A Replacement Model of Metropolitan Expansion

Urban geographers have long been interested in modeling the spatial pattern of household or population relocation within metropolitan areas. A great variety of empirical approaches have been employed; many of these are reviewed in the papers by Senior (1973, 1974). The process of residential relocation has been recognized to be complex; many economic, behavioral, and institutional factors affect the decisions of individual households to look for and choose a new dwelling. This complexity makes different empirical approaches—each with its own simplifications and emphasis—helpful in building a broader understanding of relocation and its impacts.

One empirical approach which has met with mixed popularity is that employing stationary Markov models. Among those who have used this approach are Gale (1969), Brown and Longbrake (1970), Simmons (1974), and Porell (1981). Stationarity implies a fixed probability, p , that an individual (or household) living in subarea A of the metropolitan region at a starting date will reside in subarea B at some specified later date. Thus, each such probability, p , in a stationary Markovian model is assumed to be unchanging over time: an assumption which has been criticized and found wanting by several critics including Huff and Clark (1978).¹

Why is stationarity not a good assumption in intrametropolitan relocation? There are two principal sources of nonstationarity.

1. The overall mobility rate, the propensity for an individual to move at all (regardless of destination), may change over time. For example, the overall mobility rate might be expected to vary inversely with the typical age of adults in the subarea.
2. The propensity for a mover from A to choose subarea B as a destination may change over time. Suppose B for example is a suburban area in which there is massive new housing construction in one time period followed by a period of no construction. The propensity to move from another subarea, A , into B

¹Gilbert (1972), Ginsberg (1972), Myers (1967), and Rogerson (1979) have attempted to develop or apply nonstationary Markov models.

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may well diminish from the first to the second time period reflecting these changes in the housing stock in B .

Concerns about stationarity have led several researchers to develop nonstationary models. Pickles (1980) presents an extensive review of nonstationary approaches as applied both to migration and to residential relocation. Also, one should point to Varaprasad's (1980) model which integrates population movements, household formation, and housing supply in a nonstationary model of relocation.

The organizing principles behind the model presented here are threefold. First, there is assumed to be rough balance between the total inflow and outflow of households in any subarea. The net number of households entering a zone during a period cannot exceed the sum of the net increase in dwelling units during this time plus the vacancies present at the start of the period. Gross inflows in excess of this net amount must replace a corresponding number of existing resident households, that is, a replacement principle. Second, a household which leaves one subarea is attracted to another as a function of the supply of available housing in the new subarea and is deterred by distance. Finally, flows of certain households, referred to as "lost" and "new" households, are assumed to be exogenously determined. These flows in turn determine the magnitudes of the remaining relocation flows via the replacement principle.

The model has a number of interesting policy applications. It can be used for example to assess the impact of changes in national immigration policy on residential relocation within a metropolitan region. Similar impacts can also be studied for changes in the level of housing construction in each subarea. Finally, it can also be used to show how the impact of demographic changes on residential relocation can be examined.

Let us summarize the ensuing sections. In section 1, the structure of the model is formally presented. A fundamental data problem and its resolution is discussed in section 2. The model is empirically validated for the Toronto Census Metropolitan Area (CMA) between 1966 and 1976 in section 3. Some sensitivity analyses are described in section 4. Finally, in section 5 are presented some concluding comments on the prospects for, and problems with, this kind of model.

1. THE REPLACEMENT MODEL

Suppose that a metropolitan region is divided into N zones or subareas. In each zone at a starting date, there are a number of occupied dwelling units, each with its own household, and each household with its own head. The number of heads at this date (t) in this zone (i) is $h_i(t)$, which is also the number of occupied dwellings.

At the end of a time period, say $t + 1$, several things may have happened to this household head:

1. The head at t may still be a head at $t + 1$ and still reside somewhere in zone i . Let $h_{ii}(t + 1)$ be the number of such nonmigrant heads.
2. Alternatively, the head at t may still be a head at $t + 1$ but reside in a new zone j . Let $h_{ij}(t + 1)$ be the number of these migrant heads.
3. Finally, the head at t may have by $t + 1$ died, become a nonhead, or left the region altogether. Call these "lost" heads and let their number be represented by $l_i(t + 1)$.

From this we get the following identity:

$$h_i(t) = \sum_{j=1}^N [h_{ij}(t + 1)] + l_i(t + 1). \quad (1)$$

The number of heads in zone i at $t + 1$ can be described in similar terms. It is made up of the number of nonmigrant heads from time t , $h_{ii}(t + 1)$, the number of migrant heads arriving from each other zone j , $h_{ji}(t + 1)$, and the number of "new" heads, $n_i(t + 1)$. A "new" head at $t + 1$ is either a person who was not a head at time t or is a head who resided outside the metropolitan region.

$$h_i(t + 1) = \sum_{j=1}^N h_{ji}(t + 1) + n_i(t + 1). \quad (2)$$

Let us define two additional variables; $h_i(t + 1)$ and $h_{.i}(t + 1)$. These are respectively the total number of migrant heads leaving i for, or arriving at i from, other zones in the metropolitan region.²

$$h_i(t + 1) = \sum_{\substack{j=1 \\ j \neq i}}^N h_{ij}(t + 1); \quad h_{.i}(t + 1) = \sum_{\substack{j=1 \\ j \neq i}}^N h_{ji}(t + 1). \quad (3)$$

Such definitions when used in conjunction with (1) and (2) permit us to describe components-of-change analyses for the change in households in a zone.

$$h_i(t + 1) - h_i(t) = h_{.i}(t + 1) + n_i(t + 1) - h_i(t + 1) - l_i(t + 1). \quad (4)$$

The (positive) inflow components are the in-migrating and new heads; the (negative) outflow components, the out-migrating and lost heads.

The first organizing principle of this model is that the positive and negative components of household change are in some relationship. Define $b_i(t + 1)$ as the ratio of outflow to inflow components.

$$b_i(t + 1) = \frac{h_i(t + 1) + l_i(t + 1)}{h_{.i}(t + 1) + n_i(t + 1)}. \quad (5)$$

Suppose that between t and $t + 1$ there is no net change in either the number of dwelling units or the vacancy rate in this zone. This would imply $b_i(t + 1) = 1$ since every arriving household (in the denominator) would have to displace one existing household (in the numerator) or conversely every leaving household would create a vacancy that is filled by the arrival of an in-migrating or new household.

Suppose, instead, that the housing stock or the vacancy rate does change over time. New residential buildings might be constructed and old ones demolished or converted to provide different numbers of dwelling units. In general, an increasing stock of dwelling units (or equivalently a decline in the vacancy rate) should make $b_i(t + 1)$ smaller than unity with the converse holding true. In other words, it is hypothesized that

$$b_i(t + 1) = f(HC(t + 1), HD(t + 1), HR(t + 1), V(t + 1) - V(t)), \quad (6)$$

where HC , HD , HR are respectively the number of dwelling units between t and $t + 1$: (i) added through new unit completions, (ii) lost through demolition, or (iii) net-added through conversions or reconversions of buildings. V is the vacancy rate.

²These are similar to the first two terms of (1) and (2) respectively except that nonmigrant heads $h_{ii}(t + 1)$ are excluded.

The second organizing principle in this model concerns the destination choices (within the metropolitan region) of out-migrating households. Let us define $a_{ij}(t + 1)$ to be the proportion of those heads leaving i who are in zone j at time $t + 1$.

$$a_{ij}(t + 1) = \frac{h_{ij}(t + 1)}{h_i(t + 1)} . \quad (7)$$

Factors that might influence the destination choice of an individual household include (1) availability of particular kinds of dwelling units and neighborhood amenities, including the amenity of being in close proximity to the old place of residence or to friends; (2) job site and commuting considerations; and (3) search and information-gathering strategies. We use two gross surrogates for these factors. One is $P_j(t)$, the supply of potential housing opportunities in zone j relative to other zones. This is a very crude approximation to factor (1) above. The second is d_{ij} , the distance of the destination zone from zone i . This is commonly taken to be a surrogate for factors such as search strategy and the pecuniary and nonpecuniary costs of moving. These hypotheses may be condensed as follows:

$$a_{ij}(t + 1) = f(P_j(t), d_{ij}) . \quad (8)$$

A similar argument can in part be provided for the number of lost households, $l_i(t + 1)$. Heads become lost when they become nonheads through death or some other cause or when they out-migrate from the metropolitan region. Much of this can be associated with age structure, A . Changes in headship status and long-distance migration tend to be associated with young adults while the deaths of heads are most frequent among the elderly. In addition, however, the distance argument raised earlier applies here. For households in zones at the metropolitan fringe, out-migrating from the region can mean a relatively short move. Therefore, the number of lost households should vary with the ages of zonal residents and crudely with $BORD_i$, a measure of the distance from zone i to the metropolitan boundary.

$$l_i(t + 1) = f(A_i(t), BORD_i) . \quad (9)$$

New households are treated differently. Since these new heads either were not heads in the region five years before or else in-migrated from outside, it is not clear how or why intrametropolitan distances should be important. Instead, the flow of new heads into a zone is related only to $P_j(t)$, the supply of potential housing opportunities in the zone, and $N(t + 1)$, the aggregate number of new heads arriving in the region between t and $t + 1$.

$$n_i(t + 1) = f(P_i(t), N(t + 1)), \quad \text{where } \sum_i n_i(t + 1) = N(t + 1) . \quad (10)$$

The third organizing principle is the designation of exogenous flows. In this case, l_i and n_i are the flows which determine the remainder. Let us begin by rewriting (7) as follows. In the rest of this section note that the $t + 1$ argument is implicit; it has been deleted for ease of presentation.

$$h_{ij} = a_{ij}(h_i + l_i) - a_{ij} l_i . \quad (11)$$

Substituting (5) and (3) into (11) yields

$$h_{ij} - a_{ij}b_i \left[\sum_{k \neq i} h_{ki} \right] = a_{ij}b_i n_i - a_{ij}l_i, \quad i, j = 1, 2, \dots, N; i \neq j. \quad (12)$$

There are $N(N - 1)$ equations of the form (12): one for each interzonal flow of households.

The equation system represented by (12) can be expressed in matrix form as follows:

$$\mathbf{CH} = \mathbf{D}, \quad (13)$$

where

$\mathbf{H}$ is an $N(N - 1) \times 1$ column matrix whose typical k th element is h_{ij}

$\mathbf{D}$ is an $N(N - 1) \times 1$ column matrix whose typical k th element is $a_{ij}b_i n_i - a_{ij}l_i$

$\mathbf{C}$ is an $N(N - 1) \times N(N - 1)$ matrix whose typical k, l th element is

- (a) 1 if $k = l$
- (b) $-a_{ij}b_i$ if the l th element corresponds to a flow into i , or
- (c) 0 otherwise.

If the inverse of $\mathbf{C}$ exists, a solution to the model can be found of the form

$$\mathbf{H} = \mathbf{C}^{-1}\mathbf{D}. \quad (14)$$

The structure of the model being hypothesized thus has three blocks. The first is the "lost-new" block, which models the numbers of new and lost households, n_i and l_i ; it consists of (9) and (10). The second is the "parameter" block, which models a_{ij} and b_i ; it consists of (6) and (8). Finally, the "interzonal flows" block, (14), solves for the interzonal flows on the basis of the values derived for b_i , a_{ij} , n_i , and l_i in the first two blocks.

The exogenous variables are those inputted into the first two blocks. These are interzonal distances, proximity of each zone to the metropolitan boundary, each zone's population age structure, the potential housing opportunities in each zone including changes in the stock of dwelling units and vacancy rates, and the aggregate number of new heads arriving in the CMA during a period of time.

2. THE FUNDAMENTAL DATA PROBLEM

In this model, the measurement unit is the household rather than the individual; the movement of an individual does not necessarily imply a housing stock change or replacement as does movement of a household. However, a household is not an atomistic element as is an individual; in census terms, it is defined merely as the group of individuals sharing the same dwelling unit. A household can gain or lose members over time through a variety of processes including birth, death, and in-movements or out-movements of individuals.

The first problem then is to define when an existing household ceases to exist and when a "new" household is created. To a degree, any definition is arbitrary. Here, the definition is based on the person designated as head. If a head in 1971 is no longer a head in the metropolitan region in 1976 then a household is said to

have been "lost." If a head in 1976 was not a head in the region in 1971, the household is said to be "new."

A second problem is more severe. From the census, we do not know whether a head in 1971 (or 1976) was a head five years before. Such information is fundamental, though, to the proposed model. A method is described below for estimating the probability that a head at time t was also a head five years before.

The method generates plausible probabilities. However, it is not a substitute for better census data on headship status. It is merely a crude means of estimating the flows on which the model depends. The method should not be viewed as part of the model but as an interim requirement until better census data are available. Following a detailed outline of the method, a component-of-change analysis is presented for the Toronto CMA for 1966-71 and 1971-76.

A. A Method of Estimation

Consider a five-year age-sex cohort in 1966. Let there be P_0 of these persons in total. Suppose that P_0^h are heads of household and the remainder are not. In 1971, this cohort now five years older will number P_1 of whom P_1^h are heads.

The population of the cohort may have changed over this five-year period. Let $P_1 = SP_0$. S will be greater or less than one depending on the levels of mortality, in-migration, and out-migration for that cohort.³ The numbers of heads also will have changed over this time: both because of population change and because of transitions in headship status. Assume that in the absence of headship status transitions, the number of heads and nonheads would each change by the same proportion S as the cohort total.

However, headship transitions do occur. Let us assume that a proportion s of the heads and r of the nonheads in 1966 become nonheads or heads respectively in 1971. The 1971 counts of heads and nonheads are given as follows:

$$P_1^h = (1 - s)SP_0^h + rS(P_0 - P_0^h) \quad (15)$$

$$P_1 - P_1^h = sSP_0^h + (1 - r)S(P_0 - P_0^h) \quad (16)$$

If q is the probability that a 1971 head was not also a head in 1966, it can be expressed as follows:

$$q = \frac{rS(P_0 - P_0^h)}{rS(P_0 - P_0^h) + (1 - s)SP_0^h} \quad (17)$$

Using (15) and (16), an expression can be generated for r in terms of s and the 1966 and 1971 cohort sizes. Substituting this into (17) yields

$$q = \left[1 - \frac{P_0^h/P_0}{P_1^h/P_1} \right] + \left[\frac{P_0^h/P_0}{P_1^h/P_1} \right] s \quad (18)$$

With (18), q can be estimated given 1966 and 1971 cohort sizes and some estimate of s . Since there are no ready data on s , this method would seem to leave us in the dark. There are, however, three redeeming features to this approach. First, s is not large for most cohorts. Second, one can intuitively guess which cohorts are likely to have high or low values of s . Young adult females, for example, are likely to have a high s associated with a transition from single to married

³ S will in general be different for each age-sex cohort considered.

status.⁴ The elderly may also have a high s to the extent that they move in with an offspring who assumes headship. Third, in (18), q increases linearly with s . The marginal effect of s on q is the ratio of a headship rate in the starting year to the rate for the (aged) cohort five years later. Since headship rates have generally been increasing over time and since they tend to increase with age, the marginal effect is typically substantially smaller than one. Thus, q may not be very sensitive to the level of s . For these reasons, a rough estimate of s may be quite adequate in evaluating q .

In this study, the values of q given in Table 1 are used. The 1971 and 1976 census counts of heads and nonheads by age and sex in Canada are used to estimate the intercept and slope values of (18). These intercept and slope values are reproduced in Table 2. Also presented in Table 2 are the assumed values of s and the resulting values of q by five-year age cohorts. These q values in Table 2 were weighted to arrive at the ten-year age group estimates in Table 1.

TABLE 1
Values of q Used in Model

1976 Age Group	Female	Male
15-24	0.525	0.631
25-34	0.446	0.125
35-44	0.359	0.052
45-54	0.290	0.025
55-64	0.309	0.026
65 and older	0.170	0.032

SOURCE: See text

TABLE 2
Calculation of q Values

1976 Age Group	Assumed s	Females			Assumed s	Males		
		A	B	q		A	B	q
15-19	0.06	0.882	0.118	0.889	0.05	0.954	0.046	0.957
20-24	0.18	0.378	0.621	0.490	0.03	0.605	0.395	0.616
25-29	0.08	0.436	0.563	0.482	0.01	0.166	0.834	0.174
30-34	0.04	0.379	0.621	0.404	0.01	0.064	0.936	0.073
35-39	0.02	0.316	0.684	0.330	0.01	0.068	0.932	0.077
40-44	0.02	0.368	0.632	0.380	0.01	0.017	0.983	0.026
45-49	0.02	0.276	0.724	0.291	0.01	0.023	0.977	0.033
50-54	0.02	0.275	0.725	0.289	0.01	0.006	0.994	0.016
55-59	0.02	0.338	0.662	0.351	0.02	-0.014	1.014	0.036
60-64	0.06	0.221	0.779	0.268	0.06	-0.005	1.005	0.055
65 and older	0.06	0.117	0.883	0.170	0.06	-0.030	1.030	0.032

SOURCE: See text

NOTE: A and B are intercept and slope respectively in (18)

This method generates plausible probabilities that a given head in 1976 was not also a head in 1971. These probabilities in general diminish with age (particularly among young adults) and are higher for women than for men.⁵ The sex differential reflects the tendency for males to be designated as head in married couple households.

⁴Before 1976, a male spouse, if present, was always taken to be head. In 1976, the definition was changed to permit either spouse to be head although in practice the male still typically assumed headship.

⁵Remember that these estimates are conditional on the values of s assumed. No direct attempt was made in the study to assess the sensitivity of the model to the choice of s for each age-sex cohort.

B. Components-of-Change Analysis for the Toronto CMA; 1966-71 and 1971-76

The 1976 Toronto CMA with a population of 2.8 million persons includes within its boundaries twenty municipalities (in 1971, there were twenty-nine) ranging in size from East Gwillimbury Township at 10,600 persons to the city of Toronto at 633,300 persons. Some of the less-populated outlying municipalities have been combined in this study while the larger ones are left untouched to yield a total of twelve zones (thirteen in 1971) which are described in Appendix A. The largest zone remains the city of Toronto and the least populated is the Pickering zone (at 48,700 persons). The objective in aggregating to zones is to ensure that, for the most part, the interzonal flows of heads would be large enough to make model estimation worthwhile.⁶

To estimate flows of new, lost, and migrating heads, special tabulations of the 1976 and 1971 censuses were carried out. These show the numbers of 1976 (or 1971) heads by their 1971 (or 1966) place of residence which is categorized as the same zone, another of the zones, or some point outside the CMA. Further, these data are subdivided according to the age group (15-24, 25-34, 35-44, 45-54, 55-64, and 65+) and sex of the head.

The q proportions in Table 1 were applied to the nonmigrant (same zone at start and end of a five-year period) and interzonal migrant flows by age and sex for both 1971 and 1976 census data. This permits an estimate of the number of migrant or nonmigrant heads at the end of the period who were either heads or nonheads five years before. Those who were nonheads before, are new heads according to our definition. Heads who indicate an old place of residence which was outside the CMA are all automatically new heads as well. Finally, by comparing these estimates of five years before with the actual census count of heads at the earlier date, the number of lost heads can be estimated as a residual.

Consider the city of Toronto as an example. In 1976, there were approximately 230,400 heads of whom about 46,700 (migrant heads) had resided outside that zone in 1971 including about 41,400 who had lived outside the CMA altogether. Of the $(230,400 - 46,700 =)$ 183,700 nonmigrant heads in 1976, about 155,800 were estimated, using Table 1, to also have been heads in 1971. Of the $(46,700 - 41,400 =)$ 5,300 interzonal migrants, about 3,900 were estimated, again via Table 1, to have been heads in 1971 as well. Thus the total number of new heads is $41,400 + (183,700 - 155,800) + (5,300 - 3,900) = 70,700$. In addition, by examining flows of heads into other CMA zones, it was estimated that another 39,400 heads there in 1976 had also been heads in the city of Toronto in 1971. Since there were 224,700 heads actually present according to the 1971 census, this leaves a residual of lost heads numbering $244,700 - 155,800 - 39,400 = 29,500$. The following diagram portrays the same information:

1971 Heads in Toronto		1976 Heads in Toronto	
	New	70,700	
	From Elsewhere in CMA	3,900	
		155,800	
224,700	{	230,400	
	 To Elsewhere in CMA 39,400		
	 Lost 29,500		

⁶Some boundary changes have been recorded for outlying municipalities in the CMA between 1966 and 1976. Part of the reason for aggregating to the zonal level has also been to minimize (or more correctly internalize) the impact of such changes on comparisons among 1966, 1971, and 1976 data.

This analysis was applied to each zone in the Toronto CMA for both 1971 and 1976. The results are presented in Table 3. Interesting variations are to be found both among the zones and between the two time periods. The following are noted.

1. The dominance of outflows from the city of Toronto zone to the rest of the CMA. During 1966–71, the outflow from Toronto was over one-third of total interzonal outflows in the CMA. By 1971–76, it had risen to almost one-half of the total.
2. The spatial pattern of inflows from other parts of the CMA. The inflow of heads in absolute terms has been largest in Mississauga, North York, and a collection of other new suburban areas during both 1966–71 and 1971–76.
3. Zonal and temporal variations in the arrival of new heads. In absolute terms, the city of Toronto is by far the major recipient of new heads. After comparing this pattern with that described in (2) above, it is apparent that the locational behavior of new heads is considerably different from that of locally migrant heads.
4. Zonal variations in lost heads.⁷ Although the number of lost heads tends to vary directly with the size of the zone in both time periods, there are important exceptions. Two of these in 1976 are the central zones York and East York which have very large numbers of lost heads relative to the number of heads present in 1971. Three others with similar high ratios are the fringe zones, Pickering, Oakville, and Outer York. In the former cases, this may be attributable to the zone's age structure. In the latter cases, it is more likely attributable to short-range movements to places just outside the CMA.

3. ESTIMATION OF THE MODEL FOR THE TORONTO CMA; 1966–71 AND 1971–76

In this section, the empirical form of the model is discussed. Each block of the model is analyzed in turn; first the parameter block, second the lost-new block, and finally the interzonal flows block. Model estimation is discussed in the cases of the first two blocks and a validation exercise in the case of the third.

A. The Parameter Block

In constructing Table 3, the h_i 's are found by summing the number of heads in 1966 (or 1971) in zone i who five years later were heads in any other zone of the CMA. These latter flows are the h_{ij} 's, and the a_{ij} 's are formed from them using (7). In the 1976 data set there are $12 \times 11 = 132$ such values; one for each origin-destination combination (excluding h_{ii}). In the 1971 data set, which has one additional zone, there are $13 \times 12 = 156$ such values.

The hypotheses presented in section 1 suggest the importance of the distance between i and j in affecting the level of a_{ij} . The argument made is a fairly common one, that households are more likely to move shorter distances than longer distances. In this case, we measure distance, d_{ij} , as the minimum number of zonal boundaries that have to be crossed en route from i to j . All adjacent zones thus have a $d_{ij} = 1$ and, for example, zones separated by one intervening zone will have a $d_{ij} = 2$.⁸

The second variable affecting a_{ij} is the relative supply of potential housing opportunities in zone j . A primary means by which housing opportunities are created is through the completion of new housing units. Thus, the more new

⁷These estimates of lost heads should be approached cautiously. Because they are estimated as residuals, they are subject to errors arising from a number of sources: census miscounts, boundary changes, and the method of estimation to name just three.

⁸This is a crude distance measure but perhaps is no worse than using centroid-to-centroid distances in a study where the zones are large relative to the connecting distances.

TABLE 3

Components-of-Change Analysis for Toronto CMA Zones, 1966-71 and 1971-76

Zone i	1966 Heads	1966-71 Components					1971-76 Components					1976 Heads
		h_i	n_i	h_i	l_i		h_i	n_i	h_i	l_i		
Toronto	190,100	2,600	73,400	15,400	26,000		3,900	70,600	39,400	29,500		230,400
York	42,300	500	13,200	1,900	7,500		3,700	12,600	3,400	10,400		49,100
East York	32,200	600	10,400	600	4,300		3,200	11,600	1,300	9,400		42,400
North York	107,800	4,800	40,600	4,800	2,300		12,600	46,600	10,500	18,800		176,100
Scarborough	70,600	3,700	22,100	3,900	1,000		8,500	28,300	6,800	3,400		118,100
Etobicoke	73,700	3,000	17,900	4,600	7,800		7,600	21,400	5,800	8,500		96,900
Pickering	9,400	2,400	2,800	1,000	1,900		3,000	3,100	900	3,400		13,600
Inner York	16,800	4,000	4,400	2,200	600		6,600	6,400	2,700	1,800		31,000
Outer York	12,100	2,800	3,400	900	2,300		4,400	4,500	1,400	3,200		22,000
Albion	14,900	3,900	6,300	1,400	2,400		7,500	9,400	1,700	1,100		35,400
Mississauga	27,400	8,600	14,700	2,600	1,200		15,800	21,700	4,500	5,800		74,100
Oakville	13,500	1,900	4,700	900	2,600		2,700	5,800	1,100	3,700		20,400
Halton	7,800	1,700	2,700	500	1,200		(Not part of CMA in 1976)					

NOTE: Components may not sum due to rounding

SOURCE: See text

*1971 boundary basis: according to 1976 basis, number of heads in 1971 was 17,600

housing units are completed in zone j relative to other possible destination zones, the more likely is an out-migrant from i to relocate in j .⁹

Let $HC_j(t+1)$ be the number of new housing unit completions in zone j in the five-year period leading up to the census date of June 1, 1971 (or 1976), and

$$c_{ij}(t+1) = \frac{HC_j(t+1)}{\sum_{k \neq i} HC_k(t+1)}. \quad (19)$$

Thus, $c_{ij}(t+1)$ is the proportion of all new housing unit completions (excluding those in zone i) which are found in zone j .

Canada Mortgage and Housing Corporation (CMHC) maintains (unpublished) monthly data series on new housing unit completions by municipality within each CMA. The new unit completions data used in this study are the aggregations of these monthly figures from June 1966 (or 1971) to May 1971 (or 1976). One problem arises here with respect to municipalities which were not part of the CMA at the beginning of a five-year period but were at the end. Parts of the Outer York, Albion, and Halton zones fall into this category during 1966–71 or 1971–76. The problem is that CMHC does not maintain separate municipal records until after the municipality has become part of the CMA. This forced a reduction in the number of zones for which full new housing units completion data are available to ten in both 1966–71 and 1971–76.

For each of the ten zones in each period for which HC data are available, there are thus nine other zones for which HC is also available, to which interzonal flows might occur. There are thus 90 observations of h_{ij} for each five-year time period or 180 in all for the two periods. The estimated model is as follows.¹⁰

Using 1966–71 data only,

$$\text{flog}(a_{ij}) = -2.240 + 6.058c_{ij} - 0.530d_{ij} + e_{ij}, \quad (20)$$

(5.420) (–4.613)

$$N = 89; R^2 = 0.460.$$

Using 1966–71 and 1971–76 data,

$$\text{flog}(a_{ij}) = -2.317 + 7.003c_{ij} - 0.546d_{ij} + e_{ij}, \quad (21)$$

(8.863) (–7.333)

$$N = 179; R^2 = 0.505,$$

where

$$\text{flog}(a_{ij}) = \ln[a_{ij}/(1-a_{ij})], \quad (22)$$

e_{ij} is a random error term, N is the sample size, and t -values are in parentheses.

This model provides a reasonable fit to the data when we remember that dis-

⁹In section 3B below, a somewhat broader definition of housing opportunities is used which includes housing units made available by departing lost heads. Such a formulation was also initially used when estimating (20) and (21). However, the insignificance of this latter term led to its eventual exclusion.

¹⁰The OLS method necessitates deletion of cases where $a_{ij} = 0$. There was one such case in the 1966–71 data.

tance is being crudely measured and that new housing completions are presumably only one measure of the attractiveness of a zone to potential immigrants. The R^2 is not bad in either case although the model does fit the 1971–76 period better than the earlier one. The estimations are also reassuring in that the slope coefficients are quite similar for the two time periods: the coefficient of distance (d_{ij}) remains virtually unchanged while the coefficient for relative completions (c_{ij}) increases only moderately.

The second parameter to be modeled in this block is b_i ; the ratio of gross outflows ($h_i + l_i$) to gross inflows ($h_i + n_i$) for zone i . In section 1, it is argued that b_i depends on changes in the housing stock and in the vacancy rate. There do not exist any reliable data on a municipality basis covering 1966–76 for either the vacancy rate or any components of housing stock change other than new dwelling unit completions. For this reason, a focus must be placed exclusively on the role of completions.

The other components of housing stock change, demolitions and conversions, can be crudely allowed for by looking at the ratio of new housing unit completions over a period to the number of households present at the start of the period. To the extent that demolitions and conversions vary directly with the stock of dwellings, the starting number of households should act as a proxy in deflating the effect of new housing unit completions.¹¹

Remembering that we are again restricted to ten zones in each time period because of the lack of completions data for the remaining zones, the following model has been estimated.¹²

Using 1966–71 data only,

$$b_i(t+1) = 0.929 e^{-3.044(HC_i(t+1)/h_i(t))} e_{ij}, \quad (23)$$

$$N = 10; R^2 = 0.752.$$

Using 1966–71 and 1971–76 data,

$$b_i(t+1) = 1.003 e^{-3.024(HC_i(t+1)/h_i(t))} e_{ij}, \quad (24)$$

$$N = 20; R^2 = 0.719.$$

This model also provides a reasonable fit to the observed data. The R^2 is quite good and roughly constant over the two time periods. The slope coefficient is effectively constant as well over both time periods. The principal change between 1966–71 and 1971–76 is a moderate rise in the intercept indicating a parallel upward shift in the fitted equation.

B. The Lost-New Block

Let us first consider the number of lost heads in each zone. In section 1, it is argued that the number of lost heads is a function of the age structure of a zone's population and its proximity to the CMA boundary. We here define a dummy variable $BORD_i$ which takes on a value of 1 if zone i is on the edge of the CMA and zero otherwise.¹³ Two age structure variables are included. One is the propor-

¹¹This is a crude assumption which is made necessary by a lack of data.

¹²Nonlinear least squares was used to estimate (23) and (24). No t -tests are available for the exponential coefficient. In the estimation procedure a solution was sought near the intercept value of 1.0.

¹³The bordering zones are Pickering, Albion, Outer York, Halton, and Oakville.

tion of the zone's population which is aged 20–29 at the start of the period ($P_{2,i}(t)$) and the other is the proportion aged 60 or older ($P_{6,i}(t)$). The first is to catch variance in mobility among the zones attributable to the number of young adults. The second is to capture the effect of death or aging on headship status.

Using ordinary least squares (OLS), the following model has been estimated.

Using 1966–71 data only,

$$\frac{l_i(t+1)}{h_i(t)} = -0.131 + \underset{(1.478)}{1.002P_{2,i}(t)} + \underset{(2.045)}{0.706P_{6,i}(t)} + \underset{(5.477)}{0.132BORD_i} + e_i, \quad (25)$$

$$N = 13; R^2 = 0.786.$$

Using 1966–71 and 1971–76 data,

$$\frac{l_i(t+1)}{h_i(t)} = -0.101 + \underset{(1.650)}{0.790P_{2,i}(t)} + \underset{(2.278)}{0.796P_{6,i}(t)} + \underset{(4.566)}{0.116BORD_i} + e_i, \quad (26)$$

$$N = 25; R^2 = 0.543.$$

Some success has been encountered in this estimation. The coefficients of $P_{6,i}$ and $BORD_i$ are significant in both of the above estimations and both R^2 's are moderately good. However, there are two problems. First, the 1966–71 period is apparently substantially different from the 1971–76 period in a way that is partly masked when looking at the combined regression (26). There is a substantial drop in the goodness of fit (to an R^2 of about 0.35) if one considers the 1971–76 data alone. Second, there is collinearity between $P_{2,i}$ and $P_{6,i}$ in this sample. The exclusion of one of these substantially changes the slope coefficient of the other. This makes it difficult to interpret the separate roles of young adults and the elderly in determining the number of lost households.

Consider now the second part of this block which has to do with the zonal destinations of new heads. In what follows, a distinction is drawn between new-immigrant heads and new-other heads. A new-immigrant head is one who lists his or her place of residence five years before as being outside Canada. A new-other head is any other new head. This distinction is drawn for two reasons. First, one of the purposes of the model is to enable assessment of the potential consequence of changing national immigration policies on residential relocation. Second, there is evidence that the flows of new-immigrant heads into zones is different in pattern from the flows of new-other heads.

In this model, the total number of new-immigrant (NI) and new-other (NO) heads are treated as exogenous variables. The endogenous variables are the percentages of these totals ($\%NI_i$ and $\%NO_i$) which are found in zone i at the end of the time period. Thus, if NI_i and NO_i are the actual number of new-immigrant and new-other heads in zone i , the following must hold.

$$n_i = NI_i + NO_i \quad (27)$$

$$\%NI_i = 100 \times NI_i / NI \quad \text{and} \quad \%NO_i = 100 \times NO_i / NO. \quad (28)$$

It is argued in section 1 that the incidence of new heads in a zone is related to

the relative supply of housing opportunities available there. This notion of housing opportunities has already been discussed in the parameter block in relation to the a_{ij} 's. There, the relative supply of housing opportunities has been proxied by c_{ij} as defined in (19). Here again a similar variable is used. Let $\%HC_i(t+1)$ be the percentage of all CMA new housing unit completions which occur in zone i .

$$\%HC_i(t+1) = \frac{HC_i(t+1)}{\sum_j HC_j(t+1)} . \quad (29)$$

It is argued that this variable affects both $\%NI_i$ and $\%NO_i$, although not necessarily in the same way or to the same degree.

There is, however, another facet to the opportunities argument; one that does not depend on housing supply changes. Existing occupied dwellings may be vacated during the period, augmenting the potential supply from among which the new head makes a choice. One measure of the relative incidence of the vacating of an occupied unit is the level of lost heads in a zone. The more heads are lost in a zone over a period of time, the more housing opportunities are opened up for a new head. Let $\%LST_i$ be the percentage of the total number of lost heads in the CMA who came from zone i .

$$\%LST_i(t+1) = \frac{l_i(t+1)}{\sum_j l_j(t+1)} . \quad (30)$$

The hypotheses now made are that $\%NI_i$ and $\%NO_i$ are each functions of both $\%HC_i$ and $\%LST_i$.

The above models were estimated using ordinary least squares with the following results,

Using 1966–71 data only,

$$\text{flog } (\%NI_i) = -4.352 + 0.096\%HC_i + 0.054\%LST_i + e_i , \quad (31)$$

(4.450) (3.874)

$$N = 10; R^2 = 0.865.$$

$$\text{flog } (\%NO_i) = -2.925 + 0.112\%HC_i + 0.059\%LST_i + e_i , \quad (32)$$

(11.120) (9.101)

$$N = 10; R^2 = 0.974.$$

Using 1966–71 and 1971–76 data,

$$\text{flog } (\%NI_i) = -4.662 + 0.107\%HC_i + 0.069\%LST_i + e_i , \quad (33)$$

(4.010) (3.677)

$$N = 20; R^2 = 0.696.$$

$$\text{flog } (\%NO_i) = -2.995 + 0.102\%HC_i + 0.073\%LST_i + e_i , \quad (34)$$

(8.217) (8.304)

$$N = 20; R^2 = 0.914.$$

There are several interesting aspects to these estimates. First, both immigrant and other new heads appear to be similarly sensitive to the incidences of housing completions and lost heads. The slope coefficients in these two equations are quite close for both the 1966-71 and 1966-76 data sets for each of %HC and %LST. Second, the goodness-of-fit for new immigrant heads tends to be much better for the 1966-71 period than for 1971-76. This is shown up in an R^2 for the 1966-76 data set of 0.696 compared to an 0.865 value when the 1966-71 data are considered alone. The goodness-of-fit also declines for other new heads as well but the decline is less marked. Third, in spite of a declining goodness-of-fit for the second time period, both models have only significant coefficients for both the 1966-71 and 1966-76 periods.

C. The Interzonal Flows Block

As outlined in section 1, the outputs of the parameter and lost-new blocks (a_{ij} , b_i , n_i , and l_i) are fed into the interzonal flows block to solve (14). Since this block is represented by a series of identities of the form (12), there are no additional functional relationships to be estimated. However, since this block predicts interzonal flows, these predictions can be compared to observed flows over the 1966-71 or 1971-76 periods. This provides an overall validation measure for the model which is in addition to the goodness-of-fit measures of the individual relationships already estimated in the two preceding blocks. A schematic of the model's causal structure is presented in Figure 1 as an aid in further discussion.

FIG. 1. Causal Structure of the Model

In this section, three such projections are described.

Projection P1: A forecast of the 1971 number of heads by zone using (1) 1966–71 estimated relationships, that is, (20), (23), (25), (31), and (32); (2) 1966 levels of heads, proportion of population aged over 59, and proportion of population aged 20–29; (3) 1966–71 levels of housing completions; and (4) the distance and *BORD* values for each zone.

Projection P2: A forecast of the 1976 number of heads by zone using (1) 1966–76 estimated relationships, that is, (21), (24), (26), (33), and (34); (2) 1971 levels of heads, proportion of population aged over 59, and proportion of population aged 20–29; (3) 1971–76 levels of housing completions; and (4) the distance and *BORD* values for each zone.

Projection P3: A forecast of the 1976 number of heads by zone using (1) 1966–71 estimated relationships as in P1 above; (2) 1971 levels as in P2 above; (3) 1971–76 levels as in P2 above; and (4) as in P1 and P2 above.

Projections P1 and P2 are included to show how well the model predicted household flows over its estimation periods. Projection P3 gives some idea of how well the model could have predicted 1971–76 flows given only 1971 and prior data plus 1971–76 new housing unit completions by zone.¹⁴

A summary of the projections is presented in Table 4. There, the actual number of heads in each census year (1966, 1971, and 1976) is included along with the projection (P1) for 1971 and the two projections (P2 and P3) for 1976. Some success has been encountered in these projections. The median absolute percent error (MAPE) can be calculated from Table 4 and is 3.8 percent for P1, 4.0 percent for P2, and 4.0 percent for P3. These indicate that the typical zonal forecast came within ± 3.8 to 4.0 percentage points of the actual number of heads for that year.

TABLE 4

Actual and Projected Heads by Zone; Toronto CMA, 1966–76

Zone	Actual Heads			Projected Heads		
	1966	1971	1976	1971 P1	1976 P2	1976 P3
Toronto	190,100	224,700	230,400	221,400	237,300	239,600
York	42,300	46,600	49,100	45,300	49,900	50,000
East York	32,200	38,300	42,400	38,900	41,600	41,700
North York	107,800	146,200	176,100	153,700	174,900	172,700
Scarborough	70,600	91,500	118,100	85,600	113,200	110,700
Etobicoke	73,700	82,300	96,900	79,700	91,400	90,500
Pickering	9,400	11,800	13,600	13,100	15,900	15,600
Inner York	16,800	22,500	31,000	21,700	30,300	29,100
Outer York	12,100	15,200	22,000	16,000	19,600	19,500
Albion	14,900	21,200	35,400	21,400	33,600	32,700
Mississauga	27,400	46,900	74,100	45,100	70,000	68,000
Halton	7,800	10,400	16,000	11,500	—	16,100
Oakville	13,500	16,700	20,400	17,700	21,200	21,200

SOURCE: See text

NOTE: Rounded to nearest hundred

As noted in the parameter block description earlier, the equation for the a_{ij} 's has the lowest R^2 (0.46 for the 1966–71 data, e.g.) for any of the estimated functional relationships. Therefore, one might expect larger errors when looking at individual h_{ij} projections. The mean percentage error (absolute value) in the pro-

¹⁴As noted earlier, housing completions data are unavailable for Outer York, Albion, and Halton. These values were estimated by the change in total households over the appropriate period.

jected h_{ij} 's was found to be 11.6 percent for P1, 12.6 percent for P2, and 20.8 percent in P3.¹⁵

4. TWO SENSITIVITY ANALYSES

In this section, two simple experiments are described showing some of the interesting questions which can be addressed with this model. One concerns the effect on metropolitan expansion of a decrease in the level of immigration from abroad into the Toronto CMA. The other experiment looks at the impact of a shift of housing construction toward or away from the metropolitan fringe. In both cases, we ask what the number of 1976 households in each zone would have looked like if either exogenous variable (immigration or new housing unit completions) had been different while holding other variables and all functional relationships constant at 1971 or 1971-76 values. In other words, the experiments are basically similar to projection P2 except for the change in NI or in HC_i . Let us consider these now in turn.

A. A Decline in Immigration

In the 1976 census, 67,750 household heads indicated a June 1, 1971, place of residence which was outside Canada. This was down somewhat from the corresponding figure of 74,100 in the 1971 census. Nonetheless, Toronto has been and continues to be an important destination of immigrants entering Canada from abroad.

In recent years, there have been sporadic national debates over the total amount of immigration that should be permitted or encouraged to enter Canada. These debates have been sparked by a number of issues about which the present model has nothing to say. However, one argument which has been raised concerns the potential effects that a reduction in immigration might have on the filtering process in Toronto's housing market. In this view, immigrant flows assist in maintaining a healthy stock of lower-quality, older housing by permitting the more-established current occupants of such housing to trade up to better housing elsewhere in the metropolitan region. This process of upgrading and residential relocation might well be slowed down by a reduced flow of immigrants. While the present model has nothing to say about the desirability of such impacts, it does offer basic empirical support for this kind of argument.

An experiment (projection P4) was carried out in which it was assumed that the number of immigrant heads present in 1976 had been reduced by 30 percent or 20,300 heads. Using the variables and relationships otherwise specified in projection P2, forecasts were derived of the 1971-76 flows and the 1976 total number of households by zone. Since interzonal movements are closed within the CMA, the aggregate change in both the total number of heads and the number of new heads must be this same -20,300. Table 5 records the 1976 changes in the number of new and total heads by zone. These are the changes between the immigration-reduced (P4) 1976 forecasts and those in the otherwise-similar projection P2.

To explain the data in Table 5, consider the city of Toronto as an example. In 1976, the P2 forecast was for 69,900 "new" (immigrant plus other) heads whereas in the P4 it is 62,700. This is a decline of 7,200 new heads or 35.4 percent of the total reduction of 20,325. On the other hand, the P2 forecast was for 237,000 total heads in the city in 1976 whereas in P4 it is 235,800. This is a decline of only 1,500 heads or 7.5 percent of the total change of 20,325. This means that although

¹⁵The h_{ii} terms are included in these percentage error calculations. These tend to have smaller percentage errors than do other h_{ij} 's. Therefore, the typical percentage errors for the interzonal flows ($i \neq j$) are larger.

the number of new heads fell substantially, the net number of in-migrating "old" heads ($h_{i,t} - h_{i,t-1}$) must have increased by the difference of $7,200 - 1,500 = 5,700$. This does not mean that $h_{i,t}$ increased in absolute value but rather that with the lower number of new (immigrant) heads moving into the city of Toronto in P4, $h_{i,t}$ declined faster than did $h_{i,t-1}$ thus compensating for much of the initial effect of a reduction in immigration. The implication of this is that although the city is a major recipient of new immigrants, the net reduction in total number of households after an immigration cutback would not be large. The major effect of a cutback here would be to sharply reduce the volume of interzonal flows out of the city.

TABLE 5
Reduction in Numbers of New and Total Heads by Zone
from P2 to P4; Toronto CMA, 1971-76

Zone	New Heads		Total Heads	
	Number	CMA (%)	Number	CMA (%)
Toronto	7,200	35.4	1,500	7.5
York	500	2.6	500	2.6
East York	500	2.6	500	2.5
North York	4,800	23.5	4,200	20.7
Scarborough	2,200	10.8	3,400	16.8
Etobicoke	1,100	5.4	1,400	7.0
Pickering	300	1.6	700	3.3
Inner York	500	2.3	1,500	7.2
Outer York	400	1.9	700	3.2
Albion	700	3.7	1,800	8.8
Mississauga	1,700	8.2	3,500	17.4
Oakville	400	2.0	600	2.9
Total	20,300	100	20,300	100

SOURCE: See text

NOTE: Columns may not total due to rounding

In contrast, consider Mississauga which during 1971-76 was a rapidly developing new suburban area. In P4, it is estimated that the CMA-wide reduction in immigration would reduce the number of new heads in Mississauga by about 1,700. However the reduction in the total number of heads present there is estimated to be about 3,500. Thus Mississauga is predicted to bear only 8.2 percent of the CMA-wide decline in new heads, but 17.4 percent of the decline in total heads. This latter figure is more than double the figure for the city of Toronto. Why? Mississauga receives much of its increase in households from the excess of inflows over outflows of interzonal migrant heads. The general reduction in such flows, if immigration is restricted, adversely affects Mississauga's growth. As just one example, the largest flow of old heads into Mississauga during 1971-76 was from the city of Toronto and, as argued above, the model predicts a sharp decline in this flow if immigration is restricted.

The second pair of columns in Table 5 show that the major impacts of an immigration cutback on household growth will be in the areas experiencing the greatest suburbanization during 1971-76: North York, Mississauga, and Scarborough. With the exception of North York, these are areas to which the direct flow of immigrant heads is fairly small. The major impact of reduced immigration is an indirect one on the interzonal flows that fuel suburban expansion.¹⁶

¹⁶The importance of the causality assumption is very apparent here. In the model, it is assumed that residential flows are set in motion within the CMA by the arrival of new heads. Thus, depending on the level of b_i , each 100 immigrant (or other new) heads displace a number of existing heads. If the number of immigrant heads declines, there is assumed to be a corresponding decline in the volume of relocation of old heads.

B. Altering the Spatial Pattern of Dwelling Unit Completions

For a variety of reasons, it may be of interest to look at the impact of altered spatial patterns of new dwelling unit completions. One may, for instance, want to encourage higher density development in inner metropolitan areas in order to reduce energy requirements for transportation or to reduce the consumption of valuable land in other uses at the metropolitan boundary. Controlling the locations of new dwelling unit completions is one means of achieving such objectives.

In the present model, new housing unit completions enter as an exogenous variable in several places. They help to determine (1) the number of new heads arriving in a zone, (2) the ratio, b_i , of outflows to inflows, and (3) the destinations of interzonal migrants. At the same time, new housing unit completions are an important component of housing stock change in most zones. There should be a rough correspondence between the number of such completions and the net change in total households. Although there is no constraint of this latter form directly in the model, the parameter estimates help to preserve this, as is seen below.

For the purpose of this simulation, the zones are divided into two groups: the "inner" zones which lie within the Municipality of Metropolitan Toronto (the city of Toronto, York, East York, North York, Scarborough, and Etobicoke), and the "outer" zones which are the remainder. During 1971–76 the level of completions in the inner group was in total about 50 percent larger than in the outer group.

A projection P5 for the 1971–76 period was undertaken. It was derived as follows:

Projection P5: A forecast of the 1976 number of heads by zone assuming the same data and relationships used in P2 with one exception. In the inner group of zones, the housing completions used are 10 percent larger than the actual 1971–76 levels while in the outer zones they are 15 percent smaller.

In projections P2 and P5, the CMA-wide total new housing unit completions are similar. The principal differences between them is the relative incidences of completions in different zones.

In Table 6, the forecast from P5 for each zone is presented relative to the P2

TABLE 6
Percentage Change in Forecasted 1976 Total
Households from P2 to P5

Zone	P2 to P5 (%)
Toronto	+11.2
York	+ 7.7
East York	+ 7.3
North York	+12.9
Scarborough	+12.7
Etobicoke	+10.6
Pickering	-12.4
Inner York	-14.4
Outer York	-12.4
Albion	-14.5
Mississauga	-20.8
Oakville	-12.6

projection. As one might expect, the inner zones average 10 percent increases in total households in P5 while the outer zones average -15 percent compared to P2. This simply reflects the different levels of housing completions assumed in the two projections. There are, though, marked variations for individual zones.

Mississauga appears to be the most sensitive to variations in new unit completions. Why? Mississauga is the most popular destination of interzonal migrants who leave the inner zones (principally the city of Toronto and Etobicoke). In P5, these inner zones experience a higher level of completions, which attracts migrants from elsewhere who might otherwise have gone to Mississauga. In addition, these two inner zones will also have a lower b_i , or ratio of outflow to inflow, which directly reduces the large outflows from these zones to Mississauga. For such reasons, Mississauga appears to be relatively more sensitive to the spatial incidence of new dwelling unit completions than are other outer zones.

5. SOME CONCLUDING COMMENTS

The replacement model of metropolitan expansion presented here replicates the flows of households among geographical zones in a metropolitan area. Although in certain aspects it is similar to a Markovian model of residential relocation, there are some important distinctions. The most significant of these is that the model balances the net inflow of households against changes in the stock of housing in each zone. Another important distinction is that flows of households between zones are made endogenous to a number of variables including the level of new housing unit completions, the age structure of the population, and the flows of new (immigrant and other) heads.

The model has been empirically applied using data for the Toronto CMA covering both 1966–71 and 1971–76. This necessitates a method for separating new from old heads; a distinction which is fundamental to the model. Based on these estimates of new and old heads, the model has been empirically validated. Some success has been encountered using both 1966–71 dataset and the combined 1966–76 datasets. Two experiments have been described in which the sensitivity of the model to variations in certain exogenous variables has been tested.

The model offers promise as an empirical and conceptual tool. It lets us look at some issues in residential relocation which could not be easily handled in a Markovian model. It is not being argued, however, that this replacement model is always preferable to a Markovian approach. Any model of necessity constrains the user to a particular kind of causal approach and a particular way of conceiving and measuring a process under study. If the situation being studied does not conform to these viewpoints, it would not be appropriate to employ this particular model.

The present model for example assumes that relocation can be viewed as a replacement process. Accordingly, when an interzonal migrant moves, it is as though in some sense he or she is replaced by an inflowing household. Implicit in this notion is the idea that a currently occupied dwelling unit will not tend to become vacant at some later date. This makes some amount of sense in a robust housing market where housing abandonment or long-term vacancy rate changes are not significant factors. To a certain extent, Toronto would seem to fit this requirement whereas some other cities might not.

The present model also rides roughshod over a number of considerations that some other researchers would consider to be important. It does not, for example, give very much consideration at all to housing supply aspects (other than straight completion totals), to economic issues such as the price of housing in different areas, or to microscale relocation behavior. To these other researchers, the present model might well appear to be too crude because it does not assess causality or conceive or measure the relocation process in a way which they feel is important. However, the present model does provide a particular way of looking at relocation which can be useful in certain circumstances.

APPENDIX A

Definitions of Zones

Zone Name	Municipalities Included in the Zone	
	1971 CMA	1976 CMA
Oakville	Oakville	Same
Pickering	Pickering Twp. Ajax Pickering Village	Pickering Twp. Ajax
Albion	Albion Twp. Chingacousy Twp. Toronto Gore Twp. Brampton Bolton Village	Caledon Brampton
Mississauga	Toronto Twp. Port Credit Streetsville	Mississauga
Etobicoke	Etobicoke	Same
Scarborough	Scarborough	Same
Toronto	City of Toronto	Same
York	York Borough	Same
East York	East York	Same
North York	North York	Same
Inner York	Markham Richmond Hill Vaughan	Same
Outer York	Aurora King Newmarket Whitchurch-Stouffville	Aurora King Newmarket Whitchurch-Stouffville East Gwillimbury
Halton	Esquering Twp. Acton Georgetown Milton	Not part of 1976 CMA

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