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Flow Downstream of a Cluster of Nine Jets

This work is an experimental investigation of the flow downstream of a low emission nozzle. The nozzle is a 3×3 square matrix of nine small swirling air jets, has a design swirl number of 0.8, and operates at a Reynolds number of 40,000. Particle image velocimetry (PIV) was used to map the velocity field under nonburning and atmospheric conditions for the first 18 jet diameters downstream of the nozzle exit plane. Seeding was liquid injected into the air stream and drops were sized to filter out those larger than 3×3 pixels. The results showed that the cluster blends into a single jet-like flow 12 jet diameters downstream with the axial component of the velocity displaying self-similar properties. Lateral jet interaction slows the decay of the axial component of the velocity and jet expansion in the developed region while accelerating the decay of the radial component. [DOI: 10.1115/1.2720451]

1 Introduction

The key to reducing combustion nitrogen oxides (NO_x) and carbon monoxide (CO) emissions is fast and thorough mixing of fuel and air. Multipoint lean direct injection (LDI) nozzles improve mixing and atomization by injecting fuel in small amounts through multiple injection ports, where each nozzle contains dozens of small swirling jets. This paper examines the flow downstream of a matrix of nine small swirling air jets. Combustion studies reported in the literature show multipoint lean direct injection (LDI) nozzles have lower emissions compared to ordinary nozzles [1,2]. To date, there have been no detailed investigations of the nozzle flow or the effect of lateral jet interaction. A detailed understanding of the flow is necessary to understand the role of chemistry, mixing, and atomization to emission performance and to model these nozzles. While there are many previous studies of clusters of jets reported in the literature [3–7], their results cannot be extrapolated to the flow configuration examined here as the jets are nonswirling and the arrangements are cluster specific.

Axisymmetric turbulent free jets have been widely studied. The extent of the potential core in these flows depends on the jet exit conditions. When the boundary layer upstream of the jet exit plane is thin, the core region can be substantial, whereas when the flow is turbulent upstream of the injection point the potential core shrinks considerably. The core region can be as short as $x_{3c}/D=7$, or extend beyond $x_{3c}/D=200$; where x_{3c} is the position downstream measured from the exit plane and D is the round jet diameter [8].

The growth rate of jets with differing initial conditions is not constant which is inconsistent with the assumption of asymptotic independence. It was found that by including a finite mass source and a new length scale L , the jet width δ scales as $\delta \sim x_3(x_3/L)^n$, where x_3 is the position downstream of the exit of the jet and the exponent, n , is an arbitrary constant [9]. Although these previous results were derived for nonswirling round jets, the literature shows that some of the conclusions are applicable to swirling jets as well [10–12]. The azimuthal velocity component in a swirling jet introduces a radial pressure gradient that affects the development of the shear layer, weakens the organized structures, and favors the growth of random turbulence enhancing spreading, mass entrainment, and shear layer growth rates for swirling jets compared to nonswirling ones [13].

Swirling free jets experience a sudden expansion just after the exit and the rate of initial expansion is proportional to the degree of swirl. The centerline velocity decays exponentially as, x_3^{-n} , where the exponent depends on swirl and decreases with increasing swirl intensity [10]. Axial velocity profiles of jets with swirl intensity as high as $S_N=0.68$ reached a self-preserving regime similar to nonswirling jets [12].

Multiple jet flows are widely used in combustion systems to enhance fuel atomization and mixing. Studies show that far from the jet exit plane, the flow from a cluster of round non-swirling jets has velocity profiles similar to those of a single jet issuing from a single source [3–7].

For nonswirling jet clusters, as the jets merge laterally, the peak velocity at the outer jets shifts towards the central jet axis. After merging, the velocity profiles have a single jet-like shape. Computational investigations of nonswirling jet clusters using the $k-\epsilon$ turbulence model predict a single jet-like shape of the merged flow profile but fail to correctly predict the curvature, because of the inability of the standard $k-\epsilon$ model to predict flows with strong streamline curvature [7]. The merged single jet-like flow, resulting from a cluster of nonswirling jets, reaches a spreading rate similar to the single jet, after a negligible spreading rate in the merger region [4,6]. Far downstream, the merged flow displays self-similar behavior with a slightly slower spreading rate compared to a single jet [5].

There are no studies of clusters of swirling jets reported in the literature. This work studies the flow downstream of a 3×3 square matrix of nine small swirling air jets (Fig. 1) with a design swirl number of 0.8 and a Reynolds number of 40,000 based on the air pressure drop across the nozzle and the swirl cup diameter. Particle image velocimetry (PIV) measurements were conducted under nonburning conditions at atmospheric temperature and pressure to map the velocity field. The study covered the first 21 jet diameters downstream of the cluster exit plane.

The objectives of this study were:

1. Characterize the velocity profile; and
2. Investigate the effect of lateral jet interaction on the flow field.

Because of the small size of the nozzle, seeding the airflow with solid particles for the PIV experiments was impossible. Instead, liquid was injected into the air stream and a particle sizing MATLAB routine was developed to identify liquid drop size and filter those larger than 3×3 pixels for fluid phase velocity measurements.

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Fig. 1 Macrolamination nozzle

2 Experimental Setup

The nine swirling jets matrix flow was produced using a macrolamination Parker Hannifin nozzle (Fig. 1) operated at 13 g/s of air. To evaluate the effect of adjacent jets interaction on the flow field, measurements were made for a single jet (operated at one ninth of the previous flow rate) and the full matrix of nine jets.

The nozzle was mounted in an enclosed vertical booth. The measurements were done using a LaVision Particle Image Velocimetry (PIV) system. MIL-PRF-7024 type II (nozzle calibrating fluid) was injected between 0.11 and 0.32 g/s into the air hose 1 m upstream of the nozzle inlet plane for seeding. A particle sizing MATLAB algorithm was used to preprocess the PIV images and remove large liquid droplets retaining droplets smaller than 3×3 pixels and using them to measure the gas phase velocity.

2.1 Nozzle and Vertical Test Booth. The nozzle had nine air blast fuel injectors (Fig. 1) made from a series of flat metallic sheets diffusion bonded together. The air and fuel channels were chemically etched into the metal. For each jet, the air issues into a 4 mm deep, 6 mm diameter recess acting as a radial air swirl cup. Air exhausts into three swirling layers through four 1 mm $\times$ 2.5 mm rectangular ports in each layer. All the air jets swirl in the same direction with a design air swirl number value is $S_N = 0.8$.

The nozzle was mounted in a vertical test booth 375 mm wide with an octagonal cross section. The nozzle could be positioned in the horizontal x_1 and x_2 directions as well as the vertical direction x_3 (Fig. 1). The enclosure had a rectangular Plexiglas 80 $\times$ 100 mm viewing window for optical access. Parker Hannifin Ball screw systems and Compumotor step motors with a repeatability within 0.025 mm were used for motion in the x_1 , x_2 , and x_3 directions. The overall repeatability of the translation including the railings and frame was estimated to be 0.1 mm and the system was computer controlled. The booth enclosure was operated at atmospheric temperature and pressure.

2.2 Particle Image Velocimetry (PIV) Setup. A LaVision PIV was used with a 120 mJ, 16 Hz dual cavity New wave,

Fig. 2 PIV viewing widows

Gemini PIV 15 Nd:YAG laser operated at 532 nm wavelength. The power supply and the laser rods are water cooled. The camera was a 1280 $\times$ 1024 pixel FlowMaster3S (PCO Sensicam, VGA) CCD camera operated in double-frame, single exposure mode and the images were processed with a cross-correlation algorithm. After the first laser flash, the CCD camera transfers the first image to the frame buffer (on-chip storage zone). While the first frame is read out, the CCD records the second frame and remains sensitive during the time required to transfer the first frame from the frame buffer. The readout time determines the exposure time for the second frame, 125 ms for the FlowMaster3S camera. The exposure time for the first frame is factory set to 10 μ s. The time interval Δt between two successive laser pulses was 6 μ s.

Adaptive multipass PIV with decreasingly smaller size interrogation area was used where the initial interrogation window sizes was 128 $\times$ 128 pixels and the final interrogation window size was 64 $\times$ 64 pixels. The final size was the smallest possible for the seeding density used. The cross-correlation algorithm operated with an overlap of 25% to increase the final output vector map density. A three point Gaussian correlation peak fit was used to evaluate the displacement with subpixel accuracy. Two independent Gaussian distributions were fitted to the correlation peak in the x_1 and x_3 directions; the accuracy was 0.1 pixel [14].

Figure 2 is the position and size of the viewing windows used to capture the PIV images. A specific viewing window pattern was used for each of the flow configurations (single and matrix) and the details of these windows are given in the table accompanying Fig. 2. A larger field of view imposing a lower spatial resolution was used for the full matrix case to cover the central jet as well as the corner jet. The nine jet matrix flow was probed from $x_3/D \sim 2$ to 20 with a spatial resolution $\Delta x = 2.3$ mm. The viewing windows contained the axis of the central jet. For the single jet, the flow field was mapped from $x_3/D \sim 3$ to 17.6 with a spatial resolution $\Delta x = 1.4$ mm.

3 PIV Image Preprocessing

Prior to cross correlating the raw images, the PIV images were preprocessed to filter out large drops. The algorithm allowed digital sizing of the images of the drops, the construction of filters for small and large drops, and the filtering of the original raw images to extract the images containing only small drops.

Figure 3(a) is an enlarged view of a raw PIV image with a wide range of droplet sizes. Figures 3(b) and 3(c) are the resulting large and small droplet images, respectively, obtained using a cut-off drop size of 3×3 pixel.

Fig. 3 PIV images of the seeding drops

The procedure followed and detailed in the flowchart in Fig. 4 was

1. Binary conversion;
2. Droplet identification;
3. Filtering out of large droplets; and
4. Computation of separate images of large and small drops.

To assist in droplet identification, all pixels whose intensity was smaller than a predefined threshold were set to zero. Pixels with intensities larger than the threshold are set to 1. The camera has a 10 bit dynamic range with a background noise level at 30 counts and because of this a fixed threshold of 40 counts was used to preprocess the images.

The droplet identification function uses a connectivity parameter that requires either two pixels to share an edge or a corner to be considered connected. The second connectivity parameter is more conservative when the aim is to leave out large droplets and was used in this work.

The properties of the identified droplets were evaluated. Due to diffraction a straightforward conversion of a droplet pixel size into the actual physical size for droplets smaller than $20\text{ }\mu\text{m}$ was not

possible. Particle size in the intensity matrix used for cross-correlation and its impact is discussed in detail in the literature [14]. The correlation matrix is based on the local intensity and using particle sizes smaller than 3×3 pixels results in poor dynamic and spatial resolutions and is at its worse for particle sizes below 1 pixel scale. Above 3×3 pixels, the correlation is no better; the 3×3 pixels size is an ideal balance to address these considerations. All droplets with a bounding box larger than 3×3 were defined as large. A filter set to 0 at the location of the large droplets was computed and applied to the original image to remove large particles. A similar filtering process was used to identify small droplets.

4 Experimental Setup Assessment

The number of the PIV realizations used to compute the velocity field was determined by storage limitations. Experimental checks were performed to confirm the accuracy of the mean velocity components and the statistical accuracy of the measurements. The axial component of the velocity was compared against

Fig. 4 Image preprocessing algorithm diagram

an independent measurement of the axial component of the velocity obtained using phase Doppler particle anemometry for droplets smaller than $5\ \mu\text{m}$.

For U_3 , the convergence coefficient calculated using i PIV realizations is

$$C_{U_3}^i = 100 \frac{U_3^{i+1} - U_3^i}{U_3^i} \quad (1)$$

where U_3^i ($i=1-N$, where N is the total number of PIV realizations) is the velocity average calculated using the first i realizations of the instantaneous velocity.

For both the radial and axial velocity, the maximum value of $C_{U_i}^N$ was 5%. For the mean of the axial component, the maximum value for the convergence coefficient was $C_{U_3}^N=1\%$. Figure 5 is a sample convergence curve for the mean axial velocity U_3 . The total number of PIV realizations was $N=1170$. The x axis represents the number of samples i ($1 < i < 1170$) used to compute U_3 . The y axis is the associated convergence coefficient, $C_{U_3}^i$, calculated using i samples. Figure 6 is the statistical accuracy across the matrix diagonal at different positions downstream for the axial average component and is of the order of $\sim 1\%$ at the center and increases in the outer region to 4%. Figure 7 compares the axial velocity measured with PIV and phase Doppler particle anemometry (PDPA) using $5\ \mu\text{m}$ calibrating fluid drops for the down-

stream position $x_3/D=3$ with an agreement within $\sim 2\%$ and the error on the mean axial velocity is smaller than 5%.

5 Results and Discussion

5.1 Centerline Axial Velocity Decay. A logarithmic profile of the centerline axial velocity component U_3 for the nine jets configuration is shown in Fig. 8. The horizontal axis is x_3/D from 3 to 21. The vertical axis is the centerline axial velocity for the central jet. There are two regions where the decay is linear:

1. The merger region from $x_3/D=5$ to 10; and
2. The developed region from $x_3/D=12$ to 21.

For each of these regions, the axial centerline velocity is [9]

$$U_{3,\text{max}} \sim \left(\frac{x_3}{D} \right)^{-m} \quad (2)$$

where m is the slope of the profile; $m_{m1}=0.19$ and $m_{m2}=0.46$ for the merger and the developed region, respectively. The centerline velocity decay rate is slower in the merger region than in the developed region. In the merger region, the central jet entrains high momentum air from the adjacent jets. As the lateral jets merge, more air from the surroundings is entrained and the central jet axial velocity decay rate increases consequently. The normalized lateral length scale $\delta/D \sim (x_3/D)^m$ measures jet expansion

Fig. 5 Sample convergence curve for average velocity

[9]. The surrounding eight jets from which the central jet entrains in the merging region, $x_3/D=5-10$, confines jet expansion in that region to less than $1.5D$.

While the axial velocity profile of the central jet in the matrix has a maximum at the center, the single jet axial velocity has a local minimum and maximum is off the jet axis; a common feature of swirling jets (presented later in Figs. 12 and 13). The logarithmic centerline axial velocity decay for the single jet is shown in Fig. 9. As seen with the nine jets, the logarithmic profile has two linear regions. After an initial slow linear decay rate, the

centerline axial velocity decay increases past $x_3/D=8$. In the slow decay region, the effect of the quiescent air entrainment on the axial velocity is mostly absorbed by the fast decay of the velocity maximum (at $x_r=7$ mm from the centerline) and is not seen along the centerline. Downstream of $x_3/D=8$, the centerline decay rate is faster as the decay rate of the maximum decreases and the effect of entrainment reaches the centerline. The values of m (Eq. (2)) for the single jet are $m_{s1}=0.14$ and $m_{s2}=0.69$ for the slow and fast decay regions, respectively. The initial decay rate is slow and

Fig. 6 Statistical accuracy, axial velocity

Fig. 7 Comparison of PIV and PDPA Measurements, $x_3/D=3$

similar to the rate for the nine jets. The expansion is limited to less than $1.5D$ within the first 8 diameters downstream. In the downstream region, the expansion is faster and the lateral size of the jet is almost double the jet size for the nine jets. The values of the coefficients m for the single jet and the matrix are summarized in Table 1.

Lateral jet interaction slows the spreading rate of the jet in the developed region due to the entrainment of higher momentum air from the surrounding jets. Within the first 20 diameters downstream both flows have two expansion rates. The difference between the two stages is due to jet-jet interaction for the nine jets flow and to swirl for the single jet. Similar results are reported in the literature. It was found that the spreading rate of cluster jets is

smaller than a single jet in the first 20 diameters downstream [4]. The investigation of the interaction between two jets with variable spacing showed a slower spreading rate for the combined jet [5].

5.2 Mean Velocity Field. The profiles of the axial velocity for the matrix at different x_3/D positions downstream in the merger region ($x_3/D < 10.5$) are shown in Fig. 10. The axial velocity is normalized with the corresponding maximum axial velocity $U_{3,max}$ for each downstream position x_3/D . The horizontal axis is the position along the diagonal. The central and the corner jet axis are situated at $x_r=0$ and 18 mm, respectively. The profiles at $x_3/D=3$ and 5 display, in addition to a maximum at $x_r=0$ corresponding to the central jet, a local maximum between $x_r=14$ and

Fig. 8 Exponential decay of centerline axial velocity for the cluster

Fig. 9 Exponential decay of centerline axial velocity for the single jet

12 mm. The local maximum corresponds to the corner jet exiting at the radial position $x_r=18$ mm. The outer jets are entrained into the central jet moving the local maximum towards the nozzle centerline as the flow moves downstream. The axial velocity slowly decreases in the immediate vicinity of the two maximums while for most of the region between the two maximums it increases as the flow moves downstream into the merger region

because high momentum air from the corner jet is entrained into the central region. Similar results for clusters of nonswirling air jets where the location of the local maximum moves towards the center of the main flow as the jets move downstream were reported in the literature [4,5,7].

The profiles broaden as the central velocity decreases and the velocity at the tails increases as more surrounding air is entrained. In the developed region ($10.5 < x_3/D < 21.4$), the axial velocity profiles resemble those of a single nonswirling round jet similar to the observations cited in the literature for clusters of nonswirling jets [4,5]. Figure 11 is the profile of the normalized axial velocity. The normalized radial position x_r/δ is plotted versus the normalized velocity $U_3/U_{3,\max}$. $\delta = x_3^m$ was used for scaling. The normalized profiles collapse onto a single curve within the region corre-

Table 1 Values of m ($\delta \sim x_3^m$)

Region	x_3/D	m
Merger, matrix	5–10	0.19
Slow expansion, single jet	3–8	0.14
Developed, matrix	12–21	0.46
Fast expansion, single jet	8–17	0.69

Fig. 10 Axial velocity profiles in the merging region of the cluster

Fig. 11 Cluster self-preserving profiles in the developed region

sponding to $x_r=0-18$ mm, with a slight divergence at the ends. The self-preserving behavior is similar to the results reported in the literature for twin nonswirling jets [5].

Figures 12 and 13 are the axial velocity profiles for a single jet in the slow and fast expansion regions, respectively. Unlike the profiles of the central jet in the matrix, the axial velocity profiles over the entire downstream distance examined ($x_3/D=3.7-16.8$) have a local minimum at the centerline. The profiles were normalized with the corresponding values of the velocity at the centerline $U_{3,cent}$ for each x_3/D position. The minimum at the centerline decreases from $x_3/D=3.7$ to 16.8; the decrease is initially slow while the maximum, located at approximately the same position

$x_r=8$ mm through the first phase, and drops off quickly. In the fast expansion region, the local minimum decay accelerates and the maximum decreases continuously while shifting to increasing values x_r as the flow moves downstream. At $x_3/D=16.8$, the local centerline minimum is still apparent in the profile.

The radial velocity profiles for the matrix are shown in Fig. 14. The profiles are from $x_3/D=2.3$ to 3.4. For positions further downstream, the radial velocity is less than the displacement measurement limit (0.1 pixel, corresponding to 0.5 m/s). The region between $x_r=0$ and $x_r=18$ mm has flow from the central jet directed in the positive direction of x_r and from the corner jet di-

Fig. 12 Single jet axial velocity in the near field

Fig. 13 Single jet axial velocity in the far field

rected in the negative direction of x_r .

The radial profiles display three distinct parts: a region corresponding to the central jet expansion ($x_r=0-9$ mm), a region corresponding to the outward expansion of the corner jet ($x_r=18$ mm and larger), and the region $9 < x_r < 18$ with negative radial velocity (first profile in Fig. 14). The negative radial velocity slowly decreases as the central jet expands. Figure 14 shows that by $x_3/D=2.7$ the radial velocity of the air issuing from the corner jet and directed towards the centerline is overcome by the expanding central jet and is practically zero.

6 Conclusion

Aside from combustion emission performance studies reported in the literature [1,2], there are no detailed investigations of the flow downstream of multipoint lean direct injection nozzles. These detailed studies are necessary for numerical modeling and improving our understanding of the physical mechanisms involved in the performance of these nozzles. Unlike clusters of nonswirling jets, there are no swirling jets cluster studies reported in the literature.

Fig. 14 Radial velocity profiles for the cluster

A drop sizing algorithm filtered large spray drops from the PIV images allowing measurement of the gas-phase velocity field. The mean velocity measurements agree within 2% of the same profiles collected using phase Doppler anemometry in the central region of the flow field where the PDPA measurements are most reliable.

The flow field of the cluster displays two distinct regions: the merger region ($x_3/D < 10$) and the developed region ($x_3/D > 12$). In the merger region, the characteristics of the individual jets are still visible and the expansion rate of the central jet is slowed. In the developed region the cluster blends into a single jet-like flow with the axial component of the velocity field displaying self-preserving properties. This flow behavior is similar to the behavior of clusters of nonswirling jets reported in the literature.

Lateral jet interaction:

1. Slows the expansion rate of the jet and the decay of the axial component of the velocity in the developed region; and
2. Accelerates the decay of the radial velocity and creates recirculation regions between jets just downstream of the nozzle exit.

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Nomenclature

D = air cup diameter, 6 mm
 U_r, U_3 = mean radial and axial velocities, respectively

S_N = swirl number, $S_N = T_M / RA_M$, where R is the radius of the jet, and T_M and A_M are the axial fluxes of the angular and axial momentum, respectively [15]

x_r, x_3 = radial and axial position, respectively
 $\delta \sim x_3(x_3/L)^n$ = jet length scale at the position x_3
 m = exponent of the jet growth rate, $\delta/D \sim (x_3)^m$

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