Investigating Seismic Wave Scattering in Heterogeneous Environments and Implications for Seismic Imaging

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Abstract

Inhomogeneities in the earth (fractures, layering, shape, composition) are responsible for seismic wave scattering and contribute towards amplitude, travel time, frequency and spectral fluctuations observed in seismic records. This thesis presents findings that complement our understanding of seismic scattering and imaging in heterogeneous media. Interest focused on probing the correlation between spatial variations in attributes that characterize the state (physical, chemical) of rocks and seismic waveform data with consideration towards potential implications for seismic survey design to optimize imaging, imaging with converted waves, microseismic monitoring, velocity modeling and imaging of lithological boundaries.

The highlights of the research strategy include:

- The use of stochastic methods to build realistic earth models that characterize the 1D, 2D and 3D spatial variations in rock properties. These petrophysical earth models are conditioned by experimental (“hard”) data such as geology, wave velocities and density from case study areas like the Bosumtwi impact crater and the base metal deposits in
Nash Creek (Canada) and Thompson (Canada). The distributions of the sulfide mineralization at Nash Creek and at Thompson represent two end members of the heterogeneity spectrum. While the sulfide mineralization at Nash Creek is highly disseminated in nature, the sulfide rich zones at Thompson occur as well defined volumes (lens-shaped) having a strong density contrast with respect to the host rocks.

- Analysis of modeled forward (transmitted) and backward scattered wave propagation in the heterogeneous earth models.

As a result of a study aimed at correlating resonant frequencies to scale length parameters, it is observed that the efficiency of the spectral ratio method is undermined by its sensitivity to the interference between P- and S-waves as well as the impedance contrast.

It is also demonstrated that travel time of direct arrivals (transmitted waves) can be used to infer structural heterogeneity and velocity distribution beyond borehole locations. However, the success of imaging with transmitted waves is subject to the influence of geology which must factor in the choice of acquisition geometry.

For the first time, multivariate and multidimensional (3D) heterogeneous earth models that are conditioned by hard data from multiple boreholes are constructed. The methodology requires having at least one physical rock property attribute that is sampled along the whole borehole length. This approach helped to characterize the uncertainty in the distribution of rock densities and metal content in a study region of the Nash Creek property. The density data suggests the sulfides are disseminated and this poses challenges for both gravity and seismic imaging methods. Modeling studies suggest seismic methods will not be suited for imaging zones with such disseminated mineralization. On the other hand, when dealing with massive sulfide
mineralization that has complex geology (steep dip) like the case in Thompson, the success of the seismic imaging process relies very much on the acquisition geometry as well as the variability of the physical properties of the host rock. Elastic modeling results show that a Vertical Seismic Profiling (VSP) geometry is better suited to capture the down-dip scattered wavefield from the orebody. While surface acquisition geometry with sufficient extended length in the down dip direction can also be used to detect the dipping orebody, its efficiency can however be undermined by background heterogeneity: when the scale length along the direction of dip is comparable to the dimensions of the orebody, the scattered wavefields are strong enough to mask the diffraction hyperbola generated from the ore. Moreover, the study also corroborates that converted waves generated from the scattering processes hold promise as an imaging tool for a dipping orebody as they are least affected by the scattering processes of background heterogeneity.
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Glossary of Symbols

a: Characteristic scales in general

$a_x$: Characteristic scale along x direction

$a_y$: Characteristic scale along y direction

$a_z$: Characteristic scale along z direction

$\alpha$, Vp or V: P-wave velocity

$\beta$, Vs, $V_{s1}$, and $V_{s2}$: S-wave velocity

$\nabla \phi$ and $\nabla \times \psi$: P- and S-wave displacement components

$\Phi$: A random phase between $[-\pi, \pi]$

$f_s$: Source function

F, f, f: Frequency

$u$: Vector representation of particle displacement of seismic wave

$u, U$: Scalar representation of particle displacement of seismic wave

$u^0$: Primary field

$u^1$: Scattered field

$s$: Vector component for slowness of dimensions $\mathbb{R}^n$

$\phi$ and $\psi$: Compressional and shear wave potentials.

$A$: Amplitude

$\nabla$: Del operator $-\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$P_D / P_U$: Downgoing/upgoing P-waves

$S_D / S_U$: Downgoing/upgoing S-waves

$\gamma(h)$: Variogram/Semivariogram
\( C(h) \): Autocovariance/ Autocorrelation

\( C(0) \): Covariance/ Autocovariance/ Autocorrelation value at lag distance \( h = 0 \)

\( h \): lag distance

H-: Horizontal component

V-: Vertical component

T-: Transverse component

R_d-: Radial component

\( T,t \): Travel time

\((k \text{ or } k)/k\): vector/ scalar representation of wave number

\( P(k) \): Power spectrum density function for wavenumber \( k \)

\( K_\nu(r) \): The Bessel function of the second kind

\( \Gamma(r) \): The gamma function

\( \kappa \): bulk modulus

\( \mu \): Lame constant (Shear modulus)

\( \rho \): Density

\( \lambda \): Lame constant

\( \tau \text{ or } \sigma \): Standard deviation

\( T_{ij} \text{ or } \sigma_{ij} \): Stress components

\( \delta_{ij} \): Kronecker delta

\( r \): scalar distance (hypocentral or radial)

\((x, y, z)\): Cartesian coordinate

\( \mathbf{x}, \mathbf{x} \): vector representation of spatial coordinates

\( \nu, \nu \): Hurst number

\( \delta V \): Perturbation of velocity
\( V_m, V_0, V_{\text{mean}} \): Average velocity

\( \varepsilon \): Fractional fluctuation in a rock property

\( Q^{-1}, Q^{-1} \): Intrinsic attenuation

\( E \): Energy

\( \omega \): Angular frequency

\( \varphi \): random number uniformly distributed on the interval \([0, 2\pi]\)

\( \lambda_\omega \): wavelength

\( V_p \): Phase velocity

\( L \): propagation length or the extent of the heterogeneous region

\( M \): Extent of inhomogeneity

\( Z_p \): P-wave impedance

\( (r, \psi, \theta) \): Spherical coordinates

\( (\hat{r}, \hat{\psi}, \hat{\theta}) \): Unit vectors in the spherical coordinate system

\( G(x,t) \): 3D Green’s function

\( G_c \): Universal gravitational constant

\( \{\ldots\} \): Ensemble average

\( \tilde{S} \): Tilde means the fourier transform in space

\( J \): Energy flux density

\( z \): Discrete or continuous attribute

\( z_k \): k\(^{th}\) threshold value for the discrete or continuous attribute \( z \)

\( z(x_j) \): \( z \)-datum value at location \( x_j \)

\( Z^*(x) \): Estimate of a random variable (RV) at location \( x \)
\( m(x) \): Expected value of a random variable (RV) \( Z(x) \): trend component model in the decomposition \( Z(x) = R(x) + m(x) \), where \( R(x) \) represents the residual component

\( n(x) \): Number of data values \( \{ z_j \} \) used for estimation of an attribute \( z \) at location \( x \)

\( w_i(x) \): Kriging weight associated to \( z_i \)-datum at location \( x \) for estimation of the attribute \( z \) at location \( x \)

\( E\{ \ldots \} \): Expected value

\( Cov\{ \ldots \} \): Covariance

\( Var\{ \ldots \} \): Variance

\( p\left( Z(x) | \{ Z(x_1), \ldots, Z(x_j) \} \right) \): Conditional probability distribution of the RV \( Z(x) \) given the realization of \( j \) other RVs (data).

\( \text{Prob}\{ \ldots \} \): Probability

\( \rho^* \): Correlation coefficient between two random variables (RVs)

\( Y \): Random variable representing the normal score transform of the random variable \( Z \)

\( g(x) \): Gravitational attraction at location \( x \)

1D/2D/3D: One/Two/Three dimension(s)

3C: Three-component

ACF: Autocorrelation Function

AG: Archean Gneiss

Ag: Silver

APMT: Metamafic Intrusive

AVO: Amplitude variation with offset

BMC: Bathurst Mining Camp

ccdf: Conditional cumulative distribution function
CDF, cdf: Cumulative distribution function
CMP: Common midpoint
Cu: Copper
CVRD: Compania Valé Rio Doce
CVS: Constant velocity stack
DC: Direct current
DMO: Dip moveout correction
EM: Electromagnetism
FD: Finite Difference
FFT: Fast Fourier Transform
GOCAD: Geological Object Computer Aided Design- *Software for manipulating complex objects in three dimensions.*
ICDP: International Continental Scientific Drilling Program
IODP: Integrated Ocean Drilling Program
IP: Induced polarization
KTB: Kontinentale Tiefbohrung (Continental deep drilling)
LASA: Large-aperture seismic array
LITHOPROBE: Derives from "probing the lithosphere". It is a Canadian national research project established to develop a comprehensive understanding of the evolution of the North American continent.
LVM: Locally variable mean
MAN: Manasan formation
MASU: Massive sulfide
MATLAB: Matrix laboratory (computing environment and programming language)
MPa: Megapascal (SI unit for pressure)
MSOVSP: Moving source offset VSP
MVA/MVB: Model volume A/B
NMO: Normal moveout correction
OK: Ordinary kriging
OPG: Ospwagan group
P2: Pipe formation schists
P3: Pipe formation BIF
Pb: Lead
PDF: Probability density function
PEG: Pegmatite
PS, SP: P to S converted waves, S to P converted waves
PSDF: Power Spectrum Density Function
QC: Quality control
QTS: Quasi Transfer spectra
R: Reflection coefficient
RMS: Root mean square
RTM: Reverse time migration
RV: Random Variable
SF: Setting formation
SGS: Sequential gaussian simulation
SK: Simple kriging
SNR: Signal to noise ratio
TF: Thompson formation
TNB: Thompson Nickel Belt
TWT: Two-way travel time
UM: Metamafic Ultramafics
VMS: Volcanogenic massive sulfides
VRP: Vertical Resistivity Profiling
VSP: Vertical Seismic Profiling
Zn: Zinc
Chapter 1
Introduction

For several recent decades, geophysical imaging methodologies that help our understanding of the earth’s structure have improved tremendously. The underlying principles of these geophysical imaging methods involve using measurables that are representative of the physical properties of the materials that constitute the earth. For example, seismic methods are sensitive to the velocities and density of the rock masses, whereas electromagnetic methods and gravity methods are sensitive to the electrical (resistivities) and density properties respectively. Each method has limitations in both vertical and lateral resolution of the earth’s subsurface. Except for magnetotelluric methods that image deep crustal structures (~1.5km), the bulk of electrical imaging methods have been limited to shallow crust studies. Seismic methods, on the other hand, have been more efficient in providing researchers with information of the earth’s subsurface.

The applications of seismology cover a broad range of topics that range from exploring for natural resources (oil, gas and ores) within the crust (<10km) to understanding deep earth processes occurring at tens to thousands of kilometers from the earth surface. Attributes of recorded seismic waves such as amplitudes, frequency, phase, and coda have contributed in our understanding of the earth as a heterogenous media. Heterogeneity exists at different scales as depicted by the conceptual models shown in Figure 1.1. The structures depicted in Figure 1.1 could be based on variations in geology or physical properties that are a function of the thermodynamic processes (e.g. tectonics, weathering) subjected to the rock mass. Seismic waves propagating through the respective media in Figure 1.1 behave differently due to wave scattering processes. These differences in the wave behaviour therefore motivate us primarily to better understand the characteristics of the variability in various physical rock properties. The secondary aim is then to further understand how seismic wave propagation is affected by the complex framework that characterizes all these physical rock properties. These primary and secondary aims constitute the focus of the research presented in this thesis.

Seismic wave scattering results primarily in the spatial and temporal redistribution of energy within the total wavefield causing fluctuations in amplitude and frequency information. Hence, the recorded scattered wavefield provides limited resolution (information) of the subsurface structures. Resolving subsurface heterogeneities from the scattered wavefield equally depends on
whether backscattered or forward scattered wavefields are being recorded at the sensors. This means the ability to record the total wavefield is primarily limited by the acquisition aperture. While theory pertaining to wave propagation in the homogeneous and layered models (Figures 1.1a and b) is well established in the geophysics literature (Claerbout; 1968; O’Doherty and Anstey, 1971; Aki and Richards, 2002), the case for Figure 1.1c has not been adequately resolved.

Figure 1.1: Sample earth models illustrating possible distributions of physical rock properties: a) Homogenous model; b) and c) are both heterogeneous models. Model (b) is commonly called a layered model.

Wave scattering would be more severe within Figure 1.1c and this poses serious imaging challenges especially when dealing with complex geology (e.g. overburden layers, orebodies buried in heterogeneous background) and the fact that recorded seismic data have other noise problems. In this chapter I review past and present developments in the use of seismic data for imaging the internal structure and composition of the earth. The concluding section provides details explaining the organization of my research methods and results in subsequent chapters.

1.1 Deterministic And Nondeterministic Approaches For Information On Earth’s Heterogeneity

1.1.1 Controlled source seismic methods: reflection and refraction

Seismic studies based on backscattered energy via reflection and refraction pathways have been popular in providing valuable insights into the earth’s heterogeneity. In reflection and refraction seismology, the seismic wavefields from subsurface structures are recorded by positioning sensors at the surface. Reflection imaging delineates the subsurface geologic structures based on two-way travel times though with no accurate information of subsurface velocities. Data processing involves transforming the various shot records into zero-offset sections (common-mid
point: CDP, Yilmaz, 1989). Migration is used to send the reflected wavefields recorded at the earth’s surface back to their origin at the subsurface (Chang and McMechan 1987). Reflection methods are popular in the oil and gas industry where they are used to map thin bed reservoir rocks (Hardage et al., 1998). An example of a seismic reflection application for non-hydrocarbon exploration include: studies conducted to understand the tectonic evolution of the crust in North America (Clowes et al., 1996, LITHOPROBE project); studies by Liberty (1998) to map aquifer depths relevant for geothermal applications in Idaho, USA as well as studies by Juhlin et al. (2000) to detect groundwater resources in Sweden. In the Athabasca basin (Saskatchewan, Canada), seismic reflection techniques have been applied as an exploration tool for uranium ore deposits (Hajnal et al., 2010). Reflection methods have equally played a major role in our understanding of the morphology of impact crater structures (Scholz et al., 2002).

Refraction methods provide less resolution of structures within the crust than reflection methods. However, refraction methods provide information over larger regions. Refracted waves can also be considered as transmitted waves. Nowadays, processing for refracted wavefields can be done with the use of wide-angle reflection data. A fundamental attribute that provides evidence for subsurface heterogeneity is the travel time information of direct arrivals: beyond a threshold source-receiver offset, the refracted energy arrives at the receivers before energy that has propagated within the surface layer only. In the presence of such travel time information forward modeling can be used to determine the thickness and velocities of the layered system of rocks. Refraction studies are thus relevant for near surface geophysics especially in helping to map the overburden to bedrock contact. Refraction methods can also be used to characterize the moho (Clowes et al., 1995). Demanet et al. (2001) conducted refraction seismics as part of a paleoseismological study to localize active faults in the Roer Graben, Belgium.

1.1.2 Tomography

Tomographic studies, introduced in the mid 70’s (Aki and Lee, 1976), were enhanced by efforts among the scientific community to accurately predict travel time of earthquake waves that travel through the earth. The earth can be viewed in a fundamental sense as a layered medium based on chemical stratification (core, mantle, crust). However, the scatter observed in travel time arrivals provides clues to the presence of lateral heterogeneities deep within the earth. These fluctuations in physical properties also cause waveform changes. Travel time tomography uses arrival time
information to characterize or parameterise the velocity/slowness fluctuations within a given volume of the earth’s subsurface. For example, in ray theory approximations, the path integral through the medium perturbations is equal to the travel time residual ($\Delta T$):

$$\oint_s \Delta V ds = \Delta T = T_{\text{observed}} - T_{\text{reference model}}$$ (1.1)

The major shortcoming is that the inversion results in a non-unique solution since the data is not recorded over all ray paths. Modeling for the subsurface velocities using this ray approximation provides some limitations in resolution since ray theory is a high frequency approximation\(^1\) of the full wave equation. This approximation is valid when the spatial scale of the perturbation in velocity is larger than the source wavelength (Sato and Fehler, 1998, p20). The efficiency of traveltime tomography is undermined once the complexity of the geology results in scattering effects (Wu and Toksöz, 1987). Devaney (1984) introduced the concept of diffraction tomography that takes scattering in to account for wave propagating in a weakly inhomogeneous medium (the inversion uses both amplitude and phase information). Diffraction tomography is based on the comparison of the observed wavefield with the scattered wavefield that is calculated using the Born (or Rytov\(^2\)) approximation for the perturbed media. Since the mid 70s and 80’s researchers have made considerable improvements in their transmission imaging (tomography) techniques. With the motivation for improving resolution, for example, seismic waveform tomography was introduced in the 80’s (Tarantola, 1984). Waveform tomography uses the full information from the recorded wavefield to improve upon the earth model by trying to minimize the misfit between observed and model waveform data. Pratt (1999) illustrated the methodology for implementing waveform inversion in the frequency domain and demonstrated that more

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\(^1\) Ray theory uses travel time information which is based on Fermat’s principle: the raypath between two points is that for which the travel time is a minimum with respect to nearby possible paths.

\(^2\) Just like the Born approximation (discussed in Appendix A), the Rytov approximation provides simple analytical ways for solving wave propagation (scattering) in complex media. Both approximations assume the scattered wavefield is "small" enough such that the observed field (i.e. the total wavefield in Born; the complex phase field in Rytov) is a "small" perturbation from a reference ("mean") field (Beydoun and Tarantola, 1988).
information on structure (perturbations) can be obtained via waveform information than by travel time tomography.

Within the earth science community, transmission imaging has been applied at various scales ranging from global and region scales in earthquake seismology to much localized scales such as cross borehole imaging. Traveltime tomography using teleseismic events has been applied to study the P-wave velocity structure of the mantle (Spakman et al., 1993) below Europe, the Mediterranean and a part of Asia. Toomey & Foulger (1989) used travel time information from earthquake data to study the p-wave structure of a volcano complex in Iceland. For imaging small scale structure within the crust, cross borehole, surface seismic or Vertical Seismic Profiling (VSP) acquisition configurations are used. Pratt & Shipp (1999) performed a cross-hole experiment within a layered sedimentary environment to image a fault based on the lateral offset in the slowness patterns within the derived 2D slowness model. Zelt (1998) used 3D refraction traveltime data to resolve lateral variation in slowness properties within the Faeroe basin (Scotland).

### 1.1.3 Attenuation and coda analysis

Given that the earth is not perfectly elastic, various energy loss mechanisms cause seismic waves to be attenuated: that is amplitudes of the seismic wavelet are damped. For waves propagating in a viscoelastic heterogeneous medium, amplitude damping is a combined effect of intrinsic attenuation, wave scattering, and geometric spreading. Intrinsic attenuation is generally understood to contribute significantly towards the distortion of seismic waves. Most studies determine attenuation by the quality factor \( Q \) and some models designed to explain the attenuation mechanism assume \( Q \) to be frequency independent. In these attenuation models, \( Q \) lengthens the wavelength of the dominant frequency, that is, high frequencies are easily absorbed.

For a wave propagating through an anelastic medium, the amplitude of the seismic wavelet at a given depth \( x \) (after propagation for time \( t \) ) can be written as (Hauge, 1981):

\[
A(x) = GA_o e^{ \left( -f \pi/QV \phi \right) x} = GA_o e^{ \left( -f \pi/Q \right) t} \tag{1.2}
\]
Where $G$ is the measure of the elastic losses (geometric spreading and reflectivity), $V_\phi$ is the phase velocity, $V_\phi Q$ is defined as the fractional loss of energy ($E$) per cycle of oscillation:

$$\frac{1}{Q} = \frac{\Delta E}{2\pi E}$$  \hspace{1cm} (1.3)

$A_o$ and $A(x)$ are the amplitudes of the wavefields recorded downhole at depths $x_o$ and $x$, and $f$ is the frequency of the source signal. Transforming equation 1.2 to the frequency domain and expressing it as a natural log of the amplitude ratios (i.e. $\log\left(A(f)/A_o(f)\right)$) gives a linear equation whose slope is inversely proportional to $Q$. This forms the basis for the spectral ratio method (Tonn, 1991) used for $Q$ estimation. This has been done using reflection data or VSP data (Dasgupta and Clark, 1998; Haase and Stewart, 2006).

Although seismic data observations support the idea that $Q$ is frequency independent, theoretical investigations by Knopoff and Macdonald (1958) conclude that $Q$ as well as the velocity of the seismic waves are frequency dependent. The frequency dependence of seismic propagation velocities is also known as velocity dispersion (physical dispersion\(^1\)). Liu et al. (1976) reconciled both observations and theory by demonstrating that a constant- $Q$ model (frequency independent model) exists over a defined frequency band ($F_{\text{low}}$–$F_{\text{high}}$) using a superposing series of Debye Peak Models (models in which $Q^{-1}$ peaks at a certain frequency, Zener 1948). Moreover, both velocity dispersion and intrinsic attenuation are linked by the Kramers-Kronig relation (Futterman, 1962). Velocity dispersion is strongly correlated to the petrophysical properties of rocks such as porosity, fractures, fluid mobility and scale of heterogeneities. Attenuation is equally influenced by these petrophysical properties. Hence attenuation and velocity dispersion can provide evidence for heterogeneity in the earth. Owing to the fact attenuation mechanisms influence the shape of the signal’s amplitude spectrum and velocity

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\(^1\) Physical dispersion of waves: phenomenon whereby different frequencies travel at different speeds through a material. This type of dispersion differs from surface wave dispersion often studied in global seismology. The dispersion observed in surface waves results from the fact that long period waves tend to sample regions with higher velocities (located deep within the earth’s interior). Hence long period waves travel faster than short period (high frequency) waves.
dispersion affects the phase spectrum, Sun et al. (2009) used uncorrelated vibroseis VSP data acquired in the Mallik gas hydrate research well to measure the variations in velocity dispersion and attenuation as a function of depth in the exploration seismic frequency band (~5-200Hz). Studies by Sun et al. (2009, Figure 1.2) and Huang et al. (2009) show that $Q$ in gas hydrate bearing sediments is much lower than the $Q$ values obtained in the surrounding gas hydrate free sediments (water saturated).

The contrast in $Q$ between the respective layers can result in changes in the reflection coefficient. Through numerical studies Quintal et al. (2009) argue that intrinsic attenuation observed at low frequencies (<10Hz) can enhance reflectivity between two alternating poroelastic layers even in the absence of “acoustic impedance contrast”. The changes in intrinsic attenuation at such low frequencies is explained by wave-induced fluid flow between the alternating layers, each characterized by different fluid saturations, hence a gradient in fluid pressures (White, 1975).

Amplitude attenuation observed in seismic data is not only restricted to intrinsic attenuation but also involves contributions from scattering attenuation. Attenuation due to scattering emerged from initial observation and analysis of coda waves from earthquake data (Aki and Chouet, 1975). Aki and Chouet (1975) assert that coda waves are generated as scattered waves from all directions and thus a statistical treatment of the earth’s heterogeneity can help explain the coda observed the earthquake data. Theoretical, numerical, and applied studies have shown that the quality factor estimated from coda ($Q_c$) increases with frequency. Moreover, it is thought that coda analysis hold the potential to characterize site amplification and source parameters (Aki and Chouet, 1975; Sato and Fehler, 1998; Fehler, 2003). The success of using coda for such

![Figure 1.2: The Q profiles for the Mallik data (Canada) compared with the geologic setting. $Q_{\text{lin}}$: calculated from the linear velocity dispersion relation; $Q_{\text{SR}}$: calculated from the spectral ratio of first arrivals in the correlated vibrator data (Sun et al., 2009).](image-url)
application relies on Aki and Chouet’s (1975) observation that after a certain amount of time, coda waves at a given time from the earthquake origin time tend to be independent of the epicentral distance. In other words, receivers within a certain region where energy is uniformly distributed have signals with amplitudes that are proportional to the source strength. Numerical studies using stochastic models have confirmed this observation (Frankel and Clayton, 1986, Fehler and Sato, 2003). Coda analysis can be used to characterize the random heterogeneity by quantifying and separating the effects of scattering attenuation from intrinsic attenuation. Although there is no simple linear relation between scattering and intrinsic attenuation, some researchers (Wu, 1985; Sato and Fehler, 1998; Fehler and Sato, 2003) have shown that the distinction between scattering and intrinsic attenuation is achievable using radiative energy transfer theory that accommodates multiple scattering. Radiative energy transfer theory is an energy transport theory which characterizes the transport of energy throughout a medium with multiple scatterers. This theory validates the requirement for energy conservation which is otherwise unachievable with other theories used to describe seismic wave propagation in heterogeneous medium (e.g. Born approximation). Fehler and Sato (2003) used this approach to show that scattering attenuation is dominant at low frequencies (1Hz) but at higher frequencies (~30Hz) wave attenuation is dominated by intrinsic absorption. However, recent studies (Huang et al., 2009) based on heterogeneous gas hydrate models and much higher frequencies (100Hz - 200Hz) corroborate that the analysis of amplitude loss due to intrinsic attenuation and scattering is further complicated by other amplitude loss mechanisms due to energy leakage. Energy leakage mechanisms occur during wave propagation in heterogeneous medium whereby in a region with large scale fast formation some energy ‘leaks(refracts)’ to the surrounding slow formations, leaving weak energy in the first arrivals. The results by Huang et al. (2009) further invalidate the fact that the apparent attenuation measured in such media is given by the linear relation between these respective amplitude loss mechanisms.

1.1.4 Seismic interferometry

Initially introduced in exploration seismology by Claerbout (1968), renewed interest in this domain surged with investigations published in Schuster et al. (2004). Seismic interferometry can be viewed as a process by which crosscorrelation of signals between two receivers (observation points) are used to retrieve the inter-receiver signal. In other words, the cross-correlation between the signals from the two observation points approximates the impulse
response between these two points. In some branches of science such as optics as well as in global seismology, the impulse response is also called the Green’s function whereas the term “reflectivity time series” is popularly used among exploration seismologist (Schuster et al., 2004). The roots of the central idea relating cross correlelograms to a medium’s Green function can be traced back to the initial work of Clearbout (1968). He showed that if waves emitted from a buried source in a layered medium are recorded by a receiver positioned at the surface, the autocorrelation of the receiver signal contains information about the reflectivity that characterizes the medium of propagation. Renewed interest in the idea of correlelograms for imaging surged at the beginning of the 21st century with pioneering works on the theory and application being developed in the disciplines of acoustics, ultrasonics and seismology (Weaver and Lobkis, 2001, 2004; Sneider, 2004; Campillo and Paul, 2003; Shapiro and Campillo, 2004, Wapenaar 2004; Dragonov et al., 2003).

The theory on Green’s function reconstruction has been mostly developed for diffuse wavefields (Weaver and Lobkis, 2004; Sneider, 2004). Weaver and Lobkis (2004) argue that in acoustics, diffuse wavefields can be viewed as a collection of uncorrelated and “isotropic” mix of plane waves of all propagation direction. However, this definition fails to apply to heterogeneous media (Weaver and Lobkis, 2004). Despite the lack of a unique definition for diffusive wavefields these wavefields are known to be caused by multiple scattering in heterogeneous medium, reverberations in irregular bounding surface or a random distribution of uncorrelated noise sources (Weaver and Lobkis, 2004; Sneider, 2004; Wapenaar et al., 2006). In the case of having uncorrelated sources, the signals that contribute most to the derived correlelogram come from sources that are closely aligned (parallel) to the axis joining the two receivers. When using coda, the extraction of the Green’s function (Sneider, 2004) relies on the constructive interference of scattered waves propagating parallel to the line joining the receivers at the two observation points. The Green’s function recovered contains all forms of multiple reflections and scattered wavefields (Weaver and Lobkis, 2004).

In addition to retrieving the reflectivity series of a medium, seismic interferometry can be used to retrieve information about the source distribution (Schuster et al., 2004). Given that the latter application uses deterministic wavefields to generate correlelograms, the assumption of random and uncorrelated phases is critical. These imaging methods all constitute what is called passive imaging, especially as a priori information of the source location is not always required. In
addition to constructing correlelograms, Schuster et al. (2004) uses migration to adequately retrieve information about the subsurface reflectivity.

So far, most of the reviewed methods for seismic studies of the earth’s crust are relevant to a broad spectrum of geologic settings and physical conditions. Moreover, methodologies considered are often designed to address specific problems. The research work in this thesis uses petrophysical and geological information from hardrock environment whereby geophysical investigations of interest (seismic imaging) are tailored for mineral exploration. In hardrock terrane, the challenges in using seismic methods are different from those faced in a sedimentary basin. The next section thus presents a brief review of the past and present developments of controlled source seismic imaging methods in hardrock environment.

1.1.5 Seismic imaging in crystalline environment

Seismic data in the exploration frequency band, especially in crystalline environments also suffers from effects of seismic wave scattering. Unlike earthquake data, scattering due to the heterogeneity in the rocks has a severe impact on some of the main body wave events that are used for imaging. The goals of seismic imaging conducted in hardrock environments range from basic reconnaissance purposes for mapping main geologic contacts to mineral exploration. In Canada, for example, the exploration for deep (>500m) orebodies has led to a surging need for seismic methods which are better equipped for deep imaging. Surveys (2D and 3D) exist in the literature that demonstrate the use of seismic methods for imaging sulfides (Milkerelit et al., 2000; Adam et al., 2003; Malehmir and Bellefleur, 2009a) and for defining the structural framework controlling uranium deposits (Hajnal et al., 2010). The efficiency of seismic methods for orebody (sulfides) imaging relies on the strong impedance contrast (controlled by density-Salisbury et al., 1996) between the orebody and host rock. However, it has been observed (Milkerelit et al., 2000, Bohlen et al., 2003) that the small size of orebodies, coupled with their irregular shapes as well as the heterogeneity of the host rock can result in misleading amplitude variation with offset (AVO) trends in the scattered waves generated from the orebodies. Inadequate knowledge of the variability in the velocity model also causes mislocation of imaged structures. These challenges compel the need for an integrated approach for effective geophysical imaging. Some steps like, a comprehensive petrophysical log analysis for statistical parameters of various rock properties; forward modeling that accordingly accommodates the
stochastic treatment of the earth’s heterogeneity; and the incorporation of other existing 
geological and geophysical information can be invaluable. These also have a huge impact on 
decision making and survey design. Some researchers (Milkereit et al., 2000; Malehmir et al., 
2009b; Bohlen et al., 2003) have performed modeling studies to characterize the seismic 
response of a sulfide orebody. Work done by these authors was restricted to a homogenous 
background. L’Heureux et al. (2009) show that scattering due to a heterogeneous background 
can overwhelm the diffractions from the orebody, hence rendering the imaging process more 
daunting.

1.2 Summary

Although past developments in seismic imaging methods as well as other geophysical imaging 
methods have revealed a great deal of information about the earth’s heterogeneity at depth, there 
is a continual effort geared towards a better understanding of how these heterogeneities affect 
seismic wave propagation/scattering. Finding methods that are well suited to retrieve information 
about subsurface heterogeneities (scale parameters) from attributes (amplitude, phase, frequency, 
coda) of recorded seismograms is one way of approaching this problem. This is addressed in the 
present work through studies that include:

1. The acquisition and analysis of comprehensive petrophysical, geological and 
geochemical data.

2. Integration of the petrophysical, geology, and geochemical information from multiple 
boreholes for building realistic earth models.

3. Seismic full waveform modeling using both single and multiple sources and attribute 
analysis of reflected (backscattered) and transmitted (forward scattered) waves 
propagating in these models.

The principal case studies considered include Nash Creek (Canada) and Thompson (Canada) 
where geophysical studies are relevant for mineral exploration purposes and offset VSP data 
from the Bosumtwi impact crater in Ghana.

In Chapter 2, I briefly review some of the theory underlying wave propagation in heterogeneous 
media. Approaches that range from addressing the underlying principle of wave propagation 
across a physical boundary through to more complex methods that use a stochastic treatment of 
the earth’s heterogeneity to account for seismic wave scattering are presented. A simple method
used to extract scale parameters from spectral content of forward scattered waves is also introduced.

Chapter 3 introduces the setting of the case study areas with some comment on previous research work on the geology. Particular emphasis is also given to the description of the physical rock properties that characterize the major geologic units of interest.

In Chapter 4, a stochastic method (cokriging) is introduced showing how the distribution of multiple variable rock properties can be jointly estimated within a 3D earth volume. The application focuses on using density and geochemical data that characterize the base metal deposit in Nash Creek. The second part of Chapter 4 uses the derived stochastic earth models based on Nash Creek petrophysical information to highlight implications for seismic acquisition design and imaging. A quality control test that compares modeled gravity responses with observed data in a bid to assess the validity of the stochastic earth models is also illustrated in Appendix C. In Chapter 5, a study that uses stochastic petrophysical models to evaluate implications for seismic imaging at the Thompson mine is covered.

While the first part in Chapter 6 highlights how transmitted waves (direct arrivals) can be used to image the structural heterogeneity between major geologic units within the Bosumtwi crater, the second part illustrates the pitfalls in acquiring and using transmitted waves for imaging especially for microseismic applications without due consideration for effects of geology.

Note that assumptions used for transmission imaging with seismic interferometry do not necessarily apply to the cases addressed in this work given that one deals with single active sources, coupled with strong contrasts in the inhomogeneity of the medium (e.g. impedance of orebody vs. impedance of host medium).

Finally, in chapter 7, all the results presented in this thesis are summarized and pertinent issues for future directions of research are discussed.
Chapter 2  
Wave Propagation in Heterogeneous Media

2.1 Introduction

Geological, geophysical and geochemical information provide invaluable characteristics of the earth’s heterogeneity. Borehole logs show that rock properties fluctuate over different depth scales. While some of these properties show some correlation over a certain spatial range, others may not be correlated. Geological maps show rock formations that are characterized by folded beddings, fault zones, and preferentially oriented fracture patterns. Characteristics of seismograms like variations in waveform shape (amplitude and phase), coda, as well as fluctuations in frequency content corroborate variations in the elastic properties of the earth’s interior. These characteristics of seismograms are considered to be caused by seismic energy absorption (conversion to heat) and wave scattering (redistribution of seismic energy in to different directions of propagation). In this chapter, methods used to characterize heterogeneous structure of the crust from petrophysical measurements are addressed. A theoretical and numerical basis for understanding seismic wave propagation or scattering in heterogeneous media is also covered. Finally, a simple approach demonstrating how forward scattered (transmitted) waves can be used to infer characteristics of crustal heterogeneity is examined.

2.2 Wave Propagation Across Elastic Discontinuities

Wave propagating in the earth is characterized by the process of energy distribution across interfaces or discontinuities. These discontinuities are characterized by changes in the physical properties. For example, a wave could propagate from a medium with a solid matrix with high P-wave and S-wave velocities to a matrix which is highly porous, hence a much lower P-wave and S-wave velocity. Such discontinuities are encountered at various scales within the earth ranging from the major discontinuities like the core-mantle boundary (< 200km, van der Hilst et al., 2007) to small scale discontinuities that characterize metamorphic rock intrusions (dykes), ore deposits and oil reservoirs. Hence it is important to understand the phenomenon by which wave propagates across such elastic discontinuities as it constitutes the fundamental process that characterizes wave propagation in inhomogeneous media. The underlying principles for elastic wave propagation as described by many authors (Aki and Richard, 2002; Waters, 1981) builds
upon Hooke’s law which relates stress (traction) to strain and Newton’s second law of motion which relates the force acting on a body to its mass and acceleration (F=ma). Both laws combined allow the formulation of the equation of motion (wave equation) of a body force acting at a given point as (Aki and Richard, 2002, p64):

$$\rho u_t = f_s + (\lambda + 2\mu) \nabla (\nabla \cdot u) - \mu \nabla \times (\nabla \times u)$$

(2.1)

where \( u \) represents displacement, \( \rho \) is the density, \( f_s \) is the source function, \( \lambda \) & \( \mu \) are the Lame parameters. The subscript “\( t \)” represents derivative with respect to time. The initial conditions for equation (2.1) are: \( u(x,0) = u_s(x,0) = 0 \) for \( x \neq 0 \). If one removes the dependence on direction (particle displacement is parallel to direction of propagation) for example, we then obtain a scalar equivalent for (2.1) as:

$$\rho U_t = f_s + (\lambda + 2\mu) \nabla^2 U$$

(2.2)

where the general solution (ignoring initial conditions) for wave propagation is \( U(r,t) = F(r - \alpha t) \) with \( \alpha^2 = \frac{(\lambda + 2\mu)}{\rho} \) (\( \alpha \) is the wave velocity). The simple solution obtained characterizes a spherically expanding compressional wave from the source location. However, this simple wave solution cannot apply for shear waves without consideration for vector analysis given that particle displacement is not in the same direction as the direction of wave propagation.

According to Lamé’s theorem (Aki and Richard, 2002, pp. 67-68), there exist potentials \( \phi \) and \( \psi \) for \( u \) such that

$$u = \nabla \phi + \nabla \times \psi, \quad \nabla \cdot \psi = 0$$

(2.3a)

$$\phi''_t = \frac{\partial^2 \phi}{\partial t^2} = \frac{\phi'}{\rho} + \alpha^2 \nabla^2 \phi$$

(2.3b)

$$\psi''_t = \frac{\partial^2 \psi}{\partial t^2} = \frac{\psi}{\rho} + \beta^2 \nabla^2 \psi, \text{ with } \beta^2 = \frac{\mu}{\rho}$$

(2.3c)
\( \phi' \) and \( \psi \) are the potentials for \( f_s \) (i.e. \( f_s = \nabla \phi' + \nabla \times \psi \), with \( \nabla \cdot \psi = 0 \)); \( \beta, \nabla \phi \), and \( \nabla \times \psi \) represent the S-wave velocity, P- and S-wave displacement components respectively. The forms of equations (2.3b) and (2.3c) are identical to that of equation 2.2 except that the independent variables are the potentials for P- and S-waves respectively. The numerical implementation of the elastic wave propagation in models used in this thesis is based on equation 2.3 (Bohlen, 2002).

In exploring how velocity/impedance contrasts across boundaries affect wave propagation, focus will be on using plane waves incident at a certain angle of a given boundary between two media. An example of such scenario is discussed in section 2.6 where the impedance boundaries (due to spatial fluctuations in velocities and densities) extend over short spatial distance. Also, some discussions (Appendix F) illustrating the criteria for notch frequency generation due to wave interference are based on assumption of plane wave incidence at a given elastic boundary. This formulation may not always concur with observations in the field due to the influence of geometric expansion of waves (effect on curvature- examples are shown sections 4.6 and 6.3.2). Nonetheless, the approximation for plane wave incidence at a given boundary or discontinuity can be validly used for cases where the relative distance of the discontinuity from the source location spans several wavelengths especially in global seismology. An incident plane wave at a given boundary between two half spaces, each characterized by different physical properties, results in refracted (transmitted) and reflected energy that could be either P- or S-waves. The goal here is to obtain expressions for the relative amplitudes of the refracted and reflected waves with respect to incident wave. These relative amplitude ratios are also called reflection and transmission coefficients.

It is known that plane waves are characterized by wavefronts that are normal to the direction in which the wave propagates with speed \( c \) (1/c= slowness). According to Aki and Richard (2002), a wavefront is defined as a propagating discontinuity in some dependent variable (e.g. Particle displacement, particle velocity, and particle acceleration). Thus, physical quantities like \( u \) (displacement) have dependence in space and time.

For the purpose of simplicity, consider a plane wave in the Cartesian coordinate system whereby the \( x \) and \( z \) axis are parallel to the horizontal and vertical slowness components respectively (Figure 2.1). Thus solving the wave equation (e.g. equation 2.3(b) with the first term to the right
of the equal sign dropped) for propagation in a homogenous medium by method of separation of variables gives a steady state solution for plane waves valid for all frequencies:

\[ \phi(x, y, z, t) = A \exp \left[ i (\mathbf{k} \cdot \mathbf{x} - \omega t) \right] \]  

(2.4a)

in which \( \mathbf{x} = (x, y, z) \), \( \mathbf{k} = (k_x, k_y, k_z) \) and \( |k| = \frac{\omega}{\alpha} (k - \text{wavenumber}) \). Based on the definition of slowness, (2.4a) can also be written as:

\[ \phi(x, y, z, t) = A \exp \left[ i \omega (s \cdot \mathbf{x} - t) \right], s = (s_x, s_y, s_z) \]  

(2.4b)

where \( A \) is the amplitude.

In the present analysis, the case in which particle displacement is restricted to the x-z plane i.e. \( k_y = 0 \) is considered. Hence, only two potentials for P and SV waves respectively are of interest.

Suppose a plane P-wave propagates in the direction of increasing x (Figure 2.2). Therefore, given a potential \( \phi \), the corresponding P-wave displacement and traction(stress) associated with it are \( \left( \frac{\partial \phi}{\partial x}, 0, \frac{\partial \phi}{\partial z} \right) \) and \( \left( T_{zx}, T_{zy}, T_{zz} \right) = \left( 2\mu \frac{\partial^2 \phi}{\partial z \partial x}, 0, \lambda \nabla^2 \phi + 2\mu \frac{\partial^2 \phi}{\partial z^2} \right) \).

Similarly, for a scalar potential \( \psi \), the associated SV-displacement and traction are \( \left( -\frac{\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x} \right) \) and \( \left( T_{zx}, T_{zy}, T_{zz} \right) = \left( \mu \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right), 0, 2\mu \frac{\partial^2 \psi}{\partial z \partial x} \right) \).
To fully describe wave propagation across two half spaces requires the consideration for boundary conditions which govern displacement and traction in the vicinity of the boundary. In general two boundary conditions apply:

Kinematic boundary conditions which require continuity of the components displacement through the boundary.

Dynamic boundary conditions which require continuity of the components of traction (stress) through the boundary.

The validity of these boundary conditions is also subject to the type of contact one is dealing with. For example, in a solid-fluid contact it would only be appropriate to consider continuity of the normal displacement (Aki and Richards, 2002, p129). For the present case, emphasis would be in the case where there is a solid-solid contact. The sign convention used is such that particle displacements are considered positive in the direction of increasing $x$ as well as in the direction of increasing $z$ (downwards). Given that in Figure 2.2 all waves are propagating from left to right, the sign of the displacement amplitude solely depends on the displacement component in the $z$ direction.

Consider the case for an incident downgoing P-wave ($P^1_D$) of absolute displacement amplitude of 1; the superscript represents the half space while the subscript represents the direction of wave propagation. This generates a combination of P- and SV- waves (4) that are either reflected (2) or

![Figure 2.2](image-url): a) Angular relation between an incident P-wave and derived reflected and refracted rays at the interface between two solid media. While the small arrows along the rays indicate the direction of propagation, the heavy dark arrows show the direction of particle motion. b) Complete system incident and derived plane P- and S-waves from which the matrix in A.1b (see Appendix A) can be found. Short arrows indicate direction of particle motion; long arrows show direction of propagation (modified after Aki and Richards, 2002, p141).
refracted/transmitted (2). Based on the expressions for displacement and traction as discussed above, it is evident that the expressions for the displacements in the complex system of P and SV waves all have a common factor \( \exp[i\omega(s \cdot x - t)] \). For the downgoing P-wave, the vector components for displacement and slowness are given respectively as:

\[
\left( \sin i_{1}, 0, \cos i_{1} \right), \quad \left( \frac{\sin i_{1}}{\alpha_{1}}, 0, \frac{\cos i_{1}}{\alpha_{1}} \right)
\]

Hence rewriting the full expression for the P-wave displacement following the format in Aki and Richards (2002, Table 5.3), we have:

\[
\left( \sin i_{1}, 0, \cos i_{1} \right) \exp \left[i\omega \left( \frac{\sin i_{1}}{\alpha_{1}} x + \frac{\cos i_{1}}{\alpha_{1}} z - t \right) \right]
\]

Also, If we consider a single case for the kinematic boundary condition at \( z=0 \), for example that for the horizontal displacement, we have:

\[
u_{x} = u_{x}^{2}, \quad (2.6)
\]

where displacements in the respective half spaces are a function of contributions from the potentials \( \phi \) and \( \psi \) i.e.

\[
u = B(\phi, \psi)
\]

\[
\phi = \phi_{\text{incident}} + \phi_{\text{reflected}} + \psi_{\text{reflected}}, \quad \psi = \psi_{\text{reflected}} \quad \text{(medium 1)}
\]

\[
\phi = \phi_{\text{transmitted}}, \quad \psi = \psi_{\text{transmitted}} \quad \text{(medium 2).} \quad (2.7)
\]

In evaluating (2.6), we find that each term is a sum of contributions involving factors of the type

\[
\exp \left[i\omega \left( \frac{\sin i_{1}}{\alpha_{1}} x - t \right) \right] \quad \text{or} \quad \exp \left[i\omega \left( \frac{\sin i_{1}}{\alpha_{1}} x - t \right) \right] \quad \text{or} \quad \exp \left[i\omega \left( \frac{\sin i_{1}}{\alpha_{1}} x - t \right) \right] \quad \text{or} \quad \exp \left[i\omega \left( \frac{\sin i_{1}}{\alpha_{1}} x - t \right) \right] \quad \text{or} \quad \exp \left[i\omega \left( \frac{\sin i_{1}}{\alpha_{1}} x - t \right) \right] \quad \text{or} \quad \exp \left[i\omega \left( \frac{\sin j_{2}}{\beta_{2}} x - t \right) \right]
\]
Given that the horizontal boundary condition is valid for all times and for all x we thus have that

\[ i_1 = i_1' \quad \text{and} \quad \frac{\sin i_1}{\alpha_1} = \frac{\sin i_2}{\alpha_2} = \frac{\sin j_1}{\beta_1} = \frac{\sin j_2}{\beta_2} = p \quad (2.8) \]

Equation (2.8) is consistent with Snell’s law and further highlights the fact that the horizontal slowness is conserved i.e. they share a common value \( p \) also called the ray parameter.

Expressions for the P and SV waves generated by the downgoing incident P-wave can thus be written as:

**Upgoing P-wave**

\[
\left( \sin i_1, 0, -\cos i_1 \right) P_b P_u \exp \left[ i\omega \left( px - \frac{\cos i_1}{\alpha_1} z - t \right) \right] \quad (2.9a)
\]

**Upgoing SV-wave**

\[
\left( \cos j_1, 0, \sin j_1 \right) P_b S_u \exp \left[ i\omega \left( px - \frac{\cos j_1}{\beta_1} z - t \right) \right] \quad (2.9b)
\]

**Downgoing P-wave**

\[
\left( \sin i_2, 0, \cos i_2 \right) P_b P_d \exp \left[ i\omega \left( px + \frac{\cos i_2}{\alpha_2} z - t \right) \right] \quad (2.9c)
\]

**Downgoing SV-wave**

\[
\left( \cos j_2, 0, -\sin j_2 \right) P_b S_d \exp \left[ i\omega \left( px + \frac{\cos j_2}{\beta_2} z - t \right) \right] \quad (2.9d)
\]

In which \( P_b P_u, P_b S_u, P_b P_d \) and \( P_b S_d \) represent the reflection and transmission coefficients for the derived P- and SV-waves respectively from a downgoing incident P-wave. A more general assessment for these terms can be obtained by considering other scenarios for incident P- and S-waves at the interface (Appendix A). The reflection and transmission coefficients obtained here are for displacement amplitudes and equally apply for particle velocity amplitudes. These coefficients are partly responsible for the relative difference in the amplitude strengths of the backscattered and forward scattered wavefields from targets used in the numerical experiments discussed in Chapters 4, 5 and 6. Because explosive wave sources are used in these numerical experiments, geometric spreading also contributes to the scaling (damping) of the wave amplitudes.
2.3 Autocovariances and Variograms

Physical properties of the earth vary over space and time and form the basis for our understanding of the earth as a heterogeneous medium. Although deterministic processes can describe these heterogeneities in some cases, there exist situations where we must have recourse to describing the inhomogeneities by the statistics of a sample population of measurements. Thus, any modeling (e.g. petrophysics) done in this light aims at honouring the statistical attributes of the sample population. By sample population it is understood that most measurements in the earth sciences are not completely sampled in the region or area of study i.e. it is not exhaustive. However, the sampling is done such that the derived statistics from the observed data can be considered to be representative of the true statistics if the attribute of interest is sampled everywhere in the study area. Often, mathematical tools based on two-point statistics such as autocovariances (Goff and Jordan, 1988; Sato and Fehler, 1998), and Variograms (Deutsch and Journel, 1998) are used.

Statistical analysis based on the methods mentioned above, relies on the concept of random functions (Goovaerts, 1997) whereby, the set of unknown values is regarded as a set of spatially dependent random variables. A random variable is one that can take a series of values according to some probability distribution. We can classify random variables as either continuous i.e. have a continuous range of possible outcomes e.g. velocity, porosity, permeability, density or categorical i.e. has a finite number of outcomes e.g. geology.

In describing an attribute that pertains to a rock property, for example P-wave velocities from a sonic log, two-point statistics methods are used. In these methods, it is assumed that P-wave velocity comprises a deterministic trend \( \langle V_m - \text{mean field} \rangle \) and a perturbation (stochastic) component \( \delta V(x) \). The perturbation component depends on location \( x \) such that its mean field \( \langle \delta V(x) \rangle = 0 \).

When dealing with one random variable (attribute), the two-point statistics is a univariate spatial description of the data whereby, the relationship between pairs of measurements of the same attribute at locations separated by a distance \( h \) in a certain direction are assessed. While autocovariance is a statistical measure of the spatial scale of similarity within the medium, the
variogram on the other hand is a measure of the spatial scale of variability within the medium. The general form for the autocovariance is as follows:

\[ A(h) = \langle \delta V(x) \cdot \delta V(x + h) \rangle - \langle \delta V(x) \rangle \cdot \langle \delta V(x + h) \rangle \]

The above equation reduces to \[ A(h) = \langle \delta V(x) \cdot \delta V(x + h) \rangle \] given the assumption \( \langle \delta V(x) \rangle = 0 \).

The general form for the variogram is \[ \gamma(h) = \left( \langle [\delta V(x) - \delta V(x + h)]^2 \rangle \right) \] (Goovaerts, 1997).

Despite the differences in their mathematical forms, both the autocovariance and variogram ("translated mirror image of autocovariance") are similar (Figure 2.3a). Exceptions to this rule do exist. This is the case where the variogram is not bounded (no sill, Deutsch and Journel, 1998).

In statistically describing the field measurements of rock properties (e.g. perturbation component from sonic logs) through autocovariances and variograms, it is usually assumed that the statistics is spatially homogenous and stationary. “Homogeneity” and “stationarity” in the present case mean both the autocovariance and variogram are invariant with respect to spatial translation (Goff and Jordan, 1988; Deutsch and Journel, 1998).

Although the first two moments i.e. the mean and variance can be determined directly from the observations, other parameters that fully describe the statistical character of the attribute of interest can be inferred from least squares fits of parametric functions to the experimental autocovariance/variograms. Experimental autocovariance/variograms are computed from observed (hard) data. The most popular forms of parametric function models known in the literature include the Gaussian model, the exponential model, and the von Kármán model (Sato and Fehler, 1998; Deutsch and Journel, 1998). Other models often used by geostatisticians have been documented by Deutsch and Journel (1998). The Fourier transform of these parametric functions is called the power spectral density function (PSDF). Parametric model functions also allow for determining the autocovariance and variograms at any other lag. In this thesis, focus will be on the exponential and von Kármán parametric functions whose basic mathematical forms for a random isotropic (correlation length is identical in all directions) case are given as:
Exponential

$$C(r) = \tau^2 \exp\left(-\frac{r}{a}\right)$$

(2.10)

where $\tau$ is the standard deviation, $r = \sqrt{h_x^2 + h_y^2 + h_z^2}$ is the lag distance, and $a$ is the correlation length ($a_x = a_y = a_z = a$);

von Kármán (Figure 2.3b)

$$C(r) = \frac{\tau^2}{\Gamma(\nu) \cdot 2^{\nu-1} \left(\frac{r}{a}\right)^\nu} K_\nu\left(\frac{r}{a}\right)$$

(2.11)

where $\nu$ is the hurst number which describes the roughness of the medium, $\Gamma$ is the gamma function, and $K_\nu$ is the modified Bessel function of the second kind of order $\nu$. The hurst number takes values in the range between 0 and 1. The corresponding PSDF to the von Kármán autocorrelation function for the 1D case is (Huang et al., 2009, see Figure 2.3c):
\[ P(\mathbf{k}_d) = \frac{\Gamma\left(\nu + \frac{1}{2}\right)}{\Gamma(\nu)} \frac{\varepsilon^2 (4\pi)^{\frac{1}{2}} a}{\left(1 + \mathbf{k}_d^2\right)^{\nu + \frac{1}{2}}} = a^2 k^2 \]

where \( k \) is the wavenumber vector.

The von Kármán function coincides with the exponential function when \( \nu = 0.5 \). Notice the PSDF obeys the power law for large wavenumbers (\( ak \gg 1 \), \( \text{PSDF} a^\nu(ak)^{-(2\nu+1)} \)). Similar power law characteristics have been reported for a wide range of petrophysical parameters of crystalline rocks (p-wave, s-wave, and density) measured from the KTB boreholes in Germany (Wu et al., 1994; Holliger, 1996; Leonardi and Kümpel, 1998), from deep wells in Japan (Shiomi et al., 1997), from Sudbury and the Abitibi greenstone belt in Canada (Holliger, 1996). Most of the power reported from these studies range between 0.96 and 1.6. Levander et al. (1994) also reported power law characteristics from geologic sections. Analyzing observed data such as logs with these stochastic methods helps to understand seismic wave scattering and wave attenuation in heterogeneous media such as the earth’s upper crust (Holliger, 1997). In a layered earth model, the foundation for wave propagation is adequately explained by the theory described in section 2.2. However, more advanced treatments are needed to explain wave propagation in media where the physical rock properties change rapidly over short distances. Some of the theory that considers a stochastic description of the variations in physical rock properties in order to explain seismic wave propagation is discussed in the following sections.

## 2.4 Seismic Wave Scattering: Born Approximation- Single Scattering

The notion of a seismic wave propagation in the earth as a scattering problem was fuelled by efforts to provide plausible explanations to observations from seismograms recorded from large seismic array Networks such as LASA (Large-aperture seismic array). These observations are centered around features associated to the modification of the seismic wave (seismic wave scattering) due to its interaction with the 3D heterogeneities having fluctuations in density and other elastic constants. Seismic coda from local earthquakes, for example, is attributed to backscattering or large-angle scattering of seismic waves (Aki and Chouet, 1975). Conversely, seismic waves hold the potential for allowing inference of the parameters of the inhomogeneities.
in the earth. In these applications dealing with heterogeneities of the earth, authors have resorted to using various scattering theories of wave propagation in random media. An example of a scattering theory is the single scattering formulation for a small heterogeneity/inclusion having a slightly different density and elastic properties from the surrounding medium (e.g. mafic breccia fragment hosted in a felsic rock matrix). The single scattering theory uses the Born approximation also known as first order perturbation theory. In this formulation, the total displacement field is expressed as the sum of the incident wavefield and the scattered wavefield. The latter is relatively very small compared to the incident wavefield. The formulation also assumes that the fractional perturbations of the physical properties are extremely small (<<1).

Aki and Chouet (1975) successfully used single scattering theory for scalar waves to model coda generation of local earthquakes. A more advanced formulation using a similar approach for elastic waves was developed by Wu and Aki (1985). Wu and Aki (1985) argue that for a local inclusion with a size that is very small compared to the seismic wavelength (low frequency), the scattered wavefield can be viewed as a spherical wavefield expanding outwards from a point. Moreover, their results emphasize that the frequency dependence for elastic scattering is very different from that of scalar waves. Sato and Fehler (1998, p92) used scalar wave theory to show that the scattering power is related to the Power Spectrum (Fourier transform of the ACF) of the fractional perturbation. Some of the interesting characteristics for the low frequency solutions for elastic wave scattering include (Wu and Aki, 1985):

- The scattering wavefield has a frequency dependence of \( \omega^2 (\omega = 2\pi f) \)

- The forward scattered energy, which in most contexts qualify as transmitted energy, is sensitive to velocity perturbations. On the other hand, the strength of the backscattered energy is controlled by perturbations in impedance.

A major flaw when implementing the Born approximation is that it does not account for the interference of the scattered waves with the incident waves (Sato and Fehler, 1998, pg 95). Hence, falls short of adequately explaining scattering amplitude build up in the forward direction. A way of addressing this involves using a wave scattering theory that fully accounts for the total wavefield at all times and also considers multiple scattering. This is discussed in section 2.5.
2.5 Finite Difference Methods (Multiple Scattering)

Finite difference methods constitute a numerical approach to solving for the wave equation (equation 2.1) in a given medium. Just like theoretical methods (ray tracing), finite difference (FD) methods are commonly used in seismology especially when describing wave propagation in homogenous media (McMechan, 1983). However, for heterogeneous media, numerical solutions to wave propagation are superior to their theoretical analogues especially when the strength of the velocity fluctuation in the medium exceeds 5% (Sato and Fehler, 1998). This difference stems in the treatment of the seismic wave scattering. Numerical methods account for most of the processes that affect the propagating wavefield such as multiple scattering, diffractions, intrinsic attenuation, and wave mode conversions (e.g. PS, and SP). Note that methods based on the Born approximation fall short of fully explaining scattering attenuation (Frankel and Clayton, 1986) especially when the assumption of weak scattering fails to apply. Another cause for this shortcoming lies partly in the assumption of single scattering of the primary wavefield when it intercepts a scatterer.

Many investigators have conducted finite difference modeling studies for wave propagation in random media (Frankel and Clayton, 1986; Larkin et al., 1996; Frenje and Juhlin, 1998; L’Heureux et al., 2009). Most of the research conducted was geared towards understanding effects of crustal heterogeneity (ranging from the shallow crust to the mid and lower crust) on the characteristics of observed seismograms. For example, Frankel and Clayton (1986) generated 2D synthetic random models (scale length < 50km) of physical rock properties (typically, Vp, Vs and density) and showed that the resulting seismograms from their FD solution explained the coda and travel time fluctuations observed in teleseismic data. Although the synthetics may show some similarity to the observed data, a good match may not always be guaranteed (Frenje and Juhlin, 1998) if some aspects such as intrinsic attenuation are not properly accounted for. Thus, a comprehensive numerical study that evaluates the combined effect of the fluctuations in various physical properties may be needed to fully understand observations from seismograms (Chang and McMechan, 1996).

Finite difference codes tailored to only accommodate P-wave propagation are called acoustic FD methods (McMechan, 1983). However, elastic FD codes are more comprehensive as they fully account for energy losses due to wave conversion (Bohlen, 2002; Bohlen et al., 2003). In this
thesis, all synthetic seismograms were generated using a 2D/3D viscoelastic finite difference code (Bohlen, 2002) that uses second-order centered difference approximations for time derivatives and fourth-order centred difference approximations for spatial derivatives.

2.6 Seismic Scattering Regimes

The earth’s heterogeneity as revealed by borehole logs, core analysis, and geophysical measurements span various scales. Consequently, these different scales of heterogeneity have varied effects on seismic wave propagation. These effects include changes in waveform, amplitude fluctuations, phase changes and travel time fluctuations. Wu (1989) argues that these effects allow for classifying inhomogeneous media in terms of propagation regimes. In a given heterogeneous medium characterized by an isotropic correlation (scale) length $a$, the propagation regime is described by dimensionless parameters: $ka$, $L/a$ and $m$ where $k$ is the wavenumber, $L$ is the propagation length or the extent of the heterogeneous region and $m$ is the fractional fluctuation of wave speed $\left(\frac{\delta v_p}{v_p}\right)$. These propagation regimes are as follows:

- **Quasi homogenous**: When the heterogeneities are too small to be seen by the waves ($ka<0.01$). In this case, the medium can be considered as a homogenous medium with effective parameters.

- **Rayleigh scattering**: When $ka<0.1$, the seismic wave scattering causes apparent attenuation.

- **Large-angle scattering**: Also known as mie (resonance) scattering. Given that the sizes of the heterogeneities in this regime are comparable to the seismic wavelength ($ka=1$, e.g. $0.1<ka<10$), the incident wave is scattered to different directions at large angles with respect to the incident angle. Scattering in this regime causes coda wave generation and scattering attenuation.

- **Small angle scattering**: In this regime, most of the scattered energy is concentrated near the forward directions as $ka>>1$. Seismic wave propagation in such media is characterized by focusing and diffraction problems.

Analytic solutions describing the scattered wavefield such as the Born approximation for weak scattering is valid for the Rayleigh and large-angle regimes. On the other hand, ray theory is applicable for small angle scattering regimes whereby the transverse scale of the heterogeneity is significantly larger than the average Fresnel radius along the propagation path. This scenario
constitutes a sub-class of the small angle scattering regime called the Geometric optics (GO) regime. Another subclass is the Diffraction regime. The GO regime differs from the latter in that diffractions may be neglected when characterizing the scattered wave. Figure 2.4 shows schematic plot highlighting the various regimes as well as the classification of the heterogeneities from petrophysical analysis of Nash Creek and Thompson logs.

Figure 2.4: Scattering regimes based on the spatial fluctuations of P-wave velocities (modified after Wu, 1989). Heterogeneities based on petrophysical data from some case study areas (Nash Creek, Thompson, and Bosumtwi impact crater) are classified in the plot; $a$ is the correlation (scale) length; $k$ is the wavenumber, $L$ is the propagation length or the extent of the heterogeneous medium.

Figure 2.5 shows synthetic examples of 2D heterogeneous models characterizing some of these propagation regimes. The methodology adopted for building these models, follows the steps outlined by Goff and Jordan (1988, see Appendix A). While the first model ($Ka=0.1$) can be classified in the Rayleigh regime, the other two models fall in the large angle and small angle scattering regimes respectively. Snapshots of the full propagating seismic wave computed via visco-elastic FD methods (Bohlen, 2002) for the respective models are shown in Figure 2.6. All models show that the bulk of the seismic energy is transmitted. The Rayleigh regime model produces more P-wave coda than the other two models. While the coherency of the direct wave amplitudes within this model is preserved, those for the large angle and small angle regimes are subject to the propagation distance from the source position. In the latter, the incoherency of the direct P-waves within five P-wave wavelengths (~1000m) from the source is dominated by travel...
Figure 2.5: 2D petrophysical models. $V_p/V_s = 1.8$. Mean $V_p = 6000$ m/s; Mean density = 3000 kg/m$^3$; The wavenumber ($k$) is based on the dominant frequency = 50Hz.
Figure 2.6: Snapshot of P and S-wave propagation within the respective models. The source is a plane wave with first motion in the vertical (depth direction). The dominant frequency is 50Hz. P: Direct P-wave; Pc & Sc: P- and S-wave coda.
time fluctuations. S-waves resulting from P- to S- wave conversions due to wave scattering are also present and form part of the S-wave coda. The generated S-waves are scattered in turn to give P-waves which contribute toward the total P-wave coda. Moreover, the characteristics of the S-wave coda vary significantly between each model. The S-wave coda present in the Rayleigh regime is characterized by shorter scale lengths.

On the other hand, most of the coda generated in the large angle regime is S-wave coda which has longer scale lengths. In the small angle regime, both P- and S-wave coda are almost absent. No S-waves coda is present if dealing with a homogenous background model. As the wave

![Figure 2.7: Synthetic seismograms (vertical and horizontal component) recorded by a horizontal array (60 to 940m) of receivers at 1800m depth. The source is a plane wave with first motion in the vertical (depth direction). The dominant frequency is 50Hz.](attachment:image.png)
propagation distance from the source increases (Figure 2.6), the fluctuations in the amplitudes of the direct waves are dominated by both travel time fluctuations, and interference processes from wave diffractions and coda (Figure 2.7).

Seismic energy redistribution from the vertical to the horizontal direction is prominent in the large angle regime. The differences in the waveforms of the respective component data suggests according differences in their spectral characteristics whereby some frequencies may be preferentially amplified while others are damped (resonance scattering, Milkereit et al., 2005). An approach for investigating these frequency amplifications is through relative spectral (spectral ratios) analysis between the V-component and H-component data. In this approach, the amplified frequencies are considered to hold sufficient information to enable a quantitative assessment for scale parameters in the heterogeneous medium. In section 2.7, an initial review of the spectral ratio method is presented and then an assessment based on the data in Figure 2.7 is shown.

2.7 Spectral Ratios

Changes in waveform characteristics due to the interaction of seismic waves and various heterogeneous structures in the earth suggest alterations in the spectral content of the waveform. These changes can be due to the constructive and destructive interference of various waves that are generated as a result of the seismic wave scattering process. The interference patterns result in the amplification of energy or attenuation of energy at selective frequencies called resonant frequencies. Given that ground motion is strongly affected by the existing geology, Nakamura (1989) argued that one can take advantage of such information from recorded microtremors to locally characterize the dynamic properties of the ground and structures in the vicinity of the seismic sensors. This process is known as site characterization and has been vastly used within the exploration seismology and geotechnical community (Hartzell, 1992; Hartzell, 1996; Cassidy and Rogers, 1999; Nakamura, 1989; Parolai et al., 2004) for applications relevant to earthquake microzonation, and hazard assessment. The spectral ratio approach is a common technique for estimating resonant frequencies used in site characterization. The method described by Hartzell (1992) involves dividing the spectrum (typically S-waves recorded by the horizontal component) of the site considered by that from a nearby reference site as illustrated in Figure 2.8a. A similar approach can be applied to the case where the forward scattered (transmitted) wave is generated
by a limited target having strong impedance contrast with respect to its surroundings (Figure 2.8b). The reference site is typically a sensor located on the bedrock which is claimed not to have significant amplification in the energy of the recorded seismic waves. The closeness of the two

Figure 2.8: a) Typical geological structure of sedimentary basin; b) Geologic setting in hardrock environment illustrating wave scattering from a limited (size) target (e.g. sulfide orebody). H - Horizontal component; V - Vertical component; 3C - triaxial geophone. The subscripts r, b, and f represent reference site, bedrock and free surface; $H_r = H_b; QTS = H/V$ (Adapted from Nakamura, 2000). R1, R2: Receivers.

stations allows for the isolation of the site response\(^1\) as the source and path effects are considered to be suppressed in the spectral ratio computation. For a simple illustration of the spectral ratio method, consider receiver R1 (Figure 2.8b) to be the reference site. If noise effects are ignored, the amplitude spectra of the waves recorded by R1 and R2 can be written as:

\[
A_1(f) = S_0(f) \cdot P_g(f) \cdot T_1(f) \tag{2.13a}
\]

\[
A_2(f) = S_0(f) \cdot P_g(f) \cdot T_2(f) \tag{2.13b}
\]

where, $T_1(f)$ and $T_2(f)$ represent the amplitude spectra of the transmission impulse responses characterizing the medium of propagation for receivers R1 and R2, $S_0(f)$ is the spectrum of the

\(^1\) Site response: Relative spectral amplitude ratios in a defined frequency band for a given receiver site with respect to a reference site. Any spectral amplification is considered to be associated with the geological structure in the vicinity of the receiver location (Cassidy and Rogers, 1999).
source signature, \( P_g(f) \) is the path effect (e.g. geometric spreading), and \( f \) is the frequency.

Resonant frequencies observed at R2 due to forward scattered waves from the inclusion (Figure 2.8b) can be assessed by taking the spectral ratio between expressions 2.13b and 2.13a:

\[
\frac{A_2(f)}{A_1(f)} = \frac{S_0(f) P_g(f) T_2(f)}{S_0(f) P_g(f) T_1(f)} = \frac{T_2(f)}{T_1(f)}
\] (2.14)

The simplified expression in equation 2.14 is obtained on the principal assumption that the relative distance between both receivers as well as their respective distances from the source are such that the source and path effects from both receivers are identical.

A theoretical outline of another variant of the spectral ratio method called the Quasi Transfer spectra (QTS) was proposed by Nakamura (1989; 2000). QTS is obtained by computing the spectral ratio between the horizontal and vertical spectra (H/V) of the site considered. When computing QTS, care must be taken to exclude the influence of Rayleigh waves, as Nakamura argues that the peak frequency (resonant frequency) identified through the QTS method can be explained by the vertical incident SH waves. The use of S-waves for identifying amplified frequencies is also due to the fact that P-waves (which have higher velocities) are amplified at higher frequencies. An example of a successful application of Nakamura’s method has been documented by Carniel et al. (2006). The typical frequency range for spectral ratio analysis for earthquake and microtremors is 1-15 Hz. Both methods described above do a good job at estimating the fundamental frequency. However, the H/V approach underestimates the relative amplification with respect to the reference site method (Parolai et al., 2004). Efficiency of both methods is also undermined by effects of topography.

A modeling study performed by Milkereit et al., 2003, corroborate the use of the spectral ratio method on seismic data in the exploration frequency band (30-120 Hz- short wavelengths) as a tool for extracting lateral scale length information from resonant frequencies. In this case, the spectral ratio is computed from the transverse and radial component data (T/R\(_d\)) of direct P-wave energy. The transverse and radial components are obtained from 3C seismic data. The T/R\(_d\) method is based on the redistribution of seismic energy due to lateral heterogeneity in the medium. In the event of no lateral heterogeneity, the seismic energy remains polarized in the direction of the ray (Milkereit et al., 2005).
The characteristics of recorded component data in Figure 2.7 clearly illustrate the principle of energy redistribution due to seismic scattering processes. Note that the particle displacement of the plane wave source is in the vertical (depth) direction. The spectral decomposition of the direct arrivals (fourier transform of the windowed direct wave) from the component data further illustrates resulting alteration in the frequency distribution especially that of the dominant frequency (50Hz, Figure 2.9). In the Rayleigh regime, the consistency in the spectral amplitudes of the dominant frequency can easily be observed on the horizontal (H) component data (Figures 2.9a & b). This consistency decreases in both the large angle (Figure 2.9d) and small angle regime (Figure 2.9g). For the vertical (V) component data, the consistency in the spectral amplitudes recorded at the receiver locations is higher in the small angle regime (Figure 2.9h). This is close to the spectral response obtained when using a homogeneous background model. The spectral response for a homogeneous model differs from those in Figures 2.9g & h in that there is no energy present in the horizontal component (spectral amplitude =0) and that the spectral amplitudes in the V-component are identical for all receiver locations. It can also be observed that the relative amplitude strength between the vertical and horizontal component is large.

Component data from the different models also depict significant differences in the distribution of notch frequencies: different trace records have different notch frequencies. The computed spectral ratio results (H/V) from the respective component data are shown in Figures 2.9c, f & i. In Figures 2.9c, f & i, the full dynamic amplitude range for frequencies between 0 and 150Hz is used in each case. In all three cases, no amplified frequencies, which can be associated to the respective model scale lengths, can be identified in the plots despite the strong spectral fluctuations observed in the component data. In spite of the unconvincing results, it is believed that better ways for the spectral analysis and plotting of the data need to be developed in order to assess the scattering frequencies in the exploration frequency band.

One of the main challenges in locating these resonant frequencies lies in the large spectral amplitude difference between the V- and H-component data. In Appendix F, illustrations of the spectral ratio method using more simple models suggest that other factors are critical in the analysis. For example, a 1D assessment suggests that the impedance contrast in the medium needs to be sufficiently large (Reflection coefficient (R) >=0.3) for resonant frequencies to be observed.
**Figure 2.9:** Spectral decomposition (a, b, d, e, g, h) from first breaks of the recorded wavefields in horizontal and vertical component data shown in Figure 2.7; Spectral ratio (H/V) plots - c, f, i. Green arrows indicate the distribution of notch frequencies. No resonant frequencies can be identified in the spectral ratio plots. Notch frequency: Attenuated frequency.
2.8 Summary

Information of a given rock property sampled at different spatial locations is generally characterized not by a single value but by a suite of values within a certain range. The variability in these rock properties exist over different scale lengths and lends itself towards our understanding of the earth as heterogeneous medium. Variability in physical rock properties can be readily inferred and characterized from datasets such as borehole logs. The logs are considered to comprise a deterministic trend and a stochastic component. The latter is analyzed using two point statistic methods. Derived parametric models such as those based on von Kármán functions are very useful for modeling applications as they are handy for estimating rock properties at unsampled locations. One of the major implications for the variability in physical rock properties is its effect in seismic wave propagation. Seismic wave propagation in heterogeneous media comprises processes subject to boundary conditions between areas with contrasting values (e.g P-wave impedance) in rock property and by the relative size of heterogeneities (correlation length) with respect to the seismic wavelength. Effects on seismic wave propagation range from travel time, amplitude and frequency content fluctuations (Wu, 1989). Analytic solutions such as the Born approximation can adequately describe some modifications observed on seismograms for the case where the scale of the heterogeneity is very small. The formulation considers the fractional perturbations of the physical properties are extremely less than 1 (e.g. P-wave velocity: \( \frac{\partial V}{V_m} \ll 1 \)). Hence, the resulting scattered wave is relatively very small compared to the incident wavefield.

However, the Born approximation falls short of accounting for the process of multiple scattering. A suitable approach for describing observations on seismograms caused by multiple scattering processes is by considering the total wavefield at any given time. Numerical methods based on viscoelastic finite difference routines (i.e. account for the earth’s “anelastic” behaviour: attenuation and dispersion of seismic waves; Bohlen, 2002) are most suited for characterizing the full wavefield at all times in a given propagation domain.

When P- and S-wave velocities are correlated, finite difference modeling studies corroborate that the large angle regime causes significant fluctuations in travel time especially as the fluctuations occur over small wavelength ranges. Chang and McMechan (1996) used numerical methods to show that the amount of scattering observed will be less if both P- and S-wave velocities are
uncorrelated. Through viscoacoustic FD numerical studies, Kneib and Shapiro (1995) argue that seismic wave propagation in random media (weak perturbation factor, \(ka\sim4.2\)) is characterized by two scattering regimes:

- weak fluctuation regime where incoherency in the wavefield is dominated mostly by traveltime fluctuations (amplitudes fluctuate weakly);
- strong fluctuations regime usually at large propagation distance from source where the incoherency in the seismic wavefields is caused by traveltime and amplitude fluctuations.

For this viscoacoustic media, it is argued that seismic amplitude changes due to absorption strongly dominates effects of amplitude loss due to scattering (scattering attenuation).

Due to heterogeneity in medium properties, modeling results presented in this chapter corroborate the significant presence of energy in the horizontal component. This stems from wave conversion processes. Besides wave conversions, interference mechanisms (focusing and defocusing) contribute significantly towards altering the frequency content in the respective sensor components (Section 2.7).

As indicated in section 2.7, the spectral ratio analysis did not identify any resonant frequencies from scattered waves that can be quantitatively associated with the scale length of the synthetic heterogeneous models. Results obtained so far are inconclusive as adequate methods for analysis and plotting of the data need to be researched upon. A simple way to understand the relation between resonant frequencies and scale lengths is by considering a simple earth model that characterizes the structure of a sedimentary basin. In such a model, the amplified (resonant) frequency can be obtained from the wave speed and layer thickness via the quarter-wavelength principle. Investigations in this light are further assessed for exploration seismic data in hardrock settings in Appendix F.

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© Hypothesis: Because the locations where the scattering of P- and S-waves occur are not coincident, most of the scattered fields will interfere destructively (Chang and McMechan, 1996). However, more research is needed to fully verify this explanation.
The bulk of chapter two covered the basic groundwork relevant for describing heterogeneous media and for understanding its influence in seismic wave propagation. One approach for describing crustal heterogeneity uses data measured from borehole cores (hard data). In chapter 3, I describe the petrophysical and geochemical characteristics of the sulfides and host rocks found in Nash Creek (New Brunswick, Canada) and around the Thompson mine (Manitoba, Canada).
Chapter 3
Petrophysical Study

3.1 Introduction

The rocks at Nash Creek and at Thompson bear the signature of thermal and chemical alteration owing to tectonic activity (Brown, 2007; Bleeker, 1990a, b). These changes occur at different scales (regional or local) and are thus reflected in measured physical rock properties. This chapter focuses on the collection and integrated analysis of petrophysical and geological information from rocks that make up the rock mass in the Nash Creek and Thompson study areas. In the Nash creek area, the uniqueness of the database lies in the fact that multiple rock attributes are sampled from a dense 3D network of boreholes on the property. This is in sharp contrast with most practices in the exploration industry where petrophysical data is often available in very few boreholes (<= 3). A large density database (~6000 samples) is constructed as a result of my study. Other petrophysical attributes discussed in this chapter include the compressional and shear wave velocities. At the Thompson mine these attributes are further complemented by comprehensive geologic information that characterizes the distribution of the sulfides. The sulfide distributions in the Nash Creek and Thompson areas differ significantly. Logs and other geophysical studies at Nash creek corroborate that the sulfide is highly disseminated (low grade). On the other hand, high grade sulfide zones are hosted in the igneous rocks found in the Thompson mine. The petrophysical database developed in this study forms an invaluable basis for rock physics modeling which is subsequently discussed in Chapter 4. Also, petrophysical information is critical for the assessment of geophysical imaging methods either through forward modeling or inversions. The former approach is adopted in Chapters 4, 5 and Appendix C whereby derived petrophysical models are used for seismic and gravity forward modeling studies.

3.2 Nash Creek

3.2.1 Geologic setting

The Nash Creek deposit is located along the western margin of the Jacquet River Graben in northeastern New Brunswick (Dostal et al., 1989), Figure 3.1. It is located within the Chaleur Bay syncline which is sandwiched by two Northeast trending orogenic belts namely: the
Aroostook-Percé highlands to the north and the Miramachi highlands to the south (Brown, 2007 (Figure 3.1), Walker and McCutcheon, 1995). Nash Creek is located about 50km northwest of Bathurst which is home to the Bathurst Mining Camp (BMC). The BMC, which is located in the Miramachi highlands (McCutcheon et al., 1993), has many occurrences of volcanogenic massive sulfides (VMS) that are mostly deposited in Cambro-ordovician rocks (McCutcheon et al., 1993).

The sulfide occurrences have high pyrrhotite, pyrite, and chalcopyrite content. On the other hand, the Nash Creek exploration area sits atop the BMC as it is mainly underlain by the Lower Devonian sequence of the Dalhousie Group rocks (Brown, 2007) comprising volcanic breccias, siltstones, limestones, mafic flows, rhyolites, and tuffs (consolidated volcanic ash). The main focus of mineralization is in the Dalhousie group rocks. The tectonostratigraphic setting may represent a failed continental rift system with shallow water to marginal marine environment (Brown, 2007). The rift was formed as a result of extensional tectonics. The volcanic and sedimentary rocks were deposited in a half-graben that is fault bounded to the west. It is believed that the fault boundary, which is a zone of crustal weakness, favoured hydrothermal fluid

Figure 3.1: Location map of New Brunswick and Gaspé Belt showing: green – Connecticut Valley-Gaspé Synclinorium; Ocean blue – Aroostook-Percé Anticlinorium; Light gray – Chaleur Bay Synclinorium; Red star: Nash Creek Exploration area. H – Humber Zone; D – Dunnage Zone; G – Gander Zone; C – Carboniferous rocks; RGPF – Restigouche-Grand Pabos Fault; MGF – McKenzie Gulch Fault; SF – Sellarsville Fault; RBMF – Rocky Brook-Millstream Fault; CBF – Catamaran Brook Fault. Pre-Late Ordovician inliers in the Gaspé Belt are shown. In dark grey: MM – Macquereau-Mictaw Inlier (Humber-Dunnage); E – Elmtree Inlier(Dunnage); After Wilson et al., 2005.
migration that in turn allowed for sulfide accumulation. At Nash creek, the deposit has occurrences of sphalerite, galena, and pyrite. Silver also accompanies the sulfide mineralization (Brown, 2007).

Rock physics measurements were performed on core samples to help characterize the 2D/3D distribution of the Nash Creek deposit. Understanding the variation and correlation between these physical properties is vital for every exploration project. Moreover, petrophysical data is important for rock physics modeling especially for determining how the changes in elastic properties relate to changes in mineralogy and in predicting these elastic parameters in areas with no borehole logs.

### 3.2.2 Elastic properties

Contrast in elastic properties (P- and S-wave velocities, density) is a fundamental factor governing the seismic response of an orebody. Individual velocities and densities are controlled by factors like mineralogical content, damage, pressure and fluid saturation especially in situ conditions. Various methods such as borehole logging and laboratory measurements are used to estimate these parameters. Studies indicate that the velocity-density field of sulfide ores differ distinctively from that of common silicate rocks (Salisbury et al., 1996). Though the velocities of these sulfide ores are variable, they generally have high densities. Thus, sulfide ores have high impedances \((Z = \text{velocity} \times \text{density})\), which provide ideal contrast for imaging them via seismic methods. Constraining these elastic parameters provides necessary information for adequate assessment of the potential for using seismic methods and gravity methods for imaging the shallow mineralized zones.

#### a) Density measurements

Thirty-two boreholes were logged for density information. Commonly, these small diameter boreholes were very unstable. In such circumstance, the use of a nuclear sourced gamma-logging tool is not possible; therefore all density measurements were performed by manually logging each bore core at intervals ranging from 0.5m-1.5m. The density of each core sample was computed from two measurements of mass: one with the sample suspended in air and a second with the sample suspended in water. Assuming the water has a density of 1.0 g/cm³, this simple procedure provides an estimate of the dry density of the rock. Though these rock samples were
no longer at in situ conditions, it is assumed densities would not have changed significantly over time and the measurements were a close representation of the in situ conditions of both the sulfide mineralization and the host rocks. The Nash Creek sulfides occur as matrix replacements in laterally extensive volcanic fragments and as vein mineralization in fractured (low density) zones. A total of 5981 measurements were made and the average density from this database is 2.76 g/cm$^3$ (Figure 3.2a). The low densities (<2.5 g/cm$^3$) and the high densities (>3.2 g/cm$^3$) correlate very well with alteration zones (with no mineralization) and zinc-rich mineralization zones respectively. Figure 3.2b shows the correlation between measured density and mineralization (assay data). Boreholes intersected by zones with high grade sulfides (high densities) correspond to low resistivity zones obtained from Vertical Resistivity Profiling (VRP, Milkereit et al., 2008). Detailed analysis of the borehole data suggests most of the mineralization

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The distribution of these volcanic fragments conforms to the existing sequence of host rocks in the Nash Creek area (Brown, 2007).
is hosted within felsic and mafic material (Figure 3.2c). A subset of the 32 boreholes (17 boreholes) constitutes two intersecting borehole profiles (Chapter 4, Figure 4.1). The good correlation between the density and the assay data in these 17 boreholes can be used to obtain a 3D geophysical model of the ore.

b) Velocity and porosity laboratory measurements

The velocities were determined from a suite of 18 micro cores (sampled from other boreholes on the Nash Creek property) of ore and host rock with dimensions ranging from 18-70mm in length and 25-48mm in diameter. These cores were representative of the characteristic lithology found in Nash Creek. After the micro core densities were determined, the compressional wave velocities were measured using the pulse transmission method: a pair of piezoelectric sensors placed on opposite ends of the cylindrical rock sample such that one sensor triggers an acoustic pulse that propagates through the sample and is recorded by the second sensor at the opposite end. The measurements were done under standard conditions.

Porosity was measured by using the caliper technique described by Franklin et al. (1979). The method requires measurements of the dry and the saturated masses as well as bulk volume calculated from caliper readings for dimensions of each rock sample. The rock samples were dried by putting them in an isostemp vacuum oven whereas saturation was achieved by immersing the dried cores in a container of water for a period of five days. Assuming that the total pore space volume forms a percentage of the bulk volume of the rock sample and that the air in the pore space is completely replaced by water when the rock is fully saturated, pore volume was obtained from the difference between the measured dry and saturated masses. Hence, porosity for each rock sample was measured as a percentage ratio between the pore volume and the bulk volume. The compressional and shear velocities range from ~2.91 to ~6.41 km/s and ~1.41 to ~4.20 km/s respectively. The low P-wave velocity (< 4 km/s) values correspond to rocks with a high degree of alteration. As expected, the porosity for most of these crystalline rocks is less than 10%. However, some of the measured core samples have high porosities (>=10%) and it is believed that this is strongly correlated with the low P-wave velocity values (< 4 km/s) measured for these rock samples due to fractures and brecciation. Table 3.1 summarizes the dynamic rock property measurements from the Nash Creek cores. Additional mechanical properties (Young’s modulus) were also estimated from theoretical relations between Vp, Vs, and density (Olsen et
al. 2008-Eq. 2; Schon, 1996). These mechanical properties come in handy when characterizing the rock mass for seismic stability and for mine design.

Table 3.1: Acoustic properties of Nash Creek sedimentary, felsic and mafic rocks at room temperature

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Density (g/cm$^3$)</th>
<th>$V_p$ (km/s)</th>
<th>$V_{s1}$ (km/s)</th>
<th>$V_{s2}$ (km/s)</th>
<th>Porosity n (%)</th>
<th>Young’s Modulus $E_{s1}$ (GPa)</th>
<th>$E_{s2}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.00</td>
<td>5.18</td>
<td>3.04</td>
<td>3.06</td>
<td>2.31</td>
<td>68.61</td>
<td>69.35</td>
</tr>
<tr>
<td>B</td>
<td>2.76</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>C</td>
<td>2.65</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>D</td>
<td>2.71</td>
<td>3.70</td>
<td>2.34</td>
<td>2.34</td>
<td>6.98</td>
<td>34.67</td>
<td>34.62</td>
</tr>
<tr>
<td>E</td>
<td>2.47</td>
<td>3.47</td>
<td>2.27</td>
<td>2.25</td>
<td>11.4</td>
<td>28.61</td>
<td>28.37</td>
</tr>
<tr>
<td>F</td>
<td>2.58</td>
<td>4.64</td>
<td>2.85</td>
<td>2.90</td>
<td>×</td>
<td>50.18</td>
<td>51.17</td>
</tr>
<tr>
<td>G</td>
<td>2.68</td>
<td>4.73</td>
<td>2.79</td>
<td>2.78</td>
<td>4.48</td>
<td>51.62</td>
<td>51.37</td>
</tr>
<tr>
<td>H</td>
<td>2.67</td>
<td>4.07</td>
<td>1.61</td>
<td>1.55</td>
<td>×</td>
<td>19.43</td>
<td>18.09</td>
</tr>
<tr>
<td>I</td>
<td>2.56</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>J</td>
<td>2.67</td>
<td>2.91</td>
<td>1.43</td>
<td>1.41</td>
<td>×</td>
<td>14.66</td>
<td>14.24</td>
</tr>
<tr>
<td>K</td>
<td>2.59</td>
<td>3.68</td>
<td>2.18</td>
<td>1.84</td>
<td>11.78</td>
<td>30.27</td>
<td>23.44</td>
</tr>
<tr>
<td>L</td>
<td>2.41</td>
<td>4.53</td>
<td>2.44</td>
<td>2.39</td>
<td>×</td>
<td>37.24</td>
<td>35.94</td>
</tr>
<tr>
<td>M</td>
<td>2.78</td>
<td>6.41</td>
<td>3.57</td>
<td>3.57</td>
<td>0.35</td>
<td>90.56</td>
<td>90.41</td>
</tr>
<tr>
<td>N$^*$</td>
<td>3.00</td>
<td>5.80</td>
<td>3.35</td>
<td>3.37</td>
<td>×</td>
<td>84.26</td>
<td>85.05</td>
</tr>
<tr>
<td>O$^*$</td>
<td>3.28</td>
<td>4.89</td>
<td>2.83</td>
<td>2.85</td>
<td>×</td>
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<td>66.18</td>
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<td>P$^*$</td>
<td>3.21</td>
<td>5.79</td>
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<td>×</td>
<td>86.68</td>
<td>94.97</td>
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<td>Q$^*$</td>
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<td>4.74</td>
<td>2.96</td>
<td>2.93</td>
<td>×</td>
<td>67.57</td>
<td>66.84</td>
</tr>
<tr>
<td>R</td>
<td>3.96</td>
<td>6.26</td>
<td>4.11</td>
<td>4.20</td>
<td>×</td>
<td>150.05</td>
<td>152.37</td>
</tr>
</tbody>
</table>

*Rock samples with veins of sulfide mineralization (pyrite, sphalerite, and galena)

$V_p$: Compressional wave velocity

$V_{s1}$ & $V_{s2}$: Shear wave velocities (SV & SH respectively). SV and SH velocities were measured in perpendicular directions with respect to the length of the core.

In order to further characterize the acoustic velocities, the measured values were plotted against the Nafe-Drake curve that highlights the velocity-density field (Salisbury et al., 1996) of sulfide ores and silicate host rocks at confining pressure of 200MPa (Figure 3.3). Velocity properties are severely affected by microcracks and the measured values will increase to the crack free value with pressure as the cracks are progressively closed (Schmitt et al., 2003; Salisbury et al., 1996). Thus, owing to the fact that measurements were obtained under conditions where confining pressure were slightly above 0MPa, the data have lower values than expected at the given pressure conditions (200MPa). This difference in pressure was accounted for by considering a rock physics database of acoustic velocity measurements under several confining pressures from a host of rocks samples around the world (Salisbury et al., 1999). Analysis of the rock physics database suggests that velocities should increase on average by 9% for pressures changing from 10MPa to 200MPa. Densities change very slightly over same pressure range. With the effects of pressure corrected for, it is noticed that most of the sulfide rich samples plot correctly within their respective subfield of the Nafe-Drake curve (mixed sulfides, Figure 3.3).
As compressional velocities increase with pressure, densities also increase slightly with pressure, causing acoustic impedance differences to be invariant with pressure (Salisbury et al., 1996). This suggests reflection coefficients are pressure independent even if velocities change. Thus, from our measurements, we can obtain reasonable estimates of the impedance contrast between the sulfide rich rocks and the host rocks (i.e. non mineralized rocks). A close examination of Figure 3.3 suggests the sulfide rich rocks are acoustically distinguishable from the host rocks.

**Figure 3.3:** Projected Compressional wave velocity versus densities for Nash Creek rock samples superimposed on the Nafe-Drake curve for common silicate rocks, hardrocks and selected base metal ore minerals (after Salisbury et al., 1996; Salisbury et al., 2003). Dashed lines represent lines of constant impedance; bar shows minimum impedance contrast required for strong reflection (R=0.06). Abbreviations: Bo, Bornite; Cpy, chalcopyrite; g, gangue; Ga, galena; Pn, pentlandite; Po, pyrrhotite; Py, pyrite; Sph, sphalerite; F, felsic; M, mafic; UM, ultra mafic. The red star indicates the elastic parameter for the ideal sulfide orebody used in Chapter 4 to evaluate the implications for seismic imaging at Nash Creek.

This suggests reflection coefficients are pressure independent even if velocities change. Thus, from our measurements, we can obtain reasonable estimates of the impedance contrast between the sulfide rich rocks and the host rocks (i.e. non mineralized rocks). A close examination of Figure 3.3 suggests the sulfide rich rocks are acoustically distinguishable from the host rocks.
since R=0.06 (reflection coefficient) is the threshold for producing a reasonable reflection after allowing for noise. Besides seismic methods, the density data will also be useful for gravity modeling. These are covered in Appendix C.

Measured electrical properties (resistivity and chargeability) from the rock samples corroborate that sulfides at Nash Creek are more conductive than the host rock (Appendix B.1). However, using this as a basis for interpreting geophysical data such as airborne electromagnetic data is complicated by overburden effects. The overburden at Nash Creek is weathered and thus characterized as a conductive target in the airborne EM data.

3.3 Thompson

3.3.1 Regional geologic and tectonic setting

The Thompson Nickel Belt (TNB), located in Northern Manitoba, plays host to some of the world-class nickel ore deposits which have fuelled the development of mines such as Birchtree and Thompson (Figures 3.4 and 3.5). The TNB forms a North East trending structure and forms the NorthWest boundary of the Superior province (Figure 3.4). The rockmass is highly metamorphosed and mostly consists of orthogneisses, metasediments and metavolcanic rocks (Figure 3.5). The metamorphism bears the signature of the tectonic compression that led to the development of the Trans-Hudson orogen. Some of the knowledge about the crustal architecture in the Trans-Hudson Orogen was achieved via reflection and refraction seismic studies (Lewry et al., 1994, Hammer et al., 2010) conducted as part of the Canadian LITHOPROBE program. According to Bleeker (1990b), the TNB consists mainly of reworked archean basement rocks (> 2.5ma).

Remnants of proterozoic (>2.0Ma) rocks called the Ospwagan group (OPG) overlay the bedrock (Figure 3.5). Known deposits such as Thompson and Birchtree are also confined to the western margin of the TNB (Figure 3.5). Mineralization in TNB occurs in form of disseminated, massive and breccia ores. In the Thompson mine, for example, the ore exists as massive, to semi massive

\[ \text{Gneiss formed from igneous rocks} \]
sulfide matrix breccia and stringers hosted within the Pipe 2 (P2) formation pelites that constitutes the OPG (Figure 3.6). Moreover, the distribution of the sulfides is strongly associated to the tectonic history sustained by the host rocks (Peredery, 1982; Paktunc, 1984). The basement rocks have intrusives (Peredery, 1982) such as ultramafics of lenticular shape (Figure 3.6).

Figure 3.4: General geological map of the Thompson Nickel Belt. Inset shows gravity (color overlay) and shaded magnetic map of Manitoba. The TNB forms the north-northeast trending leg of the Superior Boundary Zone (black line). See (Figure 3.5) for details of the bedrock geology of the area outlined in dashed line (modified after Layton-Mathews et al. 2007).

3.3.2 Rock physics

Given the deformation and emplacement of complex structures in Thompson, the application of geophysical imaging methods becomes more challenging. Rock physics measurements are

\footnote{Clastic rock (clay rich; very fine grained)}
important to assess the resolving (imaging) potential of targets by various geophysical imaging tools. For seismic applications the measurements of rock properties such as compressional wave velocity (Vp), shear wave velocity (Vs), density, porosity and permeability help in understanding the major structures responsible for backscattered energy (reflectivity) as well as the relative proportion of energy that is transmitted through the medium.

In Thompson, Vp, Vs and density measurements were measured from a suite of rocks that were representative of the major lithologic units in the area. One of the primary motivations was to understand the seismic response of the orebody with respect to the other lithologic contacts within the metamorphosed rock complex (Figure 3.6). The sulfide ore mineralogy is comprised of pyrrohtite, pentlandite and some minor components of chalcopyrite. Vp, Vs, and density measurements from the rock samples are summarized in Table 3.2 and Figure 3.7. Other derivatives from these elastic properties such as the Vp/Vs ratios are also included in Table 3.2. While the velocities were measured by confining the samples to a uniaxial stress ranging from 0 to 2 Kft.
86-195 Mpa, the densities were measured via the buoyancy method. From Table 3.2, a strong correlation between Vp and Vs can be inferred from the various lithologic and stratigraphic units given that the Vp/Vs and the computed poisson ratios are almost constant.

**Table 3.2:** Elastic properties of metamorphic rocks from Thompson

<table>
<thead>
<tr>
<th>Lithology/Stratigraphy</th>
<th>Density (g/cm³)</th>
<th>Vp (m/s)</th>
<th>Vs (m/s)</th>
<th>Zp (10⁵ g/cm²s)</th>
<th>Dynamic Vp/Vs</th>
<th>Dynamic Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMPT</td>
<td>3.00</td>
<td>6333.49</td>
<td>3721.08</td>
<td>18.99</td>
<td>1.70</td>
<td>0.24</td>
</tr>
<tr>
<td>PEG</td>
<td>2.67</td>
<td>5955.44</td>
<td>3539.35</td>
<td>15.93</td>
<td>1.68</td>
<td>0.23</td>
</tr>
<tr>
<td>UM</td>
<td>3.01</td>
<td>6661.08</td>
<td>3963.91</td>
<td>20.04</td>
<td>1.68</td>
<td>0.23</td>
</tr>
<tr>
<td>AG</td>
<td>2.72</td>
<td>5933.42</td>
<td>3572.41</td>
<td>16.15</td>
<td>1.66</td>
<td>0.22</td>
</tr>
<tr>
<td>MAN</td>
<td>2.67</td>
<td>5860.86</td>
<td>3552.35</td>
<td>15.65</td>
<td>1.65</td>
<td>0.21</td>
</tr>
<tr>
<td>TF</td>
<td>2.87</td>
<td>6567.59</td>
<td>3783.79</td>
<td>18.85</td>
<td>1.74</td>
<td>0.25</td>
</tr>
<tr>
<td>P2</td>
<td>2.84</td>
<td>6007.60</td>
<td>3516.54</td>
<td>17.07</td>
<td>1.71</td>
<td>0.24</td>
</tr>
<tr>
<td>MASU*</td>
<td>4.23</td>
<td>5423.95</td>
<td>3185.17</td>
<td>22.94</td>
<td>1.70</td>
<td>0.24</td>
</tr>
<tr>
<td>P3</td>
<td>3.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SF</td>
<td>2.73</td>
<td>6010.29</td>
<td>3544.52</td>
<td>16.38</td>
<td>1.70</td>
<td>0.23</td>
</tr>
</tbody>
</table>

* Massive sulfide

Reflectivity occurs at an interface provided there is an impedance (density x velocity) contrast between the two lithologic units. Although seismic wave reflection is subject to the influence of factors, such as Fresnel radius (associated to wavelength of wave and size of the target structure), rock anisotropy caused by preferably oriented fractures, a simple analysis for plane wave incidence suggests an impedance contrast of about 2.5x10⁵ g/cm²s suffices to reflect some percentage (~6%) of the incident wave propagating in a lossless medium. This amount of reflected energy is detectable by seismic reflection methods (Salisbury et al., 1996). It is clear from Figure 3.7 that the velocities of the crystalline rocks in Thompson tend to increase with density. This trend can be correlated to an increase in the metamorphic grade and content of Iron (Fe), Magnesium (Mg) and Calcium (Ca) in the crystals of the rock (Salisbury et al., 2003:- ultramafic rocks have the highest content of Fe, Mg, and Ca). With the superimposed lines of constant impedance for P and S waves, and following the geologic setting within the Thompson area (Figure 3.6) it becomes evident which combination of lithologic units will contribute to most of the backscattered (reflected) P- and S-wave energy provided other conditions such as geometry are met. Figure 3.8 shows sample petrophysical models derived by projecting these mean values on the geologic models in Figure3.6. While significant reflections will be generated
by the ultramafic intrusions (UM) within the archean basement rocks (AG), the strongest reflection would be generated from the massive sulfides irrespective of the host rock.

3.4 Conclusions and Implications

From the large density database built from a wide collection of Nash Creek core samples, it is observed that the average host rock density is 2.7 g/cm³ while the Zn-Pb-Ag sulfide ore show densities that are at least greater than 3.0 g/cm³. There is a good correlation between the high densities of the sulfide and the assay data.

Laboratory measurements of some physical and elastic wave properties under standard conditions (velocities, porosities, and dynamic Young’s modulus) have been used to predict that the sulfides and fault zones on the Nash creek property can be imaged using seismic methods. The high impedances from the sulfides is controlled by their high density contrasts with respect to the mafic, felsic and sedimentary host rocks. On the other hand, low impedances in the host rock are associated with low velocities measured from altered rock samples that could have come from fault zones. Moreover, the sulfides also show a sharp resistivity and chargeability contrast.
with respect to the host rock. Taking this into account and the fact that the deposits in the present case are shallow, it may be possible to probe the ore deposit with electrical methods such as DC resistivity surveys although noise sources such as the conductive overburden undermines the effectiveness of the method. On the other hand, seismic methods in this case will be more suitable for deep targets. This is discussed in greater detail in Chapter 4. Integrating the acquired petrophysical data from multiple drill holes as well as geologic information allows for building an adequate rock physics 3D earth model that characterizes distribution of the shallow but “blind” ore deposit. This aspect of rock physics modeling that is primarily data driven (petrophysics) will be covered in Chapter 4.

![Figure 3.8: Density (kg/m$^3$, left column) and P-wave impedance ($Z_p \times 10^5$ gcm$^{-1}$s$^{-1}$, right column) for sections in Figures 3.6a (top row) and b (bottom row).](image)

Although there is a wealth of information in this Nash Creek database, more lab measurements with a wider selection of samples from the Nash Creek property would help get a better handle
on average values for the various petrophysical properties especially the P-wave and S-wave velocities. Pressure dependent measurements of velocity values will also be useful to obtain better estimates of the dynamic elastic properties.

At Thompson the petrophysical data suggests that while the impedance contrast between crystalline rock units is controlled by velocity, the impedance contrast with respect to the sulfides is also controlled by density. It can also be inferred that the Thompson formation can generate detectable P-wave reflections if in contact with either the MAN (manasan formation) or AG (archean gneiss) rock units, hence a potential seismic marker horizon for seismic reflection methods. The next logical step in this study entails incorporating the geologic information and the petrophysical data to build constrained petrophysical models.

The petrophysical models that will be derived from the respective case studies are critical towards investigating the response of the rock mass to various geophysical methods. In Chapters 4 and 5 as well as in Appendix C, I also discuss the implications of the various rock property distributions to seismic imaging and gravity methods.
Chapter 4
Integrated 3D Modeling and Geophysical Implications: Nash Creek

4.1 Introduction

In exploration projects, a suite of datasets ranging from geology, petrophysics, and geophysical information are used to obtain an understanding of the earth’s subsurface. In most cases, subsurface information is obtained by applying processing and inversion schemes to recorded geophysical data. However, other methods can be adopted whereby the modeling is based on other types of data such as petrophysical logs and geology. At Nash Creek, the existence of large rockphysics database (e.g. density) as discussed in Chapter 3 provides an invaluable foundation for using this approach towards building a realistic 3D earth model to characterize both the host rocks and sulfides. In this chapter precedence is given to the use of geostatistical (kriging) methods (Matheron, 1963; Deutsch and Journel, 1998) for building a realistic 3D density model. This approach of building stochastic earth models differs from that presented by Goff and Jordan (1998) in that the derived models honour the information at sampled data locations (“conditional models”). Geostatistical methods for rock physics modeling have been developed and used widely for mining, hydrocarbon exploration and near surface applications (Goovaerts, 1997; Cole et al., 2003; Laine, 2003). Using geostatistics enables building a realistic petrophysical model that accounts for the variability observed in measured/sampled (hard) data. At Nash Creek, the usefulness of geostatistical tools is particularly enhanced by the dense network of borehole information which is used accordingly to condition the modeling results. Regions A and B with dimensions 300x200x100m and 495 x325.6 x280m (Figure 4.1) are those considered for building the petrophysical earth model. They shall hence be referred as Model Volume A (MVA) and Model Volume B (MVB) respectively. Figure 4.1 shows in detail the borehole distribution and availability of associated rock properties within MVA.

It is also demonstrated that the 3D density model has invaluable implications for resource evaluation of the total Zn content on the Nash Creek property.

Although primary attention is given to density modeling, the spatial distribution of other parameters such as P- and S-wave velocities are also considered. These parameters combined are useful for addressing relevant questions for geophysical investigations at the Nash Creek
property. In the second part of this chapter, I discuss the implications of the derived stochastic density model for seismic imaging methods. An overview on according implications for gravity methods is covered in section C.5 (Appendix C). The disseminated nature of the sulfides at Nash Creek renders such studies very important especially as this can be incorporated as part of the decision-making process of the exploration project.

In Chapter 5, a similar data-driven modeling approach (based principally on geologic information) and seismic investigation is adopted in the Thompson case study.

All the geostatistical analysis presented in Chapters 4 and 5 were done using MATLAB and GOCAD software packages.

![Figure 4.1](image-url)

**Figure 4.1:** Superposition of borehole distribution and gravity data (colored points) on the Nash creek property. Model volumes A (Region A) and B(Region B) used for the geostatistical studies are shown. A zoom in to MVA (top view) clearly shows the distribution of the available boreholes with available rock physics data.

### 4.2 Measure of Spatial Variability and Continuity

Measurement errors are incurred in the method used to measure the density data. This is caused in part by the relative difference in pressure and temperature conditions from those at in situ conditions. Erroneous data can affect the outcome of the geostatistical analysis negatively (Deutsch & Journel, 1998; Goovaerts, 1997). In order to minimize the errors from our data in our study, the instruments used were calibrated by using a sample metal of aluminium with known
density. Given that most of the rocks in the Nash Creek property are of mafic or felsic type, 2.3gcm$^{-3}$ was used as the cutoff value below which the density measurements were considered to be erroneous for the geostatistical study$^1$. Figure 4.2 shows histogram plots and summary statistics of the density data and other assay data collected from drilled core samples in Nash Creek.

Notice that the histogram plots of both data sets portray an asymmetric distribution which is characteristic of most data sets in the earth sciences. Despite the long tail due to high density values, the overall distribution in the density measurements approximately reflects a normal distribution. Although the sampled density measurements alone do not constitute an exhaustive data set, the large data sample size and the fact that density sampling was done from surface to bottom of most boreholes (Figure 4.2a) provide sufficient characteristic information to distinguish between rocks with and without sulfide mineralization. This provides a strong basis for estimating Zn % from density values and vice versa. However, observations based on a single borehole may introduce some bias in our stochastic models. A rigid approach warrants using all available data from all boreholes in order to obtain a spatial dependence or correlation between these properties. A measure of the spatial variability is

$^1$ Each rock sample with density below 2.3gcm$^{-3}$ had undergone significant chemical changes over time such that they would easily disintegrate when immersed in water. Wrong estimates of the rock’s mass in water are thus obtained (see section 3.2.2 for details on methodology for density measurements). Erroneous density measurements (outliers) introduce some bias in the variogram analysis of the density data (see section C.3, Appendix C).
a key step towards any geostatistical study. Spatial variability is assessed through variogram analysis (Deutsch & Journel, 1998). The variogram (semivariogram)\(^1\) is an average measure of the dissimilarity between pairs of data values separated by a distance \(h\) from which spatial lengths of correlation can be obtained. It is computed (experimental variogram) as half the average square difference between the components of every data pair separated by a distance \(h\) (see Chapter 2). The experimental variogram provides \(\gamma(h)\) only at a number of lags. In order to compute semivariogram values for any possible lag, \(h\), we fitted continuous functions to the experimental variogram\(^2\). The choice for modeling the spatial variability with such a function is to ensure existence and uniqueness of solution in the kriging matrix (Deutsch & Journel, 1998; Doyen, 2007).

At Nash Creek, local experimental vertical variograms for a few boreholes were computed to gauge the variability in local vertical scale lengths. Plots of the experimental variograms for four sampled boreholes from the Nash Creek property are shown in Figure 4.3. Local heterogeneity on the Nash Creek property is evidenced by the differences in possible variogram models that could fit the computed experimental variograms from the density logs. In order to keep the analysis simple, a regional vertical scale length (range, \(a_r\)) was computed by pooling all the data in the sample data set. Figure 4.4 shows a plot of the experimental variogram values against the lag distance \((h)\). Superimposed on the experimental semivariogram values is a fit of a basic exponential model \((a_r=3m, \text{sill}=0.045, \text{nugget}=0.033; \text{the nugget value quantifies the local erratic behaviour of a stochastic process})\). Plausible explanations for the nugget effect in this case include:

\(^1\) Although Variogram = 2* semivariogram \((2\gamma(h))\), both variogram and semivariogram terms are used in this thesis to represent \(\gamma(h)\).

\(^2\) These continuous functions i.e. \(\gamma(h) = C(0) - C(h)\) are chosen on the basis that the variance of any finite linear combination of variables from a random stationary process with a covariance function \(C(h)\) must be non-negative. Hence, the resulting covariance matrix \(\begin{bmatrix} C_{ij} \end{bmatrix}, (i, j \in 1, \ldots, n)\) must be positive definite (Goovaerts, 1997, pp. 87-88).
Figure 4.3: Experimental variograms/semivariograms of density logs from a) NC0401; b) NC0507; c) NC0515; d) NC08163. See Appendix C for logs. h:- lag distance in metres.

Figure 4.4: Histogram for density values $\geq 2.3 \text{g/cm}^3$ (Top); Vertical experimental variogram/semivariogram (crosses) and fitted exponential model (bottom).

Sample data $> 2.3 $

<table>
<thead>
<tr>
<th>Sample No</th>
<th>5951</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.76</td>
</tr>
<tr>
<td>Median</td>
<td>2.73</td>
</tr>
<tr>
<td>Min</td>
<td>2.3</td>
</tr>
<tr>
<td>Max</td>
<td>4.6</td>
</tr>
<tr>
<td>Var</td>
<td>0.9538</td>
</tr>
<tr>
<td>std</td>
<td>0.232</td>
</tr>
</tbody>
</table>
a) Sampling is too coarse to capture information on short scale (<sampling interval) variations in density

b) Measurement errors e.g. reading errors associated to the resolution of the measuring scale used in the density measurements (Figure C.6, Appendix C).

c) Outliers in the density values (>3.2 gcm\(^{-3}\)) which in the present case are associated to high grade sulfides occurring over limited spatial intervals along some boreholes (Figure C.6, Appendix C).

Figure 4.4 suggests the 0.5-1m interval used in sampling the density data fails to fully capture the spatial vertical variability in Nash Creek. Depending on “man power” as well as other field work logistics, future log measurements should be sampled at much finer scale to adequately capture the small scale variability. While such fine scale detail in the heterogeneity of the rocks is important for some applications such as resource evaluation it may not have much impact for seismic imaging analysis. In the seismic exploration frequency band such a medium with fine scale heterogeneity qualifies as a quasihomogenous medium (Figure 2.4).

Other analysis to further understand the variability in density values involved assessing the effect of lithology as well as characterizing the distribution of the density values about certain threshold values. Both analyses begin by coding each variable/property value \((z_k)\) as an indicator datum \(i(x_m; z_k)\) where

\[
i(x_m; z_k) = \begin{cases} 
1 & \text{if criterion “A” is valid} \\
0 & \text{otherwise}
\end{cases}
\]

\(x_m\) represents the spatial coordinate of the data location; criterion “A” can take various forms such as \(z_k < \text{threshold value (e.g. density)}, z_k \text{ is felsic (lithology)}\). Indicator variograms can then be computed from the indicator data. Indicator variograms generally assess how frequently two values of a property separated by a distance \(h\), are located on opposite sides of the threshold value. This implies indicator statistics are insensitive to the values of the property under consideration (e.g. extreme values), since only the position of the data with respect to the threshold value is considered. In the case of lithology, the various lithologic units at Nash Creek can be subdivided into three major rock type groups: mafic, felsic, and sedimentary. Such a classification was considered to keep the variogram analysis of lithology simple. As displayed in Figure 4.5a, the respective indicator variograms for lithology have no nugget effect. Given that
the density variogram in Figure 4.4 has a nugget effect, the plots in Figure 4.5a suggest the variation observed in the density logs (along depth direction) is not controlled by lithology. Moreover, the plots show that the felsic and mafic rocks (large range and sill) have the largest influence towards the observed vertical correlation lengths in the density property. Indicator statistics based on density threshold values (Figure 4.5b) provides supplementary information on the connectivity in the density values. The main observations are that median density values and the small extreme values are better connected in space (lower nugget effect) than the large

![Figure 4.5: Experimental indicator semivariograms conditional to: a) rock type (lithology- 21 boreholes) and b) density threshold values (32 boreholes, density>=2.3g/cm^3). Indicator semivariogram for high threshold values shows erratic behaviour.](image-url)
extreme values. Based on the erratic behavior of the indicator variogram for large extreme values, it can be argued that high density zones (highly mineralized) are scattered in space (low connectivity). However, the analysis is biased by the fact that there are a limited number of measurements with extreme values. A rigorous assessment of existing lithological data would be important in either supporting or refuting the speculative interpretation deduced from the indicator statistics. Proper characterization of the indicator statistics for extreme values is critical to the choice of methodology adopted to characterize the distribution of mineralized zones and hence resource estimates. Also, notice the range of the intermediate densities compares with those derived from the mafic and felsic indicator variograms. To some extent, these results corroborate the role of the major lithologic units in the spatial variability of the densities.

The paucity of data on the horizontal plane (few boreholes) makes analysis for horizontal spatial variability more challenging. Unidirectional variograms using large tolerance angles\(^1\) were computed for several directions in the horizontal plane. In most of the cases, there weren’t enough experimental semivariogram values to do a good model fit while in other cases (e.g. angle 90\(^\circ\)) spatial variability reached the plateau at very short lag distance (\(h = 0\)). Among all the angles used, a good model fit was obtained for the horizontal variogram/semivariogram computed at azimuths 70\(^\circ\) and 150\(^\circ\) (azimuth with respect to North direction -: directional variograms). For the purpose of simplicity, the horizontal range was considered to be isotropic (\(a_x=a_y=130m\); these range values are based on GOCAD software ~ 3 x range values obtained using MATLAB). Alternatively, horizontal spatial variability can be measured by considering a secondary property that is closely related to the property of interest. Deriving variogram parameters from secondary data has been applied successfully for some oil and gas applications (Cole et al., 2003). A suitable candidate for the case of density will be gravity data. Existing regional scale or more localized-high resolution gravity data acquired during initial exploration activities on the property can be helpful in this case. Gravity data is sensitive to both lateral and vertical changes in rock density (see section C.5 for 3D gravity modeling). However, variogram

\(^{1}\) Tolerance angles are used when computing variograms in a given direction/azimuth. This is quite handy when dealing with irregularly spaced data. The tolerance angle represents angular half-window ranges about the main azimuth considered for the variogram computation. Only data pairs within this angular domain are considered for the computation of the variogram coefficient for different lags (see Figure III.2, Deutsch and Journel, 1998).
analysis from such data would be biased if overburden effects are not properly accounted for. Other aspects of gravity measurements that could introduce errors in the variogram analysis include effects of topography, sampling methods used to acquire the gravity data, and the regional gravity field. In this study, 2D surface gravity (~20m isotropic sampling in North and East directions, see Figure 4.1) is the only data available that is sensitive to lateral variations in density of the bedrock. However, the horizontal ranges obtained from gravity data in this study, have some unconstrained margin of error given that the data has not been adequately stripped of the overburden effect (density and thickness) as well other error sources listed above. At Nash Creek, there is no adequate knowledge about the 3D structure of the overburden thickness. More details on the potential effects of overburden are discussed in Appendix C (section C.5) where 3D gravity modeling as used as a quality control tool for modeled spatial distribution of rock densities. Table 4.1 summarizes the variogram fit obtained for the bouguer gravity experimental variogram. The estimated scale lengths from the bouguer gravity data are very large compared to the borehole intervals (~25-50m). It was difficult to correlate petrophysical and geochemical information between existing boreholes despite such short borehole intervals. For this reason, the kriging study of rock densities presented in section 4.3 was implemented with horizontal scale lengths that are less than 100m. Also notice the model is anisotropic with a direction of maximum range that is comparable to the case for a subset of the Zn% data. However, this observation cannot be used to reliably argue that there is a strong correlation in the variations of the measured gravity data and the distribution of the Zn mineralization on the Nash Creek property.

**Table 4.1:** Variogram parameters of exponential model fit obtained for the bouguer gravity data and Zn% (see Appendix C).

<table>
<thead>
<tr>
<th>Data/ Variogram models</th>
<th>GOCAD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direction</td>
</tr>
<tr>
<td>Bouguer Gravity (see Figure 4.1)</td>
<td>Az: 131.3</td>
</tr>
<tr>
<td></td>
<td>Dip: 0</td>
</tr>
<tr>
<td>Zn% (1979-2008)</td>
<td>Az: 128</td>
</tr>
<tr>
<td></td>
<td>Dip: 0</td>
</tr>
</tbody>
</table>

*Elevation coordinates are set to a constant value before computation of the areal (2D) variogram.
* Semirodogram (Deutsch and Journel, 1998); Az: Azimuth with respect to North direction. The azimuths displayed represent the direction of the largest scale length (a_long, see Figure C.8 in Appendix C); a_long and a_short define the anisotropy of the horizontal scale lengths.
** Covariance (Deutsch and Journel, 1998, Goovaerts, 1997).
4.3 Density Model: Estimation and Simulation

4.3.1 Case for estimation: simple kriging and ordinary kriging

Having characterized the sample population of density and assay data, the next logical step would be to quantitatively model the spatial statistics of the various populations from the available sample data over the study area. The inherent need for such a quantitative model is due to the fact that the sample population of the property data is an inexhaustive representation of the “unknown reality”. When building the model, principal objectives include:

   a) Incorporating all relevant information present in the data such as the spatial continuity of large values (e.g. high densities, metal grades) which is relevant for mining application as is the case here.
   b) Tune the model to focus on both large and small-scale features in variations of density, or geochemical data.

Most data which quantitatively characterize rock properties in the earth sciences have sparse sampling. Considering such data aspects coupled with the limited knowledge in the physical processes that control the spatial distribution of various rock properties, the estimate of the property unknown at a given location in space can then be obtained through probabilistic models. Probabilistic models provide a set of possible values with the corresponding probability of occurrence. Unlike deterministic models which rely on the physics of the phenomenon, probabilistic models rely on the data. Spatial distributions of attributes can be modeled using either deterministic or probabilistic approaches (Goovaerts, 1997, p61). The kriging method was used to estimate the density values at unsampled locations within the volumes MVA and MVB.

   a) Kriging theory

Kriging is a generalized least squares regression technique that accounts for data solely associated to the attribute/property being estimated (Goovaerts, 1997, p125). The kriging estimator is all but a variant of the basic linear regression estimator $Z' (x)$ defined as:

$$Z' (x) - m(x) = \sum_{i=1}^{n(x)} w_i (x) [Z(x_i) - m(x_i)]$$

(4.1)
Where \( w_i(x) \) is the weight assigned to datum \( Z(x_i) \) interpreted as a realization of the random variable (RV) \( Z \); \( m(x) \) and \( m(x_i) \) are the expected values of the RVs \( Z(x) \) and \( Z(x_i) \); \( i \) indexes the datum locations \( (x) \). The number of data \( n(x) \) and associated weights is location dependent. Interpreting the unknown (unsampled) values and data values as realizations of the RVs \( Z(x) \) and \( Z(x_i) \) allows one to define the estimation error as a random variable \( (Z^*(x) - Z(x)) \). Thus, the objective of the kriging process is to minimize the estimation or error variance under the constraint of unbiasedness of the estimator, that is:

\[
\sigma^2_E(x) = \text{Var}\{Z^*(x) - Z(x)\}
\]

(4.2)

is minimized such that \( E\{Z^*(x) - Z(x)\} = 0 \)

(4.3)

The above conditions (Eqns. 4.2 & 4.3) form the basis from which the weights \( w_i \) also known as “kriging weights” are determined.

Like other stochastic modeling methods, the spatial distribution of the rock property (attribute) of interest is considered to be characterized by a second order stationary random function. This attribute can be decomposed into a trend component \( m(x) \) and a residual component \( R(x) \) with the later being a stationary random function \( (E\{R(x)\} = 0; \text{Cov}\{R(x), R(x+h)\} = E\{R(x).R(x+h)\}) \). Hence the expected value of a given attribute at a location \( x \) is the value of the trend component at that location.

Examples of two kriging variants include simple kriging and ordinary kriging. While simple kriging (SK) considers the mean to be known throughout the study area, ordinary kriging (OK) on the other hand accounts for local fluctuations in the mean by using a limited domain (usually the search neighbourhood- see section C.2, Appendix C) within which the mean is stationary. In the SK scenario, the mean is known whereas in the OK case the mean is deemed unknown.

By assuming a constant mean throughout the study area in SK, equation (4.1) translates to:
\[ \hat{Z}_s^* (x) = \sum_{i=1}^{n(x)} w_{sk}^i (x)[Z(x_i) - m] + m \]  

(4.4)

Where \( m \) : constant mean,

\( w_{sk}^i (x) \): kriging weight and the error variance given in terms of the covariances as:

\[ \sigma_{sk}^2 (x) = C(0) - \sum_{i=1}^{n(x)} w_{sk}^i C(x_i - x). \]

In ordinary kriging, the mean is assumed to be a priori unknown, especially in situations where local means may vary. Though apparent, one can observe trends in local mean variation along borehole depths in the sampled density logs (e.g. Figure 4.2a). The OK estimator is unbiased by requiring that the kriging weights sum to 1 (Doyen, 2007, p52). Hence (4.4) transforms to

\[ \hat{Z}_{ok}^* (x) = \sum_{i=1}^{n(x)} w_{ok}^i (x)[Z(x_i) - \hat{m}(x)] + \hat{m}(x) \]  

(4.5)

where \( \hat{m}(x) \) is the mean which is re-estimated implicitly from the data inside the search neighbourhood. This approach means OK is more data adaptive than SK. However, using such a method will provide flawed results if the data configuration is subject to biased sampling.

b) Data application: kriged density model

Given that the distribution of the density data is slightly skewed (~ normal distribution), SK and OK were used to interpolate/estimate the density values at unsampled locations within MVA and MVB. Before implementing the kriging algorithm, the well data was scaled up to grid dimensions of the 3D modeling volume by assigning each property (e.g. density) value to the grid cell that is closest to it. In order to preserve the variogram properties of the original input data the basic grid dimensions used were 2m x 2m x 1m (MVA) and 2.2m x 2.2m x 1m (MVB) respectively. Figure 4.6 shows plots of the kriged density from the Nash Creek logs using both SK and OK methods within volumes MVA and MVB. The choice of the two regions considered (MVA and MVB) is based on the location of the boreholes with density information. In MVA, there is no major difference in both results especially along the E-W transect of boreholes.
To View

Case MVA

Case MVB

Figure 4.6: 3D density model plots for SK and OK estimates in MVA and MVB respectively. The minimum density value is 2.3 g/cm$^3$. SK considers all the data in the computation whereas OK uses data within search neighbourhoods that depend on the maximum scale length. Variogram model: $a_x = a_y = 43.3$m, $a_z = 4$m; sill = 0.045; nugget = 0.033. These scale length values are based on MATLAB software ~ 0.333 x scale length values obtained using GOCAD. Color bar: Red- 2.3 g/cm$^3$; White: >=3.2 g/cm$^3$. 
The transect closest to the EW profile of the boreholes in MVA clearly emphasizes the variability in the subsurface densities otherwise understated when using mean densities for resource estimation. Similar results are shown within the N-S region that encapsulates the borehole locations within MVB. The results show that at locations further away from the boreholes, SK tends to the mean value which is not the case for the OK results. Departure of the estimated (OK) values from the global mean at these locations is solely due to the data configuration. The data configuration within MVA results in “unrealistic” distributions of OK density estimates in regions with no existing boreholes (see South East section of MVA). Hence subsequent kriging implementations within MVA were restricted to the SK approach. Large search ellipsoids (discussion on search ellipsoids is covered in section C.2, Appendix C) were used in order to avoid the problem of sparse data within search neighbourhoods.

While kriging interpolators provide a local sense of accuracy (“best” estimate in least square sense since error variance is minimum), such measure of local accuracy may be incomplete when several other locations are considered together i.e. there is no consideration for joint accuracy (see section 4.3.2). Another shortcoming of kriging is that it is sensitive to the data configuration. An alternative approach to stochastic modeling involves using simulation algorithms. These algorithms come in many flavours (Deutsch and Journel, 1998) and are designed to honour/reproduce the statistics representative of the area under study. Sequential Gaussian Simulation (SGS) is an example of a simulation algorithm. An overview of the methodology and its application to the stochastic modeling of the Nash Creek petrophysical data is covered in the next section.

4.3.2 Case for simulation: Sequential gaussian simulation (SGS)

a) SGS theory

Kriging algorithms have a smoothing effect when estimating a property at a distant location from the well logs (Figure 4.6). This means kriging results underestimate the fluctuations in the property (continuous random variable) within the volume of study. Typically, interpolation algorithms result in underestimation of large values and overestimation of small values (Goovaerts, 1997, p370). This is a serious flaw especially when the detection of patterns in extreme values (e.g. zones of high ore grade) is of interest. Simulations are useful in this respect because they help reproduce the geological texture (spatial variability). Often, the simulated
property has implications for decision-making processes such as risk assessment for contamination areas (Goovaerts, 1997, p259), forward modeling to optimize geophysical imaging parameters, or in the assessment of available and recoverable ore at a mine site. In such applications, it is thus useful to model the spatial uncertainty of the property at unknown locations. This makes a simulation algorithm a very powerful tool as it attempts to reproduce the spatial variability through alternative, equally probable, high-resolution realizations/models. Sequential Gaussian simulation (SGS) is an example of a simulation algorithm.

SGS provides alternative global representations where precedence is given to the reproduction of the patterns of spatial continuity with a good measure for joint accuracy, i.e. (Goovaerts, 1997, p370):

a) Realizations are conditional to their data values
b) Histograms of simulated values reproduce closely the histogram of the sample data
c) Covariance models (as well as a set of indicator covariance models for various threshold values) are reproduced.

Conceptually, the SGS algorithm implemented for a given attribute such as density on a 2D/3D grid involves sequentially visiting each grid node following a random path and deriving a simulated value for density. At each step, the density value is simulated by sampling from a Gaussian conditional probability density function (PDF) with parameters (i.e. mean and variance) given respectively by a kriging estimate and kriging variance (Doyen 2007, p80):

a) Pick on a non-simulated grid node \( j \) at random
b) Compute the kriging estimate and variance (see equation (4.4) for SK case)
c) Draw a simulated value of density \( Z(x_j) \) at random from the probability distribution:

\[
p\left(Z(x_j) \mid \{Z(x_1), \ldots, Z(x_{j-1})\}\right) \propto \exp \left\{ -\frac{1}{2\sigma_{j,sk}^2} \left[ Z(x_j) - Z_{sk}^*(x_j) \right]^2 \right\}
\]

(4.6)
d) Treat simulated \( Z(x_j) \) as additional control point

e) Go back to step a) until all nodes in the 3D grid are simulated.

**Figure 4.7:** Graphical illustration of the normal score transformation of the density data (\( \geq 2.3 \text{ g/cm}^3 \)). CDF: Cumulative distribution function. Raw data (green CDF) is mapped to the normal score domain (CDF represented as red dots). Superimposed is the background is the CDF for a normal distribution.
Notice that the looping process (step (e)) in the SGS is critical to ensuring that the realizations are spatially correlated (Doyen, 2007, p81). Moreover, the influence of limitations in search neighbourhood of conditioning data was addressed by implementing the multi-grid concept: first simulate on a coarse grid using larger search neighbourhoods then the remaining nodes are simulated using a smaller search neighbourhood.

When computer hardware requirements (processor speeds and memory) are met, SGS is computationally convenient and reasonably efficient despite inherent shortcomings: (i) SGS results in a highly disordered model (Journel and Deutsch, 1993) whereby extreme values are highly disconnected; (ii) relies on multi-variate gaussianity assumption (can hardly be checked in practice); (iii) reproduction of the variogram (usually normal score variogram) as well as indicator variograms are not guaranteed.

b) Data application

Given that the connectivity between extreme values is not evidenced by available data (assay, density, lithology) from the Nash creek property, the first shortcoming (i, see paragraph above) of the SGS method was overlooked in this study. For simulation of the density attribute conditional to well logs using the SGS approach, the following steps were followed:

Step 1: The sample density data from 29 logs are first transformed in to the Normal score domain. SGS assumes the variables are Gaussian. Unfortunately, this is not the case with the distribution of the density values measured from the Nash Creek cores. The cumulative distribution function (cdf) approach was used. Figure 4.7 shows the graphical forward transform of the density data to the normal score domain.

Step 2: The appropriateness of the multigaussian (multi-point Gaussian) assumption of the density normal scores is then verified. In the present work, the normal score data was assumed to be multi-point Gaussian..

Step 3: The SGS is then performed in the normal score space using the semivariogram model of the density normal scores (Figure 4.8). Figure 4.8 shows how different data subsets result in different vertical variogram model fits. A comparison of all cases suggests an exponential model
with a slightly larger scale length will be required to have a sill value of 1. In this study, the vertical variogram model used was an exponential function with a nugget value of 0.6, a sill of 1 and a range \( (a_z) \) of 10m. For a conservative assessment of the density distribution, isotropic scale lengths \( a_x=a_y=a_z \) were used.

**Step 4:** The final steps consist in performing a back-transform of the simulated normal scores into corresponding density values.

Three conditional realizations shown in Figure 4.9 honour the density values at the well locations. These realizations approximately reproduce the sample histogram and the input semivariogram (Figure 4.10). Unlike the SK output, the conditional SGS output depicts variability that goes beyond the vicinity of the borehole locations. Such density models with spatial uncertainty can be used to assess the uncertainty in tonnage and metal concentration.
Figure 4.9: a) Sample SGS realization for density; b) Comparison of the raw density log (RD) and the scaled log at the well region (WR) within MVA at borehole B. The scaled log is also the input data for the SGS process. The differences between realizations provide a model for the uncertainty about the distribution in space of density values. SS': 2D section along the EW transect of the boreholes in MVA.
Realizations 21 and 39 each have the smallest and largest gross cell volume with values exceeding 3.2g/cm$^3$. Thus tonnage estimates from 100 realizations range from 16,789,500 tones to 17,128,260 tones. While the ultimate goal for the rock physics modeling in this study is tailored to assess implications for geophysical exploration methods, the derived density model can also be used for other applications. Section 4.4 addresses the implications of the 3D density (stochastic) model for resource evaluation.

**Figure 4.10:** a) Experimental (realization 21) and model (solid line) variograms. Histograms of density distribution for input raw data (top panel) at well locations within MVA (b) and for SGS realization 21 (c).
4.4 Implications for Resource Estimation

4.4.1 Accounting for secondary information- Joint simulation for geochemical data (Zn%)

Given that some geophysical data measurements are subject to combined effects of more than one petrophysical property, it becomes more compelling to consider multivariate geostatistical methods such as cokriging (Deutsch and Journel, 1998; Doyen, 2007) in building probabilistic earth models for integrated studies. Cokriging is a multivariate extension of the kriging algorithm and was developed by Matheron in the 1960’s. The cokriging method has been used extensively for hydrocarbon exploration, mining, geotechnical, hydrology, geophysical and environmental studies (Xu et al., 1992; Doyen, 2007; Baskan et al., 2009; Gloaguen et al., 2005; Goovaerts, 1997).

a) Cokriging

In cokriging, the variable to be predicted (e.g. assay data: Zn %, Pb %) is called the primary variable and the other variable used to improve the estimation of the primary variable is called the secondary variable (e.g. density). For simple cokriging, the cokriging estimator is designed such that the linear estimator in (equation 4.4) is supplemented by $m(x)$ continuous secondary data values (Goovaerts, 1997, Doyen, 2007):

$$Z_{*i}(x) = m_i + \sum_{i=1}^{n(x)} w_i^{sk}(x) [Z(x_i) - m_j] + \sum_{j=1}^{m(x)} v_j^{sk}(x) [N(x_j) - m_j]$$  (4.7)

As in kriging, the regression weights are determined by minimizing the mean square prediction error. However, (4.7) is seldom used since it requires modeling of three spatial covariance functions: auto-covariance of primary and secondary variables and the cross-covariance between the primary and secondary variables. When the secondary variable is densely sampled, this may lead to instability\(^1\) in the cokriging system because (Goovaerts, 1997, p235):

\[^1\] When solving for the kriging/cokriging weights, the inverse of the covariance matrix needs to be computed and multiplied by the vector of data-to-estimation-point covariances (Goovaerts, 1997, pp. 127-209). The numerical stability depends on the covariance matrix being invertible (Ababou et al., 1994).
a) the correlation between close secondary data is much greater than that between distant primary data

b) secondary data close or even collocated with unknown primary value at estimation point tend to screen the influence of secondary data that is farther away i.e. collocated secondary data have larger kriging weights compared distant secondary data.

This instability led to the development of a variant of cokriging called collocated cokriging (Xu et al., 1992) which consists of retaining only the “single” secondary datum of any given type closest to the location \( x \) being estimated. Thus (4.7) simplifies to

\[
Z^{*}_{CK}(x) = m + \sum_{i=1}^{n(x)} w_i^{sk}(x)[Z(x_i) - m_i] + q\left[ N(x) - m_j \right]
\]

(4.8)

where \( Z(x_i) \) is the primary data, \( N(x) = N(x_i) \) is the collocated secondary data; \( w_i \) and \( q \) are the cokriging weights; \( m_i \) and \( m_j \) are the constant means of the primary and secondary variables respectively; and \( n(x) \) is the number of primary data. When the respective means for the variables are considered to be known and constant throughout the study area, the collocated cokriging process is called simple collocated cokriging. The joint simulation method used in this study is based on simple collocated cokriging. Further details on the cokriging method are covered in section C.1 (Appendix C).

b) Data Application

In mineral exploration projects, drilled cores are usually assayed for information on various metal concentrates in a bid to delineate the geometry of the orebody. Assay information is usually inexhaustive, hence interpolation schemes are applied to obtain a 3D volume of the orebody. The interpolation output is in turn used to estimate the overall metal content by taking rock density information into account. Usually average density values from a random collection of core sample specific gravity measurements are used. Although the average density characterizes the bulk property of the target rocks, it however understates the uncertainty in the resource estimates. This is further complicated by the fact that one density value alone does not characterize ore bearing rocks (Figure 4.2a). In the event that there is a dense network of
drillholes in a study area for either geophysical, geotechnical, or hydrology applications, the following steps are important for building realistic multivariate stochastic rock physics models:

a) Sample multiple rock property attributes relevant to the problem at hand in all available boreholes. Most applications often sample data at defined depth intervals within the borehole. However, a robust data framework warrants sampling at least one of the key rock attributes throughout each borehole from top to bottom. A unique characteristic of such a database is that some of the attributes considered within the sampled data space are collocated.

b) For joint estimation or simulation of more than one attribute in the model space, there are numerous approaches in the literature that can be used. Either all the N variables are simulated simultaneously (Verly, 1993) or each variable can be simulated in turn according to a predefined hierarchy (Almeida and Journel, 1994), between the rock property variables.

The approach by Almeida and Journel (1994) allows for the use of the markov-type\(^1\) approximation used in collocated cokriging (Xu et al., 1992; Doyen, 2007; Goovaerts, 1997). This approximation assumes that the dependence of the secondary variable on the primary is limited to the collocated primary datum. Hence, the cross-covariances between the primary\((Z_1)\) and secondary\((Z_2)\) variables are linearly related to the autocovariances of the primary variable:

\[
C_{12}(h) = \frac{C_{12}(0)}{C_{11}(0)} C_{11}(h) \tag{4.9}
\]

An equivalent expression for equation (4.9) in terms of semivariograms is given in Appendix C. (equation C.1). The following markov-type independence relation between the primary\((Z_1)\) and secondary \((Z_2)\) variables is a sufficient condition for such an approximation to be valid:

---

\(^1\) A markov process is a stochastic process for which different states (e.g. spatial or temporal attribute/property) are mutually exclusive (“no memory”).
The predefined classification between variables can be based on the relative strength of the correlation coefficient between various combinations of variable pairs, starting with the most important or better auto-correlated variable (Table 4.2). This approach is used in the case herein whereby possible hierarchies include:

Density → Zn(Zinc) → Ag(Silver) → Pb(Lead) or Geology → Density → Ag → Zn → Pb or Geology → Ag → Zn → Density → Pb

The outline in the first hierarchy above underscores the idea that density is used as secondary information to estimate or simulate Zn%; silver concentration (Ag%) is simulated in turn with Zn% and density as secondary information and simulations for Pb% are done with Ag%, Zn% and density as secondary information. A similar idea can be adopted for the other hierarchies.

Table 4.2: Correlation coefficients between density, and assay data from Nash Creek (all data- up to 2006).

<table>
<thead>
<tr>
<th></th>
<th>density</th>
<th>Ag</th>
<th>Pb%</th>
<th>Zn%</th>
</tr>
</thead>
<tbody>
<tr>
<td>density</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>0.32</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pb%</td>
<td>0.24</td>
<td>0.49</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Zn%</td>
<td>0.36</td>
<td>0.75</td>
<td>0.49</td>
<td>1</td>
</tr>
</tbody>
</table>

In this study, the joint simulation of density and Zn% is performed. In region MVA, there exist 14 boreholes with collocated geochemical and density information at most spatial locations (Figure 4.1). In collocated cokriging, the secondary information needs to be exhaustive i.e. exist at every location within the grid or at least densely sampled at well locations and can then be presimulated conditional to the well logs. The derived conditional SGS density model can be used as secondary information to jointly simulate Zn% distribution within MVA. The general implementation steps are outlined in section C.1 (Appendix C).

Figure 4.11 shows the normal score transform of the Zn% and its associated two-point statistics. The correlation coefficient between density and Zn% used was ~ 0.3. Figure 4.12 shows a conditional SGS realization for Zn%. Just like the raw data, MVA is dominated by areas with
extremely low Zn content (~0%). A potential spin-off application from such stochastic modeling includes the use of both density and Zn% model volumes to assess the uncertainty in resource evaluation (Figure 4.13). This can be addressed by using several joint simulations of density and the existing geochemical data. The realization in Figure 4.12 has an average Zn% of 0.78. Consequently, considering an average density of ~2.7 g/cm$^3$ this gives an average tonnage for Zn% to be 12,618 tones. This value is about half of the global estimate obtained for the whole Nash Creek property ~24,443 tones (Jankovic and Moreton, 2009). Some reasons why the tonnage estimates from the study by Jankovic and Moreton (2009) differ from those in this study include: Jankovic and Moreton (2009) considered a much larger surface area; they used the ordinary kriging (OK) method; and used a constant density value (2.76 g/cm$^3$).

With the above modeling approach, it is easier to assess the uncertainty in the metal content within the study area. It can be further extended for the assessments of the uncertainty in the content for Pb as well as Ag. However, the disseminated nature of the sulfide mineralization

![Figure 4.11: a) Normal score transformation of the Zn% data; b) Experimental (obtained from the Normal score data) and model (exponential) variograms: $a_z=25m$; Nugget=0.38; Sill=1.](image)
poses challenges for the implementation of geophysical imaging techniques. This has been observed so far in Airborne EM and DC soundings where the conductivity of the overburden is a major concern. Gravity imaging on the Nash Creek property also suffers severely from overburden effects and is evidenced in some preliminary investigation conducted with the derived stochastic models described in section 4.3 (see section C.5, Appendix C). So far, there is no adequate knowledge of the overburden structure in the area that allows these effects to be removed from gravity measurements.

**Figure 4.12:** Two views showing the distribution of the Zn% data locations within MVA: a) towards east; b) towards North; c) A sample SGS realization for Zn% in MVA. A reference location (well A) is also indicated. Well A has collocated density and Zn% information. Normal score variogram model: \( a_x = a_y = 20.6m; a_z = 7.3m; \) Nugget = 0.38, Sill = 1.
Figure 4.13: Flow chart of the joint simulation process and the application of the simulation outputs towards resource evaluation.
4.5 Implications for Seismic Imaging

4.5.1 Orebody imaging depth, shape, distribution and composition

Most of the mineralized zones identified till date at Nash Creek are shallow (<300m) and the bulk of this information is gotten from petrophysical and geochemical analysis on core. With such shallow mineralization, the efficiency of seismic imaging techniques is significantly reduced as it becomes challenging to sample diffractions or reflections from any mineralized orebody at such depths.

One of the main issues to contend with is the highly disseminated nature of the sulfide distribution at Nash Creek. So far, all seismic modeling studies have been restricted to understanding the seismic response from idealistic sulfide orebodies that are usually of high grade and with well defined shapes -lens shape (Eaton, 1999; Milkereit et al., 2000; Adam et al., 2003; Bellevue, 2004; L’Heureux et al., 2009; Malehmir et al., 2009b). Considering the complex nature of the sulfide distribution at Nash Creek, the seismic modeling studies discussed herein goes beyond the analyses based on the “ideal” orebody to consider the case for a more realistic sulfide distribution as shown in the conditioned 3D stochastic density models (sections 4.2 and 4.3). The present work is structured to first address in a general fashion the seismic imaging implications based on an idealistic lens-shaped Nash Creek ore (Table 4.3). Further seismic modeling tailored accordingly for the conditional 3D density models is later evaluated in section 4.5.3. The latter analysis provides a better assessment of challenges for seismically imaging highly disseminated, low-grade sulfides like the one in Nash Creek.

Another major challenge for imaging a shallow mineralized zone (<300m depth) with seismic reflection methods is that the reflection from the shallow orebody is masked by the direct P- and S-waves (P-wave diffraction tip is at ~ < 0.12s (two-way travel time) for Vp = 5000 m/s). This effect is further discussed in the assessment of the seismic response from a sample 2D section of the 3D stochastic density models (section 4.5.3).

For an ideal lens-shaped orebody, FD modeling (Figures 4.14 and 4.15) shows that only reflections from a deeper orebody (e.g. >500m) will be adequately sampled by the surface array of receivers without significant interference from the direct waves. Although not considered in this illustration, the seismic detectability of the shallow mineralized zone can be further
complicated by overburden effects. The weathered nature of the overburden layer causes significant alteration in the seismic wave amplitude and spectral content. Constructive and destructive interference of the seismic waves reduces the coherency of the target reflections.

### Table 4.3: Average petrophysical model parameters for Nash Creek rocks.

<table>
<thead>
<tr>
<th>Region</th>
<th>Property</th>
<th>Vp [m/s]</th>
<th>Vs [m/s]</th>
<th>Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background</td>
<td></td>
<td>5000</td>
<td>Vp /1.76*</td>
<td>2700</td>
</tr>
<tr>
<td>Nash Creek orebody*</td>
<td></td>
<td>6000</td>
<td>Vp /1.76*</td>
<td>4000</td>
</tr>
<tr>
<td>Pyrite*</td>
<td></td>
<td>7999</td>
<td>5140</td>
<td>5000</td>
</tr>
</tbody>
</table>

*From Salisbury et al., 2003; † Petrophysical parameters in chapter 3

![Figure 4.14: P-wave velocity models with lens hosted in a homogenous background: a) a_{x lens}=a_{z lens}=60m; b) a_{x lens}=200, a_{z lens}=60m; c) disseminated ore inclusion where the fluctuations in the physical rock property have a correlation length of 3m.](image)

Understanding the impact of orebody shape lends itself towards a better assessment of amplitude variation with offset (AVO) patterns in the diffractions characterizing the orebody (Bohlen et al., 2003). In this study, models with spherical and lens shaped inclusions were considered. While Figures 4.14a & b emphasize the effect of shape, Figure 4.14c emphasizes the aspect of distribution whereby a lens inclusion with a stochastic distribution of the ore property is used to represent a localized disseminated mineralized zone. This shall hence be referred to as a disseminated inclusion/orebody. The effect of composition was evaluated by using the lens
properties of a sulfide end member (pyrite). Table 4.3 summarizes the petrophysical parameters that characterize the lens and background sections in the respective models.

Figure 4.15: Scattered P- and S-wavefields generated from a spherical (a) and lens (b and c) inclusion: the orebody in a) and b) have properties of the Nash Creek ore, and the inclusion in c) has properties of pyrite. The (x, z-depth) coordinate for the source is (600, 10); T: time from t=0s.
Figure 4.15 shows snapshots of the scattered wavefields (P- and S-waves) for the spherical inclusion and lens inclusion. While the distribution of the scattered wavefields is symmetric about the spherical ore inclusion, that for the lens orebody is assymetrical. In the latter, the bulk of the P- and SP- diffractions (based on strength of wave amplitude) from the orebody are backscattered \(\text{(backscattered wavefields represent wavefields located in the region above the central horizontal axis (1000m) of the orebody)}\). This observation concurs with initial works of authors like Wu (1989) and Eaton (1999). The spherical ore inclusion has the weakest backscattered wavefield suggesting the use of surface seismic methods will be less effective in probing orebodies of such shape.

Moreover, amplitudes of the backscattered SP-diffractions are characterized by phase reversals. Phase reversals can also be identified in the S and PS diffractions. However, the degree of phase reversals especially in the backscattered wavefields is more prominent in the SS diffractions. About the transmitted waves, notice that a significant portion of the forward scattered diffractions from the orebody are masked by the transmitted direct waves. The most relevant indicator of the presence of the orebody inclusion can be identified in the local travel time deformation of wavefronts for the direct P and S- waves. This is due to the high velocities of the orebody.

The lens inclusion with characteristics of a pyrite orebody qualifies as a velocity and impedance type scatterer (Wu, 1989; see equations A.19 and A.20 in Appendix A). Such an orebody generates a considerable amount of secondary events (due to reverberations within the lens) that follow the transmitted direct waves within short periods. This can result in to interference that alters the spectral content of the primary waves.

The complex nature of the resulting scattered wavefields suggests different acquisition geometries will capture varying aspects of the orebody response. A surface acquisition geometry with receiver locations ranging from 150m to 1650m (2m depth) will sufficiently capture the backscattered waves from the orebody (Figure 4.16). While all four models show subtle differences in their respective diffraction hyperbolas, the major discrepancy is in the AVO patterns. In contrast to expectations, reflection amplitudes from the disseminated lens inclusion (Figure 4.16d) are much stronger than those from the model with a homogenous spherical inclusion (Figure 4.16a). This is particularly the case for the converted
Figure 4.16: Vertical and Horizontal component wavefield records from surface acquisition geometry for models: (a) Nash creek spherical inclusion; (b) Nash creek lens inclusion; (c) Pyrite lens inclusion and (d) disseminated orebody (lens shape). Source frequency is 100Hz. The seismic wavelength is sufficient to resolve the top (1) and bottom (2) interfaces of the ore inclusion. DP: Direct P-wave; DS: Direct S-wave. The arrows identify scattered waves generated by the ore: black: PP (P-wave reflection) ; light blue: SP converted wave; green: SS (shear wave reflection); red: PS converted waves.
Figure 4.17: Vertical and Horizontal component wavefield records from VSP (1200m) acquisition geometry for models: (a) Nash Creek spherical inclusion; (b) Nash Creek lens inclusion. Transmitted wavefield record for a horizontal geophone spread (at 1200m depth) for models: (a) Nash Creek spherical inclusion; (c) Nash Creek lens inclusion; (d) Pyrite lens inclusion. Source frequency is 100Hz. The seismic wavelength is sufficient to resolve the diffractions from top (1) and bottom (2) interfaces of the ore inclusion. DP: Direct P-wave; DS: Direct S-wave. The arrows identify scattered waves generated by the ore: black: PP (P-wave reflection); light blue: SP converted wave; green: SS (shear wave reflection); red: PS converted waves. PPP-downgoing P-wave generated by the surface reflection of the PP diffraction; PPS-downgoing converted wave generated by the surface reflection of the PP diffraction.
waves occurring at ~0.54 s. This suggests the combined effect of shape and local heterogeneity within the ore have amplitude enhancing effects. Due to the high impedance contrast of the pyrite orebody with respect to the background, the diffractions from the pyrite inclusion are relatively stronger than those from other inclusions. The respective wavefield (Vertical-V and Horizontal-H) component data provide complementary information about the target response: there are differences in the characteristics of converted (PS, SP) waves. The Horizontal components contain the bulk of the S-wave energy whereas the vertical component records contain the bulk of the P-waves. Notice that there is an overlap between the PS and SP ore diffractions especially in the range (1000m-1500m) for the surface record. It is more difficult to distinguish these events when the scattered wavefields are affected by noise effects (e.g. scattering due to background heterogeneity). Unlike the surface acquisition geometry, a VSP setup located at 1200m is a viable option for distinguishing these converted waves as these are characterized by different dips (Figure 4.17a & b). The VSP geometry records both the back scattered (upgoing waves) and forward scattered diffractions (diffracted waves located below the central horizontal axis of the orebody) from the ore. The wavefield amplitude variation with depth is such that the amplitude strength of the upgoing orebody diffractions is greater than those for the downgoing diffractions. The orebody diffractions present in both the surface and VSP records resolve both the top and bottom interfaces of the ore inclusion. However, only the SS diffraction from the shallow interface of the ore is significantly recorded in the VSP setting.

A horizontal geophone spread (at 1200m depth) equally captures forward scattered waves from the orebody. Besides local travel time changes in the primary arrivals for P and S-wave respectively, the presence of the sulfide lens can be inferred from secondary wavefields arriving within short periods of the primary events (Figure 4.17 c & d). Some of these secondary wavefields (scattered from lens) do interfere with primary waves especially for the direct P-waves (Figure 4.17d) and alter the direct P-wave amplitude and spectrum as a consequence. Rudimentary information about the location and shape of the orebody can be readily inferred from the trend of the absolute amplitude with offset (Figure 4.18). The results of the amplitude trend in Figure 4.18 have no corrections for geometric spreading.
4.5.2 Effect of background heterogeneity

The petrophysical analysis from core and borehole logs corroborate that rock properties (Vp, Vs, and density- chapter 3) of the felsic, mafic and sedimentary host rock units and the sulfide-bearing units at Nash Creek are characterized by more than one value. This motivates an understanding of how this complex setting affects the potential for using seismic techniques to probe for deep mineralized zones. This section incorporates knowledge of the heterogeneity in Nash Creek rock properties to evaluate its effect towards seismic imaging of target structures. Finite difference modeling methods (Bohlen, 2002) were used.

a) Heterogeneous petrophysical models

The heterogeneous background models were constructed following the approach by (Goff and Jordan, 1988- see Appendix A). The average petrophysical parameters (Table 4.3) characterizing the background and the orebody are based on the core studies, and borehole logs shown in Chapter 3 (e.g. Nafe-Drake plot- Figure 3.3). Given the absence of velocity logs from Nash Creek, the fluctuations in the velocity models (Figure 4.19) are considered to be normally

![Image](image-url)
distributed about the respective mean values (P- and S- wave velocities) such that the strength of perturbation (standard deviation) is \( \sim 4.5\% \). While the Vp and Vs models are correlated, these in turn are uncorrelated to the density model as a different random seed generator (in Matlab) is used when building the stochastic model (based on Nafe-Drake plot- Figure 3.3). The available density database was also considered in the process: an initial 2D simulation in the Normal score domain is mapped back to the raw data domain by using the cumulative distribution function for density values between in the range \([2.3 \, 3.0]\ \text{g/cm}^3\). This range is considered to adequately represent most of the felsic and mafic host rocks at Nash creek (Appendix C: Figure C.20). The derived stochastic models are not conditioned by well locations and their corresponding information. However, conditioning is mostly based on the two-point statistic analysis (variograms) obtained especially from the large density database. The von Kármán parametric model was used in place of the exponential model and a constant hurst number \((\nu=0.5)\) maintained for all models. The nugget effect was ignored in this case \((\text{nugget}=0)\).

**b) Background effect**

Acquisition geometries considered capture both aspects of the backscattered (reflected) and forward scattered (transmitted) waves: Surface (2m depth) and deep (1200m depth) horizontal receiver spreads as well as a VSP geometry. For a source with a dominant frequency of 100 Hz, the background heterogeneity in the respective models can be classified in two scattering regimes: large angle (Figure 4.19a, \( \kappa a = 0.75 \sim 1 \)); small angle regime (Figure 4.19 b, c and d, \( \kappa a > 7 \)). The lens can also be classified as being in the small angle scattering regime \((\kappa a >10)\). The background heterogeneity for the models considered affect the backscattered (surface shot gather) energy to varied degrees. The wave attributes that are significantly affected by background scattering are the wave amplitudes and coherence of the reflected energy (Figure 4.20). While the reflected P-waves \((\sim 0.4s)\) recorded in the vertical component can be reasonably identified in the respective models, the lateral continuity of this wavefront increases with increase in horizontal scale length. A similar pattern is depicted by PS, and SS diffractions in the horizontal component data. The detectability of the sulfide target diffractions is further compromised by the strong amplitudes of the coda.
Figure 4.19: Petrophysical models illustrating different scales of the background heterogeneity. The lens inclusion has dimensions: $a_{x}=30\,\text{m}$, $a_{z}=30\,\text{m}$. For a maximum source frequency of 200Hz the background heterogeneity can be classified in the following regimes: $k_{a_{z}}=0.75$ (a) and $k_{a_{z}}=7.5$ (b,c and d). The lens inclusion on the other hand has $k_{a_{z}}=12.5$. $k$- wavenumber.
Figure 4.20: Vertical and Horizontal component wavefield records from surface acquisition geometry for models (a) to (d) in Figure 4.19 above. Source frequency is 100Hz. The arrows identify scattered waves generated by the ore: black: PP (P wave reflection); light blue: SP converted wave; green: SS (shear wave reflection); red: PS converted waves.
On the other hand, the amplitudes and coherency of SP waves (~0.6) for the same component are more severely affected especially in the isotropic model with scale length of 3m (Figure 4.20a) and the layered model (Figure 4.20d): these waves are significantly masked by the P- and S-wave coda. Moreover, the scattering in the background diminishes the ability to resolve diffractions from the top and bottom interfaces of the orebody. Also notice the transparency of various wavefields is dependent on the recording component: the PP-waves are transparent to the horizontal component recording for all heterogeneous models and vice versa for the SS-waves.

The VSP recording aperture is sufficiently large to capture both backscattered (reflected) and forward scattered diffractions from the orebody. For the present case, backscattered energy applies to seismic waves recorded by the receivers located above the depth level of the lens (1000m). Unlike the surface shot gather for the isotropic model in (Figure 4.20a, a_x=a_z=3m), the VSP record for the same model shows no hint of the back scattered diffractions from the lens (Figure 4.21a). The seismic detectability (receivers <1000m) of the P-, S- and converted (upgoing) waves generated by the ore improves as the lateral scale length changes from a_x=30m to a_x=600m Figure 4.21. For receivers at greater depths (>1000m), the transmitted seismic waves bear no obvious signature that is indicative of the orebody. A similar observation can be inferred from the transmitted wavefield recorded by the horizontal spread of receivers at 1200m depth (Figure 4.22). No significant traveltime fluctuations are identified from the direct P-waves. Moreover, any waveform distortions in the first breaks owing to the scattering from the ore are masked by scattering processes associated to the background heterogeneity (Figure 4.19). Figure 4.19 further suggests that a 1200m (~24\lambda_0) propagation distance within a heterogeneous medium results in amplitude fluctuations that is sufficient to mask geometric spreading effects. This is in agreement with similar observations obtained for viscoacoustic plane wave propagation in random media (Kneib and Shapiro, 1995). It further highlights the challenge of using AVO as a diagnostic tool for interpretation of seismic waves in heterogeneous media.

Given that constructive and destructive interference processes occur as part of the seismic scattering process, an alternate approach for probing the orebody signature from the direct waves is to evaluate the spectral content of the respective component data. The ultimate goal is to identify any resonant frequencies caused by scattering.
Figure 4.21: Vertical and Horizontal component wavefield records from VSP(1200m) acquisition geometry for models (a) to (d) in figure 4.19 above. Source frequency is 100Hz. The arrows identify scattered waves generated by the ore: black: PP( P wave reflection); light blue: SP converted wave; green- SS (shear wave reflection); red: PS converted waves. The (x, z-depth) coordinate for the source is (600, 10).
However, the success of such a method is limited by criteria such as the impedance contrast (Reflection coefficient $>= 0.3$) in the medium: identifying resonant frequencies with respect to the “known” source signal is difficult when the reflection coefficient is $< 0.3$ (Appendix F).
Another limitation stems from the high spectral amplitudes from the vertical (V) component relative to that from the horizontal (H) component. This high amplitude ratio causes difficulties in the identification of any subtle frequency amplifications from the spectral ratio analysis of the horizontal and the vertical component (H/V) data. This aspect is further discussed in Appendix F.

4.5.3 Seismic imaging using the Nash Creek density model

Here, special concern is given to the case where the sulfide mineralization does not exist as a regular-shaped orebody (“ideal orebody”). As already suggested from variogram log analysis, the sulfide-rich zones at Nash Creek are highly disconnected in space. Moreover, the complex nature of the sulfides is enhanced by the wide range of density values (> 3.0 g/cm$^3$ - high grade) that characterize them. For a reasonable analysis, seismic wave scattering is evaluated for a sample conditional SGS density model derived in section 4.3. A 2D density section from the E-W transect of the boreholes in MVA (SS’, Figure 4.9) is ideal in this case as it is a better representation of the reality due to the dense network of boreholes with available density information (Figure 4.23). Constant Vp and Vs models with Vp/Vs=1.76 were used.

![Figure 4.23: 2D section (facing North) from sgs realization 39. Also shown are some existing boreholes along this section.](image-url)
Figure 4.24: P- and S-wavefields propagating in the homogenous (a, b, e, f) and the conditional heterogeneous (c, d, g, h) models. DP: Direct P-wave; DS: Direct S-wave, Pc & Sc: P- and S-wave coda. Notice the S-waves form the bulk of the coda. The x-coordinate of the surface source is 100m.

Seismic wave scatterers in this case can be conveniently classified as density scatterers:

\[
\frac{\delta V_p}{V_{p\text{mean}}} = \frac{\delta V_s}{V_{s\text{mean}}} = 0, \quad \frac{\delta \rho_{\text{rms}}}{\rho_{\text{mean}}} = 0.09
\]

For comparison on the characteristics of the scattered wavefields, Figure 4.24 shows snapshots of the full wavefield at two time steps for waves propagating in a homogenous and in the heterogeneous (Figure 4.23) medium. The parameters for the homogenous background model are summarized in Table 4.3. Figure 4.25 shows the V- and H- component records from the
respective models. For clarity, the plots have wider amplitude dynamic ranges due to the proximity of the receivers to the source (Source dominant frequency= 100Hz). The intrinsic difference between the homogenous and the conditional heterogeneous models is in the P- and S-wave coda generated as result of the scattering process (Figure 4.24). The total scattered field recorded by the surface receiver spread as shown in Figure 4.25c is obtained from the difference between the data in Figures 4.25a & b. Despite the presence of backscattered of P- and S-waves

Figure 4.25: Vertical and Horizontal Component surface records: a) Homogenous case ; b) Conditional heterogeneous model (Figure 4.23) ; c) Difference between (a) and (b). The coda comprises both P- and S-waves. NA: Numerical noise due to boundary conditions.
(coda), it is impossible to unambiguously identify diffractions associated to sulfide rich zones (high densities). This is in sharp contrast to the cases discussed earlier for an ideal orebody (regular lens shape). Another difficulty in identifying target diffractions from the shallow targets is caused by the presence of direct P-waves and S-waves. These direct waves have large amplitudes such that they mask the diffractions from high grade (very dense) zones. The modeling results clearly emphasize the role of sulfide distribution in the efficiency of seismic imaging at the Nash Creek property. Unlike the case for a lens-shaped orebody (“ideal orebody), the highly disseminated nature of the sulfides as depicted in the conditional density models will not be adequately imaged with seismic methods.

4.6 Summary

The present study demonstrates the application of geostatistical methods to build a 3D earth model that is based on the integration of petrophysical, geochemical and borehole geophysics information. The framework for using geostatistics in this study is primarily supported by the presence of a relatively dense spatial sampling of the study area with various borehole information which is otherwise absent or sparsely available in most exploration projects. Conditional simulation based on collocated cokriging can be used to further refine the 3D model by accommodating the spatial correlation between various rock property variables according to some predefined hierarchy. This geostatistical approach was implemented to locally characterize the 3D density and Zn% distribution of the rock mass of a shallow base metal deposit at Nash Creek. Variogram analysis of the density logs support that the high grade zones are highly disconnected and that the fluctuations in density values exist well below sampling intervals of 0.5m. These stochastic models are useful in assessing the uncertainty in resource evaluation as well as implications for various geophysical imaging methods such as seismology and gravity.

The use of seismic methods at Nash Creek is better suited for imaging deep targets. However, modeling results using a homogenous sulfide lens (“ideal orebody”) support that the detectability of the deep ore is severely undermined by seismic scattering due to background heterogeneity. Considering a source frequency of 100Hz, the orebody response will be transparent to both surface and VSP geometries provided the background heterogeneity is in the large angle scattering regime i.e. the variation in the rock property is isotropic with a correlation length of 3m. Hence, imaging of zones with disseminated sulfide mineralization will be extremely difficult
as the response from the latter is much weaker than those from the high-grade sulfide zones at Nash Creek. The difficulty in using seismic methods to image the sulfide rich zones is more severe if the disseminated sulfide has characteristic distributions similar to that depicted in the conditional 3D density volume. In this circumstance, the inability to unambiguously image the target zones is depth independent. The conditional stochastic density model thus suggests that sulfide distribution at Nash Creek is the primary factor that controls the seismic detectability of ore targets.

The seismic modeling results also highlight the advantage of using multicomponent data for imaging in heterogeneous environment. The vertical and horizontal component records are preferentially sensitive to P- and S-waves respectively. Adopting processing technology that integrates P- and S-wave processing will add value to the final stacked section. This study also shows that for a horizontally oriented orebody, the VSP technique is better suited to help in the full separation and processing of the converted waves. Analysis of the direct P-waves for the orebody response equally holds promise. However, results obtained so far support that seismic scattering owing to background heterogeneity can mask the seismic signature of the ore inclusion. Modeling results of this nature can be incorporated in decisions relevant for seismic acquisition design and source parameters to help boost the seismic resolution of deep target features at Nash Creek.

Owing to a poor knowledge of the overburden thickness, the gravity modeling results discussed in appendix C underscore the importance of adequately correcting overburden effects when performing gravity-based interpretation at the Nash Creek property.
Chapter 5
Heterogeneity and Implications for Seismic Imaging: Thompson Case Study

5.1 Introduction

In this chapter a similar modeling principle as that used in Chapter 4 is adopted. Unlike Nash Creek where petrophysical data suggest the sulfides are highly disseminated, evidence from comprehensive geologic sections in the Thompson mine (Figure 3.6) show the sulfide orebody has a well defined shape (dipping lens). In this chapter, the comprehensive geologic section forms the framework upon which petrophysical model building is based. In the case herein, petrophysical model building focuses on incorporating existing geologic models, elastic property measurements from a suite of core samples (section 3.3) and borehole logs towards building velocity and density models. Examples of these petrophysical models are shown in Figure 3.8.

These petrophysical models are used in turn to assess the applicability of seismic imaging methods at the Thompson mine. The primary objective of this seismic modeling study is two-fold: investigate the seismic detectability of

- the orebody
- the associated complex structures in the 1D zone of the Thompson mine.

The objectives of this seismic feasibility study are outlined in various sections of this chapter through considerations that range primarily from seismic acquisition design, and data processing. Emphasis is also stressed on the potential pitfalls due to heterogeneities observed both at the stratigraphic/lithologic and log scales. For the first time, a full 2D seismic acquisition and processing experiment is used to demonstrate the direct seismic detectability of the massive sulfides within the complex geology package that characterizes the mine. It is also observed that the dipping orebody generates characteristic PS converted and S-waves which can be processed with according P and S-wave separation algorithms in a bid to improve the imaging of the orebody. On the other hand, modeled effects of the observed heterogeneities on seismic wave propagation suggest these inhomogeneities can locally undermine the seismic diffractions that characterize the orebody.
5.2 Inferring Heterogeneity from Borehole Logs

Borehole logs hold primary information on how rock physical properties vary spatially. Variability in rock physical properties pose a problem for imaging especially in hardrock terrane where variability occurs over short distances. This potentially undermines interpretations in generated 2D/3D seismic sections as reflections tend to be discontinuous in hardrock terrane (Figure 5.1, Milkereit et al., 2000; Adam et al., 2003; Malehmir et al., 2009a&b). Borehole logs (vertical and/or horizontal wells) can be used to characterized medium heterogeneity. This approach is been used widely for stochastic modeling of crustal heterogeneity (Holliger, 1996; Sato and Fehler, 1998; L’ Heureux et al., 2009). Physical properties such as the P-wave velocities from a sonic log are assumed to be comprised of a deterministic component and a fluctuating component. As mentioned in Chapter 2, the fluctuating component is assumed to be random and stationary. Thus, extracting parameters that characterize the spatial variability of the physical properties suffices fitting parametric functions such as the von Kármán function (Goff and Jordan, 1988), or exponential function (Deutsch and Journel, 1998) to the autocorrelation (covariance) or variogram computed from the fluctuating component of the sampled data.

![Figure 5.1: Unmigrated seismic data for 2D line (BB', see Figure 3.5) around the Birchtree Mine in the TNB (modified after White et al., 1997).](image)

Five Vp and Vs logs from Thompson were used to estimate the vertical scale length \((a_z)\), the Hurst number \((v)\) and variance (sill). For quality information on the scale lengths, both the von Kármán and exponential function model fits were used for the experimental autocorrelation and
variograms computed from the respective logs. MATLAB was used to implement the non-linear fitting process of the parametric functions. Figures 5.2 shows the respective autocorrelation (ACF) and variogram model fits to the experimental data for Vp and Vs logs from sample boreholes in the Thompson area. The summary of all parameter estimates for these analytic functions is found in Tables D.1 and D.2 in Appendix D. A close independent assessment of results from the two approaches reveals that the scale lengths for both P- and S- wave velocities are comparable.

Figure 5.2: Raw logs and von Kármán model fits (red dashed line) to the ACF of the residuals (perturbation component) for Vp-BH116327 (top row) and Vs-BH116327 (middle row). The deterministic trend is represented by the polynomial fit of order $n \in \mathbb{Z}_+$. Bottom row-Exponential model fit to the variogram for both Vp and Vs logs from BH116327. Variogram analysis was done on the raw logs since results were insensitive to the removal of the deterministic trend. A log that samples independent geologic units in a continuous fashion does not exist.
Both the von Kármán and variogram models suggest that scale lengths for the medium velocities (both P and S) range between 2-3m on average. The average Hurst number estimate for S-waves (0.56) is lower than the estimate for P-waves (0.77). Also, the nugget effect from the variogram analysis is approximately zero. Although averaging effects due to the sampling set up of the logging tool has not been accounted, the zero nugget effect suggests the variability in the velocities logs is reasonably representative of the in situ conditions.

5.3 Seismic Modeling Studies

For this study, a finite difference (FD) viscoelastic algorithm (Bohlen, 2002) was used to model seismic wave propagation. Seismic reflections are generally controlled by the seismic impedance contrast existing between geologic units. In the Thompson 2D model sections, the impedance contrast and hence the reflection coefficients, especially from the ore body, is controlled by density (Figure 3.7 & 3.8). Performing seismic modeling in a hardrock terrane such as the complex geologic setting in Thompson provides useful information about the seismic response of the orebody amidst effects from medium heterogeneity, target geometry, and attenuation. In a region like Thompson, where the distribution of mineralization has complex structural deformations (Figure 3.6), the results from the modeling exercise lends itself to the design of acquisition geometries for adequately imaging and identifying mineralized zones of interest.

The present study focuses on the seismic response of the orebody in terms of the reflected and transmitted energy and also assesses how to effectively capture and process the seismic wavefields scattered from the target. Also, the influence of the heterogeneity due to inclusions within the archean gneiss (e.g. AMPT- see Figure 3.6) and inhomogeneities derived from well logs (BH11632300, BH1163290, BH1163330, BH1163100) is considered. Both explosive and plane wave sources were used. The sources used had dominant frequencies of ~50-110Hz which fall within the range of the exploration seismic frequency band. The two main acquisition models considered include: surface (2km receiver spread spanning positions x=2000m to x=4000m with receiver spacing of 8m) and Vertical Seismic Profiling (VSP) geometries. For the surface acquisition, shot and receiver depths were 10 m and 2m respectively. The seismic wave propagation is elastic (Quality factor \(Q\) was set to be very large (> 500), i.e. no attenuation).
5.3.1 2D- Plane wave modeling of orebody response

Studies by Bohlen et al. (2003) demonstrate that the shape and composition of an orebody hosted in a crystalline environment can generate complex scattered wavefields that have pronounced amplitude variation with offset (AVO) trends. Mikkereit et al. (2000) also report direct observations of this phenomenon from 3D seismic data. Following this line of thought, an analogue model of the orebody in Figure 3.6a was used in this investigation. The analogue model consists of a dipping (50°) lens-shaped (thickness of 44 m) massive sulfide inclusion hosted in a homogenous background with petrophysical parameters that are characteristics of the mean values obtained for the archean gneiss basement rocks (AG) (Figure 5.4-top panel). Comparing responses from these respective models provides a better understanding of the seismic response of the true orebody especially the scattered waves generated due the orebody shape. The plane wave response approximates that of an unmigrated seismic stacked section.

The seismic results show the response from the dipping lens is mostly in the down dip direction (Figure 5.3a) with the amplitude strength dominated by shear wave events (PS converted waves). Thus, reflected/diffracted P-wave and converted energy (e.g. PS conversions from ore = “finger print”) may or may not make it to the surface. Note that PS converted waves generated by the sulfide orebody are qualified as the “finger print” since PS-waves can also generated by the ultramafic (UM, Figure 3.6) inclusions in the archean basement rock. The latter also occurs as lens shaped units with identical dips as the orebody. Some of the scattered energy may only be picked up at the surface at very large offsets. On the other hand, the seismic signature from the geologically defined orebody is quite strong. Most of the diffractions from the orebody can be recorded at the surface. Unlike the lens model example, Figures 5.3b and 5.3c show that strong P-wave reflections (“foot print”) are generated from the orebody in Figure 3a&b). These strong diffractions stem from the thickest parts of the ore body as well as from locations where the ore distribution is controlled by existing complex structures. Also present are strong PS-wave conversions (“finger print”). A hint of the orientation of the orebody (dip) can also be inferred from the relative offset between the diffraction hyperbolas for all three ore models. Despite the strong diffractions observed in the recorded data, the bulk of the initial wavefield incident from the source is transmitted or forward scattered.
5.3.2 2D- Explosive wave modeling of orebody response

Although the plane wave results may serve as a proxy for the unmigrated stacked common midpoint (CMP) section, it is rather limited as it does not account for geometric spreading effects. Often, seismic data are collected using point sources (e.g. explosives) at relative offsets from receivers and then processed to obtain a migrated common- midpoint (zero offset) seismic section. Although the relative amplitude of the diffracted wavefield is much lower than for the plane wave case, identical trends for strong diffractions and wave conversion in the down dip direction are also observed for both lens and the ore model in Figure 3.6a (Figure 5.4). Scattered wavefield from the ore in Figure 3.6a is more complex due to diffractions of S-waves generated at the source as a result of the free surface boundary condition. It can be inferred from Figure 5.4 that a 3-component VSP acquisition geometry with the receiver line positioned to the left of the
oredody (e.g. 2100m) will be more efficient than a surface spread acquisition to capture most of the strong diffracted energy. Although the present modeling is 2D, the observed trend of the generated backscattered waves suggests that multicomponent recording is required to fully characterize the diffracted fields from the orebody. In addition to the advantage of limited influence from overburden effects, placing the VSP receivers close to the diffraction source helps to mitigate any amplitude attenuation effects that will be otherwise incurred if using a long surface receiver spread.

5.3.3 2D Surface seismic experiment

Based on observations from the single shot modeling (Figure 5.4), a full 2D synthetic modeling with multiple shots was done to evaluate the efficiency of the surface seismic reflection method in imaging the orebody and existing complex structure(s). For the purpose of simplicity, the model used comprised of the archean gneiss (AG), the setting formation (SF) and the orebody inclusion.

Figure 5.4: Snapshot comparing the scattered P- and S- wavefields for the ore inclusion in the Lens model (top panel) and in the model shown in Figure 3.6a respectively. DP: direct P-wave, S: S-wave; PS: P-to-S converted wave; SP: S-to- P converted wave; PP: Reflected P-wave.
a) Acquisition parameters

Owing to the fact that some of the energy diffracted from the orebody (Figure 5.3) is in the downdip direction, the shots and receivers were positioned accordingly to capture this energy. The shot and receiver positions range from x=2000m to x=4000m, with spatial sampling intervals of 40m and 8m respectively. Both shot and receiver lines were buried at a 10m depth below the model surface. Hence, the combined setup gives a total of 51 shots and 251 traces per shot. Figure 5.5 shows a sample shot gather from the synthetic data set. The explosive source is a ricker wavelet with a dominant frequency of 100Hz.

![Figure 5.5: Left panel-Raw shot gather for shot at 2920m; Right panel- Cleaned shot gather obtained by subtracting the background seismic response for model with AG unit only.](image)

b) Data processing and results

The 2D inline distribution of the shots and receivers results in a CMP bin size of 4m with maximum fold of 51 traces at the center of the acquisition line. Standard seismic processing routines commonly used for imaging hydrocarbon reservoirs were applied after muting/removing the direct P- and S-wave events in the data (Figure 5.6). Details of the processing steps and corresponding parameters are summarized in Table 5.1.
Table 5.1: Processing steps applied to synthetic 2D data set using VISTA 9.0 processing software.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Load segy data and define geometry parameters in trace headers</td>
</tr>
<tr>
<td>2.</td>
<td>Mute direct arrivals (P- and S-waves) by subtracting response from homogenous background model (AG)</td>
</tr>
<tr>
<td>3.</td>
<td>Interactive velocity analysis/Normal moveout (NMO) correction: 25% stretch mute</td>
</tr>
<tr>
<td>5.</td>
<td>CMP Sorting</td>
</tr>
<tr>
<td>6.</td>
<td>Stacking</td>
</tr>
<tr>
<td>7.</td>
<td>Amplitude Scaling (Mean)</td>
</tr>
<tr>
<td>8.</td>
<td>FX prediction filtering (time gate window: 0.2-0.6s)</td>
</tr>
<tr>
<td>9.</td>
<td>2D FK Migration</td>
</tr>
</tbody>
</table>

Given that this is a controlled experiment for elastic wave propagation in a complex geologic model, other sophisticated processing routines, often applied to real data, such as air blast muting and static corrections were not necessary. Although a dip moveout correction (DMO, Yilmaz 1989) is more appropriate for processing the diffractions from the orebody, consistent results were still obtained using NMO corrections. The choice for an interactive velocity analysis was based on the fact that the preprocessed gathers (e.g. shot/CMP) contained both P-, PS and SP

Figure 5.6: Shot gathers of background subtracted seismic response for model with AG, SF and ore (vertical component).
(converted) - wave events each distinguishable from the relative dips of the diffractions in the downdip direction (Figure 5.6). This of course translates to different velocity requirements for the different seismic events when performing constant velocity stack analysis (CVS). The CVS analysis used two velocity ranges: the first is from 1000 to 4000 m/s with an increment of 250 m/s and the second is from 4000 m/s to 6000 m/s with an increment of 100 m/s. The results in the final depth-converted brute stacked 2D image (Figure 5.7) indicate that the orebody is seismically characterized by a strong diffraction package consisting of varying amplitudes. Hence, this corroborates the potential for the application of surface seismic methods for successfully imaging the target (orebody). Reflections from the AG/SF contact on the contrary are very weak (due to weak impedance contrast). Besides using the peak of the diffraction hyperbola as an indicator for the location and orientation of the orebody, the amplitude patterns of these diffractions suggests strong ties with the shape (morphology) of the orebody whereby the strongest amplitudes are directly correlated with the thickest parts of the orebody as well as the complexly deformed section of the orebody. Given that the orebody distribution in Thompson is controlled by the tectonic configuration in place, such strong diffraction patterns hold the potential as seismic markers for a 3D lateral delineation of the target structure with 3D seismic data.

**Figure 5.7:** Brute unmigrated CMP stack (vertical component; No AGC applied).

**Figure 5.8:** Migrated section using a constant P-wave background velocity of 5933 m/s.
The diffraction pattern generated from the deformed region can be explained not only by the complex geometry of the contact between the various geologic units, but also by considering the resolving power of the seismic waves in terms of the Fresnel zone. The threshold resolving power for such a target at ~1km depth (~0.4s;background velocity- 5933m/s) is limited by the frequency content of the source signal (Table 5.2). In Thompson, the lateral (horizontal) extents of the region of complex deformation (~50-200m, Figure 3.6a) as well as the target ore fall within the large angle scattering regime. Hence, the full structure of the deformed region cannot be resolved if considering a seismic frequency band of 20-200Hz. The structure of the AG/SF contact in the deformed region also contributes towards the observed diffraction package although in relatively small amounts.

Table 5.2: Limit of seperability for a reflection based on Fresnel Radius; \( f_{\text{max}} = 2 \times \text{Dominant frequency} \)

<table>
<thead>
<tr>
<th>Dominant Frequency( (f_d) )</th>
<th>Horizontal Resolution (m)</th>
<th>Vertical Resolution (m) - ( \lambda_{\omega}/4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>179</td>
<td>13.5</td>
</tr>
<tr>
<td>70</td>
<td>225</td>
<td>21</td>
</tr>
<tr>
<td>50</td>
<td>267</td>
<td>30</td>
</tr>
<tr>
<td>20</td>
<td>426</td>
<td>74</td>
</tr>
</tbody>
</table>

\( \lambda_{\omega} \): Wavelength

Another useful tool for seismic interpretation of subsurface structures is seismic wave migration. With available knowledge of the accurate velocity information, migration helps to position seismically imaged features at their right location at depth and at the right scale (Yilmaz, 1989). Hence, migration is robust for imaging dipping structures. On the other hand, migration causes diffraction-like features to collapse to local features. Image results from the implementation of a 2D FK-Migration on the stacked section is shown in Figure 5.8. Migration was implemented considering a constant P-wave background velocity. Results (Figure 5.9) from four sample velocity models underscore the importance of accurate velocity information towards efficient imaging of the orebody. The true geologic model used is superimposed for quality control (QC) purposes. Notice the migration images the thickest part of the orebody as well as the kink around its lower limb with a background velocity of 5933m/s. Though the other velocity models sufficiently image the main features of the orebody, they are however located at the wrong position from the model surface. As expected, most of the seismic energy associated to the ore is located around areas with a high level of features associated with complex deformations. The
other parabolic/hyperbolic events in the 2D sections are the S-waves and converted waves that have been overmigrated since higher velocities (P-waves) were used for migration.

Figure 5.9: Depth migrated seismic sections using a suite of homogenous P-wave velocity models.
5.3.4 Heterogeneity and wave scattering

Inhomogeneities at Thompson exist at various scales; underscored by the geology (Figure 3.6) or the petrophysical measurements conducted in situ (velocity logs) and in the Lab (Table 3.2). Although both categories are useful for geologic and geophysical studies, it is the fluctuations in the petrophysical properties of the rock mass that has significant effects on seismic wave propagation and scattering provided other requirements for impedance contrast are met. Hence, a direct impact on the potential for seismically imaging structures of interest. Three major categories of inhomogeneities were considered and these include: the inclusions within the archean basement (e.g. AMPT); the lithologic units surrounding the host rock with emphasis on the thompson formation (TF) which is relatively contrasted (Figure 3.7) from the other surrounding hostrocks; and heterogeneity based on log-derived scale parameters.

a) Effect of granitoid inclusions and geology surrounding orebody

The mafic intrusives (AMPT) hosted within the archean basement are lenticular in shape (Figure 3.9) with characteristic scale thickness (normal to dip direction) that ranges from ~5-20m\(^{14}\) on average. The AMPT units just like other geologic contacts and the orebody dip at ~50\(^{\circ}\). Plane wave modeling results (Figure 5.10) indicate that a significant portion of the seismic reflected energy is also generated from the in situ geologic units. The major contributions of this backscattered energy are generated at contacts with either the mafic intrusives (AMPT) or thompson formation (TF). Given that the distribution of the AMPT units extends to the model surface and that their average lateral continuity in the dip direction is almost identical to that of the orebody (see Figure 3.6) the seismic response from these units may potentially cause imaging problems. However, the dips of the reflected seismic energy from the various units are slightly greater than those from orebody diffractions (“foot print”) that have almost flat to gentle dips. Data processing (e.g. FK filtering) can take advantage of this dip information to separate the response from the orebody from those of the surrounding host rocks. Figure 5.10a illustrates the seismic response for the case where the orebody inclusion is absent from the model. Although diffractions, somewhat similar to those in Figure 5.10b, can be identified the amplitudes are relatively smaller.

\(^{14}\) Manually measured from scaled versions of the geologic sections.
b) Effect of Log-scale heterogeneity

Using the information, i.e. scale (correlation) length and Hurst number (ν) derived from the ACF and variogram analysis of sonic logs in the Thompson area, 2D random models conditioned by the geologic information of the background were built following the approach outlined in Goff and Jordan, 1988. The two geologic units considered include the setting formation (SF) and the archean gneiss (AG) whereby the fluctuations in physical properties (Vp, Vs, density) were considered to be randomly distributed about their respective mean values (Table 3.2). The variances were constrained from a large pool of Lab measurements for Vp, Vs and density on cores. Figure 5.11 shows samples of the 2D Vp models constructed: Homogenous background; Isotropic random background ($a_x = a_y = a_z = 5m$); anisotropic background with the maximum correlation distance aligned in the direction of the dip ($a \perp \text{dip} = 5m$, $a_{\parallel \text{dip}} = 100m$). The models are designed with the assumption that the fluctuations in the P- and S-wave velocities as well as the density are correlated. Notice that the fluctuations in background P-wave velocity cause the AG/SF boundary to be less evident, hence potentially transparent to propagating seismic wave.

To ensure a comprehensive analysis of the effects of the background heterogeneity to the propagating wave, surface sensors and two VSP acquisition geometries at 2100m and 4200m
respectively were positioned to capture the scattered wave in both backward and forward directions (Figure 5.11a). As illustrated in Figure 5.12, most of the coda, generated as a consequence of wave scattering due to background heterogeneity, within short periods from the direct P-wave consists mainly of S-waves. For both heterogeneous cases, the lateral continuity of the wave coda is correlated to the scale lengths of the medium. A close comparison of the recorded wavefields using the various geometries (Figures 5.13, 5.14 and 5.15) corroborates that the transmitted wave recorded by the sensors in the borehole at 4200m (VSP4200) offer the least information about the orebody and the AG/SF structural contact at depth. Only the PS-converted waves provide a useful hint of the presence of inhomogeneities in the path of the propagating waves. The VSP record at 2100m (VSP2100) and the surface data contain partial information of the diffracted wavefield propagating in the opposite direction to that of the incident wave. The VSP2100 data contains most of the strong energy that is characteristic of the PS-converted wave. A surface line that extends beyond the 2000m mark in the model would be required to fully
capture this part of the PS-converted energy. All the respective component recordings provide complementary information about the reflected/diffracted wavefields.

When the background is in the quasi homogenous regime, the coherency of the diffracted waves is severely reduced (Figures 5.13b and 5.14b) as a result of the wave scattering due to fluctuations in the physical properties of the background medium. The vertical component recordings are the most affected: a section of the recording aperture (around the peak of the diffraction hyperbola) especially in the surface acquisition geometry become transparent to the orebody response. However, some significant energy that characterizes the arms of the diffracted wavefield is still present in the seismic record. This further supports the suitability of using a longer surface line or a VSP set up for imaging. Also, a significant portion of the converted waves (PS and SP) is masked by the background scattered waves. Compressional waves are more affected than the shear wave components of the orebody diffractions. Similar characteristics of the backscattered wavefield as described above are also observed in the anisotropic model. However, the effects on the orebody diffractions are less severe: the lateral extent of the diffraction wavefield is improved and some of the converted wavefields (PS and SP) between 0.45s and 0.6s are much visible (Figures 5.13c and 5.14c).

![Figure 5.12: Snapshot of wavefield propagation within the random isotropic heterogeneous model. The source position is 2600m and dominant frequency is 100Hz.](image-url)
Figure 5.13: Vertical and Horizontal component wavefield records from surface acquisition geometry for a) homogenous; b) quasi homogenous (random isotropic); and c) random anisotropic models respectively. DP: Direct P-wave, DS: Direct S-Wave, PP: P-wave reflection from AG-SF contact, PP: P-wave diffractions, SS-S-wave diffractions, PS: PS-converted waves. The bulk of the diffraction energy is generated by the orebody.
Figure 5.14: Vertical and Horizontal component wavefield records from VSP acquisition geometry (2100m) for a) homogenous; b) quasi homogenous (random isotropic); and c) random anisotropic models respectively. DP: Direct P-wave, DS: Direct S-Wave, SP\textsubscript{2}: SP-wave diffraction from thickest and deformed sections of the orebody, PP: P-wave diffractions, SS-S-wave diffractions, PS: PS-converted waves (generated by the limb of the orebody), PPP & PPS: P- and S-wave reflections of PP waves from surface boundary, PB: Upgoing P-wave reflection generated by the thin layer at the base of the 2D models (Figure 5.11).
Figure 5.15: Vertical and Horizontal component wavefield record from VSP acquisition geometry (4200m) for a) homogenous; b) quasi homogenous (random isotropic); and c) random anisotropic models respectively. DP: Direct P-wave, DS: Direct S-Wave, SP$_2$: SP-wave diffraction from thickest and deformed sections of the orebody, SS-S-wave diffractions, PS: PS- converted waves, PB: Upgoing P-wave reflection generated by the thin layer at the base of the 2D models (Figure 5.11).
5.4 Summary

Seismically probing target features in complex geology, typical of hard rock terranes, has been demonstrated in the past (Milkereit et al., 2000) to depend on an effective integration of rock property and geological databases obtained especially from boreholes. This is relevant in understanding the characteristic scattering seismic response of the orebody as well as other related mineralized zones with respect to the host geology especially deep ore targets (>700m) like the one in the Thompson mine (this study). Unlike the borehole constrained sulfide distribution at the Nash Creek property (Chapter 4), the sulfide mineralization at Thompson is a well defined 2D lens-shaped inclusion having a dip that is parallel to the lithologic package between the archean gneiss and setting formation (SF) (see Figure 3.6). Thus, the orebody can be classified as an “ideal orebody” with characteristics of an impedance and density seismic scatterer (high density contrast). For the first time, the hypothesis of the seismic detectability of the inherent strong contrast of the Thompson orebody with respect to the host geology has been tested. The approach involved the use of 2D finite difference elastic wave modeling which incorporates existing geological and rock physics (Vp, Vs, and density) information at the Thompson mine. Based on the output from the modeling exercise, the major findings and recommendations pertinent to seismic acquisition design and data processing include:

- Orebody diffractions with most of the “footprint” (P-waves) in the downdip direction can be reasonably imaged whereas the boundary of the SF produces weak reflections. Reflections from this dipping contact is boosted by the presence of the thompson formation (TF)

- A suitable approach to capture the diffracted waves is by either using an extended surface acquisition line or VSP geometry comprised of multicomponent sensors in the direction of dip.

- Processing of a full 2D synthetic seismic reflection survey data suggests the strongest diffractions stem from the thickest part of the orebody (complex deformation zone). This can be used potentially as a marker for mapping the strike of this complex region.

One of the major drawbacks for effective seismic imaging in hardrock environment both on the front of survey design and data processing are noise related problems. Noise problems could be
caused by the source used in the survey (Milkereit and Eaton, 1998) or fluctuations in the physical rock properties of the host rock (L’Heureux et al., 2009) that cause amplitude and spectral changes sufficient enough to overwhelm the seismic response of the orebody. In the Thompson mine, this study demonstrates that the AMPT inclusions (lens shaped) as well as the heterogeneities observed at the log scale can potentially undermine the effectiveness of seismically characterizing the orebody as well as complexly deformed structures at depth. While results suggest that the use of broadband seismic sources, multicomponent recording, and the classic processing routines such as dip filtering hold promise in circumventing this problem, the complex geologic setting with highly dipping structures (>50°, large dips are a limitation for migration routines) warrants the use of according sophisticated seismic processing technology for P-S separation. For example, P-S separation can be used to distinguish the characteristic body and converted waves of the orebody (“footprint” and “fingerprint”) from those generated by other lens inclusions like the ultramafics (UM).

Occasionally, when cost and access are of concern, the seismic data acquired in crystalline environment are often acquired on crooked lines. Considering the complex nature of the geology around the Thompson mine, out of plane diffractions (e.g. diffractions from structures with dips across the survey line) can be present in the data as reflection (CMP) points sample a 3D domain in the vicinity of the 2D crooked line. If straight-line processing (CMP points are considered to be located directly beneath the 2D straight-line) is applied to such data, the focus of target reflections in the final CMP stack is severely diminished. This is because the presence of out of plane reflections cause the NMO-corrected reflections to be poorly aligned (Nedimovic and West, 2003). Nedimovic and West (2003) argue that in lieu of standard processing methods, adopting processing styles that accommodate for the 3D character of the 2D crooked line can improve the resolution of the subsurface reflections in the final stack.

Another potential cause for seismic noise around the Thompson mine is the seismic scattering due to the weathered clay overburden sitting atop the bedrock. Although synthetic results (see Appendix D.1) show multiples that mask the orebody response, a look at a real seismic data set acquired in the nearby Birchtree mine corroborates that the overburden effects may not be as severe as predicted by the modeling results (Appendix D).
It is worth noting that the full 2D synthetic seismic acquisition and processing experiment conducted in this study does not provide an exhaustive solution to all possible problems or imaging challenges in the Thompson mine. This is partly because the 2D model considered is a simple one comprising the archean gneiss (AG), the orebody and the setting formation (SF) units. If the full geologic model is considered, the heterogeneity in the model results in an effective anisotropy (Wu, 1989, effective fast and slow velocity directions). This has implications for imaging surface recorded diffractions from the target given that constant velocity models are often used in the migration processing routines. An investigation on the effectiveness of migration routines using the complete Thompson model is discussed in Appendix E.
Chapter 6
Application of Transmitted Waves for Subsurface Imaging

6.1 Introduction

Information obtained from seismic data is subject to the quality of attributes such as ground displacement (amplitudes), and travel time obtained from reflections (backscattered) and first breaks. These attributes are influenced by factors like the acquisition geometry and the physical rock properties of the medium in which the seismic wavefields propagate. So far, in chapters 4 and 5, seismic modeling investigations focused primarily on using backscattered (reflections) seismic waves in order to obtain knowledge about the petrophysical model structure especially that of sulfide ore targets. Analysis of the reflection events in heterogeneous media is challenging as it often requires accommodating processing routines to boost the signal to noise ratio (SNR). This is because these reflections are weak and are also affected (interference) by wavefields generated due to scattering from the background. On the other hand, forward scattered (transmitted) waves have much stronger amplitudes and other attributes from the transmitted waves like traveltimes can be processed for subsurface information. This chapter highlights some fundamental and robust application of transmitted energy for imaging subsurface heterogeneity. Investigations considered applications tailored to use a simple two-layered model (based on Bosumtwi impact crater) and a multiple layered model (based on Mallik sonic logs).

Transmitted seismic waves can be recorded in various settings such as from earthquakes, from microseismic activities in hydrocarbon or geothermal reservoirs and from vertical seismic profiling (VSP) data recording. In this chapter, focus is in transmission imaging applications that use offset VSP data as well as microseismic data. The efficiency of two imaging methods which include a time-shift and a reverse-time migration-based approach are discussed, implemented and assessed on synthetic and real seismic (moving source offset VSP: MSOVSP) data from the Bosumtwi Impact crater. It is shown that transmitted energy contains structural information which can be relevant to image the contact between two layers with sufficient impedance contrast and also help boost the knowledge of the velocity distribution beyond any existing borehole. The later part of the chapter focuses on potential shortcomings of using transmitted waves for imaging applications. Emphasis is on the relevance of an accurate velocity model,
accurate positioning information of shots and receivers, background heterogeneity, as well as the combined effects of geology and acquisition geometry. The evaluation on the effect of geology is particularly tailored to reflect challenges faced in using microseismicity for subsurface characterization. An additional discussion on the role of geology for more complex settings like the Thompson mine is covered in Appendix E.

6.2 Bosumtwi Offset VSP-Background

The focus here is in the use of offset vertical seismic data acquired in the Bosumtwi impact crater. The Bosumtwi impact crater, located in Ghana is the largest young (~1.07 Myr) and well preserved crater in the world.

![Figure 6.1](image)

**Figure 6.1**: a) Depth (two-way travel time: TWT) section acquired via multichannel surface seismic survey (Scholz et al., 2002). Borehole LB-08A was drilled through the central uplift whereas LB-07A was drilled at the outer rim of the uplift. Note absence of reflections in the hard rock (fractured basement) environment; b) Cartoon of Offset VSP acquisition geometry; c) Scaled amplitude plots of the offset VSP data for the receiver at 450m depth. Gaps in the data represent locations with no information. The variability in breccia properties is depicted in the density (d) and P-wave velocity (e) logs from LB-08A.
The impact structure is covered by an 8km diameter Lake Bosumtwi. The base of the impact is covered by a layer of post impact sediments as revealed in the reflection section in Figure 6.1a. The Offset VSP study was done as part of the interdisciplinary Lake Bosumtwi drilling project of the International Continental Scientific Drilling Program (ICDP) of 2004 to study the subsurface of the crater structure. VSP (Zero offset and offset) studies were performed in order to understand the in situ seismic properties of the sediments and the impacted rocks in the vicinity of borehole LB-08A. Details of the drilling configuration and acquisition parameters have been discussed by Schmitt et al., 2007.

The offset VSP acquisition was done with sensors positioned at 100m, 150m, 200m and at 450m depth (Figure 6.1b) along the borehole. The deepest geophone was located within the impact-brecciated basement rocks, while the three shallower geophones were stationed within the lacustrine sediments. The airgun source was located at a water depth of ~3m. Borehole LB08A was cased from 0m to 239m through the weak sediments but left in open-hole condition from there down to the bottom-hole at 451.3m depths. Figure 6.1c shows a scaled amplitude plot of the receiver gather at 450m depth.

6.3 Imaging the Sediment-Bedrock Contact

Offset VSP data contain both direct (transmitted) and reflected energy. In order to obtain information of the subsurface structure especially in the borehole vicinity, data processing almost always only uses the backscattered (reflections) energy generated at interfaces where the impedance contrast is sufficiently large. Although analysis of reflections from seismic (both surface and offset VSP) data is often used to probe for structural heterogeneity at the subsurface, it is however not the only part of the recorded seismic wavefield with information on structure (geology). It has been demonstrated in the past (McMechan et al., 1988) that the directly transmitted energy also must carry structural information; it could be used to delineate the interface separating two geologic units with sufficient contrast in their respective elastic properties. McMechan et al. (1988) demonstrated that transmitted energy processed with same migration (Reverse Time Migration-RTM) algorithm used in more conventional processing of reflections provides a practical solution for the salt proximity problem (imaging the flanks of the salt dome). The purpose of this section is to demonstrate how direct (transmitted) waves
recorded in offset VSP setting can be used to image the contact between the sediments and the consolidated breccias rock unit below it within the Bosumtwi impact crater in the vicinity of the borehole (LB08A). The acquisition geometry for solving the salt proximity problem is similar to an offset VSP acquisition. Two methods are proposed and assessed for their ability to provide accurate information about the structure (contact) in the vicinity of the borehole using synthetic and real data.

6.3.1 Methodology

For the purpose of clarity, the two methods for imaging the sediment-breccia interface are referred to as 1) the time shift method and 2) the migration-based method. These both require extrapolation of the recorded wavefield as well as the implementation of a travel time imaging condition. Both methods mentioned above only differ in the processes by which the wavefield extrapolation and the travel time imaging condition are implemented. The wave extrapolation and the travel time imaging condition is similar to prestack migration (Chang and McMechan 1986). The only difference is that in prestack migration, the imaging condition is applied to the reflected energy. Hence, imaging applications using transmitted (direct) energy necessitate the removal of the reflected energy from the recorded data.

In the time shift method, the traveltime imaging condition is implemented by using ray tracing that assumes normal incidence at the interface to be imaged: there is no ray bending owing to velocity contrast. Thus, with accurate knowledge of the receiver and source locations and with an accurate velocity model, Pythagoras’ theorem can be used to estimate the time of flight within the sediments ($t_s$) and the breccias ($t_b$) where ($t_b + t_s$) represent the observed travel time from first break picks. Estimating the time of flight equally translates to the total distance of propagation within each layer and thus information of the offset location with respect to borehole at which each ray (for each source-receiver pair) intercepts the interface at depth can be obtained. The output image of the interface is constructed by applying to each recorded trace a corresponding time shift that represents the time of flight within the breccia unit. The output image is plot in the spatial domain whereby offset information with respect to borehole represent the estimated offset of the contact at depth. For the Bosumtwi data, approximate information on depth can then be obtained by doing a product of the one-way time axis with the velocity of the sediment layer.
The migration-based approach, first presented by McMechan et al. (1988), is on the other hand slightly more involved. In this approach, part of the wavefield extrapolation process is achieved by solving the wave equation backwards in time. An example for the 2D discrete acoustic wave equation is shown in equation 6.1.

\[
U_{i,j,l-1} = 2U_{i,j,l} - U_{i,j,l+1} + A(U_{i+1,j,l} + U_{i-1,j,l} + U_{i,j+1,l} + U_{i,j-1,l} - 4U_{i,j,l})
\]

(6.1a)

\[
A = \left(\frac{\alpha_{zz} (mn)}{h}\right)^2
\]

(6.1b)

where the displacement vector is given by the variable \(U\), \(\alpha_{zz}\) is the 2D P-wave velocity. The indices \(i, j, l \in \mathbb{Z}^+\) represent the intervals in the x, and z directions and in time. Hence, \(h = \Delta x = \Delta z; mn = \Delta t\). More details of the approach are discussed in Appendix E (E.2).

### 6.3.2 Transmission wavefield modeling and imaging

**a) Petrophysical model**

Most of the knowledge about the elastic properties of the sediments and the brecciated rocks within the Bosumtwi impact crater stem from analysis of well logs and other geophysical data sets such as zero-offset VSP data. Since borehole LB08A was cased through the unconsolidated sediments down to depths of 239m (top of the impactites, Schmitt et al., 2007), well logs only exist beyond this depth. Evidence from the seismic velocity model by Karp et al. (2002) and the first break analysis of the Bosumtwi Zero-offset VSP data (Schmitt et al., 2007) and offset VSP data (Bongajum et al., 2009) corroborate that the P-wave velocities within the sediments is approximately that of sound through water (1450 m/s). Schmitt et al. (2007) compared the sonic logs with estimated interval velocity from zero–off set data and found both velocities to be substantially lower than expected for their rock type. They posit that this velocity dispersion may be related by the presence of an extensive network of fractures. Both sonic logs and estimated seismic velocities (from Zero-offset VSP) indicate fluctuations in the P-wave velocities with a general trend that increases rapidly with depth from ~2500 m/s to ~3400m/s (Figure 6.1d). To some extent, the elastic properties of the rocks within the impact crater are bimodally distributed depending on the two lithologies: post-impact lacustrine sediments and the impactites including...
the highly damaged country rock and the associated breccias. This is quite evident in the seismic profile of Figure 6.1a. This was considered in building the petrophysical models. In the present case, the von Kármán autocorrelation function as described by Goff & Jordan (1988) is applied to characterize the perturbations (stochastic component) of the velocities and densities within the Bosumtwi impact crater.

Figure 6.2 shows the stochastic models for P- and S-wave velocities as well as density. The scale lengths within the sediments are anisotropic (a_x/a_z = 200) while the scale lengths within the brecciated unit are isotropic (a_x = a_z = 3m). The choice of these scale lengths was guided by observations of the seismic reflections patterns (horizontal layering) observed in the surface seismic data (Scholz et al. 2002, Figure 6.1a) for the case of the unconsolidated sediments. Isotropic scale lengths within the brecciated rock unit were based on L’Heureux & Milkereit’s (2007) explanation for the lack of reflections within this unit. L’Heureux and Milkereit (2007) argue that heterogeneity resulting from the impact process is responsible for the absence of reflections within the brecciated unit i.e. the scale of perturbations of the field of the elastic properties owing to damage (e.g. Vp) are smaller than the resolution limit of the seismic waves.
For the purpose of simplicity, constant background models were used to each represent the mean elastic property within the sediments and breccia respectively. However, a more realistic representation would be to have P-wave velocities, for example, to gently increase with depth due to compaction within the sediment layer. The mean P-wave velocities within both units are 1550m/s and 2888m/s respectively and are in agreement with observations from sonic logs and from seismic data analysis. To further ensure that the petrophysical model reasonably represent the in situ conditions within the impact crater, the sediment-breccia contact used in Figure 6.2 was derived by mapping the base of the reflections observed in the 2D image of the surface seismic sections (Scholz et al., 2002, Figure 6.1a).

The synthetic VSP acquisition geometry and parameters (offset range) are identical to those of the field data (receiver gather at 450m depth: R-450). The explosive source was buried at a depth of 450m while receivers were located at a depth of 3m below the surface (top boundary of the 2D model). By the principle of reciprocity, the above acquisition geometry would produce same seismograms if the positions of the source and receivers were swapped accordingly, i.e. a single receiver buried at 450m depth to record signals from several shots located at a depth of 3m below the surface of the 2D model. The later description fits the method used for acquiring the

![Figure 6.3: a) Vertical component plot of the synthetic wavefields recorded at the receiver locations (Trace interval 2.4m). Note the asymmetry of the recorded wavefield about the borehole location (0m offset). Notice presence of secondary events (e.g. diffractions (yellow circle), PS-converted wavefields). b) Scaled amplitude plot of the preprocessed synthetic data (a) with secondary events muted. Each trace is normalized to maximum amplitude of 1.](image)
Offset VSP. The synthetic data acquisition was symmetric about the borehole location with maximum offset of 600m on both sides and a trace sampling interval of 2.4m. Figure 6.3a shows the vertical component record of the shot gather. Note the asymmetry in the direct arrivals which can be associated to the irregular shape of the sediment-bedrock interface (Figure 6.2). Diffractions, also associated to the point-shaped sections of the contact, can be identified at short periods after the direct arrivals.

In processing the synthetic shot gather to image the sediment-breccia interface, some preprocessing needs to be done to mute all secondary events (Figure 6.3b). If working with noisy data, some noise filtering is also required. Filtering can also be done to adjust the spectral content of the seismic data to a desired frequency band. Moreover, imaging of the target structure using the migration-based method necessitates interpolation of the data as if the data was recorded at receiver locations identical to the grid spacing of the 2D modeling grid. Note that the implementation of the imaging algorithm in the present case relies on 1D velocity information derived principally from sonic logs and zero-offset VSP data. Thus, image reconstruction of the sediment-breccia contact with transmitted energy equally improves knowledge about the velocity field distribution beyond the borehole location i.e.

\[
V(x) \rightarrow V(x, z)
\]

(Sonic logs, Zero Offset VSP) 2D model

where \(x\), and \(z\) are spatial coordinates in the horizontal and vertical direction.

Figure 6.4a shows the predicted sediment-breccia contact using the time shift method. The average velocities for the sediments and brecciated rock units were used in the computation. The superimposed white curve represents the true contact between both units as depicted in the petrophysical model (Figure 6.2). The time shift computation either overestimates or underestimates the depth of contact. This shortcoming stems principally from the assumption that the ray path between each source-receiver pair is straight. Despite the poor agreement between the predicted and the true contact, the predicted interface does provide some hint on the asymmetry of the contact in the vicinity of the borehole location as the trend closely mimics the trend depicted in the true contact curve.

To implement the migration-based algorithm, an isotropic spatial grid spacing of 1.5m was considered to avoid problems of numerical stability and dispersion. An acoustic reverse time
migration algorithm based on the second order approximation of the wave equation (McMechan, 1983) can be used to backpropagate the synthetic data from the receivers. This approach was considered in the case herein whereby the velocities around the source and receivers (appendix E) for the constant velocity models were 2888m/s and 1550m/s respectively. The reconstructed contact image obtained following steps b) to d) (Appendix E) with Figure 6.3b as input shows a good match with the true contact from the model (Figure 6.4b). Unlike the results obtained in Figure 6.4a, the base of the central uplift to the left of the borehole is well mapped. Notice the limit of the reconstructed image depends on the acquisition aperture as well as the velocity contrast that exists between both geologic units. Figure 6.4b suggests the lateral delineation of the contact in the borehole vicinity can be done reliably within absolute offsets of 200m.

6.3.3 Transmission imaging with Bosumtwi VSP data

The Bosumtwi Offset VSP receiver gather at 450m depth (R-450) was considered (Figure 6.1c) for imaging the interface separating the sediments and the brecciated rocks. This is because the acquisition parameters are identical to those used for the synthetic model example (assuming reciprocity). Unlike the synthetic dataset where the range of surface offsets (0-600m) allows for contact imaging in a large aperture, the R450 data samples a limited aperture of the contact since
no shots could be recorded for small offsets (<100m). Thus, comprehensive contact information in the immediate vicinity of the borehole will not be achieved with the R-450 data. Moreover, some trace editing was done to mute traces at shorter offsets with incorrect positioning information. The remaining traces (Figure 6.5a) were then interpolated as if the data was recorded at grid nodes of the 2D grid used to backpropagate the recorded data. The dominant frequency of the recorded data is ~50-60Hz, hence a grid node interval of 1m was used in the interpolation.

Figure 6.5a shows the processed recorded data that is input in the backward wavefield extrapolation (2D acoustic) algorithm in a reverse time order. Without loss of generality and assuming the principle of reciprocity, the recorded wavefield from the pseudo receivers (shot positions) was backpropagated with the velocity within the loose sediment package, whereas ray tracing from the pseudo source (receiver position) was done with the velocity of the breccia unit.

The reconstructed image (Figure 6.5b) of the sediment-breccia contact, though not continuous, corroborates a structural asymmetry of the contact in the vicinity of the borehole. Based on the reconstructed image, it is interpreted that the amplitudes to the right of the borehole image the
contact within the central uplift, whereas image amplitudes to the left of the borehole provide information about the depth from surface of the contact in the vicinity of the central uplift. However, further investigations are required to validate this argument. Given that both acquisition lines for the surface and offset VSP data coincide (Figures 6.1a & b), the differences in the morphology of the contact as revealed by both data sets (Figure 6.1a and Figure 6.5b) strongly corroborates the 3D variability of the contact in the borehole vicinity. Unlike the synthetic modeling results, the reconstructed contact image using R-450 extends up to ~400m from the borehole location.

The success of the transmission imaging methods illustrated above partly relies on accurate knowledge of the velocity model from sonic logs and other seismic data sets. In addition, correct information of the acquisition parameters and the spatial locations of the sensors and receivers are equally important. Not accounting for location errors results in a flawed image reconstruction of the sediment-breccia contact. The importance of the velocity information is covered in section 6.4 in which the transmission imaging approach is further used as a quality control tool for the velocity distribution.

It is worth noting that although 2D acquisitions are commonly used to probe the subsurface, the recorded wavefield results from the wavefield interactions with the 3D heterogeneous nature of the earth. In other words, we often record the 3D response of the earth to some extent when using 2D survey lines. Out of plane diffractions can equally influence the direct wavefields recorded in the plane of the 2D spread of receivers, thus, introduce some errors in the interface (contact) image reconstruction.

6.4 Effect of Input Errors- Velocity Model and Source-Receiver Location

The success of the imaging algorithm depends on the accuracy of the input information used to obtain the final image. Sources of errors span a broad spectrum, but for the purpose of this work, focus will be on errors that are most relevant to the implementation of the imaging condition upon which seismic wavefield migration imaging is based. Thus, it can be deduced that reflection and transmission imaging based on migration are equally affected by such errors although to varying degrees owing to the differences in reflected and transmitted wavefields. In this section, the impact of errors in velocity and positioning information is assessed.
As demonstrated by Yilmaz (1989), errors in velocity information result in over migration or under migration of the seismic wavefield from an explosive source. To assess the influence of velocity errors in transmission imaging, the imaging condition using the synthetic VSP data (Figure 6.3), was implemented with 6.5% (1450 m/s) error in the average velocity of the sediments and 3.8% (3000 m/s) error in the average velocity of the brecciated rock unit. Figure 6.6a indicates that velocity model results in a contact image positioned at shallow depths and with wavelength features that are slightly larger (under migrated) than that observed for the true sediment-breccia contact. Observations from Figure 6.6a point to a logical deduction that using much higher velocities for the breccia unit leads to an over migrated contact image (shorter wavelength features compared to true contact) that is positioned below the true contact. The fact that the various velocity models result in some reconstructed image of the contact elicits concerns that question the validity of the reconstructed image. One way to address this is by performing a full migration in order to image the source location. Implementing a full migration with the derived contact information would correctly image the source position at depth.

**Figure 6.6:** a) Transmission imaging showing the effect of velocity errors; source imaging using true contact information with correct velocity model (b) and with erroneous velocity model (c); Notice the final source image in (c) is under migrated; d) Effect of receiver positioning errors on the reconstructed contact image. Receivers were considered to be located 60m to the left of their true location.
(excitation time imaging, t=0). On the other hand, if source imaging is implemented with the true contact information, correct source location can only be achieved if the right velocity model is used (Figure 6.6b and c).

Errors in positioning information (receivers/source locations) can also affect results derived from the transmission imaging process. Figure 6.6d clearly illustrates the effect of such errors in the imaging process. An offset error of 60m which is comparable to the range of shot position errors observed for the shallow receiver records of the Bosumtwi offset data (Bongajum et al., 2009) was used. Notice the derived contact image appears to have been subjected to an anticlockwise rotation with respect to the true contact. The delineated contact image to the left of the borehole position appears to be steeper whereas it is much shallower to the right of the borehole location. Figure 6.6d suggests that positioning errors severely affect the dips of the imaged contact structure.

6.5 Effect of Geology- Part I (Microseismicity)

The use of transmitted waves for information about subsurface structures is not only restricted to the imaging of lithological contacts as illustrated in the previous sections of this chapter. Other applications involve using these transmitted waves to infer information about the state of the rock mass in space and time. An example of such applications is in reservoir monitoring where interest is in probing zones depicting temporal changes in the physical properties of the rock mass. In these circumstances, changes in rock properties could be induced by active processes like fracturing and Carbon-dioxide sequestration. Fluid propagation in these respective applications is subject to rock porosity. Rock porosity can be enhanced as these high pressure fluids cause changes in pore pressure or preexisting stress reaching critical point to cause brittle failure of rocks (Shapiro et al., 2005). Fracturing processes are often characterized by zones of clustered microseisms (“small earthquakes”). Microseismicity is characterized by a continuous stream of seismic events occurring at different locations. Each event acts as a unit source generating its own wavefield. Seismic waves generated by these microseisms can be recorded as transmitted waves and used in turn to locate these microseismic zones. Figure 6.7 illustrates how adjacent wells in the field can be used to record the microseismic events. Positioning 3C-geophones in a
separate well (Well B) adjacent to the fracking/injection well (Well A) helps mitigate noise problems, otherwise encountered if the geophones were located in Well A. Nowadays, the bulk of microseismic applications focus on source location and magnitude (Maxwell, 2006 & 2010, Zhao et al., 2010, Baig and Urbancic, 2010) and this is subject to the effects of velocity distribution as well as interference effects resulting from wave scattering and interaction between waves generated from multiple microseismic events (Banerjee et al., 2002).

Subsurface geology which is strongly correlated to the macro structure of the velocity distribution affects the efficiency of characterizing the source locations (Gajewski et al., 2009-KTB Well, Germany). An understanding of the impact of the geology is relevant for designing the acquisition geometry that meets the requirements needed to solve the microseismic monitoring problem at hand. As shown in Figure 6.8, the recorded wavefield from a point source in a layered medium by geophones in a horizontal well differs from that obtained by geophones in a vertical well. The velocity model is based on logs from the Mallik gas hydrate production test well (5L-38, Satoh et al., 2005). Clearly, for a simple layered background velocity model, the 3C sensor placed in a distant vertical well will record a complex wavefield. Most of the energy propagates towards the direction of increasing depth. Also, there are lots of converted waves (PS-waves). Imaging with such data can be improved provided according processing for P-S wave separation is adopted (Huang and Milkereit, 2007). In contrast, the horizontal array above the microseismic cluster provides easy to read P- and S-wave data. In practice, however, horizontal wells are hard to find. Moreover, flawed interpretations about the source locations can be inferred from the vertical well data if minimum travel time is used (Figure 6.9). Thus, the effect of geology must be considered for the design and location of observation wells.
Pre-acquisition modeling of seismic wave propagation will help in the choice of processing to be applied on the data or in evaluating the sensitivity of the processing routine to the conditions of the data. For example, the recorded wavefield can be backward migrated in time (Chang & McMechan, 1987) to the source location(s) hence providing an image of the subsurface conditions. The process of performing the migration algorithm is further eased by the availability of the velocity field information around the target zone (derived either from surface seismic data or from well logs). Using the appropriate velocity information for the migration process is very critical. As shown in Figure 6.10, using a constant velocity model will be inadequate for migrating the wavefront (recorded in the vertical well) back to the source location.
Figure 6.9: Recorded wavefields from the respective vertical (a) and horizontal (b) receiver arrays; The dashed lines represent the axis of the source locations; c) First break picks from the respective data shown in a) and b). Note the shift (~50m) from the zero position in the apex of the travel time curve for the Vertical array.

Figure 6.10: Reverse time migration (RTM) to excitation time (t=0s) of the direct waves recorded by the horizontal geophone array (a) and the vertical geophone array (b).
6.6 Summary

Much knowledge about the earth’s subsurface and in situ conditions of the rock mass can be gained by fully recording and processing transmitted (forward scattered) seismic wavefields.

In addition to the surface seismic reflection data at the Bosumtwi impact crater, analysis of the offset VSP data provides useful information to improve the characterization of rocks underneath the lake. It is shown using synthetic models (defined on the basis of other geophysical observations) that transmitted energy (direct arrivals) can be used to provide structural information in the vicinity of the borehole. Two methods including the time shift and the migration-based methods were tested on synthetic seismograms. In the context of this study, the migration-based approach provides a reliable reconstruction of the sediment-breccia contact image. Implementation of the migration-based algorithm to the Bosumtwi offset VSP data revealed the asymmetric structure of the sediment-breccia contact in the vicinity of the borehole location despite the lack of adequate control over possible location errors associated to the surface shots.

In microseismic applications, besides event detection which is often influenced by scattering/interference mechanisms and noise, there is valuable information present in the transmission coda (e.g. converted PS waves). Modeling results show the importance of accommodating effects of geology in acquisition design for recording and processing of microseismic events. Moreover, results support that the recording of continuous wavefield data provide the basis for reverse time migration or waveform migration techniques to better image or illuminate the microseismic source region. It is generally understood that the heterogeneity within crustal rocks do exist in more complex forms that those examined in this chapter. In appendix E.4, I do a similar evaluation on the impact of more complex geology on the final migrated seismic image. Particular attention is in the case where the geologic complexity has a dip component as observed in the Thompson mine.
Chapter 7
Conclusions and Outlook

Recent developments in seismic imaging methods as well as other geophysical imaging methods have revealed much information of the heterogeneous earth structure. However, there is continuous effort geared at better understanding how these heterogeneities affect various geophysical imaging techniques. In this thesis, particular attention is directed towards seismic imaging methods. Heterogeneity in rock properties causes seismic scattering. In addition to deterministic methods, it is shown in Chapters 1 and 2 that the stochastic treatment of the physical rock properties helps provide significant explanations to trace attributes often observed from recorded seismic waves. This means probing in to trace attributes of seismic waves provides a viable option for obtaining additional information about the earth’s subsurface. Finding ways that are well suited to retrieve information about subsurface heterogeneities (e.g. scale parameters characterizing the spatial variations in Vp, Vs, and density) from attributes (amplitude, phase, frequency, coda) of recorded seismograms is one way of approaching this problem and this is the focus of the results presented in this thesis. Carrying out research in this light is primarily motivated by the fact that most exploration targets especially in hardrock environment occur in complex settings which renders their seismic detectability very difficult. Although most of these targets (sulfides) are generally characterized by sharp density contrast, complexities associated often to geology, distribution (limited size) and heterogeneities of the host rock are critical towards undermining the efficiency of imaging methods in such environments. This chapter summarizes findings obtained from various case studies considered and also discusses according developments that can be implemented in the future.

Given the motivation to find better ways for retrieving information about heterogeneities, the primary approach adopted in this work is based on petrophysical model building and the study of elastic wave propagation in these models. The methodology focuses on the:

1. Collection of petrophysical (Vp, Vs, and density), geology, and geochemical information into a comprehensive database. Information from the database is used in an integrated fashion towards building stochastic earth models.
2. Use of FD methods for elastic wave modeling in these stochastic petrophysical models. The synthetic wavefields serve as an invaluable framework to analyze attributes of reflected and transmitted (forward scattered) waves.

While the former helps in our understanding of the spatial distribution of various physical rock properties as well as the interdependence between them, the latter demonstrates the importance of seismic numerical modeling for decisions relevant in acquisition design, imaging and interpretation. Since different geologic settings result in varied seismic responses, studies were tailored to reflect the geologic settings in the following case studies: Nash Creek (Canada), Thompson (Canada) and the Bosumtwi impact crater (Ghana). The geophysical investigations in the first two case studies are relevant for mineral exploration purposes.

7.1 Nash Creek Case- Conclusions and Outlook

7.1.1 Heterogeneity in the host rock and sulfide distribution

In this area where the underlying rocks are of Devonian age, the tectonic setting has played a major role towards the distribution of sulfide rich zones as well as the distribution of the rock types (felsic, mafic, and sedimentary) that play host to these sulfides. Lab measurements on core show that there is a stronger correlation between the P-wave and S-wave velocities than there is between these respective properties with the rock densities. Most of the sulfides observed from core analysis occur as veins containing some of the sulfide end members like sphalerite, galena, and pyrite. Measured densities from such samples plot correctly on the Nafe-Drake curve as mixed sulfides. While the cross plots of P-waves and densities from the lab measurements sufficiently distinguish non sulfide-bearing (background) rocks from sulfide-bearing rocks, a better assessment along these lines is provided by density well logs.

The density well logs constitute a unique dataset. Density data from a collection of 32 boreholes, each sampled from top to bottom (~6000 measurements), show that the underlying shallow crust is heterogeneous. Fluctuations in the densities are observed both in non sulfide-bearing and sulfide-bearing rocks. This suggests that the sulfide rich rocks, which also show a strong correlation with collocated assay data (Zn% and Pb%), are not characterized by one value. On a regional scale, the background rocks have an average density of 2.7 g/cm$^3$ and the sulfide mineralized zones have densities that are generally > 3.0 g/cm$^3$. 
By using two point statistics and considering that the crustal inhomogeneity at Nash Creek is a random and stationary process in space, the variability in the observed density values as well as in the collocated assay information can be statistically characterized with appropriate parametric exponential and von kármán functions. Although variogram analysis is subject to sampling methods and measurement errors, results suggest that the heterogeneity depicted by the rock densities are not controlled by the distribution of the felsic, mafic and sedimentary rock units found in the Nash Creek study area. Moreover, indicator variogram analysis shows that the high grade zones are not connected (i.e. high grade zones are not spatially continuous).

The framework of the rock physics data favours the building of a realistic 3D earth model which can then be used to evaluate the effectiveness of geophysical imaging methods such as seismics on the Nash Creek property. The building of the 3D earth model integrates borehole petrophysical (Vp, Vs, density), and geochemical information. This is primarily eased by the presence of the relatively dense spatial sampling of the study area with multiple borehole information which is otherwise absent or sparsely available in most exploration projects. For the first time, the Nash Creek density database was used to build a stochastic density model that not only honours the statistics of the overall database but also honours the information at existing well locations within the model volume considered (MVA and MVB, Chapter 4). Depending on the problem at hand, geostatistical methods can be used to further refine 3D stochastic earth models by performing the estimation/simulation processes conditional to the well data and by accommodating the spatial correlation between various rock property variables according to some predefined hierarchy. As described in Chapter 4, for example, the joint 3D characterization of density and Zn% distribution using collocated cokriging can be performed and used in turn to assess the uncertainty in the resource estimates on the Nash Creek property.

7.1.2 Seismic imaging and gravity methods (quality control tools)

Geophysical modeling conducted with results from the stochastic modeling of petrophysical data helped to assess the implications for seismic imaging (Chapter 4) and gravity (Appendix C) methods at Nash Creek. These investigations also served as a quality control tool for the conditional 3D density model(s).

The stochastic characterization of the petrophysical heterogeneity at Nash Creek helped facilitate an assessment of seismic wave propagation effects. Multiple scattering via finite difference
methods was deemed more suitable to fully account for seismic scattering processes that are relevant especially for applications aimed at imaging sulfide targets. Simple 2D heterogeneous models (Vp, Vs, and density) considered were built to reproduce the physical rock properties and bulk distribution obtained from core measurements.

Modeling results corroborate that orebody shape, size, distribution, composition, and depth, influence the attributes of the scattered wavefields generated by the orebody. Considering the resolution limitations (due to signal wavelength) of seismic methods, seismic imaging applications are more favourable for imaging deep targets.

In the event that the sulfide target is of economic size with a regular shape (e.g. lens- “ideal orebody”), synthetic results show that detectability of backscattered waves from deep targets is severely mitigated by seismic scattering due to background heterogeneity. It is observed that high frequency seismograms (dominant frequency of 100Hz) recorded with surface and VSP acquisition geometries do not contain sufficient diagnostic signals from a homogenous ore inclusion when the background heterogeneity is in the large angle scattering regime i.e. the variation in the rock property is isotropic with a correlation length of 3m. Detectability improves once the background heterogeneity is in the small-angle regime. This means imaging of zones with similar volume and disseminated sulfide mineralization will be extremely difficult as the generated diffractions (from orebody) are much weaker. The inability to seismically identify the sulfide rich zones is more severe if the disseminated sulfide has characteristic distributions similar to that depicted in the conditional 3D density volume (Chapter 4). Moreover, the ambiguity in identifying seismic signatures of such disseminated target zones is depth independent. If analysis is based on the conditional stochastic density model alone, it can be posited that sulfide distribution at Nash Creek is the primary factor that controls the efficiency of seismic imaging methods.

Secondly, multicomponent data acquisition and processing provides a better approach for imaging in heterogeneous environment. The vertical and horizontal component records are preferentially sensitive to P- and S- waves respectively. The study also demonstrates that for a horizontally oriented orebody, the VSP acquisition geometry is more suitable for full separation and processing of the ore generated converted waves.
According to Wu (1989), forward scattered energy is sensitive to the velocity contrasts hence suggesting a suitable diagnostic tool for characterizing heterogeneities or diffractions from orebody inclusions, especially those with sufficient velocity contrast. However, analysis of fluctuations in amplitudes of the forward scattered (direct) P-waves suggest that seismic scattering due to background heterogeneity can also mask the seismic signature from an orebody. Spectral ratio methods (e.g. H/V) for identifying resonant frequencies may be another possible approach for identifying the orebody response. Nevertheless, a suitable processing method for the appropriate SNR that allows for identifying these resonant frequencies is to be developed as of yet. Synthetic results so far suggest that the success of such a method is influenced by impedance contrasts as well as the interference between the direct P- and S-waves (Appendix F).

It is generally understood that gravity measurements are sensitive to the lateral and vertical variations in density. Modeled gravity signature from the 3D stochastic density volumes show wavelength characteristics that are comparable to observed gravity from Nash Creek (Appendix C). However, there are differences in the amplitudes of the respective gravity signals. Part of the discrepancy between both data is attributed to improper accountability of contributions from the overburden structure and the regional rock mass in the observed gravity field.

Gravity modeling results also suggest only residual gravity field analyses provide firsthand information about small scale variations in density anomalies. However, the synthetic results show that such information falls short of adequately imaging deep density anomalies as the sensitivity is limited to shallow density anomalies (< 11m). Such shallow depth range falls within the range of the overburden thicknesses reported from borehole collars on the Nash Creek property. This is indicative of potential pitfalls in performing gravity-based interpretation at Nash Creek without adequately correcting for the gravity effects of the weathered overburden structure. Moreover, the highly disseminated nature of the sulfides as depicted in the conditional 3D density models further complicates the efficient applicability of gravity imaging techniques as it becomes more difficult to exactly distinguish mineralized zones from non-mineralized zones in the bouguer gravity field.

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*Residual gravity field: difference between the gravity field of the stochastic density model and that of the constant (mean) density model.*
7.1.3 Outlook I

Although the 3D earth models presented in this thesis accommodates multiple parameters (density and geochemical data), there is always need for improvement such that the derived models are more representative of the reality in place. At Nash Creek, future stochastic models could be improved by accommodating information about geology. In this case, the stochastic models that characterize the spatial distribution of density, Zn% as well as other assay information are computed conditional to a predefined geologic model.

Besides, geologic information, the probabilistic modeling can also be tailored to accommodate existing geophysical information. For example, surface gravity data stripped of overburden effects, and well log density data can be combined to invert for a 3D smooth density model. The resulting density model in this case qualifies as a deterministic trend which can be subtracted from observed well data at respective borehole locations. The residual (fluctuation) component can be considered to be random and stationary over space such that more advanced geostatistical tools such as kriging with locally variable mean (LVM, Deutsch and Journel, 1998; Doyen, 2007) can be used. For example, SGS with LVM will involve doing SGS kriging computations with the residuals as input and the derived output is added to the local trend value (density model derived from gravity inversion) to obtain the final estimate at each grid location.

So far, proposed methods for improving the stochastic models are based on accommodating secondary information, however an alternative solution should focus on improving the existing database. For example, more density data from several other boreholes on the Nash Creek property need to be acquired in order to better assess lateral variability in density values. Also, a high resolution gravity (~2m) can be conducted along the borehole transects found in the density volume A (MVA). The advantage of doing this is two-fold as it can be used to validate and improve existing conditional stochastic models of density. However, the setback of such an approach is that it is not cost effective.

Future work aimed at constraining the 2D map of the overburden thickness will also benefit integrated petrophysical and geophysical studies at Nash Creek.

With a better handle of crustal heterogeneity, there are according prospects for new ways to characterize heterogeneities from scattered seismic waves. So far, preliminary investigations
suggest spectral analysis (H/V) of forward (direct) P-waves hold significant promise. However, more research is needed to find better ways of extracting frequency and phase anomalies that are indicative of resonance scattering (Milkereit et al., 2005) processes.

7.2 Thompson Case- Conclusions and Outlook

7.2.1 Imaging a sulfide orebody in the TNB (Thompson Nickel Belt) complex

The complexity in the world class sulfide mineralization at Thompson is primarily associated to tectonic processes in the region. Zones of massive sulfide mineralization often occurring along the contact between archean gneiss (AG) and the setting formation (SF) have complexly deformed regions and are unevenly distributed (Figure 3.6). Sample geologic sections depict the orebody to be lens-shaped with a dip (>50°) that is aligned with the dip of the AG-SF contact. Unlike the Nash Creek case study, petrophysical modeling of elastic parameters in this case study used existing geologic models as the fundamental building block.

For the first time, the integration of rock property and geologic models helped in understanding the characteristic seismic scattering response of the orebody in the Thompson mine. Full 2D FD elastic wave modeling enabled the seismic detectability of the complex orebody with respect to the host geology to be successfully tested. Modeling studies based on petrophysical models comprising of the main lithologic units at Thompson provided insights relevant for seismic acquisition design and data processing that include:

- The bulk of the orebody diffractions (P-wave reflections) occurring in the downdip direction can be reasonably imaged with a broadband seismic source whereas the boundary of the SF produces weak reflections. Reflections from the latter also interfere with those generated from another geologic unit named the thompson formation (TF).

- Using an extended surface acquisition or VSP geometry with multicomponent sensors in the direction of dip offer optimal acquisition aperture for successfully imaging diffractions from the highly dipping structures. The VSP technique becomes more desirable especially when the structural dips are too steep: migration routines perform poorly on steeply dipping structures.
• Processing of a full 2D synthetic seismic reflection survey data using conventional processing routines show that the strongest diffractions stem from the thickest part of the orebody (complex deformation zone). This can be used potentially as a marker for mapping the strike of this complex region when processing 3D data.

Considering that the scattered wavefields from the orebody are complex in nature, caution must equally be taken to mitigate noise sources that undermine the signal strength of the target response. Examples of such noise sources include scattered waves caused by the background heterogeneities at the Thompson mine. These heterogeneities range from dyke shaped structures to small scale heterogeneities depicted by the fluctuations in borehole logs of various physical rock properties. In this study, the log scale fluctuations are also considered to be random and stationary. Unlike the Nash Creek case, a dip component in the log scale heterogeneities is also considered. Synthetic seismic modeling demonstrate that the metamafic (AMPT) inclusions as well as the heterogeneities observed at the log scale can potentially undermine the effectiveness of seismically characterizing the orebody and deformation structures at depth. This is in agreement with findings of other authors like L’Heureux et al., 2009. The coherence in the backscattered orebody diffractions is reduced as these are submerged in the background scattered wavefield component. Dyke-shaped heterogeneities (e.g. ultramafics- UM) also generate diffractions which interfere with those from the sulfide target. However, the characteristics of the diffraction dips from the dykes and the orebody are quite different. The differences in the diffraction dips provides vintage for improving the signal to noise ratio (SNR) of the target (orebody) diffractions. In this circumstance, selective dip filtering processing routines can be adopted to remove the scattered wavefields generated by these dyke- sized inclusions.

Multiples generated by the presence of a clay overburden can equally mask target diffractions. However, a comparison against real data suggests the numerical results tend to overestimate these effects. The synthetic results can be taken into account in acquisition design and planning as it indirectly provides a measure for the worst case scenario for seismic imaging around the Thompson mine.

Finally, it also observed that the dipping nature of the sulfide ore causes the generation of converted waves. These converted waves, though significantly affected by scattering processes
owing to the background heterogeneity, bear complementary information to help characterize the location and orientation of the orebody.

7.2.2 Outlook II

So far, 2D modeling studies at Thompson do show that the scattered wavefield from the orebody has a characteristic AVO signal that is associated to deformed and thick regions of the orebody. Hence, it suggests its potential as a marker for mapping the strike of the complexly deformed region. Future studies should focus on using realistic 3D models (Figure 7.1) to test this hypothesis. The 3D modeling should further assess the suitability of various surface and borehole acquisition geometries in providing relevant information about wavefields scattered from the orebody.

To further evaluate the impact of the log scale heterogeneities, more log data is needed in order to build stochastic models that are both conditional to the overall statistics from the petrophysical database and to any existing well data in the 2D/3D regions considered.

Figure 7.1: Diagram of a conceptual 3D model inspired from the 2D model in Figure 3.6a. Also shown are various combinations of surface and borehole acquisition geometries tailored to capture diffractions from the sulfide orebody.

Considering that the setting of the orebody at Thompson favours the generation of converted waves, future research can also be tailored to evaluate the efficiency of advanced seismic imaging methods/routines that accommodate P-S wave separation (Huang et al., 2007). The
robustness of these techniques should also be tested against the effects of seismic noise owing to seismic wave scattering from the inhomogeneities that characterize the AG and SF.

7.3 Imaging with Forward Scattered Wavefields

The combined investigations from the Nash Creek and Thompson case studies show the value of recording back scattered waves in order to infer (without inversion) information about heterogeneities despite shortcomings caused by scattering processes associated to background heterogeneities in the host rocks. While the backscattered orebody diffractions as well as associated travel times provide information about the ore, the lateral correlation length of seismic wave coda on the other hand are strongly correlated to scale lengths that characterize the background heterogeneity of the stochastic models.

Forward scattered waves are also generated as part of the seismic scattering process. Some studies in this thesis (Chapters 2 and 4) show that travel time fluctuations and other characteristics of the forward scattered (transmitted) wavefields do appear not to have significant value towards inferring information about crustal inhomogeneities especially orebody inclusions. Nonetheless, other circumstances do exist whereby transmitted waves can serve this purpose. The case examined in Chapter 6 consisted of using transmitted waves to image the structural contact between two geologic units with sufficient contrast in their impedances.

7.3.1 Structural imaging in transmission mode

Studies based on synthetic and real offset seismic data support that transmitted waves recorded in VSP mode do contain structural information about crustal heterogeneities. As demonstrated in Chapter 6, transmission imaging based on reverse time migration methods was successfully used to resolve the lateral structure of the sediment-breccia contact within the Bosumtwi Impact crater. An added advantage of this imaging methodology lies in its potential as a quality control tool for velocity: full reverse time migration helps to provide information about the velocity distribution beyond the borehole location.

In addition to VSP data, other settings by which forward scattered energy can be recorded and processed for crustal information do exist. These include passive source monitoring processes related to earthquakes and microseismic activity. Using transmitted wavefield from these processes for imaging are subject to effects such as geology, accuracy of sensor locations and
velocity information. It is demonstrated through a modeling study, tailored for microseismic applications, that the effects of geology must factor in designing the optimal acquisition geometry. A simple layered model with a local zone of high velocity results in local wavefront distortion of wavefields propagating from source to a vertical array of receivers (perpendicular to bedding). This consequently introduces flaws in any adopted source location algorithms especially if these are designed to accommodate homogenous velocity models.

7.3.2 Outlook III

Future research can focus on using the RTM transmission imaging technology for 3D elastic imaging. In this case, multiple moving source offset VSP lines (Figure 7.2) are acquired in a borehole that runs through the structure of interest. A cost effective approach will be to have single sensor fixed at depth to simultaneously record surface shots from a 3D surface reflection survey. Bongajum and Milkereit (2007) demonstrate that such data provides valuable information about the P-wave velocity anisotropy (Figure 7.3). Unlike the case in Chapter 6 which is based on the acoustic wave equation, the full wave equation would also provide improved results as the latter accounts for all scattered waveforms (P-, S-, PS-, and SP-waves).

For microseismic imaging, future work should consider effects based on different heterogeneous models (scale lengths), source parameters, and the strength of the perturbations in the rock properties. Simulated wavefields from these respective models can be evaluated to find the optimal acquisition aperture, depth and receiver sampling intervals that minimize errors in microseismic source characterization/location.
7.4 Summary

Although most of the research outlook suggested is based on individual case studies, they are nonetheless useful for addressing general issues of interest in geophysical imaging. Based on findings from the present work, future research goals can be cast in a more general sense by considering the following recommendations:

- Collect comprehensive petrophysical data such as multiple continuous borehole logs that allow sufficient characterization of the in situ heterogeneity. As demonstrated in Chapters 2, 4, 5, and 6, the character of the heterogeneity in place has important effects on how one acquires and processes seismic data.

- Research on processing techniques that lay considerable emphasis on using both reflected P and S-waves especially for imaging in hardrock environment. Given that the earth is 3D, this therefore requires the use of 3D multicomponent (3C) seismic data which ensures a more realistic imaging of subsurface heterogeneity.

- Investigate on 3D multicomponent elastic RTM methods that accommodate the use of forward scattered P-, S- and converted waves for imaging; the use of a more realistic 3D velocity model in the process should also be considered.

- Investigate on new techniques that are better adapted to investigate the suitable conditions for extracting and performing quantitative analysis (e.g. spectral ratios) that help relate resonant frequencies to the stochastic characteristics of the earth’s heterogeneity. In this light, concerted efforts should also be geared towards evaluating according implications for AVO analysis given that amplitudes as well as spectral characteristics of the seismic waves are severely affected by multiple scattering processes.
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Appendix A

A.1 Wave propagation across elastic discontinuities - general case

Analysis in section 2.2 focused on the case for a downgoing incident P-wave, which is the scenario considered in the numerical experiments shown in Chapters 2, 4, 5 and in Appendix F. However, a more general assessment requires accommodating the idea that P and S waves can be incident from above and below the interface (Figure 2.2b, Figure 6.3). All four possible types of incident P- and SV- waves each generate all four possible types of outgoing P- and SV- waves. Thus, a total of 16 coefficients are involved for a complete analysis. A detailed illustration for the different combinations of incident and derived waves of the form in equation 2.9 can be found in Aki and Richards (2002, Table 5.3, pp. 142-143).

From the continuity of \( u_x, u_z, T_{xz}, T_{zz} \) and considering that displacement amplitude is \( \omega \times \) (potential amplitude)/velocity we then obtain the following equations:

\[
\begin{align*}
\sin i_1 (P_D^1 + P_U^1) + \cos j_1 (S_D^1 + S_U^1) &= \sin i_2 (P_D^2 + P_U^2) + \cos j_2 (S_D^2 + S_U^2) \\
\cos i_1 (P_D^1 - P_U^1) - \sin j_1 (S_D^1 - S_U^1) &= \cos i_2 (P_D^2 - P_U^2) - \sin j_2 (S_D^2 - S_U^2) \\
2 \rho_1 \beta_1^2 \rho \cos i_1 (P_D^1 - P_U^1) - \rho_1 \beta_1 (1 - 2 \beta_2^2 p^2) (S_D^1 - S_U^1) &= 2 \rho_2 \beta_2^2 \rho \cos i_2 (P_D^2 - P_U^2) - \rho_2 \beta_2 (1 - 2 \beta_2^2 p^2) (S_D^2 - S_U^2) \\
\rho_1 \alpha_1 (1 - 2 \beta_1^2 p^2) (P_D^1 + P_U^1) - 2 \rho_1 \beta_1^2 \rho \cos j_1 (S_D^1 + S_U^1) &= \rho_2 \alpha_2 (1 - 2 \beta_2^2 p^2) (P_D^2 + P_U^2) - 2 \rho_2 \beta_2^2 \rho \cos j_2 (S_D^2 + S_U^2)
\end{align*}
\]

Rearranging the above system of equations such that derived waves are all on the left-hand side while the incident waves are on the right, we get:

\[
M \begin{pmatrix} P_D^1 & S_D^1 & P_U^1 & S_U^1 \end{pmatrix}^T = N \begin{pmatrix} P_D^2 & S_D^2 & P_U^2 & S_U^2 \end{pmatrix}^T
\]  

(A.1a)

Where \( M \) and \( N \) are 4x4 matrices.
Note that when \( P_D^1 = 1 \) and \( S_D^1 = P_U^2 = S_U^2 = 0 \), the reflection and transmission coefficients of the derived P- and SV-waves are simply coefficients in the first column of \( M^{-1} N \). Thus, the 16 coefficients can be obtained from the following matrix relation:

\[
\begin{pmatrix}
P_D^1 P_U^1 & S_D^1 P_U^1 & P_U^2 P_U^1 & S_U^1 P_U^1 \\
P_D^1 S_U^1 & S_D^1 S_U^1 & P_U^2 S_U^1 & S_U^2 S_U^1 \\
P_D^1 P_D^2 & S_D^1 P_D^2 & P_U^2 P_D^2 & S_U^2 P_D^2 \\
P_D^1 S_D^2 & S_D^1 S_D^2 & P_U^2 S_D^2 & S_U^2 S_D^2 \\
\end{pmatrix} = M^{-1} N \quad \text{(A.1b)}
\]

where

\[
M = \begin{bmatrix}
-\alpha_1 p & -\cos j_i & \alpha_2 p & \cos j_2 \\
\cos i_1 & -\beta_1 p & \cos i_2 & -\beta_2 p \\
2\rho_1 \beta_1^2 p \cos i_i & \rho_1 \beta_1 (1-2\beta_1^2 p^2) & 2\rho_2 \beta_2^2 p \cos i_2 & \rho_2 \beta_2 (1-2\beta_2^2 p^2) \\
-\rho_1 \alpha_1 (1-2\beta_1^2 p^2) & 2\rho_1 \beta_1^2 p \cos j_i & \rho_2 \alpha_2 (1-2\beta_2^2 p^2) & -2\rho_2 \beta_2^2 p \cos j_2
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
\alpha_1 p & \cos j_i & -\alpha_2 p & -\cos j_2 \\
\cos i_i & -\beta_1 p & \cos i_2 & -\beta_2 p \\
2\rho_1 \beta_1^2 p \cos i_i & \rho_1 \beta_1 (1-2\beta_1^2 p^2) & 2\rho_2 \beta_2^2 p \cos i_2 & \rho_2 \beta_2 (1-2\beta_2^2 p^2) \\
\rho_1 \alpha_1 (1-2\beta_1^2 p^2) & -2\rho_1 \beta_1^2 p \cos j_i & -\rho_2 \alpha_2 (1-2\beta_2^2 p^2) & 2\rho_2 \beta_2^2 p \cos j_2
\end{bmatrix}
\]

The reflection and transmission coefficients obtained here are for displacement amplitudes and equally apply for particle velocity amplitudes.

A.2 Seismic wave scattering: Born Approximation- single scattering

The section presents the Born approximation for scalar wave equation according to the method shown by Sato and Fehler (1998).

Proving for the scalar field gives us a more simple treatment to the scattering problem compared to the elastic case where it is more complex. However, the underlying principles are identical.
Assume our focus is on an incident wave being scattered by a local heterogeneity centered at the origin (Figure A.1) in a homogenous background medium. Let’s consider the wave propagation velocity within the medium is given as

\[ V(x) = V_0 \left(1 + \varepsilon(x)\right) \quad (A.2) \]

where \( |\varepsilon(x)| = \left| \frac{\partial V}{V} \right| << 1 \) is the fractional perturbation of the velocity field (e.g. P-wave velocities of the felsic and mafic host rocks at Nash Creek), \( \varepsilon(x) \neq 0 \) only for \( |x| \leq \frac{L}{2}, i \in \{1, 2, 3\} \) and \( V_0 \) is the mean velocity field.

\[ \nabla^2 u(x,t) - \frac{1}{V(x)^2} \frac{\partial^2 u(x,t)}{\partial t^2} = 0 \iff Ku(x,t) = 0 \quad (A.3) \]

where \( K = \nabla^2 - \frac{1}{V(x)^2} \frac{\partial^2}{\partial t^2} \).
Due to the localized inhomogeneity in the medium we can write the displacement field $u$ as the sum of the primary field $u^0$ and the scattered field $u^1$:

$$u = u^0 + u^1$$  \hspace{1cm} (A.4)

Equation A.3 can be solved for the scattered wavefield for a given frequency by using the first order perturbation theory whereby we assume $|u^i| \ll |u^0|$. The incident wave obeys the homogenous wave equation: $\vec{K}u^0 = 0$, where $\vec{K} = \nabla^2 - \frac{1}{V_0^2} \frac{\partial^2}{\partial t^2}$.

Substituting (A.4) in (A.3) and considering the local fluctuation in velocity we have:

$$\vec{K}u^0 + Ku^1 = 0 \iff \vec{K}u^0 + \frac{2\varepsilon(x)}{V_0^2} \frac{\partial^2 u^0}{\partial t^2} + Ku^1 = 0$$  \hspace{1cm} (A.5)

Considering the wave equation characterizing the incident wave at the inhomogeneity and neglecting the terms of the form $\varepsilon(x) \frac{\partial^2 u^1}{\partial t^2}$, we obtain a wave equation of the scattered wavefield $u^1$ as a function of the incident wavefield $u^0$:

$$\bar{K}u^1 = \frac{2\varepsilon(x)}{V_0^2} \frac{\partial^2 u^0}{\partial t^2}$$  \hspace{1cm} (A.6)

The right-hand side of equation (A.6) appears as a source term which interacts with the inhomogeneity to generate a scattered wavefield $u^1$. Taking the plane wave to be incident at the inhomogeneity along the $x$-direction (where $x_1 = x, x_2 = y, x_3 = z$, and the wavenumber $\mathbf{k} = (k_1, k_2, k_3), k_1 = k, k_2 = k_3 = 0$) with unit amplitude: $u^0 = e^{i(k \cdot x - \omega t)}$,

we have

$$\bar{K}u^1 = \frac{2\omega \varepsilon(x)}{V_0^2} e^{i(k \cdot x - \omega t)}$$  \hspace{1cm} (A.7)

Given that the inhomogeneity is small, the scattered wavefield can be viewed as a field radiating from an equivalent point source with the origin located at the centre of the inhomogeneity. Thus equation (A.7) can be transformed and solved in the spherical coordinate system $(r, \psi, \theta)$ with
unit base vectors \( \hat{r}, \hat{\psi}, \hat{\theta} \). Hence the scattered field is an outgoing spherical wavefield with wavenumber \( (k\hat{r}, 0, 0) \).

To solve (A.7), one can use the Greens function \( G(x, t) \) for a plane wave propagating in a 3D-homogenous medium with velocity \( V_0 \) which satisfies (Aki and Richards, 2002, p64)

\[
\bar{K}G(x, t) = \delta(x)\delta(t) \quad (A.8)
\]

In more elaborate terms, \( G(x, t) \) is the response of the system to a compressed force acting over a very short period of time as depicted in (A.8) by the delta functions \( \delta(x) \) and \( \delta(t) \) respectively. The solution for (A.8) is given by (Aki and Richards, 2002):

\[
G(x, t) = \frac{1}{4\pi|x|} \left( t - \frac{|x|}{V_0} \right) \quad (A.9)
\]

Given (A.9), the scattered wavefield can be written as a superposition of the basic solution of the Green’s function i.e a convolution integral:

\[
u^1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dxG(x - x', t - \tau) \frac{2\omega^2}{V_0^2} e^{i(\mathbf{k} \cdot (x' - \mathbf{x}) + i\mathbf{x} \cdot \mathbf{e})} \]

\[
u^1 = -\frac{\omega^2}{2\pi V_0^2} e^{i\mathbf{x} \cdot \mathbf{e}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 x' \frac{e^{i(\mathbf{k} \cdot (x' - \mathbf{x}))}}{|x - x'|^3} \quad (A.10)
\]

For farfield observations where \( r \gg L \) such that \( |x - x'| \rightarrow r \) in the denominator of (A.10) and \( r \gg \frac{1}{\pi} \mathbf{L}^2 k \) such that \( |x - x'| \rightarrow r - x' \hat{r} \) in the exponent \( (\hat{r} = x/r) \), we have:

\[
u^1 = -\frac{\omega^2}{2\pi V_0^2} e^{i(\mathbf{k} \cdot x') e^i(\mathbf{k} \cdot x - \mathbf{k} \cdot \hat{x})} \frac{e^{i(\mathbf{k} \cdot x - \mathbf{k} \cdot \hat{x})}}{r} A \quad (A.11)
\]

\[
A = -\frac{\omega^2}{2\pi V_0^2} e^{i(\mathbf{k} \cdot \hat{x})} \hat{e}(k\hat{r} - k\hat{i}) \quad (A.11)
\]

\( A \) is called the scattering amplitude for a localized inhomogeneity which is frequency dependent. The tild means the fourier transform in 3D space.
\[ \tilde{e}(m) = \int \int \int \tilde{e}(x) e^{imx} dx. \quad (A.12) \]

The argument of \( \tilde{e} \) in (A.11) is called the exchange wavenumber which is the difference between the wave numbers of the incident plane wave and the outgoing scattered wave (Sato and Fehler, 1998, p90). Having derived the expression for the particle displacement, we can further proceed to characterize the energy associated to the particle displacement in terms of the strain energy. For a plane wave, strain energy density is similar to the kinetic energy density \( E = \rho \dot{u}^2 / 2 \) (\( \dot{u} \) = particle velocity). It thus follows that the flux rate of energy transmission per unit area and per unit time normal to the direction of propagation is \( \rho V_0 \dot{u}^2 \). For the incident plane wave, the energy flux is given as: 

\[ J_0 = \rho V_0 \dot{u}^2 = V_0 \rho \omega^2 |u_0|^2 = V_0 E_0. \]

Hence the scattered energy flux is:

\[ J_1 = V_0 \rho \omega^2 |\dot{u}|^2 = \frac{A_1^2}{r^2} J_0 \quad (A.13) \]

Equation (A.13) therefore suggests that the energy flux of the scattered wavefield is a scaled version of the energy flux of the incident wave.

If we consider the case where the medium is random not only locally and has isotropic heterogeneity, then the scattered wavefield would also be random (Wu, 1985). Hence, it is more plausible to look at random field in the sense of its mean square amplitude. Also, if dealing with an ensemble of random functions where \( \langle \epsilon(x) \rangle = 0 \), we can characterize the ensemble average scattered energy flux due to a unit block of dimensions (\( a \) = scale length = minimum dimension for the scattering volume, Sato and Fehler, 1998, p91)

So far, the mathematical treatment derived above applies for a scalar wavefield, but then a more advance treatment of the scattering process would be required for elastic wave propagation through a heterogeneous earth. Elastic wave scattering is a more effective tool for the assessment of heterogeneities in the earth (Wu and Aki, 1985) given that heterogeneities are a combination of perturbations in the compressional wave velocity, shear wave velocity and density. Similar to the scalar wave treatment above, we shall consider the case for a single inclusion in a
homogenous elastic background medium. This forms the basis for more advanced scattering theory. The illustration presented below is guided by the method derived by the authors Wu and Aki, 1985; and Sato and Fehler, 1998.

Following the approach for scalar waves, let’s assume the local heterogeneity is described by variations in the density and the Lamé parameters:

\[
\rho(x) = \rho_0 + \delta\rho(x), \quad \lambda(x) = \lambda_0 + \delta\lambda(x), \quad \mu(x) = \mu_0 + \delta\mu(x)
\]

where the following conditions for the fractional perturbations are satisfied

\[
\left|\frac{\delta\rho}{\rho}\right| < 1, \left|\frac{\delta\lambda}{\lambda}\right| < 1, \left|\frac{\delta\mu}{\mu}\right| < 1.
\]

It is worth noting here that if the single inclusion has characteristics of the Thompson orebody while the background has the mean physical properties of the Archean gneiss (Table 3.2, Chapter 3) the above fractional perturbation condition is invalidated since \(\left|\frac{\delta\rho}{\rho}\right| \approx 0.5\).

Just as in equation (A.4), the displacement field, \(\mathbf{u}\), can be expressed as the sum of the primary field \(\mathbf{u}^0\) and the scattered field \(\mathbf{u}^1\). For consistency, the wave equation will be expressed in terms of displacement and stress field (traction \(\mathbf{\sigma}\)).

The elastic wave equation for displacement is given as:

\[
\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mathbf{\nabla} \cdot \mathbf{\sigma} = \frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial \sigma_{i1}}{\partial x_1} + \frac{\partial \sigma_{i2}}{\partial x_2} + \frac{\partial \sigma_{i3}}{\partial x_3}
\]

\[
\rho \partial_t^2 \mathbf{u}_i - \partial_j \sigma_{ij} = 0 \quad (A.14)
\]

The relation between traction and the Lamé parameters is given by the general form of the constitutive equation:

\[
\sigma_{ij} = \lambda \delta_{ij} \partial_j \mathbf{u}_i + \mu \left( \partial_i \mathbf{u}_j + \partial_j \mathbf{u}_i \right) \quad (A.15)
\]
where $δ_y$ is the Kronecker delta and $i,j \in \{1,2,3\}$ represent indices for the Cartesian coordinate system (x,y,z). The P- and S-wave velocities are written as $α(x) = \sqrt{\lambda(x) + \mu(x)/ρ(x)}$ and $β(x) = \sqrt{\mu(x)/ρ(x)}$.

Combining (A.14) and (A.15) and solving for the total wavefield using Born approximation whereby the scattered wavefield is expected to be smaller than the incident wave amplitude, we have:

\[ ρν_0 \partial_i^2 u_i = \partial_j \left[ (λ(x)δ_y, δ_j, μ(x) (\partial_i u_j + \partial_j u_i)) \right] \]

\[ ρ(x)\partial_i^2 u^0_i + (x)\partial_i^2 u^1_i = \partial_j \left[ (λ(x)δ_y, δ_j, μ(x) (\partial_i u^0_j + \partial_j u^0_i)) \right] + \partial_j \left[ (λ(x)δ_y, δ_j, μ(x) (\partial_i u^1_j + \partial_j u^1_i)) \right] \]

\[ ρν_0 \partial_i^2 u^0_i + δρν_0 \partial_i^2 u^0_i + ρν_0 \partial_i^2 u^1_i + δρν_0 \partial_i^2 u^1_i = \partial_j \left[ (λ ν_0, δ_j, μ_0 (\partial_i u^0_j + \partial_j u^0_i)) \right] + \partial_j \left[ (λ ν_0, δ_j, μ_0 (\partial_i u^1_j + \partial_j u^1_i)) \right] \]

\[ ρν_0 \partial_i^2 u^0_i + δρν_0 \partial_i^2 u^0_i + ρν_0 \partial_i^2 u^1_i + δρν_0 \partial_i^2 u^1_i = \partial_j \left[ (λ ν_0, δ_j, μ_0 (\partial_i u^0_j + \partial_j u^0_i)) \right] + \partial_j \left[ (λ ν_0, δ_j, μ_0 (\partial_i u^1_j + \partial_j u^1_i)) \right] \]

(A.16)

The expression above can be simplified by neglecting cross terms of $(δλ, δμ, δρ) \times u^1_i$ since they are considered small and the fact that the incident wave satisfies the homogenous wave equation

\[ ρν_0 \partial_i^2 u^0_i - \partial_j \sigma_{δ_j} (λ ν_0, μ_0; u^0_i) = 0 \]  

(A.17)

(A.17) becomes

\[ ρν_0 \partial_i^2 u^1_i - \partial_j \sigma_{δ_j} (λ ν_0, μ_0; u^1_i) = h_i(x,t) \]  

(A.18)

where

\[ h_i(x,t) = -δρν_0 \partial_i^2 u^0_i + \partial_j \left[ (λ ν_0, δ_j, μ_0 (\partial_i u^0_j + \partial_j u^0_i)) \right] \].
The right hand side of (A.17) is the equivalent body force and represents the interaction between the incident plane wave with the inhomogeneity. If we consider, for example, the case for an incident P-wave of unit amplitude propagating in the first direction: 
\[ u_i^0 = e^{i(k \cdot x + \omega t)}; \quad k = (k, 0, 0); \quad k = \omega / \alpha \]

we have the following corresponding equivalent force
\[
h_i(x, t) = \left[ \omega^2 \delta \rho - k^2 \left( \delta \lambda + 2 \delta \mu \right) + 2ik \delta \rho \partial_t \right] \delta_x + ik \partial_t \delta \lambda \right] e^{i(k \cdot x + \omega t)}
\]

Just like the case for the scalar wave equation, solving (A.18) for the scattered wavefield requires using the explicit form of the Greens function for elastic waves (Aki and Richards, 2002 p72). Using spherical coordinates Wu and Aki (1985) derived the expressions for the scattered P- and S-wave farfield for the case where the inhomogeneities are much smaller than the wavelength (Rayleigh scattering). In the spherical coordinate axes, the problem is symmetric to the polar axis (direction of wave incidence, Figure A.2); thus there is no latitudinal component. For the case of an incident P-wave in direction 1 (x_1) the analytic expression for the scattered P-wave \( P_{u_{r \text{P}}} \) and S-wave \( P_{u_{mer \text{S}}} \) fields are written as:
\[
P_{u_{r \text{P}}} = \frac{V_{\text{vol}} \omega^2}{4\pi \alpha_0^2} \frac{1}{r} e^{i(k_1 \cdot r + \omega t)} \left[ \delta \rho \cos \theta - \frac{\delta \lambda}{\lambda_0 + 2\mu_0} - \frac{\delta \mu}{\lambda_0 + 2\mu_0} \cos^2 \theta \right] \quad (A.19)
\]
\[
P_{u_{mer \text{S}}} = \frac{V_{\text{vol}} \omega^2}{4\pi \alpha_0^2} \frac{1}{\beta_0^2} \frac{1}{r} e^{i(k_1 \cdot r + \omega t)} \left[ \frac{\delta \rho}{\rho_0} \sin \theta + \frac{\beta_0}{\alpha_0} \frac{\delta \mu}{\mu_0} \sin 2\theta \right] \quad (A.20)
\]

where \( \omega \) is the angular frequency; \( k_p = \frac{\omega}{\alpha_0}; \quad k_S = \frac{\omega}{\beta_0}; \quad \theta \) is the scattering angle (angle between the incident and the scattering direction) and \( V_{\text{vol}} \) is the volume of the inclusion (inhomogeneity). The tild means the 3D-Fourier transform as in (A.12). The mathematical expressions derived above for the scattered wavefield help to calibrate the modeled wavefields for similar conditions using 2D/3D finite difference codes. Results from an acoustic numerical analysis performed by Jannaud et al. (1991) in order to analyze coda waves generated in a 2D random media show that the discrepancy between the numerical result and theory is negligible provided the velocity fluctuation is \( \leq 5\% \). One of the sources for the discrepancy is underscored by the fact that the
Born approximation does not consider the feedback of scattered waves into the incident waves, hence the total energy is not conserved. Although the fluctuations in velocity may be small, the cumulative effect of this feedback mechanism causes the scattering amplitude to be large enough to invalidate the smallness condition near the forward direction for high frequencies (Sato and Fehler, 1998, p95).

A.3 Building a synthetic heterogeneous 2D earth model using Fast Fourier transforms (FFT).

Once the parameters of a characteristic model fit (e.g. von Kármán function) to the experimental autocovariance function are obtained, the following steps can be used to construct 2D/3D synthetic heterogeneous models. The variations in the resulting model properties have statistical properties that are similar to those obtained from hard data (e.g. borehole logs).

**Step 1:** Use the determined parametric model parameters to compute a Power spectral density function (PSDF) on a regularly spaced wave number grid (Equation 2.12). According expressions of the PSDF for the 2D and 3D cases are given in Huang et al., 2009.

**Step 2:** Compute a Fourier spectrum by multiplying the square root of the PSDF (from step 1) by a phase factor $\exp(i\sigma)$ where $\sigma$ is a random number uniformly distributed on the interval $[0, 2\pi]$.
**Step 3**: Apply an inverse Fourier transform (depending on the dimensions used) to the output from step 2 to obtain the image in the space domain. Care must be taken to ensure the sampling requirements are met in order to avoid aliasing effects.
Appendix B

B.1 Electric and magnetic properties- Nash Creek case study

Measurements were done using 25mm-diameter by 18-24mm-long samples. The lab measurements for electric properties focused on the electric induced polarization (IP) response of the core samples. The IP response of each rock sample was derived from a record of the transient response (voltage change) induced by a 2 sec square wave electrical current. A schematic representation of an example of a measured transient response is illustrated in Figure B.1. The IP opposes the build up or collapse of the potential difference between the voltage electrodes when current is switched on or switched off. The change in voltage depends on the degree of polarizability of the sample. There is a vast literature that covers the electro-chemical mechanisms explaining the IP phenomenon as well as case histories highlighting the application of IP methods for base metal exploration (Loeb and Bertin, 1976; Fink et al., 1990).

![Figure B.1: Sample of measured induced polarization charge-discharge curve.](image)

The transient voltage response is associated with both metallic and non-metallic effects. This consequently poses a problem for the effectiveness of IP methods in ore prospecting where the IP phenomenon due to the metallic and non-metallic effects of the rock is indistinguishable. For ore prospecting, interest is usually in the IP response owing to the metallic effects of the rock.
In the experimental setup, the measureable is the transient voltage response from which chargeability and resistivity parameters can be obtained. To measure resistance, the average of the voltage values (Primary voltage) within a small time window shortly before current turn off (Figure B.2) was used. The chargeability measure was obtained from a normalized integral of the voltage values over a small time window some time after the current turn off (Figure B.2). The normalization coefficient for the integral is the primary voltage. Resistivity was obtained by using the mathematical relation:

\[ \text{Resistivity} = \frac{\text{Resistance} \times \text{Area}}{\text{length}} \]  

\( (B.1) \)

A cross plot of the chargeability and resistivity is shown in Figure B.3. Low resistivities were recorded in samples visually identified with some veins rich in sphalerite, pyrite and galena mineralization. The figure shows an inverse relation between resistivity and chargeability whereby low resistivity correlates with high chargeability and vice versa. The resistivity contrast existing between mineralized and non-mineralized rock samples provides a valuable tool for quality control on resistivities characterizing known conductors from surface DC sounding (Milkereit et al., 2008).
Nowadays, it is common practice to use IP sounding sections in constraining the interpretation of any conductors obtained in surface resistivity (DC) sounding. Based on the chargeability results from the suite of Nash Creek rock samples, IP sounding combined with surface DC sounding will be adequate for delineating the shallow sulfide-mineralized zones. Magnetic susceptibility of the rock samples was also measured, but the results were inconclusive.

Figure B.3: Cross plot of measured chargeability and resistivity from the Nash Creek rock samples.

Samples with veins hosting sulfide mineralization (pyrite, sphalerite, and galena)
Appendix C

C.1 Multivariate stochastic modeling

Considering that rock densities are reasonably associated to the geochemical properties of the rock, estimates of geochemical information at locations with no assay data but with associated density information can be improved by accommodating available density data in the kriging process. This means, the kriging of assay information is conditioned by available information on the density of the rocks. The reverse process could equally apply. An example of such a method is called collocated cokriging. The cokriging estimator works well especially when the primary variable is relatively under sampled compared to the secondary variable. The ideal scenario would be to have an exhaustive representation of the secondary variable (e.g. seismic data when used to improve the estimation of porosity in a reservoir, Doyen, 2007, p59). Non-exhaustive secondary data can equally be used.

In simple collocated cokriging, the cokriging estimator is designed such that the linear estimator (equation 4.8) is supplemented by single collocated secondary datum (Goovaerts, 1997, Doyen, 2007). As discussed in chapter 4, a markov-type screening assumption is considered in collocated cokriging. This reduces the modeling effort (computational cost) considerably as the collocated cokriging system does not call for covariance between secondary data at lags \( h > 0 \) (see Goovaerts, 1997, p236; Almeida and Journel, 1994 for proof). The markov-type assumption results in the cross-covariance between the primary and secondary variable being proportional to the auto-covariance of the primary variable i.e. the semivariogram of the primary variable is proportional to the cross semivariogram:

\[
\gamma_{yy}(h) = \gamma_{yy}(0) \gamma_{xy}(h) ; \gamma_{xy}(0) = \frac{C_{xy}(0)}{C_{yy}(0)} \tag{C.1}
\]

Thus the cokriging process requires only knowledge of:

a) auto-covariance of the primary variable

b) variance of the secondary variable

c) local correlation coefficient \( \rho^* \) between the primary and the secondary variable
Details of the local correlation coefficient existing between the densities and assay data collected on the Nash Creek property are summarized Table 4.2 (Chapter 4). Based on the data in Table 4.2, the following observations can be inferred:

1) Zn% shows the highest correlation with density
2) The highest correlation between the assay data exists between Ag and Zn
3) Pb shows same correlation to Zn and Ag

The above observations suggest the density property is a good secondary variable for cokriging Zn% distribution in the Nash Creek property. In collocated cokriging, because the secondary information needs to be exhaustive (i.e. exist at every location within the grid), a difficulty arises as the density attribute is densely sampled only at a few well locations. In this thesis, this problem is solved by prestimating/simulating the density property within the target volume conditional to the actual well log data. In the event that the multiple variables of assay data need to be jointly simulated, then the method described by Almeida and Journel (1994) can be used. They propose to simulate each variable in turn according to a pre-defined hierarchy between the variables (see section 4.4.1b for examples). The approach by Almeida and Journel (1994) uses collocated cokriging which does not require complete modeling of the covariance matrix for all the variables. Such a markov-type (coregionalization) model breaks down as soon as the number of variables involved exceeds 3. The major trade-off of using this approach is that it does not guarantee the reproduction of the spatial correlation between the variables at lags \( h \neq 0 \).

The simulation steps are as follows:

a) Define a hierarchy of variables starting with the most important or better correlated variables.

b) Transform all variables in to the normal score domain:

\[
Z_i \rightarrow Y_i, i = 1, 2, 3, \ldots,
\]

where \( Z_i \) and \( Y_i \) identify the variables in the raw data and normal score domain; \( i \) is the index identifying the various variables. Also check that the conditions for bigaussianity are met. In this present study, this condition is assumed to be valid.

c) Pre-simulate independently the secondary variable(s) conditional to information at well locations. The drawback in doing this is that dependency between secondary
variables is not accounted by the pre-simulations. However, this approximation is acceptable because of the characteristic dense sampling of secondary variables (Almeida and Journel, 1994). For the Nash Creek case, this is not of major concern since only one primary variable is considered.

d) Define a random path visiting each node of the grid only once.

e) At each node $x_i$, determine the Gaussian conditional cumulative distribution function (ccdf) of the primary variable at $x_i$ given the $n$ neighbouring primary data of the same type and the only collocated secondary data. This requires solving the simple cokriging system of dimension of $(n+1)$. Then draw a simulated value $y_i^{(1)}(x_i)$ from the ccdf and add it to the file of hard $Y_i$-conditioning data set.

f) Loop until all N nodes are simulated.

g) Back transform the simulated values for the original values.

C.2 Impact of data configuration and search ellipsoid in SGS

In kriging, the kriging estimate at any given location is obtained based on available data existing within the distance of correlation of the model variogram. Hence, the kriging weights are directly proportional to the variograms/covariances evaluated for distances and directions between the estimation location and the different data locations. In addition, a decision has to be made on how many data points need to be included in the kriging system. Often, a moving search neighbourhood or ellipsoid is used such that data points considered in the kriging system are those that fall within the local neighbourhood. Figure C.1 shows an example of a 2D search ellipsoid that is centered about the estimation location (E).

Another condition that can be used in particular for SGS applications where experimental data is sparsely distributed is to restrict the number of data considered within the search ellipsoid. In SGS, each simulated information at a given grid node can be used in turn to get an estimate of the same property at another node. Depending on the data configuration, it becomes important to restrict the number of simulated data that is used for each simulation process. Consequently, this has different implications on the resulting SGS output. For example, if at an estimation location
one considers so many surrounding simulated data such that these outnumber the available experimental (hard) data within the same search region, the resulting kriging estimate is mostly biased by the total contribution of kriging weights of the corresponding simulated data considered. For the Nash Creek data, the effect of the maximum simulated data to be used during the SGS process was evaluated within Model Volume A (MVA). Table C.1 summarizes the parameters used for two conditions considered: condition A (maximum simulated data = 16) and condition B (maximum simulated data = 40). The parameter used to check the impact of these conditions on the SGS results is the normal score variogram. The interest is in finding if the input normal score variogram model is reproduced by the SGS results. Figure C.2 suggests that the SGS output reproduces the input normal score variogram when more simulated data are being considered within the search ellipsoid (condition B). For the present case, reasonable results are obtained for:

\[
\frac{\text{vertical scale length}}{\text{(Max. # of simulated data)}} \geq 1
\]  

(C.2)

In generating the SGS outputs in Chapter 4, precedence was given over the reproduction of the normal score variogram.
Table C.1: Criteria for SGS search strategy within MVA

<table>
<thead>
<tr>
<th>Search strategy within search ellipsoid</th>
<th>Condition A</th>
<th>Condition B</th>
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<tr>
<td>Maximum simulated data</td>
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<td>40</td>
</tr>
<tr>
<td>Ratio of Dimensions Search ellipsoid to the directional correlation lengths (a_x, a_y, a_z)</td>
<td>1</td>
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Figure C.2: SGS realizations and normal score variograms using search strategy conditions A (a and c) and conditions B (b and d). The SGS process uses an isotropic variogram input model with $a_x = a_y = a_z = (30/3) \text{ m}$, Nugget = 0.6 and sill = 1.
C.3 Additional figures

Figure C.3: Map showing distribution of boreholes around in the vicinity of region 1 (MVA- see Figure 4.1). The black (broken) line represents the profile connecting some of the boreholes with available density logs. Notice there is a pseudo NS profile and an EW profile.
Figure C.4: Density logs and block assay data for boreholes along the NS profile (see Figure C.3). The trends in the respective logs suggest the deterministic from the logs can be achieved using a constant value (mean) or a linear fit to the data.
Figure C.5: a) Density logs and block assay data for boreholes along the EW profile (see Figure C.3). The trends in the respective logs suggest the deterministic from the logs can be achieved using a constant value (mean) or a linear fit to the data.
Figure C.5: b) Density logs and block assay data for boreholes along the EW profile (see Figure C.3). The trends in the respective logs suggest the deterministic from the logs can be achieved using a constant value (mean) or a linear fit to the data.
Figure C.5: c) Density logs and block assay data for boreholes along the EW profile (see Figure C.3). The trends in the respective logs suggest the deterministic from the logs can be achieved using a constant value (mean) or a linear fit to the data.
Experimental Variograms- *Effect of Outliers*

Figure C.6: Synthetic density logs without (a) and with (e) outliers; (b) and (e) represent their respective experimental semivariograms; (c) and (f) indicate the number of data pairs used in the computation of the experimental variograms at each lag ($h$). The presence of outliers causes a significant change in nugget effect and correlation length. These outliers could be due to measurement errors or due to localized zones of sulfide mineralization. $\rho_c$: correlation coefficient.
Figure C.7: Plots of directional experimental semioagram (Deutsch and Journel, 1998- blue squares) and model fits for the Bouguer gravity data.
Figure C.8: Areal anisotropy of the lateral variations in the bouguer gravity data. The blue dots represent gravity data locations. The ellipsoid results from the combined analysis of the directional semirodograms. The azimuth of maximum correlation is $131^\circ$ (clockwise with respect to the north direction: $a_{\text{short}} \approx 264$ m, $a_{\text{long}} \approx 672$ m. The input parameters for computation were: lag unit distance: 30 m; Tolerance: $30^\circ$; Bandwidth: 20 m; Number of Strata: 1.
Figure C.9: Plots of directional experimental covariances (blue squares) and model fits for Zn%. The best fits are obtained for azimuths: $0^\circ$, $90^\circ$ and $120^\circ$. 
Figure C.10: Areal anisotropy of the lateral variations in the Zn% distribution. The well locations are represented by yellow well dericks. The ellipsoid results from the combined analysis of the directional covariances. The azimuth of maximum correlation is 128° (clockwise with respect to the north direction: $a_{\text{short}} \sim 50\text{m}$, $a_{\text{long}} \sim 120\text{m}$. The input parameters for computation were: lag unit distance: 25m; Tolerance: 30°; Bandwidth: 50m; Number of Strata: 100.
C.4 Gravity- Theory and numerical modeling

a) Theory

Most gravity meters measure the vertical component of the gravitational attraction which is usually denoted \( g \). Thus following Newton’s law of gravitational attraction, and considering the Cartesian coordinate system \((x, y, z)\), the gravity observed at a point \((x_1, y_1, z_1)\) due to a three-dimensional body of arbitrary shape with density \( \rho(x, y, z) \) is

\[
g(x_1, y_1, z_1) = G_c \iiint \rho(x, y, z) \frac{z - z_1}{r^3} \, dx \, dy \, dz \tag{C.3}
\]

where \( G_c \) is the universal gravitational constant and \( r = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} \).

When dealing with an irregularly shaped body mass, the gravitational field of the body at any point can be simply approximated by discretizing the body mass as a collection of small rectangular prisms. These rectangular prisms are assumed to be of constant density such that the total gravity field is a superposition of the gravity field from the individual rectangular prisms. Given a rectangular prism with dimensions shown in Figure C.11, and considering Equation (C.3), the vertical gravity at observation point \((x', y', z')\) is given by

\[
g(x', y', z') = -G_c \rho \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \frac{z' - z}{r^3} \, dx \, dy \, dz \tag{C.4}
\]
Equation (C.4) can also be viewed as the gravity potential difference between the top and bottom boundary surfaces of the rectangular prism (Li and Chouteau, 1998). An example of another derived expression for (C.4) consisting of twenty-four terms is given as (Plouff, 1976; Li and Chouteau, 1998):

\[
g = -G_c \rho \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} (-1)^i (-1)^j (-1)^k \left[ \Delta x_i \ln \left( \Delta y_j + r_{ijk} \right) + \Delta y_j \ln \left( \Delta x_i + r_{ijk} \right) - \Delta z_k \arctan \frac{\Delta x_i \Delta y_j}{\Delta z_k r_{ijk}} \right]
\]

(C.5)

where \( \Delta x_i = x_i - x', \Delta y_j = y_j - y', \Delta z_k = z_k - z', r_{ijk} = \sqrt{\Delta x_i + \Delta y_j + \Delta z_k} \)

b) Numerical modeling

Numerical computation of the gravity field from a given density volume entails using expression C.5 above. A Matlab script “GRAV3D.m” that implements equation C.5 was written and tested against the analytic solution for the gravity effect due to a horizontal rod (Telford et al., 1990, equation 2.53, p36). A good match was obtained between the theoretical and numerical results. Forward modeling results in section C.5 (below) were done with “GRAV3D.m”.

C.5 Implications for gravity- QC on the 3D density model (Nash Creek)

The observed 3D spatial fluctuations in density values from borehole logs have direct influence in the gravity field measured at the earth surface. Given that the distribution of the sampled density values suggests that high grade material (high densities) is not highly connected, it is important to understand the efficiency of gravity imaging methods in delineating the mineralized zones. In this section, a quality control test of the conditional 3D density stochastic models is performed through computation of the gravity response. This allows a comprehensive assessment of what extent the density distribution (especially the high densities) contributes towards the total gravity field. Recall that the measured gravity field is also sensitive to effects such as drift, topography, latitude, ocean tides, and variability in overburden thickness. The latter can be very difficult to account for if the overburden layer is not properly defined. This is the case with the Nash Creek property where gravity measurements have been made with some (Ugalde et al.,
2007) or no correction for overburden thickness. The forward modeled gravity field in the present study does not consider gravity field contributions from the overburden layer. Further assessment of the modeled gravity field is done by comparing the modeled results against existing gravity data on the property.

The 3D volume used for the stochastic modeling of density distribution consisted of rectangular prisms (voxet cells) as building blocks. Hence, using expressions (C.4) and (C.5) will be valid in this case. For a more realistic analysis of the modeled gravity field, a volume that is slightly larger (by lateral extension) than the original 3D volume is considered (Figure C.12). The observation surface spans the total top surface area of the extended 3D volume.

**Figure C.12:** Top view of 3D volume used for the gravity forward modeling. The shaded part represents the dimension of the original voxet model used for the 3D stochastic modeling (W~200m, L~300m- MVA).

**Figure C.13:** Left panel- Regional Bouguer gravity field (processed with digital elevation model of region); Right panel- Bouguer gravity from a select study area (Ugalde et al., 2007). A total of 451 stations were acquired. Part of the survey involved a high-resolution 2 m spacing loop over one of the mineralized areas (red ellipse).
Initial 2D and 3D (L’Heureux et al., 2007; Ugalde et al., 2007) studies on integrated modeling with gravity data (Figure C.13), borehole densities and geology information from the Nash Creek property suggested NW-SE trending mineralized zones. These zones had densities ranging from 2.8-3.4 gcm$^{-3}$ with thicknesses ranging from 20m to 30m. However, one shortcoming with these density models is that gravity data do not have the potential to resolve the heterogeneities observed in density logs. Hence, gravity forward modeling with stochastic density models helps our understanding of how small scale fluctuations in density contribute towards the total gravity field.

![Figure C.14](image_url): Total gravity field at model surface (0m depth) due to the total volume: the inner volumes are the SGS realizations 39 (a & b) and 81(c & d). Total volume$= (L + 2\Delta L) \times (W + 2\Delta W) \times$ Thickness (100m).

The observation surface at the model top (Figure C.12) is sampled at 5m intervals in the North and East directions. A density of 2.7gcm$^{-3}$(mean density) was used to characterize the 3D extension of the modeling volume whereby $\Delta L = \Delta W = 100m$. The gravity field computed from
two realizations (39 and 81) show that the maximum amplitude is at the centre of the respective 3D volumes (Figure C.14). Within the limits of the initial 3D voxet (inner rectangle—see Figure C.12), gravity values range from 4-9 mgals. These values are slightly lower but close to observed gravity data within the same region where values range between 11mgal and 13mgal (Figure C.13, Ugalde et al., 2007). A major source for the discrepancy between the observed and the modeled gravity comes from the fact that the present model does not include deeper zones (>100m). Moreover, the observed data contains gravity contributions from the overburden layer.

At Nash Creek, the overburden thickness is poorly defined. Limited information about the overburden thickness comes from a few borehole collar data (Figure C.17). However, this information is not enough to sufficiently infer the 3D structure of the overburden layer. Another reason for the differences between observed and the synthetic gravity is the coarse sampling (~20m) used to acquire the observed data. High resolution gravity data is more suitable for probing the small scale fluctuations in the density distribution. The complex nature of the sulfide distribution (mostly low grade & disseminated) renders such analysis more challenging.

![Figure C.15: Left panel - Top view of the surface density distribution of the 3D voxet density volume (realization 81) for the bottom ~60m; Right panel – Synthetic Bouguer gravity field observed at 0m depth due to the density volume found within the bottom ~60m volume of the original density voxet (left panel). The black arrow indicates high-resolution gravity survey loop (see Figures 4.13 and 4.16).](image)

The modeled gravity field does not resolve the small scale variability of the density distribution. This observation is independent of the density volume being considered especially for the deep regions within the model (Figure C.15). Though the overall gravity field strength is different for the respective density volumes, the macrostructure of the modeled gravity field is almost identical. This further corroborates the shortcoming of gravity methods towards resolving small
scale variations in rock densities. This can pose challenges for gravity-based interpretation for mineralized zones where ore grade rocks have highly disseminated mineralization.

Figures C.16a & b show the elevation and observed Bouguer gravity field from a high resolution (2m sampling interval) gravity survey loop. The survey loop has dimensions 50m x 4m and oriented in the NW-SE direction (red ellipse in Figure C.13). Both the observed Bouguer gravity and the modeled Bouguer gravity (Figure C.16c) show comparable wavelength characteristics in the variability of the gravity field. While the order of magnitude of the modeled gravity field is less than the measured gravity, the dynamic range of fluctuation is about five times greater than the variations in the observed gravity field(< 0.12mgal). Furthermore, the trends from the

Figure C.16: a) 2D Elevation map around the high resolution survey loop (black curve).

b) Elevation, drift corrected gravity and Bouger gravity at acquisition points on the survey loop. The label “distance” is a pseudo representation of the sequence of data acquisition along the loop from the origin (OR) in the counter clockwise direction; Red Arrows: Data collected at the Base Station (BS= Borehole NC0623).

c) Observed and synthetic Bouger gravity at high resolution survey loop.
respective data sets are anti-correlated. Of course, one would not expect the synthetic gravity results to exactly match the observed since only analyses from two out of many other statistically plausible density models are considered. Moreover, the synthetic data is subject to the configuration of available density log data. In the present study, borehole A (Figure 4.1) is closest to the gravity survey loop. Better results can be achieved with more density logs from other boreholes in the vicinity. Nonetheless, these results lay further emphasis on the importance of properly accounting for the gravity field contributions of the overburden layer as well as other noise sources that overwhelm the gravity signal. In order to assess overburden effects in the gravity field, the following paragraph evaluates the extent to which shallow density distribution affects the wavelength characteristics of the gravity field.

Given that the gravity depends on the proximity of the dense material to the observation point/surface, it is expected that near surface variations in density will contribute most towards the local changes in the shape of the gravity field. The modeled gravity field due to a shallow 10m thick section from one of the stochastic density models (realization 81) depicts variations

![Histogram distribution of overburden thickness from 285 boreholes.](image)

**Figure C.17:** Histogram distribution of overburden thickness from 285 boreholes.
that are correlated with the distribution of high densities (Figure C.18 a & b). The 10m thick block contributes an average gravity field of ~ 1.1mgal. This gravity response is relatively small compared to the total gravity field contributed by the bottom 90m thick section of the density volume (Figure C.14). Hence, the short wavelength variations in the gravity field due to the density distribution within the shallow block will be masked in the gravity field due to the bottom (thicker) block. An alternate approach to assess small scale variability in the gravity field is to subtract the mean gravity field (gravity due to volume with mean density). The results reveal more details about the distribution of density anomalies within the 3D voxel volume (Figure C.18c and d). Notice the maximum residual gravity is about half the average gravity in Figure C.18b. The similarities in the distribution of the gravity anomalies between Figures 4.18b & d further corroborate the importance of the shallow distribution of density anomalies. This provides an alternate view of how much bias the characteristics the overburden layer at Nash Creek (thickness (0.45m- 25m), density distribution) can introduce in the measured gravity field.

Figure C.18: a) Top view of the surface density distribution of the 3D density volume (realization 81).; b) Gravity field due to the density volume found within 10m below the surface layer shown in (a). Residual gravity for two SGS models for density (c & d).
In summary, the amplitudes of the modeled gravity signature from the 3D stochastic density volumes compare reasonably with observed gravity from the area. Aside the fact that these density models are not exact, the identified causes of the mismatch between the theory and observed data include:

a) theoretical gravity field is computed from a limited model volume e.g. analysis precludes contributions from deeper structures

b) poor understanding of overburden effects in the observed gravity data

Only residual gravity field analyses provide firsthand information about small scale variations in density anomalies. However, the synthetic results suggest such information falls short of resolving deep density anomalies as the sensitivity is limited to shallow density anomalies (<11m). This further underscores the importance of adequately correcting overburden effects when performing gravity-based interpretation at the Nash Creek property.

C.6 Building an unconditional stochastic model for density using FFT approach (Nash Creek case study)

In order to build 2D stochastic density models such that the statistics of the density database is honoured, the FFT method (Goff and Jordan, 1988, see Appendix A) was adopted. However, rather than performing the process in the raw data domain, an initial 2D model is first created in the normal score domain. The normal score output is then backtransformed to the raw data domain conditional to the cumulative distribution function of the raw density values used. In order to characterize the background heterogeneity used in the seismic modeling study (section 4.5) the density values used fall in the range [2.3-3.0]g/cm³ (Figure C.19a). Figure C.19c shows the 2D stochastic density model output in the normal score domain. Backtransformation of normal score results to the raw data domain yields the density model in Figure C.19d. Notice the distribution of the simulated density data is in good agreement with the distribution of the raw data (Figure C.19a &b).
Figure C.19: Raw density (a) and simulated (b) density distribution; c) 2D simulated density model in normal score domain (variance= 1, nugget=0, $\nu=0.5$, $a_x=100m$; $a_z=30m$); d) 2D simulated density domain in raw data domain after backtransformation of c).
Appendix D

D.1 Effect of overburden

Amplitude, frequency and phase fluctuations observed in seismic waves can also be attributed to the presence of an overburden layer which is characteristically weathered. For seismic imaging applications, this can severely impede the effective resolution of target structures. For example, it is known that poor compensation for overburden layers result in poor resolution of the lateral extent of the sedimentary layers within a sedimentary basin.

In addition to the weathered nature of overburden layers, the efficiency of seismic imaging is further complicated by the fact that the interfaces between overburden layers and bedrock are characterized by a sharp impedance contrast with low velocities atop higher velocities (clay overburden) or high over low velocities (Basalt overburden). Such a sharp contrast can cause energy to be trapped within the overburden layer and hence severely reduces the seismic energy that reaches target structure. This is the situation at the Thompson mine where a clay-like overburden exists. In such circumstances, it becomes important to assess what influence such an overburden structure has on the seismic detectibility of orebodies at depth. This is addressed in this section by using an elastic FD modeling approach to analyze the effects of imaging seismic anomalies with 3C surface and borehole (VSP) receiver spreads.

![Figure D.1: Petrophysical model characterizing the background host rock, the lens (high grade chalcopyrite) and the overburden layer. The layer at the base of the model is for quality control purposes. A zoom into the top clay layer highlights the thickness and roughness of the different bedrock contacts used in this study.](image)

<table>
<thead>
<tr>
<th>Table D.1: Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Ore</td>
</tr>
<tr>
<td>Background</td>
</tr>
<tr>
<td>Overburden</td>
</tr>
</tbody>
</table>

Vp: P-wave velocity; Vs: S-wave velocity; ρ: Density
Table D.1 shows the input P-wave, S-wave and density parameters of the 2D (Figure D.1) models used in this study. Analysis involved comparing the responses from the inclusion (Figure D.1) in a 2D background model with and without an overburden layer.

Figure D.2: The top panel shows a snapshot of the P and S waves propagating in the model without overburden (Time=0.29s). Superimposed on the S-wave section is the surface and borehole acquisition geometries used in this study. The bottom panel shows both P and S waves for the case with clay overburden. The red star indicates the source position.

Figure D.3: The vertical component shot gathers from the surface receiver spreads for the cases without (a) and with (b) overburden (shot depth=10m). The right panel shows the worst case scenario as any reflection from the lens is masked by the multiples.

Results suggest that the overburden layer creates multiples strong enough to mask the diffractions from the dipping lens (Figures D.2 and D.3, White et al., 2010). A comparison of results from both surface and borehole data suggest the effect of the reverberations in masking
the lens diffractions is more intense for the former. Hence, the strategic VSP acquisition is more suitable for capturing the diffractions from the dipping lens (high diffraction amplitude from target). Also, notice that the diffraction amplitudes of the converted waves are relatively stronger than the P-wave diffractions in the presence of the overburden layer (Figures D.4a & b). In addition to the choice of acquisition geometry, Figure D.4c also suggests the relative location of the shot with respect to the overburden-bedrock boundary also matters. A shot located below the clay overburden mitigates the effects of multiples which undermine the detection of the orebody (Figure D.4c). These results may change significantly if effects of intrinsic attenuation and amplitude loss mechanisms are introduced. This is clearly corroborated by the seismic data acquired in a nearby mine (Birchtree mine)-Figure D.5. There is significant contrast between the modeling results and seismic data from the Birchtree mine. The Birchtree seismic data suggests that the overburden effects are much less severe than predicted from the FD modeling experiment. Thus, the synthetic results represent a worst case scenario.

**Figure D.4:** a) and b) are vertical component shot gathers from the borehole receivers (VSP geometry; see Figure D.2) with and without overburden. Notice how the amplitude from subsurface diffractions are greater than those from the surface receivers (shot depth=10m). The effect of shot depth (20m) is shown (c): the amplitude strength of generated reverberations is reduced.
D.2 Converted wave analysis

Figure D.5: Sample shot gather from the Birchtree surface seismic data set (Orsnby filter & AGC (500ms) are applied to the data).

Figure D.6: Constant velocity stack analysis for PP, PS and SS waves generated from the orebody in Figure 5.4.
As shown in Figures 5.4 and D.2, there is evidence for elastic wave conversion due to strong contrast in physical rock properties. Thus, there are implications for imaging with converted wave processing. A strong PS response from the orebody can be identified at about 550ms in Figure D.6. Isolating the PS as well as the S-wave responses may improve imaging. This can be achieved by taking advantage of the differences in the stacking velocities (Figure D.6) of the different body waves (PP, PS, and SS).

Table D.2: Estimated Von Kármán ACF model parameters

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<th>BHID</th>
<th>Poly. Fit</th>
<th>P-wave velocity</th>
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297.2* 185.2*

* Mean Variance from the variances of the respective Vp and Vs core measurements from all the lithologic units in Thompson (VALE-INCO petrophysical database (old)).

Table D.3: Estimated Exponential variogram model parameters

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Appendix E

E.1 Reverse time migration (RTM) – Background theory

Considering equation (2.1), the wave equation that describes elastic wave propagation in a given medium is written as:

\[ \rho \mathbf{u}_n = f_n + (\lambda + 2\mu)(\nabla \cdot \mathbf{u}) - \mu \nabla \times (\nabla \times \mathbf{u}) \]  

(E.1)

where \( \mathbf{u} \) represents displacement, \( \rho \) is the density, \( f \) is the source function, \( \lambda \) & \( \mu \) are the Lamé parameters, \( \nabla \) is the grad operator, subscript “\( t \)” is the derivative with respect to time. The middle term and the third term represent compression (P) and shear (S) waves respectively.

Considering the acoustic case and suppose we neglect the source parameters and rename the displacement vector with the variable \( U \), Equation (E.1) can be written as:

\[ \frac{1}{\alpha^2} U_n = \nabla^2 U, \quad \text{where} \quad \alpha^2 = \frac{(\lambda + 2\mu)}{\rho} \quad \text{(Wave velocity)} \]  

(E.2)

In order to implement equation (E.2) numerically, it is necessary to rewrite it in a discrete format. By doing this, one obtains a finite difference solver for the acoustic wave equation. Let’s consider deriving the case for a 2\textsuperscript{nd} order finite difference solver.

Writing spatial and temporal coordinates in discrete form gives \( x, y, z, t = i\Delta x, j\Delta y, k\Delta z, l\Delta t \) where \( \Delta x, \Delta y, \Delta z, \Delta t \) : spatial and temporal intervals in the x, y and z directions and in time; \( i, j, k, l \in \mathbb{Z}^+ \).

Hence the spatio-temporal coordinate of the particle displacement vector can be rewritten as:

\[ U(x, y, z, t) = U(i\Delta x, j\Delta y, k\Delta z, l\Delta t) = U(x_i, y_j, z_k, t_l) = U_{i,j,k,l} \]

For a 1D case, the first order derivative with respect to a variable (e.g. space) is given by (Taylor series approximation):

\[ U'(x) = \lim_{h \to 0} \frac{U(x + h) - U(x)}{h} = \lim_{h \to 0} \frac{U_{i+1} - U_i}{h} \quad \text{(Forward difference)} \]
\[ U'(x) = \lim_{h \to 0} \frac{U(x + h/2) - U(x - h/2)}{h}, \text{ where } h = \Delta x \text{ (central difference)} \]

Using the central difference formula for \( U'(x + h/2) \), \( U'(x - h/2) \) and for the derivative of \( U' \) at location \( x \), we can obtain the a central difference approximation for the second order derivative, \( U'' \), in space to be:

\[
U'' = \frac{\partial^2 U}{\partial x^2} \approx \frac{U(x + h) - 2U(x) + U(x - h)}{h^2} \tag{E.3}
\]

Applying (E.3) to (E.2) for a 2D case gives:

\[
\frac{1}{\alpha_{ij}^2} \left( \frac{U_{i,j,l+1} - 2U_{i,j,l} + U_{i,j,l-1}}{(mn)^2} \right) = \frac{U_{i+1,j,l} - 2U_{i,j,l} + U_{i-1,j,l}}{h^2} + \frac{U_{i,j,l+1} - 2U_{i,j,l} + U_{i,j,l-1}}{h^2} \tag{E.4}
\]

where \( h = \Delta x = \Delta z; mn = \Delta t \) and \( \alpha_{ij} = \alpha(x_i, z_j) \) is the 2D velocity field.

Further simplification of (E.4) gives:

\[
U_{i,j,l+1} - 2U_{i,j,l} + U_{i,j,l-1} = \left( \frac{\alpha_{ij}(mn)}{h} \right)^2 \left( U_{i+1,j,l} + U_{i-1,j,l} + U_{i,j,l+1} + U_{i,j,l-1} - 4U_{i,j,l} \right) \tag{E.5}
\]

\[
U_{i,j,l+1} = 2U_{i,j,l} - U_{i,j,l-1} + A \left( U_{i+1,j,l} + U_{i-1,j,l} + U_{i,j,l+1} + U_{i,j,l-1} - 4U_{i,j,l} \right) \tag{E.6}
\]

where \( A = \left( \frac{\alpha_{ij}(mn)}{h} \right)^2 \)

Equation (E.6) is the explicit form of the 2\textsuperscript{nd} order approximation of the acoustic wave equation. This allows one to solve for the wave propagation forward in time. Reverse time migration on the other hand requires solving for the wave equation backward in time (towards \( t=0 \)). This suffices rearranging (E.6) to obtain:
\[ U_{i,j,l-1} = 2U_{i,j,l} - U_{i,j,l+1} + A\left(U_{i+1,j,l} + U_{i-1,j,l} + U_{i,j+1,l} + U_{i,j-1,l} - 4U_{i,j,l}\right) \] (E.7)

The RTM implemented in this work is based on equation (E.7). In order to avoid problems with numerical stability and dispersion, the following conditions must be met:

\[
\alpha(x_i, z_j) \times \sqrt{2\Delta t} \leq h
\]

\[
\frac{\alpha_{\min}(x_i, z_j)}{f_{\text{max}}} \geq 8h \quad f_{\text{max}} - \text{Maximum frequency}
\]

The discrete form of the acoustic wave equation can also be written in implicit form. Moreover, higher order equations to solve the acoustic/elastic wave equation also exist. These have been tailored to better handle effects of numerical stability and dispersion (Loewenthal et al., 1991; Bohlen, 2002).

### E.2 Transmission imaging using reverse time migration (RTM)

This approach was initially proposed by McMechan et al. (1988). For imaging an unknown interface, ray tracing from the source using velocities observed around the source region is computed through the 2D model. This is identical to the process of forward extrapolating the wavefield through the 2D model. With the exponential increase of computational capacity over the last two decades, an alternative to ray tracing would be a forward elastic/acoustic wavefield finite difference modeling through the 2D model. The forward extrapolated wavefield is used for computing the imaging condition which was initially described by Chang and McMechan (1986). In their approach, each point in the medium is imaged by the one-way traveltime from the source to that point. The ray tracing assumes the velocity observed around the source region extends beyond the borehole location containing the receivers.

On the other hand, an elastic/acoustic finite-difference approach is used to backward propagate (extrapolate) the recorded wavefield data from the receivers and it is assumed that the velocity observed around the receiver spread is present everywhere. Given that different velocity distributions are used for ray tracing and the backward extrapolation process, the direct wave from the source and the backward extrapolated wavefield from the receivers intersect at the interface (e.g. sediment-breccia contact) to be imaged. At this interface, the sum of the ray traced
time through the velocity structure observed around the source and the wave-extrapolated time through the velocity structure around the receiver spread equal the total travel time of the direct wave observed in the recorded data. The process described above can be implemented in the following steps:

a) Preprocess the recorded data by muting all reflected wavefields. Since a finite-difference reverse time migration algorithm will be used to backward extrapolate the recorded data in time, the recorded data should be interpolated as if it were recorded using the sampling interval of the velocity model grid.

b) Use ray tracing to compute the direct arrival from the source point to each location in your velocity model grid. Create a time database for the estimated direct arrival times for all the nodes in your velocity model grid.

c) Use a reverse time migration algorithm to backward extrapolate the preprocessed recorded data.

d) The imaging condition, implemented at each time step, is done by scanning the time database for information on spatial locations (grid/nodes) where the sum of the estimated ray traced direct arrival time from the source and the backward extrapolation time gives the total recording time of the traces in the shot gather. These grid points are assigned the amplitudes at corresponding grid points in the matrix of the backward propagating wavefield. The amplitudes at each nodal point are built cumulatively with each time step.

E.3 Effect of medium heterogeneity (variability in scale length)

To investigate the effects of the scale length variability within the brecciated rock unit for the imaging process, two Vp models were considered. While the breccia unit in the first model (Figure E.1a) consists of isotropic scale lengths ($a_x=a_z=30m$) the second model (Figure E.1b) has anisotropic scale lengths ($a_x=1000m$, $a_z=30m$) for the same rock unit. The models were designed with the assumption that the fluctuations in the velocities (P- and S-waves) and densities are correlated.

As illustrated in Figures E.1c and d, the sediment-breccia contact from both models is successfully reconstructed. This suggests the main effect on the travel time fluctuations observed in the first breaks stems from the structure of the contact provided other factors such as out of plane diffractions are not present. Thus, the strength of the P-wave velocity fluctuations as
observed from the sonic log (Schmitt et al., 2007) coupled with the relative short distance of wavefield propagation between the receiver(s) and the source(s) may not be contributing sufficiently to alter the arrival times of the transmitted waves.

Figure E.1: Petrophysical models emphasizing the heterogeneity within the brecciated rock unit a) $k_a = 6, a_x = a_z = 30m$ b) $k_a = 6, a_x = 30m, a_z = 1000m$. The heterogeneity within the sediment package is such that $a_z = 5m, a_x = 1000m$. c) and d) show reconstructed contact from respective models using the transmission imaging algorithm.

E.4 Effect of geology- Part II

The illustration in section 6.5 demonstrates that heterogeneity due to geology and elastic properties play an important role in the propagation of direct waves. Just like the direct arrivals, reflection hyperbolas from scatterers are also distorted. The distribution of the various direct arrivals, reflection hyperbolas from scatterers are also distorted. The distribution of the various geologic units may cause the medium to have an effective anisotropy: preferential fast and slow directions that are strongly aligned with the orientation of the various geologic units. This affects imaging results especially when the reflections are recorded by acquisition geometries whose orientation with respect to the long axis (axis along the direction with the largest scale length) falls in the range $0^\circ < \Theta \leq 90^\circ$ ($\Theta$: angle). One of the sources of imaging errors resides in the fact...
migration routines implemented in seismic imaging, especially in hardrock environment, use constant velocity models as input. I consider the case for the Thompson mine as well as other complex models based on petrophysical logs from the Matagami mine (Quebec, Canada) to assess this problem. The approach used consists of two steps:

3. Compute the wavefield propagation from a point source at depth (Figure E.2a) that is recorded by a horizontal receiver spread at the surface of the heterogeneous model.

4. Use the acoustic reverse time migration (RTM) algorithm and a constant velocity model (e.g. mean velocity) to back propagate the direct arrivals to the source.

If the heterogeneous nature of the model has not significantly affected the seismic wavefront (direct waves), then the RTM processing will correctly focus all the seismic energy at the correct source location. Figure E.2 shows the computed wavefields and RTM results for one of the Thompson geologic sections (Figure 3.6a). Notice that the source location is well resolved when using a constant background velocity model of 6000m/s. Hence, migration of orebody diffractions will give reliable results provided an adequate constant background velocity model is used. Figure E.3 illustrates the case where some parameters like the dip and scale lengths along the direction of dip are varied. The petrophysical models are based on the stochastic features of some borehole logs from the Matagami mining camp. The fluctuations in the elastic parameters are weak (<5%). The recorded wavefields from the respective models (Figure E.3c and e) show how the shape of the wavefront is distorted when local regions with high/low velocities exist over extensive distances. Despite the distortion in the wavefront, the source locations in the respective models are well characterized using the mean P-wave velocity (6300 m/s).

So far, the success of the source characterization process in all three models can be attributed to the weak fluctuations in the background P-wave velocities. Another reason for the success is the fact that the bulk of each shot record consist of wavefields that propagated at some angle (> 0°) relative to the bedding plane, hence emphasizing the important role of the acquisition geometry. Additional modeling needs to be done for better conclusions. For example, another aspect of interest that was not considered in the present investigation include: effect of overall propagation distance.
Figure E.2: a) P-wave velocity model (Thompson mine); b) Recorded wavefield from point source located at Distance = 3200m, depth = 1250m; c) RTM results using a constant background velocity of 6000m/s. Source frequency: 110 Hz.
Figure E.3: Stochastic P-wave velocity ($V_p$) models based on sonic logs from the Matagami mine: a) $a_\parallel/a_\perp=20$, b) $a_\parallel/a_\perp=200$; $a_\parallel$ and $a_\perp$ are the scale lengths parallel and perpendicular to the dip; Mean $V_p=6300\text{m/s}$; $V_p/V_s=1.8$; Mean density=3000m/s;\[
\frac{\delta V_{RMS}}{V_{pmean}} = \frac{1}{30}; \quad \frac{\delta \rho_{RMS}}{\rho_{mean}} = \frac{1}{40}.
\]
Recorded wavefield from point source at depth=1000m; distance=3000m: c) model (a) ; e) model (b); Results from implementing acoustic reverse time migration to the direct arrivals in c (d) and e (f) respectively. The RTM algorithm was implemented with a constant velocity of 6300m/s. Source frequency: 110 Hz.
Appendix F

F.1 Investigating scale parameters from seismic transmission responses: modeling study- spectral ratio analysis

It is understood that perturbations in velocity and density are responsible for fluctuations in the amplitudes, traveltime and phase distortions observed in the seismic waves (Wu, 1989; Sato and Fehler, 1998; Milkereit et al., 2003). As shown in section 2.6 (Figure 2.7), scattering processes due to crustal heterogeneities cause redistribution of energy into different propagation directions. The energy redistribution processes coupled with the presence of multiples are the principal causes of frequency fluctuations in recorded waveforms. Milkereit et al., 2003 argue that amplification/attenuation of certain frequencies as a result of scattering are useful to estimate scale characteristics of the inhomogeneities ($\Delta V_p$, $\Delta V_s$, $\Delta \rho$) intercepted by the wavefield. The modeling study presented here examines how scattering effects for different scenarios cause alterations in frequency and phase spectra and how this could be related to scale lengths. Geometries of interest include cases with sharp impedance contrast (overburden effect) and those with defined target structures (Figure F.1).

F.1.1 Method and results

For layered medium, there exist a vast literature that emphasizes the amplitude and spectral characteristics of the transmitted (forward scattered) energy (Claerbout, 1968; O’Doherty and Anstey, 1971). The amplitude spectrum of the transmitted energy recorded at the receiver site can be formulated mathematically as:

Figure F.1: Geometry illustrating scattering by limited target (left panel), and the case of overburden (right panel, large reflection coefficient R). V: velocity
\[ A(f) = S_0(f) P(f) T(f) + n(f) \] (F.1)

and travel time delays in terms of the phase shifts as \( \delta \phi = 2\pi f \delta t \)

where, \( T(f) \) represents the amplitude spectra of the transmission impulse responses characterizing the medium of propagation, \( S_0(f) \) is the spectrum of the source signature, \( P(f) \) is the path effect (geometric spreading, reflection losses), \( n(f) \) is the background noise, \( f \) is the frequency, \( \phi \) is the phase and \( t \) is the time. In theory, note that equation (F.1) suggests scattering can be corrected for by estimating and deconvolving \( T(f) \) from recorded seismic traces. In the present study, simple forward modeling using cases for 1D and 2D distribution in physical properties are considered.

a) A simple 1D-model

The 1D-modeling helps to assess the role of multiples, magnitude of the reflection coefficient (R), and layer thickness (vertical scale length) in distorting the frequency content (e.g. by creating notches at certain frequencies) when interfering with direct arrivals. The 1D models demonstrate that layering suffices to create these multiples. Figure F.2 shows the effect of multiples created by reverberations within the overburden layer (overburden effect). Notice the amplification of frequencies other than the dominant frequency of the source wavelet (ricker wavelet).

\[ \text{Figure F.2: Amplitude spectrum of transmitted energy in two-layered medium illustrating the overburden effect: gneiss (5933 m/s, 2722 kg/m}^3 \text{) and clay overburden (2000 m/s, 2000 kg/m}^3, 20\text{m thick). Analysis is based on normal incidence. The transmitted trace consists of the direct arrival and two multiples.} \]
Though multiples distort spectral content, the overall extent of this effect depends on the kind of medium encountered. Consider the case of a simple 3-layered medium (a single layer sandwiched by a background medium—Figure F.3a). Let us assume that the ratio of the reflected to incident amplitude at the intermediate layer is $R$ (reflection coefficient). For a 1D synthetic transmission using a ricker wavelet, the ratio between the direct arrival and multiples will be $1/R^{2m}$ ($m$: $m^{th}$ multiple) with the multiples having time lags of $m\Delta t$ ($\Delta t$: two-way travel time within the intermediate layer) with respect to the direct arrivals. Significant alteration in the amplitude spectrum (e.g. creation of notches, Figure F.3b) due to interference from multiples is not apparent unless $R > 0.4$ ($R \geq 0.3$ when using spectral ratios with respect to source spectrum—Figure B.3c).

![Figure F.3: a) Schematic representation of transmitted wave (direct arrivals and multiples) through a 5ms thick intermediate layer. b) Amplitude spectrum of transmitted energy for different cases of reflection coefficients. c) Spectral ratio amplitude spectra plots in (b) with respect to source (80Hz). d) Relation between intermediate layer thickness, reflection coefficient ($R$), and notch frequency ($f_{\text{notch}}$) for a 3-layered medium. Ricker wavelet source frequencies used include: 40Hz, 50Hz, 70Hz, and 80Hz. The plot is done using $R^2$ given that the effect of the first multiple is considered to have the most impact in spectral distortion due to interference. Analysis is based on normal incidence. The transmitted trace consists of the direct arrival and two multiples.](image-url)
Notch frequencies are independent of $R$ and the source spectrum but are dependent on the thickness of the intermediate layer (Figure F.3d) whereby:

$$f_n = \frac{(2n - 1)V_z}{4a_z}$$  \hspace{1cm} (F.2)

where $V_z$: layer velocity, $a_z$: vertical scale length (layer thickness), and $f_n$: notch frequency.

Equation F.2 is also validated by numerical test results using plane wave incidence. Note that the consistent results obtained in this 1D assessment are from an idealized approach. However, this may not apply if other factors like spherical divergence, incidence angle, multiple layering, wave conversion, intrinsic attenuation and background noise are considered. Also the distribution of physical rock properties is more complex that the case examined here. The section below addresses the case for a 2D earth model.

**b) 2D- model**

Solving for the total wavefield via finite difference method provides a more realistic basis for assessing forward scattering caused by heterogeneities ($\Delta V_p$, $\Delta V_s$, $\Delta \rho$) since factors such as energy conversion and angle dependent redistribution of energy are accounted for. The 2D model used includes a single lens (sulfide ore) in a homogenous background (Figure F.4a). In order to simplify the analysis, a plane wave source with a dominant frequency of 50Hz was used. Due to waveform healing phenomenon of scattered seismic waves with distance, the scattered wavefield is measured at positions within one wavelength of the lens inclusion. Figures F.4b & c show the spectral decomposition of windowed first break events from the vertical and horizontal components of the transmitted wavefield. The presence of energy on the horizontal component (<20 %) provides key information on the presence of a scatterer. The plot of the spectral ratio (H/V) is shown in Figure F.4d. Some attenuated (notch) frequencies can be identified on the plot. Observe that the trend in the notches is subject to effects of the lens shape. Results from a similar analysis with an explosive source are almost identical to those shown in Figure F.4. Although much work needs to be done to quantitatively link these amplified/attenuated frequencies to the lens scale parameters, the output obtained suggests the H/V analysis looks promising.
Figure F.5a shows a plot of picked notch frequencies from Figure F.4d. Note that the notch values obtained at the centre of the lens (between traces 58 and 66) are erroneous as the incident angle of the wave with respect to the lens is \(\sim 90^\circ\). In this particular case, any energy present in the horizontal component for these traces is due to diffractions generated at the edges of the lens.

Figure F.5b compares estimated and true lens thickness at each receiver location. The estimated lens thickness is over estimated at receiver locations furthest from the lens centre. A major difficulty in the present analysis is due to the large discrepancy in the energy content between the respective V- and H-components. Other effects like source frequency, impedance contrast of lens inclusion as well as the first break windowing process used in the H/V analysis should be considered for future studies. For example, numerical studies (Figure F.6) corroborates that...
window size becomes critical especially when S-wave events are present within short periods of the direct P-waves. Interferences by the S-waves introduce errors in the spectral ratio analysis (H/V).

**Figure F.5:** a) Notch frequencies from the H/V plot in Figure F.4d; Black arrow indicated the centre of the lens; b) Plot of estimated and true lens thicknesses at different receiver locations. Estimated thickness = Velocity of inclusion/notch frequency.

**Figure F.6:** a) and b) show the H- and V-component VSP data recorded in a homogenous background. The explosive source is buried at 2m below the model surface and located at 50m offset from the receiver line. The interference between direct P- and S-waves (black ellipse) results in erroneous spectral fluctuations of windowed first break events (c and d:- Normalized plots). Consequently, the spectral ratio (H/V) plot (e) gives biased results. Source frequency = 90 Hz.
Copyright Acknowledgements

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