Angle illusion on a picture's surface

SHERIEF HAMMAD, JOHN M. KENNEDY *, IGOR JURICEVIC
and SHAZMA RAJANI

University of Toronto at Scarborough, 1265 Military Trail, Toronto ON M1C1A4, Canada

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Abstract—Shapes on picture surfaces are not seen accurately (Arnheim, 1954). In particular, if they depict 3-D forms, angles between lines on a picture surface are misperceived. To test four theories of the misperception, subjects estimated acute and obtuse internal angles of quadrilaterals. Each quadrilateral was shown alone or as part of a drawing of a cube. The drawings showed the tops of the cubes, tilted at various angles around a horizontal axis. This generated different acute and obtuse angles in the drawings. Compared to a quadrilateral on its own, judgments of the acute and obtuse angles in the cube drawings were biased towards 90°. The bias was present over a wide range of intermediate tilts. The results support a perspective convergence theory and run counter to 'Extreme Foreshortening', Gestalt and Cognitive theories.

Keywords: Perspective; illusion; pictures; cubes; angles; Gregory; Arnheim.

INTRODUCTION

Artists contend with three major sources of misperception. The first makes its presence known when artists judge the visual angles of objects in the scene to be portrayed. Artists resort to holding a pencil or brush at arm's length, aligned with the object. The reason is that the linear size of an object often interferes with impressions of its visual angle.

The second source becomes apparent as the artist draws more and more marks on a picture surface to show more and more of the scene. A mark that looks wrong on its own will often look right as soon as perspective shows the depths in the scene (Arnheim, 1954, p. 97; Kennedy, 1974, p. 37) — and *vice versa*! This source of error is our concern in the present paper.

To be complete, we should mention a third source. Often pictures that are perfect panoramic projections of the world give remarkably distorted impressions. To the deep concern of wealthy patrons whose goal is pictures that look realistic, since the

*To whom correspondence should be addressed. E-mail: kennedy@utsc.utoronto.ca

late Renaissance artists have found that information about the sizes and angles of parts of the scene is often used quite incorrectly by vision, despite the picture being geometrically correct (Juricevic and Kennedy, 2006).

Of these three domains of enquiry about picture misperception, in the present study we single out for attention the picture surface and illusions to do with angles. Elsewhere, perception of individual objects posed at distinctive tilts was discussed (Kennedy, 1974, 1993, 2001). Here, we test theories of an angle illusion on a picture surface by studying the effects that occur across the full range of possible tilts of a square, from close to 0° to close to 90° .

Perspective and a picture-surface angle illusion

Apparent distortions have been of interest to theory of art since the late Renaissance and the development of perspective in the 1400s. The crux of constancy research is how we perceive a 3-dimensional form despite it sending many different projections to our eye as we walk around it. How we avoid distortion was crucial to perception philosophy in the 1600s and 1700s in the writings of British Empiricists Locke, Berkeley, Hume, Bacon and Reid, and central to perception psychology since Helmholtz in the 1800s. It was debated at length in Gestalt and ecological theory of pictures in the Twentieth Century (Arnheim, 1954; Gibson, 1979).

Juricevic and Kennedy (2006) analyze distortions of apparent length in panoramic pictures. Kubovy (1986) considers perceived angles at corners of cubes. Gregory (1963) theorized about illusions resulting from unconscious use of perspective. For present purposes, a helpful case in point is two lines forming an X on the face of a perspective drawing of a cube. The acute and obtuse angles between the two lines depict right angles if the lines are parallel to contours depicting the edges of the face on which they are drawn. At times they may appear to be right angles on the picture surface (Parks, 2001). Indeed, the impetus for the current research stems from a study of EA, a congenitally blind man drawing cubes (Kennedy and Juricevic, 2006). Intriguingly, the Vs representing the corners in EA's drawings of cubes like Fig. 1 look close to 90° on the page, and equal, though in fact they are 25° apart, and the acute and obtuse angles differ from 90° by 10° to 15° .

To test theories of the misperception of angles in pictures, we presented observers with pictures of cubes at various tilts. Figure 1 may demonstrate the illusion to the reader. Two drawings of cubes in the illustration have quadrilateral tops. Two of the nine quadrilaterals below match the tops but the typical observer is unable to make the correct selections. The top on the left matches row three, column three and the one on the right matches row two, column two. Observers fail even if they describe themselves as judging the acute and obtuse internal angles of the quadrilaterals, the vertical extents of the quadrilaterals or parallel lines of the tops and the lower quadrilaterals. The obtuse topmost internal angle of the cube drawing on the left is 143° . Many observers suggest its closest match in the 9 quadrilaterals is in row one column one — a 130° angle. Matching its acute angle (41°) with row one, column one's acute angle (54°) is an error of 13° , an error of almost one-third.

Figure 1. Two of the quadrilaterals correspond to the top faces of the two cubes, but observers misidentify the matches. The top on the left matches row three, column three and the one on the right matches row two, column two.

Projection from a 3-D scene onto a 2-D surface

To introduce four theories about Fig. 1's effects, let us consider projection onto a picture surface.

Perspective projection results in convergence and foreshortening. In particular, in three-point perspective the left and right sides of the drawing of the cube converge, and verticals converge down the page, indicating the cube's tilt and revealing the top of the cube (Fig. 2).

A simple example of convergence downwards would be given in a photograph of the Empire State Building from above the front face of the building with the camera tilted to the vertical. In the photograph the contours depicting the edges of the building would narrow towards the sidewalk.

A tilt in depth produces foreshortening. In the well-known "two tables" illustration (Shepard, 1990), table-tops that are congruent on the picture surface are perceived as unequal because foreshortening changes the width of one and the length of the other. Foreshortening simultaneously depicts depth and creates apparent differences on the picture surface.

Now let us consider what might cause a change in apparent angle on a picture surface.

Figure 2. Cube drawn with extended lines to reveal three-point perspective. The left and right sides of the drawing of the cube converge to left and right, respectively. The vertical sides of the cube converge down the page, indicating that the cube is tilted (35° from horizontal).

Convergence, Foreshortening, Cognitive, and Gestalt theories

Sedgwick (2003) describes pictorial information affecting perception of the surface of a picture, and *vice versa*, as ‘crosstalk’ resulting in a compromise between true space and its projection (Arnheim, 1954; Sedgwick and Nicholls, 1993).

The compromise might be due primarily to converging lines on the picture surface. If so, its effect might be most obvious at intermediate tilts, when the converging lines are distinct. Convergence is not obvious when the cube sides are close to parallel to the picture surface, and project lines close to parallel on the picture surface. Likewise, if the cube is highly tilted, the lines for the cube sides are projected as highly foreshortened and the convergence of very short lines is hard to discern. It follows that if convergence of the lines depicting cube sides plays a major role in misperception of Fig. 1, the illusion should be most evident at intermediate angles of tilt. Let us call this Convergence theory.

Alternatively, obvious foreshortening may be the driving force. The further a depicting form is from the depicted form, the more obvious the foreshortening, and the more crosstalk, theory could contend. ‘Extreme Foreshortening’ would be ‘corrected’ in this theory. The more extreme the tilt, the larger the crosstalk because extreme tilts involve more foreshortening. If the top of a cube is pictured as close to parallel to the picture surface it projects a form close to a square, with minimum foreshortening.

Alternatively, an angle illusion could be a top-down, Gestalt effect. When physical conditions approach a ‘simpler’ shape, a percept with symmetry, equality, parallels, and orthogonality is favoured, Gestaltists argue. Visual processes respond to higher-order patterns in stimuli, albeit patterns quite incidental to perspective. “Perception transcends constantly and routinely the mere mechanical recording of sensory raw material” (Arnheim, 1980). “Projective distortions are observed only

by the select few who have been trained to watch out for them" (Arnheim, 1986). In Gestalt theory, figures that are close to a 'good form' in shape are perceived in ways related to that shape. Four 90° angles in a quadrilateral would make an especially good form. If so, angles in the input approaching 90° may be seen as 90° .

In Cognitive theory, top-down effects arise from psychological representations of familiar angles. An interpretation of a 2-dimensional drawing as a cube showing 90° corners could feed down to the perceptual input and make the acute and obtuse Vs be seen as 90° . If the V means 90° , it is seen as 90° , in a strong version of this theory. Peterson and Gibson (1993) argue perceptual input and cognitive information are processed in parallel. If recognition influences perception, angles close to 90° on the page might indeed be seen as 90° . However, it may be that angles on the page very much larger than 90° would be unaffected. They might readily be recognized to be far from 90° .

In sum, Convergence theory predicts illusions at intermediate tilts, 'obvious' foreshortening theory focuses on tilts producing 'Extreme Foreshortening', and the versions of Gestalt theory and Cognitive theory considered here both predict strong illusions as angles on the page approach 90° .

To test these four theories, we presented observers with quadrilaterals either alone, or as the top faces of cubes. They estimated the angles between the lines on the flat surface of the screen. By drawing cubes tilted around a horizontal axis, we changed the internal angles in the top face quadrilaterals (and congruent quadrilaterals presented alone).

Method

Participants. Twelve participants (8 male, mean age 20, range 2 years) were recruited from a first year psychology class at the University of Toronto at Scarborough. Each had normal or corrected-to-normal vision and was naïve to the purpose of the experiment.

Stimuli. All stimuli were generated in a bespoke program called Angle Grinder, written in Java programming language and using 'Java 3D API' to render the cubes. They were presented to all participants on a 15 in. (38.1 cm) computer monitor. Cubes were drawn in linear perspective with white lines 1 mm thick, on a black background. Three faces of each cube were shown; the top and two sides. The convex corner between the two side faces was toward the observer, and the Y intersection between the sides and the top was located in the centre of the screen; hence the top face was depicted as being tilted towards the observer. The cubes were drawn at nine different orientations. The top face was tilted vertically from the horizontal in 10° increments between 5° and 85° . At a tilt of 5° the top was highly foreshortened. It resembled a slim, highly pointed diamond, long axis left-right on the screen. The left-most V shape was highly acute, 9° . Correspondingly, the top-most V shape on the screen was highly obtuse, 171° . At a tilt of 85° the projection of the top was very close to a square. Each tilt was a rotation out of the

Figure 3. Orientation of cube stimuli through nine tilt angles from the horizontal.

picture plane about the Y intersection between the three visible faces (Fig. 3). As a result of this manipulation the upper-most angle of the quadrilateral shown at the top of the cube varied from 171° to 91°, and the left-most angle of the quadrilateral varied from 9° to 89°. Because the cubes were depicted in linear perspective, each side diminished in size as it receded away from the viewer. The resulting top-most quadrilateral does not constitute a perfect parallelogram, and hence adjacent corners do not always add up to 180°.

Participants were shown each of the 9 cube drawings five times in a randomised order. Accordingly, no two participants received the same order of tilt rotations and associated picture angles. The stimuli were presented in two blocks. In one, the quadrilateral whose angles were to be judged was shown as the top quadrilateral of a cube (cube condition). In the other, the quadrilateral was shown entirely on its own, that is, with the lines for the cube sides removed (quadrilateral condition). Half the participants received the cube condition first, and the other half were shown the quadrilateral condition first.

The quadrilaterals presented alone were the same as the ones depicting the cubes' top faces, with the same angle combinations (Fig. 4). In the trials, the order of the quadrilaterals was the same as the order of the cube tops that they copied.

Procedure. Participants were given instructions orally. For practice, they were first asked to estimate the angles between lines on a piece of paper (range 120°). Five practice drawings were presented. They were made of white lines approximately 1 mm thick, on a black background, in order to be similar to the experimental stimuli. All participants were accurate to within 10° mean error on all judgments.

Participants' heads were 77 cm from the computer screen, using a chin rest to help ensure that the eye-level of each person was approximately at the centre of the

Figure 4. Orientation of quadrilateral stimuli through nine tilt angles from the horizontal.

screen. The participants were informed that they would see pictures of quadrilaterals on the computer screen, with some being taller and some being shorter. They would be required to estimate the top-most or left-most angle (hereafter referred to as the ‘obtuse’ and ‘acute’ angles respectively) in each picture on the flat surface of the screen. Half the participants were asked for the obtuse angle in the first 45 trials, and half were asked for the acute angle first. Each stimulus was shown for 5 s, after which participants were prompted by a message on the screen to enter their estimations of these angles on the keypad. During instruction, participants were shown sample pictures (taken from the actual stimuli) of one quadrilateral and one cube presentation. To ensure the subjects attended to the correct angle in the presentations, the requisite angles were indicated with an arc. After five presentations of each quadrilateral, and a total of 45 trials, the program instructed the participant to call the experimenter, and instructions were given for the second block.

In the cube condition participants were informed that they would see pictures made up of three quadrilaterals, and that these would be of different dimensions. They were asked to estimate the obtuse or acute angle in each picture depending on their group assignment. They were shown sample pictures taken from the actual stimuli and shown which angles they were to estimate. For half of the participants, the cube and quadrilateral conditions were reversed in order (cubes first, then quadrilaterals).

At no time did the experimenter make mention of a ‘cube’ to describe the drawings. This was to discourage participants from judging the 3-D object depicted in the display, and reporting only right-angles in the stimuli. In addition, participants were asked to estimate the size of angles on the *left* of the screen, and not the angles on the *right* side of the screen, as this might have coerced some subjects to report right-angles.

Results

Judgments of the angles in cube drawings were heavily biased towards 90° (Fig. 5), with the obtuse angles' mean error 15° less than physically correct, and the acute angles 9° greater than veridical (Table 1). The maximum mean error was 31.2°, for judgments of the 157° angle, reported as a mean of 125.8°. Errors in judging quadrilaterals were much smaller (mean error 0.61° greater than veridical for acute angles and 5° less than accurate for obtuse angles). Errors in the responses were analysed for acute and obtuse angles separately.

For all statistical tests here, the criterion was $p < 0.05$.

Acute angles. In a 2 (cube vs quadrilateral) × 9 (9°, 26°, 41°, 54°, 65°, 75°, 80°, 87°, 89°) within-subjects ANOVA, acute angles invoked greater errors in drawings of cubes than in quadrilaterals alone, $F(1, 8) = 8.1$, $MSE = 367.8$.

In paired-sample t -tests, judgments of acute angles in cubes were different from those in quadrilaterals at 26° ($t(8) = 2.16$), 41° ($t(8) = 2.81$), 54° ($t(8) = 2.68$), 65° ($t(8) = 2.52$), 75° ($t(8) = 3.37$), 80° ($t(8) = 2.26$), 87° ($t(8) = 2.37$).

Figure 5. Mean errors in response to acute and obtuse angles in cube drawings and in quadrilaterals on their own.

Table 1.
Mean response errors in angle estimations

	Cube Obtuse	Quadrilateral Obtuse	Cube Acute	Quadrilateral Acute
Mean reported angle°	109.40	119.44	67.61	59.05
Mean error°	−15.04	−5.00	9.16	0.61

Obtuse angles. In a 2 (cube vs quadrilateral) $\times 9$ (presented angle: $91^\circ, 96^\circ, 103^\circ, 109^\circ, 120^\circ, 130^\circ, 143^\circ, 157^\circ, 171^\circ$) within-subjects ANOVA, judgments of obtuse angles in cubes were smaller than judgments of angles in quadrilaterals, $F(1, 8) = 14.7, MSE = 278.1$.

Paired-sample t -tests found significant differences between cube and quadrilateral conditions for 109° ($t(8) = -2.79$), 120° ($t(8) = -3.00$), 130° ($t(8) = 2.37$), 143° ($t(8) = 4.54$), 157° ($t(8) = 3.79$) 171° ($t(8) = 2.48$).

DISCUSSION

Errors are greater for the top face in a drawing of a cube than for a congruent quadrilateral. For the top face, obtuse angles are underestimated and acute angles overestimated, closer to 90° than is the case, over a wide range of angles (26° to 87° and 109° to 171°). This supports the Convergence hypothesis. The maximum error for obtuse angles is at 157° , and for acute angles is 41° , fully 116° apart, from a possible range of 162° tested. Indeed at 9° and 171° the errors were still 5° and 11° , respectively.

The Extreme-Foreshortening, Gestalt and Cognitive hypotheses are not supported. Errors are not concentrated at the extremes (neither the 5° nor the 85° tilts). Of interest, the judgments are attracted only to 90° , not to 0° or 180° , 45° and 135° , other possible components of good form. If these were attractors, angles on either side of 45° and 135° would be judged as 45° and 135° . At 41° the error is large (16°) and takes the judgment 12° above 45° . Also at 54° the judgment is 69° , which is towards 90° and not towards 45° . Likewise, at 130° , the judgment is 110° , away from 135° . At 143° , the error is large (28°) and takes the response below 135° . Similarly, the error at 157° is large (31°) and makes the judgment lower than 135° . In sum, the errors are attracted solely to 90° and distributed across a wide range.

Acute angles

To ensure the sides of the cube were visible, the largest acute angle in the top quadrilateral was 89° . With a decrease in the presented angle from 89° to 75° judgments of the quadrilateral change minimally, but there are increasingly large errors in response to the cube drawings. Indeed, at 75° , the errors are 8° towards 90° for the cube drawing but 2° away for the quadrilateral. The most erroneous overestimation (16°) of the acute angle in the cube drawing is made at 41° (and the quadrilateral at 41° yields an error of only 2°). At 26° , the error for the cube drawing is still 9° . At a presentation angle of 9° the error in the cube drawing is more than half -5° . From 41° to 9° , errors are likely decreasing smoothly to zero at an acute angle limit of 0° where the cube drawing and the quadrilateral would be identical.

Obtuse angles

To ensure side faces of the cube were evident, the largest tilt presented was 85° from the horizontal. This resulted in an obtuse angle of 91° being projected on to the screen. At this angle, there is no significant difference in angle judgment between cube and quadrilateral conditions and errors are small.

As the angle presented on the screen increased to 109° the error in judging the screen angle increased to -10° for angles in cube drawings, but was slightly positive for response to quadrilaterals, a significant difference. The error did not take the judgment all the way to 90° , failing to support Gestalt and Cognitive predictions. At 120° and beyond, judgments of quadrilaterals are still fractions of the errors induced by cube drawings. Beyond 120° , judgments of quadrilaterals are smaller than the presented angles. It may be the quadrilateral results simply reflect a response bias towards lower numbers than 130° , 143° , 157° and 171° and it may be that subjects are more accurate in judging acute angles, but for present purposes the point to note is the large discrepancy between the judgments of the quadrilateral and the cube drawing.

Errors at 171° are less than those at 157° , though still notable (11°). Likely the errors for obtuse angles approach zero smoothly as a limit at 180° at which the cube drawing would be identical to the quadrilateral.

Theories

Obvious 'Extreme-Foreshortening', Gestalt and Cognitive theories argue a form with 90° corners is imposed on lower-order perceptual processes. What is imposed is a represented form, a simple organization, or a recognized object. However, the errors do not take the form of stating 90° degrees itself. Large errors occur at intermediate tilts, and the responses are merely towards 90° . Rather than imposition of a standard form, the results suggest a compromise.

The Convergence theory focuses on a variable on the picture surface that becomes more evident as tilt increases, and then less evident as tilt increases further. This characterizes the growth and decline of error with tilt more aptly than the other theories.

Since it does not introduce ideas about representation, the Gestalt theory has a problem not faced by the others. It has trouble explaining the remarkable difference between pictures of squares and actual quadrilaterals. It has no reason for distinguishing the two.

Though the results favour the Convergence theory described here, caveats need to be mentioned. Firstly, the theory cannot predict the amount of error that converging lines may induce at various lengths or rates of convergence. As our understanding of vision's use of perspective develops (Juricevic and Kennedy, 2006), precise prediction may become possible. There should be an optimum rate of convergence (since at one extreme, lines are parallel, and at the other they form a straight line),

and an optimum length (since the convergence of very short lines is hard to see, and to be very long the lines have to be parallel).

A second caveat is that foreshortening theories are more flexible than the obvious ‘extreme-foreshortening’ one considered here. In parallel projection, faces of objects are foreshortened but not converged. Entirely in the absence of convergence, at intermediate tilts foreshortening may be sufficient to create misperception of the angles in the depicting forms.

A third caveat is that foreshortening and convergence work together and are confounded in natural optic perspective, and misperception of picture surface elements may be due to their collaboration in spatial information. If so, the issue may not be either foreshortening or convergence *per se*. Alternatively, foreshortening and convergence might work independently. Indeed, they might summate in producing misperception.

A fourth caveat is that projection rays and object tilt are independent factors. For example, parallel projection is typically taken to be orthogonal to the picture surface. But it can be oblique. The tilt of the depicted surface to the screen can be fixed while the orientation of the projection rays is varied. Different kinds of projections would repay study. They could distinguish the role each factor plays.

A fifth caveat is that we have considered particular versions of Gestalt and Cognitive theories. Alternative Gestalt and Cognitive versions might not be difficult to invent.

In conclusion, tilts intermediate between 5° and 85° produced angle illusions on a picture surface. Though Renaissance linear perspective geometry carries realistic depth information, its features affect perception of the elements on the picture surface. While there are still several factors to consider, a hypothesis about perspective convergence was supported against three rivals.

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