ESSAYS ON MONEY, TRADE AND THE LABOUR MARKET

by

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Abstract

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This dissertation consists of three essays in Macroeconomics. The first essay assesses the impact of offshoring on aggregate productivity and on labour market outcomes by developing a dynamic general equilibrium model in which workers acquire task-specific human capital. The dynamic nature of the model allows for differentiation between short and long run effects. While the welfare effects are unambiguously positive and independent of the skill-content of the offshored and inshored tasks, the distribution of the gains from trade critically depends on the time horizon. Workers with human capital specific to the inshored tasks gain over those performing offshored tasks in the short term. In the long run, the gains from trade are equally distributed among ex-ante identical agents. The model is calibrated to the U.S. economy; welfare gains from increased offshoring are found to be substantial even after taking into account losses in specific human capital for workers in the offshored occupations along the transition path.

The second essay integrates the insight that exporting firms are typically more productive and employ higher skilled workers into a directed search model of the labour market. The model generates a skill premium as well as residual wage inequality among identical workers. Trade liberalization will cause a reallocation of workers both within and across industries, which will affect both types of inequality in a way that is consistent with findings from the empirical literature on trade and inequality. A calibrated version of the model can account for much of the effect of the Canada-U.S. Free Trade Agreement
on the Canadian labour market.

The final essay incorporates a distortionary tax into the microfoundations of money framework and revisits the optimum quantity of money. An optimal policy may consist of both a positive tax rate and a positive nominal interest rate: if the buyer’s surplus share is inefficiently small, the intensive margin is distorted and the constrained optimal policy combines a sales tax with a money growth rate above that prescribed by the Friedman rule. Monetary, but not fiscal, policy alters the agent’s bargaining position, leaving a special role for a deviation from the Friedman rule.
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## Contents

1 Offshoring and Human Capital 1

1.1 Introduction ........................................................................... 1

1.2 Trade in Tasks and Specific Human Capital – Evidence .............. 7
  1.2.1 Identifying Tradable Occupations ........................................ 8
  1.2.2 Characteristics of Tradable Occupations ............................... 9
  1.2.3 Estimates of Specific Human Capital ................................. 11

1.3 A Model of Trade in Tasks with Specific Human Capital ............ 17
  1.3.1 The Environment .............................................................. 17
  1.3.2 Stationary Equilibrium ....................................................... 25

1.4 Quantitative Analysis ............................................................. 31
  1.4.1 Calibration ....................................................................... 32
  1.4.2 The Experiment ............................................................... 35
  1.4.3 Results ........................................................................... 36
  1.4.4 The Impact on the Wage Distribution ............................... 41
  1.4.5 Labour Market Frictions .................................................... 42

1.5 Conclusion ............................................................................ 44

2 Trade and Inequality 62

2.1 Introduction .......................................................................... 62

2.2 Static Model .......................................................................... 66
3.5 The Welfare Effects of Fiscal and Monetary Policy ............... 115
  3.5.1 Fiscal Policy ........................................ 116
  3.5.2 Monetary Policy ....................................... 118
  3.5.3 Optimal Policy Mix ................................... 119
3.6 Competitive Pricing ........................................... 120
3.7 Conclusion .................................................. 123

Bibliography ......................................................... 124

Appendix to Chapter 1 .............................................. 132

Appendix to Chapter 3 .............................................. 135
# List of Tables

1.2.1 Educational Attainment, by Major Occupation Group .................................. 45
1.2.2 Employment in Tradable Occupations, by Major Occupation Group ........... 46
1.2.3 Earnings Functions Estimation, IV ............................................................. 47
1.2.4 Returns to Occupational Tenure, by Occupation Group .............................. 48
1.2.5 Returns to (Potential) Experience, by Occupation Group ...................... 49
1.4.1 Responses to Offshoring: Steady State ...................................................... 50

2.2.1 Simulation Scenarios ................................................................. 95
2.4.1 Calibration Targets .............................................................................. 96
2.4.2 Parameter Values ................................................................................... 96
2.4.3 Labour Market Outcomes ................................................................. 97
2.4.4 Production Outcomes .......................................................................... 97
List of Figures

1.3.1 Fraction of Educated Working in College Occupations . . . . . . . . . . . . 51
1.4.1 Distribution of Tenure in Occupation . . . . . . . . . . . . . . . . . . . . 54
1.4.2 Transition Path of Final Output, Scenario 1 . . . . . . . . . . . . . . . . 55
1.4.3 Transition Path of Final Output, Scenario 2 . . . . . . . . . . . . . . . . 56
1.4.4 Transition Path of Final Output, Scenario 3 . . . . . . . . . . . . . . . . 57
1.4.5 Transition Path of $\mathcal{U}^E$, Scenario 1 . . . . . . . . . . . . . . . 58
1.4.6 Transition Path of Wages, Scenario 1 . . . . . . . . . . . . . . . . . . . . 59
1.4.7 Transition Path of $V^S$, 67th percentile, Scenario 1 . . . . . . . . . . 60
1.4.8 Transition Path of Final Output, Economy with Labour Market Frictions 61

2.4.1 Vacancy Filling Rate . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
2.4.2 Vacancy Filling Rate, Low-Skill Workers . . . . . . . . . . . . . . . . 99
Chapter 1

Offshoring, Trade in Tasks and Occupational Specificity of Human Capital

1.1 Introduction

Technological progress has led to considerable changes in the organization of the production process – tasks traditionally completed in close physical proximity can now be spatially separated and carried out independently, thus spurring offshoring of intermediate processes or tasks.\(^1\) Differently from past trade experiences, trade in tasks affects not only manufacturing but also high-skill service occupations.\(^2\) This has spurred a debate between two opposing viewpoints, one of which focuses on the long term gains and maintains that offshoring is productivity-enhancing. The other outlook stresses poten-

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\(^1\)Offshoring is the reallocation of production sites to foreign countries to take advantage of lower input costs. This phenomenon is often mislabelled as “outsourcing”, a term which refers to the organizational structure of the firm instead. Offshoring is the choice over where to produce, while outsourcing is the choice over what selection of tasks is to be performed outside the firm; offshoring may or may not involve outsourcing.

\(^2\)In the context of trade in tasks, an occupation is the relevant labour market counterpart; a task is the output of an occupation.
tial short term losses and warns about the disruptive effects that offshoring of high skill
tasks may bring about. Previous work evaluating claims of either side of the debate has
mostly relied on static models to address the impact of offshoring on productivity and
wages and consequently could not jointly evaluate both short and long term impacts, as
well as the transition between autarky and trade equilibria. This paper ascertains that
using a dynamic model in which workers accumulate specific human capital is imperative
for assessing the potential devaluation of human capital due to offshoring of high skill
tasks and for quantifying the magnitude of its short and long term effects on aggregate
productivity and wages.

Differentiating between specific and general human capital is particularly relevant in
the context of worker reallocation due to high-skill offshoring. Were reallocated workers’
human capital mostly general, their loss in productivity would likely be small as they
would be able to apply most of their knowledge to the new task. However, if workers
who are exposed to increased offshoring have relatively more occupation-specific human
capital, switching occupations may cause a significant loss in workers’ productivity and
wages. Motivated by this observation, I develop a dynamic general equilibrium model in
which workers acquire human capital specific to the task they are completing. Opening up
the economy to trade triggers a reallocation of workers out of offshored and into inshored
occupations, causing a loss of specific human capital. Both the increase in unemployment
during the reallocation process and the loss of human capital have a negative impact on
aggregate productivity. At the same time, increased trade allows the economy to exploit
its comparative advantage, thereby generating a positive productivity effect. In the short
run, the total effect depends on the relative magnitude of the negative reallocation and the
positive comparative advantage effects. In the long run, workers reacquire human capital
and unemployment falls to its pre-trade level, so the positive productivity effect prevails.
The magnitude of the productivity effect depends on differences between autarky and
world market price, but not on the characteristics of the traded tasks.
Chapter 1. Offshoring and Human Capital

The first part of this paper documents differences in the occupational specificity of human capital across occupations and relates this to offshoring. As such, it builds on the work of Kambourov and Manovskii (2009a), who find that returns to occupational tenure are higher than returns to job or industry tenure, indicating that workers acquire substantial amounts of occupation-specific human capital. Using occupation descriptions from the O*NET database, I first identify tradable occupations. Classifying occupations by educational attainment of their workers reveals that newly tradable occupations are more frequently high skill than low skill. More than 53% of all employment in tradable service occupations is in college occupations (managerial, professional and technical), indicating that newly exposed workers have relatively more human capital than production workers previously exposed to offshoring. Subsequently, using data from the Survey of Income and Program Participation (SIPP), I establish that workers employed in tradable service occupations have relatively high returns to occupational tenure. These high returns indicate that workers in these occupations acquire almost double as much specific human capital as workers in tradable production occupations. In other words, workers in newly tradable occupations not only accumulate general human capital, but also a significant amount of specific human capital; as a consequence, these workers may be more negatively affected by offshoring in the short run than production workers have been in the past.

Building on these findings, the second part of this paper introduces occupation specific human capital into a dynamic general equilibrium model with trade in tasks. To depict trends in globalized production, the economy consists of a large number of distinct occupations producing differentiated tasks. Workers are free to move between occupations.

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3 The Occupational Information Network (O*NET) is being developed under the sponsorship of the US Department of Labour and is designed to assist both career counsellors and the general public in the process of choosing or changing careers. However, the entire database is also available to researchers who are interested in detailed descriptions of occupations and work environments. At the centre of the database is the O*NET Content Model, which describes over 800 occupations using 277 descriptors in 6 major domains.
though labour market frictions may delay the arrival of an offer and cause the worker to remain unemployed.

Specifically, the different occupations are modeled as islands as in Lucas and Prescott (1974); workers choose an occupation to which to apply and enter the occupation with some probability or else remain unemployed. The model developed in this paper features four sources of heterogeneity in workers: educational attainment, level of occupation-specific human capital, a match-specific productivity draw, and labour market status. This structure allows the model to evaluate not only aggregate welfare effects, but also distributional effects. First, the fraction of educated workers is fixed, which allows an assessment of the possible distributional effects arising from a skill bias in trade. Second, the distribution of specific human capital is endogenous, which generates short run distributional effects which differ from the long run effects. Third, since the distribution of specific human capital is endogenous and its accumulation is explicitly modelled, the transition from short to long run can be evaluated using the calibrated model. Lastly, labour market frictions generate unemployment in equilibrium.

In the long run, trade in tasks increases overall productivity by allowing the economy to exploit its comparative advantage. The social welfare effects of the “tradability revolution” are thus unambiguously positive: their magnitude depends on differences between autarky and world relative prices (i.e. its comparative advantage), but not on the skill-content of offshored and inshored tasks. For reasonable terms of trade, the steady state welfare gains of increased offshoring are found to be between 1.8% and 4%. Yet, workers differ in their specific human capital and match-specific productivity, so increased trade does have short-run distributional effects. Moving from a state of autarky to a new trade equilibrium in which high skill tasks are also tradable, workers employed

\footnote{For brevity, “specific human capital” in the present environment always denotes occupation specific human capital.}

\footnote{The fraction of educated workers need not to be fixed; as long as workers differ in their cost of acquiring an education, distributional effects may arise.}
in import-competing occupations see their income reduced, while workers employed in exported tasks see their income increase. In the same simulation as above, the lifetime expected utility of a worker with human capital specific to the offshored occupation falls by 3.1%, while the lifetime expected utility of a worker with human capital specific to the inshored occupation increases by 3%. This change in the relative values between occupations causes workers to migrate to the exporting sector. Because of labour market frictions, unemployment increases temporarily and reallocation of skilled workers also leads to a loss in their specific human capital. Over time, such agents attain specific human capital anew, which eliminates most of the distributional effects of reallocation. In the long run, the gains from trade will be shared by all agents through the competitive nature of the labour market.

The environment most similar to that in this paper is Kambourov (2009), who assesses the impact of labour market rigidities on the success of trade reforms and calibrates the model to the Chilean and Mexican trade liberalizations. As the goal of the present paper is to examine the impact of task offshoring on the U.S. economy, the model used here introduces additional heterogeneity to capture important features of the U.S. labour market. Agents differ in their levels of education to allow the model to capture a possible skill bias in task trade. To model the lengthy search process in the labour market, agents receive idiosyncratic match-specific productivity draws upon entering an occupation. Labour market frictions, on the other hand, are modelled much more parsimoniously; most importantly, there are no firing costs in this model. An alternative approach to study the dynamic nature of the reallocation of workers is presented in Cameron et al. (2007), who develop a model with moving costs for workers; their model is estimated and the distributional effects of a trade reform are studied in Artuc et al. (2007). Also, earlier work on the dynamics of adjustment after a trade shock includes

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6A similar environment with occupation specific human capital is also used in Kambourov and Manovskii (2009b), who investigate the impact of an increase in occupational mobility on wage inequality.
Mussa (1978) and Matsuyama (1992).

This paper also touches on a variety of other literatures. On the empirical side, Amiti and Wei (2006) and Liu and Trefler (2008) have studied employment consequences of offshore outsourcing in services and found the employment effect (still) to be small. Using Swedish data, Ekholm and Hakkala (2006) find a small negative effect on employment for workers with intermediate levels of education.\(^7\) On the theoretical side, Grossman and Rossi-Hansberg (2008a) provide a model of “trade in tasks” in which production requires a continuum of tasks to be completed, an increasing fraction of which becomes tradable; Grossman and Rossi-Hansberg (2008b) extends this framework to trade in tasks between similar countries where offshoring arises as a result of increasing returns. These studies mostly aim to provide a setting which considers fragmentation and incorporates it into trade models. A related literature focuses on explicitly modelling frictions in the labour market which give rise to equilibrium unemployment and allow a consideration of the impact of trade on employment and distributional consequences of trade beyond a skill premium. Davidson et al. (1999, 2008a, 2008b), Helpman and Itskhoki (2009), Helpman et al. (2009) and Mitra and Ranjan (2007) introduce labour market search frictions into international trade models; Davis and Harrigan (2007) and Amiti and Davis (2008) generate unemployment through efficiency wages.

This paper differs from the aforementioned literature in two important ways. While previous work on trade and the labour market was mostly static in nature and typically either studied the short or the long run, this paper explicitly focuses on the dynamic nature of factor accumulation and the redistribution of workers across occupations and

\(^7\)Of course, there is a large literature on international trade and inequality, both across skill groups and residual inequality. However, most of this literature does not focus on recent developments, but rather on earlier episodes. The findings in this literature are mixed: see for example Feenstra and Hanson (1999, 2003) for evidence on the importance of trade in intermediate inputs for the increase in the skill premium. Yet, Katz and Autor (1999) and Autor et al. (2008), among others, stress the importance of skill-biased technical change in explaining the wage gap between skilled and unskilled workers. Also, see the survey by Goldberg and Pavcnik (2007) for the impact of trade liberalization on income inequality in developing countries.
skill levels. Furthermore, the goal of this paper is to provide a model which captures key features of the labour market observed in the data and can be calibrated to quantify the impact of trade in tasks on labour market outcomes. As such, it does not aim to explain the actual pattern of trade, but rather takes it as given.

The remainder of the chapter is structured as follows: Section 1.2 provides evidence that newly tradable occupations require more specific human capital compared to traditionally tradable tasks. Section 1.3 then presents a model in which the distribution of workers across occupations and skill levels is endogenously determined. The model is calibrated and several quantitative exercises are undertaken in section 1.4. Section 1.5 concludes.

1.2 Trade in Tasks and Specific Human Capital – Evidence

To analyze and discuss the labour market implications of increased trade in tasks, three questions must be addressed first. First, which occupations are actually tradable; second, what are the characteristics of workers employed in tradable occupations; and third, which of these tradable occupations face the risk of offshoring and which stand to gain from inshoring. The first and second questions are the focus of this section. The first part develops a method for identifying tradable occupations and the second provides a more detailed overview of the labour market by analyzing some informative statistics for tradable occupations. The third portion investigates whether workers in these occupations acquire comparatively more general or specific human capital, and contrasts the findings for tradable service tasks with results obtained from studying manufacturing tasks which were part of earlier waves in offshoring.
1.2.1 Identifying Tradable Occupations

To identify which occupations are tradable, I analyze the characteristics and requirements of individual occupations. Detailed descriptions of each occupation can be found in two sources: the Dictionary of Occupational Titles (DOT) and the Occupational Network Database (O*NET). For the purpose at hand, the O*NET database is the better choice; unlike the DOT, it is frequently updated and contains significantly more information on service sector occupations. Since the latest update of the O*NET database is more recent (with a first release in 1998 and the latest revision in 2007) than that of the DOT (the 4th edition was revised in 1991), it also better reflects the current conditions and requirements of each occupation. Furthermore, O*NET provides a more detailed account of each occupation through 227 distinct occupation descriptors in 6 major categories.

I first focus on the “Occupational Interest Profiles”, which describe the work environment of each occupation. Occupations labelled as “Social” or “Artistic”, for example, are unlikely to be tradable. If the work environment is social, the occupation involves a high degree of personal interaction, with examples such as teachers, therapists, and child care workers. Similarly, occupations described as artistic have a high degree of interaction with the audience or “customer” and the quality of the work output most often is highly subjective; examples of such occupations include dancers, actors, and reporters.

I then use the information provided on the typical activities performed by workers in an occupation. For every occupation, O*NET lists the “level” and the “importance”

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8Two other approaches to identify tradable occupations have been proposed by Liu and Trefler (2008), who link service import and export data (as reported by the Bureau of Economic Analysis, BEA) to the associated occupation, and Jensen and Kletzer (2004), who construct a geographic concentration index for occupations to classify tradable and non-tradable occupations. While both approaches give valuable insights into occupations potentially affected by trade in services, they both suffer from some important shortcomings. High geographic concentration of occupations can be an indication of tradability, but is not a necessary condition. Additionally, using BEA data on currently traded services does not identify every potentially tradable occupation since this type of trade is only in its early stages.
for a variety of typical activities (e.g. Monitoring Processes, Materials, or Surroundings; Analyzing Data or Information). Using criteria for tradability commonly discussed in the literature and society, such as little or no face-to-face customer interaction; standardized work output; and high information content, I define occupations as non-tradable if they involve high levels of “Assisting and Caring”, “Selling” and “Working with the Public”. More specifically, if an occupation involves delivering “standard arguments or sales pitches to convince others to buy popular product”, I regard it as potentially tradable; conversely, if it requires delivering a “major sales campaign in a new market”, I regard the degree of sophistication necessary as too high for this occupation to be tradable. The cut-offs for the other activities are similarly defined.

In the final step, I reclassify occupations which would be tradable according to the above criteria, but are unlikely ever to be traded because the cost associated with offshoring them are too high. This group consists mostly of low-skill service occupations which could be offshored in principle, but for all practical purposes cannot be - launderers, ironers and certain repair and mechanics occupations fall into this group.

This approach results in a list of 61 service occupations (see next subsection and appendix A for details) that are likely tradable. Irrespective of the rule used to assign occupations to groups according to their tradability, there will be debate about the classification of some occupations (e.g. architects are classified as non-tradable, while secretaries are classified as tradable). The direct approach allows me to classify occupations based on their characteristics alone and is hence independent of actual trade observed today, which is crucial in assessing the possible implication of an expanding and increasing trade in tasks.

1.2.2 Characteristics of Tradable Occupations

In order to depict the extent to which occupations that require different levels of (general or specific) human capital in the US economy are offshorable, the Census 2000
5% sample is used here to break down the labour force by occupation group, educational attainment (the simplest proxy for skill), and offshorability. Restricting the sample to individuals who report participating in the labour force and considering the occupational groups of the Census 1990 (for consistency with data later used in the estimation of returns to (occupational) tenure), Table 1.2.1 below presents the composition of the labour force by occupation group and highest educational level completed. Individuals are classified into four groups: high school dropouts, high school graduates, individuals with some college education, and college graduates.

Figures in columns [a] through [d] show the number of individuals in each occupation group by educational attainment. To get a rough idea of the share of employment in each broad occupation group with high human capital, I group high school graduates and dropouts into the “lower education” category and consider individuals with at least some college education “higher education”, as they arguably complete their education with a higher level of both general and specific human capital. As the last column demonstrates, workers in managerial, professional and technical occupations (high skill occupations) tend to have the highest educational attainment, while workers in production and transportation occupations, helpers, and labourers have the lowest attainment.

Table 1.2.2 breaks down the employment in tradable occupations into the same major occupation groups. The first column lists the total employment for each group and the second column the total employment within that group that is employed in tradable occupations. In total, there are about 29.6 million workers employed in occupations classified as tradable, out of a total of 133.4 million non-farm employment. The third column gives the fraction of employment that is potentially tradable in each group. Not surprisingly, technical and production occupations are the most tradable; within these categories, more than 2/3 of total employment is in tradable occupations, though technical and production occupations make up only 13.7% of overall employment. On the other end, sales, services, craft and repair and transportation occupations are generally
non-tradable and jointly represent about 42% of total employment. Overall, 22.2% of the U.S. labour force is employed in tradable occupations, though this share falls to 16.7% if only non-production occupations are taken into consideration.

Managerial, professional and technical occupations together represent 36.6% of all employment in tradable occupations, while making up about 44% of total employment. Disregarding production tasks (which have been traded in the past) these “high skill” occupations account for 53.5% of tradable employment, while making up 48% of the total non-production employment. Combining the information from Tables 1.2.1 and 1.2.2, and again disregarding production occupations, it appears that tradable service occupations are more frequently high human capital occupations than low human capital occupations. In attempting to assess the labour market implications of heightened international trade, it is important to keep in mind that these tasks can potentially be traded and that, as a consequence, the U.S. will not necessarily become a net importer of higher skill tasks.

This analysis provides a preliminary indication that workers in newly tradable occupations possess more human capital than workers previously exposed to offshoring. However, it does not distinguish between specific and general human capital. The next section addresses this question.

1.2.3 Estimates of Specific Human Capital

In order to discern whether occupations increasingly exposed to offshoring require high specific or general human capital, I investigate returns to occupational tenure using a rich dataset on survey respondents’ job, occupation and industry experience. Once I account for the contribution of observable characteristics such as age, gender, job and industry tenure and overall work experience in explaining wage levels, the remaining increase in wages over time should reflect knowledge obtained through experience in the occupation – i.e. occupation (or task) specific human capital. The extent to which occupational tenure contribution to wages, in turn, can help discern the extent to which workers in
different occupations acquire specific human capital.

A rich empirical literature studies the returns to overall labour market experience, job, and industry tenure (see for example Altonji and Shakotko, 1987; Neal, 1995; Parent, 2000; and Altonji and Williams, 2005). Recently, Kambourov and Manovskii (2009a) stressed the importance of occupation specific human capital, noting that after controlling for occupational tenure, employer and job tenure do not contribute significantly to wage growth. This finding led them to conclude that workers accumulate significant occupation-specific human capital during their careers. However, as in most of the previous analyses, the paper does not investigate how occupation-specific human capital varies across groups. Using the National Longitudinal Survey of Youth 1979, Sullivan (2008) showed that there is substantial heterogeneity across occupations in the relative importance and magnitude of occupation and industry specific human capital. Finally, Connolly and Gottschalk (2006) demonstrate that college graduates experience higher returns to general experience, while high school graduates receive higher returns to industry tenure.

The Model and Data

Following the empirical literature measuring returns to tenure, I estimate the following earnings equation:

$$\ln w_{ijmnt} = \beta_1 EmpTen_{ijt} + \beta_2 OccTen_{imt} + \beta_3 IndTen_{int} + \beta_4 WorkExp_{it} + \alpha X_{ijmnt} + \kappa_{ijmnt},$$

where $w_{ijmnt}$ is the real hourly wage of worker $i$ at employer $j$ in occupation $m$ and industry $n$. $WorkExp$ denotes overall labour market experience, while $EmpTen$, $OccTen$ and $IndTen$ denote tenure with the current employer, occupation and industry, respectively. $X$ is a set of observables which influence wages independently of tenure: gender, race, educational attainment, union status, firm size, 1-digit industry and occupation affiliation,
and state and year fixed effects. $\kappa_{ijmnt}$ is an error term decomposed as follows:

$$\kappa_{ijmnt} = \mu_i + \lambda_{ij} + \xi_{im} + \nu_{in} + \epsilon_{it},$$

where $\mu_i$ is an individual-specific component and $\lambda_{ij}$, $\xi_{im}$, $\nu_{in}$ are job-match, occupation-match, and industry-match components, respectively. These unobserved components pose a potentially serious challenge to consistently estimating the returns to tenure; workers with good employer (occupation/industry) matches, for example, may be more likely to have remained with their employer (occupation/industry) longer while at the same time receiving a higher wage due to the excellent match quality. Estimating (1.1) using Ordinary Least Squares will therefore likely result in upward-biased estimates. Following the approach developed by Altonji and Shakotko (1987), which has been widely adopted in the literature, I estimate (1.1) using an instrumental variable estimation strategy.

The standard instruments for experience and the three tenure variables are the deviations of experience/tenure for individual $i$ from the individual’s mean experience/tenure in the observed spell. If $T_{it}$ is the current tenure of worker $i$, the corresponding instrument is $\tilde{T}_{it} = (T_{it} - \overline{T}_i)$, where $\overline{T}_i$ is the average tenure of individual $i$ in the current spell. The instruments are orthogonal to their respective match components by construction. Unfortunately, they are not necessarily orthogonal to the other match components; e.g. the instrument for occupation tenure, $\tilde{\text{OccTen}}_{imt} = (\text{OccTen}_{imt} - \overline{\text{OccTen}}_{im})$, is potentially still correlated with the job-match unobserved effect $\lambda_{ij}$. For example, an individual with a good employer, but a bad occupation match might be less inclined to switch occupations than an otherwise identical individual with a bad job match because switching occupations most likely also results in losing the good employer match.

The dataset of individual employment profiles used to estimate (1) comes from the 1996 and 2001 waves of the Survey of Income and Program Participation (SIPP). The 2004 wave was recently completed and unfortunately is not yet available in its entirety.
advantage of using the SIPP is its relatively large cross-sectional sample size in comparison with other panel data sets, but it comes at the cost of having a relatively short panel length (4 and 3 years, respectively). The size of the dataset allows one to estimate the returns despite the relatively short sample and justifies departure from using data from the 1980s and early 1990s, which is advantageous for three reasons. First, many of the occupations now exposed to offshoring were neither fully developed nor common some 20 years ago; second, since there is no reason to believe that the returns to tenure are constant over time even as the returns to schooling have evolved, including earlier years of data would likely not produce estimates most relevant to current discussions on offshoring. Finally – and most importantly – the SIPP data was collected at a monthly frequency, with individuals responding to one interview every four months. This allows a much more reliable identification of job switchers – something that posed a significant challenge in previous studies using the Panel Study of Income Dynamics, PSID (Brown and Light, 1992), and the National Longitudinal Survey of Youth, NLSY. The reliability of the survey responses is also increased through an implementation of computer-assisted interviews, which reduces the risk of miscoding through dependent interviewing (i.e. questions and skip-patterns are based on the previous answers of the respondent.)

Respondents in the SIPP are asked to give the start- (and end-) dates for every job, allowing me to obtain very reliable information on employer tenure and thus circumvent the issue of initialization. In the first interview, the respondent is asked about how long she has been working in the current “line of work”, which allows me to initialize occupational tenure as well. There is, unfortunately, no information on initial industry tenure; I therefore initialize industry tenure together with occupational tenure. Finally, since I do not observe an individual from the time she enters the labour market, I have no information on her actual acquired overall work experience. However, the SIPP provides detailed information on schooling, so I use potential experience - age less 6 less numbers of years of schooling - as a proxy for actual experience. To minimize the resulting bias,
I restrict the sample to male full-time workers.

In each interview, the respondent is asked retrospectively about the past four months, and the responses are recorded for each month individually. The individual reports employer, occupation and industry classifications, hours worked, and total income. She also reports start- and end-dates for each job, which allows me to identify job switches and calculate employer tenure with comparatively high precision.\textsuperscript{10} Following Kambourov and Manovskii (2009a), occupation and industry switches are only coded as “true” switches if they coincide with employer switches. Using this convention, 20.2\% of participants switch their employers at least once per 12 months; 14.5\% switch occupations, and 13.5\% industries. These shares are somewhat lower than their PSID equivalents in Kambourov and Manovskii (2009a) and Sullivan (2008). A possible explanation is that workers who lose their job may be more likely to leave the sample. Since the SIPP has relatively high sample attrition, this could explain fewer job, occupation, and industry switches in this sample.

\textbf{Results}

Table 1.2.4 presents coefficient estimates of a specification of (1.1) which includes quadratic and cubic terms for all tenure (3-digit classification level) and experience terms. Returns to occupational tenure can then be computed from these results. First, I calculate the returns for male, full-time employees and present these in Table 1.2.4[a]. For comparison, Table 1.2.5 lists the returns to overall labour market experience. I find that staying in an occupation for two, five or ten years increases wages by about 2.0, 4.6 and 7.8\%, respectively.\textsuperscript{11}

\textsuperscript{10}Nevertheless, there is a significant seam bias in the data; more switches happen “at the seam”, or between interviews (e.g. between months 4 and 5, 8 and 9) than within interviews (e.g. between months 1 and 2, 2 and 3). However, since I am not interested in estimating a hazard function, this bias is a minor issue and causes only a small error when calculating tenure - at the most 3 months.

\textsuperscript{11}These returns are lower than those reported by Kambourov and Manovskii (2009a), where 5 years in an occupation increase wages by 12.0\%, and Sullivan (2008), who reports 5-year returns of 13.3\% if occupational tenure is computed comparably. Several factors are potentially responsible, not least
Next, I estimate the returns to occupational tenure focusing only on higher skill occupations (as defined in Table 1.2.1) and present them in Tables 1.2.4[c]-[f]. I find that the returns to tenure in these occupations are indeed significantly higher than in the full sample of occupations, indicating that individuals working in higher skill occupations not only accumulate more general human capital, but also more occupation-specific human capital. The highest returns are found for technical occupations, with 30.3% for 10 years in a technical occupation. Recall that this group also contains the highest fraction of potentially tradable occupations (see Table 1.2.2).

I also estimate returns to occupational tenure in manufacturing occupations and find that they are about the same as the returns in the full sample: 3.0%, 6.0%, and 7.4% for 2, 5, and 10 years, respectively. This is in line with the argument that workers in occupations previously exposed to offshoring acquire less specific human capital. Furthermore, the returns to tenure in manufacturing occupations that I estimated for the second half of the 1990s and early 2000s may actually be higher than the returns in already offshored manufacturing occupations – i.e. the manufacturing jobs that we still observe today are more human capital intensive than the average manufacturing job in the 1970s and 80s, which have been offshored already. This argument is consistent with conventional wisdom is that US imports have (slightly) less skill content than exports (e.g. Wolff, 2003).

The parameter estimates presented above are useful in classifying occupations as those requiring comparatively more or less specific human capital. The results provide strong indication that workers in newly tradable occupation acquire significantly more specific human capital than in previously tradable production occupations.

of which the fact that the returns to occupational tenure may have diminished since the 1980s, which represent a sizeable portion of the PSID. If the wage increase is largest for workers switching employers and not occupations, and if these switches are correlated with exiting the sample, the high attrition rate in the SIPP will cause a downward bias in the returns to tenure as well.
1.3 A Model of Trade in Tasks with Specific Human Capital

In this section, I present a model of trade in tasks (intermediate goods) which incorporates workers’ specific human capital. As a key feature of the model, the distribution of specific human capital is not exogenously fixed, but rather arises endogenously as agents choose for which task to acquire specific human capital and to produce. Every period, workers may switch occupations and forego their current specific human capital, while acquiring it again for the new task over time. Consequently, the distribution of workers across occupations and levels of specific human capital responds to shocks the economy experiences, such as technological progress and trade.

1.3.1 The Environment

The economy is populated by a measure 1 of risk-free, infinitely lived agents (workers). Thus, the agent maximizes

$$\sum_{t=0}^{\infty} \beta^t c_t,$$

where $c_t$ is the consumption of the final good in period $t$ and $\beta < 1$ is the time discount factor.

The final consumption good $Y$ is a CES-aggregate of $N$ distinct tasks:

$$Y = \left[ \sum_{i=1}^{N} \kappa_i y_i^{\rho_i} \right]^{\frac{1}{\rho}},$$

where $\kappa_i$ is a share parameter for each task.

For each task, there is a large number of producers, so both input and output market are competitive. Labour is the only variable input in the production; there is also a fixed factor for each task in which each agent holds an equal share. The fixed factor is implied by the decreasing returns technology, which is needed to assure that an occupation task will have a positive mass of workers. The representative task producer’s technology is given by:

$$y_i(z, l) = z_i(l_i)^{\alpha}, \alpha < 1,$$
where $z_i$ is a time-invariant task-specific productivity parameter and $l_i$ is the total effective labour employed in the occupation.

**Human Capital**

Ex ante, agents differ only by their general human capital – the level of education; a fraction $E$ has high education attainment and a fraction $(1 - E)$ low attainment. Highly educated workers can be employed in any occupation, while low educated workers can only be employed in some. After entering an occupation, there are two additional sources of heterogeneity between agents. First, upon entering, agents draw their worker-occupation specific productivity $\theta$ from some distribution $F_i(\theta)$; a worker provides $\theta$ units of productive time each period. Second, agents differ by their level of specific human capital. In each occupation, there are two skill-types of workers – those with acquired specific human capital (skilled workers) and those still unskilled. At the end of each period (except the first one) the worker may acquire the specific human capital necessary to become a high skill worker; the arrival rate of the skill shock for an unskilled worker is $\gamma$.\(^{12}\)

After becoming skilled, a worker remains skilled until she leaves the sector. This captures the human capital that is specific to the occupation. The increase in productivity upon becoming skilled varies between occupations, but within an occupation all agents experience the same relative increase in their productivity. While an unskilled worker has $\theta$ units of productive time each period, a skilled worker has $a_i\theta$, $a_i > 1$. A worker can either choose to leave the occupation or she can become separated exogenously at rate $\pi$; however, it is assumed that at the end of her the first period in the occupation the worker will not get separated.

\(^{12}\)For the purposes of this paper, an unskilled worker is a worker without specific human capital, whereas a non-educated worker is one with low education. The occupations that employ (high) educated workers are referred to as “high education” occupations. Incidentally, in the data, these are also the occupations in which workers acquire the most specific human capital.
At the beginning of each period, an employed worker decides whether to remain in the current occupation and keep the current productivity draw $\theta$ or become unemployed and search for a new offer (i.e. try to sample a new productivity draw). There is no time gap between quitting and searching; a worker who elects to leave her occupation begins searching in the same period. An unemployed worker chooses the sector to which to apply and with probability $(1 - \epsilon)$ receives an offer $\theta$. A worker who receives a productivity draw remains in the occupation for the current period before deciding whether or not to search again. For an educated worker, the application process consists of 2 stages. First, an educated worker applies to a high education occupation; if she receives an offer, the search has ended. However, if she does not receive an offer, she applies to a low education occupation. This structure captures the empirical observation that many college graduates start their career in a non-college occupation but stay there only for a short period of time (see Figure 1.3.1). The non-educated and unskilled worker’s problem is summarized in Figure 1.3.2; the educated and unskilled worker’s problem is summarized in Figure 1.3.3.

This structure generates a rich pattern of heterogeneity and allows the model to capture key features of the data, beyond the already discussed specific human capital. It enables me to address three key concerns regarding the distribution of the gains from trade. The partition between educated and non-educated workers generates an education premium which is potentially affected by structural changes. Because of the match-specific productivity draw, it takes time for workers to find a good match. It also introduces residual income inequality, which has been argued to be affected by increased trade, a claim that can be investigated using this model.

\footnote{While there is evidence that workers do not always begin working in the occupation they are targeting in their search process, the longer the time frame, the more likely it is that they arrive in an occupation they are targeting. Furthermore, I am interested in the worker relocation resulting from a large, permanent shock and it is more likely agents will specifically target occupations with a positive shock and avoid those with a negative one; in the steady state, agents are indifferent between all occupations, so they would be willing to apply for positions in any occupation; only along the transition path is the assumption of directed search critical.}
The labour market friction generates unemployment, both along the transition path and in equilibrium.

**The Agent’s Problem**

**a. Non-Educated Workers**

The value of being an unskilled worker in occupation $i$ with productivity shock $\theta$ at the beginning of a period is given by:

$$V^u_i(\theta, \Sigma) = \max \{J^u_i(\theta, \Sigma); U(\Sigma)\},$$

(1.2)

where

$$J^u_i(\theta, \Sigma) = \theta w_i(\Sigma) + \beta (1 - \pi) ((1 - \gamma_i)V^u_i(\theta, \Sigma') + \gamma_i V^s_i(\theta, \Sigma')) + \beta \pi U(\Sigma')$$

(1.3)

is the value of staying in occupation $i$ for an unskilled worker,

$$U(\Sigma) = \max_i \{(1 - \epsilon_i) E_{\theta} \left(J^1_i(\theta, \Sigma)\right) + \epsilon_i \beta U(\Sigma')\}$$

(1.4)

is the value of being unemployed, and

$$J^1_i(\theta, \Sigma) = \theta w_i(\Sigma) + \beta V^u_i(\theta, \Sigma')$$

(1.5)

is the value of entering the occupation $i$ with draw $\theta$. $w_i$ denotes the real wage per effective unit of labour in occupation $i$, so the worker’s income is $\theta w_i$. Wages are determined competitively and agents take them as given. $\Sigma(\theta) = (\sigma^u_1(\theta), \sigma^u_2(\theta), ..., \sigma^s_1(\theta), \sigma^s_2(\theta), ...)$ denotes the distribution of workers across sectors and productivities at the beginning of the period. $E_{\theta}$ denotes the expectation operator over the possible draws of the productivity shock $\theta$.

Similarly, the value of being a skilled worker in occupation $i$ with productivity $\theta$ at the beginning of a period is given by:

$$V^s_i(\theta, \Sigma) = \max \{J^s_i(\theta, \Sigma); U(\Sigma)\},$$

(1.6)

with

$$J^s_i(\theta, \Sigma) = \theta a_i w_i(\Sigma) + \beta (1 - \pi)V^s_i(\theta, \Sigma') + \beta \pi U(\Sigma').$$

(1.7)
Chapter 1. Offshoring and Human Capital

Search is directed, so any occupation that wishes to attract applicants must offer them the same expected value, so

$$
\bar{U}(\Sigma) \geq (1 - \epsilon_i)E_{\theta}(J_i^1(\theta, \Sigma)) + \epsilon_i \beta \bar{U}(\Sigma').
$$

(1.8)

If the value of applying to occupation $i$ is less than that of other occupations, i.e. (1.8) is not satisfied as equality for occupation $i$, no worker will apply and employment will shrink due to the exogenous separation and possible quitting. However, due to a decreasing returns technology, every sector will have a positive mass of workers and (1.8) will eventually be satisfied with equality for all occupations.

Workers are identical, so it is natural to assume that all follow the same application strategy. However, this implies that if one worker applies to an occupation with probability 1, all workers would apply to this one occupation and employment in that occupation would increase drastically while it decreases in all the others. Since wages are determined competitively, (1.8) would be violated. Therefore, in equilibrium, workers must use a mixed strategy and apply to each occupation with some probability. Let $g_A^A(\Sigma)$ denote the policy function describing this optimal application strategy and $A(\Sigma)$ the total number of applicants; then $A_i(\Sigma) = g_A^A(\Sigma)A(\Sigma)$ is the number of applicants for occupation $i$.

Since each worker takes the value of search, $\bar{U}(\Sigma)$, and the future values, $V^u$ and $V^s$, as given, the workers’ optimal quitting decision can be described by a simple reservation productivity strategy: if the productivity draw exceeds the reservation level, the worker remains in the occupation, otherwise the worker leaves and searches for a better match. These reservation productivity levels $(\hat{\theta}_u, \hat{\theta}_s)$ satisfy

$$
J_i^u(\hat{\theta}_u, \Sigma) = \bar{U}(\Sigma), \text{ and } (1.9)
$$

$$
J_i^s(\hat{\theta}_s, \Sigma) = \bar{U}(\Sigma).
$$

(1.10)

Let $g^u(\theta, \Sigma)$ denote the policy function for unskilled workers describing the optimal quitting decisions, with the convention $g_i^u(\theta, \Sigma) = 1$ if $\theta \geq \hat{\theta}_u^i$. Similarly, $g^s(\theta, \Sigma)$ denotes the
policy function for skilled workers. In a stationary equilibrium (see that definition below),
two types of workers will be employed in each occupation – temporary and permanent.
Temporary workers are those who entered at the beginning of the current period, received
a low draw and will search again in the next period, while permanent workers will remain
and only leave after an exogenous separation. As a result, in a stationary environment,
skilled workers are always permanent workers.

b. Educated Workers

A fraction $E$ of all workers is educated. Only educated workers can apply to high edu-
cation occupations. Furthermore, if an educated worker is employed in a low educa-
tion occupation, she is more productive than a non-educated worker conditional on the
occupation-specific productivity draw. An educated worker employed in a low education
occupation provides $a_c \theta$ efficiency units of labour if she is unskilled and $a_c a_i \theta$ if she is
skilled, where $a_c > 1$ is the relative productivity of an educated to an otherwise identi-
tical non-educated worker. Alternatively, one can view the educated worker as drawing
from a distribution whose mean is shifted by $a_c$ relative to that of non-educated workers.
For notational convenience, I will adopt the convention $E^E_\theta = a_c E_\theta$ for low education
occupations.\footnote{A superscript $E$ denotes educated, while no superscript denotes non-educated.}

The value of being unemployed for an educated worker is given by

$$U^E(\Sigma) = \max_{h \in H} \left\{ (1 - \epsilon_h) E_\theta \left( J^1_h(\theta, \Sigma) \right) \right.$$ \hspace{1cm} (1.11)

$$+ \epsilon_h \max_{l \in L} \left\{ (1 - \epsilon_l) E_\theta \left( J^{E,1}_l(\theta, \Sigma) \right) + \epsilon_l \beta U(\Sigma') \right\} \right\},$$

where $H$ is the set of high education occupations to which the worker applies first and $L$
is the set of low education occupations to which the worker applies if she fails to secure
an offer in a high education occupation. Using the same notation as for non-educated
workers, $J^{E,1}_h$ and $J^{E,1}_l$ denote the value of entering high and low education occupations,
respectively. Then,

\[ J_{E}^{1}(\theta, \Sigma) = \theta w_{i}(\Sigma) + \beta V_{E}^{E,u}(\theta, \Sigma'), \]  

(1.12)

with \[ V_{E}^{E,u}(\theta, \Sigma') = \max \{ J_{i}^{E,u}(\theta, \Sigma); U_{E}(\Sigma) \}, \]  

(1.13)

and \[ J_{i}^{E,u}(\theta, \Sigma) = \theta w_{i}(\Sigma) + \beta \left[ (1 - \pi) \left( 1 - \gamma_{i} \right) V_{E,u}^{E}(\theta, \Sigma') \right. \]

\[ + \gamma_{i} V_{E,s}(\theta, \Sigma') \left] + \pi U_{E}(\Sigma') \right. \].  

(1.14)

After entering a sector and drawing the specific productivity shock, the only difference between an educated and non-educated worker is the continuation value in the case of separation. As a result, the reservation productivity levels for educated and non-educated workers differ; the reservation productivity levels \( (\hat{\theta}_{E,u}^{E}, \hat{\theta}_{E,s}^{E}) \) for the educated satisfy:

\[ J_{i}^{E,u}(\hat{\theta}_{i}^{E,u}, \Sigma) = U_{E}(\Sigma), \]  

(1.15)

\[ J_{i}^{E,s}(\hat{\theta}_{i}^{E,s}, \Sigma) = U_{E}(\Sigma). \]  

(1.16)

Let \( g_{E,u}^{E}(\theta, \Sigma), g_{E,s}^{E}(\theta, \Sigma) \) denote the resulting policy functions.

Again, due to the directed nature of the search process, any high education occupation which attracts a positive number of applicants must offer at least \( U_{E}(\Sigma) \). This condition applies to high education occupations only; low education occupations which attract non-educated applicants satisfy (1.8). Since the productivity premium for educated workers, \( a_{c} \), is the same across occupations and educated and non-educated workers only differ by this constant, (1.8) also assures that educated workers are indifferent between all low-education occupations in the second stage. Since educated agents are indifferent between occupations, I assume they follow the same application strategy as the non-educated in low education occupations in the second stage.

c. Labour Supply

Let \( g_{E,A}^{E}(\Sigma) \) denote the policy function describing the optimal application strategy for
educated workers and $A^E_H(\Sigma)$ the total number of educated applicants to high skill occupations. Then the total number of educated agents applying to low skill occupations is $A^E_L(\Sigma) = \epsilon_h A^E_H(\Sigma)$.

Total labour supply in each occupation is equal to the total productive time available in the occupation,

$$l^s_i = a_i \int_{\theta} \theta g^s(\theta, \Sigma) \, d\sigma^s_i(\theta) + \int_{\theta} \theta g^u(\theta, \Sigma) \, d\sigma^u_i(\theta) + (1 - \epsilon_i) A_i \int_{\theta} dF_i(\theta) \tag{1.17}$$

Recall that $\Sigma(\theta) = (\sigma^u_1(\theta), \sigma^u_2(\theta), ..., \sigma^s_1(\theta), \sigma^s_2(\theta), ...)$ denotes the distribution of workers across sectors and productivities at the beginning of the period and $g^j_i(\theta, \Sigma), \ j = u, s$ denotes the policy function indicating whether the worker with draw $\theta$ remained employed or left the occupation in the current period.

Finally, the resulting law of motion for the distribution of workers is given by

$$\sigma^s_i' = (1 - \pi) (g^s(\theta, \Sigma) \sigma^s_i + \gamma_i g^u(\theta, \Sigma) \sigma^u_i), \tag{1.18}$$

$$\sigma^u_i' = (1 - \pi)(1 - \gamma_i) g^u(\theta, \Sigma) \sigma^u_i + (1 - \epsilon_i) A_i(\Sigma), \tag{1.19}$$

$$\sigma^{E,s}_i' = (1 - \pi) \left(g^{E,s}(\theta, \Sigma) \sigma^{E,s}_i + \gamma_i g^{E,u}(\theta, \Sigma) \sigma^{E,u}_i\right), \tag{1.20}$$

$$\sigma^{E,u}_i' = (1 - \pi)(1 - \gamma_i) g^{E,u}(\theta, \Sigma) \sigma^u_i + (1 - \epsilon_i) A^E_i(\Sigma), \tag{1.21}$$

where the prime denotes the beginning of next period’s element.

The Producer’s Problem

The producer’s problem in this environment is a simple static problem. Let $p_i$ denote the price of each task in terms of the numeraire good; then the demand for each task is given by

$$y^d_i = \left(\frac{\kappa_i P}{p_i}\right)^{\frac{1}{1-\rho}} Y, \tag{1.22}$$

where $P = \left(\sum_{i=1}^{N} p_i^{\frac{1+\rho}{\rho}} \kappa_i^{\frac{1}{\rho}}\right)^{\frac{\rho-1}{\rho}}. \tag{1.23}$
where $P$, the price index for the final good, follows from the zero-profit condition for the final good’s producer.

Labour markets in each occupation are competitive, so the real wage per effective unit of labour is equal to the value of the marginal product in terms of the numeraire good:

$$ w_i = p_i \alpha z_i (l_i)^{\alpha-1}, \quad (1.24) $$

where $p_i$ is the price of each task in terms of the numeraire good. As normalization, let $w_1 = 1$.

### 1.3.2 Stationary Equilibrium

Before studying the impact of increased trade in this environment, it is instructive to study the stationary equilibrium. A stationary equilibrium is characterized by a time-invariant distribution of workers across skill levels and occupations, i.e. $\Sigma' = \Sigma$. First, notice that the critical level of the match specific productivity is constant in a stationary environment. As a result, a worker either quits after the first period, or remains in the occupation until the match is exogenously separated. Further recall that an unskilled worker’s income is $\theta w$, and that the wage paid per effective unit of labour is a constant determined in a competitive market. Consequently, one can regard the productivity draw as an income draw as well: in a stationary environment the model reduces to a variant of the stochastic job matching model with a constant matching rate.

#### a. Non-Educated Workers

Using the fact that a skilled worker never quits in a stationary equilibrium, the steady state value of being a skilled worker in occupation $i$ with shock $\theta$ is given by

$$ J_s^i(\theta, \Sigma) = \frac{a_i \theta w_i}{1 - \beta (1 - \pi)} + \frac{\beta \pi}{1 - \beta (1 - \pi)} U(\Sigma). \quad (1.25) $$
Similarly, for an inexperienced worker in occupation \( i \), it is:

\[
J_i^u(\theta, \Sigma) = w_i \left( \frac{1 - \beta(1 - \pi)(1 - \gamma_i a_i)}{(1 - \beta(1 - \pi))(1 - \beta(1 - \pi)(1 - \gamma_i))} \right) + \frac{\beta \pi}{(1 - \beta(1 - \pi))} U(\Sigma). (1.26)
\]

Here, \( U(\Sigma) \) denotes the value of searching.

Substituting (1.5) into (1.4) and using the optimal reservation productivity strategy, the value of applying to any occupation \( i \) can be written as

\[
U_i(\Sigma) = U(\Sigma) = \frac{(1 - \epsilon)}{1 - \beta \epsilon} \left[ E_{\theta,i} w_i + \beta \left( F_i(\hat{\theta}_i) + \int_{\hat{\theta}_i}^{\bar{\theta}} J_i^u(\theta, \Sigma) dF_i(\theta) \right) \right]. (1.27)
\]

Using (1.26), the condition for the reservation productivity level (1.9) can be rearranged to yield

\[
\hat{\theta}_i w_i = (1 - \beta) U(\Sigma) \frac{1 - \beta(1 - \pi)(1 - \gamma_i)}{1 - \beta(1 - \pi)(1 - \gamma_i a_i)}. (1.28)
\]

Lastly, by substituting (1.26) into (1.27), the fundamental reservation productivity equation can be obtained:

\[
\hat{\theta}_i = (1 - \epsilon_i) \left[ E_{\theta,i}(\theta) \frac{1 - \beta(1 - \pi)(1 - \gamma_i)}{1 - \beta(1 - \pi)(1 - \gamma_i a_i)} \right.
\]

\[
+ \frac{\beta(1 - \pi)}{1 - \beta(1 - \pi)} \int_{\hat{\theta}_i}^{\bar{\theta}} (\theta - \hat{\theta}_i) dF_i(\theta)\right]. (1.29)
\]

Note that the reservation productivity level is independent of the wage rate. In a stationary equilibrium, each occupation offers a time-invariant wage per effective unit of labour. Since all sectors offer the same value to each applicant, a worker who quits after the first period is willing to resample in the same occupation again – and receive the same wage rate per efficiency unit (her income \( \theta w \) will only change because \( \theta \) changes). Therefore, the wage rate reduces to a scaling parameter and does not have an impact on the reservation productivity level.

The interpretation of (1.30) is easiest after multiplying both sides with the wage rate \( w_i \). Then, the left-hand side is the utility per period from keeping the job with the
reservation productivity, while the right-hand side is the expected utility from quitting: the expected draw in the current period plus the discounted expected improvement. The optimal reservation level equates these two values.

Finally, using that \( U_i = U_j \), (1.28) allows expressing the relative wage between two occupations as

\[
\frac{w_i}{w_j} = \frac{\hat{\theta}_j}{\hat{\theta}_i} \frac{1 - \beta(1 - \pi)(1 - \gamma_i)}{1 - \beta(1 - \pi)(1 - \gamma_j)} \frac{1 - \beta(1 - \pi)(1 - \gamma_j a_j)}{1 - \beta(1 - \pi)(1 - \gamma_j)}.
\]

(1.30)

Recall from (1.30) that the reservation levels are independent of the wage paid in the occupation. Thus, (1.30) states that the steady state relative wage between sectors depends on parameters alone; output prices only affect the overall level of wages. This is a result of the directed search in the labour market – agents will apply to the occupation with the highest expected value, driving down the wage paid and the value in that occupation until all occupations offer the same value of applying. Consequently, in the steady state, all gains from trade or technological progress are equally distributed among occupations. In the long run, trade will make all ex ante identical workers equally better off. Distributional effects arise only along the transition path and between the different educational groups, as discussed below.

b. Educated Workers

Just as with non-educated workers, the directed search assures that all high skill occupations offer the same expected value in steady state and, as a result, all occupations benefit equally from trade or technological progress. Yet, the sequential nature of the application process implies that the reservation productivity level depends on the relative wage between high and low education occupations. Following the same steps as above, the reservation productivity level for an educated worker in a high education occupation
Chapter 1. Offshoring and Human Capital

is given by

\[ \hat{\theta}^E_h = \frac{(1 - \beta)}{1 - \beta (1 - \epsilon_h) + \epsilon_h (1 - \epsilon_l) \Omega_l + \epsilon_h \epsilon_h} \left[ \frac{1 - \beta (1 - \pi) (1 - \gamma_h)}{1 - \beta (1 - \pi) (1 - \gamma_h \epsilon_h)} \right] (1.31) \]

\[
\left( (1 - \epsilon_h) E_{\theta, h}(\theta) + \epsilon_h (1 - \epsilon_l) \frac{w_l}{w_h} B_l \right) + \frac{\beta (1 - \pi) (1 - \epsilon_h)}{1 - \beta (1 - \pi)} \int_{\hat{\theta}_h}^{\theta} (\theta - \hat{\theta}_h) dF_h(\theta) \right],
\]

with \( \Omega_l = F_l(\hat{\theta}_h) + (1 - F_l(\hat{\theta}_h)) \frac{\beta \pi}{1 - \beta (1 - \pi)} \), and

\[ B_l = E_{\theta}^E(\theta) + \beta \int_{\hat{\theta}_l}^{\theta} \frac{1 - \beta (1 - \pi) (1 - \gamma_l \epsilon_l)}{(1 - \beta (1 - \pi) \epsilon_l (1 - \beta (1 - \pi) (1 - \gamma_l)) dF_l^E(\theta)}, \]

where \( \hat{\theta}_l^E \) denotes the reservation level in low skill occupation \( l \), and \( w_h \) and \( w_l \) denote the respective wage rates per effective unit of labour. Note that agents are indifferent between all sectors, so any low education sector can be used when computing (1.31). The reservation level for low education occupations, \( \hat{\theta}_l^E \), can be obtained similarly.

A non-educated worker effectively resamples from the same occupation until she receives a large enough productivity draw; an educated worker, on the other hand, might not resample from the same occupation if she quits. If an educated worker leaves a high education occupation and reapplies, she may not receive an offer and will subsequently apply to and receive an offer from a low education occupation. As a result, the relative wage between the high and low education occupation will affect her quitting decision. This, of course, has implications for the distribution of the gains from trade. While the welfare gains will be equally distributed within one group, this may not hold across groups. Depending on the terms of trade, the education premium, the relative value of being an unemployed educated to an unemployed non-educated worker, \( \frac{U^E(\Sigma)}{U(\Sigma)} \), may rise or fall; this is discussed in more detail below.

c. The Stationary Distribution

In a stationary equilibrium the productivity cut-offs are constant; consequently, the dis-
tribution across productivity levels is the underlying distribution truncated at $\hat{\theta}$. The total number of workers of each skill type follows from the skill acquisition process. Let $\Theta = E(\theta | \theta \geq \hat{\theta})$, then the steady state labour supply can be written as

$$l_i^s = \Theta_i(a_i s_i + \overline{u}_i) + E_i(\theta)(1 - \epsilon_i) A_i$$

$$+ \Theta_i^E(a_i s_i^E + \overline{u}_i^E) + E_i^E(\theta)(1 - \epsilon_i) A_i^E.$$  

$s_i$ and $u_i$ are the steady state numbers of skilled and unskilled workers in each occupation:

$$\overline{u}_i = \frac{(1 - \epsilon_i)(1 - F_i(\hat{\theta}))}{\pi + \gamma_i - \pi \gamma_i} A_i$$

$$\overline{s}_i = \frac{1}{\pi} \overline{u}_i$$

$$\overline{s}_i^E = \frac{(1 - \epsilon_i)(1 - F_i^E(\hat{\theta}_i^E))}{\pi + \gamma_i - \pi \gamma_i} A_i^E$$

$$\overline{u}_i^E = \frac{1}{\pi} \overline{s}_i^E$$

and $(1 - E) = \sum_i (\overline{s}_i + \overline{u}_i + A_i)$,

$$E = \sum_i (\overline{s}_i^E + \overline{u}_i^E + A_i^E),$$

$$\sum_{l \in L} A_l^E = \sum_{h \in H} \epsilon_h A_h^E.$$  

In order to close the model, the goods market must be cleared – the conditions for goods market clearing, however, depend on the trade regime.

**Autarky Equilibrium**

The total demand for the final consumption good is equal to the total value of the output in each occupation

$$Y^D = \sum_i \frac{p_i y_i}{P}.$$  

(1.33)
In autarky, all goods consumed must be produced domestically:

$$z_i (l_i^s)^\alpha = \left( \frac{k_iP}{p_i} \right)^{\frac{1}{1-\rho}} Y^D.$$  (1.34)

Together, the market clearing condition (1.34), the firms’ profit maximizing condition (1.24) and the conditions on relative wages from the agent’s problem solve equilibrium prices, wages and the numbers of applicants in each occupation.

**Definition**

A *stationary competitive equilibrium* for the closed economy consists of value functions $V_i^s(\theta, \Sigma)$, $V_i^u(\theta, \Sigma)$, $J_i^s(\theta, \Sigma)$, $J_i^u(\theta, \Sigma)$, $J_i^1(\theta, \Sigma)$ for non-educated and the corresponding value functions $V_i^{E,s}(\theta, \Sigma)$, $V_i^{E,u}(\theta, \Sigma)$, $J_i^{E,s}(\theta, \Sigma)$, $J_i^{E,u}(\theta, \Sigma)$, $J_i^{E,1}(\theta, \Sigma)$ for educated workers; values of search for non-educated and educated, $U(\Sigma)$ and $U^E(\Sigma)$; the associated policy functions $g_i^s(\theta, \Sigma)$, $g_i^u(\theta, \Sigma)$, $g_i^A(\Sigma)$; $g_i^{E,s}(\theta, \Sigma)$, $g_i^{E,u}(\theta, \Sigma)$ and $g_i^{E,A}(\Sigma)$; a time invariant distribution of workers across occupations and skill levels $\Sigma$; prices for each task, $p_i$; wages in each occupation, $w_i$, and sectorial and aggregate output, $y_i$ and $Y$ such that:

1. Given prices and wages, the functions $V_i^s(\theta, \Sigma)$, $V_i^u(\theta, \Sigma)$, $J_i^s(\theta, \Sigma)$, $J_i^u(\theta, \Sigma)$, $J_i^1(\theta, \Sigma)$ solve the non-educated agent’s problem and $g_i^s(\theta, \Sigma)$, $g_i^u(\theta, \Sigma)$, $g_i^A(\Sigma)$ are the optimal policy functions.
2. Given prices and wages, the functions $V_i^{E,s}(\theta, \Sigma)$, $V_i^{E,u}(\theta, \Sigma)$, $J_i^{E,s}(\theta, \Sigma)$, $J_i^{E,u}(\theta, \Sigma)$, $J_i^{E,1}(\theta, \Sigma)$ solve the educated agent’s problem and $g_i^{E,s}(\theta, \Sigma)$, $g_i^{E,u}(\theta, \Sigma)$, $g_i^{E,A}(\Sigma)$ are the optimal policy functions.
3. Individual decision rules $g_i^s(\theta, \Sigma)$, $g_i^u(\theta, \Sigma)$, $g_i^A(\Sigma)$ are consistent with the invariant aggregate distribution of types.
4. The distribution of workers across sectors and skill levels is time invariant: $\Sigma' = \Sigma$.
5. Wages are determined competitively.
6. The labour market in each occupation clears; aggregate feasibility is satisfied.
7. The task markets and the final good market clear.
Trade Equilibrium

In the trade equilibrium in which a subset \( \mathcal{T} \) of tasks are tradable, prices for tradable tasks \((p_{t1}, p_{t2}, ..)\) are taken as given and supply and demand are perfectly elastic at these prices.\(^{15}\) For simplicity, assume that there are no trade costs or tariffs. Thus, the labour market clearing conditions and the relative wage conditions, together with the market clearing conditions for the non tradable tasks, determine the stationary trade equilibrium. The stationary competitive equilibrium for the open economy differs from that of the closed economy by condition 7 and an additional condition 8:

7. The task markets for non-tradeable tasks clear; aggregate feasibility is satisfied.

8. Trade is balanced: \( 0 = \sum_{i \in \mathcal{T}} p_i (y_i^s - y_i^d) \).

1.4 Quantitative Analysis

In this section, I conduct the main quantitative experiment – predicting the time-path of key labour market outcomes resulting from increased trade in high skill service tasks. I calibrate the model to match the U.S. economy in the year 2000, around the time when trade in (high skill) services became more common. I then introduce trade in tasks by allowing the economy to import or export any quantity of some tasks (those identified in section 1.2) at given world prices and compute the resulting stationary equilibrium and the transition path.

Since trade in services remains a nascent phenomenon, it is difficult to predict the actual terms of trade. Currently, we do not know which occupations will experience import-competition and which will export, or the magnitude of the difference between autarky and world relative prices. When determining the ensuing trade equilibrium, I compute three hypothetical scenarios for the trade in tasks. The first scenario is intended

\(^{15}\)Since trade is balanced, the country really is faced with a set of international relative prices.
as the likely candidate for actual developments in trade to arise in the future, while scenarios 2 and 3 investigate the importance of the exact pattern and the terms of trade.

The key insight from these experiments is that the gains from trade almost exclusively depend on the magnitude of the comparative advantage. While the skill content of occupations has an impact on the transition path, it affects the aggregate gains from trade only marginally. The skill content of imports and exports impacts the distribution of gains between educated and non-educated workers – while all ex-ante identical agents gain equally from trade, the relative standing of non-identical agents depends on the exact pattern of trade. If trade is biased against high-skill occupations, educated workers may benefit little from trade and the college premium may fall.

1.4.1 Calibration

For the calibration, I rely on data from several sources. The information on occupational tenure is drawn from the SIPP (for more details see Section 1.2.3). Data on occupation and industry affiliation and educational attainment comes from the 5% sample of the 2000 Census and data from the national accounts (NIPA tables) is used to compute the labour share of each occupation.

The model period is chosen to be one year, as the focus of the analysis is the long-run transition from one steady state to another rather than movements at the business cycle frequency. This is also consistent with the modelling choice of directed search, as discussed in the previous section. The time discount factor, $\beta$, is taken to be 0.96, which is standard.

To be able to compute the transition path, the number of occupations cannot be too large. Therefore, I group service occupations into 6 major categories: occupations are first divided into high and low skill (or college and non-college) occupations. Each of these groups is then separated into inshored, offshored and non-traded, for a total of 6 groups. Production occupations are only assigned to inshored and offshored occupations.
The parameters of the specific human capital process, $a_i$ and $\gamma_i$, are chosen to match the occupational tenure profile identified in the data. The relative productivity of workers with specific human capital, $a_i$, varies by occupational group and ranges from 1.07 (production occupations) to 1.31 (technical occupations). The probability of becoming skilled, $\gamma$, is assumed to be constant across occupations. The data shows that the wage-occupation tenure curve flattens after 8-10 years in an occupation. Therefore, I set $\gamma$ at 0.125, which implies an average tenure of 9 years at the time of separation.

The distribution of match-specific productivity shocks is uniform; its mean is set to 1. As proposed by Menzio and Shi (2009a), the variance, $\sigma_\theta$, can be selected to match the fraction of workers in the first year of their occupational tenure. The probability of leaving an occupation after accumulating more than one year of tenure, $\pi$, is 0.079. This aligns the implied occupational tenure in the model with the average occupational tenure found in the data of 12.7 years at the time of an occupation switch, conditional on the switch occurring after year 1. Figure 1.4.1 depicts how the combination of $\sigma$ and $\pi$ can be used to match the aggregate occupational tenure distribution found in the data.

The probability of not receiving an offer, $\epsilon$, is 0.2. This implies an expected unemployment spell of 13 weeks for a non-educated worker. While the actual average unemployment duration measured in the data is higher than this (18.1 weeks in 2007, according to data from the Bureau of Labor Statistics), it is upward biased as an estimate of the expected unemployment duration because longer spells are more likely to be found in the data. In light of this fact, I use the lower estimate of 13 weeks, which is in line with estimated expected unemployment durations (e.g. Valletta, 2002). Again, as a result of the sequential search by highly educated workers, the expected length of unemployment predicted by the model for highly educated workers is shorter than in the data.

Calibrating the parameters of the production process is less straightforward due to
the lack of data available at the occupation level. For example, the labour share of output within an industry can easily be calculated from national accounts data, but there is no comparable information available for occupations as the output of an occupation on its own is not as easily measured.

To calibrate the labour share, $\alpha$, I construct an occupation-industry matrix using the 2000 Census data; each cell in this matrix represents the fraction of the occupation’s total employment working in a given industry. For example, 0.14% of all accountants are employed in cosmetic manufacturing. From the national accounts (NIPA tables), I compute the labour shares for 15 major industry groups. For each occupation, the labour share is computed as the weighted average of the labour shares in the industries in which the occupation is employed. The underlying assumption is that the labour share within an industry is the same across all occupations and differences in the labour share across occupations stem from differences across the industries in which the workers in that occupation are employed.

The productivity parameter for each task, $z_i$, and its share in the final good production function, $\kappa$, cannot be separately identified. I therefore set $\kappa$ to 1 and choose the relative magnitudes of the respective $z_i$ to match the employment share of each occupation from the 2000 Census; the level of each parameter is selected such that the autarky aggregate output $Y^A = 1$. Finally, since there is no clear target for the elasticity of substitution between tasks, I set $\rho = -2.34$, which implies an elasticity of substitution of 0.3 (i.e. tasks are complements in the production of the final good). Sensitivity analysis shows that the results are materially unaffected by the exact choice of $\rho$ as long as tasks are strong complements.

The fraction of “high-educated” workers, $E$, is calibrated as follows. Calculating the

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16The breakdown into industries is limited by the availability of “Non-farm Proprietors’ Income” by industry, which must be considered when computing the labour share for service occupations, where self-employment is more important than for manufacturing occupations.
fraction of the labour force with at least “some college” education is straightforward from the Census data. However, an educated worker may switch back and forth between college and non-college occupations in the model. Hence, that fraction does not appear to be the empirical counterpart to $E$. For consistency with the model, I therefore count all workers in high skill occupations as “high educated” irrespective of their educational background. Furthermore, all workers with college education who work in low skill occupations under the age of 30 are also counted as “high educated” since the model allows individuals with high education to be employed in high skill occupations regardless of their current employment. In the data, mostly younger workers sample low-skill occupations despite their high education; such workers search heavily for the best match, as is evident by the fraction of the high-educated employed in “high degree” occupations increasing until about age 30 and remaining constant almost until the end of the work-life. This is depicted in Figure 1.3.1. Assuming that older workers with a college education employed in a lower skill job no longer possess the qualifications for employment in a college occupation, I only include young highly educated workers employed in lower skill occupations. This results in $E = 36.7$.

1.4.2 The Experiment

In evaluating the trade equilibrium, I compute three hypothetical scenarios of trade in tasks. Since this trade is still in its early stages, it is difficult to predict the exact pattern of trade, i.e. the importing occupations’ and the exporting occupations’ terms of trade. Scenarios 1 and 3 differ with respect to the relative size of the four tradable services occupations; scenarios 1 and 2 vary with respect to the terms of trade. The scenarios are:

1. The U.S. imports and exports both high and low skill service tasks equally. For both skill groups, the autarky employment in tradable occupations is equally split between imported and exported tasks. The world market price is (on average)
20% lower for imported tasks than the domestic autarky price and 20% higher for exported tasks.

2. As scenario 1, except that the world market price is 30% lower than the domestic autarky price for imported tasks and 30% higher for exported tasks.

3. The U.S. comparative advantage is biased against high skill tasks: the autarky employment in inshored high skill occupations makes up only 30% of the total employment in tradable high skill occupations, while 70% of the workers are employed in offshored high-skill occupations. The shares are reversed for low skill occupations. As in scenario 1, the world market price is 20% lower than the domestic autarky price for imported tasks and 20% higher for exported tasks.

For all three scenarios, I assume that trade is introduced to its full extent at once and not gradually. While this assumption is not necessarily particularly realistic, it maximizes the short run adjustment cost and thus presents a useful worst case scenario. Were trade introduced very gradually, none or only few permanent workers would switch occupations and so no destruction of human capital would occur, which implies that there would be no short term distributional effects.

1.4.3 Results

Steady State Comparison

Compared to the autarky steady state, the new stationary equilibrium sees welfare (output of the final consumption good) increase in all three scenarios, as shown in Table 1.4.1. Not surprisingly, the increase is most pronounced (4.03%) in scenario 2, when the differences between autarky and trade relative prices are largest. In scenario 1, the welfare gain is 2.02%, while in scenario 3 the gain is 1.82%. The difference in outcomes between scenarios 1 and 3 can be explained by the fact that employment in occupations with high specific human capital is higher in scenario 1. As a result, the effective labour supply is higher, which causes a higher output of the comparative advantage task. Nevertheless,
the terms of trade are of first order importance from an aggregate standpoint; whether or not the offshored tasks are high or low skill is secondary.

While the terms of trade are more crucial for aggregate welfare than the economy’s particular comparative advantage occupation, the opposite is true for the distribution of the gains from trade. The directed search mechanism assures that all ex-ante identical agents benefit equally from trade in steady state. However, the gains from trade are not equally distributed across education groups, as is evident from the third and fourth rows of Table 1.4.1. If the economy has a comparative advantage in low-skill occupations (scenario 3), almost all gains from trade are reaped by the non-educated; in the more balanced case (scenario 1), the educated gain slightly more. The value of entering the labour force (the value of searching, in the context of this model) as an educated worker relative to entering the workforce while non-educated (the education premium) falls from 1.41 in autarky to 1.37 in scenario 3. In scenarios 1 and 2, where the comparative advantage is more balanced between high and low skill occupations, the education premium increases slightly to 1.419 and 1.425 respectively.

The distributional effect of trade is a result of the occupational mobility restriction for non-educated workers in the model – the non-educated cannot be employed in high education occupations, but educated workers may work in any occupation. In other words, educated workers have a comparative advantage in working in high skill occupations, or alternatively, non-educated workers are like a specific factor. As a result, college-educated workers are able to attain an education premium in autarky. However, in scenario 3, they are exposed to strong import-competition, while the non-educated see the value of their specific factor increase. It is important to point out that the number of educated workers remains constant – if agents had the choice of becoming educated at some cost, the number of educated workers would fall in scenario 3 and increase in scenarios 1 and 2, attenuating the education premium towards its autarky value.
The Transition Path

Figures 1.4.2 - 1.4.4 display the time path of aggregate output. In scenarios 1 and 2, output initially remains almost constant and then increases quickly – within 3 years, output is close to the equilibrium value. However, output then overshoots the new steady state level, staying noticeably above this level for a period of over ten years. Interestingly, the rapid increase in output and the prolonged overshooting together cause the welfare gain including transition path to be the same as the steady state gain – 2.02% steady state gain and 2.08% including the transition path for scenario 1. In scenario 3, there is no overshooting; output jumps by about 1% in the first year and after a period of rapid growth converges to the new equilibrium value.

To better understand the time dynamics of aggregate output, it is instructive to investigate the reallocation of workers first. Inspecting the time path of wages (Figure 1.4.6) for scenario 1,\(^\text{17}\) one can see that the initial response mirrors that of a specific factors model: the wage rate per unit of labour (the value of the marginal product) in the inshored occupation increases by about 4.5%, while the wage rate in the offshored occupation falls by about 21% – at the autarky reservation productivity levels, the indifference conditions on relative wages (1.30) is violated.\(^\text{18}\) This triggers a reallocations process: the value of applying to the inshored occupations exceeds the value of applying to the offshored occupations, which implies that the offshored occupations do not attract any applicants. Furthermore, because of the shift in relative wages, the value of remaining permanently in the offshored occupation is now lower than the value of searching for a worker with a low specific productivity draw. The reservation productivity in offshored occupations increases and most unskilled and even some skilled workers leave the offshored occupations for the inshored ones. This causes an increase in unemployment,

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\(^{17}\)The paths for the other scenarios are similar, so only scenario 1 is discussed in detail.

\(^{18}\)The wage in the offshored occupation need not fall; if the comparative advantage is strong, the wage could potentially increase. However, it will always be lower than the trade steady state real wage.
an increase in average worker productivity in the offshored occupations, and a decrease in average worker productivity in the inshored occupations. Note that this implies that the average income in these occupations changes less than the wage per effective unit of labour.

After the first period, some of the former applicants to inshored occupations become permanent workers and increase the effective labour supply in those occupations, which lowers the wage rate per unit of labour. The labour supply in the offshored occupations is further reduced through exogenous separation, which somewhat increases the wage rate. Thus the value of searching relative to the value of remaining permanently in offshored occupations decreases; permanent workers (skilled and unskilled) only leave their occupation in the first period after the negative shock. However, the value of applying to the inshored occupations still exceeds that for the offshored; just as in the first period, only the inshored occupations receive applicants in the second period. Over time, the effective labour force in the offshored occupations is further reduced through exogenous separation, while it keeps growing in the inshored – both through entry and acquisition of specific human capital. Eventually, the value of applying to all occupations is equalized again and both receive a positive number of applicants. The gains from trade are now equally distributed across occupations.

The evolution of the permanent workers’ values in either occupation, presented in Figure 1.4.7, is similar to that of wages; the value overshoots the steady state value for the exported task and undershoots for the imported task. However, since the value function captures all discounted future wages, the deviation from the steady state value is much smaller than for wages.\textsuperscript{19} With trade, workers who are already employed in the inshored occupations are better off unambiguously. On the other hand, skilled workers in the offshored occupations may see their value rise or fall, depending on the loss in wages.

\textsuperscript{19}This also stresses the need for a dynamic model – judging the impact of increased trade based on wage levels as in a static model overstates its impact drastically.
and the length of the transition path. For unskilled workers in the offshored occupation, the value similarly depends on the loss in wages and the length of the transition path, but also on their position in the productivity distribution. In the autarky equilibrium, a worker with a productivity shock equal to the autarky reservation level is indifferent between quitting and staying in the occupation. In the first period after the economy opens up to trade, the value of searching increases and the worker is better off. On the other hand, the worker with the highest possible productivity level sees her value decrease, just like a skilled worker.

Figure 1.4.7 shows the path of the value of being a skilled worker with a productivity draw at the 67th percentile for scenario 1; the time paths for the other productivity levels follow the same pattern. In the first year, the value of having human capital specific to the offshored occupation falls about 3% relative to its autarky value, while the value of having human capital specific to the inshored occupation increases by the same amount. The latter quickly converges to its steady state value, while the former recovers to its autarky level after 8 years and converges to the trade steady state after about 15 years. Figure 1.4.7 best exemplifies the conflict between long term gains and short term losses, while also stressing the importance of the specificity of human capital. If all human capital were general, the transition to the new equilibrium would be instantaneous and there would be no short term losses.

Returning to the dynamics of output, the initial increase in output is a result of the economy taking advantage of its comparative advantage paired with the reallocation of workers from the offshored to the inshored occupation. All but the most productive unskilled and also some of the less productive high skill workers leave the offshored occupation and apply to the inshored occupation, as discussed above. However, since some of these workers do not receive an offer in the first year and others receive a low productivity draw, aggregate output in the first period is only slightly increased – the effective labour force employed in the economy in the first year is smaller than in autarky.
Yet, at world relative prices, the value of output is higher and aggregate output does not fall. By the end of the third year, most workers who switched receive a productivity draw above the reservation level, i.e. they find a good occupation match in one of the inshored occupations, and output increases significantly.

At the same time, the average productivity of the workers who remain in the offshored occupation is very high, as only skilled and unskilled workers with a high productivity shock are left. This causes aggregate output to overshoot in scenarios 1 and 2: after three periods, the effective labour force in the inshored occupation is markedly increased, while it still remains relatively high in the offshored occupation because of the high average productivity. In all scenarios, two opposing forces affect aggregate output in the first period: the positive comparative advantage effect and the negative reallocation effect. Since the former is stronger in scenario 3 than in scenarios 1 and 2, output already increases significantly (by 1%) in the first period. Furthermore, because of the strong comparative advantage effect, output does not overshoot: the high productivity workers who separate from the offshored occupation over time do not see the value of their product decrease because the price of their output increases more than their productivity falls.

1.4.4 The Impact on the Wage Distribution

The distribution of wages is affected through three channels as wage dispersion in the model comes from three sources: agents vary by their education, their acquired specific skill and their match-specific productivity draw. In the short run, inequality will be reduced within offshored occupations and increased within the inshored occupations. Only permanent workers remain in the offshored occupations, eliminating the left tail of the productivity distribution. Furthermore, the increase in the productivity cut-off further truncates the distribution and also eliminates more unskilled than skilled workers. In other words, only good matches and mostly workers with high specific human capital remain in the offshored occupation.
Chapter 1. Offshoring and Human Capital

The inshored occupation will attract more applicants, i.e. more workers in their first year of tenure. Since workers in their first year of occupational tenure can have productivity shocks below the reservation level as well as above, both sources of within-inequality are amplified in the short term.

In the long term, however, the reservation productivity level is unchanged, and so is the relative number of first year, permanent unskilled to skilled workers within an occupation. Consequently, offshoring does not affect residual inequality within an occupation. In the long term, changes to the wage distribution can only stem from changes in the occupational composition of the economy. However, changes to the occupational composition can cause changes to the education premium. If trade is biased against high skill occupations, as in scenario 3, demand for college graduates falls, which lowers the college premium. While the competitive forces of the labour market assure that ex-ante identical workers gain equally from trade, different agents may gain differently. College educated workers have a comparative advantage in high skill occupations. If the tasks that are being offshored are produced in these occupations, college educated workers gain relatively less from trade. However, even under scenario 3, in which trade is strongly biased against high skill occupations, college graduates still gain from increased offshoring.

1.4.5 Labour Market Frictions

Finally, I conduct an experiment to investigate the potential role of flexible labour market institutions in the transition from an autarky to a trade equilibrium. In the model, labour market frictions are captured by \( \epsilon \), the probability of receiving an offer if searching in the current period. Also, one can think of \( \pi \), the arrival rate for exogenous separation, as capturing labour market institutions such as the imposition of firing costs. For this experiment, I increase \( \epsilon \) from 0.2 to 0.3, thus increasing the expected length of unemployment to 22.3 weeks. Also, I reduce \( \pi \) to 0.050 (from 0.079) which implies an average tenure of 21 years at the time of separation. I also recalibrate the task
productivity parameters $z$; all other parameters are kept unchanged to focus on the impact of labour market institutions. Together, these changes leave the steady state gains from trade almost unchanged – in steady state, the gains under scenario 1 represent a 2.07% increase in aggregate output.

The importance of strong labour market institutions for the transition can best be demonstrated by comparing the path of aggregate output to that generated in the initial experiment (Figure 1.4.8). First, output falls upon impact in the first period due to the lower job finding rate; i.e. a larger number of workers who choose to leave their occupations in response to the trade shock do not receive another job offer, thus becoming unemployed. This also causes output growth to slow down: in the economy with frictions, output takes 7 years to reach the steady state level (as opposed to 3 in the calibrated economy). The output growth is further slowed by the lower exogenous separation – a worker who decided not to quit in the first period will remain in the offshored occupation until her occupation-match is destroyed. As a consequence, these workers remain in the offshored occupation for a longer period of time.

Together, the lower job-finding rate and the lower separation rate have a noticeable impact on the transition and hence on welfare. In this simple experiment, the steady state increase in aggregate output is 2.07%, but the total welfare gain decreases to 1.79% after taking the transition path into account. This stands in contrast with the calibrated model with fewer labour market frictions in which the welfare improvement including the transition path actually exceeded the steady state gains. Although the difference is not staggering, it is larger than the difference between the steady state welfare gains for scenarios 1 and 3.

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20 One could argue that such an environment is likely to produce higher levels of specific human capital (e.g. Wasmer, 2006); an exercise such as calibrating the model to data from continental Europe is left for future research.
1.5 Conclusion

This paper develops a model of trade in tasks in which occupation-specific human capital plays a pivotal role in determining the transition path after the country begins offshoring tasks. Using this model, I demonstrate that the characteristics of the traded tasks are of secondary importance for the magnitude of the gains from trade – the key determinant of the gains from trade is the difference between the relative prices under autarky and (free) trade, not the skill content of the traded tasks. As in other models of trade, the more different trading partners are, the larger the gains from trade. The distribution of the gains from trade critically depends on the time horizon: in the short term, workers with human capital specific to the inshored occupation gain, while workers with human capital specific to the offshored occupation lose. In the long run, when the distribution of specific human capital is endogenous, the gains from trade are equally distributed among identical agents. Agents with different characteristics, e.g. ability to attend college, may gain differently if trade is biased against high or low skills.
Table 1.2.1: Educational Attainment, by Major Occupation Group

<table>
<thead>
<tr>
<th>Occupation Group</th>
<th>High School Dropout</th>
<th>High School Graduate</th>
<th>Some College</th>
<th>College Graduate</th>
<th>Low Education</th>
<th>High Education</th>
<th>Fraction High Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executive, Administrative, and Managerial Occupations (3-37)</td>
<td>579,082</td>
<td>2,343,299</td>
<td>5,056,435</td>
<td>8,694,677</td>
<td>2,922,381</td>
<td>13,751,112</td>
<td>82.5%</td>
</tr>
<tr>
<td>Professional Specialty Occupations (43-199)</td>
<td>409,595</td>
<td>1,292,446</td>
<td>4,827,643</td>
<td>15,170,575</td>
<td>1,702,041</td>
<td>19,998,218</td>
<td>92.2%</td>
</tr>
<tr>
<td>Technicians and Related Support Occupinations (203-235)</td>
<td>133,112</td>
<td>648,787</td>
<td>2,207,967</td>
<td>1,898,326</td>
<td>781,899</td>
<td>4,106,293</td>
<td>84.0%</td>
</tr>
<tr>
<td>Sales Occupations (243-285)</td>
<td>2,468,504</td>
<td>4,170,928</td>
<td>5,390,719</td>
<td>3,484,833</td>
<td>6,639,432</td>
<td>8,875,532</td>
<td>57.2%</td>
</tr>
<tr>
<td>Administrative Support Occupations, Including Clerical (303-389)</td>
<td>1,867,375</td>
<td>6,979,612</td>
<td>9,712,922</td>
<td>3,208,817</td>
<td>8,846,987</td>
<td>12,921,739</td>
<td>59.4%</td>
</tr>
<tr>
<td>Service Occupations (403-469)</td>
<td>5,417,840</td>
<td>6,330,702</td>
<td>6,091,753</td>
<td>1,668,941</td>
<td>11,748,542</td>
<td>7,760,694</td>
<td>39.8%</td>
</tr>
<tr>
<td>Craft and Repair Occupations (503-599)</td>
<td>2,512,391</td>
<td>4,455,501</td>
<td>3,510,202</td>
<td>597,736</td>
<td>6,967,892</td>
<td>4,107,938</td>
<td>37.1%</td>
</tr>
<tr>
<td>Production Occupations (603-799)</td>
<td>3,106,721</td>
<td>5,098,406</td>
<td>3,118,524</td>
<td>693,657</td>
<td>8,205,127</td>
<td>3,812,181</td>
<td>31.7%</td>
</tr>
<tr>
<td>Transportation Occupations, Helpers, and Labourers (803-889)</td>
<td>3,108,839</td>
<td>4,333,972</td>
<td>2,387,541</td>
<td>419,955</td>
<td>7,442,811</td>
<td>2,807,496</td>
<td>27.4%</td>
</tr>
</tbody>
</table>

Numbers in brackets are corresponding 1990 census occupation classification codes. Source: U.S. Census 2000, 5% sample.
<table>
<thead>
<tr>
<th>Major Occupation Group</th>
<th>Total Employment</th>
<th>Employment in Tradable Occupations</th>
<th>Fraction Tradable</th>
<th>Fraction of Total Tradable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executive, Administrative, and Managerial Occupations (3-37)</td>
<td>16,673,493</td>
<td>3,433,562</td>
<td>20.6%</td>
<td>11.6%</td>
</tr>
<tr>
<td>Professional Specialty Occupations (43-199)</td>
<td>21,700,259</td>
<td>3,970,147</td>
<td>18.3%</td>
<td>13.4%</td>
</tr>
<tr>
<td>Technicians and Related Support Occupations (203-235)</td>
<td>4,888,192</td>
<td>3,432,522</td>
<td>70.2%</td>
<td>11.6%</td>
</tr>
<tr>
<td>Sales Occupations (243-285)</td>
<td>15,514,984</td>
<td>0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Administrative Support Occupations, Including Clerical (303-389)</td>
<td>21,768,726</td>
<td>9,410,639</td>
<td>43.2%</td>
<td>31.8%</td>
</tr>
<tr>
<td>Service Occupations (403-469)</td>
<td>19,509,236</td>
<td>0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Craft and Repair Occupations (503-599)</td>
<td>11,075,830</td>
<td>0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Production Occupations (603-799)</td>
<td>12,017,308</td>
<td>9,329,378</td>
<td>77.6%</td>
<td>31.5%</td>
</tr>
<tr>
<td>Transportation Occupations, Helpers, and Labourers (803-889)</td>
<td>10,250,307</td>
<td>0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>133,398,335</td>
<td>29,576,248</td>
<td>22.2%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Numbers in brackets are corresponding 1990 census occupation classification codes. Source: U.S. Census 2000, 5% sample.
### Table 1.2.3: Earnings Functions Estimation, IV

<table>
<thead>
<tr>
<th>Table 1.2.3: Earnings Functions Estimation, IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Executive Professional Technical All High Skill Post Secondary Manufacturing Craft Administrative</td>
</tr>
<tr>
<td>Occupations Occupations Occupations Occupations Occupations Degree Occupations Occupations Occupations</td>
</tr>
<tr>
<td>Job Tenure -0.00436951 0.0055696 0.01277829 -0.01793417 -0.00114156 -0.01036377 0.00695998 -0.00431984 -0.01679791</td>
</tr>
<tr>
<td>[0.002100472]* [0.00625729] [0.00315095] [0.01081826] [0.00426079] [0.004059372] [0.004074787] [0.003950654] [0.00939503]</td>
</tr>
<tr>
<td>Job Tenure2 -0.00436951 0.0055696 0.01277829 -0.01793417 -0.00114156 -0.01036377 0.00695998 -0.00431984 -0.01679791</td>
</tr>
<tr>
<td>[0.002100472]* [0.00625729] [0.00315095] [0.01081826] [0.00426079] [0.004059372] [0.004074787] [0.003950654] [0.00939503]</td>
</tr>
<tr>
<td>Job Tenure3 -3.081E-06 3.337E-05 1.050E-05 -4.561E-05 0.00107554 -0.00079853 -0.00036829 -0.00050148 -0.000241</td>
</tr>
<tr>
<td>[0.000003821] [0.000011641] [0.000016193] [0.000021766] [0.000382462] [0.000376878] [0.000314193] [0.000343842] [0.000774614]</td>
</tr>
<tr>
<td>Occupation Tenure 0.0135746 0.00720406 0.02581094 0.04790938 0.01764079 0.0288128 0.01762865 0.01030009 0.06342592</td>
</tr>
<tr>
<td>[0.003325959]** [0.010808798] [0.013844037] [0.019524868] [0.007007010]** [0.006799191]** [0.006986100]** [0.007649082] [0.016574425]**</td>
</tr>
<tr>
<td>Occupation Tenure2 -0.00075059 0.00148627 -0.000307914 -0.00034473 -0.00010339 -0.00012528 -0.00007533 -0.00056127</td>
</tr>
<tr>
<td>[0.000280883]** [0.000902756] [0.001312258] [0.001886455] [0.000621028] [0.000646115]* [0.000653421]* [0.000616164] [0.016574425]**</td>
</tr>
<tr>
<td>Occupation Tenure3 1.2287E-05 -0.0000544 1.872E-05 5.672E-05 -9.392E-06 2.706E-05 0.00002334 0.00002654 0.00013825</td>
</tr>
<tr>
<td>[0.000005769] [0.000019083] [0.000031703] [0.000046874] [0.000013931]** [0.000014975] [0.000014875] [0.000014167] [0.000041667]**</td>
</tr>
<tr>
<td>Industry Tenure 0.0057495 0.00321059 0.00135716 0.00449829 0.00639080 0.006970862 0.007500689 0.008097873** [0.01755053]</td>
</tr>
<tr>
<td>[0.0003437900] [0.001186544] [0.000317516] [0.00024474] [0.00046874] [0.000013931]** [0.000014975] [0.000014167] [0.000041667]**</td>
</tr>
<tr>
<td>Industry Tenure2 -4.0109E-05 -0.00131386 0.00069363 0.0003767 0.0010238 0.00071408 4.939E-05 0.00023043</td>
</tr>
<tr>
<td>[0.0000027298] [0.000001303] [0.000019083] [0.000031703] [0.000046874] [0.000013931]** [0.000014975] [0.000014167] [0.000041667]**</td>
</tr>
<tr>
<td>[0.00000653]** [0.000006396] [0.000046874] [0.000013931]** [0.000014975] [0.000014167] [0.000014875] [0.000014975] [0.000041667]**</td>
</tr>
<tr>
<td>Potential Experience 0.04765768 0.05898373 0.04444721 0.04404036 0.06285707 0.05845773 0.03829994 0.0452217 0.0397313</td>
</tr>
<tr>
<td>[0.001683287]** [0.000624186]** [0.000653209]** [0.000434083]** [0.000567785]** [0.000654381]** [0.000344443]** [0.000375537]** [0.000618692]**</td>
</tr>
<tr>
<td>Potential Experience2 -0.00147254 0.00019881 0.00232992 0.00146757 -0.00220411 -0.00220411 -0.00103275 -0.00157327 -0.00109293</td>
</tr>
<tr>
<td>[0.000009560]** [0.000377403]** [0.00041886] [0.00001291] [0.000014975] [0.000014167] [0.000014975] [0.000014975] [0.000041667]**</td>
</tr>
<tr>
<td>Potential Experience3 1.6769E-05 2.473E-05 3.514E-05 1.589E-05 2.476E-05 2.413E-05 1.593E-05 1.784E-05 1.007E-05</td>
</tr>
<tr>
<td>[0.000001352]** [0.000006132]** [0.000007878]** [0.000001075] [0.00000859] [0.00000859] [0.000001075] [0.00000269] [0.000013382]**</td>
</tr>
<tr>
<td>Observations 168345 23026 16007 5452 44485 27832 25673 9366</td>
</tr>
<tr>
<td>Number of IDs 29771 4569 3298 1172 8605 5438 5533 2235</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

* Denotes statistical significance at 5%, ** significant at 1%
Table 1.2.4: Returns to Occupational Tenure, by Occupation Group

<table>
<thead>
<tr>
<th>Years in Occupation</th>
<th>All Occupations</th>
<th>College Graduates</th>
<th>Executive</th>
<th>Professional</th>
<th>Technical</th>
<th>All</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[a]</td>
<td>[b]</td>
<td>[c]</td>
<td>[d]</td>
<td>[e]</td>
<td>[f]</td>
<td>[g]</td>
</tr>
<tr>
<td>2 years</td>
<td>0.0242***</td>
<td>0.0514***</td>
<td>0.0200</td>
<td>0.0448*</td>
<td>0.0867***</td>
<td>0.0338***</td>
<td>0.0304**</td>
</tr>
<tr>
<td></td>
<td>(0.0056)</td>
<td>(0.0113)</td>
<td>(0.0184)</td>
<td>(0.0231)</td>
<td>(0.0330)</td>
<td>(0.0118)</td>
<td>(0.0120)</td>
</tr>
<tr>
<td>5 years</td>
<td>0.0506***</td>
<td>0.1074***</td>
<td>0.0664*</td>
<td>0.0879**</td>
<td>0.1884***</td>
<td>0.0784***</td>
<td>0.0597**</td>
</tr>
<tr>
<td></td>
<td>(0.0108)</td>
<td>(0.0208)</td>
<td>(0.0358)</td>
<td>(0.0433)</td>
<td>(0.0643)</td>
<td>(0.0224)</td>
<td>(0.0237)</td>
</tr>
<tr>
<td>10 years</td>
<td>0.0723***</td>
<td>0.1548***</td>
<td>0.1663***</td>
<td>0.1029*</td>
<td>0.3031***</td>
<td>0.1325***</td>
<td>0.0743**</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0243)</td>
<td>(0.0467)</td>
<td>(0.0540)</td>
<td>(0.0941)</td>
<td>(0.0281)</td>
<td>(0.0316)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

* Denotes statistical significance at 10%, ** at 5%, and *** at 1%.
Table 1.2.5: Returns to (Potential) Experience, by Occupation Group

<table>
<thead>
<tr>
<th>Years of Experience</th>
<th>All College Graduates</th>
<th>Executive</th>
<th>Professional</th>
<th>Technical</th>
<th>All</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All occupations</td>
<td>[a]</td>
<td>[b]</td>
<td>[c]</td>
<td>[d]</td>
<td>[e]</td>
</tr>
<tr>
<td>2 years</td>
<td>0.0896</td>
<td>0.1091</td>
<td>0.1103</td>
<td>0.1199</td>
<td>0.0751</td>
<td>0.1177</td>
</tr>
<tr>
<td></td>
<td>(0.0030)</td>
<td>(0.0065)</td>
<td>(0.0123)</td>
<td>(0.0124)</td>
<td>(0.0187)</td>
<td>(0.0091)</td>
</tr>
<tr>
<td>5 years</td>
<td>0.2036</td>
<td>0.2451</td>
<td>0.2488</td>
<td>0.2684</td>
<td>0.1673</td>
<td>0.2662</td>
</tr>
<tr>
<td></td>
<td>(0.0064)</td>
<td>(0.0135)</td>
<td>(0.0262)</td>
<td>(0.0259)</td>
<td>(0.0397)</td>
<td>(0.0194)</td>
</tr>
<tr>
<td>10 years</td>
<td>0.3461</td>
<td>0.4079</td>
<td>0.4177</td>
<td>0.4470</td>
<td>0.2732</td>
<td>0.4489</td>
</tr>
<tr>
<td></td>
<td>(0.0095)</td>
<td>(0.0196)</td>
<td>(0.0399)</td>
<td>(0.0385)</td>
<td>(0.0609)</td>
<td>(0.0293)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

All statistically significant at 1%.
Table 1.4.1: Responses to Offshoring: Steady State

<table>
<thead>
<tr>
<th></th>
<th>Autarky</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1</td>
<td>1.0202</td>
<td>1.0403</td>
<td>1.0182</td>
</tr>
<tr>
<td>Unemployment</td>
<td>2.97%</td>
<td>2.97%</td>
<td>2.97%</td>
<td>2.95%</td>
</tr>
<tr>
<td>$\bar{U}^{E,\text{Trade}}/\bar{U}^{E,\text{Autarky}}$</td>
<td>1.0236</td>
<td>1.0462</td>
<td>1.0032</td>
<td></td>
</tr>
<tr>
<td>$\bar{U}^{\text{Trade}}/\bar{U}^{\text{Autarky}}$</td>
<td></td>
<td>1.0172</td>
<td>1.0355</td>
<td>1.0319</td>
</tr>
<tr>
<td>$\bar{U}^{E}/\bar{U}$ (“College Premium”)</td>
<td>1.4100</td>
<td>1.4189</td>
<td>1.4247</td>
<td>1.3708</td>
</tr>
</tbody>
</table>
Figure 1.3.1: Fraction of Educated Working in College Occupations
Figure 1.3.2: The Problem of an Uneducated, Unskilled Worker
Figure 1.3.3: The Problem of an Educated, Unskilled Worker
Figure 1.4.1: Distribution of Tenure in Occupation
Figure 1.4.2: Transition Path of Final Output, Scenario 1
Figure 1.4.3: Transition Path of Final Output, Scenario 2
Figure 1.4.4: Transition Path of Final Output, Scenario 3
Figure 1.4.5: Transition Path of $U^E$, Scenario 1
Figure 1.4.6: Transition Path of Wages, Scenario 1
Figure 1.4.7: Transition Path of $V^S$, 67$^{th}$ percentile, Scenario 1
Figure 1.4.8: Transition Path of Final Output, Economy with Labour Market Frictions
Chapter 2

Trade and Inequality: A Directed Search Model with Firm and Worker Heterogeneity

2.1 Introduction

The link between international trade and income inequality has garnered much attention; a large empirical literature points to a (small) positive connection between these two developments. Concurrently, an extensive literature has studied firm dynamics in the context of international trade, documenting that exporting firms are typically more productive and employ higher skilled workers than non-exporters.\(^1\) This paper integrates these insights from the heterogeneous firms literature with a model of a frictional labour market. The presented model generates both across- and within-group wage inequality and is able to explain the findings from the empirical literature on trade and inequality.

In the model, trade liberalization causes a within-industry reallocation of workers to

\(^1\)See Bernard et al. (2007a).
higher productivity (exporting) firms and increases the relative demand for high-skilled workers, which has an increasing effect on the skill premium. At the same time, trade causes relative demand to be shifted towards the skill group that is more intensively demanded in the comparative advantage industries, which can have a positive or negative effect on the skill premium (the Stolper–Samuelson effect); the total effect therefore can be ambiguous. The model also makes predictions about within-group wage inequality – depending on the exact pattern of trade, within-inequality may increase or decrease and it also may move in opposite directions for different skill groups. Finally, this paper shows how a calibrated version of the model can be used to identify the consequences of an actual trade reform; the calibrated model can account for much of the effect of the Canada-U.S. Free Trade Agreement on the Canadian labour market.

Over the past three decades, international trade and income inequality both have increased significantly, in developed and developing countries alike. While it is challenging to establish a causal link between the two developments, most empirical evidence is suggestive of a positive relationship. However, the predictions of the standard workhorse model in trade theory, the Heckscher-Ohlin model, are inconsistent with these findings. The model can explain the increase in the skill premium in the skill-abundant country, but it predicts a fall in the skill premium in the low-skill-abundant country. A variation of the Ricardian model with trade in intermediate goods proposed by Feenstra and Hanson (1996) and extended by Zhu and Trefler (2005) can explain an increase in the skill premium for both countries: if the intermediate good the skill-rich country imports is relatively low skill intensive at home, but relatively high skill intensive in the foreign country, the skill premium can increase in both economies. However, while this model

\footnote{See Feenstra and Hanson (2003) for a U.S.-focused study and Goldberg and Pavcnik (2007) for a survey of developing countries.}

\footnote{Other approaches include Epifani and Gancia (2008), who show that if returns to scale are stronger in the skill-intensive sector and elasticity of substitution in consumption greater than one, trade can increase the skill premium in all countries. Burstein and Vogel (2008) incorporate skilled and unskilled labour in a multi-sector version of Eaton and Kortum’s (2002) Ricardian model of international trade. They find that trade liberalization increases inequality in all countries if technology is skill-biased.}
explains an increase in the skill premium, it has little to say about inequality among workers of the same skill group.

In this paper, I propose an open economy model with search frictions in the labour market that can address both within- and between-group inequality and is consistent with the empirical facts. Building on Shi (2002, 2005) and Shimer (2005), each intermediate good in the economy is produced by many firms which vary in their technology, and workers differ by their level of skill. Skills and technology are complementary: skilled workers have a comparative advantage with the high technology firms, generating a skill premium. Firms post wages to attract workers who observe the posted wages and then select a firm to which to apply. Workers can only apply to one vacancy at a time and cannot coordinate amongst each other; such search friction generates equilibrium unemployment. The competitive nature of the labour market requires that all vacancies offer workers the same expected utility. When workers apply, they trade off the probability of being chosen for a position and the wage they would receive should they are hired. Firms take this trade-off as given when they post their wage offers. More productive firms have a stronger incentive to fill their vacancies and hence try to attract more applicants by posting higher wages. Differences in firm productivity thus translate into residual wage inequality.

In order to export, firms have to incur a fixed cost. As a result, only the most productive firms engage in exporting (similarly to Melitz, 2003). Increasing openness has an effect on the allocation of resources (workers) across sectors and within sectors, as in Bernard et al. (2007a). As in a traditional trade model, the import-competing industries shrink while the exporting industries expand. Depending on the relative skill intensities, the sectorial reallocation of labour may increase or decrease both types of inequality. Within the exporting industries, firms that export increase their demand for high-skilled workers. Firms in the middle of the productivity distribution now have a lower probability of attracting high-skilled workers and begin attracting more low-skilled
workers, while firms in the bottom of the productivity distribution will cut employment. Since there are no fixed costs, these firms will not shut down but rather hire with a lower probability. The within-industry reallocation effect increases the skill premium and increases wage dispersion for low-skilled workers and reduces it for high-skilled workers. Finally, the model also highlights an important general equilibrium effect that is often ignored in empirical assessments of trade reforms. Exploiting the countries’ comparative advantages increases aggregate output and hence demand for the non-traded goods and services. Since the non-traded sector tends to be large, this will cause a reallocation of resources out of traded industries and into non-traded industries. As a consequence, overall employment in traded industries (importing and exporting combined) falls.

The calibrated model can account for much of the effect of the Canada-U.S. Free-Trade Agreement (FTA). A variety of existing empirical work concludes that the labour market effects of the FTA were small, with the noticeable exception that employment in manufacturing decreased by as much as 5%.\(^4\) Similarly, the calibrated model in this paper predicts a drop in manufacturing employment by 3.3% and little effect on inequality. The second outcome of the FTA is a significant increase in industry productivity. The calibrated model predicts a similar increase in the average firm-level productivity. However, due to a simplifying modeling assumption discussed below, the model predicts a neglectable increase in average industry output per worker.

This paper is closely related to the recent literature that incorporates search models of the labour market into trade models in order to study the link between inequality and increased trade.\(^5\) Davidson et al. (2008a) suggest a model in which heterogeneous firms and workers match in a frictional labour-market as in Albrecht and Vroman (2002). Opening the economy to international trade causes an expansion of high-tech firms which


\(^5\)This literature goes back to Davidson et al. (1999).
in turn increases the skill premium and may lower residual inequality because high skilled workers could cease seeking employment at low-tech firms under certain conditions. Helpman et al. (2009) develop a model in which ex-ante identical workers are matched with heterogenous firms. The search frictions combined with firm heterogeneity generate wage dispersion. Opening the economy to trade increases income inequality, but further increases in openness can either increase or reduce income inequality. Lastly, Costinot and Vogel (2008) present a model with positive assortative matching between workers with different skills and tasks with different skill intensities and derive comparative statics exercises for a variety of trade patterns.\footnote{Other approaches to studying the link between international trade and wage inequality include Egger and Kreickemeier (2009), who find that trade increases wage dispersion among identical workers in a fair wage model; Davis and Harrigan (2007) and Amiti and Davis (2008), who find similar results using an efficiency wages approach.}

The chapter consists of three sections. The first section lays out the static version of the model and presents a numerical example that highlights the key feature of the model. Section 2.3 extends the static model to a dynamic environment and section 2.4 provides an illustrative example of how the model can be used to study trade liberalization in the form of the Canada-U.S. Free Trade Agreement. Section 2.5 concludes with a discussion and provides an outline of future research.

2.2 Static Model

2.2.1 Environment

The model in this section is a straightforward extension of the directed search framework in Shi (2002) and Shimer (2005). The labour force consists of a large number of risk-neutral workers; the total size is normalized to 1. A fraction $h$ of these workers is high-skilled ($H$) and a fraction $(1 - h)$ low-skilled ($L$). A final consumption good $C$ is a composite good of $N$ intermediate goods (industries), $c_j$, where the subscript $j$ denotes
industry $j$. For simplicity, assume that all intermediate goods are perfect complements, so that

$$C = \min\{\sigma_1c_1, ..., \sigma_Nc_N\},$$

where $\sigma_j$ is the weight of good $j$ in the aggregation.

The economy contains a large number of potential firms, $n(j)$ of which are active in industry $j$. A firm can become active by paying a fixed cost $\kappa^e$, measured in terms of the final good. After paying the fixed cost, the firm draws its productivity, $s \geq 1$, from some distribution $F(s)$. In order to produce, a firm must hire a worker. The technology is skill-biased: if a firm hires a low-skilled worker, its output is $z_j$, independent of the firm’s productivity. However, if a firm hires a high-skilled worker, it produces $sz_j$:

$$y(s, j, i) = \begin{cases} 
    sz_j & \text{if } i = H \\
    z_j & \text{if } i = L 
\end{cases}$$

The labour market is frictional in the sense that firms and workers cannot coordinate their actions. Instead, firms and workers play the following three-stage game. After entering and observing their productivity, all firms post their offered wage for each skill level and their selection rule in case multiple workers apply to the vacancy. All firms post within the same labour market; workers are not attached to any industry and are free to apply to any firm. Then, individual workers observe the posted wages and make their application decisions. Lastly, firms select a worker out of the pool of applicants in accordance with the announced selection rule and begin producing output. Unmatched firms and workers produce 0.

### 2.2.2 Worker’s Problem

Workers cannot coordinate among each other as to which worker applies to which vacancy. Following the literature, I impose that identical workers use identical application strategies and focus on symmetric, mixed strategy equilibria. Workers can condition their
application probabilities on the type of firm (or vacancy) but not on the identity of the firm. So, all workers with the same skill level will apply to identical vacancies with the same probability. Consequently, it is possible that two workers apply to the same vacancy while another vacancy receives no applications.

Workers choose their application strategy to maximize their expected wage taking the firms’ offers and other workers’ strategies as given. Let \( q(s, i) \) denote the expected number of type \( i \in \{H, L\} \) workers at a firm with productivity \( s \); I will refer to \( q(s, i) \) as queue length. Assume that any firm that receives an application from both high and low-skilled workers prefers the high-skilled workers (I will verify below that this is the firm’s optimal strategy). If the firm receives two or more applications of the same type of worker, the firm randomly chooses one of them. Accordingly, the hiring probabilities are given by

\[
g(s, H) = \frac{1 - e^{-q(s, H)}}{q(s, H)} \quad (2.1)
\]

\[
g(s, L) = e^{-q(s, H)} \frac{1 - e^{-q(s, L)}}{q(s, L)}. \quad (2.2)
\]

The expected wage at one vacancy is denoted \( U(s, i) = g(s, i)w(s, i) \) and \( \bar{U}^i \) denotes the prevailing market expected wage for a worker of type \( i \in \{H, L\} \). Now consider the case where one type of vacancy offers a higher expected wage than all other vacancies. All workers will apply to this vacancy with probability 1, and the resulting hiring probability for each worker is zero, which is a contradiction. Similarly, any vacancy offering a lower expected wage than others will not receive any applicants. Hence, in equilibrium

\[
\bar{U}^i \geq g(s, i)w(s, i), \quad q(s, i) \geq 0; \quad i \in \{H, L\} \quad (2.3)
\]

with complementary slackness. The workers’ optimal application strategy is thus

\[
q(s, i) \begin{cases} 
0 & \text{if } U(s, i) < \bar{U}^i \\
\in (0, \infty) & \text{if } U(s, i) = \bar{U}^i.
\end{cases} \quad (2.4)
\]
From (2.3), the worker’s trade-off becomes apparent: in order to obtain a high wage, the worker must accept a lower hiring probability. This trade-off gives rise to inequality between identical workers (residual inequality). In other words, the underlying firm heterogeneity translates into income inequality: more productive firms are more eager to hire and hence attract a larger expected number of applicants. This lowers the hiring probability for each applicant, forcing more productive firms to post higher wages.

2.2.3 Firm’s Problem without Exporting

A firm in industry \( j \in N \) with productivity \( s \) chooses wages and queue length to maximize expected profits

\[
\pi(s, j) = (1 - e^{-q(s,H,j)}) (p(j) sz(j) - w(s, H, j)) + e^{-q(s,H,j)} (1 - e^{-q(s,L,j)}) (p(j) z(j) - w(s, L, j)),
\]

where \( p(j) \) denotes the price of good \( j \) and \( w(s, i, j) \) denotes the wage paid to a worker of type \( i \). Because of anonymity, firms can condition the wage on the skill level of the worker, but not on the worker’s identity. Thus, wages depend only on worker type, firm productivity and firm industry.

For the remainder of this section, I will suppress the industry index \( j \) if there is no risk of confusion. Using the expected wage condition (2.3), wages can be eliminated and the expected profits can be written in terms of queue lengths only

\[
\pi(s) = (1 - e^{-q(s,H)}) p sz - q(s, H) \UH + e^{-q(s,H)} (1 - e^{-q(s,L)}) p z - q(s, L) \UL. \tag{2.5}
\]

Taking first order conditions with respect to the queue length yields

\[
\UH \geq e^{-q(s,H)} p sz - e^{-q(s,H)} (1 - e^{-q(s,L)}) p z, \quad q(s, H) \geq 0 \tag{2.6}
\]

\[
\UL \geq e^{-q(s,H)} e^{-q(s,L)} p z, \quad q(s, L) \geq 0 \tag{2.7}
\]

with complementary slackness.
Chapter 2. Trade and Inequality

From (2.6) and (2.7), one can recognize three different types of firms: firms that attract only high-skilled workers, firms that attract only low-skilled workers and firms that attract both types of workers. Substituting (2.7) as equality into (2.6) gives

\[
\bar{U}^H \geq \bar{U}^L + e^{-q(s,H)p}z(s-1), \quad q(s,H) \geq 0. \tag{2.8}
\]

Using (2.8) and (2.7), I can solve for queue lengths and cutoff productivity levels:

\[
q(s, H) = \begin{cases}
0 & \text{if } s < \bar{s}^a \\
\log(pA) + \log(s-1) - \log(\bar{U}^H - \bar{U}^L) & \text{if } s \in [\bar{s}^a, \bar{s}^b] \\
\log(pA) + \log(s) - \log(\bar{U}^H) & \text{if } s > \bar{s}^b
\end{cases}, \tag{2.9}
\]

\[
q(s, L) = \begin{cases}
\log(pA) - \log(\bar{U}^L) & \text{if } s < \bar{s}^a \\
\log(\frac{\bar{U}^H - \bar{U}^L}{\bar{U}^L}) - \log(s-1) & \text{if } s \in [\bar{s}^a, \bar{s}^b] \\
0 & \text{if } s > \bar{s}^b
\end{cases}, \tag{2.10}
\]

where

\[
\bar{s}^a = \frac{\bar{U}^H - \bar{U}^L}{pz} + 1 \tag{2.11}
\]

and

\[
\bar{s}^b = \frac{\bar{U}^H}{\bar{U}^L}. \tag{2.12}
\]

The expected queue length of low-skilled workers is constant for low productivity firms and decreasing in firm productivity as the expected queue of high-skilled applicants gets longer. Because low-skilled workers produce the same amount of output independently of firm productivity, all firms that attract no high-skilled workers are effectively identical and hence attract the same number of low-skilled applicants. Note that this implies that workers receive the same wage at all of such firms. For firms that do attract high-skilled workers, low-skilled workers serve as “insurance” in case no high-skilled workers
apply. However, as the expected number of high-skilled applicants increases, it becomes increasingly less likely that a low-skilled worker is selected. From the worker’s trade-off, (2.3), this increases the wage the firm must offer; once the queue of high-skilled workers becomes sufficiently long, the firm ceases to attract low-skill applicants.

For high-skilled workers, the expected queue length is increasing in firm productivity. This is because more productive firms have a stronger incentive to fill a vacancy and produce output. At the same time, in order to attract a long queue of workers, a firm must offer a high wage to compensate for the resulting low hiring probability; only the most productive firms can afford to attract a long queue of high-skilled applicants.

Now I can verify that firms indeed prefer high-skilled workers. Solving (2.3) for wages and substituting (2.6) and (2.7) for $U^H$ and $U^L$ gives

$$\omega(s, H) = \frac{q(s, H)}{1 - e^{-q(s, H)}} p z ((s - 1) + e^{-q(s, L)}) < p z ((s - 1) + e^{-q(s, L)})$$

and

$$\omega(s, L) = q(s, L) \frac{p z e^{-q(s, L)}}{1 - e^{-q(s, L)}} > p z e^{-q(s, L)}.$$

From the two inequalities, it follows that

$$\omega(s, H) - p s z > \omega(s, L) - p z.$$

The left-hand side of the inequality is the profit generated by hiring a high-skilled worker, while the right-hand side is the profit generated by hiring a low-skilled worker. Since the former is strictly larger than the latter, firms prefer to hire high-skilled workers.

Finally, a firm enters an industry if the expected profits are greater than the cost of entry:

$$\int_s \pi(s) dF(s) \geq P \kappa e,$$

where $P$ denotes the price of the final good.
Chapter 2. Trade and Inequality

2.2.4 Firm’s Problem with Exporting

Now consider the problem of a firm with access to the world market at some fixed cost $\kappa^x$ (in terms of the final good). If the firm chooses to export, it will receive $p^w > p$ per unit of output. The firm makes its export decision after hiring a worker. A firm will export if the profits from exporting exceed the profits from domestic sales, i.e. if $p^w y(s, i) - P\kappa^x \geq p y(s, i)$. Consistent with the empirical evidence that the majority of firms do not export, I focus on the case where $\kappa^x$ is large relative to the difference in prices, so only firms in the top of the productivity distribution will find it profitable to export. This implies that firms that hired low-skilled workers will never export.\footnote{This is not only a natural assumption, but must also be an equilibrium outcome since some firms in the exporting industry must serve the domestic market.} Then the cut-off productivity level for exporting is given by

$$\bar{s}^c = \frac{P\kappa^x}{z(p^w - p)}. \quad (2.13)$$

Depending on the relative magnitude of $\kappa^x$ and $(p^w - p)$, this cut-off could be above or below the cut-off for firms trying to attract low-skill as well as high-skill workers, $\bar{s}^b$. However, for the remainder of the exposition in this section, I will focus on the case in which the exporting cut-off, $\bar{s}^c$, exceeds $\bar{s}^b$, as it is the case in the quantitative work discussed in section (2.4). The case where $\bar{s}^c < \bar{s}^b$ can be solved similarly.

The expected profits of a firm expecting to export if matched with a high-skill worker are given by

$$\pi(s) = (1 - e^{-q(s,H)}) (p^w sz - P\kappa^x) - q(s, H)\bar{U}^H + e^{-q(s,H)}(1 - e^{-q(s,L)})p z - q(s, L)\bar{U}^L. \quad (2.14)$$
Taking first order conditions and solving for the optimal queue lengths gives

\[ q^x(s, H) = \begin{cases} 
0 & \text{if } s < s^a \\
\log(pA) + \log(s - 1) - \log(U^H_U^L) & \text{if } s \in [s^a, s^b) \\
\log(pA) + \log(s) - \log(U^H) & \text{if } s \in [s^b, s^c) \\
\log(p^w A - P^x x) - \log(U^H) & \text{if } s \geq s^c 
\end{cases} \]  

(2.15)

\[ q^y(s, L) = \begin{cases} 
\log(pA) - \log(U^L) & \text{if } s < s^a \\
\log(U^H - U^L) - \log(s - 1) & \text{if } s \in [s^a, s^b] \\
0 & \text{if } s > s^b 
\end{cases} \]  

(2.16)

Comparing the autarky queue lengths for high-skilled workers (2.9) with those in the trade equilibrium (2.15), it is apparent that the demand for high-skilled workers is increased: substituting the exporting condition (2.13) as equality into (2.15) shows that a firm that is indifferent between being an exporter and a non-exporter has the same number of expected high-skilled applicants under autarky and under trade. However, for firms with productivity above the cut-off level, the expected queue length of high-skilled workers exceeds that which the firm would attract in autarky.

The increase in demand for high-skilled workers at exporting firms causes their expected wage to increase in order to restore the equilibrium in the labour market. Consequently, firms in the middle of the productivity distribution will reduce their demand for high-skilled workers and in turn increase their demand for low-skilled workers. This forces firms at the bottom of the productivity distribution to shorten their queue length of low-skilled applicants.

This reallocation of workers within the industry increases the skill premium as the relative demand for high-skilled workers rises. At the same time, high-skilled workers’ wages are less dispersed and those of low-skilled workers are more dispersed; the within-group (residual) inequality moves in opposite directions for the two groups, so the overall
effect is ambiguous.\textsuperscript{8} While the within-industry reallocation produces clear results, trade will also cause a reallocation of workers across industries. The effect of the across-industry reallocation is less clear and depends on the relative skill intensities of the industries, driving the possible ambiguity of the overall results. This is explored in the numerical example in section 2.2.7.

The within-industry reallocation in exporting industries also affects average firm productivity, similarly to Melitz (2003) and Bernard et al. (2007b): since the most productive firms have increased their probability of hiring and the least productive firms have decreased their probability of hiring, average firm productivity in exporting industries increases. However, due to the assumption that low-skilled workers are equally productive at all firms, average per worker output is not necessarily higher.

\subsection*{2.2.5 The Goods Market}

There are $N$ intermediate goods in the economy. In autarky, all goods consumed must be produced domestically with prices determined in equilibrium. In the free trade equilibrium in which a subset $M$ of the intermediate goods are tradeable, world prices for tradeable goods $(p^w(j))_{j \in M}$ are taken as given by the domestic agents and demand and supply at these prices is perfectly elastic. For these tradeable goods, the price cannot exceed the world price plus any potential tariff since in that case foreign firms would sell at $p^w(j)(1 + \tau(j))$, where $\tau(j)$ is the tariff imposed on the output of industry $j$. However, the price can be lower than $p^w(j)$ because of the fixed cost of exporting. If the domestic price were equal to the world market price, no firm would find it worthwhile paying the exporting cost. As the domestic price falls below the world market price, the most productive firms will start exporting.

Let $y^h(j)$ denote the amount of industry $j$ output sold in the home market at $p(j)$

\textsuperscript{8}This is similar to the result in Shi (2005). In that model, skill-biased technological progress leads to a decrease in inequality among workers who can use the new technology.
and \( y^w(j) \) the amount sold on the world market at \( p^w(j) \). Then, the income generated in industry \( j \) is given by:

\[
Y(j) = p(j)y^h(j) + I(j)p^w(j)y^w(j),
\]

where \( I(j) \) is an indicator function equal to one if \( y^w(j) > 0 \), i.e. if \( j \) is an exporting industry.

If the economy imports good \( j \) and a tariff is imposed, the tariff revenue is \( \tau(j)p^w(j)y^w(j) \).

The tariff revenue is redistributed to all individuals in the form of lump-sum transfers. Total income in the economy is:

\[
Y = \sum_{j \in N} Y(j) + \sum_{j \in M} (1 - I(j))\tau(j)p^w(j)y^w(j).
\]

Domestic demand for good \( j \) is the given by

\[
y^d(j) = \frac{Y}{P\sigma(j)},
\]

and the price of the final good is

\[
P = \sum_{j \in N} \frac{p(j)}{\sigma(j)}.
\]

### 2.2.6 Equilibrium

#### Autarky Equilibrium

The autarky equilibrium consists of the number of firms \( (n(j))_{j \in N} \), workers’ expected wages \( (U^H, U^L) \) and application strategies \( (q(s, i, j))_{i \in \{H, L\}, j \in N} \), firms’ wage posting strategies \( (w(s, i, j))_{i \in \{H, L\}, j \in N} \), and prices \( (p(j))_{j \in N} \) and \( P \) that satisfy the following conditions:

1. Given expected wages and prices, each firm’s vacancy posting strategy solves (2.5).
2. Given the firms’ vacancy posting strategies, workers’ application strategies are consistent with (2.3).
3. Entering firms’ expected profits equal the cost of entering and posting a vacancy:

\[ \int_s \pi(s,j) dF(s,j) = P \kappa^e \quad \forall j \in N. \]

4. Labour markets clear:

\[ \sum_{j \in N} n(j) \int_s q(s,H,j) dF_j(s) = h, \]

\[ \sum_{j \in N} n(j) \int_s q(s,L,j) dF_j(s) = 1 - h. \]

5. The price of the final good satisfies

\[ P = \sum_{j \in N} \frac{p(j)}{\sigma(j)}. \]

6. Goods market clear:

\[ y^d(j) = n(j) \int_s \left\{ (1 - e^{-q(s,H,j)}) s z(j) + e^{-q(s,H,j)} (1 - e^{-q(s,L,j)}) z(j) \right\} dF_j(s) \quad \forall j \in N. \]

**Free Trade Equilibrium**

The free trade equilibrium differs from that of the closed economy only by the goods market clearing condition (6):

6. Goods market clear:

\[ y^d(j) = n(j) \int_s \left\{ (1 - e^{-q(s,H,j)}) s z(j) + e^{-q(s,H,j)} (1 - e^{-q(s,L,j)}) z(j) \right\} dF_j(s) \quad \forall j \in N. \]

\[ y^d(j) = y^w(j) + n(j) \int_s \left\{ (1 - e^{-q(s,H,j)}) s z(j) + e^{-q(s,H,j)} (1 - e^{-q(s,L,j)}) z(j) \right\} dF_j(s) \quad \forall j \in M. \]
2.2.7 A Numerical Example

As discussed above, the model delivers ambiguous results for the relationship between inequality and trade. To assess the importance of the relative skill intensities between the different industries and the identity of the industries with a comparative advantage for that relationship, I now conduct a numerical exercise.

The sample economy consists of 3 industries: an export industry, an import-competing industry and a non-traded industry (services). The driver of the relative skill intensities between industries in this model is the relative dispersion of firm productivities. This is because only firms with a high productivity draw will be recruiting high-skilled workers. Increasing that dispersion without changing the average productivity will increase the number of firms above the threshold of only hiring high-skilled workers.

I will consider 4 scenarios with different combinations of skill-intensities in the 3 industries:

1. Export industry is low skill intensive, import industry is high skill intensive, and non-traded industry is high skill intensive.

2. Export industry is low skill intensive, import industry is high skill intensive, and non-traded industry is low skill intensive.

3. Export industry is high skill intensive, import industry is low skill intensive, and non-traded industry is high skill intensive.

4. Export industry is high skill intensive, import industry is low skill intensive, and non-traded industry is low skill intensive.

For this exercise, I assume firms’ productivities are exponentially distributed with parameter $\lambda$. For high-skill intensive industries, I set $\lambda = 2$ and for low-skill intensive industries is set $\lambda = 10$. The rest of the parameters are set such that the low-skill employment rate is 0.65 in all scenarios and all sectors have the same initial employment. The fraction
of high-skill workers is $h = 1/3$ and 10% of firms in the exporting industry choose to export. Without trade, scenarios 1 and 3 are identical, as are scenarios 2 and 4.

Table 2.2.1 summarizes the results for all 4 scenarios. I will focus on scenarios 1 and 3, which are empirically the more relevant cases since the exporting manufacturing sector is low-skilled relative to the rest of the economy for most trade episodes. The change in the skill-premium (expressed as the difference in average log wages between high- and low-skill workers) is not surprising. In scenario 1, in which the economy has a comparative advantage in low-skill industries, the skill premium falls after opening to trade. Conversely, in scenario 3, in which the economy has a comparative advantage in high-skill industries, the skill premium is higher with trade than in autarky. However, the impact on the skill premium is not symmetric. If the economy’s comparative advantage lies in the low-skill industries, the skill premium falls much less than it increases if the comparative advantage lies in the high-skill industries. This is because there are two forces at play: the first is the sectorial composition effect, the second is the skill bias in exporting. The sectorial composition effect arises because employment in the exporting industry will grow significantly while employment in the importing industry will shrink significantly. This benefits the worker type that is more intensively hired in the exporting industry – in essence, the Stolper-Samuelson effect. At the same time, in order to export, a firm needs a high skilled worker in order to afford the fixed cost, which increases the demand for high-skilled workers. If the exporting sector is high-skill intensive, the two effects work in the same direction and the skill premium increases unambiguously (scenario 3). However, if the import-competing industry is more high-skill intensive than the exporting industry, the two effects work in opposite directions, potentially cancelling each other out (scenario 1).

Furthermore, there is also a general equilibrium effect at play in this framework that is often overlooked in empirical work. As a result of trade, aggregate output increases, causing demand for non-traded services to increase. While this effect is small compared
to the direct effects, comparing scenario 3 to scenario 4 one can see that it can have a noticeable effect on the skill premium. If the non-traded sector is low-skill rather than high-skill intensive, the increase in the skill premium is suppressed. More importantly, this general equilibrium effect explains why overall employment in traded industry may fall as a result of a trade liberalization. In order to satisfy the increased demand for non-traded goods, which arises because of the efficiency gain in the traded industries, employment in non-traded industries must increase at the expense of the traded industries.

The effect of trade on the within-group inequality is ambiguous. For low-skilled workers, the effect of trade on inequality stems from the same two sources that also affect the skill premium – the reallocation across industries and the reallocation within an (exporting) industry. First, recall that low-skilled workers receive the same wage at all firms in the bottom of the productivity distribution which do not recruit any high-skilled workers. While this “minimum wage” is the same for all firms in an industry, it does vary across industries, which in one source of residual inequality among low-skilled workers. The reallocation of workers across industries may increase or decrease the residual inequality from this source for this group. In scenario 1, the exporting sector is low-skill intensive, so the sector that employed more low-skilled workers in autarky is growing and low-skilled workers are more concentrated as a result, which reduces inequality from this source. Conversely, in scenario 3, the exporting sector initially employed relatively few unskilled workers, so low-skilled workers are more dispersed across sectors once trade commences.

The second source of residual inequality among low-skilled workers comes from vacancies to which both high- and low-skill workers apply. These firms pay low-skilled workers above the minimum wage to compensate for the reduced hiring probability. As discussed above, the reallocation within the exporting industries has a strong effect on this source of inequality: exporting firms increase the demand for high-skilled workers,
which increases their expected utility. As a consequence, firms in the middle of the productivity shorten the queue of high-skilled workers and attract more low-skilled workers. This increases residual inequality among low-skilled workers.

For high-skilled workers, variations in hiring probabilities are the only source of inequality. Not surprisingly, inequality within high-skilled workers is thus falling as demand for them increases and their hiring becomes more focused on a small set of vacancies. This effect is most pronounced in scenario 4, where almost all high-skill workers apply at firms in the upper half of the productivity distribution of the exporting sectors as both the importing and non-traded sector are low-skill intensive. However, while this greatly reduces variation in hiring rates and hence inequality, it also greatly increases high skill unemployment. In scenario 4, the high-skill unemployment rate increases by almost 5%, from 24.8% to 29.6%.\(^9\)

As this exercise demonstrates, the model can deliver ambiguous results for the impact of trade on both across- and within-group inequality. For the skill-premium (across-group inequality), the within-industry reallocation effect resembles the effect of skill-biased technological progress, while the across-industry reallocation (toward the comparative advantage industries) causes a Stolper-Samuelson effect. For the residual (within-group) inequality, the within-industry reallocation lowers inequality among high-skilled and increases inequality among low-skilled workers; the overall effect is ambiguous. The effect of across-industry reallocation depends on the dispersion of wages within the different industries – wages in high-skill intensive industries are more dispersed, so if the comparative advantage industry is high-skilled, residual inequality is predicted to increase.

The remainder of the paper extends the simple static model to a dynamic setting that can be calibrated and used to study concrete trade experiences and to conduct policy analysis.

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\(^9\)The high unemployment rates are a result of the “one-shot” nature of the model and will be greatly reduced in the dynamic version below.
2.3 Dynamic Model

In order to implement the model quantitatively, I now extend the previous environment into a dynamic setting. Time is discrete and lasts forever. Risk neutral workers discount the future at rate $\beta \in (0, 1)$. As before, workers can be either high or low skilled and firms have access to the same skill biased technology

$$y(s, i) = \begin{cases} 
sz & \text{if } i = H \\
z & \text{if } i = L.
\end{cases}$$

The timing is as follows: a firm enters by paying a fixed entry cost, $\kappa^e$, and draws its productivity, $s$, from some distribution $F(s)$. It then posts a vacancy; if the vacancy is filled, the firm and worker stay together until either side terminates the match or the match becomes separated exogenously at rate $(\delta_i)_{i \in \{H, L\}}$. Neither the worker nor the firm can search while matched. There is a per period fixed cost, $\kappa^p$, to produce. After separation, the firm exits and the worker enters the pool of the unemployed and can start searching immediately. If the vacancy is not filled, the firm exits the market and returns to the pool of potential entrants. Exporting firms decide whether or not to export each period, paying a per-period fixed exporting cost, $\kappa^x$; firms may decide to become exporters or cease exporting each period.

2.3.1 The Firm’s and Worker’s Problems

Instead of posting a wage as in the static model, the firm posts an output sharing rule, i.e. the firm promises to pay the worker a fraction $\alpha$ of the net output for the duration of the match. For simplicity, the output share is assumed to remain constant and not depend on the state of the economy. However, the actual income of the worker may change as the value of the output changes; this is particularly the case when a firm switches its exporter status. The firm makes a forward-looking vacancy posting decision, e.g. if along the current path of the aggregate state of the economy the firm were to begin exporting in three periods, it would attract a longer queue of high-skilled
workers than if the current state were a steady state (and the firm would never export). However, this rules out the option of adjusting the worker’s output share after a shock. This assumption increases the tractability of the problem substantially, yet unfortunately may give rise to an inefficiency: as a result of a shock, either the firm or the worker may wish to unilaterally break the match although the joint surplus of the match exceeds the continuation value of separation.

Using $\Sigma$ to denote the distribution of filled vacancies across industries and using the same notation as above otherwise, the firm’s value functions are given by:\textsuperscript{10}

\begin{equation}
V^v(s, \Sigma) = (1 - e^{-q(s, \Sigma, H)}) V^{mH}(s, \Sigma) \tag{2.17}
\end{equation}

\begin{equation}
+ \left( e^{-q(s, \Sigma, H)} \right) \left( 1 - e^{-q(s, \Sigma, L)} \right) V^{mL}(s, \Sigma),
\end{equation}

\begin{equation}
V^{mH}(s, \Sigma) = (1 - \alpha_H(s)) \left( p(\Sigma)sz - P(\Sigma)\kappa^p \right) + \beta(1 - \delta_H) V^{mH}_{+1}(s, \Sigma'), \tag{2.18}
\end{equation}

\begin{equation}
V^{mL}(s, \Sigma) = (1 - \alpha_L(s)) \left( p(\Sigma)z - P(\Sigma)\kappa^p \right) + \beta(1 - \delta_L) V^{mL}_{+1}(s, \Sigma'), \tag{2.19}
\end{equation}

where $V^v(s, \Sigma)$ denotes the value of a vacancy and $V^{mH}(s, \Sigma)$ and $V^{mH}(s, \Sigma)$ denote the value of a match between a firm with productivity $s$ and a high and low skilled worker, respectively. As in the static case, I assume that firms that receive both types of workers prefer to hire the high-skilled worker.\textsuperscript{11} Again, this will be verified as true in equilibrium.

The worker’s value functions are given by:

\textsuperscript{10}Again, the industry index $j$ is omitted if there is no risk of confusion.

\textsuperscript{11}As discussed in Shi (2005), such a lexicographic selection rule is problematic in a dynamic setting. The equilibrium depends on the entire distribution of matches; to solve for the equilibrium, one must solve for the entire time-path of all matches. In particular, the threshold productivity levels may vary over time. Solving this problem is very computationally intensive. The assumptions of no on-the-job search and constant output shares are therefore made to keep the model more tractable. See Shi (2009) and Menzio and Shi (2009a, 2009b) for models of directed search on the job in deterministic and stochastic environments, respectively.
\[
U^H(s, \Sigma) = \left(1 - e^{-q(s, \Sigma, H)}\right) J^H(s, \alpha_H, \Sigma) + \beta \left(1 - \frac{1 - e^{-q(s, \Sigma, H)}}{q(s, \Sigma, H)}\right) U^H_{+1}(\Sigma')
\]

\[
U^L(s, \Sigma) = e^{-q(s, \Sigma, H)} \frac{1 - e^{-q(s, \Sigma, L)}}{q(s, \Sigma, L)} J^L(s, \alpha_L, \Sigma) + \beta \left(1 - e^{-q(s, \Sigma, H)} \frac{1 - e^{-q(s, \Sigma, L)}}{q(s, \Sigma, L)}\right) U^L_{+1}(\Sigma')
\]

\[
J^H(s, \alpha_H, \Sigma) = \max \{\alpha_H(s) \cdot (p(\Sigma)s - P(\Sigma)\kappa^p) + \beta \left(1 - \delta_H\right) J^H_{+1}(s, \alpha_L, \Sigma') + \delta_H U^H_{+1}(\Sigma') \} ; \bar{U}^H(\Sigma)
\]

\[
J^L(s, \alpha_L, \Sigma) = \max \{\alpha_L(s) \cdot (p(\Sigma)s - P(\Sigma)\kappa^p) + \beta \left(1 - \delta_L\right) J^L_{+1}(s, \alpha_L, \Sigma') + \delta_L U^L_{+1}(\Sigma') \} ; \bar{U}^L(\Sigma)
\]

where \(U^i(s, \Sigma)\) denotes the value of applying to a firm (in industry \(j\)) with productivity \(s\) and \(J^i(s, \alpha_i, \Sigma)\) denotes the value of being employed by a firm with productivity \(s\) and share \(\alpha_i(s)\).

### 2.3.2 Optimal Queue Lengths

As in the static version of the model, workers must be indifferent between all firms to which they apply. Thus, the worker’s optimal application strategy is

\[
q(s, \Sigma, i) \begin{cases} 
0 & \text{if } U^i(s, \Sigma) < \bar{U}^i(\Sigma) \\
(0, \infty) & \text{if } U^i(s, \Sigma) = \bar{U}^i(\Sigma)
\end{cases}
\]

Hence, as before, the workers’ share can be eliminated by substituting the value of searching, (2.20) and (2.22), into the value of vacancy (2.17) and taking the first order conditions with respect to the queue lengths. First, notice that (2.20) and (2.22) can be decomposed as follows:\[12\]

\[12\text{To simplify notation, the state variable } \Sigma \text{ is suppressed and subscripts } +1, 2, .. \text{ are used to denote future periods.}\]
\[ U^H - \beta U^H_{t+1} = \left(1 - e^{-q(s,H)}\right) \frac{q(s,H)}{q(s,L)} \left[ \alpha_{s,H} y_{s,H} + a_H \alpha_{s,H} y_{s,H+1} + a^2_H \alpha_{s,H} y_{s,H+2} + \ldots \right. \\
\left. + \beta \delta_H \left(U^H_{t+1} + a_H U^H_{t+2} + a^2_H U^H_{t+3} + \ldots \right) - \beta U^H_{t+1}\right], \quad (2.25) \]

\[ U^L - \beta U^L_{t+1} = e^{-q(s,H)} \frac{1 - e^{-q(s,L)}}{q(s,L)} \left[ \alpha_{s,L} y_{s,L} + a_L \alpha_{s,L} y_{s,L+1} + a^2_L \alpha_{s,L} y_{s,L+2} + \ldots \right. \\
\left. + \beta \delta_L \left(U^L_{t+1} + a_L U^L_{t+2} + a^2_L U^L_{t+3} + \ldots \right) - \beta U^L_{t+1}\right], \quad (2.26) \]

where \( a_i = \beta(1 - \delta_i) \) and \( y_{s,i} = p y(s,i) - P \kappa_p, i \in \{H, L\} \).

Similarly, the value of a vacancy, (2.17), can be decomposed into:

\[ V^v(s) = \left(1 - e^{-q(s,H)}\right) \left(1 - \alpha_{H,s}\right) \left[ y_{s,H} + a_H y_{s,H+1} + a^2_H y_{s,H+2} + \ldots \right] \\
+ \left(e^{-q(s,H)}\right) \left(1 - \alpha_{L,s}\right) \left[ y_{s,L} + a_L y_{s,L+1} + a^2_L y_{s,L+2} + \ldots \right]. \quad (2.27) \]

Finally, after cross-multiplying (2.25) and (2.26) with \( q(s,i) \) and rearranging, the resulting expressions can be substituted into (2.27):

\[ V^v(s) = \left(1 - e^{-q(s,H)}\right) \left[y_{s,H} + a_H y_{s,H+1} + a^2_H y_{s,H+2} + \ldots\right] \\
+ \left(e^{-q(s,H)}\right) \left(1 - e^{-q(s,L)}\right) \left[y_{s,L} + a_L y_{s,L+1} + a^2_L y_{s,L+2} + \ldots\right] \\
- q(s, H) (U^H - \beta U^H_{t+1}) - q(s, L) (U^L - \beta U^L_{t+1}) \\
+ (1 - e^{-q(s,H)}) \left[\beta \delta_H \left(U^H_{t+1} + a_H U^H_{t+2} + a^2_H U^H_{t+3} + \ldots \right) - \beta U^H_{t+1}\right] \\
+ e^{-q(s,H)} (1 - e^{-q(s,L)}) \left[\beta \delta_L \left(U^L_{t+1} + a_L U^L_{t+2} + a^2_L U^L_{t+3} + \ldots \right) - \beta U^L_{t+1}\right]. \quad (2.28) \]

From the viewpoint of an individual firm, the sequences of values of searching for high- and low-skilled workers, \( \{U^H_t, U^L_t\}_{t=0}^\infty \), as well as the sequence of future states of the economy, \( \{\Sigma_t\}_{t=1}^\infty \), are given. Hence, the optimal choice of queue lengths \( (q(s, H), q(s, L)) \) maximizes (2.28).

In order to obtain the optimal queue lengths for exporting firms, one need simply replace \( y_{s,H} \) with \( y^x_{s,H} = p^w s_z - P \kappa^x \).
2.3.3 Equilibrium

Let \( u(\Sigma, i) \) denote the number of unemployed workers of type \( i \) at the beginning of the period and \( n^e(\Sigma, j) \) the number of vacancies (newly entered firms) in industry \( j \). The law of motion for unemployment can then be written as:

\[
\begin{align*}
    u'(\Sigma', H) &= \delta_H h + (1 - \delta_H)(u(\Sigma) - m(\Sigma, H)) \\
    u'(\Sigma', L) &= \delta_L (1 - h) + (1 - \delta_L)(u(\Sigma) - m(\Sigma, L)),
\end{align*}
\]

with \( m(\Sigma, i) \) denoting the number of matches formed in current period:

\[
\begin{align*}
    m(\Sigma, H)) &= \sum_{j \in N} n^e(\Sigma, j) \int_s (1 - e^{-q(s,\Sigma,H)}) dF_j(s) \\
    m(\Sigma, L)) &= \sum_{j \in N} n^e(\Sigma, j) \int_s e^{-q(s,\Sigma,H)} (1 - e^{-q(s,\Sigma,L)}) dF_j(s).
\end{align*}
\]

Definition.

The equilibrium consists of sequences of

- value functions for workers \( (U^i(\Sigma), J^{ij}(s, \alpha_{ij}, \Sigma))_{i \in \{H,L\}, j \in N} \),
- value functions for firms \( (V^{w,j}(s, \Sigma), V^{m,ij}(s, \Sigma))_{i \in \{H,L\}, j \in N} \),
- the number of entering firms \( (n^e(\Sigma, j))_{j \in N} \),
- workers’ application strategies \( (q(\Sigma, s, i, j))_{i \in \{H,L\}, j \in N} \),
- firms’ wage posting strategies \( (\alpha(\Sigma, s, i, j))_{i \in \{H,L\}, j \in N} \),
- distribution of matches \( \Sigma \), and
- prices \( (p(j))_{j \in N} \) that satisfy the following conditions:

1. Given workers’ value of search and prices, each firm’s vacancy posting strategy maximizes (2.28).
2. Given the firms’ vacancy posting strategies, a worker’s application strategy is consistent with (2.24).
3. Entering firms’ expected profits equal the cost of entering and posting a vacancy:

$$\int \nu V^v(s, \Sigma) \, dF(s, j) = P \kappa^e \quad \forall j \in N.$$ 

4. Labour markets clear:

$$\sum_{j \in N} n^e(j) \int_s (q(\Sigma, s, H, j) \, dF_j(s) = u(\Sigma, H),$$

$$\sum_{j \in N} n^e(j) \int_s (q(\Sigma, s, L, j) \, dF_j(s) = u(\Sigma, L).$$

5. The price of the final good satisfies

$$P = \sum_{j \in N} \frac{p(j)}{\sigma(j)}.$$

6. The goods market clear:

$$y^d(j) = n(j) \left( \int_s sz(j) \, d\Sigma(H, j) + \int_s z(j) \, d\Sigma(L, j) \right) \quad \forall j \in N \setminus M,$$

$$y^d(j) = y^w(j) + n(j) \left( \int_s sz(j) \, d\Sigma(H, j) + \int_s z(j) \, d\Sigma(L, j) \right) \quad \forall j \in M.$$

### 2.3.4 Stationary Equilibrium

The stationary equilibrium is characterized by a time-invariant distribution of matched firm-worker pairs, i.e. $\Sigma = \Sigma'$. Since all prices and values are constant in a stationary equilibrium, (2.28) can be written as:

$$V^v(s) = (1 - e^{-q(s, H)}) \frac{p sz}{1 - \beta(1 - \delta_H)} + e^{-q(s, H)}(1 - e^{-q(s, L)}) \frac{p z}{1 - \beta(1 - \delta_L)} \quad (2.33)$$

$$-q(s, H)(1 - \beta)U^H - (1 - e^{-q(s, H)}) \left( \beta - \frac{\beta \delta_H}{1 - \beta(1 - \delta_H)} \right) U^H$$

$$-q(s, L)(1 - \beta)U^L - e^{-q(s, H)}(1 - e^{-q(s, L)}) \left( \beta - \frac{\beta \delta_L}{1 - \beta(1 - \delta_L)} \right) U^L.$$
Then, optimal queue lengths must satisfy the following first order conditions:

\[ e^{-q(s,H)} e^{-q(s,L)} \frac{pz}{1 - \beta(1 - \delta_L)} \leq -(1 - \beta) U^L \]

\[ -e^{-q(s,H)} e^{-q(s,L)} \left( \beta - \frac{\beta \delta_L}{1 - \beta(1 - \delta_L)} \right) U^L, \]

\[ q_l(s) \geq 0, \text{ with c.s.} \]

\[ e^{-q(s,H)} \frac{psz}{1 - \beta(1 - \delta_H)} - e^{-q(s,H)} (1 - e^{-q(s,L)}) \frac{pz}{1 - \beta(1 - \delta_L)} \]

\[ \leq -(1 - \beta) U^H - e^{-q(s,H)} \left( \beta - \frac{\beta \delta_H}{1 - \beta(1 - \delta_H)} \right) U^H \]

\[ + e^{-q(s,H)} (1 - e^{-q(s,L)}) \left( \beta - \frac{\beta \delta_L}{1 - \beta(1 - \delta_L)} \right) U^L, \]

\[ q_l(s) \geq 0, \text{ with c.s.} \]

The number of unemployed workers in a stationary equilibrium is given by:

\[ u(\Sigma, H) = h - \frac{(1 - \delta_H)}{\delta_H} m(\Sigma, H) \] (2.36)

\[ u(\Sigma, L) = (1 - h) - \frac{(1 - \delta_L)}{\delta_L} m(\Sigma, L), \] (2.37)

with \( m(\Sigma, i) \) denoting the number of matches formed in current period and obeying (2.31) and (2.32).

### 2.4 Application: The U.S.-Canada Free Trade Agreement

In this section, I use the model to conduct a simple and illustrative quantitative exercise studying the effects of the U.S.-Canada Free Trade Agreement (FTA) which was implemented on January 1, 1989. The agreement called for all tariffs between the two countries to be removed within 10 years.\(^{13}\) The (labour market) implications of the FTA have been subject to a series of empirical studies. Trefler (2004) finds that the FTA led...

\(^{13}\)In 1988, the effective tariff on U.S. products was 16%, Trefler (2004)
to significant labour productivity gains in Canadian manufacturing, mostly as a result of exit of low productivity plants. Furthermore, he reports substantial employment losses – 12 percent for the industries that were previously most protected. Finally, he finds a small increase in wages – about 3 percent – but no affect on income inequality. Gaston and Trefler (1997) report that the FTA had only a small effect on employment. Similarly, Beaulieu (2000) finds that the FTA had a small negative effect on low-skill employment, but did not affect earnings. Finally, Head and Ries (1999) find that the net-effect of the tariff cuts on output per plant is virtually zero, but that the FTA affected average plant-level output through industry composition.

The FTA agreement lends itself as an illustration of the model because it was a clearly defined experiment without major accompanying changes in (labour market) policies. In a developing country, trade reforms are often accompanied by other reforms, such as those promoting FDI, which will lead to changes in the production technology - these reforms in themselves are often skill-biased.

For the purpose of this exercise, I will treat the US as the sole trading partner and Canada as a small open economy. Given that the U.S. is by far Canada’s largest trading partner and given the relative size of the two economies, these assumptions seem reasonable. First, the model is calibrated to match the Canadian labour market in the advent of the FTA. The FTA is then introduced by exogenously lowering the price of import-competing sector products and increasing the price in the exporting sector; the new steady state is computed. However, because of the fixed cost associated with exporting, only a small fraction of firms will find it worthwhile to engage in trade. This, together with the requirement of balanced trade, means that the import-competing industries will shrink but not cease to produce, and the resulting domestic prices will actually differ from the world market prices.

1471.6% of all trade Canadian trade in 1988 and 79.8% in 1998 was with the U.S. (Source: Statistics Canada, CANSIM, Table 228-0003).
2.4.1 Calibration

The economy consists of three sectors: export manufacturing, import-competing manufacturing and non-traded services. I classify workers with a college or post-secondary certificate as high-skilled. The productivity distribution is an exponential one with scale parameter $\lambda$, which differs by sector. The length of a period is one quarter and all empirical moments are for male workers. Using this classification, 13 parameters are to be calibrated: the fraction of skilled workers $h$, the separation rates for both groups $\delta_L$ and $\delta_H$, the distribution parameters $\{\lambda_i\}_{i=1}^3$, the productivity parameters $\{z_i\}_{i=1}^3$, the fixed cost of entering, $\kappa_e$, producing, $\kappa_p$, and exporting, $\kappa_x$, and the discount factor $\beta$.

Three parameters can be matched directly to their counterpart in the data:

- The skill distribution in the labour force in 1988, $h = 0.304$ (Labour Force Historical Review 2000, Statistics Canada).
- The average complete job length by educational attainment (1981-1989), $\delta_H = 0.044$, $\delta_L = 0.086$ (Heisz, 1996).

After normalizing the productivity of the non-traded sector to $z_3 = 1$, and setting the per-period fixed cost $\kappa_p = 0$, and $\beta = 1.04^{-1/4}$, I can identify the remaining 7 parameters as follows:

- The college premium (difference in average log wages, adjusted for experience, between high-school and college graduates) in 1985 (Boudarbat et al., 2007).
- The skill distribution within sectors, where non-production workers are treated as high-skill and production workers as low-skill (data from Trefler, 2004).
- The increase in the fraction of exporting firms from 1984 to 1996 (Baldwin and Gu,
Table 2.4.1 lists all targets and Table 2.4.2 the corresponding parameter values. The most striking result of the calibration is the low productivity dispersion - a large $\lambda$ means the productivity distribution is tight.\textsuperscript{16} This is a result of the relatively low skill premium: a large productivity dispersion will cause a large skill premium because the most productive firms have the most incentive to fill their vacancy and will hence attract a relatively long queue and pay consequently high wages. Furthermore, an increase in dispersion increases the mean of an exponential distribution, hence a more spread-out distribution implies a higher the average high-skill worker’s productivity.

The import-competing sector (defined as the industries that saw a fall in net-exports between 1988 and 1996) is smaller (measured by employment) and more skill-intensive (measured by the ratio of non-production to production workers) than the exporting sector. Because of the assumed Leontief production structure, this results in a larger baseline productivity $z$ in the importing sector; alternatively, one could normalize all the baseline productivities to 1 and adjust the weights in the final good production function accordingly. The larger dispersion in the import-competing sector is a consequence of the higher skill-intensity; only firms with a large enough productivity draw find it profitable to attract high-skill workers. Lastly, it is worth noting that overall employment in the export- and import-competing industries is small; only a small fraction of the labour force is directly affected by the FTA.

\textbf{2.4.2 Results}

Using the calibrated parameters, I compute the pre-FTA steady state. The results are listed in the first column of Table 2.4.3. The model generates several striking results.

\textsuperscript{15}In the calibration, I treat the pre-FTA steady state as autarky, so the labour market change is induced by the increase in exporting.

\textsuperscript{16}For example, Eaton et al. (2008) and Balistreri et al. (2009) estimated parameters for a Pareto distribution imply substantially more variation in firm productivity.
First, note that the employment rate for high-skill workers is close to 1. This is a result of the hiring rule that firms always hire a high-skilled worker over a low-skilled worker. By matching the unemployment rate for low-skilled workers, the calibrated model overpredicts employment for high-skilled workers. Second, the model underpredicts within-group inequality. This is a consequence of the low dispersion in worker productivity. Inequality in the model arises from differences in job-finding probabilities. However, since the variation in the firm-/worker-level productivity is small, differences in queue length are small and the resulting wage dispersion is small. Also, the assumption that firm productivity has no impact on low-skill worker output suppresses inequality among the low-skilled workers.

Implementing the FTA increases aggregate output and causes a sectorial relocation of workers, as shown in Table 2.4.4. The model predicts an increase of 0.87% in final output. Output and employment shrink significantly in import-competing industries and expand in exporting industries. Interestingly, the net-effect on manufacturing employment is negative – employment in manufacturing is predicted to fall, both in absolute numbers as well as relative to non-traded services. Employment in the non-traded service sector increases due to the complementarities in the production of the final consumption good; as overall output increases, demand for services increases and employment in that sector increases. Prior to the FTA, 21.2% of all employment was in manufacturing; after the FTA this fraction falls to 20.5%, which translates into a drop in total manufacturing employment of 3.3%. This compares to a 5% drop in manufacturing employment that Trefler (2004) attributes to the FTA. However, it is important to stress that this loss in employment is not the consequence of import-competition “destroying jobs of hard-working Canadians” as the popular press would term it, but is rather the result of an efficiency gain caused by increased trade.

The import-competing industries are more high-skill intensive than the exporting industries (as measured by the ratio of non-production to production workers, as in
Table 2.4.3), which causes the relative demand for high-skill workers to decrease. On the other hand, in order to export, a plant must be highly productive, which increases the relative demand for high-skill workers. However, since exporting is a rare event, the skill-upgrading effect within the exporting sector is not very strong; the sectorial composition effect dominates and the skill premium falls very slightly. Similarly, the impact on within-group inequality is also small. Again, there is little variation across sectors and the import-competing sector is more dispersed than the exporting sector, so its reduction has a dampening effect on within-group inequality. Nevertheless, these predictions are in line with the empirical studies of the FTA that mostly find a small effect on labour market outcomes. While trade volumes increased substantially, the fraction of the labour force affected by them was small and importing and exporting industries were not significantly different in terms of skill content.

The most substantial shortcoming of the model is the small predicted productivity effect (as measured by the output per worker), as in Table 2.4.4. The model predicts that productivity remains nearly constant in all sectors, rather than increases as the empirical evidence suggests. This is a consequence of two modelling assumptions. First, the production technology assumes that the plant productivity has no impact on the output of a low-skilled worker. As Figure 2.4.1 shows, the probability of filling a vacancy is somewhat decreased for the plants with the lowest productivity draws and is increased for plants with high draws. However, since low-skill workers have the same productivity everywhere, the fact that they are now matched with marginally better firms actually lowers the output for these firms (see Figure 2.4.2 for the probability that a firm that attracts both high- and low-skilled applicants hires a low-skilled applicant). So while the measured match productivity does not increase, the average underlying plant productivity does. The second reason for this shortcoming is that labour is the only input in production. This prevents firms that export from increasing their capital stock and
increasing output per worker, while technology upgrading might play a role in the data.\textsuperscript{17} Incorporating these issues is left for future research.

### 2.5 Discussion

This paper presents a structural model of the labour market that generates (a) equilibrium unemployment, (b) income inequality between different skill groups, and (c) income inequality between identical workers. These features are generated by search frictions in the labour market combined with heterogeneity in firm productivity. The model is then used to study the impact of international trade on labour market outcomes such as unemployment and inequality. The model highlights two avenues for trade to affect labour market outcomes. First, trade impacts the industry structure. Not surprisingly, employment moves out of import-competing and into exporting industries. However, the model also highlights the often neglected general equilibrium effect on non-traded industries. This change in the sectorial composition has ambiguous effects on inequality, and depends on the relative skill-intensities of the industries. Second, trade changes the distribution of workers within an industry. More productive firms increase their demand for high-skilled workers, forcing medium productivity firms to recruit more less-productive workers instead of high-skilled ones and lowering the matching probability for low productivity firms. This increases the average firm productivity and the skill premium, while lowering the inequality among high-skilled workers and increasing it among low-skilled workers.

While it might be somewhat disappointing that the model does not predict a clear relationship between trade and inequality, it helps explain the varied findings in the literature – ultimately, the characterization of this relationship is an empirical one. In most developing countries, the evidence\textsuperscript{18} tends to favour an increase in inequality, suggesting

\textsuperscript{17}While I have no evidence of technology upgrading playing a role in the case of the FTA, technology upgrading and trade-induced skill-biased technological change have been mentioned as a source of increased inequality in developing countries (see for example Goldberg and Pavcnik, 2007).

\textsuperscript{18}As in Goldberg and Pavcnik (2007).
that the Stolper-Samuelson effect is not very strong. This could be explained by the fact that developing countries experience less labour reallocation across industries due to the rigidity of their labour markets.

The implications of this paper also demonstrate the need for trade models to incorporate search frictions if they seek to address labour market implications of international trade. As such, providing a structural model serves two purposes. First, it helps guide any empirical work as to the determinants of the relationship between trade and inequality. Second, the nature of the model allows it to be used for policy analysis and to conduct various counterfactual experiments.

Clearly, the model presented so far falls short along some dimensions. Most importantly, it does not generate much inequality within a group of identical workers. However, relaxing the assumption that low-skill worker and firm productivity are non-complementary (e.g. Shi, 2005) and incorporating specific skills as in Kambourov (2009) and Ritter (2008) should remedy this concern. Another advantage of the model presented above that has not been exploited thus far is its dynamic nature, which allows a study of the transition after a trade reform and weighting of short-run costs and long-run benefits (see Artuc et al. 2007, Kambourov, 2009 and Ritter, 2008). Distinguishing between short and long run effects is also desirable in understanding short run employment responses better. Finally, for a thorough investigation of the impact of a trade reform, a more detailed industry structure than in the simple example above is desirable.
Table 2.2.1: Simulation Scenarios

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ Skill Premium</td>
<td>-0.029</td>
<td>-0.022</td>
<td>0.101</td>
<td>0.090</td>
</tr>
<tr>
<td>Δ (Std. Dev. $\log(w_L)$)*100</td>
<td>-0.040</td>
<td>-0.180</td>
<td>0.530</td>
<td>0.680</td>
</tr>
<tr>
<td>Δ (Std. Dev. $\log(w_H)$)*100</td>
<td>0.150</td>
<td>-0.030</td>
<td>-0.120</td>
<td>-0.490</td>
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<tr>
<td>% Δ Y</td>
<td>0.014</td>
<td>0.009</td>
<td>0.036</td>
<td>0.032</td>
</tr>
<tr>
<td>Δ Employment Rate H-skill</td>
<td>-0.010</td>
<td>-0.024</td>
<td>-0.024</td>
<td>-0.048</td>
</tr>
<tr>
<td>Δ Employment Rate L-skill</td>
<td>-0.001</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.006</td>
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### Table 2.4.1: Calibration Targets

<table>
<thead>
<tr>
<th>Data Moment</th>
<th>Target</th>
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<tbody>
<tr>
<td>Skill Premium</td>
<td>42.1%</td>
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<tr>
<td>Unemployment Rate Low Skill</td>
<td>8.7%</td>
</tr>
<tr>
<td>Fraction of Employment in Manufacturing</td>
<td>21.2%</td>
</tr>
<tr>
<td>thereof: Fraction in Exporting Industries</td>
<td>59.4%</td>
</tr>
<tr>
<td>Ratio of Non-Production to Production Workers in Exporting Industries</td>
<td>33.3%</td>
</tr>
<tr>
<td>Ratio of Non-Production to Production Workers in Import-Competing Industries</td>
<td>42.2%</td>
</tr>
<tr>
<td>Increase in Fraction of Plants Exporting</td>
<td>9.6%</td>
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### Table 2.4.2: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$\lambda_1$</td>
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<td>$z_1$</td>
<td>6.60</td>
<td>$\kappa^e$</td>
<td>2.49</td>
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<tr>
<td>$\lambda_2$</td>
<td>5.97</td>
<td>$z_2$</td>
<td>9.40</td>
<td>$\kappa^p$</td>
<td>0.00</td>
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<tr>
<td>$\lambda_3$</td>
<td>5.29</td>
<td>$z_1$</td>
<td>1.00</td>
<td>$\kappa^x$</td>
<td>0.09</td>
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</tbody>
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### Table 2.4.3: Labour Market Outcomes

<table>
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<th>Post-FTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1]</td>
<td>[2]</td>
</tr>
<tr>
<td>Avg. log-wage H-skill</td>
<td>0.1698</td>
<td>0.1693</td>
</tr>
<tr>
<td>Avg. log-wage L-skill</td>
<td>-0.2503</td>
<td>-0.2499</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>0.4201</td>
<td>0.4192</td>
</tr>
<tr>
<td>Std. Dev. H-skill Wages</td>
<td>0.0212</td>
<td>0.0213</td>
</tr>
<tr>
<td>Std. Dev. L-skill Wages</td>
<td>0.0173</td>
<td>0.0172</td>
</tr>
<tr>
<td>Employment Rate High-skill</td>
<td>99.5%</td>
<td>99.4%</td>
</tr>
<tr>
<td>Employment Rate Low-skill</td>
<td>91.3%</td>
<td>91.3%</td>
</tr>
</tbody>
</table>

### Table 2.4.4: Production Outcomes

<table>
<thead>
<tr>
<th></th>
<th>Pre-FTA</th>
<th>Post-FTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1]</td>
<td>[2]</td>
</tr>
<tr>
<td>Output (Final Consumption Good)</td>
<td>0.864</td>
<td>0.872</td>
</tr>
<tr>
<td>Productivity Exporting Industries</td>
<td>7.313</td>
<td>7.314</td>
</tr>
<tr>
<td>Fraction of Employment in Manufacturing</td>
<td>0.212</td>
<td>0.205</td>
</tr>
<tr>
<td>thereof: Fraction in Exporting Industries</td>
<td>0.594</td>
<td>0.760</td>
</tr>
<tr>
<td>Ratio of Non-Production to Production</td>
<td>33.3%</td>
<td>33.1%</td>
</tr>
<tr>
<td>Workers in Exporting Industries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio of Non-Production to Production</td>
<td>42.2%</td>
<td>42.4%</td>
</tr>
<tr>
<td>Workers in Import-Competing Industries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of Plants Exporting</td>
<td>–</td>
<td>9.6%</td>
</tr>
</tbody>
</table>
Figure 2.4.1: Vacancy Filling Rate
Figure 2.4.2: Vacancy Filling Rate, Low-Skill Workers
Chapter 3

The Optimum Quantity of Money Revisited:

Distortionary Taxation in a Search Model of Money

3.1 Introduction

The Friedman rule has been one of the key doctrines of monetary theory for which traditional monetary economics has provided extensive support over the past decades. These traditional models rely on short-cuts, such as money-in-the-utility-function and cash-in-advance, to introduce money into their environments. Conversely, the microfoundations of money literature which does not resort to these shortcuts has pointed to a potential trade-off between the number of transactions (the extensive margin) and the quantity of goods exchanged in each trade (the intensive margin), which may render the Friedman rule a suboptimal monetary policy. However, these monetary search models have thus far mostly ignored fiscal policy, whose absence could potentially drive the
shortfall of the Friedman rule – inflation may serve as a substitute for fiscal policy.

This paper incorporates fiscal policy considerations by introducing a sales tax into the monetary search environment, and revisits the optimum quantity of money. As such, it links the microfoundations of money to the traditional money and public finance literature. The key finding is that deviating from the Friedman rule, i.e. setting a positive nominal interest rate, may result in welfare gains – inflation is not a mere substitute for omitted fiscal policy. In fact, the suboptimality of the Friedman rule arises even if no government revenue is needed; the deviation may help to improve efficiency in the number of matches, a feature nonexistent in the traditional literature, which does not model the extensive margin of trade.

The traditional money and public finance literature originates from Phelps’ (1973) critique of the Friedman rule. Phelps argues that a government which needs to resort to distortionary taxation to raise revenue should also tax money holdings. However, subsequent research has not supported this supposition. Those studies focus on the Ramsey problem, i.e. the optimal policy mix between a (sales) tax and a tax on money holdings, and tend to support the optimality of the Friedman rule: a sales tax dominates a tax on money holdings as an instrument to raise revenue (e.g. Kimbrough, 1986, Correia and Teles, 1996 and Chari et al., 1996). Faig (1988) demonstrates that the optimality of a zero inflation tax crucially depends on the assumption imposed on the utility and shopping-time functions. However, one shortfall of this literature is that it introduces money through short-cuts instead of deriving the role of money from primitives.

Conversely, the environment in the microfoundations of money literature gives rise to the importance of money as a medium of exchange – agents are anonymous and engage in random, bilateral meetings, where all trades have to be made quid pro quo. Due to the spatial and time separation of trades, agents acquire money for consumption at a later point in time, which results in a positive opportunity cost of holding money. Agents
therefore choose inefficiently low money balances unless the nominal interest rate is zero (Friedman, 1969). If the quantity exchanged is the only margin that might be distorted, following the Friedman rule is the welfare maximizing monetary policy because it ensures that agents are fully compensated for their impatience.\textsuperscript{1}

There is, however, a group of models in the microfoundations literature that points out that not only the intensive, but also the extensive margin can be distorted. Therefore, deviating from the Friedman rule may be welfare improving under certain conditions: in the large household framework (Shi, 1997), the Friedman rule assures that the intensive margin is undistorted. Nevertheless, the number of trades might be inefficient because agents are not fully compensated for the externalities their search decisions generate; for this, the agent’s surplus share must equal her contribution to the creation of the match, i.e. the Hosios rule (Hosios, 1990) also needs to be satisfied. In this case, money growth may increase the number of matches and consequently welfare.\textsuperscript{2}

Berentsen et al. (2007) further study this link between money growth and the extensive margin and show that the buyer’s match surplus becomes endogenous if she is constrained by her money balances. This gives rise to a second-best result: if the buyer’s exogenous bargaining weight is small relative to her contribution to the creation of the match, i.e. if the Hosios rule is violated, deviating from the Friedman rule will be welfare improving. However, these models do not study policy instruments other than monetary ones. If a social planner had available lump-sum taxes and transfers in the decentralized market, the Friedman rule would indeed be optimal. In the presence of distortionary taxation, however, the outcome is less clear, as the optimal taxation literature has illustrated. This issue was also brought forward by Kocherlakota (2005) and Rocheteau and

\textsuperscript{1}Berentsen and Rocheteau (2003) provide a detailed discussion of the Friedman rule in search models of money.

\textsuperscript{2}In a series of papers, Shi further demonstrates the robustness of the extensive margin effect to a number of extensions of the basic search framework: Shi (1998) introduces labour market frictions and Shi (1999) includes capital accumulation into the monetary search environment. Head and Kumar (2005) show that the suboptimality is also robust to price posting.
Wright (2005) in noting that monetary policy alone cannot be expected to fully correct for inefficiencies in an environment with multiple sources of distortions.

As this paper shows, introducing a sales tax into the money search framework (from Shi, 1997) helps to correct an inefficiency in the extensive margin. A sales tax creates a tax burden that buyers and sellers split according to their bargaining weights. Consequently, if the seller pays a larger fraction of the tax, he will reduce his search effort more than the buyer will. This improves efficiency if the seller’s search intensity exceeds the optimal level, i.e. if the seller’s bargaining weight is larger than his contribution to the creation of the match.

However, if monetary policy obeys the Friedman rule, the two surplus shares are unaffected by fiscal policy. As long as the money constraint is not binding, the buyer cannot credibly limit her offer and will only receive the fraction of the surplus equal to her Nash bargaining weight. This limits the effectiveness of the sales tax – in order to improve along the lines of the Hosios rule, the buyer’s surplus share needs to approach her contribution to the creation of the match. Thus, deviating from the Friedman rule is welfare improving even in the presence of a sales tax. By making the money constraint bind, a positive nominal interest rate allows the buyer to extract a larger fraction of the total surplus. This surplus share effect is exclusive to monetary policy – monetary and fiscal policy are not substitutes in this environment.

This key finding carries over to an environment in which buyers and sellers do not bargain, but rather take prices as given (competitive pricing). With price taking, inflation will reduce the present value of the amount received by sellers, making it less attractive to be a seller and hence reducing the seller’s search effort. At the same time, the buyer has an incentive to find a trading partner more quickly, inducing her to search more. This reduces market congestion and increases efficiency. Inflation again implicitly changes the surplus split between agents; fiscal policy cannot achieve the same – taxation simply
creates a burden on both buyers and sellers, and reduces their search efforts. Monetary policy, on the other hand, does induce the buyer to search more, because it reduces the future value of money and thus creates a surplus share effect.

The interaction between monetary and fiscal policy has also been studied by Aruoba and Chugh (2006, 2008) and Gomis-Porqueras and Peralta-Alva (2009). Aruoba and Chugh (2006) study the Ramsey problem in a simple monetary search environment; they find that deviating from the Friedman rule is optimal and that the time-path of inflation is stable across the business cycle. These results carry over to an environment with capital as Aruoba and Chugh (2008) demonstrates. Gomis-Porqueras and Peralta-Alva (2009) conduct an analysis closer to that undertaken in this paper by studying optimal fiscal and monetary policy within the Lagos and Wright (2005) framework with exogenous matching rates. The authors find that, depending on the range of fiscal policy instruments available in the centralized market, optimal policy combinations may or may not include a deviation from the Friedman rule. These findings reinforce the result presented in this paper that a microfounded environment reveals important interactions between monetary and fiscal policy – inflation is not a mere substitute for omitted fiscal policy. Rather, distortionary taxation and distortionary monetary policy function as complements.

3.2 Environment

3.2.1 Households

The environment follows Berentsen et al. (2007). The economy is comprised of a large number of households of size measure 1; each household is of a certain type, denoted $h$, with $H > 3$ different types in the economy. A household of type $h$ produces good $h$ and consumes good $(h + 1)$. Lower case letters denote household level variables, and capital

\footnote{This is a version of the large household framework by Shi (1997). The other framework commonly used in the microfoundations of money literature is the so-called LW model (Lagos and Wright, 2005). A brief discussion about possible differences in implications can be found in section 5.3.}
letters denote the corresponding aggregates. Households are made up of two types of agents: buyers and sellers. The fraction of buyers is \( n \in (0, 1) \). In each period, buyers and sellers enter a decentralized market to search for a trading partner.

All members of the household share the utility generated by the household’s consumption. Thus their common objective is to maximize the household’s utility, given as follows

\[
U = \sum_{t=0}^{\infty} \beta^t \left[ u(q^b_t) - c(q^s_t) - n\phi(\sigma_{bt}) - (1 - n)\phi(\sigma_{st}) \right], \quad \beta \in (0, 1).
\] (3.1)

Here \( \beta \) is the discount factor, and \( c(q) \) the cost of producing \( q \) in utility terms. The cost function \( c(q) \) satisfies the usual properties \( c(0) = 0, c'(q) > 0, c''(q) > 0 \) and \( c'(0) = 0 \). The utility function \( u(q) \) is assumed to be linear, i.e. the marginal utility \( u' > 0 \) is constant; this assumption is made for simplicity only.\(^4\) The number of buyers and sellers in the household is denoted by \( n \) and \( (1 - n) \) respectively. The function \( \phi(\sigma) \) represents the disutility associated with a search intensity \( \sigma \).\(^5\) For simplicity, assume \( \phi(\sigma) = \phi_0 (\sigma^\alpha - 1) \), with \( \phi_0 > 0 \) and \( \alpha > 1 \).

In each period, buyers and sellers go to the market where they engage in random bilateral matching. Since double coincidence of wants is ruled out by assumption and all members of a certain type of household are indistinguishable, a medium of exchange is needed. In each period \( t \), the total stock of money is given by \( M_t H \), where \( M_t \) is the average holding of money per household. As sellers have no use for money, all money is carried by the buyers, and each buyer carries \( m_t / n \) units of money when entering the market. Let \( \omega \) denote the value of next period’s money to the household and \( \Omega \) the value of next period’s money to other households. Finally, at the beginning of each period, the government gives a lump-sum transfer \( L_t \) to each household, so that the money stock

---

\(^4\)Differently from the Lagos and Wright (2005) structure, the assumption of constant marginal utility is not needed to generate a tractable distribution of money holdings; it solely serves to simplify the problem. It has a bearing in the case of price taking, but does not alter any of the results.

\(^5\)The expressions “search intensity” and “search effort” are used synonymously throughout this paper.
grows at a rate of \( \gamma \).

### 3.2.2 Fiscal Policy

Following the traditional money and public finance literature, the fiscal policy instrument is a simple distortionary tax. The tax is modelled as a sales tax, but in this search environment may also be interpreted as an income tax on the seller: whenever a buyer and a seller trade, the government imposes a tax at rate \( \tau \); i.e. if the buyer pays \( x \), the seller receives an after tax total of \( x/(1 + \tau) \). The tax revenue raised is returned to the agents at the beginning of the next period as a lump-sum transfer to the household. This government transfer has to satisfy

\[
L_t = (\gamma - 1)M_t + \frac{\Psi_{t-1}}{H} \tau x, \quad \gamma \geq \beta
\]  

(3.2)

where \( \Psi \) denotes the total number of matches. In this environment, this taxation scheme may also be regarded as an income tax on the seller.

An underlying assumption is that the government is able to observe matches and the amount of money exchanged although there is no sufficient record keeping technology (which makes money essential as a medium of exchange). This assumption is not as restrictive as it may seem – the monitoring technology necessary to implement this policy does not provide the type of memory that makes money dispensable (Kocherlakota, 1998). In fact, forms of sales or consumption based taxes have been used for centuries (e.g. salt taxes in Europe throughout the middle ages), with a general sales tax (“Generalkonsumakzise”) first introduced in Saxony in 1754, at a time when record keeping was neither as easy nor as advanced as today.

\[\text{It can easily be shown that this tax scheme is equivalent to the buyer paying } x(1 + \tau) \text{ and the seller receiving } x.\]
3.2.3 Matching Function

In the decentralized market, buyers and sellers meet at random, and the total number of trade matches is determined by a matching function $\Psi(B\Sigma_b, S\Sigma_s)$, where $B$ denotes the total number of buyers and $S$ the total number of sellers in the market; $\Sigma_i$ is their average search intensity. $B$ and $S$ are exogenously given as $B = HN$ and $S = H(1 - N)$. In contrast, the search intensities are optimally chosen by the households. The function $\Psi(.)$ satisfies standard assumptions, such as homogeneity of degree 1 and concavity in both arguments (following e.g. Mortensen and Pissarides, 1994 and Berentsen et al., 2007).

It is useful to define the market thickness (for buyers) as the ratio of effective sellers to buyers

$$T \equiv \frac{S\Sigma_s}{B\Sigma_b} = \frac{(1 - N)\Sigma_s}{N\Sigma_b} \quad (3.3)$$

Denote the marginal contribution of either side to the number of matches as $K_i(T)$

$$K_i(T) = \frac{\partial \Psi(B\Sigma_b, S\Sigma_s)}{\partial (i \Sigma_i)}, \quad i = B, S$$

Then, one can rewrite the number of matches as $\Psi(B\Sigma_b, S\Sigma_s) = K_b(T)B\Sigma_b + K_s(T)S\Sigma_s$. Define the share of buyer’s contribution to the total number of matches respectively as

$$\eta(T) = K_b(T) \frac{B\Sigma_b}{\Psi(B\Sigma_b, S\Sigma_s)}, \quad K_b(T) \frac{1}{K_b(T) + K_s(T)T} \quad (3.4)$$

and analogously for the seller. Finally, it will prove convenient to define the average matching rate for the buyer and seller as

$$A_b(T) = \frac{\Psi(B\Sigma_b, S\Sigma_s)}{B\Sigma_b} = \Psi(1, T)$$

$$A_s(T) = \frac{\Psi(B\Sigma_b, S\Sigma_s)}{S\Sigma_s} = \frac{\Psi(1, T)}{T}$$
3.2.4 The Bargaining Process in the Decentralized Market

The bargaining process is the key mechanism through which fiscal and monetary policy change the agents’ search behaviour: in the decentralized market, the buyer is potentially constrained by her money holdings. As a result, the bargaining outcome will depend on the buyer’s money constraint whenever it is binding, i.e. in this framework the bargaining shares of buyer and seller are endogenous. Moreover, as shown below, they depend on both monetary and fiscal policy. This channel for policy is a crucial feature of this framework and is missing in the previous literature on money and public finance.

After a buyer and a seller meet in the market, they bargain over $q$, the quantity of goods, and $x$, the amount of money to be exchanged in the trade. The bargaining process is modeled as a sequential game with an exogenous risk of breakdown. In each round, one agent proposes a pair $(q, x)$ and the respondent accepts or rejects. If the proposal is accepted, the trade takes place immediately on the agreed upon terms; if not, time $\Delta$ elapses and the respondent may make a counteroffer. During this “waiting time”, the game might break down; the probability of breakdown depends on the rejecting agent’s type. If a seller rejects the buyer’s offer, the probability of breakdown is $\theta \Delta$; if the buyer rejects the seller’s proposal the probability is $(1 - \theta) \Delta$, where $\theta \in (0; 1)$. This paper focuses on the limit case when $\Delta$ approaches 0, and there is no first-mover advantage.\footnote{In this case, the solution to the bargaining game becomes the Nash bargaining solution. For a more in-depth discussion of sequential bargaining games see Muthoo (1999).}

Assume all agents follow a stationary bargaining strategy, i.e. a buyer always proposes $(q^b, x^b)$ and a seller always proposes $(q^s, x^s)$. First, consider the buyer’s problem: When making her proposal, the buyer faces two constraints; she is restricted by her own money holdings, and she may not leave the seller less surplus than his reservation surplus. Upon accepting, the seller obtains $x^b$ units of money with a present value of $\Omega x^b$ and incurs a disutility of $c(q^b)$. Hence, his surplus from accepting is $[\Omega x^b - c(q^b)]$. If he decides to
reject, he will make a counteroffer \((Q^s, X^s)\) with probability \((1 - \theta \Delta)\) that gives him a surplus of \(\Omega X^s - c(Q^s)\). So, the buyer’s proposal must satisfy

\[
\frac{m}{n} \geq x^b \tag{3.5}
\]

\[
\Omega x^b/(1 + \tau) - c(q^b) \geq (1 - \theta \Delta) \left[ \Omega X^s/(1 + \tau) - c(Q^s) \right] \tag{3.6}
\]

Similarly, the seller’s proposal \((q^s, x^s)\) needs to satisfy

\[
\frac{M}{N} \geq x^s \tag{3.7}
\]

\[
u(q^s) - \Omega x^s \geq (1 - (1 - \theta)\Delta) \left[ u(Q^b) - \Omega X^b(1 + \tau) \right] \tag{3.8}
\]

In equilibrium, (3.6) and (3.8) will be satisfied with equality. To see why, suppose to the contrary that (3.6) holds as a strict inequality, then the buyer could increase her utility by rising \(q^b\) without increasing \(x^s\) until the constraint is satisfied with equality. Likewise, the seller could decrease \(q^s\) and hence his disutility of production if (3.8) were not satisfied with equality.

The solution to the bargaining game is summarized in Lemma 1.

**Lemma 3.2.1.** In a symmetric equilibrium with \(x^i = X^i\) and \(q^i = Q^i\), when \(\Delta \to 0\), \(x^b = x^s = x\) and \(q^b = q^s = q\) and the buyer’s surplus is given by

\[
\Theta(q, \tau) \left[ u(q) - c(q) \right] - \Theta(q, \tau) \tau c(q) \tag{3.9}
\]

The seller’s surplus is given by

\[
(1 - \Theta(q, \tau)) \left[ u(q) - c(q) \right] - (1 - \Theta(q, \tau)) \frac{\tau}{1 + \tau} u' q \tag{3.10}
\]

Where \(\Theta(q, \tau)\) is defined as

\[
\Theta(q, \tau) = \frac{\theta u'}{\theta u' + (1 + \tau)(1 - \theta)c'(q)} \tag{3.11}
\]

**Proof.** See appendix
As noted above, the buyer’s surplus share $\Theta(q, \tau)$ is endogenous as a result of the money constraint; only if $\theta = 0$ (a take-it or leave-it offer by the seller) or if $\theta = 1$ (a take-it or leave-it offer by the buyer) does it coincide with $\theta$. The buyer’s surplus depends directly on the tax rate and indirectly through the quantity exchanged in the market, on the rate of money growth. This effect of fiscal and monetary policy is critical for the results.

Fiscal policy has three effects on the agents’ surplus. The first is the effect on total surplus $[u(q) - c(q)]$, the second the tax burden levied on buyers, $\Theta(q, \tau) \tau c(q)$, and sellers, $(1 - \Theta(q, \tau)) \frac{\tau}{1 + \tau} u(q)$, and the third the effect on the surplus share the buyer receives, $\Theta(q, \tau)$. The first two effects reduce both the buyer’s and seller’s surplus because a distorting tax will reduce total surplus and impose a tax burden on both agents. The last effect goes in opposite directions for buyers and sellers, and is discussed in detail in section 3.2.

### 3.3 The Monetary Equilibrium with Fiscal Policy

#### 3.3.1 The Household’s Problem

In each period the household chooses its buyers’ and sellers’ bargaining proposals, their search intensity and next period’s money stock, taking other households’ choices as given. The problem can be written as a dynamic programming problem

$$v(m) = \max_{\{q^b, x^b, q^s, x^s, \sigma_b, \sigma_s, m+1\}} \left\{ \begin{array}{l} n\sigma_b A_b(T)u(q^b) - (1 - n)\sigma_s A_s(T)c(q^s) \\ -n\phi(\sigma_b) - (1 - n)\phi(\sigma_s) + \beta v(m_{+1}) \end{array} \right\}$$

subject to (3.5) - (3.8) and the law of motion for money

$$m_{+1} = m + (1 - n)\sigma_s A_s(T) \frac{x^s}{1 + \tau} - n\sigma_b A_b(T)x^b + L.$$
The first order conditions are given by

\[ u' = \frac{\omega + \lambda}{\Omega}(1 + \tau)c'(q^b) \] (3.13)

\[ c'(q^s) = \frac{\omega - \pi(1 + \tau)}{\Omega(1 + \tau)}u' \] (3.14)

\[ \phi'(\sigma_b) = A_b(T)\left( u(q^b) - \omega x^b \right) \] (3.15)

\[ \phi'(\sigma_s) = A_s(T)\left( \frac{\omega x^s}{1 + \tau} - c(q^s) \right) \] (3.16)

\[ \frac{\omega - 1}{\beta} = \omega + \sigma_b A_b(T)\lambda \] (3.17)

where \( \omega = \beta v_{+1}(m_{+1}) \), the discounted expected value of money next period. \( \lambda \) and \( \pi \) are the Lagrange multipliers associated with the money constraints (3.5) and (3.7) respectively.

Equation (3.13) describes the trade-off a proposing buyer faces; the monetary cost of an extra marginal unit of the consumption good is given by \( (1 + \tau)c'(q^b)/\Omega \). This amount of money is valued at \( (\omega + \lambda) \) by the buyer, where \( \omega \) is next period’s value of money and \( \lambda \) represents the tighter cash or resource constraint. Thus, the right hand side of (3.13) represents the marginal cost (in utility terms) of an extra unit of the consumption good for the buyer. The optimal proposal equalizes this marginal cost with the marginal utility. Similarly, equation (3.14) requires that the marginal gain (in utility terms) is equal to the marginal cost of producing.

Equations (3.15) and (3.16) describe the optimal choices for the search intensities. The right hand side of each of these equations is the marginal gain from increasing the search intensity. To see this, note that \( A_i(T) \) is the average matching rate per “unit of search effort”, while \( (u(q^b) - (1 + \tau)\omega x^b) \) and \( (\omega x^s - c(q^s)) \) are the gains from a trade match for a buyer and a seller respectively. A marginal increase in the search intensity increases the probability of a match by \( A_i(T) \), and increases the expected utility by \( A_i(T)\left( u(q^b) - (1 + \tau)\omega x^b \right) \). In equilibrium, this marginal gain needs to equal the marginal cost of searching.
Using the result of Lemma 1, (3.15) and (3.16) can be rewritten as

\[ \phi'(\sigma_b) = A_b(T) \Theta(q, \tau) [u(q) - (1 + \tau) c(q)] \quad (3.18) \]

\[ \phi'(\sigma_s) = A_s(T) \frac{(1 - \Theta(q, \tau))}{(1 + \tau)} [u(q) - (1 + \tau) c(q)] \quad (3.19) \]

(3.17) is the envelope condition for money.

**Stationary and Symmetric Monetary Equilibrium**

**Definition** A stationary and symmetric monetary equilibrium consists of a sequence of individual household’s choices \( \{d_t\}_{t=0}^\infty \), where \( d = (q^b, x^b, q^s, x^s, m_{t+1}, \sigma_b, \sigma_s) \), other households’ choices \( \{D_t\}_{t=0}^\infty \), and the shadow prices \( (\omega, \Omega, \lambda, \Lambda, \pi, \Pi) \). The sequence satisfies the following requirements for all \( t \):

1. optimality: \( d_t \) solves the households problem given \( D_t \),
2. symmetry: \( d_t = D_t \),
3. stationarity: \( d_t = d \),
4. \( 0 < \omega_{t-1} M_t < \infty \) and \( \omega_{t-1} M_t \) constant.

Conditions (i) - (iii) are standard. The first part of condition (iv) requires that money has a positive and finite value, where \( \omega_{t-1}/\beta \) is the period \( t \) value of one unit of money.\(^8\)

The stationary and symmetric monetary equilibrium allocation \( (q, \omega x, \sigma_b, \sigma_s) \) can be obtained from (3.18), (3.19), (3.3) and the following equations:

\[ \frac{u'}{c'(q^s)} = \left[ 1 + \frac{1}{\sigma_b A_b(T)} \left( \frac{\gamma}{\beta} - 1 \right) \right] (1 + \tau) \quad (3.20) \]

\[ u(q) - \omega x = \Theta(q, \tau) [u(q) - (1 + \tau) c(q)] \quad (3.21) \]

Equation (3.20) comes from combining (3.13) and (3.17) and imposing stationarity. Equation (3.21) follows from Lemma 1.

\(^8\)The existence of an equilibrium was established in Berentsen et al. (2007); it is necessary that \( \gamma \geq \beta \) and \( \lambda > 0 \) if and only if \( \gamma > \beta \).
From (3.20), we can see that if the economy follows the Friedman rule, \( u' = (1 + \tau)c'(q^*) \). In this case, we can solve for the equilibrium quantity without having to specify the matching function. It is obvious that only if \( \tau = 0 \), the intensive margin will be undistorted. If \( \tau \neq 0 \), the quantity \( q \) deviates from the social optimum even if \( \gamma = \beta \).\(^9\) From (3.20) it also follows that deviating from the Friedman rule will result in an inefficiency in the intensive margin. This follows directly from the fact that the buyer is constrained by his real money balances. Whenever \( \gamma > \beta \), there is an opportunity cost of holding money and hence the buyer will choose to hold inefficiently low real money holdings; \( q \) will thus also be inefficiently low.

### 3.3.2 The Buyer’s Surplus Share Revisited

Recall from Lemma 1 that the buyer’s fraction of the surplus is given by \( \Theta(q, \tau) \). Using the steady state condition (3.20) to derive \( q = q(\gamma, \tau, T) \), \( \Theta(q, \tau) \) can be rewritten as

\[
\Theta(\gamma, \tau, T) = \frac{\theta}{\theta + \frac{(1-\theta)}{1+\frac{1}{\gamma \beta (\tau)} (\frac{\tau}{\beta} - 1)}}.
\] (3.22)

From (3.22), it can be seen that the buyer’s surplus share critically depends on the rate of money growth. If monetary policy follows the Friedman rule, \( \Theta(\gamma, \tau, T) \) reduces to \( \theta \), the exogenous bargaining parameter. This implies that the surplus split is independent of the fiscal policy at the Friedman rule; the buyer will receive a fixed fraction of the total surplus regardless of \( \tau \).

The buyer’s surplus share becomes endogenous only if monetary policy deviates from the Friedman rule; from (3.22), it follows that \( \Theta(\gamma, \tau, T) \) is increasing in the rate of money growth. Equivalently, examining (3.14) shows that the higher the rate of money growth, the tighter the money constraint and the larger the Lagrange multipliers \( \lambda/\omega \) and \( \pi/\omega \). Substituting (3.14) into \( \Theta(q, \tau) \) shows that the buyer’s surplus share is increasing in \( \pi/\omega \),

\(^9\)See section 4 for the characterization of the social optimum.
i.e. the higher the rate of money growth, the larger the buyer’s surplus share.

The intuition for this result follows from the latter argument. The money constraint serves as a credible upper bound to the buyer’s offer and becomes tighter as the rate of money growth increases. This improves the buyer’s threat point in the bargaining game and allows her to extract a larger fraction of the total surplus, creating the surplus share effect of monetary policy.

The surplus share effect of monetary policy is also essential for the effectiveness of fiscal policy. At the Friedman rule, fiscal policy does not have an impact on the surplus split: \( \Theta(\gamma = \beta, \tau) = \theta \). If, however, the rate of money growth exceeds the rate of time preference \( (\gamma > \beta) \), the sales tax will further improve the buyer’s bargaining position. To see this, note that \( \sigma_b A_b(T) \) decreases as \( \tau \) increases.\(^{10}\) The intuition is the same as for monetary policy: if the buyer is constrained by her money holdings, increasing the tax rate makes this constraint even more binding, allowing the buyer to extract a larger fraction of the total surplus. However, this channel only works if the buyer’s money constraint is binding, giving a special role to monetary policy. This link between fiscal and monetary policy is a novel feature of the model and is crucial for the optimal policy as discussed below.

### 3.4 Social Optimum

Now consider the problem of a benevolent planner who seeks to maximize social welfare – the total trade surplus generated by all matches, less the cost of searching incurred to create these matches. Hence, the social welfare function can be written as

\[
W = \Psi(N\Sigma_b, (1 - N)\Sigma_s) [u(q) - c(q)] - N\phi(\Sigma_b) - (1 - N)\phi(\Sigma_s) \quad (3.23)
\]

\(^{10}\)The buyer’s search intensity falls because the reduction in total surplus and the tax burden outweigh the increase in the buyer’s surplus share. The reduction in the average matching probability for the buyer results from the reduction in the market tightness \( T \).
The planner chooses the quantity produced in each match, $q$, and agents’ search intensities $(\Sigma_b, \Sigma_s)$. The first order conditions are given by

$$
\begin{align*}
    u' &= c'(q) \tag{3.24} \\
    \phi'((\Sigma_b) &= K_b(T)(u(q) - c(q)) = \eta(T)A_b(T)(u(q) - c(q)) \tag{3.25} \\
    \phi'((\Sigma_s) &= K_s(T)(u(q) - c(q)) = (1 - \eta(T))A_s(T)(u(q) - c(q)) \tag{3.26}.
\end{align*}
$$

The social optimum $(q^*, \Sigma^*_b, \Sigma^*_s)$ is characterized by (3.24) - (3.26) and (3.3).

In comparing the social optimum characterized by (3.24) - (3.26) and the monetary equilibrium characterized above, several differences are apparent: the quantity, and the search intensities, may differ. From (3.24) and (3.20), it follows that in order for the quantity traded to be efficient, the tax rate must be zero and monetary policy needs to follow the Friedman rule.

Moreover, comparing (3.18) to (3.25) and (3.19) to (3.26), it follows that efficiency in the number of trades requires $\Theta(q, \tau) = \eta(T)$: the buyer’s surplus share needs to equal her contribution to the creation of the match. Since at the Friedman rule $\Theta(q, \tau)$ reduces to $\theta$, the Hosios rule calls for $\theta = \eta(T^*)$. However, there is no apparent reason to believe that this requirement is satisfied, since this condition links a property of the matching function to the buyer’s bargaining power. If $\theta < \eta(T^*)$, the buyer’s bargaining share is too small and her search intensity too low.\footnote{To see this, compare (3.18) to (3.25) with $\tau = 0$ and note that $\phi(\sigma)$ is a convex function.} In order to improve efficiency, $\sigma_b$ needs to increase relative to $\sigma_s$, so $T$ needs to decrease. Conversely, if $\theta > \eta(T^*)$, the equilibrium market thickness is too low and an increase in $T$ will improve efficiency.

\section{3.5 The Welfare Effects of Fiscal and Monetary Policy}

From the previous sections, it is evident that the monetary equilibrium is unlikely to coincide with the first best outcome if there is no fiscal policy and monetary policy
simply follows the Friedman rule. This section studies how fiscal and monetary policy can improve efficiency in this environment and demonstrates that monetary, but not fiscal, policy alters the agents’ bargaining position, rendering a deviation from the Friedman rule optimal whenever the buyer’s bargaining weight is small relative to her contribution to the match. This result is summarized in the following proposition.

**Proposition 3.5.1.** If $\theta < \eta(T^*)$, $\tau^* > 0$ and $\gamma^* > 0$, the optimal policy calls for a positive sales tax and a deviation from the Friedman rule.

The proof is laid out in three parts. First, section 5.1 derives the optimal tax rate at the Friedman rule. Section 5.2 then establishes that without fiscal policy, it is optimal to deviate from the Friedman rule if $\theta < \eta(T^*)$. Lastly, section 5.3 combines the previous two results and proves that an optimal policy mix consists of using the two instruments jointly.

### 3.5.1 Fiscal Policy

If the buyer’s bargaining weight is too low relative to her contribution to the match ($\theta < \eta(T^*)$), the buyer is not sufficiently rewarded for her search effort and will hence choose an inefficiently low search effort. This renders the market tight for sellers and thick for buyers, i.e. $T$ is inefficiently high. In order to improve efficiency, any policy needs to decrease the market tightness. To see that the market tightness decreases as the tax rate increases, divide (3.18) by (3.19) and impose the Friedman rule:

$$T_{|\gamma=\beta} = \left[ \left( \frac{1 - N}{N} \right)^{\alpha-1} \left( \frac{1 - \theta}{\theta} \right) \frac{1}{(1 + \tau)} \right]^{\frac{1}{\alpha}} \tag{3.27}$$

From (3.27), it follows that the market tightness at the Friedman rule is decreasing in the tax rate: $\frac{dT}{d\tau}_{|\gamma=\beta} < 0$.

As discussed above, the sales tax has three effects on the agents’ surplus. The sales tax reduces both agents’ surpluses because it distorts the quantity of goods exchanged
and levies a tax burden on them. The third effect, a change in the surplus split between buyer and seller, is not present here. At the Friedman rule, the buyer’s surplus share is equal to $\theta$, and is independent of the tax rate $\tau$. As a result, both the buyer’s and seller’s search intensity fall. However, the seller’s search effort falls relatively more than the buyer’s. This is because an increase in the tax reduces the present value of the amount of money exchanged in the match, $\omega(1 + \tau)$, making it relatively less attractive to be a seller and consequently lowering $T$.

Now, taking the derivative of the welfare function (3.23) with respect to the tax rate and evaluating at the Friedman rule gives

$$
\frac{\partial W}{\partial \tau} \bigg|_{\gamma=\beta} = \Psi \left[ c' \frac{\partial q}{\partial \tau} + \left( \frac{\theta}{\Sigma_b} \frac{\partial \Sigma_b}{\partial \tau} c(q) + \frac{1-\theta}{(1 + \tau)\Sigma_s} \frac{\partial \Sigma_s}{\partial \tau} u(q) \right) \right]
$$

$$
+ \Psi (\eta(T) - \theta) \frac{\Sigma_s}{\Sigma_b} \frac{\partial \Sigma_s}{\partial \tau} (u - c)
$$

$$
= \Psi \frac{1}{T} \frac{\partial T}{\partial \tau} \left[ -(\eta(T) - \theta)(u - c) + \tau \left( \frac{\alpha(c')^2}{c} + \frac{\theta}{\alpha-1} c(q) \left( 1 - \eta + \frac{(1+\tau)u}{u-(1+\tau)c} \right) + \left( \frac{1-\theta}{1+\tau} \right) \frac{1}{\alpha-1} u(q) \left( -\eta + \frac{(1+\tau)u}{u-(1+\tau)c} \right) \right) \right]
$$

(3.28)

Since $\frac{\partial T}{\partial \tau} < 0$, (3.28) indicates that $\tau^*_{\gamma=\beta} > 0$ if and only if $\theta < \eta(T)$.

The optimal tax rate at the Friedman rule, $\tau^*_{\gamma=\beta}$, solves

$$
(\eta(T) - \theta)(u - c) = \tau \left( \frac{\alpha(c')^2}{c} + \frac{\theta}{\alpha-1} c(q) \left( 1 - \eta + \frac{(1+\tau)u}{u-(1+\tau)c} \right) + \left( \frac{1-\theta}{1+\tau} \right) \frac{1}{\alpha-1} u(q) \left( -\eta + \frac{(1+\tau)u}{u-(1+\tau)c} \right) \right).
$$

(3.29)

If $\theta < \eta(T)$, the exogenous bargaining share of the buyer is too small and as a result her search intensity is too low and the seller’s effort too high. However, contrary to what may seem intuitive, paying a subsidy does not improve welfare in this case. As argued above, the buyer and seller split any subsidy or tax burden through the bargaining process. If the seller has a high bargaining power, he can extract most of the subsidy, further increasing his inefficiently high search effort and hence decreasing welfare.
A positive tax rate, on the other hand, reduces the seller’s search intensity relative to the buyer’s, making the market tighter for buyers and increasing efficiency. Both the buyer’s and seller’s search efforts fall, closing the wedge between social marginal benefit and social marginal cost of searching for the seller and widening the wedge for the buyer. The positive welfare effect is a result of the first effect dominating the second because the seller’s search effort falls relatively more.

3.5.2 Monetary Policy

To see how $T$ responds to an increase in the rate of money growth, combine the equilibrium conditions (3.25) and (3.20) to obtain

\[
\left(\frac{\gamma}{\beta} - 1\right)^{\alpha-1} = \left(\frac{u'}{c'(q)(1+\tau)} - 1\right)^{\alpha-1} \frac{[A_b(T)]^\alpha (u(q) - (1+\tau)c(q))}{\alpha \varphi_0 \left(1 + T^\alpha (1+\tau) \left(\frac{N}{1-N}\right)^{\alpha-1}\right)} \tag{3.30}
\]

After solving for $q = q(T)$ by dividing (3.18) by (3.19) and substituting into (3.30), the resulting expression gives $\frac{dT}{d\gamma} < 0$. This is a result of the surplus share effect described above. That is, if the rate of money growth exceeds the Friedman rule, the buyer becomes constrained by her money holdings, which allows her to credibly limit her offer to the seller, thus increasing her share of the total surplus. As a result, the buyer’s search effort increases relative to the seller’s effort and the market tightness $T$ decreases.

To analyze the welfare effect of increasing the rate of money growth above the rate prescribed by the Friedman rule, take the derivative of the welfare function (3.23) with respect to $\gamma$ and evaluate at $\gamma = \beta$. 
\[ \frac{\partial W}{\partial \gamma} \bigg|_{\gamma = \beta} = \Psi \tau \left[ c' \frac{\partial q}{\partial \gamma} + \left( \theta \frac{1}{\Sigma_b} \frac{\partial \Sigma_b}{\partial \gamma} c(q) + (1 - \theta) \frac{1}{\Sigma_s} \frac{\partial \Sigma_s}{\partial \gamma} u(q) \right) \right] \\
+ \Psi (\eta(T) - \theta) \frac{\Sigma_s}{\Sigma_b} \frac{\partial}{\partial \gamma} (u - c) \\
= \Psi \frac{1}{T} \frac{\partial T}{\partial \gamma} \left[ - (\eta(T) - \theta)(u - c) + \tau \left( \frac{\alpha (c')^2}{\alpha - 1} c(q) \left( (1 - \eta) - \alpha (1 - \theta) \right) \right) + (\frac{1 - \theta}{1 + \tau}) \frac{1}{\alpha - 1} u(q) \left( - \eta + \alpha \theta \right) \right] \\
(3.31) \]

With \( \tau = 0 \), there is a positive effect of increasing the rate of money growth at the Friedman rule if \( \theta < \eta(T) \). As argued, deviating from the Friedman rule raises the buyer's share of the surplus by tightening the money constraint, causing her to exert a higher search intensity and increasing social welfare. If, however, \( \theta > \eta(T) \), the Friedman rule is still constrained optimal. In this case, the Hosios rule demands a negative nominal interest rate which is not a feasible policy option.

The welfare improvement in deviating from the Friedman rule results from trading off the efficiency of the quantity traded against reducing the inefficiency of the search intensity. This is a typical second best/ optimal taxation result – the second order loss in the intensive margin is smaller than the first order gain in the extensive margin.

3.5.3 Optimal Policy Mix

It is apparent that both fiscal and monetary policy in isolation will be welfare improving, at least if \( \theta < \eta(T) \). However, the key question remains: is the Friedman rule optimal in the presence of fiscal policy, and how does the optimal policy mix \((\gamma^*, \tau^*)\) compare to the optimal rate of money growth in an environment without a sales tax?

To answer the first question, set \( \tau = \tau^* \big|_{\gamma = \beta} \) in the first order condition for the rate of money growth evaluated at the Friedman rule (3.31). Using (3.29), the resulting
expression can be simplified to

\[ W_\gamma(\tau^*_|\gamma=\beta, \beta) = \frac{1}{T} \frac{\partial T}{\partial \gamma} \tau^* \left( \frac{\theta}{\alpha-1} c(q) \left( -(1-\theta) - \frac{(1+\tau^*)c}{u-\tau^*c} \right) + \left( \frac{1}{1+\tau^*} \right)^{\frac{\alpha}{\alpha-1}} u(q) \left( \theta - \frac{(1+\tau^*)u}{u-\tau^*c} \right) \right) > 0 \text{ if } \theta < \eta \]

\[ < 0 \text{ if } \theta > \eta \] (3.32)

Since both \( \frac{\partial T}{\partial \gamma} \) and the expression in round brackets are negative, the sign of (3.32) is the same as \( \tau^*_|\gamma=\beta \). As shown above, \( \tau^*_|\gamma=\beta > 0 \text{ if } \theta < \eta \); hence with distortionary taxation, increasing the rate of money growth above the rate of time preference is still welfare improving if the buyer’s bargaining power is lower than her contribution to the match creation. Thus, the conditions for the suboptimality of the Friedman rule are the same as without fiscal policy.

To understand this optimal policy, it is important to remember the surplus share effect of monetary policy and how it affects the effectiveness of fiscal policy. At the Friedman rule, fiscal policy does not have an impact on the bargaining weights: the buyer receives a \( \theta \) fraction of the surplus, independent of the tax rate. If, however, the rate of money growth exceeds the rate of time preference, the sales tax will further improve the buyer’s bargaining position and the buyer’s surplus share increases in \( \tau \). This channel of fiscal policy only works if the buyer’s money constraint is binding, leaving a special role for monetary policy.\(^\text{12}\)

### 3.6 Competitive Pricing

While the analysis in section 5 demonstrates that monetary policy can play a special role in achieving efficiency and cannot be replaced by distortionary taxation, it is impor-
tant to note that this conclusion carries over to different pricing mechanisms. As this section demonstrates, a deviation from the Friedman rule is optimal with competitive pricing.\(^{13}\)

With price taking, the search friction is modelled as entry into a specific market for good \(h\). Once in the market, buyers and sellers observe the price and the buyer demands \(q^b\) while the seller offers \(q^s\). To make the two models comparable, let the entry function be the matching function described earlier; in this case, the number of buyers equals the number of sellers in the market. Thus, this setup can also be interpreted as bilateral matching with the buyers and sellers taking prices as given. So, the probability of a buyer entering the market is the same as her finding a trading partner in the environment with bilateral matching and bargaining, \(A_b(T)\sigma_b\); similarly, the probability of trading for the seller is \(A_s(T)\sigma_s\).

The household’s problem is now given by

\[
v(m) = \max_{\{q^b, q^s, \sigma_b, \sigma_s, m+1\}} \left\{ \begin{array}{c} n\sigma_b A_b(T) u(q^b) - (1 - n)\sigma_s A_s(T) c(q^s) \\ -n\phi(\sigma_b) - (1 - n)\phi(\sigma_s) + \beta v(m+1) \end{array} \right\}
\]

subject to

\[
pq^b \leq \frac{m}{n}
\]

and the law of motion for money

\[
m_{+1} = m + (1 - n)\sigma_s A_s(T) \frac{pq^s}{(1 + \tau)} - n\sigma_b A_b(T)pq^b + L.
\]

\(^{13}\)In monetary search models, a variety of pricing mechanisms other than bargaining have been used. Head and Kumar (2005) introduce price-posting with random search into the large household framework. Rocheteau and Wright (2005) study several different pricing mechanisms (bargaining, competitive equilibrium, competitive search) in the LW framework (Lagos and Wright, 2005). They show that competitive search leads to an efficient monetary equilibrium – fiscal policy is unnecessary and deviating from the Friedman rule is not welfare improving. However, in the case of price taking, the Friedman rule may not be optimal in their environment.
Imposing stationarity and symmetry, the equilibrium condition is given by

$$\frac{u'(q)}{c'(q)} = \left[ 1 + \frac{1}{\sigma_b A_b(T)} \left( \frac{\gamma}{\beta} - 1 \right) \right] (1 + \tau). \quad (3.34)$$

which is the same as the one with bargaining in (3.20). Solving for the buyer’s surplus gives

$$u(q) - \omega pq = u(q) - u'q + \frac{1}{\sigma_b A_b(T)} \left( \frac{\gamma}{\beta} - 1 \right) (1 + \tau)c'(q). \quad (3.35)$$

Evaluating (3.35) at the Friedman rule and imposing the Hosios rule gives

$$\frac{(u(q) - u'q)}{(u(q) - c(q))|_{\gamma=\beta}} = \eta. \quad (3.36)$$

This condition in general need not to be satisfied. With linear utility, it furthermore reduces to $\eta = 0$, which is not admissible. However, it is obvious that this condition need not be satisfied with a concave utility function either.

Imposing a sales tax at the Friedman rule reduces both the buyer’s and seller’s surpluses, with the seller’s surplus falling more than the buyer’s. Hence, imposing a sales tax will improve welfare if the market thickness at the Friedman rule was too high, i.e. if the market was congested by sellers.\(^\text{14}\) However, just as with bargaining, increasing the rate of money growth is the more efficient policy: from (3.35), it is apparent that the buyer’s surplus increases as the rate of money growth increases (in the neighborhood of the Friedman rule), which grows the buyer’s search intensity. Similarly, the seller’s surplus and search intensity fall, reducing the congestion and increasing efficiency.

The rationale behind this result is similar as with bargaining. Deviating from the Friedman rule makes it less attractive to be a seller. The discounted value of the amount of money paid for one unit of goods, $\omega p$, falls; the seller’s market power is not sufficient to increase the price such that $\omega p$ stays constant, which is essentially the surplus share

\(^\text{14}\)This is a result of the constant marginal utility. It makes the demand very elastic, hence the seller has to bear most of the burden. With decreasing marginal utility, the buyer bears a greater share of the burden, making a sales tax even less effective.
effect from above. On the other hand, the buyer will try to spend the money more quickly because it looses value over time, while the present value of her expenditure falls.

3.7 Conclusion

This paper introduces a sales tax into the monetary search framework. A departure from the Friedman rule is optimal whenever there is a thick market on the buyer’s side, i.e., if there are too many sellers relative to buyers in the market. If the buyer’s bargaining weight is too small, the buyer’s search effort will be inefficiently low, causing a congestion in the market. By restricting the buyer’s money balance, monetary policy changes the agents’ bargaining position, allowing the buyers to extract a larger fraction of the trade surplus. A sales tax cannot reproduce this surplus share effect, leaving a role for a deviation from the Friedman rule as an optimal monetary policy.

Moreover, this finding does not depend on bargaining as the pricing mechanism; the surplus share effect of monetary policy carries over to an environment with competitive pricing. Key to this effect is the change in the future value of money due to inflation, which makes it less attractive to be a seller and induces buyers to search more. This reduces the market tightness, which cannot be achieved by a sales tax.

This paper focuses only on the scenario in which the buyer’s contribution to the creation of the match exceeds the exogenous Nash bargaining weight. In that case, deviating from the Friedman rule can improve efficiency because it increases the buyer’s surplus share above her exogenous bargaining parameter. In the opposite situation, the exogenous bargaining parameter is the lower bound for the buyer’s surplus share; a negative nominal interest rate is not a feasible policy option, so the Friedman rule is the constrained optimal policy. However, fiscal policy can achieve some welfare improvement by setting a negative tax rate – paying a subsidy to the agents.
Bibliography


Appendices
Appendix to Chapter 1

A.1.1 Algorithm to Compute Transition

1. Compute autarky and trade steady states.

2. Guess the number of periods for the transition path $T$.

3. Guess the time path of value functions $\{\bar{U}^0(\Sigma_t), \bar{U}^{E,0}(\theta, \Sigma_t), J^0_i(\theta, \Sigma_t)\ldots\}_{t=0}^T$.

4. Starting with the autarky distribution of workers and given trade prices and next period’s values, compute the first period equilibrium.

5. Using the resulting distribution and the future values, compute the following period’s equilibrium. Continue until period $T$.

6. Using the sequence of equilibria, compute the resulting sequence of value functions $\{\bar{U}^1(\Sigma_t), \bar{U}^{E,1}(\theta, \Sigma_t), J^1_i(\theta, \Sigma_t)\ldots\}_{t=0}^T$.

7. If $\bar{U}^0(\Sigma_t) \approx \bar{U}^1(\Sigma_t)$, $\bar{U}^{E,0}(\Sigma_t) \approx \bar{U}^{E,1}(\Sigma_t)$, $\ldots$ $\forall t$, we have convergence; if not, redo (4)-(6).
### A.1.2 Offshorable Service Occupations

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<th>Occupation Title</th>
</tr>
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<tbody>
<tr>
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<td>Financial managers</td>
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<tr>
<td>23</td>
<td>Accountants and auditors</td>
</tr>
<tr>
<td>24</td>
<td>Underwriters</td>
</tr>
<tr>
<td>26</td>
<td>Management analysts</td>
</tr>
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<td>Aerospace engineers</td>
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<tr>
<td>45</td>
<td>Metallurgical and materials engineers</td>
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<td>46</td>
<td>Mining engineers</td>
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<td>Petroleum engineers</td>
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<td>Chemical engineers</td>
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<td>49</td>
<td>Nuclear engineers</td>
</tr>
<tr>
<td>53</td>
<td>Civil engineers</td>
</tr>
<tr>
<td>54</td>
<td>Agricultural engineers</td>
</tr>
<tr>
<td>55</td>
<td>Electrical and electronic engineers</td>
</tr>
<tr>
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<td>Industrial engineers</td>
</tr>
<tr>
<td>57</td>
<td>Mechanical engineers</td>
</tr>
<tr>
<td>58</td>
<td>Marine engineers and naval architects</td>
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<tr>
<td>59</td>
<td>Engineers, n.e.c.</td>
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<tr>
<td>63</td>
<td>Surveyors and mapping scientists</td>
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<tr>
<td>64</td>
<td>Computer systems analysts and scientists</td>
</tr>
<tr>
<td>65</td>
<td>Operations and systems researchers and analysts</td>
</tr>
<tr>
<td>66</td>
<td>Actuaries</td>
</tr>
<tr>
<td>67</td>
<td>Statisticians</td>
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<tr>
<td>68</td>
<td>Mathematical scientists, n.e.c.</td>
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<td>69</td>
<td>Physicists and astronomers</td>
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<td>Chemists, except biochemists</td>
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<td>78</td>
<td>Biological and life scientists</td>
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<tr>
<td>166</td>
<td>Economists</td>
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<td>Typists</td>
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<td>Mail clerks, exc. postal service</td>
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<td>363</td>
<td>Production coordinators</td>
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<tr>
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<td>General office clerks</td>
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<td>385</td>
<td>Data-entry keyers</td>
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<tr>
<td>386</td>
<td>Statistical clerks</td>
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</table>
Appendix to Chapter 3

A3.1 Proof of Lemma 3.2.1.

The constraints are given by
\[
\begin{align*}
\Omega x^b / (1 + \tau) - c(q^b) &= (1 - \theta \Delta) \left[ \Omega X^s / (1 + \tau) - c(Q^s) \right] \\
u' Q^s - \omega X^s &= (1 - (1 - \theta) \Delta) \left[ u' q^b - \omega x^b \right] \\
x^b &= X^s = m/n
\end{align*}
\]

Rearranging gives
\[
\begin{align*}
Q^s &= Q^s(q^b, \Delta) = \frac{1}{u'} \left[ \omega \frac{m}{n} + (1 - (1 - \theta) \Delta) \left( u' q^b - \omega x^b \right) \right] \\
\frac{\partial}{\partial \Delta} Q^s(q^b, \Delta) &= -\frac{1 - \theta}{u'} \left[ u' q^b - \omega \frac{m}{n} \right]
\end{align*}
\]

In equilibrium \( Q^s(q^b, 0) = q^b \), so
\[
\Omega \frac{m/n}{1 + \tau} - c(q^b) = (1 - \theta \Delta) \left[ \Omega \frac{m/n}{1 + \tau} - c \left( Q^s(q^b, \Delta) \right) \right],
\]
which can be rearranged to
\[
\theta \Omega \frac{m/n}{1 + \tau} = \frac{1}{\Delta} \left[ c(q^b) - (1 - \theta \Delta) c \left( Q^s(q^b, \Delta) \right) \right].
\]

Take limit \( \Delta \to 0 \)
\[
\theta \Omega \frac{m/n}{1 + \tau} = \frac{1}{\theta u' + (1 - \theta)(1 + \tau) c'} \left( \theta c(q) + (1 - \theta) c'(q) q \right),
\]
which can be rearranged to
\[
\frac{\omega}{n} = \frac{(1 + \tau) u'}{\theta u' + (1 - \theta)(1 + \tau) c'} \left( \theta c(q) + (1 - \theta) c'(q) q \right)
\]

So, the seller’s surplus is given by
\[
\frac{\omega}{n} = \frac{(1 - \theta) c'}{\theta u' + (1 - \theta)(1 + \tau) c'} \left( u' q - (1 + \tau) c(q) \right),
\]
and the buyer’s surplus is given by

\[ u'q - \omega \frac{m}{n} = \frac{\theta u'}{\theta u' + (1 - \theta)(1 + \tau)c'} (u'q - (1 + \tau)c(q)). \]