i.e., ~30-80 keV x-rays, corresponding to radiographic energies.

Figure 1.1: (Left y-axis) Solid lines plot the linear attenuation coefficient as a function of x-ray energy for background and lesion, taken to be water and polyethylene, respectively. (Right y-axis) Dashed line plots the x-ray transmission through 10 cm of water (approximating average chest thickness). (Data adapted from NIST database, http://physics.nist.gov)

Another challenge in radiography arises from quantum noise, which is observed as random fluctuations in the image and can diminish the conspicuity of low-contrast lesions as illustrated in Fig 1.2(a-c). The effect of quantum noise can be mitigated in at least two ways: 1) use of a
greater number of x-rays; or 2) increased detection efficiency. The first solution implies more x-ray radiation to the patient, which should always be kept to a minimum. The second solution involves the development of more efficient detectors, an area of ongoing medical imaging research.

Figure 1.2: Illustration of quantum noise and anatomical noise in the detection of a subtle abnormality. All images contain a Gaussian sphere in the center simulating a lesion. The quantum noise increases from left to right and overlying anatomy (corresponding to the lung region of a chest radiograph) is increased from top to bottom.

X-ray scatter also significantly affects image quality and is primarily caused by x-rays that undergo Compton and Raleigh scattering and lose their linear trajectory, resulting in detected x-rays that yield incorrect attenuation information and thereby increase noise and reduce contrast. There are several ways to mitigate scatter, including: an air gap between the patient and the detector; an anti-scatter grid; and various software-based methods of subtracting estimates of the scatter contribution from an image, such as Monte-Carlo simulations.
A fourth factor affecting conspicuity arises from overlying anatomical clutter. Depending on the imaging task, a given anatomical structure can represent either the structure of interest or can constitute a confounding background of the image. For example, if a radiologist is searching for a fractured rib, then the rib trabeculae constitute the structures of interest, and the lungs and vessels are background structures that may confound detection of the fracture. Conversely, if a radiologist is searching for lung nodules, the ribs form the background and may confound detection of the nodule. Variations in the image arising from anatomy not associated with the structures of interest are denoted **background anatomical noise**. Recent work seeking to compare the relative influence of quantum noise and background anatomical noise in the chest has demonstrated that anatomical noise can be a far more confounding factor than quantum noise.\(^8\) Figure 1.2 illustrates this with simulated images illustrating of quantum and anatomical noise on lesion conspicuity.

Other factors include spatial resolution which is limited by spatial spreading of the detection quanta in the detector, the size of the detector pixel, and the size of the x-ray focal spot. Artifacts caused by non-uniformities in the detection device can also reduce conspicuity if not appropriately corrected for.

## I.2 Increasing Conspicuity by Reducing Background Anatomical Noise

Based upon the principles and observations mentioned above, it is clear that background anatomical noise caused by overlying clutter in the 2D projection is a significant factor affecting lesion conspicuity, and we hypothesize that the reduction of background anatomical noise will significantly improve lesion conspicuity in medical images. Several prevalent methods for reducing background anatomical noise are presented below, categorized in terms of: 1) spatial
discrimination [e.g., CT reconstruction, as shown in Fig 1.3(b)]; and 2) material discrimination [e.g., DE image decomposition, as shown in Fig 1.3(c)].

Figure 1.3. Illustration of the challenges posed by anatomical background in (a) a projection radiograph of the chest which are reduced through spatial discrimination [as in (b) a coronal CT image] and/or material discrimination [as in (c) a dual-energy image (with bones removed)].

I.2.1 Spatial discrimination in medical imaging

Background anatomical noise in a projection image results from the compression of 3D volumetric information \( \mu(x,y,z) \) into a 2D image, as in the integral form of the Beers-Lambert law:

\[
I_{\text{proj}}(x,y) = \int \mu(x,y,z)dz
\]

where \( I_{\text{proj}}(x,y) \equiv \log(N_0 / N(x,y)) \), as in the radiograph of Fig. 1.2(a). Recovery of the 3D information amounts to rejection of overlying anatomy from a given image in that the radiologist is able to visualize individual “slices” of the 3D anatomy without the confounding influence of anatomy in neighbouring slices [as illustrated in Fig 1.3(b)]. A brief introduction to some
imaging technologies that allow partial or full recovery of 3D volumetric information is given below. Prevalent imaging technologies not employing x-rays are also described to provide a brief overview of other clinical 3D imaging modalities providing improved conspicuity through spatial discrimination.

Tomosynthesis imaging systems acquire multiple x-ray projections at a varying angles around the patient. Several acquisition modes are possible, including linear and circular trajectories. 3D reconstruction yields a stack of slices at varying depths within the patient, providing an estimate of the 3D volumetric data:

\[ I_{\text{tomo}}(x, y, z) \approx \mu(x, y, z) \]  \hspace{1cm} (1.3)

Common methods for tomosynthesis include “geometric tomography” for which a single slice is imaged in focus (and structures from neighbouring slices blurred out) and more current approaches, such as filtered backprojection and algebraic reconstruction.

b. **Computed tomography**

Developed in the early 1970s, computed tomography (CT) provides an accurate means of reconstructing 3D tomographic images and has gained widespread application in medical imaging. The invention earned G. Hounsfield and A. Cormack the Nobel Prize for Medicine in 1979. CT systems acquire x-ray projections [Eq. (1.2)] at all angles, \( \theta \), around the patient:

\[ I_{\text{proj}}(x', y'; \theta) = R\{\mu(x, y, z)\} \]  \hspace{1cm} (1.4)

also known as the Radon transform denoted, \( R \). In the limit of infinitely many projections over 360°, the inverse of the Radon transform yields the 3D volumetric image:

\[ I_{\text{CT}}(x, y, z) = \mu(x, y, z) = R^{-1}\{I(x', y'; \theta)\} \]  \hspace{1cm} (1.5)
Various algorithms, such as filtered back-projection and iterative algebraic methods, have been developed. Figure 1.3(b) shows an example CT coronal reconstruction of the chest, where overlapping ribs and other anatomical structures are removed from the image.

c. **Other 3D imaging modalities**

Another imaging modality providing spatial (volumetric / tomographic) discrimination is magnetic resonance imaging (MRI), developed in the 1980s and earning P. Lauterbur and Sir. P. Mansfield the 2003 Nobel Prize for Medicine. MRI uses the principle of nuclear magnetic resonance (NMR) to localize in 3D the parameters associated with NMR (e.g., proton density and T1 and T2 relaxation times), which depend not only on the material properties but also the chemical environment, and thereby distinguish anatomical structures. Imaging is achieved by placing the patient in a strong, uniform magnetic field (e.g., 0.5 - 3 T), inducing a net magnetization of the proton nuclear spins in the direction of the field. Weaker transverse gradient magnetic fields excite the nuclear spin, and as the transverse magnetic fields change, the nuclear spins emit a radio-frequency (RF) signal that is detected using a RF coil. By applying three-dimensional gradients the location of the signal is encoded in the signal, and the information can be recovered using Fourier techniques similar to CT reconstruction to yield a 3D volumetric MR image [for example, as in Fig 1.4(a)]. MRI can provide exquisite soft-tissue contrast, particularly in distinguishing various types of nervous tissues, but is prone to susceptibility artifacts in regions where proton density is low, as in air, which is why MRI is not typical for lung imaging.

Positron emission tomography (PET) provides another 3D imaging technique used to estimate the 3D distribution of radio-labeled molecules, for example glucose, which can indicate areas of increased metabolism such as malignant tumors. PET employs isotopes that are positron emitters, resulting in pair annihilation that in turn produces two 511 keV gamma rays at ~180°
from each other. A ring of detectors around the patient is used to detect coincident gamma rays. The line between joining two coincident events provides information on the location of the isotope and is used to reconstruct a 3D distribution of the radio-labeled molecule. Due primarily to the positron range the resolution of PET scanner has a limit of 1-2mm. Most PET scanners built today are hybrid systems that combine a PET gantry with a CT scanner to provide improved anatomical registration between the radioisotope distribution (as imaged in PET) and the structural anatomy (as imaged in CT). Figure 1.4(b) shows an example image from a PET-CT scanner of non-Hodgkin’s lymphoma in the mediastinum.

Finally, ultrasound is a widely used technique that provides 3D imaging by measuring mechanical properties of tissues through the use of a transducer placed on the surface of the patient to emit ultrasonic pulses (1-10 MHz) and detect echoes (known as pulse-echo operation).
Spatial discrimination is achieved by measuring the time between the pulse and the echo, while tissue discrimination is achieved by accounting for the different tissue interfaces which have different echo properties (i.e., pressure and reflection coefficients). The transducer can be moved along the surface and angled to obtain various slices through the patient [for example, Fig 1.4(c)].

### 1.2.2 Material discrimination in medical imaging

Rewriting Eq. (1.2) by regrouping all the different materials composing a volume in terms of $\mu_i$, and the total path length of the $i$th material across that volume, denoted $t_i$, yields:

$$ I_{\text{proj}}(x, y) = \sum_i \mu_i(x, y)t_i $$

(1.6)

The ability to distinguish various materials [i.e., $\mu_i(x, y)t_i$] is the basis for material discrimination as shown in the following examples.

a. **Contrast-enhanced projection imaging**

One means of boosting conspicuity is to enhance the contrast of the structures of interest. This can be achieved by injecting a contrast agent with high attenuation (typically, high $Z$) intravenously to the organ of interest. Denoting the contrast agent material, $a$, with attenuation properties such that $\mu_a(x, y) >> \mu_i(x, y)$ for $i \neq a$ in Eq. (1.6), then $I(x, y)$ for the CE image can be roughly approximated by:

$$ I_{\text{CE}}(x, y) \approx \mu_a(x, y)t_a $$

(1.7)

Therefore, the contrast-enhanced projection provides an image predominantly of the organ of interest. Example applications of contrast-enhanced imaging include: coronary angiography
(which uses iodine injected intravenously to significantly increase the contrast of arteries of the heart) and cerebral angiography (for identifying tumors and arteriovenous malformations of the central nervous system).

b. **Digital subtraction angiography**

Improvement on CE imaging can be achieved using digital subtraction angiography (DSA) by acquiring an image before contrast injection, giving a mask image, $I_m$. Before injection, the mask image displays a very low contrast representation of the structure of interest and no contrast agent. The mask image can therefore be written as:

$$I_m(x, y) = \sum_{i=1}^{n} \mu_i(x, y)t_i$$  \hspace{1cm} (1.8)

After contrast injection the projection image is given by:

$$I_c(x, y) = \sum_{i} \mu_i(x, y)t_i,$$  \hspace{1cm} (1.9)

and after subtraction:

$$I_{DSA}(x, y) = I_c(x, y) - I_m(x, y)$$

$$= \mu_a(x, y)t_a$$  \hspace{1cm} (1.10)

which yields an image showing only the structures of interest associated with the contrast agent. The DSA image therefore shows the CE structure conspicuously by minimizing the anatomical background.

c. **Dual-energy x-ray imaging**

Dual-energy (DE) x-ray imaging improves material discrimination by cancelling a given material from the image:
where, \( b \), denotes the cancelled material. Tissue cancellation is obtained by subtracting a low- and high-energy x-ray projection, with details of DE image acquisition and decomposition algorithms described more fully in Sec. II.

DE imaging can generate a ‘soft-tissue’ image by cancelling bone:

\[
I_{\text{soft}}(x, y) \propto \left[ \sum_i \mu_i(x, y) t_i \right] - \mu_b(x, y) t_b
\]

(1.12)

and a ‘bone’ image by cancelling soft-tissue:

\[
I_{\text{bone}}(x, y) \propto \left[ \sum_i \mu_i(x, y) t_i \right] - \mu_{\text{soft}}(x, y) t_{\text{soft}}
\]

(1.13)

where the bone and soft-tissue are cancelled from the projection image, respectively, as shown in Fig. 1.5. The nomenclature assumes a two-component model of the anatomy as described below.
DUAL-ENERGY X-RAY IMAGING

II.1 Historical and Physical Foundations of DE Imaging

The first application of DE imaging can be attributed to Jacobson\textsuperscript{14,15} in 1953 for attempting to increase the contrast of iodine in angiographic images by using low-power monoenergetic x-ray sources on either side of the K-edge of iodine. Unfortunately, the technology was not viable due to the low-power sources requiring very long exposures, causing blurry images due to respiratory
and cardiac motion. To increase x-ray tube output, the use of a fluoroscopic x-ray tube with a rotating filter wheel was investigated by Mistretta in the 1970s.\textsuperscript{16} The two filters produced a narrow spectrum on each side of the iodine K-edge, but the approach also proved to be difficult because the attenuation caused by the patient reduced the spectral separation, while highly attenuating the signal; the resulting images were strongly limited by image noise and low contrast. Further developments were pursued by Alvarez and Macovski\textsuperscript{17} who formulated a generalized framework for cancelling specific materials in radiographic images by exploiting differences in photoelectric absorption and Compton scattering components. This approach is essentially the one still used in DE imaging systems today.

As mentioned above, we conceptualize the anatomy (e.g., the chest) very simply to consist of two components: soft-tissues and bony structures. At low energies, bone contrast relative to soft-tissue is greater than at high energies as can be seen in Fig.1.6(a). Note that at diagnostic energies it is generally safe to assume that the total attenuation consists primarily of either photoelectric absorption or Compton scattering. (Effects such as coherent scatter and pair-production are negligible.) Figure 1.6(b) plots the fraction of absorption due to the photoelectric effect. At low energies (<40 keV) x-rays interact with bone mostly via photoelectric absorption and with soft-tissues mostly via Compton scattering. At high energies, however, (>70 keV) most x-ray interactions are via Compton scattering in both types of tissues. It is this large difference in energy response that allows the cancellation of either soft-tissues or bone even when using polyenergetic x-ray sources.
I.1.1 Log subtraction

The log subtraction technique for DE image decomposition assumes two material types (denoted \(a\) and \(b\)) and two x-ray energies, low and high, denoted \(L\) and \(H\). From the Beers-Lambert law [Eq. (1.1)] it follows that the transmitted number of x-rays at low- and high-energy is given by:

\[
N_L = N_0 \left[ \exp(-\mu_a(E_L)\tau_a - \mu_b(E_L)\tau_b) \right] \quad \text{(1.14a)}
\]

\[
N_H = N_0 \left[ \exp(-\mu_a(E_H)\tau_a - \mu_b(E_H)\tau_b) \right] \quad \text{(1.14b)}
\]

Spatial coordinates \((x, y)\) are assumed implicitly in the following sections; therefore, defining the low- and high-energy images as \(I_L = -\ln(N_L / N_0)\) and \(I_H = -\ln(N_H / N_0)\), and subtracting \(I_L\) and \(I_H\) weighted by a parameter, \(w\), called the tissue cancellation parameter, yields a dual-energy image:

\[
I_{DE} = I_H - wI_L = \left[ \mu_a(E_H)\tau_a + \mu_b(E_H)\tau_b \right] - w\left[ \mu_a(E_L)\tau_a + \mu_b(E_L)\tau_b \right] \quad \text{(1.15)}
\]
Furthermore, defining the tissue cancellation parameter as follows:

\[ w = \frac{\mu_b(E_H)}{\mu_b(E_L)} \]  

(1.16)

simplifies Eq. (1.15) to:

\[ I_{DE} = \mu_a(E_H)t_a - \frac{\mu_b(E_H)}{\mu_b(E_L)} \mu_a(E_L)t_a \]

\[ = \Delta \mu t_a \]  

(1.17)

where

\[ \Delta \mu \equiv \mu_a(E_H) - \frac{\mu_b(E_H)}{\mu_b(E_L)} \mu_a(E_L) \]  

(1.18)

This yields a signal that cancels the attenuation (i.e., signal contribution) from material \( b \); therefore, the resulting image presents material \( a \) alone. The method provides a general means for cancelling any material depending on the choice of \( w \), determined by the cancelled material attenuation coefficient at low and high energies [Eq. (1.16)]. For the soft-tissue and bone image, the DE images are given by:

\[ I_{soft} = I_H - w_s I_L \text{ where } w_s = \frac{\mu_{bone}(E_H)}{\mu_{bone}(E_L)} \]  

(1.19)

and

\[ I_{bone} = w_b I_L - I_H \text{ where } w_b = \frac{\mu_{soft}(E_H)}{\mu_{soft}(E_L)} \]  

(1.20)
denoted the standard log subtraction (SLS) decomposition algorithm. Note that the change in sign for Eq. (1.20) ensures that the bone image exhibits white bone and dark background (which is the same polarity as a conventional radiograph).

<table>
<thead>
<tr>
<th>Interaction type</th>
<th>Atomic # dependence</th>
<th>Energy dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compton scattering</td>
<td>$Z^0$</td>
<td>$E^0$</td>
</tr>
<tr>
<td>Photoelectric absorption</td>
<td>$Z^1$</td>
<td>$E^{-3}$</td>
</tr>
</tbody>
</table>

Table 1.1: Summary of dependence of x-ray attenuation on atomic number and energy for Compton scattering and photoelectric absorption in the diagnostic x-ray energy range.

An important condition for producing a useful, non-trivial DE images (i.e., a non-zero image) relies on two important conditions: 1) $E_L \neq E_H$, and 2) the Compton scattering attenuation coefficient [$\mu_{CS}(E)$] and the photoelectric attenuation [$\mu_{PE}(E)$] are linearly independent functions as shown in Table 1.1. This can be demonstrated by rewriting Eq (1.14) in terms of $I_L$ and $I_H$ in matrix form:

$$
\begin{pmatrix}
I_L \\
I_H
\end{pmatrix} =
\begin{pmatrix}
\mu_a(E_L) & \mu_b(E_L) \\
\mu_a(E_H) & \mu_b(E_H)
\end{pmatrix}
\begin{pmatrix}
t_a \\
t_b
\end{pmatrix}.
$$

(1.21)

Assuming that the attenuation coefficient for any material (at diagnostic energies) can be written as a weighted sum of the Compton scattering (CS) attenuation and photoelectric (PE) attenuation, the weights ($\alpha_i$ and $\beta_i$) form a transition matrix from the ‘material matrix’ to the ‘CS and PE matrix’:

$$
\begin{pmatrix}
\mu_a(E_L) & \mu_b(E_L) \\
\mu_a(E_H) & \mu_b(E_H)
\end{pmatrix} =
\begin{pmatrix}
\mu_{CS}(E_L) & \mu_{PE}(E_L) \\
\mu_{CS}(E_H) & \mu_{PE}(E_H)
\end{pmatrix}
\begin{pmatrix}
\alpha_a & \beta_a \\
\alpha_b & \beta_b
\end{pmatrix}.
$$

(1.22)

Therefore, the low and high-energy image can be re-written in terms of the ‘CS and PE’ matrix:
where \( t_a' \) and \( t_b' \) are the path length associated with the Compton and photoelectric attenuation coefficients, respectively. The values of \( t_a' \) and \( t_b' \) can be solved for [and consequently for \( t_a \) and \( t_b \) via Eqs (1.21-1.23)] if and only if the matrix is invertible (i.e., forms a basis). This implies:

\[
\det \begin{pmatrix}
\mu_{cs}(E_L) & \mu_{pe}(E_L) \\
\mu_{cs}(E_H) & \mu_{pe}(E_H)
\end{pmatrix}
= \mu_{cs}(E_L)\mu_{pe}(E_H) - \mu_{pe}(E_L)\mu_{cs}(E_H) \neq 0
\]  

(1.24)

Provided that \( E_L \neq E_H \), the above equation holds by the fact that Compton scattering and photoelectric absorption are linearly independent functions of energy – i.e., Compton scattering and photoelectric absorption are proportional to \( E_0^0 \) and \( E^{-3} \), respectively, in the diagnostic energy range (as shown in Table 1.1).

Finally, it follows from Eqs. (1.22) and (1.24) that the ‘material matrix’ also forms a basis:

\[
\det \begin{pmatrix}
\mu_a(E_L) & \mu_b(E_L) \\
\mu_a(E_H) & \mu_b(E_H)
\end{pmatrix}
= \mu_a(E_L)\mu_b(E_H) - \mu_b(E_L)\mu_a(E_H) \neq 0
\]  

(1.25)

which is equivalent to Eq(1.18) for \( \Delta\mu \neq 0 \). Therefore, \( I_{DE} \) yields a non-zero image (i.e., \( \Delta\mu \neq 0 \)) if and only if: 1) \( E_L \neq E_H \), and 2) Compton scattering and photoelectric absorption are linearly independent functions of energy, as postulated above.

An interesting corollary is that if one assumes only two types of x-ray interactions (e.g., Compton and photoelectric), then acquiring more than two images (e.g., three image at three different energies) does not provide more information, since the images only span a 2D basis formed by the two x-ray interactions; hence, the third image would be a linear combination of the first two. This also implies that it is not possible to cancel more than one material at a time.
However, if the applied x-ray energies were shifted to a region where more than two x-ray interactions occur, then it is theoretically possible to cancel more than one material type at a time, as in mammography where Raleigh scattering becomes significant (and linearly independent to Compton scattering and photoelectric absorption). One triple-energy application was studied in the mid 1970s by Kelcz and Mistretta\textsuperscript{18} investigating k-edge absorption to obtain a third x-ray interaction linearly independent to the first two. Initial results showed that both soft-tissue and bone could be cancelled from an image of soft-tissue + bone + iodine, yielding an ‘iodine-only’ image. Triple-energy imaging remains an interesting but challenging approach due to stronger requirements for quasi-monoenergetic spectra as well as increased image noise and radiation dose.

\section*{II.1.1 Material basis decomposition}

Expressing a DE image in terms of another basis yields a useful framework for decomposing DE images. It was shown in the previous section that the set of attenuation coefficients forms a basis; therefore, instead of employing the low-and high-energy images as bases, material basis decomposition, uses the fact that any two sufficiently different materials can form a linearly independent basis:\textsuperscript{17}

\begin{equation}
\begin{pmatrix}
I_L \\
I_H
\end{pmatrix} =
\begin{pmatrix}
A_1(E_L) & A_2(E_L) \\
A_1(E_H) & A_2(E_H)
\end{pmatrix}
\begin{pmatrix}
t_1 \\
t_2
\end{pmatrix}
\end{equation}

where \(A_1\) and \(A_2\) are the attenuation coefficients of the two basis materials, and \(t_1\) and \(t_2\) are path-lengths corresponding to each basis material. The material bases generally used are PMMA and Al, because they have similar attenuation properties to soft-tissue and bone, respectively. Most clinical imaging systems do not provide monoenergetic x-rays; therefore, the relationship
between $t_1$ and $t_2$ and the low- and high-energy signal is generally nonlinear. Such systems therefore require calibration, where Eq. (1.26) is solved empirically for the thicknesses by imaging several known thicknesses of PMMA and Al and fitting a polynomial function to solve for $t_1$ and $t_2$, the material basis images. In the case of chest imaging, a linear transformation (i.e., weighted summation) is then applied to the basis images (PMMA and Al images) to yield a soft-tissue and bone image. Material basis decomposition requires additional steps due to the calibration process and the solution to a complex inversion problem to obtain the material basis images, however the generated images are potentially more quantitative (i.e., material thickness information) than the simpler log subtraction algorithm. In terms of image quality, however, it has been shown that both techniques yield equivalent image quality (contrast, noise, and spatial resolution).\textsuperscript{12}

I.1.2 Equivalent radiographs

In addition to decomposing DE images, the low- and high-energy projection can also be used to form an “equivalent radiograph” at any x-ray energy (typically at energies between the low and high energy). This yields an image of nearly identical appearance to a conventional radiograph that can be viewed in combination with the soft-tissue and bone images. The equivalent radiograph is given by the addition of the low- and high-energy image weighted by a parameter:

$$I_{\text{equiv}} = I_L + w_e I_H$$

(1.27)

where $w_e$ is chosen qualitatively such that the contrast of soft-tissue and bone is similar to that in a conventional radiograph (typically $w_e \sim 0.6-0.9$, depending on patient thickness). Figure 1.7 compares an equivalent radiograph with a DR, where both images were acquired at the same
total dose and present comparable image quality. The equivalent radiograph removes the need for acquiring a standard radiograph in addition to the low- and high energy projections.

![Figure 1.7: (a) DR and (b) equivalent radiograph obtained from the low- and high-energy projection (see Fig. 1.5). Both images were acquired at the same total dose and present similar image quality characteristics.](image)

Furthermore, in the case of material basis decomposition, solving the inverse problem can also yield synthesized monoenergetic radiographs at any keV, which has potentially useful applications in CT for reducing beam-hardening artifacts.¹⁹

### II.2 X-ray Radiographic and DE Image Acquisition Systems

A significant advantage of DE imaging compared to various x-ray tomographic technologies is that it requires fairly minor changes in imaging hardware and can be implemented on a wide variety of familiar x-ray imaging platforms – e.g., upright chest stands, C-arms, and mobile imaging systems – without major changes to the tube or detector, patient support, etc. The sections below provide an overview of prevalent x-ray imaging modalities in medical imaging, followed by a description of their extension to DE x-ray imaging.
II.2.2 X-ray detector technologies in medical imaging

a. Screen-Film Radiography

The first radiograph ever acquired was of Mrs. Roentgen’s hand in 1895 using a photographic plate. Over the course of the 20th century, the technology for film radiography has improved tremendously, but the principle remains the same – x-rays are detected directly or indirectly (via a phosphor) using a film emulsion. Modern screen-film imaging systems employ a light-tight screen-film cassette, which consists of one or two intensifying screens and a sheet of film. The intensifying screens consist of a phosphor layer (typically Gd₂O₂S:Tb, YTaO₄, or CaWO₄) held by a support layer. X-rays interact with the phosphor to emit visible light, which in turn interacts with the film emulsion. X-rays also interact directly with the film emulsion but account for only ~5% of the film exposure. After exposure, the film is chemically processed and displayed on a light-box.

b. Computed radiography

Another radiographic acquisition system uses storage phosphor in place of the screen-film combination, termed computed radiography (CR). CR was developed later in the 1970s and became common in most medical centers by the end of the century. It employs a similar cassette as screen-film, but contains only a photo-stimulable phosphor plate, e.g., BaFBr and BaFI. Exposure of a storage phosphor plate excites atoms therein to metastable states rather than creating an immediate emission of light as with the phosphors used in screen-film cassettes. The exposure thereby forms a “latent” image via the excited electron states that undergo a transition from the valence band to the conduction band. In the process some electrons are trapped in slightly lower energy states called F-centers. After exposure, the storage phosphor plate is
scanned using a red laser that stimulates trapped electrons to the conduction band, followed by transition to the valence band, resulting in the emission of blue-green light, with intensity proportional to the absorbed x-ray energy. The emitted light intensity is recorded on a pixel-by-pixel basis during laser scanning to produce a radiographic image.

c. Flat-panel detectors

Developments in active matrix technologies (e.g., active matrix liquid crystal displays, AMLCDs) in the 1990s gave rise to flat-panel detectors (FPDs), which typically consists of an active matrix array of sensors and switching elements that convert incident x-rays or optical photons into a digital signal to produce 2D radiographic images. For example, a-Si:H TFTs act as a switching element on each pixel to read the signal integrated in each sensor element. The conversion from x-rays to electrons is either indirect (using a phosphor) or direct (using a photoconductor to convert x-rays directly to electron-hole pairs). Both technologies have been the focus of recent research\textsuperscript{20-30} and a brief description of their operation and application is provided below.

Figure 1.8 (a) Indirect detection FPD, in which a scintillator is coupled to photodiodes. (b) Direct detection FPD, in which a photoconductor directly converts x-rays to electron-hole pairs, which are collected and constrained against lateral spread by an applied electric field.
For indirect conversion FPDs, [Fig. 1.8(a)] each pixel element consists of an overlying scintillator layer (typically CsI:TI or Gd₂O₂S:Tb) to convert x-rays into optical photons which are in turn detected by the photodiode and converted to electron-hole pairs. Each photodiode is coupled to a TFT (Fig. 1.9), and each row of pixels is connected to gate driver circuitry to supply switching voltage to the gate of each TFT. Initially in the readout sequence, all rows have negative voltage so that the TFTs are non-conducting and the photodiodes integrate incident signal. The gate driver then switches a single row of TFTs to a conducting state, and the current from each photodiode passes along data lines to external amplifiers. The TFTs on the gate line are then turned back to negative, and the process is repeated for each row until the entire matrix is read out. For direct conversion FPDs, [Fig. 1.8(b)] the readout method is typically the same (an active matrix of TFTs), but instead of a phosphor and photodiode combination, signal is generated directly in a photoconductor (typically a:Se or PbI₂) by converting the incident x-ray directly into electron-hole pairs, which are in turn collected by an electric field applied across the photoconductor layer. The charge is then stored in a storage capacitor and read out by the TFT array.

Figure 1.9: Diagram of an FPD imaging array. Horizontal gate lines control the TFT switch and the signal is read via the vertical data lines to external amplifiers. PD = Photodiode. TFT = thin-film-transistor. Amp = current-to-voltage integrating amplifier.
Physical properties of the scintillator and other components of the FPD affect the conversion efficiency, conversion noise, and spatial resolution. These factors need to be taken into account when designing FPDs for medical applications where an optimal gain and resolution is required in order to minimize radiation to the patient while maximizing image quality. The results below pertain primarily to indirect-detection FPDs (CsI:Tl scintillator), although the methods are sufficiently general to apply to either detection technique.

II.2.3 DE imaging systems

There are two main techniques for DE image acquisition: 1) single-shot DE image acquisition, in which the low- and high-energy projections are acquired simultaneously, and 2) double-shot DE image acquisition, in which the low- and high-energy projections are acquired sequentially.

a. Single-shot DE imaging

In a single-shot DE system, the cassette is modified by including an additional detector (e.g., a film-screen combination or CR plate), with an x-ray filter sandwiched between the two detectors. The filter generally consists of a Cu sheet or other highly attenuating metal to harden the beam by preferentially attenuating low-energy x-rays, resulting in a relatively low-energy image captured by the front detector, and a relatively high-energy image captured by the back detector. A low-attenuation phosphor screen (e.g., Y2O3:S) and a high-attenuation phosphor screen (e.g., Gd2O2S:Tb) can also be used for the front and rear screens, respectively, to obtain increased spectral separation and higher detection efficiency. Typical x-ray energies vary from 100 to 140 kVp. Following exposure, the two detector images are processed as described above (e.g., log subtraction) to yield a DE image. Most modern single-shot imaging systems use passive storage phosphor detectors due to the fact that the images are inherently digital and do not require an
extra digitization step as in film radiography;\textsuperscript{32} furthermore, comparable image quality can be obtained with a decreased dose in CR.

\textit{b. Double-shot DE imaging}

Double-shot DE imaging involves a pair of projections acquired in successive exposures at low- and high-energy. Such can be accomplished, for example, using active storage phosphor systems or FPDs. Active storage phosphor technology\textsuperscript{33} employs a storage phosphor plate without the use of an inter-detector filter. The first exposure uses a high-kVp beam which yields a latent image in both phosphors. A high-intensity flash lamp is used to illuminate the front plate, thereby erasing the latent image without erasing the second screen (which is optically insulated). A second exposure uses a low-energy beam to form a low-energy latent image on the front plate. The transmission to the second plate is assumed to be minimal. The application of a filter wheel on the x-ray tube synchronized to the low- and high-energy exposures further increases the spectral separation between low- and high-energy spectra.

DE imaging using FPDs involves sequential delivery of a low-energy beam, readout of the low-energy projection, delivery of a high-energy beam, and readout of the high-energy projection. Similar to the active phosphor system the use of a filter wheel on the x-ray tube synchronized to the low- and high-energy exposures further increases spectral separation between the low- and high-energy projections. After double-shot acquisition, the projections can be rapidly decomposed into DE images and displayed for viewing. The use of FPDs for DE imaging has been investigated by many authors\textsuperscript{12,34,35} and appears to offer a very promising approach to DE imaging.
c. Other DE image acquisition systems

In the past 40 years numerous x-ray imaging technologies were developed and extended to DE imaging. Some work was focused on developing sources exhibiting monoenergetic characteristics. Examples include the use of synchrotron radiation,\textsuperscript{36} which provides monoenergetic x-ray sources, but requires highly specialized facilities. Also, the use of k-fluorescence to provide quasi-monoenergetic x-rays has been investigated using x-ray filters or anode material with specific k-edge energies to increase iodine contrast for angiography.\textsuperscript{37} Other techniques have focused on reducing x-ray scatter, as in linearly scanned projections using a xenon ionization detector\textsuperscript{38} with a front and rear region for detection of low and high-energy x-ray, similar to the single shot-technique discussed above. Other developments include the use of double-shot acquisition techniques with CCD-based x-ray detectors.\textsuperscript{39}

d. Comparison of double-shot and single-shot DE imaging

The main advantage of single-shot DE imaging is that the low- and high-energy images are temporally coincident and, therefore, perfectly registered (with a possible – fairly small – difference in magnification factor). However, obtaining good spectral separation using an inter-detector x-ray filter is challenging and often results in a significant spectral overlap between low- and high-energy projections that reduces the quality of DE image by reducing contrast compared to double-shot DE imaging systems as demonstrated by Alvarez et al.,\textsuperscript{33} Ho et al.,\textsuperscript{40} and Shaw et al.\textsuperscript{41} Furthermore, the inter-detector filter absorbs a significant fraction of image quanta – representing a large reduction in system efficiency and increasing image noise.

On the other hand, one of the main challenges in double-shot DE imaging is that the finite time interval between low- and high-energy projections, during which anatomical motion caused by
respiration or cardiac motion can introduce significant artifacts in the DE decomposed image. For the active storage phosphor system, time delays are on the order of 50 ms. For FPD-based systems, inter-exposure delays can range from ~20 ms to several seconds. However, motion artifacts can be mitigated through patient breath-hold, cardiac-triggering, deformable image registration, and improved readout speed, which are the focus of ongoing research.¹²,³⁵,⁴²-⁴⁴

III CLINICAL APPLICATIONS OF DUAL-ENERGY X-RAY IMAGING

With the advent of inherently digital radiographic technologies such as FPDs, DE imaging presents a natural extension of such modalities to improve conspicuity through material discrimination. Such implies a potentially broad range of applications and – to the extent that a DE image may be acquired with speed, workflow, and radiation dose equivalent to that of a conventional radiograph – the potential for DE imaging to become a new normal in projection imaging. Below, is a brief overview of the various potential applications for DE imaging in a wide range of clinical contexts, ranging from projection imaging to tomographic imaging – some of which are well underway and nearing clinical implementation (e.g., chest) and some of which form the subjects of future research.

III.1 DE Radiography

III.1.1 Thoracic imaging

Thoracic imaging typically entails the visualization of lungs, mediastinum, heart, and chest wall, and according to the Canadian Association of Radiologists, (www.car.ca) approximately half of all x-ray radiography exams obtained in medical institutions are thoracic exams. In the United States, this equals 70-100 million chest radiographs per year (www.atp.nist.gov). The potential to improve on this extremely prevalent imaging technology – and considering the challenges to x-ray projection imaging associated with anatomical background structure described above –
makes DE imaging a very promising technology. For example, Odagiri et al.\textsuperscript{45} conducted a study on the use of DE imaging and found that tracheobronchial abnormalities are better visualized using DE images than in CR images. Kamimura and Takashima also showed that DE imaging improves detection of pulmonary vascular abnormalities.\textsuperscript{46} DE imaging also improves on conventional radiography in the evaluation of asbestos-related pulmonary and pleural abnormalities.\textsuperscript{47,48} The soft-tissue image removes pleural calcifications to better allow the evaluation of the lung parenchyma, while the bone image enhances the detection of pleural calcifications, thus potentially eliminating the need for CT and providing a confident diagnosis for asbestos-related disease. Furthermore, DE soft-tissue images permit better visualization of tracheal and airway abnormalities by removing the overlying thoracic spine. Whereas the bone image improves characterization of bone lesions such as bone metastases, primary bone tumors, rib fractures, rib erosion and bone islands are all better visualized in DE bone images than in conventional radiography.\textsuperscript{49}

\textbf{III.1.2 Early-stage lung cancer detection}

One of the most promising applications of DE imaging is early detection of lung nodules for lung cancer screening and diagnosis. Lung cancer is the leading cause of cancer mortality in both men and women, accounting for 30\% of cancer deaths. It is the second most common cancer in North America, but kills more Canadians than the next three cancers (breast, colon, and prostate) combined. In 2004, approximately 22,000 Canadians were diagnosed with lung cancer, and approximately 19,000 died from the disease (www.cancer.ca). Overall lung cancer has a very poor prognosis, and early detection is the most important factor in surviving lung cancer. For example, 5-year survival for a capsulated Stage I non-small cell lung cancer is \(~60-70\%\), but 5-year survival plummets to approximately 1\% if the disease has metastasized (Stage IV).
Therefore, a major goal in lung cancer screening of at-risk populations is to detect cancer at Stage I, when it is potentially curable and the tumors are typically less than 3 cm with no nodal or metastatic involvement.

An effective screening system for early detection is therefore essential in reducing mortality. Traditional chest radiography has proven to be inadequate, missing ~30% of nodules on first read and rarely indentifying nodules <10 mm. By comparison, more sensitive modalities, such as low-dose CT, can detect six times as many Stage I tumors. In CT, the sensitivity of nodal disease detection (75%) exceeds specificity (66%) and accuracy (69%). Therefore a challenge remains in reducing the rate of false positives (false alarms). DE imaging has shown promising application in the detection of early-stage lung cancer. Several studies have investigated the performance of DE imaging compared to conventional radiography for the detection of small pulmonary nodules and calcifications. Significant improvement was found in both cases, where an increase in the area under the receiver operating characteristic (ROC) curve (i.e., a plot of sensitivity and
specificity, for which a value of 1 corresponds to perfectly sensitive and accurate detection) was improved from 0.82 for conventional radiography to 0.96 for DE imaging.\textsuperscript{47,50} An illustration of the improved conspicuity of an isolated nodule behind a rib is shown in Fig. 1.10.

III.1.3 Musculoskeletal imaging

![Figure 1.11: (a) DR and (b) DE bone image displaying a rib fracture.](image)

Another potential application for DE imaging is musculoskeletal imaging, which involves the examination of the skeleton, ligaments, muscles, and joints. DE imaging can be used to improve material differentiation, such as visualizing fractures in areas with overlying soft-tissue (for example, as in Fig. 1.11), differentiating rib metastases from fractures, detecting swelling vs. tears in ruptured patella tendon, and removal of soft tissue obscuring visualization of the C7T1 vertebral junction. DE imaging can furthermore improve on existing imaging techniques in terms of material classification: subtle scaphoid (occult) fractures, ligaments in the knees, tendons, fat
and muscle in the rotator cuff, thoracic spine, calcification of cartilage, and diagnosis of diseases such as gout, triangular fibrocartilage complex injuries, and chondrocalcinosis - calcium pyrophosphate dihydrate crystal deposition.

III.1.4 Mammography

X-ray mammography is the predominant image-based tool for screening and diagnosis of breast cancer. Compared to general radiography, mammography typically involves lower x-ray energies due to smaller tissue volumes and the necessity for higher soft-tissue contrast. In principle, low- and high-energy x-ray projections of the breast can be acquired using similar radiographic technology as described in Sec II.2 but with lower energies, typically 15-25 kVp and >40 kVp, for the low- and high-energy projections, respectively, and current developments are also focusing on the application of FPD technologies and contrast-enhanced breast imaging. An important application for DE mammography is in early breast cancer detection. The detection of micro-calcification is often hindered by background structures such as adipose tissue, glandular tissue, vessels, and ducts, similar to the situation found in chest radiography and lung nodule detection. Moreover, contrast-enhanced DE imaging of the breast offers the potential to better visualize blood perfusion abnormalities associated with breast disease.

III.2 Real-Time DE Imaging

X-ray fluoroscopy involves the rapid, real-time acquisition of sequential projection images, important to a broad range of applications throughout interventional guidance. Extending such capability to DE images requires rapid acquisition of low- and high-energy projections and decomposition of DE images (each <~30-100 ms). Initial work has employed x-ray image intensifiers and a double-shot acquisition mode. Recent focus has shifted to using FPDs, with
fluoroscopic DE systems reported to provide 30 fps low- and high energy projection acquisition, resulting in 15 fps real-time DE imaging.

### III.2.1 Cardiac imaging

Coronary artery disease is the second cause of mortality (after cancer) in Canada, accounting for approximately 60,000 deaths (www.statcan.ca) and was responsible for over 12 million deaths worldwide in 2001 (www.who.int). Diagnosis of atherosclerotic heart disease can be improved through quantitative visualization of coronary artery calcification, which is difficult to detect in x-ray projections due to background anatomical noise. CT offers vastly improved 3D and 4D visualization of the coronary artery, and the development of a more cost-effective alternative to CT imaging could make the detection of coronary artery calcification more widely available. The use of real-time DE imaging for coronary artery calcium detection and quantification has been investigated by Molloi et al.\textsuperscript{54,35} and results showed that a low-dose clinical system is feasible.

### III.2.2 DE image-guidance

The ability to better visualize the placement of interventional tools used during medical intervention could significantly improve the precision of image-guided procedures. Examples include: intra-aortic and biliary stent placement (e.g., as shown in Fig. 1.12), stent integrity evaluation, stent fragment retrieval, and catheter placement (e.g., Swan-Ganz, NG tubes, PICC lines, etc.). Applications also include needle tracking, (e.g., thoracic or breast biopsies) and foreign object detection (e.g., airway obstruction, esophageal obstruction (fish-bones), and pre-MRI screening of the orbits for patients with a history of metal-work).
Figure 1.12: (a) Digital radiograph of an anthropomorphic phantom and (b) DE image showing improved visibility of a stent.

Another potential application of DE image guidance is in radiation therapy, which requires precise patient positioning to deliver a prescribed dose to the tumor while sparing the surrounding healthy tissues. Megavoltage portal imaging is often used but suffers from poor tissue contrast in the MeV x-ray range. More recently, cone-beam CT (CBCT) systems employing FPDs have been used to provide 3D imaging at the time of treatment. However, DE imaging may offer a faster, low-dose adjuvant to CBCT in improving the visibility of implanted markers used for patient positioning and visualizing target volumes directly – particularly in the lung.

III.3 Volumetric DE Imaging

DE imaging can be extended to volumetric imaging, where the low- and high-energy projections information is processed to yield a DE image used for volumetric reconstruction. Below are brief descriptions and examples of how the acquisition of separate low- and high-energy x-ray
projections can be used in tomographic imaging. Such approaches potentially resolve the challenges associated with both spatial and material discrimination that limit conspicuity.

**III.3.1 DE Tomosynthesis**

DE tomosynthesis, as mentioned in Sec I, acquires projections at multiple angles around the patient to provide spatial discrimination and be extended to DE imaging by acquiring low- and high-energy projections at each view to yield a set of DE images that can be reconstructed using conventional tomosynthesis techniques. A potential application of DE tomosynthesis includes lung nodule detection for early cancer screening, which has been investigated by Sone et al.\textsuperscript{56} for the examination of calcifications in pulmonary lesions. Preliminary results indicated that use of the bone tomogram improved the recognition of calcification and provided the ability to differentiate between calcification and dense fibrotic lesions. Similarly, DE tomosynthesis could also have applications in diagnosis of coronary artery disease by providing improved vessel imaging in intravenous angiography.\textsuperscript{57}

**III.3.2 DE CT**

CT combined with DE imaging can be used to obtain improved quantitative 3D imaging compared to standard CT reconstruction. For example, DE CT can provide physical density information in addition to electron density (i.e., Hounsfield units). Furthermore, beam-hardening artifacts that are sometimes a problem in conventional CT can be reduced through the use of synthesized mono-energetic projections.\textsuperscript{19} DE CT images can be reconstructed with a given material cancelled from the image and improve the contrast of other materials as illustrated in Fig 1.13. In this example (obtained on the imaging bench of Fig. 1.13 in Chapter 2), bone-cancelled and soft-tissue-cancelled reconstructions are shown in comparison with standard CT
reconstructions. In Fig 1.13(c) the bone signal is removed from the reconstruction (and thereby replaced with air). Similarly in Fig 1.13(d) the bone weighted image cancels the soft-tissue material (and replaces soft-tissues with air). DE CT can therefore improve contrast (e.g., liver contrast with respect to water is significantly increased in Fig 1.13(d) compared to Fig 1.13(a)-(b)) and provide additional information for improved tissue differentiation.

Figure 1.13: Example of (a) low- and (b) high-energy CT reconstructions and DE CT reconstruction with (c) bone and (d) soft-tissue removed.
Clinical applications for tissue differentiation include differentiating focal fatty infiltration of the liver from neoplastic hypodense masses,\textsuperscript{58} where results have shown a significant increase in the ability to differentiate the two compared to traditional CT. Another application includes kidney stone identification, where symptomatic kidney stones affect almost 100,000 people in Canada each year. Conventionally, kidney stone diagnosis has been performed using CT or x-ray projection imaging with or without intravenous contrast agents. Over the past recent years CT has been used more frequently due to its increased sensitivity, speed, and lack of intravenous contrast; however, CT does not differentiate stone composition.\textsuperscript{59} Therefore, improved differentiation of uric-acid and non-uric acid kidney stones has been investigated using DECT with very promising results, where sensitivity and accuracy were found to be >95\% in most cases.\textsuperscript{60}

III.4 A Better Use of X-Rays

X-ray radiography is the most common medical imaging examination in Canada. It is often the first and only diagnostic test performed on a patient, and the ability to improve on this highly prevalent imaging system could be significantly beneficial to patient care over a broad range of clinical applications. The advent of digital detectors enables rapid acquisition and decomposition of DE images, and improved detection efficiency permits DE image acquisition at doses equivalent to conventional radiography. These advances have resulted in a revival of interest in an imaging technology that is over 50 years old. It stands to reason that high-performance DE imaging systems using FPDs have the potential to significantly improve lesion conspicuity through the more knowledgeable application of x-ray energies applied to the patient, analysis (decomposition) of the resulting images, and the ability to boost the conspicuity of subtle abnormalities through the reduction of confounding background anatomical noise.
IV OVERVIEW OF THESIS

IV.1 Hypothesis and Aims

This thesis presents a theoretical and experimental investigation of DE x-ray imaging performance with the following general hypothesis:

The imaging performance of dual-energy x-ray radiography can be maximized through evaluation and optimization of Fourier-based metrics of spatial resolution and noise (including NPS, NEQ, and detectability index) combined with a quantitative description of imaging task.

The work was carried out in terms of four specific aims. These include:

Aim 1) Extension of theoretical modeling of system performance to DE imaging

Aim 2) Optimization of DE imaging acquisition parameters and decomposition techniques

Aim 3) Extension of framework to DE image noise reduction algorithms

Aim 4) Validation of the theoretical model in comparison to human observer performance where completion of each aim resulted in a published work as described below and in Table 1.2.

IV.2 Outline for Thesis

As shown in Table 1.2, the core chapters of this work are adapted versions of published work. Specifically, Chap. 2 provides an introduction and review of prevalent image quality metrics, including Fourier-domain metrics such as the modulation transfer function (MTF), noise power spectrum (NPS), detective quantum efficiency (DQE), and noise equivalent quanta (NEQ). Such metrics are essential to performance characterization in radiography and present experimental and theoretical (cascaded systems analysis) methods for system evaluation. Chapter 3 extends these Fourier-based metrics to DE imaging, yielding figures of merit for image performance in
terms of a detectability index. Chapter 4 employs the theoretical foundations developed in Chapters 2 and 3 to optimize the DE image acquisition and decomposition parameters. Chapter 5 extends cascaded systems analysis to DE imaging noise reduction algorithms, which are higher-performance variations of the basic log subtraction technique. Chapter 6 applies the work to a variety of imaging tasks (including detection and discrimination) and different model observers in DE imaging and compares the predicted performance to that of real human observers. Finally, Chapter 7 provides a summary of results and concluding remarks.

<table>
<thead>
<tr>
<th>Aim</th>
<th>Work presented in Chapter:</th>
<th>Published in whole or in part in:</th>
</tr>
</thead>
</table>

Table 1.2 : List of published work most relevant to each Chapter.
CHAPTER 2: RADIOGRAPHIC IMAGING PERFORMANCE

1 INTRODUCTION

The ability to quantify imaging performance is essential to the development of medical imaging systems, particularly in the early stages of system design and pre-clinical evaluation. It provides a means of understanding the factors that limit image quality and optimizing performance. Ultimately, imaging performance metrics should answer the question of how well the imaging system allows an observer to perform a given task.

This chapter provides an overview of experimental and theoretical metrics and methods of imaging performance evaluation. First, a brief overview of prevalent imaging performance metrics employed in radiographic imaging is presented. Spatial domain and Fourier domain metrics for quantifying image quality are introduced, covering basic theory and empirical method of assessment. Secondly, cascaded systems analysis (CSA) for modeling system performance is presented, providing a theoretical framework for computing Fourier performance
metrics over a wide range of imaging conditions. Finally, the detectability index is derived as a metric that includes a description of imaging task in performance evaluation, laying the foundation for work in later chapters.

II SPATIAL DOMAIN METRICS OF IMAGING PERFORMANCE

II.1 Contrast, Noise and Signal-to-Noise Ratio

As discussed in Chapter 1, contrast is an important factor affecting conspicuity and is defined for 2D images as:

\[ C = \langle I_1(x, y) \rangle - \langle I_2(x, y) \rangle \] (2.1)

where \( I(x,y) \) denotes the two-dimensional signal, \( \langle \rangle \) denotes the mean signal over a 2-D region, and 1 and 2 denote the regions of interests (ROI) for the signal and the background, respectively (see Fig 2.1). Written this way, contrast represents a mean signal difference (and carries units of signal).

![Figure 2.1: Illustration depicting the region of interest (ROI) for evaluation of contrast, noise, and SDNR.](image)

Image noise refers to signal fluctuations that are not associated with the structure of interest. Sources of such fluctuation include stochastic variations in signal owing to Poisson-distributed (i.e., quantized) incident x-rays as well as random variations in signal arising from readout

41
electronic. The magnitude of noise in an image can be characterized by the pixel standard deviation across a 2D region of interest:

$$\sigma_i = \sqrt{\langle I_i^2(x, y) \rangle - \langle I_i(x, y) \rangle^2}$$

(2.2)

where $i$ denotes the $i$th ROI. Defined this way, the image noise has the same units as the signal.

The ratio of contrast and noise gives a useful (unitless) metric [called the contrast-to-noise ratio (CNR) or, more precisely, the signal-difference-to-noise ratio (SDNR)] that is often used for assessment of image quality in medical images:

$$SDNR = \frac{C}{\sigma} = \frac{\langle I_1(x, y) \rangle - \langle I_2(x, y) \rangle}{(\sigma_1 + \sigma_2)/2}$$

(2.3)

where $\sigma$ denotes the average noise for the two regions. The SDNR provides a simple, intuitive metric describing the conspicuity of a structure of interest that is large (with respect to the correlation lengths of the imaging system) and presents a small signal difference to the background.

II.2 Limitations of Large-Area Spatial-Domain Metrics

Large-area spatial-domain image quality metrics such as contrast and SDNR are often simple to compute and provide practical figures of merit in certain cases; however, they generally do not convey the whole picture. For example, the displayed contrast of digital images can be set to any arbitrary value (through adjustment of the digital window and level of the display), and SDNR has been shown to poorly describe image quality when low- or high-pass filters are applied to the image.\(^6^1\) Furthermore, large-area spatial-domain metrics are not amenable to the inclusion of a more general definition of imaging task\(^6^2\) as described in Sec. III.5; therefore; a more
comprehensive set of metrics is required to better describe the image statistics, the correlations within the image, and ultimately the detectability of a structure of interest.

III  FOURIER-DOMAIN METRICS OF IMAGING PERFORMANCE

Fourier-based metrics are a prevalent approach to characterizing medical imaging performance. Initially developed in the field of electronic engineering to analyze 1-D signals in communication systems in terms of temporal frequencies and frequency-dependent transfer characteristics of an electronic circuit, these Fourier-based techniques were later applied to 2-D signals in optical and imaging systems in terms of spatial frequencies. Below is an overview of the various Fourier-based metrics that have been broadly used in evaluating radiographic imaging performance and a brief mathematical foundation for the theoretical work presented in subsequent chapters.

III.1  Modulation Transfer Functions

III.1.1  Linear and shift-invariant systems

An imaging system can be described in terms of Fourier-based metrics only if the system is linear and shift-invariant (LSI). The response of a system is said to be linear as a function of input quanta if it obeys superposition of two or more inputs. Specifically, let $T\{\}$ describe the transfer characteristics of a given system such that $f(x, y)$ is the input and $T\{f(x, y)\}$ is the output. Therefore a system is linear if and only if:

$$T\{f_1(x, y) + f_2(x, y)\} = T\{f_1(x, y)\} + T\{f_2(x, y)\}$$  \hspace{1cm} (2.4)

For some nonlinear systems, it may be possible to linearize the system by a nonlinear transformation; alternatively, linearity may hold only over a certain range of signal values corresponding to a range of linear operation of the imaging system.
A system is shift-invariant if the output is independent of the location of the input. Therefore for any spatial shift $x_0$ and $y_0$ it follows that for a shift-invariant system:

$$T\{f(x, y)\} = T\{f(x + x_0, y + y_0)\}$$

(2.5)

For real imaging systems, such as FPDs, the LSI assumptions are not strictly obeyed. For example, for sufficiently high exposures, detector response saturates (corresponding to the signal capacity of the photodiodes). However, FPDs have been shown to be highly linear up to a certain signal level (e.g., ~50% of sensor saturation) and are certainly “linearizable” within a given, narrow range of exposures. Shift-invariance is somewhat more difficult to justify for discretely sampled digital systems. Cunningham has argued that such systems are “cyclically” invariant (i.e., invariant under shifts equal to the pixel spacing) and that such is sufficient for the application of Fourier-based characterization. Further, Albert and Maidment have shown that the degree to which shift-invariance is violated is fairly minor over a broad range of conditions and for a very broad class of tasks. Most recently, Badano et al. have suggested that such detectors exhibit highly shift-variant response depending on the angle of x-ray incidence (i.e., oblique rays) such that the shift invariance assumption would only be expected to hold over a limited spatial range. All considered, the extent to which FPDs obey LSI assumptions is a subject of some debate, but can be considered a reasonable assumption over narrow ranges of exposure level and spatial extent.

### 3.1.2 Impulse response and Transfer functions

The output signal is related to the input signal as governed by the impulse response function as follows:
\[ I(x, y) = \int \int_{-\infty}^{\infty} f(x-x', y-y') PSF(x', y') dx' dy' \]  \hspace{1cm} (2.6)

where \( PSF(x, y) \) is defined as the point spread function (PSF) such that:

\[ \int \int_{-\infty}^{\infty} PSF(x, y) dx dy = 1 \]  \hspace{1cm} (2.7)

The Fourier representation of the PSF is the optical transfer function (OTF):

\[ OTF(u, v) = F\{PSF(x, y)\} \]  \hspace{1cm} (2.8)

where \( F\{\} \) denotes the Fourier transform. The modulation transfer function (MTF) given by the modulus of the OTF:

\[ MTF(u, v) = |OTF(u, v)| \]  \hspace{1cm} (2.9)

which provides a useful metric describing the magnitude of transferred signal at each spatial frequency. Note that: 1) Eq. (2.7) implies that \( MTF(0)=1 \) (DC signal magnitude is unchanged by the transfer function); and 2) an ideal system has \( MTF=1 \) at all spatial frequencies. The MTF is often employed to characterize the resolution of an imaging system in terms of \( MTF(f) \) or the frequency \( (f_{50} \text{ or } f_{10}) \) at which \( MTF(f) \) reduces to 0.50 or 0.10, respectively.

Experimentally, it can be difficult to measure the PSF directly, since such would require imaging of a 2D delta function so that \( I(x, y)=PSF(x, y) \) in Eq. 2.6, therefore requiring the imaging of an infinitesimally small object (e.g., a pinhole placed on the detector surface), for which the mean signal magnitude would be correspondingly small (and the resulting image dominated by noise).

More convenient impulse functions are typically used in such measurements, such as the line spread function (LSF, which is the Radon transform of the PSF):

\[ LSF(x) = \int PSF(x, y) dx \]  \hspace{1cm} (2.10)
which can be measured from the image of a narrow slit\(^66\) or the edge spread function (ESF, given by the anti-derivative of the LSF):

\[
ESF (x) = \int_{-\infty}^{x} LSF (x') dx'
\]  \hspace{1cm} (2.11)

which can be measured from the image of a sharp edge.\(^66\) This can be rewritten:

\[
LSF (x) = \frac{\partial}{\partial x} ESF (x)
\]  \hspace{1cm} (2.12)

Figure 2.2: Illustration of the relationship between the PSF, LSF, ESF, and MTF.

Hence, the PSF, LSF, ESF, and MTF are all related. For example, for a radially symmetric MTF, Eqs. [(2.7)-(2.11)] together with the Fourier slice theorem suggest that the 1-D MTF (i.e., a slice of the full 2-D MTF) is given by:\(^67\)

\[
MTF(f) = \left| \frac{\int \frac{\partial}{\partial x} ESF(x) \exp(-2\pi f x) dx}{\int \frac{\partial}{\partial x} ESF(x) dx} \right|
\]  \hspace{1cm} (2.13)
where the denominator ensures that $\text{MTF}(0)=1$ [equivalent to Eq. (2.7)]. Therefore, the MTF can be experimentally assessed by imaging a sharp edge object to yield an estimate of the ESF, which in turn is related to the LSF (by derivative) and MTF as in Eq. (2.13). Figure 2.2 summarizes the relationship among the three impulse functions and the MTF in terms of the 1D and 2D Fourier transform.\textsuperscript{63}

III.2 Stochastic Processes and the Noise-Power Spectrum (NPS)

III.2.1 Random processes

A stochastic process defines a series of random variables that fluctuate about a statistically defined mean and standard deviation of the random variable. For example, fluctuations in pixel signal (i.e., image noise) can be described as a stochastic process. Consider a random variable $a(x)$, which could represent the signal in a radiographic image by extending it to 2D [i.e., $I(x, y)$]. The expected value of $a(x)$ is given by:

$$E\{a(x)\} = \int_{-\infty}^{\infty} \lambda P_{a(x)}(\lambda) d\lambda$$

(2.14)

where $P_{a(x)}(\lambda)$ is the probability density function (PDF) of $a(x)$, often assumed to be Poisson or Gaussian in radiographic images. Defining $\Delta a(x) = a(x) - E\{a(x)\}$, the autocovariance, which describes the correlation of $a(x')$ with itself at a location displaced by $x$ about the expected value, is given by:

$$K_a(x) = E\{\Delta a(x')\Delta a^*(x'+x)\}$$

(2.15)

where $*$ denotes the complex conjugate. The autocovariance is more informative and generalizes the pixel standard deviation, since:\textsuperscript{67}
Thus, the scale value of the autocovariance – i.e., the pixel variance – characterizes the magnitude of signal fluctuations in the spatial-domain.

### III.2.2 Stationarity

Note that characterization of a stochastic process in such terms requires not only that the system is LSI as discussed above but also that it is stationary (i.e., that the mean and variance do not depend on the spatial location in the image). There are several factors that limit the stationarity of real medical imaging systems – e.g., spatial dependence of the mean energy of incident x-rays (e.g., Heel effect); however, as with linearity and shift-invariance, the assumption is taken to hold at least over a reasonable spatial extent. Highly non-stationary systems could alternatively be considered in terms of spatially varying statistical measures – i.e., 2D space-frequency analogous to 1D time-frequency of temporally non-stationary processes. 67

Moreover, there are different degrees of stationarity that a system may obey, the most relevant of which for the work presented in this thesis is termed ‘wide-sense stationarity’ (WSS), which implies that the expected value (mean) and autocovariance function (variance) are independent of the spatial location. Further, ergodicity implies that the image statistics can be determined from either an ensemble average (over multiple images) or a spatial average (over multiple samples within a single image). An ergodic system can be shown to be WSS. Therefore, it follows that for an ergodic WSS system, the expected signal for a 2D image, \( E\{I(x,y)\} \), is equivalent to \( \langle I(x,y) \rangle \), the mean signal over a 2D region, corresponding to the notation in Sec. II. A more complete discussion on stationary random processes in medical images can be found in the Handbook of Medical Imaging. 67
III.2.3 Fourier description of noise

Given a stationary stochastic process, the noise-power spectrum (NPS) provides an equivalent Fourier-based representation of the autocovariance function:

\[
NPS_a(x) = F[K_a(x)]
\]  \hspace{1cm} (2.17)

which describes the variance contribution at each spatial frequency, \( f \). This can be rewritten in terms of an integral (assuming a WSS process):\(^67\)

\[
NPS_a(f) = \lim_{X \to \infty} \frac{1}{X} \mathbb{E} \left\{ \int_X \Delta a(x) \exp(-2\pi f x) dx \right\}^2
\]  \hspace{1cm} (2.18)

which provides a practical form for experimentally determining the NPS of an image, rewritten for a finite area measurement on a 2D projection as follows:

\[
NPS(u, v) = \left\langle \frac{1}{A} \left| \iint_A (I(x, y) - \langle I(x, y) \rangle) \exp(-2\pi i(ux + vy)) dx dy \right|^2 \right\rangle
\]  \hspace{1cm} (2.19)

where \( A \) denotes the area over which the NPS is computed. Note that the NPS has units of signal-squared times space-squared (for a 2D image). The relative or normalized NPS, denoted NNPS, is given by the absolute NPS divided by the mean signal squared:

\[
\text{NNPS}(u, v) = \frac{NPS(u, v)}{\langle I(x, y) \rangle^2}
\]  \hspace{1cm} (2.20)

Finally, it can be shown that the NPS is related to the pixel standard deviation using Parseval’s theorem and Eqs (2.2 and 2.16):

\[
\sigma^2 = \int \int NPS(u, v) du dv
\]  \hspace{1cm} (2.21)
The NPS therefore provides a description of not only the magnitude of pixel fluctuations (i.e., the variance) but also the spatial-frequency content of the noise – i.e., the extent to which fluctuations between pixels are correlated.

### III.2.4 Anatomical background NPS

The NPS as described above is effective in describing quantum noise in an imaging system but must be slightly adapted to characterize background anatomical noise as described in Chapter 1. As discussed, background anatomical noise may far outweigh quantum noise and electronic noise in an image; therefore, background anatomical noise can be the most important factor in limiting detectability\(^8\) – for example, in the detection of lung nodules in the presence of overlying ribs in a chest radiograph as discussed in Chapter 1. For this reason, it becomes worthwhile to include an additional NPS term, \(NNPS_B(u,v)\), corresponding to image fluctuations associated with background anatomical structure.\(^{68,69,70}\) As with numerous processes that do not lend themselves to a simple statistical description (e.g., fluctuations in certain electrical circuits, the frequency of sunspots, cardiac arrhythmia, and precession of the earth’s axis), a common empirical form for the background anatomical NPS involves frequency dependence in proportion to a \(1/f\) characteristic:\(^{71-74}\)

\[
NNPS_B(u,v) \approx \frac{K}{f^{\beta}}
\]  

(2.22)

where \(f\) is a radial spatial frequency term \([f^2=u^2+v^2]\), and \(K\) and \(\beta\) quantify the magnitude and frequency dependence of the background anatomical noise, respectively. Such an empirical description of anatomical clutter has shown to provide a reasonable description of background anatomical noise in x-ray mammography.\(^{75}\) It is now possible to define a figure of merit that accounts for both the quantum noise and the background anatomical noise.
III.3 Detective Quantum Efficiency (DQE)

The detective quantum efficiency (DQE) is an important figure of merit in detector performance analysis\textsuperscript{67} and can be defined in several equivalent ways (see Table 2.1). The first is in terms of the MTF and NPS along with the incident quantum fluence, all measurable quantities as described above, yielding the “experimental” form of the DQE:

$$DQE_{\text{exp}}(u, v) = \frac{MTF^2(u, v)}{\bar{q}_0 \cdot \text{NNPS}(u, v)}$$ \hspace{1cm} (2.23)

where $\bar{q}_0$ is the mean fluence of quanta at the detector. The fluence carries unit of [x-rays/mm\(^2\)] and therefore the DQE carries no units.

This description of the DQE can be reformulated in a “stochastic” form,\textsuperscript{76} in which the actual (measured or predicted) NPS, $\text{NPS}(u, v)|_{\text{actual}}$, is related to the deterministic NPS, $\text{NPS}(u, v)|_{\text{det}}$, (i.e., the NPS for an idealized system that is identical to the actual system except that each process in image formation is deterministic and not stochastic). The “stochastic” form of the DQE is then:

$$DQE_{\text{sto}}(u, v) = \frac{\text{NPS}(u, v)|_{\text{det}}}{\text{NPS}(u, v)|_{\text{actual}}} = \frac{\text{NNPS}(u, v)|_{\text{det}}}{\text{NNPS}(u, v)|_{\text{actual}}}$$ \hspace{1cm} (2.24)

Note that this description is equivalent to the empirical DQE [Eq. (2.23)], where:

$$\text{NNPS}|_{\text{det}}(u, v) = \frac{MTF^2(u, v)}{\bar{q}_0}$$ \hspace{1cm} (2.25)

The stochastic formulation often provides useful insight for understanding what information is conveyed by the DQE, illustrating that it is the stochastic nature of the imaging system that limits the system to less than ideal performance.
A third definition is a ‘conceptual’ form of the DQE expressed as a ratio of signal-to-noise ratio (SNR) squared at the output of the detector to that at the input:

$$DQE_{\text{con}}(u, v) = \frac{SNR_{\text{out}}^2(u, v)}{SNR_{\text{in}}^2(u, v)}$$  \hspace{1cm} (2.26)$$

where $SNR_{\text{in}}$ reflects a Poisson stochastic process with mean signal equal to $\bar{q}_0$ and noise equal to $\sqrt{q_0}$. For the output $SNR_{\text{out}}$, the signal is equal to $G\bar{q}_0MTF(u, v)$, where $G$ is the gain of the system and the noise is given by $\sqrt{\text{NPS}(u, v)}$. Therefore Eq. (2.26) reduces to:

$$DQE_{\text{con}}(u, v) = \frac{\frac{G\bar{q}_0MTF(u, v)}{\sqrt{\text{NPS}(u, v)}}}{\frac{\bar{q}_0}{\sqrt{q_0}}} = \frac{MTF^2(u, v)}{q_0\text{NPS}(u, v)}$$  \hspace{1cm} (2.27)$$

where $G\bar{q}_0$ is equivalent to $\langle I(x, y) \rangle$, and equivalent to the empirical form of DQE.

The final formulation is the “generalized” DQE (GDQE), defined by the empirical DQE [Eq. (2.23)] modified to include the background anatomical noise, $\text{NNPS}_B(u, v)$:

$$GDQE(u, v) = \frac{MTF^2(u, v)}{q_0(\text{NPS}(u, v) + \text{NNPS}_B(u, v)MTF^2(u, v))}$$  \hspace{1cm} (2.28)$$

where $\text{NPS}(u, v)$ denotes noise in the projection (i.e., the NNPS due to quantum and electronic noise) and $\text{NNPS}_B(u, v)$ denotes the background anatomical noise. In the denominator, $MTF(u, v)$ appears separately to $\text{NNPS}_B(u, v)$ so that the latter describes the frequency content of the overlying anatomy in the object (not the image), taken as a source of stochastic variation.
independent of the imaging system, and the product \( NNPS_{\beta}(u,v)MTF^2(u,v) \) reflects the anatomical background fluctuations as presented in the image.

<table>
<thead>
<tr>
<th>DQE Formulation</th>
<th>Mathematical Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>( \frac{MTF^2(u,v)}{q_0NNPS(u,v)} )</td>
</tr>
<tr>
<td>Stochastic</td>
<td>( \frac{NNPS(u,v)</td>
</tr>
<tr>
<td>Conceptual</td>
<td>( \frac{SNR_{\text{out}}^2(u,v)}{SNR_{\text{in}}^2(u,v)} )</td>
</tr>
<tr>
<td>Generalized</td>
<td>( \frac{MTF^2(u,v)}{q_0(nnPS(u,v) + NNPS_{\beta}(u,v)MTF^2(u,v))} )</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of DQE definitions

### III.4 Noise Equivalent Quanta (NEQ)

The noise equivalent quanta (NEQ) describes the SNR of the output image and can be rewritten in terms of the DQE and incident fluence:

\[
NEQ(u,v) = SNR_{\text{out}}^2(u,v) = \frac{MTF^2}{NNPS} = \frac{1}{q_0DQE(u,v)} \tag{2.29}
\]

This expression provides a useful image quality metric in terms of absolute image quality rather than the efficiency of the system as described by the DQE. It also follows that the NEQ is analogous to the large-area spatial-domain SDNR. Furthermore, the NEQ can be “generalized” (GNEQ) by including the background anatomical noise:

\[
GNEQ(u,v) = \frac{MTF^2(u,v)}{NNPS(u,v) + NNPS_{\beta}(u,v)MTF^2(u,v)} \tag{2.30}
\]
yielding a figure of merit for image quality that accounts for both quantum and background anatomical noise.

III.5 Imaging Task and Detectability Index

The Fourier transform of an object relative to background defines the imaging task in the form of an idealized task function that describes the spatial frequencies of interest in performing a given task. Given an imaging task, such as a detection or discrimination task, a spatial-frequency-dependent task function, $W_{\text{Task}}(yu,v)$, can be defined. The DQE or NEQ may in turn be combined with the task function to yield the detectability index, $d'$, providing a metric for task-based system evaluation, optimization, and investigation of model observer performance:77-79

$$d'^2 = \iint_{Nyq} \text{NEQ}(u,v)W_{\text{Task}}^2(u,v)dudv.$$ (2.31)

Alternatively, in this work the detectability index can be written in terms of the DQE rather than NEQ, thus providing of useful figure of merit for detector performance. Furthermore, in the work below, the generalized NEQ and DQE were also used to compute the detectability index. Note that it will be indicated explicitly whether the detectability index refers to DQE or NEQ and if it describes the quantum-noise-only or ‘generalized’ case. Written as in Eq. (2.31), the detectability index neatly separates properties of the imaging system (i.e., \(\text{NEQ}\)) from aspects of the imaging task ($W_{\text{Task}}$). In the sections below and in Chapter 3, the detectability index is computed from the DQE and task function integrated along the $u=v$ diagonal up to the Nyquist frequency, $f_{Nyq}$. A variety of task functions are considered in greater detail below. For example, a simple delta-function detection task was considered as a hypothesis-testing task, corresponding to differentiation between signal-present (a delta function) versus signal-absent (noise-only) cases, giving a constant task function,$^62$ $W_{\text{Task}}(u,v) = \text{constant}$. In Chapters 4, 5, and 6, the detectability
index is computed in terms of somewhat more complicated and realistic imaging tasks (e.g., detection and discrimination tasks).

IV EXPERIMENTAL METHODS OF RADIOGRAPHIC PERFORMANCE CHARACTERIZATION

IV.1 Experimental Setup

Figure 2.3: Experimental bench. 1) Kilovoltage x-ray tube with collimator. 2) Anthropomorphic phantom 3) Flat-panel detector (Trixell Pixium 4600 shown). The imaging system provides an adjustable framework and motion control system mounted on an optical bench and was developed as an experimental proving ground for imaging performance in DE imaging, tomosynthesis, and cone-beam CT.

An imaging bench was constructed (Fig. 2.3) as an experimental platform for advanced applications of FPDs, including DE imaging, tomosynthesis, and CBCT. The imaging bench consists of an x-ray tube (Rad 94 in a Sapphire housing; W target; 0.4-0.8mm focal spot; 14° angle; Varian Medical Systems, Salt Lake City, UT) powered by a constant potential generator (CPX 380, EMD Inc, Montreal, QC). Two detectors were employed in this work: 1) the first FPD was an RID-1640A, from PerkinElmer Optoelectronics (Santa Clara, CA); 2) and the second detector was a Trixell Pixium 4600 (Moirans, France). See Table 2.2 for detailed specifications. A motion control system (6K series with Gemini drives, Parker Daedal, Harrison
PA) allowed precise and reproducible adjustment of system geometry. The work in Chapters 2 and 3 employs the PerkinElmer detector, and the work in Chapters 4, 5, and 6 employs the Trixell detector.

<table>
<thead>
<tr>
<th>Panel Specifications</th>
<th>PerkinElmer RID 1640A</th>
<th>Trixell Pixium 4600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillator screen</td>
<td>CsI</td>
<td>CsI</td>
</tr>
<tr>
<td>Pixel number</td>
<td>1024x1024</td>
<td>3000x3000</td>
</tr>
<tr>
<td>Pixel Pitch</td>
<td>400 µm</td>
<td>143 µm</td>
</tr>
<tr>
<td>Total area</td>
<td>41x41cm²</td>
<td>41x41cm²</td>
</tr>
<tr>
<td>ADC</td>
<td>16 bit</td>
<td>14 bit</td>
</tr>
<tr>
<td>Frame rate</td>
<td>3.5 fps</td>
<td>0.2 fps</td>
</tr>
</tbody>
</table>

Table 2.2: Specifications for the two FPDs employed in this work.

IV.2 System Gain and Linearity

The system gain, $\Gamma$ (describing the mean detector signal per unit incident exposure), was determined by measuring the mean signal in 50 “flood” images. A silicon diode (R100 detector with Barracuda exposure meter; RTI Electronics, Molndal Sweden) placed on the detector (SDD=144cm) was used to measure the exposure. $\Gamma$ was determined from linear fits to mean signal versus exposure. The region of interest (ROI) was 128x128 pixels at the center of the FPD, and fits were performed on measurements up to 50% of sensor saturation, for which the detector response is highly linear.

IV.3 Measurement of the MTF

The MTF was measured by placing a precision-machined straight Pb edge (2mm thick) on the detector at a slight angle (~5°). Fifty projections at ~50% detector signal saturation were acquired and averaged to reduce the effect of x-ray quantum noise. Approximately 50 realizations (10
rows x 500 columns) were obtained in each measurement. For each realization, a shift and add technique was performed to oversample the step function produced by the edge, as described by Fujita et al.\textsuperscript{66} This technique generates an oversampled ESF from which the derivative yielded the LSF. A median filter [sliding window, finite impulse response (FIR) filter] was applied to smooth the tails of the LSF, removing some of the high-frequency noise introduced by differentiation. For each realization, the Fourier transform of the 1-D LSF yielded the MTF [Fig. 2.2 and Eq. (2.13)]. The average of all MTFs (~50) for each realization gave the final measured MTF.

IV.4 Measurement of Quantum Noise

The NPS was measured from 100 “flood” images, which were gain and offset corrected to account for stationary variations by the mean of 50 “flood” and “dark” fields acquired with and without exposure of x-rays to the FPD, respectively. The FPD was read 15 times between flood projections in order to minimize correlations due to image lag. Each non-overlapping realization was 100x100 pixels in size. Approximately 8000 realizations formed the ensemble. The relative NPS was computed by normalizing the FFT squared of each realization by the physical area, \( A \), of the realization and dividing by the mean signal squared [Eq. (2.19)]. The mean of the 8000 NPS estimates gave the final NPS, with error bars provided by the sample standard deviation. The NPS was measured at 60, 80, 100, 120, and 140 kVp, with filtration by 2mm Al + 0.6mm Cu. The NPS was characterized as a function of exposure by varying the mAs (12.5, 2.5, 0.8, 0.5, and 0.4 mAs) such that all measurements were below \( \sim 50\% \) sensor saturation. The R100 photodiode was placed directly on the detector (SDD=144cm) near the center of the panel and just above the FOV for measurement of exposure.
IV.5 Measurement of Background Anatomical Noise

The background anatomical NPS was measured in a manner similar to that of the flood-field NPS except that real patient or anthropomorphic phantom radiographs formed the image data. Analysis was restricted to a region of interest in the lung. The two sets of data included clinical patient data (University Health Network, Toronto ON), where 90 patients were chosen randomly from the clinical database. The chest radiographs were acquired at 140–150 kVp using a Siemens FD-X digital chest unit (~200 μm pixel pitch). A second set of data was acquired on the imaging bench described above using an anthropomorphic chest phantom (Fig. 2.4). The phantom consists of a modified Rando™ phantom with a natural human skeleton, a custom-formulated lung material formed of microbubble-infused polyurethane, and embedded spheres of various diameter and composition (intended to simulate lung nodules). The background anatomical NPS measured in the patient population was compared to that in the chest phantom as a means to evaluate the appropriateness of the phantom for realistic anatomical NPS estimates. The stationarity of the background anatomical NPS within each patient was compared to the variations observed across the patient population as a check on stationarity relative to population variations.

The total anatomical NPS measurement was performed directly on the lung region of the images as illustrated in Fig. 2.5. The regions of the lungs were manually delineated and automatically divided into non-overlapping realizations. The clinical CR data were ~2100 x 2100 pixel format, and the size of the realizations was 100 x 100 pixels. The number of realizations varied from approximately 100 to 140 realizations per patient. The format of the image data acquired using the chest phantom was smaller (1024 x 1024 pixels); therefore, to obtain realizations corresponding to approximately the same size with respect to the anatomy as in the CR data, the
realizations were 40 x 40 pixels [see Fig. 2.5(b)]. Fifty radiographs of the chest phantom were acquired with ~2500 realizations used to compute the background anatomical NPS.

Figure 2.4: On the left is a photograph of the anthropomorphic chest phantom. On the right are CT images of the anthropomorphic phantom. The segmented region in red corresponds to the simulated lung. The purple spheres simulate lung nodules of various contrast and size.

Figure 2.5: (a) Example CR chest image of a patient (140 kVp). (b) Example radiograph of the Princess Margaret Hospital (PMH) chest phantom acquired on the imaging bench (Fig. 2.3). The highlighted regions correspond to
realizations from which the NPS analysis was performed. The realizations were each 100 x 100 pixels and 40 x 40 pixels for the CR patient images and the phantom radiographs, respectively.

To extract a semi-empirical estimate of $S_b(u,v)$ the total measured relative background anatomical NPS was fit to the following empirical form:

$$NNPS_{tot}(u,v) = NNPS_Q(u,v) + NNPS_B(u,v)MTF^2(u,v) + NNPS_{add}$$

$$\cong k_1 MTF^2(u,v) + \frac{K}{f^\beta} MTF^2(u,v) + k_2$$

(2.32)

where $k_1$, $k_2$, $K$ and $\beta$ are fitting parameters. The $k_1MTF^2(u,v)$ term approximates the quantum noise as being proportional to $MTF^2(u,v)$. The $K/f^\beta$ term models the background anatomical noise, with the measured MTF of the FPD included outside $NNPS_B(u,v)$ so that the measurements of $K$ and $\beta$ are independent of the imaging system (i.e. a property of the background anatomical structures alone). The $k_2$ term approximates the electronic noise (taken as white). The terms $K$ and $\beta$ are thus derived as empirically determined parameters in the generalized DQE:

$$GDQE(u,v) = \frac{MTF^2(u,v)}{\overline{q_0}(NNPS_Q(u,v) + \frac{K}{f^\beta} MTF^2(u,v) + NNPS_{add})}$$

(2.33)

where $NNPS_Q(u,v)$ was computed using cascaded systems analysis, $NNPS_{add}(u,v)$ was modeled as a white NPS from measurements of pixel dark noise and dark-field NPS, $\overline{q_0}$ was determined using the Spektr toolkit, and the MTF, $MTF(u,v)$, was obtained from a single-parameter Lorentzian fit\textsuperscript{26} to the measured MTF.
V THEORETICAL METHODS OF RADIOGRAPHIC PERFORMANCE CHARACTERIZATION

V.1 Cascaded System Analysis

Cascaded systems analysis (CSA) provides a powerful analytical tool that describes the signal and noise transfer characteristics of imaging systems in a manner that is physically intuitive and identifies the factors that limit imaging performance. Investigators have studied such analysis extensively for FPDs, and other imaging systems, showing that the NPS, DQE, and NEQ can be predicted analytically over a broad range of detector configurations and imaging conditions. Assumptions inherent to CSA include linearity, shift-invariance, and stationarity as described in Sec. III above. CSA represents each physical process in the imaging chain as a gain stage (e.g., conversion of x-rays into optical photons), a spatial spreading (blurring) stage (e.g., spreading of optical photons in the scintillator), or a sampling stage (e.g., readout of the detector signal at locations according to the pixel matrix). At each stage of the imaging process the output is determined as a function of the input by transfer equations described by Rabbani et al. The CSA modeling of FPD performance has been reported for a variety of detector designs and imaging applications. A brief description of the CSA model employed herein is provided below, with a more detailed description of each stage given in Appendix A. The model is similar to previously reported approaches, modified in two important respects [account of variation in optical gain in branches of the parallel cascade, and extension to DE image decomposition (Chapter 3)].

V.2 Imaging Stages For Indirect-Detection FPD

Figure 2.6 shows a flow-chart illustrating the 7 stages of image formation in an indirect-detection FPD. Stage 0 describes the mean fluence of quanta, $\bar{q}_0$, incident on the detector. The incident x-
ray energy spectrum, \( q_0(E) \), was generated for any kVp (50-150 kVp) and added filtration using the Spekt\textsuperscript{88} toolkit, based on data from the TASMIP method of Boone and Seibert\textsuperscript{89}. The fluence per unit exposure (\( \bar{\phi}/X \)) is computed by integrating the energy-dependent fluence per unit exposure\textsuperscript{90} with the normalized incident spectrum. Stage 1 involves the quantum detection efficiency, \( \bar{g}_1 \), describing the probability that an x-ray will interact in the scintillator or photoconductor (indirect or direct-detection, respectively; henceforth, we consider the former.). Stage 2 represents the conversion of x-rays to optical photons, characterized by a mean optical gain, \( \bar{g}_2 \). K-Fluorescence in the scintillator was included in the model by means of parallel-cascaded systems analysis as described by Yao and Cunningham.\textsuperscript{91} The description was extended in this work to include variance in the conversion gain, as opposed to a deterministic or Poisson-distributed gain.\textsuperscript{83,91,92} (See Appendix A for details.) Stage 3 is a stochastic spreading stage, characterized by the scintillator MTF, \( T_3(u,v) \), and describing the blur of optical photons in the scintillator. Stage 4 represents the conversion of optical photons to electrons in the photodiode, described by the quantum efficiency, \( \bar{g}_4 \). Stage 5 is a presampling stage corresponding to the integration of quanta by the photodiode and described by the presampling pixel MTF, \( T_5(u,v) \). Stage 6 represents the sampling of the detector signal and is characterized by the pixel pitch, \( a_{pix} \), photodiode aperture, \( a_{pd} \) and fill factor, \( f = a_{pd}^2 / a_{pix}^2 \). Finally, Stage 7 represents electronic readout with an additive noise term due to the pixel dark noise, amplifier noise and digitization. Table 2.3 provides a summary and glossary of CSA parameters.
Figure 2.6 Flow-chart representation of cascaded systems analysis for a FPD. Each stage represents a physical process in the imaging chain. At stage 2, a parallel cascade models K-fluorescence: branch A represents the case in which all the energy of the incident x-ray is deposited locally in the scintillator, branch B is the case in which only a fraction of the energy is deposited locally after K-x-ray production, and branch C corresponds to remote deposition of the K-x-ray.
### Table 2.3: Glossary and summary of parameters used in the cascaded systems analysis model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>60 kVp</th>
<th>80 kVp</th>
<th>100 kVp</th>
<th>120 kVp</th>
<th>140 kVp</th>
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<tr>
<td>$\phi_0/X$</td>
<td>2.59 x 10^5</td>
<td>2.77 x 10^5</td>
<td>2.83 x 10^5</td>
<td>2.74 x 10^5</td>
<td>2.64 x 10^5</td>
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<tr>
<td>$g_1$</td>
<td>0.94</td>
<td>0.82</td>
<td>0.72</td>
<td>0.64</td>
<td>0.59</td>
</tr>
<tr>
<td>$\zeta$</td>
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<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>$g_{2A}$</td>
<td>1528</td>
<td>1823</td>
<td>2019</td>
<td>2159</td>
<td>2259</td>
</tr>
<tr>
<td>$g_{2B}$</td>
<td>521</td>
<td>819</td>
<td>1011</td>
<td>1151</td>
<td>1251</td>
</tr>
<tr>
<td>$g_{2C}$</td>
<td>1117</td>
<td>1117</td>
<td>1117</td>
<td>1117</td>
<td>1117</td>
</tr>
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<td>1371</td>
<td>1670</td>
<td>1860</td>
<td>2000</td>
<td>2099</td>
</tr>
<tr>
<td>$\varepsilon_{g_{2B}}$</td>
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<td>129</td>
<td>142</td>
<td>178</td>
<td>216</td>
</tr>
<tr>
<td>$\varepsilon_{g_{2B}}$</td>
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<td>169</td>
<td>234</td>
<td>299</td>
</tr>
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<td>$\varepsilon_{g_{2C}}$</td>
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<td>65</td>
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<td>40</td>
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<td>0.78</td>
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</tr>
<tr>
<td>$g_4$</td>
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<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
</tbody>
</table>

V.3 Experimental Validation of CSA

Since DE imaging is the main focus of this work and employs images acquired over a broad range of energies, it was necessary to validate CSA calculations of NPS and DQE in comparison to measurements over a similarly broad range of techniques. Also, the parallel cascaded systems model described in Appendix A extends the modeling of K-fluorescence to include variation in the optical gain as an energy-dependent phenomenon, also requiring validation of theory in comparison to measurement.

V.3.1 K-fluorescence and MTF

To consider the impact of K-fluorescence on the DQE, the MTF was calculated with and without K-fluorescence, as shown in Fig. 2.7. Measurements on the experimental bench gave the total system MTF [$T_3(u,v)T_5(u,v)T_{K_{fluores}}(u,v)$], with the pixel aperture MTF [$T_5(u,v)$] described well by a sinc function. The measured MTF divided by $T_5(u,v)$ gave the scintillator MTF [$T_3(u,v)T_{K_{fluores}}(u,v)$],
with $T_{Ktot}(u,v)$ computed using the model of Que et al.\textsuperscript{93} (See Appendix A.) The curves in Fig. 2.7 are Lorentzian fits to $T_3(u,v)$ and $T_3(u,v)T_{Ktot}(u,v)$. In calculation of the DQE (below), cases with and without K-fluorescence were modeled with the MTF as $T(u,v) = T_3(u,v)T_3(u,v)T_{Ktot}(u,v)$ and $T(u,v) = T_3(u,v)T_3(u,v)$, respectively.

**V.3.2 NPS and DQE**

Figure 2.8 shows the typical agreement between the NPS and DQE measurements and calculations using cascaded systems analysis. Results with and without K-fluorescence can be appreciated by considering the quantum gain in Branch A, $\bar{g}_2$, and the effective quantum gain, $\bar{g}_2$, respectively. The NPS and DQE measurements shown in Fig. 2.8 were extracted from 2D measurements along a diagonal in the Fourier domain up to the Nyquist frequency $(u_{Nyq}, v_{Nyq})$. Theoretical calculations demonstrated that the influence of K-fluorescence on the NPS is subtle but non-negligible, increasing the NPS particularly at low frequencies. The DQE is degraded at all frequencies because of the increase in the NPS at low frequencies and the degradation of the MTF at higher frequencies. Each column shows the NPS and DQE at a given kVp. For the NPS (top row), the results appear non-monotonic with kVp due primarily to differences in the exposure, X, required to give ~50% detector signal saturation; the NPS computed at $X = 1.0$ mR is included for comparison, showing monotonic increase in the NPS with kVp. Correspondingly, the DQE (bottom row) shows monotonic reduction with kVp. The agreement between measured and calculated NPS and DQE is good across all frequencies, giving confidence that the extended CSA model provides a useful theoretical tool for analysis of imaging performance across the broad range of conditions considered below.
Figure 2.7: The solid line shows the scintillator MTF with K-fluorescence ($T_3$, $T_{Ktot}$), and the dashed line shows the scintillator MTF without K-fluorescence ($T_3$). Curves shown are Lorentzian fits based on measurements of the system MTF and modeling of $T_{Ktot}$ as in Que et al.93

Figure 2.8: NPS (top row) and DQE (bottom row) at various kVp. Measurements were performed on the imaging bench using a FPD with a 250 mg/cm² CsI:TI converter at 400 μm pixel pitch. The solid and dashed lines show theoretical calculations with and without K-fluorescence, respectively. Inclusion of K-fluorescence improves the agreement of the theoretical model, especially for the DQE. NPS measurements at various kVp had slightly different exposure levels due to limitations in technique selection, but show reasonable agreement with CSA calculations. For comparison at various kVp, calculations are shown for the case of 1.0 mR exposure, where the NPS is seen to increase with kVp.
VI  EXAMPLE RESULTS: THE DETECTIBILITY INDEX FOR CONVENTIONAL RADIOGRAPHY

VI.1  Background Anatomical NPS

Figure 2.9(a) shows the total anatomical NPS measurements obtained from 90 CR patient chest images. The background anatomical NPS differs markedly from the quantum NPS in that the former has a very strong low-frequency characteristic. Also a large amount of variability was observed across the population data, consistent with the observation of Samei et al. Figure 2.8(b) shows the variability (i.e., the stationarity of the background anatomical NPS) within one patient in comparison to the population variation. The error bars represent two standard deviations in NPS estimates across ~100 realizations from a single patient image. This variation is important to examine because it quantifies the non-stationarity within an image, which is of the same order as the non-stationarity across the population. Therefore the validity of the measured NPS estimate with respect to non-stationarity is about the same as the validity of the empirical model across the population average. Finally, Fig. 2.8(c) compares the patient population background anatomical noise with the background anatomical noise measured in the chest phantom (60-140 kVp). Although the overall magnitude of the NPS is less in the chest phantom, due primarily to imperfect simulation of real anatomy, the frequency dependence is similar, and the chest phantom provides a reasonable approximation for thoracic background anatomical noise.
Figure 2.9: (a) Measurements of total anatomical NPS on 90 CR chest patients acquired at 140-150 kVp (b) The shaded area is the region spanned by the 90 CR chest patients. The error bars represent the variation within one patient showing that NPS variations across one radiograph are of the same order as that of the population variation. (c) Total anatomical NPS for the patient population (shaded area) in comparison to that measured using the chest phantom, which provides a reasonable approximation to real thoracic anatomy for anatomical noise measurements.

Figure 2.10: (a) Anatomical noise parameters, $K$ and $\beta$, measured as a function of kVp (60-140kVp). The solid line (left axis; $K$) quantifies the magnitude of background anatomical noise. The dashed line (right axis; $\beta$) quantifies the frequency content in the background anatomical noise. (b) Radiographs of lung regions at various kVp illustrate the reduction in magnitude of background anatomical noise (i.e., reduction in contrast of overlying bone) as kVp increases.

Measurements of background anatomical noise were performed using the chest phantom, and empirical estimates of $K$ and $\beta$ were derived as a function of kVp [see Fig. 2.10(a)]. The factor, $K$, which quantifies the magnitude of the anatomical noise, was observed to decrease considerably as the kVp was increased. On the other hand, $\beta$, which quantifies the frequency content of the anatomical noise, did not appear to vary significantly across the same range of
kVp. This can be observed qualitatively in the images of Fig. 2.10(b): as the kVp increases, the rib contrast (related to K) is reduced, while the “clumpiness,” $\beta$, of the anatomical noise does not change.

VI.2 Generalized DQE

Figure 2.11(a) shows the calculated DQE for a 250 mg/cm$^2$ CsI:TI-based FPD with a 400$\mu$m pixel pitch and 80% fill factor at an exposure of 0.1 mR and various kVp (4mm Al and 0.6 mm Cu added filtration). Theoretical calculations were performed at 60, 80, 100, 120 and 140 kVp, showing monotonic reduction in DQE with kVp. However, if background anatomical noise is included in the total NPS by taking the empirically determined background anatomical parameters $K$ and $\beta$ from Fig. 2.10(a), the effect changes entirely as shown in Fig. 2.11(b). The GDQE is severely degraded by the presence of low-frequency anatomical noise and peaks at mid-frequencies. Even more interesting, the GDQE increases significantly with kVp, corresponding to the reduction in background anatomical noise at higher kVp.

Figure 2.11: (a) Theoretical calculations of DQE at various kVp showing monotonic reduction in DQE as the kVp increases. (b) Theoretical calculations of the generalized DQE, using $K$ and $\beta$ values measured at various kVp, showing improved in GDQE with kVp. The distinction highlights the importance of the generalized approach.
VI.3 Detectability Index

Figure 2.12 summarizes these results in terms of the detectability index computed as a function of kVp using conventional and generalized formulations of DQE (without and with background anatomical noise, respectively, as in Eqs. 2.23 and 2.28). According to the conventional description, detectability index decreases with kVp. However, generalization of the DQE tells the opposite story: the detectability index increases with kVp when background anatomical noise is included. This is observed qualitatively in Fig. 2.10, showing reduced rib contrast and increased nodule conspicuity as the kVp is increased. This analysis is compelling because it brings image theory in line with techniques used clinically in chest radiography, where the techniques are typically in the higher kVp range.\textsuperscript{94}

![Graph showing detectability index vs. kVp](image)

Figure 2.12: The conventional and generalized detectability index (for a delta-function detection task) for radiographic chest images as a function of kVp. While detectability based on the conventional description of DQE degrades monotonically with kVp, the generalized detectability index (which includes background anatomical noise in GDQE) increases with kVp, bringing imaging theory in line with typical chest imaging techniques.\textsuperscript{95}
Cascaded systems analysis was shown to be a reasonable model for computing Fourier-based metrics of radiographic imaging performance. Background anatomical noise was included in a generalized DQE analysis of FPDs, yielding descriptions of spatial-frequency-dependent imaging performance that are distinct from conventional approaches to quantum noise alone. The analysis reveals a quantitative and intuitive description of increasing detectability index with increasing kVp for chest radiography when the “generalized detectability” is considered, thus bringing imaging theory in line with clinical techniques.

More importantly, this work provides the theoretical foundation for the work in this dissertation. The following chapters build on this framework by extending these metrics of imaging performance to DE imaging and also investigate more realistic tasks and sophisticated model observers. Such analysis will provide a valuable means to configure DE imaging systems and investigate imaging performance across a broad range of detector configurations, imaging conditions, and imaging tasks. Incorporation of generalized DQE and NEQ analysis in the evaluation of detectability index provides a quantitative tool that begins to bridge the gap between detector performance and observer performance while accounting for the reduction of anatomical background noise in DE images.
CHAPTER 3: DUAL-ENERGY IMAGING PERFORMANCE METRICS

I  INTRODUCTION
In this chapter, theoretical cascaded systems analysis (CSA) is extended to DE imaging, modeling weighted log-subtraction DE decomposition as a deterministic operation performed on two independent projections combined in proportion to the tissue cancellation parameter, \( w_s \). The CSA model for DE imaging is also extended to include anatomical noise, yielding the generalized DQE and NEQ for a DE imaging system. These theoretical methods are in turn extended to evaluation of task-based figures of merit – specifically, the model observer detectability index – by considering simple spatial-frequency-dependent imaging task functions in combination with the generalized DQE.

Weighted log subtraction, derived from straightforward manipulation of Beer’s law, was taken as the basic means of image decomposition in this work. While somewhat more sophisticated decomposition techniques are available, as discussed in Chapter 1, log subtraction is considered
in the present analysis because it lends itself well to CSA and has been shown to provide roughly equivalent DE image quality to basis decomposition.\textsuperscript{12} It was shown in Chap. 1 that a “soft-tissue” image can be generated from a low- and high-energy projection in the following way:\textsuperscript{12}

\[
\ln[I_{DE}(x, y)] = -w_s \ln[I_L(x, y)] + \ln[I_H(x, y)]
\] (3.1)

where subscripts $DE$, $L$, and $H$ denote the “soft-tissue” DE image and the low- and high-energy projections, respectively. Note that in this chapter, for notational convenience, $I$ denotes the projection and rather than the log of the projection as in Chap 1. The tissue cancellation parameter ($w_s$) acts as a weighting parameter between the two projections. A similar equation can be written to yield the DE bone image:

\[
\ln[I_{bone}(x, y)] = -\ln[I_H(x, y)] + w_b \ln[I_L(x, y)]
\] (3.2)

The current chapter focuses on image quality in the DE soft-tissue image, in which the conspicuity of soft-tissue lesions is increased through the reduction of anatomical background noise (e.g., the ribs and clavicles). Investigation of image quality in the DE bone image is addressed in Chapter 5 and 6.

II THEORETICAL METHODS

As discussed in Chapter 2, the NEQ and DQE are frequency-dependent figures of merit widely used in describing imaging performance and identifying factors limiting image quality. Derivation of the (generalized) DE NEQ ($GNEQ_{DE}$) and DE DQE ($GDQE_{DE}$) extends conventional definitions of NEQ and DQE for radiography by formulation of the DE image noise-power spectrum ($NPS_{DE}$) and modulation transfer function (MTF).
II.1 The DE Modulation Transfer Function (MTF)

The MTF for a parallel process is given by the weighted sum of the optical transfer function (OTF) for each process:

\[
MTF_{DE}(u,v) = \frac{k_L OTF_L(u,v) + k_H OTF_H(u,v)}{k_L + k_H}
\]

where \(k_L\) and \(k_H\) are the respective weights determined from the low- and high-energy signals. At radiographic energies it is often reasonable to make the following assumption:

\[
OTF_L(u,v) = OTF_H(u,v)
\]

since stochastic spreading in the scintillator (stage 3 in CSA) exhibits a low dependence on the incident x-ray energy in the radiographic energy range (60-150 kVp). Therefore it follows that the DE MTF is given by:

\[
MTF_{DE}(u,v) = |OTF_L(u,v)| = |OTF_H(u,v)| = MTF(u,v)
\]

where MTF without subscripts denotes the detector MTF equivalent to the projection or radiographic MTF. The assumption of equivalent OTF (MTF) in the low- and high-energy projections is invoked to simplify the analytical formulation of NEQ. It is a simplifying – not a necessary – assumption, and as shown in Chapter 5, the analysis may be carried forth in the general case, \(MTF_L \neq MTF_H\).

II.2 The DE Noise-Power Spectrum (NPS)

This section provides a derivation of the normalized dual-energy NPS, \(NNPS_{DE}(u,v)\), in terms of the NPS of the low- and high-energy images and the tissue cancellation parameter, \(w_s\). We first derive the autocovariance for a general random variable, \(c(x)\), arbitrarily comprised of random
variables $a(x)$ and $b(x)$. By Fourier transform, we show that the NPS of $c(x)$ is related to the NPS and first-derivative of $a(x)$ and $b(x)$. From this general analytic approach, we treat the specific case in which $c(x)$ represents the dual-energy image, $I_{DE}$, and $a(x)$ and $b(x)$ are the low-and high-energy images as in Eq. 3.1.

Consider $c(x) = f[a(x), b(x)]$ where $a(x)$, $b(x)$ and $c(x)$ are random variables, and $a(x)$ and $b(x)$ have absolute errors, $\partial a$ and $\partial b$, respectively. From a Taylor expansion:95

$$\Delta c(x) = \Delta a(x) \left( \frac{\partial c}{\partial a} \right) + \Delta b(x) \left( \frac{\partial c}{\partial b} \right) + \ldots,$$

(3.6)

where $\Delta a(x) = a(x) - E\{a(x)\}$ and similarly for $\Delta b(x)$ and $\Delta c(x)$, where $E\{a(x)\}$ is the expectation value of $a(x)$. From the definition of the autocovariance (see Chap. 2), which describes the correlation of $\Delta a(x')$ with itself at a location displaced by $x$, we have:67

$$K_a(x) = E\{\Delta c(x')\Delta c^*(x'+x)\}$$

$$= E\{\Delta a(x')\Delta a^*(x'+x)\left( \frac{\partial c}{\partial a} \right)^2 + \Delta b(x')\Delta b^*(x'+x)\left( \frac{\partial c}{\partial b} \right)^2 +$$

$$\left( \frac{\partial c}{\partial a} \right) \left( \frac{\partial c}{\partial b} \right) \left( \Delta a(x')\Delta b^*(x'+x) + \Delta a^*(x'+x)\Delta b(x') \right) + \ldots \}$$

(3.7)

Which can be rewritten as:

$$= \left( \frac{\partial c}{\partial a} \right)^2 K_a(x) + \left( \frac{\partial c}{\partial b} \right)^2 K_b(x) +$$

$$\left( \frac{\partial c}{\partial a} \right) \left( \frac{\partial c}{\partial b} \right) \{ E\{\Delta a(x')\Delta b^*(x'+x)\} + E\{\Delta a^*(x'+x)\Delta b(x')\} \} + \ldots$$

(3.8)

where * indicates the complex conjugate. Under the assumption that $a$ and $b$ are uncorrelated, the cross-terms are zero and the expression reduces to:
The NPS, \( NPS_a(u) \), for the random variable \( a(x) \) is given by the Fourier transform of the autocovariance:

\[
NPS_a(u) = F\{K_a(x)\}
\]  

(3.10)

where \( F\{\} \) denotes the Fourier transform. Taking the Fourier transform of Eq. 3.9, we obtain:

\[
NPS_c(u) = \left( \frac{\partial c}{\partial a} \right)^2 NPS_a(u) + \left( \frac{\partial c}{\partial b} \right)^2 NPS_b(u)
\]  

(3.11)

Taking \( a(x) \), \( b(x) \), and \( c(x) \) as the low-energy, high-energy, and dual-energy images, respectively, and extending to two spatial-frequency dimensions \((u, v)\):

\[
NPS_{DE}(u, v) = \left( \frac{\partial I_{DE}}{\partial I_L} \right)^2 NPS_L(u, v) + \left( \frac{\partial I_{DE}}{\partial I_H} \right)^2 NPS_H(u, v)
\]  

(3.12)

where the 2D DE “soft-tissue” image is defined as in Eq. (3.1).

The assumption of statistical independence between \( a(x) \) and \( b(x) \) – that is, \( I_L(x, y) \) and \( I_H(x, y) \) – is reasonable, since they represent independently acquired projection images. This results for the DE NPS is not limited to double-shot acquisitions, but also applies to single-shot acquisition systems, for which it was shown that the low- and high-energy projections were uncorelated\(^{96}\). However, correlation between the two projections could be introduced by image lag,\(^{97}\) representing a possible limitation to this assumption.
We rewrite Eq. 3.12 as the relative (or “normalized”) NPS, \( NNPS \), which is defined as the NPS divided by the mean signal squared, and we substitute the derivatives after linearizing the signal [i.e. exponentiating both sides of Eq. (3.1)]:

\[
NNPS_{DE}(u,v) = \frac{NPS_{DE}(u,v)}{I_{DE}^2}
\]

\[
= \left( -w_s \frac{I_H}{I_{w_s+1}} \right)^2 NPS_L(u,v) + \left( \frac{1}{I_{w_s}} \right)^2 NPS_H(u,v)
\]

\[
= w_s^2 \frac{NPS_L(u,v)}{I_L^2} + \frac{NPS_H(u,v)}{I_H^2}
\]

\[
= w_s^2 NNPS_L(u,v) + NNPS_H(u,v) \tag{3.13}
\]

Therefore, the NPS for a DE image (e.g., the soft-tissue image) is given by the NPS of the high-energy image plus the NPS of the low-energy image weighted by the square of the tissue cancellation parameter and provides the framework for extending cascaded systems analysis to DE imaging as shown in Figure 3.1.
Figure 3.1 Flow-chart representation of NPS propagation for dual-energy imaging. The high- and low-energy NPS are computed independently and combined as a function of the DE decomposition tissue cancellation parameter, $w_s$, to yield the dual-energy image NPS. The gray boxes correspond to the cascaded systems processes in Fig. 2.9, computed for low- and high-energy images.

II.3 The Dual-Energy Detective Quantum Efficiency (DQE)

As described in Chapter 2, the DE DQE is given by the ratio of “deterministic” and “actual” NPS:

\[
DQE_{DE}(u, v) = \frac{NNPS(u, v)_{DE}^{det}}{NNPS(u, v)_{DE}^{actual}}
\]  

(3.14)

Where the numerator and denominator are defined in the following subsections.

II.3.1 The DE “deterministic” NPS:

From Eq. 3.13, for a DE soft-tissue image:
Substituting the low- and high-energy deterministic NPS gives:

\[
NNPS_{DE}(u,v) = w_s^2 \frac{MTF_L^2(u,v)}{q_{0L}} + \frac{MTF_H^2(u,v)}{q_{0H}}
\]  

(3.16)

Rearranging terms, we obtain:

\[
NNPS_{DE}(u,v) = \frac{MTF_H^2(u,v)}{q_{0DE}}
\]  

(3.17)

where an “effective” DE image fluence is defined as:

\[
-q_{0DE}(u,v) = -q_{0H} \frac{1}{1 + w_s^2 \frac{MTF_L^2(u,v)}{MTF_H^2(u,v)} q_{0H} q_{0L}}
\]  

(3.18)

This term should not be considered a fluence in the usual sense; it is a notational convenience that is, in general, spatial-frequency-dependent, but offers a form for the resultant fluence that is obtained when the low- and high-energy images are combined in DE image decomposition. The spatial-frequency-dependence in \( q_{0DE}(u,v) \) arises due to possible differences in the spatial-frequency transfer characteristics (i.e., MTF) at low and high energy. Under the simplifying assumption that the MTF for the high- and low-energy images are the same:

\[
MTF_L(u,v) = MTF_H(u,v) = MTF(u,v)
\]  

(3.19)

the effective DE fluence becomes:

\[
-q_{0DE} = -q_{0H} \frac{1}{1 + w_s^2 \frac{q_{0H}}{q_{0L}}}
\]  

(3.20)
Note that $\overline{q}_{0,DE}$ reduces to $\overline{q}_{0,H}$ in the special case $w_s = 0$ [i.e., nullification of the low-energy component in $I_{DE}(x,y)$].

II.3.2 The DE “actual” NPS:

The DE NPS can be written as:

$$ NNPS_{DE}(u,v)_{\text{actual}} = w_s^2 NNPS_{L}(u,v)_{\text{actual}} + NNPS_{H}(u,v)_{\text{actual}} + \ldots $$

where $NNPS(u,v)_{\text{actual}}$ contains both quantum and electronic noise for the low- or high-energy image, and $NNPS_{DE,B}(u,v)$ is the background anatomical noise for the DE image. If we assume that the additive electronic noise is the same in both the high- and low-energy images:

$$ NNPS_{DE}(u,v)_{\text{actual}} = w_s^2 \left( NNPS_{L,Q}(u,v) + NNPS_{add}(u,v) \right) + \ldots $$

$$ = NNPS_{DE,Q}(u,v) + NNPS_{DE,add}(u,v) + NNPS_{DE,B}(u,v) $$

where we have grouped terms such that the DE NPS is expressed in terms of a DE quantum noise component:

$$ NNPS_{DE,Q}(u,v) = w_s^2 NNPS_{L,Q}(u,v) + NNPS_{H,Q}(u,v) $$

and a DE additive noise component:

$$ NNPS_{DE,add} = NNPS_{add}(w_t^2 + 1) $$

Therefore, substituting the deterministic and actual NNPS into Eq. (2.22) yields the DE DQE:
\[
DQE_{DE}(u,v) = \frac{NNPS(u,v)_{DE}|_{det}}{NNPS(u,v)_{DE}|_{actual}}
\]

\[
= \frac{1}{q_{0,DE}} \frac{MTF^2}{NNPS_{DE,G}(u,v) + NNPS_{DE,add}(u,v)}
\] (3.26)

II.4 The Dual-Energy Noise-Equivalent Quanta (NEQ)

It was shown in Chapter 2 [Eq. (2.24)], that the NEQ can be defined in term of the spatial-frequency dependent output SNR as defined in Chap 2. Therefore it follows that for a DE imaging system the DE NEQ is given by:

\[
NEQ_{DE}(u,v) = SNR_{out,DE}^2 (u,v)
\]

\[
= \frac{MTF^2(u,v)}{NNPS_{DE,G}(u,v) + NNPS_{DE,add}(u,v)}
\] (3.27)

Equations (3.26) and (3.27) although useful in characterizing quantum and electronic noise in the DE image, provide only a partial characterization of DE image quality because they ignore variations associated with overlying anatomical structures, termed “anatomical noise”, which have been identified as one of the most important factors limiting detectability of subtle lesions. Generalized forms of the DQE and NEQ, described below, which incorporate background anatomical noise, provide a more complete characterization in relation to conspicuity.

II.5 Generalization of the DE DQE and NEQ

Investigators have incorporated descriptions of anatomical noise as an additional source of noise in the DQE and NEQ, as in mammography. The approach has more recently been extended to radiographic and DE imaging of the chest, with a spatial-frequency-dependent noise term,
\( NNPS_b(u,v) \), associated with background anatomical NPS, included in the denominator of the DE DQE to yield the generalized DE DQE:

\[
GDQE_{DE}(u,v) = \frac{MTF^2(u,v)}{q_{0,DE}(NNPS_{DE,0}(u,v) + NNPS_{DE,b}(u,v) + NNPS_{DE,add}(u,v))}
\]  

(3.28)

The MTF term adjacent to the anatomical noise term distinguishes \( NNPS_b(u,v) \) as independent of the detector. Including anatomical noise as a stochastic noise term in the GDQE poses some consideration in general radiography and can be limited in cases where anatomical noise is, in fact, deterministic structure (e.g., overlying ribs) that expert radiologists are able (at least to some extent) to “see through.” In describing the performance of DE imaging, however, including anatomical noise in this manner is reasonable in that deterministic structure (viz., the ribs) are removed in the soft-tissue-only image, leaving \( NNPS_b(u,v) \) to describe primarily the residual “clumpy background” associated with overlying soft-tissue structures, similar to such analysis in mammography.\(^{62,73}\) It is also possible to define the generalized DE NEQ for a DE soft-tissue image:

\[
GNEQ_{DE}(u,v) = \frac{MTF^2(u,v)}{NNPS_{DE,0}(u,v) + NNPS_{DE,b}(u,v)MTF^2(u,v) + NNPS_{DE,add}(u,v)}
\]  

(3.29)

Thus for DE imaging, including anatomical noise in \( GNEQ_{DE} \) in this manner is a reasonable approach to describing the first- and second-order image statistics, and the resulting DE GNEQ should give a meaningful description of the spatial-frequency-dependent signal-to-noise ratio in DE images. This expression lends itself to experimental and/or theoretical analysis in a manner similar to the conventional or generalized DQE for radiography. Each of the terms may be experimentally determined, as shown below, or estimated theoretically via CSA (or semi-
empirically, in the case of $NPS_{DE,b}$). Moreover, it is generalized by inclusion of anatomical noise, which captures the improvements in imaging performance for a DE image.

III EXPERIMENTAL METHODS

III.1 Dual-Energy Image MTF

The DE image MTF was measured to verify the validity of the assumption made in deriving Eq. (3.5). The method used for measuring the DE MTF was similar to that used for the projection MTF described in Chapter 2. However, in this case, fifty low- and high-energy projection images of the edge were acquired and combined according to Eq. (3.1) to yield a DE ESF from which the DE MTF was computed (Sec. VI.3 in Chap. 2). Results were also compared with the MTF measured using either the low- or high-energy projections.

III.2 Dual-Energy Image NPS

The DE image NPS was analyzed in two ways to verify Eq. (3.13). The first was by computing the NPS of the low- and high-energy images separately and then combining them according to the tissue cancellation parameter as in Eq. (3.13) to obtain the DE NPS. The second method was to decompose DE flood images from low- and high-energy flood images [Eq. (3.1)] and compute the NPS directly from the DE floods. The high-energy kVp technique was 120 kVp with 1.1 mm Cu filtration added to increase the separation between the mean energies. The low-energy technique was 60 kVp with no added filtration. The mAs (1.0 and 1.6, respectively) was chosen to give approximately 50% saturation signal level in the bare beam as measured on the FPD.

III.3 Generalized NEQ and DQE

The DE anatomical NPS was measured using the imaging bench (See Fig. 2.3 in Chap. 2) and the anthropomorphic phantom (Fig. 2.4) across a variety of kVp, filtration, and exposure
conditions. The total anatomical NPS in DE images of the phantom was measured in the same manner as described above for plain radiography (patient and/or phantom images; Fig. 2.4), where realizations were formed within a ROI in the lung, and the anatomical parameters, $K$ and $\beta$, were measured as fitting parameters to yield estimates of anatomical noise. These results were taken as semi-empirical input to CSA of the generalized DQE for a DE image. The effective fluence [Eq. (3.21)] was computed using the measured exposure and fluence-per-unit exposure for the low- and high-energy images. The MTF for the low- and high-energy image were assumed equivalent, supported by measurements in our laboratory that show the MTF to vary only slightly (within experimental error) for the two energies used.

IV RESULTS

IV.1 The DE MTF

Figure 3.2: Measurement of low-, high-, and dual-energy image MTF.

Figure 3.2 plots measurements of the low-, high-, and dual-energy MTF. Results indicate that there is no significant energy dependence at these energies and the DE MTF corresponds closely to the detector MTF. Therefore, the assumptions made in deriving the DE MTF [Eq. (3.5)] and DE fluence [Eq. (3.21)] are reasonable assumptions for the presented theoretical analysis.

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IV.2 The DE NPS and Anatomical NPS

The NPS for DE flood-field images was computed in two ways as described in Sec. III.2. Both results were observed to produce the same results for the DE NPS, thus verifying the expression in Eq. (3.13). Figure 3.3(a) shows the measured and calculated NPS for a DE image, showing the amplification of quantum noise due to DE image processing. The measured exposure at the detector was 1.8 and 0.8 mR in the low- and high-energy images, respectively. The solid line with squares corresponds to the NPS for the DE image computed using CSA, where reasonable agreement is observed between the theory and measurements (and discrepancies are attributed primarily to the Lorentzian fit to the MTF). Figure 3.3(b) shows the measured total anatomical noise and shows the strong reduction in anatomical noise achieved in DE image decomposition. Taken together, these results clearly show that DE imaging slightly increases quantum noise, while significantly reducing anatomical noise.

Figure 3.3: (a) Plots of the low- and high-energy NPS with the DE NPS measured using two different techniques to verify Eq. 3.13. The line is the NPS computed on the floods processed according to Eq. 3.1 with $w_s = 0.4$. The square symbols represent the NPS calculated using the high- and low-energy NPS and Eq. 3.13 with $w_s = 0.4$. The straight line with shows the theoretical prediction for the DE image. (b) Total anatomical NPS measurements for low-energy, high-energy, and DE images. Measurements were performed on the lung region of the PMH chest phantom (See Fig. 2.4). While DE processing increases the quantum NPS (a) it reduces the total NPS (b) due to reduction in anatomical noise.
Figure 3.4: (a) Measurement of anatomical noise parameters, $K$ and $\beta$, as a function of the DE tissue cancellation parameter, $(w_s = 0.2 - 0.6)$, showing a strong dependence on $w_s$. (b) DE images at various $w_s$, demonstrating removal of the ribs from the image where $K$ is minimized and $\beta$ is maximized. Note: The darker spheres evident in the $w_s = 0.6$ (black-bones) image are acrylic. The lighter spheres in the $w_s = 0.4$ image represents the simulated nodules.

Figure 3.4(a) shows the strong dependence observed between the anatomical noise parameters, $K$ and $\beta$, and the tissue cancellation parameter, $w_s$, with minimization of $K$, corresponding to reduction of bone contrast [Fig 3.2(b)]. However, close inspection of the images in Fig. 3.2(b) reveals an increase in quantum noise at the same level of $w_s (~0.4)$. This suggests a tradeoff between the reduction of anatomical noise and the amplification of quantum noise. Hence analysis such as the generalized CSA, which takes into account the quantum noise and anatomical noise, is essential in the analysis DE imaging performance.

IV.3 The DE NEQ and DQE

Figure 3.5 (a) and (b) plot the DE DQE and DE NEQ, respectively, at various values of $w_s$. At $w_s = 0$ the DE image is simply the high-energy image; therefore, the DQE corresponds to the DQE for the high-energy image. As $w_s$ increases, there is a greater contribution of the low-energy
signal in the DE image, and the DE DQE converges to the low-energy DQE. For the DE NEQ, \( w_s = 0 \) corresponds again to the high-energy NEQ and as \( w_s \) increases the DE NNPS increases [Eq. (3.13)], and consequently the DE NEQ decreases according to Eq. (3.27). Results illustrate how DE decomposition decreases image quality in terms of the NEQ if only quantum and electronic noise are accounted for in the NPS.

Calculations of the generalized DQE are shown in Fig. 3.6(a) for selected values of \( w_s \). Figure 3.6(b) shows a grayscale image plot of the generalized DQE across a range of tissue cancellation parameters. The generalized DE DQE was computed using CSA with measured values of \( K \) and \( \beta \) [Fig. 3.4(a)]. The generalized DE DQE exhibits a strong dependence on the tissue cancellation parameter and is maximized when the anatomical noise in minimized. While a slight spatial-frequency-dependence is noted in the optimal value of \( w_s \) in Fig. 3.6(a), as shown in Fig. 3.6(b), a value of \( w_s \approx 0.4 \) essentially maximizes GDQE at all spatial frequencies.

Figure 3.5: Plots of (a) DE DQE [(Eq. (3.26)] and (b) DE NEQ [(Eq. (3.27)] computed using CSA for various values of \( w_s \).
Figure 3.6: (a) Grayscale image plot of the generalized DQE as function of DE tissue cancellation parameter. Maximum GDQEDE occurs at the optimal tissue cancellation parameter. (b) Generalized DQE plots for selected values of $w_s$. GDQEDE is maximized near $w_s = 0.4$, corresponding to minimization of the anatomical NPS [Fig. 3.4(a)] and cancellation of overlying ribs [Fig. 3.4(b)].

IV.4 The DE Detectability Index

The generalized DE detectability index computed for a delta-function detection task is shown in Fig. 3.7, calculated using the generalized DQE for DE images at various tissue cancellation parameters. The results demonstrate a distinct optimum at $w_s = 0.4$. This result agrees well with the value that was chosen qualitatively by a single observer viewing a series of DE images decomposed using various tissue cancellation parameter values. Also, the detectability index for this idealized detection task is found to increase considerably for DE imaging compared to plain radiography. Specifically, in the DE images detectability is $\sim 0.35$ at optimal tissue cancellation, whereas in plain radiography detectability varied from $\sim 0.10 - 0.18$ (Fig. 2.11), depending on beam energy. Therefore, DE imaging increased detectability by more than a factor of two.
Figure 3.7: Generalized, dual-energy detectability index computed for a delta-function detection task as a function of tissue cancellation parameter. Detectability was computed using the DE generalized DQE of Fig. 3.6(a), demonstrating a distinct optimum at $w_s = 0.4$.

V CONCLUSIONS

For DE imaging, extension of cascaded systems analysis provides a powerful theoretical framework for evaluation of the DE DQE, NEQ, and detectability index. Just as such analysis has proven invaluable to the development and optimization of flat-panel detector systems for radiography, application in DE imaging allows identification of the factors limiting DE imaging performance and offers a guide to the development of high-performance systems. The generalized, dual-energy detectability index gives a quantitative tool to optimize the tissue cancellation parameter, with quantitative account of the tradeoffs between quantum and anatomical noise. A conventional description (ignoring anatomical noise) would be insufficient in describing DE imaging performance, because the tradeoffs between quantum noise and anatomical noise are central to optimal DE decomposition. The ability of DE imaging to improve diagnostic performance is apparent in the analysis above, where the detectability index for a
simple, idealized detection task was a factor of two greater for DE imaging compared to radiography, in agreement with the improved conspicuity offered by DE techniques.\textsuperscript{34,47}

These results are promising because they point to other important areas of investigation, such as optimal selection of dual kVp settings in DE imaging and optimal allocation of dose between high and low-energy images. Also, the Fourier-based analysis of anatomical noise in DE images provides a fast, practical technique for optimal selection of tissue cancellation parameter through minimization of the anatomical NPS, corresponding to minimization of bony contrast.
CHAPTER 4: DUAL-ENERGY IMAGING SYSTEM OPTIMIZATION

I INTRODUCTION
Understanding the physical factors that limit imaging performance is important in developing FPD-based DE imaging to its full potential. Chapter 4 applies previously reported experimental and theoretical methodology\(^{99,100}\) that extends descriptions of detective quantum efficiency (DQE) and noise-equivalent quanta (NEQ) to DE imaging and includes anatomical background noise to yield the generalized DQE (GDQE) and NEQ (GNEQ) as discussed in Chap. 3. In the sections below a figure of merit (detectability index\(^{62}\)) is obtained by integrating the DE GNEQ with the task function and used as an objective function for system optimization. Cascaded systems analysis (CSA)\(^{26,81}\) was employed as a theoretical framework for calculating the NEQ of FPD-based DE imaging systems. We examine optimal acquisition and decomposition parameters – including kVp, dose allocation between low- and high-energy projections, and tissue cancellation parameter – each evaluated as a function of dose across a broad range, varying from
fluoroscopic to radiographic dose levels. Finally, the framework is applied to compare: \( i. \) the low-dose limits of imaging performance achieved with DE imaging as compared to radiography and fluoroscopy; and \( ii. \) the DQE performance of single-shot versus double-shot DE imaging systems.

II METHODS

II.1 Task-Based Figure of Merit

II.1.1 The detectability index

As described in Chapters 2 and 3, the detectability index may be written as the integral of the NEQ weighted by a task function [Eq. (2.32)]:

\[
d^{12} = \int_{-\frac{\nu_{Nyq}}{2}}^{\frac{\nu_{Nyq}}{2}} \int_{-\frac{\nu_{Nyq}}{2}}^{\frac{\nu_{Nyq}}{2}} GNEQ_{DE}(u,v)W_{\text{task}}^2(u,v)dudv \tag{4.1}
\]

where \( GNEQ_{DE} \) denotes the generalized NEQ [Eq. (3.28)]:

\[
GNEQ_{DE}(u,v) = \frac{MTF^2(u,v)}{NNPS_{DE,Q}(u,v) + NNPS_{DE,B}(u,v)MTF^2(u,v) + NNPS_{DE,\text{add}}(u,v)} \tag{4.2}
\]

The detectability index provides a figure of merit that incorporates detector performance and imaging task, offering an objective function for system optimization.\(^{77,101}\)

II.1.2 Task function

The task function may be written as the Fourier-difference between two hypotheses,\(^62\) for example, “object present” and “object absent”. Although several tasks are relevant to chest imaging (e.g., discrimination, size estimation, and localization) a nodule detection task was used below for simplicity, with more complex task functions investigated in Chap. 6. The task
function was mathematically separated into two terms to differentiate the size of the nodule [described by $F_{\text{task}}(u,v)$] from its contrast (relative to background), $C_{DE}$. Hence:

$$W_{\text{Task}}(u,v) = C_{DE} \cdot F_{\text{Task}}(u,v)$$  \hspace{1cm} (4.3)

$C_{DE}$ is the contrast in a DE image between the nodule and surrounding lung tissue, and was estimated as derived in Appendix B for the absolute contrast between two objects in a DE image:

$$C_{DE} = -[\mu_{H,\text{Lung}} - \mu_{H,\text{Nodule}} - \frac{\mu_{H,\text{Bone}}}{\mu_{L,\text{Bone}}} (\mu_{L,\text{Lung}} - \mu_{L,\text{Nodule}})] N_{\text{Nodule}}$$  \hspace{1cm} (4.4)

The effective attenuation coefficients were computed by integrating the linear attenuation coefficient with the x-ray spectrum:

$$\mu_{\text{eff}} = -1/t \ln \left[ \frac{\int_{E=1}^{150} q_0(E)e^{\mu(E)t}dE}{\int_{E=1}^{150} q_0(E)dE} \right]$$  \hspace{1cm} (4.5)

where $t$ is the thickness of the material (1 mm nodule, 50 mm lung, and 3 mm cortical bone), $\mu_{\text{eff}}$ is the effective linear attenuation coefficient for lung, bone, or nodule tissue material appearing in Eq. (4.4) and $q_0(E)$ is the low- or high- energy spectrum incident on the detector after filtration by 10 cm of water to account for beam hardening. The values for the attenuation coefficient of lung were taken from ICRU Report 44,\textsuperscript{102} and bone (cortical) and nodule (approximated by polystyrene) were taken from Hubbell.\textsuperscript{103}

The spatial frequency content of the task function, $F_{\text{task}}(u,v)$, is determined by the Fourier transform of the difference between the object function and the background function\textsuperscript{62}:

$$F_{\text{Task}}(u,v) = N[F(\text{hyp}_1(x,y) - \text{hyp}_2(x,y))]$$  \hspace{1cm} (4.6)
where \( hyp_1(x,y) \) and \( hyp_2(x,y) \) are hypotheses corresponding to the object and background functions. The object function herein approximated a nodule as a two-dimensional gaussian, with background (object absent) approximated by a constant. \( N \) is a normalization factor such that an object function of unit contrast has signal power [i.e., area under \( F_{Task}^2(u,v) \)] equal to \( \hat{P}_{Nodule} \), (the area of the nodule). Although a simple gaussian detection task was used, this general formulation can also be applied to describe more complex imaging tasks (e.g., discrimination and size estimation\(^\text{104}\)).

### II.1.3 Cascaded systems analysis for the modeling of DE NEQ

As described in Chapters 2 and 3, cascaded systems analysis (CSA) provides a method for calculating analytically the signal and noise performance of imaging systems by separating the imaging chain into a series of stages. The framework is applied in this chapter to model the DE NPS and MTF combined with the experimentally determined anatomical background noise to yield a semi-empirical estimate of \( GNEQ_{DE}(u,v) \) and to compute the detectability index. Below is an overview of the CSA stages corresponding to the DE imaging system modeled in this chapter.

#### a. Stage 0: Incident fluence

Stage 0 corresponds to the fluence of quanta incident on the detector denoted \( \overline{q}_0 \). Fluence was calculated from the normalized spectrum \( q_{0,norm}(E) \), fluence per unit exposure \( (q/X)(E) \), and exposure \( X_{det} \) at the detector:\(^\text{26}\)

\[
\overline{q}_0 = X_{det} \int_{E} q_{0,norm}(E) \left( \frac{q}{X} \right)(E)dE
\]  

(4.7)

Spectra were computed using the Spektr toolkit,\(^\text{105}\) based on the TASMIP method of Boone and Seibert.\(^\text{89}\) Exposure at the detector, \( X_{det} \), was calculated from the entrance surface dose (ESD,
measured at the entrance surface of the phantom) as described below. Other dose metrics can be used, such as effective dose, imparted energy\(^{106}\) or dose at a specified depth, but ESD is often reported in chest radiography and is simple to compute and verify experimentally. Exposure at the detector, \(X_{\text{det}}\), was computed by assuming a chest thickness in the region of the lung equivalent to 10 cm of water\(^7\):

\[
X_{\text{det}} = \int_{E} q_p(E) e^{-\mu_{\text{lung}}l_{\text{lung}}} \left(\frac{q}{X}(E)\right) dE
\]  

(4.8)

where \(q_p(E)\) denotes the spectrum incident upon the patient and was calculated from the entrance surface exposure (ESE):\(^{107}\)

\[
\text{ESE} = \frac{\text{ESD}}{\bar{f}_{\text{water}} \cdot \text{BSF}}
\]  

(4.9)

where \(\bar{f}_{\text{water}}\) is the f-factor for water (from Hubbell\(^{108}\) averaged over the x-ray spectrum) and BSF is the backscatter factor as function of half-value layer (HVL) Cu taken from Johns et al.\(^{109}\)

b. Stages 1-7: Signal and noise propagation in the projection image

Stages 1 to 7 correspond to: 1) interaction of x-rays in the scintillator; 2) conversion of x-rays to optical photons; 3) spatial blurring of optical photons; 4) conversion of optical photons to e-h pairs by the photodiode; 5) integration of charge by the photodiode; 6) sampling of detector pixels; and 7) additive electronic noise due to pixel dark noise, amplifier noise, and digitization. Calculation of the parameters for each stage was the same as described previously\(^99\) but with a 200 \(\mu\)m pixel pitch detector. The MTF was computed using a parameterized Lorenztian model for the scintillator MTF, \(T_3(u,v)\), that agreed well with the measured MTF, multiplied by a sinc
function, \( T_s(u,v) \), corresponding to the sampling aperture. The quantum NPS was computed for a 250 \( \text{mg/cm}^2 \) CsI:Tl-based FPD, with K-fluorescence included as described by Yao and Cunningham.\(^9\) Previous work has shown good agreement between CSA and measured NPS and DQE, across a wide range of energies.\(^9\) The NPS for the low- and high-energy projections were computed at the appropriate low and high kVp, filtration, and ESD and added filtration to yield the DE NPS.

**II.1.4 Measurements of the anatomical background NPS**

Measurements of the anatomical NPS were obtained using an experimental imaging bench and an anthropomorphic chest phantom as described in Chap. 3 using the PekinElmer detector. The anthropomorphic chest phantom was described in a previous article\(^9\),\(^1\) and is a modified Rando phantom consisting of simulated lung material (custom formulations of polyurethane and micro-balloons). The lung contains nodule-simulating spheres of various contrast (polystyrene, polyurethane and polypropylene) and diameter (1.6-12.7 mm). Use of the phantom allowed measurement of anatomical noise across a wide range of dose and kVp, prohibitive of real patient studies. Low-energy projections of the phantom (60-90 kVp) were acquired with 2 mm Al added filtration, while high-energy projections (120-150 kVp) were acquired with 2 mm Cu added filtration. The mAs was adjusted so that approximately 50% signal saturation was obtained inside the lung region, corresponding to (5-2) mAs for the low-energy projections and (4-1) mAs for the high-energy projections. An antiscatter grid was not employed in the current study. The choice of filters is an important factor in producing low- and high-energy x-ray spectra that have minimal energy overlap in order to optimize bone cancellation. The identification of optimal beam filtration for both spectra is presently under investigation, and the filters selected herein are comparable to the optima suggested by initial studies.\(^1\)

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The anatomical background NPS in the DE images was measured as previously described in Chap. 2, using a technique that minimizes possible effects of non-stationarity on the NPS: first, ROIs forming the data ensemble were chosen to be of extent larger than fluctuations associated with anatomical noise; second, the location of the ROIs in the image data was randomized in order to randomize the phase of the anatomical noise estimates with respect to background clutter. It should be noted that the simulated nodules were present in the projections when computing the anatomical noise, however the effect on the resulting background NPS estimate is small since the area covered by the nodules is small compared to that of the entire lung regions; therefore, the contribution to the background NPS from anatomical clutter arising from the ribs and bronchial structures outweighed that of the nodule. Anatomical NPS in the lung field, $NNPS_B(u,v)$, was extracted from the measured NPS (i.e., corrected for quantum and electronic noise contributions) and modeled as $1/f$ noise:

$$NNPS_B(u,v) \approx \frac{K}{f^\beta} \quad (4.10)$$

where:

$$f = \sqrt{u^2 + v^2} \quad (4.11)$$

The fitting parameters $K$ and $\beta$ were measured for 49 separate kVp combinations (7 low and 7 high kVp) and 101 values of $w_s$ (0 to 1 in steps of 0.01). The data were interpolated to 1 kVp bins, generating a $K$ and $\beta$ look-up table for any kVp pair and $w_s$ value. The term $S_B(u,v)$ was computed from $K$ and $\beta$ as a two-dimensional, rotationally symmetrical function. The anatomical NPS was measured up to the Nyquist frequency (1.25 mm$^{-1}$ for 400 μm pixel pitch) and extrapolated to model a higher resolution detector (to 2.5 mm$^{-1}$ for 200 μm pixel pitch) more
consistent with diagnostic chest imaging. Such extrapolation is appropriate under the assumption that $NNPS_{DE}(u,v)$ is independent of dose and detector configuration.

II.2 System Design and Performance Optimization

II.2.1 Optimal dose allocation and kVp pairs

The detectability index was computed as a function of the low- and high-energy kVp, the tissue cancellation parameter ($w_s$), and dose allocation ($A$), defined as:

$$A = \frac{ESD_L}{ESD_L + ESD_H}$$ \hspace{1cm} (4.12)

where a dose allocation of 0.5 corresponds to equal ESD between the low- and high-energy images.

Figure 4.1: Flowchart for multivariate calculation of peak detectability as a function of the tissue cancellation parameter ($w_s$), dose allocation ($A$), and kVp. The inserted graph illustrates the detectability index as a function of the tissue cancellation parameter

Figure 4.1 depicts the framework for multivariate analysis involved in the optimization process. The detectability index was computed by first initializing the parameters to their minimal values – i.e. $w_s=0$, $A=0$, $kVp_L=60$, and $kVp_H=120$. Secondly, $C_{DE}$ and $F_{Task}(u,v)$ were computed to yield the task function, $W_{Task}(u,v)$. Thirdly, $NNPS_{DE,E}(u,v)$, $NNPS_{DE,B}(u,v)$, and $NNPS_{DE,add}(u,v)$ were computed to yield the $GNEQ_{DE}(u,v)$. Finally, $GNEQ_{DE}(u,v)$ and $W_{Task}(u,v)$ were integrated [Eq.
(4.1)) to generate the DE detectability index, \( d'^2 \). The tissue cancellation parameter and dose allocation were then incremented and \( GNEQ_{DE}(u,v) \) was recomputed using the new \( NNPS_{DE,q}(u,v), NNPS_{DE,b}(u,v), \) and \( NNPS_{DE,add}(u,v) \). The detectability index was calculated over the range of \( w_s \) and \( A \) for one energy-pair, and the peak detectability \( (d'^2_{\text{peak}}) \) corresponding to optimal tissue cancellation parameter \( (w_s^*) \) and optimal dose allocation \( (A^*) \) were identified. This process was repeated for every kVp pair until matrices of the peak detectability \( d'^2_{\text{peak}}(kVp_L,kVp_H) \), optimal tissue cancellation parameter \( w_s^*(kVp_L,kVp_H) \), and optimal dose allocation \( A^*(kVp_L,kVp_H) \) were generated. The optimal kVp pair for each dose level was identified at maximum \( d'^2_{\text{peak}} \). To investigate the effect of patient dose on the optimal parameters, the calculation described above was computed at three levels of ESD\text{DE} (defined as the sum of ESD\text{L} and ESD\text{H}) – viz., “High-Dose” (5 mGy, to investigate the upper theoretical limit of DQE), “Radiographic” (0.5 mGy, corresponding approximately to an ESD for a conventional anterior-posterior chest exam)\(^{112} \), and “Low-Dose” (0.05 mGy, corresponding approximately to an ESD per frame comparable to fluoroscopy). This wide range in dose levels was motivated to demonstrate tradeoffs in dual-energy imaging systems between various sources of noise (i.e. electronic, quantum and anatomical noise) as the dose was varied.

**II.2.2 Application of the theoretical framework: Low-dose DE imaging performance**

The detectability index for radiographic and DE imaging systems was computed to compare imaging performance as a function of dose. The radiographic system was modeled as a 250 mg/cm\(^2\) CsI:Tl-based FPD with 200 \( \mu \)m pixel pitch (0.80 fill factor), operated at 130 kVp (with 2mm Al added filtration) with 10 cm of water to simulate a patient. Detectability index was computed as a function of ESD for the radiography system (with parameters held fixed at the
II.2.3 Application of the theoretical framework: Comparison of single-shot and double-shot DE imaging

In single-shot DE imaging, the low- and high-energy images are acquired in a single exposure by use of a double (or double-sided) detector. Such technologies include for example, photon counting detectors with energy discrimination and sandwiched CR plates with an inter-detector beam filter. Single-shot techniques may also employ pre-patient k-edge filters to modulate the x-ray spectrum and reduce quantum noise in parts of the image. The main advantages of single-shot DE imaging are the absence of motion artifact and operation without cardiac gating. Typically, a single-shot DE imaging system has a filter placed between two detectors to harden the beam and produce a high-energy image on the second detector. Of course, the inter-detector filter causes a significant loss of image quanta. Also, depending on the detector geometry, accurate registration can be challenging due to slightly different magnification factors between the two projections. A hypothetical detector was modeled to compare the performance of single-shot and dual-shot DE systems. The single-shot system does not model any particular system but rather is a hypothetical model illustrative of a basic sandwiched single-shot system. An analogous practical embodiment of such, for example, could involve a sandwiched pair of flat-panel detectors (although the performance of such a system would be somewhat reduced from the idealized case considered, due to attenuation in the glass substrate and metal readout lines). Flat-panel detectors utilizing a low-attenuation (e.g., flexible polymer) substrate would ameliorate such effects. For purposes of this paper, the idealized
sandwich is a hypothetical design with x-ray absorption only in the front and back detectors and the inter-detector filter, representing a best-case configuration.

![Diagram of DE systems](image)

Figure 4.2: Illustration of (a) the double-shot DE imaging FPD employed in this work and (b) a hypothetical single-shot DE imaging system. The former involves a single 250 mg/cm² CsI scintillator, whereas the latter consists of a 150 mg/cm² CsI:Tl front-detector, a 0.8 mm Cu inter-detector filter, and a 250 mg/cm² CsI:Tl back-detector. For the single-shot case, absorption in the inter-detector layer results in a low- and high-energy image acquired on the front and back detectors, respectively. The plotted spectra illustrate the difference in low- and high-energy separation between the double- and single shot DE systems.

The DQE\textsuperscript{DE} served as a figure of merit to compare single-shot and double-shot DE imaging performance, where the double-shot DQE, \( DQE_{DE,DS}(\mu,\nu) \), was computed using Eq. (3.26) and the same system parameters described in Sec. II.3 of Chap.3 at [70/130] kVp with 2 mm Cu added filtration in the high-energy beam. A diagram of the hypothetical single-shot system is depicted in Fig. 4.2(b) consisting of two CsI:Tl scintillator layers (150 mg/cm² and 250 mg/cm²) separated by 0.8 mm Cu. A 120 kVp beam with 2 mm Al added filtration was considered,
consistent with a typical single-shot DE imaging configuration.\textsuperscript{32,33} The DQE was derived from the “stochastic definition”\textsuperscript{76} as the ratio of the deterministic and actual NPS [Chap 2, Eq. (2.24)]:

\[
DQE_{DE,SS}(u,v) = \frac{NNPS_{DE,Det}}{NNPS_{DE,Actual}}
\]

\[
= \frac{(MTF_H(u,v))^2}{q_{H,0}} + w_s^2 \frac{(MTF_L(u,v))^2}{q_{L,0}}
\]

\[
= \frac{NNPS_{DE,Q}(u,v)}{NNPS_{DE,Q}(u,v) + NNPS_{DE,add}(u,v)}
\]

(4.13)

where \textit{SS} denotes the single-shot DE DQE. The low-energy fluence, \( \tilde{q}_{L,0} \) (equivalent to \( \tilde{q}_{\text{Front},0} \) in Fig. 4.2(b), is the fluence incident on the first detector, whereas the high-energy fluence, \( \tilde{q}_{H,0} \), is the fluence incident on the inter-detector Cu layer. \( MTF_L(u,v) \) and \( MTF_H(u,v) \) are the MTFs for the front and back detectors, respectively. The zeroth stage in the CSA calculation for \( S_{H,Q}(u,v) \) corresponds to the fluence incident on the second detector, (i.e \( \tilde{q}_{\text{Back},0} \) in Fig. 4.2(a). For simplicity and to allow a basic comparison of DE imaging performance in terms of quantum noise alone, anatomical background noise was not included in this comparison (i.e., taken as equivalent between SS and DS), and the tissue cancellation parameter was estimated using the definition of \( w_s \) and the attenuation coefficient of bone:\textsuperscript{12}

\[
w_s = \frac{\mu_{H,\text{Bone}}}{\mu_{L,\text{Bone}}}
\]

(4.14)

The effective values for the attenuation coefficients were calculated using Eq. (4.5) and low- and high-energy incident spectra. Both \( DQE_{DE,SS}(u,v) \) and \( DQE_{DE,DS}(u,v) \) were computed with a
total entrance surface dose of 0.5 mGy, and a dose allocation, A=0.5, was set for the double-shot
DE imaging system.

III  RESULTS

III.1  Anatomical NPS and the Generalized Dual-Energy NEQ

Figure 4.3: (a) Anatomical NPS measurements in the anthropomorphic chest phantom at [70/130] kVp plotted as a
function of the tissue cancellation parameter and spatial frequency. (b) DE GNEQ at [70/130] kVp plotted as a
function of the tissue cancellation parameter and spatial frequency. The superimposed black line depicts the
maximum DE GNEQ at each spatial frequency. An optimal value for the tissue cancellation parameter is suggested
in the range ws ~ 0.33 - 0.42, with the optimum depending on the imaging task.

Figure 4.3(a) plots $NPS_{DE,B}(f)$ as a function of the tissue cancellation parameter at [70/130]
kVp. The overall magnitude of the anatomical background noise (quantified by $K$) varies
significantly as a function of $w_s$ and is minimized at $w_s=0.40$. The anatomical background NPS is
also flatter (reduced values of $\beta$) at $w_s=0.40$ corresponding to complete bone cancellation and
reduced low-frequency anatomical noise. Therefore, reduction of anatomical noise corresponds
to reduction of overlying bone and suggests optimal values for the tissue cancellation parameter
where anatomical background noise is minimized. Figure 4.3(b) plots a 1-D diagonal strip (45° in
the Fourier domain) of the $GNEQ_{DE}(u,v)$ at [70/130] kVp as a function of $w_s$. The exposure was
set to 0.5 mR in both the low- and high-energy image such that tradeoffs between quantum noise and anatomical noise are revealed. The 1/f anatomical noise is the cause for the strong reduction in $GNEQ_{DE}(u,v)$ at low spatial frequencies. The superimposed black line depicts the maximum $GNEQ_{DE}(u,v)$ at each spatial frequency, showing that there is no single value of $w_s$ that maximizes the $GNEQ_{DE}(u,v)$; rather, a range of $w_s$ (~0.3 to 0.4 over the range 0 to 2.5 mm$^{-1}$) is suggested. Hence, a description of what frequencies are most important given a specific imaging task is necessary to find the optimal tissue cancellation parameter.

### III.2 The Task Function and Detectability Index

Figure 4.4(a) plots DE contrast as a function of energy difference (kVp) between the low- and high-kVp image. DE contrast increases with the kVp difference because of the differential energy dependence in the linear attenuation coefficients of nodule and lung tissue [Eq. (4.4)]. The inset figure illustrates the 1 mm gaussian object function approximating the nodule. While more complex task functions are considered in Chapter 6, in the current study the size of the nodule was chosen to correspond to a scale of interest (1 mm) such that the task function weighs mid- and high-frequencies important for edge detection and discrimination by way of a simple detection task. Larger nodules in a simple detection task weigh only the lowest spatial-frequencies of the $GNEQ_{DE}(u,v)$, dominated by anatomical noise. Choice of a 1 mm gaussian provides higher weighting at mid-frequencies that are likely important in more realistic, complex tasks. Figure 4.4(b) plots the task functions, where the magnitude of $W_{Task}(u,v)$ varies with kVp according to the nodule contrast and spatial-frequency extent is determined by $F_{task}(u,v)$. The detection task function preferentially weights lower frequencies of the $GNEQ_{DE}(u,v)$ where anatomical noise is generally greater in magnitude than quantum or electronic noise. Therefore, the influence of anatomical noise plays an important role in determining optimal DE
decomposition and acquisition parameters when using the detectability index as an objective function.

![Figure 4.4](image)

Figure 4.4: (a) Nodule contrast relative to lung as a function of the difference in kVp in DE images. The insert graph shows a 1D gaussian approximating a nodule and background function approximated by a constant. (b) Task function for nodule detection in DE images at various energy pairs. The magnitude of the task function scales with the nodule-to-lung contrast plotted in Fig 4.4(a).

To illustrate the correspondence between nodule conspicuity and the detectability index, images were decomposed at optimal $w_s$ predicted for each energy pair and shown in comparison to calculated $d''^2$ in Fig. 4.5. The detectability index is seen to correlate qualitatively with nodule conspicuity. For example at reduced low-kVp, nodule contrast is increased and at greater high-kVp quantum noise is amplified, suggesting [60/120] kVp as the technique that qualitatively maximizes nodule conspicuity, which agrees with quantitative values of detectability index. The ESD varied somewhat in this illustration (from 0.08 mGy at [60/150] kVp to 0.14 mGy at [90/120] kVp) because exposure was held fixed at the detector for each energy in order to study tradeoffs between quantum noise, DE contrast, and anatomical noise. Still, the qualitative results of Fig. 4.5 support the notion that detectability index correlates reasonably with perceived image quality.
Figure 4.5: Dual-energy images acquired across a range of low- and high kVp. The detectability index was calculated for each image for a nodule detection task taking into account anatomical noise, contrast, and exposure at the detector. Correlation between nodule detection task taking into account anatomical noise, contrast, and exposure at the detector. Correlation between nodule conspicuity and detectability index provides qualitative verification that detectability index provides a reasonable surrogate for image quality.

Figure 4.6 illustrates a typical optimization of the tissue cancellation parameter and dose allocation for a given energy pair, in this case shown for [70/130] kVp. A fairly strong dependence was observed on both independent variables with a clear optima at $w_s^*=0.37$ and $A^*=0.32$, meaning that approximately one-third of the ESD should be allocated to the low-energy image. The majority of dose is allocated to the high-energy image because $NNPS_{DE,Q}(u,v)$ is more sensitive to $NNPS_{H,Q}(u,v)$ than to $NNPS_{L,Q}(u,v)$. This dose allocation is consistent with the work of Johns and Yaffe, who found that in DE mammography more
dose should be allocated in the high-energy image. It should be noted that the optimal tissue cancellation parameter, \( w_s^* \), is slightly lower than the value that simply minimizes anatomical noise, [Fig 4.3(a)], because quantum noise and imaging task have been taken into account.

![Detectability index computed at [70/130] kVp as a function of the tissue cancellation parameter \( w_s \) and dose allocation \( A \) between the low- and high-energy image. The optimal parameters, \( w_s^* = 0.37 \) and \( A^* = 0.32 \), are identified at peak detectability, \( d^{2}_{\text{peak}} \).

III.3 Optimal Tissue Cancellation, Dose Allocation, and kVp

Figure 4.7 plots the peak detectability, \( d^{2}_{\text{peak}} \), optimal tissue cancellation parameter, \( w_s^* \), and optimal dose allocation, \( A^* \), as a function of kVp pairs at three levels of ESD_{DE}. At lower dose (0.05 mGy) the greatest value of \( d^{2}_{\text{peak}} \) was achieved at [60/150] kVp because quantum noise, the dominant source of noise at low dose, is minimized at lower kVp and nodule contrast is greatest at larger energy differences [Fig 4.4(a)]. The corresponding optimal tissue cancellation parameter and dose allocation for [60/150] kVp were 0.15 and 0.20, respectively [Fig 4.7(d) and 4.7(g)]. As ESD_{DE} is increased, anatomical noise dominates over quantum noise, and greater kVp are suggested, as shown for the case ESD_{DE} = 0.5 mGy, where the optimum is again at [60/150] kVp, but where the trend suggests a greater high-energy kVp. At this dose level the
optimal tissue cancellation parameter and dose allocation for [60/150] kVp were 0.25 and 0.30, respectively [Fig 4.7(e) and 4.7(h)]. When ESD$_{DE}$ increases further, the optimal $d^{-2}_{\text{peak}}$ is observed at [90/150] kVp [Fig 4.7(c)], corresponding to conditions for which anatomical noise, the dominant source of noise at high dose, is minimized. At this higher dose the corresponding optimal tissue cancellation parameter increased to 0.4 [Fig. 4.7(f)], and optimal dose allocation increased to 0.4 [Fig. 4.7(i)].

Overall the optimal tissue cancellation parameter varied with dose, because quantum noise increases relative to anatomical noise at extremely low dose levels; therefore, $w_s^*$ decreases to reduce the magnitude of $NNPS_{DE,Q}(u,v)$ in Eq. (3.23). At all dose levels, $w_s^*$ is greatest at [90/120] kVp because of smaller kVp separation; therefore, a greater degree of subtraction of the low-energy image is required to cancel bony structures in the high-energy image. Also, $A^*$ is seen to correlate with $w_s^*$ throughout because noise arising from $NNPS_{L,Q}(u,v)$ is multiplied by $w_s^2$; therefore, if $w_s^*$ increases it becomes advantageous to increase $A$, which is proportional to $ESD_L$ and inversely proportional to $NNPS_{L,Q}(u,v)$. These results illustrate the tradeoffs between, contrast, quantum noise, and anatomical noise in DE decomposition that need to be taken into account for system optimization.
Figure 4.7: Optimal peak detectability index, $d'^2_{peak}$, optimal tissue cancellation parameter ($w_s^*$), and optimal dose allocation ($A^*$) computed as a function of low- and high-kVp and shown for three levels of DE entrance surface dose: “High-Dose” (5 mGy), “Radiographic” (0.5 mGy), and “Low-Dose” (0.05 mGy). The optimal kVp pairs were found at [90/150], [60/150], and [60/150], respectively.

Figure 4.8(a) summarizes the above results by plotting the optimal dose allocation and tissue cancellation parameter corresponding to the kVp pair with the greatest $d'^2_{peak}$ as a function of ESD$_{DE}$. At very low dose, $w_s^*$ is very low and hence most of the dose goes to the high-energy image. As ESD$_{DE}$ increases, $w_s^*$ increases because quantum noise becomes less dominant and hence $A^*$ increases. Furthermore, once a certain dose level is achieved (ESD ~1 mGy), $A^*$ and $w_s^*$ become asymptotic, implying that values close to maxima have been reached.

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III.4 Application of the Theoretical Framework: Low-dose DE Imaging Performance

Figure 4.8(b) compares the performance of radiography and DE imaging as a function of the total ESD in terms of the peak detectability index for both systems. The $d_{\text{peak}}^2$ values for the DE system at each point along the curve were calculated at the optimal $w_s^*$ and $A^*$ values shown in Fig. 4.8(a). DE imaging performs significantly better than radiography at doses greater than 0.05 mGy ESD. Also, the detectability index for radiography becomes asymptotic at ~1 mGy, whereas the DE detectability index continues to increase. This asymptotic limit in the former is because radiographic image quality is fundamentally limited by anatomical noise, whereas DE imaging can reduce the amount of anatomical noise and therefore makes better use of the applied radiation to the patient.

Figure 4.8: (a) Optimal tissue cancellation (dashed line) and optimal dose allocation (solid line) calculated as a function of total entrance surface dose. At very low doses $w_s^*$ and $A^*$ converge to zero, corresponding to acquiring a single projection. (b) Comparison of detectability index for radiography and DE imaging. The solid line depicts the peak detectability index computed for a DE imaging system at optimal kVp, dose allocation, and $w_s^*$ [see Fig. 4.8(a)] as a function of the total entrance surface dose ($E_{\text{SD}}$). The dashed line depicts the detectability index computed for a radiographic system at 130 kVp. The performance of the DE imaging system for a detection task increases significantly with respect to the radiographic system at higher ESD, where anatomical noise limits conventional radiographic performance.
III.5 Application of the Theoretical Framework: Comparison of Single-Shot and Double-Shot DE Imaging

Figure 4.9: Single-shot vs double-shot DE imaging. The solid line depicts the DQE\textsubscript{DE} computed for a double-shot DE system. The dashed line depicts the DQE\textsubscript{DE} computed for a single-shot DE system (illustrated in Fig. 4.2). Both systems were modeled with an ESD of 0.5 mGy. The double-shot DE imaging system provides a factor of two increase in both DQE\textsubscript{DE} and nodule contrast.

Figure 4.9 compares the DQE of single-shot and double-shot DE imaging systems. The single-shot DQE\textsubscript{DE} is approximately half that of the double-shot DQE\textsubscript{DE} due primarily to the filter between the two detectors, which amplifies the quantum sink\textsuperscript{81} and degrades signal and noise performance for the single-shot system. Moreover, quantum noise is greater in the single-shot DE system because the Cu filter is in front of the high-energy detector and, as discussed above, quantum noise in DE images is most sensitive to quantum noise arising from the high-energy projection. The DQE is furthermore increased in double-shot DE imaging because $w_s$ is smaller (0.51 compared to 0.73 in single-shot) due to greater separation of x-ray spectra possible with different kVp and (pre-patient) added filtration. Moreover, soft-tissue contrast is increased for the double-shot case ($C_{DE}$=0.0128 for double-shot compared to 0.0053 for single-shot) due to improved energy separation. These results are consistent with Alvarez et al., who observed that double-shot images have less quantum noise than single-shot DE images.\textsuperscript{33} Although single-shot
DE imaging offers advantages such as simultaneous acquisition of low- and high-energy images, the tradeoffs in increased quantum noise needs to be considered. Also, new FPD technologies that allow rapid frame read-out in combination with precise cardiac gating systems offer means to circumvent this tradeoff.

**IV CONCLUSIONS**

The framework employed above provides a description of DE imaging performance accounting not only for quantum and electronic noise as factors for system optimization but also anatomical noise and imaging task. Other factors that warrant consideration are x-ray scatter\textsuperscript{101,120-123} and noise reduction algorithms\textsuperscript{124-126} in description of the DQE and NEQ. Such noise reduction algorithms include filtering of the high-energy imaging, correlated noise reduction, and noise clipping – and is the focus of the work in Chapter 5 and 6.

In light of this work, DE energy imaging is seen as a generalization of conventional radiography in that there is a continuum between radiography and DE imaging through variation of $w_s$ and $A$. Optimal system performance in this framework, corresponding to peak detectability, is found on that continuum for any given dose without dose penalty associated with the second projection, a drawback often associated with DE imaging but no longer an impediment thanks to high-performance FPDs that are quantum limited at a fraction of conventional radiographic dose. Therefore, by combining projections acquired at different x-ray energies it is possible to obtain images that make better use of the applied radiation to the patient, reduce dose, and increase conspicuity in structures of interest.

In conclusion, a theoretical framework was developed to understand and optimize DE imaging performance. Description of quantum and anatomical noise in DE images was combined with a
Fourier-based description of imaging task to yield an objective function (detectability index) that correlated qualitatively with nodule conspicuity and was used to optimize acquisition parameters (dose allocation and kVp) and tissue cancellation parameter, \( w_s \). It also was demonstrated that a task function that weights spatial frequencies of the \( GNEQ_{DE}(u,v) \) was necessary to uniquely select the optimal system parameters – i.e., GNEQ alone is insufficient to determine optimal system configurations. The optimal tissue cancellation parameter, dose allocation, and kVp were found to depend on the imaging dose due to tradeoffs among nodule contrast, anatomical noise, and quantum noise. For example, at ESD comparable to a conventional radiograph (ESD ~0.5 mGy) the optimal kVp was [60/150] kVp and the optimal dose allocation was ~0.30. However, at higher dose (ESD ~5 mGy) the optimal kVp was [90/150] with an optimal dose allocation of ~0.4. Similarly at low dose levels (ESD ~0.05 mGy) optimal kVp depended primarily on the low-kVp setting (viz., 60 kVp), was less sensitive to high-kVp, and suggested optimal dose allocation of ~0.20. Thus, dose is clearly as an important consideration for optimal system design and acquisition. Two further applications of this framework were demonstrated: the theoretical potential of DE imaging performance compared to traditional radiography as a function of dose (showing the relative boost in detectability for DE imaging down to low-dose extremes); and the significant DQE advantage of double-shot over single-shot DE imaging in terms of noise performance (a factor of two improvement in DQE for the double-shot system).
CHAPTER 5: DUAL-ENERGY IMAGING NOISE REDUCTION

ALGORITHMS

I INTRODUCTION

As discussed previously, a common means of decomposing soft-tissue and bone DE images is weighted log-subtraction of low- and high-energy projections.\textsuperscript{12,124} While this process minimizes anatomical noise associated with a given overlying material component (e.g., ribs), it amplifies the quantum noise component of the resulting DE images.\textsuperscript{127} The development of FPDs with high detective quantum efficiency (DQE) permits DE imaging at total patient dose comparable to that of a conventional chest radiograph.\textsuperscript{128} However, the increased quantum noise in DE images remains a challenge and has motivated the development of noise reduction algorithms that mitigate quantum noise amplification relative to standard log-subtraction (SLS), including:
In this chapter, we present a general formulation of linear DE image decomposition. Such formulation reduces to well-known linear algorithms (e.g., SLS, SSH, and ACNR) as special cases, allows examination of more general forms of noise reduction, and is amenable to cascaded systems analysis, which has proven a valuable methodology for understanding and optimizing signal and noise transfer characteristics. Non-linear noise reduction algorithms (e.g., NC and EAS) were not investigated in this work since they were not amenable to linear systems analysis. For the same reason the basis decomposition method, another common algorithm for dual-energy decomposition, was not investigated in this work, although previous work by Sabol et al. showed negligible difference in image quality between basis decomposition and SLS.

There are several assumptions associated with the theoretical approach that deserve to be acknowledged. The first involves linearity, shift-invariance, and stationarity as discussed below. X-ray scatter is also an important factor in image quality but was not included in the current model. Furthermore, anatomical background noise has been shown to be an important factor affecting observer performance and was described in Chapters 3 and 4 by including a $1/f^\beta$ characterization of anatomical noise in the total NPS. Since the purpose of this chapter is to develop a theoretical framework for algorithms intended to reduce quantum noise, anatomical noise was assumed to be independent of the choice of noise reduction algorithm and was therefore not included. (I.e., all algorithms were considered equivalent in terms of tissue cancellation, and the focus herein is on the mitigation of quantum noise.) First, the general linear formulation is described, showing its relation to various special case algorithms. Second, the
imaging performance of each algorithm is analyzed theoretically and experimentally in terms of the noise-power spectrum (NPS), modulation transfer function (MTF), noise-equivalent quanta (NEQ), and detectability index. Finally, detectability index is used as an objective metric to optimize an important technique factor in DE imaging (viz., the fraction of dose allocated between low-and high-energy projections) for various decomposition algorithms, with theoretical results examined in comparison to DE images of a simple chest phantom. The results and implications for DE chest imaging are discussed in terms of multi-function optimization, weighing tradeoffs among multiple soft-tissue and bone imaging tasks in the choice of technique factors and decomposition algorithms.

II THEORETICAL METHODS

II.1 Generalization of linear dual-energy decomposition algorithms

A general method for linear DE image decomposition is illustrated in Fig. (1), in which the low- and high-energy images are individually convolved with a filter and then added to form a DE image. Consider the combination of two log projections:

\[ I_{DE}(x, y) = I_L(x, y) * h_L(x, y) + I_H(x, y) * h_H(x, y) \]  \hspace{1cm} (5.1)

where \( I_L \) and \( I_H \) denote low- and high-energy (log) projections, and \( h_L \) and \( h_H \) denote convolution filters. This form emphasizes that the weighted combination of low- and high-energy images is, in general, a function of spatial frequency, rather than a simple constant-weighted sum (or difference). This form also provides a general formulation for the application of cascaded systems analysis in describing the noise (NPS) and spatial resolution (MTF) characteristics of DE imaging systems. In the sections below, it is shown that well-known decomposition algorithms reported in the literature; including weighted log subtraction and various noise reduction algorithms, arise as special cases of Eq. (5.1).
II.1.1 Standard log subtraction (SLS)

The simplest and most prevalent form of DE image decomposition involves a weighted subtraction of low- and high-energy images, referred to below as “standard log subtraction” (SLS):

\[ I_{SLS}(x, y) = -wI_L(x, y) + I_H(x, y) \]  
\[ = I_L(x, y) * [-w\delta(x, y)] + I_H(x, y) * \delta(x, y), \]  

where \( w \) is the tissue cancellation parameter \(^{12}\), given ideally by the ratio of the effective linear attenuation coefficient of the cancelled material at low and high energy, \( \mu_L/\mu_H \). Note that convolution with the delta function, \( \delta(x,y) \), has no effect on the image Equation (5.2a) is written in a form recognizable as SLS, whereas Eq. (5.2b) is slightly recast analogous to the general form of Eq. (5.1). Note that Eq. (5.2) yields a soft-tissue-only decomposition with conventional “polarity” (white soft-tissue on black background), for which \( w = \mu_{\text{bone}}^L/\mu_{\text{bone}}^H \). To yield the complementary bone image with conventional polarity (white bones on black background) the right-hand side of the equation is multiplied by –1, and \( w = \mu_{\text{soft}}^H/\mu_{\text{soft}}^L \). Since the factor (-1) does not affect the noise or spatial resolution (the subjects of interest herein) only the form in Eq. (5.2) is used below, and the change in sign is explicit only in specifically describing the bone image.

II.1.2 Simple smoothing of the high-energy image (SSH)

It has been demonstrated that under typical imaging conditions, the greater contributor of quantum noise in a DE image formed by SLS is the high-energy projection, rather than the low-energy projection.\(^{128}\) As described by Johns and Yaffe,\(^{129}\) a low-pass filter (LPF) applied to the
high-energy projection prior to DE decomposition can mitigate this effect, while the unfiltered low-energy projection preserves high spatial-frequency information. The resulting algorithm is termed “simple smoothing of the high-energy image” (SSH), expressed as:

\[
I_{SSH}(x, y) = -wI_L(x, y) + I_H(x, y) * h_{LPF}(x, y) \\
= I_L(x, y) * [-w\delta(x, y)] + I_H(x, y) * h_{LPF}(x, y)
\]  

(5.3a)

(5.3b)

where \(h_{LPF}\) denotes the low-pass filter. As above, Eq. (5.3a) is written in a more familiar form, whereas Eq. (5.3b) is recast analogous to the general form of Eq. (5.1). Similarly, the bone image of conventional polarity (white bone) is yielded by multiplying the right-hand side by \(-1\).

II.1.3 Anti-correlated noise reduction (ACNR)

The anti-correlated noise reduction (ACNR) algorithm was originally described by Kalender,\textsuperscript{130} Ergun,\textsuperscript{113} and McCullough\textsuperscript{125} and takes advantage of the fact that quantum noise in the soft-tissue image and bone image is anti-correlated. In decomposing a soft-tissue (or bone) image, the ACNR algorithm applies a high-pass filter to the complementary image (i.e., the bone image or the soft-tissue image, respectively). This effectively removes gross anatomical structures from the complementary image, leaving only quantum noise and some residual edge artifacts, where the quantum noise is anti-correlated to the quantum noise in the original DE image. The original DE image and the filtered complementary image are then added, weighted by a parameter, \(w_a\). The soft-tissue image decomposed by ACNR is thus:

\[
I_{ACNR}(x, y) = I_{SLS}(x, y) + w_nI_{SLS}'(x, y) * h_{HPF}(x, y) \\
= I_L(x, y) * [w_nwI_{HPF}(x, y) - w\delta(x, y)] + I_H(x, y) * [\delta(x, y) - w_nh_{HPF}(x, y)]
\]  

(5.4a)

(5.4b)

where

\[
I_{SLS}'(x, y) = w_nI_L(x, y) - I_H(x, y)
\]  

(5.4c)
is the complementary image, $w_c$ is the tissue cancellation parameter for the complementary image, and $h_{HPF}$ denotes the high-pass filter. The value of $w_n$ can be determined qualitatively or quantitatively through the minimization of quantum noise. As above, a bone image of conventional polarity involves an overall change in sign for both $I_{SLS}$ and $I_{SLS}^c$, resulting in an overall change of sign in $I_{ACNR}$.

II.1.4 A general linear noise reduction algorithm (GLNR)

An example of the general algorithm [Eq. (5.1)] is given by the following combination of low and high-pass filters:

$$I_{GLNR}(x,y) = I_L(x,y) * \left[ w_L h_{L,HPF}(x,y) - w h_{L,LPF}(x,y) \right] + I_H(x,y) * \left[ h_{H,LPF}(x,y) - w h_{H,HPF}(x,y) \right]$$

(5.5)

This equation can reduce to Eqs. (5.2), (5.3), or (5.4) with the appropriate choice of $w_L$, $w_H$, $h_{L,LPF}(x,y)$, $h_{L,HPF}(x,y)$, $h_{H,LPF}(x,y)$, and $h_{H,HPF}(x,y)$. While a full multivariate optimization of the parameters in Eq. (5.5) is beyond the scope of the current manuscript, the form is presented to demonstrate that the flexibility in the weighting factors and convolution filters of the GLNR algorithm will, under certain conditions, yield DE imaging performance superior to SLS, SSH, or ACNR.

Equations (5.2), (5.3), (5.4) and (5.5) illustrate that SLS, SSH, ACNR and GLNR are each special cases of the general form in Eq. (5.1), differing only in the form of the convolution filters, $h_I(x,y)$ and $h_H(x,y)$. This paper focuses firstly on SLS, SSH and ACNR, examining GLNR as a point of discussion for the potential merit of more general, multivariate decomposition. Table 5.1 summarizes the convolution filters for the four algorithms, along with the respective 2D Fourier transforms of the convolution filters, $H_L(u,v)$ and $H_H(u,v)$.
Table 5.1: Summary of low- and high-energy convolution filters for four DE decomposition algorithms (SLS, SSH, ACNR, and GLNR), shown to be special cases of the general form in Eq. (5.1).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$h_L(x, y)$</th>
<th>$h_H(x, y)$</th>
<th>$H_L(u, v)$</th>
<th>$H_H(u, v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLS</td>
<td>$-w \delta(x, y)$</td>
<td>$\delta(x, y)$</td>
<td>$-w$</td>
<td>1</td>
</tr>
<tr>
<td>SSH</td>
<td>$-w \delta(x, y)$</td>
<td>$h_{HPP}(x, y)$</td>
<td>$-w$</td>
<td>$H_{HPP}(u, v)$</td>
</tr>
<tr>
<td>ACNR</td>
<td>$w_z w_z h_{HPP}(x, y) - w \delta(x, y)$</td>
<td>$\delta(x, y) - w_z h_{HPP}(x, y)$</td>
<td>$w_z w_z H_{HPP}(u, v) - w$</td>
<td>$1 - w_z H_{HPP}(u, v)$</td>
</tr>
<tr>
<td>GLNR</td>
<td>$w_z h_{L,HPP}(x, y) - w h_{H,LPP}(x, y)$</td>
<td>$h_{H,LPP}(x, y) - w h_{H,HPP}(x, y)$</td>
<td>$w_z H_{L,LPP}(u, v) - w H_{H,LPP}(u, v)$</td>
<td>$H_{H,LPP}(u, v) - w H_{H,HPP}(u, v)$</td>
</tr>
</tbody>
</table>

II.2 Cascaded Systems Analysis

The image noise and spatial resolution of DE imaging systems were described in terms of the NPS and MTF, theoretically modeled using cascaded system analysis (CSA). Chapters 3 and 4 (previous work\textsuperscript{128}) extended CSA to FPD-based DE imaging systems using SLS and investigated optimal DE image acquisition and decomposition parameters. The current chapter extends analysis to the general linear decomposition formalism described above, with noise reduction algorithms such as SSH and ACNR emerging as special cases. The usual assumptions and limitations of CSA apply, including linearity, shift invariance, and stationarity, with caveats described by Albert,\textsuperscript{64} Barrett,\textsuperscript{134} and others fully acknowledged.

II.2.1 The projection image NPS and MTF

The formation of a projection image in an indirect-detection FPD can be described by a serial and parallel cascade of stages as described in Chapter 3. Calculation of the gain and spreading parameters for each stage was the same as described previously.\textsuperscript{127} The model considered a detector design corresponding to the Pixium-4600 FPD (Trixell, Moirans, France), incorporating a 143 μm pixel pitch detector in combination with a 250 mg/cm\textsuperscript{2} CsI:Tl scintillator and 80% fill.
factor. Previous work has shown good agreement between CSA and measured NPS and detective quantum efficiency (DQE), across a wide range of energies and exposure conditions.\textsuperscript{127}

### II.2.2 The DE image NPS

To derive the normalized NPS (NNPS, equal to the NPS divided by the square of the mean signal) for the DE image described by Eq. (5.1), we first consider the NNPS of the low- or high-energy projection \([NNPS_{L,H}(u,v)]\) and that following convolution with the low- or high-energy filter, \(h_{L,H}(x,y)\), described by a simple, deterministic transfer of the image noise:\textsuperscript{135}

\[
NNPS_{L,H}^{filt}(u,v) = NNPS_{L,H}(u,v)H_{L,H}^2(u,v)
\]

Therefore the application of a filter may increase or decrease the NNPS of the filter image depending on the choice of filter parameters. Second, we note that the NNPS for an image produced by the sum of the two uncorrelated log projections is the sum of the two NNPS, where the NNPS is the relative NPS of the linear projection.\textsuperscript{127}

Note that anatomical NPS as included in the generalized NEQ in previous work\textsuperscript{127} would be correlated and would need to be treated differently. As discussed in Chap.3, in DE imaging the quantum noise between the two images is independent and uncorrelated, irrespective of whether the system involves sequential acquisition of low- and high-energy images (“double-shot” DE imaging) or simultaneous capture of such via a detector sandwich (“single-shot” DE imaging). Therefore, it follows that the NNPS for a DE image decomposed according to Eq. (5.1) is the sum of the (filtered) low- and high-energy NNPS:

\[
NNPS_{DE}(u,v) = NNPS_{L}(u,v)H_{L}^2(u,v) + NNPS_{H}(u,v)H_{H}^2(u,v)
\]

Substituting the filters (Table 5.1) corresponding to the three special cases discussed above, the NNPS for DE images decomposed by SLS is therefore:

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\[ NNPS_{SLS}(u,v) = NNPS_L(u,v)w^2 + NNPS_H(u,v) \]  
\[ \text{(5.8a)} \]

and similarly for SSH:

\[ NNPS_{SSH}(u,v) = NNPS_L(u,v)w^2 + NNPS_H(u,v)H_{LPP}(u,v) \]  
\[ \text{(5.8b)} \]

and ACNR:

\[ NNPS_{ACNR}(u,v) = NNPS_L(u,v)[w_Lw_uH_{LPP}(u,v) - w] + NNPS_H(u,v)[1 - w_uH_{LPP}(u,v)]^2 \]  
\[ \text{(5.8c)} \]

Equation (5.7) is therefore a generalization of the equation previously derived for SLS\textsuperscript{127,128} and provides a general form for the NNPS for DE imaging systems employing linear decomposition algorithms.

### II.2.3 The DE image MTF

The MTF is derived below for the general case in which the MTF in the (filtered) low-and high-energy images are not necessarily equal. Three transfer functions contribute to the low- or high-energy image MTF: the spread of quanta in the scintillator, \( [\text{Stage } 3, T_{3,L,H}(u,v)] \), computed by deconvolving the photodiode aperture from the measured MTF; the photodiode aperture, \( [\text{Stage } 5, T_{5,L,H}(u,v)] \), a two-dimensional sinc function; and the applied low- or high-energy filter \( [H_{L,H}(u,v)] \). The optical transfer function (OTF) of the filtered low- or high-energy projection is given by:

\[ OTF_{L,H}(u,v) = T_{3,L,H}(u,v)T_{5,L,H}(u,v)H_{L,H}(u,v) \]  
\[ \text{(5.9)} \]

For a parallel system (e.g., image summation), the resultant MTF is given by the modulus of the weighted sum of the constituent OTFs, normalized to unity at zero frequency.\textsuperscript{63} Hence for the DE image:
\[
MTF_{DE}(u,v) = \frac{\frac{k_L OTF_L(u,v) + k_H OTF_H(u,v)}{k_L OTF_L(0,0) + k_H OTF_H(0,0)}}
\]

\[
= \frac{k_L T_{3,L}(u,v)T_{5,L}(u,v)H_L(u,v) + k_H T_{3,L}(u,v)T_{5,L}(u,v)H_H(u,v)}{k_L H_L(0,0) + k_H H_H(0,0)}
\] (5.10)

where \(k_L\) and \(k_H\) are weights associated with the low- and high-energy OTFs, determined by the signal in the low- and high-energy projection. The normalization of the MTF arises from the fact that the decomposition algorithm does not change the mean signal or zero-frequency component of the DE image. For example, considering low- or high-energy projections in which the signal behind an object is \(I_{L,H}^{\text{obj}}\), and the signal in the background is \(I_{L,H}^{\text{back}}\), we have for \(k_L\) or \(k_H\):

\[
k_{L,H} = I_{L,H}^{\text{obj}} - I_{L,H}^{\text{back}}
\] (5.11)

Equation (5.13) can be simplified under the assumption that the scintillator MTF is independent of energy over the range of kVp considered \([T_{3,L}(u,v)=T_{3,H}(u,v)=T_3(u,v)]=[T_{5,L}(u,v)=T_{5,H}(u,v)=T_5(u,v)]\) and that the photodiode aperture is the same for the low- and high-energy projections \([T_{3,L}(u,v)=T_{3,H}(u,v)=T_3(u,v)]=MTF(u,v)\). The detector MTF is therefore the product \(T_3(u,v)T_5(u,v)=MTF(u,v)\). It is also convenient to define a relative weighting parameter:

\[
k_{rel} = \frac{k_L}{k_H}
\] (5.12)

For the typical case in which object contrast is greater in the low-energy image, \(k_{rel} \geq 1\), and the lower bound, \(k_{rel} = 1\), corresponds to the limiting case where the low- and high-energy images are identical (i.e., kVpL=kVpH). Equation (5.10) therefore reduces to:

\[
MTF_{DE}(u,v) = MTF(u,v)\frac{k_{rel}H_L(u,v) + H_H(u,v)}{k_{rel}H_L(0,0) + H_H(0,0)}
\] (5.13)
The value of \( k_{rel} \), determined by the low- and high-kVp and material type, may have a significant effect on the weighting of the filters and consequently on the DE MTF. For the SLS algorithm, this gives:

\[
MTF_{SLS}(u,v) = MTF(u,v) \quad (5.14a)
\]

which is simply the detector MTF, independent of \( k_{rel} \). For the SSH algorithm, on the other hand:

\[
MTF_{SSH}(u,v) = MTF(u,v) \left\{ \frac{H_{LPF}(u,v) - w_k}{1 - w_k k_{rel}} \right\} \quad (5.14b)
\]

and for the ACNR algorithm:

\[
MTF_{ACNR}(u,v) = MTF(u,v) \left\{ \frac{k_{rel}(w_n w_c H_{HPF}(u,v) - w) + (1 - w_n H_{HPF}(u,v))}{1 - w_k k_{rel}} \right\} \quad (5.14c)
\]

each obtained by substituting the low- and high-energy filters from Table 5.1. Also note that \( MTF(0) = 1 \) throughout, since SSH and ACNR do not change the mean signal compared to SLS.

### II.2.4 The DE noise-equivalent quanta (NEQ)

The NEQ for a radiographic image can be written in terms of the MTF and normalized NPS:

\[
NEQ(u,v) = \frac{MTF^2(u,v)}{NNPS(u,v)} \quad (5.15)
\]

and provides a useful spatial-frequency-dependent metric for imaging performance. A well-known property of the NEQ for radiography is that a deterministic filter applied to the projection image will simply cancel out in Eq. (5.13) and will not change the NEQ. In DE images, however, the filters \( H_L(u,v) \) and \( H_H(u,v) \) in general do not cancel out, as seen by combining Eqs. (5.7) and (5.13) to yield the DE NEQ:
It is therefore possible to modify the NEQ in the resultant DE image by differentially filtering the low- and high-energy image by $H_L(u,v)$ and $H_H(0,0)$, where a special case includes: $\text{NEQ}_{DE} = \text{NEQ}_H$ for $H_L(u,v) = 0$ and $H_H(0,0) = 1$. A similar derivation for the DE DQE could be derived, accounting for the DE fluence as in Chapter 3. NEQ is preferred herein as a better indicator of image quality and the term that is explicit in the detectability index, below.

**II.3 Imaging Task and Detectability Index**

The detectability index provides a figure of merit computed by integrating the NEQ with a task function, $W_{\text{task}}(u,v)$, which describes the spatial frequencies of importance in performing a given imaging task:

$$d^2 = \int \int \text{NEQ}(u,v)W_{\text{task}}^2(u,v)dudv$$

which can be rewritten explicitly in term of the MTF and NNPS using Eq (5.16):

$$d^2 = \int \int \frac{MTF^2(u,v)k_{rel}H_L(u,v) + H_H(0,0)k_{rel}H_L(0,0) + H_H(0,0)}{NNPS_L(u,v)H_L^2(u,v) + NNPS_H(u,v)H_H^2(u,v)}dudv$$

Equation (5.17)

The task function weighs spatial frequencies of the NEQ according to structures of interest and can be described as the Fourier transform of the difference in two hypotheses, $\text{hyp}_1$ and $\text{hyp}_2$:

$$W_{\text{task}}(u,v) = \left|3[\text{hyp}_1(x,y) - \text{hyp}_2(x,y)]\right|$$

Equation (5.18)

Although numerous tasks are relevant to chest imaging (e.g., shape discrimination, size estimation, and localization) a nodule detection task was used below for simplicity, which considers two simple hypotheses: 1) object-present and 2) object-absent. The definition of the
detectability index assume a low-contrast (small signal difference) imaging task such that the NNPS is the same in both hypotheses (i.e., in the region of the object and the background). The object-present function was computed from the simulated projection of a spherical nodule for DE images. The object signal was computed separately in the low- and high-energy projections, accounting for differences in attenuation coefficient in each case (but the nodule diameter was held fixed). For all cases below, the nodule was modeled as a 3 mm diameter sphere with linear attenuation coefficient equal to high-density polyethylene (HDPE, simulating a soft-tissue nodule pertinent to soft-tissue-only DE images) or polytetrafluoroethylene (Teflon, simulating a bony sphere pertinent to bone DE images). Also the function was normalized by setting its peak to the DE signal behind the nodule, $I_{DE}^{obj}$, and the region outside the nodule to the background DE signal, $I_{DE}^{back}$. The object-absent hypothesis function was taken as a constant and set to the background DE signal, $I_{DE}^{back}$. The values for $I_{DE}^{obj}$ and $I_{DE}^{back}$ were calculated by CSA, which accounts for energy-dependent absorption in the low- and high-energy projection signal, where the DE signal was calculated using Eq. (5.2). Note that noise reduction does not affect the DC signal.

**II.4 Optimization of DE Image Dose Allocation**

The detectability index provides a useful objective function for optimization of various image acquisition and decomposition parameters. One such factor, shown in Chapter 4 to be an important parameter in DE image quality, is the allocation of dose between the low- and high-energy projections. Defined as the fraction of total entrance surface dose, ESD, imparted by the low-energy image, the dose allocation, $A$, can be varied freely while keeping the total dose to the patient fixed:
\[ A = \frac{\text{ESD}_L}{\text{ESD}_L + \text{ESD}_H} \]  

Allocation is bounded between 0 and 1, with a value of 0.5 corresponding to equal ESD imparted by the low- and high-energy images. The optimal dose allocation for various decomposition algorithms was investigated theoretically by computing the detectability index as a function of dose allocation – i.e., determining the value of \( A \) yielding maximum detectability. The relative performance of various decomposition algorithms was evaluated from the peak detectability.

### III EXPERIMENTAL METHODS

#### III.1 Experimental Setup

**III.1.1 DE imaging systems**

*a. Experimental imaging bench*

The imaging bench shown in Fig. 2.2 was used for DE MTF and NPS measurements. Components of the bench were described in Chapter 2 and employed the Trixell Pixium 4600 FPD.

*b. Clinical imaging prototype*

A prototype DE imaging system (Fig. 5.1) is under development at our institution for clinical studies, based on a Kodak RVG-5100 digital radiography chest stand, with components similar to those of the experimental bench (Fig. 2.2), including the same FPD (Pixium 4600). The clinical prototype incorporates a 10:1 anti-scatter Bucky grid (Advanced Instrument Development Inc., Melrose Park, NJ) and was used to acquire all phantom images shown below. Processing and decomposition of DE images obtained with the clinical prototype was identical to that on the experimental bench.
DE image acquisitions

DE image acquisition parameters were the same for both the experimental and clinical system. The low- and high-energy tube potential were set to 70 and 130 kVp (denoted [70/130] kVp) with (2 mm Al + 0.6 mm Ag) added filtration to the high energy-image. The resulting tube output was 7.62 and 0.82 mR/mAs, with HVLs of 2.01 mm and 13.9 mm, respectively. In all experiments, the total ESD to the phantom (ESD_{tot} = ESD_L + ESD_H) was set to ~0.1 mGy, corresponding to the average value for a DR chest exam determined by the 2001 NEXT study. The ESD was computed from the entrance surface exposure as described previously. The dose allocation [Eq. (5.20)] was varied through adjustment of low- and high-energy mAs, while keeping the total ESD constant.

<table>
<thead>
<tr>
<th>Decomposition parameter</th>
<th>Soft-tissue Image</th>
<th>Bone Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>0.27</td>
<td>0.60</td>
</tr>
<tr>
<td>w_c</td>
<td>0.60</td>
<td>0.27</td>
</tr>
<tr>
<td>d_{LPF}</td>
<td>0.90</td>
<td>0.15</td>
</tr>
<tr>
<td>w_n</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>d_{HPF}</td>
<td>0.50</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 5.2 Summary of DE image decomposition parameters.
III.1.2 DE image decomposition

DE images were formed using the noise reduction algorithms described in Eqs. (5.1-5.5). The low- and high-pass filters were Gaussians characterized by parameters $d_{LPF}$ and $d_{HPF}$, respectively, as in:

$$ H_{LPF} (u,v) = \exp \left( - \frac{(u^2 + v^2)}{2 d_{LPF}^2} \right) $$

(5.21a)

and

$$ H_{HPF} (u,v) = 1 - \exp \left( - \frac{(u^2 + v^2)}{2 d_{HPF}^2} \right) $$

(5.21b)

The decomposition parameters ($w, w_c, w_b, d_{LPF}$, and $d_{HPF}$) were determined by imaging an anthropomorphic chest phantom (Model 55-8PL; Radiology Support Services, Long Beach, CA) on the clinical prototype and qualitatively selecting the parameters that provided tissue cancellation, minimization of quantum noise, and management of edge artifacts and/or image...
blur. While these selections do not necessarily represent optimal values, they are nonetheless reasonable choices that yielded high-quality DE decompositions suitable for the studies reported below. The resulting decomposition parameters are summarized in Table 5.2, with corresponding example phantom images shown in Fig.5.2.

III.2 Measurement of the DE Image NPS

To examine the noise transfer characteristics associated with SLS, SSH, and ACNR image decomposition, the DE NNPS was measured using 50 low-energy (70 kVp) and 50 high-energy (130 kVp+2 mm Al + 0.6 mm Ag added filtration) “flood-field” images acquired on the experimental bench and processed according to Eqs. (5.2), (5.3), and (5.4). To provide a linear DE signal suitable for NPS analysis, the exponential of the processed signal was employed and a small signal approximation was made. Each flood-field image was gain and offset corrected to account for stationary variations by the mean of 25 flood and dark fields acquired with and without x-ray exposure. The panel was read 10 times between projections to minimize correlations due to image lag. Approximately 7000 realizations, each 100x100 pixels, formed the basis for NPS analysis. To remove low-frequency artifacts, a plane was fit to each realization and subtracted, providing zero-mean noise-only realizations. The NNPS was computed from the magnitude-squared of the FFT of each realization, divided by the physical area of the realization and by the square of the mean DE signal. The final estimate of the NNPS was taken as the mean of the 7000 individual NNPS, with error bars given by the sample standard deviation.

III.3 Measurement of the DE Image MTF

Measurements of the DE MTF were performed to verify the dependence of $k_{rel}$ on material type. Measurements of the edge-spread function (ESF) were performed for various material types and were compared with theoretical results. We constructed edges using commonly available
materials, including Cu (1 mm thick), Al (6 mm thick), and polymethylmethacrylate (PMMA, 10 mm thick) to permit validation of $k_{\text{rel}}$ over a large range of x-ray attenuation coefficients. The precision-machined straight edges were placed at a slight angle (~3°) relative to the FPD matrix on the experimental bench. Furthermore, each edge was carefully aligned with the center of the beam in order to mitigate possible distortion of the MTF caused by incident x-rays impinging obliquely to the edge. As discussed above, the MTF associated with decomposition algorithms involving spatial-frequency-dependent filters is expected to depend on the signal magnitude presented between low- and high-energy projections – i.e., upon the material being imaged. Measurements with a Pb edge (2 mm thick) characterized the basic MTF characteristics of the FPD and are reported as a basis of comparison for the MTF associated with the various noise reduction algorithms.

The thickness of the various edge materials was chosen to give high contrast and contrast-to-noise ratio in the ESF images. As a result (particularly for the 10 mm PMMA edge), x-ray scatter presented a potentially confounding factor to the MTF measurements. Therefore, an antiscatter grid (6:1 grid ratio; 103 lpi; Soeye Products, Sungnam, Korea) was employed and the collimators were constrained to a 5x5 cm$^2$ field of view. The grid was placed between the edge and the FPD, with gridlines perpendicular to the edge. To eliminate gridline artifacts from the projection data, “flood-field” calibration was performed with the grid in place and unperturbed between flood-field and edge measurements. The above precautions assured that the grid and scatter had little or no effect on the MTF measurements but successfully mitigated the influence of x-ray scatter. This was verified by comparing: i.) the MTF measured using a PMMA edge and a grid, with ii.) the MTF measured using a 2 mm thick Pb edge and no grid. Fifty low- and high-energy projections with mean bare-beam signal ~50% detector saturation were acquired, averaged to
reduce x-ray quantum noise, processed according to Eqs. (5.2), (5.3), and (5.4), and exponentiated to linearize the signal. Again, a small signal approximation was made to approximate a linear signal.

Approximately 20 ROIs (10 rows x 500 columns) were obtained in each ESF measurement. For each ROI, a shift and add technique was performed to provide an oversampled ESF as described by Fujita et al.\textsuperscript{66} and Samei et al.\textsuperscript{137} The derivative of the oversampled ESF yielded the line-spread function (LSF), the tails of which were smoothed by a median filter, removing high-frequency noise introduced by differentiation. For each ROI, the FFT of the LSF (normalized to unit area) yielded the MTF. The average obtained over the 20 ROIs provided the final MTF measurement, with error bars determined by the sample standard deviation.

Measurements were compared to the DE image MTF computed using Eqs. (5.13) and (5.14), with values of $k_{rel}$ empirically determined as in Eqs. (5.11) and (5.12) from the low-and high-kVp edge projection data as described above:

$$k_{rel} = \frac{I_{L}^{obj} - I_{L}^{back}}{I_{L}^{obj} - I_{L}^{back}}$$

\begin{equation}
(5.22)
\end{equation}

where the values $I_{obj}^{ob}$ and $I_{obj}^{back}$ denote the mean log signal measured in a 25x25 pixel region with and without the edge present, respectively.

**III.4 Performance Evaluation in Phantom Images**

The theoretical analysis described in Sec. I provided calculations of NEQ and $d'$ for various noise-reduction algorithms across a range of exposure conditions (viz., dose allocation). To provide a qualitative test of the correlation between increases in $d'$ and perceived image quality, DE images of a PMMA slab were acquired with the clinical prototype under conditions
corresponding to the theoretical calculations, e.g., [70/130] kVp and total ESD=0.1 mGy. The mAs in low- and high-energy projections were varied to provide a broad range of dose allocation ($A=0.12, 0.39, 0.57, 0.79, \text{ and } 0.97$).

The phantom presented nodule- and bone-simulating materials – HDPE and Teflon spheres, respectively, each 3 mm diameter (see Chap. 5 Sec. II.3) – selected to simulate nodule and calcification detection tasks in soft-tissue and bone images. The spheres were placed within a uniform 10 cm thick slab of PMMA representing attenuation in the chest of an average patient.$^7$ Low- and high-energy projections of the phantom were decomposed according to Eqs. (5.1-5.5) with the decomposition parameters shown in Table 5.2. Values of $k_{rel}$ for HDPE and Teflon were empirically determined using Eq. (5.22) from low- and high-energy projections of each sphere. The detectability index was computed according to Eq. (5.18) and qualitatively compared to phantom images acquired at varying levels of dose allocation.

IV RESULTS

IV.1 The DE Image NPS

Figure 5.3 plots the measured and theoretical NNPS in soft-tissue and bone images for the three decomposition algorithms. The plots are 1D diagonal profiles of NNPS($u, v$) along $u = v$ in the Fourier domain. The noise reduction algorithms, SSH and ACNR, significantly reduce the NNPS below that of SLS. For both the soft-tissue and bone images, the ACNR exhibits the lowest NNPS, with steep suppression of quantum noise below $\sim 1 \text{ mm}^{-1}$. The results are similar to those reported by Warp and Dobbins,$^{124}$ who demonstrated sharp reduction in the NPS at low to mid spatial frequencies for these and other noise reduction algorithms. The theoretical NPS computed using CSA exhibits excellent agreement with measurements and provides experimental validation of Eqs. (5.7) and (5.8).
Figure 5.3 Measured (data points) and theoretical (straight lines) NNPS for (a) “soft-tissue” and (b) “bone” flat-field images formed using the three decomposition algorithms. SSH and ACNR are seen to reduce noise significantly compared to SLS. Theoretical calculations using cascaded systems analysis show excellent agreement with measurements.

Figure 5.4 Flood-field images depicting quantum mottle under different DE image decomposition algorithms corresponding to the NPS plots shown in Fig. 5.3. The algorithms are seen to dramatically affect the magnitude and spatial-frequency content of the noise.
The magnitude and spatial-frequency content (i.e., ‘texture’) of quantum-noise-only images for the algorithms are illustrated in Fig. 5.4. The grayscale window and level are equivalent for each image to facilitate intercomparison. Clearly, SSH and ACNR impart a dramatic reduction in the magnitude of random noise; moreover – particularly for ACNR – the effect on noise ‘texture’ overall is that of a low-pass noise transfer characteristic (i.e., low-frequency noise correlation), which may be beneficial or deleterious to the NEQ, depending on the corresponding signal transfer characteristic (MTF), discussed below.

IV.2 The DE Image MTF

Measurements of the MTF were performed to provide experimental validation of the signal transfer characteristics described by Eq. (5.14). For the SLS algorithm, $MTF_{SLS}(u,v)$ measured using the Cu, Al, and PMMA edge was found to be equal to $MTF(u,v)$ (measured with a Pb edge) as expected, since the algorithm does not involve spatial-frequency-dependent decomposition, see Eq. (5.14a). For $MTF_{SSH}(u,v)$ and $MTF_{ACNR}(u,v)$, on the other hand, the values of $k_{rel}$ for Cu, Al, and PMMA influenced the results and were measured according to Eq. (5.22), yielding, $k_{rel} = 5.4$, 2.8, and 1.6, respectively. The resulting MTFs [i.e., $MTF_{SSH}(u,v)$ and $MTF_{ACNR}(u,v)$] are shown in Fig. 5.5. The data points are measurements and the solid lines are calculations based on Eq. (5.14). The dashed line is the detector MTF, $MTF(u,v)$, included for purposes of comparison. Figures 5.5(a) and 5.5(b) plot $MTF_{SSH}(u,v)$ for soft-tissue and bone decomposition, respectively. It is interesting to note that although this algorithm involves blurring of the high-energy image, in some cases the DE image is effectively sharpened due to edge-enhancement associated with subtraction of blurred and un-blurred images – resulting in an MTF that may exceed 1 at higher frequencies, as might be expected from an unsharp mask or sharpening filter. For the bone image especially, SSH imparts a significant, material-specific
“sharpening” of the image. Figure 5.5(a) suggests that for the soft-tissue image, SSH entails fairly severe blurring of structures of interest (e.g., PMMA, similar to solid pulmonary nodules), whereas for the bone image, shown in Fig. 5.5(b), structures of interest are appreciably sharpened (e.g., Al, similar to bone).

Figure 5.5 Measured and theoretical MTF associated with DE images formed using various noise reduction algorithms. The dashed line is the MTF of the detector as determined from ESF measurements using an angled Pb edge. For the SSH and ACNR algorithms, the MTF depends on the signal differences presenting in low- and high-energy images. (a,b) The MTF for SSH in soft-tissue and bone images. (c,d) The MTF for ACNR in soft-tissue and bone images. The data points with error bars are measurements, and the solid lines are theoretical calculations.

Figures 5.5(c) and 5.5(d) plots $MTF_{ACNR}(u,v)$ for soft-tissue and bone decompositions, respectively. Similar to Fig. 5.5(a) for the soft-tissue image, ACNR entails moderate blurring of
structures of interest (e.g., PMMA), while for the bone image [Fig. 5.5(d)], the algorithm imparts a degree of edge-enhancement on structures of interest (e.g., Al). Taken together, Figs. 5.5(a-d) suggests that the $MTF_{ACNR}(u,v)$ performs better for the soft-tissue image using ACNR, whereas for the bone image, $MTF_{SSH}(u,v)$ is significantly better. Excellent agreement between theory and measurements is exhibited throughout and this provides confidence in the derivation of Eq. (5.10).

![Figure 5.6 Calculations of the DE MTF for various decomposition algorithms. (a) MTF for the soft-tissue image, for which the structure of interest is HDPE ($k_{rel} = 1.7$). (b) MTF for the only image, for which the structure of interest is a Teflon sphere ($k_{rel} = 2.4$).](image)

To examine these results further and provide a basis for theoretical calculations of NEQ, the DE MTF was computed for materials more closely simulating structures of interest in soft-tissue and bone images. As shown in Fig. 5.6, the MTF for HDPE and Teflon (with $k_{rel}$ measured to be 1.7 and 2.4, respectively) was found to depend markedly on choice of decomposition algorithm. In the soft-tissue decomposition, SSH reduces the MTF significantly, whereas ACNR provides a slight boost in MTF. In the bone image, SSH significantly increases the MTF. For the bone image $MTF_{SSH}(u,v)$ was found to be greater than one near 0.5 mm$^{-1}$ due to an effective sharpening effect (similar to an unsharp mask) in which subtracting the original image from the
blurred image yields an edge-enhanced image with an MTF >1 at some spatial frequencies. This effect is dependent on the decomposition parameters and the material-dependent value of $k_{rel}$.

An additional point of interest in these results is the possible ‘zeroing’ of the MTF, depending on the material type and choice of decomposition parameters. For example, SSH decomposition of the soft-tissue image suggests an MTF tending to zero at $\sim1.1$ mm$^{-1}$. This effect was experimentally verified by imaging a radiographic line-pair pattern. As predicted by Eqs. (5.14b) and (5.14c), contrast transfer of line-pairs in the DE image could be nulled at any particular spatial frequency by variation of the decomposition parameters.

### IV.3 The DE Image NEQ

Figure 5.7 plots the NEQ for soft-tissue and bone images decomposed by SLS, SSH, and ACNR. The material of interest is HDPE (simulating a solid pulmonary nodule) for the soft-tissue image, and Teflon (simulating bone) for the bone image. Note that the algorithms are equivalent at zero frequency, since the scale value of filters appearing in Eqs. (5.1-5.5) is the same for all algorithms. For the soft-tissue image [Fig. 5.7 (a)], SSH imparts a significant reduction and zeroing of the NEQ at mid-frequencies associated with the MTF of Fig. 5.6 (a). The ACNR algorithm provides improved NEQ at all frequencies compared to SLS. For the bone image [Fig. 5.7 (b)], both SSH and ACNR boost NEQ significantly compared to SLS, with the former providing slightly improved low-frequency performance by virtue of significantly improved low-frequency MTF, as shown in Fig. 5.6 (b).

Overall, the NEQ imply superior performance for ACNR decomposition of the soft-tissue image, and SSH decomposition of the bone image. In light of the strong spatial-frequency-dependent influence on NEQ imparted by these noise reduction algorithms, it is important to consider the spatial frequencies of interest in accomplishing a given imaging task. Furthermore, since the
choice of algorithm and decomposition parameters may be selected in post-processing, DE image decomposition can be “tuned” in a task-specific manner to optimize imaging performance. Such results provide strong motivation for understanding and validating Fourier-based descriptions of imaging task for DE imaging system optimization.

Figure 5.7 NEQ calculated from the NNPS and MTF of Figs. 5.3 and 5.6, respectively. (a) NEQ of the soft-tissue image, taking HDPE (similar to a solid pulmonary nodule) as the material of interest in the decomposition. (b) NEQ of the bone image, taking Teflon (similar to cortical bone) as the material of interest in the decomposition.

IV.4 Detectability Index and Optimal Dose Allocation

Calculation of NEQ [Fig. 5.7] and imaging task [Eq. (5.19)] provides the basis for analysis of detectability index [Eq. (5.18)] as an objective function for system optimization. Figure 5.8 plots the detectability index for the detection of a 3 mm HDPE or Teflon sphere (in soft-tissue or bone images, respectively) computed as a function of dose allocation. The superior algorithm in each case is immediately apparent: ACNR for the soft-tissue image, and SSH for the bone image. For the soft-tissue image, ACNR achieves peak detectability ~3 times greater than that of SLS or SSH. The optimal dose allocation for SLS, SSH and ACNR in soft-tissue decomposition was $A^* = 0.22, 0.28, \text{ and } 0.60$, respectively. For the bone image, SSH provided highest detectability overall, with optimal dose allocation significantly increased compared to SLS ($A^* = 0.84$ and
0.92 for ACNR and SSH, respectively, compared to $A^* = 0.42$ for SLS). These results suggest improved performance for a lower proportion of dose delivered by the high-energy image in the case of SLS and demonstrate that the SSH and ACNR algorithms successfully mitigate the correspondingly higher levels of quantum noise in the high-energy image demonstrated by the higher dose allocations values for SSH and ACNR compared to SLS. The tendency toward increased dose allocation combined with high-performance noise reduction algorithms is clear.

![Figure 5.8](image)

Figure 5.8 Detectability index computed as a function of dose allocation for the detection of: (a) a 3mm HDPE sphere in a soft-tissue image and (b) a 3mm Teflon sphere in a bone image.

As seen in Fig. 5.8, optimal soft-tissue imaging performance is obtained via ACNR decomposition (with $A^* = 0.60$) and optimal bone imaging performance via SSH decomposition (with $A^* = 0.92$). Note that at the extreme values of $A=0$ and $A=1$, $d'$ tends to zero because the NNPS for either the low or the high-energy image, respectively, goes to infinity while $w$ is held fixed (thus still using the low- and high-energy image, despite the divergent NNPS); therefore, the DE NEQ and $d'$ tend to 0 at these extremes of allocation. That the optimal noise reduction algorithm is different for the two types of DE image is interesting, but poses no major impediment to clinical implementation, since the algorithms can simply operate in parallel. The
discrepancy in optimal dose allocation, however, presents somewhat of a dilemma, since dose allocation must be determined before data acquisition and is not subject to modification between the two types of DE image. As discussed in Sec. V, selection of an optimum appropriate to both tasks should consider the tradeoffs according to multi-function optimization. Simply averaging the two optima \[ A_{\text{eff}}^* = \frac{(0.92 + 0.60)}{2} = 0.76 \] is a reasonable guess if the objective functions exhibit equal sensitivity (slope). As discussed below, a more comprehensive, multi-function method that minimizes the total tradeoff in detectability index suggests an overall optimum of \[ A_{\text{eff}}^* = 0.79. \]

Figure 5.9 Soft-tissue DE images of a 3mm HDPE sphere at various dose allocation and decomposition algorithms corresponding to the calculations of Fig. 5.8(a). The detectability index (soft-tissue sphere detection task) computed using Eq. (5.18) is superimposed. Qualitative agreement is observed between the trends in \( d' \) and conspicuity of the sphere.
Figure 5.10 Bone DE images of a 3mm Teflon sphere at various dose allocation and decomposition algorithms corresponding to the calculations of Fig. 5.8(b). The detectability index (bony sphere detection task) computed using Eq. (5.18) is superimposed. Qualitative agreement is observed between the trends in $d'$ and conspicuity of the sphere.

Qualitative verification of these trends in detectability is illustrated in Figs. 5.9-5.10, showing images of a 3 mm diameter HDPE or Teflon sphere acquired at multiple levels of dose allocation. The corresponding detectability index in each case is superimposed and is observed qualitatively to correlate well with nodule conspicuity. For example, in Fig. 5.9 (soft-tissue images of the HDPE sphere) detectability overall is highest for ACNR in the range $A \sim 0.6-0.8$. For SLS, conspicuity is highest at lowest allocation, consistent with Fig. 5.10(a). Similarly for SSH, conspicuity peaks in the range $A \sim 0.1-0.4$, as does detectability in Fig. 5.8(a). As shown in Fig. 5.10, bone conspicuity also correlates well with $d'$. The sphere is best seen with the SSH algorithm at higher levels of allocation, consistent with detectability index computed in Fig. 5.8(b). Similarly, the trends in conspicuity of the sphere for SLS and ACNR are consistent with detectability index shown in Fig. 5.8(b). While overall correlation between sphere conspicuity
and detectability index appears to be good, some discrepancy between the magnitude of $d'$ and the conspicuity evident among decomposition algorithms can be expected, since a real observer is susceptible to low-frequency ‘coloring’ of the noise imparted by these algorithms that may deleteriously affect the performance of human observers. For example in Fig. 5.9, the top-left (SLS decomposition at $A=0.12$) and bottom-right (ACNR decomposition at $A=0.97$) each imply $d'\sim 1$; however, conspicuity to the human eye is diminished in the latter, likely due to the significantly greater low-frequency component (‘coloring’) of the NPS.

V DISCUSSION

V.1 Optimization in the Presence of Multiple Tasks

An imaging system is generally used to accomplish more than one imaging task – e.g., nodule detection and shape discrimination, or detection of a nodule in the soft-tissue image and detection of calcifications in the bone image. If different optimal parameters are identified for each imaging task, a dilemma is presented in identifying a single optimal value, particularly if the optimal parameter involves a technique factor that cannot be varied after data acquisition (e.g., the low- or high- kVp or dose allocation). Multi-function optimization helps to address this problem by minimizing the total reduction in the objective function for all tasks considered. Therefore, let $f_i(p)$ be the objective function (e.g., the detectability index) for the $i^{th}$ task where $p$ denotes the parameter being optimized and $p_i^*$ denotes the value which maximizes $f_i(p)$. Further, let $a_i$ denote an optional weight proportional to the relative “importance” of the $i^{th}$ task with respect to all other tasks. Such weighting may be subjectively determined, quantitatively evaluated by independent cost-benefit analysis, or simply taken equal for all tasks. The overall optimum is denoted $p_{\text{eff}}^*$ such that:
\[ \sum_i a_i \left[ f_i(p_i^*) - f_i(p_{eff}^*) \right] = \min_p \left( \sum_i a_i \left[ f_i(p_i^*) - f_i(p) \right] \right) \]  

(5.23)

This equation balances tradeoffs among all tasks, where for the \( i \)th task, \( p_{eff}^* \) gives performance that is maintained in proportion to \( a_i \). For all tasks weighted equally (\( a_i = 1 \ \forall \ i \)), \( p_{eff}^* \) gives performance that minimizes the total diminishment in the objective function. Note that for the \( i \)th task, employing the multi-task optimum value, \( p_{eff}^* \), instead of \( p_i^* \), may significantly reduce the ability to perform the task; nonetheless, it is the best choice in light of all tasks considered (even if no task is performed particularly well). Thus, the term “optimal” does not necessarily equate to “good” or “ideal” for each imaging task, but it indicates the best the system can perform given competing objectives. Therefore some consideration must clearly be applied in cases where competing tasks suggest vastly different individual optima. For the case of two tasks (soft-tissue and bone detection, as in Fig. 5.8) equally weighted, the \( d' \) objective functions are such that the overall optimal allocation from Eq. (5.23) is \( A_{eff}^* = 0.79 \), which appears to impart a small, reasonable tradeoff in detectability for each task.

V.2 Optimal DE Image Acquisition and Decomposition

The results above, reported in terms of detectability index, can also be interpreted in terms of the area under the receiver operating characteristic (ROC) curve, \( A_z \). Assuming symmetric, binormal distributions in positive and negative cases, \(^{139}\) detectability is related to \( A_z \) by the error function:\(^{140}\)

\[ A_z = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{d'}{2} \right) \right) \]  

(5.24)
Therefore a value of \( d' = 0 \) corresponds to \( A_Z = 0.5 \) (pure guessing), and \( A_Z \) goes to 1 as \( d' \) tends to infinity (perfect response). The typical range for confident observer response is from \( A_Z \sim 0.70 \) (i.e., \( d' = 0.69 \), corresponding to “just-noticeable”) to \( A_Z \sim 0.92 \) (i.e., \( d' = 2.0 \), corresponding to “fairly conspicuous”).

Considering the results in terms of \( A_Z \) further illustrates that determination of optimal DE image acquisition technique is inextricably linked to the choice of decomposition algorithm. For example, consider the soft-tissue imaging performance shown in Fig. 5.9(a). Conventional SLS yields peak soft-tissue imaging performance of \( d' = 1.0 \) \( (A_Z = 0.76) \) at \( A^* = 0.22 \). ACNR on the other hand yields peak performance of \( d' = 2.7 \) \( (A_Z = 0.97) \) at \( A^* = 0.60 \). Thus, knowledgeable choice of acquisition technique and decomposition algorithm improves nodule visibility from barely detectable \( (A_Z = 0.76 \text{ with SLS}) \) to clearly conspicuous \( (A_Z = 0.97 \text{ with ACNR}) \) without increasing patient dose.

Similarly for the bone imaging performance shown in Fig. 5.8(b). Conventional SLS yields peak performance of \( d' = 0.9 \) \( (A_Z = 0.73) \) at \( A^* = 0.42 \). SSH on the other hand yields peak performance of \( d' = 4.3 \) \( (A_Z = 0.99) \) at \( A^* = 0.92 \). Again, knowledgeable choice of acquisition technique and decomposition algorithm improves nodule visibility from barely detectable \( (A_Z = 0.73 \text{ with SLS}) \) to clearly conspicuous \( (A_Z = 0.99) \).

Considering soft-tissue and bone tasks equally, the multi-function effective optimum may be determined by Eq. (5.23). For conventional SLS, \( A_{\text{eff}*} = 0.32 \). Taking instead ACNR for the soft-tissue case and SSH for the bone case, we have \( A_{\text{eff}*} = 0.79 \). For the soft-tissue image, then, performance increases from \( d' = 0.7 \) \( (A_Z = 0.69) \) for SLS to \( d' = 2.6 \) \( (A_Z = 0.97) \) for ACNR. Similarly for the bone image, performance increases from \( d' = 0.8 \) \( (A_Z = 0.72) \) for SLS to \( d' = 4.1 \) \( (A_Z = 0.99) \) for SSH. In each case, multi-function optimization of acquisition technique \( (A_{\text{eff}*}) \)
combined with knowledgeable selection of decomposition algorithm (ACNR or SSH) increases visibility from “barely visible” to “conspicuous” without penalty in dose.

Finally, the GLNR technique [Eq. (5.5)] provides an example implementation of the general linear decomposition [Eq. (5.1)], the performance of which was investigated in terms of NEQ, $d'$, and image quality in comparison to SLS, SSH, and ACNR. The convolution filters in Eq. (5.5) were manually adjusted to maximize NEQ, with allocation fixed at $A = 0.79$. Figure 5.12 shows the resulting DE images of HDPE and Teflon spheres decomposed using SLS, SSH, SSH, and GLNR. The SLS, SSH, and ACNR images correspond to the cases in Figs. 5.9-5.10 exhibiting highest detectability. The GLNR image is seen to provide increased detectability index and conspicuity compared to the other algorithms, demonstrating the potential for improved decomposition by virtue of the general linear form. A complete optimization of GLNR
and other noise reduction algorithms is a subject of future work that will include not only maximization of NEQ (as described here) but also minimization of edge artifacts, which can be a problem for any algorithm employing the high-pass filter. For example, both ACNR and GLNR can introduce bone edge artifacts in the soft-tissue image. Methods developed to minimize the effect of such artifacts have been reported\textsuperscript{121} but can impart non-stationary frequency response characteristics across the DE image, introducing a challenge to the application of CSA modeling. Future work (contained in Chapter 6) includes optimization of the decomposition parameters for ACNR and SSH. Still, the initial results in Fig. 5.11 are encouraging, suggesting a three- to five-fold increase in detectability for GLNR compared to SLS and a \(~10\text{-}15\%\) increase in detectability compared to the next-best noise reduction algorithm (ACNR or SSH). Other future work also includes the description of anatomical background noise\textsuperscript{127} as described in Chapter 3 in formulation of the generalized NEQ. X-ray scatter is also an important factor in image quality and the ability to perform an imaging task. Previous work by Kyprianou et al.,\textsuperscript{141} Siewerdsen,\textsuperscript{78} and Cunningham,\textsuperscript{142} included x-ray scatter in imaging performance models. These approaches could be combined with this work where anatomical noise and scatter would be expected to reduce the detectability index. Furthermore, the detectability index was computed with a basic ideal observer model, although there is a vast collection of models in the literature. Chapter 6 extends the analysis to more sophisticated observer models, including non-prewhitening and eye-filter models.

\section*{VI CONCLUSIONS}

Generalization of linear DE image decomposition algorithms provides a useful framework for extending cascaded systems analysis to describe the spatial resolution and noise performance of DE imaging systems. Theoretical analysis was validated by the excellent agreement observed
between calculations and measurements of NNPS and MTF for all decomposition techniques. The NEQ was found to demonstrate a strong dependence on the choice of decomposition algorithm as well as a strong dependence on the decomposition parameters therein. Integrating the spatial-frequency-dependent NEQ with a simple analytical description of the imaging task yielded a detectability index objective function for the identification of optimal DE image acquisition and decomposition techniques. Optimal acquisition parameters in DE imaging were shown to depend on the choice of decomposition algorithm, particularly those involving noise reduction, which impart complex, spatial-frequency-dependent effects in the DE NEQ. For the tasks and nominal imaging conditions considered above, the results suggest optimal DE imaging performance achieved using ACNR decomposition of the soft-tissue image, SSH decomposition of the bone image, and two-task optimal dose allocation, $A_{\text{eff}}^* = 0.79$. Such optimization was found to elevate task performance from the regime of “just detectable” ($A_z \sim 0.7$) to “fairly conspicuous” ($A_z > 0.92$). Finally, the general linear form for spatial-frequency-dependent DE image decomposition [Eq. (5.1)] was found to provide a useful framework for the development and optimization of new, task-specific decomposition algorithms for high-performance DE imaging.
CHAPTER 6: BRIDGING THE GAP: COMPARISON OF THEORY AND OBSERVER PERFORMANCE

I INTRODUCTION
The development of imaging systems benefits tremendously from the ability to model observer performance from first principles. It enables the development and optimization of medical imaging systems without the requirement of costly prototypes and time consuming human observer studies. In a recent anniversary paper\textsuperscript{143} for the Medical Physics Journal, Krupinski and Jiang argue that the evaluation of medical imaging system needs to go beyond simple metrics and must ultimately address the impact on diagnostic performance and patient care. However, there is generally a gap between physical metrics that describe detector performance (such as MTF, NPS and NEQ) and those that describe the performance of real observers. There has been considerable research in the development of model observers\textsuperscript{144-146}, giving analytical models that describe human observer performance to varying degrees, depending on the characteristics of the image and the imaging task. The work described below seeks to relate physical signal and noise
characteristics (MTF and NPS) with human observer performance by employing the MTF and NPS derived from first principles as described in Chap. 5, combining them with a Fourier description of imaging task according to various observer models, and correlating the resulting detectability index with measurements of human observer performance.

Dual-energy (DE) imaging provided the platform for this work, involving both acquisition and decomposition parameters (specifically, dose allocation and noise reduction parameters, respectively) that significantly affect the MTF and NPS as discussed in Chap. 5. Three imaging tasks were considered - detection, shape discrimination, and texture discrimination - each modeled theoretically as Fourier domain templates. The detectability index was computed using the theoretically derived MTF, NPS, and imaging task, considering four model observers. A DE imaging phantom was constructed to present structures corresponding to each imaging task, and the resulting DE images were used in MAFC tests of human observer performance. The extent to which theoretically derived detectability index agreed with actual human observer performance was evaluated across a broad range of acquisition and decomposition parameters.

II THEORETICAL METHODS
Four model observers were investigated in this study: the Fisher-Hotelling (FH) observer; the FH observer with an eye filter and internal noise (FHE); the non-prewhitening (NPW) observer; and the NPW observer with an eye filter and internal noise (NPWE). A brief description of each is presented below, with notation based on that of Burgess et al.\textsuperscript{73} and additional details found in previous chapters. Each of the terms appearing in Eqs.(6.1-6.8) below, were computed theoretically, providing an approach to observer performance that is based on first principles of spatial-frequency-dependent signal and noise transfer characteristics yet begins to bridge the gap to image quality as assessed by real human observers.
II.1 The Fisher-Hotelling Observer (FH)

The Fisher-Hotelling (FH) observer is modeled as a prewhitening matched filter, by means of a detection template, $T_{FH}$, that decorrelates the noise, defined in the Fourier domain as: \[ T_{FH}(u,v) = \frac{MTF(u,v)W_{\text{Task}}(u,v)}{NNPS(u,v)} \] (6.1)

where $u$ and $v$ are the spatial frequencies, $MTF(u,v)$ denotes the modulation transfer function, $NNPS(u,v)$ denotes the normalized noise-power spectrum (i.e., the NPS divided by the square of the mean signal), and $W_{\text{Task}}(u,v)$ denotes the task function (which describes the spatial frequencies of interest for a given imaging task, as detailed in Chapter 2). The cross-correlation of the detection template [Eq. (2)] yields a decision variable used to derive the detectability index for the FH observer: \[ d_{FH}^2 = \iint \frac{MTF^2(u,v)W_{\text{Task}}^2(u,v)}{NNPS(u,v)} dudv \] (6.2)

where the 2D integration is performed over the Nyquist region. In this form, the detectability index can also be understood as a weighted sum of the task function with the noise equivalent quanta (NEQ) and represents perhaps the simplest observer model for assessment of performance. The resulting detectability index has been used as a figure of merit for system optimization in a variety of applications. This model observer is also equivalent to the one introduced in Chapter 2, which is given by the sum over the weighted combination of noise equivalent quanta (NEQ) and task function.
II.2 The Fisher-Hotelling Observer with Eye Filter (FHE)

The FH observer can be modified to include an eye filter and internal noise to better incorporate factors that potentially degrade performance in real human observers:

\[
d^2_{FHE} = \int \int \frac{MTR^2(u,v)W^2_{task}(u,v)}{NNPS(u,v)E^2(u,v) + N_i} du dv
\]  \hspace{1cm} (6.3)

where \(E(u,v)\) is the eye filter and \(N_i\) denotes the internal noise. The eye filter employed in this work was the same as that used by Burgess\textsuperscript{149} and is modeled on the contrast sensitivity function curve measured by Barton for the human visual system:\textsuperscript{150}

\[
E(f) = f^n \exp(-cf^2)
\]  \hspace{1cm} (6.4)

where \(f\) denotes the radial spatial frequency, \(f^2 = u^2 + v^2\). A value of \(n=1.3\) has been shown to yield good agreement with measurements and the value of \(c\) is determined such that \(E(f)\) peaks at 4 cycles/deg for a given viewing distance (set to 50 cm in this work, thus yielding a value of \(c \approx 3\)) as shown in Fig. Error! Reference source not found.

The internal noise was assumed to be uncorrelated (constant NPS) and set to a fraction (0.02 at 100cm viewing distance) of the zero-frequency NNPS scaled to the observer’s viewing distance, \(D\):

\[
N_i = 0.02 \left( \frac{D}{100} \right)^2 NNPS(0,0)
\]  \hspace{1cm} (6.5)

The value of 0.02 was the same as that used by Burgess et al.,\textsuperscript{149} estimated from published human observer internal noise measurements (typical values for \(N_i\) ranged from 1x10\(^{-6}\) mm\(^2\) to 1x10\(^{-5}\) mm\(^2\)). The FHE observer has been shown to provide an improved model for human observer performance compared to the FH observer in sphere detection tasks.\textsuperscript{149}
II.3 The Non-Prewhitening Observer (NPW)

The non-prewhitening observer (NPW) initially proposed by Wagner does not decorrelate the noise in the image. It uses a template matched to the expected signal to form a detection template, $T_{\text{NPW}}(u,v)$, defined in the Fourier domain as:

$$T_{\text{NPW}}^2(u,v) = MTF^2(u,v)W_{\text{Task}}^2(u,v) \quad (6.6)$$

Therefore variations in the signal or background are not compensated by this observer in comparison to the FH observer [Eq. (3)]. Similarly, the detectability index for the NPW observer is given by:

$$d_{\text{NPW}}^2 = \frac{\iint MTF^2(u,v)W_{\text{Task}}^2(u,v) du dv}{\iint NNPS(u,v)MTF^2(u,v)W_{\text{Task}}^2(u,v) du dv} \quad (6.7)$$

The NPW observer better models human observers in cases where humans are believed not to be able to prewhiten the noise, as in high-pass-filtered images (e.g., CT). The NPW observer reduces to the FH observer in the case of uncorrelated (white) noise.
II.4 The Non-Prewhitening Observer with Eye Filter (NPWE)

The NPW observer may be similarly modified, as shown by Burgess\textsuperscript{152}, to include an eye filter and internal observer noise:

$$d_{\text{NPWE}}^2 = \frac{\iiint MTF^2(u,v)W_{\text{task}}^2(u,v)E^2(u,v)dudv}{\iiint NNPS(u,v)MTF^2(u,v)W_{\text{task}}^2(u,v)E^4(u,v) + MTF^2(u,v)W_{\text{task}}^2(u,v)N_d dudv}$$ (6.8)

The same eye filter and internal noise values as described above for the FHE observer were used for the NPWE observer.

II.5 Imaging Tasks

The work below investigates more complex task functions than the simple (delta-function or gaussian sphere) detection tasks previously reported in Chaps. 2-5. Task functions were defined, as in earlier work, as the Fourier difference between two hypotheses (e.g., signal absent vs. signal present):

$$W_{\text{task}} = |F[h_1(x,y) - h_2(x,y)]|$$ (6.9)

where $h_1(x,y)$ and $h_2(x,y)$ are the spatial domain representations of the two hypotheses, and $F$ is the Fourier transform operator. Below, hypothesis 1 and hypothesis 2 are denoted "normal" and "abnormal", respectively.

II.5.2 Sphere detection task

The sphere detection task considered hypotheses of a uniform background and the projection of a sphere. The 2D projection profile was obtained by computing the attenuated signal through a 3D sphere. Linear attenuation coefficients (www.nist.gov) for polyethylene and Teflon were used to model soft-tissue and bony lesions, respectively. Figure 6.2(a) illustrates the sphere detection
hypotheses and the resulting task function, which predominantly weighs very low spatial frequencies. A subtle ring at low-mid frequencies associated with the edge of the sphere is also evident.

Figure 6.2: Hypotheses and task functions for the three tasks investigated

### II.5.3 Shape discrimination task

The shape discrimination task considered hypotheses of the projection of a sphere and that of a cylinder, as shown in Fig. 6.2(b). This task describes the discrimination of subtle shape differences that might present between various types of lesions (e.g., a solid nodule vs. a vessel). The signal for each task was normalized such that both tasks had the same signal power:

\[
\iint h_1^2(x, y) \, dx \, dy = \iint h_2^2(x, y) \, dx \, dy
\]  

(6.10)

Equivalent signal power was achieved by setting the height of the cylinder profile equal to the mean of projected sphere profile and by setting the cylinder diameter equal to its profile height.

### II.5.4 Texture discrimination task

The final task was a texture discrimination task, which considered the projection of a smooth and textured cylinder as the two hypotheses as shown in Fig. 6.2(c). This task roughly corresponds to the discrimination of various textures that might be exhibited by various lesions (e.g., a solid
nodule vs. a spiculated mass or ground-glass opacity). Rings in the task function correspond to the crenellation in the textured cylinder. Again, equal signal power [Eq. (6.10)] was achieved by setting the diameters equal and setting the height of the smooth cylinder equal to the mean height of the textured cylinder.

II.6 Comparison of $d'$ with Human Observer Performance

The detectability index can be related to the area under the ROC curve, denoted $A_Z$, under the assumption that the distributions of normal and abnormal cases are Gaussian and homoscedastic (equal variance):$^{67}$

$$A_Z = \frac{1}{2} \left( 1 + \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-t^2) dt \right)$$

$$= \frac{1}{2} \left( 1 + \text{erf} \left( \frac{d'}{2} \right) \right)$$

(6.11)

In the experiments below, MAFC tests were used to evaluate human observer performance, recognizing that while MAFC tests are less efficient than receiver operating characteristic (ROC) tests (requiring more images to achieve a given level of statistical error)$^{67}$ they are well suited to phantom studies involving a large number of images and are well tolerated by observers. The resulting figure of merit for human observer performance in MAFC tests is the “proportion correct,” denoted $P_C$. For an MAFC test with $M$ choices, $M$ is related to the detectability index by$^{67}$

$$P_C(d', M) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left( -\frac{(x-d')^2}{2} \right) \Phi(x)^{M-1} dx$$

(6.12)

where $\Phi(x)$ is the cumulative Gaussian distribution function, $\Phi(x) = \int_{-\infty}^{x} \exp(-y^2/2) dy$. Note that for $M=2$ (a 2AFC test), the proportion correct is simply equal to $A_Z$ (under the assumption of...
an ideal ROC test in which observer response does not vary over the course of the test). All results below are reported in terms of $A_Z$, with theoretically derived $d'$ converted to $A_Z$ via Eq. (6.11) and with experimentally determined $P_C$ (from MAFC tests) converted to $A_Z$ by numerical inversion of Eq. (6.12) (a reverse look-up table between $P_C$ and $A_Z$ computed numerically).

III EXPERIMENTAL METHODS

III.1 Imaging Setup

III.1.1 DE imaging system

Figure 6.3: (a) Image of the flat-panel detector and the task phantom used to acquire DE images of the various stimuli corresponding to the different imaging tasks. (b) Projections of the various stimuli used for the task phantom, which were placed on a 10 cm slab of PMMA.

Figure 6.3 illustrates the prototype DE imaging system developed in our laboratory and the task phantom constructed to mimic the idealized tasks of Fig. 6.2. The system employed the Trixell (Moirans, France) FPD described in Chapter 2 (Pixium 4600, 250 mg/cm², 3000x3000 pixels, 0.143 mm pixel pitch, and ~0.8 fill factor). The system also included a 10:1 antiscatter bucky grid. The low- and high-energy kVp were 60 kVp and 120 kVp, with 2.5 mm Al filtration and 2.5 mm Al + 0.6 mm Ag total filtration, respectively. As covered in previous chapters and other
experimental measurements, these acquisition techniques were believed to provide optimal DE image quality.

III.1.2 Task phantom

A phantom was built using a 10 cm slab of PMMA, simulating an average chest thickness, on which were placed stimuli corresponding to the various hypotheses as shown in Fig 6.3 Polyethylene and Teflon material were used to simulate soft-tissue and bony lesions in the soft-tissue and bone images, respectively. The stimuli were precisely machined to ensure that the objects corresponded closely to the idealized hypotheses. The sphere detection task used 3 mm diameter spheres. The shape discrimination task used a 6 mm diameter sphere and a cylinder with height and diameter equal to 87% of the sphere’s diameter to ensure equal signal power between the two hypotheses [Eq. (11)]. For the texture discrimination task, both cylinders had a diameter of 11 mm. The smooth cylinder had a height of 14 mm, and the textured cylinder had minimum and maximum crenellation heights of 12 mm and 16 mm, respectively, ensuring equal signal power.

III.2 Human Observer Study

III.2.1 Multiple-alternative forced Choice (MAFC) tests

In an MAFC study, the observer is required to identify which of $M$ images contains a specified stimulus, where $M - 1$ images are “normal” (i.e., the first hypothesis), and the other image contains an abnormality (i.e., the second hypothesis). The fraction of correct responses, $P_c$, gives a figure of merit for task performance that can be related to the area under the ROC curve and detectability index as discussed in Section II. D. To choose an appropriate value of $M$ for the studies herein, the dependence of the measurement sensitivity of $A_z$ on $M$ was analyzed from
Eqs. 6.11 and 6.12, with the aim of obtaining $A_Z \sim 0.8$ in the observer studies, yielding a value of $M = 9$ for all studies below.

**III.2.2 Dependence of performance on acquisition technique**

The first study investigated performance as a function of dose allocation, defined as the fraction of dose imparted by the low-energy image:

$$A = \frac{ESD_L}{ESD_L + ESD_H}$$  \hspace{1cm} (6.13)

Five values of dose allocation were chosen for the study, selected from technique settings that were deliverable by the x-ray generator capabilities such that the total entrance surface dose (i.e., $ESD_L + ESD_H$) was constant – 0.1 mGy corresponding to the average dose for a posterior-anterio (PA) DR chest image. The resulting allocation values were $A = 0.06, 0.28, 0.45, 0.67, \text{ and } 0.85$.

**III.2.3 Dependence of performance on decomposition technique**

The second study investigated performance as a function of the various decomposition parameters for the SSH and ACNR algorithms. The low- and high-pass filters were chosen as Gaussian filters as in Chap. 5 characterized by $d_{LPF}$ and $d_{HPF}$ for the SSH and ACNR algorithms, respectively. The nominal decomposition parameters listed in Table 6.1 were selected based on qualitative examination of patient images acquired in an ongoing clinical study such that the decomposed image provided best tissue cancellation and reduction of noise without introducing significant artifacts.

Images were decomposed by independently varying, the $d_{LPF}$, $d_{HPF}$, and $w_n$ parameters while fixing the other parameter values to the nominal values shown in Table 6.1. Dose allocation was
held at 0.5 throughout the decomposition study, and the total dose was reduced to 0.05 mGy to reduce the number of cases for which the task was too conspicuous (i.e., $AZ > 0.95$). The SSH smoothing parameter, $d_{LPF}$, and the ACNR high-pass filter parameter, $d_{HPF}$, were each varied from 0.1 to 1 in increments of 0.1. The ACNR weighting parameter, $w_n$, was varied from 0.2 to 2 in increments of 0.2.

<table>
<thead>
<tr>
<th>Decomposition parameter</th>
<th>Soft-tissue Image</th>
<th>Bone Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>0.31</td>
<td>0.59</td>
</tr>
<tr>
<td>$w_c$</td>
<td>0.59</td>
<td>0.31</td>
</tr>
<tr>
<td>$d_{LPF}$</td>
<td>0.90</td>
<td>0.15</td>
</tr>
<tr>
<td>$w_n$</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>$d_{HPF}$</td>
<td>0.50</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 6.1: Nominal values for the decomposition parameters associated with each of the decomposition techniques.

### III.2.4 Experimental setup

The 9AFC study was conducted on a dedicated viewing station in a radiology reading room under controlled lighting. Images were displayed on a diagnostic workstation (Dell Precision-380, 3GHz Pentium 4 with dual-head monochrome Barco displays, 1530 x 2048 resolution, 8-bit grayscale). Each image was 300x300 pixels in size (900 x 900 pixels for the 9AFC, with a 10-pixel border between each image), and each image pixel corresponded to a single pixel on the display. The background of the display was masked to medium grey, given by the mean pixel value of the images. Observers were not allowed to adjust window, level, or magnification. A comfortable viewing distance of 50 cm was encouraged but not enforced.

Six physicists were used as observers, considered reasonably expert observers for purposes of a simple phantom experiment. The study was separated into three tests, corresponding to the three tasks, conducted in random order, and with images obtained using variable dose allocation and decomposition parameters presented randomly within each test. Observers were instructed as to
the defined “abnormal” (stimulus) and “normal” hypotheses for each task according to the illustrations in Fig. 6.1. For each case, 9 images were presented in a 3x3 matrix (Fig 6.4), 8 representing “normal” and 1 representing the stimulus, and observers selected the stimulus image by mouse click. Each observer was trained on ~70 cases before each test, with training data completely distinct from the actual test data. Normal and abnormal cases were randomly selected from fifteen images acquired at each combination of allocation and decomposition parameter, ensuring that no set of 9 images were repeated twice for the same observer. Each repeat, including the training set, was therefore statistically independent. A total of 1400 cases were presented to each observer. Short breaks were allowed between each test. An average time of ~3 h was required to complete all three tests, with an average of ~8 s per case.

![Example images presented in 9AFC human observer tests: (a) sphere detection, (b) shape discrimination, and (c) texture discrimination. For purposes of illustration, the stimulus image is in the bottom left corner.](image)

**IV RESULTS**

**IV.1 Dependence of Performance on Acquisition Technique**

Figure 6.5 plots theoretical and measured observer performance in terms of $A_Z$ as a function of dose allocation. Results pertain to the sphere detection task, with separate plots shown for the three decomposition algorithms (SLS, SSH, and ACNR) as well as the soft-tissue [Fig. 6.5(a-c)] and bone [Fig. 6.5(d-f)] image. The data points represent measurements obtained from the 9AFC
study, with error bars corresponding to the 95% confidence interval computed across the 6 observers. The curves in each plot correspond to four observer models (FH, FHE, NPW, and NPWE). Results agree with the previously observed dependence of image quality on dose allocation, with optimal performance exhibited in the range $A \sim 0.3-0.7$ depending on the applied decomposition algorithm and task. As expected from results in Chap. 5 ACNR yielded the greatest performance for the soft-tissue sphere detection tasks, and SSH is best for the bone image.

Figure 6.5: Sphere detection performance as a function of dose allocation in DE imaging (i.e., the fraction of dose allocated to the low-energy projection). Data points represent measurements from human observers, and curves represent theoretical model observers. (a-c) Soft-tissue and (d-f) bone image. Results are shown for the 3 decomposition algorithms (SLS, SSH, and ACNR).

The FH and NPW models perform similarly in the sphere detection task, since the task function weighs mostly low spatial frequencies [(Fig.6.5(a)] for which there is little correlation in the image data and therefore little difference between a prewhitening and non-prewhitening
observer. Both of these models appear to overestimate the performance of the human observers. The addition of an eye filter and internal noise in the FHE and NPWE models yields better agreement with human observer performance. For the soft-tissue image, although the experimental error bars are fairly large, the NPWE model is seen to agree reasonably well with measurement in terms of both overall magnitude and trend for all three decomposition algorithms. For the bone image, the FHE model appears to give the best agreement with measurement, although the fairly high conspicuity ($A_z > 0.95$) for the SSH and ACNR algorithms make it difficult to differentiate results.

Figure 6.6: (a-c) Shape and (d-f) texture discrimination performance as a function of dose allocation for the soft-tissue image. Data points represent measurements from human observers, and curves represent theoretical model observers. Results are shown for the 3 decomposition algorithms (SLS, SSH, and ACNR).
Figure 6.6 shows similar results for the shape and texture discrimination tasks. Results are shown for the soft-tissue images only, since (similar to Fig. 6.5), the conspicuity in the bone images was fairly high ($A_z > 0.95$, as predicted by theory and borne out in measurement). Despite fairly large error bars, the results suggest that the NPWE model exhibits the closest correspondence with measurements for both the shape and texture discrimination tasks. Again, the ACNR algorithm provides the highest performance for the soft-tissue images. For the bone images (not shown), the SSH algorithm provided the highest performance, in agreement with theory.

![Figure 6.6: Shape Discrimination (Soft-tissue)](image1)

![Figure 6.6: Texture Discrimination (Soft-tissue)](image2)

IV.2 Dependence of Performance on Decomposition Technique

Figure 6.7 plots theoretical and experimental performance for the SSH and ACNR algorithms evaluated as a function of decomposition parameters in the SSH and ACNR algorithms.

IV.2 Dependence of Performance on Decomposition Technique

Figure 6.7 plots theoretical and experimental performance for the SSH and ACNR algorithms evaluated as a function of decomposition parameter. Results are shown only for the soft-tissue
image, since the bone images were too conspicuous to differentiate trends ($Az > 0.95$), although theory and measurement were seen to agree. For the SSH algorithm [Fig. 6.7(a,c)], the $d_{LPF}$ parameter was seen to impart a significant effect on observer performance for both imaging tasks, showing a fairly sharp performance drop at $d_{LPF} \sim 0.5$. (See Discussion Section below). Both the overall magnitude and trend in performance was seen to agree with the NPWE model.

For the ACNR algorithm [Fig 6.7(b,e)], the parameter $w_n$ exhibits a clear optimum in the region $w_n \sim 0.8-1.0$. In contrast with results of the allocation study (Figs. 6.5-6.6), the application of an eye filter and internal noise not only reduced the overall magnitude of $Az$ (compared to the FH and NPW models) but also significantly affected the trend (shape) of the curves in a manner that brings model observer performance more in line with that of human observers.

V DISCUSSION AND CONCLUSION

Perhaps the most interesting example that the theoretical model combining Fourier metrics of imaging performance (NEQ) and imaging task in calculation of detectability index does describe the performance of real human observers is seen in Fig. 6.7(d) – the texture discrimination task under the SSH decomposition algorithm. In that case, as shown in previous work,$^{155}$ the algorithm effectively zeroes the NEQ at a given spatial frequency, depending on the choice of $d_{LPF}$. As illustrated further in Fig. 6.8(b), we see that if $d_{LPF}$ is chosen such that the “zeroing” of the NEQ coincides with the spatial frequencies of interest in the imaging task (i.e., the frequency of the crenellation, as shown in Fig. 6.2), then the detectability index plummets. Such is predicted theoretically and borne out experimentally, demonstrating that Fourier-based modeling from first principles of signal and noise transfer characteristics provides a valid description of real human observer performance.
In this chapter, the performance of human observers was compared to several model observers computed from a theoretical framework using cascaded systems analysis. Overall, results demonstrated that image acquisition parameters, decomposition parameters, and the imaging task all affect the performance of real observers. Furthermore, this work provides initial validation that the theoretical, Fourier-based modeling of NEQ, imaging task, and simple model observers can reasonably predict the performance of real observers. Results also identified that inclusion of
an eye filter and internal noise yielded higher correspondence to human observer performance
due to the removal of low-spatial frequencies – i.e., the FHE and NPWE models agreed best with
measurements. Such findings help to bridge the gap between metrics of detector performance
(e.g., NEQ, either computed theoretically or measured experimentally) and the performance of
real observers, linked by a Fourier description of the imaging task.
CHAPTER 7: SUMMARY AND CONCLUSIONS

I MODELING AND OPTIMIZATION OF DE IMAGING SYSTEMS

X-ray radiography is a widespread imaging modality that is indispensable to a broad spectrum of medical imaging procedures, so the ability to improve on its performance through the development and optimization of DE imaging techniques with FPDs could have a significant impact on the field. Since DE images can be acquired at dose that is equivalent to a conventional radiograph yet provide improved conspicuity of subtle abnormalities through material discrimination, DE imaging suggests a new normal means of radiographic examination in which a knowledgeable application of the x-ray energies achieves improved detection and characterization of disease.

I.1 An Overview of Results

This dissertation presented a theoretical framework for modeling and optimizing the performance of DE x-ray imaging systems. Chapters 2 and 3 provided the theoretical foundation for modeling DE imaging performance using cascaded systems analysis. Chapter 4 applied the
theoretical framework to chest imaging and quantified the tradeoffs between quantum noise and background anatomical noise in the optimization of DE imaging techniques. Chapter 5 extended the framework further to include noise reduction algorithms by developing a generalized decomposition algorithm for DE imaging. Finally, Chapter 6 compared the performance as predicted for a variety of model observers to actual human observer performance for a variety of imaging tasks, acquisition techniques, and decomposition algorithms. Overall, the theoretical modeling developed in this work identified optima across the entire DE imaging chain, from factors related to the x-ray tube output (e.g., kVp pair and dose allocation) to those associated with the decomposition algorithms (e.g., noise reduction algorithms). The analysis was based upon first principles of Fourier-based imaging performance and was shown to agree with real human observer performance.

Results in Chapter 6 concluded that the detectability index computed for the NPWE observer was the most accurate predictor of human performance overall. Below is a summary of theoretical results computed using the NPWE observer for the detection of a polyethylene sphere and Teflon sphere in the soft-tissue and bone images, respectively. The detectability index was computed as a function of the low- and high-energy kVp and dose allocation [i.e., $d'(kVp_L, kVp_H, A)$], similar to the optimization results described in Chapter 4 (see Fig. 4.1). Note that the model also included 2.5 mm Al and 2.5 mm Al + 0.6 mm Ag total filtration for the low- and high-energy image, respectively, and the total ESD was fixed to 0.1 mGy throughout. For each energy pair, an optimal dose allocation (denoted $A^*$) was identified at peak detectability (denoted $d'_{peak}$) yielding surfaces of $d'_{peak}(kVp_L, kVp_H)$ and $A^*(kVp_L, kVp_H)$. 

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Figure 7.1: Detectability index as function of low- and high-energy kVp for a (a)-(c) Polyethylene sphere detection task and (d)-(f) Teflon sphere detection task. Optima are identified at [45/80] kVp using ACNR and at [40/150] kVp using SSH for the polyethylene and Teflon detection tasks in the soft-tissue and bone image, respectively.

Figure 7.2: Optimal dose allocation as a function of low- and high-energy kVp for a (a)-(c) Polyethylene sphere detection task and (d)-(f) Teflon sphere detection task. Circles identify the corresponding optimal dose allocation for each maximum d' peak.
Figures 7.1 and 7.2 plot the theoretical results for $d'_{\text{peak}}$ and $A^*$, respectively, under different imaging tasks and decomposition algorithms. An optimum is identified (i.e., a global maximum in $d'_{\text{peak}}$) at $[45/80]$ kVp for the ACNR algorithm, with a corresponding dose allocation of 0.4 [Fig 7.2(c)] for the Polyethylene detection task. For the bone (Teflon) image, peak detectability is found at $[40/150]$ kVp with the SSH algorithm, with a corresponding dose allocation of 0.6 [fig 7.2(f)]. While the bone image result is not strictly an optimum (i.e., the maximum is in the “corner” of the plot), the range investigated corresponds to that typically available in radiographic x-ray equipment.

Furthermore, note that the two optima are mutually incompatible – i.e., the parameters of kVp pair and dose allocation cannot be different within one DE image acquisition. Therefore, to account for both the soft-tissue and bone detection tasks, we turn to the multiple task optimization technique described in Chapter 5. Considering each task to be of equal importance, the multi-task optimization [Eq. (5.23)] yields an overall optimum of $[43/136]$ kVp, with a dose allocation of 0.5. The results furthermore demonstrate a steeper dependence on the low-kVp than on the high-kVp. Therefore (assuming optimal added filtration and total dose equivalent to a conventional radiograph), we arrive at the following overall take-home points regarding optimal DE image acquisition technique:

1.) the low-kVp should be as low as possible;

2.) the selection of high-kVp is of secondary importance (although lower is better for the soft-tissue image, and higher is better for the bone image); and

3.) the dose should be allocated as to give roughly equal ESD between low- and high-energy projections.
Furthermore, regarding DE image decomposition:

4.) the ACNR algorithm is optimal for the soft-tissue image; and
5.) the SSH algorithm is optimal for the bone image.

I.2 From Blackboard to Bedside: Application to a Prototype DE Imaging System

The work presented in this dissertation proceeded in parallel to the development of a high-performance prototype DE imaging system for preclinical testing in patient volunteers at our institution. The work helped to motivate the case for double-shot imaging (as opposed to single-shot imaging; see Chapter 4) and the selection of differential filtration between the low- and high-energy beams. The optima provided important theoretical understanding in the development of optimal acquisition techniques with total dose equivalent to that of a conventional PA chest radiograph. Furthermore, the investigation of noise reduction algorithm provided a useful guide for optimal implementation. The experimental prototype has entered clinical trials at our institution, with acquisition and decomposition techniques guided in part by the findings of this dissertation. Analysis of the clinical images in future work, including various sub-arms of the clinical study in which kVp and dose allocation were varied, will provide invaluable validation of the optima presented in this thesis beyond the various phantom studies described in each chapter.

II IMPROVING CONSPICUITY AND FUTURE DIRECTIONS

II.1 Dual-Energy X-Ray Imaging

This dissertation pursued the hypothesis that the ability to discriminate image information based on material content would improve the conspicuity of subtle lesions through the reduction of overlying anatomical background noise. This premise motivated the investigation of DE imaging
performance by means of a theoretical model. The work yielded a fairly general theoretical framework for analyzing DE imaging performance and demonstrated the boost in generalized NEQ associated with the minimization of anatomical clutter. It identified optimal DE image acquisition techniques, decomposition techniques, and validated the approach in comparison to real human observer performance. The analysis further demonstrated that – given a high-performance FPD and optimal acquisition / decomposition techniques – DE image acquisition is possible at the same dose as conventional CR and DR systems, while significantly improving conspicuity of subtle lesion by the reduction of overlying background noise [Fig. 4.8(b)].

The development and availability of FPDs that provide high DQE and rapid readout of digital x-ray projections have made DE imaging a practical reality for clinical implementation. A commercially available DE chest imaging system has recently become available (Definium 8000, GE Healthcare, USA). The system provides fast DE image acquisition (200 ms inter-exposure delay) although the implementation appears somewhat suboptimal in that the system does not employ differential filtration between the low- and high-energy projections, and the radiation dose is fairly high (equal to the combined PA and LAT dose of conventional DR). The experimental prototype under development at our institution is a step toward higher-quality, lower-dose DE imaging achieved via differential added filtration, optimal kVp selection and dose allocation, and high-performance noise reduction algorithms. The main drawback of the initial experimental prototype compared to the commercially available system is the long inter-exposure delay (~6 sec) associated with the FPD, which will presumably be resolved in clinical configurations that incorporate a faster detector.

A variety of challenges for DE imaging using FPDs remain to be fully resolved. Results in Chapter 4 demonstrated that double-shot DE imaging offers a factor of two improvement in DQE
compared to single-shot DE imaging systems; however, double-shot imaging is prone to motion artifacts and is the focus of ongoing research, where possible solutions include breath-hold, cardiac gating, and deformable image registration.\textsuperscript{12,35,43} Other challenges include x-ray scatter, which remains a significant factor of image quality by reducing contrast and increasing noise and is also the focus of recent research with specific solutions being developed for DE imaging.\textsuperscript{52,156}

Future work could also investigate the extension of DE imaging to 3D imaging technologies such as DE tomosynthesis, which could furthermore improve lesion conspicuity by addressing both material and spatial discrimination as discussed in Chapter 1. The development of DE CT also presents interesting diagnostic value by providing improved quantitative imaging such as in kidney stone differentiation.\textsuperscript{60} Current clinical implementations of DE CT involve decomposition in the domain of reconstructed images; however, reconstruction of decomposed projections may offer a potential image quality advantage. For example, there are several methods under investigation for the acquisition of the low- and high-energy projections in CT, including: dual-tube systems for simultaneous acquisition, which may be costly to build; or sequential acquisitions, which are prone to motion artifacts similar to double-shot DE imaging systems.

\textbf{II.2 Bridging the Gap: From Fourier Metrics to Human Observers}

The ability to predict human observer performance from first principles is somewhat the “brass ring” of medical imaging physics. Such allows evaluation of system performance prior to the development of costly prototypes and without time-consuming human observer studies. As discussed in Chapter 6, the ability to predict human observer performance helps to bridge the gap between metrics of image performance computed theoretically (e.g., NEQ) and the performance of real human observers.
Clearly, there remain challenges in accurately predicting human observer performance in a real clinical setting. The tasks considered in this work were simple, highly idealized detection and discrimination tasks, and future work should consider more diagnostically relevant imaging tasks and conditions. Similarly, although reasonable agreement was found between model and human observer performance in Chapter 6, there is considerable variation and room for improvement, suggesting a need for more sophisticated model observers – for example, a channelized model observer that applies weights ‘channels’ differently across the spatial-frequency spectrum.\cite{67}

Future work could also include extending this theoretical framework of modeling observer performance to other DE imaging systems, including DE fluoroscopy, mammography, angiography, tomosynthesis, and CT. Such would naturally combine recent work in CSA modeling\cite{23,83,157,158} of real-time and 3D imaging systems with that of DE imaging.
APPENDICES
APPENDIX A: CASCADED SYSTEMS ANALYSIS

I  INTRODUCTION

This Appendix provides details regarding cascaded systems analysis as employed in this work, covering the “RSV transfer relations” and the 7 stages of image formation in an indirect-detection flat-panel detector. The analysis includes a parallel cascade model of K-fluorescence as studied by several authors,\textsuperscript{83,91,157} however, the work presented below extends the analysis to include a general account of variance in the conversion gain by inclusion of the Poisson excess (Swank factor).

II  SIGNAL AND NOISE TRANSFER

The transfer functions of Rabbani, Shaw, and Van Metter (herein referred to as the “RSV relations”) describe the NPS transfer for each stage.\textsuperscript{85} The relations conventionally refer to propagation through a stage corresponding either to “gain” (a change in the mean number of quanta due to amplification or attenuation) or “spreading” (a change in the spatial distribution of
quanta due to stochastic scatter or deterministic integration across an aperture). The process of “sampling” is described by an additional relation as described below.

II.1 Gain Stage

For a stage in which the mean number of quanta changes (but the spatial distribution does not), the NPS at the output of the stage, \( S_i(u, v) \), is given by that at the input, \( S_{i-1}(u, v) \), as:

\[
S_i(u, v) = g_i^2 S_{i-1}(u, v) + \sigma_{g_i}^2 \bar{q}_{i-1}
\]  
(A.1)

where \( \sigma_{g_i}^2 \) is the variance in the gain. For example, \( \sigma_{g_i}^2 = 0 \) for a deterministic gain, and \( \sigma_{g_i}^2 = g_i \) for a gain with a Poisson distribution. The term \( \sigma_{g_i}^2 \) can also be rewritten in terms of the Poisson excess \( \varepsilon_{g_i} \) as:

\[
\sigma_{g_i}^2 = g_i (\varepsilon_{g_i} - 1)
\]  
(A.2)

II.2 Deterministic Spreading Stage

For a deterministic spreading stage the NPS is transferred by the square of the MTF:

\[
S_i(u, v) = S_{i-1}(u, v) T_i^2(u, v)
\]  
(A.3)

where \( T_i^2 \) is the MTF that describes the blurring of quanta by integration across a pixel aperture.

II.3 Stochastic Spreading Stage

For a stochastic spreading stage the NPS is transferred according to the RSV relation:

\[
S_i(u, v) = (S_{i-1}(u, v) - \bar{q}_{i-1}) T_i^2(u, v) + \bar{q}_{i-1}
\]  
(A.4)
The output NPS is seen to consist of two components – an uncorrelated (white noise) component, \( q_{i-1} \), that is transferred without modulation and a correlated noise component, \( S_{i-1}(u,v) - q_{i-1} \), that is modulated by the MTF squared. Thus, uncorrelated quanta are randomly scattered and remain uncorrelated, whereas correlated quanta are further correlated according to system blur.

### II.4 Sampling Stage

For a sampling stage the output is given by the convolution of the presampling NPS with the comb function, \( III(u,v) \), corresponding to the sampling grid:

\[
S_j(u,v) = S_{i-1}(u,v) ** III_i(u,v)
\]  

(A.5)

where

\[
III_i(u,v) = \sum_{j,k=-\infty}^{\infty} \delta(u - \frac{j}{a_{\text{pix}}}, v - \frac{k}{a_{\text{pix}}})
\]  

(A.6)

and \( a_{\text{pix}} \) is the pixel pitch.

### III CASCADED SYSTEMS MODEL FOR FLAT-PANEL DETECTORS

The sections below summarize the cascaded systems model for FPDs, similar to that originally developed and validated by Siewerdsen et al.\textsuperscript{25,26} and extended to include a parallel cascade description of K-fluorescence based on the work of Yao and Cunningham.\textsuperscript{91} The resulting model provides a description of signal and noise properties for a given kVp (and dose, detector configuration, etc.) which provides the basis for modeling of DE imaging as described throughout the thesis.
III.1 Stage 1: Quantum Detection efficiency

The fluence incident on the detector, $\overline{q}_0$, is computed by integrating the fluence per unit exposure, $\frac{q_0(E)}{X}$, over the normalized incident x-ray spectrum, $\overline{q}_{\text{norm}}$:

$$\overline{q}_0 = X \int_0^{E_{\text{max}}} \frac{q_0(E)}{X} \overline{q}_{\text{norm}}(E) dE$$  \hspace{1cm} (A.7)

Where ‘norm’ denotes to the normalization such that $\overline{q}_{\text{norm}}$ is area normalized to 1 (i.e., provides a probability density function for the incident spectrum) and $\frac{q_0(E)}{X}$ is a fixed, specific function arising from definition of the unit exposure.

Similarly, $\overline{g}_1$ is computed by integrating $\overline{g}_1(E)$ over the normalized incident spectrum:

$$\overline{g}_1 = \int_0^{E_{\text{max}}} \overline{g}_1(E) \overline{q}_{\text{norm}}(E) dE$$  \hspace{1cm} (A.8)

where

$$\overline{g}_1(E) = 1 - e^{-\frac{\mu(E)}{\rho} s}$$  \hspace{1cm} (A.9)

where $\mu(E)$ is the attenuation coefficient of the scintillator, $\rho$ is the density (4.51 g/cm$^2$ for CsI:Tl) and $s$ is the surface density of the scintillator (given by the thickness times density).

Detection of x-rays follows a binomial distribution, with $\sigma_{g_1}^2 = \overline{g}_1 (1 - \overline{g}_1)$. The mean fluence and NPS at Stage 1 is therefore:

$$\overline{q}_1 = \overline{q}_0 \overline{g}_1$$  \hspace{1cm} (A.10)
\[ S_1(u,v) = \bar{g}_1 S_0(u,v) + q_0 \bar{g}_1 (1 - \bar{g}_1) = q_0 \bar{g}_1 \]

(A.11)

since \( S_0 = \bar{q}_0 \) for Poisson incident photons.

### III.2 Stage 2: Conversion gain

Stage 2 describes conversion of x-rays into optical photons in the scintillator. A secondary parallel process may occur in which an x-ray interacts with an electron in the K-shell of an atom in the scintillator and produces a K x-ray that may be re-absorbed in the scintillator and produce light at a remote location. This process causes reduction in sensitivity (due to K x-ray escape), increases the system blur, and increases noise. Figure 2.1 illustrates the three possible scenarios whereby an x-ray produces optical photons. Branch A corresponds to the case in which all of the energy of the x-ray is converted to optical photons locally (at the site of interaction). Branch B represents the case in which a fraction of the energy is deposited locally and produces optical photons when a K-shell interaction occurs. Branch C corresponds to the resultant energy in the K x-ray being deposited remotely to produce optical photons.

#### III.2.1 K-fluorescence parameters

a. **Probability of K x-ray Production (\( \xi \) and \( \omega \))**

The probability that an incident photon interacting in the scintillator will undergo a K-shell interaction is given by \( \xi \), and was determined for CsI from published work.\(^{159}\) The fluorescent yield of K-shell photoelectric interactions (i.e., the fraction of K-x-rays that are emitted) is denoted \( \omega \); hence the product, \( \xi \omega \), quantifies the probability that a photoelectric interaction will result in production of a K x-ray. For the detector considered (CsI:Tl), Cs and I have similar
physical properties, such as density, $\xi$, $\omega$, and K-edge energies ($E_K$); therefore, an effective value for each was computed by considering the respective fractional weights of each element in CsI to give $[\xi=0.834, \omega=.870, \text{and } E_K =35\text{keV}]$. Since no K x-ray can be expected to be produced below the K-edge energy, an energy dependent probability of producing K x-rays is defined:

$$\xi\omega(E) = \begin{cases} 0 & \text{for } E < E_K \\ $\xi\omega$ & \text{for } E > E_K \end{cases}$$  \tag{A.12}

b. Probability of K-x-ray re-absorption ($f_K$)

The probability of re-absorption somewhere of K x-rays in the screen for each photoelectric interaction producing a K x-ray is denoted $f_K$ and was computed analytically using a multi-layer model initially developed by Vyborny (extended by Shuping and Judy, and Chan and Doi) for a mono-energetic normally-incident x-ray. An effective value, $\overline{f_K}$, was computed by integrating $f_K(E)$ over the normalized incident spectrum at that stage:

$$\overline{f_K} = \frac{\int_E f_K(E)q_1(E)\xi\omega(E)dE}{\int_E q_1(E)\xi\omega(E)dE}$$  \tag{A.13}

c. Spatial Spreading of K-x-rays ($T_K$)

The MTF corresponding to the spread of K x-rays in the phosphor is denoted $T_K$. The point-spread function (PSF) of the K x-ray was computed using a multilayer model as described by Que.

$$\text{PSF}_K(r, E) = C \int_0^L \left[ 1 + e^{-\mu(E)\eta} - e^{-\mu(E)L} (1 - e^{-\mu(E)\eta}) \right] \frac{e^{-\rho(E_k)\eta}}{r^2 + \eta^2} E_K \mu_\rho(E_k)d\eta$$  \tag{A.14}
where \( r \) is the radial distance, \( \mu_{pe} \) is the attenuation coefficient due to photoelectric interactions only, \( L \) is the thickness of the scintillator, \( d\eta \) is an infinitesimal layer thickness, and \( C \) is a normalization constant [ensures unit area as in Eq. (2.7)]. The Radon transform of the PSF yields the line-spread function (LSF) from which the FFT was taken as \( T_K \). For CsI the dependence on the energy of the incident x-ray was found to be negligible, because the dominating term is the attenuation coefficient due to the K x-ray, \( \mu(E_K) \), which is independent of the incident energy. Therefore \( T_K \) was simply computed numerically at the mean incident x-ray energy.

### III.2.2 Conversion gain fluence and NPS

The fluence in each branch of the parallel cascade is \( \overline{q} \) multiplied by the respective gain:

\[
\overline{q}_{2A} = \overline{q}_0 \overline{g}_1 (1 - \xi\omega) \overline{g}_{2A} \\
\overline{q}_{2B} = \overline{q}_0 \overline{g}_1 \xi\omega \overline{g}_{2B} \\
\overline{q}_{2C} = \overline{q}_0 \overline{g}_1 \xi\omega \overline{f}_K \overline{g}_{2C}
\]

where the mean gain appearing in these equations is determined by integrating the energy-dependent gain over the normalized spectrum at that stage:

\[
\overline{g}_{2A} = \frac{\int_E W \cdot E \cdot \overline{q}_1(E)(1 - \xi\omega(E))dE}{\int_E \overline{q}_1(E)(1 - \xi\omega(E))dE} \cdot \overline{g}_{esc}
\]

\[
\overline{g}_{2B} = \frac{\int_E W \cdot E \cdot (1 - \frac{E_K}{E}) \cdot \overline{q}_1(E)\xi\omega(E)dE}{\int_E \overline{q}_1(E)\xi\omega(E)dE} \cdot \overline{g}_{esc}
\]

\[
\overline{g}_{2C} = W \cdot E_K \cdot \overline{g}_{esc}
\]

where \( W \) is the mean number of optical photons produced per keV absorbed and was taken to be 56 photons/keV.83
The fraction of optical photons escaping the scintillator is denoted \( g_{esc} \). This value was empirically determined for FPDs employed in this work by measuring the mean gain for the system (i.e., the mean signal per unit exposure, sometimes termed the gamma or sensitivity) and comparing to the theoretical expectation:

\[
\Gamma = \alpha^2 \left( q_0 / X \right) g_1 g_2 g_3
\]  

(A.17)

Since all of the terms in Eq. (A.16) are known escape \( g_{esc} \) (contained within \( g_2 \)), the escape efficiency could be solved for semi-empirically under the simplifying assumption that it was energy independent. The results yielded a value \( g_{esc} = 0.55 \), similar to that of Hillen et al.\(^{163}\) and others.\(^{25,164}\)

Writing the gain-variance in terms of the Poisson excess, \( \sigma^2_{g_i} = g_i (1 + \varepsilon_{g_i}) \), the NPS for the parallel branches of the cascade can be derived from the RSV relations as (taking \( i=2A, 2B, 2C \)):

\[
S_{2A}(u, v) = q_0 \bar{g}_2 (1 - \xi \omega) g_{2A} (g_{2A} + 1 + \varepsilon_{g_{2A}})
\]

\[
S_{2B}(u, v) = q_0 \bar{g}_2 \xi \omega g_{2B} (g_{2B} + 1 + \varepsilon_{g_{2B}})
\]

\[
S_{2C}(u, v) = q_0 \bar{g}_2 \xi \omega \bar{F}_k \bar{g}_{2C} (g_{2C} + 1 + \varepsilon_{g_{2C}})
\]

(A.18)

The Swank factor \( I_{2i} \) was computed using the moments of the absorbed energy distribution (AED):

\[
I_{2i} = g_{2i}^2 \frac{\int_E p_1(E) dE}{\int_E M_{2i}(E) p_1(E) dE}
\]

(A.19)

where:

\[
M_{2i}(E) = \frac{M_{2i}^2(E)}{M_{0i}(E) I(E)} = \frac{g_{2i}^2(E)}{I(E)}
\]

(A.20)
where $M_{0i}$, $M_{1i}$, and $M_{2i}$ are the zeroth, first, and second moments of the AED. Finally, the Poisson excess for each branch of the system was computed using the Swank factor:

$$
\varepsilon_{gi} = \bar{g}_{2i} \left( \frac{1}{L_{2i}} - 1 \right) - 1
$$

(A.21)

Also, the NPS cross term was as described by Cunningham:

$$
S_{2BC}(u, v) = q_0 g_1 f_K \bar{g}_{2B} \bar{g}_{2C} T_K(u, v)
$$

(A.22)

Hence by substitution:

$$
S_2(u, v) = \bar{q}_0 \bar{g}_1 \left[ (1 - \xi \omega) \bar{g}_{2A}(\Gamma_{2A} + 1 + \varepsilon_{g_{2A}}) + \xi \omega \bar{g}_{2B}(\Gamma_{2B} + 1 + \varepsilon_{g_{2B}}) + \xi \omega f_k \bar{g}_{2C} T_K(u, v) \right]
$$

Similarly for the total fluence at stage 2:

$$
\bar{q}_2 = \bar{q}_{2A} + \bar{q}_{2B} + \bar{q}_{2C}
$$

$$
= q_0 \bar{g}_1 \left[ (1 - \xi \omega) \bar{g}_{2A} + \xi \omega \bar{g}_{2B} + \xi \omega f_k \bar{g}_{2C} \right]
$$

(A.23)

Therefore an effective gain at stage 2 can be defined,

$$
\bar{g}_2 = (1 - \xi \omega) \bar{g}_{2A} + \xi \omega \bar{g}_{2B} + \xi \omega f_k \bar{g}_{2C}
$$

(A.24)

To make the rest of the analysis more tractable, a term describing the effect of K-fluorescence can be defined:

$$
\begin{equation}
P_K(u, v) = \frac{(1 - \xi \omega) \bar{g}_{2A}(\bar{g}_{2A} + \varepsilon_{g_{2A}}) + \xi \omega \bar{g}_{2B}(\bar{g}_{2B} + \varepsilon_{g_{2B}}) + \xi \omega f_k \bar{g}_{2C}(\bar{g}_{2C} + \varepsilon_{g_{2C}}) + 2 f_K \xi \omega \bar{g}_{2B} \bar{g}_{2C} T_K(u, v)}{\bar{g}_2}
\end{equation}
$$

(A.25)

It is the spatial-frequency-dependent cross-term in $P_K(u, v)$ that is mainly responsible for the increase of the NPS and goes as $T_K(u, v)$; therefore, the effect of K-fluorescence is greater at lower frequencies as can be seen in Fig. 2.10. Note the special case when $\xi \omega = 0$ – i.e. when no K x-rays are produced:
\[ P_k(u,v) \big|_{\omega=0} = \bar{g} + \xi \]  

(A.26)

Finally, \(S_2(u,v)\) can be rewritten in terms of \(P_k(u,v)\):

\[ S_2(u,v) = Q_0 \bar{g}_1 \bar{g}_2 [P_k(u,v) + 1] \]  

(A.27)

III.3 Stage 3: Spreading of Optical Quanta

This stochastic blurring stage quantifies the spreading of the optical photons in the scintillator:

\[ \bar{q}_3 = \bar{q}_0 \bar{g}_1 \bar{g}_2 \]  

(A.28)

and according to Eq. A3:

\[ S_3(u,v) = Q_0 \bar{g}_1 \bar{g}_2 (1 + P_k(u,v)T_3^2(u,v)) \]  

(A.29)

where \(T_3\) was determined from measurements of the system MTF [representing the product \(T_3(u,v)T_5(u,v)T_{Ktot}(u,v)\)], where \(T_5(u,v)\) is a sinc function (see stage 5) and \(T_{Ktot}(u,v)\) is the weighted sum of the MTFs at the 3 branches normalized to unity at zero frequency:

\[ T_{Ktot}(u,v) = \left[ \frac{(1-\xi \omega)g_{2A} + \xi \omega g_{2B} + \xi \omega f_k \bar{g}_2 T_k(u,v)}{\bar{g}_2} \right] \]  

(A.30)

where \(T_k(u,v)\) was modeled according to Que et al.93

III.4 Stage 4: Optical Coupling to Photodiode

This gain stage describes the coupling of optical photons to the intrinsic layer of the a-Si:H photodiode and the subsequent (one-to-one) conversion of optical photons to electrons. The mean fluence is dictated by the coupling efficiency, \(g_4\):
and the NPS is given by the RSV relations as:

\[
S_4(u,v) = q_0 g_1 g_2 g_4 \left[ 1 + g_4 P_k(u,v) T_3^2(u,v) \right]
\]

(A.32)

### III.5 Stage 5: Pre-Sampling Pixel MTF

This deterministic spreading stage describes blurring of the signal due to integration across the pixel aperture. The mean signal is given by:

\[
q_5 = q_0 a_{pd}^2 g_1 g_2 g_4
\]

(A.33)

where we note a change in units associated with integrating the fluence (quanta / mm²) to a mean number of quanta for a given pixel aperture. The NPS is given by:

\[
S_5(u,v) = q_0 a_{pd}^4 g_1 g_2 g_4 \left[ 1 + g_4 P_k(u,v) T_3^2(u,v) \right] T_5^2(u,v)
\]

(A.34)

Where \( T_5(u,v) = \left| \text{sinc}(u \cdot a_{pd}) \cdot \text{sinc}(v \cdot a_{pd}) \right| \) and \( a_{pd} \) is the size of the pixel aperture (assumed to be square for the sake of simplicity).

Stage 6: Image Sampling

This stage represents the sampling of the image at discrete pixel locations and quantifies the effect of aliasing on the NPS. The mean fluence is:

\[
q_6 = q_0 a_{pd}^2 g_1 g_2 g_4
\]

(A.35)
and the NPS is increased by convolution of the presampling NPS at stage 5 with the Fourier transform of the sampling grid:

$$S_6(u,v) = \overline{q_0} a^4_{pd} \overline{g_1} \overline{g_2} \overline{g_4} \left(1 + \overline{g_4} P_k(u,v) T^2_3(u,v) \right) T^2_5(u,v) * \text{III}(u,v)$$  \hspace{1cm} (A.36)

### III.6 Stage 7: Electronic Noise

Finally, electronic noise associated with detector readout electronics is included. The mean fluence is unchanged, except by an arbitrary factor converting image quanta (electrons) to analog-digital units (ADU), denoted $k_{ADU}$ and taken as 600 e/ADU as specified by the FPD manufacturer:

$$q_7 = q_0 a^2_{pd} \overline{g_1} \overline{g_2} \overline{g_4}$$  \hspace{1cm} (A.37)

The NPS is increased through addition of the additive electronics NPS, assumed to be uncorrelated (white) and independent of the incident signal:

$$S_7(u,v) = q_0 a^4_{pd} \overline{g_1} \overline{g_2} \overline{g_4} \left[1 + \overline{g_4} P_k(u,v) T^2_3(u,v) \right] T^2_5(u,v) * \text{III}(u,v) + S_{\text{add}}$$  \hspace{1cm} (A.38)

Equation (A.38) represents the final result for the theoretical NPS. Note that if there is no K-fluorescence, then $P_k(u,v)$ is replaced by $\overline{g_2} + \varepsilon_{g_2}$ in the above expressions, in which case the expressions reduce to familiar forms of the NPS as described by Siewerdsen et al.\textsuperscript{25} and others\textsuperscript{142,157}.
APPENDIX B: SOFT-TISSUE CONTRAST IN DE IMAGES

Figure B.1: Diagram of the simple pillbox geometry used to compute contrast between nodule and lung in DE images

To evaluate the effect of kVp selection in DE imaging on subject contrast, we consider the simple case of a uniform slab (Fig. B.1) of lung with thickness $l_{\text{Lung}}$ and attenuation $\mu_{\text{Lung}}$ containing a pillbox nodule of thickness $l_{\text{Nodule}}$ and attenuation $\mu_{\text{Nodule}}$. From the Beers-Lambert Law, it follows that:

$$I_1 \bigg|_{L,H} = \log(\Phi_1 / \Phi_0) = -\mu_{\text{Lung}} I_{\text{Lung}} \bigg|_{L,H}$$ \hspace{1cm} (B.1)

$$I_2 \bigg|_{L,H} = \log(\Phi_2 / \Phi_0) = -\left[\mu_{\text{Nodule}} I_{\text{Nodule}} - \mu_{\text{Lung}} (l_{\text{Lung}} - l_{\text{Nodule}})\right] \bigg|_{L,H}$$ \hspace{1cm} (B.2)
where $I$ denotes the projection image, $\Phi$ denotes x-ray fluence, subscripts $L$ and $H$ stand for low- or high-energy, and subscripts 1 and 2 describe regions of the image behind lung-only and behind lung-plus-nodule, respectively. From the definition of a soft-tissue DE image [Eq. (3.1)] it follows that:

$$I_{DE,1} = -I_{L,1} + w_s I_{H,1}$$

$$= (\mu_{H,Lung} - w_s \mu_{L,Lung})I_{Lung}$$

$$I_{DE,2} = -I_{L,2} + w_s I_{H,2}$$

$$= (\mu_{H,Nodule} - w_s \mu_{L,Nodule})I_{Nodule} - (\mu_{H,Lung} - w_s \mu_{L,Lung})(I_{Lung} - I_{Nodule})$$

Similar equations may be written for the DE bone image according to Eq. 3.2 To estimate the tissue cancellation parameter, $w_s$, the theoretical value derived from the log-subtraction algorithm can be used [see Eq. (1.16)]. It follows that:

$$C_{DE} = I_{DE,1} - I_{DE,2}$$

$$= -[\mu_{H,Nodule} - \mu_{H,Lung} - \frac{\mu_{H,Bone}}{\mu_{L,Bone}}(\mu_{L,Nodule} - \mu_{L,Lung})]I_{Nodule}$$

Hence the contrast between the nodule and lung in a DE image depends on the difference in energy-dependence of the attenuation coefficient between lung, nodule and bone. Correspondingly, contrast is increased for greater degrees of energy separation (reduced spectral overlap) between the low- and high-energy projections.
Figure B.2: (a) Plots of attenuation coefficients for nodule (polyethylene), lung (water), and bone, depicting the dependence of contrast on the difference in attenuation between the nodule and water at low and high energy. (b) Calculation of $C_{DE}$ for a 3 mm polyethylene sphere (simulating a nodule) as a function of low and high kVp (with 2.5 mm Al and 2.5 mm Al+ 0.6 mm Ag total filtration respectively).
REFERENCES


