Node Selection in Cooperative Wireless Networks

by

Elzbieta Beres

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Electrical and Computer Engineering
University of Toronto

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Abstract

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2009

In this thesis, we argue for node selection in cooperative decode-and-forward networks. In a single-hop network with multiple relays, we show that selecting a single node to aid in the transmission between a source and a destination outperforms both traditional orthogonal transmissions and distributed space-time codes. In networks where sources transmit information over multiple hops and relays can communicate with each other, we study the relationship between cooperation and channel-adaptive routing. We show that cooperation is only beneficial if designed jointly with a routing scheme. This motivates a search for optimal algorithms in generalized relay networks.

In networks without restrictions on the relays in terms of whom they can communicate with, we study the problem of optimal resource allocation in terms of transmission time. The resource allocation selects the relays to participate in the transmission and optimally allocates time resource between the selected relays. To implement this resource allocation algorithm, we propose a recursive solution which reduces the computational complexity of the algorithm.

For large networks, the resulting computational complexity of implementing the algorithm is exponential in the size of the network and is likely to preclude its implementation.
We thus propose that the resource allocation be implemented sub-optimally through node selection: a subset of the nodes in the network should be selected and used as input to the optimal resource allocation algorithm. We provide guidelines for selecting the nodes and propose four heuristics which offer various complexity-performance trade-offs. Compared to the optimal resource algorithm, all four heuristics significantly decrease the required computation complexity of the optimal algorithm.
Acknowledgements

The years over which I completed this PhD have been the best and happiest of my life, and I feel very lucky and thankful for all I have learned and experienced.

I am most indebted to my supervisor, Prof. Ravi Adve, who has guided my intellectual development throughout graduate school. From him I have learned to think precisely and critically, and his innumerable insights on many topics will stay with me forever. I am grateful for the support, encouragement and intellectual freedom he gave me, and for having believed in me.

I would like to thank my Ph.D. committee, Prof. TJ Lim, Prof. Ben Liang and Prof. Elvino Sousa for having guided this process and contributed invaluable comments and ideas. A big thank you to the external examiner of my thesis, Prof. Halim Yanikomeroglu, who was very generous with his time and provided me with more feedback, suggestions and comments than I ever could have hoped for.

I would also like to thank Darlene Gorzo and Judith Levene in the graduate office and Jayne Leake in the undergraduate office for their good humour and kind and efficient help. Such support really cannot be overstated.

The group and office were and are full of wonderful people who contributed to making this a fun and learning experience. Special thank you to Azadeh Kushki for the years of close friendship and very generous help in all aspects of my life, including font design. Also special thank you to Adam Tenenbaum, for his good spirits and very frequent technical and
academic help. Thanks to all past and current group members, including Josephine Chu and Andrew Eckford, whose conversations helped me look at the other side of just about any argument.

Any success requires a healthy, encouraging and stimulating living environment, and I would like to thank all who have offered me one: the community at Massey College for giving me the chance to develop in so many directions and to meet so many spectacular people; Sister Agnes and the Carmelite Sisters, for taking me in for the summer and providing me with more peace and tranquility that I have ever known; and my roommate and friend Sapna Sharma, who added daily joy and laughter to my life, and helped me in every single thing I did, large and small. It would not have been the same without her.

My deepest gratitude to all my friends who have supported me, cared for me, and helped me develop intellectually, emotionally, and spiritually: Lidia Radi, Sebastian Schulleiter, Ester Macedo, George Kovacs, Simon Watson, Jorge Torres, Paul Medvedev, and Ramy Farha and Ali Khachan. Their impact and influence on me has been tremendous, and I feel so lucky to have been blessed with their friendship.

And finally, a huge thank you to my family: Mom, Dad, and Greg. They worried about me, supported my every decision and helped when no one else could. Their love has been my inspiration.
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Chapter 1

Introduction

The designers of wireless systems face the challenge of increasing data rates to meet the requirements of future generations of networks. The signal at the transmitter, however, suffers from channel fading caused by the superpositioning of multiple reflected and refracted copies of a signal arriving from different directions. The destination therefore receives a random copy of the signal transmitted by the source. Fading results in an inherent unreliability of the wireless channel and, in turn, this unreliability is the fundamental limitation to increasing data rates. Techniques to increase the reliability of the wireless channel are thus a priority.

A popular technique successful in addressing the adverse effects of channel fading is diversity. Diversity can be implemented in the temporal dimension through the use of channel coding and an interleaver, the frequency dimension through frequency hopping, or in the spatial dimension through multiple antennas. In all cases, diversity allows a user to average the “good” and “bad” fades such that a user sees a channel with lower variance. This thesis focuses on exploiting the spatial diversity available in distributed, mesh-like, networks.

Traditionally, spatial diversity is implemented with multiple antennas at the transmitter and receiver [1]. Spatial diversity, however, can also be implemented in a distributed fashion by employing other terminals in the system as virtual antennas. This technique, referred to
as relaying or cooperative diversity, requires a terminal to act as a relay for a source terminal in its transmission to the destination. Research in this area has been motivated by two main considerations: (1) it is still unclear whether size and economical considerations will permit the installation of multiple antennas on wireless nodes. Relaying would provide diversity even in situations where this is not possible; (2) even if multiple antennas can be installed on terminals, they can be used either to provide additional diversity, or for other purposes such as multiplexing data streams or beamforming. Cooperative diversity can be applied in any networks with sufficient number of nodes willing to act as relays, including mesh, ad-hoc, sensor, and potentially, cellular networks.

In this thesis, we focus on static, distributed, cooperative diversity systems comprising multiple relaying nodes which implement cooperative diversity using decode-and-forward. We assume that all nodes in this network are relatively simple, inexpensive and limited in circuitry and computational complexity. Specifically, for our purposes, this implies that we do not consider power control. Our goal is to maximize the data rate between a source and its destination. These assumptions are most relevant to mesh networks of access points. Early examples of these networks are the Toronto Ontario Hydro network recently installed in the downtown core in Toronto [2]. Such nodes derive their power not from batteries but from a “regular” power supply. We focus on two variants of these networks: parallel-relay networks and generalized-relay networks. In parallel-relay networks, we assume that relays cannot communicate with each other, i.e., the only allowable connections are between each relay and a source and destination. These networks are now well-studied in the literature, and serve as a building block for more complex, generalized-networks. In generalized-relay networks, no constraints are imposed on the connectivity between nodes; thus any relay may communicate with any other relay in the process of transmitting source data to the destination. These networks offer potentially improved performance over that achievable in the parallel-relay network.
In this thesis, we make three main assumptions. First, each transmission occurs over a frequency-flat channel [1]. Second, each source transmission occurs on a dedicated multiple-access channel, which resolves the issue of poor scalability of random access protocols [3]. This also allows us to eliminate the MAC layer from our analysis, hence simplifying the problem at hand. Because cooperation imposes a causality requirement, we consider the use of time-division-multiple-access (TDMA), where nodes transmit information using orthogonal time-slots. This method is particularly well suited, and often proposed, for systems employing cooperation [4].

Third, we assume that all nodes transmit at constant power levels, and thus do not allow for power adaptation. This is motivated by two main considerations. First, because the nodes considered in this thesis are stationary and attached to a fixed power supply, their life-time is not a consideration. Individual power is therefore not constrained and each node is allowed to transmit at its maximum available power of $P$ Watts. Second, avoiding power allocation significantly simplifies the nodes themselves, an important consideration for the simple and inexpensive nodes considered in this thesis.

In the following section, we describe and motivate the problems addressed in this thesis and, very briefly, describe the available literature relevant to these problems. The reader is assumed to have some knowledge of the research in cooperation networks [5]. A more detailed literature survey is provided in Chapter 2.

\section{Background and Motivation}

In a single-relay network, data transmitted by the source is captured and processed by the relay node, which forwards this data to the destination of the source. The majority of research in such a three-node network has focused on the forwarding strategy of the relay. The relay strategies can be categorized into several groups, include processing strategies
(selecting between Decode-and-Forward (DF) [4], Amplify-and-Forward (AF) [4], Estimate-and-Forward (EF) [6] and Demodulate-and-Forward (DemF) [7]), coding strategies for DF systems, and resource allocation strategies (determining the fraction of available power and system bandwidth to allocate between the source and relay).

Because the terminology in the literature is not always consistent, we briefly define the relevant terms as they are used in this thesis. Although cooperative diversity and cooperation is sometimes used to refer to two users relaying for one another, here we use the terms cooperation and relaying inter-changeably to represent the scenario where a relay node forwards the source data to the destination which, in turn, combines the data arriving from both source and relay. We use the term multihop for a scenario where the destination does not process the information arriving directly from the source, i.e., the information flow is strictly between the source and relay and relay and destination.

The existence of multiple relays adds a further complication: in addition to relaying strategies, we need a strategy to decide how many and which relays should participate in helping the source. The assumption that all relays participate in the transmission has motivated research into the coordination of the participating relays.

Organizing multiple relays to aid in the transmission of information between a source and destination raises the problem of coherent reception of multiple independently-faded copies of the transmitted signal at the destination. This combination can be achieved

\[ \text{1.1. Parallel-Relay Networks} \]

A large body of work related to multiple relays in cooperative diversity has neglected the question of how many and which relays should participate in helping the source. The assumption that all relays participate in the transmission has motivated research into the coordination of the participating relays.
through pre-coding [8], orthogonal transmissions and a maximal-ratio-combiner (MRC) [4,9],
a RAKE receiver for CDMA-based systems [8,10], or through distributed space-time codes
(DSTC) [11–15].

Each of these methods, however, suffers from serious drawbacks. Pre-coding requires
exact channel phase knowledge at the transmitter; such information is difficult to obtain with
a high degree of accuracy, and this method is generally regarded as impractical. Assigning
orthogonal channels to each transmission suffers a significant performance penalty resulting
from the decreased spectrum assigned to the direct source transmission [11]. A technique
commonly considered to overcome this problem is that of distributed space-time coding
(DSTC). DSTC is applicable to multiple-source systems, whereby data from the multiple
relays of the multiple sources are transformed in the temporal and spatial domains such that
multiple transmissions can be carried out without a bandwidth penalty. DSTC, however,
requires symbol and carrier synchronization between relays which renders the implementation
of DSTC difficult, if not impractical. Furthermore, as we will see, the performance of DSTC
actually suffers from a power penalty which degrades its performance.

In the first phase of this thesis, we solve the problem of coordinating multiple relays
transmissions in a parallel-relay, multi-source network by proposing to use Selection Cooper-
ation – selecting a single node to aid in the transmission between the source and destination.
In addition to being simple to implement, we show that Selection Cooperation can actually
outperform DSTC for most network sizes.

1.1.2 Generalized-Relay Networks

The work in Selection Cooperation focuses on communication between source and destination
over a single hop. In a large network with many nodes, considering single-hop parallel-relay
networks is clearly sub-optimal: if the end-to-end SNR is very low, transmission between
source and relay may be impossible even with the possibility of relaying, and incorporating multi-hops may be necessary. In any SNR-regime, in fact, channel conditions may be such that allowing transmissions between relays may be beneficial; in fact, communication in a mesh-network generally involves multiple hops. Allowing for multiple hops and transmissions between relays leads to a fully-connected network, where any node may transmit to any node.

In bridging the gap to generalized multiple relay networks, we consider selection in multi-hop relay networks. In this context, the term selection refers to choosing a subset of nodes in the network to optimize a chosen metric. However, selecting relays to create a path between source and destination is related to routing; in fact, routing can be interpreted as a special case of selection. In the second phase of this thesis, we consider the interaction between cooperative diversity and routing, and illustrate how little cooperation helps if treated independently from routing. This conclusion motivates a search for an algorithm that optimally combines routing and cooperation, the theme of the third phase of this thesis.

We consider a single source communicating with the destination with the help of multiple relays. The source communicates with the destination using a dedicated channel which must be split and allocated to all relays participating in this transmission. Each relay transmission, therefore, occurs on a dedicated time-slot. In the context of generalized-relay networks comprising nodes transmitting with fixed power levels, we seek an optimal, in terms of outage rate, relaying strategy. This strategy consists of two parts: (1) optimal selection of relays; (2) allocation of channel resources to the selected relays. For a system with no power allocation, a solution to this problem provides an upper bound on cooperative performance in multi-relay network where dedicated channels are assigned for each source transmission.

To the best of our knowledge, this problem has not been addressed in the literature. In [16], Boyer et al. introduce the framework of generalized-relay networks and study the relationship between system constraints and achievable combination of communication links between cooperating nodes. In general, works in the area of multi-relay systems with gen-
eralized links and dedicated multiple access generally neglect the bandwidth penalty arising from multiple hops by assuming either full-duplex nodes, a bandwidth-unconstrained system, or the availability of channel phase information at the transmitter [9,17–28].

In the context of generalized-relay networks with constant power transmissions and no interference, we provide a solution to the optimal transmission-time allocation problem for a decode-and-forward network of arbitrary size. We further suggest relay selection schemes which significantly reduce the complexity of the optimal resource allocation scheme. These contributions are discussed in the following section.

1.2 Thesis Contributions

The main contributions of the thesis are:

• We propose Selection Cooperation in cooperative, decode-and-forward, bandwidth-limited, parallel-relay networks where relays transmit under individual power constraints and do not communicate with each other (Chapter 3).
  
  – We derive the high-SNR outage probability approximation of Selection Cooperation for a single source-destination pair, considering all source-relay, relay-destination, and source-destination channels.
  
  – We give an approximation of the outage probability of Selection Cooperation for all SNR levels and arbitrary channel distributions.
  
  – We generalize Selection Cooperation to network with multiple sources (and thus flows), an issue largely avoided in the literature. We present three different relay selection schemes based on available centralization and complexity requirements.
  
  – For the most simple scheme, we derive the high-SNR outage probability and Diversity-Multiplexing Tradeoff, and show that even the simple selection scheme
outperforms DSTC for networks with more than three relaying nodes. The use of selection answers an important question - in a parallel relay network with multiple sources, how many nodes should one cooperate with? The answer is, apparently, one (best) node.

This work has spawned research on selection in AF networks [29], low-complexity DemF networks [30], the performance evaluation of selection using other performance metrics [31–33], and others [34].

- In networks with multihop and cooperation, we study the relationship between cooperation and routing according to instantaneous channel conditions. We show that cooperation offers little or no benefit if implemented independently from routing. This result motivates a search for an optimum cooperation and routing strategy in multihop cooperative networks (Chapter 4).

- In generalized networks, we study the problem of transmit time allocation between the relays. The resource allocation answers the question of which nodes should cooperate and how to maximize performance with complexity constraints (Chapter 5).

  - We present the optimal resource allocation, consisting of optimal relay selection and optimal allocation of transmission time. The optimization is obtained by maximizing over the rates obtained for each particular set of active relays; the unique time allocation for each particular set can be obtained by solving a linear system of equations.

  - We derive a recursive algorithm for the optimization problem which reduces the number of required iterations and reduces the computation load of each iteration from $O(N^3)$ to $O(N^2)$, where $N$ is the number of relays.
• In the context of generalized-relay networks, we propose sub-optimal relay selection as a way of achieving the gains of resource allocation at a reasonable computational complexity, and develop guidelines for node selection. Through simulations, we show that the sub-optimal selection algorithm should select nodes that maximize the rate on the direct path between source and destination, as well as the sum of rates between the source and relay and relay and destination (Chapter 6).

The predominant theme in this thesis is that of selection. We argue that both in parallel-relay networks and generalized relay networks, selecting a sub-set of nodes (one node in parallel relay networks and a small set in generalized-relay networks) out of all available nodes offers excellent trade-offs in terms of performance and complexity. In the case of parallel-networks, the selection has a rigorous and optimal strategy. Unfortunately, in multi-hop networks, obtaining the optimal sub-set of nodes is computationally complex and we have to resort to heuristics. We also highlight that in networks where information is transmitted through multiple relays, routing can be interpreted as a special case of selection. This observation motivates the introduction of traditional routing algorithms to the problem of node selection in cooperative networks.

1.3 Thesis Outline

The rest of this thesis is organized as follows. In Chapter 3, we introduce Selection Cooperation as an alternative to DSTC in parallel-relay networks. In Chapter 4, we introduce a simplified version of generalized networks and illustrate that for cooperation to add extra benefit over channel-adaptive routing, routing and cooperation must be designed jointly in some optimal manner. In Chapter 5 we solve the optimal resource allocation problem for generalized-relay networks, and in Chapter 6 we suggest heuristics for sub-optimal relay
1.3. Thesis Outline

as practical method of reducing the complexity of the resource allocation algorithm. We conclude this thesis in Chapter 7.
Chapter 2

Literature Survey

In this chapter, we survey the available literature in the area of cooperative diversity relevant to this thesis. We begin with single-relay networks in Section 2.2 and focus on contributions which center on the relay forwarding strategy, resource allocation between the source and relay, and partner selection in single relay networks. This section presents the main challenges in the area of cooperative diversity and serves as an introduction to the problems encountered in multiple-relay cooperative networks. The main works, as they relate to the contributions in this thesis, are discussed in Section 2.3. Because this thesis deals uniquely with Decode-and-Forward networks, we mostly limit the presented results to the Decode-and-Forward protocol. For clarity of exposition, we begin with a background on performance metrics.

2.1 Performance Metrics

Before delving into the current literature on cooperative diversity, we begin with a discussion on the performance metrics used to evaluate different schemes.
2.1. **Performance Metrics**

2.1.1 **BER and SER**

The bit error rate (BER) and symbol error rate (SER) are important metrics in communications systems, as they directly and accurately measure algorithm performance. These metrics, however, are very difficult to write in closed-form. Often, these metrics are used as the final evaluation of the system design, rather than the starting point.

2.1.2 **Outage Probability**

Another important metric in communications is the system capacity: the maximum transmission rate at which error-free transmission is possible. For fading channels, non-zero capacity is achievable for ergodic channels only, where channels fluctuate rapidly as compared to the coding interval. In the case of non-ergodic channels, as are many of those considered in the literature, the theoretical capacity is zero, as error-free transmission cannot be guaranteed at any transmission rate. For fading channels, a common metric is the outage probability $P_{\text{out}}$, defined as the probability that the mutual information $I_{sd}$ between a source $s$ and its destination $d$ drops below a specified target rate, $R$:

$$\begin{align*}
P_{\text{out}} &= \Pr[I \leq R].
\end{align*}$$

(2.1)

Errors are guaranteed once the channel mutual information drops below the transmission rate; the outage rate is thus a lower bound on the system error rate [35]. Because outage probability is generally easier to compute in closed-form than the BER/SER, it is a popular metric used to evaluate cooperative diversity schemes where the existence of relays adds significant complexity to the calculation of error probabilities.
2.1.3 Diversity order

A popular metric for spatial diversity systems is diversity order, defined as the negative of the slope of the outage probability or error rate curve as a function of the signal-to-noise ratio (SNR) in a log-log plot, i.e.,

\[ d = - \lim_{\text{SNR} \to \infty} \frac{\log_2 P_{\text{out}}(\text{SNR})}{\log_2(\text{SNR})}. \]  

(2.2)

Diversity order measures the asymptotic decrease of the error rate. In a physical sense, diversity order is a measure of the number of independent copies of the source signal received by the destination. Irrespective of its performance in the low-SNR regime, a scheme with higher diversity order will eventually outperform one with lower \( d \); a scheme with higher diversity order is thus considered better than one with lower diversity order.

Schemes with the same diversity order may, however, exhibit very different behaviours in the low-SNR regime. In addition, this metric does not tell us where, as a function of SNR, this asymptotic behaviour dominates. Two schemes with the same diversity order may, in practice, have very different performance. Furthermore, a scheme with lower diversity order may outperform a scheme with higher diversity order in the low-SNR regime, which may prove important depending on the operating point. Diversity order, although informative, should therefore always be used in conjunction with other metrics.

2.1.4 Outage Rate

The error rate and outage probability are meaningful metrics for physical-layer considerations where the source and destination are abstracted from the rest of the network. In systems where the data can be transmitted through multiple nodes, a meaningful metric is the
2.1. Performance Metrics

maximum rate, \( R_{\text{out}} \), at which transmission is possible given a target outage probability \( P_0 \):

\[
R_{\text{out}} = \arg \max_R \quad \text{s.t.} \quad \Pr[I_{sd} \leq R] \leq P_0.
\] (2.3)

Over many hops, outage rate is easier to calculate than the end-to-end outage probability, and is thus a popular metric in multi-hop networks.

2.1.5 Diversity-Multiplexing Trade-off

In [35], Zheng and Tse define the Diversity-Multiplexing Trade-off (DMT) which describes how a diversity order is affected as the transmission rate is varied with SNR. The DMT tradeoff is defined as the highest attainable multiplexing gain \( r \) for a constant diversity gain \( d \) (and vice-versa), where

\[
r \doteq \lim_{\text{SNR} \to \infty} \frac{R(\text{SNR})}{\log_2(\text{SNR})}, \quad d \doteq \lim_{\text{SNR} \to \infty} \frac{P_{\text{out}}(\text{SNR})}{\log_2(\text{SNR})},
\]

where \( R(\text{SNR}) \) is the transmission rate in bits per channel use, and \( P_{\text{out}} \) is the outage probability. The DMT generalizes the concept of diversity order: a scheme achieves full diversity order when the rate, as a function of SNR, is constant, while the diversity order is zero when the transmission rate is always at its maximum. Like diversity order, DMT masks the performance in the low and medium SNR regime, and should therefore be used in conjunction with other metrics to evaluate system performance.

In this thesis we will generally use the outage probability and outage rate as performance metrics.
2.2 Single-Relay Networks

In this section, we discuss networks where a single relay aids in the transmission of information between a source and a relay, as in Figure 2.1. This relay may be one of many relays available in the overall network.

The classical relay channel was first introduced and analyzed by van der Meulen [36]. In their seminal work, Cover and El Gamal derived the upper and lower bounds on the capacity of the general relay channel, as well as the exact expression for the capacity of the degraded relay channel (where the source-relay channel is better than the relay-destination channel) [37]. This marked the beginning of what is now a large and growing field of research on relaying and cooperative diversity. Note that the exact expression for the capacity of the general relay channel is still an open problem.

Sendonaris et al. [8] are credited with first proposing the concept of user cooperation diversity, where two users employ the relay channel in parallel: user one transmits to the destination with user two acting as a relay, and vice-versa. The two main assumptions made by the authors are: (1) full-duplex relay nodes, i.e., relays can receive and transmit simultaneously on the same channel; (2) each node knows the phase of its channel to the destination. All transmissions to the destination can thus be made on the same channel.
through phase-cancellation at the source and coherent combining at the destination. There is, therefore, no cost associated with relaying, as no new channels need to be assigned for the relay to forward data to the destination;

Using the above assumptions, the authors show that cooperative diversity increases the achievable rate region, and that this increase depends on the quality of the inter-user channel. The better this channel, the greater the improvement. In non-ergodic environments, cooperation improves outage probability, even when the increase of the achievable rate region due to cooperation is minimal (in cases where the inter-user channel is bad). They also show that cooperation can significantly increase coverage by decreasing the received power each terminal needs to receive information.

Although theoretically interesting, the above analysis is based on assumptions which are difficult to implement in practice. First, channel phase information at the transmitter can be difficult to obtain via feedback and thus coherent reception at the destination is difficult to achieve without orthogonal transmissions. This problem is amplified in networks with multiple relays. We will thus revisit this problem in Section 2.3.

Second, present day system designs preclude full-duplex operation, which requires interference cancellation between received and transmitted data at the relay, a particularly difficult task given that the power of the transmitted signal is usually orders of magnitude greater than the received signal. The majority of the works on cooperative diversity have thus focused on half-duplex relay transmissions, i.e., each node either transmits or receives at any one time. In addition, to avoid interference between transmissions, relays are assigned orthogonal channels in time, frequency or code. Although all three are equivalent from a theoretical perspective, time-division includes a natural causality and thus may be easier to implement.

The issues discussed here introduce new questions, such as (1) given that relaying with orthogonal channels incurs a cost in terms of bandwidth, when is relaying beneficial? (2)
2.2. Single-Relay Networks

how should the orthogonal channels be allocated between the source and relay? (3) what strategies should the relay employ to process and re-transmit the information? (4) in a large network, how should node partnering be determined?

The available literature can be roughly classified according to the type of problem addressed: forwarding strategy by the relay, resource allocation between the source and relay, and partnering algorithms. We begin with the work of Laneman et al. [4] who introduced practical versions of the relaying strategies of [8, 36, 37] and sparked further research into cooperative diversity.

Laneman et al. analyze the diversity order and outage probability of Amplify-and-Forward (AF) and Decode-and-Forward (DF) relaying under the assumption of half-duplex relay transmissions: the source and relay transmit using orthogonal time slots of equal duration [4]. As will be discussed further in Section 2.2.1, in DF the relay fully decodes the source codeword and re-encodes the information stream before re-transmitting to the destination; in AF, the relay simply amplifies the source information. In [4], for both cases, the relay simply repeats the codeword received from the source; in the case of DF, this coding scheme is referred to as repetition coding.

The authors discuss and evaluate the classic Fixed Relaying scheme and introduce two variants: Selection Relaying and Incremental Relaying. In Fixed Relaying, the relay always forwards the source information to the destination. This strategy results in order-2 diversity for AF, and order-1 diversity for DF. The loss of diversity order for DF is due to the requirement for error-free decoding at the relay, which limits the system performance to that of direct transmission between source and relay. In Selection Relaying, the forwarding of the information depends on the reliability of information received at the relay. In [4], the relay forwards the information only if the relay has correctly decoded the source information, i.e., the capacity of the source-relay channel exceeds the required rate $2R$ (the factor of 2 models
the bandwidth expansion required for relaying with orthogonal channels):

\[
\log_2(1 + 2|a_{s,r}|^2\text{SNR}) \geq 2R,
\]

(2.4)

where \( |a_{s,r}|^2 \) is the instantaneous source-relay channel power. If this condition is not satisfied, the source continues to transmit its information to its destination, using either a repetition or another code, and the relay terminal does nothing. Both the AF and DF versions of this protocol achieve order-2 diversity, as the probability of making an error in the source-relay channel is effectively eliminated.

Incremental Relaying builds on Selection Relaying in that it uses limited feedback from the destination, which sends an acknowledgement (ACK) to the source and relay, if the SNR of the received signal is sufficiently high. If the ACK is positive, the relay does nothing and transmission from the source continues. If the ACK is negative, the relay transmits the message from the source in an attempt to gain spatial diversity. Both the AF and DF versions of this protocol also achieve order-2 diversity.

In Sections 2.2.1 - 2.2.3, we discuss the remaining literature dealing with single-relay networks, classified according to contribution.

### 2.2.1 Forwarding Strategy

Forwarding encompasses a wide range of strategies developed for the relay channel: relay processing of the information from the source, the coding of the source and relay information, and the degree of broadcast and receive collision incurred by the relay and destination. In what follows, we discuss each relaying strategy in detail.
2.2. Single-Relay Networks

Processing of source information at the relay

- Amplify-and-Forward (AF) - Like analog repeaters, the relay simply amplifies the received signal and forward this amplified signal to the destination;

- Decode-and-Forward (DF) - Also referred to as Smart Repeaters. The relay decodes, re-encodes and transmits this re-encoded data,

- Compress-and-Forward (CF) - Also referred to as Estimate-and-Forward. As in AF, relay does not decode data from the source. Instead, the relay estimates and compresses (quantizes) the source data before transmitting to the destination.

- Demodulate-and-Forward(DemF) - The relay demodulates, but does not decode, the data from the source and forwards to the destination.

The performance of AF and DF was compared in [38] where the authors show that the relative performance of each scheme depends on the position of the relay: DF outperforms AF if the relay is closer to the source, and AF outperforms DF if the relay is closer to the destination. In general, DF requires a strong source-relay channel. The difference between the two schemes arises from their treatment of potential errors: DF, when implemented using Selection Relaying discussed in Section 2.2, discards erroneous information received by the relay. AF, on the other hand, suffers from noise amplification, with the possibility of contributing to errors made by the destination as a result of an active relay.

CF, on the other hand, can be used for all channels, and always results in a rate greater than that obtained with direct transmission. In comparison, DF only improves the rate when \( |a_{s,r}|^2 \geq |a_{s,d}|^2 \) and \( |a_{s,r}|^2 \geq |a_{r,d}|^2 \), where \( |a_{s,r}|^2 \), \( |a_{r,d}|^2 \) and \( |a_{s,d}|^2 \) are the source-relay, relay-destination and source-destination channel powers. As \( |a_{s,r}|^2 \) grows, however, the DF rate becomes greater than the CF rate [6].
2.2. Single-Relay Networks

From a practical view, it is still unclear which scheme is easier to implement. DF may offer higher complexity due to the decoding requirement at the relay, but affords more flexibility by allowing the relay to transmit a codeword different from the one received. AF, on the other hand, may prove problematic in terms of storing data in analogue format. It is possible that each system will implement a method most suitable to its specific requirements. In this thesis, we consider DF networks exclusively; therefore for the most part, we focus this survey on DF networks.

Demodulate-and-Forward was proposed for very low-complexity and inexpensive nodes such as those used for sensor networks. At the expense of performance, demodulating instead of decoding the source information at the relay significantly reduces the required computational complexity of the relay [7]. The technique is thus not in competition with DF, as they are used in systems with different constraints and trade-offs.

Coding Strategy and Code Design

From an information theoretic perspective, in DF mode the relay can do one of two things:

- Transmit the entire decoded message using repetition coding [4];

- Use an independent codebook to code the source message. The codeword transmitted by the relay helps the destination refine the information from the source, which it would otherwise be unable to decode [37].

The choice between the two strategies depends on the trade-offs defined by complexity and performance requirements. The decoding of the source and relay data for repetition coding is simple - the destination uses maximal ratio combining (MRC), while the decoding for relays employing independent codebooks is significantly more complex [39]. At the same time, due to inefficiencies of repetition coding, using independent codebooks results in higher rates.
than repetition coding [4, 6]. It is still unclear which way the trade-offs will sway and which method will become preferable in practice.

In practice, messages are encoded using channel codes and a significant body of work addresses the merging of cooperation and channel coding. The majority of the available literature on coded cooperation involves either coding to implement repetition coding, whereby the relay repeats only a fraction of the coded source bits [40–43], or coding for implementation of the independent codebooks at the relay, where the channel codes attempt to reach system capacity [44–46]. In addition, there is work on coded cooperation which reduces the complexity of relay decoding [7, 30, 47–49].

Additional complexity is created when considering the problem of relaying from a practical perspective by assuming channel coding and the possibility of errors at transmission rates even below the mutual information. In this case, it is not immediately clear when the relay should forward the information from the source and how potential error propagation will affect system performance. This problem is studied in [50, 51], where the authors evaluate system performance when the relay makes forwarding decisions based on received SNR.

**Degree of receive and broadcast collision**

The half-duplex DF and AF protocols suggested by Laneman et al in [4] are orthogonal: once the relay begins transmission, the source is silent. This design is simple to implement; the performance, however, is sub-optimal. It is possible, in fact, for the source to continue transmission on the same channel as the relay, with the destination decoding the source and relay transmission using successive decoding. This scenario is proposed by and evaluated, in terms of the DMT, by Azarian et al. [52].

The authors introduce the non-orthogonal Amplify-and-Forward diversity scheme (NAF) which differs from Laneman's AF only in that the source does not cease to transmit when the relay is relaying its data to the destination. Instead, it continues transmission of new data,
Table 2.1: Half-Duplex Transmission Protocols for Single Relay Networks

<table>
<thead>
<tr>
<th>Protocol</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Slot 1</td>
<td>$S \rightarrow R, D$</td>
<td>$S \rightarrow R, D$</td>
<td>$S \rightarrow R$</td>
</tr>
<tr>
<td>Time Slot 2</td>
<td>$S \rightarrow D, R \rightarrow D$</td>
<td>$R \rightarrow D$</td>
<td>$S \rightarrow D, R \rightarrow D$</td>
</tr>
</tbody>
</table>

on the same channel as the relay. The relay thus repeats only half the information arriving from the source. Not surprisingly, the authors find that the algorithm achieves a better diversity-multiplexing trade-off than the basic AF strategy, i.e., for a given multiplexing gain, a higher diversity order is achievable. This result is obtained, of course, at the price of complexity, as the destination must be able to separate the information arriving from the source and relay.

For DF, the authors introduce DDF, an adaptation of Laneman’s selection diversity DF scheme. In this scheme, the relay listens to the source until it collects enough information to correctly decode the message, at which point it stops listening and begins re-transmitting the message to the destination. This re-transmission occurs on the same channel as the source transmission and is performed using an independent Gaussian codebook. Similarly to NAF, the complexity of this scheme is very high, but the algorithm achieves a very high multiplexing-diversity tradeoff. For rates less than 0.5, the algorithm is optimal, as the trade-off is equivalent to that obtained by the ‘genie-aided’ diversity, where the relay receives the source information error-free.

The half-duplex orthogonal and non-orthogonal AF and DF protocols of [4,52] are unified in a common framework and evaluated, in terms of ergodic and outage capacity, by Nabar et al. [53]. The protocols are summarized in Table 2.2.1. $S$, $R$, and $D$ denote the source, relay and destination nodes, and $X \rightarrow Y$ indicates a transmission from node $X$ to node $Y$. $S \rightarrow R, D$ for example, indicates that the relay and destination both process the source transmission, while $S \rightarrow R$ indicates that the destination does not listen to the information
from the source, while the relay does.

Using broader performance metrics, the authors of [53] arrive at the same conclusions as [52]. Protocol I, corresponding to NAF and DDF in [52] and in the general non-orthogonal cooperation scenario considered in [6], achieves the best performance, resulting from maximum broadcasting and receive collision, i.e., the destination receives the maximum amount of information. Protocol III has the worst performance, since the destination does not decode the source information in the first time slot. Such a scenario, although clearly suboptimal, may be applicable to large networks where the destination may be busy in the first time slot transmitting its own data or listening to the data of other nodes. Protocol II corresponds to the orthogonal protocol of [4]. This protocol exhibits full broadcast, i.e., the source transmits to both the source and the relay, but avoids receive collision at the destination. The protocol is popular because of the simplicity of its implementation; it is the protocol we consider in this thesis. The trade-offs involved in broadcasting and receive collision are further discussed in [16].

The protocols discussed thus far have been evaluated using the DMT or outage probability. The achievable rate, however, is an important performance metric. For fading relay channels, Host-Madsen and Zheng develop the upper capacity bounds and achievable rate for DF and CF for full-duplex, half-duplex, synchronous and asynchronous transmissions [6]. The channel model is shown in Figure 2.1. We describe here the non-orthogonal capacity-achieving DF protocol and the resulting achievable rate. We will use this result, generalized to multiple relays, in Chapter 5.

The source divides the information it transmits to the destination into two independent messages, \( x_1 \) and \( x_2 \). In the first time slot of duration \( t \), the source transmits its first message \( x_1 \), with power \( P_s^{(1)} \). This message is received by the relay and destination as \( v_1 \) and \( y_1 \), respectively. In the second time-slot, of duration \( (1 - t) \), the source transmits its second message \( x_2 \), with power \( P_s^{(2)} \). In addition, the relay decodes the source message, re-encodes
into codeword $w_2$ using an independent code, and retransmits using power $P_r^{(2)}$. At the end of the transmission period, the destination first decodes the messages $x_2$ and $w_2$ using successive decoding, then decodes $x_1$ using the information carried in $x_2$ as side information.

The received signals at the relay and destination are given by

$$v_1 = \sqrt{P_s^{(1)}} h_{sr} x_1 + n_r, \quad y_1 = \sqrt{P_s^{(1)}} h_{sd} x_1 + n_{d_1}, \quad y_2 = \sqrt{P_s^{(2)}} h_{sd} x_2 + \sqrt{P_r^{(2)}} h_{rd} w_2 + n_{d_2},$$

and the achievable rate, per complex dimension, for fixed source and relay transmission powers and for unit noise power is given by

$$R = \max_{0 \leq \beta \leq 1} \min \{ C_1(\beta), C_2(\beta) \}, \quad (2.5)$$

where

$$C_1(\beta) = t \log_2 \left( 1 + |h_{sr}|^2 P_s^{(1)} \right) + (1 - t) \log_2 \left( 1 + (1 - \beta) |h_{sd}|^2 P_s^{(2)} \right),$$

$$C_2(\beta) = t \log_2 \left( 1 + |h_{sd}|^2 P_s^{(1)} \right) + (1 - t) \log_2 \left( 1 + |h_{sd}|^2 P_s^{(2)} + |h_{rd}|^2 P_r^{(2)} + 2 \beta P_s^{(2)} P_r^{(2)} \right). \quad (2.6)$$

The above rate assumes synchronous transmission, i.e., all nodes know the phases and magnitudes of all channels, and all nodes are perfectly synchronized. The parameter $\beta^2$ represents the correlation between the source and relay signals in the second time-slot. For asynchronous transmission, the rate can be obtained by setting $\beta = 0$. As discussed above, in this thesis we consider orthogonal DF, where the source remains silent during the second
time slot, and thus $P_s^{(2)} = 0$. In this case, $C_1$ and $C_2$ reduce to

$$C_1 = t \log_2 \left( 1 + |h_{sr}|^2 P_s^{(1)} \right)$$

(2.7)

$$C_2 = t \log_2 \left( 1 + |h_{sd}|^2 P_s^{(1)} \right) + (1 - t) \log_2 \left( 1 + |h_{rd}|^2 P_r^{(2)} \right).$$

(2.8)

### 2.2.2 Resource Allocation

Where some feedback is possible, resource allocation can have significant performance benefits by adapting transmission parameters to the channel. For the single-relay case, several works have dealt with various aspects of resource allocation, in terms of power and/or bandwidth and time. In the rest of this thesis, we refer time or bandwidth resource allocation as channel resource allocation, which precludes power adaptation. As mentioned in Chapter 1, thesis does not consider power adaptation. The problems and solutions presented in the literature differ in the assumptions and constraints imposed on the system. Using instantaneous or average channel conditions (depending on availability of channel state information), the power allocation problem can be solved under per-node power constraints or short- or long-term average power constraints, where the total power is shared between the source and relay node. The same is true for energy-constrained nodes such as mobile phones or sensor nodes.

The channel resource allocation problem, in terms of system bandwidth or time, results from the half-duplex constraint: given that the source and relay must share resources to transmit data to the destination, these resources can be allocated to the source and relay in an adaptive fashion according to channel conditions. Note that systems employing repetition coding cannot perform channel resource allocation, since repeating the source symbol necessitates the same amount of resources as those used by the source. Resource allocation is thus possible only in systems employing independent codebooks, as the codeword chosen
by the relay can originate from a codebook of different alphabet. The following is a brief summary of the main works dealing with resource allocation for the single-relay network.

Yao et al. determine the optimal power and time allocation for relayed transmissions specifically in the low-power regime [54]. Larsson and Cao present various strategies for allocating power and channel resources under energy constraints [55]. For the channel resource allocation problem, however, the authors consider selection combining only and do not address the scenario of joint decoding of the source and relay signals. The works in [6, 56, 57] address the problem of power and channel resource allocation under joint average power constraints. Optimal time and bandwidth allocation using instantaneous and average channel conditions is obtained using power control in [58]. Channel resource allocation under fixed power is developed in [59].

2.2.3 Partner Selection

We conclude this section on single-relay networks by discussing partner selection. The problem of partner selection occurs in networks with many nodes, where each node must select a cooperating partner, i.e., the nodes cluster themselves into groups of two. The available literature in partner selection schemes includes [40, 41, 60, 61]. These works assume that channels are changing quickly in time, and partner selection can only be made once, or rarely, based on average channel conditions. The authors of [40] also suggest distributed schemes where relays choose their partners based on instantaneous source-relay channels, but the crucial source-destination channels are not taken into account. These approaches are all suitable for systems with fast-varying channels, not allowing for an adaptation of cooperating partners according to channel conditions. Clearly, such an approach results in at most order-one diversity, and could significantly be improved by adapting the partner choice to the changing channel conditions. We consider this approach in Chapter 3 where we propose Selection
2.3 Multiple-Relay Networks

In this section, we generalize the single-relay network to multiple-relay networks, where every relay can potentially aid the source with its transmission to the destination. As in Section 2.2, we focus on DF networks. The use of multiple relays is clearly preferable to single-relay networks, as each relay may add to the diversity order of the system. Here, the challenges stem from the difficulty and costs, in terms of bandwidth or power, of organizing multiple relay transmissions.

We discuss the available literature in the area of multiple-relay networks in two parts: that dealing with parallel-relay networks, where relays do not communicate with one another, and the generalization of these networks, where relays may listen to and decode the information transmitted by other relays. This is the most general system model, but has received very little attention in the literature.

2.3.1 Parallel-Relay Networks

Consider a network where a source communicates with the destination with the help of (potentially) multiple relays employing DF. The existence of multiple relays raises the question: which relays should transmit, and how? The problem arises from the necessity of combining the relay transmissions at the source.

Clearly, the optimal strategy is for relays which have correctly decoded the source information to form a virtual antenna and to jointly transmit the source information using beamforming [8]. This approach, however, requires exact channel state information of the relay-destination channel, of both magnitude and phase, at each relay. Note that, although the power allocation between relays could be computed at some central node and only the
quantized value for each relay fed-back to that relay, the relay would still require very precise phase information. Because the feedback of precise channel phase information is considered difficult and impractical, the authors of [8] also suggested that each relay use a CDMA code to transmit the information on the same channel (frequency band or time-slot), and the information be collected coherently at the destination using a RAKE receiver. This approach was further studied in [10]. This implementation, however, is limited to CDMA systems; clearly, a more general approach to combining relay transmissions is necessary.

Two other approaches have been suggested in the literature: assigning each relay an orthogonal channel and decoding the multiple relay transmissions using MRC or, for a network with multiple-sources, coordinating all relay transmissions using a distributed space-time code (DSTC). These approaches are compared in [11], which we discuss below.

**Orthogonal transmissions and Distributed Space-Time Coding**

In [11], the authors analyze a parallel-relay network of $m$ nodes, where cooperative diversity is obtained with orthogonal transmissions or with distributed space-time codes. The authors consider a set $\mathcal{M}$ of $m$ source nodes, $s$, where each node in the set has information to transmit to its destination, $d(s)$, and can use the other nodes in $\mathcal{M}$ to potentially act as relays.

For both protocols, due to the half-duplex constraint, the transmission period is divided into two phases of equal duration. In phase one, each node transmits its own data to the destination, and each relay attempts to decode the data of all other $m - 1$ nodes. At the end of the phase one, each source has a set $\mathcal{D}(s)$, consisting of those relays which have correctly decoded its data. The protocols differ in phase two.

In the case of orthogonal transmissions, each node uses repetition coding to forward the data of those nodes for which it belongs to the decoding set, i.e., those nodes whose data it has correctly decoded. Each of the maximum $m - 1$ codewords is transmitted using an orthogonal time-slot. With $\mathcal{D}(s)$ the decoding set of $s$, the mutual information between
source $s$ and its destination $d(s)$ is

$$I_{rep} = \frac{1}{m} \log_2 \left( 1 + \text{SNR} |a_{s,d(s)}|^2 + \text{SNR} \sum_{r \in D(s)} |a_{r,d(s)}|^2 \right). \quad (2.9)$$

Note the $\frac{1}{m}$ pre-log factor arising from the penalty of orthogonal transmissions. This term is replaced by $1/2$ with DSTC. In phase two of the DSTC protocol, the decoding set for each node encodes its information using a distributed space-time code; each relay thus transmits its encoded information using one channel only. The space-time code design is not presented, though orthogonal codes are suggested. A relay encodes the source information using its column of the orthogonal space-time matrix - each row of the column is reserved for one of the cooperating terminals. Because the receiver can estimate the channels, it can determine which relays are involved in the second phase and thus adapt its decoding appropriately. In the second phase, all cooperating terminals transmit on the same channel, adding an extra row for each cooperating user in the orthogonal channel matrix. In a case where $N - n$ users do not transmit, possibly because they failed to decoded the source message correctly, the columns of the other users remain orthogonal and diversity order $n$ with $n$ cooperating terminals is possible, even if the code is designed for $N$ users, where $N > n$.

The authors derive the outage probability of the repetition and DSTC schemes as

$$\Pr[I_{rep} < R] \propto \left[ \frac{2^{mR} - 1}{\text{SNR}} \right]^m, \quad (2.10)$$

$$\Pr[I_{DSTC} < R] \propto \left[ \frac{2^{2R} - 1}{2\text{SNR/m}} \right]^m, \quad (2.11)$$

where, for the sake clarity, we have omitted the dependence on average channels parameters. The decrease in the exponent from $m$ with repetition to 2 with DSTC is derived from the inherent bandwidth efficiency of DSTC, which employs two channels instead of $m$ to relay
the data from one source. The factor of \(2/m\) appearing in the denominator of the DSTC expression stems for the energy constraint, and the need for the node to divide its power between the relays for which it is transmitting. We will see in Chapter 3 that this power-splitting can be improved through Selection Cooperation.

Both strategies thus provide diversity order \(m\), i.e., the number of users cooperating, not just the number or users decoding. Repetition codes, however, are bandwidth inefficient, and the SNR loss due to bandwidth inefficiency is exponential in \(m\). DSTC thus significantly outperform orthogonal repetitions.

Since DSTC provide significant gains, their performance in networks has been analyzed in several works \([12–15]\) under various scenarios and using different performance metrics. DSTC, however, requires accurate synchronization between all nodes, a task difficult to accomplish. More discussion about implementation issues of DSTC is provided in Chapter 3.

**Selection Cooperation and Opportunistic Relaying**

In this thesis, we propose Selection Cooperation as an alternative to DSTC in multi-source DF parallel-relay networks. A similar approach is suggested independently by Bletsas et al. \([62, 63]\). The authors consider a selection scheme as an alternative to DSTC and argue for its simplicity of implementation, but consider only a single source-destination pair, i.e., a network with a single source with multiple potential relays. The authors assume that the simplicity of the scheme comes at the price of a performance loss as compared to MRC-based schemes. As implemented, the scheme of \([62]\) results in a non-zero probability of two relays being selected for the same source and the analysis presented focuses on quantifying this probability. In \([63]\), on the other hand, the selection is performed in a manner similar to selection proposed in this thesis; the analysis, however, apart from not considering a network setting, assumes no connection between a source and its destination. Although this assumption significantly simplifies the analysis (complicated only by the necessity of
including the potentially strong source-destination channel), it does not provide a framework e.g., for improving reliability only as required (wherein nodes relay only when the source-destination channel is in outage [4, 56]). In comparison, in our framework one could first select a relay and then decide exactly how and when to relay, as in [56].

Inspired by the work in [64], selection has also been proposed by Zhao et al. for AF networks in [29]. The authors analyzed selection as a special case of beamforming with limited feedback of \(\log_2(m - 1)\) bits of feedback and beamforming vectors as columns of an \((m - 1) \times (m - 1)\) identity matrix. In a traditional MIMO system with co-located antennas, the performance achievable with such an identity matrix is identical to any unitary matrix, i.e., any rotation of the codebook of beamforming vectors results in the same performance. In AF schemes, on the other hand, this does not hold true.

Unsurprisingly, selection suffers significant performance loss in terms of outage probability and bit error rate as compared to optimal beamforming with unlimited feedback. However, optimal beamforming requires knowledge of the coherent channel values at the transmitter. This information is usually fed-back and the relays use one beamforming vector from a codebook. The authors show that this codebook design is extremely complicated, and random codebooks are the only practical solution. With a similar number of feedback bits, selection outperforms beamforming with random codebooks in terms of outage probability and diversity order. Although it is possible to improve the performance of the random codebooks beamforming by adding more feedback, selection is optimal under the constraint of \(\log_2(m - 1)\) bits of feedback, i.e., amongst the class of unitary \((m - 1) \times (m - 1)\) matrices, the identity matrix is optimal. Because selection exhibits robust performance under few bits of feedback and, unlike all other beamforming schemes, has the significant benefit of not requiring synchronization, the authors argue that selection is the most attractive currently available beamforming scheme for distributed networks.
2.3. Multiple-Relay Networks

**Resource Allocation for the Parallel-Relay Network**

The orthogonal transmission scheme, as presented in [11], is a special case of a resource allocation problem where each orthogonal transmission is allocated equal power and time-slot duration. This approach could be generalized, however, with each relay assigned a power level and/or channel resource, either in terms of time or bandwidth. Although resource allocation in this case requires feedback to the relays, this feedback is limited to channel magnitudes only, and is significantly simpler to implement than that required for beamforming with phase information. This problem has been considered by several works for the parallel-relay channels. Ibrahimi and Liang develop the optimal power allocation for the multi-relay cooperative OFDMA AF system [65]. By maximizing the channel mutual information, Anghel et al. find the optimal power allocation for multiple parallel relays in AF networks [66, 67]. A more general solution is given in [68] where the authors give the optimal power and channel resource allocation for a parallel-relay network with individual power constraints on the nodes.

### 2.3.2 Generalized-Relay Networks

From an information theoretic perspective, most literature relating to generalized relay networks seeks to determine capacity assuming full-duplex relays [69–74]. For half-duplex relays, achievable rates are derived for random access networks [75]. In an interesting work, in [76] the authors derive the DMT for generalized-relay AF networks [76].

A theme that runs throughout this thesis is the notion that selection, i.e., a specific implementation of cooperation, can be interpreted as a version of routing according to instantaneous channel conditions. This observation suggests that if routing is channel-adaptive, it will benefit from the same gains as cooperation. The same conclusion is being reached from various directions in the research community, where there is increasing consensus that
a cross-layer, channel-adaptive approach to routing is necessary for the continued evolution of networks. This is particularly true for static networks where channels change slowly, and such channel adaptation is possible. In [77], the authors approach this problem in an energy-limited, bandwidth-unlimited scenario. Similarly, Haenggi [78] analyzes various routing strategies under Rayleigh fading conditions and energy constraints. He argues that when various practical network and physical layer issues are considered, long hops are often preferable to short hops. Similarly, in [79] Chen et al. propose suboptimal algorithms to finding routes that maximize spectral efficiency in a multihop.

A subject more related to this thesis is the problem of combining routing with cooperative diversity. Interest in this problem has recently started gaining momentum. Boyer et al. find the optimal power allocation that minimize the BER in a multihop cooperative AF and DF network [9]. In a similar work, Sadek et al. derive exact SER expressions in generalized-relay networks [17]. Yang and Host-Madsen consider routing and power allocation in asynchronous networks [18]. In [19] the authors evaluate the BER performance of a joint multihop cooperative network. In [20–28] system performance is optimized by minimizing energy consumption or maximizing throughput. In all cases, the authors either assume full-duplex nodes, a bandwidth-unconstrained system, or the availability of exact channel knowledge at the transmitting cooperative nodes that allows them to precode data and not suffer bandwidth expansion.

Although not directly related to this thesis, we note that in random access networks, a significant body of work deals with channel-adaptive routing [80–84] and resource allocation [85–92].

In this thesis, we approach the problem of routing and cooperation in dedicated-access, bandwidth-constrained networks where each hop carries a penalty in terms of bandwidth. For a simple network with no resource allocation, this problem is considered in Chapter 4. We note that this work has been used as a basis for further study [93–95]. For more advanced
networks which permit channel resource allocation, the problem is addressed in Chapters 5 and 6.
Chapter 3

Selection Cooperation

We begin this thesis with considering a parallel-relay network with *multiple sources*, where each node has data to transmit and is also available to act as a relay for other nodes. The data traverses only one hop and the relays are precluded from communicating with one another. This model is shown in Figure 3.1. In this context, we propose Selection Cooperation, whereby the source communicates with the destination using a *single* relay. We show that in slow-fading channels, for most network sizes and rates, and with some limited overhead, selection outperforms the distributed space-time scheme (DSTC) of [11] in terms of outage probability. This advantage arises from a more efficient use of power. Throughout this chapter we compare Selection Cooperation to DSTC since, with multiple simultaneous transmissions, the DSTC scheme in [11] makes far more efficient use of resources compared to the other published approaches, such as maximal ratio combining (MRC) based schemes.

In a distributed network with multiple simultaneous transmissions, our scheme places relay selection at the core of system design. The importance of relay selection derives from a power constraint: the power available at a relay node depends on the number of sources supported by that relay, and the relay selection for one source may impact the choice of another. A large portion of this chapter is thus devoted to relay selection: we analyze three...
Figure 3.1: Parallel-Relay Network where relay nodes communicate only with the source and destination, but not to each other.

different relay selection schemes based on varying degrees of centralization and tolerance for complexity, and show that even the simplest of these schemes outperforms DSTC for most network sizes. This chapter is structured as follows. Section 3.1 presents and analyzes the selection cooperation algorithm in a system with a single source. Section 3.2 extends this work to multiple transmissions, discussing three possible implementations of the scheme in a distributed network with multiple sources. Detailed derivations are deferred to appendices. Section 3.3 concludes the chapter.

We begin with the simplified scenario of a single source network.

3.1 Single Source-Destination Pair

3.1.1 System Model and Selection Cooperation

To introduce Selection Cooperation, consider a single source node $s$ communicating with a destination $d$ with the help of $N$ potential relays, $r_1 \ldots r_N$. The relays satisfy a half-duplex constraint. The channel $a_{i,j}$ between nodes $i$ and $j$ is modeled as flat and slowly-fading Rayleigh with variance $1/\lambda_{i,j}$, i.e., $|a_{i,j}|^2 \sim \lambda_{i,j} \exp \left[ -\lambda_{i,j} |a_{i,j}|^2 \right]$. Because nodes are static, inter-node channels change very slowly. Each node transmits with constant and maximum
3.1. Single Source-Destination Pair

power $P$. This communication between the source and destination targets an end-to-end data rate of $R$. Due to the half-duplex constraint, communication is performed in two time slots. The source distributes its data in the first time slot, while the destination and each of the $N$ relays decode this information. The decoding set $\mathcal{D}(s)$ for the source is the set of relays that decoded the information correctly, i.e., a node $r_k \in \mathcal{D}(s)$ if the source-relay channel has a capacity above the required rate $R$:

$$\frac{1}{2} \log_2 \left( 1 + |a_{s,r_k}|^2 \frac{P}{N_0 W} \right) = \frac{1}{2} \log_2 \left( 1 + |a_{s,r_k}|^2 \text{SNR} \right) \geq R, \quad (3.1)$$

where the factor of 1/2 models the two time slots required for relaying given the half-duplex constraint, $N_0$ is the noise power spectral density and $\text{SNR} = P/N_0 W$ is the non-faded signal-to-noise ratio at the receiving node. The approach of allowing only a subset of all $N$ nodes to relay is referred to in [11] as “selection relaying” and ensures full diversity order in decode-and-forward schemes. In [11] the nodes in the decoding set either each forward the source data in a round robin fashion (requiring $N + 1$ time slots and allowing for MRC) or use a DSTC (requiring only two time slots).

Our scheme differs from selection relaying in that the destination chooses a single relay with the best instantaneous relay-destination channel to forward the information to the source in the second time slot. Relaying is thus performed on orthogonal channels but, as with DSTC, because only one relay is chosen for each source, the pre-log factor is only 1/2. Choosing one of $N$ relays requires $\log_2(N)$ bits of feedback from the destination.

We should note here the potential confusion between the terms “selection relaying” and “Selection Cooperation”. In selection relaying all nodes in $\mathcal{D}(s)$ forward data for the source, while Selection Cooperation restricts relaying to only one node from this decoding set.

1Note that the mutual information expression assumes Gaussian codebooks. For our purposes, constant power $P$ implies constant variance of transmitted symbols.
3.1.2 Performance Analysis and Simulation Results

This chapter generally uses outage probability as the figure of merit to compare various cooperation schemes. In this section, we evaluate the outage probability of Selection Cooperation in the high and low SNR regimes. High-SNR approximations are very common and are, in fact, the main tool for evaluation.

Analysis for High SNR

If node $r_j$ is selected as the relay, the destination combines the transmissions from source and relay. As in [4], this mutual information is given by

$$I_{\text{sel}} = \frac{1}{2} \log_2 \left( 1 + \text{SNR} |a_{s,d}(s)|^2 + \text{SNR} |a_{r_j,d}(s)|^2 \right). \quad (3.2)$$

**Proposition 1:** For a single source-destination pair, with $N$ potential nodes, the outage probability of the selection scheme in the high-SNR regime, is

$$P_{\text{out}} \simeq \left[ \frac{(2^{2R} - 1)}{\text{SNR}} \right]^{N+1} \lambda_{s,d} \sum_{|D(s)|} \frac{1}{|D(s)|} + 1 \times \prod_{r_i \in D(s)} \lambda_{r_i,d} \prod_{r_i \notin D(s)} \lambda_{s,r_i}, \quad (3.3)$$

where $1/\lambda_{i,j}$ is the variance of the channel between node $i$ and $j$.

**Proof:** The proof is presented in Appendix A.

Note that, as expected, Selection Cooperation provides the full diversity order of $N + 1$. This formulation is similar to that presented in [63], with the important difference that here, the source-destination channel is taken into account. This significantly complicates the analysis and we focus on the high-SNR regime. The authors of [63] neglect the source-destination channel (decreasing the diversity order by one), and present the exact outage probability.

In Figure 3.2, this high-SNR approximation is verified for increasing numbers of relays.
3.1. Single Source-Destination Pair

Figure 3.2: Outage probability for Selection Cooperation with a single source-destination pair. $R = 1$ bps/Hz, $\lambda_{i,j} = 1$, $N + 1 = 3, 4, \ldots, 6$

with $\lambda_{i,j} = 1$ and $R = 1$ bps/Hz. The approximation in (3.3) is very good for SNR levels above 15 dB.

Analysis in the Low to Medium SNR Regime

The approximation in (3.3) may not hold in the low and medium SNR regime, in systems with a high diversity order or where rate is increased as function of SNR. We thus develop the outage probability approximations for Selection Cooperation for arbitrary SNR. The approximations are based on the Taylor series and third-order approximations of the exponential function. The closed-form expressions can be used in various stages of system design: to predict or verify outage probabilities, for example, or to maximize rates given a target outage probability.

Proposition 2: In the low- and medium-SNR, for a single source-destination pair with $N$
potential nodes, the outage probability of the selection scheme conditioned on the decoding set $\mathcal{D}(s)$ can be approximated as

$$\Pr[I_{sel} < R|\mathcal{D}(s)] \approx \lambda_{s,d} \left[ \prod_{r=1}^{[\mathcal{D}(s)]} f_r e^{-\lambda_{r,d} b} \right] \sum_{i=0}^{[\mathcal{D}(s)]} \binom{[\mathcal{D}(s)]}{i} b^{3([\mathcal{D}(s)]-i)} (-1)^i$$

where

$$f_r = \frac{-7}{2L\lambda^4_{r,d}(7b^3L^3 + 2L^6 + 14b^6)} \left[ \frac{\lambda^4_{r,d}Le^{\lambda_{r,d} b}(-4b^3 + L^3)}{L^{3i+1}} + \frac{4\lambda^3_{r,d}(e^{\lambda_{r,d} b^3} - e^{\lambda_{r,d} L} L^3 - b^3)}{3i+2} + \frac{12\lambda^2_{r,d} L^2 e^{\lambda_{r,d} L}}{3i+1} - 24\lambda_{r,d} Le^{\lambda_{r,d} L} + 24 e^{\lambda_{r,d} L} - 24 \right]$$

and

$$b = \frac{2^{2R} - 1}{\text{SNR}}, \quad L = \min(y_0, b), \quad y_0 = \frac{(1 + \sqrt{2})^2 - 1 + (1 + \sqrt{2})^\frac{1}{3}}{\lambda_{s,d}(1 + \sqrt{2})^\frac{1}{2}}.$$

**Proof:** The proof is presented in Appendix B.

We now we test the low-SNR approximation by simulating the outage probability of Selection Cooperation in two scenarios. We compare the approximations developed here with the high-SNR approximations in (3.3). To obtain the high-SNR approximation, we still sum over the exact expression for the probability of a decoding set, using (B.2), but replace (3.4) with

$$\Pr[I_{sel} < R|\mathcal{D}(s)] = b^{N+1} \lambda_{s,d} \prod_{r=1}^{[\mathcal{D}(s)]} \lambda_{r,d}.$$
3.1. Single Source-Destination Pair

Figure 3.3: Outage Probabilities vs. SNR. $R = 1$ bps/Hz, $N = 4$, $\lambda_{r,d} = [1, 1, 1, 1]$, $\lambda_{s,r} = [1, 1, 1, 1]$, and $\lambda_{s,d} = 1$.

Figure 3.4: Outage Probabilities vs. SNR. $R = 1$ bps/Hz, $N = 6$, $\lambda_{r,d} = [1, 2, 3, 4, 5, 6]$, $\lambda_{s,r} = [6, 5, 4, 3, 2, 1]$, $\lambda_{s,d} = 5$ and $\lambda_{s,d} = 0.5$. 
rather than as a direct comparison. The simulated results are presented in Figures 3.3 and 3.4. Both plots show the simulated and analytical outage probability for a Selection Cooperation system with 6 relays and rate $R = 1 \text{ bps/Hz}$. Although this rate is kept constant, we note that for the purpose of the approximation, increasing the rate has the same effect as decreasing the SNR (decreasing $b$); it is thus sufficient to evaluate the approximations by considering the low-SNR regime.

Figure 3.3 plots the outage probabilities for the case where all inter-node channels have unit power. With the exception of using (3.4), this plot is similar to the one given in [64]. The approximations developed in this section are accurate for all SNR values, while the high-SNR approximations are accurate in the range of outage probabilities $P_{\text{out}} = 10^{-2} - 10^{-3}$.

As shown in Figure 3.4, the importance of the approximation in (3.4) is seen clearly in a more realistic setting. In this figure, the inter-channel quality is set to $\lambda_{r,d} = [1, 2, 3, 4, 5, 6]$, and $\lambda_{s,r} = [6, 5, 4, 3, 2, 1]$. We provide the results both for a strong and weak source-destination channel, i.e., $\lambda_{s,d} = 0.5$ and $\lambda_{s,d} = 5$, respectively. In each one, the approximations developed in this section track the simulated plots at all SNRs, and crucially, at the practical values of outage probability, $P_{\text{out}} = 10^{-2} - 10^{-3}$. The high-SNR approximations, however, only begin to converge at much lower outage probabilities.

In this section, we have introduced the idea of node selection in the context of a single-relay network, and evaluated its performance in the high- and low-SNR regimes. In the following section, the selection scheme is implemented in a network with multiple source-destination pairs. The performance of Selection Cooperation is compared to the DSTC scheme of [11], chosen as one of the few efficient schemes that specifically analyzes networks with simultaneous multiple flows.
3.2 Selection in Multi-Source Networks

3.2.1 System Model

In this section, we extend the concept of Selection Cooperation to network settings, largely using the notation of [11]. The network comprises a set $\mathcal{M}$ of $N$ nodes. Each node $s \in \mathcal{M}$ has information to transmit to its own destination, $d(s) \notin \mathcal{M}$, and acts as a potential relay for other nodes in $\mathcal{M}$. While we use the notation $s$ for a source node and $r$ for a relay node, we emphasize that every node in $\mathcal{M}$ is a source node and is, potentially, also a relay node. The channel between any two nodes is assumed independent of all other channels. This model is appropriate for networks where each node may have its own destination.

Each node transmits to its destination using an orthogonal channel. To simplify the exposition, we assume these channels are orthogonal in frequency, though theoretically the channels can be orthogonal in any dimension. A node relaying for a specific source-destination pair uses the frequency band of that assigned to that pair.

Because the network comprises multiple sources, each node can potentially relay for several other nodes. This raises the question of relay selection and power allocation and motivates the various selection schemes discussed below in Section 3.2.2. Each node has a power constraint of $P$ Watts. In DSTC, every node expects to relay for all other $N-1$ nodes and thus uses $2P/N$ per source in both phases. In our case, in the first phase, each source sends its data using full power $P$. In the second phase, each relay divides its power evenly between the source nodes it is supporting. A relay node supporting $n$ source nodes will thus use $P/n$ Watts for each source. Note that a relay node does not know \textit{a priori} how many nodes it will relay for and does not know the channel to the destination for these sources; it thus cannot pre-compute a better power distribution. Clearly with additional feedback to the relays, a better power distribution or another form of resource allocation would be
3.2. Selection in Multi-Source Networks

possible.

We consider both centralized and decentralized versions of the network. A centralized network is governed by a central unit (CU) with knowledge of all network parameters. A CU makes all assignment decisions. A CU could also be used to optimize resource allocations, but this is not investigated until Chapter 5. In the absence of a CU, the network is decentralized and decisions are made locally by the nodes, with limited information regarding the rest of the network.

3.2.2 Selection Cooperation Schemes

The concept of selection in this multi-source network is identical to that presented in Section 3.1. For each node, transmission is divided into two phases of equal duration. In phase one, all nodes use their orthogonal channels to transmit information to their respective destinations, and each node decodes the information from the other \( N - 1 \) sources. Each node determines if it has decoded the information correctly. If node \( s_j \) has decoded the information from source \( s_i \) correctly, it declares itself as a member of the decoding set \( D(s_i) \) of nodes eligible to relay for node \( s_i \). Such a decoding set, \( D(s_i) \), is formed for each source node \( s_i \in \mathcal{M} \). In phase two, for each source \( s_i \), a relay is chosen from its decoding set \( D(s_i) \), and each relay forwards the information using the orthogonal channel of that source. Note that we assume that each node can receive and transmit simultaneously in multiple frequency bands. The activity of a node \( s_i \) can thus be summarized as follows: in phase one, it transmits its information and decodes the information of the other \( N - 1 \) nodes; in phase two, it forwards the information of those nodes for which it was chosen as a relay. The summary of the protocol along with examples of channel allocations are shown in Figures 3.5 and 3.6. In Figure 3.5, relays are uniformly distributed between the source-destination pairs, i.e., each relay transmits for only one source. In Figure 3.6, one relay is chosen to transmit
Figure 3.5: Selection in a network with $N$ users. Example channel allocations for the best-case scenario where relay nodes are distributed uniformly across source-destination pairs. Here, node 1 relays for source-destination pair $N$ and node $k$ for source-destination pair $k-1$, $k = 2 \ldots N$.

for other nodes in the network, and must split its power between all transmissions. This “power-splitting” problem motivates a search for effective relay selection schemes. In this section we present three relay selection schemes based on varying degrees of centralization and tolerance for numerical complexity. While in Section 3.1 the relay was selected as the one with the best instantaneous channel to the destination, in a network setting the per-node power constraint motivates search for a more sophisticated scheme. For example, suppose two source nodes, $s_1$ and $s_2$, are assigned to the same relay $r$ with the best instantaneous channel to both $d(s_1)$ and $d(s_2)$. The power available at node $r$ for each source is $P/2$. However, performance could potentially be improved by assigning one of the source nodes to a different “free” relay node with available power $P$. The problem is thus to assign relays to source nodes to minimize some figure of merit which depends on channel conditions as well as available power at the relays. However, as we shall see, the simple assignment scheme of
3.2. Selection in Multi-Source Networks

Figure 3.6: Selection in a network with $N$ users. Example channel allocations for the worst-case scenario where one node, here node 2, relays for all other nodes. In this example, node 1 relays for node 2.

Section 3.1, extended to multiple sources, remains an effective tradeoff between complexity and performance.

Optimal Relay Assignment

We state here the optimal relay choice in a network setting. The mutual information between source $s_i$ and destination $d(s_i)$, if using $r_j$ as the relay is:

$$I_{s_i,d(s_i);r_j} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}|a_{s_i,d(s_i)}|^2 + \frac{\text{SNR}}{N_j}|a_{r_j,d(s_i)}|^2 \right),$$

(3.7)

where $N_j$ is the number of sources that choose node $r_j$ as their relay. One optimal approach, in max-min sense, calculates the mutual information between source and destination of all $N$ transmissions for all possible relay assignments, and picks the relay assignment which
3.2. Selection in Multi-Source Networks

maximizes the minimum mutual information of these $N$ transmissions:

$$\{r(s_1), \ldots, r(s_N)\} = \arg\max_{\forall i_1 \in \mathcal{D}(s_1), \ldots, i_N \in \mathcal{D}(s_N)} \min \{I_{s_1d(s_1); r_{i_1}}, \ldots, I_{s_Nd(s_N); r_{i_N}}\}, \quad (3.8)$$

where, for example, $I_{s_1d(s_1); r_{i_1}}$ denotes the mutual information between source $s_1$ and its destination $d(s_1)$, with node $r_{i_1}$, taken from $\mathcal{D}(s_1)$, used as a relay.

This optimal scheme requires a CU with global knowledge of all channels to make $N^{N-1}$ comparisons and choose the best one in max-min sense. Hence, though optimal, this scheme is clearly impractical.

**Sequential Relay Selection**

The optimal algorithm described above can be simplified considerably by performing this search sequentially for each source node in $\mathcal{M}$. This sub-optimal algorithm thus works as follows. The relay of the first node, $r(s_1)$, is chosen from $\mathcal{D}(s_1)$, the decoding set of $s_1$, independently of the other sources. This relay, $r(s_1)$, is the node with the highest channel power to the destination of $s_1$, $d(s_1)$. For the second source node $s_2$, two potential relaying nodes from $\mathcal{D}(s_2)$ are picked for consideration: nodes $r_j$ and $r_k$, with the best and second-best channels to the destination, respectively. If $r_j$ is not already used as a relay for $s_1$, i.e., $r_j \neq r_{s_1}$, it is automatically chosen as the relay for $s_2$. If $r_j$ is already relaying, however, the CU decides between $r_j$ and $r_k$ by considering that $r_j$ would need to halve its power to accommodate both source nodes $s_1$ and $s_2$. The CU thus compares $I_{s_2d(s_2); r_k}$ to
\[ \min \{ I_{s_1 d(s_1); r_j}, I_{s_2 d(s_2); r_k} \}, \]

where

\[ I_{s_2 d(s_2); r_k} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}|a_{s_2, d(s_2)}|^2 + \text{SNR}|a_{r_k, d(s_2)}|^2 \right), \] \hspace{1em} (3.9)

\[ I_{s_1 d(s_1); r_j} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}|a_{s_1, d(s_1)}|^2 + \frac{\text{SNR}}{2}|a_{r_j, d(s_1)}|^2 \right), \] \hspace{1em} (3.10)

\[ I_{s_2 d(s_2); r_j} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}|a_{s_2, d(s_2)}|^2 + \frac{\text{SNR}}{2}|a_{r_j, d(s_2)}|^2 \right). \] \hspace{1em} (3.11)

If \( I_{s_2 d(s_2); r_k} \) is the larger value, \( r_k \) is chosen as the relay for \( s_2 \); otherwise, \( r_j \) is chosen.

This process repeats until each source has been assigned a relay, with potentially one more relay added to the comparison for each source node considered. For each source node \( s_i \), the CU begins with the relay \( r_j \) with the best channel to \( d(s_i) \). If that relay is free, it is automatically assigned to \( s_i \). Otherwise, the CU calculates the mutual information for \( s_i \) and for all \( N_j \) source nodes already supported by \( r_j \), using \( P/(N_j + 1) \) as the available relay power. The minimum of all the \( N_j + 1 \) values of mutual information is associated with relaying node \( r_j \). The CU repeats this process for the relay with the next best channel, until it finds a relay that is not claimed by any source node. It then assigns the source \( s_i \) to the relay node with the highest relay-destination mutual information.

The iterative scheme is significantly simpler than the exhaustive search of the optimum scheme, but the resulting complexity of \( O(N!) \) is still significant. Also, like the optimum scheme, it requires a centralized node with knowledge of all inter-node channels. Therefore, while more efficient than the optimum scheme, this scheme is also likely to be impractical.

**Distributed Relay Assignment**

The simple selection scheme extends the selection approach in Section 3.1 to the network setting considered here. The destination of \( s_i, d(s_i) \) picks as its relay the node with the
highest instantaneous relay-destination channel power, \( r(s_i) \):

\[
r(s_i) = \arg \max_{r_k \in \mathcal{D}(s_i)} \{|a_{r_k}|^2\}; \quad k = 1 \ldots |\mathcal{D}(s_i)|, \tag{3.12}
\]

i.e., each relay is picked independent of the other source-destination pairs. The only penalty is that a node relaying for \( N_j \) sources uses power \( P/N_j \) for each forwarding link. As the numerical results given below show, this scheme is extremely effective, achieving near optimum performance for small network sizes. The rest of this section is largely focused on this scheme.

This simple scheme is of \( O(N^2) \) complexity in the worst case, where each of the \( N \) destinations must make \( N - 1 \) comparisons to find a maximum out of (at most) \( N - 1 \) channels. The scheme can also be implemented in fully decentralized manner, as each destination picks its relay independently of all other nodes.

Analytical results for the optimal and sub-optimal selection schemes are difficult to obtain. Figure 3.7 compares the outage probability of all three selection schemes via simulation. Not surprisingly, performance improves with increasing system intelligence. Finally, the difference in performance between the three schemes also increases with increasing network sizes. This is to be expected, since with more nodes there is a higher potential for power splitting, and the optimal and sub-optimal approaches reduce this problem. Note, however, that even for a network size of \( N = 5 \), the performance loss of the simplest scheme is not very large. This is clarified by (C.2) in Appendix C, which gives the probability that a relay will be chosen by \( n \) nodes other than the source node for which it was already chosen. The probabilities of \( n = 0 \) or 1 (the relay relaying for one and two nodes) is similar and significant, but falls off quickly with increasing \( n \). Most of the time, therefore, a relay will transmit either with full power or half its power. For smaller networks, the power-splitting problem of simple selection is thus not very significant, and the performance of the simple
3.2. Selection in Multi-Source Networks

Figure 3.7: Outage probabilities of simple, sub-optimal and optimal selection $R = 1$ bps/Hz, $\lambda_{i,j} = 1$, $N = 3, 5$

scheme closely tracks that of the optimal scheme. Given the complexity of the selection process, it proves impossible to simulate larger network sizes for the optimal and iterative schemes; it is thus difficult to make definitive assertions about the relative performance of the optimal, suboptimal and simple schemes. Our conclusion is that the simple selection scheme is a very good and practical choice, but could nonetheless be improved with further optimization. Due to its simplicity, however, the rest of this chapter focuses on this scheme.

It is worth noting that recently the authors of [96] have formulated the joint power and relay allocation problem. The optimization is solved approximately by eliminating the integer programming component and solving the resulting convex optimization problem.
3.2.3 Performance Analysis

Outage Probability

This section presents the analytical and simulated outage probability for the simple selection scheme described above, including a comparison to the outage probability of the DSTC protocol of [11]. The details of the derivations are deferred to the appendix.

The outage probability is evaluated for a particular source-destination pair, \( s_j - d(s_j) \), with arbitrary average channels both between the destination and all other relay nodes, i.e., \( r_i - d(s_j) \), \( r_i, s_j \in \mathcal{M} \), as well as between all the relay nodes themselves, i.e., \( r_i - r_k \), \( r_i, r_k \in \mathcal{M} \). To simplify the analysis, we assume equal average channels between all other destinations and relay nodes, i.e., \( r_i - d(s_k) \), \( r_i, s_k \in \mathcal{M} \), \( r_i, s_k \neq r_j \). (Recall that denoting a node \( s \) or \( r \) is done purely to highlight its purpose in the second phase, but that in fact \( r_k = s_k \) denote the same node. Furthermore, by definition, the destination nodes are not in the set \( \mathcal{M} \); thus \( r_k \neq d(s_i) \), \( \forall r_k, s_i \in \mathcal{M} \).

**Proposition 3:** In this scenario, the high-SNR outage probability for a specific source-destination pair using simple selection is

\[
\Pr[I_{\text{simple-sel}} < R] \approx \left[ \frac{2^R - 1}{\text{SNR}} \right]^N \lambda_{s,d} \sum_{|\mathcal{D}(s)|} \frac{1}{|\mathcal{D}(s)| + 1} \prod_{r_i \in \mathcal{D}(s)} \lambda_{r_i,d} \prod_{r_i \not\in \mathcal{D}(s)} \lambda_{s,r_i} \sum_{n=0}^{N-2} K_m(n + 1)^{|\mathcal{D}(s)|},
\]

where

\[
K_m = \binom{N-2}{n} \left[ \frac{1}{N-2} \right]^n \left[ \frac{N-3}{N-2} \right]^{N-2-n}.
\]

**Proof:** See Appendix C.

Following the approach in [11], we bound (3.13) to eliminate its its dependence on the
3.2. Selection in Multi-Source Networks

Figure 3.8: Simple Selection Cooperation in a network. $R = 1 \text{ bps/Hz}, \lambda_{i,j} = 1, N = 3, 4, 5$

average channels, $\lambda_{i,j}$.

\[
\left[ \frac{2^{2R} - 1}{\text{SNR}/\bar{\lambda}} \right] N \sum_{|\mathcal{D}(s)|} \frac{1}{|\mathcal{D}(s)|} + 1 \sum_{n=0}^{N-2} K_m(n + 1)^{|\mathcal{D}(s)|} \leq \text{Pr}[I_{\text{simple--sel}} < R] \leq \left[ \frac{2^{2R} - 1}{\text{SNR}/\bar{\lambda}} \right] N \sum_{|\mathcal{D}(s)|} \frac{1}{|\mathcal{D}(s)|} + 1 \sum_{n=0}^{N-2} K_m(n + 1)^{|\mathcal{D}(s)|}, (3.15)
\]

where $\lambda_r = \min \{\lambda_{r,d(s)}, \lambda_{s,r}\}$, $\bar{\lambda}_r = \max \{\lambda_{r,d(s)}, \lambda_{s,r}\}$, and $\bar{\lambda}$ is the geometric mean of all the $\lambda_r$ and $\lambda_{s,d(s)}$, and $\bar{\lambda}$ is the geometric mean of all the $\bar{\lambda}_r$ and $\lambda_{s,d(s)}$. As discussed in Appendix C, the upper bound assumes that the relay chosen in the communication between a source and its destination is in the decoding set of all other source nodes. This is a worst-case assumption, since ideally this relay would not be available to relay for other nodes. Clearly, from (3.15), Selection Cooperation scheme achieves full diversity order of $N$ for each source-destination pair.

The high SNR approximation in (3.13) is verified in Figure 3.8, which compares the analytical and simulated results for the simple selection scheme. The analytical results are obtained by calculating (3.13) for increasing network sizes with equal average channels with $\lambda = 1$ and $R = 1 \text{ bps/Hz}$. The approximation appears valid at SNR levels above 12dB.

An analytical comparison of the DSTC and simple selection scheme is complicated by the difficulty in writing (3.13) with $N$ sources in closed-form. The comparisons are thus presented numerically in Figures 3.9 and 3.10, in both cases for channels with $\lambda = 1$. Figure 3.9 demonstrates the outage probability of both schemes for $R = 1 \text{ bps/Hz}$ by calculating (3.13) and comparing to the outage of DSTC. Note that even the simple scheme always outperforms the DSTC scheme, and that the improvement increases for increasing $N$. This is due to the
3.2. Selection in Multi-Source Networks

Figure 3.9: Outage probabilities of DSTC and simple selection combining. $R = 1$ bps/Hz, $\lambda_{i,j} = 1$, $N = 3, 5, 8$

Figure 3.10: Ratio of selection outage probability to DSTC outage probability for $R = 1, \ldots, 5$ and $N = 3, 4, \ldots, 10$
3.2. Selection in Multi-Source Networks

Figure 3.11: Outage probabilities of DSTC and simple selection with unequal average channels and $R = 1 \text{ bps/Hz, } N = 3, 4$

increasingly efficient use of power as $N$ increases.

The results above demonstrate that selection outperforms DSTC for all network sizes when $R = 1$. To test how DSTC compares to selection for different rates and network sizes, we compute the ratio (independent of SNR) of the outage probability using selection and using DSTC for various values of $R$ and $N$, and present the results in Figure 3.10. Clearly, Selection Cooperation outperforms DSTC when this ratio is less than one, as is the case for all shown values of $R$ when $N \geq 3$. DSTC, on the other hand, perform better for the small network size of $N = 3$ when $R > 2$. We note that the entire function falls off sharply with increasing $N$, while for larger values of $N$ the dependence on $R$ is negligible.

Equation (3.13) is valid when the channels between all destinations other than the one being analyzed and the relay nodes are equally strong on average. Clearly, this is not the case in practical systems, where channel power is attenuated by distance and affected by
3.2. Selection in Multi-Source Networks

shadowing. Although analytical results for such a case and general network sizes are difficult to obtain for the simple Selection Cooperation scheme, Figure 3.11 presents the results of simulations that compare the simple selection scheme to the DSTC in this scenario.

In this simulation, the average channel power, $E \{ |a_{i,j}|^2 \}$, is itself an exponential random variable with parameter $\lambda = e^z$, where $z$ is a zero mean normally distributed random variable with unit variance. This model approximates large-scale, log-normal fading. The simulations demonstrate the improvement in performance of simple Selection Cooperation over DSTC. Although compared to the equal average channel gain scenario presented in Figure 3.9 both selection and DSTC exhibit worse performance, the gap between the two schemes increases. This suggests that DSTC is more sensitive to scenarios with asymmetrical average channel gains, an unsurprising observation given that space-time codes are designed for, and perform best, when all channels are independent and have equal average power. Selection, on the other hand, does not inherently have this built-in condition; the performance loss as compared to equal channel gain situation is due only to an uneven distribution of power in a network scenario. It is thus expected that this loss would be smaller than that obtained by DSTC.

**Diversity Multiplexing Trade-off**

The discussion so far has dealt with outage probability exclusively. In this section we compare the diversity multiplexing trade-off curves of Selection Cooperation and DSTC. Using the standard definitions of diversity order $\Delta$ and normalized rate $R_{\text{norm}}$ [35], this trade-off can easily be derived from (3.15) as

$$\Delta_{\text{simple--sel}} R_{\text{norm}} = N(1 - 2R_{\text{norm}}).$$  (3.16)
3.2. Selection in Multi-Source Networks

In [11], the authors determine the lower and upper bounds for the trade-off as

\[ N(1 - 2R_{\text{norm}}) \leq \Delta_{\text{DSTC}} R_{\text{norm}} \leq N(1 - \left[ \frac{N - 1}{N} \right] 2R_{\text{norm}}). \] (3.17)

The selection diversity-multiplexing trade-off is thus exactly the lower bound of the DSTC trade-off. This difference comes from the coding at the relays: because in selection the data is simply repeated by the relay, the corresponding mutual information is a logarithmic function of the sum of the source-destination and relay-destination channels. For DSTC, the lower bound is achieved in this manner. If the relays use an independent codebook the upper bound is achieved. In this chapter we do not explore the use of independent codebooks at the relay in the context of selection. For increasing network sizes, however, the upper and lower bound converge, as does the performance of Selection Cooperation and DSTC. Furthermore, the flat curves as a function of \( R \) for higher \( N \) in Figure 3.10 suggest that DSTC only reach the lower bound of the trade-off curve for higher \( N \), and the performance difference between the two schemes is dominated by the SNR-gain of their outage probabilities.

3.2.4 Overhead and System Requirements

Due to the geographical distribution of nodes, any cooperative transmission scheme suffers some overhead. In this section, we discuss the overhead, centralization and complexity requirements of Selection Cooperation.

After the first time slot, each of the \( N \) relays attempts to decode \( N - 1 \) messages. For each message, the relay must indicate its success or failure with one bit, resulting in a total of \( N(N - 1) \) feed-forward bits. For each source, the destination (or the CU) must select one of the \( N - 1 \) relays using \( \log_2(N - 1) \) feedback bits, resulting in a total of \( N \log_2(N - 1) \) feedback bits. The overhead is thus \( N \left[ \log_2(N - 1) + (N - 1) \right] \) bits per network, and \( \left[ \log_2(N - 1) + (N - 1) \right] \) bits per source-destination pair.
The simple selection scheme is fully decentralized (i.e., the destination only needs to know the channels from its relays), and thus the above calculated overhead is sufficient to implement this scheme. The transmission of the overhead bits must occur on orthogonal channels, decreasing the available bandwidth for the transmission and affecting system performance. We assume, however, that cooperation will be implemented in low-mobility environments, such as mesh networks, where channels change slowly in time. In such a case, the channel coherence time spans many code blocks, and the $N \log_2(N - 1) + (N - 1)$ overhead bits are assumed negligible in comparison to the transmitted information bits.

It is difficult to compare the implementation overhead of DSTC and Selection Cooperation because DSTC requires symbol-level synchronization. However, in both schemes, only the relay nodes that have correctly decoded the message proceed to encode and transmit the data. Because the relays must inform each other whether they have correctly decoded the message of each of the sources, a feed-forward overhead also exists in DSTC systems, and is identical to that of Selection Cooperation: $(N - 1)$ bits per source-destination pair. The incremental overhead of the simple selection scheme is thus only $\log_2(N - 1)$ bits.

In a traditional MIMO system, space-time coding may be preferable to selection diversity because its implementation does not require feedback. In a distributed system, however, DSTC incurs a significant overhead, generally not accounted for in the analysis. DSTC requires overhead for synchronization and, although the quantification of this overhead is beyond the scope of this chapter, it is likely that it will be larger than the $\log_2(N - 1)$ bits required to implement simple Selection Cooperation. Additionally, there may be feedback required to organize the participating nodes into indices of the DSTC matrix. Given that the overhead for synchronization and this organization is difficult to quantize, for this purposes of this chapter, we do not assume any further centralization or overhead requirements of DSTC. As a summary, the network requirements of each scheme are given in Table 3.1.
### 3.3. Conclusions

In this chapter, we have presented Selection Cooperation in distributed, multi-source networks. Through analysis and simulation, we have shown that Selection Cooperation achieves full diversity order and significantly outperforms DSTC in a distributed system for all net-

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Complexity</th>
<th>Centralization</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>$N^2$</td>
<td>No</td>
<td>$N\log_2(N - 1) + (N - 1)$</td>
</tr>
<tr>
<td>Sub-Optimal</td>
<td>$N!$</td>
<td>Yes</td>
<td>$N\log_2(N - 1) + (N - 1) + LN^{N-1}$</td>
</tr>
<tr>
<td>Optimal</td>
<td>$N^{N-1}$</td>
<td>Yes</td>
<td>$N\log_2(N - 1) + (N - 1) + LN^{N-1}$</td>
</tr>
<tr>
<td>DSTC</td>
<td>—</td>
<td>No</td>
<td>$N(N-1)$</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of network requirements for Simple, Sub-Optimal, Optimal Selection and DSTC

### 3.2.5 Discussion

The superior performance of Selection Cooperation over DSTC in network setting is not surprising: with $N$ transmit antennas, selection transmit diversity (with $\log_2(N)$ bits of feedback) also outperforms space-time codes (no feedback required) in traditional MIMO systems. The performance loss of STC results from the power constraint: with $N$ antennas, the received signal power in each time slot with STC is $\sum_{i=1}^{N} |h_i|^2$, while the received power with selection is $|h_{\text{max}}|^2$, where $|h_{\text{max}}|^2 = \max(|h_1|^2, |h_2|^2, \ldots, |h_N|^2)$. Note that, as described above, in a distributed network DSTC also requires significant feedback.

Although it could be deduced that selection will outperform DSTC, the extent and the region of this performance gain is not clear a priori since, as previously stated, simple selection carries a power penalty. Furthermore, although in a decode-and-forward system with a single source-destination pair the optimal solution is clearly beamforming, this is not necessarily true in a network setting where each node has a peak power constraint. As stated previously, this problem has thus far been explored only in [96].

### 3.3 Conclusions

In this chapter, we have presented Selection Cooperation in distributed, multi-source networks. Through analysis and simulation, we have shown that Selection Cooperation achieves full diversity order and significantly outperforms DSTC in a distributed system for all net-
works with more than three potential relays. This is because the relays do not have to “waste” energy transmitting to destinations over poor channels. Furthermore, simulations show that this improvement is greater in the practical case of geographically distributed nodes with channel qualities impacted by distance attenuation and shadowing.

The one significant drawback with Selection Cooperation, in a network setting, is the overhead involved in choosing a relay. Relay nodes must declare themselves eligible to cooperate and the destination must feedback the choice of relay. The analysis in this chapter neglects the small overhead to choosing a relay on the overall throughput. However, it does not change the straight-forward manner in which the scheme could be applied or the central claims made here.

The parallel-relay network considered in this chapter is a theoretical construct. In practical networks, data may be transmitted through multiple hops and several relays. Before discussing the optimal cooperative strategy in a generalized network in Chapter 5, in the following chapter we consider a simplified scenario of generalized networks where relays are also allowed to communicate with each other. In this context, we investigate the role of cooperation diversity when the transmission path from the source to destination is chosen dynamically.
Chapter 4

Selection in Multi-hop Networks

In the preceding chapter, we considered the scenario of multi-source parallel-relay networks, where data is transmitted from the source to the destination via one hop. In this context, we proposed Selection Cooperation. A parallel-relay network, however, is a simplified scenario which provides insights about the nature of protocols; on the other hand, it is not a very good model for a practical network. Practical networks will almost certainly involve some communications over multiple hops, implying that relays will have the ability to communicate with one another.

To bridge the gap between parallel-relay and generalized-relay networks, in this chapter we consider multihop networks, where each relay receives information from an adjacent relay (note that in a generalized network, any relay may communicate with any other relay, not only adjacent ones). We observe that in a multi-hop setting, routing, when implemented according to instantaneous channel conditions, can be interpreted as a version of cooperative diversity. Within a single hop, Selection Cooperation differs from optimal routing only in the presence of a source-destination link (which provides for one additional diversity path). This observation naturally leads to the question: if routing is a version of cooperation, what is the role of cooperation diversity in a system where routing is adapted to changing
channel conditions? The answer to this question is not immediately obvious, since we are suggesting the implementation of cooperation on top of a scheme that is already cooperative in nature (adaptive channel routing). For the sake of clarity, we point out here that any future reference to “routing” will refer to “channel adaptive routing”, i.e., routing based on instantaneous channel conditions.

To address this question, we focus on the problem of combining routing and cooperative diversity in a multi-hop mesh network with static nodes. In this chapter, we accomplish this combining by allowing cooperative diversity on each link between two adjacent relays. We note, however, that this strategy is highly sub-optimal, as this chapter does not attempt to propose new routing algorithms; its main goal is simply to offer insights into the effect of cooperation when applied in a straightforward manner on top of a routing algorithm. The insight is the following: cooperative diversity does not provide any rate gains over an optimal routing algorithm. By decreasing the amount of resulting hops, these rate gains can only be achieved when cooperative diversity is incorporated into the route selection algorithm. The chapter thus underscores the importance of integrating routing and the selective use of cooperative diversity. The actual design of optimal and sup-optimal cooperative and multihop schemes is undertaken in Chapters 5 and 6.

This chapter is structured as follows. Section 4.1 describes the system model. Section 4.2 describes the routing and cooperative diversity schemes considered in the paper, and the results of simulations evaluating these schemes are presented in section 4.3. Section 4.4 concludes the work.

4.1 System Model

In this section, we discuss the network layout, channel model, the multiple access scheme, and the constraints applied throughout the chapter. In a large part, the same system model
Figure 4.1: $N \times M$ Network Layout. $r_{(x,y)}$ denotes a node in position $(x,y)$. The horizontal and vertical distance between each node is $d$.

will be used in the rest of this thesis.

### 4.1.1 Network Model

We consider a distributed system, as shown in Figure 4.1, consisting of a source $S$, destination $D$, and $NM$ stationary nodes aligned in a uniformly placed grid. The distance between a node and its four nearest neighbors is $d$. This simple structure approximately models mesh and other networks where nodes are static with known locations.

The channel between nodes in locations $(i,j)$ and $(k,l)$, $a_{(i,j)-(k,l)}$, is modelled as flat and slowly-fading Rayleigh with variance $1/\lambda_{(i,j)-(k,l)}$; $|a_{(i,j)-(k,l)}|^2$ is exponential with parameter $\lambda_{(i,j)-(k,l)}$. This channel is independent of all other channels between remaining nodes. $\lambda_{(i,j)-(k,l)}$, which is inversely proposal to the average channel power, is a function of internode distance, $d_{(i,j)-(k,l)}$, through the attenuation exponent $p_a$, i.e., $\lambda_{(i,j)-(k,l)} \propto d_{(i,j)-(k,l)}^{p_a}$. This model does not include shadowing, although this can easily be incorporated. Note that assuming static nodes and very slowly-fading channels is crucial to the discussion of routing protocols which adapt to changing channels conditions since an adaptation is only possible if the channels are changing slower than the possible rate of adaptation.

A source node $S$ transmits data to destination node $D$ with the help of the $NM$ nodes in the grid. The data can be routed from the source to destination over multiple hops, and the nodes may use Selection Cooperative within each hop. We denote the communication
4.1. System Model

Figure 4.2: Illustrating multi-hop communications with cooperative diversity.

between the source and destination as a flow. Figure 4.2 illustrates a flow over three hops: cooperative diversity is used in the first and third hops only.

4.1.2 Multiple Access

Transmissions between nodes are accomplished using orthogonal time-slots; since in this thesis we consider a bandwidth-limited network, each transmission results in a bandwidth penalty, i.e., the amount of information communicated is inversely proportional to the number of hops. In particular, we model the multihop rate loss as follows. The source transmits data to the destination using a time slot of duration normalized to 1. The half-duplex constraint precludes nodes from transmitting on the same channel simultaneously; in a flow with $N_h$ hops, therefore, each hop uses a time slot of duration $(1/N_h)$. This model explicitly accounts for the increased traffic due to splitting large hops or implementing cooperative diversity. Although it is possible to optimize the length of each time-slot allocated to each hop, this analysis is beyond the scope of this chapter; the issue of optimal time allocation is the topic of Chapter 5.

As described in Chapter 1, the nodes considered in this chapter are stationary and each node transmits at its maximum available power of $P$ Watts. Furthermore, the MAC protocol eliminates the need for strict power control, and the total power is also unconstrained: a flow through $N_h$ hops, for example, would consume $N_hP$ Watts. Although it might at first
glance seem unfair to compare two scenarios with different hop numbers, each using different power levels, we argue that such a scenario is applicable to systems with stationary nodes with dedicated power supplies.

4.2 Routing and Cooperation Algorithms

In this section, we discuss the various schemes used to transmit information from the source S to the destination D. All nodes use DF repetition coding, although we highlight that using independent codebooks would not change the conclusions of this chapter. Note that in most of these approaches, the transmitted requires knowledge of forward channel power, which requires either centralization or a scheme to distribute channel information throughout the network. We assume, however, that because the nodes are stationary, the channel is changing slowly enough to allow for such a distribution of information. At the same time, we stress that the algorithms discussed in this section serve as performance bounds only; their implementation would require the development of more efficient routing algorithms.

The channel between nodes \((i,j)\) and \((k,l)\), \(|a_{(i,j)-(k,l)}|\) implies a mutual information between the nodes, \(I_{(i,j)-(k,l)} = \log_2(1 + \text{SNR}|a_{(i,j)-(k,l)}|^2)\). As in Chapter 3, the signal to noise ratio is defined as \(\text{SNR} = \frac{P}{N_0W}\), with \(P\) the power transmitted by each node, \(W\) the bandwidth available for each transmission, and \(N_0\) the noise power spectral density. To be sustainable, the data rate over this channel, \(R_{(i,j)-(k,l)}\) must be less than the mutual information.

4.2.1 1-Hop Smart Cooperation

The 1-Hop Smart Cooperation algorithm optionally implements cooperative diversity without routing. Motivated by Chapter 3, we consider Selection Cooperation and always choose at most one “best” cooperative partner. Unlike the scheme in 3, however, the Smart Coop-
4.2. Routing and Cooperation Algorithms

The operation algorithm implements cooperation only if it is advantageous in terms of rate. The destination coherently combines the two transmissions.

Without and with cooperation, the mutual information between the source and destination is

\[ I_{\text{no-coop}} = \log_2(1 + \text{SNR}|a_{s,d}|^2), \]
\[ I_{s-r(i,j)-d} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}(|a_{r(i,j),d}|^2 + |a_{s,d}|^2) \right), \]

where \( r_{(i,j)} \) is the relay chosen for cooperation, and the pre-log factor of \( (1/2) \) indicates the further halving required to implement cooperative diversity with a half-duplex constraint. To prevent error propagation, the maximum achievable rate of the source transmission through relay \( r_{(i,j)} \) is the minimum of the rate obtained on the source-relay channel and the cooperating source-relay-destination channel,

\[ I_{\text{coop}_{(i,j)}} = \min \{ I_{s-r(i,j)-d}, I_{sr(i,j)} \}, \]
\[ I_{sr(i,j)} = \frac{1}{2} \log_2(1 + \text{SNR}|a_{s,r(i,j)}|^2). \]

The selected relay maximizes the cooperative mutual information over all nodes, i.e.,

\[ I_{\text{coop}} = \max_{i,j} I_{\text{coop}_{(i,j)}}. \]

To ensure that cooperation always helps, the source uses cooperative diversity only if it increases the achievable rate. The final rate of the Smart Cooperation algorithm, \( R_{SC-1} \), is therefore the maximum between the cooperative and non-cooperative mutual information:

\[ R_{SC-1} = \max\{I_{\text{coop}}, I_{\text{non-coop}}\}. \]
4.2. Routing and Cooperation Algorithms

4.2.2 1-Hop Simple Cooperation

The Simple Cooperation scheme differs from the Smart Cooperation scheme only in the last step. Whereas in Smart Cooperation the relay is active only if cooperation improves the rate, in Simple Cooperation the relay is always active. This is the traditional model for cooperative diversity. The resulting rate is simply the cooperative mutual information: \( R_{SiC-1} = I_{coop} \). Clearly, \( R_{SiC-1} \leq R_{SC-1} \). Comparing this scheme to 1-Hop Smart Cooperation tells us how often the increased signal-to-noise ratio (SNR) due to maximal ratio combining outweigh the pre-log factor of \((1/2)\).

4.2.3 2-Hop Smart Cooperation

The 2-Hop Smart Cooperation algorithm is very similar to the 1-Hop Smart Cooperation algorithm discussed above, with the exception that the algorithm always uses two hops. This scheme is the simplest from those combining Smart Cooperation and multi-hop communications. The source transmits to the intermediary node \( n_I \) with index \( (\lfloor N/2 \rfloor, \lfloor M/2 \rfloor) \), where \( \lfloor \cdot \rfloor \) indicates rounding to the nearest lower integer. 2-Hop Smart Cooperation is implemented to increase the rate \( R_{S-n_I} \) between the source and \( n_I \), and the rate \( R_{n_I-D} \) between \( n_I \) and the destination. For both hops, all \( \frac{NM}{2} - 1 \) are used as potential cooperative nodes (the intermediary node is precluded from cooperating with itself). The final rate, \( R_{SC-2} \) is the minimum of the rates achieved on both hops, \( \min\{R_{S-n_I}, R_{n_I-D}\} \). The two-hop transmission implies that transmission on each hop occurs using a time-slot of duration \( 1/2 \).

4.2.4 Dynamic Routing

Dynamic routing searches for the rate-maximizing path between the source and destination. To ensure forward progress, we constrain the maximum number of hops to \( M + 1 \), i.e., there can never be hopping backwards or along the same vertical axis. The rate \( R_{DR} \) is the
maximum of all rates achieved using any number of hops less than $M + 1$. For a specific $N_h$-hop flow, the flow rate is $(1/N_h)$-th of the minimum of the mutual information achieved on each hop. As discussed in Section 4.1.2, the $(1/N_h)$ factor accounts for the half-duplex constraint and penalizes multiple-hop routes. The algorithm can be summarized as follows:

1. Determine the one-hop rate: $R_1 = I_{SD}$.

2. For $2 \leq N_h \leq (M + 1)$, determine the rate achieved on each of the possible $N_h$-hop flows. This rate is the maximum of the minimum of the $N_h$ mutual information terms:

$$R_{N_h} = \max \min_{N_h} \frac{1}{N_h} \{ I_{S-(i,j)}, I_{(i,j)-(k,l)}, \ldots, I_{(m,n)-D} \}$$

where the optimization is over indices $(i, k, \ldots m)$ and $(j, l, \ldots n)$, i.e., for each $N_h$, the maximization is over all possible flows consisting of $N_h$ hops, and thus over all node-indices. The strict inequality in the $y$-coordinate ensures forward progress by eliminating the possibility of routing backwards or along a vertical line (note that in our definition, the $S - D$ link is along the $y$-direction).

3. Find the maximum over all the flows:

$$R_{DR} = \max \{ R_1, R_2, \ldots, R_{M+1} \}. \quad (4.7)$$

### 4.2.5 Dynamic Routing with Cooperation

The Dynamic Routing with Cooperation algorithm *sequentially* combines Dynamic Routing and Smart Cooperation. The optimal dynamic route is chosen using the Dynamic Routing algorithm. Given this route, cooperation is used to increase the rate along this path, i.e.,
Smart Cooperation is applied between the nodes chosen by the Dynamic Routing algorithm.

### 4.2.6 Dynamic Cooperative Routing

Dynamic Cooperative Routing *simultaneously* combines Dynamic Routing with Cooperation: the optimal path is chosen together with the cooperative partners. As in Dynamic Routing, the Dynamic Cooperative Rate $R_{DCR}$ is the maximum of the rates achieved with different hops:

$$R_{DCR} = \max\{R_1, R_2, \ldots, R_{M+1}\}.$$  \hspace{1cm} (4.8)

Unlike Dynamic Routing, however, $R_{DCR}$ is achieved with cooperation potentially included in each of the $N_h$ hops, i.e., for each possible combination of hops, the algorithm implements Smart Cooperation along each hop. With the exception of the cooperation included in the search for optimal route, this algorithm is very similar to Dynamic Routing. In the interest of brevity, we provide only an outline of this algorithm below:

1. Perform the Smart Cooperation algorithm. The result is $R_1$, the maximum rate achieved with one hop.

2. For $N_h \leq M + 1$, find a set of all possible $N_h$-hop routes between source and destination. For each route, perform Smart Cooperation along all hops, i.e., determine if any cooperating partner can increase the rate along any of the $N_h$ hops. $R_{N_h}$, the rate of a $N_h$-hop route, is the minimum rate along each of the $N_h$ hops (including a factor of $1/N_h$ for bandwidth expansion).

3. The final rate $R_{DCR}$ is the maximum of the rates obtained using all possible routes and cooperating partners.
Note that dynamic cooperative routing is an extremely complex scheme with large computation overhead.

In summary, the routing and cooperative schemes considered include, in increasing order of complexity, the traditional model for cooperative diversity, Smart Cooperation, cooperation within multi-hop communications, and also “optimal” schemes such as dynamic routing (without cooperation) and the combination of dynamic routing with cooperation (in both sequential and simultaneous flavors). The first two schemes choose a fixed number of hops independent of SNR, and the other two are dynamic, based on SNR and channel conditions.

4.3 Results

In this section, we present results of the schemes discussed in Section 4.2. For the purposes of this chapter, we present simulation results only; a detailed analysis for a generalized relay network with time allocation is presented in Chapter 5.

To decrease algorithmic complexity, we implement the routing algorithms presented in this chapter using Dijkstra’s algorithm. The metrics used by these algorithms, however, are not isotonic, and thus the algorithm does not guarantee optimal paths [97]. Because our goal here is to investigate the relationship between routing and cooperation and not to determine the optimal routing algorithm, the sub-optimality of this implementation is not of great consequence and does not change our main conclusions. We address the problem of optimal routing in Chapter 5.

In the simulations, we choose a $9 \times 9$ network (i.e., $N = M = 9$) and an attenuation exponent of $p_a = 2.5$. This choice is motivated by the scenario of planned static mesh-nodes installed on posts; transmissions between such nodes should undergo little shadowing effects and a lower attenuation exponent.

The results presented are obtained from 60,000 fading realizations and the resulting
4.3. Results

![Graph](image)

Figure 4.3: Nodes arranged on a grid. Rate obtained for $P_{out} = 10^{-3}$ and $N = M = 9$.

cumulative density function, $F_R(r)$, of the instantaneous rate, calculated according to the methods described in Section 4.2. The rate presented for a specific SNR is that for which the probability of outage is $10^{-3}$, i.e., $F_R^{-1}(10^{-3})$. The main results are displayed in Figure 4.3. The rates, as defined in the above section, are displayed as a function of receive SNR at the destination. This model was chosen for fair comparisons between various network sizes, as it is independent of the distances between the nodes. This figure is thus the central contribution of this chapter. To gain intuition about the behavior of the algorithms, we simulate their performance for SNRs as high as 20 dB. From Figure ??, several interesting conclusions can be drawn. The very poor results of the 2-hop Smart Cooperation scheme of Section 4.2.3 demonstrate the loss of performance of any routing algorithm that makes routing decisions “off-line”. The poor performance of this scheme is due to bandwidth expansion: even for a high instantaneous SNR, where it might be advantageous to route directly to the destination
and not undergo the bandwidth penalty of splitting the transmission in two, the scheme forces
the two hops and always undergoes this penalty. Similarly, always enforcing cooperation, as
in the simple 1-hop cooperation scheme of Section 4.2.2, also provides poor performance.

The following conclusion is an optimistic one: focusing on the single hop with smart
cooperation scheme of Section 4.2.1, even without a routing protocol and a with direct
transmission between the source and destination, cooperative diversity significantly improves
the flow rate. Thus with minimal complexity, very good performance can be obtained with
a simple 1-hop scheme and a Smart Cooperation diversity scheme. The results of the 1-hop
Simple Cooperation scheme, however, indicate the importance of cooperating only when
necessary, an observation verified in Figure 4.4. Here, we plot the percentage of hops which
use cooperation (in the Smart Cooperation scheme) as a function in SNR. As the SNR
increases, the use of cooperation becomes more sporadic and the direct source-destination
link outperforms a cooperative approach.

Returning to Figure 4.3, in the low-SNR regime, the 1-Hop Smart Cooperation scheme
is outperformed by the Dynamic Routing algorithm which always chooses the best non-
cooperative route. This suggests that at low SNR, multiple hops are advantageous in reaching
the destination. It is important to realize that, although we have referred to this scheme as
non-cooperative, it is not devoid of diversity: optimally selecting the best route ensures that
data is always sent on high-SNR links, which is conceptually a selection diversity notion. At
high SNR, the outage rates of Dynamic Routing and 1-Hop Smart Cooperation converge,
suggesting that in this regime Dynamic Routing rarely selects more than one relay.

Perhaps more surprisingly, this algorithm has essentially identical performance to one
where cooperative diversity is applied directly on top of the predetermined, dynamically
chosen, route, i.e., where cooperative diversity and routing are treated as disjoint problems.
This has interesting implications: cooperative diversity offers no benefit when a good path has
already been chosen. This is consistent with the work of [98]. However, this result does not
imply that cooperative diversity cannot help. If cooperation is included within the routing protocol figures of merit, as proposed in Section 4.2.6, higher data rates are possible. This scheme corresponds to the curve marked ”Dynamic Cooperative Routing” in Figure ??.

The intuition for this can be obtained from Figure 4.5, where we plot the average number of hops - averaged over all Monte Carlo runs - obtained with the Dynamic Routing (Section 4.2.4) and Cooperative Dynamic Routing (Section 4.2.6) schemes. The performance improvement of the Cooperative Dynamic Routing scheme thus comes from the influence of the cooperative node which can, on average, encourage longer hops. Cooperative Dynamic Routing decreases the number of hops, thus decreasing bandwidth expansion. This also explains why cooperative diversity applied onto the predetermined dynamic routes offers little help: because the routes are predetermined, this method cannot decrease the number of hops.

To verify the above results, Figure 4.6 shows the outage rate for a square network with 81 nodes distributed randomly using a uniform distribution. As in the grid-pattern, the per-
Figure 4.5: Nodes arranged on a grid. Average number of hops obtained when \( P_{out} = 10^{-3} \) and \( N = M = 9 \).

Figure 4.6: Randomly distributed nodes. Rate obtained for \( P_{out} = 10^{-3} \) and \( N = M = 9 \).
formance of Dynamic Routing with cooperation is essentially identical to Dynamic Routing. This result is thus due to the relationship between routing and cooperation algorithms, not to the particular distribution of the nodes. An interesting result is the outage rate obtained by 1-Hop Smart Cooperation, which is very close to that obtained with Dynamic Routing even in the low-SNR regime. This strong performance of 1-Hop Smart Cooperation in the random network, as compared to the grid network, points to the efficacy of a high-order diversity system with many available nodes, as expected when the nodes are not confined to distances far away from the source and destination.

4.4 Conclusions

The available literature in cooperative diversity generally assumes cooperation is always a net positive. This is mainly because network issues such as routing have generally been ignored. Consequently, we are motivated by such an analysis in mesh networks wherein data rate is a primary figure of merit. Given this lack of research into the interdependence between routing and cooperative diversity in such bandwidth-limited, power-unlimited networks, we have simulated several schemes which combine various degrees of optimality of routing and diversity implementation.

Although our simulations are based on Selection Cooperation and repetition coding, we expect these conclusions to hold for any cooperative scheme. The results point to a careful and discriminate approach to multihopping: in general, fewer hops are preferred, and a route should only be split if the high quality of the multi-hop channels can compensate for the loss of bandwidth penalty incurred from multihopping. Breaking routes into several hops off-line results in a dramatic performance loss in terms of rate; interestingly, cooperative diversity by itself, added onto the individual hops, does not compensate for this loss. Dynamically choosing routes according to changing channel conditions performs very well and, once again,
applying cooperative diversity on the chosen nodes has no benefit. Cooperative diversity is beneficial only if the search for the cooperating nodes is included into the dynamic routing algorithm; this benefit is due to longer hops and reduced bandwidth expansion.

These results are obtained in a sub-optimal way: nodes involved in routing (multihopping) do not communicate with non-adjacent relays, and relays selected to aid in the transmission on a particular path between nodes cannot transmit to nodes/relays on other paths. Furthermore, no resource allocation is applied on the transmissions. Because the routing algorithm in this chapter requires some degree of centralization or organization (in terms of implementing a routing algorithm), the information required for this organization could be used for the purposes of resource allocation. While the work in this chapter has investigated the interaction between cooperative diversity and routing in a mesh network, the natural question to ask is: what is the optimum strategy for organizing relay transmissions in a multihop, cooperative, generalized-relay network? This is the topic of the next chapter.
Chapter 5

Resource Allocation for Cooperative Diversity

In this chapter, we investigate resource allocation for large scale cooperative wireless networks. We consider the most general setting with both multihop and cooperative links, and address the joint problem of optimal relay selection and allocation of time resources to the selected relays.

The resource allocation is framed in the context of mesh networks of relatively simple and inexpensive nodes. We concentrate on resource allocation in terms of transmission time only, removing power allocation from the optimization; we further simply the problem by considering orthogonal transmissions. This is motivated by the need to reduce complexity, allowing for nodes which can implement the resource allocation simply by switching on and off. The networks considered here, however, are more complex than those studied in Chapters 3: because resource allocation requires knowledge of instantaneous channel powers, we assume either the existence of a central unit, or the knowledge of the network at each node through distributed algorithms. In either case, as in Chapter 4, this scenario involves feedback. Furthermore, unlike the nodes considered in both Chapter 3 and 4 where the nodes
used repetition coding, the nodes here use independent codebooks, which are necessary for the implementation of resource allocation in the context of cooperative diversity. The use of independent codebooks adds additional coding and decoding complexity at the nodes.

With these assumptions, this chapter provides the solution to the resource allocation problem for a decode-and-forward network of arbitrary size while combining multihop and cooperative diversity communication paths. Inherent to the optimal resource allocation problem is optimal relay selection. To the best of our knowledge, no other work provides a solution to time-allocation for an arbitrarily connected cooperative network. The solution can be interpreted as a generalization of the opportunistic protocol presented by Gunduz and Erkip, where the relay is active only when it increases the outage rate [56]. In terms of the resource allocation solution, it is also a generalization of the solution in [59], where channel resource allocation is determined under fixed power for a three-node decode-and-forward network. This chapter is structured as follows. Section 5.1 describes the system model. In Section 5.2 and 5.3 we develop the proposed resource allocation scheme and present a significantly simplified practical implementation. Simulation results are presented in Section 5.4, and concluding remarks are presented in Section 5.5.

5.1 System Model

We consider a mesh network of static nodes comprising a source and destination node and $N$ potential relays. The inter-node channel powers are denoted as $|a_{ij}|^2$, where $i$ and $j$ represent the source node $s$, relay nodes $r_k$, $k = 1 \ldots N$, or the destination node $d$. They are assumed independent of each other and are modelled as flat and slowly-fading exponential with parameter $\lambda$. $\lambda$ is inversely proportional to the average channel power and is a function of inter-node distance, $d_{ij}$, through the attenuation exponent $p_a$, e.g., $1/\lambda_{sd} = (1/d_{sd}^{p_a})$, and $1/\lambda_{r_kd} = (1/d_{r_kd}^{p_a})$. We do not include shadowing into the fading model, although this
can easily be incorporated on an instantaneous basis. Because the nodes are static, the channels are assumed to change very slowly with time; we thus assume knowledge of all channel gains (although not channel phases) at the nodes. This knowledge is essential to our resource allocation scheme. With the aim of designing simple and cheap nodes, we assume half-duplex channels and orthogonal transmissions, which greatly simplifies receiver design. The relays are assumed to be numbered in some convenient order such that relay $r_j$ transmits after $r_i$ if $j > i$. For example, the relays may be in a linear constellation as shown in Figure 5.1. We also assume the decode-and-forward cooperation strategy and utilize independent codebooks, which allow for the optimization of system resources (see [39] for an overview on current coding methods for nodes using decode-and-forward). Note that repetition coding does not allow for this resource allocation. However, since we do not expect the essential conclusions to change, we did not reopen the problems investigated in Chapters 3 and 4.

With these assumptions, the cooperation framework for the $N$-relay fully-connected network is as follows. The half-duplex constraint precludes the relays from transmitting and receiving simultaneously on the same channel, and the unavailability of forward-channel phase information at the nodes precludes the nodes from simultaneous transmissions. The transmission between the source and destination is thus divided into $N + 1$ time-slots, of duration $t_0$, $t_1$, $\ldots$, $t_N$, with $t_0 + t_1 + \ldots + t_N = 1$. In the first time-slot, the source transmits its information to all the nodes. The first relay, $r_1$, decodes this information and the

Figure 5.1: Location of the relays with respect to the source and destination.
remaining \( N \) relays and the destination store the information for future processing. In the second slot, of duration \( t_1 \), the first relay re-transmits the information using an independent codebook, the second relay decodes the information from the first relay and the source, and the remaining \( N - 1 \) relays and the destination store the information for further processing. In general, each relay \( r_k \) decodes information from the source and from the previous relays \( r_1 \ldots r_{k-1} \) up to and including time-slot \( t_{k-1} \). This process continues until all relays have transmitted and the destination attempts to decode the information.

Assuming that each node uses power \( P \) and \( W \) Hz per transmission (noting that although each node transmits for a different amount of time, the symbol durations and thus the corresponding bandwidth used by each node is the same), the signal to noise ratio (SNR) at node \( j \) resulting from transmission from node \( i \) can be written as \( \text{SNR}_{ij} = \frac{P}{N_0 W} |a_{ij}|^2 \), where \( N_0 \) is the noise density. In the rest of the thesis, we use the short-hand notation \( L_{ij} \) to denote \( \log_2(1 + \text{SNR}_{ij}) \), the capacity of the corresponding channel.

## 5.2 Optimal Resource Allocation and Relay Selection

In this section, we solve the joint problem of resource allocation and relay selection for the network discussed above. Essentially, we give the optimum values of \( t_i, i = 0 \ldots N \), such that the achievable rate between source and destination is maximized. We begin here with a fully-connected network, where each node is linked to all other nodes through a non-zero channel.
5.2.1 Fully Connected Network

Consider a source-destination pair communicating with the help of \( N \) relays. Assuming that each relay is active, the mutual information at each relay and destination can be written as

\[
I_1(t_0) = t_0 L_{sr_1},
\]

\[
I_k(t_0, t_1, t_2, \ldots, t_{k-1}) = t_0 L_{sr_k} + t_1 L_{r_1 r_k} + \ldots + t_{k-1} L_{r_{k-1} r_k},
\]

\[
I_D(t_0, t_1, t_2, \ldots, t_k, \ldots, t_{N-1}, t_N) = t_0 L_{sd} + t_1 L_{r_1 d} + \ldots + t_{k-1} L_{r_{k-1} d} + \ldots + t_N L_{r_N d},
\]

where \( I_k \) and \( I_D \) denote the mutual information at relay \( r_k \) and the destination, respectively.

With all \( N \) relays cooperating, the achievable rate is the minimum of the mutual information obtained at each individual relay node:

\[
R_N = \max_{t_0, \ldots, t_N} \min \{ I_1(t_0), I_2(t_0, t_1), \ldots, I_{k-1}(t_0, \ldots, t_{k-2}), I_k(t_0, \ldots, t_{k-1}), I_{k+1}(t_0, \ldots, t_k), \ldots, I_N(t_0, \ldots, t_k, \ldots, t_{N-1}), I_D(t_0, \ldots, t_k, \ldots, t_{N-1}, t_N) \},
\]

such that \( t_i \geq 0, \forall i \),

\[
t_0 + t_1 + \ldots t_N \leq 1.
\]

For reasons that will soon become clear, let us consider the case with with relay \( r_k \) removed from the network. The maximum achievable rate \( R_{N-1}^k \) becomes

\[
R_{N-1}^k = \max_{t_0, \ldots, t_{k-1}, t_{k+1}, \ldots, t_N} \min \{ I_1(t_0) \ldots, I_{k-1}(t_0, \ldots, t_{k-2}), I_k(t_0, \ldots, t_{k-1}), I_{k+1}(t_0, \ldots, t_k), \ldots, I_D(t_0, \ldots, t_{k-1}, t_{k+1}, \ldots, t_N) \},
\]

such that \( t_i \geq 0, \forall i \),

\[
t_0 + \ldots t_{k-1} + t_{k+1} + \ldots t_N \leq 1.
\]
Removing relay $r_k$ is thus equivalent to removing $t_k$ and $I_k$ from the optimization. [We use the subscript in $R^k_{N-1}$ to denote the maximum number of potentially active relays, and the superscript to denote the relay removed]. The maximum rate at which the source can transmit to the destination can thus be written as the maximum of the rate obtained by using all $N$ relays, and the rate obtained by successively removing each relay:

$$R_T = \max\{R_N, R^1_{N-1}, R^2_{N-1}, \ldots, R^N_{N-N-1}\}.$$  

(5.6)

If $R_T = R^k_{N-1}$, the maximum rate can be obtained by iterating through (5.4) and (5.5), successively removing a relay each step. Note that obtaining $R^k_{N-1}$ includes the cases where two or more relays are removed. In theory, therefore, all $2^N$ possible cases must be checked.

Let $(t^*_0, t^*_1, \ldots, t^*_N)$ denote the resource allocation that solves the optimization problem. We begin an outline of the solution to the optimization problem in (5.4), (5.5) and (5.6) with the following proposition.

**Proposition 1:** With a maximum number of potential relays $N$, the maximum achievable rate $R_T = R_N$ only if $t^*_k \neq 0, \forall k$. Otherwise, if $t^*_k = 0$, $R_T = R^k_{N-1}$.

**Proof:** With exactly $N$ active relays, and with $k < n < N$, the resulting rate can be written explicitly as:

$$R_N = \max_{t_0, \ldots, t_N} \min \{(t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1r_2}), \ldots, (t_0 L_{sr_k} + \ldots + t_{k-1} L_{r_{k-1}r_k}),$$

$$(t_0 L_{sr_n} + \ldots + t_{k-1} L_{r_{k-1}r_n} + t_k L_{r_kr_n} + t_{k+1} L_{r_{k+1}r_n} \ldots t_{n-1} L_{r_{n-1}r_n}), \ldots),$$

$$(t_0 L_{sd} + \ldots + t_{k-1} L_{r_{k-1}rd} + t_k L_{r_krd} + t_{k+1} L_{r_{k+1}rd} + \ldots t_N L_{r_Nrd})\}.$$  

(5.7)
Setting $t_k = 0$ gives

$$R_N = \max_{t_0, \ldots, t_{k-1}, t_k, \ldots, t_N} \min \left\{ \left( t_0 L_{sr_1}, (t_0 L_{sr_2} + t_1 L_{r_1 r_2}), \ldots, (t_0 L_{sr_k} + \ldots + t_{k-1} L_{r_k-1 r_k}), \right) \right. \\
(t_0 L_{sr_n} + \ldots + t_{k-1} L_{r_k-1 r_n} + t_k + 1 L_{r_k+1 r_n} \ldots t_{n-1} L_{r_{n-1} r_n}), \ldots, \\
(t_0 L_{sd} + \ldots + t_{k-1} L_{r_k-1 r_d} + t_k + 1 L_{r_k+1 r_d} + \ldots t_N L_{r_N r_d}) \right\} \quad (5.8)$$

$$\leq \max_{t_0, \ldots, t_{k-1}, t_k, \ldots, t_N} \min \left\{ \left( t_0 L_{sr_1}, (t_0 L_{sr_2} + t_1 L_{r_1 r_2}), \ldots, \\
(t_0 L_{sr_n} + \ldots + t_{k-1} L_{r_k-1 r_n} + t_k + 1 L_{r_k+1 r_n} \ldots t_{n-1} L_{r_{n-1} r_n}), \ldots, \\
(t_0 L_{sd} + \ldots + t_{k-1} L_{r_k-1 r_d} + t_k + 1 L_{r_k+1 r_d} + \ldots t_N L_{r_N r_d}) \right\} \quad (5.9)$$

$$= R_N^{k-1}, \quad (5.10)$$

since (5.9) has one fewer term in the minimization than (5.8). ■

To solve the optimization problem of (5.4) we thus require only the critical points for which $t_k^* \neq 0, \forall k$. In the following proposition, we show that for each $R_N$, i.e., given a set of potential relays, only one solution satisfies $t_k^* \neq 0, \forall k$.

**Proposition 2:** The unique solution to the optimization problem in the inner loop of (5.4) for which $t_k^* \neq 0, \forall k$ is given by $I_1(t_1) = I_2(t_1, t_2) = \ldots = I_N(t_1, \ldots, t_N) = I_D(t_1, \ldots, t_N)$.

**Proof:** We consider all possible critical points obtained from the optimization in (5.4). The points are obtained either by maximizing each individual term in (5.4) or by intersecting all possible combinations of the terms in (5.4). We show that the only solution leading to non-zero solutions results from intersecting every term in (5.4).

The critical points for the optimization problem can be obtained by solving the following:

1. Maximize the individual terms in (5.4) except $I_d(t_0, \ldots, t_N)$:

$$\forall k \leq N, \max_{t_0, \ldots, t_{k-1}} I_k(t_0, \ldots, t_{k-1}) \quad \text{s.t.} \quad t_0 + \ldots + t_{k-1} \leq 1. \quad (5.11)$$
5.2. Optimal Resource Allocation and Relay Selection

Because the optimization is not over \( t_m, \forall k \leq m \leq N \), the solution to this problem clearly has all \( t_m = 0, \forall k \leq m \leq N \), and thus cannot be a solution to the overall optimization problem.

2. Maximize \( I_d(t_0, \ldots, t_N) \):

\[
\max_{t_0, \ldots, t_N} I_d(t_0, \ldots, t_N) = \max_{t_0, \ldots, t_N} \{ t_0L_{sd} + \ldots + t_NL_{rd} \}, \quad \text{s.t.} \quad t_0 + \ldots + t_N \leq 1. \quad (5.12)
\]

In this case, all variables are included in the optimization. It is easy to show, however, that this function is maximized by selecting the largest \( L \) value, i.e., evaluating the Kuhn-Tucker conditions leads to a solution of the form \( t_m = 1, t_k = 0, \forall k \neq m \), where \( m = \arg \max_k \{ L_{sd}, L_{r_1d}, \ldots, L_{r_kd}, \ldots, L_{r_Nd} \} \). Therefore, this solution is also not a solution to the overall optimization problem.

3. Maximize the function that results from the intersection of all possible combinations of the functions \( I_k \). Let \( \mathcal{M} \) denote all possible subsets of \( \{1 \ldots N\} \). \( \mathcal{M} \) then contains \( 2^N \) such subsets, i.e., \( |\mathcal{M}| = 2^N \). Consider one such subset \( \delta_k = (m_1, m_2, \ldots, m_k) \), with \( m_1 < m_2 < m_k \). One critical point then is

\[
\max_{t_0, \ldots, t_{m_k-1}} I_{m_k}(t_0, \ldots, t_{m_k-1}) \quad (5.13)
\]

such that

\[
I_{m_1}(t_0, \ldots, t_{m_1-1}) = I_{m_2}(t_0, \ldots, t_{m_2-1}) = \ldots = I_{m_k}(t_0, \ldots, t_{m_k-1}). \quad (5.14)
\]

This optimization then gets repeated for all sets \( \delta_k \in \mathcal{M} \). In all but one combination, this optimization is not over all the variables \( \{t_0, \ldots, t_N\} \). As in point (1), this maximization also leads to \( t_k = 0 \) for some value of \( k \).
4. Maximize the intersection of all terms in (5.4):

\[ I_1(t_0) = I_2(t_0, t_1) = \ldots = I_N(t_0, \ldots, t_{N-1}) = I_d(t_0, \ldots, t_N). \]  

(5.15)

This is the only case that leads to \( t_k \neq 0, \forall k = 0 \ldots N \). ■

Essentially, this proposition shows that if all \( N \) relays are to contribute, all terms in the minimization in (5.4) must be equal. This proposition applies to any value of \( N \). Therefore, if the optimal solution has \( k < N \) relays, an expression like (5.4) can be written for those \( k \) relays.

The linear system of equations in (5.15) has a simple solution. Setting each equation to a constant, solving for the vector of unknowns \( t = (t_0 \ldots t_N) \) and normalizing, we obtain

\[ \mathbf{L}_{N+1} \mathbf{t}_{N+1} = \mathbf{1}_{N+1}, \Rightarrow \mathbf{t}_{N+1} = \frac{\mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}}{||\mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}||_1} = \frac{\mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}}{\mathbf{1}_{N+1}^T \mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}}, \]  

(5.16)

where \( ||\mathbf{v}||_1 \) denotes the sum of the elements of \( \mathbf{v} \), i.e., the 1-norm. \( \mathbf{1}_{N+1} \) is the length-(\( N+1 \)) vector of ones and \( \mathbf{L}_{N+1} \) is the (\( N+1 \)) × (\( N+1 \)) channel matrix

\[
\mathbf{L}_{N+1} = \begin{bmatrix}
L_{sr} & 0 & 0 & \ldots & 0 \\
L_{sr} & L_{r_1 r_2} & 0 & \ldots & 0 \\
L_{sr} & L_{r_1 r_3} & L_{r_2 r_3} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & 0 \\
L_{sd} & L_{r_1 d} & L_{r_2 d} & \ldots & L_{r_N d}
\end{bmatrix}.
\]  

(5.17)

The solution in (5.16) does not guarantee that the constraint \( t_k > 0, \forall k = 0 \ldots N \) is satisfied. To ensure that only solutions for which this constraint is satisfied are considered, we again consider the set \( \mathcal{M} \). Each entry in the set corresponds to a channel matrix, \( \mathbf{L}_m \), similar to that in (5.22), formed using the relays in that entry of the set. Furthermore, let
$|m|$ denote the size of the matrix $L_m$. A relay set and its corresponding solution, denoted as $t_m$, is included as a potential solution if $t_m$ satisfies the constraint, i.e.,

$$t_m > 0_{|m|}, \quad (5.18)$$

where $0_{|m|}$ is the all-zero vector of size $|m|$, $0_{|m|} = [0, 0, 0, \ldots, 0]^T$ and the inequality operates on an element-by-element basis. Let the set $K$ form the subset of $M$ that comprises all potential solutions. Let $L_k$, $t_k$ and $|k|$ denote the channel matrix, its corresponding solution and size, respectively, for each entry of the set $K$. Note that the number of active relays being considered in each entry is $|k| - 1$. Finally, the optimum solution can be obtained by solving (5.16) for all possible combinations of active relays in the set $K$ i.e.,

$$t^* = \max_K \frac{L_k^{-1}1_{|k|}}{1_{|k|}^T L_k^{-1}1_{|k|}}, \forall k = 1, \ldots, |K|. \quad (5.19)$$

Given that entry $k^*$ corresponds to $t^*$, the maximum achievable rate vector can thus be written as

$$L_{k^*} t^* = L_{k^*} \frac{L^{-1}_{k^*}1_{|k^*|}}{1_{|k^*|}^T L^{-1}_{k^*}1_{|k^*|}} = \frac{1_{|k^*|}}{1_{|k^*|}^T L_{k^*}^{-1}1_{|k^*|}}, \quad (5.20)$$

and the maximum achievable rate, $R^*$, is

$$R^* = \frac{1}{1_{|k^*|}^T L_{k^*}^{-1}1_{|k^*|}}, \quad (5.21)$$

Note that the solution described above is equivalent to the iterative maximization in (5.6), and that removing a relay $r_k$ translates to removing the $k^{th}$ row and $(k+1)^{th}$ column from the channel matrix in (5.22). Removing the first relay, for example, reduces the channel
matrix in (5.22) to

\[ L_N = \begin{bmatrix} L_{sr_2} & 0 & \ldots & 0 \\ L_{sr_3} & L_{r_2r_3} & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ L_{sd} & L_{r_2d} & \ldots & L_{r_Nd} \end{bmatrix}. \] (5.22)

Since |M| = 2^N, there are therefore at most 2^N possible solutions which are to be tested to find the global optimum.

### 5.2.2 Numbering

In Section 5.2.1, we gave the solution to the optimization problem for a network with nodes numbered as in Figure 5.1. The numbering of the relay nodes impacts performance through causality: relay r_k decodes information from relay r_{k-1}, but not vice-versa.

A complete solution to the optimization problem must therefore take into account an optimal numbering scheme. In the worst case (in terms of computational power), an optimal solution can be obtained for a specific numbering scheme, and the truly optimal solution can be maximized over all possible numbering schemes.

Clearly, such an approach is impractical. Although a search for an optimal or effective sub-optimal solution is beyond the scope of this thesis, we study the effects of numbering on the solution and resulting rate by considering some numbering schemes based on heuristics. We consider two approaches: numbering based on average channel conditions, and numbering based on instantaneous channel conditions.
5.2. Optimal Resource Allocation and Relay Selection

In the case of the square network in Figure 5.1, the numbering is trivial: node numbers increase away from the source and towards the destination. In the case of square network with nodes arranged in a grid, we consider two numberings which we refer to as **Average Descending Numbering** and **Average Linear Numbering**, shown in Figures 5.2 and 5.3, respectively, for a $4 \times 4$ network.

- **Average Descending numbering**: node numbers increase towards the destination and downwards,

- **Average Linear numbering**: node numbers increase towards the destination but vertical numbering ensures that nodes closest to each other retain close numbering.

**Numbering based on instantaneous channel conditions**

- **Instantaneous $S - R_k$ numbering**: node numbers increase with increasing source-relay channels. The first node has the best source-relay channel, the second node has...
the second-best source-relay channel, etc.

- **Instantaneous $R_k - R_m$ numbering**: nodes are numbered to maximize the channel between adjacent nodes. The first relay has the best source-relay channel. The second relay has the strongest $r_1$-relay channel. Numbers are assigned in this process to unoccupied relays. This heuristic is based on the notion that we should maximize the capacity of each $(R_k, R_{k+1})$ hop.

- **Random numbering**: nodes are numbered randomly. This case evaluates the worst-case scenario and tests the robustness of the optimization to numbering.

These schemes are evaluated in Section 5.4. As we will see, the achievable rate is remarkably robust to the chosen numbering scheme.

### 5.2.3 Partially Connected Network

In this section we briefly discuss the more practical case of a partially connected network in which some links between the nodes in the network are unavailable. This is a generalization of the fully-connected network discussed in Section 5.2.1 above. Such a network is more likely to represent a large scale network where, in any case, the solution in (5.19) would be computationally impossible.

As an example, consider the two-relay network with the link between $r_1$ and $r_2$ is removed. The channel matrix thus becomes

$$L_3 = \begin{bmatrix} L_{sr_1} & 0 & 0 \\ L_{sr_2} & 0 & 0 \\ L_{sd} & L_{r_1d} & L_{r_2d} \end{bmatrix}.$$  \hfill (5.23)

Removing the link thus reduces the rank of this matrix by one, and the matrix is now
non-invertible, eliminating the solution defined by \( I_1 = I_2 = I_3 \), where both relays are active. The optimal solution in this case is thus to select \( r_1, r_2 \), or not to relay. Note, however, that removing a link does not automatically lead to a non-invertible channel matrix. Consider, for example, the three-relay network with the link between \( r_1 \) and \( r_3 \) removed. The corresponding channel matrix

\[
L_4 = \begin{bmatrix}
L_{sr_1} & 0 & 0 & 0 \\
L_{sr_2} & L_{r_1r_2} & 0 & 0 \\
L_{sr_3} & 0 & L_{r_2r_3} & 0 \\
L_{sd} & L_{r_1d} & L_{r_2d} & L_{r_3d}
\end{bmatrix}
\] (5.24)

is full-rank and invertible.

In general, the approach to the optimization problem for the case of the arbitrary connected network is that the same as for the fully-connected network, with the exception that the matrix \( L_{N+1} \) may not be invertible, in which case the corresponding solution obtained from its inverse is, of course, inadmissible. The remaining steps remain unchanged.

### 5.3 Implementation with Reduced Complexity

The optimization problem in (5.4), (5.5) and (5.6) involves checking \( 2^N \) potential solutions. Although the process is conceptually simple, each solution involves the inverse of a matrix. In this section, we show how the optimization problem in the previous section can be significantly simplified using a recursive solution. This solution, which exploits the special structure of the channel matrix, greatly simplifies the matrix inversion, as well as reduces the number of possible solutions to check. Essentially, while the solution in Section 5.2.1 was a top-down approach, the approach we suggest here is bottom-up.

Consider a set of \( p \) relays, \( \mathcal{P} = \{r_1, r_2, \ldots, r_p\}, p \geq 0 \), and its corresponding channel
5.3. Implementation with Reduced Complexity

matrix \( L_{p+1}^P \), solution vector \( t_{p+1}^P \) and maximum rate (if available) \( R_P \). We note that if \( p = 0 \) and the set has no relays, the channel matrix and solution vector are constants, \( L_{sd} \) and \( 1 \), respectively. Denote as \( \mathcal{P}' \) the set \( \mathcal{P} \) appended with another relay, i.e., \( \mathcal{P}' = \{ r_1, r_2, \ldots, r_p, r_{p+1} \} \). Denote as \( L_{p+2}^P, t_{p+2}^P \) and \( R_P' \) the channel matrix, solution vector and rate corresponding to set \( \mathcal{P}' \).

**Proposition 3:** Given \( (L_{p+1}^P)^{-1} \), \( (L_{p+2}^{P'})^{-1} \) can be obtained with complexity order \( O(p^2) \)

**Proof:** For \( p \geq 0 \), the channel matrix \( L_{p+2}^{P'} \) can be written as

\[
L_{p+2}^{P'} = \begin{bmatrix}
L_{p+1}^P(1 : p, 1 : p) & 0_{p \times 2} \\
F_{2 \times p} & T_2 
\end{bmatrix},
\]  

(5.25)

where \( L_{p+1}^P(1 : p, 1 : p) \) denotes the first \( p \) rows and columns of the matrix \( L_{p+1}^P \), \( 0_{p \times 2} \) is a \((p \times 2)\) matrix of zeros, \( T_2 \) is \((2 \times 2)\) lower- triangular square matrix, and \( F_{2 \times p} \) is a \((2 \times p)\) fully-loaded matrix. Note that \( L_{p+1}^P(1 : p, 1 : p) \) is triangular. Using the inverse of a partitioned matrix [99], \((L_{p+2}^{P'})^{-1}\) can be written as

\[
(L_{p+2}^{P'})^{-1} = \begin{bmatrix}
(L_{p+1}^P(1 : p, 1 : p))^{-1} & 0_{p \times 2} \\
-T_2^{-1}F_{2 \times p}(L_{p+1}^P(1 : p, 1 : p))^{-1}T_2^{-1} 
\end{bmatrix}.
\]  

(5.26)

Note that \((L_{p+1}^P(1 : p, 1 : p))^{-1}\) is the inverse of a partition of the triangular matrix \( L_{p+1}^P \). Using the inverse of a partitioned matrix one more time, however, it is easy to see that

\[
(L_{p+1}^P(1 : p, 1 : p))^{-1} = (L_{p+1}^P)^{-1}(1 : p, 1 : p),
\]  

(5.27)
and thus

\[
(L_{p+2}^{p'})^{-1} = \begin{bmatrix}
(L_{p+1}^p)^{-1}(1 : p, 1 : p) & 0_{p \times 2} \\
-T_2^{-1}F_{2 \times p}(L_{p+1}^p)^{-1}(1 : p, 1 : p) & T_2^{-1}
\end{bmatrix}.
\]

(5.28)

Using this above proposition, the solution vector \(t_{p+2}^{P'}\) of \(L_{p+2}^{p'}\) can be obtained from the solution vector \(t_{p+1}^P\) of \(L_{p+1}^P\):

\[
t_{p+2}^{P'} = \frac{(L_{p+2}^{p'})^{-1}1_{p+2}}{1_{p+2}(L_{p+2}^{p'})^{-1}1_{p+2}} = \begin{bmatrix}
t_{p+1}^P(1 : p) \\
t_{p+2}^P(p + 1) \\
t_{p+2}^P(p + 2)
\end{bmatrix},
\]

(5.29)

where \(t_{p+1}^P(1 : p)\) represent the first \(p\) entries of the already-calculated solution vector \(t_{p+1}^P\), and \(t_{p+2}^P(p + 1)\) and \(t_{p+2}^P(p + 2)\) are the last two entries of the solution vector \(t_{p+2}^{P'}\) that remain to be calculated. \(R^{P'} = \frac{1}{1_{p+2}(L_{p+2}^{p'})^{-1}1_{p+2}}\) is the maximum achievable rate obtained using the set \(P'\) of relays. The last two entries of the solution vector \(t_{p+2}^{P'}(p + 1)\) and \(t_{p+2}^{P'}(p + 2)\) can be written as

\[
\begin{bmatrix}
t_{p+2}^{P'}(p + 1) \\
t_{p+2}^{P'}(p + 2)
\end{bmatrix} = R^{P'} \begin{bmatrix}
-T_2^{-1}F_{2 \times p}(L_{p+1}^P)^{-1}(1 : p, 1 : p) & T_2^{-1}
\end{bmatrix} 1_{(p+2) \times 1},
\]

(5.30)

With a corresponding achievable rate \(R^{P'}\)

\[
R^{P'} = \frac{1}{1_{p+2}(L_{p+2}^{p'})^{-1}1_{p+2}} = \left(\sum_{i,j} (L_{p+2}^{p'})^{-1}(i, j)\right)^{-1},
\]

\[
= \left(\sum_{i,j} (L_{p+1}^p)^{-1}(i, j) - \sum_{i,j} T_2^{-1}F_{2 \times p}(L_{p+1}^p)^{-1}(1 : p, 1 : p)(i, j) + \sum_{i,j} T_2^{-1}(i, j)\right)^{-1},
\]

(5.31)
where we use $\sum_{i,j} A(i,j)$ to denote the summation over all the elements of matrix $A$.

Using the above, the optimization problem for a network of $N$ potential relays can be solved recursively as follows:

1. Determine the set of all potential relay combinations. Sequence the set with an increasing number of relays, for example:

$$
\mathcal{M} = \{(r_1), (r_1, r_2), (r_1, r_2, r_3), \ldots (r_1, r_2, \ldots, r_N),
(r_1, r_3), (r_1, r_3, r_4), \ldots, (r_1, r_3, \ldots, r_N),
\ldots
(r_1, r_N),
(r_2), (r_2, r_3), (r_2, r_3, r_4), \ldots (r_2, r_3, \ldots, r_N),
(r_2, r_4), (r_2, r_4, r_5), \ldots, (r_2, r_4, \ldots, r_N),
\ldots
(r_2, r_N),
\ldots
(r_{N-1}, r_N)\}.
$$

Note that each “row” of $\mathcal{M}$ is a subset of relay combinations in which each element is formed from the previous element by adding a relay.

2. In each “row”, obtain the channel matrix, its respective optimized time allocation vector and achievable rate for each element (i.e., relay combination) recursively using (5.28), (5.29), (5.30) and (5.31).
3. Check that for each particular set $\mathcal{P}$ of $p$ relays, the solution $t_p$ and achievable rate $R_p$ satisfies the constraints:

$$R^{\mathcal{P}} \geq 0,$$

$$t_{p+1}^P > 0_{p+1}. \quad (5.33)$$

- If both constraints are satisfied, place the solution in the potential set of valid solutions $\mathcal{K}$, advance elements and return to step (1).

- If (5.33) is not satisfied, check which element of the the allocation vector $t_p$ does not satisfy the constraint.
  - If any of the first $(p-1)$ entries of $t_p$ are less than zero, i.e., $t_p(1:p-1 < 0_{p-1})$, this constraint will not be satisfied for any other relay combinations in this “row”. Advance rows and return to item (1).
  - If the constraint is not satisfied by either of the last two items in the solution vector, discard the solution but check the other elements in the “row”.

4. From the set $\mathcal{K}$, pick the highest achievable rate and its corresponding time allocation.

The recursive algorithm given above simplifies the optimization problem in two ways:

1. It reduces the computation load of determining successive matrix inverses by writing each matrix inverse as a function of another, already known, matrix inverse, and two other matrices obtained through simple matrix multiplication.

2. It may eliminate some potential solutions by discarding relay combinations which do not satisfy constraints. For example, if the relay combination $(r_1, r_2, r_3)$ does not satisfy the constraints, the combination $(r_1, r_2, r_3, r_4)$ may be automatically discarded.
5.3.1 Complexity and Number of Operations

In the next chapter we will compare relay selection schemes partly on computational complexity. In this section we calculate this complexity, which also quantifies the computational savings of the recursive scheme presented above in Section 5.3.

The complexity of the recursive scheme is bounded by complexity of matrix multiplication. The number of operations (multiplications and additions) required in the product of two matrices of size \((m, n)\) and \((n, p)\) is

\[
2mpn - mp.
\] (5.34)

and the number of operations required for the product of a matrix of size \((m, n)\) with a square, size-\(n\) diagonal matrix is

\[
m \left( \sum_{k=0}^{n-1} k + \sum_{k=1}^{n} k \right) = mn^2.
\] (5.35)

We now calculate the number of operations required for each channel matrix of size \((q + 1)\), corresponding to the set \(Q'\) of \(q\) relays. The calculation of the matrix fundamental to the recursive algorithm,

\[
\left( L_{q+1}^{Q'} \right)^{-1} = -T_2^{-1}F_{2 \times (q-1)} \left( L_{q}^{Q} \right)^{-1} (1 : q-1, 1 : q-1).
\] (5.36)

requires a total of

\[
2q^2 + 2q + 1
\] (5.37)

operations, broken down as:

1. \(-T_2^{-1} \rightarrow 5\) operations,
2. \(-T_2^{-1}F_{2\times(q-1)} = A_{2\times(q-1)} \rightarrow 6(q-1)\) operations using (5.34),

3. \(A_{2\times(q-1)} (L_q^Q)^{-1} (1: q-1, 1: q-1) \rightarrow 2(q-1)^2\) operations, using (5.35).

From (5.31), the number of operations required to calculate \(R^Q\) is

\[q^2 + 2q + 4.\]  \hspace{1cm} (5.38)

Using (5.30), the number of operations required to update the solution vector is

\[1 + 2(q + 1) = 2q + 3.\]  \hspace{1cm} (5.39)

Summing (5.37), (5.38) and (5.39), we obtain the total number of operations required in one iteration of the resource allocation algorithm:

\[\text{Op}(q) = (2q^2 + 2q + 1) + (q^2 + 2q + 4) + (2q + 3) = 3q^2 + 6q + 8.\]  \hspace{1cm} (5.40)

Note that the complexity order of calculating each rate and solution vector is \(O(q^2)\). Without the recursion, this complexity is of order \(O(q^3)\), resulting from the inverse of the channel matrix. The recursion thus introduces significant savings in terms of complexity.

We now calculate the worst-case total number of operations required by the resource allocation algorithm. In the worst case, the algorithm cycles through \(2^N\) operations consisting of \(\binom{N}{q}\) sets of \(q\) relays which require \(3q^2 + 6q + 8\) operations. The total worst case number of operations is therefore

\[\sum_{q=1}^{N} \binom{N}{q} (3q^2 + 6q + 8).\]  \hspace{1cm} (5.41)

This calculation could be rendered more precise by accounting for the savings obtained in
Section 5.3 which eliminate some potential solutions a priori by discarding relay combinations known to not satisfy the constraints. The probability of this occurring for a particular channel constellation is unfortunately very difficult to compute, and we thus show only the worst-case result. We will use this result in the following chapter.

5.4 Simulations

In this section, we present results of the resource allocation scheme discussed in Section 5.2 for networks with 1 to 6 relays arranged linearly, and 4 and 9 nodes arranged in a grid. The figure of merit is the achievable rate with an outage probability of $10^{-3}$. A closed form expression for the outage probability of optimized cooperation is very complicated and beyond the scope of the thesis. The outage probability and rate are thus obtained numerically.

The relays are equispaced on a line between the source and destination, as in Figure 5.1, and we use an attenuation exponent of $p_a = 2.5$. This choice is motivated by the application of static mesh-nodes installed on posts; transmissions between such nodes should undergo little shadowing and a lower attenuation exponent. From 60000 fading realizations we obtain the cumulative density function of the instantaneous rate $F_R(r)$. The outage rate, as a function of the average end-to-end SNR, $\frac{P}{N_0W}$, is the rate for which the probability of outage is $10^{-3}$, i.e., $F_R^{-1}(10^{-3})$.

We begin with the outage rates for optimized cooperation in Figure 5.4, and non-optimized cooperation in Figure 5.5. The rate for the optimized cooperation is obtained from (5.19). Non-optimized cooperation uses equal time allocation, i.e., the rate for a particular relay set is simply the minimum of the mutual information at each node. Non-optimized cooperation, however, does optimally select relays by choosing the best, in terms of rate, of the $2^N$ relay combinations.
5.4. Simulations

Figure 5.4: Outage rate vs. SNR using $1, \ldots, 6$ potential relays and with resource allocation.

Figure 5.5: Outage rate vs. SNR using $1, \ldots, 6$ potential relays and without resource allocation.
5.4. Simulations

Comparing Figure 5.4 and Figure 5.5 shows that optimizing resources increases rates significantly, as expected. The outage rate increases as a function of nodes available to relay. We also note the typical phenomenon of decreasing marginal returns: the gains of adding each additional relay decreases with increasing number of relays.

Figures 5.6 and 5.7 show the average number of relays that are active from the set of potential relays for optimized and non-optimized cooperation. For each network size, this number is a decreasing function. Interestingly, the number of active relays decreases much faster for non-optimized as compared to optimized cooperation, suggesting that optimizing resources distributes the relaying burden more effectively.

To test the effect of geometry on the outage rate, we compare the rates obtained by optimizing resources and the placing relays on a line, as in Figure 5.4 to those obtained by placing the relays on a regular square grid. We number the relays in the grid in ascending order downwards and towards the source; a derivation of the optimal numbering is beyond the scope of this thesis. The results are demonstrated in Figure 5.8, where we place 4 and 9 relays on a $2 \times 2$ and $3 \times 3$ square grid. As shown in the figure, the rate for the linear constellation is significantly higher than that obtained by the grid constellation, suggesting that the path-loss incurred by traversing all the nodes laterally results in non-negligible performance loss.

We evaluate the performance of the numbering schemes discussed in Section 5.2.2 in Figure 5.9. The four schemes, including two based on average channel conditions and two based on instantaneous channel conditions, exhibit indistinguishable performance in terms of rate. There is an expected drop in rate with random numbering, though we note that this drop is no more than approximately 0.25 bits/channel use. The algorithm is thus quite robust to numbering schemes.

In Figure 5.9 we also compare the effect of numbering when used without resource allocation, and show only the case of instantaneous $S - R_k$ numbering and random numbering.
5.4. Simulations

Figure 5.6: Average number of active relays with 1,..., 6 potential relays and with resource allocation.

Figure 5.7: Average number of active relays with 1,..., 6 potential relays and without resource allocation.
Figure 5.8: Outage rate vs. SNR using resource allocation and for 4 and 9 relays arranged in a grid and in a line.

The improvement from instantaneous over random numbering in this case is less than 0.1 bits/channel use. The robustness of the numbering scheme thus increases by eliminating time optimization. To gain insight into this phenomenon, in Figure 5.10 we plot the average number of active users for the instantaneous and random numbering schemes with and without resource allocation. We first observe that the instantaneous numbering scheme uses more relays than the random numbering scheme when resource allocation is used, and that this difference is constant over the SNR region of interest. Without resource allocation, on the other hand, the number of relays used when using instantaneous and random numbering decreases quickly and is constant for SNR values higher than 10 dB. It is clear from this figure that the difference in rate performance between instantaneous and random numbering is an increasing function of the number of selected relays. Because so few relays are selected without resource allocation, the effect of the numbering scheme is negligible. The
5.5 Conclusions

In this chapter, we determined the optimal channel resource allocation, in terms of time allocation, for the $N$-node cooperative diversity, multihop network using decode-and-forward and independent codebooks. For a particular network, i.e., set of potential relays, the unique solution for a particular relay numbering scheme is obtained by taking the inverse of the triangular channel matrix, and the optimal solution is found by maximizing over the rate for each possible network, given its maximum size. Through simulations, however, the influence of the numbering scheme increases when time allocation is introduced, increasing the number of relays used for both numbering schemes and increasing the sensitivity to the numbering scheme. This sensitivity increases slowly, however, and is negligible for the various numbering schemes based on heuristics.

Figure 5.9: Outage rate vs. SNR using resource allocation and for various numbering schemes for 9 potential nodes arranged in a grid
5.5. Conclusions

Figure 5.10: Average number of active relays using resource allocation and with various numbering schemes for 9 potential nodes arranged in a grid

optimization is shown to be robust to the numbering scheme. We show that by exploiting the special structure of the channel matrix, the optimization can be performed in a recursive fashion which decreases the computation load of the channel matrix inverse and the number of required iterations. Node selection is inherent to the optimization strategy. Simulation results show significant gains in achievable rate due to resource allocation, but diminishing marginal returns as a function of network size. Furthermore, we show a significant benefit to arranging the nodes in a linear, as opposed to a grid, constellation. The next chapter in this thesis deals with decreasing the complexity of the optimal resource allocation algorithm through sub-optimal node selection.
Chapter 6

Relay Selection in Multihop Networks

In Chapter 5, we presented an optimal resource allocation algorithm which allocates time slots between \( N \) potentially cooperating nodes. This algorithm requires computationally intensive matrix inversions; in Section 5.3, however, we reduced the complexity of the resource allocation algorithm by eliminating matrix inversions of matrices of size greater than 2. Although successful in reducing the number of operations required at each iteration, the algorithm must still cycle through \( 2^N \) iterations, resulting exponential complexity in the size of the network. Since the resource allocation must be performed in real time on instantaneous channel conditions, for real networks with a large \( N \) it is thus reasonable to expect that this exponential complexity will render the implementation of the optimization algorithm impractical.

We thus propose a sub-optimal strategy which significantly reduces this complexity. This strategy is based on the idea that high rates, close to those obtained from the optimal algorithm, can be obtained by selecting only a subset of the nodes in the network and performing resource allocation only on those nodes. The proposed strategy is thus composed of two independent steps: (1) node selection, and (2) resource allocation on the selected nodes. Note that the second step may further reduce the final number of nodes selected.
for relaying. In this chapter, we address the issue of node selection for optimal resource allocation. In particular, we attempt to answer the questions: which and how many nodes should be selected, and what criteria should be used to select them. Note that here, unlike Chapter 5, we do not consider optimal selection - optimal selection is essentially a by-product of the fully optimal resource allocation algorithm in the previous chapter. The goal here is to reduce the computation load associated with implementing the resource allocation.

The answer to the node selection question is highly dependent on complexity requirements; clearly, the more nodes selected, the better the results. Each additional node, on the other hand, adds exponential complexity. The proposed node selection algorithms are thus evaluated in terms of performance-complexity trade-offs.

We propose four different relay selection schemes which are necessarily based on heuristics. The heuristics arise from the observation, discussed in Chapter 4, that in multihop networks with cooperation and sufficient network knowledge, Selection Cooperation is essentially routing using instantaneous channel conditions; conversely, routing is nothing more than node selection, with each particular routing scheme and corresponding metric selecting nodes according to different criteria. We therefore suggest implementing of the selection schemes proposed in this chapter using traditional routing methods such as Dijkstra’s algorithm [97].

The first two schemes are based on optimizing different metrics which emphasize sets of different channels in the network. These schemes offer good performance in the low/high SNR-regime, and poor performance in the high/low SNR-regime, thus motivating a search for a scheme which combines the principles of the two schemes and exhibits good performance over all SNR values. With this goal, we suggest two such schemes and find that both compare very favourably with optimal resource allocation. The rate-complexity trade-offs of the two schemes are very implementation-dependent, and thus we do not unequivocally suggest the use of one over the other, particularly since both are based on heuristics which
could potentially be improved with further research. We do, however, stress that selection of nodes prior to resource allocation is imperative in large networks, and that the metrics presented in this chapter offer good guidelines to further development of node selection algorithms.

The rest of the chapter is organized as follows. Section 6.1 presents the system model. In Section 6.2 we present the selection algorithms, and in Section 6.3 we discuss the measures of complexity used to evaluate the schemes. Section 6.4 presents the simulation results, and we conclude the chapter in Section 6.5.

6.1 System Model

Consider again a square network of $N$ nodes arranged in grid between the source and destination. The nodes are numbered in increasing order from source to destination, and in increasing order downwards. An example of such a network and its numbering for 16 nodes is shown in Figure 5.2. All other assumptions and parameters are the same as in Chapter 5.

6.2 Node Selection

As in all parts of this thesis, node selection is performed on instantaneous channel conditions. As in Chapter 5, we assume the existence of a central unit with knowledge of all channel states, or the ability to implement distributed algorithms using quantized feedback of channel states. We assume that this feedback can be accomplished using existing control packets, and that the differences in feedback between various node selection schemes are negligible.

Recall from Section 5.2.1 that the optimal solution is obtained by maximizing the rate
6.2. Node Selection

$R_k$ over the set of relays $\mathcal{R}_k$ in $\mathcal{K}$

$$R^* = \max_{\mathcal{R}_k \in \mathcal{K}} R_k,$$

(6.1)

where $\mathcal{K}$ is the subset of all potential relay combinations with valid solutions, and $\mathcal{R}_k = \{r_1, r_2, \ldots, r_{|k|}\}$ is a set of relays of size $|k|$, with $|k| \leq N$. In this chapter, we seek a metric whose optimization is less complex than the maximization of the rate $R_k$. We begin by considering a particular set $\mathcal{R}_p$ of $|p|$ relays, $\mathcal{R}_p = \{r_1, r_2, \ldots, r_{|p|}\}$. From all the channel capacities in the network of the relays in $\mathcal{R}_p$, we consider three sets:

1. Inter-relay capacities on the ‘direct path’ between source and destination through the relays in $\mathcal{R}_p$,

$$\mathcal{L}_{DP} = \{L_{sr_1}, L_{r_1r_2}, \ldots, L_{r_{p-1}r_p}\}.$$

(6.2)

2. For each relay in $\mathcal{R}_p$, the capacity between the source and relay,

$$\mathcal{L}_{SR} = \{L_{sr_1}, L_{sr_2}, \ldots, L_{sr_p}\}.$$

(6.3)

3. For each relay in $\mathcal{R}_p$, the capacity between the relay and destination,

$$\mathcal{L}_{RD} = \{L_{r_1d}, L_{r_2d}, \ldots, L_{r_rd}\}.$$

(6.4)

The above sets, chosen through extensive testing of the influence of various channel capacities on the final rate, represent the key channel capacities in the network. They include all capacities contributed by the relays in $\mathcal{R}_p$ except the inter-relay capacities not on the direct path, such as $L_{r_1r_3}$.

In the following sections, we derive four metrics and corresponding selection algorithms
using the channel capacities in the three sets. One observation in this thesis is that in multihop cooperative networks, routing and selection are equivalent, provided that routing is performed using instantaneous channel conditions. We thus propose to use routing concepts and algorithms, such as Dijkstra’s algorithm, to select nodes according to different metrics and selection criteria.

6.2.1 Direct-Path (DP) Selection

We first consider a selection scheme that emphasizes the importance of strong channels on the direct path between the source and destination. This is the path a multihop communication would take without cooperative diversity. Using relays exclusively in set $\mathcal{L}_{DP}$ in (6.2), we derive the metric which maximizes the non-cooperative direct-path rate, obtained when information is transmitted using multihop only.

**Proposition 1:** With resource allocation, $R_{NC-max}$, the maximum non-cooperative rate obtained using relays $(r_1, r_2, \ldots r_p)$ in $\mathcal{R}_p$, is

$$R_{NC-max} = \left( \frac{1}{L_{sr_1}} + \frac{1}{L_{r_1r_2}} + \ldots + \frac{1}{L_{r_{p-1}r_p}} \right)^{-1}. \quad (6.5)$$

**Proof:** From equation (5.21) in Chapter 5, the maximum achievable rate $R_{max}$, given a set of $p$ relays, is

$$R_{max} = \frac{1}{1^T L_{p+1}^{-1} 1_{p+1}}. \quad (6.6)$$
Here, because the relays are not cooperating and only the channels on the direct path between source and destination are considered, the channel matrix $L_{p+1}$ has the form

$$L_p = \begin{bmatrix}
L_{sr_1} & 0 & 0 & 0 \\
0 & L_{r_1r_2} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & L_{r_{p-1}r_p}
\end{bmatrix},$$  \hspace{1cm} (6.7)

and (6.6) reduces to (6.5).

Without cooperation between the relays but resource allocation along the direct path, therefore, the achievable rate $R_{NC-max}$ is maximized by maximizing (6.5). The goal of Direct-Path (DP) Selection is to maximize $R_{NC-max}$; the algorithm therefore attempts to find the set of relays that maximizes (6.5). To facilitate implementation, we note that maximizing (6.5) is equivalent to minimizing the metric

$$g_{DP}(\mathcal{R}_p) = \left( \frac{1}{L_{sr_1}} + \frac{1}{L_{r_1r_2}} + \ldots + \frac{1}{L_{r_{p-1}r_p}} \right).$$  \hspace{1cm} (6.8)

Because the metric (6.8) is additive while (6.5) is not, this observation is helpful in that it easier to implement a distributed algorithm on the basis of an additive metric. More details about implementation issues and complexity are given at the end of this chapter.

Direct-Path Selection is thus the minimization of the metric $g_{DP}(\mathcal{R}_p)$ over the relay sets $\mathcal{R}_p$ in $\mathcal{M}$, where $\mathcal{M}$ is the set of all $2^N$ relay combinations.

$$\mathcal{R}_{DP} = \arg \min_{\mathcal{R}_p \in \mathcal{M}} g_{DP}(\mathcal{R}_p)$$  \hspace{1cm} (6.9)

The search for the minimum path is over all $2^N$ relay combinations of the set $\mathcal{M}$, instead of the reduced set $\mathcal{K}$ with valid solutions. This is because it is impossible to tell a priori
which solutions will be valid. Once the chosen relay set \( R_{DP} \) is input into the optimization algorithm, however, the invalid solutions will be discarded.

Because this metric maximizes the achievable rate over all relay sets, it selects the optimal number of relays by naturally penalizing for the number of hops. In [79], Chen et al. address the problem of multihop routing in a non-cooperative network without resource allocation. They propose the metric

\[
(1 + \alpha \frac{1}{L_{sr_1}}) + \sum_{R_p} \left( 1 + \alpha \frac{1}{L_{r_k r_{k+1}}} \right) + \left( 1 + \alpha \frac{1}{L_{rd}} \right),
\]

(6.10)

to find the suboptimal multihop route between source and destination. Note that unlike here, in their work the path chosen by the metric is the final path, without possibility of discarding relays later on. The metric \( g_{DP} \) proposed here is a special case of the metric proposed in [79], with \( \alpha = 1 \). The proposition, therefore, sheds light on why the metric in [79] performs well: it is a generalization of the metric that maximizes the non-cooperative, multihop rate with resource allocation.

### Implementation Complexity of DP Selection

Here, we briefly discuss the implementation complexity of DP Selection. Since the motivation to explore selection is to reduce the complexity of optimal resource allocation, we also calculate the number of operations required by the algorithm.

As in [79], the optimization in (6.9) can be efficiently implemented through an algorithm such as Dijkstra’s algorithm. The algorithm requires each node to determine the best path from itself to the destination by searching through the best path to the destination of other
6.2. Node Selection

nodes. The metric in (6.8) is isotonic, i.e.,

\[ g_{\text{DP}}(\mathcal{R}_p) < g_{\text{DP}}(\mathcal{R}_k) \implies g_{\text{DP}}(r_u \cup \mathcal{R}_p) < g_{\text{DP}}(r_u \cup \mathcal{R}_k), \quad \forall \mathcal{R}_p, \mathcal{R}_k \in \mathcal{M}, r_u \notin \mathcal{R}_p, \mathcal{R}_k, \quad (6.11) \]

and thus Dijkstra’s algorithm is guaranteed to yield optimum path in DP sense [97].

In general, the algorithm is of \( O(N^2) \) complexity [101]. In this thesis we implement a simplification of Dijkstra’s algorithm by numbering nodes as in Figure 5.2 and allowing node \( n_j \) to search only through other nodes \( n_k, j < k \leq N \). This is justified because, on average, channel rates increase with increasing node numbers, and the probability of a node \( n_j \) finding a path through node \( n_k, n < j \), is small. Once every node has the best path to the destination, the source finds the best path by searching through the best path of all the nodes. Labeling the metric (6.8) at each node \( n_k \) as \( g_{\text{DP}_k} \), the update step of each node can be described as

1. Node \( n_j \) calculates the metric in (6.8) obtained using its own rate to the destination, i.e., \( g_{\text{DP}_j} = 1/L_{n_j d} \).

2. For each node \( n_k, j < k \leq N \), if \( 1/L_{n_j n_k} + g_{\text{DP}_k} < g_{\text{DP}_j} \implies g_{\text{DP}_j} = 1/L_{n_j n_k} + g_{\text{DP}_k} \).

Each node \( j \) thus performs \( N - j \) additions and \( N - j \) comparisons, and the source performs \( N \) additions and comparisons. The total number of operations performed is thus

\[ 2 \sum_{j=1}^{N} (N - j) = 2 \sum_{j=1}^{N} j = N(N + 1). \quad (6.12) \]

6.2.2 Source-Relay-Destination (SRD) Selection

In this section, we neglect the contribution of the direct-path channels and propose a metric using relays from sets \( \mathcal{L}_{\text{SR}} \) and \( \mathcal{L}_{\text{RD}} \), defined in (6.3) and (6.4), respectively.
6.2. Node Selection

Ideally, this metric should be derived from (5.21), the maximum rate obtained by using all $N$ relays in the network. Unfortunately (5.21) involves a large matrix inverse and cannot be implemented without excessive complexity. We therefore propose a heuristic with little theoretical foundation but which, as we will see, performs very well in high-SNR conditions.

Given the set $\mathcal{R}_k$ comprising each relay in the network, i.e.,

$$\mathcal{R}_k = (r_1, r_2, \ldots, r_{2^N}), \quad (6.13)$$

we propose the metric

$$g_{\text{SRD}} = L_{sr_k} + L_{rd_k}, \quad \forall r_k \in \mathcal{R}_k. \quad (6.14)$$

The metric in (6.14) emphasizes the contribution of the capacities between each relay and the source and destination. This metric cannot be implemented in the same way as the DP metric in (6.8), since simply maximizing (6.14) over all relays would select every relay in the system. We therefore propose to first fix the number of relays to select, $L$, then simply to select the $L$ relays which maximize (6.14). SRD Selection can thus be described as follows:

1. Calculate $g_{\text{SRD}}$ in (6.14) for each relay $r_k$ in the network,

2. Select $L$ relays which maximize $g_{\text{SRD}},$

3. Sort the $L$ relays in increasing order of source-relay channel. These relays form the set $\mathcal{R}_{\text{SRD}}.$

Before discussing the sorting in step 3, it is worth commenting on the number of relays to select. The parameter $L$ will determine the trade-off between performance and complexity, and will be investigated further in Section 6.3 and Section 6.4. Unfortunately, choosing a good value for $L$ has little justification beyond experimentally examining trade-offs. The
optimal resource allocation algorithm always performs better with increasing \( L \); on the other hand, increasing \( L \) adds exponential computational complexity to the algorithm which performs \( 2^L \) iterations over all combinations of relays. The parameter \( L \) thus determines the performance of SRD Selection as well as the resulting complexity of the resource allocation algorithm.

We now return to the issue of sorting. Because the structure of the channel matrix imposes causality in the transmission of the nodes, it is reasonable to expect that the numbering of the selected relays will have an impact on rate. The results in Section 5.4 show that this impact is negligible for various “reasonable” numbering schemes of nodes arranged in a grid, and small even for a random numbering scheme. As we will see in Section 6.4, this is not the case when only a subset of \( L \) relays is selected, in which case sorting the \( L \) relays becomes important, such that the \( L \) selected relays are “reasonably” numbered at the input to the resource allocation algorithm. This dependence on numbering in the selection algorithm is not surprising: if only a small subset of nodes from the network is available, it is important that the nodes with the best rate to the source participate in the transmission to other nodes. If the selected nodes are not “reasonably” numbered, this may not happen.

**Complexity**

One advantage of SRD Selection is the simplicity of its distributed implementation, as compared to DP Selection. In DP Selection, if implemented using a distributed algorithm like Dijsktra, each relay must know the best path to the destination and the list of relay nodes to get there, and this process involves iterating through all other (allowable) nodes; in SRD Selection, each relay needs knowledge only of its rate to the source and destination, which it sums, resulting in a total of \( N \) operations. The centralizing unit then selects \( L \) nodes with the best metric, in \( \sum_{k=0}^{L}(N - k) = NL - \frac{1}{2}L^2 + L \) operations. The total number of operations required is thus \( N(L + 1) - \frac{1}{2}L(L + 1) \).
6.2.3 Combining DP and SRD: Simple-Combination (SC)

Direct-Path Selection and SRD Selection both choose relays based on different channel sets. It is natural, then, to simply combine both sets of relays into one. The goal here is to combine the benefits of both DP and SRD Selection. Recalling that \( R_{DP} \) and \( R_{SRD} \) are the sets of relays chosen by DP Selection and SRD Selection algorithms and accounting for the possibility that DP and SRD Selection may choose the same relay(s), SC gives

\[
R_{SC} = R_{DP} \cup R_{SRD}.
\]  

(6.15)

The size of the new relay set, \( |R_{SC}| \), depends on the size of the sets \( R_{DP} \) and \( R_{DP} \), as well as the probability of overlap. We define the parameter \( L_s \) as the maximum size of the set \( |R_{SC}| \). The number of relays selected by SRD Selection, \( |R_{SRD}| \), when used for the purposes of SC-Selection, is then \( \max\{0, L_s - |R_{DP}|\} \). Because the SC uses relays from DP and SRD, the overhead and complexity is the sum of both schemes. The number of operations required is thus

\[
N^2 + N + N(L + 1) - \frac{1}{2}L_s(L_s + 1) = N^2 + N(L_s + 2) - \frac{1}{2}L_s(L_s + 1).
\]  

(6.16)

6.2.4 Combining DP and SRD: Integrated-Combination (SC)

In this section, we propose a metric which combines DP Selection and SRD Selection in an integrated fashion, i.e., instead of selecting relays of both schemes, we select relays that jointly maximize the direct-path rate and the source-relay and relay-destination rates. As in SC Selection, we aim to obtain the benefits of both DP and SRD Selection. The underlying motivation to the integrated metric is the potential inefficiency of SC Selection which simply uses the relays selected by both schemes; by integrating the DP and SRD metrics, it may be
possible to select fewer relays with similar performance.

For example, suppose that DP Selection choses relays \{r_1, r_2\} with strong direct-path channels, and SRD Selection chosen relays \{r_3, r_4\} with strong source-relay-destination channels. SC Selection then selects relays \{r_1, r_2, r_3, r_4\} and shifts responsibility onto the resource allocation algorithm to allocate resources between these relays (potentially discarding some) in such a way as to maximize the benefit of the channel strength contributed by each relay. The goal of IC Selection is to design a metric that will result in a selection of fewer relays than SC Selection, but with similar characteristics. This set may or may not include some relays chosen by SC Selection. IC may, for example, result in the set \{r_1, r_6, r_4\}, with \(r_1\) and \(r_4\) representing relays with a strong direct-path and source-relay-destination channel, respectively, and \(r_6\) with somewhat strong direct-path and source-relay-destination channels.

In this regard, this propose a metric similar to the DP metric in (6.8), in that it is used in what would traditionally be referred to as routing with the purpose of node selection. The proposed metric, \(g_{IC}\), is thus a weighted sum of the variations of the two metrics \(g_{DP}\) and \(g_{SRD}\):

\[
g_{IC}(\mathcal{R}_p) = \left\{ \frac{1}{L_{sr_1}} + \frac{1}{L_{r_1r_2}} + \ldots + \frac{1}{L_{r_{p-1}r_p}} \right\}^{-1} + \frac{\beta}{\sqrt{|\mathcal{R}_p|} + 1} \sum_{\mathcal{R}_p} (L_{sr_p} + L_{r_{p}d}) \right\}, \quad (6.17)
\]

and the set of relays \(\mathcal{R}_{IC}\) is selected by maximizing the metric over all subsets of relays, \(\mathcal{R}_p\), in the set \(\mathcal{M}\),

\[
\mathcal{R}_{IC} = \arg \max_{\mathcal{R}_p \in \mathcal{M}} g_{IC}(\mathcal{R}_p). \quad (6.18)
\]

To justify (6.17), re-write the metric as

\[
g_{IC}(\mathcal{R}_p) = g'_{DP} + g'_{SRD}. \quad (6.19)
\]
where $g'_\text{DP}$ and $g'_\text{SRD}$ are

$$
g'_\text{DP} = (g_{\text{DP}})^{-1},
$$

$$
g'_\text{SRD} = \frac{1}{\sqrt{|R_p| + 1}} g_{\text{SRD}}.
$$

The IC Selection algorithm maximizes its metric; DP Selection, on the other hand, minimizes its metric. The optimization of DP Selection, then, implemented in IC Selection, requires the inverse of the DP metric, as in (6.21).

The metric $g'_\text{DP}$ naturally penalizes for the number of hops and self-selects the number of relays. The metric $g_{\text{SRD}}$, on the other hand, does not: it is maximized by using all relays in the network. Note that in SRD Selection this is not an issue because the controller simply selects $L$ relays; here, on the other hand, we require that the number of relays selected be controlled by the algorithm through the metric. $g_{\text{SRD}}$ should therefore be a decreasing function of the number of relays used in the path. This function is, unfortunately, very difficult to obtain theoretically. We thus choose the weighing by the inverse of the square root of number of hops on the path defined by the relays in $R_p$, and show in Section 6.4 that it performs well in simulations. Note, however, that this metric is a heuristic and could be improved through further study.

Finally, the metrics $g'_\text{DP}$ and $g'_\text{SRD}$ are summed through the coefficient $\beta$ which controls their relative importance. To simplify implementation, $\beta$ should be a constant for all values of SNR, and can be chosen through simulations. Because $g'_\text{SRD}$ selects more relays than $g'_\text{DP}$ (this fact is not intuitively obvious but can be ascertained through simulations), increasing $\beta$ increases performance and complexity by selecting more relays. Choosing $\beta$ is thus a question of trading off performance and rate.
6.2. Node Selection

Implementation and Complexity

Like DP Selection, the IC Selection scheme can be implemented with an algorithm such as Dijkstra’s algorithm. As the metric \( g_{IC} \) does not satisfy (6.11), however, it is not isotonic and thus Dijkstra’s algorithm is not guaranteed to yield the optimum path. Because this algorithm is only the first step in an optimization procedure, this is not necessarily a problem: the algorithm picks “good” nodes that result in good performance after optimization by resource allocation; in this respect, the metric sets the first steps and finding the optimal solution in terms of the metric is not the goal. At the same time, note that IC-algorithm is a heuristic, and better heuristics can certainly be developed with further research. Low-complexity implementation such as distributed implementation through Dijksstrra, however, is important: implementing this algorithm by testing all \( 2^N \) paths would result in complexity comparable to the complexity of the resource allocation algorithm, and defeat the purpose of this preliminary step.

The distributed algorithm can be described as follows. Node \( n_j \) calculates the metric \( g_{IC_j} \) in (6.17) obtained using the rate from \( n_j \) to the destination, i.e., \( g_{IC_j} = 1/L_{n_jd} \). For each node \( n_k, j < k \leq N \):

1. Determine \( g_{DP'_{jk}} \) in (6.21) by taking the inverse of the stored metric \( g_{DP_k} \) at node \( n_k \) and adding \( 1/L_{n_jn_k} \).

2. Calculate the square root of the potential number of hops when selecting node \( n_k \), potentially using a look-up table to decrease the number of operations.

3. Determine \( g_{SRD'_{jk}} \) in (6.21) by calculating \( L_{sn_k} + L_{n_kd} \), adding to stored metric \( g_{SRD_k} \) and scaling by \( \beta \) and the square root of the potential number of hops.

4. Determine \( g_{IC_{jk}} = g_{DP'_{jk}} + g_{SRD'_{jk}} \).

5. If \( g_{IC_{jk}} > g_{IC_j} \) then \( g_{IC_j} = g_{IC_{jk}} \).
After iterating through all the nodes, node $n_j$ inverts and stores the path metric $g_{DP,j}$. The number of operations required by each node $n_j$ is thus $1 + 7(N - j)$ and the total number of operations required by the algorithm is

$$N + 7 \sum_{j=1}^{N} j = N + \frac{7}{2}(N^2 + N) = \frac{7}{2}N^2 + \frac{9}{2}N$$

(6.22)

### 6.3 Complexity of Selection Algorithms combined with Resource Allocation

The algorithms presented in this chapter reduce the computational complexity of the optimal resource allocation algorithm. To evaluate these algorithms, we quantify this complexity in terms of number of additions and multiplications. The complexity is derived from two steps:

1. The complexity required to select a set of $m$ relays. The number of additions and multiplications is given at the end each of each section describing the algorithm, and depends on the size of the network, $N$ and, if relevant, $L$.

2. The complexity, resulting from computing the optimal resource allocation using $m$ relays selected by each algorithm. This complexity differs for each selection algorithm only through the number of selected nodes $m$, and can be obtained from (5.41) in Chapter 5, with $N$ replaced by $m$. We note here that (5.41) counts the worst-case number of operations, as it counts all $2^m$ possibilities, ignoring the potential savings through elimination of certain relay combinations. However, because the $m$ chosen relays constitute a small subset of the network of $N$ nodes and are already strong “candidates”, it is unlikely that many of these relay combinations would be discarded. Equation (5.41) is thus a close approximation to the time allocation complexity.
The algorithms presented in this chapter are thus evaluated according to the complexity requirements arising from both steps. This is further discussed in Section 6.4.

### 6.4 Simulations

In this section, we evaluate the performance of the selection algorithms discussed in Section 6.2 for networks with 9, to 16 relays and 25 nodes. Each set of relays is arranged in a square grid. For each algorithm, we obtain the set of selected relays, which we then use as input to the resource allocation algorithm described in Chapter 5. The simulation set-up is identical to that of Section 5.4. We summarize the set-up below for convenience.

The figure of merit is the achievable rate with an outage probability of $10^{-3}$. From 60,000 fading realizations we obtain the cumulative density function of the instantaneous rate $F_R(r)$. The outage rate, as a function of the average end-to-end signal to noise ratio, is the rate for which the probability of outage is $10^{-3}$, i.e., $F_R^{-1}(10^{-3})$. The relays are arranged and numbered on a grid between the source and destination, as in Figure 5.2, and we use an attenuation exponent of $p = 2.5$. For networks with 9 and 16 nodes, we compare the outage rates to those obtained from optimal resource allocation in (5.19). For the network with 25 nodes, the implementation of the optimal resource allocation algorithm is too complex, and we compare the performance of the selection algorithms only.

Figure 6.1 shows the outage rate as a function of SNR in a $4 \times 4$ network for DP Selection and SRD Selection with 6 nodes selected and sorted according to source-relay rates. It is immediately clear that SRD Selection performs very well at high SNR, and DP Selection at low SNR. At first glance, it may seem that SRD performs well even at low-SNR; its rate loss, however, is approximately 30% at $-10$ dB. Because our goal here is to maximize performance in the entire SNR regime (recall that “SNR” here refers to the signal-to-noise-ratio at the destination; it is thus quite possible that the SNR may be low in certain transmissions),
6.4. Simulations

These results motivate our search for a metric which combines the good performance of DP and SRD Selection in the low and high SNR regimes.

Before moving on to SC and IC Selection, we first discuss the effects of sorting relays on the performance of SRD Selection, as demonstrated in Figure 6.2. Here, we compare the outage probability obtained with SRD Selection using sorted and unsorted relays, with the sorting performed using Instantaneous $S - R_k$ numbering, as discussed in Section 6.2.3. This plot illustrates that sorting has a very significant impact on performance. This result is not surprising, as SRD Selection does not naturally sort the relays in any way, as does DP Selection; as discussed Section 6.2.3, with only a few nodes selected it is imperative that nodes with good channels to the source can transmit first and help the relays with good channels to the destination.

We compare the performance of SC, IC and SRD Selection in Figures 6.3 and 6.4, which
6.4. Simulations

Figure 6.2: Outage rate vs SNR for Optimal Resource Allocation, SRD Selection with sorted and unsorted relays in a network of 16 nodes

Figure 6.3: Outage rate vs SNR for Optimal Resource Allocation, SRD Selection, SC and IC Selection in a network of 16 nodes
6.4. Simulations

Figure 6.4: Average number of operations vs SNR for Optimal Resource Allocation, SRD Selection, SC and IC Selection in a network of 16 nodes

Figure 6.5: Outage rate vs. average number operations for SC Selection and IC Selection at 20 dB, 15 dB, 10 dB and −10 dB
show the outage rate and average number of operations required to obtain the outage rate. SRD and SC Selection use 6 and a maximum of 6 relays, respectively.

The main conclusions from these plots arise from a comparison of IC and SC Selection. First, however, we revisit the issue of sorting. We point to the discrepancy in the outage rate of the SRD SC Selection, a somewhat unexpected result given that the relays chosen by SC include all relays chosen by SRD. It would thus be reasonable to expect SC to perform at least as well as SRD for the same number of selected relays. Figure 6.3 demonstrates, however, that SRD actually outperforms SC for all SNR values greater than 0 dB. The explanation for this reverts back to sorting.

In SRD, all selected relays are sorted according to their source-relay rates. In SC, however, only the relays making up the set formed from SRD Selection are sorted; this set is then appended to the set obtained from DP Selection. This in itself would not cause a performance degradation were it not for the fact that DP and SRD sometimes select the same relays; a decision regarding where to "put" this relay in relation to other relays is not straightforward. Note also that sorting the entire set would increase the performance at high SNR, but lower the performance at low SNR. We conclude, then, that without a more detailed study of optimized sorting, the performance of SRD Selection cannot be reached using any scheme which simply appends relays which may contain duplicates.

We now compare the outage probabilities obtained with SC and IC Selection with parameter $\beta = 0.17$. (More details about setting $\beta$ can be found in the paragraph below). IC Selection tracks the performance of SC-Selection up to SNR of 10 dB and at 20 dB IC-Combination is 4.7% from SC-Selection. The complexity of IC Selection, on the other hand, is $2-3$ times lower than that of SC-Selection over the SNR range 5-20 dB. This low computational complexity of IC Selection may be attractive when considering implementation in real-time applications. We highlight, however, that both IC and SC-Selection perform very favorably compared to the Optimal Resource Allocation algorithm: the outage rate
of both suboptimal algorithms tracks that of the optimal algorithm closely at significantly lower computation load.

The above results were obtained by IC and SC Selection for a fixed $\beta$ ($= 0.17$) and fixed maximum number of relays ($= 6$) selected by SC-Selection. In Figure 6.5, we verify this result over various $\beta$s and number of relays. For a fair comparison, we show the outage rates for both schemes as a function of average number of operations for SNR values of $-10$ dB, $0$ dB, $10$ dB and $20$ dB. The results are a function of $\beta$ for IC, and the number of selected relays for SC. For IC, the line is obtained using a starting value of $\beta = 0.1$, and increased by $0.05$. For SC, the line is obtained for number of relays $2, 3, \ldots, 7$. From the figure, IC Selection outperforms SC-Selection at low complexity and SNR values, and vice-versa. The selection between one scheme and the other is thus highly dependent on complexity constraints and data requirements in various SNR regimes.

To verify the above conclusions for other networks, we demonstrate the rate and complexity obtained using the selection algorithms in networks of 9 and 25 nodes. Figure 6.6 demonstrates the outage rate as a function of SNR for Optimal Resource Allocation, SRD (again using 6 relays), SC and IC. Because SRD Selection selects 6 out of 9 nodes, its performance in terms of rate is almost identical to that of the optimal resource allocation algorithm. This suggests that for smaller networks, it is advisable to choose a small subset of nodes based only on relay-destination and source-relay channels.

We demonstrate the performance of the selection algorithms in a network of 25 nodes in Figures 6.7, 6.8 and 6.9. As with the 16-node network, SRD and SC select 6 relays. IC Selection uses $\beta = 0.14$ (obtained experimentally). Figure 6.7 shows the effectiveness of SC, which closely tracks the outage rate of SRD Selection. IC and SC outage rates are comparable until $10$ dB, and IC starts to under-perform SC in the higher SNR-regime. Unlike in the 16-node network, however, IC exhibits higher computational complexity in the lower SNR-regime, as is seen in Figure 6.8. This suggests that in larger networks, SC-
Selection may offer better performance-complexity trade-offs. This is verified in Figure 6.9, which shows the outage rates as a function of average number of operations required to achieve a specific rate for IC and SC Selection. For IC Selection, the curve is obtained with $\beta = [0.1, 0.15, 0.2, 0.25, 0.3]$, and for SC-Selection, for maximum number of relays between 2 and 7. The figure demonstrates that in the higher SNR-regime, SC can always offer higher rates at a specific computational complexity, and thus offers a better trade-off than IC Selection. This conclusion is reversed in the lower SNR regime, where for 0 dB, it is IC that offers higher rates for a fixed number of operations. A performance comparison of IC and SC selection is thus rendered difficult by its dependency on SNR and network size, as well as the particular parameters used in the algorithms. We stress however, that both algorithms drastically decrease the computational load of the resource allocation algorithm: with SC and IC Selection, for 25-node networks the average number of operations is on the
order of $10^3$; without selection, the number of operations required by the resource allocation algorithm, computed using (5.41), is $1.9 \times 10^{10}$.
Figure 6.8: Average number of operations vs SNR for Optimal Resource Allocation, SRD Selection, SC Selection, IC Selection and DP Selection in a network of 25 nodes

Figure 6.9: Outage rate vs. average number operations for SC Selection and IC Selection Selection at 20 dB, 15 dB, 10 dB and $-10\text{ dB}$
6.5 Conclusions

The goal of this chapter is to develop node selection heuristics to avoid the huge computation load of the resource allocation algorithm in Chapter 5. In this chapter, we presented four node-selection heuristics which select relays based on various channel metrics. The resource allocation algorithm is performed on the selected relays only, significantly reducing the complexity of resource allocation by decreasing the number of operations required to run the algorithm. The use of heuristics, unfortunately, makes this chapter largely a simulation exercise.

Our results show that in selecting channels, the inter-relay path is important in the low-SNR regime, and the channels between the relays and the source and destination nodes are important in the high-SNR regime. Consequently, the first two schemes, DP and SRD Selection, which emphasize the direct-path and the source-relay-destination channels, perform well in the low- and high-SNR regime, respectively.

To design an algorithm that performs well in both regimes, we proposed two selection heuristics: SC- and IC Selection. SC selects relays by combining the relays selected by DP and SRD Selection, while IC selects the relays by integrating the combining of DP and SRD into one metric. Both algorithms perform very well, in that they closely track the performance of the optimal resource allocation algorithm at significantly lower computational loads. The choice between each scheme, however, is very implementation dependent and the best choice varies depending on the computational complexity limitations and performance requirements. In addition, both SC- and IC Selection require determining the algorithm parameters: the maximum number of relays to select for SC-Selection, and the $\beta$ parameter for IC Selection. We therefore do not advocate for one scheme versus another, since both schemes offer advantages and disadvantage in various scenarios. Furthermore, both schemes are based on heuristics which could, potentially, be improved with further study. We do
stress, however, that the complexity of the resource allocation algorithm, particularly in large networks, requires the implementation of a selection scheme, and this scheme should be based on some form of combination of relays which maximize the direct-path and source-relay-destination channels.
Chapter 7

Conclusions

In this thesis we have addressed the issue of implementing cooperative diversity in decode-and-forward networks with multiple relays. The theme of this work is selection in large-scale networks.

We began with addressing the problem of organizing relay transmissions in parallel-relay multi-source networks, where a source communicates with a destination with the help of multiple relays (which do not communicate with each other). Previous works have assumed that all available relays in the network participate in the transmission. Here, we have proposed Selection Cooperation, whereby a single (best) relay is chosen for the transmission. We have shown that, in addition to being easier implement, Selection Cooperation outperforms DSTC for most network sizes.

In bridging the gap to generalized-relay networks where relays can communicate with each other, we explored the relationship between cooperation and channel-adaptive routing in a network where only adjacent relays communicate with each other through multihops. Through simulations, we illustrated that cooperation has no effect if implemented independently from a routing algorithm, and that gains can only be achieved if cooperation and routing are jointly designed. This has motivated a search for optimal multihop and cooper-
For a generalized network, where each relay can potentially communicate with all other relays, we proposed a resource allocation scheme which optimally selects relays and allocates transmission times between the selected relays. To reduce the computational complexity of the algorithm, we proposed a recursion which reduces the complexity of each step from $O(k^3)$ to $O(k^2)$, with $k$ the size of the relay set considered in each step.

The complexity of the optimal resource allocation is likely to render its implementation impractical. To overcome this problem, we proposed selecting only a subset of nodes to input into the resource allocation algorithm. In this regard, we presented several selection heuristics based on emphasizing different channels in the network. Observing that selection and channel-aware routing are equivalent concepts, we proposed implementing the heuristics using traditional routing algorithms such as Dijkstra’s algorithm. The heuristics can almost certainly be improved with further research; we highlight, however, that they are successful at significantly reducing the complexity of the optimal resource allocation algorithm while exhibiting good performance in terms of outage rate.

The over-arching theme of this thesis is selection. Throughout this thesis, we argue that selection is a practical and effective means of implementing cooperative diversity in all networks where implementing optimal algorithms is impractical. For parallel-relay networks, the selection algorithm should select a single relay; in generalized-relay networks, the algorithm should select a small set of relays.

The work in this thesis could be extended in several ways.

- The conclusions reached in Chapters 4, 5 and 6 were based on simulations. The channel model used in the simulations was Rayleigh fading with an attenuation exponent $p_a = 2.5$. We believe that this model is a good approximation for the channels found in mesh networks, where nodes are located on lampposts. It would be informative,
however, to simulate the schemes and algorithms proposed in this thesis using more realistic channel models, determined using experimentation on mesh networks.

- The resource allocation scheme in Chapter 5 is developed for peak-power constraints. This is appropriate for nodes transmitting at the peak of their power capabilities. It might be advantageous, however, to generalize the resource allocation to include power, using either short or long-term power constraints.

- Similarly, the work could be generalized by allowing multiple users in the system to share system resources and solving the resource allocation problem to maximize sum rates.

- The node selection algorithms developed in Chapter 6 are based on heuristics and could be improved with further research. The best parameter $\beta$ should be chosen for each network size, or for all networks. Furthermore, unlike the optimal algorithm, the selection schemes are sensitive to node numbering. We therefore suggest the design of an effective numbering scheme that improves the performance node-selection.

- This thesis makes a connection between node selection and routing. A similar connection could be made between selection and scheduling, as in OFDMA carrier-assignment allocation problems. This connection could be further investigated to determine whether routing decisions could be made by adapting known scheduling algorithms such as the Hungarian algorithm.
Publications

The publications arising from this thesis are:

**Published**


**Submitted**

Appendix A

Outage probability of selection: single source-destination pair

In this section, we develop the high-SNR approximation to the outage probability $\Pr[I_{sel} < R]$ for Selection Cooperation in a single user system. As in [11], the derivation uses the total probability law:

$$\Pr[I_{sel} < R] = \sum_{\mathcal{D}(s)} \Pr[\mathcal{D}(s)] \Pr[I_{sel} < R|\mathcal{D}(s)]. \quad (A.1)$$

Probability of the Decoding Set

This derivation is given in [11] and is summarized here for completeness. Relays are in the decoding set if the source-relay channel satisfies

$$\Pr[r \in \mathcal{D}(s)] = \Pr[|a_{s,r}|^2 > (2^{2R} - 1)/\text{SNR}] = \exp\left[-\lambda_{s,r}(2^{2R} - 1)/\text{SNR}\right].$$

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Because each relay makes independent decisions and the channel fading realizations are independent, the probability of a specific decoding set is

\[
\Pr[D(s)] = \prod_{r_i \in D(s)} \exp[-\lambda_{s,r_i}(2^2R - 1)/\text{SNR}] \times \prod_{r_i \notin D(s)} (1 - \exp[-\lambda_{s,r_i}(2^2R - 1)/\text{SNR}])
\]

\[
\approx \left[\frac{2^2R - 1}{\text{SNR}}\right]^{N - |D(s)|} \times \prod_{r_i \notin D(s)} \lambda_{s,r_i}, \quad (A.2)
\]

where the final approximation in (A.2) is valid at high-SNR.

**Outage Probability Conditioned on the Decoding Set**

This section develops the outage probability for a single source node \(s\) communicating with destination \(d\) using a relay chosen among \(N\) relays from the decoding set. The chosen relay has the best relay-destination channel, i.e., largest \(|a_{r,d}|\).

Define the random variables \(X\) and \(Y\) as

\[
X = \max_{r_i \in D(s)} \{|a_{r_i,d}|^2\}; \quad i = 1 \ldots |D(s)|, \quad (A.3)
\]

\[
\Rightarrow F_X(x) = \prod_{i=1}^{|D(s)|} (1 - \exp[-\lambda_{r_i,d}y]), \quad (A.4)
\]

and

\[
Y = |a_{s,d}|^2, \quad (A.5)
\]

\[
\Rightarrow f_Y(y) = \lambda_{s,d} \exp[-\lambda_{s,d}y], \quad (A.6)
\]

where the cumulative distribution function of \(X\) in (A.4) is derived using the independence of \(|a_{r_i,d}|^2 \forall i\). For the proposed selection scheme, the channel mutual information is thus

\[
I_{\text{sel}} = \frac{1}{2} \log(1 + \text{SNR}(X + Y)). \quad (A.7)
\]

Let \(b = (2^2R - 1)/\text{SNR}\). Since \(b \to 0\) as \(\text{SNR} \to \infty\), coupled with the approximation
\( e^x \simeq (1 + x) \) as \( x \to 0 \), using the binomial expansion yields:

\[
\Rightarrow \Pr[I_{sel} < R|D(s)] = \Pr[(X + Y) < b] = \int_0^b \prod_{i=1}^{\left|D(s)\right|} (1 - \exp[-\lambda_{r_i,d}(b - y)]) \lambda_{s,d} \exp[-\lambda_{s,d}y]dy
\]

\[= \lambda_{s,d} \prod_{i=1}^{\left|D(s)\right|} \lambda_{r_i,d} \int_0^b (b - y)^{\left|D(s)\right|} (1 - \lambda_{s,d}y)dy\]

\[= \lambda_{s,d} \prod_{i=1}^{\left|D(s)\right|} \lambda_{r_i,d} \times \int_0^b \sum_{j=0}^{\left|D(s)\right|} \left( b^{\left|D(s)\right|} - j(-y)^j (1 - \lambda_{s,d}y)dy \right), \]

where \( \binom{n}{p} = \frac{n!}{p!(n-p)!} \). After solving the integral, some manipulations and the use of identity 0.155 from [102], this expression reduces to

\[
\Pr[I_{sel} < R|D(s)] = b^{\left|D(s)\right|+1} \lambda_{s,d} \left[ \sum_{j=0}^{\left|D(s)\right|} \binom{\left|D(s)\right|}{j} (-1)^j \prod_{i=1}^{\left|D(s)\right|} \lambda_{r_i,d} \right]
\]

\[= b^{\left|D(s)\right|+1} \lambda_{s,d} \frac{1}{\left|D(s)\right| + 1} \prod_{i=1}^{\left|D(s)\right|} \lambda_{r_i,d}. \quad (A.8)\]

Outage Probability

The total outage probability is obtained by substituting (A.2) and (A.8) into (A.1)

\[
\Pr[I_{sel} < R] \simeq b^{N+1} \lambda_{s,d} \sum_{|D(s)|} \frac{1}{|D(s)| + 1} \times \prod_{r_i \in D(s)} \lambda_{r_i,d} \prod_{r_i \notin D(s)} \lambda_{s,r_i}
\]

\[= \left[ \frac{2^{2|R| - 1}}{\text{SNR}} \right]^{N+1} \lambda_{s,d} \sum_{|D(s)|} \frac{1}{|D(s)| + 1} \times \prod_{r_i \in D(s)} \lambda_{r_i,d} \prod_{r_i \notin D(s)} \lambda_{s,r_i}. \quad (A.9)\]

Hence the proposition in Section 3.1 is proved. \( \blacksquare \)
Appendix B

Outage probability of selection in the low and medium-SNR regime

In this section, we develop the low-SNR approximation to the outage probability \( \Pr[I_{sel} < R] \) for Selection Cooperation in a single user system. As in Appendix A, we write the probability of outage of Selection Cooperation as

\[
\Pr[I_{sel} < R] = \sum_{\mathcal{D}(s)} \Pr[D(s)] \Pr[I_{sel} < R | D(s)],
\]

(B.1)

where \( \mathcal{D}(s) \) is the decoding set of source \( s \). Without resorting to high-SNR approximation, the probability of a decoding set is simply [11, 64]

\[
\Pr[D(s)] = \prod_{r \in \mathcal{D}(s)} e^{-\lambda_{s,r}a} \times \prod_{r \not\in \mathcal{D}(s)} (1 - e^{-\lambda_{s,r}a}),
\]

(B.2)
where \( a = (2^R - 1)/\text{SNR} \). Next, the outage probability of Selection Cooperation given a decoding set \( D(s) \) is [64]

\[
\Pr [I_{sel} < R|D(s)] = \int_0^a \left[ \prod_{r=1}^{[D(s)]} (1 - e^{-\lambda_{r,d}(a-y)}) \right] \lambda_{s,d} e^{-\lambda_{s,d}y} dy
\]

\[
= \lambda_{s,d} \int_0^a \left[ \prod_{r=1}^{[D(s)]} e^{-\lambda_{r,d}a} \left( e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y} \right) \right] e^{-\lambda_{s,d}y} dy. \tag{B.3}
\]

The evaluation of this integral is difficult due to the product of the exponential. We thus make two approximations:

1. Taylor Series expansion of

\[
e^{-\lambda_{s,d}y} \approx \left( 1 - \lambda_{s,d}y + \frac{1}{2} \lambda_{s,d}^2 y^2 - \frac{1}{6} \lambda_{s,d}^3 y^3 \right), \quad 0 \leq y \leq y_0,
\]

\[
\approx 0 \quad y > y_0. \tag{B.4}
\]

Because \( \lambda_{s,d} \) is always positive, the expression on the right side of (B.4) is a decreasing function of \( y \). This is readily shown by examining its derivative with respect to \( y \):

\[
\frac{d}{dy} \left( 1 - \lambda_{s,d}y + \frac{1}{2} \lambda_{s,d}^2 y^2 - \frac{1}{6} \lambda_{s,d}^3 y^3 \right) = -\frac{1}{2} \lambda_{s,d} \left( y - \frac{1}{\lambda_{s,d}} \right)^2 - \frac{1}{2} \lambda_{s,d} < 0, \quad \forall y, \lambda_{s,d} \geq 0.
\]

\( y_0 \) is the point where this expression becomes negative, i.e.,

\[
(1 - \lambda_{s,d}y + \frac{1}{2} \lambda_{s,d}^2 y^2 - \frac{1}{6} \lambda_{s,d}^3 y^3) = 0.
\]

\[
\Rightarrow y_0 = \frac{(1 + \sqrt{2})^\frac{1}{2} - 1 + (1 + \sqrt{2})^\frac{1}{2}}{\lambda_{s,d}(1 + \sqrt{2})^\frac{1}{2}}. \tag{B.5}
\]
B. Outage probability of selection in the low and medium-SNR regime

2. Third-order approximation of

\[ e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y} \approx f_r(a^3 - y^3), \quad (B.6) \]

where \( f_r \) is determined below.

The second term could also be expanded using Taylor series; the product of the terms, however, would significantly increase the number of terms in the approximation. Here, we choose order-three approximations which, as we show in Section 4.3, yield good results. Clearly, the accuracy of the approximations could be further increased by increasing the approximation order.

Using the approximation in (B.4), the upper limit of the integral in (B.3) is \( L \), where \( L = \min(y_0, a) \). The parameter \( f_r \) is then obtained by minimizing \( E(f_r) \), the total squared error \( es(y, f_r) = |(e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y}) - f_r(a^3 - y^3)|^2 \) over the range \((0, L)\).

\[
E(f_r) = \int_0^L es(y, f_r) dy, \\
= \int_0^L |(e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y}) - f_r(a^3 - y^3)|^2 dy, \\
= \frac{1}{14\lambda_{r,d}^4} \left[ f_r \left( -28e^{\lambda_{r,d}a}a^3\lambda_{r,d}^4L + 7e^{\lambda_{r,d}a}\lambda_{r,d}^4L^4 - 28a^3\lambda_{r,d}^3L^4 - 168 + 28e^{\lambda_{r,d}L}a^3\lambda_{r,d}^3L^3 \\
- 28e^{\lambda_{r,d}L}\lambda_{r,d}^3L^3 + 84e^{\lambda_{r,d}L}\lambda_{r,d}^2L^2 - 168e^{\lambda_{r,d}L}\lambda_{r,d}L + 168e^{\lambda_{r,d}L} \right) \\
+ f_r^2 \left( 14a^6\lambda_{r,d}^4L - 7a^3\lambda_{r,d}^4L^4 + 2\lambda_{r,d}^4L^7 \right) + 28e^{\lambda_{r,d}a}\lambda_{r,d}^3L^3 - 7\lambda_{r,d}^3 \\
+ 14e^{2\lambda_{r,d}a}\lambda_{r,d}^4L - 28e^{\lambda_{r,d}L}\lambda_{r,d}^3 + 7e^{2\lambda_{r,d}L}\lambda_{r,d}^3 \right]. \quad (B.7)
\]
To minimize $E(f_r)$, we set the derivative of this expression with respect to $f_r$ to zero and obtain

$$f_r = \frac{-7}{2L\lambda_{r,d}(7a^3L^3 + 2L^6 + 14a^6)} \left[ \lambda_{r,d}^4 L e^{\lambda_{r,d}a} (-4a^3 + L^3) + 4\lambda_{r,d}^3 (e^{\lambda_{r,d}L} a^3 - e^{\lambda_{r,d}L} L^3 - a^3) + 12\lambda_{r,d}^2 L^2 e^{\lambda_{r,d}L} - 24\lambda_{r,d} L e^{\lambda_{r,d}L} + 24 e^{\lambda_{r,d}L} - 24 \right]. \quad \text{(B.8)}$$

Using Binomial Expansion, the resulting outage probability approximation can thus be written as

$$\Pr [I_{sel} < R|\mathcal{D}(s)] \\ \approx \lambda_{s,d} \int_0^L \left[ \prod_{r=1}^{\lvert \mathcal{D}(s) \rvert} e^{-\lambda_{r,d}a f_r (a^3 - y^3)} \right] \left( 1 - \lambda_{s,d} y + \frac{1}{2} \lambda_{s,d}^2 y^2 - \frac{1}{6} \lambda_{s,d}^3 y^3 \right) dy,$$

$$= \lambda_{s,d} \left[ \prod_{r=1}^{\lvert \mathcal{D}(s) \rvert} f_r e^{-\lambda_{r,d}a} \right] \int_0^L \left( a^3 - y^3 \right)^{\lvert \mathcal{D}(s) \rvert} \left( 1 - \lambda_{s,d} y + \frac{1}{2} \lambda_{s,d}^2 y^2 - \frac{1}{6} \lambda_{s,d}^3 y^3 \right) dy,$$

$$= \lambda_{s,d} \left[ \prod_{r=1}^{\lvert \mathcal{D}(s) \rvert} f_r e^{-\lambda_{r,d}a} \right] \int_0^L \sum_{i=0}^{\lvert \mathcal{D}(s) \rvert} \binom{\lvert \mathcal{D}(s) \rvert}{i} \left( a^3 \right)^{\lvert \mathcal{D}(s) \rvert - i} (-y^3)^i \left( 1 - \lambda_{s,d} y + \frac{1}{2} \lambda_{s,d}^2 y^2 - \frac{1}{6} \lambda_{s,d}^3 y^3 \right) dy,$$

$$= \lambda_{s,d} \left[ \prod_{r=1}^{\lvert \mathcal{D}(s) \rvert} f_r e^{-\lambda_{r,d}a} \right] \sum_{i=0}^{\lvert \mathcal{D}(s) \rvert} \binom{\lvert \mathcal{D}(s) \rvert}{i} \left( a^3 \right)^{\lvert \mathcal{D}(s) \rvert - i} (-1)^i$$

$$\int_0^L \left( y^{3i} \right)^i - \lambda_{s,d} y^{3i+1} + \frac{1}{2} \lambda_{s,d} y^{3i+2} - \frac{1}{6} \lambda_{s,d} y^{3i+3} \right) dy,$$

$$= \lambda_{s,d} \left[ \prod_{r=1}^{\lvert \mathcal{D}(s) \rvert} f_r e^{-\lambda_{r,d}a} \right] \sum_{i=0}^{\lvert \mathcal{D}(s) \rvert} \binom{\lvert \mathcal{D}(s) \rvert}{i} \left( a^3 \right)^{\lvert \mathcal{D}(s) \rvert - i} (-1)^i$$

$$\left( \frac{L^{3i+1}}{3i + 1} - \lambda_{s,d} \frac{L^{3i+2}}{3i + 2} + \frac{1}{2} \lambda_{s,d} L^{3i+3} - \frac{1}{6} \lambda_{s,d} L^{3i+4} \right). \quad \text{(B.9)}$$

$$\left( \frac{L^{3i+1}}{3i + 1} - \lambda_{s,d} \frac{L^{3i+2}}{3i + 2} + \frac{1}{2} \lambda_{s,d} L^{3i+3} - \frac{1}{6} \lambda_{s,d} L^{3i+4} \right). \quad \text{(B.10)}$$
Appendix C

Outage Probability of Simple Selection: multiple source-destination pairs

In this section, we develop the outage probability bounds of Selection Cooperation in network where the relay node is chosen in a “simple” manner - the chosen relay is the one that can decode the source message correctly and has the best relay-destination channel.

We consider a source node \( s_j \in \mathcal{M} \) communicating with its destination \( d(s_j) \) using a relay \( r_j \). We use the random variable \( N_r \) to denote the number of other source-destination pairs that have also selected relay \( r_j \). We assume that all other destination nodes have equal average channels to all the relay nodes. This assumptions makes \( N_r \) independent of the average channel values and consequently of the node selected by \( d(s_j) \), thus significantly simplifying the analysis.

The outage probability for this source-destination pair is again obtained by using the
C. Outage Probability of Simple Selection: multiple source-destination pairs

The total probability law, averaged over $N_r$,

$$\Pr[I_{sel} < R] = \sum_{n=0}^{N-2} \Pr[I_{sel} < R|N_r = n]p(N_r = n), \quad (C.1)$$

where $p(N_r = n)$ denotes the probability of relay $r_j$ supporting $n$ nodes other than the chosen source node $s_j$.

To further simplify the analysis, we use the high-SNR approximation that all relay have correctly decoded the data of all source nodes except for source $s_j$. The relay $r_j$ is thus included in the decoding set of all source nodes (note that this approximation will give an upper bound on the outage probability, as it increases the average value of $N_r$). At best, no other source-destination pairs have selected relay $r_j$ and $N_r = 0$. At worst, all other source-destination pairs have selected $r_j$, in which case $N_r = N - 2$ (a source cannot relay for itself, and thus $r_j$ cannot relay its own data). Thus $0 \leq N_r \leq N - 2$. In the case of equal average channels between all destination nodes other than $d(s_j)$ and the relay nodes, the random variable $N_r$ has binomial density independent of the average channels

$$p_{N_r}(n) = \binom{N-2}{n} \left[ \frac{1}{N-2} \right]^n \left[ 1 - \frac{1}{N-2} \right]^{N-2-n}. \quad (C.2)$$

The other term in (C.1), $\Pr[I_{sel} < R|N_r = n + 1]$, is the outage probability of the $s_j - d_j$ communication aided by relay $r_j$, which is also supporting $N_r$ other source nodes, and thus expending $P/(N_r + 1)$ Watts for each supported source node. The mutual information, given that $N_r = n$, is thus

$$I_{sel} = \frac{1}{2} \log \left( 1 + \text{SNR} \times Y + \frac{\text{SNR}}{n+1} X \right), \quad (C.3)$$
where \( Y \) and \( f_Y(y) \) are defined in (A.5) and (A.6) and

\[
X = \frac{1}{(n + 1)} \max_{r_i \in D(s)} \{|a_{r_i,d}|^2\}; \quad i = 1 \ldots |D(s)|
\]

\[
F_X(x) = \prod_{i=1}^{(n + 1)} \left(1 - \exp\left[-(n + 1)\lambda_{r_i,d}y]\right) . \tag{C.4}
\]

The development of \( \Pr[I_{sel} < R|N_r = n]\) is very similar to that shown in Appendix A, and we thus give only the final result:

\[
\Pr[I_{sel} < R|N_r = n] = \left[\frac{2^{2R} - 1}{\text{SNR}}\right]^N \lambda_{s,d} \sum_{|D(s)|} (n + 1)^{|D(s)|} \frac{1}{|D(s)|} + 1 \times \prod_{r_i \in D(s)} \lambda_{r_i,d} \prod_{r_i \notin D(s)} \lambda_{s,r_i}, \tag{C.5}
\]

where the \( n + 1 \) sources supported by the relay account for the factor of \( (n + 1)^{|D(s)|} \) within the summation, and the last simplification stems from the assumption that all nodes have equal average channels to their destinations, i.e., \( \lambda_{r_i,d} = \lambda_{r_k,d} = \lambda_{r,d}; \forall i, k \).

Combining (C.1), (C.2) and (C.5), we obtain

\[
\Pr[I_{simple\text{-}sel} < R] \approx \left[\frac{2^{2R} - 1}{\text{SNR}}\right]^N \lambda_{s,d} \sum_{|D(s)|} \frac{1}{|D(s)|} + 1 \prod_{r_i \in D(s)} \lambda_{r_i,d} \prod_{r_i \notin D(s)} \lambda_{s,r_i} \sum_{n=0}^{N-2} K_m(n + 1)^{|D(s)|}, \tag{C.6}
\]

where

\[
K_m = \binom{N-2}{n} \left[\frac{1}{N-2}\right]^n \left[\frac{N-3}{N-2}\right]^N . \tag{C.7}
\]

Hence the proposition in Section 3.2.3 is proved.
Bibliography


