Aero-Structural Optimization of Divergence-Critical Wings

by

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Abstract

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This study investigates the use of the divergence speed as an additional constraint to a multi-disciplinary optimization (MDO) problem. The goal of the project is to expand the MDO toolbox by adding an aeroelastic module used where the aeroelastic characteristics present a possible safety hazard. This paper examines aeroelastic theory and MDO disciplines. The divergence constraint function is developed on a BAH wing. The optimization problem is executed on the HANSA HFB 320 transport jet using the FEAP structural solver and a Vortex Lattice Method as the aerodynamic solver. The study shows that divergence speed can function as a safety constraint but the stress constraints determine the optimum design. Furthermore, obtaining a true divergence constraint will require a finer mesh, a more efficient aerodynamic solver and non-finite difference approach to gradient determination. Thus, the addition of the divergence constraint does not yet directly benefit this MDO framework.
Dedicated to my family Geoff, Gwen, Todd and Otis, who always believed in me,

Dr. Joaquim Martins, who supported me,

and all the friends along the way.
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Chapter 1

Introduction

1.1 Aero-Structural Aircraft Design

1.1.1 Aircraft Design and MDO

Multi-disciplinary design optimization, or MDO, is one of the new emerging sciences that can provide optimal solutions to problems that require more than one field of expertise, or 'discipline' to solve. This quality alone has resulted in the early adoption of MDO strategies by aircraft designers. The analysis of aircraft is by its very nature coupled by the aerodynamic and structural interactions. Aerodynamic and structural computational analyses can be solved efficiently using modern computers. While on-going development in fluid flow and structural solution methods has by no means stopped, computational based analysis techniques been developed to the point where it can be reasonably called a mature field, in terms of the methods, although the details of more specialized cases still require further research. Notwithstanding, the availability of computational fluid dynamics (CFD) and computational structural mechanics (CSM) give us powerful tools that enable engineers to perform the numerical analyses of the individual disciplines effectively. Multi-disciplinary analysis is used to couple the analysis of these individual disciplines together efficiently to produce an optimized solution based on a objective cho-
sen by the designer or engineer. These powerful numerical tools also pave the way for the development and widespread use of MDO in academic and industrial institutions, as the availability of these tools allows both researchers and professionals to explore the benefits of multi-disciplinary optimization.

Coupling the aerodynamic and structural disciplines together defines the basic aero-structural optimization problem. In fact, the tight coupling between the two primary disciplines, makes aircraft design suitable to the application of MDO methodologies. While two disciplines poses a multi-disciplinary design problem that is easier to conceptualize, aircraft are incredibly complex and contain many more disciplines such as propulsion, flight control, performance and aeroservoelasticity [25]. The theoretical basis that dictates MDO strategies is expandable up to an infinite number of disciplines, if the engineer is willing to pay the increased calculation cost. In order to accommodate these additional disciplines, which may be added at the engineer’s discretion, any implementation of an MDO strategy in aircraft design is generally a modular scheme. A modular scheme promotes the simple insertion (or removal) of disciplinary analyses, thus reducing the amount of reprogramming required when incorporating new disciplines into a MDO problem. However, implementing an optimization scheme that is able to balance the advantages and disadvantages of many disciplines is a major design challenge, and can result in an arduous iterative procedure. The purpose of researching MDO techniques is to overcome the computational challenge to produce accurate reliable results that may possibly produce configurations not previously considered by the designers.

The application of MDO to an aero-structural problem takes place in two stages. The first is a low-fidelity stage, and the second is high-fidelity. During the low-fidelity phase, which is generally early in the design process, aircraft are analyzed for general characteristics. These stages have a low processing time, and the various disciplines involved may be using reduced levels of detail, or ignoring certain design factors. The purpose of low-fidelity analysis and optimization is to give the designers a general idea of the
design space of the vehicle, and important metrics such as lift-to-drag ratio, weight or range. High-fidelity optimization occurs when the general design of the aircraft has been resolved, and the designers are searching for smaller improvements that affect more esoteric characteristics of the aircraft, such as optimizing the final wing shape for a certain characteristic. High-fidelity optimization uses much finer models than low-fidelity, and the disciplinary analysis is generally more complex and rigorous. Naturally, the computational time required for these analyses is much greater, yet provide a much more detailed optimum.

Multi-disciplinary analysis has also been used in aircraft design for more than aerostuctural optimization. MDO techniques have been successfully applied to conceptual and mission designs by incorporating the optimization of various performance criteria such as range, fuel weight and fuel balancing. FLOPS [33] (FLight OPtimization System) is an example of this type of code, as is the recently developed MAGIC [42].

1.1.2 Purpose of Multi-disciplinary Optimization

Overall, the purpose of both low- and high-fidelity optimization is to circumvent the engineers intuition about the design problem. The optimization process will chose a path based upon the results of the individual analyses of the disciplines (and according to a certain MDO strategy), and may result in an aircraft design that the engineer had not previously considered. Naturally, another aim of MDO is to achieve this optimization in as little time as possible, thus increasing the efficiency of the optimization procedure. To this purpose, different MDO strategies exist which can be more appropriate to different problems. One such factor is the amount of coupling in an MDO problem. Reducing the amount of coupling between the disciplines grants the optimizer more freedom to explore the design space. The consequence of this is that the optimizer may explore unfeasible points where the individual discipline analysis is valid, but the coupled analysis is not. The benefit is that the optimizer is more likely to provide a non-intuitive solution,
however the drawback is a possibly unreasonable solution. Research has already been performed on the advantages and disadvantages of particular MDO strategies, such as by Sobieszczanski [52] or Martins and Tedford [56]. The purpose of this paper is not to do this, however issues concerning multi-disciplinary analysis will appear during the course of the investigation.

1.1.3 Description of Aeroelasticity

Aeroelasticity is the interaction of inertial, aerodynamic and elastic forces. The results of these interactions are well described by Collar’s triangle, shown in Figure 1.1. The interaction of at least two of these forces produces various aeroelastic phenomena, which are described below.

1. **Divergence:** Divergence is a static aeroelastic effect that involves the interaction of aerodynamic and elastic forces. Wing torsional divergence is the most common divergence, that occurs when the local angle of the wing increases to the point where structural failure occurs. This happens when the lifting forces on a wing induce a torsional moment that twists the wing about its elastic axis. This twist results in an increase of the local angle attack of the wing. This in turn produces more lift, which in turn induces further twist. Eventually, the aerodynamic forces and the elastic reactive forces will either reach an equilibrium, or the reactive forces will be overcome. The local angle of attack will continue to increase, as will the aerodynamic forces, until structural failure occurs. Thus at precisely the divergence speed, the increase in aerodynamic torsional moment due to an increment in twist angle is equal to the corresponding increment in elastic restoring torque. If the aircraft exceeds this speed, the aerodynamic moment increase will be greater than the elastic restoring torque. Divergence speed is primarily affected by the coupling between the torsional and bending degrees of freedom. Thus, swept back wings are less susceptible to divergence than forward swept wings.
2. **Flutter:** Flutter is a dynamic aeroelastic phenomena which occurs at the flutter speed. An example of flutter can be seen when an aircraft’s wings are seen to oscillate up and down during flight. There are actually many kinds of flutter phenomena: classical flutter is associated with potential flow and coupling of degrees of freedom, while non-classical flutter can involve periodic breakaway and attachment of flows. Principal factors affecting flutter are wing torsional stiffness, aspect ratio and sweep. Flutter speed is increased by increasing sweep and decreasing the aspect ratio of a wing. Higher torsional stiffness will also reduce flutter speeds. In the past, flutter has been handled by adjusting the mass distribution in the wing to increase stiffness or change the natural frequency of the wing structure. However, increasing the stiffness of a wing means usually results in a mass increase, which means a compromise between flutter speed and performance.

3. **Control Reversal:** Control ineffectiveness is defined as a static aeroelastic phenomena that occurs when the intended effects of a control surface is nullified by the elastic deformation of the structure. When the control input produces the opposite of the intended motion, it is termed reversal. For example, a banking aircraft displaces one aileron up to reduce lift in the wing, and the other down to increase lift. However, the down aileron produces a twisting moment on the wing that reduces the local angle of attack of the wing, as the up aileron increases the local angle of attack of its wing. These twisting moments are proportional to the square of the aircraft speed, while the elastic restoring torques remain constant. Once the control reversal speed is reached, the twisting moment will negate any change in the lift of the wing induced by the aileron deflection, leaving the aircraft unable to roll. At a faster speed, the effects of aileron displacement will be reversed. This is the aileron reversal speed and is quite a serious problem for swept back wings. To prevent the problem, wing torsional stiffness can be increased, but this will increase the aircraft weight.
4. **Buffeting**: Buffeting is a dynamic aeroelastic vibration that occurs in the tail or fuselage due to impulses from the wing wake. This effect is difficult to compensate for as the wake behind wings in certain maneuvers (such as stall) are not easily predicted. Buffeting can be reduced by good placement of the tail assembly on the aircraft.

5. **Dynamic Stability**: This general category includes dynamic aeroelastic effects that are due to dynamically changing loads on an aircraft, such as gusting. Any elastic deformation and resulting aerodynamic force changes that affect the stability of the aircraft are contained within this category. For example, the response of a wing to a dynamic gust load is important in determining the wing loading properties during the design of the aircraft.
Figure 1.1: Collar’s Triangle
1.2 Project Overview

As described in the previous section, a basic multi-disciplinary optimization of an aircraft consists of the analysis of two disciplines; aerodynamics and structures. This coupled analysis, by its nature, is an aeroelastic analysis. This means that it is susceptible to aeroelastic phenomena such as divergence and flutter. Aeroelastic phenomena are important to the design of an aircraft, as they result in reduced control of the aircraft, undesirable performance, and often cause damage, sometimes catastrophic, to the vehicle. Aircraft design to avoid undesirable aeroelastic behavior for traditional aircraft is well documented, particularly in the civilian jet transport industry. However, with the emergence of new designs and roles for jet aircraft, as well as new speed regimes that they operate in, the possibility of unfavorable aeroelastic behavior must be considered in the aircraft design. The earlier these considerations are incorporated into the design process, the less time and money is wasted analyzing flawed designs that are susceptible to wing divergence or flutter. By incorporating aeroelastic constraints into an early design optimization, the design space of the aircraft will be reduced such that undesirable aeroelastic effects do not occur during the range of the aircraft’s mission profile.

This project uses divergence speed as an example to prove this concept. Using an iterative multi-disciplinary analysis (MDA) within a multi-disciplinary feasible optimization strategy, the use of a divergence speed as an optimizer constraint to produce a low-fidelity optimized wing shape for an aircraft will show that accounting for aeroelastic phenomena early on in the design stage is a beneficial strategy for wing configurations prone to divergence problems. The optimization objective is the range of the aircraft, which is a function that takes into account both the weight and lift-to-drag ratio of the aircraft. These variables are directly affected by the wing thickness and the shape of the wing, which also affects divergence speed. Thus range is optimized with respect to wing thickness, and constrained by the divergence speed, and yield stress of the material. The assumption is that if this criteria is useful in these cases, then it will be also be useful for
unconventional wing shapes where aeroelastic behavior may be unpredictable, or poorly understood. While there are many aeroelastic phenomena, divergence speed is used for two reasons. First, wing divergence is a much easier programming task than flutter or buffeting, which are time dependent phenomena. Furthermore, iterative analysis schemes tend to suffer from numerical divergence as well. Situations can arise when the aeroelastic analysis does not converge, or converges to a non-physical result. Both wing divergence and numerical divergence will be prevented by the measures implemented in this project. Wing divergence theory is covered in § 2.1. A plate model of a wing approximated from a wing box is used in the structural analysis, and this model is reduced to a beam to calculate the structural stiffness matrix for the purpose of divergence analysis. Aerodynamic forces are calculated from the full plate model, and reduced to the beam’s position, although the effect of the plate’s thickness is ignored during aerodynamic calculations. The optimizer, SNOPT, uses a sequential quadratic algorithm to solve the optimization problem.

1.2.1 Motivation: Aeroelastic Analysis in MDO

The current method of dealing with aeroelastic phenomena is to address any aeroelastic problems during the latter stages of the development and design cycle. Typically this is handled during a scale model or prototyping stage, where an actual aircraft is placed in a wind tunnel or flown, and any aeroelastic problems that result are noticed and addressed. This method has been very successful in the last 30 years, largely due to the fact that the basic design of subsonic civilian transport aircraft has not changed, and the engineers designing them have more than a decade of experience with these wing configurations. The basic wing shape used on these aircraft has already been thoroughly analyzed and well optimized for the mission profile. Thus, the aeroelastic behavior of these aircraft is well known and generally predictable, especially by engineers who have a considerable amount of experience in this design field. However, new aircraft designs such as the blended-wing
body (BWB) are coming to light for which, should a similar approach be used, would be too time consuming and expensive to be viable method to properly design the next generation of aircraft. In order to insert the consideration of aeroelastic constraints earlier into the design process, the use of multidisciplinary optimization (MDO) is put forward as an alternative to the traditional design process. In MDO, aeroelastic considerations in aircraft design have been ignored, represented only by simple linear corrections, or addressed only in the prototype phase of a project. Recent research into the inclusion of aeroelastic considerations earlier into the aircraft design process has shown that while feasible, the cost of running the detailed analysis for these constraints is prohibitive. However, the increasing performance demands of modern aircraft require the inclusion of these considerations as early as possible in the design process. Recent research into the inclusion of aeroelastic considerations into the preliminary aircraft design process has shown that while feasible, the cost of running the detailed analysis for these constraints is very high [23, 4]. With conventional aircraft design, this particular problem is fairly irrelevant. The aeroelastic behavior of conventional aircraft is well known, and even initial designs of traditional aircraft can be designed by knowledgeable engineers to avoid most aeroelastic phenomena. However, this would not be the case for unconventional aircraft designs. The traditional “build and test” approach is too time consuming and expensive to be a viable method to properly design this next generation of aircraft. This has been characterized in the literature as a spiral development [38], where experimental testing gradually advances an aircraft design in small steps. While the final product may be a revolutionary aircraft, the process is time consuming and expensive. There is a further wrinkle in the general aero-structural design problem for next generation designs; some of the new aircraft are designed to be flown at supersonic speeds. While supersonic flight is well understood, there is little experience designing supersonic passenger airliners, and there has not been much emphasis on optimizing the design in the past few years, particularly for commercial viability. This is especially important as the standards of safety
and comfort increase substantially when passengers are carried. Furthermore, these high speeds may bring previously unexpected aeroelastic interactions into play. The flight characteristics of trans-, super-, and hypersonic airplanes are relatively new territories that require rigorous aerodynamic and aeroelastic treatment if there are to be successful aircraft designs, and the increased performance demands of modern aircraft can require more detailed analysis, which only increases the demands on computer resources throughout the design process. However, if some of the restrictions on these new aircraft designs can be determined early in the design phase through the use of aeroelastic constraints to reduce the design space, then computer resources can be conserved. Thus, the main motivation for this research is to use MDO to optimize aircraft by including aeroelastic effects directly into aero-structural optimization via constraints on the optimization problem. Optimization with aeroelastic constraints will allow engineers to reduce the design space of next generation aircraft, reduce the amount of computer resources and the time needed to perform the preliminary design, and allow for early evaluation of new aircraft designs for critical flight problems such as divergence. In addition, many new nonconventional designs, such as the BWB and joined wing, are designs for which there a large amount of prior engineering experience does not exist. By using an MDO framework to assist the design an aircraft, extensive previous experience regarding the design problem is not required. The final result of the optimization will comply with the physics of the analysis, and be constrained appropriately. By performing low fidelity design optimization in this manner, unconventional aircraft designs can be efficiently designed, and the engineers will be able to take the design in a direction which is safe from aeroelastic phenomena within the projected operational envelope. Implementation of this project will add another tool to the MDO framework at the University of Toronto MDO Laboratory, as a module to calculate the aeroelastic properties of wing shapes will be available for future projects.

Overall, inclusion of aeroelastic constraints in low-fidelity optimization early in the design
cycle will free up time to explore more design options and evaluate them, save overall design costs, or alternative a high-fidelity analysis can be used to fine tune the detailed design near the end of the design process. These factors justify the inclusion of aeroelastic constraints earlier in the aircraft design and optimization process. However, at this time, most implementation of low- or high-fidelity MDO does not take into account either static or dynamic aeroelastic effects. Including the analysis of these aeroelastic phenomena, and the subsequent use of the results as constraints, will not only be useful to engineers, but will give MDO another valuable tool in its repertoire [23]. It is hoped that expanding the MDO toolbox will not only make the University MDO framework more powerful and versatile, but also promote the use of MDO in the private aviation industry as a valuable tool and an efficient, effective complement to the traditional design process.

1.3 Survey of Aeroelastic Optimization

For many years, the field of aeroelastic analysis has been considered to be a mature field, with no further advancements in the fundamental theory expected. Recent work has focused more on aerodynamic or structural analysis techniques and not aeroelastic analysis. Livne [30] states that in “...1999 that the perception of aeroelasticity as mature and somewhat stagnant was challenged head on...”. What was realized by the industry, and voiced here by Livne, is the same thing stated by Krammer [23]; that the increasing performance and cost requirements of next generation aircraft necessitate the re-consideration of the role of aeroelastic interactions in aircraft design. In addition, the current economy and market, as well as the military, have created an opportunity for supersonic and hypersonic aircraft, as well as unmanned aerial vehicles (UAV) [32]. MDO approaches to tackle these designs with aeroelastic considerations will be essential to the
timely design of efficient and safe aircraft. In addition, if active and adaptive controls are to be implemented on the next generation of aircraft, it is essential that the aircraft is optimized for aeroelastic and aeroservoelastic behavior.

For example, in the past, the treatment of aeroelastic interactions in aircraft design was to simply ignore the aeroelastic problems, or by making simple assumptions [40] about aeroelasticity. However, “increasing performance requirements and pressure to reduce the cost to design and operate aircraft can no longer be met by traditional design processes” [23]. The method used by Niu in his textbook contrasts with the opinion put forth by Krammer et al. Furthermore, if such a practice can be put into effect in an MDO framework, the potential for the optimization of next generation aircraft would be quite substantial.

Aeroelasticity has been studied by many organizations. Early aeroelastic studies performed in Germany by the Daimler–Benz Aerospace AG Military Aircraft Division (DASA-M) developed LAGRANGE, a structural optimization package that included static and dynamic aeroelastic effects. During the study, technology needs for MDO were identified. The results of the DASA-M studies spurred the designers to produce a list of needs for aeroelastic consideration in MDO: efficient methods for cross-discipline sensitivities, suitable MDO frameworks, automated model generation that is also robust, and aeroelastic effects and phenomena in performance and flight dynamics [23].

LAGRANGE itself was successful in implementing constraints for aeroelastic effectiveness and flutter in problems such as the simplified ACA-fin model and for a full aircraft model of JPATS ‘Ranger 2000’. The ACA-fin test showed that buckling constraints required consideration. The Ranger 2000 model and optimization was successful in detecting the potential for control surface flutter. These results show the need for buckling constraints, as well as the integration of aerodynamic analysis and control system design became more pressing. However, addition of these extensions to LAGRANGE were considered complex and costly. Subsequent studies on the ACA-Fin coupled LAGRANGE to
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aerodynamic solver packages, which were then driven by a separate optimization package iSIGHT. This combination led to an acceptable global design if the approximations on the problem were good, but it also highlighted the lack of suitable MDO framework and robust models in the industry at that time. During the last two decades, investigations into aeroelasticity have generally been organized into a computational analysis portion and an experimental portion.

An example of such a two-part study was conducted by Heeg [21], who compared the computational analysis of a computer model of a 21 inch span NACA-0012 wing to a scale experimental model. Using a linear panel code consisting of 20 vortices, 180 wake modeling vortices and derived structural properties, good agreement between the analysis and experiment data was reached. This work provided insight into the sub- and super-critical behavior of divergent wings. Further work on both divergence and flutter was performed on airfoils [47, 48] and cantilevered wings [15] to investigate divergence characteristics of a wings and airfoils in wind tunnels. Edwards [15] also used the MSC/NASTRAN [49] finite element tool and a generalized doublet lattice method as part of their generalized aeroelastic analysis method (GAAM) to calculate flutter and divergence speeds of the BAH jet transport wing (and airfoils), and found good agreement with published results. It is clear from these papers that the research performed on aeroelastic phenomena, particularly divergence, is thorough. Flutter of different configurations has also been studied, as in the case of Lee–Rausch [28], who used a Navier–Stokes algorithm to analyze the AGARD Wing 445. Rausch used the Navier–Stokes equations in conjunction with the V-g method for flutter analysis in order to determine the flutter response of the wing. The V-g method is a recognized flutter speed method, and as such this kind of analysis has been thoroughly investigated.

With many analysis tools now available, the challenge now remains to incorporate aeroelastic analysis into an MDO framework such that the designer benefits from the increased computational load, and any organizational challenges presented by the increased com-
plexity of the problem are overcome. However, it must not be forgotten that aeroelastic analysis techniques are still being investigated today. Recent studies in aeroelasticity [13, 9] have attempted to develop more complex non-linear models with which to analyze conventional and unconventional wing shapes.

A prime example of such an analysis under investigation for aeroelastic phenomena is presented by G. Romeo et al [16], in which the non-linear aeroelastic behavior of a high altitude long endurance (HALE) wing is investigated, by using a computer simulation involving both non-linear aerodynamics and structures, and two wind tunnel models. The simulation showed fairly good agreement with a MSC/NASTRAN simulation of the same wing. The goal of the research is to understand the aeroelastic behavior of the HALE wing by predicting (and verifying) flutter speed, and modeling pre- and post-flutter behavior. It has been shown in previous studies that non-linear effects produce coupling between the bending and torsion of the HALE wing, which can induce limit cycle oscillation even below the flutter speed. In the same way, the current project seeks to do the same with divergence speed, and unconventional wing designs, though the analysis components are linear and do not take time marching into account. There is also a major difference between an HALE model and the models under consideration here, in that the analysis of a HALE wing is centered around a beam model, while a plate model is more appropriate for the wings studied in the current research.

While there has been research, as demonstrated, in the analysis of more complex aero-dynamic and structural models, there has also been research in the reduction of complex structural models to a simpler equivalent form for less computationally intensive analysis. Where as Livne [13] and others [9, 16] investigated the use of non-linear structural analysis tools, Giles [19] and Navarro [31] investigated the use of equivalent plate models to represent wing boxes. Research is also being carried out on composite wing models, such as the work present by Chedrik’s group in [57]. The papers concerning equivalent plate models attempt to formulate an equivalent plate for a wing box consisting of thin
skeins, spars and ribs and compare them to a previously existing finite element codes (EAL [58] for Giles [19], and ADINA [11] for Navarro [31]). Giles uses a polynomial series to model the height of the wing box and thickness of the skins of the wing box, up to a degree specified by the analyst. The ribs and spars have linearly varying stiffness, and are continuously attached to the skin in the appropriate axial directions. Using these characteristics, equivalent stiffness, mass and damping matrices are formed using the total potential energy method, and solved using a Ritz solution technique. Navarro builds on this by including time dependant variables in the formulation, and makes a point of distinguishing between the linear and non-linear portions of the Ritz solution. The implementation of very stiff virtual springs are used by Navarro to enforce boundary conditions at points where it is difficult to choose a Ritz polynomial to automatically satisfy the boundary conditions.

Both studies show very good agreement with the respective FEM programs, even though different test cases, and slight variations in the model were used. Giles, however, described that stresses calculated in the region of trailing edge of the wing box could differ by more than 5%. It should be noted that both studies attempted to generalize the wing box coordinates so that the implementation was non-specific. It is interesting to note that in Navarro’s results, the maximum discrepancy between only the linear equivalent plate and the FEM program is generally only about 25%, and is more accurate if the overall deflection of the model is smaller. Giles was able to show a substantial computational time saving with his technique in terms of analysis time, however the time required for the construction of the equivalent plate model was not available.

These researches demonstrate an principle used in the current research, in that a box model is approximated as a plate in order to reduce both the analysis time taken by pyFEAP, and to facilitate the iterative analysis procedure used by the author’s model, without sacrificing accuracy due to the reduced complexity of the plate model. The time savings in this case can be substantial, as thousands of structural analyses may be car-
ried out in the course of the aero-structural optimization, but the calculations for the construction of the equivalent plate model are only carried out once.

As mentioned previously, the current research uses shell elements to form a plate model in the pyFEAP program from a wingbox shape via parameterization. This is along the same lines as the more complex technique used by Kennedy and Striz [26] in their structural model generation software. The use of shells as elements in a plate model, particularly for the Reissner–Mindlin [51] model, is also a topic under investigation [45]. Although the work in Polli [45] covered a very specific effect of thickness extensions and material fibers on dynamics of shells in a Reissner-Mindlin formulation, the conclusion is that shell models can reach the analytical solution of plate models. This fact is taken advantage of in the current research to justify the use of shells in the finite element model. Naturally, independent justification of this conclusion was carried out by the author during studies with FEAP.

Multi-disciplinary frameworks are usually based around a particular MDO strategy, although there is research continually underway to produce frameworks that are able to adapt the MDO strategy to the current design problem. In his survey paper on recent MDO developments, Sobieski states that the “main challenges of MDO are computational expense and organizational complexity” [52]. Independent of the design strategy however, Sobieski clearly states that the computational cost of the MDO problem may be greater than the sum of its parts, even if the disciplines involved are only solving linear systems, such as the MDO problem in this paper. Sobieski goes on to label MDO problems as falling into three categories: Those requiring multiple disciplines but only one analyst, MDO of an entire system (for instance, performance analysis of an entire aircraft) and the third being MDO problems that deal with organizational and computational challenges. The problem that is analyzed in this paper falls quite clearly into the first category. The study of the interaction of the aerodynamic and structural disciplines creates a new discipline, aeroelasticity, and in this case the author is the single analyst
This single analyst approach has been implemented by other groups as well with variations based on test model, software used and available hardware and software at the time of the study. A few of these cases and how they relate to the current project are outlined below. An overview of different MDO strategies can be found in references [37] and [27].

Martins [36] created an MDO approach that considered static aeroelastic solutions to a supersonic problem, using FESMEH [22] for structural analysis, a parallel multi-block flow solver for aerodynamic analysis, and the nonlinear constrained optimizer NPSOL [43] optimization package. NPSOL is the precursor to SNOPT [41], the optimization suite that is used in the current project. The goal of the research was to provide an integrated aero-structural optimization framework, and provide different techniques for sensitivity analysis within that framework. During this research, numerical divergence was encountered if the optimizer attempted to make the wings too thin, and the iterative aero-structural solver could not converge. The numerical divergence is possibly a symptom of actual divergence in the aero-elastic model, as it tends to occur for extremely small wing thicknesses. However, aeroelastic divergence was not included as an aeroelastic constraint since Martins’ approach focused on developing and implementing a coupled sensitivity analysis could be used for MDO. The current project attempts to build on this effective MDO framework presented by Martins by attempting to prevent physical divergence from occurring in the model, as well as numerical divergence from occurring during the aero-structural analysis, while producing an integrated aero-structural optimum demonstrated by the results of Martins’ research.

Maute et al. [4] developed a similar approach to Martins, but using a Schur–Krylov scheme to compute the design sensitivities. This research was carried out on a NACA-0012 three dimensional plate model, as opposed to the frame and shell model used by Martins. Maute’s goal was to show that the global sensitivity equations (GSE) could
be solved more efficiently by a Schur–Krylov solver than a linear block Gauss–Seidel solver, while improving robustness and convergence rates. Maute also showed that a Schur–Newton–Krylov algorithm could be used to compute the steady state aeroelastic response. The analysis was carried out in the context of an aeroelastic optimization problem, but only analysis was performed, not optimization. By analyzing a fairly complex model consisting of 2,550 structural degrees of freedom (DOF) and a 58,152-DOF fluid mesh, Maute concluded that staggered algorithms like a linear Gauss–Seidel scheme would show an unsatisfactory performance particularly on tightly coupled aero-structural problems that were “soft”; that is, if the structural model possessed a low stiffness. This is an important characteristic, as the current project frequently encounters structures characterized as soft if the divergence constraint is in effect, since the optimizer can be searching the design space for an optimum point near the boundary where the aeroelastic model is considered divergent. This boundary is set manually in the constraint set-up. Searches in this area may very well cause performance problems for a linear-analysis based iterative aero-structural analysis scheme.

Other approaches that only dealt with analysis included the MOB Blended Wing Body (BWB) project in Europe [53]. The MOB BWB project was significant in that it modeled a variety of behaviors for a Blended Wing Body aircraft using ZAERO’s [55] DLM method. However, it was not an optimization study, merely the determination of the flutter onset frequency. The study used Saab’s in-house SALSA package and the flutter prediction module of LAGRANGE (which can calculate aeroelastic flutter from ZAERO data) to compare their results. This builds on the work performed by Holinger [23] (discussed in more detail later), who first pointed out the fidelity gap between aerodynamics and aeroelasticity, and the deficiencies in the aeroelastic optimization and analysis toolbox. Both the MOB BWB and Karbel included aeroservoelastic analysis in their treatment of the flutter frequencies. Both papers showed that flutter speed and frequency is calculable for different, sometimes radical, configurations of aircraft. However, both
studies used rather low fidelity models on which to base their analysis. MOB mentions only that high fidelity coupling and CFD analysis expensive. They then base their high fidelity analysis on the low fidelity model. The results are not what was expected, which shows that new aircraft configurations are providing new challenges to the aeroelastician of the 21st century [30].

Within MDO frameworks, there exist various strategies with which to perform the optimization. A paper with a similar aim to Martins was a project intended to validate bi-level integrated systems synthesis (BLISS) decomposition method as a valid tool to define design spaces for aircraft design [38]. By using flutter and divergence speeds as constraints, Mavris was able to produce a converged vehicle within a design space that could be explored by more traditional aircraft design techniques. The aeroelastic and aerodynamic analysis was performed by ZONA Technology’s ZAERO program [55], structural analysis was performed by ANSYS [2] and performance criteria evaluation by FLOPS [39], developed at NASA Langley. The sample problem and structural model were heavily scaled down in order to solve the MDO problem in less time, using a maximum of eleven design variables, some of which were fixed. A response surface is used to preserve inter-disciplinary coupling at the system level. In this case, the aeroelastic effects could not be optimized by the aerodynamic or structural-FE optimizers. As a result, the aeroelastic constraints were pushed up into the system level optimizer, which was FLOPS. The scaled down model produced a number of important conclusions: foremost among these was that aeroelastic phenomena such as divergence speed could be successfully implemented in an optimization scheme.

By running the optimization scheme at different theoretical (but not realistic) conditions, Mavris showed that the design space can be mapped by this approach, a technique that could be used early in the design process. This is a similar approach to Farhat’s study of flutter characteristics of an F-16 [17] jet aircraft, however that study showed disagreement with flight test data despite a complex model. That particular study also used
inconsistent Mach numbers relative to the experimental data, which may also explain some of the inaccuracy.

The BLISS approach showed evidence that detailed analysis of a multi-disciplinary system will be expensive, but does produce an improved result compared to the baseline. However, the research represents an attempt to tackle a truly multi-disciplinary problem. However, the lack of design variables, and the use of only a highly swept back model does leave some doubt as to the usefulness of this approach on non-conventional, or more complex models. Yet it does provide a foundation upon which to base the current investigation, which attempts to do much the same thing, but with fewer, and linear, disciplinary analysis, and without considering flutter speed.

BLISS is not the only optimization strategy implemented in recent studies. In 1995, Braun and Kroo applied collaborative optimization (CO) to an multi-disciplinary design environment. CO can minimize the amount of information transferred between disciplines, and can permit the removal of large iteration loops between disciplines. This strategy, while not applied to an aero-structural problem, was successfully implemented on a lunar ascent trajectory optimization problem, and was able to reproduce the tightly constrained solution.

Gradient based algorithms are not the only MDO strategies that has been put into use to solve aero-structural problems. Arizono and Isogai [3] successfully applied a genetic algorithm (GA) to a composite supersonic transport cranked wing optimization problem. During optimization with respect to composite ply thickness and orientation variables, only a highly swept design was considered and therefore flutter velocity was the only aeroelastic consideration used. The GA was chosen because it could deal with a discontinuous objective function and was, in the opinion of the researchers, robust. A commentary on robust optimization can be found in reference [29]. The result of the GA study is interesting in that only the optimums that only satisfied the static strength constraint or both the static and buckling strength constraints violated the aeroelastic
flutter constraint.

Related to application of different optimization strategies and numerical divergence is the work of Ajmera et al [20, 6]. This group created an aero-structural optimization framework 'WingOpt', which is similar to the framework used in this thesis research. This aerodynamic equations are solved by a private vortex lattice method (VLM) code which is linear, as is the VLM module used by the author. The structural equations solved are also linear, however the structural model is not, as a wingbox with MSC/NASTRAN or a non-linear equivalent plate model is used to perform the structural analysis. Optimization is performed by either FFSQP or NPSOL, both of which use sequential quadratic algorithms much like SNOPT. Ajmera’s group tested six different MDO architectures that were variations on the multi-disciplinary feasible (MDF), individual discipline feasible (IDF) and all-at-once (AAO, a.k.a. SAND) algorithms. The program used an iterative MDA procedure, and contained semi-empirical routines to calculate the drag divergence Mach number. However, the divergence speed was not used as an explicit constraint. However, because only a highly swept, conventional transport airplane was used as a baseline design (a Boeing 737-200), the divergence speed was never a factor in the design optimization. All of the optimization algorithms produced similar results, and the MDF2 formulation was the quickest, having the least number of analysis calls. As mentioned, numerical divergence, termed as aeroelastic instability by the designers, was encountered in the WingOpt program. In order to communicate the non-convergence of the aero-structural iterations, a divergence constraint parameter (DCP) was formed that terminates the iteration loop if three consecutive iterations show non-convergent trends. This solution worked well for constant angles of attack throughout the aeroelastic iterations, however it was not robust if the angle of attack was updated constantly. The program then either failed to predict divergence or resulted in very slow convergence of the MDA. Treatment of numerical divergence in this way can result in excessive major iterations in the optimizer levels. A non-converging MDA leaves no gradient data for
the SQP based optimizer, and a feasible design point cannot be found. For these optimization problem formulations, the DCP worked as an implicit divergence constraint. Aeroelastic studies performed in Germany by the Daimler-Benz Aerospace AG Military Aircraft Division (DASA-M) developed LAGRANGE, a structural optimization package that included static and dynamic aeroelastic effects. During the study, technology needs for MDO were identified, that do not seem to yet be met today. The results of the DASA-M studies produced the following needs for aeroelastic consideration in MDO: efficient methods for cross-discipline sensitivities, suitable MDO framework, automated/robust model generation, and aeroelastic effects in performance and flight dynamics [23].

Clearly, while the calculation of aeroelastic effects is possible at this stage, the use of these phenomena as constraints in optimization is a technique which is not yet mature enough to use in industry. However, as a community, the aircraft design world is inching closer to the goal of a reliable, robust optimization framework with aeroelastic constraints.
1.3.1 Significance of Proposed Work

At the moment, the aeroelastic behavior of an aircraft is considered only when most of the aircraft design is complete. However, predicting dynamic and static aeroelastic behavior is essential in designing the next generation of aircraft. If an MDO framework can be constructed that successfully accounts for aeroelastic phenomena that an aircraft is susceptible to, then aeroelastic behavior can be considered at the inception of design rather than at the end, where further design investigation and modification can be complicated, time consuming and costly. Even for a simple geometry, the problems encountered in coupled aeroelastic modeling are common to all MDO approaches. Thus, a reliable, accurate aeroelastic optimization process that requires a reasonable amount of computer resources and time to complete represents an advance for the entire MDO problem solving approach and possibly the aircraft industry. Furthermore, the industry lacks the appropriate software and MDO algorithms for aeroelastic and aeroservoelastic optimization [23], and the completion of a successful aeroelastic MDO algorithm will be beneficial to developing an general purpose MDO framework. This framework would be beneficial to analyzing both unconventional aircraft designs and new airspeed regimes. As long as the aero-structural tools are appropriate to the airspeed regime being studied (hyper, super, trans- and subsonic), and the structural solver can appropriately model the wing being studied, then this code can be applied to any framework as long as the appropriate inputs can be provided to the aeroelastic prediction module.
Chapter 2

Theory

This chapter deals with the theory of behind each phase of the project. There are many ways to analyze mechanical systems, and this section will give a brief overview of which equations were used to analyze the wing system and how the equations were developed.

2.1 Aeroelastic Theory

Aeroelasticity is the study of systems where the inertial, aerodynamic and structural forces of a given system interact to effect a non-negligible change in the motion of a body situated within a flow. Aeroelastic effects can be static or dynamic in nature, depending on the degree of coupling of these three forces. The various phenomena produced by aeroelastic coupling in a system is best illustrated by Collar’s triangle (Fig. 1.1). Elastic, aerodynamic and inertial forces are placed at the vertices of the triangle. Phenomena produced by coupling involving all three types of forces are located within the triangle, and phenomena involving only two forces are placed along the sides.

Divergence, control stability, and control reversal are the only static phenomena, and thus involve only terms relating to the elastic and aerodynamic forces. These phenomena are termed static because they do not involve inertial forces which are dependent up acceleration within the differential equation of motion, as will be seen in § 2.1.1. The
other aeroelastic effects, such as flutter, buffeting and vibrations, are all dependent on the inertial characteristics of the aircraft, and thus are called dynamic phenomena, as they require time-dependent analysis. The phenomenon that is being studied in this case is divergence.

However, as all aeroelastic phenomena involve the motion, whether oscillatory or not, of the aircraft or wing. Hence, it is necessary to develop the equations of motion for an airfoil, and then extend the equations to a wing with finite span. This can be done in general for airfoils, after which the specific details of the analysis can be chosen based on the aero-structural model being analyzed.

### 2.1.1 Development of the Aeroelastic Equations of Motion

Consider a general case wing with six degrees of freedom as shown in Fig. 2.1. Like any mechanical system, the motion of a wing or airfoil can be described in terms of the types of energy present: the kinetic energy $T$, and the potential energy $U$. For a simple
system with one degree of freedom, the Newtonian equation of motion can be developed via Newton’s law or by use of the Lagrange equations. Defining \( m \), and \( k \) as constants of mass and stiffness respectively, and \( x \) as the single linear displacement degree of freedom, the kinetic and potential energy can be defined as shown below:

\[
T = \frac{1}{2} m \dot{x}^2 \quad \text{(2.1)}
\]

\[
U = \frac{1}{2} k x^2 \quad \text{(2.2)}
\]

Lagrange’s equation (Eq. (2.3) can be applied to the energy expressions, and after adding a viscous damping term \( c \dot{x} \) the result is the equation of motion Eq. (2.5)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 1, 2, \ldots n \quad \text{(2.3)}
\]

\[
L = T - V \quad \text{(2.4)}
\]

\[
m \ddot{x} + c \dot{x} + k x = X(t) \quad \text{(2.5)}
\]

Where \( X(t) \) represents a time dependent forcing function. The vector \( x \) and the constants \( m, c \) (the damping coefficient), and \( k \) can be extended to all degrees of freedom illustrated in Fig. 2.1, provided that the appropriate extensions to the diagrams are made. If the airfoil degrees of freedom of interest heave \( z(t) \) and pitch \( \theta_y(t) \) are considered to be uncoupled from each other, and the other degrees of freedom are assumed to be steady and uncoupled from pitch and heave (and hence ignored), then Eq. (2.5) can be written as:

\[
\begin{bmatrix}
M_{zzij} & 0 \\
0 & M_{\theta_y\theta_yij}
\end{bmatrix}
\begin{bmatrix}
\dot{z}(t) \\
\dot{\theta}_y(t)
\end{bmatrix}
+ \begin{bmatrix}
C_{zzij} & 0 \\
0 & C_{\theta_y\theta_yij}
\end{bmatrix}
\begin{bmatrix}
\ddot{z}(t) \\
\ddot{\theta}_y(t)
\end{bmatrix}
+ \begin{bmatrix}
K_{zzij} & 0 \\
0 & K_{\theta_y\theta_yij}
\end{bmatrix}
\begin{bmatrix}
z(t) \\
\theta_y(t)
\end{bmatrix}
= \begin{bmatrix}
A_{zzij} & A_{\theta_y\theta_yij} \\
A_{\theta_yzi} & A_{\theta_y\theta_yij}
\end{bmatrix}
\begin{bmatrix}
z(t) \\
\theta_y(t)
\end{bmatrix} \quad \text{(2.6)}
\]

where the coefficient \( A \) describes the forcing function with the same units to the stiffness \( K \). The forcing function is independent of the structural system, so there is a possibility
that coupling will exist between the degrees of freedom in the forcing function, but not in the structure. This equation describes the motion of a single cross section of the wing, but can be extended to multiple stations along the span of the wing (see § 2.1.5).

Eq. (2.6) represents the types of forces acting on a wing in fluid flow. The types of forces can be classified as follows:

\[
\begin{bmatrix}
\ddot{z}(t) \\
\ddot{\theta}_y(t)
\end{bmatrix} + \begin{bmatrix}
\dot{z}(t) \\
\dot{\theta}_y(t)
\end{bmatrix} + \begin{bmatrix}
z(t) \\
\theta_y(t)
\end{bmatrix} = \begin{bmatrix}
\ddot{z}(t) \\
\ddot{\theta}_y(t)
\end{bmatrix} + \begin{bmatrix}
\dot{z}(t) \\
\dot{\theta}_y(t)
\end{bmatrix} + \begin{bmatrix}
z(t) \\
\theta_y(t)
\end{bmatrix}
\]

This can be written as:

\[
M \ddot{x} + C \dot{x} + K x = A x
\] (2.9)

Where \( M, C, K, A \) are the square matrices of coefficients pertaining to inertial, damping, elastic and aerodynamic forces. In order to describe the motion of the wing in the flow through numerical simulation, it is now necessary to find a method to determine the elements of the coefficient matrices. This subject will be addressed in sections § 2.1.4 and § 2.1.5.
From this point forward in this paper, a bold $K$ represent a two dimensional matrix of elements which are denoted by $K_{xy}$. This applies to $M$, $C$ and $A$.

### 2.1.2 Aeroelastic Divergence

Divergence is a static aeroelastic phenomena that occurs due to the interaction of the elastic forces of the wing structure and the aerodynamic forces incurred by the flow over the wing. Divergence occurs when the aerodynamic forces produce an elastic displacement in the structure that changes the effective angle of attack of the wing (that is, the angle of the wing with respect to the freestream velocity) such that the change in angle of attack, $\delta\alpha$, produces an additional lift force on the wing, $\delta L$. This additional lift force produces a further structural displacement, another $\delta\alpha$, which in turn produces additional lift. This cycle has two possibilities: either the consecutive incremental increases in lift and $\alpha$ decrease until a stable equilibrium is reached, or the increments continue to grow larger in size. Naturally, von Mises stresses in the wing induced by the continually growing angle of attack will cause structural failure of the wing. Clearly, the case of divergence is one where elastic restoring force of the structure is insufficient to compensate for the aerodynamic forces and moments on the wing, resulting in an interactive feedback loop between aerodynamic loads and aircraft deformations [21].

To determine the divergence speed, consider the equation of motion of the wing, where the degrees of freedom are $h$ and $\theta_y$, such as in Eq. (2.8). Since divergence is a static phenomenon, as discussed above, any terms involving the first or second derivatives of the degrees of freedom can be ignored. Thus, the equation is reduced to:

$$
\begin{bmatrix}
K_{zzij} & K_{z\theta_{yij}} \\
K_{\theta_{y}z_{ij}} & K_{\theta_{y}\theta_{yij}}
\end{bmatrix}
\begin{bmatrix}
z(t) \\
\theta_y(t)
\end{bmatrix}
=
\begin{bmatrix}
A_{zzij} & A_{z\theta_{yij}} \\
A_{\theta_{y}z_{ij}} & A_{\theta_{y}\theta_{yij}}
\end{bmatrix}
\begin{bmatrix}
z(t) \\
\theta_y(t)
\end{bmatrix}
$$

(2.10)

If we assume an exponential solution to this equation of motion, we can write,

$$
z(t) = \dot{z}e^{pt} \text{ and } \theta_y(t) = \dot{\theta}_y e^{pt}
$$
where the values $p_j$ represent the eigenvalues of this system and the corresponding eigenvectors represent the mode shapes of the system [8]. Representing the structural and aerodynamic coefficients as a single matrix symbol, defining the vector of all degrees of freedom as the vector $x$, and dividing out the constant $q$ from the aerodynamic matrix $A$, such that

$$[K - qA_0] \dot{x} = 0 \quad (2.11)$$

where $q$ is the dynamic pressure. $A$ is generally referred to as the generalized aerodynamic forces (GAF) matrix, where the subscript $0$ represents the static portion of the GAF matrix. The time dependent portion of the Eq. (2.11), $e^{pt}$ has been dropped as divergence is a static phenomenon, so time dependency is not relevant. The dynamic pressure is defined as:

$$q = \frac{1}{2} \rho u^2 \quad (2.12)$$

where $\rho$ is the density of air, and $u$ is the velocity of the air (which is equal to the velocity of the aircraft). The dynamic pressure represents the magnitude of the aerodynamic loading on the structure.

Obviously Eq. (2.11) cannot be satisfied by $\dot{x} = 0$, as that would represent a trivial solution where all of the degrees of freedom were fixed to be zero, and no motion of the wing would occur, and neither would divergence. Physically, divergence occurs when the rotational or linear displacements of the wing result in a structural failure. Mathematically, this can be represented as the displacement vector (or an element of it) of the wing going to infinity. Generally, this definition is applied to either the pitch degree of freedom, or the vertical displacement. With either definition of divergence, $\dot{x}$ is not zero. Thus, in order to solve for the divergence, it is assumed that $\dot{x}$ is non-zero, therefore the divergence speed must be solved by a solution of

$$[K - qA_0] = 0 \quad (2.13)$$
Eq. (2.13) is a generalized eigenvalue problem, where \( q \) represents the eigenvalues of the problem. Thus, the lowest positive real eigenvalue of the set \( q \) represent the dynamic pressure at which the system is divergent, which we call \( q_{\text{div}} \). The corresponding eigenvectors of \( q_{\text{div}} \) describe the shape of the system, thus at \( q_{\text{div}} \) we expect the system to have the shape described by its eigenvector. Furthermore, if the all the eigenvalues of system are negative, then the wing will not diverge. This generally occurs in the case of swept-back, where the bending deformations of the wing tend to pitch the leading edge of the wing down, reducing the local angle of attack of the wing and preventing divergence. All sweptback aircraft, in fact, have a sweep angle past which divergence will not occur for the wing at any speed.

The matrix \( A_0 \) is usually singular, so an alternative formulation for solving the eigenvalue problem is suggested by Borglund [8], which involves rearranging the eigenvalue equation to produce the inverse of the eigenvalues \( q \). As the eigenvalues are the diagonals of a square matrix in the decomposition of the eigenvalue equation, each \( q \) can be found simply by inverting each eigenvalue. In addition, the eigenvectors of the system are unchanged by the this process, as the rearrangement of the eigenvalue equation is equivalent to multiplying both sides of the eigenvalues equation by \( 1/\lambda \).

The divergence speed is found from \( q \) using the formula

\[
V_{\text{div}} = \sqrt{\frac{2q_{\text{div}}}{\rho}}
\]

When analyzing Eq. (2.11), it must be remembered that the equation from which it is derived is based upon the application of Lagrange’s equation to the energies of the system. Thus, each term in the equation, when multiplied by the displacement vector \( \hat{x} \), must result in a force. When viewed in this manner, the terms \( K \) and \( A_0 \) in Eqs. (2.11) and (2.13) can be viewed as matrices of sensitivities of the structural and aerodynamic forces with respect to the degrees of freedom of the system. We use this view of the system to determine the values of the two matrices. We will now describe the two methods used to determine the aerodynamic forces and the structural displacements of the system.
2.1.3 Generation of GAF for a Panel Method

In this particular implementation of an aero-structural analysis, a panel code will be utilized to compute the aerodynamic forces. This means that a linear theory is being used to compute the lift forces on the wing. As such, what is required in order to solve Eq. (2.11) is the sensitivity of the lift forces with respect to the components of \( \hat{x} \). In this particular case, these components are the heave \((z)\) and pitch \((\theta_y)\) degrees of freedom.

Since the stiffness matrix can be viewed as the sensitivity of the structural reaction forces to the perturbed degrees of freedom of the finite element mesh, the divergence equation can be thought of to look like this:

\[
\begin{bmatrix}
\frac{\partial F_{s1}}{\partial z_1} & \frac{\partial F_{s1}}{\partial \theta_1} & \frac{\partial F_{s1}}{\partial z_2} & \frac{\partial F_{s1}}{\partial \theta_2} \\
\frac{\partial T_{s1}}{\partial z_1} & \frac{\partial T_{s1}}{\partial \theta_1} & \frac{\partial T_{s1}}{\partial z_2} & \frac{\partial T_{s1}}{\partial \theta_2} \\
\frac{\partial F_{s2}}{\partial z_1} & \frac{\partial F_{s2}}{\partial \theta_1} & \frac{\partial F_{s2}}{\partial z_2} & \frac{\partial F_{s2}}{\partial \theta_2} \\
\frac{\partial T_{s2}}{\partial z_1} & \frac{\partial T_{s2}}{\partial \theta_1} & \frac{\partial T_{s2}}{\partial z_2} & \frac{\partial T_{s2}}{\partial \theta_2}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
\theta_1 \\
z_2 \\
\theta_2
\end{bmatrix}
- q
\begin{bmatrix}
\frac{\partial L_1}{\partial z_1} & \frac{\partial L_1}{\partial \theta_1} & \frac{\partial L_1}{\partial z_2} & \frac{\partial L_1}{\partial \theta_2} \\
\frac{\partial M_1}{\partial z_1} & \frac{\partial M_1}{\partial \theta_1} & \frac{\partial M_1}{\partial z_2} & \frac{\partial M_1}{\partial \theta_2} \\
\frac{\partial L_2}{\partial z_1} & \frac{\partial L_2}{\partial \theta_1} & \frac{\partial L_2}{\partial z_2} & \frac{\partial L_2}{\partial \theta_2} \\
\frac{\partial M_2}{\partial z_1} & \frac{\partial M_2}{\partial \theta_1} & \frac{\partial M_2}{\partial z_2} & \frac{\partial M_2}{\partial \theta_2}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
\theta_1 \\
z_2 \\
\theta_2
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\tag{2.15}
\]

where \( F_{sn} \) and \( T_{sn} \) represent the structural reaction (denoted by the subscript \( s \)) force and torque at node \( n \), \( L_n \) and \( M_n \) represent the aerodynamic lift and moment at node \( n \), and \( w \) and \( \theta \) represent the nodal displacements in the heave and pitch directions respectively.

The above example is shown for two nodes, which is really only useful for illustrating the structure of the matrices. A real analysis case consists of more than two nodes.

The first matrix is simply the stiffness matrix, as the derivative of the governing structural equation \( K u - f = 0 \) with respect to the displacements \( u \) is the stiffness matrix \( K \).

However, the aerodynamic sensitivities are not so straightforward, as the degrees of freedom of the model are not explicit in the linear equation solved by the aerodynamics. In order to calculate the sensitivities, the aerodynamic mesh must be perturbed in the appropriate direction first. This is a computationally intensive process if done using
finite differences, as it must be performed as many times as there are nodes (less the constrained nodes) by the number of degrees of freedom of interest. Once the GAF matrix is computed, all that remains to put the matrix in the proper form is to divide out the dynamic pressure $q$ that the lift sensitivities were calculated at. This non-dimensionalizes the lift sensitivities with respect to the airspeed, which allows the sensitivities to be used in the eigenvalue problem to produce the dynamic divergent pressure, $q_{\text{div}}$.

2.1.4 The Vortex Lattice Method

There are multiple ways to determine the generalized aerodynamic forces (GAF) matrix. Various methods of representing the aerodynamic forces have been covered in the literature review. Options include fully real GAF matrices that involve some variation on the Theodorsen function [12, 7], imaginary GAF matrices [10] that involve the derivative of the GAF itself, and panel methods such as the doublet lattice method [8, 15]. For this project, we have employed a vortex lattice method (VLM) to determine the aerodynamic forces on the wing.

Panel codes consist of solving the equation

$$A \Gamma - v = 0 \quad (2.16)$$

where $A$ is the matrix of aerodynamic influence coefficients (AIC), $\Gamma$ is a vector of the horseshoe vortex strengths, and $v$ is the vector of panel boundary conditions.

Panel codes are an application of potential theory, which is governed by Laplace’s equation (Eq. (2.17)).

$$\nabla^2 \Phi = 0 \quad (2.17)$$

Laplace’s equation is a linear, second order partial differential equation, and to use it in this context, one must assume that the flow in question is irrotational. In addition, compressibility effects must be able to ignored in the flow; that is, the flow must be incompressible. Furthermore, the flow must neglect viscous effects, since they are not
Chapter 2. Theory

accounted for in the Laplace equation. A panel method also assumes that there are no non-linearities in the flow field. The result of these assumptions is that a panel method cannot properly model skin friction drag, flow separation, or shockwaves in the transonic region. It is, however, able to quickly calculate lift and induced drag for problems where compressibility effects are negligible. It is also possible to modify panel codes to be valid for supersonic speeds, and calculate vortex drag at these speeds.

The panel code uses Laplace’s equation to define the flow field on the wing given a set of boundary conditions. These boundary conditions, in this case, is that the inviscid flow past a solid body has a velocity component normal to the surface of zero. Mathematically,

\[ \nabla \Phi \cdot \hat{n} = 0 \]  

(2.18)

As a general rule, a boundary value problem associated with Laplace’s equation is well posed if \( \Phi \) or \( \partial \Phi / \partial n \) is defined at every point of the surface, so the boundary condition used will result in a solvable flow field for the panel code. The final product of a panel code is the singularity strength produced by the solution of Laplace’s equation. Panel codes are defined by the type of singularity distribution used to assemble the influence coefficient matrix \( A \) in Eq. (2.16). A singularity type is assumed, though the strength is unknown, and is used to construct \( A \). The three main distribution schemes are source, doublet and vortex.

A source consists of a point where the strength of the singularity is infinite, and decreases with the inverse square of the distance from the source. A doublet configuration is based upon a positive and negative source approaching each other until they are infinitesimally close. The vortex configuration models a rotating flow about a axis, and declines as the inverse of the square of the distance from the source. The vortex lattice method constructs a horseshoe vortex by laying one finite vortex source and two infinite vortex sources connected at their endpoints in a U-shape, where the open side is between the two infinite lengths of vortex singularity. The resulting U-shape is a called a horseshoe vortex, and the integrated streamwise form can be seen in Eq. (2.19)
\[ \Phi(\vec{r}_1, \vec{r}_2) = \int \frac{\Gamma}{4\pi R(\vec{r}_1, \vec{r}_2)} dx \] (2.19)

where the scalar \( R \) is the distance from the vortex singularity and \( \Gamma \) is the strength of the singularity in general.

The VLM functions by placing a series of horseshoe vortexes on the wing, one per trapezoidal panel. The front of the horseshoe, which is the only finite vortex segment, is located at the one-quarter chord point of each panel, aligned with the leading edge of the panel. The other two segments extend to infinity in the streamwise direction from either end of the finite section, which are located at the spanwise edges of each panel. The AIC matrix is formulated by determining the relative influence of the singularities of the wing based on their geometric positions. By determining the relative positions of the control points on each panel to every section of every vortex (of unknown singularity strength) on the entire wing, and collecting their respective influence on the resulting velocity vector, the influence coefficients are calculated. During this process, the influence of both wings on each other must be considered in order to get a proper lift distribution across the wing. This can be handled via symmetry, for conventional wing configurations, or asymmetrically, for cases such as the oblique wing [24]. At this point assumptions can also be made about the singularity strengths \( \Gamma \) themselves, as to whether they are constant, linearly varying across the panel, or follow some other mathematical function. In order to determine the AIC matrix and solve for the vortex strengths \( \Gamma \), the downwash boundaries (or boundary conditions) of each panel are required. These boundary conditions are applied at the control point of each panel, which is located at the three-quarter chord, midspan location on each panel. This location can be determined by analyzing linear lifting line theory for a thin section of wing, and finding a point at which the normal velocity of the flow to the wing is zero. This is the boundary condition of the panel code. Simply stated, at the control point, there can be now flow through the wing. Thus, at this point, the array of singularities, (whose strengths are not known beforehand) produce
a resultant flow at the control point whose normal component to the wing of induced velocity balances the normal component of the free stream velocity. This tangential flow can be calculated based on the geometry parameters of the wing, such that

\[- u_m \sin \delta \cos \phi - v_m \cos \delta \sin \phi + w_m \cos \phi \cos \delta + U_\infty \sin (\alpha - \delta) \cos \phi = 0 \tag{2.20}\]

where \(u_m, v_m\) and \(w_m\) are all components of the induced velocity in the flow \(x, y, z\) coordinates respectively. \(\delta\) and \(\phi\) are the mean camber angle and dihedral angle, \(\alpha\) is the local angle of attack and \(U_\infty\) is the freestream velocity. For a planar wing, such as the one being used in this model, this equation reduces to

\[w_m = -U_\infty \alpha \tag{2.21}\]

for small angles of attack \(\alpha\). The solution of Eq. (2.16) provides the strength of the horseshoe vortices located at each panel. From these strengths, the downwash velocities, coefficient of lift \((C_L)\) and coefficient of drag \((C_D)\) can be calculated. In addition, the distributed lift per until area over each panel can be obtained.

As stated earlier, the vortex lattice method makes quite a few assumptions about the flow that is being analyzed. The VLM neglects both thickness effects of the airfoil, and viscous effects of the flow, and treats the airfoil as a series of flat panels. However, for most cases, the these effects offset each other, producing good agreement between the VLM values and experimental data [5]. The assumption has also been made in Eq. (2.21) that the angle of attack of the aircraft is small, so large angles of attack can call the results of the panel code into question. Additionally, in a completely rigorous treatment of the aerodynamics of a wing, the vortices should be placed at the mean camber surface of the wing, and the trailing vortices should follow a curved path to more accurately represent flow over a wing. However, a flat plate approximation (linearized theory) to these vortexes is suitably accurate for engineering applications [5], and matches the plate model used for structural model. The trade off is that at high angles of attack, or for cases where the gradients of the circulation strengths are high, the numerical calculation provided by the
VLM loses accuracy due to the possible separation of the flow at high angles of attack and the use of linear lifting theory. VLM methods are also susceptible to a problem known as leading edge suction, where the resultant pressure distribution calculated from the VLM results in an extremely large force at the leading edge. This can beget errors within the lift and drag calculations, and is caused by the flat plate assumption mentioned above. In order to avoid this problem, a mesh should be chosen that does not contain small panels at the leading edge. This spreads the suction problem out over a greater area when calculating pressure, and reduces the effect on the lift and drag calculated. This is avoided during this study by using large panels evenly distributed over the wing.

2.1.5 The Finite Element Method

The finite element method (FEM) is used to approximate the reaction of a continuous structure based upon the response of discrete points of the structure. Of importance to this project is the stiffness matrix of the plate that is being used to model the wing. This matrix is used to solve the equation

\[ K u - f = 0 \]  \hspace{1cm} (2.22)

Where \( K \) is the stiffness matrix, \( u \) is the vector of nodal displacements, and \( f \) is the vector of nodal forces.

The stiffness matrix of the plate is found by discretizing both the spanwise and chordwise directions into a number of nodes. The plate is simulated by an arrangement of shell elements, which approximates solid plates. This holds true as long as the thickness dimension is small in relation to the other two dimensions. The model is formulated in terms of force resultants which are computed via integration of von Mises stress components over the cross-sectional thickness of the shell [54]. This model is able to reproduce bending and in-plane deformations that other plate based finite element programs, and approach mathematically exact answers to simple plate problems as the grid mesh is
refined.

As stated in § 1.2, the code uses the plate model only to carry out the structural portion of the multi-disciplinary analysis. Analysis of the divergence speed is carried out by reducing the plate model to a beam located at the aeroelastic axis. The beam model stiffness is not taken directly from the plate stiffness, but instead formulated based on the physical parameters of the plate. The beam model is formulated such that it retains the inertial parameters of the plate. Use of the parallel axis theorem with respect to a beam located at the aeroelastic axis allows the calculation the bending and torsional inertias. Ashley [7] has developed equations for the coupled flexibility of a swept wing with the degrees of freedom of heave and rotation about the pitch axis based on slender beam theory. They are:

\[
C^{zz}(y, \eta) = \int_0^{\min(\frac{y}{\cos \Lambda}, \frac{\eta}{\cos \Lambda})} \left( \frac{y}{\cos \Lambda} - \bar{\lambda} \right) \left( \frac{\eta}{\cos \Lambda} - \bar{\lambda} \right) \partial \bar{\lambda} \frac{EI}{GJ} + \int_0^{\min(\frac{y}{\cos \Lambda}, \frac{\eta}{\cos \Lambda})} \partial \bar{\lambda} \frac{\eta}{\cos \Lambda} - \bar{\lambda} \right) \frac{EI}{GJ} \partial \bar{\lambda} \tag{2.23}
\]

\[
C^{\theta \theta}(y, \eta) = \int_0^{\min(\frac{y}{\cos \Lambda}, \frac{\eta}{\cos \Lambda})} \left( \frac{\cos^2 \Lambda}{GJ} + \frac{\sin^2 \Lambda}{EI} \right) \partial \bar{\lambda} \tag{2.24}
\]

\[
C^{\theta z}(y, \eta) = \begin{cases} 
- \sin \Lambda \int_0^{\frac{y}{\cos \Lambda}} \left( \frac{\eta}{\cos \Lambda} - \bar{\lambda} \right) \partial \bar{\lambda} & y \leq \eta \\
- \sin \Lambda \int_0^{\frac{\eta}{\cos \Lambda}} \left( \frac{\eta}{\cos \Lambda} - \bar{\lambda} \right) \partial \bar{\lambda} & \eta \leq y 
\end{cases} \tag{2.25}
\]

where \( y \) is the point of the elastic response due to a unit load or torque (depending on the equation) at \( \eta \). The \( \bar{\lambda} \) variable represents positions along the swept elastic axis, and is defined as \( \bar{\lambda} = \lambda / \cos \Lambda \), where \( E \) is Young’s modulus, and \( G \) is the shear modulus of the material. \( \Lambda \) is the sweep angle of the elastic axis. The stiffness matrix is found by inverting the total flexibility matrix.

\[
\mathbf{K} = \mathbf{C}^{-1} = \begin{bmatrix} C^{zz} & C^{z\theta} \\ C^{\theta z} & C^{\theta \theta} \end{bmatrix}^{-1} \tag{2.26}
\]

Application of a pressure field to the plate model, and the collected equivalent forces and moments of the same pressure field to the beam has produced the similar displacements along the spanwise stations in both models. This justifies the use of the beam model in solving the divergence equation for the plate-model wing.
2.1.6 Wingbox Approximation

The original structural model used to simulate an aircraft wing is a rectangular tube or wing box which consists of just the skin of the wing. While a normal wing box will also contain additional members such as spars, ribs, and spar caps, this model was kept simple in order to facilitate conversion from wingbox to equivalent plate. The wingbox parameters are the height $h$, the chord $c$ and the skin thickness $t$ as shown in Fig. 2.2. The wingbox is converted to the plate by determining the plate thickness needed, at the midspan of each panel in the spanwise direction, to give the plate an equivalent 2nd moment of area ($I_{xx}$) in the bending direction. This is accomplished with the formula
shown in Eq. (2.27)

\[ t_{\text{plate}} = \left( \frac{(I)_{\text{i,box}}}{c_{i,\text{plate}}} \right)^\frac{1}{3} \]  

(2.27)

The second moment of area of the box \( I_{\text{box}} \) is calculated in the standard manner, and \( t_{\text{plate}} \) and \( c_{\text{plate}} \) denote the thickness and chord of the plate. The chord of the plate and the chord of the box are assumed to be equivalent, which greatly increases the simplicity of the calculation. The height of the box is based upon the airfoil used in the aircraft, and is defined in the model as a percentage of the total chord at that spanwise point on the wing. The subscript \( i \) indicates the spanwise panel \( i \) for which the thickness is being calculated.

This calculation alone is not sufficient. The model is completed by deriving a torsional stiffness modification factor (\( G_{\text{factor}} \)) at each spanwise station by comparing the given value from the model to the calculated stiffness, given the previously calculated thickness at each station. This is accomplished by Eq. (2.28) and gives the plate the same torsional stiffness \( GJ \) as the wingbox.

\[ G_{\text{factor}} = \frac{J_{\text{box}}}{12t_{\text{plate}}(c_{\text{plate}})^3} \]  

(2.28)

Since the box and plate are assumed to be made of the same material, the shear modulus \( G \) is not present in the equation.

Thus, the equivalent plate is formulated in the structural solver by inputting the plate thicknesses, the wing geometry (in terms of panel nodes, which define the chord and span), and the Young’s and shear moduli \( (E,G) \). The Young’s modulus is the same for the box and plate, but the shear modulus of the plate is multiplied by the \( G_{\text{factor}} \) during the input stage. This is accomplished by defining the plate as an orthotropic material and defining \( E \) the same in all three coordinate directions, and the modified \( G \) in all three coordinate directions to produce an isotropic plate that does not obey Eq. (2.29)

\[ G = \frac{E}{2(1+\mu)} \]  

(2.29)
The approximation of the wingbox to a flat plate also requires one further modification to the analysis which plays a factor in the optimization. The stress in the wing is calculated for the plate model. However the plate and the wing box have thicknesses, thus the distance of the loaded material from the neutral axis is different. The plate is thinner than the height of the box; therefore the same moment applied to the box, which has the same second moment of inertia, will produce a greater stress. Thus, the yield stress of the material, when applied to the optimization as a constraint boundary, is reduced by a geometric factor that is the ratio of the box thickness to the plate thickness. This ratio is determined at each panel along the span, then the largest ratio is used. This ratio is recalculated when there is a change greater than 5 mm in a panel thickness. In order to keep the divergence speed as accurate as possible while reducing the number of computational operations, the beam stiffness parameters denoted in §2.1.5, particularly the torsional rigidity, is recalculated every structural iteration. This is because $G_{\text{factor}}$ (the wingbox-to-plate equivalency factor) is only recalculated if a plate thickness changes by more than 5 mm. This comparatively less intensive beam calculation keeps the divergence speed accurate while the plate model moves changes from the initial wingbox approximation. When $G_{\text{factor}}$ is recalculated, the entire wing is remade in pyFEAP.
Chapter 3

Analysis and Optimization Procedures

This chapter describes the interaction of the multi-disciplinary analysis (MDA) that is carried out, and further describes the optimization problems that use the MDA.

3.1 Multi-Disciplinary Analysis

The general traits of the MDA carried out here are as follows: using a panel code for the aerodynamics, and a plate model composed of shell elements for structures, iterative multidisciplinary analysis is carried out until the coefficient of induced drag for the wing has become static to a tolerance of $1 \times 10^{-6}$.

3.1.1 Interaction of the Disciplines

In this multi-disciplinary analysis scheme, there are essentially three disciplines. They are Aerodynamics, Structures, and Aeroelasticity. While an aero-structural analysis is itself an aeroelastic analysis, it is differentiated in this respect in that the Aeroelastic module is used solely to compute the divergence speed of the wing being studied. Be-
fore describing the specific components, a diagram showing the interaction of the three disciplines to produce a converged state will best illustrate the overall aero-structural iteration scheme. The aerodynamic and structural disciplines send converged state data to the optimizer, however these arrows are not shown to preserve clarity. In this particular MDA, the aerodynamics is the first discipline analyzed in the sequence, as the initial data set for the system contains no initial forces, only airspeed, altitude and angle of attack. The wing is considered to be undeflected at the commencement of the MDA; this condition plus the initial data is sufficient to compute the boundary conditions (the local total angle of attack of the control point with respect to the freestream velocity) for the vortex lattice method at the control points. Thus the aerodynamics can be solved for the strength of the horseshoe vortices, $\Gamma$. The pressure force on the panels can be computed from these values. Evenly distributed panels are used to prevent leading edge suction. The pressure forces are passed to the structures discipline via an intermediary function that converts the pressure forces into forces on the nodes of the structure via a Gaussian interpolation [59] function. This is accomplished easily as the structural has an exactly equal planform to the aerodynamic mesh. The structural program FEAP [54] uses the nodal forces as input forces, and solves the structural plate model for the displacements of the nodes of the structure. The nodal displacements are passed to the aerodynamic discipline, and the new boundary conditions (which now include a summation of the freestream angle of attack and the relative twist of the panel due to the nodal perturbations) are calculated, at which point the aerodynamic system is solved again. This system avoids the computationally intensive process of recalculating the AIC matrix in the VLM after every iteration. If only the vertical perturbations are considered, the boundary conditions can be recalculated and the system is re-solved for the original planar mesh. The new forces are then reapplied to the non deflected structures model, and this give the new deformations. This algorithm is much faster than recalculating the AICs over again and leaving the structures mesh deformed, as (for n unconstrained nodes), only an $n \times 1$
Figure 3.1: Flow chart showing the interaction of the disciplines in the MDA
vector is recalculated, instead of an $n \times n$ matrix, and the coordinate perturbations of the previous iteration do not need to be stored by the structures discipline. The former process, however, is still equivalent to the latter one. Thus, the former process iterates until the change in the coefficient of drag for the entire wing, which is calculated during the aerodynamic solution, has converged to a predetermined value, in this case $1 \times 10^{-6}$. This process differs from a computational fluid dynamics (CFD) based MDA in that the aerodynamic and structural disciplines only iterate once to solve for their respective state variables. However, the MDA is still iterative, in that each discipline must be solved multiple times in order to reach a converged state. While this is not unusual behavior for the structural discipline, a CFD solver is usually iterated multiple times to converge the aerodynamic solution before the coupling variables are passed to the structural discipline. However, the VLM solves in one step, as it solves a linear equation to determine $\Gamma$.

The purpose of the *Aeroelasticity* discipline, as mentioned above, is solely to compute the divergence speed of the wing. The results of the discipline are used only at the optimizer level, does not affect the convergence of the MDA. It therefore does not refer any coupling variables back to the structures or aerodynamics.

Throughout the MDA, the Python language is used as a coding language. Python is a high level scripting language that contains a simple syntax, and supports object oriented programming. The ease of importing object modules with Python makes it ideal for multi-disciplinary analysis, as it is relatively easy to implement even complicated algorithms that take advantage of imported components. In this way, multi-disciplinary programs can be quickly put together only by programming the intermediate steps that take place between disciplines, such as unit conversion or matrix rearrangement. Although Python is a relatively new language and lacks some of the higher level mathematical solvers or additional modules [44], it does contain sufficient linear algebra modules needed to solve the posed divergence speed equation. Furthermore, new modules, that can deal with objects such as sparse matrices [18], which may be useful at a later stage,
are currently in development, or being released.

### 3.1.2 Aerodynamic Solver: KVLM

The tool used to solve the aerodynamic discipline is a vortex lattice method code called KVLM [14]. KVLM is an implementation of a vortex lattice method described in section 2.1.4 that models a thin lifting surface traveling at subsonic using quadrilateral panels that can be adjusted for twist, dihedral angle and sweep. This code was originally written by Kristian Dixon and later adapted by Alan Yu, who converted it to complex numbers and fixed a number of bugs. The program is written in the Python. In its current form, KVLM has been verified against Tornado [46], which is a multiblock Navier-Stokes computational fluid dynamics code. The comparison was carried out by Alan Yu, and his results can be seen below in Fig. 3.2

Like most panel codes, KVLM’s accuracy decreases if the total local angle of attack rises above 15 degrees, since flow separation may occur at that angle of attack that will not be modeled by the code. As KVLM is a subsonic code, it will also suffer accuracy problems if the airspeed of the lifting surface approaches Mach 1.0, the transonic speed regime. Like all panel codes, KVLM cannot effectively model bluff bodies and can suffer from leading edge suction; thus, the wing model being analyzed must be chosen such that its initial analysis, and subsequent iterative analysis in the MDA, are reasonable. This means a plausible initial shape to the airfoil, freestream angle of attack, airspeed, and panel mesh. Currently, KVLM’s mesh setup is handled automatically, and is setup to symmetrically distributed panels about the midchord, so avoiding leading edge suction simply means keeping the number of chordwise panels moderately low. This is both beneficial and detrimental: holding a aspect ratio of unity with a low chordwise panel count means that the analysis is kept strictly low fidelity, but this does speed up computational time.
Figure 3.2: Comparison of KVLM to Tornado
KVLM’s input parameters for mesh generation are leading edge sweep, span, taper ratio and aspect ratio. Any of these parameters can be set as input variables to KVLM, and it will generate an appropriate mesh. This facilitates the use of these variables later as design variables, in order that the planform shape, as well as the thickness of the wing, may be optimized. During iterations, the perturbations of the mesh nodes are also an input variable, as these are coupling variables from the finite element code. The nodal perturbations deform the mesh, and allow KVLM to calculate the lift and drag of the deflected wing shape.

KVLM’s contribution to the MDA is through the calculated singularity strengths, but not directly. KVLM contains internal subroutines to convert the singularity strengths $\Gamma$ to a pressure distribution, and subsequently a lift distribution with respect to the areas of each panel. This is KVLM’s primary output. The pressure and lift distribution calculated for each panel is assumed constant across each panel, thus the solution becomes more accurate as the number of panels increases, and their size decreases. During the MDA, KVLM also produces values for the total lift and the total drag generated by both wings, as well as the spanwise and total lift coefficient, and total induced drag coefficient. The latter is used as the convergence parameter for the multi-disciplinary analysis. Once the $C_D$ has been determined to be static up to a certain tolerance for two iterations, the MDA solution is considered to be converged.

### 3.1.3 Finite Element Solver: pyFEAP

The FEM program used to solve the structures discipline is the Finite Element Analysis Program (FEAP) [54] developed at the University of California at Berkeley. This program has been wrapped in Python to create pyFEAP [1] as part of the pyMDO multi-disciplinary framework [35]. FEAP itself a general purpose finite element analysis (FEA) program created in the Fortran programming language to serve as a tool for solving problems in the field of solid mechanics. Extensions exist that can apply FEAP to thermal
problems and problems dealing with the contact of solid bodies. pyFEAP is able to access all of the functionality of FEAP from a script written in the Python programming language. The Python language itself fulfills the role of a wrapper language well, and the extra computing overhead added to the program by the inclusion of Python is not appreciable. pyFEAP was created with aid of the program f2py [44], which is a program developed to automatically a Python interface script to a Fortran program. This Python script that would allow users to access the functions of the Fortran-coded program from within a Python-coded program.

pyFEAP is quite versatile in that it is able to model a variety of elements. One of

![Diagram of a FEAP-constructed wing](image)

Figure 3.3: Example of a FEAP-constructed wing
these elements is the shell model, which may be curved, or planar. Since a panel code is employed in the MDA, a plate element is used to model the wing, and this is handled in pyFEAP with a shell element. The plate model being studied is also cantilevered, so the boundary nodes at the root of the model are constrained. pyFEAP formulates the shell element via force resultants, which are calculated from the integration of the components of stress in the shell throughout the cross sectional thickness. Shell elements contain six free degrees of freedom; that is, translation along the Cartesian coordinates \((u_x, u_y, u_z)\) and rotations about the Cartesian axes \((\theta_x, \theta_y, \theta_z)\). Plates, on the other hand, only have \(u_z, \theta_x\) and \(\theta_y\) as free degrees of freedom. However, it has been confirmed that pyFEAP shell elements will mimic the behavior of plates if the thickness of the shell is small compared to the other two dimensions. This was verified by the author by comparing the solution by pyFEAP of a standard constrained plate problem located in the textbook “Energy and Finite Element Methods in Structural Mechanics” by I. Shames [51]. As expected, the pyFEAP solution approached the analytical answer as the grid mesh was refined.

In the MDA, FEAP takes as input the nodal forces generated by the aerodynamics discipline at each of its nodes. This includes the constrained nodes, though the forces there are ignored by pyFEAP. An intermediary subroutine takes the lift distributions from the aerodynamics and converts them into nodal forces via Gaussian interpolation [59] for a quadrilateral having four sampling points, thus solving the integral formulation of

\[
I = \int_{-1}^{1} f(x)dx = \sum_{j=1}^{n} H_i f(a_j)
\]  

for \(n = 4\). The lift distribution per area now converted to nodal forces, the structural equation is solved in pyFEAP and the displacements of the nodes are the result. In addition, pyFEAP also provides the stresses in each shell that composes the wing model. pyFEAP is also capable of providing the weight of the finite element model under analysis, given the proper data, however this is handled by a Python subroutine programmed by the author. The reason for this is that while a cantilevered plate is an approximation
of a wing it is a very bad approximation of wing weight, as most wings are hollow to contain fuel and control surface actuators. Thus, the wing weight is modeled not as a solid plate, but as a box whose inner dimensions are equivalent to the outer dimensions of the plate. This results in a weight model that has identical bending and torsional inertia, but approximately 40% of the weight of the plate. In the MDA, only the node coordinate perturbations in the vertical (z-axis) direction are returned to the aerodynamics discipline as coupling variables.

### 3.1.4 Aeroelastic Divergence

The divergence module is included in the multi-disciplinary analysis, but it is not strictly part of the iterative part of the MDA that produces the converged deformed shape of the wing. The aeroelastic divergence module is called only when the wing has fully converged. As such, the flag to enter the divergence code is evaluated after the aerodynamic solution for an MDA iteration is performed. There are three steps to producing the divergence speed.

The divergence speed is determined by reducing the plate wing model to a simple beam model. This operation is performed every structural iteration; that is, the torsional stiffness \( G \) of the beam is recalculated every time a divergence calculation is required. This is done so that the approximate divergence speed for the plate can be found for the two degrees of freedom of interest (heave and pitch) because the equivalent wing formulation for these two degrees of freedom is known and relatively easy to compute. The beam model is calculated as described in \( \text{\S} \ 2.1.5 \), after the inertial properties of the plate at the beam node points are calculated. The beam points are located at the node points of the plate, in the spanwise direction. Thus, the equivalent thickness used to calculate the inertia at each span station, chordwise point is taken to be the average of the adjacent panels at that particular chordwise location. The inertial properties of the plate is calculated for the span station located between each shell, midway from each
node point in the chordwise direction, so that the thickness used to generate the inertia calculation for that section of the plate is always the average. The inertias about the bending axis and the torsional inertia are summed at the beam point via the parallel axis theorem to get the total equivalent inertia.

Once the stiffness matrix for the beam is obtained, the GAF matrix is required. Since we are using a panel code, which utilizes linear theory to calculate lift, what we are searching for is the sensitivity of the lift on the wing to perturbations of the VLM mesh, as described in §2.1.3. In this model, the full plate and VLM solver is used to determine the sensitivities. For the heave degree of freedom, the entire line of nodes along the spanwise station of the wing (the beam node points are all located at the spanwise stations, i.e. the lines of nodes, of the plate) are perturbed upward in the $z$-direction (heave direction) and the new lift is calculated. A forward finite difference formula is used to calculate the sensitivity. In order to compute the sensitivities with respect to the pitch degree of freedom, the finite difference step is considered to be in radians, and the vertical displacements of the plate nodes along the span station necessary to effect that change in the local angle of twist about the beam location on the wing are computed, carried out, and the lift and moment is recalculated. Again, the sensitivity is a forward finite difference type.

The forces and moments on the beam used to calculate these sensitivities are determined simply by properly summing the nodal forces along the wing span station, and multiplying by the appropriate moment arm.

\[ F_{\text{beam}}^j = \sum_i F_{\text{node}}^i \]  
\[ F_{\text{beam}}^j = \sum_i e_i F_{\text{node}}^i \]

where $e$ is the distance between the beam and the node $i$ (from 1 to the number of chord-wise panels) in the $x$-direction (i.e. chord-wise), $F_{\text{beam}}^j$ and $M_{\text{beam}}^j$ are the summed forces and moments at beam station $j$ respectively. As stated earlier, the beam point is
located at the midpoint of a panel’s span dimension and the midchord.

All the sensitivities being obtained, all that remains is to solve the divergence eigenvalue problem as described earlier, and take the lowest positive real eigenvalue as the dynamic divergence pressure, and compute the divergence speed from it. The divergence speed is the only output of the divergence module.

3.1.5 Optimization Package: SNOPT

The SNOPT optimization package is implemented in Fortran. For the purposes of this MDA, it has been compiled with a Python interface, or a 'wrapper'. In this form it is known as pySNOPT. pySNOPT implements a sequential quadratic programming (SQP) algorithm. Gradients are generally provided by the user using a method of their choice, but pySNOPT can calculate them automatically using the finite differences method. The search directions obtained by pySNOPT result from the SQP programming; i.e. a sequence of quadratic subproblems are solved that provide the search vector [41]. SNOPT is able to solve non-linear problems and can even deal with discontinuities in the design space unless they are near the optimum.

This paper is not concerned with the method of SQP, so it will not be discussed in any detail. The reader is referred to bibliography entry [41] for more information.

3.2 Optimization Problem

3.2.1 Reference Case

As the goal of the project is to use the divergence to change the optimization design space, a reference case is useful in order to compare the effect of adding the constraint. This reference case is henceforth referred to as C1. The reference case consists of the typical aero-structural optimization for range. The subsonic Breguet range equation for civil jets is used as an objective function, with the thickness of the wing as design variables.
The yield stress serves as a constraint, as does the requirement that the lift generated is equal to the weight of the aircraft. The optimization problem can be written as:

\[
\text{minimize} \quad R \\
\text{w.r.t.} \quad t_{\text{root}}, t_2, \ldots t_{\text{tip}}, \alpha \\
\text{s.t.} \quad \sigma_n \leq \sigma_{\text{yield}} \\
L - W = 0
\]

where \( t_1, t_2 \) and etc. are the equivalent plate thicknesses. These are initially specified as the wingbox skin thicknesses; the equivalent plate thickness are calculated during the MDO setup. It is also possible to use the wingbox corner thicknesses; the MDA program performs a bilinear interpolation on the corner thicknesses in order to generate the thickness for the entire plate, and these values are reprogrammed into FEAP. For this project, the thickness is determined a set of points along the span corresponding the middle of panel at that span station.

The final design variable \( \alpha \) is the freestream angle of attack of the wing. \( \sigma \) indicates the von Mises stress in the shell elements that compose the plate model, and \( \sigma + \text{yield} \) is the yield stress. The von Mises stress are an appropriate constraint to use as they are scalar and always positive for a plate. \( R \) is the subsonic Breguet range equation for jet powered aircraft, which is:

\[
R = \left( \frac{V}{sfc \cdot D} \right) \ln \left[ \frac{W_i}{W_i - W_f} \right]
\]

where \( V \) is the cruise velocity, \( sfc \) is the specific fuel consumption, \( L \) is the total lift, \( D \) is the total drag, \( W_i \) is the initial weight, and \( W_f \) is the fuel weight. The second constraint \( L - W = 0 \) gives the optimizer the condition that the lift generated by the wings must be enough to support the mass of the aircraft, thereby maintaining level flight. This is accomplished by changing the freestream angle of attack. The VLM code used generates lift with a linear proportionality to the freestream angle of attack of the wing.

As with all optimization, bounds must be set on all of the variables. Due to the nature
of the MDA in use, the thickness constraints cannot simply be set to a value of zero. The
MDA has difficulty converging very thin models with high degrees of forward or rear-
ward swept. The thickness bound is thus set dependent on the type of aircraft wing being
modeled, and the speed that the aircraft is flying at, since the magnitude of the loads on
the wing are a factor in the non-convergence. The characteristic of this non-convergence
is that the MDA rotates between two displaced states. The first state is deformed, and
produces very high aerodynamic forces. The resulting structural displacement is great
enough to produce a negative lift situation. These forces produce displacements that
return the wing to a deformed state that produces very high positive lift. This situation
perseveres infinitely, as the MDA is unable to resolve the two states into a converged
solution. The limit on the freestream angle of attack is $\pm 20^\circ$.

The optimization problem requires the calculation of several gradients: that of the ob-
jective function with respect to the design variables, the constraints with respect to the
design variables as well as the aerodynamic forces with respect to nodal displacements
for the divergence speed calculation. These gradients are calculated using a Taylor series
finite differences (FD), such that the first derivative is calculated using Eq. (3.5).

$$
\left( \frac{u_{i+1} - u}{\Delta x} \right) = \left( \frac{\partial u}{\partial x} \right)_i, \tag{3.5}
$$

### 3.2.2 Divergence Constrained Optimization

The revised optimization problem includes the divergence speed as a constraint. This
case is referred to during this paper as C2. The optimization problem can then be written
as:

\[
\begin{align*}
\text{minimize} & \quad R \\
\text{w.r.t.} & \quad t_{\text{root}}, t_2, \ldots t_{\text{tip}}, \alpha \\
\text{s.t.} & \quad \sigma_n \leq \sigma_{\text{yield}} \\
& \quad L - W = 0 \\
& \quad v_{\text{div}} \geq v_{\text{target}}
\end{align*}
\]

The problem is now constrained so that the divergence speed must be above what the target divergence speed is set to. For a swept back wing, with the above constraints, there should be very little difference between the above optimization problem and the reference case, since the divergence speed should not be an active constraint for a swept back wing, particularly if the amount of sweep back is large. However, for a forward swept wing, a significant amount of difference is expected.

### 3.2.3 Divergence Constrained Optimization Without Stress Constraints

The revised optimization problem includes the divergence speed as a constraint, but omits the stress constraints. This case is not realistic; it is run only to more clearly observe the effect of the divergence constraint on the optimization. This case is referred to during this paper as C3. The optimization problem can then be written as:

\[
\begin{align*}
\text{minimize} & \quad R \\
\text{w.r.t.} & \quad t_{\text{root}}, t_2, \ldots t_{\text{tip}}, \alpha \\
\text{s.t.} & \quad L - W = 0 \\
& \quad v_{\text{div}} \geq v_{\text{target}}
\end{align*}
\]
Chapter 4

Results

4.1 Accuracy of Divergence Speed Calculation

The accuracy of the divergence speed calculation is gauged from a standard analytical test case taken from Ashley et al. [7]. The test case is a BAH civil jet wing, tapered and unswept about the 35% chord location. The reference data is given in terms of the bending rigidity ($EI$) and torsional rigidity ($GJ$), which is used to construct an approximation of the wingbox as a plate model based on the parameters used by the author in this study.

This equivalent plate is based on matching the bending inertia of the plate to the bending stiffness value provided in Ashley at points along the span of the wing. The plate thickness is determined by the formula:

$$t_{\text{plate}, i} = \left( \frac{12(EI)_{i, \text{model}}}{Ec_{i, \text{plate}}} \right)^{\frac{1}{3}}$$

(4.1)

where the subscript model denotes values taken from Ashley’s text, and $t_{\text{plate}}$ and $c_{\text{plate}}$ denote the thickness and chord of the plate. The subscript $i$ indicates the spanwise station $i$, with unique value of $EI_{\text{model}}, c_{\text{plate}}$ and $t_{\text{plate}}$. As before for the box, the plate and the published model are made equivalent by determining a $G_{\text{factor}}$ as before, though
with a slightly different formula, show in Eq. (4.2)

\[ G_{\text{factor}} = \frac{GJ_{\text{model}}}{GJ_{\text{plate}}} \]  \hspace{1cm} (4.2)

where

\[ GJ_{\text{box}} = 12t_{\text{plate}}c_{\text{plate}}^3 \]  \hspace{1cm} (4.3)

These values are inserted into the model input file generation program to form a plate with identical bending inertia and torsional stiffness as the BAH model. The BAH planform is shown in Fig. 4.1. Note the jet nacelle located at about the one-third span location. The nacelle has the effect of changing the inertia of the wing at and around its location. The coordinate locations are given in meters in the figure. Ashley’s text includes a range of divergence speeds based on the type of aerodynamic theory used.
### Table 4.1: Divergence speeds of BAH jet transport wing for different aerodynamic approaches

<table>
<thead>
<tr>
<th>Method of Analysis</th>
<th>Divergence Speed $U_D$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip theory (no finite span correction)</td>
<td></td>
</tr>
<tr>
<td>$\frac{dC_L}{d\alpha} = a_0 = 5.5$</td>
<td>462.3</td>
</tr>
<tr>
<td>Strip theory:</td>
<td></td>
</tr>
<tr>
<td>Glauert’s substitution finite span correction</td>
<td></td>
</tr>
<tr>
<td>$\frac{dC_L}{d\alpha} = a_0 \frac{AR}{AR+2} = 4.1497$</td>
<td>532.2</td>
</tr>
<tr>
<td>Strip theory:</td>
<td></td>
</tr>
<tr>
<td>Diederich and Budiansky finite span correction</td>
<td></td>
</tr>
<tr>
<td>$\frac{dC_L}{d\alpha} = a_0 \frac{AR}{AR+4} = 3.325$</td>
<td>593.8</td>
</tr>
<tr>
<td>Lifting-line theory:</td>
<td></td>
</tr>
<tr>
<td>Symmetric</td>
<td>555.7</td>
</tr>
<tr>
<td>Lifting-line theory:</td>
<td></td>
</tr>
<tr>
<td>Antisymmetric</td>
<td>582.2</td>
</tr>
</tbody>
</table>

For a case where the structural model is inconclusively defined, as the text produces the appropriate flexibility matrices where needed. Using the iterative MDA architecture, with an airspeed of 100.0 m/s at a sea level dynamic pressure of 6125.00 Pa, a number of results for divergence speed were produced, which are shown in Table 4.2. At higher fidelity, a converged divergence speed should be reached, however this resolution was not found. It is likely that at an MDA with more than 1000 panels will yield a divergence speed converged to 0.1 m/s. Given the available data, it does not appear that satisfactory convergence is reached, even at 625 panels. Based on the trend shown in Fig. 4.2, there will be a convergence speed, and it is likely above or within the range described in Table 4.1.

The second column in Table 4.2 shows the time to run the MDA iteration and sub-
Table 4.2: Divergence speeds of BAH jet transport wing for varying mesh sizes calculated using the MDA

<table>
<thead>
<tr>
<th>Number of panels, sw × cw</th>
<th>$U_D$ (m/s)</th>
<th>Run Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 × 5</td>
<td>853.7</td>
<td>39.2</td>
</tr>
<tr>
<td>8 × 8</td>
<td>728.6</td>
<td>143.5</td>
</tr>
<tr>
<td>10 × 10</td>
<td>708.4</td>
<td>242.2</td>
</tr>
<tr>
<td>15 × 15</td>
<td>672.5</td>
<td>788.00</td>
</tr>
<tr>
<td>20 × 20</td>
<td>659.9</td>
<td>1899.7</td>
</tr>
<tr>
<td>25 × 25</td>
<td>643.9</td>
<td>3900.3</td>
</tr>
<tr>
<td>30 × 30</td>
<td>642.1</td>
<td>7432.7</td>
</tr>
</tbody>
</table>

sequent divergence speed calculation, collectively referred to as the divergence analysis. It should be noted that while the divergence speed calculation takes less time than the multi-disciplinary analysis (involving multiple runs of the VLM panel code and FEM analysis), it is still non-trivial, as the generalized aerodynamic force matrix $A_0$ requires the calculation of the sensitivities of the aerodynamic forces to perturbation inputs on each node. As one would expect, this becomes time consuming for a wing with 900 panels. In this column, it is clear that the time required for the divergence analysis does not scale linearly with the coarseness of the aerodynamic and structural mesh used; thus it is preferable to use a mesh that will provide reasonable accuracy without a prohibitive cost.

This convergence study also indicates that the divergence model used tends to overestimate the divergence speed, since the result of the analysis decreases towards an answer as the mesh becomes finer. This trend has been verified independently for meshes up to 400 panels on other wing models; thus there is good confidence that the divergence result converges at a speed near the published range.
4.2 Optimization Results

As stated in section § 3.2, the model used in the optimization problem is the Hamburger Flugzeugbau HFB 320 HANSA, or HFB320. This aircraft was designed as a forward swept aircraft, thus the baseline configuration is swept forward by $15^\circ$ at the quarter-chord. A model was constructed for four distinct configurations, summarized in Table 4.3. These configurations were constructed for the purpose of testing and comparing the results to the baseline configuration. The goal of the project is to optimize only the baseline configuration. The optimization are as follows:

**von Mises Stress:** The yield stress of Aluminum is $250 \times 10^6$ Pa, which is the upper limit for this constraint. The bending von Mises stress constraint has a lower limit of 0 Pa. This is an inequality constraint in the optimization problem.

**Divergence Speed:** The divergence speed has an upper limit of 310 m/s, or 1,116 km/h, the cruising speed of the HANSA. This is an inequality constraint in the
optimization problem.

**Lift-Weight:** The lift generated by the HANSA wings is constrained to be equal to the weight of the wings, the standard cargo weight (7 passengers) and the full fuel load of (3,310 kg). This is an equality constraint in the optimization problem that is satisfied through modification of the angle of attack $\alpha$.

**Plate Thickness:** The thickness lower limit is set to the minimum thickness of 2024-T3 aluminium available, which is $1.984 \times 10^{-4}$ m, or 1/128 in. This is not an active constraint, but is set to prevent the optimizer from straying outside of a realistic design space. This limit was set after observing the behavior of early iterations of the aeroelastic module, where the optimizer reduced the thickness so far that the displacements of the wing were orders of magnitude greater than the semi-span of the wing. Since a 6.7 m wing with a 50 m tip displacement is unrealistic, the minimum thickness was constrained. An attempt to decrease the panel thickness below this value results in the optimization proceeding with the minimum thickness value.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\Lambda_{c/2}$ ($^\circ$)</th>
<th>$\Lambda_{c/4}$ ($^\circ$)</th>
<th>AR</th>
<th>$\lambda$</th>
<th>$b_{ss}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>−19.4</td>
<td>−15.0</td>
<td>6.0</td>
<td>0.333</td>
<td>6.72</td>
</tr>
<tr>
<td>Unswept</td>
<td>0.0</td>
<td>4.76</td>
<td>6.0</td>
<td>0.333</td>
<td>6.72</td>
</tr>
<tr>
<td>Rearward Swept</td>
<td>5.0</td>
<td>9.7</td>
<td>6.0</td>
<td>0.333</td>
<td>6.72</td>
</tr>
<tr>
<td>Forward Swept</td>
<td>−30.0</td>
<td>−26.3</td>
<td>6.0</td>
<td>0.333</td>
<td>6.72</td>
</tr>
</tbody>
</table>

Table 4.3: HANSA configurations for the optimization

The dihedral and tip twist parameters have not been set as design variables during this investigation in order to keep the design variables. Within each test case, the optimization problem was run with four different starting points that varied the initial distribution
of the skin thicknesses of the wing, and the initial angle of attack. The initial thickness distribution between the root and the tip varied linearly across each spanwise station. The case number serves to identify which case was run on which configuration, and with which set of constraints, as every configuration and constraint arrangement was not run with each test case. Repeated tests of these cases would not have produced any informative results. All test cases were conducted at an airspeed of 229.2 m/s (875 km/h), the max cruising speed of the HFB 320 at an altitude of 7,600 m, resulting in a dynamic pressure of 14,445.5 Pa. The minimum value of the thickness design variable was set

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Root Thickness ($\times 10^{-3}$ m)</th>
<th>Tip Thickness ($\times 10^{-3}$ m)</th>
<th>FS $\alpha$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>8.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>9.0</td>
<td>9.0</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>8.0</td>
<td>10.0</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>9.0</td>
<td>9.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Table 4.4: HANSA configurations for the optimization

to 1/128 in (1.984 × 10^{-4} m), and the angle of attack was not to exceed ±20.0°. The divergence speed criteria was set to $v_{div} \geq$ Mach 1. The test cases were run with mesh sizes of 8 spanwise panels by 4 chordwise panels. It was decided that this mesh would produce usable results within a reasonable time\(^1\). A safety factor of 2 and a geometric factor of 5 was applied to the stress constraint, which was set to a yield stress of 250 × 10^6 Pa for 6061-T6 aluminum alloy. Each test case was run in three different constraint scenarios: stress and lift constraints only (C1), stress, divergence speed and lift constraints (i.e. all constraints) active (C2), and divergence and lift constraints only (C3).

\(^1\)Later studies with finer meshes (12 × 6) panels would take 5 to 10 times as long, if they finished at all.
4.2.1 Standard Configuration

The standard HFB320 configuration uses the baseline parameters described in Table 4.3. This configuration was tested in all three constraint scenarios in order to gain an understanding of the behavior of the baseline configuration. The scenarios involving stress and lift constraints only, and all constraints, resulted in similar optimization points, with an agreement of one decimal place in the range function. Some of these results are detailed in Table 4.5 below. All values are optimum values unless otherwise stated. Where $\alpha_{fs}$ is the optimum angle of attack, $v_{div}$ is the divergence speed, Time is the duration in hours required to complete the optimization and Major It. and Minor It. are the number of major iteration steps and line searches respectively. The case number CX-Y indicates the constraint scenario X and starting point Y, as described in Table 4.4.

<table>
<thead>
<tr>
<th>Case</th>
<th>Range (km)</th>
<th>L/D</th>
<th>Weight (kg)</th>
<th>$v_{div}$ (m/s)</th>
<th>$\alpha_{fs}$ (°)</th>
<th>Major It.</th>
<th>Minor It.</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 − 1</td>
<td>1,168.66</td>
<td>2.89</td>
<td>2,003.12</td>
<td>509.49</td>
<td>2.11</td>
<td>98</td>
<td>240</td>
<td>24.82</td>
</tr>
<tr>
<td>C1 − 2</td>
<td>1,168.62</td>
<td>2.89</td>
<td>2,004.33</td>
<td>508.09</td>
<td>2.11</td>
<td>97</td>
<td>182</td>
<td>34.28</td>
</tr>
<tr>
<td>C2 − 2</td>
<td>1,168.62</td>
<td>2.89</td>
<td>2,004.33</td>
<td>508.03</td>
<td>2.11</td>
<td>43</td>
<td>123</td>
<td>10.87</td>
</tr>
<tr>
<td>C2 − 3</td>
<td>1,168.58</td>
<td>2.89</td>
<td>2,005.86</td>
<td>506.72</td>
<td>2.11</td>
<td>42</td>
<td>76</td>
<td>9.21</td>
</tr>
<tr>
<td>C2 − 4</td>
<td>1,168.66</td>
<td>2.89</td>
<td>2,003.12</td>
<td>509.43</td>
<td>2.11</td>
<td>56</td>
<td>99</td>
<td>13.12</td>
</tr>
<tr>
<td>C3 − 1</td>
<td>1,183.97</td>
<td>2.72</td>
<td>1,470.88</td>
<td>355.41</td>
<td>1.22</td>
<td>101</td>
<td>279</td>
<td>29.42</td>
</tr>
</tbody>
</table>

Table 4.5: Comparison between various standard configuration cases.

All of the tests using either C1 or C2 constraints went to similar optima. This is due to the active constraints in this test cases. During C1 and C2, the active constraints at the optimum were the yield stress constraints and the lift constraint. During C1, the divergence constraint was not enforced, and during C2, it did not become at any point during the optimization. Fig. 4.3 shows the stress values across the panels at the optimum of C1-1 and C2-2. The C3 constraint case, which does not include stress constraints, came to a different optimum than the other constraint scenarios. In the C3 cases, the minimum
skin thickness bounds were reached by all the thickness design variables, and the divergence constraint was still not active, which was evidenced by the Lagrange multipliers. Thus, for the standard configuration, the HFB 320 does not experience divergence below Mach 1. However, the stress values for the C3-1 case, shown in Fig. 4.2.1 show that for the minimum thickness, the stress constraints will be violated. The C3-1 case has a larger range than the other cases. This is not unexpected as the equivalent plate thickness are at minimum; in addition the number of constraints has been reduced from 10 in the C2 case to 2 in the C3 case. At this optimum, 8 of the 32 panels are at maximum stress, and this stays true for the other cases that came to the same optimum. The two particular cases shown above have almost identical stress distributions. The final thickness distributions for the cases C1-1 to C2-4 are similarly identical. Fig. 4.2.1 shows that at the minimum thickness, the stress constraints are highly violated. This evidence indicates that only using divergence speed as a constraint will produce an infeasible optimum.

The values of the thickness design variables show a similar trend. The C1-1, C2-2 and C3-1 optimum skin thicknesses are shown in Table 4.6. “Station” represents the spanwise location of the panel, where 1 is the panel next to the root, and (in this example) up to 8 at the tip.

From the table, the general trend for the C1 and C2 constraints is a decreasing skin

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 - 1, t(m) x 10^{-4}</td>
<td>9.850</td>
<td>8.058</td>
<td>5.987</td>
<td>4.544</td>
<td>2.455</td>
<td>3.953</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>C2 - 2, t(m) x 10^{-4}</td>
<td>9.941</td>
<td>8.120</td>
<td>6.002</td>
<td>4.528</td>
<td>2.473</td>
<td>3.776</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>C3 - 1, t(m) x 10^{-4}</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>6.295</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
</tbody>
</table>

Table 4.6: Skin thickness design variable values at optimum, standard configuration.

thickn ess from root to tip, with a small increase near the tip edge. Unfortunately, the C3 constraint case provides no information with regards to thickness, as all of the design
Figure 4.3: Stress distributions at optima for C1-1, C2-2 and C3-1 cases
variables save one are at the minimum. It is likely that the lone variable not at the
minimum is the result of poor gradient information near the thickness boundaries. This
hypothesis is reinforced by the C3-2 through C3-4 cases (not shown), which were unable
to achieve converged solutions, even though the design variables were at the thickness
boundary. The optimizer appears to be 'stuck', making small changes (steps on the or-
der of $10^{-3}$ or less) to the 5th thickness variable, and adjusting the angle of attack in
an attempt to satisfy the lift constraint. However, the optimizer is unable to satisfy the
optimality tolerances, despite being very close to the same design point as the C3-1 case.
The C3-2 objective function history demonstrates the poor gradient information near
the minimum thickness boundary. As can be seen in Fig. 4.4, the optimizer has reached
the same approximate range as case C3-1 by iteration 200, and is stuck there for the
following 300 iterations without satisfying optimality conditions.

In this case, the C3 constraint scenarios provide almost no information. However, it
should be remembered that the divergence constraint result provided by the analysis is
larger in magnitude than the result would be for a finer mesh. Thus, the actual divergence speed at the minimum thickness bounds will be less than 355.41 m/s. In this manner, it is possible to write off the entire model as being unrealistic, since using a realistic divergence speed constraint provides no useful optimization results. Increasing the divergence speed constraint is not a valid solution to this problem, as the value chosen would be only a 'ballpark' number and fairly arbitrary. A finer mesh can be run to determine if the divergence constraint actually becomes active in the C3 case for the stock configuration.

<table>
<thead>
<tr>
<th>Case</th>
<th>Range (km)</th>
<th>L/D</th>
<th>Weight (kg)</th>
<th>$v_{div}$ (m/s)</th>
<th>$\alpha_{fs}$ ($^\circ$)</th>
<th># Major</th>
<th># Minor</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C2 - 1$</td>
<td>1,166.30</td>
<td>2.91</td>
<td>2,062.69</td>
<td>535.63</td>
<td>2.22</td>
<td>44</td>
<td>114</td>
<td>41.52</td>
</tr>
<tr>
<td>$C2 - 3$</td>
<td>1,166.25</td>
<td>2.91</td>
<td>2,061.47</td>
<td>525.75</td>
<td>2.17</td>
<td>46</td>
<td>181</td>
<td>43.97</td>
</tr>
<tr>
<td>$C3 - 1$</td>
<td>1,183.6901</td>
<td>2.73</td>
<td>1,481.35</td>
<td>359.76</td>
<td>2.24</td>
<td>145</td>
<td>310</td>
<td>227.36</td>
</tr>
</tbody>
</table>

Table 4.7: Comparison between various standard configuration cases using a 12x6 mesh.

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C2 - 3, t(m) \times 10^{-4}$</td>
<td>11.760</td>
<td>10.635</td>
<td>8.540</td>
<td>6.848</td>
<td>5.379</td>
<td>4.309</td>
<td>2.405</td>
<td>5.100</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>$C3 - 1, t(m) \times 10^{-4}$</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
</tbody>
</table>

Table 4.8: Skin thickness design variable values at optimum, standard configuration, 12x6 mesh.

This course of action was carried out, with the results shown in Tables 4.7 and 4.8, along with high fidelity cases for the C2-1 and C2-3 cases. The finer mesh used was 12 panels spanwise by 6 chordwise. A finer mesh was not deemed to be a useful result with the stress-only (C1) constraints, since it would generate the same answer as in the previous tests, only with more resolution along the span. The stress distributions are shown in Fig. 4.5.

We see that going to a higher fidelity gives a more resolution to the thickness of each
Chapter 4. Results

(a) C1-1 stress distribution at optimum for 12x6 mesh
(b) C2-3 stress distribution at optimum for 12x6 mesh
(c) C3-1 stress distribution at optimum for 12x6 mesh

Figure 4.5: Stress distributions at optima for cases C1-1, C2-3 and C3-1 cases for 12x6 mesh
span station, but the thickness shape is unchanged. The exception seems to be the 12x6 C1-1 case, but pySNOPT indicates that in this case an optimum solution was found, but without the accuracy requested in the convergence tolerance. There then could be a small error in the thickness distribution. Given that the other two were unchanged, it is more likely that the higher fidelity optimization came to a more resolved thickness distribution. As before, the stress constraints are violated in Case 3.

One more factor that needs to be considered when reviewing any results produced with this particular model and architecture is that the torsional stiffness modification factor, referred to previously as the \( G_{\text{factor}} \), is set at the beginning of the optimization and is not changed after each design iteration unless there is a large change. Therefore if the initial design point is far from the optimum point, the divergence speed calculation will be inaccurate. Recall from § 2.1.6 that the torsional rigidity of the beam is recalculated every structural iteration to keep the divergence speed calculation as accurate as possible. However, these small inaccuracies could accumulate and may be contributing to the problems in the C3 tests near the minimum skin thickness bounds. This was observed if the starting point was far from the minimum plate thickness. Setting the \( G_{\text{factor}} \) at the beginning of the optimization, when the wing-box is taller, means that the wing will be stiffer than it should be if the thickness design variables are decreased in magnitude, resulting in a higher divergence speed than would normally be calculated. Generally it has been found that a change of up to 25 m/s can occur, depending on how far from the optimum the initial point was set at. The difference can be verified by running a stand-alone divergence analysis using the final design variables produced by the optimizer. In the case of C3-1, for an 8x4 mesh, the MDA produced a divergence speed of 335.52 m/s compared to the 355.41 m/s, a difference of 19.8936. This factor does not have enough of an effect in the 8x4 case to make a difference to the constraint boundary.

Fig. 4.6 shows the final displaced states of the C2-1 case and the C3-1 case. As expected, the C3 case has been deflected further vertically, and has a greater local positive angle
Figure 4.6: Comparison of displaced optimized wings, C2-1 and C3-1 cases

of attack than the thicker C2 wing. As the higher fidelity mesh provided no further insight into the divergence constraint, three more test cases were run at with a 32 panel mesh, but with a doubled divergence speed, to gauge the effect. The results are shown in Tables 4.9 and 4.10. As expected, there is no change in the C1-1 case, but the C2 and C3 case come to new, constrained optimum. The thickness distribution is quite changed from the stress-constrained optimum, as the C2 case contains 2 'bumps' from root to tip, and the C3 case a very large bump. The divergence constraint is clearly reached, so the stress constraint must be examined as well.
Figure 4.7: Stress distributions at optima for cases C1-1, C2-2 and C3-1 cases with divergence constraint speed at 620 m/s
### Table 4.9: Comparison between three standard configuration cases using a 8x4 mesh and a doubled divergence speed constraint of 620 m/s.

<table>
<thead>
<tr>
<th>Case</th>
<th>Range (km)</th>
<th>L/D</th>
<th>Weight (kg)</th>
<th>( v_{div} ) (m/s)</th>
<th>( \alpha_{fs}(^\circ) )</th>
<th># Major</th>
<th># Minor</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 - 1</td>
<td>1,168.66</td>
<td>2.89</td>
<td>2003.12</td>
<td>509.49</td>
<td>2.11</td>
<td>98</td>
<td>240</td>
<td>24.74</td>
</tr>
<tr>
<td>C2 - 2</td>
<td>1,164.97</td>
<td>2.93</td>
<td>2,134.10</td>
<td>620.00</td>
<td>2.33</td>
<td>55</td>
<td>175</td>
<td>15.01</td>
</tr>
<tr>
<td>C3 - 1</td>
<td>1,165.80</td>
<td>2.92</td>
<td>2,134.08</td>
<td>620.00</td>
<td>2.31</td>
<td>96</td>
<td>292</td>
<td>22.41</td>
</tr>
</tbody>
</table>

### Table 4.10: Skin thickness design variable values at optimum, standard configuration, 8x4 mesh with doubled divergence speed constraint.

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 - 1, ( t(m) \times 10^{-4} )</td>
<td>9.850</td>
<td>8.058</td>
<td>5.987</td>
<td>4.544</td>
<td>2.455</td>
<td>3.953</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>C2 - 2, ( t(m) \times 10^{-4} )</td>
<td>9.001</td>
<td>6.832</td>
<td>4.912</td>
<td>10.051</td>
<td>5.879</td>
<td>8.733</td>
<td>5.090</td>
<td>1.984</td>
</tr>
<tr>
<td>C3 - 1, ( t(m) \times 10^{-4} )</td>
<td>5.437</td>
<td>6.590</td>
<td>7.525</td>
<td>8.094</td>
<td>9.235</td>
<td>7.628</td>
<td>7.154</td>
<td>1.984</td>
</tr>
</tbody>
</table>

### 4.2.2 Experimental Configuration

The effect of the divergence constraint can also be tested by making the standard HFB 320 configuration more susceptible to divergence, which is accomplished by further sweeping the wing forward. Given the results in § 4.2.1, the experimental configuration was only run all three constraint cases, but only using starting points 1, 2 and 3. Scenario 4 was dropped since it did not provide any additional information; the four starting points came to the same optimum design point in each constraint scenario. These optimum points were reached without any indication of encountering local maxima. One result from each constraint case is shown in the tables and figures below.

In order to ensure that this configuration will provide additional insight into the use of the divergence constraint to achieve an improved optimum design, an isolated MDA of the experimental configuration was carried out with the minimum skin thicknesses. This produced a divergence speed of 268.53 m/s, so a divergence constrained optimum was
expected. The results of the optimization are shown in Tables 4.11 and 4.12.

The stress distributions are shown in Fig. 4.8, and a comparison of the displaced opti-

<table>
<thead>
<tr>
<th>Case</th>
<th>Range (km)</th>
<th>L/D</th>
<th>Weight (kg)</th>
<th>$v_{div}$ (m/s)</th>
<th>$\alpha_{fs}$ (°)</th>
<th># Major</th>
<th># Minor</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC1 – 2</td>
<td>1,166.14</td>
<td>2.91</td>
<td>2,233.54</td>
<td>396.48</td>
<td>2.29</td>
<td>54</td>
<td>348</td>
<td>98311.99</td>
</tr>
<tr>
<td>FC2 – 2</td>
<td>1,161.14</td>
<td>2.96</td>
<td>2,233.74</td>
<td>464.69</td>
<td>2.31</td>
<td>43</td>
<td>130</td>
<td>38383.95</td>
</tr>
<tr>
<td>FC3 – 2</td>
<td>1,179.01</td>
<td>2.76</td>
<td>1,609.17</td>
<td>310.00</td>
<td>1.10</td>
<td>106</td>
<td>288</td>
<td>174159.84</td>
</tr>
</tbody>
</table>

Table 4.11: Comparison between various experimental configuration cases using a 8x4 mesh.

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC1 – 2, $t(m) \times 10^{-4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FC2 – 2, $t(m) \times 10^{-4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FC3 – 2, $t(m) \times 10^{-4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.12: Skin thickness design variable values at optimum, experimental configuration, 8x4 mesh.

This configuration produces quite different results than the standard configuration. The
C1 – 2 and C2 – 2 cases are not identical this time, although in both cases the diver-
gence speed constraint is not active. It is quite possible that from these starting points,
the C1 and C2 cases have found local maxima. Thus we see that the inclusion of the
divergence constraint can have an effect on the design space searched. However, it is not
always a positive one, as the resulting optimum is heavier and less stressed than the C1
case, and the divergence speed is high enough such that divergence should not be a factor.
Figure 4.8: Stress distributions at optima for cases C1-2, C2-2 and C3-2 cases for 8x4 mesh, experimental configuration
Figure 4.9: Comparison of displaced optimized wings for experimental configuration
4.2.3 Unswept Configuration

The unswept configuration does not produce any data that is beneficial towards proving the usefulness of divergence speed as a constraint in optimization. Since the divergence speed increases as the amount of sweepback increases, the divergence constraint is not a factor for the unswept configuration. The thickness distribution is always defined by the stress constraint and the lift constraint, and in the C3 case, the thicknesses go to the minimum value. The results of the unswept configurations are shown in Table 4.13 to 4.14. The C1 and C2 constraint scenarios produce identical answers, as expected.

It is interesting to note at this point that during the last three configurations, the C3 constraint scenario has always taken the most major and minor iterations in order to converge, and in some cases, has not converged at all, or has not been able to completely satisfy the optimality tolerances. This indicates that using only the divergence and lift constraints does not provide pySNOPT with good gradient information. Comparison of the value of the range objective function of cases \( C2 - 2 \) and \( C3 - 2 \) in Fig. 4.10 indicates the only difference between the two cases occurs early in the optimization. The \( C3 - 2 \) case takes tiny steps between iterations 5 and 30, before moving towards the minimum thickness. This seems to indicate a lack of search direction information being supplied to the optimizer, as opposed to the \( C2 - 2 \) case where there was additional information provided by the additional constraints.

<table>
<thead>
<tr>
<th>Case</th>
<th>Range (km)</th>
<th>L/D</th>
<th>Weight (kg)</th>
<th>( v_{\text{div}} ) (m/s)</th>
<th>( \alpha_{f_s} ) (°)</th>
<th># Major</th>
<th># Minor</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( UC1 - 2 )</td>
<td>1,179.7049</td>
<td>2.79552</td>
<td>8,711</td>
<td>2032.87335</td>
<td>2.14</td>
<td>29</td>
<td>124</td>
<td>5.33</td>
</tr>
<tr>
<td>( UC2 - 2 )</td>
<td>1,179.7049</td>
<td>2.79552</td>
<td>8,711</td>
<td>2032.87335</td>
<td>2.14</td>
<td>44</td>
<td>109</td>
<td>10.09</td>
</tr>
<tr>
<td>( UC3 - 2 )</td>
<td>1,188.7901</td>
<td>2.70918</td>
<td>8,430</td>
<td>1592.32543</td>
<td>1.85</td>
<td>90</td>
<td>241</td>
<td>21.60</td>
</tr>
</tbody>
</table>

Table 4.13: Comparison between various unswept configuration cases using a 8x4 mesh.
Chapter 4. Results

Figure 4.10: Range vs. number of iterations for the C2-2 and C3-2 unswept configurations

(a) Value of range function at each iteration, unswept C2-2

(b) Value of range function at each iteration, unswept C3-2
Chapter 4. Results

(a) C1-2 stress distribution at optimum, unswept configuration

(b) C2-2 stress distribution at optimum, unswept configuration

(c) C3-2 stress distribution at optimum, unswept configuration

Figure 4.11: Stress distributions at optima for cases C1-2, C2-2 and C3-2 cases for 8x4 mesh, unswept configuration
### Table 4.14: Skin thickness design variable values at optimum, unswept configuration, 8x4 mesh.

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$UC1 - 2, t(m) \times 10^{-4}$</td>
<td>4.226</td>
<td>4.679</td>
<td>3.972</td>
<td>3.129</td>
<td>2.141</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>$UC2 - 2, t(m) \times 10^{-4}$</td>
<td>4.226</td>
<td>4.679</td>
<td>3.972</td>
<td>3.129</td>
<td>2.141</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>$UC3 - 2, t(m) \times 10^{-4}$</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
</tbody>
</table>

Figure 4.12: Comparison of displaced optimized wings for unswept configuration.
4.2.4 Swept-Back Configuration

<table>
<thead>
<tr>
<th>Case</th>
<th>Range (km)</th>
<th>L/D</th>
<th>Weight (kg)</th>
<th>$v_{div}$ (m/s)</th>
<th>$\alpha_{fs}(^\circ)$</th>
<th># Major</th>
<th># Minor</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RC1 - 2$</td>
<td>1,182.73</td>
<td>2.77382</td>
<td>8,635</td>
<td>4401272.88</td>
<td>2.34</td>
<td>27</td>
<td>107</td>
<td>2.24</td>
</tr>
<tr>
<td>$RC2 - 2$</td>
<td>1,182.73</td>
<td>2.77382</td>
<td>8,635</td>
<td>4401272.88</td>
<td>2.34</td>
<td>94</td>
<td>276</td>
<td>22.61</td>
</tr>
<tr>
<td>$RC3 - 2$</td>
<td>1,189.66</td>
<td>2.71116</td>
<td>8,430</td>
<td>2676432.87</td>
<td>2.23</td>
<td>47</td>
<td>82</td>
<td>10.08</td>
</tr>
</tbody>
</table>

Table 4.15: Comparison between various swept-back configuration cases using a 8x4 mesh.

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RC1 - 2, t(m) \times 10^{-4}$</td>
<td>3.753</td>
<td>3.510</td>
<td>3.440</td>
<td>2.784</td>
<td>1.989</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>$RC2 - 2, t(m) \times 10^{-4}$</td>
<td>3.753</td>
<td>3.510</td>
<td>3.440</td>
<td>2.784</td>
<td>1.989</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
<tr>
<td>$RC3 - 2, t(m) \times 10^{-4}$</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
<td>1.984</td>
</tr>
</tbody>
</table>

Table 4.16: Skin thickness design variable values at optimum, swept-back configuration, 8x4 mesh.

The results of the swept-back configuration are not helpful in determining the use of the divergence constraint. As expected, and similar to the unswept wing, the divergence constraint was not active, and the stress constraint defined the optimum thickness distribution. The C3 scenario went quickly to the minimum thickness bounds. In an attempt to make the swept-back case useful, only a small amount of sweepback was applied, thus the deflected shape in Fig. 4.14 has a positive deflected angle of attack, though it is near zero degrees. For greater sweep, a negative angle of attack due to the deflection of the wing would be apparent. In general, it was found that this configuration is not useful with respect to proving the efficacy of the divergence speed constraint. However, it does provide a small insight into current aircraft design; wings are lighter, less stressed and have greater range than forward swept wings. The value of forward swept wings lies in
improved maneuverability and location of the initial stall point on the wing. In case of the HFB 320, the placement of the wing root further aft allowed construction of a passenger cabin without a wing spar running through it.
(a) C1-2 stress distribution at optimum, swept-back configuration

(b) C2-2 stress distribution at optimum, swept-back configuration

(c) C3-2 stress distribution at optimum, swept-back configuration

Figure 4.13: Stress distributions at optima for cases C1-2, C2-2 and C3-2 cases for 8x4 mesh, swept-back configuration
Figure 4.14: Comparison of displaced optimized wings for swept-back configuration
Chapter 5

Conclusions and Further Directions

5.1 Conclusions

5.1.1 Divergence Speed Calculation

The accuracy of the divergence calculation is one of the key elements needed in order to realistically solve the problem stated in § 1.2. The divergence speed acts as the unique constraint that enables the proposed constrained optimization. As there are a number of published analytical cases, this appeared to be the most reliable way to verify the accuracy of the divergence calculation routine. This is a minor but important step; though the equation to be solved is fairly straightforward (Eq. (2.13)), the variables within are not trivial to calculate, as shown in § 2.1. There can only be confidence in the results of the design optimization if the calculation of the constraints and the constraint gradients are reliable. The elements of the variables that make up the calculation were compared for the BAH wing drawn from Ashley [7]. Ashley has given a number of analytical divergence speeds for this wing, shown in § 4.1. As we can see, the various calculation methods do not agree. Each of these approaches, as well as the one used in the MDA analysis presented in earlier chapters, assume incompressible aerodynamics in order to keep the modeling approach simple to implement. Regarding the code written for this thesis,
during the creation of the experiments, it was thought that the use of the VLM code and its underlying assumptions, such as incompressible aerodynamics, would reduce the amount of time and resources used to complete the multi-disciplinary optimization. This is a reasonable assumption, as the purpose of the project was to show that divergence speed, used as a constraint in optimization, will produce a wing with longer range that is not prone to divergence.

This use of a fixed point iteration implementation, however, did not work as planned by the author. Performing the MDA is still a computationally intensive process as it requires a large number of sensitivity calculations, thus taking a large amount of time. In order to show that the divergence calculation is at least a reasonable approximation, simulations were run with progressively larger meshes, as demonstrated in Table 4.2. The convergence was analyzed in Fig. 4.2, and showed a converging trend.

The convergence, however, is quite slow; between 295 and 400 panels, there is a difference of approximately 13 m/s as opposed to 26 m/s between the previous steps. Some degree of convergence is reached when moving from 625 to 900 panels. However, this is convergence only on the order of 10 m/s, and at the cost of almost double the amount of time required to run the analysis. This is fairly important when considering the total cost to run the optimization problem. Due to the cost of running a divergence analysis for even 100 panels, a much lower mesh refinement is used in the optimization problem in order to keep the program run-time reasonable. During the full optimization problem, one must consider that the sensitivities required for the divergence calculation, sensitivities for the optimization gradients of the objective function with respect to the design variables, and of the sensitivities of the constraints with respect to the design variables are calculated. It is evident from the results that this requires a great deal of processor time. As each gradient evaluation requires the divergence analysis to be rerun, this means that the divergence analysis must be called \( n_d + 1 \) times per optimization iteration, where \( n_d \) is the number of design variables. The time required for the divergence analysis does not scale
linearly with the coarseness of the aerodynamic and structural mesh used. Hence it is preferable to use a mesh that will provide reasonable accuracy without a prohibitive cost. The results shown in the previous chapter indicate that due to the repeated sensitivity calculations and the difficulty of finding an optimum for these constraints shows that even a coarse mesh results in long computation times. This shall be discussed in § 5.1.2. Nonetheless, from these results, we can conclude that the divergence calculation is providing a somewhat accurate value, though consistently higher than the value expected from theory. Knowing this limitation, it is possible to move onto the full optimization problem, using the divergence speed as a constraint.

5.1.2 HANSA 320 HFB Wing Optimization

The baseline configuration of the HANSA resulted in a consistent range result from the C1 and C2 cases. The cases that were run, including those not presented for reasons of brevity, resulted in a range result between 1,161.14 km and 1,189.66 km. Both extreme ranges occurred for HANSA wing shapes with a swept-back elastic axis. The stated range is 2420 km. We therefore conclude that the simulation does not accurately model the aircraft. The optimization of the limited range can be investigated to determine the efficacy of the aeroelastic constraint on the aerostructural analysis. Given the small amount of variation (approximately 1.3%) between the results for these numerical simulations, it is likely that the differences in ranges given by each individual case are due to slight differences in the optimum point found in the design space by pySNOPT. By investigating the constraint behavior at the optimum point, we can confirm this conclusion.

For the C1 case, where only the stress constraints were active, and C2, where the stress constraints and the divergence constraint were active, the optimizer came to nearly identical optima; there was only small differences in the final design variables and objective function. Only the duration of the optimization was significantly different. In fact, for
these cases, the Lagrange multipliers recorded at the optimum point show that the stress
constraints on the same panels were active for each simulation. This indicates that the
optimizer came to similar optima since the same panels were active upon the yield stress
constraint boundary. For these cases and all of the starting thickness values, the stress
constraint dictates the behavior of the simulation. This indicates one of two things: that
the starting point is too thick, and that an optimum solution with regard to the maxi-
mum stress of a few panels is found before the divergence speed becomes a crucial factor,
or that for this particular configuration divergence speed is not a dominant constraint.
Recall that the divergence speed and the panel stress were both set as inequality con-
straints. The divergence speed is restricted to be greater than 310 m/s, or 825 km/h, and
the stress constraint is set to be between zero and the yield stress of 2024-T3 Aluminium,
which is $250 \times 10^6$ Pa.

Determining the role of the divergence constraint in the optimization requires an exami-
nation of the thickness of the panels and the value of the divergence condition. Regarding
the thickness values for the panels, all of the cases above came to the same thickness val-
ues for the same panels. Two of the panels, in this case at stations 6 and 7 consistently
went to the minimum thickness boundary of $1.98 \times 10^4$ m. The C3-1 case, however, is
different. In this case, only the divergence constraint was active and all of the panel
thicknesses but one went to the minimum. Clearly, removing the stress constraints from
the optimization problem has a significant effect on its behavior.

While the behavior of the C3-1 case with respect to the others is different, the optimizer
comes to near the same point in terms of range. From Table 4.6, we see that while the
thickness design variables are not identical, they have the same order of magnitude. As
a result, the range objective is not vastly different. Given the thickness of the C3-1 case,
this indicates that 1183.97 km is the closest result with respect to the published HFB 320
range. This is due to the low wing weight with respect to the other optimizations. When
looking at all variations of the HANSA case, the swept-back elastic axis case produces the
longest range due to this minimum thickness bound. We know then that this is not truly an optimum point, but simply a bound on the reality of the simulation. Furthermore, the aeroelastic constraint will never be beneficial for (i.e. extend) the range; the addition of a divergence speed constraint to the reference problem should reduce the range if it is active. The benefit of adding the aeroelastic constraint is to improve the safety of unconventional wing shapes, such as forward swept wing.

Thus, we conclude that for this MDA, the parameters involved do not calculate a realistic range; the cause of this being either insufficient lift-to-drag ratio calculated, or an improper initial-to-final weight ratio. Since most of the weight is fixed and based on the standard passenger and maximum fuel loads of the HANSA, it is likely there is an error with the weight ratio. Though inaccurate, as mentioned earlier, the range equation can still be used to determine if the aeroelastic constraint is providing a beneficial influence on the aerostructural problem.

We now consider the other constraint, the divergence speed. Examination of the divergence constraint shows that for all cases, this constraint never reaches the constraint boundary — i.e. it is not an active constraint. For all of the C1 and C2 constraint cases, the final divergence speed is within 1 m/s, an expected result as the thickness design variables come to the same optimum for these cases regardless of initial conditions. What is interesting is that the C3 cases are resolved without the divergence constraint becoming active either; pySNOPT comes to an optimum solution approximately 45 m/s above the constraint boundary. The optimum solution is likely reached because the minimum thickness is reached on almost all panels. Thus we conclude that the divergence constraint for these studies lies outside the permitted design space. In retrospect this is expected. The designers of the HANSA would have designed it for a divergence speed well above the cruise speed.

Recall from § 5.1.1 that the divergence speed decreases towards convergence for higher fidelity meshes, but at the cost of large increase in processing time. What this indicates is
that a realistic solution to the HANSA optimization can not be found with a low fidelity mesh, in this case on the order of 32 panels. From Table 4.2, this puts the cases run in the lowest time bracket, yet the total time for the optimization was between 10.70 and 48.38 s. Thus, we find that this optimization program as a whole is not efficient. Certainly, part of the reason for this is that, as discussed in § 5.1.1, the aeroelastic constraint only achieves a degree of convergence when the total number of panels is on the order of 625 panels. The total cost of calculating the sensitivities required for the divergence calculation, objective function and constraint gradients with respect to the design variables. As each gradient evaluation requires the divergence analysis to be rerun, this means that the divergence analysis must be called \( n_d + 1 \) time per optimization iteration. Recall that for each divergence analysis, the panel code must have its sensitivity calculated, as the \( A_0 \) matrix must be recalculated for the new displaced wing shape in that iteration. The \( A_0 \) matrix is linked to the current vertical and rotational (or twist) displacement of the wing. The vertical displacement is known and the new twist is retained the boundary conditions for each panel, thus in each major iteration, \( A_0 \) is recalculated. This is an expensive operation and appears to greatly increase the time for the optimization to complete.

When run with a higher fidelity (a 12x6 mesh as opposed to 8x4), there is a surprising result: the divergence speed computed increases. The C1 case was not run as the divergence speed is not an active constraint in that case; it is a reference case and invariably produces a similar result to C2 case, as long as the divergence constraint is not active. As can be seen from the results in Table 4.7, the divergence speed constraint is still not active. The optimizer shows no overall change in behavior; it progresses steadily, if slowly, to an optimum solution. This is expected, as the case that converged before should do so at just over double the mesh resolution. There is a more plausible physical explanation. While the fidelity has been increased, it has not been done to a great extent. The fidelity increase was less than in the divergence speed tests, so the overall result was simply to
shift the span stations to different co-ordinates along the pitch axis. As a result, we see
greater thicknesses in the wing thickness profile, and as a result, higher divergence speed.
A likely conclusion is that in order to truly see the effect of a decrease in the divergence
speed as fidelity increases, the increases must be on a greater scale than Table 4.2. The
increase in this case was on a similar scale, however it seems that the optimization pro-
cess, with additional constraints, may affect the convergence trend seen earlier.
In order to test the efficacy of the divergence constraint, the Standard configuration was
run with the divergence speed constraint set to double the cruise speed (620 m/s as op-
posed to 310 m/s), effectively introducing a safety factor of two. The results are shown
in Tables 4.9 and 4.10 for the optimum solution results and stress respectively. Here we
clearly see that the divergence speed constraint is now active. As expected, there is no
effect on case 1 as the divergence constraint is not active. Case 2 comes to a thicker,
heavier optimum, although the calculated range is not decreased much, most likely owing
to the higher $L/D$ ratio. Case 3 comes to an optimum as well, thinner and with a higher
range, as expected, but with a single stress constraint violated on the inner most panel.
These results can be linked to the optimizations performed with the HANSA configu-
ration using an even greater forward sweep. This was referred to as the experimental
configuration. This configuration was run for the same reason as increasing the value of
the divergence speed constraint: to bring the constraint into play and observe the effects.
The results of the forward sweep provided one additional piece of information: for the
same wing shape and starting point, the stress constraint had more influence than the di-
vergence constraint. Thus we see for the case FC2-2 that, although the divergence speed
is closer to the boundary than in the standard configuration, it does not become active
before an optimum point which is constrained by the active yield stress constraints.
The experimental configuration also makes clear a conclusion that is fairly plain from the
outset: a divergence constraint alone is not sufficient for the design of an aircraft wing,
as yield stress violations occur if the constraint is set too low. While it certainly has a
longer range, the final design is unsafe and will fail. We saw in the previous configuration that a doubled divergence constraint value produced only one panel with the yield stress exceeded; however this safety factor would have to be determined by trial and error. A yield stress constraint is therefore integral to producing an efficient optimization. This is reinforced by the shorter optimization time for case 2 for all configurations, including trials not shown in this report.

Unswept and Swept-back Conclusions

The HFB 320 configuration was also run with the quarter chord elastic axis at $0^\circ$ and swept backwards to $9.69^\circ$. While the use of a divergence constraint here is redundant, especially in the case of back sweep, it was hoped that the results would provide insight into the behavior of the optimization problem.

The convergence history of the UC3-2 and the RC2-2 case actually does provide some insight into the behavior of the optimization problem. It demonstrates a type of problem encountered frequently when the divergence-only constraint was used. The optimizer can become 'stuck' near the optimum. For the unswept and back sweep cases, this is clearly due to only the $L = W$ constraint being in effect, and the optimizer taking small steps in different directions to find the maximum range. Looking at the pySNOPT output, one sees that the feasibility and optimality tolerances (i.e. exit conditions) cycle through orders of magnitude as pySNOPT takes small steps around the optimum, until it abruptly exits when suddenly satisfying the optimality conditions. The result is a 'cycle' of similar displacements and boundary conditions in the output. However, there is inconclusive evidence to classify this as concrete behavior, since it did not occur for every starting point. It seems that pySNOPT occasionally found the correct step on the first or second attempt and exited. As a result, the swept-back C2 case (RC2-2) took the longest to converge to the optimum. In all other test scenarios the C3 case (divergence
and \( L = W \) constraints only) took longer to converge at the optimum. Hence, we see that this cycling near the optimum in RC2-2 can add a considerable extension to the optimization time.

**Comments on Flutter-like Behavior**

During the development of this code and the initial optimization runs, a phenomenon was seen where the optimizer encountered a mathematical representation of a physical effect: flutter. This was encountered when aerodynamic forces were large or the wing thickness was near the minimum. Flutter, as a constraint or calculation, was not considered in this project. The initial aerostructural modules used, however, did not contain a minimum thickness bound, or a trigger to recalculate the plate model if a thickness was reduced by more than 5 mm. Only a factor intended to scale back the aerodynamic forces if the displacements became unrealistic (i.e. exceeded the half-span). The factor, referred to as \( \beta \), took on many forms, such as geometric progression (as the realism boundary was repeatedly encountered), natural exponent progression and even a backtracking step which used the last ‘real’ result of the aerodynamic forces and forced the optimizer to proceed. The result was that the MDA portion of the optimization became stuck in a flutter situation where one iteration would have a large positive displacement and a negative local angle of attack, and the subsequent iteration would have the opposite. It was observed that these displacements grew in magnitude until hitting a mathematical maximum imposed by the progression of \( \beta \). The end result is the same; the optimization never converged and had to be manually terminated.

The cause of this phenomenon can be pinned on two reasons: firstly, in the initial stages, there was no wingbox to plate approximation, a plate was simply used and the resulting torsional stiffness was low relative to the wingbox, resulting in large local angles of attack, and secondly, the minimum thickness bound was too low. Implementing a more realistic wingbox to plate conversion and minimum thickness seems to have alleviated this flutter
problem, though small instabilities can be seen around the optimum points in certain cases. The instabilities seem to have decreasing rather than increasing magnitudes, as most trials found an optimum, though not always with the optimality accuracy specified.

**Final Conclusion**

This study has shown that the divergence constraint can be implemented with this MDO framework. The divergence constraint returned is, according to tests, converging towards an accurate result and some confidence can be had in the value provided that the optimization uses a fine enough mesh. With respect to the HANSA, the divergence constraint did not improve the optimum, though according to the simulation, the aircraft should be free from divergence effects. That stated, the constraint could be useful for aircraft configurations where less is known about the aircraft and its divergent behavior. In the HANSA case, it was known beforehand that it was divergence free; evidenced by some 100 successful test flights and commercial service.

The components used to construct the MDO framework are not incredibly efficient. When run even as MDA, the duration required is quite long. Thus, the conclusion must be drawn that in its current form, this aeroelastic module is not useful to the MDO user due to the high computational cost, a problem encountered by previous research by others. The MDO toolbox for this framework has only been slightly expanded, and further work is necessary to develop a truly useful aeroelastic module for constraint implementation. These future directions are discussed briefly in §5.2

**5.2 Future Direction**

**5.2.1 Program Efficiency**

The goal of this project was to build-up the toolbox for performing multidisciplinary optimization. As a result, an aeroelastic module was written and integrated into the
optimization suite. The aeroelastic module relied upon determination of the GAF sensitivities (the $A_0$ matrix) using finite differences, which caused $n^2$ calculations for $n$ panels. This operation becomes expensive for a system with a fine mesh. The same can be said of the overall system. The VLM-code used in this project was not particularly fast, but was required to be run multiple times, specifically to perform the optimization minor iterations as well as to determine the $A_0$ matrix. As a result, the tools are too slow to be used in serious, high fidelity optimization experiments. Future work should endeavor to make the routines more efficient, if possible.

One avenue of future work in this area is to use a Newton iteration or a trust-region method. A Newton iteration would converge quadratically, which could substantially improve the optimization duration; however a method would be needed increase the likelihood of convergence. A Newton iteration works only where the first derivatives are available; that is certainly the case here, but calculation of those derivatives may be time consuming, thus the overall duration may not be significantly shortened, especially if finite differences are used to calculate the derivatives. One method of overcoming this would be with a trust region method, restricting the Newton method from entering a part of the design space for which the quadratic model of the problem is no longer appropriate. Coupled adjoint sensitivity (CAS) analysis could also be useful in solving this problem. When using CAS, the solution of a coupled system is not required as the partial derivatives of the state variables of each discipline (structural or aerodynamic) with respect to the design variables take into account the solution of that system. In this way the two systems can be decoupled. The number of calculations required to obtain the gradients is reduced as only coupling variables are considered; this would be beneficial for the aerodynamic discipline, which has a large number of state variables.

Part of this improvement will be in resolving the conflict sometimes encountered by the optimizer near the optimal solution. The sensitivity of the objective function and constraints can be great near the optimum. This may, however, simply be in the nature
of the problem when an aeroelastic constraint is introduced. A solution that was not attempted during this study was the introduction of an interior penalty method, such as a logarithmic barrier. This would increase the penalty as the optimizer neared the constrained optimum. This penalty could force the optimizer to settle on optima with less iterations. One drawback is that the objective function must be twice continuously differentiable.

5.2.2 Range, Wing Weight and L/D Accuracy

One notable flaw of the current implementation is that the range is far less than what is expected. A possible sources of error is the weight extrapolation (either the estimate or the fixed weight assumption). The other terms in the equation fall into expected ranges. The $L/D$ ratio is consistently between 3 and 2.5, which is lower than expected for this aircraft. The cruise velocity and specific fuel consumption are also within the expected tolerances, which leaves the weight estimation as the only candidate for inaccuracy in the range. A more accurate estimation of the weight may change the ratio of initial to final weights, resulting in a more accurate range. In fact, this is indicated by the wing weight; assuming that the estimation of the fuselage weight is accurate, the wings should have a mass of about half of what has been indicated by the experimental data. It is suspected that much of this inaccuracy lies with the use of a plate to box conversion within the program, resulting in inaccuracies within the amount of lift generated due to approximations of the torsional stiffness. Furthermore, the weight calculation is based on the lift force generate by the plate; the actual wing would be a box; i.e. hollow. This implies that a more complex structural model will result in a more accurate wing weight. Regarding the $L/D$ ratio, the error may be due to the estimated friction drag included in this ratio. The friction drag was set as a constant at the beginning of the optimization, and is based on geometric shapes with similar dimensions to the HANSA oriented properly with respect to the airflow. If these estimations were high, then the $L/D$ ratio
may be too low. However, it was observed that modifying the friction drag constant only affected the value of the range and the $L/D$ ratio itself; the design variables at the optimum point remained the same for the same case. It was also noted that removal of friction drag increased the $L/D$ by a factor of $30 -$ a much less realistic than the $2.8$ to $3.0$ seen during the study. However, the author preferred to use the friction drag based on geometry as opposed to guessing the $C_L$ and back solving for the friction weight.

5.2.3 Wingbox Approximation

As mentioned in the previous paragraph, the use of a true wing-box structure throughout the MDO program could result in more accurate wing weight, and hence a more accurate range. The justification for the plate model here was that the conversion from wingbox to plate only took place once at the beginning of the optimization, and that doing so would permit a simpler FEAP implementation. In future iterations, however, a more complete grasp of FEAP’s capabilities is recommended along with a wingbox structure throughout the code.

Use of the actual wingbox would reduce the amount of equivalency factors used in the current implementation of the aerodynamic, structural and aeroelastic disciplines. This would eliminate a number of possible sources of error within the code. For example, the use of an equivalent plate model is affecting the calculated weight of the wings, and hence the range of the HFB 320.

The use of a true wingbox will also require an expansion of the stress constraint to include negative bending stresses, which will occur at the top surface of the wingbox. The yield stress constraint must be redefined such that excessive stress all surfaces of the wingbox are considered. This will increase the number of stress constraints that must be processed when checking the stress constraint.
5.2.4 New Design Variables

One further item that could be added in the future is the addition of more design variables, notably the structural jig twist. In the simulations, the twist of the wing is calculated and absorbed by the boundary conditions to account for the aerodynamic effects of the twisted wing, but the jig twist is initially set to $0^\circ$. These new design variables were attempted but due to time constraints were not implemented to the satisfaction of the writer. An increase to the efficiency of the full optimization suite used is also suggested before the incorporation of a new set of design variables in order to keep computational time reasonable.

The addition of jig twist would allow for a more accurate simulation of the wingbox model. Jigtwist is a design variable that can be translated back into the beam model used for the divergence speed calculation, which means that effects of the addition will have an impact on the aeroelastic constraints.
References


