FDTD Characterization of Antenna-Channel Interactions via Macromodeling

by

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Abstract

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Modeling of radio wave propagation is indispensable for the design and analysis of wireless communication systems. The use of the Finite-Difference Time-Domain (FDTD) method for wireless channel modeling has gained significant popularity due its ability to extract wideband responses from a single simulation. FDTD-based techniques, despite providing accurate channel characterizations, have often employed point sources in their studies, mainly due to the large amounts of resources required for modeling fine geometrical details or features inherent in antennas into a discrete spatial domain. The underlying influences of the antenna on wave propagation have thus been disregarded. This work presents a possible approach for the efficient space-time analysis of antennas by deducing FDTD-compatible macromodels that completely encapsulate the electromagnetic behaviour of antennas and then incorporating them into a standard FDTD formulation for modeling their interactions with a general environment.
To my family
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Chapter 1

Introduction

Radio propagation in indoor and outdoor wireless channels forms the basis of wireless communication systems. The evolution of wave propagation in a channel can be attributed to reflection, diffraction and scattering from the objects present in the channel. The performance of wireless communication systems is greatly affected by the above mechanisms of wave propagation and other factors such as the path between the transmitter and receiver and their relative speed of motion and also the kind of antennas that are associated with the transmitter and receivers.

The antenna radiation properties influence the overall link budget of a wireless channel. In addition, the action of an antenna in its receiving state to incident fields that arise due to sources elsewhere, is also an important factor in the overall characterization of the wireless channel system. The fields generated by the transmitter, arrive at the receiver either directly or through reflection, refraction and diffraction while the receiver itself may re-scatter part of them back into the domain (Figure 1.1). The latter usually has a secondary-effect on the wave propagation in the channel. Antennas, in general, have two possible modes of scattering: the structural mode of scattering and the antenna mode of scattering [1]. The structural mode of scattering is mainly due to the given shape, size and material of the antenna regardless of its transmitting and receiving properties. The
antenna mode of scattering, on the other hand, is due to the specific transmitting and radiating properties of the antenna. The fields scattered by the antenna are in general the result of the combination of both scattering modes. Despite the above classification, the scattering from an antenna is precisely due to the induced currents on the terminals and body of the antenna by an incident field.

In the presence of multiple antennas, the interference arising between the antenna elements becomes an issue. Also, in situations incorporating antenna arrays, such as a MIMO based communication system which consists of an array of transmitters and receivers, there exists mutual coupling between the individual antenna elements that has to be taken into account. Hence the treatment of antennas in wireless channel modeling deserves due attention.

1.1 Motivation

Characterization and modeling of wireless channels has been a very active area of research for many years. Proper modeling of wave propagation in a channel is essential for the design of transmitters and receivers. Traditionally radio channel models were developed
Chapter 1. Introduction

statistically that were often calibrated though measurements \[^2\]. Statistical models such as the path-loss model \[^3\], however, are site-specific and would often require repeated sets of on-site measurements for different transmitters and receivers. On the other hand there exists numerical techniques which with the aid of advanced computational resources provide a more rigorous alternative for channel modeling. One such technique is ray tracing where interactions with objects are modeled using equations governing geometric optics \[^4\],\[^5\]. The ray tracing approach, while capable of handling full channel models easily, assumes electrically large objects and is thus not suitable for including discontinuities or small structures such as antennas in its simulations. Other techniques have also been developed including time-domain based numerical electromagnetic solvers such as the Finite-Difference Time-Domain method \[^6\] which numerically solves Maxwell’s equations in a discrete manner in both space and time. The FDTD method has gained significant popularity as a channel solver due to its many advantages such as simplicity in its implementation and its ability to extract wideband responses from a single simulation run. The FDTD method also enhances channel simulations due its ability to model small geometrical details and hence capture rich multi-path propagation.

With emphasis on channel modeling, it has to be noted that many of the previous FDTD based techniques developed \[^7\],\[^8\], despite providing accurate channel characterizations, have often employed point sources in their studies on the effect of wave propagation in a wireless channel. The underlying influences of the antenna on wave propagation and in the case of multiple antenna systems, the issue of mutual coupling between the antenna elements thus have been disregarded. The choice of the point source as the transmitter arises from the huge amounts of computational resources required for modeling fine geometrical details or features inherent in antennas accurately into a discrete spatial domain and this is a major drawback of the FDTD scheme. There do exist techniques that have tried to incorporate the impact of antennas and these will be presented in chapter 2.
A proper assessment of the antenna’s operation and behaviour in a channel can sub-
sequently lead to an improvement in the antenna’s design to optimize its performance. Accordingly, this work will focus on developing suitable antenna models using the FDTD algorithm that can be easily incorporated into realistic channel simulators which in turn can provide a complete characterization of the effects of the antenna in the wireless channel performance. This will allow for proper space-time analyses of systems incorporating different antennas including the newly emerging class of metamaterial based small antennas as well as MIMO communication systems. This will ultimately assist in the development of communication systems to meet the high demands for improved channel capacity and communication reliability.

1.2 Objectives

The main objective of this thesis is to develop an accurate and efficient means for characterizing antenna-channel interactions using the FDTD method. In order to accomplish this, FDTD-compatible macromodels of the antenna will be developed that completely encapsulates the transmitting, scattering and the receiving properties of the antenna. The channel model along with the antenna macromodel must provide a complete characterization of the system and enable the extraction of system-level parameters.

1.3 Outline

Chapter 2 discusses previous and current techniques that are employed for modeling of antennas. This is followed by numerical results that are derived from simulations of an indoor wireless channel incorporating the previous approaches for antenna modeling. In addition to time domain results, the impact of such techniques on system level channel parameters is also briefly evaluated.

A novel time domain scheme for macromodeling antennas is developed in chapter 3.
A general formulation is first presented for constructing a macromodel for an arbitrary antenna and incorporating it into a standard mesh formulation. Numerical validation results are presented in 2D-FDTD for common antenna geometries.

Finally, chapter 4 presents a reciprocity-based receive macromodel formulation for characterizing receivers. The formulation is first presented through a simplified reciprocity-based macromodeling scheme for macromodeling minimum-scattering antennas and is validated in 2D-FDTD for a short dipole antenna. The receive macromodel is further extended and combined with the generic macromodeling scheme for characterizing general antenna systems as receivers and numerical results from the simulations of transmitter and receiver systems are presented.

This work presents a complete FDTD formulation for the characterization of antenna-channel interactions using the concept of macromodels. The formulation, through various simulations in 2D-FDTD, is shown to be accurate in trying to capture the EM behaviour of antennas when placed in its environment and also to be more computationally efficient than full wave simulations. Since the work in this thesis is based entirely on the FDTD scheme, a brief overview of its formulation is presented in Appendix A. For all the FDTD simulations performed in this thesis, the computational domain was terminated using the Perfectly Matched Layer (PML) absorbing boundary conditions.
Chapter 2

FDTD Techniques for Modeling Antennas

This chapter will provide an overview of current techniques used for modeling antennas using the Finite-Difference Time-Domain algorithm. Wideband numerical simulations were performed to analyse and compare the numerical techniques and the computed time-domain results are presented. The impact of these techniques on system level parameters is also discussed.

2.1 Introduction

The Finite-Difference Time-Domain method has been widely used to model electromagnetic wave propagation in indoor wireless channels, mainly due to its capability to provide wide-band channel responses from a single simulation. However, the computational resources required for integration of elements with fine geometrical features into large-scale simulations becomes quite extensive in the FDTD scheme. This problem is commonly encountered when trying to model antenna-channel interactions where the antenna and the channel differ significantly in scale and require different rates of spacial discretization in the FDTD scheme. Antennas typically used in Ultra-Wide Band (UWB) communication
systems often have complex geometries associated with them and thus require highly fine
FDTD grids to model them with reasonable accuracy. This consequently leads to a fine
discretization of the entire wireless channel problem thus resulting in high computational
costs. Using coarse meshing rates on the other hand would greatly distort the antenna
geometry and in the case of electrically small antennas, would even make it impossible
to model them. Hence, efficient and accurate antenna modeling techniques need to be
developed and employed in the FDTD algorithm at both the transmitting and receiving
ends.

In addition to obtaining accurate time-domain results, it is also necessary to obtain
system level parameters that are very useful for analysing the performance of commu-
nication systems. As a result the time-domain numerical techniques used for modeling
antennas and simulating them in a multi-path environment such as an indoor channel,
should also be able to provide accurate information on the wireless channels’ perform-
ances such as path loss exponents and signal fading statistics from the time-domain
results. Such information is also useful for evaluating the accuracy of these wireless
channel models and for their improvement.

2.2 Overview of Current Antenna Modeling Tech-
niques in FDTD

In the FDTD algorithm, the most straightforward way of modeling antennas involves
approximating the antenna geometry using a fine discretization of the FDTD grid and
specifying the material parameters that are associated with the antenna. This model of
the antenna is then excited by applying a suitable source such as a Gaussian pulse to
the terminals of the antenna. This approach however results in huge computational costs
especially if the antenna is to be embedded in a large scale simulation. Approximating
the antenna using a coarse grid discounts for small geometrical details and often results in
stair-casing errors, especially when approximating curved boundaries, which may significantly contaminate the solution to the problem. Hence more efficient approaches need to be developed for modeling antennas in FDTD which are more appropriate for embedding antennas in large-scale simulations.

The problem of meshing small or complex structures within a larger domain has been dealt with previously where different FDTD cell sizes have been used to model the entire geometry of the problem as shown in Figure 2.1. This approach is popularly known as sub-gridding in the literature \[6\]. As can be seen from Figure 2.1 in the FDTD sub-gridding scheme a fine mesh (sub grid) is used to model the structure of interest which is placed within the actual computational domain (primary grid). In the case of multiple complex structures, multiple sub grids are placed as required within the primary grid. This approach avoids spatial oversampling due to dense meshing of the entire domain and results in better computational efficiency. However, since the scheme requires data transfer between the two grids, the process requires both interpolation as well as extrapolation operations to be performed at the boundary. Such operations generate numerical errors that often render the scheme unstable after a finite number of time steps \[6,9\]. Sub-gridding schemes are also subject to reflections arising from the abrupt coarse-fine mesh transitions. Although many techniques have been developed
to improve the performance of sub-gridding schemes in terms of numerical stability and boundary reflections \cite{10},\cite{11},\cite{12}, the general applicability of such techniques may not be guaranteed \cite{6}. Moreover, the objects in the dense mesh regions may often have to be re-discretized for simulations involving different coarse-fine mesh aspect ratios, making the process tedious.

In this section current FDTD techniques used to model and simulate antenna structures will be discussed. Several techniques such as the hard-box excitation method, the total-field/scattered-field (TF/SF) formulation \cite{6}, the Dual-grid FDTD (DG-FDTD) method \cite{13} and a hybrid FDTD/MoM technique \cite{14} among others have been developed and used widely. All these techniques are aimed at trying to provide efficient ways of modeling antennas in the FDTD grid along with their environment in order to achieve accurate results with reduced computational time. In the following sections the hard-box excitation method, the total-field/scattered-field method and the Dual-grid FDTD method will be discussed in detail. These methods have simple implementation procedures for modeling antennas using the FDTD algorithm and will form a basis for the development of two novel antenna modeling schemes which will be discussed in chapters 3 and 4.

\subsection{2.2.1 Hard-box Excitation Method}

The hard-box excitation method makes use of the equivalence principle to model the antenna in terms of its equivalent or near-field sources on a closed Huygens surface \cite{15} surrounding the antenna. For a rectangular Cartesian FDTD grid, the simplest closed surface is that of a box that completely encloses the antenna. In a simulation implementing the hard-box excitation scheme, the antenna is completely replaced by its equivalent tangential electric and magnetic fields that are directly defined on the closed Huygens surface. These equivalent or near-field tangential electric and magnetic field sources are the fields radiated by the antenna in free-space and are generally found from
an FDTD simulating the antenna alone and by recording the time-domain fields on a closed Huygens surface until they have decayed. The tangential field sources are specified at every time step in the Yee’s update algorithm and are used to update the fields outside the box. The hard-box excitation method, although a simple and straightforward way of modeling antennas, is not very suitable in situations where multiple scattering objects are present or when antenna elements are placed at close proximity to each other such as in an array. This is due to the simple reason that fields reflected from scattering objects are back scattered by the hard box rather than the antenna. As a result field perturbations that occur in the near-field regions of the antenna are not properly accounted for in this scheme thus discounting for any effects arising from mutual.

2.2.2 Total-field/Scattered-field Excitation Method

The total-field/scattered-field formulation (TF/SF) is a technique that was developed by Taflove and Umashanker and is one of the most popular excitation schemes used in FDTD [6]. The field values evaluated in the FDTD grid are that of the total fields which are a superposition of the incident fields and scattered fields and thus can be expressed
Chapter 2. FDTD Techniques for Modeling Antennas

\[ \vec{E}_{\text{tot}} = \vec{E}_{\text{inc}} + \vec{E}_{\text{scat}} \]  
(2.1a)  
\[ \vec{H}_{\text{tot}} = \vec{H}_{\text{inc}} + \vec{H}_{\text{scat}} \]  
(2.1b)

Based on the above, the FDTD grid in the TF/SF scheme is decomposed into two regions: the total field region and the scattered field region and the corresponding fields are evaluated in each region as shown in Figure 2.3. In this scheme, first the incident fields, \( \vec{E}_{\text{inc}} \) and \( \vec{H}_{\text{inc}} \), are applied on a closed surface which also acts as an interface between the total and scattered field regions. These incident fields are the equivalent sources obtained on the closed Huygens surface resulting from the radiation of the antenna in free-space. Unlike the hard-box excitation method in which the sources are directly specified on the near-field surface, in this scheme they are used as a correction term in the FDTD update equations as explained below.

Let us consider the case shown in Figure 2.4. The update of the total field \( E_{x,\text{tot}}(i,j,k) \) requires a special treatment since it incorporates a scattered field component, \( H_{z,\text{scat}}(i,j,k) \), in its update equation as per the field arrangement in the grid instead of the required total field. In order to correct this the incident field, \( H_{z,\text{inc}}(i,j,k) \), has to be added to \( H_{z,\text{scat}}(i,j,k) \) and the corresponding update equation for \( E_{x,\text{tot}}(i,j,k) \) becomes (assuming...
Figure 2.4: Field updates for the Total-field/Scattered-Field formulation.

\[ E_{x,\text{tot}}^{n+1}(i, j, k) = E_{x,\text{tot}}^{n-1}(i, j, k) + \frac{\Delta t}{\varepsilon} \left[ \frac{H_{y,\text{tot}}^{n+\frac{1}{2}}(i, j, k - 1) - H_{y,\text{tot}}^{n+\frac{1}{2}}(i, j, k)}{\Delta z} \right. \]
\[ + \frac{H_{z,\text{scat}}^{n+\frac{1}{2}}(i, j, k) - H_{z,\text{tot}}^{n+\frac{1}{2}}(i, j, k - 1)}{\Delta y} \] \left] + \frac{\Delta t}{\varepsilon} \left[ \frac{H_{z,\text{inc}}^{n+\frac{1}{2}}(i, j, k)}{\Delta y} \right] \right) \] (2.2)

A similar treatment is applied to the update of the scattered field \( H_{z,\text{scat}}(i, j, k) \) and its update equation is given by

\[ H_{z,\text{scat}}^{n+\frac{1}{2}}(i, j, k) = H_{z,\text{scat}}^{n-\frac{1}{2}}(i, j, k) + \frac{\Delta t}{\mu} \left[ \frac{E_{x,\text{scat}}^n(i, j + 1, k) - E_{x,\text{scat}}^n(i, j, k)}{\Delta y} \right. \]
\[ + \frac{E_{y,\text{scat}}^n(i, j, k) - E_{y,\text{scat}}^n(i + 1, j, k)}{\Delta x} \] \left] + \frac{\Delta t}{\mu} \left[ \frac{E_{x,\text{inc}}^n(i, j, k)}{\Delta y} \right] \right) \] (2.3)

The scattered fields here refer to those fields which are generated as a result of the scattering from the environment outside the TF/SF boundary as well as the fields generated as a result of re-scattering by the antenna or any other scattering object that is present inside the TF/SF boundary due to scattered fields from the environment incident.
upon it. Typically the source region or the region inside the TF/SF boundary is treated as the scattered field region and the total fields are evaluated elsewhere. Nevertheless, due to the above method of injecting the incident fields into FDTD grid, the TF/SF interface is transparent to incoming scattered fields that approach the source region as well as outgoing fields scattered by the source itself. As a result any perturbations arising from the source are properly accounted for in this scheme. This is a major advantage of the TF/SF method over the hard-box excitation method.

2.2.3 Dual-Grid FDTD method

The Dual-Grid FDTD technique is an extension to the TF/SF scheme discussed previously in that it uses the TF/SF formulation to model the interaction of the antenna with its surrounding environment using an FDTD mesh that is coarser compared to the mesh required to discretize the antenna. This follows from the notion that a fine discretization of the environment is not required in order to capture the effects of multi-path wave propagation in the channel. The DG-FDTD simulation is implemented in two steps: (1) a fine FDTD simulation of the antenna and (2) a coarse FDTD simulation of the antenna with its environment. First the antenna structure is simulated in free-space in a very fine grid and the tangential electric and magnetic fields on a closed rectangular surface are saved at every time step until they have decayed. In the second step the antenna is simulated with its environment using a coarse mesh. The fields saved in the first step are used as the excitation in the coarse simulation consisting of a coarse model of the antenna and its environment. These fields are excited using the TF/SF formulation as described in the previous section in which they are used as a correction term in the field update equations. However, since the simulation with the environment in done using a coarse mesh, the saved field values need to be interpolated in order to match spatially and temporally with the coarse simulation. Let us consider, for example, a fine mesh to coarse mesh ratio of $k=2$ where the spatial discretizations of the coarse simulation
are twice compared to that required for the antenna simulation. For the discretization shown in Figure 2.4, the required interpolations for the $E_x$ and $H_y$ field components are as shown below.

\[ E_{x,inc}^{\text{coarse}}(i, j, k) = \frac{1}{2} \left( E_{x}^{\text{fine}}(i, j, k) + E_{x}^{\text{fine}}(i + 1, j, k) \right) \]  
(2.4a)

\[ H_{y,inc}^{\text{coarse}}(i, j, k) = \frac{1}{4} \left( H_{y}^{\text{fine}}(i, j, k) + H_{y}^{\text{fine}}(i + 1, j, k) + H_{y}^{\text{fine}}(i, j, k + 1) + H_{y}^{\text{fine}}(i + 1, j, k + 1) \right) \]  
(2.4b)

The coarse grid also presents a higher limit on the maximum time step value that can be used for the simulation of the antenna with the environment. It can be easily observed that for a mesh interpolation ratio of $k$, if the maximum time step imposed by the fine simulation is $\Delta t_{\text{max}}$, the maximum allowable time step based on the CFL limit for the coarse simulation is $k\Delta t_{\text{max}}$. This subsequently reduces the number of time steps required for the actual simulation by $k$ and also leads to a considerable reduction in the amount of memory required for the storage of the incident field values.
2.3 Comparison of Computational Requirements

The hard-box, TF/SF and DG-FDTD schemes all require the storage of the incident electric and magnetic field values that are radiated by the antenna and obtained on the closed Huygens’ surface, S. The required storage can be expressed as,

\[
Storage\{IncidentFields\} = 2 \times i \times M \times \frac{T}{\Delta t}
\]  

(2.5)

where \(i\) denotes the number of field components tangential on each surface comprising \(S\), \(M\) is the number of mesh cells on \(S\) and \(T\) is the length of the incident fields.

Of the three schemes, the hard-box scheme is the most efficient since it does not require the simulation of the region inside the box and can be used for arbitrary mesh aspect ratios. Moreover, the hard-box scheme can be implemented with incident tangential electric fields alone. The TF/SF and DG-FDTD schemes come next and present similar computational efficiencies. The use of the DG-FDTD scheme for high mesh aspect ratios is greatly limited since it may not be possible to represent the antenna geometry accurately in highly coarse meshes.

2.4 Numerical Simulations and Results

The numerical antenna modeling schemes discussed in the previous section were applied for the simulation an indoor channel geometry in 2-D FDTD. The simulations performed using the antenna modeling techniques were compared against a standard full wave simulation of the antenna with the channel to analyse their accuracy and efficiency in characterizing wave propagation across the channel. Results were computed both in the time and frequency domains. The latter was used to extract and analyze the impact of the different modeling schemes on system-level channel parameters.
2.4.1 Wideband Simulation of an Indoor Channel Geometry

A 2-D simulation of an office room was carried out in the $TE_z$ mode. The dimensions and layout of the room are shown in the figure below. Scattering objects such as desks and shelves were placed in the room in order to account for scattering from multiple objects.

![Figure 2.6: Layout of the 4 m × 2.5 m office room used in the wideband simulations. The transmitter was located at positions $T_x_1$ and $T_x_2$ and the temporal received signals were computed at $Rx_1$ and $Rx_2$ for each transmitter position.](image)

Approximate material properties were used for the walls and the scattering objects. For the walls a relative permittivity of $\varepsilon_r = 6.0$ and a conductance of $\sigma = 0.0814 \, \text{S/m}$ were used. For the door $\varepsilon_r = 5.0$ and $\sigma = 0.0084 \, \text{S/m}$ were used and for the window $\sigma = 2.4$ and $\sigma = 0.0 \, \text{S/m}$ were used. The desks and shelf were assigned an $\varepsilon_r$ of 3.0 and a $\sigma$ of 0.001 S/m.

A bow-tie antenna of length 84 mm and a flare angle of 90° was used as the transmitting device. The antenna was excited with a hard modulated Gaussian pulse source.
Chapter 2. FDTD Techniques for Modeling Antennas

with a maximum amplitude of 1V by specifying the $E_x$ component of the field as follows,

$$E_x \Delta x = e^{-\left(\frac{n \Delta t - t_0}{\Delta x}\right)^2} \cdot \sin(2\pi f_0 n \Delta t)$$  \hspace{1cm} (2.6)

where $f_0$ is the centre frequency of interest. The simulations were carried out for a source pulse with centre frequency at 900 MHz and having a bandwidth from 0 - 1.8 GHz for two different positions of the antenna, $Tx_1$ and $Tx_2$ as shown in Figure 2.6.

The equivalent sources of the antenna were first found from a fine $\lambda_{min}/34$ mesh discretization. These sources were then used to model the antenna according to the previously discussed schemes. First the antenna was simulated in the presence of the room using the hard-box excitation scheme with a $\lambda_{min}/34$ FDTD grid discretization. Next a TF/SF simulation was carried out at a discretization rate of $\lambda_{min}/17$ first without the antenna and then with a coarse model of the antenna inside the Huygens surface. The latter is the implementation of the DG-FDTD scheme. Finally a coarse model of the antenna was used to simulate the indoor channel at a mesh size of $\lambda_{min}/17$. A full wave simulation of the antenna at a $\lambda_{min}/34$ discretization rate was also performed and was used as a reference to compare the different antenna modeling schemes. For the fine simulations, the FDTD mesh cell size was chosen as $\Delta x = \Delta y = 2.0 \times 10^{-3}$ s which is about 0.96 of the CFL limit. The FDTD parameters used for the coarse simulations were twice that of the fine simulations. All the simulations were performed for both antenna positions and were allowed to run for a total time of 45 ns.

The time domain waveforms of the received fields at $Rx_1$ and $Rx_2$ were obtained from each simulation for comparison. Figure 2.7 shows the received waveforms of the $E_x$ component of the field at $Rx_1$ for both antenna positions. The accuracy of the different schemes were quantitatively analysed by computing the numerical relative errors according to equation (2.7) and the corresponding error values are tabulated in Table 2.1.
Figure 2.7: Temporal waveforms of the electric field component $E_x$ obtained from a 2D simulation of an office room using different antenna modelling schemes at $Rx_1$ for antenna positions a) $Tx_1$ and b) $Tx_2$.

\[
\epsilon = \sum_{n=1}^{n=N} \frac{(e_{ref}(n) - e_{model}(n))^2}{e_{ref}(n)^2}
\]  

(2.7)

Based on the results it can be observed that the different numerical schemes do present certain levels of inaccuracies in trying to model the antenna behaviour in the channel. The inaccuracies are visible both from the plots of the temporal waveforms as well as the computed relative errors. The results indicate varied levels of inaccuracies between
Table 2.1: Computed relative errors in the temporal waveforms extracted from a 2-D simulation of an office room

<table>
<thead>
<tr>
<th>Antenna Position</th>
<th>Antenna Modeling Scheme</th>
<th>Error % (Rx1)</th>
<th>Error % (Rx2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx1</td>
<td>Hard-box</td>
<td>2.04</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>TFSF (w/o antenna)</td>
<td>0.73</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>DG-FDTD</td>
<td>0.30</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>Coarse</td>
<td>1.29</td>
<td>0.80</td>
</tr>
<tr>
<td>Tx2</td>
<td>Hard-box</td>
<td>2.08</td>
<td>2.35</td>
</tr>
<tr>
<td></td>
<td>TFSF (w/o antenna)</td>
<td>4.38</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>DG-FDTD</td>
<td>0.77</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Coarse</td>
<td>0.30</td>
<td>0.38</td>
</tr>
</tbody>
</table>

the different schemes for the two different positions of the antenna. The DG-FDTD and coarse simulations present the same scenario in terms of scattering from the antenna but differ in the incident fields that each inject into the grid. Since the incident fields for the DG-FDTD are obtained via interpolation they are more accurate than the fields radiated by the distorted antenna in the coarse simulation. For the coarse simulation, even a small distortion in the geometry impacted the incident fields that are radiated by the antenna which are quite visible in the time-domain plots of the field waveforms. This will definitely become worse if the antenna is discretized in a much coarser mesh wherein significant distortion will occur in the antenna shape and excitation terminals. The hard-box scheme presents slightly higher errors which are mainly due to the scattering of fields from the Huygens surface rather than the antenna. The TF/SF simulation without the antenna presents even higher errors and emphasizes the fact that the antenna does have a significant secondary effect on the solution to the problem. There is obvious increase in errors for the hard-box scheme and the TF/SF scheme without the antenna for transmitter position $T_x_2$ due to the stronger interaction between the antenna and the surrounding objects which both schemes discount for totally. It should be noted here that, since the errors are calculated at two specific points in the room, the magnitude of the errors are highly dependent on the nature of the reflected fields that arrive at those points.
2.4.2 Impact of Modeling Schemes on System-Level Parameters

As mentioned earlier, the attractiveness in applying time-domain methods for wireless channel modeling and simulations relies in its capability to provide wide-band channel responses in a single simulation run. This is useful in obtaining frequency dependent parameters and also system level parameters. Wireless channel parameters of particular interest for the analyses of communication systems include the path-loss exponent and signal fading characteristics such as small-scale/large-scale fading among others. Such parameters have a critical role in signal propagation models and hence it is important to study the impact of the time-domain numerical electromagnetic solvers on them. Traditionally, signal fading profiles for different channel geometries were extracted through extensive measurement-based data. While measurement-based data are always useful, with the development of numerical electromagnetic simulators such profiles can be easily computed by sampling the field values in the computational domain and fitting them to known propagation models. The latter is explored in the following section for computing the path loss exponent value for the indoor channel geometry in section 2.4.1.

Computation of Path-Loss for Indoor Channel Simulation

Theoretical and measurement-based propagation models have indicated that the average received signal power decreases logarithmically with distance in both indoor and outdoor channels [2]. This leads to the simplified log-distance path loss model which is widely used in characterizing wireless channels and is based on the following equation,

\[ 10 \log \left( \frac{p_0}{p} \right) = n 10 \log \left( \frac{d}{d_0} \right) \]  

(2.8)

where, \( p_0 \) is the average received signal power at a small reference distance \( d_0 \) from the transmitter, \( p \) is the averaged received signal power at an arbitrary distance \( d \) from the transmitter and \( n \) is the path loss exponent. The path loss exponent, \( n \), in the above
equation specifies the rate of increase of the path loss with distance and is an important indication of the wireless channel performance. The value of $n$ depends on the type of objects present in the surroundings and their material properties.

Equation (2.8) was applied to the numerical simulation of the office room in order to determine the value of the path loss exponent at the centre frequency of 900 MHz. The time domain electric field values were sampled throughout the room and the corresponding frequency domain values were obtained at 900 MHz via discrete Fourier transform. The steady-state distributions of the magnitude of the $E_x$ field component in the room at 900 MHz for the antenna position $T x_1$ obtained using the different antenna modeling schemes are plotted in Figure 2.8. The corresponding field distribution obtained from a point source simulation is also shown for comparison.

The power values were computed at free-space regions in room and were then averaged over a uniform square mesh of area of $1.5\lambda_{min} \times 1.5\lambda_{min}$. The local average power $p_0$ was calculated at a distance, $d_0 = 1.5\lambda_{min}$. Shown in Figure 2.9 are the scatter plots obtained for the antenna position $T x_1$ using the different antenna modeling schemes. In each figure the path loss is plotted as a function of distance according to equation (2.8) and the corresponding path loss exponent is estimated as the slope of the best-fitting line to the data set for each case.

As a first observation one can note the significant difference in the magnitude of the fields radiated by a point source excitation and those from the antenna from the field distribution plots in Figure 2.8. Channel simulations incorporating a point source are thus not suitable when trying to determine actual signal power levels across the channel which is strongly dependent on the radiation characteristics of the antennas involved. The field distributions from the different antenna modeling schemes also show slight dissimilarities in the field magnitude levels compared to the full-wave simulation. The PLE values computed using the different schemes including the point source simulation are interestingly very close to the value from the fine full wave simulation. In equation (2.8),
Figure 2.8: Steady-state distribution of the magnitude of the $E_x$ component of the field at 900 MHz computed using the following schemes: (a) Hard-box excitation scheme, (b) TF/SF scheme without the antenna, (c) Dual-Grid scheme, (d) Coarse ($\lambda_{\text{min}}/17$) simulation (e) Fine ($\lambda_{\text{min}}/34$) simulation and (f) Point Source simulation.
Figure 2.9: Scatter plots for 765 points sampled uniformly over the room computed using the following schemes: (a) Hard-box excitation scheme, (b) TF/SF scheme without the antenna, (c) Dual-Grid scheme, (d) Coarse ($\lambda_{\text{min}}/17$) simulation (e) Fine ($\lambda_{\text{min}}/34$) simulation and (f) Point Source simulation. The corresponding path loss exponent values are shown for each case.
the power loss is calculated with respect to a local averaged power value corresponding to the particular source and as a result it may be possible to determine the path loss exponent values based on the simplified path-loss model to a certain degree of accuracy using any field distribution. In this regard, one should also note the slight differences in the local averaged power values relative to the different schemes. However this may not always be the case and a simple example would be of a case where a highly directive antenna, such as an array, is used as the transmitting element. The nature of power loss in the channel for such a case would be contrastingly different from that of a point source which radiates nearly omni-directionally. The PLE is also extracted by means of a best-fit line to data sets of power samples that, as can be seen from the plots, are highly scattered thus revealing the inadequate nature of this simplified model. These and also the low level of interpolations involved in the coarse schemes can be attributed to the similarities in the computed PLE values.

2.5 Conclusions

This chapter presented a brief insight into the techniques commonly used to model antennas in the FDTD algorithm. The different schemes were analysed by performing an indoor channel simulation and the errors in the time-domain results were computed based on results obtained from a reference full wave simulation. The inaccuracies present in the different schemes were clearly visible both in the temporal plots of the field waveforms as well as in the field distribution plots in the frequency domain. The numerical schemes, however, provided fairly accurate results in predicting the system channel parameter studied, namely the path loss exponent.

The numerical example in section 2.4 was studied mainly to compare the different existing antenna modeling schemes and involved only a small level of interpolation from the fine to coarse FDTD grid and thus did not have a significant impact on the antenna
geometry. There are numerous applications, however, which involve the simulation of electrically small antennas and antennas with complex geometries that differ greatly in scale compared to the surroundings and wherein a small level of interpolation would not suffice to provide substantial computational efficiency along with accurate results. Moreover, a major disadvantage of these techniques relies in their inability to properly model antennas as receivers due to either the absence of the antenna characteristics in the model or the subsequent distortion of the antenna geometry and its terminals when modeled in a coarse grid. This may also result in inaccuracies when trying to account for mutual coupling effects between multiple antennas. With the goal to tackle such problems, a novel antenna modeling scheme which utilizes the concept of macromodels will be developed and evaluated in the next chapter.
Chapter 3

FDTD Macromodeling of Antennas

Current modeling schemes such as those discussed in the previous chapter generally require simulation of the antenna region, except for the hard-box scheme, in order to account for back-scattering from the antenna when placed in an environment such as a wireless channel. For antennas with complex geometries, small geometrical details or features are often lost when the problem at hand is scaled to that of the environment, thus impacting the accuracy when trying to capture the antenna’s interaction with the environment. This is apparent in schemes that rely on a coarse model of the antenna for the simulation and the inaccuracy is bound to increase especially when the antenna is close to a strong scatterer. This chapter introduces the concept of macromodels and its adaptation to the FDTD algorithm for the efficient simulation of antennas.

3.1 Introduction to Antenna Macromodeling in FDTD

In order to overcome the difficulties involved in discretizing antennas and embedding them in large-scale channel simulations, a suitable approach that avoids the need to discretize the antenna involves macromodeling the antenna or the antenna system. In the context of electromagnetics, a macromodel is a model that is created for a subset of the equations defining an electromagnetic system in state-space form [16]. A macromodel
may be regarded as a technique of encapsulating the electromagnetic behavior of the region consisting of the EM structure, such as an antenna, in the form of a transfer function matrix. This technique of representing the antenna in terms of its macromodel is very useful when the problem at hand involves the modeling of small geometrical figures that would require extensive resources in the FDTD scheme.

Although a recent and emerging technique, the generation of macromodels has been investigated by a number of researchers and has been reported in the literature regarding its adaptation to FDTD simulations. Macromodels have been demonstrated for one and two dimensional FDTD in [17], [18] and [19]. In three dimensions, the development of a fast high-resolution FDTD scheme with macromodels has been discussed and demonstrated in [16] in which the technique was applied for the analysis of waveguide filters using the FDTD algorithm. A similar scheme has been reported in [20] with applications for waveguide analysis. However, not much has been discussed regarding its adaptation to the analysis of antennas in FDTD and that calls for an interesting research into this area. Macromodels for time domain analyses can be constructed directly in the time domain or in the frequency domain which can later be transformed to the time domain via inverse Fourier transform. In this thesis, discussion on the construction of macromodels will be focused on the time domain.

Once a suitable macromodel that completely defines the antenna system has been developed, it can be incorporated into a standard FDTD mesh formulation which can then be used to model the antenna with the environment. The macromodel will essentially serve as an interface between the antenna region and the environment. Since the antenna and the environment differ significantly in scale, by employing appropriate spatial and temporal interpolation schemes, the macromodel can be incorporated in a coarse FDTD mesh for the efficient simulation of the antenna and its environment. Several efforts have also been taken to reduce the amount of variables required for developing and processing macromodels particularly by using model-order reduction (MOR) techniques [17]. These
attractive features make macromodeling a very promising tool for modeling antennas in the FDTD algorithm.

In this chapter a novel generic macromodeling scheme for the simulation and analysis of all classes of antennas will be presented and discussed. A simplified reciprocity-based macromodeling scheme for the analysis of minimum-scattering antennas using the FDTD algorithm will be discussed in the next chapter.

### 3.2 Formulation of a generic Antenna Macromodeling scheme in FDTD

In this section a general scheme for macromodeling antennas will be developed for applications involving the FDTD algorithm. This scheme was developed in the view of eliminating the need to simulate the region consisting of the antenna in the presence of its environment. Since the antenna domain typically requires a fine discretization to capture its behaviour accurately, eliminating this need will result in a better efficiency when the antenna is simulated in an environment such as a wireless channel where a fine discretization of the entire domain would either not be a feasible option or be unnecessary.

The proposed macromodeling scheme consists of two main stages. The first stage consists of generating the macromodel of the antenna under test. This macromodel will contain all information regarding the EM behaviour of the antenna (transmitting, scattering and receiving) and will be used to represent the antenna in the main simulation with the environment. The second stage involves an iterative simulation of the developed antenna macromodel for a specific problem at hand. These two stages will be explained in detail in the following sections.
3.2.1 Construction of Antenna Macromodel

The construction of a macromodel for a generic antenna consists of two steps: a) obtaining the equivalent sources of the antenna and b) deriving the impulse responses of the antenna. The equivalent sources of the antenna represent the transmitting properties of the antenna and are used to excite the antenna in the simulation with the environment. As mentioned in the previous chapters, the antenna re-interacts with the fields that are scattered from the environment and propagate back towards the antenna. This phenomenon has a secondary effect on the solution to the problem. Hence, in order to obtain a full solution to the problem, it is necessary to capture this secondary interaction of the antenna with the scattered fields from the environment. This is accomplished by means of the impulse responses of the antenna which will be used to characterize the scattering properties of the antenna. The main idea behind the derivation of the impulse responses of the antenna is for them to serve as a means of encapsulating the scattering behaviour of the antenna through transfer functions that are defined at particular ports located on a closed surface enclosing the antenna. These transfer functions or impulse responses can then be used to find the corresponding scattered fields that are produced for any incident field on the antenna as they would be generated when scattered by the antenna itself. These will be discussed in detail below.

A) Equivalent Sources of the Antenna - Transmit Macromodel

The first step in constructing a macromodel of an antenna deals with representing the antenna in terms of its equivalent sources or currents on a closed Huygens surface, $S$, according to the equivalence principle. The equivalent sources of the antenna are obtained by either exciting the antenna in free-space with a source $g(t)$, such as a Gaussian or modulated Gaussian pulse, having a specified frequency bandwidth as required by the application or problem at hand or from its pattern. For the former approach, the excitation is typically done using a hard excitation scheme and the time domain fields on $S$ are
recorded at every time step until the fields have decayed or converged, as shown in Figure 3.1. The equivalent source fields transmitted by the antenna in free-space, denoted by $\vec{E}_T$ and $\vec{H}_T$, can now be used to excite and simulate the antenna in the presence of its environment.

**Figure 3.1:** Configuration for obtaining the equivalent sources of an antenna.

---

**B) Impulse Responses of the Antenna - Scattering Macromodel**

The first step in the derivation of the impulse responses of the antenna is to define ports around the antenna on a closed surface surrounding the antenna as shown in Figure 3.2. For simplicity this surface can be chosen as the same closed Huygens surface on which the equivalent sources of the antenna were calculated. Each port is now excited by an impulse function, $s(t)$, that satisfies the following criterion,

\[
\int_{-\infty}^{\infty} s(t) dt = 1 \tag{3.1}
\]

The impulse function in FDTD is approximated by a Gaussian pulse which is suf-
sufficiently more broadband than the source excitation in order to achieve a nearly flat spectrum for \( s(t) \) over the bandwidth of \( g(t) \). The impulse excitation is applied to field components lying tangential to the surface, \( S \), at every port. By applying a field equivalence principle on \( S \), it will be shown later on that it is sufficient to extract the impulse responses of the antenna by exciting only the tangential electric field components at each port with an impulse function. This reduces the complexity of the scheme and the following discussions will deal with the derivation of the impulse responses due a tangential electric field impulse excitation alone.

The challenge here is to properly obtain the fields scattered by the antenna at all ports including the excitation port. A hard source excitation of \( s(t) \) will result in the impulse function itself at the excitation node. A soft or a transparent excitation scheme will be able to capture the scattered fields from the antenna. However, for a 2D-FDTD scheme this would restrict the impulse function to that of a modulated Gaussian pulse as the DC component of a baseband Gaussian pulse would interfere with the impulse response values. One way to overcome this problem is to model the tangential electric

Figure 3.2: Configuration for obtaining the impulse responses of an antenna.
field sources in terms of their equivalent magnetic current sources as,

\[ \vec{M}_s = \vec{E} \times \hat{n} \]  \hspace{1cm} (3.2)

where, \( \hat{n} \) is the surface normal vector directed into the closed Huygens box. These sources can be directly applied to the field update equation using the discretized form of equation (A.14). It is important to note that \( \vec{M} \) in equation (A.14) refers to a volume current and hence \( \vec{M}_s \) has to be divided by the unit cell’s length in the direction normal to the tangential surface. Another issue to note here is that the magnetic currents are collocated in space with the electric field nodes as a result of Faraday’s law. Since the location of the magnetic current sources coincides with the location of the electric fields they are dislocated by half a cell corresponding to the nearest magnetic field nodes as shown in Figure 3.3.

Figure 3.3: Dislocation of the magnetic field nodes with the electric impulse source.

Hence, in order to properly include the magnetic current sources in the update equations, the sources have to be distributed as follows. Now let us consider a port located on a surface \( y = j_0 \Delta y \) that forms a part of the Huygens surface enclosing the antenna with surface normal vector \( \hat{n} = \hat{y} \) as shown in Figure 3.3. Furthermore, let the impulse
source, $s(t)$, be located at the position of one of the tangential components on that port, which as shown in Figure 3.3 is $E_x$, and the corresponding equivalent magnetic current source due to it is,

$$M_z = \frac{E_{x,source}}{%y}$$

The presence of the source will now affect the updates the magnetic fields $H_z(i, j_0, k)$ and $H_z(i, j_0 - 1, k)$. Noting that the discrete field update equations for the FDTD algorithm are derived based on a centered finite difference approximation (Appendix A), the source can be split equally and distributed between the two adjacent magnetic field nodes, $H_z(i, j_0, k)$ and $H_z(i, j_0 - 1, k)$. In this way, the update equation for the magnetic field node located inside the surface $S$ can be deduced as,

$$H_z^{n+1/2}(i, j_0, k) = H_z^{n-1/2}(i, j_0, k) + \frac{\Delta t}{\varepsilon} \left[ \frac{E_y^n(i, j_0, k) - E_y^n(i + 1, j_0, k)}{%x} + \frac{E_y^n(i, j_0 + 1, k) - E_y^n(i, j_0, k)}{%y} \right]$$

$$+ \frac{E_{x,source}^n(i, j_0, k)}{2%y}$$

$$H_z^n(i, j_0 - 1, k) = H_z^{n-1}(i, j_0 - 1, k) + \frac{\Delta t}{\varepsilon} \left[ \frac{E_y^n(i, j_0 - 1, k) - E_y^n(i + 1, j_0 - 1, k)}{%x} + \frac{E_y^n(i, j_0 - 1, k + 1) - E_y^n(i, j_0 - 1, k)}{%y} \right]$$

$$+ \frac{E_{x,source}^n(i, j_0 - 1, k)}{2%y}$$

A similar update equation is obtained for the update of the magnetic field node half a cell outside the surface $S$. Thus by equally distributing the source, it can be directly incorporated into the regular FDTD update equations and the impulse function $s(t)$ can now be easily injected at each port as a magnetic current source according to the above equations. This method serves as a convenient means of extracting the scattered fields generated by the antenna at all ports compared to a soft source excitation.

Each port defined around the antenna is now excited sequentially using the magnetic current sources and the corresponding electric and magnetic scattered field responses, denoted by $\vec{E}_{imp}$ and $\vec{H}_{imp}$ at all ports are recorded at all time steps. Assuming there
are $P$ ports, this will result in two $P \times P$ matrices, $A_E$ and $A_H$, which consist of the electric and magnetic impulse responses respectively, of the antenna system. The electric impulse response matrix, $A_E$, can be expressed as,

$$
A_E = \begin{bmatrix}
\hat{E}_{1,\text{imp}}^1 & \cdots & \hat{E}_{1,\text{imp}}^P \\
\vdots & \ddots & \vdots \\
\hat{E}_{m,\text{imp}}^1 & \hat{E}_{m,\text{imp}}^m & \hat{E}_{m,\text{imp}}^P \\
\vdots & \ddots & \vdots \\
\hat{E}_{P,\text{imp}}^1 & \cdots & \hat{E}_{P,\text{imp}}^P
\end{bmatrix}
$$

(3.5)

The magnetic impulse response matrix, $A_H$, can be expressed in a similar way. In the above matrix $\hat{E}_{\text{imp}}^{m,p}$ refers to the impulse response of the antenna at port $m$ due to an impulse excitation at port $p$. Each of the responses denoted in the form $\hat{E}_{\text{imp}}^{m,p}$ in the above matrix are tensors by themselves and consist of the responses of all tangential electric and magnetic field components at a particular port due to the excitation of each tangential component at any port. Hence, in 3D FDTD with discretization based on an orthogonal cartesian coordinate system, there will be two components of the electric field that are tangential to any side of the Huygens surface. The matrix $\hat{E}_{\text{imp}}^{m,p}$ would thus consist of four different response values and can be represented as,

$$
\hat{E}_{\text{imp}}^{m,p} = \begin{bmatrix}
h_{E_i,E_i}^{m,p} & h_{E_i,E_j}^{m,p} \\
h_{E_j,E_i}^{m,p} & h_{E_j,E_j}^{m,p}
\end{bmatrix}
$$

(3.6)

where the superscripts $m$ and $p$ are used to indicate coupling between ports while the subscripts $E_i$ and $E_j$ are used to indicate coupling between the different tangential field components. The term $h_{E_i,E_j}^{m,p}$ thus represents the impulse response of the field component $E_j$ at port $m$ due to the impulse excitation of the field component $E_i$ at port $p$. These impulse responses represent both radiation and the structural scattering properties of
the antenna.

The matrices consisting of the equivalent sources and the impulse responses of the antenna comprise the transmit and scattering macromodels of the antenna. In addition, there exists the receive macromodel which is used to characterize the receiving properties of the antenna. This will be explained in the next chapter. The behavior of the antenna is thus fully defined by the quantities stored in these two matrices. For a specific antenna, its equivalent sources and impulse response values are calculated only once and are saved as part of the antenna’s history. The antenna will now be completely replaced by its macromodel in subsequent simulations incorporating the environment.

3.2.2 Simulation of Antenna Macromodel

This section will discuss how the antenna macromodel can be incorporated into a standard FDTD simulation to model the interaction of the antenna with its environment.

A) Simulation of the Antenna with the Environment

Once the macromodel is developed for a specific antenna, it can now be used to embed the antenna in a simulation with its environment. The first step in this process consists of exciting the antenna using its equivalent sources that were obtained previously as part of the macromodel of the antenna according to the total-field/scattered-field formulation described in chapter 2. For consistency in the discussions to follow, the region inside the Huygens surface will be treated as the scattered field region and the region outside consisting of the environment will be treated as the total field region. The simulation is carried out until the fields scattered by the environment have decayed at all nodes on the Huygens surface, $S$ (Figure 3.2). The scattered components of the resulting tangential electric and magnetic fields, denoted by $\vec{E}_R(t)$ and $\vec{H}_R(t)$ respectively, at all nodes on the Huygens surface are recorded at every time step during the time-stepping loop. It is important to note here that the TF/SF simulation of the antenna with the environment
does not include a discretization of the antenna inside the closed Huygens surface. This feature is unique in comparison to the DG-FDTD technique discussed in the previous chapter. The scattered fields will now give rise to secondary scattered fields that are generated by the antenna structure. The procedure for calculating this is explained next.

B) Calculation of the Secondary-scattered fields generated by the Antenna

The next step involves the calculation of the secondary scattered fields that are generated by the antenna. This is accomplished via convolution. From LTI system theory, it is well known that the response of a system to an arbitrary input can be calculated from its impulse response via convolution, as shown below.

\[ y(t) = h(t) * x(t) \]

![Figure 3.4: Response of an LTI system to an arbitrary input.](image)

The same principle is employed here in order to calculate the fields scattered by the antenna. The antenna’s impulse response values obtained in the first step and the new scattered fields obtained in the previous step are used to accomplish this. The new scattered electric fields at each port can be considered as new separate excitations to the antenna domain just as in the case of deriving the antenna’s impulse responses. Now, the surface supporting the new electric field sources can be considered a perfect magnetic conductor (pmc) and hence by image theory the electric field sources are doubled and the magnetic fields are disregarded as shown in Figure 3.5. This surface equivalence principle was not applied while deriving the impulse responses and is instead applied here since the impulse source, \( s(t) \), is thought of as a normalized function satisfying equation 3.1. As a result, the factor of two arising from the doubling of the electric field sources is applied to the new sources on the Huygens surface rather than the impulse source. Hence it
is sufficient to record only the tangential electric fields, $\vec{E}_R(t)$, on the Huygens surface during the TF/SF simulation of the antenna. A similar surface equivalence principle can be applied if one were to deal with only magnetic fields or electric current sources in which case the surface supporting the sources can be considered to be a perfect electric conductor.

Now instead of re-simulating the antenna domain with the new sources, the new fields scattered by the antenna can be calculated by applying the principle of convolution. The new source at each port is an input to the system incorporating the antenna and the new response at each port is obtained by convolving the source with the corresponding impulse responses at each port due to an impulse excitation at the port corresponding to the location of the new source. The total secondary scattered electric and magnetic fields at a particular port on the Huygens surface will then be a superposition of the responses at that port due to the new sources at all ports on the Huygens surface. As a result the secondary scattered electric and magnetic fields generated by the antenna, at say port $m$, can be calculated according to the following set of equations,

$$E_{ss,j}(m, t) = \sum_{p=1}^{P} \sum_{i} h_{E_j,E_{R,i}}^{m,p} \ast (2 \cdot E_{R,i}(p, t))$$ \hspace{1cm} (3.7a)

$$H_{ss,j}(m, t) = \sum_{p=1}^{P} \sum_{i} h_{H_j,E_{R,i}}^{m,p} \ast (2 \cdot E_{R,i}(p, t))$$ \hspace{1cm} (3.7b)
where $i$ and $j$ represent $x$, $y$, $z$ components of fields tangential on $S$ and the factor of two comes from the application of the surface equivalence principle.

The secondary-scattered fields are thus obtained without the need for an FDTD simulation of the antenna region. Once the secondary scattered fields, $\vec{E}_{ss}(t)$ and $\vec{H}_{ss}(t)$ are obtained at all ports, they will be used as new sources for the antenna which are again excited using the total-field scattered-field formulation. The antenna macromodel is then re-simulated with the environment to find the new secondary scattered fields, $\vec{E}_{ss}'(t)$ and $\vec{H}_{ss}'(t)$ and the entire algorithm is repeated in an iterative fashion, until the secondary scattered fields have decayed. A summary of the iterative macromodel scheme is shown in Figure 3.6 as a flowchart.

An important issue regarding the derivation of the antenna’s impulse responses and the results of the convolution deserves attention here. In linear systems theory, the impulse response of a system is obtained from an impulse excitation, $\delta(t)$ which is applied at time $t=0$ as shown in Figure 3.7(a). In FDTD, however, this is not the case since the impulse function, being Gaussian in nature, is applied smoothly over few time steps in order to avoid the evolution of spurious fields, as shown in Figure 3.7(b). The peak time of the Gaussian pulse can be regarded as an approximation of the time instant at which the impulse excitation is applied in the FDTD algorithm. The impulse responses of the antenna derived using the Gaussian approximation of the impulse function are thus shifted by this time delay. Based on this argument, the impulse responses have to be shifted in time by the peak time of the pulse, $t_0$. However, since the Gaussian function in FDTD evolves smoothly with time before $t_0$, there may be instances where the impulse response values at particular ports begin to evolve even before $t_0$. Instead, it can be seen that the time-shift in the responses in turn affects secondary-scattered fields as is shown by the following derivation.

Let $h(t)$ be a time domain impulse response of the antenna system for the impulse excitation $\delta(t)$ and $h_g(t)$ be the same impulse response obtained from an impulse excita-
tion approximated by a broadband Gaussian pulse with peak time $t_0$. The two responses can be related as follows,

$$h(t) = h_g(t + t_0)$$  \hfill (3.8)

By substituting the above in (3.7) and applying the transformation $t' = t + t_0$, the following
Figure 3.7: Impulse functions in (a) LTI system theory and (b) FDTD.

can be deduced,

\[ E_{ss,j}(m, t' - t_0) = \sum_{p=1}^{P} \sum_{i} h_{g,E,j,E_R,i}^{m,p} \ast (2 \cdot E_{R,i}(p, t' - t_0)) \]  \hspace{1cm} (3.9a)

\[ H_{ss,j}(m, t' - t_0) = \sum_{p=1}^{P} \sum_{i} h_{g,H,j,E_R,i}^{m,p} \ast (2 \cdot E_{R,i}(p, t' - t_0)) \]  \hspace{1cm} (3.9b)

Hence, it would be more appropriate to time-shift the secondary scattered fields, \( \vec{E}_{ss}(t) \) and \( \vec{H}_{ss}(t) \), or the scattered fields from the environment, \( \vec{E}_{R}(t) \), by \( t_0 \), rather than time-shifting the impulse response values.

### 3.3 Numerical Validation Results

The macromodel algorithm developed in the previous section was incorporated into a 2D-FDTD formulation in the \( TE_z \) mode. The algorithm was first validated for a simple antenna geometry of a dipole placed close to a PEC corner as shown in the Figure 3.8. The length of the dipole was chosen to be \( \frac{\lambda}{5} \) at 900 MHz. The FDTD parameters used for discretizing the computational domain were: \( \Delta x = \Delta y = 4.166 \times 10^{-3} \) m and \( \Delta t = 9.5 \) ps. The FDTD mesh dimensions of the working volume were 336 x 372 cells in the \( x \) and \( y \) directions respectively. The dipole was excited with a Gaussian pulse with \( f_{max} = 900 \) MHz to obtain its equivalent sources on a closed Huygens surface.
The impulse responses of the dipole antenna were obtained by exciting the tangential electric field components on each node on the Huygens surface with a more broadband Gaussian pulse having a bandwidth from $0 - 2.7$ GHz. As discussed in the previous section, the impulse sources were modeled in terms of their equivalent magnetic current sources and incorporated into the FDTD update equations. The impulse function was excited in such a way as to ensure that the peak-time, $t_0$, of the pulse occurred at some multiple of the time step value, $\Delta t$, in order to facilitate the time-shifting of the results of the convolution easily. For the dipole, the size of the macromodel was $25 \times 4$ cells.

The macromodel simulation was carried out for 5 iterations. Figure 3.9 shows the corresponding time domain waveforms of the $E_x$ component of the total field that were obtained at $Rx_1$ and $Rx_2$ compared with that obtained from a full wave simulation incorporating the dipole antenna. For any field component in the total field region, which in this case is the region outside the Huygens surface, the waveform obtained during each iteration of the macromodel simulation comprise the total field that is generated due to the effect of the radiation and scattering from the antenna and the scattering from the environment. The total field waveform then for any field component at any node in the total field domain is reconstructed by summing the waveforms obtained from all iterations over all time steps. As can be observed from the above results, there is very
good agreement between the results from the macromodel simulation and the full wave simulation.

![Figure 3.9: Plot of the $E_x$ component of the total field at (a) $Rx_1$ and (b) $Rx_2$ obtained from a simulation of a dipole close to a PEC corner.]

The algorithm was then validated for a bow-tie antenna of length 75 mm and flare angle of $90^\circ$. The bow-tie antenna due to its geometry presents a stronger interaction with the PEC corner compared to the dipole and hence the fields produced and scattered by it are larger in magnitude. The bow-tie antenna was positioned in the same way as that of the dipole close to the PEC corner. The size of the macromodel in this case was $21 \times 21$ cells.

Again the time domain $E_x$ component of the total field after 5 iterations of the macromodel algorithm at $Rx_1$ and $Rx_2$ were obtained and are shown in Figure 3.10 along with a reference solution obtained from a full wave simulation. Again it can observed that there is very good agreement between the results from the macromodel simulation and the full wave simulation.

For any numerical electromagnetic solver, an important factor characterizing the effectiveness of the solver relies on the convergence of the scheme or algorithm being executed. Being an iterative scheme, it is thus important to see how well the solution to the problem converges with each iteration of the macromodel algorithm. One way to check this is in our case is to observe how the time domain fields converge to the full solution to
Figure 3.10: Plot of the $E_x$ component of the total field at $Rx_1$ and $Rx_2$ obtained from a simulation of a bow-tie close to a PEC corner.

the problem after each iteration. Since the fields scattered by the bow-tie are larger in magnitude compared to the dipole, the evolution of the scattered fields are easily visible and this is plotted in Figure 3.11 for the $E_x$ component of the received field at $Rx_1$ for the first four iterations of the algorithm.

It can be observed from the plots that the results converge very well with a reasonably accurate result being obtained with just two iterations and a very accurate solution obtained with four iterations indicating that the algorithm is indeed convergent. The above can be quantitatively analyzed by plotting the error in the total received field obtained after each iteration. A more interesting aspect however would be to plot the error in the scattered portion of the received fields since the calculation of the scattered fields is our main focus here. This is shown in Figure 3.12 and was obtained by subtracting the incident pulse that would arrive at the observation points in the absence of the environment and then calculating the relative error based on equation (2.7). The error in the scattered fields is plotted for 10 iterations of the macromodel algorithm for received fields at $Rx_1$ and $Rx_2$. In general the relative errors are found to decrease with each iteration with nearly constant values observed after 5 iterations of the algorithm.

The bow-tie antenna used for the previous results was used in a second simulation with a lossy dielectric wall. The FDTD parameters were the same as in the previous
Figure 3.11: Waveform of the $E_x$ component of the total field from the bow-tie simulation after the (a) first iteration, (b) second iteration, (c) third iteration and (d) fourth iteration. The dashed waveform represents the corresponding field obtained from a full wave simulation.

The bow-tie was surrounded by a wall with relative dielectric permittivity of 5.0 as shown in Figure 3.13. The simulation was carried out for two different values of wall conductivities of 0.005 S/m and 0.07 S/m. The resulting relative errors in the scattered component of the received field $E_x$ at Rx is plotted for 9 iterations in Figure 3.14.

Convergence of the algorithm is again observed from the plots. It can also be noted that the simulation incorporating the wall with a higher loss present lower error values compared with that having a lower loss value. This is consistent with what is generally
Figure 3.12: Plot of the relative error in the scattered components of the temporal received fields at $Rx_1$ and $Rx_2$ obtained from a macromodel simulation of a bow-tie close to a PEC corner for 10 iterations.

Figure 3.13: Simulation layout of a bow-tie antenna surrounded by a lossy dielectric wall. Expected and further validates the accuracy of the scheme. Another implication of this is that the convergence in the total fields will be faster if the environment is lossy. Since real-time simulations often incorporate losses that are present due to the different materials available in the environment, a few iterations of the macromodel scheme is sufficient to provide useful and reasonably accurate results.
Figure 3.14: Plot of the relative error in the scattered component of the temporal received field at $Rx$ obtained from the macromodel simulation of a bow-tie antenna surrounded by dielectric wall with two different loss values.

In the plots of the error versus the number of iterations one would expect the error to keep decreasing as the number of iterations of the macromodel algorithm increases. However, there exists a so called 'error floor' region after a few iterations where the error remains more or less constant. This can be attributed to two main sources of errors. The first source of error arises from the TF/SF approach to exciting the antenna macromodel, where the region inside the Huygens’ surface, $S$, is free-space. The resulting scattered fields from the environment that pass through $S$ may produce small ripples which may contaminate the scattered fields on $S$ that are used to calculate the secondary scattered fields via convolution. The second is in regards to the derivation of the impulse responses where the impulse function is approximated by a sufficiently broadband Gaussian pulse whose spectrum is not perfectly flat over the bandwidth of interest. Also there exists approximations in the way the impulse function is excited in the FDTD grid which may contribute to the error. However these errors as observed in the previous simulations are very small and do not impact the accuracy of the results significantly.
3.3.1 Macromodel Results with Interpolation

The previous section presented a validation of the macromodel scheme and did not involve any interpolations when simulating the macromodel with the environment. The computational efficiency of the scheme relies in its ability to simulate the antenna macromodel with its environment in a mesh that is coarser than what is required to discretize the antenna. Since the equivalent sources and impulse responses of the antenna are computed using a really fine discretization rate, these values have to be interpolated in order to incorporate the macromodel into a coarse FDTD mesh that models the environment such as a wireless channel.

Usually when the coarse to fine mesh ratio is odd, it may be sufficient to obtain the values from the fine mesh corresponding to the cell lying at the centre with respect to the coarse cell. However this process of down-sampling in space has shown to be inaccurate for higher mesh aspect ratios as will be seen later. A simple and accurate interpolation scheme for odd coarse-fine mesh aspect ratios has been presented in [21] and is depicted in Figure 3.15. The scheme, depicted in Figure 3.15, interpolates the fields from a fine mesh based on field positions in the coarse mesh. The electric field components are interpolated by averaging the fields along the plane of the surface $S$ while the magnetic

![Figure 3.15: Interpolation of field values for odd coarse-fine mesh aspect ratios.](image)
field components are interpolated by averaging the fields along the plane located at a distance of $k\Delta/2$, $k$ being the coarse-fine mesh aspect ratio and $\Delta$ the cell size of the fine mesh. Based on this, for a mesh aspect ratio of $k = 3$ as shown in Figure 3.15, the field values for the coarse mesh can be expressed as,

$$E'_x(i, j, k) = \frac{1}{3} (E_x(i, j, k) + E_x(i + 1, j, k) + E_x(i + 2, j, k))$$  \hspace{1cm} (3.10a)

$$H'_z(i, j, k) = \frac{1}{3} (H_z(i, j - 2, k) + H_z(i + 1, j - 2, k) + H_z(i + 2, j - 2, k))$$  \hspace{1cm} (3.10b)

For even coarse-fine mesh aspect ratios, the interpolation scheme explained in section 3.2.3 can be used. Also with respect to time, the field values are either down-sampled or interpolated if required to correspond with the time step of the coarse simulation.

The simulation of the bow-tie antenna close to a PEC corner was performed for different mesh aspect ratios using the macromodel algorithm. The equivalent sources and the impulse responses of the bow-tie antenna were computed at a discretization rate of $\lambda_{\text{min}}/80$ with the same FDTD parameters as used previously. The scattering macromodel of the bow-tie antenna was characterized using the impulse responses that corresponded to the impulse excitation of the middle cell for each mesh aspect ratio. In the case of mesh aspect ratios of 3 and 5 the fields and impulse responses were simply down-sampled in both space and time. For $k = 7$, the simulation was first performed by using down-sampled values and later by using values interpolated in space according to the scheme described above. In order to evaluate the performance of the macromodel scheme against the current standard schemes, the simulations were also performed by modeling the bow-tie antenna using the hard-box and the DG-FDTD schemes. Again the received fields at $Rx_1$ and $Rx_2$ were obtained from each simulation. The results were compared to a reference fine simulation of the antenna at a discretization rate of $\lambda_{\text{min}}/80$ and the relative errors computed using (2.7) are tabulated in 3.11. The size of each port for the coarse macromodel simulations increase by the mesh aspect ratio factor, $k$, and
the total number of ports on $S$, consequently decreases. For the bow-tie antenna, the size of the macromodel for the fine simulation was $21 \times 21$ cells, while that for the coarse simulations with mesh aspect ratios 3, 5 and 7 were $7 \times 7$ cells, $5 \times 5$ cells and $3 \times 3$ cells respectively.

Table 3.1: Computed relative errors in the temporal waveforms extracted from a 2-D simulation of Bow-tie close to a PEC corner for different antenna modeling schemes and mesh interpolations.

<table>
<thead>
<tr>
<th>Coarse-Fine Mesh Ratio</th>
<th>Antenna Modeling Scheme</th>
<th>$Rx_1$ Total</th>
<th>$Rx_1$ Scattered</th>
<th>$Rx_2$ Total</th>
<th>$Rx_2$ Scattered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 3$</td>
<td>Hard-box</td>
<td>0.36</td>
<td>1.35</td>
<td>0.14</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>DG-FDTD</td>
<td>0.52</td>
<td>1.95</td>
<td>0.34</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Macromodel</td>
<td>0.13</td>
<td>0.49</td>
<td>0.11</td>
<td>0.21</td>
</tr>
<tr>
<td>$k = 5$</td>
<td>Hard-box</td>
<td>1.00</td>
<td>3.73</td>
<td>0.48</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>DG-FDTD</td>
<td>1.13</td>
<td>4.21</td>
<td>0.55</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>Macromodel</td>
<td>0.34</td>
<td>1.26</td>
<td>0.31</td>
<td>0.59</td>
</tr>
<tr>
<td>$k = 7$</td>
<td>Hard-box</td>
<td>1.42</td>
<td>5.76</td>
<td>1.04</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td>DG-FDTD</td>
<td>0.95</td>
<td>3.88</td>
<td>0.79</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>Macromodel</td>
<td>0.88</td>
<td>3.59</td>
<td>0.62</td>
<td>1.25</td>
</tr>
<tr>
<td>$k = 7$ (with interpolation)</td>
<td>Hard-box</td>
<td>0.41</td>
<td>1.65</td>
<td>0.22</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>DG-FDTD</td>
<td>0.70</td>
<td>2.86</td>
<td>0.40</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>Macromodel</td>
<td>0.23</td>
<td>0.95</td>
<td>0.09</td>
<td>0.19</td>
</tr>
</tbody>
</table>

The macromodel algorithm consistently presents lower numerical errors than the hard-box and DG-FDTD schemes. Figure [3.16] shows the convergence of the macromodeling algorithm for different mesh aspect ratios for the field sampled at $Rx_1$. For completeness, the convergence of the results from the macromodel simulations for mesh aspect ratios of 3 and 5 with interpolation are also shown. For the mesh aspect ratio of $k = 7$, the simulation with interpolated values presents lower errors compared to the results obtained without interpolation and significantly improves the convergence of the macromodeling scheme.
Figure 3.16: Convergence of the macromodeling algorithm for the $E_x$ field at $Rx_1$ for different mesh aspect ratios.

### 3.4 Memory and Computational Requirements

The macromodel approach for simulation of antennas requires the storage of three matrices: one for the equivalent sources and one each for the electric and magnetic impulse responses. The storage required for each of the impulse response matrices depend on the total number of macromodel ports and the length of the impulse responses and can be expressed as,

$$
Storage \{ImpulseResponses\} = 2 \times i^2 \times P^2 \times \frac{t_{imp}}{\Delta t}
$$  \hspace{1cm} (3.11)

where $i$ denotes the number or tangential field components at each port, $P$ is the total number of ports and $t_{imp}$ is the length of the impulse responses.

In general, the computational load scales in terms of FDTD-related multiplications as $N_v^2$ for 2D FDTD and $N_v^3$ for 3D FDTD, $N_v$ being the FDTD working volume as well as the number of time steps required for the simulation. Hence significant reductions in
computational times are readily observed when the antenna macromodel is incorporated in coarse meshes. Despite being an iterative procedure, the algorithm needs to be executed for only a few iterations to produce useful results and is still computationally more efficient compared to a brute-force approach. Apart from the time required to run the main FDTD loop implementing the TF/SF formulation, the convolution loop also contributes to the total computation time whose execution time increases with the number of ports on the macromodel and the total number of simulation time steps. The impulse responses of an antenna decay quite rapidly in time compared to the scattered fields and as a result the length impulse responses will usually be a small fraction of the total simulation time. Figure 3.17 illustrates the convolution between an impulse response and a scattered field in the time domain where the convolution process is windowed over the length of the impulse responses, \( t_{\text{imp}} \).

![Figure 3.17: Windowing of the convolution of the antenna’s impulse response with the scattered field in the time domain.](image)

It can be seen that the computational time for the convolution loop increases until the first \( t_{\text{imp}}/\Delta t \) time step loops and remains constant thereafter. The computational load per time step of the convolution loop can be inferred from the number of convolution-related multiplications, \( M_{\text{conv}} \), which can be expressed as,

\[
M_{\text{conv}} = 2 \times i^2 \times P^2 \times n, \quad n = 1, 2, \ldots \frac{t_{\text{imp}}}{\Delta t} \tag{3.12}
\]
In order to increase the efficiency of the time domain convolutions, it is necessary to store the impulse responses of the antenna using as few time domain samples as possible. In the general case, when the behaviour of the antenna is unknown, an algorithm such the one presented in [22], could be used to terminate the simulation after the impulse responses have decayed sufficiently at all ports around the antenna and thus store them using minimum number of time domain samples. The execution time of the convolution loop, as seen from (3.12), increases quadratically with respect to the number of macromodel ports. Convolution in the time domain thus may not be an efficient approach when there are a large number of ports on the closed surface, $S$. In such cases, a more efficient approach would be to multiply the impulse responses with the scattered fields in the frequency domain. This process, however, requires an IDFT operation to be performed on the results at all ports, in order to allow for the re-injection of the new fields into the FDTD grid.

Since the macromodel algorithm is iterative, a suitable criterion for the automatic termination of the simulation needs to be incorporated into the algorithm. This can be used to avoid unnecessary subsequent iterations of the algorithm and obtain useful and complete results efficiently. A suitable approach would be to observe the field at a particular point or the voltage that is induced or received at the antenna terminals. The calculation of the latter will be explained in the next chapter. In the time domain, a criterion based on the deviation in the norm of the total field or voltage values, $E$, between each iteration, $i$, and the previous iteration, $i - 1$, can be set as,

$$ \| E_i(t) \| - \| E_{i-1}(t) \| \leq \delta $$

(3.13)

where $\delta$ is a pre-defined error value based on the desired accuracy level. Generally a $\delta$ value of the order of $10^{-4}$ or less for the above criterion indicates the solution has converged and subsequent iterations will not impact the solution greatly. The convergence
of the macromodel scheme based on (3.13) is shown for the macromodel simulation results with interpolation in section 3.3.1 in Figure 3.18. A similar criterion can also be established in the frequency domain, as also stated in [22], which measures the maximum deviation between the total fields or voltages obtained after two successive iterations as,

\[
\max \left[ 20 \log_{10} \left( | E_i(\omega) - E_{i-1}(\omega) | \right) \right] \leq \delta
\]

(3.14)

where \( \delta \) is the maximum deviation and a value of -40 dB or less would indicate the solution has converged.

\section{3.5 Conclusions}

In this chapter, a method for the accurate incorporation of antenna models in an FDTD domain has been proposed. The scheme utilizes the concept of macromodels to represent the electromagnetic behaviour of antennas in terms of their equivalent sources and impulse responses. In addition to the construction of FDTD-compatible antenna macro-
models, an iterative approach for embedding and simulating the antenna macromodels with the environment using the FDTD algorithm has been outlined. The interaction of the antenna with its environment is fully captured through an iterative process. Numerical results showed very good accuracy when compared to a brute-force approach. The main efficiency of this approach is that it does not require the antenna to be discretized. The antenna macromodel was also shown to perform with better accuracy compared to the hard-box and DG-FDTD schemes when incorporated into coarser FDTD meshes. A discussion on the computational requirements of the scheme as well as possible criteria for the automatic termination of the macromodel algorithm have been presented.
Chapter 4

Characterization of Receivers: The Reciprocity-based Receive Macromodel

In the previous chapter a generic scheme for macromodeling and simulating antennas was developed which characterizes the transmitting and scattering properties of the antenna. This chapter presents the reciprocity-based receive macromodel for characterizing the receiving properties of the antenna.

4.1 Introduction

In addition to characterizing the transmitting and scattering properties of the antenna, it is essential to analyze the performance of the antenna at the receiving end by determining received signals, available power and antenna factor among other parameters. In the ideal case, that is when the antenna is modeled accurately in a fine grid, the field or voltage value at the antenna terminals can be directly used to determine what the antenna receives and thus characterize its receiving properties. In a coarse simulation, however,
the antenna features can get significantly distorted and hence it may not be possible to accurately compute received signals. The reciprocity theorem for antennas can be used in such a case to compute the signals received by the antenna at its terminals.

The formulation of the reciprocity theorem is stated in the next section. The implementation of the reciprocity theorem for calculating received voltages using the FDTD algorithm is demonstrated through a novel reciprocity-based macromodeling scheme for the analysis of minimum-scattering antennas. Since the proposed algorithm deals with the concept of minimum-scattering antennas, a brief discussion on them is first provided followed by the FDTD implementation of the reciprocity-based macromodeling scheme. The reciprocity theorem is then extended for characterizing general antenna systems as receivers and its application is demonstrated through numerical results.

4.1.1 The Reciprocity Theorem

The reciprocity theorem is one of the most well known theorems in electromagnetic theory and a very useful tool for analyzing antenna systems. This theorem inter-relates the sources and fields of the antenna system in the transmitting and receiving states linearly and time-invariantly. Consider a linear and isotropic medium having volume $V$ and enclosed by surface $S$, as shown in Figure 4.1. Consider two sets of electric and magnetic current sources with phasors $\vec{J}_a, \vec{M}_a$ and $\vec{J}_b, \vec{M}_b$.

Assuming the surface $S$ becomes a large sphere at infinity, the fields, $\vec{E}_a, \vec{H}_a$ and $\vec{E}_b, \vec{H}_b$, produced by the two sets of sources respectively satisfy the following relationship [23]

$$\int_V (\vec{E}_b \cdot \vec{J}_a - \vec{H}_b \cdot \vec{M}_a)dv = \int_V (\vec{E}_a \cdot \vec{J}_b - \vec{H}_a \cdot \vec{M}_b)dv$$ \hspace{1cm} (4.1)

This is known as the Lorentz Reciprocity Theorem in integral form in the frequency domain.
Chapter 4. Characterization of Receivers

4.2 Minimum-scattering Antennas and Equivalent Circuits

The concept of minimum-scattering antennas stems from an early work by Montgomery and Dicke on the scattering properties of antennas [24]. Minimum-scattering antennas are a special class of antennas that do not re-radiate an incoming electromagnetic field when the input current is zero. Antennas that fall under this category exhibit the property that the scattered power is exactly equal to the absorbed power under matched load conditions [25]. A short dipole is an example of a minimum-scattering antenna, while a resonant half-wave dipole, array of short dipoles and small helices behave as approximate minimum-scattering antennas [26].

The traditional equivalent circuits for an arbitrary antenna in the receiving and transmitting modes are shown in Figure 4.2. However, it has been shown in [25] that the above equivalent circuits generally do not provide complete information about the scattered fields generated by the antenna, except in the cases of minimum-scattering antennas. This is due to the fact that in general, antennas in the receiving case have more than one
source, and thus they scatter more than they absorb. In the case of minimum-scattering antennas, the above circuit in the receiving case is valid and power is distributed between the antenna and the load, the two powers being equal under matched conditions. As a result the power scattered or re-radiated by a minimum-scattering antenna can be found from the open circuit voltage received at the input terminals of the antenna. This voltage can be obtained using the reciprocity theorem stated earlier.

4.3 Formulation of a Reciprocity-Based Macromodeling Scheme in FDTD

The macromodeling algorithm discussed in the previous chapter can be simplified for the special class of minimum-scattering antennas (MSA) due to the existence of a direct linear relation between the input of the MSA and the fields scattered by it as previously explained. This is made possible by invoking the reciprocity theorem for calculating the voltage induced at the antenna terminals which will in turn be used to find the fields scattered or re-radiated by the antenna. Although the proposed scheme is applicable to the analysis of minimum scattering antennas, it may also be possible to analyze weakly scattering antennas using this scheme.

The following sections will discuss in detail the implementation of the reciprocity-
based antenna macromodeling scheme for minimum scattering antennas in the FDTD algorithm. The first step in this scheme involves developing a suitable macromodel for the MSA. The macromodel is then used to embed and simulate the MSA in a general environment using the FDTD algorithm. The reciprocity theorem combined with linearity (convolution) is used to find the fields scattered by the antenna.

4.3.1 Construction of the MSA Macromodel

As in the generic macromodeling scheme, the first step in the reciprocity-based macromodeling algorithm is the construction of the MSA macromodel which consists of obtaining the equivalent sources and impulse responses of the MSA.

The equivalent sources of the minimum-scattering antenna on a surface enclosing the antenna are first obtained by exciting the antenna in free-space with a source $g(t)$, such as a Gaussian or modulated Gaussian pulse, covering the frequency bandwidth required for the simulation of the problem at hand. The time domain fields are recorded on a closed Huygens’ surface, $S$, as shown in Figure 4.3(a), at every time step until the fields have decayed or converged.

Unlike the generic case, the derivation of the impulse responses is simplified for the case of an MSA on the basis of its scattering properties which is mainly re-radiation due to an induced voltage at the antenna terminals as a result of fields incident upon the antenna. The impulse responses of the MSA are thus obtained in exactly the same way as its equivalent sources, except now the source $s(t)$, being an impulse function satisfying the criterion in equation (3.1), is much more broadband than $g(t)$. The fields radiated by the MSA due to $s(t)$ correspond to its time domain impulse response values and are also saved on the closed Huygens’ surface, $S$, as shown in Figure 4.3(b).

$\vec{E}_T$ and $\vec{H}_T$ in Figure 4.3(a) refer to the equivalent sources of the MSA while $\vec{E}_{imp}$ and $\vec{H}_{imp}$ in Figure 4.3(b) refer to the impulse responses of the MSA on the closed Huygens’ surface, $S$. This set of equivalent sources and the impulse responses together form the
macromodel of the minimum-scattering antenna and will be used to simulate the MSA with its environment.

4.3.2 Simulation of the MSA Macromodel

The MSA macromodel is now used to simulate the minimum-scattering antenna with its environment. As in the generic case, the antenna is first excited using its equivalent sources according to the total-field/scattered-field formulation without the antenna. Again the region inside the Huygens’ surface will be treated as the scattered field region and the region outside consisting of the environment will be treated as the total field region. The simulation is carried out until the fields scattered by the environment have decayed at all nodes on the Huygens’ surface, $S$. The resulting scattered components of both the tangential electric and magnetic fields, denoted by $\vec{E}_R(t)$ and $\vec{H}_R(t)$ respectively, at all nodes on the Huygens’ surface, are recorded at every time step during the time-stepping loop as shown in Figure 4.4.

The scattered fields will now generate an induced voltage at the terminals of the MSA which will give rise to secondary scattered fields from the antenna. These secondary scattered fields are obtained in two steps: a) by using reciprocity to calculate the induced voltage and b) by using convolution to calculate the new fields radiated by the MSA.
Figure 4.4: Configuration of the TF/SF Simulation of the Antenna with the environment and the derivation of the scattered fields.

A) Reciprocity - Calculation of the MSA Induced Voltage

The reciprocity theorem is first invoked on the antenna domain in order to calculate the voltage that is received at the antenna terminals as a result of the scattered fields from the environment that are incident on the antenna. The reciprocity theorem can be formulated either in the frequency domain or in the time domain. Here the frequency domain formulation is discussed and as a result, the equivalent source fields of the antenna, \( \vec{E}_T(t) \) and \( \vec{H}_T(t) \), and the scattered fields, \( \vec{E}_R(t) \) and \( \vec{H}_R(t) \), both need to be transformed to the frequency domain as will be explained below. This is easily done on the fly in the FDTD time-stepping loop during the course of the TF/SF simulation of the antenna with environment using the Discrete Fourier Transform (DFT).

In order to calculate the induced voltage, \( V_{ind}(t) \), using the reciprocity theorem, let us consider the two reciprocal cases of the antenna system shown in Figure 4.5. The configuration in Figure 4.5(a) with a Norton equivalent circuit representation of the antenna system corresponds to the antenna in the transmitting state. The configuration in Figure 4.5(b), on the other hand, corresponds to the antenna in its receiving state. In Figure 4.5(b) \( \vec{M}_R \) and \( \vec{J}_R \) denote the current densities due to the scattered fields \( \vec{E}_R \) and \( \vec{H}_R \) respectively. These above two states can be interrelated according to the reciprocity theorem as discussed in section 4.1.1. Applying equation (4.1) to the above system, one
Figure 4.5: Two reciprocal states of the antenna: (a) the transmitting state and (b) the receiving state

arrives at the following:

$$\int_V (\vec{E}_R \cdot \vec{J}_T - \vec{H}_R \cdot \vec{M}_T) dv = \int_V (\vec{E}_T \cdot \vec{J}_R - \vec{H}_T \cdot \vec{M}_R) dv$$  \hspace{2cm} (4.2)

where $\vec{M}_T$ and $\vec{J}_T$ denote the surface current densities due to the equivalent source fields $\vec{E}_T$ and $\vec{H}_T$ respectively. It should be noted that, $\vec{M}_T$, $\vec{J}_T$, $\vec{M}_R$ and $\vec{J}_R$ are modeled as sources in the macromodeling scheme and hence they can be used in the formulation of the reciprocity theorem. Now, in the configuration shown in Figure 4.5, due to the assumption of an electric current source $\vec{J}_T$, we have $\vec{M}_T = 0$, which reduces the left-hand side of (4.2) to $V_{ind} I_T$ as follows,

$$\int_V (\vec{E}_R \cdot \vec{J}_T) dv = (\vec{E}_R \cdot \hat{u} L_g) (\vec{J}_T \cdot \hat{u} S_g) = V_{ind} I_T$$  \hspace{2cm} (4.3)

where $L_g$ is the excitation gap length and $S_g$ is the area of the surface element over which $\vec{J}_T$ exists and $\hat{u}$ is the unit vector in the direction of the source. This results in the following equation for the induced voltage, $V_{ind}$

$$V_{ind} = \frac{1}{I_T} \int_V (\vec{E}_T \cdot \vec{J}_R - \vec{H}_T \cdot \vec{M}_R) dv$$  \hspace{2cm} (4.4)
In this scheme, however, the integral in the equation for the induced voltage is evaluated only on the surface of the Huygens’ box and hence (4.4) reduces to

\[ V_{\text{ind}} = \frac{1}{I_T} \oint_S (\vec{E}_T \cdot \vec{J}_{s,R} - \vec{H}_T \cdot \vec{M}_{s,R}) \, ds \]  

(4.5)

where \( \vec{J}_{s,R} \) and \( \vec{M}_{s,R} \) denote surface electric and magnetic currents respectively. Also, assuming the FDTD cell size is small compared to the operating wavelength, which is generally the case given the \( \lambda_{min}/10 \) restriction on the maximum cell size, the tangential electric and magnetic fields and currents on the Huygens’ surface can be assumed to be constant over the surface area of the cell, and the integral in (4.5) can be further simplified and expressed in the following summation form

\[ V_{\text{ind}} = \frac{1}{I_T} \sum_{(i,j) \in S} \left( \vec{E}_T(i,j) \cdot \vec{J}_{s,R}(i,j) - \vec{H}_T(i,j) \cdot \vec{M}_{s,R}(i,j) \right) \triangle s(i,j) \]  

(4.6)

where \( \triangle s \) represents the tangential surface area of a unit cell lying on the Huygens’ surface. The summation in (4.6) is done for every unit cell surface lying tangential to the Huygens’ box. The expression in (4.6) can now be used to calculate the voltage induced at the antenna terminals in the frequency domain. The corresponding time domain values for \( V_{\text{ind}}(t) \) are extracted via IDFT. The induced voltage will now be used to determine the secondary scattered fields generated by the antenna and this is explained in the next section.

B) Linearity - Calculation of the Secondary Scattered Fields

The final step involves the calculation of the secondary scattered fields that are generated by the MSA. The induced voltage, \( V_{\text{ind}}(t) \), can be considered as a new excitation for the antenna domain. Subsequent simulation of the MSA with the new voltage source will produce secondary scattered fields that are re-radiated by the antenna. But in this scheme, instead of re-simulating the MSA with the new source, the fields re-radiated or
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scattered by it are found by applying linear system theory. The antenna’s impulse response values and the induced open-circuit voltage obtained in the previous step are used to accomplish this. Observing that the antenna is a minimum-scattering element, with equivalent circuit in the receiving state as shown in Figure 4.2(b), the fields radiated by it can be directly correlated to the voltage induced at its terminals. Hence, the secondary scattered electric and magnetic fields, denoted by $\vec{E}_{ss}(t)$ and $\vec{H}_{ss}(t)$ respectively, at every node on the Huygens’ surface are obtained by convolving $V_{ind}(t)$ with the corresponding impulse response values at that node. Simply,

$$\{ \vec{E}_{ss}(t), \vec{H}_{ss}(t) \} = V_{ind}(t) \ast \{ \vec{E}_{imp}(t), \vec{H}_{imp}(t) \}$$

(4.7)

As in the generic macromodel algorithm, the secondary-scattered fields have to shifted in time by the peak time of the Gaussian impulse function. Equivalently, if the impulse response values of the antenna are available in the frequency domain, the secondary scattered fields can be found in the frequency domain from $V_{ind}$ by simply multiplying it with the impulse responses. This would generally be faster than convolution in the time domain but the reduction in computational time is traded off later by the time required to perform IDFT for the secondary scattered fields on every node on the Huygens’ surface to extract the corresponding time domain values.

The secondary-scattered fields are thus obtained without the need for an FDTD simulation of the minimum-scattering antenna. These fields are now used to obtain the new equivalent sources on the Huygens’ surface enclosing the antenna. The total-field scattered-field simulation of the MSA macromodel with the environment is repeated to obtain the new induced voltage which will in turn be used to find the new secondary scattered fields from the antenna. The TF/SF simulation of the MSA macromodel and the steps incorporating the calculation of the secondary-scattered fields generated by the
antenna are thus carried out in an iterative fashion, until the secondary scattered fields have decayed beyond a certain threshold. Due to the direct correlation between the secondary scattered fields and the induced open-circuit voltage, a threshold based on the amplitude of the induced voltage can be predetermined and set to automatically terminate the algorithm. A summary of the iterative reciprocity-based macromodel scheme is depicted as a flowchart in Figure 3.6

4.4 Numerical Validation Results

In order to validate the above reciprocity-based macromodel scheme 2-D FDTD numerical simulation was performed in the $TE_z$ mode. A short dipole surrounded by a lossy dielectric wall was used as the test case for the simulation as shown in Figure 4.7. The length of the dipole was $\lambda/5$ at 900 MHz. The wall was assigned a relative permittivity value of 3.0 and a conductivity value of 0.05 S/m. The FDTD parameters were: $\Delta x = \Delta y = 4.166 \times 10^{-3}$ m and $\Delta t = 9.5$ ps. The FDTD mesh dimensions were 336 $\times$ 372 cells in the $x$ and $y$ directions respectively. The dipole was excited with a Gaussian pulse with $f_{max} = 900$ MHz to obtain the equivalent sources on a closed Huygens’ surface. The impulse responses of the dipole antenna were obtained by exciting it with a more broadband Gaussian pulse having a bandwidth from 0 - 2.7 GHz. The impulse function was excited in such a way as to ensure that the peak-time, $t_0$, of the pulse occurs at some multiple of the time step value, $\Delta t$, in order to facilitate the time-shifting of the results of the convolution easily. Three iterations of the macromodel algorithm were performed to achieve convergence in the field values.

Shown in Figure 4.8 is the induced voltage at the dipole terminals calculated using the reciprocity theorem at the end of the first iteration of the macromodel algorithm and compared with that obtained from a simulation of the antenna domain consisting of the dipole. The latter was performed by obtaining the fields scattered by the envi-
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Figure 4.6: Flowchart of the Reciprocity-Based Antenna Macromodeling Scheme.

Ronon which were received on the Huygens’ surface during the first iteration of the macromodel algorithm and using them to excite the antenna domain according to the TF/SF formulation. Results show very good agreement between the two schemes indicating the effectiveness of the reciprocity theorem to accurately calculate the induced voltage received at the antenna’s terminals.
Figure 4.7: Simulation layout for validation of the Reciprocity-Based Macromodel Scheme.

Figure 4.8: Plot of the induced voltage obtained from the first iteration of the macromodel scheme.

Figure 4.9 shows the induced voltages obtained at the end of the first, second and third iterations. The results indicate that the back-scattered voltage indeed converges very well with every iteration. The reciprocity-based macromodel scheme is thus convergent.

Finally, Figure 4.10 shows the waveform of the $E_x$ component of the field scattered by the dipole at a node on the Huygens’ surface (shown in Figure 4.7) due to the induced voltage from the first iteration compared with that obtained from an equivalent TF/SF simulation with the dipole antenna discretized in the region inside the closed Huygens’
Figure 4.9: Plot of the back-scattered voltage obtained from (a) the first iteration (b) the second iteration and (c) the third iteration of the macromodel scheme.

As in the generic case, the total field waveform for any field component at any node in the total field domain is reconstructed by summing the waveforms obtained from all iterations over all time steps.

Figure 4.10: Plot of the $E_x$ component of the total field at a node on the Huygens’ surface.

As can be observed from the above results, there is very good agreement between the
4.5 Numerical Simulation Results and Discussions

A second 2-D simulation was performed with the same dipole placed close to a PEC corner as shown in Figure 4.11. Compared to the dielectric wall in the previous simulation, the PEC corner accounts for stronger interaction with the antenna and thus produces stronger scattered fields. The FDTD parameters are the same as in the previous simulation and the mesh size was 300 × 300 cells in the x and y directions respectively. A Gaussian pulse with $f_{\text{max}} = 900$ MHz was used to excite the dipole antenna.

The $E_x$ component of the total field at $Rx_1$ and $Rx_2$ were obtained and are shown in Figure 4.12. A reference solution was obtained using a TF/SF simulation of the same problem and is also shown. Figure 4.13 is a magnified plot of the field waveform at $Rx_1$ showing the evolution of the scattered fields for the first two iterations of the algorithm. It can be seen that the solution converges within just two iterations of the algorithm.

The same simulation was repeated with a bow-tie antenna with length 75 mm and

![Figure 4.11: Simulation layout of a short dipole close to a PEC corner.](image-url)
Figure 4.12: Plot of the $E_x$ component of the total field at $Rx_1$ and $Rx_2$ for the dipole simulation.

Figure 4.13: Waveform of the $E_x$ component of the total field from the dipole simulation after the (a) first iteration and (b) second iteration. The waveform in red represents the corresponding field obtained from a full wave simulation.

The corresponding received waveforms of the $E_x$ field component at $Rx_1$ and $Rx_2$ after 5 iterations of the algorithm are shown in Figure 4.14 and compared against a TF/SF result.

It can be seen from the plots of the time domain waveforms that the scattered fields in the case of the bow-tie antenna obtained from the reciprocity-based macromodel simulation even after 5 iterations do not agree well with the reference results from the TF/SF simulation and subsequent iterations were found to have no impact on the solution. The scattered fields evolving from the bow-tie from the TF/SF simulation are found to be significantly larger in magnitude than those from the corresponding macromodel simula-
Figure 4.14: Plot of the $E_x$ component of the total field at $Rx_1$ and $Rx_2$ for the bow-tie simulation.

tion. This can be attributed to the fact that, unlike the short dipole, the bow-tie antenna is not a minimum scattering antenna and thus the power scattered or re-radiated by the bow-tie cannot be completely determined from the open circuited voltage induced at its terminals. It can be seen that the reciprocity-based macromodel scheme is not able to properly capture the scattered fields accurately as in this case the secondary scattered fields are a result of both the radiated power resulting from the induced voltage as well as direct reflections of incident fields from the geometrical features of the bow-tie antenna.

The reciprocity theorem has been employed previously in FDTD simulations, notably in [27]. This paper presents a hybrid MoM/FDTD numerical formulation for the calculation of Specific Absorption Rate (SAR) in the human body by utilizing the reciprocity relation to determine the induced voltage which would serve as the excitation for the MoM simulation of the antenna. The scheme was applied for simulations involving a dipole antenna close to a human head. A time domain reciprocity relation for pulsed-field multiport antenna system and its application to indoor wireless communication performance analysis has also been briefly discussed in [28]. The above do not clearly state the fact that such an approach is limited to the analysis of minimum scattering antennas. However the results from the bow-tie simulation clearly reflect this point.
4.6 Memory and Computational Requirements

The reciprocity-based macromodeling approach for simulation of minimum-scattering antennas requires the storage of the equivalent sources and the impulse responses of the MSA. The storage required for the impulse responses are similar to that required for the equivalent sources and can be expressed as,

\[ \text{Storage}\{\text{ImpulseResponses}\} = 2 \times i \times P \times \frac{t_{imp}}{\Delta t} \]  

(4.8)

where \( i \) denotes the number of tangential field components at each port, \( P \) is the total number of ports and \( t_{imp} \) is the length of the impulse responses. Compared to the generic macromodeling scheme, the computational load is greatly reduced for the reciprocity-based macromodeling algorithm especially with regards to the execution of the convolution loop for the calculation of the secondary-scattered fields. In the current scheme, only one set of convolutions need to be carried out consisting of the scattered fields and the induced voltage as opposed to the generic scheme which requires multiple sets of convolutions consisting of the scattered fields and the impulse responses corresponding to each macromodel port excitation. The number of convolution-related multiplications, \( M_{\text{conv}} \), for the reciprocity-based macromodeling scheme can be expressed as,

\[ M_{\text{conv}} = 2 \times i \times P \times n, \quad n = 1, 2, \ldots, \frac{t_{imp}}{\Delta t} \]  

(4.9)

4.7 Extension of the Receive Macromodel for General Antennas

The reciprocity-based receive macromodel formulation outlined previously in the MSA macromodeling scheme can also be applied for characterizing general antenna systems as receivers by directly incorporating it with the generic macromodeling scheme discussed
in the previous chapter. For a receiver consisting of an antenna system with only one accessible port, the reciprocity relation is applied in the same way as explained in section 4.3.2 to determine the received voltage at the antenna terminals. The formulation can also be extended to antenna systems with more than one accessible port, such as arrays and MIMO systems, to determine the voltage received at each port. Here the reciprocity relation relates the receiving properties of each antenna port to the transmitting properties of the antenna system due to the excitation of that particular port alone. The transmitted fields corresponding to the excitation of each port are thus obtained when all the other accessible ports are not excited and are either open-circuited or terminated by a load. For an array with, say $K$ ports, the voltage received by the $i^{th}$ element can be expressed as (based on equation (4.6)),

\[
V_{R,i} = \frac{1}{I_{g,i}} \sum_{(i,j)\in S} \left( \bar{E}_T(i,j)|_{V_{g,k}=0,k\neq i} \cdot \bar{J}_{s,R}(i,j) - \bar{H}_T(i,j)|_{V_{g,k}=0,k\neq i} \cdot \bar{M}_{s,R}(i,j) \right) \Delta s(i,j) \quad (4.10)
\]

where $V_{g,k}$ denotes excitation at port $k$ and $I_{g}$ is the Norton equivalent current. The receive macromodel will thus consist of $K$ sets of transmitted field values corresponding to the excitation of each port of the antenna system.

4.7.1 Numerical Simulation Results

Numerical results from the simulation of transmitter and receiver systems using the macromodeling scheme are presented in this section. First a simple case of a single element antenna as the transmitter and receiver is presented. Thereafter results from the simulation of an array of transmitters and receivers in an indoor channel are presented.
A) Simulation of a Meander Dipole antenna

A 2D simulation of a meander dipole antenna \[29\] surrounded by a lossy dielectric wall was performed over the frequency range 0-1.8 GHz in the $TE_z$ mode. The simulation layout is shown in Figure 4.15. The material parameters for the wall were: $\varepsilon = 2.75$ and $\sigma = 0.02$ S/m and a modulated Gaussian pulse with centre frequency at 900 MHz was used to excite the transmitting antenna. The meander dipole antenna requires a minimum free-space discretization rate of approximately $\lambda_{\text{min}}/80$ to accurately describe its geometry. A coarser discretization rate would distort the antenna geometry greatly which would result in inaccuracy when trying to compute received voltages. To demonstrate this, the simulation was performed using the macromodeling scheme and the DG-FDTD scheme and the voltages induced at the terminals of the receiver antenna for each case were computed and compared. Both simulations were performed using a $\lambda_{\text{min}}/10$ mesh modeling the environment and the results were compared against those from a fine $\lambda_{\text{min}}/50$ simulation. The sub-gridding scheme in \[21\] was used to interpolate the equivalent sources and impulse responses of the meander dipole. For the fine $\lambda_{\text{min}}/50$ simulation, the FDTD mesh parameters were: $\Delta x = \Delta y = 2.0 \times 10^{-3}$ m and the time step $\Delta t$ was chosen to be 4.6 ps. The mesh sizes for the coarse and fine simulations

![Figure 4.15: Simulation layout of a meander dipole transmitter and receiver.](image)
Figure 4.16: Voltages received by a meander dipole antenna computed using different antenna modeling schemes.

were 84×84 and 420×420 respectively and the number of time steps were 600 and 3000 respectively. For the macromodel simulation, a complete solution was obtained in just one iteration of the algorithm. The received voltages are plotted in Figure 4.16. The voltage received by a point receiver is also plotted for comparison. The voltage received by a coarse model of the antenna is shown to be extremely inaccurate while that computed using the reciprocity theorem agrees well with the reference voltage from the fine simulation.

B) Wideband simulation of an array of transmitters and receivers in a hallway

A 2D wideband numerical simulation of a long hallway in the $TE_z$ mode was performed in order to demonstrate the application of the macromodeling algorithm for modeling arrays. The layout of the hallway along with the material parameters is shown in Figure 4.17. An array of two collinear bow-ties with centres separated by a distance of approximately $0.4\lambda$ at 900 MHz was used for both the transmitter and receiver. Each bow-tie antenna element in the array was of length 58 mm and with a flare angle of $53^\circ$. For the transmitter array, each antenna element was excited with a 1V Gaussian pulse source modulated at a centre frequency of $f_0 = 900$ MHz and with bandwidth from 0-1.8 GHz.

A coarse-fine mesh ratio of 3 was used for the macromodel simulation. The array
Figure 4.17: Layout of the hallway geometry used in the wideband simulation along with the positions of the transmitter (Tx) and receiver (Rx) arrays. The temporal received signals were obtained at positions $A$ and $B$.

macromodel was constructed using a fine $\lambda_{min}/30$ mesh discretization and was used to simulate the array in a coarser $\lambda_{min}/10$ discretization of the hallway geometry. For obtaining the impulse responses, a Gaussian pulse having a bandwidth from 0-5.4 GHz was used as the impulse excitation. The values from the $\lambda_{min}/30$ simulation were approximated for the $\lambda_{min}/10$ simulation by selecting those field and impulse response values from the former that coincided with the centre of the coarse mesh’s cell. The computed results were compared against a reference fine ($\lambda_{min}/30$) simulation of the entire problem. For the fine $\lambda_{min}/30$ simulation, the FDTD mesh parameters were: $\Delta x = \Delta y = 2.77 \times 10^{-3}$ m and the time step $\Delta t$ was chosen to be 6.2 ps. The mesh sizes for the macromodel and fine simulations were $1456 \times 432$ and $4368 \times 1296$ respectively and the number of time steps were 4000 and 12000 respectively. For the macromodel simulation, convergence was obtained after 3 iterations of the algorithm. The corresponding simulation times were 5403 s for the macromodel simulation and 39236 s for the fine simulation.

The temporal waveforms of the $E_y$ field component at points $A$ and $B$ were extracted and are plotted in Figure 4.18. The voltages received by the receiver array, $V_{out,1}$ and $V_{out,1}$, are plotted in Figure 4.19. Very good accuracy is observed in the time domain plots from the macromodel simulation when compared with the fine FDTD simulation. Figure 4.20 shows the steady-state distribution of the $E_y$ field component in the hallway at 900 MHz for the macromodel and fine FDTD simulations. Both plots of the field
Figure 4.18: Temporal waveforms of $E_y$ at points (a) $A$ and (b) $B$ in the hallway.

Figure 4.19: Temporal waveforms of the voltages (a) $V_{out,1}$ and (b) $V_{out,2}$ received by the two bow-tie antenna array receiver.

distribution are in good agreement with each other.

Finally, the channel response matrix was computed for the Tx/Rx system in the hallway. The channel response matrix relates the transmit (input) and receive (output)
Figure 4.20: Steady-state distribution of the magnitude of the $E_y$ component of the field in the hallway at 900 MHz computed from the (a) Macromodel ($\lambda_{\text{min}}/10$) simulation and (b) Fine ($\lambda_{\text{min}}/30$) simulation.

parameters of a MIMO system and gives important information about the wireless channel’s performance and its capacity. Such knowledge is critical in the design of efficient transmitters and receivers for MIMO systems. For a $2\times2$ Tx/Rx system, such as the one used in the hallway simulation, the MIMO system model can be represented as,

$$\begin{bmatrix} V_{\text{out,1}} \\ V_{\text{out,2}} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} V_{\text{in,1}} \\ V_{\text{in,2}} \end{bmatrix}$$

(4.11)

The channel response parameters $h_{11}$, $h_{12}$, $h_{21}$ and $h_{22}$ were computed for the same positions of the transmitter and receiver and are plotted in Figure 4.21. Again, very good agreement is observed in the results from the macromodel and fine FDTD simulations indicating the accuracy of the macromodeling scheme.
Figure 4.21: Channel Responses for the $2 \times 2$ Tx/Rx system in the hallway: (a) $h_{11}$, (b) $h_{12}$, (c) $h_{21}$ and (d) $h_{22}$.

4.8 Conclusions

In this chapter, the characterization of receivers using the FDTD algorithm is presented by utilizing the reciprocity theorem to macromodel the receiving properties of the antenna in terms of its transmitting properties. The formulation of the receive macromodel is first demonstrated through a reciprocity-based scheme for macromodeling minimum scattering antennas in which the induced voltage at the MSA terminals, calculated using the reciprocity theorem, is directly used to determine the fields scattered by the MSA. Compared to the generic macromodeling scheme, the reciprocity-based macromodeling scheme presents a simplified three step iterative approach for embedding and simulating minimum-scattering antennas with the environment using the FDTD algorithm and has
been validated for the case of a short dipole.

The reciprocity-based receive macromodel is then extended to model general antenna systems as receivers. The receive macromodel combined with the transmit and scattering macromodels completely characterizes the electromagnetic behaviour of any antenna system. The reciprocity theorem was shown to be more accurate than previous antenna modeling schemes in computing received voltages in the simulation of a transmitter and receiver consisting of a meander dipole antenna. Numerical results from the simulation of a 2×2 Tx/Rx system in an indoor channel also showed very good accuracy and efficiency when compared with a full wave simulation.
Chapter 5

Conclusions

5.1 Summary

This thesis focuses on the characterization of antenna-channel interactions using the FDTD method. Given the extensive computational resources that would be required to model complex antennas along with the channel in the FDTD method, the development of a more efficient approach to incorporate the EM behaviour of antennas into channel simulations became a necessity. An analysis of previous FDTD based schemes for modeling antennas revealed inaccuracies in their performances even for small fine-coarse mesh interpolations. The inaccuracies arising from a coarse representation of the antenna in simulations due to distortion of the antenna terminals and geometry can be readily understood. The approach that has been considered in this work is to develop macromodels of antennas and thus completely eliminate the need to include the antenna in the simulations with the environment.

A systematic approach for the time domain macromodeling of general antenna structures and their subsequent simulation with the environment was first developed. The antenna’s radiated fields in free-space were used to represent its transmitting properties while its impulse responses were used to calculate fields scattered by it via convolution.
By exploiting the scattering properties of minimum-scattering antennas, a simplified reciprocity-based macromodeling algorithm was then developed for simulating MSAs with the environment. In this scheme a complete formulation for calculating the voltage induced or received at the antenna terminals is presented through the application of the reciprocity theorem for antennas which relates the receiving properties of an antenna to its transmitting properties. Both, the generic and reciprocity-based macromodeling schemes, were validated through 2D-FDTD simulations for common antenna geometries and the interaction of the antenna was shown to be accurately captured through an iterative process.

The reciprocity theorem was further extended and combined with the generic macromodeling scheme to effectively model antennas at the receiving end to enable calculation of received signals at its terminals while simultaneously accounting for its scattering or radiating properties. This would not be possible if a coarse model of the antenna was used in the simulation since the distortion of the antenna terminals in addition to the distortion of its geometrical features would greatly affect the receiving properties of the antenna. This was demonstrated through a simulation consisting of a meander dipole antenna transmitter and receiver, in which the received voltage computed using the DG-FDTD scheme was shown to be extremely inaccurate. A more realistic simulation was performed in which an array of transmitters and receivers were placed in a indoor channel modeled as a long hallway. The transmitter and receiver can be viewed as a 2×2 MIMO system. In addition to very good agreement between the macromodel and full wave simulations in the time domain results, the macromodel simulation was also able to accurately compute the channel impulse response parameters.

The two novel FDTD-based antenna macromodeling schemes developed in this thesis provide a complete tool for modeling antenna-channel interactions due to their capability to account for the transmitting, scattering and receiving properties of antennas. The schemes do not involve any sub-gridding in its implementation and hence are not sus-
ceptible to the limits of sub-gridding techniques such as the usability for arbitrary mesh aspect ratios and long term instability. Since the interaction of the antenna in both schemes is preserved through its radiated fields and impulse responses rather than its geometry, the antenna macromodel can be incorporated into coarser FDTD meshes for the efficient simulation of the antenna and its environment, with possible gains increasing in the case of small antenna systems.

5.2 Future Work

Although a complete formulation highlighting the key concepts required for macromodeling antennas and incorporating them into channel simulations has been presented, an implementation of the formulation in the 3D domain remains. This is essential in order to enable characterization of real-world antenna-channel interactions. The computational time and resources required, however, to simulate full 3D channel models in FDTD are quite extensive and not practical. A more efficient approach can be derived by coupling macromodels developed in FDTD with channel models solved by ray tracing. It is worth mentioning that previous work combining the FDTD method with ray tracing has been done \[30],\[31] where ray tracing has been exploited to study wide areas and FDTD has been used to study areas with complex discontinuities. However, the approaches have employed omnidirectional sources located in the ray tracing modules and are limited to one interaction between the FDTD and ray tracing modules. The ultimate goal and a practical application would be to develop a software test bench for the analysis of end-to-end communication systems that can account for antenna behaviour in the calculation of bit error rates (BER) and other parameters.
Appendix A

The Finite-Difference Time-Domain Algorithm

FDTD Fundamentals: Maxwell’s Equations

The FDTD algorithm is a numerical method for solving Maxwell’s equations in the time domain. Maxwell’s equations form the basis of electromagnetism. These equations relate the electric and magnetic fields, \( \vec{E} \) and \( \vec{H} \), to the electric and magnetic flux densities, \( \vec{D} \) and \( \vec{B} \), and the electric and magnetic sources, \( \vec{J} \) and \( \vec{M} \) as follows:

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M} \tag{A.1}
\]

\[
\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \tag{A.2}
\]

\[
\nabla \cdot \vec{D} = \rho \tag{A.3}
\]

\[
\nabla \cdot \vec{B} = 0 \tag{A.4}
\]

where \( \rho \) is the total charge density. Equations (A.1) and (A.2) are termed the Faraday’s Law and the Ampere’s Law respectively. Equations (A.3) and (A.4) are called the Gauss’s Law. In addition to the above four equations, there exist the constitutive relations which
relate the field quantities to the properties of the medium:

\[
\vec{D} = \varepsilon \vec{E} \quad (A.5)
\]
\[
\vec{B} = \mu \vec{H} \quad (A.6)
\]
\[
\vec{J} = \sigma \vec{E} \quad (A.7)
\]
\[
\vec{M} = \sigma^* \vec{H} \quad (A.8)
\]

where, \( \varepsilon = \varepsilon_r \varepsilon_0 \) (\( \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \)) is the electric permittivity of the medium, \( \mu = \mu_r \mu_0 \) (\( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)) is the magnetic permeability of the medium, \( \sigma \) is the electric conductivity of the medium and \( \sigma^* \) is the magnetic resistivity of the medium. In free-space \( \varepsilon_r = \mu_r = 1 \) and \( \sigma = \sigma^* = 0 \).

FDTD Equations and the Yee Algorithm

The formulation of the FDTD method is based on the two Maxwell’s curl equations (A.14) and (A.2). Assuming an isotropic medium with \( \rho = 0 \), the above two curl equations can be expressed in the Cartesian coordinate system with \( E_x, E_y, E_z \) and \( H_x, H_y, H_z \) as the field components in the x, y and z directions respectively as follows:

\[
\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon} \left[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x \right] \\
\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left[ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y \right] \\
\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left[ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z \right] \quad (A.9)
\]
Appendix A. The Finite-Difference Time-Domain Algorithm

\[
\begin{align*}
\frac{\partial H_x}{\partial t} &= \frac{1}{\mu} \left[ \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma^* H_x \right] \\
\frac{\partial H_y}{\partial t} &= \frac{1}{\mu} \left[ \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma^* H_y \right] \\
\frac{\partial H_z}{\partial t} &= \frac{1}{\mu} \left[ \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma^* H_z \right]
\end{align*}
\] (A.10)

These equations can be transformed to their corresponding discrete forms by applying the centered finite-difference approximation on both the time and space first-order partial derivatives according to the following equations,

\[
\begin{align*}
\frac{\partial F^n(i, j, k)}{\partial x} &= \frac{F^n(i + \frac{1}{2}, j, k) - F^n(i - \frac{1}{2}, j, k)}{\Delta x} + O(\Delta x^2) \quad (A.11a) \\
\frac{\partial F^n(i, j, k)}{\partial t} &= \frac{F^{n+\frac{1}{2}}(i, j, k) - F^{n-\frac{1}{2}}(i, j, k)}{\Delta t} + O(\Delta t^2) \quad (A.11b)
\end{align*}
\]

where \(i, j, k\) represent the discrete spatial indices in the \(x, y, z\) directions respectively and \(n\) represents the temporal index. In order to apply the finite difference approximations, the six field components \(E_x, E_y, E_z, H_x, H_y, H_z\) can be placed according to the Yee cell [32] as shown in Figure [A.1]. In this arrangement, the electric and magnetic field components are dislocated in space by half a cell. This forms a single unit cell in the FDTD grid with dimensions \(\Delta x \times \Delta y \times \Delta z\). The entire grid can be formed by stacking these unit cells in the \(x, y\) and \(z\) directions. Based on the above arrangement of fields in the unit cell with the node \((i,j,k)\) defined as shown in the figure, and assuming \(\sigma^* = 0\), the finite difference approximations (discarding the error terms) to \(E_x\) and \(H_x\) in
Appendix A. The Finite-Difference Time-Domain Algorithm

Figure A.1: A Yee cell.

equations (A.9) and (A.10) are,

\[
E_x^{n+1}(i, j, k) = \left( 1 - \frac{\sigma \Delta t}{2 \varepsilon} \right) E_x^{n-1}(i, j, k) + \frac{\Delta t}{1 + \frac{\sigma \Delta t}{2 \varepsilon}} \left[ \frac{H_z^{n+\frac{1}{2}}(i, j, k) - H_z^{n+\frac{1}{2}}(i, j - 1, k)}{\Delta y} \right.
\]
\[
+ \frac{H_y^{n+\frac{1}{2}}(i, j, k - 1) - H_y^{n+\frac{1}{2}}(i, j, k)}{\Delta z} \left. \right] \tag{A.12}
\]

\[
H_z^{n+\frac{1}{2}}(i, j, k) = H_z^{n-\frac{1}{2}}(i, j, k)
\]
\[
+ \frac{\Delta t}{\mu} \left[ \frac{E_y^n(i, j, k + 1) - E_y^n(i, j, k)}{\Delta z} + \frac{E_z^n(i, j, k) - E_z^n(i, j + 1, k)}{\Delta y} \right] \tag{A.13}
\]

Similar equations can be derived for the remaining field components and can be found in [6]. The six discrete field equations form the basis of the FDTD method and are used in the Yee algorithm in a time marching fashion to update all the electric and magnetic field nodes in the grid. In the time-domain algorithm all fields are initially set to zero.
A source is first specified at particular cells in the FDTD grid. The magnetic fields are then updated using the finite difference equations at a time step of \((n + 1/2)\Delta t\) from their previous value at \((n - 1/2)\Delta t\). This is then followed by the update of the electric fields at \((n + 1)\Delta t\) based on their value at \(n\Delta t\). The update of the electric and magnetic fields alternately at \(n\Delta t\) and \((n + 1/2)\Delta t\) time steps respectively arises from the centered difference approximation used in deriving the field update equations. This process is repeated until required or until all field values in the grid have converged or decayed to zero. Since the error terms in the finite-difference approximations in Appendix A.11 approach zero as the square of the spatial and temporal increments, the time-stepping scheme is second order accurate in both space and time [6].

**Numerical Stability Conditions**

Like most numerical techniques the Yee algorithm is conditionally stable. The maximum time step \(\Delta t_{max}\) that can be chosen for stable operation is given by the Courant-Friedrichs-Lewy (CFL) condition,

\[
\Delta t_{max} = \frac{1}{c\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}
\]  

(A.14)

where, \(c\) is the maximum speed of light in the FDTD grid. A derivation of the above condition can be found in [6]. The cell sizes \(\Delta x\), \(\Delta y\) and \(\Delta z\) are chosen to be less than or equal to \(\frac{\lambda_{min}}{10}\), where \(\lambda_{min}\) is the smallest wavelength in the FDTD grid based on the maximum frequency contained in the source.

**Source Excitations in FDTD**

Source excitations in the FDTD method are usually done by applying a source function \(g(t)\) such as a Gaussian or modulated Gaussian pulse to particular cells in the FDTD grid.
The two most common methods of applying the source excitations are hard excitation and soft excitation. In the hard excitation scheme, the source function is directly enforced on a particular field component as follows,

\[ E(n\Delta t) = g(n\Delta t) \]  

(A.15)

The disadvantage of the hard source is that it acts as a 'hard' boundary and thus creates spurious retro-reflections. In the soft or transparent excitation scheme on the other hand, the source function is applied by means of superposition as follows,

\[ E(n\Delta t) = E(n\Delta t) + g(n\Delta t) \]  

(A.16)

Unlike the hard source, in the soft excitation scheme incoming waves are allowed to pass through the source without producing any reflections.

**Absorbing Boundary Conditions**

Since computational resources are limited, the FDTD mesh has to terminated with appropriate absorbing boundary conditions (ABCs) in order to prevent reflections of fields back into the computational space. Initially ABCs were realized using analytical techniques. One such ABC is the Mur’s ABC \[^{[33]}\] in which the tangential fields on the outer boundaries were made to obey the one-dimensional wave equation in the direction normal to the mesh wall. These were however outperformed by an alternate implementation of the ABCs, realized by terminating the outer boundary of the computational domain in an absorbing material medium. Theoretically, to have no reflections, the wave impedance of the absorbing medium has to be matched with that of the medium in the computational domain. This can be achieved by introducing a magnetic loss component, \( \sigma^* \), into the
medium and enforcing the condition,

$$\frac{\sigma}{\varepsilon} = \frac{\sigma^*}{\mu}$$  \hspace{1cm} (A.17)

However for such an absorber, matching holds only for waves impinging the absorber at normal incidence. This problem was treated by Berenger [34] who introduced the Perfectly Matched Layer (PML) ABCs, in which outgoing waves were attenuated using a lossy medium. This is realized in FDTD by adding a certain number of cells with artificially defined losses outside the computational domain. Various implementations of the PML medium were proposed in order to enhance its performance. Of these, the uniaxial PML (UPML) presents a realization that corresponds with Maxwell’s equations and has been one of the more popular choices. The UPML absorber is a uniaxial isotropic medium having permittivity and permeability tensors incorporating losses and of the form

$$\bar{\varepsilon} = \bar{\mu} = \begin{bmatrix}
    s_y s_z s_x^{-1} & 0 & 0 \\
    0 & s_x s_z s_y^{-1} & 0 \\
    0 & 0 & s_x s_y s_z^{-1}
\end{bmatrix}$$  \hspace{1cm} (A.18)

with a standard choice for the stretching factors $s_x$, $s_y$ and $s_z$ being

$$s_k = 1 + \frac{\sigma_k}{j\omega\varepsilon}, \hspace{0.5cm} k = x, y, z$$  \hspace{1cm} (A.19)

where $\sigma_k$ represents the artificially introduced losses. The PML losses are specified such that they gradually increase along the direction normal to the interface and outward in order to reduce spurious wave reflections at the PML surface. More detailed discussions on the various types of PML and their implementations as well as the appropriate choices of the PML conductivity profiles can be found in [6].
References


REFERENCES


