ISSN 0723-4864, Volume 48, Number 5

REVIEW ARTICLE	
S. Kim · K.D. Kihm · T. Thundat:	Fluidic applications for atomic force microscopy (AFM) with microcantilever sensors 721
RESEARCH ARTICLES	
D. Bond · H. Johari:	Impact of buoyancy on vortex ring development in the near field 737
A. Agrawal · L. Djénidi · R.A. Antonia:	Near-field measurements and development of a new boundary layer over a flat plate with localized suction 747
M.J. Burin · E. Scharfman · H. Ji:	Local measurements of turbulent angular momentum transport in circular Couette flow 763
J.-L. Aider · J.-F. Beaudoin · J.E. Wesfreid:	Drag and lift reduction of a 3D bluff-body using active vortex generators 771
N. Benard · E. Moreau · J. Griffin · L.N. Cattafesta III:	Slope seeking for autonomous lift improvement by plasma surface discharge 791
C. Goeppfert · J.-L. Marié · D. Chareyron · M. Lance:	Characterization of a system generating a homogeneous isotropic turbulence field by free synthetic jets 809

(Continuation on cover page IV)

Indexed in Current Contents

**This article was published in the above mentioned Springer issue.
The material, including all portions thereof, is protected by copyright;
all rights are held exclusively by Springer Science + Business Media.
The material is for personal use only;
commercial use is not permitted.
Unauthorized reproduction, transfer and/or use
may be a violation of criminal as well as civil law.**

An empirically validated model of the pressure within a droplet confined between plates at equilibrium for low Bond numbers

M. J. Schertzer · S. I. Gubarenko · R. Ben Mrad ·
 P. E. Sullivan

Received: 19 June 2009 / Revised: 30 September 2009 / Accepted: 14 October 2009 / Published online: 5 November 2009
 © Springer-Verlag 2009

Abstract An analytical model is presented that describes the equilibrium pressure within a confined droplet for small Bond numbers without prior knowledge of the interface shape. An explicit equation for the pressure was developed as a function of the gap height, surface tension, and contact angle. This equation was verified empirically. The shape of the interface was found based on the pressure predicted by both the proposed model and a model commonly used in electrowetting on dielectric (EWOD) investigations. These shapes were compared against experimentally observed interfaces for aspect ratios between 3.5 and 18. The pressures and shapes predicted by the proposed model were at least an order of magnitude more accurate than those predicted with a more commonly used model. At an aspect ratio of 3.5, the average error in the predicted shape was almost 4%, but decreased below the experimental error at an aspect ratio of 6. An aspect ratio of

15 is required for an EWOD device to split water droplets in air. The error in the model pressure and its predicted interface in this case were approximately 0.3%. The analytical pressure model proposed here can be used to increase the accuracy of models of practical EWOD devices. Better accuracy can be attained for small aspect ratios by iteratively calculating pressure using the model proposed here.

List of symbols

g	Acceleration due to gravity, $[g] = \text{m/s}^2$
$\bar{h}, h$	Half height of the gap, $[\bar{h}] = \text{m}$
N_B	Bond number
$\mathbf{n}$	Outward facing normal vector
$\bar{p}, p$	Pressure within the droplet, $[\bar{p}] = \text{Pa}$
$\bar{R}$	Droplet radius in the horizontal plane, $[\bar{R}] = \text{m}$
$\bar{r}, r$	Droplet radius in the vertical plane, $[\bar{r}] = \text{m}$
$\bar{z}, z$	Position on the vertical axis, $[\bar{z}] = \text{m}$
γ	Surface tension between the droplet fluid and the surrounding medium, $[\gamma] = \text{N/m}$
η	Change in ξ with respect with z
θ	Angle between tangent to interface and horizontal plane, $[\theta] = \text{rad}$
$\theta _{-h} \equiv \theta_-$	Interface angle at $z = -h$ (also π minus the contact angle)
$\bar{\kappa}, \kappa$	Total local curvature of the interface, $[\bar{\kappa}] = 1/\text{m}$
$\bar{\xi}, \xi$	Horizontal distance between the interface and the contact line, $[\bar{\xi}] = \text{m}$
$\bar{\rho}, \rho$	Circular-cylindrical coordinate, $[\bar{\rho}] = \text{m}$
ρ_F	Fluid density, $[\rho_F] = \text{kg/m}^3$
φ	Circular-cylindrical coordinate, $[\varphi] = \text{rad}$
$\bar{\phi}, \phi$	Level set function, $[\bar{\phi}] = \text{m}$
$\bar{\nabla}, \nabla$	Gradient operator, $[\bar{\nabla}] = 1/\text{m}$

To separately denote dimensional and corresponding nondimensional quantities, a dash “-” is added above the dimensional values. For all the lengths, scale equals to $\bar{R}$; for curvatures— $1/\bar{R}$; for pressures— $\gamma/\bar{R}$

M. J. Schertzer · S. I. Gubarenko · R. Ben Mrad (✉) ·
 P. E. Sullivan
 Department of Mechanical and Industrial Engineering,
 University of Toronto, Toronto, ON M5S 3G8, Canada
 e-mail: rbenmrads@mie.utoronto.ca

M. J. Schertzer
 e-mail: schertmjs@mie.utoronto.ca

S. I. Gubarenko
 e-mail: sigubar@mie.utoronto.ca

1 Introduction

The completion of the human genome project has resulted in significant interest in the human proteome. Conventional tools used in proteomics have relatively long cycle times due to inefficient mass transport of biological material from solution to the reaction site (Lai et al. 2004). They also suffer from the overuse of expensive reagents (Lee and Lee 2004). These disadvantages are particularly limiting in proteomics, where high throughput is necessary and reagents cannot be easily synthesized (Lee and Lee 2004).

Micro total analysis systems (μ -TASs) automate macroscale processes by integrating a series of modules onto a single microscale device. In many applications, the development of these systems increases throughput by reducing cycle times and the consumption of raw materials (Lee and Lee 2004). The majority of μ -TASs for biological applications are based on continuous flow microfluidics (i.e. Lai et al. 2004; Lee and Lee 2004; Endo et al. 2005). These systems have reduced cycle times and raw material consumption. Despite these advantages, continuous flow devices tend to be difficult to fabricate because of the need for complex components like valves.

More recently, μ -TAS devices have been developed that can individually control the motion of multiple droplets of fluid without using channels or other complex components (i.e. Wixforth et al. 2004; Pollack 1999; Fan et al. 2003; Chatterjee et al. 2006; Cho et al. 2003). Because of their ability to control discrete volumes of fluid, this class of devices is often referred to as digital (or discrete) microfluidics. One of the most promising digital microfluidic platforms uses electrowetting on dielectric (EWOD) to control droplet motion. In these devices, droplets with diameters on the order of 1–2 mm are confined between parallel plates separated by a distance on the order of 50–150 μm (Pollack 1999; Fan et al. 2003; Chatterjee et al.

2006; Cho et al. 2003). Bond numbers for water droplets in these devices range between 3.5×10^{-4} and 3.1×10^{-3} . These devices have the capability to create, move, split, and mix droplets of fluid. They also have low power consumption, high reversibility, and wide applicability to different fluids (Pollack 1999; Fan et al. 2003; Chatterjee et al. 2006; Cho et al. 2003).

It is maintained that EWOD devices manipulate droplets through surface tension effects (i.e. Pollack 1999; Cho et al. 2003; Ren et al. 2002; Walker and Shapiro 2006). The shape of the droplet in equilibrium is a function of the surface tension force acting at the interface and the pressure drop across the interface. EWOD devices manipulate the interface shape of a confined droplet so that the radius of curvature in the vertical plane is increased. This weakens the surface tension at the leading edge of the droplet. The greater surface tension at the trailing edge of the droplet then forces the droplet toward the weakened interface above the active electrode. This assumption is the basis for models that have been used to predict droplet motion (Ren et al. 2002; Walker and Shapiro 2006) and a criterion for splitting water droplets (Cho et al. 2003).

Most investigations in EWOD assume the interface in the vertical plane is semi-circular and can be defined as a function of the contact angle and the gap height (Cho et al. 2003; Ren et al. 2002; Walker and Shapiro 2006). This assumed shape is then used to find the pressure drop across the interface so that

$$p = \frac{1}{h} \cos \theta|_{-h}, \quad \bar{p} = \frac{\gamma}{h} \cos \theta|_{-h}, \quad (1)$$

where $\theta|_{-h}$ is 180° minus the contact angle, $\bar{p}$ is the excess pressure within the droplet, and $\bar{h}$ is half the gap distance between the confining plates (Fig. 1). Dimensionless values of $\bar{p}$ and $\bar{h}$ are found using the scales $\gamma/\bar{R}$ and $\bar{R}$, respectively, where γ is the surface tension between the

Fig. 1 Static equilibrium of a droplet confined between two parallel plates; aspect ratio is $\bar{R}/\bar{h} = 1/h$

fluid and the surrounding medium, and $\bar{R}$ is the droplet radius in the horizontal plane (Fig. 1).

The retarding forces on the droplet in both Ren et al. (2002) and Walker and Shapiro (2006) were increased to agree with experimental observations. In the study by Ren et al. (2002), a contact line friction coefficient of 0.4 was added to reduce droplet speed. In the study by Walker and Shapiro (2006), the retarding force caused by contact angle hysteresis was effectively increased above what would be expected for the contact angles observed. This was done to eliminate the formation of satellite droplets in the simulation of droplet splitting. These models may have overestimated the driving forces on the droplet because the surface tension forces were determined by assuming the shape of the interface. Even if this were not the case, it is valuable to determine the uncertainty associated with the semi-circular interface approximation.

When determining the accuracy of (1), it is interesting to examine the case of a cylinder, where the contact angles on the top and bottom plate are 90° . The limit of (1) as $\theta \rightarrow 90^\circ$ is zero. This suggests that the pressure in the droplet in this case would be atmospheric pressure. However, from the Laplace law (Berthier 2008) and an expression for curvature (do Carmo 1976), the pressure difference across a curved three-dimensional interface is

$$\bar{p} = \gamma(1/\bar{r} + 1/\bar{R}), \quad (2)$$

where $\bar{R}$ and $\bar{r}$ are the radii of curvature in the horizontal and vertical planes, respectively. As $\theta|_{-h} \rightarrow 90^\circ$, $\bar{r} \rightarrow \infty$ and the pressure in the droplet becomes $\gamma/\bar{R}$ which approaches zero if $\bar{R} \rightarrow \infty$. This suggests that (1), which is derived by assuming the shape of the interface, should be replaced by a more phenomenologically accurate model for pressure and surface tension forces in EWOD devices.

Despite the base assumptions, (1) is a useful model for EWOD investigations that provides an explicit equation for the pressure difference that drives droplet motion as a function of parameters relevant to EWOD devices (i.e. γ , h , and θ). Earlier work has studied confined or surface tension driven droplets but was focused on uncovered droplets (Brochard 1989), or geometries at scales not suitable for microfluidic devices (Carter 1988). Investigations of geometries seen with EWOD devices are found in the liquid bridges community (i.e. Montanero et al. 2002; Acero et al. 2005; Verges et al. 2001). An explicit equation for the pressure across the interface of a liquid is given by Montanero et al. (2002), and a model to determine this pressure was presented by Acero et al. (2005). However, neither model finds pressure as an explicit function of the contact angle. The shape of a liquid bridge was derived by Verges et al. (2001) and could be used to determine pressure across the interface. The interface shape derived by Verges et al. (2001) is dependent on a parameter related to

contact angle, but no relationship was provided. Thus, an explicit equation for the pressure dependent on the contact angle cannot be determined. Since none of these models result in a pressure across the interface as a function of the contact angle, they cannot be easily integrated into models of the driving force of a droplet in EWOD devices.

This study analytically determines the change in pressure across the interface of a droplet confined between two plates in the absence of an applied electrical field. An explicit equation for pressure is derived as a function of the surface tension, gap height, and contact angle. The predicted pressure within the droplet will be used to predict the interface shape. These predictions will be compared against the observed interface shape for aspect ratios between 3.5 and 18.

2 Experimental facility

The experimental facility used in this investigation is shown in Fig. 2. The facility consists of a confined droplet device and an optical measurement system. The confined droplet device is made up of two silica glass slides measuring approximately $25 \text{ mm} \times 38 \text{ mm}$. A 50-nm layer of Teflon was added to the surface of both substrates to make them hydrophobic. This was accomplished by spin coating a 1:2 Teflon and Flourinert FC-75 solution onto the substrates at 2,000 RPM for 60 s. The substrates were first baked at 175°C for 20 min and then at 330°C for 10 min. The spacing between the two substrates is achieved using eight pieces of 3M double-sided Scotch tape. This tape has a nominal thickness of approximately $100 \mu\text{m}$, but the gap distance between the two plates was measured optically in each case. Droplets were deposited onto the bottom substrate using a pipette before being covered by the upper plate. Since the droplet volume and gap height are known, the droplet radius was determined by assuming a cylindrical droplet. This assumes that the maximum deformation of the interface is much smaller than the droplet radius. The change in radius due to evaporation for aspect ratios of 3.5 and 18 is presented in Fig. 3. To minimize the evaporative loss in the droplet, imaging of the interface was performed within 3 min of the droplet deposition in all cases. In this time frame, the change in the radius for all cases is less than 2%. Even after 500 s, this change is less than 4% for all cases examined here.

The optical measurement system used in this investigation consisted of a Leica MZ16F fluorescence stereo-microscope fitted with a $2\times$ lens and a MS5K black and white camera from Canadian Photonics Laboratories (CPL). The CPL camera has a CCD array of $1280 \text{ pixels} \times 1020 \text{ pixels}$. All images were taken at $16\times$ magnification so that the resolution of the optical measurement

Fig. 2 **a** Sketch and **b** photograph of the experimental facility

Fig. 3 Change in the radius due to evaporation for aspect ratios of 3.5 (*square*) and 18 (*circle*)

system was $1.3 \mu\text{m}/\text{pixel}$. There is no distortion in the image due to pixel aspect ratio as each pixel is $12 \mu\text{m} \times 12 \mu\text{m}$. All images presented here were back lit using a Schott DCR III light source.

The optical measurement system was mounted on an x – y stage, which was used to focus the image and to adjust the field of view in the horizontal direction. The confined droplet device was mounted on a custom machined Delrin block, which was mounted on a Melles Griot manual jack. The jack allowed for vertical adjustment of the field of view. The alignment of the confined droplet device and the

microscope was controlled using custom-made fixtures. The shape of the droplet interface was then determined using software developed in MatLab.

Calibration of the optical measurement system was performed using an in-plane calibration target similar to that used by Giardino et al. (2008). An array of squares with a side length of $75 \mu\text{m}$ was printed on a transparent screen. The horizontal and vertical distance between the centers of the squares was $250 \mu\text{m}$ (Fig. 4). The pitch of the Delrin block was adjusted with a 0.3-mm-thick shim that was added to maximize the number of squares that appeared in focus during the calibration. An error in the pitch would cause the interface near the substrate to appear unfocused. The location of the center of each square was determined using the image processing toolbox in MatLab and calibrated against their known positions. Misalignment in the roll angle of the image was corrected by measuring the slope between squares on the calibration target in MatLab and rotating the image appropriately. The measurement area for this investigation was approximately $120 \mu\text{m} \times 1000 \mu\text{m}$ (shown in Fig. 4). This area is approximately 50% wider and 25% longer than the interface. The interface was in this region for all images presented here. Projected points of interest on the edge of the measurement area were determined by assuming that the error on both $\bar{\rho}$ and $\bar{z}$ axes was linear. The average error in both $\bar{\rho}$ and $\bar{z}$ was on the order of 0.1 pixels on the edge of the region of interest. The average absolute error in both $\bar{\rho}$ and $\bar{z}$ was approximately 0.5 and 0.9 pixels, respectively. On the edge of the region of interest, the error in $\bar{\rho}$ and $\bar{z}$ was always below 1 and 2 pixels, respectively.

CPL imaging software was used to create bitmap images, which were analyzed using the image processing toolbox in MatLab. A typical droplet image is shown in Fig. 5. The initial grayscale image was vertically divided into five equal sections. The grayscale threshold in each section was found using the Otsu method (Otsu 1979), and the initial image was converted into 5 binary images. The curves from the region of interest in each binary image were used to reconstruct the shape of the interface (i.e. Fig. 5c, d).

3 Analytical model

A static axisymmetric three-dimensional model was developed to predict the pressure drop and surface tension forces across the interface of a confined droplet at low Bond numbers in the absence of an electric field. An explicit expression for the pressure across the interface as a function of the gap height, surface tension, and contact angle was derived. A unique feature of this model is that

Fig. 4 Image of the calibration target used in this investigation. The outlined area is the area of interest and known (+) and measured (circle) points of interest are shown

Fig. 5 Images of the (a) left and (b) right interface of a droplet with an aspect ratio of 3.5. Images of the (c) left and (d) right interface with the reconstructed plot of the interface

the pressure within the droplet is found without assuming the final shape of the interface in the vertical plane.

The model was developed in a circular-cylindrical coordinate system $\{\bar{\rho}, \phi, \bar{z}\}$. The angular and vertical coordinates are bounded by π and the gap between the plates, respectively. Since no electric field is applied and the surface properties are independent of position, it is assumed that the droplet is symmetric around the $\bar{z}$ -axis, which runs vertically through the droplet's center of mass

A (Fig. 1). The Bond number (N_B) gives the relationship between gravitational and surface tension forces as

$$N_B = 2g\rho_F h^2/\gamma \quad (3)$$

where ρ_F is the fluid density, and g is gravitational acceleration. In a typical EWOD device, the Bond number is between 3.5×10^{-4} and 3.1×10^{-3} (Pollack 1999; Fan et al. 2003; Chatterjee et al. 2006; Cho et al. 2003). Thus, surface tension forces are dominant over gravitational forces. It is therefore assumed that gravitational effects do not play a role in the shape of the interface, and that the droplet is symmetric about the horizontal plane that runs through the droplet's center of mass.

With these assumptions, a stress equilibrium on the interface shows that its shape can be determined by equating the excess pressure within the droplet and the surface tension between the liquid and the surrounding medium (Fig. 1). This can be expressed as

$$\gamma \bar{\nabla} \cdot \mathbf{n} = \bar{p}, \quad (4)$$

where $\bar{\nabla}$ is the gradient operator, and $\mathbf{n}$ is the outward facing normal. The term $\bar{\nabla} \cdot \mathbf{n}$ represents the total local curvature of the interface (Moon and Spencer 1971).

The total droplet curvature can be determined using the level set function (Osher and Fedkiw 2003)

$$\bar{\phi}(\bar{\rho}, \bar{z}) \equiv \bar{\rho} - \bar{R} - \bar{\xi}(\bar{z}), \quad (5)$$

where $\bar{\rho}$ is the position on the radial axis, $\bar{R}$ is the distance between the droplet center and the three phase contact line along the plate, and $\bar{\xi}(\bar{z})$ is the horizontal distance between the contact line and the interface at a given vertical position (Fig. 1). The surface $\bar{\phi} = 0$ describes the droplet interface. The total curvature of a surface is the divergence of its outward facing normal, which can be described as $\mathbf{n} = \nabla \phi / \|\nabla \phi\|$ (do Carmo 1976; Osher and Fedkiw 2003) for dimensionless gradient ∇ and level set function ϕ . The total curvature of the surface described by (5) is then

$$\kappa = \left(\phi_{\rho}^2 \phi_{zz} + \phi_z^2 \phi_{\rho\rho} - 2\phi_{\rho} \phi_z \phi_{\rho z} + \|\nabla \phi\|^2 \phi_{\rho} / \rho \right) / \|\nabla \phi\|^3, \quad \|\nabla \phi\| = \sqrt{\phi_{\rho}^2 + \phi_z^2}, \quad (6)$$

where ϕ_i is the derivative of the dimensionless level set function with respect to i . If the origin is translated to the contact line, and the partial derivatives of (6) are evaluated, it can be shown that the deformation of the interface ξ is given by,

$$\xi_{zz} + p(1 + \xi_z^2)^{3/2} - (1 + \xi_z^2)/(1 + \xi) = 0, \quad \xi(-h) = 0, \quad \xi_z(-h) = ctg\theta|_{-h}. \quad (7)$$

This differential equation of interface shape is of the second order and shows dependence on $\{h, \theta, p\}$. Eq. 7 can be rewritten as two-first-order differential equations

$$\begin{aligned} \frac{d\xi}{dz} &= \eta, & \xi(-h) &= 0 \\ \frac{d\eta}{dz} &= \frac{1+\eta^2}{1+\xi} - p(1+\eta^2)^{3/2}, & \eta(-h) &= \text{ctg}\theta|_{-h}, \end{aligned} \quad (8)$$

where $\eta \equiv d\xi/dz = \text{ctg}\theta$. The assumption that the interface shape is symmetric about the horizontal plane leads to the condition that $\theta(0) = \pi/2$. Using this condition and realizing that $\xi < 1$, it can be shown by integrating the second Eq. in (8) that,

$$\begin{aligned} \theta|_{-h} - \frac{\pi}{2} &+ \frac{2p}{\sqrt{p^2-1}} \left\{ \arctg\left(\frac{p-1}{\sqrt{p^2-1}}\right) - \arctg\left[\frac{p\text{tg}(\theta|_{-h}/2)-1}{\sqrt{p^2-1}}\right] \right\} \\ &= h, \end{aligned} \quad (9)$$

where p is the dimensionless pressure $p = \bar{p}/(\gamma/\bar{R})$. The criterion for using EWOD devices to split water droplets in air is that the aspect ratio must be greater than 15 (Cho et al. 2003). Since $\bar{r}$ and $\bar{h}$ are of similar magnitude, it is reasonable to consider the case where $\bar{r} < \bar{R}$. If $\bar{r} < \bar{R}$, then $p > 1$. Using a Taylor series for the arctangent and eliminating terms of $O((1/p)^3)$ or higher, the pressure in the static droplet can be approximated as

$$\begin{aligned} p &= \frac{1}{h} \cos\theta|_{-h} + \frac{\pi - 2\theta|_{-h} + \sin 2\theta|_{-h}}{4 \cos\theta|_{-h}}, \\ \bar{p} &= \frac{\gamma}{h} \cos\theta|_{-h} \left(1 + \frac{\bar{h}}{\bar{R}} \frac{\pi - 2\theta|_{-h} + \sin 2\theta|_{-h}}{4 \cos^2\theta|_{-h}} \right). \end{aligned} \quad (10)$$

This pressure can be used in (8) to determine the interface shape of the confined droplet. Unlike (1), the limit of p as $\theta|_{-h} \rightarrow 90^\circ$ is 1 and not 0. This is the true value of pressure for a cylindrical droplet with principal dimensionless curvatures 0 and 1. This expression provides an approximation of the pressure within the droplet and accounts for the surface tension force that arises from both principal curvatures of the interface. In some cases examined here, the shape predicted by (10) did not satisfy the condition $\xi(h) = 0$. As such, the pressure that does satisfy this condition was found iteratively using (8) with (10) as the initial value. This pressure is referred to as the iterative pressure solution.

The iterative dimensionless pressure and those predicted by (1) and (10) are presented in Fig. 6 for a contact angle of 112° and aspect ratios between 3.5 and 18. All three functions increase monotonically with aspect ratio. The prediction from (1) is always lower than that from (10) by a value of 0.98. This value varies between 0.79 and 1 for contact angles of between 180° and 90° , respectively. The difference between the iterative pressure and that predicted from (10) is 0.04 at an aspect ratio of 3.5 and asymptotically approaches 0.02 as the aspect ratio increases. The

pressures predicted by (1) and (10) are shown as a percentage of the iterative pressure solution in Fig. 6b. The error decreases with aspect ratio for both predictions. However, the error in the pressure from (10) is more than an order of magnitude lower than that from (1) for all aspect ratios examined here. At an aspect ratio of 15, the difference in the iterative pressure and those predicted by (1) and (10) are 14% and 0.3%, respectively. This case is practically important as the aspect ratio must be greater than 15 to use EWOD devices to split water droplets in air (Cho et al. 2003).

The difference between the pressure from (1) and (10) is significant for practical EWOD devices. To determine which prediction is more accurate, (1), (10), and the iterative pressure was used in (8) to predict the shape of the droplet interface in the vertical plane. A comparison of the three predicted interface shapes for an aspect ratio of 15 and a contact angle of 112° is presented in Fig. 7. In all cases, $\xi(-h) = 0$ is a boundary condition and is met exactly. The difference between the shape predicted for the

Fig. 6 Plots of (a) the dimensionless pressures from (1) (filled triangle), (10) (filled diamond), and the iterative pressure (filled square), and (b) the difference between the iterative pressure and those predicted by (1) (open triangle), and (10) (open diamond)

iterative pressure and those predicted by (1) and (10) increases with $\bar{z}$. However, the shape predicted using (10) is clearly more accurate. The values of $\bar{\xi}(\bar{h})$ for interface shapes predicted by (1) and (10) were approximately 58 and 0.5 μm , respectively. The error in both cases also decreases with increasing aspect ratio (Fig. 8). As expected, the error associated with (1) is much greater than that predicted by (10) in all cases. This suggests that using the apparent geometry to determine the pressure drop across the interface is not reasonable for EWOD devices. To further examine this result, the predicted interface shapes will be compared to experimentally measured interfaces.

4 Results and discussion

Optical measurements of the interface were used to verify the following assumptions made in the analytical model:

- the interface shape is axisymmetric,
- the undeformed interface is symmetric about the horizontal axis that passes through the center of the droplet, and
- the maximum value of $\bar{\xi}$ is much less than the droplet radius $\bar{R}$.

The shape of the interface was determined by analyzing images of a droplet confined between two hydrophobic plates. The difference in the shape of the left and right side interfaces is presented in Fig. 9. The average difference between the profiles was 0.2 μm . This is lower than the experimental resolution (1.3 μm). Over 91% of the data

Fig. 8 The average difference between the iterative pressure and those predicted by (1) (triangle) and (10) (diamond)

agreed within one pixel, and over 99% was within two pixels. This suggests that the droplet is axisymmetric.

Assumption (b) was tested by comparing the interface shape from $\bar{z}[0, -\bar{h}]$ to the shape from $\bar{z}[0, +\bar{h}]$. A typical case is shown in Fig. 10. Here, the average difference in $\bar{\xi}$ between the upper and lower halves of the interface was 0.3 μm . Over 93% of the data agreed to within one pixel, and over 99% was within two pixels. This suggests that the interface is symmetric about the horizontal plane for this gap distance and that surface tension forces are of greater importance than gravitational effects in this case. This suggests that the Bond number here is low enough (0.06) to provide a useful comparison for EWOD devices with smaller gap distances where $N_B \approx 1.5 \times 10^{-3}$.

Fig. 7 Plots of (a) the interface shapes predicted by (8) using the iterative pressure —, and those predicted by (1) ····· and (10) - - - for an aspect ratio of 15, as well as (b) the difference between the shape predicted with the exact pressure and those predicted by (1) ····· and (10) - - -

Fig. 9 Profiles of the (a) *left* and (b) *right* side of the droplet interface at $16\times$ magnification for an aspect ratio of 3.5 and (c) a histogram of the difference between the profiles

Fig. 10 Profile of (a) the droplet interface at $16\times$ magnification for an aspect ratio of 15, (b) a histogram of the difference between the *upper* and *lower* portion of the interface

Assumption (c) was tested by examining $\xi_{\max}$ ($=\xi_{\max}/\bar{R}$) over the range of aspect ratios examined here (Fig. 11). The criterion for splitting water droplets in air in EWOD devices is that the aspect ratio of the droplet must be greater than 15 (Cho et al. 2003). For a droplet with an aspect ratio of 15, the maximum deformation of the interface was $81\text{ }\mu\text{m}$ or 1.4% of the droplet radius. In this investigation, $\xi_{\max}$ was found to be between 6.2% and 1.2% for aspect ratios of 3.5 and 18, respectively. These

results suggest that it is reasonable to assume that $\xi_{\max} < \bar{R}$ for practical EWOD devices.

The contact angle of the measured interface shapes must be known in order to predict the shapes with (1) or (10). These angles are difficult to measure directly from the image due to uncertainty that arises from the image resolution. The contact angle was found by fitting a second-degree polynomial to the measured data near the substrate. The derivative of this function at the wall is taken to be the

Fig. 11 The maximum measured deformation of the interface as a percentage of the droplet radius in the *horizontal* plane

contact angle. The average contact angle for the images examined here was 111.9° . This result was verified by checking the data against simulations with contact angles between 111° and 113° . The simulations with contact angles of 112° were the best fit for the data.

Equations (1) and (10) were used with (8) to predict the shape of the interface in the vertical plane. These shapes are compared with measured interface shapes for aspect ratios of 3.5, 9, and 15 in Fig. 12. The shape predicted by (10) is clearly more accurate than that predicted by (1). The

error in the interface shape predicted by (1) increases with $\bar{z}$ in all cases. Although the overprediction of $\bar{\xi}(\bar{z})$ decreases with aspect ratio, the average error is still $20 \mu\text{m}$ for an aspect ratio of 15. The shape predicted by (10) is more accurate than (1) in all cases. For an aspect ratio of 3.5, the value of $\bar{\xi}(\bar{z})$ is zero at $-\bar{h}$, but it increases to approximately $-10 \mu\text{m}$ at $\bar{h}$. The average error in this case was $-3.0 \mu\text{m}$. This error decreases with aspect ratio, and is below the resolution for aspect ratios of 9 and 15. This result shows that the proposed model for pressure across the interface offers a significant improvement over the model that is used in the majority of EWOD investigations.

A detailed comparison of the experimental interface shapes and those predicted by (10) is shown in Fig. 13. The distribution of the error ($G(T)$) is given in (Fig. 13d–f), each bar in the histogram corresponds to an error of one pixel. For an aspect ratio of 3.5, the error in the predicted shape increases with $\bar{z}$. The predicted value of $\bar{\xi}(\bar{h}) \approx -10 \mu\text{m}$. The distribution of the error in this case is broad spanning from $+2$ pixels to -6 pixels. The distribution narrows as the aspect ratio is increased. For aspect ratios above 6, the majority of the data agrees to within 1 pixel and virtually all the data is within two pixels. The same comparison was also made for the shapes predicted with the iterative pressure solution (Fig. 14). For this pressure, the agreement with the experimental data is consistent for all aspect ratios. The distribution of the difference between

Fig. 12 Comparison of — the measured and - - predicted interface shapes and the . . . . . difference between the two for the pressure predicted by (1) at aspect ratios of (a) 3.5, (b) 9, and (c) 15 and that predicted by (10) for aspect ratios of (d) 3.5, (e) 9, and (f) 15

Fig. 13 Comparison of the interface shapes - - of the pressure predicted by (10) and — experimental data for (a) 3.5, (b) 9, and (c) 15, and histograms of the difference as a percentage of ξ_{max} for aspect ratios of (d) 3.5, (e) 9, and (f) 15

Fig. 14 Comparison of the interface shapes predicted by the - - iterative pressure and the — experimental data for (a) 3.5, (b) 9, and (c) 15, and histograms of the difference as a percentage of ξ_{max} for aspect ratios of (d) 3.5, (e) 9, and (f) 15

predicted and measured values of $\bar{\xi}(\bar{z})$ were centered about 0 μm in all cases. The majority of the data agreed to within 1 pixel, and virtually all the data agreed to within two pixels. There is little difference between the accuracy of the shapes predicted by (10) and the iterative pressure for aspect ratios greater than or equal to 9.

The average difference between the experimental data and the shapes predicted using (1), (10), and the iterative pressure is presented in Fig. 15. As expected, the error associated with (1) is much larger than that for the other two predictions (Fig. 15a). The error associated with the iterative pressure was always less than half of the experimental resolution (Fig. 15b). When (10) is used to predict the interface shape at an aspect ratio of 3.5, the average error is on the order of 2 pixels. This error decreases to approximately 1 pixel at an aspect ratio of 6. For aspect ratios greater than or equal to 9, the shape predicted using (10) agrees with the experimental data to within half a pixel.

This study has shown that Eq. (10) provides excellent accuracy for modeling the droplet pressure in confined

droplets with no applied electric field. However, in cases where the aspect ratio becomes small (i.e. droplet splitting in EWOD devices), better accuracy can be obtained by determining the pressure iteratively using (8) until $\xi(h) = 0$. The predicted value of (10) should be used as an initial value in this process.

5 Conclusions

An analytical model was developed to describe the equilibrium pressure within a confined droplet for small Bond numbers. An explicit equation for the pressure across the interface was developed as a function of the gap height, surface tension, and contact angle. No assumptions about the final shape of the droplet interface were required to predict this pressure. The assumptions made in the development of the analytical model were verified empirically. The pressure determined from the model proposed here predicted the shape of the droplet interface in the vertical plane. Predicted shapes were also determined using the previous model, and the pressure that satisfied the condition that the contact line of the predicted interface shape occurs at the same radius on both substrates. The shapes predicted from these pressures were compared with experimentally observed interface shapes for aspect ratios between 3.5 and 18.

It was found that the equilibrium droplet interface is axisymmetric and symmetric about the horizontal plane for geometries that are relevant to electrowetting on dielectric devices. It was also found that the maximum deformation of the interface is less than 1.5% of the droplet radius for devices that are capable of splitting water droplets in air. These findings support the validity of the approximations made in the analytical model presented in this investigation.

The dimensionless pressure from (1) consistently underpredicted the iterative pressure. This resulted in errors of 44% and 14% at aspect ratios of 3.5 and 18, respectively. Although (10) consistently overpredicted the pressure, it consistently increased the accuracy from (1) by more than an order of magnitude. The maximum difference between the predicted and iterative pressure was 1.9% at an aspect ratio of 3.5, but it decreased asymptotically to 0.25% for an aspect ratio of 18.

The error associated in the predicted interface shapes also decreased by more than an order of magnitude with the use of the proposed model. The average error in the shape predicted by the proposed model was on the order of two pixels for an aspect ratio of 3.5, but was below half a pixel for aspect ratios greater than 6. This accuracy was comparable to the predictions made with the iterative pressure for practical aspect ratios.

Fig. 15 The average measured and predicted values of ξ as a percentage of the measured value of ξ_{max} as a function of aspect ratio for the interface shapes predicted by the iterative pressure (filled square), and those predicted by (1) (filled triangle) and (10) (filled diamond). The full scale is shown in (a). A much smaller scale is provided in (b) to better compare the iterative solution and that predicted by (10)

The results of this investigation show that the explicit equation for pressure proposed here can be used in EWOD investigations to increase accuracy. However, in cases where the aspect ratio becomes small (i.e. droplet splitting), better accuracy can be attained by iteratively solving for the pressure using (8) so that the contact line of the predicted shape occurs at the same radius on both substrates.

Acknowledgments The support of the Canadian Foundation for Innovation and Engineering Services Inc. (ESI) and National Sciences and Engineering Research Council (NSERC) of Canada through Discovery grants and PGS D scholarships is greatly appreciated.

References

- Acero FJ, Ferrera C, Cabezas MG, Montanero JM (2005) Liquid bridge equilibrium contours between non-circular supports. *Micrograv Sci Technol* 17:8–20
- Berthier J (2008) Microdrops and digital microfluidics. William Andrew Pub Norwich, NY
- Brochard F (1989) Motions of droplets on solid surfaces induced by chemical or thermal gradients. *Langmuir* 5:432–438
- Carter WC (1988) The forces and behavior of fluids constrained by solids. *Acta Metall* 36:2283–2292
- Chatterjee D, Hetayothin B, Wheeler AR, King D, Garrell RL (2006) Droplet-based microfluidics with non-aqueous solvents and solutions. *Lab on a Chip* 6:199–206
- Cho S, Moon H, Kim C (2003) Creating, Transporting, Cutting, and Merging liquid droplets by electrowetting-based actuation for digital microfluidic circuits. *J Microelectromech Syst* 12:70–79
- do Carmo MP (1976) Differential geometry of curves and surfaces. Prentice-Hall, Inc, Englewood Cliffs
- Endo T, Okuyama A, Matsubara Y, Nishi K, Kobayashi M, Yamamura S, Morita Y, Takamura Y, Mizukami H, Tamiya E (2005) Fluorescence-based assay with enzyme amplification on a micro-flow immunosensor chip for monitoring coplanar polychlorinated biphenyls. *Analytica Chimica Acta* 53:7–13
- Fan SK, Hashi C, Kim CJ (2003) Manipulation of multiple droplets on NxM grid by cross-reference EWOD driving scheme and pressure-contact packing. In: *Proceedings of the IEEE Conference on Micro Electro Mechanical Systems (MEMS '03)*, Kyoto, Japan, Jan 2003, pp 694–697
- Giardino J, Hertzberg J, Bradley E (2008) A calibration procedure for millimeter-scale stereomicroscopic particle image velocimetry. *Exp Fluids* 45:1037–1045
- Lai S, Wang S, Luo J, Lee L, Yang S, Madou M (2004) Design of a compact disk-like microfluidic platform for enzyme-lined immunosorbent assay. *Anal Chem* 76:1832–1837
- Lee S, Lee S (2004) Micro total analysis system (μ -TAS) in biotechnology. *Appl Microbiol Biotechnol* 64:289–299
- Montanero JM, Cabezas G, Acero J (2002) Theoretical and experimental analysis of the equilibrium contours of liquid bridges of arbitrary shape. *Phys Fluids* 14:682–693
- Moon P, Spencer DE (1971) Field theory handbook (Including coordinate systems, Differential equations and their solutions), 2nd edn. Springer, New York
- Osher S, Fedkiw R (2003) Level set methods and dynamic implicit surfaces. Springer, New York
- Otsu N (1979) A threshold selection method from grey-level histograms. *IEEE Trans Syst Man Cybern* SMC-9:62–66
- Pollack M (1999) Electrowetting-based microactuation of droplets for digital microfluidics, Ph.D. thesis, Duke University, North Carolina
- Ren H, Fair R, Pollack M, Shaughnessy E (2002) Dynamics of electro-wetting droplet transport. *Sensors and Actuators B* 87:201–206
- Verges MA, Larson MC, Bacou R (2001) Forces and shapes of liquid bridges between circular pads. *Exp Mech* 41:351–357
- Walker SW, Shapiro B (2006) Modeling the fluid dynamics of electro-wetting on dielectric (EWOD). *J Microelectromech Syst* 15:986–1000
- Wixforth A, Strobl C, Gauer C, Toegl A, Scriba J, Guttenberg ZV (2004) Acoustic manipulation of small droplets. *Anal Bioanal Chem* 379:289–291