THE DESIGN OF COMPACT PLANAR ANTENNAS FOR LAPTOP APPLICATIONS
BASED ON METAMATERIAL CONCEPTS

by

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Graduate Department of Electrical Engineering
University of Toronto

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Abstract

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Master of Applied Science
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University of Toronto
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Two laptop antennas are presented using two different designs based on metamaterials. The first design consists of planar monopole loaded with an electric-LC resonator (ELC). This novel topology allows for the realization of a multi-band antenna by using the ELC to add multiple resonances. This structure is analyzed using full-wave simulations. A circuit model is also developed to gain further understanding. This technique is then used to design a Wi-Fi antenna. The second design uses a modified double-tuned matching network to create a single-band match for a planar monopole antenna. The matching network is implemented using a complementary-split-ring-resonator (CSRR). The design is once again analyzed using full-wave simulations and a circuit model is also developed. This technique is then applied to design a WiMax antenna. Both the Wi-Fi and WiMax antennas are fabricated and show good agreement between the simulated and measured results.
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No thesis is a solitary endeavour despite the work being attributed to a single author. This thesis was no exception and I was aided and encouraged down this road by many people.

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March 2010
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<tr>
<td>ELC</td>
<td>Electric-LC</td>
</tr>
<tr>
<td>CSRR</td>
<td>Complementary Split Ring Resonator</td>
</tr>
<tr>
<td>NRI-TL</td>
<td>Negative Refractive-Index Transmission-Line</td>
</tr>
<tr>
<td>EBG</td>
<td>Electromagnetic Band Gap</td>
</tr>
<tr>
<td>SRR</td>
<td>Split Ring Resonator</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
<tr>
<td>USB</td>
<td>Universal Serial Bus</td>
</tr>
<tr>
<td>VSWR</td>
<td>Voltage Standing Wave Ratio</td>
</tr>
<tr>
<td>PIFA</td>
<td>Planar Inverted-F Antenna</td>
</tr>
<tr>
<td>FR-4</td>
<td>Flame Retardant 4 (A type of substrate)</td>
</tr>
<tr>
<td>HFSS</td>
<td>High Frequency Structural Simulator</td>
</tr>
<tr>
<td>SMA</td>
<td>Sub-Miniature version A</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>TEM</td>
<td>Transverse Electric Magnetic</td>
</tr>
<tr>
<td>PEC</td>
<td>Perfect Electric Conductor</td>
</tr>
<tr>
<td>PMC</td>
<td>Perfect Magnetic Conductor</td>
</tr>
<tr>
<td>CPW</td>
<td>Co-Planar Waveguide</td>
</tr>
<tr>
<td>LC</td>
<td>Inductor-Capacitor</td>
</tr>
<tr>
<td>RC</td>
<td>Resistor-Capacitor</td>
</tr>
<tr>
<td>RL</td>
<td>Resistor-Inductor</td>
</tr>
<tr>
<td>RLC</td>
<td>Resistor-Inductor-Capacitor</td>
</tr>
<tr>
<td>ADS</td>
<td>Advanced Design System</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyzer</td>
</tr>
<tr>
<td>AUT</td>
<td>Antenna Under Test</td>
</tr>
<tr>
<td>GHz</td>
<td>Gigahertz ($10^9$Hz)</td>
</tr>
<tr>
<td>MHz</td>
<td>Megahertz ($10^6$Hz)</td>
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Chapter 1

Introduction

1.1 Goal

Metamaterial antennas and laptop antennas are two areas of research which have produced a variety of different techniques for designing antennas. Over the past 6-7 years metamaterial antennas have garnered a lot of attention for their promising capabilities. Laptop antennas meanwhile, are a more mature area where a lot of work has been done in creating and refining a variety of practical designs. With these two separate areas, the question that this thesis attempts to answer is as follows: can metamaterial design techniques be successfully applied to design laptop antennas? The first step in answering this question is to understand the specifications of such a project.

1.1.1 Specifications

All laptop antennas have a general set of criteria that have to be met. These are summarized in [1], pp. 1113-1114, and some of the requirements are listed here as follows:

- Multi-Band/Broadband Operation: Depending on the communication protocol being used, the antenna must work at multiple frequencies or have enough bandwidth to transmit and receive data.

- Input Impedance Matching: At the operating frequencies, the input impedance of the antenna must be matched below a given VSWR for a system impedance of $Z_o = 50 \ \Omega$. In this thesis this is given by a VSWR of 2:1, or equivalently an $|S_{11}| \leq -10 \ dB$

- Omnidirectional Radiation Pattern: The pattern must be omnidirectional to ensure reliable connectivity independent of the location of the laptop relative to the base station
Figure 1.1: A diagram showing possible locations for an antenna on a laptop computer. For the antennas discussed in this thesis, 4A, 4B, and 4C are the desired locations, placing constraints on the geometry of the antenna. Reprinted with permission from [1], p. 1123. © 2010 Wiley.

- Polarization: Polarization purity is not important as the laptop operates in an environment with a lot of scattering

- Geometry: The antenna must fit into the space allocated in the overall system and must be low-profile.

For the specifications governing the antennas presented in this thesis more specific guidelines are given regarding the geometry and the multi-band operation. These are elaborated on below.

Geometry

The first specification regarding the geometry of a laptop antenna is the location of the antenna. This will determine the available volume for the antenna. Fig. 1.1 from [1], p. 1123, shows possible locations for an antenna to be placed on a laptop computer. Through a discussion with Intel [2], it was decided that the laptop antenna would be designed to fit on the frame of the monitor as shown in Fig. 1.1. This location is an optimal location to place a laptop antenna, a result that has also been confirmed in the literature as seen in [3], [4].

Placing the antenna on the frame of the laptop monitor also places constraints on the geometry of the antenna. To place the antenna on the frame of the monitor, the antenna must have a narrow width and a low profile. This translates into the width of the antenna being kept less than 10 mm and the height of the antenna being kept less than 1 mm. There are no constraints along the length of the antenna, however this dimension should be minimized as much as possible to keep the antenna compact. In the designs that will be presented in this thesis, the length of the antenna will be kept less than 50 mm long. These constraints are illustrated in Fig. 1.2.
An important constraint related to the geometry of the antenna is the size of the ground plane. An ongoing trend in laptop design is to remove the ground plane from behind the screen, thus removing the large ground plane for the antenna [2]. This implies that any ground plane that the antenna relies on must be contained in the space shown in Fig. 1.2. Thus the antenna designs developed in this thesis must compensate for this and the ground plane must be very compact and its role in the design of the antenna must be mitigated. This coupled with the volume that is available for the antenna shows that the antenna design must be kept compact to meet these geometric constraints.

Another consideration is the substrate that the antenna is placed on. The antennas presented in this thesis will be designed on FR-4 as it is a low-cost substrate that is commonly used in printed-circuit board designs. FR-4 has a dielectric constant of $\varepsilon_r = 4.34$, and a loss tangent of $\tan\delta = 0.016$. The height of the substrate is 0.4 mm, allowing the antenna to maintain a height less than 1 mm. The substrate also has metallization layers on both sides. These layers are colour-coded in the schematics with one layer being orange and the other blue. This legend can be seen in Fig. 1.3.
Table 1.1: Wi-Fi and WiMax antenna specifications

<table>
<thead>
<tr>
<th></th>
<th>Frequency Bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wi-Fi</td>
<td>2.40 GHz-2.485 GHz</td>
</tr>
<tr>
<td></td>
<td>5.15 GHz-5.85 GHz</td>
</tr>
<tr>
<td>WiMax</td>
<td>2.30 GHz-2.70 GHz</td>
</tr>
<tr>
<td></td>
<td>3.30 GHz-3.80 GHz</td>
</tr>
</tbody>
</table>

Multi-Band/Broadband Operation

For the multi-band operation of the antenna, the designs presented in this thesis will target the Wi-Fi or WiMax specifications which dictate the design frequencies of the antenna. The frequency bands for both these specifications are given in Table 1.1. It is noted that both specifications are multi-band and that the WiMax specifications require a lot of bandwidth in each band at lower frequencies, a difficult challenge.

1.2 A Review Of Some Relevant Antennas

With the specifications for the antenna designs laid out, it is now useful to review the work that has already been published in the literature to give some context to the antenna designs that will be presented in the following chapters. Some relevant antenna designs will be reviewed from the literature in metamaterial antennas and laptop antennas.

1.2.1 Metamaterial Antennas

The use of metamaterials in antenna design took place in four main areas:

1. Leaky Wave Antennas
2. Artificial Substrates
3. Small Antennas
4. Multi-band and Broadband Antennas

Leaky Wave Antennas

One-dimensional periodic leaky wave antennas radiate due to the fast wave propagation excited along the periodic structure. Metamaterials made from one-dimensional negative-refractive-index transmission-lines (NRI-TL) are periodic structures which can radiate a fast wave over a certain range of frequencies as shown in the dispersion diagram in Fig. 1.4(a). Another
interesting property of NRI-TL metamaterials is their ability to excite a backward wave [5]. Combining these two properties in a leaky-wave antenna gives a backward fast-wave that can be excited on a 1-D NRI-TL. If a leaky-wave antenna is constructed from unit cells exhibiting this property, the radiation beam formed from the metamaterial leaky wave antenna emerges at a negative angle as first demonstrated in [6]. A radiation pattern from [6] can be seen in Fig. 1.4(b) where the main beam can be observed to be emerging at a negative angle. An interesting note about these metamaterial leaky wave antennas is that the fast wave is excited in the fundamental mode of the NRI-TL as opposed to 1-D periodic leaky-wave antennas where the $n = -1$ mode is the fast-wave mode [1], p. 329. This design of NRI-TL leaky-wave antennas can scan from endfire to broadside and back to endfire by closing the stopband between the left-handed and right-handed bands as demonstrated in [7]. The stopband is closed by enforcing the condition given in [8] which allows for continuous scanning of the leaky-wave antenna.

Another interesting design of a metamaterial leaky-wave antenna is given in [9] where an NRI-TL leaky-wave antenna is designed to reduce the beam squinting of the antenna. This was done by designing the NRI-TL unit cell to minimize the difference between the phase velocity and group velocity which subsequently reduces the beam squint in the beam formed by the antenna. Unlike other NRI-TL leaky-wave antennas, this antenna was designed to operate in the right-handed band of the NRI-TL dispersion relation where the beam squint was minimized.

Other metamaterial leaky-wave designs included those found in [10], [11], [12].
dimensional metamaterial leaky-wave antennas can also be constructed and are discussed in [13] and [14]. Overall it can be seen that a metamaterial approach to leaky wave antenna design leads to unique and interesting properties in the performance of the antenna.

### Artificial Substrates

Another approach is the use of artificial substrates for antennas. The kind of antennas that benefit from this approach are often patches or printed dipoles which normally sit on dielectric substrates. This is an area of research that was preceded by the work done using electronic band gap (EBG) structures [15] [1], pp. 737-779. The use of EBG’s as a substrate for a patch or dipole antenna is done to increase the gain of the antenna. However by using metamaterial substrates, other features can be engineered such as miniaturization of the antenna itself. Two examples of this technique are discussed below which show how a patch antenna lying on a metamaterial substrate can be miniaturized.

The first example is the design of a subwavelength patch antenna sitting on a metamaterial substrate as discussed in [16]. The design of the antenna consists of a patch antenna sitting on an inhomogeneous substrate as shown in Fig. 1.5. The substrate consists of two concentric discs. The center disc of the substrate has effective material parameters \( \varepsilon_2, \mu_2 \) and the outer disc has effective material parameters given by \( \varepsilon_1, \mu_1 \). By analyzing the modes excited under the patch lying on such a substrate it is found that by making \( \mu_2 \) negative and \( \varepsilon_1 \) positive, the dimensions of the patch can be shrunk while still exciting the fundamental mode of the patch (\( TM_{110} \)) to radiate. While [16] shows theory and simulated results only, it can be seen that physical realization of this antenna would rely on a substrate composed of very electrically small split-ring resonators (SRR) embedded underneath a patch antenna, posing a difficult fabrication challenge.

The second method for miniaturizing a patch antenna using a metamaterial substrate is seen in [17], [18], where an artificial high-\( \mu \) substrate is designed using SRR’s. The SRR is a disper-
sive metamaterial element that achieves a negative $\mu$ above resonance. Just below resonance however, the $\mu$ increases rapidly giving a large effective permeability. A patch antenna sitting on a magneto-dielectric substrate with a large $\mu$ can be miniaturized (due to the decreasing effective wavelength). Thus by creating a metamaterial substrate that exhibits a large $\mu$, (a task that is very difficult at microwave frequencies using ‘normal’ materials) the patch can be miniaturized. A picture of the patch antenna on a metamaterial substrate is shown Fig. 1.6. It should be noted that the bandwidth of the patch is small and is constrained by the dispersion of the metamaterial substrate.

**Small Antennas**

The application of metamaterials to small antenna design found promise through the use of the subwavelength unit cells that make up a metamaterial. By creating antennas using a small number of electrically small unit cells, a variety of different approaches emerged to miniaturize antennas. A sampling of the approaches taken using metamaterials are discussed below.

One approach is using the dispersion relation of an NRI-TL unit cell. The dispersion relation of NRI-TL metamaterials can be engineered to achieve a desired phase shift across a unit cell, at a given frequency. This engineering of the dispersion relation can allow for the realization of large or small phase shifts using a small number of unit cells. Because the unit cells of NRI-TL metamaterials are very much subwavelength, a small number of these unit cells properly configured can form an antenna that is also electrically small. An example of this design approach was proposed in [19] and analyzed more thoroughly in [20] where an antenna is formed to feed four closely spaced monopoles in phase using four NRI-TL unit cells. An example of the antenna is shown in Fig. 1.7. Here, each unit cell is designed to obtain a $0^\circ$ phase shift, feeding the monopoles in phase. A folded monopole effect allows the small radiation
resistance of the electrically small monopole to be increased to 50 Ω creating a narrow-band, well matched, efficient radiator.

Another example of this design technique is applied to patch antennas where a subwave-length patch antenna is realized by constructing the patch out of NRI-TL unit cells [21]. A normal patch antenna can be viewed as a $\frac{1}{4}$ resonant transmission-line, which implies a $180^\circ$ phase shift from one end of the patch to the other. In the metamaterial case the unit cells used in the patch are designed to incur a total $180^\circ$ phase shift to create a resonance at a desired frequency. When designing the metamaterial antenna using the left-handed band, the ability to achieve a larger and larger phase shift at lower and lower frequencies for a fixed number of unit cells is what allows the antenna to be miniaturized. This is illustrated in Fig. 1.8, where a dispersion relation for an NRI-TL unit cell can be seen. In [21] examples are demonstrated using two and four unit cells respectively.

A variation of this idea can be seen again in Fig. 1.9 where a dipole antenna, using metamaterial unit cells is built [23]. Ten unit cells are designed to achieve a total $180^\circ$ phase shift in an electrical length of $\frac{1}{4}$, halving the size of the dipole. While this antenna is not electrically small it demonstrates the design concept of shrinking the size of the antenna.

In many of the NRI-TL antenna designs, multiple resonant bands can also be achieved in the antenna by taking advantage of the $\pm180^\circ$, $\pm360^\circ$, $\pm540^\circ$, etc., phase shifts that occur at different frequencies in the left-handed and right-handed behaviour of the antenna [24] [25] [22]. This is illustrated in the dispersion relation from [22] in Fig. 1.8 where multiples of $\pm180^\circ$ phase shifts can be achieved using a fixed number of NRI-TL unit cells. By designing the antenna to be matched at these points, small multi-band antennas can be created [25].

Another conceptual approach in the design of small metamaterial antennas can be seen in [26] where a Split-Ring Resonator (SRR) is used to create an electrically small antenna.
Chapter 1. Introduction

Figure 1.8: The dispersion relation of an NRI-TL unit cell. It can be seen that at lower frequencies, larger phase shifts can be achieved. This plot also shows how $\pm 180^\circ$, $\pm 360^\circ$, $\pm 540^\circ$ phase shifts can be achieved using $N$ unit cells. Reprinted with permission from [22]. © 2010 IEEE.

Figure 1.9: A schematic of a dipole antenna using NRI-TL unit cells. Reprinted with permission from [23]. © 2010 IEEE.

the NRI-TL unit cells which are subwavelength, the SRR is also a subwavelength structure. In [26] an electrically small monopole antenna is used to feed a single SRR over a large ground plane as shown in Fig. 1.10. The antenna is narrowband with a 2.42% bandwidth and an efficiency of 43.6%.

Finally another approach for designing small antennas using metamaterials is to use metamaterial shells surrounding an electrically small dipole as shown in Fig. 1.11 [27] [28]. It is
well known that electrically small antennas are inefficient due to their small input resistance and large reactance. Here by using a metamaterial shell, consisting of a dispersive negative-ε material, the reactive near field of a dipole is canceled out. This allows the antenna to achieve narrowband matching and a high efficiency.

The difficulty with designing electrically small metamaterial antennas is that the fundamental limits given by Chu [29], [30] do not allow for bandwidths that can be used in practical antenna design. While the metamaterial design techniques discussed above can be applied to help match the antenna to 50 Ω and/or maintain a good efficiency they do not increase the bandwidth. As discussed in Section 1.1.1, the specifications for the antennas presented in this thesis preclude the antenna from being electrically small as large bandwidths with multiple
bands are needed.

Multi-Band and Broadband Antennas

Other attempts to design metamaterial antennas included designs that are multi-band and broadband, though not electrically small, breaking away from the paradigm above. An example of this is a broadband metamaterial antenna demonstrated in [31] where a monopole antenna is loaded with an NRI-TL unit cell as shown in Fig. 1.12(a). Here the metamaterial unit cell increases the bandwidth of the monopole by adding another resonance to the antenna at a lower frequency. This can be seen in the measured $|S_{11}|$ in Fig. 1.12(b) of the antenna with and without the metamaterial loading. This design can also be seen as a planar version of the ring antenna shown in Fig. 1.7 as a folded monopole effect is achieved again at higher frequencies when the metamaterial unit cell is at its $0^\circ$ phase point.

Another broadband metamaterial design uses SRR’s that are directly fed by a monopole antenna as shown in Fig. 1.13 [32]. Here, along with the metamaterial loading, a slot in the ground plane is included to act like a matching network which helps match the antenna over a broad range of frequencies. A 63% -10 dB bandwidth is achieved by exciting different parts of the antenna at different frequencies (the monopole at some frequencies and the SRR’s at others). A multi-band metamaterial antenna is also demonstrated in [33] and is shown in Fig. 1.14. Here a dipole antenna is made to work at multiple, frequencies through the addition of metamaterials. By coupling SRR’s to the arms of the dipole, the SRR adds resonances to the antenna, along with the dipole resonance itself. By controlling the geometry of the
SRR, different resonant frequencies can be achieved and a multi-band antenna is formed. This technique is demonstrated for a dual-band and tri-band case. Thus, by avoiding the small antenna paradigm that is common in metamaterial antenna design, some promising design techniques can be demonstrated.

1.2.2 Laptop Antennas

Antennas are integrated into laptops to access communication networks such as wireless local area networks (WLAN). As other emerging communication standards such as WiMax become more popular, laptop systems will also be forced to adapt to include coverage for those frequency bands as well. Currently, many of the laptop antennas presented in the literature are targeted
for covering the Wi-Fi frequency bands. The many different designs can be broken down into the classification shown in Fig. 1.15 from [1], p. 1085. There are two main categories of antennas, external and internal. External antennas are found on wireless cards, or USB keys, and can consist of dipoles, monopoles, slots, patches, inverted-F, planar inverted-F and chip antennas. Internal antennas tend to consist of slots, patch antennas, monopoles, microstrip and inverted-F antennas. With WLAN’s more prevalent today, internal antennas are becoming standard on laptops. A review of some of the more common design approaches for internal antennas follows.

One of the more common designs found in laptop antennas is the inverted-F antenna [34] [35] [36]. A picture of an inverted-F antenna for a laptop can be seen in Fig. 1.16(a) from [36]. The inverted-F antenna is a planar design and consists of a quarter wavelength arm parallel to a ground plane. The arm is shorted at one end and open at the other, with the open end radiating. Because of the ground plane it can be viewed from image theory as a two-wire $\frac{1}{4}$ transmission-line resonator. By choosing the appropriate feed point along the length of the arm, the antenna can be matched to 50 $\Omega$. In general, the inverted-F antenna for laptop computers is designed to be multi-band to cover the WLAN/Wi-Fi specifications. In the inverted-F antenna shown in Fig. 1.16(b) this is done by providing different current paths for the low and high-frequency bands to create multiple resonances.

Another common design is the monopole antenna which can take on many different configurations. The basic monopole is a quarter wavelength long arm over a ground plane, fed by a coaxial cable. However these are rarely used as an internal laptop antenna [1], p. 1120. Instead, variations such as tapering the monopole, meandering the monopole, or adding multiple arms can be used to create a broadband and/or multi-band effect. An example of a tapered
monopole which provides a broadband response can be seen in [37]. By tapering the width of the monopole, the bandwidth of the monopole is increased to create a broadband response with a 2:1 VSWR bandwidth greater than 50%.

A meander-line antenna consists of a radiating wire that has been repeatedly bent. This allows for an electrically long current path in a physically small space giving a compact antenna. By adjusting the length of the meandered line the resonant frequency can be adjusted. Also by adjusting the amount of bending or meandering in the line along with other parameters such as the thickness of the line, the input impedance of the antenna can be matched [1], p.492. A meandered-line monopole is seen in [38] and is shown in Fig. 1.17. This meander-line monopole achieves a single resonance in the 2.4 GHz band.

Similarly, a monopole with multiple arms is demonstrated in [39], [40]. This antenna creates
Chapter 1. Introduction

Figure 1.18: A picture of a dual-band branch antenna. Each arm provides a different current path to create a resonance at different frequency. Reprinted with permission from [39]. © 2010 IEEE.

current paths with different lengths to create multiple resonances in the antenna. As shown in Fig. 1.18 from [39], by adjusting the length of each arm the resonances of the antenna can be controlled at different frequencies creating a multi-band antenna.

Microstrip or patch antennas are another common element in laptop antenna design. Here, a simple half-wavelength patch can be placed at the back of a laptop screen. However, this does not provide omnidirectional radiation which is a desired characteristic in laptop antenna. To fix this issue, the patch can also be wrapped around the top of the screen to achieve a more omni-directional radiation pattern [41]. This also makes the antenna more compact. A variation of the patch that is similar to the inverted-F antenna is the planar-inverted-F antenna (PIFA) which consists of a quarter-wavelength patch with one side of the patch shorted. By cutting slots in a patch or PIFA or placing shorting pins or other reactive elements on the patch the antenna can be modified to create multiple bands [42] [43].

It is noted that a common theme in all of these designs is the large ground plane that is implicit in the design of these antennas. This is due to the assumption of a system ground that is located on the laptop behind the screen or under the keyboard. For example looking at the antennas presented in [35], [40], each antenna relies on a ground plane that is 45 mm × 93 mm and 260 mm × 200 mm respectively. However as stated above in Section 1.1, an ongoing trend is to remove the large ground plane that the antenna relies on. An example of this trend can be seen in Fig. 1.19 from [44], which consists of a dual-band Wi-Fi meander-line dipole antenna with no ground plane.
Figure 1.19: A dual band meandered line dipole. The antenna has no ground plane making it compact. The meandered lines provide current paths to match the antenna at two different frequencies. Reprinted with permission from [44]. © 2010 IEEE.

1.3 Motivation

Given the specifications listed above in Section 1.1 and the discussion of metamaterial and laptop antennas in Section 1.2, the reasons for using metamaterials to design a laptop antenna can be traced to their ability to miniaturize the size of the antenna. This is a major motivating factor in using the metamaterial design techniques to try and design a laptop antenna, especially to decrease the size of the ground plane as discussed in Section 1.1. However, it should be emphasized again that given the limits governing electrically small antennas by Chu [29], the metamaterial approach will not be used to create an electrically small antenna, but a physically compact one. This gives the antenna the possibility to also meet the bandwidth specifications given above.

Along with the size, the multi-band and bandwidth constraints of the antenna must be met. As already shown, metamaterials have shown promise in creating multi-band and/or wideband antennas, thus they also provide a means to achieving this constraint.

Other constraints such as the impedance matching, radiation pattern and polarization can be met through careful design of the antenna.

With the variety of metamaterial design approaches the question is which design technique to use. Leaky-wave antennas and artificial substrates cannot meet the geometrical constraints. Using the dispersion engineering techniques of transmission-line metamaterials is one possibility that can be both compact and multi-band. However, as shown in the literature, it is an approach that is difficult to achieve the desired bandwidth required at multiple bands even if the antenna is not electrically small. This is due to the dispersion relation of transmission-line metamaterials, where the phase shift per unit cell varies as a function of frequency, making it difficult to match
the antenna over a broad range of frequencies as required for some of the Wi-Fi and WiMax specifications.

One approach that is attractive is that reported in [26], [33] using metamaterial particles such as the SRR, to load an antenna such as a dipole or a monopole. This is an approach used in this thesis to design an antenna. It should be stated that the approach taken in this thesis using metamaterial particles was developed independently of the work presented in [26], [33]. However the approach that will be presented in this thesis has some similarities that are acknowledged here. Nonetheless, this approach has potential because the metamaterial particles themselves are physically compact and can be designed to fit in the geometrical constraints given above. Also, integrating these metamaterial particles into an antenna design allows for multi-band operation.

The other approach that will be taken, and one that has not been seen in the literature is the use of ‘resonant’ transmission-line metamaterial unit cells in the design of a matching network to increase the bandwidth of an antenna. For the Wi-Max specifications, a large bandwidth is required at the operating bands at lower frequencies which can be a difficult to meet with the antenna alone. By using a simple matching network that is implemented using ‘resonant’ transmission-line metamaterial networks, the bandwidth of the antenna can be increased. The matching network will also fit into the geometrical constraints given above keeping the antenna compact. While this approach does not yield a multi-band antenna it does demonstrate a simple technique for increasing the bandwidth of an antenna.

It should be noted that the designs presented in this thesis cannot be strictly considered to be made out of metamaterials since they do not have any effective index of refraction, permittivity or permeability. Instead these antennas are inspired by the properties of the unit cells that can be used to make up metamaterials. These unit cells are then appropriated to create an antenna that meets the specifications listed above, making the antenna based on metamaterial concepts.

It should also be emphasized that while the work presented throughout this thesis is motivated by and applied to a specific problem of designing laptop antennas, the design techniques can be applied to the design of compact, multi-band antennas in general. Thus the concepts themselves are not limited to the problem that is being solved but will hopefully be seen as one more tool that the antenna designer can use to tackle the larger task of designing compact, multi-band antennas for any application.

1.4 Overview

The rest of this thesis proceeds in the following manner.

Chapter 2 gives an overview of the non-metamaterial concepts used in this thesis. This
involves the planar monopole and double-tuned matching networks.

Chapter 3 gives an overview of the metamaterial concepts used in this thesis. This involves metamaterial particles, such as the Electric-LC resonator (ELC), and ‘resonant’ transmission-line metamaterials.

Chapter 4 goes over the concept and design of laptop antennas using metamaterial particles such as the ELC resonator to load a planar monopole antenna. This technique will then used to design a Wi-Fi antenna.

Chapter 5 demonstrates how to design a double-tuned matching network using a ‘resonant’ transmission-line metamaterial as a matching network. This technique is applied to design an antenna that covers the 2.3 GHz-2.7 GHz band of the WiMax spectrum.

Chapter 6 will cover the fabrication and measurement of the antennas presented in the previous two chapters.

Finally, Chapter 7 will give some concluding remarks along with some ideas for future work using metamaterials in compact antenna design.
Chapter 2

Theory - Monopoles and Matching Networks

Outside of the metamaterial theory discussed in Chapter 3, there are two main concepts used in the formulation and design of the antennas presented in this thesis. These concepts are:

1. Planar monopole antennas.

Each topic will be discussed in detail in the following sections.

2.1 The Planar Monopole

Before introducing any metamaterial concepts, a simple planar monopole antenna is first developed. The monopole antenna is a \( \frac{\lambda}{4} \) wire over a large ground plane [45], p.191. The planar monopole is a variation of the monopole with the ground plane and wire lying in the same plane. This configuration has been extensively studied in the literature and many different variations can be found such as in [46], [47]. It will be seen that this antenna serves as a starting point for the metamaterial designs to be presented in the following sections. It will also serve as a reference point to compare the performance of the metamaterial designs.

A schematic of the planar monopole antenna is shown in Fig. 2.1 along with some typical values for the dimensions. It can be seen that the antenna as shown in Fig. 2.1 meets the geometry requirements given in Chapter 1. The important features of this antenna include a ground plane on layer 1 and a microstrip feedline and monopole on layer 2. The length of the monopole is approximately \( \frac{\lambda_g}{4} \) at 2.6 GHz, where \( \lambda_g \) is the effective wavelength on the FR-4 substrate. The microstrip feedline is 50 Ω and was designed using Agilent’s Linecalc tool and optimized in Ansoft’s High-Frequency Structure Simulator (HFSS). At the base of the
Figure 2.1: Planar monopole antenna with typical dimensions

microstrip line the width of the ground plane is extended from 6 mm wide to 15 mm wide. This extension is to simply make room for an SMA connector and it has a minimal effect on the performance of the planar monopole.

2.1.1 HFSS Setup

The planar monopole antenna is simulated using Ansoft’s HFSS. HFSS is a commercial finite element method (FEM) code which solves Maxwell’s equations in a given geometry [48]. Setting up the computational domain in HFSS to simulate an antenna involves two important issues, exciting the antenna and terminating the computational domain. The setup discussed here applies to all the antennas discussed in this thesis.

The setup for the antenna can be seen in Fig. 2.2, with the excitation shown more closely in Fig. 2.2(b). The domain is terminated using HFSS’s radiation boundary conditions, as shown in Fig. 2.2(a), and are placed approximately \( \frac{\lambda}{4} \) away from the radiating object. In the simulation setup shown in Fig. 2.2(a) the radiating boundary conditions are placed at \( \frac{\lambda}{4} = 37.5 \text{ mm} \), which corresponds to a frequency of 2.0 GHz. This allows the antenna’s far-field to be accurately simulated down to 2.0 GHz which covers the low-frequency spectrum of the Wi-Fi and WiMax bands.

The excitation is shown in Fig. 2.2(b) and consists of an HFSS ‘wave-port’ which excites a 50 \( \Omega \) coaxial line. The coaxial line feeds an HFSS model of a generic SMA connector which then excites the microstrip feedline of the antenna. This setup was developed by Dr. Marco Antoniades at the University of Toronto and allows for the antenna, the SMA connector, and cable to be included in the simulation model.

Also included in the simulation are the losses of the metal and substrates. The losses of the metal are included by using HFSS’s finite conductivity boundary condition, which allows
Figure 2.2: HFSS setup of the antenna simulation. (a) Entire HFSS setup, including the HFSS radiation boundary condition, (b) Close up of the wave port, coaxial line and SMA connector.

Figure 2.3: $|S_{11}|$ and input impedance of planar monopole antenna. (a) $|S_{11}|$, (b) Input Impedance

the user to specify the conductivity, $\sigma$, of the metal. A value of $\sigma = 5.8 \times 10^7$ is used which corresponds to the conductivity of copper. The losses of the FR-4 substrate were given in Chapter 1 and are included in the HFSS material parameters of the substrate.

### 2.1.2 Simulation Results

Using the HFSS setup described above, the planar monopole antenna is simulated with the dimensions given in Fig. 2.1. The $|S_{11}|$ and input impedance are plotted in Fig. 2.3. It is noted that the input impedance is taken at the base of the microstrip feedline and is always referred to $50 \, \Omega$.

Looking at the $|S_{11}|$ in Fig. 2.3(a), there is a slight dip that occurs at approximately
2.65 GHz. This corresponds to the electrical length of the monopole being $\frac{\lambda_0}{4}$, referred to as a monopole mode. However this monopole mode is poorly matched, for reasons which will be discussed below. This monopole resonance can also be seen in the input impedance in Fig. 2.3(b), where there is a weak resonance at approximately 2.65 GHz. There is another mismatched resonance in the input impedance at approximately 5.50 GHz which corresponds to a dipole resonance of the antenna as the electrical length of the monopole is $\sim \frac{\lambda_0}{2}$.

Two sets of simulations are run to further characterize the planar monopole antenna:

1. vary $W_g$ from 6 mm-30 mm
2. vary $l_{\text{monopole}}$ from 15 mm-30 mm

By varying $l_{\text{monopole}}$, the change in the resonant frequency of the monopole is shown and by varying $W_g$, the effects of the ground plane size on the matching of the antenna can be characterized as well. Looking at Fig. 2.4, where $l_{\text{monopole}}$ is varied from 15 mm-30 mm, there is a dip in the $|S_{11}|$ that increases with frequency from approximately 2.15 GHz to approximately 4.00 GHz as $l_{\text{monopole}}$ decreases. This shows the monopole resonance shifting in frequency as the length of the monopole is varied.

It can be seen in Fig. 2.5(a) that by increasing the size of the ground plane, $W_g$, the matching in the $|S_{11}|$ at the monopole resonance improves. This is because a monopole relies on a large ground plane to function correctly. This is also seen by looking at the input impedance of the antenna in Fig. 2.5(b) and Fig. 2.5(c). Plotting the input impedance for different values of $W_g$, it is shown that as the ground plane gets smaller, the monopole resonance gets weaker, and the input match degrades. Thus, it is demonstrated that the monopole resonance ends up being suppressed by the narrow ground plane. This shows that by designing the planar monopole antenna to fit the geometry constraints, specifically by keeping the width of the antenna $\leq 10$ mm, the monopole resonance of the antenna cannot be used to create a useable
Figure 2.5: $|S_{11}|$ and input impedance of planar monopole antenna. (a) $|S_{11}|$, (b) Input impedance, (c) Input impedance from 2 GHz-4 GHz

frequency band in the antenna. Instead other methods must be looked at to make the planar monopole antenna work while still maintaining a compact geometry.

### 2.2 Double-Tuned Matching Networks

The design of double-tuned matching networks is a well known matching technique and the following explanation draws from [49] [50] [51], pp 43-27 - 43-29. The goal of the double-tuned matching network is to match a frequency dependent load below a given VSWR level over a range of frequencies (in this case, a VSWR of 2:1, or inside the $S = 2$ circle on the Smith Chart). The most common frequency dependent load is the antenna for which the application of double-tuned matching networks has been demonstrated in [52]. This is a different paradigm then the matching techniques given in classic microwave textbooks such as [53] [54], which describe how to design a matching network to match a load to the system impedance at a
single frequency. In the double tuned case, the load is never perfectly matched but is kept below a desired VSWR for the frequency span in consideration. The frequency dependent load can be modeled by any part of a series RLC load (i.e. a resistance R, a series RC, a series RL, or a series RLC circuit), or any part of a parallel RLC load (i.e. a resistance R, a parallel RC, a parallel RC, or a parallel RLC circuit). To bring this frequency dependent load below the given VSWR, the double-tuned matching network uses two stages. The first stage sets up the impedance locus for the second stage by conforming the impedance locus on the Smith Chart to a specific set of constraints, while it is the second stage of the matching network itself that brings the impedance locus below the given VSWR level. In the following explanation the double-tuned matching network is developed for a series-type load though the results can be easily extended for a parallel-type case which will be briefly described in Section 2.2.3. A modified double-tuned matching network will also be developed in Section 2.2.5 to better fit the matching network for a compact antenna design.

2.2.1 The First Stage

The first stage of the double-tuned matching network sets up the impedance locus of the frequency dependent load to meet three constraints. The first constraint is that at a frequency \( f_l \), the impedance locus intersects the negative imaginary axis of \( \Gamma \) on the Smith Chart, where \( \Gamma \) is the reflection coefficient. The second constraint is that at a frequency \( f_h \), where \( f_h > f_l \), \( f_h \) intersects the positive imaginary axis of \( \Gamma \) on the Smith Chart. In fact, \( f_l \) and \( f_h \) designate the span of frequencies that are matched by the matching network. The last constraint is that at a frequency \( f_m \), where \( f_h > f_m > f_l \), the load at \( f_m \) be purely real and that it lie on the edge of the \( S = 2 \) circle.

The first two constraints are met by adding series reactive components, depending on the load. If the load is purely resistive, adding a series inductor and capacitor can meet the first two constraints. Here the inductance and capacitance are chosen to place \( f_l \) and \( f_h \) at the desired frequencies. Likewise, for a series RC or a series RL circuit, adding an appropriately valued series inductor or capacitor respectively would help meet the first two constraints where values of the added reactive components can be adjusted so that \( f_l \) and \( f_h \) are at the desired frequencies. For a series RLC load, no extra reactive components are needed as the first two constraints are already met, however additional series inductances or capacitances can be added to adjust \( f_l \) and \( f_h \).

At the third constraint the load is purely real at a frequency \( f_m \). This is because the reactive components of the load and the first stage cancel out. An ideal impedance transformer can then be used to adjust the impedance locus such that the load at \( f_m \) lies on the edge of the \( S = 2 \) circle.
Figure 2.6: The input impedance looking into the first stage on the Smith Chart with all three constraints met. The dashed circle is the $S = 2$ circle and the vertical line is the imaginary axis of $\Gamma$.

Figure 2.7: A schematic of the matching network after the first stage. The $R, L, C$ elements can be part of the load or 1st stage depending on the load. The impedance transformer bring the impedance locus to the edge of the $S = 2$ circle.

Thus, the input impedance looking into the first stage of the matching network is a series RLC load whose impedance locus has been adjusted to lie on the edge of $S = 2$ circle through an impedance transformer. This is seen in Fig. 2.6 on the Smith Chart along with a schematic of the load and first stage in Fig. 2.7. It can be shown that by enforcing all three constraints, that the impedance locus after the first stage at $f_l$ and $f_h$ lies on the $|\Gamma|^2_l$ circle, where $|\Gamma|_l$ is the magnitude of the reflection coefficient when $S = 2$. This is illustrated in Fig. 2.8 where the $|\Gamma|^2_l$ circle is drawn on the Smith Chart. Because $f_l$ and $f_h$ mark the span of frequencies that will be brought inside the $S = 2$ circle, the $|\Gamma|^2_l$ circle marks the maximum improvement in the reflection coefficient that is achieved by a double-tuned matching network.
2.2.2 The Second Stage

The second stage of the double-tuned matching network brings the impedance locus inside the $S = 2$ circle. This happens in the frequency range $f_l < f < f_h$. To bring the impedance inside the $S = 2$ circle, a shunt inductor, $L_m$, and shunt capacitance, $C_m$, are added in the second stage. The values of $L_m$ and $C_m$ can be found by looking at the admittance of the first stage at $f_l$ and $f_h$. Looking at the frequency $f_l$ on the Smith Chart in Fig. 2.6 the load is capacitive with a susceptance $jB_l$. At the frequency $f_h$, the load is inductive with a susceptance $jB_h$. Using $L_m$ and $C_m$ the susceptance at both these frequencies can be canceled out. This is expressed as:

\[
\begin{align*}
    j\omega_l C_m - \frac{j}{\omega_l L_m} &= -jB_l, \\
    j\omega_h C_m - \frac{j}{\omega_h L_m} &= -jB_h 
\end{align*}
\]

(2.1) (2.2)

where $\omega_l = 2\pi f_l$ and $\omega_h = 2\pi f_h$. Solving equations (2.1) and (2.2) together for both $L_m$ and $C_m$ gives the following expressions:

\[
\begin{align*}
    L_m &= \frac{\omega_l^2 - \omega_h^2}{\omega_l^2 \omega_h B_h - \omega_l B_l \omega_h^2}, \\
    C_m &= \frac{\omega_h B_h - \omega_l B_l}{\omega_l^2 - \omega_h^2}.
\end{align*}
\]

(2.3) (2.4)

By using the values of $L_m$ and $C_m$ from equations (2.3) and (2.4), the impedance locus is brought onto the real axis of the Smith Chart at the frequencies $f_l$ and $f_h$. This forms a loop as shown in Fig. 2.9 that lies within the $S = 2$ circle. Note that for equations (2.3) and (2.4) to give positive values for $L_m$ and $C_m$, $jB_h$ must always be inductive and $jB_l$ must always be
Chapter 2. Theory - Monopoles and Matching Networks

Figure 2.9: The input impedance looking into the second stage on the Smith Chart. The dashed circle is the $S = 2$ circle. The input impedance is now contained in the $S = 2$ circle.

![Diagram](image)

Figure 2.10: A schematic of the matching network after the second stage.

capacitive. The entire matching network can be seen in Fig. 2.10. It can be seen graphically that by designing the first stage of the matching network to place the load on the imaginary $\Gamma$ axis at $f_l$ and $f_h$, the second stage is able to bring these points on the impedance locus to the edge of the $S = 2$ circle. This can be understood by plotting the impedance locus after the first stage on a Y-Smith Chart in Fig. 2.11. It is seen that at $f_l$ and $f_h$, the impedance locus intersects the same conductance circle (shown in green). This conductance circle also intersects the edge of the $S = 2$ circle on the real axis of the Smith Chart. Thus by adding shunt susceptive components in the second stage, the points on the impedance locus at $f_l$ and $f_h$ move on this conductance circle. This is also how the ‘loop’ on the Smith Chart is formed.

2.2.3 Matching A Parallel Load

The same steps used to match the series load can be applied to match a parallel load. Once again the parallel load can be either a resistor, a shunt RL, or RC circuit or a parallel RLC
Figure 2.11: The impedance locus after the first stage as plotted on a Y-Smith Chart. The green circle represent the constant conductance circle that intersects the load at \( f_l \) and \( f_h \). By adding a shunt inductance and capacitance the two points move on the constant conductance circle to the real axis of the Smith Chart, which forms the loop seen in Fig. 2.9.

Circuit. Here the first stage of the matching network is again used to meet the constraints listed in Section 2.2.1. However this is done by adding a parallel inductance or capacitance where necessary instead of series components. The second stage also brings the impedance locus on the Smith Chart inside the \( S = 2 \) circle. For a parallel load, the second stage consists of a series inductance and capacitance whose values are given by:

\[
L_m = \frac{\omega_l X_h - \omega_l X_l}{\omega_l^2 - \omega_h^2} \quad (2.5)
\]

\[
C_m = \frac{\omega_l^2 - \omega_h^2}{\omega_l^2 \omega_h X_h - \omega_l X_l \omega_h^2} \quad (2.6)
\]

where \( X_l \) and \( X_h \) are the reactance’s at \( f_l \) and \( f_h \) respectively and \( f_l \) and \( f_h \) are the points on the impedance locus which intersect the imaginary \( \Gamma \) axis. Here the series components move the impedance locus on a constant resistance circle to form a loop on the Smith Chart. This is illustrated in Fig. 2.12.

2.2.4 \( n \)-Tuned Matching Networks

In [55], \( n \)-tuned matching networks are given as an extension of a double-tuned matching network. Just as double-tuned matching networks increase the bandwidth of a load over a single-tuned matching network (i.e. simply bringing the load to the center of the Smith Chart), an \( n \)-tuned matching continues increasing the bandwidth of the load by adding multiple stages to the matching network. Each stage that is added alternates between a series/parallel LC network. For example, for the matching network shown in Fig. 2.10, the third stage would be a
Figure 2.12: Matching a Parallel RLC load using a double-tuned matching network (a) Schematic of the Parallel RLC load and matching network. (b) The impedance locus after the first stage (blue curve) and after the double-tuned matching network (black curve). The red circle is the $S = 2$ circle.

In [55] the increase in bandwidth that each stage of the matching network provides when matching an antenna is developed analytically. This is plotted in Fig. 2.13 where the final VSWR of the load is plotted versus the bandwidth (specifically the $Q$ bandwidth product, where $Q$ is the quality factor of the antenna itself). Here it can be seen that a double-tuned matching network gives the largest single stage increase in bandwidth. While subsequent improvements in the bandwidth can be made for each extra stage added, they are subject to diminishing increases in the improvement of the bandwidth, converging to the case of an infinitely-tuned matching network. Another consideration is also the added complexity of the matching network itself as $n$ increases. Thus, a double-tuned matching network often offers a good trade-off between the bandwidth that can be achieved and complexity.
Figure 2.13: A plot of the VSWR vs. the $Q$ bandwidth product. The $Q$ referenced is that of the load being matched which in this case is the $Q$ of an antenna. The bandwidth is that achieved by the matching network for a given VSWR. It can be seen that for a given VSWR as the number of stages is increased the $Q$ bandwidth product increases. Reprinted with permission from [55]. © 2010 IEEE.

2.2.5 A Modified Double-Tuned Matching Network

As already shown the implementation of a double-tuned matching network allows for a large bandwidth improvement to be achieved. When applying this concept to compact antenna design further trade-offs between size and complexity must be made. For example a double-tuned matching network can take up a lot of space due to the use of an impedance transformer. The implementation of the impedance transformer in a planar microstrip setting can also add a lot of complexity to the design. In this section it will be demonstrated that the double-tuned matching network can be modified to a simpler form by removing the impedance transformer which allows for the compact design of the matching network while reducing its complexity. This however comes at the expense of a reduced bandwidth in the match. This modified double-tuned matching network can be seen in Fig. 2.14. The modified second stage will be described initially, followed by the modified first stage.

Looking at Fig. 2.14 the second stage of the modified double-tuned matching network can be seen to be matching a series RLC load. By removing the impedance transformer this series RLC load no longer lies on the edge of the $S = 2$ circle, breaking the constraint given in Section 2.2.1. However it is assumed that the real part of the series RLC circuit is $0 \, \Omega < R < \frac{Z_0}{2}$ forcing the series RLC impedance locus to lie on the left-half of the Smith Chart and outside the $S = 2$ circle as shown in Fig. 2.15.
Figure 2.14: A schematic of the modified double-tuned matching network without an impedance transformer

Figure 2.15: A series RLC impedance locus on the Smith Chart in between the first stage and second stage of the modified double-tuned matching network (blue curve). The highlighted green sections of the impedance locus indicate the range of frequencies of $f_l$ and $f_h$ that can be brought inside the $S = 2$ circle by the second stage of the matching network. This is because these frequencies on the impedance locus intersect constant conductance circles that partially lie inside the $S = 2$ circle.

The main idea behind the second stage of the double-tuned matching network described in Section 2.2.2 is that it forms a ‘loop’ on the Smith Chart by canceling out the inductive and capacitive components of a series RLC impedance locus as given by equations (2.1), and (2.2). A close look at equations (2.1) and (2.2) shows that they are not dependent on the constraints imposed by the first stage of the matching network given in Section 2.2.1. Thus the design of the modified second stage relies on choosing appropriate values for $f_l$ and $f_h$ on the series RLC impedance locus to match the load at a desired frequency. The frequencies $f_l$ and $f_h$ on the impedance locus no longer conform to the constraints given in Section 2.2.1. The appropriate
choices for \( f_l \) and \( f_h \) are shown in Fig. 2.15 where the highlighted green sections show the appropriate range of frequencies that can be brought inside the \( S = 2 \) circle. This range of frequencies on the impedance locus shown in Fig. 2.15 lie on constant conductance circles that intersect the interior of the \( S = 2 \) circle. Thus the shunt inductance and capacitance of the second stage can bring these points inside the \( S = 2 \) circle by canceling out the susceptance found at \( f_l \) and \( f_h \) respectively. This is how the modified second stage matches the series RLC load.

It can be seen that two different cases arise from this situation:

1. **Single Band Match** A single band match can be formed by choosing either \( f_l \) or \( f_h \) to lie in the regions marked in Fig. 2.15. With one frequency chosen from the highlighted region on the impedance locus, the other frequency is chosen from outside the highlighted region on the impedance locus. By choosing the other frequency point outside of the highlighted range of frequencies only one part of the impedance locus will be brought into the \( S = 2 \) circle while the other part is kept outside. With both frequency points chosen equations (2.3) and (2.4) can be used to find values for \( L_m \) and \( C_m \). This forms a loop on the Smith Chart with one branch of the loop inside the \( S = 2 \) circle. It is noted here that this gives less bandwidth than the double-tuned matching shown in Section 2.2.2 as only part of the loop is contained in the \( S = 2 \) circle instead of the whole loop.

2. **Dual Band Match** A dual band match can also be formed by choosing both \( f_l \) and \( f_h \) from the highlighted regions marked in Fig. 2.15. With both frequencies chosen equations (2.3) and (2.4) can once again be used to find values for \( L_m \) and \( C_m \). This forms a loop on the Smith Chart with two branches of the loop inside the \( S = 2 \) circle separated by a section outside the \( S = 2 \) circle to create a dual band match.

By not using an impedance transformer, the design of the first stage is modified by no longer having to meet the three constraints, given in Section 2.2.1. Instead the goal of the first stage is two-fold. The first goal is to modify the load if necessary to form a series RLC circuit. Thus, depending on the load, series reactive components must be added to modify the impedance as necessary. For example if the load is purely resistive, a series inductance and capacitance needs to be added to create a series RLC circuit. The second goal is to adjust the impedance locus such that the desired range of frequencies lie in the highlighted regions given in Fig. 2.15. This allows for the desired design frequencies to be matched by the second stage. This is done by choosing the appropriate values for the series inductance and capacitance of the first stage to form an impedance locus with the appropriate frequencies in the highlighted regions.

To demonstrate the modified double-tuned matching network, two examples are developed in the following sections using a series RLC load as the frequency dependant load. First a single
band matching example is shown followed by a double-tuned matching example.

2.2.6 Single-Band Matching

For the single-band case, the second stage of the matching network can be used to bring one part of the impedance locus inside the $S = 2$ circle. This is demonstrated for a series RLC load on the Smith Chart in Fig. 2.16(a). The series RLC load is given by $R = 15\ \Omega$, $L = 2.65\ \text{nH}$, $C = 1.5\ \text{pF}$. In this example, the load is integrated with the first stage. It is noted that if a different range of frequencies were to be matched than those given by the series RLC load used in this example, more series inductance or capacitance could be added as necessary to adjust the load so that the desired frequencies lie in the appropriate region.

In this example, the second stage of the matching network is designed to bring the capacitive section of the load inside the $S = 2$ circle, while the inductive section of the load is kept outside of the $S = 2$ circle. This is done through a judicious choice of $f_l = 1.9\ \text{GHz}$ and $f_h = 4.43\ \text{GHz}$ on the impedance locus as shown in Fig. 2.16(a). Here $f_l$ is chosen such that it lies on a constant conductance circle that intersects the interior of the $S = 2$ circle. This allows for the shunt inductance and capacitance of the second stage to move the impedance at $f_l$ into the $S = 2$ circle (and onto the real axis of the Smith Chart), matching the load. Meanwhile the point at $f_h$ is chosen such that it lies on a constant conductance circle that does not intersect the interior of the $S = 2$ circle. Once again the shunt inductance and capacitance move the impedance at $f_h$ so that it is kept outside of the $S = 2$ circle.

Using the values of the susceptance at $f_l$ and $f_h$ and plugging them into equations (2.1)-(2.4) the values for $L_m$ and $C_m$ are found to be $1.81\ \text{nH}$ and $1.37\ \text{pF}$ respectively. A schematic of the matching network and load can be seen in Fig. 2.16(b) and the matched load can also be seen on the Smith Chart in Fig. 2.16(a) where a loop can be found on the Smith Chart that is partially contained within the $S = 2$ circle.

This matching technique is implemented in Chapter 5 using a CSRR-transmission-line network to match the planar monopole antenna of Section 2.1 at a single band.

2.2.7 Dual-Band Matching

For the dual-band case, the second stage of the matching network can be used to bring two parts of the impedance locus inside the $S = 2$ circle. Using the same load as in the previous section, the frequencies $f_l$ and $f_h$ are chosen to bring part of the capacitive and inductive branches of the impedance locus inside the $S = 2$ circle as shown in Fig. 2.17(a). The point on the impedance locus at $f_l = 1.9\ \text{GHz}$ is chosen once again because it lies on a constant conductance circle that intersects the interior of the $S = 2$ circle. Likewise the impedance locus at $f_h = 3.29\ \text{GHz}$ is
Figure 2.16: Using the second stage of a double-tuned matching network to match a series RLC load at a single band. (a) The series RLC load on the Smith Chart (blue Curve). The points $f_l$ and $f_h$ are marked on the Smith Chart. The impedance locus after the matching network is also shown (black curve), with a loop formed on the Smith Chart. Note that part of the loop now lies within the $S = 2$ circle (red curve) (b) Schematic of the load and matching network. The matching network is much simpler, consisting only of a shunt LC network.

chosen because it also lies on a constant conductance circle that intersects the interior of the $S = 2$ circle. The shunt inductance and capacitance of the modified second stage bring both these points within the $S = 2$ circle.

Plugging in the susceptance at $f_l$ and $f_h$ into equations (2.1)-(2.4), the values for $L_m$ and $C_m$ are found to be 1.17 nH and 3.48 pF respectively. A schematic of the matching network and load can be seen in Fig. 2.17(b) and the matched load can also be seen on the Smith Chart in Fig. 2.17(a). The matched load in Fig. 2.17(a) now forms a loop on the Smith Chart with two separate parts of the loop within the $S = 2$ circle. It is noted that the part of the matched impedance locus between $f_l$ and $f_h$ is outside the $S = 2$ circle, showing the dual-band match.

Due to time constraints the dual-band matching technique is not implemented to match an antenna but is described here for the sake of completeness.
Figure 2.17: Using the second stage of a double-tuned matching network to match a series RLC load at two different bands. (a) The series RLC load on the Smith Chart (Blue Curve). The points $f_l$ and $f_h$ are marked on the Smith Chart. The impedance locus after the matching network is also shown (black curve) with a loop formed on the Smith Chart. Note that two separate parts of the loop now lies within the $S = 2$ circle (red curve), (b) Schematic of the load and matching network.
Chapter 3

Theory - Metamaterials

Metamaterials are synthesized structures whose electromagnetic properties can be engineered. The basis of a metamaterial is a unit cell that is much smaller than a wavelength. The individual properties of the unit cell aggregate to determine the effective material properties of the metamaterial as a whole. The design of the unit cell can be tailored to synthesize a variety of material parameters such as a positive or negative permittivity [56], [57], a positive or negative permeability [18], [58], a negative refractive index [8], [59], or anisotropy [60]. This list of material parameters extends outside the realizable parameters found in nature. This leads to many interesting phenomena that can be realized such as, backward wave propagation [5], negative refraction [61], and sub-diffraction focusing [62], [63]. The unit cell itself can be repeated periodically [8] or aperiodically [64], to form a homogenous or inhomogeneous metamaterial respectively. Thus at the heart of every metamaterial design is the unit cell which bestows the effective material properties on the metamaterial.

There are two main approaches that have arisen to synthesize metamaterials at microwave frequencies. They are the bulk approach and the transmission-line approach. Each approach is discussed in more detail below.

3.1 Bulk Approach

In the bulk approach to metamaterial design, the unit cell is formed through the inclusion of ‘particles’. These particles, which are much smaller than a wavelength, create an electric and/or magnetic polarization that can produce a negative permittivity or permeability respectively. The first metamaterials made using the bulk approach used a wire and split-ring resonator (SRR) unit cell to create a negative refractive index [59]. The wire grid gave a negative permittivity, while the SRR gave a negative permeability.

The use of these particles in the design of the unit cell of the metamaterial is an area of
Figure 3.1: The Split-Ring Resonator. (a) Geometry of the SRR (b) The excitation of the SRR by a magnetic field, $H$ (blue arrow) along with the induced currents (black arrows). The magnetic moment, $m$, is also shown (red arrow).

ongoing research. The work in this thesis is derived from the bulk approach in metamaterial design by using these particles in the design of compact antennas. Thus this section details the functionality and properties of some relevant metamaterial particles, and their potential application to antenna design.

3.1.1 The Split-Ring Resonator

Any discussion of metamaterial particles must begin with the SRR. The SRR was introduced as a metamaterial element by Pendry et al. in 1999 in [58]. The SRR is non-magnetic but collectively induces a magnetic response in the electromagnetic field generating an effective negative permeability. It was a fundamental building block in initial metamaterial designs and continues to be used extensively [59], [61]. The SRR is shown in Fig. 3.1(a) and consists of two concentric closely spaced loops with a gap in either loop. The loops provide an inductance while the spacing between the loops provides a distributed capacitance. An SRR is excited by a time-varying axial magnetic field as shown in Fig. 3.1(b). The magnetic field induces circulating currents on the loops themselves. This current sets up a magnetic dipole moment, which opposes the applied magnetic field. When the magnetic dipole moment overcomes the applied magnetic field, due to the resonance of the SRR, the effective permeability becomes negative [65].
Figure 3.2: A SRR consisting of a single loop and a capacitive gap in a parallel-plate waveguide. The parallel-plate waveguide is modeled as a box with PEC and PMC walls filled with air ($\mu_0$, $\epsilon_0$). The PEC boundaries are the shaded faces, and the PMC boundaries are the faces with an ‘X’. The excitation is also shown, with the magnetic field perpendicular to the loop.

Analysis

To analyze the SRR in more detail a simple circuit model can be built following [65], [66], and [67]. This model considers only the simplest case of exciting the SRR with an axial magnetic field and does not investigate the coupling between SRRs or the losses in the SRR.

The SRR is placed in a parallel-plate waveguide as shown in Fig 3.2. The dimensions of the parallel-plate waveguide and SRR are both assumed to be much smaller than $\lambda$, the free space wavelength. The parallel-plate waveguide environment allows the periodicity of the unit cell to be taken into account by only having to model one unit cell. The SRR is excited by the TEM mode of the waveguide, with the magnetic field, $H$, perpendicular to the SRR as described above. Without any loss of generality the SRR is modeled as a single loop with the distributed capacitance between the inner and outer rings lumped into the gap capacitance as shown in [65].

The first part of the model is found by examining the parallel-plate waveguide by itself.
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The well known circuit model for a parallel-plate waveguide is shown in Fig. 3.3 and consists of a series inductance and shunt inductance. The series inductance, $L_{TL}$, is given by,

$$L_{TL} = \frac{\mu_0 A}{d} \quad (3.1)$$

where $A = lh$ and the shunt capacitance, $C_{TL}$ is given by,

$$C_{TL} = \frac{\epsilon_0 ld}{h} \quad (3.2)$$

The inductance and capacitance models the permittivity and permeability, respectively of the material inside the parallel-plate waveguide, which in this case is a vacuum ($\mu_o, \epsilon_o$).

As stated above, the SRR has an inductance from the loop and a capacitance from the gap. This can be modeled as a tank LC resonator as shown in Fig. 3.4 with $L_{SRR}$ modeling the inductance of the SRR from the loop and $C_{SRR}$ modeling the lumped capacitance.

The magnetic field of the TEM mode in the parallel-plate waveguide induces a current on the SRR which sets up a voltage across the capacitive gap on the SRR. The voltage excited on the SRR by the magnetic field can be understood by applying Faraday’s law,

$$V_{SRR} = \frac{d\Phi_B}{dt} \quad (3.3)$$
Figure 3.5: A circuit model of the SRR in the parallel-plate waveguide

where $V_{SRR}$ is the voltage across the gap of the SRR that is induced by the magnetic field, $H$, and $\Phi_B$ is the magnetic flux through the SRR. This can be written in time-harmonic form as,

$$V_{SRR} = j\omega A_{SRR} \mu_0 H,$$

(3.4)

where $\Phi_B$ is replaced by $A_{SRR} \mu_0 H$ and $A_{SRR}$ is the area of the loop which is given by $A_{SRR} = l_{SRR} w_{SRR}$. This interaction between the magnetic field of the parallel-plate waveguide and the SRR can be captured in the circuit model by a mutual inductance, $M$, between the inductance of the SRR and the inductance of the parallel-plate waveguide as shown in Fig. 3.5. The dot convention is chosen such that Lenz’s law is enforced (i.e. the induced current on the SRR opposes the applied magnetic field.). The voltage induced on $L_{SRR}$ by the mutual inductance is given by $j\omega MI_1$, where $I_1$ is the current through $L_{TL}$. By comparing this expression to Equation (3.4), and noticing that the current through $L_{TL}$ is given by $I_1 = H d$ (Amperes Law), the mutual inductance can be found and is given by

$$M = \frac{\mu_0 A_{SRR}}{d}. \quad (3.5)$$

Applying basic circuit analysis to the circuit model of Fig. 3.5 the following set of equations can be derived:

$$V_1 = j\omega L_{TL} I_1 + j\omega M I_2, \quad (3.6)$$
where $V_1$ is the voltage across $L_{TL}$, $V_2$ is the voltage across $L_{SRR}$, and $I_2$ is the current on the SRR. The impedance of the series branch $Z_{\text{series}} = \frac{V_1}{I_1}$ can then be found by simplifying the above equations, which gives the following result:

$$Z_{\text{series}} = j\omega L_{TL} - \frac{j\omega^2 M^2}{\omega L_{SRR} - \frac{1}{\omega C_{SRR}}}.$$  \hfill (3.9)

To extract the effective permeability, $\mu_{\text{eff}}$, of the SRR, it can be shown that the series impedance of a parallel-plate waveguide filled with a fictitious material with permeability $\mu_{\text{eff}}$ is given by,

$$Z_{\text{ppw}} = \frac{j\omega \mu_{\text{eff}} A}{d} = \frac{j\omega L_{TL} \mu_{\text{eff}}}{\mu_o},$$  \hfill (3.10)

where $\frac{4d}{A}$ has been replaced using equation (3.1). Thus by equating $Z_{\text{series}}$ with $Z_{\text{ppw}}$, the effective permeability can be found:

$$Z_{\text{ppw}} = Z_{\text{series}},$$  \hfill (3.11)

$$\frac{\mu_{\text{eff}}}{\mu_o} = \frac{Z_{\text{series}}}{j\omega L_{TL}}.$$  \hfill (3.12)

Substituting for $Z_{\text{series}}$ gives,

$$\frac{\mu_{\text{eff}}}{\mu_o} = 1 - \frac{\omega M^2}{L_{TL}(\omega L_{SRR} - \frac{1}{\omega C_{SRR}})}.$$  \hfill (3.13)

This gives the effective permeability of a periodic array of SRRs, which can be seen to have a Lorentzian-type dispersion [68], p.29. From this expression some important values from the dispersion of $\mu_{\text{eff}}$ can be extracted. First, it can be seen that the SRR has a resonant frequency given by,

$$\omega_o = \frac{1}{\sqrt{L_{SRR} C_{SRR}}},$$  \hfill (3.14)

where the SRR is capacitive below $\omega_o$ and inductive above. Due to the capacitive nature of the SRR, as $\omega \to \omega_o$, the effective permeability becomes increasingly larger, and at the resonant frequency, $\omega = \omega_o$, $\mu_{\text{eff}}$ switches signs and becomes negative due to the resonant nature of the SRR. As $\omega$ moves away from $\omega_o$, $\mu_{\text{eff}}$ becomes less negative until $\omega = \omega_{mp}$, where $\omega_{mp}$ is the plasma frequency of the SRR and is given by

$$\omega_{mp} = \frac{\omega_o}{\sqrt{1 - \frac{M^2}{r_{SRR} L_{TL}}}}.$$  \hfill (3.15)

At this frequency $\mu_{\text{eff}} = 0$. This shows that the reactive and resonant loading of the SRR on the magnetic field generates an effective permeability which becomes negative for a certain band
of frequencies. This can also be seen by simulating an SRR inside a parallel-plate waveguide in HFSS and using the S-parameters to extract the effective permeability as given in [69] [70]. The HFSS setup is shown in Fig. 3.6(a) with the given dimensions. The extracted permeability is plotted in Fig. 3.6(b), where a Lorentzian type dispersion can be seen as calculated above. It can also be seen in Fig. 3.6(b) that for a small range of frequencies the permeability becomes negative.

From the circuit model shown in Fig. 3.5 it is seen that the SRR does not affect the shunt branch of the parallel-plate waveguide which implies that the SRR has no electric response. This however is not entirely true as a more in depth analysis of the SRR shows that the SRR has other responses. This can be seen in [71] where the SRR is analyzed by examining the magnetic and electric dipole moments of the SRR. From the derivation in [71] this is given by the following set of equations:

\[
m_z = a_{zz}^{mm} B_z - a_{zy}^{em} E_y, \tag{3.16}
\]

\[
p_y = a_{yy}^{ee} E_y - a_{zy}^{me} H_z, \tag{3.17}
\]

\[
p_x = a_{xx}^{ee} E_x. \tag{3.18}
\]

Here, \(m_z\), \(p_y\) and \(p_x\) are the magnetic and electric dipole moments respectively in the directions given by the subscripts. \(B_z\), \(E_y\) and \(E_x\) are the external magnetic and electric fields in the directions given by the subscript. The remaining terms are the magnetic and electric polarizabilities.
The term $\alpha_{zz}^{mm}$ implies that a z-directed magnetic field produces a magnetic polarizability in the z-direction. The same convention applies for the other polarizabilities.

The magnetic response derived above using the circuit model is given by the $\epsilon_{zz}^{mm} B_z$ term which causes the magnetic dipole moment $m_z$. However it is seen that a magnetic dipole moment is also excited by the $E_y$ field, showing that the SRR is bianisotropic. An electrical dipole moment in the $x$ and $y$ directions shows that an electric response in the SRR is also excited by the external fields. It is noted however that only the $p_y$ dipole moment is resonant. Thus, equations (3.16)-(3.18) show other possible ways that an SRR can be excited.

While the analysis so far has discussed the response of the SRR at a single frequency it is worth noting that other resonances exist. An examination of the SRR at higher frequencies, shows that other resonances in the SRR can be found [72]. These resonances occur due to higher order, non-uniform current distributions supported on the SRR, either on the whole SRR itself or on the inner and outer rings separately. These higher order resonances can also create an effective permeability and permittivity and can be used separately.

3.1.2 The Electric-LC resonator

The previous section showed a simple analysis for the well known metamaterial particle, the SRR. The discussion now turns to a lesser-known metamaterial particle, the Electric-LC resonator (ELC). The ELC resonator is a key metamaterial particle used in this thesis in the design of a compact, multi-band antenna. Using the analysis of the previous section as a template, an analysis of the ELC follows in this section, concluding with the application of the ELC to antenna design.

The ELC was introduced in 2006 by Schurig et al. in [73]. The goal of the ELC was to introduce a planar, resonant particle that created a negative permittivity instead of a negative permeability. Until the introduction of the ELC, bulk negative permittivity metamaterials were made using wire grids as introduced in [57] by Pendry, a structure that was analyzed first by Rotman in [74] in the context of modeling plasmas. The wire grids create a negative permittivity by creating an electric polarization in the wires that is antiparallel to the incident electric flux density ($D = \epsilon_0 E$). This is due to currents along the wire that can be modeled as an inductance. When this electric polarization overcomes the incident electric flux density, a negative permittivity is produced [65]. However the fabrication of these wire grids is difficult with electrical connections required between wires, and any break or termination of the wire causes variations in the effective permittivity [73]. It is also difficult to create wire lattices with spacing much smaller than a wavelength. For propagation along the length of the wire the wire grids are spatially dispersive due to the large extent of the wire in that direction [75]. Thus came the motivation to create a particle that was planar, easy to fabricate (using standard
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Figure 3.7: The Electric-LC Resonator. (a) Geometry of the ELC (b) The Excitation of the ELC by an electric field, $E$ (blue arrow) along with the induced currents (black arrows).

lithographic techniques), and could make unit cells much smaller than a wavelength. One attempt was a wire lattice with periodic capacitive gaps along the wire [76]. However the bulk parameter values were very sensitive to the manufacturing tolerances of the capacitive gaps and the size of the wires with respect to the wavelength were not reduced. After these attempts came the ELC, which is shown in Fig. 3.7(a). The ELC consists of two loops with a capacitive gap between the two loops. This compact structure can be made much smaller than a wavelength due to its inductance and capacitance which reduces its resonant frequency.

The working idea behind the ELC first involves creating a metamaterial particle to couple to the incident electric field. This is accomplished by the capacitive gap in the ELC which couples to an incident electric field directed across the gap as shown in Fig. 3.7(b). This induces a current on the ELC which flows around the inductive loops and causes the ELC to resonate, due to its capacitive and inductive nature. This resonance sets up an electric polarization along the ELC which like the wire media overcomes the incident electric flux density to produce an effective negative permittivity.

**Analysis**

To further analyze the ELC, a simple circuit model can be built following the analysis shown in Section 3.1.1 by placing the ELC in a parallel-plate waveguide. This model considers the case where an incident electric field is incident on the ELC, with the electric field directed across the gap. The ELC is placed in a parallel-plate waveguide with the dimensions shown in Fig. 3.8. The dimensions of the parallel-plate waveguide and ELC are both assumed to be much less
Figure 3.8: An ELC placed within a parallel-plate waveguide. The fringing capacitance of the capacitance is modeled for simplicity as a parallel-plate capacitor with plate separation $g$ and plate area $A_{cap}$. The parallel-plate waveguide is modeled as a box with PEC and PMC walls filled with air ($\mu_0, \epsilon_0$). The PEC boundaries are the shaded faces, and the PMC boundaries are the faces with an ‘X’. The excitation is also shown, with the electric field aligned along the capacitive gap.

than $\lambda$. Once again, the parallel-plate waveguide allows the periodicity of the unit cell to be taken into account by only having to model one unit cell.

The model begins by looking at the parallel-plate waveguide alone. The parallel-plate waveguide was already modeled as shown in Fig. 3.3 with $L_{TL}$ and $C_{TL}$ given by equations (3.1) and (3.2).

The ELC shown in Fig. 3.7(a) has an inductance from both loops and a capacitance from the gap. This can be modeled as a tank LC resonator as shown in Fig. 3.9 with $L_{ELC}$ as the total inductance of the ELC and $C_{ELC}$ as the total capacitance. To simplify the analysis, the capacitive gap of the ELC is modeled as a parallel-plate capacitor with a plate separation $g$. 
and area $A_{cap}$ even though the ELC itself is a planar structure and the capacitance itself is a fringing capacitance.

The electric field of the TEM mode in the parallel-plate waveguide couples to the capacitive gap on the ELC which induces a current on the center arm of the ELC. The current excited on the ELC by the electric field, $E$, can be understood as a displacement current density, $J_D$,

$$J_D = \varepsilon_0 \frac{\partial E}{\partial t}. \quad (3.19)$$

This can be written in time-harmonic form as,

$$J_D = j\omega \varepsilon_0 E. \quad (3.20)$$

This interaction between the electric field of the parallel-plate waveguide and the capacitive gap of the ELC can be modeled in a circuit by a mutual capacitance, $M_c$ between $C_{TL}$ and $C_{ELC}$ as shown in Fig. 3.10. The current induced on the capacitance of the ELC by the mutual capacitance is given by $j\omega M_c V_1$, where $V_1$ is the voltage across $C_{TL}$. By comparing this expression to Equation (3.20), and noticing that $V_1 = \frac{E}{h}$, where $h$ is the height of the parallel-plate waveguide and that $J_D = \frac{J_0}{A_{cap}}$, the mutual capacitance can be given by

$$M_c = \frac{\varepsilon_0 A_{cap}}{h}. \quad (3.21)$$

Applying basic circuit analysis again, the following series of equations can be applied to the ELC circuit model:

$$I_1 = j\omega C_{TL} V_1 + j\omega M_c V_2, \quad (3.22)$$

$$I_2 = j\omega M_c V_1 + j\omega C_{ELC} V_2, \quad (3.23)$$

$$I_2 = \frac{j}{\omega L_{ELC}} V_2, \quad (3.24)$$

where $I_1$ is the current through $C_{TL}$, and $V_2$ and $I_2$ are the voltage across and current through $C_{ELC}$. The admittance of the shunt branch $Y_{shunt} = \frac{I_1}{V_1}$ can then be found by simplifying the
To extract the effective permittivity, \( \epsilon_{\text{eff}} \), of the ELC it can be shown that the shunt admittance of the parallel-plate waveguide filled with a fictitious material with permittivity \( \epsilon_{\text{eff}} \) is given by,

\[
Y_{\text{ppw}} = j\omega \epsilon_{\text{eff}} \frac{ld}{h} = \frac{j\omega \epsilon_{\text{TL}} \epsilon_{\text{eff}}}{\epsilon_o},
\]

(3.26)

where \( \frac{ld}{h} \) is replaced using equation (3.2). Thus by equating \( Y_{\text{shunt}} \) with \( Y_{\text{ppw}} \), the effective permittivity is given by,

\[
Y_{\text{ppw}} = Y_{\text{shunt}}
\]

(3.27)

\[
\frac{\epsilon_{\text{eff}}}{\epsilon_o} = \frac{Y_{\text{shunt}}}{j\omega \epsilon_{\text{TL}}}
\]

(3.28)

Substituting for \( Y_{\text{shunt}} \) gives,

\[
\frac{\epsilon_{\text{eff}}}{\epsilon_o} = 1 - \frac{\omega M_e^2}{C_{\text{TL}}(\omega C_{\text{ELC}} - \frac{1}{\omega L_{\text{ELC}}})}
\]

(3.29)

This gives the effective permittivity of a periodic array of ELC’s, which is shown to have a Lorentzian-type dispersion. This is different than wire media which has a Drude-type dispersion. As shown for the SRR, some relevant values can be extracted from equation (3.29) by examining some key points. The ELC has a resonant frequency given by,

\[
\omega_o = \frac{1}{\sqrt{L_{\text{ELC}} C_{\text{ELC}}}}
\]

(3.30)

where the ELC is inductive below \( \omega_o \) and capacitive above. Due to the inductive nature of the ELC, as \( \omega \to \omega_o \), the effective permittivity becomes increasingly larger, and at its resonant
Figure 3.11: Using HFSS to extract the permittivity of the ELC. (a) Geometry of the HFSS simulation for a 2-dimensional array of ELC’s. The same boundary conditions apply as discussed in Fig. 3.8. The ELC also sits on a 0.4 mm FR-4 substrate. (b) The extracted permittivity of an array of ELC’s.

At higher frequencies, \( \omega = \omega_o \), \( \epsilon_{eff} \) switches signs and becomes negative. As \( \omega \) moves away from \( \omega_o \), \( \epsilon_{eff} \) becomes less negative until \( \omega = \omega_{cp} \), where \( \omega_{cp} \) is the plasma frequency of the ELC and is given by

\[
\omega_{cp} = \frac{\omega_o}{\sqrt{1 - \frac{M^2}{C_{ELC}C_{TL}}}}.
\]

At this frequency \( \epsilon_{eff} = 0 \). Above \( \omega_{cp} \) the effective permittivity becomes positive again. This shows that the reactive and resonant loading of the ELC on the electric field generates an effective permittivity which becomes negative for a certain band of frequencies. This can also be seen by simulating an ELC inside a parallel-plate waveguide in HFSS and using the S-parameters to extract the effective permittivity as shown in [73] [77]. This is shown in Fig. 3.8 where the HFSS setup is shown with the given dimensions. The extracted permittivity is plotted in Fig. 3.11(b). From the full-wave simulation, the effective permittivity of the 2-D array of ELC’s again shows a Lorentzian type dispersion which becomes negative for a range of frequencies.

Unlike the SRR, the ELC is not bianisotropic and an electric response is not produced when excited by a uniform magnetic field or vice versa [73]. This is because a uniform magnetic field does not excite the ELC due to the orientation of the loops relative to one another.

At higher frequencies, higher order resonances can be found in the ELC as well [73] [77]. These resonances occur due to the longer electrical length of the ELC and the non uniform current distributions that are supported on the ELC. In the antenna design discussed in Chapter...
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Figure 3.12: (a) A different topology for an ELC. This specific ELC will be used in the antenna designs presented below. (b) The current distribution on the ELC at its electric resonance.

4, the higher order resonance of the ELC can be used to add multiple bands to the antenna.

Topology

The ELC shown in Fig. 3.7(a) is not the only topology of ELC. A variation of the ELC topology was introduced in [75] and is seen in Fig. 3.12 along with its resonant current distribution. It consists of two back to back loops with a capacitive gap in either loop. This specific topology will be used extensively in the designs presented in this thesis because of its ability to be excited by a monopole antenna. This topology has all the same properties as the ELC discussed above, except for a lower resonant frequency due to the presence of two capacitive gaps. Other topologies with similar properties as discussed above were introduced in [77]. Many of these topologies were used in the design of terahertz metamaterials. Fig. 3.13 reproduced from [77] shows four other kinds of ELC resonators other than the two already mentioned.

Usage in Antenna Design - The ELC-monopole antenna

Once again, an understanding of the ELC is important in the design of a compact antenna. Using the planar monopole antenna discussed in Chapter 2 as a starting point, the ELC can be integrated into the geometry of the planar monopole antenna. A monopole antenna with an ELC will be referred to as an ELC-monopole antenna. A schematic of the ELC-monopole antenna is shown in Fig. 3.14 where the ELC shown in Fig. 3.12 is used. The ELC is placed directly beneath the monopole using the entire space beneath the monopole.
By placing the ELC below the monopole, the goal is to excite the resonance of the ELC using the monopole. As discussed above, the ELC is excited by a uniform electric field oriented across the capacitive gap and has no response to a uniform magnetic field. However, from the configuration shown in Fig. 3.14 it can be seen that the monopole is not configured to excite the capacitive gap of the ELC. What the monopole does do is induce currents on the center arm of the ELC. This occurs in the following manner and is illustrated in Fig. 3.15:

1. A current is excited on the monopole antenna
2. This current induces a current on the center arm of the ELC in the opposite direction.
3. The current on the center arm of the ELC flows onto each loop causing the ELC to resonate.

This shows that the monopole antenna can excite the electric resonance of the ELC without using a uniform electric field to couple into the ELC. Another way of understanding this inter-
Figure 3.14: Schematic of ELC-monopole antenna. It is implied for now that antenna meets the geometry specifications of Chapter 1. (a) Front, (b) Back.

Figure 3.15: Diagram showing how currents on the monopole excite the ELC to resonate. (a) A current is excited on the monopole, (b) This induces a current on the center arm of the ELC in the opposite direction, (c) The currents flow around the ELC at resonance.

action is by looking at the magnetic field of the monopole. The magnetic field of the monopole is concentric with the monopole itself, and threads each loop of the ELC in an opposing manner. Because of the oppositely directed magnetic field applied to each loop of the ELC, a current flows in each loop that opposes the magnetic field of the monopole. This produces the current distribution seen in Fig. 3.15(c) exciting the electric resonance of the ELC.

This is the major difference between the free-space configuration of the ELC and its use
in a planar monopole that must be noted. In the free space configuration the ELC is excited by a uniform electric field. However in a planar monopole antenna configuration, the ELC is excited by inducing currents on the ELC and/or by the oppositely directed magnetic field of the monopole applied to each loop. Both of these configurations cause the ELC to resonate with the same current distribution, but are excited differently. This is also different than the SRR-monopole antenna where the same coupling mechanism is used for both the free space and monopole excitations. Because the monopole can be seen as inducing currents on the ELC and/or coupling into the loops of the ELC, this interaction can be thought of as a ‘magnetic’ coupling between the ELC and monopole, a fact that will become relevant in Chapter 4 when developing a circuit model.

Fig. 3.15 also shows why the specific topology of ELC was chosen. The ELC used in this antenna fits in well geometrically with the monopole antenna by allowing the monopole to induce currents on its center arm of the ELC. This is opposed to the topology of ELC shown in Fig. 3.7(a) where the currents on the monopole would have to excite the capacitive gap along the center arm which does not provide good coupling between the monopole and ELC.

Once again, by understanding the resonance and excitation of the ELC and its use as a metamaterial particle, the ELC can be properly integrated into a monopole antenna. As it will be shown in Chapter 4, the resonance excited in the ELC by the monopole can be used to create a resonance in the antenna itself that is matched at a desired frequency in the antenna. The higher-order resonances of the ELC that are excited by the monopole will also be discussed in Chapter 4.

### 3.2 Transmission-Line Approach

The transmission-line approach to metamaterial design stems from the modeling of dielectrics as a distributed LC network. The design of metamaterials involves extending this approach by modifying the configuration of the LC network.

A transmission-line can be viewed as a model for the propagation of a TEM wave through a homogenous, isotropic medium. This model is represented by a unit cell of length $d \ll \lambda$ with its per unit length series inductance and shunt capacitance shown in Fig. 3.16. This can be thought to represent the magnetic and electric response of the medium respectively, i.e. its permittivity and permeability. This is given by:

\[
\mu_0 = \frac{L'}{g} \quad (3.32)
\]

\[
\epsilon_0 = C'g \quad (3.33)
\]
where $L'$ and $C'$ are per unit length inductances and capacitances, and $g$ is the ratio between the free space impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ and the characteristic impedance $Z_0 = \sqrt{\frac{L'}{C'}}$.

By modifying the transmission-line model to include other reactive elements, a negative permittivity and/or permeability can be realized. This was the approach taken by Professor G.V. Eleftheriades’s group at the University of Toronto in [8]. In the basic case the series inductance and shunt capacitance of the transmission-line are interchanged forming a transmission-line with a series capacitance and shunt inductance. This is intuitive from an impedance perspective as the per unit length series impedance of a capacitor is negative as given by $-\frac{j}{\omega C_d}$ and the per unit length shunt admittance of an inductor is also negative as given by $-\frac{j}{\omega L_d}$. This gives a unit cell as shown in Fig. 3.17 with the following material parameters:

$$\mu = -\frac{1}{\omega^2 C_d g}, \quad (3.34)$$

$$\epsilon = -\frac{g}{\omega^2 L_d}, \quad (3.35)$$

This unit cell simultaneously realizes a negative permeability and permittivity respectively and is referred to as a negative-refractive-index transmission-line (NRI-TL) medium. The guided wave propagation on the NRI-TL is also referred to as left-handed as the electric field, magnetic field and Poynting vector form a left-handed triplet.

Practical realizations of an NRI-TL medium involve placing the series capacitance with the series inductance of a host transmission-line and the shunt inductance with the shunt capacitance of the transmission-line as shown in Fig. 3.18. This is because any implementation of an NRI-TL medium must be done on a host transmission-line which naturally has a series inductance and shunt capacitance. This implements a negative permittivity and permeability when the impedance and admittance of the series and shunt branches are both negative, and a positive permittivity and permeability when the impedance and admittance of the series and

\[ L / 2 \]
\[ C \]
\[ L / 2 \]
Figure 3.17: A negative-refractive-index transmission-line (NRI-TL) unit cell. The series inductance has been interchanged with a series capacitance, and the shunt capacitance has been interchanged with a shunt inductance. This gives an effective $\varepsilon$ and $\mu$ that are negative.

Figure 3.18: A 'practical' implementation of a negative-refractive-index transmission-line (NRI-TL) unit cell. The series capacitance $C$ and shunt inductance $L$ are integrated into a transmission-line with a series inductance $L_{TL}$ and shunt capacitance $C_{TL}$. When the impedance of the series branch and admittance of the series branch are negative the material parameters are negative

shunt branches are both positive. When compared to bulk metamaterials, transmission-line metamaterials offer two significant advantages, low-loss and a larger bandwidth over which the index of refraction is negative.

### 3.2.1 ‘Resonant’ Transmission-Line Metamaterial Design

A variation on the transmission-line metamaterial is the ‘resonant’ transmission-line metamaterial proposed in [78] and [79]. As opposed to the transmission-line metamaterials described above which loaded the transmission-line unit cell with a series capacitor and shunt inductor respectively, resonant transmission-line metamaterials load the transmission-line unit cell with
metamaterial particles such as the SRR or the complementary split-ring resonator (CSRR), which will be discussed below.

For example, Fig. 3.19(a), reproduced from [78] shows a schematic of an SRR coupled to a co-planar waveguide (CPW) transmission-line. This loads the series inductance of the CPW line with an LC resonator, as shown in Fig. 3.19(b) also from [78]. Once again when the reactance of the series branch is negative due to the capacitance of the LC resonator, the permeability is negative. This is caused by the magnetic field of the CPW line exciting the resonance of the SRR. A negative permittivity can be achieved in this example by adding a shunt inductance as in the transmission-line metamaterial example. Loading the transmission-line with an SRR can be seen as implementing a Lorentzian type of dispersion [68], p. 31. This gives a more narrowband response in the effective negative permeability of the resonant transmission-line metamaterial. This is useful in realizing miniaturized narrowband filters such as in [80] and [81] where a bandpass and bandstop filter are demonstrated respectively using resonant metamaterial transmission-lines.

The Complementary Split-Ring Resonator (CSRR) and CSRR-Transmission-Line Networks

One key particle in the design of resonant metamaterial transmission-lines is the CSRR. The CSRR was first introduced in [79] and was derived by applying duality to the SRR, replacing metal with dielectric and vice versa. An example of the CSRR can be seen in Fig. 3.20. Thus, since the SRR produces a magnetic response from an axial magnetic field, creating a negative permeability, the CSRR can be thought of as driving an electric response by creating a negative permittivity from a axial electric field. Likewise, a circuit model for the CSRR is the dual of
Figure 3.20: The Complementary Split-Ring Resonator (CSRR). The CSRR is formed by taking a SRR and applying duality by replacing the metal with dielectric and vice versa. The shaded part is metal and the white dielectric.

the SRR, with the capacitance being replaced by an inductance and vice versa, though this still reduces to a tank LC resonator. It should be noted that unlike the SRR, the CSRR does not produce a negative material response when used as a bulk metamaterial. This is because of duality and symmetry when applied to the metal surface that the CSRR is embedded on [71]. As shown in Fig. 3.21 from [82], the electric field on either side of the CSRR flips sign and thus there is no net electric response from the CSRR. Thus in free space the CSRR is not an effective bulk metamaterial particle.

However, by loading a transmission-line with a CSRR such that the electric field of the transmission-line is axial with the CSRR, a negative permittivity can be achieved in the context of a transmission-line metamaterial. For a microstrip line this can be seen in Fig. 3.22 where a single unit cell can be seen. In this unit cell configuration the CSRR is etched into the ground plane of the microstrip line, directly below the microstrip itself. Here the electric field of the microstrip line is axial to the CSRR and excites the CSRR resonance. When the inductance of the CSRR (from the metal loops in the CSRR), dominates the admittance of the shunt branch the effective permittivity of the transmission-line network becomes negative. To create a negative permeability in the unit cell a series capacitance can be added to the microstrip line [82], thus creating a negative refractive index.

Once again like the SRR loaded transmission-line, a more narrowband negative permittivity is achieved when compared to non-resonant transmission-line metamaterials, which is useful in various filter designs. Another advantage that the CSRR implementation has over the transmission-line metamaterial approach is that the shunt loading can be achieved without any
Figure 3.21: The electric and magnetic fields surrounding a CSRR. The perspective is from in the plane of the CSRR. Note that for the electric field, the direction of the field is reversed on either side of the CSRR resulting in a net electric polarization of zero. Reprinted with permission from [82]. © 2010 IEEE.

Figure 3.22: A schematic of an CSRR placed beneath a microstrip line. The capacitive flare improves the coupling between the CSRR and the microstrip line.

vias or shunt components, just a patterned ground plane. This can be attractive for certain applications where a via-less design is desired.
Figure 3.23: Simulating a CSRR-microstrip unit cell in HFSS. (a) Schematic of the CSRR-microstrip network. (b) Simulated S-Parameters. Note the stop-band that forms due to the inductance of the CSRR.

A full-wave two-port simulation in HFSS is carried out for the CSRR microstrip network shown in Fig. 3.23(a) and the S-parameters are plotted in Fig. 3.23(b). Because this transmission-line network only implements a negative permittivity, a stop band forms when the CSRR loading causes the shunt branch to become inductive. This can be seen in the simulated S-parameters where the $S_{21}$ dips below -20 dB at approximately 1.4 GHz and the $|S_{11}|$ is at a maximum of -0.67 dB. (A CSRR loaded transmission-line with a series capacitance would instead form a pass band, provided that the negative permittivity from the series capacitance overlapped in frequency with the negative permeability provided by the CSRR [82].)

From a circuit model perspective the CSRR loading is seen as loading the shunt capacitance of the transmission-line with an LC tank circuit as shown in Fig. 3.24 [83]. This circuit model models the complete two-port S-parameters of the CSRR-transmission-line network. A method for extracting values for the circuit model from the full-wave data is given in [83] which the interested reader is referred to. A closer look at the shunt admittance gives the following expression:

$$Y_{\text{shunt}} = (Z_{\text{shunt}})^{-1}$$

$$Y_{\text{shunt}} = \left( \frac{-j}{\omega C} + \frac{j\omega L_{\text{CSRR}}}{1 - \omega^2 L_{\text{CSRR}}C_{\text{CSRR}}} \right)^{-1}. \quad (3.37)$$

Equation (3.37) shows how the shunt admittance of the CSRR loaded transmission-line evolves with respect to frequency. For very small values of $\omega$ the $-j/\omega C$ term dominates as the second term on the right is small and inductive. As $\omega$ increases the inductance from the second term
Figure 3.24: Circuit model of CSRR-transmission-line network derived in [83]

increases and at a frequency $\omega = \omega_z$, the admittance becomes infinite as the inductance of the second term cancels out the capacitance $\frac{j}{\omega C}$. For frequencies larger than $\omega_z$ the admittance is inductive and the permittivity is negative. At a frequency $\omega = \omega_0 = \frac{1}{\sqrt{L_{CSRR}C_{CSRR}}}$ the inductance is canceled out by the capacitance of the CSRR and the admittance becomes zero. Above $\omega = \omega_0$ the admittance becomes capacitive again, restoring a positive permittivity.

**Using the CSRR-Transmission-Line As A Matching Network**

In Chapter 5, the CSRR-transmission-line network is used to implement a matching network to match a planar monopole antenna. It is the shunt loading of the CSRR given by equation (3.37) which implements the matching network (the theory of which is described below in Chapter 2). Specifically it is the shunt loading, described above, that is provided by the CSRR for frequencies greater than $\omega_z$ that matches the antenna. This is a novel application of the CSRR-transmission-line network that takes advantage of its compact size and via-less design to implement a printed shunt matching network.
Chapter 4

The ELC-Monopole Antenna

With the concept of the ELC-monopole antenna introduced in the last chapter, the goal now is to develop a fuller understanding of the antenna by using full-wave simulations to characterize the antenna. The antenna will be simulated using HFSS to confirm that the antenna works as intended and to get a better understanding of the antenna using measurements such as $|S_{11}|$, input impedance, current density distributions, radiation patterns and efficiency. To further the understanding of the ELC-monopole, a circuit model is also developed to model the input impedance of the antenna to explain how the ELC and monopole interact at a single resonance. A parametric study of the antenna is also done and the results interpreted using the circuit model to give a sense of the design intuition of the antenna. Finally this understanding of the ELC-monopole antenna will be applied to the design of a Wi-Fi antenna for a laptop computer.

4.1 Simulated Results

The ELC-monopole antenna with the dimensions shown in Fig. 4.1 is simulated in HFSS. The $|S_{11}|$ and input impedance are plotted in Fig. 4.2 with the $|S_{11}|$ compared to the $|S_{11}|$ of an equivalently-sized planar monopole. There are two resonant dips in the $|S_{11}|$ of the ELC-monopole antenna below -10 dB while there are none in the planar monopole antenna. The two resonant dips occur at 3.31 GHz and 6.08 GHz respectively. This simple comparison shows the effect of placing the ELC resonator below the monopole, and its ability to add resonances in a compact space. These resonances can also be seen in Fig. 4.2(b) where the input impedance of the ELC-monopole antenna is plotted. Once again two resonances are seen at 3.31 GHz and 6.29 GHz which are due to the ELC, along with a resonance near 2.60 GHz and a resonance near 5.20 GHz. These last two resonances correspond to the resonances of the planar monopole itself, with the resonance at 2.6 GHz corresponding to the monopole mode and the resonance
Figure 4.1: Schematic of ELC-monopole antenna with relevant dimensions. The antenna meets the geometry specifications given in Chapter 1. (a) Front, (b) Back.

Figure 4.2: Simulated $|S_{11}|$ and input impedance of the ELC-monopole antenna. (a) $|S_{11}|$ compared to the $|S_{11}|$ of the planar monopole antenna, (b) input impedance at 5.2 GHz corresponding to the dipole mode as discussed in Chapter 2.

To give a more detailed picture of the resonances added by the ELC, the surface current density, $J_{surf}$, is plotted on the antenna at 3.31 GHz and 6.08 GHz in Figures 4.3 and 4.4 respectively. These are the frequencies where the antenna is best matched. In Fig. 4.3 at 3.31 GHz, the current density on the monopole is traveling in one direction, while the current density on the center arm of the ELC is traveling in the opposite direction. This current
density then wraps around the two loops of the ELC and travels down the capacitive arms. This demonstrates that the current density on the ELC-monopole antenna is the same as the current distribution predicted in Fig. 3.15. It can also be seen from the strength of the current density distribution that the monopole is primarily exciting the center arm of the ELC and not the capacitive gap. This demonstrates the ‘magnetic’ nature of the coupling, a fact that will be used in the circuit model in Section 4.2. This resonance is termed the fundamental resonance of the ELC-monopole antenna.

In Fig. 4.4 at 6.08 GHz the electrical length of the antenna is longer, where the current density along the length of the monopole resembles a $\frac{1}{2}$ dipole. This is due to the dipole mode resonance at 5.20 GHz. The current density on the monopole continues to induce currents on the ELC at this frequency. The current distribution that is induced on the ELC is a higher order mode with nulls on the center arm of the ELC and the capacitive gaps. When the ELC is electrically longer, another resonance occurs due to this higher-order mode of the ELC where its
resonant frequency is approximately twice that of the fundamental resonant frequency \cite{77}. As mentioned above, this resonance is seen in the input impedance at 6.29 GHz. Thus at 6.08 GHz, the higher-order ELC resonance, combined with the dipole mode on the monopole causes the antenna to be matched to 50 Ω. This mode will also be referred to as the higher-order mode of the ELC-monopole antenna. The higher-order mode in the ELC-monopole antenna is a more complicated interaction between the monopole and the ELC. Thus while the frequency of the fundamental resonance is controlled by the ELC, the higher-order mode is affected by both the length of the monopole and the ELC. This will be elaborated on more in Section 4.3 for both resonances.

The radiation properties of the ELC-monopole antenna are also investigated. Before looking at the radiation patterns, the unbalanced currents on the antenna are examined to show the radiating currents on the antenna. In Fig. 4.5 the current density is plotted again at 3.31 GHz with the currents on the ground plane included. This is because the ground plane is small and
Figure 4.5: Unbalanced currents on ELC-monopole antenna at 3.31 GHz including the currents on the ground plane

contributes to the radiation produced by the antenna [51]. The unbalanced currents are easily identified and include the currents on the capacitive arms of the ELC, the currents on the top of the monopole and the currents on the edge of the ground plane. The currents on the center arm of the ELC and the monopole tend to cancel out as they flow in opposite directions. From this we can conclude that the radiation properties of the antenna will be dipolar like with a null in the pattern along the length of the antenna. The patterns are plotted at 3.31 GHz for three different cuts in Fig. 4.6, with the coordinate system also shown in Fig. 4.6(d). It can be seen from all the cuts that the radiation pattern resembles a dipolar like pattern with the ‘xz’ and ‘xy’ cuts being the E-plane of the antenna and the ‘yz’ cut the H-plane. It is interesting to note that there is a tilt in the pattern, towards the space behind the antenna ($-90^\circ < \phi < 90^\circ$, $0^\circ < \theta < 180^\circ$). This is due to the currents on the ELC and ground plane radiating slightly out of phase to create a pattern that is tilted as shown.

The patterns at 6.08 GHz also follow a similar trend with dipolar like radiation expected due to the currents along the length of the antenna. This can be seen in Fig. 4.7 where a similar dipolar like radiation pattern occurs at 6.08 GHz, however the pattern has more lobes due to the longer electrical length of the antenna.

The efficiency at each resonant frequency is reported in Table 4.1. The addition of the ELC resonator still allows the ELC-monopole antenna to maintain a respectable efficiency. The lower efficiency at the fundamental resonance can be explained qualitatively by the fact that
Figure 4.6: Simulated Radiation Patterns for three cuts of the antenna. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) XZ plane, (b) XY plane, (c) YZ plane, (d) the coordinate system.

Figure 4.7: Simulated Radiation Patterns for three cuts of the antenna. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) XZ plane, (b) XY plane, (c) YZ plane.

the monopole is not radiating on its own but coupling energy into the ELC to be re-radiated. Because the coupling is not ideal (i.e. not all the power incident on the monopole reaches the ELC and is radiated), the antenna’s efficiency decreases. At the higher-order mode a high efficiency is maintained.
### Table 4.1: Efficiency of ELC-monopole antenna at its resonant frequency

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.31 GHz</td>
<td>70%</td>
</tr>
<tr>
<td>6.08 GHz</td>
<td>&gt;95%</td>
</tr>
</tbody>
</table>

By placing an ELC below the planar monopole antenna, multiple resonances have been added to the antenna creating multiple bands. The resonance at 3.31 GHz is a resonance that was purposely designed for in the placement of the ELC while the higher-order mode at 6.08 GHz was not. However both resonances demonstrate the usefulness of the ELC-monopole antenna in compact antenna design and the ability of the ELC to add multiple resonances to the antenna.

### 4.2 Circuit Model

In this section a circuit model is developed to model the input impedance of the ELC-monopole antenna at its fundamental resonance. The circuit model will be used to give insight into how the geometrical parameters of the ELC-monopole antenna affect the input impedance at the fundamental resonance. What the circuit model does not do is model the higher order resonances in the antenna, the radiation properties, efficiency or other quantities.

The model is developed in two parts, with the circuit model of the ELC at its fundamental resonance developed first, followed by the circuit model of the planar monopole antenna. The two parts will then be combined to model the ELC-monopole antenna as a whole. To formulate the circuit model, approximate values will be extracted where possible but a heuristic approach will also be used to fit the model to the full-wave simulation.

A circuit model for an ELC resonator at its fundamental resonance was shown in Fig. 3.9 as an LC resonator where $L$ is the inductance of the loops and $C$ is the capacitance of the gaps in the ELC. This circuit model does not include any losses of the ELC (either radiation or Ohmic)\(^1\). Thus this circuit model must be modified to include losses since the ELC is being used in an antenna.

For the ELC pictured in Fig. 4.8(a), each loop of the ELC can be thought of as an inductor with inductance $L_{loop}$. On each loop there is a capacitive gap $C_{gap}$. In series with each inductor and capacitor there is a resistance $R_{ELC}$, which represents the ohmic loss due to the current flowing in the loop and the radiation resistance in each loop. This circuit can then be simplified into a series RLC circuit as shown in Fig. 4.8(b) where,

\(^1\)For SRR’s it is known that there are both ohmic and radiation losses [84]. This concept can be extended to ELCs.
Figure 4.8: Circuit model for the ELC used in the ELC-monopole antenna, including its resistance (a) Circuit model of the ELC, (b) Simplified Circuit

\[ R = \frac{R_{ELC}}{2}, \]  
\[ L = \frac{L_{\text{loop}}}{2}, \]  
\[ C = 2C_{\text{gap}}. \]

The frequency of the fundamental resonance, \( \omega_o \), and the \( Q \) of the ELC resonator are given by:

\[ \omega_o = \sqrt{\frac{L_{\text{loop}}}{2}} \frac{1}{2C_{\text{gap}}} = \sqrt{\frac{1}{LC}}, \]  
\[ Q = \frac{1}{R} \sqrt{\frac{L}{C}}. \]

Using this circuit model for the ELC, values for \( L_{\text{loop}} \) and \( C_{\text{gap}} \) can be found by making some approximations. The loop inductance, \( L_{\text{loop}} \), can be estimated by using the equation for the inductance of a rectangular wire loop, which is given in [85] as:

\[ L = \frac{\mu_0}{\pi} \left[ -2(w + l) + 2\sqrt{w^2 + l^2} - l \ln \left( \frac{l + \sqrt{w^2 + l^2}}{w} \right) - w \ln \left( \frac{w + \sqrt{w^2 + l^2}}{l} \right) + l \ln \left( \frac{2l}{a} \right) + w \ln \left( \frac{2w}{a} \right) \right] \]

where the length, \( l \), and width, \( w \), of the loop are given by the length of the ELC, \( l_{ELC} \), and half
the width of the ELC, \( \frac{w_{ELC}}{2} \). The variable, \( a \), is the radius of the wire forming the loop. Since the loops of the ELC are made up of planar strips of copper, the value of \( a \) is approximated by the formula given in [45], p. 514 for modeling planar strips as wires,

\[
a = 0.25 t_{ELC}. \tag{4.7}
\]

For the ELC used in Fig. 4.1, Equation (4.6) gives an inductance of \( L_{\text{loop}} = 22.54 \text{ nH} \), and a total inductance of \( L = 11.27 \text{ nH} \).

To approximate the capacitance, the frequency of the fundamental resonance, \( \omega_0 \), along with the inductance, \( L \), are used where,

\[
C = \frac{1}{L \omega_0^2}. \tag{4.8}
\]

From Section 4.1 the frequency of the fundamental resonance is \( f_0 = 3.31 \text{ GHz} \), and the total capacitance of the ELC is found to be \( C = 0.21 \text{ pF} \).

The values of \( L \) and \( C \) are reasonable approximations for the geometry of the ELC as the loops in the ELC are large implying a large inductance and the capacitive gap, \( g_{ELC} \), is also large implying a small capacitance.

The total resistance of the ELC, \( R \), includes both ohmic and radiation loss. This is a difficult parameter to extract as there are no known analytic expressions for the resistance of an ELC due to its radiation loss\(^2\). Thus the value of the resistance, \( R \), will be found by using \( R \) as a fitting parameter in the circuit model for the ELC-monopole antenna.

With the circuit model of the ELC in place, the next step is to create the circuit model of the planar monopole antenna. This procedure is detailed in Appendix A. The final model and the values of each component can be seen in Fig. 4.9 and Table 4.2. The main detail regarding the circuit model of the planar monopole is that the series RLC circuit consisting of \( L_s \), \( R_s \) and \( C_s \), which represent the planar monopole’s intrinsic inductance, resistance and capacitance respectively outside of its \( \frac{\lambda}{4} \) and \( \frac{\lambda}{2} \) resonances. These resonances are represented by the two parallel resonators respectively.

Using the circuit model of the ELC resonator and the planar monopole antenna, the two models can be combined to create a circuit model for the ELC-monopole antenna. Chapter 3 showed that the monopole couples into the ELC by inducing a current on the ELC through the magnetic field of the monopole. This interaction can be modeled as a mutual inductance between the series inductance of the monopole, \( L_s \), and the inductance of the ELC, \( L \), which

\[^2\text{The resistance due to the loss, can be modeled by the sheet resistance, } R_s \text{ of the copper at which is given by}
\]

\[
R_s = \sqrt{\frac{\omega_0 j \sigma}{2 \sigma}}, \tag{4.9}
\]

where \( \sigma \) is the conductivity of copper. However this alone is not sufficient to model the total resistance.
Figure 4.9: A circuit model of the planar monopole antenna

Table 4.2: Values of the planar monopole antenna circuit model without any ELC loading.

<table>
<thead>
<tr>
<th>Circuit Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>4.6 (\Omega)</td>
</tr>
<tr>
<td>$L_s$</td>
<td>0.53 nH</td>
</tr>
<tr>
<td>$C_s$</td>
<td>2.88 pF</td>
</tr>
<tr>
<td>$R_p$</td>
<td>373 (\Omega)</td>
</tr>
<tr>
<td>$L_p$</td>
<td>0.52 nH</td>
</tr>
<tr>
<td>$C_p$</td>
<td>1.56 pF</td>
</tr>
<tr>
<td>$R_m$</td>
<td>11.67 (\Omega)</td>
</tr>
<tr>
<td>$L_m$</td>
<td>0.41 nH</td>
</tr>
<tr>
<td>$C_m$</td>
<td>8.74 pF</td>
</tr>
</tbody>
</table>

is given by:

\[
M = k\sqrt{LL_s},
\]

(4.10)

where \(k\) is a coupling coefficient between the monopole and ELC with \(0 < k < 1\). The \(k\) factor will also be used as a fitting parameter in the ELC-monopole antenna circuit model along with the resistance of the ELC, \(R\). The circuit model of the ELC-monopole antenna with the mutual coupling between the ELC and monopole is shown in Fig. 4.10(a).

Using some simple circuit analysis, the ELC resonator can be represented as a parallel RLC circuit in series with the planar monopole antenna as shown in Fig. 4.10(b), where \(R', L'\) and \(C'\) are given by:

\[
R' = \frac{\omega^2 M^2}{R},
\]

(4.11)

\[
L' = \omega^2 M^2 C,
\]

(4.12)

\[
C' = \frac{L}{\omega^2 M^2}.
\]

(4.13)

This analysis can be seen in more detail in Appendix A. The parallel RLC circuit representing the ELC in Fig. 4.10(b) has the same resonant frequency, \(\omega_o\), and \(Q\) as the series RLC circuit.
Chapter 4. The ELC-Monopole Antenna

Figure 4.10: Circuit model for the ELC-monopole antenna (a) Circuit model showing the inductive coupling between the ELC and the monopole, (b) The simplified circuit.

model of the ELC in Fig. 4.8 where:

\[
\omega_0 = \frac{1}{\sqrt{LC'}} = \frac{1}{\sqrt{LC}},
\]

(4.14)

and

\[
Q = R'\sqrt{\frac{C'}{L'}} = \frac{1}{R}\sqrt{\frac{L}{C}}.
\]

(4.15)

This simplified circuit model shows that through the parallel RLC resonator representing the ELC, the fundamental resonance of the ELC-monopole antenna is controlled by the ELC itself. This model also shows that the fundamental resonance is independent of the other resonances in the antenna as well.

The circuit model of the ELC-monopole antenna is simulated in Matlab and the coupling, \(k\), and ELC resistance, \(R\), are used to fit the circuit model to the full-wave simulation. The ELC resistance, \(R\), controls the width of the fundamental resonance and the \(k\) factor controls the peak value of the resistance at resonance. These are adjusted until they fit the full-wave simulation of the input impedance and are found to be \(k = 0.37\) and \(R = 9\ \Omega\) respectively. The input impedance of the circuit model is plotted in Fig. 4.11 along with the full-wave simulation of the input impedance. It can be seen that the circuit model captures the behaviour of the ELC-monopole antenna at the fundamental resonance. At higher frequencies however, the circuit
Chapter 4. The ELC-Monopole Antenna

Figure 4.11: Input Impedance of the ELC-monopole antenna circuit model compared to HFSS simulations (a) From 2 GHz-4 GHz, (b) From 1 GHz-7 GHz

The model does not capture the behaviour of the higher order resonances of the ELC, showing the limitations of the model.

Ultimately this circuit model is useful in understanding how to design the ELC-monopole antenna at its fundamental resonance. For example, to control the frequency of the fundamental resonance in the ELC-monopole antenna, the inductance and capacitance of the ELC have to be varied, which leads to an understanding of which geometrical parameters to change such as $l_{ELC}$ and $g_{ELC}$. Other parameters such as the $Q$ of the ELC, and the mutual inductance, $M$,
can also affect the matching at the fundamental resonance, giving further insight into designing the antenna. As stated above, the circuit model also shows that the fundamental resonance is controlled by the ELC only, allowing the designer to focus on the ELC to shape the fundamental resonance, simplifying the design of the antenna. These ideas will continue to be expanded upon in the next section.

4.3 Parametric Analysis

With an understanding of how the ELC-monopole antenna works, different geometric parameters can be varied in HFSS to shed further understanding on the behaviour of the antenna, as well as an intuition into the design of the antenna. The following parameters which are referred to in Fig. 3.14, will be varied:

1. The size of the ELC, specifically the length of the ELC, $l_{ELC}$ from 12 mm-24 mm.
2. The capacitive gap of the ELC, $g_{ELC}$ from 0.4 mm-2 mm.
3. The width of the monopole, $w_{monopole}$ from 0.2 mm-1 mm.
4. The length of the monopole, $l_{monopole}$ from 22 mm-30 mm.

The first two variables modify the dimensions of the ELC and show how the ELC affects the antenna. The last two variables vary the geometry of the monopole itself and show how the monopole affects the antenna. Some of these variations will be mapped to the circuit model of the previous section to highlight an intuitive understanding of the geometrical variation. The parameters examined here are not an exhaustive list as other geometrical parameters of the ELC-monopole antenna such as the thickness of the ELC, and the length of the ground plane affect its performance.

1. Varying $l_{ELC}$

The simulated $|S_{11}|$ and input impedance for different values of $l_{ELC}$ can be seen in Fig. 4.12. Starting with the fundamental resonance of the antenna, it is shown in Fig. 4.12(a) that $l_{ELC}$ is inversely proportional to the frequency of the fundamental resonance of the antenna. By varying $l_{ELC}$ over a range of 12 mm, the frequency of the fundamental resonances shifts over a range of 1.40 GHz, while maintaining the $|S_{11}| < -10$ dB.

Varying $l_{ELC}$ changes the inductance, $L$, of the ELC because $l_{ELC}$ affects the size of the loop. Increasing the inductance causes the frequency of the fundamental resonance to decrease in the ELC-monopole antenna.
Figure 4.12: $|S_{11}|$ and input impedance of the ELC-monopole antenna for different values of $l_{ELC}$. (a) $|S_{11}|$ (b) Input impedance. The input impedance is shown from 2.0 GHz-4.5 GHz for clarity’s sake.

It can also be seen that changing $l_{ELC}$ changes the matching of the fundamental resonance. This is seen in the input impedance in Fig. 4.12(b) where Re($Z_{in}$) broadens for larger values of $l_{ELC}$ and narrows for smaller values of $l_{ELC}$. Using the circuit model of Section 4.2 this is explained using the $Q$ of the ELC resonator which is given by Equation (4.5). Since the resonance is broadening as $l_{ELC}$ increases, it is assumed that the $Q$ of the ELC resonator is
Figure 4.13: The input impedance of the circuit model for increasing inductance and resistance from 2-4.5 GHz. A shift in the resonance frequency along with a broadening of the resonance is seen for increasing inductance and resistance.

decreasing and the resistance $R$ of the ELC resonator is increasing.\textsuperscript{3} Using the circuit model of Section 4.2, the inductance $L$ and resistance $R$ are varied together and are plotted in Fig. 4.13. Increasing the inductance, $L$, and the resistance, $R$, of the ELC, lowers the frequency of the fundamental resonance and broadens the resonance respectively. This is the same trend seen in Fig. 4.12(b). Thus as $l_{ELC}$ increases, the inductance, $L$, and resistance, $R$, of the ELC also increases which causes the fundamental resonance of the ELC-monopole antenna to decrease in frequency and broaden.

From a design perspective, increasing $l_{ELC}$ too much will cause the resonance to be poorly matched by broadening the resonance, while decreasing $l_{ELC}$ too much will also affect the matching as the resonance narrows. Therefore when designing the fundamental resonance of the antenna using $l_{ELC}$, attention must be paid to the tradeoff between controlling the frequency of the fundamental resonance and its matching.

Turning to the higher-order resonance, the $|S_{11}|$ in Fig. 4.12(a) shows that the higher-order mode also decreases in frequency as $l_{ELC}$ increases. Thus attention must also be paid to higher-order resonance, if it is be used, when varying $l_{ELC}$ as $l_{ELC}$ affects both the fundamental resonance and the higher-order resonance.

For $l_{ELC} = 20$ mm and $l_{ELC} = 24$ mm another resonance can be seen at 6.10 GHz and

\textsuperscript{3}This increase in $R$ could be from one of two sources, the ohmic loss or the radiation loss. Assuming that $R$ is dominated by the radiation resistance, it can be assumed that this increase in $R$ is mainly due to the radiation resistance.
6.65 GHz respectively in Fig. 4.12(a). This resonance is not related to the higher-order resonance of the ELC. However it is clear that the presence of a ELC helps match the antenna to 50 Ω as this resonance is not present when the ELC is removed. Looking at the current distribution in Fig. 4.14 at 6.10 GHz for $l_{ELC} = 24\, mm$, a strong current distribution can be seen on the monopole while the currents on the ELC are weaker. This current distribution on the monopole is slightly larger than $\frac{\lambda}{2}$. It was found in simulation that changing $l_{monopole}$ also changes the resonant frequency of this mode. Given the strong currents on the monopole and its dependance on $l_{monopole}$, this mode will be referred to as the ELC-dipole mode of the antenna. This mode will become useful in Section 4.4 in the design of a Wi-Fi antenna.

2. Varying $g_{ELC}$

The parameter $g_{ELC}$ shown in Fig. 4.1, varies the capacitance of the ELC by changing the spacing between the capacitive flare of the ELC. Fig. 4.15 shows the $|S_{11}|$ and the input impedance for different values of $g_{ELC}$. From Fig. 4.15(a), increasing $g_{ELC}$ decreases the capacitance and increases the frequency of the fundamental resonance and vice versa. In the input impedance...
Figure 4.15: $|S_{11}|$ and input impedance of the ELC-monopole antenna for different values of $g_{ELC}$. (a) $|S_{11}|$ (b) Input impedance. The input impedance is shown from 2.0 GHz-4.5 GHz for clarity’s sake.

shown in Fig. 4.15(b), the fundamental resonance broadens or narrows as the capacitance increases or decreases respectively, affecting the matching of the resonance. This can be explained by the circuit model by looking again at the $Q$ of the ELC resonator. As the resonance broadens by increasing $g_{ELC}$, the $Q$ of the ELC resonator is decreasing and the resistance $R$ must be increasing. This is shown in Fig. 4.16 where the input impedance of the circuit model is plotted as $C$ and $R$ vary. An increase in $C$ and $R$ in the circuit model lowers the resonant fre-
Figure 4.16: The input impedance of the circuit model for different values of \( C \) and \( R \).

frequency and broadens the resonance, showing the same trend as the full-wave input impedance in Fig. 4.15(b).

This sweep demonstrates that \( g_{ELC} \) can also control both the frequency of the fundamental resonance and its matching. This makes \( g_{ELC} \) another useful parameter in the design of the fundamental resonance.

3. Varying \( w_{\text{monopole}} \)

The width of the monopole is given by the parameter \( w_{\text{monopole}} \) as shown in Fig. 4.1. The simulated \( |S_{11}| \) and input impedance for different values of \( w_{\text{monopole}} \) are shown in Fig. 4.17. From the \( |S_{11}| \), the frequency of the fundamental resonance is relatively unchanged, however the matching of the fundamental resonance varies for different values of \( w_{\text{monopole}} \). This can be seen more clearly in the input impedance in Fig. 4.17(b) where the peak value of the real part of the input impedance at the fundamental resonance is inversely proportional to \( w_{\text{monopole}} \).

Varying \( w_{\text{monopole}} \) can be thought of as varying the inductance of the monopole, where increasing \( w_{\text{monopole}} \) decreases the monopole inductance. In the circuit model of Section 4.2, the inductance of the monopole is modeled by \( L_s \). The inductance \( L_s \) is also the inductance through which the ELC couples to the monopole in the circuit model. Thus from the circuit model perspective, changing \( w_{\text{monopole}} \) changes \( L_s \), which affects the coupling of the fundamental resonance.

This is demonstrated in equation (4.10) where the mutual inductance between the monopole and ELC depends on the inductance of the monopole itself. An increase in the monopole inductance, \( L_s \), will result in a higher mutual inductance and vice versa, without affecting the
resonant frequency of the fundamental resonance. The input impedance of the circuit model is shown in Fig. 4.18 for different values of $L_s$ to demonstrate this trend. Smaller values of $L_s$ cause the peak value of $\Re(Z_{in})$ of the circuit model to decrease at the fundamental resonance, the same trend as in Fig. 4.17(b). This shows that changing $w_{\text{monopole}}$ changes the inductance of the monopole which changes the coupling between the ELC and monopole.

From a design point-of-view, $w_{\text{monopole}}$ is a very useful parameter for matching the fundamental resonance of the ELC monopole antenna to 50 Ω without changing the resonant frequency. It is also noted that the matching at the higher-order mode is affected by $w_{\text{monopole}}$ as well, as shown in Fig. 4.17(a) which shows that attention must be paid to the higher-order mode when varying $w_{\text{monopole}}$.

4. Varying $l_{\text{monopole}}$

The length of the monopole is given by $l_{\text{monopole}}$ as shown in Fig. 4.1. The $|S_{11}|$ and input impedance for different values of $l_{\text{monopole}}$ are shown in Fig. 4.19. The $|S_{11}|$ shows that there is little variation in the frequency of the fundamental resonance, demonstrating that the fundamental resonance of the ELC-monopole antenna is not strongly dependant on the length of the monopole. However at the higher-order mode, increasing $l_{\text{monopole}}$ causes the frequency of the higher-order mode to decrease and vice-versa. This is also seen in the input impedance plotted in Fig. 4.19(b), where the dipole resonance and the higher-order resonance are decreasing in frequency.

Changing $l_{\text{monopole}}$ changes the dipole mode on the monopole with respect to frequency. This also affects the frequency at which the higher-order mode of the ELC is excited, as seen in Fig. 4.19(b). It can be seen that the higher order resonance of the ELC is affected by the monopole much differently than the fundamental resonance of the ELC.

From a design perspective, varying $l_{\text{monopole}}$ allows control over the placement of the higher-order mode, which can be useful in the design of a multi-band antenna.

4.4 Wi-Fi Antenna Design

The proposed design techniques using ELC resonators can be applied to the task of designing a Wi-Fi antenna for a laptop computer. The design procedure will be walked through step-by-step to show how a Wi-Fi antenna can be designed in HFSS by integrating ELC resonators into a planar monopole antenna.
Figure 4.17: $|S_{11}|$ and input impedance of the ELC-monopole antenna for different values of $w_{\text{monopole}}$. (a) $|S_{11}|$ (b) Input impedance. The input impedance is shown from 2.0 GHz-4.5 GHz for clarity’s sake

4.4.1 Substrate

In this section a multi-layer substrate is used to allow for the addition of multiple ELC’s to the antenna. This is different from the single layer substrate used throughout the rest of this thesis. The substrate consists of two layers of FR-4 with three metallization layers. The metallization layers are referred to as the bottom, middle and top layer and are colour-coded as shown in Fig. 4.20. The total height of the substrate is 0.8 mm with each FR-4 layer having a thickness
Figure 4.18: Input Impedance of the circuit model for different values of \( L_s \). At the fundamental resonance the peak in \( \Re(Z_{in}) \) decreases as \( L_s \) decreases.

of 0.4 mm. This satisfies the height requirement given in Chapter 1 and allows the antenna to maintain a low profile. It is important to note that the microstrip feedline still sits on a two layer substrate with no top layer present, as also shown in Fig. 4.20. This is to facilitate the placement of an SMA connector.

4.4.2 Step 1 - Planar Monopole Antenna

The first step in the design of the Wi-Fi antenna is to design the planar monopole antenna. To fit the geometry constraints, the Wi-Fi antenna is chosen to be 6 mm wide and the length of the monopole is 25 mm long. This is shown in Fig. 4.22 along with the rest of the dimensions of the monopole. At this stage, the Wi-Fi antenna only uses the bottom two layers of the substrate with the top layer left unused. The antenna is simulated in HFSS and the \( |S_{11}| \) is plotted in Fig. 4.22. The \( |S_{11}| \) is similar to the \( |S_{11}| \) shown in Fig. 2.3(a) and the second layer of FR-4 has little effect on the performance of the planar monopole. However the second layer of FR-4 does decrease the electrical length of the monopole.

4.4.3 Step 2 - Adding an ELC

The second step in the design process is to add an ELC to the antenna. The goal of integrating a single ELC resonator with the planar monopole is to cover part of the Wi-Fi spectrum. The ELC added in this step will be designed to specifically cover the low-band of the Wi-Fi spectrum. However, keeping in mind that the ELC causes other resonances such as the higher-order mode
Figure 4.19: $|S_{11}|$ and input impedance of the ELC-monopole antenna for different values of $l_{\text{monopole}}$: (a) $|S_{11}|$ (b) Input impedance. The input impedance is shown from 4.0 GHz-7.0 GHz for clarity’s sake.

or the ELC-dipole mode, attention will be paid to those resonances to see if they can be used to partially cover the high-band.

To design the ELC to cover the low-band, the geometrical parameters of the ELC have to be adjusted to place the fundamental resonance of the ELC near 2.40 GHz. The geometrical parameters of the antenna also have to be adjusted so that the fundamental resonance is matched to 50 Ω. The process of tuning the ELC is as follows:
Chapter 4. The ELC-Monopole Antenna

Figure 4.20: Diagram of a 3-layer substrate with colour-coded metallization layers. The feedline sits on a two layer substrate to facilitate the placement of an SMA connector.

Figure 4.21: Schematic of the planar monopole antenna on the new substrate.

Figure 4.22: $|S_{11}|$ of planar monopole antenna on the new substrate. The results are similar to those of Chapter 2.

1. Adjust ELC parameters such as $l_{ELC}$, $t_{ELC}$, and $g_{ELC}$ as shown in Fig. 4.1 until the fundamental resonance is at the correct frequency.
2. Adjust parameters such as $l_{ELC}$, $g_{ELC}$ and $w_{monopole}$ also shown in Fig. 4.1 until the resonance is matched.

3. If the fundamental resonance is at the correct frequency and is matched continue, otherwise repeat steps 1 and 2 until both conditions are met.

It should be stated that there is not a singular solution to this problem and that a variety of solutions using the geometrical parameters of the antenna are possible.

Using HFSS, the above process is carried out until an appropriate solution is found. A potential solution is shown in Fig. 4.23, along with the relevant dimensions. The $|S_{11}|$ and input impedance are plotted in Fig. 4.24. In these plots the fundamental resonance of the ELC lies near the low-band of Wi-Fi spectrum with the $|S_{11}| < -10 \text{ dB}$ from 2.45 GHz to 2.55 GHz.

Looking at the rest of the $|S_{11}|$ and input impedance in Fig. 4.24, two other bands are seen, one at 4.81 GHz and another at 5.83 GHz. From the previous sections it is known that the band at 4.81 GHz is caused by the higher order mode of the ELC-monopole antenna, while the band at 5.83 GHz is caused by the ELC-dipole mode. The higher order mode near 4.81 GHz can be ignored in this design as it falls out of band, however the ELC-dipole mode at 5.83 GHz conveniently falls within the Wi-Fi spectrum with an $|S_{11}| < -10 \text{ dB}$ from 5.40 GHz-6.05 GHz. It will be used to cover part of the high-band.

**4.4.4 Step 3 - Adding Another ELC**

The final task is to cover the rest of the high-band of the Wi-Fi spectrum. To accomplish this, another resonance can be introduced into the antenna by placing another ELC on the top metal layer and using its fundamental resonance to create another band. The fundamental resonance of this second ELC resonator is placed approximately around the 5.10 GHz-5.40 GHz.

---

$^{4}$As stated in Section 4.3, the location of this band is controlled by varying $l_{monopole}$. 
range. This resonance has to merge with the existing resonance at the high-band to maintain an $|S_{11}| < -10$ dB across the entire high-band. This can be done because the resonance introduced by the second ELC is independent from the existing resonances at the high-band. This technique of merging resonances is a simple way to increase the bandwidth of the antenna without a large increase in complexity.

A schematic of the Wi-Fi antenna can be seen in Fig. 4.25. One parameter that is unique to the second ELC is its location relative to the monopole, $l_{ELC_2}$, as shown in Fig. 4.25. The location of the second ELC affects both the matching of this resonance and its merging with the existing resonances. At around 5 GHz the monopole is electrically longer and the currents on the monopole are no longer uniform. Thus, the location of the second ELC is important because it affects how the currents on the monopole will couple into the second ELC which affects how well the fundamental resonance of the second ELC is matched and how well it merges with the existing resonances at the high-band. An example of this is shown in a plot of the $|S_{11}|$ for different values of $l_{ELC_2}$ in Fig. 4.26. This plot demonstrates that the resonance of the second ELC is excited for some values of $l_{ELC_2}$ but not for other values.

Thus the second ELC is designed using HFSS as follows:

1. Adjust the parameters of the second ELC such as $l_{ELC_2}$, $t_{ELC_2}$, and $gap_{ELC_2}$ until the fundamental resonance of the second ELC is at the correct frequency.

2. Adjust $l_{ELC_2}$ until the resonance is matched to 50 Ω and has merged with the existing resonance at the high-band.

3. If the resonance is at the correct frequency and is matched continue, otherwise repeat steps 1 and 2 until both conditions are met.
Chapter 4. The ELC-Monopole Antenna

Figure 4.25: Schematic of the Wi-Fi antenna with two ELC’s. The dimensions of the second ELC are shown.

Figure 4.26: $|S_{11}|$ of the Wi-Fi antenna for different values of $loc_{ELC_2}$.

A plot of the $|S_{11}|$ and input impedance of the Wi-Fi antenna is shown in Fig. 4.27 for an optimal configuration of the second ELC. (The dimensions can be seen in Fig. 4.25). It is seen in the $|S_{11}|$ that in this configuration, the two resonances at the high-band have merged together covering the entire high-band of the Wi-Fi spectrum with an $|S_{11}| < -10$ dB.

Looking at the low-band of the $|S_{11}|$ in Fig. 4.27(a) it is seen that the fundamental resonance of the first ELC has shifted down to 2.38 GHz. This slight shift in frequency is due to the presence of the second ELC affecting the resonant frequency of the first ELC. Generally speaking, from a zeroth-order perspective, the fundamental resonance due to the first ELC and the fundamental resonance of the second ELC are independent of each other and can be designed as such. This is because they work a different frequencies and do not couple to one another. Also the amount of physical overlap is minimized between the first and second ELC. However due to the higher-order effects and parasitics there is still some interaction that must be accounted for. To counter-act this frequency shift, the last step of the design procedure is
Figure 4.27: Simulated $|S_{11}|$ and input impedance of the Wi-Fi antenna with two ELC’s added, (a) $|S_{11}|$, (b) input impedance.

Table 4.3: Dimensions of the Wi-Fi Antenna. The schematic can be seen in Fig 4.25.

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{\text{monopole}}$</td>
<td>25 mm</td>
</tr>
<tr>
<td>$w_{\text{monopole}}$</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>$l_{ELC}$</td>
<td>18.25 mm</td>
</tr>
<tr>
<td>$w_{ELC}$</td>
<td>5.8 mm</td>
</tr>
<tr>
<td>$t_{ELC}$</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>$g_{ELC}$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$l_{ELC_2}$</td>
<td>7.3 mm</td>
</tr>
<tr>
<td>$w_{ELC_2}$</td>
<td>5.0 mm</td>
</tr>
<tr>
<td>$t_{ELC_2}$</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>$gap_{ELC_2}$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$loc_{ELC_2}$</td>
<td>7.6 mm</td>
</tr>
</tbody>
</table>

to optimize the geometrical parameters in HFSS to fit all the bands more closely to the Wi-Fi specifications.

### 4.4.5 Final Design and Results

After the last step of optimizing the antenna in HFSS, a final design is reached. The schematic is the same as in Fig. 4.25 but the dimensions of the final design are given in Table 4.3. The $|S_{11}|$ and input impedance are plotted in Fig. 4.28 with the bandwidth of the antenna given in Table 4.4.
Figure 4.28: Simulated $|S_{11}|$ and input impedance of the final design of the Wi-Fi antenna. (a) $|S_{11}|$, (b) input impedance.

Table 4.4: Simulated bandwidth of the Wi-Fi antenna.

<table>
<thead>
<tr>
<th>Wi-Fi Antenna Simulated Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Band</td>
</tr>
<tr>
<td>2.41 GHz-2.51 GHz</td>
</tr>
<tr>
<td>High-Band</td>
</tr>
<tr>
<td>5.14 GHz-5.92 GHz</td>
</tr>
</tbody>
</table>

Table 4.5: Simulated bandwidth of the Wi-Fi antenna

<table>
<thead>
<tr>
<th>Wi-Fi Antenna Simulated Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Efficiency</td>
</tr>
<tr>
<td>2.44 GHz</td>
</tr>
<tr>
<td>69%</td>
</tr>
<tr>
<td>5.23 GHz</td>
</tr>
<tr>
<td>55%</td>
</tr>
<tr>
<td>5.60 GHz</td>
</tr>
<tr>
<td>87%</td>
</tr>
</tbody>
</table>

The simulated radiation patterns are given in Figs. 4.29, 4.30 and 4.31 for the three different frequencies where the $|S_{11}|$ is at a local minimum. These frequencies are 2.44 GHz, 5.23 GHz and 5.60 GHz respectively. The coordinate system shown in 4.6(d) is used again here. The patterns at each frequency show dipolar like radiation due to the currents along the length of the antenna. The simulated efficiency is also given in Table 4.5 for the same frequencies. The efficiency maintains reasonable values at all three resonances, showing the Wi-Fi antenna to be an efficient antenna.

This design exercise demonstrates how adding metamaterial resonators to a planar monopole can be used to design a multi-band antenna. The ELC resonator offers independent control of its resonances, allowing the designer to control where each resonance is placed. This allows for the systematic design of a Wi-Fi antenna.
Figure 4.29: Simulated Radiation Patterns for three cuts of the antenna at 2.44 GHz. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) XZ plane, (b) XY plane, (c) YZ plane. (d) Coordinate System.

Figure 4.30: Simulated Radiation Patterns for three cuts of the antenna at 5.23 GHz. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) XZ plane, (b) XY plane, (c) YZ plane.
Figure 4.31: Simulated Radiation Patterns for three cuts of the antenna at 5.60 GHz. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) XZ plane, (b) XY plane, (c) YZ plane.
Chapter 5

A Modified Double-Tuned CSRR-Microstrip Matching Network

The previous chapter showed how loading a monopole with an ELC can create a multi-band antenna. However it is worth noting that the bandwidth at the low-band is constrained by the resonance of the ELC. Here a single-band antenna is developed to increase the bandwidth at the lower frequencies. The use of double-tuned matching networks to increase the bandwidth of a frequency dependant load, has been shown in Chapter 2. Also in Chapter 2 a modified double-tuned matching network was developed to allow the matching network to be integrated into a compact planar antenna. In this chapter a modified double-tuned matching network is designed to match a compact planar monopole antenna at a single band. A ‘resonant’ metamaterial unit cell is used to implement the second stage of the matching network as described in Section 5.2. A circuit model is also developed and parametric analysis is also carried out in Sections 5.3 and 5.4. This knowledge will then be applied to the goal of matching the planar monopole antenna to the 2.3 GHz-2.7 GHz WiMax band, covering 400 MHz of bandwidth.

5.1 The Planar Monopole Antenna Without A Matching Network

The antenna being matched is the planar monopole antenna shown in Chapter 2. The specific dimensions of the antenna used in this section are shown in Fig. 5.1. The major difference between the planar monopole of Chapter 2 and the planar monopole shown in this section is the longer ground plane to create space for the matching network that will be placed on the
Figure 5.1: Schematic of the reference antenna that will be matched using the second stage of a double-tuned matching network. The antenna sits on an FR-4 substrate that is 0.4 mm thick. The microstrip line is de-embedded when plotting the $|S_{11}|$ of the antenna on the Smith Chart.

The planar monopole antenna of Fig. 5.1 is then simulated in HFSS. The simulated $|S_{11}|$ and input impedance of the planar monopole are shown in Fig. 5.2. The input impedance is shown on the Smith Chart with part of the microstrip feed-line de-embedded. The transmission-line that is de-embedded allows the input impedance to be shown at the approximate location where the matching network will be placed. As already shown, the $|S_{11}|$ is never matched below -10 dB. However, the input impedance of the planar monopole antenna shown on the Smith Chart with a section of transmission-line de-embedded, crudely resembles a series RLC circuit in the 1 GHz-5 GHz range. It is capacitive at low frequencies and inductive at high frequencies, with a zero in the input impedance at 3.1 GHz. It should be noted that the input impedance does not maintain a constant real part, a deviation from the ideal case. This can be tolerated as it is the susceptance of the load that is adjusted by the second stage of the modified double-tuned matching network. From this simulation, it can be seen that the length of transmission line following the monopole acts as the first stage of a double-tuned matching network. This allows for the matching network to be designed without adding any extra series reactance, as it
Figure 5.2: Simulated $|S_{11}|$ and input impedance of the reference antenna with some of the microstrip transmission-line de-embedded. (a) $|S_{11}|$, (b) Input Impedance on the Smith Chart. Notice that the input impedance somewhat resembles a series RLC circuit, despite the real part of the input impedance varying with frequency. Because of the resemblance, the reference antenna is a good candidate for matching using the second stage of a double-tuned matching network to increase its -10 dB bandwidth.

is the length of transmission line that controls the shape of the impedance locus. As it will be shown later, this parameter also controls the frequency that the antenna is matched at. Thus using the theory discussed in Chapter 2, the second stage of a modified double-tuned matching network, consisting of a shunt LC network, can be used to increase the -10 dB band to the antenna by bringing part of the impedance locus within the $S = 2$ circle.

5.2 Implementation of the Embedded Matching Network

The second stage of the matching network could be implemented in a number of ways. Chip components could be used though these are difficult to place in shunt with microstrip lines and cannot scale to higher frequencies. Printed components are another option. In this design a microstrip line with a complementary-split-ring-resonator (CSRR) will be used to implement the matching network. The CSRR-microstrip network can be seen in Fig. 3.22 and consists of a CSRR embedded in the ground plane along with a capacitive ‘flare’ in the microstrip line. As described earlier, integrating a microstrip network with a CSRR loads the shunt capacitance of the transmission-line with an LC resonator. Thus, the CSRR microstrip network is used to implement the shunt LC network needed to match the planar monopole. The shunt inductance of the CSRR-microstrip matching network is from the inductive loop of the CSRR and the
Table 5.1: Dimensions of the Matching Network

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{CSRR}$</td>
<td>16 mm</td>
</tr>
<tr>
<td>$w_{CSRR}$</td>
<td>5 mm</td>
</tr>
<tr>
<td>$t_{CSRR}$</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>$gap_{CSRR}$</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>$loc_{flare}$</td>
<td>6 mm</td>
</tr>
<tr>
<td>$w_{flare}$</td>
<td>3 mm</td>
</tr>
<tr>
<td>$l_{flare}$</td>
<td>8 mm</td>
</tr>
</tbody>
</table>

shunt capacitance is from the capacitive gap of the CSRR along with the capacitive flare in the microstrip line. Because the CSRR-microstrip network is compact, it fits easily within the narrow ground plane of the planar monopole, keeping the antenna compact. It also avoids the use of vias, keeping the antenna completely planar.

### 5.2.1 Initial Full-Wave Simulations

The matching network is placed on the microstrip feed line of the planar monopole as shown in Fig. 5.3. The relevant geometrical parameters of the entire antenna are also shown in Fig. 5.3. Because the matching network has to be designed using its geometry, the design is carried out using full-wave simulations, with the results being confirmed with a circuit model in Section 5.3. The relevant geometrical parameters of the CSRR include its length, $l_{CSRR}$, its gap size, $gap_{CSRR}$, and the thickness of the loop, $t_{CSRR}$. Other relevant geometrical parameters include the length and width of the flare on the microstrip line, $l_{flare}$ and $w_{flare}$ respectively, and the location of the matching network relative to the edge of the ground plane, $loc_{flare}$. Using HFSS, some initial values for the geometry of the CSRR-microstrip network are arrived on and are given in Table 5.1.

The $|S_{11}|$ and input impedance of the antenna and matching network, with the dimensions given in Table 5.1, are shown in Fig. 5.4, with the input impedance plotted on the Smith Chart. With the matching network in place, the input impedance is taken from the base of the antenna with no transmission-line de-embedded. For comparison, the $|S_{11}|$ of the planar monopole antenna shown in Fig. 5.2 is also plotted in Fig. 5.4(a). The simulated $|S_{11}|$ shows a band below -10dB from 2.06 GHz to 2.56 GHz, demonstrating the effect of the matching network on the $|S_{11}|$ of the planar monopole antenna. The Smith Chart also shows the loop that has formed due to the matching network, with part of the loop falling inside the $S = 2$ circle. From the theory presented in Chapter 2, the ‘series RLC-like’ input impedance of the
A Modified Double-Tuned CSRR-Microstrip Matching Network

Figure 5.3: Schematic of the reference antenna with a CSRR-microstrip matching network with dimensions (a) Top view with coordinate system, (b) Bottom view.

planar monopole is transformed into a loop due to the shunt capacitance and inductance added by the microstrip-CSRR network. Here the capacitive arm of the impedance locus of the planar monopole antenna is brought into the $S = 2$ circle by the shunt inductance and capacitance provided by the CSRR. The inductive arm of the impedance locus is kept outside of the $S = 2$ circle by properly tuning the shunt inductance and capacitance of the CSRR matching network via its geometry. It should be noted that the impedance locus has also been rotated clockwise on the Smith Chart due to the presence of the transmission-line between the matching network and the base of the antenna. This accounts for the orientation of the loop on the Smith Chart in Fig. 5.4(b).

5.3 Circuit Model of the Matching Network

To demonstrate that the CSRR-microstrip network is acting as a shunt LC network, the values of the shunt inductance and capacitance need to be extracted from the CSRR-microstrip network.
Figure 5.4: The $|S_{11}|$ and input impedance, plotted on the Smith Chart, for the planar monopole with the CSRR-microstrip matching network. (a) $|S_{11}|$, (b) Smith Chart, the dashed circle is the $S = 2$ circle

given in the previous section. Unlike the circuit model presented in Chapter 3 which models the complete two-port S-parameters of the CSRR-microstrip line a simpler model is used here consisting of a shunt inductor and capacitor only. When using the CSRR-microstrip network as the second stage of a double-tuned matching network, the model only needs to capture the behaviour of the CSRR as a shunt LC network.

To extract the values of the shunt inductance and capacitance of the CSRR-microstrip matching network a two-port simulation is run in HFSS on the matching network only, with the geometrical parameters given in table 5.1. A diagram of a 2-port simulation of the matching network is shown in Fig. 5.5. The simulation setup consists of the microstrip matching network, with HFSS ‘waveports’ to excite the structure. An airbox (not shown) surrounds the structure to terminate the simulation space. The ports are de-embedded up to the capacitive flare in the microstrip transmission-line.

From this two-port simulation the $Z$-parameters are extracted. It is known that for a shunt network, the impedance of the shunt network can be extracted by interpreting the simulated $Z$-parameters as a T-model, where $Z_{21}$ of a T-model gives the impedance of the shunt elements [54]. It is important to note that the values of the other $Z$-parameters are not relevant as they don’t affect the shunt susceptance since $Z_{11}$ and $Z_{22}$ are series elements in the T-model. Thus, the effect of these parameters can be assumed to be minimal as the circuit model’s purpose is to capture the behaviour of the CSRR-microstrip network as a shunt LC circuit. Therefore the imaginary part of $Z_{21}$ is the only relevant parameter and is plotted in Fig. 5.6 which models the shunt reactance of the CSRR-microstrip network. From the imaginary part of $Z_{21}$, it is
Figure 5.5: The HFSS model of a 2-port simulation of the matching network. The ports have been de-embedded up to the capacitive flare on the microstrip line.

Figure 5.6: The imaginary component of $Z_{21}$ from a two-port full-wave simulation of the CSRR-microstrip network.

shown that the CSRR-microstrip particle behaves as a shunt LC circuit for frequencies above 1.1 GHz (above the frequency of the zero, $\omega_z$ as described in Chapter 2). There is a resonant frequency $f_o$ at 3.28 GHz, which corresponds to the resonant frequency of the CSRR itself, with the impedance inductive below the resonant frequency and capacitive above. At frequencies below $f_o$ it can be seen that the inductive nature of the CSRR-matching network cancels out the capacitive susceptance of the planar monopole and brings this branch inside the $S = 2$ circle. Above $f_o$ the capacitive nature of the matching network cancels out the inductive susceptance
of the planar monopole and keeps this branch outside of the $S = 2$ circle.

To extract approximate values for $L_m$ and $C_m$ a few approximations can be made. At a frequency $f \gg f_o$ it can be assumed that the load is mostly capacitive and that $\text{Im}(Z_{21}) \approx \frac{-1}{2\pi f C_m}$. Therefore the capacitance is given by:

$$C_m \approx \frac{-1}{2\pi f \text{Im}(Z_{21})}.$$ \hspace{1cm} (5.1)

From Fig. 5.6, $f=8.10$ GHz, and $\text{Im}(Z_{21}) = -10.43\Omega$. From these values an approximate value for $C_m$ is found to be $C_m \approx 1.88$ pF. To find the inductance, $L_m$, the resonant frequency $f_o$ is used and $L_m$ is given by:

$$L_m = \frac{1}{4\pi^2 f_o^2 C_m}.$$ \hspace{1cm} (5.2)

The value of $L_m$ is then found to be $L_m=1.25$ nH. A schematic of the circuit model is shown in Fig. 5.7. In Fig. 5.8, $\text{Im}(Z_{21})$ from the two-port simulation of the CSRR-microstrip network and the circuit model are plotted. From Fig. 5.8 it can be seen that the shunt LC circuit approximates the shunt reactance of the CSRR-microstrip network fairly well.

To further demonstrate the validity of the circuit model, the simulated input impedance of the planar monopole, with its microstrip feed-line de-embedded by the appropriate amount, can
Figure 5.9: Schematic of the reference antenna combined with the Circuit model of the matching network

be placed in series with the shunt-LC circuit model of the CSRR, allowing the planar monopole to act as the load and first stage for the circuit model of the second stage of the double-tuned network. This setup is seen in Fig. 5.9. The $|S_{11}|$ and input impedance of the entire network can then be found using a circuit simulator (Agilent’s Advanced Design System) and compared to the full-wave simulation. A 50 Ω transmission-line is also added after the matching network. The transmission-line is needed because there is a length of transmission-line between the microstrip-CSRR network and the base of the antenna where the input impedance is measured in the full-wave simulation. The electrical length $l$ is used as a fitting parameter to simply rotate the impedance on the Smith Chart until it aligns with the full-wave results and was found to have an electrical length of $l = 70^\circ$ at 2.28 GHz.

Fig. 5.10(a) shows the $|S_{11}|$ from the circuit simulation compared to the full-wave simulation of the antenna with the CSRR matching network. They both show good agreement with a band below -10 dB at similar frequencies. In Fig. 5.10(b) there is a plot of the input impedance on the Smith chart for the full-wave simulation and the circuit simulation. There is also good agreement between the full-wave simulation and the circuit model with a loop forming on the Smith chart from the circuit model. This shows that the circuit model of the CSRR captures the behaviour of the CSRR as the second stage of a double-tuned matching network.

5.4 Parametric Analysis Of The Matching Network

To further understand the effect of the geometry of the matching network on the input impedance of the antenna, some relevant parameters will be swept using HFSS. These results will be understood in light of the theory of modified double-tuned matching networks. The parameters being swept are:

1. The location of the matching network, $\text{loc}_{\text{flare}}$, from 6mm to 1mm.
2. The width of the flare on the microstrip line, $w_{\text{flare}}$, from 1.6mm to 4mm.
Figure 5.10: $|S_{11}|$ and input impedance of the reference antenna with the circuit model matching network compared to the full-wave simulation. (a) $|S_{11}|$, (b) Input Impedance on the Smith Chart. The solid curve is the full-wave simulation of the reference antenna and CSRR matching network and the dashed curve is the circuit model with the reference antenna.

3. The length of the CSRR, $l_{CSRR}$, from 8mm to 14mm.

4. The thickness of the loop in the CSRR, $t_{CSRR}$, from 0.2mm to 0.5mm.

The first parameter affects the length of transmission-line between the monopole and the matching network. The last three parameters affect the shunt inductance and capacitance of the matching network.

1. **Varying loc\_flare**

The parameter loc\_flare is the distance between the capacitive flare in the microstrip line and the edge of the ground plane. It is a unique parameter because it does not affect the shunt capacitance or inductance of the matching network. A sweep of loc\_flare is done in HFSS and the $|S_{11}|$ and input impedance are shown in Fig. 5.11. The $|S_{11}|$ shows that for increasing values of loc\_flare the -10 dB band shifts to lower frequencies. The Smith Chart shows the loop formed by the matching network rotating counter clockwise which corresponds to moving the -10 dB band to lower frequencies.

The parameter loc\_flare affects the length of transmission-line between the monopole and the matching network. In Chapter 2 it was shown that the first stage of a modified double-tuned matching network, which consists of a series inductance and capacitance, can be used to adjust the load to match the desired frequency bands. However since the load being matched in this chapter is the frequency dependent input impedance of an antenna, adding series inductances
or capacitances is not the most effective way to adjust the impedance locus. Instead the length of transmission-line between the matching network and the monopole rotates the impedance locus on the Smith Chart, providing the role of the first stage in the modified double-tuned matching network. This is demonstrated in Fig. 5.12 where the input impedance of the planar monopole antenna is plotted for different lengths of de-embedded transmission-line sections. It can be seen that this impedance locus rotates for different values of de-embedded transmission-line sections, adjusting what frequencies can be matched. This allows the designer to control what frequencies are matched by the second stage of the matching network, by choosing the proper length of transmission-line between the matching network and the planar monopole.

2. Varying $w_{\text{flare}}$

The antenna and matching network are simulated for different values of $w_{\text{flare}}$ in HFSS. The $|S_{11}|$ and input impedance are plotted in Fig. 5.13. The $|S_{11}|$ shows that the frequency band below -10 dB shifts slightly for different values of $w_{\text{flare}}$, however the major change is that the matching of this band varies, with the matching improving for smaller values of $w_{\text{flare}}$ and vice versa. On the Smith Chart in Fig. 5.13(b) the loop formed by the matching network moves closer to the center of the Smith Chart for smaller values of $w_{\text{flare}}$. It should be noted that if $w_{\text{flare}}$ were to keep decreasing, the loop would continue to decrease in size and move away from the center of the Smith Chart.

As stated earlier, the parameter $w_{\text{flare}}$ is related to the shunt capacitance. Increasing $w_{\text{flare}}$ increases the total shunt capacitance. Looking at equation (2.1), an increase in the shunt capacitance of the matching network causes the left-hand side of the equation to increase.
Figure 5.12: Smith Chart plot of the input impedance of the monopole antenna for different lengths of transmission-line sections de-embedded. The length of the transmission-line section that is de-embedded is given in terms of the electrical length from the base of the monopole.

Figure 5.13: The $|S_{11}|$ and input impedance for different values of $w_{\text{flare}}$, (a) $|S_{11}|$, (b) Smith Chart.

which increases $jB_l$, the capacitive susceptance being canceled out. Since the matching network in this Chapter is bringing the capacitive arm of the impedance locus of the planar monopole antenna inside the $S = 2$ circle, it can be seen that adjusting the shunt capacitance of the matching network will vary what part of the capacitive branch is brought into the $S = 2$ circle as given by $jB_l$. Thus if increasing the capacitance too much moves $jB_l$ outside of the appropriate range given in Fig. 2.15 then the match will degrade. In the full-wave simulation
Figure 5.14: The $|S_{11}|$ and input impedance for different values of $t_{CSRR}$, (a) $|S_{11}|$, (b) Smith Chart.

shown in Fig. 5.13(b) this can be seen by the changing size of the loop that is formed on the Smith Chart. From a design perspective this shows that a careful adjustment of the shunt capacitance is needed to help match the load to 50 Ω.

3. Varying $t_{CSRR}$

The next parameter that is varied is the thickness of the CSRR, $t_{CSRR}$. The result of this variation in HFSS is shown in Fig. 5.14. The $|S_{11}|$ shows that the matching of the -10 dB bands varies for different values of $t_{CSRR}$. On the Smith Chart this is seen with the loop formed by the matching network moving towards the center of the Smith Chart and beyond as $t_{CSRR}$ decreases. This is similar to the results of varying $w_{flare}$.

The parameter $t_{CSRR}$ controls the inductance of the CSRR which affects the shunt inductance of the matching network. Larger values of $t_{CSRR}$ correspond to a smaller inductance and vice versa. Looking at equation (2.1) again, an increase in the shunt inductance cancels out a smaller susceptance, $jB_l$, where the load is capacitive. This leads to a similar result seen when varying $w_{flare}$, where by varying the shunt inductance different values of the load susceptance are canceled out which affects the matching. Thus the matching of the load can be improved by bringing part of the load inside the $S = 2$ circle through a judicious choice of $t_{CSRR}$.

4. Varying $l_{CSRR}$

The simulated $|S_{11}|$ and input impedance are shown in Fig. 5.15 for different values of $l_{CSRR}$. The $|S_{11}|$ shows the matching of the -10 dB band varying for different values of $l_{CSRR}$ along
with a small frequency shift. On the Smith Chart this corresponds to the loop moving closer or further away from the center of the Smith Chart.

As shown with varying $w_{\text{flare}}$ and $t_{\text{CSRR}}$, varying the shunt inductance and capacitance of the CSRR can control the -10 dB match of the planar monopole. The parameter $l_{\text{CSRR}}$ controls both the shunt inductance and capacitance. As $l_{\text{CSRR}}$ increases the inductance and capacitance of the CSRR increases, and vice versa. Thus similar to the last two variations, $l_{\text{CSRR}}$ controls the matching of the antenna by adjusting both the capacitance and inductance to cancel out the appropriate values of the load susceptance. This makes $l_{\text{CSRR}}$ another useful design parameter to tune both the inductance and capacitance to achieve a good match.

5.4.1 Design Procedure

From these parametric sweeps, a further understanding of the design of the matching network has been demonstrated. These parameters can be used to optimize the matching network to match the planar monopole antenna to a desired frequency and maximize the bandwidth. Other geometric parameters such as $g_{\text{PCSR}}$, and $l_{\text{fla}}$, also affect the net shunt capacitance and inductance. While these parameters are not discussed here, they follow trends similar to the other variations.

A simple, heuristic, design methodology can also be derived from this parametric analysis.

1. Simulate the antenna being matched without the matching network. De-embed the length of transmission-line to the approximate initial location of the matching network. Examine the input impedance to check that the impedance locus meets the appropriate constraints.

Figure 5.15: The $|S_{11}|$ and input impedance for different values of $l_{\text{CSRR}}$, (a) $|S_{11}|$, (b) Smith Chart.
2. (a) Add a CSRR-microstrip network to the feedline and vary the location, $loc_{flare}$ to adjust the frequency that the match occurs at

(b) Tune parameters such as $w_{flare}$, $gap_{CSRR}$ or $l_{CSRR}$ which control the matching of the load.

3. Iterate steps 2a and 2b where necessary to match the antenna at the desired frequency and to optimize the bandwidth.

5.5 Final Design - A Single-Band WiMax Antenna

The matching network can now be used to match the planar monopole antenna to the WiMax band of 2.3 GHz-2.7 GHz. Using the design procedure of the previous section the parameters of the matching network are adjusted and optimized in HFSS to meet the WiMax specification. The final dimensions are given in Table 5.2. Fig. 5.16 shows the $|S_{11}|$ and input impedance from the full-wave simulation of the antenna and its matching network. The input impedance is taken from the base of the antenna with no transmission-line de-embedded. From the $|S_{11}|$ a -10 dB band from 2.25 GHz-2.93 GHz is seen which covers the WiMax band from 2.3 GHz-2.7 GHz. On the Smith chart a loop is formed that is partially contained within the $S = 2$ circle. Thus the matching network has allowed a wide-band match to be achieved at the desired frequency range.

The second stage of the matching network for the WiMax antenna can also be modeled by following the procedure given in Section 5.3. The extracted values for $L_m$ and $C_m$ are found after running an HFSS two-port simulation of the WiMax matching network with the dimensions given in Table 5.2. The extracted values are $L_m = 1.19$ nH and $C_m = 1.91$ pF respectively. The results of the full-wave two port simulation and the circuit model are shown in Fig. 5.17.

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{CSRR}$</td>
<td>12.25mm</td>
</tr>
<tr>
<td>$w_{CSRR}$</td>
<td>5mm</td>
</tr>
<tr>
<td>$t_{CSRR}$</td>
<td>0.2mm</td>
</tr>
<tr>
<td>$gap_{CSRR}$</td>
<td>0.2mm</td>
</tr>
<tr>
<td>$loc_{flare}$</td>
<td>5.8mm</td>
</tr>
<tr>
<td>$w_{flare}$</td>
<td>3.5mm</td>
</tr>
<tr>
<td>$l_{flare}$</td>
<td>6.1mm</td>
</tr>
</tbody>
</table>
Figure 5.16: Simulated $|S_{11}|$ and input impedance of the reference antenna with the CSRR-microstrip matching network. (a) $|S_{11}|$, (b) Input impedance on the Smith Chart. Notice that a loop has formed in the input impedance. This is due to the effective shunt inductance and capacitance of the CSRR.

Following Section 5.3 again, the circuit model can also be used to match the input impedance of the planar monopole antenna with the microstrip line de-embedded as shown in Fig. 5.9. As shown in Section 5.3 a transmission-line with an electrical length of $95^\circ$ at 2.54 GHz is added in series with the circuit model and load as shown in the schematic in Fig. 5.7. The $|S_{11}|$ and Smith Chart of the circuit model and full-wave simulation are plotted in Fig. 5.18. A good agreement can be seen in both the $|S_{11}|$ and on the Smith Chart, showing that the circuit model captures the behaviour of the matching network correctly and demonstrates the modified
Figure 5.18: $|S_{11}|$ and input impedance of the planar monopole antenna with the circuit model, modeling the matching network at the WiMax band. (a) $|S_{11}|$, (b) Input Impedance on the Smith Chart. The solid curve is the full-wave simulation of the planar monopole antenna and CSRR matching network and the dashed curve is the circuit model with the planar monopole antenna. The dashed circle is the $S = 2$ circle.

double-tuned matching.

The simulated radiation patterns of the antenna are found at 2.54 GHz, which corresponds to the minimum in the $|S_{11}|$. The patterns are shown for three different cuts in Fig. 5.19. The coordinate system is shown in Fig. 5.3. The radiating currents on the antenna are from the currents on the monopole, and the currents on the edge of the small ground plane. This gives rise to dipolar like radiation patterns as shown in Fig. 5.19. The simulated efficiency at 2.54 GHz is 97% showing that the antenna with the matching network is very efficient, with the losses mainly due to the losses of the matching network.
Figure 5.19: Simulated radiation patterns. The co-polarization is the solid line and the cross-polarization is the dashed line. The coordinate system can be seen in Fig. 5.3(a). The xz and xy cuts are the E-plane of the antenna and the yz cut is the H-plane. (a) xz cut, (b) xy cut, (c) yz cut.
Chapter 6

Fabrication and Measurement

In this chapter, the fabrication and measurement of the Wi-Fi antenna designed in Chapter 4 and the WiMax antenna discussed in Chapter 5 is discussed. First the process of fabricating both antennas is described, followed by a discussion of the measured results for both antennas.

6.1 Fabrication

To fabricate both antennas a few considerations about their geometry must be taken into account. Looking at the schematic of the Wi-Fi antenna given in Fig. 4.25, it is shown that there are three metal layers and two substrate layers. This can be broken down into two ‘boards’, as shown in Fig. 6.1. The first board is a double-sided board consisting of the ground plane and first ELC on one side, and the monopole and microstrip feed on the other. The second board is single-sided and contains the second ELC. Each board is a 0.4 mm thick FR-4 substrate, as stated earlier.

The WiMax antenna on the other hand, consists of one double-sided, 0.4 mm, FR-4 board. On one side is the microstrip feedline and monopole and on the other side is the ground plane embedded with a CSRR.

Thus to fabricate the boards making up each antenna, copper clad FR-4 substrates are used. The copper cladding consists of 1 oz. rolled copper, which translates into a thickness of 35 μm.

The fabrication facilities for printed circuit boards at the University of Toronto consist of two methods, a milling process and a chemical wet etching process. The milling process uses an LPKF Laser and Electronics AG ProtoMat H100 machine with different sized routing and drill bits to remove copper. The wet etching procedure uses a Ferric Chloride solution to remove the copper after the board has been developed using a photolithographic process.

It was decided to use the milling process to fabricate each board. This is for three reasons,
first the minimum feature size of either antenna is $\geq 200 \, \mu m$ which is within the tolerances of the milling machine. Secondly, a double-sided board can be fabricated more precisely on the milling machine as it uses fiducial markers to align both sides of the board. Finally, the unique shape of the board requires the use of the milling machine to cut the boards out.

One extra detail regarding the Wi-Fi antenna is that it consists of two boards which have to be physically held together. At the University of Toronto there is a board press which can press two substrates together using a prepreg (a thin sheet of glue) along with varying cycles of heat and pressure to bond the boards together. This was used in [86] to bond multiple boards together. However, since the substrates are only 0.4 mm thick, there was a chance that the boards may not be able to stand the pressure. Thus, it was decided to avoid the board press. Instead to hold the two layers together, the substrates are extended to form ‘tabs’ as shown in Fig. 6.2. On each of these tabs, holes are drilled using the milling machine and plastic screws are used to hold the boards together. (To verify that the tabs did not alter the performance of the antenna, the antenna was simulated in HFSS with the tabs in place and no substantial changes were found.)

Thus the procedure to build the Wi-Fi antenna is as follows:

1. Create a mask using Agilent’s ADS for each metal layer, the holes for the screws, the fiducial’s, and the outline of the antenna.
2. Using the masks mill the double-sided board on the milling machine.
3. Repeat for the single sided board.
4. Solder a SMA connector onto the microstrip feedline of the double sided board.
5. Insert plastic screws to hold the two layers together.
Figure 6.2: Schematic of the Wi-Fi antenna with tabs to hold both boards together. The screws will be placed where the black circles are.

For the WiMax antenna the procedure is simpler:

1. Create a mask using Agilent’s ADS for each metal layer, the holes for the screws, the fiducial's, and the outline of the antenna.

2. Using the masks mill the double-sided board on the milling machine.

3. Solder a SMA connector onto the microstrip feedline of the double sided board.

6.1.1 Fabrication of The Wi-Fi Antenna

The Wi-Fi antenna is fabricated using the geometry given in Table 4.3 and is shown in Fig. 6.3. To initially characterize the antenna, the $|S_{11}|$ is measured using an Agilent E8364B Vector Network Analyzer (VNA). The VNA is calibrated for the frequency range of interest using a 1-port short-open-load calibration procedure with the calibration standards provided in the Agilent 85033D calibration kit. (This procedure is repeated every time the $|S_{11}|$ is measured for any antenna). The measured $|S_{11}|$ is plotted in Fig. 6.4 along with the simulated data from Chapter 4. There is a noticeable discrepancy between the measured and simulated results. One discrepancy is that a frequency shift has occurred, shifting the resonance at the low-band and the high-band up in frequency by $\sim 400$ MHz. The second discrepancy is that at the high-band there is only one resonance present.
Chapter 6. Fabrication and Measurement

Figure 6.3: The initial fabrication of the Wi-Fi antenna. (a) Top View, (b) Both fabricated boards

Figure 6.4: $|S_{11}|$ of the fabricated Wi-Fi antenna compared to simulation.

Reasons For The Measured Frequency Shift

The frequency shift can be traced to two possible causes. The first ties in with the milling process. As stated above, the milling machine uses different routing bits to remove the unwanted copper. These bits also inadvertently remove some of the substrate in the range of approximately 100 $\mu$m.\(^1\) For most substrates this is not a major problem if the height of the substrate is $> 1$ mm. However, for a substrate of 0.4 mm this is a problem as a large percentage of the substrate is being removed. This is confirmed by measuring the thickness of each board using calipers. It was found that the thickness of the board varied from approximately 0.25 mm-0.35 mm depending on the fabrication.

\(^1\)When each routing bit is used the height of the bit is re-calibrated manually to make sure that only the copper is being removed. However, the milling machine does not give any information as to how much of the substrate is removed along with the copper.
The other possible cause is an air gap between the two boards that compromise the antenna. A visual inspection of the antenna revealed that an air gap is present between both boards. This is despite the use of the plastic screws. This air gap is hard to eliminate as the screws can only be tightened so much due to the brittleness of the boards.

The ‘missing’ resonance at the high-band can be explained by these reasons above. The air gap and thinner substrate affect how the second ELC couples to the monopole and thus affects the resonances at the high-band.

These observations can also be confirmed in simulation. In Fig. 6.5(a) it can be seen that adding an air gap between each board in simulation causes a frequency shift at both the low-band and high-band resonances in the $|S_{11}|$. The matching of the resonances at the high-band is also affected. Fig. 6.5(b) also shows a small frequency shift occurs at both bands in the $|S_{11}|$ when the height of the substrate is decreased to 0.3 mm.

**Overcoming the Substrate Issues**

To solve the problems encountered during the fabrication of the antenna, the geometry of the antenna can be modified to better match the simulated results. To do this the knowledge gained from the parametric analysis and design procedure in Chapter 4 is used. Since the resonances have shifted up in frequency, the parameters of the ELC antenna will be modified to compensate and shift the resonances down in frequency. This requires increasing the value of parameters such as $l_{ELC}$ to move the fundamental resonance of the first ELC down in frequency. The missing resonance at the high-band will also be restored by increasing $l_{ELC_2}$ and varying $loc_{ELC_2}$. To accomplish this task, different versions of the Wi-Fi antenna are built. The process is outlined as follows:
1. Fabricate the first board, increasing $l_{ELC}$ to move the fundamental resonance down in frequency and $l_{monopole}$ to move the ELC-dipole mode down in frequency. Attach an SMA connector to the first board and measure the $|S_{11}|$.

2. Compare the measured results of the first board to the simulated results of the first board alone. Repeat the previous step, re-fabricating designs until the measured results match the simulated results.

3. With the first board in place, fabricate the second board, increasing $l_{ELC_2}$ to move the resonance down in frequency, and adjusting $loc_{ELC_2}$ to match the resonance. Use the screws to attach the second board to the first board and measure the $|S_{11}|$ of the antenna.

4. Compare the measured results with the simulated results of Fig. 4.28(a). Repeat the previous step until the measured results match the simulated results.

The final results of this process are discussed in Section 6.2.1.

**6.1.2 Fabrication of The CSRR-Matching Network Antenna**

The WiMax antenna is fabricated using the dimensions given in Table 5.2 and a fabricated prototype can be seen in Fig. 6.13. The $|S_{11}|$ is measured using the VNA and the measured and simulated $|S_{11}|$ are shown in Fig. 6.7. It can be seen that the measured $|S_{11}|$ shows an approximately 300 MHz shift in frequency with the minimum in the $|S_{11}|$ shifting from 2.54 GHz to 2.84 GHz. The -10 dB bandwidth of the measured antenna is 700 MHz, from 2.6 GHz to 3.30 GHz, which only partially overlaps with the WiMax spectrum. On the Smith Chart it can be seen that the matching network has formed a loop, however the loop is rotated clockwise due to the frequency shift.
As in Section 6.1.1, this shift in frequency can be tied to the fabrication process. Once again the milling process, has milled away some of the substrate which causes a shift in frequency. To compensate for this shift in frequency the design can be altered by varying some of the design parameters experimentally. The intuition into what parameters to change is once again drawn from the parametric analysis done in Chapter 5. For example, since the -10 dB band has shifted up in frequency, a parameter such as $loc_{\text{flare}}$ could shift this -10 dB band back down in frequency. To accomplish this the following process is used, similar to Section 6.1.1:

1. Fabricate the antenna, increasing $loc_{\text{flare}}$ to move the -10 dB band down in frequency. Attach an SMA connector to the antenna and measure the $|S_{11}|$.

2. Compare the measured results to the simulated results. Repeat the previous step, re-fabricating designs and increasing $loc_{\text{flare}}$ until the measured results match the simulated results.

The final results of this process are discussed in Section 6.2.2.

### 6.2 Measurements

To characterize the final design for each antenna, the $|S_{11}|$ is measured on the VNA as described previously and compared to the simulated results. Another set of measurements to characterize each antenna is the radiation pattern. For each antenna the radiation patterns will be measured.

Figure 6.7: Measured $|S_{11}|$ and input impedance of the antenna with the CSRR matching network. (a) $|S_{11}|$, (b) Input Impedance on the Smith Chart. The solid curve is the measured input impedance and the dashed curve is the simulated.
at the frequency corresponding to the local minimums found in the $|S_{11}|$. The measurement of the radiation patterns is done using the procedure described below.

The radiation patterns of both antennas are measured using the anechoic chamber at the University of Toronto. The anechoic chamber allows for the antenna’s far-field radiation pattern to be characterized. The antenna under test (AUT) is characterized by placing the AUT on a rotating pedestal and measuring the received power from a known transmitting horn antenna to calculate the gain as a function of the angle of rotation. The first step however involves calibrating the chamber using another known horn antenna on the receive end. After calibration, the AUT can then be measured. For each antenna, the co-polarization and the cross-polarization of the xy, xz, and yz cuts are measured. (See the coordinate system in Fig. 4.6(d) as the same coordinate system is used for both antennas.)

Measuring the antennas in the anechoic chamber brings up an important issue regarding spurious currents on the feed cable. The ground plane of either antenna is small, a feature that makes them attractive for laptop systems where a compact antenna is desired. However the downside of this feature is that this small ground plane does not prevent currents from flowing on the outside of the feed cable. These currents occur from the coaxial to microstrip transition, where the currents from the ground plane of the antenna end up flowing on both sides of the outer conductor of the coaxial cable connected to the antenna. If the ground plane were larger, this would not be a significant problem as the currents on a larger ground plane would dissipate and not reach the outside of the coaxial cable. However on a smaller ground plane this is not possible and spurious radiation occurs from the feed cable. To prevent this from happening there are two possible solutions. The first is to use a sleeve balun, which acts like an open circuit at a specific frequency to prevent currents from flowing on the outside of the cable. It is a narrow-band solution. The other option are ferrite beads which act as a choke by preventing the currents from flowing on the outside of the cable through a large inductive and lossy impedance. The ferrite beads work over a range of frequencies but become less effective at higher frequencies. The ferrite beads can be seen in Fig. 6.8(a) and a sleeve balun designed to work at 5.5 GHz by Dr. Macro Antoniades of the University of Toronto is shown in Fig. 6.8(b). In this thesis both options are used. At lower frequencies such as the low-band of the Wi-Fi and WiMax specifications, the ferrite beads are used as they are effective at that frequency. At higher frequencies such as the high-band of the Wi-Fi specification the 5.5GHz balun is used.

Along with the radiation pattern, the efficiency of the antenna is measured using the modified Wheeler cap method [87], [88], pp. 239-249 at the frequency of the local minimum in the $|S_{11}|$. This procedure involves taking two measurements of the antenna using the VNA. The first is a measurement of the antenna in free space as done when measuring the $|S_{11}|$. The second measurement of the antenna involves placing the antenna in a spherical metal enclosure
Chapter 6. Fabrication and Measurement

6.2.1 Final Design and Measurement of The Wi-Fi Antenna

Following the process given in Section 6.1.1 the fabricated design is optimized to better match the simulated results. A final design is reached with the dimensions given in Table 6.1. A picture of the antenna can be seen in Fig. 6.9. The measured $|S_{11}|$ is given in Fig. 6.10 along with the simulated $|S_{11}|$ from Section 4.4.5. The measured $|S_{11}|$ shows that all three resonances are present and are close to the same frequencies as in the simulated $|S_{11}|$. The minimums in the $|S_{11}|$ occur at 2.44 GHz, 5.23 GHz and 5.60 GHz respectively. It can be seen that the measured results are close to the Wi-Fi specifications as shown in Table 6.2. The low-band bandwidth has slightly narrowed in the measured $|S_{11}|$, a consequence that could be overcome by further experimental optimization of the antenna. At the high-band, the -10dB band has shifted to a slightly lower frequency and the $|S_{11}|$ at approximately 5.50 GHz is slightly above -10 dB. This discrepancy could also be overcome with further experimental optimization. However it can be seen that the main concept is demonstrated, with the antenna covering most of the Wi-Fi spectrum.

Radiation Patterns And Efficiency

To measure the radiation patterns of the Wi-Fi antenna, ferrite beads are used for the measurements that take place at 2.44 GHz as stated previously. The 5.50 GHz sleeve balun in Fig. 6.8(b) is used to measure the antenna at 5.60 GHz. Unfortunately, there was not enough time to build
### Table 6.1: Dimensions of the Fabricated Wi-Fi Antenna

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Simulated Value</th>
<th>Fabricated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{\text{monopole}}$</td>
<td>25 mm</td>
<td>31 mm</td>
</tr>
<tr>
<td>$w_{\text{monopole}}$</td>
<td>0.2 mm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>$l_{\text{ELC}}$</td>
<td>18.25 mm</td>
<td>21.75 mm</td>
</tr>
<tr>
<td>$w_{\text{ELC}}$</td>
<td>5.8 mm</td>
<td>5.8 mm</td>
</tr>
<tr>
<td>$t_{\text{ELC}}$</td>
<td>0.35 mm</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>$g_{\text{ELC}}$</td>
<td>1.0 mm</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>$l_{\text{ELC}_2}$</td>
<td>7.3 mm</td>
<td>9.0 mm</td>
</tr>
<tr>
<td>$w_{\text{ELC}_2}$</td>
<td>5.0 mm</td>
<td>5.0 mm</td>
</tr>
<tr>
<td>$t_{\text{ELC}_2}$</td>
<td>0.35 mm</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>$\text{gap}_{\text{ELC}_2}$</td>
<td>1 mm</td>
<td>1 mm</td>
</tr>
<tr>
<td>$\text{loc}_{\text{ELC}_2}$</td>
<td>7.6 mm</td>
<td>4 mm</td>
</tr>
</tbody>
</table>

Figure 6.9: The final fabrication of the Wi-Fi antenna. (a) Top View, (b) Both fabricated boards.

### Table 6.2: Simulated and Measured bandwidth of the Wi-Fi antenna

<table>
<thead>
<tr>
<th></th>
<th>Simulated Value</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Band</td>
<td>2.41 GHz-2.51 GHz</td>
<td>2.41 GHz-2.47 GHz</td>
</tr>
<tr>
<td>High-Band</td>
<td>5.14 GHz-5.92 GHz</td>
<td>5.06 GHz-5.75 GHz</td>
</tr>
</tbody>
</table>

a balun in the frequency range of 5.23 GHz, which prevented the accurate measurement of the Wi-Fi antenna at that frequency.

The measured results at 2.44 GHz with the ferrite beads can be seen in Fig. 6.11. There is a good agreement between both the measured patterns and simulated patterns shown in Fig. 4.29.

A similar result can be seen at 5.60 GHz in Fig. 6.12 where the Wi-Fi antenna is measured
Figure 6.10: $|S_{11}|$ of the fabricated Wi-Fi antenna compared to simulation after the geometry of the antenna has been adjusted.

Figure 6.11: Measured Radiation Patterns for three cuts of the antenna at 2.44 GHz. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) xz plane, (b) xy plane, (c) yz plane.

with a sleeve balun. A good agreement between the measured patterns and simulated patterns of Fig. 4.31 can be seen again, with the dipolar like nature of the patterns being captured in measurement. One discrepancy however is that the cross-polarization of the measured patterns is much higher than the simulated patterns. But, as stated in Chapter 1 the polarization purity of the antenna is not a design constraint.

The need for chokes or baluns to measure the antenna properly also raises the question of how this antenna would be integrated into a laptop computer as it is common to find a coaxial cable running from the base of the laptop to the display where the antenna is located. Ultimately this kind of antenna with a small ground plane is best suited for tight integration on a circuit board with the power amplifier/low-noise amplifier physically close. This would
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(a)

(b)

(c)

Figure 6.12: Simulated Radiation Patterns for three cuts of the antenna at 5.60 GHz. The solid blue line is the co-polarization and the dashed red line is the cross-polarization. (a) xz plane, (b) xy plane, (c) yz plane.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Simulated Efficiency</th>
<th>Measured Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.44 GHz</td>
<td>69%</td>
<td>70%</td>
</tr>
<tr>
<td>5.23 GHz</td>
<td>55%</td>
<td>58%</td>
</tr>
<tr>
<td>5.60 GHz</td>
<td>87%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Table 6.3: Measured and Simulated Efficiency of the Wi-Fi Antenna

avoid the need for a coaxial cable to connect the antenna to the rest of the system. Instead the antenna would be directly fed by the microstrip feed line. This places limitations on the usefulness of the antenna for certain types of systems, showing an inherent trade-off between the ground plane size and practicality.

Finally the efficiency of the Wi-Fi antenna is measured. The results, using the modified Wheeler cap method, can be seen in Table 6.3 where the simulated and measured efficiencies are compared. The measured and simulated results show good agreement, demonstrating that the fabricated Wi-Fi antenna maintains a respectable efficiency at all its frequency bands.

6.2.2 Final Design and Measurement of The CSRR-Matching Network Antenna

Following the procedure given in Section 6.2.2 the antenna is optimized to better fit the simulated results. A picture of the final design is shown in Fig. 6.13 and the dimensions of the final design are shown in Table 6.4. Fig. 6.14 shows the measured $|S_{11}|$ compared to the simulated $|S_{11}|$. As shown, there is good agreement between the measured and simulated $|S_{11}|$ and the input impedance. The measured -10 dB bandwidth is from 2.18 GHz to 2.74 GHz for a total bandwidth of 560 MHz that is centered at 2.54 GHz, giving a 22% bandwidth. This is slightly
less than the simulated bandwidth but this could also be overcome by further experimental optimization by adjusting the amount of shunt susceptance provided by the CSRR. On the Smith Chart the loop that is formed due to the matching network can be seen. It is noted that the loop in the measured \(|S_{11}|\) is still slightly rotated clockwise compared to the simulated results.

Table 6.4: The final dimensions of the Matching Network for the WiMax antenna compared to the dimensions given in simulations.

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Simulated Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>l</em>&lt;sub&gt;CSRR&lt;/sub&gt;</td>
<td>12.25 mm</td>
<td>12.25 mm</td>
</tr>
<tr>
<td><em>w</em>&lt;sub&gt;CSRR&lt;/sub&gt;</td>
<td>5 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td><em>t</em>&lt;sub&gt;CSRR&lt;/sub&gt;</td>
<td>0.2 mm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>gap&lt;sub&gt;CSRR&lt;/sub&gt;</td>
<td>0.2 mm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td><em>l</em>&lt;sub&gt;flare&lt;/sub&gt;</td>
<td>5.8 mm</td>
<td>7.5 mm</td>
</tr>
<tr>
<td><em>w</em>&lt;sub&gt;flare&lt;/sub&gt;</td>
<td>3.5 mm</td>
<td>5.8 mm</td>
</tr>
<tr>
<td><em>l</em>&lt;sub&gt;flare&lt;/sub&gt;</td>
<td>6.1 mm</td>
<td>6.1 mm</td>
</tr>
</tbody>
</table>

**Radiation Patterns And Efficiency**

The measured radiation patterns are shown for three different cuts in Fig. 6.15 at 2.54 GHz, which correspond to the minimum in the measured \(|S_{11}|\). As stated previously to make sure that the cable does not radiate in the measurement setup, ferrite beads are used to prevent any currents on the cable from radiating. The measured patterns show a dipole-like radiation pattern, which arises from the current distribution on the monopole. These results show good agreement with the simulated results in Fig 5.19.
Figure 6.14: Measured $|S_{11}|$ and input impedance of the antenna with the CSRR matching network. (a) $|S_{11}|$, (b) Input Impedance on the Smith Chart. The dashed curve is the measured input impedance and the solid curve is the simulated. The dashed circle is the $S = 2$ circle.

Figure 6.15: Measured radiation patterns. The co-polarization is the solid line and the cross-polarization is the dashed line. The coordinate system can be seen in Fig. 5.3(a). (a) Measured xz cut, (b) Measured xy cut, (a) Measured xz cut.

The efficiency is measured at the minimum in the $|S_{11}|$ at 2.54 GHz. The measured efficiency is 89% while the simulated efficiency was 97%, showing good agreement between the measurement and simulated results.
Chapter 7

Conclusion

Chapter 1 posed a question about using metamaterials in the design of compact planar antennas. The question was:

- Can metamaterials be successfully applied to the design of compact laptop antennas?

This question has been answered through the work presented in this thesis. Here, two antennas have been developed, a Wi-Fi antenna using ELC resonators and a single-band WiMax antenna using a modified double-tuned matching network implemented with a CSRR.

The Wi-Fi antenna shows the development of a design technique where a host antenna is loaded with metamaterial particles to create resonances that can be controlled by the geometry of the metamaterial particle. It was shown how to use these metamaterial particles to create multiple resonances in the antenna. This technique was also extended to create multi-band antennas by adding multiple metamaterial particles. These metamaterial particles are independent of each other allowing for each band to be independently controlled. The antenna was fabricated and measured and showed good agreement with the simulated results.

The WiMax antenna applied the concept of modified double-tuned matching to increase the bandwidth of a planar monopole antenna at a single band to cover the 2.3 GHz-2.7 GHz WiMax band. The implementation of the matching network was done by using the unit cell of a resonant transmission-line metamaterial to create a via-free, compact shunt LC network. Thus while the matching network is developed and modified from a well known matching technique, its implementation is dependent (by choice) on a metamaterial understanding. This antenna was also fabricated and measured showing good agreement with the simulated results.

It is noted that both antennas meet the specifications listed in Chapter 1 for both size and bandwidth, though the WiMax antenna only has as single band. This is accomplished through the careful design of the antenna using the metamaterial elements.
With the successful use of metamaterial concepts to design compact antennas the question is raised regarding the advantage of using metamaterials in the design of compact antennas over other techniques. One answer is that this metamaterial approach offers a unique conceptual route for coming up with novel antenna topologies. In addition to this unique advantages can arise in this process. For example, one advantage of the specific design techniques developed in this thesis is that the large ground plane has been shrunk to a much smaller size. This is a major differentiator between these antennas and many of the laptop antennas in the literature discussed in Chapter 1. For the Wi-Fi antenna this is because the resonances of the antenna are now controlled via the ELC itself. And for the WiMax antenna the matching network is what allows the antenna to achieve a large bandwidth (though this technique is not necessarily metamaterial specific). In both cases this allows for the physical footprint of the antenna to decrease due to their novel topology.

It can also be claimed that these designs allow for the independent control of the frequency bands or resonances of the antenna. For the Wi-Fi antenna, this is because the resonances are controlled by the ELC, as stated above. For the WiMax antenna the matching network controls the band as also stated above. This is different than many meander-line or inverted-F antennas where each resonance or frequency band is integrated into the same geometry making it difficult to separate one from the other. This is a desirable characteristic from the antenna designers perspective as it allows for a simple way to create multi-band/wideband antennas.

Overall, the performance of these metamaterial antennas is on par with the antennas presented in the literature in terms of a good efficiency (> 70%), omni-directional radiation patterns and good bandwidth/multiple bands. Ultimately, these metamaterial techniques offer another tool in the antenna designer’s toolbox to design compact antennas and gives the designer simple techniques to create multiple bands and/or large bandwidths.

The major drawback in the design of the antennas in this thesis is that a small ground plane requires the necessity of a balun and/or ferrite beads due to the cable radiating. This prevents the antenna from being used in a laptop that uses a coaxial cable to connect the antenna to the rest of the system. However there is a design trend in laptop computers that allows for the antenna to be directly connected to the receiver or transmitter [2]. In these scenarios the antennas presented in this thesis would be well suited to such an application.

7.1 Future Work

The design techniques presented in this thesis opens the door for further improvement, refinement and innovation.

First, given the two working laptop antennas presented in this thesis, the next step would be
to integrate these antennas into a laptop computer. A substantial of work has been carried out in this area such as in [89], and this work could be used to investigate how the $|S_{11}|$, efficiency, and radiation patterns of the antennas presented in this thesis would work in the presence of a laptop computer. The designs could be refined to ensure compatibility with a laptop and further experimental optimizations could be carried out.

Looking at the specific designs again, one technique presented in this thesis used a monopole antenna to couple to the ELC itself. This idea can be extended to a differential antenna such as a printed dipole antenna where the constant current on a small dipole can be used to couple into an ELC. An example of this is shown in Fig. 7.1. This would be advantageous as the small ground plane would no longer be present, removing the need for ferrite beads or baluns. This a relatively simple way to extend this technique while circumventing the issue of parasitic currents on the cable.

For the matching network design using resonant metamaterial transmission-lines, further work on the development of a modified double-tuned dual-band matching network as described in Chapter 2 would be very beneficial as it could create a multi-band and at the same time a wideband antenna.

An interesting synthesis would be to combine the two ideas presented in this thesis into one antenna. Since using the matching network leaves the area around the monopole empty it would be easy to integrate an ELC with the monopole to create an additional band separate from the
Figure 7.2: A printed monopole antenna with a modified double-tuned matching network using a CSRR-microstrip network loaded with an ELC to add an extra resonance.

matching network. This is pictured in Fig. 7.2. This would be one way perhaps to realize an antenna that meets both the Wi-Fi and WiMax specifications as the matching network could be used to cover the lower band Wi-Max band and the ELC’s could be used to cover the other higher-frequency Wi-Max and Wi-Fi bands. From a design perspective, this would also give the antenna designer more degrees of freedom in the design of the antenna, provided that the interaction between the matching network and metamaterial particle is minimal.

Ultimately, the continued refinement and development of the techniques developed in this thesis would provide for a greater understanding of antenna design and would continue to open doors for new ways and ideas to apply metamaterial concepts whether directly or indirectly to the field of antenna design.
Appendix A

The ELC-Monopole Circuit Model

This appendix gives further insight into the planar monopole circuit model shown in Chapter 4 and the simplification of the ELC-monopole circuit model also from Chapter 4.

A.1 The Planar Monopole Circuit Model

The planar monopole circuit model was shown in chapter 4. The development of the model was done as follows. The input impedance of the planar monopole antenna in Fig. 2.3(b) can be viewed as a 1-port circuit over the frequency span of interest (in this case it is 1-7 GHz). The resonances at 2.67 GHz, and 5.58 GHz can be modeled using a parallel RLC resonator and the zero in the reactance at 3.44 GHz can be modeled as a series RLC resonator. The series RLC circuit can be thought of as the planar monopole’s inductance, capacitance and resistance outside of its quarter-wavelength and half-wavelength resonances. These three resonators can then be combined in series to form the circuit model for the planar monopole antenna as shown in Fig. A.1.

The values for the parallel RLC resonator modeling the resonance at $f_{po}=5.58$ GHz can be found by examining the real and imaginary parts of the input impedance near $f_{po}=5.58$ GHz. The resistance $R_p$ is found by looking at the value of $Re(Z_{in})$ at $f_{po}$, with $R_p = Re(Z_{in})|_{f_{po}} = 373 \, \Omega$. The inductance, $L_p$, and capacitance, $C_p$, can also be modeled by looking at the $Q$ of the resonance at 5.58 GHz. The $Q$ of the resonance can be estimated by looking at the bandwidth of the resonance:

$$Q = \frac{f_{po}}{\Delta f}, \quad (A.1)$$

where $\Delta f = f_1 - f_2$, and $f_1$ and $f_2$ mark the frequency span between $Re(Z_{in})|_{f_1..f_2} = \frac{1}{2} Re(Z_{in})|_{f_{po}}$. From examining Fig. 2.3(b), $\Delta f = 0.27 \, GHz$ and $Q = 20.67$. Since the $Q$ is also given as,

$$Q = \omega_{po}R_pC_p, \quad (A.2)$$
Appendix A. The ELC-Monopole Circuit Model

Figure A.1: A circuit model of the planar monopole antenna

$C_p$ can then be found by rearranging the above equation:

$$C_p = \frac{Q}{\omega_p R_p}, \quad (A.3)$$

with $C_p$ found to be $C_p = 1.56 \, \text{pF}$. With $C_p$ known, $L_p$ can then be found from the resonant frequency $f_{op}$ by using

$$L_p = \frac{1}{\omega_p^2 C_p}. \quad (A.4)$$

From this equation, $L_p = 0.52 \, \text{nH}$.

The same procedure can then be repeated for the parallel RLC resonator modeling the resonance at $f_{mo} = 2.67 \, \text{GHz}$, where $C_m = 8.74 \, \text{pF}$, $L_m = 0.41 \, \text{nH}$ and $R_m = 11.67 \, \text{Ω}$.

The series RLC circuit is modeled by looking at the frequency of the zero in the reactance at $f_{so} = 3.44 \, \text{GHz}$. The capacitance $C_s$ is found by looking at $f \ll f_{so}$, when the reactance is negative. At $f \ll f_{so}$ it is assumed that the reactance is dominated by the series capacitance of the series RLC resonator. This is expressed as,

$$C_s \approx -\frac{1}{2\pi f \text{Im}(Z_{in})}, \quad (A.5)$$

where $f = 1.25 \, \text{GHz}$ and $\text{Im}(Z_{in}) = -44.2 \, \text{Ω}$ and $C_s \approx 2.88 \, \text{pF}$.

The inductance $L_s$ can then be found by summing the reactance at $f_{so}$ from $C_s$, $L_s$, and both parallel RLC resonators. This is expressed as

$$\omega_{so}L_s - \frac{1}{\omega_{so} C_s} + X_p + X_m = 0, \quad (A.6)$$

where,

$$X_p = \frac{\omega_{so} L_p}{1 - \frac{\omega_{so}^2}{\omega_p^2}}. \quad (A.7)$$
Table A.1: Values of the planar monopole antenna circuit model without ELC loading.

<table>
<thead>
<tr>
<th>Circuit Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>4.6 Ω</td>
</tr>
<tr>
<td>$L_s$</td>
<td>0.53 nH</td>
</tr>
<tr>
<td>$C_s$</td>
<td>2.88 pF</td>
</tr>
<tr>
<td>$R_p$</td>
<td>373 Ω</td>
</tr>
<tr>
<td>$L_p$</td>
<td>0.52 nH</td>
</tr>
<tr>
<td>$C_p$</td>
<td>1.56 pF</td>
</tr>
<tr>
<td>$R_m$</td>
<td>11.67 Ω</td>
</tr>
<tr>
<td>$L_m$</td>
<td>0.41 nH</td>
</tr>
<tr>
<td>$C_m$</td>
<td>8.74 pF</td>
</tr>
</tbody>
</table>

and

$$X_m = \frac{\omega_{ao} L_m}{1 - \frac{\omega^2}{\omega_{mo}^2}}.$$  \hspace{1cm} (A.8)

Solving for $L_s$ from equation (A.6) gives $L_s = 0.53$ nH.

Finally the resistance, $R_s$, can be found by looking at $Re(Z_{in})|_f$ where $f$ is far away from the resonant frequency of either parallel RLC resonator. Setting $f = 1.25$ GHz, $R_s \approx 4.6$ Ω.

The circuit-element values for the planar monopole antenna circuit model are summarized again in Table A.1. The input impedance of the model is plotted along with the input impedance of the full-wave simulation of the planar monopole antenna in Fig. A.2. The circuit model shows good agreement with the full-wave simulation.

**A.2 The Simplified ELC-Monopole Circuit Model**

In Chapter 4 the circuit model for the ELC-monopole antenna shown in Fig. 4.10(a), was simplified into the form shown in Fig. 4.10(b). This was done by replacing the series RLC resonator representing the ELC that was mutually coupled to the series inductance of the planar monopole with a parallel RLC resonator in series with the rest of the circuit. The values of the parallel RLC resonator are given by equations (4.11), (4.12), (4.13). This derivation is described below.

The circuit being analyzed is shown in Fig. A.3. Note that only the series inductance, $L_1$, is shown. The other reactive components are ignored for the sake of simplicity. Here an inductance $L_1$ is mutually coupled through a mutual inductance $M$ to an inductance $L_2$. The inductance $L_2$ is also part of a series RLC tank circuit. For this system the following set of
Figure A.2: Comparing the input impedance of the HFSS simulation of the planar monopole antenna with the circuit model

Equations can be written:

\[ V_1 = j\omega L_1 I_1 + j\omega M I_2, \quad (A.9) \]
\[ V_2 = j\omega M I_1 + j\omega L_2 I_2, \quad (A.10) \]
\[ V_2 = (R - j\omega C)(-I_2). \quad (A.11) \]

Here \( V_1 \) is the voltage across the inductor \( L_1 \), and \( I_1 \) the current through it. The voltage \( V_2 \) is the voltage across the inductor \( L_2 \), and \( I_2 \) the current through it. Equation (A.9) gives the voltage across inductor \( L_1 \) from its self and mutual inductance. Likewise, equation (A.10) gives the voltage across inductor \( L_2 \) from its self and mutual inductance. Equation (A.11) gives the voltage and current relationship across the resistance and capacitance of the series RLC tank circuit. Substituting equation (A.11) into (A.10) and rearranging gives the following result:

\[ I_2 = \frac{-j\omega M I_1}{R + j\omega L_2 - \frac{j}{\omega C}}, \quad (A.12) \]

which can then be substituted into equation (A.9),

\[ V_1 = j\omega L_1 I_1 + j\omega M \left[ \frac{-j\omega M I_1}{R + j\omega L_2 - \frac{j}{\omega C}} \right]. \quad (A.13) \]

From this equation the impedance of the circuit looking into the inductance \( L_1 \) can be found and is given by:

\[ Z = \frac{V_1}{I_1} = j\omega L_1 + j\omega M \left[ \frac{-j\omega M}{R + j\omega L_2 - \frac{j}{\omega C}} \right]. \quad (A.14) \]
Appendix A. The ELC-Monopole Circuit Model

Figure A.3: Circuit schematic of a series RLC resonator inductively coupled to an inductance, $L_1$.

Here the second argument on the right hand side represents the contribution from the series RLC tank mutually coupled to the inductance $L_1$. Calling this term $Z'$, it can be rearranged as follows:

\[ Z' = j\omega M \left[ -\frac{j\omega M}{R + j\omega L_2 - \frac{j}{\omega C}} \right], \quad (A.15) \]

\[ Z' = \frac{\omega^2 M^2}{R + j\omega L_2 - \frac{j}{\omega C}}, \quad (A.16) \]

\[ Z' = \frac{\omega^3 M^2 C}{\omega RC + j\omega L_2 C - j}, \quad (A.17) \]

\[ Z' = \frac{j\omega^3 M^2 C}{1 - \omega^2 L_2 C + j\omega RC}. \quad (A.18) \]

It is well known that the impedance of a parallel RLC resonator is given by:

\[ Z_{||} = R' || \frac{-j}{\omega C} || j\omega L' = \frac{j\omega L'}{1 - \omega^2 L'C' + j\omega \frac{L'}{R'}} \quad (A.19) \]

Noticing that equations (A.18) and (A.19) are similar in form, the coefficients can be equated as follows:

\[ L' = \omega^2 M^2 C, \quad (A.20) \]

\[ \omega^2 L'C' = \omega^2 L_2 C, \quad (A.21) \]

\[ \omega RC = \frac{L'}{R'}. \quad (A.22) \]
Appendix A. The ELC-Monopole Circuit Model

Figure A.4: Circuit schematic of the simplified circuit consisting of a parallel RLC in series with inductance $L_1$.

This reduces to the following expressions for $R'$, $C'$, $L'$:

\[
L' = \omega^2 M^2 C, \quad (A.23)
\]
\[
C' = \frac{L_2}{\omega^2 M^2}, \quad (A.24)
\]
\[
R' = \frac{\omega^2 M^2}{R}, \quad (A.25)
\]

which are the same expressions given in equations (4.11),(4.12),(4.13). The total impedance looking into the inductance $L_1$ can then be expressed as follows:

\[
Z = j\omega L_1 + \frac{j\omega L'}{1 - \omega^2 L'C' + j\omega \frac{L_1}{R'}}, \quad (A.26)
\]

where $R'$, $L'$ and $C'$ are given as above.

Thus it can be concluded from equation (A.26) that the series RLC tank mutually coupled to the inductance $L_1$ can be reduced to a parallel RLC circuit in series with the inductance $L_1$ as shown in Fig. A.4.
Appendix B

The Modified Wheeler Cap Method

The radiation efficiency of an antenna is defined as: “the ratio of the total power radiated by the antenna to the total power accepted by the antenna at its input terminals during radiation” [45], p.1036. This is expressed mathematically as

\[ \eta_{\text{rad}} = \frac{R_r}{R_r + R_l} \]  \hspace{1cm} (B.1)

where \( R_r \) represents the radiation resistance of the antenna that models the radiated power that is dissipated, and \( R_l \) is the loss resistance that models the power dissipated in the conductor and dielectric losses. The expression \( R_r + R_l \) models the total power accepted into the antenna. The radiation efficiency can also be defined as the ratio between the gain of the antenna and its directivity.

From the expression given in equation (B.1) measuring the efficiency of an antenna would require a measurement technique that would separate the loss resistance, \( R_l \), from the radiation resistance, \( R_r \). A technique that accomplishes this is the Wheeler Cap method and is described below, along with a ‘modified’ method better suited for the antennas described in this thesis. This discussion draws from [87] and [88], pp. 239-249 and the references therein to which the interested reader is referred to. In the last section of this Appendix the efficiencies of the Wi-Fi and WiMax antennas are found using this technique.

B.1 The Wheeler Cap Method

Harold Wheeler introduced the eponymous Wheeler Cap Method in [90] to measure the efficiency of a small dipole. Here he described enclosing an antenna in a ‘radiation shield’ or cavity which is called the ‘Wheeler Cap’. The Wheeler Cap is placed on the radiansphere which is the boundary between the near-field of the antenna, where energy is stored and the far-field where the power is radiated. By enclosing the antenna in this cavity, the radiation resistance
is shorted out leaving only the loss resistance. The assumption is that there is no interaction or coupling between the antenna and the Wheeler Cap.

It is also assumed that the input impedance of the dipole can be modeled as a series RLC circuit, with the capacitance due to dipole itself and the inductance due to an external tuning inductance added in series with the dipole. This external tuning inductance causes a resonance by canceling out the capacitance of the dipole. At resonance, the reactance of the antenna is zero and only the radiation and loss resistance remain.

Thus by measuring the input impedance at the resonance of the small dipole enclosed in the Wheeler Cap the loss resistance of the antenna can be found, $R_{wcap} = R_l$. By also measuring the input impedance of the dipole at resonance in free space, the total resistance of the antenna is measured, $R_{fs} = R_l + R_r$. From these two measurements the efficiency of the antenna can be calculated and is given by:

$$\eta_{rad} = 1 - \frac{R_{wcap}}{R_{fs}}.$$  \hspace{1cm} (B.2)

This demonstrates how the Wheeler Cap method, using two simple measurements, can isolate the loss resistance of an antenna and find the efficiency of the antenna at a single frequency.

### B.2 The ‘Modified’ Wheeler Cap Method

The measurement procedure described above is specific to a dipole antenna, whose input impedance is modeled as a series RLC circuit at resonance. For many antennas, this limits the applicability of this technique to only antennas where the input impedance resembles a series RLC circuit at resonance. In [87], McKinzie proposed a modification to the Wheeler Cap method where the complex input impedance of an antenna at the frequency of its minimum $|S_{11}|$ can be used to extract the efficiency of the antenna. This modified method proceeds as follows.

1. The same measurements described in Section B.1 are carried out at the frequency $f_{\text{min}}$ corresponding to the minimum in the $|S_{11}|$ of the antenna.

   (a) First the input impedance of the antenna is measured in free space giving a value of $Z_{fs}(f_{\text{min}})$.

   (b) Secondly the input impedance of the antenna is measured inside the Wheeler Cap giving a value of $Z_{wcap}(f_{\text{min}})$.

2. After these measurements, the impedance locus of the free space measurement is rotated on the Smith Chart using an ideal lossless transmission-line with an electrical length $\theta$. The value of $\theta$ is chosen until the impedance at $f_{\text{min}}$ becomes purely real, where $Z_{fs}(f_{\text{min}}) \rightarrow R_{fs}(f_{\text{min}})$. 

By adding this section of transmission-line the impedance locus of the antenna becomes like a series RLC resonator as it is tangent to a constant resistance circle, is capacitive above resonance and is inductive below. It should be noted that the impedance locus does not have to exactly form a series RLC locus but just resemble its general form.

3. Using the same ideal, lossless transmission-line with an electrical length $\theta$, the impedance locus from the Wheeler Cap measurement is rotated on the Smith Chart. The intersection of this locus with the real axis of the Smith Chart gives $R_{\text{wcap}}$. Many times the frequency of $R_{\text{wcap}}$ is not the same as the frequency of $Z_{\text{wcap}}$ but is slightly different.

4. Using the values of $R_{\text{fs}}$ and $R_{\text{wcap}}$ the efficiency of the antenna can be calculated at single frequency, $f_{\text{min}}$, using equation (B.2).

This is the measurement technique that will be used to characterize the Wi-Fi and WiMax antennas at the frequency of their local minimums in their $|S_{11}|$. As it was shown in Chapters 4 and 5 the input impedance of these antennas is complex but as it will be shown, the input impedance can be made to resemble a series RLC circuit in the vicinity of $f_{\text{min}}$.

### B.3 Measuring the Antennas

Using the measurement procedure described above, each antenna is measured in free space and in the Wheeler Cap using a VNA. The Wheeler Caps that were used in measurement can be seen in Fig. B.1 and consist of two metal bowls placed together to form a spherical-like cavity. Copper foil is used to join the bowls together to create a sealed enclosure (from an electrical viewpoint). The bottom of one bowl has a hole for the placement of an SMA connector to allow for the placement of the antenna in the Wheeler Cap. The diameter of the large bowl in Fig. B.1 is 38 cm and the diameter of the smaller bowl is 26 cm. These Wheeler Caps were built by Dr. Marco Antoniades at the University of Toronto. Practically the size, shape, and material of the Wheeler Caps is not critical. However, the main concern when using the bowls is to avoid any cavity resonances that coincide with $f_{\text{min}}$ of the antenna. Thus the need for different sized bowls to allow for multiple measurements in the case of any cavity resonances.

After measuring the antenna in free space and inside the Wheeler Cap the data from both measurements is imported into Agilent’s Advanced Design System (ADS). Here the ideal transmission-line is added and the data re-calculated to give the value of the efficiency.
Appendix B. The Modified Wheeler Cap Method

Figure B.1: Large and small Wheeler cap used in the measurement of the efficiency of the antenna. Reproduced from [88], p. 247.

B.4 Data

The data from measuring the Wi-Fi and Wimax antennas is shown in this section. The first example shows the Wi-Fi antenna measured at 2.44 GHz, which corresponds to the local minimum in the $|S_{11}|$. The second example shows the efficiency of the WiMax antenna measured at 2.54 GHz, which also corresponds to this antennas local minimum in the $|S_{11}|$.

B.4.1 Wi-Fi Antenna

Fig. B.2(a) shows the free space measurement and Wheeler Cap measurement from the VNA on the Smith Chart in the vicinity of 2.44 GHz without any transmission-line added. Here it can be seen that the impedance of the antenna in free space is complex. Following steps 2 and 3 in the procedure described in Section B.2 an ideal transmission-line is added using Agilent’s ADS. The transmission-line has an electrical length of $\theta = 55^\circ|2.44 \text{ GHz}|$. The result of this can be seen in Fig. B.2(b) where the impedance locus has been rotated with $R_{fs}$ and $R_{wcap}$ as shown, with $R_{wcap}$ now at 2.42 GHz. From this rotation it can be seen that free space impedance locus is capacitive below 2.44 GHz, has no reactance at 2.44 GHz and is inductive above 2.44 GHz, crudely resembling a series RLC circuit despite the variable real part. The values of $R_{fs}$ and $R_{wcap}$ from Fig. B.2(b) are found to be $R_{fs} = 44.1 \ \Omega$ and $R_{wcap} = 13.1 \ \Omega$ respectively giving an efficiency of 70% as shown in Chapter 6. This process was repeated at the 5.23 GHz and 5.60 GHz to measure the efficiency at those bands.

B.4.2 WiMax Antenna

Fig. B.3(a) shows the free space measurement and Wheeler Cap measurement from the VNA on the Smith Chart for the WiMax antenna in the vicinity of 2.54 GHz without any transmission-
Appendix B. The Modified Wheeler Cap Method

Figure B.2: (a) The free space measurement (blue) and wheeler cap measurement (red) without any transmission-line. (b) The free space measurement (blue) and wheeler cap measurement (red) with a $\theta = 55^\circ \|_{2.44\,GHz}$ ideal transmission-line. $R_{\text{wcap}}$ is at 2.42 GHz, a slight shift in frequency.

Following steps 2 and 3 in Section B.2 an ideal transmission-line is added using Agilent’s ADS. The transmission-line has an electrical length of $\theta = 122^\circ \|_{2.54\,GHz}$. The result of this can be seen in Fig. B.3(b) where the impedance locus has been rotated with $R_{fs}$ and $R_{wcap}$ as shown, with $R_{wcap}$ now at 2.52 GHz. Once again, this rotation shows that the free space impedance locus is capacitive below 2.54 GHz, has no reactance at 2.54 GHz and is inductive above 2.54 GHz, resembling a series RLC circuit. The values of $R_{fs}$ and $R_{wcap}$ from Fig. B.2(b) are found to be $R_{fs} = 50.8$ Ω and $R_{wcap} = 5.5$ Ω respectively giving an efficiency of 89% as shown in Chapter 6.
Figure B.3: (a) The free space measurement (blue) and wheeler cap measurement (red) without any transmission-line. (b) The free space measurement (blue) and wheeler cap measurement (red) with a $\theta = 122^\circ \omega_{2.54\text{GHz}}$ ideal transmission-line. $R_{\text{wcap}}$ is at 2.52 GHz, a slight shift in frequency.
Bibliography


