Structural Characterization, Optimization, and Failure Analysis of a Human-Powered Ornithopter

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M.A.Sc. Thesis

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Abstract

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The objective of this work was to develop an analysis framework for the structural design of the Human-Powered Ornithopter (HPO). This framework was used in a kinematic-aerostructural optimizer for flapping-wing flight (Ornithia), as well as analytically to design the HPO, and focused on three goals. First was the development of an accurate and computationally inexpensive finite-element method, to be integrated with Ornithia, which would capture the geometric nonlinearity of the aerostructural interaction of the wing when subjected the large deformations in flight. Second was the assembly of a model by which the aircraft primary structure, the wing main spar especially, could be exactly characterized and designed. Third was the establishment of a process and toolbox for failure analysis which could be applied universally in the design of the HPO. The validation and tuning of these models involved extensive testing on prototype carbon fiber composite components.
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<tr>
<td>BF</td>
<td>Buckling Factor</td>
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<tr>
<td>CNC</td>
<td>Computer Numerically Controlled</td>
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<tr>
<td>CF</td>
<td>Carbon Fiber</td>
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<tr>
<td>DOF</td>
<td>Degree-of-Freedom</td>
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<td>E</td>
<td>Young’s Modulus</td>
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<tr>
<td>EI</td>
<td>Bending stiffness of a structure, ([Nm^2])</td>
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<tr>
<td>EPO</td>
<td>Engine-Powered Ornithopter</td>
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<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
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<td>FEM</td>
<td>Finite Element Method</td>
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<tr>
<td>G</td>
<td>Shear Modulus</td>
</tr>
<tr>
<td>GJ</td>
<td>Torsional stiffness of a structure, ([Nm^2])</td>
</tr>
<tr>
<td>HALE</td>
<td>High-Altitude Long-Endurance</td>
</tr>
<tr>
<td>HPA</td>
<td>Human-Powered Aircraft</td>
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<tr>
<td>HPO</td>
<td>Human-Powered Ornithopter</td>
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<tr>
<td>(\nu)</td>
<td>Poisson’s Ratio</td>
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<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
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<td>Prepreg</td>
<td>Pre-Impregnated Composite Material</td>
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<td>TR</td>
<td>Thickness Ratio</td>
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<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
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Chapter 1

Introduction

1.1 Motivation

The Human-Powered Ornithopter (HPO) Project was started in 2007 by a group of undergraduate and graduate students of the Faculty of Applied Science and Engineering at the University of Toronto. The project was initiated as a spin-off of the flapping-wing vehicle research previously conducted by Dr. J. D. DeLaurier’s research group (Flight Dynamics and Aircraft Design/Analysis) at the University of Toronto Institute for Aerospace Studies. The project’s ambitious goal was the successful controlled flight of a human-powered ornithopter by the fall of 2009. The dream of a successful human-powered flapping-wing aircraft has been pursued over the past 500 years, since Da Vinci’s ornithopter was first conceptualized in 1490. If successful, this flight vehicle would be:

1. The first successful human-powered flapping-wing aircraft in the world;

2. The first successful piloted flapping-wing aircraft in the world;

3. The first successful human-powered aircraft in Canada.

Previous attempts at building and flying human-powered ornithopters have included most notably Masashi Harada’s “Karura” in Japan [4], and Yves Rousseau in France [5]. The design of the HPO is fundamentally different from either of these aircraft, but much more similar to previous propeller-driven HPAs. For the HPO, as with previous human-powered aircraft (HPAs), only the most carefully designed edge-of-the-envelope machines can be successful. The direct impact of this thesis and research will be to enable that dream to become a reality.

For commercial and further-reaching purposes, the current investigation is valuable for its application to High-Altitude Long-Endurance (HALE) UAV design. HALE aircraft are
meant to fly extended stationkeeping missions at high-altitude, generally with high-efficiency wing planforms, low wing-loading, and low-power propulsion systems. These ultralight, high-aspect ratio aircraft have much in common with HPAs, and the technology and tools developed in one field are extremely compatible with the other [6]. For example, the composite-tube primary structure of the HPO is very similar to that used in the ERAST research aircraft including Pathfinder and Helios (see Figure 1.1), developed by NASA and AeroVironment [7,8].

![Helios UAV](image)

Figure 1.1: The Helios UAV, developed by AeroVironment for NASA’s ERAST program

HALE and stationkeeping UAVs in general have been investigated as early as the 1980s in Canada at UTIAS, and are becoming more pertinent today for use in missions such as atmospheric monitoring, telecommunications relay, and military surveillance. This research is therefore worthwhile not only in its immediate and narrow application to the design of the HPO but also in the broader context of adaptation to an exciting and growing field of aircraft development in the form of HALE UAVs.

### 1.2 Overview of the HPO Configuration and Design

It is pertinent for an understanding of this investigation to provide a brief overview of the configuration and design of the HPO, which will be further broken-down as is necessary when analysis on particular structural members is elaborated on. Figure 1.2 provides a good overview of the configuration of the HPO from an earlier design stage, before all fabrication methods were finalized.
1.2.1 Propulsion and the Nature of the Ornithopter Design Problem

By virtue of its propulsion method the HPO is different from most previous HPAs. Flapping-wing propulsion integrates what are generally two separate systems, aerodynamic lift generation and propulsion. In conventional design, the fact that both of these systems are separate gives them a lesser degree of interdependence. However, for a flapping-wing aircraft and the HPO in particular, the aerostructural design of the wing determines both the lift and propulsive characteristics. Thus adjusting parameters to suit one need also directly effects the other. In general, a flapping-wing aircraft produces thrust by allowing the wing to twist and utilizing several strategies to ensure that the time-averaged normal-force vector of the wing is a thrust rather than a drag [9]. This kind of complex relationship makes an ornithopter an ideal application for an aerostructural optimization scheme (this point will be revisited later).

As an illustration, for an ornithopter, often the competing design requirements for lift-generation and thrust-generation can directly contradict. For example, the Daedalus HPA team (arguably the most successful to date) was able to design an aircraft with a main spar which was very stiff in bending and very stiff in torsion: this placed desirable limits on, for example, the wing’s dihedral, washout, and failure characteristic [6]. However for the
HPO, though the same limits on mean-dihedral and failure (due to lift loads) were desired, the wing needed to be torsionally compliant in order to twist for thrust production. This torsional compliance induces risk of failure in the main spar from bending, but also from the aerodynamic twist itself.

One key feature by which the HPO is differentiated from any previous piloted ornithopter attempt is its flapping mechanism. Instead of hinging the wing at one centre point (as with "Karura" or Yves Rousseau’s Ornithopter) or at multiple points (as with U of T’s Engine-Powered Ornithopter), the HPO’s wing achieves its flapping deflection entirely by the elastic bending of the main spar. The wing is actuated by the pilot on the downstroke, and allowed to return upwards at its natural aerostructural frequency during the pilot’s recovery stroke. The means of power transmission from the pilot/powerplant to the wings is similar to a rowing mechanism, based on that developed by Derk Thys in the Netherlands for his Rowingbike [10]. The pilot’s legs push out on each downstroke, while his centre-of-gravity remains stationary (an ideal situation for aircraft stability). The legs pull on two drive-wires, one for each wing, which pull the outboard sections downwards. The wing is designed such that the downward motion produces the desired twist in the structure for ideal thrust generation.

1.2.2 Primary Structure Design Considerations

The wing of the HPO was designed similarly to that for many aircraft, given either stiffness or failure constraints for each degree of freedom or structural member. The driving design considerations in the case of the HPO were:

1. Ensure optimal torsional compliance for the wing to generate sufficient thrust (stiffness constraint);
2. Limit bending deflection in the chordwise (in-plane) direction of the wing to control flapping kinematics (stiffness constraint);
3. Limit bending deflection in the out-of-plane direction of the wing to maintain thrust efficiency (stiffness constraint);
4. Design for optimal flapping kinematics while mitigating relevant failure modes (failure constraint).

While it is not possible to truly optimize a design for multiple objectives, iteration can be used to target a design on several objectives while ensuring satisfaction for others. In general with the HPO, kinematic design considerations (e.g. those which governed stiffness
characteristics) were given precedence, and failure constraints were mitigated via available means afterwards.

1.2.3 Primary Structure Fabrication

The wing design being one of the focuses of this work, its structure bears some preliminary description. The primary structure fabrication method was chosen to provide optimal design flexibility, allowing characteristics of each component to be tailored in some cases nearly independently. The wing main spar was fabricated from prepreg carbon fibre composite tubes. The tubes each consisted of an axially-wrapped tubular section, which bore bending and torsional shear forces, and a unidirectional "cap" section, which bore the bending compressive and tensile forces (see Figure 1.3). In this way the spars were designed very similarly in function to a structural I-beam. Fortunately, this fabrication method also allows tailoring of the torsional and bending properties of the tube nearly independently. For example, the torsional properties of the spar could be changed without significantly changing the bending properties by modifying the angle at which the axial layers were wrapped (because these layers made a small contribution to bending stiffness), and the bending properties could be changed by adjusting the cap geometry (which had little shear stiffness and thus made an insignificant contribution to torsional stiffness). The main spar was designed to resist the torsional and out-of-plane bending forces imposed on the wing, whereas the in-plane bending forces were resisted using a truss structure (see Figure 1.4). For each truss, the main spar and several smaller carbon fibre tubes comprised the compression members, and pre-tensioned Kevlar lines contributed bending shear stiffness. The pre-tensioned Kevlar cross-bracing

Figure 1.3: A section of the HPO’s carbon fibre spar, with the cap centerline indicated by the yellow strip and the angle of the axially-wrapped plies visible on the right side of the tube.
lines were able to prevent the carbon tube frame from deforming like a parallelogram while undergoing both thrust and drag forces.

Figure 1.4: An experimental test section of the in-plane truss structure. Note the mock-spar section at the top of the truss frame, and the smaller tubes on the remaining 3 sides. The yellow lines crossing the frame structure are the pre-tensioned Kevlar cross-bracing lines. Note the spacing of the ribs (12”) for scale.

1.2.4 Secondary Structure

For completeness, the design of the aerodynamic surfaces will be described here, in the order of the load path as carried from the air to the primary structure. This design is similar to that used by several HPAs previously. The wing’s skin is 48 gauge Mylar polyester film, heat-shrunk to high tension to ensure a wrinkle-free surface. For about 70% of the top surface and about 15% of the bottom surface, the skin was supported by 3mm thick polystyrene “leading-edge sheeting”, which helped the skin maintain the ideal airfoil shape when under tension. The sheeting and skin were bonded to lightweight ribs, spaced at 12” intervals. The rib shapes were manufactured from polystyrene foam using a CNC hot-wire cutter, with Basswood cap strips were glued to the top and bottom surfaces to provide bending stiffness and a good bonding surface for the wing skin. The aerodynamic loads were transferred from the ribs to the main spar via thin Balsawood plates. See Figure 1.5 for a good view of the completed wing structure (without Mylar skin).
1.3 Objectives

The goal of this investigation was to develop an array of tools for the structural design and engineering of the HPO, and ultralight high-aspect ratio aircraft in general. The design of the HPO was done using an aero-structural-kinematic optimizer, "Ornithia", developed by Todd Reichert at UTIAS for the design of flapping-wing aircraft (ornithopters) in general [11]. The optimizer initially used a vortex-lattice method for the aerodynamic discipline and a dynamic linear finite-element method (FEM) for the structural discipline to determine the optimal flapping kinematics for efficient thrust production. In moving toward refining and finalizing the design of the aircraft, two areas of extreme interest were: more accurate structural models that could provide better predictions about the static and dynamic behavior of the loaded wing; and a failure analysis framework for predicting the performance of the primary structure in flight. With these interests in mind, the objectives of this investigation were:

1. Develop an accurate but computationally-inexpensive nonlinear structural model of the HPO primary structure (suitable for high-aspect ratio wings in general);

2. Formulate analytic equations to predict the structural properties of HPO primary structure components, and adjust with empirical results using as-built parts;

3. Develop a complete failure analysis framework for the HPO primary structure, and validate with as-built parts.
1.4 Significance of Proposed Work

In the area of structural modeling, a nonlinear model is necessary because a linear model would fail due to the large displacements and rotations in the structure. These cause geometric nonlinearity in calculating, for example, the loading case of the structure throughout the dynamic time frame. Fully nonlinear shell-element FEMs for use in tubular structures similar to those in the HPO have been developed, but these are high-fidelity and computationally intensive. Therefore an appropriate alternative would be a medium-fidelity computationally inexpensive model for this type of structure.

The second mentioned area where an information gap exists is the availability of analytic equations for the design of a tubular composite tube. The advantage of the composite construction techniques being used in the HPO project is that they are extremely efficient and tailorable. However, an obvious result of the use of laminate-based non-isotropic materials is that representative equations need to be developed to design parts for the desired properties. This involves the use of material directionality and lamina theory without equivalent implementation in high-fidelity commercial FEA packages. Regardless, fine-tuning and empirical correlation of models using validation testing with as-built parts is critical for each application.

Failure of composite structures is a research area that has gained prominence in recent years as these materials have become ubiquitous in aerospace, automotive, and recreational industries. The likely failure mode of a composite part is often uncertain, and a thorough analysis must be done for each type of structural component. Theory for analyzing the failure of cylindrical shells in bending- and torsional-buckling has been developed, for example, by NASA for rocket vehicles, but no HPO/HALE-applicable equivalent exists.
Chapter 2

Nonlinear Structural Modeling

2.1 Literature Review and Early Work

This literature review describes previous efforts to model, in a geometrically-nonlinear sense, the structural deformation and dynamics of a highly-flexible aircraft wing. As an outline of the problem, the HPO wing was expected to exhibit very large deformations out-of-plane (up to 3m in amplitude), and in twist (up to 30 degrees from root to tip). The in-plane deformations were expected to be an order-of-magnitude smaller, due to the high-rigidity of the in-plane truss structure and the relatively small loads in that direction. Thus one of the greatest problems that would arise when dealing with a strictly linear model is that the loads would not always act in the same direction relative to the deformed structure. For example, drag loads on an upwardly-displaced outboard section of wing are likely to create a large torsional moment at the root, which would not be captured by a linear FEM. The end goal was to create a finite-element model that would integrate with Todd Reichert’s Ornithia aero-structural-kinematic optimizer for flapping-wing aircraft, which had previously used a linear FEM with 10 degrees-of-freedom per element (in particular no axial deformations). This would yield a better degree of accuracy necessary for the design and construction of a successful HPO.

Patil and Hodges have shown the significant effect of structural geometric nonlinearities on structural dynamics and aeroelastic characteristics of HALE aircraft [12]. They focused on the nonlinearities introduced due to large (non-trivial) deformations, and the dominant effect of these on the dynamic properties of high-aspect ratio wings. Their conclusion was that the geometrically exact calculation of angle of attack and proper application of airloads was critical for proper prediction of aeroelastic response. This was an extension on Patil’s use of a simplified mixed variational formulation to model an example slender wing structure and solve the resulting coupled PDEs, forgoing an finite-element approach because of the
high initial development costs when applied to an aircraft with non-fixed configuration [13]. Thus conclusions were reached similar to those in the initial feasibility research conducted for the HPO.

In several cases, high-fidelity finite-element methods for aerostructural optimization have been implemented (using quadrilateral or shell elements to discretize the structure), but these have generally been for steady-state solutions only [14–16]. Cesnik, Hodges, and Patil have advocated the use of a reduced-approach to structural modeling, as opposed to a high-fidelity FEM [17]. However they advise caution, noting that in some cases simplified models will not represent the material distribution and geometry with sufficient accuracy. Fortunately in the case of the HPO, both of these concerns are avoided. First, the deformations expected should fall well within the elastic range for the materials involved (namely carbon fibre composite, which fails before it encounters the strains at which plastic deformation would occur). As for an accurate geometric representation, a single reduced-beam formulation is a very realistic method for modeling the wing primary structure. The wing itself is high aspect-ratio, and thus the structure’s dimensions perpendicular to the long axis are small compared to the length of the long axis itself. Thus it satisfies the requirements for the engineering beam assumption to hold.

Mark Drela of M.I.T.’s Daedalus HPA team developed a complete aerostructural solver and load-prediction program for arbitrary high aspect-ratio wings, using a nonlinear energy-method formulation with 6 degree-of-freedom nodes and an aerodynamic panel code [1]. His original program only solved for steady-state forces and displacements of the wing structure, but has since been adapted to the unsteady case. The solution method involves assembling the entire coupled system of equations describing the aerostructural state of the aircraft, and using a Newton method to solve the system at each time step. Though this method does not use an FEM formulation and thus is not an exact analog to the situation in Ornithia, it represents an ideal in terms of the concurrent aerostructural solution method.

In fact, at the outset of this work, a method with only non-linearities in the out-of-plane bending and torsion degrees-of-freedom was desired in order to improve computational efficiency. In addition, only 10 degrees-of-freedom were desired for the element, with axial deformation being forgone for simplicity (due to the small expected displacements in that direction). These were simplifications imposed on the prevalent methodology, which as mentioned previously for Drela [1] and Crisfield [18] consisted of fully-non-linear 12-DOF methods.

Crisfield developed a suitable finite-element formulation (using a ”standard” and ”geometric” tangential stiffness matrix), achieving nearly quadratic convergence using a Newton-Raphson method for arbitrarily large displacements [18]. This was done by using a co-
rotational dynamic framework to represent the deformed geometry of the structure when undergoing large displacements, and using the kinematically exact translations to determine the change in structural stiffness and applied loads in the global frame [19]. Essentially, Crisfield’s approach assumes that in the local reference-frame the structural deformation is in the linear regime and the non-linearities are introduced entirely by the large motions of the element in the global frame. In this case, the derived Euler-Bernoulli beam model is appropriate and known to be valid. Crisfield’s dynamic derivations are based on a frame external to the element itself, which is roughly the mean-frame between the frames at each of the endpoints. At each time step, the equations of motion are assembled from the nodal internal (elastic), external (applied), and inertial forces, which are then solved using an iterative approach similar to the Newton-Raphson method. In addition, Crisfield has shown that in the dynamic sense, a simple time-integrating algorithm (suggested by the author) can be shown to be “approximately energy conserving”, and in his validation the method was shown to be universally convergent. In this way the formulation can be suited to accept any element formulation for which the elastic stiffness matrix is known.

During an earlier stage of this investigation, Crisfield’s method was chosen. However, the dynamic formulation proved to be too involved to adapt to Ornithia’s computational method and to integrate in a timely manner. In fact, it seems likely that the kinematic exactness of this formulation would have been unnecessary for the time invested. What was instead deemed time-critical was a method by which the tangential forces of the displaced beam structure of the wing could be summed into the torsional load contribution applied to the inboard section of spar. This was the geometrical nonlinearity deemed most critical for the accuracy of the wing kinematics. The intent to drop the axial deformation terms was also revised. Though the expected deformation in this DOF was small, it was realized that the compression forces due to the drive wires (which actuate the wings’ flapping motion) would be critical for failure prediction and hence stress determination in the main spar.

An appropriate non-linear formulation due to Garcia [20] was investigated, which included the general elastic stiffness matrix of the linear FEM and a non-linear geometric stiffness matrix that contributed a bending stiffening (or softening) effect due to axial forces. This bending or stiffening was in fact a translation of these axial forces to the appropriately cross-coupled degrees of freedom on which they would act (e.g. by creating a bending moment) under large deformation. In addition, Garcia’s work included the implementation an additional ”kinematic nonlinear twist contribution” from helicopter blade dynamics,

\[
\Theta = \frac{1}{2}(-\theta_2 + \theta_1)(\Lambda_2 + \Lambda_1) + \frac{(-\Lambda_2 + \Lambda_1)(v_2 + v_1)}{L} + \frac{(\theta_2 - \theta_1)(w_2 + w_1)}{L}
\]

11
where \( v \) and \( \theta \) are the transverse displacement and rotation of the element and \( w \) and \( \Lambda \) are the lateral displacement and rotation of the element, where subscripts indicate the element node. This correction was shown to more accurately predict (in fact slightly over-predict) the torsional deformation induced by combined bending displacement in the two orthogonal directions to the element’s longitudinal axis. Dramatically, without this term, the torsional deformation given by Garcia’s non-linear FEM under-predicted the torsional deformation by half in his validation cases. Garcia’s method was developed for the static case, whereby an incremental solution method would be used. However, for the dynamic case, the nonlinear twist correction appears to be energetically non-conservative, as the additional twisting deformation imposed by the correction term (outside of the FEM) is not accounted for in the in the structure’s stored elastic energy at the next time step. This is compounded by the numerical instability introduced by the rotation required to keep the nonlinear geometric stiffness matrix in the correct orientation to the displaced structure. These shortcomings were found during the validation of a dynamic, time-marching finite-element method, using several variations of Newmark’s method, coded by the author. The case used was similar to Garcia’s static load case, in which a single end-point load is applied to a representative beam.

The time integration algorithm initially chosen was the Newmark method (Constant Average Acceleration Variant) already implemented in Ornithia for the linear FEM, which had been shown to be universally stable and convergent for flapping-wing problems in that application [21]. In addition, consultation showed that the Newmark method should be stable for problems without heavy non-linearities (e.g. those without massive deformation, structural failure, snap-through, etc.) [22]. The method was adapted to include the incremental displacements as a sum-over-history stored-elastic force, such that the equilibrium equation takes the following form:

\[
\sum_{i=0}^{n} (K^i \Delta q^{i+1}) + D^n q^{n+1} + M^n \ddot{q}^{n+1} = F^{n+1}
\]

where \( K^i \) denotes the global stiffness matrix at a given time step, \( \Delta q \) the incremental displacement and \( q \) the displacement (with dots indicating time derivatives), \( D \) the structural damping matrix, \( M \) the consistent mass matrix, \( F \) the applied external forces, and \( n \) the current time-step. This method was inspired by that used by Garcia, which carried out an incremental approach to the stored elastic energy within the same time-step, but not in the dynamic time-stepping case [20]. However, in using this method, uncertainty arose from questions of which time-step the mass matrix should be taken from, given accelerations at the current time-step, etc. Thus errors visible in the steady-state solution and stability with
the non-linear method invited doubt. At this point a fully-incremental equilibrium approach was adopted, consistent with the expression given by Boswell,

\[ K^n \Delta q^{n+1} + D^n \Delta \dot{q}^{n+1} + M^n \Delta \ddot{q}^{n+1} = \Delta F^{n+1} \]

where each quantity can be seen to be taken clearly from the same instant in time and fully time-history independent (except for the rotation of the stiffness matrix) [23]. Initially this method seemed more promising because it was clearly self-consistent. However, when implemented in the Newmark algorithm, it was again found to yield inconsistent results. At this point the fault was realized to lie primarily with the rotation of the non-linear structural model. In fact, as can be seen below in Figure 2.1, both of the above methods agree exactly with the original Newmark method given a linear FEM for terms of steady-state and time history.

The conclusion of this review and the concurrent investigation was that no suitable model existed that could be adapted and integrated to this particular case. Thus a medium-fidelity model, including finite-element formulation and geometrically-nonlinear kinematic method, needed to be devised from the current linear model.

### 2.2 Research Methodology

The process for devising an appropriate finite-element representation for Ornithia will proceed in three steps as follows:

1. Implement a 12-DOF linear finite-element formulation for the HPO wing;
2. Devise and implement a nonlinear, kinematically-accurate adaptation for bending-torsional coupling;
3. Code and integrate the resulting method with Ornithia’s aerostructural iteration and dynamic time-integration scheme.

#### 2.2.1 12-DOF Linear Finite-Element Formulation

A linear 12 degree-of-freedom finite-element formulation was sought for Ornithia, to be implemented in \texttt{strCalc2} (whereas \texttt{strCalc1} was an earlier FEM used in Ornithia with the fewer degrees of freedom). The 12-DOF element consists of a two-node element, each node having the full 6 degrees-of-freedom. Explicit formulations and derivations for an element with the isolated degrees of freedom (rotation and displacement in each of 2 bending
directions, 1 axial, and 1 torsional) are found in J. R. R. A. Martin’s undergraduate FEM course at the University of Toronto, taken by the author [24]. The element formulations for each of these DOFs were assembled in a consistent fashion to form the fully general 3-D frame element. It was necessary for the Newmark method to obtain the elastic stiffness matrix and consistent mass matrix for the element, as well as the consistent force vector for determining the nodal loads from the aerodynamic method. The elastic stiffness matrix has the form:

$$
k_e = \begin{bmatrix}
12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 & 6\frac{EI_x}{l^2} & -12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 & 6\frac{EI_x}{l^2} \\
0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 \\
0 & 0 & 12\frac{EI_z}{l^3} & -6\frac{EI_z}{l^2} & 0 & 0 & 0 & 0 & -12\frac{EI_z}{l^3} & -6\frac{EI_z}{l^2} & 0 & 0 \\
0 & 0 & -6\frac{EI_z}{l^2} & 4\frac{EI_z}{l} & 0 & 0 & 0 & 0 & 6\frac{EI_z}{l^2} & 2\frac{EI_z}{l} & 0 & 0 \\
0 & 0 & 0 & 0 & GJ & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 \\
0 & 6\frac{EI_z}{l^2} & 0 & 0 & 0 & 0 & 0 & 4\frac{EI_z}{l} & -6\frac{EI_z}{l^2} & 0 & 0 & 0 & 2\frac{EI_z}{l} \\
-12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 & -6\frac{EI_x}{l^2} & 12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 & -6\frac{EI_x}{l^2} \\
0 & 0 & -12\frac{EI_x}{l^3} & 6\frac{EI_x}{l^2} & 0 & 0 & 0 & 0 & 12\frac{EI_x}{l^3} & 6\frac{EI_x}{l^2} & 0 & 0 \\
0 & 0 & -6\frac{EI_x}{l^2} & 2\frac{EI_x}{l} & 0 & 0 & 0 & 0 & 6\frac{EI_x}{l^2} & 4\frac{EI_x}{l} & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & GJ & 0 \\
6\frac{EI_x}{l^2} & 0 & 0 & 0 & 0 & 2\frac{EI_x}{l} & -6\frac{EI_x}{l^2} & 0 & 0 & 0 & 0 & 4\frac{EI_x}{l}
\end{bmatrix}
$$

Here \( l \) is the undeformed length of the element, \( EI_x \) is the in-plane bending stiffness, \( EI_z \) is the out-of-plane bending stiffness, \( EA \) the extensional stiffness, and \( GJ \) the torsional stiffness of the complete wing primary structure (including main spar and in-plane truss). \( EI_x, EI_z, EA, \) and \( GJ \) were determined as will be explained in the structural characterization portion.
of this work. The consistent mass matrix is as follows:

\[
m_e = \begin{bmatrix}
\frac{156\rho_{Al}}{420} & 0 & 0 & 0 & 0 & \frac{22\rho_{Al}^2}{420} & \frac{54\rho_{Al}}{420} & 0 & 0 & 0 & 0 & -\frac{13\rho_{Al}^2}{420} \\
0 & \frac{\rho_{Al}}{3} & 0 & 0 & 0 & 0 & 0 & \rho_{Al} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{156\rho_{Al}}{420} & -\frac{22\rho_{Al}^2}{420} & 0 & 0 & 0 & 0 & \frac{54\rho_{Al}}{420} & \frac{13\rho_{Al}^2}{420} & 0 & 0 \\
0 & 0 & -\frac{22\rho_{Al}^2}{420} & \frac{4\rho_{Al}^3}{420} & \frac{13\rho_{Al}^2}{420} & 0 & 0 & 0 & -\frac{13\rho_{Al}^2}{420} & \frac{3\rho_{Al}^3}{420} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\rho_{Al}}{3} & 0 & 0 & 0 & 0 & 0 & \frac{\rho_{Al}}{6} & 0 \\
\frac{22\rho_{Al}^2}{420} & 0 & 0 & 0 & 0 & \frac{4\rho_{Al}^3}{420} & \frac{13\rho_{Al}^2}{420} & 0 & 0 & 0 & 0 & -\frac{3\rho_{Al}^3}{420} \\
\frac{54\rho_{Al}}{420} & 0 & 0 & 0 & 0 & \frac{13\rho_{Al}^2}{420} & \frac{156\rho_{Al}}{420} & 0 & 0 & 0 & 0 & -\frac{22\rho_{Al}^2}{420} \\
0 & \frac{\rho_{Al}}{6} & 0 & 0 & 0 & 0 & 0 & \frac{\rho_{Al}}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{54\rho_{Al}}{420} & -\frac{13\rho_{Al}^2}{420} & 0 & 0 & 0 & 0 & \frac{156\rho_{Al}}{420} & \frac{22\rho_{Al}^2}{420} & 0 & 0 \\
0 & 0 & \frac{13\rho_{Al}^2}{420} & \frac{3\rho_{Al}^3}{420} & \frac{13\rho_{Al}^2}{420} & 0 & 0 & 0 & \frac{22\rho_{Al}^2}{420} & \frac{4\rho_{Al}^3}{420} & 0 & 0 \\
0 & 0 & 0 & \frac{\rho_{Al}}{6} & 0 & 0 & 0 & 0 & 0 & \frac{\rho_{Al}}{3} & 0 & 0 \\
\frac{-13\rho_{Al}^2}{420} & 0 & 0 & 0 & 0 & \frac{-3\rho_{Al}^3}{420} & \frac{-22\rho_{Al}^2}{420} & 0 & 0 & 0 & 0 & \frac{4\rho_{Al}^3}{420}
\end{bmatrix}
\]

Here \( \rho \) is the lineal-density of the structure over the element, and \( A \) is the cross-sectional area (both taken to be constant over the length of the element). The consistent force vector for a uniformly distributed load is given by

\[
f_e = \begin{bmatrix}
F_c/2 \\
F_s/2 \\
F_n/2 \\
M_c/2 + F_n/12 \\
M_s/2 + eF_n/2 \\
M_n/2 - F_c/12 \\
F_c/2 \\
F_s/2 \\
F_n/2 \\
M_c/2 - F_n/12 \\
M_s/2 + eF_n/2 \\
M_n/2 + F_c/12
\end{bmatrix}
\]

where the forces imposed on their structure maintain their aerodynamic naming convention, with the subscript \( c \) denoting chord-wise forces, \( s \) denoting span-wise forces, and \( n \) denoting normal forces. In addition, \( e \) is the distance between the elastic axis and the quarter chord of the wing, positive when the elastic axis is aft of the quarter chord.

As a final component, in the case of structural failure analysis, it is a useful feature of
the FEM that the strains and stresses due to deformation in each degree of freedom of the structure can be determined given the local nodal displacement of the element. The matrix equations for each of the strains are [24]:

\[
\epsilon_{\text{bending}} = \left[ \begin{array}{cccc}
-\frac{6y}{l^2} & \frac{4y}{l^2} & \frac{2y}{l^2} & \\
\delta_{z1} & \psi_1 & \delta_{z2} & \psi_2
\end{array} \right]^T
\]

\[
\epsilon_{\text{axial}} = \left[ \begin{array}{cc}
-\frac{1}{l} & \\
\delta_{x1} & \delta_{x2}
\end{array} \right]^T
\]

\[
\gamma_{\text{torsion}} = \left[ \begin{array}{cc}
-\frac{1}{l} & \\
\theta_1 & \theta_2
\end{array} \right]^T
\]

where

- \( y \) is the distance of the element’s exterior face from the centroid;
- \( l \) is the length of the element;
- \( \delta_{zi} \) is the local transverse bending displacement of node \( i \);
- \( \psi_i \) is the local rotation due to bending of node \( i \);
- \( \delta_{xi} \) is the local axial displacement of node \( i \);
- \( \theta_i \) is the local torsional rotation of node \( i \).

The stresses can thus be determined using the constitutive relation

\[
\sigma_{\text{bending}} = E\epsilon_{\text{bending}}
\]

\[
\sigma_{\text{axial}} = E\epsilon_{\text{axial}}
\]

\[
\sigma_{\text{torsion}} = G\gamma_{\text{torsion}}
\]

where \( E \) is the Young’s modulus and \( G \) the shear modulus of the material.

### 2.2.2 Nonlinear Modification and Integration with Ornithia

The goal of the nonlinear modification to the linear FEM was to provide a means of coupling large displacements in the bending degrees of freedom with the torsional degrees of freedom. In the application of the HPO, this is necessary because the out-of-plane bending deformations are sufficiently large that the chord-wise and chord-normal aerodynamic forces are acting out of the plane of the original undisplaced structure, and hence exert a cross-coupling torque where, in a linear regime, none would exist. What follows is an explanation of how the method is derived and implemented.
It is first important to note that the forces acting on the structure are generated via a vortex-panel aerodynamic method, which in Ornithia is in the form of aeroCalc2 [11]. This code determines the chord-wise and normal forces acting on each discretized wing panel, where the panels correspond to the discretized elements in the FEM formulation. The aerodynamic forces and moments, which are computed over the entire panel, are then distributed across the corresponding nodes of the structure. This includes the important modification that the moments and forces are transformed such that they are all taken as acting about the elastic axis of the structure, as opposed to the quarter-chord of the airfoil. It is also necessary to note that the approximation has been made that the chord-wise and normal axes of the airfoil (the directions in which the forces are computed) correspond to the principal structural axes in which the in-plane ($EI_x$) and out-of-plane ($EI_z$) properties of the spar are known.

The determination and imposition of the nonlinear force contributions proceeds as follows, within nonlinForces. First, the aerodynamic force vector as applied at the elastic axis (called the effective force vector), is taken from aeroCalc2. The nonlinear force distribution algorithm then proceeds from the inboard (i.e. nearer to root) to outboard nodes, performing the nonlinear correction at each node. The correction begins with the determination of the initial undeformed spar geometry, which is that used by strCalc2 and aeroCalc2 to determine the global effective force vector. nonlinForces then uses the information of the undeformed structure in the global frame to subtract from the current node the force contribution from each node outboard, in the coupled degrees of freedom only. The step of subtracting these components before performing the nonlinear correction prevents "double-counting" of these forces and moments. nonlinForces then uses the true deformed position of each of the outboard nodes, taking the cross-product with the current node to determine the moment-arm at which the outboard forces are imparting a torque on the inboard section. The contributions from each outboard node are then summed in the global frame and reapplied at the current node.

This inclusion of the deformed geometry at the current time-step, as opposed to the standard use of the undeformed, initial geometry to compute nodal forces and moments, is the critical adaptation. It is in this way that Ornithia is able to use the large deformations encountered within the linear FEM to determine, in a kinematically-correct fashion, the actual cross-coupling between degrees of freedom. As mentioned, in conventional application this cross-coupling is not accounted for precisely because a linear FEM is based on the assumption that the deformations encountered will be small and hence no cross-coupling exists. It is important to note, however, that the use of a linear FEM to represent the structure is still valid, because for each slender element the deformations encountered are indeed small.
(and thus within the engineering and Euler-Bernoulli beam theory assumptions of the linear FEM), and only when a large number of these small deformations are accumulated does the overall deformed geometry become substantial. Hence a method has been effectively boot-strapped onto a linear FEM that accounts for the important geometric nonlinearities of the structure without adding significant complication or computational expense.

For completeness, it is important to mention that the final time-integration scheme used in this method is still that due to Newmark, with the choice of parameters corresponding to the Constant Average Acceleration Method [25]. Previously mentioned research by Lari-jani had shown that Newmark’s method proved to be robust and universally-convergent for flapping-wing problems in the linear regime, and this work demonstrates that the standard method shares those same characteristics given the introduced nonlinearity [21]. This is in accordance with earlier assurances made by Hodges to that effect [22].

2.2.3 Validation and Results

The aerostructural solver in Ornithia is Newmark’s Constant Average Acceleration Method for the dynamic time integration scheme, and a Newton-Raphson method is used to solve the coupled aerostructural problem within each time step. The Newton-Raphson method uses a sensitivity matrix of the combined aerostructural system of equations to compute a satisfactory solution.

The validation of the aerostructural solver was begun with the validation of each of the two methods separately. The vortex-panel method was validated by Todd Reichert and Thomas Veitch (a member of the HPO project team, and original author of the vortex-panel code used) by comparison with experimental test cases for 2-D and 3-D unsteady flows [11,26]. The linear FEM was validated against an experimental case used by Garcia [20]. A slender test beam was point-loaded at the tip, with the load being applied through a sweep of angles $\theta$ between fully lateral (e.g. sideways, $\theta = 0^\circ$) and fully transverse (e.g. vertical, $\theta = 90^\circ$). Garcia has experimental data for the steady-state tip displacement and rotation as a function of $\theta$. In addition, the static solution was compared with the Euler-Bernoulli beam-theory analytic solution. The entire combined aerostructural solution was very difficult to validate and compare with known solutions except by inspection for simplified cases, as has been noted by Mark Drela of the Daedalus team. However, the Newmark method has been found in simulation to be consistently convergent, with no energy loss or dissipative numerical errors in the system. In addition, the Newton aerostructural iteration has been found to be at least quadratically convergent, with the residuals vanishing for each time step. This implies a mutually-acceptable and correct solution to the system of equations for
each of the aerodynamic and structural methods separately.

It remains to be seen what the in-flight performance of the HPO will be. The test flights in late September and early October will be documented with high-definition video footage throughout. Similarly to Daedalus, it should be possible to extract bending and torsional deformation information from this footage with reasonable time accuracy. This would allow a comparison of Ornithia’s predicted kinematic performance with actual. In addition, as the aircraft will be launched under tow and glide during landing, there should be ample static-deformation imagery of the wing against which the FEM results can be compared.
Figure 2.1: Time-evolution of total displacement and final steady-state displacement of linear, partially-incremental, and fully-incremental Newmark methods for a linear FEM analysis of Garcia’s test beam case, in one-dimension.
Chapter 3

Structural Characterization

3.1 Literature Review

The structural design of historical HPA structures has not been very well documented, and consequently the literature is relatively sparse, especially concerning detailed analysis. The goal of this literature review was to determine three points:

1. The objectives used by previous HPA teams to guide structural characterization and design;

2. Means by which an exact model of the composite laminate structure of a tube in general could be obtained;

3. Test methods used by previous HPA teams to validate and adjust their empirical models.

3.1.1 Design Objectives to guide Structural Characterization

On the first point of design objectives, M.I.T.’s Daedalus Team (which designed and built its HPAs in the U.S.A. and flew the final aircraft in Greece) had the most accumulated and documented experience. Two of the structural design leaders on that team were Juan Cruz (currently at NASA Langley Research Centre) and Mark Drela (currently Professor of Aircraft Design at M.I.T.). Cruz was largely responsible for developing the tube-and-cap configuration for the main spar, after having experimented with several other more complicated and less robust fabrication methods. Cruz and Drela determined that primary structure design should be dominated by strength requirements, so as to achieve the maximum weight savings [27]. They devised an elaborate scheme by which an alternative to the conventional
V-n loading diagram could be used to determine the limit loads placed on the primary structure. This allowed the wing and other primary structural members to be designed with especially small safety factors, and thus with only just the required strength and stiffness to carry their load. In addition, the Daedalus team found that materials with the highest specific stiffness available were the most desirable, as a significant and crucial change between the prototype aircraft (the Michelob Light Eagle) and the final record-setting HPA was the higher-modulus carbon fibre composite used in the latter [6]. On another project, Peer Frank of the Velair (built and flown in Germany) instead focused on designing his primary structure for the desired stiffness as opposed to strength (regardless his sectioned survived to 3.34G loading, versus 1.75G on the Daedalus) [6, 28]. The Velair wing, for example, was designed to have a tip deflection not exceeding 2m in flight. The reason for this constraint is unclear, and such a constraint would be unreasonable for the HPO (with the wing’s thrust-dominated design tailoring). Finally, John McIntyre of the Airglow HPA (built and flown in England) claimed that the wing’s torsional stiffness dominated design considerations [29]. This may imply that the shear/torsion tube, which accounted for 70% of the spar’s weight, was the most significant structural member; however the driving need behind this design consideration is unclear. Besides this, the secondary design considerations in the case of the wing were for failure, which was designed for a failure load of 2G. Also, in their document on the general design of carbon fibre tubes, the McIntyre brothers recommend that capped tubes be designed to a limit load of 2.5G in order to obtain sufficient stiffness [2]. This figure may, again, apply to HPA wings in general, but is not especially applicable here.

3.1.2 Development of Analytic Characterization Models for the Primary Structure

An accurate analytical model should, by necessity, exist for each HPA, as the edge-of-the-envelope design makes such exactitude critical. Aircraft which have had tube-and-cap spars similar to that of the HPO have included the Daedalus, the Airglow, and the Velair [2, 28]. As the section properties in torsion and bending are most important for the characterization of the wing’s structure during design, analytic equations for the properties of the spar sections are especially important. This detailed step was not very well documented in historical HPA literature. John and Mark McIntyre (designers of the Airglow) used section-appropriate analytic engineering equations for moments of area and polar moments of inertia in the design of their spar structures, as did Peer Frank (of the Velair) [2, 28]. The equations used and level of accuracy in the analytic models (in mimicking the actual fabricated configuration) were not specified. However the use of such a straightforward analytical approach seems to
have been successful and will thus be followed here.

### 3.1.3 Testing and Validation

The refinement and validation of any analytic predictive model is a critical step before design and fabrication are carried out. Mark Drela had written a complete aerostructural computational model for the Daedalus wing, which determined torsional, out-of-plane bending, and in-plane bending deflections [1]. For simple cases the computational model was validated first with an analytic model, and shown to provide perfect agreement. For the complete design case, used to compute the aircraft construction parameters, the model was validated using in-flight photos and observations of the deformed wing. Again, the agreement was shown to be perfect to within plotting accuracy (see Figure 3.1.

![Figure 3.1: Computationally-predicted deflection of Daedalus HPA wing compared to deflection observed in flight. [1]](image)

The available Airglow literature doesn’t specify what kind of structural testing, if any, was performed to validate the empirical model for structural deformation. For Velair, however, there seems to have been extensive testing and validation performed [28]. All of the primary structural members had prototypes made and tested to failure. During these tests, measurements were made of the structural deformation to compare with the empirical model for structural flexibility. A long 6.5m section of spar was tested on a cantilever mount with a single point load at the end, which provided a shear and moment loading condition similar to that encountered in flight. In addition, coupon tests were performed on samples of the carbon fibre used to manufacture the tubes, laminated and cured in exactly the same way as the spars in order to mimic manufacturing conditions. Once as-built figures were obtained for the carbon fibre, the empirical model was tuned to account for this. The final observa-
tions in-flight showed that the deflected wing shape agreed closely with that predicted, and the tip deflection was in actuality 1.95m compared with the 2m for which it was designed.

3.2 Research Methodology

This phase of research saw the development of methodologies for the characterization and design of the composite primary structure of the aircraft. Existing and previously-developed analytic equations for moments of area and polar moments of inertia were refined and implemented in a convenient form for structural design and also post-analysis (for components already built). In order to remove additional sources of error, material properties (e.g. Young’s modulus and failure strength) were accurately determined with empirical testing, after which refinement and final validation of the tools was carried out. Thus the required steps to develop an accurate predictive model for the structural properties of the laminate carbon fibre tubes used in the HPO’s primary structure were:

1. Formulate the equations to analytically predict the section properties of the tubular structures, for both capped and non-capped sections;

2. Carry out tests to determine the material properties of the composites involved for as-built specimens (versus their manufacturer-supplied catalog values);

3. Carry out tests on capped and non-capped specimens to tune parameters for the predictive model;

4. Finalize and validate the predictive model using the results of structural testing.

Each of these points was attacked somewhat concurrently; but for each, an explanation and the organization will be expounded in a linear fashion.

3.2.1 Determination of Section Properties

Achieving exactly the desired wing properties of the HPO is more critical than for any human-powered aircraft. Whereas HPA structures have previously been designed simply as light as possible, and hence generally failure-case dominated, the efficiency of flapping-wing flight is very sensitive to the wing’s kinematics, which are tailored almost exclusively through the structural properties. Hence especially for the wing, but for the mass efficiency of the HPOs primary structure in general, the refinement and implementation of a structural model for the design of tubular composite structures is indispensable. Each of the primary structural members of the HPO is fundamentally a hollow cylindrical shell: a tubular composite section.
As mentioned previously, these consist of sections with and without additional unidirectional plies (colloquially "caps") on the top and bottom surfaces of the tube. The tubular sections are fabricated by winding a strip of unidirectional carbon fibre pre-impregnated (prepreg) tape around a cylindrical aluminum mandrel. The strips are cut to width such that given the diameter of the tube, the plies will wrap onto the mandrel at exactly the designed laminate angle desired. For example, a tube for the fuselage structure (where torsional stiffness is required) might be wrapped at the optimal 45°, whereas the tubes for the main spar were wrapped at 30° to give the maximum practical torsional compliance. For spar sections, the unidirectional caps were laid up as one set of stacked plies, in which the layer closest to the tube would be widest, and the width of each subsequent stacked layer would be reduced to mitigate stress concentrations at the edges. See Figure 3.2 for a cross-sectional view of the spar section and cap layup process.

![Figure 3.2](image)

Figure 3.2: Tapered capstrip layup, profile view of tubular spar arrangement, and exploded view of finished spar, as constructed for the HPO. [2,3]

The most accurate means by which to model the properties of these composite sections has been found to be by adhering exactly to the true geometry of the tube laminate. The tubular structural cross-sections were satisfactorily modeled on the assumption that they are a radially-symmetric cylinder. By inspection of the structures used this is true, providing the laminate is correctly wrapped and consolidated on the mandrel during fabrication. Modeling the capped sections required iteration in the representations to develop better accuracy. Options considered previously for modeling these, in order of simplest to most complex, were:

- Approximation of the caps as a single rectangular section, with the thickness being the sum of the ply thicknesses and the width being the average of the plies;

- Approximation of each cap ply as an independent rectangular cross-section, at the same or increasing distances from the structural centroid;
• Exact representation of each cap ply as an independent circular-arc cross-section, displaced exactly the correct distance from the centroid using the ply thickness to determine the geometry of each ply in the stacked laminate.

The final method was found to be the most accurate for modeling the caps with acceptable accuracy. The second moment of area of an arbitrary cross-section is evaluated through the integral

\[ I = \int_A y^2 \, dA. \]

If each cap ply is modeled in polar coordinates as a circular arc segment of radius \( r \) and thickness \( t \), subtended by an angle \( \alpha \) in radians, then

\[ y = r\cos\theta, \, dA = rtd\theta. \]

By proceeding with the integral and using a trigonometric substitution, the final equation for the lamina second moment of area is:

\[ I = [\alpha + \frac{1}{2}\sin(2\alpha)]r^3t. \]

The second moment of area for the entire cap is thus obtained via a summation of the contribution of each cap, ensuring exactly the correct radius is obtained by summing the thickness of all previous plies. In addition, the \( \alpha \) used is exactly that of each cap, given that the bottom ply has the maximum width (often with \( \alpha = 90^\circ \)), and each subsequent layer tapers in width by as much as 2mm.

For the torsional properties of the spar, which has a non-radially-symmetric cross-section, the GJ must be computed. The equation for the GJ of a thin-walled, closed section composed of discrete segments is [30]

\[ \frac{1}{GJ} = \frac{1}{4A^2} \sum_{i=1}^{n} \frac{l}{Gt}. \]

This formula can be re-arranged as

\[ GJ = \frac{4A^2}{\sum_{i=1}^{n} \frac{l}{Gt}}. \]

Therefore, for the tubular cross section of inner radius \( r \) having two non-capped segments bounded by two capped segments of width \( w_{caps} \) and lamina thicknesses \( t \), the equation for GJ is

\[ GJ = 4\pi^2 r^4 \times \left[ \frac{2\pi r - 2w_{caps}}{G_{\text{tube}}t_{\text{tube}}} + \frac{2w_{caps}}{G_{\text{tube}}t_{\text{tube}} + G_{\text{caps}}t_{\text{caps}}} \right], \]
The primary remaining difficulty was in the determination of the sectional properties of the in-plane truss. The contributing members of this cross section were the main spar and rear spar, but both sections were comprised of different moduli of carbon fibre at different wrap angles, etc, such that although the moment of area of each section could be computed about its own centroid, locating the centroid of the entire structure in order to apply the parallel axis theorem proved difficult. For a composite section including material of different elastic moduli, Hibbeler [31] shows that the centroid can be located using

$$\bar{y} = \frac{\sum E_i \bar{y}_i A_i}{\sum E_i A_i}$$

where $E_i$ is the elastic modulus of each section, $E_{ref}$ is a chosen non-dimensionalizing reference elastic modulus, $\bar{y}_i$ is the location of the centroid of each section, and $A_i$ is the area of each section. Through substitution of the known quantities for each in-plane truss component and choosing the elastic modulus of the main spar tube and rear spar as $E_{ref}$, the centroid location can be determined:

$$\bar{y} = \frac{\frac{E_{caps}}{E_{ref}} \bar{y}_{caps} A_{caps} + \bar{y}_{spartube} A_{spartube} + \bar{y}_{rearspar} A_{rearspar}}{\frac{E_{caps}}{E_{ref}} A_{caps} + A_{spartube} + A_{rearspar}}.$$  

The remaining sectional properties required for the wing design (e.g. areas, moment of area for a cylinder, etc) were either found explicitly elsewhere or could be obtained through trivial derivations of the information already obtained [32].

### 3.2.2 Determination of Material Properties

The determination of composite laminate material properties is very different from most materials for several reasons, including, but not limited to:

- The materials are non-isotropic, that is their properties are not the same in all directions due to their fibrous composition;
- The material properties are highly dependent on the rotational orientation of the laminate plies;
- The strengths and stiffnesses of composites often differ in compression and tension, even in the same axes, due to dramatically different failure modes and mechanical forces in action;
The material properties depend highly on the conditions under which the layup is cured.

These issues were the ones most commonly encountered in the HPO project, and all had to be dealt with. That most prevalent from the outset was the angle-ply problem. For example, given a unidirectional tape it may have relatively well-known material properties in the axes parallel to and perpendicular to the fibres. However, often it is desirable and necessary to rotate the unidirectional tape in order to blend its properties in the material axes into more desirable properties in the axes of the laminate (or completed structure). It is thus necessary to transform the material properties from the known material axes to the new structural axes. The course materials in Professor Hansen’s Composites Short Course at UTIAS provided a valuable resource for this purpose [33]. The elastic properties of the composite in the material axes (e.g. the principle axes of the anisotropic material) are referred to as the elastic constants, $Q$, and for a laminate where the principal axes of the layup align with the structural axes (e.g. the axes in which the structural stresses are determined), as with all laminates in the HPO project, the layup is orthotropic and hence the matrix of orthotropic elastic constants takes the form:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

with

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12} \nu_{21}},$$
$$Q_{22} = \frac{E_{22}}{1 - \nu_{12} \nu_{21}},$$
$$Q_{12} = \frac{\nu_{12} E_{22}}{1 - \nu_{12} \nu_{21}} = \frac{\nu_{21} E_{11}}{1 - \nu_{12} \nu_{21}},$$
$$Q_{66} = G_{12},$$

where the subscripts 1 and 2 correspond to the axes in the plane of the lamina along the fibre and perpendicular to the fibre, and 6 corresponds to the shear direction. The transformation matrix which is used to rotate stresses from the material axis to the structural axis through
an angle $\theta$ is given by

$$ T = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} $$

The matrix to transform the engineering strains from the material to structural axes takes the form

$$ \tilde{T} = R T^{-1} R $$

where $R$ is the Reuter matrix,

$$ R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} $$

Thus, given that $Q$ satisfies the constitutive relation $\sigma = Q\epsilon$, the rotated laminate-properties matrix $\tilde{Q}$ takes the form

$$ \tilde{Q} = T^{-1}QRT^{-1} = T^{-1}QT^{\text{T}} $$

By this method, if one can assemble the elastic material constants in the material axes, one can determine the material properties of the orthotropic laminate in the structural axes for an arbitrary orientation. This transformation was programmed into Matlab in order to allow the properties to be transformed for any of the tube laminates being fabricated. This was convenient in many cases where the transformation only needed to be applied once, for example in the spar where each tubular section is fabricated from the same prepreg material at the same orientation.

Of the problems listed above, the gamut of material properties make designing with composites very difficult at times. It was challenging throughout the project to find all the required properties necessary to carry out a fully-analyzed design process given a particular composite product. In fact, only a fraction of the material properties could be obtained from the manufacturer in many cases. Some properties had to be estimated from those provided by another manufacturer for a similar material, or calculated for the carbon/epoxy matrix using the rule of mixtures based on the volume fraction of each material in the composite.

The three materials most often used in this project were:

- Newport NCT301-HS40: A high-modulus, high-strength prepreg used to fabricate all capped spar sections during the summer of 2008 (the 3” and 2” sections of which are used in the current aircraft);
• ACG MTM28-M46J: A high-modulus, high-strength prepreg used preferentially for all non-capped primary structure members during the summer of 2009, and also for the caps of the 2.5” and 1.5” spar sections currently on the aircraft;

• ACG VTM264-HTR40: A standard modulus, high-strength prepreg used for all spar tubular sections on the HPO due to the low shear stiffness of the laminate (provided optimal torsional compliance).

The appendix contains tables of the properties used to design with each of these three products, and the means by which each property was obtained.

Initially, coupon testing of each of the used laminates had been intended, in order to establish a rigorous list of accurate material properties for as-built specimens. Four samples of each of the encountered materials were laid up under the same conditions used for the curing of the primary structure components in which they are used. The samples were prepared as specified by the ASTM "Dogbone" Sheet-size test specimen, in a thickness of about 3 mm, with one being destroyed accidentally during the difficult machining process. However, for the stiffnesses of the materials used, there existed no apparatus at the University of Toronto that could accurately gauge the stress-strain curve of the coupons manufactured. The Department of Materials Science and Engineering generously donated time on their Instron testing machine, but the grips could not sufficiently bind the coupons and the accuracy of the machine’s measurement was insufficient on the scales witnessed during tensile testing. Given a laser-interferometer equipped Instron machine and better preparation of the specimens, such testing might have been possible. In addition, the other testing rigs required for flexural modulus, compression failure, shear failure, etc., could not be located at U of T and hence these tests were abandoned. All design has therefore occurred with the provided and estimated properties noted above.

3.2.3 Bending and Torsional Testing of Spar Specimens: Test Setup

To validate the characterization and modeling equations derived, it was necessary to carry out a number of tests on several of the different structural specimens used in the HPO project, including capped and non-capped sections. For capped sections, an early spar layup in which the middle 20cm of an 8m long 3” diameter section had failed during the laminate cure provided a number testing specimens, as the rest of the tube had cured perfectly (the problem in the middle was due to an improperly sealed gap between two lengths of oven). All of the testing was carried out on specimens mounted on a cantilever attachment, built on a
truss structure mounted securely to the ground as visible in Figure 3.3. A 3” outer-diameter steel tube was mounted on the truss, onto which all carbon fibre tubes were secured such that a length of between 6” and 12” of steel tube was inserted.

Measurements were taken using dial-gauges placed along the length of the test specimen. Error from the possible displacement of the cantilever mount was eliminated by mounting a stiff aluminum bar to the steel tube by means of 4 bolts (see Figure 3.4. After the dial-gauges had been mounted to the aluminum and the bar allowed to settle, any deformation in the cantilever mount was matched by the attached bar, and thus the dial gauges. For bending tests, the dial gauges were mounted to measure the beam deflection at the centerline of the spar cap (see Figure 3.5).

For torsion tests, in order that only twisting and not combined bending-twisting be measured, an additional support was placed at the tip of the tube which would allow nearly frictionless twisting but no bending (see Figure 3.6). To measure torsional displacement, lever arms were bonded to the sidewall of the tube such that, given the radius at which the dial gauge measured, it was possible using the displacement measured by the gauge to determine the angular displacement at the tube (see Figure 3.7). It should be noted that for bending tests, the tube-mount is assumed to provide a perfect cantilever base with respect to the aluminum bar, and hence only one measurement is necessary, as far from the tube root as possible. However, previous experience with U of T’s Engine-Powered Ornithopter’s (EPO) structural testing has shown that a structure is much more difficult to constrain in torsion. Hence for torsion tests one angular measurement was taken at the root, and one near the tip, between which was uninterrupted tubular section. Thus the root displacement could be subtracted from the tip displacement for an exact measure of the torsional flexure in the intervening tube. (NB: For the test of spar 2008-1, the mount as mentioned above was not yet fabricated. The specimen was cantilever mounted to a tube, in turn fastened to a heavy wooden barn-beam. The measurements taken were taken using dial gauges on stands mounted to the floor. The error in those tests is thus expected to be greater.)

Loads were applied during bending tests using a sling on the end of the beam section, with which a point-load could be incrementally applied (using iron weights). Preparation for torsion tests was more difficult. Curved aluminum plates, each with a 3/4” hole drilled, were bonded to opposite sides of the tube. After curing, a matching hole was drilled through the carbon tube on each side. In this way, a steel moment arm could be used to apply torques (by hanging weights off the moment arm) to the carbon spar without risk of local material failure (see Figure 3.8).
3.2.4 Bending and Torsional Testing of Spar Specimens: Experimental Results

Figures 3.9, 3.10, 3.11, and 3.12 below show the results of the bending tests, to determine EI, and the torsion tests, to determine GJ, for several 3” diameter spar sections. These sections were each cut from the first failed 8m spar section. As each section was cut from a longer tube, on which the caps were tapered, it was important to record the number of caps on each tube, their length, etc., for the predictive model. The tests are each named according to the specimen, in order of specimens tested that year.

The error for each test was determined from the reading error on each dial gauge used during the test, generally $\pm 0.005”$. In the case of Spar 2008-1, the error also includes an additional allowance for the possible displacement of the gauge mount, which was observed during testing. The plots show a line of best fit through all of the collected data in that run, to give a sense of the consistency of the measurement and deformation. The line of best fit has not been forced to an x-intercept of 0, in order to better capture the slope of the data set. Where the intercept is not zero, it is due to either one or more gauges being set incorrectly at the beginning of the test, or the settling of the test specimen onto the mount. In any
case, both these errors are nullified after the first few points, and would not be present in the more consistently linear sections of the plot.

The EI and GJ properties of each spar section were determined as follows. The displacement of an arbitrary point along a cantilevered beam with an applied end load is given by:

\[ \Delta = \frac{P x^2}{6EI} (3L - x), \]

where \( P \) is the applied load, \( x \) is the distance of the displaced point from the cantilever, and \( L \) is the distance from the applied load to the cantilevered root [32]. The angular displacement \( \theta \) of a uniform cylinder subjected to a torque at one end and fixed at the other is:

\[ \Theta = \frac{TL}{GJ}, \]

where \( T \) is the applied torque, and \( L \) the length of the bar over which the displacement is measured [30]. With these formulas, given either individual data points, the average of a number of points, or the slope of a line of best fit to a number of points, one can determine the average EI or GJ of the section. For EI and GJ, the average values were taken over a set of data points for which the mean-squared-error of the fit was closest to zero. This was to ensure the most consistently measured and linear portion of the structure’s deformation was analyzed for the relevant property. Table 3.1 shows the experimentally determined properties for each length of spar.
Figure 3.5: Dial gauge mount used during bending testing, making contact on the cap centerline, mounted to the aluminum bar, and thereby attached to the root of the test stand.

3.2.5 Parametric Tuning of Predictive Model and Validation

Given an analytic predictive model and experimental results, it was possible to impose and adjust a parameter $\rho$ on the analytic formulas. The most current adjusted formula was then used at each point in the design. There were three major design phases for the main spar:

1. Summer 2008, early spar design: A test spar of length 3m was built, which was designed with minimum number of tube layers (4) at a wrap angle of $30^\circ$ (thought to be the minimum angle from which the CF tubes could be pulled from the mandrels) for maximum torsional compliance. The number of cap plies and widths were chosen to give the section a predicted bending stiffness of about $30,000 \text{Nm}^2$. This section was intended to be used to adjust the model before the design of the main spars.

2. Summer 2008, main spar fabrication: Tests from section 2008-1 were the first used to adjust the predictive model. At this time successive refinements in the accuracy of the analytic equations were carried out first, bringing the model closer to the experimental result. When, as mentioned, the model was an exact representation of the spar geometry, $\rho$ was adjusted to bring the models to exactly the experimental values. At this
point each of the main spar sections was designed and fabricated for the ideal stiffness based on Ornithia’s kinematic optimization.

3. Early 2009 to Present, Model adjustment and design characterization: The aircraft was not completed by the end of summer 2008, which allowed a full additional school term to refine and validate the design. Each of the 2009-x sections was tested as explained previously, and the experimental results used to refine the model. It was possible to determine with each additional test an updated and more accurate $\rho$.

It is important to note that after the summer of 2008, all sections of the main spar (3”, 2.5”, and 2” diameter tubes) had been fabricated and thus their design was fixed. However, in the summer of 2009, due to design changes that had occurred in the interim, it was possible to redesign and newly fabricate several smaller sections of the wing. The lengths which were used from the previous design included two 3” inner-diameter, 7.5m sections, and two 2” I.D., 1.5m sections. Two 2.5” I.D. 5m long sections and two 1.5” I.D. 1.5m long sections were designed and built from scratch with the benefit of refined design. It is also

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$EI_{Exp.} \text{[Nm}^2]\text{]}$</th>
<th>$GJ_{Exp.} \text{[Nm}^2]\text{]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar 2008-1</td>
<td>28,263</td>
<td>6,509</td>
</tr>
<tr>
<td>Spar 2009-1</td>
<td>26,651</td>
<td>6,091</td>
</tr>
<tr>
<td>Spar 2009-2</td>
<td>30,392</td>
<td>6,563</td>
</tr>
<tr>
<td>Spar 2009-3</td>
<td>30,702</td>
<td>5,900</td>
</tr>
</tbody>
</table>

Table 3.1: Tubular spar experimental results for EI and GJ.
important to note that although several spar sections were fabricated, it was less important what the actual properties were so much as they were known as exactly as possible. Given exact knowledge of the structural properties, it was still possible, with the remaining design parameters (e.g. chord, sweep, jig-twist, elastic axis location, etc.), to configure the wing for optimal kinematics.

Table 3.2 shows the progress of each round of testing for EI and GJ. It displays the theoretical analytic (or adjusted analytic) value prior to the test, the experimental value, the error in the experimental versus predicted value, and the adjusted parameter $\rho$ after the result of that test. The coefficients for the 2009 spars were adjusted independently of the 2008 spar primarily because of the improved accuracy of the bending and torsional tests.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$EI_P$ [Nm$^2$]</th>
<th>$EI_E$ [Nm$^2$]</th>
<th>$\Delta$ [%]</th>
<th>$\rho$</th>
<th>$GJ_P$ [Nm$^2$]</th>
<th>$GJ_E$ [Nm$^2$]</th>
<th>$\Delta$ [%]</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar 2008-1</td>
<td>37,549</td>
<td>28,263</td>
<td>-24.7</td>
<td>0.753</td>
<td>4,560</td>
<td>6,509</td>
<td>+42.7</td>
<td>1.427</td>
</tr>
<tr>
<td>Spar 2009-1</td>
<td>28,198</td>
<td>26,651</td>
<td>-5.5</td>
<td>0.732</td>
<td>6,262</td>
<td>6,091</td>
<td>-2.7</td>
<td>1.200</td>
</tr>
<tr>
<td>Spar 2009-2</td>
<td>30,178</td>
<td>30,392</td>
<td>+0.7</td>
<td>0.734</td>
<td>6,303</td>
<td>6,563</td>
<td>+4.1</td>
<td>1.147</td>
</tr>
<tr>
<td>Spar 2009-3</td>
<td>31,166</td>
<td>30,702</td>
<td>-1.5</td>
<td>0.731</td>
<td>6,328</td>
<td>5,900</td>
<td>-6.7</td>
<td>1.053</td>
</tr>
</tbody>
</table>

Table 3.2: Final results of spar testing, with predicted values (based on current best estimate), experimental values, prediction error, and averaged adjustment coefficient after the results of that test.
3.3 Integration

The design model was implemented in a form that, given the fabrication details of a structure (e.g. number of caps, width of caps, number of tubular layers, angle of tubular wrap, types of carbon used), the exact structural properties are determined. This was used to either back-out properties of structures as-built already for testing or the aircraft itself, or with a discrete iterative method for new part design (which, given constraints of minimum cap size, taper ratio of subsequent stacked cap plies, fabrication accuracy, etc, was a rapid design process).

This predictive model has been fully integrated with the wing design and structural predictive tool, in the form of an Excel spreadsheet containing the breakdown of all the required structural properties of the wing at each rib station.

The wing design process was an iterative exchange between Ornithia and the wing design tool spreadsheet. An example iteration would begin with a specification from Ornithia for the kinematically desired properties at nodal points along the span, generally correlating with the locations of tube joints (where the diameter changes) along the span. The wing design tool was then used to match as closely as possible the ideal properties of the wing through adjustment of fabrication variables. In this way a feasible tube structure could be devised and passed back to Ornithia for a check of optimality. The iterative process was necessary because the nature of composite material fabrication, namely the requirement that they be laid-up in plies of discrete thickness. This mandated that the design be done in a topologically discrete fashion. As topological optimization is generally an involved and time-intensive process, it was deemed easier to carry out this process using the aforementioned
iterative and intuitive process with a team member, namely the author, using the wing design tool [34].

In addition, the wing design tool has been integrated with a comprehensive linear FEM model of the full aircraft structure, programmed as an undergraduate thesis at the University of Toronto by HPO team member Tatiana Chiesa [35]. The model uses a wireframe depiction of the aircraft structure to determine in-flight limit-case stresses and deflections in the structure. The equations assembled in the current work for determining structural properties have been adapted and implemented directly in that program, and the properties of the wing structure are input directly from the wing design spreadsheet. Chiesa’s FEM analysis of the aircraft was used with the author’s failure prediction codes (to be outlined shortly) in order to design the 6.5m long tailboom section, the 3m downtube (which connects the tailboom to the fuselage), and the bracing wires used throughout the aircraft structure: namely the static wing wires and downtube support wires.
Figure 3.9: Displacement vs. Load plots for Spar 2008-1
Figure 3.10: Displacement vs. Load plots for Spar 2009-1
Figure 3.11: Displacement vs. Load plots for Spar 2009-2
Figure 3.12: Displacement vs. Load plots for Spar 2009-3
Chapter 4

Failure Analysis

4.1 Literature Review

As mentioned previously, the design of historical HPA structures has not been as extensively documented as might be desired from a research viewpoint; consequently the literature is relatively sparse, especially concerning detailed analysis methods. The goal of this literature review was to determine what previous work had been done in three areas pertaining to failure analysis:

1. The objectives and methodology used by previous HPA teams to guide structural design, especially in the case of failure analysis;

2. The framework and tools used by previous HPA projects (with a structure similar to the HPO) to analyze their primary structural components for failure;

3. Testing methods and the regimes used by previous HPA teams to validate and adjust their failure models.

4.1.1 Design Objectives and Methodology for Failure

As with the structural characterization mentioned previously, the Daedalus team had the most extensively documented experience with structural design and analysis. As discussed, Cruz and Drela determined that primary structure design should be dominated by strength requirements, to achieve the maximum weight savings [27]. A full analysis of the limit cases for the airframe structure was carried out, as opposed to the traditional use of a V-n diagram and constant safety factors. For example, the worst load cases imaginable, such as an extreme steady sideslip, or a strong wind gust, would be used to design a given component like the wing as opposed to designing for a constant safety factor above and beyond flight
load. These loads were therefore for extreme cases, and would often imply an instantaneous and catastrophic failure, but were nonetheless within the realm of static failure analysis methods. This much more specific and detailed load-factor analysis is reckoned to have saved a significant amount of weight on the final aircraft versus the prototype, potentially as much as 25% of the airframe’s empty weight [6,36]!

The designer of the Velair, Peer Frank, focused on stiffness-based design, and seems to have done no detailed failure analysis of the primary structure (at least from what is explained in literature). Instead, all of that aircraft’s tubes were load-tested to failure, and as long as the desired flight loads were survived the component was deemed sufficient [28]. Some of his components, for example the main spar, survived loading to as high as 3.34G of the expected flight load. Such a part could be considered grossly over-designed for an HPA, and the weight scarcely afforded. This design by failure no doubt proved sufficient for the aircraft’s operation however, considering that in many cases Frank mentions the lack of available resources to determine failure of certain components.

John McIntyre of the Airglow makes no mention of a systematic design scheme for failure analysis. As noted previously, the only mentioned concern for design was that the torsional stiffness of the spar was the driving design consideration. McIntyre also comments that the primary structure (main spar, rear spar, in-plane truss, etc) was designed to carry the entirety of the airframe load, and that the secondary structure (ribs, etc) serves only to maintain the aerodynamic shape. This is a crucial methodology for design of lightweight structures, as load-path redundancy cannot be afforded.

4.1.2 Analysis Framework and Failure-Prediction Tools

Cruz determined, during work on the Daedalus HPA, that two prominent failure modes of the composite tube structure were bending and torsion-induced buckling [37]. As a result, Cruz used NASA SP-8007, a design document concerning the buckling failure of thin-walled circular cylinders. Cruz coded this report’s failure-prediction equations (which themselves were analytically developed and empirically tuned). This program was used to determine the spacing of foam/balsa bulkheads, which stabilized the tubular spars and other tubular structural elements against buckling. The entire primary structure was thus designed with failure checks from this code. Other failure methods (material failure primarily) were assessed with stress outputs from the finite-element codes written by Drela. In addition to SP-8007 used by Cruz, NASA published a document during the Saturn rocket development program, CR-912, which serves as a general manual for analysis of shell structures (stiffened and unstiffened). This framework was recommended by Jorn Hansen, professor of composites.
analysis and laminate failure at UTIAS, but treats only isotropic materials. A combination of CR-912 and SP-8007 seem to be the best collection of analytic/empirical equations for tubular shell analysis for an HPA.

Peer Frank used primarily testing, and very little explicit analysis, for failure prediction of the Velair. For example, Frank knew of no accurate analytic methods that existed to predict failure for the cap-stiffened spars in torsional buckling, so failure testing was Frank’s only option. In some cases there is a complete absence of engineering design, as in one case in particular where during the spar cap layup additional plies were added “on the fly”, “to be safe”.

Again, McIntyre makes no mention of the particular failure analysis tools used for the Airglow. McIntyre notes that additional layers were added to the tubular spar in areas of local stress concentration, such as hardware connection points, but makes no mention of the means used to determine the number of layers required or the stress concentration determination. Empirical methods were used to determine the propensity of the spar tubes to fail in torsional buckling. Sections of tube were built and tested, and due to the laminate stiffness the problem was determined to be a non-issue.

4.1.3 Testing and Validation

Daedalus’ project team carried out extensive proof-loading and failure-prediction validation during the construction of both the prototype Michelob Light Eagle and the final Daedalus aircraft [6, 37]. Spars for both aircraft were proof-loaded, using loads placed at the exact rib locations to reproduce the in-flight shear and moment distributions. For the buckling-prediction codes used, Cruz made and tested cap-reinforced spar sections to adjust empirical coefficients for his predictive model in a fashion similar to that done by the authors of the original SP-8007 report [38]. These were supplemented by spar section failure testing on a 4-point bending rig. The validated code was then used for component design.

While working on the Velair, Frank used 3- and 4-point bending tests to fail each smaller prototype component during the preliminary design stages (e.g. tail surface spars, outboard wing spar segments) [28]. Final parts were then built with suitable adjustments to yield approximate and satisfactory load factors. The Velair’s largest single structural component, the 6.5m main wing spar, was proof-loaded using the fuselage mount as a cantilever point, simulating the flight end-condition. This was then failed with a single point-load at the endpoint, and the spar rebuilt exactly as before following a satisfactory test to 3.34G.

McIntyre used similar methods to Velair, forgoing analytic predictions and instead relying on proof-testing [2, 29]. All primary structures were stressed to the design figure of 2G. In
addition, again having forgone analysis, testing alone was used to determine the necessity of bulkheads to prevent torsional buckling. The aircraft was however clearly over-designed, as the airframe survived a high-speed ground loop (during which a wing made violent contact with the runway) during an aborted take-off roll [29].

4.2 Research Methodology

The goal of this portion of the work was to develop a comprehensive failure analysis framework for the HPO, which would be sufficiently general to cover all tubular members in the primary structure. As determined by the literature review, and from the author’s structural testing experience gained during previous research projects concerning the HPO (see [7]), the three limiting failure modes encountered in the wing and primary structure are:

1. Composite laminate failure, a risk present in all of the prepreg carbon tube structures (analogous limiting case to material failure in isotropic materials);

2. Euler buckling, a risk in long slender sections without a high EI, primarily the 3/4” rear spars;

3. Torsional buckling failure, primarily a risk in the main spar, but also other sections with the potential to undergo high torques.

The final comprehensive failure code included these three failure modes, and an explanation will follow for the development and process behind each of these analyses.

4.2.1 Laminate Failure Analysis

Capped Spar Sections

Composite laminate failure modes were addressed first because of the availability of proven analytic methods. Professor J. Hansen covered lamina failure analysis extensively in his advanced composites at UTIAS, and the course slides proved a valuable resource for adapting and implementing such a scheme for the HPO [33]. The detailing of this analysis will begin with capped spar sections, as the different (e.g. unidirectional and axial) layers of the laminate require a more complex analysis than for a purely axially-wrapped non-capped tube.

The tubular laminates used in the HPO fall within the assumptions of classical plate theory, because of their extremely-thin characteristic in one dimension compared with the others, their lack of significant stresses perpendicular to the plane of the laminate, and the
proximity of the in-plane strains to zero [33]. Working from this theory, the constitutive relation for a laminate can be assembled in the form

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
A & B \\
B & D
\end{bmatrix} \begin{bmatrix}
\epsilon^o \\
\kappa
\end{bmatrix}
\]

where

- \( N \) is the stress resultant vector through the laminate;
- \( M \) is the moment resultant vector through the laminate;
- \( A \) is the membrane "stretching" stiffness matrix;
- \( B \) is the membrane bending/stretching coupling matrix;
- \( D \) is the flexural rigidity matrix;
- \( \epsilon^o \) is the vector of the membrane strains;
- \( \kappa \) is the vector of the membrane curvatures.

In the situation where there are no flexural stresses on the laminate (which will also be the case here, even though in the gross structure there may be flexure), the assumption can be made that no bending or curvature are present in the laminate. Therefore, the situation simplifies to:

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
A
\end{bmatrix} \begin{bmatrix}
\epsilon^o
\end{bmatrix}
\]

The form of the stress resultant and plate strain vectors are

\[
N^T = [N_x, N_y, N_{xy}]
\]

\[
\epsilon^{oT} = [\epsilon^o_x, \epsilon^o_y, \gamma^o_{xy}]
\]

where \( x \) and \( y \) refer to the longitudinal and lateral dimensions of the laminate, and \( xy \) refers to shear. Each stress resultant component is obtained by taking the average of the stress over the thickness of the lamina. The membrane stiffness matrix \( A \) is determined for a laminate comprised of \( n \) uniform-thickness layers using the sum

\[
A = \sum_{i=1}^{n} (\bar{Q})_i t_i
\]

where \( \bar{Q} \) is the transformed orthotropic elastic constants matrix, introduced previously.
The laminate failure analysis proceeds as follows for a capped tube. First, \( \mathbf{A} \) is assembled using the contributions from the cap and the tube laminate, taken at the point of maximum thickness in the cap (which is the point of maximum compressive stress due to bending, that being the point furthest away from the centroid). The structural stresses are then taken from the FEM, and combined such that the total maximum compressive force and tensile forces are found. As for a beam, the bending stress will act as a compression on one capped surface and a tension on the other capped surface. These are combined with any axial force to compute the aforementioned maximums in the spar. The stress resultant vector \( \mathbf{N} \) is then determined using the maximum stresses. The constitutive relation above, given \( \mathbf{A} \) and \( \mathbf{N} \), is now inverted to find the plate strain vector, \( \epsilon^\circ \), which is uniform through the laminate. The \( \bar{Q} \) vector is then taken for each of the spar and cap laminates, and the stress-strain relation

\[
\sigma = \bar{Q}\epsilon^\circ
\]

multiplied out. Now note that the unidirectional caps already have their material axes aligned with the longitudinal axes of the structure, and hence the computed laminate stresses can be directly compared against the material strengths. However, the stresses in the tube laminate are displaced off of the material axes by the wrap angle \( \theta \) of the tube. The stresses thus need to be rotated using the transformation

\[
\sigma_{\text{material}} = T\sigma_{\text{structure}}
\]

where \( T \) is the transformation matrix given for computing the orthotropic elastic constants [39]. Now, the tube material stresses can also be directly compared with their determined material strengths. It is important that the compressive and tensile stresses both be compared with their corresponding material strengths, as these are often dramatically different for composite materials. The failure proximity of the spar was then computed via the maximum stress conditions. The conditions are that in no direction can the material stress exceed the failure stress in that axis, given the imposed safety factor. The method of maximum stress conditions is in common use in industry [33]. Thus it is possible to determine whether a laminate structure is sufficiently strong in each direction, and by what margin.

Non-capped Sections

The process for a non-capped section is identical to that above, except for two simplifications.

1. The assembly of \( \mathbf{A} \) does not include any unidirectional cap layers;
2. The stresses need only be calculated for the rotated tube laminate.

4.2.2 Euler Buckling Analysis (for Rear Spar)

Euler buckling is induced when an axial compressive load on a member, which may not be sufficient to cause a material failure in that member, causes an instability and forces the member to bend and deform in a premature collapse. The equation for determining the Euler buckling load $P$ in a slender structural member is

$$P = \frac{\pi^2 EI}{k l^2}$$

where $l$ is the unsupported length of the column and $k$ is the effective length factor, a measure of the stabilizing nature of the structure’s end supports [31]. In the case of the rear spar, the unsupported length is taken as that between the ends of each in-plane truss section. As for the end conditions, because the joint of each rear spar section is only stabilized against out-of-plane motion by the frame, and not solidly against rotation, the end condition is treated as a pin (for which $k = 1$). In reality this is a conservative choice (as will be shown later during validation), as in actuality the spar is somewhat stabilized by the intervening ribs and the rear spar is somewhat restrained at the end of each frame by its inherent bending stiffness.

The rear spar was checked for the possibility of Euler buckling failure by computing the in-plane drag loads on the wing. During thrusting motion, the rear spar is under tension and hence buckling is not a concern. As the in-plane truss was modeled similarly to a beam structure, the compressive force in the rear spar was determined using the maximum compressive stress found for the wing structure in the in-plane direction. Given the load in the wing, the proximity to failure of the rear spar in Euler buckling could be easily determined.

4.2.3 Torsional Buckling Analysis

As mentioned, consultation with Juan Cruz, who was the structures specialist of M.I.T.’s Daedalus HPA project gave appreciation for the buckling failure modes to be encountered. The torsional buckling exhibited in these tubular sections is again an instability problem, where the shear stresses in the thin tube wall cause a snap-through and collapse under load. The failure is analogous to the twisting of an aluminum pop can, when the thin sidewall buckles and allows the can to be crushed. This mode is most frequently encountered in thin shell structures. Rocket bodies are fabricated in this fashion. Therefore, to be able to predict
their failure modes under torsion, axial, and bending induced thin-wall buckling, NASA has contracted several studies. Two of these were "NASA Contractor Report 912: Shell Analysis Manual", recommended by Professor Hansen and "NASA Space Vehicle Design Criteria (Structures) SP-8007: Buckling of Thin-Walled Circular Cylinders", recommended by Juan Cruz. SP-8007 was coded by Cruz for this purpose in the Daedalus project, and was intended for use with orthotropic materials as used here; and Cruz noted that the results were very consistent for the composite carbon fibre tubes used in that project [37]. SP-8007 gives two crucial formulas: first, a formula for the critical torque that will cause a thin-walled cylinder to buckle; and second, a formula that determines the applicability of the buckling analysis [38]. The formula for the critical buckling torque is given as

$$\tau_{cr} = (BF)\bar{D}_y (E_x E_y - E_{xy}^2)^{\frac{3}{8}} \frac{R_2^\frac{\gamma}{l}}{l^2}$$

where, with $x$ and $y$ being the longitudinal and circumferential axes of the tube laminate as before and $xy$ being the shear axes,

- $BF$ is the "Buckling Factor", an empirically-adjusted coefficient;
- $D_y = \frac{E_y t_{tube}^3}{12(1-\nu_{12}^2)}$;
- $\bar{E}_x = \frac{E_x s_{tube}}{1-\nu_{12}^2}$;
- $\bar{E}_y = \frac{E_y s_{tube}}{1-\nu_{12}^2}$;
- $\bar{E}_{xy} = \nu_{12} \frac{(E_x + E_y)}{2} t_{tube}$;
- $l$ the unsupported length of tube.

The shear flexibility coefficient $R$ is given by

$$R = \frac{\pi^2 D}{l^2 D_q}$$

with $D$ the wall flexural stiffness per unit width

$$D = \frac{(E_x + E_y) t_{tube}^3}{12(1-\nu_{12}^2)}$$

and $D_q$, the transverse shear-stiffness parameter,

$$D_q = \frac{G_{xz} h^2}{h - t_{tube}};$$
where $h$ is the distance between the exterior and interior face plies. The formula for the thickness ratio of the tube is

$$TR_{\text{tube}} = \left( \frac{\bar{D}_y}{\bar{D}_x} \right)^{\frac{5}{6}} \left( \frac{\bar{E}_x \bar{E}_y - \bar{E}_{xy}^2}{12 \bar{E}_y \bar{D}_x} \right)^{\frac{1}{2}} \frac{l^2}{r_{\text{tube}}}$$

where

$$\bar{D}_x = \frac{E_x t_{\text{tube}}^3}{12(1 - \nu_{12}^2)}$$

and for the buckling analysis to be accurate, it is suggested that $TR$ be roughly $\geq 500$.

### 4.3 Bending and Torsional Failure Testing

#### 4.3.1 Test Setup

The same bending and torsion test stand used in the structural characterization experiments was also used for failure testing purposes, having been designed for the large loads expected in these tests. The method by which bending loads (via a sling around the spar tip) and torques (moment arm through loading plates, with rolling end-support) were applied also remained the same. Figure 4.1 gives a sense of the deformations and loads (in this case about 250lbs) encountered in many of these failure tests. A notable exception to the pure-bending or pure-torsion loading cases were proof-loading tests. Figure 4.2 shows a 7.5m section of main spar undergoing a proof-loading test, during which the member was subjected to 110\% of flight load, in each of the bending and torsion directions. These proof-load tests were less to verify the design of the member than to ensure that it was free of any dangerous defects in manufacturing.

#### 4.3.2 Results

**Bending Failure Tests**

In all beam-bending tests, the dominant mode of failure was compression in the composite laminate. The failure was always very near to the root of the tube, where the bending moment loads were highest. This was true for both spar sections, where the initial failure was in the unidirectional caps, and non-capped sections. Inspection and post-analysis of the failed structures reveals a catastrophic fracture that generally propagated rapidly and completely through the tube shear wall. However, high-speed video footage reveals that this shear-wall failure was secondary. Figure 4.3 shows a typical root compressive failure in the spar caps. Figure 4.4 shows the type of shear/compressive failure encountered in the
Figure 4.1: Loaded spar test section immediately before bending failure
tailboom during a bending failure test. Table 4.1 summarizes the results of the bending
tests conducted, giving the section description, predicted bending moment, failure bending
moment, and error.

<table>
<thead>
<tr>
<th>Test Description</th>
<th>Predicted [Nm]</th>
<th>Actual [Nm]</th>
<th>∆ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar 2009-2</td>
<td>2,373.8</td>
<td>2,253.2</td>
<td>-5.0</td>
</tr>
<tr>
<td>Spar 2009-3</td>
<td>2,373.8</td>
<td>2,015.3</td>
<td>-15.0</td>
</tr>
<tr>
<td>2008 Tailboom 2” Section</td>
<td>145.7</td>
<td>168.0</td>
<td>+15.3</td>
</tr>
<tr>
<td>2008 Tailboom 3” Section-1</td>
<td>321.4</td>
<td>346.0</td>
<td>+7.6</td>
</tr>
<tr>
<td>2008 Tailboom 3” Section-2</td>
<td>321.4</td>
<td>328.6</td>
<td>+2.2</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of bending failure results conducted for the HPO.

Rear Spar Buckling Failure Test

An in-plane truss test section was constructed in the summer of 2009 in order to measure
the stiffness properties of the Kevlar cross-bracing. After sufficient data had been gathered,
the truss was failed in order to test the Euler buckling prediction. The section was loaded
in bending similarly to spar sections before, with the main spar cantilevered and the rear
spar pinned at the root. The predicted compressive failure load in the rear spar was 370N,
whereas the rear spar actually buckled at 456N. This discrepancy was likely due to the
stabilization of the ribs and the inherent stiffness of the spar end joints (e.g. not true pins).
**Torsion Failure Tests**

As expected from the literature review, the torsional failure modes were buckling-dominated. In fact, there was not a single primary failure in shear, and use of the laminate failure analysis described earlier shows that the test sections were never within 50% of the laminate shear failure load when buckling occurred. Such a commanding failure mode took a great deal of iteration and investigation to effectively mitigate. As noted previously, for the Daedalus project buckling failure was mitigated by the gluing of stabilizing circular biscuits inside the tubular spar at intervals of $4 \times Diameter_{spar}$, a spacing empirically determined by Cruz as early as 1987 [37]. As shown by the analysis in SP-8007, buckling failure is influenced predominantly by the stiffness of the tube laminate. In the case of the Daedalus, the spar was made from the highest-modulus carbon fibre available [6]. Unfortunately, for HPO, the spar tubes are all made such that they possess as little wall stiffness as possible. Thus buckling failure has been a more persistent problem on this project than for any HPA before. It is likely
that if tube wall buckling, while elastic and non-catastrophic in and of itself, were to occur in flight, it would induce a shear wall collapse of the tube and a subsequent compressive failure in the spar. For this reason, the design philosophy for the HPO used the tuned failure model, and kept stresses within the range such that if a single biscuit were improperly installed, the doubling of the unsupported length of tube would still not cause buckling under flight load. Thus the wing was designed such that the peak aeroelastic torque would not exceed that required to buckle biscuits at 12” spacing. This case-based design, as opposed to typical factors of safety, is in the spirit that was typical of the Daedalus team [27].

The biscuit spacing in the HPO was estimated to be between 6” and 12”, meaning there could be as many as 200 biscuits in the main spar alone. Given that each biscuit would weigh approximately 5g, the resulting mass could comprise as much as 5% of the wing. Thus it was critical to find as lightweight a solution as possible by both maximizing the spacing of the biscuits and fabricating them as light as possible. Initially, a 3/8” solid balsa- ply biscuit was used, but during installation it was discovered that the biscuits were prone to rotation (“barn-dooring”) inside the tube, in which case the effect of the biscuit was nullified. A second iteration saw a polystyrene core between two solid faces (the additional depth prevented barn-dooring), but these were prone to crushing across the grain of the faces. At this point external stiffeners were designed, one in which a balsa hoop was glued to the exterior of the spar and wrapped circumferentially with carbon to form a sandwich structure.
The second used 1/64” plywood reinforcing rings glued to the rib plates. Unfortunately, failure occurred at torques below that for internal biscuits due to failure and delamination of the stiffeners from the tube sidewall. Eventually, a biscuit was designed with an ultra-low-density structural foam core and very thin balsa-plywood face plates. This biscuit proved consistent to fabricate, lightweight compared with previous solutions, and robust in testing. This is the type installed throughout the aircraft, as shown in Figure 4.5. Figure 4.6 shows a typical buckling failure in the spar, with the diagonal buckled crease running the length of the spacing between adjacent biscuits, as well as a spar in which successive buckles were induced under small incremental loading.
Figure 4.6: A typical buckle between two biscuits in a spar section, and section with multiple induced buckles (creases indicated).

Most of the torsional failure tests conducted were for the purpose of refining biscuit design and finally establishing a buckling factor $BF$ for the coded analysis from SP-8007. As such, table 4.2 gives the failure torque of each section by biscuit type and spacing, along with the percent of maximum flight load (given the predicted flight load in the summer of 2008).

<table>
<thead>
<tr>
<th>Test &amp; Biscuit Description</th>
<th>$\tau_{\text{failure}} [N\text{m}]$</th>
<th>% Flight Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar 2008-1, 3/8” Balsa/Polystyrene 12”</td>
<td>109.25</td>
<td>67</td>
</tr>
<tr>
<td>Spar 2009-1, 3/8” Balsa/Polystyrene 6”</td>
<td>206.45</td>
<td>126</td>
</tr>
<tr>
<td>Plywood Rib-rings (External) 6”</td>
<td>153.04</td>
<td>94</td>
</tr>
<tr>
<td>Balsa/Carbon Hoop (External) 6”</td>
<td>134.97</td>
<td>82</td>
</tr>
<tr>
<td>3/32” Balsa/Rohacell 6”</td>
<td>193.60</td>
<td>118</td>
</tr>
<tr>
<td>3/16” Balsa-ply/Rohacell 6”</td>
<td>277.48</td>
<td>170</td>
</tr>
</tbody>
</table>

Table 4.2: Spar buckling test results for each type of biscuit.

### 4.4 Parametric Tuning of Predictive Model and Validation

#### 4.4.1 Composite Laminate Failure

The composite laminate failure analysis was used to predict the failure of spars in bending. As can be seen from the results in table 4.1, three of the four tests were very close to their predicted values. The test of spar section 2009-3 appears to be an exception. Although there
was no obvious precipitating factor, it is possible that an inconsistency in that particular layup (the tube having been part of the first failed spar) could have caused premature material failure. For this reason, the composite laminate properties code was left without the imposition of an adjustment parameter. The potential inconsistency of the spar layup process was accounted for by designing the wing, in bending, for a factor of safety of 2.

4.4.2 **Euler Buckling Failure**

From the test results, the actual Euler buckling load was 1.23 times that of the predicted load, due to the stabilizing factors mentioned above. This figure was chosen as a coefficient for the Euler buckling formula when used in design and failure analysis of the rear spar in the context of the in-plane truss.

4.4.3 **Torsional Buckling Failure**

As had been done by NASA when performing the testing involved with SP-8007, experimentation was required to determine a buckling factor, \( BF \), in order to adjust the analysis to the particular situation at hand. The spars were a special situation for the application of this analysis because, although the thin sidewall was prone to buckling, the tube was stiffened longitudinally by the unidirectional caps. In addition, the thickness ratio \( TR \) for the 3” spar sections was determined to be roughly 344. This is on the low (e.g. thicker) end of the window for the SP-8007 analysis to be valid. However, as the analysis had proved satisfactory historically (e.g. with the Daedalus), it was decided to likewise account for this with the \( BF \) [37]. \( BF \) for the 3” spar sections was thus determined to be 288,953. This figure was used for the remaining sections of spar, which all shared similar proportionality in the cap/tube perimeter ratio and thickness ratio.

An empirically tailored \( BF \) was not determined for a non-capped section during the course of this project, for three reasons:

1. The only two other sections subjected to torque loads are the downtube and tailboom; and in those two cases the torques predicted were not nearly as high as that of the main spar;

2. Both the tailboom and downtube were designed to be much stiffer than the spar, having much thicker tubewalls, which would have departed from the applicability of the methodologies in SP-8007;

3. The tubewalls being considerably thicker and stiffer ensures that these two members are much less at risk of torsional buckling failure.
Regardless, to prevent tube oblation under bending loads and potential torsional buckling issues, in both of these members biscuits were installed at 12” intervals (this was the distance chosen by the Daedalus team specifically to prevent bending oblation, and double that selected for the spar [37]).

4.5 Integration

This failure analysis methodology was written into a MATLAB code. The analysis described above made the code applicable for both capped and non-capped sections. This was integrated with both Ornithia and Tatiana Chiesa’s FEM model of the HPO [11,35]. In Ornithia, the bending failure and torsional buckling predictions were used as constraints for tailoring the aerodynamic performance of the wing such that with the already-built spars, the bending stress was limited to a factor of safety of 2 and the torsional stress was tailored such that the spar would see the buckling torques required for 12” biscuit spacing. The failure code was used with Chiesa’s thesis work to design the downtube and tailboom, given appropriate factors of safety.

The Euler buckling prediction was used extensively in the design of the in-plane truss. A 2-D truss FEM was written to design each of the truss frames in order to tailor the stresses in the tubular members (chordwise “compression members” and the spanwise rear spar) and Kevlar cross-bracing, and determine/design the exact stiffness properties of the truss. The rear spar was subjected to the Euler buckling formula with the stabilization factor included.
Chapter 5

Future Work

Having completed the entire design process from start to finish, it is clearly possible to evaluate areas where still-better design tools and analyses would have been desirable. This is not to diminish the enormity of the current task, given the foundation from which it was undertaken. Given that design and engineering are iterative sciences, there is always more to do. That more was not done in this work was a testament to the ambitious timeline of the project, and the fact that the aircraft itself needed building. Future work that could stem from this research falls roughly into two categories: refinements and functionality that could be added to the current design framework; and further validation and testing within the current framework.

5.1 Refined and Improved-Functionality Methods for Design

The structural FEM code implemented in Ornithia at the conclusion of this work could be accurately termed a \textit{geometrically nonlinear method for determining applied aerodynamic forces} bootstrapped onto a linear finite-element method. Pending full validation from in-flight observation, it is of course conceivable that a more rigorous and consistent framework exists to account for the geometric nonlinearity of the structural deformations encountered. This could better capture the exact structural kinematics and coupling of forces and structural deformations. It was the author’s original intention to develop and program such a method, but in the time allotted and with the other tasks at hand, it has proven infeasible to do so. The co-rotational dynamic method developed by Crisfield seemed the most promising framework available, and either this or Drela’s exact formulation would provide the best starting point in the future for a computationally-inexpensive nonlinear method.
In the vein of improving modeling accuracy and consolidating design tools, one option would be a better FEM model of the wing itself. In this work, the wing was modeled as a 1-D beam, with its sectional properties a combination of those of the main spar as well as the other components of the wing primary structure. As the in-plane truss detailed design was carried out with a separate FEM, it is conceivable that the two could be combined. For instance, within the same FEM program, the main spar could be modeled as a 12-DOF frame element, the rear spar as a 6-DOF frame element (out-of-plane bending and axial DOFs), the compression member as a 4-DOF grid element (axial compression and axial torsion), and the cross-bracing wires as 2-DOF bar elements (axial extension only). This would allow the exact contribution of each structural element in each direction to be more accurately captured.

It is worth noting, given that a full-aircraft frame element model was programmed as part of an undergraduate thesis, that it should be possible to integrate such a model with Ornithia. This would allow complex interactions such as body-heaving, wing induced downwash-effects on the tail, and dynamic tail-surface response to be captured. Part of the reason for this not having been done was to preserve the workability of Ornithia as an optimizer for iterative design, as the computational expense inherent in such a comprehensive framework would be prohibitive.

Given the potential for an optimization-based design methodology for incremental improvement, it would be beneficial to keep the entire design process available to the optimizer. It would be possible and desirable therefore to program all of the structural characterization formulas into MATLAB, such that given the fabrication parameters for a part on the aircraft, its section structural properties could be determined automatically. This would negate the step of iterative feedback between Ornithia and an Excel spreadsheet, as is currently done. Therefore, it would be possible for faster and more effective design iteration and development.

Given tubular members for which the design is not integrated with a holistic model, it would be useful to have a composite-tube design tool at hand, potentially integrated with stand-alone optimization functionality. Given the structural characterization model and failure analysis framework, it should be possible to program a simple optimization methodology that would carry out the design of a tubular structure. Thus, subjected to failure and stiffness constraints, and given a sufficiently defined objective (e.g. minimum weight for a required stiffness or a required strength) the optimizer could instantaneously design a structure for the desired load. This would save considerable time in design and analysis, and be accessible to any project member.

In the realm of failure prediction, one of the shortcomings of the available resources
was that no analytical formulation existed that exactly modeled the kind of longitudinally-stiffened cylinders represented by the main spar. Experimentation showed clearly that the current method used would be sufficient, but improved accuracy is clearly possible. The trouble may be that few other structures in general are assembled quite like those of the HPO, with wide-flange reinforcement on an otherwise very thin tube. Alternatively, it is likely that a full multidimensional membrane-element-based FEA could more accurately predict failure than any analytically-derived model.

Finally, one constant roadblock throughout the project has been the lack of the full range of material properties required for design, especially given the high-performance composites available to the team. It would be desirable in the future to locate the appropriate facilities and spend the necessary time to experimentally determine as-built properties for each of the materials used, as this would instill additional confidence and accuracy in the design process which, with a closer-to-limit-load design capability, would translate into significant weight savings over the entire aircraft. Alternatively, it would be possible to contrive a series of tests, with axially-wrapped tube specimens built as for the aircraft, which would determine each of the material properties required for their design. By constructing a number of tubes at different wrap-angles, it would be possible to use the composite orthotropic elastic constants algorithm described earlier to back out the laminate properties. This has the advantage that a "fudge-factor" would already be included, and the properties determined would be for the exact laminate geometry in which they would eventually be used.

5.2 Further Validation of Existing Analyses

In general, it would have been desirable to have a greater number of specimens tested for every experiment run during the project. There are so few data points throughout the development of the HPO models that it is difficult to get a sense for the statistical trends or accuracy of the testing and specimen fabrication. The dearth of data is primarily influenced by 
a) the extremely limited timeline of the project and b) the difficulty, time, and expense in fabricating each and every test specimen. It is difficult to conceive of means by which the fabrication and testing process might have been streamlined. This remark is applicable for each of the tests carried out, including EI and GJ measurement, and failure testing.

Given additional room in the timeline before the aircraft needed to be designed and built, there would have been three priorities for further experimentation:

1. accumulating additional spar and non-capped tube bending failures, especially with shear forces accurately accounted for in laminate failure prediction;
2. testing of remaining spar diameters (2.5”, 2”, and 1.5”) to see if torsional buckling analysis departs or holds accurately;

3. torsion failure in thicker non-capped tubes of the kind used in the remainder of the aircraft primary structure, to obtain torsional failure cases with which to further validate the laminate prediction code and to determine whether failure in these kinds of tubes is buckling or material dominated.

Finally, given more time, it would have been desirable to carry out more combined-loading and more elaborate proof-loading cases. In general it was time-consuming, and thus infrequent, to set up a complete testing rig for a given part. Therefore approximations, simplifications, and modifications would be made. It would be clearly desirable to be more thorough and precise in this sense, with a rigorousness similar to that seen in industry, as some of the parts are subjected to in-flight loads near to their limit-design.
Conclusion

A suite of design and analysis tools has been developed and programmed for use in the Human-Powered Ornithopter Project as follows: a finite-element method with an adaptation for geometrically nonlinear displacements has been developed and coded in a way that is comprehensive, accurate, and computationally inexpensive:

- comprehensive because of its ability to capture deformation and stress data;
- accurate because of the exactness of the 12-DOF FEM for linear structural displacements;
- computationally inexpensive because of the quadratic convergence of the structural iteration method and the minimal additional computation required to capture nonlinear effects.

This structural method was crucial for the success of the aero-structural-kinematic optimizer with which the HPO wing was designed.

An accurate characterization of the tube structures used in the HPO has been carried out:

- a set of formulas has been derived that completely characterizes the section properties of each structural member in the HPO;
- a comprehensive database of the material properties required for design has been gathered;
- the tools required to design with high-performance composite laminates have been implemented;
- a complete design program to obtain the exact structural properties and fabrication requirements of the wing structure has been devised and utilized.

Without these tools it would not have been possible to design and build the structure of the HPO with sufficient accuracy to obtain the desired performance.
A framework for a thorough failure analysis of the HPO (and HPA structures in general) has been assembled:

- a process to evaluate the stresses and proximity to failure of a tubular composite laminate under combined bending, axial, and torsional loads;
- a corrected formula to predict the buckling of slender tubular compression members;
- a satisfactory prediction tool for the torsional buckling of thin-walled tubular structures of non-uniform cross-section fabricated with an orthotropic composite.

This framework is applicable not only to the current work, but also to other aircraft such as HALE UAVs with a similar structural configuration. Additionally, each of these tools has been tested, adjusted, and validated using test specimens and prototype parts built as they would be for the experimental aircraft itself. Sufficient experimentation has been done to instill a healthy sense of confidence in the aircraft’s design team, despite the entire design’s very slim margin for failure and the ubiquitous use of cutting-edge and unorthodox aerospace technologies. Structural properties and failure modes can be consistently predicted to within 5%, which is critical given the meagre margins of the limit load cases used for design. Noteworthy is the fact that the characterization and failure analysis framework developed here has been used in the design of every primary structural component in the aircraft. This is especially so because of the design complexity of structures like the wing. Using these tools throughout the project to generate a carefully engineered flying machine has brought this project closer than ever before to achieving the age-old dream of flight, for man to fly like a bird. At the time of this writing the aircraft has been designed and built, and the team waits on the weather for a perfect day for flight testing and a chance at historical success for the world’s first human-powered flapping-wing aircraft.
### Appendix: Material Properties of Carbon Fibre Prepregs

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, ( \rho )</td>
<td>Kg/m³</td>
<td>1.648 ( \times ) 10³</td>
<td>Calculated from Newport Properties</td>
</tr>
<tr>
<td>Ply Thickness</td>
<td>m</td>
<td>1.420 ( \times ) 10⁻⁴</td>
<td>Newport Technical Staff</td>
</tr>
<tr>
<td>( E_{11} ), Tension</td>
<td>Pa</td>
<td>2.199 ( \times ) 10¹¹</td>
<td>Newport Properties</td>
</tr>
<tr>
<td>( E_{11} ), Compression</td>
<td>Pa</td>
<td>2.103 ( \times ) 10¹¹</td>
<td>Est. from Flexural Mod., Newport Properties</td>
</tr>
<tr>
<td>( E_{22} ), Tension</td>
<td>Pa</td>
<td>9.000 ( \times ) 10⁹</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>( E_{22} ), Compression</td>
<td>Pa</td>
<td>1.100 ( \times ) 10¹⁰</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>( \sigma_{u11} ), Tension</td>
<td>Pa</td>
<td>2.282 ( \times ) 10⁹</td>
<td>Newport Properties</td>
</tr>
<tr>
<td>( \sigma_{u11} ), Compression</td>
<td>Pa</td>
<td>9.609 ( \times ) 10⁸</td>
<td>Est. from Flexural Strength, Newport Properties</td>
</tr>
<tr>
<td>( \sigma_{u22} ), Tension</td>
<td>Pa</td>
<td>8 ( \times ) 10⁴</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>( \sigma_{u22} ), Compression</td>
<td>Pa</td>
<td>2.500 ( \times ) 10⁹</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>G</td>
<td>Pa</td>
<td>4.4 ( \times ) 10⁹</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>( \sigma_\text{u, Shear} )</td>
<td>Pa</td>
<td>8.067 ( \times ) 10⁴</td>
<td>Newport Properties</td>
</tr>
<tr>
<td>( \nu_{12} )</td>
<td>N/A</td>
<td>0.300</td>
<td>MATS324 Course Material [41]</td>
</tr>
</tbody>
</table>

Table 1: Material properties used for Newport NCT301-HS40, with methods of determining each property noted. NB: These properties have not been approved by the manufacturer or any other commercial body for use.
<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$</td>
<td>$Kg/m^3$</td>
<td>$1.611 \times 10^4$</td>
<td>Calculated from ACG Properties</td>
</tr>
<tr>
<td>Ply Thickness</td>
<td>$m$</td>
<td>$1.470 \times 10^{-4}$</td>
<td>ACG Technical Staff</td>
</tr>
<tr>
<td>$E_{11}$, Tension</td>
<td>$Pa$</td>
<td>$2.640 \times 10^{11}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$E_{11}$, Compression</td>
<td>$Pa$</td>
<td>$2.100 \times 10^{11}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$E_{22}$, Tension</td>
<td>$Pa$</td>
<td>$6.910 \times 10^{9}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$E_{22}$, Compression</td>
<td>$Pa$</td>
<td>$7.550 \times 10^{9}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$\sigma_{u11}$, Tension</td>
<td>$Pa$</td>
<td>$2,108 \times 10^{9}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$\sigma_{u11}$, Compression</td>
<td>$Pa$</td>
<td>$7.250 \times 10^{8}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$\sigma_{u22}$, Tension</td>
<td>$Pa$</td>
<td>$2.310 \times 10^{7}$</td>
<td>ACG Properties</td>
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<tr>
<td>$\sigma_{u22}$, Compression</td>
<td>$Pa$</td>
<td>$1.550 \times 10^{8}$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$G$</td>
<td>$Pa$</td>
<td>$4.4 \times 10^9$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$\sigma_u$, Shear</td>
<td>$Pa$</td>
<td>$1.240 \times 10^8$</td>
<td>ACG Properties</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>N/A</td>
<td>0.270</td>
<td>ACG Properties</td>
</tr>
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Table 2: Material properties used for ACG MTM28-M46J, with methods of determining each property noted. NB: These properties have not been approved by the manufacturer or any other commercial body for use.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$</td>
<td>$Kg/m^3$</td>
<td>$1.910 \times 10^4$</td>
<td>Calculated from ACG Properties</td>
</tr>
<tr>
<td>Ply Thickness</td>
<td>$m$</td>
<td>$1.170 \times 10^{-4}$</td>
<td>Measured</td>
</tr>
<tr>
<td>$E_{11}$, Tension</td>
<td>$Pa$</td>
<td>$1.300 \times 10^{11}$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$E_{11}$, Compression</td>
<td>$Pa$</td>
<td>$1.150 \times 10^{11}$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$E_{22}$, Tension</td>
<td>$Pa$</td>
<td>$9.000 \times 10^{9}$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$E_{22}$, Compression</td>
<td>$Pa$</td>
<td>$1.000 \times 10^{10}$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$\sigma_{u11}$, Tension</td>
<td>$Pa$</td>
<td>$2.000 \times 10^{9}$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$\sigma_{u11}$, Compression</td>
<td>$Pa$</td>
<td>$1.300 \times 10^{9}$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$\sigma_{u22}$, Tension</td>
<td>$Pa$</td>
<td>$8.000 \times 10^8$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
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<tr>
<td>$\sigma_{u22}$, Compression</td>
<td>$Pa$</td>
<td>$2.500 \times 10^8$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$G$</td>
<td>$Pa$</td>
<td>$4.4 \times 10^9$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$\sigma_u$, Shear</td>
<td>$Pa$</td>
<td>$9.5 \times 10^8$</td>
<td>Est. from Hexcel’s “Prepreg Technology” [40]</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>N/A</td>
<td>0.300</td>
<td>MATS324 Course Material [41]</td>
</tr>
</tbody>
</table>

Table 3: Material properties used for ACG VTM264-HTR40, with methods of determining each property noted. NB: These properties have not been approved by the manufacturer or any other commercial body for use.
Bibliography


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[38] “NASA Space Vehicle Design Criteria (Structures) SP-8007: Buckling of Thin-Walled Circular Cylinders.” August 1968.

