THE PROPERTIES AND EFFECTS OF METRO NETWORK DESIGNS

by

Sybil Jean-Marie Derrible

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Civil Engineering
University of Toronto

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Abstract

Since 2008, more than half of the world population lives in cities. To cope with this rapid urbanization in a sustainable manner, transit systems all around the world are likely to grow. By studying 33 networks in the world, this thesis identifies the properties and effects of metro network designs by using a graph theory approach.

After the literature review, a new methodology was introduced to translate networks into graphs; it notably accounts for various transit specificities (e.g., presence of lines). Metro networks were then characterised according to their State, Form, and Structure; where State relates to the development phase of metros; Form investigates the link between metros and the built environment; Structure examines the intrinsic properties of metros, by notably looking at their connectivity. Subsequently, the complexity and robustness of metros were studied; metros were found to possess scale-free and small-world features although showing atypical topologies; robustness emphasizes on the presence of alternative paths. Three network design indicators (coverage, directness and connectivity) were then related to ridership (annual boardings per capita), and positive relations were observed, which suggests that network design plays an important role in their success. Finally, these concepts were applied to the Toronto metro plans announced by the
Toronto regional transportation authority, Metrolinx; it was found that the grid-pattern nature of the plans could hinder the success of the metro; seven possible improvements were suggested.

Overall, the topology of metro networks can play a key role in their success. The concepts presented here can particularly be useful to transit planners; they should also be used along with conventional planning techniques. New transit projects could benefit greatly from an analysis of their network designs, which in turn may play a relevant role in the global endeavour for sustainability.
Acknowledgments

First and foremost, I would like to thank my parents, siblings and family for their support and guidance, not only during this doctorate, but throughout my entire education. Special gratitude goes to my father who has always been a role model in my life and to my mother who is unfortunately not with us anymore; I owe it all to you two. As this thesis represents the end of my education, I could not have wished for a better upbringing.

I would like to thank my thesis supervisor, Professor Chris Kennedy, for his support, advice and constant optimism in this journey. Chris has the particular skill of seeing the potential in students and prepares them effectively for their life journey. At the very beginning, Chris told me “the Ph.D. is all about the student”, and I can only aspire to adopt a similar philosophy in my future academic career.

In addition, I would like to thank the members of my committee, Eric Miller, Amer Shalaby, Matt Roorda and Brendon Hemily for their invaluable support. Their expertise of the transportation realm was of significant help for this thesis. I can only wish to see you for many years to come at the Transportation Research Board annual meetings. Furthermore, special thanks go to the external examiner Vukan Vuchic of the University of Pennsylvania.

Moreover, I would like to give many thanks to my colleagues and friends from the Sustainable Infrastructure Group, the Civil Engineering Department, the University and all around the world. I am sorry if I wish not to name anyone, but the list would be much too long. We have shared a lot of experiences these past few years, and this thesis would not be the same without you.

Last but far from least, immense gratitude goes to my partner Marie-Agathe. This is a journey we have done together. You have always been there for me, and I can only hope to help you as much in your future endeavours. Not only have you inspired me greatly, notably by entering me in the world of Art History, but your consistent and constant support, and your invaluable loving nature and warmth are much appreciated. My gratitude cannot be described by words, thus all I can say is thank you from the bottom of my heart.
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List of Symbols

$\alpha$ The $\alpha$-index or degree of cyclicity [-].

$\beta$ The $\beta$-index or complexity [-].

$\gamma$ The $\gamma$-index or degree of connectivity or clustering coefficient (in this thesis) [-].

$\delta$ Maximum number of transfers [-].

$\varepsilon$ Scaling factor [-].

$\eta$ The $\eta$-index [km].

$\theta$ The $\theta$-index. Dimensions depend on the variable used.

$\iota$ The $\iota$-index [km].

$\kappa$ Segment of transit line [-].

$\lambda$ Line overlapping index [-].

$\mu$ Cyclomatic number [-].

$\pi$ The $\pi$-index [-].

$\rho$ Connectivity or structural connectivity [-].

$\sigma$ Coverage [-].

$\tau$ Directness [-].

$A$ Average line length [km].

$B_{pc}$ Annual boardings per capita [trips].

$\bar{B}$ Average number of connections [-].

$b$ Number of connections [-].

$\{b\}_i$ Matrix of connections [-].

$C$ Average clustering coefficient [-].

$C_v$ Clustering coefficient of neighbourhood of $v$ [-].

$d$ Length of diameter of graph [km].
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<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$d_{ij}$</td>
<td>Shortest-path length between vertex $i$ to $j$. Dimensions depend on the variable used.</td>
</tr>
<tr>
<td>$E$</td>
<td>Total number of edges in a graph [-].</td>
</tr>
<tr>
<td>$E^{\text{max}}$</td>
<td>Potential number of edges in a completely connected graph [-].</td>
</tr>
<tr>
<td>$e$</td>
<td>One edge unless otherwise defined [-].</td>
</tr>
<tr>
<td>$e_{ij}$</td>
<td>Number of edges connecting vertex $i$ to $j$ [-].</td>
</tr>
<tr>
<td>${e_{ij}}$</td>
<td>Matrix of edges [-].</td>
</tr>
<tr>
<td>$E^m$</td>
<td>Number of multiple edges [-].</td>
</tr>
<tr>
<td>$e^m$</td>
<td>One multiple edge [-].</td>
</tr>
<tr>
<td>$E^s$</td>
<td>Number of single edges [-].</td>
</tr>
<tr>
<td>$e^s$</td>
<td>One single edge [-].</td>
</tr>
<tr>
<td>$E_{\text{glob}}$</td>
<td>Global efficiency. Dimensions depend on the variable used.</td>
</tr>
<tr>
<td>$E_{\text{loc}}$</td>
<td>Local efficiency. Dimensions depend on the variable used.</td>
</tr>
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<td>$G$</td>
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</tr>
<tr>
<td>$G_r$</td>
<td>Random graph [-].</td>
</tr>
<tr>
<td>$i$</td>
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<td>$k_v$</td>
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<td>$L$</td>
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<td>$M$</td>
<td>Total length of network [km].</td>
</tr>
<tr>
<td>$N_{\text{iss}}$</td>
<td>Number of inter-station spacings [-].</td>
</tr>
<tr>
<td>$N_L$</td>
<td>Number of lines [-].</td>
</tr>
<tr>
<td>$N_S$</td>
<td>Number of stations [-].</td>
</tr>
<tr>
<td>$n_s$</td>
<td>One station [-].</td>
</tr>
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$P$ Average shortest-path length. Dimensions depend on the variable used.

$p$ Number of sub-graphs of $G$ [-].

$R$ Route length of network [km].

$R_T$ Ratio of harmonic means of ideal $\bar{T}$ and real $\bar{t}$ trip times [-].

$r^T$ Robustness of metro [-].

$S$ Average inter-station spacing [km].

$T$ Traffic flow. Dimensions depend on the variable used.

$\bar{T}$ Harmonic mean of ideal trip times [min].

$\bar{t}$ Harmonic mean of real trip times [min].

$V$ Total number of vertices in a graph [-].

$v$ One vertex unless otherwise defined [-].

$V^e$ Number of end vertices [-].

$v^e$ One end-vertex [-].

$V'$ Number of transfer vertices [-].

$v'$ One transfer vertex [-].

$V_c'$ Number of transfer possibilities [-].

$W$ Observed number of vertices weighted by their functions [-].
Chapter 1
Introduction & Philosophy

1.1 Introduction

The year 2008 was a milestone in the history of the world as more than 50% of the world population was urbanized (i.e. living in cities), according to a report by the United Nations (2008a). With more than 6.7 billion people in the world as this document is being written (2010), there are more than 3.4 billion people who live in cities. Furthermore, this urbanization is also rapid; from 1950 to 2050 (projected), while the total world population grows by an average of 1.29% per year, the urban population grows on average by 2.16% annually, compared to 0.44% for the rural population (United Nations 2008b). Figure 1.1 shows the evolution of the world population (total, urban and rural) from 1950 to 2050 (projected).

![Figure 1.1 World, urban and rural populations from 1950 to 2050 (projected); data source: (United Nations 2008b).](image)

In developed countries, 75% of the population already lives in cities, with a relatively slow urban population growth rate (around 0.92% annually). In developing countries, the urban population
only accounts for 45% of the total population at the moment, but with an annual growth rate of 2.85%, they are urbanizing rapidly (United Nations 2008b).

One impact of this rapid urbanization is the growth of cities all around the globe. In particular, the emergence of mega-cities and mega-regions is becoming more frequent, where population levels are reaching new heights. As a consequence, the planning and management of such mega-cities and mega-regions can prove to be challenging. Indeed, more infrastructure is required to provide essential services in spatially-constrained areas. Moreover, the resources necessary to provide for this world population and the methods to extract these resources are presenting environmental issues, which may be the determining challenge of the 21st century. Not only are these issues tied with large cities, they also apply to the multitude of smaller cities that are now seeing a sudden growth in population but are not equipped with the necessary tools and knowledge to face them. While conventional planning and management methods cannot cope with these contemporary issues, it is necessary to find new and innovative solutions to address these challenges.

One umbrella, under which these novel solutions can be grouped, is sustainable development. Nonetheless, cities are complex systems that are evolving constantly, and planning for a sustainable future is no easy task. Furthermore, while the scale of these issues is considerable, the scope is no less significant, including disciplines ranging from finance and agriculture, to construction, water management, energy generation and many more. In particular for this work, the realm of transportation will have to undergo major structural and behavioural changes to adapt for the future.

This work therefore falls within the topic of sustainable urban transportation. One method to make urban transportation systems more sustainable is by promoting public transportation. Indeed, as Figure 1.2 shows, not only do transit systems address spatial problems, by moving people more efficiently (i.e. less road-space is needed, which relieves congestion), they normally also address environmental issues by emitting less greenhouse-gas (GHG) emissions per trip than the private automobile, particularly for electric transit modes (electric buses, streetcars, light rail transit systems, metros, electric regional rail transit systems, etc.).
One particular research area has emerged recently as being promising to address numerous problems across disciplines, including urban transportation. This area focuses on networks. Although the concept of networks is well-known in the scientific community, new findings in the past 12 years have given a new momentum to the subject (which is now referred to as Network Science), and many researchers all around the world are currently studying networks. Viewing systems as networks can have many advantages. A network being a collection of nodes (also called vertices in conventional graph theory) joined by links (also called edges in conventional graph theory), it is easier to identify general patterns occurring in systems, regardless of system specificities; in fact, many systems in the world have been found to share similar network properties and their understanding can be relevant.

### 1.2 Network Properties and Effects

In the context of this work, a network property can be seen as a general attribute of a network, whether planned for or not; such properties include the scale-free feature as we will see later on,
but they do not include specific features such as the number of nodes and links. The identification of network properties can be significant in the study of networks, regardless of subject area (e.g., biology, physics, engineering, etc.). Not only can the solutions found in one type of network help design another type of network, they can also be relevant to finding and/or characterising common properties of networks in one single discipline (e.g., metro networks). By definition, networks only have nodes and links; it is therefore logical to expect common properties in a wide range of networks. This is especially relevant as topologies of networks can be complex in nature. For a fairly exhaustive list of network properties, see (Bornholdt and Schuster 2003).

More particularly, in the past 12 years, two properties have emerged to be relevant and predominant in the literature and seem to be shared by many complex networks; these are scale-free and small-world properties. Scale-free networks were introduced by Barabási and Albert (1999). In these networks, the distributions of number of links per node (i.e. degree distribution) follow a power law; in other words, few nodes have many links and many nodes have few links. Small-worlds were introduced by Watts and Strogatz (1998). These networks are locally well connected, whilst being globally relatively close to each other in terms of degrees of separation (i.e. number of steps required to go from one node to another); this property, present in many real-life networks, coincides with the sociological concept of “glocalization” developed by Wellman and Hampton (1999). These two properties will be explained thoroughly in the literature review (chapter 2).

The finding of these two network properties of complex networks has revolutionized the study of networks. As a result, scholars all around the world have started to look at many types of networks, ranging from sociological (Albert and Barabási 2000; Newman et al. 2002; Kossinets and Watts 2006) and ecological networks (Sole and Montoya 2001; Dunne et al. 2002; Banavar et al. 2007), to physical (Rodríguez-Iturbe 1997; Sachtjen et al. 2000; Arenas et al. 2001), virtual networks (Albert et al. 1999; Tadic 2001) and many more.

Moreover, by using a network approach, it is possible to see how changes in one part of the system can affect the rest of the system. For instance, initial properties may change by adding new nodes and links to a network. These phenomena are called network effects, and their identification and understanding can help in the goal towards findings new solutions. The
concept of network effects is inherently related to the concept of network itself. In general, two types of effects have been defined: direct and indirect. Direct effects are direct consequences of an action. For example, when the telephone was first invented, it became more valuable as more people used it. Social networking websites such as Facebook offer a more contemporary example; the registration of initial users influenced the registration of their friends, engendering a snowball effect, which increased the value of using the website for all parties. On the other hand, indirect effects are indirect consequences of an action. For the example, the sale of toner cartridges is an indirect effect of the sale of printers. A more contemporary example is the familiarization of social networking websites such as Facebook, which facilitated the popularization of other similar websites such as Twitter.

Defining the boundaries of the system studied is also of paramount importance to be able to identify the effects. Indeed, depending on the system boundaries, effects can be direct or indirect. For example, if we study the specific effects of Facebook on other social networking websites, the popularization of Twitter will be a direct effect, while the effect of these websites on the media is indirect; this example has been chosen since postings on Twitter and Facebook have been a direct source of news on several occasions this past year. This statement is particularly true for transportation systems; for example, economic effects can be direct or indirect depending on the definition of the system boundaries.

Although the theory of network effects is relatively well understood, quantifying these network effects presents a real challenge. One possible method to estimate these effects is through the use of input-output tables, where the changes in one component can be fairly easily transferred to the other components. These changes have been referred to as ripple effects, which are essentially akin to network effects. Such input-output tables are not available in all disciplines, however; they are especially useful in fairly well-known applications such as relatively simple economic analyses, but are seriously limited when it comes to more complex systems. The identification of network effects is not only a problem of parameter calculation (to estimate the magnitude of the effects), it is also a topological problem (in a network, what are the nodes and how are they connected with each other); for instance, investing in renewable energies has effects on power generation and distribution (i.e. the network partially consists of the power grid), but it also has effects on employment, financial markets (i.e. less pressure on oil reserves), etc., and the simple mapping of this network is non-trivial.
Nevertheless, the value of identifying network properties and the value of understanding and quantifying network effects remain substantial. To be able to predict more accurately the impacts of specific network topologies is a significant step forward in the goal towards finding sustainable solutions. Here again, the scale is large, and ramifications can be imagined in many disciplines. In particular for transportation, the prediction of travel demand has been a core topic of study in the community. Or more specifically in the context of this work, the identification of the network properties and network effects of public transportation systems can be of practical use, notably for their planning and design.

1.3 Public Transportation Network Properties, Effects, and Designs

Before discussing the properties of public transportation networks, it should be mentioned that transit has a myriad of network effects on urban systems, whether they are economic (e.g., effects of transit investment), social (e.g., effects on accessibility, equality) or even environmental (e.g., effects on transportation greenhouse-gases emissions). Table 1.1 offers one example of network effects of transport infrastructure investment.

In this instance, the effects of investment are considered; naturally, direct effects include construction and time savings effects, while indirect effects include spatial distribution expenditure and crowding-out effects (i.e. less traffic congestion). Again here, the categorisation of the effects depends on the system boundaries. For instance, if the system was to look at the effect of investment on traffic congestion, the “crowding-out” effects would become direct. Moreover, it should be mentioned that the list of network effects presented in Table 1.1 is not exhaustive, which relates back to the problem of mapping the network effects of public transportation systems. The fact that some of these effects are temporary while others are permanent further complicates the task. In addition, the relationships between these effects have rarely been quantified. Several studies exist on the network effects of transport investment, but they are either qualitative or depend on case studies entirely (Banister 2000). This is easily understandable given many determining factors are location and time specific (e.g., costs of tunnelling for economic effects, nature of the energy grid for environmental effects), which further increases the number of obstacles to surmount.
Table 1.1 Permanent and Temporary effects of transport infrastructure investment; adapted from (Oosterhaven and Knaap 2003).

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<th>Temporary</th>
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<td>Direct</td>
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<tr>
<td>via markets:</td>
<td>Construction effects</td>
<td>Transportation costs and time savings effects</td>
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<td>external effects:</td>
<td>Environmental, safety effects</td>
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<td>Indirect</td>
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<td>via demand:</td>
<td>Spatial distribution of</td>
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<td>expenditure effects</td>
<td>expenditure effects</td>
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<td></td>
<td>via supply:</td>
<td>Crowding-out effects</td>
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<tr>
<td></td>
<td>external effects:</td>
<td>Productivity and location effects</td>
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<tr>
<td></td>
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<td>Indirect emissions, etc</td>
</tr>
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</table>

Consequently, we see through this example that public transportation systems are in fact complex systems, and solutions to network effects problems such as quantifying the impact of transport investment are not trivial. The research potential is therefore significant, and much work lies ahead. The first step forward, in the undertaking of such a study, is therefore to delineate and clearly define the system studied and its boundaries.

In the context of this work, we choose to focus on topological network properties; that is the geometry of the physical network itself. In other words, we are rather interested in identifying the network properties and network effects of the way stations and lines are connected together, organized and integrated in a city. As mentioned, the study of networks has recently emerged as a particularly strong area of interest, and transit systems, by being real-life applications of networks, are particularly interesting in that respect. Historically, transit systems have typically been implemented to respond to a certain demand; this demand is most often a pre-requisite for the financial stability of transit systems, although economic returns are not only collected from the fare box, but also from the ability of transit to enhance economic growth. It should also be mentioned that the implementation of transit has also been a response to other criteria that can have a social nature (e.g., to make transit available to specific neighbourhoods in a city), an economic nature (e.g., to decrease the negative impacts of congestion) or even simply to respond to a political agenda. In other cases, transit has even been built because of its reputation to enhance economic development, the success of which depends heavily on its integration in the city. The history of public transportation planning, not being part of this thesis, will not be
discussed here; accounts, reviews and analyses can be found in (Schaeffer 1975; Vuchic 2007; Cervero 1998; Mees 2000; Rodrigue 2006).

Typically, the study of the network feature of transit systems is limited to their operational characteristics and ability to move people (e.g., transit assignment stage of a four-stage model (Le Clerq 1972; Spiess and Florian 1989)). What is clear, however, is that the topological aspect of transit network (i.e. their geometry) has seldom been a determining factor in the planning of public transportation; at most, a general appreciation of the integration of the network is considered. Even in master-planned cities like Stockholm in Sweden, the transit network was designed to link the “satellites towns” with downtown Stockholm; it is highly unlikely that the master-planned transit system was built to possess specific network properties as defined here.

Many cities in the world now have transit systems that were planned to achieve specific goals, and designing specific network properties was not one of these goals. Consequently, there now exist many transit networks in the world, and one may ask whether these networks share common characteristics despite the fact they were planned and built discretely. In other words, is it possible to see emerging patterns in network design? Moreover, are these characteristics common in other networks as well (e.g., biological networks)? How relevant are they and are there design best practices? These are some of the objectives that were set at the beginning of this work, and answering these questions offers a real contribution.

1.4 Contribution and Objectives

The planning and design of transit systems is a complex process that requires much input from many different fields. Traditionally, demand-side variables such as origin-destination flows are first examined to find appropriate corridors. Emphasis can also be put into servicing a specific class of people (e.g., lower-income people). As was mentioned in the previous section, little attention is paid to the topological network feature of the public transportation systems (Musso and Vuchic 1988). Since cities are growing rapidly, transit systems are also likely going to grow rapidly, and as a result, acquiring an understanding of the network properties and network effects of transit systems may be a favourable practice to help transportation planners. This work essentially attempts to seek whether transit networks possess properties that were not specifically
designed at the planning phase, but are nonetheless inherently present in their topologies, and whether these properties can be controlled and planned for in the future to meet the municipal and/or regional transportation vision.

For instance, Figure B.1 shows the map of the Paris metro system. This system is one of the most extensive in the world with 14 lines and 297 stations. What properties can be identified by the way the lines are integrated within this network? Are these properties shared with other metro networks in the world? What is the impact of the number of transfer stations, and ratio of transfer to total number of stations? Does the design of this network have any effects on ridership, future growth and robustness of the network? Moreover, this system is fairly centralised as it mainly serves the City of Paris, which covers about 106 km²; how does this affect its integration within the urban environment?

These are some of the questions this thesis is addressing. Broadly speaking, the main objective of this thesis is to identify and analyze the topological network properties and effects of public transportation systems, which then leads to the development of transit network design recommendations.

In this thesis, topological/geometric network properties can include characteristics that are shared by several metro networks. Moreover, these properties also include properties found in many real-life networks (e.g., biological), such as scale-free and small-worlds features (which will be defined thoroughly in the next chapter). On the other hand, network effects include the impact of these properties on future growth (e.g., ways an existing metro can grow considering current network topologies), robustness of the network (e.g., how many alternative paths are available), and ridership (e.g., how network design indicators influence ridership). Furthermore, the impact of network size on these network properties and effects is also studied. Most transit systems were built independently of each other, identifying common properties and effects is non-trivial but could potentially be of substantial help to transit planners.

To study these transit network properties and effects, this work uses graph theory concepts to effectively “translate” a system into a graph (defined as a collection of vertices linked by edges); the use of graph theory is common to study networks. Nevertheless, graph theory can be applied in several different ways, and the methodology used here is novel and offers a new contribution both to the transportation and network science communities. Transit systems have specificities in
their network topologies that must be considered when analyzing them; the specificities include the presence of lines, overlapping between the lines (tracks shared by two or more lines), and the presence of stations with different attributes (transfer-stations vs. termini vs. other non-transfer stations). They are also planar networks (i.e. two lines crossing will create a new transfer station), thus they cannot have links joining two distant stations (e.g., two stations located far apart cannot have simply one link connecting them, but a number of stations and possible line transfers). The methodology presented here is elaborated to capture essential network properties, whilst addressing these specificities and being useful to planners. Most notably, stations are differentiated according to their attributes, which does not seem to have been done previously in the present literature. Overall, the methodology developed here is one of the main contributions of this work.

Public transportation systems are also composed of different modes (e.g., bus, streetcar, light rail, metro, etc.). Differentiating between the modes using graph theory can be challenging. Although the links between the stations can be given different weights to reflect their influence, values to weight these links can be hard to determine. As a result, this work only deals with metro systems; here metro refers to urban rail systems with exclusive right-of-way (ROW A, as categorised by Vuchic (2005)), whether it is underground, at grade or elevated.

The choice of metro systems is particularly relevant. First of all, metros are closed systems (i.e. tracks are not shared with other transit modes), thus their network structures are not directly affected by external factors such as availability of tracks. Second, as they are relatively costly to build, metros are typically designed in such a way to achieve specific goals (e.g., to bring people from the satellite towns to the city centre as in Stockholm, or in contrast to offer an extensive service in the city centre alone as in Paris). Third, as they are often underground, they do not have to follow existing road patterns, which is much more interesting to study from the network viewpoint as it enables a wider diversity of patterns; buses simply run on roads, their network topologies are not as original (i.e. many bus networks follow a typical grid-pattern). Furthermore, fewer metros exist in the world, notably because they require much funding, as a result a wide range of sizes exists (from one line in Lausanne, Switzerland, to 14 lines in Paris, France); it is therefore possible to study a wide range of topologies and analyze the evolution of such networks. Additionally, by being generally smaller in size compared to other networks (up
to 422 stations in New York City, US compared to thousands of stops for bus networks), existing design properties are easier to identify.

Additionally, it should be mentioned that much effort was put into analyzing metros from all over the world so that properties identified are not location-specific (e.g., North-America vs. Europe vs. Asia) or dependent on land-use characteristics (e.g., low-density vs. high-density). On the other hand, by only studying metro networks, this work does not directly analyze problems of modal integration (i.e. feeder modes are not included in this analysis), it also partially hinders the quantification of the effects (i.e. impact on ridership); integrating the other transit modes would, however, require a different mindset, which would not reveal the network properties that are at the core of this work.

The fundamental questions behind this work are therefore related to network properties and effects of metro systems. What type of networks are metros? Are they random networks, or are there some recognizable patterns in their design? Do they share characteristics with networks from other fields, such as river networks, biological networks or even virtual networks such as the World Wide Web? If so, what lessons can we learn from these other networks? More specifically, this work attempts to achieve the following objectives, that are each a chapter of this thesis.

- **How can metro networks be studied using a graph theory approach?**
- **Can metro networks be characterized, depending on factors such as their size, integration with the urban form, their structure?**
- **How complex are metro networks in the network science context? Do they follow general network properties such as the presence of scale-free and small-worlds patterns? What effects do these properties have on the robustness of metros?**
- **Can these network properties of metros affect ridership? What recommendations can be made?**
- **How can these networks properties and effects be used in practice?**
This work is therefore most valuable to transit planners, and is most applicable at the strategic/conceptual planning phase, used along with more conventional techniques. Figure 1.3 shows the four levels of transportation planning as presented by Vuchic (1999).

Level IV: Individual Facilities
Planning, Design, Operation of Single Facility

Level III: Single Mode Network or System
Coordination of Single Mode Network/System

Level II: Multimodal Coordinated System
Integration of Multimodal Network/System

Level I: City-Transport Relationship
City-Transport Balance

Figure 1.3 The Four Levels of Transportation Planning; adapted from (Vuchic 1999).

Level I deals with the relationship between transportation and the urban system as a whole, thus at the macroscopic level; a particular study by Shin et al. (2009) shows the impact of land consumption on transportation systems. Level II deals with the integration of all transport modes in a city, to effectively control the role of each mode in a city, which is still at the macroscopic level. Level III looks at the network level for each mode, so that each road or transit line is integrated in a way that facilitates travelling, which could be placed at a mesoscopic level. Finally, Level IV looks at every single road or transit line, which can be related to the microscopic level.

The key word in this thesis is network, and since only public transportation systems are analysed, and in particular metros, this work is primarily useful for level III planning, although it can also address some level I issues (notably related to regional transportation visions).
For this work, we chose to consider a representative pool of 33 worldwide metro systems shown on Figure 1.4.

These systems range from 2-line systems (e.g., Rome, Italy) to the Paris 14-line system. Figure 1.4 shows each system on a world map as well as a list of the 33 networks. Out of the total of 33 metros, nine have 2-3 lines, ten have 4-6 lines, six have 7-9 lines, and eight have 10-14 lines. Note that no one-line system was considered since they do not possess network properties as
defined in this work. Also, only chapters 4 and 5 analyse the total 33 networks; chapters 6 and part of 7 analyse a smaller set of 19 metros for reasons that will be mentioned in chapter 6.

Moreover, when reading this work, two major factors should be kept in mind. First, the intention of this work is to provide transit planners with new tools to better design transit networks. The fundamentals of transit planning are not refuted; in fact, the work presented here should be systematically used with conventional planning methods. For instance, this work does not address the problems of finding optimal transit corridors; this particular exercise is invariably related to factors such as population density, which are not directly considered here. On the contrary, one of the main contributions of this work lies within the integration of transit lines into transit networks and within the integration of transit networks in the built environment.

Second, this work does not directly consider the management and operation of transit systems. For instance, this work does not deal with such problems as scheduling. Although these problems are fundamental to run efficient transit systems, this thesis solely looks at the properties and effects of metro network designs. Furthermore, operational characteristics related to network design such as the presence of overlapping lines or the issue of having one circle line versus two circumferential lines are not examined, notably because these characteristics are mainly operational and also because they do not necessarily relate to network properties and effects.

About terminology, it should be mentioned that while the terms nodes and links are more common to describe the components of networks, we prefer the terms vertices and edges. These terms are essentially identical (i.e. nodes are vertices, and links are edges); vertices and edges seem to be mostly used for graphs, while nodes and links are used to describe networks. In this work, we use the terms vertices and edges when both describing graphs and networks.

This thesis is organised as follows. First, a review of the existing literature that applied graph theory and network science concepts to study transit network design is given in chapter 2. Then, in chapter 3, the novel methodology that was developed for this work is explained thoroughly. In chapter 4, three characteristics (State, Form, and Structure) are developed to identify and analyse the properties and effects of metro networks. Afterwards, in chapter 5, the complexity of metros is studied (i.e. by identifying scale-free and small-world properties), and their effects on the robustness of metros are analysed (where a specific indicator of robustness is developed). Subsequently, in chapter 6, the effects of these properties on ridership are examined by using
three specific network design indicators (coverage, directness, and connectivity). In chapter 7, all the concepts presented in this thesis are applied to the specific case of Toronto metro network and future plans. A brief conclusion is then offered in chapter 8, which notably lays the path for future research on the topic of transit network design. Finally, appendix A contains the definition of most terms introduced in this thesis, and appendix B contains popular maps of 14 metros that are referred to in the text.
Chapter 2
Literature Review

2.1 Introduction

The problem of comprehensive urban transportation planning and network design is relatively old and may have originated in the town of Miletus in Asia Minor, with the works of Hippodamus (Castagnoli 1971). The streets of Miletus followed a grid pattern.

![Grid structure of the street network of Miletus; adapted from (Castagnoli 1971).](image)

In the Roman Empire, the process of street planning took a larger scale and was standardized (Vitruvius Pollio 1914); the Romans typically built two standard roads in cities, serving as main corridors for the mass movement of people and goods. An account for the history of city planning can be found in (Hugo-Brunt 1972).
Public transportation planning, on the other hand, is relatively recent and emerged more strongly during the industrial revolution and at the turn of the 20th century. Originally, public transportation was purely a means to provide mobility to city residents; planning objectives were notably financial, and transit lines and systems were most often run by private companies. With the automobile revolution in mid-20th century, however, city residents found a new means of mobility, and public transportation lost its initial vigour in various parts of the world (Schaeffer 1975; Vuchic 2007). Nevertheless, transit constitutes an essential component of transport systems in cities and a myriad of benefits exist, from economic and social to environmental. The main objectives of public transportation planning therefore seems to have shifted from a system operator perspective (i.e. to make profit), to a community perspective having impacts on many levels, from the economic activity of the city to its livability (Vuchic 1999), which can be related back to the concepts of sustainability. Even today, more specific objectives of transit planning differ depending on decision-making authorities as mentioned in chapter 1.

In the pursuit of better planning transit systems, this work aims to help transit planners understand the topological network features of public transportation systems and provide them with relevant knowledge. As the title of this thesis suggests, three main areas are approached here: public transportation, network effects and design, and graph theory. Concepts of the last, however, were first applied to road networks, and even though the study of such networks is not part of this thesis, it remains important to recall these studies; it is all the more important considering relevant network indicators emerged from these studies and are still used today. Then, significant attention is put into reviewing the study of transit networks from the transportation community; the study of transit network is a broad term, it is worth noting that only studies using a graph theory approach are considered; other studies notably related to the transit network assignment problem (e.g., (De Cea et al. 1989; Spiess and Florian 1989; Wilson and Nuzzolo 2009)) are not discussed here. Finally, relevant studies on transit and the new field of network science are reviewed; network science seems to hold a great potential to analyse transit systems, and this thesis particularly uses concepts from this field. But first, a brief account on the origins of graph theory is presented.

This chapter is based on the article “Applications of Graph Theory and Network Science to Transit Network Design” by Derrible and Kennedy (2010a).
2.2 Origins of Graph Theory

The origins of graph theory date back to 1741 in Königsberg, Prussia, which is now Kaliningrad in Russia. Inspired by the city’s characteristics, the famous mathematician Leonhard Euler made a simple observation. From Figure 2.2, which is an engraving of Königsberg around Euler’s time, it is possible to see four land masses (i.e. four nodes/vertices) including the island in the centre, and seven bridges (i.e. seven links/edges). Euler (1741) simply showed that it was not possible to cross all seven bridges consecutively only once because the difference between the number of edges and vertices is an odd number (i.e. 7 – 4 = 3); this problem is now famously known as “The Seven Bridges of Königsberg”. While graph theory is presently used in many disciplines, from computer sciences to physics and chemistry, it actually originated from an urban transportation problem.

Figure 2.2 Engraving of former Königsberg, now Kaliningrad, Russia (von Merian-Erben 1652).
After more than a century, in the 1880-90’s, the British mathematician Cayley used a graph theory approach to study one particular type of graphs: trees, which led to the famous Cayley trees (Cayley 1889). Trees are essentially graphs that do not have any cycles or loops. In particular for Cayley-trees, each vertex has an identical number of connections, except for “leaf-vertices” (i.e. vertices at the end of the graphs); an example is offered on Figure 2.3 for 3 and 4-Cayley trees.

![Figure 2.3](image) Examples of Cayley trees. (a) 3-Cayley tree. (b) 4-Cayley tree.

Later, in the second half of the 20th century, the French mathematician Berge made significant contributions to the field, particularly on hyper-graphs. He also recalled and used the cyclomatic number $\mu$ (Berge 1962), which is a simple, yet relevant measure. This indicator is also called the 1st-Betti number in the literature. It basically counts the number of loops or cycles in a graph and is defined as:

$$\mu = e - (v - p) = e - v + p$$

(2.1)

where $e$ is the number of edges (links), $v$ is the number of vertices (nodes), and $p$ is the number of sub-graphs. Transportation networks are rarely disjoint, thus $p$ normally equals 1. Essentially, $(v-I)$ is the number of edges in a tree graph with $v$ vertices. Therefore, subtracting $(v-I)$ to $e$ simply counts the number of “extra-edges” in the graph, and it is these “extra-edges” that create cycles. The presence of cycles is important in transportation networks, notably to offer alternative paths, i.e. for robustness as we will see later on. To illustrate this measure, Figure 2.4 compares three different tree networks to cyclic networks (i.e. networks with cycles).
Many graph theory problems are also famous in the scientific community; they often consider coloring and route problems such as the “four colour theorem”, the “road colouring problem”, the “shortest path problem”, and the “spanning tree problem” (Copes 1987; Gould 1988).

Graph theory is now used by many researchers around the world and is not confined within the realm of mathematics anymore. It notably inspired the field of network science which will be reviewed in the final section of this chapter.

It should also be mentioned that the brief review of the three mathematicians cited here is by no means comprehensive, for instance, other relevant mathematicians include Cauchy, Hamilton, Kirchhoff, and Pólya. Moreover, many other researchers further contributed to the field, but are not cited here; for more information, see (Biggs 1976; Andrásfai 1977).

### 2.3 Early Works on Transportation

The application of graph theory to road transportation systems emerged in the late 1950’s and lasted through the 1970’s. Avondo Bodino (1962) was one of the first to formulate an application of the theory of graphs to transport systems. He notably applied a few already developed indicators to road transport systems, but did not consider network design, which is more relevant here.
At the time, the application of graph theory to transport systems was mainly tied to economics; notably as an attempt to forecast the regional economic impacts of the construction of the US Interstate Highway system and various freeways built in cities. Afterwards, with the development of computers, research was then redirected towards exploiting more intensive models, e.g., the four-stage model. This section therefore reviews the contributions made to the road transportation sector prior the computerization era; relevant indicators are also enlisted and explained thoroughly. The application of graph theory to public transportation will be dealt with in the next section.

2.3.1 The first round of indicators: Garrison and Marble

Garrison and Marble (1962; 1964; 1965) significantly contributed to the field by introducing three graph theory measures/indicators directly linked to network design. Consider a graph with $e$ edges and $v$ vertices, the first indicator, $\alpha$-index is related to the cyclomatic number (equation (2.1)) and is defined as:

$$
\alpha = \frac{e - v + 1}{\frac{1}{2} \cdot v \cdot (v-1) - (v-1)}
$$

(2.2)

In equation (2.2), the numerator is the cyclomatic number $\mu$, and the denominator can be associated with the maximum cyclomatic number possible as it is the maximum number of possible edges $\frac{1}{2} \cdot v \cdot (v-1)$ minus the number of vertices in a tree graph $(v-1)$; i.e. $e_{\text{max}} - v + 1$. It can be referred to as a degree of “cyclicity”, since it is the ratio of actual to potential number of cycles in a graph. For planar networks (i.e. two crossing edges necessarily create a new vertex), the maximum number of possible edges is $3(v-2)$, and therefore the maximum cyclomatic number is greatly reduced to $(2v-5)$; hence:

$$
\alpha = \frac{e - v + 1}{3(v-2) - (v-1)} = \frac{e - v + 1}{2v - 5}
$$

(2.3)

As transportation networks are mostly planar, perhaps with the exception of airline networks, using the planar version of $\alpha$ is favourable. A second indicator by Garrison and Marble was developed with the same logic. Instead of looking at the ratio of cycles, however, it examines the
ratio of actual to potential edges. It is the \( \gamma \)-index that is sometimes referred to as connectivity; it is defined as:

\[
\gamma = \frac{e}{\frac{1}{2} \cdot v \cdot (v-1)}
\]  

(2.4)

In equation (2.4), the numerator is the number of edges \( e \), while the denominator is the maximum number of possible edges \( e_{\text{max}} \); thus \( \gamma \) is the ratio of actual to potential number of edges. As the term connectivity has become generic in the literature, we prefer to refer to \( \gamma \) as the degree of connectivity (note that in the literature, the inverse of \( \gamma \) is also sometimes referred to degree of connectivity). As for the previous indicator, the planar version of \( \gamma \) may be more useful; \( \gamma \) therefore becomes:

\[
\gamma = \frac{e}{3(v-2)} = \frac{e}{3v-6}
\]  

(2.5)

Finally, the last indicator, the \( \beta \)-index, is derived differently. Instead of providing a measure of actual-to-potential property, it simply is the ratio of edges to vertices; in mathematical form:

\[
\beta = \frac{e}{v}
\]  

(2.6)

The \( \beta \)-index is therefore the average number of connections per vertex; it is widely used in the scientific community. It has also been referred to as an indicator of complexity (i.e. the more connections per vertex the more complex). Interesting observations can be made from calculating this indicator, notably when compared to network size (as we will see in chapter 4).

Overall, these three indicators enable the understanding of different network characteristics and they have been used in various instances in the transport literature. The two latter indicators (\( \gamma \) and \( \beta \) indices) seem to be more relevant, however; in particular for this thesis, we use them to investigate the state of metro networks (chapter 4).
2.3.2 The second round of indicators: Kansky

In the early 1960’s as well, Kansky (1963) worked on similar issues and tried to relate road transportation network features with economic development. He first recalled the $\alpha$, $\gamma$ and $\beta$ indicators, and then introduced four specific indicators ($\eta$, $\pi$, $\theta$, $\iota$) incorporating specific properties of transportation networks (e.g., length of system, traffic flow).

The first indicator $\eta$ essentially calculates the average edge length. In equation (2.7), $M$ is the total length of the network; this indicator therefore simply divides total network length by the number of edges.

$$\eta = \frac{M}{e} \quad (2.7)$$

The second indicator $\pi$ is more conceptually challenging but it offers valuable insights. It is called $\pi$ because it relates to the calculation of the circumference of a circle ($\pi=C/d$). In this case, the circumference is simply the total length of the transport system $M$, and for the diameter, the length $d$ of the network diameter is used; for networks, the diameter is the largest shortest-path to reach the two extremities of the network (i.e. take the two furthest nodes, the diameter is the shortest-path to go from one node to the other).

$$\pi = \frac{M}{d} \quad (2.8)$$

Conceptually, $\pi$ is rather an indicator of system spread; a more compact network will have many kilometres of roads, whilst having a small diameter to facilitate travelling and reduce travel time. For instance from Figure 2.5, imagine the vertices are identical warehouses, network $b$ is doing better than network $a$, because in general, vehicles travel less to link any two warehouses; in this example, vehicles linking the two furthest warehouses of network $a$ have to travel a longer distance (2.2 units) than to link to two furthest warehouses of network $b$ (1.5 units), which is reflected by a smaller value of $\pi$. 
Figure 2.5 Difference in \( \pi \) indicator from two networks. (a) Network a. (b) Network b.

The third indicator \( \theta \) relates to the traffic flow. Instead of focusing on edges, however, it emphasizes the role of vertices. In equation (2.9), \( T \) is the total traffic flow, which can be expressed in tons (or total transit passengers for instance) and \( v \) is the number of vertices. This indicator therefore calculates the average flow per vertex (i.e. average flow per station for instance).

\[
\theta = \frac{T}{v} \quad (2.9)
\]

The final indicator \( \iota \) applies to networks where no travel flow data is available. In equation (2.10), while \( M \) is the total length of the system, \( w \) is the observed number of vertices weighted by their functions; i.e. heavily connected vertices are given heavier weights, the underlying assumption is that these vertices likely handle larger amounts of traffic flow. If all vertices carry the same weight, then \( \iota \) becomes an indicator of distribution (i.e. it counts the track length associated to stations).

\[
\iota = \frac{M}{w} \quad (2.10)
\]

Kansky also introduced a variation of the \( \iota \) indicator. Instead of using the weighted number of vertices, he uses the total traffic flow \( T \). In this instance, \( \iota \) becomes the average track length per passenger, equation (2.11), and the inverse of \( \iota \) is the average number of transit passengers per unit of track length (e.g., per kilometre of track).
2.3.3 Other notable contributions

Using graph theory to analyse transport networks seemed to have been relatively popular in the 1960’s and 1970’s. In fact, Morlok (1970) dedicated a book to the different aspects of transport technology and network structures, where he reviews all the above indicators. His primary objective was to develop a method to be able to compare different transport modes (air, rail, and bus); assessing these modes according to their network properties and other variables related to costs.

One study by Bon (1979) looked at the specific properties of road networks in 13 islands around the world. He also analyzed the evolution of the degree of connectivity, $\gamma$, of these road networks along the dimensions of time (by looking at the evolution of each networks in 1958, 1967 and 1972) and size (by comparing the different networks). Overall, he argues that while several researchers suggest $\gamma$ increases with network size, and other suggest it decreases with size, the degree of connectivity $\gamma$ in fact stays constant around a value of 0.5. This notion of evolution of degree of connectivity is non-trivial, and we will see later on the impact of size on $\gamma$ for public transportation networks.

The topic of network design has also been advanced greatly by the work of Newell (e.g., (Newell and Daganzo 1986b; Newell and Daganzo 1986a)); however, he mainly addressed transportation network design objectives as a function of traffic flow, as opposed to identifying the properties and effects of current transportation networks as done here. Following a similar logic, Daganzo (2010) studied the competitiveness of grid versus hub-and-spoke transit networks and found that a hybrid network (having a grid pattern in the central city coupled with a hub-and-spoke
structure) can be more competitive in terms of accessibility (expressed in minutes but accounting for user and agency costs).

More recently, Black (2003) reviewed the early stages of the research, where he specifically recalls the indicators introduced by Garrison and Marble, explaining clearly all the different concepts. His work focused on the geographical feature of transport systems. In fact, the relevance of the network feature of transportation systems seems more predominant in geographical studies; other works include Taaffe (1996) and Rodrigue (2006).

Finally, the use of graph theory to study transportation networks seems to re-emerge strongly at the moment, notably due to the strong interest in network science. New studies are undertaken to study the impact of network features of transportation networks on travel distance, trip assignment and even mode choice (Parthasarathi et al. 2010).

2.4 Public Transportation and Graph Theory

The use of graph theory concepts to study public transportation networks came later, in the 1980’s and 1990’s. The fact that different modes exist in public transportation can be a hurdle to effectively study transit networks; indeed, how can edges of buses, streetcars, LRTs or metros be compared? Moreover, the goal of transit is not only to move people from their origins to their destinations, but also to do it in a way that minimizes travel time and avoids unnecessary transfers, whilst being convenient to use, reliable, and safe to name a few. There are therefore many more aspects to consider when studying transit networks. Moreover, public transportation systems have certain specificities (e.g., presence of line, overlapping between the lines, etc), which either require that present indicators be adapted or even that new indicators be developed.

2.4.1 Trip time as connectivity: Lam and Schuler

The first study that was found using graph theory on transit systems was produced by Lam and Schuler (1981; 1982). They first applied Garrison and Marble’s indicators to transit networks. They then introduced a new indicator $R_T$ related to trip time as an indicator of connectivity; it is
the ratio of potential to actual reciprocal harmonic means of trips times. Consider a region serviced by transit with \( n \) representative trips; it therefore has \( n \) origin-destination pairs. First, the ideal trip time \( T \) for each trip ‘\( i \)’ is calculated in the ideal case (i.e. the network is completely connected); the harmonic mean of the ideal trip times \( \overline{T} \) is calculated as:

\[
\overline{T} = \frac{1}{\frac{1}{n} \left( \frac{1}{T_1} + \frac{1}{T_2} + \ldots + \frac{1}{T_n} \right)} = \frac{1}{\frac{1}{n} \sum_{i} \frac{1}{T_i}}
\]

Then, the real trip times \( t \) for each trip ‘\( i \)’ are calculated. In this case, if there are no connections between individual origin-destination pair, the trip time becomes infinite (i.e. \( t_i = \infty \)). The mean of the real trip times is calculated as:

\[
\overline{t} = \frac{1}{\frac{1}{n} \left( \frac{1}{t_1} + \frac{1}{t_2} + \ldots + \frac{1}{t_n} \right)} = \frac{1}{\frac{1}{n} \sum_{i} \frac{1}{t_i}}
\]

As a result, the connectivity indicator \( R \) is defined as:

\[
R = \frac{\overline{T}}{\overline{t}}
\]

Note that since \( t_i \) is the real trip time, it is necessarily greater than or equal to \( T_i \), which is why the potential trip is in the numerator. Consequently, the connectivity indicator \( R \) is bound between 0 and 1 (i.e. \( 0 \leq R \leq 1 \)). To be computed, this indicator requires much information on operating characteristics and travel patterns, which may not be available at first. Moreover, we choose to focus on network design solely, and as it was mentioned in the introduction, problems of operations are not considered in this thesis.

### 2.4.2 Transit network design: Vuchic and Musso

The most significant contribution came from Musso and Vuchic (1988) and Vuchic and Musso (1991) in the late 1980’s and 1990’s; Vuchic also dedicated an entire chapter on this topic in his book *Urban Transit: Operations, Planning and Economics* (Vuchic 2005). However, their
approach seems to be slightly different from traditional methods. They tackle networks as computational systems as opposed to mathematical, following the general trend towards computerisation. In this thesis, both approaches are used.

Figure 2.6 shows a transit network with three lines; line AC (solid), line AD (dash) and line EF (dot). Lines AC and AD are overlapping from stations A to B. Let us assume that each spacing between stations is 1 km long to illustrate some of the coming concepts.

![Transit network with overlapping lines](image)

**Figure 2.6** Transit network with overlapping lines; adapted from Figure 1.1 in (Vuchic 2005).

To study transit networks, Vuchic and Musso have first divided the type of network properties into two groups. The network on Figure 2.6 will be used to illustrate these properties. First, the *network size and form* indicators are:

- Number of stations on line \(i\) (e.g., line AC has seven stations, line AD has six stations, and line EF has eight stations)
- Number of inter-station spacings (arcs) on line \(i\) (e.g., line AC has six inter-station spacings, line AD has five inter-station spacings, and line EF has seven inter-station spacings)
- Length of line \(i\) (e.g., assuming each spacing between stations is 1 km long, line AC is six km long, line AD is five km long, and line EF is seven km long)
- Number of transfer stations, (e.g., three transfer stations)
- Number of lines in the network \(N_L\) (e.g., three lines)
- Number of stations in the network \(N_S\) (e.g., 15 stations)
- Number of inter-station spacings $N_{iss}$ (e.g., 15 inter-station spacings)
- Route length of network $R$ (e.g., 15 km)
- Number of circles (i.e. cyclomatic number $\mu$; e.g., 1 cycle)
- Number of station-to-station travel paths: total, direct, with transfer (e.g., total 105, direct 58, with transfer 47)

Second the *network topology* indicators are defined:

- Average inter-station spacing $S$ (e.g., 1 km)
- Line overlapping $\lambda$ (e.g., 1.2)
- Circle availability (i.e. the $\alpha$-index; e.g., 0.011 for non-planar and 0.04 for planar networks)
- Network complexity (i.e. the $\beta$-index; e.g., 1)
- Network connectivity (i.e. the $\gamma$-index; e.g., for 0.14 non-planar and 0.38 for planar networks)
- Directness of service (e.g., 0.55)

The *network size and form* indicators are self-explanatory. The *network topology* indicators, on the other hand, are directly related to the topic of this thesis. It is possible to see that out of the six indicators, three are taken from Garrison and Marble’s work. The three others are specific to transit networks, and it is worth spending time on each of them.

First, the average inter-station spacing $S$ is defined as the ratio of the route length of network $R$ and the number of inter-station spacings $N_{iss}$; the number of inter-station spacing is the total number of stations per line minus one terminal for each line to count the effective number of edges in the network and minus to the number of spacings that have overlapping lines.

$$S = \frac{R}{N_{iss}}$$  \hspace{1cm} (2.15)

The second indicator relates to the presence of overlapping lines. It calculates the ratio of total lengths of the lines by the length of the network as a whole; mathematically, it is defined as:
In equation (2.16), \( R_i \) is the length of line ‘\( i \)’ and \( R^k_m \) is the length of all overlapping segments; ‘\( k \)’ designates the number of lines overlapping and ‘\( m \)’ is an identification number. Transit systems without overlapping lines have a \( \lambda \) equal to 1, and the more line overlaps the greater the value of \( \lambda \) is. This indicator is indeed valuable to capture the overlapping line property. It should be noted that overlapping lines (i.e. two or more lines sharing segments of tracks) and trunk-and-branches lines (i.e. one line having branches in either or both ends of the line) share similar operating characteristics such as matters of scheduling. To differentiate these two types of lines, the line overlapping indicator, \( \lambda \), can be used. In this thesis, the typology of lines was chosen in accordance with the transit agencies’ maps (i.e. whether transit agencies’ maps show one line with branches and two or more overlapping lines).

Finally, the directness of service indicator calculates the ratio of paths (origin to destination, and accounting for all possible paths) that can be done without transferring to the total number of paths possible \( \frac{1}{2} \cdot v \cdot (v-1) \); if all stations are considered as vertices. While this indicator captures the concept of directness, it does not, however, account for network size (i.e. larger networks may show a smaller value whilst being better overall). In this thesis, we have created another directness indicator that accounts for network size and is simpler to compute.

Overall, the main advantage of these three new indicators lies in the fact they account for several specificities of public transportation networks. This is a necessary step to be able to effectively study the network feature of transit systems.

Vuchic and Musso have also recalled the different types of transit lines. Transit lines can be categorized qualitatively depending on their size, form, and location within the city. This categorization is important here as this terminology will be kept in the thesis. A detailed description, in terms of geometry and operation, of each of these line categories can be found in (Vuchic 2005). Here, a short review of each is presented; the different line categories are illustrated on Figure 2.7.
Radial lines (line 1 on Figure 2.7) systematically have a terminal in the city centre and then go outside the city into the suburbs. They typically follow high-demand corridors and serve commuters who live in the suburbs and work in the city centre. They sometimes have branches. Branches often have lower frequencies of service, which then integrates with other branches into a higher level of service in the main section of the line; branches of high-order transit systems (e.g., metro) can be, however, more costly to build and operate and are sometimes substituted by feeder modes (e.g., bus, bus rapid transit (BRT), light rail transit (LRT)).
Diametrical lines (lines 2, 3 and 6 on Figure 2.7) have both termini outside the city but systematically pass through the city centre; in some respect, they are analogous to having two connected radial lines. They are sometimes called transverse or through lines. They also follow high-demand corridors and serve commuters who work in the city centre (from both extremities of the line) and commuters who live in one extremity and work in another. They do not necessarily link to opposite extremities of a city; for instance, they can have an L shape depending on local characteristics or planning objectives.

Tangential lines (line 4 on Figure 2.7) have both termini outside the city centre and can have the main section passing by the central area, but not through the city centre. They normally have lower demand and connect commuters who live and work outside the city centre as well as distributing trips to radial and diametrical lines. They are often implemented to relieve demand on existing diametrical and radial lines. They can be significantly valuable in terms of network design as we will see later on.

Circle or ring lines (line 5 on Figure 2.7) do not have termini since they form a circle. They normally follow high and medium-demand areas and provide a direct connection between these areas. They can also distribute trips to radial and diametrical lines as well as serve commuters who live and work outside the city centre, similar to tangential lines. There also exist circumferential lines that geometrically resemble semi-circle lines (e.g., lines 2 and 6 of the Paris metro), and hence possess similar characteristics (e.g., distribute trips to radial and diametrical lines). These circumferential lines possess operating benefits but may add transfers to transit users compared to circle lines. Moreover, it should be mentioned that circumferential lines are sometimes assimilated to circle lines in the literature.

As part of their study, Musso and Vuchic (1988) collected and calculated the different network indicators as well as identified the line category for 10 systems in the world: Chicago, IL; Hamburg, Germany; London, UK; Milan, Italy; Munich, Germany; Osaka, Japan; Paris, France; Tokyo, Japan; and Washington, DC.
2.4.3 Other notable contributions

More recently, a study by Gattuso and Miriello (2005) applied Garrison and Marble’s as well as Kansky’s indicators to 13 metro networks (through out Europe, plus New-York). Their main contribution lies in the fact they applied weights to vertices in order to account for the transfer property of transit systems and difference in attractiveness of stations. They notably developed two new indicators.

The first indicator is the node’s range of influence, which is dependent on three variables. The first variable is the geographic position of a station; they define three positions in a city, the “centre”, the “first corona” and the “second corona”. The second variable is the relative vertex weight (ratio of vertex weight to total weights of all vertices, where the weight is dependent on the number of connections at the vertex). The final variable is related to the directness variable from Musso and Vuchic (1988); instead of having the total number of trips in the denominator, they only consider the total number of direct trips for all transit lines; essentially, it is the ratio of direct trips of a station divided by the total number of possible direct trips from all the stations in the system.

The second indicator is the network’s covering. It simply measures the area coverage of a system by using the node’s range of influence as the radius; overlapping ranges of influences are then subtracted to compute the effective covering area of the network.

This study by Gattuso and Miriello (2005) therefore emphasizes on the property of metro stations relative to their location in the city as opposed to looking at the network feature of metro systems. In their article, they also provide a relatively detailed analysis of the cross-correlations between the indicators as well as carrying out a comparative analysis between the metro systems studied.

Finally, a few studies exist that examine several systems’ characteristics such as the length of systems. For instance, Levinson (2000) studied metro and light rail transit systems in the world at the continent and country levels. In particular, he looks at the relationship between ridership with several network characteristics such number of lines per urban area, line lengths, station spacing, and rail kilometres per million people. By doing so, he is able to make some relevant observations and speculate qualitatively about the future of rail transit systems in the world.
Another study includes the work of Hass-Klau and Crampton (2002) who have looked scrupulously at 24 light rail systems, mostly in Europe, a few systems in North-America, and Melbourne in Australia, and ranked them. Although they look at only a few network characteristics (length of system and network density relative to population), they have done an extensive analysis, incorporating variables of demography (population, density, etc), economy (GDP, fares, etc), operations (headways, speed, use of monthly passes, etc), and even levels of pedestrianization.

The use of graph theory to study public transportation is relatively scarce in the literature despite its potential. This may be due to the fact that little attention has been put into analysing the network feature of transit systems by transit planners. It may also be due to the increasing use of computers in the research community, which may have shifted efforts into elaborating complex transportation models; e.g., ILUTE (Salvini and Miller 2005). Nevertheless, as the field of network science is emerging strongly, many network scientists use public transportation systems as real-life examples of networks, and this may be beneficial to the transportation community. This thesis particularly uses network science concepts, and a review of the existing studies is presented in the next section.

2.5 Network Science

The field of network science is relatively young and is linked to graph theory. From its origin, graph theory dealt almost exclusively with regular graphs; regular graphs are graphs with specifically defined structures (e.g., lattices, molecules, or common shapes such as polygons), and typically be relatively small in size (i.e. not in thousands of vertices). Early forms of “complexity” included mostly dimensional aspects (i.e. hyper-graphs, not reviewed here) and mathematical proofs. Overall, little attention was brought to irregular forms of graphs until the emergence of what is now called network science. Network science notably deals with graphs showing non-trivial patterns that are typically more structurally complex and can have an unpredictable nature; for instance, lattices have easily identifiable patterns, while random graphs have to be studied more thoroughly. This discipline first emerged in the late 1950’s from the work of Erdös and Rényi (1959) on random graphs, although it truly became popular in the late

In this section, we will first go through and explain the concepts of random graphs, small-worlds and scale-free networks. We will then review the contributions to date that have used network science concepts to study public transportation systems.

2.5.1 Random graphs: Erdős and Rényi

The work of Erdős and Rényi essentially constitutes the birth of network science (although the term was coined and became popular much later). Prior to their contribution, only regular graphs were studied as mentioned. Their approach was rather different as they introduce a random process in the formation of networks (Erdős and Rényi 1959). One well-known example is the formation of social ties during a cocktail party (Buchanan 2002).

Moreover, they have also studied the evolution of random graphs (Erdős and Rényi 1960). Here, in a random graph, an equal probability is given to the formation of edges between vertices; i.e. all vertices have an equal chance to be connected by an edge (the actual number of existing edges must be defined at the beginning). They noted that by following this process, most vertices ended up having a similar number of connections, which can be represented by a Poisson’s distribution in some instances; more information can be found in (Bollobás 2001). A Poisson distribution is defined as:

\[
f(x, \lambda) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}
\]  

where \( x \) is the number of occurrences of an event, and \( \lambda \) is the expected number of occurrences. Take a graph with 30 vertices (here, \( x=30 \)) and assign 30 edges randomly, if we assume a Poisson distribution applies, most vertices will have 2 connections (i.e. \( \lambda=2 \) on Figure 2.8) as opposed to having one vertex with 30 connections for instance. On Figure 2.8, an expected number of occurrences \( \lambda \) of 5 and 15 are also represented. Essentially, if the average distribution of connections is 5 (i.e. \( \lambda=5 \), on average vertices have 5 connections), it is likely that higher proportion of vertices will have 5 connections as opposed to having vertices with 10 connections.
on the one hand and others with only 1 connection on the other hand. As a result, it appears that the number of connections per vertex follows a bell-shaped distribution, tending towards normal distributions for large number of occurrences.

![Figure 2.8 Poisson distribution for 30 occurrences and $\lambda = 2, 5$ and 15.](image)

Normal distributions are common in real life. Consequently, finding such distributions here seemed reasonable at the time. This type of distribution is also present in several transportation networks, in particular for planar networks such as road networks (one example is the US Interstate Highway system). Later, however, researchers actually found that most real-life networks were not random and did not follow such distributions.

### 2.5.2 Small-Worlds: Watts and Strogatz

In 1998, Watts and Strogatz (1998) introduced a new concept related to networks, that of highly-clustered small-worlds. While a network may be large, the shortest distance (i.e. the number of degrees) to go from any node to another is relatively short. For instance, if we consider a fully connected network with 1,000 vertices (i.e. there is always a path to go from one vertex to
another), it can require up to 999 degrees (i.e. steps) to go from the two most distantly spaced vertices (i.e. imagine all vertices are along a line). In small-world networks, however, only few degrees are needed (e.g. 6 degrees). This concept is now popular and is sometimes associated with the saying “six degrees of separation” (i.e. all humans can be linked through six degrees). For further information, Watts (2003) published a popular book entitled “Six Degrees: the science of a connected age”.

Notable networks having this property are random networks as introduced in the previous section. As long as there is a possibility to go from one node to another (i.e., the graph is not disjoint), the average number of degrees should be relatively small.

The contribution by Watts and Strogatz (1998) lies in the fact that their networks are also highly clustered, which is a common characteristic in real-life networks. Here, clustering refers to the fact that nodes clique together forming a highly connected neighbourhood. On Figure 2.9, \( p \) can be seen as the degree of randomness; with \( p=0 \), nodes are connected to their four neighbours, with \( p=1 \) nodes are connected randomly (i.e. all pairs of vertices have an equal probability to be connected). The regular network is highly clustered; i.e. looking at one node, it is connected to four neighbours, and these four neighbours are also almost all connected with one another, thus the average shortest path is fairly large. The random network on the right has very little clustering. The small-world network is a hybrid of the two latter networks; i.e. most nodes form a cluster, and a few edges are connected randomly.

![Figure 2.9](image)

*Figure 2.9* Random wiring of ring lattice; adapted from (Watts and Strogatz 1998).
Watts and Strogatz (1998) found that even though the small-world network had little randomness, the average shortest path was small. This type of network can notably be related to friendship networks, where most of one’s friends know each other, but that person also has acquaintances, and it is these acquaintances that enable the small average short path. In the rest of this work, small-world networks will refer to this type of network only.

Mathematically, small-world networks obey two basic rules:

1. High clustering $C$
2. Small average shortest-path length $P$

Rule 1 requires high clustering compared to same size (i.e. same number of vertices and edges) random networks. As mentioned, clustering refers to the number of connections between the vertices of a given neighbourhood. Clustering is measured through the clustering coefficient. For instance, take an undirected graph $G$ with $V$ vertices and $E$ edges, and consider one particular vertex $v$, the neighbourhood $k_v$ of $v$ contains all the vertices that are connected to $v$; there are therefore as many neighbourhoods as there are vertices although they are not all distinct. The clustering coefficient $C_v$ calculates the total number of existing edges within this neighbourhood $k_v$, say $e_v$, to the maximum number of edges that could exist, $e_v^{\text{max}}=k_v(k_v-1)/2$ (i.e. if all the vertices of the neighbourhood were connected with one another).

$$C_v = \frac{e_v}{e_v^{\text{max}}} = \frac{2 \cdot e_v}{k_v(k_v-1)} \quad (2.18)$$

For instance on Figure 2.9 for the regular network, each vertex is connected to four neighbours (i.e. the neighbourhood is composed of 4 neighbour vertices); and for this graph, all vertices are identically connected. Per neighbourhood, there could therefore be a total of $e_v^{\text{max}}=(4 \cdot 3)/2=6$ edges, and from looking at the figure, we see that each neighbourhood only contains three edges (i.e. $e=3$); therefore the clustering coefficient of each vertex is $3/6=0.5$.

The average clustering of a network is calculated as the average of all the clustering coefficients:

$$C = \frac{1}{V} \sum_{i} C_i \quad (2.19)$$
where $V$ is the total number of vertices in the graph, and each vertex is attributed a number ‘i’. For the regular network on Figure 2.9, with $C_v=0.5$ for each of the 20 vertices, $C = (0.5+0.5...+0.5)/20 = 0.5$.

We can see that $C_v$ is relatively close to the $\gamma$ index introduced by Garrison and Marble. In fact, they are mathematically very close, except that $C_v$ considers neighbourhoods while $\gamma$ looks at the entire network at once. For transportation networks, either $C_v$ or $\gamma$ can be used depending on the application; for transit networks, we prefer to use $\gamma$ for reasons that will be mentioned later on.

Rule 2 requires small average shortest-path lengths compared to same size regular networks. Small-world networks actually have shortest-path lengths characterised by a logarithmic scaling (i.e. as the number of vertices increases linearly, the average shortest-path increases logarithmically; for more information, see (Newman 2003b)). As a result, rule 2 is assessed by verifying that the average of all the shortest-path length $P$ is smaller than the log of the total number of vertices, i.e. $P \leq \ln V$. To calculate the shortest-path length, several methods exist, such as the Floyd algorithm and the Dijkstra algorithm that are not explained here; for more information, see (Teodorovic 1986; Taylor 2007). For transportation networks, this concept is valuable as origin-destination trips should be as short as possible. In particular for transit networks, the sole number of transfers required to go from the origin to the destination of a trip may be preferable to use (since each station would otherwise add a degree of separation even without a line transfer).

In the literature, to verify whether a network is a small-world, $C$ and $P$ are compared with a random network that has the same number of vertices and edges (i.e. rewiring the network randomly). For instance, a graph $G$ has 1,000 vertices and 5,000 edges, and a random graph $G_r$ is created having 1,000 vertices and 5,000 edges also. $G$ is a small-world only if its clustering coefficient is reasonably larger than the clustering coefficient of $G_r$, and both average path lengths are relatively similar.

Overall, the concept of small-world networks can be valuable in the transportation community. More specifically for network design, a small-world topology may have impacts on ridership and robustness for instance.
2.5.3 Scale-free networks: Barabási and Albert

The concept of scale-free network emerged in the late 1990’s also, from the work of Barabási and Albert (1999). In their case, they started to investigate the distribution of edges in real-life networks (i.e. number of connections per vertex) and noticed that instead of having bell-shapes, the distributions followed power laws; they notably looked at the structure of the World Wide Web and the US power grid. As introduced in section 2.5.1, until then, most complex networks were thought to have random properties; the discovery of scale-free networks was substantial.

Mathematically, consider a graph $G$ with $V$ vertices and $E$ edges, it has a scale-free pattern if the probability $f$ that a vertex $v$ has $b$ connections (number of edges connected to vertex) follows a power law:

$$f(b) \propto b^{-\varepsilon}$$ (2.20)

In other words, when randomly sampling vertices, the distribution of connections is found to follow a power law. Such a distribution is clearly different from a Poisson distribution. In Poisson distribution, most vertices end up having a similar number of connections, which can be affiliated with a certain scaling. However, power-law distributions are not limited in the number of connections per vertex, hence the absence of this scaling, which explains the term scale-free.

In equation (2.20), the exponent $\varepsilon$ is called the scaling factor. The value of the scaling factor can add much information about the property of the network. To illustrate the impact of the scaling factor, Figure 2.10 shows its effect on normalized frequency distributions; in this case, we simply plotted the function $f(x)=x^{-\varepsilon}$, for $x=1..6$ and $\varepsilon=1..5$.

On Figure 2.10, as the scaling factor increases (e.g., from 2 to 3), the curve becomes steeper. In other words, a small $\varepsilon$ (e.g., 1) implies a “fat-tail” distribution (slower decay); hence a more important presence of vertices with many connections. Buchanan (2002) describes this type of network as “aristocratic” since a number of nodes have many connections. Larger $\varepsilon$ values (e.g., 5) are described as “egalitarian” networks since only a few vertices have many connections (i.e. edges are more evenly distributed). In general, large scale-free networks have a scaling factor in the range $2<\varepsilon<3$ (Barabási and Bonabeau 2003).
Networks tend to follow two patterns to become scale-free (Barabási and Albert 1999):

1. Continuous expansion by the addition of new vertices

2. Preferential attachment of new vertices to existing vertices that are already well connected

The second pattern is the key here. Instead of assigning edges randomly, new edges preferably attach to already well connected vertices. This theory is not new (Yule 1925; Simon 1955) and can be witnessed in many systems, including transportation systems; it was revealed, however, more strongly in scale-free networks. Another example is the number of links on the World Wide Web; my personal webpage (http://individual.utoronto.ca/sderrible) may have only one or two links pointing to it, which is common, compared to other popular websites such as Google, Amazon or Facebook that may have thousands of links pointing to them. This concept of preferential attachment is now commonly associated with the phrase “the rich get richer”, since already well connected vertices become even more connected.
Figure 2.11 shows the US Interstate highway system on the bottom left and the US Air Traffic system on the bottom right. The US Interstate Highway system was built to link the different States in the US; the highways were therefore spatially evenly distributed, which resulted in most vertices (here, cities) having a similar number of connections; thus following a random network structure. The US Air Traffic system, on the other hand, follows passenger demand, which led to the creation of hubs such as the New York JFK airport and the Chicago O’Hare airport; these hubs now have a disproportionate number of connections compared to smaller airports (i.e. they are in the right-hand end of the tail in a power law distribution). Transit systems can also show scale-free properties as we will see in the next section.

![Diagram of transportation networks](image)

**Figure 2.11** Examples of transportation networks. The US Interstate highway system on the bottom left has a random network structure, compared to the US Air Traffic system on the bottom right that has a scale-free network structure; adapted from Figure 6.1 in (Barabási 2003).

Overall, the discovery of scale-free networks was a substantial contribution in network science, and myriad of systems were found to show scale-free features in their network topologies; ranging from sociological (Albert and Barabási 2000; Newman et al. 2002; Kossinets and Watts 2006) and ecological networks (Sole and Montoya 2001; Dunne et al. 2002; Banavar et al. 2007), to physical (Rodríguez-Iturbe 1997; Sachtjen et al. 2000; Arenas et al. 2001), virtual networks (Albert et al. 1999; Tadic 2001) and many more; reviews can be found in (Albert and Barabási 2002; Newman 2003b; Newman et al. 2006). In addition, Barabási (2003) wrote a popular book
on the topic entitled “Linked : how everything is connected to everything else and what it means for business, science, and everyday life”.

2.5.4 Applications to transit systems

The recent findings in the study of networks are quite promising for the future of the network science community. This community is growing rapidly and includes researchers spanning a wide range of disciplines (e.g., biology, sociology, physics, economics, engineering, etc). While myriad of networks have been studied, the study of transportation systems, by taking a network science approach, seems relatively limited. Moreover, transportation networks are most often studied by network scientists seeking real-life examples of networks as opposed to transportation engineers and planners. As a result, the existing studies do not necessarily contribute to the transportation community. This phenomenon is even more acute for transit systems. This section presents a fairly exhaustive list of past studies.

In 2002, Latora and Marchiori (2001; 2002) noted that the clustering coefficient $C$ and average shortest-path length $P$ from Watts and Strogatz (1998) were ill-defined to study transit systems; notably because termini have only one neighbour which offset the clustering coefficient (i.e. termini are only connected to one station, thus $C=0/0$). Moreover, during the creation of the random network (to be able to compare $C$ and $P$), they found that most stations actually ended up not being connected (because in transit networks $E \sim V$, whereas in most networks $E >> V$); the path length between many nodes therefore became infinite, which also offset the results. Consequently, they defined two new indicators that use the inverse of the shortest path length as a measure of efficiency (if the path length is infinite, i.e. $L=\infty$, then the efficiency is 0); mathematically, both indicators are essentially the same, except that the global efficiency $E_{glob}$ is applied to the entire network (to substitute for $P$ which is a global property), and the local efficiency $E_{loc}$ considers the station level (to substitute for $C$ which considers neighbourhoods).

$$E = \frac{1}{V \cdot (V-1)} \sum_{ij} \frac{1}{d_{ij}}$$ (2.21)
where $V$ is the total number of vertices, and $d_{ij}$ is the shortest path length between vertex ‘$i$’ and ‘$j$’. By applying these measures to the Boston subway system, they found that small-worlds properties existed in this network at the global level (i.e. rule 2 in section 2.5.2).

A similar procedure was applied to the Seoul, Tokyo, Boston and Beijing systems by Chang et al. (2006); these systems were also found to have small-world properties. Moreover, they focused on triangular-formed sub-graphs to analyze the response to “incidents of disconnection”, which can be related to robustness here (i.e. a higher number of “triangles” offers more alternatives). In addition, Vragović and Díaz-Guilera (2005) also studied and compared the network efficiencies of the Madrid, Barcelona and Boston systems by proposing a variant of local efficiency.

Due to the definition of the local efficiency indicator, all these studies categorized transit systems as declustered networks (i.e. poor local efficiency). In reality, however, transit users do not have to transfer at each station (i.e. the number of degrees to go from one station to another does not depend on the number of stations, but rather on the number of line transfers). Moreover, they did not account for the planarity of transit systems; i.e. when two edges cross each other, they systematically create a new vertex, or for transit, when two lines cross each other, a new station is systematically built to offer the possibility to transfer between the two lines.

Other researchers noted that transit systems could have different graph characteristics depending on the method used to translate a network into a graph. Alternative representations of transit systems can also remediate problems seen in the above paragraphs. Figure 2.12 shows four different possible methods. Figure 2.12 (a) shows a simple transit system. Figure 2.12 (b) shows its representation in L-space; only connections are considered regardless of lines. Figure 2.12 (c) shows its representation in B-space (bipartite representation), where stations and lines become nodes so as to account for the presence of lines. Figure 2.12 (d) shows its representation in P-space, which directly connects all stations part of the same line; thus accounting for the presence of lines here as well. Figure 2.12 (e) shows its representation in C-space, where lines are represented as nodes and the links represents the presence of a direct transfer between two lines (this method is particularly useful to find the maximum number of line transfers in a transit system, see chapter 3).
In 2004, by using a bipartite graphical representation (i.e. B-space), Seaton and Hackett (2004) calculated the clustering coefficient, the average path length and average degree vertex of the Boston and Vienna rail transit systems. Overall, they found that these systems were in fact small-worlds, obeying both rules of section 2.5.2.

Later, Xu et al. (2007), and Huapu and Ye (2007) further analyzed the complexity of several bus networks in China and found that scale-free and small-worlds features were present in these systems (findings varied depending on the graph theoretic methodology applied). By looking at the Nagoya transit network in Japan, Shi et al. (2008) also studied the complex properties of such networks, notably by differentiating the transit modes. Such studies can also be found on larger systems such as the China Railway Network (Li and Cai 2007). However, these studies remain small in scale (few networks), and limited in properties examined.

On a larger scale, by looking at 22 transit systems (bus and tramway) in Poland, Sienkiewicz and Holyst (2005) found that some systems appeared to show a scale-free property, with scaling factors ranging from 2.4 to 4.1; with 15 systems out of the 22 had a scaling factor greater than three. Moreover, most systems also appeared to be small-worlds. Additionally, they acknowledged that effort should be put into accounting for the presence of lines as opposed to
solely considering the stations, which is why they seem to favour P-space representation. Nevertheless, they did not account for the planar property of transit systems.

Another significant contribution was elaborated by von Ferber et al. (2007; 2009), where they studied 14 public transportation networks in the world, looking at all technologies and accounting for the overlapping property of transit systems (which they associate to a harness effect). They found scaling factors ranging from 1.24 to 4.99 depending on the fit (i.e. power law or exponential). It should be noted that for scaling factors greater than the 4, the degree distribution can either be fitted to a power law or exponential distribution (Cohen et al. 2002). von Ferber et al. found that six transit systems followed an exponential distribution out of the fourteen studied using an L-space representation (three with values below 4), and eleven systems out of the fourteen using a P-space representation (three with values below 4). Additionally, they also observed small-world properties in all systems.

Nevertheless, these studies still lack several properties that are characteristic of transit systems such as the planarity of networks, and the presence of lines is also not always accounted for. Moreover, the ability to transfer is another relevant property of transit systems that is missing in these studies.

The topology of transit networks is also only briefly considered. In these existing studies, the transit systems were either seen as a collection of nodes (i.e. no transit lines) or studied using various representations (e.g., L and P-space), which may not reflect reality and thus hinder the usefulness to public transportation planners and practitioners.

2.6 Conclusion

Problems of urban transportation planning can be dated back to the Roman Empire, and despite two millennium of experience, they remain present today. One field that appears particular fitted to address such problems is graph theory.

The origin of graph theory is relatively old, dating back to the 18th century from the works of Euler. Applications to transportation systems, however, came much later in the 1950’s and
1970’s with the development of specific network indicators; notably from the work of Garrison and Marble, and Kansky.

The study of transit networks using a graph theory approach emerged in the 1980’s with substantial contributions from Vuchic and Musso. For instance, nowadays, transit lines can usually be categorized according to their typology and location in the city (e.g., radial, diametrical lines). Various indicators have also been developed to account for transit specificities such as the presence of lines and overlapping between the lines.

The field of network science is also emerging strongly and seems promising to analyse transit networks. In particular, the discovery of small-worlds and scale-free characteristics gave significant momentum to the field. Several works have also applied these concepts to transit networks, but their usefulness to transit engineers and planners may be limited.

It appears that the methodology used to translate transit networks into graphs plays an instrumental role in their successful analysis. It is therefore of paramount importance to define a method that accounts for transit specificities whilst allowing for the study of network science concepts. The method used in this thesis differs from those presented in this chapter, and much time is dwelt on explaining this methodology in the next chapter.
Chapter 3
Methodology – Metros as graphs

3.1 Introduction

Physically, metro systems are real-life networks, where the vertices are stations and the edges are rail tracks. The mathematical field of graph theory is typically used to collect information on networks, which considers networks as graphs. The process of creating a graph from a transit network is an instrumental component of this work. As we have seen in chapter 2, there are different ways to achieve this, and finding an appropriate method is not trivial.

For this work, we put a significant amount of effort into making the method as useful as possible for the transportation community. Moreover, since we are applying this method to 33 metro networks in the world, the method needs to be consistent and applicable to all types of metro networks. As a result, we differentiate between transfer stations, termini, and other stations so as to better identify the network properties and effects. We also account for the presence of overlapping lines, which means that not all edges are equivalent. This methodology was applied manually to the 33 networks introduced in chapter 1.

First, we can represent a transit network as an undirected graph \( G \) with \( V \) vertices and \( E \) edges; \( G=\{V,E\} \); undirected because most transit lines have two-way traffic. In this chapter, we will first assign line attributes to each station before defining the vertices, the edges, and another essential measure: the maximum number of transfers. At the end of this section, we will finally recapitulate this information in a summary and give a detailed procedure to apply the methodology.

Moreover, to better illustrate the method, practical examples of the concepts introduced are presented throughout the chapter by referring to the Toronto metro system. Figure 3.1 (a) presents the actual map of the Toronto system, and Figure 3.1 (b) presents its graphical representation. Overall, the Toronto metro has a number of lines, \( N_l \), of four that are colour-coded here (Figure 3.1 (b) also identifies each line with a number, from 1 to 4), and a total number of stations, \( N_s \), of 69 (note that not all the stations are shown on Figure 3.1 (b) for reasons that will be given in the next section). The route length of the network \( R \), which is the
total one-way (i.e. one direction) operating track length, is also collected; for Toronto, \( R \) is 68.75 km.

(a) Map of the Toronto metro; adapted from (TTC 2009).

(b) Graphical representation of the Toronto metro.

**Figure 3.1** The Toronto metro network. (a) Map of the Toronto metro. (b) Graphical representation of the Toronto metro.

### 3.2 Lines and line attributes

Before defining the vertices and edges as shown on Figure 3.1 (b), the number of lines for each metro needs to be collected. Most metro networks studied in this thesis are composed of independent lines only (i.e. no overlapping between lines as discussed in section 2.4.2) such as the Toronto (Figure 3.1 (a)) and Paris (Figure B.1) metros. Nevertheless, the presence of branches and overlapping lines can complicate this task, as in the cases of London (Figure B.2)
and New York City (Figure B.3) in particular. Therefore, for all 33 metro systems, we have decided to follow data provided by transit agencies, whether the information is present on their websites, reports or even popular maps (i.e. number of coloured lines).

In parallel, data on the number of lines hosted per station is collected. Let a station \( n_s \) be an element of the total number of stations \( N_S \), the first step is to attribute a value ‘\( \ell \)’ of the number of lines each station is hosting. Essentially, we count the number of lines passing through each station \( n_s \), resulting in a one-dimensional matrix \( \{ \ell_i \} \) per transit system. From the 33 networks studied, stations host from 1 to a maximum of 6 lines (i.e. \( 1 < \ell < 6 \)). A table \( L \) with the frequency distribution of number of stations for each line attribute \( \ell \) can then be computed; this table essentially counts the number of occurrences for \( \ell \) equals 1, 2, etc. Table 3.1 shows \( L \) for the Toronto metro. A similar undertaking was produced for the 33 networks studied in this work.

<table>
<thead>
<tr>
<th>( \ell )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stations</td>
<td>64</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Lastly, the number of connections \( b \) at a station \( n_s \) should be defined; although this measure is not collected in practise, it will be referred to later to explain how the concept of scale-free networks was applied to transit systems. Essentially, all stations \( n_s \) are attributed a value \( b \) of their number of connections; i.e. termini have only one connection (\( b=1 \)), all other non-transfer stations have two connections (\( b=2 \)), etc. Using this information, it is possible to create a one-dimensional matrix \( \{ b_i \} \), and the average number of connections for the entire network is denoted \( \overline{B} \).

### 3.3 Vertices

In a graph, we define the vertices \( V \) as only the transfer stations and the termini; i.e. all other stations are not vertices. In this method, we also differentiate between these two types of stations. The transfer stations are referred to as transfer vertices \( V' \), and the termini are referred to as end vertices \( V^e \). To better view how the stations are being differentiated, Figure 3.2 shows a Venn diagram; we notably observe that vertices remain part of the total number of stations \( N_S \).
Additionally, each vertex is assigned an arbitrary identification number (for example, on Figure 3.1 (b), vertices 1 to 10).

Transfer vertices $V^t$ are therefore the transfer stations (black circles on Figure 3.1 (b)), where it is possible to switch lines without exiting the system; whether the transfer is a simple cross platform interchange (e.g., Mong Kok station in Hong-Kong), a short walk (e.g., St George station in Toronto) or a longer walk (e.g., lines 4 and 13 at the Montparnasse Bienvenüe station in Paris, or between the Central and Bakerloo line at Oxford Circus station in London).

End vertices $V^e$ are the line termini (white circles on Figure 3.1 (b)), where it is not possible to switch to another metro line. Note that if a line terminal actually hosts two lines, it is considered a transfer-vertex (e.g., vertex 5 on Figure 3.1 (b)); this is also true if two termini connect (e.g., vertex 4 on Figure 3.1 (b), the resulting station is considered to be a transfer vertex as opposed to two end vertices); in other words, the ability to transfer is the determining factor here.

For the example of Toronto, the metro has a total of 10 vertices (5 end and 5 transfer-vertices) as shown on Figure 3.1 (b).

Mathematically, the total number of vertices $V$ is equal to the sum of the number of transfer vertices $V^t$ plus the number of end vertices $V^e$:

$$V = V^t + V^e$$  \hspace{1cm} (3.1)

where,

$$V = \sum_i v_i$$  \hspace{1cm} (3.2)
In equations (3.2), (3.3), and (3.4), \( v_{i,\ell} \) is a vertex (numerically, each vertex has a value of 1) with identification number ‘\( i \)’ and line attribute ‘\( \ell \)’ as defined above.

As mentioned in section 3.2, two one-dimensional matrices can be defined, one with line attribute \( \ell_i \) and one with number of connections \( b_i \).

<table>
<thead>
<tr>
<th>Vertices ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line attribute ( \ell_i )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Connections ( b_i )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.2 shows these two matrices \( \ell_i \) and \( b_i \) for the vertices \( i \) only, not the total number of stations \( N_S \); by definition, the other stations invariably host only one line (e.g., \( \ell=1 \)) and have two connections (i.e. \( b=2 \)). For this work, we did not need to collect matrices \( \ell_i \) and \( b_i \). The information used is the line attribute distribution tables \( L \); for Toronto, \( L \) was already shown in Table 3.1.

### 3.4 Edges

The vertices are connected with one another by edges to compose a graph. For metro networks, edges \( E \) are non-directional links; again here because of most lines have two-way traffic. In graph theory, they are typically defined as connecting vertices ‘\( i \)’ with vertices ‘\( j \)’. As a result, it is possible to build a \( V \times V \) matrix of edges \( e_{ij} \).

We also define two types: single and multiple edges, to account for the overlapping line specificity of transit networks.
Single edges $E^s$ are the typical rail tracks with no overlapping. For instance, on Figure 3.1 (b), the Scarborough line (line 3) has a single edge connecting vertex 4 to vertex 6 (i.e. $e_{46}=1$).

Multiple edges $E^m$ are defined as follows. If two consecutive vertices are linked by two edges, then we consider there is one single edge and one multiple edge. For instance in Toronto, the edge between vertices 10 and 1 is single; however, there are two edges connecting vertices 1 to 2, one is considered single and the other multiple (the order is arbitrary). The introduction of multiple edges accounts for the overlapping property of certain transit lines. Nevertheless, since we are not dealing with all stations, but transfer-stations and termini only, it may not be obvious from the graph whether one edge is multiple or not. For the Toronto example, vertices 2 and 3 are connected by two edges; however here, although the vertices are consecutive, the stations are not, which is why we do not define one of these two edges as multiple.

It is therefore possible to compute a $V \times V$ matrix of edges $\{e_{ij}\}$, where $e_{ij}$ equals 1 for single edges and is greater than one for multiple edges. Table 3.3 shows this matrix for the Toronto network; it also shows the number of single edges and multiple edges. Additionally, it can be seen that the table was adjusted to account for the fact two consecutive edges were not considered multiple between vertex 2 and 3.

<table>
<thead>
<tr>
<th>${e_{ij}}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total edges $E$</th>
<th>Single edges $E^s$</th>
<th>Multiple edges $E^m$</th>
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<tbody>
<tr>
<td>1</td>
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<td>2</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>4</td>
<td>3</td>
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<td>4</td>
<td>3*</td>
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<td>4</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>22</td>
<td>20</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

* adjusted for non-multiple edges
Mathematically, the total number of edges $E$ is equal to the sum of the number of single edges $E^s$ and the number of multiple edges $E^m$.

$$E = E^s + E^m$$  \hspace{1cm} (3.5)

where,

$$E = \frac{1}{2} \sum_{ij} e_{ij}$$  \hspace{1cm} (3.6)

$$E^s = \frac{1}{2} \left( \sum_{ij} \frac{e_{ij}}{e_{ij}} \right), \text{ for all } e_{ij} \neq 0$$  \hspace{1cm} (3.7)

$$E^m = \frac{1}{2} \left( \sum_{ij} \left( e_{ij} - \frac{e_{ij}}{e_{ij}} \right) \right), \text{ for all } e_{ij} \neq 0$$  \hspace{1cm} (3.8)

In equations (3.6), (3.7), and (3.8), $e_{ij}$ is the number of edges linking vertex ‘$i$’ to ‘$j$’ (i.e., 0, 1, 2 edges, etc.). In these equations, edges are also counted twice, hence the need to divide the sum by two (i.e. Figure 3.1 (b), $e_{12} = 2$ and $e_{21} = 2$ though there are the same).

Equations (3.7) and (3.8) are atypical in the way that we divided one item by itself; this is to make sure that we add only ‘one’ edge when considering a possible link between two vertices. The downside is that most of the matrix is filled with zeroes (Table 3.3), and this can complicate the computing task. Also, note that equation (3.8) basically calculates the remaining number of edges that were not included in $E^s$. Finally, the fact these equations need to be adjusted for non-multiple edges further hinders an easy application. It should be noted that since we collected the data manually, we did not produce this matrix and use these equations for the 33 networks, but counted the single and multiple edges directly; this procedure was nevertheless included for future users who may prefer to collect the data computationally.

### 3.5 Maximum number of transfers

Another important measure that is invariably collected for this work is the maximum number of transfers $\delta$ to go from one vertex to another by taking the shortest path; it is commonly referred
to as the \textit{network diameter} in graph theory. The \textit{network diameter} is the total number of edges (or degrees) linking the two “furthest” vertices, using the shortest path. However, metro networks have lines. In Figure 3.1 (b), the \textit{network diameter} is 5, e.g., to go from vertex 10 to 6 ($e_{101}$ to $e_{12}$ to $e_{23}$ to $e_{34}$ to $e_{46}$), but four of these edges are part of the same metro line and no transfers are needed. In fact, the maximum number of transfers is only 3, to go from vertex 4 to vertex 7. Note that we also do not count the number of degrees (i.e. edges) to go from one vertex to another, but the number of passenger transfers. Essentially, $\delta$ is the smallest maximum number of transfers to link any two lines.

The easiest method to find $\delta$ is to represent the graph in C-space as seen in chapter 2. Essentially, a new graph is built, where the lines are the vertices and the transfer stations are the edges. Figure 3.3 shows an example for the Toronto metro network.

\begin{center}
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{c-space_representation_toronto_metro.png}
\caption{C-space representation of the Toronto metro.}
\end{figure}
\end{center}

The Sheppard line (line 4) is connected to the Yonge-University-Spadina line (line 1) at vertex 5. The Yonge-University-Spadina line (line 1) is connected to the Bloor line (line 2) at vertices 1, 2, and 3. And finally, the Bloor line (line 2) is connected to the Scarborough RT line (line 3) at vertex 4. At this point, a shortest-path algorithm (e.g., Dijkstra) can be used to find the maximum number of transfers $\delta$; metro networks are typically small in size, consequently, a simple visual inspection may be enough. In the case of Toronto, it is clear that the value of $\delta$ is three. Note that Dong and Chen label graphs in C-representation as \textit{transit line network} models.
3.6 Summary and Procedure

In this section, we summarize the information that was presented above. More importantly, we provide a detailed, step-by-step, procedure to apply the methodology to other networks in the world.

The first step to use this methodology is to collect basic functional information on the metro studied. In this work, we particularly used the book *Transit Maps of the World* by Ovenden (Ovenden 2007); we also visited each agency website of the 33 metro networks. First, the following indicators are collected:

- Route length of network $R$
- Number of Lines $N_L$
- Number of Stations $N_S$

Afterwards, each metro is redrawn as a graph with transfer stations and termini being the vertices and edges connecting these vertices. Although we performed this step manually, computer software may be available for this purpose. From these graphs, we collected the following information:

- Total number of vertices $V$
- Number of transfer vertices $V^t$
- Number of end vertices $V^e$
- Total number of edges $E$
- Number of single edges $E^s$
- Number of multiple edges $E^m$

Then, either by using a shortest-path algorithm (from $\{e_{ij}\}$ or from the graph directly using a computer software), by redrawing the network in a $C$-space representation, or even by simple visual inspection of the graph, it is possible to collect:

- Maximum number of transfers $\delta$

Finally, all line attributes $\ell$ for each vertex is first collected; the remaining stations, part of $N_S$, invariably host only one station by definition (i.e. $\ell=1$). As a result, it is possible to compute:
- Frequency distribution table $L$ of the line attribute

A summary of all the measures collected and computed can be found in Table 3.4. The first column contains the measures; the second column contains the symbols defined for each measure; the third column contains the values of the measure for the specific case of Toronto; and the fourth column contains the images that were used on Figure 3.1 (b).

**Table 3.4** Summary of data collected for each network, with symbol and graphical image used and values for the Toronto metro; note that the colours are not essential here; we simply kept the original colours from Figure 3.1 (a) to facilitate understanding of the methodology.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Symbol</th>
<th>Size or Value</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route Length (km)</td>
<td>$R$</td>
<td>68.75</td>
<td></td>
</tr>
<tr>
<td>Lines</td>
<td>$N_L$</td>
<td>4</td>
<td><img src="image" alt="Lines" /></td>
</tr>
<tr>
<td>Stations</td>
<td>$N_S$</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>Vertices</td>
<td>$V$</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>end</td>
<td>$V^e$</td>
<td>5</td>
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</tr>
<tr>
<td>transfer</td>
<td>$V^t$</td>
<td>5</td>
<td><img src="image" alt="transfer" /></td>
</tr>
<tr>
<td>Edges</td>
<td>$E$</td>
<td>11</td>
<td><img src="image" alt="Edges" /></td>
</tr>
<tr>
<td>single</td>
<td>$E^s$</td>
<td>10</td>
<td><img src="image" alt="single" /></td>
</tr>
<tr>
<td>multiple</td>
<td>$E^m$</td>
<td>1</td>
<td><img src="image" alt="multiple" /></td>
</tr>
<tr>
<td>Max number of transfers</td>
<td>$\delta$</td>
<td>3</td>
<td><img src="image" alt="Max number of transfers" /></td>
</tr>
<tr>
<td>Line attribute frequency table</td>
<td>$L$</td>
<td>See Table 3.1</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4
Characterising Metro Networks

4.1 Introduction

Metro transit systems are composed of stations all linked by tracks; they are in essence physical networks. The study of networks is emerging as an area of fundamental focus in the 21st century as mentioned in chapter 1 and 2. Whether it is the human body, the World Wide Web or even the stock market, all can be studied using analytical tools dedicated to networks. Metro networks, although being topologically simpler than many other networks, present some specific challenges, notably due to the existence of lines, as well as overlapping between the lines.

While traditional transit planning methods consider such characteristics as demography, geography, demand, cost and others, none seem to address the network design in a direct manner, which becomes increasingly important as systems grow in size and popularity, notably to move people in a city.

The analysis presented in this chapter studies networks from a holistic perspective and can prove to be particularly useful at the strategic planning phase; however, it does not consider the operations (e.g., local vs. express lines, capacity, etc.) or quality of service of the systems, but focuses solely on the design on the network (e.g., number of lines, transfers, etc.) as discussed in chapter 1. The purpose is not to comprehensively plan a new metro system (e.g., find an optimal corridor), but to reveal new and substantial information on the network aspect of transit systems that can supplement more traditional planning tools.

This analysis also offers a means to compare systems in the world, as well as help planners in their decision-making and target-setting processes. Developing characteristics that can be applied to all metro networks can prove to be particularly useful for planners and engineers. Not only is it possible to evaluate possible design scenarios, it is also possible to emulate specific characteristics found in other exemplary systems. For instance, if a city aspires to have a metro akin to the London system, it can purposefully design a metro to contain the same characteristics as the London system, whilst addressing other fundamental objectives (e.g., location of transit in a specific corridor, etc). Moreover, results from this analysis can be relevant to inform transportation professionals about the current status of a metro and identify which aspects of the
design need to be improved. Overall, the broad goal of this analysis is to create a systematic method to establish the characteristics of metro networks.

For this research, we use principles of graph theory to define three particular characteristics of transit networks: *State*, *Form*, and *Structure*. *State* describes the development of networks, whether they are relatively simple or have grown to be more complex (with respect to the indicators studied). *Form* describes the spatial relationship between transit networks and the built environment, looking at network design and identifying the type of uses (regional or local). *Structure* describes networks through two indicators: connectivity and directness; connectivity refers to the influence and importance of transfer stations in the network; directness determines the ease to travel within a network so as to avoid multiple transfers that can be inconvenient to passengers. All three characteristics are particularly relevant to transit network design, planning, and also to acquire a more comprehensive understanding of networks.

More specifically, the objectives of this chapter are to:

- Adapt graph theory indicators according to transit specificities to acquire an enhanced understanding of the three network characteristics: *State*, *Form* and *Structure*
- Characterize transit systems according to their network properties, which enables a clearer understanding of the present state and highlights potential growth scenarios

The methodology to translate a metro into a graph, as introduced in chapter 3, is used extensively in this chapter. Each graph $G$ is therefore composed of $V$ vertices and $E$ edges; $G = \{V, E\}$.

Moreover, each characteristic uses two specific indicators. Several of these indicators were already presented in chapter 2 (literature review). New indicators are also developed; they will be explained comprehensively.

By studying 33 metro systems in the world, over six major world areas (North and Latin America, Eastern and Western Europe, Africa and Asia), we notably account for different cultures and city specificities. Additionally, by including networks of all sizes (from 2 lines to 14 lines) we are also able to get a sense of the patterns of development of metro systems.
The rest of this chapter proceeds as follows. First, emphasis is put into explaining the three characteristics and applying them to the 33 networks presented in chapter 1. Second, it is possible to depict and characterise metro systems by plotting two-dimensional graphs with respect to the characteristics’ indicators. Finally, the three characteristics are integrated into one diagram to better examine the relationship between them and identify paths of development.

This chapter is based on the article “Characterizing Metro Networks: State, Form, and Structure” by Derrible and Kennedy (2010b).

4.2 Network Characteristics

In this section, the three network characteristics, State, Form, and Structure, are defined sequentially. Each characteristic is composed of two network indicators. All previously introduced indicators will be reviewed in this section and new indicators will be explained comprehensively.

Moreover, it should be mentioned that these three characteristics were developed to identify relevant patterns of metro networks. Although it is possible to think of other characteristics, the scope of the State, Form, and Structure characteristics is fairly large, which enables a relatively comprehensive view of metro networks.

4.2.1 State

State refers to the current development phase of a network, whether it is a relatively simple network or a more complex one in terms of topology (with respect to the indicators studied); naturally, it is strongly related to the size of the network, and yet relevant patterns can be identified, notably in the development of metro systems when building new stations and lines.

The first step is to identify the number of different possible phases in the State of a metro. To do this, we use two network indicators that were developed by Garrison and Marble (1962) and introduced in chapter 2; the first one relates to complexity, it is the $\beta$-index; the second indicator was referred to as degree of connectivity, it is the $\gamma$-index.
4.2.1.1 Complexity $\beta$

Despite its name, the $\beta$ indicator is fairly simple; it was first introduced in equation (2.6); it expresses the ratio of edges to vertices:

$$\beta = \frac{E}{V}$$  \hspace{1cm} (4.1)

where $E$ is the number of edges and $V$ is the number of vertices in a graph $G=\{V,E\}$.

From $\beta$, we are able to extract some significant properties by looking at how the ratio evolves as networks grow in size, i.e. how many edges are created when introducing new vertices. Predicting the behaviour of the number edges as we increase the number of vertices (also described as the average number of edges per vertex) is not a trivial exercise. In fact, it depends greatly on the type of network (e.g., road, rail, air, electric, social, etc); there are no systematic methods to determine it.

4.2.1.2 Degree of connectivity $\gamma$

The degree of connectivity $\gamma$ indicator calculates the degree of connectivity as opposed to what we call structural connectivity (section on Structure later on). It describes how much a network is connected relative to how much it could be connected. Therefore, $\gamma$ calculates the ratio between the actual number of edges to the potential number of edges; that is if the network is 100% connected. In a multi-dimensional graph, the potential number of edges is calculated as $\frac{1}{2}V(V-1)$; it simply calculates all the possible links between the vertices, and then halves the result not to count the potential edges twice since we are dealing with undirected graphs. Nevertheless, metro networks are almost always planar. This means that two edges crossing each other will automatically create a new vertex (with a few rare exceptions, e.g., London); this is understandable as creating transfer stations offer more transfer possibilities to transit users, which is favourable.
Therefore, for planar networks with $V \geq 3$, the potential number of edges becomes $3V-6$. As a result, $\gamma$ becomes:

$$
\gamma = \frac{E}{E_{\text{max}}} = \frac{E}{3V-6}
$$

(4.2)

where $E$ is the actual number of edges, $E_{\text{max}}$ is the potential number of edges and $V$ is the number of vertices in a graph $G=\{V,E\}$.

This variant of the degree of connectivity was first introduced in equation (2.5).

4.2.2 Form

Form illustrates how networks are integrated in the built environment. Metro networks can play a different role depending on how they were planned. Some networks act at the regional level, by bringing people from residential areas to employment areas (e.g., suburbs to CBD), while others tend to focus on servicing the people living in one specific location, so most trips are made via the metro. To take another perspective, Form can be related to the strategy that was envisioned when the network was conceptually created. It is particularly related to Level I planning as introduced in chapter 1.

In this section, we look at three typical metro network indicators: number of lines $N_L$, total number of stations $N_S$ and total route length $R$; these measures are usually the first to be mentioned when analysing transit networks. They can be rather indicative of the integration of a metro within the urban environment. The goal is to identify whether a network is regionally or locally focused.

4.2.2.1 Average Line Length $A$

The first indicator for the Form characteristic is the average line length $A$, which is defined as the ratio between the total route length of network $R$ and the total number of lines $N_L$. 
Typically, longer lengths imply the lines are trying to reach further out in the suburbs, therefore representing a regionally-oriented network. On the contrary, shorter lines serve a reduced area, hence a tendency towards locally-oriented network. Although issues of jurisdictions may exist, it is clear that lines reaching further away from the city centre attract potential users living in the suburban areas.

4.2.2.2 Number of Stations $N_S$

The second indicator relevant for the Form characteristic is simply the number of stations $N_S$. Indeed, a network with small average line length and many stations will tend to increase local coverage, compared to a network with long average line length and few stations. In particular, a metro may have few stations with long average line length to offer a faster service to suburban trips (i.e. regionally-oriented network).

To some respect, this concept can be analysed by looking at the inter-station spacing $S$; it is only briefly discussed here. The inter-station spacing is directly related to the number of inter-station spacings $N_{iss}$ as defined in section 2.4.2, which was not collected; this number of inter-station spacings $N_{iss}$ is notably related to the number of stations and the total number of cycles in the network. Here, as an approximation, we simply define the inter-station spacing as the ratio of total route length of network $R$ and the number of stations $N_S$.

$$S = \frac{R}{N_S}$$

Although this definition of inter-station spacing is not exact, it offers a fairly indicative approximation; errors generated are insignificant for this analysis.

The interpretation of the inter-station spacing may be limited since it says very little on the expansion level of the network; nevertheless $S$ adds a level of detail. Although, there is one well-
known trade-off related to inter-station spacing (shorter spacings usually imply lower speeds, hence longer travel time), values calculated may be informative for the *Form* characteristic.

### 4.2.3 Structure

For the *Structure* characteristic, two new indicators of network are developed: connectivity and directness. They are explained comprehensively in this section.

#### 4.2.3.1 Structural Connectivity $\rho$

For the *Structure* characteristic, we talk about structural connectivity as opposed to degree of connectivity as presented in section 4.2.1.2. In other words, the $\rho$ indicator measures the influence and importance of connections (i.e. transfers) in the system. But first, we have developed a new indicator that is central to this research; it is called the number of transfer possibilities $V'_c$ and is defined as:

$$V'_c = \sum_{i} (\ell - 1) \cdot v_{i,\ell}$$

(4.5)

where $v_{i,\ell}$ is a vertex (numerically, each vertex has a value of 1) with identification number ‘$i$’ and line attribute ‘$\ell$’ as introduced in chapter 3; here, the subscript ‘c’ stands for ‘connectivity’.

Essentially, $V'_c$ counts all the transfer possibilities in a network; it is the sum of the number of lines at a vertex minus one. For instance, from Figure 3.1 (b), if traveling along the Bloor line (line 2), starting from vertex 10 and arriving at vertex 1, there is one transfer possibility. In total, the Toronto metro has a total $V'_c$ of 5; possibilities are present at vertices 1, 2, 3, 4, and 5.

Nevertheless, vertices 1 and 2 offer the exact same transfer possibility, which falsely overestimates the number of transfer possibilities. To address this problem, the number of multiple edges $E^m$ is subtracted from the number of transfer possibilities.
Moreover, larger networks tend to have more transfer stations, thus a larger $V'_c$. As a means to standardize this indicator, the number of transfer vertices is incorporated. Considering these issues, the connectivity indicator $\rho$ is defined as:

$$\rho = \frac{V'_c - E^m}{V'}$$

(4.6)

where $E^m$ is the number of multiple edges; not to double count possibilities (e.g., vertices 1 and 2 on Figure 3.1 (b)); and $V'$ is the total number of transfer vertices.

In equation (4.6), the numerator calculates the total number of net transfer possibilities; the net value was preferred to avoid false information due to the overlapping line property of metro networks (e.g., northern part of the Circle line for the London Underground). The denominator is used as a means to standardize the indicator; in practise, it actually provides information about the structure of the network itself, which in turn makes the indicator independent of network size; this is a rather valuable property. Another advantage of this indicator is that it emphasizes “hubs” (i.e. here defined as transfer-stations hosting more than two lines). Indeed, adding one edge between two new vertices will not improve connectivity since the two new transfer-possibilities at the numerator will be offset by the two new transfer-vertices at the denominator. It therefore becomes clearer that $\rho$ effectively measures a rather structurally focused connectivity.

To better illustrate this indicator, Figure 4.1 shows examples of networks with their respective connectivity indicator values. The first network on the left is a centralised network, which produces high connectivity. The third network on the right is a grid network; even though more transfer stations are present, it is not as “convenient” to transfer, hence a lower connectivity. The second network in the middle can represent a real-life network, having three transfer stations; it also contains a circle line. Each network on this figure has four lines of identical lengths (i.e. same average line length). Overall, the $\rho$ indicator favours centralised systems since they offer more transfer possibilities at only a few stations (i.e. presence of “hubs”).
Figure 4.1 Examples of networks to illustrate the connectivity indicator $\rho$. Each network has four lines of identical length. The first network on the left is centralised, which produces a high connectivity. The third network on the right is a simple grid network and more transfers are required on average, hence lower connectivity. The second network in the middle can be seen as a real-life network. Note that none of these systems were given multiple edges.

4.2.3.2 Directness $\tau$

The second indicator for the Structure characteristic is directness $\tau$. Directness is certainly the hardest indicator to conceptualize. In a world where transit has to be as attractive as possible, avoiding transfers is crucial. The concept of directness is also relatively specific to transit networks; i.e. a similar indicator could be used for road networks, but switching roads using an automobile does not carry the same impact to the user as switching transit lines.

A measure of directness obviously has to be related to the maximum number of transfers $\delta$. However, $\delta$ cannot be considered as a raw measure since larger networks tend to have larger $\delta$ but still perform adequately given their structure. We therefore need to derive an appropriate expression.

We first saw a parallel between directness and the indicator $\pi$ introduced by Kansky (1963) and introduced in equation (2.8) in chapter 2. He explains $\pi$ as being “a number expressing the relationship between the circumference of a circle and its diameter … let us assume that the total mileage of a transportation system is analogous to the circumference of a circle and the total
mileage of all edges of the diameter of a network is analogous to the diameter of the circle”. Here, \( \pi \) does not deal with line type (circle, radial, tangential, etc); it measures the ratio between the total route length and the length of the network diameter.

For transit, the total route length \( R \) is used as the numerator. For the denominator, the length of the network diameter is concomitant with the maximum number of transfers \( \delta \) since it follows the “shortest” route that requires the highest number of transfers. Therefore, we can assume that the “longest” route is equal to \( \delta \) times a given length. This given length can be taken as a portion \( \kappa \) of the average line length \( A = R/N_L \). The denominator therefore is \( \kappa \cdot R/N_L \cdot \delta \), and the directness indicator becomes:

\[
\tau = \frac{R}{\kappa \cdot \frac{R}{N_L} \cdot \delta} = \frac{N_L}{\kappa \cdot \delta}
\] (4.7)

Equation (4.7) shows how directness simplifies into a measure of number of lines and maximum number of transfers. The portion \( \kappa \) can be omitted; first because there are only arbitrary values for it (half, a third of a line, etc); second because it is a constant (i.e. it does not add information) and it simply gets absorbed by the slope of a line during a regression analysis. This measure of directness remains simple whilst accounting for network size (number of lines). Consequently, the directness indicator can be defined as:

\[
\tau = \frac{N_L}{\delta}
\] (4.8)

### 4.3 Results

The data collected and results calculated are shown in Table 4.1. By studying 33 metro systems in the world, ranging all sizes (from 2 to 14 lines), we can effectively scrutinize metro networks and draw some conclusions on their characteristics. Table 4.1 includes all the data used and the various indicators calculated. Data was mostly collected from each individual transit authority website; the book *Transit Maps of the World* by Ovenden (2007) was also useful for that purpose.
Table 4.1 Results containing the basic indicators and the characteristics calculated for the 33 networks studied.

<table>
<thead>
<tr>
<th>Metro Networks</th>
<th>Route Length $R$ (km)</th>
<th>Stations $N_S$</th>
<th>Lines $N_L$</th>
<th>Vertices $V$</th>
<th>End $V^e$</th>
<th>Transfer $V^t$</th>
<th>Hosting $x$ lines</th>
<th>Total Edges $E$</th>
<th>Single $E^s$</th>
<th>Multiple $E^m$</th>
<th>Max. number of transfers $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brussels</td>
<td>39.50</td>
<td>59</td>
<td>3</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>3 1 0 0 0 0</td>
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<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Washington DC</td>
<td>171.14</td>
<td>86</td>
<td>5</td>
<td>17</td>
<td>9</td>
<td>8</td>
<td>4 3 1 0 0 0</td>
<td>25</td>
<td>19</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Tokyo</td>
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<td>69</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>5 0 0 0 0 0</td>
<td>11</td>
<td>10</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Montreal</td>
<td>60.86</td>
<td>68</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>3 1 0 0 0 0</td>
<td>11</td>
<td>11</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Boston</td>
<td>102.56</td>
<td>117</td>
<td>5</td>
<td>29</td>
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<td>32</td>
<td>31</td>
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<td>3</td>
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<tr>
<td>Marseille</td>
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<td>24</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>2 0 0 0 0 0</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Delhi</td>
<td>68.00</td>
<td>59</td>
<td>3</td>
<td>8</td>
<td>6</td>
<td>2</td>
<td>2 0 0 0 0 0</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Singapore</td>
<td>89.40</td>
<td>64</td>
<td>4</td>
<td>12</td>
<td>6</td>
<td>6</td>
<td>6 0 0 0 0 0</td>
<td>13</td>
<td>12</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Cairo</td>
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<td>53</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>2 0 0 0 0 0</td>
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<td>1</td>
</tr>
<tr>
<td>Rome</td>
<td>34.94</td>
<td>47</td>
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<td>5</td>
<td>4</td>
<td>1</td>
<td>1 0 0 0 0 0</td>
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It should be noted that the New York City metro presents a particular problem due to the importance of branches within the system as discussed in section 3.2, although the value of nine was chosen for the number of lines (i.e. there are nine different coloured lines on the New York City metro map). This issue is dealt with separately in the following sections.

4.3.1 State

As introduced in equation (4.1), the measure $\beta$ calculates the ratio between the number of edges and the number of vertices. When performing a regression analysis between $E$ and $V$ (see Figure 4.2), we calculate a slope of 1.94 and an intercept of -10, a goodness-of-fit (adjusted $R^2$) of 0.99, with t-tests of 46.55 and -6.92 respectively, both being higher than the 95% confidence value. The value 1.94 of the slope is particularly interesting. It means that by adding one vertex to a network, almost two edges are created, thus close to a 1:2 relationship; this is not trivial.

![Figure 4.2](image_url) Relationship between edges $E$ and vertices $V$. 
First of all, the relationship found is linear and of the form:

\[ E = a \cdot V - b \]  \hspace{1cm} (4.9)

where \( a \) and \( b \) are constants.

To take a closer look at the ratio \( \beta \), equation (4.9) becomes:

\[ \beta = a - b / V \]  \hspace{1cm} (4.10)

It is obvious that as the number of vertices increases, \( \beta \) tends to the value \( a \). We find empirically, through the regression analysis, that the value of \( a \) is just under two. Figure 4.3 strengthens the important character of the value “two”.

![Figure 4.3 Relationship between \( \beta \) and \( V \).](image)

Theoretically, the value of \( a \) could differ from two. We recall that for planar graphs with \( V \geq 3 \), the maximum number of edges in a planar graph is: \( 3V - 6 \). If we therefore substitute \( E \) by \( 3V - 6 \) in equation (4.10), we get \( \beta_{\text{max}} = (3V - 6) / V \). Consequently, as the number of vertices increases, \( \beta \)
tends to the value three (i.e. \( a = 3 \)). Relating this phenomenon to the degree of connectivity \( \gamma \) (equation (4.2)), the actual to potential number of edges tend to \( \frac{2}{3} \), or 66% completely connected.

\[
\gamma = \frac{e}{3v-6} \to \frac{2}{3}
\]  

(4.11)

We therefore see that as networks grow, they seem to adopt a 1:2 relationship between their number of vertices and edges; this results in networks that are 66% completely connected. The reason for this phenomenon is unclear; nevertheless we can try to tentatively explain how edges are formed.

Taking the example of a small two-line network (e.g., Rome), the two lines are likely to cross only once, hence the number of edges equals the number of vertices minus one, a rough 1:1 relationship. This phenomenon slowly decreases as the number of edges and vertices increase (for small networks, the regression is influenced by the intercept). Taking the example of a larger network, there are three possible scenarios. Firstly, the end-vertices solely create one edge, a 1:1 relationship. Secondly, if a new line crosses an already existing vertex, then two edges are created, but not a vertex, a 0:2 relationship. Thirdly, if a new line crosses an already existing line and thus create a new transfer-vertex, then three edges are created (four edges minus one existing), a 1:3 relationship. A certain number of variations between these three scenarios are also possible. Furthermore, about half of these edges are actually created; since edges connect two vertices, they should not be counted twice. As a result, it is possible to see that edges can be created without creating vertices. Moreover by adding a line, new vertices do not only add edges to this new line, but also to pre-existing lines. This is why the 1:1 relationship rapidly evolves.

Overall, \textit{State} appears to approximately follow a three-phase process (Figure 4.4). The first phase consists of creating a network and developing it (up to \( \beta=1.3 \) and \( \gamma=0.5 \)). For instance the Prague Metro (Figure B.4) has a small number of vertices (9) and edges (9), hence \( \beta=1 \), and \( \gamma=0.43 \) since all lines intersect each other forming a triangular shape in the centre.

In this first phase, no patterns seem to emerge, most networks are scattered, and the relationship between \( \beta \) and \( \gamma \) is relatively weak.
Figure 4.4 State of Metro Networks.
Then, new vertices are added to the network, which gradually increases the number of edges to the second phase (up to $\beta=1.6$ and $\gamma=0.6$); also the relationship between $\beta$ and $\gamma$ seems to strengthen. Nevertheless, although metros in the second phase can already be fairly expansive such as the Moscow metro, they can still become more complex and connected, and reach the third phase.

Once the network is significantly expanded, we see the third and final phase (up to $\beta=1.96$ and $\gamma=0.7$) where the ratio of edges to vertices seems to stay constant (a bit lower than 2) and the degree of connectivity approximates 66% fully connected. The Madrid Metro offers an example (Figure B.5). The Madrid metro has 13 lines, 46 vertices and 82 edges, hence $\beta=1.78$. Its highly inter-connected lines render a degree of connectivity $\gamma$ of 0.62. It should be mentioned here that the Chicago metro has the highest $\beta$ and $\gamma$; however, this is due to the presence of the “loop” in the downtown that has many overlapping lines which are not accounted for by the $\beta$ and $\gamma$ indices.

4.3.2 Form

*Form* is assessed using three typical attributes of metro networks: route length $R$, number of lines $N_L$ and number of stations $N_S$. First, it is obvious that the three measures are inter-dependent (as can be shown by single and multiple regressions). Indeed, as networks grow, so do the number of lines, stations and route length. However, not all networks evolve in the same way. Figure 4.5 shows the relationship between the average line length ($A$) and number of stations ($N_S$).

The figure shows three zones. The first one is labelled “Regional Accessibility”. By increasing the average line length and the distance between the metro stations, some networks focus on connecting people living in the suburbs to the city core, acting more like a regional rail system; this therefore enhances regional accessibility by allowing the suburban population to use the metro. By accessibility, here, we are more interested in the presence of a metro at the regional level (whether it is accessible by foot, transit, Park & Ride, Kiss & Ride, etc.), rather than its walking accessibility by transit users. It should be noted that the term accessibility is widely used in the literature; for a short review, see Geurs and van Wee (2004).
Figure 4.5 Form of Metro Networks.
The second zone is labelled “Local Coverage”. By having many stations and keeping lines short, networks focus on servicing the city core; hence making the metro a prime travel mode downtown. As a result, most types of trips (e.g., work, commercial, recreational, etc) can be made using the metro as opposed to a few selected types of trips (e.g., work) for the “Regional Accessibility” zone.

The third zone is a mix of the other two and is labelled “Regional Coverage”. The emphasis is two fold; the city core is generally well serviced, while still connecting the surrounding regions. Essentially, the metro is expansive locally and covers a significant land area, whilst having long lines reaching the suburbs.

From Figure 4.5, it is possible to see that networks such as the Washington DC (which has five long lines reaching the suburbs, Figure B.6) and St Petersburg (Figure B.10) metros emphasize on connecting people, who live further away, to the city core; Stockholm also is notably known for connecting its satellite towns to the downtown. Meanwhile, other cities such as Paris and Barcelona have an extensive network in the city core; Paris for instance covers only 106 km$^2$ as mentioned in chapter 1 and has 14 lines compared to Toronto that covers about 600 km$^2$ and has 4 lines. Finally, metros seeking to be regionally accessible (long lines) whilst well covering the centre tend to have a dense network extending to the whole region as are Tokyo, Moscow and London (which has 13 long lines that mostly interconnect in the centre, hence providing high local coverage; Figure B.2). Also, note that New York City was omitted from Figure 4.5 due to its large number of branches (the average line length $A$ cannot be calculated accurately); however, we can safely assume that it is in the regional coverage zone.

There are no optimal systems here. Regional accessibility may be neglecting important areas of the city core. Local coverage may not link all inhabitants of the region; even if it has a regional rail service, transfers between modes can be burdensome. Moreover, regional coverage may hinder speed of service by making frequent stops. Last, it should be noted that design of metro systems is also concomitant to other existing transit technology (e.g., bus, streetcar, light rail, etc), which is why it is not possible at this stage to favour one of the three zones.

Finally, the average station spacing should also be considered; values are given in Table 4.1. A bar chart with the inter-station spacing $S$ of all metros is shown in Figure 4.6.
Figure 4.6 Bar chart of inter-station spacing $S$ for each network.
The average station spacing between all the networks was found to be 1.13km. We can automatically link it with the previous analysis. The Washington DC and St Petersburg metros have spacings of 1.99 and 1.93km respectively, again sustaining the fact they act as regional rail systems, while Paris’ and Barcelona’s are significantly smaller with 0.71 and 0.83km respectively.

On the other hand, the London (1.43km), Tokyo (1.45km) and Moscow (1.63km) networks have higher than average spacings accounting for the fact they extend into the suburbs whilst still servicing locally.

Again, the interpretation of the average inter-station spacing S is limited, since in many cities, S is shorter downtown to maximize coverage and longer in the suburbs, notably to offer a faster service and also to limit capital investment linked to the costs of building stations. Another trade-off exist between coverage and speed of service due to such factors as dwell times. This is, however, not discussed here; for more information, see (Vuchic 2005).

### 4.3.3 Structure

Structure is represented by two different indicators: structural connectivity $\rho$ and directness $\tau$. Figure 4.7 shows the relationship between these two indicators.

Connectivity $\rho$ was introduced in equation (4.6), being related to transfer possibilities. Table 4.1 shows the values of $\rho$ for each network. It seems the most connected network is Buenos Aires metro (Figure B.7), since it has only five lines, but manages to have two transfer stations with $\ell=3$ (i.e. “hubs”), hence $\rho=1.50$.

The least connected is the Brussels metro, with a value of 0.75 (Figure B.8). The Brussels metro has three lines and four transfer-vertices, and although it has one transfer vertex with $\ell=3$, it also has two multiple edges that impact its connectivity. Also of note, the Delhi metro also has three lines but only two transfer-vertices, and yet it appears to be more connected than Brussels, with $\rho=1$ since it does not have multiple edges.
Figure 4.7 Structure of Metro Networks.
This is due to the nature of the connectivity indicator and the configuration of the transfer-vertices; again, the presence of “hubs” is favourable for this indicator.

Directness $\tau$ was introduced in equation (4.8). Tokyo and London have the highest values ($\tau=6.50$ each) since they have a large number of lines but are able to sustain a small maximum number of transfers (Table 4.1). On the other hand, Singapore, Lyon and Toronto do quite poorly in terms of directness ($\tau=1.33$ each) given they do not have many lines but a large number of maximum transfers. As a matter of comparison, London has 13 lines and $\delta=2$ as opposed to Toronto that has four lines and $\delta=3$.

From Figure 4.7, which is a plot of connectivity against directness, it is possible to denote three zones; some networks are directness-oriented; others are connectivity-oriented; and finally others are, what we call, “integrated”, they couple both properties. It seems most directness-oriented networks are also regionally-oriented. The relationship is not as clear for connectivity-oriented networks. Nonetheless, it seems that the Buenos Aires metro is well connected but has poor directness, which can be spotted when looking at its metro map (Figure B.7); it has five lines, four well located transfer-stations, but a maximum number of transfers of two.

In comparison, Mexico City (Figure B.9) achieves high connectivity and directness; it is not surprising when considering the fact it has 11 metro lines that are dense and well connected while keeping a maximum number of transfers to a value of two as well.

Arguably, it may be preferable for metro networks to have an integrated structure since this offers both properties of directness and connectivity. This is also achievable for all sizes of metros. Yet, this decision must agree with the vision adopted by the municipality/region.

4.4 Characterizing Metro Networks

4.4.1 Network Characteristics, Line Type and Land-Use

At this point, we have been able to look at State, Form and Structure individually for all metro networks. We will now relate these characteristics to line types as defined in chapter 2 and identify possible land-use impacts.
From *State*, it seems that networks tend towards a 1:2 relationship between the number of vertices and edges. At the same time, the degree of connectivity seems to increase to a maximum of 66% completely connected. This is all the more interesting when we consider that Bon (1979) dedicated an entire paper to the γ ratio, but looking at road networks (highways) of 13 islands. He found that road networks tended to be 50% connected. This means that transit networks have more actual edges than road networks. This may be due to the fact that the road networks Bon looked at were covering entire islands instead of cities. Or it could also be because transfers on roads are not considered negative as with transfers in transit systems. It would also be interesting to study the importance of transferring in metro, and how much a role the design plays, by looking at the ratio of linked to unlinked trips (i.e. the number of transfers made in a network relative to ridership); however, this data is rarely available.

Our analysis of *Form* has identified three types of networks: regional accessibility, local coverage and regional coverage. A regional accessibility strategy will prefer long lines connecting people with major employment centres; these networks can also extend outside the city administrative boundaries, thus acting also as a regional rail system. The lines are most often radial and diametrical. On the contrary, local coverage strategies will keep lines relatively short and focus on providing a comprehensive level of service in a defined part of the city (e.g., Barcelona), hence the emergence of (semi)-circle and tangential lines (e.g., Paris). Finally, some systems couple both strategies to design a region-wide and extensive metro system (e.g., Seoul). This type of network also depends on the availability of other transit technologies: bus, bus rapid transit, light rail transit, and especially regional rail. The type of strategy can have a great impact on some city components such as land use. Enhancing regional accessibility favours the development of a few corridors (e.g., Toronto and North York Centre) that can strengthen the city core (Cervero 1998). However, having long lines reaching the suburbs can also enhance other negative effects such as urban sprawl, resulting in heavily used metro systems during peak hour only (Vuchic 2005). In contrast, local coverage tends to develop larger areas of a city; it concentrates population and employment density whilst not being contained in one corridor.

We have also analysed metro networks in relation with their *Structure*. Networks emphasising directness can have long radial and diametrical lines that are well connected in the downtown so as not to create many unnecessary transfers, thus keeping the travel times reasonably short (e.g., Washington DC). Directness-oriented networks can also have tangential and/or (semi)-circle...
lines to reduce the number of transfers. On the contrary, other networks can be well connected but do poorly in terms of directness. This is especially true when, despite the presence of “hubs”, networks do not avoid transfers. For instance, the Buenos Aires metro lines are of the radial and diametrical type but do not meet at a central point, which makes it difficult to go from one terminal to the terminal of another line. One systematic approach to increase directness is to plan circle (e.g., Moscow) or tangential (e.g., Mexico City) lines. Furthermore, connectivity and directness can be coupled effectively without impeding each other, hence integrated networks. In Figure 4.7, we see that networks achieving high connectivity and directness all have radial and/or diametrical lines that are inter-connected by (semi)-circle and/or tangential lines. In addition, achieving integrated networks does not depend on Form. Longer lines can still be connected to other long lines via tangential lines.

4.4.2 Network Characteristics and Development

Figure 4.8 displays all strategies taken from the 33 networks. First networks are divided according to their Form and then according to their Structure. Finally, we list networks according to their State.

We see that most relatively simple networks (first phase) belong to the regional accessibility zone. Simple networks normally have a small number of lines emphasising moving people to one particular area of the downtown (e.g., CBD, central station, etc). As they grow, they are likely to be integrated and/or move to the regional coverage zone. Nevertheless, three simple networks (Buenos Aires, Lyon and Lisbon) have managed to emphasize local coverage.

This can be due to the geographical features of the city; for instance, Lyon’s CBD is located by the Gare Part-Dieu but its major commercial and recreational sectors are located on the Presqu’ile between the Place des Terreaux and Bellecour. If the initial vision/strategy is kept, as they grow, these networks are likely to become integrated whilst remaining in the local coverage zone.

Only Washington DC belongs to the second phase but still is part of the regional accessibility zone; for future expansions, it may be preferable to include tangential and/or (semi)-circle lines (i.e. build new lines, create many new stations and keep the same maximum number of
transfers), which would increase local coverage and hence possibly move it to the regional coverage zone either as integrated or directness-oriented. Here again, any decision must agree with the vision set by the region.

Figure 4.8 Characteristics of Metro Networks studied.

Some more mature networks (in the second/third phase of State) are in the local coverage zone; they all have an integrated structure. This is almost automatic. Indeed, as networks grow in a
restricted area, lines are likely to inter-connect more often; this normally enhances connectivity (by creating and reinforcing stations hosting more than two lines) and keeps the number of transfers to a minimum relative to number of lines (directness). Future development is likely to remain limited unless they take another approach and extend their lines into the suburbs to become more regional-coverage oriented.

Finally, other mature networks are in the regional coverage zone. Those focusing on either directness or connectivity already have extensive lines. New lines are likely to be tangential and/or (semi)-circle, which would increase connectivity and result in an integrated structure. Networks that are integrated and part of the regional coverage zone can only remain in this zone no matter how much more they grow; these networks are also likely to develop other transit technologies, potentially to act as “feeder” to the metro, or to serve corridors with lower-demand.

4.5 Conclusion

For this chapter, we studied 33 metro networks located throughout the world. The main objective was to adapt graph theory to transit-specific applications and use this method to characterize networks. We used the methodology introduced in chapter 3 as a way to translate transit networks into graphs, using the concepts of edges and vertices. We also used or developed a set of indicators to measure the characteristics: State, Form and Structure.

The State of a network is determined by the ratio of edges to vertices $\beta$ and its degree of connectivity $\gamma$. There are three existing phases in the formation of metro networks; networks in the third and final phase have a 1:2 relationship between vertices and edges and tend to be 66% completely connected.

Form was then analysed. We found that a trade-off sometimes exists between regional accessibility and local coverage. Networks emphasizing regional accessibility have longer lines reaching to the outer layers of cities, while local coverage is enhanced by shorter lines being present throughout the city core. A combination of the two resulted in a regional coverage type network. There are no preferred practises on this matter; it is rather dependent on other transit technologies and strategies set by transit planning authorities.
The *Structure* of metros was then defined and analysed by using indicators of connectivity and directness. It appears that while some networks tend to be directness-oriented and others connectivity-oriented, networks can possess both properties and become what we called “integrated”. The key to this success may be held by the presence of (semi)-circle and/or tangential lines.

This study of networks can be a valuable component of transit network design and planning. It should be incorporated in future projects as it can add information along with demand-side variables such as the Origin-Destination pairs.

Overall, graph theory seems to grant the tools to effectively study metro networks, and it could be used to further exploit the field. In particular, we use the tools presented here in chapter 7 to evaluate the proposed Toronto plans for the next 15 and 25 years.

In the next chapter, we further use graph theory to investigate other properties of networks (i.e. scale-free and small-world networks), and learn whether metro networks share characteristics with other real-life networks.
Chapter 5
The Complexity of Metro Networks

5.1 Introduction

The planning of transportation systems is a multifaceted process that can prove to be extremely complex. One relevant feature of transportation systems is that they are physical networks. While many traditional approaches for analyzing transport networks exist, the new and emerging field of network science could be particularly valuable to help transportation planners and operators as discussed in chapters 1. Nevertheless, the link between network science and transportation is not clearly defined at the moment (chapter 2). One goal of this chapter is therefore to adapt concepts of network science and statistical mechanics to transportation. As transportation planners, we put effort into proposing research methods that can be applied first-hand to the transportation industry, which in turn can bring valuable insights to the field of network science.

Two properties of complex networks have emerged to be particularly relevant and were presented in chapter 2; these are scale-free patterns and small-worlds effects. Scale-free networks, as introduced by Barabási and Albert (1999), follow a power law distribution between the number of nodes and their number of connections; i.e. few nodes have many links and many nodes have few links. Small-worlds were introduced by Watts and Strogatz (1998); these networks have the particularity of being locally well connected while remaining close, with respect to degrees of separation, to all other parts of the network thanks to existence of a few supra-regional links. Following the finding of these two properties of complex networks, scholars all around the world have started to look at many types of networks, ranging from sociological (Albert and Barabási 2000; Newman et al. 2002; Kossinets and Watts 2006) and ecological networks (Sole and Montoya 2001; Dunne et al. 2002; Banavar et al. 2007), to physical (Rodríguez-Iturbe 1997; Sachtjen et al. 2000; Arenas et al. 2001), virtual networks (Albert et al. 1999; Tadic 2001) and many more; reviews can be found in (Albert and Barabási 2002; Newman 2003b; Newman et al. 2006).

Transportation networks, being real life examples of networks, are of particular interest. In this chapter, we study the 33 metro networks enlisted in chapter 1 and characterised in chapter 4. The methodology from chapter 3 is also used here.
This chapter offers a novel and pragmatic approach to look at transit networks, as well as locating transit systems in the realm of network science. Much can be learned from identifying the typology of transit networks. Indeed, applications can be broad, from transportation planning and network design to scheduling and evacuation planning. One analysis, which is invariably performed on networks, is that of robustness, which we provide here. Robustness is a fundamental indicator to consider for all networks, and transportation systems are no exception; applications to transit systems include (Angeloudis and Fisk 2006; Scott et al. 2006; Berche et al. 2009). In this chapter, robustness deals more specifically with alternative paths offered to transit users and likelihood of accidents/failures. Performing such an analysis is all the more important considering it fits in the broader context of resilience (i.e. how cities can respond to major disruptions), which is a relevant aspect of urban sustainability.

By studying 33 metro networks in the world, across a wide range of sizes (from two to 14 lines), the broad goal of this chapter is to effectively analyze the complexity of metro systems by taking a network science approach.

It is then possible to look at the effects of complexity on the robustness of metro networks. More specifically, the chapter attempts to answer these three questions.

- How complex are metro systems; i.e., are they scale-free networks and small-worlds?
- What is the impact of network size on complexity?
- What are the implications of complexity on the robustness of the systems?

At first, we briefly re-explain the concepts of scale-free and small-world networks (as first introduced in chapter 2), and more importantly we adapt them to metro systems. We also introduce a robustness indicator corresponding to the properties of transit systems. In the results and discussion section, we learn that most metros are indeed scale-free and small-world networks; however they show atypical behaviours, which have substantial impacts on their robustness.

This chapter is based on the article “The Complexity and Robustness of Metro Networks” by Derrible and Kennedy (2010c).
5.2 Properties of Metro Network

In this chapter, the notion of complexity is defined by the scale-free feature of networks as well as the small-world properties. Consequently, this section recalls and adapts these two concepts to metro systems. We also develop our own measure of robustness.

5.2.1 Scale-free networks

The scale-free property of some networks, as defined by Barabási and Albert (1999), is one of the most significant contributions to the field. In this section, we first briefly recall the concept, then adapt it to metro networks and finally present the statistical process that was undertaken to assess the potential scale-free nature of metros.

5.2.1.1 The concept of scale-free networks

Scale-free networks essentially have many nodes with few connections and few nodes with many connections. Consider a network with \( N \) nodes; it has a scale-free pattern if the probability density function \( f \) that a node \( n \) has \( b \) connections (as defined in chapter 3) follows a power law:

\[
f(b) \propto b^{-\varepsilon}
\]  

(5.1)

where the exponent \( \varepsilon \) is called the scaling factor.

In other words, when randomly sampling vertices, the distribution of connections is found to follow a power law.

As mentioned in chapter 2, scale-free networks tend to follow two general patterns (Barabási and Albert 1999):

3. Continuous expansion by the addition of new nodes
4. Preferential attachment of new nodes to existing nodes that are already well connected
The value of the scaling factor is of primary importance here. The influence of the scaling factor was shown in Figure 2.10 in chapter 2.

Briefly, a high $\varepsilon$ (e.g., 3 or 4) relates to an aggressive and sudden decay, thus a “thin-tail” distribution, which was characterised as an “egalitarian” network (Buchanan 2002) since most nodes have the same number of connections. Essentially, the vast majority of nodes have few connections and only a few selected nodes (if any) have more connections.

A small $\varepsilon$ (e.g., 1) implies a “fat-tail” distribution (slower decay); hence a relatively larger number of nodes have more connections, which was characterised as an “aristocratic” network (Buchanan 2002).

For metro networks, however, equation (4.1) cannot be applied in its current condition as we are about to see.

### 5.2.1.2 Applying the scale-free concept to metro networks

To investigate the potential scale-free property of metro networks, we look at all stations $N_S$ of a metro network as opposed to the vertices $V$ solely. In previous chapters, the vertices $V$ were defined such that only transfer stations and termini were studied. Since we are looking at the scale-free feature of networks, we are rather interested in the total distribution of transfers, hence the need to consider $N_S$.

Additionally, relative to existing network science studies, including those on transit systems (chapter 2), we decided to concentrate on the distribution of the line attribute $\ell$ rather than the number of connections $b$; this is an innovative approach. Indeed, the determining factor in transit is the availability to transfer lines rather than the physical number of connections. For instance, non-transfer, non-terminal stations have 2 connections (i.e. $b=2$), while termini have only 1 connection (i.e. $b=1$). A scale-free topology dictates that there should be more stations with 1 connection (so here termini) than stations with 2 connections (so here non-transfer, non-terminal stations), which clearly does not reflect transit network topologies. By using the line attribute $\ell$, termini and non-transfer, non-terminal stations therefore both have $\ell=1$, which better addresses transit network specificities.
As a result, for public transportation systems, we are more interested in the number of lines $\ell$ passing through a station $n_s$; therefore, the relationship in equation (4.1) becomes:

$$f(\ell) \propto \ell^{-\varepsilon} \quad (5.2)$$

In other words, if we take all stations $N_S$ of a system and identify those that host one line, two lines, etc, and if the frequency plot decays following a power law, then it is a scale-free network. This translates into having many stations hosting only one lines, and few stations hosting more than one line.

Considering the two basic rules for the creation of scale-free networks, it is relatively evident that metros obey both of them. By building new lines, new stations are constantly added to the network, and these new lines often intersect with existing lines in high-demand areas where previous stations already exist (e.g., CBD), hence the “preferential attachment”.

Metro systems have many more termini and non-transfer, non-terminal stations than transfer-stations (i.e. many more stations with $\ell=1$ than with $\ell>1$). Therefore, intuitively, the distribution could fit a power law (i.e. many stations host one line, and few host many lines). Nevertheless, the value of the exponent $\varepsilon$ can add important information. There are no predictable values for it, although large scale-free networks typically have a scaling factor in the range $2<\varepsilon<3$ (Barabási and Bonabeau 2003).

### 5.2.1.3 Statistical Analysis

It is also important to discuss the statistical analysis that was undertaken in this chapter. Due to the small number of data points (from two to six for each metro system since there is one data point per line attribute $\ell$), it is challenging to compute a statistically rigorous value for $\varepsilon$.

Goldstein et al. (2004) argues that the use of a maximum likelihood estimation (MLE) approach, combined with a Kolmogorov-Smirnov (KS) test is preferable to fit power-law distributions. In our case, the MLE method is not usable due to the small number of observations; early tests clearly showed odd results. A more typical approach, extensively used in the field, is to perform
an ordinary least-squared (OLS) regression to log-log values of the data points. The frequency function $f$ essentially takes the form:

$$
\ln(f(\ell)) = -\varepsilon \cdot \ln(\ell) + \ln(a)
$$

(5.3)

In equation (5.3), the scaling factor $\varepsilon$ is the slope of the equation, and the multiplying coefficient $a$ is the intercept; the parameter $a$ is not discussed in this analysis as it is highly dependent on the total number of stations rather than the topology of metro networks. The statistical significance of the goodness-of-fit (adjusted $R^2$) and the t-test values are then examined extensively to assess the fit of the regression.

From the literature, however, it seems this method can produce inaccurate results despite the significance of the statistical indicators (Newman 2005). We therefore decided to perform an additional test to validate whether the fitted models followed power law distributions. Essentially, we applied a $\chi^2$-test that is particularly suited to examine frequency distributions as it is the case here. This test is defined as:

$$
\chi^2 = \sum_i \frac{(\text{Expected}_i - \text{Observed}_i)^2}{\text{Expected}_i}
$$

(5.4)

where ‘Expected,’ is calculated values using the results from the regressions, and ‘Observed,’ is the data points collected; here, actual values are used, not the log-form.

Using this two-step process provides a means to rigorously assess the statistical significance of the calculated scaling factors.

5.2.2 Small-Worlds

The small-world property of some networks, as defined by Watts and Strogatz (1998), is another significant contribution to the field of network science as described in chapter 2. In this section as well, we first briefly recall the concept, then adapt it to metro networks. Unlike the previous section, no major statistical analysis is undertaken here.
5.2.2.1 The concept of small-world networks

Small-worlds refer to the common saying that we are all connected through six degrees of separation. Small-world networks were well mathematically defined by Watts and Strogatz (1998). They obey two basic rules:

1. High clustering
2. Small average shortest-path length

In rule 1, clustering refers to the number of connections between the vertices of a given neighbourhood. For instance, take an undirected graph $G=\{V,E\}$ and consider a vertex $v$, the neighbourhood $k_v$ of $v$ contains all the vertices that are connected to $v$; there are therefore as many neighbourhoods as there are vertices although they are not all distinct. The clustering coefficient $C_v$ then calculates the ratio number of existing edges within this neighbourhood $k_v$, say $e_v$, to the maximum number of edges that could exist, $e_v^{\text{max}}=k_v(k_v-1)/2$ (i.e. if all the vertices of the neighbourhood were connected with one another).

$$C_v = \frac{e_v}{e_v^{\text{max}}} = \frac{2 \cdot e_v}{k_v(k_v-1)}$$  \hspace{1cm} (5.5)

The overall clustering of a network is calculated as the average of all the clustering coefficients:

$$C = \frac{1}{V} \sum_i C_i$$  \hspace{1cm} (5.6)

This measure presents various problems for metro networks as we will see next.

In rule 2, the average shortest-path length is calculated as the average of all the maximum number of degrees of separation by taking the shortest path to connect each vertex. To follow the small-world property, the average shortest-path length must be below the log of the number of stations: $\ln N_S$.

5.2.2.2 Applying the small-world concept to metro networks

As opposed to the scale-free section, here, we apply this concept to the vertices $V$ only (not all stations $N_S$). For rule 1, the presence of non transfer non terminal stations offers no relevance
since they do not relate to clustering; an additional argument is presented below. For rule 2, the maximum number of transfers is more useful than the number of stations.

Considering the methodology to be applied, the measures presented in equations (5.5) and (5.6) presents two problems in the particular case of metro networks; some of which were already introduced previously in chapter 2.

First, it does not account for the planar property of metro networks (i.e., two edges crossing will create a new vertex). Indeed, the total number of potential edges is calculated as $k_v(k_v - 1)/2$, whereas for planar graphs the number of potential edges is $3k_v - 6$ (for graphs having more than three vertices) as we have seen many times previously.

More critically, as we are dealing with small graphs, only a few neighbourhoods exist; as a result, transfer stations hosting many lines may be part of many neighbourhoods. For instance from Figure 3.1 (b), we see that vertex 3 is part of three neighbourhoods out of a total of ten neighbourhoods. Moreover, termini have only one connection; therefore the neighbourhood is composed of only one station, thus $C_v = 0/0$, which presents obvious problems. Overall, the main purpose of the clustering coefficient as defined above is to appreciate how vertices cluster together (e.g., in social networks), and this measure is mainly applied to larger networks ($1,000+$ vertices, and $E >> V$). For transportation systems, we are rather interested in calculating the level of development; i.e. how much metros are developed compared to their potential development. For a similar size, more developed metros tend to have shorter trip distances, i.e. more clustering, hence small-world in this context.

As a result, we prefer to use the degree of connectivity (the $\gamma$-index) introduced to transportation by Garrison and Marble (1962), and already used fairly extensively in this work. Mathematically, it is essentially identical to the clustering coefficient defined above (i.e. by calculating the ratio of actual to potential number of edges), except that it is applied to the entire network as opposed to neighbourhoods within the network. The conceptual framework established by Watts and Strogatz is therefore kept intact.

By accounting for the planar property, i.e., for graphs $G$ with $V \geq 3$ and $E$ edges, the maximum number of edges is $E_{\text{max}} = (3 \cdot V - 6)$, the degree of connectivity $\gamma$ is defined as:
\[ \gamma = \frac{E}{E_{\text{max}}} = \frac{E}{3 \cdot V - 6} \] (5.7)

A high \( \gamma \) relates to a higher clustering. It is impossible, however, to determine the threshold value for \( \gamma \) to become a small-world. In the literature, studied networks are compared with random networks, but again for transit networks, since there are relatively few edges, a random network with similar characteristics invariably ends up having segregated stations (i.e. not connected to the rest of the network), which hinders this analysis. In this work, values for \( \gamma \) are therefore simply compared to other existing small-world networks.

Finally, in rule 2, small-world networks have average shortest-path lengths that are smaller than the log of the number of vertices: \( \ln N_S \); as discussed in section 2.5.2. Naturally, public transportation networks have to be dealt with differently due to the presence of lines. As a result, we will use the maximum number of transfers \( \delta \) and verify that \( \delta \leq \ln N_S \).

### 5.2.3 Robustness

The concept of robustness is of fundamental importance when studying networks. For instance, the robustness of metro systems impacts their ability to offer alternative routes during the occurrence of failures, accidents or even targeted attacks. It is defined in this work as an indirect effect of network design topology.

In this section, we first briefly describe an existing robustness indicator that is used in several network analyses and find that it is not applicable to metro networks. Consequently, we develop a new indicator related to the cyclomatic number as seen in chapter 2.

#### 5.2.3.1 Existing robustness indicators

In the network science literature, one robustness indicator is to the concept of assortativity that was introduced by Newman (2002; 2003a). It basically measures how much nodes with connection indices \( b \) are linked to other nodes with similar connection indices \( b \); where \( b \) is the number of edges per vertex as described in section 3.2. The procedure to compute the
assortativity of a network is as follows. Convert the matrices \( \{ e_{ij} \} \) and \( \{ b_i \} \) (introduced in chapter 3) into a matrix \( \{ f_{ij} \} \) of connection index so that each cell is a fraction of the sum such that:

\[
\sum_j f_{ij} = 1, 
\]

\[
\sum_j f_{ij} = a_i, 
\]

\[
\sum_i f_{ij} = c_j 
\]

where \( a_i \) are the row sums and \( c_j \) are the column sums.

The matrix \( \{ f_{ij} \} \) therefore has as many rows and columns as there are discrete values of \( b \). The assortativity coefficient \( r \) is calculated as:

\[
r = \frac{\sum f_{ii} - \sum a_i c_i}{1 - \sum a_i c_i} 
\]

The value of \( r \) ranges from -1 (completely disassortative) to 1 (completely assortative). In an assortative network, nodes are linked to other nodes having a similar connection index; in equation (5.11), note that it is the diagonal values \( f_{ii} \) of the matrix \( \{ f_{ij} \} \) that are being used (i.e. the nodes that have similar connection indices). For a practical example, see (Newman 2003a).

This type of network is more robust since taking away a highly connected node will leave the path length relatively intact. Here again, however, this concept cannot be applied to transit networks.

In metro networks, most stations \( n_s \) are non-transfer, non-terminal stations (i.e. most stations along a line have 2 connections and are connected to other stations having 2 connections), which means that most metro networks are assortative regardless of network size. In fact, smaller metro networks often are more assortative because they have fewer possible connection indices (i.e. their transfer stations offer one or two transfers compared to larger networks that can have up to six transfers). Whereas in reality, larger metros tend to be more reliable since they offer more alternatives to get from point A to point B; they therefore tend to be more robust. While a
measure of robustness of metro networks needs to reflect this property, the concept of assortativity clearly does not. Even if only the vertices $V$ are considered (i.e. transfer stations and termini), the same effect occurs since larger metros have many transfer stations with different connection indices. Consequently, a different indicator is needed.

5.2.3.2 Developing a new robustness indicator for metro networks

Instead of looking at stations with similar connection indices, we are rather interested in calculating the total number of paths available or alternative routes. One way to do so is by using the cyclomatic number $\mu$ as defined in chapter 2. The cyclomatic number $\mu$ calculates the number of cycles (i.e. circuits or loops) in a graph by subtracting the number of edges in a tree, $V-1$, from the number of edges $E$.

$$\mu = E - V + 1 = \frac{1}{2} \sum_{i} e_{ij} - \sum_{i} v_{i} + 1$$  (5.12)

Note that we are using the vertices $V$ and not total stations $N_S$ since there cannot be more cycles by using $N_S$ by definition.

Nevertheless, we still need to calculate the ‘net’ number of cycles by subtracting the total number of multiple edges $E^m$. Indeed, on Figure 3.1 (b), transferring at vertex 1 or 2 is identical; i.e. the extra edge does not add an alternative path if either vertices is temporarily compromised.

In addition, the probability of failures/accidents should be accounted for, which can be related to the size of the network. Therefore, our robustness indicator $r_T$ is:

$$r_T = \frac{\mu - E^m}{N_S}$$  (5.13)

where ‘T’ stands for “transit”.

In equation (5.13), the number of stations $N_S$ was used as a “propensity to fail” property; i.e. a higher number of stations can induce a higher probability of random failure. It should be noted that ideally we would have taken into account track crossovers, which are of paramount
importance for transit robustness. In reality, when a failure occurs, only line segments where trains can change directions are kept in operation. Therefore, even if one failure affects only one station, an entire segment is most often disrupted. Moreover, other indicators such as the operating budget allocated to maintenance of vehicles, tracks and stations (i.e. to avoid failures) could offer a more realistic approximation of robustness. Nevertheless, this information was not available, and more importantly it does not lie within the network science context of this chapter.

At this point, we have explained the concepts of scale-free and small-world networks, and we have introduced a new and adapted robustness indicator for metro networks. We can therefore start to analyze the results and examine the complexity and robustness of metro systems.

### 5.3 Results and Discussion

In this section, we investigate whether metro networks are in fact scale-free and small-worlds, and how results vary with network size in order to identify potential growth patterns. Additionally, we look at the impact of these properties on the robustness of metro networks.

Effectively, by examining metro networks using the method presented in chapter 3, it is possible to acquire an understanding of their complexity. Table 5.1 shows all results that were calculated for the 33 networks considered in this research.

The first set of data is the basic indicators as introduced in chapter 4. Then, the line distributions (which are essentially the $L$ matrices as seen in chapter 3) are enlisted for each network. In the third set of data, properties, we included the values for the scaling factor $\epsilon$ (metros marked with a ‘*’ were not found statistically significant), the degree of connectivity or clustering coefficient $\gamma$, our robustness indicator $r^T$, and a ratio that will be useful for the analysis, it is the percentage of transfer to total vertices ($V_t / N_S$); we discuss these values in this section.
Table 5.1 Results containing the basic indicators, the line distributions $L$ and network properties calculated for the 33 network studied.

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<th>Lines $N_L$</th>
<th>Stations $N_S$</th>
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<th>End Vertices $V^e$</th>
<th>Transfer Vertices $V^t$</th>
<th>Total Edges $E$</th>
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* Not statistically significant and discarded
** Not statistically significant but kept
5.3.1 Scale-free networks

To assess whether metro networks follow the scale-free feature, we plot the frequency distributions of the line attributes and test whether they have power-law fits as described in section 5.2.1. Figure 5.1 shows a log-log plot of all systems, where lines simply connect the data points and are not linear regressions.

![Log-Log Frequency Distribution of Line Attributes for 33 Metro Networks.](image)

**Figure 5.1** Log-Log Frequency Distribution of Line Attributes for 33 Metro Networks.

5.3.1.1 Statistical Analysis; are metro networks scale-free?

Table 5.2 shows the regression results (scaling factor $\varepsilon$ and multiplying coefficient $a$) of all metro systems. The statistical analysis could only be performed for all systems having more than two data points (i.e. systems having stations hosting more than two lines), which concerns a total of 19 metros out of the 33 studied.
First, we reported the adjusted $R^2$, the t-test values for $\varepsilon$ and the root mean square of the regression as collected from the ordinary least squared (OLS) analysis. Then, we added the degrees of freedom ($\ell_{\text{max}} - 1$), which are the values used for the $\chi^2$ tables, the $\chi^2$ values calculated and the 95% confidence values from the tables. A final column stipulates whether the regression results passed the $\chi^2$-test or not using the 95% confidence values.
It is possible to see that out of these 19 metros, six failed the $\chi^2$-test; five are marked with a ‘*’ and one (London) is marked with ‘***’ in Table 5.1 and Table 5.2. From the line distributions $L$ of these six systems, it is understandable why the test failed. First, for London and Chicago, the values for $\ell_5$ and $\ell_6$ are identical, which disagrees with the nature of the fit, thus increasing the $\chi^2$ value. London, however, remains fairly close to the 95% confidence value, and a visual inspection suggests the presence of a power law distribution, which is why the value is still considered in the following analyses (marked as ‘***’ in Table 5.1 and Table 5.2). Chicago, on the other hand, has significantly poor statistical results. This is explained by the rapid decay from $\ell_1$ to $\ell_2$ and then a certain levelling, which decreases the value of the scaling factor artificially; this is representative of its network structure (few transfers except for one area downtown referred to as the “loop”). A similar observation can be made with Shanghai. For the other three metros (Madrid, Osaka and Seoul), the decay from $\ell_1$ to $\ell_2$ is rapid, as well as the decay from $\ell_2$ to $\ell_3$, which creates a discontinuous transition. Out these six metros, only London is kept, increasing the total number of systems to 14.

Seeing as how the majority of the metro systems follow a power-law distribution (14 of out of the 19 considered), the calculated scaling factor of the other networks having only two data points can be assessed using equation (5.3). We therefore performed a similar OLS method to log-log values of the remaining systems; values are reported in Table 5.1 and Table 5.2 as well.

The values computed for the scaling factors $\varepsilon$ can be informative on the structures of the metro networks. From looking at these values (Table 5.1), we can already see that most networks have a scaling factor greater than three, thus not falling in the range $2<\varepsilon<3$ as introduced in section 5.2.1.2. This is somewhat expected considering the low number of possible connections (i.e. a maximum of 6). As a result, relatively high values of $\varepsilon$ are found, which means that metro networks are more of the “egalitarian” type having mostly nodes with similar line attributes (due the presence of stations with $\ell=1$).
5.3.1.2 Scale-free and metro network size

To further examine these networks, we can look at the influence of network size (here, number of stations $N_S$) on $\varepsilon$. Figure 5.2 shows the results; the horizontal line represents the median value (3.30); the five networks marked with ‘*’ in Table 5.1 and Table 5.2 were not included.

It seems that $\varepsilon$ varies significantly for smaller networks (left of Figure 5.2), from 2.10 (Hong-Kong) to 5.52 (Rome). This result is quite unanticipated. Initially, we expected all small networks to show large value of $\varepsilon$ that would decrease as $N_S$ increases, while in fact a few metro networks show a surprising behaviour. Indeed, several small networks (Hong-Kong, Bucharest, Lyon and St-Petersburg) have an unusual number of transfer-vertices whilst having a limited number of stations. Considering these networks have no transfer-stations hosting more than two lines ($\ell \leq 2$), this information suggests their ratio of transfer to total vertices ($V^t/N_S$) is relatively high compared to other networks of similar size. As a result, for smaller networks, there does not appear to be universal patterns in terms of scaling factor.

![Figure 5.2 Scaling factor $\varepsilon$ as a function of number of stations $N_S$ for 28 Metro Networks.](image-url)
In contrast, as size increases (further right of Figure 5.2), the scaling factor $\varepsilon$ seems to converge to values between 2.80 and 3.30. The network at the far right is New York City ($N_S=422$); the networks to the left of New York City are London ($N_S=306$), Paris ($N_S=297$) and Tokyo ($N_S=202$). Larger networks therefore show a relatively “thin” tail, i.e. large scaling factor. This result is counter-intuitive. Initially, we expected that large networks would have many transfer-stations, hence have a low scaling factor (i.e. “fat-tail”). However, this is not the case. In practice, these networks have a selected few hubs ($\ell$ up to 6) that offer most transfer possibilities, which can actually be a favourable network design practice (Derrible and Kennedy 2010d). These networks therefore have relatively few transfer stations, whilst offering many transfers; these transfers are simply only offered by very few stations. One might characterize these networks as “oligarchic” due to the presence of these hubs.

The phenomenon of “preferential attachment” is clearly present and particular strong here. We will later see the implications of this behaviour on robustness.

### 5.3.2 Small-Worlds

As we have seen, small-worlds obey two basic rules. The first relates to the clustering or cliquishness of the network, so that nodes are connected with one another. The second rule considers the path length to go from one section of the network to another, to confirm that all nodes are reachable through a reasonably short path (i.e. making the least number of transfers possible).

#### 5.3.2.1 Analysis; are metro networks small-worlds?

By looking at the degree of connectivity $\gamma$, we can examine the first rule identified by Watts and Strogatz. From Table 5.1, all networks appear to be relatively clustered compared to other networks from the literature, with values ranging from 0.39 (Delhi) to 0.71 (Chicago), contradicting the previous finding by various studies including Vragović and Díaz-Guilera (2005) who did not account for the planar specificity of transit networks.
The second basic rule deals with the diameter of the network. As mentioned, the presence of transit lines drastically changes the problem. Therefore, we can use the maximum number of transfer $\delta$. We also recall that the value of $\delta$ should be below the log of the total number of stations: $\ln N_s$. From Table 5.1, values of $\delta$ are systematically smaller than the corresponding values of $\ln N_s$. This is further shown on Figure 5.3; all $\ln N_s$ are systematically greater than the maximum number of transfer $\delta$. It is therefore clear that all metros studied satisfy rule 2.

![Figure 5.3](image)

**Figure 5.3** Maximum number of transfers $\delta$ vs. log of number of stations $N_s$; i.e. all points below the $45^\circ$ line have an $x$-coordinate greater than the $y$-coordinate.

5.3.2.2 Small-worlds and metro network size

By plotting the degree of connectivity $\gamma$ versus an indicator of network size (we choose the number of stations $N_s$ here again), Figure 5.4 shows yet another surprising result. As networks grow, they tend to become more connected.
From equation (5.7), we recall that the number of potential edges increases linearly with the number of vertices, and by a factor of roughly three; larger networks will therefore need to build more edges (i.e. new transfer possibilities) to remain evenly connected. Not only do current systems satisfy this criterion, larger networks tend to become even more clustered.

![Figure 5.4](image)

**Figure 5.4** Degree of connectivity $γ$ as a function of number of stations $N_S$ for 33 Metro Networks.

This is an interesting feature that is entirely reliant upon the planar property of metro networks. For non-planar networks, the potential number of edges, $E(E-1)/2$, increases in a quadratic manner. If one were to plot Figure 5.4 using the non-planar number of potential edges, the result would actually be inverted (i.e. an increase in size would imply a decrease in clustering). We see here yet another reason to account for specificities of metro networks. From chapter 4, we recall that metro systems actually seem to asymptotically tend to being 66% completely connected.

As a result, we find that metro networks are small-worlds. Moreover, they also follow an unexpected behaviour by becoming more clustered as they grow in size. As we are about to see,
the coupling of both properties (small-world and scale-free features) has a significant impact on robustness.

5.3.3 Implications on Robustness

Robustness is an important aspect that is often studied when analyzing networks. For the case of metro networks, we have developed a robustness indicator that addresses some specific transit properties. The indicator $r^T$ is related to number of cycles in a network, and the total number of stations $N_S$ represents the probability a failure occurs. Values for $r^T$ can be found in Table 5.1.

In this section, we will first look at the impact of metro network size on robustness and then at the impact of the scale-free and small-world feature on the robustness of metro networks.

5.3.3.1 Robustness and metro network size

Naturally, as networks grow, they tend to generate more cycles (i.e. the cyclomatic number increases), which is a positive element of robustness; this feature can be linked to the increase in clustering as seen above (section 5.2.2). This is particularly true considering smaller networks may have a null cyclomatic number $\mu$. Nevertheless, larger networks also possess more stations (which we used as a “propensity to fail” property), which make the robustness indicators strongly reliant on its number of stations as well.

From Table 5.1, we see that the least robust systems are Rome and Delhi since they have no cycles (they also have a high $\varepsilon$ and a low $\gamma$); the most robust system is Tokyo with $r^T=0.25$. Figure 5.5 shows the influence of network size (i.e. number of stations $N_S$) on the robustness of metro systems. In this case, there is no general pattern. First, as networks grow, they develop more cycles, which in turn increase robustness; therefore, for smaller networks, the cyclomatic number $\mu$ is the determining factor. However, after a certain size, both components of $r^T$ become relevant. For instance, the system with the largest numerator ($\mu-E^m$) is Seoul (64), although Tokyo (51) is the most robust system. In addition, New York City seems to maintain a low numerator (37), while having a total of 422 stations, negatively impacting its robustness.
An increase in size therefore does not automatically translate into an increase in robustness. Here again, the number of transfers may be the determining factor, which we can relate back to scale-free and small-world networks.

![Figure 5.5 Robustness $r_T$ versus number of stations $N_S$ for 33 Metro Networks.](image)

5.3.3.2 Impacts of scale-free and small-world features on the robustness of metro networks

Both, scale-free and small-world features, have an impact on the robustness indicator $r_T$; it appears that a high clustering coefficient $\gamma$ (to generate cycles) and a low scaling factor $\varepsilon$ (to have more transfer stations) are preferred.

As networks grow in size, however, they tend to become more clustered ($\gamma$ increases), whilst the scaling factor $\varepsilon$ settles to a relatively high value (between 2.80 and 3.30, see section 5.3.1), thus impacting robustness (because of the proportion of transfer to total stations becomes relatively low). In practice, stations that serve main areas of a city tend to acquire more transfers,
potentially following a process of “preferential attachment”, undermining the importance of smaller transfer stations; although this seems to be preferable to generate a high connectivity indicator $\rho$ (chapter 4), it appears new transfer stations should also be created with smaller line attributes to increase the robustness of the network. Nevertheless, there are existing networks that seem to have a large $\gamma$ and a small $\varepsilon$. A key aspect to achieve this seems to rely partly on the proportion of transfer vertices in the network.

Figure 5.6 shows the relationship between robustness and the ratio of transfer vertices to total number of stations $V_t/N_S$ (only black-coloured data points are labelled on Figure 5.6); the relationship is strong and seems to best apply to networks that have already reached a certain size (i.e., networks that have generated enough cycles).

![Figure 5.6 Robustness $r_T$ versus percentage of transfer-vertices $V_t$ (i.e., $V_t/N_S$).](image)

Indeed, the Hong-Kong system, which despite having favourable indicators, remains smaller in size and has multiple edges that seriously impact its robustness.
While it appears that a high clustering coefficient $\gamma$ and a low scaling factor $\varepsilon$ are preferable, these are non-trivial properties that can be challenging to emulate in new transit network design. Figure 5.7 is a graph showing the scaling factor $\varepsilon$ on the $y$-axis and the clustering coefficient $\gamma$ on the $x$-axis.

First of all, it is interesting to see that no networks achieve high clustering $\gamma$ and have a significantly high scaling factor $\varepsilon$, (top-right part of Figure 5.7), simply because high clustering $\gamma$ relates to larger metros. Networks in the top-left part of the figure (low $\gamma$ and high $\varepsilon$) seem to do poorly in terms of robustness; they are particularly small in size and have a simple topology. Networks in the bottom-left part of the figure (low $\gamma$ and low $\varepsilon$) can do better in terms of robustness, but these networks are mostly relatively small networks, which, as we have seen in section 5.3.1.2, do not have any identifiable patterns in terms of scaling factor, but appear to have a high proportion of transfer to total number of stations. Finally, it is possible to see that in general but not systematically, networks in the right-bottom part of the figure are doing relatively well in terms of robustness. These networks are larger in size, but it should be remembered that in this case, the proportion of transfer to total stations becomes a determining factor due to the definition of the robustness indicator (i.e. it is heavily dependent on the total number of stations).

In this analysis, we therefore further account for the importance of transfer stations, and in particular the percentage of transfer stations to total number of stations. It appears that although the presence of transfer hubs is generally favourable, other transfers (with smaller line attribute, i.e. $\ell=2$ or 3) are also of relevant.

As a result, it is worth spending time on a selected few metro networks and identify what properties are favourable in terms of robustness. If we recall from chapter 4, we have defined three phases in the evolution of a metro system. This was part of the $State$ characteristic, which relied on indicators of complexity $\beta$ and degree of connectivity $\gamma$ (which is used as the clustering coefficient here). We will explore one system from each phase (from simple to more complex) and identify the properties that render greater robustness. These properties can therefore become guidelines in the design of metro systems to address objectives of robustness.
Figure 5.7 Scaling factor $\varepsilon$ vs. the clustering coefficient (or degree of connectivity) $\gamma$ for 33 metro networks studied.
5.3.3.3 Robustness and the network characteristic State

The metros for each of the three phases of the State characteristic that seem to do particularly well in terms of robustness are St Petersburg (1st phase), Barcelona (2nd phase), and Tokyo (3rd phase).

In this section, we go through each metro network design, identify the line types and other relevant properties (including network science properties), compare these properties to similar-sized metros, and present a map of the systems.

The St Petersburg metro (Figure B.10) has four lines of the diametrical and radial types. It has a total of 54 stations, of which six are transfer stations, hence a ratio of 11.11%. These properties translate into a scaling factor of 3.00, which is low compared to the size of this network. All lines intersect with one another (δ=1) at different stations in the downtown, which created a certain number of cycles, and thus allowed for a relatively high clustering coefficient (0.45). Overall, this structure lead to a robustness indicator of 0.0556, which is high compared to similar sized metros, e.g., Toronto (0.0145), Lyon (0.0256), and Lisbon (0.0227).

The Barcelona metro (Figure B.11) has nine lines of the radial, diametrical and tangential types. It has a total of 123 stations, of which 18 are transfer stations (14 with \( \ell=2 \), 2 with \( \ell=3 \), 2 with \( \ell=4 \)), hence a ratio of 14.63%. These properties translate into a scaling factor of 3.08, which is lower than most similar sized metros (note, however, that it is higher than the St Petersburg metro, which means the Barcelona system could include more transfer stations if the system is to expand to lower \( \varepsilon \)). Metro lines seem to intersect in the downtown, forming hubs (e.g., Passeig de Gràcia station), but also outside the city to create simpler transfer stations (e.g., Maragall station), hence generating cycles. It has a clustering coefficient of 0.54, which is relatively high for its size. Therefore, considering its structure, it has a robustness indicator of 0.1138, which is high compared to similar sized metros, e.g., Berlin (0.0706), Mexico City (0.0861).

The Tokyo metro (Figure B.12) is certainly the best example of a large network offered in this dataset. It has 13 lines (of all types), with 202 stations; 45 of these stations are transfers, hence a ratio of 22.28%, which is significantly high. These properties translate into a low scaling factor of 2.80. All lines intersect at multiple points, thus keeping a low maximum number of transfers
(2). Tokyo managed to remain fairly well connected by creating hubs downtown, whilst having many transfer vertices with $\ell=2$ or $3$ in the outer parts of the city core; this characteristic seems to be key to create a robust system. Indeed, it has a robustness indicator of 0.2525, which is significantly high compared to similar sized metros, e.g., Paris (0.1684), London (0.1405), New York City (0.0877).

From this analysis, it seems networks can adopt a certain strategy to be robust as they increase in size. First, smaller networks should emphasize on building a few transfer stations, which is natural in the development of smaller metros. Indeed, for smaller networks lines that are built most often serve and connect at main areas (e.g., financial districts, commercial centers, central train stations, etc). This has the property to generate cycles, which logically creates alternative routes in case of failures/accidents. With an increase in size, these transfer stations should become hubs, hosting three or more lines. Simultaneously, effort should be put into creating new and smaller transfer stations again to further generate cycles, lower the scaling factor $\varepsilon$ and increase the ratio of transfer to total stations; this can notably be done by connecting segments of networks that are outside of city center through the construction of tangential and/or (semi)-circle lines.

Concurrently, this should ameliorate the overall network design component. First, creating transfer stations should increase the connectivity $\rho$ and directness $\tau$ of the network. The initial phase of a new network is likely going to result from the construction of radial lines servicing these main areas. Stations in these areas are then likely going to become hubs, thus reducing the number of maximum transfers $\delta$ and further creating potential positive network effects (direct and indirect). At which point the construction of a tangential or (semi)-circle line may create new transfer-vertices that will further increase many favourable direct network effects of metro networks (e.g., connectivity) as well as indirect effects (here, robustness).

5.4 Conclusion

The field of network science is a rapidly emerging area of study as we have seen multiple times in this work. Metro networks, by being real-life applications, are particularly interesting systems to study and certainly add a significant contribution to the field. In the past 12 years, two types of
networks have been discovered and have radically changed the field of network science; these are scale-free and small-world networks as introduced by Barabási and Albert (1999), and Watts and Strogatz (1998) respectively.

In this research, we sought to effectively identify which type of networks metro systems belonged to in order to grasp a sense of their complexity. We used the graph theory methodology that was presented in chapter 3 to collect the different topological properties of 33 metro networks worldwide.

The goals of the chapter were therefore to analyze the complexity of metro systems within the realm of network science, look at the impact of network size, and discuss the implications on robustness. By studying the various properties of metro networks, we adapted some of the existing indicators as well as introducing new ones. We found that most metro systems were scale-free networks since their line attributes follow a power law distribution; scaling factors for five systems out of the 33 studies could not be computed. We also noticed a wide array of behaviours existed for smaller networks; as these systems grow, however, they actually tend towards similar values of scaling factor $\varepsilon$ (between 2.80 and 3.30). Furthermore, we also found that metro networks obeyed the two small-worlds rules, most notably due to their planar property and the presence of lines, which significantly reduces the need to transfer. They also appear to become more clustered as they grow in size. These network features of metro systems are relatively atypical, and within the realm of network science, it may be interesting to investigate whether other real-life networks possess this property.

We then looked at the impacts of the scale-free and small-world features, as well as network size, on the robustness of metro systems. Robustness was measured using the number of cycles present in the system (i.e. number of loops) and its propensity to fail (dependent on size; here the number of stations). Overall, more clustered networks generally do better in terms of robustness because they have generated a significant number of cycles (i.e. alternative routes). Nevertheless, as metro systems grow, they also create new stations that may impede robustness. It is therefore important to create new transfer stations outside or at the periphery of the city core, which lowers the scaling factor $\varepsilon$ and further contributes to the robustness of larger systems; smaller networks first need to become more clustered by creating transfer stations.
Globally, much can be learned from the study of transit networks, and the consideration of the scale-free and small-world aspects is instrumental. Robustness is one example but applications could range from transportation planning and scheduling to dispatch optimization and emergency evacuation planning. Nevertheless, present measures and indicators cannot be applied systematically, they first have to be updated and adapted to account for specificities that can otherwise change the outcome of an analysis significantly. The use of novel techniques can also be of substantial help to the transport community.

To illustrate the usefulness of the findings presented here, we use the scale-free and small-world concepts in chapter 7 to evaluate the proposed Toronto plans for the next 15 and 25 years.

In the next chapter, we further use graph theory to investigate the importance of three network properties on ridership by looking at indicators of coverage, directness and connectivity.
Chapter 6
Relating Network Properties to Ridership

6.1 Introduction

Metros contribute greatly to metropolitan regions around the world. They are part of the identity of cities and are an essential economic component (Banister 2000). As we saw in chapter 1, their extent and use are likely to increase considering the present challenges the world is facing in terms of sustainable development (growth of urban areas) and global warming (lower GHG emissions of mass transit and electric-powered technologies). The analysis of metro systems is often associated solely with demand characteristics, either geographic features (density) and/or cultural features (Europe vs. Asia vs. North-America). The supply side is habitually briefly considered (e.g., by comparing the number of lines of transit systems), leaving only a general appreciation of the network complexities as it has been mentioned multiple times in the previous chapters.

This research focuses rather on the topologies of metro systems, bringing attention to network design as a whole. This is done specifically by incorporating some updated graph theory concepts, using such information as number of lines, stations, transfers and others. The methodology introduced in chapter 3 is also used here.

In this chapter, we study the impact of three network design indicators on ridership. The choice of ridership is particularly relevant here and can be seen as an indicator of performance. While transit planning objectives may differ (chapter 1), ultimately, all aspire to serve as many people as possible, which can be translated into high ridership levels. As a result, the broad goal of this chapter is to investigate the relationship between network design and ridership, and identify positive elements of metro network design.

Up to this point, much effort focused on identifying the properties of metro networks. These properties do not necessarily relate to ridership but rather to the vision/strategy adopted by each city; for instance, a direct relationship between ridership and metros that emphasise regional accessibility or local coverage cannot be determined. In this chapter, it is therefore important to use network indicators having a strong relationship with ridership regardless of location specific
characteristics. By considering a variety of indicators, three seem to possess such a relationship; these three are coverage, directness and connectivity.

Coverage $\sigma$ is based on the total number of stations and land area. Directness, as introduced in chapter 3, relates to the maximum number of transfers necessary to go from one station to another. Connectivity, as introduced in chapter 3 as well, attempts to describe an overall view on the transfer possibilities to travel in the network so as to appreciate a sense of mobility. Ridership is used in the form of boardings per capita per year. As a result, we can see that data on the city characteristics have to be collected as well. In particular, we use data of population, area, and total boardings.

By considering 19 worldwide cities and metros, and by performing single and multiple regression analyses, we develop and discuss empirical relationships between three network indicators and ridership. More specifically, the objectives of this chapter are:

- Collect data on ridership and city characteristics, and compute this data in a manner such that it can be appropriately used
- Develop adequate indicators of metro network design that have a positive impact on ridership
- Establish empirical relationships between ridership and network characteristics.

In this chapter, 19 metro networks around the world were studied as opposed to 33 networks in the previous chapters. Initially, more networks were considered but collecting information on city characteristics and ridership revealed to be significantly challenging. In particular for this analysis, we developed a specific method to calculate population and land area data as we will see in section 6.2.

The 19 cities considered and their metro networks are: Toronto, ON; Montreal, QC; Chicago, IL; New York City, NY; Washington D.C.; San Francisco, CA; Mexico City, Mexico; London, UK; Paris, France; Lyon, France; Madrid, Spain; Berlin, Germany; Athens, Greece; Stockholm, Sweden; Moscow, Russia; Tokyo, Japan; Osaka, Japan; Seoul; South Korea; and Singapore.
From this list, only one city (San Francisco) was not present in the dataset of 33 metros used in the previous chapters. First, it should be added that this work was produced before the work presented in chapters 4 and 5. The San Francisco metro system was taken out of the dataset in later studies due to its highly irregular network topology; the San Francisco metro (BART) has four lines, yet it has a significantly high proportion of overlapping lines, which in some respect could be assimilated to a metro with one or two lines and many branches (Figure B.13).

In this chapter, we present these three objectives sequentially. For the first objective, we explain how the data on city characteristics and ridership is collected, and how this information is used. In the second objective, we first introduce and explain the coverage indicator extensively, and then review briefly the concepts of the two other network design indicators (connectivity and directness). For the third objective, we first investigate the impact of each indicator separately, and then compute a ridership model, which can potentially be used as a fairly simple testing/sketching tool. Finally, we present a brief discussion of potential future research avenues and other potentially relevant network design indicators.

This chapter is based on the article “A Network Analysis of Subway Systems in the World using Updated Graph Theory” by Derrible and Kennedy (2009c).

6.2 Data on Ridership and City Characteristics

Extensive data was collected throughout this study. First two city characteristics are needed in this analysis: population and served area by transit of each of the 19 metro systems studied; in this section, we explain thoroughly how this information was collected. Second, we present the type of ridership data collected and how it is used in this study.

6.2.1 City characteristics

For this analysis, two city characteristics are considered: population and served area by transit. Population is required first and foremost to be able to calculate per capita ridership figures.
Served area by transit is required in the coverage network indicator as we will see in section 6.3.1.

The information for the latter characteristic (served area by transit) was not readily available for any of the 19 networks. Values of total land areas were available but consisted of administrative areas (at various levels as we will see), which could not be assimilated to served area by transit; moreover, these areas also included uninhabited areas such as lakes, parks, etc, which can seriously impact the analysis. To go around the problem, we decided to calculate served area by dividing population by population density. This is a rather pragmatic approach to handle the issue of defining the area served by metro networks. Nevertheless, collecting these two city characteristics turned out to be a substantially challenging process, as previously acknowledged (Miron 2003).

Historically, most cities had rather small area boundaries that have not been updated with urban growth at the administrative level (e.g., city of Lyon). Naturally, such population and population density values cannot be considered given they do not reflect the real effects of public transportation networks.

The next administrative level with available data is the “urban area”. The concept of urban area is not defined in a similar fashion in every country, and it normally encloses a much bigger area than that served by transit networks (e.g., Grand Lyon).

Moreover, a few urban areas actually have separate transit systems (e.g., Tokyo-Yokohama). This information was even more important as we are dealing with international cities; hence there was a need to acquire balanced standardized values.

To illustrate this issue using population figures, London has 7.5 million inhabitants, with an urban area population of 8.3 million, compared with the city of Osaka that has 2.6 million inhabitants, with an urban area population of 15.5 million. Clearly, either city population or urban area population cannot be used consistently for the 19 cities. Figure 6.1 shows an example of a metro network in a city within its urban area. The light shaded grey area is the served area that is desired for the analysis.
Out of the 19 cities, only Stockholm, Moscow and Tokyo had readily available information on population. Data on Stockholm and Tokyo was available from their transit agencies. For Moscow, we used city official data that encompass true levels of population.

For most of the remaining networks (all but Paris), we found that the arithmetic mean of the city population and urban area population gave the most practical measure of population. Not only does this approach seem realistic in terms of served area by transit, it also offsets other uncertainties concomitant to the fact we are dealing with international cities (different administrative systems). Data on population and density data were collected mostly from [www.citypopulation.de](http://www.citypopulation.de) (Brinkhoff 2008) at the city level and from Demographia (2007) at the urban area level.

Nonetheless, Paris had to be tackled differently due to the unique geographic feature of population. Data was collected for the city of Paris, its three surrounding administrative regional municipalities, plus two further regional municipalities that had most of their inhabitants located in the same overall region.

All the values collected and computed are shown in the results section, as well as the methods and sources used.
6.2.2 Ridership

In this chapter, we study the impact of network properties on ridership, where ridership can be seen as a performance indicator. The term ridership is relatively generic and can have different definitions depending on the context of the analysis or the data source. In particular for this study, we chose to define ridership as annual boardings per capita $B_{pc}$. In this section, we briefly discuss the process that was taken to choose this particular definition of ridership.

Typically, transit systems are assessed in comparison to other transport modes by using mode split figures (auto, transit, walk/cycle). Such figures, however, typically emphasize the homework trips, where regional rail may be determinant, but not part of the analysis. Moreover, we expect transit users of large networks not only to use public transportation to go to work, but also for different purposes. Therefore, a better indicator may be the numbers of transit trips.

Transit trips are usually defined as either linked or unlinked. The American Public Transportation Association (APTA) defines unlinked trips as:

“Unlinked Transit Passenger Trip is a trip on one transit vehicle regardless of the type of fare paid or transfer presented. A person riding only one vehicle from origin to destination takes ONE unlinked passenger trip; a person who transfers to a second vehicle takes TWO unlinked passenger trips; a person who transfers to a third vehicle takes THREE unlinked passenger trips.” (APTA 2010)

A linked trip is therefore the combination of the unlinked trips per passenger.

Nevertheless, figures of number of transit trips (linked or unlinked) are relatively challenging to acquire and validate on the international scale, which partially explain why only 19 networks were studied. In addition, the main sources that include this information are the Millennium Cities Database (UITP 2001) and the Mobility Cities Database (UITP 2006). Though comprehensive, the Millennium Cities Database contains data from 1995 (not appropriate for recent networks - e.g., Madrid) and the Mobility Cities Database does not contain data for the 19 transit networks studied here. These two databases, however, have been useful to compare/validate the data collected.
Moreover, as we are dealing with the metro mode only, including all transit trips biases the results in favour of public transportation systems with extensive bus and other non-metro transit modes.

Consequently, it became logical to use the number of boardings (metro only) as the principal indicator. Average daily ridership of metro systems is most often cited on transit authority websites; however, metro use in the week-end is relevant, again not to over-emphasize the homework trip and to account for “shopping” or “entertainment” trips. As a result, annual figures were preferred. Though they were not always readily available, we managed to collect reasonable values by looking up transit financial reports and statistical agencies data or communicating directly with transit authorities. All the data collected on annual boardings date between 2005 and 2007. They are therefore reasonably recent (especially considering the publication related to this chapter was written in spring of 2008). These values were then validated using the two sources mentioned above (Millennium and Mobility Cities databases). It remained, however, impossible to identify whether the annual boardings figures collected considered linked or unlinked trips (due to lack of information from the sources).

Finally, this data should not be used as a raw measure since cities with higher populations are likely to have more annual boardings regardless of network size. It is therefore clear that this data needs to be standardized. In this study, we chose to standardize annual boardings by population, thus generating annual boardings per capita $B_{pc}$ figures. By using population, it is therefore possible to appreciate the net attractiveness of a network design as opposed to its sole ridership. The values chosen for population were discussed in the previous section (6.2.1).

At this point, the city characteristics and ridership have been defined and collected. We will now introduce or review the three network design indicators that are central to this study.

### 6.3 Network Design Indicators

While several indicators were evaluated (notably the Garrison and Marble, and Kansky indicators introduced in chapter 2), three stood out as having particularly strong positive correlations with ridership as defined in section 6.2.2. In this section, we develop and explain these three network design indicators; they are coverage, directness and connectivity.
The two latter indicators were already presented earlier in this work (chapter 4). Their definitions are, nevertheless, briefly reviewed here. But first, the coverage indicator is introduced and presented comprehensively.

6.3.1 Coverage

The concept of coverage is relatively simple. The purpose is to measure the coverage area accessible by metro networks, which can be a measure of accessibility.

This can be done in two different ways. Either transit lines are defined to cover the total areas around them, which essentially corresponds to the area of a rectangle plus the area of two semi-circles for the termini (Figure 6.2 (a)); or, alternatively, a circular area (with a given radius) around each station is considered (Figure 6.2 (b)). These two methods may encounter problems of overlapping, which results in double counting portions of the coverage area (Vuchic 2005). This is especially true for method a in which the basis is the transit line and not the station (i.e. areas around transfer stations are systematically double counted). As a result, we preferred to use method b.

By taking a threshold value of 500m radius for coverage area, where $N_S$ is the number of stations, coverage $\sigma$ is defined as:

$$\sigma = \frac{N_S \cdot \pi \cdot 0.5^2}{Served \ Area}$$

(6.1)

where $Served \ Area$ is the area calculated in section 6.2.1 using population and population density.
One comment should be made at this point about the radius used. The value of 500m was chosen semi-arbitrarily. In reality, it could be anywhere from 400m to 800m or even 1km depending on the context (e.g., accessibility). To avoid problems of overlapping, however, we chose a radius smaller than the smallest inter-station spacings of the 19 metro networks studied in this chapter (chapter 4).

Moreover, the actual value for the radius is actually of low relevance in the analysis. As we are more concerned with the relationship of each network with one another, the term “$\pi \cdot 0.5^2$” in equation (4.1) is simply a constant, and it actually gets absorbed in the slope of the equation during the regression analysis. Nevertheless, we left it in the original equation so as to be able to look at each system individually.

6.3.2 Directness

The concept of directness $\tau$ was first introduced in chapter 4 as an indicator for the Structure characteristic. It is briefly reviewed here.

The directness indicator $\tau$ is related to the concept of the indicator $\pi$ as introduced by Kansky (1963) in chapter 2. He explains $\pi$ as being “a number expressing the relationship between the circumference of a circle and its diameter … We can apply this notion to transportation networks … let us assume that the total mileage of a transportation system is analogous to the circumference of a circle and the total mileage of all edges of the diameter of a network is analogous to the diameter of the circle”. In other words, $\pi$ does not deal with line type (circle, radial, spinal, etc) but instead it measures the ratio of the total track length to the length of the “longest” route.

To apply the concept to public transportation, the numerator can be the total route length $R$. As the denominator is related to the diameter of the system, from a transit network point of view, it is concomitant to the maximum number of transfers needed to go from any vertex to another, introduced as $\delta$ previously. In fact, this is one measure of the longest route possible in the network and can be associated with a diameter. As a matter of consistency, $\delta$ should be multiplied by some sort of length so as to keep the ratio dimensionless. One suggested solution is to multiply it by a segment $\kappa$ of the average line length. If $N_L$ is the total number of lines of a
network, \( R/N_L \) is the average line length; hence \( R/ (\kappa \cdot N_L) \) is a segment of a line. Directness \( \tau \) is therefore defined as:

\[
\tau = \frac{R}{\delta} = \frac{\kappa \cdot N_L}{\delta}
\]  

(6.2)

We have therefore switched from an overall network-size feature to a maximum transfer appreciation.

As a result, in this study, we simply kept the directness indicator \( \tau \) as:

\[
\tau = \frac{N_L}{\delta}
\]  

(6.3)

### 6.3.3 Connectivity

Similar to directness, the connectivity indicator was first introduced in chapter 4 as an indicator for the Structure characteristic. It is also briefly reviewed here.

Connectivity (or structural connectivity) in this case relates to ability to travel freely within the network. It could also be associated with degree of mobility or density of transfer possibilities, hence the symbol \( \rho \). In other words, it shows the degree by which a network is connected, allowing for greater travel path choices.

It may be determined by summing all the transfer possibilities \( V'_c \), which is the sum of the number of metro lines going through a transfer station minus one. It differs from the traditional definition of degree of a node; we do not count the total number of edges at a vertex, but simply the number of possibilities to switch transit lines. For instance, a transfer station sharing two transit lines offers one transfer possibility, another sharing three lines offers two possibilities, etc.

The number of transfer possibilities \( V'_c \) is mathematically defined as:

\[
V'_c = \sum_i (\ell - 1) \cdot v_{i,t}
\]  

(6.4)
where $v_{i\ell}$ is a vertex (numerically, each vertex has a value of 1) with identification number ‘$i$’ and line attribute ‘$\ell$’ as introduced in chapter 3; here, the subscript ‘c’ stands for ‘connectivity’.

In order to account for the net sum of transfer possibilities (avoid redundancies), the number of multiple-use edges $E^m$ needs to be subtracted from $V'_c$. This accounts for the net number of transfer possibilities but is not yet an index. It still needs to satisfy the international context criteria, the wide range of network topologies existing. Consequently, to standardize connectivity, the equation further needs to be divided by an appropriate variable, here the total number of transfers $V'$; as a result, the connectivity indicator $\rho$ is:

$$\rho = \frac{V'_c - E^m}{V'}$$

(6.5)

The denominator essentially allows for the ratio to become an index. A dispersed network can have a high number of transfer stations while still having a relatively low connectivity feature. For instance, the London subway system has a higher net sum of transfer possibilities than the Madrid system; nevertheless Madrid’s network is more highly connected and has a higher number of boardings per capita. In other words, $\rho$ measures connectivity regardless of network size.

We will now analyse the results.

### 6.4 Results

In this section, results for the city characteristics (collected and calculated), the ridership (collected and calculated), the basic network indicators (as previously reported) and the three network design indicators (calculated) are shown in Table 6.1. In cells with “ - ”, data was not available; footnotes were provided when this impacted the results, showing alternate data sources or methods applied. In general, sources for the data (when collected) were provided earlier in sections 6.2 and 6.3 or in chapter 3 for the basic and network design indicators.
Table 6.1 Results containing the city characteristics, the ridership data, the basic network indicators, and the three network design indicators calculated.

<table>
<thead>
<tr>
<th>Metro Networks</th>
<th>City Population</th>
<th>Urban Area Population</th>
<th>City Population Density (pop/km²)</th>
<th>Urban Area Population Density (pop/km²)</th>
<th>Population data used</th>
<th>Served Area data used (km²)</th>
<th>Boardings (millions per year)</th>
<th>Annual Boardings per capita</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toronto</td>
<td>2,503,281</td>
<td>4,367,000</td>
<td>3,972</td>
<td>2,643</td>
<td>3,435,141</td>
<td>1,039</td>
<td>265.3</td>
<td>77.23</td>
</tr>
<tr>
<td>Montreal</td>
<td>1,620,693</td>
<td>3,216,000</td>
<td>4,439</td>
<td>1,851</td>
<td>2,418,347</td>
<td>769</td>
<td>278.2</td>
<td>115.05</td>
</tr>
<tr>
<td>Chicago</td>
<td>2,833,321</td>
<td>8,308,000</td>
<td>4,816</td>
<td>1,511</td>
<td>5,570,661</td>
<td>1,761</td>
<td>186.8</td>
<td>33.53</td>
</tr>
<tr>
<td>New York City</td>
<td>8,214,426</td>
<td>17,800,000</td>
<td>10,457</td>
<td>2,050</td>
<td>13,007,213</td>
<td>2,080</td>
<td>1804</td>
<td>138.69</td>
</tr>
<tr>
<td>Washington DC</td>
<td>581,530</td>
<td>3,934,000</td>
<td>3,657</td>
<td>1,313</td>
<td>2,418,347</td>
<td>909</td>
<td>259.4</td>
<td>114.88</td>
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<tr>
<td>San Francisco</td>
<td>744,041</td>
<td>3,229,000</td>
<td>6,152</td>
<td>2,366</td>
<td>1,986,521</td>
<td>466</td>
<td>99.3</td>
<td>49.97</td>
</tr>
<tr>
<td>Mexico City</td>
<td>8,463,906</td>
<td>17,400,000</td>
<td>-</td>
<td>9,736</td>
<td>12,931,953</td>
<td>2,657²</td>
<td>1417</td>
<td>109.57</td>
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<td>London</td>
<td>7,517,700</td>
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<td>5,100</td>
<td>7,897,850</td>
<td>1,451</td>
<td>1078</td>
<td>136.49</td>
</tr>
<tr>
<td>Paris</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8,403,000¹</td>
<td>1,021¹</td>
<td>1860.9</td>
<td>221.46</td>
</tr>
<tr>
<td>Lyon</td>
<td>467,400</td>
<td>1,349,000</td>
<td>9,764</td>
<td>1,414</td>
<td>908,200</td>
<td>162</td>
<td>96.5</td>
<td>106.28</td>
</tr>
<tr>
<td>Berlin</td>
<td>3,404,037</td>
<td>3,880,000</td>
<td>3,820</td>
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<td>3,642,019</td>
<td>1,044</td>
<td>475.0</td>
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<td>Madrid</td>
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<td>4,900,000</td>
<td>5,171</td>
<td>5,255</td>
<td>4,016,232</td>
<td>770</td>
<td>690</td>
<td>171.81</td>
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<tr>
<td>Athens</td>
<td>3,072,922</td>
<td>3,685,000</td>
<td>-</td>
<td>5,270</td>
<td>3,378,961</td>
<td>1,282²</td>
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</tr>
<tr>
<td>Stockholm</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1,900,000¹</td>
<td>1,050¹</td>
<td>297</td>
<td>156.32</td>
</tr>
<tr>
<td>Moscow</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10,521,000³</td>
<td>653³</td>
<td>2475.6</td>
<td>235.30</td>
</tr>
<tr>
<td>Tokyo</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>15,070,000³</td>
<td>9,837³</td>
<td>2974</td>
<td>197.35</td>
</tr>
<tr>
<td>Osaka</td>
<td>2,635,420</td>
<td>15,450,000</td>
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<td>5,681</td>
<td>9,042,710</td>
<td>1,028</td>
<td>912</td>
<td>100.85</td>
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<td>9,820,171</td>
<td>17,600,000</td>
<td>16,205</td>
<td>14,773</td>
<td>13,710,086</td>
<td>885</td>
<td>2264</td>
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</tr>
<tr>
<td>Singapore</td>
<td>4,553,000</td>
<td>4,000,000</td>
<td>6,669</td>
<td>9,593</td>
<td>4,276,500</td>
<td>526</td>
<td>434.9</td>
<td>101.68</td>
</tr>
</tbody>
</table>

¹ Data gathered from transit agencies (Stockholm and Tokyo), official city websites (Moscow), or calculated specifically (Paris, see section 6.2.1)
² Since city population density was not available for Mexico City and Athens from the http://www.citypopulation.de website, half the urban area population density was used
³ Population density used was taken from http://www.demographia.com/db-lonlanypar.htm
⁴ Population density used was taken from the Millennium Cities Database (2001)
⁵ Population density used for the 23 wards of Tokyo
<table>
<thead>
<tr>
<th>Metro Networks</th>
<th>Route Length</th>
<th>Stations</th>
<th>Lines</th>
<th>Vertices</th>
<th>Edges</th>
<th>Line Attribute</th>
<th>Network Design Indicator</th>
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<tbody>
<tr>
<td></td>
<td>$R$ (km)</td>
<td>$N_S$</td>
<td>$N_L$</td>
<td>Total</td>
<td>End</td>
<td>Transfer</td>
<td>Total</td>
</tr>
<tr>
<td>Toronto</td>
<td>68.747</td>
<td>69</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>Montreal</td>
<td>60.858</td>
<td>65</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>Chicago</td>
<td>173.075</td>
<td>151</td>
<td>7</td>
<td>24</td>
<td>11</td>
<td>13</td>
<td>47</td>
</tr>
<tr>
<td>New York City</td>
<td>368.046</td>
<td>422</td>
<td>11</td>
<td>73</td>
<td>26</td>
<td>47</td>
<td>130</td>
</tr>
<tr>
<td>Washington DC</td>
<td>171.143</td>
<td>86</td>
<td>5</td>
<td>17</td>
<td>9</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>San Francisco</td>
<td>182.252</td>
<td>43</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>Mexico City</td>
<td>177.1</td>
<td>151</td>
<td>11</td>
<td>35</td>
<td>12</td>
<td>23</td>
<td>47</td>
</tr>
<tr>
<td>London</td>
<td>438.725</td>
<td>307</td>
<td>13</td>
<td>83</td>
<td>27</td>
<td>56</td>
<td>155</td>
</tr>
<tr>
<td>Paris</td>
<td>256.8</td>
<td>306</td>
<td>19</td>
<td>108</td>
<td>42</td>
<td>66</td>
<td>178</td>
</tr>
<tr>
<td>Lyon</td>
<td>29.3</td>
<td>39</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Berlin</td>
<td>216.967</td>
<td>192</td>
<td>12</td>
<td>74</td>
<td>17</td>
<td>57</td>
<td>126</td>
</tr>
<tr>
<td>Madrid</td>
<td>226.7</td>
<td>190</td>
<td>13</td>
<td>46</td>
<td>10</td>
<td>36</td>
<td>82</td>
</tr>
<tr>
<td>Athens</td>
<td>52.003</td>
<td>44</td>
<td>3</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Stockholm</td>
<td>109.48</td>
<td>100</td>
<td>3</td>
<td>20</td>
<td>11</td>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>Moscow</td>
<td>282.5</td>
<td>173</td>
<td>12</td>
<td>42</td>
<td>14</td>
<td>28</td>
<td>67</td>
</tr>
<tr>
<td>Tokyo</td>
<td>292.376</td>
<td>202</td>
<td>13</td>
<td>61</td>
<td>16</td>
<td>45</td>
<td>119</td>
</tr>
<tr>
<td>Osaka</td>
<td>125.419</td>
<td>121</td>
<td>8</td>
<td>36</td>
<td>12</td>
<td>24</td>
<td>53</td>
</tr>
<tr>
<td>Seoul</td>
<td>287</td>
<td>286</td>
<td>11</td>
<td>71</td>
<td>17</td>
<td>54</td>
<td>135</td>
</tr>
<tr>
<td>Singapore</td>
<td>89.4</td>
<td>64</td>
<td>4</td>
<td>12</td>
<td>6</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>
From Table 6.1, we can see that values for coverage are always much lower than one; they range from 0.027 in Athens to 0.254 in Seoul (i.e. 25.4% of the served area calculated is located within 500m of a station). On the other hand, values for directness are always higher than one and range from 1.33 (Toronto and Singapore) to 6.50 (London and Tokyo). As for connectivity, values lower than one can be generated, e.g., 0.60 for San Francisco compared to 1.62 for Paris, due to the significant presence of multiple edges in the San Francisco metro.

Each indicator is first analysed separately and then grouped in a multiple linear regression, using an ordinary least squared (OLS) method. To assess the statistical significance of the parameters calculated, we use a measure of goodness-of-fit (adjusted $R^2$) and t-test results. Considering there are 19 cities, the t-test values (with right tail probabilities) should be at least 1.328 for 90% confidence and 1.729 for 95% confidence.

### 6.4.1 Coverage

As we have seen in equation (4.1), the coverage indicator calculates the percentage of station area coverage to what we have defined as served area.

In this instance, coverage can be linked to concepts of accessibility. Naturally, as accessibility increases, ridership is also likely to increase. Figure 6.3 shows a graph of annual boardings per capita versus the coverage indicator. Although there appears to be much spread, we can see from the figure that coverage in fact tends to have a positive relationship with ridership.

The existing relationship as shown on Figure 6.3, however, does not appear to be linear; in fact, it was found statistically that a logarithmic regression better fitted the data than a linear regression.

Consequently, this means that the marginal effect of adding a station to the system decreases. In other words, an already large network will not generate as many additional boardings per capita as a smaller network by building new stations. This is quite interesting and perhaps revealing considering the nature and potentials of network effects. Theoretically, it could be argued that adding new stations to a large network should attract a significant number of new boardings.
while in fact the opposite seems to be true. This phenomenon can have many ramifications, one of which is on the energy use per ride (Derrible and Kennedy 2009a, 2009b). Overall, this phenomenon seems to follow the theory of diminishing marginal returns; the work of Tainter (1988) on the collapse of complex societies can be referred to for further information on the concept of diminishing returns linked to infrastructure.

![Annual boardings per capita as a function of coverage $\sigma$.](image)

**Figure 6.3** Annual boardings per capita as a function of coverage $\sigma$.

Moreover, another important point should be discussed. One well-known trade-off exists between speed and coverage, stipulating that higher coverage induces lower speeds (due notably to longer dwell times and lower operating speeds), hence longer travel times, thus impacting ridership. In this work, however, we decided not to account for this phenomenon. First speed cannot be compared alone, provided it notably also depends on technology used; i.e. a system with low coverage could have fairly high speeds, though not attracting many passengers. Second, we assumed that there is no “universal” inter-station spacing to effectively include the variable in the analysis; i.e. the inter-station spacing is specific to the city demographics, not to the mode. Third, for the study of network properties and effects, it was felt that measuring coverage
(accessibility) was more sensible than, for instance, measuring the travel time to the city centre. It could, nevertheless, be included in further studies, notably by considering the indicator introduced by Lam and Schuler (1981; 1982) that relates time and connectivity as introduced in chapter 2.

In order to quantify the relationship between coverage and ridership, Figure 6.3 is re-plotted using the natural logarithm of coverage (Figure 6.4). As a result of using logarithmic values of coverage, all values (on the x-axis) calculated are negative; indeed we are now dealing with the logarithmic function and all coverage values fall between 0 and 1.

Performing a linear regression analysis (ordinary least squared method) results in a slope value of 67.30 and an intercept of 278.66 with t-test statistics being 4.37 and 7.71 respectively, and a goodness-of-fit (adjusted $R^2$) of 0.502. The model parameters are therefore statistically significant; they meet the 95% criteria.

$y = 67.303x + 278.66$

Adjusted $R^2 = 0.502$

Figure 6.4 Annual boardings per capita as a function of the natural log of coverage $\sigma$. 

$\ln \sigma$.
6.4.2 Directness

The directness indicator, equation (6.3), is related to the number of lines and maximum number of transfers as introduced previously in chapter 4 and reviewed in section 6.3.2.

Figure 6.5 illustrates the annual boardings per capita as a function of directness. This figure shows a comparatively good relationship as well. Moreover, note that in this case, the relationship seems to be linear, unlike coverage.

Three networks (bottom three) do not seem to follow the general pattern. These networks are from left to right: Chicago, Athens and San Francisco. All three have the particularity of having low annual boardings per capita figures. The Chicago subway system has a low value of $\delta$, due to the presence of ‘the loop’ (almost all the lines interconnect downtown). The Athens Metro (Figure B.14), though being well connected, is fairly small in comparison to population (3 lines for almost 3.4 million inhabitants). The San Francisco BART (Figure B.13) is often associated with a regional rail system (average inter-station spacing of more than 3km); its design is ‘linear’
and contains many multiple-use edges. The two latter networks seem to lack accessibility that translates into low ridership. Nevertheless, the lack of accessibility is already accounted for in the coverage indicator.

When performing an OLS regression analysis, the slope has a value of 22.53 and the intercept 41.17 with t-test statistics of 3.62 and 1.61 respectively. The goodness-of-fit (adjusted $R^2$) was calculated to be 0.401. The model parameters are therefore statistically significant as well.

Note that by omitting Chicago, Athens and San Francisco, the slope becomes 19.33 and the intercept 67.50 with a goodness-of-fit (adjusted $R^2$) of 0.547. This further demonstrates the strong relationship between directness and boardings per capita as well as showing the reasonable consistency of the slope.

### 6.4.3 Connectivity

The connectivity indicator, equation (4.6), is concomitant to the number of transfer possibilities in a network, the number of multiple edges (to avoid redundancies) and the total number of transfer vertices (as means to standardize the indicator); it was introduced previously in chapter 4 and reviewed in section 6.3.3.

Figure 6.6 shows the annual boardings per capita versus connectivity. It is possible to observe that despite a steeper slope, a relationship is present. The lowest point away from the regression line is the Athens Metro (Figure B.14), for the reasons mentioned above.

Again here, the relationship seems to be linear, unlike coverage. Essentially, this means that adding connectivity generates proportionally similar effects on either small or large networks.

The OLS regression analysis results a value for the slope of 159.15 and -42.34 for the intercept with t-test statistics of 3.81 and -0.93 respectively. The goodness-of-fit (adjusted $R^2$) was calculated to be 0.428.

In this case, the t-test statistic of the intercept is low. Theoretically, the value of the intercept should be positive; i.e. existence of boardings even without connectivity (for one-line networks for instance). This negative intercept can, however, be understood by the steepness of the curve.
Overall, the relationship appears to be fairly strong, and no modifications were made. Moreover, the value of the intercept is not relevant insofar as more emphasis is put on the multiple regression analysis coming in the next section.

![Graph showing annual boardings per capita as a function of connectivity ρ.](image)

\[ y = 159.15x - 42.336 \]

*Adjusted R² = 0.428*

**Figure 6.6** Annual boardings per capita as a function of connectivity ρ.

### 6.4.4 Multiple Regression Analysis

Up to this point, the three indicators were considered separately. None of them seem to be more influential than the others. Indeed, they all show similar values of goodness-of-fit.

As a result, the next step is to perform a multiple linear regression analysis, similarly by using an ordinary least squared (OLS) method. The results are shown in Table 6.2; Table 6.2 a shows the goodness-of-fit results; Table 6.2 b shows the various parameter statistics; Table 6.2 c shows the actual and predicted annual boardings per capita \( B_{pc} \), as well as the residuals.
Table 6.2 Multiple Regression Statistics. a: Results of goodness-of-fit. b: Parameter statistics. c: Results with actual and predicted annual boardings per capita $Bpc$, as well as residuals

a: Results of goodness-of-fit.

<table>
<thead>
<tr>
<th>Goodness of fit</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.784</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.741</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b: Parameter statistics.

<table>
<thead>
<tr>
<th>Parameter Coefficients</th>
<th>Standard Error</th>
<th>t-test values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>97.242</td>
<td>50.472</td>
</tr>
<tr>
<td>Ln of Coverage $\sigma$</td>
<td>44.609</td>
<td>12.549</td>
</tr>
<tr>
<td>Directness $\tau$</td>
<td>8.594</td>
<td>4.996</td>
</tr>
<tr>
<td>Connectivity $\rho$</td>
<td>92.406</td>
<td>32.549</td>
</tr>
</tbody>
</table>

c: Results with actual and predicted annual boardings per capita, $Bpc$, as well as residuals.

<table>
<thead>
<tr>
<th>Metro Networks</th>
<th>Actual $Bpc$</th>
<th>Predicted $Bpc$</th>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toronto</td>
<td>77.23</td>
<td>50.899</td>
<td>26.335</td>
</tr>
<tr>
<td>Montreal</td>
<td>115.05</td>
<td>108.911</td>
<td>6.143</td>
</tr>
<tr>
<td>Chicago</td>
<td>33.53</td>
<td>68.030</td>
<td>-34.499</td>
</tr>
<tr>
<td>New York City</td>
<td>138.69</td>
<td>143.162</td>
<td>-4.467</td>
</tr>
<tr>
<td>Washington DC</td>
<td>114.88</td>
<td>105.118</td>
<td>9.762</td>
</tr>
<tr>
<td>San Francisco</td>
<td>49.97</td>
<td>69.929</td>
<td>-19.954</td>
</tr>
<tr>
<td>Mexico City</td>
<td>109.57</td>
<td>118.624</td>
<td>-9.051</td>
</tr>
<tr>
<td>London</td>
<td>136.49</td>
<td>165.436</td>
<td>-28.946</td>
</tr>
<tr>
<td>Paris</td>
<td>221.46</td>
<td>236.944</td>
<td>-15.487</td>
</tr>
<tr>
<td>Lyon</td>
<td>106.28</td>
<td>132.407</td>
<td>-26.130</td>
</tr>
<tr>
<td>Berlin</td>
<td>130.42</td>
<td>149.038</td>
<td>-18.615</td>
</tr>
<tr>
<td>Madrid</td>
<td>171.81</td>
<td>176.766</td>
<td>-4.953</td>
</tr>
<tr>
<td>Athens</td>
<td>27.23</td>
<td>54.219</td>
<td>-26.992</td>
</tr>
<tr>
<td>Stockholm</td>
<td>156.32</td>
<td>89.504</td>
<td>66.812</td>
</tr>
<tr>
<td>Moscow</td>
<td>235.30</td>
<td>194.249</td>
<td>41.052</td>
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<tr>
<td>Tokyo</td>
<td>197.35</td>
<td>171.045</td>
<td>26.300</td>
</tr>
<tr>
<td>Osaka</td>
<td>100.85</td>
<td>114.035</td>
<td>-13.184</td>
</tr>
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<td>Seoul</td>
<td>165.13</td>
<td>159.984</td>
<td>5.151</td>
</tr>
<tr>
<td>Singapore</td>
<td>101.68</td>
<td>80.962</td>
<td>20.721</td>
</tr>
</tbody>
</table>

The mathematical model relating the network characteristics to annual boardings per capita ($Bpc$) was found to be:
\[ Bpc = 44.61 \cdot \ln{\sigma} + 8.59 \cdot \tau + 92.41 \cdot \rho + 97.24 \]  \hspace{1cm} (6.6)

where \( Bpc \) is the annual boardings per capita, \( \sigma \) is the coverage indicator, \( \tau \) is the directness indicator, and \( \rho \) is the connectivity indicator.

In equation (6.6), note that the intercept of 97.24 is not the lowest annual boardings per capita since the log version of the coverage is negative. Moreover, from looking at the values for the parameters and the t-stats of the three network design indicators, it seems coverage and connectivity share relatively equal weight, while directness is a bit weaker.

The goodness-of-fit (adjusted \( R^2 \)) of 0.741 is reasonably high. Three t-stats reach the 95% confidence level, the remaining value (for directness) falls close to the 95% confidence range. Most model values were found to be within \( \pm 25\% \) of the reported ridership figures. These statistical results suggest that the three indicators have a pertinent impact of ridership and that maximizing them may increase annual boardings per capita. A separate analysis further showed little correlation between the three variables. The strongest occurs with directness and connectivity, having an adjusted \( R^2 \) of 0.26, which is understandable. When increasing the number of transfer possibilities (i.e. connectivity), directness should logically increase also since more paths exists to go from one vertex to another one, thus potential lowering the maximum number of transfers.

Figure 6.7 shows the predicted annual boardings per capita versus actual \( Bpc \); 45° line (i.e. \( y=x \)) was added to show the desired results.

The predictions rendered by this model should be interpreted cautiously. They may not be accurate, and yet, they offer valuable guideposts. Most importantly, the model should be used to compare network design proposals and determine which design is likely going to result in a higher increase in ridership. Following this logic, relative results between scenarios may be more important than the absolute ridership values calculated. This model is thought to be more of a testing, screening or sketching tool rather than a conventional travel demand forecasting model. Which is why, results can be most useful during the strategic/conceptual planning phase of a transit system. In addition, such predictions allow transit planners to compare their designs with
other systems in the world, which can be used to set effective targets or even as marketing tool to the public. Further information on this topic will be presented in the next chapter.

![Figure 6.7 Predicted versus Actual number of boardings per capita.](image)

### 6.5 Discussion

The statistical analysis showed high correlation between ridership per capita and coverage, directness and connectivity.

In future studies, other network design indicators could be included, such as line type. Vuchic (Vuchic 2005) identified four main types of lines as seen in chapter 2: radial, diametrical, tangential, and circle; most of which can also have branches and/or loops, etc (see chapter 2). Including such indicators, however, may create problems depending on the way line types are modeled. The use of dummy variables (e.g. 1 if line is radial, 0 otherwise) could be introduced but the low number of networks studied makes it difficult to account for.
Moreover, we did not include the presence of other transit modes in each city. For instance, the bus mode often acts as a feeder to the metro. This is also true for Light Rail Transit (LRT). The performance of other transit modes is most likely significant and related to the performance of the metro. Here, we tried to overcome this discrepancy by considering boarding levels for the metro mode only. Furthermore, we mainly chose cities with extensive and predominant metro systems to avoid this problem. Indeed, the presence of a large and predominant network may minimize the impacts of these other modes; i.e. we assume most transit riders use the metro only. As a matter of fact, the Rome metro was purposely discarded for this reason. Nevertheless, future work could include the other transit modes, but is likely to be time-consuming; notably because the different transit technologies would have to be carefully modeled. Alternatively, an option could be to include an “inter-modality” indicator.

6.6 Conclusion

In this chapter, we have analysed 19 metro networks located around the world. The purpose was to investigate the role of network design on ridership. To do so, updated graph theory concepts were used as introduced in chapter 3. The 19 metro networks were all compared/assessed using annual boardings per capita as an indicator of ridership.

First, coverage $\sigma$ was introduced as a way to capture concepts of accessibility to transit. This was done by considering the number of stations present in a system over served area. Second, directness $\tau$ deals with the maximum number of transfers to go from one station to another; a high number of transfers can be seen as an inconvenience to transit riders. Third and final, connectivity $\rho$ relates to the number of transfer possibilities in a network. The three indicators showed a relatively strong relationship with respect to annual boardings per capita.

Performing a multiple regression analysis showed a significantly strong relationship, with a goodness-of-fit (adjusted $R^2$) of 0.741. In addition, all t-test statistics were statistically significant. It is apparent that the three components introduced play a key and relatively equal role in network design. They, therefore, can have an important impact on ridership. As a result, for future transit projects, the aim should be to adequately maximize coverage (without impeding speed) and maximize connectivity while keeping trips as direct as possible.
The role of metro network design therefore seems to be particularly important to generate positive indirect effects, and in particular to attract people to transit and generate higher ridership. To further strengthen this analysis, other relevant indicators could be developed, notably by adding line type characteristics, travel times, and also by considering other existing transit modes.

The work presented in this chapter, as well as the works presented in chapters 4 and 5, will now be applied to the proposed Toronto plans for the next 15 and 25 years.
Chapter 7
Application of Network Design Concepts

7.1 Introduction

Population growth in the Greater Toronto Area (GTA) has overstressed the transportation system. The regional transportation agency, Metrolinx, issued 15-year and 25-year transit plans to significantly extend the present transit system. In the next 15 years, about 42 transport infrastructure projects are envisioned, 38 of which are directly related to public transportation; capital investments are in the order of CA$30 billion. Furthermore, the 25-year plan includes another 10 projects, one of which is the Queen subway line, adding another CA$20 billion of investments (Metrolinx 2008). In the past three decades, relatively little has been done to improve the GTA public transportation service; however, the coming three decades will likely witness major changes and innovations in the entire transportation realm of the region. But are these plans effective and suited for the needs of the region? Throughout the chapter, the 15 and 25-year plans are referred to as Toronto 15 and Toronto 25 respectively.

For this chapter, we apply the concepts presented in chapters 4, 5 and 6; the methodology presented in chapter 3 is again used here. During the planning of transit, these concepts should be used along with conventional planning tools. They are especially useful at the strategic/conceptual planning phase, notably to test different network design scenarios. Moreover, by comparing the proposals with other transit systems in the world, these concepts can help find possible improvements and set new targets.

Considering the novelty of these concepts, we wanted to apply them to a practical case study (here, the plans for Toronto) to show their relevance and potential uses. Essentially, the overall goal of this chapter is to demonstrate how an analysis of network properties and effects can help the strategic planning of public transportation networks.

Due to the scale of the plans, we restrict our analysis to the projects proposed for the city of Toronto only, which is the major economic and population centre of the Greater Toronto Area. These projects incorporate the Transit City Plan (TTC 2007) (seven light rail transit lines), one new subway line, two subway lines extensions and the extension of the Scarborough RT (Metrolinx 2008).
More specifically, the objectives of this chapter are:

- Define the existing Toronto metro, as well as the 15 and 25-year plans using the methodology presented in chapter 3

- Study the impact of these plans on the characteristics of the metro using the concepts from chapter 4

- Study the impact of these plans on the complexity and robustness of the metro using the concepts from chapter 5

- Predict future ridership and propose potential improvements to maximize three network design indicators using the concepts from chapter 6

It should be remembered that the model developed in chapter 6 is more of a sketching or testing tool rather than a travel demand forecasting tool. As such, relative predictions (comparing the current Toronto metro with the plans) may be more informative than absolute ridership.

Here, the plans are first presented in detail, and a graphical representation is shown for each plan. Then the results are presented, containing values for the concepts introduced in chapters 4, 5 and 6; these results solely include values for the existing Toronto metro, as well as for Toronto 15 and 25; results for the other metros can be found in the previous chapters. Afterwards, the three last objectives are achieved. That is, the characteristics of both plans are presented and compared to other metros; a similar analysis is performed on the complexity and robustness of the plans; and finally, future ridership predictions are given, and possible improvements enlisted.

This chapter is partially based on the article “Evaluating, Comparing, and Improving Metro Networks: an application to the Toronto proposed plans” by Derrible and Kennedy (2010d), specifically for the fourth objective.

### 7.2 Toronto Metro Network

In this section, we apply the graph theory methodology to the Toronto’s existing metro network and the proposed 15-year and 25-year plans. Due to the scale of the plans, only the transit
projects proposed for the city of Toronto are studied; these incorporate the seven Light Rail Transit (LRT) lines from the Transit City Plan (TTC 2007), the two subway extensions, the Scarborough RT extension, and the proposed subway line on Queen St.

We recall that the concepts that were presented in this thesis were developed for metro lines only, whilst we need to apply them to LRT lines as well in this chapter; i.e., we consider the proposed LRT lines to be akin to metro lines; in other words, we assume the proposed LRT lines are metro lines. Although light rail and heavy rail do not share the same technology, they share other relevant characteristics, e.g., (semi)-exclusive right-of-way (ROW A vs. ROW B), medium-to-high capacity, frequent service, high speed, comparable ‘image’ to the public, etc. The underlying assumption here is that transit riders will not favour using one mode over the other. Nonetheless, the use of the ridership model (equation (6.6)) may inflate the predictions.

More particularly, the ridership model requires values of population to calculate $B_{pc}$. Initially, population data was used to standardize annual boardings and compute $B_{pc}$ values to avoid biases related to population size; here, however, it is used during sketching/forecasting to estimate total boardings. In addition, coverage requires population to calculate served area, along with population density. In chapter 6, we identified the Toronto served area to be 1,038 km². Although the administrative City of Toronto covers about 630 km² of land, some people living outside the city boundaries still use the metro on a daily basis (e.g., City of Vaughan residents). Considering the scale of the Metrolinx plans (GTA wide), even more people are likely to use the system. In addition, the current served population was calculated to be 3.435 million inhabitants, which is likely to increase in the 15 and 25-year frame. For example, the Ontario Ministry of Finance estimated an increase in the GTA population from 5.9 million in 2006 to 6.9 million by 2016 and 8.3 million by 2031 (Ontario Ministry of Finance 2007). We prefer, however, to fix the Toronto population and population density to current levels. This results in smaller total boarding values and thus offers a balancing effect to the assimilation of LRT as metro.

Finally, the coverage indicator (for the ridership model) also requires a value for the total number of stations. The present average spacing between the stations is 1 km. We therefore assume a similar inter-station spacing will be applied to the proposed network.

We now apply the graph theory methodology presented in chapter 3 to the Toronto existing system (as previously shown in chapter 3) and the 15 and 25-year transit plans.
7.2.1 Existing System

The existing Toronto metro network is composed of four lines (Bloor, Spadina-University-Yonge, Scarborough RT, and Sheppard East), with a route length of 68.75 km, and 69 stations. It has a total of 10 vertices (five transfer and five end-vertices), 11 edges (10 single and one multiple-edge), and a maximum number of transfers $\delta$ of three. The total number of transfer possibilities $V'_c$ is five. Figure 7.1 shows the existing four lines along with their vertices and edges.

![Graphical representation of the existing Toronto metro network](image)

Figure 7.1 Graphical representation of the existing Toronto metro network

Note that the Spadina and St Clair segregated streetcar lines were not included in agreement with the Metrolinx document (Metrolinx 2008). In this case, the streetcar technology cannot be assimilated to heavy rail technology because of their significant difference; moreover, these two streetcar lines have shorter inter-stop spacing (i.e., frequent stops), which significantly lowers their operating speed.

7.2.2 15-year Plan

The proposed 15-year plan includes six of the seven LRT lines present in the Transit City Plan (TTC 2007), the extensions of the two Spadina-University-Yonge termini, and the extension of the Scarborough RT. Several options are proposed for the latter. We will assume the
Scarborough line will extend to Markham Rd and Sheppard Ave East (3.7 km); hence joining the Sheppard East LRT line.

The new Light Rail Transit (LRT) lines are:

- Don Mills (line 5 on Figure 7.2) – 17.6 km
- Eglinton Crosstown (line 6) – 30.8 km
- Etobicoke – Finch West (line 7) – 17.9 km
- Jane (line 8) – 16.5 km
- Sheppard East (line 9) – 13.6 km
- Waterfront West (line 10) – 11 km

The subway and RT Extension are:

- Downsview Station to Vaughan Corporate Centre (ext. 1 on Figure 7.2) – 8.6 km
- Finch to Langstaff (ext. 2) – 6.8 km
- Kennedy to Sheppard (ext. 3) – 3.7 km

Figure 7.2 shows a graphical representation of the proposed 15-year plan; the new lines and extensions are represented as dashes.

The new network would therefore have a total of 10 lines, with a route length of 198.85 km (note that we added 3.6 km to link the Exhibition place terminal of the Waterfront West line to Union
Station, i.e., the current 509 streetcar line), and 199 stations (assuming a 1 km inter-station spacing). It would have a total of 28 vertices (19 transfer and nine end-vertices), 38 edges (37 single and one multiple-edge). The maximum number of transfers \( \delta \) would remain three. The total number of transfer possibilities \( V'_c \) would leap to 21 as opposed to five presently.

### 7.2.3 25-year Plan

The 25-year plan includes the seventh LRT line from the Transit City plan (TTC 2007); it also proposes a new downtown subway line on Queen St. from Dundas West and Bloor to Greenwood and Danforth.

The new Light Rail Transit (LRT) line is:
- Scarborough Malvern (line 11 on fig.3) – 15 km

The new Subway line is:
- Queen St. (line 12 on fig.3) – 15 km

Figure 7.3 shows a graphical representation of the 12 lines present in the 25-year plan along with their vertices and edges. The two new lines are represented by dashes.

![Figure 7.3 Graphical representation of the 25-year Toronto metro plan](image)
At the completion of the 25-year plan, the Toronto rapid transit network would consist of 12 lines, with a route length of 228.85 km, and 229 stations (assuming a 1 km inter-station spacing). There would be a total of 33 vertices (23 transfer and 10 end-vertices), 46 edges (45 single and one multiple-edge), and an identical maximum number of transfers $\delta$ of three. The total number of transfer possibilities $V'_c$ would increase further by six possibilities, totalling 27.

### 7.3 Results

At this point, the existing Toronto system has been defined graphically following the methodology presented in chapter 3. A similar process was applied to the Toronto 15 and Toronto 25 plans, where LRT lines were considered metro lines. In this section, we provide the results, for these three systems. Table 7.1 shows these results from the application of the concepts presented in chapter 4, 5 and 6. First the basic indicators can be seen, followed by the values for the State, Form and Structure characteristics.

<table>
<thead>
<tr>
<th>Metro Networks</th>
<th>Toronto Existing</th>
<th>Toronto 15</th>
<th>Toronto 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route Length</td>
<td>$R$ (km)</td>
<td>68.75</td>
<td>198.85</td>
</tr>
<tr>
<td>Stations</td>
<td>$N_S$</td>
<td>69</td>
<td>199</td>
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<tr>
<td>Lines</td>
<td>$N_L$</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Vertices</td>
<td>$V$</td>
<td>10</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>$V^e$</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>$V^t$</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>Basic Indicators</td>
<td>Line Attribute</td>
<td>Hosting $x$ lines</td>
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</tr>
<tr>
<td></td>
<td>$\ell$</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0</td>
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<td></td>
<td></td>
<td>5</td>
<td>0</td>
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<tr>
<td></td>
<td></td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Edges</td>
<td>$E$</td>
<td>11</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>$E^e$</td>
<td>10</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>$E^m$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Max. number of transfers</td>
<td>$\delta$</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7.1 Results for the existing Toronto metro, as well as for the 15 (Toronto 15) and 25-year (Toronto 25) plans.
<table>
<thead>
<tr>
<th>Characteristics</th>
<th>State</th>
<th>Complexity $\beta$</th>
<th>Degree of Connectivity $\gamma$</th>
<th>Form</th>
<th>Av. line length $A \ (km)$</th>
<th>Number of Stations $N_S$</th>
<th>Structure</th>
<th>Structural Connectivity $\rho$</th>
<th>Directness $\tau$</th>
<th>Complexity and Robustness</th>
<th>Scaling Factor $\varepsilon$</th>
<th>Clustering Coefficient $\gamma$</th>
<th>Robustness Indicator $r^T$</th>
<th>Ratio of Transfer to Total Vertices $V^T / N_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complexity</td>
<td>$\beta$</td>
<td>1.10 1.36 1.39</td>
<td>0.46 0.49 0.49</td>
<td>Av. line length $A \ (km)$</td>
<td>17.19 19.89 19.07</td>
<td>Number of Stations $N_S$</td>
<td>69 199 229</td>
<td>Structural Connectivity $\rho$</td>
<td>0.80 1.05 1.13</td>
<td>Directness $\tau$</td>
<td>1.33 3.33 4.00</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
</tr>
<tr>
<td>State</td>
<td></td>
<td></td>
<td></td>
<td>Form</td>
<td></td>
<td></td>
<td></td>
<td>Structural Connectivity $\rho$</td>
<td>0.80 1.05 1.13</td>
<td>Directness $\tau$</td>
<td>1.33 3.33 4.00</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
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<tr>
<td>Degree of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Directness $\tau$</td>
<td>1.33 3.33 4.00</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
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<tr>
<td>Connectivity</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Connectivity $\rho$</td>
<td>0.80 1.05 1.13</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coverage</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Coverage $\sigma$</td>
<td>0.052 0.151 0.173</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
<td></td>
<td></td>
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<tr>
<td>Directness</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Directness $\tau$</td>
<td>1.33 3.33 4.00</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connectivity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Connectivity $\rho$</td>
<td>0.80 1.05 1.13</td>
<td>3.68 4.02 3.99</td>
<td>0.0145 0.0503 0.0568</td>
<td>0.0725 0.0955 0.1004</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Then, the complexity and robustness indicators are shown, and finally the values for the indicators of coverage, directness and connectivity, as well as the annual boardings per capita $B_{pc}$ and total annual boardings. In general, by looking at the various results, we can see that the metro plans significantly modify the current state of the metro design. For instance, the complexity $\beta$ of the Toronto metro is to evolve from 1.10 to 1.39, and the scaling factor $\varepsilon$ is to shift from 3.68 at the moment to 3.99 for Toronto 25.

The following sections analyses these results in detail, notably by comparing them to other metro networks in the world, and finding potential avenues for improvements.
7.4 Characterizing the Metro Plans

The characteristics of metro networks were defined with respect to their *State*, *Form* and *Structure* as developed in chapter 4. In this section, we analyse these three characteristics for the metro plans and look at their evolution compared to other metro systems in the world.

Figure 7.4 shows the *State* of the initial 33 networks plus Toronto 15 and Toronto 25. Overall, we see that the Toronto metro would move from phase 1 to phase 2 for both plans and start to follow the quasi-linear trend that appears in phase 2 and 3 metro networks. More specifically, the complexity $\beta$ evolves by 24% for Toronto 15 (1.36) and by 26% for Toronto 25 (1.39). The degree of connectivity $\gamma$ evolves by roughly 6.5% for both Toronto 15 and 25. The smaller evolution of the degree of connectivity $\gamma$ can be explained by the topological nature of the plans, which appears to follow a grid-pattern. As a result, the *State* of the Toronto metro network would compare to the Mexico City and Berlin metro.

Figure 7.5 shows the *Form* of the initial 33 networks plus Toronto 15 and Toronto 25. Overall, we see that the Toronto metro would move from the regional accessibility zone at the moment, to the border between the regional coverage and local coverage zones. More specifically, the average line lengths $A$ evolve by 15.7% for Toronto 15 (19.89km) and 10.9% for Toronto 25 (19.07km); this is a relatively insignificant increase considering the overall increase in route length of 189% for Toronto 15 (198.85km) and 233% for Toronto 25 (228.85km). A similar evolution is observed for the number of stations $N_S$ since the average inter-station spacing was kept constant at 1km. Consequently, the *Form* of the Toronto metro network should compare loosely to Berlin, Madrid and Tokyo. Such a development may be beneficial for the metro and the city; a local coverage *Form* may induce more compact land-use.

Figure 7.6 shows the *Structure* of the initial 33 networks plus Toronto 15 and Toronto 25. Overall, we see that the Toronto metro would move from being connectivity-oriented to becoming integrated. More specifically, the directness $\tau$ evolves by 150% for Toronto 15 (3.33) and by 200% for Toronto 25 (4.00); and connectivity $\rho$ evolves by roughly 32% for Toronto 15 (1.05) and by 41% for Toronto 25 (1.13). Once again, the smaller evolution of connectivity can be explained by the grid-pattern design of both Toronto 15 and Toronto 25. As a result, the *Structure* of the Toronto metro network would compare to Seoul and Berlin.
Figure 7.4 State of Planned Toronto Metro Networks.
Figure 7.5 Form of Planned Toronto Metro Networks.
Figure 7.6 Structure of Planned Toronto Metro Networks.
Although the *State* of the Toronto metro would compare to the Mexico City metro, we can see that the Mexico City metro is achieving better in terms of its *Structure*, while having a similar number of lines. Essentially, this means that the Toronto plans could focus on maximizing their connectivity and directness.

Moreover, although the *Form* of the Toronto metro would compare to the Madrid and Tokyo metro, both systems are actually in phase 3 of *State* and are achieving better in terms of their *Structure*. A similar argument can be made by comparing the *Structure* of the Seoul metro to the Toronto plans.

From this analysis, it seems that the connectivity and directness of the Toronto plans could be improved, which would render the Toronto metro as more integrated.

We will now analyse the complexity and robustness of the Toronto plans.

## 7.5 The Complexity and Robustness of the Metro Plans

The complexity of metro network was defined by the scale-free and small-world properties of metro networks.

By performing the statistical procedure presented in section 5.2.1.3, we find that both Toronto 15 and Toronto 25 are scale-free networks; both have statistically significant indicators (adjusted $R^2$ and t-test) and pass the $\chi^2$-test. The scaling factors $\varepsilon$ were calculated to be 4.02 for Toronto 15 and 3.99 for Toronto 25; both scaling factors are actually larger than the scaling factor of the existing system, which was 3.68.

Essentially, this means that the Toronto plans are more of the “egalitarian” type, having a “thin-tail” distribution; i.e. few transfer stations exist. Figure 7.7 shows the scaling factors of the previously studied metro networks and Toronto 15 and 25 as a function of total number of stations.
While we noticed that metro network tend to follow a pattern as they grow in size, we can see that the Toronto plans do not follow this pattern, and have significantly larger scaling factors than their peer networks on Figure 7.7. This phenomenon is likely going to be reflected in the robustness analysis.

With respect to the small-world property, both plans obey the two rules presented in section 5.2.2.1. The first rule related to the average shortest-path length; by keeping the maximum number of transfers constant while building up to eight new lines, this rule is easily met by the Toronto plans. The second rule related to the clustering coefficient (or degree of connectivity) $\gamma$, which was already introduced in the previous section for the State characteristic. Here again, the Toronto plans met the requirements.

An interesting phenomenon is observable concerning the evolution of the clustering coefficient $\gamma$. By comparing the evolution of the clustering coefficient to other networks in the world (Figure
5.4), we see that the Toronto plans are under-achieving compared to most peer networks. Here again, this is due to the grid-pattern design of the plans. To increase $\gamma$, the plans should incorporate more edges between existing vertices, which would simultaneously increase the number of transfer stations hosting more than two lines (i.e. transfer vertices with $\ell > 2$).

As seen in chapter 5, metros with higher clustering coefficients and low scaling factors tend to be more robust. For the Toronto plans, the clustering coefficients remains particularly low, while the scaling factor actually increases; this should translate into poor robustness.

Numerically, the robustness $r^T$ evolves by 247% for Toronto 15 (0.0503) and by 292% for Toronto 25 (0.0568), compared to 0.0145 for the existing Toronto metro. Although this change appears significant singularly, it is worth remembering that the existing system is doing relatively poorly in terms of robustness.

**Figure 7.8** Degree of connectivity $\gamma$ as a function of number of stations $N_S$ for 33 Metro Networks plus Toronto 15 and Toronto 25. Only black-coloured points are labelled.
Figure 5.5 shows the robustness of the previously studied metro networks and Toronto 15 and 25 as a function of total number of stations. It is clear from this figure that the Toronto plans are under-achieving compared to most peer networks. The Barcelona metro, which was the example of a phase 2 metro in section 5.3.3.3, has a robustness of 0.1138 (a 100% difference).

This poor robustness seems to be directly correlated with the grid-pattern topological nature of the plans as already mentioned multiple times.

As a result, few transfer stations exist compared to the total number of stations, which notably affects the connectivity of the plans. Similarly, this feature seriously impacts the robustness as illustrated on Figure 5.6.

Regarding the network design of the Toronto plans, it appears that there is one significant issue that could be addressed at the planning phase. To address this issue, more lines should be intersecting each other, which would either create new transfer stations or provide new transfer
possibilities to existing transfer stations. Such changes are likely to improve the *Structure* of the metro, possibly the *State* as well; in addition, it should increase the clustering coefficient, lower the scaling factor, and thus improve robustness.

![Figure 7.10 Robustness $r^T$ versus percentage of transfer-vertices $V_t$ (i.e. $V_t/N_S$) for 33 Metro Networks plus Toronto 15 and Toronto 25. Only black-coloured points are labelled.](image)

At this point, we will examine the potential impacts of the Toronto plans network design on future ridership levels.

### 7.6 Future Ridership Predictions of the Metro Plans

In this section, the three network design indicators of chapter 6 (coverage, directness and connectivity) are calculated for Toronto 15 and Toronto 25. Consequently, we can provide an estimate of future ridership potentials by using the model presented in chapter 6 (equation (6.6)).
Secondly, we can compare the Toronto plans to other metro networks in the world (here 18 other systems), identify peer networks and offer a more detailed comparison analysis.

7.6.1 Future Ridership

The existing Toronto metro has values of coverage, directness and connectivity of 0.052, 1.33 and 0.80 respectively. By using equation (6.6), the predicted annual boardings per capita \( B_{pc} \) is 54.15.

The actual total annual number of boardings for the metro was recorded to be 265.3 million in 2006 (APTA 2009) or 77.23 boardings per capita. With 54.15, the predicted \( B_{pc} \) is 30% lower than the actual value. Although this discrepancy seems large, it is still a reasonable approximation considering the limited number of inputs. In addition, it appears that the ridership values calculated for Toronto are conservative. As a result, future ridership values calculated may be conservative as well, which could be another balancing effect to the assimilation of LRT as metro.

With six new lines, the 15-year plan achieves a coverage of 0.151, a directness of 3.33 and a connectivity of 1.05, which represents increases of 190%, 150% and 32% respectively. This design is estimated to generate annual boarding per capita \( B_{pc} \) of 140.26, or a total of 482 million boardings per year, which is an extra 216.7 million boardings over current levels (82% increase).

With two additional lines, the 25-year plan has an overall coverage of 0.173, a directness of 4.00 and a connectivity of 1.13 respectively, which represents increases of 232%, 200% and 41% respectively to the existing system, and increases of 15%, 20% and 7.4% respectively to Toronto 15. Using equation (6.6), the value of predicted annual boarding per capita \( B_{pc} \) is 158.80, or a total of 546 million boardings per year. This adds 280.7 million boardings to current ridership levels (105% improvement), or an additional 64 million boardings per year to the 15-year plan, which represents 13.3% improvement.
It is obvious that both the 15-year and 25-year plans significantly increase the three network indicators and ridership. We can further analyze the proposed plans by comparing them to other existing metro networks in the world.

### 7.6.2 International Comparison

The proposed Toronto transit improvements can be compared to other transit systems in the world. In the first instance, we look at 18 other networks in the world, hence taking a global vision. We then refine the analysis to four networks (Berlin, Madrid, Singapore and Mexico City) that have similar city characteristics, number of lines or route length.

Table 7.2 shows values of the annual boardings per capita and the three network indicators for the current Toronto metro system, the two proposed plans and 18 other systems in the World. It also shows respective values for the 25th, 50th, and 75th percentile.

From Table 7.2, we see that the current system is doing poorly with respect to $B_{pc}$, coverage, directness, and connectivity, being close or below the 25th percentile. The proposed plans, however, would significantly increase the three network indicators and, in-turn, ridership.

With respect to boardings per capita, $B_{pc}$, Toronto 15 (140.26) and Toronto 25 (158.80) are comparable with New York City (138.69), London (136.49), Berlin (130.42), Stockholm (156.32) and Seoul (165.13). The proposed plans would therefore put Toronto among some of the major transit systems in the world, although not reaching the most used systems present in this dataset (Moscow, Paris, Tokyo and Madrid).

The three network indicators also show significant improvements from the current system. Coverage $\sigma$ would reach 15% and 17.3% of the served area for Toronto 15 and 25 respectively, which is larger than Berlin and Tokyo, although smaller than Madrid, Paris and Seoul. As neither plans increase the maximum number of transfers $\delta$, directness $\tau$ (which is the lowest at the moment) would increase to 3.33 and 4.00; hence placing it along side with New York City, Mexico City, Berlin and Seoul. Similarly, connectivity $\rho$ would increase to 1.05 and 1.13, thus becoming comparable to New York City, Berlin and Osaka. Overall, the Toronto metro system would move up from the 25th to the 50th percentile for all measures.
Table 7.2 City characteristics, ridership and network indicators for 19 Metro Networks plus Toronto 15 and 25.

<table>
<thead>
<tr>
<th>Metro Network</th>
<th>Population (million)</th>
<th>Served Area (km²)</th>
<th>Annual Boardings (million)</th>
<th>Boardings per capita</th>
<th>Coverage σ</th>
<th>Directness τ</th>
<th>Connectivity ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toronto</td>
<td>3.435</td>
<td>1,038</td>
<td>265.3</td>
<td>77.23</td>
<td>0.052</td>
<td>1.33</td>
<td>0.80</td>
</tr>
<tr>
<td>Toronto 15</td>
<td>3.435</td>
<td>1,038</td>
<td>482.5*</td>
<td>140.26*</td>
<td>0.151</td>
<td>3.33</td>
<td>1.05</td>
</tr>
<tr>
<td>Toronto 25</td>
<td>3.435</td>
<td>1,038</td>
<td>546.3*</td>
<td>158.80*</td>
<td>0.173</td>
<td>4.00</td>
<td>1.13</td>
</tr>
<tr>
<td>Montreal</td>
<td>2.418</td>
<td>770</td>
<td>278.2</td>
<td>115.05</td>
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<td>0.144</td>
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<tr>
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<td>171.81</td>
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<td>Moscow</td>
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<td>653</td>
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<td>235.30</td>
<td>0.208</td>
<td>6.00</td>
<td>1.25</td>
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<td>15.070</td>
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<td>2,974.0</td>
<td>197.35</td>
<td>0.104</td>
<td>6.50</td>
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<tr>
<td>Osaka</td>
<td>9.043</td>
<td>1,028</td>
<td>912.0</td>
<td>100.85</td>
<td>0.092</td>
<td>2.67</td>
<td>1.08</td>
</tr>
<tr>
<td>Seoul</td>
<td>13.710</td>
<td>885</td>
<td>2,264.0</td>
<td>165.13</td>
<td>0.254</td>
<td>3.67</td>
<td>1.00</td>
</tr>
<tr>
<td>Singapore</td>
<td>4.277</td>
<td>526</td>
<td>434.9</td>
<td>101.68</td>
<td>0.096</td>
<td>1.33</td>
<td>0.83</td>
</tr>
<tr>
<td>25th percentile</td>
<td>3.579</td>
<td>633</td>
<td>518.6</td>
<td>76.49</td>
<td>0.076</td>
<td>2.540</td>
<td>0.830</td>
</tr>
<tr>
<td>50th percentile</td>
<td>6.250</td>
<td>1,103</td>
<td>945.3</td>
<td>125.75</td>
<td>0.126</td>
<td>3.75</td>
<td>1.06</td>
</tr>
<tr>
<td>75th percentile</td>
<td>10.730</td>
<td>1,872</td>
<td>1,966.0</td>
<td>180.51</td>
<td>0.190</td>
<td>5.162</td>
<td>1.341</td>
</tr>
</tbody>
</table>

*predicted values of annual boardings and $Bpc$

Figure 7.11 (a), (b), and (c) show boardings per capita $Bpc$ as a function of coverage $σ$, directness $τ$ and connectivity $ρ$ respectively for Toronto, Toronto 15, Toronto 25, Berlin, Madrid, Singapore and Mexico City. These metros are used in the more detailed comparison to follow.

In this research, the Berlin metro includes the U-bahn system as well as several lines from the S-bahn system that are located in the city center and adopt the same fare system as the U-bahn. Berlin seems to be the most similar to Toronto, having 3.64 million inhabitants covering 1,044 km² of land; its metro is 151.7 km long. Both Toronto 15 and Toronto 25 would exceed Berlin in annual boardings and $Bpc$ by as much as 17%. Both plans would also outgrow Berlin in terms of coverage. Nevertheless, Berlin is doing fairly well in terms of directness and connectivity, which explains the high number of boardings despite the lower coverage. Toronto 25 seems to do comparatively well in directness and connectivity; hence performing better than Berlin.
(a) Boardings per capita $Bpc$ vs. Coverage $\sigma$.

(b) Boardings per capita $Bpc$ vs. Directness $\tau$. 

Annual boardings per capita $Bpc$

Coverage $\sigma$

Directness $\tau$
The Madrid Metro has grown to be known as a success story in the past 15 years, doubling its route length to a total of 226.7 km. Toronto 15 would almost triple the current route length, and Toronto 25 would increase the current route length by a factor of 3.3 to almost 230 km in total. Nevertheless, it would still not reach current ridership levels experienced in Madrid. In terms of served area, Madrid is significantly smaller than Toronto’s, explaining the discrepancy in the coverage indicator. However, by taking a closer look at directness and connectivity, we see that Madrid is outperforming the two Toronto proposed plans; despite the fact Madrid has comparable number of lines (13 in total). The Madrid network has a total of 46 transfer possibilities $V^i_c$ compared to 21 for Toronto 15 and 27 for Toronto 25.

Compared to Singapore, the Toronto plans perform better. Nevertheless, Singapore is presently carrying out significant improvements to its current system. At its completion (within the next 15 years), the Singapore system will compare to other major metro networks in the world. Finally, Mexico City is a large city, both in terms of population and land area. Considering coverage, the Toronto proposals would cover a higher percentage of served area. Nonetheless, Mexico City’s
current directness and connectivity indicators are strong, reinforcing the possibility to increase these two indicators for Toronto.

As a whole, it seems both the 15 and 25-year plans would carry the Toronto metro system to the same network design levels as metros in other international cities. As a result, Toronto could be doing better than other cities like Berlin. Nevertheless, it still lacks several network properties and characteristics to become comparable to other peer networks as seen as in the previous sections. As a result, there appears to be room for more developments; this is certainly reflected when comparing the directness and connectivity indicators to the Madrid and Mexico City systems. In the next section, we will therefore attempt to identify possible leeway for improvements.

7.7 Possible Improvements

By using the ridership model (equation (6.6)), further possible improvements can be designed. In this section, we propose seven potential improvements that could enhance the Toronto proposed networks substantially. Note that these improvements address the three network design indicators (coverage, directness, and connectivity) primarily; metro characteristics, and complexity and robustness are only briefly discussed.

Considering coverage, the Toronto plans already addressed this feature significantly. As a result, we chose to focus on the two indicators of directness and connectivity (and in particular the latter), which seem to be able to benefit most from this process.

Directness is directly proportional to the number of lines and the maximum numbers of transfers $\delta$. Toronto 15 includes a total of 10 lines and Toronto 25 proposes 12 lines, which is fairly large already. Consequently, the one property that should be dealt with is $\delta$. The maximum number of transfers $\delta$ can be decreased in two ways; either change the current plans to reduce the number of transfers or join two lines together. Presently, three transfers are required to go from the Waterfront West line (line 10) to the Sheppard East line (line 9) or from the Etobicoke-Finch West line (line 7) to the Sheppard East line (line 9); see Figure 7.2 and Figure 7.3. For Toronto 15, if the Sheppard East line was a part of the current Sheppard subway line (lines 4 and 9 are
continuous), $\delta$ would effectively be reduced to two, and directness $\tau$ would become four (Imp. 1 on Figure 7.12). For the Toronto 25 plan, the introduction of the Scarborough Malvern line (line 11) would also render $\delta$ to be three. By joining the Eglinton Crosstown line with the Scarborough Malvern (Imp. 2, lines 6 and 11 are continuous), $\delta$ would be reduced to a value of two (assuming Sheppard and Sheppard East lines are continuous). Therefore, there would be a total of 10 lines; hence the directness indicator $\tau$ would increase to five.

Connectivity is related to the number of transfer possibilities $V'_c$, the number of multiple edge $E^m$ and the total number of transfer stations $V'$ (equation (6.5)). The current number of multiple edges is one, which is sufficiently low. Consequently, two parameters can be evaluated: $V'_c$ and $V'$. The purpose is to increase the former and possibly decrease the latter. By having more transfer possibilities, the network becomes more connected, which in-turn attracts more riders. The second alternative is to eliminate transfer stations; while this may seem illogical at first to enhance connectivity, the rationale here is to promote selective transfers that create “hubs” (transfer stations hosting more than two lines). For instance, the Etobicoke-Finch West line could cross the Spadina-University-Yonge line at Yonge-Sheppard station (Imp. 3 on Figure 7.12). Also, the Jane line (line 8) could join the Etobicoke-Finch West line where it intersects with the Spadina-University-Yonge line (Imp. 4). In addition, the Queen subway line (line 12) could cross the Bloor line (line 2) at Jane station (Imp. 5, where Jane and Bloor lines intersect). We almost see here the emergence of a pseudo-circle line (Queen – Jane – Etobicoke-Finch – Sheppard – Don Mills). Furthermore, the Bloor line could connect with the Eglinton Crosstown line, creating one more transfer possibility (Imp. 6). Another option is to connect the Etobicoke-Finch West line (line 7) with the Eglinton Crosstown line as well (Imp. 7).

Figure 7.12 shows the possible improvements, where changes are represented by dashes (this includes the lines proposed to become continuous).

The collected and calculated properties and characteristics for the proposed Toronto network improvements are shown in Table 7.3. The calculation of the scaling factor rendered statistically significant indicators (adjusted $R^2$ and t-test) and passed the $\chi^2$-test.
With these seven improvements, the State of the metro would remain in phase 2 but move slightly closer to Barcelona. The Form of the metro would move to the regional coverage zone. The Structure of the metro would be fully integrated and closer to Paris.

The clustering coefficient $\gamma$ would increase to 0.56 and the scaling factor $\varepsilon$ lowers to its original value of 3.68. Although the ratio of transfer to total vertices decreases to 0.0788 (compared to 0.1004 for Toronto 25), the robustness actually increases to 0.0622, which is almost a 10% increase compared to Toronto 25.

![Figure 7.12](image)

**Figure 7.12** Graphical representation of possible improvements to the 25-year Toronto Network plan

By assuming the addition of 12km of rail (Imp. 6 and 7), coverage would increase slightly by 5% since the Bloor line and Etobicoke-Finch West line would be extended to the Eglinton Crosstown line; note that the impact on coverage was minimized.

Furthermore, directness $\tau$ would increase to five, which represents a 25% increase to Toronto 25. Moreover, even though the number of transfer stations would decrease from 23 to 19 and shift $V^t_c$ from 27 to 24, connectivity increases to 1.21, which is 7% increase to Toronto 25.

Overall, by using the ridership model, the expected annual boardings per capita would reach a total of 176.07, which is a 12% increase compared to Toronto 25; thus the network design improvements would place Toronto among some of the most effective metros on the planet. This translates into a total annual number of boardings of about 605 million, which is a 128% increase.
to current annual boardings. Compared to Toronto 25, this represents 59.4 million additional boardings.

Finally, even though some of these improvements may appear costly at first, performing such a network design analysis may actually reduce capital investment needed; for instance these improvements reduce the number of stations to be built by 4. Having continuous lines (instead of two separate ones) may also be preferable with regards to capital funding (although transit technology plays an important role in the case of Toronto).

Table 7.3 Results for the Toronto metro with seven proposed improvements.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>State</th>
<th>Complexity $\beta$</th>
<th>Degree of Connectivity $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form</td>
<td>Av. line length $A$ (km)</td>
<td>24.09</td>
<td></td>
</tr>
<tr>
<td>Number of Stations $N_S$</td>
<td>241</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structure</td>
<td>Structural Connectivity $\rho$</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Directness $\tau$</td>
<td>5.00</td>
<td></td>
</tr>
<tr>
<td>Complexity and Robustness</td>
<td>Scaling Factor $\varepsilon$</td>
<td>3.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Clustering Coefficient $\gamma$</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Robustness Indicator $r_T$</td>
<td>0.0622</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ratio of Transfer to Total Vertices $V^t / N_S$</td>
<td>0.0788</td>
<td></td>
</tr>
<tr>
<td>Future Ridership Predictions</td>
<td>Coverage $\sigma$</td>
<td>0.182</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Directness $\tau$</td>
<td>5.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Connectivity $\rho$</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Annual Boardings per capita $B_{pc}$</td>
<td>176.07$^1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Boardings (millions per year)</td>
<td>605.7</td>
<td></td>
</tr>
</tbody>
</table>

$^1$ predicted values of annual boardings and $B_{pc}$
Overall, it is therefore possible to see how these concepts of network design can be applied most practically. Moreover, these seven proposed improvements account for the analyses already produced into elaborating the two plans, i.e., no substantial modifications (i.e. change of corridor location, addition of a significant number of lines, etc.) have been proposed. Corridor location, for instance, is assumed to have been previously studied thoroughly, notably by looking at population density, origin-destination pairs, etc., and is not jeopardized by this network design analysis. In fact, the resulting network design is only marginally modified.

In addition, although a number of these proposals may not be practical, possibly due to micro-level constraints (e.g., too few lanes on particular streets), an appreciation of network design remains desirable and can lead to significant improvements that notably increase ridership.

### 7.8 Conclusion

The existing Toronto transportation system is overstressed, and considering the future population growth forecasts, the situation is not likely to improve on its own. As a result, the Toronto regional transportation agency, Metrolinx, has issued two plans to address these challenges. The scale of the plans is large, adding up to eight lines and more than tripling the current route length. The goal of this chapter was to offer an application of the concepts presented in chapters 4, 5 and 6, thus study the network feature of the plans.

Firstly, we translated the 15-year and 25-year plans into graphs using the methodology introduced in chapter 3.

We then examined the metro characteristics of the plans, and found significant improvements in the network feature of the metro. The plans would move the Toronto metro to phase 2 of the State characteristic. With respect to Form, the Toronto metro would move to the border between the regional coverage and local coverage zones. Finally, it would an integrated typed Structure. Nevertheless, it does not necessarily compare to peer networks, notably because of the topological nature of the plans which follow a grid pattern. In particular, indicators of connectivity and directness show relatively poor results that should be maximized.
Subsequently, we studied the complexity and robustness of the plans. Both Toronto 15 and Toronto 25 were scale-free and small-worlds. Although the Toronto metro is to significantly grow in size, it seems to have a particularly low clustering coefficient (0.49 for Toronto 25) and significantly high scaling factor (4.02 for Toronto 25), which seriously impact its robustness. Moreover, the evolution of the Toronto network counters several of the trends that were identified in chapter 5, notably concerning the magnitude of the scaling factor as a function of the number of stations. Once again, the grid-pattern design seems to be partially responsible for these poor results.

After, we calculated the three network design indicators (coverage, directness and connectivity) and estimated future ridership. While the existing Toronto metro shows particularly low values for the three network indicators, both the 15-year and 25-year plans seem to address them effectively, which would place Toronto at the same level as other renowned metro systems like Madrid and Berlin. The 15-year plan would cover 15.10% of the served area with a directness $\tau$ of 3.33 and a connectivity $\rho$ of 1.05. The 25-year plan would further improve the network by covering 17% of the served area, having a directness $\tau$ of 4.00 and a connectivity $\rho$ of 1.13. As a result, annual boardings per capita $Bpc$ were predicted to reach 140.3 and 158.8, or total annual boardings of 482 and 546 million for Toronto 15 and Toronto 25 respectively. While the existing Toronto system ranked in the lower 25th percentile when compared to 18 other metro networks in the world, the two plans would leap Toronto to the 50th percentile.

Finally, seven possible improvements were proposed, notably to further increase ridership, but also to lower the scaling factor. For instance, the Jane line could connect with the Etobicoke-Finch West line and the western segment of the Spadina-University-Yonge line at a single station. The eastern terminal of the Etobicoke-Finch West line could reach the Sheppard-Yonge station, hence facilitating transfers with the Sheppard subway line. Moreover, the Etobicoke-Finch West line and the Bloor subway line could intersect with the Eglinton Crosstown line. In addition, the Queen subway line could terminate at Jane station instead of Dundas West as presently planned. Finally, the Sheppard subway line and the Sheppard East LRT line could be continuous as well as the Eglinton Crosstown line and the Scarborough Malvern line. Overall, these proposed improvements would decrease the scaling factor to 3.68 (which is the value of the existing system), further increase directness to 5.00 and connectivity to 1.26; they would render a
total of 180 annual boardings per capita, or 619 million annual boardings, which is a 134% increase from current levels, or 13.3% from Toronto 25.

No major changes were proposed to modify the plans. As mentioned, the concepts presented in this thesis are most useful if applied in complement to traditional planning techniques.

Overall, network design is a significant component of transit planning. It is therefore important to analyze the possible impacts of different scenarios. In the case of Toronto, although the Metrolinx plans would significantly improve the existing Toronto metro system, we identified key components in their network design that could be improved, and hence provide a better transit system to the Toronto Region population.
Chapter 8
Conclusion

8.1 Introduction

The year 2008 was a landmark in the history of the world as more than 50% of the world population lived in cities. Moreover, cities all around the planet are growing fast. Naturally, this rapid urbanization is likely to create significant problems in cities, and considering the contemporary environmental issues related notably to the substantial emission of carbon in the atmosphere, new solutions need to be innovative whilst being effective and efficient. These solutions can be categorized under the umbrella of sustainable development. Nevertheless, planning for a sustainable future is not easy task, and the realm of transportation is not excluded.

Transportation holds a major stake in this global endeavour, and conventional planning techniques are no longer suitable to address growing problems. One particular transport mode that seems to address challenges of sustainability presented to cities is public transportation. By being a provider of mass transportation, public transit not only has the potential to relieve road congestion levels in cities (which has multiple positive ramifications), but also to transport people in a more environmentally friendly manner. As a result, public transportation systems all over the world are likely to grow substantially, and the planning of such systems is of key concern.

Public transportation systems are physical networks, where the nodes/vertices are stations or stops, and links/edges are roads or rail tracks. It is therefore logical to examine this network feature of transit systems and develop guidelines to help planners. Nevertheless, the study of the topology of transit networks is relatively scarce at the moment, which is reflected in planning practices and designs. The potential for contribution is therefore substantial.

In addition, the study of networks has emerged strongly in the past 12 years, and researchers all over the world have started to examine and scrutinize many different types of networks. From this research, two major types of networks have been discovered: scale-free networks and small-worlds. Scale-free networks have degree distributions that follow a power law; i.e., many nodes have few degrees (i.e. connections) and few nodes have many degrees. Small-worlds have the
properties of having high clustering (i.e. nodes clique together), whilst having relatively small average shortest-path length. Many networks, across various disciplines (e.g., biology, physics, etc.), show scale-free and small-world features. The identification of network properties, their understanding and control, offers much potential.

As transit systems are likely to grow substantially, finding the properties and effects of transit networks can be helpful to transportation planners and engineers. In this thesis, we decided to focus on metro systems only; partially because they offered a more interesting topology to study. Effectively, the main goal of this thesis was to study the properties and effects of metro network designs. By using a graph theory approach, we translated 33 metro networks in the world and studied their network topologies. More specifically, the objectives were:

- How can metro networks be studied using a graph theory approach?
- Can metro networks be characterized, depending on factors such as their size, integration with the urban form, their structure?
- How complex are metro networks? Do they follow general network properties such as the presence of scale-free and small-worlds patterns? What effects do these properties have on the robustness of metros?
- Can the inherent network properties of metros affect ridership? What structural recommendations can be made?
- How can this information be used in practice?

In the next section, we provide a chapter by chapter summary of the findings of this thesis.

### 8.2 Summary of chapters

In chapter 1, we introduced the philosophy and objectives of this thesis. We notably identified the need to study transit network design, listed the objectives (as given above) and defined the scope (network properties and effects) and scale (33 networks) of this study.
In chapter 2, we first recalled the origins of graph theory (Euler and the seven bridges of Königsberg) and reviewed significant graph theory concepts and indicators. We then reviewed the contributions that were made on the study of transport networks (road systems in particular) using a graph theory approach, and recalled the various indicators introduced, notably those by Garrison and Marble, and Kansky. Afterwards, we focused on the study of transit networks using a graph theory approach, notably by presenting a relatively comprehensive review of the contributions by Vuchic and Musso; we also found that transit networks hold network specificities (e.g., planarity, presence of lines, etc) that require a different set of indicators. Subsequently, we presented several concepts from the emerging field of network science, with a particular emphasis on the scale-free and small-world properties of networks.

In chapter 3, we introduced a new methodology to translate metro networks into graphs. The method presented in this thesis is novel and innovative. First, only transfer stations and termini are considered as vertices $V$ of a graph $G$ as opposed to all stations $N_S$; this is significant as it facilitates the understanding of the “transferring” properties of transit networks, as well as the development of a relevant robustness indicator. Moreover, all stations $N_S$ were assigned an attribute $\ell$ relative to the number of lines they host, which becomes relevant to study the connectivity of metro networks as well as their potential scale-free nature. Overlapping lines were also accounted for by the creation of single and multiple edges; and the maximum number of line transfers required to join to the furthest edges (in terms of number of transfers) was defined and collected for 33 networks. The nature of the methodology used is determining since it needs to reflect relevant specificities of transit network. Much emphasis was put into developing a method that reveals useful properties in order to help transit planners and engineers.

In chapter 4, we characterised metro networks along their State, Form and Structure. The State characteristic was based on indicators of complexity ($\beta$) and degree of connectivity ($\gamma$). We notably identified three phases in the evolution of metro networks; phase 1 networks were relatively simple in topology and no trends were apparent; phase 2 networks were relatively more complex and larger in size; phase 3 networks including some of the largest metros in the world. We notably found that a quasi-linear trend started to appear between phase 2 and 3 metro networks. Moreover, we also found that metros tended to be 66% completely connected. The Form characteristic was based on the average line length ($A$) and the number of stations ($N_S$). We identified three zones existed: regional accessibility (long lines reaching the suburbs), local
coverage (short lines covering the downtown extensively) and regional coverage (downtown well covered while having long line reaching the suburbs). No preferred practices could be determined here although metros in the local coverage zone may offer denser land-use patterns in their core. The Structure characteristic was based on indicator of directness ($\tau$) and connectivity ($\rho$). We found that three types of networks existed; those that are directness-oriented, connectivity-oriented, or integrated. The latter type may be preferable as many transfers are available while being convenient to travel (not many transfers required to join any two stations).

Finally, we briefly discussed the impacts of these characteristics on line types and land-use, and offered potential paths for development. This analysis also provides powerful tools to compare metro networks in the world.

In chapter 5, we investigated the complexity of metro networks, where complexity referred to the potential scale-free and small-world feature of metros. First, we recalled and adapted these two concepts to metro. For instance, while scale-free networks have degree (i.e. connection) distributions that follow power laws, we preferred to use the distribution of line attributes; not only does it better suit the study of metro networks (since only termini can have one connection), but it also captures relevant information on the transferring properties of metros, which is more useful to planners. Overall, by performing a rigorous statistical analysis, we found that most metro networks were in fact scale-free; their scaling factors (i.e. exponent of the power law), however, were found to be atypical. No general patterns were found for smaller networks, and yet, as they grow in size, metros tend to have scaling factors between 2.8 and 3.3, which is relatively large. Essentially, we found that metro networks are likely to have transfer “hubs” offering most transfer possibilities. By investigating the small-world property, we found that metro networks obey both rules (clustering and small average shortest-path length). They also have the property to become more clustered as they grow in size, which is atypical.

Subsequently, we studied the impact of these properties on the robustness of metro networks; where a robustness indicator was developed to capture metro network specificities (it is related to the number of alternative paths in metro networks). Overall, we found that a high clustering coefficient and a low scaling factor were favourable, although the total number of stations also played a key part. Moreover, we may conclude that the design of metro networks should be concentrated on building transfer hubs, notably at the city core, as is common; however, as metro networks further grow in size, they should also have smaller transfer stations at the periphery to
generate more alternative paths, notably by building (semi)-circle or tangential lines, which can further generate multiple positive impacts.

In chapter 6, we looked at three specific network indicators and their impact on ridership. In this case, however, we only looked at 19 metros. The three indicators are coverage, directness and connectivity; ridership was measured in annual boardings per capita. Coverage measures the amount of land covered by the metro network relative to the served area. Directness (as introduced in chapter 4) relates to the number of transfers to go from one station to another; it can be associated with a measure of “convenience” to travel, as transferring is often seen as negative. Connectivity (as introduced in chapter 4 as well) is also referred to as structural connectivity; by looking at the number of transfer possibilities, while accounting for redundancies, it appreciates the transfer properties of metro networks. By performing individual linear regressions, we found that the three indicators had a positive and statistically significant effect on ridership. Nevertheless, the nature of the relationship between ridership and coverage appeared to be logarithmic, perhaps following the law of diminishing returns. Afterwards, we performed a multiple regression analysis (using the log of coverage) and developed a fairly simple ridership model, which can be used notably to estimate ridership growth potentials in relations with network design, as a testing, screening or sketching tool. In general, in terms of network design, it seems that coverage and connectivity should be maximized, while trips should remain as direct as possible. Finally, it should be mentioned that results from this study should systematically be used along with conventional techniques. Although we realise the ridership predictions may not be entirely accurate, they can still offer useful information about the nature of the network designs.

In chapter 7, we offered an application of the concepts developed in chapters 4, 5 and 6 to the metro of Toronto. The Toronto regional transportation authority, Metrolinx, developed a 15-year and a 25-year plan that would significantly increase the current metro system. These plans include seven light rail lines (that were considered as metro lines), one new subway line, two subway extensions, and the extension of the Scarborough RT line. Overall, we found that the grid-pattern design of the network has negative impacts. Regarding metro characteristics, the Toronto plans would not necessarily compare to its peer networks. Regarding its complexity, we found that the plans would achieve a relatively low clustering coefficient and generate a significantly high scaling factor, which would negatively affect the robustness of the system.
Although ridership is likely to grow by a significant amount, the directness and connectivity of the plans could be improved, which would simultaneously address some of the problems previously presented. We further made seven possible structural recommendations. Although they may not all be feasible, considering the level of capital investment involved, performing an analysis of the network design may be desirable.

Overall, these analyses of metro network design revealed interesting properties and effects on the topological nature and typology of metro systems. Such analyses are recommended in future projects; not only can they inform planners on the nature of their network designs, they can also help planners compare various scenarios with one another, compare these scenarios with other international metro networks, as well as reveal particular aspects of the designs that may need to be improved.

8.3 Future research

The analysis of transit network design can be of significant help to transportation planners and engineers all around the world. In this thesis, relevant properties and effects of existing metro networks were identified, and the different designs were characterised and assessed. The concepts and analyses presented here are novel and innovative, and future research can be imagined.

In this section, potential research avenues for future work are outlined. In particular, four such avenues are identified: modal integration and coordination, land-use development, travel demand forecasting, and transit network design beyond graph theory. Each avenue is discussed here.

8.3.1 Modal Integration and Coordination

One of the first steps to enhance the study of transit networks is to incorporate other modes of transit. By considering the metro mode only, this thesis successfully identified specific properties of metro networks, and several effects were also analysed; nevertheless, the integration of other
modes of transit could be useful to planners. At this point, it is uncertain how this could be pursued, and solutions to this problem will need to be creative.

At first, new indicators could be developed to incorporate intermodal properties to the existing model introduced in chapter 6. For instance, this could be modelled by having an indicator of number of transfer possibilities with other modes of transit divided by the number of transfer stations. Moreover, dummy variables could be introduced to account for the presence of other modes (i.e. value of 1 is mode is present, and 0 otherwise). A myriad of other indicators can be imagined. Nevertheless, graphs will then have to be adapted to model the presence of other transit networks.

Graphs are composed of vertices and edges, and although we have defined different types of each for the metro mode, new techniques will need to be developed to integrate other transit modes. Perhaps it would be preferable to start by integrating other modes sharing similar characteristics such as light rail or regional rail systems, subsequently add the bus rapid transit mode (BRT) which enjoys a semi-exclusive right-of-way, and finally carry on by adding bus and other present transit modes that have shared rights-of-ways.

One potential solution is to start by constructing disjointed graphs (each mode having its own graph). This would provide a platform to first study each network topology discretely (and hence find potential useful properties). Figure 8.1 shows how the transit network is integrated in Greater Zurich. From Figure 8.1, it is clear that the three different “nets” possess different properties. In particular, it is possible to relate this figure with the structural connectivity indicator (see Figure 4.1). In this particular case, the “primary net” (which serves suburban trips mostly) appears to have a high structural connectivity (being completely radial), potentially to offer a fast service from the suburbs to the city centre at the expense of coverage (in terms of spatial distribution). In contrast the “supplementary net” has a grid structure, which results in low connectivity; here the grid-structure may be maximizing coverage (or minimizing walking distance) at the expense of speed. The “secondary net” seems to be to have a hybrid topology (i.e. neither completely radial nor grid). As a result, we see that different modes can have antonymic properties that serve different purposes. Gaining a better understanding of the network properties and effects of each mode can be desirable.
To then integrate the various transit modes using a graph theory approach, a new type of edge could be defined to simulate the possibility to transfer. For example, using the Toronto transit system as a case study, a new type of edge could be created to connect the Spadina streetcar station with the Spadina metro station; although physically there is only one station, this new type of edge would differentiate between the modes while showing the possibility to transfer. Using this methodology may result in a better understanding of public transportation as multimodal systems.
Alternatively or complimentarily, it may be useful to weight the different edges so that different transit modes have different network characteristics. Otherwise, using a bi-partite representation may be useful (i.e. B-space representation, see section 2.5.4), even if it may present integration limitations.

Additionally, vertices may need to be given specific attributes depending on the type of infrastructure present for transferring, etc. A transfer where one has to exit the station may need to be considered differently. Similarly, the fare system may need to be accounted for, whether it is an integrated system or not. Note that adding this information shifts the emphasis of the research from a “macro” perspective to a more detailed, “micro”, perspective.

Furthermore, flows of passengers can be modelled, which can help to coordinate different transit modes. The coordination of transit modes is essential to offer an efficient system to the public; timed transfer systems offer one example of coordination.

Finally, it may be worth incorporating operating characteristics directly so that transit systems are not only characterised by their topological properties but also by their operating properties. This could potentially be done by weighting edges according to transit capacity (e.g., in terms of passenger per hour), speed, frequency, etc.

### 8.3.2 Land-Use Development

It is commonly accepted now that land-use and transportation systems are intrinsically related, with a particularly strong relationship between land-use and public transportation systems (Polzin 1999). In fact, a co-evolution can be observed in many cases whereby a transit line is implemented on a specific corridor having sufficient population density; the presence of this transit line then attracts new residents/businesses and intensifies the land-use, which may then create a need for a higher order transit service on this corridor. Nevertheless, such a relationship is particularly challenging to quantify and outcomes can be diverse (Derrible and Farooq 2010).

Typically, land-use is more built up around transit stations (note that this greatly depends on physical accessibility to the transit station as well as other factors); this phenomenon is often more acute around transfer stations. It can be speculated that transit network properties may have
an effect on the level of land-use development in a city or a neighbourhood. For instance, a highly radial transit network could engender a dense land-use in the downtown core (where transfer stations are located), as opposed to a ubiquitous transit network (e.g., Paris metro) that may create a more evenly distributed, yet fairly dense, land-use.

Similarly, land-use patterns can depend on the type of transit mode present. A metro line has a much higher passenger capacity than a BRT line for instance; therefore, land-use around metro stations may be much more developed than around BRT stations. Nevertheless, BRT lines have generally shorter inter-station spacings; thus land-use around BRT lines as whole (as opposed to stations) may be more evenly distributed than around a metro line.

Figure 8.2 shows an illustrative distribution of population depending on availability of modes. On Figure 8.2, the city at the top has two transit lines; people can also walk but the automobile mode is absent. As a result, we can see that most land-use development happens around the transit lines and in particular where the two lines cross. For the city in the middle, half of the residents have access to the automobile, resulting in a much more evenly distributed, less dense, land-use. Finally, the city at the bottom does not have transit, resulting in an even more dispersed land-use.

The topology of the transportation network can have an impact on the development of land-use in the city. An analysis of transportation network properties and effects could therefore useful to predict future land-use development. Potentially, vertices could be given attributes according to the land-use around them (e.g., commercial, mixed, residential land-use) as well as their location within the city (e.g., city centre, inner suburb, outer suburb). When planning for new lines and in agreement with zoning policies, economic forecast and geographic location, attributes of new vertices could be matched with attributes of existing vertices, hence making it possible to forecast land-use development (as well as travel demand as we will see in the next section). Alternatively, edges could be given such attributes to capture land-use around lines as opposed to stations. In addition, it should be mentioned that this methodology is not restricted to transit networks only but could be applied to all transport modes.

As the study of transportation / land-use interactions is becoming increasingly important in the literature, adopting a network approach could reveal useful properties and effects.
Figure 8.2 Illustrative distribution of population; adapted from Figure 1.12 in (Pushkarev 1982).
8.3.3 Travel Demand Forecasting

Another research area that is important in transportation is travel demand forecasting. Figure 8.3 shows one schematic example of the interactions between urban activity and transportation systems. First of all, we see that the “Transportation Network” has a direct impact on “Land Development” and “Location Choice”, which was discussed in the previous section. Second, we also see the “Transportation Network” appears to have an impact on “Travel Demand” as well. It is therefore possible to study this relationship more closely.

Traditionally, travel demand is forecasted by highly-developed mathematical models (e.g., multinomial logit, nested logit, etc.) using utility theory concepts.

These models use many endogenous and exogenous factors present in national censuses and travel surveys. Endogenous factors include mostly socio-economic characteristics at the household level such as income, number of children, number of automobiles, number of people with drivers’ and many others. Exogenous factors used include trip distance, in-vehicle travel
time, other transit related times (e.g., access, egress, wait times), as well as number of stops (i.e. dropping the children at day care) and many others.

Nevertheless, many factors included in these models are not determining for mode choice. For instance, Cervero (1996) showed that urban form had a strong impact on mode choice regardless of factors such as income. It is therefore legitimate to assume that network properties and effects also have an impact (and perhaps a strong impact) on mode choice.

As a result, it can be imagined that existing travel demand models could consider factors and characteristics of transportation networks such as route length of network, number lines (for transit), number of possible transit origin-destination pairs (i.e. to measure the effect of network spatial distribution of travel demand), etc. In addition, new indicators could be developed such as the ratio of length of bus lines by the total length of roads. Moreover, several network properties such as structure of network (as defined in chapter 4) or scaling factor (chapter 5) could also become integral for travel demand forecasting.

Furthermore, by using utility theory concepts, these models assume that people maximize their utility, and that they have perfect information of the system, while this is not necessarily true. Perhaps future methods could take a network approach. For instance, places of residence, employment and other could be modelled as vertices and be given attributes related to their urban form for instance. Each trip per person could be modelled by an edge; as a result people would only have information available for this particular edge as opposed to the entire system. To determine mode choice, new models could be strongly based on network characteristics (e.g., topology of transit network versus road network), while endogenous factors could have a relatively smaller impact. As a result, mode choice models could evolve from being person-specific (i.e. depending on socio-economic factors) or trip-specific (i.e. home-work trip) to rather location-specific (i.e. trip-ends being located in neighbourhoods with transit accessibility supported by transit-oriented development). Nevertheless, much work lies ahead to be able to improve current travel demand models; looking at network properties has the potential to offer one contribution, but others can be imagined.
8.3.4 Transit Network Design beyond Graph Theory

Conceptually, this thesis takes a top-down approach to study transit networks; i.e. properties of existing networks were identified and analysed; one assumption was therefore that existing network topologies addressed the needs of the residents of each city. In contrast, one can imagine taking a bottom-up approach where a transit network evolves from the current needs of residents; for instance by using agent-based models.

Agent-based models have emerged strongly in the past two decades, notably from the works of Epstein and Axtell (1996); an extensive review can be found in (Batty 2005). Agent-based models have also grown in popularity in the transportation community; e.g., TASHA (Roorda et al. 2008), used in ILUTE (Miller et al. 2004; Salvini and Miller 2005). An application to transit network design could offer a substantial contribution.

By developing an agent-based model, for instance, one could create transit networks built organically from the needs of the agents; various constraints would need to be applied such as travel time, transfer minimization, minimum ridership, number or length of lines, etc. It could first be applied to mock cities (e.g., even distributions of housing and employment vs. radial distribution), and then to real cities (which will also allow to study the impacts of geographical characteristics such the presence of a lake, mountains, etc.). These organically built networks could then be analysed and compared to real networks, perhaps by using the concepts presented in chapter 4, 5, and 6.

Using such agent-based models, it may be possible to determine optimal transit network topologies, where transit systems could be seen as a network of networks for instance; Figure 8.4 offers an illustration. As a result, it would be possible to consider, integrate and coordinate the various transit modes. Moreover, a study of land-use development could be added, as well as an analysis of travel demand.

Overall, we see that there is much potential for the study of transit design and using a network approach can help to find creative solutions that are needed to plan for a sustainable future. In this thesis, we used principles of graph theory for this purpose, but we expect the field of transit network design to grow substantially in the coming decades, hence providing a myriad of benefits for the future generations living in cities all around the world.
Figure 8.4 Multimodal Transportation System; adapted from Figure 1.1 in (Meyer and Miller 2001).
References


Biggs, N. 1976, Graph theory 1736-1936, Clarendon Press, Oxford [Eng.].


City Planning 2000, *ROW Shared*, City of Toronto, Toronto, ON.


Copes, W. 1987, *Graph theory: Euler's rich legacy*, Janson Publications, Providence, R.I.


Euler, L. 1741, "Solutio problematis ad geometriam situs pertinentis", *Commentarii Academii Scientiarum Imperialis Petropolitanae*, vol. 8, pp. 128-140.


Gattuso, D. and Miriello, E. 2005, "Compared Analysis of Metro Networks Supported by Graph Theory", *Networks and Spatial Economics*, vol. 5, no. 4, pp. 395-414.


Metrolinx 2008, *Draft Regional Transportation Plan*, Metrolinx, Toronto, ON.


Miron, J.R. 2003, "Urban Sprawl in Canada and America: just how dissimilar?", *Cities Lab @ UTSC*.


TTC 2007, *Toronto Transit City - Light Rail Plan*, Toronto Transit Commission, Toronto, ON.


von Merian-Erben 1652, *Königsberg (engraving)*, Kaliningrad, Russia.

Vuchic, V.R. 2007, Urban transit systems and technology, John Wiley & Sons, Hoboken, NJ.


Appendix A
Definition of Terms

\(\alpha\) The \(\alpha\)-index or degree of cyclicity [-]. It is the ratio of the actual number of cycles (see cyclomatic number \(\mu\)) and the potential number of cycles in a completely connected graph. It is bound between 0 and 1 but can be expressed as a percentage. See section 2.3.1.

\(\beta\) The \(\beta\)-index or complexity [-]. It is the ratio of the number of edges \(E\) and the number of vertices \(V\), which can be interpreted as the average number of edges per vertex. It is bound between 0 and \(+\infty\). In the dataset, values range from 0.80 in Rome to 1.96 in Chicago. See sections 2.3.1 and 4.2.1.1.

\(\gamma\) The \(\gamma\)-index or degree of connectivity or clustering coefficient (in this thesis) [-]. It is the ratio of actual number of edges \(E\) and the potential number of edges \(E_{\text{max}}\) in a completely connected graph. It is bound between 0 and 1 but can be expressed as a percentage. In the dataset, values range from 0.39 in Delhi to 0.71 in Chicago. See sections 2.3.1 and 4.2.1.2

\(\delta\) Maximum number of transfers [-]. It is the smallest number of transfers between the two furthest lines. For instance, \(\delta=3\) for the Toronto metro to link the Scarborough RT line with the Sheppard line (essentially, there are three transfer stations: Kennedy, Bloor/Yonge, and Sheppard/Yonge station). It can be computed manually or by using a shortest-path algorithm in a C-representation as shown in section 2.5.4. It is bound between 1 and the total number of lines in the metro. In the dataset, values range from 1 to 3 in multiple metros. See section 3.5.

\(\epsilon\) Scaling factor [-]. It is the exponent of the power law distribution in scale-free
networks. Although this exponent is a negative number (e.g., -1, -3), the absolute value is generally quoted (e.g., 1, 3). Most complex networks tend to have scaling factors between 2 and 3. In the dataset, values range from 2.10 for Hong-Kong to 5.52 for Rome. See sections 2.5.3 and 5.2.1.

\( \eta \) The \( \eta \)-index [km]. It is the average edge length. See section 2.3.2.

\( \theta \) The \( \theta \)-index. It is the average traffic flow per vertex. Dimensions depend on the variable used for traffic flow; for instance, in transit systems, number of yearly passengers per vertex could be used. See section 2.3.2.

\( \iota \) The \( \iota \)-index [km]. It is the ratio of network length \( M \) and weighted vertices \( w \). Alternatively, it can be the ratio of network length \( M \) and traffic flow \( T \) (e.g., for transit, this could be the average number of passengers per km of track). See section 2.3.2.

\( \kappa \) Segment of transit line [-]. For instance, it could be half of a line (i.e. \( \kappa=0.5 \)) or one third of a line (i.e. \( \kappa=1/3 \)). See sections 4.2.3.2 and 6.3.2.

\( \lambda \) Line overlapping index [-]. It is the ratio of the sum of all line lengths \( R_L \) and the total route length of network \( R \). Transit systems without overlapping lines have a \( \lambda \) equal to 1, and the more line overlaps the greater the value of \( \lambda \) is. See section 2.4.2.

\( \mu \) Cyclomatic number [-]. It is the number of cycles/circuits in the network and is determined by subtracting the number of edges in a tree network (i.e. \( V-I \)) to the total number of edges \( E \). It is bound between 0 and \(+\infty\). See section 2.2.

\( \pi \) The \( \pi \)-index [-]. It is defined as the ratio of total route length \( M \) and the length of the
network diameter $d$. See section 2.3.2.

Connectivity or structural connectivity [-]. It measures the importance and influence of transfer stations in the network. It is defined as the net number of transfer possibilities $(V'_c - E^m)$ divided by the number of transfer stations $N_S$. It is bound between 0 and the total number of lines in the metro minus one (for a completely radial system having only one transfer station in the centre). In the dataset, values range from 0.60 in San Francisco to 1.50 in Buenos Aires. See sections 4.2.3.1 and 6.3.3.

Coverage [-]. It is the proportion of area covered by transit stations and total served area (see section 6.2.1 for a definition of served area). It is bound between 0 and 1 but can be expressed as a percentage. In the dataset, values range from 0.027 in Athens to 0.25 in Seoul. See section 6.3.1.

Directness [-]. It is the ratio of the number of transit lines $N_L$ and the maximum number of transfers $\delta$. It is bound between 1 and the total number of lines in the metro. In the dataset, values range from 1.33 in Toronto, Lyon and Singapore to 6.50 in London and Tokyo. See sections 4.2.3.2 and 6.3.2.

Average line length [km]. In this thesis, it is defined as the ratio of the route length of network $R$ and the number of lines $N_L$. Note that it does not take in account properties of overlapping lines (more appropriately, the sum of all line lengths $R_L$ could be used, but this information was not available). In the dataset, where values are calculated, they range from 7.33km in Lyon to 36.49km in Stockholm. See section 4.2.2.1.

Annual boardings per capita [trips]. It is the total number of metro trips made in a year divided by population (see section 6.2.1 for a definition of population). In the dataset, it
is uncertain whether trips are linked or unlinked as it is often not described by transit agencies. Values were, however, validated using the Millennium Cities Database (UITP 2001) and the Mobility Cities Database (UITP 2006). In the dataset, values range from 27.23 in Athens to 235.30 in Moscow. See section 6.2.2.

\[ \bar{B} \] Average number of connections [-]. See sections 3.2 and 5.2.1.1.

\[ b \] Number of connections [-]. See sections 3.2 and 5.2.1.1.

\[ \{b_i\} \] Matrix of connections [-]. See sections 3.2 and 5.2.1.1.

\[ C \] Average clustering coefficient [-]. See clustering coefficient below and sections 2.5.2 and 5.2.2.

\[ C_v \] Clustering coefficient of neighbourhood of \( v \) [-]. It is the ratio of the actual number of edges in the neighbourhood of \( v \) (see definition below at \( k_v \)) and the potential number of edges in a completely connected neighbourhood \( v \). It is bound between 0 and 1 but can be expressed as a percentage. See sections 2.5.2 and 5.2.2.

\[ d \] Length of diameter of graph [km]. It is the length of the path that requires the smallest maximum of transfers. If multiple paths require have similar smallest maximum number of transfers, then the average of all these path lengths is calculated. This is the definition of the term as defined by (Kansky 1963). See section 2.3.2.

\[ d_{ij} \] Shortest-path length between vertex \( i \) to \( j \). Dimensions can vary depending on what is measures (e.g., number of edges, km). See section 2.5.4.
$E$  Total number of edges in a graph [-]. It is bound between 1 and $+\infty$. In the dataset, values range from 4 in Rome to 155 in London. See section 3.4.

$E_{\text{max}}$  Potential number of edges in a completely connected graph [-]. A completely connected graph can refer to a planar ($E_{\text{max}} = 3V-6$) or a non-planar ($E_{\text{max}} = 0.5 \cdot V \cdot (V-1)$) graph. In the dataset, metros are considered planar graphs, values range from 9 in Rome to 243 in London. See section 2.3.1, 4.2.1.2, and 6.3.3.

$e$  One edge unless otherwise defined [-]. See section 3.4.

$e_{ij}$  Number of edges connecting vertex $i$ to $j$ [-]. Values can be 0 if no edge exists, 1 if there are no overlapping lines, 2 or more if at least two lines overlap. See section 3.4.

$\{e_{ij}\}$  Matrix of edges [-]. See Table 3.3 in section 3.4.

$E^m$  Number of multiple edges [-]. It is the total number of edges $E$ minus the number of single edges $E^s$. It is bound between 0 and $+\infty$. In the dataset, values range from 0 in many metros to 30 in London. See section 3.4.

$e^m$  One multiple edge [-]. See section 3.4.

$E^s$  Number of single edges [-]. It is the number of edges in a graph considering no overlapping (if two consecutive vertices are joined by two edges, we consider one is single and the other is multiple). It is bound between 1 and $+\infty$. In the dataset, values range from 4 in Rome to 134 in Seoul. See section 3.4.

$e^s$  One single edge [-]. See section 3.4.
$E_{glob}$ Global efficiency. Dimensions can vary depending on what is measured (e.g., one over number of edges, one over km). See section 2.5.4.

$E_{loc}$ Local efficiency. Dimensions can vary depending on what is measured (e.g., one over number of edges, one over km). See section 2.5.4.

$G$ Graph [-]. It is a collection of vertices $V$ and edges $E$; $G=\{V,E\}$. See section 3.1.

$G_r$ Random graph [-]. It is a collection of vertices $V$ and edges $E$, where the edges $E$ are randomly distributed. See section 2.5.2.

$i$ Identification number [-].

$k_v$ Neighbourhood $k_v$ [-]. The neighbourhood contains all vertices that have a direct edge with vertex $v$, but it does not contain $v$ itself. See sections 2.5.2 and 5.2.2.

$L$ Distribution of line attributes [-]. See Table 3.1 in section 3.2.

$\{\ell_i\}$ Matrix of line attributes [-]. See Table 3.2 in section 3.3.

$\ell$ Line attribute [-]. It is the number of lines hosted by a station. It ranges from 1 for termini and non-termini non-transfer stations to a maximum of 6 lines in the dataset (one occurrence in London and one in Chicago). See section 3.2.

$M$ Total length of network [km]. Defined by (Kansky 1963), it is conceptually close to the
route length of network $R$ but typically applied to road networks. See section 2.3.2.

$N_{iss}$: Number of inter-station spacings [-]. It is the sum of all inter-station spacings in a network. See section 2.4.2.

$N_L$: Number of lines [-]. In the dataset, values range from 2 lines in Marseille, Rome and Cairo to 14 lines in Paris. See section 3.2.

$N_S$: Number of stations [-]. In the dataset, values range from 24 in Marseille to 422 in New York City. See section 3.3.

$n_s$: One station [-]. See section 3.3.

$P$: Average shortest-path length. Dimensions can vary depending on what is measured (e.g., number of edges, km). See section 2.5.2.

$p$: Number of sub-graphs of $G$ [-]. For metro networks, there are no disjointed sub-graphs; thus $p=1$. See section 2.2. The letter “$p$” was also used once on Figure 2.9 in section 2.5.2 as the probability of randomness for the distribution of edges in a graph.

$R$: Route length of network [km]. It is the total one-way (i.e. one direction) operating track length. In the dataset, values range from 19km in Marseille to 438.73km in London. See sections 2.4.2 and 3.1.

$R_T$: Ratio of harmonic means of ideal $\bar{T}$ and real $\bar{t}$ trip times [-]. It is bound between 0 and 1 but can be expressed as a percentage. See section 2.4.1.
Robustness of metro [-]. In this thesis, it is defined as the net number of cycles (i.e. cyclomatic number $\mu$ minus number of multiple edges $E^m$) divided by the number of stations $N_S$. In the dataset, values range from 0 in Rome and Delhi to 0.2525 in Tokyo. See section 5.2.3.2.

Average inter-station spacing [km]. It is the ratio of the sum of all line lengths $R_L$ and the number of inter-station spacings $N_{iss}$; this information, however, can be challenging to collect or not available. In this thesis, as an approximation, we simply divide the route length of network $R$ by the number of stations $N_S$. In the dataset, values range from 0.67km in Brussels to 1.99 in Washington DC. See sections 2.4.2 and 4.2.2.1.

Traffic flow. Dimensions depend on the variable used; for instance, in transit systems, number of yearly passengers could be used. See section 2.3.2.

Harmonic mean of ideal trip times [min]. See section 2.4.1.

Harmonic mean of real trip times [min]. See section 2.4.1.

Total number of vertices in a graph [-]. It is bound between 2 and $+\infty$. In the dataset, values range from 5 in Rome to 83 in London. See section 3.3.

One vertex unless otherwise defined [-]. See section 3.3.

Number of end vertices [-]. It is the number of termini (that do not offer transfers) and is bound between 0 and $+\infty$. In the dataset, values range from 4 in Marseille, Rome and Cairo to 27 in London. See section 3.3.
$v^e$ One end-vertex [-]. See section 3.3.

$V$ Number of transfer vertices [-]. It is the total number of transfer stations; in other words, it is the sum of all vertices having a line attribute $\ell$ greater than or equal to 2. It is bound between 0 and $+\infty$. In the dataset, values range from 1 in Rome to 56 in London. See section 3.3.

$V_c^t$ One transfer vertex [-]. See section 3.3.

$V_c^t$ Number of transfer possibilities [-]. It is the sum of the number of lines (i.e. line attribute $\ell$) at a vertex minus one. In the dataset, values range from 1 in Rome to 86 in London. See sections 4.2.3.1 and 6.3.3.

$W$ Observed number of vertices weighted by their functions [-]. See section 2.3.2.
Appendix B
Metro Maps

This appendix contains popular metro maps that are referred to throughout the thesis.

Figure B.1 Paris metro map; adapted from (RATP 2010).
Figure B.2 London metro map; adapted from (TfL 2010).
Figure B.3 New York City metro map; adapted from (MTA 2010).
Figure B.4 Prague rail transit system map showing the three metro lines; adapted from (prague-tourism 2010).
Figure B.5 Madrid metro map; adapted from (Metro 2010).
Figure B.6 Washington DC metro map; adapted from (WMATA 2010).
Figure B.7 Buenos Aires rail transit system map, showing the five metro lines; adapted from (Subte 2009).
Figure B.8 Brussels metro map; adapted from (STIB 2008).
Figure B.9 Mexico City metro map; adapted from (STC 2010).
Figure B.10 St Petersburg metro map; adapted from (on-line.spb.ru 2009).
Figure B.11 Barcelona metro map; adapted from (TMB 2010).
Figure B.12 Tokyo metro map; adapted from (Tokyo Metro 2010).
Figure B.13 San Francisco BART metro map; adapted from (BART 2010).
Figure B.14 Athens metro map; adapted from (AttikoMetro 2010).
Copyright Acknowledgements

List of publications related to this thesis:

Chapter 2: “Applications of Graph Theory and Network Science to Transit Network Design” (Derrible and Kennedy 2010a)

Chapter 4: “Characterizing Metro Networks: State, Form, and Structure” (Derrible and Kennedy 2010b)

Chapter 5: “The Complexity and Robustness of Metro Networks” (Derrible and Kennedy 2010c)

Chapter 6: “A Network Analysis of Subway Systems in the World using Updated Graph Theory” (Derrible and Kennedy 2009c)

Chapter 7: “Evaluating, Comparing, and Improving Metro Networks: an application to the Toronto proposed plans” (Derrible and Kennedy 2010d)