ACCESS, ECONOMIC GROWTH, AND SPATIAL DISPERSION*

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1. INTRODUCTION

An important question in regional planning concerns the effect of economic growth, at one place, on the spatial distribution of population and economic activity elsewhere. In the development literature, spread and backwash hypotheses have been proposed. The spread hypothesis asserts that areas near an initial center of economic development experience increased activity, for example because the initial expansion means an increase in their output markets. The backwash hypothesis alternatively suggests that nearby places decline as that center strips away their labor supply and invades local product markets. Rather than being mutually exclusive, these two effects of growth may occur at the same time. It is the purpose of this paper to describe a spatial equilibrium model which establishes the simultaneous occurrence of both the spread and backwash effects.

It is useful to consider access in terms of varying prices through space. Models of spatially-variant prices and their effects on production and land use are commonly associated with von Thünen. There is a fault, however, with the models of von Thünen and his intellectual descendants which make it difficult to apply any previously developed model to the issues of spread and backwash effects.

Economic growth needs to be considered at several levels. At one level, growth is an act of foregone consumption in one period leading to increased consumption in later periods. At another, economic growth is associated with substitution in inputs, production techniques, outputs and consumption. The typical von Thünen model can be used only at the former level. The realization of growth in a city, as represented by increased demands for rural goods or lower relative prices for the city's goods, in that model, increases rents and land-use intensity in peripheral rural areas. These increased activity levels are a spread effect. Since, in the typical von Thünen model, there is neither an explicit substitution of rural for city-made goods nor an explicit shift of labor from the periphery to the city, there can be no backwash effects. To describe growth effects at the substitution level, at least two inputs to production and two outputs are required in the rural periphery. Then,

* Research was supported by the Transport Development Agency (Canada) through a grant from the University of Toronto-York University Joint Program in Transportation. The author is grateful to R. MacKinnon and J. Simmons, among others, for helpful comments on an earlier draft.

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Date received: November, 1973; revised, February, 1975.

growth in the city can result in a backwash effect as individuals in the periphery substitute the city good for one of their two locally produced goods.¹

2. SETTING FOR THE MODEL

We assume the existence of a large region filled with many farms. A farm is characterized by the fact that each has the same amount of resident homogeneous labor. It may allocate this labor among three activities; on-farm food or soap production and factory labor. In this two-good spatial economy, the farm produces a gross quantity of food requiring labor and land inputs and a quantity of soap requiring merely a labor input.²

Each farm has the same technology and the same preference functions for food and soap. Each is utility-maximizing, occupies everywhere-homogeneous land, and pays a given rent per unit land (the rent accruing to an agency outside the realm of the model). Each farm chooses to consume an amount of soap obtained either from on-farm production or purchased from the factory. Food is obtained either from net on-farm food production (net of rental payments) or as wages from the factory (net of commuting costs) and is reduced by the amount used to make soap purchases.

Finally the factory offers to exchange soap for food at a fixed mill price. Also the factory is assumed to have an infinitely elastic demand for labor at a given mill wage. This wage is paid in the commodity food. The delivered price of soap at the farm and the net unit wage increase and decrease with distance respectively reflecting transport and commuting costs.

3. OVERVIEW OF ISSUES

Characteristics of the Solution

There is, for the utility-maximizing farm, a limited labor market area within which the net wage is greater than the labor marginal-value-product when all labor is allocated to farm work. Outside this area, the farm chooses to allocate no labor to the factory while, within it, some labor is so allocated.

There will also be a limited output market area within which the delivered price of factory soap is less than the farm's marginal opportunity cost of producing it in autarky. Outside this market area, each utility-maximizing farm will choose not to trade while within it some trade will take place.

¹ This shortcoming of the usual von Thünen model has another ramification. Economic growth has typically been associated with transitions from a general self-sufficient agriculturally-based sector to specialized manufacturing, service, and agricultural sectors. A model of access and growth should describe the extent and rate at which farms give up home production of their generally nonpriced goods in favor of city-made ones. In empirical growth work where the measurement of home production is often infeasible, some have argued that estimates of actual growth are too high. In spatial terms, estimates of the intensity of land use may be similarly misleading if aggregate farm outputs are not completely measured. Therefore, a comparison of spread effects from the usual von Thünen model with observation may incorrectly imply an inefficient spatial pattern of activity densities.

² Hymer and Resnick [4] present a similar model of an agrarian economy except that rents and the distance factor are ignored.

Outside the output market area where each farm is in autarky, all farms will be of identical size and have identical outputs, consumption, and distribution of labor between food and soap production. This arises because tastes, technology, labor resources and land are identical everywhere.

Inside the output market area, there will be several effects altering the pattern of production and consumption. Since the farm faces a lower price for soap, it is expected to substitute food in favor of soap consumption. Secondly, the lower soap price leads to substitution of soap in favor of food production. Thirdly, the land rents change to leave the farm indifferent about location with respect to the factory. This suggests that land rents change to eliminate the income effect in the soap price change. Finally, differential rents on land suggest that farms will alter the intensity of land-use by substitution of land in favor of greater labor input. This implies a smaller labor input in soap production and a larger input in food production.

Generalized Application

The model is developed essentially as a parable. A simplified but not unrealistic analysis is possible using this guise. To what extent can such an analysis be generalized? In particular can the definitions of commodities and actors be fruitfully generalized?

The two commodities, food and soap, are readily extended. The commodity soap is produced by both factory and farm in the model. Since the factory sells its output over a finite market area with associated transportation costs, its production costs are lower than those of the farm in that area. However, the farm by assumption can achieve only constant returns to scale in soap production. The factory could not exist if it were limited to the same technology as the farm. There must be lumpy production indivisibilities such that the factory has significantly lower unit costs while the farm cannot generate increasing returns to scale. Food, on the other hand, represents a class of commodities which are not subject to quantum changes in production technology—at least not at the scale of operation used here.

The concept of the two actors in the model, the factory and the farm, can now be generalized in terms of these commodity classes. The most elementary notion of a "farm" in this model is a unit of labor which could be viewed at several levels: (i) a group of one or more individuals operating under the name of a single farm, (ii) a farming district consisting of all the farm labor working therein, or (iii) a farming region consisting of all labor employed on farms, in local service towns, or by associated businesses. Thus, one can think of the farm in a central place context as a lower-order urban center together with all its hinterland markets.

The notion of a "factory" can also be taken in different contexts. It may be just the single productive facility envisaged in the name, a set of productive facilities, or, in a central place context, a higher-order urban center.³ All that is necessary

³ In the central place framework, the farm can represent any level of urban center other than the highest-order center and the factory could be the next-highest level of center. It is therefore possible to use the model outlined above to describe the spatial relationship between any level of urban center and its lower-level satellites. The findings of the model are as applicable to the spacing and size distribution of a system of cities as they are to the size and spacing of farms.

is that the factory represent an activity carried on at a sufficient scale to achieve an indivisibility not possible on a smaller farm.

4. FORMAL DELINEATION OF THE MODEL

Detailed Setting

Begin by assuming that the common preference function for farms is expressed as a utility level, U , for each farm given by a log-linear relationship with soap (X) and food (Y) consumption⁴

$$(1) \quad U = X^a Y^{1-a} \quad (0 < a < 1)$$

Gross food production (Q) for each farm is determined by a common Cobb-Douglas production function involving the land input (L) and the food-labor input (h_y)

$$(2) \quad Q = b h_y^\beta L^\gamma \quad (b, \beta, \gamma > 0; \beta + \gamma < 1)$$

It is assumed that there are decreasing returns to scale in food production.

The total land rental payment by each farm is the product of the rental rate, $R(s)$, and the farm land input, L . Net food production (Y_1) is thus gross food production less total land rental payments

$$(3) \quad Y_1 = Q - R(s)L$$

We assume that soap production (X_1) is proportional to soap labor input (h_x)

$$(4) \quad X_1 = c h_x$$

The factory offers to trade soap for food at the given mill price P_b . Transportation costs are assumed to be proportional to distance so that the delivered price, $P(s)$, at a distance s from the factory is⁵

$$(5) \quad P(s) = P_b + ts \quad (t > 0)$$

The factory also offers employment at a given mill wage of w_b . Commuting costs are assumed to be proportional to distance so that the delivered wage is given by⁶

$$(6) \quad w(s) = w_b - rs \quad (r > 0)$$

⁴ The amount of land occupied by the farm does not enter the farm's utility level. Such an extension does not imply a significant change in the results of the model except in the corner solution case where farms totally give up food production in favor of factory work. Information about this extreme case has been given up in order to maintain simplicity in the model.

⁵ The assumption that the delivered price ratio is a linear function of distance may be troublesome to those who see two kinds of freight charges; the cost of shipping food to the factory and the cost of shipping the soap back. Suppose that the farm gives up an amount of food at a mill site price of P_1 dollars per unit before freight charges of t_1 dollars per unit distance. Soap is purchased at P_2 dollars per unit plus freight charges of t_2 dollars per unit distance. The relative price ratio is now $P(s) = (P_2 + t_2 s) / (P_1 - t_1 s)$, which can be expressed as

$$P(s) = P_2/P_1 + (P_2/P_1 + t_2/t_1)[(s t_1/P_1) + (s t_1/P_1)^2 + \dots]$$

Since $P_1 - t_1 s > 0$, we approximate $P(s)$ by $(P_2/P_1) + [(P_2/P_1) + (t_2/t_1)](t_1/P_1)s$ which is the linearly-increasing function of s assumed in Equation (5).

⁶ There are assumed to be no time costs to commuting here. Introduction of a time element complicates the solution without significantly changing the interpretation of it.

The farm chooses to allocate an amount of labor, h_z , to factory work. The farm operates under the constraint that its total labor supply, h , is fixed in amount

$$(7) \quad h = h_x + h_y + h_z$$

Total consumption of soap on the farm is given by the sum of on-farm soap production, X_1 , and soap purchases, from the factory

$$(8) \quad X = X_1 + X_2$$

Total food consumption of the farm, Y , is given by the sum of net farm food production and factory wage earnings less the food traded to obtain soap⁷

$$(9) \quad Y = Y_1 + w(s)h_z - P(s)X_2$$

Utility Maximization Conditions

Each farm seeks to maximize its utility using five instrument variables: location (s), food and factory labor inputs (h_y, h_z), land input (L), and soap purchases (X_2). Five first-order conditions are thus derived. Two assert that the value-marginal-product of labor in all three activities must be the same

$$(10) \quad cP(s) = \beta Q/h_y$$

$$(11) \quad w(s) = \beta Q/h_y$$

Another asserts that consumption of soap is a fixed proportion of total income, both valued at the delivered price

$$(12) \quad P(s)X_2 = \alpha[Y + P(s)X_2]$$

Another asserts that the land input is such that the marginal product of land is equal to its rent

$$(13) \quad R(s) = \gamma Q/L$$

The final condition asserts that the change in rental payments associated with a marginal change in location must be offset by the sum of the changes in soap expenditures and in net factory wage earnings

$$(14) \quad R'(s)L + tX_2 + rh_z = 0$$

Inconsistency and Resolution

There are two inconsistencies within this model. Equations (10) and (11) are both satisfied only at one distance, s^A , such that⁸

$$(15) \quad s^A = (w_b - cP_b)/(r + ct)$$

assuming $w_b > cP_b$. Therefore, it is not generally possible to have the value-marginal-product of labor equal in all three allocations simultaneously. This incon-

⁷ It is assumed that factory wages are paid in food since food is the only convertible commodity in the model as far as the farms are concerned.

⁸ Assuming that some labor is allocated to at least one of either on-farm soap production or factory labor.

sistency occurs because the value-marginal-products of labor in on-farm soap production and factory labor are constant at a given distance, s . At any given distance, no labor can be allocated to the activity with lower value-marginal-product.

A further inconsistency arises from the requirement that soap purchases be nonnegative. The point where $X_2 = 0$ defines the output market boundary, s^* . We limit our analysis to the case where the labor market area is a subset of the output market area ($s^\Delta < s^*$). Therefore, s^* defines the distance beyond which the farms are totally independent of the factory.

This model's solution is a composite of solutions to three separate problems: the autarky solution when $s > s^*$, the output market solution with no factory labor (termed the M_1 market solution) when $s^\Delta < s < s^*$, and the labor and output markets solution with no on-farm soap production (termed the M_2 market solution) when $0 < s < s^\Delta$.

Autarky Solution

The farm in autarky neither trades with nor allocates labor to the factory

$$(16) \quad h_z = 0 \quad X_2 = 0$$

The utility-maximizing conditions, Equations (10) to (14), now reduce to three

$$(17) \quad c[\alpha Y / (1 - \alpha)X] = (\beta Q / h_y)$$

$$(18) \quad R(s) = \gamma Q / L$$

$$(19) \quad R'(s) = 0$$

when $L \neq 0$. From (19) land rents are constant over space. Denote this fixed rent by $R(s) = R^*$.

Using (2), (4), (17), and (18), it is derived that the efficient farm allocates its work time-budget between food and soap roughly in relation to the relative utility of each commodity^{9, 10}

$$(20) \quad h_y^* = k_1 h \quad h_z^* = (1 - k_1)h$$

$$(21) \quad k_1 = \beta(1 - \alpha) / [\beta(1 - \alpha) + \alpha(1 - \gamma)]$$

Note that $0 < k_1 < (1 - \alpha)$ and $(1 - k_1) > \alpha$. Using (20), (21), and (18) the, efficient lot-size and aggregate food output levels may be derived.

$$(22) \quad L^* = (k_1 h)^{\beta/(1-\gamma)} (\gamma b / R^*)^{1/(1-\gamma)}$$

$$(23) \quad Q^* = b(k_1 h)^{\beta/(1-\gamma)} (\gamma b / R^*)^{\gamma/(1-\gamma)}$$

The efficient consumption (or net production) of food and soap can now be established as well as the utility level associated with this consumption.

$$(24) \quad X^* = c(1 - k_1)h$$

⁹ This being strictly true in the case of constant returns to scale in food production.

¹⁰ An asterisk superscript is used to denote the efficient autarky solutions for each variable.

$$(25) \quad Y^* = (1 - \gamma)b(\gamma b/R^*)^{\gamma/(1-\gamma)}(k_1 h)^{\beta/(1-\gamma)}$$

$$(26) \quad U^* = X^{*\alpha} Y^{*(1-\alpha)}$$

Finally, the implicit price of soap in autarky is given by the slope of the production possibility function at the optimal production combination.

$$(27) \quad P^* = (b\beta/c)(\gamma b/R^*)^{\gamma/(1-\gamma)}(k_1 h)^{-(1-\gamma-\beta)/(1-\gamma)}$$

M₁ Market Solution

A farm lies within the market area of the factory when the delivered price is less than the marginal opportunity cost, P^* , of producing it in autarky. The market boundary in this case is the distance s^* from the factory to the farm such that

$$(28) \quad s^* = (b\beta/ct)(\gamma b/R^*)^{\gamma/(1-\gamma)}(k_1 h)^{-(1-\beta-\gamma)/(1-\gamma)} - P_b/t$$

Thus, the market boundary depends on the structures of preference and technology as well as on f.o.b. price and transportation costs.

The M_1 market lies between the distances s^A and s^* . There is no factory labor input ($h_s = 0$). The first-order maximization conditions are the following:

$$(29) \quad P(s)X = \alpha[Y + P(s)X]$$

$$(30) \quad cP(s) = \beta Q/h_y$$

$$(31) \quad R'(s) = -tX_2/L$$

$$(32) \quad R(s) = \gamma Q/L$$

In this solution, we do not seek a specific location, s , but a rent function which, according to (31), leaves the farm indifferent as to marginal changes in location. After some manipulation of (2), (3), (4), (7), (8), (9), and (29) through (32), we obtain the following differential equation:

$$(33) \quad \partial R(s)/\partial s = -[t/\gamma P(s)][\alpha(1 - \gamma) + (1 - \alpha)\beta]R(s) \\ + t(1 - \alpha)ch(1/\gamma b)^{(1-\beta)/(1-\beta-\gamma)}[b\beta/cP(s)]^{-\beta(1-\beta-\gamma)}R(s)^{(1-\beta)/(1-\beta-\gamma)}$$

One solution to (33), which is unique up to a constant of integration, is the following:¹¹

$$(34) \quad R^a(s) = b^{1/\gamma}\gamma[cP(s)/\beta]^{-\beta/\gamma}g_a(s)^{-(1-\beta-\gamma)/\gamma}$$

where

$$(35) \quad g_a(s) = [U^*(\alpha/1 - \alpha)^{1-\alpha}P(s)^\alpha - \alpha chP(s)]/\alpha(1 - \beta - \gamma)$$

The particular constant of integration chosen here is the one which guarantees that $R(s) = R^*$ when $s = s^*$.

The efficient consumption of soap (X^a) and food (Y^a) can now be determined

¹¹ The letter a is used as a superscript or subscript to denote an optimal solution in the M_1 market.

FIGURE 1: Optimal Land Input as a Function of Distance from the Factory.

for any price ratio, $P(s)$

$$(36) \quad X^a = U^*[\alpha/(1 - \alpha)P(s)]^{1-\alpha}$$

$$(37) \quad Y^a = U^*[(1 - \alpha)P(s)/\alpha]^\alpha$$

Since the farm is left indifferent with respect to location when utility is maximized, changes in the price ratio merely enable the farm to attain different combinations at the same level of indifference. Thus, (36) and (37) represent compensated demand curves where a decrease in the price ratio leads to an increase in soap consumption and a decrease in food consumption.

The efficient land input (L^a) may also be derived

$$(38) \quad L^a = P(s)^{\beta/\gamma} g_a(s)^{(1-\beta)/\gamma} (c/\beta)^{\beta/\gamma} (1/b)^{1/\gamma}$$

The derivative of land input with respect to delivered price casts some light on the behavior of L^a

$$(39) \quad dL^a/ds = [L\beta t/\gamma P(s)] \{1 + [(1 - \beta)/\beta][(X^a - ch)/(X^a - \alpha ch)]\}$$

For this derivative to be negative, it is necessary that

$$(40) \quad X^a < [1 - (1 - \alpha)\beta]ch$$

Since $X^a \geq X^* = (1 - k_1)ch > \alpha ch$, for X^a sufficiently close to X^* (i.e., at distance s close to s^*), this derivative will always be negative because (40) can always be satisfied there. In other words, at s^* we have $X^a = X^* = (1 - k_1)ch$ but $\alpha < (1 - k_1) < 1 - (1 - \alpha)\beta$ so that (39) is negative. As s is decreased, X^a increases smoothly from (36) and the land input derivative increases (i.e., toward a zero then positive value).¹²

Two observations now follow. First, if the M_1 market is defined over a sufficiently large range of s values, the optimal land input in (38) is a nonmonotonic function of distance. As shown in Figure 1, it takes potentially an inverted U shape with a maximum at distance s^x . Secondly, even if the function is monotonic, which would occur if s^A were greater than s^x (i.e., truncating the inverted U shape), optimal land input decreases as one approaches s^* . The notion that the physical area of a farm decreases as one moves away from the factory is an unusual one.

¹² I am indebted to G. Holtzclau for a clarifying comment here.

The optimal food-labor input can now be shown to be a monotonically-decreasing function of the delivered price

$$(41) \quad h_y^a = \beta g_a(s)/[cP(s)]$$

Since $dh_y^a/dP(s)$ is negative, food-labor input decreases steadily from s^A to s^* . Soap-labor input is the difference between the fixed aggregate labor resource and the variable h_y^a . Thus, $dh_x^a/dP(s)$ is always positive.

We can derive an expression for Q^a , gross food output, from (2), (36), and (41)

$$(42) \quad Q^a = g_a(s)$$

Gross food output is not a monotonic function of the delivered price. Its derivative is

$$(43) \quad dQ^a/dP(s) = (X^a - ch)/(1 - \beta - \gamma)$$

Thus, $dQ^a/dP(s) < 0$ in some finite region near the market boundary, s^* .

Finally, one may derive an expression for net food production

$$(44) \quad Y_1^a = (1 - \gamma)g_a(s)$$

The derivative of Y_1^a is proportional to that for gross food output and thus shares its possible nonmonotonic form.

A graphical interpretation of the system of equations (34) through (44) is possible. The production functions (2) and (4) together with the labor constraint (7) and a strictly positive rent imply the existence of a production possibility frontier in net food (net of rental payments) and soap.¹³ Such a production possibility frontier for the autarky rent, R^* , is displayed as the curve PP' in Figure 2. The indifference curve corresponding to the highest level of utility (U^*) possible in autarky is U^*ade . In autarky, the farm chooses to consume and produce, at a , X^* units of soap and Y^* net units of food. The line faf' is a tangent common to PP' and U^*ade and has a slope of $-P^*$.

Within the M_1 market area, the production possibility curve changes shape because changes in rent through space alter the level of net food production possible given the level of soap production. Since rent is payable in food and since soap production requires no land, the maximum soap production remains at ch units regardless of the level of rent. The maximum net food output possible with any level of soap output decreases as the rent increases.

In equilibrium, rents change to ensure that the associated budget line permits the farm to trade back to exactly the same utility level regardless of location. Given the utility level U^* and the delivered price at a location, the rent changes to re-define the production possibility curve so that it is tangential to a line of slope $-P(s)$ which is also tangent to U^* .

Two loci of points can be identified when the delivered price is permitted to vary continuously from location to location. A locus of consumption combinations begins at a at s^* . Moving from this boundary toward the factory, the locus traces out ade , a segment of the indifference curve, as $P(s)$ decreases.

¹³ There is no finite production possibility frontier in gross food and soap as there is no constraint on the amount of land which a farm may use in food production.

Again, the solution rests on our ability to solve the differential equation (48). After some substitution, (48) can be reduced to

$$(49) \quad dR(s)/ds = 1/\gamma b[-\alpha b(1 - \beta - \gamma)/P(s) + rb/w(s)]R(s) \\ - [\alpha tw(s)/P(s) + r]h(\gamma b)^{-(1-\beta)/1-\beta-\gamma}[\beta b/w(s)]^{-\beta/1-\beta-\gamma}R(s)^{1-\beta/1-\beta-\gamma}$$

We choose that solution, unique up to a constant of integration, such that $R(s^\Delta) = R^a(s^\Delta)$. Solutions for the efficient level of each variable can now be derived in a manner analogous to that used in the M_1 market model.¹⁴

$$(50) \quad X^b = X_2^b = U^*[\alpha/(1 - \alpha)P(s)]^{1-\alpha}$$

$$(51) \quad Y^b = U^*[(1 - \alpha)P(s)/\alpha]^\alpha$$

$$(52) \quad L^b = [w(s)/\beta]^{\beta/\gamma} b^{-1/\gamma} g_b(s)^{(1-\beta)/\gamma}$$

$$(53) \quad g_b(s) = 1/\alpha(1 - \beta - \gamma)[U^*(\alpha/1 - \alpha)^{1-\alpha}P(s)^\alpha - \alpha w(s)h]$$

$$(54) \quad h_v^b = \beta g_b(s)/w(s)$$

$$(55) \quad h_z^b = [(1 - \gamma)/(1 - \beta - \gamma)]h \\ - [\beta/(1 - \beta - \gamma)][(U^*/\alpha)[\alpha(1 - \alpha)]^{1-\alpha}P(s)^\alpha/w(s)]$$

$$(56) \quad Q^b = g_b(s)$$

$$(57) \quad Y_1^b = (1 - \gamma)g_b(s)$$

$$(58) \quad R^b(s) = \gamma b^{1/\gamma}[w(s)/\beta]^{-\beta/\gamma} g_b(s)^{-(1-\beta-\gamma)/\gamma}$$

Solutions for several of these variables show spatial patterns similar to those derived in the M_1 market solution. Efficient soap consumption (X^b) decreases monotonically with distance from the factory as did X^a . Also, $R^b(s)$ declines monotonically as did $R^a(s)$. Efficient soap purchases (X_2^b) decline monotonically as in the M_1 market area although there is a quantum change at the common boundary, s^Δ . Food consumption (Y^b) increases monotonically with distance as did Y^a .¹⁵

¹⁴ The superscript b is used to denote an optimal M_2 solution.

¹⁵ The M_2 market solution is based on the implicit assumption that some nonzero and nonunity share of a farm's total labor is allocated to the factory. There exists the possibility that all or none of the farm's labor supply will be so allocated and that this will occur over broad ranges of distances. All the possibilities for and implications of such corner solutions cannot be discussed here but it is useful to determine the critical distances in the model.

There exists a maximum wage and associated minimum distance, s_1 , such that $h_v^b = 0$ if $s \leq s_2$. The distance s_1 satisfies the nonlinear condition that

$$w(s_1) = (U^*/\alpha h)[\alpha/(1 - \alpha)]^{1-\alpha}P(s_1)^\alpha$$

A corner solution exists only if s_1 is positive. There also exists a minimum wage and associated maximum distance, s_2 , such that $h_v^b = h$ if $s \geq s_2$. The distance s_2 satisfies the nonlinear condition that

$$w(s_2) = [\beta U^*/(1 - \gamma)\alpha h][\alpha/(1 - \alpha)]^{1-\alpha}P(s_2)^\alpha$$

A corner solution exists if $\max(0, s_1) < s_2 < s^\Delta$ in this case.

Similarly, the M_1 market solution implicitly assumes that a nonzero and nonunity share of labor is allocated to farm food production. At the outer boundary, s^* , this assumption has

The remaining variables display spatial patterns in the M_2 market area which are significantly different from those in the M_1 market area. Food-labor input (h_y^b) increases monotonically with distance in contrast to the decreasing monotonic form for h_y^a . Gross and net food output (Q^b, Y_1^b) both display an increasing monotonic relationship with distance to the factory unlike the inverted- U relationship found in the M_1 market area. Land input (L^b) also displays an increasing monotonic relationship unlike the inverted U form possible in the M_1 market area.

What is the source of this difference between the M_1 and M_2 market solutions? The answer lies in the substitution between output combinations. In the M_1 market solution, the farm has a choice of producing any combination of food and soap, both of which enter into its utility level. In the M_2 market, there can be no substitution of outputs because only one good is produced. The M_2 model is quite similar to the typical von Thünen model in this respect. The potential for substitution among output goods is the critical difference between the solutions to the two market areas.

5. IMPLICATIONS OF THE MODEL

Transportation Planning

In the planning of transportation facilities and system improvements, an important concern is with the long-run impact of changes in access on spatial population density and activity patterns.

Each farm represents the same fixed amount of labor (h). Thus, if L represents the land input per farm, then $(1/L)$ is the worker density by place of residence and (h/L) is a proxy for population density.¹⁶ Population density, being the inverse of land input, takes on a U -shaped (if discontinuous at s^A) relationship with respect to s . An improvement in the transportation network implies a possible increase in population density close to the factory and a decrease in density further away. An illustration is given in Figure 3 where $(h/L)_0$ is the initial density and $(h/L)_1$ is the density after the improvement. This permits changes in access to have both positive and negative effects on population growth and density.¹⁷ In Figure 3, those areas lying within s_s units of the factory experience population growth while beyond that boundary areas lose population.

This latter effect is consistent with an observation that improvements in a

already been shown to hold (i.e., $0 < k_1 < 1$). However, moving from s^* toward s^A , h_y^a increases in value and may even exceed h . There exists a distance s_3 at which $h_y^a = h$.

$$s_3 = [\beta U^*/\alpha ch(1 - \gamma)]^{1/(1-\alpha)} [\alpha/l(1 - \alpha)] - (P_b/l)$$

A corner solution exists if $s_3 > s^A$.

There is a relationship between s_3 and s_2 . If $s_3 < s^A$, then for a value of s arbitrarily close to but greater than s^A , $h_z > 0$. This means that for any s arbitrarily close to but less than s^A , $h_z > 0$. Thus, assuming that $s_3 < s^A$ is sufficient to ensure no corner solutions in either the M_1 or M_2 market near s^A . Assuming $s_3 < s^A$ and $s_1 < 0$ (i.e., assuming sufficient curvature of production and preference functions to ensure these two results) is adequate to ensure that no corner solutions exist anywhere in either model.

¹⁶ Knowledge of h_z as a function of s also indicates the worker density by place of employment.

¹⁷ That is, existence is conditional on (40) being satisfied.

FIGURE 3: Population Density on Farms as a Function of Distance to Factory.

regional transportation network create benefits mainly for the largest city and its most immediate satellites.

Economic Growth

There is a symmetry between the effects of economic growth and the effects of transportation cost changes. Economic growth tends to imply an improvement in efficiency. In this model, such improvements may take the form of price reductions or wage increases. This affects the spatial distribution of activity and population through changes in delivered prices and wages through space which is the same way in which general transportation improvements operate. Thus, Figure 3 can be used to indicate how economic growth, as so manifested, affects population spatially.

Economic growth in terms of price decreases and wage increases leads to an increase in population and activity near the factory but to a decrease further away. In terms of the two curves in Figure 3, economic growth leads to population growth within s_0 units of the factory corresponding to the spread concept. However, from s_0 to s^* , economic growth at the center has led to a decline in population or to the backwash effect. Thus, the simultaneous validity of the backwash and spread effects are established. The spread hypothesis operates near the factory while the backwash hypothesis is valid in the outer reaches of the factory's market area.

In concluding, some caution is to be noted in the interpretation of backwash in this model. The usual conception of backwash effects involves a number of equity aspects not considered here: the flight of youths to the larger cities, discrimination in capital markets or government policies, and the unemployability of those who cannot out-migrate. The present model describes only a redistribution of activity which occurs in equilibrium such that no farm is either better or worse off after a change. The depopulation of certain areas occurs because (i) the city takes over

some of the previously local production and (ii) each farm uses more land for food production thus spreading out population more thinly. In other words, backwash effects related to equity issues can not be described by the model since everyone is assumed to be identical and, in equilibrium, equally well-off.

Nonetheless, the model has policy relevance. From an economic perspective, the model describes an efficient equilibrium so that little can be said in terms of efficiency goals. If, however, the society has social goals concerning the preservation of smaller towns and rural areas, the model indicates important market relationships against which feasible policies must be matched. The particular effect of changes in the transportation system can also be nicely outlined with this model.

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