Unemployment Insurance Eligibility and the Dynamics of the Labor Market

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy

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Abstract

This thesis examines a number of issues regarding the Mortensen-Pissarides search and matching model’s empirical performance. Chapter 1 documents the volatility puzzle with the Canadian data. The combined data from both Canada and the United States present an additional difficulty. Even if the unobserved value of leisure is allowed to be as high as required to fit the business cycle in the United States or in Canada, the model cannot reconcile the similar labor cycles with the large policy differences in the UI benefits and income taxes in the two countries when the value of leisure is assumed to be the same in both countries.

Chapter 2 takes into account the realistic institutional features of the UI system and investigates the impacts of the UI benefits on the labor market outcomes. If entitlement to UI benefits must be earned with employment, generous UI is an additional benefit to an employment relationship, so it promotes job creation. If individuals are risk neutral, UI is fairly priced, and the UI system prevents moral-hazard unemployed workers, the generosity of UI has no effect on unemployment.

Chapter 3 shows that the Mortensen-Pissarides search and matching model can be successfully parameterized to generate observed large cyclical fluctuations in unemployment and modest
responses of unemployment to changes in the UI benefits. The key features behind this success are the endogenous eligibility for UI benefits and the heterogeneity of workers. With the linear utilities commonly assumed in the Mortensen-Pissarides model, a fully rated UI system designed to prevent moral hazard has no effect on unemployment. However, the UI system in the United States is neither fully rated nor able to prevent workers with low productivity from quitting their jobs or rejecting employment offers to collect benefits. As a result, an increase in UI generosity has a positive, but realistically small, effect on unemployment. This chapter answers the Costain and Reiter (2008) criticism with the Mortensen-Pissarides model.
Acknowledgments

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Chapter 3

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Chapter 1
Cyclical Behavior of Unemployment and Job Vacancies: A Comparison between Canada and the United States

1.1 Introduction

The Mortensen-Pissarides (1994) search and matching model has become the standard model of the labor market presented in most macroeconomics textbooks. One of the reasons for this popularity is that, with simple productivity shocks, the model correctly predicts the key empirical regularities in the cyclical fluctuations of unemployment and job vacancies. Despite this success, however, recent work by Shimer (2005a) has raised a serious question about the validity of this model by showing that, as long as unemployed workers care little about their leisure, the predicted variability of unemployment and job vacancies is much lower than that observed in the United States.

Although a large number of related studies has emerged to address this challenge, there has been little systematic work to check if this failure of the Mortensen-Pissarides model can be observed in other countries as well. This chapter fills this gap by examining the business cycle in the Canadian labor market. Although the Canadian labor market is similar to that in the United States in many respects, Canadian data are particularly interesting given the differences in the generosity of unemployment insurance (UI) benefits and in tax rates between the two countries. These variables affect the opportunity cost of employment, which has proved to be a crucial variable to determine the cyclical predictions of the Mortensen-Pissarides model.

The dynamics of the Canadian labor market are found to be similar to...
those observed in the United States. Over the business cycle, both unemploy-
ment and job vacancies are volatile and persistent, and these two variables
have a strong negative correlation (Beveridge curve). Workers find jobs more
easily in booms than in recessions, while firms fill their vacancies more easily in
recessions than in booms. Consistent with the way the matching is modelled in
the Mortensen-Pissarides model, the job-finding and the vacancy-filling rates
correlate closely, and with opposite signs, with the vacancy-unemployment
ratio. Qualitatively, all these observations are correctly predicted by the stan-
dard Mortensen-Pissarides model with productivity shocks. However, as in
the United States, when the model is calibrated assuming that workers do not
value their time much while they are unemployed, the model predicts only a
small fraction of the observed variation in unemployment and job vacancies in
Canada.

The empirical data also show that in both Canada and the United States
shocks affecting the job-finding rate are the main driving force of cyclical
fluctuations in unemployment. However, the relative importance of job sepa-
rations differs in the two countries. In the United States, Shimer (2005a) finds
that the separation shocks account for a small fraction of the cyclical fluctu-
ations in unemployment. In contrast, separations are important contributors
in Canada. Yet, the introduction of separation shocks in the calibrations of
the model does not significantly improve the model’s ability to replicate the
high fluctuations in unemployment and job vacancies observed in reality.

A comparison of the data from Canada and the United States uncovers an
additional difficulty in explaining the observed cyclical variations in the labor
market with the Mortensen-Pissarides model. Although it is easy to make the
cyclical fluctuations in the vacancy-unemployment ratio as large in the model
as observed in the United States or in Canada by simple parameterization
of the opportunity cost of employment, no calibration permits the model to
reconcile the similar labor cycles in both countries as long as workers in these
two countries share the same value of leisure. More specifically, when the
value of leisure in the United States obtained from targeting the American
business cycle data is imposed on the model of the Canadian economy, the
model generates unrealistic predictions for unemployment and job vacancies: the unemployment rate rises to 100 percent and the number of vacancies drops to zero. These results are driven by the fact that Canada provides much more generous UI benefits and has higher income taxes relative to the United States. This, together with the large common value of leisure, substantially raises the opportunity cost of employment and results in a negative match surplus. A similar failure presents when the value of leisure determined by matching the cyclical fluctuations in the Canadian labor market is imposed on the model of the American economy. The predicted standard deviation of the vacancy-unemployment ratio accounts for only about 20 (40) percent of its empirical unconditional (conditional) counterpart in the United States. An opposite intuition applies here: the lower income tax, and much more stingy UI benefits in the United States, accompanied by the low common value of leisure, greatly lower the worker’s opportunity cost of employment, which enlarges the match surplus and destroys the amplification mechanism argued by Hagedorn and Manovskii (2008a) (to be explained in Section 1.5). These findings are robust to several variations of the model, such as adding training costs, deviating from the Hosios rule to generate smoother real wages, and fitting conditional responses to productivity shocks instead of overall cyclical fluctuations.

Allowing for different preferences for leisure in the two countries proves crucial in resolving this failure. However, the value of leisure required to fit the United States cycles has to be about 1.6 times larger than the corresponding value of leisure in Canada after taking into account the difference in productivity between the two countries. Such a large gap is implausible. This finding is also robust to the variations of the model listed above. Although all these features allow for calibrations of the model with opportunity costs of employment not as high as those in Hagedorn and Manovskii (2008a), they have little impact on the implied gap between the values of leisure in Canada and the United States.

The literature most related to this Chapter is Costain and Reiter (2008), which criticizes the Hagedorn and Manovskii’s calibration by pointing out that if the non-market returns are high, the response of unemployment to changes
in labor market policy, particularly unemployment insurance, is unrealistically large. Indeed, when the Canadian UI benefits and income taxes are introduced into the model of the American economy, the model predicts a dramatic rise in unemployment, to the point where all workers become unemployed. Similarly, when this exercise is reversed by imposing the American policies on the Canadian economy, the predicted unemployment declines by 50 percent. These predicted large reactions of unemployment suggest that the attempts to fix the volatility puzzle by simple parameterization of the opportunity cost of employment open up the door to other problems, such as unrealistic effects of changes in labor policy. Despite the similar results of Costain and Reiter (2008) and this study, the two works differ in their methodology; Costain and Reiter examine the policy effect through reduced-form regressions, while this chapter pursues this question by studying the data in two specific countries.

The rest of this chapter is organized as follows. Section 1.2 documents the key facts characterizing the Canadian business cycle. Section 1.3 briefly describes the stochastic version of the Mortensen-Pissarides model with training costs and taxes. Section 1.4 calibrates the model using Canadian data, and discusses the performance of the model in explaining the observed business cycles in the Canadian labor market with a low opportunity cost of employment. Section 1.5 examines the model’s empirical performance in simultaneously accounting for the cyclical variations in Canada and the United States with a high opportunity cost of employment. The additional difficulty is found by studying the effects of imposing one country’s policy on the other. The role of the value of leisure in improving the model’s fit is explored and the gap between the implied values of leisure in the two countries is discussed. Section 1.6 concludes with a summary and suggestions for further research.

1.2 Canadian Labor Market Facts

This section documents the movements of the main variables in the Canadian labor market: unemployment, vacancies, job-finding rate, separation rate, and labor productivity. For comparison purposes, the construction of these vari-
ables follows Shimer (2005a).

The first variable of interest is unemployment, which is measured as the number of workers who are able to work, available for work, actively seeking jobs, and yet not working. To highlight the business-cycle fluctuations, the raw series in unemployment is detrended as in Shimer (2005a), using the Hodrick-Prescott filter with a smoothing parameter of $10^5$. (The same transformation is applied to the rest of the variables.) The evolution of unemployment in Canada is shown in Figure 1. Over the sample period of 1962 to 2003, unemployment climbed gradually and exhibited strong fluctuations; the cyclical component, the difference between the log of unemployment and its trend, has a standard deviation of 0.162. Hence, unemployment fluctuates as much as 32 percent above or below its trend over the cycles. Moreover, the cyclical component of unemployment also shows a large persistence as evidenced by its autocorrelation of 0.956.

![Figure 1: Quarterly Canadian Unemployment (In Thousands) and Trend, 1962 - 2003](image)

The flip side of unemployment is job vacancies, reflecting the willingness by a firm to hire workers. The conventional measure of job vacancies is the help-wanted index elaborated from ads in major newspapers. Until recently, there was little question about the validity of this standard proxy for vacancies, but in the last few years many firms have increasingly relied on the Internet to post
their vacancies. Therefore, the help-wanted index has become less useful, and Statistics Canada stopped compiling it in 2003 (but it has not yet introduced a substitute). For this reason, the whole set of time series in this study ends in that year. Similarly to unemployment, job vacancies display remarkable variations. The cyclical component of job vacancies has a standard deviation of 0.237, and it also exhibits a large persistence over the sample period with an autocorrelation of 0.956.

Figure 2 displays simultaneously the cyclical components of unemployment and vacancies. Throughout 1962 - 2003, the two series are negatively correlated with a correlation coefficient of −0.689. Since unemployment is countercyclical, while vacancies are procyclical, the vacancy-unemployment ratio is strongly procyclical. The empirical data show that the standard deviation for the cyclical component of the vacancy-unemployment ratio is 0.367. Figure 2 also shows that vacancies lead unemployment, and that the cycles of the former have slightly larger amplitudes.

NOTE: Both unemployment and vacancy in Figure 2 are expressed in logs as deviations from the Hodrick-Prescott trend with a smoothing parameter $10^5$.

The job-finding rate, $f_t$, is a measure of the rate at which an unemployed worker finds a job. This rate plays a key role in the Mortensen-Pissarides model as it determines, together with the separation rate, the dynamics of
unemployment. Assuming, as in Shimer (2005a), a fixed labor force, unemployment at $t + 1$ is the sum of the workers who lose their jobs from $t$ to $t + 1$ (short-term unemployed at $t + 1: u_{t+1}^s$) plus the unemployed workers at $t$ who remain unemployed at $t+1$. That is, $u_{t+1} = u_{t+1}^s + u_t(1 - f_t)$.\footnote{A limited cyclical fluctuation of the Canadian labor force is observed over the sample period as evidenced by its low standard deviation of 0.016.} Using the number of workers who have been unemployed for less than 4 weeks to measure $u_t^s$, the resulting average monthly job-finding rate is 0.309.\footnote{The alternative measure used in Hall (2005b), based on job-duration data, yields a monthly job-finding rate of 0.302.} That is, over the sample period, close to one third of the unemployed workers found jobs within one month. Figure 3 plots the evolution of the quarterly average of the monthly job-finding rate and its trend from 1962 to 2003. The job-finding rate displayed considerable variations as evidenced by the standard deviation of 0.105 for its cyclical component. Compared with its counterpart in the United States (see Figure 5 in Shimer 2005), the rates in Canada showed similar trends except for the period of 1990-2000, when they steadily rose in Canada while remaining fairly stable in the United States. Figure 4 displays simultaneously the cyclical components of both the vacancy-unemployment ratio and the job-finding rate, revealing a strong positive relationship between these two variables, with a correlation of 0.753. This high correlation is consistent with a fairly stable matching function, as assumed by the Mortensen-Pissarides model.

Another important determinant of fluctuations in unemployment is job separation. The separation rate, $s_t$, measures the departure rate of workers from their employing firms when it is no longer in their mutual interest to continue their relationship. With a constant labor force, short-term unemployment at $t + 1$, $u_{t+1}^s$, is the group of workers who have separated from their job at $t$. That is, $u_{t+1}^s = e_t s_t (1 - \frac{1}{2} f_t)$ where $e_t$ is employment in period $t$. The term in parenthesis reflects the fact that unemployment is measured in a survey date (middle of each month in Canada), so a newly unemployed worker has, on average, half a month to find a new job before he or she is recorded as unemployed. Using the job-finding rate constructed above, the average monthly
separation rate is about 0.03. That is, over the sample period, an average of three percent of workers separated from their jobs each month, so jobs lasted on average 2.8 years.

Figure 3: Monthly Job-Finding Rate for Unemployed Workers, 1962 - 2003

Figure 4: Comparison of Cyclical Components of V-U Ratio and Job-Finding Rate, 1962 - 2003

NOTE: Both the V-U ratio and job-finding rate in Figure 4 are expressed in logs as deviations from the Hodrick-Prescott trend with a smoothing parameter 10^5.

Figure 5 shows the evolution of the quarterly average of the monthly separation rate and its trend. The difference between the log of the separation rate and its trend has a standard deviation of 0.096. Of particular note is that

^6An alternative measure in Hall (2005b), using the job tenure data, yields a monthly separation rate of 0.031.
the trends of the separation rates in Canada and the United States after 1980 were different: the trend in Canada was roughly stable, but it declined substantially in the United States (see Figure 7 in Shimer 2005). This difference can be partly explained by the fact that Canada, since the 1971 liberalization of the UI program, provides much more generous UI benefits relative to the United States.\footnote{The 1971 liberalization of Unemployment Insurance broadened coverage to most (93 percent) of the labor force, compared to 42 percent in 1940 when it was created. Program changes also included easier work requirements; increased level (two-thirds of insurable earnings replacement rate), duration and range of benefits (adding sickness, maternity and retirement benefits).\footnote{Five criteria are widely used to assess the generosity of UI, namely, the replacement rate, the maximum duration of benefits, the fraction of the work force covered by the UI program, the weeks of employment required to qualify for UI, and the categories of unemployed workers who qualify for UI. The Canadian UI system is far more liberal by any of these criteria. See Table 2 in Moorthy (1989) for more details.}} Because UI benefits essentially subsidize unemployment, they increase the worker flows from employment to unemployment, thus raising the job separation rate. Green and Riddell (1997) examine the response of employment durations to the change in the UI entitlement requirement in 1990 in Canada. They find strong evidence of the moral hazard effect: the labor market participants tailor their behavior to adjust to the change in the eligibility requirement, and many jobs terminate when workers approach the duration that would permit a UI entitlement.\footnote{See also Andolfatto and Gomme (1996), Christofides and Mckenna (1996) and Moorthy (1989) for the discussion of the effects of the UI system on job duration and unemployment.}

It is time to examine the relative contributions of the finding rate and the separation rate to the cyclical fluctuations in the unemployment rate. For this purpose, it is useful to notice that over the sample period the actual unemployment rates almost coincided with the implied "steady-state unemployment rates" constructed by using the time series of the separation and the finding rate: \( u^{ss}_t = \frac{s_t}{s_t + f_t} \) (see Figure 6).\footnote{At steady state, the flows out of unemployment equal the flows into unemployment. That is \( e_t s_t = u_t f_t \). In Figure 6, the actual rates were observed to be lower than the constructed steady-state ones. One possible explanation is that in constructing the steady-state unemployment rate, I rule out the possibility of staying out of the labor force, which leads to overestimation of the unemployment rates.} Therefore, the contributions of varia-
tions in the job-finding rate and in the separation rate to the fluctuations in the unemployment rate can be easily decomposed by constructing two theoretical unemployment rates: one with the actual separation rate and the mean job-finding rate, denoted as $u^1_t \ (u^1_t = \frac{s_t}{s_t + E(f_t)})$, and the other with the actual job-finding rate and the mean separation rate, denoted as $u^2_t \ (u^2_t = \frac{E(s_t)}{E(s_t) + f_t})$.

Figures 7 and 8 compare the cyclical components of $u^1_t$ and $u^2_t$, respectively, with the cyclical component of $u^{ss}_t$. Unlike what Shimer observes in the United States labor market, both figures show pronounced co-movements between the
two series, suggesting that not only the job-finding rate but also the separation rate is an important determinant of the cyclical behavior of unemployment. In particular, the job-finding rate accounts for 62 percent of the observed fluctuations in unemployment, while the separation rate accounts for 54 percent of those fluctuations. These two percentages add up to more than 1 since the finding rate and the separation rate are correlated.

NOTE: All the unemployment rates in Figures 7 and 8 are expressed in logs as deviations from the Hodrick-Prescott trend with a smoothing parameter $10^5$.

Labor productivity is the last variable examined in this section. It is measured as real output per worker in all industries excluding agriculture and the
public sector. Figure 9 depicts the cyclical components of labor productivity and the vacancy-unemployment ratio. The vacancy-unemployment ratio is procyclical throughout the whole sample period, with a correlation of 0.52 with labor productivity. The most important message in this figure, however, is that the vacancy-unemployment ratio fluctuates much more than labor productivity. The vacancy-unemployment ratio displays remarkable variation, deviating above or below its trend by more than 0.5 log points eight different times, and reaching 1 log point below the trend in the recession of 1982. In contrast, labor productivity is relatively stable, never fluctuating beyond 6 percent. The overall fluctuations in the vacancy-unemployment ratio are over ten times larger than those of labor productivity in the period from 1962 to 2003.10

![Figure 9: Quarterly Cyclical Components of Canadian V-U Ratio vs. Labor Productivity, 1962 - 2003](image)

NOTE: Both the V-U ratio and productivity in Figure 9 are expressed in logs as deviations from the Hodrick-Prescott trend with a smoothing parameter 105.

Table 1 collects the key statistical moments describing the Canadian labor market and compares them to their analogs from the United States. In summary, this table documents the following facts: 1) Unemployment and job

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10When labor productivity is defined as output per hour worked, the resulting standard deviation is still low, equal to 0.041 at the annual frequency, close to the standard deviation of output per worker, which is 0.034 at the annual frequency.
vacancies display considerable variations over the sample period, and both are about 10 times more volatile than labor productivity. Moreover, the vacancy-unemployment ratio is strongly procyclical, with a standard deviation almost 20 times larger than that of labor productivity. 2) All variables show remarkable persistence. 3) Both job creation and job destruction are critical factors in explaining the cyclical movements in unemployment. 4) The cycles of job vacancies slightly lead those of unemployment. Finally, of particular note in Table 1 is the similar data moments in the United States and Canada, which implies that the labor markets in these two countries share similar dynamics over the business cycles.

<table>
<thead>
<tr>
<th></th>
<th>(u)</th>
<th>(v)</th>
<th>(v/u)</th>
<th>(f)</th>
<th>(s)</th>
<th>(p)</th>
</tr>
</thead>
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<tr>
<td><strong>Standard deviation</strong></td>
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<td>0.237</td>
<td>0.367</td>
<td>0.105</td>
<td>0.096</td>
<td>0.021</td>
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<td>0.190</td>
<td>0.202</td>
<td>0.382</td>
<td>0.118</td>
<td>0.075</td>
<td>0.020</td>
</tr>
<tr>
<td><strong>Quarterly autocorrelation</strong></td>
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<td>0.956</td>
<td>0.959</td>
<td>0.791</td>
<td>0.795</td>
<td>0.876</td>
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<td></td>
<td>0.936</td>
<td>0.940</td>
<td>0.941</td>
<td>0.908</td>
<td>0.733</td>
<td>0.878</td>
</tr>
</tbody>
</table>

**NOTE:** All variables in Table 1 are expressed in logs and deviations from the Hodrick-Prescott trends. The numbers in the upper line are the empirical data moments in the Canadian labor market, while the ones in the lower line are the counterparts in the United States. The U.S. data are from Shimer (2005a).
1.3 The Mortensen-Pissarides Search and Matching Model with Training Costs and Taxes

In this section, I lay out a variation of the discrete time version of Shimer's (2005) model with the following three extensions. A general linear income tax is introduced in a way that labor income (wages and UI compensations) and corporate income (sales minus wages) are taxed at a common rate $\tau$. The value of the opportunity cost of employment $z$ is decomposed into three components: UI benefits $b$, taxes $t$ and leisure $l$. Lastly, a one-time training cost $k$ is introduced. Workers and firms pay a respective tax-deductible training cost $k^w$ and $k^f$ upon forming a match. The total training cost, $k = k^w + k^f$, is split between the two parties in a match according to the generalized Nash bargaining solution. This cost captures in a simplified fashion the fact that firms incur hiring and training costs when they recruit new employees, while workers typically suffer human capital losses when they undergo a spell of unemployment. Earlier studies have found that this cost is important to improve the fit of the model to the business cycle data. Moreover, an additional benefit from this consideration is that it increases the opportunity cost of the match without resorting to a high value of the opportunity cost of employment for the worker, which will be shown in Section 1.5. To facilitate comparability, I use Shimer's notation whenever possible.

In the model, both workers and firms are identical, infinitely-lived, risk-neutral and discount the future income at a common rate $r$. In each period, an employed worker earns an endogenous wage net of taxes, $w^*_t (1 - \tau)$, which is contingent on the realization of labor productivity (the aggregate state of the economy), while an unemployed worker receives a utility value from both the after-tax UI benefits and leisure, $b (1 - \tau) + l$, and searches for a job at no cost. Each firm has access to a constant returns to scale production technology.

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11See Mortensen and Nagypál (2007), Silva and Toledo (2007), and Yashiv (2006) for discussions about the importance of training costs (or, more generally, turnover costs) for the dynamics of unemployment.
producing output $p_t$ with one unit of labor in each period. There is free entry of firms. In each period, after the realization of the productivity shock, firms decide whether to post a vacancy or not. The firm that desires to hire a worker posts a vacancy at a tax-deductible cost $c$ (units of output). When the vacancy is filled, the firm yields a net profit $(p_t - w_t)(1 - \tau)$.

Unemployed workers and firms are brought together pairwise by a matching technology, which is assumed to be Cobb-Douglas in unemployment $u_t$ and vacancies $v_t$: $m(u_t, v_t) = \mu u_t^{1-\eta} v_t^{\eta}$. Symmetry across the workers implies that the job-finding rate $f_t$ at which each worker finds a job at $t$ is equal to the matches formed in that period divided by unemployment. Likewise, the vacancy-filling rate $q_t$ at which a vacancy is filled at $t$ is equal to the matches formed in that period divided by the measure of vacancies:

$$f(\theta_t) = \frac{m(u_t, v_t)}{u_t} = m(1, \theta_t) = \mu \theta_t^{\eta} = \theta_t q(\theta_t),$$

(1)

where $\theta_t$ is the vacancy-unemployment ratio (also called market tightness).

Once a vacancy is taken by an unemployed worker, the match remains until an exogenous separation occurs (no job-to-job transition), which takes place at a rate $s_t$. The match surplus is split up between the worker and the firm according to a generalized Nash bargaining rule in each period, through which the wage is determined and continuously updated. The bargaining power for the worker is $\beta \in (0, 1)$.

For the time being, the only shock in the economy generating business cycles is a productivity shock. Labor productivity $p_t$ is assumed to be stochastic and follows a first-order Markov process. Contingent on productivity being $p$, the values of being an employed worker and an unemployed worker, $W_p$ and $U_p$, and the values of a firm matched with a worker and posting a vacancy, $J_p$ and $V_p$, are recursively defined by the following discrete-time Bellman equations:
\[ W_p = w_p(1 - \tau) + \frac{1}{1+r}[sE_p U_{p'} + (1-s)E_p W_{p'}], \tag{2} \]
\[ U_p = b(1 - \tau) + l \]
\[ \quad + \frac{1}{1+r} \{ f(\theta_p) [E_p W_{p'} - k^w(1 - \tau)] + [1 - f(\theta_p)] E_p U_{p'} \}, \tag{3} \]
\[ J_p = (p - w_p)(1 - \tau) + \frac{1}{1+r}(1-s)E_p J_{p'}, \tag{4} \]
\[ V_p = -c(1 - \tau) + \frac{1}{1+r}q(\theta_p) [E_p J_{p'} - k^f(1 - \tau)] = 0, \tag{5} \]

where the expression \( E_p X_{p'} \) denotes the expected value of a variable \( X \) (\( W \), \( U \), \( J \), or \( V \)) conditional on the aggregate state \( p' \) next period. The free entry condition drives \( V_p \) to zero for all values of \( p \).

The total surplus from the match is defined as:

\[ \gamma_p = (J_p + W_p - U_p) / (1 - \tau). \tag{6} \]

The Nash bargaining rule implies that the firm pays a fraction \((1 - \beta)\) of the total training cost and obtains a fraction \((1 - \beta)\) of the total surplus:

\[ k^f = (1 - \beta)k. \tag{7} \]

\[ J_p / (1 - \tau) = (1 - \beta)\gamma_p. \tag{8} \]

The opportunity cost of employment is defined as:

\[ z = b + \frac{l}{1 - \tau}. \tag{9} \]

Notice that since leisure is not taxed, income taxes can be considered as part of the opportunity cost of employment. Defining \( t = \tau l / (1 - \tau) \), the opportunity cost of employment can then be decomposed into three components: the value of leisure, the value of UI benefits, and a term that captures the effect of taxes \((z = l + b + t)\).
The equilibrium values of \( J_p, W_p, U_p, \gamma_p, w_p, \) and \( \theta_p \) are determined by the system of equations (2) to (9). As pointed out by Mortensen and Nagypál (2007), this system of equations can be easily solved by finding first \( \gamma_p \) and then the remaining equilibrium functions. Substituting (2) to (4) into (6) and using (7) to (9), it yields:

\[
\gamma_p = p - z + \frac{1}{1 + r} [(1 - s - \beta f(\theta_p)) E_p \gamma_{p'} + \beta f(\theta_p) k].
\]

(10)

This equation determines the dynamic behavior of the surplus from a match. Intuitively, the total value of a match is the net surplus in the current period, \( p - z \), plus the expected discounted value of the match next period. The match survives next period with probability \( 1 - s \), but (10) contains the term \( [1 - s - \beta f(\theta_p)] \) to take into account that when the match dissolves, the value of unemployment is not zero due to the expected gains received by the worker from forming an employment elsewhere next period. The term \( \beta f(\theta_p) k \) captures the training costs paid by the worker in the future employment.

Combining (1) with (5), (7) and (8), the stochastic equilibrium of the vacancy-unemployment ratio must satisfy:

\[
\theta_p = \left[ \frac{\mu(1 - \beta)}{c(1 + r)} \max\{0, E_p \gamma_{p'} - k\} \right]^{\frac{1}{1 - \eta}}.
\]

(11)

The \( \max \) operator in (11) represents the firm’s optimal behavior in the job creation activity. When the training costs are too large compared to the realized productivity in downturns, firms would optimally choose not to open up vacancies in those periods.

The equilibrium values of \( \gamma_p \) and \( \theta_p \) are the solution to equations (10) and (11). Once \( \theta_p \) is obtained, the dynamics of unemployment follow from the law of motion: \( u_{t+1} = [1 - f(\theta_p)] u_t + s(1 - u_t) \). As long as \( p \) does not change, this unemployment (rate) converges to a conditional steady-state unemployment (rate): \( u_p^{ss} = \frac{s}{s + f(\theta_p)} \), which is contingent on \( p \).
1.4 The Cyclical Behavior of Unemployment and Job Vacancies in Canada

This section calibrates a simplified version of the previous model, where the training costs, the income taxes and the value of leisure are all set to be zero, to match the Canadian business cycle facts. Given this simplicity, the opportunity cost of employment equals the UI benefits. The purpose is, in the same setup as Shimer (2005a), to gauge to what extent the model explains the observed volatilities in unemployment and job vacancies with a low opportunity cost of employment. Labor productivity is assumed to follow a stochastic process that satisfies: \( p = z + e^y(p^* - z) \), where \( p^* \) is a parameter normalized to one. The total net surplus \( (p^* - z) \) is assumed to be positive, which implies \( p > z \). So, for all values of \( p \), there are bilateral gains from the match. The underlying variable \( y \) is an exogenous random variable with a zero mean. It follows an eleven-state Markov process in which transitions only occur between contiguous states. As detailed in the Appendix, the transition matrix governing this process is fully determined by two parameters: \( \Delta \) (the step size in a transition) and \( \lambda \) (the probability that a transition occurs).

To capture the fact that job destruction is also an important determinant of the fluctuations in unemployment, a second simulation extends the model of Section 1.3 by adding separation shocks. In this case, the separation rate, instead of being a constant, follows a first-order Markov process that satisfies: \( s = e^y s^* + \epsilon \), where \( s^* \) is calibrated to the average monthly separation rate, and \( \epsilon \) is an i.i.d. truncated normal random variable with a zero mean and a \( \sigma^2 \) variance.\(^{12}\)

\(^{12}\)Since the distribution functions of \( p' \) and \( s' \) depend only on \( y \) (and so \( p \)), equations (10) and (11) describing an equilibrium remain the same with the qualification that \( s \) is now the realization of a stochastic process.
In the simulations of the model, the period frequency is set to be one month, although consecutive periods are aggregated to match productivity and real wage data, which are only available at quarterly frequencies. Table 2 summarizes the calibration targets and their values. The separation and the finding rates are those constructed in Section 1.2. The elasticity of the finding rate with respect to market tightness $\eta$ is estimated using the same method as Mortensen and Nagypál (2007) explained in the Appendix. The Hosios condition is used to pin down the worker’s bargaining power, so $\beta = 1 - \eta$. The opportunity cost of employment $z$ is chosen to fit the Canadian statutory replacement rate of UI benefits (see the Appendix for details), and this sets $z/w = 0.6$. Finally, following Shimer (2005a), the monthly real interest rate $r$ is set to be consistent with an annual rate of 4.8 percent; the standard deviation and the autocorrelation of $p$ are aimed to be consistent with the observed moments of quarterly productivity; and the mean of market tightness $\theta$ is normalized to one, which implies that the value of $\mu$ in the matching function equals the monthly finding rate. In the second simulation with separation shocks, the correlation between $s$ and $p$ is targeted to their empirical counterpart at a quarterly frequency, and following Shimer (2005a), the standard deviation of $s$ is aimed to be the same as the standard deviation of quarterly productivity.\(^{14}\)

\(^{13}\)Shimer (2005) proposes regressing the log of the finding rate on the log of the vacancy-unemployment ratio to find $\eta$. However, this yields a value of $\eta$, which is outside the plausible range proposed by Petrongolo and Pissarides (2001).

\(^{14}\)When the observed standard deviation of separation is chosen as the target, similar to

<table>
<thead>
<tr>
<th>Calibration Targets for the Canadian Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average monthly separation rate ($s$)</td>
<td>0.03</td>
</tr>
<tr>
<td>Average monthly finding rate ($f$)</td>
<td>0.309</td>
</tr>
<tr>
<td>Elasticity of the finding rate with respect to market tightness ($\eta$)</td>
<td>0.54</td>
</tr>
<tr>
<td>Opportunity cost of employment ($z/w$)</td>
<td>0.6</td>
</tr>
<tr>
<td>Annual real interest rate ($r$)</td>
<td>0.048</td>
</tr>
<tr>
<td>Standard deviation of labor productivity (quarterly in logs)</td>
<td>0.021</td>
</tr>
<tr>
<td>Autocorrelation of labor productivity (quarterly in logs)</td>
<td>0.876</td>
</tr>
<tr>
<td>Normalization units of $\theta$</td>
<td>1</td>
</tr>
<tr>
<td>Correlation between productivity and separation (quarterly in logs)</td>
<td>–0.396</td>
</tr>
</tbody>
</table>
TABLE 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Source of Shocks</th>
<th>Productivity</th>
<th>Productivity and Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity (p)</td>
<td>Stochastic</td>
<td>Stochastic</td>
<td></td>
</tr>
<tr>
<td>Separation rate (s)</td>
<td>0.03</td>
<td>Stochastic</td>
<td></td>
</tr>
<tr>
<td>Step size (Δ)</td>
<td>0.032</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>Probability parameter (λ)</td>
<td>0.312</td>
<td>0.329</td>
<td></td>
</tr>
<tr>
<td>Variance of ε (σ₂)</td>
<td>—</td>
<td>0.00086</td>
<td></td>
</tr>
<tr>
<td>Parameter (α)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Cost of posting a vacancy (c)</td>
<td>0.404</td>
<td>0.414</td>
<td></td>
</tr>
<tr>
<td>Matching function (μ and η)</td>
<td>0.309μ0.46η0.54</td>
<td>0.309μ0.46η0.54</td>
<td></td>
</tr>
<tr>
<td>Bargaining power of workers (β)</td>
<td>0.46</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>UI benefits (z)</td>
<td>0.573</td>
<td>0.578</td>
<td></td>
</tr>
<tr>
<td>Real interest rate (r)</td>
<td>0.004</td>
<td>0.004</td>
<td></td>
</tr>
</tbody>
</table>

The values of \{s, r, η, β, μ\} directly follow from the stated targets in Table 2, and the values of \{z, c, Δ, λ, φ, σ₂\} are obtained by simulating the model and revising their values until the targets in Table 2 are matched. The outcome of this calibration process is summarized in Table 3.

Table 4 compares the predicted standard deviations of unemployment, vacancies, and the vacancy-unemployment ratio with those observed in the Canadian economy. The unconditional standard deviations are those calculated from the cyclical components of these variables constructed in Section 1.2. The conditional standard deviations are obtained using the formula: 

\[
\text{conditional stdv}(X) = \text{stdv}(X) \cdot corr(p, X),
\]

where \(X\) is the variable of interest. As argued by Mortensen and Nagypál (2007), this conditional criterion allows for the evaluation of the performance of the Mortensen-Pissarides model in predicting the response to productivity shocks without having to make the strong assumption that other shocks are not affecting labor market fluctuations. In any case, as the table reports, the standard deviations obtained from the simulations of the model are far from those observed in the Canadian economy, both conditional and unconditional. For example, the model with only the results in the model with only separation shocks shown in Shimer (2005), the model predicts a positive correlation between unemployment and the vacancies, which is counterfactual.
productivity shocks generates standard deviations of unemployment and the 
vacancy-unemployment ratio that are only 12 percent and 13 percent of their 
respective empirical unconditional counterparts. Even using the conditional 
criterion, the model can explain only 37 percent and 24 percent of the observed 
conditional standard deviations. Adding separation shocks increases the stan-
dard deviation of unemployment moderately, but it has almost no effect on 
the standard deviations of vacancies and the vacancy-unemployment ratio.\textsuperscript{15}

| TABLE 4 |
| Simulation Results for the Canadian Model |

<table>
<thead>
<tr>
<th>Standard Deviations</th>
<th>Model</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One Shock</td>
<td>Two Shocks</td>
</tr>
<tr>
<td>( u )</td>
<td>0.019</td>
<td>0.029</td>
</tr>
<tr>
<td>( v )</td>
<td>0.034</td>
<td>0.035</td>
</tr>
<tr>
<td>( v/u )</td>
<td>0.046</td>
<td>0.047</td>
</tr>
</tbody>
</table>

\textbf{1.5 The Model’s Implications from a Comparison between Canada and the United States}

The above results depend on the opportunity cost of employment \( z \) being 
low. As argued by Hagedorn and Manovskii (2008a), the Mortensen-Pissarides 
model generates such low standard deviations of unemployment and job vacan-
cies as found in Shimer (2005a) because it is calibrated to match a relatively 
large net surplus from the match \((p - z)\). They also show that for values of 
\( z \) around 97 percent of the marginal product of labor \((p)\), the model fits the 
cyclical labor market movements well.\textsuperscript{16} The important channel through which 
this amplification operates is the percentage changes in a firm’s net profits: 
When \((p - z) \simeq 0\), even a small percentage change in labor productivity \( p \) in-
duces a very large percentage change in the net profit \( p - z\), which provides the 
firm with incentives to hire more workers. In the case of Canada, \( z \) has to be 

\textsuperscript{15} When the elasticity of finding rate with respect to market tightness \( \eta \) is estimated using 
the method in Shimer (2005), the model’s explanatory power is even lower. 

\textsuperscript{16} A similar point was made by Costain and Reiter (2008).
0.953 for the unconditional standard deviation of the vacancy-unemployment ratio to match its empirical counterpart in the model with zero training costs.

The similarity between Canada and the United States in the value of $z$ brings up another question: what would happen if the different policy changed in the two countries? Since the UI policy and taxation would alter the value of $z$, it is of interest to study: 1) how workers and firms respond to the policy changes; 2) when the Canadian policy is introduced into the United States, whether the model can generate the cyclical variations observed in the labor market in Canada, and vice versa. The findings in this section uncover an additional difficulty with the Mortensen-Pissarides model: Simple parameterization for $z$ can fix the volatility puzzle with the model in the United States or in Canada, as argued by Hagedorn and Manovskii (2008a), but as long as workers value leisure in the same way in both countries, it cannot fix the model’s failure in reconciling the similar cyclical variations and the large policy disparities in UI benefits and income taxes between the two economies. In addition, this section shows that the above failure can be resolved by relaxing the assumption of the common value of leisure, but the required value of leisure in the United States would need to be 1.6 times larger than the level in Canada. Such a gap is too large to be plausible.

1.5.1 Effects of Imposing the Canadian (U.S.) Policies on the U.S. (Canadian) Model

In this part, the model in Section 1.3 is calibrated to fit the data from Canada and the United States. The main purpose is to evaluate the effects of imposing one country’s policy on the other. I set out by examining the impact of the Canadian policy on the United States economy. To this end, the model is first calibrated to fit the data in the United States, including the labor market variability, under the American policies. Then the Canadian policies are introduced into the calibrated model of the American economy to find out: if the model is able to generate the cyclical fluctuations of the
vacancy-unemployment ratio observed in Canada, and how unemployment in the American model economy reacts to the policy changes? Motivated by the recent literature, the exercise is conducted in several different ways, such as adding training costs (Mortensen and Nagypal 2007, Silva and Toledo 2007), departing from the Hosios rule to targeting the cyclical fluctuations in real wage (Hagedorn and Manovskii 2007), and matching the conditional variability in the labor market (Mortensen and Nagypal 2007).

The calibration strategy is similar to the one employed in Section 1.4, except for the separation rate \( s \) and the opportunity cost of employment \( z \). In the model of the American economy, the separation rate \( s \) is calibrated to match the average monthly unemployment rate over the period of 1962-2001, which is \( u_{US} = 0.0567 \). With respect to the value of \( z \), instead of targeting the statutory replacement rate of UI benefits as in Shimer (2005a), it is set to be composed of the values of leisure, UI benefits, and taxes, with the value of leisure as a free parameter. The tax rate is chosen to match the average general tax burdens relative to GDP, so \( \tau_{US} = 0.30 \). With respect to UI benefits \( b \), the statutory UI benefits replacement rate tends to overstate the generosity of UI benefits because not all unemployed workers are paid UI benefits, and not all recipients of UI get the statutory replacement benefits. To adjust for these factors, \( b \) is calibrated to fit the actual replacement rate, which is measured as the ratio of the average weekly UI benefits paid to unemployed workers to the average weekly earnings paid to employed workers. This yields: \( (b/w)_{US} = 0.111 \) (see the Appendix for details). Lastly, the value of leisure \( l \) is picked to match the standard deviation of the vacancy-unemployment ratio in the United States. The model of Section 1.3 introduces one-time training costs \( k \). In the calibrations where these are positive, they are calibrated as

\(^{17}\)If the average monthly separation rate constructed in Section 1.2 is used, the predicted average unemployment rate is slightly larger than the average rate observed in the United States, which makes it inappropriate to conduct the experiment regarding the reaction of unemployment to policy changes.

\(^{18}\)See Figure 1 in "The Economic and Fiscal Update 1999" by the Department of Finance Canada for rates in Canada and the United States.
follows. As in Silva and Toledo (2007), the training costs in the United States are measured using data from the 1982 Employer Opportunity Pilot Project (EOPP). According to these data, the total average cost of the training in the first three months is approximately equivalent to 55 percent of the quarterly wage of a newly-hired worker.\textsuperscript{19} This implies a calibration target of $(k/w)_{US} = 0.55$. It is worth noting that the value of $\eta$ in the United States happens to be the same as that in Canada by using the calibration method in Mortensen and Nagypál (2007). In the model of the Canadian economy, the counterpart targets for the policy parameters $\tau$ and $b$ are $\tau_{CA} = 0.35$ and $(b/w)_{CA} = 0.265$.

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
 & U.S. & Canada \\
\hline
Monthly real interest rate ($r$) & 0.004 & 0.004 \\
Monthly finding rate ($f$) & 0.452 & 0.309 \\
Average monthly unemployment rate ($u$) & 0.0567 & 0.778 \\
Actual UI benefits replacement rate ($b/w$) & 0.111 & 0.265 \\
Average income tax rate ($\tau$) & 0.30 & 0.35 \\
Standard deviation of labor productivity (quarterly in logs) & 0.020 & 0.021 \\
Autocorrelation of labor productivity (quarterly in logs) & 0.878 & 0.876 \\
Elasticity of the finding rate with respect to $\theta$ ($\eta$) & 0.54 & 0.54 \\
Normalization units of $\theta$ & 1 & 1 \\
Normalization of labor productivity ($p^*$) & 1 & 1 \\
Standard deviation of $\theta$ (quarterly in logs) & 0.382 or 0.151 & 0.367 or 0.191 \\
Conditional standard deviation of real wage $w$ (quarterly in logs) & free or 0.012 & free or 0.016 \\
Ratio of training costs to quarterly wage rate ($k/w$) & 0 or 0.55 & 0 or 0.37 \\
\hline
\end{tabular}
\caption{Calibration Targets for the U.S. and the Canadian Models}
\end{table}

The calibration targets in the United States are summarized in Table 5 (the first column). In the simulations that serve the purpose as mentioned earlier, all parameters except the UI benefits $b$ and tax rate $\tau$ are calibrated to fit the targets in the United States and are forced to be the same in the two countries. In a baseline calibration, the value of leisure is calibrated to match the unconditional standard deviation of $\theta_{US}$, using the Hosios rule to

\textsuperscript{19}The 1992 Small Business Administration Survey (SBA) suggests that 70 percent of training spells are finished in the first three months. Using also the 1982 EOPP project, Barron, Berger and Black (1997) and Dolfín (2006) provide a detailed discussion of the measure of training costs.
determine $\beta$, and no training costs. In subsequent calibrations, these targets are changed to match the standard deviation of $\theta_{US}$ conditional on $p$, the standard deviation of the real wage conditional on $p$ and the observed training costs in the United States.

### TABLE 6
Effects of the Canadian Policies on the U.S. Model

**A. U.S. Policy**

<table>
<thead>
<tr>
<th>Calibration Targets in the U.S.</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s.d.(\theta)$</td>
<td>0.382</td>
<td>0.382</td>
<td>0.151</td>
<td>0.151</td>
<td>0.151</td>
</tr>
<tr>
<td>$s.d.(w)$</td>
<td>$free$</td>
<td>$free$</td>
<td>$free$</td>
<td>$free$</td>
<td>0.012</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.25</td>
</tr>
<tr>
<td>$k/w$</td>
<td>0</td>
<td>0.55</td>
<td>0</td>
<td>0.55</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter values in the U.S.</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.051</td>
<td>0.054</td>
<td>0.131</td>
<td>0.130</td>
<td>0.621</td>
</tr>
<tr>
<td>$s$</td>
<td>0.026</td>
<td>0.026</td>
<td>0.027</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>$k$</td>
<td>0</td>
<td>1.605</td>
<td>0</td>
<td>1.594</td>
<td>1.513</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.284</td>
<td>0.295</td>
<td>0.297</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.263</td>
<td>0.131</td>
<td>0.104</td>
<td>0.074</td>
<td>0.082</td>
</tr>
</tbody>
</table>

**B. U.S. vs Canadian Policy**

<table>
<thead>
<tr>
<th>Decomposition of opportunity cost of employment $z$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>0.591</td>
<td>0.591</td>
<td>0.556</td>
<td>0.556</td>
<td>0.536</td>
</tr>
<tr>
<td>$t$</td>
<td>0.253</td>
<td>0.318</td>
<td>0.238</td>
<td>0.299</td>
<td>0.230</td>
</tr>
<tr>
<td>$b$</td>
<td>0.111</td>
<td>0.292</td>
<td>0.108</td>
<td>0.285</td>
<td>0.110</td>
</tr>
<tr>
<td>$z$</td>
<td>0.955</td>
<td>1.201</td>
<td>0.902</td>
<td>1.140</td>
<td>0.876</td>
</tr>
</tbody>
</table>

| Predicted response of $u$ (%)                      | 100     | 100     | 100     | 100     | 100     |

The calibration results in the model of the American economy under the American policies are reported in Section A of Table 6. The upper part describes the targets in the calibrations. Model 1 is the baseline model. Model 2 adds training costs to this model. Models 3 and 4 target the conditional standard deviation of $\theta$ with and without training costs, respectively. Finally, Model 5 targets the standard deviation of the real wage (conditional on $p$) instead of using the Hosios rule to determine $\beta$. The lower part shows the parameter values that fit the American model to the observed target values.
before the policy changes take place. When the Canadian UI benefits and income taxes are introduced, the policy parameters \( b \) and \( \tau \) adjust to fit the Canadian targets while the rest of the parameters remain the same. Section B of Table 6 decomposes the calibrated value of the opportunity cost of employment \( z \) into its three components: \( l \) (value of leisure), \( t \) (taxes), and \( b \) (UI benefits) under the American and Canadian policies, respectively. The predicted response of unemployment to the policy changes is reported at the bottom of Section B.

The targets listed in Section A are exactly matched by the model if the policies are set to the American values, and the remaining variables take the values shown in the second part of Section A (some parameter values are omitted to conserve space). Notice that prior to the policy changes, the calibrated values of \( l \) are robustly large, over 50 percent of labor productivity in all cases, and the resulting values of \( z \) are close to 1. However, since the values of \( z \) are still smaller than 1, the current profits received by the firms \((p - z)\) remain positive in all models. When the much more generous UI benefits and the higher income taxes prevalent in Canada replace their counterparts in the United States, Section B shows that the model economy reaches a corner solution where all workers become unemployed and firms post no vacancies. The results are robust to the introduction of training costs, the adoption of the conditional criterion, and the alternative calibration for the critical parameter \( \beta \). Intuitively, given the large value of leisure, in response to the pronounced rises in the UI benefits and income tax rate, the opportunity cost of employment increases considerably and surpasses labor productivity. The resulting negative match surplus makes the market activity no longer attractive, which induces both firms and workers to stay inactive in the labor market.

In the simulations presented in Table 6, the observable parameters \( \{\lambda, \Delta, k\} \) and the unobservable parameters \( \{c, \mu\} \) are picked by targeting the United States data moments and assumed to be the same before and after the policy changes.\(^{20}\) Can the model’s fit be improved if these parameters are calibrated

\(^{20}\)When the values of \( \mu \) and \( \eta \) are the same before and after the policy changes, the
to match the Canadian targets after the introduction of the Canadian policies? I explore the answer in what follows. When the parameters \( \{\lambda, \Delta, k\} \) are allowed to be different, they are targeted to the stochastic process of labor productivity and the ratio of training costs to wages observed in Canada. As to the training costs in Canada, based on the "Learning and Development Outlook 2005" by the Conference Board of Canada, Goldenberg (2006) estimates that in Canada the training costs as a fraction of wages are around two-thirds of those in the United States, which implies \( (k/w)_{CA} = 0.37 \). When the parameters \( \{c, \mu\} \) are allowed to be different, they are calibrated to the fit the normalized mean of \( \theta_{CA} \), and the average monthly finding rate \( f_{CA} \), respectively. Given the intuition explained above, one can easily see that as long as the high value of leisure obtained in the United States is imposed in Canada, recalibrating the parameters \( \{\lambda, \Delta, k, c, \mu\} \) does not alter the model’s prediction for unemployment; the corner solution still remains.

What would happen if the above exercise is reversed? To find out the answer, following the same strategy, the model is calibrated to fit the Canadian targets. In particular, the value of leisure \( l_{CA} \) is determined by targeting the standard deviation of \( \theta_{CA} \), and the separation rate is set to fit the average unemployment rate \( u_{CA} \). The Canadian targets are summarized in Table 5 (see the second column). In the second step, the policies and stochastic process for productivity observed in the United States are imposed on the preferences estimated for the Canadians. A similar failure appears! In all models considered above, the bottom part of Table 7 shows that the predicted standard deviation of \( \theta \) accounts for only about 20 (40) percent of its empirical unconditional (conditional) counterpart in the United States. Not surprisingly, the policy matching function is implicitly assumed to be the same in both countries. This assumption is relaxed when the value of \( \mu \) is set to match its Canadian target.

\(^{21}\)If the values of parameters \( \lambda \) and \( \Delta \) calibrated to the stochastic process of productivity observed in Canada are retained in the model of the American economy, the simulated productivity is about 4 times more volatile than what is observed in the United States, which gives rise to a much larger predicted standard deviation of \( \theta \).

\(^{22}\)Allowing the parameters \( \{c, \mu, k\} \) to take their American values does not affect the results for the standard deviations of \( \theta \).
changes also evoke sharp reactions of the unemployment rates (drop of about 50 percent) in the model of the Canadian economy. An opposite intuition applies here: the lower tax rate and the much more stingy UI benefits, along with the low common value of leisure, significantly lower the non-market returns, which not only enlarges the net profit $p - z$, destroying the amplification channel argued by Hagedorn and Manovskii, but also reduces the attractiveness of unemployment.

| TABLE 7 |
| Effects of the U.S. Policies on the Canadian Model |
| A. Canadian Policy |
| Model 1 | Model 2 | Model 3 | Model 4 | Model 5 |
| Calibration Targets in Canada |
| $s.d.(\theta)$ | 0.367 | 0.367 | 0.191 | 0.191 | 0.191 |
| $s.d.(w)$ | free | free | free | free | 0.016 |
| $\beta$ | 0.46 | 0.46 | 0.46 | 0.46 | 0.25 |
| $k/w$ | 0 | 0.37 | 0 | 0.37 | 0.37 |
| Parameter values in Canada |
| $c$ | 0.051 | 0.053 | 0.099 | 0.100 | 0.214 |
| $s$ | 0.025 | 0.025 | 0.026 | 0.026 | 0.026 |
| $k$ | 0 | 1.089 | 0 | 1.083 | 1.063 |
| B. Canadian vs the U.S. Policy and Stochastic Productivity Process |
| Model 1 | Model 2 | Model 3 | Model 4 | Model 5 |
| CA | U.S. | CA | U.S. | CA | U.S. | CA | U.S. |
| $\lambda$ | 0.290 | 0.300 | 0.300 | 0.300 | 0.301 | 0.300 | 0.302 | 0.300 | 0.303 | 0.300 |
| $\Delta$ | 0.260 | 0.052 | 0.159 | 0.046 | 0.135 | 0.044 | 0.102 | 0.040 | 0.106 | 0.041 |
| Decomposition of opportunity cost of employment $z$ |
| $l$ | 0.447 | 0.447 | 0.427 | 0.427 | 0.415 | 0.415 | 0.396 | 0.396 | 0.402 | 0.402 |
| $t$ | 0.241 | 0.192 | 0.230 | 0.183 | 0.223 | 0.178 | 0.213 | 0.170 | 0.217 | 0.173 |
| $b$ | 0.265 | 0.110 | 0.260 | 0.108 | 0.263 | 0.109 | 0.258 | 0.107 | 0.254 | 0.104 |
| $z$ | 0.953 | 0.749 | 0.917 | 0.718 | 0.901 | 0.702 | 0.867 | 0.673 | 0.873 | 0.679 |
| Predicted response of $u$ and variability of $\theta$ |
| $s.d.(\theta)$ | 0.367 | 0.077 | 0.367 | 0.077 | 0.191 | 0.064 | 0.191 | 0.065 | 0.191 | 0.064 |
| $u$ (%) | 7.78 | 3.13 | 7.78 | 3.23 | 7.78 | 4.25 | 7.78 | 4.30 | 7.78 | 4.10 |

These negative findings show that the model with a high opportunity cost of employment cannot simultaneously explain the data in the two countries that are observed over the business cycles and in the long run. More precisely, the observed differences in the various features of environments and the policy
disparities in UI benefits and taxes do not help the model explain the average behavior of unemployment and job vacancies in the two countries.

1.5.2 Independent Calibrations for Canada and the United States

The explanation behind the difficulty shown in Section 1.5.1 suggests that an easy fix would be to allow for different preferences of leisure in the two countries. Then, two questions arise immediately: How different would the values of leisure in these two countries have to be in order to be in line with the observed business cycles data and the big policy disparities? How plausible is the gap in the implied values of leisure in the two countries? To pursue the answer, I undertake two independent calibrations for Canada and the United States. That is, in the following simulations, all the parameters in the United States (Canadian) model economy, regardless of whether they are observable or not, are determined to fit the United States (Canadian) targets listed in Table 5. Since labor productivity in Canada is around 80 percent of that in the United States, the values of $l_{CA}$ are calculated both taking or not taking into account these productivity differences.

Table 8 displays the calibration results in both countries that correspond to the various models discussed above. The ratios of $l_{US}/l_{CA}$ are reported at the bottom of the table to facilitate the comparison. The main message delivered by Table 8 is that in all cases the values of leisure in Canada are much lower than those in the United States. Without taking into account the productivity differences between the two countries, the value of leisure in the United States is around 30 percent higher than the value of leisure in Canada. Once the productivity differences are considered, this gap climbs to over 60 percent. Fitting conditional responses to productivity shocks, adding training costs, and deviating from the Hosios rule to generate less volatile real wages help reduce the required values of $z$ relative to those in Hagedorn and Manovskii (2008a), but these changes have little impact on the implied gap between the values of leisure in the two countries.
1.6 Concluding Remarks

This chapter aims to investigate how well the standard Mortensen-Pissarides search model explains the business cycle fluctuations observed in the Canadian labor market. Although the model is successful in predicting many of the observed qualitative features, a key quantitative implication is unrealistic. As in Shimer (2005a), the simulation results reveal that the model lacks the ability to reproduce the observed high variability in unemployment and job vacancies with low values of leisure. In particular, using a similar calibration methodology to the one used by Shimer, the response of the vacancy-unemployment ratio to labor productivity shocks is less than $1/3$ of the estimated response in the Canadian data.

The similar performance of the model in both Canada and the United States, and the large differences in the UI generosity and in taxation between
the two countries uncover an additional difficulty for the model. For example, when the value of the opportunity cost of employment $z$ is allowed to be as high as needed to duplicate the observed large variations in the vacancy-unemployment ratio in the United States, the implied large value of leisure (the unobserved component of $z$), together with the relatively high UI benefits and income taxes in Canada, induces the non-market returns to surpass labor productivity, which results in counterfactual predictions about unemployment and job vacancies in the model of the Canadian economy. This failure can be resolved by allowing for different values of leisure in the two countries. To examine how different the values of leisure between these two countries have to be, this chapter calculates the values of leisure in Canada and the United States that would be required to generate a realistic variability of the vacancy-unemployment ratio, and finds that the value of leisure in Canada would have to be around 60 percent of the value of leisure in the United States. It is an unrealistically large disparity between the two countries. Moreover, this chapter represents the Costain and Reiter criticism by showing the exaggerated response of unemployment when the Canadian UI benefits and income tax rate are put in place in the model of the American economy. All the negative findings in this chapter imply that simple parameterization cannot fix the volatility puzzle with the standard version of the Mortensen-Pissarides model.

Although some authors have successfully resolved the volatility puzzle along the line of wage rigidity (see Hall 2005a, Hall and Milgrom 2008), there are some problems with this approach. Mortensen and Nagypál (2007) point out that a rigid wage per se is not enough to fix the puzzle. To this end, the level of wages must also be close to productivity (the opportunity cost of employment $z$ is implicitly required to be sufficiently large). To show the importance of this point, they demonstrate that given a small value of $z$, the model still lacks the ability to produce the observed large cyclical variations even if the worker’s bargaining power is set to be zero.\footnote{With zero bargain power for workers, the model is simplified into the one with fixed wages equal to the opportunity cost of employment $z$.} Intuitively, firms’
incentive to post vacancies is driven by the net profits \( p-w \) or net match surpluses \( p-z \). With wage or opportunity cost of employment close to productivity, the net profits or net match surpluses respond strongly to changes in productivity, which gives rise to larger amplitudes in the job creation behavior over the cycles and, therefore, to larger cyclical labor market movements. More recently, a similar point has been made by Pissarides (2007), who emphasizes that it is the expected wage rather than the volatility of wages over the spells of employment that motivates job creation. He also advocates that a better proxy for the Nash wages in the model would be the wages in the new matches, while the microeconometric evidence shows that wages for the new hires are indeed procyclical in the United States and main countries in Europe.\(^\text{24}\) Moreover, the empirical data on real wages in Canada and the United States do not support this solution. Although Canada has stronger union power and a more generous social security system (such as unemployment insurance, health care, and pension plan), the standard deviation of the real wage conditional on productivity in Canada is about 0.016, even larger than its counterpart in the United States, which is 0.012.

The additional difficulty uncovered in this chapter suggests that it would be useful to explore some features of UI institutions that might simultaneously explain the observed cycles in these two countries. In particular, it would be interesting to examine the effects of the rules of the UI provisions, such as the length of employment required to gain UI eligibility, the duration of benefits, and the rating of UI contributions, which affect the labor market flows over the cycles yet differ in the two countries. Also, given that job separation, another important factor for unemployment fluctuation, is insufficiently explored in the literature, it would be worthwhile endogenizing job separation by considering that some workers quit (or are temporarily laid-off) in order to collect UI benefits, or do not search while they are collecting UI benefits.

\(^{24}\)See also Bils et al. (2007), and Haefke, Sonntag and van Rens (2007) for the empirical evidence for procyclicality of wages in the new hires.
Chapter 2
Labor Market Cycles and Unemployment Insurance Eligibility

2.1 Introduction

Most models of employment flows in the labor market assume that workers automatically qualify for unemployment insurance (UI) benefits while they are searching for a job. As pointed out by Mortensen (1977), Burdett (1979), and Hamermesh (1979), this simplistic view of how a UI system operates may lead to highly misleading conclusions about its impact on the labor market. To avoid this criticism, several papers taking into account more realistic features of the UI systems have emerged. However, because of the institutional complexities of actual UI systems, these models rely exclusively on numerical methods for their analyses, and, they either assume an exogenous distribution of real wages (Andolfatto and Gomme, 1996) or a non-standard mechanism for its determination (Brown and Ferrall, 2003). In this chapter, we advance an analytically tractable version of the standard Mortensen-Pissarides search and matching model in which workers are not always entitled to UI benefits because such an entitlement must be earned with prior and not too distant employment, and it can be lost if workers quit their jobs voluntarily or refuse job offers.

If UI benefits are unconditionally received while searching for a job, they represent an opportunity cost of employment, and improve the bargaining position of workers while negotiating over wages with their employers. As a result, UI benefits reduce the expected profits of filling a vacancy, and hurt firms’ incentives for job creation and therefore employment. In contrast, if UI benefits are conditional on prior employment, they are no longer an opportunity cost, but an indirect benefit of employment. True enough, once workers

25Chapter 2 is joint work with my thesis supervisor, Professor Miquel Faig.
become eligible for UI their bargaining position improves and so their salaries rise. However, this is well anticipated by all involved, so UI benefits reinforce the bargaining position of firms dealing with workers who are not yet eligible for UI. Consequently, UI benefits promote the value of filling a vacancy and stimulate job creation. This is the entitlement effect stressed by Mortensen (1977), Burdett (1979), and Hamermesh (1979) but operating through a new channel. In those papers, the desire to earn UI entitlement reduces the reservation wages of workers searching for jobs, which in turn reduces unemployment. In our model, the entitlement effect operates through the bargaining positions of firms and workers. The UI benefits, making the employment match more attractive to workers, enable firms to appropriate a larger fraction of the match surplus, which translates into a stronger incentive to post vacancies.

Even if generous UI benefits encourage job creation, they may hurt employment when we take into account the financial costs of the UI system. When the UI program is funded by the UI contribution fees paid by employed workers, a generous UI system is also an expensive one, and the large fees needed to maintain it lower the workers’ desire of being employed, and so the value of filling a vacancy. Therefore, an expensive UI system imposes a downward pressure on employment. Based on these two competing effects of UI benefits, we obtain the following analog to Ricardian Equivalence: If UI rules can prevent the moral hazard behavior of becoming or remaining unemployed, each employed worker is charged a fair unemployment insurance fee, and utilities are linear, then the generosity of UI benefits, the duration of these benefits, and the time it takes to become eligible for UI are all irrelevant to the determination of output, vacancies, and unemployment.

Like Ricardian Equivalence, this irrelevance result should be a useful benchmark to pinpoint the economic effects of a UI system as violations of its premises. That is, the economic relevance of a UI system must be found on the risk aversion of workers, the "unfair" pricing of UI services, and moral hazard. If workers are risk averse, UI provides the valuable service of smoothing consumption fluctuations in the presence of employment shocks. The
Mortensen-Pissarides model typically abstracts from this purpose by assuming linear utilities, and we follow this tradition in this chapter. If UI contributions, or equivalently taxes that ultimately fall on employed workers, do not match the expected present discounted value of the UI benefits to be received during unemployment spells, the entitlement effect of UI benefits does not offset the opportunity cost of financing these benefits. So, the UI system may either increase or decrease employment depending on if the insured workers in question are subsidized or not from other sources of government revenue. Finally, workers may alter the hazard of being or remaining unemployed by changing their search intensity, refusing job offers, or strategically quitting jobs once they are eligible for UI. In a balance between tractability and realism, the present contribution focuses on the third of these hazards; that is, the assumptions of the model allow for moral hazard quits, but abstract from the effect of UI on search intensity and the acceptance of job offers.\footnote{The moral hazard job rejections will be studied in Chapter 3. Alexopoulos and Gladden (2004) examine how the UI system affects unemployment spell. They find changes in UI benefits significantly alter reservation wage, while search intensity does not change much.}

The possibility of moral hazard quits opens the door to an interesting form of multiple equilibria. For a generic set of parameters values, two types of equilibria coexist: A "good" equilibrium where workers do not quit once they are eligible for UI and a "bad" equilibrium where such quits occur. In the good equilibrium, few workers collect UI benefits and many are employed, so the UI contributions required to finance the UI program can be low, which in turn makes it undesirable to quit a job to collect UI benefits. In the bad equilibrium, many workers collect UI and few contribute to the UI system. Hence, UI contributions need to be high, which induces workers to quit as soon as they can collect UI. This multiplicity of equilibria is a reminder that fully funded UI contributions is not enough to curtail the moral hazards induced by a UI system.

When our model is confronted with data from Canada and the United States, it offers the following insights on the current debate about the appro-
priateness of the Mortensen-Pissarides model in explaining the cyclical fluctuations in the labor market. First, the eligibility rules of these two countries show a major concern to avoid moral hazard quits and such concern is only meaningful if the value of leisure is not too low. For example, in our baseline calibration with United States data, we calculate that for values of leisure below 80 percent of labor productivity, workers would never quit to collect UI even if they knew they would be able to collect the statutory UI replacement rate (40 percent) with probability one. Hence, even if the obvious political and social concern about unemployment implies that the value of leisure cannot be close to labor productivity (see Mortensen and Nagypál, 2007), the further concern to avoid moral hazard unemployment implies that the value of leisure cannot be too low either. Second, our calibrations of the model to cyclical data from Canada and the United States require similar values of leisure as a percentage of labor productivity (around 54 percent) even if the levels of generosity of UI systems in these two countries are quite different. Third, our calibrations are able to generate realistically large labor market cycles in response to productivity shocks, even if unemployment responses little to correlated changes in taxes and UI benefits.

The rest of this chapter is organized as follows. Section 2.2 describes our stochastic version of the Mortensen-Pissarides model with a UI system in which individuals need to earn their UI eligibility. Section 2.3 analyzes a deterministic version of the model. Section 2.4 fits the model to data on the labor market cycles in Canada and the United States. Finally, section 2.5 concludes.

2.2 The Model

Our model is a stochastic version of Pissarides (1985) search model. To simplify algebraic expressions, we use continuous time in the analysis of this and the following section, although a discrete time version of the same model will be employed in the numerical simulations of Section 2.4.
2.2.1 Basic Environment

In the economy, there is a continuum of measure one of workers, and a large measure of potential firms with free entry into the labor market. Both workers and firms are infinitely lived, risk neutral, maximize their expected utilities, and discount future utility flows at the common rate $r$. Production requires the cooperation of one worker and one firm. For this cooperation to take place, workers and firms must first enter the labor market and search for a suitable partner. Once a match has been formed, it produces a flow of output $p$ until it breaks down. The productivity $p$, common to all matches in the economy, follows a Markov jump stochastic process with a constant arrival rate $\lambda$ and takes values in a finite support $P \in \mathbb{R}_+^n$. The surplus from a match is split between the two parties according to a generalized Nash bargaining solution. Finally, employment matches dissolve either exogenously as a result of separations which come at an arrival rate $s$, or endogenously if breaking the match is in the interest of either of the two parties.

The key feature we introduce to this standard environment is that workers do not always collect unemployment insurance benefits (UI) while they are searching for jobs. For workers to be eligible for UI, they must first be employed for a while, and benefits do not last forever. Furthermore, UI benefits are meant to be collected for workers who lose their jobs involuntarily, although, to be realistic, we allow some workers who quit to successfully pretend to have lost their jobs involuntarily.

To capture these features in a tractable way, the following assumptions are made. Newly employed workers are not eligible for UI, and eligibility is the outcome of a jump stochastic process with an arrival rate $g$. Eligible workers always collect UI if they suffer an exogenous separation from their jobs, but if they quit, they collect UI with probability $\pi \in [0, 1]$. Finally, unemployed workers collecting UI lose eligibility either when they are offered a job or as a result of a jump stochastic process with an arrival rate $d$.\(^{27}\)

\(^{27}\)To avoid the complexity that involves the multiple labor markets in equilibrium, we
Unemployment insurance is provided by a government, which finances the UI system with a mandatory state dependent contribution fee $\tau_p$ collected from all employed workers. Since the government can borrow and save at the interest rate $r$, the UI program can run deficits or surpluses over time. Later on, we will allow for permanent deficits or surpluses by introducing general taxation and a public good.

All workers are identical in terms of preferences and abilities, and supply labor inelastically. The only difference across workers lies in the UI eligibility, which is indicated by the individual state variable $i$:

$$i = \begin{cases} 1, & \text{if the worker is eligible for UI, and} \\ 0, & \text{otherwise.} \end{cases}$$

Net of the UI contribution fee, employed workers earn a state dependent wage rate $w^i_p$, where the superscript $i$ denotes the UI eligibility state, and the subscript $p$ denotes the productivity state. The wage rate $w^i_p$ depends on UI eligibility because UI benefits raise the opportunity cost of employment, so they improve the worker's bargaining position in the negotiations to split the match surplus. Unemployed workers receive a flow utility from leisure $\ell$, and, if eligible for UI, they also receive UI benefits $b$. To avoid uninteresting possibilities, both $\ell$ and $b$ are assumed to be positive, and $\ell$ is assumed smaller than the production in a match net of the UI contribution fee: $\ell < p - \tau_p$ for all $p \in P$. However, the total opportunity cost of employment for a worker who is entitled to collect UI, $\ell + b$, may surpass production net of the UI fee for some realizations of $p$, which raises the possibility of moral hazard quits.

All firms possess the same production technology and preferences. Each one of them chooses to either stay idle or be active in the labor market. An active firm searching for a worker posts a vacancy at a constant flow cost $c$, and an active firm paired up with a worker gains an output flow $p$ and incurs a

---

*assume here that unemployed workers lose the UI entitlement upon accepting job offers. Chapter 3 explores the case where this assumption is relaxed and it shows that the main qualitative results in this Chapter remain the same. See Chapter 3 for greater detail.*
labor cost $w_p^t + \tau_p$. In addition to the flow costs of posting vacancies, we follow Mortensen and Nagypál (2007) in assuming that there is a one time hiring and training cost $k$ (training costs for short) when a worker and a firm meet. We assume that this cost is transferable, and split between the two parties by the same type of generalized Nash bargaining as in the wage negotiations. As a result, a firm and a worker end up incurring the respective costs $k^f$ and $k^w$ to start an employment relationship. Although most properties of our model do not depend on $k$ being strictly positive, we believe that a successful numerical implementation of the model requires taking into account the full labor turnover costs.\footnote{Also see page 14 for the importance of training costs.}

The search frictions in the labor market are characterized by a constant returns to scale matching technology: $M(v, u)$. The function $M$ maps vacancies posted $v$ and unemployment $u$ onto the number of successful matches formed. Let $\theta$ be the vacancy-unemployment ratio ($v/u$, also called market tightness). The constant returns to scale of $M$ implies that the rate at which workers find jobs (finding rate) is just a function of $\theta$: $f(\theta) = M(v, u)/u = M(\theta, 1)$. Likewise, the rate at which firms fill vacancies (filling rate) satisfies:

$$q(\theta) = \frac{M(v, u)}{v} = M(1, \theta^{-1}) = \frac{f(\theta)}{\theta}. \quad (12)$$

The function $M$ is assumed continuously differentiable, increasing in both arguments, and concave. Furthermore, it satisfies the terminal conditions: $M(1, 0) = M(0, 1) = 0$, and $M_1(0, 1) = M_2(1, 0) = \infty$. Therefore, workers find it easier to find jobs when vacancies are abundant relative to unemployment (in booms), while firms find it easier to fill their vacancies when the reverse is true (in recessions).

\subsection*{2.2.2 Bellman Equations}

Workers may be in four possible states depending on whether they are em-
ployed or not and whether they are eligible for UI benefits or not. Analogously, firms paired with a worker may be in two possible states depending on the worker’s UI eligibility state. Contingent on productivity being $p$, let the values of being an employed worker and an unemployed worker, respectively, be $W_p^i$ and $U_p^i$, where superscript $i$ denotes the worker’s UI eligibility state. Similarly, let the values of a firm matched with a worker with UI eligibility state $i$ be $J_p^i$. Finally, when the economy experiences a productivity change ($p \rightarrow p'$), let the expression $E_{p'}X_p^i$ denote the expected value of $X$ ($W$, $U$, or $J$) conditional on $p$. Using this notation, the utility values $W_p^i$, $U_p^i$, and $J_p^i$ for $i = 0, 1$ are recursively determined by the following Bellman equations.

The value of an unemployed worker who does not collect UI is the flow value of the utility from leisure plus the expected gains from transitions to employment, which comes with an arrival rate $f(\theta_p)$, or to a different productivity state, which comes with arrival rate $\lambda$. When a transition to employment happens, the worker incurs the training costs $k_w$:

$$rU_p^0 = \ell + f(\theta_p)(W_p^0 - U_p^0 - k_w) + \lambda(E_pU_p^0 - U_p^0).$$

(13)

The analog equation for the value of an unemployed worker collecting UI includes the flow value of the utility from both leisure and UI benefits and the expected gains or losses from transitions to employment, UI ineligibility, and a different productivity state. These transitions come at the arrival rates $f(\theta_p)$, $d$, and $\lambda$, respectively. Note that since workers lose UI entitlement upon taking job offers, they yield the expected present discounted value of being employed $W_p^0$ rather than $W_p^1$:

$$rU_p^1 = \ell + b + f(\theta_p)(W_p^0 - U_p^0 - k_w) + d(U_p^0 - U_p^1) + \lambda(E_pU_p^1 - U_p^1).$$

(14)

The value of an employed worker ineligible for UI is the present discounted value of wages plus the expected gains or losses associated with exogenously losing the job, becoming eligible for UI, and experiencing a productivity change.
The arrival rates of these events are \( s, g, \) and \( \lambda \), respectively:

\[
rW_p^0 = w_p^0 + s (U_p^0 - W_p^0) + g (W_p^1 - W_p^0) + \lambda (E_p W_p^0 - W_p^0), \quad \text{and} \quad (15)
\]

A worker eligible for UI can choose to quit the job to collect UI with probability \( \pi \) instead of continuing with the match. The expected present discounted value of utilities attained with these two choices are respectively the first and second arguments of the max operator in the following equation:

\[
rW_p^1 = \max \left\{ (1 - \pi) rU_p^0 + \pi rU_p^1, \ w_p^1 + s (U_p^1 - W_p^1) + \lambda (E_p W_p^1 - W_p^1) \right\}. \quad (16)
\]

Upon quitting, the worker gets the expected utility of being unemployed, otherwise the worker gets the flow value of utility from wage plus the expected capital gains or losses associated with losing the job exogenously or experiencing a transition to a different productivity.

Because of free entry, the value of an unmatched firm is zero. The value of a firm employing a worker is the flow value of current profits plus the expected gains or losses associated with the worker becoming eligible for UI, the match exogenously dissolving, and productivity changing, which occur with arrival rates \( g, s, \) and \( \lambda \). At any time, a firm can terminate the match, so the values of a matched firm cannot be negative. Given our assumptions, the value of a firm employing a worker ineligible for UI is always positive,\(^{29}\) but the same cannot be assured if the worker is eligible for UI. Consequently,

\[
rJ_p^0 = p - \tau_p - w_p^0 + g (J_p^1 - J_p^0) - sJ_p^1 + \lambda (E_p J_p^0 - J_p^0). \quad (17)
\]

\[
rJ_p^1 = \max \left\{ 0, \ p - \tau_p - w_p^1 - sJ_p^1 + \lambda (E_p J_p^1 - J_p^1) \right\}. \quad (18)
\]

Unmatched firms do not post vacancies if there are no expected gains in filling them, so \( \theta_p = 0 \) if \( J_p^0 - k_f \leq 0 \). Otherwise, unmatched firms post vacancies until the flow costs of posting a vacancy is equal to the expected gains.

\(^{29}\)As proven in Proposition 2.1, \( V_p^0 > 0 \). Therefore, (22) implies \( J_p^0 > 0 \)
of filling it, which occurs with an arrival rate $q(\theta_p)$, so $c = q(\theta_p) \left( J_p^0 - k^f \right)$. Using (12) and $f(\theta_p) \geq 0$, these two relations can be summarized as follows:

$$c \theta_p = f(\theta_p) \max \left\{ 0, J_p^0 - k^f \right\}$$

(19)

Since we abstract from the possibility of workers carrying UI eligibility earned from past employment to a new job, firms get the same value from matching with a worker who is collecting UI as the one who is not. Hence, it is consistent to assume that there is a single labor market where all workers and all firms interact. We leave the complexities derived from a dual labor market that differentiates workers depending on if they are eligible for UI or not to Chapter 3.

2.2.3 Nash Bargaining

The surplus of an employment match depends on the worker’s entitlement to receive UI in case the match were dissolved. If the worker is not eligible for UI, the match surplus is defined as:

$$V_p^0 = W_p^0 - U_p^0 + J_p^0.$$  

(20)

If the worker is eligible for UI, the match surplus depends on if the potential dissolution would be considered a quit or not by the UI agency. Because the UI agency imperfectly monitors why employment separations occur, we assume that if a match were to break down while bargaining, the worker would be able to collect UI with probability $\pi$. This is the same probability of collecting UI after a voluntary quit because a worker who quits can be considered as one who cannot successfully negotiate a suitable pay raise. This assumption implies that the worker’s opportunity cost of employment is $(1 - \pi) U_p^0 + \pi U_p^1$. Consequently, the match surplus when a firm bargains with a worker eligible
for UI is:

\[ V^1_p = W^1_p - (1 - \pi) U^0_p - \pi U^1_p + J^1_p. \]  

(21)

The generalized Nash solution to the bargaining problem maximizes the weighted product of the match surpluses of the two parties: 

\[ (J^i_p)^{1-\beta} (V^i_p - J^i_p)^{\beta}, \]

where \( i \) takes values 1 or 0 depending on the worker’s UI eligibility state, and \( \beta \) denotes the worker’s bargaining power. The solution to this problem leads to the familiar sharing rule:

\[ J^i_p = (1 - \beta) V^i_p, \quad \text{for } i = 0, 1. \]

(22)

Similarly, when a firm and a worker first meet, the surpluses of both parties must subtract the training costs required for employment relationship to commence, so generalized Nash bargaining implies:

\[ J^i_p - k^f = (1 - \beta) (V^i_p - k), \quad \text{for } i = 0, 1. \]

(23)

The combination of (22) and (23), together with \( k^f + k^w = k \), results in the following split of the training costs:

\[ k^f = (1 - \beta) k, \quad \text{and } k^w = \beta k. \]

(24)

### 2.2.4 Equilibrium

A recursive stochastic equilibrium is a set of eleven functions \( \theta_p, w^0_p, w^1_p, U^0_p, U^1_p, W^0_p, W^1_p, J^0_p, J^1_p, V^0_p, V^1_p \) that satisfy the Bellman equations (45) to (50), the free entry condition (19), the match surplus definitions (20) and (21), and the Nash bargaining solutions (22) to (24). This system of equations can be reduced to the following four functional equations (see the Appendix):

\[ c \theta_p = f (\theta_p) (1 - \beta) \max \{0, V^0_p - k\}, \]

(25)
\begin{equation}
\hat{U}_p = \frac{b + \lambda(E_p \hat{U}_{p'} - \hat{U}_p)}{r + d + f(\theta_p)}, \quad (26)
\end{equation}

\begin{equation}
\hat{B}_p = \max \left\{ \frac{s \hat{U}_p + \lambda(E_p \hat{B}_{p'} - \hat{B}_p)}{r + s + g}, \pi \hat{U}_p - V^0_p \right\}, \text{ and} \quad (27)
\end{equation}

\begin{equation}
V^0_p = \frac{p - \ell - \beta f(\theta_p) (V^0_p - k) + g \hat{B}_p - \tau_p + \lambda (E_p V^0_{p'} - V^0_p)}{r + s}, \quad (28)
\end{equation}

Equation (25) is just the free entry condition (19) combined with the Nash bargaining rules (22) to (24). Equation (26) states that the value of UI eligibility for an unemployed worker, $\hat{U}_p \equiv U^1_p - U^0_p$, is equal to the expected present discounted value of the UI benefits received by an eligible worker during a spell of unemployment. Equation (27) calculates the incremental value of achieving UI eligibility, which depends on if the match breaks down or not because of such eligibility. If the match survives UI eligibility (first term in 27), $\hat{B}_p$ is the difference between the expected present discounted values of the UI benefits to be received upon an exogenous separation of the current match if the worker is eligible for UI ($i = 1$) or not ($i = 0$), which are equal to:

\begin{equation}
B^1_p = \frac{s \hat{U}_p + \lambda(E_p B^1_{p'} - B^1_p)}{r + s}, \quad (29)
\end{equation}

\begin{equation}
B^0_p = \frac{g (B^1_p - B^0_p) + \lambda(E_p B^0_{p'} - B^0_p)}{r + s}. \quad (30)
\end{equation}

If UI eligibility kills the employment match (second term in 27), then $\hat{B}_p$ is the expected value of UI eligibility for an unemployed worker minus the value of the match. Finally, equation (28) states that the value of the match between a firm and a worker ineligible for UI is the expected present discounted value of the benefits resulting from the match. These benefits include the labor productivity net of both the value of leisure and the workers’ expected value of finding a new job if the match breaks down, $p - \ell - \beta f(\theta_p) (V^0_p - k)$, the net benefits from the UI system, $g \hat{B}_p - \tau_p$, and the expected gains from a productivity change, $\lambda (E_p V^0_{p'} - V^0_p)$. Notice that the UI contribution $\tau_p$ detracts
from the value of the match exactly in the same way as the value of leisure does, but UI benefits have exactly the opposite effect. This implies that instead of reducing the value of a match, UI benefits make the match more attractive at least before workers become eligible to collect them.

The equilibrium functions \( \theta_p, \bar{U}_p, \bar{B}_p, \) and \( V^0_p \) solve (25) to (28), and the remaining functions that define an equilibrium follow recursively from these four. The following proposition establishes the existence and some basic properties of an equilibrium.

**Proposition 2.1** An equilibrium exists and has the following properties: \( U^1_p > U^0_p \) and \( V^0_p > 0 \) for all \( p \in P \). Furthermore, if \( \pi \leq s/(r + s + g + \lambda) \), then \( V^1_p > 0 \) for all \( p \in P \). (see the proof in the Appendix).

As one would expect that an unemployed worker benefits from being eligible for UI, and the match surplus is always positive if the worker is not eligible for UI. Also, if the probability at which an eligible worker quits the job to collect UI benefits is low, then the match surplus remains positive once worker’s UI eligibility is achieved. The following two propositions state additional properties of this equilibrium.

**Proposition 2.2** If workers are always denied benefits after quitting a job voluntarily (\( \pi = 0 \)) and the UI system is fully funded by UI contribution fees (each worker is charged the expected present discounted value of expected UI benefits), then the level of UI benefits, the duration of these benefits, and the time it takes to become eligible for UI are irrelevant for the determination of output, vacancies, and unemployment. In particular, the introduction or elimination of a fully funded UI system with \( \pi = 0 \) has no effect on these variables.
Proof: Since \( \pi = 0 \), moral hazard quits never occur. Consequently, the expected present discounted value of UI contributions from a newly employed worker is:

\[
T_p = \frac{\tau_p + \lambda(E_pT_p' - T_p)}{r + s}.
\]  

(31)

For the UI system to be fully funded, \( T_p \) must be equal to \( B_p^0 \), which characterizes the expected present discounted value of UI benefits received by a newly hired worker. Comparison of equations (30) and (31) implies that this equality holds if and only if \( \tau_p = gB_p \). Substituting this into equation (28), it follows immediately that \( V_p^0 \) is independent of \( b, d, \) and \( g \). Therefore, neither \( \theta_p \), nor output, unemployment, or vacancies depends on the UI policy parameters.

Proposition 2.3 As long as there are no moral hazard quits, the equilibrium paths of vacancies and unemployment are independent of the probability of collecting UI after quitting a job voluntarily.

Proof: Workers have no incentive to quit after they become eligible for UI if and only if the first argument in the \( \max \) operator in (27) does not fall short of the second one, and if this holds for all \( p \in P \), \( \pi \) drops out from the system of equations (25) to (28), which determines \( \theta_p \) and so output, unemployment, and vacancies.

Propositions 2.2 and 2.3 taken together provide a set of conditions that render a UI system irrelevant. Like other irrelevance results, such as Ricardian Equivalence, these propositions should be useful to pin point the economic effects of a UI system as violations from their stated premises. In this vein, the effects of a UI system have to be found in incorrectly pricing its insurance services, moral hazard, and risk aversion. More precisely, the adverse effects of UI program on output and employment have to be found either in the way it is financed, which may distort job creation, or in the rules for the
provision of benefits, which may engender strategic behavior such as quitting once eligibility is achieved or not searching while benefits last. Also, with risk aversion, the benefits of reducing income uncertainty with UI benefits affect the willingness to work and save in the ways that are beyond the scope of the present contribution.

2.3 Deterministic Equilibrium

To obtain sharp results, this section follows Shimer (2005a) and Mortensen and Nagypál (2007) and analyzes the special case where \( p \) is deterministic. As argued by Mortensen and Nagypál (2007), the comparative statics analysis of this deterministic model provides a good approximation for the dynamics of the stochastic model if productivity shocks are rare \( \lambda \to 0 \), or they occur frequently but their changes are small.\(^{30}\)

As we will see, the predictions of how the economy reacts to shocks, such as a rise in productivity or an increased generosity of UI benefits, depends crucially on the assumptions we make about the UI contribution fee \( \tau \). On one extreme, we can assume that \( \tau \) is an endogenous variable that adjusts to maintain the UI system fully funded. On the other extreme, we can assume that \( \tau \) is an exogenous parameter not affected by the shocks considered. For this second assumption to be logically consistent in a general equilibrium context, we need to extend the model and assume that \( \tau \) includes both UI contributions and general taxes, and that the government provides a public good, which yields separate utility. With this extension, when \( \tau \) is kept constant while other parameters change, we are implicitly assuming that the government adjusts the provision of the public good endogenously to balance its budget.

\(^{30}\)Shimer (2005) documents that the standard deviation of productivity over the period of 1951-2003 is small, only 0.020 in magnitude.
2.3.1 Exogenous $\tau$

With the simplification that $p$ is deterministic, the system of equations (25) to (28) that characterizes an equilibrium can be reduced to the crossing of the two schedules depicted in Figure 10. These schedules relate the value of a newly formed match $V^0$ with the vacancy-unemployment rate $\theta$ as follows. Schedule JC (job creation) represents the free entry condition (25). Its upward sloping shape captures that firms respond to a rise in the expected profits associated with a rise in $V^0$ by posting more vacancies until the filling rate becomes sufficiently low so that the value of posting a vacancy falls back to zero. Schedule MV (match value) is the representation of the mapping from $\theta$ to $V^0$ implied by the remaining equilibrium equations (26) to (28). Using (25), the absence of productivity shocks and the equilibrium properties that $V^0 > 0$, these equations simplify into:

$$\hat{U} = \frac{b}{r + d + f(\theta)},$$  \hspace{1cm} (32)

$$\hat{B} = \max \left\{ \frac{s\hat{U}}{r + s + g}, \pi\hat{U} - V^0 \right\},$$ \hspace{1cm} and \hspace{1cm} (33)

$$V^0 = \frac{p - \ell - \tau - \beta(1 - \beta)^{-1}c\theta + g\hat{B}}{r + s}. \hspace{1cm} (34)$$

Equation (34) implies that there are two reasons why the value of a match $V^0$ falls with $\theta$ as represented in Figure 10. First, as indicated by $\beta(1 - \beta)^{-1}c\theta$, workers find jobs easier if there are more vacancies posted, which pulls up the wage due to the improved bargaining power and lowers the match surplus. Second, as captured by $g\hat{B}$, the expected present discounted value of the UI benefits received by an eligible worker in (32) falls with the job finding rate and so with $\theta$. As a result, the value of the jobs needed to gain this eligibility falls as well.
Figure 10 is useful to analyze the qualitative implications of the model. For example, an increase in training costs $k$ shifts the JC schedule up, which leads to a rise in $V^0$ and a fall in $\theta$. Intuitively, the rise in $k$ makes it more costly to open up a vacancy, so market condition becomes less favorable to workers, which reduces workers’ bargaining position, but increases the value of UI entitlement. So wage falls and profits received by firms rise. In contrast, an increase in labor productivity or a fall in the value of leisure shift the MV schedule up. The rise in the match productivity makes a job offer more attractive and therefore raises both $V^0$ and $\theta$. A more generous provision of UI benefits (a rise in $b$ or $g$, or a fall in $d$) also shifts the MV schedule up because the matches that would allow workers to earn UI eligibility become more valuable (entitlement effect). Meanwhile, a more expensive UI contribution (a rise in $\tau$) has the opposite effect on the value of matches because it is equivalent to an increase in the value of leisure $\ell$ (financial cost effect). Consequently, in contrast with models where workers do not need to accumulate the employment time to gain UI eligibility, a more generous UI system and a more expensive one have competing effects on the vacancy-unemployment ratio $\theta$. Therefore, as we will show in Section 2.4, our model is able to reconcile the sharp response of $V^0$, $\theta$, $v$, and $u$ to productivity improvements with a
mild response of them to changes in \( b \) and \( \tau \) if these changes tend to happen together.\(^{31}\)

**Types of Equilibrium**

Depending on whether firms post vacancies in equilibrium or not, and whether workers eligible for UI quit their jobs or not, we can distinguish four possible types of equilibria:

- Normal: \( V^0 > k \) \( \bar{V}^1 \geq 0 \),
- Strategic: \( V^0 > k \) \( \bar{V}^1 \leq 0 \),
- Phase-out: \( V^0 \leq k \) \( \bar{V}^1 \geq 0 \),
- Autarky: \( V^0 \leq k \) \( \bar{V}^1 \leq 0 \),

where \( \bar{V}^1 \) is the value of the match in which worker gains UI eligibility through employment:

\[
\bar{V}^1 = V^0 + \left[ s / (r + s + g) - \pi \right] \hat{U}
\]

Since upon achieving the UI entitlement, dissolving the match is an option. The value of the match can be defined as: \( V^1 = \max \{0, \bar{V}^1\} \). To facilitate the exposition in what follows, it is useful to write down an alternative expression of \( V^1 \): \( V^1 = V^0 + \hat{B} - \pi \hat{U} \). The sum of \( V^0 + \hat{B} \) measures the value of continuing a match upon workers’ gaining UI, while the term \( \pi \hat{U} \) reflects the expected gains from quitting.

Vacancies are posted in equilibrium if and only if the value of a newly formed match exceeds the training costs \( (V^0 > k) \), and employed workers have no incentive to strategically quit a job if the value of continuing the match is not exceeded by the expected gains from quitting \( (V^0 + \hat{B} \geq \pi \hat{U}) \). In the normal equilibrium, both of these inequalities hold, so new jobs are created and matches survive when workers become eligible for UI. In the strategic equilibrium, the first inequality holds but not the second. So, new employment

\(^{31}\)In Chapter 3, I consider the case where only change in \( b \) occurs.
matches are formed, but they break down as soon as workers become eligible for UI. Finally, in the phase-out equilibrium and the autarky equilibrium, no new jobs are created. If the value of initial employment is positive, the worker-firm pair maintains until an exogenous separation comes in the phase-out equilibrium, while workers quit as soon as they become eligible for UI in the autarky equilibrium.

Figure 11: Types of Equilibrium

Figure 11 depicts how $k$ and $\pi$ interact in the determination of the various types of equilibria (see the Appendix for its construction). Denote $\tilde{k}$ as the critical value of training costs such that firms are indifferent about whether to post a vacancy or not at $k = \tilde{k}$. Denote $\tilde{\pi}$ as the critical value of probability of quitting to collect UI such that workers recently gaining UI eligibility are indifferent between keeping and quitting a job at $\pi = \tilde{\pi}$. As stated in Propositions 2.1 and 2.3, as long as $\pi$ is sufficiently low ($\pi < \tilde{\pi}$), the value of $\bar{V}^1$ would be positive and $\pi$ has no effect on $V^0$. So the $V^0 = k$ line is horizontal at the value $\tilde{k}$ for $\pi < \tilde{\pi}$. Once $\pi$ is sufficiently high for workers to quit upon receiving UI eligibility ($\pi \geq \tilde{\pi}$ or $\bar{V}^1 \leq 0$), an increase in $\pi$ makes an employment match more valuable. Hence, the $V^0 = k$ line is upward sloping for $\pi \geq \tilde{\pi}$.

Likewise, when $k$ is sufficiently large ($k \geq \tilde{k}$ or $V^0 \leq k$), since $k$ has no local effect on the value of $\bar{V}^1$, the line $\bar{V}^1 = 0$ is vertical at the value of $\tilde{\pi}$ for
Once $k$ is sufficiently low for new jobs to be created ($k < \bar{k}$ or $V^0 > k$), a reduction in $k$ (downward shift of JC line in Figure 10) reduces $V^0$ and increases $\theta$. As more vacancies are created, the job finding rate $f(\theta)$ goes up, which reduces the value of UI eligibility $\hat{U}$ and so $\hat{B}$. Consequently, the reduction in $k$ reduces both the value of continuing a match ($V^0 + \hat{B}$) and the expected gains from quitting ($\pi\hat{U}$), so it has an ambiguous effect on the value of $\pi$ needed to maintain the equality $\bar{V}^1 = 0$. This implies an ambiguous slope for the line $\bar{V}^1 = 0$ for $k < \bar{k}$.\(^{32}\)

Since $V^0$ is guaranteed to be positive and finite, and moral hazard quits are ruled out if $\pi < s/(r + s + g)$, on regions in Figure 11 where the normal and the phase-out equilibria exist are never empty. However, depending on how productive matches are and how generous the UI system is, moral hazard quits may not happen even if $\pi = 1$, in which case there is no strategic or autarky equilibria for all admissible values of $\pi$.

Figure 11 is useful to show how changes in match productivity or UI generosity alter the position of the lines $V^0 = k$ and $\bar{V}^1 = 0$, and therefore affect the nature of equilibrium. For example, increasing the net productivity of a match $(p - \ell - \tau)$ or the generosity of UI benefits (a rise in $b$ or $g$, or a fall in $d$) makes a newly formed match more valuable, so it shifts up the $V^0 = k$ line in Figure 11. Likewise, even if the worker is eligible for UI, the value of $V^1$ increases with net productivity, so a rise in $(p - \ell - \tau)$ shifts the $\bar{V}^1 = 0$ line to the right. However, the rise in UI generosity has an ambiguous effect on the location of the $\bar{V}^1 = 0$ line because a more generous UI system raises both the value of continuing the match and the expected gains from quitting. This implies that paradoxically increasing the generosity of the UI system may prevent quits in some regions of the parameter space, while encourage quits in other regions. To see this, in the region where $V^0 > k$ (normal or strategic equilibrium), the increase in the expected gains from quitting is always dominant. So if $b$ or $g$ rise or $d$ falls, the value of $\bar{V}^1$ declines and the line $\bar{V}^1 = 0$ declines.

\(^{32}\)As depicted in Figure 11, in a neighborhood of $(\bar{k}, \bar{\pi})$ the slope of this frontier must be negative (see Appendix).
shifts to left. This means that if the rise in UI generosity is sufficiently large, the economy could move from the normal equilibrium to the one with strategic quits where all workers choose to quit once they become eligible for UI. However, we cannot be certain that a shift in the same direction occurs in the region where \( V^0 \leq k \).

### Impacts of Productivity Shocks and Changes in UI Benefits

To study the quantitative predictions of the model, we can apply the standard comparative statics methodology to the equilibrium system of equations (25) and (32) to (34). Of particular interest is the elasticity of the finding rate with respect to labour productivity because it gives a good indication of the amplitude of the labor market cycles generated by productivity shocks (see Mortensen and Nagypál, 2007). As long as \( \theta > 0 \), this elasticity is derived as follows (see the Appendix for derivations):

\[
\frac{df}{dp} = \begin{cases} \frac{p}{p-z} \left[ \frac{(1-\eta)(r+s)+\beta f}{\eta(r+s+\beta f)} + \frac{f}{r+d+f}\frac{g\hat{B}}{p-z} \right]^{-1} & \text{if } \bar{V}^1 > 0, \text{ and} \\ \frac{p}{p-z} \left[ \frac{(1-\eta)(r+s+g)+\beta f}{\eta(r+s+g+\beta f)} + \frac{f}{r+d+f}\frac{g\pi\hat{U}}{p-z} \right]^{-1} & \text{if } \bar{V}^1 < 0. \end{cases}
\]

(35)

where \( \eta \) is the elasticity of \( f \) with respect to \( \theta \), and

\[
z = \begin{cases} \ell + \tau + (r+s)k - g\hat{B} & \text{if } \bar{V}^1 > 0, \text{ and} \\ \ell + \tau + (r+s+g)k - g\pi\hat{U} & \text{if } \bar{V}^1 < 0. \end{cases}
\]

(36)

In the absence of training costs and the UI program, the second term in the square brackets in (35) drops and \( z = \ell \), so the finding rate responds to changes in productivity in the same way as derived in Mortensen and Nagypál (2007). As Shimer (2005a) pointed out, for reasonable parameter values and a low value of leisure, this response is too small to generate the pronounced cycles in the United States labor market. A high value of leisure, by making the profits
margin $p - z$ small, leads to a sufficiently large elasticity of $f$ with respect to $p$ to rationalize the observed responses over the business cycle. For example, Hagedorn and Manovskii (2008a) found that when $\ell$ equals 0.97, model is able to generate the variance of $\theta$ observed in the United States business cycles.

Once training costs and the UI system are introduced into the model, as long as the equilibrium remains normal or strategic, equation (36) implies that the profit margin falls with $\tau$ and $k$, so a small profit margin may be compatible with a relatively small value of $\ell$ if UI contributions, taxes, and training costs are large. However, the effect of UI benefits on the profit margin works in the opposite directions. Positive UI benefits also add the second terms inside the square in (35), which further decreases the elasticity of $f$ with respect to $p$. Hence, if one takes into account the opportunity cost of financing the UI benefits, the presence of the UI system can increase and decrease the labor market cycles to productivity shocks.

Since $p$, $\ell$, and $\tau$ only enter the equilibrium system of equations (25) and (32) to (34) in the determination of the profits, an increase in $\ell$ or $\tau$ affects $f$ in the same way as a reduction in $p$ of the same magnitude does.

To assess the impact of the UI on the labor market outcomes, since UI benefits enter the determination of the profits through the term $g\hat{B}$, differentiating equations (32) to (34) yields:

$$\frac{df}{db} = \left\{ \begin{array}{ll} \frac{df}{dp} + \frac{ds}{s} & \text{if } \bar{V}^1 > 0, \text{ and} \\ \frac{df}{dr} + \frac{dg}{g} & \text{if } \bar{V}^1 < 0. \end{array} \right. \quad (37)$$

As pointed out in the qualitative analysis of Figure 10, a rise in $b$ unambiguously increases $\theta$ and so the job finding rate $f$ (MV schedule shifts up in Figure 10). As long as the increase in $b$ does not trigger moral hazard quits, this implies that, paradoxically, a more generous UI system reduces unemployment. However, a move towards a more generous UI system may trigger a shift from the normal to the strategic equilibrium in which case the level of steady state unemployment will experience a discontinuous jump. Indeed, for the stock of
unemployment to be constant, the flows into unemployment must equal the flows out of unemployment. Denoting $u^{ss}$ as the steady state unemployment, we have:

$$(1 - u^{ss}) s = u^{ss} f$$ in the normal equilibrium, and

$$(1 - u^{ss}) (s + g) = u^{ss} f$$ in the strategic equilibrium. (38)

Solving for $u^{ss}$ from these equations yields:

$$u^{ss} = \frac{s}{s + f}$$ in the normal equilibrium, and

$$u^{ss} = \frac{s + g}{s + g + f}$$ in the strategic equilibrium. (39)

Consequently, this model shows that the effect of UI generosity on unemployment is non-monotonic. Even though a rise in $b$ reduces $u^{ss}$ through the rise in $f$ (entitlement effect), it may also trigger moral hazard quits in which case the effective separation rate is $s + g$ instead of $s$. The discontinuous jump in $u^{ss}$ predicted in this model is an artifact of workers being homogenous (moral hazard effect is dominant). With heterogeneity as introduced in Chapter 3, a rise in $b$ could increase or decrease $u^{ss}$ depending on whether entitlement effect dominates moral hazard effect.

2.3.2 Fully Funded UI System

If the UI system is fully funded, then using an argument analogous to the one in the proof of Proposition 2.2, the value of $\tau$ has to adjust to satisfy:

$$\tau = \begin{cases} 
\hat{g} \hat{B} & \text{if } \bar{V}^1 \geq 0, \text{ and} \\
g \pi \bar{U} & \text{if } \bar{V}^1 \leq 0.
\end{cases}$$ (40)
Therefore, the value of a new match simplifies into:

\[ V^0 = \begin{cases} \frac{p - \ell - \beta (1 - \beta)^{-1} c \theta}{r + s} & \text{if } \bar{V}^1 \geq 0, \text{ and} \\ \frac{p - \ell - \beta (1 - \beta)^{-1} c \theta}{r + s + g} & \text{if } \bar{V}^1 \leq 0. \end{cases} \]  

(41)

As depicted in Figure 12, the endogenous adjustment of \( \tau \) to maintain the UI system fully funded induces two different MV schedules depending on whether moral hazard quits occur (\( MV^N \)) or not (\( MV^S \)). The schedule \( MV^N \) lies above \( MV^S \) because moral hazard quits are costly to the UI system, so high UI contributions need to be imposed. Consequently, both the equilibrium value of a new match and the vacancy-unemployment ratio are higher in a normal equilibrium relative to their counterparts in a strategic equilibrium with the same parameter values: \( (V^0N, \theta^N) > (V^0S, \theta^S) \). Equation (41) shows that \( b, \pi, \) and \( d \) have no effect on neither \( (V^0N, \theta^N) \) nor \( (V^0S, \theta^S) \), and \( g \) has no effect on \( (V^0N, \theta^N) \) and is inversely related to \( V^0S \) and \( \theta^S \).
Multiplicity of Equilibria

By equations (32) and (33), the conditions for these equilibria to be consistent with the incentive to quit or not are:

\[
\left(\pi - \frac{s}{r + s + g}\right) b \leq V^{0N} \left[r + d + f \left(\theta^N\right)\right] \quad \text{no incentive to quit, (42)}
\]
\[
\left(\pi - \frac{s}{r + s + g}\right) b \geq V^{0S} \left[r + d + f \left(\theta^S\right)\right] \quad \text{no incentive to continue, (43)}
\]

Therefore, if the UI system is very generous (\(b, g, \pi\) are high, and \(d\) is low), then (43) is satisfied and (42) is violated, so moral hazard quits occur. On the other extreme, if the UI system is very stingy (\(b, g, \pi\) are low, and \(d\) is high), then (42) is satisfied and (43) is violated, so moral hazard quits do not occur. Finally, since \(V^{0N} > V^{0S}\), there is a generic set of intermediate UI systems such that both (43) and (42) are satisfied, in which case two different equilibria coexist: In one of them, workers quit once they become eligible for UI, while they do not do so in the other.

The intuition for the generic multiplicity of equilibria is the following. In the "good" equilibrium, UI contributions are relatively low because workers eligible for UI do not quit, and workers have no incentive to quit because the UI contributions they have to pay if they remain employed are low. In sharp contrast, in the "bad" equilibrium UI contributions need to be large to finance the expensive UI payments since all workers quit upon earning UI eligibility. The large UI fees, in turn, encourage the workers to quit since they are burdened with large UI contributions if they remain employed.

Figure 13 illustrates the regions of the coexistence of the various types of equilibria for some intermediate values of \(\pi\). For low values of \(k\), new jobs are created, so the normal equilibrium may coexist with the strategic equilibrium. For high values of \(k\), no new jobs are created, so the phase-out equilibrium may coexist with the autarky equilibrium. Finally, for intermediate values of
$k$, the normal equilibrium may coexist with the autarky equilibrium. In this case, the high UI contributions need to finance the expensive UI system not only give workers incentives to quit once they are eligible for UI, but also shut down the creation of new jobs since the profits received by firms are reduced by the large UI fees.

2.4 Labor Cycles in Canada and the United States

This section calibrates the model to data from Canada and the United States allowing for the value of leisure in these two countries to be as high as needed to generate a realistic large volatility in the unemployment-vacancy ratio. In particular, we examine if the similar labor market cycles experienced in the two countries can be generated with reasonably similar values of leisure. This proves to be an insurmountable challenge for the standard version of the Mortensen-Pissarides model where entitlement to UI does not need to be earned (see Chapter 1, or Zhang, 2008). The reason for this difficulty is that Canada has both higher taxes and UI benefits than the United States, which
is inconsistent in those models with the similar amplitude of the cycles experienced by unemployment and vacancies. In the present model, taxes and UI benefits affect the opportunity cost of employment in opposite directions, so there is hope that this challenge can be met. This section also examines if the calibrated values of leisure are neither too high for unemployment not to be a major social concern nor too low to make the UI rules trying to prevent moral hazard behavior nonsensical. Finally, we enquire about the response of unemployment to increases in UI benefits and taxes.

The numerical simulations in the calibration use a discrete time version of the model analyzed in Sections 2.2 and 2.3 with the following specializations. The matching function is assumed to be Cobb-Douglas: 
\[ M(v, u) = u^{1-\eta} v^\eta, \]
where \( \eta \) is the elasticity of the finding rate with respect to the vacancy-unemployment ratio: 
\[ f(\theta) = \mu \theta^\eta. \]
Also, consistent with Shimer (2005a), labor productivity is assumed to follow a stochastic process that satisfies:
\[ p = \ell + \tau + \epsilon y(p^* - \ell - \tau), \]
where \( p^* \) is normalized to one, and \( y \) is a zero mean random variable that follows an eleven-state symmetric Markov process in which transitions only occur between contiguous states. As detailed in the Appendix, the transition matrix governing this process is fully determined by two parameters: the step size of a transition, \( \Delta \), and the probability that a transition occurs, \( \lambda \).

The model period in the simulations is chosen to be one month, so the real interest rate is set to the conventional monthly rate of 0.4 percent. The calibration targets, summarized in Table 9, aim to replicate the main rates, the labor market flows and, in a stylized way, the key features of the taxation

\[^{33}\] The same failure regarding the large difference in the required values of leisure between Canada and the United States would present when risk-aversion preference is introduced into the model. To achieve the endogenous adjustment in the value of leisure that is required by the small-surplus calibration, one can consider a model with worker saving and a high intertemporal elasticity between leisure today and tomorrow as employed in Krusell et al. (2007) or Costain and Reiter (2007). The introduction of saving leads to a strategic saving incentive for workers, who accumulate the assets to improve their outside options in future wage bargaining. In that model, workers save up to a point where the match surplus is small enough at least for the short-term unemployment. The difference in the required value of leisure between these two countries arises due to the same reasoning stated in the text.
and UI systems in the two countries. The data sources and methodological
details in calculating these targets can be found in the Appendix.

<table>
<thead>
<tr>
<th>Table 9</th>
<th>U.S.</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly real interest rate ($r$)</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>Average monthly finding rate ($f$)</td>
<td>0.452</td>
<td>0.309</td>
</tr>
<tr>
<td>Average monthly unemployment rate ($u$)</td>
<td>0.0567</td>
<td>0.0778</td>
</tr>
<tr>
<td>Elasticity of finding rate with respect to $\theta$ ($\eta$)</td>
<td>0.544</td>
<td>0.540</td>
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<tr>
<td>Average vacancy-unemployment ratio $\theta$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Standard deviation of $\theta$ (quarterly in logs)</td>
<td>0.151/0.382</td>
<td>0.191/0.367</td>
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<tr>
<td>Standard deviation of labor productivity (quarterly in logs)</td>
<td>0.020</td>
<td>0.021</td>
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<tr>
<td>Autocorrelation of labor productivity (quarterly in logs)</td>
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<tr>
<td>Average weeks of employment needed for UI eligibility ($1/g$)</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Average weeks before UI benefits expire ($1/d$)</td>
<td>24</td>
<td>33</td>
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<tr>
<td>Average actual UI benefits replacement rate ($bu^t/wu$)</td>
<td>0.111</td>
<td>0.265</td>
</tr>
<tr>
<td>Average tax rate inclusive of UI contributions ($\tau$)</td>
<td>0.300</td>
<td>0.350</td>
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<tr>
<td>Ratio of training costs to quarterly wage rate ($k/w$)</td>
<td>0.550/0</td>
<td>0.370/0</td>
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<tr>
<td>Standard deviation of real wage $w$ (quarterly in logs)</td>
<td>free/0.012</td>
<td>free/0.016</td>
</tr>
</tbody>
</table>

From the labor market, the calibrations aim to replicate the standard devi-
ation and autocorrelation of detrended labor productivity, the standard devi-
ation of the vacancy-unemployment ratio $\theta$, the average monthly finding and
unemployment rates, and the elasticity of the finding rate with respect to the
market tightness $\theta$. Detrended labor productivity and the finding rates were
calculated using the same methodologies as in Shimer (2005a). The average
unemployment rates are directly calculated using standard data from both
countries over the sample periods 1951-2003 for the United States, and 1962-
2003 for Canada. The average vacancy-unemployment ratio $\theta$ is normalized
to be one, which implicitly defines the units in which vacancies are measured
and sets the value of $\mu$ to be the average monthly finding rate. The standard
deviation of $\theta$ is used as the gauge of the amplitude of the cyclical fluctuations in the labor market. In the baseline calibration, we follow Mortensen and Nagypál (2007) and target the standard deviation of $\theta$ conditional on $p$, recognizing in this way that productivity shocks are not the only source of cyclical variations. However, to check how much our results depend on this choice, we also report calibrations using the unconditional standard deviation of $\theta$ as the target. Finally, the elasticity of the finding rate with respect to the market tightness $\eta$ is estimated using the method proposed by Mortensen and Nagypál (2007), which uses the law of motion of unemployment at the steady state and sets $\eta_{US} = 0.544$ in the United States, while $\eta_{CA} = 0.540$ in Canada.

From the UI programs, the calibrations aim to be consistent with the average time it takes for a worker to gain UI eligibility, the average duration of UI benefits, and the average actual replacement rates of UI benefits in the two countries. In the United States, UI eligibility takes around 20 weeks of work and the maximum duration of benefits is around 24 weeks. In Canada, both the time needed for eligibility and the maximum duration of benefits have changed over time and currently depend on the unemployment rate in the region of residence. The targets used in the calibration, 15 weeks to gain eligibility and 33 weeks for the maximum duration of benefits, are representative figures over the sample period. The average actual replacement rate

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34 Card and Riddell (1992) documents that in most states in 1989, UI eligibility requires 20 weeks of work, or the earnings equivalent of 20 weeks of full-time work at the minimum wage, and the maximum duration of benefits lasted around 24 weeks. Similarly, Osberg and Phipps (1995) compares the UI eligibility requirements across states and finds that Texas (relatively less generous state) and New York (relatively more generous state) both set 20 weeks as the minimum employment weeks to qualify in 1992.

35 As to the entitlement UI weeks in Canada, under the UI Act of 1971, regular UI eligibility required a minimum of 8 employment weeks during the base year. In 1977, the minimum employment weeks was replaced by variable entrance requirement (VER) and increased to 10-14 weeks in 1977, then to 10-20 weeks in 1990. Effective in 1997, the VER based on employment weeks was replaced by an entrance requirement based on hours of work. The minimum hours for regular UI benefits ranged from 420-700 hours. We link these two VERs by converting hours of work to full-work weeks. For example, 420 hours is equivalent to 10.5 weeks of full-time work.

36 With respect to the maximum duration of benefits, as reported in Table 4 in “EI Reform
of UI benefits is defined as the ratio of the average weekly UI benefits paid to unemployed workers over the average weekly insurable earnings paid to employed workers. As explained in the Appendix, these rates are obtained as the product of two ratios. The first ratio is the average weekly UI benefits paid to UI recipients over the average weekly insurable earnings paid to employed workers \((b/w)\). The second ratio is the fraction of unemployed workers receiving UI benefits \((u^1/u)\). Finally, the parameter \(\tau\) is interpreted not just as UI contributions but also as a general tax, so the government is using a large fraction of \(\tau\) to finance a public good, which yields separable utility to the constituents of the economy\(^{37}\).

The final parameter that characterizes the UI programs in our model is the probability of collecting benefits after a voluntary quit, \(\pi\). Because of Proposition 2.3, this probability is irrelevant in the determination of output, unemployment, and vacancies as long as moral hazard quits do not occur in equilibrium. In our model with homogeneous workers, if moral hazard quits occurred for some realizations of \(p\), it would generate the strongly counterfactual prediction that occasionally all employed workers eligible for UI would quit at the same time. To avoid this prediction, we set \(\pi\) to the maximum probability that prevents moral hazard quits for all realizations of \(p\). Because of Proposition 2.3, all probabilities lower than this maximum have identical predictions for output, unemployment, and vacancies.\(^{38}\)

In the baseline calibration, the costs of training a worker in the United States are targeted to match the costs reported in the 1982 Employer Oppor-

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37See Annex 4 (Tax Relief: Issues and Options) of "The Economic and Fiscal Update 1999" by the Department of Finance Canada.
38The probability \(\pi\) does affect the real wage, and so its standard deviation. However, this effect turns out to be fairly small in our simulations.
tunity Pilot Project as fraction of the quarterly wage rate: \((k/w)_{US} = 0.55\). This is the same total cost used in Silva and Toledo (2007). In Canada, Goldemberg (2006) estimates that training costs as a fraction of wages are around two-thirds of those in the United States, which implies \((k/w)_{CA} = 0.37\). To check how much our results depend on the presence of these training costs, we also report calibrations without them. Finally, to determine the bargaining weight of workers, our baseline calibration uses the Hosios rule: \(\beta = 1 - \eta\). To check the robustness of this choice, we also conduct a calibration where \(\beta\) is chosen to match the standard deviation of the real wage conditional on \(p\).

The values of \(\{r, \mu, \eta, \beta, \tau, g, d\}\) follow directly from the stated targets in Table 9. The values of the remaining parameters \(\{s, b, c, \Delta, \lambda, k, l\}\) are obtained with the following iterative procedure. First, an initial guess about the values of these parameters is formed. Using this guess the model is simulated for a long horizon (24,000 months), and the initial guess is then revised. This process continues until the predictions of the model match the targets of Table 9. In this procedure, the probability of collecting UI after a voluntary quit \(\pi\) is set tentatively to 1. If with this probability moral-hazard quits occur for some values of \(p\) \((V_1^p < 0 \text{ for all } p \in P)\), then it is revised downward to the maximum value that prevents moral hazard for all possible values of \(p\).

Table 10 displays the calibration results. The upper part of the table describes the specific targets for each particular calibration. Model 1 is our baseline calibration that targets the standard deviation of \(\theta\) conditional on \(p\), uses the Hosios rule to determine the bargaining weight \(\beta\), and incorporates positive training costs. Model 2 deviates from model 1 by targeting the unconditional standard deviation of \(\theta\). Model 3 removes the training costs from the model. Finally, model 4 departs from the Hosios rule and calibrates \(\beta\) by targeting the conditional standard deviation of the real wage. The middle part of the table reports the parameter values that fit the model to the observed target values for the two countries. Finally, the lower part of the table shows some of the implications for each model.
<table>
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<th>TABLE 10 Calibration Results</th>
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All the models are successfully calibrated to our intended targets with similar values of leisure for the United States and Canada. In the baseline Model 1, these values of leisure are respectively 53 and 54.2 percent of average labor productivity. The major reason that these values are so much lower than those in Hagedorn and Manovskii (2008a) is that taxes, which are assumed to be 30 percent in the United States and 35 percent in Canada, are a separate variable in our model. Also, as demonstrated by the calibrations of Models 2 to 4, the value of leisure increases if the amplitude of the business cycle is
targeted to match the unconditional standard deviation of $\theta$, training costs are removed, or the standard deviation of real wages is targeted instead of imposing the Hosios rule. However, as long as the same model is used in both countries, the values of leisure in the United States and Canada are fairly similar.

All our calibrations are consistent with the widespread social concern about unemployment. For each extra worker unemployed, society as a whole loses the gap between labor productivity and the value of leisure $(p - l)$, which in our calibrations is on average between 40 and 47 percent of labor productivity. In addition, for each extra unemployed worker, the cost of posting vacancies increases by $c\theta$ (which averages $c$), and the cost of training workers increases by $fk$. As Tables 9 and 10 imply, these turnover costs vary widely across our simulations, but in all cases they are important. For example, in the Model 1 calibration to data from the United States, for each extra unemployed worker, the average extra cost of posting vacancies increases by 13 percent of labor productivity, and the average extra cost of training workers increases by a further 72 percent of labor productivity. Therefore, in this particular instance, the overall social costs of having an extra worker unemployed adds up to 132 percent of the output the worker would produce if employed.

The costs of unemployment as born by workers are reported in the first two lines of the lower part of Table 10. The first one of these lines reports how many months of pay a worker who is not eligible for UI would be willing to sacrifice to avoid a spell of unemployment. The second line reports the analog number of months for an eligible worker. In the baseline Model 1, an American worker is willing to sacrifice 1.05 months of pay to avoid becoming unemployed before earning UI eligibility and 0.9 months of pay after this eligibility is earned. The analog figures for a Canadian worker are 0.91 months of pay before earning UI eligibility and 0.38 months of pay after eligibility is earned. These costs are substantial but not catastrophic; in particular if the worker falls into the soft safety net of the Canadian UI system. In judging these figures, one must take into account that on average a spell of unemployment lasts only 2.2 months.
in the United States and 3.2 months in Canada. Also, while unemployed, the worker avoids payroll and income taxes and assigns a significant value to the extra leisure. The main reason that the cost of becoming unemployed remains substantial is that upon losing a job the worker incurs a significant loss in human capital, paid in the model through the one-time training costs. As Table 10 reflects, the costs of losing a job are much lower in Model 3 where there are no training costs. They are also lower in Model 4 where the worker appropriates a much smaller fraction of the employment-match surplus.

The design of the UI systems in Canada and the United States shows a clear concern to avoid workers choosing to become or remain unemployed to collect UI. For example, both systems require a long prior employment period to gain UI entitlement and terminate UI when unemployed workers experience a long unemployment spell, which is when they need UI support the most. Because of the homogeneity of workers, our simulations rule out moral-hazard quits to avoid the untenable prediction that all workers entitled to collect UI quit for some realizations of $p$. Allowing for sufficient heterogeneity to induce moral-hazard quits for a small but significant fraction of employed workers is beyond the scope of the present contribution. However, Table 10 (third line of the lower part) reports the minimum critical value of leisure that a deviant worker would need to quit once UI is earned assuming that the worker expects to collect the UI statutory replacement rate (0.4 for the United States and 0.55 for Canada) with certainty\textsuperscript{39,40}. In the baseline Model 1, American workers are only willing to quit to collect UI if their value of leisure is no less than 80 percent of their labor productivity. Therefore, the

\textsuperscript{39}In Canada, according to the 1955 Employment Insurance Acts and the subsequent amendments, the statutory UI replacement rate averaged, over the period 1962-2003, 55 (60 for claimants with dependents) percent of the average yearly insurable earnings in the qualifying period. The value of 0.4 in the United States is from Shimer (2005).

\textsuperscript{40}Since eligible workers in the rest of the economy receive the actual UI replacement rate, which is lower than the statutory rate received by a deviant worker, the resulting critical value of leisure for the deviant worker can be smaller than the value of leisure for the rest of the economy that is calibrated to fit the volatility of the vacancy-unemployment ratio.
concern built in the UI system to avoid moral-hazard unemployment only makes sense if some American workers have values of leisure not too distant from this level. In the alternative models, these critical values of leisure are lower, but except for the model without training costs these critical values are at least 70 percent of labor productivity. In Canada, with a more generous UI system, the analog critical values are much lower, which is consistent with the perception that "abusing" the UI system by some groups of the labor force is a common occurrence in this country.

The last two lines of Table 10 report the semi-elasticities of unemployment with respect to UI benefits and taxes; that is, they report the average percentage increase in unemployment if \( b \) or \( \tau \) increases by 1 percent of labor productivity. As Proposition 2.2 implies, if \( b \) and \( \tau \) increase in tandem to keep the bottom line of the UI finances untouched, unemployment would not be affected. However, if only one of these variables changes, it does have an effect on unemployment. If \( b \) increases, unemployment falls because the jobs needed to earn UI eligibility become more attractive, so the employment-match surplus, and therefore, the number of vacancies posted by firms increase. The bulk of empirical evidence contradicts this negative response. One could realign the model with reality by allowing the increase in \( b \) to induce moral-hazard quits. Unfortunately, to accomplish this without falling in the absurd prediction that once all workers eligible for UI quit, it would require to introduce worker or match heterogeneity which is beyond the scope of the present contribution. We leave this interesting extension to future work. If \( \tau \) increases by 1 percent of labor productivity, unemployment increases. In the Model 1 calibration, this increase is 4.5 percent in the United States and 5.5 percent in Canada. These semi-elasticities are higher than the values typically found in the empirical literature (see Costain and Reiter, 2008), which finds values around 2. However, the empirical estimates are not uncontroversial because the response of unemployment to increases in \( \tau \) depends crucially on how the extra money collected from taxes is used. If it is used, at least in part, to raise UI benefits, then the overall responses of unemployment would be predicted to be smaller.
than the elasticities reported in Table 10. Theoretically, it is easy to perfectly control the increases in $\tau$ while keeping $b$ constant, but in empirical work it is difficult to identify increases in $\tau$ which are uncorrelated with variables that might affect the attractiveness of finding or keeping jobs.

2.5 Conclusion

Once workers have to earn their entitlement to UI benefits with prior employment, a generous UI system is an additional benefit to an employment relationship and as such promotes job creation. This positive entitlement effect counteracts the negative effect derived from the high cost of financing a generous UI system. If individuals are risk neutral, the UI system is fairly priced, and the rules of the UI system prevent moral-hazard unemployment, then the presence and generosity of the UI system have no effect on output, unemployment, and vacancies. As with Ricardian Equivalence, this irrelevance result should be useful to pinpoint the effects of a UI system to violation of its premises. That is, the economic effects of a UI system arise from three sources: The insurance it provides to smooth the income fluctuations experienced by risk-averse workers. The potential financial unbalance if the UI provisions for a segment of the labor force are subsidized or taxed. And, the potential moral-hazard effects on search behavior, acceptance of job offers, and quit decisions.

In itself, the endogenous entitlement to UI benefits as modelled in this chapter does not resolve the current debate about the suitability of the Mortensen-Pissarides model in generating realistic labor market cycles. However, it does bring some insights into the debate. The obvious concern to prevent moral-hazard quits in the design of the UI systems is meaningful only if workers have a sufficiently high value of leisure. For example, in our baseline calibration, we obtain that if all the workers in the United States had values of leisure below 80 percent of their labor productivity, then the UI system would generate no moral-hazard quits. So, at the margin, the concern about moral-hazard
quits would be misplaced. Also, since the generosity of a UI system has an ambiguous and potentially small effect on unemployment, one can reconcile a high response of unemployment to changes in labor productivity with a small response of unemployment to changes in UI benefits. In particular, this is important to resolve why Canada and the United States have similar labor cycles even though taxes and UI benefits are considerably higher in Canada.
Chapter 3  
Unemployment Insurance Eligibility, Moral Hazard and Equilibrium Unemployment

3.1 Introduction

Empirical studies regarding the impact of unemployment insurance (UI hereafter) program on workers’ incentive to work document that changes in UI benefits have significant but modest effect on unemployment (see Solon, 1985; Moffitt and Nicholson, 1982; Meyer, 1990). However, a challenge has been posted in accounted for this observation in various models. For example, Ljungqvist and Sargent (2006) argue that, when the publicly-provided benefits ignored in Prescott (2002) is taken into account, with Prescott’s calibration of the parameters, the standard growth model generates larger movements in employment in Europe than it has experienced. More recently, a similar difficulty is found in the Mortensen-Pissarides search and matching model. Several authors, including Hornstein et al. (2005), Costain and Reiter (2008), and Zhang (2008), criticize that the calibration strategy argued by Hagedorn and Manovskii (2008) brings up some problems with the standard model although it fixes the volatility puzzle. Particularly, the high value of non-market activities required to generate labor market cycles observed in the United States induces dramatic responses of unemployment to labor policy changes. This chapter resolves this problem by introducing some realistic institutional features of the unemployment insurance (UI) system and worker heterogeneity into the Mortensen-Pissarides model.

The consideration of eligibility for UI benefits proves crucial to this success. In the standard model, workers automatically qualify for UI benefits while they are searching for a job. Thus, the UI benefits represent the opportunity cost of employment and, therefore, hurt employment. However, in reality, the UI entitlement must be earned by prior employment and the UI benefits do not last forever. When such features are taken into account, the UI benefits create
a positive effect on the incentive to work, which imposes a downward pressure on unemployment.

The central insight for this positive effect lies in the workers’ desire to gain or retain UI entitlement. Job seekers who are not eligible for UI are eager to be hired in hope of earning UI entitlement through employment. This entitlement effect is stressed in Mortensen (1977). In addition, this contribution extends the entitlement effect to job seekers receiving UI benefits. Due to the positive possibility of losing UI entitlement and the option of retaining it by taking a job, a more generous UI system makes the UI recipients value the UI entitlement more and, thus, more willing to accept a job.

The UI system has distortion effects on workers’ job retention and acceptance behavior. When the government lacks information on job offers and reasons for match dissolutions, workers eligible for UI might quit their jobs or reject job offers to collect UI. Worker heterogeneity is introduced into the model to capture the fact that the UI-seeking behavior largely occurs among low-skilled workers. In line with reality, for a given productivity, the model shows that strategic quits happen to workers who are engaged in low (match-specific) productivity matches. Likewise, job rejections occur when a UI recipient in contact with a firm draws a low match-specific productivity. These moral-hazard effects are reinforced by the presence of the UI contribution fees. Intuitively, a large UI contribution fee required to finance the system reduces a worker’s desire to be employed.

Given the competing effects of the UI system on employment, this chapter

\[41\] The entitlement effect is also studied by Burdett (1979), Hamermesh (1979), van Den Berg (1990), Albrecht and Vroman (2005), and Coles and Masters (2006).

\[42\] Most papers look at the entitlement effect either on the side of the UI nonrecipients, such as that of Mortensen (1977), or on the side of the recipients, for example, that of Albrecht and Vroman (2005).

\[43\] For instance, Green and Riddell (1997) find that many jobs terminate when workers approach the duration that permits a UI entitlement and the strategic terminations are most likely to happen among the low-skilled. Katz and Meyer (1990) report that a sharp increase in the escape rate from unemployment is observed among UI recipients when the benefits are likely to expire. A similar UI seeking behavior is also documented in Alexopoulos and Gladden (2004).
generalizes the validity of the irrelevance effect of the UI system established in Faig and Zhang (2008) (also Chapter 2) in a setting with worker heterogeneity. When the UI system is fully funded in a particular way, the rules of the UI provisions can prevent the moral hazard behavior in job retention and job acceptance, and workers are risk-neutral as commonly assumed in the Mortensen-Pissarides model, the entitlement effect can be perfectly neutralized by the financial cost effect. Consequently, under these premises, the presence and generosity of the UI system are irrelevant to the determination of vacancies, unemployment and output.

However, the UI system prevalent in the United States is neither fully rated nor able to prevent workers with low productivity from quitting their jobs or rejecting job offers. When the model is confronted with the data from the United States and the value of leisure is allowed to be as high as needed to reproduce the observed labor market cycles, the results show that moderate increases in the UI replacement rate lead to increases in the unemployment rate similar to those observed in the U.S. economy. For example, with UI benefits (measured in units of productivity) 1 percent more generous than its current level, the predicted log unemployment rises by 1.4 percent, which squares well with the estimate of 2 in Costain and Reiter (2008). Intuitively, more generous UI benefits raise the disutility of working, which reduces the eligible worker’s expected gains from a match and triggers more moral hazard. Moreover, this effect is amplified by the endogenous job-creation decision made by firms. The rise in job refusals lowers the firms’ expected profits from matches with eligible workers. The eagerness to work by ineligible workers increases the firms’ expected profits. As a result, the firm posts fewer vacancies for the eligible workers and slows down their transitions out of unemployment.\footnote{This mechanism is consistent with the incentive of firms to delay rehiring workers who receive UI.} However, the predicted reaction of unemployment remains realistically modest primarily because these positive effects on unemployment are partially offset by the entitlement effect. More specifically, the ineligible workers’ desire to gain UI
entitlement encourages the firm to create more jobs for UI nonrecipients. The
eligible workers’ incentive to maintain UI entitlement curtails the degree of
job rejections, which improves the firm’s profit and translates into more job
opportunities for UI recipients.

To examine the respective contributions of the entitlement effect and the
moral hazard effect, I shut down UI eligibility rules and moral hazard one at
time from the model. With model parameters recalibrated to fit the same
targets used in the benchmark economy, a comparative statics is conducted
to examine reactions of unemployment to the same changes in benefits. The
results show that both effects are quantitatively important. When the UI
benefits are assumed to be universal and to last forever, a 50 percent rise
in the UI benefits leads the model economy to a corner solution where all
workers become unemployed. However, when the entitlement requirements are
restored, the same change induces only a 5 percent increase in unemployment.
Likewise, when workers are not allowed to decline job offers or quit their jobs,
the moral hazard effect is removed from the model. The calibrated model
shows that unemployment reacts quite little to the UI benefit changes, and
the semielasticity of unemployment drops to 0.8, less than half the size of its
empirical counterpart.

Allowing for strategic quits opens up a door to examine the contribution of
job separation to determining the overall level of unemployment. A large body
of literature, such as Sider (1985), Davis (2005), Fujita and Ramey (2008), and
Menzio and Shi (2008), emphasizes the importance of job separation in explain-
ing the business cycle and long-term movements in unemployment. Agreeing
with this view, I find that strategic quits account for one-third of the reac-
tions of unemployment to the increase in UI benefits although the effect of the
changes in finding rate dominates.

The paper most related to this chapter is Faig and Zhang (2008). How-
ever, that work considers homogeneous workers, which leads to counterfactual
predictions about strategic quits.\footnote{In Faig and Zhang (2008), because of the homogeneity in workers, if one worker decides to quit the job after gaining UI entitlement, all workers would do so simultaneously.} Also, that paper hinges on the positive correlation between the UI benefits and income taxes to generate a positive response of unemployment to the rise in UI benefits. Several recent papers propose alternative ways to reconcile the cyclical and policy-related variations in unemployment. For example, Costain and Reiter (2008) fix the problem with the help of match-embodied technological change. Hagedorn et al. (2008b) reach a similar conclusion by exploring worker heterogeneity in skills.

The rest of the chapter proceeds as follows: Section 3.2 lays out a stochastic version of the model. Section 3.3 studies the main properties and existence of a stationary equilibrium. An irrelevance theorem for a UI program is established. Section 3.4 calibrates the model to data in the United States. Finally, Section 3.5 concludes.

3.2 The Model

3.2.1 Model Environment

Consider a continuous time model economy with an infinite horizon, populated by a continuum of measure one of workers and a large measure of potential firms with free entry into the labor market. Both the workers and the firms are risk-neutral and discount future income at a common rate $r$.\footnote{The reason for choosing risk neutrality preference is to make sure that different results in the quantitative analysis are caused by the introduction of the UI eligibility rule and worker heterogeneity. The main quantitative results are even stronger if risk aversion preference is imposed since risk averse workers care more about UI entitlement.} Firms with vacancies are identical in all respects, while workers searching for jobs differ in UI eligibility. Denote $e$ as the UI eligibility state: some of the unemployed workers are eligible for UI and receive benefits ($e = 1$), while others are not ($e = 0$). To balance the tradeoff between realism and tractability, the UI eligibility is introduced in a stylized way: Ineligible workers would earn UI
entitlement by working for a while and this transition is modeled as a jump stochastic process with an arrival rate $g$. For unemployed workers who are currently collecting UI benefits, they face a risk of losing entitlement over a spell of unemployment, which is also modeled as a jump stochastic process with an arrival rate $d$.\footnote{As will explained in Section 3.4, the parameters $g$ and $d$ are chosen in a way such that the average time required for an ineligible worker to gain UI entitlement and the average duration of benefits predicted from the model equal their empirical counterparts in the United States.}

A firm-worker pair is required to form to carry out production. For this purpose, workers and firms with vacancies search in the labor market to look for a suitable partner. Following the standard directed-search theory, the search process is summarized as a two-stage game: all firms simultaneously post wage contracts that stipulate that worker’s wage is contingent on productivity of a formed match and the worker’s eligibility state over the spell of employment; after observing all wage contracts in the market, workers decide on which firm to apply to.\footnote{The reasons for the choice of the directed search are as follows: First of all, it has become standard in the search theory to adopt the directed search approach to deal with heterogeneity issue. See Acemoglu and Shimer (1999), Shi (2002), Shimer and Wright (2004), Shimer (2005b) and Moen and Rosen (2007). Secondly, the directed search approach is very attractive in that it explicitly models the tradeoff between the wage schedule and the match frequency associated with that wage. This mechanism brings some important benefits: It assures efficient allocations in equilibrium. In addition, it provides an interesting microfoundation for the wage determination. In contrast, the Nash-bargaining wage in the random search framework is determined by splitting the match surplus according to an exogenous bargaining power.} Then, firms randomly pick workers from applicants and commit to the wage contracts posted. Any formed match produces a flow output until it dissolves. The productivity $\bar{p}_p(\epsilon) = \bar{\bar{p}}_p + \epsilon$ consists of two parts. One part $\bar{\bar{p}}_p$ is common to all matches in the economy and the other part $\epsilon$ is assumed to be match-specific, which is realized when workers and firms meet and characterizes the difference in worker’s ability. The subscript $p$ in the common component of productivity denotes the state of the economy, which follows a Markov jump stochastic process with a constant arrival rate $\lambda$ and takes values in a finite support $P \in \mathbb{R}_+^n$.}
Matches are formed as follows. Upon being paired up with firms, workers
draw $\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]$ from an exogenous cumulative distribution function $H(\epsilon)$ and
decide whether to take the job or not. If the workers turn down the offer, they
are allowed to collect UI benefits as long as they are entitled to UI. Otherwise,
an employment relationship is formed and $\epsilon$ stays constant during the spell
of employment. Denote $\epsilon^e_p$ as the critical value of $\epsilon$. The ineligible (eligible)
workers would take a job only when the realized match-specific productivity
exceeds the critical value, $\epsilon \geq \epsilon^0_p \left( \epsilon^1_p \right)$. As long as the UI entitlement is valuable
to workers, the value of $\epsilon^1_p$ is not smaller than $\epsilon^0_p$ simply because the option of
rejecting an offer and continuing receiving benefits leads to a higher outside
option for the eligible workers than for the ineligible ones. Job seekers of type
$e$ are matched with firms as a result of a jump stochastic process with an
arrival rate $f^e_p$. Thus, the effective job-finding rate for workers of type $e$ is
$f^e_p \left( 1 - H \left( \epsilon^e_p \right) \right)$. Workers are assumed to retain their UI eligibility state upon
taking a job.

Quitting jobs, or moral hazard quits, is allowed. Matches dissolve either
due to exogenous separation shocks that come at an arrival rate $s$ or because
of voluntary separations initiated by workers. Since the government cannot
observe the reason for the match separations, workers leaving a job voluntarily
are able to collect full benefits if they are entitled to UI. More realistically,
workers who quit jobs or reject job offers could be caught by the UI agency
with a positive possibility. When taking this into account, the main qualitative
results remain the same. For a given $\epsilon$, an employed worker who recently
becomes eligible for UI is identical in all respects to the one who is eligible for
UI at the time of forming a match, which implies that for a given productivity
state $p$, moral hazard quits only happen among workers who have recently
earned their entitlement and have the match-specific productivity $\epsilon$ falling in
the interval $[\epsilon^0_p, \epsilon^1_p]$.

All unemployed workers receive a flow utility from leisure $l$ regardless of
the UI eligibility state, while eligible workers also gain a flow utility from flat
UI benefits $b$. UI benefits are provided by a government-run UI program that
is financed with UI contribution fees $\tau_p^e$ paid by employed workers. The UI fees depend on both the aggregate state of the economy $p$ and the employed workers’ eligibility state $e$. The government can borrow or save at the interest rate $r$, so the UI program can run temporary deficits or surpluses for the time being. Later on, I will allow for permanent deficits or surpluses by introducing a public good and general taxation.

To facilitate the exposition, I assume here that active firms searching for workers post time-(or tenure)-independent wage contracts $\phi_p$ that specify wages contingent on the productivity state and matched workers’ individual states $(e, \epsilon) : \phi_p = \{w_p^0(\epsilon), w_p^1(\epsilon)\}$. More generally, the wage could be tenure-dependent in the contracts. However, as proved in the Appendix, the optimal time-independent wage is optimal among a wider class of time-dependent contracts as well. Under contract $\phi_p$, eligible workers with $\epsilon \geq \epsilon^1_p$ accept the job and receive the wage $w_p^1(\epsilon)$ until the match breaks down. Analogously, ineligible workers with $\epsilon \geq \epsilon^0_p$ accept the job and receive the wage $w_p^0(\epsilon)$ until the match dissolves exogenously or until they gain the UI entitlement. Workers with a newly earned eligibility receive the $w_p^1(\epsilon)$ over the rest of the spell of employment if they choose not to quit; otherwise, they become unemployed and collect UI benefits $b$. An active firm posts a vacancy at a flow cost $c$. When a production process starts, the firm gains a flow profit $\hat{p}_p(\epsilon)$ net of the labor costs $w_p^e(\epsilon) + \tau_p^e$.

For tractability purpose, both $l$ and $b$ are assumed to be positive, and $l$ is assumed to satisfy: $\bar{p}_p + \epsilon - \tau_p^0 \geq l$, for all $p \in P$. That is, for any realized values of $\epsilon$ and $p \in P$, the surplus from the match with ineligible workers would never fall below zero. This implies that the ineligible workers are always willing to take a job. Hence, $\epsilon^0_p = \epsilon$ for all $p \in P$ (the critical value is at the frontier of admissible values).

There are $m \geq 1$ submarkets. Suppose $\phi_{jp}$ is the wage contract posted in the $j$th submarket for a given productivity state $p$. Workers choose from the set of posted wage contracts $\{\phi_{jp} : j = 1, 2, ..., m, \text{ and } p \in P\}$. I refer to the set of firms posting $\phi_{jp}$ and the set of workers who direct their search to this
wage contract as submarket $j$. In this particular submarket, for a given $p \in P$, denote $u^e_j$ and $v^e_j$ as the respective measure of searching workers of type $e$ and vacancies to be filled by type-$e$ unemployed workers, and $\theta^e_j$ as the vacancy-unemployment ratio (also called market tightness). Workers and firms are paired up together by a constant returns to scale matching function, which is Cobb-Douglas in the measure of type-$e$ unemployment $u^e_j$ and vacancies $v^e_j$:

$$M(v^e_j; u^e_j) = \mu (u^e_j)^\eta (v^e_j)^{1-\eta}.$$  

The symmetry across the workers in the submarket implies that the matching rate, $f(\theta^e_j)$, at which the workers are matched with jobs is equal to the number of matches divided by unemployment of type $e$. Likewise, the rate $q(\theta^e_j)$, at which the firms have the vacancies paired up with workers, is equal to the number of matches divided by the measure of vacancies.\footnote{I suppress for convenience the dependence on $p$ in the notations of the unemployment, vacancies, market tightness, and turnover rates.} The elasticity of the matching rate with respect to the market tightness, $1 - \eta$, satisfies $\eta \in (0, 1)$.\footnote{Since the matching function is Cobb-Douglas in $u$ and $v$, the value of $\eta$ is independent of $\theta$ and thus constant for any $p \in P$ and $e \in \{0, 1\}$.} The rates $f(\theta^e_j)$ and $q(\theta^e_j)$ have the following relationship:

$$f(\theta^e_j) = \mu \cdot (\theta^e_j)^{1-\eta} = \theta^e_j \mu \cdot (\theta^e_j)^{-\eta} = \theta^e_j q(\theta^e_j). \quad (44)$$

### 3.2.2 Bellman Equations

In this part, I focus on a particular submarket $j$. In equilibrium, there exists a single market for each type of unemployed workers. To save on notation, I drop $j$ in the subscript hereafter. Workers may be in one of four possible states depending on their employment state and UI eligibility state. Likewise, firms paired with workers may be in one of two possible states depending on their worker’s eligibility for UI. Contingent on the aggregate state $p$ and the realized match-specific productivity $\epsilon$, denote $W^\epsilon_p(\epsilon)$ and $U^\epsilon_p$ as the values of being an employed worker and an unemployed worker of type $e$, respectively.
Similarly, denote $J^e_p(\epsilon)$ and $V^e_p$ as the values of a firm hiring a worker of type $e$ and a firm with a vacancy, respectively. Denote $E_p X_p$ as the expected values of $X$ ($W(\epsilon), U, J(\epsilon),$ and $V$) conditional on $p$ when the economy experiences a change in the productivity state. The utility values are recursively defined by the following Bellman equations.

**Workers’ Problem**

An unemployed worker ineligible for UI receives a flow utility from leisure plus the expected gains or losses from being matched with a firm and a change in productivity, which happen with arrival rates $f^0_p$ (or $f^0(\theta^0_p)$) and $\lambda$, respectively. Given $\epsilon^0_p = \epsilon$ for all $p \in P$, any ineligible worker is willing to take a job.

$$rU^0_p = l + f^0_p \left[ \int_{\epsilon^0}^{\tilde{\epsilon}} W^0_p(\epsilon) dH(\epsilon) - U^0_p \right] + \lambda \left( E_p U^0_p - U^0_p \right).$$  \hspace{1cm} (45)

An unemployed worker receiving UI earns a flow utility from both leisure and UI benefits. The expected gains or losses come from being matched with a firm, losing UI entitlements, and experiencing a productivity change. The associated arrival rates are $f^1_p, d,$ and $\lambda$, respectively. Upon being paired with a firm, the worker with $\epsilon \geq \epsilon^1_p$ accepts the job. Otherwise, the worker rejects the offer and continues collecting UI benefits.

$$rU^1_p = l + f^1_p \left[ \int_{\epsilon^1_p}^{\tilde{\epsilon}} W^1_p(\epsilon) dH(\epsilon) + H(\epsilon^1)U^1_p - U^1_p \right] + d \left( U^0_p - U^1_p \right) + \lambda \left( E_p U^1_p - U^1_p \right).$$ \hspace{1cm} (46)

An employed worker ineligible for UI receives a wage $w^0_p(\epsilon)$ plus the expected gains or losses from exogenously losing the job, becoming eligible for UI and experiencing a change in productivity, which occur with the respective arrival rates $s, g$ and $\lambda.$
An employed worker with UI eligibility chooses whether to quit the job or not. The worker becomes unemployed and collects full benefits after quitting the job. Otherwise, the worker receives a wage $w_p^1(\epsilon)$ plus the expected gains or losses from exogenously losing the job at an arrival rate $s$ and a productivity change at an arrival rate $\lambda$.

$$rW_p^0(\epsilon) = w_p^0(\epsilon) + s \left[ U_p^0 - W_p^0(\epsilon) \right] + g \left[ W_p^1(\epsilon) - W_p^0(\epsilon) \right] + \lambda \left[ E_p W_p^0(\epsilon) - W_p^0(\epsilon) \right], \forall \epsilon. \quad (47)$$

Firms’ Problem

A firm hiring an ineligible worker obtains the flow profits $(\hat{p}_p(\epsilon) - w_p^0(\epsilon) - \tau_p^0)$ plus the expected gains or losses from the exogenous match dissolution, the worker’s gaining UI eligibility and a productivity change. The associated arrival rates for these events are $s$, $g$ and $\lambda$.

$$rJ_p^0(\epsilon) = \left[ \hat{p}_p(\epsilon) - w_p^0(\epsilon) - \tau_p^0 \right] - sJ_p^0(\epsilon) + g \left[ J_p^1(\epsilon) - J_p^0(\epsilon) \right] + \lambda \left[ E_p J_p^0(\epsilon) - J_p^0(\epsilon) \right], \forall \epsilon. \quad (49)$$

A firm with an eligible worker either gains nothing if the worker quits the job, or receives the flow profits $(\hat{p}_p(\epsilon) - w_p^1(\epsilon) - \tau_p^1)$ plus the expected gains or losses from an exogenous match separation and a productivity change that occur at arrival rates $s$ and $\lambda$, respectively.

$$rJ_p^1(\epsilon) = \max \left\{ 0, \left[ \hat{p}_p(\epsilon) - w_p^1(\epsilon) - \tau_p^1 \right] - sJ_p^1(\epsilon) + \lambda \left[ E_p J_p^0(\epsilon) - J_p^1(\epsilon) \right] \right\}, \forall \epsilon. \quad (50)$$
The value of a firm with a vacancy $V_p^e$ is defined by

$$rV_p^e = -c + q_p^e \int_{\epsilon_p^e}^{\tau} J_p^e(\epsilon)dH(\epsilon) = 0, \text{ for } e = 0, 1. \quad (51)$$

A firm posts vacancies in the submarket with workers of type $e$ until the flow cost of posting a vacancy equals the expected gains from filling it, which occurs at an arrival rate $q_p^e \left(1 - H(\epsilon_p^e)\right)$. The free entry condition drives the value of $V_p^e$ to be zero.

### 3.2.3 Competitive Search Equilibrium

In equilibrium, if a worker of type $e$ enters the $j$th submarket, this submarket must yield the worker the highest $U_p^e$. Let $U_p^e$ denote the equilibrium utility of being a type-$e$ unemployed worker conditional on $p$, then it must satisfy: $U_{p,j}^0 = U_p^0$ and $U_{p,j}^1 = U_p^1$, for $j = 1, 2, \ldots, m$.

For expositional purposes, conditional on $p$ and $e$, denote $R_p^e(\epsilon)$ as the worker’s \textit{ex post} gains from a match for a given $\epsilon$ and $R_p^e$ as the worker’s \textit{ex ante} gains from a match. Analogously, conditional on $p$ and $e$, denote $S_p^e(\epsilon)$ and $S_p^e$ as the firm-worker pair’s \textit{ex post} match gains for a given $\epsilon$ and \textit{ex ante} match gains, respectively. Note that due to strategic quits, $R_p^1(\epsilon) = 0$ and $S_p^1(\epsilon) = 0$ for $\epsilon \in [\xi, \epsilon_p^1]$. Hence, $R_p^1 \equiv \int_{\epsilon_p^0}^{\tau} R_p^1(\epsilon) \ dH(\epsilon) = \int_{\xi}^{\tau} R_p^1(\epsilon) \ dH(\epsilon)$ and $S_p^1 \equiv \int_{\epsilon_p^0}^{\tau} S_p^1(\epsilon) \ dH(\epsilon) = \int_{\xi}^{\tau} S_p^1(\epsilon) \ dH(\epsilon)$. Therefore, in equilibrium, $R_p^e$ and $S_p^e$ can be defined as:

$$R_p^e = \int_{\xi}^{\tau} R_p^e(\epsilon) \ dH(\epsilon) = \int_{\xi}^{\tau} \left[W_p^0(\epsilon) - U_p^e\right] \ dH(\epsilon), \text{ for } e = 0, 1. \quad (52)$$

$$S_p^e = \int_{\xi}^{\tau} S_p^e(\epsilon) \ dH(\epsilon) = \int_{\xi}^{\tau} \left[R_p^e(\epsilon) + J_p^e(\epsilon)\right] \ dH(\epsilon), \text{ for } e = 0, 1. \quad (53)$$
Substituting (52) into (45) and (46) gives:

\[ rU_p^0 = l + f_0^0R_p^0 + \lambda \left( E_pU_p^0 - U_p^0 \right). \]  (54)

\[ rU_p^1 = l + \frac{r}{2}b + \frac{r}{d} \left[ f_1^1R_p^1 + \lambda \left( E_pU_p^1 - U_p^1 \right) \right] + \frac{d}{r + d} \left[ f_0^0R_p^0 + \lambda \left( E_pU_p^0 - U_p^0 \right) \right]. \]  (55)

The critical value of \( \epsilon_p^1 \) is determined by:

\[ S_p^1(\epsilon_p^1) = 0, \text{ if } \epsilon_p^1 \in [\xi, \tau], \text{ or} \]
\[ \epsilon_p^1 = \xi, \text{ if } S_p^1(\xi) > 0, \text{ or} \]
\[ \epsilon_p^1 = \tau, \text{ if } S_p^1(\tau) < 0. \]  (56)

For a given \( p \), if \( \epsilon_p^1 \) falls into the range \( [\xi, \tau] \), an eligible worker is indifferent between taking a job and declining it. When \( S_p^1(\xi) > 0 \), any eligible worker receives positive gains from forming a match, so \( \epsilon_p^1 = \xi \). Similarly, when \( S_p^1(\tau) < 0 \), any eligible worker suffers losses from forming a match, so \( \epsilon_p^1 = \tau \).

Let \( \Phi \) denote the set of wage contracts \( \phi_p \) in all submarkets for any \( p \in P \), and \( \Phi^f \) the set of feasible wage contracts that satisfy the participation constraints of the worker and the firm. From a worker’s perspective, a worker of type \( e \) enters the submarket that offers the highest expected utilities \( U_p^e \). Equations (54) and (55) imply that the attractiveness of a submarket (or a wage contract) can be summarized by the expected gains \( R_p^e \). From a firm’s perspective, taking \( U_p^e \) as given, a firm chooses wage contract \( \phi_p \) to maximize \( V_p^e \). Hence, contingent on \( p \), the firm’s maximizing problem can be expressed as

\[
\max_{R_p^e(U_p^e)} \left\{ \max_{\phi_p \in \Phi^f(R_p^e)} \left[ -c + q_p^e(f_p^e(R_p^e)) \int_{\epsilon_p^e} \int_{\epsilon_p^e} J_p^e(\epsilon)dH(\epsilon) \right] \right\}, \text{ for } e = 0, 1, \text{ and } p \in P. \]  (57)

The resulting value of a firm with a vacancy can be written as \( V_p^{e\max}(U_p^e) \). The
free entry condition assures that

\[ V^{e_{\text{max}}}(U^e_p) = 0, \text{ for } e = 0, 1. \]  \hfill (58)

**Definition 3.1** The competitive search equilibrium is a vector of \((U^e_p, f^e_p, R^e_p)_{p \in P, e \in \{0, 1\}}\), and a wage contract \(\phi^*_p\) which solve the maximization problem (57) and satisfy (58).

**Proposition 3.1 (Validity of the Hosios Rule)** In the competitive search equilibrium in the submarkets with workers of type \(e\), the Hosios condition holds.

The validity of the Hosios rule in each submarket implies that the optimal wage contract in the competitive search equilibrium is equivalent to the Nash bargaining wage in an economy with undirected search. In that economy, unemployed workers are separated into two labor markets according to their eligibility state; firms and workers search randomly in each labor market; and the wage is determined bilaterally by a generalized Nash bargaining rule upon forming a match. The Hosios rule suggests that a worker’s bargaining power in wage negotiation equals his contribution to contacting a firm, which is characterized by the parameter \(\eta\) in the matching function. Suppose the worker’s bargaining power is \(\beta\), then \(\beta = \eta \in (0, 1)\). Therefore, in the competitive search equilibrium, conditional on productivity state \(p\) and the employed worker’s individual state \((e, \epsilon)\), the worker and firm share the match surplus according to the following rule:

\[ R^e_p(\epsilon) = \beta S^e_p(\epsilon), \text{ and } J^e_p(\epsilon) = (1 - \beta) S^e_p(\epsilon), \text{ for } e = 0, 1. \]  \hfill (59)
3.2.4 Characterization of Equilibrium

**Definition 3.2** For a given productivity \( p \), the competitive search equilibrium is a set of functions \( (w^e_p(\epsilon), \theta^e_p, U^e_p, W^e_p(\epsilon), J^e_p(\epsilon), S^e_p(\epsilon))_{\epsilon=0,1} \) and \( \epsilon^1_p \), which satisfy the Bellman equations (45)-(50), the free entry condition (51), the definitions of match surplus (52)-(53), the equation (56) determining the critical value \( \epsilon^1_p \) and the surplus sharing rule (59) with \( s \) satisfying the Hosios’ rule. Define \( \hat{\rho}^e = \frac{\int_{x=0}^{x=\epsilon^1_p} \hat{\rho}^e_p(x) \, dx}{1-H^e_p(\epsilon)}, \) \( H^e_p = H(\epsilon^1_p), \) \( \hat{U}_p \equiv U^1_p - U^0_p \) and \( \hat{B}_p \equiv S^1_p - S^0_p + \hat{U}_p. \) This system of equations can be rewritten by the following functional equations:

\[
c\theta^e_p = (1-\beta) f^e_p S^e_p, \text{ for } e = 0, 1. \tag{60}
\]

\[
\hat{U}_p = \frac{b + c\beta (1-\beta)^{-1} \left[ \theta^1_p(S^1_p) - \theta^0_p(S^0_p) \right] + \lambda \left( E_p \hat{U}_p - \hat{U}_p \right)}{r + d}. \tag{61}
\]

\[
\epsilon^1_p = \frac{1}{\tau^1_p} + \frac{b + c\beta (1-\beta)^{-1} \theta^1_p - d\hat{U}_p - \hat{p}_p}{\epsilon^1_p \in [\epsilon, \bar{\epsilon}].} \tag{62}
\]

\[
S^1_p = \frac{1 - H^1_p}{H^1_p} \left[ \hat{p}^1_p - l - c\beta (1-\beta)^{-1} \theta^1_p(S^1_p) \right] + (1 - H^1_p) \left[ d\hat{U}_p - b - \tau^1_p \right] + \lambda \left( E_p S^1_p - S^1_p \right). \tag{63}
\]

\[
S^0_p = \frac{\hat{p}^0_p - l - c\beta (1-\beta)^{-1} \theta^0_p(S^0_p) \right] + \left( g\hat{B}_p - \tau^0_p \right) + \lambda \left( E_p S^0_p - S^0_p \right). \tag{64}
\]

(see Appendix for the derivation of 61-64).

The expected present discounted value of the UI eligibility for an unemployed worker, \( \hat{U}_p \equiv U^1_p - U^0_p \), is defined by equation (61). For a given \( p \), the value of \( \hat{U}_p \) consists of two components: one is the expected present discounted value of the UI benefits received by an eligible worker over the spells of unemployment; the other is the worker’s expected present discounted value of the
incremental gains or losses from future employment. The term "incremental" means the difference in the expected gains from an employment elsewhere if the worker is eligible for UI \((e = 1)\) or not \((e = 0)\). Equation (60) suggests that that \(\theta^e (S^e)\) is increasing in \(S^e\), so the two components have competing effects on \(\hat{U}_p\) for \(S^p_0 \geq S^p_1\). In this case, the market condition is more favorable for the ineligibles than the eligibles \((f^0_p > f^1_p)\). Therefore, the worker who considers gaining a UI entitlement by taking a job is aware that if he accepts the offer, he would be less likely to be matched with a firm than he would be otherwise. As a result, the concern over the future employment opportunity makes the UI eligibility less desirable.

Equations (63) and (64) define the ex ante surplus from the match with an eligible worker and an ineligible worker, respectively. In both types of matches, the ex ante surplus equals the expected present discounted value of benefits from the match, which contains three parts: labor productivity net of the value of leisure and the worker’s expected gains from future employment if the current match breaks down exogenously, the expected net benefits from the UI system (also called opportunity benefits of employment), and the expected gains from a productivity change. These benefits are obtained conditional on establishing an employment relationship after the contact of a worker and a firm, which happens with probability \((1 - H^1_p)\) for the eligible worker, while \(1\) for the ineligible worker.

Notice that upon accepting a job, the expected net benefits from the UI system for an eligible worker are \(\left(d\hat{U}_p - b - \tau^1_p\right)\). The eligible worker benefits from avoiding the potential losses of UI entitlement which comes at an arrival rate \(d\), gives up the UI benefits he has been receiving and starts to pay the UI contribution fees. The term \(d\hat{U}_p\) positively enters (63) simply because the possibility of losing UI entitlement creates an incentive for the eligible worker to take a job to retain the UI entitlement, which lowers the worker’s bargaining position and gives rise to a higher match surplus. This is the entitlement effect for eligible workers. In contrast, the UI benefits and UI fees \(b + \tau^1_p\) represent the disincentive (or moral hazard) effect of the UI system and the cost of financing
Likewise, the expected net benefits from the UI system for an ineligible worker are \( (g\hat{B}_p - \tau^0_p) \). The term \( g\hat{B}_p \) captures the expected gains from earning the UI entitlement. Upon earning UI eligibility, a worker obtains the value of \( \hat{U}_p \) and starts to extract his gains from surplus \( S^1_p \) instead of \( S^0_p \). The term \( g\hat{B}_p \) represents the *entitlement effect for the ineligible workers* in Mortensen (1977).

The equilibrium values of \( \{\theta^e_p, S^e_p\}_{e \in \{0,1\}} \) and \( \{\hat{U}_p, \epsilon^1_p\} \) can be solved from the system of equations (60)-(64). Given \( S^e_p \), the values of \( U^e_p \) can be solved by combining (54)-(55) with the surplus-sharing rule (59). Given \( \hat{U}_p \) and \( \theta^e_p \), the equilibrium wages \( w^e_p(\epsilon) \) can be solved by combining (49)-(50) with (53) and (59).

### 3.3 Deterministic Equilibrium

To analyze how the UI system affects the equilibrium objects in the labor market, following Mortensen and Nagypál (2007) and Pissarides (2007), this section studies the deterministic version of the economy described above.\(^{51}\) To simplify the analysis, the \( H(\epsilon) \) is assumed to be the uniform distribution.

Since the worker has a desire to reject job offers or quit their jobs to collect UI benefits if and only if \( \hat{U} \geq 0 \), I limit myself to the case where \( \hat{U} \geq 0 \) in what follows. In addition, the critical value \( \epsilon^1 \) could be lower than \( \epsilon \) or larger than \( \bar{\epsilon} \) depending on the degree of UI generosity relative to productivity of a formed match.\(^{52}\) To avoid uninteresting results, I focus on the case in the

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\(^{51}\) All authors argue that the matching flows are large, and productivity is rather persistent, which suggests that unemployment quickly converges to its steady state. Hence, there is no loss of generality to derive the cyclical results from comparative statics with a continuous-time model that compare steady states at different realizations of labor productivity.

\(^{52}\) When the UI benefits are stingy and the UI fees are small compared to match productivity, \( \epsilon^1 = \bar{\epsilon} \) and the model economy ends up in an equilibrium without moral hazard unemployment. On the contrary, if the UI benefits are quite generous and the UI fees are large relative to the match productivity, \( \epsilon^1 = \bar{\epsilon} \) and all eligible unemployed workers would choose not to work.
middle where $\epsilon^1 \in [\underline{\epsilon}, \bar{\epsilon}]$ in the following analysis.$^{53}$

**Proposition 3.2 (Property and Existence of Equilibrium)** If the UI contribution fees $\tau^e$ satisfy: (a) $\tau^0 \leq \overline{p} + \underline{\epsilon} - l$; (b) $\tau^1 \leq \overline{p} + \underline{\epsilon} - l - b$; and (c) $\tau^1 \leq \tau^0 \leq \tau^1 + m$, where $m = \frac{r + g}{r + d}b - \frac{\tau^e}{2} \geq 0$, then a stationary equilibrium exists and has the following properties: $0 \leq S^1 \leq S^0$ and $0 \leq \hat{B} \leq \hat{U} \leq \frac{b}{r + d}$.

The value of the UI fees $\tau^e$ plays an important role in shaping the properties of the equilibrium. To assure a positive value of $\hat{U}$, the UI-eligibles pay lower UI fees relative to the UI-ineligibles. Intuitively, this helps keep the surplus $S^1$ from falling too low relative to $S^0$. If the relative value of $S^1$ is tiny, as pointed out in Section 3.2.4, the concern about future employment opportunities would outweigh the expected value of the UI benefits for ineligible workers with job offers in hand. Consequently, the job offers become no longer attractive and the value of $\hat{U}$ goes negative. On the other hand, to establish the existence of equilibrium, the UI fees paid by the UI-eligibles should not be too small compared to those contributed by the UI-ineligibles. If $\tau^0$ is considerably larger than $\tau^1$, the match surplus $S^0$ would become considerably smaller than $S^1$, which makes the UI entitlement extremely attractive and leads to an unbounded $\hat{U}$.$^{54}$

The bounds of $\hat{U}$ imply that the presence of the UI system has different effects on the worker’s willingness to take a job without the consideration of the effect of the financial costs of the UI system $\tau^e$. For eligible workers, since $\hat{U} \leq \frac{b}{r + d}$, the entitlement effect $d\hat{U}$ is dominated by the disincentive effect $b$. This translates into expected losses upon accepting a job. In contrast, the ineligibles workers always benefit from earning UI eligibility through taking a job since $g\hat{B} \geq 0$.

$^{53}$See Appendix for the derivation of the condition for $\epsilon^1 \in [\underline{\epsilon}, \bar{\epsilon}]$.

$^{54}$The restriction of $\tau^0 \leq \tau^1 + m$ is a sufficient condition that assures $\hat{U}$ to be bounded from above.
3.3.1 Irrelevance of the UI System

In this part, I study a case where the UI fee \( \tau^e \) is an endogenous variable such that it adjusts to fully finance the UI system. In this theoretical case, I show that the UI generosity would have no effect on the labor market outcomes if the rules of UI provisions can eliminate moral hazard from becoming or remaining unemployed, and the UI system is fairly priced in the way that each type of employed workers receives zero expected gains from the system.\(^{55}\)

**Definition 3.3** A fully funded UI system is one in which the expected present discounted value of net benefits from the UI system for a worker who is newly hired but not yet entitled to UI is zero.

Since in the absence of the UI system there is a single market, for the UI system to be irrelevant it must be the case that firms are indifferent between hiring an eligible and an ineligible worker, which implies that the markets for these two types of workers have the same finding rate (essentially becoming a single merged market). The following Lemma establishes the condition on UI fees that leads to this indifference.

**Lemma 3.1** In a fully funded UI system, the markets for eligible and ineligible workers will have identical finding rate \( f \) if the UI contribution fees \( \tau^e \) paid by employed workers of type \( e \) satisfy the following condition:

\[
\left[ r + s - \frac{sf}{r + d + f} \right] \tau^0 + g \tau^1 = \frac{gsb}{r + d + f}.
\] (65)

\(^{55}\)An irrelevance result for the UI system with homogeneous workers is established in Faig and Zhang (2008).
Proposition 3.3 (Irrelevance of the UI System) For an economy defined in Lemma 3.1, if the UI contribution fees \( \tau^e \) are defined as:

\[
\begin{align*}
\tau^0 &= g\hat{B}, \\
\tau^1 &= d\hat{U} - b,
\end{align*}
\]

(66) \hspace{1cm} (67)

where \( \hat{B} = \frac{d\hat{U} + r^p - \tau^1}{r + s + g} \), and \( \hat{U} = \frac{b}{r + d} \), then the level of UI benefits, the duration of UI benefits, and the time required to gain UI entitlement are all irrelevant for the determination of output, unemployment, and vacancies.

**Proof:** It is easy to check that the UI fee \( \tau^e \) defined by (66)-(67) satisfies the condition of (65), so the UI system is fully funded under such a scheme of UI fees. Next, I prove the irrelevance result.

Substituting the UI fee scheme characterized by equations (66)-(67) into equations (63)-(64) for a given \( p \), the terms measuring the expected net benefits from the UI system for eligible and ineligible workers, \( \left(d\hat{U} - b - \tau^1\right) \) and \( \left(g\hat{B} - \tau^0\right) \), become zero. As a result, all the UI policy parameters \( g, d, b \) drop from equations that determine the equilibrium outcomes. In addition, as defined in Lemma 3.1, the two markets in this economy share the same finding rate \( f \). This implies that firms are indifferent between hiring eligible workers and ineligible workers, so \( S^0 = S^1 \). Applying the condition \( \tau^0 \leq \bar{p} + \xi - l \) in Proposition 3.2 and comparing equation (63) with (64) for a given \( p \), one can easily prove that \( e^1 = \xi \), which means that moral hazard unemployment disappears from the model.

To understand why the moral hazard behavior can be fully eliminated under the above UI fee scheme, it is worth noting that this irrelevance result requires that the UI system gives a subsidy to the UI-eligibles, and collects the UI fees only from UI-ineligibles since \( 0 = g\hat{U} > 0, \tau^1 = -r\hat{U} < 0 \).

\[\text{Note that since two markets merge into one, } S^0 = S^1, \text{ which implies } \hat{B} \equiv \hat{U}.\]
presence of the subsidy makes job offers more attractive for the UI-eligibles. Hence, with everything else equal, eligible workers become more willing to accept job offers, which moves the critical value $\epsilon^1$ towards $\epsilon$ and reduces the incidence of moral hazard.\textsuperscript{57} In addition, since the UI fee scheme is determined in a way that both types of workers gain nothing from the UI system and face the identical finding rate. Therefore, the resulting economy resembles the one with a single market and no UI system. It then follows that $\epsilon^1 = \epsilon$.

The intuition behind the irrelevance result is straightforward. By imposing the UI fee scheme characterized by equations (66)-(67), the financial cost effect perfectly neutralizes the (net) entitlement effect within each group of the employed. By offering subsidy to the UI-eligibles and collecting the UI fees from the UI-ineligibles so as to assure zero expected gains from the UI system and the same finding rate for both types of workers, the moral hazard effect of the UI system is eliminated. As a result, the UI system has no impact on the labor market outcomes.

It is interesting to explain why this irrelevance result requires a subsidy to the UI-eligibles. Proposition 3.2 shows that eligible workers incur expected losses from taking a job. So they are choosier about job opportunities. In response, firms provide high wages, which reduces their profits and hurts job creations. To remove this distortion and restore an efficient number of vacancies, the UI system subsidizes the firms’ hiring activities by offering financial support to the eligible worker to offset the net disincentive effect. The amount of subsidy is determined by the rule that the expected net benefits from the UI system must be zero, so $\tau^1 = d\hat{U} - b$.\textsuperscript{58}

However, the conditions under which the irrelevance result holds seem too strong to be satisfied in reality. It is unlikely that the UI provisions can completely rule out moral hazard behavior, and the prevalent UI system in the

\textsuperscript{57}The movement toward $\epsilon$ is also reinforced by the rise in the value of $\hat{U}$ as indicated by the equation (61) for a given $p$.

\textsuperscript{58}If the probability of collecting UI upon job rejection were lower than one, the subsidy to eligible workers could be reduced to the point that these workers end up paying positive UI fees.
United States may not fully funded by the UI fees in the way stated in Proposition 3.3. Hence, the realistic UI system does affect the key variables in the labor market, such as output, vacancies, and unemployment; and the final result depends on whether the entitlement effect dominates the two disincentive effects: moral hazard effect and financial costs effect.

3.4 Quantitative Analysis

This section calibrates a discrete time version of the model laid out in Section 3.2. The calibration targets aim to replicate the main rates and flows in the labor market and, in a stylized way, the key features of the taxation and UI systems in the United States. The model period in the simulations is set to be one week. The consecutive periods are aggregated to construct monthly or quarterly series to match the implications of the model with properties of empirical series observed at those frequencies.

3.4.1 Parameterization

The interest rate is set to target the annual rate of 4.8 percent. As to the technology and matching parameters, the elasticity parameter in the matching function $\eta$ is set to match the average unemployment rate over the period 1962-2001: $U^{ss} = 5.55\%$. In the model, at the steady state, the flows out of employment due to exogenous separations and moral hazard quits equal the flows into employment by finding jobs with success. The aggregate unemployment $U^{ss}$ is expressed as:

$$U^{ss} = \frac{s}{s + f^0Z}, \text{ and } Z = \frac{gf^1(1 - H^1) + gd(1 - H^1) + sd}{gf^0 + gd + sd}.$$  (68)

The value of $\mu$ is chosen by matching the aggregate monthly job-finding rate to its empirical counterpart 0.452 given in Shimer (2005a). In the model, the aggregate finding rate conditional on $p$ is calculated as the number of workers
who find a job within a week divided by the number of the total unemployment in that week. The value of $\beta$ is pinned down by applying the Hosios rule, so $\beta = \eta$.

With respect to the flow turnover cost $c$, Silva and Toledo (2007) report that the total recruiting costs account for 14 percent of the quarterly pay per hire, which implies that the total cost of maintaining a vacancy $C$ equals 1.68 times the weekly pay per hire. The value of $C$ can be measured as the weighted average of the turnover costs in the two markets for a given $p$: 

$$C = \frac{V^1}{V^1 + V^0} \frac{q^\varepsilon q^1 (1-H^1)}{q^1} + \frac{V^0}{V^1 + V^0} \frac{c^0}{q^0},$$

where $V^e$ measures the number of vacancies posted for unemployed workers of type $e$.

It remains to specify the parameters in productivity and the separation rate $s$. The match-specific productivity $\epsilon$ is assumed to be drawn from a uniform distribution with the lower bound $\xi$ normalized to zero (so $\overline{p}_p$ is the lowest productivity in a match). As to the upper bound $\overline{\tau}$, since the spread of the match-specific productivity affects the degree of moral hazard, and then the spell of employment, the value of $\overline{\tau}$ is chosen to fit the job tenure distribution constructed by Diebold et al. (1997) as shown in Figure 14. More specifically, following Menzio and Shi (2008), the separation rate $s$ and the upper bound $\overline{\tau}$ are determined by minimizing the distance between the tenure distribution generated by the model and its empirical counterpart. The aggregate productivity, for a given $p$, is the weighted average of the expected productivity of two types of matches: 

$$\hat{p} = \frac{E^0}{E^1 + E^0} \left( \overline{p} + \frac{\epsilon}{2} \right) + \frac{E^1}{E^1 + E^0} \left( \overline{p} + \frac{\epsilon + \epsilon^1}{2} \right),$$

where $E^e$ measures the number of employed workers of type $e$. The mean of the weekly aggregate productivity $\hat{p}$ is normalized to one. Following Shimer (2005a), the common part of productivity $\overline{p}$ is assumed to follow a stochastic process that satisfies: 

$$\overline{p} = l + \tau^0 + e^y (\overline{p}^x - l - \tau^0),$$

where $\overline{p}^x$ is the mean of $\overline{p}$ and is determined by targeting the average productivity $\hat{p}$ that is normalized to one; $y$ is a zero mean random variable that follows an eleven-state symmetric Markov process in which transitions only occur between contiguous states.

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59 The choice of the distribution does not affect the main qualitative results in this paper.
As detailed in Zhang (2008), the transition matrix governing this process is fully determined by two parameters: the step size of a transition $\Delta$, and the probability that a transition occurs $\lambda$. The parameters $\nabla$ and $\lambda$ are picked to fit the moments of the quarterly average of monthly productivity, namely the standard deviations 0.020 and the autocorrelation 0.878.

<table>
<thead>
<tr>
<th>Table 11</th>
<th>Baseline Parameterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Parameterization</td>
</tr>
<tr>
<td>Preference Parameters</td>
<td></td>
</tr>
<tr>
<td>Interest rate $r$</td>
<td>0.001</td>
</tr>
<tr>
<td>The value of leisure $l$</td>
<td>0.472</td>
</tr>
<tr>
<td>Technology and Matching Parameters</td>
<td></td>
</tr>
<tr>
<td>Exogenous separation rate $s$</td>
<td>0.005</td>
</tr>
<tr>
<td>Elasticity parameter for matching function $\eta$</td>
<td>0.466</td>
</tr>
<tr>
<td>Scale parameter for matching function $\mu$</td>
<td>0.360</td>
</tr>
<tr>
<td>Bargaining power for workers $\beta$</td>
<td>0.466</td>
</tr>
<tr>
<td>Vacancy posting cost $c$</td>
<td>0.703</td>
</tr>
<tr>
<td>Average productivity $\bar{p}$</td>
<td>0.951</td>
</tr>
<tr>
<td>Step size $\nabla$</td>
<td>0.052</td>
</tr>
<tr>
<td>Transition parameter $\lambda$</td>
<td>0.076</td>
</tr>
<tr>
<td>Lower bound of match-specific productivity $\xi$</td>
<td>0</td>
</tr>
<tr>
<td>Upper bound of match-specific productivity $\tau$</td>
<td>0.078</td>
</tr>
<tr>
<td>Policy parameters</td>
<td></td>
</tr>
<tr>
<td>General tax $\tau$</td>
<td>0.300</td>
</tr>
<tr>
<td>Actual UI replacement rate $b$</td>
<td>0.249</td>
</tr>
<tr>
<td>Arrival rate of gaining UI eligibility $g$</td>
<td>0.050</td>
</tr>
<tr>
<td>Arrival rate of losing UI eligibility $d$</td>
<td>0.042</td>
</tr>
</tbody>
</table>

For the parameters of the UI program, the calibrations aim to be consistent with the average time it takes for a worker to gain UI eligibility, the average duration of UI benefits and the average actual replacement rates of UI benefits. In the United States, UI eligibility takes around 20 weeks of work and the maximum duration of benefits is around 24 weeks.\(^60\) The actual replacement

\(^60\)See a detailed explanation in Faig and Zhang (2008).
ratio \( (b/w) \) is measured as the ratio of the average weekly UI benefits paid to the eligible unemployed workers over the average weekly insurable earnings paid to the employed workers. Since not all eligible unemployed workers receive UI benefits, the actual replacement ratio is calculated as a product of the takeup rate and the ratio of the average UI benefits received by the UI recipients to the average earnings for the employed. The former averaged 0.70 over the period of 1977-1987 as documented by Blank and Card (1991) while the latter is persistently around 0.357 over the period of 1972-2003 as reported in Zhang (2008). So \( b/w = 0.25 \). For the value of leisure \( l \), since the sum of the value of UI benefits \( b \) and \( l \) is important for the model’s business cycle property, I pick \( l \) to fit the observed volatility of the aggregate vacancy-unemployment ratio. The standard deviation of the quarterly average of the weekly vacancy-unemployment ratio conditional on productivity is 0.102 as reported in Hagedorn and Manovskii (2008a).

Finally, the values of \( \tau^e \) are assumed to be the same and be proportionate to wages in all the simulations for reasons of simplicity, so \( \tau = \tau^0 = \tau^1 \). The parameter \( \tau \) is interpreted as a general tax including the UI contribution fees and is determined to target the general tax burden relative to GDP, which is \( \tau = 30\% \). So, the government is using a large fraction of \( \tau^e \) to finance a public good, which yields separable utility to the constituents of the economy.

The values of \( \{r, \xi, g, d, \tau\} \) follow directly from the stated targets described above. The values of the parameters \( \{l, \mu, \eta, \beta, c, s, \Delta, \lambda, \tau, \varphi^*, b\} \) are obtained with the following iterative procedure. First, an initial guess about the values of these parameters is formed. Using this guess the model is simulated for a long horizon (144,000 weeks), and the initial guess is then revised. This process continues until the predictions of the model match the

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\(^{61}\)Notice that since leisure is not taxed, income taxes can be considered as part of the opportunity cost of employment. Defining \( t = \tau l/(1 - \tau) \), the opportunity cost of employment can then be decomposed into three components: the value of leisure \( l \), the value of UI benefits \( b \), and a term that captures the effect of taxes \( t \).

targets. Table 11 reports the calibrated values of the parameters.

### 3.4.2 Benchmark Results

The first column of Table 12 summarizes the main results from the benchmark model. Given the current UI system, any eligible workers searching for jobs receive benefits worth 0.249 units of productivity.

The following four lines show some results that the parameterization was chosen to match, which shows the benchmark parameterization are well-behaved. The standard deviation of $\theta$ conditional on productivity $\hat{p}$ is 0.102. The monthly finding rate is 0.452, yielding a 5.55 percent unemployment rate or, conversely, an average duration of unemployment of 9.56 weeks. Figure 14 compares the simulated job tenure distribution to its empirical counterpart. With the benchmark parameterization, the model’s prediction squares quite well with the realistic distribution.

![Figure 14: Job Tenure Distribution](image)

The next four lines report the weekly job finding rates for ineligible and eligible job seekers. The weekly matching rate for the UI-nonrecipients is much higher than that for the UI recipients ($f^0 > f^1$). This, along with the
positive possibility of refusing a job offer, makes the UI recipients, on average, much less likely to form an employment relationship with firms. This sharp contrast reflects various effects of the UI system on firms’ optimal job-creation behavior. The presence of job rejections in the market with the UI-recipients reduces the firm’s profit and discourages job creation activities. The desire to earn UI entitlement by the UI-nonrecipients raises the firm’s profits and promote job openings. Consequently, the entry into the market with the UI-ineligibles is more attractive to the firms.

<table>
<thead>
<tr>
<th>Table 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results in the Benchmark Parameterization, and for the Alternative UI Schemes</td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Unemployment benefit ( b )</td>
</tr>
<tr>
<td>Standard deviation of ( \theta ) conditional on ( \hat{\rho} ) (weekly)</td>
</tr>
<tr>
<td>Monthly job-finding rate ( f ) (aggregate)</td>
</tr>
<tr>
<td>Average unemployment duration (weeks)</td>
</tr>
<tr>
<td>Unemployment ( U^{ss} ) (%)</td>
</tr>
<tr>
<td>Weekly finding (or matching) rate for ineligible unemployed ( f^0 )</td>
</tr>
<tr>
<td>Weekly finding rate for eligible unemployed ( f^1(1 - H^1) )</td>
</tr>
<tr>
<td>Weekly matching rate for eligible workers ( f^1 )</td>
</tr>
<tr>
<td>Fraction of eligible unemployed who reject offer ( H^1 )</td>
</tr>
<tr>
<td>Fraction of employment with eligibility ( E^1/E^{ss} ) (%)</td>
</tr>
<tr>
<td>Fraction of unemployment with eligibility ( U^1/U^{ss} ) (%)</td>
</tr>
</tbody>
</table>

Table 12 is closed with the model implications for the compositions of the employed and the unemployed. First, the majority of the employed are entitled to UI benefits. The reason is that the weekly aggregate separation rate, a sum of the exogenous separation rate \( s \) and the quitting rate measured
as the incidences of strategic quits within one week to the total employment in that week \( \frac{\rho H^3}{E^2 + E_\tau} \), is small (0.0061) so that workers can, on average, work long enough to gain the UI entitlement. Second, the majority of the unemployed are eligible for UI. The predicted share of unemployment made up of the UI-eligibles is about 84.5 percent, broadly in line with the empirical evidence. 

3.4.3 The Impacts of the Changes in the UI Replacement Rate

In the previous section, I establish that the model accounts well for the data in the United States labor market. Now I am in a position to conduct comparative statics to study if the model is able to generate a positive and moderate response of unemployment to the rise in benefits. Meanwhile, I examine if the model can still reproduce the observed large fluctuations in unemployment over the business cycles. To this end, I start with increasing the benefit payment by 1 percent (measured in units of productivity), and proceed by making the benefit 10 percent more generous than its current level. The remaining columns of Table 12 deliver the results from these two UI policy changes.

Predicted Response of Unemployment to UI Policy Changes

It is widely recognized that the effect on unemployment of a rise in benefits is significant but modest. For example, Costain and Reiter (2008), based on cross-country regressions, estimates that the semi-elasticity of unemployment

\[^{63}\text{Blank and Card (1991) estimate that 43 percent of the unemployed workers were eligible for UI over the period of 1977-87. However, in reality workers who claim to have quit jobs are not eligible for UI. Also, workers need to wait usually for 1 week before receiving benefits and are regarded as being ineligible for UI if this period has not expired. Some workers might be disqualified for UI because of the insufficient earnings in the base period or having previous employment in the uncovered sector. Blank and Card (1991) look at the most common reasons for UI ineligibility by using the data in March CPS. Their study shows that about 13 – 42 percent of the unemployed are considered as UI-ineligibles for these reasons (see p. 1165, Table 1). Taking into account this adjustment, my result is broadly in line with the empirical evidence.}\]
with respect to the UI replacement rate is around 2.\textsuperscript{64} My results are in line with this conventional view. When the UI benefit rises by 0.01 to 0.259, unemployment in logs rises by 1.4 percent. In addition, in face of a 10–percent rise in benefits, the third column shows that the average duration of unemployment increases by 0.77 weeks, falling in the range of 0.5 – 1.5 weeks established by the existing empirical findings.\textsuperscript{65} Meanwhile, as shown in Table 12, the predicted standard deviation of $\theta$ remains nearly as large as what is observed in the data in both cases (in line 2).

Several papers, including Hornstein \textit{et al.} (2005), Costain and Reiter (2008), and Zhang (2008), criticize that the calibration method for the value of leisure proposed by Hagedorn and Manovskii (2008a) causes dramatic reactions of unemployment to the labor policy changes in the standard model, although it resolves the volatility puzzle. By using a calibration strategy in the spirit of their argument, this model not only nicely preserves the business cycle properties of the standard Mortensen-Pissarides model, but also fixes the overreaction problem. The main reason for this success is because the entitlement effect curbs the rise in unemployment induced by the moral hazard effect and the cost of financing the UI system.

More specifically, unemployment reacts through the following channels. More generous UI benefits raise the disutility of working, which reduces the eligible worker’s expected surplus from a match and triggers more moral hazard (higher $H^1$). Moreover, this effect is magnified by the change in the firm’s job-creation incentive. The rise in job refusals lowers the firm’s expected profits and leads to fewer vacancies for the UI-eligibles, which slows down their transitions out of unemployment (lower $f^1$). This mechanism is quantitatively

\textsuperscript{64}Like most cross-country regressions, the estimate in Costain and Reiter (2008) is subject to the endogeneity problem. See Hagedorn et al. (2008) for further discussion.

\textsuperscript{65}Solon (1985) concludes that a 10–percentage point increase in the replacement rate leads to an increase in unemployment between 0.5 – 1 weeks. Moffitt and Nicholson (1982) offer estimated responses of about 1 week to the same change, which is regarded as the most reliable estimate. Considering more institutional details of the UI system, Meyer (1990) indicates that the same amount of rise in the replacement ratio would be associated with an increase of around 1.5 weeks in duration.
important since the majority of the unemployed are receiving UI benefits. However, the predicted reaction of unemployment remains realistically modest because these positive effects on unemployment are partially offset by the entitlement effect. The increase in benefits makes a job offer more attractive to the UI-ineligibles, and urges them to take a job at even lower wages. This promotes the job creation for the UI nonrecipients and speeds up their escape from unemployment (higher $f^0$). In addition, the larger UI benefits make it more costly for the UI-eligibles to lose entitlement and restrain the job rejections. This mitigates the rise in $H^1$ and, therefore, reverses the negative effect of moral hazard on the job openings in the market with UI recipients to some degree.

**Contributions of Moral Hazard Effect and Entitlement Effect**

How does unemployment react if the moral hazard effect or the entitlement effect is removed from the model? To find out the answer, I undertake the following two experiments. In Experiment 1, I shut down the moral hazard effect by forbidding workers to reject job offers or to quit their current job. In Experiment 2, I shut down the entitlement effect by allowing unemployed workers to receive UI entitlement unconditionally and the UI benefits to last forever. In both experiments, with the parameters recalibrated to the same targets used in the benchmark economy in Section 3.4.1, a comparative statics is conducted to examine how much unemployment reacts to the same changes in the level of UI benefits.

The calibration strategy follows closely to what is employed in Section 3.4.1 with slight differences. In Experiment 1, in the absence of the moral hazard behavior, the match-specific productivity $\epsilon$ is set to be zero. The exogenous separation rate $s$ is then determined to match the aggregate monthly separation rate, which is 0.026 as reported in Menzio and Shi (2008). In Experiment 2, in the absence of the entitlement qualification requirements,

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66Alternatively, one can assume perfect information on job offers and job separations by the UI agency.
the parameters $g$ and $d$ disappear from the model, the two labor markets reduce into a single one. The calibrated values of the parameters in these two experiments are shown in Table A.3.1 in the Appendix. Figure A.3.1 displays the job tenure distributions from both the model without the entitlement effect and the data. Table 13 compares the responses of unemployment to changes in the UI benefits in the benchmark model to the ones in the models without moral hazard effect and entitlement effect, respectively.

<table>
<thead>
<tr>
<th>Changes in the UI Benefits</th>
<th>Benchmark</th>
<th>No Moral Hazard</th>
<th>No Entitlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta b = 0.01$</td>
<td>5.78</td>
<td>5.68</td>
<td>5.78</td>
</tr>
<tr>
<td>$\Delta b = 10%$</td>
<td>6.13</td>
<td>5.88</td>
<td>6.17</td>
</tr>
<tr>
<td>$\Delta b = 20%$</td>
<td>6.84</td>
<td>6.26</td>
<td>6.98</td>
</tr>
<tr>
<td>$\Delta b = 30%$</td>
<td>7.76</td>
<td>6.73</td>
<td>8.11</td>
</tr>
<tr>
<td>$\Delta b = 40%$</td>
<td>8.90</td>
<td>7.28</td>
<td>9.80</td>
</tr>
<tr>
<td>$\Delta b = 50%$</td>
<td>10.55</td>
<td>8.00</td>
<td>100</td>
</tr>
</tbody>
</table>

The results show that both of these two effects are quantitatively important. As shown in the second column, when the moral hazard effect is missing from the model, although unemployment still rises on account of the increase in benefits, the magnitude is rather small. For instance, the semielasticity of unemployment with respect to the UI benefits is about 0.8, less than half the size of what is observed in data. When the moral hazard rejections and quits are allowed, the rise in benefits greatly retards the transitions from unemployment to employment, and gives rise to more transitions from employment to unemployment, which moves the model implication towards the reality.

In the third column, one sees that in the absence of the entitlement effect, the model tends to generate stronger responses of unemployment compared to the ones in the benchmark case. For example, when $b$ rises by 50 percent, the unemployment rate in the model without entitlement effect surges to 100
percent. Intuitively, with the UI benefits 50 percent more generous than its current level, the eligible worker’s opportunity cost of employment greatly increases and surpasses the match productivity for any realization of $p$, which makes the market activity no longer desirable. In sharp contrast, when the entitlement requirements are imposed, unemployment goes up to only 10.55 percent in response to the same change. One thing to note is that the quantitative importance of the entitlement effect increases with $b$. The effect is negligible when benefits are moderate, but enormous when benefits are generous. This reflects the relative tradeoff faced by the UI-ineligibles between the expected gains from receiving UI benefits after obtaining the UI entitlement and the expected losses from fewer job opportunities in future after an exogenous job loss. When benefits get larger ($\Delta b = 50\%$), the expected losses become tiny relative to the expected gains, which makes the job offer quite attractive to the UI-ineligibles, and leads to a large entitlement effect. The reverse intuition holds when benefits get smaller.

3.4.4 Relative Importance of Inflows and Outflows in Determining Unemployment

What is the mechanism by which UI benefits raise the unemployment rate? The findings in the literature are mixed and inconclusive. For instance, Topel (1983, 1984) highlights the role of the entry rate into unemployment while Hall (2005a) and Shimer (2005a) emphasize the opposite dimension—the exit rate from unemployment. I find that the effect of the change in outflows is dominant, but the inflows do play an important role in determining the overall level of unemployment, echoing the arguments in Sider (1985), Davis (2005), Fujita and Ramey (2008) and Menzio and Shi (2008).

Given the exogenous parameters $\{s, g, d\}$, the steady state unemployment $U^{ss}$ shown in (68) is fully determined by the matching rate $f^e$ and the degree of moral hazard $H^1$. The matching rate determines the ease of exit from unemployment (outflows) for job seekers. And the severity of moral hazard $H^1$
approximately measures the entry into unemployment (inflows) given that the majority of moral hazard unemployment takes the form of strategic quits. Following Shimer (2005a), the relative contributions of outflows and inflows to the reactions of unemployment can be easily decomposed by constructing two theoretical unemployment rates: one has the actual job matching rate and assumes the same degree of moral hazard before and after the policy change, which is denoted by $U_{1ss}$. The other has the actual degree of moral hazard, but fixes the matching rate to the one in the benchmark, which is denoted by $U_{2ss}$. The results are summarized in Table 14.

<table>
<thead>
<tr>
<th>Table 14</th>
<th>The Importance of Outflows and Inflows in Determining Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark</td>
</tr>
<tr>
<td>Unemployment rate $U^{ss}$ (%)</td>
<td>5.55</td>
</tr>
<tr>
<td>Theoretical unemployment $U_{1ss}$ (%)</td>
<td>5.55</td>
</tr>
<tr>
<td>Theoretical unemployment $U_{2ss}$ (%)</td>
<td>5.55</td>
</tr>
<tr>
<td>Incremental contribution of outflows $\frac{\Delta U^{ss}}{U^{ss}}$ (%)</td>
<td>–</td>
</tr>
<tr>
<td>Incremental contribution of inflows $\frac{\Delta U^{ss}}{U^{ss}}$ (%)</td>
<td>–</td>
</tr>
</tbody>
</table>

A comparison shows that the deterred outflows from unemployment is the major reason for the rise in $U^{ss}$. For a given severity of moral hazard, about half of the increase in $U^{ss}$ can be explained by the decline in the matching rates. As explained above, the slower escape from unemployment is mainly caused by a significant fall in the finding rate for UI recipients $f^1 (1 - H^1)$, and the fact that most of the unemployed in the economy are receiving UI benefits reinforces this effect. Although the effect of the changes in outflows dominates, the inflows operating through strategic quits prove quantitatively

\[\text{67} \text{The calibrated model shows that moral hazard behavior in job rejections and job quits plays quite different roles in accounting for the rise in unemployment: only around 3 percent of the increase in unemployment is explained by job rejections. In contrast, moral hazard quits account for about one-third of the variations. See Table A.3.2 in the Appendix for more details.}\]
important; nearly 30 percent of the rise in unemployment is accounted for by the channel of entry. This finding is not sensitive to the magnitude of the rise in benefits.68

3.5 Concluding Remarks

This chapter investigates the effects of UI generosity on the labor market outcomes in the Mortensen and Pissarides search and matching model where the realistic UI eligibility rules are imposed and worker heterogeneity is introduced. This work illustrates the variety of effects that the UI system may have on unemployment. The entitlement effect arises since the presence of the UI system creates the desire for the UI nonrecipients to gain UI entitlement and the incentive for the UI recipients to retain UI eligibility, which facilitates forming employment relationships and reduces unemployment. The UI system has two unintentional effects. A more expensive UI system hurts employment due to the burden of the UI contribution fees required to finance the program. Also, a more generous UI system aggravates the moral hazard problem since the improved outside option induces more workers engaged in the low-productivity matches to quit their jobs and more workers paired up with bad jobs to turn down offers as long as they are entitled to UI.

These offsetting effects of the UI system on unemployment imply that under some conditions the irrelevance of the UI system emphasized in Faig and Zhang (2008) holds with heterogeneous workers. Like Ricardian Equivalence, this irrelevance result hinges on specific conditions that do not necessarily hold in reality and therefore it is not meant to characterize the UI system as irrelevant in reality. However, it can be used as a benchmark to pinpoint the economic effect of the UI system on the labor market. That is, if the system does have some effects on the labor market outcomes, it must be related to the way it is financed since it would distort the firm’s job creation behavior. Or, it might

\footnote{68The contributions of outflows and inflows do not sum to one because of the interaction between the match rate \( f^\nu \) and the degree of moral hazard \( H^1 \).}
be due to the rules of the UI provisions since it would trigger moral hazard quits or rejections. Lastly, it might be because workers are not risk-neutral.

Introducing the realistic institutional details of the UI system is crucial to improving the model’s empirical performance. With a large value of leisure, as argued by Hagedorn and Manovskii (2008a), the model successfully reproduces different cyclical and UI policy-related variations in unemployment. This proves to be an insurmountable challenge in the standard model where unemployed workers receive UI unconditionally. This chapter can meet this challenge mainly because the entitlement effect attenuates the rise in unemployment caused by the moral hazard and financial cost effects. However, this mechanism is absent from the standard model.

This work can be extended in several ways. For example, it can provide a framework to study to what extent the generosity of the UI system itself can explain the large disparity in the level and duration of unemployment between the United States and the European countries. It is well known that the European countries provide much longer UI benefits relative to the one in the United States. The model suggests that with everything else equal, the extension of the UI benefits from 24 weeks to 52 weeks raises unemployment from 5.55 percent to 6.86 percent. Also, in this chapter the labor market transitions are limited to the changes over employment and unemployment. However, some empirical evidence shows that the UI generosity causes substantial flows into and out of the labor force (see Moothy 1989; Atkinson and Micklewright 1991; Andolfatto and Gomme 1996b). Since the driving forces underlying these flows could be entitlement and moral hazard effects as stressed in this chapter, it is interesting to consider the state of being out of labor force, which is missing in this contribution, but likely important in enhancing our understanding in the behavioral effects of the UI system for labor market participants.
References


Davis, Steven J. (2005) "Comment on ‘Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years’," *NBER/Macroeconomics annual* 20, 139-157.


Wagner, Don (2001) "Do Tax Differences Contribute Toward the Brain Drain from Canada to the U.S.?" Mimeo.


Appendix

1. Proofs in Chapter 2

1.1 Derivation of the System (25) to (28)

Substitute the value functions $U^0_p$, $U^1_p$, $W^0_p$, $W^1_p$, $J^0_p$ and $J^1_p$ from (45) to (50) into (20) and the definition of $\hat{U}$. Use (22) to (24) to simplify terms and obtain (26) and the following equation:

$$(r + s)V^0_p = p - \ell - \beta f(\theta_p) (V^0_p - k) + g (W^1_p + J^1_p - J^0_p - W^0_p) - \tau_p + \lambda(E_p V^0_p - V^0_p).$$

(69)

Define $\hat{B}_p = W^1_p + J^1_p - J^0_p - W^0_p$ to obtain (28). Finally, to obtain (27), substitute (47) to (50) into the definition of $\hat{B}_p$ and simplify using that because of Nash bargaining a worker is willing to quit a job if and only if the employing firm is also willing to terminate the match.

1.2 Proof of Proposition 2.1

For this proof, it is convenient to rewrite the system of equations characterizing an equilibrium as follows. Define $\theta(V^0)$ to be the real function that satisfies:

$$c\theta = f(\theta) (1 - \beta) \max\{0, V^0 - k\}.$$ Also, define

$$\Gamma(V^0) = V^0 + \frac{\beta c\theta(V^0)}{(1 - \beta) (r + s + \lambda)}.$$

The assumed properties of the matching function imply that $\theta/f(\theta)$ is a strictly increasing function of $\theta$ such that $\lim_{\theta \to 0} [\theta/f(\theta)] = 0$, so $\theta(V^0)$ is well defined, continuous and increasing, and $\theta(0) = 0$. Therefore, $\Gamma(V^0)$ has the same properties, and $\Gamma(V^0)$ is positive if and only if $V^0$ is positive. Using the above definitions, $z_p = \ell + \tau_p$ and (25), the system of equations (26) to (28) can be rewritten as:

$$\hat{U}_p = \frac{b + \lambda E_p \hat{U}_{p'}}{r + d + f(\theta(V^0_p)) + \lambda}.$$
\[
\hat{B}_p = \max \left\{ \pi \hat{U}_p - V^0_p, s\hat{U}_p + \lambda E_p\hat{B}_p \right\}, \quad \text{and} \quad (71)
\]

\[
\Gamma (V^0_p) = \frac{p - z_p + g\hat{B}_p + \lambda E_p V^0_p}{r + s + \lambda}. \quad (72)
\]

For a set of functions \(\{V^0_p, \hat{B}_p, \hat{U}_p\}\), let \(x \in R^3_n\) be the vector \((V^0_1, ..., V^0_n, \hat{B}_1, ..., \hat{B}_n, \hat{U}_1, ..., \hat{U}_n)\), and \(F (x) \in R^3_n\) be the values of \(\{V^0_p, \hat{B}_p, \hat{U}_p\}_{p \in P}\) on the right-hand-side of (70) to (72) when the left-hand-side of these equations is evaluated at \(x\). Define \(X\) as the subset of \(R^3_n\) that satisfies the following bounds: \(x_i \in [0, p_i - z_i + b/(r + d)]\) for \(i = 1\) to \(n\), and \(x_i \in [0, b/(r + d)]\) for \(i = n + 1\) to \(3n\). The set \(X\) is non-empty, closed, bounded, and convex. Also, using \(V^0 \leq \Gamma (V^0)\) and \(p > z_p\) for all \(p \in P\), one can easily check that \(F\) maps \(X\) onto itself. Consequently, Brower’s fixed point theorem implies that \(F\) has a fixed point in \(X\).

Given the bounds from the previous paragraph, equations (70) and (71) imply that \(\hat{U}_p, \hat{B}_p > 0\) if \(b > 0\). Similarly, since \(\Gamma (V^0)\) is positive if and only if \(V^0\) is positive, (72) implies that \(V^0_p > 0\). Finally, since \(V^1_p > 0\) is equivalent to \(\hat{B}_p > \pi \hat{U}_p - V^0_p\), and \(\hat{B}_p > s\hat{U}_p\) \((r + s + g + \lambda)\) because \(E_p \hat{B}_p > 0\). \(V^1_p\) can be guaranteed to be positive if \(\pi \leq s/(r + s + g + \lambda)\).

### 1.3 Comparative Statics Derivations

Combining (25) to (28), the following equations determine \(\theta\):

\[
\begin{align*}
\frac{c\theta}{1 - \beta} \left( \frac{r + s}{f (\theta)} + \beta \right) &= p - \ell - \tau - (r + s) k + \frac{sg}{r + s + g} \frac{b}{r + d + f (\theta)}, & \text{if } V^1 \geq 0, \\
\frac{c\theta}{1 - \beta} \left( \frac{r + s + g}{f (\theta)} + \beta \right) &= p - \ell - \tau - (r + s + g) k + \frac{\pi g b}{r + d + f (\theta)}, & \text{if } V^1 \leq 0.
\end{align*}
\]

(73)
Applying the Implicit Function Theorem, 
\[
\frac{\partial \theta}{\partial p} = \begin{cases} 
\frac{c}{1 - \beta} \left[ \frac{1 - \eta}{f} (r + s) + \beta \right] + \frac{s}{r + s + g (r + d + f)^2} \pi bgf' \left[ \frac{c}{1 - \beta} \left( \frac{1 - \eta}{f} (r + s + g) + \beta \right) + \frac{\pi bgf'}{(r + d + f)^2} \right]^{-1}, & \text{if } \bar{V}^1 \geq 0, \\
\frac{c}{1 - \beta} \left[ \frac{1 - \eta}{f} (r + s) + \beta \right] + \frac{s}{r + s + g (r + d + f)^2} \pi bgf' \left[ \frac{c}{1 - \beta} \left( \frac{1 - \eta}{f} (r + s + g) + \beta \right) + \frac{\pi bgf'}{(r + d + f)^2} \right]^{-1}, & \text{if } \bar{V}^1 \leq 0.
\end{cases}
\] (74)

Therefore, using \( \frac{\partial f}{\partial p} = f'(\partial \theta/\partial p) \), and the definitions of \( \hat{U} \) in (32), \( \hat{B} \) in (33), and \( \eta \), we obtain 
\[
\frac{\partial f}{\partial p} = p \left[ \frac{c\theta}{1 - \beta} \left( \frac{r + s}{f} + \beta \right) \frac{(1 - \eta)(r + s) + \beta f}{\eta (r + s + \beta f)} + \frac{fg\hat{B}}{r + d + f} \right]^{-1}, \quad \text{if } \bar{V}^1 \geq 0,
\]
\[
\frac{c\theta}{1 - \beta} \left( \frac{r + s + g}{f} + \beta \right) \frac{(1 - \eta)(r + s + g) + \beta f}{\eta (r + s + g + \beta f)} + \frac{fg\pi \hat{U}}{r + d + f} \right]^{-1}, \quad \text{if } \bar{V}^1 \leq 0.
\] (75)

Finally, using (73), (32), (33) and \( \hat{z} \) in (36), and simplifying yields (35).

If the UI system is fully funded, equation (73) simplifies into:
\[
\begin{cases} 
\frac{c\theta}{1 - \beta} \left( \frac{r + s}{f} + \beta \right) = p - \ell - (r + s) k, & \text{if } \bar{V}^1 \geq 0, \\
\frac{c\theta}{1 - \beta} \left( \frac{r + s + g}{f(\theta)} + \beta \right) = p - \ell - (r + s + g) k, & \text{if } \bar{V}^1 \leq 0.
\end{cases}
\] (76)

Therefore, the second terms inside the square brackets in (75) and the effect of UI in the definition of \( \hat{z} \) drop out.

1.4 Existence of Different Types of Equilibria

If \( V^0 \leq k \), then \( \theta = f = 0 \). Therefore, (32) to (34) imply that \( V^0 \leq k \) is equivalent to
\[
k \geq \begin{cases} 
\frac{p - \ell - \tau}{r + s} + \frac{g}{r + s} \frac{s}{r + s + g} \frac{b}{r + d + f} \equiv \bar{k}, & \text{if } \bar{V}^1 \geq 0, \\
\frac{p - \ell - \tau}{r + s + g} + \frac{\pi g}{r + s + g} \frac{b}{r + d + f}, & \text{if } \bar{V}^1 \leq 0.
\end{cases}
\] (77)
Furthermore, if $V^0 \leq k$, using (32) to (34), the condition for workers not to have incentives to remain in their employment matches once they are eligible for UI simplifies with the help of (32) to (34) into:

$$\pi \geq \frac{(p - \ell - \tau)(r + d)}{b(r + s)} + \frac{s}{r + s} \frac{g}{r + s + g} \equiv \bar{\pi}.$$  \hfill (78)

If $V^0 > k$, for an equilibrium to be consistent with no strategic quits, it must satisfy: $\theta c = f(\theta)(1-\beta)(V^0-k)$, (32) to (34), and $\bar{V}^1 \geq 0$. Once the values for $V^0$, $\bar{B}$, and $\bar{U}$ are substituted into the definition $\bar{V}^1$, these conditions reduce to:

$$c\frac{(r + s)}{1 - \beta} f(\theta) \theta + \frac{\beta c \theta}{1 - \beta} \left[ b \left( r + s + g \right) \frac{gsb}{r + d + f(\theta)} \right] = p - \ell - \tau - k \left( r + (\bar{\pi}) \right)$$

$$\frac{s}{r + s + g} + \left[ \frac{\theta c}{1 - \beta} \frac{r + d + f(\theta)}{b} \right] \geq \pi.$$  \hfill (80)

Therefore, the $V^0 + \bar{B} = \pi \bar{U}$ line must satisfy (79) and (80) with equality, or, after simplifying, (79) and the following equation:

$$\pi = \frac{\theta}{r + s} \frac{g}{r + s + g} + \left( p - \ell - \tau - \frac{\beta c \theta}{1 - \beta} \right) \frac{r + d + f(\theta)}{b}.$$  

Since $\theta/f(\theta)$ is increasing with $\theta$, (79) defines $\theta$ as an implicit decreasing function of $k$. Therefore, in Figure 11, the line $\bar{V}^1 = 0$ has a negative slope in the region where $V^0 > k$ if the expression inside the first parenthesis in (79) is positive, which must be true around the point $(\bar{k}, \bar{\pi})$ where $\theta = 0$. In other regions, the slope of this line is ambiguous.

If the UI system is fully funded, (77) simplifies to

$$k \geq \begin{cases} 
\frac{p - \ell - \beta (1-\beta)^{-1} \theta N}{r + s} = V^0_N, & \text{if } V^0_N + \bar{B}^N \geq \pi \bar{U}^N, \\
\frac{p - \ell - \beta (1-\beta)^{-1} \theta S}{r + s + g} = V^0_S, & \text{if } V^0_S + \bar{B}^S \leq \pi \bar{U}^S. 
\end{cases} \quad \hfill (81)$$

While the conditions for strategic quits to occur or not simplify into (42) and
2. Proofs in Chapter 3

2.1 Proof of Tenure-Independent Contract

Tenure-independent contract: The optimal dynamic contract repeats the static contract, provided that the firm can commit to not renegotiate the contract.

Proof: when wage contracts are assumed to be increasing with tenure, firms offer deferred compensation. However, the firm does not benefit from such a compensation. Because the worker’s opportunity cost of employment is time-invariant, the deferred compensation does not influence the worker’s participation constraint at the hiring margin (i.e., the incentive to take a job), but loosens the participation constraints in the firm’s optimal contract decision. Consequently, there is no loss for the firm to restrict attention to tenure-independent contracts. An alternative proof is that given the linear preference, in the model what workers (firms) care about is the expected present discounted value of wages (profits) at the hiring margin. Hence, how wages evolve over the spell of employment does not matter for the equilibrium outcomes.69

2.2 Proof of Proposition 3.1

I first show proof in a deterministic version of the above model (λ = 0), and relax this restriction later on. Mathematically, Step 2 can be formalized as:

\[
\max_{R^e} - c + q^e(f^e(R^e))(S^e - R^e), \ s.t. \ U^e.
\]

FOCs with respect to \( R^e \) lead to

\[
q^e_{f^e} \cdot f^e_{R^e} \cdot (S^e - R^e) = q^e, \ \text{for } e = 0, 1. \tag{82}
\]

69See Pissarides (2007) for a similar argument with discussion in greater detail. A formal mathematical proof is available upon request.
Rearranging (82) yields
\[
\left( \frac{\partial q^e}{\partial f^e} \cdot \frac{f^e}{q^e} \right) \cdot \left( \frac{\partial f^e}{\partial R^e} \cdot \frac{R^e}{f^e} \right) (S^e - R^e) = R^e, \quad \text{for } e = 0, 1. \tag{83}
\]

The first term in brackets in (83) is the elasticity of the vacancy filling rate with respect to the job finding rate in a submarket with workers of type \( e \), denoted by \( \varepsilon_{q^e f^e} \). The second term in (83) is the elasticity of the job finding rate with respect to the worker’s expected gains from a match, denoted by \( \varepsilon_{f^e R^e} \).

Recalling (54) and (55), it is easy to check that for a given \( U^e, \varepsilon_{f^e R^e} = -1, \quad \forall \ e = 0, 1 \). Besides, since \( f^e = \theta^e q^e, \varepsilon_{q^e f^e} = \frac{\varepsilon_{f^e q^e}}{\varepsilon_{f^e \theta^e}} \) where \( \varepsilon_{f^e \theta^e} \) is the so-called elasticity of the finding rate with respect to the submarket tightness \( \theta^e \). Since \( \varepsilon_{f^e \theta^e} = -\eta \in (-1, 0) \), it is easy to check that \( \varepsilon_{f^e \theta^e} = 1 - \eta \). Hence, (83) can be expressed as:
\[
\frac{R^e}{S^e - R^e} = \frac{\eta}{1 - \eta}, \quad \text{for } e = 0, 1. \tag{84}
\]

Equation (84) suggests that in the submarket with workers of type \( e \), the Hosios condition holds. That is, the fraction of the total surplus from a match that goes to a worker is equal to the worker’s contribution to forming a match.

When \( \lambda = 0 \) is relaxed, equations (54) and (55) show that \( \varepsilon_{f^e R^e} = -1 \) still holds for \( e = 0, 1 \). Given that \( \varepsilon_{q^e \theta^e} = -\eta \), it follows \( \varepsilon_{f^e \theta^e} = 1 - \eta \). Hence, a stochastic version of (83) can be expressed as: \( \frac{R^e}{S^e - R^e} = \frac{\eta}{1 - \eta}, \forall \ p \in P \) and \( e = 0, 1 \). So the Hosios rule holds in a stochastic version of the model.

### 2.3 Derivation of Equations (61)-(64)

**Equation (61):** Subtracting (54) from (55) and combining with the surplus sharing rule (59) leads to (61).

**Equation (62):** By definition, if the marginal worker who is indifferent between accepting and rejecting job offers exists \( (\epsilon^1_p \in [\xi_p, \Upsilon]) \), then it must have \( S^1_p (\epsilon^1_p) = 0 \). Equation (62) follows immediately from \( S^1_p (\epsilon^1_p) = 0 \).

**Equation (63):** For \( \epsilon \in [\xi_p, \epsilon^1_p] \), \( S^1_p (\epsilon) = 0 \); otherwise, the *ex post* value of \( S^1_p (\epsilon) \) can be derived by substituting equations (46), (48) and (50) into
(52)-(53) with \( e = 1 \) and combining with the sharing-rule (59):

\[
S_1^p(\epsilon) = \frac{\dot{p}_p(\epsilon) - l - b - \tau_1^p - \beta f_1^1 S_1^p + d\dot{U}_p + \lambda (E_p S_1^p(\epsilon) - S_1^p(\epsilon))}{r + s}, \forall \epsilon \geq \epsilon_1^p.
\]

Substituting out the term \( \beta f_1^1 S_1^p \) in (85) by the free entry condition \( (c\theta_1^1 = (1 - \beta) f_1^1 S_1^p) \) and integrating \( \epsilon \) over the interval \([\epsilon_1^p, \overline{\epsilon}]\) gives:

\[
S_1^p = \frac{(1 - H_1^1)(\dot{p}_p^1 - l - c\beta (1 - \beta)^{-1}\theta_1^p) + (1 - H_1^0)(d\dot{U}_p - b - \tau_1^1) + \lambda (E_p S_1^p - S_1^p)}{r + s}.
\]

**Equation (64):** Substituting equations (45), (47) and (49) into (52)-(53) with \( \epsilon = 0 \), and combining with the sharing-rule (59) yields:

\[
S_0^p(\epsilon) = \frac{\dot{p}_p(\epsilon) - l - \beta f_0^0 S_0^p + g \left[ S_1^p(\epsilon) - S_0^p(\epsilon) + \dot{U}_p \right] - \tau_0^p + \lambda (E_p S_0^p(\epsilon) - S_0^p(\epsilon))}{r + s}, \forall \epsilon.
\]

Substituting out the term \( \beta f_0^0 S_0^p \) in (86) by the free entry condition \( (c\theta_0^0 = (1 - \beta) f_0^0 S_0^p) \) and integrating \( \epsilon \) over the interval \([\epsilon, \overline{\epsilon}]\) gives:

\[
S_0^p = \frac{(\dot{p}_0^0 - l - c\beta (1 - \beta)^{-1}\theta_0^p) + \left[ g \left( S_1^p - S_0^p + \dot{U}_p \right) - \tau_0^p \right] + \lambda (E_p S_0^p - S_0^p)}{r + s}.
\]

2.4 Proof of Proposition 3.2

To avoid uninteresting results, I focus on the case where \( \epsilon_1^1 \in [\epsilon, \overline{\epsilon}] \) in the following proof. However, Proposition 3 holds for \( \epsilon_1^1 > \tau \) but under slightly different conditions and the proof is available upon request.

In the deterministic version of the model, the system of equations (60)-(64) simplifies into:

\[
c\theta^e = (1 - \beta) f^e S^e, \text{ for } e = 0, 1. \tag{87}
\]

\[
\dot{U} = \frac{b + c\beta (1 - \beta)^{-1} \left[ \theta_1 (S_1^1) - \theta_0 (S_0^0) \right]}{r + d}. \tag{88}
\]

\[
\epsilon_1^1 = \tau_1^1 + l + b + c\beta (1 - \beta)^{-1} \theta_1 - d\dot{U} - \overline{p}, \forall \epsilon_1^1 \in [\epsilon, \overline{\epsilon}]. \tag{89}
\]
$S^1 = \frac{(1 - H^1) \left[ \left( \hat{p}^1 - l - c\beta (1 - \beta)^{-1} \theta^1 (S^1) \right) + \left( d\hat{U} - b - \tau^1 \right) \right]}{r + s}$. \hfill (90)

$S^0 = \frac{\left[ \hat{p}^0 - l - c\beta (1 - \beta)^{-1} \theta^0 (S^0) \right] + \left[ g \left( S^1 - S^0 + \hat{U} \right) - \tau^0 \right]}{r + s}$. \hfill (91)

**A. Show that $S^0 \geq S^1$ holds if $\tau^0 \leq \tau^1 + \frac{r+g}{r+d}b - \frac{r-\epsilon}{2}$.**

**A.1. Derive $S^0 - S^1$.**

Subtracting (90) from (91) and combining with (88) yield:

$$S^0 - S^1 = \frac{\left( \frac{\epsilon^1 (\theta^1 (S^1) - \theta^0 (S^0))}{1 - \beta} \right) + \left( \frac{r+g}{r+d}b + \tau^1 - \tau^0 - \frac{(\epsilon^1 - \epsilon)^2}{2(\tau - \epsilon)} \right)}{r + s + g}, \ \forall \epsilon^1 \in [\xi, \bar{\xi}].$$ \hfill (92)

Note that $\epsilon^1 \in [\xi, \bar{\xi}]$ can happen in two cases. One case is $S(\epsilon^1) = 0$. That is, workers with $\epsilon^1$ are indifferent between taking and rejecting offers. The other case is $S(\epsilon) > 0$. That is, workers with $\epsilon$ receive positive expected gains from a match, which implies eligible workers are always willing to accept a job. So $\epsilon^1 < \epsilon$. Since $\epsilon \in [\xi, \bar{\epsilon}]$, it implies that $\epsilon^1 = \epsilon$ and $H^1 = 0$. The resulting expression of $(S^0 - S^1)$ can be regarded as a special case for $\epsilon^1 \in [\xi, \bar{\xi}]$ with the term $\frac{(\epsilon^1 - \epsilon)^2}{2(\tau - \epsilon)} = 0$. In both cases, $S^1 \geq 0$.

**A.2. Show $S^0 \geq S^1$ if $\tau^0 \leq \tau^1 + \frac{r+g}{r+d}b - \frac{r-\epsilon}{2}$.**

It is easy to see that $\frac{(\epsilon^1 - \epsilon)^2}{2(\tau - \epsilon)} \in \left[ 0, \frac{\epsilon - \epsilon^1}{2} \right]$ for $\epsilon^1 \in [\xi, \bar{\xi}]$. The condition $\tau^0 \leq \tau^1 + \frac{r+g}{r+d}b - \frac{r-\epsilon}{2}$ implies that the second term in the RHS of the equation (92) is non-negative. Suppose $S^0 < S^1$, it must be true that the first term in the RHS of the equation (92) is strictly negative, which implies that $\theta^1 (S^1) < \theta^0 (S^0)$ given $\beta \in (0, 1)$. However, the free entry condition (60) shows that $\theta^0 (S^0)$ is increasing in $S^0$, which suggests that $S^1 < S^0$. A contradiction arises and the supposition cannot hold. So $S^0 \geq S^1$ holds if $\tau^0 \leq \tau^1 + \frac{r+g}{r+d}b - \frac{r-\epsilon}{2}$.

**B. Show $0 \leq \hat{U} \leq \frac{b}{r+d}$ if $S^1 \leq S^0$, $\tau^1 \leq \tau^0$.**
B.1. Show $\hat{U} \geq 0$.
Combining (88) with (91) and rearranging (92) yields:

$$
(r + s + g) \left( S^0 - S^1 \right) = (r + g) \hat{U} + \tau^1 - \tau^0 - \frac{(\epsilon^1 - \epsilon)^2}{2(\bar{\tau} - \underline{\tau})}. \tag{93}
$$

Given $S^0 \geq S^1$ and $\frac{(\epsilon^1 - \epsilon)^2}{2(\bar{\tau} - \underline{\tau})} \geq 0$, equation (93) implies that the sufficient condition for $\hat{U} \geq 0$ is $\tau^1 \leq \tau^0$.

B.2. Show $\hat{U} \leq \frac{b}{r+d}$ if $S^0 \geq S^1$.
Since $S^0 \geq S^1$ and $\theta^0 \geq \theta^1$ ($\theta^e (S^e)$ is increasing in $S^e$), equation (88) implies $\hat{U} \leq \frac{b}{r+d}$.

C. Show $0 \leq \hat{B} \leq \hat{U}$ if $S^0 \geq S^1$, $0 \leq \hat{U} \leq \frac{b}{r+d}$.

Given $\hat{B} \equiv S^1 - S^0 + \hat{U}$, and $S^0 \geq S^1$, one can easily see that $\hat{B} \leq \hat{U}$. To prove $\hat{B} \geq 0$ is equivalent to prove $\hat{U} \geq S^0 - S^1$. Substituting (93) into $\hat{U} \geq S^0 - S^1$ simplifies the question in hand into $s\hat{U} \geq \tau^1 - \tau^0 - \frac{(\epsilon^1 - \epsilon)^2}{2(\bar{\tau} - \underline{\tau})}$. Given $\tau^0 \geq \tau^1$, $\frac{(\epsilon^1 - \epsilon)^2}{2(\bar{\tau} - \underline{\tau})} > 0$, and $\hat{U} \geq 0$, the inequality of $s\hat{U} \geq \tau^1 - \tau^0 - \frac{(\epsilon^1 - \epsilon)^2}{2(\bar{\tau} - \underline{\tau})}$ always holds.

D. Proof of Existence.

Recall the free entry condition, which states that $\theta^e (S^e)$ is the real function satisfying: $c\theta^e = (1 - \beta) f(\theta^e)S^e$, for $e = 0, 1$. Define

$$
\Gamma_S (S^e) = S^e + \frac{c\beta \theta^e (S^e)}{(1 - \beta)(r + s)}, \quad e = 0, 1. \tag{94}
$$

The properties of the matching function imply that $\theta^e / f^e(\theta^e)$ is an increasing function of $\theta^e$ such that $\lim_{\theta^e \to 0} \theta^e / f^e(\theta^e) = 0$, so $\theta^e (S^e)$ is well defined, continuous and increasing in $S^e$ with $\theta^e (0) = 0$, for $e = 0, 1$. Therefore, $\Gamma_S (S^e)$ has the same properties and $\Gamma_S (S^e)$ is positive if and only if $S^e$ is positive.
Substituting (94) into the system of equations (88)-(91) yields:

\[
\hat{U} = \frac{b + c\beta(1 - \beta)^{-1}[\theta^1(S^1) - \theta^0(S^0)]}{r + d},
\]

(95)

\[
\varepsilon^1 = \tau^1 + l + \frac{rb}{r + d} + \frac{c\beta(1 - \beta)^{-1}[r\theta^1(S^1) + d\theta^0(S^0)]}{r + d} - \bar{p},
\]

(96)

\[
\Gamma_S(S^1) = \frac{\bar{p} - \tau^1 - l - b + d\hat{U} + \frac{(\varepsilon^1 - \bar{\varepsilon})^2}{2(\tau - \bar{\varepsilon})} + \frac{\tau + \bar{\varepsilon}}{2}}{r + s},
\]

(97)

\[
\Gamma_S(S^0) = \frac{\bar{p} - \tau^0 - l + g\hat{B} + \frac{\tau + \bar{\varepsilon}}{2}}{r + s},
\]

(98)

where \( \hat{B} \equiv S^1 - S^0 + \hat{U}, \) and \( 0 \leq \hat{B} \leq \hat{U} \leq \frac{b}{r + d}, \) \( \varepsilon \leq \varepsilon^1 \leq \bar{\varepsilon}, \) \( \frac{\bar{p} + \varepsilon - \tau^1 - l - b}{r + s} < \Gamma_S(S^1) \leq \frac{\bar{p} + \varepsilon - \tau^1 - l - b + \frac{bd}{r + s}}{r + s}, \) and \( \frac{\bar{p} + \varepsilon - \tau^0 - l}{r + s} < \Gamma_S(S^0) \leq \frac{\bar{p} + \varepsilon - \tau^0 - l + \frac{bd}{r + s}}{r + s} \). Since \( \bar{p} + \varepsilon - \tau^1 - b \geq 1 \)

and \( \bar{p} + \varepsilon - \tau^0 \geq 1, \) it is easy to see that \( \Gamma_S(S^1) \in \left[0, \frac{\bar{p} + \varepsilon - \tau^1 - l - b + \frac{bd}{r + s}}{r + s}\right], \) and \( \Gamma_S(S^0) \in \left[0, \frac{\bar{p} + \varepsilon - \tau^0 - l + \frac{bd}{r + s}}{r + s}\right]. \) By equations (95) and (96), one can see that both \( \hat{U} \) and \( \varepsilon^1 \) are continuous in \( S^e \) given that \( \beta \in (0, 1) \) is constant.

For a set of functions \( \{\hat{U}, \varepsilon^1, S^1, S^0\} \), let \( x \in R^4_+ \) be the values \( \{\hat{U}, \varepsilon^1, S^1, S^0\} \) on the right-hand side of (95)-(98) when the left-hand sides of these equations are evaluated at \( x \). Define \( X \) as the subset of \( R^4_+ \) that satisfies the following bounds: \( x_1 \in [0, \frac{b}{r + d}]; x_2 \in [\varepsilon, \bar{\varepsilon}]; x_3 \in \left[0, \frac{\bar{p} + \varepsilon - \tau^1 - l - b + \frac{bd}{r + s}}{r + s}\right]; \) and \( x_4 \in \left[0, \frac{\bar{p} + \varepsilon - \tau^0 - l + \frac{bd}{r + s}}{r + s}\right] \). The set \( X \), therefore, is non-empty, closed, bounded and convex. Also using \( S^e \leq \Gamma_S(S^e) \), it is easy to show that \( F \) maps \( X \) into itself. Consequently, the Brower’s fixed point theorem implies that \( F \) has a fixed point in \( X \).

2.5 Derivation of Conditions for \( \varepsilon^1 \in [\varepsilon, \bar{\varepsilon}] \)

A. Derive the condition for \( \varepsilon^1 \geq \varepsilon \).

For \( \varepsilon^1 \in [\varepsilon, \bar{\varepsilon}] \), the inequalities of \( S^1(\varepsilon) \leq 0 \) and \( S^1(\bar{\varepsilon}) \geq 0 \) must hold. Equation
(85) implies that these two inequalities are equivalent to:

\[
\begin{aligned}
\bar{p} + \varepsilon - \tau^1 - l - b - c\beta (1 - \beta)^{-1}\theta^1 + d\bar{U} & \leq 0. \quad (99) \\
\bar{p} + \bar{\varepsilon} - \tau^1 - l - b - c\beta (1 - \beta)^{-1}\theta^1 + d\bar{U} & \geq 0. \quad (100)
\end{aligned}
\]

Given \( \bar{U} \leq \frac{b}{r+a} \) (see Proposition 3) and \( \theta^1 \geq 0 \), the maximum of the LHS of the equation (99) equals \( \bar{p} + \varepsilon - \tau^1 - l - \frac{rb}{r+a} \). Equation (99) shows that a sufficient condition for \( S^1(\varepsilon) \leq 0 \) (or \( \epsilon^1 \geq \varepsilon \)) is \( \tau^1 \geq \bar{p} + \varepsilon - l - \frac{rb}{r+a} \).

**B. Derive the condition for \( \epsilon^1 \leq \bar{\varepsilon} \).**

First, I find out the maximum of \( S^1 \) given the condition for \( \epsilon^1 \geq \varepsilon \) is satisfied. Rearranging (90) gives:

\[
S^1 + \frac{c\beta (1 - \beta)^{-1}\theta^1 (S^1)}{r + s} = \frac{\bar{p} - \tau^1 - l - b + d\bar{U} + \frac{(\varepsilon^1 - \varepsilon)^2}{2(\tau^1 - \varepsilon)} + \frac{\bar{\varepsilon} + \varepsilon}{2}}{r + s}, \quad (101)
\]

where \( S^1 = \frac{c(\theta^1)^\eta}{\mu(1-\beta)} \) and \( \epsilon^1 = \tau^1 + l + b + c\beta (1 - \beta)^{-1}\theta^1 - d\bar{U} - \bar{p} \). Equation (101) shows that \( \theta^1 \) is a function of \( \bar{U} \). Since \( \frac{\partial \theta^1}{\partial \bar{U}} = \frac{d(1-H^1)}{r+\beta} (\frac{r+\beta}{r+\beta+1-H^1}) \geq 0 \), and \( \theta^1(S^1) \) is increasing in \( S^1 \), both \( S^1 \) and \( \theta^1 \) reach their maximum when \( \bar{U} \) equals its maximum value \( \frac{b}{r+a} \). Denote \( \theta^1_{\text{max}} \) as the maximum value \( \theta^1 \), and denote \( \varepsilon^1 \) as the value of \( \varepsilon^1 \) associated with \( \theta^1_{\text{max}} \). It follows that \( \theta^1_{\text{max}} \) can be determined by:

\[
\frac{c}{1 - \beta} \left( \frac{(\theta^1_{\text{max}})^n}{\mu} + \frac{\beta \theta^1_{\text{max}}}{r + s} \right) = \frac{\bar{p} - \tau^1 - l - \frac{rb}{r+a} + \frac{(\tau^1 - \varepsilon)^2}{2(\tau^1 - \varepsilon)} + \frac{\bar{\varepsilon} + \varepsilon}{2}}{r + s},
\]

where \( \bar{\varepsilon} = \tau^1 + l + \frac{rb}{r+a} + c\beta (1 - \beta)^{-1}\theta^1_{\text{max}} - \bar{p} \).

Recall equation (100), the minimum of the term \( (\bar{p} + \bar{\varepsilon} - \tau^1 - l - b - c\beta (1 - \beta)^{-1}\theta^1_{\text{max}}) \) in (100) is \( \bar{p} + \bar{\varepsilon} - \tau^1 - l - b - c\beta (1 - \beta)^{-1}\theta^1_{\text{max}} \). Given \( 0 \leq \bar{U} \leq \frac{b}{r+a} \), the sufficient condition for \( S^1(\tau) \geq 0 \) (or \( \epsilon^1 \leq \tau \)) is \( \tau^1 \leq \bar{p} + \bar{\varepsilon} - l - b - c\beta (1 - \beta)^{-1}\theta^1_{\text{max}} \).

Combining the conditions for \( S^1(\varepsilon) \leq 0 \) and \( S^1(\tau) \geq 0 \) gives the condition for \( \epsilon^1 \in [\varepsilon, \bar{\varepsilon}] \). To assure the validity of this inequality, one needs \( \bar{\varepsilon} - \varepsilon > 125 \).
That is, the spread of the heterogeneity in productivity $\epsilon$ needs to be sufficiently larger than the UI benefits $b$.

2.6 Derivation of Lemma 3.1

Denote $A_i^e$ as the expected present discounted value of net benefits from the UI system, where $i$ denotes the worker’s employment state $i = \{E, U\}$, and $e$ denotes the UI eligibility state $e = \{0, 1\}$. The value of $A_i^e$ can be recursively characterized by the following Bellman equations:

\[
\begin{align*}
ra_E^0 &= -\tau^0 + g(A_E - A_E^0) + s(A_U^0 - A_E^0), \quad ra_E^1 = -\tau^1 + s(A_U^1 - A_E^1). \\
ra_U^0 &= f(A_E^0 - A_U^0), \quad ra_U^1 = b + f(A_E^1 - A_U^1) + d(A_U^0 - A_U^1).
\end{align*}
\]

An ineligible worker pays UI contribution fees $\tau^0$ and receives the expected net benefits or losses from the UI system from gaining UI eligibility and an exogenous match dissolution, which happen with the arrival rates $g$ and $s$, respectively. An eligible worker pays UI contribution fees $\tau^1$ and suffers the expected losses from an exogenous match dissolution at an arrival rate $s$. An unemployed worker without UI entitlement receives no benefits, and gains the expected benefits from finding a job at an arrival rate $f$. An unemployed worker who is eligible for UI collects benefits $b$ plus the expected benefits or losses from finding a job and losing UI entitlement, which happen with the respective arrival rates $f$ and $d$.

A fully rating UI system requires $A_E^0 = 0$, that is, workers who are newly hired and have not earned UI eligibility yet receive zero expected benefits from the UI system. The condition for $A_E^0 = 0$ can be expressed as a function of $\tau^e$ and $b$ as shown in (65).

2.7 Derivation of Equation (68)

\footnotesize{$A_i^e$ is independent of the match-specific productivity $\epsilon$ in that neither UI benefits or UI contribution fees depend on $\epsilon$.}
Denote $U^e(E^e)$ as the number of the unemployment (employment) of type $e$. At the steady state, the flows out of $U^e(E^e)$ must equal the flows into $U^e(E^e)$.

\[
\begin{align*}
E^0s + U^1d &= U^0f^0, \\
E^1s + E^0H^1g &= U^1(d + f^1(1 - H^1)), \\
U^0f^0 &= E^0(g + s), \\
E^0(1 - H^1)g + U^1f^1(1 - H^1) &= E^1s.
\end{align*}
\]

Solving the system gives $\frac{U^0}{E^0} = \frac{g + s}{f^0}$, $\frac{U^1}{E^0} = \frac{g}{d}$, $\frac{E^1}{E^0} = \frac{gd(1 - H^1) + gf^1(1 - H^1)}{sd}$, which leads to (68).

2.8 Figures and Tables in Chapter 3

![Figure A.3.1: Job Tenure Distribution - No Entitlement Effect](image)
### Table A.3.1
Parameterization in the Two Experiments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameterization</th>
<th>No Moral Hazard</th>
<th>No Entitlement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate $r$</td>
<td></td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>The value of leisure $l$</td>
<td></td>
<td>0.498</td>
<td>0.387</td>
</tr>
<tr>
<td><strong>Technology and Matching Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exogenous separation rate $s$</td>
<td></td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>Elasticity parameter for matching function $\eta$</td>
<td></td>
<td>0.459</td>
<td>0.508</td>
</tr>
<tr>
<td>Scale parameter for matching function $\mu$</td>
<td></td>
<td>0.149</td>
<td>0.382</td>
</tr>
<tr>
<td>Bargaining power for workers $\beta$</td>
<td></td>
<td>0.459</td>
<td>0.508</td>
</tr>
<tr>
<td>Vacancy posting cost $c$</td>
<td></td>
<td>0.302</td>
<td>0.591</td>
</tr>
<tr>
<td>Average productivity $p$</td>
<td></td>
<td>1</td>
<td>0.929</td>
</tr>
<tr>
<td>Step size $\nabla$</td>
<td></td>
<td>0.044</td>
<td>0.038</td>
</tr>
<tr>
<td>Transition parameter $\lambda$</td>
<td></td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td>Lower bound of match-specific productivity $\xi$</td>
<td></td>
<td>–</td>
<td>0</td>
</tr>
<tr>
<td>Upper bound of match-specific productivity $\tau$</td>
<td></td>
<td>–</td>
<td>0.094</td>
</tr>
<tr>
<td><strong>Policy parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General tax $\tau$</td>
<td></td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>Actual UI replacement rate $b$</td>
<td></td>
<td>0.249</td>
<td>0.247</td>
</tr>
<tr>
<td>Arrival rate of gaining UI eligibility $g$</td>
<td></td>
<td>0.050</td>
<td>–</td>
</tr>
<tr>
<td>Arrival rate of losing UI eligibility $d$</td>
<td></td>
<td>0.042</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table A.3.2
Relative Contributions of Strategic Quits and Job Rejections

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark</th>
<th>$\Delta = 0.01$</th>
<th>$\Delta = 10%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of unemployed workers who have quit jobs $U^q$</td>
<td>0.00830</td>
<td>0.00910</td>
<td>0.01037</td>
</tr>
<tr>
<td>The number of unemployed workers who have refused offers $U^r$</td>
<td>0.00347</td>
<td>0.00354</td>
<td>0.00366</td>
</tr>
<tr>
<td>The share of the unemployed for job quitters $U^q / U^{**}$ (%)</td>
<td>14.96</td>
<td>15.74</td>
<td>16.92</td>
</tr>
<tr>
<td>The share of the unemployed for job rejecters $U^r / U^{**}$ (%)</td>
<td>6.26</td>
<td>6.14</td>
<td>5.97</td>
</tr>
<tr>
<td>Incremental contribution of strategic quits $\Delta U^q / \Delta U^{**}$ (%)</td>
<td>–</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>Incremental contribution of job rejections $\Delta U^r / \Delta U^{**}$ (%)</td>
<td>–</td>
<td>3.04</td>
<td>3.28</td>
</tr>
</tbody>
</table>

**NOTE:** $U^q$ and $U^r$ measure the numbers of the eligible unemployed workers who have quit jobs and refused job offers in the past, respectively.
3. Calibration Strategy and Methodology

3.1 The Stochastic Process of Productivity (Separation)

With respect to the random variable $y$, the initial value of $y$ lies on a finite-ordered (11 states) set of points, which is called the feasibility set $Y$.

$$y \in Y \equiv \{-5\Delta, -4\Delta, \ldots, 0, \ldots, 4\Delta, 5\Delta\}, \: \Delta > 0,$$

where $\Delta$ is the step size, measuring the change in the cyclical components of labor productivity in logs.

At the beginning of each period, the economy is hit by an exogenous labor productivity shock. The value of $y$ responds by either taking a new value, $y'$, with probability $\lambda$ or staying unchanged otherwise. The new value $y'$ moves up or down by one step $\Delta$, but still remains within the feasibility set $Y$.

$$y' = \begin{cases} y + \Delta, \text{ with prob } \frac{\lambda}{2} (1 - \frac{y}{5\Delta}), \\ y - \Delta, \text{ with prob } \frac{\lambda}{2} (1 + \frac{y}{5\Delta}). \end{cases}$$

The probability of moving up is decreasing in the current value of $y$, which ensures the mean reversion. The parameters $\Delta$ and $\lambda$ are calibrated to match the empirical moments of the quarterly productivity, namely the standard deviation and autocorrelation.

3.2 Elasticity of Finding Rate with respect to the Market Tightness

Following Mortensen and Nagypál (2007), $\eta$ is estimated using the law of motion of unemployment at steady state: the flows out of unemployment (also the number of successful matches) equal the flows into unemployment. It gives that:

$$m(u, v) = s(1 - u). \tag{102}$$

Taking logarithms on both sides of equation (102) and combining with the specification of the matching function, it follows that

$$\ln \mu + \eta \ln v + (1 - \eta) \ln u = \ln s + \ln(1 - u).$$

Thus, the coefficient in an OLS regression of $ln(v \mid u)$ on $lnu$ can be derived as

$$\frac{\partial E \ln(v \mid u)}{\partial \ln u} = -\frac{1}{\eta} \left( \frac{u}{1 - u} + 1 - \eta \right).$$
The data moments stated in Table 1 imply that

$$\frac{\partial E \ln(v \mid u)}{\partial \ln u} = \rho_{vu} \frac{\sigma_v}{\sigma_u}.$$

Using the data on the average monthly unemployment rates over the period of 1962-2001, which are 7.78 percent in Canada and 5.67 percent in the United States, the estimated value of $\eta$ turns out to equal 0.54 in both countries.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_v$</th>
<th>$\sigma_u$</th>
<th>$\rho_{vu}$</th>
<th>$u$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.237</td>
<td>0.162</td>
<td>-0.689</td>
<td>0.0778</td>
<td>0.540</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.202</td>
<td>0.190</td>
<td>-0.894</td>
<td>0.0567</td>
<td>0.544</td>
</tr>
</tbody>
</table>

### 3.3 Statutory UI Benefits Replacement Rate

According to the 1955 Employment Insurance Acts and the subsequent amendments, the Canadian statutory UI replacement rate averaged, over the period of 1962-2003, 55 (60 for claimants with dependents) percent of the average yearly insurable earnings in the qualifying period. The simulations in Section 1.4 use $z/w = 0.6$ (the median for the whole sample of claimants).

### 3.4 Actual UI Benefits Replacement Rate

The actual UI benefits replacement rate is measured as the ratio of the average weekly UI benefits paid to an unemployed worker to the average weekly earning paid to an employed worker (term (1) in the table below). To facilitate the measure, the indicator of interest is decomposed into two parts shown as terms (2) and (3). Term (2) is the ratio of the average weekly benefits paid to an UI recipient to the average weekly insurable earnings paid to an employed worker on a gross basis. Term (3) is the eligibility rate, the ratio of the average weekly number of UI recipients to the average weekly number of unemployed workers.

<table>
<thead>
<tr>
<th>Actual UI Benefits Replacement Rate, 1972-2003</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Average UI benefits (the unemployed)</td>
</tr>
<tr>
<td>Average earnings (the employed) (1) =</td>
</tr>
<tr>
<td>Average UI benefits (UI recipients)</td>
</tr>
<tr>
<td>Average earnings (the employed) (2) ×</td>
</tr>
<tr>
<td>Eligibility rate (3)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Canada</td>
</tr>
<tr>
<td>0.265</td>
</tr>
<tr>
<td>0.406</td>
</tr>
<tr>
<td>0.653</td>
</tr>
<tr>
<td>U.S.</td>
</tr>
<tr>
<td>0.111</td>
</tr>
<tr>
<td>0.357</td>
</tr>
<tr>
<td>0.310</td>
</tr>
</tbody>
</table>
4. Data Source

4.1 Variables in the Labor Market


7. Real wage: Measured as the average nominal wage per worker in all industries divided by the implicit GDP deflator. The implicit GDP deflator is
measured as nominal dollar GDP divided by real GDP. In Canada, the data required are from Statistics Canada, CANSIM II, V500266 and V1996471 for the nominal wage, V498943 for the real GDP and V498086 for the nominal GDP over the period of 1962-2003. In the United States, the data are from the Bureau of Labor Statistics, Series PRS85006063 for the nominal compensation, PRS85006013 for employment, PRS85006043 for the real GDP and PRS85006053 for the nominal GDP.

4.2 Indicators of Generosity of Unemployment Insurance System

1. UI benefits replacement rate (for UI recipients): Measured as the ratio of the average weekly regular UI benefits paid to UI recipients to the average weekly earnings paid to employed workers on a gross basis. In Canada, the data required are from Statistics Canada, CANSIM II, V384494 for the average weekly regular UI benefits, and V75249, V729405, V1597104 for the average weekly earnings over the period of 1972-2003. In the United States, this ratio is directly from the U.S. Department of Labor Employment and Training Administration Annual Report and Financial Data (Taxable and Reimbursable Claim, Column 27) over the period of 1972-2003.

2. UI eligibility rate: Measured as the ratio of the monthly number of regular UI recipients to the monthly number of unemployment. In Canada, the data on the monthly regular UI recipients are from Statistics Canada, CANSIM II, V384652 and V2062814 over the period of 1976-2003. In the United States, this ratio is directly from Table C.1 in Wayne Vroman (2004) over the period of 1967-2003.

3. Minimum employment weeks to qualify for the regular UI benefit: In Canada, the data required are from Statistics Canada, Table 1 in Publication 11F0019MPE No.125 over the period of 1962-1994. After 1994, the minimum employment hours (or equivalently weeks) became depending on the regional unemployment rate. According to the relevant information from Human Resources and Skills Development Canada (see example from http://srv200.services.gc.ca/iws/EIRegions/toronto.aspx?rates=1), matching the long run unemployment rate 7.78%, 15 weeks is picked to be the approximate estimate. In the United States, see detailed discussion in Card and Riddell (1992) and Osberg and Phipps (1995): Publication IN-AH-223E-11-95 from Human Resources and Social Development Canada (HRSDC hereafter).

4. Entitlement weeks of the regular UI benefit: In Canada, the data required are from Table 4 in “EI Reform and Multiple Job-Holding - November 2001” released by the HRSDC. The same estimate can be seen in Belzil (2001), "Unemployment Insurance and Subsequent Job Duration: Job Matching ver-