CONSTRUCTING MATHEMATICAL KNOWLEDGE USING MULTIPLE REPRESENTATIONS: A CASE STUDY OF A GRADE ONE TEACHER

by

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Abstract

This study examined how an elementary teacher fostered student mathematical understanding and the strategies that she used to help students learn mathematical concepts. A case study of a Grade 1 teacher is described based on qualitative data from interviews and classroom observation sessions using a peer coaching model.

The evidence from the study suggests that this teacher benefited from professional development opportunities to gain deeper insights regarding her teaching practices. There were five major findings: (1) enthusiasm for improving her practices was necessary to successfully meet her goals; (2) this teacher’s role in the classroom was important to facilitate the construction of knowledge; (3) the classroom was an environment where her students felt safe; (4) a variety of tasks and strategies that students of varied abilities, interests and aptitudes can enjoy were used; and (5) multiple representations (including the use of manipulatives) were used to scaffold the construction of knowledge.
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Chapter One: Introduction

1.1 Introduction

The purpose of this thesis is to determine how an elementary mathematics teacher can facilitate the construction of knowledge in her classroom. I chose this topic because of my own personal and professional interest in how to best teach to all types of learners. I have been lucky to be in environments which nurtured and stimulated me as a learner and created a spirit of life-long learning. This thesis is an examination of this notion and how, in an elementary classroom, a teacher can create an environment that elicits the enthusiasm for quality learning just as I had felt at that age. This chapter outlines the research context and questions as well as the significance and my personal connection to the study. The plan of the thesis is also shared.

1.2 Research Context

The standards of the National Council of Teachers of Mathematics (NCTM, 2000) urge teachers to create an environment where students learn mathematics with understanding. This is particularly important at the elementary level where students are developing basic mathematically skills which they will rely upon both in future studies and in life. The elementary classroom is also where students develop their knowledge of how to learn mathematics.

A teacher’s personal idea bank is rich with creative and effective strategies which they believe will help their students to learn mathematics. With the constant development of new initiatives and changing provincial and state policies, directives and curriculum guidelines, teachers can benefit from learning from other sources. Professional development opportunities and pre-existing frameworks for professional growth can help
educators add to their idea bank or focus their attention to specific areas of their teaching for areas of improvement (Guskey, 2000; McDougall, 2004).

The Ontario Achievement Chart (Ontario Ministry of Education, 2005) for elementary mathematics identifies four categories of knowledge and skills. They are: knowledge and understanding, thinking, communication and application.

In order for students to meet the knowledge and understanding standards set by the Ministry, teachers must foster both procedural and conceptual knowledge within their students. While procedural knowledge may be easier to create, conceptual knowledge requires the learner to create relationships between various concepts and have a deeper understanding of the mathematics material (Cobb, Wood, & Yackel, 1991). A teacher who uses a constructivist approach ensures that students will indeed consider relationships and form links between pre-existing and new knowledge thereby cementing the understanding of a mathematical concept (Reys, Suydam, Lindquist, & Smith, 1998).

The category of thinking focuses on the “use of critical and creative thinking and/or processes” (Ontario Ministry of Education, 2005, p. 22). The suggested role of the teacher in the classroom is that of a guide and as such, teachers should guide their young student on how to tackle mathematics (Zack & Graves, 2001). Young students may not intuitively know how to learn something that they may not have ever encountered before, so teachers can aid students in their journey or model proper learning behaviour. Teachers should be provided with opportunities to allow students to construct their own knowledge as well as develop a set of thinking skills to help them be successful mathematics learners.
The third category is communication and suggests that teachers should provide opportunities for students to express what they know. Teachers who allow their students to learn in cooperative groups inherently provide them with an outlet to communicate (Johnson & Johnson, 1991; Vygotsky, 1978). Student communication also strengthens the constructivist approach as it requires students to sort through their ideas and reflect upon the connections that they have made (Vygotsky, 1978). Another way students can be exposed to various forms of communication is through the use of multiple representations (Pape & Tchoshanov, 2001). These representations can be oral, written or visual as students explore mathematics using pictures, diagrams, concrete materials (often in the form of manipulatives) or develop a familiarity with the language of mathematics by using symbols as they progress through the different representation forms.

The final category of application ties together all of the ideas already mentioned. A mathematics learner is constantly linking ideas within and between various experiences. The teacher should view part of their role in the classroom as creating a safe learning community where students can take risks in new contexts (Pape, Bell, & Yetkin, 2003). Therefore, asking them to apply their knowledge to a new context will not seem as intimidating as they will merely be seeking out another linkage to be made. A student who is fluent in the use of multiple representations will most definitely meet the criteria of “making connections within and between various representations” (Ontario Ministry of Education, 2005, p. 23).

The Western and Northern Canadian Protocol (2006), albeit using different terminology, emphasizes the same key components that students should master to achieve mathematical success. Instead of four categories, the Western and Northern Canadian
Protocol (WNCP) highlights seven key processes for mathematics learning. They are: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology and visualization.

While the Ontario Achievement Chart’s (OAC) knowledge and understanding category collectively asks teachers to allow opportunities for students to develop both procedural and conceptual knowledge, the WNCP has specifically created processes for each, mental mathematics and estimation, and connections, respectively. The WNCP’s reasoning, technology and visualization processes are a more detailed breakdown of the OAC’s thinking category. The WNCP has also decided to give a general mathematical reasoning process along with two more specific processes to give teachers something to focus their teaching efforts towards as tools and skills for student learning. Needless to say, the communication process directly parallels the Ontario Ministry of Education’s rationale and the final category of application, is interpreted as problem solving to the WNCP. Although framed with problem solving in mind, this process still asks students to “develop and apply new mathematical knowledge” (Western and Northern Canadian Protocol, 2006, p. 6).

The exposure to professional development opportunities will expose the teacher to new teaching strategies as well as hone their thinking regarding their professional practices. These teachers will be encouraged to reflect on how students learn and whether there is a learning philosophy which parallels the goals of mathematics education today. Teachers will actively refine their role as a teacher and employ teaching strategies which develop well-rounded learners who adhere to the NCTM (2000), Ontario Ministry of Education (2005) and Western and Northern Canadian Protocol (2006) standards.
1.3 Research Questions

In this study, I explored how a Grade 1 teacher creates an environment which fosters students’ construction of mathematical knowledge. This thesis focuses on the following research questions:

1. How do elementary school teachers foster student mathematical understanding?
2. What strategies do teachers use to help students learn mathematics concepts?

I answer these questions by carrying out a detailed case study on the practices of a Grade 1 teacher as she engages in professional development and on her change processes as she improves her ability to teach mathematics.

1.4 Significance of the Study

This study is significant because the case study of this teacher may give administrators and mathematics teachers insights into how to improve the quality of the construction of students’ knowledge about mathematics. Although this study takes place at the Grade 1 level, the findings and benefits span across all grade levels.

Regardless of how motivated a teacher may be to change, working alone in a classroom limits a teacher’s opportunities for professional growth and so teachers need to look for ways to dialogue with colleagues and others in the field to improve their teaching practices. Teachers cannot be expected to create new ideas on their own given the limited amount of time and resources that are available to them, so professional development opportunities in the form of workshops, seminars or creating support groups within the school with other teachers and/or administrators is of paramount importance.

In the reform-based classroom, teachers move away from the traditional role of knowledge-provider and transform their classroom into a student-centered environment.
This adjustment is difficult for most teachers as their role in the classroom is being altered. Teachers, especially those who have been teaching for many years or those who were taught using traditional methods, may not know what the current role of the teacher should be and how they fit into today’s student-centered learning environment.

The composition of students in today’s classroom is diverse. A class comprised of students with different backgrounds, ability levels and learning styles dictates that teachers must be resourceful and employ a variety of effective teaching strategies to help their students in the learning process. It is not just the use of a variety of teaching strategies which is important, but the efficacy of each of the strategies as well. The teacher must know at which time they should use specific strategies to match up with the developmental level of their students as well as being able to select the strategies which will exploit their students’ strengths.

The findings can be a resource to teachers looking to better their teaching practices. The detailed information about the importance of professional development opportunities for the study participant, her role in the classroom and how she employed effective teaching strategies is invaluable for teachers who are seeking to improve their teaching.

1.5 Background of the Researcher

My personal interest in this topic derives from my own experiences and goals as a mathematics teacher. As a novice secondary school mathematics teacher, I would like to find new ways to improve my teaching practices and introduce concepts in different ways. I was taught using traditional teaching methods and have, for the most part, taught in the same way.
As a student teacher, I discovered that I could come up with more creative ideas teaching at the middle school level than I could when put in the secondary school classroom. It could have been my comfort with the simpler curriculum content at the lower levels or the fact that my mentor teachers at the middle school level had a background in the arts and humanities where they were used to being more creative in their lessons. My mentors at the secondary school level were more traditional in their approach, and so I was able to come up with more interactive lessons when preparing my lessons in middle school mathematics classrooms.

My first professional teaching position was as a Grade 12 mathematics teacher at Neuchâtel Junior College in Neuchâtel, Switzerland where I taught all three Grade 12 academic mathematics courses for the first time. My first year of teaching was spent relearning concepts that I had not dealt with during my undergraduate studies in university and in some cases, learning material that I clearly did not have a grasp of when I was a student myself. This process took a long time, but forced me to break down each concept, examine the material in different ways and deal with challenges that some of my students may face themselves. I was putting myself in my students’ shoes and ensuring that I understood the material inside and out. During that first year, I created lessons much like the lessons that I had been presented as a student. First, I conducted a sort of review of previous concepts, next, an overview of the concept to be examined for that lesson, then, a series of examples using the concept, followed by a chance to clarify any misunderstandings and concluding with opportunities to practice the material. Each lesson was clear, precise and orderly, but lacked creativity and a chance for students to discover the material for themselves. I knew how I could best learn the material, and felt
that I could predict how the students would also learn the material, so I set up my lessons in a way that they would go through the same motions as I did as a student so that they could be as successful as I was.

My second and third years at Neuchâtel Junior College were steps in a different direction. By that point, I had fine-tuned my original lesson plans so that the students would not have any challenges with the way that I presented the material, but most lessons were still of a traditional approach. Each semester, however, I would add an activity to as many lessons as I could to move away from my traditional teaching methods. As the only mathematics teacher at the school, and not having a network of mathematics colleagues to share ideas with or professional development opportunities to learn new ideas, it was difficult to come up with ideas on my own, especially for the more abstract concepts in the course. I slowly started incorporating technology, manipulatives and exploration activities in my lessons. However, many of my lessons were still traditional in nature. Although my students’ assessments showed that they were learning the material, these traditional teaching practices are not in line with the current mathematics education trends.

While my experiences are at the intermediate and secondary levels, I see the value of determining how students construct knowledge at the elementary level when children are first exposed to basic mathematical concepts. I believe that, by understanding how students naturally learn and how a teacher can foster quality learning, I can use the same principles at the higher-grade levels with more complex and abstract concepts. I have also found that many teachers at the secondary level are still using traditional teaching methods, while more elementary teachers are using reform-based practices in the
classroom, so by going into elementary classrooms, I would see the strategies that I
would like to adopt being used.

1.6 Plan of the Thesis

There are five chapters in my thesis organized to describe the study in detail.
Chapter One provides an overview of the thesis describing the research questions,
background and significance of the study.

A review of existing literature is found in Chapter Two and examines previous
research conducted in this area. Professional development is discussed with particular
attention to the peer coaching model. The Ten Dimensions of Mathematics Education
(McDougall, 2004) is described as a theoretical framework for teaching improvement.
Constructivism is explored to further understand how students learn and develop
knowledge. Different facets of the role of the teacher are identified for teachers to be
aware of their place in today’s classroom. Finally, cooperative learning and multiple
representations are discussed as teaching strategies which address the needs of today’s
diverse student population.

Chapter Three describes the methods used to carry out the study. The participants
in this study are part of a larger research project on School Improvement in Mathematics
(McDougall, 2009) and I discuss how this study uses the larger study as a base. I also
discuss the data collection and analysis methods and the ethical considerations of the
study.

In Chapter Four, I present the case study of my participant teacher. I use this case
to illustrate how a Grade 1 mathematics teacher facilitates the construction of student
knowledge. Finally, Chapter Five takes the insight from the case study to answer the
research questions posed in Chapter One. I conclude with suggestions for further research.
Chapter Two: Literature Review

2.1 Introduction

As teachers continue on their journey to become the best educator that they can possibly be, they need to have the right tools to help them to improve. An enthusiastic attitude is a good first step, but is not nearly enough as teachers need to seek out professional development opportunities in order to find out more about who they are as educators, who they would like to become and strategies that they can use to get them there. The area of teacher improvement is vast and so, the use of an existing framework, such as the Ten Dimensions of Mathematics Education (McDougall, 2004), can help focus a teacher’s efforts to where they most need to improve. Learners are, constructing knowledge and so teachers must turn their attention to their role in the classroom and what strategies they can use so that their students can construct knowledge effectively. Two such strategies include cooperative learning and the use of multiple representations. In this chapter, I will discuss all of these components, which ensure exemplary student achievement.

2.2 Professional Development

Guskey (2000) defines professional development as a process with three defining characteristics: it is intentional, ongoing and systemic. Designed to promote change and improvement, professional development has a clear purpose and participants must subscribe to these goals in order to move forward. Guskey (2000) suggests three steps to ensure that professional development is intentional. The first step is to clearly state goals and expectations. Being explicit in the purposes and goals of the process aids participants to be focused appropriately. All entities of the process will work together with the same
purpose as opposed to working against each other towards divergent goals. The second step is to create worthwhile goals. An educator can create a personal interest by making the goals meaningful and applicable to their mission. All parties involved will feel the importance of the goal and put forth a deeper investment towards the process. The third and final step is to create an assessment scheme. Assessment of the learning and the program must be used to determine whether or not the process was successful. Without these three steps, the professional development process is seen to be random and thereby seems to proceed without any clear purpose. If professional development is to be worthwhile, there must be a set goal that all participants have a vested interest in and can be measured to determine its success.

As our knowledge about education continues to change, so too does our vision of how we should teach. To keep up with this ever-changing view, professional development also needs to be increased. We know that we will never reach the ultimate understanding of education and that there is always something more to discover. Due to its expansive nature, one cannot hope to learn all there is to know in just one in-service session. Professional development is a journey that ties together in-services, literature and experience (Guskey, 2000). The lack of time looms as another barrier to teachers seeking out professional growth (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). A constant effort in improving practice needs to be made for positive change to occur (Desimone, Porter, Garet, Yoon, & Birman, 2002).

The implementation of a new idea into a mathematics classroom is not usually effective if a teacher does not have the tools to do so. However, even with these tools but without the support of others, these ideas will never be sustainable (Guskey, 2000;
Weasmer, 2008). All stakeholders, whether they are teachers, administrators, families or school officials, need to embrace the change and to employ practices that can support and encourage the new ideas to grow and develop (Weasmer, 2008). Making a change in our educational system affects more than just our students and therefore, for our society to reap the benefits, all stakeholders must play a part in nurturing this change.

2.2.1 Peer Coaching

One model of professional development is peer coaching (Arnau, Kahrs, & Kruskamp, 2004; Bruce & Ross, 2008; Loucks-Horsley et al., 2003; Slater & Simmons, 2001). This method involves pairing two colleagues in a session with classroom observation, giving feedback and discussion to allow both members a chance to learn from one another (Loucks-Horsley et al., 2003). Both participants have a chance to reflect on what they observe along with their own teaching practices in order to grow. This reciprocal gain is one of the major benefits of peer coaching. It is important to have an extra set of eyes in the classroom to give teachers another perspective that they may not have come up with in their own reflections (Guskey, 2000). The colleague can help identify both strengths and weaknesses for the teacher to focus on along with seeing practices that they may not have had exposure to beforehand.

In order for peer coaching to be successful, participants must be eager to learn from the experience (Arnau et al., 2004). As this model requires an investment of time by both colleagues that may be difficult in an already busy schedule, enthusiasm towards the process is a must.

At first, there may be hesitation to engage in the process as the teacher may be uneasy at the prospect of being observed and evaluated, but being paired up with a
colleague that they trust will calm some fears (Arnau et al., 2004). An environment of “trust, collegiality, and continuous growth” (Loucks-Horsley et al., 2003, p. 208) will ensure that participants feel more comfortable. As long as both participants focus on the goal of professional development and maintain a level of respect for their peers, the advantages will certainly outweigh the disadvantages (Slater & Simmons, 2001).

To help professional development to become more effective in mathematics, a theoretical framework is necessary. One theoretical framework that has been used in elementary and secondary education is the Ten Dimensions of Mathematics Education.

2.3 The Ten Dimensions of Mathematics Education

The Ten Dimensions of Mathematics Education (McDougall, 2004) was chosen as the theoretical framework for this study. The framework was developed through multi-year research projects aimed to identify areas of teaching in which they can focus their attention in order to improve (McDougall, Ross, & Ben Jaafar, 2006; McDougall, 2004; McDougall, 2007; McDougall & Fantilli, 2008; Ross, McDougall, & Hogaboam-Gray, 2002).

The first dimension is Program Scope and Planning and focuses on the inclusion of all strands of mathematics in the classroom. A highlight of this dimension is the use of mathematical processes along with the integration and connection of a variety of strands within a unit and/or lesson. Teachers are expected to use a variety of resources to strengthen their delivery of the curriculum.

The second dimension, named Meeting Individual Needs, involves the use of different teaching techniques and strategies to cater to the needs of each individual in the classroom. In this dimension, teachers should consider balancing lessons styles and using
differentiated instruction techniques including: scaffolding, open-ended tasks, varying tools and time, and varying physical and grouping arrangements.

In the next dimension, Learning Environment, teachers are to focus on two key themes: classroom organization and cooperative groups, and teacher feedback and student input/choice. Teachers are asked to create cooperative groups for student work while considering group size and composition. The teacher should aim to create a learning environment where both students and teachers reciprocate feedback.

The highlight of Student Tasks is the use of rich tasks within the mathematics classroom. Teachers should aim to make tasks engaging whether they are skill or procedural based. The use of rich tasks set in real-life contexts and tasks that allow for students to choose from multiple representation forms leads to an increased student achievement.

The fifth dimension is Constructing Knowledge and this dimension urges teachers to use a variety of instructional strategies and to apply effective questioning techniques. Teachers should reflect on whether their questioning elicits mathematical thinking.

Parental involvement in academics can play a large part into the success of the student. The Communicating with Parents dimension encourages teachers to acknowledge the power of family interaction and asks teachers to communicate about student performance, the mathematics program and mathematics education with parents. Communication with parents can be in many forms and messages should be about content and performance as well as giving parents strategies and tools regarding how to help their children learn and improve.
The next dimension is titled Manipulatives and Technology. As we move towards a reform-based classroom, evidence of the use of manipulatives and technology should be present. The regular and varied use of these tools can develop a better conceptual understanding of the material and encourages students to make connections through exploration.

The eighth dimension is Students’ Mathematical Communication. Students should use both oral and written forms of communication to express their understanding and discover new ideas and, teachers should assign tasks that incorporate both methods of communication as well as develop the skill level and efficacy of using each of the forms.

The ninth dimension focuses on the role of Assessment in the classroom. Teachers should use diagnostic, formative and summative assessment to report on student achievement. Through their assessment, teachers should clearly express levels of student achievement both in content and processes and use a variety of assessment strategies.

The final dimension deals with Teacher’s Attitude and Comfort with Mathematics. A teacher who shows a genuine interest in mathematics and values the importance of the subject will have a greater chance of passing along these beliefs to their students. Having a secure comfort level with the subject allows the teachers to make more connections between concepts and fosters a deeper sense of inquiry. This strong knowledge base leads to a greater self-confidence of the teacher’s ability to promote student learning in mathematics.

The Ten Dimensions of Mathematics Education framework allows teachers to focus on key areas that will generate higher levels of student achievement. Teachers will be more effective at improving their teaching practices if they focus on one or two
dimensions as opposed to trying to improve their practices in a general fashion. Although teachers generally pick one or two areas of focus for improvement, the dimensions naturally overlap and improvement in one area will inevitably improve other areas of their teaching. If teachers and administrators work towards progressing in one or more of these specific areas, schools will get closer to meeting standards set out by the NCTM (2000).

The Ten Dimensions of Mathematics Education comes with a variety of teaching tools to help the teacher fine-tune their teaching beliefs and practices. The Continuum (McDougall, 2004), along with the Attitudes and Practices for Teaching Math Survey (McDougall, 2004), can be used as self-assessment tools for the teacher to determine which dimensions they should focus on to improve their teaching practices. The continuum can also be used by colleagues to mentor one another in observation sessions and comes in the form of a distinct rubric for each dimension with criteria and four levels which range from a traditional approach to a reform-based method of teaching. Guiding questions are provided to prompt the teacher to consider key ideas within the chosen dimension and points for possible evidence are also given to highlight what an exemplary teacher should be doing in their classroom. For those who are unsure of how to use the continuum, a guide with further questions and discussion points is provided to make the Ten Dimensions more accessible. The Attitudes and Practices for Teaching Math Survey is a 20-question survey, which highlights the dimensions that the teacher may want to focus on as areas of improvement and also highlights their strengths as a teacher.

Using the Ten Dimensions of Mathematics Education as a framework for teacher improvement is an effective way to increase student achievement. For my thesis, I have
decided to focus on Dimension 5 – Constructing Knowledge and examine how a primary teacher focuses on this dimension to improve her teaching practices. The constructivist approach to teaching encompasses the key ideas within almost all of the other dimensions so I felt that it was a strong dimension to focus on. I will also discuss Dimension 7 – Manipulatives and Technology as the teacher uses these tools to facilitate a deeper construction of knowledge.

2.4 Construction of Knowledge

Construction of knowledge has been studied by psychologists and educational theorists for many years (Alagic, 2003; Gardner, 1993a; Perkins, 1993; Piaget, 1995; Reys at al., 1998; Vygotsky, 1978). Researchers have examined how students learn and highlight strategies and environments that students should experience in order to make their construction of knowledge most effective and meaningful (Gardner, 1993a; Kim & Baylor, 2006; Mevarech & Kramarski, 1997; Van de Walle & Folk, 2005; Vygotsky, 1978). In this section, I will first describe what knowledge is and what it means for understanding to occur, then discuss some of the major theories as to how students can learn most effectively.

Theorists believe that there are two types of knowledge: procedural and conceptual. Procedural knowledge is based on a sequence of actions, often involving rules and algorithms (Reys et al., 1998). Specifically for mathematics, it is the understanding of the rules, procedures and symbols used to carry out mathematical tasks (Van de Walle & Folk, 2005).

By contrast, conceptual knowledge is based on connections between discrete pieces of information. This type of knowledge requires the learner to think about
relationships, make connections and adjust any previously made links that may have proved to be faulty (Reys et al., 1998). This type of knowledge is long-term and creates a deeper understanding where the learner has made connections for themselves (Cobb et al., 1991).

True understanding occurs when a student is able to use what they know and apply it to new situations (Gardner, 1993a; Perkins, 1993). A student demonstrates understanding by “being able to carry out a variety of actions or performances with the topic by the ways of critical thinking: explaining, applying, generalizing, representing in new ways, making analogies and metaphors” (Alagic, 2003, p. 384). For this reason, teachers must create situations where students are given the opportunity to show their understanding in these different ways to ensure that they are successfully constructing knowledge. It is important for educators to encourage effective learning and foster an environment for deeper understanding at a young age. By instilling students with a goal of deeper understanding at all times is to prepare them for the future, as other lessons in their academic career and beyond, will be easier to grasp (Alagic, 2003).

2.4.1 Learning Theories

Researchers have studied how students learn in order to deepen the knowledge pool so that educators can effectively encourage the construction of knowledge. Constructivism is a theory which highlights the importance of allowing opportunities for students to make their own connections and researchers have discovered strategies that teachers can use to facilitate and encourage these links.
2.4.1.1 Constructivism

Under the view of constructivism, students construct their own knowledge (Reys et al., 1998). They cannot gain it through transmission, as the student needs to create their own knowledge by reflecting on their own personal experiences, finding patterns and connecting them with other ideas in their knowledge base. In order to create these connections, the students need to have the chance to dialogue with others (peers or the teacher) to discuss the thoughts that they are trying to sort through. They can take information from others, however, they must synthesize it with their own mixture of ideas to form a more complete understanding of a concept. It is a process that takes time and goes through different stages of adding and taking away new linkages that may or may not be correct.

A foundational theorist in this area, Piaget highlighted that new concepts are only truly understood after connections with previous concepts have been constructed and further explained that these connections are required for meaningful learning to occur (Reys et al., 1998). He explained that, for true learning to occur, one needs to have conceptual knowledge, a type of knowledge he referred to as logico-mathematical knowledge (Van de Walle & Folk, 2005).

Although the connections that a student makes are internal, interactions in both physical and social contexts are needed to making learning occur (Kim & Baylor, 2006). These interactions uncover ideas that the student may not have had before and give the student another outlet to sort through their ideas. The use of manipulatives and other concrete objects is just one way that students can learn in a physical context. The idea of interaction through social context falls in line with the work of Vygotsky.
Vygotsky (1978; 1986) says that learners construct knowledge by interacting with people and through carrying out activities. To him, learning is a social activity where the student needs to be exposed to a variety of stimuli in order to reach their full potential. Once the student has absorbed the information from their interactions with people and experiences, they are then able to internalize it for themselves so that it becomes part of their knowledge base. Vygotsky (1978) termed his idea of learning through interaction as the zone of proximal development (ZPD). He defined ZPD as: “The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86).

The ZPD calls for the learner to interact with their peers. As students work together, they have the chance to both share their own understanding and hear about the understanding of others. This dialogue can also include talk about confusions and dilemmas where students can seek out the source of their confusion, find the errors in their assumptions and create corrections to construct a more accurate understanding. They can then take all of this information and internalize it to help them make meaning of the concept. Borrowing and building upon a peer’s ideas is encouraged. If the classroom is to become a learning community, all members will attribute their success to not only themselves but credit their peers as well (Zack & Graves, 2001).

There have been conflicting views as to how students should be grouped. Piaget (1995) believed that, if possible, students should work with peers with a similar ability level as the dialogue that can occur from this type of grouping uncovers ideas to let students re-visit their own thoughts and stimulates personal growth. On the other hand,
some theorists, like Vygotsky, believe that, by grouping students of varying capabilities, peers can uncover ideas that they might not have been able to come up with on their own and would therefore, have a chance to expand their understanding base in a way that they would not have been able to do if they were working alone or with other like-minded students.

According to Owen, Perry, Conroy, Geoghegan and Howe (1998), students learn in a four-phase, cyclic process. The first phase is experiencing, where students are participating in activities to stimulate learning. The second phase is discussing, where students discuss moments that came from the experiencing phase with their peers. This give-and-take of ideas will bring to light any ideas and confusions that the student and their peers may have experienced in the activities. The third phase of the cycle is generalizing, and it is during this time that the learner takes the ideas from the discussions, internalizes them and forms a stronger framework for their own understanding. The final phase, applying, asks the learner to take their new understanding and test it in different situations to ensure that their framework is indeed correct, leading to a new experiencing phase for the cycle to continue.

2.4.1.2 Introducing Students to New Knowledge

When constructing new knowledge, students make sense of new information by linking new and existing knowledge. The student refers to “strategies, tactics, or principles that are already in memory and compare the problem at hand with problems that have been solved before” (Mevarech & Kramarski, 1997, p. 367), and thus, exposing students to a variety of teaching tools early in their education will help them in future studies.
Students are naturally most comfortable with concrete ideas over those that are abstract. In mathematics, when first exposing students to new ideas, getting the students to interact with concrete tools (such as manipulatives) can ease them into the concept and develop the basic connections needed for them to progress to abstract ideas and the use of more complex mathematical language (Reys et al., 1998). It is a step-by-step progression as learning does not happen instantaneously and takes time to develop.

When introducing students to new knowledge, there are key components that a teacher can integrate to make learning more successful. The need to actively involve students as well as giving them the opportunity to build upon and connect to previous learning has already been discussed, as well as the benefits of the use of physical tools such as manipulatives. Teachers should also develop good communication skills within their students and ask questions that facilitate learning. Learning is a developmental process and teachers need not be concerned if a student is having difficulty obtaining or retaining a concept. By keeping a positive and encouraging attitude, teachers can create a safe environment and provide positive experiences which help students progress in their journey of learning. Not only are the students learning mathematical concepts, but they are also in the process of learning about themselves as learners. Teachers need to do both, expose their students to as many experiences as possible, and ensure that their role in the classroom is productive in aiding their students to develop all the tools necessary to be effective learners.

2.5 Role of the Teacher

In the reformed classroom, the role of the teacher is even more important. Instead of solely being the knowledge provider, they are now seen to be guides helping navigate
their students in the discovery of knowledge (Zack & Graves, 2001). As students need to create their own understanding by linking new ideas to previous personal experiences, teachers cannot impose their own comprehension of ideas onto their students. As guides, they can take a step back and encourage students to reach back to their prior knowledge to form the connections for themselves (Alagic, 2003).

Teachers need to be creative and flexible to create a diverse range of learning opportunities for their students. Even in the classroom, teachers are also continuing on their own journey and should not discount the potential to learn from and at the same time as their students (Zack & Graves, 2001). There are a myriad of ideas that students may come up with that teachers may not have uncovered themselves, which they can add to their knowledge bank and even draw from in future lessons to deepen content delivery.

Teachers must also facilitate student thinking by encouraging dialogue (Alagic, 2003). They should ask students to talk through their ideas, sort through confusions and explain their understanding of a concept. If students have difficulty with this, teachers should be able to give suitable prompts or even model the appropriate behaviour themselves. Students should also be expected to listen to other students and reflect upon the comments that are shared. Teachers can encourage students by asking probing questions and leading students to look for patterns (Reys et al., 1998).

Even indicating disapproval with students who give short explanations can be effective. This discussion stresses to the students the importance of the process and takes some of the focus off of getting the correct answer. In traditional classrooms, students tend to focus on a straightforward approach to solving problems and often rely on one technique to getting an answer. When this method does not work, these students freeze
and do not have the resources to be creative, take risks and try other methods of solving
the problem (Pape et al., 2003).

The teacher needs to develop a sense of community within the walls of their
classroom, so that the students feel safe and able to explore freely. Without this climate,
students will not reach out to the teacher to ask for help nor will they be able to
completely immerse themselves in the variety of tasks presented to them (Zack, 1993). If
the students do not feel comfortable in the classroom or in classroom discussions, they
will not be as willing to take risks and try different approaches to solving problems (Pape
et al., 2003).

While the student is interacting with their environment and/or peers, they may
experience a mixture of emotions that could affect their ability to learn (Kort, Reilly, &
Picard, 2001). The quality of their experiences could affect the quality of their learning.
Positive emotions and feelings of self-efficacy accompany the creation of new
connections, whereas negative emotions tend to correlate with incorrect assumptions
(Owens et al., 1998). The teacher needs to be able to monitor the learning environment so
that these natural emotions do not hinder the student’s ability to learn (Kort et al., 2001).

By setting up a positive learning environment, teachers are encouraging positive
learning experiences which will lead to more successful learning situations. Teachers can
better understand how the student may be feeling and how their experiences may affect
their learning by getting to know each and every student. Teachers who encourage
students to respect one another and who have developed a bond also minimize the
negative effects of these emotions (Owens et al., 1998). The teacher needs to emphasize
to the students that sometimes mathematics can be difficult and sometimes it takes a
couple attempts before a correct solution is found (Pape et al., 2003). These hurdles in learning and having students deal with failure as opposed to running from it prepares them for other challenges that they may face outside of the walls of the classroom.

A teacher is an integral part in ensuring that a student is successful. By being cognizant of their role in the classroom they can create an environment that fosters effective learning. Learning, not only of the subject matter, but that of skills that will be beneficial to them in the rest of their lives.

2.6 Cooperative Learning

The National Council of Teachers of Mathematics (NCTM) has created a set of standards that propose to change mathematics teaching procedures and hope to “enhance students’ understanding of mathematics and to help them become better mathematical doers and thinkers” (Henningsen & Stein, 1997, p. 524). The NCTM standards (2000) suggest that teachers should create instructional programs in which students can communicate their mathematical thinking coherently and clearly to others thereby solidifying the place for cooperative learning in the mathematics classroom.

In order to follow current trends in education, teachers need to expand their teaching toolkit and create a learning environment that gives them the chance to gain academic knowledge and life skills. Implementation of cooperative learning meets these goals and can prove to be most effective if the teacher examines the class, lesson and strategies from all angles along with being optimistic and persevering through some inevitable challenging moments.

“Cooperative learning is the instructional use of small heterogeneous groups of students who work together to maximize their own and each other’s learning” (Vaughan,
This learning can focus on both academic and social development (Lopata, Miller, & Miller, 2003). The instructional processes used in cooperative learning can range from simple to complex. Bennett and Rolheiser (2001) name the simple processes as tactics and the more complex processes as strategies. Many researchers have and continue to study the use of cooperative learning in the classroom and the variety of strategies and tactics available for teachers to apply in the classroom.

### 2.6.1 Benefits of Cooperative Learning

Cooperative learning can be lauded for its “constructivist potential because individual students need to generate and share personal understandings of parts of a topic” (Vermette & Foote, 2001, p. 32). The diversity of levels within the classroom only strengthens the case for cooperative learning. By combining the varied experiences and prior knowledge of the students, there is a larger knowledge base for the students to construct their knowledge from (Schoenfeld, 1987). NCTM standards are designed to encourage teachers to create student-centered learning environments and student tasks that engage students in “problem solving, modeling and constructively building conceptual understanding” (McClintock, O’Brien, & Jiang, 2005, p. 139) which suggests constructivist ideals, so cooperative learning is an excellent response to this call.

Many educators employ cooperative learning techniques (Bennett & Rolheiser, 2001; Ke & Grabowski, 2007; Stevens & Slavin, 1995; Webb, Farivar, & Mastergeorge, 2002; Van de Walle & Folk, 2005; Vaughan, 2002; Vermette & Foote, 2001). The collaborative nature of cooperative learning gives students a chance to complete tasks and attain concepts that they may not have been able to compete themselves because they were too difficult (Paradis & Peverly, 2003). The benefits of cooperative learning parallel...
Vygotsky’s (1978) zone of proximal development, which is the difference between what a learner can do on their own versus what they can do with help from others. Vygotsky (1978) believes that adults or a child’s peers can help their development and in our current world of constructivist, student-centered learning, teachers can use cooperative learning to increase the understanding of mathematics.

A variety of studies have supported the use of cooperative learning in the mathematics classroom as a way of “improving achievement, attitudes, higher-order thinking skills and self-concept outcomes” (Ke & Grabowski, 2007, p. 250). Ke and Grabowski’s study solidified this notion as their investigation of the team-games-tournament strategy encouraged positive mathematics attitudes in fifth-grade students.

In a mathematics classroom, students gain proficiency in the subject area and develop good life skills. Cooperative learning is a teaching model that is beneficial in both areas, increasing student achievement and developing social skills (Siegel, 2005). In a two-year study, it was found that students from schools that employed cooperative learning practices had more friends than those students in traditional classrooms (Stevens & Slavin, 1995). By working in a group, students are more willing to take risks as they have an immediate group of peers which act as a support system (Ke & Grabowski, 2007).

The evidence shows that there is a significant benefit to the use of cooperative learning in the mathematics classroom as it creates a positive effect on mathematics achievement, students’ reasoning skills, student interaction and attitudes towards mathematics. Although teachers can experiment and try to implement cooperative learning strategies into their classroom by their own accord, there exist a variety of
implementation strategies and considerations that researchers and other educators have already shared with the education community.

2.6.2 Implementation of Cooperative Learning

Teachers think about, among other things, covering the curriculum, needs of the individual learners, assessment and time management. In addition, when creating a varied learning environment and including cooperative learning practices into lessons, teachers should consider various frameworks.

Tschannen-Moran and Hoy (2000) propose five components to a teacher’s preparation of using cooperative learning in the classroom. First, teachers are to consider the group characteristics of their class. This component requires teachers to assess the social, emotional, achievement and cognitive levels of their class. The teacher may need to vary the type of activity used depending on the skill level of their students or focus the task more on developing the cooperative learning skills of the group as opposed to focusing on the academic result.

The second component is goal setting. Here, teachers create the lesson with the student’s level of performance in mind. They create a lesson that works to meet achievable goals (social or academic) set out by the teacher. Assuming that all students understand the nature of group dynamics and how to work effectively in cooperative learning situations is naïve; so teachers must create situations where students can develop the skills needed for cooperative learning.

Getting there, is the next step of the process. This component addresses the assignment of students to specific groups depending on their characteristics, developing the roles for each student within the task and what type of cooperative learning task or
strategy is to be used in the lesson. It is important for teachers to know their students and to create groups that will work in a positive manner.

One concern for teachers is that not all members of the group participate equally (Johnson and Johnson, 1991). Parr (2007) suggests assigning roles to each member of the group to alleviate this problem. As a Grade 7 teacher, she found that groups of up to four students work best and so created four different types of roles. The first role is that of an encourager. This student encourages their peers and keeps the group on task. The second role is that of a researcher or recorder. This student records any information during the task and is in charge of submitting any requirements to the teacher. The third role is that of the equipment manager. Ms. Parr is a science teacher, so for her classroom this role primarily deals with managing and maintaining lab supplies. However, in the mathematics classroom, this student could collect rulers, protractors, calculators, paper and writing utensils. The equipment manager could also be in charge of any manipulatives. The final role is that of assignment director. This student is to oversee the group and ensure that all members are fully aware of all parts of the group activity.

After roles are assigned, teachers need to consider which type of tactic to use during the activity. Teachers may reflect on the ability level of their students on working in a cooperative manner and choose a simpler tactics that does not need as much expertise. Other tactics may be more natural for the teacher to use depending on the curriculum to be covered.

The next component is guiding the process. In this part, teachers must consider the decisions made up to this point and as the students work, determine whether there are adjustments that need to be made. The teacher should also consider what their specific
role is depending on the needs of the students. As the groups are working, teachers should remain active and monitor the progress and dynamics of each group (Webb et al., 2002). Teachers can identify misconceptions and guide students toward the goals already set out. It is also easy for students to get off-task during cooperative learning sessions, so if there are no students in the group urging members to stay on-task, teachers are vital in maintaining focus within the group (Lopata et al., 2003). Teachers can also model proper behaviour during lessons. Not only are they enforcing their rules and expectations but they are also training students on proper etiquette (Johnson & Johnson, 1991; Lightner, 2007; Webb et al., 2002).

Gazing backwards and glimpsing ahead, is the final component to Tschannen-Moran and Hoy’s (2000) mnemonic. Simply put, this is a reflection on the process and a chance to make changes to improve on the process for the next time around. Parr (2007) also suggests that reflecting on the process is integral in the success of cooperative learning. Not only does it give the students an extra opportunity to think about the task that was completed and how they worked with their group, but it also gives the teacher useful feedback about how the groups are functioning and the students’ understanding of the process thereby letting teachers prepare more accurately for the next cooperative learning session.

One component not included in Tschannen-Moran and Hoy’s (2000) study was the importance of teachers establishing rules, objectives and expectations for the task to be worked on in their cooperative groups (Johnson & Johnson, 1991). Teachers may choose their own set of expectations depending on their comfort level and the type of task being presented. One study outlined the importance of clearly stating activity objectives
to the students (McClanahan & McClanahan, 2002). Without these objectives, it was found that the students deemed the activity to be a waste of their time.

Careful considerations of these elements create a well thought-out approach to cooperative learning. A mathematics teacher who actively reflects upon the tasks/strategies that they will be implementing, the characteristics of their target audience and their teaching goals will surely institute a more effective lesson.

2.6.3 Challenges When Implementing Cooperative Learning

The job of a teacher is a challenging one. There are many factors to consider when creating an effective lesson. Whether adding new teaching strategies to your repertoire or just pulling out comfortable ideas from your mathematics teaching toolkit, there are always decisions and choices that make the teacher’s job easier and hopefully, at the same time helps the students to learn.

One common complaint with teachers is the lack of time (Cannon, 2006). This could take the form of lacking the time needed to cover everything on the curriculum guidelines. Cooperative learning takes time for students to feel comfortable with and to carry out. A teacher led, lecture style lesson is controlled and does not allow for much time variation. By contrast, a lesson that involves cooperative learning needs time for students to discuss and as it is student centered, students may decide or just naturally need more time to fully carry out the task than initially allotted by the teacher.

Incorporating active learning strategies into the classroom also requires the teacher to put more time into planning than traditional teaching methods (Yazedjian & Kolkhorst, 2007).
Experienced teachers who have never used cooperative learning strategies may find it difficult to integrate it into their regular teaching practices because they may not feel that their created lessons lend themselves to cooperative strategies (Cannon, 2006). Sometimes the solution to this problem is for teachers to adjust research-based models of cooperative learning to a framework that they are more comfortable with and can smoothly fit with their teaching style (Siegel, 2005).

New teachers may find “learning the essentials of teaching math…as well as the rudiments of classroom management and organization” (Baker, Gerstein, & Dimino, 2004, p. 19) to be enough of a struggle, so to ask them to adopt complex teaching models such as cooperative learning may be a lot to ask. These inexperienced teachers may find it easier to get comfortable with their job before implementing cooperative learning techniques. After getting to know their colleagues, figuring out what curriculum to cover and familiarizing themselves with the student and parent community, teachers may finally be able to turn their attention to cooperative learning strategies.

Mathematics educators might not see any immediate positive effects of the use of cooperative learning in their classroom. This could be discouraging. Yazedjian and Kolkhorst (2007) stress the importance of not letting this stop teachers from continuing to use the strategy. Students need time to settle and get used to working with their peers, especially if they have never been asked to do so in a manner such as this. At the beginning, students may be tentative in actively participating in activities and even have a difficult time adjusting to student-centered ones if they are used to those that are teacher-directed. Teachers must have faith in their initiatives to see the documented benefits of cooperative learning to shine through.
These challenges can easily dissuade mathematics teachers from implementing cooperative practices into their lessons, however, through careful preparation and knowing the existing research on its benefits, teachers must continue with their efforts to eventually see the rewards in their students. As educational theorist Paulo Freire (2004) wrote, “changing is difficult but it is possible” (p. 77) and so teachers need to remember that the benefits of cooperative learning outweigh the challenges they will face.

After implementation of cooperative learning into the mathematics classroom, studies have shown that several factors exist that can encourage teachers to continue to use these practices in their regular teaching routine. A study showed that the more teachers know about a teaching strategy, the greater the chance they will continue to use it over time (Baker et al., 2004). If teachers have the opportunity to gather more information about cooperative learning from workshops, reading existing literature and/or pursuing advanced degrees, there will be a greater chance that teachers will not only have a procedural knowledge of cooperative learning, but a conceptual one as well. The study found three key reasons for the ongoing use of cooperative learning in participating teachers’ classes: (1) a workshop introducing cooperative learning and different strategies that could be implemented into their classes; (2) an expert in the field (often a graduate student) providing on-site assistance and feedback; and (3) the school and/or school board’s support to the teachers in the form of monetary funds that could be allocated for any variety of resources.

Teachers can also benefit from discussing the seen benefits of cooperative learning in their classroom with their colleagues (Cannon, 2006). Not only does this give
a chance for teachers to share in their success, but it builds staff morale and creates a support system for the teacher.

Cannon’s study (2006) echoes Baker et al. (2004) in that teachers who are just beginning to introduce cooperative learning into their teaching practices need to have reinforcement in order for them to maintain its use. This reinforcement can come in the form of “feedback from a peer or a person knowledgeable in the new strategies or innovations” (Cannon, 2006, p. 10).

As previously mentioned, sometimes students need time to get accustomed to cooperative learning and so, if teachers have the knowledge and skills in place to ensure a continued use of cooperative learning, the most promising effects will surface. Cooperative learning is a proven strategy to encourage students to develop a more meaningful understanding of the concepts being presented.

2.7 Multiple Representations

Although social interactions in the form of cooperative learning are beneficial, teachers look for more ways to give their students an opportunity to construct knowledge. The use of various representation forms, which help students to make connections and communicate their mathematical understanding, is another effective strategy.

There are many such types of representation forms and can be grouped into two categories: internal and external (Pape & Tchoshanov, 2001). Internal representations are mathematical ideas developed by the learner through experience, whereas external representations come in the form of symbols, equations, pictures, charts and graphs. A student is said to have representational thinking if they are able “to interpret, construct,
and operate (communicate) effectively with both forms of representations, external and internal, individually and within social situations” (Pape & Tchoshanov, 2001, p. 120).

Hiebert (1988) gives four steps in developing fluency of representation forms. First, the student must connect the representation form with the mathematics that they will be using. Second, the student must develop algorithms and procedures on how to use the representation form. Third, the student must develop enough familiarity with the representation form that they feel comfortable using it and finally, the student must be able to apply their use of the representation form to more complex ideas.

By using multiple representations, students can deepen their mathematics understanding, especially if they can easily transfer from representation form to representation form (Suh & Moyer, 2007). If a student is only capable of using one form, it means that the student has only made a limited number of connections associated to that type of form or even worse, they may have memorized the procedure of how to use the form but not truly understand the concept that the form represents. Using multiple representations should get the student to think about how the concept relates to the real world, justify their thought processes and clarify their thinking (Pape & Tchoshanov, 2001). The representation form should be used to strengthen the connections that they have made in the construction of knowledge.

The use of multiple representations can increase a student’s representational fluency as well as develop their problem solving skills (Kahveci & Imamogu, 2007; Pape & Tchoshanov, 2001). As students are generally more comfortable using one form over another, having them use other forms that they may not feel as comfortable with, forces them to deepen their understanding of the concept. By doing so, they can organize their
ideas in a way that they can use the various forms, as well as sort through the various forms to determine how they can best be used to demonstrate their understanding (Herman, 2007).

Gardner (1993b) describes the importance of teaching using different strategies to cater to the variety of learning styles within the classroom. He believes that all individuals have multiple intelligences, some of which are stronger than others, so by using multiple strategies in the classroom, teachers can ensure that students will have a chance to show off their strengths as well as develop those intelligences which are weaker. Gardner initially had seven intelligences (bodily-kinesthetic, interpersonal, verbal-linguistic, logical-mathematical, intrapersonal, visual-spatial and musical), but he later added two more intelligences to the list, naturalist and existential (Gardner, 1999; Gibbs, 2006).

When students use various forms, they have more opportunities to communicate their thinking. A representation form should be used to stimulate dialogue with peers and teachers and students should be able to discuss the merits of their chosen representation form and be able to compare it to other forms (Pape & Tchoshanov, 2001). In the beginning phases of using representations, students may not be as eloquent with their representation form and this discussion should help them refine their understanding and their ability to represent their knowledge (DiSessa, Hammer, Sherin, & Kolpakowski, 1991). Once a student is able to represent a concept in different ways and can explain how each of the representation forms relates to one another, they can say that they truly understand the concept (Lesgold, 1998).
The NCTM (2000) states that students should “create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representation to solve problems; and use representations to model and interpret physical social and mathematical phenomena” (p. 67). The NCTM’s representation standard states that representations can be used to strengthen a student’s mathematics conceptual understanding and their ability to communicate mathematical principles. It is mentioned that representations are both a means of processing as well as a format of showing a product.

The purpose of representations is to stimulate cognitive understanding rather than just as a vehicle for demonstrating knowledge (Pape & Tchoshanov, 2001). By working with different representation forms, students have a chance to make connections and use their representation form “to explain or justify an argument” (p. 124). Saving representations until the end of a unit or lesson as an evaluative tool robs the student of their experience. As the forms provide for a source of discussion and exploration, students often discover more about a concept than they would otherwise if they had only used standard algorithms to learn a concept.

Teachers need to use all forms of representation equally and not show bias towards a certain form as students pick up on this and tend to favor the form chosen by the teacher (Herman, 2007). At the elementary level, students are still being exposed to the basics of mathematics and have not already made any generalizations on mathematical concepts, so it is important for the teacher to keep an open mind and engage their students in many different representation forms and learning strategies. The more that a student is exposed to different representation forms and the more time that
they have to learn to use those forms, the more confident and comfortable they will feel with constantly having to change their chosen method of representation. The students will grow up with the idea that there is not one correct method and that they should be able to use any method at any time (although there are some forms which lend itself to being used at certain times over others).

Teachers can use multiple representations to lead their students to make connections between concrete and abstract representation forms. By scaffolding the forms and guiding students to see the progressive lean towards abstract representation forms, students may be able to make the connections more easily (Alagic & Palenz, 2006).

Using multiple representation forms also provide a chance for a group of students with a diverse ability level to become engaged. As the types of forms are varying in abstractness, students with different levels of understanding will always have a type of representation form which they understand and may move to higher levels of understanding from that point (Alagic, 2003). Teachers can also use this fact when they are assessing their students. If a student chooses a simpler representation, chances are the student has a more basic understanding of the concept.

In Herman’s (2007) study, students explained that, although they did not use all of the representation forms when solving a problem, they liked the idea of having “a repertoire of strategies from which to pick” (p. 48). The students found that they would pick the form that was the most efficient for the given problem and used the other forms to check the answer that they came up with using the first method. It was a way to verify
their work and students found it reassuring to have an alternative method of solving the problem to see if their answers matched.

A further benefit of cooperative learning is that students would have exposure to seeing their peers use representation forms that they, themselves, may not have naturally chosen to use. The student would have a chance to see the mathematical problem solved in different ways, build on other students’ knowledge and could even add some of these foreign representation forms to their knowledge base (Ahmed, Clark-Jeavons, & Oldknow, 2004; Mevarech & Kramarski, 1997). The positive benefits of using both cooperative learning and multiple representation forms will surely produce a more successful student.

2.7.1 Forms of Representation

The work of Lesh, Landau and Hamilton (1983) highlights five representation forms which teachers should use in their teaching: real life experience, manipulative models, pictures or diagrams, spoken symbols and written symbols. The combination of using all of these representation forms, along with encouraging the students to make connections between the types of representations, deepens mathematics understanding, specifically conceptual understanding. When a student is able to demonstrate their knowledge using all of these forms, it shows that the student has a secure understanding of the material.

Each type of representation has its own strength and encourages different connections to be made (Alagic, 2003; Suh & Moyer, 2007). Teachers can select a specific type of representation form to focus on if there is a specific part of the concept that they wish to highlight (Alagic & Palenz, 2006). Open-ended representation forms
lend themselves to having students create their own strategies for solving the problems (Suh & Moyer, 2007). Representation forms can also be ranked in order of accessibility and teachers can choose specific types of representation forms to match the ability level of individual students in their class.

### 2.7.2 Structure

Bruner (1966) declared that students need three levels of engagement for students to fully build a complete understanding of a mathematics concept. The first is enactive, where students use manipulatives and other concrete materials to construct their understanding. The second level is iconic, where students represent their understanding using pictures and graphs and the final level is symbolic, where the students use numerals to represent what they know.

Similarly, Dienes (1960) created levels which students move through to create a thorough mathematical understanding. He created five levels in total: free play, generalization, representation, symbolization and formalization. During free play, students work with physical materials and manipulatives to discover basics about the concept. In generalization, the student notices patterns and commonalities and then takes these ideas to be represented by images in the representation level. Next, the student describes their representation using mathematic language and symbols and finally, they create a set of rules and algorithms to match with their understanding of the concept.

With both structural levels set by Bruner and Dienes, one can imagine them to be rungs on a ladder, where the first rung (level) is the most basic and the further up the ladder you get, the more abstract the form and its associated connections becomes. In
both structures, the first level involves the use of concrete material, often in the form of manipulatives.

2.7.3 Manipulatives

Although one of the more basic representation forms, manipulatives provides a good base for the students as children naturally use concrete objects to model mathematical situations (Moch, 2001). Manipulatives have been found to improve conceptual knowledge and help students to visualize abstract mathematics concepts (DeGeorge & Santoro, 2004; Green, Piel, & Flower, 2008; Suh & Moyer, 2007).

The use of manipulatives is transferrable for many types of learners. Due to its accessible nature, many learners who would otherwise be struggling can benefit from the use of this representation form. In line with Gardner’s (1993b) theory of multiple intelligences, manipulatives give students with a disposition towards tactile/kinesthetic learning a chance to shine. The students can touch and feel it instead of just hearing and reading mathematics (Moch, 2001).

Manipulatives also aids students with limited English-language skills as they can focus on an object and how it relates to the mathematical concept rather than deciphering through the language even before getting to uncover the concept they are to learn (DeGeorge & Santoro, 2004). Manipulatives are a representation form which is common to all cultures and languages and this is yet another reason why they are a fundamental form to be used in the construction of knowledge.

The use of manipulatives not only enriches the learning of average ability leveled students, but also helps lower-leveled students develop their understanding without special modifications to the lesson specifically for them. Manipulatives gives students a
chance to work with concrete ideas, and slowly develop their abstract understanding, therefore scaffolding their learning at their own pace (Moch, 2001).

Manipulatives also promote positive learning characteristics within students. When given hands-on tasks, an increase “in student motivation, willingness to ask questions and volunteer information, enthusiasm, and attention to assigned tasks” is seen (DeGeorge & Santoro, 2004, p. 28). Perhaps it is their accessibility that makes manipulatives so successful. Students, no matter their level, will be able to make connections when using manipulatives and so are more willing to engage in classroom activities. Students also use concrete objects in play and using manipulatives in class may trigger similar positive emotions that they would have outside of the classroom.

The most successful lessons with manipulatives require teachers to guide the student’s lessons rather than leading the students step-by-step. There needs to be some room for the students to explore and discover for themselves. Students need to be told the goals of the lesson and how the manipulatives can help them reach that goal and the role of the teacher is to make sure that the students remember this link throughout the process. Teachers also need to guide their students towards the overall goal and prompt them to make the correct connections to get to this goal (Stein & Bovalino, 2001).

Manipulatives allow students to make “important linkages between conceptual and procedural knowledge” (Balka, 1993, p. 22). Students do not make these connections naturally and therefore, one of the roles of the teacher, when using manipulatives, is to guide the student to see these links (Kaput, 1989). Often working with manipulatives focuses the students’ attention solely on the activity at hand and so the teacher needs to remind the student about the big picture of the concept (Suh & Moyer, 2007).
2.7.3.1 Ensuring a Successful Manipulatives Lesson

There are three characteristics of teachers who created effective lessons using manipulatives (Stein & Bovalino, 2001). The first was that the teachers had previous experience with the manipulatives. The teachers had attended workshops or took specific courses on using manipulatives where they were able to practice creating, implementing and trouble-shooting a lesson using these teaching tools. They had both theoretical and practical knowledge about the manipulatives. The second characteristic was that the teacher spent a significant amount of time preparing their lesson. Regardless of whether they prepared the lesson themselves or used a pre-existing lesson, the teachers walked through the lesson just as their students would to increase their familiarity with the process and predicting possible challenges for students. The final characteristic that teachers showed was the fact that they prepared the classroom for the lessons themselves. They selected the manipulatives and organized them and the students into groups. All of these characteristics exhibit themes that the teachers are conscientious, have thought through the lessons and put themselves in the place of their students.

The activity should be student-centered and builds upon the student’s current knowledge. As learning is a progressive process, the manipulatives will help the student develop connections at their current level and more abstract ideas (Ahmed et al., 2004). These connections will strengthen their construction of knowledge and move them towards more complex representation forms.

2.8 Summary

Recent research shows that it is possible to implement cooperative learning and multiple representations into the mathematics classroom, and that it provides benefits to
students’ learning. Cooperative learning enables students to construct their own knowledge and with NCTM standards asking for teachers to promote learning in a constructivist environment where effective and long-term learning occurs, it seems to be a perfect fit for our mathematics classrooms (Ding, Li, & Piccolo, 2007). The use of multiple representations also fosters a deeper understanding level in our students and should also be used in the classroom (Pape & Tchoshanov, 2001; Suh & Moyer, 2007). Although challenges exist when implementing these strategies, teachers need to push themselves to better their teaching practices by seeking out more professional development opportunities and concentrate on their role as a teacher to create an environment where these strategies can flourish.

A framework such as the Ten Dimensions of Mathematics Education (McDougall, 2004) can be used as a guide to focus the teacher in developing both their own potential. This potential may be enhanced by paying particular attention to Dimension 5 – Constructing Knowledge, where these strategies can be most effective.
Chapter Three: Methodology

3.1 Introduction

In this thesis, I discuss the journey of a Grade 1 teacher as she undergoes a change in her teaching practices. She wants to ensure that her students succeed at constructing conceptual and procedural knowledge in the mathematics classroom.

This chapter discusses the research context and introduces the participant of the study. The methods of data collection and analysis are described as well as the ethical considerations of the study.

3.2 Research context

For my thesis, I use a grounded theory approach in this qualitative study (Glaser & Strauss, 1967). A qualitative study was chosen to explore “naturally occurring, ordinary events in natural settings” (Miles & Huberman, 1994, p.10) and gain a better understanding of the phenomenon of knowledge construction in the mathematics classroom. The study was conducted to determine both how an educator constructs knowledge as well as the reasons why an educator would do so.

I have carried out a case study, focusing on a Grade 1 teacher and the teaching practices and strategies that she uses to produce successful mathematics students. The purpose of a case study is to carry out a deep investigation and analysis of a single entity where multiple factors can be explored (Flyvbjerg, 2006; Merriam, 1988; Smith, 1978; Stake, 1995). The study is an observational case study where “the major data-gathering technique is participant observation and the focus of the study is on a particular organization” (Bogdan & Biklen, 1992, p.63). Instead of selecting the school as the organization of study, my thesis investigates the teacher participant and the focus of the
case study is to determine how this specific participant fosters student mathematical understanding.

As part of the grounded theory approach, a series of interviews and observation sessions were conducted to further examine emerging themes uncovered from the data. Initial interviews were more general of nature whereas subsequent and final interviews asked more detailed questions specific to the area of study and relevant to the emerging themes. From the data collected, I have discovered themes that have guided my findings to determine how teachers can facilitate an environment where effective construction of knowledge occurs.

3.3 Participant

Sabrina started her teaching career in Central Canada, teaching at the elementary level and continued to teach in this division when she moved to Western Canada. She has been teaching for 25 years and has experience teaching Kindergarten, Grade 1 and Grade 2. She also has experience of being a part-time resource teacher. During the study, she was the only teacher teaching Grade 1 at the school.

3.3.1 School context

Founded in 1926, St. Brendan is an elementary school located in a large urban centre in Western Canada. The school’s population is just over 200 students and encompasses Kindergarten through to Grade 7. There is also an on-site pre-school. St. Brendan is an independent Catholic school. Most of the students are of Italian descent and there are some students of Chinese or Philippino origin. The staff at St. Brendan consists of the principal, two secretaries, nine teachers, a physical education teacher, two library teachers, two resource teachers and thirteen educational assistants.
The 2006-2007 Foundational Skills Assessment (FSA) results show that the majority of students at St. Brendan are meeting or exceeding expectations of the province in all components (reading, writing and numeracy) and are above the provincial average. The following table gives a detailed breakdown of the 2007-2008 FSA results according to grade and component. The provincial averages for 2007-2008 had not been released at the time this thesis was written.

<table>
<thead>
<tr>
<th>2007-2008 Foundation Skills Assessment results</th>
<th>Unknown performance level</th>
<th>Not yet meeting provincial expectations</th>
<th>Meeting provincial expectations</th>
<th>Exceeding provincial expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 4 - Reading comprehension</td>
<td>7%</td>
<td>27%</td>
<td>60%</td>
<td>7%</td>
</tr>
<tr>
<td>Grade 4 - Writing</td>
<td>7%</td>
<td>0%</td>
<td>93%</td>
<td>0%</td>
</tr>
<tr>
<td>Grade 4 - Numeracy</td>
<td>7%</td>
<td>13%</td>
<td>70%</td>
<td>10%</td>
</tr>
<tr>
<td>Grade 7 - Reading comprehension</td>
<td>0%</td>
<td>7%</td>
<td>73%</td>
<td>20%</td>
</tr>
<tr>
<td>Grade 7 - Writing</td>
<td>0%</td>
<td>3%</td>
<td>90%</td>
<td>7%</td>
</tr>
<tr>
<td>Grade 7 - Numeracy</td>
<td>0%</td>
<td>7%</td>
<td>60%</td>
<td>33%</td>
</tr>
</tbody>
</table>

Table 1: 2007-2008 Foundation Skills Assessment results.

3.4 Data Collection

Data collection for the study took place in three different ways: a survey, interviews and in-class observations. The data was collected between November 2006 and November 2008 and arrangements to meet with the participant were scheduled based on her availability.

Sabrina is involved in the School Improvement in Mathematics study (McDougall, 2009). As part of this study, she learned about the Ten Dimensions of
Mathematics Education framework in a workshop and took an Attitudes and Practices for Teaching Math survey which asked questions about the teacher/administrator’s current attitudes and practices about mathematics education (Appendix A). The 20-question survey gave the participant quantitative data regarding her alignment to reform-based teaching qualities categorized by dimension (McDougall, 2004). After reviewing the data, the school improvement team selected dimensions which the school would work to improve and individual teachers could choose the same dimensions or decide to focus on a different dimension of their own choosing. Sabrina selected Dimension 5 – Constructing Knowledge and Dimension 7 – Manipulatives and Technology for personal growth.

The first visit on November 21, 2006 consisted of an initial interview with the participant, prior to the start of the peer coaching sessions, to determine her thoughts about success in the classroom and what her goals for her students were. Questions were also asked to find out more about the background of the teacher, the school’s culture, challenges that she faced as an educator and general opinions about mathematics education (Appendix C).

As part of the School Improvement in Mathematics study (McDougall, 2009), Sabrina was paired up with a colleague to engage in peer coaching sessions. Sabrina’s partner was Paige, the Grade 3 teacher who has taught her entire career at St. Brendan. In her 19 years of teaching, Paige has taught at both the Primary and Junior level. Paige is also part of the School Improvement in Mathematics study and had chosen Dimension 5 – Constructing Knowledge and Dimension 10 – Teacher’s Attitude and Comfort With Mathematics to focus on for professional development.
During the peer coaching sessions, the partners were given observation templates to guide them through the process (Appendix B). Each pair took turns teaching and observing each other in the classroom setting. Prior to entering the class, the pair would engage in a pre-conference where the observer would ask a series of questions to guide the teacher to consider important qualities in the dimension that the teacher had chosen to focus on. The observer would also ask the teacher what they were to look for during the lesson and find out more information about the lesson that was to be taught and the chosen teaching strategies of the teacher. During the teaching time, the observer would sit in on the lesson and take notes in the style of their choice. The observation templates also had key ideas for the observer to look for corresponding with the specific dimension chosen by the teacher. They could also make other notes if they wished.

After the lesson, the pair would have a post-conference to debrief the lesson and during this time, the observer would state their observations and if asked, could share suggestions of what could be changed the next time around. The teacher was also encouraged to share their ideas about the lesson and what they would like to focus on for the next session. After the pair completed this process, the researchers interviewed the pair about the peer coaching process (Appendix D). Four peer coaching sessions with the pair were completed, where each teacher had a chance to observe their partner twice. The sessions where Sabrina was the teacher occurred on January 23 and May 12, 2008.

A final interview on November 28, 2008, was carried out with Sabrina to gather more data about what she learned from the process and her focus on the construction of knowledge within her students (Appendix E). All interviews and peer coaching sessions
were audio recorded and observations of classroom sessions were recorded in handwritten field notes.

3.5 Data Analysis

All interview and peer coaching session recordings were transcribed and the transcripts were reviewed against the recordings to ensure accuracy. As this study used a grounded theory methodology, an inductive approach was used where there was no pre-existing hypothesis to answer the research questions (Glaser & Strauss, 1967). The data analysis followed a series of coding cycles. For this study, an open coding format was used where data was examined, compared and categorized (Strauss & Corbin, 1990).

In the open coding process, the first step involved the reading of transcripts and key ideas were highlighted. These key ideas were labeled in order to make the data more manageable. Next, these smaller data snippets which Glaser and Strauss (1967) refer to as concepts were grouped by similarity to form sub-categories. The sub-categories were regrouped by similarity and labeled as categories (Strauss & Corbin, 1990). The categories are the over-arching themes extracted from the data.

Four categories were found: professional development, constructing knowledge, role of a teacher and multiple representations. Within the professional development category, a sub-category of identifying a focus was coded. The role of teacher category had two sub-categories (positive reinforcement and engaging lessons) and the multiple representation category had three sub-categories (purpose, how to introduce/integrate them and manipulatives).

In keeping with the grounded theory approach, the data collected proved to be the source of the major findings of the study, however, categories and subcategories were
created with components of existing literature in mind to be able to generate a possible hypothesis which could be relevant to current research and practices (Glaser & Strauss, 1967).

3.6 Ethical Considerations

The ethical review process for this thesis was completed as part of McDougall’s (2009) research project. Pseudonyms for teachers and the school are used in this study to ensure confidentiality. Specific details about the location of the school have been omitted.

Participants first verbally agreed to be part of this study and then signed a formal consent letter to confirm their participation. The participants were selected for their enthusiasm for the project and their willingness to collaborate, participate, share their experiences and spend time with the project. It was stressed to the participants that they could participate as much as they wanted and could stop at any time. Participants were reminded that pseudonyms would be used and all interviews would be confidential.
Chapter Four: Findings

4.1 Introduction

In this thesis, I explore how an elementary mathematics teacher fosters student mathematical understanding. In examining the case study of Sabrina, four themes emerged from the data analysis. They were: professional development, constructing knowledge, role of a teacher and multiple representations.

4.2 Professional Development

Sabrina has a clear purpose as an educator. She says, “I would like to see all of the kids have a good understanding of what I am trying to teach them, or the concepts that are being taught” (Teacher interview, November 21, 2006). Every decision that she makes as an educator, whether it be the professional development opportunities that she seeks or the teaching strategies that she uses, are selected with this goal in mind. She wants her students to succeed and ensures that she has the right tools in place to make this happen.

Sabrina is a teacher with the desire to improve. Sabrina stated that a challenge as an educator is the “range of abilities and the range of understanding” in the students (Teacher interview November 21, 2006). Sabrina says she wants to have the tools to “do a good job of meeting all of their needs and all of those ranges” (Teacher interview, November 21, 2006). She is constantly seeking out professional development opportunities, and looks for ways to become a better teacher. Sabrina also feels encouraged to improve because of the support that she receives from her principal. She says, “Mathematics is important to [the principal] and he is encouraging, so you are willing to take it a step further” (Teacher interview, November 21, 2006). Her focus to
improve stems from her dedication to her students. In discussions with Sabrina, it is clear that she wants her students to succeed and wants to improve her teaching to meet this goal.

Although she is a veteran teacher, Sabrina continues to evolve and improve her teaching practices. She describes her journey:

For me, I want to get better. How can I get better at teaching kids, particularly math? I really like math, and with the changing programs, there has certainly been a change towards the exploration, towards a different way of teaching it. For me personally and professionally, I wanted to see what else I can do…I want to do things differently. (Teacher interview, November 28, 2008)

Sabrina’s main method of professional development is by attending workshops. She says that she has been lucky to have the opportunity to attend a number of workshops over her teaching career and that her principal is supportive of these ventures. The workshops, however, are not as frequent as Sabrina would like nor are they always effective. Sabrina says, “You take a stab in the dark going to workshops because some of them are not good” (Teacher interview, November 21, 2006). After mentioning to her principal that she is still looking for ways to improve her mathematics teaching practices, the principal suggested that she participate in the School Improvement of Mathematics Project as another professional development opportunity.

Sabrina believes that learning from various contexts and professional development outlets can improve her teaching and therefore make her a better teacher to help her students succeed. Sabrina has attended many workshops throughout her career and values the importance of reading educational research literature, but likes the opportunity to engage in professional development opportunities such as the peer
coaching because, as she says, “It is nice to see it in action” (Teacher interview, November 28, 2008).

4.2.1 Identifying a Focus

As a teacher who is striving to meet the standards of the NCTM (2000), Sabrina is seeking out new ways to deliver her course content in the most appropriate fashion. She discusses the importance of keeping up with the current trends in mathematics education and sees that, for the moment, student exploration is valued in reform-based teaching practices and says that she wants her students to “have fun” and be able to apply their mathematics understanding “to a game or something like that” (Teacher interview, November 21, 2006). Sabrina understands the need to continue to evolve as an educator and current teaching practices encourage student-centered learning with exploration and discovery-based activities.

When deciding which dimension she wanted to focus on, Sabrina thought the decision was simple: Dimension 5 – Construction of Knowledge. This dimension is directly linked to her wanting to improve upon her teaching strategies:

To me, Dimension 5 seemed very important. How do we present this, how do we allow them to explore, how do we allow them to tell us what they know already and go from what they know already to where they are going. (Teacher interview, November 28, 2008)

Sabrina also highlights the importance of student communication and the power of being able to gage their knowledge level by the way that they express themselves. She believes that, by learning more about Dimension 5, she will be able to get students to express themselves more eloquently and help alleviate the challenge of how “some [students] can explain so much and some cannot explain at all” (Teacher interview, November 21, 2006).
Sabrina mentioned a second dimension which branches off from the Construction of Knowledge. She says “we use manipulatives so much because it helps them in their concept development” (Teacher interview, November 28, 2008). Although her main focus is Dimension 5, Sabrina also stresses the strength of manipulatives to foster deeper mathematical understanding and chooses to spend some of her time with Dimension 7 – Manipulatives and Technology as well.

4.3 Constructing Knowledge

Sabrina aims for her students to have a basic understanding of mathematical concepts and once they hit that point, she pushes them to reach their full potential. Sabrina says, “I want to see how far they can take it” (Teacher interview, November 21, 2006). She would like to see her students have fun in the mathematics classroom. If the students can apply their knowledge to a game or real-world situation, Sabrina believes that she has done her job and that her students have a firm grasp of the mathematical concepts.

In order to have successful students, Sabrina believes that they need to fully understand the concepts being presented and can apply them to a variety of situations. She believes that this is a view shared with her fellow colleagues and even tried to encourage this idea among the parent community by hosting parent evenings “where we can introduce them to games like boxcars and jacks and talk about the program a little bit” (Teacher interview, November 21, 2006). It is the construction of knowledge, which Sabrina values as being important to her students’ success because she wants her students to be able to “talk about what they understand” (Teacher interview, November 21, 2006). Sabrina believes that she has a very important and specific role as a teacher in order to
ensure that her students can succeed. In order for her students to fully achieve to their ability level, Sabrina acts as a facilitator, creating a safe environment and nudges her students by means of a variety of teaching strategies.

4.4 Role of the Teacher

Sabrina sees herself as a guide in her students’ journey of gaining mathematical knowledge. She believes that it is unrealistic to expect students to be able to make all the necessary connections on their own without any form of prompting, so she often manipulates the situation so that her students are forced to make the appropriate connections to develop their mathematical understanding.

Sometimes students need help to make connections because, at this early stage in their mathematical education, they are still learning the language of mathematics and need help to communicate their understanding. Often it is not just their lack of knowledge of the correct terminology, but the complete inability to express (using any form of communication) what they know due to the fact that the information is so new and abstract to them. Sabrina says, “They have trouble describing things, but if they can somehow show you and you can help them along with some of the language, then it comes along” (Peer coach interview, January 23, 2008). The teacher acts as an aid and as a model to develop mathematical communication.

Teacher modeling is beneficial in other ways as well. If students are having difficulty coming up with strategies on how to solve a problem, Sabrina may choose to model one strategy to get them started on the right track. Students may be at a loss on how to deal with a completely foreign mathematical situation, so having their teacher demonstrate one way to tackle the problem may be enough of a push to get them to think
creatively and to come up with their own ideas. Sabrina says that she needs to act as a monitor in these situations. She says, “I have got to talk to them, they have got to show me what they are doing and if they do not have any ideas maybe I can demonstrate something” (Peer coach interview, January 23, 2008).

Students can model as well. If Sabrina sees that a student has come up their own connection or strategy, she will often ask the student to share their idea with the rest of the class. Sabrina relates that, by having one student share with the entire group, “you may have some more encouragement to get other kids to give their ideas” (Peer coach interview, January 23, 2008).

Having students share their ideas often has many positive repercussions. It opens the lines of communication, encourages other classmates to come up with and share their ideas, and also gives the students a sense of ownership in their work. Sabrina believes that it is important to empower her students and to give them a feeling that “I contributed to that, that was my contribution” (Peer coach interview, January 23, 2008). In a student-centered environment, it is important that students feel that they have a sense of power and that their opinion is important and essential to the success of the classroom community.

One specific example of how Sabrina gave her students ownership involves a lesson where Sabrina teaches her students subtraction through storytelling. At the beginning of the year, Sabrina asked each student to draw a variety of objects on slips of paper. In many lessons, Sabrina incorporates these objects, but, in this specific case, the object on the slip of paper is the subject of the subtraction story. Sabrina chose a slip of
paper with a pirate ship drawn on it, and the student who drew the object, even before any learning has begun felt a sense of ownership over the lesson.

Sabrina created a story involving a pirate ship and asked the student how many pirates should be on the boat. Once the suggestion was made, Sabrina asked students to play the part of pirates and had them act out the scenario of sailing on a pirate ship, going through a storm and having a set number (through suggestion of another student in the class) fall overboard. The subtraction problem that arose from the story was the number of pirates who started in the boat less the number of pirates who fell out of the boat equals the number of pirates left in the boat.

Once the story was complete and the students answered the subtraction question, Sabrina gave students the chance to express the subtraction problem in other ways. Examples of representation forms included using numerals and symbols, pictures and words.

All of the students who participated, whether it be through acting, giving suggestions or showing their representation form, gained a sense of ownership of the situation. They were all vital parts of the lesson and Sabrina tried to involve as many students as possible.

4.4.1 Positive Reinforcement

As a teacher, it is quite obvious that Sabrina considers the emotions of her students. On top of giving her students ownership, she creates a positive environment and a community atmosphere in her classroom. In her classroom, Sabrina was observed encouraging her students by giving them praise and or doing something as simple as giving them a handshake for doing good work. Sabrina also models behaviour for the
students in the class to follow. She will often applaud a student after they have shared
their idea and towards the end of a lesson the students often applaud their peers without
being prompted to do so.

Sabrina admits that, although she always tries to be encouraging and create a
positive experience in the classroom, things do not always go as planned. She does not let
this take over the environment that she has created in the classroom and tries to ensure
that the final feeling in the classroom is a positive one. She tells of one situation where
she needed to fix a negative sentiment:

Well, I had a meltdown yesterday in math, so I was very aware that I had done
some really bad things. After I calmed down, I had my lunch, and we went back
for a few minutes after lunch and we did a different little activity but in a much
more positive way, because I do not want to leave them ever with a bad feeling
about math. It did end on a much more positive note. (Peer coach interview,
January 23, 2008)

The sense of community and comfort within the classroom along with the positive
feelings towards mathematics felt by Sabrina’s students helped them to take risks and to
think outside of the box and be creative to come up with new connections to deepen their
mathematical understanding.

4.4.2 Engaging Lessons

Sabrina shares that, making the lessons meaningful to the students ensures that
they are more engaged, take more ownership and even facilitates a deeper understanding
of their learning. She says:

You cannot imagine. It is sometimes hard to find kids who will want to tell an
addition or subtraction story, so all I did was make little blank cards and I said,
“You can draw whatever you want on the card. Just one object on the card,
whatever you want.” And then we shuffle them then we say, “Okay, it is story
time”. Boom. Pick out a card. They love it. (Peer coach interview, January 23,
2008)
In this example, the students have a personal connection to the activity without having to create the intimidating, mathematical portion of the story.

During a lesson where students used toy dinosaurs as manipulatives, two students were observed discussing the details of the story and how the dinosaurs would play a part of their scenario. The students felt that they needed a story that was of interest to both of them before they could engage in the mathematical component of the situation.

Sabrina carefully selects the type of activity used in her lessons, as she wants something that can keep the attention of the students and to show her students that mathematics is fun. She says:

They really enjoy anything to do with games. Anything where they can use things with their hands, they really like that. They like trying to figure out, if you put it into perspective of a game or a puzzle and I talk a lot about detective work. They like that. (Peer coach interview, January 23, 2008)

For the students who are afraid of math, turning their learning into a game or something else that the students naturally enjoy can distract the students from their fears. Games and puzzles are activities that Sabrina’s young students do at home and outside of the classroom. Sabrina uses games and puzzles so that the students do not feel like mathematics is something very different or abstract.

Even grouping students together is part of Sabrina’s strategy to get her students to have fun and enjoy learning. The children are always in groups in the playground and this playtime in groups is extended into the classroom. As social beings, having the students work together in groups ensures that “they have lots of fun…they think it is all fun” (Peer coach interview, January 23, 2008).

One way that Sabrina fosters success in her students is by placing them in groups. Sometimes, Sabrina starts her lessons by having the students work by themselves, but
often she finds that her weaker students have problems making the connections on their own, so by “pairing them with a student who is more advanced, hopefully they can learn from them” (Peer coach interview, January 23, 2008). She believes that, by having students exploring and discovering together, they can learn from each other and gain insights that they may not have been able to reach if they were working individually.

Sabrina also considers the composition of the groups. She sometimes lets her students select their grouping themselves, in order to increase productivity, however, for the most part, she creates mixed-ability groupings in her lessons. If Sabrina sees that a student is struggling, she “puts them with a really strong partner who knows to show them” (Peer coach interview, January 23, 2008). Sabrina says, “Instead of [the answer] coming from me all the time, coming from another student is good” (Peer coach interview, January 23, 2008).

Although Sabrina creates a safe and engaging learning environment for her students in order to make the mathematical learning process easier, she finds that there are still additional steps that need to be taken to guide her students towards success. For her students, mathematics is a new language and Sabrina believes that the students need to be ready to represent mathematics in an abstract way. She says:

You want to move them. You do not want it just to be an abstract thing that they can see that real things can be represented in the drawing, so you are trying to move them in their ability to think that way. (Teacher interview, November 28, 2008)

Sabrina understands that each student is unique in his or her journey of understanding. She knows that students start at different levels, move at different speeds and need different types of tasks and activities to spark the necessary connections for
learning a concept. Her goal is to help each student move from the concrete to the abstract, in whatever method is best for the student’s development.

Sabrina is careful to monitor their progress and evaluate their learning. She incorporates strategies which ask her students to record their learning so that she has something to go back to. She can also use this to decide if the student needs more attention and remediation to clarify a misconception or if they have fully grasped the concept and can move on to the next topic. Sabrina shares that this is one of the challenges that she faces as a teacher: “How do they record their learning? It is really hard for me to always be going to every desk to do their evaluation” (Teacher interview, November 28, 2008). With a class of around 30 students, she finds it difficult to monitor the progress of each student as she moves around the class and observes the lesson unfold.

However, developing her student’s ability to record and represent their learning in different forms helps her as it gives concrete tools that she can use on her own time. For Sabrina, the ideal would be for her students to be able to express their learning on paper so that, as she says, “I can take it home and look at it. It is a manageability thing too” (Teacher interview, November 28, 2008). According to Sabrina, this is the most difficult skill for students to attain. The jump from using manipulatives to putting their thoughts on paper requires the most time and attention.

It is clear that Sabrina is conscious and cognizant of where her students are at in their learning journey. Always watching and collecting data, she shares that “some kids are not there yet” (Teacher interview, November 28, 2008).

Although some students learn differently, for the most part, Sabrina says:
Young students need a lot of concrete materials so that they can connect to the abstract concept. It helps them to understand what it means and what they have to do with these materials. It helps them to solidify their understanding, they go from a model and then they will go to representational drawings. Help them to gain skills and increase their skills. (Peer coach interview, May 12, 2008)

For Sabrina, these concrete materials come in the form of manipulatives that students can hold, move and use to visualize a concept.

4.5 Multiple Representations

When starting a concept, Sabrina always asks her students what they already know about the topic. This way, she can find out more about the background of her students and where they are starting their journey. She will ask her students to represent their knowledge in any way possible, whether it is using words, pictures, or numbers. Knowing this information also lets Sabrina know what representation forms the students have used previously or may be inclined to use naturally. She can determine the course of action for how she wants to present her lesson to best meet the needs of her students.

In her lessons, Sabrina incorporates a variety of strategies to help her students construct mathematical knowledge and to give them an outlet to express what they have learned. She is pleased with the power of manipulatives and has seen how engaged her students are when they use them. Sabrina believes that, by exposing her students to more representation forms and incorporating as many as possible into individual lessons, they will have a deeper understanding of mathematics concepts than they would otherwise.

In the following sections, I shall discuss the purpose of multiple representations, how to introduce and integrate them into the classroom and the use of manipulatives as one type of representation form.
4.5.1 Purpose of Multiple Representations

Using multiple representations in a lesson gives Sabrina’s students a chance to work with the form that they are most comfortable with. Each student has a natural strength and by incorporating many forms into her lessons, Sabrina gives all of her students a chance to feel like her teacher strategies are geared just for them.

Although students may feel more inclined to use a specific form, Sabrina stresses the importance of being able to use all of the forms. She believes that, being able to represent a concept in multiple ways fosters a deeper understanding for the students. By using just one form, there is the potential that students are just learning a rote formula and so, being able to maneuver between the forms ensures a true understanding of the concept. As she says:

I think for some kids, the hardest is when you are changing how you record or present or show your work, because it is the concept they are working at, whereas they just want to work on that, “Oh yeah, you put your this and there and there and there”. They do not have to understand why they are doing it. By doing it different ways, they have to show some more understanding. (Teacher interview, November 28, 2008)

4.5.2 How to Introduce/Integrate Them

Sabrina has found that the textbook that St. Brendan uses is a great support to her teaching. The Math Makes Sense 1 textbook (Jackson, 2004) also encourages a variety of representation forms and this resource gives Sabrina ideas on how to present each form and templates of activities to use in lessons. Sabrina takes cues from the Teacher’s Manual about the order in which to present the representation forms. The first time that she follows the guide, Sabrina uses the same order as the Manual. However, if she notices the students struggling with certain representation forms or if the order does not seem to quite work, she will change it for future lessons. She also mentions that she will adapt the
lessons presented in the manual. She may add her own ideas that she would like to try or have found to have worked in the past or take out elements that she is skeptical about.

When presenting a new representation form for the first time, Sabrina models how to use the representation form. Sabrina refrains from individually demonstrating how to use the form by always trying to involve as many students as possible when possible. She will often ask a volunteer from the class to help her demonstrate, or ask students to model the form as she prompts them on how to use it. If Sabrina does demonstrate herself, she tries to include the students during simple procedural steps, for example, counting out loud as a class.

Sometimes Sabrina will introduce a representation form by means of scaffolding. First, she will demonstrate how to use a form, and then ask her students to try to use the form for themselves. Once the students have practiced for a while, she will have the student compare their work to her own exemplars to see if they match. She explains that she will ask the students, “Did yours look like mine?” (Teacher interview, November 28, 2008) and if they do, then she lets the students do it on their own. Next, the entire class, including Sabrina, will continue to use the representation form until Sabrina thinks that they have grasped the form. As the students become comfortable with the form and have used it for a while, Sabrina may start to introduce a new form. As the students keep using the first representation form, Sabrina will demonstrate the new form until she believes the students are used to observing it. From there, the students will test out the new form, abandoning the original representation form (but retaining the ability to use it for the future) and practice the new form until Sabrina asks them to match their work with hers. By this point, the cycle has recommenced, and continues for as many forms as Sabrina
wants to introduce. Sabrina says that it is a “step by step” process where you “move them from the concrete to the abstract” (Teacher interview, November 28, 2008).

After introducing the representation form for the first time, Sabrina lets her students experiment with the form and practice using it in small groups or pairs. During this time, Sabrina walks around the room to monitor the students’ progress and watches if the students are using the representation form correctly. If she sees that there are problems, she may re-group the students based on strength, help specific groups individually, or address the entire class to clarify any common mistakes that are being made. During this time, she models the form again or asks students to demonstrate what they have been working on.

Sabrina says that the subsequent lesson will always begin with a review, not only of the mathematics concepts that were studied previously, but also a refresher on the representation form. Especially with new forms, she believes that the more practice the students can get, the better.

Sometimes Sabrina will ask the students if they have any representation forms that they have come up with themselves that they would like to share with the class. During one lesson, a student was sharing his strategy and Sabrina acted as the scribe to put his words onto paper. Sabrina was careful to clarify what the student was trying to express so that she could accurately represent the student’s thoughts. She also asked the student probing questions about his method to both understand what he did so that she could scribe more effectively, and to get both the student and his peers to fully understand the process of his method. Having the student verbalize further his representation triggered more connections for the student and the rest of the students.
could grasp the steps that he took and maybe make additional connections for themselves. These extra connections could spark another strategy that they could, in turn, share with the class.

### 4.5.3 Manipulatives

One strategy that Sabrina uses to help students construct their mathematical knowledge is through the use of manipulatives. Sabrina uses manipulatives in her lessons almost every day and so her students have become accustomed to using these tools as a way of making connections in the mathematical context.

Sabrina believes that being comfortable with the manipulatives is important for the students in order for them to have the most effect. Without that comfort level, the students will concentrate more on the tool rather than how it can be used to facilitate the construction of knowledge.

At the beginning of the year, when Sabrina was first introducing the manipulatives to her students, she felt that it was important for her to demonstrate the proper procedure on how to use the objects. This modeling, however, did not always come from just her. Sabrina felt that it was important to incorporate the students as much as possible and after she models how to use an object, she “will pick a volunteer partner and we will show the class how it should look and how it should sound when we use these things” (Peer coach interview, May 12, 2008).

For her young students, using the manipulatives properly also teaches the students life skills on how to respect tools and care for them. Sabrina prepares the manipulatives for the lesson and takes the time to pre-package them in sets for groups in the class. However, just like a child would be taught by their parents to take care of their toys and
other personal effects at home, her students are taught how to handle the objects, share
them with their peers and are expected to put them away in an orderly manner after use.
Sabrina mentions that it is sometimes difficult to get all of the manipulatives that she
would like for her class. Sometimes, as with other resources, she needs to share them
with other classes, so, in order for them to be reused over and over again, they need to
stay in good condition and the students are directly responsible for that.

According to Sabrina, the use of manipulatives increases students’ understanding
and knowledge of the concept and acts a tool to:

Show what they know about it and sometimes it helps them to organize, organize
their things, organize the way they approach the problem to solve it, it helps with
their counting skills and their focus, because a lot of little kids lose count easily.
(Peer coach interview, May 12, 2008)

Manipulatives are just one way for students to demonstrate their understanding and gives
them the chance to explore mathematical concepts in a different fashion.

4.6 Summary

Sabrina’s story tells us how, as a veteran teacher, an educator should always be
looking for more ways to improve their teaching practices. Educational trends change and
it is important to keep up with current ideas. With the goal of student success always in
the back of her mind, Sabrina believes that, as the Grade 1 teacher at her school, she is
responsible for building a foundation of mathematical knowledge for her students and
takes this role seriously. She aims to have her students construct their knowledge by
giving them a variety of strategies and representation forms which they can use to make
connections and create a deep understanding of mathematical content.
Chapter Five: Discussion and Interpretation of Findings

5.1 Introduction

In this chapter, I revisit the research questions posed in Chapter One and explore how the case study of Sabrina answers those questions. Next, I discuss the major findings from the study and link them to the current literature. Finally, I suggest areas for further research in the realm of constructing mathematical knowledge.

5.2 The Research Questions

My thesis focused on the research questions posed in Chapter One. Those questions were:

1. How do elementary school teachers foster student mathematical understanding?
2. What strategies do teachers use to help students learn mathematics concepts?

I will examine each research question based on the findings of the case study of Sabrina.

5.3 Discussion of Each Research Question

5.3.1 How do elementary school teachers foster student mathematical understanding?

The case study of Sabrina shows that she considers herself throughout all parts of her teaching in order to be the most capable professional to foster student mathematical understanding. By getting to understand herself as an educator, she can equip herself with all of the necessary tools to deliver an effective lesson where students develop mathematical understanding. She can create an environment which meets the needs of her students and gives them the opportunity to learn by getting to know her students as learners.
5.3.1.1 Enthusiasm for Professional Growth

The case study of Sabrina illustrates that teachers are looking for ways to improve. Although a veteran teacher, Sabrina is still seeking out professional development opportunities to improve her teaching practices and learn new teaching strategies that she can use to help her students learn mathematics concepts more efficiently and effectively. By attending many workshops and in-service sessions, Sabrina has a wealth of teaching strategies that she may not have learned when she was a pre-service teacher or could have come up with on her own.

Sabrina understands that education is changing and that teachers need to continually educate themselves to keep up with the current trends and is therefore enthusiastic about finding outlet for professional growth. This supports the findings of Desimone et al. (2002). Her journey to improve as an educator continues regardless of the number of years that she has been teaching and she is encouraged by the support that she gets from her fellow teachers and her administrators. Although Sabrina’s enthusiasm shows that she would persevere without their support, being part of a unified group who are all looking to improve the mathematics program at St. Brendan leads to increased results (Weasmer, 2008).

Although she has attended many workshops and seminars, Sabrina is intrigued to reap the benefits of the peer coaching process and dialoguing with her peers in a more structured setting compared to quick hallway conversations or lunchtime staffroom chats. She knows that other teachers in her school have effective teaching strategies and she welcomes the opportunity to learn from them and learn more about her own current teaching practices through their eyes (Loucks-Horsley et al., 2003). By using the Ten
Dimensions of Mathematics Education (McDougall, 2004) as a framework for the peer coaching model, she was able to select a dimension that specifically focused on teaching strategies and had tools, including the Ten Dimensions continuum and observation templates, which allowed her to hone her thinking in this area.

One of Sabrina’s teaching goals is to have all of her students understand the mathematics concepts being taught and so it comes as no surprise that she focuses on ways to foster mathematical understanding. As an educator, Sabrina has a clear goal and parallels Guskey’s (2000) notion that professional development needs to be intentional. Attending workshops help focus Sabrina’s professional beliefs and got her to reflect on her own teaching practices. Sabrina understands whom she wants to be as a teacher, has a goal of becoming that educator and seeks out opportunities to help her to meet those goals. She uses the peer coaching model to strengthen her chances of growth as the model gives her a tangible way to assess her practices. Although the observer’s comments are not to be evaluative, hearing from another person what she has done in her classroom gives Sabrina a chance to reflect on her practices and self-evaluate.

Sabrina’s peer coaching partnership encourages professional growth. Her partner, Paige, is also an educator who is looking to improve and both have decided to invest their already precious time into the model to help themselves, each other and ultimately the mathematics program to improve (Arnau et al., 2004). Although both teachers expressed nervousness about the process, they pushed on as they knew that their partner was a successful educator with great insight to share. They had both taught at the school for many years and had built a level of trust and respect for one another.
Although Sabrina has taught for many years, she is still seeking out professional development opportunities to improve her teaching practices and learns new strategies to help foster student mathematical understanding. Her ability to foster mathematical understanding has increased and continues to increase as Sabrina enthusiastically adds strategies and fine-tunes her teaching practices with each professional development opportunity.

5.3.1.2 Role of the Teacher and Impact on Students

The professional development opportunities that Sabrina has been exposed to let her clarify her beliefs as an educator and self-reflect about her practices. As such, Sabrina is cognizant of her role as a teacher in the classroom and has come to realize the effect that she has on her students. She is aware that she is a role-model to her student and that they will pick up on any cues that she gives regarding how to interact with materials and learn a concept. She actively models appropriate behaviour to foster mathematical understanding among her students. Sabrina’s view is that students create their own mathematical understanding and so she merely acts as a guide in their journey.

During her classroom routine, Sabrina moves from student to student and observes how they learn alone and with others. She has built a level of understanding of her students and is familiar with who they are as individuals and as learners. The work she has put in to monitor her students and to learn about the individuals in her classroom has been valuable as now she knows how to group her students to maximize their academic potential and minimize their emotional stress (Owens et al., 1998).

Sabrina believes that students can make their own connections. However, when dealing with new concepts which may be viewed as abstract, her students need to be
nudged in the right direction and this is where she steps in to guide her students in their learning. This supports the findings of Zack and Graves (2001). She can demonstrate, lead and model learning until her students feel more comfortable and/or can make connections for themselves. It is the students that need to make the connections for themselves and connect their current understanding with prior knowledge (Alagic, 2003).

Sabrina creates an environment in her classroom where students can concentrate on learning mathematics. She has developed a level of respect amongst the students by positively reinforcing good behaviour and modeling appreciation statements. The students feel safe to experiment and take risks to create their mathematical understanding.

Lessons in Sabrina’s classroom are engaging. Students are involved in their learning and are seen to smile, laugh and actively participate in tasks. Sabrina creates a feeling that mathematics is fun and worthwhile, and students subscribe to this and fully engage in lessons to succeed in gaining mathematical understanding.

The use of multiple representations fosters mathematical understanding by giving students a variety of strategies to learn concepts. Sabrina uses a variety of forms and students of all ability levels can find a form that they feel comfortable using. The various forms challenge the students to develop a stronger mathematical understanding of the concept and act as a vehicle to communicate their knowledge to their peers and Sabrina herself.

This case study depicts an elementary school teacher whose global awareness of herself as an educator fosters student mathematical understanding. Sabrina constantly improves herself as a teacher and puts into place a stable base for learning to occur. She
is aware of her role in the classroom, how she can create an environment where student can learn and provides a variety of opportunities for her students to explore mathematics.

5.3.2 What strategies do teachers use to help students learn mathematics concepts?

The case study of Sabrina illustrates a variety of strategies used to help students learn mathematics concepts. These strategies help to create a more diverse learner and give all students a chance to participate in the lesson and learn to their capabilities.

Sabrina creates an environment which is conducive to learning mathematics. She allows her students to feel safe, models appropriate behaviour and nudges her students to make connections in their learning. She encourages her students to use their own voice and praises students who create their own understanding of concepts and asks them to share their knowledge with the rest of the class. Students who have not gained a mathematical understanding of the material themselves will now have yet another method of grasping the concept. They are able to hear about the concept from a different voice (instead of always hearing it from the teacher) and may gain new insight to the material.

Positive reinforcement also encourages Sabrina’s students to continue to progress. When making appropriate connections, Sabrina praises her students and instills a feeling that the steps they are taking are reflective of how students learn mathematics. Owens et al. (1998) propose that praise is a positive motivator for students. Sabrina constantly tries to instill positive emotions within her students. She wants her students to view mathematics and learning mathematics as something fun.

Sabrina gets to know her each of her students and their learning styles. She determines their ability level and how she can best help her students to improve as
individuals. She is aware of the choices that she makes when creating her lessons and tries to use a variety of methods which will speak to each one of her students at different times (Gardner, 1999).

5.3.2.1 Cooperative Learning

Cooperative learning is a strategy that Sabrina often implements to help her students to construct knowledge. While working with their peers, her students are exposed to more ideas that they may not have been able to come up with on their own and will be forced to explain their own understanding of the concept thereby making them further rationalize their thoughts (Vygotsky, 1978; 1986). Sabrina capitalizes on Vygotsky’s (1978) zone of proximal development by putting her students in different groups to learn from as many people as possible.

The cooperative task where students worked together to create subtraction scenarios helped the students to learn by having the chance to discuss their scenario with a respected peer to create a topic of mutual interest to the group. This allowed each member to feel a connection to the scenario, and consequently, was eager to continue with the mathematical task. Students learned the concept by creating a subtraction scenario, which they could both be proud of and could share with the class. If the students were working individually, they may not have been as excited to learn, as they did not have another colleague to share their excitement with and feed off of each other’s enthusiasm. This supports the findings of Ke and Grabowski (2007) as well as Stevens and Slavin (1995). The students have more fun when they are working in groups and learning is easier and faster if students commit to the process rather than if they are skeptical or fearful of it.
Having her students work in groups gets Sabrina’s students interacting with one another and elevates the sense of community in her classroom. The students bond, grow stronger as a unit when they need to work together and can learn from one another to develop their individual mathematical understanding. Students can rely on one another as a support and Sabrina instills this by modeling supportive behaviour within and among her students. With this continued effort, students in Sabrina’s classroom will celebrate the successes of not only themselves, but of their peers as well, similar to Zack and Graves’ (2001) findings.

5.3.2.2 Student Voice

Giving students a voice is a teaching strategy that Sabrina often applies to help her students learn mathematics concepts. By empowering students, students take ownership of their learning and are eager to learn mathematical concepts. They are proud to learn and want to so that they can demonstrate to the teacher and their peers what they have learned. Sharing also gives students a chance to hear the ideas of their peers. These ideas may be ones that the students have not considered themselves. Activities where student tell stories, act out scenarios and incorporate their own drawings into the activity are all examples of how Sabrina involves her students and gives them ownership of their learning.

Regardless of the mode of delivery, having students share their ideas also helps them to clarify their learning. By getting the students to express their ideas, the students will have to organize and clarify their thoughts and this creates a deeper understanding of the material (Alagic, 2003). The students will have had to truly learn the mathematics concepts to be able to share and voice their ideas.
5.3.2.3 Providing Learning Opportunities

Sabrina creates an environment where students feel a sense of responsibility and accountability for both their own learning and of their peers. She also creates lessons which are engaging to highlight the fact that mathematics is relevant and a component of life. Drawing out the importance of mathematics and connecting it to situations outside of the classroom, Sabrina’s students do not see mathematics as a discrete entity and view it as something in which they like to engage.

Sabrina fosters student mathematical understanding by creating real-world situations where they can incorporate mathematics concepts. This way, the concepts will not be trivial and abstract as they have a place in their every day lives. The students will connect mathematics with real-world experiences and create connections based on the amalgamation of these two entities (Reys et al., 1998). Even efforts to incorporate mathematics into the home environment by educating parents about mathematics games and the concepts that are being taught in the classroom can help students feel like the mathematics that they learn in school is applicable outside of the classroom. By having more opportunities to practice their mathematics, the students will have a greater chance of developing a rich mathematics understanding.

Sabrina creates a multitude of opportunities for her students to construct their knowledge. By having her students work in groups and exposing them to multiple representation forms, her students have chances to make connections to solidify their mathematical understanding. Their exposure to a variety of stimuli helps them to fully develop their learning potential (Vygotsky, 1978; 1986).
As Sabrina is a Grade 1 teacher, it is important to highlight the number of strategies and learning opportunities that she uses in her classroom. Similar to Mevarech and Kramarski’s (1997) findings, exposing her students to these teaching tools prepares Sabrina’s students for their future studies as the will already be comfortable with the strategies or tactics being used and will only need to concentrate on learning the more advanced concepts.

Students are naturally inquisitive and enjoy completing activities to feel success. They like to have fun and young children are often playing and exploring their surroundings. Teachers should create tasks that students have experience with or frame their lessons around something of personal interest to them, as it helps the students to learn because they can link their mathematics learning to previous experiences (Holton, Ahmed, & Williams, 2001; Moch, 2001; Vygotsky, 1978). Tasks which are engaging also foster learning as the students are hooked and continue to push forward with the task, naturally picking up new ideas as they actively engage in the task.

If a teacher turns a mathematics lessons into a game or a puzzle, they are integrating the curriculum with activities that the students enjoy and are used to doing in their lives (Burns, 2009, Cavanagh, 2008; Jordan, 2007). They can enjoy their learning and have fun at the same time. Some students may not even realize that they are learning mathematics and weaker students or those with mathematics anxiety may have an easier time learning the material because they are distracted by the task rather than focusing on the content.

Children are naturally inquisitive and enjoy solving problems. By incorporating mathematics into activities which students are eager to engage in, students will learn the
mathematics concepts much quicker as they race to solve the puzzle or mystery. These tasks keep the students having fun while learning and engaged on the task at hand.

**5.3.2.4 Use of Multiple Representations**

The use of multiple representations is an important way that Sabrina fosters the construction of knowledge. By incorporating a variety of strategies to learn mathematics, her students are given a variety of opportunities to create, experiment and demonstrate their knowledge.

Her students were exposed to a variety of strategies. Some of these strategies came naturally to the students and different students had an affinity to different strategies. However, it was important for the students to be exposed to a variety of tasks including those with which they may struggle. The students learned more about themselves as learners, finding out their strengths and weaknesses and hopefully improving their weaknesses so that they would become more well-rounded learners (Pape et al., 2003).

This exposure to a variety of strategies strengthened Sabrina’s students’ aptitude for representational thinking. She asked her students to work with and between the different forms to stimulate them to understand mathematics concepts at a deeper level (Lesgold, 1998; Pape & Tchoshanov, 2001). Her students learned to interpret the different forms and to develop a fluency to use different forms in different contexts.

Sabrina’s scaffolding of the multiple representation forms that she uses in her class parallels Bruner’s (1966) levels of engagement. Sabrina uses manipulatives as a starting point with her students, no matter their ability level and they move on to more abstract representations in the form of drawings and pictures until finally her students are ready to represent their mathematical understanding using symbols and more advanced
mathematical terminology. By going through Bruner’s three levels, Sabrina’s students have a chance to build their mathematical understanding in a progressive manner so that they are not expected to make the drastic jump to understanding abstract concepts all of a sudden. While some students may not need the first level to be competent at the second level, exposing her students to all levels gives the students of varied abilities a safe zone where they can join the group and move up to the most abstract level together.

Multiple representations give students of all ability levels somewhere to start. The type of representation form used will both draw out certain types of learners and also draw out certain levels of learners (Gardner, 1993b). No matter the ability level, the student has an access point to start their learning that is appropriate for them. Sabrina has put into place a model of progression where all of her students can use these multiple representation forms to build their mathematical understanding (Alagic & Palenz, 2006).

By giving her students various outlets for expressing themselves, Sabrina’s students are able to explore different ways to demonstrate their understanding. Students will not feel limited by the constraints of how they are able to communicate their learning and can experiment with different formats which will help them to grow as a learner. As Sabrina’s students are used to experimenting with different representation forms, they do not get frustrated as easily when they do not understand how to deal with a difficult concept and will deepen their understanding by using their creativity to take risks and try other routes to solving their problems (Pape et al., 2003).

Manipulatives are a fundamental representation form as they act as a basic starting point for learners of all ability levels. These tools can act as a distraction for
tentative or fearful learners or even focus the attention of those students who may need extra assistance to keep them on task and working towards learning the material.

Manipulatives give students something concrete to help them understand abstract concepts (Moch, 2001). They bring the mathematics back to something that they have experience with outside of the classroom as these manipulatives can come in the form of toys or household objects (among other things).

Some manipulatives are fun toys that the students can play with, but they are also functional tools to facilitate the students in making connections to further their learning. These tools are often the first step in the path towards a fluent understanding of a mathematical concept (Reys et al., 1998). Sabrina realizes the importance of a strong foundation and often uses manipulatives to aid her developing learners. She uses manipulatives as one of the first representation forms to be introduced to her students and she will move on to more advanced forms such as drawings and symbols once her students have mastered the concept using manipulatives. Manipulatives can also act as a common ground for students of varied abilities working together in cooperative groups who aim to improve their knowledge level.

Manipulatives can act as a positive distraction for the students in Sabrina’s class. Playing and experimenting with manipulatives is fun and students often learn the concepts by accident. Sabrina does not worry about having to deal with students who fear mathematics as they are constantly at play. The manipulatives are a wonderful starting point for the students’ learning as they use these tools which are already a part of their every day life and students can create knowledge by connecting new mathematics concepts to these pre-existing experiences. They are used as an access point to abstract
mathematical concepts where students can take an already familiar object and translate it into mathematical thought (DeGeorge & Santoro, 2004; Green et al., 2008; Moch, 2001; Suh & Moyer, 2007).

For some students, the manipulatives act as a distraction to keep their minds off of the fact that they are learning new and at times, difficult material. For other students, manipulatives act as something that they can focus their attention on to help guide them in their learning. They know that the manipulative has a function to help them make connections and understand the mathematics concept and holding and interacting with the manipulative keeps this focus. The manipulative is used as a tool to create understanding and organize the students’ thoughts (Gardner, 1999). It may also be a vehicle to help the student explain their thought processes to the teacher and their peers.

Sabrina clearly uses a variety of strategies to help her students learn mathematics concepts. She takes into consideration her students as people (not just as learners) by empowering and engaging them and exposing them to strategies which will make the learning of the material more accessible. The steps that she took to foster student mathematical understanding provide a good base for student learning.

5.4 Major Findings

This thesis examined the case study of a Grade 1 teacher and how she facilitated the construction of mathematical knowledge in her classroom. The study showed that the teacher took many steps to create a learning environment that lead to the successful construction of knowledge amongst her students and that she carefully examined different facets of her teaching to ensure that her students had the best possible chance of
being successful. The teacher had a professional goal to deliver lessons which would help students construct their knowledge efficiently and effectively.

The major findings of the case study can be summarized as follows:

1. Enthusiasm for improving her practices to become an effective practitioner is necessary for this teacher to successfully meet her goals. She reflects honestly and as open to change.

2. This teacher’s role in the classroom and the impact that she has on her students is important. This teacher acts as a guide to facilitate in the construction of knowledge and models appropriate learning behaviours.

3. The classroom is an environment where her students feel safe. There is a level of respect within the classroom community and her students have the opportunity to work in various group situations. Students are given opportunities to take ownership of their learning and share their ideas with the classroom community.

4. A variety of engaging tasks that are relevant to the lives of their students and various tasks and strategies that students of varied abilities, interests and aptitudes can enjoy are used. Although not all tasks are of interest to all students, there is enough variety that each student finds something that they are successful at or enjoys.

5. Multiple representations are used to scaffold the construction of knowledge and expose students to various forms of representation. Multiple representations facilitate the students’ progression from concrete ideas to those which are more abstract. One fundamental representation form is the use of manipulatives.
5.5 Implications for Further Research

This study shows an elementary teacher on a journey to create the ultimate pieces of the puzzle in constructing student mathematical knowledge. She seeks out learning opportunities, creates a specific classroom environment and employs a variety of strategies which will target different types and ability levels of learners. With all of these puzzle pieces in place, her students are observed to enjoy learning mathematics and construct knowledge. However, further research can investigate more strategies that a teacher can put in place to meet the diverse needs of today’s student community.

With the push for inclusive classrooms, another area of additional research could be whether or not the findings of this particular study are effective with students with learning disabilities or behavioural problems. The present study offers strategies which meet the needs of learners of varying ability level and types of learners. However, are there any additional considerations that need to be addressed in classrooms involving students with more specific needs? Are these strategies appropriate and engaging for these types of learners?

The case study examines a Grade 1 teacher. Although her case shows an exemplary model of how a teacher facilitates the construction of knowledge, it is just one case. As this study followed a grounded theory approach, the themes and findings that emerged from this study are specific to this particular case study and as such, more research and evidence must be collected to create a more general picture of how teachers can foster effective mathematics learning. A future study involving more elementary teachers and completing a cross-case analysis of their findings would provide a stronger model of how elementary teachers promote the construction of mathematical knowledge.
By comparing the practices of a variety of teachers, it could be discovered if there are strategies that have proved to be successful in different classrooms.

In a similar vein, I wonder if the findings would be different at a different grade level or if similar strategies could be applied to a secondary classroom. At the secondary level, the concepts are even more abstract, so what tools can a teacher use as a foundation for learning for these types of concepts? Could similar representation forms be used to scaffold a student’s mathematics learning at these higher levels?

This study has provided a wealth of information about how the construction of mathematical knowledge at the elementary level can occur and the power of using multiple representations to enhance this process. Further research in this area will shed more light on how to best meet the needs of our learners at all age, grade and ability levels and what strategies and considerations teachers should examine to promote satisfaction and success among mathematics learners.
References


Appendix A

Attitudes and Practices to Teaching Math Survey (McDougall, 2004, pp. 87-88)

Instructions:
Circle the extent to which you agree with each statement, according to the A to F scale below.
Then, use the charts at the top of the next page to complete the Score column for each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Extent of agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I like to assign math problems that can be solved in different ways.</td>
<td></td>
</tr>
<tr>
<td>2. I regularly have all my students work through real-life math problems that are of interest to them.</td>
<td></td>
</tr>
<tr>
<td>3. When students solve the same problem using different strategies, I have them share their solutions with their peers.</td>
<td></td>
</tr>
<tr>
<td>4. I often integrate multiple strands of mathematics within a single unit.</td>
<td></td>
</tr>
<tr>
<td>5. I often learn from my students during math because they come up with ingenious ways of solving problems that I have never thought of.</td>
<td></td>
</tr>
<tr>
<td>6. It’s often not very productive for students to work together during math.</td>
<td></td>
</tr>
<tr>
<td>7. Every student should feel that mathematics is something he or she can do.</td>
<td></td>
</tr>
<tr>
<td>8. I plan for and integrate a variety of assessment strategies into most math activities and tasks.</td>
<td></td>
</tr>
<tr>
<td>9. I try to communicate with my students’ parents about student achievement on a regular basis as well as about the math program.</td>
<td></td>
</tr>
<tr>
<td>10. I encourage students to use manipulatives to communicate their mathematical ideas to me and to other students.</td>
<td></td>
</tr>
<tr>
<td>11. When students are working on problems, I put more emphasis on getting the correct answer rather than on the process followed.</td>
<td></td>
</tr>
<tr>
<td>12. Creating rubrics is a worthwhile exercise, particularly when I work with my colleagues.</td>
<td></td>
</tr>
<tr>
<td>13. It is just as important for students to learn probability as it is to learn multiplication.</td>
<td></td>
</tr>
<tr>
<td>14. I don’t necessarily answer students’ math questions, but rather ask good questions to get them thinking and let them puzzle things out for themselves.</td>
<td></td>
</tr>
<tr>
<td>15. I don’t assign many open-ended tasks or explorations because I feel unprepared for unpredictable results and new concepts that might arise.</td>
<td></td>
</tr>
<tr>
<td>16. I like my students to master basic operations before they tackle complex problems.</td>
<td></td>
</tr>
<tr>
<td>17. I teach students how to communicate their math ideas.</td>
<td></td>
</tr>
<tr>
<td>18. Using technology distracts students from learning basic skills.</td>
<td></td>
</tr>
<tr>
<td>19. When communicating with parents and students about student performance, I tend to focus on student weaknesses instead of strengths.</td>
<td></td>
</tr>
<tr>
<td>20. I often remind my students that a lot of math is not fun or interesting but it’s important to learn it anyway.</td>
<td></td>
</tr>
</tbody>
</table>
# Attitudes and Practices to Teaching Math Survey Scoring Chart

For statements 1–5, 7–10, 12–14, and 17, score each statement using these scores:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

For statements 6, 11, 15, 18, 19, and 20, score each statement using these scores:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

To complete this chart, see instructions below:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Related Statements</th>
<th>Statement Scores</th>
<th>Sum of the Scores</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program Scope and Planning</td>
<td>4, 8, 13</td>
<td>6, 4, 5</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>2. Meeting Individual Needs</td>
<td>2, 6, 7, 15, 16</td>
<td>6</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>3. Learning Environment</td>
<td>3, 5, 6</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4. Student Tasks</td>
<td>1, 2, 11, 15, 16</td>
<td>6</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>5. Constructing Knowledge</td>
<td>5, 11, 14, 15, 16</td>
<td>6</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6. Communicating With Parents</td>
<td>19, 9</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>7. Manipulatives and Technology</td>
<td>10, 18</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8. Students’ Mathematical Communication</td>
<td>3, 6, 10, 17</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>9. Assessment</td>
<td>8, 11, 12, 19</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>10. Teacher’s Attitude and Comfort with Mathematics</td>
<td>4, 7, 13, 15, 20</td>
<td>6</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

**Total Score (All 10 dimensions)**

**Overall Score** (Total Score ÷ 38)

**Step 1** Calculate the **Average Score** for each dimension:

1. Record the score for each Related Statement in the third column.
2. Calculate the **Sum of the Scores** in the fourth column.
3. Calculate the **Average Score** and record it in the last column.

For example:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Related Statements</th>
<th>Statement Scores</th>
<th>Sum of the Scores</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program Scope and Planning</td>
<td>4, 8, 13</td>
<td>6, 4, 5</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

**Step 2** Calculate the **Overall Score**:

1. Calculate the **Total Score** of the sums for all 10 dimensions in the fourth column.
2. Calculate the **Overall Score** by dividing the Total Score by 38.

For example:

**Total Score (All 10 dimensions)** | 152
--- | ---
**Overall Score** (Total Score ÷ 38) | 4

**Step 3** Interpret the results:

<table>
<thead>
<tr>
<th>Average Score for Each Dimension</th>
<th>Overall Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average scores will range from 1 to 6. The higher the average score, the more consistent the teacher’s attitude and teaching practices are with current mathematics education thinking, with respect to the dimension. A low score indicates a dimension that a teacher might focus on for personal growth and professional development.</td>
<td>The overall score will range from 1 to 6. The higher the overall score, the more consistent the teacher’s attitude and teaching practices are with current mathematics education thinking and the more receptive that teacher will likely be to further changes in his or her practice.</td>
</tr>
</tbody>
</table>
Appendix B

Ten Dimensions: Observation Template (Adapted from McDougall, 2004)

Pre-observation Conference
1. What dimension are you working with today?
2. What topic will you teach today?
3. What did you do prior to today in that topic?
4. Is there anything specific that you want me to look for?

Post-observation Conference
1. How do you think it went?
2. Did you accomplish what you planned to do in the lesson?
3. What would you do differently?
4. Give feedback from your notes. This discussion should centre on what you saw and heard.
5. Now that you have had this feedback, what are you going to do next to improve or change your teaching practice? Give the teacher an opportunity to talk about what he/she might do in the future.

Dimension 5: Construction of Knowledge

Refers to how the teacher helps students develop their mathematical understanding

Guiding Questions (Pre-conference)
Instructional Approach
1. How do you try to acknowledge the different prior knowledge students have?
2. How do you decide when to guide rather than deliberately focus on students’ approaches?
3. What different instructional strategies do you use in teaching mathematics?

Questioning
4. How do you encourage more students to respond to questions?
5. How do you decide how long to wait for an answer?
6. Are you aware of your tone and body language when you respond to student answers?

Guiding Questions (Observation)
Instructional Approach
1. How is prior knowledge determined and acknowledged?
2. Does the teacher focus on the students’ approaches or his or her own approach?
3. Does the teacher focus on building understanding?

Questioning
4. Do the teacher’s questions elicit prior knowledge in a deliberate way?
5. How many questions are asked?
6. How does the teacher treat wrong answers?
7. Are the teacher’s tone and body language appropriate?

**Possible Evidence**

**Instructional Approach**
- Teacher makes deliberate connections to prior knowledge.
- Student questions drive the lessons and tasks
- Teacher provides significant blocks of time for student exploration of concepts using a variety of materials and strategies
- Teacher listens to student answers and encourages exploration of errors and misconceptions.
- Teacher appears reasonably knowledgeable about constructivism and what it looks like in a mathematics classroom.

**Questioning**
- Teacher asks probing questions to start the lesson, deepen thinking and understanding
- Students are regularly asked to clarify their understanding so the teacher can support their learning
- Teacher asks fewer, but deeper, questions which require student thinking and provides sufficient wait time after asking those questions.
- Teacher does not judge answers to questions too quickly.
- Teacher’s tone and body language does not influence student responses negatively.

**Observations**

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**Dimension 7: Manipulatives and technology**

*Concerns how the teacher uses manipulatives and technology to teach math.*

**Guiding Questions (Pre-conference)**

**Manipulative use / Technology (Use appropriate term based on teacher’s goals)**
1. How do manipulatives/technology assist your students in their learning?
2. Do you regularly use manipulatives/technology?
3. Are any manipulatives/technology always available to all students?
4. How you been able to use manipulatives/technology to enhance the curriculum in any way?
5. What do you hope to achieve through the use of manipulatives/technology?
6. How do you teach students to use the manipulatives/technology correctly and efficiently?
7. What professional development have you undertaken to integrate manipulatives/technology?
Guiding Questions (Observation)

Manipulative use
1. Are students familiar with manipulatives/technology?
2. Is the focus mainly on how to use the physical materials or on how the materials represent mathematical ideas?
3. Is exploration of mathematical ideas with materials encouraged?

Technology
4. Do students use calculators as a regular tool in problem solving situations?
5. Do many students misuse calculators, for example, to add single-digit numbers?
6. Does the software used reflect the spirit and intentions of the curriculum?

Possible Evidence

Manipulative use
- Students are not distracted by manipulatives but use them as meaningful learning tools.
- Students use language that clearly connects mathematical vocabulary to physical actions with the manipulatives.
- Students refer to the manipulative materials by name.
- Manipulatives are used by all students, not just struggling students.
- Manipulatives are easily accessed and in sufficient quantity for all students to use them effectively.

Technology
- Students are provided with access to computers and calculators.
- Students access technology in problem solving contexts.
- The teacher focuses on the value of technology in learning ideas rather than as an end in itself.
- Software used is curriculum appropriate.
- A schedule for regular access to computers is posted.
- Students show good judgment about when technology is used.
- Students show a level of comfort with technology.
- Students appear to know procedures for using technology correctly and efficiently.
Instructions for using the technology are posted and/or reviewed.

Observations
Appendix C

School and District Improvement in Elementary Mathematics
Principal and Teacher Questions

Background questions:
1. What is your name?
2. What grade do you teach or what is your role in the school?
3. How long have you been here at this school?
4. Where did you teach before and what grades have you taught?
5. How many years have you been teaching?
6. Why did you become a teacher?
7. Where did you go to university?

Versions of success:
1. For you, what counts as success for students in this school?
2. What are your goals in education?
3. How widely accepted are your goals with other teachers in the school? Among parents?
4. How does your school improvement plan incorporate your goals for students?
5. How is the school improvement plan created in this school (principal)?

Challenging circumstances:
1. What are the most challenging things for you as you go about your work in this school?
2. Do you think this school is different from other schools in its challenges?
3. How would you describe the community of parents with whom you work?
4. How has the school context changed over the past few years, and what changes are going on now?

Mathematics:
1. How would you describe your goals in mathematics?
2. How widely accepted are these views in the school? Among the parents?
3. How would you describe the provincial ministry’s vision of mathematics?
4. How do you meet the mathematics goals of the province?
5. Which of the Ten Dimensions have you selected for your personal growth? Why did you select those dimensions?
6. Which of the Ten Dimensions have you selected for your school improvement plan? Why did you select those dimensions?

School culture:
1. How do you create an environment, which supports success in mathematics?
2. What challenges have you faced in trying to create a culture that supports student achievement in mathematics?
3. How do you work with staff and administration to develop the goals/vision of the school? To develop mathematics improvement?
4. How were the issues resolved?

Overall:
1. What are the programs that support success in mathematics outside of the classroom?
2. What do you think we should say in our report about how schools can be more effective in supporting mathematics improvement?
3. Do you have a mathematics implementation team? If so, what is their role and what do they do?
Appendix D

School and District Improvement in Elementary Mathematics
Peer Coaching Process Questions

Questions for the observer:
1. How do you think the process went? (pre-interview, lesson, debrief)
2. What was the most challenging part of your task today?
3. How did you decide what order to ask your questions?
4. Do you think your should state your observations or evaluate?
5. When is the best time to do the conference?
6. Do you think the process would be different if the partnering was teacher/teacher or teacher/administrator?
7. Do you feel like you should have been trained on how to be an effective observer?
8. Do you have suggestions/what might be done differently the next time?

Questions for the teacher:
1. Is this your first time to do this process? (pre-interview, lesson, debrief)
2. How do you think the class went?
3. Have you done peer coaching before? Having another colleague visit your classroom?
4. What was scary about this process? (Consider who is the observer…teacher or administrator.)
5. What do you think about the process?
6. What was scary/challenging about having me/us in the classroom?
7. How did you address Dimension _____ in your lesson?
8. What did you do differently today?
9. What was the most interesting about the process?
10. How helpful was the feedback that you got?
11. Do you think that you should be given feedback (vs. observation)?
12. Was it okay for you to hear evaluation/feedback from ______? What if it was somebody else?
Appendix E

Final Interview Questions

Observation template/guiding questions:
1. Do you use the observation template when observing in the classroom?
2. What kind of notes do you take in the classroom when observing? Is it teacher or student focused?
3. Prior to receiving the guiding questions, what kinds of things did you discuss in the pre- and post-observation sessions?
4. Have the guiding questions been useful to you?
5. Have there been certain questions that you’ve found more helpful for you in this process?
6. What recommendations do you have for improvement to the observation template?
7. What recommendations do you have for improvement to the guiding questions?

Peer coaching process:
1. Why did you decide to participate in this project?
2. What have you learned from going through this experience?
3. Has your teaching changed in any way?
4. What is/was the biggest challenge of this process?
5. What is/was the greatest benefit of this process?
6. Will you continue with this process in the future?
7. What recommendations do you have for improvement to this process?

Survey/dimensions focus:
1. Which dimension(s) did you select as areas for improvement?
2. How did you decide on this/these dimensions? (Did they select the same dimensions as highlighted as a weakness in the survey?)

Questions for Sabrina about her teaching:
1. What was your focus coming into the project?
2. What area of your teaching did you want to examine/improve?
3. Have you noticed any changes?
4. You use many different forms of representation in your classroom. Pictures, drama, manipulatives, etc…
   a. How do you approach the activity when the students encounter these representation forms for the first time? (Modeling, training, let students discover for themselves…)
   b. Is there a reason why you incorporate all of these ideas into your classroom?
   c. Is there one that you find your students respond to the best? Least?
   d. Do you present these representation forms in a specific order? (Easiest to more difficult…)}