Nonlinear FEA of the Crush Behaviour of Functionally Graded Foam-Filled Columns

by

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A thesis submitted in conformity with the requirements for the Degree of Master of Applied Science,
Department of Mechanical and Industrial Engineering
University of Toronto

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Abstract

The use of metallic foams as a filler in thin-walled structures can enhance their crashworthiness characteristics. It is believed that, tailoring the properties of the foam filler would enhance the effectiveness of these characteristics. This view is also supported by recent works in the literature. It is the objective of this study to examine the crush behaviour of functionally graded foam-filled tubes and evaluate the effect of discretely graded density upon the specific energy absorbed. Nonlinear parametric finite element simulations of the foam-filled tube were developed to estimate the most favourable foam density gradient in the lateral and axial directions. The effect of various design parameters such as density grading, number of grading layers, and thickness of the interactive layer upon the resulting specific energy absorption was investigated. The results show that the specific energy absorption of a tube filled with functionally graded foam is better than uniform density foam.
Acknowledgements

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<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_l$</td>
<td>Cross sectional area of foam layer</td>
<td>$N_G$</td>
<td>Number of grading layers</td>
</tr>
<tr>
<td>$A_{Total}$</td>
<td>Total cross sectional area of FGF</td>
<td>$N$</td>
<td>Gradient exponent</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Area of the removed core</td>
<td>$n$</td>
<td>Strain hardening exponent</td>
</tr>
<tr>
<td>$c$</td>
<td>Width of the column</td>
<td>$P$</td>
<td>Property of interest</td>
</tr>
<tr>
<td>$C, K$</td>
<td>Geometrical and material constant</td>
<td>$SEA$</td>
<td>Specific energy absorption</td>
</tr>
<tr>
<td>$C_{avg}$</td>
<td>Constant for interaction</td>
<td>$S_e$</td>
<td>Stroke efficiency</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Mean width of the column</td>
<td>$\Delta SEA$</td>
<td>Variation of SEA</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Mean width of the foam filler</td>
<td>$\Delta SEA_{DFGF}$</td>
<td>Variation of SEA of DFGF filled column</td>
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<tr>
<td>$d$</td>
<td>Distance</td>
<td>$TEA$</td>
<td>Total energy absorption</td>
</tr>
<tr>
<td>$d_{max}$</td>
<td>Effective crushing distance</td>
<td>$t_o$</td>
<td>Thickness of interactive layer</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>$t$</td>
<td>Thickness of grading layers</td>
</tr>
<tr>
<td>$E_e$</td>
<td>Energy efficiency</td>
<td>$V_{initial}$</td>
<td>Initial volume of foam</td>
</tr>
<tr>
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<td>Absorbed energy of column</td>
<td>$\Delta V$</td>
<td>Volume difference of compacted foam</td>
</tr>
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<td>$E_{DFGF}$</td>
<td>Absorbed energy of DFGF</td>
<td>$\varepsilon$</td>
<td>Engineering strain</td>
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<tr>
<td>$E_f$</td>
<td>Elastic modulus of foam</td>
<td>$\varepsilon_T$</td>
<td>True strain</td>
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<td>$E_{int.}$</td>
<td>Interaction energy</td>
<td>$\varepsilon_v$</td>
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<td>Response surface error</td>
<td>$\varepsilon_D$</td>
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<td>Instantaneous collapse load</td>
<td>$\mu$</td>
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<td>Maximum load</td>
<td>$\xi$</td>
<td>Response surface variable</td>
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<td>Mean collapse load of column</td>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
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<tr>
<td>$F_{m}$</td>
<td>Mean collapse load of foam-filled column</td>
<td>$\rho$</td>
<td>Density of column wall material</td>
</tr>
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<td>Density of outer layer</td>
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<td>$h$</td>
<td>Column wall thickness</td>
<td>$\rho_{max}$</td>
<td>Maximum foam density</td>
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<tr>
<td>$L$</td>
<td>Length of the column</td>
<td>$\rho_{min}$</td>
<td>Minimum foam density</td>
</tr>
<tr>
<td>$M_o$</td>
<td>Fully plastic moment</td>
<td>$\rho$</td>
<td>Relative density</td>
</tr>
<tr>
<td>$m_{tot.}$</td>
<td>Total mass of the component</td>
<td>$\Delta \rho$</td>
<td>Density range</td>
</tr>
<tr>
<td>$H$</td>
<td>Half of fold length</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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\[ \rho_f(y) \] Density grading function
\[ \sigma \] Engineering stress
\[ \sigma_N \] Normal stress
\[ \sigma_o \] Flow stress of column wall material
\[ \sigma_{os} \] Flow stress of the parent material
\[ \sigma_T \] True stress
\[ \sigma_u \] Ultimate strength of column wall material
\[ \sigma_{ys} \] Yield strength of parent material of foam
\[ \sigma_f \] Flow stress of foam
\[ \sigma_t \] Tensile strength of foam
\[ \sigma_y \] Yield strength of column wall material
\[ \tau_s \] Shear stress
\[ \tau_f \] Shear strength of foam
Chapter 1: Introduction and Justification

Summary: In this chapter, the definition of the problem, justification of the study, and the method of approach are outlined and presented.

1.1 Introduction

Innovative concepts in the design of light weight and the development of environmentally friendly vehicles are increasingly sought. The solution for the above requirements is to minimise the weight of the vehicle so that fuel consumption is reduced and the emission of gasses is within a specific limit.

Light weight alloys, such as aluminium and magnesium, are gradually finding their place in vehicle designs ([1] and [2]). Automobile manufacturers, such as Mercedes-Benz, Audi, GM, and Ferrari, have developed aluminium space frames with high weight saving ratio. However, the concern for the crashworthiness of lighter vehicles must be addressed. According to the Insurance Institute for Highway Safety of United States (IIHS), there were 96 fatalities per million registered vehicles for the small cars, compared to 62 fatalities for the midsize class of cars and 64 fatalities per million for large sedans in 2007 [3] (Figure 1.1).

![Figure 1.1 Crashworthiness and occupant safety of light weight vehicle frame (after [4])](image)

Progressive folding collapse of thin-walled structures is known to be an effective energy absorbing mechanism in crashworthiness applications that protect passengers from serious
injuries. Therefore, modern vehicle frames are made from thin-walled tubes. Furthermore, it has been proven that crashworthiness of thin-walled sections can be further improved by filling these tubes with uniform density foams [5]. Metallic foams are good candidates for crashworthiness applications, since they can undergo large plastic deformation at an approximately constant stress ([6] and [7]).

The proper selection of tube geometry and metallic foam density dictate the amount of energy to be stored by the foam-filled tube. Increasing the foam filler’s density will result in increasing the total energy absorption (TEA) of the filled component [8]. Although utilising high-density metallic foam could increase the TEA of thin-walled tubes, some researchers suggest that the specific energy absorption (SEA, J/kg) may be reduced ([9] and [10]). Consequently, to further enhance the crashworthiness of foam-filled thin-walled sections, other alternatives should be considered.

1.2 Justification of the Study

In nature, porous materials such as wood, bamboo or human bones are most often “functionally graded” [11]. Recently, more attention has been devoted to a new concept known as functionally graded foam material (FGFM). In FGFM, the characteristics of the foam, e.g. the density, are varied continuously in a predefined manner through the thickness in order to improve the energy absorption characteristics over and above those offered by uniform density foam [12].

The ultimate objective in a collision is to maximize the safety of the passengers and minimize the damage to the vehicle. It has been shown in references [8] and [12] that FGFM is a suitable candidate for improving SEA over traditional uniform density foam. However, producing FGFM is more complex than uniform density foam (UDF), and requires advanced technologies to manufacture a given gradient under laboratory conditions [8]. In this study, a discrete form of functionally graded foam (DFGF) is introduced. DFGF can be constructed from existing foam layers with different densities in the axial or lateral directions. Density grading, number of grading layers, and thickness of the interactive layer (interactive layer is the outermost layer of the DFGF) are likely to affect the energy absorption characteristics of DFGF. The effect of these parameters should be explored to provide a better understanding of the tailoring properties of the DFGF.
1.3 Research Objectives

It is, therefore, the objective of the current study to:

(i) evaluate the effect of DFGF as opposed to UDF upon the crush behaviour of foam-filled tubes, and

(ii) evaluate the effect of density grading, number of grading layers, thickness of the interactive layer, and interface condition (contact and friction) upon the resulting specific energy absorption.

1.4 Method of Approach

Figure 1.2 outlines the method of approach adopted. It consists of two sections. The first includes the details of the FE model as well as validation of the FE simulation results with closed form solutions and available experimental results [13]. In the second, efforts are devoted to study the effect of density grading, number of grading layers, thickness of the interactive layer and other relevant design parameters of DFGF filled column upon the resulting specific energy absorption using response surface models (RSMs). Details of the RSM and interactive layer are provided in Chapters 2 and 3, respectively.

![Crush Behaviour of Functionally Graded Foam-Filled Columns](image)

**Figure 1.2** Method of approach
1.5 Thesis Layout

This thesis is divided into five chapters. **Chapter One** justifies the undertaking of the study and outlines the method of approach adopted in this work. **Chapter Two** presents a review of existing literature on thin-walled columns, metallic foams, and foam-filled columns. **Chapter Three** provides a detailed account of the quasi-static finite element modelling of the axial collapse of empty and foam-filled columns. The design parameters of DFGF filled columns in the axial and lateral directions are introduced. The effect of layered model parameters on the FE predictions is also presented. **Chapter Four** outlines the major findings of the research. This Chapter summarises the quasi-static FE simulations results, including the validation of FE predictions. The effect of design parameters on DFGF filled column is also determined in this chapter. **Chapter Five** concludes the work and outlines areas for possible future exploration.
Chapter 2: Literature Review

Summary: This chapter presents an overview of existing literature pertaining to crashworthiness of foam-filled columns and response surface approach. The review related to foam-filled columns is divided into three areas: energy absorption by plastic collapse of thin-walled columns, metallic foams, and the crush behaviour of foam-filled columns.

2.1 Axial Collapse of Thin-Walled Columns

Buckling of columns was first introduced by Euler in 1744. Rhodes [14] summarised the early work on buckling of rectangular tubes. However, since the progressive plastic collapse of aluminium square columns is the primary concern in this study, the concept of buckling is not directly relevant to this work.

Kitagawa et al. [15] studied the axial crushing behaviour of thin-walled columns. The mechanism of axial crush of thin-walled columns was described as follows:

(i) during the collapse, local buckling occurs at the weakest point of the column and slight wave appear on the column wall,

(ii) as the collapse continues stress concentrations rise at the edges of the column wall,

(iii) the column wall edges yield, and

(iv) Progressive collapse occurs along the length.

The deformation stages and progressive plastic collapse of an axially crushed thin-walled column are illustrated in Figures 2.1 and 2.2, respectively. The collapse behaviour of thin-walled columns can be evaluated by terms defined in Appendix A.
2.1.1 Progressive Plastic Collapse

Wierzbicki and Abramowicz [17] presented a theory that describes the mechanics of progressive folding of thin-walled structures under axial loading. In order to develop their theory, they used the following assumptions:

(i) the structure consists of planar surface elements,

(ii) the material is assumed to be rigid-perfectly plastic,

(iii) the length of the local buckling wave $2H$ remains constant during the formation of each fold, and

(iv) the constraints imposed on the crushing process by the boundary and symmetry conditions force the fold lines to move through the material.
As a special case, they calculated the crushing load based on energy balance assumption for the symmetrical collapse mode of a square tube. Abramowicz and Wierzbicki [18] improved their solution by assuming an arbitrary angle between the adjacent plates of the structure. Abramowicz and Jones [19] conducted around 84 dynamic tests on various lengths of steel columns with width to thickness ratio \((c/h)\) of 30.25 and 32.18. Based on the results, they developed theoretical predictions for the axial progressive collapse of square box columns and introduced two basic collapse elements as shown in Figures 2.3(a) and 2.3(b).

![Figure 2.3 Basic collapse elements: (a) type-I, and (b) type-II (after [19])](image)

Based on these two elements, four deformation modes of collapse were predicted for the collapse behaviour of square columns. These deformation modes are as follows:

**Symmetric mode**

This mode of deformation, developed by a collapse layer, consists of four type-I folding elements. The mean collapse load \((F_m^c)\) and half fold length \((H)\) for symmetric mode of collapse can be calculated from [20]:

\[
F_m^c = M_o \left[ 52.22 \left( \frac{c}{h} \right)^{\frac{1}{2}} \right] \tag{2.1}
\]

\[
H = 0.99 c^{\frac{2}{3}} h^{\frac{1}{3}} \tag{2.2}
\]

where \(c\) is the width and \(h\) is the thickness of the column wall and \(M_o\) is the fully plastic moment of the wall per unit length, and it is given by [20]:

\[
M_o = 0.25 \sigma_o h^2 \tag{2.3}
\]
where \( \sigma_o \) is the flow stress of the column material. For an ideal elastic-perfectly plastic material, the flow stress is considered to be equal to the yield stress. However for materials showing strain hardening, Hanssen et al. [21] suggested the following equation for calculating the flow stress:

\[
\sigma_o = 2.23^n \frac{\sigma_u}{n+1} \left[ \frac{2}{n+2} \right]^{\frac{2}{3}} \left[ \frac{h}{c} \right]^{\frac{4n}{9}}
\]

(2.4)

where \( n \) is the strain hardening exponent and \( \sigma_u \) is the ultimate strength of the column wall material.

**Extensional mode**

This mode of deformation is developed by a collapse layer consisting of four type-II folding elements. The \( F_m^c \) and \( H \) for the extensional mode of collapse can be calculated from [20]:

\[
F_m^c = M_o \left[ 32.64 \left( \frac{c}{h} \right)^{\frac{1}{2}} + 8.16 \right]
\]

(2.5)

\[
H = c^2 h^2
\]

(2.6)

**Asymmetric mixed mode A**

This mode of deformation is developed by two collapse layers with six type-I and two type-II folding elements. The \( F_m^c \) and \( H \) for the asymmetric mixed mode A of collapse can be calculated from [20]:

\[
F_m^c = M_o \left[ 42.92 \left( \frac{c}{h} \right)^{\frac{1}{2}} + 3.17 \left( \frac{c}{h} \right)^{\frac{2}{3}} + 2.04 \right]
\]

(2.7)

\[
H = 0.78 c^2 h^3
\]

(2.8)

**Asymmetric mixed mode B**

This mode of deformation is developed by two layers with seven type-I and one type-II basic folding elements. The \( F_m^c \) and \( H \) for the asymmetric mixed mode B of collapse can be calculated from [20]:

\[
F_m^c = M_o \left[ 45.90 \left( \frac{c}{h} \right)^{\frac{1}{2}} + 1.75 \left( \frac{c}{h} \right)^{\frac{2}{3}} + 1.02 \right]
\]

(2.9)

\[
H = 0.86 c^2 h^3
\]

(2.10)
Langseth and Hopperstad [22] studied the energy absorption and the collapse behaviour of square thin-walled columns made of aluminium alloy 6060 subjected to dynamic and quasi-static loading. In their study, three different tempers and wall thicknesses of the column were tested. The impact velocity was varied from 8 m/s to 20 m/s with a 56 kg projectile. They carried out quasi-static tests to develop a relationship between the dynamic and quasi-static collapse behaviour.

In all the quasi-static crushed specimens, the progressive symmetric mode of deformation was observed and the mode of collapse was independent of the temper condition of the column wall material. However, the number of folds formed along the length of the column was a function of the column wall temper condition. In dynamic cases, a mixture of symmetric, asymmetric, and extensional modes was observed. The experimental results also showed that the dynamic mean load was higher than the corresponding quasi-static load for the same axial displacement, which indicates the existence of the inertia effect.

2.1.2 Finite Element Modelling of Thin-Walled Columns

Langseth and Hopperstad [23] performed finite element analysis using LS-DYNA on the quasi-static and dynamic axial collapse of thin-walled aluminium columns. They modelled the column using shell elements and introduced a small trigger in the column wall as a representation of the initial geometrical imperfections. They compared the FE predictions with the experimental data in order to validate the model as well as the assumptions made regarding the material input data and the modelling of the initial geometry of the column. In addition, a parametric study was conducted to investigate the response of aluminium column, varying the mass of the projectile and the impact velocity. They determined that, $F_m^c$ of the column is an increasing function with respect to an increase in the impact velocity, and the mass ratio between the projectile and the specimen had no effect on $F_m^c$.

In another study, Langseth et al. [24] investigated the crash behaviour of thin-walled aluminium columns in order to improve the energy absorbed by thin-walled sections. Dynamic simulations were carried out using LS-DYNA for of T4 temper aluminium columns. Only one quarter of the columns were modelled using symmetry planes, since symmetric deformation mode was observed for T4 temper specimens during the experiments. The columns were modelled using
the Belytschko-Lin-Tsay shell element, while the loading platen was modelled as a rigid body. They found good agreement between the shape of the final profile and the instantaneous load-displacement curve.

2.2 Metallic Foams

Metallic foams are cellular structures that can absorb impact energy regardless of the direction of the impact. Furthermore, they have unique mechanical, electrical, thermal, and acoustic properties [25]. This combination of specific properties cannot be achieved simultaneously from other conventional materials. The mechanical properties of this group of material can be related to their density [26].

Open-cell and closed-cell structures of metallic foam are shown in Figures 2.4(a) and 2.4(b), respectively. In open-cell foams, the cells are created by metal struts that are connected at vertices and formed a web of interconnected voids. In case of closed-cell foams, the cells are closed, and separated by thin solid walls.

![Cellular structure of metallic foams](image)

**Figure 2.4** Cellular structure of metallic foams (a) closed cell (after [26]), and (b) open cell (after [27]).

2.2.1 Manufacturing Processes

The first attempt to create metallic foams was done by Sosnik in 1943 [28]. He added mercury to molten aluminium to create pores in the material. In 1956, Elliot [29] replaced mercury by agents that could generate gas bubbles by thermal decompositions.
Numerous attempts have been undertaken in 1960s to produce metallic foams from aluminium, magnesium and zinc or alloys based on one of these metals ([30]-[32]). In 1969, Allen [33] introduced the basic processing technique for the manufacturing of the aluminium foam from aluminium powder and titanium hydride powder (TiH₂) as the foaming agent. Reviews of the production processes for producing open-cell and closed cell metallic foams from aluminium, zinc, nickel, and steel alloys are presented in references [34]-[37].

Foaming temperature, heating rate and the type of foaming agent are some of the essential factors that affect structure of metallic foams ([38] and [39]). Metallic foams used in structural crashworthiness and automotive applications are predominantly fabricated using the direct gas injection technique since this method is economical for mass production process of metallic foams [25].

Cymat Aluminium Corp. utilises this technique, as shown in Figure 2.5, for manufacturing closed-cell metallic foams with a production rate of 1000 kg/h. In order to prepare the metal matrix composite, a variety of aluminium alloys such as casting alloy ALSi10Mg can be used.

The aluminium melt viscosity is increased by uniform distribution of stabilising agents such as silicon carbide (SiC), aluminium oxide (Al₂O₃) or magnesium oxide (MgO) particles. Air, nitrogen (N₂) or argon (Ar) is then injected uniformly through a rotating impeller into the melt to form bubbles. The bubbles rise through the liquid metal matrix composite and create a foam structure. The resultant mixture floats to the surface where it cools down and solidified on conveyor belts to form the final product.

The cell size of the final product and the foam density could be controlled by varying the operational conditions of the injection system. By changing the rate and the amount of injected gas into the liquid metal, aluminium foams with relative densities between 4% and 20% could be produced.
2.2.2 Mechanical Properties

Figure 2.6 illustrates a typical compressive stress-strain curve of aluminium foam. The compression response consists of three regions: a linear elastic deformation at small strain values (less than 0.05), a long plateau with nearly constant stress value, and the densification region at large strain values (typically between 0.6-0.9, depending on the foam density). In the plateau region, progressive elastic buckling, yielding and fracture are the dominant failure modes of the cell walls. In the densification region, all the cell walls have been collapsed and further cell collapse cannot occur after this stage.

Figure 2.5 Manufacturing of aluminium foam (a) a schematic illustration of direct foaming of melts by gas injection (after [40]), and (b) horizontal continuous casting process (after [41])
The area underneath the stress-strain curve of the metallic foam is equal to the amount of absorbed energy by the metallic foam per unit volume and is given by:

\[ TEA = \int_{\varepsilon_i}^{\varepsilon_f} \sigma(\varepsilon) \, d\varepsilon \]  \hspace{1cm} (2.11)

Ashby and Gibson [43] conducted extensive research on the mechanical behaviour of foam materials, and found that the mechanical properties of foam materials follow a power law function of the following form:

\[ \frac{P_{\text{Foam}}}{P_{\text{Solid}}} = C(\bar{\rho})^n \]  \hspace{1cm} (2.12)

where \( P \) is the property of interest, \( C \) is a constant of proportionality, \( \bar{\rho} \) is the relative density of foam, and the exponent \( n \) usually has a numerical value between 1.5 and 2. The relative density of foam is given by:

\[ \bar{\rho} = \frac{\rho_f}{\rho_s} \]  \hspace{1cm} (2.13)

where \( \rho_f \) is the foam density, and \( \rho_s \) is the density of the parent material of the foam. In crashworthiness applications, the flow stress (plateau stress) is the quantity of interest. Since only closed-cell metallic foams were considered in this study, some of the uniaxial flow stress relations of the closed-cell metallic foam are presented in this section.

Ashby and Gibson [43] introduced the following relation for the flow stress of closed-cell metallic foams:

\[ \sigma_f = 0.3\sigma_{ys}(\bar{\rho})^{1.5} \]  \hspace{1cm} (2.14)

where \( \sigma_{ys} \) is yield strength of the parent material of the metallic foam. In another study, Santosa and Wierzbicki [44] suggested that the flow stress of a closed-cell foam structure can be calculated from:

\[ \sigma_f = 1.05\sigma_{os}(\bar{\rho})^{1.52} \]  \hspace{1cm} (2.15)

where \( \sigma_{os} \) is the flow stress of the parent material of the foam. The dependence on \( \bar{\rho} \) in both equations is nearly the same. In spite of this, the proportionality constant of equation (2.14) is almost three times smaller than equation (2.15). The flow stress of foam in equation (2.14) is calculated based on the yield strength of the cell wall material. However, the initiation of plastic
collapse is due to the formation of plastic hinges at the corners while the cell edges remain rigid [44]. Hanssen et al. [45] introduced a simple empirical relationship for the flow stress of aluminium foams:

\[ \sigma_f = 3640(\bar{\rho})^3 \]  

(2.16)

In another study, Reyes et al. [46] suggested the following expression for the flow stress of aluminium foams:

\[ \sigma_f = 800(\bar{\rho})^{2.38} \]  

(2.17)

### 2.2.3 Finite Element Modelling of Foam Materials

Santosa and Wierzbicki [44] developed a truncated cube model for FE modelling of the crush behaviour of the closed-cell aluminium foam. Figure 2.7 depicts the truncated cube model for closed-cell aluminium foam. The truncated cube model consisted of a system of collapsing cruciform and pyramidal sections and was capable of presenting the basic folding mechanism of cells. In their study, theoretical analysis was based on energy consideration combined with the minimum principle in plasticity. They showed that analytical formulation for the crushing resistance of the truncated cube cell was correlating accurately with their numerical results. They also developed closed form solutions for crushing resistance of closed-cell aluminium foam in terms of relative density with excellent agreement with the experimental results.

![Figure 2.7](image)

**Figure 2.7** Truncated cube model for closed-cell foam: (a) an assembly of foam cell, and (b) isometric view of the basic folding element (after [44])

Meguid et al. [47] employed a modified unit cell model to model the localization of deformation in cellular materials due to existence of density variations and material defects. The modified
unit cell model was based on the unit cell proposed earlier in reference [44] as illustrated in Figure 2.8. They assumed that the mean through-thickness density variation was a function of the casting process of the foam and the in-plane density variation was assumed to follow a statistical probability distribution of the Gaussian type. Their modified model exhibited deformation localization patterns comparable to those observed in the experimental compression testing of both transverse and in-plane crushing. They also found that if the proper density distribution was taken into account, there is a good correlation between the FE predictions and the experimental results.

Figure 2.8 Modified unit cell (a) top view, and (b) isometric view (after [47])

Czekanski et al. [48] developed a new 3D unit cell based on the complex geometry of closed-cell metallic foams. The geometry of the new unit cell was obtained from careful study of the foam morphology. The new unit cell model consisted of a central ellipsoid connected through three perpendicular planes. These planes were attached to the skeletal of the cubic structure. The foam model with the new unit cell structure could have geometrical asymmetry and anisotropy up to a certain degree. They used the new unit cell model to study the effect of several geometric parameters including cell size, shape, and aspect ratio on the peak crush load, TEA, and SEA. Their model was validated with the experimental in-plane and transverse crush test results of 10% density aluminium foam. Good agreement was obtained in the overall behaviour of the foam. However, the theoretical predictions overestimated experimental peak collapse loads. This inconsistency was mainly attributed to the idealized geometric features and mechanical properties of the foam under consideration. Figure 2.9 illustrates the graphical representation of the new unit cell.
2.2.4 Functionally Graded Foam Materials

Kieback et al. [49] developed a metallurgical method for producing functionally graded foam materials through graded metal powder compacts followed by melt processing. Brother and Dunand [50] produced density-graded aluminium foam from polyurethane foam precursors using an investment casting method in order to improve the mass efficiency of the graded foam.

Mortensen et al., ([11] and [51]) fabricated porous structures in which outer layers of dense metal encase a central part made of foam with graded porosity. They produced specimens containing up to five layers of porous aluminium foam, with different densities, located between two outer skins of pure aluminium.

Matsumoto et al. [52] introduced an alternative technique for fabricating density-graded aluminium foams. Their method is based on chemical dissolution of uniform density foams. The uniform density foam is immersed within a sodium hydroxide (NaOH) bath of controlled pH and then NaOH is drained by gravity at a constant rate.

Recently, Gupta et al. ([53] and [54]) fabricated functionally graded syntactic foam. Syntactic foams are composites filled with hollow particle known as “microballoons”. Their experimental investigations confirmed that functionally graded syntactic foam can improve the energy absorption.

Cui et al. [12] presented functionally graded polymeric foam in which the size of micro-scale cells is varied continuously in a specific gradient pattern. They also showed that functionally graded polymeric foam materials are suitable candidates for improving SEA over uniform
density foams. Kiernan et al. [55] investigated the stress wave propagation through a virtual functionally graded foam material. They showed that the stress wave profile and amplitude can be determined by the gradient function that defines the variation in density.

Brother and Dunand [56] examined the mechanical properties of uniform and graded aluminium alloy 6061 foams. As it was expected, the uniform density foam shows compressive response with an extended and approximately a constant plateau region followed by densification at high strain (0.75-0.8). By contrast, the density-graded foam exhibits a smoothly rising plateau stress with an early densification (0.35-0.4). Figure 2.10 demonstrates the compressive response of both the uniform density and density-graded foam.

![Figure 2.10 Compressive stress-strain curve of uniform density foam vs. density-graded foam (after [56])](image)

**2.3 Foam-Filled Columns**

**2.3.1 Crush of Uniform Density Foam-Filled Columns**

The comparison between an empty and foam-filled column under quasi-static loading is presented in Figure 2.11. As observed, the number of folds of the filled column is increased compared to empty column due to the interaction between the foam filler and the column wall.
The improvement in axial collapse of columns filled with foam material can be expressed as:

\[ F_{m}^{\text{tot.}} = F_{m}^{c} + F_{m}^{f} + F_{m}^{\text{int.}} \]  

(2.18)

where \( F_{m}^{\text{tot.}} \) is the mean collapse load of foam-filled column, \( F_{m}^{c} \), \( F_{m}^{f} \), and \( F_{m}^{\text{int.}} \) are the mean loads for empty column, foam, and the interaction effect between the foam-filler and the column wall, respectively. Figure 2.12 shows the typical collapse response of foam-filled columns and the interaction effect between the foam-filler and the column wall. Santosa and Wierzbicki [58] studied the effect of filling aluminium square columns with foam and honeycomb materials on axial crushing. They estimated the \( F_{m}^{\text{tot.}} \) by the following expression:

\[ F_{m}^{\text{tot.}} = 14 \sigma_o \frac{l}{3} \frac{s}{h^3} + K \sigma_f c^2 \]  

(2.19)

where \( K = 1 \) for honeycomb and \( K = 2 \) for aluminium foam.
Hanssen et al. [59] studied the collapse behaviour of foam-filled columns. In their experiments, aluminium square columns made of 6060-T4 and 6082-T4, with $80 \text{ mm} \leq c \leq 160 \text{ mm}$, and $41 \leq c/h \leq 80$ were used. The aluminium foam with $5\% \leq \rho \leq 18\%$ was also used for filling the columns. They found that the interaction effect between the foam filler and the column wall could decrease the size of folds. Accordingly, the foam filler can affect the number of developed folds during the crush and the total number of developed folds is an ascending function with respect to foam density.

Hanssen et al. [21] expanded their studies by experimentally investigating the axial collapse of triggered square aluminium foam-filled columns under dynamic and quasi-static loading. All the specimens were indented close to the top on two opposite sides of the column wall in order to ensure localized progressive axial collapse. The specimens consisted of aluminium square columns made of 6060-T4, and 6060-T6 with an $80 \text{ mm} \times 80 \text{ mm}$ cross section and three column wall thicknesses of $1.5 \text{ mm}$, $1.95 \text{ mm}$, and $2.46 \text{ mm}$. Three densities of $0.17 \text{ g/cm}^3$, $0.34 \text{ g/cm}^3$, and $0.51 \text{ g/cm}^3$ were used for the aluminium foam filler. They found good agreement between the quasi-static tests and their previous work. They observed that the transition from the symmetric mode to asymmetric and extensional modes is dependent on the foam density as well as the column wall thickness. Local ruptures occurred at the corners of some of the specimens regardless of their material temper condition. However, these local ruptures had insignificant influence on the TEA of the foam-filled columns. In spite of this, global rupturing that occurred for some of the T6 temper specimens filled with high density foam significantly reduced the TEA of the filled columns. In addition, they developed an empirical expression for predicting the $F_m^{\text{tot}}$. Their expression is the sum of the mean loads of the empty column, the axial contribution of foam filler, and the interaction between the foam filler and the column wall as given by:

$$F_m^{\text{tot}} = 13.06\sigma_o c_m \frac{1}{3} h^3 + c_f^2 \sigma_f + C_{\text{avg}} c_m h \sqrt{\sigma_f \sigma_o}$$

where $c_m = c - h$, $c_f = c - 2h$ and $C_{\text{avg}}$ is a linear increasing function of deformation that is determined experimentally.
2.3.2 Finite Element Modelling of Foam-Filled Columns

Multiple cell models can accurately simulate the collapse behaviour of closed-cell metallic foams. However, these models are not time efficient in modelling of foam-filled components. In order to model the collapse behaviour of foam-filled components, continuum modelling of the metallic foams is the preferred choice. Continuum models can provide a reasonably accurate prediction of the collapse behaviour and can be easily used for design and optimization purposes.

Chen and Wierzbicki [60] examined the axial crushing of hollow and foam filled multi-cell columns and developed a theoretical solution for the mean crushing load of multi-cell sections. Quasi-static finite element analyses were also carried out using PAM-CRASH to simulate the axial crushing of foam-filled double-cell and triple-cell columns. They found that the interaction effect between the foam core and the column wall could increase the total crushing resistance of the double cell and triple-cell significantly.

Meguid et al. [61] conducted extensive experimental and FE simulations using LS-DYNA in order to verify the crush behaviour of PVC foam-filled aluminium circular column. Furthermore, they investigated the effect of different geometrical properties on the collapse mechanism and energy absorption characteristic of foam-filled columns. Figure 2.13 illustrates a schematic representation of the model geometry that was developed for FE simulations.

The circular aluminium column was modelled by a linear elasto-plastic material model with isotropic hardening and von-Mises criteria, and material model 26 in LS-DYNA was used for modelling the crush behaviour of PVC foam. Belytschko-Lin-Tsay shell element with six through thickness integration pointes were used to mesh the tube. Eight-node solid elements with one-point reduced integration were used to mesh the foam material. Hourglass control was employed to eliminate spurious zero-energy modes arising due to reduced integration elements.

In FE simulations, the foam filler’s stiffness was varied by changing the cross section dimensions through introducing concentric through-thickness holes. The tube stiffness was also controlled by changing the wall thickness.
They found that the relative axial stiffness of the component has a major role in collapse behaviour. In addition, they suggested that there is an optimum geometrical configuration in which the maximum value of SEA can be obtained without changing the mode of collapse.

Figure 2.13 Model geometry and contact conditions (after [61])

Recently, Song et al. [62] performed experimental, theoretical, and numerical studies on the axial crushing of aluminium foam-filled hat sections. They used LS-DYNA for numerical modelling and developed a theoretical crush model for the hat sections.

Mirfendereski et al. [63] studied the effect of quasi-static and dynamic loading on the energy absorption characteristics of empty and foam-filled tapered thin-walled rectangular tubes. They also investigated the effect of density of foam, wall thickness, number of tapered sides and tapered angle on response of foam-filled tapered tubes. In addition, they compared the collapse behaviour of tapered rectangular tube to that of straight rectangular tube in order to determine their relative performance as impact energy absorbers.
2.4 Response Surface Approach

Response surface approach was first introduced by Box and Wilson in 1951 [64]. It is a collection of effective statistical and mathematical methods for developing and optimizing processes.

The applications of this approach are in problems with a number of input parameters that have the potential to affect the performance or quality characteristic of the process [65]. The general form of response surface model can be expressed as [66]:

\[ y = f(\xi_1, \xi_2, \xi_3, ..., \xi_k) + e \]  \hspace{1cm} (2.21)

where \( y \) is the response variable of interest, \( \xi_1, \xi_2, \xi_3, ..., \xi_k \) are independent variables, \( f \) is the true response function, and \( e \) includes effects such as random and measurement errors on the response. Although there are quite a number of publications on the crashworthiness of empty and filled columns, only a few attempts have been made to optimize the performance of empty and filled columns.

Yamazaki and Han [67] used crashworthiness maximization techniques for tubular structures. They used response surface approach to construct an approximation RSM of the design sub-problem based on the FE simulation results. In addition, they implemented an optimization technique based on the developed RSM to maximize the absorbed energy for the circular and square columns under impact loading.

Zarei and Kroger ([68]-[71]) conducted a comprehensive study on finding more efficient and lighter crash absorbers as well as achieving maximum energy absorption through using design optimization techniques and RSM. The general procedure for the implementation of the optimization technique and RSM is explained in Figure 2.14.

Recently, Sun et al. [8] developed RSMs based on axial collapse simulation results of FGFM and uniform density foam-filled columns in order to examine the effect of density grading on the energy absorption characteristics of the filled column. They used the constrained single objective and multi-objective optimizations to discover the optimal design of FGFM filled columns.
Figure 2.14 Flowchart of the optimization process (after [68])
Chapter 3: Finite Element Modelling

Summary: This chapter presents the main aspects of the finite element model developed for the simulation of quasi-static axial crushing of discrete functionally graded foam-filled columns. Geometry of FE model and the design parameters of discrete functionally graded foam-filled column are presented. Utilisation of the layered technique for modelling of discrete functionally graded foam is also explained in detail.

3.1 Material Modelling

3.1.1 Column Wall Material

In view of the ductility, low density, and cost effectiveness, aluminium alloys are commonly used for shock absorber components ([22] and [72]). In general, aluminium alloys with T4 condition have higher ductility compared to T6 condition. Accordingly, aluminium alloy 6061 with T4 temper condition was selected for the square column. The mechanical properties of this alloy are summarized in Table 3.1 and Figure 3.1 presents the engineering stress-strain curve of this alloy [73].

The hardening model with von-Mises yield criterion described by material model 24 in LS-DYNA was used to represent the aluminium alloy considered. In order to obtain the true stress-strain curve required by material model 24, the following equations were used for converting the engineering stress-strain data of aluminium alloy 6061-T4 [74]:

\[ \varepsilon_r = \ln(\varepsilon + 1) \]  \hspace{1cm} (3.1)
\[ \sigma_r = \sigma(\varepsilon + 1) \]  \hspace{1cm} (3.2)

where \( \varepsilon, \sigma \) are the engineering strain and stress, \( \varepsilon_r \) is the true strain, and \( \sigma_r \) is the true stress.
Table 3.1 Mechanical properties of AA6061-T4 (after [73])

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$ [GPa]</td>
<td>70</td>
</tr>
<tr>
<td>Yield strength, $\sigma_y$ [MPa]</td>
<td>145</td>
</tr>
<tr>
<td>Ultimate strength, $\sigma_u$ [MPa]</td>
<td>245</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.334</td>
</tr>
<tr>
<td>Density, $\rho$ [kg/m$^3$]</td>
<td>2770</td>
</tr>
</tbody>
</table>

Figure 3.1 Engineering stress-strain curve for aluminium alloy 6061 T4 (after [73])

3.1.2 Foam Filler Material

Constitutive material models that account for pressure dependence and denoted material model numbers 26, 63, 75, and 126 are available in LS-DYNA for representing the structural properties of foams. The ability of these models to predict the response of metallic foams under different loading conditions has been investigated by Hanssen et al. [75]. They found that the performance of the foam material models is very much related to the type of loading. They suggested that material models 63 and 75 are time efficient and accurate in case of loading of the foam alone. In a foam-filled structure, either of the material models 26 and 126 can be used. To model the foam-filled structure, these two models (26 and 126) describe the elasto-plastic behaviour of foam with six independent stress components, which allow the user to simulate a completely nonlinear anisotropic foam material.
In this study, material model 26 was selected to model the foam filler. In this model, the yield surface is given as a function of volumetric strain defined based on the following equation [76]:

$$
\varepsilon_v = \frac{\Delta V}{V_{initial}}
$$

(3.3)

where $\Delta V$ is the volume difference of the foam after compaction and $V_{initial}$ is the initial volume of the foam. A typical yield curve required by material model 26 is illustrated in Figure 3.2. If the foam material is compressed up to the densification region, material model 26 switches to an isotropic elastic-perfectly plastic material with von-Mises yield criterion.

Specifically in the case of aluminium foam, the expansion of foam in the lateral direction under compression is negligible [73]. Therefore, the volumetric strain in material model 26 is considered to be the same as the uniaxial strain. As a result, the experimental uniaxial compression stress-strain curve can be used as the normal stress components of material model 26.

Figure 3.2 Schematic representation of stress-strain behaviour used in LS-DYNA material model 26 (after [76])

Figure 3.3 illustrates a uniaxial compressive stress-strain curve for 10% aluminium foam. After the densification stage, the collapsed foam has the following mechanical properties as solid aluminium: densified elastic modulus $E = 70$ GPa, densified yield strength $\sigma_y = 98.3$ MPa, densified Poisson’s ratio $\nu = 0.334$. 

26
Figure 3.3 Compressive stress-strain curve for 10% aluminium foam (after [73])

The mechanical behaviour of aluminium foam is described by elastic modulus $E_f$, shear modulus $G_f$, yield strength (flow stress) $\sigma_f$, tensile strength $\sigma_t$, shear strength $\tau_f$, and densification strain $\varepsilon_D$. All these parameters can be related to the relative density of aluminium foam through the following equations ([43] and [44]):

\[
E_f = \alpha E_s (\bar{\rho})^2 \quad (3.4)
\]

\[
G_f = 0.5E_f \quad (3.5)
\]

\[
\sigma_f = 1.05\sigma_{os} (\bar{\rho})^{1.52} \quad (3.6)
\]

\[
\sigma_t = 1.1\sigma_f \quad (3.7)
\]

\[
\tau_f = 0.5\sigma_f \quad (3.8)
\]

\[
\varepsilon_D = 1 - 1.4(\bar{\rho}) \quad (3.9)
\]

where $E_s$ is Young’s modulus and $\sigma_{os}$ is the flow stress of the parent material of the aluminium foam. Since Cymat aluminium foam was selected for this study, the coefficients $\alpha$ and $\sigma_{os}$ in equations (3.4) and (3.6) should be adjusted to represent the properties of Cymat foam. Based on characterisation of Cymat foams, $\alpha$ and $\sigma_{os}$ are selected to be 0.69 and 98.3 MPa ([41] and [60]), respectively. In addition to the mechanical properties of foam, which was defined based on the relative density, a stress-strain relationship is also required to build up the yield curve for
various densities of aluminium foam in material model 26. This relationship is defined based on the elastic modulus $E_f$ and the plateau stress $\sigma_f$ of the metallic foam, as summarized in Table 3.2 and Figure 3.4, respectively. Accordingly, the uniaxial stress-strain curve obtained from the relationship deduced from Table 3.2 can be used for all the six yield curves governing the normal stress and shear stress components in material model 26. A volumetric strain at densification is also required by material model 26. The value of volumetric densification was calculated from equation (3.9) [76].

**Table 3.2** Stress-strain corresponding relationship for arbitrary foam density (after [77])

<table>
<thead>
<tr>
<th>Point</th>
<th>Strain</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(2)</td>
<td>$\sigma_f / E_f$</td>
<td>$\sigma_f$</td>
</tr>
<tr>
<td>(3)</td>
<td>0.6</td>
<td>$\sigma_f$</td>
</tr>
<tr>
<td>(4)</td>
<td>0.7</td>
<td>1.35 $\sigma_f$</td>
</tr>
<tr>
<td>(5)</td>
<td>0.75</td>
<td>5.0 $\sigma_f$</td>
</tr>
<tr>
<td>(6)</td>
<td>$\varepsilon_D$</td>
<td>0.05 $E_f$</td>
</tr>
</tbody>
</table>

**Figure 3.4** Construction of stress-strain curve for aluminium foam based on plateau stress $\sigma_f$ (a) engineering stress-strain curve, and (b) power law relation for plateau stress

\[ \sigma_f = 1.05 \sigma_{\text{os}} (\bar{\rho})^{1.52} \]
3.2 Layered Model

In order to model the foam-filled column, a layered approach was first introduced by Heyerman [13]. Based on experimental investigations, he found that aluminium foam is incapable of withstanding high plastic shear flow. Accordingly, he concluded that modelling of the foam as a solid continuum in foam-filled tubes cannot be supported experimentally. He then developed the layered model.

In his layered model, the foam was divided into a series of horizontal layers. Each layer was constructed from a single layer of solid elements. A tie-break failure criterion was then applied on the coincident nodes of the solid elements in adjacent layers of the foam in order to simulate the local slippage of aluminium foam. This tie-break criterion can be described as follows [76]:

\[
\left[ \frac{\text{max}(\sigma_N, 0)}{\sigma_t} \right]^2 + \left[ \frac{\tau_S}{\tau_f} \right]^2 \geq 1
\]

where \(\sigma_N, \tau_S\) are the normal and shear stress values, respectively. If the failure criterion is met, the tied nodes are released and then a surface-to-surface contact algorithm is activated between the solid elements of the adjacent layers. Since the foam with the weaker material properties always fails first, the values of \(\sigma_t\) and \(\tau_f\) are calculated based on the weaker foam using equations (3.7) and (3.8), respectively.

Figure 3.5 illustrates the deformation of a quasi-static axially crushed filled column with 10% aluminium foam for the two cases of non-layered and layered model at 70% of the collapse. In the non-layered model, the aluminium foam deforms in a global pattern and the folds of the column do not fully interact with the foam filler. To develop the first fold of the collapse, the column wall should shift the entire solid continuum of the foam filler rather than the surrounding foam around the fold. Hence, as illustrated in Figure 3.6, higher forces are required for the formation of the first fold compared with the layered model. In addition, the folds in the non-layered model are smaller in size compared with the folds in the layered model. The layered model illustrates better localized deformation and folds could penetrate locally in the foam around the folds.
Figure 3.5 Axial crush of filled column (ρf =10%) at 70% of the crush distance (a) non-layered model, and (b) layered model

Figure 3.6 Load-displacement response of the non-layered model vs. that of the layered model

3.3 Column Geometry

Figure 3.7 illustrates the pertinent geometrical features of the column under consideration. The dimensions of column were selected based on a practical range for manufacturing the aluminium square columns and with the intention of preventing the global buckling of the column under
quasi-static loading. Based on existing work in the literature, ranges of $2.4 \leq L/c \leq 8$ for the length to width ratio and $25 \leq c/h \leq 37$ for width to thickness ratio can provide a stable progressive collapse for axial crushing of square columns [78]. Accordingly, the following dimensions were selected for the aluminium square column: length of column $L=300$ mm, width of column $c=78$ mm, and column wall thickness $h=3$ mm. A gap size of 0.1 mm between the column wall and the foam filler was considered to represent the actual physical configuration of the test specimens in the FE model.

![Figure 3.7 Details of the column under consideration](image)

### 3.4 Finite Element Mesh

The Belytschko-Lin-Tsay shell element with five through thickness integration points was used for modelling of the column wall. Convergence checks showed that an element size of $3 \text{ mm} \times 3 \text{ mm}$ is sufficient to capture the physics of the problem. This element size is in agreement with earlier results ([22] and [79]). In total, 2400 shell elements are required to mesh a quarter of the column with this element size.
The foam filler was modeled with eight node solid elements with one-point reduced integration to prevent volumetric locking of the solid elements. Furthermore, stiffness-based hourglass control is needed to minimize the effect of spurious zero energy deformation modes resulting from one-point reduced integration ([13] and [41]). The size of the solid elements was again determined through convergence tests. The solid element is about $4.16 \text{ mm} \times 4.16 \text{ mm} \times 10 \text{ mm}$. The height of the solid element was determined by the number of horizontal layers used to define tie-break. In total, 2430 solid elements are needed to mesh the DFGF with the selected solid element size.

3.5 Boundary and Contact Conditions

In the current study, the length-to-width ratio ($L/c$) and width-to-thickness ($c/h$) ratio of the column wall are 3.85 and 26, respectively. Based on these ratios, axial collapse is anticipated to occur ([22] and [23]). This symmetric mode of collapse can be simulated by a quarter of the column. In the FE model, two symmetry planes along the vertical x-y and y-z planes were used to represent this symmetry. The nodes that were located in the x-y symmetry plane were constrained in the z-direction and the nodes that were located in y-z symmetry plane were constrained in the x-direction. Figure 3.8 illustrates the boundary and contact conditions applied to the DFGF.

**Figure 3.8** Details of boundary and contact conditions applied to the DFGF
From a design point of view, the shock absorber column is required to be welded to the actual structure. Accordingly, it is necessary to restrain the nodes of the box column on the top and bottom surfaces. The nodes of the column on the top surface were restrained in all directions except the direction of the axial crush. The column nodes on the bottom surface were completely restrained in all degrees of freedom. The nodes of the DFGF on bottom surface of the structure were only confined in the crush direction.

Single-surface contact was applied to the column walls to prevent self-penetration of the developed folds during axial collapse. The interaction between the DFGF and the column wall was modelled with automatic surface-to-surface contact.

Material model 20 in LS-DYNA was used to represent the loading platen, which was meshed with one eight-node solid element. Since one element was employed for modelling the loading platen, a node-to-surface contact was used as the contact between the loading platen and the DFGF. All mentioned contact algorithms use a penalty formulation in order to calculate the contact force between the components.

### 3.6 Triggering Mechanism

The objective of the triggering mechanism is to account for the material and geometrical imperfections of the column, at which axial collapse would initially start. Without the triggering mechanism, numerical errors may initiate an unrealistic folding process [80]. This response is in contradiction to the previous experimental studies [13], where initial imperfections in the column wall material develop symmetric mode of collapse.

Meguid et al. [73] has effectively applied sinusoidal shaped trigger for quasi-static axial crushing of empty and foam-filled columns. This type of triggering has no effect on the total number of folds produced during the axial collapse of square columns.

A sinusoidal shaped trigger with amplitude of 0.05 mm that could initiate the first fold of the collapse was used in the FE model. The amplitude of the trigger was selected to be less than 0.09% of the column width. Values above this range could affect the collapse behaviour of the column [73].
Figure 3.9 illustrates the location of the trigger which is at a distance of $2H$ from the top of the column and has a length of $2H$, where $H$ is the half fold length for a symmetrical fold in a square column. The value of $H$ is obtained from equation (2.2).

![Diagram of a square column with trigger and dimensions labeled](image)

Figure 3.9 Exterior view of the square column with exaggerated trigger

### 3.7 Details of Discrete Functionally Graded Foam

#### 3.7.1 Axial Grading

In order to produce an axially graded DFGF, the mechanical properties of foam should be controlled through the length of the column from one layer to another. Since the mechanical properties of the foam are related to its density, grading of the foam layers can be obtained through varying the density of the foam layers in a certain fashion. A number of design parameters that are required to define the DFGF are as follows:

**Density grading function ($\rho_f(y)$)**

In DFGF, the density gradient defined through a power law function. The power law function has either of the following forms [12]:

---

34
Ascending Pattern : \( \rho_f(y) = \rho_{\text{min}} + (\rho_{\text{max}} - \rho_{\text{min}}) \left( \frac{y}{L} \right)^N \)  
(3.11)

Descending Pattern : \( \rho_f(y) = \rho_{\text{max}} - (\rho_{\text{max}} - \rho_{\text{min}}) \left( \frac{y}{L} \right)^N \)  
(3.12)

where \( L \) is the length of column, \( y \) is the distance from the top of the column, and \( N \) is the grading exponent which varies between 0 and 10. The two patterns of the power law function are shown in Figures 3.10.

In ascending pattern of density distribution, the density of foam layers increases from the minimum value on the top surface of the DFGF to the maximum value at the bottom surface. However, in descending pattern of density distribution, density of foam layers decreases from the maximum value on the top surface toward the minimum value at the bottom surface. It is essential to identify the most efficient grading pattern by examining both forms of the power law function.

**Maximum and minimum foam density** \( (\rho_{\text{max}}, \rho_{\text{min}}) \)

Cymat cooperation produces aluminium foam with maximum density of 20% [41]. In order to obtain the maximum potential of UDF, this value was selected as an upper ceiling of the foam density range for DFGF.

The effect of the density difference, between the maximum and minimum values of the foam density, on the resulting SEA of DFGF was also investigated. Since foam densities below 8% do not contribute significantly to crashworthiness. As a result, \( \rho_{\text{min}} \) was varied from 8% to 15%.

**Number of grading layers** \( (N_G) \)

It is also necessary to find the optimum number of grading layers to ensure that the discrete pattern of grading reflects the physical response of the DFGF.
Figure 3.10 Variation of density vs. normalised distance for axial grading (zero is top surface and one is at the bottom of the column) (a) ascending grading pattern, and (b) descending grading pattern.
3.7.2 Lateral Grading

In order to produce laterally graded DFGF, the mechanical properties of foam should be controlled through the width of the column from one layer to another. Grading of the foam in the lateral direction is more complex than the axial direction due to space limitations and the minimum feasible layer thickness. Figure 3.11 illustrates the cross section design parameters considered of the DFGF in case of lateral grading. These design parameters are as follows:

Density grading function \( \rho_f(y) \)

In order to define the density profile of DFGF in the case of lateral grading, equations (3.11) and (3.12) were used. The variables \( y \) and \( L \) were replaced by \( x \) and \( w \), respectively, where \( x \) is the distance from the inner core of the DFGF and \( w \) is the half width of the column.

Density of the outermost foam layer \( \rho_o \)

The density of the outermost foam layer has a significant effect on the interaction between the DFGF and column wall. The effect of \( \rho_o \) should be examined not only on the resulting SEA but also on the optimum value of the density that provides the highest interaction with the column wall.

Density of the innermost foam layer \( \rho_i \)

The inner foam core of the DFGF contributes to both the mass and the TEA of the DFGF. Hence, it is necessary to examine the effectiveness of the inner core from SEA perspective. The value of \( \rho_i \) was varied from 0 to 10%. The intention is not merely to evaluate the SEA of the DFGF, but also to compare the resulting SEA to that of an equivalent UDF.

Thickness of the outermost foam layer \( t_o \)

The outermost layer of DFGF is in direct contact with the column wall. Effectively, it is the interacting layer of DFGF. It is necessary to consider the thickness of this layer as an independent design variable to examine its effect on the SEA levels of the DFGF.
Number of grading layers ($N_G$)

Similar to the axial grading case, it is important to investigate the effect of number of grading layers on the resulting SEA. However, care has to be taken since a large number of layers can result in unrealistically thin foam layers. In fact, an excessively thin layer may result in foam layers with less than one through-thickness cell [25].

![Diagram of DFGF with design parameters](image)

**Figure 3.11** Design parameters for the DFGF in lateral grading

Core removal ($A_c$)

Based on the findings related to the innermost density effect, it is reasonable to examine the effect of removing the innermost foam core. The area of the removed core gradually increases from 5% to 50%, while the remaining foam area is used to construct the DFGF. Figure 3.12 illustrates the cross section of the DFGF with removed area of $A_c$. 

![Diagram of DFGF with core removal](image)
3.8 Quasi-static Modelling using LS-DYNA

LS-DYNA explicit finite element solver is a multipurpose FE package and it is commonly used for the crashworthiness analysis in automotive industry [76]. LS-DYNA was chosen as an effective FE solver for the simulations of this study. The input data file for LS-DYNA solver was generated using a custom-written MATLAB pre-processor.

Crush behaviour is related to static deformation, where the inertia and strain rate effects are insignificant. Since aluminium alloys are generally light weight and not strain rate sensitive [81], the inertia effect of a DFGF filled column is limited due to the usage of light weight metallic foams. Most research in this area uses the crush behaviour of aluminium based structures as an approximate representation of dynamic loading of the component.

For quasi-static modelling, the solution time can be reduced by either increasing the speed of the striker or by scaling up the mass of the model. Increasing the speed of the striker reduces the total number of time steps required by LS-DYNA to calculate the solution. Artificially increasing the mass scale reduces the CPU cost and amplifies the performance. However, too much increase in the mass of the model or the speed of the loading platen increases the kinetic energy significantly and the results cannot be accepted. It is important to verify that the load-displacement response of the model is not affected by implementing either of the mentioned techniques. The overall effect of these two techniques can be controlled by ensuring that the kinetic energy of the model is negligible (<0.15%) when compared with the total internal energy.
of the model [82]. In all simulation cases, the speed of the striker was set to 200 mm/s. In order to prevent a high value of initial collapse load on the foam-filled column, the speed of the loading platen was gradually increased from 0 to final value of 200 mm/s over 1ms. Three mass scaling ratios of 10, 100, and 1000 were examined and the results are shown in Figures 3.13 and 3.14, respectively.

![Figure 3.13 Load-displacement responses of the FE model for various mass scaling](image)

**Figure 3.13** Load-displacement responses of the FE model for various mass scaling

![Figure 3.14 Internal and kinetic energy of the FE model for various mass scaling](image)

**Figure 3.14** Internal and kinetic energy of the FE model for various mass scaling
Mass scaling ratio of 1000 caused some oscillations in the load-displacement response. However, in all the three cases the kinetic energy of the model was nearly zero. As a result, a mass scaling of 100 was selected for the quasi-static analysis.

### 3.9 Sensitivity Analysis of the Layered Model

In this section, the effects of foam-column friction, inter-layer friction, and the number of layers are investigated. The effect of the above parameters on the quasi-static axial crush behaviour using a filled column was examined. In order to facilitate comparison with experiments, the 10% aluminium foam was selected.

#### 3.9.1 Effect of Friction between Column Wall and Foam Filler

Automatic surface-to-surface contact pairs were defined between the column wall and the foam filler, where the coefficient of friction could be directly assigned for this contact algorithm. To examine the sensitivity of the model, three values of $\mu = 0.0$ (no friction), $\mu = 0.5$, and $\mu = 1.0$ were considered. Comparison of $F_{m}^{\text{tot}}$ for the three cases are presented in Figure 3.15.

![Figure 3.15 Mean collapse loads ($F_{m}^{\text{tot}}$) for varying coefficients of friction between column wall and foam-filler](image)

**Figure 3.15** Mean collapse loads ($F_{m}^{\text{tot}}$) for varying coefficients of friction between column wall and foam-filler
The maximum difference in the $F_{m}^{\text{tot}}$ between the three cases is around 4% and the maximum peak load of the collapse stayed unaffected. Hence, that for quasi-static axial collapse of foam-filled columns the effect of friction between the foam and the column wall is negligible. Similar results were obtained from a previous study by Santosa et al. [44].

3.9.2 Effect of Friction between Layers in Layered Model

It is required to investigate the sensitivity of the layered model to the friction between the adjacent horizontal layers. Three cases of $\mu = 0.0$ (no friction), $\mu = 0.5$, and $\mu = 1.0$ were considered. Comparison of $F_{m}^{\text{tot}}$ for the three cases are presented in Figure 3.16.

The results show that the friction between the layers affects the $F_{m}^{\text{tot}}$. For example, when $\mu = 1.0$, the value of $F_{m}^{\text{tot}}$ increases by 11%. The maximum peak load of the model is also increased by 7% with the $\mu = 1.0$.

![Figure 3.16](image_url)

**Figure 3.16** Mean collapse loads ($F_{m}^{\text{tot}}$) for various coefficients of friction between adjacent horizontal layers
3.9.3 Effect of Number of Layers in Layered Model

In an ideal FE model, the foam should be divided to an unlimited number of layers to allow the shear fracture criteria to take place at any segment of the model [13]. This would lead to very costly computational time. Furthermore, increasing the number of horizontal layers could increase the risk of numerical instability in the model resulting from the use of smaller element sizes.

A convergence test was performed for three different layer arrangements 20, 30, and 60 to find the optimum number of layers. Comparison of $F_{m}^{\text{tot}}$ and the load-displacement response for various configurations of layers are presented in Figures 3.17 and 3.18, respectively.

The $F_{m}^{\text{tot}}$ reduces with increasing the number of horizontal layers. This response can be justified due to the fact that with higher number of layers, the column wall folds should displace thinner layers of foam for penetration.

The model with 60 horizontal layers became unstable at a crush distance of 150 mm. This instability can be related to the fact that at crush distance of 150 mm, the solid elements at the interface between the foam and column wall were extremely distorted. As a result, the model could not handle the numerical errors and the model became unstable.

The maximum difference between the $F_{m}^{\text{tot}}$ of 30 and 60 horizontal layer configurations is about 2%. There is no significant difference between the maximum peak loads of these two configurations. Therefore, it was decided to use 30 horizontal layers for modelling the foam filler.
Figure 3.17 Mean collapse loads ($F_{m}^{\text{tot}}$) for various numbers of horizontal layers

Figure 3.18 Load-displacement responses for various numbers of horizontal layers
Chapter 4: Results and Discussions

Summary: In this chapter, the results obtained from extensive crush simulations of the DFGF are presented. It is divided into two sections. In the first, the results of the FE predictions for empty and uniform density foam-filled columns are validated using existing experimental data. In the second, the effect of various geometrical and material parameters pertaining to the design of the DFGF is evaluated and discussed.

4.1 Validation of FE Simulations

4.1.1 Quasi-static Axial Crushing of Empty Columns

FE model validation was performed using an empty square column made of aluminium alloy 6063-T4. The column was 300 mm long and had cross section dimensions of 76 mm × 76 mm with 3 mm wall thickness.

Quasi-static crush test of the specimen was conducted by Heyerman [13] using electro-hydraulic test facility with a 250 kN load-cell. The specimen was mounted between two 50 mm thick steel plates and was crushed up to 225 mm (75% of column height) at a constant loading rate of 2 mm/min to ensure quasi-static loading [13].

Figure 4.1 illustrates the progressive collapse of the empty column under quasi-static loading. The first collapse fold initiated at the location of the trigger. Symmetrical progressive collapse was observed during the crush due to proper selection of geometrical dimensions for the column. In total, three inward and three outward folds were developed along the length of the column.

Figure 4.2 depicts a comparison of the deformed shape between FE prediction and experimental result at crush distance of 225 mm. The overall shape of FE model in terms of the number of collapse folds and general appearance is in good agreement with the crushed column.
Figure 4.1 Axial crush of empty column (FE prediction)

Figure 4.2 Comparison of the collapse behaviour of empty column at crush distance of 225 mm
(a) FE prediction, and (b) experiment [13]

Figure 4.3 illustrates the load-displacement response of the FE prediction and the experimental results of the empty column. There is good overall agreement between the experimental data and FE results. FE prediction of the initial peak load is 6% higher than the experimental results. This may be due to the fact that the first fold in the FE model was initiated by the triggering mechanism.
In contrast, the first fold in the experimental specimen was initiated as a result of imperfections existing in the column wall. The amplitude of the trigger in the FE model was selected to be less than 0.09% of the column width. This value could be adjusted to match the initial collapse load between the FE prediction and experimental results. However, higher amplitudes could affect the subsequent collapse behaviour and energy absorption characteristics of the column [73].

![Figure 4.3 Comparison of load-displacement response of empty column of experiment [13] vs. FE prediction](image)

Figure 4.3 Comparison of load-displacement response of empty column of experiment [13] vs. FE prediction

Figure 4.4 depicts a comparison between FE predictions and experimental results of the $F_m^c$, where a good agreement is observed between the two approaches. The flow stress of aluminium alloy 6063-T4 based on equation (2.4) is 116.8 MPa, where $\sigma_u = 155$ MPa and $n = 0.15$. The theoretical value of $F_m^c$ of the empty column based on equation (2.1) at final crush distance of 225 mm is 40 kN. Maximum difference between the FE prediction and theory at the plateau region is 5.7% and between the FE prediction and the experiment is 7.6%.

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4.1.2 Axial Crushing of Uniform Density Foam-Filled Columns

In order to validate the FE model of foam-filled column, the thin-walled column used in the previous section was filled with 10% foam and axially crushed under the same conditions described earlier for empty column.

Figure 4.5 presents the crush behaviour of 10% foam-filled column at different stages of crushing. The presence of the 10% aluminium foam inside the column reduced the length of the folds. Consequently, more folds developed during the collapse along the length of the column. The number of folds increased from six in the case of empty column to eight for foam-filled column.

Figure 4.6 illustrates the discrepancy between the FE prediction and the experimental results [13] at the final crush distance of 225 mm. The overall shape of FE model, in terms of the number of collapse folds and general appearance is in good agreement with the experimental results.
Figure 4.5 Axial crush of foam-filled column ($\rho_f = 10\%$)

Figure 4.6 Comparison of the collapse behaviour of foam-filled column ($\rho_f = 10\%$) at crush distance of 225 mm (a) FE prediction, and (b) experiment [13]

Figure 4.7 illustrates the load-displacement response of the FE prediction and the experimental results of the 10% foam-filled column. There is good overall agreement between the experimental data and FE results. FE prediction of the initial peak load is 4% higher than the experimental results.
A discrepancy in the subsequent peak-to-peak load values in FE results compared with the experiment is observed. This discrepancy can be justified based on the fact that the FE model is symmetric and homogeneous and there are no imperfections or defects in the column wall except for the trigger.

Figure 4.7 Load-displacement response of foam-filled column ($\rho_f = 10\%$) for experiment [13] vs. FE prediction.

Figure 4.8 depicts a comparison between FE predictions and experimental results of $F_m^{\text{tot}}$, where a good agreement is observed between the two approaches.

The theoretical value for the $F_m^{\text{tot}}$ based on equation (2.20) is 80 kN. Maximum difference between the FE prediction and theory at the plateau region is 5%, and between the FE prediction and the experiment is 12.5%. Hence, the FE model can simulate the quasi-static axial collapse of foam-filled columns with a reasonable accuracy.
4.2 Axial Grading of the DFGF Filled Column

In this section, the effect of various geometrical and material parameters on the resulting energy absorption characteristics of the axially graded DFGF filled column is explored.

4.2.1 Effect of Density Grading

The density variation along the length of the column for both equations ((3.11) and (3.12)) was illustrated in Figure 3.10. In both equations, the value of grading exponent ($N$) was varied between 0 and 10, while all the other design variables were fixed (e.g. $N_G = 30$, $\rho_{\text{max}} = 20\%$, $\rho_{\text{min}} = 8\%$, and $\Delta \rho = 12\%$).

Since the total mass of the DFGF filled column was not fixed, the UDF of the same weight as the DFGF was selected for comparing the energy absorption characteristics between the two cases. The equivalent density of the UDF in the case of axial grading can be calculated from:

\[
\rho_{\text{avg}} = \frac{\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_{N_G}}{N_G}
\]  

The comparison of the TEA and SEA of the DFGF filled, and those of the corresponding UDF
filled columns are illustrated in Figures 4.9 and 4.10, respectively. The size of the fold length of the DFGF filled column and that of its equivalent UDF filled column for two patterns of grading function are presented in Figures 4.11.

Based on Figure 4.9, the TEA of the DFGF filled column with ascending grading pattern is higher than that of the corresponding UDF filled column for \( N \leq 2 \). Increasing the value of \( N \) above 2 did not affect the TEA of the DFGF filled column. The TEA of the DFGF filled column with descending grading pattern is higher than that of the corresponding UDF filled column for \( 0.5 \leq N \leq 5 \). Setting \( N \) equal to values less than 0.5 resulted in reduction of the TEA, and negligible improvement was observed for \( N > 5 \). The maximum SEA improvement of the DFGF filled column with ascending grading pattern compared with that of the UDF filled column is 6% at \( N = 0.2 \). This value is 2% higher than the SEA of the DFGF filled column with descending grading pattern (Figure 4.10). Hence, the ascending pattern of the grading function has some advantages over the descending pattern for improving the crashworthiness of the DFGF.

The size of the fold length of the DFGF filled column with ascending grading pattern is smaller than that of the corresponding UDF filled column for \( N \leq 2 \). The maximum reduction in the fold length of the DFGF filled column with respect to the UDF filled column is 4% at \( N = 0.2 \). Setting \( N \) equal to values greater than 2 resulted in larger fold length. The size of the fold length of the DFGF filled column with descending grading pattern is smaller than that of the equivalent UDF filled column for \( N \geq 0.5 \). The maximum reduction in the fold length of the DFGF filled column with respect to the UDF filled column is 3% at \( N = 2 \). Setting \( N \) equal to values less than 0.5 resulted in larger fold length (Figure 4.11). Based on Figures 4.9-4.11, it can be concluded that \( N \) has a significant impact on the TEA, SEA, and the fold length of the DFGF column. Although the improvement in the SEA of the DFGF over that of the UDF is somewhat marginal, the crashworthiness of the DFGF can be improved over the UDF with proper selection of grading exponent(\( N \)) through an optimization process.

Similar results were obtained from a previous study by Sun et al. [8]. They obtained 7.5% improvement in the SEA of axially graded FGFM filled column with ascending grading pattern at \( N = 0.2 \) under dynamic loading.
Figure 4.9 TEA of the axially graded DFGF filled column vs. the equivalent UDF filled column (a) ascending grading pattern, and (b) descending grading pattern
Figure 4.10 SEA of the axially graded DFGF filled column vs. the equivalent UDF filled column (a) ascending grading pattern, and (b) descending grading pattern
Figure 4.11 Fold length of the axially graded DFGF filled column vs. the equivalent UDF filled column (a) ascending grading pattern, and (b) descending grading pattern
4.2.2 Effect of Number of Grading Layers

In order to investigate the effect of the number of grading layers ($N_G$) on energy absorption characteristics of the axially graded DFGF filled column, the values of $N_G$ were varied (10, 15, and 30). The changes in total mass of the DFGF with variation of $N_G$ were due to numerical discretization errors, which are negligible. Since it was determined that the ascending grading pattern could provide more improvements for the SEA of the DFGF filled column, this pattern was used. Value of $N$ was varied between 0 and 10, while all the other design variables were fixed (e.g. $\rho_{\text{max}} = 20\%$, $\rho_{\text{min}} = 8\%$, and $\Delta\rho = 12\%$).

Figure B.1 illustrates the response surface obtained based on FE simulation results for the SEA of the DFGF filled column for various values of $N_G$. Based on the obtained response surface, a comparison of the SEA of the DFGF filled column with three different values of $N_G$ is presented in Figure 4.12.

![Figure 4.12](image-url)  
*Figure 4.12* Comparison of the SEA of the axially graded DFGF filled column for various values of $N_G$.

The maximum deviation of the SEA for 10 and 15 layers compared with 30 layers is 2\% at $N = 0.2$. Since the variation of $N_G$ has a negligible effect on the total mass of the DFGF filled
column, the TEA follows the same trend as the SEA, and the maximum deviation of the TEA for 10 and 15 layers compared with 30 layers is also 2% at $N = 0.2$. In addition, the size of the fold length and the total number of folds of the DFGF filled column for various values of $N$ remained unchanged. Hence, $N_g$ does not have a marked effect on the energy absorption level of the DFGF.

However, increasing the $N_g$ beyond a certain limit can result in unrealistic thin foam layers that are not practical from manufacturing and strength point of view. In view of that and in order to simplify the FE model, $N_g = 30$ was selected to match the total number of layers used in the layered model.

### 4.2.3 Effect of Density Range

Earlier experimental investigations indicate that foam densities below 8% do not provide significant improvement of the crashworthiness performance of foam-filled columns [13]. On the other hand, Cymat Aluminium Corp. produces metallic foams up to a maximum density of 20% [41]. Accordingly, 8% and 20% were selected as the representative of the lower and upper values of the density range for this study.

The values of $\rho_{\text{min}}$ were varied (8%, 12%, and 15%) in order to investigate the effect of different density range ($\Delta \rho$) upon the SEA. For each case, equation (3.11) was used to evaluate the density profile. Value of $N$ was varied between 0 and 10, while all the other design variables were fixed (e.g. $N_g = 30$ and $\rho_{\text{max}} = 20\%$).

Figure 4.13 illustrates the response surface for $\Delta \text{SEA}_{\text{DFGF}}$ for various $\Delta \rho$ values. The maximum improvement of the SEA of the DFGF filled column among the three cases is 6% at $\Delta \rho = 12\%$ and $N = 0.2$. Hence, the DFGF with larger density difference is more effective at maximizing the SEA of the DFGF filled column. Accordingly, $\Delta \rho = 12\%$ was selected for investigating the effect of other design variables of axial grading of the DFGF.
4.3 Lateral Grading of the DFGF Filled Column

In this section, the effect of various geometrical and material parameters on the resulting energy absorption characteristics of the laterally graded DFGF filled column is investigated.

4.3.1 Effect of Density Grading

In order to investigate the effect of the density grading in lateral direction, equations ((3.11) and (3.12)) were modified by replacing $y$ and $L$ with $x$ and $w$ (Figure 3.11), respectively. The value of grading exponent ($N$) was varied between 0 and 10, while all the other design variables were fixed (e.g. $N_G = 4$, $\rho_{\text{max}} = 20\%$, $\rho_{\text{min}} = 10\%$, $\Delta \rho = 10\%$, $t_o = 9.5$ mm, and $A_c = 0\%$).

The comparison of the TEA, interaction energy ($E_{\text{int.}}$), and SEA of the DFGF filled column and those of the corresponding UDF filled columns are illustrated in Figures 4.14, 4.15 and 4.16, respectively.
The equivalent density of the UDF for lateral grading of the DFGF can be calculated from:

\[ \rho_{\text{avg.}} = \frac{\rho_1 A_1 + \rho_2 A_2 + \ldots + \rho_{N_0} A_{N_0}}{A_{\text{total}}} \] (4.2)

where \( A_1, A_2, \ldots, A_{N_0} \) are the cross sectional area of different grading layers and \( A_{\text{total}} \) is the total cross sectional area of the DFGF. The following equation was used for calculating the \( E_{\text{int.}} \):

\[ E_{\text{int.}} = \text{TEA} - E_c - E_{DFGF} \] (4.3)

where \( E_c \), \( E_{DFGF} \) are the absorbed energy by the column and the DFGF, respectively. It is observed in Figure 4.14 that the TEA of the DFGF filled column with ascending grading pattern is higher than that of the corresponding UDF filled column for \( N \geq 1 \). Setting \( N \) equal to values less than 1 resulted in negligible improvement of the TEA of the DFGF filled column.

The TEA of the DFGF filled column with a descending grading pattern is lower than that of the corresponding UDF filled column for \( 0 \leq N \leq 10 \). This can be justified by the fact that the interaction of the foam filler and the column wall material is a function of the relative stiffness of the foam filler and the column wall material [21]. Since the descending grading pattern offers the minimum density foam for the interactive layer (outermost layer of the DFGF), the interaction between the column wall and the DFGF is reduced. On the other hand, the ascending grading pattern offers the maximum density foam for the interactive layer. Consequently, the interaction between the column wall and the DFGF is enhanced (Figure 4.15).

The maximum SEA improvement of the DFGF filled column with ascending pattern of the grading function compared with that of the corresponding UDF filled column is 3% at \( N = 10 \). The maximum reduction of SEA of the DFGF filled column with a descending grading pattern compared to the UDF filled column is about 6% at \( N = 10 \) (Figure 4.16). Accordingly, it can be concluded that the ascending grading pattern has some advantages over the descending grading pattern for improving the crashworthiness of the DFGF.
Figure 4.14 Comparison of the TEA of the DFGF filled column for two forms of grading

Figure 4.15 Comparison of $E_{int}$ of the DFGF filled column for two forms of grading
Based on the presented results in Figures 4.14-4.16, it can be concluded that the value of $N$ has a significant influence on the TEA, $E_{int}$, and SEA of the DFGF filled column. However, improvement in energy absorption characteristics of the DFGF was only observed for values of $N \geq 1$. Hence, for determining the effect of other design variables, the value of $N$ was limited to vary between 1 and 10.

### 4.3.2 Effect of Number of Grading Layers

In order to investigate the effect of number of grading layers ($N_G$) on energy absorption characteristics of the laterally graded DFGF filled column, the value of $N_G$ was varied from 3 to 5. In each case, the spacing layers were selected to have an equal thickness. The thickness of each layer was set to 12.5 mm, 9.5 mm, and 7.5 mm for the cases of 3, 4 and 5 grading layers, respectively.

Figure 4.17 depicts the changes in total mass of the DFGF based on different values of $N_G$. The observed change in the total mass of the DFGF filled column due to variation of $N_G$ is as a result of numerical discretization errors.
Since it was determined that the ascending grading pattern provides more improvements for the SEA of the DFGF filled column, equation (3.11) was used to evaluate the density profile. The value of $N$ was varied between 1 and 10, while all the other design variables were fixed (e.g. $\rho_{\text{max}} = 20\%$, $\rho_{\text{min}} = 10\%$, $\Delta \rho = 10\%$, and $A_c = 0\%$).

Figure B.2 illustrates the response surface for the TEA of the DFGF filled column with three different values of $N_G$. The comparison of the TEA and SEA of the DFGF filled column and those of the corresponding UDF filled columns are illustrated in Figures 4.18 and 4.19, respectively.

The maximum deviation of the TEA for 4 and 5 grading layers compared with that of the 3 grading layers at $N = 10$ is 5%, while the maximum deviation of the SEA for 4 and 5 grading layers compared with that of the 3 grading layers at $N = 10$ is 4%. For this reason, it can be concluded that number of grading layers above 3 does not have a marked effect on the energy absorption characteristics of the DFGF. However, increasing the number of grading layers beyond a certain limit can result in unrealistic thin foam layers. In order to simplify the FE model in the case of lateral grading, $N_G = 4$ was selected for all the simulation cases.
Figure 4.18 Comparison of the TEA of the DFGF filled column for various values of number of grading layers ($N_G$)

Figure 4.19 Comparison of the SEA of the DFGF filled column for various values of number of grading layers ($N_G$)
4.3.3 Effect of Density Range

The values of \( \rho_{\text{min}} \) were varied (8%, 12%, and 15%) in order to investigate the effect of different density range (\( \Delta \rho \)) upon the SEA. For each case, equation (3.11) was used to evaluate the density profile. Value of \( N \) was varied between 1 and 10, while all the other design variables were fixed (e.g. \( N_g = 4 \), \( \rho_{\text{max}} = 20\% \), \( t_o = 9.5 \text{ mm} \), and \( A_e = 0\% \)).

Figure 4.20 illustrates the response surface for \( \Delta \text{SEA}_{\text{DFGF}} \) with different \( \Delta \rho \) values. The maximum improvement in the SEA of the laterally graded DFGF filled column among the three cases is 3.5% at \( \Delta \rho = 12\% \) and \( N = 10 \). Accordingly, the DFGF with larger density difference is more effective at enhancing the SEA of the DFGF filled column.

Based on the fact that the number of grading layers in the lateral grading of the DFGF is limited, the effect of the innermost and outermost layer on the interaction between the DFGF and column wall should be investigated in order to find the most favourable value of the density and the thickness for these layers.

![Figure 4.20 Response surface of \( \Delta \text{SEA}_{\text{DFGF}} \) for various values of \( \Delta \rho \)](image-url)
4.3.4 Effect of the Outermost Layer Density

Since the outermost layer of the DFGF is in direct contact with the column wall, the effect of the density of this layer ($\rho_o$) on the SEA of the DFGF should be investigated. FE simulations based on axial crushing of the UDF filled column with densities within the range of 10% to 20% were carried out. The stiffness of the foam filler was reduced by removing the inner core of the foam filler gradually from 0% to 60% of the total cross-sectional area purposely, in order to find which density of foam could maintain the most interaction with the column wall.

Similar tests were conducted by Meguid et al. [61], and they suggested that an optimum geometrical configuration with the maximum value of the SEA could be found for hollowed core foam filler without affecting the mode of collapse of the filled column.

The TEA and SEA of the UDF for 10%, 15%, and 20% with different core removal lengths are shown in Figures 4.21 and 4.22, respectively. The TEA for all cases reduces as the removed core area increases.

![Figure 4.21](image)

**Figure 4.21** TEA of the UDF filled column with various length of removed core

However, the SEA for the three cases of the UDF increases up to a maximum point, and then decreases as the removed core area increases. The highest value of the SEA among the three
cases is 9%. This value was obtained from 20% density foam with 30% removed area from the inner core.

Accordingly, 20% foam density is the best candidate for the interactive layer based on the selected geometry and material properties of the column wall since it has the highest SEA among the densities from 10% to 20%. As a result, for all the simulation cases the value of $\rho_{\text{max}}$ was fixed to 20%.

![Figure 4.22 SEA of the UDF filled column with various length of removed core](image)

**4.3.5 Effect of the Innermost Layer**

Based on earlier work [62], the crushed foam filler in the column can be divided into two distinguishable regions, as illustrated in Figure 4.23. These regions are (a) densified region at the core of the foam filler in which the foam was crushed in the axial direction, and (b) interaction region adjacent to the column wall in which the foam was crushed in both the axial and lateral directions. In view of this, it was decided to examine the effect of innermost layer of the DFGF on the interaction between the foam filler and the column wall.
The density of innermost layer ($\rho_i$) was gradually varied from 0% to 10%. Two values of $\rho_i = 0\%$ (hollow core) and $\rho_i = 10\%$ were selected in order to present two extreme cases of the results. For each case, equation (3.11) was used to evaluate the density profile. Value of $N$ was varied between 1 and 10, while all the other design variables were fixed (e.g. $N_c = 4$, $\rho_{\text{max}} = 20\%$, $\rho_{\text{min}} = 0\%$ and 10%, and $t_o = 9.5\text{ mm}$). In each case, the spacing layers were selected to have equal thickness. The TEA, $E_{\text{int}}$, SEA, and the fold length of the DFGF filled column and that of the corresponding UDF filled column are presented in Figures 4.24-4.27, respectively.

As a result of removing 7% of the foam material from the core of the DFGF ($\rho_i = 0\%$), the TEA of the DFGF filled column was reduced compared to that of the case where $\rho_i = 10\%$. It is also observed from Figure 4.24 that the TEA for both cases is higher than that of the corresponding UDF filled column. This is practically visible for the case of a hollow core ($\rho_i = 0\%$), which can be justified based on the comparison of $E_{\text{int}}$ presented in Figure 4.25. Since the highest density foam was located at the outermost layer through density grading function, the DFGF had a higher interaction with the column even when the core was hollowed. On the other hand, the
corresponding UDF foam lost its effectiveness in interacting with the column wall, when the total mass of the component was reduced. Maximum improvement of the SEA of the DFGF filled column for $\rho_i = 0\%$ was 9% at $N = 10$. For the case where $\rho_i = 10\%$, the SEA of the DFGF filled column was marginally improved by 3% at $N = 10$ compared to that of the UDF filled column, as illustrated in Figure 4.26. This can be justified since the inner core of the DFGF for the case where $\rho_i = 10\%$ increases the total mass without affecting the $E_{int}$.

The size of the fold length in both cases is smaller than that of the corresponding UDF (Figure 4.27). The maximum reduction in the fold length is around 11% at $N = 10$ for $\rho_i = 10\%$. As a result, the DFGF is superior, from the SEA point of view over the UDF for lightweight applications.

![Figure 4.24](image-url)

**Figure 4.24** Comparison of the TEA of the DFGF filled column for $\rho_i = 0\%$ and $\rho_i = 10\%$
**Figure 4.25** Comparison of $E_{int}$ of the DFGF filled column for $\rho_i = 0\%$ and $\rho_i = 10\%$

**Figure 4.26** Comparison of the SEA of the DFGF filled column for $\rho_i = 0\%$ and $\rho_i = 10\%$
4.3.6 Effect of Thickness of the Outermost Layer

In order to investigate the effect of thickness of the outermost layer \( t_o \) on the energy absorption characteristics of the DFGF, the value of \( t_o \) was varied gradually from 5 mm to 20 mm for the two cases where \( \rho_i = 0\% \) and \( \rho_i = 10\% \). The remaining area in the DFGF was equally divided between the other grading layers. As a result, the thicknesses of the remaining grading layers can be calculated as follows:

\[
t = \frac{w - t_o}{N_G - I}
\]

(4.4)

Three values of \( t_o \) were selected: 8 mm, 12 mm, and 17.5 mm. In order to evaluate the density profile, equation (3.11) was used. Value of \( N \) was varied between 1 and 10, while all the other design variables were fixed (e.g. \( N_G = 4, \rho_{\text{max}} = 20\%, \rho_{\text{min}} = 0\% \) and 10\%, and \( A_c = 0\% \)).

Figure B.3 illustrates the response surface for the TEA of the DFGF filled column for various values of \( t_o \) and \( \rho_i \). The comparison of the TEA, \( E_{\text{int}} \), SEA and the fold length of the DFGF
filled column with those of the corresponding UDF filled column are presented in Figures 4.28-4.31, respectively.

By increasing the thickness of the interactive layer \((t_o)\) from 8 mm to 17.5 mm, the TEA of the DFGF filled column increased by 26% at \(N = 10\) for \(\rho_i = 0\%\). However, for the case where \(\rho_i = 10\%\) at \(N = 10\) only a 12% increase was observed.

For the two cases where \(\rho_i = 0\%\) and \(\rho_i = 10\%,\) the TEA and the \(E_{int}\) of the DFGF filled column were higher than those of the corresponding UDF filled column. However, when the total mass of the DFGF filled column \((m_{tot})\) was above 1.4 kg, regardless of the value of \(t_o\), the improvements in the TEA and \(E_{int}\) were marginal.

The maximum improvement of SEA for the case where \(\rho_i = 10\%\) is approximately 2.5% at \(t_o = 8\) mm and \(N = 10\) is insignificant compared to the 13% improvement of SEA for \(\rho_i = 0\%\) at \(t_o = 8\) mm and \(N = 10\). The size of the fold length for the two cases where \(\rho_i = 0\%\) and \(\rho_i = 10\%,\) with various values of \(t_o\) was smaller than that of the corresponding UDF.

The maximum reduction in the fold length is about 8% for the case where \(\rho_i = 0\%\) and \(t_o = 8\) mm at \(N = 10\). The presented results showed that if \(t_o \geq 8\) mm, this could only add to the total mass of the DFGF without improving the \(E_{int}\) of the DFGF filled column. Consequently, the improvement in the SEA diminishes for both \(\rho_i = 0\%\) and \(\rho_i = 10\%\) for values above \(t_o = 8\) mm. Furthermore, the effect of \(t_o\) is more pronounced for the case where \(\rho_i = 0\%\) because of the small mass.
Figure 4.28 Comparison of the TEA of the DFGF filled column for various values of $t_o$: (a) $\rho_i = 0\%$, (b) $\rho_i = 10\%$
Figure 4.29 Comparison of $E_{\text{int}}$ of the DFGF filled column for various values of $t_o$ (a) $\rho_i = 0\%$, and (b) $\rho_i = 10\%$. 

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Figure 4.30 Comparison of the SEA of the DFGF filled column for various values of $t_0$ (a) $\rho_i = 0\%$, and (b) $\rho_i = 10\%$
Figure 4.31 Comparison of the fold length of the DFGF filled column for various values of $t_0$

(a) $\rho_i = 0\%$, and (b) $\rho_i = 10\%$
4.3.7 Effect of Core Removal

Based on the findings related to the effect of innermost density on the SEA of the DFGF, it is important to find the proper value of core removal ($A_c$). Accordingly, the value of $A_c$ was varied gradually from 5% to 50% of the total cross-sectional area of the DFGF. The remaining area was used to construct the DFGF. Since it was found that $t_o$ could affect the energy absorption characteristics of the DFGF filled column, the value of $t_o$ was also varied gradually from 4 mm to 11 mm to examine this effect on the DFGF with removed core area of $A_c$. Based on the values $A_c$ and $t_o$, the thicknesses of the remaining grading layers can be calculated using the following equation:

$$t = \frac{\sqrt{(1 - A_c/100)w^2}}{N_G - 1} - t_o$$  \hspace{1cm} (4.5)

Insignificant change was observed in the TEA, the SEA or the mode of collapse of the DFGF filled column for $5\% \leq A_c < 20\%$ and $30\% < A_c < 50\%$, compared with that of the UDF filled column. Accordingly, the simulation results of $A_c = 20\%$ and $A_c = 30\%$ with three values of 4 mm, 8 mm and 11 mm were selected. In order to evaluate the density profile, equation (3.11) was used. Value of $N$ was varied between 1 and 10, while all the other design variables were fixed (e.g. $N_G = 4$, $\rho_{max} = 20\%$, $\rho_{min} = 10\%$ and $\Delta\rho = 10\%$). Figure B.4 illustrates the response surface for the TEA of the DFGF filled column for various values of $A_c$ and $t_o$. The comparison of the TEA, $E_{int}$, SEA and the fold length of the DFGF filled column and those of the corresponding UDF filled column are presented in Figures 4.32-4.35, respectively. The TEA of the DFGF filled column increases by increasing the value of $t_o$ from 4 mm to 11 mm for both $A_c = 20\%$ and $A_c = 30\%$. Furthermore, the TEA and the $E_{int}$ of the DFGF filled column were higher than those of the corresponding UDF filled column for both $A_c = 20\%$ and $A_c = 30\%$. This can be justified, since the total mass of the DFGF filled column ($m_{tot}$) is below 1.4 kg and the corresponding UDF foam is not as effective as the DFGF in interacting with the column wall. The maximum improvement of the SEA for the case where $A_c = 30\%$ is 12%, which is 5% more than the case where $A_c = 20\%$ at $t_o = 4$ mm and $N = 10$. The size of the fold length in both cases
where \( A_c = 20\% \) and \( A_c = 30\% \), with various values of \( t_o \), is smaller than that of the corresponding UDF. The maximum reduction in the fold length is about 5\% at \( t_o = 4 \) mm and \( N = 10 \) for the case where \( A_c = 30\% \).

**Figure 4.32** Comparison of the TEA of the DFGF filled column for various values of \( t_o \) (a) \( A_c = 20\% \), and (b) \( A_c = 30\% \)
Figure 4.33 Comparison of $E_{\text{int}}$ of the DFGF filled column for various values of $t_o$ (a) $A_c = 20\%$, and (b) $A_c = 30\%$.
Figure 4.34 Comparison of the SEA of the DFGF filled column for various values of $t_o$ (a) $A_c = 20\%$, and (b) $A_c = 30\%$
Figure 4.35 Comparison of the fold length of the DFGF filled column for various values of $t_o$ (a) $A_c = 20\%$, and (b) $A_c = 30\%$
Based on presented results in Figures 4.32-4.35, it can be concluded that $A_c$ and $t_o$ are two important design parameters that can be used for designing the optimized DFGF for lightweight applications.
Chapter 5: Conclusions and Future Work

Summary: In this chapter, the problem is stated, the objectives are identified, and the contributions resulting from this work are outlined. A brief description of areas that would benefit from future work is also provided.

5.1 Problem Statement

In view of the fact that new innovative concepts are being designed for light weight vehicles, there are always new challenges to further improve their crashworthiness. The ultimate objective in a collision is to maximize the safety of the passengers and minimize the damage to the vehicle.

It has been shown that functionally graded foam material, with graded density, is a suitable candidate for improving the specific energy absorption over uniform density foam. However, producing functionally graded foam is more challenging than producing uniform density foam and requires advanced technologies to produce a given gradient. In this study we used, discrete functionally graded foam. Discrete functionally graded foam can be constructed from foam layers with varied densities that are manufactured using existing technologies. Both axial and lateral grading can be developed using discrete approach.

Density grading, number of grading layers, and thickness of the interactive layer are likely to affect the energy absorption characteristics of the discrete functionally graded foam. The effect of these parameters should be explored to have a better understanding of the tailored properties of the discrete functionally graded foam.

5.2 Objectives

The objectives of this study were to:

(i) evaluate the effect of discrete functionally graded foam as opposed to uniform density foam upon the crush behaviour of foam-filled tubes, and
(ii) evaluate the effect of density grading, number of grading layers, thickness of the interactive layer, and interface considerations upon the resulting specific energy absorption.

### 5.3 Conclusions

Given below is a summary of the findings of the work:

(i) The developed FE model was able to simulate the deformation and collapse behaviour of a discrete functionally graded foam-filled column accurately under quasi-static loading.

(ii) Response surfaces were developed for quasi-static axial crushing of discrete functionally graded foam-filled columns. The collapse behaviour and energy absorption characteristics of discrete functionally graded foam-filled column with various grading exponents were compared with those of uniform density foam-filled columns of the same weight. It was shown that density grading function, which controls the variation of foam density, has a significant effect on the performance of discrete functionally graded foam regardless of the direction of grading.

(iii) The ascending grading pattern was shown to have some advantages over the descending grading pattern for improving the crashworthiness of the discrete functionally graded foam.

(iv) The number of grading layers does not have a significant effect on the energy absorption characteristics of discrete functionally graded foam under quasi-static loading.

(v) Increasing the density range improves both the energy absorption and specific energy absorption of the discrete functionally graded foam.

(vi) In lateral grading of the discrete functionally graded foam, the thickness and density of the interactive layer play an important role. Both of these parameters were considered as independent variables and optimum values for these two parameters
were approximately estimated to improve the interaction between the discrete functionally graded foam and the column wall.

(vii) The effect of the inner core on the energy absorption characteristics of foam-filled columns was investigated. The FE results confirmed that the inner core of the filled column does not have a marked effect on the interaction between the foam filler and the column wall.

(viii) The effect of core removal on energy absorption characteristics of the discrete functionally graded foam was examined. Removing the core of the discrete functionally graded foam reduced the total energy absorption of the filled column. However, the specific energy absorption of the discrete functionally graded foam was improved over the uniform density foam of the same weight.

5.4 Thesis Contribution

The main contributions of the current work can be summarised as:

(i) developed and validated a nonlinear quasi-static FE model of discrete functionally graded foam-filled columns using the commercial LS-DYNA software package,

(ii) developed response surfaces for axial and lateral grading of discrete functionally graded foam-filled columns,

(iii) investigated the effect of various design parameters on the quasi-static crush behaviour of discrete functionally graded foam-filled columns, and

(iv) determined the optimum grading pattern for discrete functionally graded foam in the axial and lateral directions.
5.5 Future Work

Further research can be extended to:

(i) study the dynamic collapse of discrete functionally graded foam-filled columns in order to investigate the effect of inertia on mode of collapse and maximum crippling force at the time of impact,

(ii) examine the effect of continuous grading on mode of collapse and energy absorption,

(iii) utilise optimization techniques for designing functionally graded foam-filled components for automotive applications, and

(iv) investigate the effect of oblique loading on the performance of functionally graded foam-filled columns.
References


[16] D. Pjevac, Crush behaviour of foam-filled columns subjected to oblique loading, in: Mechanical & Industrial Engineering, Toronto: University of Toronto, 2004


[67] K. Yamazaki, J. Han, Maximization of the crushing energy absorption of tubes, Structural Optimization, 16 (1998) 37-49.

[68] H. R. Zaeri, M. Kroger, Crashworthiness optimization of empty and filled aluminium crash boxes.


Appendix A: Characteristics of thin-walled columns

The design of thin-walled columns for crashworthiness applications is a challenge due to the highly nonlinear large deformation and the need to predict the mode of collapse under different impact conditions [13]. A good design should be able to dissipate the impact energy of collision through plastic collapse in a controlled manner within a certain distance and minimizes the maximum peak force and acceleration of the impact [83]. The collapse behaviour of thin-walled columns can be evaluated using the following terms [84]:

( i) Instantaneous load-displacement response

( ii) Total energy absorption

( iii) Specific energy absorption

( iv) Mean collapse load

( v) Stroke efficiency

( vi) Energy efficiency

The instantaneous load-displacement response is obtained by measuring the load and displacement responses of the component subjected to axial collapse. In the load-displacement curve, the first peak load is defined as initial collapse load. This value is most probably the maximum load that the component experiences during the collapse.

The total energy absorption (TEA) is calculated based on the area under the load displacement curve, and is given by:

\[
\text{TEA}(d) = \int_0^d F(x)dx \quad \text{(A.1)}
\]
Typical responses of load and internal energy versus displacement of thin-walled columns are illustrated in Figures A.1(a) and A.1(b), respectively.

![Figure A.1](image)

**Figure A.1** Typical response of thin-walled columns (a) instantaneous load vs. displacement, and (b) internal energy vs. displacement

The specific energy absorption (SEA) is defined as the energy absorbed per unit mass of the component and is given by:

$$\text{SEA}(d) = \frac{1}{m} \int_0^d F(x) dx$$  \hspace{1cm} (A.2)

where $m$ is the mass of the component. In order to compare the energy absorption characteristics of different components, it is appropriate to normalise the calculated energy by the mass and use SEA.

The mean collapse load ($F_m$) can be determined by dividing the energy absorption by the displacement $d$, and is given by:

$$F_m(d) = \frac{1}{d} \int_0^d F(x) dx$$  \hspace{1cm} (A.3)

A typical mean collapse load versus displacement response of an empty aluminium square column is illustrated in Figure A.2.
The stroke efficiency \( (S_e) \) is defined as the ratio between the maximum useful deformation \( d_{\text{max}} \) and the original length of the component \( L \) as given by:

\[
S_e = \frac{d_{\text{max}}}{L}
\]  
(A.4)

Beyond \( d_{\text{max}} \), the component behaves as a rigid mass and a significant increase in collapse load, as illustrated in Figure A.1(a), is observed. As a result, the usable stroke length of the component is limited to \( d_{\text{max}} \).

The energy efficiency \( (E_e) \) is defined as the ratio between the energy absorbed by the component up to \( d_{\text{max}} \) and the energy absorbed by an ideal energy absorber. For an ideal energy absorber, a peak load would be developed instantly and its value would remain constant while the component is being fully crushed over its original length. As a result, the energy efficiency can be calculated from:

\[
E_e = \frac{TEA(d_{\text{max}})}{F_{\text{max}} \cdot L}
\]  
(A.5)

For ideal energy absorbers, the maximum value of stroke efficiency and energy efficiency is considered to be 1.
Appendix B: RSMs of DFGF Filled Column

Figure B.1 Response surface for the SEA of the DFGF filled column for various values of $N_G$

Figure B.2 Response surface for the TEA of the DFGF filled column for various values of $N_G$
Figure B.3 Response surface for the TEA of the DFGF filled column for various values of $t_o$, (a) $\rho_i = 0\%$, and (b) $\rho_i = 10\%$
Figure B.4 Response surface for the TEA of the DFGF filled column for various values of $t_o$, (a) $A_c = 20\%$, and (b) $A_c = 30\%$