ON IMPROVING SPECTRUM UTILIZATION THROUGH COOPERATIVE DIVERSITY AND DYNAMIC SPECTRUM TRADING

BY

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Abstract

The prime wireless spectrum is inherently a critical yet scarce resource. As the demand of wireless bandwidth grows exponentially, it becomes a crucial issue to improve the spectrum utilization for the development and deployment of any new wireless technologies. In this thesis, we seek to address this problem through cooperative diversity and dynamic spectrum trading, in the context of the envisioned primary-secondary dynamic spectrum sharing paradigm. For an OFDMA-based cellular primary network which owns an exclusive right to access a certain spectrum band, we propose XOR-assisted cooperative diversity to improve the spectral efficiency of the allocated band, as well as an optimization framework to address the resource allocation problem. For the secondary network that utilizes cognitive radios to opportunistically exploit the spectrum white spaces, we establish a spectrum secondary market, design the market institution based on double auctions, and solve the decision making problem using reinforcement learning, to improve spectrum utilization via trading among secondary users.
To my parents
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Chapter 1

Introduction

1.1 Background

While the radio spectrum is inherently scarce, the number of wireless technologies, devices, and services is growing unprecedentedly. As a result, the discrepancy between the unabated growth of wireless bandwidth demands and the severely limited availability of wireless spectrum becomes a crucial issue to address for the development and deployment of any new wireless services and technologies. The White House, for example, has been concerned enough to start a Presidential Spectrum Policy Initiative and create a multi-agency Federal Government Spectrum Task Force [1] to focus on interagency initiatives to use spectrum more efficiently.

Generally, the problem of spectrum scarcity has two distinct facets. First of all, the proliferation of bandwidth-intense delay-sensitive multimedia contents and applications, such as teleconferencing, mobile video streaming and distributed gaming, requires larger and larger wireless access network capacity. However, the wireless spectrum has been regulated under a static assignment policy, where very long-term (on the order of 10s
of years) spectrum leases covering large geographical regions are sold. Hence, the total amount of spectrum resources allocated to the wireless service providers is fixed. Further, purchasing additional spectrum from centralized spectrum management authority, such as the FCC in the U.S., often requires years to conclude and entails huge amounts of capital. Improving the spectral efficiency of the wireless access technologies therefore becomes imperative to the capacity improvement of the allocated spectrum and the realization of wireless broadband access.

Second, as becoming more and more evident, the static spectrum assignment policy itself also leads to underutilization of the scarce spectrum resource. National Science Foundation (NSF)-funded spectrum usage measurements have shown that the usage of licensed bands remain highly sporadic both geographically and temporally, especially in the cellular and TV broadcast bands [2]. A similar conclusion can be found in extensive spectrum measurements conducted at several major U.S. cities [3]. In other words, not all the allocated spectrum resources are utilized at all times in all regions. The static spectrum assignment results in *spectrum white spaces* that undermine the utilization and availability of the spectrum.

Driven by this observation, the idea of *dynamic spectrum sharing* [4, 5] has emerged as a revolutionary technology to improve the spectrum utilization by allowing secondary users to dynamically access the unused spectrum resources vacated by the primary users for delay-insensitive applications, such as file transfers and web browsing. Dynamic spectrum sharing is enabled by the development of *cognitive radios*, which utilize advanced radio and signal-processing technologies to sense the environment and adapt the transmission parameters [6]. Indeed, the FCC has recently established rules to allow devices to operate in broadcast television spectrum on a secondary basis, which represents the
first step to permit secondary operation in the TV white spaces [7].

With dynamic spectrum sharing and access, the next generation wireless networking paradigm is envisioned to be consisted of two major components: the primary network and the secondary network [8]. The primary network is an existing network infrastructure that owns an exclusive right to access a certain spectrum band, e.g. the common cellular and TV broadcast networks. The secondary network (or cognitive radio network) does not have the spectrum license, and opportunistically access the unused portion of the spectrum bands held by primary networks.

Corresponding to the two forms of spectrum scarcity above, our quest of how to improve the spectrum utilization in this thesis is then pursued in both the primary and secondary networks. In the primary network regime, we consider an Orthogonal Frequency Division Multiple Access (OFDMA) [9] based cellular network and focus on cooperative diversity, which serves as an emerging and powerful technology that holds tremendous promise to improve the spectral efficiency of wireless systems. For the secondary network, market-based spectrum sharing mechanism is increasingly recognized as the choice of solution framework, due to its operational simplicity and allocative efficiency. We are interested in designing efficient spectrum trading mechanism to enable dynamic and adaptive allocation of the spectrum and thus improve the spectrum utilization.
1.2 Cooperative Diversity in Cellular Primary Networks

Traditional wireless networks have predominantly used direct transmission in either a one-hop or multi-hop fashion. In contrast to conventional direct communications, cooperative communication allows the destination to listen to both the source and a relay that overhears and repeats the transmission due to the broadcast nature of the wireless medium. It is a new communication paradigm that promises significant capacity and multiplexing gain in wireless networks [10, 11]. It also provides a form of spatial diversity to combat severe channel fading, which is more commonly referred to as cooperative diversity.

Recent years have witnessed a significant amount of research in the field of cooperative diversity. In [12, 13], Sendonaris et al. introduced the concept of user cooperation diversity. Generally speaking, in a wireless network where a relay or a group of relays are located between the source and destination, the relay can facilitate the transmission by employing Amplify-and-Forward or Decode-and-Forward strategy [14]. In both cases, the combination of the directly transmitted signal and relayed signal provides a form of cooperative diversity, which has been shown to improve the throughput and/or power efficiency [12, 13, 14]. Cooperative techniques have also been considered in various IEEE 802 standards. For example, the IEEE 802.16j standard is concerned about adopting multi-hop relay to enhance coverage, throughput, and system capacity of the IEEE 802.16 fixed and mobile wireless metropolitan-area network [15].

Network coding, another interesting technique to allow coding capability in exchange for network capacity gain, has spurred a plethora of research attention. It is shown
to improve performance in multi-hop wireless networks [16] and information exchange paradigm [17], when used beyond traditional routing. In the context of cooperative diversity, network coding has also been leveraged at the relay to mix packets from different cooperative sessions, provided that the relay overhears and successfully decodes multiple transmissions and these transmissions share a common destination [18, 19, 20].

In this thesis, we investigate cooperative diversity with the use of network coding to improve spectral efficiency from a new perspective. Previous works rely heavily on the single shared channel model which lays down the groundwork for the overhearing ability. We instead seek to understand network coding aided cooperative diversity in multi-channel networks, which has not yet been touched upon. We assume the context of OFDMA [9] based cellular primary networks, such as WiMAX [21] and LTE [22] networks. Multi-channel networks impose unique challenges of using network coding aided cooperative diversity. In these networks, overhearing is no longer naturally available. Users can hear each other only when tuned to the same channel. Coding opportunities are therefore to be carefully invented and engineered, rather than opportunistically harvested. Moreover, network coding entails that the broadcast rate is confined to the worst rate of all links involved, aggravating the task of finding profitable coding opportunities.

To address these challenges, we propose XOR-CD, a cooperative diversity scheme with XOR network coding on bidirectional traffic of a mobile station. Through information theoretical analysis and computer simulations, we show that it can dramatically improve the spectral efficiency of relaying and thereby boost throughput performance. In addition, we formulate an optimization framework that jointly considers channel assignment, relay assignment, relay strategy selection and power allocation, to tackle the resource allocation problem with XOR-CD. Efficient optimal and approximation algorithms are proposed.
based on set packing and bipartite matching and are thoroughly evaluated via computer simulations.

1.3 Dynamic Spectrum Trading in Secondary Networks

In the secondary network regime, substantial research efforts have been undertaken for users to dynamically share the spectrum. In the context of primary-secondary sharing model, how to incentivize primary users to open up their underutilized spectrum, and how to ensure the secondary transmissions will not cause harmful interference, are two important questions to address for any meaningful sharing method. Among others, market-based mechanisms, where blocks of abundant spectrum are “rented” out in exchange for payments, are generally believed by policy makers and technologists to be simple yet effective solutions in the sense that secondary users in need can gain access and primary ones can obtain financial benefits [23]. Auctions are perceived to be fair and efficient market mechanism to achieve dynamic spectrum sharing through trading [4, 24, 25, 26, 27].

State-of-the-art spectrum auctions, to our knowledge, are proposed under a primary market paradigm. Specifically, these auctions are performed weekly or daily with legacy spectrum owners on the selling side and secondary users on the buying side. Channels are often modeled to be homogeneous, buyers are assumed to be static and have fixed demands, and interference information such as conflict graphs are globally available. From an economics perspective, such an approach parallels a primary market of the capital markets, and is only suitable to deal with the issuance of relatively long-term
spectrum leases from legacy owners to large secondary entities [28].

In contrary, we mainly focus on dynamic spectrum trading among small secondary users themselves, *e.g.* mesh routers of small wireless networks, APs of home networks, mobile devices. By shifting from a “macro” to a more “micro” perspective, we observe that the underlying assumptions of the primary market paradigm no longer hold.

*First*, due to the bursty nature of individual traffic demand [29], secondary users have a natural desire to trade their spectrum in an ad hoc fashion *among themselves* to adapt to their instantaneous demands. Specifically, as the demand for spectrum varies over time for a particular user, it is beneficial to *sell* excessive channels it currently holds when traffic demand is reduced. Future purchases can then be made with the windfall from the sale when traffic demand increases, and other users in need may bid for the channels to satisfy their unfulfilled demands. However, conventional monolithic primary market auctions miss such potential benefits of redistributing resources dynamically from those with excessive supplies to those with outstanding demands, undermining the efficiency of using the wireless spectrum.

*Second*, due to fading and mobility, the quality of a wireless channel varies significantly over time. As such, the optimal spectrum allocation at the time of an conventional auction may become detrimental before the next one takes place. With such time-varying channel qualities, users have the inherent tendency to *sell* their channels in exchange for better ones. Both time-varying channel qualities and user demands call for a “trading” platform among users, so that spectrum as a resource becomes more liquid and easier to obtain and relinquish, leading to more efficient utilization. Such spectrum trading among users is not feasible in the spectrum primary market.

*Third*, for a given wireless channel, different users experience independent fading and
have different channel gains, generally referred to as multi-user diversity. Such diversity gains can be effectively leveraged to improve spectrum utilization, by allocating channels to users with the best channel gains. Conventional spectrum auctions only support homogeneous channels and do not exploit potential benefits of multi-user diversity.

Therefore, efficient market mechanisms need to be developed for dynamic spectrum trading among secondary users themselves. Moreover, despite the vast amount of literature on establishing spectrum market to enable dynamic spectrum access, there is still a lack of understanding on users’ strategic interaction with such complex market environments, being informationally decentralized and heterogeneous. The use of microeconomics in dynamic spectrum trading is mainly concerned about the design of game or system to achieve a specific outcome (mechanism design), or the characterization of the equilibrium outcomes of given games (game theory). As pointed out in [30], it most often neglects how an individual user, being selfish and autonomous, acquires information through its interactions with the market and observations of the relevant outcomes, and how such information impacts its behavior in cooperating and competing with other users to dynamically acquire resources in the spectrum market. Therefore, further understanding and elegant solution are imperatively needed for the user rationale problem, especially on how to value the channels, and how to make intelligent trading decisions in the spectrum market, which is important towards the efficient operation of market-based dynamic spectrum access.

As an early attempt to unravel these questions, we propose a spectrum secondary market that complementary to the primary market, thus completing the big picture of spectrum trading. The secondary market mechanism is based on double auctions that
allow bilateral trading of the spectrum among secondary users. We prove critical properties of the double auction mechanism, most importantly its truthfulness and efficiency, and show its versatility and robustness in supporting dynamic spectrum trading through computer simulations. To characterize user rationale in the market, we formulate it as a dynamic program, and adopt a reinforcement learning framework to design efficient learning algorithms to gradually form an optimal trading policy.

1.4 Structure of the Thesis

The reminder of the thesis is structured as follows. In Chapter 2, we present related work. In Chapter 3 we introduce a novel XOR-assisted cooperative diversity and an optimization framework for the resulted resource allocation problem in OFDMA cellular primary networks. In Chapter 4, we establish a spectrum secondary market based on double auctions, and present a comprehensive characterization of the market in terms of market mechanism and user rationale. Finally, we summarize the thesis and discuss future directions in Chapter 5.
Chapter 2

Related Work

In this chapter, we summarize related works on the fields of cooperative diversity and dynamic spectrum access respectively.

2.1 Cooperative Diversity

In wireless networks, multi-path fading can be effectively combated through the use of diversity—redundant transmissions of the same signal over essentially independent fading channels in conjunction with appropriate receiver combining techniques. Cooperative diversity is a particular form of antenna diversity that utilizes distributed antennas belonging to different users through relaying. The receiver combines the directly transmitted signal with the relayed signal from other helping users to average out the impairing channel effect.

The roots of cooperative diversity can be traced back to the relay channel model studied in [10]. The popularity of cooperative diversity is owed to [12, 13, 14], where different cooperation strategies such as Amplify-and-Forward and Decode-and-Forward
are developed. Performance characterizations in terms of outage probability are provided which reveal that large power savings are resulted from the use of cooperative diversity. More complex relay protocols, including distributed space time coded cooperations, are considered in many following papers [31, 32].

Network coding, introduced in [33], is originally proposed in information theory to achieve the cut-set capacity bounds of unicast flows from the source to each destination in a multicast scenario. The key idea is to combine incoming packets through coding at intermediate nodes. Li et al. [34] show that linear codes are sufficient to achieve the maximum rate. Along with theoretical explorations, network coding has also been shown to practically improve throughput in wireless multi-hop networks when combined with routing [16]. A similar conclusion is made for the information exchange paradigm in which two nodes exchange data with each other via a relay [17]. In these works, cooperative diversity is not leveraged as a mechanism to combat fading, since the destination only processes the transmission from the relay.

With regards to the use of network coding in cooperative diversity, in [19, 20] simple network coding based cooperative diversity schemes are proposed for networks with single and multiple source-destination pairs, and are shown to lead to higher spectral efficiency and lower outage probability. In these analysis, it is generally assumed that there is always at least one node willing to relay, and the message can always be successfully overheard at the relaying node. These assumptions implicitly postulate that the network is very dense and the user-relay channel is perfect, which are artificial and self-defeating considering the interference in such a relay-enabled dense network. Another line of research focuses on developing information theoretically efficient channel coding schemes with the aid of network coding [35]. This requires complicated encoder and decoder
2.1. COOPERATIVE DIVERSITY

Table 2.1: Related studies to XOR-assisted cooperative diversity.

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design which is often impractical.

We emphasis here that, in general, most existing works entail the overhearing assumption with a single shared channel model, which significantly simplifies the relay protocol design and performance analysis, while we assume an OFDMA based multi-channel setting in which the joint optimization of resource allocation and relay strategy selection becomes a unique challenge to address.

Optimization in cooperative networks has received some research attention recently [36, 37, 38, 39]. [36] considers power allocation for a simple triangle network with one pair of source-destination and one relay. [37] considers multi-hop ad hoc networks and proposes a framework that jointly considers routing, relay selection and power allocation. [38, 39] consider OFDMA based networks and are most related to our work. [38] studies channel assignment and power allocation for multi-hop OFDMA networks. [39] proposes solutions for joint optimization of channel assignment, relay strategy selection and power allocation in OFDMA cellular networks based on conventional AF and DF.

Different from previous works on network coding aided cooperative diversity and optimization of cooperative networks (summarized in Table 2.1), to our knowledge, this work represents the first attempt to study cooperative diversity in multi-channel networks with the use of network coding. We propose a novel diversity scheme with XOR that can
dramatically improve the resource efficiency of relaying and thereby boost throughput performance. More importantly, we formulate an optimization framework that jointly considers network coding, channel assignment, relay strategy selection and power allocation, which has not yet been discussed.

2.2 Auction Based Dynamic Spectrum Sharing

In the second part of the thesis, we focus on auction-based dynamic spectrum sharing to improve spectrum utilization in the secondary network. Auctions are generally believed to be beneficial to the bidders as they assign goods to those who value them most. They are beneficial to the spectrum regulators as well because they get a higher revenue than that obtainable through static pricing [40]. FCC has been conducting spectrum auctions to sell licenses of wireless spectrum over large geographical areas since 1994 [41]. However, so far, no auctions have been conducted for the secondary networks. Comprehensive literature related to the traditional spectrum auctions is available [42, 43], which is only concerned with allocating primary rights of the spectrum with time periods of years to large service providers.

Efficiency is one of the most important goals researchers usually pursue when designing auction mechanisms. An efficient market maximizes the total profit obtained by all participating users [44]. To achieve efficiency, the ideal mechanism is to let users report their true valuations of the goods and the desired quantities. Then the auctioneer solves an optimization problem based on this information to maximize the total profit of all users. However, moment’s of reflection tells us that such method is infeasible, because in practice one cannot prevent users from lying about their valuation of the goods, provided
that doing so can improve their own profit. Therefore, \textit{truthfulness} becomes another critical design goal of auction mechanism, which ensures no one can expect a higher utility gain then reporting its true valuation. Other than efficiency and truthfulness, budget balance and individual rationality are necessary to ensure that the auctioneer and users have incentives to participate in the auction since doing so will result in non-negative utility gain for them.

Existing works on dynamic spectrum auctions invariantly adopted a primary market based approach between primary networks as sellers and secondary networks as buyers. Single-sided auctions are usually considered in which only the buyers compete in price for the spectrum being auctioned. In [4], spectrum band auctions have been proposed, where bidders obtain different spectrum channels to minimize interference. [45] proposes a multi-winner auction to prevent user collusion and optimize spectrum efficiency, while the truthfulness is compromised. [46] proposes approximation allocation algorithms to approach the optimal revenue for primary networks subject to interference constraint, while the truthfulness is not taken into account as well.

A truthful auction design is proposed in [26] based on the classical VCG auction [47] that addresses the spatial reuse of spectrum. Recently, another interesting work [48] considers another possibility of auctioning both the rights of being primary and secondary networks on each channel together, and designs a truthful auction framework with different assumptions of bid independence. For all existing works, channels are often modeled to be homogeneous, buyers are assumed to be static and have fixed demands, and interference information such as conflict graphs are globally available. In contrast to these works that essentially mimic a spectrum primary market based on mostly single-sided auctions with only competitive buyers, we establish a novel spectrum secondary
Auction based Dynamic Spectrum Sharing

2.2. AUCTION BASED DYNAMIC SPECTRUM SHARING

A double auction is a process of buying and selling goods with potential buyers and sellers submitting their bids and asks simultaneously to an auctioneer, who sets some price to clear the market. The bilateral nature of trading clearly sets it apart from single-sided auctions. Real-world examples of double auctions include the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX). Note that Zhou et al. [27] have recently proposed to use double auctions together with channel allocation algorithms for spectrum trading. Although [27] adopts a double auction mechanism, it is still designed as a primary market, and only supports homogeneous channels and single-unit trading.

The literature on mechanism design of double auctions is limited. In economics, theoretical and empirical studies [49, 50, 51] have been conducted to show that double auctions are more efficient than single auctions. [52] proposes a single-unit double auction that is truthful and efficient when the number of users becomes large, while [53] advocates a VCG double auction with truthfulness and efficiency. However, the former cannot support multi-unit trading, and the latter is not ex-post budget balanced and requires the auctioneer to have infinite financial incentive to hold the auction. [50] proposes a multi-unit double auction without theoretical analysis on its economic properties. Therefore these schemes cannot be applied for spectrum trading in secondary market directly. Our work is motivated by these existing works, but differs in a number of important aspects. We design the mechanism to enforce users to bid truthfully according to their instantaneous evaluations of spectrum, while supporting multi-unit trading. Second, our design asymptotically maximizes the social welfare from trading, thus optimizes the utilization
of the spectrum resources. These and other critical economic properties, including budget balance and individual rationality, are rigorously proved. To our knowledge, no prior work has studied the characterization of the spectrum secondary market, including both mechanism design and user rationale.

A vast literature exists for the decision making problem in autonomous systems (see [54] and references therein). Users are often modeled as strategic players from a game theoretic perspective. Some specific games are proposed that lead to the preferable equilibrium point. This requires strong assumptions, for instance, on the exact information about the entire game, including private information of other players. In the context of spectrum secondary markets, these assumptions do not apply. For sequential decision making with incomplete information, reinforcement learning is a well-suited solution framework that has been applied to complex practical scenarios such as robot control and chess playing [55]. To our knowledge, however, there is no related work on using reinforcement learning in spectrum trading.
Chapter 3

XOR-Assisted Cooperative Diversity in OFDMA Cellular Networks

Cooperative diversity, and network coding, are arguably two of the most promising technologies to improve the spectral efficiency in the primary network regime. In this chapter, we investigate the use of network coding in cooperative diversity from a new perspective. Previous work relies heavily on the single shared channel model which lays down the groundwork for the overhearing ability. We instead seek to understand network coding aided cooperative diversity in the context of Orthogonal Frequency Division Multiple Access (OFDMA) [9] based networks. Multi-channel networks impose unique challenges of using network coding aided cooperative diversity. In these networks, overhearing is no longer naturally available. Users can hear each other only when tuned to the same channel. Coding opportunities are therefore to be carefully invented and engineered, rather than opportunistically harvested. Moreover, network coding entails that the broadcast rate is confined to the worst rate of all links involved, aggravating the task of finding profitable coding opportunities.
3.1 Main Contributions

Our first contribution in this chapter is a novel XOR-assisted cooperative diversity scheme in OFDMA based primary networks, referred to as XOR-CD. It exploits coding opportunities on bi-directional traffic on the uplink and downlink of a Mobile Station (MS). Bi-directional traffic is profoundly available in OFDMA cellular networks, providing abundant network coding opportunities. Further, the channel conditions of the uplink and downlink of the same MS do not differ substantially, especially when path loss is dominant over random fading. This remedies the shortcoming of coding to the lowest rate of the spatially apart links that may differ significantly [56].

Fig. 3.1 illustrates an example to show the basic idea of XOR-assisted cooperative diversity (XOR-CD). Bi-directional traffic exists between MS and the BS. Assume that the Relay Station (RS) helps through cooperative relaying using orthogonal channels. By performing maximum ratio combining (MRC) over the two received copies of the same content, both MS and BS can enjoy a lower outage probability and thus higher throughput [14]. XOR network coding can be used here to mix packets A and B at the
3.1. MAIN CONTRIBUTIONS

RS and multicast to MS and BS an re-encoded packet \((A \oplus B)'\) using only one single subchannel. Assume that channel coding and modulation are linear, \((A \oplus B)' = A' \oplus B'\). The MS and BS can still receive the intended information by XOR-ing the combined packet with one that is known \textit{a priori} to itself. Therefore, MRC can still be performed and cooperative diversity can still be capitalized.

The benefits of XOR-CD are intuitive. In the ideal case where channels are symmetric, and the BS-RS and MS-RS channel qualities are the same, XOR-CD achieves the same transmission rate for both cooperative sessions involved with a saving of one sub-channel and the power of one transmission compared to the conventional DF. The saved sub-channel and power can be used to accommodate more cooperative sessions, thereby further improving the network throughput.

Our second contribution is a unifying optimization framework to practically reap the promising gains of XOR-CD in OFDMA networks with reasonable complexity. We note that three kinds of gains can be exploited: (i) \textit{multi-user diversity gain}: for a given data/relay subchannel, different MS experience independent fading processes, allowing us to assign the subchannel to the MS with the largest channel gain; (ii) \textit{cooperative diversity gain}: RS helps the intended receiver to combat fading and improve SNR through cooperative relaying; (iii) \textit{network coding gain}: bi-directional traffic between BS and MS is amenable to network coding which is utilized at RS to make relaying more resource efficient, increasing the network capacity. In light of different forms of gains available, our framework jointly considers relay assignment, relay-strategy selection, and subchannel assignment for both MS and RS in a single cell (hereafter referred to as RSS-XOR problem). We prove that RSS-XOR is NP-hard, and propose efficient polynomial-time algorithms based on packing with constant approximation factor.
Finally, we also extend our model to consider power allocation among cooperative sessions the RS supports. We propose subgradient-based algorithms to solve the power allocation problem in the dual domain, and show that XOR-CD consistently performs better in power-limited settings.

Note that we only consider the simplest form of network codes, XOR here. Random linear network coding may be applied to encode packets from different MS together. However, since the broadcast rate is confined to the worse rate of all links involved, whether doing so can outperform the simplest XOR codes is doubtful, not to mention the extra computational overhead introduced by linear network codes. To simplify the analysis, in this work we focus on XOR network codes only.

3.2 System Models

In this section, we introduce the underlying system models for the RSS-XOR optimization framework.

3.2.1 Network Models

We consider a single-cell OFDMA wireless network. BS is communicating with each MS with bi-directional traffic. The system operates in FDD mode, meaning that the uplink and downlink of an MS are assigned orthogonal sets of data subchannels. A small number of RS are employed in the cell to provide cooperative diversity. They may help some MS for transmissions on some of their data subchannels, using relay subchannels from a relay channel pool orthogonal to the data channel pool. One relay subchannel is used to support only one data subchannel of the MS in conventional cooperative diversity.
(CD) scheme. In the case of XOR-CD, one relay subchannel is used to support two data subchannels, one for uplink and one for downlink, as we illustrated. We further assume that the BS and MS always have frames to send and the OFDM frames are synchronized. In this case, it can be conceived that cooperative transmission progresses in parallel with direct data transmission in the long run. DF is used as the conventional CD scheme.

3.2.2 Channel Models

We model the wireless fading environment by large scale path loss and shadowing, along with small scale frequency-selective Rayleigh fading. Fading between different subchannels are independent. Note that we do not assume that the up and downlink are symmetric and has the same fading statistics. The network operates in a slow fading environment, so that channel estimation is possible and full channel side-information (CSI) is available, which makes the optimization possible. Such assumptions about the fading environment are commonly used as in [36, 39, 38].

An equal amount of power is allocated for data and relay transmissions across all data and relay subchannels. In the extended models with relay power allocation, however, RS can adjust the power level for each of the relay subchannels they use in order to confine themselves to their power budget.

3.3 An Optimization Framework for XOR-CD

We present our optimization framework in this section.
3.3.1 Notations

Denote $\zeta$, $\psi$, $\Omega$ and $\Phi$ as the set of data subchannels, relay subchannels, MS, and RS, respectively. $s \in \Omega$ denotes an MS and $r \in \Phi$ denotes an RS $r$, respectively. $l \in \mathcal{L}$ denotes a directed link from the source $S(l)$ to the destination $D(l)$ where $\mathcal{L}$ denotes the set of all links. Each link, being uplink or downlink, has a corresponding MS $s$ such that $D(l) = s$ or $S(l) = s$. Let $M(l) = s$ denote this relationship between $l$ and $s$. Each link can operate in one and only one of three modes, namely the direct transmission mode, conventional CD mode and XOR-CD mode, depending on the choice of relay strategy. Define function $R(c_i, l)$ as the achievable direct transmission rate of link $l$ when it is assigned with subchannel $c_i \in \zeta$. For conventional CD, $R(c_i, c_r, r, p^r_{c_i, c_r})$ is defined as achievable rate function of $l$, when RS $r \in \Phi$ is assigned to be the relay for transmission on data subchannel $c_i$, with allocated power $p^r_{c_i, c_r}$ on relay subchannel $c_r$. For XOR-CD, $R(c_i, c_j, c_r, r, p^r_{c_i, c_j, c_r})$ denotes the achievable rate function if $r$ is the relay of $s$ for its uplink transmission on $c_i$ and downlink transmission on $c_j$, with allocated power $p^r_{c_i, c_j, c_r}$ on relay subchannel $c_r$.

3.3.2 Information Theoretic Analysis

We first seek to provide an information theoretical analysis for the XOR-CD scheme in OFMDA wireless networks, in order to derive the rate functions for three transmission modes. The complex channel gains for different links are denoted as shown in Fig. 3.2. The noises are modeled as i.i.d. circularly symmetric complex Gaussian noises $\mathcal{CN}(0, N_0 W)$. 
3.3. AN OPTIMIZATION FRAMEWORK FOR XOR-CD

Figure 3.2: The channel models for DF and XOR-CD, where $h_{l,c}$ denotes channel gain of link $l$ when it is assigned subchannel $c$.

Direct Transmission

For direct transmission, the achievable rate is found using the well-known formula (in b/s/Hz):

$$R(AB,c_1) = \log_2 \left(1 + \frac{P \cdot |h_{AB,c_1}|^2}{\Gamma N_0 W}\right), \quad (3.1)$$

where $\Gamma$ is the gap to capacity and $P$ denotes the direct transmission power. For notational convenience we denote $\frac{|h_{AB,c_1}|^2}{\Gamma N_0 W}$ as $CNR_{AB,c_1}$, where $CNR$ represents channel gain-to-noise ratio. Then the rate function can be expressed as:

$$R(AB,c_1) = \log_2(1 + P \cdot CNR_{AB,c_1}). \quad (3.2)$$

Conventional CD

For the DF relay channel for transmission from $A$, first $R_0$ attempts to decode $A$’s message. Assuming decoding is successful, $R_0$ transmits to $B$ with power $p_{AB}^{R_0,c_1,c_4}$ using
3.3. AN OPTIMIZATION FRAMEWORK FOR XOR-CD

the relay subchannel \( c_4 \) as depicted in Fig. 3.2. Therefore, the maximum rate for this mode can be readily found to be

\[
R(c_1, c_4, R_0, p_{AB}^{R_0, c_1, c_4}, AB) = \min \{ \log_2(1 + P \cdot CNR_{AR_0, c_1}), \\
\log_2(1 + P \cdot CNR_{AB, c_1} + p_{AB}^{R_0, c_1, c_4} \cdot CNR_{R_0 B, c_4}) \}. \tag{3.3}
\]

Note that compared to result from [14], our result does not have \( \frac{1}{2} \) before the expression. The reason is that a two-slot implementation is assumed in [14] with a shared channel, whereas we assume relay transmission progresses in parallel with data transmission, which is valid in the long run in our OFDMA-based multi-channel model.

Inspecting the rate function, we can see that increasing the relay power will first increase the rate, but not any more after reaching a threshold since pumping a higher rate will make the relay unable to decode. The threshold value of the relay power is such that

\[
R(c_1, c_4, R_0, \tilde{p}_{AB}^{R_0, c_1, c_4}, AB) = \log_2(1 + P \cdot CNR_{AR_0, c_1})
\]

\[
= \log_2(1 + P \cdot CNR_{AB, c_1} + \tilde{p}_{AB}^{R_0, c_1, c_4} \cdot CNR_{R_0 B, c_4}), \tag{3.4}
\]

which gives us

\[
\tilde{p}_{AB}^{R_0, c_1, c_4} = \frac{CNR_{AR_0, c_1} - CNR_{AB, c_1}}{CNR_{R_0 B, c_4}} P. \tag{3.5}
\]

Similar analysis can be given for \( R(c_2, c_3, R_0, p_{BA}^{R_0, c_2, c_3}, BA) \) and the threshold power \( \tilde{p}_{BA}^{R_0, c_2, c_3} \).
3.3. AN OPTIMIZATION FRAMEWORK FOR XOR-CD

XOR-CD

The relay transmissions from $R_0$ to $A$ and $B$ are done by performing XOR over the two messages and multicasting using a single relay subchannel $c_3$. Therefore, the rate of this mode, for each of the two links involved, can be shown to be

$$R(c_1, c_2, c_3, R_0, p_A^{R_0,c_1,c_2,c_3}, A) = \min\{\log_2(1 + P \cdot CNR_{AR_0,c_1}), \log_2(1 + P \cdot CNR_{BR_0,c_2}), \log_2(1 + P \cdot CNR_{AB,c_1} + p_A^{R_0,c_1,c_2,c_3} \cdot CNR_{R_0B,c_3}), \log_2(1 + P \cdot CNR_{BA,c_2} + p_A^{R_0,c_1,c_2,c_3} \cdot CNR_{R_0A,c_3})\},$$

(3.6)

assuming $A$ is an MS and $B$ is the BS. The first two terms in (3.6) represent the maximum rate at which the relay can reliably decode the source messages from both $A$ and $B$, while the last two terms in (3.6) represent the maximum rate at which $A$ and $B$ can reliably decode the intended message given repeated transmissions from $R_0$’s multicast. Again the threshold value of relay power at $R_0$ is such that

$$R(c_1, c_2, c_3, R_0, p_A^{R_0,c_1,c_2,c_3}, A) = \min\{\log_2(1 + P \cdot CNR_{AR_0,c_1}), \log_2(1 + P \cdot CNR_{BR_0,c_2})\}.$$ 

The expression for threshold relay power $p_A^{R_0,c_1,c_2,c_3}$ can then be derived.

3.3.3 RSS-XOR Problem

Our main objective is to optimize the strategies of assigning appropriate sets of relay subchannels to RS and data subchannels to MS, and pairing RS to data subchannels of MS with different choices of relay strategies, in order to maximize the aggregated
3.3. AN OPTIMIZATION FRAMEWORK FOR XOR-CD

throughput subject to a fairness model. We consider proportional fairness model, given its ability to strike a good balance between utilization and fairness and its robustness with respect to changes in topology and power constraints [57]. Denote the throughput of link $l$ as $\lambda_l$, then the long-term objective function under proportional fairness model can be expressed as $\max \sum_l \ln \bar{\lambda}_l$, where $\bar{\lambda}_l$ is the long-term average throughput for link $l$. The optimization needs to be done periodically as MS may move and channel qualities change over time. It is proved in [57] that the instantaneous optimization maximizing the marginal utility $\sum_l U_l'(\bar{\lambda}_l) \cdot \lambda_l$ at each epoch (interval) leads to a long-term maximization of $\sum_l U_l(\bar{\lambda}_l)$, if we assume link $l$ has a concave utility function $U_l(\bar{\lambda}_l)$. Therefore, our objective at each epoch $t$ is:

$$\max \sum_{l \in \mathcal{L}} \frac{\lambda_l}{\bar{\lambda}_l(t)}$$ (3.7)

For both uplink and downlink, traffic can be classified into three classes corresponding to three transmission modes, namely direct traffic, conventional CD traffic and XOR-CD traffic. Introduce three 0–1 decision variables $x_{ci}^l$, $y_{r,c_i,c_r}^l$, and $z_{r,c_i,c_j,c_r}^s$. $x_{ci}^l$ indicates whether link $l$ is assigned data subchannel $c_i$ for direct transmission. $y_{r,c_i,c_r}^l$ indicates whether link $l$ is operating in conventional CD mode with RS $r$ and data-relay subchannel pair $(c_i, c_r)$. Each MS may be assigned multiple such channel pairs depending on the instantaneous channel quality. $z_{r,c_i,c_j,c_r}^s$ indicates whether MS $s$ is assigned with RS $r$ and relay subchannel $c_r$ for its uplink on data subchannel $c_i$ and downlink on $c_j$ for XOR-CD.

Since an equal amount of power $P$ is used for every direct and relay transmission, the
3.3. AN OPTIMIZATION FRAMEWORK FOR XOR-CD

throughput constraint of link \( l \) can be characterized as follows:

\[
\lambda_l = \sum_{c_i} R(c_i, l) x_{c_i}^l + \sum_{c_i, c_r, r} R(c_i, c_r, r, P, l) y_t^{r,c_i,c_r} + \sum_{c_i, c_j, c_r, r} R(c_i, c_j, r, P, s) z_s^{r,c_i,c_j,c_r},
\]

(3.8)

where \( s = M(l), \forall l \in L \).

For data subchannels, we dictate that each data subchannel can only be assigned to one link which operates in one of the three modes. Therefore,

\[
\sum_{l \in L} \left( x_{c_i}^l + \sum_{c_r \in \psi, r \in \Phi} y_t^{r,c_i,c_r} \right) + \sum_{s \in \Omega, r \in \Phi, c_j \in \zeta, c_r \in \psi} \left( z_s^{r,c_i,c_j,c_r} + z_s^{r,c_j,c_i,c_r} \right) \leq 1, \forall c_i \in \zeta,
\]

(3.9)

where the first term accounts for the possibility that \( c_i \) is assigned for direct and conventional CD modes, and the second term accounts for the possibility of XOR-CD mode. Notice that this constraint also implicitly takes into consideration that each link can only operate in one of the three modes.

Similarly, each relay subchannel can be assigned to only one cooperative session, be it conventional CD session or XOR-CD session. Therefore,

\[
\sum_{l \in L} \sum_{r \in \Phi} \sum_{c_i \in \zeta} y_t^{r,c_i,c_r} + \sum_{s \in \Omega} \sum_{r \in \Phi} \sum_{c_i \in \zeta} \sum_{c_j \in \zeta} z_s^{r,c_i,c_j,c_r} \leq 1, \forall c_r \in \psi.
\]

(3.10)

Consequently, the RSS-XOR problem becomes a mixed-integer program, with the objective (3.7), subject to constraints (3.8), (3.9), and (3.10).
3.3.4 NO-XOR Problem

We also provide a framework for the joint optimization with only conventional cooperative diversity, i.e., the NO-XOR problem. It is studied as a baseline comparison. The NO-XOR problem can be formulated in a similar way as the RSS-XOR problem, with $z_{s}^{r,c_{i},c_{j},c_{r}}$ equal to zero for any $c_{i}, c_{j} \in \zeta$, $c_{r} \in \psi$, $s \in \Omega$, $r \in \Phi$. Formally,

\text{NO-XOR: } \max \sum_{l \in L} \frac{\lambda_{l}}{\lambda_{l}(t)} \quad \text{s.t.} \quad \lambda_{l} \leq \sum_{c_{i} \in \zeta} R(c_{i}, l)x_{l}^{c_{i}}
\quad + \sum_{c_{i} \in \zeta} \sum_{c_{r} \in \psi} \sum_{r \in \Phi} R(c_{i}, c_{r}, r, P, l)y_{l}^{r,c_{i},c_{r}},
\quad \sum_{l \in L} x_{l}^{c_{i}} + \sum_{l \in L} \sum_{r \in \Phi} \sum_{c_{r} \in \psi} y_{l}^{r,c_{i},c_{r}} \leq 1, \ \forall \ c_{i} \in \zeta,
\quad \sum_{l \in L} \sum_{r \in \Phi} \sum_{c_{i} \in \zeta} y_{l}^{r,c_{i},c_{r}} \leq 1, \ \forall \ c_{r} \in \psi. \quad (3.11)

3.3.5 Power Allocation

We extend the two models by considering the scenario wherein each RS has a power budget constraint. RS then have to allocate the right amount of power across all cooperative sessions so as to maximize marginal utility. Mathematically, the throughput constraints of both problems are updated by replacing $R(c_{i}, c_{r}, r, P, l)$ with $R(c_{i}, c_{r}, r, p_{l}^{r,c_{i},c_{r}}, l)$ and $R(c_{i}, c_{j}, c_{r}, r, P, s)$ with $R(c_{i}, c_{j}, c_{r}, r, p_{s}^{r,c_{i},c_{j},c_{r}}, s)$ in (3.8) and (3.11). Moreover, the constraint that the total power used at the RS cannot exceed its power budget can be
expressed as follows for the RSS-XOR problem:

\[
\sum_{l \in \mathcal{L}} \sum_{c_i \in \zeta} \sum_{c_r \in \psi} p_{l}^{r, c_i, c_r} + \sum_{s \in \Omega} \sum_{c_i \in \zeta} \sum_{c_j \in \zeta} \sum_{c_r \in \psi} p_{s}^{r, c_i, c_j, c_r} \leq P_r, \forall r
\]  

(3.12)

where \( P_r \) denotes the power budget of RS \( r \). Power allocation version of RSS-XOR can be formulated by adding constraint (3.12) into the original formulation.

For NO-XOR, the power constraint is simply:

\[
\sum_{l \in \mathcal{L}} \sum_{c_i \in \zeta} \sum_{c_r \in \psi} p_{l}^{r, c_i, c_r} \leq P_r, \forall r \in \Phi. \tag{3.13}
\]

The power allocation version of NO-XOR is similarly formulated by adding constraint (3.13) into (3.11).

### 3.4 Approximation Algorithms

Conventional approaches to solve mixed-integer problems, such as branch and bound, are computationally formidable. Our solution algorithms need to be called at each epoch, making the task of deriving efficient heuristic algorithms imperative. We propose algorithms that can be applied to the BS and RS in real OFDMA-based wireless networks for the RSS-XOR and NO-XOR problems. Specifically, we first prove that RSS-XOR is NP-hard and show it can be solved in polynomial-time with an approximation ratio of 1.5 using our algorithm. We then show that NO-XOR can be optimally solved by transforming to weighted bipartite matching. Finally we propose a subgradient algorithm to solve power allocation of the two problems.
3.4. APPROXIMATION ALGORITHMS

3.4.1 A Set Packing Algorithm for RSS-XOR

Solving the seemingly prohibitive RSS-XOR problem hinges on transforming to a weighted set packing problem. We first establish the equivalence and prove the NP-hardness, then propose our algorithm with a constant approximation factor.

![Figure 3.3: Set construction and transformation into an intersection graph with 2 data subchannels and 2 relay subchannels. Vertices in $G_C$ correspond to sets in $C$. Edges are added between vertices whose corresponding sets intersect.](image)

**Proposition 3.1** The RSS-XOR problem is equivalent to a maximum weighted 3-set packing problem, and is NP-hard.

**Proof** Construct a collection of sets $C$ from a base set $\zeta \cup \psi$ as shown in Fig. 3.3. There are three kinds of sets, representing three transmission modes respectively. $(c_i)$, where $c_i \in \zeta$ represents the direct transmission mode with data subchannel $c_i$. $(c_i, c_r)$ where $c_i \in \zeta, c_r \in \psi$ corresponds to the conventional CD mode with data subchannel $c_i$ and relay subchannel $c_r$. The third kind, $(c_i, c_j, c_r)$ corresponds to the XOR-CD mode with data subchannel pair $(c_i, c_j)$ and relay subchannel $c_r$, where $c_i, c_j \in \zeta, c_r \in \psi$. Sets intersect if they share at least one common element, and are otherwise said to be disjoint. Each set has a corresponding weight, denoting the maximum marginal utility found across all
possible assignments of this set to different combinations of RS and links. Specifically,

\[
w(c_i) = \max_l \frac{R(c_i, l)}{\bar{\lambda}_l(t)},
\]

\[
w(c_i, c_r) = \max_{l,r} \frac{R(c_i, c_r, r, P, l)}{\bar{\lambda}_l(t)}.
\]

For \((c_i, c_j, c_r)\), the weight is found over all possible assignments of this set to combinations of RS and uplink-downlink of an MS, since it can only be assigned to one MS. Formally,

\[
w(c_i, c_j, c_r) = \max_{s,r} \sum_{l:s = M(l)} \frac{R(c_i, c_j, c_r, r, P, s)}{\bar{\lambda}_l(t)}.
\]

Note that \(w(c_i, c_j, c_r)\) essentially sums up rates of uplink and downlink of \(s\) since one XOR-CD session incorporates two cooperative transmissions. The optimization is to find the optimal strategy to choose the transmission mode and assign RS and channels to each link in order to maximize the aggregated throughput. The maximization is over all links. Equivalently, we can also interpret it as to find the optimal strategy to select disjoint channel combinations and assign RS and links to them so as to maximize the objective. In this alternative interpretation, the maximization is done over all possible channel sets by matching them to the best possible links and RS without duplicate use of channels. The solution found must exhaust all subchannels since we can always improve the total weight by adding sets corresponding to unassigned data and relay subchannels. The number of elements in a set is at most 3, therefore the problem is equivalent to weighted 3-set packing [58], which is NP-complete.

The assignment corresponding to the set weight is recorded down in a table \(T_{assign}\). We see that, for sets \((c_i)\), the size of weight search space is \(|L|\); for sets \((c_i, c_r)\) and
(c_i, c_j, c_r), the search space size is |Φ| |L|. Thus, the weight construction process is of
polynomial time complexity, given the number of three kinds of sets are also polynomials
of |ζ| and |ψ|.

To propose a good approximation algorithm with reasonable time complexity, first we
construct an intersection graph G_C of the set system C with set of vertices V_C and set of
undirected edges E_C as shown in Fig. 3.3. Weighted set packing then can be generalized
as a weighted independent set problem, the objective of which is to find a maximum
weight subset of mutually non-adjacent vertices in G_C [59]. The size of sets is at most
3, therefore G_C is 3-claw free. Here a d-claw c is an induced subgraph that consists
of an independent set T_c of d nodes. The best known approximation for the weighted
independent set problem in claw-free graph is proposed in [59] and then acknowledged
in [58], which we extend to form our algorithm ALG2.

First we introduce a greedy algorithm, called Greedy, that prepares the groundwork
for ALG2. Define N(K, L) to be the set of vertices in L that intersect with vertices in K,
\( i.e. N(K, L) = \{u \in L : \exists v \in K \text{ such that } \{u, v\} \in E \text{ or } u = v\}. \) Greedy is a natural
heuristic that repeatedly picks the heaviest vertex from among the remaining vertices
and eliminate it and the adjacent vertices as shown below.

\begin{algorithm}
1: \( S \leftarrow \emptyset \)
2: while \( V_C - N(S, V_C) \neq \emptyset \) do
3: choose \( u \in V_C - N(S, V_C) \) with the maximum weight
4: \( S \leftarrow S \cup \{u\} \)
5: end while
\end{algorithm}

A natural thought to improve on the maximal independent set found by Greedy is
to do local search and replace a set with its claw with larger weight, which motivates
ALG2 summarized as in Algorithm 2. From [59], local improvements on the square of
total weights solve the weighted independent set problem with a constant approximation factor of 1.5, which is the best result known so far [58]. Therefore we have the following:

**Proposition 3.2** ALG2 provides at least $\frac{2}{3}$ of the optimum of RSS-XOR problem. This is the best performance guarantee one can have unless a better algorithm can be found for the weighted independent set problem.

**Algorithm 2 ALG2.**

1: Construct the collection of weighted sets $C$ and transform into the weighted undirected graph $G_C$.
2: Obtain a maximal independent set $S$ using Greedy.
3: while $\exists$ claw $c$ such that $T_c$ improves $w^2(S)$ do
4: $S \leftarrow S - N(T_c, S) \cup T_c$
5: end while
6: Assign channels to RS and links by searching the entries in $T_{\text{assign}}$ corresponding to the sets present in $S$.

3.4.2 A Matching Algorithm for NO-XOR

We consider NO-XOR problem now. Surprisingly, we find that it can be optimally solved in polynomial time.

**Proposition 3.3** The NO-XOR problem is equivalent to weighted bipartite matching over all data and relay channels, and can therefore be solved optimally.

**Proof** Construct a bipartite graph $A = (V_1 \times V_2, E)$ where $V_1$ and $V_2$ correspond to the set of data subchannels $\zeta$ and relay subchannels $\psi$ respectively, as shown in Fig. 3.4. We patch void vertices to $V_2$ to make $|V_2| = |V_1| = |\zeta|$. The edge set $E$ corresponds to $|\zeta|^2$ edges connecting all possible pairs of channels in two vertex sets. Each edge $(k, j)$ carries
three attributes, \((w_{kj}, l_{kj}, r_{kj})\), where

\[
\begin{align*}
  w_{kj} &= \max_{l,r} \frac{R(k, j, r, P, l)}{\bar{\lambda}_l(t)}, \\
  (l_{kj}, r_{kj}) &= \arg \max_{l,r} \frac{R(k, j, r, P, l)}{\bar{\lambda}_l(t)}.
\end{align*}
\] (3.17)

For edges connecting data subchannels to void relay subchannels that we patched, the edge weights become \((w_{kj}, l_{kj}, 0)\) where \(l_{kj}\) is the link providing maximum marginal utility if data subchannel \(k\) is used. This essentially captures the maximum marginal utility given by direct transmission.

Observe that \(A\) is bipartite, we can see the NO-XOR problem is equivalent to finding the maximum weighted bipartite matching on \(A\). The second attribute of an edge \((k, j)\) in the maximum matching represents the link assigned with this data-relay subchannel pair \((k, j)\), while the third attribute dictates the transmission mode or the corresponding RS. A 0 in the third attribute simply means the link should work in direct transmission mode. Hence, the maximum matching found represents the comprehensive assignment of RS, data and relay subchannels, as well as the transmission strategy decision, therefore optimally solves the NO-XOR problem.

Several good polynomial-time algorithms exist for solving the bipartite matching problem, of which the Hungarian algorithm [60] is a popular choice. Since the graph construction is \(O(|\zeta|^2 \cdot |L| \cdot |\psi|)\), the whole algorithm is polynomial time.

### 3.4.3 A Power Allocation Algorithm

Finally, we turn our focus to the power allocation problem. The power limited versions of RSS-XOR and NO-XOR are proposed in Sec. 3.3.5. They are non-convex problems
Figure 3.4: The graphical model to show the equivalence of NO-XOR problem and weighted bipartite matching. Dotted vertices are void vertices patched. Not all links are shown here.

because of the integer constraints. Duality gap for non-convex problems is non-zero in general. However, in an OFDMA system with many narrow subchannels, the optimal solution of RSS-XOR and NO-XOR is always a convex function of $P_r$, because if two sets of throughputs using two different channel-RS-link assignments and relay strategies are achievable individually, their linear combination is also achievable by a frequency-division multiplexing of the two sets of strategies. This idea for non-convex problems of multi-carrier systems is discussed earlier in [61]. In particular, using the duality theory of [61], the following is true:

**Proposition 3.4** The power limited RSS-XOR and NO-XOR problems have zero duality gap in the limit as the number of OFDM subchannels goes to infinity, even though the discrete selection of channels, RS and relay strategies are involved.
3.4. APPROXIMATION ALGORITHMS

A detailed proof can be constructed along the same line of argument as in [61]. This proposition allows us to solve the non-convex problems in their dual domain. Note that although the proposition requires number of channels to go to infinity, in reality the duality gap is very close to zero as long as number of channels is large [39]. Consider the power limited RSS-XOR problem. The dual method for power limited NO-XOR problem can be developed in a similar way. Introduce an Lagrangian multiplier vector $\mu$ to the power constraint (3.12) and the dual problem becomes

$$\min_{\mu \geq 0} \ g(\mu)$$

(3.18)

where

$$g(\mu) = \max \left\{ \sum_{l \in L} \frac{\lambda_l}{\lambda_l(t)} + \sum_{r \in \psi} \mu_r \left( P_r - \sum_{l,c,c_r} p_{l,c,c_r} - \sum_{s,c_l,c_j,c_r} p_s^{r,c_l,c_j,c_r} \right) \right\}$$

subject to (3.8)–(3.10).

(3.19)

Since each MS $s$ corresponds to two links, we can equally split power used for XOR-CD $p_{r,c_i,c_j,c_r}$ to these two links without violating the power constraint. Mathematically, we let

$$p_{l,c_i,c_j,c_r} = \begin{cases} \frac{1}{2} p_{s}^{r,c_i,c_j,c_r} & \text{if } s = M(l); \\ 0 & \text{otherwise}. \end{cases}$$

(3.20)

The objective function of (3.19) can then be written as

$$\max \sum_l \left( \frac{\lambda_l}{\lambda_l(t)} - \sum_{r,c,c_r} \mu_r p_{l,c,c_r} - \sum_{r,c_i,c_j,c_r} \mu_r p_{l}^{r,c_i,c_j,c_r} \right) + \sum_r \mu_r P_r.$$  

(3.21)
\[ \sum_r \mu_r P_r \text{ is constant in (3.21) since } \mu \text{ is given for each instance of } g(\mu). \] So solving the optimization (3.19) is equivalent to solving the following:

\[
\max \sum_l \left( \frac{\lambda_l}{\lambda_l(t)} - \sum_{r,c,r} \mu_r P_{l,r,c,r} - \sum_{r,c_i,c_j,c_r} \mu_r P_{l,r,c_i,c_j,c_r} \right)
\]
subject to (3.8)–(3.10).

Compared with the original RSS-XOR problem, the only difference is the objective function which now includes the cost of power. \( \mu \) can be interpreted as a pricing variable vector for relay power. (3.22) can be thought of as maximizing the total marginal utility minus the total cost of relay power used, given the current prices of power at RS. This is easily decomposed into revenue maximization over every possible set of data and/or relay channels. Therefore, it can be solved using the approximation algorithm \( ALG2 \) in Sec. 3.4.1, with the weights being the maximum marginal revenue instead of the maximum marginal utility. For ease of exploration, we dictate that the relay power for each cooperative session is set to the threshold value as derived in Sec. 3.3.2 and 3.3.2. Then,

\[
w_{(c_i,c_r)} = \max_{l,r} \frac{R(c_i, c_r, r, P_{l,r,c_i,c_r}, l)}{\lambda_l(t)} - \mu_r P_{l,r,c_i,c_r},
\]
\[
w_{(c_i,c_j,c_r)} = \max_{s,r} \sum_{l:s=M(l)} \frac{R(c_i, c_j, c_r, r, P_{s,r,c_i,c_j,c_r}, s)}{\lambda_l(t)} - \mu_r P_{s,r,c_i,c_j,c_r}.
\]

After solving (3.22), the dual problem (3.18) can be readily solved via a subgradient method which repeatedly updates the power prices according to the demand/supply relationship at RS to regulate the power consumption. To summarize, the algorithm for solving the power limited RSS-XOR problem, referred to as \( ALG3 \), works as shown in
Algorithm 3. Notice that the subgradient algorithm is suitable for distributed imple-
mentation across RS. Each RS is able to verify its power consumption, and update its 
own relay power price autonomously according to $\nu^{(k)}$ informed by BS. The updated 
prices can be transmitted to the BS to solve the revenue maximization problem with a 
negligible amount of overhead.

3.5 Performance Evaluation

3.5.1 Simulation Setup

The key of our experiment settings is to derive the achievable data rate of a subchannel 
when it is allocated to an arbitrary MS. This requires computing the corresponding 
SNR value. To generate realistic results, we adopt empirical parameters to model the 
wireless fading environment. The subchannel bandwidth is set to be 312.5 kHz. Data 
subchannels are centered around 5GHz, while relay subchannels are centered around 2.5 
GHz. Channel gain between two nodes at each subchannel can be decomposed into
a small-scale Rayleigh fading component and a large-scale log normal shadowing with standard deviation of 5.8 and path loss exponent of 4. The inherent frequency selective property is characterized by an exponential power delay profile with delay spread 15 µs. The time selective nature is captured by the Doppler spread, which depends on the MS’s speed (throughout the simulation the MS are moving with speed uniformly distributed from 1 to 5 m/s according to random waypoint model with 0 pause period). The gap to capacity $\Gamma$ is set to 1, which corresponds to perfect coding. The power constraint for each transmission is such that $\frac{P}{N_0W} = 23$ dB. This corresponds to a medium SNR environment. Such an experimental setup is commonly used in related studies \[38, 39\].

### 3.5.2 Performance of XOR-CD

![Throughput comparison](image1.png)

(a) Throughput comparison

![Power efficiency comparison](image2.png)

(b) Power efficiency comparison

Figure 3.5: Performance of XOR-CD against conventional CD.

We first evaluate performance of XOR-CD compared with conventional CD and direct transmission. We focus on the scenario where 10 MS are uniformly located in a cell with 100m radius. To reduce the computation load we set the number of data subchannels to
be 100, and that of relay subchannels to be 30. 1 RS is deployed in the cell. ALG2 and ALG3 as proposed in Sec. 3.4.1 and 3.4.2 are implemented to obtain the optimization results. Fig. 3.5(a) plots the time averaged throughput for a one-second period of time, with a sampling period of 5 ms. The optimization therefore is done for 200 times. We can see that the average throughput is slowly decreasing over time. The reason is that our objective takes fairness into account which makes the optimization favor a “slower” MS as time goes, resulting in a slowly decreasing trend. We can clearly see that XOR-CD outperforms the conventional diversity scheme by around 30%. This is as expected, because XOR-CD conserves relay channels that can be utilized to support more cooperative sessions. To further illustrate its superiority in this aspect, we study XOR-CD’s relay resource efficiency. We define power efficiency to be the ratio of throughput improvement obtained over direct transmission, and the amount of relay power used. As we see in Fig. 3.5(b), XOR-CD’s power efficiency is significantly, mostly over 100%, better than that of conventional CD. Finally, notice that the conventional diversity scheme alone provides over 20% improvement compared with simple direct transmission. This diversity gain is similar to the network coding gain, which further confirms the advantage of XOR-CD to “double” the diversity gain without any costs.

3.5.3 Effects of Relay Resources

Next, we study effects of relay resources. We focus on number of RS and number of relay subchannels. Fig. 3.6 shows the results. Intuitively, more RS provides a better chance for MS to find a nearby RS with better relay channel qualities. More relay subchannels enables more cooperative sessions to happen. Clearly, these two factors contribute to the increasing trend reflected in Fig. 3.6. XOR-CD consistently maintains a 20 – 30% gain
3.5. PERFORMANCE EVALUATION

(a) Effect of RS. Number of relay subchannels is 20.

(b) Effects of relay subchannels. Number of RS is 3.

Figure 3.6: Effects of Relay Resources. Number of data channels is 100, number of MS is 10, and cell radius is set to be 100m.

over conventional CD. We also note that the marginal gains are gradually diminishing for both factors. This can be explained as the optimization always tries to harvest the largest performance gains first. The diminishing trend suggests that we could use a small amount of relay resources to obtain a reasonably satisfactory improvement.

3.5.4 Effects of Path Loss

For XOR-CD, it is preferable to encode the uplink and downlink of an MS with similar channel conditions, and assign the same relay subchannel to them. In case the channel conditions of uplink and downlink differ significantly, the link with worse channel condition will undermine the shared multicast rate. In Fig. 3.7 we explore the influence of path loss on the average throughput improvement over time. Path loss diversity across MS is increased when we increase the cell radius, since MS’s differences in distances to BS are increased. For XOR schemes that encode packets from different links [16], the
network coding gain diminishes and researchers proposed methods such as superposition codes to deal with this issue [56]. In contrast, XOR-CD takes advantage of bi-directional traffic on the same MS instead of encoding across two different MS. Though channel conditions of different MS differ severely as cell radius increases, uplink and downlink conditions of the same MS become more similar, since path loss becomes dominant over random fading. This explains the increased throughput improvement of XOR-CD when cell radius is increased. Therefore, XOR-CD remedies the shortcoming of coding to the worse rate, and achieves better throughput improvement in large path loss case, which is substantially different from previous approaches of using XOR.

3.5.5 Performance of Power Allocation

Finally, we evaluate performance of XOR-CD when relay power is constrained. We implement the subgradient based power allocation algorithm ALG3, as well as its counterpart
3.5. PERFORMANCE EVALUATION

Table 3.1: Throughput values of different relay power profiles.

<table>
<thead>
<tr>
<th>Relay power profile:</th>
<th>Throughput (Mbps)</th>
<th>Improvement (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS1 10P RS2 10P RS3 10P</td>
<td>6.69</td>
<td>—</td>
</tr>
<tr>
<td>RS1 15P RS2 9P RS3 6P</td>
<td>6.93</td>
<td>3.58</td>
</tr>
<tr>
<td>RS1 15P RS2 6P RS3 9P</td>
<td>6.91</td>
<td>3.29</td>
</tr>
<tr>
<td>RS1 18P RS2 7P RS3 5P</td>
<td>7.14</td>
<td>6.73</td>
</tr>
<tr>
<td>RS1 18P RS2 5P RS3 7P</td>
<td>7.11</td>
<td>6.28</td>
</tr>
</tbody>
</table>

for the NO-XOR problem. We enforce a uniform power constraint for each of the RS. The per RS power constraint is such that each RS has relay power $NP$, where $P$ is the power used for one direct transmission as in Sec. 3.3.2. We vary $N$ from 10 to 100 and obtain Fig. 3.8. We observe that XOR-CD is less sensitive to power constraints as reflected by the marginal improvement compared to conventional CD. This is because XOR-CD utilizes power more efficiently, resulting in a lower power demand at RS. Therefore, pumping more power does not improve its performance substantially.

We also study non-uniform power constraints. For the same configuration and network topology, we fix the total power constraint to be 30$P$ and vary different RS’s individual power constraint. Table 3.1 summarizes the results of different relay power profiles. We observe that allocating more power into RS1 has a positive effect on the average throughput, while adjusting the constraints at RS2 and RS3 has little impact. The reason is that in our simulation RS are randomly located inside the cell. RS1 is located closest to the BS, providing a much better relay channel for cooperative transmissions. Allocating more power into RS1 boosts its relaying capacity and improves throughput. It also suggests that power allocation at RS needs to be location-adaptive to best utilize resources.
3.6 Summary and Discussions

We represent an early attempt to study XOR-assisted cooperative diversity to improve the spectrum utilization in OFDMA primary networks in this chapter. As our first contribution, we design XOR-CD, a novel cooperative diversity scheme with XOR in multi-channel networks. It capitalizes bi-directional traffic and is shown to be able to greatly improve relay efficiency and throughput by both information theoretical analysis and realistic simulations. As our second contribution, we propose a unifying optimization framework to exploit multi-user diversity, cooperative diversity and network coding jointly. We establish the NP-hardness of the problem, and propose efficient approximation algorithms with provably the best performance guarantee. Extensive simulation corroborates the effectiveness of our algorithms.

Our scheme is centralized and the optimization problem is solved at the BS. Although full CSI at the BS is a common assumption to be made in many existing works, in practice it is difficult to obtain and causes extra communication overhead on the BS. One may consider opportunistic cooperative diversity protocols, where RS observe the CSI of its neighboring MS and opportunistically help the transmission in a decentralized fashion.
Chapter 4

Dynamic Spectrum Trading: A Spectrum Secondary Market

Having discussed using XOR-assisted cooperative diversity to improve the spectrum utilization in the primary network, now we shift our focus to the secondary network. Secondary networks, or cognitive radio networks (CRNs), have emerged in recent research to efficiently use white spaces by secondary devices [8]. One of the important challenges in CRNs is how temporally unused spectrum may be allocated from primary users (PUs) to secondary users (SUs). Market-based approaches enable such dynamic redistribution of spectrum such that SUs in need can gain access and PUs can obtain benefits by leasing their abundant spectrum [4, 24, 25, 26, 27]. Auctions [26, 27] are perceived to be fair and efficient candidate solutions to achieve such dynamic spectrum access.

Conventional spectrum auctions, to our knowledge, are proposed under a primary market paradigm, as we discussed in Chapter 1. In contrary, we mainly focus on dynamic spectrum redistribution among small heterogeneous cognitive users themselves, e.g. mesh routers of small wireless networks, APs of home networks, which may differ
4.1. MAIN CONTRIBUTIONS

We push the state-of-the-art to the next level by going beyond a primary market. We advocate that a spectrum secondary market complementary to the primary market is in their technologies, sensing abilities, and transmission capabilities. By shifting from a “macro” to a more “micro” perspective, we observe that the underlying assumptions of the primary market paradigm no longer hold. For small users, traffic demand is extremely bursty as widely observed by many existing works [29]. Moreover, channel bandwidth is of a finer granularity now, exhibiting significant time and frequency selectivity due to fading and user mobility as reported by extensive measurements [62]. The monolithic primary market paradigm designed for long-term spectrum redistribution becomes inherently inefficient, if not detrimental, when applied to this scenario.

4.1 Main Contributions

Figure 4.1: The conceptual spectrum market structure for cognitive users. Legacy owners lease unused spectrum to spectrum brokers, each of which represents the aggregated demands of users of a certain area. The leased channels are then traded among cognitive users (solid links in the bottom) dynamically in secondary market, with their spectrum broker as the auctioneer (dotted links in the middle).
4.1. MAIN CONTRIBUTIONS

to be established, as shown in Fig. 4.1. The secondary market works in harmony with the primary market through “spectrum brokers.” The primary market, operating in a large time scale (e.g. weekly or daily), is the marketplace where each spectrum broker, with multiplexed demands across users of a certain area [4], bids for relatively long-term spectrum leases from legacy owners based on existing solutions [4, 24, 25, 26, 27]. The leased spectrum resources are then traded dynamically amongst cognitive users in the same area through the secondary market in a much finer time scale (e.g. several minutes), to adapt to time-varying demands and channel conditions, with their spectrum broker as the market maker. The spectrum brokers can be easily implemented in a similar infrastructure as the base stations of IEEE 802.22 based cognitive radio networks [63], and are anticipated to evolve in the near future.

In the secondary market, a virtual currency, such as the lightweight currency in [64], serves as the medium of transaction. A user can sell some of the channels to others when traffic demand is reduced, or when the channels are in deep fade. The windfall from the sales can be used for future purchases when demand increases, or exchanged for channels with better conditions. Other users may bid for these channels to satisfy their unfulfilled demands, or to better utilize them as they may perceive much better conditions due to multi-user diversity. By establishing the secondary market among cognitive users, spectrum as a resource becomes more liquid and easier to obtain and relinquish, leading to more efficient utilization, as implied by fundamental principles of economics and finance [28].

As our first contribution towards this end, we design a periodic double auctions mechanism. Buyers and sellers submit bids and asks simultaneously in each period and a trade is made if a buyer’s bid exceeds a seller’s ask. The double auction is one of the most
common market institutions in real-world markets (e.g., the New York Stock Exchange). In our spectrum secondary market, each of the different channels is analogous to different stocks in the stock market, with dynamic prices to incorporate multi-channel and multi-user diversity. Through trading, channels are dynamically and efficiently redistributed to maximize resource utilization. Our design is proven to be truthful, so users cannot expect a higher utility gain by cheating. Therefore, the dominant strategy is to report the true valuations in the bids or asks. It is also asymptotically efficient, in the sense that it maximizes the total utility gains obtained by all participating users asymptotically.

In a recent work [27], a truthful spectrum double auction has been proposed. We comment that, as discussed above, it is designed for spectrum primary markets with homogeneous channels and fixed identical user demands. We adopt double auctions for secondary market trading, with significantly different assumptions and design requirements, such as heterogeneous channels and multi-unit trading. In this regard, our double auction is not a mere extension of [27]. To our knowledge, this is the first spectrum auction design tailored for secondary market that guarantees truthfulness and near-optimal economic efficiency.

As our second original contribution, we analyze the user rationale in such a market, as they make sequential decisions on what channels and how many to trade, according to dynamic traffic demands and link conditions. We characterize this as a dynamic programming problem, and propose a practical reinforcement learning framework to solve it. Through systematic trial-and-error interactions with the market, users receive reinforcements as their utilities, and incrementally learn the optimal policy. We carefully engineer the learning algorithm to reduce dimensions of the state and action spaces, making it feasible to be implemented on each user. This contribution further differentiates
our work from existing spectrum auctions, including [27], that simplify the user decision making problem as submitting uniformly random bids [26, 27]. Our work completes the characterization of the secondary market in terms of both market mechanism and user rationale, representing a step further towards enabling efficient spectrum redistribution in different scenarios.

4.2 Preliminaries

We start by presenting an overview of the spectrum secondary market design problem, and discuss critical economic properties required to implement efficient spectrum auctions.

4.2.1 Challenges

The challenges of designing a spectrum secondary market are two-fold. First, given all the multi-unit bids and asks, how do we determine the winning sellers and buyers, match the total demand with supply, and determine the prices for the winners? The goal of the auction design is to maximize the total utility gain, which essentially represents the spectrum utilization. To achieve this goal and to maintain feasibility, the auction is required to satisfy certain economic properties outlined in Sec. 4.2.2. We address this challenge in Sec. 4.3 by a new multi-unit double auction design, with rigorous theoretical analysis.

Second, besides having a robust and efficient auction in place, users in the secondary market also need to strategically exploit possible trading opportunities and make trading decisions in order to maximize their utility. They can only observe the history of trading
prices, their own traffic demands, and perceived channel qualities. They have no knowl-
edge on the decisions others have made in the market, which significantly affects the
auction outcome. How do they find possible trading opportunities, and make intelligent
trading decisions considering the dynamics of all the variables, such that optimal utility
gain can be achieved?

4.2.2 Economic Properties for Spectrum Auction

Truthfulness, individual rationality and budget balance are the three critical properties
that are required to design economically robust double auctions [65]. To formally define
these properties, we need to introduce a set of notations first.

The reservation price is defined as the true highest price \( p^b_i \) per unit a buyer \( i \) is willing
to offer, and the lowest price \( p^a_j \) a seller \( j \) is willing to accept. Its value is equal to each
user’s private valuation of the channel, and is unknown to other users and the spectrum
broker, serving as the auctioneer here. Bid (ask) price \( b_i \) (\( a_j \)) is the reported highest
(lowest) price buyer (seller) \( i \) (\( j \)) is willing to trade. Along with \( b_i \) (\( a_j \)) a buyer (seller)
also submits \( q^b_i \) (\( q^a_j \)) indicating the maximum volume she intends to trade. \( \hat{p}^b \) and \( \hat{p}^a \) are
the transaction prices at which winning buyers and sellers trade. The utility gain for a
winning buyer is \( u^b_i(b_i) = (p^b_i - \hat{p}^b)\hat{q}^b_i \), and for a winning seller is \( u^a_j(a_j) = (\hat{p}^a - p^a_j)\hat{q}^a_j \),
where \( \hat{q}^b_i \) and \( \hat{q}^a_j \) denote the volumes of units traded. This implies quasi-linear utility
functions [28], which will be rigorously defined in Sec. 4.4.1.

With properly defined utility functions and prices, if all the users bid truthfully, the
4.2. PRELIMINARIES

double auction is essentially solving the following social welfare maximization problem:

$$
\max_{\hat{p}^b, \hat{p}^a, \hat{q}^b, \hat{q}^a} \sum_i u^b_i(b_i) + \sum_j u^a_j(a_j) + \hat{q}(\hat{p}^b - \hat{p}^a)
$$

s.t. \( \hat{q} = \sum_i \hat{q}^b_i = \sum_j \hat{q}^a_j \), \hspace{1cm} (4.1)

where \( \hat{q}(\hat{p}^b - \hat{p}^a) \) is the trading surplus collected by the auctioneer to compensate its efforts. The difficulty of this problem lies in the facts that the utility function of each user, hence \( p^b_i, p^a_j \), are private, and users may cheat by setting \( b_i \neq p^b_i, a_j \neq p^a_j \), provided that they can benefit from such behavior.

Definition 4.1 (Truthfulness) A double auction is truthful if no buyer \( x \) or seller \( y \) can obtain a higher utility gain \( u^b_x(b_x) \) or \( u^a_y(a_y) \) by cheating, i.e., setting \( b_x \neq p^b_x, a_y \neq p^a_y \).

Truthfulness is crucial to our spectrum double auction design. Users in the auction have strong incentives to lie to obtain outcomes that favor their own interests. To resist such manipulations, other users have to strategize on how to submit their own prices, resulting in heavy overhead. Since users do not have complete information about the market, the mistrust would discourage them from participating in trading, and the market itself may collapse.

Definition 4.2 (Ex-post Individual Rationality) A double auction is ex-post individual rational if the expected utility gain of any user participating in the auction truthfully is non-negative for all possible outcomes. Equivalently, ex-post individual rationality holds if \( \hat{p}^b \leq p^b_x, \hat{p}^a \geq p^a_y \) for any winning buyer \( x \) and seller \( y \).

Ex-post individual rationality ensures that a user has non-negative utility gain if it
reports its true reservation price, therefore provides incentives for it to participate in the auction.

**Definition 4.3 (Ex-post Budget Balance)** A double auction is ex-post budget balanced if the auctioneer’s payoff (total payments from buyers, less the revenues of the sellers) is non-negative for all possible outcomes.

Ex-post budget balance motivates the auctioneer to hold the auction since the auction will never run into deficit. In practice, the auctioneer can charge a transaction fee to (winning) users. For simplicity, we do not include this charge in our proofs.

Other than these properties, we wish that our double auctions are economically efficient so spectrum resources are fully utilized, and the auction can be conducted periodically in a practical way.

**Definition 4.4 (Economic Efficiency)** A double auction is economically efficient if the auction maximizes total utility gains feasible with complete information from all users.

By the impossibility theorem [66], it is clear that no mechanism can achieve all properties simultaneously. We therefore relax the economical efficiency constraint to an asymptotic one. *Asymptotic efficiency* means that the welfare loss under the auction compared to the maximum feasible social welfare converges to zero as the number of buyers and/or sellers approaches infinity. As the auction becomes large enough, almost all the feasible social welfare will be realized.

### 4.3 The Spectrum Market Institution

Our first contribution is a new spectrum secondary market mechanism that involves periodic double auctions, similar to real-world stock markets. Our use of periodic double
auctions marks a clear departure from conventional wisdom in spectrum auction design. In this section we first introduce the organization of the market and then the main design and characterization of the spectrum double auction. We rigorously prove all the required properties for our spectrum double auction in this section. To our knowledge, this is the first spectrum auction design that guarantees truthfulness and near-optimal economic efficiency. In addition, we show that the double auction can be executed in polynomial time, which justifies its practicality.

4.3.1 Spectrum Secondary Market Organization

We consider a micro-level ad hoc cognitive radio network covered by one spectrum broker. We assume channel reuse is possible in a macro level across micro networks represented by many spectrum brokers, and is taken care of by spectrum auctions in primary market [26, 27]. Users of the micro-level network may differ in their technologies, sensing abilities and transmission capabilities, that renders a centralized DSA approach costly and infeasible. The only assumption about the users is that they all use OFDMA as the multi-access technology as recommended by IEEE 802.22 draft [63]. As a matter of fact, OFDMA has been already implemented in various technologies including IEEE 802.16, 802.20, LTE and etc [67]. It ensures that every user has a set of orthogonal channels, each of which is divided into a number of orthogonal subcarriers with identical bandwidth. We model channels by frequency-selective fading, with coherence bandwidth in the order of the bandwidth of a few channels. This implies that fading between channels far away from each other is uncorrelated, and each subcarrier of the same channel has the same fading statistics.

Intrinsically, the market mechanism among cognitive users is a double auction with
multiple divisible *commodities*, as channels are heterogeneous across users. For each channel, it is a single-commodity double auction, and the subcarriers are the smallest trading unit. W.L.O.G., we focus on an arbitrary round of a single-commodity double auction in the subsequent analysis.

### 4.3.2 Designing Periodic Double Auctions

We illustrate in details the working of the spectrum double auction in this section. It consists of the following two phases.

**Winner Determination**

The spectrum broker sorts all orders so that

\[
 b_1 > b_2 > \cdots > b_n \tag{4.2}
\]

and

\[
 a_1 < a_2 < \cdots < a_m, \tag{4.3}
\]

where \(m\) and \(n\) denote the number of bids and asks in this round, respectively. Strict ordering relations are assumed, since if two buyers/sellers report the same price their volumes can be combined to form an equivalent bid/ask. The demand volumes are arranged according to the descending price order as shown in (4.2), and the supply volumes according to the ascending price order (4.3). There exists a critical point \(q^*\)
where there are $K$ bids and $L$ asks such that:

$$a_{L+1} \geq b_K \geq a_L, \quad \text{and} \quad \sum_{i=1}^{K-1} q_i^b \leq \sum_{j=1}^{L} q_j^a \leq \sum_{i=1}^{K} q_i^b, \quad \text{or} \quad (4.4)$$

$$b_K \geq a_L \geq b_{K+1}, \quad \text{and} \quad \sum_{i=1}^{L-1} q_i^a \leq \sum_{j=1}^{K} q_j^b \leq \sum_{i=1}^{L} q_i^a \quad (4.5)$$

The first case — corresponding to Eq. (4.4) — is shown in Fig. 4.2. If Eq. (4.4) holds, $q^* = \sum_{j=1}^{L} q_j^a$; in case Eq. (4.5) holds, $q^* = \sum_{i=1}^{K} q_i^b$. If such a point cannot be found, in other words the supply and demand curves do not intersect, $q^*$ is simply the minimum of total supply $\sum_{i=1}^{n} q_i^a$ and demand $\sum_{i=1}^{m} q_i^b$.

![Figure 4.2: Spectrum double auction design.](image)

The total transaction volume $\hat{q}$ is then set as follows:

$$\hat{q} = \min \left( \sum_{i=1}^{L-1} q_i^a, \sum_{i=1}^{K-1} q_i^b \right) .$$

Each bid with index less than $K$ and each ask with index less than $L$ will be involved in
a trade, and are thus the winners of the auction. **Algorithm 4** summarizes the winner determination solution.

**Algorithm 4 Winner Determination.**

1. Sort \( \{b_i\}_1^n \) according to Eq. (4.2), plot the demand curve
2. Sort \( \{a_j\}_1^m \) according to Eq. (4.3)
3. \( j = 0 \)
4. **repeat**
5. \( j = j + 1 \)
6. Plot \((a_j, q_{a_j}^a)\) as shown in Fig. 4.2
7. **until** Eq. (4.4) or Eq. (4.5) is satisfied, or \( j = m \)
8. Set \( L = j \), set \( K \) according to Eq. (4.4) or Eq. (4.5), set the winning set of bids and asks \( W = (i)_{1}^{K-1} \cup (j)_{1}^{L-1} \)
9. Set **winners** to the set of users corresponding to the winning set of orders \( W \)
10. **RETURN** **winners**

**Payment Determination**

First we set the transaction price \( \hat{p}_b \) per subcarrier to be \( b_K \) for winning buyers, and \( \hat{p}_a \) to be \( a_L \) for winning sellers. Since \( b_K < b_i, \forall \ i < K \) and \( a_L > a_j, \forall \ j < L \), trading is profitable for both sides. Then we need to determine the trading volume for each winner. Note that because our mechanism supports multi-unit trading, we need to match the supply and demand volumes exactly. Hence it is possible that some orders can only be **partially** satisfied. Specifically, when \( \sum_{i=1}^{K-1} q_{i}^b > \sum_{j=1}^{L-1} q_{j}^a \), as the case in Fig. 4.2, every ask with index less than \( L \) is satisfied. The bid \( b_{K-1} \), however, cannot be fully executed. Instead of letting buyers corresponding to bid \( b_{K-1} \) suffer the shortage, we dictate that \( \sum_{i=1}^{K-1} q_{i}^b - \sum_{i=1}^{L-1} q_{j}^a \) winning buyers are randomly chosen to sacrifice one subcarrier each to absorb the shortage. The purpose of doing so is to ensure bid-independence as we will show in Sec. 4.3.3.

Similarly, when \( \sum_{i=1}^{K-1} q_{i}^b < \sum_{j=1}^{L-1} q_{j}^a \), every bid with index less than \( K \) is satisfied,
Algorithm 5 Payment Determination.

1. \( \hat{p}^b = b_K, \hat{p}^a = a_L, \hat{q} = \min \left( \sum_{i=1}^{L-1} q_j^a, \sum_{i=1}^{K-1} q_i^b \right) \)
2. if \( \sum_{i=1}^{K-1} q_i^b > \sum_{j=1}^{L-1} q_j^a \) then
3. Pick a random set of \( \sum_{i=1}^{K-1} q_i^b - \sum_{j=1}^{L-1} q_j^a \) buyers from winners, set the trading volume for each of the chosen buyers \( i' \) to be \( \hat{q}_i^b = q_i^b - 1 \)
4. else if \( \sum_{i=1}^{K-1} q_i^b < \sum_{j=1}^{L-1} q_j^a \) then
5. Pick a random set of \( \sum_{j=1}^{L-1} q_j^a - \sum_{i=1}^{K-1} q_i^b \) sellers from winners, set the trading volume for each of the chosen sellers \( j' \) to be \( \hat{q}_j^a = q_j^a - 1 \)
6. end if
7. for all \( x, y \in \text{winners}, x \neq x', y \neq y' \) do
8. \( \hat{q}_i^b = q_i^b, \hat{q}_j^a = q_j^a \)
9. end for
10. RETURN \( \hat{q}_i^b \hat{p}^b \) for winning buyer \( i \) to pay, \( \hat{q}_j^a \hat{p}^a \) for winning seller \( j \) to receive, \( \hat{q}(\hat{p}^b - \hat{p}^a) \) for the spectrum broker as its payoff

while a random set of \( \sum_{j=1}^{L-1} q_j^a - \sum_{i=1}^{K-1} q_i^b \) sellers is chosen to sacrifice one subcarrier of supply each. Each winning buyer (seller) pays (receives) \( \hat{p}^b (\hat{p}^a) \) times its total trading volume. The spectrum broker collects the trading surplus, \( \hat{q}(\hat{p}^b - \hat{p}^a) \). This clears the market for a given channel. Note that we sacrifice the potential trading values from \( b_K \) and \( a_L \), denoted by \( D \), as well as the volume mismatch between supply and demand, \( C \), in Fig. 4.2. Algorithm 5 summarizes the payment determination algorithm.

By the same mechanism, the spectrum broker clears the remaining orders for other channels. Transaction prices for each channel are also announced to every user. All outstanding orders are then removed. This concludes one round of trading.

4.3.3 Ex-post Individual Rationality, Ex-post Budget Balance, and Truthfulness

We first prove the ex-post individual rationality and budget balance properties.
Theorem 4.1 The periodic double auction mechanism as described in Sec. 4.3.2 is ex-post individual rational and ex-post budget balanced.

Proof All winning buyers have their bids larger than $b_K$ and hence $\hat{p}^b$, and all winning sellers have asks less than $a_L$ and hence $\hat{p}^a$. Therefore by Definition 4.2, our double auction is ex-post individual rational. Since $b_K \geq a_L$, $\hat{p}^b \geq \hat{p}^a$. Since the total buying and selling quantities are guaranteed to match, the auctioneer’s payoff, $\hat{q}(\hat{p}^b - \hat{p}^a)$, is no less than zero. Therefore by Definition 4.3 the double auction is ex-post budget balanced.

Our main result in this section is the proof of truthfulness of our double auction. It consists of three steps: (1) We prove that the winner determination is monotonic. (2) We show that the payment determination algorithm is bid-independent for both buyers and sellers. (3) Based on these lemmas, we finally prove truthfulness, i.e. no user can improve its utility by setting its bidding price other than the reservation price, by considering all possible outcomes of bidding truthfully and untruthfully.

Monotonic Winner Determination

Lemma 4.1 If any buyer $i$ wins by bidding $b_i^1$, it will also win if it bids $b_i^2$, where $b_i^2 > b_i^1$, provided all the other bids and asks remain the same.

Proof We prove this lemma by contradiction. Consider two sorted lists of bids, $B_1$ and $B_2$. The bids of $B_1$ and $B_2$ are the same except in $B_1$ buyer $i$ bids $b_i^1$, and in $B_2$ it bids $b_i^2$. Define the position of $i$ in $B_1$ and $B_2$ as $\text{pos}(b_i^1)$ and $\text{pos}(b_i^2)$ respectively. Since $b_i^1 < b_i^2$, $\text{pos}(b_i^1) > \text{pos}(b_i^2)$. Since all the other bids and asks remain the same, the demand curves for $B_1$ and $B_2$ after $\text{pos}(b_i^1)$, and before $\text{pos}(b_i^2)$ are the same. Assume now that $i$ loses by
bidding \( b_i^2 \). Then \( \hat{q} \) must be smaller than \( \sum_{i=1}^{\text{pos}(b_i^2)} q_i^b \), and hence smaller than \( \sum_{i=1}^{\text{pos}(b_i^1)} q_i^b \), which means \( i \) also loses by bidding \( b_i^1 \). This is a contradiction. Hence it cannot be that \( i \) wins in \( B_1 \) and not win in \( B_2 \). \( \square \)

Similarly we can prove the following lemma for sellers.

**Lemma 4.2** If any seller \( j \) wins by asking \( a_j^1 \), it will also win if it asks \( a_j^2 \), where \( a_j^2 < a_j^1 \), provided all the other bids and asks remain the same.

**Bid-independent Payment Determination**

**Lemma 4.3** If any buyer \( i \) wins by bidding \( b_i^1 \) and \( b_i^2 \), the expected total payment charged to \( i \) is the same, provided all the other bids and asks remain the same. If any seller \( j \) wins by asking \( a_j^1 \) and \( a_j^2 \), the expected total payment received by \( j \) is the same, provided all the other orders remain the same.

**Proof** We prove the case for buyer. The case for seller can be proved in a similar spirit. Without loss of generality, assume that \( b_i^1 < b_i^2 \). As in the proof of **Lemma 4.1**, we have \( \text{pos}(b_i^1) > \text{pos}(b_i^2) \), and the demand curves for \( B_1 \) and \( B_2 \) after \( \text{pos}(b_i^1) \) are the same. Since the supply curves are the same, they intersect at the same point with the demands curves for \( B_1 \) and \( B_2 \), and have the same set of winning buyers and sellers. Therefore the transaction price \( \hat{p}^b \) is the same for \( B_1 \) and \( B_2 \), which is independent of \( i \)'s bid. The bid volume is also the same for \( B_1, B_2 \). Further, \( i \) has the same probability to be chosen to sacrifice one unit of demand surplus, if any, since the total number of winning buyers does not change. Hence, by **Algorithm. 5** the expected payment charged to \( i \) is the same. \( \square \)
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Truthfulness

Based on the above lemmas, we can prove the truthfulness of spectrum double auction.

**Theorem 4.2** The spectrum double auction mechanism is truthful with respect to the reservation price.

**Proof** Suppose a buyer $i$ with reservation price $p_b^i$ reports $b_i \neq p_b^i$. Consider the outcomes of bidding $p_b^i$ and $b_i$. There are four possible scenarios. (1) $i$ loses for both cases. Then $i$ has zero utility gain in both cases. (2) $i$ wins by bidding $p_b^i$ but loses by bidding $b_i$. This happens only if $b_i < p_b^i$ by Lemma 4.1. Then its utility gain is clearly non-zero and is no less than that when it bids truthfully (zero). Our claim holds. (3) $i$ wins by bidding $b_i$ but loses by bidding $p_b^i$. This happens only if $b_i > p_b^i$ by Lemma 4.1. In this case, let $\hat{p}^b(b_i)$ and $\hat{p}^b(p_b^i)$ be the transaction prices when $i$ wins and loses. Immediately we have $\hat{p}^b(p_b^i) > p_b^i$. For $i$ to win by bidding $b_i$, $b_i$ must be at least larger than $\hat{p}^b(p_b^i)$, because if $b_i = \hat{p}^b(p_b^i)$, $i$ still loses. It is easy to show that $\hat{p}^b(b_i) \geq \hat{p}^b(p_b^i)$ must hold since all other bids and asks remain the same. Therefore $\hat{p}^b(b_i) \geq \hat{p}^b(p_b^i) > p_b^i$, and hence $i$ has negative utility gain when it bids $b_i$, which is no more than when it bids truthfully (zero). Our claim holds. (4) $i$ wins in both cases. By Lemma 4.3, $i$ is charged same payment, leading to same utility gain.

From the above we can see no buyer can obtain higher utility gain by bidding $b_i \neq p_b^i$. In a similar spirit we can show the same for all sellers. Then we conclude that no user has incentive to bid untruthfully. \qed

The essential reason for our mechanism being truthful is that every winner always buys or sells at the prices proposed by someone else, since simply letting each winner buy or sell at its own price will violate bid-independent payment determination, and cannot
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4.3.4 Asymptotic Economic Efficiency

An efficient market maximizes the total social welfare obtained by all participating users. The double auction is generally believed by economists to be highly efficient, especially when the number of users is large in empirical studies [49, 50, 51]. We prove that, our periodic double auction is asymptotically efficient when the number of users is large. Notice that the trading surplus collected by the auctioneer, \( \hat{q}(\hat{p}_b - \hat{p}_a) \), is also counted as the social welfare realized, because it is essential to ensure ex-post budget balance and motivate the auctioneer to hold the auction. It also compensates the auctioneer’s efforts to process the bids/asks and clear the market. We first prove a lemma to bound the expected difference between two consecutive bids/asks in the sorted lists. It is later used to bound the efficiency loss factor and prove the asymptotic result.

**Lemma 4.4** Assume that the reservation prices of the \( n \) buyers are i.i.d. random variables with continuous density function \( f(\cdot) \) defined on the interval \([b, \overline{b}]\), and reservation prices of \( m \) sellers are also i.i.d. random variables with PDF \( g(\cdot) \) on \([a, \overline{a}]\). Denote the minimum and maximum of \( f \) and \( g \) as follows:

\[
\phi = \min\{f(x) : b \leq x \leq \overline{b}\} > 0 \quad (4.6)
\]

\[
\gamma = \min\{g(x) : a \leq x \leq \overline{a}\} > 0 \quad (4.7)
\]

\[
\psi = \max\{f(x) : b \leq x \leq \overline{b}\} > 0 \quad (4.8)
\]

\[
\delta = \max\{g(x) : a \leq x \leq \overline{a}\} > 0 \quad (4.9)
\]
Assume that $\phi, \gamma, \psi$ and $\delta$ are bounded away from zero. Then we have the following result:

\[
\frac{1}{\psi(m+1)} \leq E[b_i - b_{i+1}] \leq \frac{1}{\phi(m+1)} \tag{4.10}
\]
\[
\frac{1}{\delta(n+1)} \leq E[a_{i+1} - a_i] \leq \frac{1}{\gamma(n+1)} \tag{4.11}
\]

**Proof** We prove inequality (4.11). The case for inequality (4.10) is similar. From the result of Pearson’s work [68] a century ago, we have the following:

\[
E[a_{i+1} - a_i] = C^j_n \int_0^\pi G(y)^j (1 - G(y))^{n-j} \, dy
\]

where $G(\cdot)$ is the CDF corresponding to the PDF $g(\cdot)$. Let $u = G(y)$, we can write $E[a_{i+1} - a_i]$ as

\[
E[a_{i+1} - a_i] = C^j_n \int_0^1 g^{-1}(u) u^j (1 - u)^{n-j} \, du
\]

where

\[
g^{-1}(u) = \frac{dG^{-1}(u)}{du} = \frac{1}{g(y)}.
\]

Since $g(y)$ is continuous on $[a, \pi]$, using assumption (4.7) and (4.9), we have

\[
\frac{1}{\delta} \leq g^{-1}(u) \leq \frac{1}{\gamma}.
\]

Noting that

\[
\int_0^1 u^j (1 - u)^{n-j} \, du = \frac{j!(n-j)!}{(n+1)!},
\]

we have

\[
\frac{1}{\delta(n+1)} \leq E[a_{i+1} - a_i] \leq \frac{1}{\gamma(n+1)}
\]
which completes the proof.

Note the assumption that $\phi, \gamma, \psi$ and $\delta$ are bounded away from zero is reasonable for most distributions. Moreover, as users’ private valuations typically fall within a small region in almost all existing works [26, 69], the intervals on which $f(\cdot)$ and $g(\cdot)$ are defined are small, which further guarantees the validity of this assumption.

We are now in a position to introduce our efficiency result and the proof. As noted above, some potential trading volume is sacrificed to make our mechanism truthful. The trading volume loss results in a loss in economic efficiency, since it could have been successfully cleared by our price setting rules. We prove that although there does exist a loss in efficiency, it is asymptotically insignificant.

**Theorem 4.3** Our periodic double auction mechanism achieves 100% economic efficiency when the number of trading users scales to infinity. It is therefore asymptotically efficient.

**Proof** We prove for the case when Eq. (4.4) holds. When $\sum_{i=1}^{K-1} q_i^b > \sum_{j=1}^{L-1} q_j^a$, as shown in Fig. 4.2, the efficiency loss can be represented by the sacrificed trading quantities from buyer $K$ and seller $L$, denoted as $D$ and the volume of supply shortage $C$. We have

$$D \leq (b_K - a_L)q_L^a \leq (a_{L+1} - a_L)q_L^a$$  (4.12)

and the value of $C$ is bounded by

$$C \leq (b_1 - b_K)q_L^a.$$  

Let $\Delta(K, L)$ denote the efficiency loss of our mechanism. Then by Lemma 4.4 we have
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the following:

\[ E[\Delta(K, L)] = E[D + C] \leq \left( \frac{1}{\gamma(n+1)} + \frac{K - 1}{\phi(m+1)} \right) E[q^a]. \]  

(4.13)

We take expectation since the reservation prices and trading quantities are all random variables. The maximum total market value, denoted as \( \Theta(K, L) \), can be expressed as follows:

\[ \Theta(K, L) = \sum_{i=1}^{K} q^b_i (b_i - b_K) + \sum_{j=1}^{L} q^a_j (a_L - a_j) + (b_K - a_L) \sum_{j=1}^{L} q^a_j. \]

Using Lemma 4.4, assuming \( q^b_i \) and \( q^a_j \) are i.i.d random samples equal in distribution to random variables \( q^b \) and \( q^a \) respectively, we can bound its expectation as

\[ E[\Theta(K, L)] > \sum_{i=1}^{K-1} i \cdot E[q^b] + \sum_{j=1}^{L-1} j \cdot E[q^a]. \]  

(4.14)

We then can bound the efficiency loss factor \( \rho(K, L) \) can be bounded as follows:

\[ \rho(K, L) = E \left[ \frac{\Delta(K, L)}{\Theta(K, L)} \right] < E[q^a] \left( \frac{1}{\gamma(n+1)} + \frac{K - 1}{\phi(m+1)} \right) \frac{2\psi(m+1)}{K(K-1)E[q^b]} \left( \frac{2\psi(m+1)}{K(K-1)\gamma(n+1)} + \frac{2\psi}{K\phi} \right). \]  

(4.15)

Therefore, if \( K \), the number of buyers who successfully trade, is large, the market inefficiency converges to zero at a rate no slower than \( 1/K^2 \). In other words,

\[ \rho(K, L) = O \left( 1/K^2 \right). \]  

(4.16)

For the case when \( \sum_{i=1}^{K-1} q^b_i \leq \sum_{j=1}^{L-1} q^a_j \) holds, we can prove that \( \rho(K, L) = O \left( 1/L^2 \right). \) Same conclusions can be proved when Eq. 4.5 holds. Hence we conclude that our double
auction design is asymptotically efficient when the number of traders scales to infinity.

\[
\begin{align*}
\text{4.3.5 Computational Efficiency} \\
\text{We analyze the computational complexity here for a given set of } n \text{ distinct bids and } m \text{ distinct asks for a particular channel. For the winner determination algorithm, sorting the bids and asks, plotting demand and supply curves, and finding } K \text{ and } L \text{ require } O(n \log n + m \log m) \text{ time. Compute the corresponding winners require } O(K + L). \text{ Thus the winner determination takes } O(n \log n + m \log m) \text{ since } K < n \text{ and } L < m. \text{ For the pricing algorithm, since } K, L \text{ and the set of winners are given by winner determination algorithm, the complexity comes only from setting the trading quantity for each winner, which takes } O(\max(K, L)). \text{ Therefore the pricing algorithm runs in } O(\max(K, L)). \text{ Together, the overall complexity of the spectrum double auction for a specific channel is } O(n \log n + m \log m). \\
\textbf{Theorem 4.4} \text{ The spectrum double auction runs in } O(n \log n + m \log m), \text{ where } n \text{ and } m \text{ is the number of distinct bids and asks for a particular channel respectively. Therefore it is computationally efficient.}
\end{align*}
\]

\[
\text{4.4 User Rationale: A Reinforcement Learning Framework} \\
\text{With the presence of the spectrum secondary market as described in Sec. 4.3, in each trading round users have to make decision on what channels to buy or to sell, and how many. This problem is further complicated because a user has only limited information}
\]
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about the market. In this section, we show that the seemingly formidable problem can be tackled by a reinforcement learning framework, which essentially adopts a systematic trial-and-error way to derive the optimal decision.

4.4.1 Problem Formulation

In our market, the consistent objective of any selfish user is to maximize its expected long-run self-interest, which can be represented by a utility function. It needs to be quasi-linear to ensure that users can compensate each other with payments [28]. As such, in our problem the utility function includes the benefits of satisfying traffic demand, which is non-linear, as well as the amount of virtual currency held that can potentially be used to purchase more spectrum, which is the linear part.

For round \( t \), let \( X_i(t) \) denote the vector of channel holdings for user \( i \) after trading, and \( R_i(t) \) denote its current achievable rates on all channels. \( B_i(t) \) represents the amount of funds it possesses after trading and \( d_i(t) \) the traffic demand. The utility function can then be expressed as:

\[
U_i(X_i(t), B_i(t)) = \epsilon_i \min \left\{ \frac{(X_i(t))^T R_i(t)}{d_i(t)}, 1 \right\} + B_i(t),
\]

where \( \epsilon_i \) is a positive parameter that indicates the relative importance of current demand satisfaction in comparison with future trading potential. We assume that all nodes have the same form of utility functions, but they may have different \( \epsilon \)'s that are only privately known to characterize their preferences. Such definition of utility function motivates users to trade among themselves in order to improve their utilities dynamically.

At the start of each round before trading happens, users have to make trading decisions. Before doing so, first they need to evaluate the channels and demands to decide
their reservation prices. To be consistent with its definition in Sec. 4.2.2, reservation price of buying a channel per subcarrier should be equal to the additional utility it can bring:

$$p^b_{i,c}(t) = \epsilon_i \left[ \min \left\{ \frac{(X_i(t-1)R_i(t) + R^c_i(t)}{d_i(t)}, 1 \right\} - \min \left\{ \frac{(X_i(t-1)R_i(t)}{d_i(t)}, 1 \right\} \right]$$

(4.18)

where \(X_i(t-1)\) is the channel portfolio before trading at \(t\). If \(i\) buys one unit of \(c\) at price higher than \(p^c_i(t)\), net utility gain will be negative, which violates the rationality of users. Likewise reservation price of selling a channel can be defined as the utility loss due to demand satisfaction decrease. With such definitions, it can be readily verified that the utility gained from trading is exactly the difference between transaction price and reservation price as defined in Sec. 4.2.2.

Then, the decision making rationale of maximizing the expected long-run utility can be modeled as follows:

$$\max_{D_i(t)} \mathbb{E}\left[ \sum_{t=0}^{\infty} \epsilon^t U_i(X_i(t), B_i(t)) \right]$$

(4.19)

s.t. \(X_i(t-1), D_i(t), p_i(t) \xrightarrow{M(t)} X_i(t), p(t) \)

(4.20)

\(B_i(t-1) + (X_i(t-1) - X_i(t))^T p(t) = B_i(t), \) \hspace{1cm} (4.21)

where \(D_i(t)\) is the volume vector indicating how many subcarriers to buy or sell denoted by positive or negative values. Since the auction mechanism is proved to be truthful, user \(i\) will report its true valuation of the channels \(p_i(t)\) according to Eq. (4.18). \(p(t)\) is the transaction price vector at \(t\) after trading, and \(X_i(-1)\) and \(B_i(-1)\) are given boundary conditions.
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Constraint (4.20) summarizes the dynamic interaction between the user and the environment \( M(t) \), including the double auction mechanism, and other users’ trading decisions. Specifically, user \( i \) submits its bids/asks with volume and price vectors \( \mathbf{D}_i(t), \mathbf{p}_i(t) \). The environment \( M(t) \) determines which parts of \( \mathbf{D}_i(t) \) can be executed, updates \( i \)'s channel portfolio accordingly, and outputs the transaction price \( \mathbf{p}(t) \). \( i \)'s cash holding is also updated according to Eq. (4.21).

Solving the dynamic program above is extremely difficult. First, the curse of dimensionality renders the design of efficient algorithm to search the entire solution space overwhelmingly difficult, as the complexity grows exponentially [70]. Second, even if such an efficient algorithm can be found for our problem, the expectation operation requires \textit{a priori} knowledge of the interaction between a single user and the environment \( M(t) \). This knowledge cannot be obtained due to the unpredictable dynamics of traffic demands and link qualities, as well as the fact that users only have \textit{incomplete information} about the market. They are not able to determine how many users are currently in the market, what are their utility functions, and what are their bids/asks. It is also infeasible for a user to observe the previous actions of other players and perform backward induction because of the sealed-bid mechanism. Moreover, it is impractical to assume that such interaction between the user and the environment can be appropriately modeled without participating in the market in the first place.

However, since users are still capable of observing their \textit{own} actions and utility, as well as relevant market outcomes in the history, we may design an appropriate solution for them to gradually \textit{learn} the optimal strategies through past experience. Being iterative and incremental, the \textit{reinforcement learning} algorithms have become our choice that our proposed framework is based upon. To our knowledge, we are the first to apply
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reinforcement learning to derive optimal trading policy in spectrum market.

4.4.2 Reinforcement Learning

Reinforcement learning (RL) is a branch of machine learning that enables a decision maker, or an agent, having a set $A$ of alternative actions to involve an optimal decision policy through systematic trial-and-error interactions with the external environment characterized by a set $S$ of states. It has been successfully applied for constructing autonomous systems that improve themselves with experience [55].

In RL, as shown in Fig. 4.3, the agent observes the current state $s(t)$ and chooses an action $a(t)$ according to some policy $\pi$, which is a probability distribution on the state-action mapping, i.e. $\pi : (s, a) \rightarrow \pi(s, a)$. The agent’s job is to find the optimal policy $\pi$ based on feedback from the environment, known as reinforcement $r$, that maximizes accumulated reinforcement values. Suppose the environment is at state $s(t)$ in round $t$, after the agent performs action $a(t)$ it shifts to $s(t+1)$ in the next round with probability $P_{ss'} = P\{s(t+1) = s'|s(t) = s, a(t) = a\}$. The agent receives reinforcement $r(t+1) \in R$, the expected value of which can be expressed as $R_{ss'} = E\{r(t+1)|s(t+1) = s', s(t) = s, a(t) = a\}$. A decision policy is incrementally improved as the agent chooses an optimal action following the current policy, and then makes corrections by adjusting it based on the most recent observation $\langle s(t-1), a(t-1), s(t), r(t) \rangle$.

For improved efficiency, we adopt the $Q$-learning method [55] that improves decision policies with the aid of $Q$-value functions, $Q : (s, a) \rightarrow Q(s, a)$, where $Q(s, a)$ represents the expected return when taking action $a$ in state $s$ and then following the current policy $\pi$ thereafter.

$$Q^\pi(s, a) = E \left[ \sum_{k=0}^{\infty} \varepsilon^k r_{t+k} | s(t) = s, a(t) = a, \pi \right].$$
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Figure 4.3: Interaction between RL agent and the environment. $Z^{-1}$ denotes a unit time delay.

The standard update rule for $Q$-learning is given as:

$$Q(s(t-1), a(t-1)) \leftarrow Q(s(t-1), a(t-1)) + \eta \left[ r(t) + \varepsilon \max_a Q(s(t), a) - Q(s(t-1), a(t-1)) \right]$$

where $\eta$ is the learning rate. The iteration of a $Q$-value function is considered to have converged to the optimal $Q^*$ when its value for each state-action pair is no smaller than that of any other value functions.

The above $Q$-learning algorithm only updates the $Q$-value for one state-action pair after each iteration with the environment, which could be very slow to converge. To greatly improve the convergence speed, we adopts the Dyna-$Q$ architecture [71] which after each real interaction, performs $k$ hypothetical interactions simulated by the learned system model, i.e. $\{R_{ss'}^a\}$ and $\{P_{ss'}^a\}$. The update rule for the hypothetical interactions is

$$Q(s, a) \leftarrow Q(s, a) + \eta \left[ \sum_{s'} \left( \hat{R}_{ss'}^a + \varepsilon \max_{a'} Q(s', a') \right) \hat{P}_{ss'}^a - Q(s, a) \right]$$

where $\hat{R}_{ss'}^a$ and $\hat{P}_{ss'}^a$ are the estimates of $R_{ss'}^a$ and $P_{ss'}^a$. The details of Dyna-$Q$ are given in Algorithm 6.
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Algorithm 6 Q-Learning Algorithm: Dyna-Q.
1: For all $s \in S$, $a \in A$, $Q(s, a) = 0$
2: while true do
3: Perform one interaction $s(t - 1) \xrightarrow{a(t-1)} s(t)$
4: Update $\hat{P}_{ss'}$
5: Receive reinforcement value $r(t)$
6: Update $\hat{R}_{ss'}$
7: Update $Q(s(t - 1), a(t - 1))$ by Eq. (4.22)
8: Randomly choose $k$ state-action pairs $\{\pi, \bar{a}\}$
9: for all $\{\pi, \bar{a}\}$ do
10: Update $Q(s, a)$ by Eq. (4.23)
11: end for
12: end while

Given all the $Q$-values of state-action pairs associated with the current state $s$, the probability of taking action $a$ follows the Boltzmann distribution given by:

$$P(a|s) = \frac{e^{\alpha Q(s,a)}}{\sum_{a'} e^{\alpha Q(s,a')}},$$

where $\alpha$ is a positive constant that controls the “sharpness” of differentiating actions corresponding to different $Q$-values.

4.4.3 Decision Making By RL

We wish to leverage reinforcement learning to solve the trading decision making problem (4.19). Before deploying the Dyna-Q algorithm, there are a few outstanding issues that we need to address.

1) Dividing the state and action spaces: To apply RL framework we need to define our state and action spaces properly for users. We define the state of environment $S$ as the tuple $\langle \frac{(X_i(t-1))^T R_i(t)}{d_i(t)}, B_i(t-1) \rangle$. The first term denotes demand satisfaction by the current channel portfolio before trading, while the second term is simply the budget.
remained from the previous time slot. They account for users’ desire and capability to trade respectively, and are therefore naturally to be included to characterize the state space. However they are continuous variables, hence the state space potentially contains an infinite number of points. Solutions to this problem involve either dividing continuous states into sections, or generalizing \( Q \)-functions to the continuous domain. Generalization methods usually require a neural network to approximate the continuous function, while discrete solutions only require table-based mapping. Hence we choose to discretize the state space.

In our solution, two thresholds are used to divide the values of the two variables into two sections respectively, namely \textit{GOOD} and \textit{BAD}. The state space hence consists of \( 2^2 \) distinct states. We note that such a quantization may not achieve the optimal performance-complexity tradeoff and alternatives are possible, which are beyond the scope of this chapter. As we show in Sec. 4.5.2, such design achieves excellent performance with reasonably complexity and memory consumption.

By the same token we also need to discretize the action space. We first categorize all the channels into three groups, represented by \textit{GOOD}, \textit{MEDIUM} and \textit{BAD}, according to the perceived instantaneous rate \( R_c^i(t) \) on each channel \( c \). We define two \textit{atomic} actions for each category of channels, namely \textit{BUY} and \textit{SELL}. The action space \( \mathcal{A} \) can then be defined as the tuple \( \langle a_{GOOD}, a_{MEDIUM}, a_{BAD} \rangle \), representing actions for each category of channels. One may notice at this point that, the action space defined is still not clear on exactly how many subcarriers to trade for each channel. We solve this issue without adding extra complexity by further refining the \textit{BUY} and \textit{SELL} atomic actions
4.4. USER RATIONALE: A REINFORCEMENT LEARNING FRAMEWORK

In the trading mechanism, we assume the trading scale is a fraction of the current state, which is treated as uniform distribution. Users only have to choose 1 out of 2\(^3\) combinations of atomic actions for channels. The trading quantity is then generated by the corresponding uniform distribution and is equally shared on channels of the same group. We summarize that this definition of action space is amenable to implementation with little complexity and reasonable resolution of the solution domain.

2) Convergence of learning: RL models require that the environment is stationary, meaning that \(R_{ss'}\) and \(P_{ss'}\) do not change over time. Our spectrum secondary market does not satisfy this requirement, since both quantities are affected by unpredictable traffic and channel dynamics. However, as long as \(R_{ss'}\) and \(P_{ss'}\) are only varying mildly and gradually, RL methods are still effective solutions to sequential decision making problems, where we have difficulty to rigorously characterize the dynamics of the environment [55].

Formally, we define our RL-based solution for the trading decision making as follows. A user is represented by an RL agent locally observing the environment state as defined previously at the very start of the current round \(t\). It categorizes all channels into three groups according to \(R_i^c(t)\), and chooses actions from the action space \(\mathcal{A}\). This forms its volume vector \(D_i(t)\). When \(D_i(t)\) together with \(p_i(t)\) determined by (4.18) as its prices are exposed to the environment, the double auction runs and execution result is obtained as well as the transaction price \(p(t)\). The channel portfolio \(X_i(t)\) and amount of cash
4.5. EXPERIMENTAL RESULTS

$B_i(t)$ are updated accordingly. The agent receives a reinforcement of

$$r(t) = U_i(t) = \epsilon_i \min \left\{ \frac{(X_i(t))^T R_i(t)}{d_i(t)}, 1 \right\} + B_i(t).$$

The objective of the agent is to obtain an optimal decision policy $\pi^* : \{P(a|s)\}$ so that $\sum_{k=1}^{\infty} E[\epsilon^k r(t+k)]$ is maximized at any time $t$. The Dyna-Q algorithm as in Algorithm 6 is applied to learn the optimal policy iteratively.

4.5 Experimental Results

We are now ready to resort to extensive simulations to study the performance of our spectrum secondary market design. As no previous work has been done for secondary market, we rely on the double auction in [27] to serve as our performance benchmark, which represents state-of-art spectrum allocation in primary market paradigm. Be reminded that the double auction in [27] only supports homogeneous channels and single-unit bids and asks, and therefore bidding and asking prices are randomly generated.

4.5.1 Simulation Settings

In the simulations, we adopt the following models.

We use practical settings of an OFDMA cognitive radio network, including channel frequency, bandwidth, and adaptive modulation and coding schemes, as specified in IEEE 802.22 draft [63]. There are a total of 48 channels, each of which contains 128 orthogonal subcarriers. Channel gain can be decomposed into a large-scale log normal shadowing with standard deviation of 5.8 and path loss exponent of 4 and a small-scale Rayleigh fading component. The inherent frequency selectivity is characterized by an exponential
power delay profile with delay spread $1.257\mu s$. The time selectivity is captured by the doppler spread, which depends on the user’s speed. We assume every user moves around the network area according to the random waypoint model with its speeds (in km/h) following a uniform distribution $U[0, 10]$. The combined complex gain is generated using an improved Jakes-like method [72].

We assume that data packets arrive at users following an asymptotically self-similar model, ARIMA process, to model the bursty traffic [29]. All packets have the same size. The buffer is assumed to be sufficiently large, and the amount of data in it reflects user’s demand. We use two metrics to evaluate performance. 1) Average User Throughput. 2) Spectrum Utilization as the average utility from all users.

### 4.5.2 Overall Performance of the Secondary Market

We first evaluate the effectiveness of our spectrum secondary market. The simulation is performed for 200 minutes with carefully selected parameters and trading interval which we will explain in detail in later sections. Each user runs the learning algorithm while the trading proceeds.
Fig. 4.4 shows the results. We observe that secondary market outperforms primary market based approach significantly for spectrum allocation among cognitive users. A 30% throughput gain and a 35% spectrum utilization gain is achieved on average, demonstrating the benefits provided by sensible secondary market design. Another interesting result is that the performance margin becomes more substantial along the time axe, which reflects the effect of improved trading decisions through learning. The results clearly indicates that with the secondary market, every channel is traded as a different stock with dynamic prices across users, and is more efficiently utilized as time goes by, albeit the temporal and spatial variation of user demands and link qualities.

4.5.3 Tradeoff Between Performance and Trading Overhead

Figure 4.5: Investigation of the sweet spot of the market.

As seen, the secondary market helps to significantly improve the performance. However, it is by no means a free lunch, as users have to search its best trading strategies by learning, and the spectrum broker has to consume energy to manage trading. Thus, we wish to obtain the optimal tradeoff between trading frequency and performances.
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To address this question, we vary the length of the trading period in the spectrum market, and Fig. 4.5 shows the results. We observe that there is indeed a sweet spot. No matter what the market size is, we have a preferable tradeoff between the overhead and performance if trading is conducted around every 200 seconds. On one hand, performance decreases dramatically with longer trading intervals. On the other hand, more frequent trading substantially increases the overhead, with only marginal improvement. Thus, we may wish to set the market to operate every three minutes.

4.5.4 Convergence of the Learning Algorithm

The convergence of $Q$-values is critical to the performance of reinforcement learning algorithm. Finally, we investigate this issue in this section. We carefully choose the learning rate $\eta = 0.5$ and discounting factor $\varepsilon = 0.9$ as in Eq. 4.22 for updating $Q$-value. First, we set the number of hypothetical iterations $k$ equal to 5, and examine $Q$-values of three randomly selected state-action pairs at an arbitrary user in the network. Fig. 4.6(a) shows $Q$-value curves for a 200-minute simulation. All values converge after around 150 minutes (50 trading rounds). We then vary $k$ to exploit potential performance benefits. As shown in Fig. 4.6(b), the curves converge more quickly and change more sharply with
higher number of hypothetical iterations \((k = 15)\).

The two simulations above are performed under fixed \(\alpha\) in Boltzmann distribution. We next study the impact of this value on \(Q\)-value convergence. Fig. 4.6(c) shows the learning curves with \(k = 10\) and \(\alpha = 0.001\) instead of 0.01 as in the previous simulations. We observe the \(Q\)-value curves can not always converge after 200 minutes. This is because a very small \(\alpha\) offers little discrimination among different state-action pairs, making the learning progress extremely slow. We also test a large \(\alpha\) value (2.0). The result shows that learning curves do not converge either, because a large \(\alpha\) essentially prevents reasonable exploration in the state-action space. Therefore, convergence of learning algorithm heavily depends on the parameter settings. With careful hand-tuning, it converges reasonably fast with satisfactory performance.

4.6 Discussions

We discuss some issues that may affect the practicality of our secondary market approach when it is put into use in reality.

First, we assume in our analysis that users do not collude, meaning that they do not try to collectively cheat and game the mechanism. Though in the literature of mechanism design, such assumption is commonly made to simplify analysis, in reality users have strong incentive to collude if doing so is beneficial to them. Preventing collusion is a challenging task itself and is beyond the scope of this thesis. Interested readers are directed to [45] for a more detailed discussion.

Second, users in reality can “spy” on other users by tuning into their channels and overhear the bid/ask transmission. Applying existing cryptographic technologies can effectively prevent such activities. Moreover, our mechanism relies heavily on the security
of the micropayment scheme used for the transactions. Though a lot of micropayment schemes have been developed, we are not aware of any scheme that is made specifically for the use in wireless networks. We borrow a micropayment scheme for P2P networks [64] in our work. Again developing a micropayment for wireless networks is beyond the scope of the thesis, but it is worth clarifying this point.

4.7 Summary

We consider improving spectrum utilization in the cognitive radio network regime. We moved beyond the state-of-the-art that considers a primary market paradigm to establishing a secondary market for spectrum redistribution among cognitive users. In this chapter, we have presented a novel market mechanism based on dynamic double auctions, which makes it possible for users to bilaterally trade their channel holdings. The double auction is provably truthful and asymptotically efficient, and solves the dual challenges of temporal and spatial variation of traffic demands and channel conditions. We further presented the first reinforcement learning algorithm in the literature to enable users to incrementally learn the optimal trading strategy amid unpredictable market dynamics.
Chapter 5

Concluding Remarks

In this thesis, we have addressed the problem of improving spectrum utilization in both the primary and secondary network regimes. We assume a context of an OFDMA based primary network in the first part of the thesis, and focus on using cooperative diversity and network coding as the building blocks for new transmission protocols with improved spectral efficiency in multi-channel networks. We seek to unravel two questions that have not yet been explored: First, how to effectively exploit network coding aided cooperative diversity when overhearing cannot be opportunistically harvested; Second, how to reap different forms of gains available in such networks when relay assignment, relay strategy selection, channel assignment and power allocation intricately interplay with each other. We have devised XOR-CD, a first-of-its-kind XOR-assisted cooperative diversity scheme that capitalizes bi-directional traffic and is shown to be able to greatly improve relay efficiency and throughput by both information theoretical analysis and realistic simulations. Moreover, a unifying optimization framework to exploit multi-user diversity, cooperative diversity and network coding jointly has been developed, with which we established the NP-hardness of the resource allocation problem, and proposed efficient approximation
algorithms with provably the best performance guarantee.

In light of its simplicity and versatility, we plan to investigate XOR-CD in different networking scenarios in the future. For example, it may be exploited in multi-hop multi-channel networks, where the difficult routing and spatial reuse pose new challenges to unleash its full potential. In a multi-hop scenario, routing becomes tightly coupled with the resource allocation problem and is considerably more difficult than the single-hop scenario we have considered. Further, the resource allocation problem alone deserves more in-depth investigation, as now the channels can be spatially reused across hops of communications. Nevertheless, we argue that the simple single-hop OFDMA network we have focused is fairly realistic to existing primary networks such as the WiMAX or LTE based networks, and may correspond to the most immediate implementation possibility of XOR-CD.

In the second part of the thesis, we shift our focus to the secondary network. Given that existing auction-based solutions are inevitably proposed under a primary market paradigm, which is suitable only to deal with issuance of relatively long-term spectrum leases from legacy owners to large cognitive entities, we turn our focus to heterogeneous small secondary users. We push the state-of-the-art to the next level by going beyond a primary market. We advocate that a spectrum secondary market complementary to the primary market is to be established. The secondary market works in harmony with the primary market through spectrum brokers. In the secondary market, a virtual currency, such as the lightweight currency in [64], serves as the medium of transaction. A user can sell some of the channels to others when traffic demand is reduced, or when the channels are in deep fade. The windfall from the sales can be used for future purchases when demand increases, or exchanged for channels with better conditions. Other users may
bid for these channels to satisfy their unfulfilled demands, or to better utilize them as they may perceive much better conditions due to multi-user diversity. By establishing the secondary market among cognitive users, spectrum as a resource becomes more liquid and easier to obtain and relinquish, leading to more efficient utilization, as implied by fundamental principles of economics and finance [28].

Towards this end, we characterized such a spectrum secondary market in terms of market mechanism and user rationale. We devised a novel multi-unit double auction to support bilateral trading dynamically among secondary users. The critical economical properties of the mechanism, including truthfulness, efficiency, budget balance and individual rationality, have been rigorously established. We further contributed a reinforcement learning algorithm to address the channel pricing and trading decision making problem for users amid unpredictable market dynamics, which to our knowledge, has not yet been discussed in the literature.

To enable auction-based dynamic spectrum access, many open questions need to be addressed. As it represents a radical change to the existing spectrum management policies and spectrum market structure, dynamic spectrum access is essentially a clean-slate approach towards enabling more efficient utilization of the spectrum which has far-reaching impact on almost all players of the game, from the policy-makers to the service providers to numerous end users. Under the primary/secondary network paradigm, a lot of future network architectures have been proposed with different targeting application scenarios and underlying spectrum sharing models [8]. Whether these architectural designs can be efficient, whether they can co-exist and work well with the current static spectrum management structure and be effectively deployed in the future, call for the devotion of even more extensive research attention and efforts. Whether our double auction based
secondary market approach can be a feasible and effective solution, despite the challenge of the implementations of the spectrum broker infrastructure and the common control channel, is an interesting topic of future research. Though implementation calls for future research, we believe our design and characterization of the spectrum secondary market, in terms of market mechanism and user rationale, further the understand of and shed light on efficient spectrum allocation among cognitive users.
Bibliography


