Nonlinear Parametric Generation in Birefringent Poled Fibers

by

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Abstract

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Conventional step-index silica fibers do not possess a second-order optical nonlinearity due to symmetry concerns. However, through the process of poling, the generation of a frozen-in DC field $E^{DC}$, and in turn, a non-zero second-order nonlinearity $\chi^{(2)} = 3\chi^{(3)}E^{DC}$, can be created in optical fibers. In this thesis, I measure the individual $\chi^{(2)}$ tensor elements of birefringent periodically poled fiber via second-harmonic generation and sum-frequency generation experiments. The symmetry of the $\chi^{(2)}$ tensor is consistent with that of the $\chi^{(3)}$ for isotropic media. This is the first study that characterizes all the $\chi^{(2)}$ tensor elements in birefringent poled fiber. Furthermore, I investigate the intermix of the $\chi^{(2)}$ tensor elements by twisting the fiber, which results in the generation of new second-harmonic signals not observed in untwisted fiber. The conversion efficiencies and spectral positions of these new signals can be varied by twisting the fiber.
Acknowledgments

First, I want to thank my supervisor, Professor Li Qian, for guiding me through the maze of experiments that is this thesis. This work would not have been possible without her patience, knowledge, and encouragement.

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Special thanks to Professor Lacra Pavel and Professor Joyce Poon for their advice, time, and willingness to serve on my committee.

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## Glossary

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<tr>
<td><strong>ASE</strong></td>
<td>amplified spontaneous emission</td>
</tr>
<tr>
<td><strong>CW</strong></td>
<td>continuous wave</td>
</tr>
<tr>
<td><strong>DGD</strong></td>
<td>differential group delay</td>
</tr>
<tr>
<td><strong>DFG</strong></td>
<td>difference-frequency generation</td>
</tr>
<tr>
<td><strong>EDFA</strong></td>
<td>erbium-doped fiber amplifier</td>
</tr>
<tr>
<td><strong>ES</strong></td>
<td>Electrostriction</td>
</tr>
<tr>
<td><strong>EO</strong></td>
<td>electro-optic</td>
</tr>
<tr>
<td><strong>FPC</strong></td>
<td>fiber-based polarization controller</td>
</tr>
<tr>
<td><strong>OPO</strong></td>
<td>optical parametric oscillator</td>
</tr>
<tr>
<td><strong>PBS</strong></td>
<td>polarizing beam-splitter</td>
</tr>
<tr>
<td><strong>PDL</strong></td>
<td>polarization-dependent loss</td>
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<tr>
<td><strong>PPLN</strong></td>
<td>periodically-poled lithium niobate</td>
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<tr>
<td><strong>PPSF</strong></td>
<td>periodically-poled silica fibers</td>
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<tr>
<td><strong>QPM</strong></td>
<td>quasi-phase-matched</td>
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<tr>
<td><strong>SF</strong></td>
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SFG  sum-frequency generation
SH   second-harmonic
SHG  second-harmonic generation
SMF  single-mode fiber
SON  second-order nonlinearity
SPDC spontaneous parametric downconversion
TLS  tunable laser sources
VOA  variable optical attenuator
WDM  wavelength division multiplexer
Chapter 1

Introduction

1.1 Motivation

Step-index fibers are made of fused silica, an amorphous compound, and therefore lack a second-order optical nonlinearity due to reasons of symmetry [6]. However, it is possible to induce a second-order nonlinearity in fused silica fibers and silica waveguides via the processes of thermal [20, 30] and UV [14, 15] poling.

The reasons for wanting a second-order nonlinearity (SON) ($\chi^{(2)} \neq 0$) in fiber are numerous. For one, it enables frequency generation through such three-wave mixing processes as second-harmonic generation (SHG), sum-frequency generation (SFG), difference-frequency generation (DFG), and optical parametric oscillation.

Another reason is that the presence of a $\chi^{(2)}$ also allows for the generation of non-classical light through spontaneous parametric downconversion (SPDC), a process where it is possible to generate single photon pairs with correlated properties [23].

For example, photon pairs generated from type-II SPDC can share polarization correlations that cannot be explained classically [3, 7, 23]. For this reason, SPDC is considered an important and useful process in quantum optics [18] and quantum information. Applications of SPDC-derived photon pairs include quantum key distribution [10] and quantum computation [21].
Figure 1.1: Three nonlinear optical processes that are achievable in a medium with a non-zero $\chi^{(2)}$. These include a) SHG, where two beams of the same frequency ($\omega_0$) interact in the medium to generate a SH ($2\omega_0$) beam, b) SFG, where two non-degenerate beams ($\omega_1, \omega_2$) generate an SF ($\omega_{SF} = \omega_1 + \omega_2$), c) DFG, where the generated beam has a frequency ($\omega_2$) that is the difference of the two input beams.

All of these processes (SHG, SFG, DFG, SPDC) have been demonstrated before in crystals [6, 13, 23]. The processes have also been demonstrated in periodically-poled lithium niobate (PPLN) waveguides [9, 33], where the SON is very large ($\chi^{(2)} > 10$ pm/V) when compared to that of poled fiber (usually < 0.2 pm/V, and in my case, < 0.1 pm/V). But there are certain advantages to having such processes in-fiber, and these advantages are what motivates the work in this thesis.

First, although the efficiencies of the above stated processes are all proportional to $(\chi^{(2)})^2$, they are also proportional to the square of the interaction length $L^2$. While it is difficult and expensive to grow crystals and fabricate other waveguides to be more than 10 mm in length, poled fiber can be made with lengths in excess of 10 cm (my poled fibers samples are > 20 cm in length) with relative ease, or as long as 1 m [12].
Figure 1.2: a) Energy level diagram for the SPDC process. b) A block diagram of the process with all the input and output optical fields. The $\Delta (\neq 0)$ signifies that the process needn’t be degenerate.

Secondly, fiber-to-fiber coupling is much less labour-intensive than fiber-to-crystal or fiber-to-waveguide coupling. Thus, parametric processes generated in-fiber are readily compatible with current optical telecommunications technology. This is especially true if the generated wavelengths also happen to lie in the telecom band ($\lambda \approx 1.5 \mu m$).

1.2 The Problem

While poled fiber holds the promise of fiber-based parametric generation, there are also problems. Although the origin of the SON in poled fiber has been well-studied [2, 19, 32], the symmetry of the $\chi^{(2)}$ tensor is still not understood.

The literature [19, 22] suggests that the process of (thermal) poling induces a space-charge DC field $E^{DC}$ in the fiber. This space-charge field acts through the Kerr nonlinearity ($\chi^{(3)}$) of the fiber to produce an effective SON: $\chi^{(2)} = 3 \chi^{(3)} E^{DC}$.

The elements of the $\chi^{(2)}$ tensor may be labeled by subscripts: $\chi^{(2)}_{ijk}$. The $i, j, k$ represent the polarizations of the three interacting optical waves in the above second-order processes (Figure 1.1, Figure 1.2). For crystals, the subscripts can take on values of all three cartesian coordinates $x, y, z$. This results in 27 tensor elements. With poled fiber, however, the direction of propagation ($z$ by convention) need not be considered, as the field amplitude $E_z$ of the propagating field is
orders of magnitude smaller than $E_x$ or $E_y$, the field amplitudes in the transverse directions of the fiber.

So there are only eight $\chi^{(2)}$ tensor elements we need to worry about in poled fiber. As we shall see in Section 2.1, not all of these tensor elements can be the same or even non-zero. This means that the efficiencies of the processes involving the SON should be dependent upon the polarizations of the interacting fields.

However, prior to this work, such polarization dependence in poled fibers has not been observed definitively. Measurements of the electro-optic (EO) coefficient ($r \approx \chi^{(2)}E^{DC}$) of poled fiber have observed no polarization dependence [17, 39] within error, while experiments based upon SHG in poled fibers have yielded inconclusive evidence [27, 34] of polarization dependence.

### 1.3 My Work

In this thesis, I demonstrate the ability to measure the magnitudes of the individual $\chi^{(2)}$ tensors of the fiber through spectrally-separated SHG. This spectral separation arises from the fiber birefringence, which causes the SHG signals to be phase-matched at different wavelengths when the polarizations of the interacting fields ($\omega_0$ and $2\omega_0$ in Figure 1.1a) are varied. I can relate the conversion efficiencies of the different SHG processes to the different tensor elements because the polarizations of the interacting fundamental and SH fields are also monitored.

I go a step further to analyze the symmetry of the tensor elements by twisting the fiber. Fiber twisting causes the eigenmodes of a birefringent fiber to change from linearly-polarized to elliptically-polarized. The phase-matching of elliptically-polarized fields results in additional SHG signals not found in an untwisted fiber. Experimentally, the results agree well with what is expected of a fiber that has a SON described by $\chi^{(2)} = 3\chi^{(3)}E^{DC}$.

With these two experiments, I believe that I have completely characterized the $\chi^{(2)}$ tensor in poled fiber, a feat that (to date) no one else has been able to accomplish.

The poled fibers that I use for this work have been fabricated and provided by
Professor P. G. Kazansky’s group at the University of Southampton.

1.4 Organization

Chapter 2 provides background material upon which the rest of the thesis is based. It contains a primer on nonlinear optics, how the poling process is performed, and a review of the body of work that concerns the polarization dependence of various nonlinear processes in poled fiber.

Chapter 3 gives an account of how I perform my SHG experiments, and the results that were obtained.

In Chapter 4, I utilize a linear-optical technique to demonstrate that my fiber is indeed birefringent. Using this linear-optical technique also provides me with the ability to check that the polarizations involved in the SHG experiments are indeed the principal polarization state of the fiber. The chapter also provides a new experimental method of quantifying the fiber birefringence through fiber twisting.

Chapter 5 follows in the same vein as Chapters 3 and 4, where I incorporate fiber twisting and SHG experiments to generate SHG signals not present in untwisted fiber. Comparison with theory leads to a good agreement with the current model of the SON in poled fiber.

In Chapter 6, I show that the SFG process in twisted fiber is broadband. The experimental SFG spectrum provides us with a tuning curve that also works for other three-wave mixing processes such as DFG and SPDC.

And finally in Chapter 7, I wrap up by summarizing my work and pointing to new directions for future work.
Chapter 2

Background

2.1 Nonlinear Optical Processes

The equation of motion for the electric field in bulk non-magnetic dielectric matter is the inhomogeneous wave equation:

\[ \nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \]  

(2.1)

where the vector polarization \( \vec{P} \) describes completely the response of the material. In the simplest case of the material response, \( \vec{P} \) is a linear function of the \( \vec{E} \)-field: \( \vec{P}_i = \epsilon_0 \chi^{(1)}_{ij} E_j \). In the realm of nonlinear optics, \( \vec{P} \) is no longer a linear function of \( \vec{E} \). In such nonlinear media, the \( \vec{P} \) is written as a perturbative expansion involving powers of \( \vec{E} \):

\[ \vec{P}_i = \vec{P}_i^{(L)} + \vec{P}_i^{(NL)} = \vec{P}_i^{(1)} + \left( \vec{P}_i^{(2)} + \vec{P}_i^{(3)} + \cdots \right) \]

\[ = \epsilon_0 \left( \chi^{(1)}_{ij} E_j + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \cdots \right) \]  

(2.2)

where we sum over all repeated indices \( j,k \). As \( \vec{P} \) and \( \vec{E} \) are 3-dimensional vectors, \( i, j \) and \( k \) take on values of \( x,y,z \) in general. The \( \chi^{(1)}, \chi^{(2)}, \chi^{(3)}, \ldots \) are
tensors of rank 2, 3, 4, . . . with $3^2, 3^3, 3^4, \ldots$ elements respectively.

Nonlinear optical processes can be roughly divided into two categories:

- parametric: these processes conserve the energy of the incident lightwaves with respect to the scattered lightwaves.
- non-parametric: where the energies of the incident lightwaves are not equal to the output lightwaves.

From this point onward, I will deal exclusively with parametric nonlinear processes.

The $\chi^{(2)}$ tensor allows for second-order parametric processes such as SHG (Figure 1.1a), in which a lightwave of frequency $\omega$ interacts with itself in the medium to generate a lightwave at $2\omega$: $(\omega) + (\omega) \rightarrow (2\omega)$.

Optical fibers are made of fused silica, an amorphous material that possesses inversion symmetry. This in turn means that optical fibers lack a second-order nonlinearity, which can be shown by applying an inversion transformation to $\mathcal{P}$ (Equation 2.4):

$$\mathcal{P}^{(2)}_i = \chi^{(2)}_{ijk} E_i E_j$$  \hspace{1cm} (2.3)

$$\Rightarrow -\mathcal{P}^{(2)}_i = \chi^{(2)}_{ijk} (-E_j)(-E_k)$$  \hspace{1cm} (2.4)

As the material is invariant under inversion, the two expressions (Equations 2.3 and 2.4) must be equal. Equating these two expressions for $\mathcal{P}^{(2)}_i$ yields $\chi^{(2)} = 0$. Therefore, the $\chi^{(3)}$ tensor is the lowest order nonlinearity in standard silica fiber.

As was mentioned earlier, the $\chi^{(3)}$ tensor has $3^4 = 81$ tensor elements. That is because each of the indices $i, j, k, l$ can take on any of the three cartesian axes $x, y, z$. However, for fiber, only the two directions ($x$ and $y$) transverse to the direction of propagation $z$ are relevant, as the the field amplitude $E_z$ (in the direction of propagation) is negligible. This reduces the number of $\chi^{(3)}$ tensor elements we need to deal with in fiber to 16.

Inversion symmetry tells us that the $\chi^{(3)}$ tensor element (in fused silica) with
an odd number of $x$ or $y$s must be zero, i.e., $\chi^{(3)}_{xxxy} = \chi^{(3)}_{xxyx} = \cdots = \chi^{(3)}_{yyxx} = \chi^{(3)}_{yyyy} = 0$. We now only have 8 non-zero tensor elements:

$$\chi^{(3)}_{ijkl} = \begin{pmatrix} \chi^{(3)}_{xxxx} & \chi^{(3)}_{xxyy} & \chi^{(3)}_{xyyx} & \chi^{(3)}_{yxyx} \\ \chi^{(3)}_{yyyy} & \chi^{(3)}_{yyxx} & \chi^{(3)}_{yxxy} & \chi^{(3)}_{yxxy} \end{pmatrix}$$

(2.5)

Additionally, for an amorphous material such as fused silica, the $\chi^{(3)}$ tensor should look the same regardless of our choice of $x$ and $y$, so long as they lie in the transverse plane of the fiber. Swapping the $x$ direction with the $y$ and vice versa gives $\chi^{(3)}_{xxxx} = \chi^{(3)}_{yyyy}$ and $\chi^{(3)}_{xyyx} = \chi^{(3)}_{yxyx}$. So, each element in the top row of Equation 2.5 is now equal to the corresponding element in the bottom row. By the same logic, the magnitudes of the 4 tensor elements in the top row of Equation 2.5 can be related to each other [6]:

$$\chi^{(3)}_{xxxx} = \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyyx} + \chi^{(3)}_{yxyx}$$

(2.6)

We can write a similar expression relating the elements of the bottom row of Equation 2.5 by swapping the $x$’s with the $y$’s in Equation 2.6.

Consider the relabeled tensor element $\chi^{(3)}_{ijkl}(\omega_i; \omega_j, \omega_k, \omega_l)$. We may interpret this tensor element intuitively (but somewhat incorrectly) as an $i$-polarized lightwave at frequency $\omega_i$ produced by the interaction of three beams polarized in the $j$, $k$, and $l$ direction, with frequencies $\omega_j$, $\omega_k$, and $\omega_l$ (respectively). As we are dealing strictly with parametric processes, the energies of the fields must be conserved. That is: $(-\omega_i) + (\omega_j) + (\omega_k) + (\omega_l) = 0$.

In the previous paragraph, we have written the $\chi^{(3)}$ tensor as a function of not only polarization, but of frequency as well. However, the nonlinear susceptibility can be approximated as being dispersionless [6] when the nonlinearity arises from electronic interactions, and when all the frequencies we are dealing with $(\omega_i, \omega_j, \omega_k, \omega_l)$ are far away from the bandgap energies (phononic and electronic) of the material. For fused silica, when the lightwaves are all in the visi-
ble and infrared ($\lambda \approx 400$-1600 nm) range, full permutation symmetry is present among the indices of the tensor elements, which simplifies Equation 2.6 even further:

$$
\chi^{(3)}_{xxx} = 3\chi^{(3)}_{xyy} = 3\chi^{(3)}_{yxx} = 3\chi^{(3)}_{xxy} \\
\chi^{(3)}_{yxy} = 3\chi^{(3)}_{yx} = 3\chi^{(3)}_{xyx} = 3\chi^{(3)}_{yxy} \\
(2.7)
$$

Kleinman symmetry is said to be observed when there is full permutation symmetry among all the tensor elements of the nonlinear susceptibility.

As we shall see in the next section (Section 2.2), an effective $\chi^{(2)}$ can arise in a fiber that has been subjected to an intense DC electric field. This poling field causes the migration of impurity ions in the fused silica, in turn setting up a space-charge field $E^{DC}$ [19]. An effective second-order nonlinearity arises now through the $\chi^{(3)}$ nonlinearity described above for amorphous material:

$$
\chi^{(2)}_{ijk} (-\omega_i; \omega_j, \omega_k) = 3\chi^{(3)}_{ijkx} (-\omega_i; \omega_j, \omega_k, 0)E^{DC}_x \\
(2.8)
$$

In Equation 2.8 and the rest of the thesis, I will use the convention that the direction of poling is in the $x$-direction. With this in mind, and looking back at Equation 2.5, we find that Equation 2.8 gives us 4 non-zero $\chi^{(2)}$ tensor elements:

$$
\chi^{(2)}_{ij} = \begin{pmatrix}
\chi^{(2)}_{xxx} & \chi^{(2)}_{xy} & \chi^{(2)}_{yx} & \chi^{(2)}_{xyy} \\
\chi^{(2)}_{yxx} & \chi^{(2)}_{yxy} & \chi^{(2)}_{xyx} & \chi^{(2)}_{yxx} \\
\chi^{(2)}_{yx} & \chi^{(2)}_{yxy} & \chi^{(2)}_{xyx} & \chi^{(2)}_{yxy} \\
\chi^{(2)}_{xyy} & \chi^{(2)}_{yxy} & \chi^{(2)}_{xyx} & \chi^{(2)}_{yxy}
\end{pmatrix} = \begin{pmatrix}
\chi^{(2)}_{xxx} & 0 & 0 & \chi^{(2)}_{xyy} \\
0 & \chi^{(2)}_{yxy} & \chi^{(2)}_{xyx} & 0
\end{pmatrix} \\
(2.9)
$$

If we are considering second-harmonic generation (SHG) ($\chi^{(2)}_{ijk} = \chi^{(2)}_{ijk} (-2\omega; \omega, \omega)$) or any other three-wave phenomena where the interacting fields are at optical (visible, infrared) wavelengths, the $\chi^{(2)}$ should be dispersionless, and Kleinman symmetry should hold. That is:

$$
\chi^{(2)}_{xyy} = \chi^{(2)}_{yxy} = \chi^{(2)}_{yxy} \\
(2.10)
$$
To relate the elements of Equation 2.10 to \( \chi^{(2)}_{xxx} \), we must appeal to Equation 2.6. Using Equation 2.10 and assuming that \( \chi^{(2)}_{ijk} = 3\chi^{(3)}_{ijk}E_{DC} \), we have:

\[
\chi^{(2)}_{xxx} = 3\chi^{(2)}_{yyy} = 3\chi^{(2)}_{yxx} = 3\chi^{(2)}_{yx} \quad (2.11)
\]

One may question whether the presence of the non-uniform distribution of ions that sets up \( E_{DC} \) may destroy the symmetry of the \( \chi^{(3)} \) tensor described above. We can be assured that this is not the case. First of all, the experiments described later in this thesis demonstrate that the \( \chi^{(2)} \) symmetry in our poled fiber follows well from the \( \chi^{(3)} \) symmetry. Secondly, because the concentration of ions involved in setting up this field are low, typically on the order of 1 ppm (10^{16} ions cm^{-3}) \([1]\), the \( \chi^{(3)} \) symmetry will only be altered by a perturbatively small amount.

## 2.2 Origins of the \( \chi^{(2)} \) in Poled Fiber

A specially-modified step-index fiber consisting of two large air holes sandwiching the core is used for the process of poling in my samples. I will call this the twin-hole fiber \([4]\). An electron micrograph of the fiber cross-section is shown in Figure 2.1.

Tungsten electrodes are inserted into the air holes, the fiber is heated to 600 K, and a voltage in excess of 2 kV is applied across the electrodes \([4, 39]\), resulting in an applied DC electric field \( E_{app} \approx \frac{2 \text{kV}}{20 \mu \text{m}} \). Due to the increased temperature, the ionic impurities (mainly Na\(^+\)) in the fiber will have higher mobilities and will migrate due to the presence of \( E_{app} \).

Positive charges such as Na\(^+\) will be pushed away from the anode (+), while the negative charges are immobile, creating a region depleted of positive ions surrounding the airhole where the anode is inserted \([31]\). If the fiber is poled for longer and longer time periods, the depletion region will extend beyond an annulus surrounding the anodic airhole, to as much as 10 microns into the fiber itself \([39]\). When the temperature is lowered back to room temperature, and the applied field \( E_{app} \) is turned off, the positive ions stop moving and are frozen in. The charge
Figure 2.1: Electron micrograph of twin-hole fiber used for poling. The large black holes are the air holes through which electrodes are inserted for poling. The small white dot is the core of the fiber. Photo Credit: Kazansky Group, University of Southampton, 2006.

separation in turn sets up a frozen-in DC-field $E^{DC}$.

The evidence for this space-charge model of thermal poling is overwhelming. The region depleted of positive charges can be observed under a microscope via HF-etching (ionically depleted glass is less reactive than normal glass [25]), as evidenced by [5]. The spatial profile of the SON in poled fiber can also be observed directly via second-harmonic spectroscopy [2]. The etching and SON profiles were found to be in good agreement.

So, there is a consensus as to how the SON arises in poled fiber. The separation of charge caused by the poling process produces a frozen-in DC field $E^{DC}$, which in turn produces an effective SON through the $\chi^{(3)}$ nonlinearity. However, the jury is still out on how the various tensor elements of this effective $\chi^{(2)}$ relate to each other. A review of the experiments reported in the literature regarding the polarization dependence of various second-order nonlinear processes (in poled fiber) is given in the next section.
2.3 Polarization Dependence of Parametric Processes in Poled Fiber: A Review

Measurements of the polarization dependence of the $\chi^{(2)}$ tensor can be divided into two subsets. One subset involves measuring the electro-optic coefficient through interferometric means, and then relating that to the $\chi^{(2)}$ tensor values. The other subset involves measuring polarization-dependence of SHG in the poled fibers. Let us discuss the electro-optic experiments first.

Indirect measurements of the $\chi^{(2)}$ tensor elements through the electro-optic effect in poled fibers [17, 39] using Mach-Zehnder interferometer (MZI) setups have shown a ratio closer to 1:1.

\[ \Delta n \propto \chi^{(3)}(-\omega; \omega, 0, 0)E^{DC}E^{AC}_{app} \]  (2.12)

**Figure 2.2:** The MZI setup for measuring the index change due to the electro-optic effect in poled fiber. The electrodes are left inside the twin-holed fiber after the thermal poling process. An applied AC potential generates the $E^{AC}_{app}$ in the fiber. Taken from [39] with permission.

The electro-optic effect in poled fiber involves the interplay of an applied AC field $E^{AC}_{app}$ in addition to the frozen-in field $E^{DC}$:
An increase in \( E_{app}^{AC} \) results in a linear change in the phase \( \delta \phi = L \Delta n \) experienced by the optical field at \( \omega \). By launching various polarizations into the fiber, it was found that that this phase-change was only weakly polarization-dependent.

However, in these electro-optic experiments, there is another mechanism at play. The polarization-independence of the electro-optic measurements can be attributed to the contribution to \( \Delta n \) by electrostriction [17]. Electrostriction (ES) is the refractive index change that is induced by the strain of the DC fields \( \Delta n_{ES} \propto E^{DC}E_{app}^{AC} \). It turns out that \( \Delta n_{ES} \) is comparable in magnitude to \( \chi_3^{(2)}E^{DC}E_{app}^{AC} \), and that \( \Delta n_{ES} \)'s own polarization dependence can cancel out the polarization-dependence of the \( \chi_3^{(2)}E^{DC}E_{app}^{AC} \) contribution [17, 24].

Electrostriction, however, does not affect the values of the \( \chi_2^{(2)} \) tensors (Equation 2.11). Therefore, measuring SHG efficiencies for different input polarizations in poled fibers is a more reliable way of determining the ratio of \( \chi_2^{(2)} \) tensor elements.

However, though polarization dependence of SHG has already been observed by several groups in poled bulk silica [19, 30] and poled fiber [27, 34], the measured ratio for \( \chi_{xxx}^{(2)}/\chi_{xyy}^{(2)} \) was found to vary from 1.6:1 [27] to 7:1 [30]. Only one of these studies was able to show the expected 3:1 ratio; but that work [19] probed only the \( \chi_{xxx}^{(2)} \) and \( \chi_{xyy}^{(2)} \) tensor elements. The studies done in poled fiber [27, 34] were not able to obtain this 3:1 ratio; we suspect that they did not properly take into account the wavelength-dependence of SHG brought about by birefringent phase-matching. Additionally, none of the above works measured all \( \chi_2^{(2)} \) tensor elements. Later in this thesis, I measure each of the \( \chi_2^{(2)} \) tensor elements present in thermally-poled fiber via type I and type II SHG, verifying Equation 2.11 experimentally.

### 2.4 Phase-Matching

The presence of a non-zero \( \chi_2^{(2)} \) in a material is a necessary but insufficient condition for nonlinear second-order parametric generation in that material. The interacting optical fields must also satisfy a phase-matching constraint (see Appendix
Consider the three-wave mixing processes in poled fiber. The phase-matching condition looks like this:

$$\beta_i^{(2\omega)} = \beta_j^{(\omega-\Delta)} + \beta_k^{(\omega+\Delta)} + K$$

(2.13)

where $\beta_i^{(2\omega)}$ is the propagation constant of the $i^{th}$ polarized field at frequency $2\omega$. Frequency detuning between the two lightwaves at the fundamental frequency $\omega$ is represented by $\Delta$. The indices $i, j, k$, representing the polarization of the field, are constrained to be one of two transverse directions ($x$ or $y$) of the fiber ($z$ is the direction of propagation).

The $K$ term is a crystal momentum contribution from the medium due to a periodic grating. If we wish to have phase-matched second-harmonic generation of $\lambda_{SH} \approx 775$ nm light, a periodic grating must be written on the fiber to modulate the $\chi^{(2)}$ profile along the direction of propagation: $\chi^{(2)}(z) = \chi^{(2)}(z + \Lambda)$. Such a grating, of period $\Lambda$ (on the order of tens of microns), is written along the length of the fiber using UV erasure methods [8]. The crystal momentum $K$ attained from this grating is $K = \pm \frac{2\pi}{\Lambda}$.

Poled fibers that are quasi-phase-matched (QPM) in this manner are said to be periodically-poled, and are called periodically-poled silica fibers (PPSF). From this point onward, I will use the terms PPSF and poled fiber interchangeably.

The phase-matching condition written as it is in Equation 2.13 can help us categorize and differentiate between the different three-wave mixing processes that are possible. In the case of SHG, where two fields of degenerate frequency interact in the $\chi^{(2)}$ medium to generate a frequency-doubled light field (Figure 1.1a), $\Delta = 0$. In the case of $\Delta \neq 0$, where two nondegenerate lightwaves near the fundamental frequency generate a second-harmonic light-field, the process is called SFG (Figure 1.1b).

Cases where $j = k$ are called type-I processes, while a process is called type-II when $j \neq k$. In the case of, say, type-II SPDC (Figure 1.2), I mean that the down-converted photons (with polarizations $j, k$) will be of different ($j \neq k$) polarizations.
Our poled fiber samples have an additional property: they are birefringent. I attribute the birefringence to the fiber geometry (Figure 2.1), as the fiber core is believed to be elliptical. So, for some wavelength, there exist two polarization eigenmodes of the fiber, which must clearly be the $x$ and $y$ directions, that would necessarily have different propagation constants $\beta_x \neq \beta_y$. This $x$ is also aligned with the direction of the frozen-in DC field (Figure 2.1).

As we shall see, the fiber is birefringent at both $2\omega$ (775 nm) and at $\omega$ (1550 nm). This has the nice consequence that should the SHG conversion efficiencies for all possible combinations of polarizations $i, j, k$ be non-zero, there will be no spectral overlap between the signals.

In the remainder of this thesis, we will concern ourselves with experiments in SHG and SFG. In these cases, we use a shorthand notation to denote a particular signal: $j + k \rightarrow i$. For example, $y + y \rightarrow x$ denotes a type-I process in which a $y$-polarized field at the fundamental wavelength generates $x$-polarized light in the second-harmonic, while $x + y \rightarrow y$ involves the type-II generation of a $y$-polarized SH beam.

Putting the phase-matching condition together with Equation 2.11, we find that three spectrally separated SHG signals should be observable in poled fiber, two of type-I, and one of type-II (Table 2.1).

**Table 2.1:** The Expected SHG Signals in a Thermally-Poled Birefringent Fiber

<table>
<thead>
<tr>
<th>Polarizations</th>
<th>Tensor Element</th>
<th>Type</th>
<th>Relative Conversion Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y + y \rightarrow x$</td>
<td>$\chi^{(2)}_{yxy}$</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>$y + x \rightarrow y$</td>
<td>$\chi^{(2)}<em>{yxy} + \chi^{(2)}</em>{yyx}$</td>
<td>II</td>
<td>4</td>
</tr>
<tr>
<td>$x + x \rightarrow x$</td>
<td>$\chi^{(2)}_{xxx}$</td>
<td>I</td>
<td>9</td>
</tr>
</tbody>
</table>

In Chapters 3 and 4, I show that Table 2.1 is indeed true for my poled fiber.

Finally, it is important to note that $i, j, k$ need not be restricted to linear polarizations such as $x$ and $y$. In Chapter 5, I will show that twisting a birefringent
fiber results in its polarization eigenmodes becoming elliptically-polarized. In that case, we can use those eigenmodes as the labels for $i$, $j$, and $k$. 
Chapter 3

Second Harmonic Generation

3.1 Introduction

In this chapter, I set out to measure the individual $\chi^{(2)}$ tensor elements of the fiber via second-harmonic generation. By monitoring the fundamental and second-harmonic polarizations, I can elucidate which $\chi^{(2)}$ tensor element a particular SHG signal is associated with, and by measuring the conversion efficiency of that signal the magnitude of the tensor element can be obtained.

3.2 Experimental Setup

The PPSF that I use is a twin-hole fiber (Figure 2.1) that was thermally poled. The direction of poling ($x$) is parallel to one of the principal (birefringence) axes of the fiber. The poled fiber is QPM for the generation of SH light at $\lambda_{SH} \approx 775$ nm in the LP$_{01}$ mode. For my experiments, the poled fiber is packaged by having both its ends fusion-spliced to connectorized single-mode fiber (SMF) (Corning SMF28) pigtails. The length ($L$) of the poled portion is 23.5 cm, while the composite fiber (PPSF and SMF28 pigtails) is 161 cm long.

All measurements are performed with continuous wave (CW) tunable laser sources (TLS): HP 8168F, and Agilent 8164A. Because the maximum power of
these lasers is less than $7 \text{ dBm}$ (5 mW) and the normalized conversion efficiencies for SHG in my fiber are on the order of $10^{-4} \text{ W}^{-1}$, I will need to amplify this light. For that purpose, an erbium-doped fiber amplifier (EDFA) is used.

The wavelength range for my experiments are then limited by the wavelength range within which the EDFA operates, because both TLSes have larger tunable ranges (1440-1590 nm for the HP, and 1510-1640 for the Agilent) than the EDFAs. If not otherwise stated, the primary EDFA used is the Pritel SPFA-22, which has excellent wavelength coverage extending slightly beyond the C-Band (1535-1565 nm); it can amplify CW laser light with wavelengths as short as 1520 nm, and as long as 1570 nm with less than 10% amplified spontaneous emission (ASE) in the total power (at 1 mW input power, as measured with an optical spectrum analyzer). With a 1 mW input from the above TLSes, I use the amplifier to provide $\approx 50 \text{ mW}$ of output power.

Should an amplified signal ($\leq 50 \text{ mW}$) at wavelengths to the red of 1565 nm be required, an L-Band (1565-1605 nm) amplifier (Pritel FA-18L) can be used. In later experiments, such as sum frequency generation, an additional C-Band amplifier may also be required. For that, I shall use the Pritel LNHPA-30.

Because the SHG process I am studying is polarization-dependent, I must have good control over the polarization state of the lightfield at the fundamental wavelength (which I shall refer to as the *fundamental polarization* for short). Although the polarization state of the light at the output of the TLS is constant over its tunable range (Figure 3.1a), that of the amplified light coming out of the EDFAs is not (Figure 3.1b). That is, the EDFAs do not maintain the polarization of the input light.

I can remove this unwanted polarization wandering by adding a fiber-based polarizing beam-splitter (PBS) after the EDFA (Figure 3.1c). However, there is a drawback; although the polarization state is now constant when I sweep in the 1520-1570 nm range, the output power is not. As the PBS acts like a polarizer, the wandering polarization at the output of the EDFA means that the power will vary as I vary the wavelength. This problem is remedied by introducing a fiber-
Figure 3.1: a) The polarization state of the HP8168F TLS is monitored with a polarimeter (Thorlabs Pat 9000b) as I sweep the wavelength from 1520-1570 nm. The polarization remains fixed, as indicated by the small dark dot traced out on the Poincare sphere. b) The output polarization of the SPFA-22 EDFA as I sweep the TLS’s wavelength from 1520-1570 nm. The polarization states trace out a limacon-like shape as the wavelength is swept. c) The output of the SPFA-22 EDFA is then sent into a fiber-based polarizing beam splitter (PBS). The wavelength scanning range is again from 1520-1570 nm. The polarization states trace out a small striplike path.

Based polarization controller (FPC) between the EDFA and the PBS (Figure 3.2a), and scanning a small wavelength range ($\approx$ 10 nm) at a time. After scanning each range, the FPC is manually readjusted so that the average power in the next wavelength range (to be scanned) is maximized. The variation in the power as we sweep through the wavelength range is usually less than $\pm$30%. Also, note that the fundamental power is monitored throughout the experiment (Figure 3.2b), and the SHG efficiencies are normalized to it in later calculations.

Before the light is launched into the poled fiber, I must have a way of adjusting the polarization. This is done by placing a second FPC after the PBS (FPC2 in Fig-
Figure 3.2: The fiber-based setup for measuring the SHG spectrum. a) The dotted line denotes the launch apparatus for the fundamental wavelength. The first fiber-based polarization controller (FPC1) is adjusted to maximize the fundamental power, while FPC2 is used to adjust the launch polarization of the fundamental wavelength before it enters the poled fiber. FPC2 can be a manually controlled or a computer-controlled polarization controller (Agilent 11896A).

b) The monitoring apparatus at the output end of the poled fiber. A WDM splits the fundamental (≈ 1550 nm) from the SH (≈ 775 nm); the polarization state and power of the fundamental is monitored with a polarimeter, while the SH power is measured with a silicon detector. Alternatively, the output of the poled fiber can go directly into an optical spectrum analyzer (OSA), if I am worried about the polarization-dependent loss (PDL) of the WDM and power meter at 775 nm.

Figure 3.2a). The fundamental lightwave is then launched into the PPSF, generating second-harmonic light at $\lambda_{SH} \approx 775$ nm.

Recall that both ends of the poled fiber are fusion-spliced with connectorized SMF pigtails, making it convenient to perform fiber-based measurements. However, this makes it impossible to monitor the polarization at the immediate output end of the poled fiber. I can only monitor the polarization (of the SH and the
fundamental) after it has traversed 70 cm or so of SMF.

At the output end of the poled fiber, a WDM splits the fundamental ($\approx 1550$ nm) from the SH ($\approx 775$ nm). The polarization state and power of the fundamental is monitored with a polarimeter, while the SH power is measured with a silicon detector.

The WDM allows for $> 20$ dB extinction of SH light in the fundamental (1550 nm) leg. The silicon detector that is used to measure the power at 775 nm also insensitive to 1550 nm light. For example, 60 mW of CW 1550 nm light shone directly onto the silicon detector yields only a reading of 0.8 nW. The use of the WDM further reduces the reading to 0.01 nW, which becomes the noise floor for the silicon detector.

The polarimeter cannot measure power beyond 10 dBm (10 mW). I place a variable optical attenuator (VOA) (JDS Uniphase HA9) in between the WDM and the polarimeter to lower the power entering the polarimeter.

The insertion losses for the various devices in Figure 3.2b, as well as the poled fiber (PPSF) itself, are measured. The results are shown in Table 3.1. For the PPSF, the insertion loss also includes the poled-fiber to SMF28 splice losses.

**Table 3.1**: Insertion Losses (IL) of Various Devices in Experimental Setup

<table>
<thead>
<tr>
<th>Device</th>
<th>IL 1550 nm (dB)</th>
<th>IL 775 nm (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPSF</td>
<td>2.7</td>
<td>6.0</td>
</tr>
<tr>
<td>Attenuator</td>
<td>1.5</td>
<td>N/A</td>
</tr>
<tr>
<td>WDM</td>
<td>1.3</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The polarization-dependent loss (PDL) of a device can be gauged by using the setup shown in Figure 3.3. It is obtained by finding the ratio of the maximum to minimum output power readings, as the polarization of the input is swept while keeping the input power constant. The PDL at 1550 nm of the devices shown in Table 3.1 is at most $0.2 \pm 0.1$ dB.

Measuring the PDL at 775 nm is more difficult; both the poled fiber and the SMF28 fiber are multi-moded at this wavelength. Ideally, I will need to use a
coherent light source at 775 nm that couples light only into the LP$_{01}$ mode, as that is the mode whose PDL I wish to assess. I use the SH light generated from another poled fiber. An added challenge is that the FPC (Figure 3.3) works via strain-induced birefringence; this may also cause the SH light to be coupled to a higher-order mode, say LP$_{11}$ or LP$_{02}$, which would make my measurements meaningless (as I would then be measuring the PDL for those modes). The PDL of the PPSF at 775 nm is measured to be 0.6 dB. However, given the above problems with the measurement, this value is most likely an overestimate of the PDL.

![Diagram](image)

**Figure 3.3:** The setup for measuring polarization-dependent loss (PDL). The launch polarization of the coherent source is varied with an FPC while the power is monitored. The error in measurement may be gauged by removing the device, and having the output of the FPC directly connected to the power meter; i.e., measuring the PDL of the FPC itself.

### 3.3 The SHG Spectrum

To get the SHG spectrum I first need to find the approximate wavelengths at which the various SHG signals are maximized. I need to sweep the polarization as well as the wavelength. The CW laser sources I am using at the fundamental frequency are wavelength-tunable, so sweeping the wavelength is straightforward. The experimental setup in Figure 3.2 is used, with FPC2 being a computer-controlled polarization controller (Agilent 11896A), while FPC1 is a regular paddle-based polarization controller. I sweep the wavelength from 1520-1580 nm (in 0.2 nm steps), and at each wavelength step, FPC2 scans the entire Poincare sphere of polarization. The maximum SH power ($P_{SH}$) is recorded at each wavelength step,
as is the fundamental power $P_F$ and polarization state of the fundamental $\hat{P}$. This gives 3 peaks in the 1540-1555 nm range. A finer resolution (0.1-nm) wavelength-scan is performed for each of the 3 peaks. The results are shown in Figure 3.4.

![Figure 3.4: The SHG spectrum for the poled fiber. The nonlinear transmittances $\left( \eta_{SH} = \frac{P_{SH}}{(P_F)^2} \right)$ are plotted against the fundamental wavelength $\lambda_F$. The type-II signal at 1552.4 nm is normalized differently: $\left( \eta_{SH} = \frac{P_{SH}}{P_{FX}P_{FY}} \right)$](image)

From Appendix A, I know that the $P_{SH}$ is proportional to $(P_F)^2$. As a verification that I am actually observing SHG, I shall see if this power dependence is true for the middle peak (labeled $X + X \rightarrow X$) in Figure 3.4 by performing a log-log plot of $P_{SH}$ against $P_F$ (Figure 3.5). The plot clearly indicates that $P_{SH} \propto (P_F)^2$.

At the wavelength where the first of these peaks occur (1542.2 nm), the nonlinear transmittance $\eta_{SH} = \frac{P_{SH}}{(P_F)^2}$ is found to be maximized at a particular polarization state of the fundamental; call this the $Y$ polarization. And call the fundamental polarization state that maximizes $\eta_{SH}$ at the second peak location, 1549.4 nm, the $X$ polarization; $X$ is found to be orthogonal to $Y$. The third peak (1552.4 nm) is maximized when the polarization state of the fundamental is in a 50/50 coherent superposition of the $X$ and $Y$ polarizations.
Figure 3.5: A log-log plot of the SH power $P_{SH}$ versus the fundamental power $P_F$ for the 1549.4-nm peak ($X + X \rightarrow X$). The slope of the line (2.01) clearly indicates that $P_{SH} \propto (P_F)^2$.

To illustrate what is meant by $X$, $Y$, and coherent superpositions of $X$ and $Y$, I can plot the $\eta_{SH}$ as a function of the fundamental polarization state ($\hat{p}$) for each of the three wavelengths (1542.2, 1549.4, 1552.4 nm). The Agilent 11896A gives me the ability to vary the fundamental polarization over the entire Poincare sphere; I monitor the fundamental power ($P_F$) and polarization ($\hat{p}$) at the output end of the SMF-pigtailed poled fiber with a polarimeter, while $P_{SH}$ is monitored by the silicon detector. The results of the scans are shown in Figure 3.6.

Figure 3.6a shows the raw data points $\eta_{SH}$ plotted on the Poincare sphere, while in Figure 3.6b, I smooth the data out over the whole Poincare sphere using a Gaussian averaging function.

So now I have identified the 1542.2 and 1549.4 peaks as type-I SHG signals (the interacting fundamental beams are of one polarization) and the the 1552.4 peak as a type-II signal (it requires a coherent superposition of both $X$ and $Y$ polarizations). For a type-II SHG signal, a portion of the the fundamental power
Figure 3.6: (color online) The nonlinear transmittance at the three peaks is plotted against the fundamental state of polarization. The data on all three peaks has been rotated such that the X polarization coincides with the north pole ($s_3 = 1$) of the Poincare sphere. Naturally, this means that the Y polarization now corresponds to the south pole ($s_3 = -1$).

a) shows the nonlinear transmittances as discrete data points.

b) shows the data points averaged out over the spheres. A Gaussian weighting function is used to do this.

Although, I have measured the polarizations of the fundamental for each signal, I have yet to measure the polarization states at the SH. To do so, I connect the output end of the poled fiber into the free-space setup shown in Figure 3.7.

I find that the polarization at the second-harmonic for two peaks (1542.2, and 1549.4 nm) is orthogonal to the SH polarization of the third (1552.4) peak. I can
relate the SH polarization to the $X$ and $Y$ polarizations of the fundamental by using the same free-space setup to measure the polarization state at the fundamental. I simply need to replace the quarter wave plate (QWP) and power meter in Figure 3.7 with ones that work at 1550 nm. What I find is that the SH polarization at 1542.2 and 1549.4 nm is $X$, while the SH polarization at 1552.4 nm is $Y$.

### 3.4 Summary

I am able to determine the fundamental and SH polarizations present in each of the three SHG signals. Table 3.2 summarizes the results of this chapter. The signals are denoted in the shorthand described in Section 2.4.

**Table 3.2:** The Observed SHG Signals in a Thermally-Poled Birefringent Fiber

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Polarizations</th>
<th>Type</th>
<th>Relative Conversion Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1542.2</td>
<td>$Y + Y \rightarrow X$</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>1552.4</td>
<td>$Y + X \rightarrow Y$</td>
<td>II</td>
<td>4.0</td>
</tr>
<tr>
<td>1549.4</td>
<td>$X + X \rightarrow X$</td>
<td>I</td>
<td>9.9</td>
</tr>
</tbody>
</table>

Figure 3.7: The output of the poled fiber is collimated into free-space, where a quarter-wave-plate (QWP) and polarizer setup are used to determine the SH polarization at each of the three peaks. The polarizer is achromatic and the collimator also works at the fundamental wavelength.
Several questions remain. Are $X$ and $Y$ linear polarizations? If not, how do they relate to the principal polarization axes $x$ and $y$ of the poled fiber? If I can relate $X$ and $Y$ to $x$ and $y$, I will be able to know which tensor element is responsible for each of the SHG signals in Table 3.2. From the efficiencies of the signals, then, I will be able to obtain the magnitudes of the $\chi^{(2)}$ tensor elements.

Additionally, is there a way to quantify the birefringence of the fiber? It is possible to relate the SHG peak separations to the birefringence (Appendix B), but this requires information about the fiber geometry. In the next chapter, I will demonstrate experimentally a method to measure the birefringence of the poled fiber.
Chapter 4

Linear Birefringence Measurements

4.1 Introduction

In this chapter, I will experimentally relate the X-Y polarizations (the principal polarizations for SHG which I found in Chapter 3) to the principal polarization states of the fiber (x-y). The experiment will also provide a measurement of the polarization mode dispersion (PMD) of my poled fiber. Finally, I show that it is possible to measure the birefringence of the poled fiber at the fundamental wavelength by twisting it.

4.2 A simple model of fiber birefringence

A birefringent fiber supports two orthogonal polarization eigenmodes (| ±⟩) with different propagation constants $\beta_+^{(o)} \neq\beta_-^{(o)}$. In the parlance of the previous chapters, these are the principal polarization states of the fiber: x and y. Here, though, I try to be more general; I do not assume that these eigenmodes are linearly polarized. By writing in the bra-ket notation, I am implying | ±⟩ exist in Jones space, with $\langle + | + \rangle = 1 = \langle - | - \rangle$ and $\langle + | - \rangle = 0$.

If I launch (monochromatic) polarized laser light into this birefringent fiber, I can write the input polarization state | in⟩ as a superposition of | +⟩ and | −⟩:
with $\alpha$ and $\gamma$ as complex numbers. The $|z = 0\rangle$ reminds us that this is the polarization state at the input end of the fiber.

Now, because $|\pm\rangle$ are non-degenerate eigenmodes, the polarization state of the laser light will evolve as it travels along the length of the fiber. After traveling through a birefringent fiber of length $L$, the polarization becomes:

$$|\text{out}\rangle = |z = L\rangle = |+\rangle + \gamma e^{i(\beta_+ - \beta_-)L} |-\rangle$$

(4.1)

So, a relative phase $\phi = (\beta_+(\omega) - \beta_-(\omega))L$ is incurred between the $|\pm\rangle$ eigenmodes. (If I had access to the immediate input and immediate output ends of the poled fiber, a measurement of this relative phase $\phi$ would then yield the fiber birefringence. However, the poled fiber is fusion-spliced to SMF fiber, so I cannot do this.)

By making the following constraints:

- the launch polarization $|\text{in}\rangle$ is frequency independent
- the eigenstates are frequency independent [38], i.e., $\frac{d}{d\omega} |\pm\rangle = 0$

I can write $|\text{in}\rangle$ and $|\text{out}\rangle$ as Jones vectors

$$|\text{in}\rangle = \begin{bmatrix} \langle + |\text{in}\rangle \\ \langle - |\text{in}\rangle \end{bmatrix} = \begin{bmatrix} \alpha \\ \gamma \end{bmatrix}, \quad |\text{out}\rangle = \begin{bmatrix} \langle + |\text{out}\rangle \\ \langle - |\text{out}\rangle \end{bmatrix} = \begin{bmatrix} \alpha \\ \gamma e^{i\phi(\omega)} \end{bmatrix}$$

(4.2)

Take note that I have chosen to write the Jones vectors in the $\pm$ basis rather than in the $x$-$y$ linear polarization basis (Figure 2.1) (or the $H/V$, as is the normal convention). I may write the output polarization $|\text{out}\rangle$ as a Stokes vector (note that these definitions also deviate from convention):
\[ \vec{s}_{\text{out}}(\omega) = \left( \begin{array}{c} \langle \text{out} | \sigma_x | \text{out} \rangle \\ \langle \text{out} | \sigma_y | \text{out} \rangle \\ \langle \text{out} | \sigma_z | \text{out} \rangle \end{array} \right) = \left( \begin{array}{c} 2|\alpha||\gamma|\cos(\phi(\omega) + \theta) \\ 2|\alpha||\gamma|\sin(\phi(\omega) + \theta) \\ |\alpha|^2 - |\gamma|^2 \end{array} \right) \] (4.3)

with \( \sigma_i \) being the Pauli matrices:

\( \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)

and \( \theta = \text{arg}(\frac{\gamma}{\alpha}) \). In this definition of the Stokes parameters, the eigenmodes of the fiber \( \pm \) are just the north and south poles (respectively) of the Poincare sphere:

\[ \hat{p}_\pm = \begin{pmatrix} 0 \\ 0 \\ \pm 1 \end{pmatrix} \]

What happens to the output Stokes vector \( \vec{s}_{\text{out}}(\omega) \) as I scan the wavelength? Well, I can answer that by taking the derivative of Equation 4.3 with respect to \( \omega \):

\[ \frac{d}{d\omega} \vec{s}_{\text{out}}(\omega) = \frac{d\phi(\omega)}{d\omega}(\vec{s} \times \hat{p}_+) \] (4.4)

For a constant input polarization \( |\text{in}\rangle \), Equation 4.4 tells me that the output polarization \( \vec{s}_{\text{out}} \) will precess about \( \hat{p}_+ \) as I sweep the wavelength. So by monitoring \( \vec{s}_{\text{out}} \), I can determine the position of \( \hat{p}_+ \) and, additionally, the differential group delay (DGD) \( \tau_{\text{DGD}} \) between the two principal polarization states of the fiber:

\[ \tau_{\text{DGD}}(\omega_0) = \left( \frac{d\phi(\omega)}{d\omega} \right)_{\omega=\omega_0} \] (4.5)

The radius of precession is given by \( r_\perp \equiv 2|\alpha||\gamma| \)

30
4.3 The Experiment

Here I show that the $X$ polarization I observed in Chapter 3 is in fact a principal polarization state of the poled fiber. I do so by first determining the position of the $X$ polarization with SHG. I then use the wavelength scanning technique discussed in Section 4.2 to see if the axis of precession ($\hat{p}_\perp$) coincides with $X$.

The experimental setup is shown in Figure 4.1. I launch amplified CW laser light at the fundamental wavelength into the poled fiber. This time, however, I use a free-space polarization controller consisting of achromatic waveplates (one half-wave $\lambda/2$ and one quarter-wave $\lambda/4$ plate) and a polarizer. I find the $X$ polarization by adjusting the waveplates (which are on rotatable mounts) to maximize the SH power at the 1549.4 nm peak Figure 3.4. The output end of the poled fiber is then disconnected from the SH power meter (Figure 4.1) and reconnected directly to the polarimeter via a 50-cm-long SMF patchcord. In this way, I am able to measure the relative Stokes parameters of the $X$ polarization. The $X$ polarization is measured to be at $\vec{s}_{\text{out}} = (-0.04, -0.999, 0.01)^T$.

![Figure 4.1: Experimental setup for determining the $X$ and $x$ polarizations. To find $X$, I maximize the SH power at the 1549.4 nm peak by adjusting the waveplates. To find $x$ (or more precisely, $\hat{p}_\perp$), the EDFA is removed and I monitor the fundamental polarization at the output end of the poled fiber with a polarimeter as I sweep the fundamental wavelength.](image)

Now, I am able to proceed with my wavelength-scanning measurements. The
EDFA is removed from the setup. Removing the EDFA allows for the wavelength to be scanned from 1510-1590 nm. (EDFAs will attenuate laser light at wavelengths outside their gain bandwidth, so it is best to remove the EDFA from the setup). I then adjust the waveplates so that the polarization at 1550 nm lies halfway between X and Y; with \(|\alpha| \approx |\gamma| \approx 1/\sqrt{2}\), Equation 4.3 tells me that the precession path of the wavelength scanning will trace out a great circle on the Poincare sphere. (I want to do this, because the larger the precession radius \(r_\perp\), the less significant the contribution from the SMF birefringence).

I scan the wavelength, while monitoring the output polarization. I re-adjust the waveplates so that the output polarization at 1550 nm is different from what it was before, and I perform another wavelength scan. This is done repeatedly.

The SMF in the system may also contribute some birefringence. This is the major source of noise in these measurements. To reduce the effects from the SMF, I ensure the SMF in the system is not twisted, coiled, or otherwise stressed. I also perform multiple wavelength scans at different initial input polarizations so that the effects from SMF may be averaged out.

The results of the wavelength scans are shown in Figure 4.2. The colored circles represent the precession paths extrapolated from the wavelength scan data. The experimental data are the small red dots that lie on the circles. And the large colored dots at the centre of the image represent the extrapolated axes of precession for each of these wavelength scans.

Figure 4.2 tells me that the X polarization I observed in Chapter 3 is in fact the principal state of polarization \(x\) for my poled fiber. With that in mind, I can now relate the nonlinear transmittances of the various SHG signals (Table 3.2) to the corresponding elements of the \(\chi^{(2)}\) tensor (Table 4.1). This is done by appealing to the fiber geometry (from which I can calculate the fiber dispersion) and the length \(L\) and QPM period \(\Lambda\) of the PPSF. The procedure is documented in Appendix A.

Additionally, from these wavelength scanning experiments, I obtain the differential group delay \(\tau_{DGD} = L \left( \frac{1}{v_{g,y}} - \frac{1}{v_{g,x}} \right)\) at 1550 nm to be around 14.6 fs (over 23.5 cm of poled fiber). This will be important information should we choose...
Figure 4.2: The different-colored rings represent the different precession paths of various wavelength scans. Each wavelength scan has a different starting point, but the range is the same for all scans (from 1510-1590 nm). The dots in the middle are the calculated rotational axes for their respective wavelength scan. These dots also pinpoint the principal polarization (birefringence) axis of the fiber. Above, I determined that the $X$ polarization was at $\vec{s}_{out} = (-0.04, -0.999, 0.01)^T$. So, the principal polarization state of the fiber $x$ is equal to the $X$ polarization.

Table 4.1: The Observed $\chi^{(2)}$ Signals in Our Poled Birefringent Fiber

<table>
<thead>
<tr>
<th>Polarizations</th>
<th>Tensor Element</th>
<th>Value of Element ($\times 10^{-2}$ pm/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y + y \rightarrow x$</td>
<td>$\chi^{(2)}_{xyy}$</td>
<td>$2.1 \pm 0.1$</td>
</tr>
<tr>
<td>$y + x \rightarrow y$</td>
<td>$\chi^{(2)}<em>{xyy} + \chi^{(2)}</em>{yyx}$</td>
<td>$4.2 \pm 0.1$</td>
</tr>
<tr>
<td>$x + x \rightarrow x$</td>
<td>$\chi^{(2)}_{xxx}$</td>
<td>$6.7 \pm 0.3$</td>
</tr>
</tbody>
</table>

to use the PPSF in pulsed excitation, where walkoff between differently-polarized pulses may require some sort of compensation.

4.4 Twisting and a Measurement of Birefringence

(The following is inspired by [29]. Note that from this section onward, $\omega$ denotes optical frequency, while $\Omega$ denotes the rate of twist (measured in units of radians
per metre) of the fiber.

Is it possible to measure the birefringence \( \delta \beta = \beta_y - \beta_x \) at the fundamental wavelength? In my situation, where I cannot monitor the polarization at the immediate input/output of the poled fiber (it is fusion-spliced to SMF pigtails on both ends), the answer to this question seems to be no. However, all is not lost.

It turns out that in a uniformly twisted birefringent fiber \([26, 28]\), the principal polarization states \((x, y)\) of the untwisted fiber are no longer polarization eigenmodes of the twisted fiber. Consider the field \( \vec{E}(z) = \hat{x} E_x e^{-i(\beta_x)z} + \hat{y} E_y e^{-i(\beta_y)z} \). The \( E_x \) and \( E_y \) field amplitudes become coupled through twisting and their energies will beat as they travel along the fiber (in the \( z \) direction). In Jones space, the linear coupled mode equations look like this \([28]\):

\[
\frac{d}{dz} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = i \begin{bmatrix} \beta_x \\ i\Omega (1 - \frac{g}{2}) \beta_y \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = iC \begin{bmatrix} E_x \\ E_y \end{bmatrix} \tag{4.6}
\]

where \( \Omega \) is the twist rate (measured in radians per metre), and \( g \) is the elasto-optic constant \((g = 0.14 - 0.16 \) for fused silica step-index fibers \([28, 36]\)).

I can find the new polarization eigenmodes of the twisted fiber by diagonalizing matrix \( C \) in Equation 4.6. The (unnormalized) eigenmodes \(| \tilde{x} \rangle, | \tilde{y} \rangle\) are:

\[
| \tilde{x} \rangle = \begin{bmatrix} 1 \\ -i\xi \frac{1}{1+\sqrt{1+\xi^2}} \end{bmatrix}, \quad | \tilde{y} \rangle = \begin{bmatrix} -i\xi \frac{1}{1+\sqrt{1+\xi^2}} \\ 1 \end{bmatrix}, \quad \xi = \frac{2\Omega (1 - \frac{g}{2})}{\beta_y - \beta_x} \tag{4.7}
\]

with associated eigenvalues \( \tilde{\beta}_x, \tilde{\beta}_y \):

\[
\tilde{\beta}_x = \frac{\beta_x + \beta_y}{2} - \frac{1}{2} \sqrt{(\beta_y - \beta_x)^2 + \left(2\Omega \left(1 - \frac{g}{2}\right)\right)^2}
\]

\[
\tilde{\beta}_y = \frac{\beta_x + \beta_y}{2} + \frac{1}{2} \sqrt{(\beta_y - \beta_x)^2 + \left(2\Omega \left(1 - \frac{g}{2}\right)\right)^2} \tag{4.8}
\]

where we’ve made a simplifying assumption that \( \beta_y > \beta_x \) (which is true for our case).
Notice that the birefringence $\delta \tilde{\beta} = (\tilde{\beta}_y - \tilde{\beta}_x)$ is now modified:

$$
\delta \tilde{\beta} = \sqrt{(\beta_y - \beta_x)^2 + \left(2\Omega \left(1 - \frac{g}{2}\right)\right)^2}
$$

(4.9)

In the limit where $\Omega \rightarrow 0$, the eigenmodes $\tilde{x}, \tilde{y}$ reduce to $x$ and $y$, the principal polarization states of the untwisted fiber:

$$
|\tilde{x}\rangle \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix},
|\tilde{y}\rangle \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\tilde{\beta}_x \rightarrow \beta_x,
\tilde{\beta}_y \rightarrow \beta_y
$$

But that is not so interesting. What is interesting is that according to Equation 4.9, the birefringence $\delta \tilde{\beta}$ increases as a function of the twist $\Omega$. Meanwhile, taking the derivative of Equation 4.9 with respect to the frequency $\omega$:

$$
\frac{d}{d\omega} \delta \tilde{\beta} = \frac{(\beta_y - \beta_x)}{\sqrt{(\beta_y - \beta_x)^2 + \left(2\Omega \left(1 - \frac{g}{2}\right)\right)^2}} \cdot \frac{d}{d\omega} (\beta_y - \beta_x)
$$

(4.10)

tells me that the $\left| \frac{d}{d\omega} \delta \tilde{\beta} \right|$ actually decreases with increasing $\Omega$. But I know how to measure $\frac{d}{d\omega} \delta \tilde{\beta}$. It is simply just the differential group delay $\tau_{DGD} = L \frac{d}{d\omega} \delta \tilde{\beta}$, where $L$ is the length of the poled fiber.

So, simply by measuring the $\tau_{DGD}$ at various twists $\Omega$, I can then extract the value of the birefringence $(\beta_y - \beta_x)$ by using Equation 4.10 to do a maximum-likelihood fit. To effect a twist in the poled fiber, I first uncoil the fiber, and place it straight on a flat platform. One end is held fixed, while the other end is twisted with the help of a rotation mount (Figure 4.3). The mount is placed on a translation stage, allowing for the fiber to be held taut.

The apparatus to measure $\tau_{DGD}$ is the same as the one shown in Figure 4.1. Multiple measurements are performed for a particular twist. For each twist, I plot the (arithmetic) mean value of the $\tau_{DGD}$, with the accompanying error bars being the standard deviation of that collection of measurements. The greater the twist, the larger the error bars; this may be due to the increased contribution from the
Figure 4.3: To effect a twist, the SMF-pigtailed poled fiber sample is held straight on a flat surface, one end fixed securely, while the other end is free to twist as it is fastened to a rotational mount.

SMF fiber, or because the poled fiber is not uniformly-twisted.

The data is shown in Figure 4.4. Fitting the data to Equation 4.10 yields a birefringence of $(\beta_y - \beta_x) = (7.4 \pm 2.8) \times 10^{-5} \, \mu m^{-1}$ at the fundamental wavelength ($\lambda_F \approx 1550 \, nm$). In terms of effective indices $n_{eff,i} (\beta_i = \frac{2\pi n_{eff,i}}{\lambda_{eff,i}})$, the value of the birefringence is $\delta n^{(\omega)} \triangleq (n_{eff,y} - n_{eff,x}) = (1.82 \pm 0.70) \times 10^{-5}$.

I cannot measure the birefringence at the SH, because the poled fiber is multimode at 775 nm, and I do not have access to a stable tunable source at that wavelength. However, I can estimate the birefringence at 775 nm by observing the peak separation between the three SHG signals Table 3.2. See Appendix B for details.

4.5 Summary

In this chapter, I have shown experimentally that the X polarization is in fact one of the principal polarization states ($x$) of the fiber. I have also observed experimentally a $\tau_{DGD}$ of 14.6 fs for our poled fiber at the fundamental wavelength.

Finally, I demonstrated experimentally that twisting a birefringent fiber reduces its DGD, and twisting the fiber even more can provide an estimate of the fiber birefringence (which in this case is $(\beta_y - \beta_x) = (7.4 \pm 2.8) \times 10^{-5} \, \mu m^{-1}$ ($\delta n^{(\omega)} \triangleq (n_{eff,y} - n_{eff,x}) = (1.82 \pm 0.70) \times 10^{-5}$ at the fundamental wavelength).

It would be logical to ask, then, what happens to the SHG signals when the
Figure 4.4: The experimental data for the \( \frac{d}{d\omega} \delta \tilde{\beta} = \tau_{DGD}/L \) at various twists \( \Omega \). A maximum-likelihood fit (solid line) is performed using Equation 4.10. Fitting parameters are \((\beta_y - \beta_x) (7.4 \pm 2.8 \times 10^{-5} \; \text{\(\mu\)m}^{-1})\) and \(\frac{d}{d\omega} (\beta_y - \beta_x) (60.7 \pm 1.1 \; \text{fs/m})\), the birefringence and DGD (respectively) for the untwisted fiber.

fiber is twisted? Do the relative strengths of the signals diminish or increase? Are there other phase-matched SHG signals that are possible? This is what I hope to answer in Chapter 5.
Chapter 5

Twisting and Nonlinear Parametric Phenomena

5.1 Introduction

In this chapter, I show theoretically and demonstrate experimentally that it is possible to generate SHG signals that are not explicitly supported by the $\chi^{(2)}$ tensor in (untwisted) poled fiber. This is done simply by twisting the fiber.

5.2 The Theory

From Chapter 3 and Chapter 4, I know that the tensor elements of the poled fiber are:

$$\chi_{ijk}^{(2)}(-2\omega; \omega, \omega) = \begin{pmatrix}
\chi_{xxx}^{(2)} & \chi_{xxy}^{(2)} & \chi_{xyx}^{(2)} \\
\chi_{yxx}^{(2)} & \chi_{yxy}^{(2)} & \chi_{yyx}^{(2)} \\
\chi_{yxy}^{(2)} & \chi_{yyx}^{(2)} & \chi_{yyy}^{(2)}
\end{pmatrix} = \chi^{(2)} \begin{pmatrix}
1 & 0 & 0 & \frac{1}{3} \\
0 & \frac{1}{3} & \frac{1}{3} & 0
\end{pmatrix} \tag{5.1}
$$

The non-zero tensor elements shown in Equation 5.1 give rise to three SHG signals ($y + y \rightarrow x, x + x \rightarrow x, y + x \rightarrow y$) with normalized conversion efficiencies in the ratio 1:9:4, as seen in Figure 3.4 and Table 4.1.

But if the fiber is twisted, what sort of other SH signals do I avail myself
of? To find out, I should write the $\chi^{(2)}$ tensor (Equation 5.1) in terms of the eigenmodes of the twisted fiber. From Section 4.4, we saw that uniformly twisting a birefringent fiber results in the (initially) linearly-polarized eigenmodes of the fiber becoming elliptically polarized (Equation 4.7).

In general, the birefringence $\delta\beta = (\beta_y - \beta_x)$ of the untwisted fiber at the fundamental wavelength ($\omega_0$) will not be equal to the birefringence at the SH ($2\omega_0$). So the polarization eigenmodes of the twisted fiber at $\omega_0$ will not be the same as the polarization eigenmodes at $2\omega_0$ (Equation 4.7). I can write the eigenmodes of the twisted fiber at $\omega_0$ as: $X^{(\omega_0)} = \begin{bmatrix} 1 \\ \epsilon \end{bmatrix}$ and $Y^{(\omega_0)} = \begin{bmatrix} -\epsilon^* \\ 1 \end{bmatrix}$, while for the SH, I replace $\epsilon$ with $\delta$: $X^{(2\omega_0)} = \begin{bmatrix} 1 \\ \delta \end{bmatrix}$; $\epsilon$ and $\delta$ are complex numbers, and $\epsilon \neq \delta$ in general.

As the SHG process in the fiber is phase-matched (Equation 2.13) for the polarization eigenmodes of the fiber, we can calculate the relative efficiencies of the various SHG signals by writing Equation 5.1 in terms of the $\left(X^{(\omega_0)}, Y^{(\omega_0)}\right)$ and $\left(X^{(2\omega_0)}, Y^{(2\omega_0)}\right)$ bases (specifically $\epsilon$ and $\delta$). (Table 5.1) shows that it is possible for all six phase-matching combinations to have non-zero conversion efficiencies.

<table>
<thead>
<tr>
<th>Table 5.1: Relative Efficiencies of SHG Signals in Twisted Fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal</strong></td>
</tr>
<tr>
<td>$X^{(\omega)} + X^{(\omega)} \rightarrow X^{(2\omega)}$</td>
</tr>
<tr>
<td>$Y^{(\omega)} + Y^{(\omega)} \rightarrow X^{(2\omega)}$</td>
</tr>
<tr>
<td>$Y^{(\omega)} + X^{(\omega)} \rightarrow Y^{(2\omega)}$</td>
</tr>
<tr>
<td>$Y^{(\omega)} + Y^{(\omega)} \rightarrow Y^{(2\omega)}$</td>
</tr>
<tr>
<td>$X^{(\omega)} + X^{(\omega)} \rightarrow Y^{(2\omega)}$</td>
</tr>
<tr>
<td>$Y^{(\omega)} + X^{(\omega)} \rightarrow X^{(2\omega)}$</td>
</tr>
</tbody>
</table>
By plugging into Equation 4.7 and Table 5.1 the measured value of the birefringence at the fundamental wavelength \( \delta \beta^{(\text{0\omega})} = 7.4 \pm 2.8 \times 10^{-5} \mu m^{-1} \) and the calculated birefringence (Appendix B) at the SH \( \delta \beta^{(2\text{\omega})} = 13.4 \times 10^{-5} \mu m^{-1} \) I can plot the theoretical conversion efficiencies for each of the signals (Figure 5.1) as a function of the amount of twist (Figure 5.1).

\[ \frac{40}{2} \]

**Figure 5.1:** The relative conversion efficiencies of all six SHG signals in twisted poled fiber. The twisting results in all possible phase-matching combinations of SH/fundamental polarizations having non-zero conversion efficiencies. The \( X + X \rightarrow Y \) signal is not labeled, as it is more than two orders of magnitude smaller than the other signals.

In Figure 5.1, the relative conversion efficiencies of the six SHG signals are plotted against the number of twists \( \left( \frac{\Omega L}{2\pi} \right) \) experienced by the poled fiber (which is of length \( L \)). I take note of the new type II signal \( Y + X \rightarrow X \). For large enough twist \( \left( \frac{\Omega L}{2\pi} > 2.5 \right) \), the conversion efficiency for this signal can be larger than the \( X + X \rightarrow X \) signal. On the other hand, the relative conversion efficiency of the \( X + X \rightarrow Y \) signal is too small to be observed on the plot (it is two orders of magnitude smaller than all the others signals).

For any twist rate \( \Omega \), I am also able to predict the positions of the six peaks.
That is because the peak separations $\Delta \omega$ are proportional to the fiber birefringence ($\Delta \omega \propto \delta \tilde{\beta}$, see Appendix B), and the birefringence of the twisted fiber is a function of the twist rate $\Omega$ (Equation 4.9). Figure 5.2 shows the theoretical SHG spectrum for 3 different values (red = 0 turns, green = 0.66 turns, blue = 0.95 turns) of twist.

Figure 5.2: Theoretical SHG spectrum for various twists; (red = 0 turns, green = 0.66 turns, blue = 0.95 turns) As a result of the twisting, (see the green and blue spectra), two new signals ($X + Y \rightarrow X$, $Y + Y \rightarrow Y$) now have non-zero conversion efficiency. The positions of the peaks for each value of twist is estimated by considering the birefringence of the twisted fiber at both the second-harmonic and fundamental wavelengths.

5.3 The Experiment

The same setup as described in Section 3.2 and Figure 3.2 is used to measure the SHG, and the fiber is twisted in the same manner as shown in Figure 4.3 and described in Section 4.4.

For each value of the twist, the positions of the six peaks are predicted in the...
same way as was shown in Figure 5.2. For a given signal, I set the tunable laser to be at the predicted (fundamental) wavelength, and I sweep the polarization of the fundamental wavelength until the SH power is maximized. At that point I measure the polarization of the fundamental with the polarimeter; after doing so, I scan the wavelength of the fundamental beam about this peak to obtain its spectrum.

Just as in Figure 5.1, I can plot the experimentally-obtained peak efficiencies of the six signals at various values of twist (Figure 5.3). The data points are accompanied by errorbars, while the solid lines are the efficiencies predicted by the model shown in Figure 5.1. The good agreement between experiment (Figure 5.3) and theory (Figure 5.1) is further indication that the model of the $\chi^{(2)}$ tensor (Equation 5.1) holds true for our fiber.

![Figure 5.3](image)

**Figure 5.3:** The experimental nonlinear transmittances of the various SHG signals are plotted against $|\Omega L/2\pi|$, the number of revolutions the poled fiber is subjected to along its whole length $L$.

I may also plot the experimental SHG spectrum of the various signals at varying twists (Figure 5.4), just as I did for the theoretical spectrum in Figure 5.2. The $Y + Y \rightarrow Y$ SHG signal is not plotted because its spectrum overlaps with the much stronger $X + X \rightarrow X$ signal.

Again, Figure 5.4 demonstrates the fact that with increasing twist, the peaks
**Figure 5.4:** The experimental SHG spectrum for the various signals are plotted at different values of twist. *Red* = 0 revolutions, *Green* = 0.66 revolutions, *Blue* = 0.95 revolutions. The arrows on top of each peak denotes the direction in which that particular peak will evolve with increasing twist.

will shift. This is because the birefringence of the fiber (at both fundamental and SH wavelengths) has been modified (increased) by the twisting (Equation 4.9). However, because the fundamental and SH birefringences of the untwisted fiber are unequal \( \left( \beta_y^{(2\omega)} - \beta_x^{(2\omega)} \right) \neq \left( \beta_y^{(\omega)} - \beta_x^{(\omega)} \right) \), this results in the different signals shifting in different directions (either to higher or lower wavelengths) at varying rates.
5.4 Another Estimation of Fiber Birefringence

Consider the four signals I observed in Figure 5.4. I can label their phase-matched wavelengths as in Table 5.2.

Table 5.2: The Four Observed SHG Signals in Twisted Poled Fiber

<table>
<thead>
<tr>
<th>Fundamental Polarization</th>
<th>SH Polarization</th>
<th>Peak Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X + X$</td>
<td>$X$</td>
<td>$\lambda_a$</td>
</tr>
<tr>
<td>$Y + Y$</td>
<td>$X$</td>
<td>$\lambda_b$</td>
</tr>
<tr>
<td>$Y + X$</td>
<td>$X$</td>
<td>$\lambda_c$</td>
</tr>
<tr>
<td>$Y + X$</td>
<td>$Y$</td>
<td>$\lambda_d$</td>
</tr>
</tbody>
</table>

The peak separation $|\lambda_a - \lambda_b|$ is directly proportional to the modified birefringence at the fundamental wavelength (see Appendix B). I recall from Equation 4.9 that the modified birefringence $\delta \tilde{\beta}^{(\omega)} = \tilde{\beta}_y^{(\omega)} - \tilde{\beta}_x^{(\omega)}$ of the twisted fiber is a function of the twist rate $\Omega$. So, I can write the peak separation $|\lambda_a - \lambda_b|$ as a function of the twist rate $\Omega$ (Equation 5.2).

$$|\lambda_a - \lambda_b|\Omega = |\lambda_a - \lambda_b|\Omega=0 \cdot \sqrt{1 + \left( \frac{2\Omega(1 - \frac{g}{2})}{\beta_y^{(\omega)} - \beta_x^{(\omega)}} \right)^2}$$  
(5.2)

where $|\lambda_a - \lambda_b|\Omega$ is the peak separation when the twist rate is $\Omega$.

In a similar vein, the peak separation $|\lambda_c - \lambda_d|$ is directly proportional to the modified birefringence at the SH. A similar equation equates the twist to the peak separation

$$|\lambda_c - \lambda_d|\Omega = |\lambda_c - \lambda_d|\Omega=0 \cdot \sqrt{1 + \left( \frac{2\Omega(1 - \frac{g}{2})}{\beta_y^{(2\omega)} - \beta_x^{(2\omega)}} \right)^2}$$  
(5.3)

A maximum likelihood (chi-square) fit using Equations 5.2 and 5.3 gives us the birefringence of the untwisted poled fiber at the fundamental and SH wavelengths (respectively).

From Figure 5.5, I am able to extract values of the birefringence at the funda-
Figure 5.5: a) The peak separation $|\lambda_d - \lambda_b|$ plotted against the twist $\Omega$. The fit of this data with Equation 5.2 gives another estimate at the fundamental wavelength for the untwisted fiber. b) The peak separation $|\lambda_c - \lambda_d|$ plotted against the twist $\Omega$. The fit of this data with Equation 5.2 gives an estimate of the birefringence at the SH wavelength for the untwisted fiber.

For the untwisted fibers. Comparing this to the experimental value of the birefringence at the fundamental wavelength ($7.4 \pm 2.8 \times 10^{-5} \mu m^{-1}$) obtained in Chapter 4, I find that the values yielded from both linear (Section 4.4) and nonlinear (Figure 5.5a) experiments are consistent.

5.5 Summary

By twisting the fiber I was able to demonstrate the ability to control both the strengths and the peak positions of new SHG signals, albeit not independently of
each other. The experimental twist data is in good agreement with the theory, and so is consistent with the model of the $\chi^{(2)}$ in poled fiber (Section 2.2). This is the first analysis and demonstration of frequency conversion in twisted birefringent fiber.

Finally, I have found yet another way to assess the fiber birefringence, this time by looking at the peak separations of the various SHG signals and plotting them as a function of the twist. The results are consistent with what I found in Chapter 4.
Chapter 6

Sum-Frequency Generation

6.1 Introduction

So far, I have only performed SHG experiments on our poled fibers. These SHG experiments tell us little about the bandwidth of their phase-matching. Suppose I wish to use the poled fiber to generate parametrically down-converted photon pairs (Figure 1.2), or use the fiber as part of an optical parametric oscillator (OPO). What would be bandwidth of the SPDC, or the tuning curve of the OPO? Sum-frequency generation (SFG) gives the answer to both those problems.

6.2 Experimental Setup

From Section 2.4, the phase-matching condition for which $\Delta \neq 0$ and $j \neq k$ results in type-II SFG. I wish to observe this process.

As the fundamental lightwaves are no longer degenerate, I will need two amplified TLS sources. These fundamental lightwaves are also orthogonally polarized, and will have to be combined into one fiber before being launched into the poled fiber. It turns out that this is easily done with the PBS I used in Figure 3.2, as it has three ports (2 inputs, 1 output). The modified setup is shown in Figure 6.1.

There is a problem with the setup in Figure 6.1. I cannot monitor the individual
Figure 6.1: a) Two amplified laser sources at the fundamental wavelength are combined with a PBS. But as they enter the splitter from different ports, their polarizations are automatically orthogonal to each other. FPC1 and FPC3 are used to maximize the individual beam powers, and FPC2 is used to adjust the polarizations being launched into the PPSF so that one beam is X polarized, and the other is Y polarized.

b) At the output of the poled fiber, the same 1550/775 nm WDM separates the fundamental (1550 nm) from the second-harmonic (775 nm), with meters monitoring their respective powers.

powers of the fundamental beams during the SFG experiment. I circumvent this problem by measuring only a small part of the SFG spectrum at a time; between each of these runs, I measure the powers of the fundamental beams separately. I then stitch together the spectrum (Figure 6.2).

6.3 Results

The efficiency of the type-II SFG process is measured by the nonlinear transmittance \( \eta_{SF} \): 
\[
\eta_{SF} = \frac{P_{SH}}{P_{FX} P_{FY}},
\]
where \( P_{Fj} \) is the power of the \( j \)th-polarized field at the fundamental wavelength.

I perform the SFG experiment on our twisted poled fiber sample \( \frac{|\Omega L|}{2\pi} \approx 0.43 \) turns. Shown in Figure 6.2 are the type-II SFG spectra for the \( Y + X \rightarrow X \) and \( Y + X \rightarrow Y \) signals. The fundamental wavelengths are swept from 1520-1580 nm,
in 0.1 nm steps. This range is dictated by the operating wavelength range of the EDFAs that I use (Section 3.2). The nonlinear transmittance $\eta_{SF}$ of the $Y + X \rightarrow X$ process has been scaled by a factor of 4 so that its spectrum looks comparable to the $Y + X \rightarrow Y$ signal.

$\eta_{SF}$ of the $Y + X \rightarrow X$ process has been scaled by a factor of 4 so that its spectrum looks comparable to the $Y + X \rightarrow Y$ signal.

Both of the signals in Figure 6.2 have missing parts in their spectra. In these missing parts, the SFG signals were drowned out by the type-I SHG signals ($X + X \rightarrow X, Y + Y \rightarrow X$).

The results seem to suggest that the $Y + X \rightarrow X$ SFG signal, which arises only in twisted poled fiber, is also broadband. And by broadband, I really mean that the fundamental wavelengths can be detuned far away from each other. However, the SH wavelengths at which these two signals are phase-matched are not broadband, as seen in the tuning curve (Figure 6.3). I can extend the tuning curve by applying...
what I know of the fiber geometry (core radius $a \approx 2.5 \, \mu m$, index contrast $\Delta n \approx 0.014$). From that information, I can calculate the fiber dispersion, and applying the measured fiber birefringence and positions of the peaks as they appear in the experimental spectrum, a predicted extension of the tuning curve emerges (Figure 6.3).

![Tuning curve for the two type-II SFG signals](image)

**Figure 6.3:** Tuning curve for the two type-II SFG signals, where the nonlinear transmittance $\eta_{SF}$ is plotted against the wavelength of the $Y$-polarized fundamental field, and the sum-frequency wavelength $\lambda_{SF}$. The experimental tuning curve is extended with a predicted tuning curve. The prediction is based upon dispersion calculations for a particular fiber geometry.

### 6.4 Summary

By demonstrating that type-II SFG in poled fiber is possible, this gives us confidence that type-II SPDC (the opposite process) is also possible. I have shown that the SFG phase-matching at the fundamental wavelength is at least 60 nm wide. This leads us to believe that the bandwidth of SPDC generated in our poled fiber will be at least 60 nm.
A tuning curve has also been obtained from the SFG data, which may be useful information when constructing an OPO.
Chapter 7

Conclusions

7.1 Summary

In this thesis, I have determined that the $\chi^{(2)}$ tensor elements relevant to a poled fiber have magnitudes as predicted by the space-charge model of the second-order nonlinearity [19]:

$$\left| \chi^{(2)}_{xxx} \right| = 3 \left| \chi^{(2)}_{yxy} \right| = 3/2 \left| \chi^{(2)}_{yyx} + \chi^{(2)}_{yxy} \right|$$

This was done in Chapter 3, where I measured the magnitudes of the individual $\chi^{(2)}$ tensor elements directly with the SHG process.

Furthermore, by effecting a twist in the fiber (Chapter 5), and generating SH signals that involved 5 of the 6 possible permutations of fundamental and second-harmonic polarizations, comparison with the theory has shown that my experimental data is in agreement with:

$$\chi^{(2)}_{xxx} = 3 \chi^{(2)}_{yxy} = 3 \chi^{(2)}_{yxy} = 3 \chi^{(2)}_{yyx}$$

To demonstrate the broadband phase-matched nature of these nonlinearities in the poled fiber, I performed type-II sum-frequency generation (Chapter 6).
And finally, I was able to develop two different approaches to measuring the fiber birefringence based upon fiber twisting: a completely linear-optical method, and another method based upon observing the peak separations in the SHG spectra of the twisted fiber. The two methods were found to give values of the fiber birefringence that were in good agreement.

7.2 Significance
To date, we are the only group to have definitively observed spectrally-resolved type-I and type-II SHG in poled fiber. Consequently, I have been able to map out the $\chi^{(2)}$ tensor elements in poled fiber; their values support the conventional model (Section 2.2) of the SON in poled fiber.

Additionally, I showed experimentally the ability to control both the strengths and the peak positions of all possible SHG signals in the poled fiber simply by the application of a twist. To our knowledge, this is the first demonstration and analysis of parametric generation in twisted nonlinear fiber.

7.3 Future Work
The groundwork has now been laid for observing type-II SPDC in periodically poled fibers. This process is akin to time-reversed SFG, where pump light at $2\omega$ in a $\chi^{(2)}$ medium is used to generate light beams of frequency $\approx \omega$ (Figure 1.2). SPDC is an important process for the generation of single photons [11] and entangled photon pairs [23], which are important resources in quantum optics and quantum information.

We now know the magnitudes of the $\chi^{(2)}$ tensor elements (Chapter 3), with which we can use to estimate the efficiency of the SPDC process [37]. We believe a type-II SPDC generation rate of $> 2.5 \times 10^6$ pairs/s with a 50-mW CW pump is possible.

The tuning curve I observed for the SFG spectrum (Chapter 6) can be used to give us a sense of the SPDC bandwidth.
The DGD measurements (Chapter 4) gives the walkoff between the $x$- and $y$-polarized fields, which can be important information in the generation of entangled photon pairs. If the walkoff is too large (which it is not in this case), the entangled photon pairs will be distinguishable from one another and the photons generated via SPDC will no longer be entangled.
Appendices
Appendix A

The SHG Formalism

In this Appendix, we find the equations of motion for SHG. We start from basic principles (nonlinear wave equation), and by making judicious approximations (slowly-varying envelope, non-depleting pump approximation), we can relate the second-harmonic power \( P_{SH} \) to the fundamental power \( P_F \) through such things as the poled fiber length \( L \), the value of the second-order nonlinearity \( \chi^{(2)} \), and the phase-matching condition.

Let us consider the type-I SHG signal \( x + x \rightarrow x \) due to the \( \chi^{(2)}_{XXX} \) tensor element. The SH field can be written as this:

\[
\vec{E}(2\omega_0)(\vec{r},t) = \hat{x} \cdot E_x^{(2\omega_0)}(z) \cdot \cos(\beta^{(2\omega_0)}z - 2\omega_0 t + \phi^{(2\omega_0)}(z))
\]

(A.1)

where we’ve separated the field amplitude \( E_x^{(2\omega_0)} \) into two parts, the transverse eigenmode \( u^{(2\omega_0)}(x,y) \) and longitudinal envelope function \( A^{(2\omega_0)}(z) \). We can write a similar expression (Equation A.2) for the field at \( \vec{E}(\omega_0) \).

\[
\vec{E}(\omega_0)(\vec{r},t) = \hat{x} \cdot E_x^{(\omega_0)} \cdot \cos(\beta^{(\omega_0)}z - 2\omega_0 t + \phi^{(\omega_0)}(z))
\]

(A.2)
The \( \{ \beta_{lm}^{(\omega)}, u_{lm}^{(\omega)}(x,y) \} \) satisfy the eigenvalue equation for the weakly-guiding fiber waveguide:

\[
\left( \nabla_t^2 + \frac{(n_{l}^{(\omega)})^2 \omega^2}{c^2} \right) u_{lm}^{(\omega)}(x,y) = \left( \beta_{lm}^{(\omega)} \right)^2 u_{lm}^{(\omega)}(x,y) \tag{A.3}
\]

where \( \nabla_t^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \), and \( lm \) represents the LP \( \text{lm} \) mode \([16]\) of the fiber. For convenience, we will drop the \( lm \) subscripts.

We plug in the net field \( \vec{E}_{\text{net}} = \vec{E}^{(2\omega_0)} + \vec{E}^{(\omega_0)} \) into the nonlinear wave equation (Equation 2.1). However, we only care about the \( \vec{E}^{(2\omega_0)} \), which simplifies matters:

\[
\nabla^2 \vec{E}^{(2\omega_0)} - \left( \frac{n^{(2\omega_0)}}{2} \right)^2 \frac{\partial \vec{E}^{(2\omega_0)}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{\mathcal{P}}_{\text{NL}}^{(2\omega_0)}}{\partial t^2} \tag{A.4}
\]

As we worry only about SHG, the nonlinear polarization \( \vec{\mathcal{P}}_{\text{NL}} \) is truncated to second order:

\[
\vec{\mathcal{P}}_{\text{NL}} = \hat{x} \mathcal{P}_{\text{NL},x} = \hat{x} \varepsilon_0 \chi_{xxx}^{(2)} \left( \begin{array}{c} \varepsilon_x^{(\omega_0)} \cos \left( \beta^{(\omega_0)} z - \omega_0 t + \phi^{(\omega_0)}(z) \right) + \\
\varepsilon_x^{(2\omega_0)} \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)}(z) \right) \end{array} \right)^2 \tag{A.5}
\]

from which we can extract the only non-zero term that is at frequency \( 2\omega_0 \):

\[
\mathcal{P}_{\text{NL},x}^{(2\omega_0)} = \frac{1}{2} \varepsilon_0 \chi_{xxx}^{(2)} \left( \varepsilon_x^{(\omega_0)} \right)^2 \cos \left( 2\beta^{(2\omega_0)} z - 2\omega_0 t + 2\phi^{(2\omega_0)}(z) \right) \tag{A.6}
\]

We may then write the right-hand side of Equation A.4 like this:

\[
\mu_0 \frac{\partial^2 \vec{\mathcal{P}}_{\text{NL}}^{(2\omega_0)}}{\partial t^2} = -\frac{(2\omega_0)^2}{c^2} \left( \frac{1}{2} \chi_{xxx}^{(2)} \left( A^{(2\omega_0)}(z) \right)^2 \left( u^{(2\omega_0)}(x,y) \right)^2 \right) \times \cos \left( 2\beta^{(2\omega_0)} z - 2\omega_0 t + 2\phi^{(2\omega_0)}(z) \right) \tag{A.7}
\]

As for the left-hand-side of Equation A.4, we can insert Equation A.1 and
\[
\left( \nabla^2_t + \frac{\partial^2}{\partial z^2} + \frac{n^2(2\omega_0)^2}{c^2} \right) \left[ e^{(2\omega_0)} \cdot \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} (z) \right) \right] = \\
A^{(2\omega_0)}(z) \cos(\beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)}) \left( \nabla^2_t + \frac{n^2(2\omega_0)^2}{c^2} \right) u^{(2\omega_0)}(x,y) \\
+ u^{(2\omega_0)}(x,y) \frac{d^2}{dz^2} \left( A^{(2\omega_0)} \cos(\beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)}) \right)
\]

(A.8)

The \( \frac{d^2}{dz^2}(\cdot) \) term in Equation A.8 is going to cause us some trouble. Let us expand it:

\[
\frac{d^2}{dz^2} \left( A^{(2\omega_0)} \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right) \right) = \\
\left( - \left( \beta^{(2\omega_0)} + \frac{d\phi^{(2\omega_0)}}{dz} \right)^2 A^{(2\omega_0)} + \frac{d^2 A^{(2\omega_0)}}{dz^2} \right) \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right) \\
- \left( 2 \left( \beta^{(2\omega_0)} + \frac{d\phi^{(2\omega_0)}}{dz} \right) \frac{dA^{(2\omega_0)}}{dz} + A^{(2\omega_0)} \frac{d^2 \phi^{(2\omega_0)}}{dz^2} \right) \sin \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right)
\]

If we apply the slowly-varying envelope approximation:

\[
\left( \beta^{(2\omega_0)} \right)^2 \gg \beta^{(2\omega_0)} \left| \frac{d\phi^{(2\omega_0)}}{dz} \right| \gg \left| \frac{d^2 \phi^{(2\omega_0)}}{dz^2} \right| \\
\left( \beta^{(2\omega_0)} \right)^2 A^{(2\omega_0)} \gg \beta^{(2\omega_0)} \left| \frac{dA^{(2\omega_0)}}{dz} \right| \gg \left| \frac{d^2 A^{(2\omega_0)}}{dz^2} \right|
\]
to the $\frac{d^2}{dz^2} (\cdot)$ term,

$$
\frac{d^2}{dz^2} \left( A^{(2\omega_0)} \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right) \right) \approx
- \left( \beta^{(2\omega_0)} \right)^2 A^{(2\omega_0)} \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right)
- 2\beta^{(2\omega_0)} \frac{dA^{(2\omega_0)}}{dz} \sin \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right)
- 2\beta^{(2\omega_0)} A^{(2\omega_0)} \frac{d\phi^{(2\omega_0)}}{dz} \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} \right)
$$

(A.9)

Plugging Equation A.9 back into Equation A.8 and appealing to Equation A.3, we eventually end up with:

$$
\left( \nabla_i^2 + \frac{\partial^2}{\partial z^2} + \frac{n^2(2\omega_0)^2}{c^2} \right) \left[ \phi^{(2\omega_0)} \cdot \cos \left( \beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} (z) \right) \right]
\approx - 2\beta^{(2\omega_0)} \frac{dA^{(2\omega_0)}}{dz} \cdot u^{(2\omega_0)} \cdot \sin(\beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} (z))
- 2\beta^{(2\omega_0)} \frac{d\phi^{(2\omega_0)}}{dz} \cdot A^{(2\omega_0)} \cdot u^{(2\omega_0)} \cdot \cos(\beta^{(2\omega_0)} z - 2\omega_0 t + \phi^{(2\omega_0)} (z))
$$

(A.10)

Replacing the left- and right-hand side of Equation A.4 with Equation A.10 and Equation A.6, and discarding all the ’positive frequency’ terms ($\sin (\omega t + \Phi (z)) = \frac{1}{2\pi} (e^{i(\omega t + \Phi (z))} + e^{-i(\omega t + \Phi (z))}) \to \frac{1}{2\pi} e^{-i(\omega t + \Phi (z))}$) we eventually end up with:

$$
2i\beta^{(2\omega_0)} \cdot u^{(2\omega_0)} \cdot \frac{d\tilde{A}^{(2\omega_0)}}{dz} = - \frac{1}{2} \frac{(2\omega_0)^2}{c^2} \chi^{(2)}_{\text{xxx}} \left( \tilde{A}^{(2\omega_0)} \right)^2 \left( u^{(2\omega_0)} \right)^2 e^{i(2\beta^{(2\omega_0)} - \beta^{(2\omega_0)}) z}
$$

(A.11)

where $\tilde{A}^{(2\omega_0)} \equiv A^{(2\omega_0)} e^{i\phi^{(2\omega_0)}}$.

We simplify further by multiplying both sides by $\left( u^{(2\omega_0)} \right)^* \cdot i$ and integrating
over the fiber cross-sectional area $\mathcal{A}$:

$$
(2\pi a^2) \frac{d\tilde{A}^{(2\omega_0)}}{dz} =
$$

$$
\frac{i}{4} \frac{(2\omega_0)^2}{\beta^{(2\omega_0)} c^2} \chi^{(2)}_{xxx} \left( \tilde{A}^{(\omega_0)} \right)^2 \left( \int d\mathcal{A} \left( u^{(\omega_0)} \right)^2 \left( u^{(2\omega_0)} \right)^* \right) e^{i(2\beta^{(\omega_0)} - \beta^{(2\omega_0)})z} \tag{A.12}
$$

where $a$ is the core radius of the fiber. Defining the effective area as:

$$
\mathcal{A}_{eff} \equiv (2\pi a^2) \left( \int d\mathcal{A} \left( u^{(\omega_0)} \right)^2 \left( u^{(2\omega_0)} \right)^* \right)^{-2} \tag{A.13}
$$

we can simplify further:

$$
\frac{d\tilde{A}^{(2\omega_0)}}{dz} = \frac{i}{4} \frac{(2\omega_0)^2}{\beta^{(2\omega_0)} c^2} \chi^{(2)}_{xxx} \left( \tilde{A}^{(\omega_0)} \right)^2 \left( \int \frac{2\pi a^2}{\mathcal{A}_{eff}} \right) e^{i(2\beta^{(\omega_0)} - \beta^{(2\omega_0)})z} \tag{A.14}
$$

In the undepleted pump approximation, the fundamental field $\tilde{A}^{(\omega_0)} \equiv A^{(\omega_0)} e^{i\phi^{(\omega_0)}}$ is taken to be independent of $z$. So, by integrating Equation A.14 subject to the initial condition $\tilde{A}^{(2\omega_0)}(z = 0) = 0$, we can solve for $\tilde{A}^{(2\omega_0)}(z)$.

An added complication arises because of quasi-phase-matching. The $\chi^{(2)}_{xxx}$ is a periodic function of $z$:

$$
\chi^{(2)}_{xxx}(z) = \frac{\chi^{(2)}_{xxx}}{2} \left( sgn \left( \cos \left( \frac{2\pi}{\Lambda} z \right) \right) + 1 \right) \tag{A.15}
$$

with $\Lambda$ the quasi-phase-matching period, and $sgn(x) = \begin{cases} 
\frac{x}{|x|}, & \text{for } x \neq 0 \\
0, & \text{for } x = 0
\end{cases}$. To first order (in the Fourier series), we may approximate $sgn(\cos \left( \frac{2\pi}{\Lambda} z \right)) \approx \frac{2}{\pi} \cos \left( \frac{2\pi}{\Lambda} z \right)$ and accordingly, $\chi^{(2)}_{xxx}(z) \approx \frac{1}{2\pi} (\chi^{(2)}_{xxx})_{max} e^{\pm iKz}$, where $K \equiv \frac{2\pi}{\Lambda}$.
Integrating Equation A.14 with respect to $z$ and squaring the result, we have

$$
|\tilde{A}(2\omega_0)(z)|^2 = \frac{z^2}{64\pi^2} \left(\frac{2\omega_0}{n_{eff}(2\omega_0)/c}\right)^2 \left(\frac{2\pi a^2}{\omega_{eff}}\right)^2 \left(|\tilde{A}(\omega_0)|^4 (\chi_{xxx})_{\text{max}}(2)\right) \times
sinc^2\left(\frac{(2\beta(\omega_0) - \beta(2\omega_0) \pm K) z}{2}\right) \tag{A.16}
$$

where we define $sinc(x) \equiv \frac{\sin(x)}{x}$, and $n_{eff}(2\omega_0) \equiv \frac{\beta(2\omega_0) c}{2\omega_0}$. The term inside the $sinc$ function $\left(2\beta(\omega_0) - \beta(2\omega_0) \pm K\right)$ is what is known as the phase-matching condition.

Taking Equation A.14 and replacing $z$ with $L$ (the length of the poled fiber), and writing everything in terms of power $\left(\frac{P(\omega_0)}{n_{eff}(2\omega_0)/c} \equiv |A(\omega_0)(z)|^2\right)$ (see Section A.2):

$$
P(2\omega_0) = \left(\frac{\omega_0}{c}\right)^2 \frac{(\chi_{xxx})_{\text{max}}(2)^2}{8\pi^2 \varepsilon_0 \omega_{eff}(2\omega_0) n_{eff}(2\omega_0)^2} \times
sinc^2\left(\frac{(2\beta(\omega_0) - \beta(2\omega_0) \pm K) z}{2}\right) \tag{A.17}
$$

we get an expression that relates the fundamental power $P(\omega_0)$ to the second-harmonic power $P(2\omega_0)$ through the poled fiber length ($L$), fiber dispersion ($\beta(2\omega_0)$, $\beta(\omega_0)$), and nonlinearity strength ($\chi^{(2)}$).

By taking a particular fiber geometry ($a = 2.5 \, \mu m$, index contrast $\Delta n = 0.014$), we arrive at a value of 0.067 pm/V for $(\chi_{xxx})_{\text{max}}^{(2)}$ (see Figure A.1).

Note that the values of the $\chi^{(2)}$ tensor elements given in Table 4.1 are of $(\chi_{xxx})_{\text{max}}^{(2)}$, $(\chi_{xyy})_{\text{max}}^{(2)}$, etc.

And what of the $y+x \rightarrow y$ SHG signal? Here, the $\chi_{xyy}^{(2)}$ tensor is used. Beginning
Figure A.1: We fit the experimental SHG data (red) to Equation A.17 (black). We arrive at a value of 0.067 pm/V for \((\chi_{\text{xx}}^{(2)})_{\text{max}}\).

again with the polarization vector:

\[
\mathcal{P}_{NL,y} = \varepsilon_{0} \chi_{yxy}^{(2)} \left( \varepsilon_{y}^{(2\omega_{0})} \cos \left( \beta_{y}^{(2\omega_{0})} z - 2\omega_{0} t + \phi_{y}^{(2\omega_{0})} \right) \right) ^{2} \\
+ \varepsilon_{x}^{(\omega_{0})} \cos \left( \beta_{x}^{(\omega_{0})} z - \omega_{0} t + \phi_{x}^{(\omega_{0})} \right) \\
+ \varepsilon_{y}^{(\omega_{0})} \cos \left( \beta_{y}^{(\omega_{0})} z - \omega_{0} t + \phi_{y}^{(\omega_{0})} \right)
\]

\[
\mathcal{P}_{NL,y}^{(2\omega_{0})} = \varepsilon_{0} \chi_{yxy}^{(2)} \left( \varepsilon_{x}^{(\omega_{0})} \varepsilon_{y}^{(\omega_{0})} \cos \left( \beta_{x}^{(\omega_{0})} \beta_{y}^{(\omega_{0})} z - 2\omega_{0} t + \phi_{x}^{(\omega_{0})} + \phi_{y}^{(\omega_{0})} \right) \right) \\
(A.18)
\]
Going through the same motions as above, we arrive at:

\[
P_y^{(2\omega_0)} = \left( \frac{\omega_0}{c} L \right)^2 \frac{ \left( X_{\text{max},\text{max}}^{(2)} \right)^2 P_x^{(\alpha_h)} \cdot P_y^{(\alpha_h)}}{2\pi^2 n_{\text{eff},x} n_{\text{eff},y} \cdot c E_0^2} \times \frac{\text{sinc}^2 \left( \left( \beta_y^{(\alpha_h)} - \beta_x^{(\alpha_h)} + \beta_y^{(\alpha_h)} \pm K \right) \frac{L}{2} \right)}{(A.19)}
\]

The fundamental power is now split into two parts, one for the \(x\)-polarized field \((P_x^{(\alpha_h)})\) and one for the \(y\)-polarized field \((P_y^{(\alpha_h)})\). The \(A_{\text{eff}}^{xy}\) is the effective area, defined similarly to Equation A.13:

\[
A_{\text{eff}}^{xy} \equiv (2\pi a^2)^{-2} \left( \int dA u^{(\alpha_h)} \cdot u^{(\alpha_h)} \right)^{-1}
\]

where we’ve written the mode profiles \(u^{(\alpha_h)}\) as functions of polarization. Although in actuality we calculate them from the infinite-cladding approximation [16] without regard for polarization dependence.

So, we find that for the type-I SHG signals, \(P^{(2\omega_0)} \propto \left( P^{(\omega_0)} \right)^2\), and for the type-II SHG signals, \(P^{(2\omega_0)} \propto \left( P_x^{(\alpha_h)} \cdot P_y^{(\alpha_h)} \right)\).

### A.1 Mode Normalization

The guiding modes \(u_{im}^{(\alpha_h)}(x,y)\) of a step-index fiber in the weakly-guiding approximation are determined by the following waveguide eigenvalue equation [16]:

\[
\left( \nabla_i^2 + \frac{\left( n^{(\alpha_h)} \right)^2 \omega^2}{c^2} \right) u_{im}^{(\alpha_h)}(x,y) = \left( \beta_{im}^{(\alpha_h)} \right)^2 u_{im}^{(\alpha_h)}(x,y)
\]

By convention, I take the mode profile to be normalized like this:

\[
\int dA \left| u_{im}^{(\alpha_h)} \right|^2 = 2\pi a^2
\]
Note that both in Equation A.21 and the way in which we define the fields (Equation A.1, Equation A.2) means that $u_{lm}(x,y)$ is unitless.

### A.2 Power Normalization

Consider the electric field inside the fiber at frequency $\omega$:

$$\vec{E}^{(\omega)}(\vec{r}, t) = \hat{x} \cdot E^{(\omega)}(\vec{r}) \cos \left( \beta^{(\omega)}z - \omega t + \phi^{(\omega)}(z) \right) = \hat{x} \cdot A^{(\omega)} u^{(\omega)} \cos \left( \beta^{(\omega)}z - \omega t + \phi^{(\omega)}(z) \right)$$  (A.22)

The intensity $I^{(\omega)}$ is:

$$I^{(\omega)} = \frac{cn_{eff}^{(\omega)} \varepsilon_0}{2} |E^{(\omega)}(\vec{r})|^2$$  (A.23)

We can show this by writing down the definition of the intensity $I^{(\omega)}$:

$$I^{(\omega)} \equiv \langle |\vec{E}^{(\omega)} \times \vec{H}^{(\omega)}| \rangle$$  (A.24)

The $\vec{E}$-field is related to the $\vec{H}$ field via Faraday’s law. Under the assumption of the slowly-varying envelope approximation again ($\left| \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial z} \right|, \left| \frac{\partial \phi}{\partial z} \right| \ll \beta$) the $\vec{H}$-field will look like:

$$\vec{H}^{(\omega)} = -\hat{y} \left( \frac{\beta^{(\omega)}}{\mu_0 \omega} \right) E^{(\omega)} \cos \left( \beta^{(\omega)}z - \omega t + \phi^{(\omega)} \right)$$  (A.25)

So, $I^{(\omega)} = \frac{cn_{eff}^{(\omega)} \varepsilon_0}{2} |E^{(\omega)}|^2$.

The total power (after integrated over the cross-sectional area $\mathcal{A}$ of the fiber) and using Equation A.21 is:

$$P^{(\omega)} = \frac{cn_{eff}^{(\omega)} \varepsilon_0}{2} \left| A^{(\omega)}(z) \right|^2 \int d\mathcal{A} \left( |u^{(\omega)}|^2 \right) = cn_{eff}^{(\omega)} \varepsilon_0 \pi a^2 |A^{(\omega)}|^2$$  (A.26)
Appendix B

Estimating Birefringence from Peak Separations

In this Appendix, we relate the birefringence $\delta \beta := \beta_y - \beta_x$ of the fiber to the fiber dispersion and the spectral separation between the various SHG signals.

Experimentally, in the untwisted fiber (Chapter 3), we observed three spectrally-separated peaks, each corresponding to a different combination of the fundamental and second-harmonic polarizations:

<table>
<thead>
<tr>
<th>Fundamental Polarization</th>
<th>SH Polarization</th>
<th>Phase-Matched Fundamental Freq ($\omega_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x+x$</td>
<td>$x$</td>
<td>$\omega_1$</td>
</tr>
<tr>
<td>$y+y$</td>
<td>$x$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td>$x+y$</td>
<td>$y$</td>
<td>$\omega_3$</td>
</tr>
</tbody>
</table>

We can relate these peak separations to the birefringence at the fundamental frequency ($\omega_0$):

$$\beta_y^{(\omega_0)} - \beta_x^{(\omega_0)} = (\omega_2 - \omega_1) \left( \frac{d\beta^{(2\omega_0)}}{d\omega} - \frac{d\beta^{(\omega_0)}}{d\omega} \right)$$

(B.1)

By $\frac{d\beta^{(\omega_k)}}{d\omega}$, we mean $\frac{d\beta}{d\omega}$ evaluated at $\omega_k$ using the weakly-guiding solutions [16].
B.1 Derivation

Consider the phase-matching conditions for the two type-I signals in Table B.1:

\[ \beta^{(2\omega_1)} = \beta^{(\omega_1)} + \beta^{(\omega_1)} + K \quad (B.2) \]
\[ \beta^{(2\omega_2)} = \beta^{(\omega_2)} + \beta^{(\omega_2)} + K \quad (B.3) \]

Subtracting equations B.2 from B.3:

\[ \beta^{(2\omega_2)} - \beta^{(2\omega_1)} = 2(\beta^{(\omega_2)} - \beta^{(\omega_1)}) \]

and expanding the left (LHS) and right (RHS) sides:

\[ LHS = \beta^{(2\omega_2)} - \beta^{(2\omega_1)} \]
\[ = 2(\omega_2 - \omega_1) \left( \frac{d\beta_x}{d\omega} \right)_{\omega=2\omega_1} + O((\omega_2 - \omega_1)^2) \]
\[ RHS = 2 \left( \beta^{(\omega_2)} - \beta^{(\omega_1)} \right) \]
\[ = 2 \left( (\beta^{(\omega_2)} - \beta^{(\omega_1)}) + (\beta^{(\omega_2)} - \beta^{(\omega_1)}) \right) \]
\[ = 2 \left( \beta^{(\omega_2)} - \beta^{(\omega_1)} \right) + 2 \left( (\omega_2 - \omega_1) \left( \frac{d\beta_x}{d\omega} \right)_{\omega=\omega_1} + O((\omega_2 - \omega_1)^2) \right) \]

we get the following expression if we ignore the 2nd and higher order terms \( O((\omega_2 - \omega_1)^2) \):

\[ \beta^{(\omega_2)} - \beta^{(\omega_1)} = (\omega_2 - \omega_1) \left( \frac{d\beta_x^{(2\omega_1)}}{d\omega} - \frac{d\beta_x^{(\omega_1)}}{d\omega} \right) \quad (B.4) \]
By \( \frac{d\beta}{d\omega}^{(\omega_0)} \), we mean \( \frac{d\beta}{d\omega} \) evaluated at \( \omega_0 \). If we make the assumption that \( \frac{d\beta}{d\omega}^{(\omega_0)} \approx \frac{d\beta}{d\omega} \), can be approximated by the \( \frac{d\beta}{d\omega} \) that is calculated for an infinite-cladding weakly-guiding step-index fiber [16], then we arrive at the expected result, Equation B.1.

### B.2 Application

Because the birefringence \( \delta\beta \) is proportional to the peak separation \( \Delta\omega \), we can also estimate where (in the spectrum) the peaks associated with other combinations of the second-harmonic and fundamental polarizations may occur:

**Table B.2: Predicted Positions of Other SHG Signals Not Observed in Untwisted Poled Fiber**

<table>
<thead>
<tr>
<th>Fundamental Polarization</th>
<th>SH Polarization</th>
<th>Phase-Matched Fundamental Freq (( \omega_0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y + x )</td>
<td>( x )</td>
<td>( \frac{\omega_1 + \omega_2}{2} )</td>
</tr>
<tr>
<td>( x + x )</td>
<td>( y )</td>
<td>( \omega_3 - \frac{\omega_2 - \omega_1}{2} )</td>
</tr>
<tr>
<td>( y + y )</td>
<td>( y )</td>
<td>( \omega_3 + \frac{\omega_2 - \omega_1}{2} )</td>
</tr>
</tbody>
</table>

By taking the \( y + x \to x \) (Table B.2) and \( y + x \to y \) (Table B.1) peak positions, we can then find the birefringence at the second-harmonic:

\[
\beta_y^{(2\omega_0)} - \beta_x^{(2\omega_0)} = 2 \left( \omega_3 - \frac{\omega_1 + \omega_2}{2} \right) \left( \frac{d\beta^{(2\omega_0)}}{d\omega} - \frac{d\beta^{(\omega_0)}}{d\omega} \right) \tag{B.5}
\]

The derivation of Equation B.5 follows the same line of reasoning as the derivation of Equation B.1.
Appendix C

Relating the Inner Product in Jones Space to the Dot Product in Stokes Space

It is possible to relate the Jones and Stokes vector views of the polarization through a simple equation:

\[ |\langle a | b \rangle|^2 = \frac{1 + \vec{s}_a \cdot \vec{s}_b}{2} \]  

(C.1)

The derivation follows in the body of this appendix.

The polarization state \(| p \rangle\) of a light beam in fiber can be written as the coherent superposition of two orthogonal polarizations \(| \pm \rangle\):

\[ | p \rangle = \alpha | + \rangle + \gamma | - \rangle \]

where \(\alpha\) and \(\gamma\) are complex numbers with the constraint \(|\alpha|^2 + |\gamma|^2 = 1\). The Jones vector is constructed by representing \(| p \rangle\) as a column vector:
Consider the outer product $|a\rangle\langle a| b\rangle\langle b|$. The trace of this operator can be written in the Jones space (bra-ket notation):

$$
tr(|a\rangle\langle a| b\rangle\langle b|) = \sum_i \langle i| a\rangle\langle a| b\rangle\langle b| i\rangle = \sum_i \langle b| i\rangle\langle i| a\rangle\langle a| b\rangle = |\langle a| b\rangle|^2
$$

where the $\{|i\rangle\}$ are an orthonormal basis (e.g., $|\pm\rangle$).

We can also expand $tr(|a\rangle\langle a| b\rangle\langle b|)$ in another way. But first, we make the following observations about the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ (as defined in Section 4.2) and the identity matrix $I$: [35]

- $tr(\sigma_i) = 0$ for $i = x, y, z$
- $\frac{1}{2}tr(\sigma_i \sigma_j) = \delta_{ij}$ where $i, j = x, y, z$, and $\delta_{ij}$ is the Kronecker delta
- all $2 \times 2$ matrices with complex elements are in the linear span of $\{I, \sigma_x, \sigma_y, \sigma_z\}$ over the field of complex numbers
- all $2 \times 2$ Hermitian matrices are in the linear span of $\{I, \sigma_x, \sigma_y, \sigma_z\}$ over the field of real numbers

Consider $|a\rangle$, which is written as $\vec{s}_a = (s_{x,a}, s_{y,a}, s_{z,a})$ in Stokes space. The Jones matrix $|a\rangle\langle a|$ (which is obviously Hermitian) is easily related to $\vec{s}_a$ when we expand it in the basis of the Pauli matrices $\sigma_i$:
\[ |a\rangle\langle a | = \frac{1}{2} (I + \vec{s}_a \cdot \vec{\sigma}), \quad \vec{\sigma} \equiv \Sigma_i \hat{\sigma}_i \sigma_i \quad \text{(C.3)} \]

with \(s_{i,a} \equiv tr(|a\rangle\langle a |)\sigma_i\).

So, applying Equation C.3 (and a similar expression for \(|b\rangle\langle b |\)), we have:

\[
tr(|a\rangle\langle a | b\rangle\langle b |) = tr\left(\frac{1}{4} (I + \vec{s}_a \cdot \vec{\sigma})(I + \vec{s}_b \cdot \vec{\sigma})\right)
\]
\[
= \frac{1}{4} tr(I + \vec{s}_a \cdot \vec{\sigma} + \vec{s}_b \cdot \vec{\sigma} + (\vec{s}_a \cdot \vec{\sigma})(\vec{s}_b \cdot \vec{\sigma}))
\]
\[
= \frac{1}{4} tr(I + (\vec{s}_a \cdot \vec{\sigma})(\vec{s}_b \cdot \vec{\sigma})) \quad \text{(C.4)}
\]

Making use of the fact that [35]:

\[(\vec{s}_a \cdot \vec{\sigma})(\vec{s}_b \cdot \vec{\sigma}) = (\vec{s}_a \cdot \vec{s}_b)I + i(\vec{s}_a \times \vec{s}_b) \cdot \vec{\sigma} \quad \text{(C.5)} \]

we have:

\[
tr(|a\rangle\langle a | b\rangle\langle b |) = \frac{1}{4} tr(I (1 + \vec{s}_a \cdot \vec{s}_b))
\]
\[
= \frac{1}{2} (1 + \vec{s}_a \cdot \vec{s}_b) \quad \text{(C.6)}
\]

By setting equations C.6 and C.2 equal, we arrive at Equation C.1.
Bibliography


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[27] T. Mizunami, Y. Sadakane, and Y. Tatsumoto. Second-harmonic generation from thermally-poled twin-hole silica-glass optical fiber and enhancement


