THREE WAVE MIXING IN PERIODICALLY QUANTUM-WELL-INTERMIXED GaAs:AlGaAs SUPERLATTICES: MODELING, OPTIMIZATION, AND PARAMETRIC GENERATION

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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Abstract

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The three wave mixing process was modeled in GaAs:AlGaAs superlattices using two new modeling tools that were developed in the course of this work: A 2D beam propagation tool for optimizing quasi-phase matching gratings, and a 1D iterative beam propagation tool for determining the output powers and threshold of optical parametric oscillators of arbitrary geometries. The 2D tool predicts close to 80% enhancement of conversion efficiency by phase matching near 800 nm compared to 775 nm, which was the originally designed operation wavelength. The model also predicts resonant behaviour for an abrupt grating profile. The 1D tool was used to determine the threshold conditions for parametric oscillation for different geometries. The performances of different phase matching approaches in AlGaAs were quantitatively compared. The model also indicated the need for pulsed operation to achieve reasonably low threshold powers in AlGaAs waveguides.
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Chapter 1

Introduction

This chapter aims to introduce the motivation and relevant background information for this work, including basic concepts of nonlinear optics (NLO), quantum well intermixing (QWI), and to provide examples of the different approaches to achieve nonlinear frequency conversion in AlGaAs.

1.1 Background

Recent advances in optical communication systems have driven the need for cost-effective, compact, multifunctional all-optical signal processing devices, which include active components such as lasers and amplifiers, and passive components such as agile wavelength routers, channel converters, switches, modulators and filters. These devices have the potential to overcome the bandwidth limitations of conventional electronics in the ultra high-speed communications industry, where currently the need to perform Optical-to-Electrical-to-Optical conversion limits the systems’ bandwidth. Integrated all-optical devices present an attractive platform to perform many of the signal processing operations that are required for realizing next-generation optical communications systems. These photonic integrated circuits (PICs) are not directly limited in performance by carrier dynamics, electromagnetic interference, and the bandwidth bottleneck of electronic
interconnects, currently around 40 Gbps [11].

Nonlinear optics plays an important role in realizing these PICs. High speed optical switches have been realized using nonlinear Mach-Zender interferometers (NMZI) [12] and nonlinear directional couplers (NDC) [13], which employ the instantaneous Kerr nonlinearity, a third order effect. Channel conversion in wavelength division multiplexing (WDM) systems has also been demonstrated using difference frequency mixing, a second order effect [14]. Advances in nonlinear-optic device efficiency are instrumental in realizing next-generation PICs for switching, routing and signal processing applications.

Beyond optical communication, nonlinear optical devices such as optical parametric oscillators (OPOs) are instrumental in realizing tunable sources of coherent radiation in the mid and far infrared. Vibrational resonances of many organic molecular bonds fall in that wavelength window, which makes a tunable bright source of radiation in that wavelength range highly desirable for applications such as trace gas sensing [15] and detection of toxins [16], environmental research [17], quality control in manufacturing, as well as in-vivo biosensing [18]. Lasers exist in this wavelength range, but have a number of limitations. Lead salt tunable lasers [19] are able to cover the wavelength range of 3-25 µm, however they require cryogenic cooling for operation. Quantum cascade lasers that rely on inter-band radiative transitions have been demonstrated in the wavelength ranging between 2 and 10 µm and require complicated wafer growth, however they offer little tunability [20]. Efficient commercial table-top optical parametric oscillators offer a wide tunability range from near- to mid-infrared, can be operated at room temperature, and are a common item in optics research labs. However these devices are bulky and require careful cavity mirror alignment since the interacting pulses have to propagate synchronously in the parametric gain medium. OPOs that operate in continuous wave (CW) can also found in research labs, however these devices operate with very high optical pump power, and thus experience stability issues due to temperature effects. Having a low-threshold, portable, on-chip OPO will not only enable the commercialization of in-
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Integrated PICs for optical communications applications, but also the realization of diverse highly-sensitive lab-on-a-chip type devices for label-free optical biosensing by means of absorption spectroscopy.

The realization of an efficient integrated chip-based OPO is challenging, since materials that efficient diode lasers are made of, such as AlGaAs, are not birefringent. The lack of birefringence, for reasons outlined in the next section, makes it more challenging from an engineering standpoint to perform the frequency conversion efficiently. Since the efficiency of a frequency mixing process scales as the intensity of the interacting beams, tight beam confinement in waveguides is necessary of low threshold operation. However, tightly confining waveguides suffer from comparatively high scattering losses which limit the nonlinear interaction length (the length at which the beam intensities are sufficiently high for efficient nonlinear interaction to take place), which in turn hinders the conversion efficiency. Parametric gain in nonlinear media is typically rather small, and hence careful optimization of the gain cavity and mode profiles while minimizing losses is of paramount importance. Continuous improvements in processing and growth technology, coupled with improved modeling capabilities, have resulted in the creation of several methods for nonlinear frequency mixing that have the potential of becoming the parametric gain medium in an integrated on-chip OPO, which are described briefly in section 1.4.

1.2 Quantum Well Intermixing

The key to realizing on chip integrated optical devices is working with a low-loss, highly-nonlinear material system that allows the creation of both active and passive photonic devices. III-V group semiconductors, and GaAs and its alloys in specific, have mature fabrication technology that allows the creation of low-loss waveguides, possess very high second and third order nonlinearities [21], and can posses direct band gaps that enable the
fabrication of bright lasing sources. In fact, the figure of merit for second order nonlinear interaction in GaAs, defined as \( \frac{\chi^{(2)}^2}{n_{\omega_1} n_{\omega_2} n_{\omega_3}} \), where \( \chi^{(2)} \) is the second order susceptibility and \( n_{\omega_i} \) are the refractive indices of the interacting beams, is approximately 10× larger than that of LiNbO_3. Furthermore, the mature AlGaAs fabrication technology, which was spawned by the telecom boom in the 90’s, now presents a feasible platform for creating low-loss high-confinement waveguides for efficient nonlinear interaction, which can subsequently be integrated on-chip with diode lasers to create cost-effective miniature devices. Moreover, AlGaAs based heterostructures such as superlattices (SLs) and multiple quantum wells (MQWs) allow for tailoring the linear and nonlinear optical and electronic properties of the devices to suit the given application requirements. GaAs:AlGaAs MQW structures can efficiently confine free carriers in the lower-energy gap layers, and hence enable the fabrication of efficient laser sources. Superlattices allow control over the devices’ band gap and tailoring it to the required application [22].

The band structure of a superlattice is different than that of its constituents. This band-structure is characterized by the formation of minibands of allowed energy levels that are dependent on the composition of the superlattice constituents and the thickness of the barriers and wells [6]. The band gap (the absorption edge) change directly alters the macroscopic refractive index through the Kramers-Kronig relations. It has also been shown theoretically and demonstrated experimentally that the change in bandgap alters the magnitude of the tensor elements of the second and third order nonlinear susceptibilities [23] [24].

The aforementioned properties of MQWs and SLs are highly desirable for implementing all-optical integrated devices. In order to implement a cost-effective multifunctional optical device on-chip a direct control over the optical properties of the material, and specifically the band gap, is necessary. Quantum well intermixing (QWI) is a post-growth fabrication technique that allows modifying the band gap of a SL or MQW structure on the same wafer, and thus allowing the integration of smaller-bandgap lasers with larger-
bandgap, lower-loss linear and nonlinear waveguides. QWI, in the simplest terms, is induced disorder in a MQW structure that happens due to the diffusion of MQW constituents between the barriers and wells. Even a small amount of such diffusion breaks the structure symmetry and changes the bandgap of the structure back to its bulk value. The disordering, in turn, can be achieved by means of a mechanism called impurity-free vacancy disordering (IFVD), where no foreign impurities are introduced into the wafer and losses are minimized. Disorder is instead achieved by shallow As$^+$ ion implantation, which creates defects in the lattice. The implantation is followed by rapid thermal annealing (RTA) which enables the defects to diffuse across the SL and render the material as bulk. The controlled ion implantation and subsequent RTA allows patterning the wafer before the etching process, making appropriate modifications to the wafer’s bandgap (and linear and nonlinear properties) to suit the application at hand. Such fabrication process has proven to introduce lower scattering losses than the Etch-and-Regrowth process for shallow-contrast waveguides.

1.3 Nonlinear Optics

The field of nonlinear optics, and nonlinear optical frequency generation specifically, blossomed shortly after the advent of the laser in the early 60’s. The synergy between the laser and nonlinear frequency mixers produced sources of coherent radiation in regions that were not readily accessible by available gain media. Frequency doubling enabled the creation of high-power solid-state laser sources at 532 nm by generating the second harmonic of a Nd:YAG laser. This, in turn, allowed for the implementation of tabletop Ti:sapphire oscillators which were mode-locked using a Kerr-lens. The Ti:sapphire oscillators would then be used to pump table-top optical parametric oscillators (OPOs) to generate tunable coherent optical pulses in the near- to mid-infrared regions of the electromagnetic spectrum.
Nonlinear optics is the manifestation of a non-harmonic material response to an applied electromagnetic field. In the two level atom approximation, this is equivalent to an atom interacting with an applied harmonic field and a reservoir that is characterized by a random Hamiltonian. On a macroscopic level, this is represented by a Taylor expansion of the polarization vector to include contributions of higher powers of the electric field:

\[
P_i = \epsilon_0(\chi^{(1)}_{ij} E_j + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + ...)
\]  

The second order ($\chi^{(2)}$) interactions are responsible for the effects of three-wave mixing, second harmonic generation, and optical rectification. The second order effects are governing the three-wave mixing (TWM) processes, and constitute the primary focus of this work. The third order ($\chi^{(3)}$) interactions are responsible for the Kerr effect, self- and cross-phase modulation, and two-photon absorption. Higher order terms are only evident under the influence of very high field strengths, and are beyond the scope of this work.

1.3.1 Phase matching

Let us consider two monochromatic waves, with frequencies $\omega_1$ and $\omega_2$, propagating in a second-order nonlinear medium. Expansion of the second order term in 1.1 results in the appearance of new frequencies in the electric polarization, namely $2\omega_1$, $2\omega_2$, $0$, $\omega_1 \pm \omega_2$. New frequencies will be generated efficiently when the electric dipoles at incremental lengths $\Delta z$ are oscillating coherently, and hence interfering constructively over a large propagation distance. In order for this constructive interference to take place the phase velocities, $v_{pi} = c/n_{wi}$, of the interacting frequencies must be equal. Chromatic dispersion prevents this condition from being satisfied: waves of different frequencies will propagate while sequentially going in- and out-of phase with respect to one another, thus interfering constructively and destructively in succession over the course of the interaction length.
The nonlinear polarization in equation 1.1 exhibits a cyclical behaviour due to a $e^{i\Delta k z}$ term that appears in the second order expansion, where $\Delta k = k_3 - k_1 - k_2$ is the wavevector mismatch term. This phase mismatch limits the length at which efficient nonlinear wave mixing takes place, which is called the coherence length, equal to

$$L_c = \frac{\pi}{|\Delta k|} \tag{1.2}$$

In Al$_{0.5}$Ga$_{0.5}$As for a second harmonic generation process this length is $\sim 2 \, \mu$m, which leads to very low conversion efficiencies in bulk AlGaAs. Birefringence in certain crystals, such as calcite or KDP, may be exploited to overcome the phase mismatch by having the interacting beams propagate at appropriate angles with respect to the optical crystal axis. Unfortunately, most birefringent crystals have relatively low second-order susceptibility and are not suitable for monolithic integration with diode lasers. III-V semiconductors such as GaAs, on the other hand, possess very large magnitudes of $\chi^{(2)}$, and are readily integrable with conventional laser diodes. Since they belong to the cubic Zinc-blend crystal type they are not naturally birefringent, and hence careful waveguide engineering must take place in order to satisfy the phase matching condition.

### 1.4 Approaches to phase matching in cubic semiconductors

Two distinct approaches exist for achieving the phase-matching condition in non-birefringent crystals. One approach aims to achieve equal effective propagation constants $\beta_i = 2\pi n_i/\lambda_i$ for the three interactive beams by means of waveguide dispersion engineering. Specific methods include artificially induced form birefringence [25] and modal phase matching in multi-mode [26] and Bragg-reflection waveguides [27]. These are discussed and mutually compared in greater depth in chapter 4. The second approach is termed
“Quasi Phase-matching”, and it involves a periodic modulation of the $\chi^{(2)}$ tensor along the direction of propagation of the interacting beams. A periodic modulation of $\chi^{(2)}$ every coherence length ensures positive field intensity buildup over the entire interaction length. The ideal (lossless) efficiency of this process is lower than that of birefringence phase matching, and for long ($>\sim 10^2 L_c$) samples can be characterized by an “effective” $\chi^{(2)}$:

$$\chi_{eff}^{(2)} = \chi_{max}^{(2)} \left(1 - \frac{\gamma}{\pi} \sin(\pi \theta)\right)$$ (1.3)

where $\chi_{max}^{(2)}$ is the maximal nonlinear susceptibility, $\gamma = \frac{\chi_{min}^{(2)}}{\chi_{max}^{(2)}}$, and $\theta$ is the duty cycle, that is the portion of the period at which the susceptibility is maximal. Equation 1.3 suggests that the optimal quasi-phase matched grating will have a modulation from $\chi_{max}^{(2)}$ to $\chi_{min}^{(2)}$, i.e. by inverting the sign of the nonlinear tensor element every coherence length. In this case the effective susceptibility becomes:

$$\chi_{eff}^{(2)} = \frac{2}{\pi} \chi_{max}^{(2)}$$ (1.4)

for a 50:50 duty cycle. Smaller modulations will produce yet smaller effective susceptibilities. It is hence desirable to operate under conditions of full tensor inversion to achieve the highest parametric gain.

1.4.1 Artificially induced form birefringence phase matching

Artificially induced birefringence, or form birefringence, has been proposed by van der Ziel [28] in 1975 as a means to achieve phase matching in cubic media. The method proposes the creation of a multi-layered structure, where the layers have a large index contrast. This will ensure different propagation constants for two orthogonal polarizations, provided that the index contrast is sufficiently large. Obtaining the large index contrast has been challenging. The first demonstration of this method came in 1995 by Berger et. al. [25], where a single-polarization waveguide was demonstrated by designing
the TM mode to reach cutoff at 0.9 µm. The authors employed selective oxidation of an AlAs layers embedded in between GaAs layers, thus transforming AlAs with $n = 2.9$ to Al$_2$O$_3$ with $n = 1.6$. The authors have then proceeded to realize a difference frequency generation (DFG) device using a 4-layer stack of GaAs-Al$_2$O$_3$ [2], shown in figure 1.1. 5.3 µm idler beam in the TM polarization was observed from mixing 1.32 µm Nd:YAG pump beam in the TE polarization with 1.058 µm Ti:Sapphire signal in the TE polarization. Second harmonic generation was also demonstrated using Al$_{0.3}$Ga$_{0.7}$As-Al$_2$O$_3$ multilayered waveguides with reported normalized conversion efficiencies $\eta_{\text{norm}} = 4.2% W^{-1} cm^{-2}$ [29].

The main drawback of this approach is the difficulty of monolithic integration of the passive wavelength conversion waveguides with active gain components since the Al$_2$O$_3$ layers render the waveguide as insulator, and hence unsuitable for electrical pumping. Vertical integration may be required to realize an integrated OPO using this technology.

### 1.4.2 Modal Phase Matching

Another method that aims to achieve the phase matching condition via engineering the waveguide in the transverse direction is modal phase matching [3]. The waveguides are designed such that the fundamental beam propagates in the TE$_{00}$ and TM$_{00}$ modes, while
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Figure 1.2: Modal phase matching [3]. a) Effective index diagram showing the phase matching of the fundamental to a higher order mode. b) Cross-section of the waveguide. Note the poor spatial overlap

the second harmonic is generated in the third-order TE$_{20}$ mode. Waveguide dispersion renders the higher order mode to possess a lower effective propagation constant than the fundamental waveguide mode, assuming a non-dispersive medium. This offsets the higher effective propagation constant of the higher order mode due to material dispersion, thus achieving phase matching.

Unlike form birefringence described in the previous section, modal phase matching does not incorporate insulating layers in between the heterostructure layers, and hence is readily integrable with laser diode on a single chip. In fact, a third-order laser diode that can be used for integration has been demonstrated on this platform [30]. However there is an inherent limitation to this method in the poor overlap integral between the fundamental and the second harmonic mode profiles, which limits the conversion efficiency. Nevertheless, the measured internal conversion efficiency for the device was $30 \pm 5\% W^{-1} cm^{-2}$ [3].

An alternative modal phase matching mechanism employs Bragg-reflection waveguides [31], where the heterostructure dispersion can be tailored to match the effective indices of a fundamental beam propagating in a total-internal-reflection (TIR) mode and a second harmonic beam propagating in the fundamental Bragg-reflection mode. Highly-confining waveguides are realized using this design, and a modification of the layer structure can allow for a tailored far-field profile for the Bragg mode, which in turn
allows for better coupling efficiency. Diode lasers were also demonstrated on the same platform [32], which further proves the potential for integration of passive and active components into a multi-functional device. The main challenges with this method remain the poor coupling efficiency and the high losses due to multiple barrier-well interfaces. The top reported SHG conversion efficiency remains at $6.8 \times 10^{-3}\% W^{-1}cm^{-2}$, which is substantially lower than comparable efficiencies of other methods.

### 1.4.3 Quasi Phase Matching via Domain Inversion on Orientation Patterned Substrates

The most efficient quasi-phase matching method involves the inversion of the sign of $\chi^{(2)}_{eff}$ every coherence length. Physically this is done through inverting the crystal axis in the waveguide every coherence length, which presents a challenge in crystal growth, and which has been accomplished by Fejer et. al. [4]. The authors grew a thin layer of Ge on top of a (001) GaAs substrate, upon which a thin layer of single-phase (00\mbar) GaAs was grown. The QPM grating was lithographically patterned and etched, and the subsequent regrowth of AlGaAs through MBE ensured that the domains that were grown on top of the substrate were oriented at the (01\mbar) and the ones grown on top of the GaAs corrugations were oriented at (01\mbar) direction. The resulting waveguide structure is depicted in figure 1.3. These waveguides suffer from moderately high losses at the fundamental wavelength: in the order of 6-7 dB/cm at 1.55\m, and 13 to 15 dB/cm for the second harmonic. The reported internal conversion efficiency was $23\% W^{-1}cm^{-2}$.

### 1.4.4 Quasi Phase Matching via Periodic Domain Disordering

In section 1.3 MWQ structures were introduced as a viable platform for linear and nonlinear device integration. Band calculations for a GaAs/AlAs superlattice have shown a strong modulation of the second and third order nonlinear susceptibilities in proximity to
Figure 1.3: Phase matching via orientation patterned substrates. a) Rib Waveguide structure showing the mode profile in the inset b) Waveguide cross-section showing the corrugation and material composition [4]

Figure 1.4: SEM cross-section of the superlattice before annealing showing the ion implantation trajectories
the structure’s band gap [33] [23]. If one were to tune the band gap from its intact state to its fully intermixed state every coherence length, one would essentially accomplish quasi phase matching with 50% modulation of the second order susceptibility. The effective susceptibility, as per equation 1.3, becomes 0.2 of its maximum (intact) value, which leads to lower ideal (lossless) conversion efficiencies compared to the etch and regrowth method. Domain disordered requires to work close to the structure band gap, which leads to higher losses at the second harmonic (or OPO pump) wavelengths. However QWI has three distinct advantages over all aforementioned methods. Firstly, QWI is a post growth fabrication process that allows for selective intermixing of different regions on a single wafer as per application requirements. This leads to the second advantage, which is the possibility of creating multi-functional devices on a single wafer that includes active, linear, and nonlinear components. Thirdly, the intermixing process does not induce prohibitively high losses at the telecom wavelengths, with reported losses of 4.2 dB/cm for 1.55 µm [23].

CW second harmonic generation and difference frequency generation has been reported using periodic quantum well intermixing, with an efficiency of 0.18%W$^{-1}$cm$^{-2}$ for type-I SHG [34]. The waveguide structure consisted of a 0.6 µm-thick core layer of 14:14 monolayer GaAs:Al$_{0.85}$Ga$_{0.15}$As superlattice. AlGaAs buffer layers were grown using MBE to enhance the end-fire coupling efficiency. An implantation mask was formed by electroplating of a 2.3-m-thick gold layer for a desired QPM period of 3.5 m and duty cycle. Ion implantation was carried out using 4.0 MeV As$^{2+}$ ions at a dosage of 2.0 × 10$^{13}$ cm$^{-2}$ followed by rapid thermal annealing at 775º C for 60 s. An SEM of pre-annealed SL core and the implantation trajectories are shown in figure 1.4. The relatively low efficiency is attributed to imperfections in the intermixing process, as well as to higher losses at the second harmonic wavelength (20-30 cm$^{-1}$) due to the necessary operation close to the band gap of the structure.

Table 1.1 summarizes the comparison between the phase matching techniques in
<table>
<thead>
<tr>
<th>Technique</th>
<th>Efficiency [%W$^{-1}$cm$^{-2}$]</th>
<th>Fundamental Loss [cm$^{-1}$]</th>
<th>SH Loss [cm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periodic Domain Disordering [34]</td>
<td>0.18</td>
<td>0.9</td>
<td>20-30</td>
</tr>
<tr>
<td>Periodic Domain Inversion [4]</td>
<td>23</td>
<td>1.4</td>
<td>3</td>
</tr>
<tr>
<td>Modal Phase Matching [3]</td>
<td>30</td>
<td>3.5-6</td>
<td>-</td>
</tr>
<tr>
<td>Modal Phase Matching in BRW [31]</td>
<td>$6.8 \times 10^{-3}$</td>
<td>7.8</td>
<td>41</td>
</tr>
<tr>
<td>Form birefringence [35]</td>
<td>1500</td>
<td>0.4-0.6</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1.1: Comparison of phase matching techniques in GaAs/AlGaAs material system. Note that the values for the internal conversion efficiency for the Form Birefringence technique account for conversion from 2 to 1 μm. Conversion from 1.55 μm will be considerable lower. The normalized efficiency reported in [1] is 0.12 %W$^{-1}$cm$^{-2}$

1.5 **Thesis objective and outline**

In this chapter the basic concepts regarding frequency conversion in semiconductors have been outlined. Chapter 2 presents a detailed derivation of the wave equations that are used to model frequency conversion in waveguides. Chapter 3 discusses the numeric algorithm used to model the nonlinear waveguides, and presents the optimization results for two design parameters: the dependence of the conversion efficiency of the second harmonic generation process on the fundamental wavelength, and the effect of a diffused grating profile on conversion efficiency of SHG.

A major objective of this thesis is to determine the conditions for optical parametric oscillation in SL waveguides, as well as to pinpoint to the design parameters that need to be optimized in order to achieve low-threshold oscillation. Chapter 4 presents the theory of optical parametric oscillation and the numerical scheme that is used to determine the OPO threshold. The finding in this thesis will indicate the feasibility of producing an OPO on the SL platform, and quantitatively assess the required optimization of the different design parameters of the OPO. Chapter 5 summarizes this work and introduces avenues for further research to be conducted on the topic.
Chapter 2

Three wave mixing in

Semiconductor Waveguides

This chapter delves into the details of parametric interaction in superlattice waveguides. The equations that govern the three wave mixing process in waveguides are derived in section 2.1. These equations will be used to create the numeric model, described in detail in chapter 3, to optimize for the frequency conversion process. They will also be used in chapter 4 to create a model for determining the OPO threshold.

The derivations of the coupled propagation equations found in this chapter are not novel, and can be found in similar notation in existing papers and textbooks. Nevertheless they are presented here in full such that the reader may familiarize himself with the notation used throughout this thesis, as the consistency will ensure the accuracy of the simulations’ results. Thus, the derivations are provided in full starting from Maxwell’s equations. First, the general paraxial wave equation that governs three wave mixing (TWM) in our waveguides will be derived. These will serve as the basis for deriving the 2D and 1D waveguide approximation equations for the TWM process. The 2D approximation to TWO will serve as the basis for the 2D modeling tool that will be developed in chapter 3. The 1D approximation will serve as the basis for the iterative
modeling tool that will be used to determine the threshold of an on-chip OPO, where computational speed is of greater importance. This is followed by derivation of two simple (limiting) analytic solutions that will be used to test the modeling tools.

2.1 Derivation of the nonlinear wave equation

In this section we derive the three coupled wave equations that govern three wave mixing. We start with Maxwell’s equations for a source free medium:

\[ \nabla \cdot \vec{D} = 0 \]  \hspace{1cm} (2.1)

\[ \nabla \cdot \vec{B} = 0 \]  \hspace{1cm} (2.2)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  \hspace{1cm} (2.3)

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \]  \hspace{1cm} (2.4)

For a non-magnetic medium the following constitutive relations hold:

\[ \vec{B} = \mu_0 \vec{H} \]  \hspace{1cm} (2.5)

\[ \vec{D} = \epsilon_0 \vec{E} + \vec{P} \]  \hspace{1cm} (2.6)

Substituting equation 2.6 into 2.1 results in

\[ \nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = 0 \]  \hspace{1cm} (2.7)

We now take the curl of equation 2.3 and perform the arithmetic:

\[ \nabla \times \nabla \times \vec{E} = -\nabla \times \left( \frac{\partial \vec{B}}{\partial t} \right) \]
\[
\begin{align*}
\frac{\partial}{\partial t} (\nabla \times \vec{B}) &= -\frac{\partial}{\partial t} \mu_0 (\nabla \times \vec{H}) \\
&= -\mu_0 \frac{\partial}{\partial t} \left( \frac{\partial \vec{D}}{\partial t} \right) = -\mu_0 \frac{\partial^2 \vec{D}}{\partial t^2} \\
&= -\mu_0 \left( \epsilon_0 \frac{\partial E^2}{\partial t^2} + \frac{\partial P^2}{\partial t^2} \right) .
\end{align*}
\]

(2.8)

We employ the following vector identity:

\[
\nabla \times \nabla \times \vec{E} = -\nabla \left( \nabla \cdot \vec{E} \right) - \nabla^2 \vec{E} .
\]

(2.9)

Combining the results of 2.9 and 2.8 results in:

\[
\nabla \left( \nabla \cdot \vec{E} \right) - \nabla^2 \vec{E} = -\mu_0 \left( \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{\partial^2 \vec{P}}{\partial t^2} \right) .
\]

(2.10)

We now make the assumption that \( \nabla (\nabla \cdot \vec{E}) = 0 \) (a source-free medium), which essentially states that the nonlinear polarization term is a small perturbation to the linear polarization. We are now left with:

\[
-\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} .
\]

(2.11)

We proceed by rearranging equation 2.11, and making the substitution \( c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \), to obtain the wave equation:

\[
\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} .
\]

(2.12)

Decomposing the polarization term into its linear and nonlinear components results in:

\[
\vec{P} = \vec{P}^{lin} + \vec{P}^{NL} .
\]

(2.13)
Which renders the linear displacement as follows:

\[
\vec{D}^{lin} = \epsilon_0 \vec{E} + \vec{P}^{lin}
\]  

Taking \( \vec{P}^{lin} = \chi^{(1)} \vec{E} \) and \( n = \sqrt{1 + \chi^{(1)}} \) allows us to write the wave equation 2.11 as follows:

\[
\nabla^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}^{NL}}{\partial t^2}
\]  

At this point we decompose the fields into their time-harmonic components:

\[
\vec{E}(\vec{r}, t) = \frac{1}{2} \left\{ \sum_n \vec{E}_n(\vec{r}) e^{-i\omega_n t} + c.c. \right\}
\]  

\[
\vec{P}^{NL}(\vec{r}, t) = \frac{1}{2} \left\{ \sum_n \vec{P}^{NL}_n(\vec{r}) e^{-i\omega_n t} + c.c. \right\}
\]  

This turns the wave equation into the inhomogeneous Helmholtz equation for every \( \omega_n \) frequency component:

\[
\nabla^2 \vec{E} + \frac{\omega^2 \epsilon_r(\omega)}{c^2} \vec{E} = -\omega^2 \mu_0 \vec{P}^{NL}
\]  

We proceed by expressing the electric field frequency components and waveguide normal modes:

\[
E_i(x, y, z, t) = \frac{1}{2} \left\{ E(x, y, z) \cdot e^{i(k_i z - \omega_i t)} + c.c. \right\}
\]  

where \( k_i = n_{eff,i} k_{0,i} = n_{eff,i} \frac{2\pi}{\lambda_i} \) are the effective propagation constants in the waveguide for wave \( i \). We would now like to expand the spatial gradient term, while making two approximations. First, assume that the wavevector \( k \) is independent on the \( z \) coordinate. This is strictly speaking not true for a waveguide where the refractive index undergoes a modulation in the direction of propagation, but we assume that such change is small. The second approximation we make is the slowly-varying envelope approximation (SVEA), i.e. \( \frac{\partial^2 E}{\partial z^2} \ll k \frac{\partial E}{\partial z} \). This approximation is justified for pulse lengths > 1ps. This results in
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the following nonlinear Helmholtz equation:

\[
2ik \frac{\partial E(x, y, z)}{\partial z} + \frac{\partial^2 E(x, y, z)}{\partial y^2} + \frac{\partial^2 E(x, y, z)}{\partial x^2} + k_0(n^2 - n_{\text{eff}}^2)E = -\omega^2 \mu_0 P^{NL}
\]

(2.20)

where \( n = \sqrt{\epsilon_r(w, x, y, z)} \) is the local refractive index at location \( \vec{r}' = (x, y, z) \) at frequency \( \omega \). We have dropped the time and space harmonic dependence, along with the complex conjugates of the fields, in equation 2.20 for brevity. We now turn our attention to expanding the nonlinear polarization source term. We deal specifically with a three-wave mixing process, and adopt the common notation by terming the three interacting frequencies as signal, idler, and pump, while noting that they are mathematically identical. Hence, the nonlinear polarization is expanded as follows:

\[
P_{NL}^{\text{sig}} = \frac{1}{2} \chi^{(2)}_{\text{eff}} \epsilon_0 \kappa E_p E_{\text{idl}}^* \]

(2.21)

\[
P_{NL}^{\text{idl}} = \frac{1}{2} \chi^{(2)}_{\text{eff}} \epsilon_0 \kappa E_p E_{\text{sig}}^* \]

(2.22)

\[
P_{NL}^{\text{pmp}} = \frac{1}{2} \chi^{(2)}_{\text{eff}} \epsilon_0 \kappa E_{\text{sig}} E_{\text{idl}} \]

(2.23)

where \( \kappa = 1 \) for a degenerate three wave mixing process, and \( \kappa = 2 \) for the non-degenerate case. Expanding the three coupled equations using the waveguide mode notation gives:

\[
P_{NL}^{\text{sig}} = \frac{1}{2} \chi^{(2)}_{\text{eff}} \epsilon_0 \kappa E_{\text{idl}}^*(x, y, z)e^{-i(k_i z - w_i t)} E_{\text{pmp}}(x, y, z)e^{i(k_p z - w_p t)}
\]

(2.24)

\[
P_{NL}^{\text{idl}} = \frac{1}{2} \chi^{(2)}_{\text{eff}} \epsilon_0 \kappa E_{\text{sig}}^*(x, y, z) E_{\text{pmp}}(x, y, z)e^{i(k_p z - k_i z - w_p t + w_i t)}
\]

(2.25)

\[
P_{NL}^{\text{pmp}} = \frac{1}{2} \chi^{(2)}_{\text{eff}} \epsilon_0 \kappa E_{\text{sig}}(x, y, z) E_{\text{idl}}(x, y, z)e^{i(k_i z + k_p z - w_i t - w_p t)}
\]

(2.26)
Since we are dealing with an energy-matched parametric process, \( \omega_p = \omega_s + \omega_i \). Going back to 2.20 and cross-dividing by the exponential terms, while introducing the wave mismatch term \( \Delta k = k_{pmp} - k_{sig} - k_{idl} \), we arrive at:

\[
2ik_{sig} \frac{\partial E_{sig}}{\partial z} + \frac{\partial^2 E_{sig}}{\partial y^2} + \frac{\partial^2 E_{sig}}{\partial x^2} + k_0(n_{sig}^2 - n_{eff,sig}^2)E_{sig} =
\]
\[
= \frac{1}{2} \frac{\chi^{(2)}_{eff} k \omega_{sig}^2}{c^2} E_{idl}^* E_{pmp} e^{i\Delta kz} \tag{2.27}
\]

\[
2ik_{idl} \frac{\partial E_{idl}}{\partial z} + \frac{\partial^2 E_{idl}}{\partial y^2} + \frac{\partial^2 E_{idl}}{\partial x^2} + k_0(n_{idl}^2 - n_{eff,idl}^2)E_{idl} =
\]
\[
= \frac{1}{2} \frac{\chi^{(2)}_{eff} k \omega_{idl}^2}{c^2} E_{sig}^* E_{pmp} e^{i\Delta kz} \tag{2.28}
\]

\[
2ik_{pmp} \frac{\partial E_{pmp}}{\partial z} + \frac{\partial^2 E_{pmp}}{\partial y^2} + \frac{\partial^2 E_{sig}}{\partial x^2} + k_0(n_{pmp}^2 - n_{eff,pmp}^2)E_{pmp} =
\]
\[
= \frac{1}{2} \frac{\chi^{(2)}_{eff} k \omega_{pmp}^2}{c^2} E_{idl} E_{sig} e^{-i\Delta kz} \tag{2.29}
\]

The three equations above are the coupled inhomogeneous Helmholtz equations under the SVEA for a quadratic nonlinear medium. The \( e^{i\Delta kz} \) terms suggests a cyclic behaviour of the power conversion between the interacting beams that would hinder the efficiency of the mixing process. This system of equations can only be solved analytically for the case where no pump depletion or losses occur, and the waveguides' profile is homogeneous along the direction of propagation of the beams. A numerical method to solve for the general case is developed in the next chapter.

### 2.2 Reduction to two dimensions

Numeric evaluation of 2.27-2.29 presents a computational challenge since the inclusion of terms \( \frac{\partial^2 E}{\partial x^2}, \frac{\partial^2 E}{\partial y^2}, \) and \( \frac{\partial E}{\partial z} \) invokes the need to model in a three-dimensional space. The
problem can be simplified by a plane wave approximation in the y-axis \(\frac{\partial^2 F}{\partial y^2} = 0\). In this case we choose to expand the waveguide modes as follows:

\[
E_i(x, y, z, t) = \frac{1}{2} F(y) \left\{ \mathcal{E}(x, z) \cdot e^{i(k_i z - \omega_i t)} + \text{c.c.} \right\}
\]

(2.30)

Where \(F(y)\) is the transverse power distribution in the y axis for a given cross-section in the x axis. We normalize \(F(y)\) as follows:

\[
\int_{-\infty}^{\infty} |F(y)|^2 dy = 1
\]

(2.31)

Using this notation the three coupled equations 2.27-2.29 become

\[
2ik_{\text{sig}} \frac{\partial \mathcal{E}_{\text{sig}}}{\partial z} F_{\text{sig}} + \frac{\partial^2 F_{\text{sig}}}{\partial y^2} \mathcal{E}_{\text{sig}} + \frac{\partial^2 \mathcal{E}_{\text{sig}}}{\partial x^2} F_{\text{sig}} + k_0(n_{\text{sig}}^2 - n_{\text{eff,sig}}^2) \mathcal{E}_{\text{sig}} F_{\text{sig}} =
\]

\[
= \frac{1}{2} \chi_{\text{eff,sig}}^2 \omega_{\text{sig}} F_{\text{idl}} F_{\text{pmp}} \mathcal{E}_{\text{sig}}^* \mathcal{E}_{\text{pmp}} e^{i\Delta kz}
\]

(2.32)

\[
2ik_{\text{idl}} \frac{\partial \mathcal{E}_{\text{idl}}}{\partial z} F_{\text{idl}} + \frac{\partial^2 F_{\text{idl}}}{\partial y^2} \mathcal{E}_{\text{idl}} + \frac{\partial^2 \mathcal{E}_{\text{idl}}}{\partial x^2} F_{\text{idl}} + k_0(n_{\text{idl}}^2 - n_{\text{eff,idl}}^2) \mathcal{E}_{\text{idl}} F_{\text{idl}} =
\]

\[
= \frac{1}{2} \chi_{\text{eff,idl}}^2 \omega_{\text{idl}} F_{\text{sig}}^* F_{\text{pmp}} \mathcal{E}_{\text{sig}}^* \mathcal{E}_{\text{pmp}}^* e^{i\Delta kz}
\]

(2.33)

\[
2ik_{\text{pmp}} \frac{\partial \mathcal{E}_{\text{pmp}}}{\partial z} F_{\text{pmp}} + \frac{\partial^2 F_{\text{pmp}}}{\partial y^2} \mathcal{E}_{\text{pmp}} + \frac{\partial^2 \mathcal{E}_{\text{sig}}}{\partial x^2} F_{\text{sig}} + k_0(n_{\text{pmp}}^2 - n_{\text{eff,pmp}}^2) \mathcal{E}_{\text{pmp}} =
\]

\[
= \frac{1}{2} \chi_{\text{eff,pmp}}^2 \omega_{\text{pmp}} F_{\text{idl}} F_{\text{pmp}} \mathcal{E}_{\text{idl}} \mathcal{E}_{\text{sig}} e^{-i\Delta kz}
\]

(2.34)

The next step is to multiply the equations above by the complex conjugate of the field envelope \(F(y)\):

\[
F_{\text{sig}}^* F_{\text{sig}} \left[ 2ik_{\text{sig}} \frac{\partial \mathcal{E}_{\text{sig}}}{\partial z} + \frac{\partial^2 \mathcal{E}_{\text{sig}}}{\partial x^2} + k_0(n_{\text{sig}}^2 - n_{\text{eff,sig}}^2) \mathcal{E}_{\text{sig}} \right] =
\]
Chapter 2. Three wave mixing in Semiconductor Waveguides

\[
\frac{1}{2} \frac{\chi_{e f f}^{(2)} K \omega^2}{c^2} F_{\text{sig}}^* F_{\text{idl}}^* F_{\text{pmp}} \mathcal{E}_{\text{idl}}^* \mathcal{E}_{\text{pmp}} e^{i \Delta k z} = (2.35)
\]

\[
F_{\text{idl}}^* F_{\text{idl}} \left[ 2i k_{\text{idl}} \frac{\partial \mathcal{E}_{\text{idl}}}{\partial z} \right] + \frac{\partial^2 \mathcal{E}_{\text{idl}}}{\partial x^2} + k_0 (n_{\text{idl}}^2 - n_{e f f, \text{idl}}^2) \mathcal{E}_{\text{idl}} = \frac{1}{2} \frac{\chi_{e f f}^{(2)} K \omega^2}{c^2} F_{\text{idl}}^* F_{\text{sig}}^* F_{\text{pmp}} \mathcal{E}_{\text{idl}}^* \mathcal{E}_{\text{pmp}} e^{i \Delta k z} = (2.36)
\]

\[
F_{\text{pmp}}^* F_{\text{pmp}} \left[ 2i k_{\text{pmp}} \frac{\partial \mathcal{E}_{\text{pmp}}}{\partial z} \right] + \frac{\partial^2 \mathcal{E}_{\text{pmp}}}{\partial x^2} + k_0 (n_{\text{pmp}}^2 - n_{e f f, \text{pmp}}^2) \mathcal{E}_{\text{pmp}} = \frac{1}{2} \frac{\chi_{e f f}^{(2)} K \omega^2}{c^2} F_{\text{pmp}}^* F_{\text{idl}}^* F_{\text{sig}}^* \mathcal{E}_{\text{idl}} \mathcal{E}_{\text{pmp}} e^{-i \Delta k z} = (2.37)
\]

and integrate both sides of the equation along y-axis. Assuming the modes are non-leaky in the y-axis, we introduce the term

\[
\left[ A_{e f f, y}^{(2)} \right]^{-1} = \int_{-\infty}^{\infty} dy F_{\text{sig}} F_{\text{idl}} F_{\text{pmp}}
\]

(2.38)

And since \( \int_{-\infty}^{\infty} F^* F dy = 1 \) we are left with

\[
2i k_{\text{sig}} \frac{\partial \mathcal{E}_{\text{sig}}}{\partial z} + \frac{\partial^2 \mathcal{E}_{\text{sig}}}{\partial x^2} + k_0 (n_{\text{sig}}^2 - n_{e f f, \text{sig}}^2) \mathcal{E}_{\text{sig}} = \frac{1}{2} \frac{\chi_{e f f}^{(2)} K \omega^2}{c^2 A_{e f f, y}^{(2)}} \mathcal{E}_{\text{idl}}^* \mathcal{E}_{\text{pmp}} e^{i \Delta k z} = (2.39)
\]

\[
2i k_{\text{idl}} \frac{\partial \mathcal{E}_{\text{idl}}}{\partial z} + \frac{\partial^2 \mathcal{E}_{\text{idl}}}{\partial x^2} + k_0 (n_{\text{idl}}^2 - n_{e f f, \text{idl}}^2) \mathcal{E}_{\text{idl}} = \frac{1}{2} \frac{\chi_{e f f}^{(2)} K \omega^2}{c^2 A_{e f f, y}^{(2)}} \mathcal{E}_{\text{sig}}^* \mathcal{E}_{\text{pmp}} e^{i \Delta k z} = (2.40)
\]

\[
2i k_{\text{pmp}} \frac{\partial \mathcal{E}_{\text{pmp}}}{\partial z} + \frac{\partial^2 \mathcal{E}_{\text{pmp}}}{\partial x^2} + k_0 (n_{\text{pmp}}^2 - n_{e f f, \text{pmp}}^2) \mathcal{E}_{\text{pmp}} = \frac{1}{2} \frac{\chi_{e f f}^{(2)} K \omega^2}{c^2 A_{e f f, y}^{(2)}} \mathcal{E}_{\text{idl}} \mathcal{E}_{\text{sig}} e^{-i \Delta k z} = (2.41)
\]

The equations above are the 2D approximation to the nonlinear coupled equation for modeling TWM. These equations will be discretized and numerically solved in Chapter 3.
2.3 Reduction to one dimension

Equations 2.39 to 2.41 present a base for accurate modeling of rib waveguides, particularly those that have low index contrast between the rib and the etched parts of the waveguide. A desktop computer should be able to simulate a 1 mm long waveguide using these equations within several minutes. If, however, one were to optimize for several parameters, or simulate an oscillator cavity where many round trips are taking place before a steady-state solution is reached, it is desired to reduce the computational effort further. Also, we would like to obtain a simple analytical solution for simple thought experiments to test the code, and such analytical solutions are unobtainable for the abrupt 2D waveguide geometry. Hence, we expand the waveguide modes similarly to the 2D case:

\[ E_i(x, y, z, t) = \frac{1}{2} F(x, y) \{ \mathcal{E}(z) \cdot e^{i(k_i z - \omega_i t)} + \text{c.c.} \} \]  

(2.42)

Where both \( F(x, y) \) is normalized as follows:

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(x, y)|^2 \, dx \, dy = 1 \]  

(2.43)

are similarly normalized as in equation 2.31. Furthermore, we make the plane wave approximation, such that \( \frac{\partial F}{\partial x} = 0 \) and \( \frac{\partial F}{\partial y} = 0 \). Following a similar procedure as in 2.32 to 2.41, we arrive at the following equations that only depend on the \( z \) coordinate for TWM:

\[ 2ik_{\text{sig}} \frac{\partial \mathcal{E}_{\text{sig}}}{\partial z} = \frac{1}{2} \frac{\chi^{(2)}_{\text{eff}} k_{\text{sig}}^2 \mathcal{E}_{\text{sig}}^* \mathcal{E}_{\text{idl}} \mathcal{E}_{\text{pmp}}}{c^2 A_{\text{eff}}^{(2)}} e^{i\Delta k z} \]  

(2.44)

\[ 2ik_{\text{idl}} \frac{\partial \mathcal{E}_{\text{idl}}}{\partial z} = \frac{1}{2} \frac{\chi^{(2)}_{\text{eff}} k_{\text{idl}}^2 \mathcal{E}_{\text{idl}}^* \mathcal{E}_{\text{pmp}}}{c^2 A_{\text{eff}}^{(2)}} e^{i\Delta k z} \]  

(2.45)

\[ 2ik_{\text{pmp}} \frac{\partial \mathcal{E}_{\text{pmp}}}{\partial z} = \frac{1}{2} \frac{\chi^{(2)}_{\text{eff}} k_{\text{pmp}}^2 \mathcal{E}_{\text{idl}} \mathcal{E}_{\text{sig}}}{c^2 A_{\text{eff}}^{(2)}} e^{-i\Delta k z} \]  

(2.46)
Where
\[
[A_{eff}^{(2)}]^{-1} = \int_{-\infty}^{\infty} F_{sig} F_{idl} F_{pmp} dxdy \tag{2.47}
\]
is the spatial overlap integral. In the equations above the refractive index \(n\), and hence the propagation constant \(k\), are implicitly complex. In order to make the analytic solution more explicit, we would like to separate the real and complex parts of \(k\) in the equations above, and at the same time perform the coordinate transformation \(A = E \sqrt{\frac{(A_{eff}^{(2)})^2 n c \varepsilon_0}{2}}\), such that \(A = \sqrt{P}\), where \(P\) is the optical power in each beam. The equations 2.44 to 2.46 now take the following form:

\[
\frac{dA_{sig}}{dz} = -\frac{1}{2} \alpha_{sig} A_{sig} - i\kappa_{sig} A_{idl}^* A_{pmp} e^{-i\Delta kz} \tag{2.48}
\]

\[
\frac{dA_{idl}}{dz} = -\frac{1}{2} \alpha_{idl} A_{idl} - i\kappa_{idl} A_{sig}^* A_{pmp} e^{-i\Delta kz} \tag{2.49}
\]

\[
\frac{dA_{pmp}}{dz} = -\frac{1}{2} \alpha_{pmp} A_{pmp} - i\kappa_{pmp} A_{idl} A_{sig} e^{i\Delta kz} \tag{2.50}
\]

where \(\alpha_i\) and the linear power loss coefficients and

\[
\kappa_{sig} = \sqrt{2\pi^2 (\chi_{eff}^{(2)})^2 \frac{n_{eff,sig} n_{eff,idl} n_{eff,pmp} c \varepsilon_0 \lambda_{sig}^2 (A_{eff}^{(2)})^2}{\lambda_{sig}(A_{eff}^{(2)})^2}} \tag{2.51}
\]

\[
\kappa_{idl} = \sqrt{2\pi^2 (\chi_{eff}^{(2)})^2 \frac{n_{eff,sig} n_{eff,idl} n_{eff,pmp} c \varepsilon_0 \lambda_{idl}^2 (A_{eff}^{(2)})^2}{\lambda_{idl}(A_{eff}^{(2)})^2}} \tag{2.52}
\]

\[
\kappa_{pmp} = \sqrt{2\pi^2 (\chi_{eff}^{(2)})^2 \frac{n_{eff,sig} n_{eff,idl} n_{eff,pmp} c \varepsilon_0 \lambda_{pmp}^2 (A_{eff}^{(2)})^2}{\lambda_{pmp}(A_{eff}^{(2)})^2}} \tag{2.53}
\]

are the nonlinear coupling coefficients that are explicitly real. Equations 2.48-2.50 are a convenient set of equations to optimize the performance of complex devices with long interaction lengths.
Chapter 2. Three wave mixing in Semiconductor Waveguides

2.4 Simple analytic solutions

In this section we concentrate on providing some specific analytic solutions to equations 2.48-2.50. These equations can be solved easily in the lossless limit, and such solutions are provided in textbooks such as [21] and [36]. In this section we explore the solution to the aforementioned equations for slightly more complicated cases: second harmonic generation while all three beams experience loss, and difference frequency generation with a lossless pump.

2.4.1 Second Harmonic Generation

We look at the following set of equations:

\[
\frac{dA_{\omega_1}}{dz} = -\frac{1}{2} \alpha_{\omega_1} A_{\omega_1} \quad (2.54)
\]

\[
\frac{dA_{\omega_2}}{dz} = -\frac{1}{2} \alpha_{\omega_2} A_{\omega_2} \quad (2.55)
\]

\[
\frac{dA_{\omega_3}}{dz} = -\frac{1}{2} \alpha_{\omega_3} A_{\omega_3} - i\kappa A_{\omega_3} A_{\omega_3} e^{i\Delta kz} \quad (2.56)
\]

This is the case for sum frequency generation where the frequencies \(\omega_1\) and \(\omega_2\) may be identical, but their polarization may be different. It is clear that the solution of equations 2.54 and 2.55 take the form of a decaying exponential. We rewrite the propagation equation for \(A_{\omega_3}\) in the following form:

\[
\frac{dA_{\omega_3}}{dz} + \frac{1}{2} \alpha_{\omega_3} A_{\omega_3} = -i\kappa A_{\omega_3,0} A_{\omega_3,0} e^{-\frac{\alpha_{\omega_1} + \alpha_{\omega_2}}{2} + i\Delta kz} \quad (2.57)
\]

We can solve the equation above using the method of integrating factors. The solution takes the form:

\[
A_{\omega_3}(z) = \int_0^L -i\kappa A_{\omega_3,0} A_{\omega_3,0} e^{-\frac{\alpha_{\omega_1} + \alpha_{\omega_2}}{2} + i\Delta kz} e^{-\frac{\omega_3}{2} z} \quad (2.58)
\]
Which after algebraic simplification becomes

\[
A_{\omega_3}(z) = -i\kappa A_{\omega_3,0} A_{\omega_3,0} \left[ e^{-\alpha_{\omega_1} + \alpha_{\omega_2} + \alpha_{\omega_3} z + i\Delta k z} \right]^L_{0} = -i\kappa A_{\omega_3,0} A_{\omega_3,0} \frac{e^{-\alpha_{\omega_1} + \alpha_{\omega_2} + \alpha_{\omega_3} L + i\Delta k L}}{e^{-\alpha_{\omega_1} + \alpha_{\omega_2} + \alpha_{\omega_3} z + i\Delta k z}} (2.59)
\]

Multiplying the numerator and denominator of equation 2.59 by \( L \) allows us to clearly see the limiting lossless case where

\[
A_{\omega_3}(z) = -i\kappa A_{\omega_3,0} A_{\omega_3,0} L \quad (2.60)
\]

And since the \( A = \sqrt{P} \), the power grows as the square of the propagation distance.

### 2.4.2 Difference Frequency Generation

Let us now consider the case of difference frequency generation for a lossless pump in the case of no phase mismatch. The power of the pump stays constant at \( A_p = A_{p,0} \), while the signal and idler equations are as follows:

\[
\frac{dA_s}{dz} = -\frac{1}{2} \alpha_s A_s - ig_s A_i^* \quad (2.61)
\]

\[
\frac{dA_i}{dz} = -\frac{1}{2} \alpha_i A_i - ig_i A_s^* \quad (2.62)
\]

where \( g_{i,s} = \kappa_{i,s} A_{3,0} \) We transform the equations above into a matrix form:

\[
\frac{d}{dz} \begin{pmatrix} A_s \\ A_i^* \end{pmatrix} = \begin{pmatrix} -\alpha_s \quad -ig_s \\ -ig_i \quad -\alpha_i \end{pmatrix} \begin{pmatrix} A_s \\ A_i^* \end{pmatrix} \quad (2.63)
\]
The solution of this system of equations is straightforward. We find the eigenvalues:

\[ \gamma_{1,2} = -\frac{\alpha_s + \alpha_i}{4} \pm \sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i} \]  

(2.64)

and the eigenvectors:

\[ \zeta_{1,2} = \begin{pmatrix} \frac{\alpha_s - \alpha_i}{4} \pm \sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i} \\ \mp ig_i \end{pmatrix} \]  

(2.65)

If the initial conditions for the fields are \( A_s(0) = A_{s,0} \) and \( A_i(0) = A_{i,0} \), then the unique solution can be written in the form:

\[
A_s = e^{-\frac{(\alpha_s + \alpha_i)z}{4}} \left[ A_{s,0} \cosh\left(\sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i} z\right) + \frac{ig_s A_{s,0} + \frac{\alpha_s - \alpha_i}{4} A_{i,0}^*}{\sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i}} \sinh\left(\sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i} z\right) \right]
\]  

(2.66)

\[
A_i^* = e^{-\frac{(\alpha_s + \alpha_i)z}{4}} \left[ A_{i,0}^* \cosh\left(\sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i} z\right) + \frac{ig_i A_{i,0}^* + \alpha_s - \alpha_i}{4} A_{s,0} \sinh\left(\sqrt{\frac{\alpha_s + \alpha_i}{16} + g_s g_i} z\right) \right]
\]  

(2.67)

Which are the solutions for a difference frequency generation with an undepleted pump. This solution may be directly applied to solve for the behaviour of an OPO where the pump does not experience loss, i.e. when the parametric process takes place in the laser cavity itself. Modal-phase-matching approaches may take direct use of this result to model their OPO devices.

### 2.5 Conclusion

This chapter accomplishes two objectives: first, it shows the derivation of all required equations for modeling our devices, along with limiting-case solutions that were used
to test the code in subsequent chapters. Secondly, it sets a consistent notation to be followed for the remainder of this thesis.
Chapter 3

Numerical Modeling of Superlattice Waveguides

3.1 Introduction

To date, the bulk of effort in optimization and modeling of our frequency conversion devices has been focused on optimizing second harmonic generation for pulsed pump [37] [38]. The reason for this has been the low internal conversion efficiency of the initial devices which did not allow the detection of measurable energies with a CW pump, and other second order effects such as difference frequency generation were not observed at the time. The high peak powers of pico- and femtosecond pulses allowed the detection of sizable SHG conversion efficiencies, however the use of high peak powers of picosecond pulses came with a number of drawbacks, namely group velocity mismatch (GVM) between the pump and the second harmonic waves, and third order nonlinear effects such as self- and cross-phase modulation and two photon absorption that hindered the devices’ efficiency. Moreover, the short pulses’ bandwidth often exceeded the conversion bandwidth of our devices, leading to only a small fraction of the input pulse power to be phase-matched with it’s second harmonic. All these effects imposed limits on the
conversion efficiency, and hence effort was made to understand these phenomena and to quantify their effects on our devices. Group velocity mismatch places an upper limit to the efficiency by limiting the device length to $\sim$ 3 mm for a lossless waveguide. Self-phase modulation would shift the phase matching wavelength as a function of power, while nonlinear absorption has been shown to inhibit the parametric gain of the devices above peak powers of 50 W.

Advances in the fabrication process allowed measurable CW conversion efficiencies to be recorded [34], which opened the potential for operating devices with low peak powers, thus limiting the role that the aforementioned effects play in hindering efficiency. We hypothesize that the interplay between linear absorption and the modulation of $\chi^{(2)}$ are the main effects that limit efficiency in CW operation. In all previous simulation work the fundamental wavelength was set to 1.55 $\mu$m, and little consideration was given to linear absorption and the modulation of $\chi^{(2)}$ as the fundamental wavelength was changed. It is desired to explore how the combined effect of change in linear absorption and modulation of $\chi^{(2)}$ affect the efficiency as the fundamental wavelength is tuned away from the half band-gap. Common logic dictates that since both the modulation and absorption at the second harmonic wavelengths decrease when the fundamental is red-tuned from the half-band-gap there must be an extremum wavelength of optimal conversion. Using that wavelength as the pump of an optical parametric oscillator (OPO) will result in lower threshold powers while still allowing for wide wavelength tunability.

Furthermore, in all previous simulation work a square duty cycle profile of first and second order susceptibilities was assumed. We know that this profile is not square, but diffused due to lateral ion straggle during the ion implantation process, and the thermal diffusion of induced defects between masked regions and ones exposed to ion implantation during the rapid thermal annealing process. Such diffusion may be modeled as thermal diffusion, and it is interesting to quantify to what extent it affects the conversion. We do not know the diffusion length of such process, but we suspect it is between 100 nm
Chapter 3. Numerical Modeling of Superlattice Waveguides

3.2 The model

Various numerical algorithms have been created for modeling both linear and nonlinear waveguides of arbitrary geometry [39] [40] [41] [42] [43] [44] [45] [46] [47] [48] [49] [50]. The complexity of these algorithms varies depending upon the number of physical effects that are included in the model. The simplest algorithm involves solving a one-dimensional scalar equation with the undepleted pump approximation (UPA). More complicated (and realistic) models include the vectorial and semi-vectorial Beam Propagation Methods (BPM) [51] [52]. These methods are better suited for accounting for the phase transformation that takes place in a waveguide, and therefore yield more accurate results for coherent processes such as TWM, and self- and cross-phase modulation (SPM and XPM).

The inclusion of any physical effect within the numerical model increases the computational time. Hence, several approximations that are outlined below are included in the
model. Two approximations were implicit in the derivation of the propagation equations in the previous chapter: the slowly varying envelope approximation (SVEA) and the omission of the transverse second-order spatial derivative in the y-axis in equations 2.39 to 2.41. The (paraxial) SVEA eliminates the need to compute the second-order derivative in the z-axis, which effectively transforms the numerical problem into a linear PDE in \( z \), thus eliminating the need to compute the time-intensive iterative solution of higher order Padé approximants [52]. The paraxial approximation is accurate for shallow (\(<\sim 4\%\)) index contrast waveguides. The deeply etched SL waveguides push the boundaries of the paraxial approximation, with index contrast of \(\sim 3.5\%\) in the C-band. The penalty for the relatively high contrast is a small error in the phase matching wavelength for a given coherence length. The effective area approximation of the second order derivative in the y-axis does not allow for computation of the full radiative loss due to mode mismatch, however the inclusion of at least one transverse spatial derivative allows us to gain insight into the effect of mode mismatch between as-grown and the intermixed sections of the waveguide, while at the same time reducing the computational order from \( O(n^3) \) to \( O(n^2) \).

Other approximations are conditional on the test cases explored. Wagner et. al. measured the intensity-dependent refractive indices and nonlinear absorption of the superlattice waveguides. The typical damage threshold for our samples is \(\sim 100 \text{ mW} \) of average CW power near the bang-gap, and \(\sim 300 \text{ mW} \) near the half-band-gap. The nonlinear absorption was measured to be as high as 4 cm/GW, which for powers of 100 mW and computed third order effective areas of \(\sim 6 \mu \text{m}^2 \) translate into a change of \(< 0.01 \text{cm}^{-1} \) in the absorption coefficient. This change is over an order of magnitude smaller than the uncertainty in loss measurements. Likewise, the change in the refractive index for \(\sim 100 \text{mW} \) beam travelling through the waveguide is \(< 1^{-6} \), which causes a shift of \(< 0.1 \text{ nm} \) in the phase matching wavelength. This change is far smaller than the one caused by the inhomogeneity of the implantation period. Hence third order effects
are justifiably omitted from the model. The omission reduces the computation order by a linear factor.

As previously mentioned, our waveguides consist of a shallow refractive index grating, which invites the inclusion of the effects of reflections at interfaces. Even though the grating is shallow, the large (~270 periods/mm) number of interfaces in a sample results in narrow stop bands. A semi-analytical transfer matrix method was used to determine whether a stop band coincides with any wavelength of operation. The results showed a 1-nm wide stop band at 770 nm, and otherwise full transparency at the wavelengths 1.5 \( \mu \)m to 1.7 \( \mu \)m and 0.75 \( \mu \)m to 0.85 \( \mu \)m. This result suggests that unidirectional propagation may be considered for any wavelength of operation while excluding any data at ~770 nm. This saves the need for computation of bidirectional propagation, which requires storage of field profiles at every interface for stability purposes for a nonlinear PDE, and iterative computation of the fields [49]. The inclusion of bidirectional propagation would not have been feasible to perform using a desktop computer for the necessary (>100) number of periods, especially while dealing with a diffused grating profile which renders every computational step as an interface.

To summarize, the following assumptions were made while developing the numeric algorithm:

1. SVEA, due to the shallow index contrast

2. Effective area approximation, due to the guided structure

3. No \( \chi^{(3)} \) effects, due to low-power CW operation

4. Unidirectional propagation, due to propagation in the grating’s transparency range with the exception of (770 ±0.5) nm.

With these assumptions and their limitations in mind we proceed to discretize the propagation equations.
3.3 The algorithm

A straightforward unconditionally stable and accurate algorithm to discretize equations 2.39 to 2.41 is the Crank-Nicholson (CN) [53] centred-difference method, where the differential operators are discretized as:

\[
\frac{\partial E(x,z)}{\partial z} = \frac{E_x^{z+1} + E_x^{z-1}}{\Delta z} \tag{3.1}
\]

\[
\frac{\partial^2 E(x,z)}{\partial x^2} = \frac{E_x^{z+1} - 2E_x^z + E_x^{z-1} + E_{x+1}^{z+1} - 2E_{x+1}^z + E_{x+1}^{z-1}}{(2\Delta x)^2} \tag{3.2}
\]

where \(\Delta x\) and \(\Delta z\) are the discrete spatial step sizes in the \(x\) and \(z\) axes respectively, and \(E_x^z\) is the electric field value at location \((x, z)\) on the computational mesh. The CN method is an implicit method that requires knowledge of the field values at locations \(z + 1\) and \(z\) along the waveguide in order to calculate the fields at the location \(z + 1\), and is therefore implicit.

The DuFort-Frankel (DFF) method is a modification of the standard centred-difference CN approach, whereby the field value at location \(z\) is approximated by \(E_x^z = \frac{1}{2}(E_x^{z+1} + E_x^{z-1})\). The DFF method provides lower truncation error than the CN method, and is equivalent to discretizing the following paraxial problem for linear media using the simple Euler method [54]:

\[
\left(2ik_0n_{eff} \frac{\partial}{\partial z} + k_0(n^2 - n_{eff}^2) + \frac{\partial^2}{\partial x^2} - \left(\frac{\Delta z}{\Delta x}\right)^2(1 - \Delta x^2)\frac{k_0(n^2 - n_{eff}^2)}{2} \frac{\partial^2}{\partial z^2}\right)E(x,z) = 0 \tag{3.3}
\]

for \(\frac{\partial^2 E}{\partial z^2} \ll k \frac{\partial E}{\partial z}\). For a longitudinally invariant structure the eigenvalue problem in 3.3 has solutions defined by the following characteristic equation:

\[
2ik_0n_{eff} \gamma + k_0(n^2 - n_{eff}^2) + \left(\frac{\Delta z}{\Delta x}\right)^2(1 - \Delta x^2)\frac{k_0(n^2 - n_{eff}^2)}{2} \gamma^2 = 0 \tag{3.4}
\]
Solving the quadratic equation 3.4 for $\gamma$ provides two solutions; one is the desired paraxial solution:

$$\gamma_p \approx \frac{k_0(n^2 - n_{\text{eff}}^2)}{2n_{\text{eff}}} \tag{3.5}$$

and one is a parasitic ghost solution:

$$\gamma_g = \frac{\pi}{\Delta z} - 2\phi - \gamma_p \tag{3.6}$$

where $\tan(\phi) = \frac{\Delta z(1-\Delta x^2)(n^2-n_{\text{eff}}^2)}{2\Delta x^2 k_0 n_{\text{eff}}}$. The root cause of this ghost solution is the numerical error of the algorithms, and can be mitigated (but not overcome) by the methods outlined in [46]. The DFF algorithm for a lossless medium is subject to the stability condition:

$$\Delta z < \Delta x \frac{n_{\text{eff}}}{\sqrt{n^2 - n_{\text{eff}}^2}} \tag{3.7}$$

It is clear that when losses are introduced, $\Im\{\gamma_p\} < 0$, and therefore $\Im\{\gamma_g\} > 0$. This means that for a lossy medium the ghost solution will experience gain, and therefore the lossy DFF scheme is unstable. To combat this instability a dissipation parameter $\Gamma$ is introduced into equation 3.3, such that it reads as:

$$\left(2ik_0 n_{\text{eff}} \frac{\partial}{\partial z} + k_0(n^2 - n_{\text{eff}}^2) + \frac{\partial^2}{\partial x^2} - \left(\frac{\Delta z}{\Delta x}\right)^2 (1 - \Delta x^2) \frac{k_0(n^2 - n_{\text{eff}}^2)}{2} \left(\frac{\partial^2}{\partial z^2} - \frac{2}{\Delta z} \Gamma \frac{\partial}{\partial z}\right)\right) E(x, z) = 0 \tag{3.8}$$

Where $\Gamma > \Delta z k_0 \frac{3\{n^2\}}{4n_{\text{eff}}^4}$. This ensures that the dissipation parameter $\Gamma$ is at least as large as the gain coefficient of the ghost solution. The equation 3.8 is trivially discretized to yield the following explicit expression for the electric field propagation for a linear medium:

$$E_{x=p+1}^{z=\ell+1} = E_{x=p}^{z=\ell+1} + \frac{i k_0 n_{\text{eff}}^p}{\Delta z} - (1 + \Gamma) \left(\frac{k_0^2 (n_p^2 - n_{\text{eff}}^p)^2}{2} - \frac{1}{\Delta x^2}\right) + \frac{1}{\Delta x^2} \left(E_{z=p}^{\ell+1} + E_{z=p}^{\ell-1}\right) \tag{3.9}$$
Where $E_{i}^{p+1} = E(x = l\Delta x, z = (p + 1)\Delta z)$, $n_{t}^{p}$ is the refractive index in the location $x = l\Delta x, z = (p + 1)\Delta z$, and $n_{eff}^{p}$ is the effective index for the spline $z = (p + 1)\Delta z$.

Adding the second order nonlinear term is trivial, however it introduces a higher degree of instability to the model, and requires a higher dissipation constants $\Gamma$ to be used. This constant for nonlinear propagation was empirically determined to be $\Gamma^{NL} = 4\Gamma^{lin}$. The increase in dissipation is still very small ($\sim 10^{-2}$) compared to the lineal loss, and with its introduction the algorithm becomes stable for lengths exceeding 1mm, and for values of $\chi^{(2)}$ of as high as $10^5$ pm/V. The three nonlinear equations for the TWM process become:

\[
\begin{align*}
(E_{x=l}^{z=p+1})_{\text{sig}} &= (E_{l}^{p+1})_{\text{sig}} + \frac{ik_{0,\text{sig}}(n_{eff,\text{sig}}^{p})}{\Delta z} + (1 - \Gamma_{\text{sig}})\left(\frac{k_{0,\text{sig}}^{2}(n_{t}^{p})^{2} - (n_{eff,\text{sig}}^{p})^{2}}{2} - \frac{1}{\Delta x^{2}}\right) \\
&+ \frac{1}{\Delta x^{2}}\left((E_{l+1}^{p})_{\text{sig}} + (E_{l-1}^{p})_{\text{sig}}\right) \\
&+ 2k_{0,\text{sig}}(A_{eff,p}^{(2)})^{-1}\chi_{eff,p}(E_{l}^{p})_{\text{idl}}(E_{l}^{p})_{\text{pmp}} \cdot \exp^{i\Delta k_{-p}\Delta z} \\
& \tag{3.10}
\end{align*}
\]

\[
\begin{align*}
(E_{x=l}^{z=p+1})_{\text{idl}} &= (E_{l}^{p+1})_{\text{idl}} + \frac{ik_{0,\text{idl}}(n_{eff,\text{idl}}^{p})}{\Delta z} + (1 - \Gamma_{\text{idl}})\left(\frac{k_{0,\text{idl}}^{2}(n_{t}^{p})^{2} - (n_{eff,\text{idl}}^{p})^{2}}{2} - \frac{1}{\Delta x^{2}}\right) \\
&+ \frac{1}{\Delta x^{2}}\left((E_{l+1}^{p})_{\text{idl}} + (E_{l-1}^{p})_{\text{idl}}\right) \\
&+ 2k_{0,\text{idl}}(A_{eff,p}^{(2)})^{-1}\chi_{eff,p}(E_{l}^{p})_{\text{sig}}(E_{l}^{p})_{\text{pmp}} \cdot \exp^{i\Delta k_{-p}\Delta z} \\
& \tag{3.11}
\end{align*}
\]

\[
\begin{align*}
(E_{x=l}^{z=p+1})_{\text{pmp}} &= (E_{l}^{p+1})_{\text{pmp}} + \frac{ik_{0,\text{pmp}}(n_{eff,\text{pmp}}^{p})}{\Delta z} + (1 - \Gamma_{\text{pmp}})\left(\frac{k_{0,\text{pmp}}^{2}(n_{t}^{p})^{2} - (n_{eff,\text{pmp}}^{p})^{2}}{2} - \frac{1}{\Delta x^{2}}\right) \\
&+ \frac{1}{\Delta x^{2}}\left((E_{l+1}^{p})_{\text{pmp}} + (E_{l-1}^{p})_{\text{pmp}}\right) \\
&+ 2k_{0,\text{pmp}}(A_{eff,p}^{(2)})^{-1}\chi_{eff,p}(E_{l}^{p})_{\text{sig}}(E_{l}^{p})_{\text{idl}} \cdot \exp^{i\Delta k_{-p}\Delta z} \\
& \tag{3.12}
\end{align*}
\]

These coupled mode equations may be used for any case of degenerate or non-degenerate TWM process for arbitrary waveguide geometry.

Since the effective-index approach has been taken to transform the 3D ridge waveguide into a 2D slab waveguide, it is appropriate to use the lowest order (symmetric) slab mode as an excitation source. This field distribution may be obtained analytically by following
a procedure in [21]. The field profile is then normalized for the required input powers.
The DFF algorithms requires knowledge of two initial fields at steps \( z = 0 \) and \( z = \Delta z \).
The initial field is the calculated slab mode. The field at the second step may be obtained
by forward difference discretization of the propagation equations. There exists a small
mode mismatch between the analytic mode profile and the numerical mode, whereby the
power lost due to mode mismatch is \( \sim 0.5\% \).

The domain is terminated in the transverse direction by perfectly matched layers
(PMLs) using the coordinate transformation \( x \rightarrow (1 + i\sigma)x \), where \( \sigma \) is piecewise continuous along the PML, and is given by the relation:

\[
\sigma(x) = \sigma_{\text{max}}x^2
\]  

(3.13)

where \( x = 0 \) at the boundary between the PML and the propagation (lossless) domain,
and \( x = d \) at the edge of the computational domain, and \( d \) is the PML width. The
computational domain is terminated by a perfect electric conductor (PEC). The PML
width was chosen to be 10 \( \mu m \) on each side of the propagation domain, while the \( \sigma_{\text{max}} \)
parameter was chosen to be 0.1 \( \Omega^{-1} \). Both the PML width and \( \sigma_{\text{max}} \) parameters were
determined empirically.

The modeling tool has undergone extensive testing for stability and accuracy. A linear
directional coupler was tested using the model and the coherence length of the coupler
differed by 2.5\% from the same coupler modeled using RSoft Beam Prop software. The
results of modeling lossless nonlinear waveguides compared to analytic solutions within
10\% for several test geometries. The reason for this large error is a mode mismatch
between the fundamental mode of the mesh to the calculated excitation field, causing a
loss of \( \sim 1\% \) of power in the first 300\( \mu m \) of propagation, and a “breathing” effect that
is caused by the mode mismatch.
3.4 Waveguide structure and fabrication

Figure 3.1 shows the GaAs:Al\textsubscript{0.85}Ga\textsubscript{0.15}As SL core wafer. The wafer is grown on (100) semi-insulating GaAs substrate using Molecular Beam Epitaxy (MBE). The 600 nm GaAs:Al\textsubscript{0.85}Ga\textsubscript{0.15}As superlattice guiding layer is surrounded from top and bottom with 300 nm Al\textsubscript{0.56}Ga\textsubscript{0.44}As buffer layers that have a similar refractive index to the SL core to enhance the coupling efficiency. The buffer layers are in turn surrounded by lower-refractive index Al\textsubscript{0.6}Ga\textsubscript{0.4}As cladding layers. A base layer of lower-index Al\textsubscript{0.85}Ga\textsubscript{0.15}As separates the lower cladding from the substrate, thus minimizing substrate leakage. A 100 nm GaAs cap is grown on top of the wafer to prevent oxidation of the high-aluminum content buffer. The Gehrsitz [55] model was used to calculate the effective indices of the AlGaAs layers. This model was favoured over because it was empirically determined by measuring the refractive indices of AlGaAs over a broad composition range in the telecom band, which makes it more suitable than either the Adachi or the Aframowitz models, both of which are semi-empirical models that were derived from the refractive indices of AlGaAs around the band edge. The refractive indices of the intact and intermixed SL core layer was determined by Wagner [5] using a quadratic-regression back-
calculation of the empirical measurements of the intact and intermixed slab effective indices using the grating coupler technique by Kleckner [6], that were then adjusted for the GaAs:Al$_{0.85}$Ga$_{0.15}$As (as opposed to GaAs:AlAs) superlattice by reducing the bandgap parameter for the AlAs layer. Figure 3.2 shows the polarization-dependent dispersion of the intact and intermixed GaAs:Al$_{0.85}$Ga$_{0.15}$As superlattice core. These dispersion curves were loaded into the Lumerical MODE Solver package which solved for mode distribution and effective indices in the waveguides. Both the 2D effective indices of the waveguides and cross-section indices of the rib and cladding layers were determined using Lumerical Mode Solutions. The deeply-etched waveguides were multimoded at wavelengths $< 0.85 \mu$m, and are single-moded in the C-band. As per equations 2.39 - 2.41, in order to optimize the conversion efficiency it is necessary to maximize the overlap integral in equation 2.38 within the SL core, where the phase matching condition is satisfied. Figure 3.3 illustrates the confinement of the fundamental and the second harmonic modes. The fraction of the fundamental (1550 nm) mode that is confined in the core is $\sim 50\%$, whereas that value for the mode at 775 nm is $\sim 90\%$. The TE mode shows better confinement, and is therefore expected to exhibit lower linear losses due to leakage and surface roughness. The wafers were grown in the MBE facilities in the University of Sheffield, UK. The waveguide fabrication is carried out by our collaborators from the University of Glasgow, UK: Prof. David Hutchings, Dr. Barry Holmes and Usman Younis. The fabrication process is summarized in the following steps:

1. 200 nm thick protective SiO$_2$ cap was deposited on top of the wafer

2. A thin 5-10 nm seed layer of Ti was deposited on top of the SiO$_2$ cap to serve as an adhesion layer for Au.

3. 40-60 nm Au layer was deposited on the Ti seed layer.

4. Poly(methyl methacrylate) (PMMA) was spun on top of the gold layer as a resist for EBL.
Figure 3.2: Polarization-dependent dispersion of the intact and intermixed SL core. Values of the SL core were obtained through back-calculation by Wagner [5] using slab effective index data from Kleckner [6]. The lower effective indices for the TM polarization compared to the TE polarization suggest lower field confinement for TM polarized light.
5. QPM periods varying from 3.1µm to 3.8µm were written in PMMA via electron beam lithography (EBL). Three different duty cycles, 40:60, 50:50, and 60:40 (open/closed) were drawn in PMMA.

6. A thick 2.0µm to 2.3µm layer of Au was electroplated on the exposed Au layer, followed by stripping the remaining PMMA. This forms the implantation mask.

7. The wafers were then sent for ion implantation in University of Surrey, UK, where As$^{2+}$ ions were implanted in the sample at the dosage of 2$^{13}$ ions/cm$^2$, at implantation energies of 4 MeV.

8. Remaining Au, Ti, and SiO$_2$ were stripped, and a fresh 200 nm SiO$_2$ layer was deposited on top of the wafer to protect it from oxidation during the rapid thermal annealing (RTA), and to suppress interdiffusion of Ga$^{2+}$ ions via the impurity-free vacancy disordering (IFVD) process [56].

9. RTA was carried out for 60 s at 775° C. The dielectric cap was subsequently removed.

10. Waveguides of widths varying from 2 µm to 4 µm in 0.5 µm steps were lithographically patterned in HSQ and etched using Reactive Ion Etching (RIE) in SiCl$_4$
11. The wafer was cleaved along the (110) crystal direction at different lengths for testing.

A deep 1.3 \( \mu \text{m} \) etch is required to confine the second harmonic mode near 775 nm in the lateral direction, since the wafer was originally designed to guide light in the telecom wavelengths. The waveguides were cleaved along the (110) crystal direction to take full advantage of the modulated effective values of \( \chi^{(2)}_{zxy} \) and \( \chi^{(2)}_{xyz} \) tensor elements along the direction of beam propagation.

### 3.5 Modulation of \( \chi^{(2)} \)

The second and third order tensor elements were determined through band calculations by Hutchings [24], and \( \chi^{(2)} \) dispersion for a QPM grating is presented in figure 3.4. A modulation of up to 50\% between the intact and intermixed superlattice has been predicted. It is evident that these values are significantly lower than the bulk values of \( \chi^{(2)} \) for AlGaAs, yet they are equivalent to these of PPLN under full intermixing [21].

Bulk Zinc-blend (crystal class \( \bar{4}3m \)) has 6 identical \( \chi^{(2)} \) tensor elements: \( \chi^{(2)}_{xyz} = \chi^{(2)}_{xzy} = \chi^{(2)}_{yzx} = \chi^{(2)}_{zxy} = \chi^{(2)}_{zyx} = \chi^{(2)}_{zyx} \). The SL breaks the inversion symmetry along the z-axis, turning the SL into a \( \bar{4}2m \) crystal. In this case, there are two independent \( \chi^{(2)} \) tensor elements: \( \chi^{(2)}_{xyz} = \chi^{(2)}_{xzy} = \chi^{(2)}_{yzx} = \chi^{(2)}_{zxy} \) and \( \chi^{(2)}_{zyx} = \chi^{(2)}_{zyx} \). The effective \( \chi^{(2)} \) along the direction of propagation is calculated according to the formula:

\[
\chi^{(2)}_{\text{eff}} = \sum_{ijk} \chi^{(2)}_{ijk} \hat{e}_i \hat{e}_j \hat{e}_k \tag{3.14}
\]

where \( \hat{e} \) are the polarization unit vectors of the interacting beams. The wafers were cleaved along the (110) crystal direction. In type-I phase matching the fundamental (or signal/idler) propagates in the TE polarization, while the second harmonic (or pump)
Figure 3.4: Dispersion of $\chi^{(2)}(-2\omega; \omega, \omega)$ tensor elements as a function of a fundamental frequency $\omega$. a) Dispersion of the $\chi^{(2)}_{xyz}$ tensor element for type-II phase matching. b) Dispersion of the $\chi^{(2)}_{zxy}$ tensor element for type-I phase matching. For a given frequency the modulation for type-II phase matching is greater than for type-I phase matching, making type-II phase matching the more efficient process given equal losses and mode overlap.

propagates in the TM polarization. In type-II phase matching, the fundamental propagates in both the TE and TM polarization, while the second harmonic propagates in the TE polarization. Type-I phase matching utilizes the $\chi^{(2)}_{zxy}$ tensor element, while type II phase matching utilizes the $\chi^{(2)}_{xyz}$ tensor element. It is evident from figure 3.4 that type-II phase matching is a more efficient process due to the larger modulation in the utilized $\chi^{(2)}$ tensor element. This is illustrated in the schematic of the crystal directions in figure 3.5. Due to the dual-degeneracy in the $xyz$ tensor element permutation, and the quadruple-degeneracy in the $zxy$ tensor element permutation, $\chi^{(2)}_{eff} = \chi^{(2)}_{zxy}$ for type-I phase matching, and $\chi^{(2)}_{eff} = \chi^{(2)}_{xyz}$ for type-II phase matching. The modulation due to intermixing may be directly observed by measuring the photoluminescence (PL) spectrum of the superlattice waveguides. The peak PL wavelength shift directly corresponds to the bad gap shift of the superlattice from the intact to intermixed regions. The measurement was carried out using a JobinYvon Horiba LabRam spectrometer. An optical
microscope integrated with the spectrometer was used to focus a 632.8 nm HeNe laser at an intermixed section of the wafer. The focusing objective is used to collect the scattered light in the back-scattering configuration and direct it at a spectrometer. The band gap of the intermixed superlattice has been measured at 700 nm, while that of the intact superlattice has been measured at 747 nm (figures 3.7 and 3.6). The resultant blue shift of 47 nm is significantly lower than the previously reported shift of 75 nm for the same implantation dose of $2 \cdot 10^{13}$ ions/cm$^2$ and annealing temperature of 775° C. We can extrapolate the modulation in $\chi^{(2)}$ from [24] to be $\sim 20$ pm/V, and a $\chi^{(2)}_{eff}$ of $\sim 8$ pm/V, leading to a material figure of merit $\chi^{(2)}_{eff}/n^3$ an order of magnitude lower than PPLN.

### 3.6 Measurements of linear loss dispersion

In order to perform the optimization of the wavelength detuning from the band gap the values of loss coefficients need to be determined over a range of wavelengths near the band edge. Previous transmission measurements estimated the loss of the second harmonic wavelengths to be between 20 to 30 cm$^{-1}$, however no accurate measurements
Figure 3.6: Photoluminescence spectrum of a fully intermixed SL using 632.9 nm excitation. The PL peak has shifted by $\sim 45$ nm compared to the as-grown section.

Figure 3.7: Photoluminescence spectrum of the SL waveguide using 632.9 nm excitation. The PL cross-section is in the order of a few microns, and shows both the PL peaks from as-grown (marked AG) and intermixed (marked QWI) sections in a waveguide. Intermediate peaks show the existence of a diffused grating profile.
have been performed over that range. The refractive indices exhibit strong dispersion in that wavelength range, thus we expect a similarly strong dispersion in the loss coefficients at these wavelengths due to the Kramers-Kronig relations.

A popular method for measuring propagation (distributed) losses is the Fabry-Perot (FP) technique. By tuning the wavelength of a narrow-linewidth coherent source around the desired measurement wavelength, and by knowing the facet reflectivity and sample length one can deduce the distributed propagation loss by measuring the contrast of the FP fringes. The following formula is used to determine the loss coefficient [57]:

$$\alpha = \frac{1}{L} \frac{1}{(1 - R)} \ln \left( \frac{1 + \sqrt{\zeta}}{1 - \sqrt{\zeta}} \right)$$

where $L$ is the length of the sample, $R$ is the facet reflectivity, $\zeta = P_{\text{min}}/P_{\text{max}}$ is the fringe contrast, and $\alpha$ is the loss coefficient in units of $[L]^{-1}$. The technique works best for low loss ($< 2\text{cm}^{-1}$), low dispersion, straight waveguides with a known facet reflectivity. The method is less accurate for shorted samples where the distributed losses are not as dominant as the mirror losses. Moreover, leaky modes hinder the accuracy of the results in shorter samples. An implicit requirement for the FP technique is a light source with a linewidth significantly narrower than the FP fringe width, which is $\sim 0.1$ nm in our case.

The experimental setup used to perform both the FP and transmission measurements are shown in figures 3.9 and 3.11 respectively. In both setups an end-fire rig was used to couple light in and out of the waveguides. The rig consisted of two $\times 40$ microscope objectives mounted on 3-axes translation stages. A CW source (HP 1385) laser was used to perform the FP measurements at 1550 nm. 4% of the input light was directed at a calibrated photodiode in order to monitor the fluctuations in the laser output. A second calibrated photodiode was used to collect the out light from the waveguide. Both diodes were connected to a LabView computer program that swept the laser wavelength
Figure 3.8: Measured loss coefficient vs. Wavelength for TE polarized light. The value at \(1.55 \, \mu m\) was measured using the FP method. All other data points are transmission measurements scaled to the FP data point. The inset shows the dispersion between 800 nm and 770 nm, the axes are the same as in the main graph. The TM polarization exhibits the same dispersion, but the losses are 1-2 \(cm^{-1}\) higher.

Figure 3.9: Schematic of the experimental setup for FP loss measurements using a CW laser source at the C-band and 980 nm.
and recorded a time-averaged output of both diodes for every given laser wavelength. A typical FP trace is depicted in figure 3.10. The output curve was normalized by the input, and the slope was eliminated using a linear fit. The average fringe contrast was taken over several periods. The procedure was repeated for six waveguides on the same wafer, all 4 ± 0.05 mm long, and the resulting loss coefficients for each waveguide were averaged. The dominant source for error is the facet reflectivity, since we do not have an accurate method of measuring it, and damaged facets significantly alter the frequency response of the cavity. The facet reflectivity was assumed to be 28%. The error in measurements is 1 cm\(^{-1}\) determined from the standard deviation of the individual waveguides’ loss values. The results for the loss coefficient at 1550 nm are 4.1 ± 1 cm\(^{-1}\) for the TE polarization, and 5 ± 1 cm\(^{-1}\) for the TM polarization.

A second CW source (New Focus Velocity 6300) was used to read the transmission on the same setup at 980 nm. The value of the loss at 980 nm was determined through the ratio in transmission between 1550 nm and 980 nm. The transmission readings were scaled by the ratio of power coupling between a Gaussian beam with a beam waist equal
Transmission measurements were also performed on a different setup using a Ti:Sapphire (Coherent Mira) oscillator operating in CW mode. Powers were kept below 10 mW such that nonlinear losses did not inhibit the measurements. Transmission measurements were taken across 5 different waveguides between 770 nm to 980 nm on a shorter (1.1 ± 0.05 mm long) sample of the same wafer, where the 980 nm measurement was used to scale the loss coefficient with the other measurements. There is an added error, estimated between 1-2 cm$^{-1}$ between 800 nm and 770 nm due to the variation of the mode profile of the oscillator that caused variance in the coupling efficiency. The resultant loss coefficient dispersion for the TE polarization is shown in figure 3.8. The TM polarization’s loss coefficients are 1-2 cm$^{-1}$ higher than that of the TE polarization.
3.7 Simulation results

3.7.1 Wavelength dependence of the conversion efficiency

Since both the modulation of $\chi^{(2)}$ and the linear losses for the second harmonic decrease as the detuning of the fundamental from the half band-gap increases we expect to see an optimal detuning point where the conversion efficiency is maximized. This expectation goes in line with the experimental evidence that suggests that the most efficient wavelength for SHG conversion is between 1580-1590 nm, as opposed to the originally designed 1550 nm. The purpose of the simulation is to quantify the increase in efficiency that we can expect to achieve in type-I and type-II phase matching by working farther away from the band edge. Figures 3.12 and 3.13 show the conversion efficiency $\eta = P_{2\omega}/P_{\omega}^2$ and the normalized conversion efficiency $\eta_{\text{norm}} = P_{2\omega}/(P_{\omega}^2 L^2)$ for type-I and type-II phase matching as a function of wavelength.

The results show an interesting trend, as the interplay between absorption and $\chi^{(2)}$ modulation does not result in a sharp increase in efficiency for a particular wavelength, but rather allows for -3dB bandwidth of over 50 nm in conversion efficiency. This result shows the ability to realize devices that utilize the generated second harmonic light with longer propagation lengths at longer wavelengths for applications involving cascaded second order effect and quantum optics applications where twin photons are generated by parametric down-conversion. Furthermore, the longer propagation lengths for the second harmonic light facilitates the design of an integrated OPO, as there is less power lost in coupling between the laser and parametric gain sections.

Figure 3.14 shows the tuning curves for type-I and type-II phase matching. These curves are off by $\sim 40$ nm from the experimentally reported values in [58]. This translates into an error in the superlattice refractive index of $\delta(\Delta(n_{2\omega} - n_\omega)) = 0.01$, which is an order of magnitude above the precision of the refractive index measurements. This change is partially accounted for by the deeper etch resulting in different effective indices for both
Figure 3.12: a) Conversion efficiency for type I phase matching as a function of wavelength. b) Normalized conversion efficiency for type I phase matching.
Figure 3.13: a) Conversion efficiency for type II phase matching as a function of wavelength. b) Normalized conversion efficiency for type II phase matching.
Figure 3.14: Tuning curve for type-I and type-II phase matching for a 3.5 μm wide rib and 1.3 μm deep etch
the fundamental and second harmonic waves compared to previously reported data. A small portion of the error may be accounted for by wafer-to-wafer variations. To reiterate, the superlattice refractive indices were back-calculated using Lumerical Mode Solver by Wagner [5] from Kleckner’s data [6]. The refractive indices for the AlGaAs cladding layers were derived from Gersitz’ model, which is the most accurate such model to the author’s knowledge.

The iterative solution suggests that the quantum-well-intermixed superlattice exhibits negative group velocity dispersion (GVD) in the TM polarization at the fundamental wavelengths, while exhibiting positive GVD at the SH wavelengths, implying the existence of a resonance in between the two wavelengths. This resonance should not physically exist, which leads us to believe that there is an error in Kleckner’s measurements. Also, Kleckner’s data was taken for GaAs:AlAs superlattice, while the refractive index values in the simulation is for GaAs:Al$_{0.85}$Ga$_{0.15}$As SL. The GaAs:AlAs data was modified in accordance with the change in photoluminescence wavelength shift, but that may not account for the full shift at the SH wavelength, where the dispersion is high. The results imply that the SL refractive index information in our possession is inaccurate.

### 3.7.2 The effect of diffused grating profiles

Lateral ion straggle that occurs during the implantation process and thermal diffusion of defects that occurs during the rapid thermal annealing process promotes SL layer interdiffusion in the masked regions of the superlattice, causing a gradual change in the optical properties of the SL as one transitions from the intermixed to the masked regions. We make the first-order approximation that such change also obeys the linear diffusion equation. This approximation may very well be far-fetched, however it is still valuable to explore how a smoothly-varying first and second order susceptibility profile affects the conversion efficiency.

Since we are dealing with a periodic grating it is convenient to look at the problem
in the frequency domain, where the effect of thermal diffusion exponentially damps the high spatial frequency components of the grating according to [6]:

\[
c(x) = c_0 + \sum_{n=1}^{\infty} c_n \cos \left( \frac{n\pi x}{\Lambda} \right) \exp \left( -\frac{n^2\pi^2 l_d^2}{4\Lambda^2} \right) \tag{3.16}
\]

\[
c_0 = \theta \chi_{AG} + (1 - \theta) \chi_{QWI}
\]

\[
c_n = \frac{2}{n\pi} (\chi_{AG} - \chi_{QWI}) \left[ \sin \left( \frac{n\pi \theta}{2} \right) - \sin (n\pi(1 - \theta/2)) \right]
\]

where \(\chi_{AG}\) and \(\chi_{QWI}\) represent the physical quantity, be it the first or second order susceptibility, in the masked (as-grown) and intermixed sections of the waveguide respectively, \(\Lambda\) and \(\theta\) are the grating period and duty cycle respectively, and \(l_d\) is the diffusion length. Since we cannot compute an infinite sum we only take into account the first 15 Fourier elements. Computing more Fourier elements alters the conversion powers by \(\sim 1\%\) for a given period and duty cycle.

Figures 3.15 and 3.16 are the results of varying the duty cycle for different diffusion lengths: 5 nm, 100 nm, 200 nm and 300 nm for type-I and type-II phase matching respectively, using the fundamental wavelength 1.6 \(\mu\)m. Strong peaks that are superimposed on the gain curve are seen at low diffusion lengths. These peaks are separated by \(\sim 0.08 \cdot L_c = 0.29 \ \mu\)m, which is not far removed from a half-wavelength of the fundamental beam in the waveguide, which is \(\sim 0.26 \mu\)m. We hypothesize that the sharp transitions between the as-grown and intermixed regions are numerical artifact of the unidirectional propagation can only treat reflections at a single interface.

The theoretically optimal QPM duty cycle can be written as

\[
\theta_{opt} = \frac{L_{c,AG}}{L_{c,AG} + L_{c,QWI}} \tag{3.17}
\]

where \(L_{c,AG}\) and \(L_{c,QWI}\) are the coherence lengths in the as-grown and intermixed regions, and is equal to 55:45 for type-I phase matching in our case. When the diffusion
lengths are very low, the fringe effect masks this value, leading to lower efficiency. At higher diffusion lengths the optimum duty cycle is getting closer to 80:20, but the maximum efficiency is not compromised. This happens because diffusion damps high spatial frequency components, thus making the written duty cycle look closer to 50:50. In order to maintain the optimum 55:45 duty cycle we need to artificially introduce more high frequency spatial components, and the result of this is the higher optimal duty cycles. Experiments indicate that 60:40 duty-cycle performs better than 50:50 or 70:30 duty-cycles, which leads us to believe that the diffusion length in our samples is close to 100 nm. As expected, if we were to look at any optimal duty cycle, and measure the lengths of regions that are higher and lower than \((\chi_{AG} + \chi_{QWI})/2\) we would arrive at the optimal duty cycle of 55:45.

3.8 Conclusion

Three accomplishments were outlined in this chapter. The first one is the development of a robust and stable modeling tool for modeling QPM waveguides of arbitrary configuration. This tool can be used to accurately model not only the uniform gratings that were studies in this chapter, but also chirped gratings of arbitrary profiles. The second accomplishment is the verification of the hypothesis of the existence of an optimal conversion wavelength for a given dispersion of \(\chi^{(2)}\) and \(\alpha\). This will allow us to tailor the wavelength of an integrated laser to achieve optimal conversion in an on chip optical parametric oscillator or amplifier. The third accomplishment is the study of the effect of a diffused grating profile on QPM waveguides, which adds to our extant body of knowledge on the conversion mechanism in QPM waveguides. The results of this chapter help explain why we observe the optimal second harmonic and difference frequency generation to be most efficient for waveguides that are phase-matched around 1590 nm even though the wafers were originally designed for 1550 nm conversion, and why the 60:40 written
Figure 3.15: Efficiency vs. Duty Cycle for type-I phase matching. The diffusion length is a) 5 nm b) 100 nm c) 200 nm d) 300 nm
Figure 3.16: Efficiency vs. Duty Cycle for type-II phase matching. The diffusion length is a) 5 nm b) 100 nm c) 200 nm d) 300 nm
duty cycle produces the highest efficiencies. With this understanding in mind we can go forward and look at how we can use the QWI technology to realize a functional Optical Parametric Oscillator.
Chapter 4

Optical Parametric Oscillators in GaAs:AlGaAs Superlattices

Following the findings of the previous chapter regarding the conversion efficiency that can be achieved in superlattice waveguides, we aim to evaluate the feasibility of different designs for optical parametric oscillators utilizing GaAs:AlGaAs superlattices. First, the basic theory of optical parametric oscillation is outlined, followed by a description of the numerical model used to evaluate the different proposed designs. This is followed by the numeric evaluation of several different OPO designs, and concluded with a comparison between the different technologies for parametric frequency conversion that were discussed in chapter 1. The findings in this chapter allow us to gauge the state of the art in the industry regarding the feasibility of producing an integrated OPO, and pinpoint the technology most likely to be used for an integrated AlGaAs OPO.

4.1 Introduction

It has been shown in chapters 2 and 3 that nonlinear processes can give rise to amplification of different frequencies under the presence of intense fields. It was also demonstrated in chapter 3 that such amplification is rather small, typically $< 2 \text{ cm}^{-1} \text{W}^{-0.5}$. As a con-
sequence, high input powers in two of the three interacting beams is commonly used to achieve amplification of the third beam - in such case, the device is called an Optical Parametric Amplifier (OPA). In order to lower the power required for the generation of new frequencies of sizable intensity while only using one input frequency one may use a technique borrowed from the laser - positive feedback. By placing the nonlinear medium inside an optical cavity with reasonably high mirror reflectivities and low losses one can gain simultaneous amplification of both the signal and idler frequencies within the cavity over the course of many round trips.

From a conceptual standpoint, the device is similar to a laser in the conditions required for operation (oscillation) - the parametric gain inside the cavity must overcome the propagation and mirror losses, and the amplified waves’ frequencies must coincide with the normal modes of the optical cavity. Unlike a laser, though, there are three waves involved in the parametric interaction within a cavity, and either one, two, or three waves may be simultaneously resonant. The simplest case is the Singly Resonant OPO (SROPO), where only the signal or idler experience feedback and whose frequency coincides with a normal mode of the cavity. A more complex case is the Doubly Resonant OPO (DROPO), where both the signal and idler resonate in the cavity, while the pump experiences little back-reflection. This configuration is more constrained than SROPO, since both the signal and idler have to coincide with cavity normal modes, but it allows for significantly lower powers to be used to reach threshold. Finally, the Triply Resonant OPO (TROPO) is the most efficient, but at the same time a highly over-constrained, configuration that is rarely used in commercial products. The decrease in threshold power of TROPO over DROPO is not significant in our case, since the losses for the pump are significantly higher than the losses for the signal and idler, and most of the pump is depleted due to loss and parametric conversion in a single round trip in the waveguide cavity.
4.2 Motivation and recent developments

To reiterate from the introduction chapter, the motivation for creating an on-chip OPO is to first and foremost be able to generate frequencies in the mid-IR for sensing purposes. Specifically, remote sensing of pollution and toxic gasses is possible using absorption spectroscopy. Alternatively, local on-chip sensing or detection can be made possible using the evanescent tail of the waveguide mode. In order to realize these devices commercially we require powers in the order of 10 mW for remote sensing [59]. Lower powers are required for local detection on a lab-on-a-chip setup where material under study is exposed to the evanescent tail of a waveguide mode.

To date, no truly-integrated OPO has been demonstrated. Lipson et. al. have developed a SiN micro-ring "parametric oscillator" that is based on degenerate four-wave-mixing process, which can be used to amplify the micro-ring normal modes in close vicinity of a strong pump. However, such a device offers no control over the amplified spectrum, and would require a strong mid-IR pump to generate mid-IR frequencies. Fejer et. al. have developed an external-cavity quasi-phase matched OPO for generating 10µm radiation by epitaxially growing thin films of alternating (100) and (100) GaAs. However this OPO was pumped using a LiNBO$_3$ oscillator, which in turn was pumped using a Nd:YAG laser.

Significant work has been done by Rosencher et. al. on creating models for specific (linear) OPO geometries for pulsed and quasi-CW operation [60]. Various schemes for mode-locking a potential semiconductor laser have also been proposed, including the use of large group-velocity-dispersion SESAMs to passively mode-lock a parametric gain medium [61], as well as the study of an inter-cavity oscillator which incorporates an idler amplifier [62]. However, the experimental realization of any of these schemes on a semiconductor chip remains a challenge. Hence, in this chapter we purposefully limit our discussion to proven technologies for realizing an OPO in semiconductor waveguides.
4.3 Theory of Optical Parametric Oscillation

We start by repeating the coupled mode equations that govern the three wave mixing process in waveguides, reduced to one effective dimension 2.48-2.50:

$$\frac{dA_{\text{sig}}}{dz} = -\frac{1}{2} \alpha_{\text{sig}} A_{\text{sig}} - i\kappa_{\text{sig}} A_{\text{idl}}^* A_{\text{pmp}} e^{-i\Delta k z}$$ (4.1)

$$\frac{dA_{\text{idl}}}{dz} = -\frac{1}{2} \alpha_{\text{idl}} A_{\text{idl}} - i\kappa_{\text{idl}} A_{\text{sig}}^* A_{\text{pmp}} e^{-i\Delta k z}$$ (4.2)

$$\frac{dA_{\text{pmp}}}{dz} = -\frac{1}{2} \alpha_{\text{pmp}} A_{\text{pmp}} - i\kappa_{\text{pmp}} A_{\text{idl}} A_{\text{sig}} e^{i\Delta k z}$$ (4.3)

where $\alpha_i$ and the linear power loss coefficient, and the nonlinear coupling coefficients are given by:

$$\kappa_{\text{sig}} = \sqrt{\frac{2\pi^2(\chi_{\text{eff}}^{(2)})^2}{n_{\text{eff,sig}} n_{\text{eff,idl}} n_{\text{eff,pmp}} c \epsilon_0 \lambda_{\text{sig}}^2 (A_{\text{eff}}^{(2)})^2}}$$ (4.4)

$$\kappa_{\text{idl}} = \sqrt{\frac{2\pi^2(\chi_{\text{eff}}^{(2)})^2}{n_{\text{eff,sig}} n_{\text{eff,idl}} n_{\text{eff,pmp}} c \epsilon_0 \lambda_{\text{idl}}^2 (A_{\text{eff}}^{(2)})^2}}$$ (4.5)

$$\kappa_{\text{pmp}} = \sqrt{\frac{2\pi^2(\chi_{\text{eff}}^{(2)})^2}{n_{\text{eff,sig}} n_{\text{eff,idl}} n_{\text{eff,pmp}} c \epsilon_0 \lambda_{\text{pmp}}^2 (A_{\text{eff}}^{(2)})^2}}$$ (4.6)

where

$$[A_{\text{eff}}^{(2)}]^{-1} = \left( \int_{-\infty}^{\infty} F_{\text{sig}} F_{\text{idl}} F_{\text{pmp}} dx dy \right)$$ (4.7)

is the overlap integral. Here we also add the nonlinear absorption term $\alpha_2$ to the equations since the inter-cavity powers in an OPO may reach values where third order effects are not negligible [37]. This coefficient is of the order of 1 cm$^2$/GW [23] for our superlattices, and is normalized by the third-order effective area that gauges how confined a mode is,
and is defined by the equation:

\[ A_{\text{eff}}^{(3)} = \left( \frac{\int_{-\infty}^{\infty} E^2(x,y) dx\,dy}{\sqrt{\int_{-\infty}^{\infty} E^4(x,y) dx\,dy}} \right)^2 \]  

(4.8)

Thus, the coupled mode equations take the following form:

\[
\frac{dA_{\text{sig}}}{dz} = -\frac{1}{2} \alpha_{\text{sig}} A_{\text{sig}} - \frac{1}{2} \alpha_{2,\text{sig}} \frac{|A_{\text{sig}}|^2}{A_{\text{eff}}^{(3)}} A_{\text{sig}} - i\kappa_{\text{sig}} A_{\text{idl}} A_{\text{pmp}} e^{-i\Delta k z} 
\]

(4.9)

\[
\frac{dA_{\text{idl}}}{dz} = -\frac{1}{2} \alpha_{\text{idl}} A_{\text{idl}} - \frac{1}{2} \alpha_{2,\text{idl}} \frac{|A_{\text{idl}}|^2}{A_{\text{eff}}^{(3)}} A_{\text{idl}} - i\kappa_{\text{idl}} A_{\text{sig}}^* A_{\text{pmp}} e^{-i\Delta k z} 
\]

(4.10)

\[
\frac{dA_{\text{pmp}}}{dz} = -\frac{1}{2} \alpha_{\text{pmp}} A_{\text{pmp}} - \frac{1}{2} \alpha_{2,\text{pmp}} \frac{|A_{\text{pmp}}|^2}{A_{\text{eff}}^{(3)}} A_{\text{pmp}} - i\kappa_{\text{pmp}} A_{\text{idl}} A_{\text{sig}} e^{i\Delta k z} 
\]

(4.11)

The equations above are adequate for modeling OPO operation in either CW or quasi-CW operation, where the pulse length greatly exceeds the cavity round-trip.

### 4.4 Numerical model for determining the threshold power for parametric oscillators

The algorithm for modeling the three coupled equations in chapter 3 albeit accurate, is not suitable for modeling beam propagation in an OPO cavity due to the relatively high computational time and memory required per unit distance. Hence, a discretized 1-dimensional algorithm will be used for this purpose. The three equations 4.9-4.11 can be discretized to yield a numerical model to accurately predict the threshold powers required for parametric oscillation to take place. The model will track the power in the signal, idler, and pump beams through a large number of round trips in a potentially multi-segmented waveguide with lengths of up to several millimeters.

As mentioned in the previous section, there are two conditions for parametric oscilla-
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The parametric gain should exceed the loss in the cavity, and the oscillating field(s) are to be normal modes of the cavity. The second condition is trivial to satisfy in a cavity that is significantly longer than the wavelength, which supports many longitudinal modes. Thus, any cavity phase effects are omitted from the model - this reduces the computational effort to propagating a single data point that contains the power information for every interacting wave.

Equations 4.5 to 4.7 are discretized using the standard Euler explicit centred-difference explicit method, resulting in the following expression for the three coupled wave equations:

\[ A_{\text{sig}}^{z+1} = A_{\text{sig}}^{z-1} + 2\Delta z \left[ -\frac{1}{2} \alpha_{\text{sig}} A_{\text{sig}}^z - \alpha_{2,\text{sig}} \frac{|A_{\text{sig}}^z|^2 A_{\text{sig}}^z}{2A_{\text{eff}}^{(3)}} - i\kappa_{\text{sig}} (A_{\text{sig}}^z)^* A_{\text{pmp}}^z e^{-i\Delta k z} \right] \] (4.12)

\[ A_{\text{idl}}^{z+1} = A_{\text{idl}}^{z-1} + 2\Delta z \left[ -\frac{1}{2} \alpha_{\text{idl}} A_{\text{idl}}^z - \alpha_{2,\text{idl}} \frac{|A_{\text{idl}}^z|^2 A_{\text{idl}}^z}{2A_{\text{eff}}^{(3)}} - i\kappa_{\text{idl}} (A_{\text{sig}}^z)^* A_{\text{pmp}}^z e^{-i\Delta k z} \right] \] (4.13)

\[ A_{\text{pmp}}^{z+1} = A_{\text{pmp}}^{z-1} + 2\Delta z \left[ -\frac{1}{2} \alpha_{\text{pmp}} A_{\text{pmp}}^z - \alpha_{2,\text{pmp}} \frac{|A_{\text{pmp}}^z|^2 A_{\text{pmp}}^z}{2A_{\text{eff}}^{(3)}} - i\kappa_{\text{pmp}} A_{\text{idl}}^z A_{\text{sig}}^z e^{i\Delta k z} \right] \] (4.14)

No artificial dissipation term is necessary in this case since the loss is not introduced through a complex variable but rather as a term in the equation. The reflections are taken into account by multiplying the \( A \) variable by the field reflectivity at a location \( z = nL \), where \( L \) is the length of the cavity, and \( n \in N \). Numerous round trips in the cavity can be simulated by propagating the amplitude variables through different cavity sections in a loop.

The initial conditions for the problem require a non-zero input for both the signal and idler beams, otherwise the trivial solution will emerge. The initial power values are chosen to be the typical powers of parametric fluorescence in AlGaAs, on the order of \( 10^{-11} \)W [63]. Changing this value to within 3 orders of magnitude does not have an effect on the threshold power, and varies the number of round trips it takes to reach threshold by approximately 10%. To determine the threshold condition for a given geometry, power is
gradually increased until the evolution of the beam power reaches a non-zero steady-state solution.

The modeling tool was tested to reproduce simple analytic results for the TWM process in chapter 2 as well as to reproduce the threshold condition for low-mirror loss cavities derived in [21], and the threshold power was typically 5% higher of these analytic solutions. The reason for these typically higher powers lies in the difficulty in accurately gauging the power at which amplification is observed.

### 4.5 Evaluation of a Coupled-Ring OPO design

As a major milestone for in our project we would like to demonstrate either an integrated DROPO which relies on type II phase matching, or a degenerate SROPO which relies on type I phase matching using a SL laser operating near 800 nm which generates signal and idler waves near 1.6 µm. The results in chapter 3 indicate that there is little difference between the two configurations in terms of efficiency. Figure 4.1-(a) shows a schematic of our proposed integrated OPO design. The active laser ring serves as the pump, with an emission band centred at ∼ 800 nm, and has tunability of ∼ 2 nm (please refer to Appendix A for detailed information regarding the laser). The laser is coupled to a passive ring using two dichroic multi-mode interference (MMI) couplers. The experimentally measured "cross"- and "bar"-cross talk behaviour of the coupler is depicted in figures 4.1-(b) and 4.1-(c) respectively [7]. The straight passive section between the two couplers contains the quasi-phase matched (QPM) grating, where the signal and idler beams experience parametric gain. Once the beams reach the end of the QPM section, the remaining pump light is diverted back to the active cavity, while the signal and idler propagate along the passive ring, and a while fraction of the beam is diverted as output. The typical ring diameters are 0.4 mm.

Such a coupled ring design has a number of advantages and compromises. The design
Figure 4.1: Proposed double-ring OPO design. a) Schematic of the design depicting the coupling between the two rings. b) "Cross" cross-talk operation of the dichroic MMI coupler at the telecom wavelengths. A little as 10% of optical power is lost via coupling losses. c) "Bar" cross-talk operation of the dichroic coupler. [7]
allows for pulsed operation of the laser: designing the rings to be of different lengths allows synchronizing the arrival of signal, idler, and pump pulses at every round trip to counteract the effects of group velocity mismatch between the beams. Another design advantage lies in the lack of need to fabricate high-reflectivity dielectric gratings to serve as cavity mirrors, which simplifies the fabrication process considerably. The disadvantage of the design lies in the relatively long passive section where the generated signal and idler beams are allowed to attenuate. This is equivalent to a straight cavity with relatively lossy mirrors, but one that allows for more close-spaced longitudinal modes and hence somewhat relaxes the constraint on having coinciding normal modes for both the signal and idler.

Figure 4.2 depicts the cavity length dependence of the threshold power for different loss coefficients for the signal and idler: 4.1 cm\(^{-1}\) is the loss coefficient for our current waveguides, 1.5 cm\(^{-1}\) is the loss coefficient for waveguides with a shallow 0.8 µm etch, and 0.5 cm\(^{-1}\) is the state of the art for Si nanowires, where the etch is deeper than the
guiding layer. The pump losses were kept constant at 25 cm$^{-1}$, and optimal modulation of $\chi^{(2)}$ was assumed at 20 pm/V. It is clear from the results that operating such OPO in CW will melt the device. We could potentially Q-switch the lasers using a multisectioned diode schemes according the method in references [64], [65], [66], and [67]. Nevertheless, the round trip duration of light in the laser cavity dictates a repetition rate of several $\sim 10$ GHz, which does not allow for peak powers larger than several watts to be generated while keeping the average powers under 20dBm (100 mW), which is close to the damage threshold of the devices. Hence, the propagation losses throughout the devices need to be minimized in order to lower the threshold power.

### 4.6 Proposed alternative design

The proposed coupled-ring design is somewhat inefficient for CW or quasi-CW operation since the parametric gain takes place over a very short portion of the OPO loop. Linear losses in the remainder of the loop greatly reduce the gain over a single round trip. Hence, a straight-forward method to reduce the propagation losses is to eliminate as much of the loop as possible, as depicted in figure 4.3. In this design the wafer is cleaved at the end of the QPM section, and a high-reflectivity dielectric coating is deposited on the cleaved edge. In addition, gold is deposited on the output edge of the dichroic coupler to serve as a mirror. This forms a dual cavity for the laser and the OPO. As figure 4.4 shows, the threshold power of this design is almost three times lower than that of the dual ring design, however it still makes an integrated OPO not feasible with the current loss values.

It is also worthwhile to look at the limitations of the material technology itself by examining the most efficient cavity design - a straight waveguide section with 99% reflective mirrors on either end, as depicted in figure 4.5. Figure 4.6 shows the oscillation threshold of such cavity for different values of modulation and signal and idler loss. The
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Figure 4.3: Proposed alternative OPO design. The cleaved edge on the right has high reflectivity coating, the etched edge of the dichroic coupler has gold deposition to serve as the second mirror.

Figure 4.4: Threshold power of the proposed alternative design for different values of signal and idler loss coefficients. The threshold values are almost 3 times lower than those of the double ring design.
results show that it is possible to operate the OPO under 30 dBm of CW power with full modulation of $\chi^{(2)}$ and using the lowest experimentally determined loss coefficient for an intact shallow-contrast (0.8\,\mu m etch depth) superlattice waveguide. This shows that it is possible to make a proof-of-concept device that is pumped with a table-top Ti:Sapphire laser while using an acousto-optic modulator (AOM) to lower the average power arriving at the facet of the device. In order to reach a desired 20 dBm threshold the losses will need to be lowered to $\sim 0.1$ cm$^{-1}$ for the signal and idler 10 cm$^{-1}$ for the pump given the currently used waveguide structure and optimal intermixing. These loss values are comparable to state of the art for AlGaAs waveguides.

4.7 Comparison between different phase matching technologies with GaAs/AlGaAs materials

Now that we have explored the limitations of our material technology, it is interesting to see how it compares performance-wise to other technologies for frequency conversion that have been reported in the literature. We restrict ourselves to considering only technologies for frequency conversion in AlGaAs, as they are the ones that have the potential for integration with on-chip lasers. In chapter 1 four different frequency conversion technologies utilizing AlGaAs were listed, and all have the potential to serve as a basis of an integrated OPO. The modal phase matching technique is readily integrable with laser
diodes, as demonstrated in [30]. On the other hand, the form birefringence method, which promises to be most efficient, cannot be integrated with an active component on the same guiding layer due to the presence of oxide layers. However, a vertical integration scheme may present a viable solution to this problem. It is interesting to see which technology has the best potential to serve as the passive parametric conversion component in an on-chip OPO. The 1D model developed here can provide a quantitative comparison of the threshold powers required for OPO operation. The results of the simulations, that are shown in figure 4.7, suggest that the two most promising technologies for low-threshold OPO operation are quasi phase matching using orientation patterned substrates, and form birefringence phase matching. Both approaches may potentially allow the creation of OPOs with under-20 dBm threshold. The two main advantages of these methods over the remaining three are the low propagation losses and high $\chi^{(2)}_{eff}$ - OPS employs the full swing of the tensor element (where $\chi^{(2)}_{eff} = 0.64\chi^{(2)}_{max}$), while FBPM utilizes the full tensor element.
Figure 4.7: Threshold power of a straight cavity OPO with 99% facet reflectivities for different technologies. BRW - Bragg reflection waveguides [8], MPM - modal phase matching [3], OPS - quasi phase matching using orientation-patterned substrates [9], FBPS - form birefringence phase matching [10]

4.8 Conclusion

In this chapter we have discussed the basic theory behind optical parametric oscillation, along with the description of the numerical model used to determine the threshold power of arbitrary OPO chip geometries. The modeling tool was then applied to different on-chip OPO geometries to compare their relative efficiency. The different approaches for phase matching in AlGaAs were evaluated for serving as the building block of an on-chip OPO. Form-birefringence phase matching is currently the most promising technology for this task due to the low linear losses combined with a large value of the utilized $\chi^{(2)}_{eff}$. 
Chapter 5

Conclusion

5.1 Summary

The main contribution of this work is the enhancement of our modeling capabilities. Two new modeling tools were developed: A 2D beam propagation tool for optimizing QPM gratings, and a 1D iterative beam propagation tool for determining the output powers and threshold of optical parametric oscillators of arbitrary geometries. The 2D tool was used to evaluate the change in conversion efficiency as a function of detuning from the half bad-gap and as a function of diffusion length for a diffused grating profile. Enhancement of the conversion efficiency was seen for 1 mm long samples near 800 nm. Reflections at sharp interfaces along the grating were observed for low diffusion lengths which were superimposed on the gain curve. These resonances were not observed for diffusion lengths over 200 nm. The 1D tool was used to determine the threshold conditions for parametric oscillation for different geometries and approaches to phase matching. It allows us to quantitatively compare the performance of the different technologies and OPO geometries, to determine the kind of improvements we need to make to the material technology to reduce threshold, and to observe the effect of such improvements on the threshold power.
5.2 Future research directions

A main deficiency of the QWI technology compared to other approaches for phase matching in AlGaAs is the utilization of small effective second order susceptibility. In order to increase efficiency, and decrease the threshold power for parametric oscillation the figure of merit

$$M = \frac{(\chi^{(2)}_{eff})^2}{n_1 n_2 n_3 (A^{(2)}_{eff})^2}$$

needs to be maximized, along with minimizing the linear losses and the value of $A^{(3)}_{eff}$. If we are to continue with the current in-plane waveguide design, the following needs to take place:

1. Optimization of the intermixing process. Current results show sub-optimal modulation of $\chi^{(2)}$, which is approximately 30% of the theoretically predicted maximum value. Process optimization needs to take place in order to both minimize defect-induced scattering losses and maximize the intermixing.

2. Mode-locking of integrated lasers. The results of chapter 4 suggest that even the most efficient approach for making an OPO that was reported in the industry requires threshold powers higher than 100 mW. To reduce this value, mode-locking may allow us to operate the devices with higher peak power while maintaining average powers below 100 mW.

3. Minimization of the second-order effective area. This may be achieved by fabricating high-contrast nanowire-type waveguides, where the etch depth is greater than the depth of the guiding layer. This may reduce the $A^{(3)}_{eff}$ by as much $\sim \times 2$, leading to $\sim \times 4$ improvement in the conversion efficiency. These waveguides are highly sensitive to surface roughness, which would require special attention given to the etch recipe. Also, minimizing $A^{(2)}_{eff}$ invariantly minimizes $A^{(3)}_{eff}$, and third-order effects that hinder conversion efficiency will become evident at lower powers.
4. Use the nanowires’ birefringence that can be controlled by controlling the geometry [68] to assist the phase matching process. Strong birefringence will enable the use of longer-period gratings to cover the same phase mismatch, which will reduce the required spatial resolution for the grating, thus easing the fabrication process by increasing the feature size and lowering fabrication tolerances. This will also allow the use of sputtered silica caps to be used to promote and suppress intermixing [56]. Sputtered silica caps allow for larger band gap modulation compared to ion implantation, however they have lower spatial resolution.

We also aim to demonstrate integrated active and passive devices on the same chip. Planar integration may not be the best avenue to proceed in that direction, as it does not allow for separate optimization of the laser and OPO cavities. Moreover, SL lasers are not as efficient as MQW laser. Hence, vertical integration may be a better approach for realizing multi-functional devices. A new wafer along with vertical couplers will have to be designed and optimized.
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