IS TWO A PROPERTY?

"Whatever is, is one...whatever are, are many."
—Bertrand Russell (1905)¹

Bill and Hillary live together in Washington. Bill is one human being. So is Hillary. But they are two humans; they are different humans. What is it for them to be two humans? It is for them to instantiate a property, namely, being two humans. This is the view that I aim to clarify, defend, and develop in this paper. In doing so, I reject the standard conception of property (and relation) and defend a broader conception that allows nonstandard properties like the one in question. The standard conception retains the long-standing bias against the many in favor of the one; the conception contains principles to the effect that, roughly, many things as such cannot instantiate a property. The broader conception results from rejecting those principles.

I begin by clarifying the notion of the many in section I. In section II, I formulate and argue against the crucial principles of the standard conception. In section III, I clarify the language used to attribute nonstandard properties to the many. Then I elaborate on the broader conception, with the aid of the resources available in the language, in section IV. Section V is an epilogue that places the conception in a historical perspective.

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Here is the contrast between what is one and what are many. Bill is one. He is some one thing, specifically, one human being. Likewise with Hillary. But Bill and Hillary are many things, namely, more than one thing. They are, specifically, two humans. Moreover, the two humans are many things only, not some one thing. By contrast, Bill is one thing only, not many things. He (or his body) has many parts: one heart, two eyes, four limbs, and the like. But this does not make Bill himself the many parts, any more than his having two parents makes him the two parents. Surely, the contrast does not rest on the fact that Bill and Hillary are both humans. Anything, any object, is one thing. It is irrelevant to this what kind of thing it is, whether it is an adult or a child, a human or a city, material or immaterial, spatiotemporal or not, concrete or abstract, and so on. Likewise, Bill and Hillary are two simply because they are not identical with each other. Whether they are similar or not, how closely related they are, and so on are immaterial to their being many.

Some might object that the contrast somewhat breaks down because there are complex objects, such as sets or classes, groups or collections, aggregates or mereological sums, or the so-called totalities or pluralities. They agree that such an object, too, is one thing, but object that it is many things as well. This is not correct. Consider, for example, the set {Bill, Hillary}, an object that in some sense comprehends both Bill and Hillary (but nothing else). The set is two, the objectors argue, because it is (namely, is identical with) the two humans (though it is not either of them). But this assumption does not hold. Bill and Hillary are two humans; by contrast, the set is not two humans. The set in a sense has two humans; it is related to them as its members. But this does not make the set itself two humans; it does so no more than Bill's having two Americans as parents makes Bill himself two Americans. Thus a complex object, too, is one thing only, not many things. How many members, elements, or parts it has is as irrelevant to this as is how many parents a human has.

1 One might object that one cannot get ‘(Bill, Hillary) is two humans’ from ‘Bill and Hillary are two humans’ by replacing ‘Bill and Hillary’ with ‘(Bill, Hillary)’. Although I think the sentences have the same predicate ‘to be two humans’, which takes different forms, we can bypass this issue by considering a contrast immune to the objection: Bill and Hillary, and Chelsea are three humans; (Bill, Hillary) and Chelsea are not three humans (they are only two things: one set and one human).

2 In Parts of Classes (Cambridge: Blackwell, 1991), David Lewis holds that the mereological sum of Bill and Hillary is in a sense identical with the two humans (so it is in a sense two humans). I have argued against this view in “Is Mereology Ontologically Innocent?” Philosophical Studies, xciii, 2 (February 1999): 141-60.
Some might respond by replacing, in effect, the talk of being two (or being many) with the talk of related properties that complex objects instantiate, such as being two-membered, a property instantiated by any set (or class or whatever has members) with exactly two members. One cannot identify this latter property with the former, they concede; being two-membered is instantiated by the set [Bill, Hillary], which it is not correct to say instantiates being two. Still, they argue, one can hold that being two-membered is the real predicative component of facts like the following:

(1) Bill and Hillary are two.

That is, on their view, the sentence ‘Bill and Hillary are two’ states the same fact that the sentence ‘[Bill, Hillary] is two-membered’ states, that is, the fact that:

(2) [Bill, Hillary] is two-membered.

but the former sentence is misleading about the nature of the fact because there is no property indicated by the apparent predicate ‘to be two’.

This view rests on an incorrect analysis of facts like (1). To see this, consider facts like the following:

(3) Bill is not identical with Hillary [and Bill is a thing and Hillary is a thing].

Bill and Hillary must be two different things, insofar as Bill is not identical with Hillary (while Bill exists as a thing and so does Hillary). So:

Thesis I: Fact (1) is a logical consequence of fact (3).

But just because Bill is not identical with Hillary (while both Bill and Hillary exist) does not mean:

\footnote{For an account of (natural) number based on this analysis, see Penelope Maddy, \textit{Realism in Mathematics} (New York: Oxford, 1990). For example, she identifies the number two as the property of being two-membered. Fregean accounts that identify numbers with a kind of objects (for example, [\(\emptyset\), [\(\emptyset\]}] can also be seen to be based on analyzing facts like (1) as facts like (2) that contain \textit{relational properties} characterized with reference to the objects (for example, being equinumerous with [\(\emptyset\), [\(\emptyset\]}]). For Frege’s account, which differs from the contemporary Fregean accounts in identifying numbers as extensions of concepts, see, for example, his \textit{Die Grundlagen der Arithmetik} (Breslau: Köhner, 1884), translated by J. L. Austin as \textit{The Foundations of Arithmetic} (New York: Blackwell, 1950). In \textit{A System of Logic} (New York: Harper, 1891, 8th edition), J. S. Mill identifies, for example, two as a property (having two parts) instantiated by an aggregate. This account, too, must rest on a similar analysis of facts like (1).}
(4) There is something (for example, a set) of which Bill is a member and Hillary is a member. Thus, fact (4) is not a logical consequence of fact (3). Because, however, fact (4) is a logical consequence of fact (2):

Thesis II: Fact (2) is not a logical consequence of fact (3).

Now, it follows from theses I and II that fact (1) cannot be reduced to fact (2).

Underlying this argument is the thought that fact (1) is a fact which concerns both Bill and Hillary but which concerns nothing else; Bill and Hillary are two (consequently, many) things simply because they are things not identical with each other. The fact that Bill is identical with himself does not concern the singleton [Bill], which is not Bill, because the fact concerns nothing but Bill. Likewise, fact (1) does not concern the set [Bill, Hillary], which is neither Bill nor Hillary, no matter how closely it might be related to both Bill and Hillary. It is the same with any complex object, be it the aggregate of the humans or the so-called plurality thereof. Thus, fact (1) cannot be analyzed as involving a complex object.

The proper analysis of facts like (1), I shall argue, is the one implicit in the above distinction between the one and the many. Fact (1) is composed of (a) Bill and Hillary, on the one hand, and (b) being two, on the other; its predicative component, being two, is a property instantiated by many things (for example, Bill and Hillary) but not by any one of them. Similarly, the fact that Bill and Hillary are many is formed by being many, a property instantiated by no object, simple or complex. Accepting this analysis requires a radical departure from the standard conception of property and relation.

II. BEING TWO AS A PLURAL PROPERTY

On the standard conception of property and relation, there is no property like being two. I shall now explain two related principles in

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5 One might object to this by identifying logical consequence with modal entailment, defined via necessity (namely, metaphysical necessity, as it is commonly called). On the orthodox view on the modal status of sets, fact (3) modally entails fact (4); see, for example, Kit Fine, “First-order Modal Theories: Sets,” Nous, xv (1981): 177-205. This, however, gives no relief for the analysis in question. The fact that Bill is a human and the fact that (Bill) has exactly one member that is a human modally entail each other, on the orthodox view, but the two facts cannot be identified, because the property of having exactly one member that is a human cannot be taken to be the predicative component of the former fact. Thus, it is necessary to draw a distinction among facts finer than the one available through modal entailment. My view is that logical consequence, among others, yields such a distinction. For more on this or related issues, see Fine, “Essence and Modality,” Philosophical Perspectives, viii (1994): 1-16; and Joseph Almog, “The What and the How,” this JOURNAL, lxxxviii, 5 (May 1991): 225-44.
the conception which rule out such properties, and argue that facts like (1) cannot be given a proper analysis without rejecting the principles.

II.1. The principle of singular instantiation. On the standard conception, a property cannot be instantiated by many things as such; there is no such property as being two, which would be instantiated by two things as such. For a better understanding of reasons for rejecting such properties, it is necessary to be clearer about the content of the phrase ‘as such’. The phrase is necessary because the conception accommodates some properties that are in a sense instantiated by many things: for example, being a human, which is in a sense instantiated by, for example, Bill and Hillary. What is the difference between the standard properties like this and those ruled out by the conception?

One might attempt to explain it as follows. Being a human, instantiated (in a sense) by Bill and Hillary, is instantiated by every one of them; being two, by contrast, is not instantiated by every one of those, by whom it is supposed to be instantiated. But this explanation does not get at the heart of the matter. Being one or more humans would be instantiated by any one of those who instantiate it; either of Bill and Hillary, who are one or more humans (specifically, two humans), is one or more humans (specifically, one human). But the standard conception rules out such a property as well. This is due to an important feature which it shares with, for example, being two and distinguishes them from the standard properties.

Being a human must occur more than once to be instantiated by Bill and Hillary—whether to form two different facts, such as the fact that Bill is a human and the fact that Hillary is a human, or as different components of one complex fact, such as the fact that:

Bill is a human [the first occurrence], and Hillary is a human [the second occurrence].

By contrast, being two can (or could) occur just once to be instantiated by many things; it should be taken to occur only once to be instantiated by the two humans and, thus, form fact (1). It is the same with being one or more humans; this property can (or could) occur just once to be instantiated by Bill and Hillary—though it can also occur two more times to be instantiated by Bill and Hillary, respectively.

Thus, the standard conception contains the following principle:

The principle of singular instantiation: there is no property that can occur just once to be instantiated by many things (that is, more than one thing).
II.2. The principle of singularity. The principle of singular instantiation is, I think, a consequence of an underlying principle that formulates the more general constraint that the standard conception imposes on relations as well as properties. To state this principle, it is useful to clarify some basics about objects, properties, and relations.

Bill, for example, is an object; being a human, a property; living in, a relation. There is a categorical difference between objects and properties or relations. Very roughly, properties or relations are predicative; objects are not. Being a human is predicated of Bill (specifically, Bill instantiates the property); Bill is a human. Similarly, living in is predicated of Bill and Washington, so to speak, in the given order (specifically, Bill bears the relation to Washington); Bill lives in Washington. Bill, by contrast, cannot be predicated at all. He occurs in the fact that Bill is Bill, but not as a predicable; he occurs in it to bear the relation of identity, indicated by ‘is’ in the sentence ‘Bill is Bill’, to himself.7

Properties are distinguished from relations by the number of argument places. A property has one argument place; a relation, two or more. Being a human has only one argument place; it can be filled in with Bill to yield the fact that Bill is a human. Living in has two argument places; they can be filled in with Bill (the first) and Washington (the second), respectively, to yield the fact that Bill lives in Washington.

The distinction between properties and relations, we can see, is not as fundamental as the one between objects and properties or relations. The difference between a property and a two-place relation is no greater than the one between the relation and a three-place relation, such as the one indicated by the predicate ‘to give...to’. So I shall henceforth use the term ‘relation’ in a broader sense to apply it to properties as well: properties are one-place relations, relations with one argument place.

Now, consider what can fill in argument places of relations. The argument place of being a human can be filled in with (in short, admits of) Bill because he instantiates the property. In general, whatever instantiates a property can fill in its argument place. But the converse does not hold. The argument place of a property admits of

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6 I use ‘indicate’ widely for the relation between any linguistic expression and its metaphysical counterpart, and reserve ‘refer’ for the more specific relation applicable to nonpredicable expressions.

7 There is also the property of being Bill (composed of identity and Bill), but Bill differs from the property.
whatever instantiates its complement as well, because a property and its complement have the same kind of argument places. London, for example, can fill in the argument place of being a human, whose complement, the property of not being a human, is instantiated by the city.

If so, what cannot fill in the argument place of a relation? The argument place of being a human does not admit of many things as such. It (in a sense) admits of Bill and Hillary, but for this to happen, the property must occur twice; its argument place admits of only one thing (that is, Bill) as the property occurs just once. And it is the same with relations like living in. The first argument place of this relation (in a sense) admits of Bill and Hillary to yield the fact that:

Bill and Hillary live in Washington.

But the relation must occur twice for this to happen, as can be seen in the following analysis of the fact:

Bill lives in Washington, and Hillary lives in Washington.

Now, on the standard conception, this is the general situation with any argument place of any relation whatsoever; that is:

The principle of singularity: an argument place of a relation does not admit of many things as the relation occurs just once.

We can see that this explains why, on the conception, the principle of singular instantiation holds; a property cannot occur just once to be instantiated by many things unless its argument place admits of them as the property occurs just once.

II.3. The ground for plural relations. I shall now argue that the principle of singularity does not hold. Given the argument, it is straightforward to see that the principle of singular instantiation does not hold, either.

Call an argument place of a relation plural if it admits of many objects as such; singular otherwise. Accordingly, call a relation singular if all of its argument places are singular; plural otherwise. On the principle of singularity, there is no plural relation. I argue that there are plural relations because there are facts like the following:

Bill and Hillary are two. (Fact (1))
Bill and Hillary are two humans.
Bill and Hillary live together.
Russell and Whitehead coauthor a book.
Some humans love only one another.
The coauthors of Principia Mathematica (in short, PM) cooperate.
Bill is not one of the coauthors of PM.

The proper analysis of such facts, I claim, involve plural properties like:

being two
being two humans
living together
cooperating
loving only one another

and two-place relations like:

coauthoring
being one of

whose first and second argument places, respectively, are plural.

My argument for the claim runs as follows. (A) Being two, for example, is the predicative component of fact (1) (so it is a relation), indicated by the predicate 'to be two', that it shares with facts like the following:

(5a) Dolly and Washington are two.
(5b) Washington and Neptune are two.

(B) And it has only one argument place (so it is a property), which admits of Bill and Hillary to yield fact (1). (C) Finally, the argument place is a plural one because the property occurs only once in, for example, fact (1), in which the argument place is filled in with many things, for example, Bill and Hillary.

One might essentially agree with (A) but object that the predicative component of fact (1) is the singular two-place relation indicated by the phrase ‘... and —— are two’. This view might seem plausible if one considers only facts like (1) or (5a), but it fails to square with facts like the following:

(6a) Bill, Hillary, and Washington are not two.
(6b) Bill, Hillary, Washington, and Neptune are not two.

These facts cannot be seen to contain a singular two-place relation. Their common predicative component, the relation of not being two, must be taken to have only one argument place that admits of Bill, Hillary, and Washington (as such) to yield fact (6a); and Bill, Hillary, Washington, and Neptune (as such) to yield fact (6b). That is, not

* For an account of number that rests on this view, see John Bigelow, *The Reality of Numbers* (New York: Oxford, 1988), pp. 49-54. (I have added 'essentially' because he might object that the phrase, strictly speaking, does not indicate being two.)
being two is a plural property. Now, because this is the complement
of the predicative component of fact (1), being two, this must also
be a plural property.

Some might concede that being two is not a two-place relation,
one with two argument places built into it, but object that it is a singu-
lar relation of a special kind, one that Henry Leonard and Nelson
Goodman⁹ call a multigrade relation. On this view, being two (or its
complement) is a relation that can occur in one fact with, for exam-
ple, two argument places filled in while occurring in another fact
with another number of argument places filled in. But this view
meets difficulty with facts like the following:

(7) There are some things that are not two.

which shares the complement of being two with facts like (6a). How
many, and what kind of, argument places does the relation occur
with in fact (7)? The right answer to this question is: it occurs with
only one argument place, a plural one. Then there is no reason to
take the relation to occur with other, singular argument places in,
for example, fact (6a).

How else can one object to the above argument for plural prop-
ties? It is hard to reject (C) while granting (A)-(B). One cannot
argue that being two occurs twice in fact (1), which cannot be analyzed
as a conjunctive fact. Thus, to reject (C), one would have to argue
that Bill and Hillary are in a sense one complex object, but this,
we have seen, is not a correct view. Some might object that there is no
fact corresponding to the construction ‘Bill and Hillary are two’, be-
cause this is not grammatically correct. For, on their view, ‘two’ used
as an adjective must be complemented with nouns (or noun
phrases). But this view does not help much in defending the princi-
ple of singularity. One can apply the above argument to, for exam-
ple, the fact that:

(8) Bill and Hillary are two humans.

and conclude that being two humans is a plural property. Because
they cannot challenge this argument with the grammatical objec-
tion, they might attempt to do so by defending alternative analyses
of fact (8). But the analyses they might propose can be seen to result

⁹ See their “The Calculus of Individuals and Its Uses,” Journal of Symbolic Logic, v,
2 (June 1940): 45-55. The linguistic counterparts of multigrade relations are
called variably polyadic predicates. For more discussions of the views examined in
this paragraph and the previous, see my “Numbers and Relations,” Eirenitos, xlix, 1
from different ways of analyzing fact (1), because fact (8) can be analyzed as containing fact (1) as a conjunct as follows:

Bill and Hillary *are two*, and they are humans.

Can one then defend a way to analyze facts like (1) or (8) without plural relations? One might hold that such facts contain only singular properties instantiated by complex objects. On her view, fact (1) can be analyzed as follows:

(2) [Bill, Hillary] is two-membered.

which results from the set [Bill, Hillary] filling in the argument place of the singular property of being two-membered. But such an analysis cannot be right, we have seen, because fact (1) concerns nothing (that is, no object) but Bill or Hillary.

To avoid this problem, one might attempt to analyze such facts in terms of properties rather than complex objects. They, one might argue, are second-order facts like the one composed of (i) the singular (first-order) property of *being identical with Bill or Hillary* and (ii) a second-order property that somehow corresponds to ‘two’, one that is instantiated by any singular property instantiated by two things.\(^\text{10}\)

For one can argue (correctly, on my view) that this fact does not logically entail the existence of any *object* except Bill or Hillary; the existence of a *property*, which it logically entails, does not logically entail the existence of an object that one might use as its representative.

Because properties are not determined by their instances, however, the scheme of second-order analysis meets difficulties in preserving logical relations among facts like the following:

(9) Russell and Whitehead cooperate.
(10) Russell and Whitehead are the authors of PM.
(11) The authors of PM cooperate.

Consider the second-order facts that the scheme renders them as:

(9a) cooperate (is identical with Russell or Whitehead)\(^\text{11}\)
(10a) ∀a[a is identical with Russell or Whitehead ↔ a is an author of PM]
(11a) cooperate (is an author of PM)

\(^{10}\) This is the kind of analysis that would be proposed by, for example, Michael Dummett, who argues that “a plural noun-phrase...under a correct analysis...is seen to figure predicatively”—*Frege: Philosophy of Mathematics* (Cambridge: Harvard, 1991), p. 93.

\(^{11}\) The sentence used here is a second-order predication whose predicate is ‘cooperate’, whose argument place is filled in with the predicate ‘is identical with Russell or Whitehead’.
Although the properties indicated by the phrases ‘is identical with Russell or Whitehead’ and ‘is an author of PM’ are co-extensional (so, fact (10a) holds), they are not the same property; to be identical with Russell or Whitehead is not the same as to be an author of PM. If so, one of the properties may instantiate a second-order property—for example, the one indicated by ‘cooperate’—that the other does not. Thus, (11a) is not a logical consequence of (9a) and (10a). But, clearly, (11) is a logical consequence of (9) and (10). To avoid this problem, one might analyze facts like (9) not as second-order predications but as second-order existential generalizations like:

(9b) \(\exists X [\forall \alpha (X(\alpha) \leftrightarrow \alpha \text{ is identical with Russell or Whitehead}) \land \text{cooperate}(X)]\)\(^{14}\)

But this modified scheme fails other logical relations. Consider the following:

(12) Russell and Whitehead are philosophers who coauthor PM.
(13) There are some philosophers that cooperate and coauthor PM.

The scheme renders fact (12) as follows:

(12a) \(\exists X [\forall \alpha (X(\alpha) \leftrightarrow \alpha \text{ is identical with Russell or Whitehead}) \land (\forall \alpha (X(\alpha) \rightarrow \alpha \text{ is a philosopher}) \land \text{coauthor}(X, \text{PM})]\)

This fact and fact (9b) do not logically entail the second-order counterpart of fact (13):

\(^{14}\) One cannot defend the scheme by adopting the extensional conception of relation (which is implausible given properties like the above-mentioned) that identifies co-extensional relations. (11a) is not a logical consequence of (9a) and (10a) unless the “identity” between the first-order properties is a logical consequence of their extensional equivalence. But it is one thing to hold the extensional conception; quite another to hold (more implausibly) that its truth relies on logic alone.

\(^{15}\) One might analyze (9) as the fact that: Russell cooperates-with Whitehead, and Whitehead cooperates-with Russell. This analysis is not applicable to fact (11), which has the same predicative component. One can avoid this problem by analyzing facts like (9) and (11) in terms of the second property indicated by ‘\(\forall \alpha [...\alpha... \rightarrow \exists \beta (\alpha \# \beta \land \alpha \text{ cooperates-with } \beta)\)’, whose argument place (indicated by ‘...\alpha...’) admits of a first-order property (for example, being identical with Russell or Whitehead). But this analysis fails because pairwise cooperation of, for example, three humans does not logically entail their cooperation: any two of them may cooperate (for example, to write a book between them while excluding the other, or to kill the other) while not all three of them cooperate.

\(^{16}\) This is the key idea in Russell’s “No-Class Theory,” on which the talk of sets (or “classes”) is reducible to the talk of properties (or “propositional functions”); see, for example, his Introduction to Mathematical Philosophy (London: Allen and Unwin, 1919), p. 187ff.
But, clearly, facts (9) and (12) logically entail fact (13).15

Now, some people might concede that fact (9) contains a plural property (and thus, reject the principle of singularity) but still object that fact (1) should be analyzed as a second-order fact whose predicative component is a special kind of second-order property, the one that corresponds to the quantifier phrase ‘∃α∃β(α≠β ∧ ∀γ[...γ... ↔ (γ=α ∨ γ=β)])’. On their view, the phrase ‘Russell and Whitehead’ functions ambiguously in ‘Russell and Whitehead are two’ and ‘Russell and Whithead cooperate’; and the predicative components of the facts stated by them cannot combine to yield a complex property, because they do not have the same kind of argument places. But there is nothing wrong with contracting the two sentences as if the phrase functions unambiguously therein, and the resulting sentence states a logical consequence of the two facts:

Russell and Whitehead are two and cooperate.

which logically entails the fact that:

There are some things that are two and cooperate.

Being two, like cooperating, must be seen to occur as a plural property in this fact—as a component of the complex property: being two and cooperating. Then facts like (1), too, must be analyzed in terms of the plural property given their connection to the more complex facts like the above.

This completes my argument against the principle of singularity. It should be clear from the argument that the principle of singular instantiation must also be rejected. Not every property that violates the former principle violates the latter principle; being one has an argument place that admits of, for example, Bill and Hillary as such, but is not instantiated by them as such. But the properties mentioned in the argument are counterexamples to the latter principle as well. Be-

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15 One might consider analyzing, for example, fact (9) as follows:

∃X[∀α(X(α) ↔ α is identical with Russell or Whitehead) ∧ ∀Y[∀α(X(α) ↔ Y(α) → cooperate(Y))]]

This relies on the assumption that the property indicated by ‘cooperate’ is instantiated by any property co-extensional with a property that instantiates it. But one cannot assume this without analyzing the second-order property. For more on second-order treatments, see subsection III.2.C below.
ing two, for example, is instantiated by any two things as such; its complement, by any things more than two.

Notice also that this last point establishes that the argument place of being two admits of any two or more things as such—no matter how many they are, whether they are finitely many or infinitely so. Moreover, the argument place, like that of being one, admits of any one thing—for example, Bill—as well. For any one thing instantiates the property of not being two because there are facts like the following:

Bill is not two [in particular, not two humans].
Bill is not two, but only one [namely, one human].

III. The Language and Logic of Plurals

In the next section, I shall elaborate on the nature of plural relations and confirm the intuitions adduced against alternative analyses of facts like (1), such as the intuition that fact (1) is a logical consequence of fact (3). I prepare for the task here by clarifying the syntax, semantics, and logic of the language I shall use for that purpose. The language contains plural, as well as singular, constructions, which allows one to use it to refer to many things as such and indicate and analyze plural, as well as singular, relations. We are familiar with the language. It is English (or its relevant fragment).

III.1. Informal Syntax and Semantics. English has: singular terms, such as the proper name ‘Bill’, the definite description ‘the author of Academica’, or the pronoun ‘it’; singular predicates, predicates all of whose argument places are singular (namely, they admit of only a singular term), such as ‘to be a human’; and singular quantifiers, such as ‘something’ (or ‘There is something that’). In addition, it has: plural terms, such as the conjunctive plural term ‘Bill and Hillary’, the plural definite description ‘the authors of PM’, or the plural pronoun ‘they’; plural predicates, predicates some of whose argument places are plural (that is, they admit of a plural term\(^\text{18}\)), such as one-place predicates like ‘to be two humans’ or ‘to cooperate’, or two-

\(^{18}\) Infinitely many things (for example, the natural numbers) cannot (as such) be indicated by a plural term resulting from joining their names using ‘and’, but the absence of such a term no more makes the things unsuitable for the plural property than the absence of a term referring to a pebble makes the pebble unsuitable for a singular property.

\(^{19}\) For elaborations of discussions in this section, see my “The Language and Logic of Plurals,” Synthese (forthcoming).

\(^{18}\) Plural argument places may admit of a singular term as well; recall, for example, ‘to be two’. (One might distinguish them further into neutral and exclusively plural ones.) Most English predicates, on my view, have such an argument place; Cicero writes Academica and ‘Russell and Whitehead write PM’ have the same predicate.
place predicates like ‘to be one of’, ‘to write’, or ‘to carry’; and *plural quantifiers*, such as ‘some things’ (or ‘There are some things that’). Thus, the language contains sentences like ‘Bill and Hillary are two humans’, ‘Russell is one of the authors of PM’, and ‘There are some things that are two humans, and they cooperate and write a book’, as well as sentences like ‘Bill is a human’, ‘Cicero is the author of *Academica*’, and ‘There is something that is a human, and he writes a book’.

Plural terms must be sharply distinguished from (first-order) predicates. These are *predicable* expressions that can combine with non-predicable expressions to form a sentence. The predicate ‘to be a human’ combines with the singular term ‘Bill’ to yield the sentence ‘Bill is a human’. By contrast, plural terms, like singular terms, are nonpredicable expressions. The plural term ‘Bill and Hillary’ cannot combine with ‘Bill’ to yield a sentence, just as the singular term ‘Bill’ cannot combine with ‘Bill’ to yield a sentence. Nonpredicable expressions can constitute a sentence—for example, ‘Bill is one of Bill and Hillary’ or ‘Bill is Bill’—only by combining with predicables—expressions—for example, ‘to be one of’ or ‘to be’.

Accordingly, different kinds of semantic relations are appropriate for plural terms, on the one hand, and predicates, on the other. The relation of *reference* is appropriate for nonpredicable expressions; *being true of*, for predicables (of one argument place). The predicate ‘to be a human’ (or ‘to be one of Bill and Hillary’) is true of Bill, but does not refer to him; the singular term ‘Bill’, by contrast, refers to Bill, but is not true of him. How about plural terms? They are referential expressions. The plural term ‘Bill and Hillary’, for example, is not true of anything (or any things). It refers to some things, namely, Bill and Hillary (as such). This does not mean that the plural term refers to Bill and also to Hillary; it refers to neither of them. A typical plural term refers to some things without referring to any one of them.

We can characterize the truth of plural, as well as singular, predications in terms of the semantic functions of terms and predicates.

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19 There is another semantic function of predicables: they indicate relations. We need to consider this function to characterize the truth of sentences with, for example, second-order predicates (except first-order quantifiers), but the fragment of English here in question contains no such predicate.

20 ‘To refer to’ as used here is a plural predicate, which indicates a plural relation; likewise with ‘to be true of’ as used below to be applicable to plural predicates.

21 This qualification is necessary because some plural terms refer to only one thing. ‘Cicero and Tully’ is a plural term, but refers to only one thing: Cicero, that is, Cicero and Tully, who are not many.
The singular predication ‘Bill is a human’ is true if its predicate is true of Bill, to whom the term ‘Bill’ refers. To give a similar account of plural predications like ‘Bill and Hillary are two’, we need to consider the semantics of plural predicates like ‘to be two’. One-place plural predicates can be true of some things: the predicate ‘to be two humans’ is true of any two humans—for example, Bill and Hillary; ‘to be two’, any two different things—for example, Washington and Dolly; ‘to coauthor a book’, any writers that write a book together—for example, Russell and Whitehead; ‘to be one’, any one thing—for example, Cicero. Now, the plural predication ‘Bill and Hillary are two’ is true if its predicate ‘to be two’ is true of Bill and Hillary, to whom the term ‘Bill and Hillary’ refers. We can extend this account to define the satisfaction and truth of sentences in the fragment of English in question, using plural constructions in the metalanguage, but it is not necessary to do so for the present purpose.

III.2. Regimentation and logic. To give a proper treatment of logical relations among sentences in the fragment of English that contains plural constructions, it is useful to regiment the fragment. Regimenting facilitates formulation of logical relations among sentences in it by imposing austere syntax. Let me call the regimented fragment of English the language of plurals, and the logic that pertains to sentences in the language (and, thus, their English counterparts) the logic of plurals. The language extends elementary language, the regimented language developed in tandem with elementary logic, by including (counterparts of) plural constructions; accordingly, the logic of plurals extends elementary logic to do justice to logical relations involving plural constructions.

III.2.A. The language of plurals. The primitive expressions of the language of plurals are:

I. Primitive Terms

   Singular Constants: for example, ‘Bill’ (‘b’), ‘Hillary’ (‘h’)

   Singular Variables: for example, ‘a’, ‘b’

   Plural Variables: for example, ‘a’s’, ‘b’s’

II. Term Connective: ‘©’ (‘AND’)

III. Predicates

   Singular Predicates

      logical: ‘=’ (‘is-identical-with’)

      nonlogical: for example, ‘is-a-human’ (‘H’), ‘admires’ (‘A’), ‘is-a-member-of’ (‘ε’)

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22 Plural variables correspond to plural pronouns as used anaphorically in sentences like ‘There are some humans, and they cooperate’. I put ‘©’ to make the variables look like nouns of English in plural forms, but the variables are simple symbols.
Plural Predicates

logical: 'I' ('IS-ONE-OF')
nonlogical: for example, 'COOPERATE', 'WRITE', 'SING-TOGETHER'

IV. Sentential Connectives: ¬ ('it-is-not-the-case-that'), ∧ ('and')
V. Quantifiers

The singular existential quantifier '∃' ('there-is-something...such-that')
The plural existential quantifier 'Σ' ('there-are-some-things...such-that')

The language officially contains only the existential quantifiers and two sentential connectives. But the universal quantifiers '∀' ('anything...is-such-that') and '∀' ('Any-things...are-such-that') and other sentential connectives that I shall use can be introduced via the usual definitions.

It is not necessary to elaborate on the rules of formation except for some clarifications. First, the plural argument places of predicates admit of any singular, as well as plural, term. Thus 'Bill IS-ONE-OF Bill' is grammatically correct, though the second argument place of its predicate is plural. Second, the term connective '∈' accepts any two (occurrences of) terms to form a complex plural term. Thus '[Cicero∈Tully]' and '[Cicero∈Cicero]' are plural terms. Third, the first argument place of the plural (or singular) quantifier, marked by '...', admits of only a plural (or singular) variable.

In addition to the predicate '∈', familiar from elementary logic, the language of plurals has another logical predicate: 'IS-ONE-OF'. This predicate, as a regimentation of the predicate 'to be one of' in English, counts as a logical predicate for the same reason as the predicate '∈' does so: some logical relations rely on the specific nature of the predicate (or its English counterpart). For example, 'Cicero is one of Cicero and Caesar' is a logical truth; 'Cicero is one of Tully and Caesar' and 'Cicero is one of the Roman philosophers' are logically equivalent with 'Cicero is Tully or Caesar' and 'Cicero is a Roman philosopher', respectively. The logical relations can be lost if the predicate is replaced with another.

---

25 Its first argument place is singular. One might object that the English predicate accepts only a plural term in its second argument place. If so, we can define a new predicate 'is-one-of' with the old predicate:

s is-one-of t =_V ∀a[s is one of (t AND a)], where t is any term

and introduce the counterpart of the new predicate as a primitive in the regimented language.
Given the predicate φ, we can define a plural predicate, whose ith argument place is plural, on the basis of a predicate φ whose ith argument place is singular as follows:

\[ φ(\xi_1, \ldots, \xi_n, a_1, a_2, \ldots, a_l) \equiv \forall \beta \forall \alpha (β(\beta, \alpha) \rightarrow φ(\xi_1, \ldots, \xi_n, \beta, \ldots, \xi_l)) \]

where \(i = 1, \ldots, n\) are terms appropriate for the relevant argument places of φ and β occurs free in none of φ and \(\alpha, \ldots, \alpha_l\).

Call the predicate \(φ^{\text{PL}}\) (or simply \(φ\)) if φ is one-place the (\(i\)th) plural expansion of the base predicate φ. Sentences like ‘Bill and Hillary are humans’ and ‘They are humans’ can be seen to be formed by the plural expansion ‘-s-a-human’ (in symbols, \('H\)') of the singular predicate ‘is-a-human’. We can then explain the logical equivalence between, for example, the plural predication ‘Bill and Hillary are humans’ and the sentenceal conjunction ‘Bill is a human, and Hillary is a human’ as resulting from the definition of the plural expansion; the plural expansion is true of any things (for example, Bill and Hillary) each of which the singular base is true of.

Now, plural definite descriptions like ‘the humans’ (in symbols, \('\text{the people}\)\(H(\text{the people})\)’) can be seen to result from applying a description operator to a plural expansion. This description, for example, can be defined in contexts in which it occurs primarily as follows:

\[ \Phi(\text{the people}H(\text{the people})) = \forall \forall \forall (\Phi(\beta, \alpha) \rightarrow H(\beta)) \land \Phi(\alpha) \]

III.2.B. The logic of plurals. I shall now present logical truths in the language of plurals in an axiomatic way. To do so, it is convenient to use schematic letters for different kinds of expressions (or sets thereof) in the language: ‘σ’ for singular terms; ‘τ’ and ‘υ’ for any terms; ‘σ’ and ‘υ’ for singular and plural variables, respectively; ‘ϕ’ and ‘φ’ for sentences; and ‘τ’ for sets of sentences. (They are all used with or without numerical subscripts.) Say that φ is a plural universal generalization of ϕ if and only if either φ = ϕ or φ = \(\forall \tau_1 \forall \tau_2 \forall \tau_3 \tau_4 \).

\[ \Phi(\text{the people}H(\text{the people})) \]

is used for sentence φ in which term τ occurs primarily. There is no other kind of plural definite description—for example, ‘the uncles of PM’ or ‘the things such that something is a human if, and only if, it is one of them’ (which the humans can be taken to abbreviate). Such descriptions, too, can be paraphrased onto the language of plurals, though somewhat differently. ‘The cousins of PM cooperative’ can be paraphrased onto ‘There are some things that are cousins of PM, and they cooperate’.

Although I shall also talk of other logical relations (for example, logical implications), such talks are reducible to talks of logical truth in the usual way.
...$Q_u\phi^\top$, where $Q_1$, $Q_2$, ..., $Q_n$ are the singular or plural universal quantifiers whose first argument places admit of the respective variables $u_1, u_2, ..., u_n$. Then the axioms of the system are the plural universal generalizations of instances of the following schemas:

**Group A**

I. Truth-functional tautologies
   II. $\phi(s) \rightarrow \exists v \psi(v)$, where $s$ is substitutable for $v$ in $\phi(v)$
   III. $\forall v(\phi(v) \rightarrow \psi(v)) \rightarrow [\forall v \psi(v) \rightarrow \forall v \psi(v)]$
   IV. $\phi \rightarrow \forall v \phi$, where $v$ does not occur free in $\phi$
   V. $s^\approx s_1 \rightarrow [\phi \rightarrow \phi_1]$, where $\phi$ is an atomic sentence and $\phi_1$ is obtained from $\phi$ by replacing $s$ in some of its occurrences by $s_1$

**Group B**

VI. $\phi(t) \rightarrow \Sigma w \phi(w)$, where $t$ is substitutable for $w$ in $\phi(w)$
    VII. $\Pi w[\phi(w) \rightarrow \psi(w)] \rightarrow [\Pi w \phi(w) \rightarrow \Pi w \psi(w)]$
    VIII. $\phi \rightarrow \Pi w \phi$, where $w$ does not occur free in $\phi$

**Group C**

IX. $H(s, [t@u]) \leftrightarrow [H(s, t) \lor H(s, u)]$
    X. $H(s, s_1) \leftrightarrow s^\approx s_1$
    XI. $\exists v H(v, u)$

**Group D**

XII. $\forall v[H(v, t) \leftrightarrow H(v, u)] \rightarrow [\phi \rightarrow \phi_1]$, where $v$ does not occur free in $t$ or $u$, $\phi$ is an atomic sentence, and $\phi_1$ is obtained from $\phi$ by replacing $t$ in some of its occurrences by $u$
    XIII. $\exists v \psi(v) \rightarrow \Sigma w \forall v [H(v, w) \leftrightarrow \phi(v)]$, where $w$ does not occur free in $\phi(v)$

The schemas of **Group A** are familiar from elementary logic. Those of **Group B** are plural counterparts of schemas II-IV. **Group C** contains schemas that concern mostly the logic of the predicate ‘H’. The rest are contained in **Group D**.

We can do with one rule of inference, modus ponens: from $\phi$ and $[\phi \rightarrow \psi]^\top$, we may infer $\psi$. There are logical truths of the language we cannot obtain by applying the rule repeatedly to the axioms. How can we then distinguish such logical truths from its sentences that are not logically true? I do not give a full answer to this question

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28 Recall that the universal quantifiers are defined as follows:

$\forall v \phi(v) \equiv_{uf} \exists v \sim \phi(v); \Pi w \phi(w) \equiv_{uf} \Sigma w \sim \phi(w)$

27 The definitions of ‘$t$ is substitutable for $u$ in $\phi(u)$’ and ‘$t$ occurs free in $\phi$’ (where $t$ and $u$ are variables) are straightforward.

29 They are taken from Herbert Enderton, *A Mathematical Introduction to Logic* (New York: Academic, 1972), pp. 104ff. (The schema $s^\approx s$ is derivable given $\forall$ and $\exists\forall$.)
here, because the logic of the language cannot be given a complete
axiomatization.20 For present purposes, it suffices to give a partial
answer: the logic is a conservative extension of elementary logic, that is:

Conservation principle: a sentence \( \phi \) that belongs to an elementary
language (or the singular fragment of the language of plurals) is logi-
cally true if and only if it is logically true on elementary logic.

Thus, sentences like

\[
\forall \gamma (A(\varepsilon, \gamma) \to \gamma = 0) \\
\wedge \forall \gamma (A(\varepsilon, \gamma) \to \gamma = 0) \\
\to \exists \alpha [\varepsilon \lnot \varepsilon \land \gamma = 0]
\]

are not logically true even in the language of plurals; they are not so on elementary logic.

III.2.C. Remarks on the logic. The conservation principle helps to ar-
gue against the prevalent view that to assert plural quantifications
like the following, known as the Geach-Kaplan sentence, is to com-
mit one to the existence of sets (or other complex objects):20

\[
\exists \alpha \forall \beta [(\beta \in \alpha_\varepsilon \land C(\beta) \land \forall \gamma (A(\beta, \gamma) \to \beta \neq \gamma \land \gamma \epsilon \alpha)].
\]

The view rests on paraphrasing such sentences into singular quanti-
fications like the following:

\[
\exists \alpha (\exists \beta \exists \varepsilon \in \beta \land \forall \beta [\beta \epsilon \varepsilon \land C(\beta) \land \forall \gamma (A(\beta, \gamma) \to \beta \neq \gamma \land \gamma \epsilon \alpha)].
\]

But this is not a correct paraphrase; (14) and (15) are not logically
equivalent. To see this, consider the following:

\[
\exists \alpha (\exists \beta \exists \varepsilon \in \beta \land \forall \beta \exists \varepsilon \in \beta \land (C(\gamma) \wedge C(\gamma) \wedge \forall \gamma (A(\gamma, \gamma) \to \gamma = 0) \wedge
\forall \gamma (A(\gamma, \gamma) \to \gamma = 0)).
\]

20 The language contains sentences like ‘Some positive natural numbers are succes-
sors of only one another’, which amounts to the negation of the full induction
principle that helps to yield a categorical characterization of natural numbers. The logic
of languages that contain the logical resources of the language presented above and
a two-place predicate (for example, ‘is a successor of’ or ‘admirers’), which helps to
state the simulacrum of a sufficient fragment of the theory of finite sets, cannot be
given a full axiomatic characterization. For a full model-theoretic characterization of
the logic, see my “The Language and Logic of Plurals,” section IV.

20 For this view, see W. V. Quine, Methods of Logic (London: Routledge, 1972, 3rd
edition), p. 239, and his The Roots of Reference (La Salle, Ill.: Open Court, 1973), p. 111. See
also Michael D. Resnik, “Second-order Logic Still Wild,” this JOURNAL, lxxxv, 2
(February 1988): 75-87; and Allen P. Hazen, “Against Pluralism,” Australasian Journal
of Philosophy, lxxi, 2 (June 1993): 132-44. For other, less direct criticisms of the view,
see George Boolos, “To Be Is To Be a Value of a Variable (or to Be a Value of Some
Variable),” this JOURNAL, lxxx, 8 (August 1984): 430-49, and his “Nominalist Platon-
ism,” Philosophical Review, xlv, 3 (July 1985): 327-44; see also Lewis, Parts of Classes.
(16) does not logically imply (15), by the conservation principle, because it does not do so on elementary logic. But (16) logically implies the following, which logically implies (14), by schema VI:

\((17) \text{Ezra and Thomas are critics who admire only each other (in symbols, } \forall \beta (H(\beta, [e@tl]) \rightarrow C(\beta) \land \forall \gamma (A(\beta, \gamma) \rightarrow \beta \neq \gamma \land H(\gamma, [e@tl])))\).

We can see some important features of the logic of plurals through schema VI, which we can use to show that (17) logically implies (14). Sentence (14) can be seen to result from replacing the italicized plural terms in the following sentences with the plural quantifier:

\textit{Ezra and Thomas are critics who admire only each other (sentence (17)).}

\textit{Ezra, Thomas, and Bill are critics who admire only one another.}

\textit{Ezra, Thomas, Bill, and Hillary are critics who admire only one another.}

Thus, by the schema, any of these sentences logically implies (14). Other instances of its antecedent include the following (or their paraphrases):

\textit{John and Carol are children who carry a piano.}

\textit{John and Carol are children who are healthy.}

\textit{Russell, Moore, and Whitehead are famous humans who are three philosophers.}

\textit{Bill, Boris, Tony, and Howard are politicians who cooperate.}

Thus, we can use the schema to show that they logically imply the following, respectively:

\textit{There are some children who carry a piano.}

\textit{There are some children who are healthy.}

\textit{There are some famous humans who are three philosophers.}

\textit{There are some politicians who cooperate.}

Notice that sentences like the following, too, are instances of the schema, because the schematic letter ‘\(t\)’ can be replaced with a singular term as well:

If Cicero is a Roman who is a philosopher, some Romans are philosophers (in symbols, \([ R^S(\alpha) \land P^S(\alpha) ] \rightarrow \Sigma \alpha [ R^S(\alpha) \land P^S(\alpha) ]\)).

\textsuperscript{3} Thus, ‘\(\Sigma \alpha [ R^S(\alpha) \land P^S(\alpha) ]\)’, ‘\(\exists \alpha [ R^S(\alpha) \land P^S(\alpha) ]\)’, and ‘\(\exists \alpha [ R(\alpha) \land P(\alpha) ]\)’, which can be taken to paraphrase ‘Some Romans are philosophers’, are logically equivalent.
Some might object that these are not logically true, because ‘Some Romans are philosophers’ implies the existence of at least two Romans. But this is not a correct view. The plural quantification cannot logically imply ‘There are at least two Roman philosophers’; it is logically implied by, for example, ‘Cicero and Tully are Romans who are philosophers’, which is logically implied by ‘Cicero is a Roman who is a philosopher, and Tully is a Roman who is a philosopher’.

Schemas XI-XIII help to distinguish the logic of plurals from second-order logic. Schema XII is the plural counterpart of schema V, which states the principle of substitutivity of identity (for singular terms). Schema XIII states the principle that, if some things are the same things as some things and the former are so-and-so, then the latter, too, must be so-and-so; its antecedent gives the condition for some things (for example, Russell and Whitehead) to be the same things as some things (for example, the authors of PM). Thus, we can use the schema to show that sentences like the following are logically true:

If Russell and Whitehead are the authors of PM, and Russell and Whitehead cooperate, then the authors of PM cooperate.
If the children who carry a piano are John and Carol, they carry it if and only if John and Carol carry it.

Consider schema XIII. The following sentences, for example, are instances of its antecedent and consequent, respectively:

(18) There is a philosopher who writes something (in symbols, $\exists \alpha (P(\alpha) \land \exists \beta W(\alpha, \beta))$).
(19) There are some things such that something is one of them if and only if it is a philosopher who writes something (in symbols, $\Sigma \gamma \forall \alpha (H(\alpha, \gamma) \leftrightarrow (P(\alpha) \land \exists \beta W(\alpha, \beta)))$).

Thus, by the schema, (18) logically implies (19). Using the schema, we can also show that ‘There is a politician in Washington’, for example, logically implies ‘There are some politicians in Washington such that any politician in Washington is one of them’; this sentence is logically equivalent with the consequent of its appropriate instance.

Notice that sentence (19) can be abbreviated as follows:

(19a) There are the philosophers each of whom writes something.

for if some things are such that something is one of them if and only if it is a philosopher who writes something, they are the philosophers.
each of whom writes something (and vice versa). In general, the consequent of schema XIII can be abbreviated as \[ \sum w_i[w_i = (\mu w)\phi'(w)] \],\(^{32}\) where the predicate \(\equiv\) is defined as follows:

\[ t = u = \sigma \forall v[H(v, t) \leftrightarrow H(v, u)], \] where \(v\) does not occur free in \(t\) or \(u\).

Thus, we can take the schema to state a principle giving a logically sufficient condition for there to be some things (for example, the philosophers each of whom writes something): if there is something that is so-and-so, there are \textit{the} things each one of whom is so-and-so. Notice that the necessary condition is also sufficient; the converse of the schema holds as well, which we can show using schema XI:

\[ \text{XIV. } \sum w \forall v[H(v, w) \leftrightarrow \phi(v)] \rightarrow \exists v \phi(v), \text{ where } w \text{ does not occur free in } \phi(v) \]

Thus, the antecedent of XIII is essential to the schema; we cannot do without the antecedent to get a stronger schema of logic. Most instances of its consequent are not logical truths; sentence (19) is not a logical truth. Moreover, the instances include logical falsities, such as \[ \Sigma \alpha \forall \beta[H(\beta, \alpha) \leftrightarrow \beta \neq \dot{\beta}], \] which logically implies \(\exists \beta \beta \neq \dot{\beta}\).

One might attempt to preserve schema XIII by treating plural terms and quantifiers as predicates and second-order quantifiers, respectively, because instances of the following schema, the second-order counterpart of XIII, are logically true:

\[ \text{XIIIa. } \exists X \forall v[X(v) \leftrightarrow \phi(v)], \text{ where } X \text{ does not occur free in } \phi \]

But this is not sufficient to reduce the logic of plurals to second-order logic. The converse of XIIIa, that is, the second-order counterpart of XIV, does not hold: \[ \exists X \forall v[X(v) \leftrightarrow \alpha \neq \dot{\alpha}] \], which is logically implied by \[ \forall v[\alpha \neq \dot{\alpha} \leftrightarrow \alpha \neq \dot{\alpha}] \] on second-order logic, is true; but \(\exists v \alpha \neq \dot{\alpha}\) is false. We cannot derive the converse of XIIIa, notice, because the second-order counterpart of schema XI (namely, \(\exists v X(v)'\)) does not hold; \(\exists v \alpha \neq \dot{\alpha}\) is logically false. Moreover, notice that second-order logic contains a schema stronger than XIIIa:

\[ \text{XIIIb. } \exists X \forall v[X(v) \leftrightarrow \phi(v)], \text{ where } X \text{ does not occur free in } \phi \]

which we can use to get the negation of \(\forall X \exists \alpha X(\alpha)\), a second-order universal generalization of an instance of the schema \(\exists v X(v)\). Thus,

\(^{32}\) To see this, apply the definition of plural definite descriptions.
one cannot take plural quantifiers as second-order quantifiers in an alternative notation.\textsuperscript{35}

It is usual to circumvent this well-recognized discrepancy between the logic of plural quantifiers and the logic of second-order quantifiers by paraphrasing plural quantifiers like ‘Some things are such that...’ into second-order notation using restricted quantifier phrases like ‘\(\exists X[\exists \alpha X(\alpha) \land \ldots]\)’. This scheme of paraphrase is objectionable on syntactic grounds, because applying the scheme to sentences like ‘John and Carol carry a piano’ or ‘A book is written by Russell and Whitehead’ fail to preserve the fact that they share predicates with sentences like ‘John carries a piano’ or ‘A book is written by Cicero’. Still, some might defend the scheme by paraphrasing ‘A book is written by Cicero’, for example, by the second-order sentence ‘\(\exists \beta(B(\beta) \land \exists X[\exists \alpha X(\alpha) \land \forall \alpha (X(\alpha) \leftrightarrow \alpha = e) \land \text{write}(X, \beta)]\)’. They might argue that this suffices to account for the logic of plurals, if not their syntax, because, for example, this paraphrase, like their paraphrase of ‘A book is written by Russell and Whitehead’, logically implies their paraphrase of ‘A book is written by some things’.

But the second-order scheme cannot preserve the logical relations among plurals encapsulated in schema XII. Consider arguments like the following, whose logical validity is captured by the schema:

\begin{enumerate}
  \item[(20)] Russell and Whitehead cooperate.
  \item[(21)] Russell and Whitehead are the authors of PM.
\end{enumerate}

Therefore,

\begin{enumerate}
  \item[(22)] The authors of PM cooperate.
\end{enumerate}

To account for their logical validity by applying the scheme to sentences like (20) and (22), it is necessary to appeal to the second-order counterpart of schema XII:

\begin{enumerate}
  \item[(XIIa)] \(\forall \phi \forall[\exists Y(\phi) \leftrightarrow Y(\phi)] \rightarrow [\phi \rightarrow \phi],\) where \(X\) and \(Y\) are second-order variables, \(\phi\) an atomic sentence, and \(\phi,\) obtained from \(\phi\) by replacing \(X\) in some of its occurrences by \(Y\)
\end{enumerate}

\textsuperscript{35} Neither can second-order languages, governed by second-order logic, be taken to provide alternative notations for plural constructions—pace Boolos, who proposes to interpret the formulas in second-order “languages” by rendering them into the language of plurals. He reads the second-order quantifier ‘\(\exists X\)’ as ‘there are some things such that...the...’, and argues that this leads to a defense of the logical status of XIIIb—see his “Reading the Begriffsschrift,” *Mind*, xciv, 375 (July 1985): 331-44, here p. 340. But the plural reading falls short of justifying XIIIb, whose instances, on that reading, include logical falsities.
But this schema cannot be considered a schema of logical truths in languages that contain second-order predicates like ‘cooperate’, which one needs to obtain the suggested paraphrases of sentences like (20). Let the second-order predicate ‘$\exists_e$’ indicate the relation of “identity” between properties, and the first-order predicates ‘$A$’ and ‘$B$’ the property of being identical with Russell or Whithead and that of being an author of PM, respectively. Then ‘$\exists_e(A, B)$’ is false, though both ‘$\exists_e(A, A)$’ and ‘$\forall_e [A(a) \equiv B(a)]$’ are true. Other second-order predicates give rise to the same problem: because the extensional equivalence between, for example, the properties indicated by the predicates ‘$A$’ and ‘$B$’ does not logically imply their “identity,” the predications that attribute, for example, the relation indicated by ‘cooperate’ to the properties are not logically equivalent even granting their extensional equivalence.

To defend schema XIIa, one might argue that second-order predications generate opaque contexts. Schema XIIa cannot be taken to warrant substituting ‘is Tully’ (solum veritate) for the co-extensional predicate ‘is an author of Academica’ in the following sentence:

(23) Bill thinks that Cicero is an author of Academica.

just as schema VI does not warrant substituting ‘Tully’ for ‘Cicero’ in the sentence. But it is hard to appeal to restrictions like this to defend schema XIIa. Sentence (23) cannot be analyzed as a predication composed of ‘Cicero’ and a first-order predicate (or ‘is an author of Academica’ and a second-order predicate) because ‘Cicero’

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"In Plurals and Events (Cambridge: MIT, 1993), Barry Schein presents a second-order scheme that does without such predicates. The scheme renders, for example, ‘they cooperate’ to a singular quantification over events: ‘$\exists_e (e \in$ a cooperation event $\land \forall_e (A(e, a) \equiv T(e))$’, where ‘$T$’ is a second-order variable corresponding to the pronoun ‘they’ and ‘$A(e, a)$’ amounts to, roughly, ‘$a$ is an agent of $e$’. This scheme is subject to a version of the so-called Russell’s paradox. The scheme renders ‘$X$s are as many as $Y$s’ to ‘$\exists_e (AMA(e) \land \forall_e (A(e, a) \equiv X(a)) \land \forall_e (O(e, a) \equiv Y(a))$’, where ‘$AMA(e)$’ and ‘$O(e, a)$’ amount to, roughly, ‘$e$ is an event whose agents are as many as its objects’ and ‘$a$ is an object of $e$’ respectively. Thus, Schein must take the following, which amounts to the true sentence ‘Any things are as many as themselves’, to hold:

$$\forall X (\exists_0 X(a) \rightarrow \exists e (AMA(e) \land \forall_e (A(e, a) \equiv X(a)) \land \forall_e (O(e, a) \equiv X(a)))$$

This leads to a contradiction, assuming ‘$\exists_0 \equiv \forall$’. Its instance obtained by the predicate ‘$F$’ defined as follows leads to a contradiction, because ‘$\exists_0 F(a)$’ holds:

$$F(\beta) \equiv \exists_e (X(\beta) \land AMA(\beta) \land \forall_e (A(\beta, a) \equiv X(a)) \land \forall_e (O(\beta, a) \equiv X(a)))$$

‘$[AMA(e) \land \forall_e (A(e, a) \equiv F(a)) \land \forall_e (O(e, a) \equiv F(a))$’, which implies ‘$\forall X [\forall_e (A(e, a) \equiv X(a)) \rightarrow \forall_e (X(a) \equiv F(a))]$’, implies ‘$F(a) \equiv F(e)$’
(or 'is an author of *Academica*') occurs in an opaque context, generated by 'thinks', in (23); but the second-order sentences considered in the above paragraph are all second-order predication. Some people might perhaps argue that opaque contexts are more prevalent, that they are generated by all second-order predicates (except first-order quantifiers), but this defense renders the schema useless in accounting for the logical relation in question among sentences like (20)-(22).

IV. VARIETIES OF PLURAL RELATIONS

I shall now discuss various kinds of plural relations, with concentration on plural properties. In particular, I shall analyze a special kind of plural properties, the pure numerical properties, in terms of logical resources only, and confirm the intuitions to which I have appealed.

Being two, recall, is instantiated by some things, but not by anything. It is the same with most of the plural properties discussed above. It is useful to focus on such properties in arguing against the principle of singularity. But, of course, there are other kinds of plural properties. Being one (or being one human), for example, is instantiated by something, for example, Bill, but not by any two or more things (as such). And there are properties instantiated by some one thing as well as by some two or more things (as such): for example, not being two, or writing a book, a property instantiated by Cicero and also by Russell and Whitehead (as such). Finally, some plural properties are not instantiated at all; as the singular property of being a Martian philosopher is not instantiated by anything, the plural property of being two Martian philosophers is not instantiated by any one or more things.

Next, recall the plural expansions like 'is-a-human^' (in symbols, "H^") of singular one-place predicates like 'is-a-human'. The plural expansions indicate plural properties that I call the plural expansions of their singular bases, the properties indicated by the singular predicates. For example, the property that I call being human[s], indicated by 'is-a-human^', is the plural expansion of being a human. Now, any plural expansion is distributive, that is, it is instantiated by some

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5 Some case can be made of this—'the author of *Academica* cannot be substituted for 'Cicero' in 'α (is Cicero, is Cicero)' (Notice, however, that the language of plurals officially contains no definite descriptions.) But the problem with Schema XIIa arises primarily because it differs from the correct formulation of the principle of substitutivity for predicates:

\[ \varphi(X, Y) \rightarrow [\phi \rightarrow \phi'] \], where \( \phi' \) is obtained from \( \phi \) by replacing \( X \) in some places by \( Y \).
things as such if and only if it is instantiated by every one of them. We can see this by considering their analysis in terms of their singular bases. The following definition of the predicate ‘being-a-human’ for example, can be seen to give an analysis of the property of being human[s]:

\[ H''(\alpha s) \equiv_{df} \forall \beta [H(\beta, \alpha s) \rightarrow H(\beta)] \]

That is, for some things to be human[s] is for them to be such that every one of them is a human. Thus, the plural property is instantiated by, for example, Bill and Hillary (as such) if and only if it is instantiated by Bill and by Hillary; for ‘\(H''([b \circ h])\)’ and ‘\(H''(h) \land H''(b)\)’ are logically equivalent. But this logical equivalence relies on the nature of the plural expansion; sentences of the forms ‘\(\phi(t \circ u)\)’ and ‘\(\phi(t) \land \phi(u)\)’ are not in general logically equivalent. Accordingly, most of the other plural properties, such as being two, are nondistributive.

Let me now focus on a special kind of plural properties: numerical properties. I distinguish two kinds of them: impure ones like being two humans or being two-membered; and pure ones like being two. I shall analyze the impure ones in terms of the pure ones, and the pure ones in terms of logical resources only.

Being two humans, for example, can be analyzed in terms of (i) being two and (ii) being human[s] according to the following definition:

\[ \text{BE-TWO-HUMANS}(\alpha s) \equiv_{df} \text{BE-TWO}(\alpha s) \land H''(\alpha s) \]

Thus, for some things to be two humans is for them to be two and also humans. The analysis separates the numerical component of the impure numerical property from its non-numerical component. This clarifies that Bill and Hillary’s being of the same kind is irrelevant to their being two.

Similarly, we can distil the numerical component of being two-membered, for example, by analyzing it as follows:

\[ \text{is-two-membered}(\alpha) \equiv_{df} \Sigma \beta s (\text{BE-TWO}(\beta s) \land \forall \delta [H(\delta, \beta s) \rightarrow \delta c \alpha]) \]

For something to be two-membered is, roughly, for it to have as its members some things that are two. This analysis conforms to, and clarifies, the idea that, for example, the set [Bill, Hillary] instanti-

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\(^{36}\) I use ‘some things’, and so on, to mean the same as ‘some one or more things’, and so on. Thus, the above sentence implies ‘it is instantiated by something if and only if it is instantiated by every one of it’.

\(^{37}\) There are exceptions like being at least one.
ates being two-membered because Bill and Hillary as such instantiate being two, not vice versa. The relation of membership, indicated by the predicate ‘ε’, plays only the subsidiary role of relating the set to the people in question, its members. Thus, the relation, too, is extrinsic to the numerical feature proper of being two-membered.

It is the same with other impure numerical properties like having one offspring, having two parts, or having three elements. They inherit their numerical character from the pure numerical properties that underlie them as their common components.

Next, we can analyze the properties of being one and being two as follows:

$BE\text{-ONE}(\alpha s) \equiv _{def} \exists \beta \forall \delta (H(\delta, \alpha s) \leftrightarrow \delta = \beta)$

$BE\text{-TWO}(\alpha s) \equiv _{def} \exists \beta \exists \gamma (\beta \neq \gamma \land \forall \delta (H(\delta, \alpha s) \leftrightarrow \delta = \beta \lor \delta = \gamma))$

It is straightforward to give similar analyses of the other properties corresponding to natural numbers (greater than one).

Now, we can confirm the logical equivalence of facts like (1) and (3), which can be stated by the following sentences:

$(1a) \ BE\text{-TWO} \ ([b @ h])$

$(3a) \ b \neq h \land \exists \beta \beta = b \land \exists \beta \beta = h$

These sentences are logically equivalent, given the definition of the predicate ‘BE\text{-TWO}’, because the following is a logical truth:

$\forall \delta (H(\delta, [b @ h]) \leftrightarrow \delta = b \lor \delta = h)^{98}$

We can also confirm that the existence of, for example, the set [Bill, Hillary] is not a logical consequence of Bill and Hillary’s being two. For sentence (1a) does not logically imply the singular construction ‘∃α[αα ∧ hα]’, because, as we can see using the conservation principle, sentence (3a) does not do so.

Finally, notice that the definition of, for example, the predicate ‘BE\text{-TWO}’ can be modified as follows, using only bounded quantifiers:

$BE\text{-TWO}(\alpha s) \equiv _{def} \exists \beta (H(\beta, \alpha s) \land \exists \gamma (H(\gamma, \alpha s) \land \beta \neq \gamma \land \forall \delta (H(\delta, \alpha s) \leftrightarrow \delta = \beta \lor \delta = \gamma)))$

$^{98}$ Or we can analyze them recursively in terms of their respective predecessors:

$BE\text{-}(N+1)(\alpha s) \equiv _{def} \exists \beta \exists \gamma (BE\text{-}N(\gamma s) \land H(\beta, \alpha s) \land \forall \delta (H(\delta, \gamma s) \leftrightarrow H(\delta, \alpha s) \land \delta \neq \beta))$

$^{99}$ We can show this using schemas IX-X.
This helps to confirm and clarify the idea that instantiating a pure numerical property is a matter intrinsic to the things in question (for example, Bill and Hillary); whether or not there is any other object (for example, the set [Bill, Hillary]) is irrelevant to their being two.\textsuperscript{10}

\section*{V. Epilogue}

The standard conception of relation is the natural ally of the Fregean conception of language and logic. The Fregean language extends the Aristotelian language by including, among others, predicates with more than one argument place; and the Fregean logic extends the Aristotelian logic to the extended language. The development of the Fregean conception, which helped to make clear the limitations of the Aristotelian conception of language and logic, led to the establishment of the standard conception, which improves on the Aristotelian conception of relation that does not allow the existence of relations even in the usual sense. Similarly, we can regiment plural constructions and formulate their logic as in section \textsc{iii}, which helps to clarify the limitations of the Fregean conception and replace the standard conception with a broader conception that allows plural relations. Such work, however, does no more than help to remove blindfolds, without which we can directly see the limitations of the standard conception. It is, after all, two things that are two.

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\footnote{On my view, natural numbers are the above-mentioned numerical properties. See my \textit{Understanding the Many} (Ph. D. Dissertation, University of California/Los Angeles, 1995), chapter 4; and "Numbers and Relations."}