EVOLUTION OF URBAN BUILT SPACE

MARKETS AND DECISIONS

By

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for the degree of Doctor of Philosophy

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Abstract

To understand the factors that influence the spatio-temporal distribution of built space, and thus population in an urban area, play an extremely important role in our greater understanding of urban travel behaviour. Existing location of activity centres, especially home and work, strongly influences the short-term individual-level decisions such as mode of transportation, and long-term household-level decisions such as change in job and residential location. Conditions in the built space market also affect households’ and firms’ location and relocation decisions, and hence influence the general travel patterns in an urban area. This research addresses two very important, but at the same time, not very widely investigated dimensions that play a key role in the evolution of built space and population distribution: Markets and Decisions. A disequilibrium based microsimulation modelling framework is developed for the built space markets. This framework is then used to operationalize the Greater Toronto and Hamilton Area’s owner-occupied housing market within Integrated Land Use Transportation and Environment (ILUTE) modelling system. Simulation results captured heterogeneity in the transaction prices, due to type of dwellings and different market conditions, in a very disaggregate fashion. On the decisions side, this research first developed a generic multidimensional modelling framework that captures
the behaviour of builders in terms of the supply of new built space. The where, when, how much, and what type of supply decisions were incorporated within a single framework. This modelling framework was then applied to estimate a model for the supply of new office space in the Greater Toronto Area (GTA). Estimation results indicated a risk taker behaviour on the builders’ part, while market conditions and supply of resources (labour, construction cost etc.) were also found to be important factors in decision making. In addition to that, this research also developed a comprehensive hedonic analysis for the asking rent of office space in the Greater Toronto Area. The effects of accessibility, quality, location, and market conditions on rent were explored. Data indicated a high degree of spatial heterogeneity and clustering effects. Spatial analysis techniques were incorporated within the hedonic framework to capture these effects. Estimation results indicated that access to transport infrastructure, distance from CBD, and vacancy rate were significant in explaining the variation in the rent.
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\(^{1}\) In South Asian culture, the concept of debt that a person owes to his ancestors and is obliged to repay in his life
# TABLE OF CONTENTS

Abstract........................................................................................................................................ii
Acknowledgment...........................................................................................................................iv
Table of Contents..........................................................................................................................v
List of Figures.................................................................................................................................ix
List of Tables..................................................................................................................................xii

**PART I: Urban Built Space...........................................................................................................1**

1 Introduction..................................................................................................................................2
   1.1 Background and Motivation.................................................................................................2
   1.2 Scope of Research................................................................................................................8
   1.3 Research Significance..........................................................................................................9
   1.4 Thesis Structure..................................................................................................................10

2 Conceptual Framework for Modelling Urban Built Space Evolution.................................12
   2.1 Introduction.........................................................................................................................12
   2.2 Integrated Land Use, Transportation, and Environment Modelling Framework.............12
   2.3 Built Space Evolution within ILUTE System......................................................................15

**PART II: Markets..........................................................................................................................18**

3 Built Space Markets...................................................................................................................19
   3.1 Introduction........................................................................................................................19
   3.2 Literature Review..............................................................................................................23
   3.3 Concluding Statement.......................................................................................................34

4 Market Clearing Model Formulation.......................................................................................36
   4.1 Introduction........................................................................................................................36
   4.2 Problem Statement.............................................................................................................36
   4.3 Key Assumptions and Definitions.....................................................................................36
   4.4 Theoretical Foundation.....................................................................................................37
4.5 Mathematical Structure..............................................................................................................41
  4.5.1 Root Finding.......................................................................................................................44
  4.5.2 Transaction Prices Search: Pseudo-code.............................................................................47
5 Application and Operationalization: Owner-Occupied Housing Market in ILUTE........48
  5.1 Introduction............................................................................................................................48
  5.2 Conceptual Model for Owner-Occupied Housing Market..................................................48
  5.3 Operationalization of the Owner-Occupied Housing Market............................................51
    5.3.1 New Housing Supply.........................................................................................................54
    5.3.2 Asking Price, Mobility, Location Choice Decisions......................................................55
    5.3.2.1 Choice Set Generation................................................................................................55
    5.3.3 Synthetic Population.......................................................................................................56
    5.3.4 Space................................................................................................................................57
    5.3.5 Time....................................................................................................................................57
    5.3.6 Price Set Search................................................................................................................57
6 ILUTE Housing Market: Software, Simulation and Results..................................................59
  6.1 Introduction............................................................................................................................59
  6.2 Definitions................................................................................................................................60
  6.3 Software Design.....................................................................................................................61
  6.4 Simulation...............................................................................................................................63
  6.5 Selected Results.....................................................................................................................65
  6.8 Discussion and Concluding Remarks.....................................................................................73

PART III: Decisions......................................................................................................................76

7 Built Space Valuation and Supply............................................................................................77
  7.1 Introduction............................................................................................................................77
  7.2 Literature Review...................................................................................................................83
    7.2.1 Supply Modelling.............................................................................................................83
    7.2.2 Valuation..........................................................................................................................87
LIST OF FIGURES

1.1 Greater Toronto Area: Regions and Boundaries.........................................................3
1.2 Population Growth Trend in the Greater Toronto Area.................................................4
1.3 In-Migration Trend in the Greater Toronto Area..........................................................5
1.4 Global Spatial Trends of In-Migration to Toronto.......................................................5
1.5 Employment Trend in the Greater Toronto Area..........................................................6
1.6 Housing Price Trend in the GTA..................................................................................7
1.7 Research Themes and Thesis Chapters........................................................................11
2.1 ILUTE Structure and Current Implementation............................................................14
2.2 A Conceptual Framework of Built Space Evolution....................................................17
3.1 Family of Bit Rent Curves............................................................................................25
3.2 Zero Profit/Utility Surfaces with Different Steepness.................................................25
4.1 Matrix Representation of the Built Space Market.........................................................38
4.2 Probability Summations for Buyers and Sellers.........................................................38
4.3 Two-stage directed and bounded search for potential transaction prices.....................46
5.1 Conceptual Model for the Owner-Occupied Housing Market.......................................49
5.2 Operational Housing Market in ILUTE.......................................................................51
6.1 Extensive Class Diagram of ILUTE............................................................................61
6.2 World Class Diagram – Relationships........................................................................62
6.3 Price Distributions for the Simulation Year 2001.......................................................68
6.4 Price Distributions by Type for the Simulation Year 2001..........................................69
6.5 New Housing Supply.................................................................................................71
6.6 Population Distribution by Year..................................................................................72
7.1 Various Stages of Construction....................................................................................80
7.2 Various Markets and Agents involved in the Built Space Supply.................................81
9.1 Study area and approximate location of the business nodes.......................................104
9.2 Yearly supply of office floor space in GTA...............................................................107
9.3 Yearly supply of floor space by type for the GTA .................................................................108
9.4 Yearly supply of floor space by regions in the GTA ..........................................................109
9.5 Spatial distribution of floor space type in the GTA ..............................................................109
9.6 Distribution of buildings by number of floors ......................................................................110
9.7 Number of Construction workers .......................................................................................111
9.8 Wage Rates for Construction Workers (in 2001 CAN Dollars) .........................................112
9.9 Construction Cost for Square Foot of Office Space by Number of Floors in the Building (in 2001 CAN Dollars) ........................................................................................................113
9.10 Gross Rent per Sq. Ft. for Office Space (in 2001 CAN $) ................................................114
10.1 Office space rent variation with reference to the distance from CBD .................................125
10.2 Share of Office Space Type in 2005 .....................................................................................127
10.3 Average Asking Rent by Type of Building .........................................................................129
10.4 Spatial Description of Office Space and Transportation Network .......................................130
10.5 Asking rate estimate as a function of distance from CBD ..................................................140
A.1 Household, Family, and Person Class Diagram – Inter-Relationships .................................177
A.2 Household Class Diagram – Relationships .......................................................................178
A.3 Family Class Diagram – Relationships ...............................................................................178
A.4 Person Class Diagram – Relationships ...............................................................................179
A.5 Household Class Diagram – Attributes and Operations ....................................................180
A.6 Family Class Diagram – Attributes and Operations ..........................................................180
A.7 Person Class Diagram – Attributes and Operations ..........................................................181
A.8 Location, SpatialObject, Area, CensusZone, and TTSZone Class Diagram – Relationships ..........................................................182
A.9 SpatialObject Class Diagram – Relationships ....................................................................183
A.10 TTSZone Class Diagram – Relationships .......................................................................183
A.11 CensusZone Class Diagram – Relationships ....................................................................183
A.12 Location Class Diagram: Attributes and Operations .......................................................183
A.13 SpatialObject Class Diagram: Attributes and Operations.................................183
A.14 Area Class Diagram: Attributes and Operations...........................................184
A.15 TTSZone Class Diagram: Attributes and Operations....................................184
A.16 CensusZone Class Diagram: Attributes and Operations............................184
A.17 DwellingUnit Class Diagram – Attributes and Operations...........................185
A.18 SpaceBuilder Class Diagram – Attributes and Operations...........................185
A.19 DwellingUnit Class Diagram – Relationships...............................................186
A.20 HousingMarket Class Diagram – Relationships............................................187
A.21 BidderSet Class Diagram – Attributes, Operations, and Relationships...........187
A.22 DwellingChoiceSet Class Diagram – Attributes, Operations, and Relationships187
A.23 HousingMarket Class Diagram – Attributes and Operations........................188
B.1 Simulated World – System Sequence Diagram.............................................189
B.2 Housing Market Clearing – System Sequence Diagram..................................190
B.3 Household class – State Chart Diagram of Housing Market Interaction...............191
B.4 DwellingUnit class – State Chart Diagram of Housing Market Interaction..........192
B.5 Price Finding – State Diagram of Housing Market Interaction........................193
B.6 New Housing Supply – System Sequence Diagram.......................................194
LIST OF TABLES

6.1 List of Main ILUTE Classes...........................................................................................................62
6.3 Transaction Prices by Type for ILUTE and Toronto Real Estate Board.............................71
9.1 Business Nodes in the study area..............................................................................................106
9.2 Summary Statistics..................................................................................................................115
9.3 Model Parameter Estimates....................................................................................................117
9.4 Correlation Matrix between the error terms.......................................................................117
10.1 Business Nodes in the study area.........................................................................................126
10.2 Descriptive statistics for Asking Rent by Type of Building.............................................127
10.3 Summary Statistics.................................................................................................................131
10.4 Hedonic Analysis of Asking Rent..........................................................................................139
A.1 List of Decisions Implemented in the Population Classes......................................................180
PART I: URBAN BUILT SPACE
CHAPTER 1
INTRODUCTION

1.1 Background and Motivation

Since 2008, for the first time in the history of mankind, urban areas of the world are host to more than 50% of its population\(^2\). In Canada the share of urban areas is 80% of the total population, while in the province of Ontario, this share is 85%\(^3\). Due to the rapid urbanization and the resulting population and economic growth, cities all over the world are becoming more important than countries (Pronk and Mimica, 2008). Urban regions are analogous to “concentrated mass”. It is where the growth, technological progress, and intellectual vibrancy are concentrated and most importantly, where the majority of votes for politicians are cast. Particularly, in Canada, the rapid urbanization process that it has experienced in the past few decades, has given rise to pressure on the infrastructure of the cities and a high demand for liveable communities characterized by sustainable options for urban transportation and housing (Miller, 2008). For a city to stay competitive, maintain its growth, and keep attracting good businesses and creative people, it has to be able to offer great living conditions, growth opportunities, affordable housing, good quality commercial and other activity-centric built spaces\(^4\), environmental sustainability, high accessibility through its transportation system, and various other important urban infrastructure systems.

The Greater Toronto Area (GTA), which is located in the Greater Golden Horseshoe region in Ontario, is composed of City of Toronto and York, Durham, Peel, and Halton Regions (Figure 1.1). It is the largest and one of the most rapidly growing urban areas of Canada that has experienced a very high degree of growth in the past few decades. According to the Global Power Index 2009 (GCPI), economically Toronto is the world’s 15\(^{\text{th}}\) most powerful city. It is ranked number 3\(^{\text{rd}}\) among the North American cities. The population of Toronto has increased

\(^2\) Source: United Nations Population Fund (UNFPA)
\(^3\) Source: Statistics Canada
\(^4\) Built space is a generic term, used throughout this dissertation, to represent various types of spaces in an urban area that have a physical structure and associated monetary value; can be identified as individual quasi-unique units (based on their attributes and location); and provide opportunities for various activities. These spaces include: dwelling units, office spaces, retail spaces, industrial spaces, etc.
fivefold in the past fifty years. In 2006, there were more than 5 million people living in the GTA and according to the Ontario Government’s recent forecasts, this number will become almost double in next 25 years (figure 1.2).

![Greater Toronto Area: Regions and Boundaries](image)

**Figure 1.1**
Greater Toronto Area: Regions and Boundaries

In recent years, the growth in population is primarily driven by the influx of new immigrants to Canada, coming and settling in the GTA. GTA is one of the leaders in the immigrant receiver regions of the world. In the past 20 years, it is estimated that an average about 90,000 new Canadian immigrants each year came and settled in the GTA (figure 1.3). These immigrants are highly skilled and come from various different parts of the world (figure 1.4). A high percentage of these immigrants comes from China and India as skilled labour. This

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5 Source: Planning and Development Services, York University
consistently high rate of highly skilled immigration to the GTA is indicative of the fact that the GTA is a growing urban area offering competitive living conditions and good economic opportunities.

On the downside, the population growth in the GTA and search for affordable housing has resulted in urban sprawl and extension of the urban boundaries further and further into rural areas surrounding the GTA. Evidence also suggests that the region is experiencing increase in the stress on the transportation infrastructure, environment deterioration, increase in energy consumption, and decrease in affordability.

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Figure 1.2
Population Growth Trend in the Greater Toronto Area

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*Future estimates generated by Ontario Ministry of finance*
Figure 1.3
Immigration Trend in the Greater Toronto Area\textsuperscript{7}

Figure 1.4
Global Spatial Trends of In-Migration to Toronto\textsuperscript{8}

\textsuperscript{7} Estimates are generated using the historic data from Statistics Canada
\textsuperscript{8} Source: Toronto Real Estate Board (TREB)
The economic growth of the GTA has been phenomenal in the past few decades and is the major driver of population growth in the area. The region produces one-fifth of the nation’s GDP and is considered the financial capital of Canada. The employment in the GTA is dominated by the office based employment sector (about 45% in the City of Toronto\(^9\)). The forecast for the next twenty years by the Government of Ontario shows a sharply increasing growth trend in the employment in the GTA (figure 1.5). To accommodate this increase, the region will require not only growth and affordability in terms of residential, commercial, and industrial built space, but also in its transportation system that can provide quick, efficient, and congestion free movement of people and goods in the GTA.

![Figure 1.5](image)

**Figure 1.5**

*Employment Trend in the Greater Toronto Area*

With the growth of population in the region, the GTA has experienced a high demand for affordable housing. On the other hand, if one looks at the housing prices for the single family housing over the past 25 years, a steady increase in prices is noticed. Figure 1.6 compares the yearly average prices in 2009 dollar values. Even after the recent major recession, prices seem to have an increasing trend. Evidence suggests that the supply of affordable housing seems to be trailing the ever increasing demand in the GTA.

\(^9\) Source: City of Toronto
To maintain a sustainable growth of economy and population in an urban area, it is very important that the planning and policy making for the supply and management of major urban infrastructure, including transportation, housing, commercial space, and other forms of built spaces, not only take into account the future demand related to individual systems, but also the interactions between these infrastructures, that directly influence the spatio-temporal patterns of travel and distribution of the population and activities. While planning a sustained urban growth necessitates the use of integrated frameworks for modelling and analysis of the various systems and their two-way interactions, there seems to be a serious lack in terms of their usage by the urban areas, including the Greater Toronto Area.

Miller (2008) pointed out that, in practice, the transportation system is typically modelled using some form of four-stage travel demand model that takes population and employment forecasts for the zoning system as exogenous. Built space evolution is treated as a separate activity involving trend extrapolation, professional judgement, and various ad hoc methods. This general lack of proper built space evolution models and connectivity between built space evolution and transportation modelling has been very common in North America. Lee (1994) criticized large scale urban models arguing that the models developed since his original criticism in “Requiem for Large-Scale Models” (Lee, 1973), represented neither good science nor good
engineering. Not a good science because, they lacked good theoretical and behavioural foundation, and not good engineering, because they were not very practical and useful. In the past 15-20 years, however, there is a renewed interest in this regard (Wegner, 1995; Miller et al., 2004; Hunt et al., 2005). The new wave of modeling effort is predominantly based on microsimulation models of individual agents’ behaviour. Microsimulation (originally named, “micro-analytic” by Orcutt) approach provides a comprehensive and flexible framework to model the behaviour of individual agents and the representation of various processes that drive the urban demographic and economic evolution (Orcutt, 1957, 1990).

The Integrated Land Use, Transportation, and Environment (ILUTE) system is a microsimulation based framework for urban systems modelling that is under development at University of Toronto (Miller and Salvini, 1998; Miller et al., 2010). In recent years, significant progress has been made in developing activity based travel demand modelling, greenhouse gases emission and dispersion modelling, operational prototyping, and the demand aspect of the built space evolution within the ILUTE system (Salvini, 1998, 2003; Elgar, 2007; Roorda, 2008; Hatzopoulou and Miller, 2008; Habib, 2009; Habib and Miller, 2009). However, many serious gaps still remain, in terms of theory, models, and operationalization, to represent a comprehensive built space evolution, completely integrated with other modules within the ILUTE system. This research is motivated by the need to fill these gaps, particularly, in the behavioural modelling of the built space markets and the new built space supply and valuation decisions. This effort will thus result in a comprehensive and integrated tool for the policy makers so as to assess the sustainability of various policies related to transportation, land use, environment, and energy use.

1.2 Scope of Research

The core motivation of this research is to enhance our collective understanding of the behaviour of the major agents that are involved in the evolution of urban built space. The evolution of the built space is directly influenced by the decisions these agents make and their mutual interactions in the built space markets. To achieve this objective, the scope of this research involves the following key tasks:
a) Development of a conceptual framework that comprehensively represents various processes that are occurring in the built space evolution and is integrated with various other components of ILUTE system.

b) Development of a behavioural theory and microsimulation modelling framework that comprehensively represents the interactions of agents in the built space markets.

c) Investigation of various processes and agents’ behaviour in the context of the supply of new built space.

d) Development of econometric models that capture the agents’ behaviour of decision making in terms of the supply of new built space and valuation of the built space.

e) Design and development of a full-scale operationalization of built space evolution within ILUTE system.

f) Validation of the predicted output of the operationalization with the historic data for an extended period of time (1986–2006).

1.3 Research Significance

The ability to forecast the spatio-temporal distribution of population and built space, resulting from various policy scenarios, is critical for the assessment of policies in the context of urban sustainability. By filling in very important gaps in the built space evolution literature and developing and validating a full scale operational microsimulation of built space evolution, this research is a significant step forward towards achieving this goal. The market-disequilibrium based behaviourally rich model of market clearing developed in this dissertation has strong scientific foundations and is very easily operationalizable. The dynamic econometric models of the supply of new built space and valuation, developed in this dissertation, capture the interplay between various dimensions of decision making; incorporate various spatial phenomena; investigate the role played by accessibility in the evolution of the built space; and are highly disaggregate. The full scale operational microsimulation of the built space evolution for the GTA, that is developed here, is unique in the sense that it is validated with an extended historic data and is a highly detailed and effective policy-testing tool for the urban planners. This research not only provides the ILUTE system the capability to microsimulate the evolution of
built space, but also extents the framework, so that various new components (energy use, environmental impact, etc) can be incorporated within ILUTE.

1.4 Thesis Structure

Based on the scope defined in section 1.2, this dissertation is divided into four parts: Part I, which is composed of chapters 1 and 2, presents the introduction of the thesis and a conceptual model for the evolution of urban built space, within the integrated land use, transportation, and environment modelling framework.

In Part II which is composed of chapters 3, 4, 5, and 6, this dissertation covers the first major theme of the built space evolution, Markets. Chapter 3 discusses the state of research in the context of built space markets. A theoretical formulation of the microsimulation clearing process for the built space markets is proposed in chapter 4. The application of this formulation to the owner-occupied housing market is presented in Chapter 5. Chapter 6 discusses the operationalization of owner-occupied housing market and the validation of the simulation results.

In Part III, which is composed of chapters 7, 8, 9, and 10, this dissertation shifts the focus to the second major theme, Decisions. In chapter 7, the dissertation discusses the state of research in the context of the supply of new built space and valuation of the built space. The model formulation for the decisions related to supply of new built space is presented in chapter 8. The application of this model formulation to new office space supply is presented in chapter 9. Chapter 10 presents models of valuation of office space rent in the Greater Toronto Area.

Finally, part IV, which consists of chapter 11, presents the summary, conclusions, limitations, and future directions of the research are discussed. Figure 1.7 illustrates the various parts of this dissertation.
Figure 1.7
Research Themes and Thesis Chapters
CHAPTER 2

CONCEPTUAL FRAMEWORK FOR MODELLING URBAN BUILT SPACE EVOLUTION

2.1 Introduction

This chapter presents the basic structure of the ILUTE system. Within the ILUTE system, a conceptual framework for built space evolution is then introduced. This conceptual framework acts as a template for the various components that are then developed in parts II and III of this dissertation.

2.2 Integrated Land Use, Transportation, and Environment Modelling Framework

The Integrated Land Use, Transportation, and Environment (ILUTE) modelling framework is an agent-based, comprehensive, and integrated microsimulation that dynamically evolves urban spatial form, demographics, travel behaviour, and environmental impacts over time for the Greater Toronto-Hamilton Area (Miller et al., 2010). It has been under constant development at the University of Toronto since the mid-nineties. During this time it has gone through various iterations of updates and changes, the details of which can be found in: Miller and Salvini, 1998, 2001; Miller et al., 2004; Salvini, 1998, 2003; Miller et al., 2008a.

Figure 2.1 shows the most up-to-date structure and process execution sequence of the ILUTE system. It is designed as a time driven serially executing set of modules. The state of the system at any point is defined in terms of individual persons, households, dwellings, firms, office spaces, etc. that collectively form the state of the urban region. The ILUTE system is initialized using the base year population and other input datasets (e.g. economic data, spatial zonal definitions, demographic data, etc) that are required to define the initial state of the system. During the simulation, each year, the population demographics are updated. This update includes the evaluation of decisions related to in-migration and out-migration, education, marriage, divorce, birth, aging, etc. at household, person, and family levels. The ripple effects of these decisions among the persons, families, and households are maintained. The labour market module evaluates the decisions by the labours in terms of entering/exiting the labour market, and work mobility. It also performs the matching process of jobs with the labour, in addition to determining and maintaining the wage rates of the labours.

Built space market module manages the dynamic evolution of the built space and changes in the spatial and temporal distribution of the population due to their participation in the built space markets. In

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10 This section is primarily based on the description of ILUTE system in Miller et al. (2010)
the following sections and chapters, the built space module is discussed in more detail. The auto
ownership module dynamically evolves the urban vehicle stock using the vehicle transactions and vehicle
type/vintage choice models for the households, developed by Mohammadian and Miller (2002a, 2002b,

TASHA (Travel/Activity Scheduler for Household Agents), which is a microsimulation
framework for travel demand modelling, generates the activity/travel patterns for each person within each
household for a typical weekday. The details of the theory, design and operationalization of TASHA can
be found in: Miller and Roorda (2003); Roorda and Miller (2006), Roorda et al. (2006), Roorda et al.
(2008), Miller et al., (2006, 2008). TASHA is connected to a network traffic assignment model (e.g.
EMME or MatSim) so as to generate the urban spatio-temporal travel patterns. These patterns are then
used by the environment module to generate and analyse the emissions produced on road network, their
dispersion across the urban area, and effects on the urban population. The details on the environment
module can be found in: Hatzopoulou et al. (2007); Hatzopoulou and Miller (2008); Miller et al. (2009);
and Hao et al. (2010).

Recently, a major effort has been undertaken to develop the energy module in the ILUTE system.
This ongoing effort is focused on generating and analysing the spatio-temporal patterns of urban energy
use based on the population, built space, and traffic distributions. As a first step, a model for household’s
heating equipment selection and usage is under development (Chingcuanco, 2010a).
FIGURE 2.1
ILUTE Structure and Current Implementation (Miller et al., 2010)
2.3 Built Space Evolution within ILUTE System

Built space evolution is conceptualized as the outcome of decisions and interactions among various agents (including, households, firms, builders, etc.) in the urban built space markets (for instance, owner-occupied housing market, rental market, office space market, etc.). Based on their role, these agents can be divided into two categories: *demanders* and *suppliers*. Demanders represent the demand-side of the market while suppliers represent the supply-side.

As shown in figure 2.2, the build space evolution module within ILUTE system is divided into three identifiable sub-modules. The demand sub-module encapsulates the decision making behaviour of the demanders. Existing demanders in the urban area decide on becoming active in the market. A mobility decision model can be used to evaluate this decision. Once the demanders are active in the market, they start looking for the available built space options. This behaviour of choice formation is captured using a separate choice set generation mechanism. Based on the available choice set, demanders determine their preferences. The preferences are represented by a location choice preferences model. New demanders of built space are created by various other modules (e.g. in-migration sub-module, firmography module, etc.) within ILUTE system. These demanders also go through the choice set generation and location choice preferences models. The existing demanders may also act as suppliers, by bringing in the built space they currently own for resale or rental in the market.

On the supply side, builder agents decide on maintaining a certain level of new supply to the already existing stock of the built space. They are faced with decisions of when, where, what type, and how much of the built space to build. These decisions are modelled using a multidimensional decision model that can maintain the interplay between all these dimensions of decision making. Another decision that suppliers have to make when listing the built space in the market is the setting up of asking price/rent for the built space. Asking price/rent represents the expectation of the suppliers regarding how much profit they can get from selling or renting the built space in the market. A separate model is needed to capture the behaviour of suppliers in term of setting asking price/rent for the built space they own. Other sources of the suppliers may come from out-migration, death of a firm/household, etc.

The built space market is the place where suppliers and demanders interact with each other so as to transact a built space at certain monetary value. In the microsimulation context, the interactions for each built space are individually managed. The demanders show their level of interest in the built space and the supplier decides which demander to sell/rent the space to. It is very important that the behaviour of the agents in the models of this interaction is properly represented. Based on the behaviour of the demanders in terms of price/rent, the market can be classified as *price-taker* or *price-formation* markets.
In price-taker markets, the demanders accept the asking price as non-negotiable, while in price-formation markets, demanders negotiate the price in a bidding process, thus the transaction price may be different than asking price.

The built space evolution module gets feedback from various other modules of ILUTE systems through their influence on the agents’ decisions making, and lagged changes in the valuation and stock of built space due to changes in those systems. These changes are captured by various models implemented in the built space. For instance, the construction of a major commercial centre may result in high traffic volumes and thus deteriorating environment in the neighbourhood. This may influence a household to decide on moving to another neighbourhood. Or, the introduction of a new streetcar line in the neighbourhood may increase the asking lease rates for the nearby office spaces.
FIGURE 2.2
A Conceptual Framework of Built Space Evolution
PART II: MARKETS
CHAPTER 3

BUILT SPACE MARKETS

3.1 Introduction

The second part of the dissertation is focused on a very important modelling dimension in the context of urban built space evolution, Markets. A market encapsulates the interaction of two different types of agents (seller and buyers) in the exchange of services, quasi-unique goods, and monetary transactions. The effect of interaction in the markets on the agents is the change in their utility and profit levels. Sellers are interested in selling/leasing/renting their services and goods, so as to achieve a gain in their profit. Buyers/Renters are interested in buying/leasing/renting a space so as to achieve a gain in their total utility. The behaviour of a seller in the market is usually modelled using a profit function, while utility function represents the behaviour of buyer in the market. Modelling built space markets is very important in the context of understanding the evolution of urban systems in general and built space in particular, as they drive the pattern of population and space distribution in an urban area and represent the economic health of the region.

Based on the price determination mechanism, built space markets in an urban area can be divided into two categories: Price-Taker and Price-Formation market. In a Price-Taker market, a seller lists its built space at a certain asking price. The buyer is assumed to be a price-taker, that is, it accepts the asking price as is and determines the gain in its utility at that price. Based on the utility gains from various built space choices available to it, the buyer chooses an option. In terms of microsimulation modelling of such a market, the price-taker market clearing problem thus becomes a matching problem in which the modeller is interested in finding out “who gets what” (Farooq et al., 2010a). The price determination and choice set formation models are exogenous to the clearing process. The agents are assumed to have limited information about the market and are individually profit/utility maximizers. At a given exogenously determined price surface for the built space stock and choice-sets of the buyer agents, the sequence of clearing in the market, guides the matching process.
The most common example of a price-taker market is the rental housing market. Rent levels for the listed dwellings in the markets are fixed and are heavily regulated by the government (at least in the case of the Greater Toronto Area). The owners list their dwellings at a list price which they usually determine based on the quality of the space, location, and most importantly, the previous rent level. Each year, there is a steady rise in the rent of occupied dwellings which is based on the maximum rent increase allowed by the government. In rental housing markets, there is less dynamics in terms of the rent levels. In terms of market size, Giroux-Cook (2010) reported that there were 40,000 to 60,000 households active each year in the rental market in the GTA, between 1990 and 2006.

In a price-formation market, a seller lists its built space at a certain asking price, but, unlike the price-taker market, this price does not necessarily remain fixed during the clearing process. Each buyer generates its choice set based on factors including the asking price, minimum quality requirements from a built space, location, and various other needs. Buyers in the market bid for the built space so that they can outbid each other and at the same time achieve a maximum gain in their utility levels. Sellers, on the other hand, try to maximize their profit by accepting the highest possible bid. If the buyer cannot find a unit on which it can bid, so as to achieve a gain in its total utility, it may decide to leave the market. Similarly, if the seller doesn’t get a good bid for its space, it may either leave the market or lower the asking price to generate more interest from the active buyers in the market. Thus, the resulting transaction price is a function of market interaction between the buyers and sellers and market conditions. In terms of microsimulation modelling of such market, the price-formation market clearing problem is a matching problem in which the modeller is interested in finding out “who gets what at what price” (Farooq et al., 2010a).

Given the current market conditions, the asking price captures the perception of a seller about the value that it can achieve from the built space it owns. Asking price is only a reference point for the final transaction price. Transaction price on the other hand, is an outcome of the market and is expected to be within certain range of the asking price. The buyers and sellers in price-formation markets are utility and profit maximizers, respectively. They are assumed to have limited information about the market and are noncooperative agents.
The most common example of price-formation market in the urban systems modelling context is the owner-occupied housing market. Builders and households list their new and existing dwellings in the market at certain asking prices. Based on their knowledge of the market, households that are looking for a dwelling in the market, choose a certain set of dwellings according to their needs and expectations from the dwellings listed in the market. Households bid on the dwellings, based on the maximum utility they can gain from the dwellings. This may result in the household with the highest bid becoming the new owner of the dwelling. At any time both buyers and sellers can leave the market if their expectations are not met. It should be noted that builders selling the stock of newly built dwellings may behave more flexible and be better informed about the whole market than that of a household reselling its dwelling.

While the two types of housing markets discussed above are usually modelled separately due to the differences in the price mechanism, there is a strong two way interaction going on between both markets. At any time, these two markets are operating in parallel and households switch between the two as they learn more about each market. In some cases, given that the rental market is more active than owner occupied-market, a household that was initially interested in selling the second dwelling may decide to rent it. Thus, in any microsimulation modelling framework of built space evolution, one should carefully incorporate the two-way interactions between the two markets.

In terms of a solution for the clearing of these two markets, the urban economics and integrated land use and transportation modelling literature is dominated by the approaches that impose some degree of strong equilibrium assumption, so as to generate a unique price surface for the market (Anas, 1982, 1992, 1994, 1995; Putman, 1983; Echenique et al., 1990; Martínez, 1992, 1996a, 1996b; Anas and Arnott 1993, 1994; de la Barra, 1995). While these approaches are easily operationalizable and exhibit well defined and well investigated properties, in my view, the equilibrium assumption is an oversimplification of the market characteristics and behaviour of the agents in the market. In actual markets, agents have limited information about the market; they are individually utility/profit maximizers; they are noncooperative among each

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11 Such a situation may arise when a household who is looking to change its current dwelling, finds a new dwelling in the market, buys it, but has not yet been able to sell the current dwelling.
others in the market; and their decisions are conditioned upon the sequence of decisions taken by other agents.

Based on the above observations, in a microsimulation clearing of housing market, potentially there could be infinite price surfaces, with each surface representing the sequence in which the market is cleared. The sequence itself is stochastic, depending on the previous and starting state of the market (as a system) and at any point, during the clearing, phenomena like who-gets-active-when and who-leaves-when. Moreover, there will always be households left in the active market that are still looking for a dwelling; there will always be dwellings left that are to be sold; and households will be continuously entering and exiting the market. The clearing process of the market is thus a continuous process in time rather than a discrete step process.

In this dissertation, the market equilibrium assumption is dropped and it is assumed that the market is in constant disequilibrium. A new price formation mechanism is proposed that works within the disequilibrium assumption. The market clearing solution that is formulated and operationalized here is able to incorporate the market characteristics and behaviour of agents in the market in a more detailed fashion. On the downside, though, it is heavily dependent on the accuracy of the models of choice set generation, asking price, and location choice preferences, and the assumption that the stochastic clearing sequence used in the microsimulation clearing is representative of the actual market clearing sequence.

A microsimulation modelling framework for price-formation market has already been suggested by Farooq et al. (2010a) and applied for rental housing market of the GTA, by Giroux-Cook (2010). The proposed model reduced the clearing problem to a graph theoretic problem and proposed a stochastic clearing mechanism that, in each iteration, tries to maximize individual agent’s utility. In this part of the dissertation, however, I have concentrated the efforts on developing a disequilibrium based microsimulation market clearing process for price formation markets. The next section of this chapter discusses the state of the literature related to built space markets. In chapter 4 I propose a theoretical framework for the microsimulation clearing of price formation markets. Chapter 5 discusses the application and operationalization details of the proposed framework to owner-occupied housing market within the ILUTE modelling
framework. In chapter 6, which is the last chapter of part II of the thesis, I discuss the software design, simulation, and results of the operationalized owner-occupied housing market in ILUTE.

3.2 Literature Review

Central Place Theory (CPT), originally proposed by a German geographer, Walter Christaller (1933) and then later modified by a German economist, August Lösch (1944), is one of the earliest classic theories of spatial competition that explained the reasons behind the distribution patterns, size, and number of cities and towns in various countries, around the world. Later it was also used to explain the size and spacing of centres within polycentric metropolitan areas. CPT assumed that the space to be an isotropic (flat) surface with evenly distributed population and resources. Consumers were assumed to have similar purchasing power and their transportation cost being proportional to the distance from centre and equal in all directions. The demand for goods was expressed as a function of total cost (including the distance that has to be travelled to purchase the good) and thus the supply as the function of distance. It was assumed that there is a perfect market competition, so as many firms can enter into the market as long as their sales are more than certain threshold ($S_{\text{min}}$) that keeps a firm in profit. Christaller proposed that at market equilibrium, firms will be packed together to form a hexagonal lattice of market areas, with each market area generating sales of exactly $S_{\text{min}}$. For consumers, this solution will minimize the cost associated with the average travel distance.

Goods could also be categorized into a hierarchy that is based on the range over which consumers will travel to purchase the good and a minimum profitability threshold. The lower order goods have smaller ranges and thresholds while the higher order goods have larger ranges and thresholds. This hierarchy will result in layers of hexagonal lattices with each having its own equilibrium. In an urban area one generally observes hierarchies of centres, but the concept of an abstract hexagonal lattice is too far from reality and is unable to capture the behaviour of suppliers and demanders. The model was able to capture the role of accessibility in shaping up of the urban form. CPT fails to describe various processes going on within the urban built space markets that form these central spaces. The model explains the urban form as a result of the dynamics in the goods supply, while assuming the population to be uniformly distributed. This assumption severely limits the scope of the model, even in explaining the size and distribution of the urban areas. In terms of game theory, the equilibrium solution proposed in the CPT is
equivalent to the outcome of a cooperative game. This contradicts with the basic perfect competition assumption of the CPT. A cooperative behaviour of the firms is usually only observed in the case of oligopoly. Moreover, the perfect competition assumption in the market for all the different types of goods is also not very representative. Due to the nature of the good and the associated market, oligopoly or monopoly may exist in certain goods.

Hotelling (1929) assumed a different behaviour of firms in terms of location choice strategy in the urban space market. He argued that a far more common location strategy of firms is to locate immediately next to each other, so as to capture as big of a market share as possible. This assumption explains the phenomena like spatial competition and agglomeration of economies in a more consistent way. In terms of game theory, the solution proposed by Hotteling is an outcome of noncooperative game. The assumption of perfect competition is more consistent with the noncooperative games.

Alonso’s (1964) bid rent model was the first major effort towards modelling the underlying processes that are going on in the urban built space markets. Just like central place theory, Alonso also abstracted the urban area as isotropic surface with one central location and the travel cost being equal in all directions around the centre. The model assumed that in terms of choosing the location in space and the quantity (square footage of floor space, number of rooms), the firms and households are maximizers of profit and utility, respectively. They compete with each other in the built space market by bidding for the desired locations. The location owner, who is also a profit maximizer, sells the location to highest bidder.
Figure 3.1
Family of Bit Rent Curves (Alonso, 1964)

Figure 3.2
Zero Profit/Utility Surfaces with Different Steepness (Alonso, 1964)
For every user of the land, a bit rent function can be derived in terms of the distance from centre of an urban area. For a firm this bid rent function represents the willingness to pay (bid) at various locations that are indifferent to the firm, since they all return the same profit levels to it. This means that for different profit levels there will be different bit rent curves that are all parallel to each other. The shape and slope of these bit rent curves are a function of revenue and cost function of the firm. Different firms will thus have different bit rent curves for the same levels of profit. In the built space market, firms will try to outbid each other. The firm that desires a certain location the most will try to lower its target profits as low as possible. This will enable the firm to bid higher under current revenue and cost levels. In a perfect market conditions, a bidding war will occur between such firms and the location will be sold to a firm that has highest bit at zero profit. No one will be able to outbid this firm without going into loss. So, at equilibrium all the locations will be sold to the interested firms that can pay the highest, while operating at zero profit levels. Using the zero-profit bid rent curves for all the successful firms, one can establish equilibrium bid surface and associated type of (firm) activity for the entire urban area. The transaction prices can then be endogenously established from the equilibrium bid surface.

Similarly, in the case of households, there will be a family of bid rent curves for each household at different levels of desired utility. The shape and slope of the curves will be different for the households based on different levels of income and total cost. Here also, one can derive an equilibrium bid surface for the entire urban area. Bid rent model explains the evolution of a monocentric city well. Business firms usually are located at a more central and visible location (e.g. financial institutions). They have higher willingness to pay levels and steeper bid rent curves. Households have a less steep curve and are located around the centre of the city and in the suburbs. Agriculture, with the least steep curve, is located outside the city boundaries. This model can be extended to polycentric cities by relaxing some of the basic assumptions.

A very important contribution of the bid rent model towards our understanding of the urban system evolution is its ability to endogenously determine the prices within the clearing process. The prices are modelled as a direct result of the interaction of supply and demand. The effect of different market conditions on the price is thus captured by the model. By imposing a strong urban area level equilibrium, a unique surface is computed. This seems to be an over
simplified assumption and is not a very true representation of the inherent stochasticity in a system as complex as the built space market in an urban area. Another interesting aspect of the bid rent model is that the use of land (in terms of type of built space) and the intensity of use are also endogenously determined. This enables the bid rent model to forecast the evolution of the city and the built form as a function of built space market.

Alonso’s original bid rent model used a hedonic approach to estimate the bid rent functions for the buyers in the built space market. That involved estimation of revenue, cost, and utility functions. Ellickson (1981) on the other hand, applied random utility theory (RUM) to the bid rent model and proposed a stochastic bid rent model. This resulted in a very clear interpretation of the results and a better representation of the stochastic nature of the built space market. The bid \( B_h(z) \) of a household \( h \) for dwelling \( z \) was assumed to have a deterministic \( (\Psi_h(z)) \) and a stochastic \( (\varepsilon_h) \) component.

\[
B_h(z) = \Psi_h(z) + \varepsilon_h \quad [3.1]
\]

If stochastic (error) term was assumed to be independently, and identically extreme value distributed. Then the probability that a household \( h \) will be the highest bidder for dwelling \( z \) is determined by:

\[
P(h|z) = \frac{\exp(\Psi_h(z))}{\sum_{h'} \exp(\Psi_{h'}(z))} \quad [3.2]
\]

Where, \( h' \) is the set of all household that are interested in the dwelling \( z \). The deterministic part of the bid was modelled as a function of the attributes of a dwelling and its neighbourhood. The model was estimated for the San Francisco area. Households were stratified into 24 types and separate models were estimated for each type.

Martinez (1992, 1996a, and 1997) developed the Bid-Choice theory, which is a location choice and multi-locators equilibrium theory for urban land markets, with a strong microeconomics foundation. The willingness-to-pay (WP) for a certain type of dwelling \( v \) in a zone \( i \) by the household \( h \) at certain level of utility \( u_h \) and income level \( y_h \) is derived from the inverse of the indirect unity \( V \).

\[
WP_{hvi} = y_h - f(d_v, z_i; u_h) \quad [3.3]
\]
Martinez shows that the function $f$ represents the expenditure function and the difference between WP and rent represents the consumer surplus (CS) for the household. Similar to the utility maximization behaviour, the household’s consumer surplus maximization behaviour will dictate that it chooses the dwelling that maximized its CS.

$$CS^* = \max_{z \in \Omega} (WP_s - r_s), \text{ where } \Omega \text{ is the set of choices available to the household}$$ [3.4]

The maximum bid that a household is willing to put on a dwelling is derived from WP as:

$$B_{hs} = WP_s - w_h(A, H)$$ [3.5]

Where, $w_h$ is the speculation factor as a function of auction conditions $A$ and number of bidders $H$.

The sellers were assumed to be profit maximizers, so they will sell the dwelling to the highest bidder

$$r^*_s = \max_{h \in H} (WP_{hs} - w_h)$$ [3.6]

This shows that the rent is driven by the consumer behaviour (CS maximization) and indirectly by the availability of the type of dwelling. The parameter estimation for the WP function required that the model is simultaneously replicating the location choices as well as rents. To do so, three equilibrium conditions were imposed. First, every household must be location somewhere. Second, supply of dwelling is the function of rent levels and is adjusted based with demand. Third, the allocated land cannot exceed the available land. So, at equilibrium in the urban area:

$$\max_{s \in S} CS_{hs} = \max_{s \in \Omega} (WP_{hs} - \max_{h \in H} (WP_{hs} - w_h))$$ [3.7]

Where, $S$ is the total available stock in the urban area. Martinez also showed that under these conditions the Bid-Rent (Alonso, 1964) and the utility maximization approach for residential location choice (McFaden, 1978), are equal. The willingness to pay function was divided into an observable and stochastic part. The stochastic part was assumed to be identically and independently Gumbel distributed. Under such assumption, the probability of a household choosing a location (choice version) in its choice set and a household making the highest bid for a given dwelling (bid rent version) among all the bidders were reduced to a logit model.
The Bid-Choice model was operationalized in MUSSA, an urban evolution modelling framework for Santiago, Chile (Martinez, 1996b). MUSSA simulates the housing and commercial space market. It divides the consumers in both markets into categories separate estimation for the parameters in the willingness to pay function. It is also connected to a four-stage travel demand model, ESTRAUS.

Bid-Choice has a strong microeconomic foundation and it gives a good basis for microsimulation of the urban evolution. The rent is a direct result of the interaction of supply and demand and current market conditions. The strong equilibrium assumption however, limits it in terms of representing the complex interactions, stochasticity, and the dynamics going on in the built space market. It is hardly the case that all the households in an urban area who are looking for new dwelling are able to find one. Households enter the housing market, but if after a certain time they couldn’t find a dwelling according to their expectations, they might go out of the market or defer their plans to move. Similarly, owners might also take their dwelling out of the market if they couldn’t find a good price or might rent rather than sell the dwelling.

The strong equilibrium assumption also means that for the clearing problem of a market, there will be a unique solution with a single price surface. In reality, this is hardly the case. The agents in the market, both buyers and sellers, have limited information. Moreover, the decision of one agent affects the decision of many other agents. For instance, if initially, a household is not the highest bidder on a dwelling, but all the bidders higher than it leaves the auction, then suddenly this household becomes the highest bidder and might become the owner of the dwelling. There is an inherent stochasticity and path dependence in the housing market, which is not represented in the case of a strong equilibrium assumption.

In the context of microsimulation of urban systems, analysts are interested in tracking the changes for every household, through the simulation horizon. Furthermore, they are interested in knowing about the decision pattern along the lifespan of households for better policy making. Another shortcoming that results directly because of equilibrium assumptions is that bid-choice model fails to track the life timeline of individual households. At each time step, MUSSA does the matching of households and dwellings at certain endogenously determined rent, without any
reference to the state of the household in the previous time step. The uniqueness of the sellers, buyers, and built space stock is not conserved throughout the simulation horizon.

Other important, utility maximization, supply-demand equilibrium based models of built space market clearing includes: Smith (1969); Anas (1982, 1992, 1994, 1995); MEPLAN (Echenique et al., 1990); Anas and Arnott (1993, 1994); DRAM/EMPAL (Putman, 1983); Landis and Zhang (1997); PECAS (Hunt and Abraham, 2003); TRANUS (de la Barra, 1995); Bayer, McMillan, and Rueben (2004); Leishman and Bramley (2005); and Wood and Ong (2008).

UrbanSim which is an operational microsimulation land use and transportation modeling framework, is based on bid rent, random utility, and hedonic theory (Waddell and Ulfarsson, 2003; Waddell et al., 2008). Unlike bid rent theory, it relaxes the strong equilibrium assumption by making the prices exogenous to the clearing process. Buyers in the built space markets are assumed to be price-takers. The transaction prices are determined through a separate hedonic function which has a feedback coming from the current market conditions. The hedonic function also takes into account the built space characteristics, neighbourhood amenities and accessibility to determine the transaction rents.

At each time step, household and firm agents’ mobility decisions are evaluated (Waddell et al., 2003). The location choice decision for the active agents in the market is then evaluated using a multinomial logit model with random sampling. The resulting probabilities of a household to choose a location (choice version) and the price levels computed from the hedonic model are used in a Monte Carlo simulation to match the agents and space in the market. The relocation of agents thus becomes more like a yearly allocation process, rather than the outcome of their “mobility careers”.

The market clearing mechanism in the UrbanSim is not as strongly based on equilibrium assumptions as is the case in the bid rent, bid choice, or other similar approaches. Waddell et al. (2003) argued that the stochasticity that was introduced in the mobility and location choice decision, and the separation of prices determination from the clearing mechanism make it a disequilibrium based approach. However, in my view, even in disequilibrium, the transaction
price determination mechanism should still be endogenous to the clearing process, as it is the outcome of the supply and demand interaction.

Waddell et al. (2003) argued that individual consumers and producers (except in an oligopoly) do not affect the prices and they are assumed to be price-takers. This assumption is an over simplification of the complex interactions that are going on within the built space markets, especially in the case of the housing market. While this assumption is applicable to markets like the rental housing market, this assumption cannot be generalized. When the market is buyer driven (i.e. more built space options than buyers), the buyers will be able to dictate the lowering of the prices, while in case of seller driven market (i.e. more buyers than built space options), the seller will be able to get a higher price due to the potential bidding war between the buyers.

Moreover, the use of hedonic theory to determine the transaction price is an over simplification of the built space market clearing processes. The transaction prices are the outcome of the dynamic interaction between supply and demand in the built space market. On the other hand, asking price is the price demanded by the supplier-agent on the basis of its perception of the value offered by the product’s attributes and the existing market conditions (Farooq et al., 2010). While asking price is a good reference for determining the transaction price, it doesn’t mean that the seller will be able to get this price in various possible market conditions.

The ILUMASS microsimulation modelling framework models the agents’ relocation as a search process in which agents evaluate dwellings one by one and accept a dwelling if it provides a significant improvement over their current dwelling (Moeckel et al., 2005). Just like UrbanSim, it also determines the transaction price from a hedonic function.

Miller and Haroun (2000) proposed a conceptual framework for dynamic microsimulation disequilibrium based clearing of housing market with endogenous price generation mechanism. The life-time utility $U$ at time $t$ for household $h$ living in dwelling $d$ was represented as:

$$U_h(t, d) = V_h(t, d) + \alpha_h \log(Z_h(t, d)) + \delta_h \log(S_h(t)) + \epsilon_{htd}$$

[3.8]

Where:
\[ V_h(t,d) \] is the utility derived from living in dwelling \( d \) at time \( t \)

\[ Z_h(t,d) \] is the expenditure on all the goods except dwelling

\[ S_h(t) \] is the savings and other investments of the household \( h \) at time \( t \)

\( \varepsilon_{htd} \) is the error term

Non-housing expenditure was expressed as a function of income of the household, operating cost, capital cost, and mortgage. Once the household decides to get active in the market, it forms a choice set using a spatial search mechanism. At this point the household decides on whether to bid for a dwelling and how much it should bid.

Miller and Haroun (2000) modelled the clearing and price formation process as a multi-player game in which the potential buyers for the dwelling decide on the maximum bid they are going to offer and the sellers decide on the minimum bid they are willing to accept. All the bids are based on the expected change in the total utility of the households, if the potential transaction is successful. The households bid on the dwellings that give them highest positive change in their total utility. Sellers drop all the bidders whose bids are less than the minimum expected bid for their dwellings. If there is no bidder left then the seller decides on whether to stay in the market. In case of only one bidder, the dwelling is sold to the household at its bid price. For more than one bidder, the dwelling is sold to the highest bidder at the transaction price equal to the bid of second highest bidder. The households that are unsuccessful in the bidding decide on whether to stay in the market.

This clearing process captures the complex interactions that are going on the markets in a behaviourally consistent and representative way. There is no weak or strong market level equilibrium assumptions imposed for the solution. The price formation is totally endogenous and is driven by the supply and demand interaction. The solution is path dependent and is dependent on the sequence in which the dwellings are cleared and the decisions of buyers and sellers to stay in the market. However, formulation of the rules and the sequence for the exact execution of the bidding process in an operational microsimulation will not only be hard to work out, but will also be computationally very expensive. This conceptual framework also involves maintaining the total utility of households during their lifespan. In terms of operationalization, that will require rich retrospective survey of households for a long duration. Moreover, maintaining the total
utility of households in the simulation will be expensive both in terms of memory and computation.

PUMA, which is an operational microsimulation framework of integrated land use and transportation, also uses the concept of positive change in the total utility of households for market clearing (Ettema et al., 2006). The sellers set a list price for the dwelling based on their perception of the market. The buyers compute their expected gain in the utility based on their perception of the market. The perception of both buyers and sellers is updated based on market conditions. That will result in change in the list price and bids. The clearing involves rounds of searching, placing a bid, bargaining, and accepting/rejecting offers. At the end of each round, households decide on whether to stay in the market. An operational prototype based on this theory was developed for the Randstad region in the Netherlands (Ettema et al., 2006).

The earliest attempt to formulate the housing market as a game theoretic problem can be found in the work by Shapley and Scarf (1974). They formulated the clearing problem as a trading problem of an indivisible and single good (dwelling unit). It was assumed that traders (households) are cooperative agents with full information of the market. Based on the utility that it gets from them, every household sorted the dwelling units (including the existing dwelling, it owned) in the market. It is not clear if prices were used in the computation of the utility. For clearing, a linear programming based solution, called “Top Trading Cycle” was proposed. For each cycle the price level was set, using some exogenous mechanism. In each cycle, the household with highest utility for a dwelling was assigned that dwelling at the set price levels. The first cycle had the highest price level, while the last one had the lowest. Shapley and Scarf proved that their formulation of the housing market as a game theoretic problem has a non-empty (i.e., there is at least one solution possible) and balanced (Nash equilibrium exists) core. They also proved that, due to the concept of trading cycles and different price levels, market competition exists.

While the game theoretic approach of modelling the housing market has a great potential to capture the behaviour and complex interactions of the agents and various characteristics of the housing market in more detail (Miller and Haroun, 2000), because of the overly simplified assumptions, the formulation of Shapley and Scarf failed to do so. At any time in the market
there never are equal number of sellers and buyers. The agents (both sellers and buyers) never have complete information of the housing market and during the clearing process they behave as non-cooperative agents interested in maximizing only their own individual utility/profit.

Despite these over simplifications, Shapley and Scarf provided a strong theoretical basis for modelling the housing market as a game theoretic problem. Later, various efforts focused on making it more representative of the behaviour of agents and market characteristics. Quinzii (1984) brought the concept of exchanging two goods: money and dwelling. Wako (2005) extended the “Trading Cycle” framework to \( n \times m \) game with non-cooperative agents and exogenous price formation. It was proved that for such game a coalition-proof Nash equilibrium exists. Other major efforts in this regards include: Dubey and Shublik (1978), Owen (1992), Takamiya (2001) and Klaus (2008). All most all of these efforts primarily focused on exploring the properties of the resulting games.

### 3.3 Concluding Statement

Earlier attempts of modelling built space markets extended theories first proposed for the agricultural land development problem and applied them to understand the evolution and development of cities in a spatio-temporal context. These approaches were very aggregate (working at the city level), but were based on sound economic theory. The second wave of built space modelling efforts was primarily based on the microeconomic theory of consumption and production and consumer choice theory. These models worked at a more disaggregate level both in terms of space and decision makers. The predominant assumption used in these models to clear the market was having a strong equilibrium between supply and demand of the built space. Various microsimulation integrated land use and transportation frameworks were developed and operationalized, based on these models. While these models have firm theoretical foundation, the equilibrium assumption is an oversimplification of the clearing problem and falls short to completely represent the built space market’s characteristics and behaviour of the consumers and producers in terms of decision making and interactions in the market.

In parallel to the equilibrium based microeconomic approaches, the game theoretic literature also developed a few market clearing approaches by formulating the problem as a game and using linear programming to find the solution for these games. While game theory has a
great potential in representing the behaviour of consumers and producers and the complex interaction that is going on in the market, the existing game theoretic literature over simplifies the behaviour of agents and market characteristics in terms of representation in the game.

In recent years, the trend in integrated land use and transportation literature is moving towards disequilibrium based microsimulation models of market clearing. The approaches developed and operationalized in this wave however, are at best quasi-disequilibrium based. One shortcoming, in particular, is that the price formation mechanism in these models is not completely a part of a clearing mechanism. Usually a separate hedonic model is used to determine the transaction prices. The behavioural representation of the agents in the market is also limited.

The main goal of this part of the dissertation is to extend the state of the art by developing a microsimulation-disequilibrium based price-formation market clearing approach, which takes its foundation from both game theory and random utility theory of consumer choice, so as to richly represent the agents’ behaviour and market characteristics in the clearing model. This approach includes a price formation mechanism that is truly embedded (rather than coupled) within the clearing process.
CHAPTER 4
MARKET CLEARING MODEL FORMULATION

4.1 Introduction
This chapter introduces the theoretical foundation for a microsimulation-disequilibrium based approach for the clearing of price formation built space markets. The chapter is organized as follows: first, a formal problem statement is presented, which is followed by the key assumption made in the formulation of the model. Then a formal theoretical formulation of the model is presented. Finally, the resulting mathematical structure of the model is discussed.

4.2 Problem Statement
At any time $t$ the price-formation, built space market has $n$ sellers and $m$ buyers. The clearing of such a market requires matching a buyer to its most desired and available built space offered by a seller, at an endogenously generated transaction price. All the transactions in the market satisfy the expectations and represent the behaviour of the agents involved.

4.3 Key Assumptions and Definitions
There are two types of agents in the market: buyer agents (households and firms) and seller agents (households, builders, and landlords). Assumptions concerning each of these agents are listed below.

Buyer agent assumptions are:

- All buyers are utility maximizers.
- A buyer is looking for a single “unit” of built space to purchase (e.g., a single dwelling unit or a given amount of commercial floor space).
- The utility function for the buyer is known.
- There is an exogenous mechanism defined that generates a choice set for the buyer.
- Differences among the buyers’ behaviour are captured in the utility function and the choice set generation mechanism.
- At any time in the market, each buyer has the option to either keep looking or leave the market.
**Seller assumptions are:**

- All sellers are profit maximizers.
- Each seller is offering a single “unit” of built space (e.g. a dwelling unit) for sale.
- The asking price function for seller is known.
- All the sellers are behaviourally the same.
- At any time in the market, the seller has the option to keep the built space in the market or leave the market.

**Other assumptions include:**

- Both buyers and sellers are non-cooperative agents with limited information of the market.
- Market perception (information) for both sellers and buyers is updated as they spend more time in the market. Based on these changes in market perceptions:
  - Sellers may adjust their asking prices.
  - Buyers may include more built space alternatives in their choice sets (i.e., they may update their choice sets over time).

### 4.4 Theoretical Foundation

Let us represent the built space market with a \((m \times n)\) matrix in figure 4.1. The rows in the matrix represent the active buyers in the market while the built spaces are represented by the columns. For buyer \(b_k\) the available choices (choice set) are already defined and represented by the blue colour cells, while the red colour represents the bidder set for built spaces \(s_j\). Note that the bidder set for \(s_j\) is the result of formation of the choice sets for all the buyers in the market that are interested in \(s_j\).

Now, let us assume that there is a mechanism available by which one can compute the probability \(P_{ij}\) of a buyer \(b_i\) to buy a particular built space \(s_j\) among its choice set \(C_i\). This probability should be a function of the utility that a buyer expects to obtain from the built space, conditional upon the asking prices of these built spaces.
**Figure 4.1**
Matrix Representation of the Built Space Market

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**Figure 4.2**
Probability Summations for Buyers and Sellers

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<td>$\sum_i P_{ij}$</td>
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</tbody>
</table>

38
The summation of all the probabilities in the choice set for \( b_i \) will be equal to one. To generalize the statement, the row sums for the matrix in figure 4.2 will all be unity and the sum of all row sums will be equal to the number of buyers \( m \) that are active in the market.

\[
\sum_j p_{ij} = 1 \quad [4.1]
\]

\[
\sum_i \sum_j p_{ij} = m \quad [4.2]
\]

At a certain price level, the column sum for each built space represents the expected demand for it, in the market. Depending on this level, the column sum may or may not be equal to one. If more buyers have a higher probability of buying this particular built space, it will have a higher column sum (that may be more than one) compared to the built space with not a very high number of interested bidders. Another important aspect of this value is that, at the right price level, the column sum is expected to have a value of unity, representing the right level of demand. Thus, at the micro-equilibrium state for a built space the supply equals expected demand and only one bidder gets to be the new owner of that built space at a transaction price.

The probability of selection of a built space by a buyer is a function of its price \( r \), among other attributes of the buyer, built space, neighbourhood, and market. During the bidding process, all the other attributes except the price, remain constant. With the change of price level the attractiveness of the built space increases or decreases. This results in bidders entering or leaving the bidding process of the built space. The same phenomenon is going on, simultaneously for all the active built spaces. As these built spaces are connected to each other by some degree of adjacency (as evident in figure 4.1), through the choice sets of bidders, the increase or decrease in price of one built space will have a ripple effect on the demand of other built spaces.

Consider a situation in which buyers \( b_1, b_2, \) and \( b_3 \) are bidding for built space \( s_j \). Their choice sets are represented by \( C_1, C_2, \) and \( C_3 \) respectively. Suppose that the probability of selection of \( s_j \) for the bidders can be ranked as \( P_{ij} > P_{2j} > P_{3j} \). In addition to that: \( P_{ij} + P_{2j} + P_{3j} \gg 1 \). This means that ceteris paribus, bidder \( b_1 \) is the most interested bidder. Moreover, the built space \( s_j \) has a very high demand. Now, the owner of \( s_j \) will react to that by demanding a higher price. This increase in the price will result in the decrease in the values of \( P_{ij}, P_{2j}, P_{3j} \), while an increase in the \( P_{jk}, P_{2k}, P_{3k} \) for all \( s_k \neq s_j, s_l \neq s_j, s_m \neq s_j \) in \( C_1, C_2, \) and \( C_3 \) respectively. Due to
this increase, some other built space $s_m$ in the choice set of buyer $b_2$ might become more attractive than $s_j$ and it might decide to buy that. The price increase may also result in $s_j$ becoming unaffordable and having no other attractive built space in the choice set, $b_3$ decides to leave the built space market.

Now consider the same case as above, but here $P_{1j} + P_{2j} + P_{3j} \ll 1$. Here, for ceteris paribus, the price may be too high, resulting in a lower demand. The owner of $s_j$ will either remove the dwelling out of the market as it thinks that the market conditions are not right for selling it or lower the asking price for the built space. This decrease in the price will result in the increase in the values of $P_{1j}, P_{2j}, P_{3j}$, while a decrease in the $P_{1k}, P_{2l}, P_{3m}$ values. This might also result in generating interests from other buyers and thus $b_4$ may also enter into bidding process.

The above mentioned processes may be going on simultaneously for every built space active in the market. The built spaces are influenced by the state of bidding on other spaces through the common bidders in the bidder-set, while buyers are affected from each other’s decision via common built spaces in their choice-set. Suppose that one of the built spaces $s_k$ that $b_1$ was interested had a very high asking price, but there is a decrease in the price of that built space due to readjustment of price by its owner (owner realized that the price was too high). This will increase the probability of that built space to be selected for $b_1$ and decrease the probability for $s_j$ which was previously the top choice of $b_1$, and thus $b_1$ being the highest bidder. This may result in $b_1$ becoming the highest bidder for $s_k$, thus buying it and leaving $b_2$ to become the highest bidder for $s_j$.

This dynamic process will go on simultaneously and at the point of micro-equilibrium for a single built space, the demand will be equal supply and thus the column sum for the cleared built space should be equal to unity (equation [4.3]). The price level that will cause this will be the potential transaction price and the bidder with highest probability of selection will become the new owner of the built space. Any further increase or decrease in the price will result in instability and the bidding process will continue ad infinitum. By bringing in micro-equilibrium concept at built space level, it is ensured that the clearing price problem has a feasible solution.

$$\sum_i P_{ij} = 1 \quad [4.3]$$
In the built space market, the sellers have only limited information about the market based on which they decided on changing the demanded price or leaving the market. They do not know in addition to their built space, what other built spaces, the bidders are bidding for. On the other hand, buyers also have only limited information about the market trend and all the bids that have been placed on the built space, they are bidding for. A bidder does not know about the maximum willingness to pay of other bidders or what other options they have available in their choice sets. Bidders are only interested in maximizing their own profit, while the sellers are only interested in maximizing their own profit.

The characteristics of the market and behaviour of agents defined above are very similar to the limited information, noncooperative game of trading. In this game, there are two types of players (buyers and sellers) with two different goals. Buyers have the option to stay in the market and keep bidding for built spaces they are interested in or leave the market. Similarly, sellers have the option to stay in the market and keep on changing the asking price or leave the market. The micro-equilibrium condition introduced above makes sure that the goals of both types of agents are met. It ensures that the price is formed endogenously from within the market. Moreover, it guarantees that there is a solution for the clearing game at individual dwelling level. Note that the solution may or may not be feasible for all the players. If the solution is feasible, it will result in a successful transaction. Otherwise, the players may decide to leave the market or look for other options in the market. These conditions ensure that the market clearing game has a non empty core (i.e. a solution exists for the clearing process). Although, there is no guarantee that the core of this game will be unique. With the change of sequence in which individual built spaces are clear out, the possible end solution may change.

4.5 Mathematical Structure

In the past 35 years, discrete choice modelling literature has extensively studied the location choice decisions of firms and households in the urban context. The probability of selection of a built space by a buyer is predominantly modelled using the frameworks defined under the Random Utility Theory (RUM), which was developed in the seminal work of McFadden (1973), Manski (1977), and Williams (1977). RUM is a consumer choice theory that assumes that the decision maker decides between choices based on the utility gained from the choices. The inconsistencies between the observed and predicted choice behaviour are assumed to be the
result of analyst’s observational deficiency (Ben-Akiva and Lerman, 1985). An error term is thus introduced in the utility function.

Based on the characteristics of the error term and the functional form of observable utility, various types of choice models can be defined. The most common assumptions used in RUM based model is a linear utility function and an error term which is independently and identically Gumbel distributed (Koppelman and Bhat, 2006). These assumptions result in a logit model. Other models include: probit (normal error terms), nested logit (choices divided into hierarchy, with shared error terms in the nests), and mixed logit (correlated parameters treated as random variables).

Suppose that the indirect utility (the maximum utility that is achievable under the given prices and income level (Ben-Akiva and Lerman, 1985) is represented by a linear function defined in equation [4.4]. The completely observable part \( V \) of the utility \( U \) is a function of characteristics of the seller (including his income), built space (potential price), and neighbourhood in which the space is located.

\[
U(b, s) = V(X_b, X_s, X_n; \beta_b, \beta_s, \beta_n) + \epsilon_{bs}
\]  

[4.4]

Where

\( U(b, s) \) = Utility that buyer \( b \) associates with built space \( s \)

\( V(b, s) \) = Observable part of the utility function

\( X_b, \beta_b \) = Characteristics of the buyer and the associated estimated parameter value

\( X_s, \beta_s \) = Characteristics of the built space and the associated estimated parameter value

\( X_n, \beta_n \) = Characteristics of the neighbourhood and the associated estimated parameter value

\( \epsilon_{bs} \) = The unobservable portion of the utility

If the observable utility \( V \) is assumed to be linear in parameters and divided into the price utility and non-price utility then:

\[
V(b, s) = V^r + V^r' = \beta_r r_s + \beta^T X
\]

[4.5]

Where
\( r_s, \beta_r \)  = Price of the space and the associated estimated parameter

\( X, \beta \)  = Vector representing the non-price characteristics of buyer, space, and neighbourhood and the associated estimated parameter vector

Let’s assume that the assumption that \( \epsilon_{bs} \) is independently and identically Gumbel distributed, then the probability that buyer \( b_i \) will chose built space \( s_j \) is given by (Ben-Akiva and Lerman, 1985):

\[
P(b_i, s_j) = P_{ij} = \frac{e^{V_{ij}}}{\sum_{l \in C_i} e^{V_{il}}}
\]

[4.6]

Where \( C_i \) is the choice set of buyer \( b_i \)

Equation 4.3 thus results in:

\[
\sum_{i} P_{ij} = \sum_{i \in B} \left[ \frac{e^{V_{ij}}}{\sum_{l \in C_i} e^{V_{il}}} \right] = \sum_{i \in B} \left[ \frac{e^{(V_{ij} + V_{ij}')}}{\sum_{l \in C_i} e^{(V_{il} + V_{il}')}} \right] = 1
\]

[4.7]

Where \( B \) is the bidder set for the built space \( s_j \)

Note that all the non-price utilities \( V_{ij}' \) and \( V_{il}' \) in equation [4.7] remain constant, during the clearing process. If it is also assumed that the prices remain constant for the built spaces other than the one which is going through the clearing process then equation [4.7] can be rearranged as:

\[
\sum_{i \in B} \left[ \frac{Y_{R}}{\left( \sum_{l \in \left( C_i - j \right)} V_{il} + Y_R \right)} \right] - 1 = 0
\]

[4.8]

Where

\[ R = e^{V_{ij}'} \]
\[ Y = e^{V_{ij}'} \]
\[ \Psi = e^{(V_{il} + V_{il}')} \]

The solution to our clearing problem for a built space \( s_j \) is thus reduced to finding the value of \( R \) in equation [4.8]. This equation however, is a \( B \)th degree polynomial with a maximum of \( B \) roots. This means that the number of possible solutions to the clear problem equals the
number of bidders. This observation makes sense as every bidder will have an acceptable maximum price (or willingness to pay) at which the bidder is willing to buy \( s_j \). That price will be reflective of its socio-economic status. Not all of these prices though will be acceptable to the seller. The seller on the other hand, will only be interested in the prices that are in the vicinity of its asking price, especially the potential transaction prices that are higher than the asking price.

### 4.5.1 Root Finding

Finding the root of polynomials is one of the oldest mathematical problems. The solution of quadratic equation was known to ancient Babylonians (McNamee, 2007). In the modern age, numerical solutions for the polynomial root finding problem has been extensively investigated in the mathematics and computer science literature, due to its vast applications in engineering, physics, computer science, economics, finance and various other fields (Pan, 1997). These algorithms vary from finding just one root to finding all the possible roots. Pan reported that the recent progress in the algorithm development for root finding has focused on considerably reducing computational complexity and associated computational times. Root finding algorithms can be grouped into: simultaneous, Newton’s and related, and Matrix methods (McNamee, 2007).

Newton-Raphson method is perhaps the most widely employed algorithm for root finding. It is a gradient based method that defines an iterative function (equation [4.10]). This function is based on the approximation of Taylor expansion of the polynomial \( f(r) \).

\[
f(r_0 + \epsilon) = f(r_0) + f'(r_0) \cdot \epsilon + \frac{1}{2} f''(r_0) \cdot \epsilon^2 + \cdots \tag{[4.9]}
\]

or

\[
f(r_0 + \epsilon) \approx f(r_0) + f'(r_0) \cdot \epsilon \tag{[4.10]}
\]

Where

- \( r_0 \) = initial value
- \( \epsilon \) = The offset from the initial value

Equation [4.10] represents the tangent line to the curve at coordinates: \((r_0, f(r_0))\), while \((r_0, 0)\) is the point at which this tangent meets the horizontal axis. The amount of offset \( \epsilon \) that is needed to get closer to the nearest root from \( r_0 \) can be given by equating the equation to zero:
An iterative function can be defined by starting from an initial value of $r$ and repeatedly adding the new offset to the previous value of $r$. The convergence criterion is achieved when the offset becomes negligibly small.

\[
\varepsilon = -\frac{f(r_0)}{f'(r_0)}
\]  

[4.11]

One of the main reasons of its wide use is the simplicity of the algorithm, fairly good accuracy, and quickness, if given proper initial values. However, Newton-Raphson method iterations are haphazard and uncontrolled within the search space. The algorithm is extremely sensitive to the initial value and if the initial values are not fairly close to the root it cannot guarantee convergence. It is recommended to be used in conjunction with an algorithm that can guarantee global convergence (McNamee, 2007).

In the context of built space clearing, analyst is not interested in all the possible prices that could result from finding roots for equation [4.8]. The seller sets an asking price for the built space and expects a bid that is more or less near that price. If it doesn’t get any good bid then seller either decides on leaving the active market or reasserts the price. So, for equation [4.8], analyst is only interested in finding only those roots that are within a certain range of the asking price set by the seller. As stated before, Newton-Raphson haphazardly searches for roots and may return any root, depending on the starting condition. It can also get stuck in a local critical point. For searching the potential transaction prices, a search mechanism is needed, which is bounded and which can be directed.

For this purpose, a two stage bounded and directed search process was developed that used Laguerre’s root finding method at the top level and the Newton-Raphson method at the lower level. Laguerre’s root finding method guarantees convergence regardless of the starting value, although the accuracy is not as high as Newton-Raphson (Press et al., 2007). At any point in the search, Laguerre’s method gives the distance $d$ (equation [4.13]) to the nearest root by assuming all the other roots to be at equidistant from that point. This assumption introduces approximation in the value. That is why, here a combination of Laguerre’s and Newton-Raphson method is used. At upper level the Laguerre method guarantees that search is fairly near to the
next potential transaction price. At this point, the value from Laguerre method is used as a starting value in the Newton-Raphson method, which converges to the nearest root fairly quickly.

\[ d = \frac{n}{G \pm \sqrt{(n-1)(nK-G^2)}} \]  \hspace{1cm} [4.13]

Where

\[ G = \frac{f'(r)}{f(r)} \]

\[ K = \left\{ \frac{f'(r)}{f(r)} \right\} - \left\{ \frac{f''(r)}{f(r)} \right\} \]

\[ f(r) = \text{Value of the polynomial at } r \]

\[ f'(r) = \text{First order derivative value for the function at } r \]

\[ f''(r) = \text{Second order derivative value for the function at } r \]

The search process assumes that the asking price for a built space is already known from an exogenous model. Based on that, it defines the range of the search: \([r_{\text{min}}, r_{\text{max}}]\). It starts searching from \(r_{\text{min}}\) and iteratively move towards \(r_{\text{max}}\) in small discrete steps. Suppose that the dotted line in figure 4.3 represents the potential transaction prices that one is interested in computing. Then the distance \(d\) is computed using Laguerre’s method. If this distance is lower than certain threshold value \(d_{\text{acc}}\), it moves to the Newton-Raphson method that gives the exact value of the nearest price. The algorithm iteratively search for the values until it reaches the \(r_{\text{max}}\). The exact steps of the developed algorithm are as follow.

![Figure 4.3](image)

**Figure 4.3**

Two-stage directed and bounded search for potential transaction prices
4.5.2 Transaction Prices Search: Pseudo-code

Given: $r_{\text{min}}$, $r_{\text{max}}$, $d_{\text{acc}}$, and $\Delta r$

1. Set initial value of $r = r_{\text{min}} + \Delta r$

2. If $r > r_{\text{max}}$ then go to 5

3. Find the distance $d$ to the nearest root $r_i$ using equation 4.13

4. If $d < d_{\text{acc}}$ then apply Newton-Raphson to find the exact price,
   Else $r = r + \Delta r$ and go to 2.

5. End

Where:

$r_{\text{min}}$: Minimum price

$r_{\text{max}}$: Maximum price

$d_{\text{acc}}$: Max acceptable distance to the nearest root (potential transaction price), before moving to the Stage 2 i.e. Newton-Raphson procedure

$\Delta r$: Increase in the price, as the process moves to next iteration
CHAPTER 5
APPLICATION AND OPERATIONALIZATION: OWNER-OCCUPIED HOUSING MARKET IN ILUTE

5.1 Introduction

The theoretical framework suggested in chapter 4 is used to operationalize the owner-occupied housing market in the Integrated Land Use Transportation and Environment (ILUTE) modelling framework. This chapter first describes the conceptual model that is used to implement the owner-occupied housing market. It is followed by the description of architectural details. In the end the software design and implementation that were needed for the operationalization is discussed.

5.2 Conceptual Model for Owner-Occupied Housing Market

A detailed discussion of the general conceptual model for built space markets within ILUTE has been presented in chapter 1. Based on specific features of the price formation markets, here a modified version of the same conceptual model for the owner-occupied housing market is presented. Figure 5.1, outlines the various components of the model.

At any time $t$, on the demand side, households decide to get active in the housing market and start looking for the potential dwellings. This process is exogenous to the market clearing mechanism. Another important source that feeds households to the housing market is the immigration process, in which new households migrate to the urban area and are looking for a dwelling to own. Marriages, divorce, and moving out of parent’s place are other sources that result in creation of households that may be looking to own a dwelling and thus become active in the housing market. Note that most of these new households (including in-migrants) will probably first enter the renter market and very few will have financial means to consider owning a dwelling immediately. The decision by these households, on which market to enter is exogenous to the clearing process.

On the supply side, the major source of active dwelling stock comes from the households that already have a dwelling, but are looking for a new dwelling. For instance, a couple that decided to expand the family may be trying to sell their apartment and looking for a bigger
dwellings. In most of the cases, mobility decisions of a household that already have a dwelling will result in addition of their dwelling to the active dwelling pool in the owner-occupied housing market, unless the household decides to rent one of the dwellings that it will own at the end of clearing. The out-migration decision of the households will also result in their dwellings getting active in the owner-occupied or rental housing market.

Figure 5.1
Conceptual Model for the Owner-Occupied Housing Market

Depending on the market and economic conditions, each year, builders build certain quantities of new dwellings. This new stock becomes active in the market. Note that the proposed framework does not differentiate between the new or existing dwellings, in terms of their owners’ behaviour and age of the dwelling. It is assumed that every dwelling has a single owner who is interested in maximizing his profit out of that dwelling. The fact that a builder may be introducing a batch of 20 dwellings in the market and is interested in maximizing the profit from the batch has no affect on the searching mechanism for the transaction prices.
The housing market has a pool of active households that are searching for new dwellings. These households have their choice set defined using a choice set generation process that is exogenous to the market clearing process. These households also have a well defined utility assigning mechanism that helps the clearing process to represent their utility maximization behaviour during the clearing of individual dwellings. On the housing stock side, the housing market has a pool of dwellings that have been listed in the market for selling by their owners. At the time when a dwelling is added to the active dwelling pool of the housing market, the owner sets an asking price for it. This price represents the perception of a seller about the profit that it can get from the dwelling, given its knowledge of the existing market conditions, quality of the dwelling, and neighbourhood characteristics. The asking price determination mechanism is exogenous to the market clearing.

The market clearing mechanism is a microsimulation model that finds the potential buyer and the associated transaction price, one by one for all the dwellings in the active pool. The seller has the option to accept or reject the transaction price or even leave the housing market. The clearing process uses the price finding mechanism defined in chapter 4. It uses the asking price set by the seller as the reference point and searches for the potential transaction prices in the vicinity. Sellers can accept or reject the price. Sellers have the option to readjust the prices based on their updated market perception. Similarly, the households have the option to accept or reject the dwelling at a transaction price or even leave the market.

The market clearing process is disequilibrium based and is highly path dependent. The end prices and the matching of dwellings to households depend on various dimensions that are representative of the housing market characteristics and behaviour of various agents involved in the process. These dimensions include: individual choice set generation by the households; asking price setting by the sellers; the stochastic sequence in which active dwellings are cleared; decisions of the buyers and sellers regarding staying in the market; buyer and sellers accepting or rejecting the offers; initial and stopping conditions for the clearing; and the housing market size. All these micro and macro level dimensions influence and shape up the individual prices and matching thus effectively evolving the distribution of population and housing in the urban area.
5.3 Operationalization of the Owner-Occupied Housing Market

The current operational version of owner-occupied housing market is described in figure 5.2. For the operational purpose, a concept of sub-market cycle is introduced in the clearing process. Instead of clearing the whole market at once, only a portion (sub-market) of active households and dwellings is taken and the clearing process is ran on that. As the size of sub-market decreases more active households and/or dwellings are introduced into the sub market.

Figure 5.2
Operational Housing Market in ILUTE
The major reason behind doing that is the limitation that the current version of ILUTE simulation serially updates its modules (including the housing market) in a yearly time step. Clearing a year’s worth of stock in the market in a single iteration will not be very representative of the actual housing market. Instead, the yearly stock is cleared in twelve sub-market cycles that vaguely represents the twelve months of a year. The sub-market cycle size is arbitrary and can be set to a size smaller or larger than the current size in the ILUTE implementation. Though, a smaller size may result in having too few available options for the active households’ choice set generation. While a larger size (e.g. quarterly step), may have the same issues as clearing the whole market in one cycle. It is however, very important to conduct a sensitivity analysis on the market with different sub-market cycle sizes.

At the time of initialization of the housing market update process, one-twelfth of the dwelling stock from the active dwelling pool and one-twelfth of the households from the active household pool are first randomly chosen. After that the choice-set and utilities are generated for all the households and asking prices are determined for all the dwellings currently in the sub-market. The clearing algorithm then randomly chooses one dwelling at a time and looks for the potential transaction prices for the selected dwelling. One of the potential transaction prices is then chosen randomly and based on that the probability-of-selection values are updated for all the bidders, one last time. The household with highest probability-of-selection is assigned the dwelling at the selected transaction price. Note that the randomness is introduced in the selection of the final transaction price, and not in the selection of the highest bidder. As the potential transaction prices are a direct outcome of adjustments in the deterministic part of the bidders’ utility, one can expect that the random selection of the successful bidder (based on selection probabilities), instead of transaction price, will have the same average effect.

Once the household and transaction price is selected, the dwelling and household are removed from the market and bidder sets of all the dwellings to which this household was bidding, are updated. Choice sets of all the other households who were interested in this dwelling but failed to buy it, are also updated. The one-by-one clearing process is continued until the sub-market becomes half of its original size, either in terms of number of households or dwellings (whichever happens first). At that point the sub-market is updated with new households and dwellings, so that it is restored to its initial size. The choice set of new households added to the
sub-market is updated. Households who were already in the sub-market and own a dwelling, based on the number of their failed attempts in bidding, decide on whether to stay in the market or leave the market. If the choice set has very few dwellings in it, it is updated for all the households that decide to stay in the market. Similarly, for the dwellings that are already in the sub-market, based on their failed attempts to find a buyer, their owners decide on whether to stay in the market or leave. Depending on the number of clearing attempts, the asking price is lowered to reflect the changed market perception of the owner, for the dwellings that stay in the market.

This process goes on until the housing market is reduced to one-twenty-fourth of its original size either in terms of households or dwelling units. This means that the sub-market is updated twenty three times in total. The last one-twenty-fourth of the households and dwellings are rolled over to the next year’s clearing process. This process and the concept of a pseudo-month is more representative of what goes on in the urban owner-occupied housing markets, where households and dwellings enter the market, stay in it for few months and leave if they couldn’t find a suitable deal. Also, at any one time, a household has only limited choices available. It might be that the choices that were available to the household in the first month were not very attractive to it, but in the second month, with the addition of new dwelling stock, more attractive options became available to the household.

This clearing mechanism is also able to represent the changing expectations of the sellers with the increase in awareness about the market conditions. If a seller sets the asking price of the dwelling too low, the resulting transaction price will be higher and will reflect the high demand of the dwelling. Similarly, if the price is too high, the demand will be low and may result in few failed attempts to clear it. This will result in lowering of the asking price or decision by the seller to leave the market. The clearing process can manage the case where, for instance, a household who is out migrating, or ends up with two dwellings, must sell the dwelling, even at a fairly low price that does not maximize the owner’s profit.

If the active household pool is large relative to the active dwelling pool, the situation will represent a seller driven market. In this case, the demand for each dwelling on average will be high because of relatively large bidder sets for these dwellings. This will result in a higher transaction prices for the dwellings. Similarly, in case of the active dwelling pool being larger
than the active household pool, the market will be driven by the buyers resulting in (on average) lower demand for individual dwellings. The lower demand will thus generally result in lower transaction prices.

One of the limitations of the current clearing mechanism is that there is no differentiation between the behaviour of a builder and a household as a seller. Builders usually introduce batches of dwellings in the market rather than just a single dwelling at a time. They can be assumed to be better informed about existing market conditions. They are also interested in maximizing profits from the batch rather than a single dwelling. Due to their better knowledge of the market and ability to quickly update the final features of dwelling (e.g. type of tiles used in the kitchen), one would expect that the asking prices set by the builders will have more room for negotiation. This may result in builders typically being more flexible in terms of negotiating the transaction price. On the other hand a household, who is reselling its dwelling in the market, might not be as well informed about the market conditions as the builder and will be interested in maximizing the profit on a single dwelling only. This will result in household being less flexible in terms of the difference between its asking price and the final transaction price. Datasets describing builders’ behaviour are extremely rare. A major data collection effort will be needed, so as to represent more of the builders’ behaviour in the market.

5.3.1 New Housing Supply

A Monte Carlo simulation based process was developed to operationalize the new housing stock supply in ILUTE. The supply process used the econometric models of new housing stock and location choice models for new dwellings that were estimated by Haider (2003). The location choice model is estimated for Traffic Analysis Zones (TAZ) defined by Transportation Tomorrow Survey’s (TTS) 1996 zoning system. The econometric model of total new stock developed by Haider (2003) forecasted the housing starts. Depending on the type of housing and economic and market conditions, the time between start and completion of the construction project may vary. However, due to lack of proper statistics on construction duration, it was assumed that the construction for all the housing starts, completed in a fixed time of one year.

As a first step in the simulation, the total new stock by type of housing is computed using the econometric models. The probabilities of selection for each TAZ by all the types of housing
are computed using the logit based location choice model developed by Haider. The probability values that are extremely low are reduced to zero. The cumulative probabilities for all the TAZs for each type are then computed. The location (TAZ) for each dwelling in the new stock is then decided by drawing a random number between 0 and 1 and comparing to the cumulative distribution values corresponding to the dwelling type.

5.3.2 Asking Price, Mobility, Location Choice Decisions

The asking price, household mobility, and location choice decision models developed by Habib (2009) are used in the current ILUTE operationalization. The asking price model is based on hedonic analysis theory that evaluates the dwellings using the value bearing attributes including structural and neighbourhood attributes. It doesn’t differentiate between a household and a builder or a new dwelling and a resale of existing dwelling. The model takes into account the existing market conditions by using the average prices in the neighbourhood. Thus, there is a lagged feedback coming in from the clearing process of previous year to the model in determining the asking prices of the current year.

Each year, the mobility model evaluates the decision of household on whether to change the existing dwelling or not. The model is based on a binary logit formulation with normally distributed random parameters. The households that decide to move are added to active household pool in the housing market. Their dwellings are added to the active dwelling pool.

For finding the potential transaction prices for a dwelling, the probabilities of selection by the bidders are needed. Here, the location choice model estimated by Habib (2009) to compute these probabilities is used. This model is based on gain and loss concept where the utility gain due to the change from existing to potential situation is used in the logit formulation based probabilities.

5.3.2.1 Choice Set Generation

An important aspect of the location choice model is the generation of choice set for the decision maker. In situations where the universal choice set is very large, a mechanism is needed to reduce the choice set that are considered by the decision maker. In reality a decision maker doesn’t have information about the full choice set and it only considers a very small sample of the choice universe. Various approaches have been developed to model the choice set generation
process. These methods could be divided into deterministic and probabilistic approaches (Bierlair et al., 2010). In the deterministic choice set generation, based on the context and/or characteristics of decision makers, the choice set is formed using some deterministic rules. While in a probabilistic approach, the choice set generation process is explicitly modelled as a stochastic process and its effects are included in the choice model. Various examples of this approach includes Manski (1977), Swait and Ben-Akiva (1987), Morikawa (1995), Ben-Akiva and Boccara (1995), Swait (2001), Frejinger et al. (2009), and Martinez et al. (2009).

McFadden (1978) in his model of residential location choice suggested adjustments to the likelihood function in the estimation process in the cases where the choice universe is impractically large to be considered by the decision maker. In one of the proposed cases where a small number of choices are randomly chosen from the choice universe, McFadden proved that no adjustments were needed to the likelihood function so as to estimate consistent parameters. Elgar et al. (2009) modified the random sampling approach and introduced spatial anchor points in the sampling processes. They oversampled around those points due to a strong evidence from dataset that decision makers were giving more importance to the locations around these anchor points.

In the operationalization of the housing market in ILUTE, approach similar to the one used by Elgar et al. (2007) is used. Pushkar (1998) in the residential mobility survey for households in the Greater Toronto Area (GTA), reported that 90% of the households relocated within 20km of their previous location. In the operational choice set generation process for relocating households, 75% of the choices are sampled from the dwellings that are within 15km of the previous location and the remaining 25% from dwellings that are greater than 15km away. It will be interesting to see the effects of adding other dimensions (for instance, size of the household, social network, work/school location etc) in the choice set generation on the improvement of the model. Currently due to lack of data, a more sophisticated choice set generation process is infeasible.

5.3.3 Synthetic Population

The agents that are maintained in the current operational version of ILUTE are: Household, Family, and Person. The kinship between the persons and their status in the household and
family is also maintained. Maintaining these relationships gives us the leverage to be able to implemented more complicated models that need information like changes in the total salary of the household, social network of the household, etc.

The initial population was synthesized using the synthesis procedure developed by Pritchard (2008). This process is an adaptation of the iterative proportional fitting method that can manage a very high dimension of attributes of agents and can also maintain the relationships between person, family, and household agents.

5.3.4 Space

The location choice model for new housing estimated by Haider (2003) uses Traffic Analysis Zones (TAZ) as the choice set. The asking price, mobility, and location choice model in Habib (2009) uses the Census zonal data. To avoid inconsistencies, a space management system is operationalized that recognizes both Census Tract (CT) and TAZ. It also manages the mapping between the two systems. Every household and dwelling in the ILUTE simulation is associated to both CT and TAZ using this space management system.

5.3.5 Time

Due to the fact that the implemented models were estimated using different magnitudes of time steps, the ILUTE simulation maintains three different times: yearly, quarterly, and pseudo-monthly. The mapping between lower and higher time-steps is maintained. In cases where the models are using the aggregated values of certain variables (for instance, average price of dwelling in the CT, last year), a separate mechanism is developed that aggregates various attributes of the agents, both spatially and temporally to the desired level.

5.3.6 Price Set Search

During the clearing process, a dwelling is randomly chosen from the active dwellings in the sub-market. Starting from the lower bound which is exogenously set as a percentage of the asking price, the algorithm starts computing the distance from nearest potential transaction price using the Laguerre method operationalized in ILUTE. The algorithm proceeds in small steps towards the upper bound for the transaction price. Both increment and upper bound are exogenously provided to the housing market clearing mechanism. If the distance is less than or equal to the
step size, the algorithm switches to the implementation of Newton-Raphson method so as to find the exact value of the price. The set of potential transaction prices found in this search are then used to decide the final buyer and transaction price.

Note that the algorithm doesn’t take into account the number of failed attempts to sell the dwelling. The adjustment in the price reflecting the changed perception of the seller due to failed attempts is managed one level above, at a point where sub-market is updated. Cases where dwelling must have to be sold may be because of the fact that the household is out-migrating or has already bought a new dwelling and must sell its old one, are also managed at the time of sub-market update.
CHAPTER 6
ILUTE HOUSING MARKET: SOFTWARE, SIMULATION AND RESULTS

6.1 Introduction

Salvini (1998 and 2003) designed and developed a comprehensive operational prototype for ILUTE modelling framework that provided a proof of concept and strong basis for the operational microsimulation of urban systems. The effort mainly focused upon the basic architectural design, essential class structure, and features that are required for a time-driven microsimulation. The software was designed using the Unified Modeling Language (UML) and implemented in the C++ programming language using the Object Oriented Paradigm (OOP).

To operationalize a full scale version of ILUTE modelling framework, a newer software version of ILUTE (v1.0) is developed that revisited and extended the prototype version initially developed by Salvini (1998 and 2003). ILUTE v1.0 is also using UML for design and C++ as the programming language. Major features of the new versions include: comprehensive and operational demographic evolution, price-taker market abstraction and application, owner-occupied housing market, housing supply, spatial object management, probability and other utility-functions, data management, and visualization modules. The detail discussion of the design and implementation of these modules and the theory behind them can be found in Miller et al. (2008a), Miller et al. (2008b), Farooq et al. (2008), Chingcuanco (2010b), Giroux-Cook (2010), and Farooq et al. (2010). A significant amount of time and effort has been invested in the validation of the output from ILUTE simulation with historic data. Details can be found in Miller et al. (2010).

This chapter focuses on the design and implementation details that are specific to the owner-occupied housing market. It first gives an overview of the ILUTE class design. Important classes that are involved in the housing market are discussed. Next, the system sequence design for the housing market and associated processes are presented. This chapter then discusses the simulations and results. At the end of the chapter, conclusions and future research directions are presented.

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12 A newer version, ILUTE is currently under development, which is programmed in the C# language
6.2 Definitions

Agent: An object that has decision making and/or learning capabilities built into it (Jennings and Wooldridge, 1998).

Aggregation: A whole/part relationship between the objects. Same as Association, except, there is no cyclic relationship between the objects.

Association: The ability of one object to be able to send a message to another object.

Class: An entity that serves as an abstraction for describing the attributes, operations, relationships, and semantics of a set of objects (Jacobson et al., 1998).

Inheritance: The phenomena in which a child class has all the attributes and associations of the parent class. Moreover, the child class can have attributes and associations of its own. Two children from single parent class may be different from each other.

Instantiation: The process of creation of an object from a certain type of class.

Object: The realization of a class in the software system. An object is an entity that can be created, updated, interacted, and deleted during the execution of the software system.

Serialization: The conversion of an object to a format so that it can be stored or retrieved from a storage medium.

Subsystem: A collection of atomic tasks that offers a specific behaviour to other subsystems (Jacobson et al., 1998).

System: A collection of subsystems that accomplishes a specific purpose (Jacobson et al., 1998).

System configuration: A collection of element-value pairs that defines the composition of a system at any time during its execution.

System Sequence Diagram: A diagram which shows a particular sequence of a use case, the events, their sequence and implications.

Unified Modeling Language (UML): A language for visualizing, specifying, constructing and documenting the artifacts of a software system.

Use Case: A response sequence of the system to some external event.
6.3 Software Design

As part of the current design, there are approximately 50 classes that are identified and implemented in the design of ILUTE v1.0. Table 6.1 lists the major classes and figure 6.1 shows the relationships between various classes of the system. At the top level, the Application class object instantiates and manages the World class object. World is the abstraction in which the urban system evolves through ILUTE simulation (Salvini, 1998 and 2003). All the agents in the simulation are inherited from the SimulatedObject class that acts as an abstraction for the basic attributes and operations needed in them. The World class object manages time within the simulation and at each time step updates all the agents and modules. It manages the loading or saving of the data. Moreover, it also maintains the collections of agents, spatial objects, and various helper objects in the simulation.

Figure 6.1
Extensive Class Diagram of ILUTE
Table 6.1 List of Main ILUTE Classes

<table>
<thead>
<tr>
<th>Class</th>
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<tbody>
<tr>
<td>Application</td>
</tr>
<tr>
<td>Area</td>
</tr>
<tr>
<td>AutoTransactionModel</td>
</tr>
<tr>
<td>BidSet</td>
</tr>
<tr>
<td>BirthDataManager</td>
</tr>
<tr>
<td>CensusZone</td>
</tr>
<tr>
<td>DeathDataManager</td>
</tr>
<tr>
<td>DemographicDataManager</td>
</tr>
<tr>
<td>DivorceDataManager</td>
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<tr>
<td>DriverILicenceDataManager</td>
</tr>
<tr>
<td>DwellingUnit</td>
</tr>
<tr>
<td>EducationDataManager</td>
</tr>
<tr>
<td>Family</td>
</tr>
<tr>
<td>FileSystem</td>
</tr>
<tr>
<td>Household</td>
</tr>
<tr>
<td>HousingMarket</td>
</tr>
<tr>
<td>IluteException</td>
</tr>
<tr>
<td>InMigrationDataManager</td>
</tr>
<tr>
<td>Job</td>
</tr>
<tr>
<td>JobMarket</td>
</tr>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Logger</td>
</tr>
<tr>
<td>MarriageDataManager</td>
</tr>
<tr>
<td>MarriageMarketModerator</td>
</tr>
<tr>
<td>MarriageMarketPool</td>
</tr>
<tr>
<td>Matrix</td>
</tr>
<tr>
<td>MonetaryValue</td>
</tr>
<tr>
<td>OutMigrationDataManager</td>
</tr>
<tr>
<td>Person</td>
</tr>
<tr>
<td>SimulatedObject</td>
</tr>
<tr>
<td>simulationDate</td>
</tr>
<tr>
<td>SpaceBuilder</td>
</tr>
<tr>
<td>SpatialObject</td>
</tr>
<tr>
<td>SysCompatibility</td>
</tr>
<tr>
<td>TemporalDataManager</td>
</tr>
<tr>
<td>TravelTimes</td>
</tr>
<tr>
<td>TTSZone</td>
</tr>
<tr>
<td>Utils</td>
</tr>
<tr>
<td>Vehicle</td>
</tr>
<tr>
<td>World</td>
</tr>
</tbody>
</table>

Figure 6.2 shows the relationships between the World class and other classes in the ILUTE software. A detailed description of this class diagram can be found in Farooq et al. (2008).
Appendix A describes the important classes that were designed and implemented in ILUTE 1.0, whereas, appendix B discusses the major system sequences that were developed in the software that are related to the operationalization of owner-occupied housing market.

6.4 Simulation

The ILUTE v1.0 software has been successfully used to simulate the evolution of population and built space for the Greater Toronto and Hamilton Area. The software is generic enough that theoretically, it could be ported for any urban area in the world. Any application other than the GTA will require: a) Synthesis of the initial population with similar specifications as the one used in ILUTE v1.0 b) Re-estimation of the various models implemented in the ILUTE 1.0 (any variable used in the new models that is not maintained in current version will require additional programming work) c) Input data files that are used by the software.

A major effort in terms of a historic validation of ILUTE simulation output for the duration of 1986–2006 is in progress. This effort has been divided into two phases. In first phase the focus is upon getting the trends, directions, and scale from the various modules in ILUTE right. More importance is given to the qualitative than quantitative validation with the existing data. We are also looking at the effects of changes in various modules on other modules. A detailed description of this effort and the validation results have been published in Miller et al. (2010). In the second phase, it is planned that a more systematic validation with more rigorous quantitative tests are conducted that are based on formal statistical and other validation methods.

ILUTE v1.0 simulates the entire population (4.2 million in 1986) of the Greater Toronto and Hamilton Area. During the execution, the software maintains approximately 10 million objects and evaluates hundreds of millions of decisions. Due to the enormous memory and time requirements, the software runs on high performance hardware (8GB of RAM with 2.4GHz multi-core Intel processor). The execution of the twenty year simulation (1986–2006) takes approximately 1 week on these machines. A smaller sample simulation however can run in a reasonable time (few hours) on a desktop or a laptop.

The current version of the software runs serially on a single core, which means that the performance gain due to the use of multi-core computer architecture, that is readily available

\textsuperscript{13} For details of the models, inputs, and synthesized population, please see Farooq \textit{et al.} (2010b)
these days, is not completely exploited. In this context we are currently engaged in an effort to
design and implement a parallelized version of ILUTE software to exploit these available
technologies to significantly reduce run times. The parallelization of individual agent decisions
in ILUTE is trivial, but the parallelization of markets (both price-taker and –formation markets)
requires dealing with issues like objects consistency, shared memory access management, and
process scheduling. We have implemented a prototype for parallel version of the price-taker
market that takes advantage of the generalization of the problem to a graph-theoretic problem;
details can be found in Farooq et al. (2010a). For the price-formation market, specifically the
housing market, we are investigating the use of Graphic Processing Unit (GPU) to achieve
speedup. A GPU has hundreds of threads running in parallel. Here we exploit the scarce nature
of the housing market matrix (figure 4.1) to run clearing of hundreds of dwellings in parallel.
The details of the process could be found in Luu et al. (2010). A full scale implementation of
these parallelization efforts in ILUTE software is in progress.

The interaction of built space and population in ILUTE with the transportation system
microsimulation (for instance TASHA) in the current version of the software is very ad hoc. The
data from TASHA (or, for that matter, any suitable travel demand model) is exogenously fed to
ILUTE. An ongoing effort jointly taking place at University of Toronto and Universiteit Utrecht
however is focusing on making this integration more seamless (Farooq et al., 2010c). As a first
step the coupling is achieved by unifying the class structure of ILUTE and TASHA and making
TASHA as a module implemented within the ILUTE software. For a true integration, a project-
based approach is being explored that was initially proposed by Miller (2005a, 2005b). In this
approach the life of an agent is modeled as various projects running parallelly and serially.
Projects are the means to achieve the goals that results from the needs of an agent. These projects
can be frequent (work project) or infrequent (marriage project) in terms of their realization.
Travel in this context becomes an episode of many activities in which an agent engages in to
achieve the projects’ goals. The work on design and implementation of the software based on
this concept is in progress.

In the next section, results specific to the housing market in the current ILUTE v1.0
implementation are presented. The reported results are generated using the averages from 10
simulation runs. The sample size, for the simulated population in these runs, is 10% of the total
population of the Greater Toronto and Hamilton Area. Twenty year ILUTE simulation from 1986 to 2006 is ran using different seeds and then aggregated for the results. Due to the current high time requirement for the running of ILUTE simulation, at present only a more qualitative analysis of the results is presented. In future I plan to conduct more rigorous tests that look into various scenarios and quantitative assessment of the results in a more systematic fashion.

6.5 Selected Results

ILUTE generate a very wide array of outputs related to the spatial and temporal evolution of the population and built space stock of the Greater Toronto and Hamilton Area (GTHA), but here the focus is specifically on the results that are directly related to the housing market and its performance. Figure 6.3 shows the distribution of the asking and transaction prices for the dwellings that were active in the housing market for the simulation year 2001. The shape of the distribution is covering a wide price range with a high percentage of dwellings in the low to medium prices and a decreasing trend in the share as the prices become extremely high. One can see occasional dwellings with prices in million dollars. The simulation in general is capturing the difference in the prices due to features of the dwelling, neighbourhood characteristics, accessibility level, and market conditions.

The average asking price is $380,000 while the average for transaction price is $392,000 with a standard-deviation of $180,000. The average asking price seems to be very high and thus resulting in a higher transaction price. Unfortunately, access to any data source on asking prices for the year 2001 in the GTHA is not available, but the TREB reports that the average transaction price in 2001 was $222,000. Clearly, both the average asking and transaction prices generated in ILUTE simulation are higher than the value reported by TREB. The computation of the right asking price is very important in the market clearing process as the price search algorithm only searches for the prices that are in proximity of the asking price set for a dwelling. More investigation is needed in terms of calibration of the asking price model, so to make it more representative of the real estate prices.

If one looks at the transaction price distributions for the individual types (Figure 6.4 and Table 6.2), it can be notes that the average price for detached dwelling is the highest, followed by semi-detached, attached, and apartment housing. The difference between the average prices from
the ILUTE simulation and TREB data for semi-detached, attached, and apartments are less than $50,000. Moreover, for these types, the TREB prices are within one standard-deviation of the ILUTE simulation. However, in case of the detached dwellings, the difference in price is $173,000 with a very high ILUTE simulation price. As the detached housing has a high share in the total dwelling stock, the resulting difference in the average transaction prices for the total dwelling stock between ILUTE simulation and TREB data becomes very high.

The asking price model for dwellings that is implemented in the current version of the ILUTE simulation treats detached-dwellings differently than other types of dwellings. For the detached-dwelling, all else been equal, the asking price is bumped up by about 20% regardless of its spatial location. This results in, at an average, higher asking and transaction prices for detached-dwellings. It is suspected that this step up might be the reason for the higher transaction prices and might need to be revisited and calibrated. Moreover, there is an accumulation of this effect, as the simulation progresses from 1986 to 2001.

The shapes of the transaction price distributions for the individual types seem to represent the characteristics of the market for these types. Detached dwellings have a wider distribution with high share for a larger range of prices. This represents the effect of wider variation in terms of size, location, and other features of the dwelling. The same phenomenon is visible in the case of semi-detached dwellings, but to a lesser extent. In the case of attached and apartment dwellings the distribution has a narrower range for prices with high peak. This represents the fact that there is lesser variety available in the market in terms of attached and apartment type dwellings.

Figure 6.5 reports the total new stock of dwellings generated in the ILUTE simulation from 1986 to 2006, compared with Census data. The simulation results closely match the census data. In general, one can observe that the forecasted values are lower than those reported in the Census. ILUTE simulation seems to perform better in the bust than boom cycles of the new housing construction.

Figure 6.6 shows the spatial and temporal variation of population density during the ILUTE simulation run from 1986 to 2001. In general the simulation is capturing the dynamics in the Greater Toronto and Hamilton Area. The development and densification of the Yonge Street
corridor in North York, Markham, Richmond Hill, and Mississauga are clearly visible as the simulation moves from 1986 to 2001. There are, however, a few spatial anomalies that one can observe in the suburbs, where the densification is too high and doesn’t make much sense. A more thorough investigation and validation by comparison with the data from Census and other sources is needed.
Figure 6.3
Price Distributions for the Simulation Year 2001
Figure 6.4
Price Distributions by Type for the Simulation Year 2001
Figure 6.4
Price Distributions by Type for the Simulation Year 2001
Table 6.2 Transaction Prices in 2001 by Type for ILUTE and Toronto Real Estate Board

<table>
<thead>
<tr>
<th></th>
<th>ILUTE</th>
<th>TREB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>St. Dev.</td>
</tr>
<tr>
<td>Detached</td>
<td>480,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Semi-Detached</td>
<td>280,000</td>
<td>130,000</td>
</tr>
<tr>
<td>Attached</td>
<td>260,000</td>
<td>110,000</td>
</tr>
<tr>
<td>Apartment</td>
<td>226,000</td>
<td>96,400</td>
</tr>
<tr>
<td>Total</td>
<td>392,000</td>
<td>180,000</td>
</tr>
</tbody>
</table>

Figure 6.5
New Housing Supply
Figure 6.6
Population Distribution by Year (Darker shades of green represents higher densities)
6.8 Discussion and Concluding Remarks

The clearing mechanism proposed for price-taker markets presented in this part of the thesis is an innovative, behaviourally rich, and readily operationalizable solution. The proposed mechanism is applied to the owner-occupied housing market clearing problem. The behaviour of individual agents and the characteristics of the active market determine transaction prices and the associated matching between the households and dwellings. The endogenous price formation is achieved without imposing the oversimplified assumption of strong equilibrium on the market.

The clearing problem has been posed as a multiplayer non-cooperative limited-information game. The buyers only have limited information about the market conditions and available options. They are trying to maximize their utility and outbid other bidders for their preferred locations. The sellers on the other hands are trying to maximize their profits. Note that there are actually two different types of sellers in the market: builders and households. Builders have better information of the market than households reselling their dwellings. In this version of the market clearing they are treated the same. In the future I plan to represent the behaviour of the two agents differently in the clearing process. Note that both buyers and sellers have the option to leave the market if they do not find a dwelling or price in accordance with their expectations.

The resulting game seems to have a non-empty and non-unique core. The various possible solutions are conditioned to the clearing sequence of the individual dwellings and the decisions of the individual agents. One can observe that this is very similar to what happens in real life. There is an intrinsic stochasticity in the market that strong equilibrium based clearing mechanisms fail to capture. Another interesting property of this game is that the resulting matching is a stable matching. That is: “There will not be any pair of agents (household-dwelling) in the simulation that are assigned an unacceptable match (based on their utility and profit maximization behaviour)” and “There will not be any potential pair left in the active-market that are not matched to each other, but will mutually prefer to be matched to one another.” Future work is required to more formally establish the mathematical properties of this game.
The proposed clearing mechanism is able to represent various market conditions, including: buyer- and seller-driven markets. Due to the microsimulation nature of the mechanism, one can also represent such behaviour, disaggregated by dwelling type and neighbourhood. For instance, one can simulate the situation where there are more demanders for a certain type of dwelling or dwellings in certain neighbourhoods. This will result in a seller-driven clearing in those segments of the market. Households that are not able to afford the higher prices will thus shift their choice-set and look for dwellings in other neighbourhoods and of other types.

The lagged interaction between new housing supply and the market activity is also represented in the clearing mechanism. The market conditions in previous years influence the start of new housing construction projects. Similarly, the asking price model also takes into account the transaction prices of previous years in the neighbourhood. The mechanism is highly flexible in the sense that the new housing supply model, location choice model, asking price model and choice-set generation process are completely separate from the core clearing. It can accommodate any kind of variation in these models without changing the core clearing mechanism.

One of the dimensions of housing market analysis that researchers and planners are interested in, is the ability to formulate a Social Welfare Function (SWF) for the households that are active in the market. Such a function helps in evaluating the impact of various policy scenarios on the general welfare of society (Cowell, 2004). Such a function requires knowledge of preferences of each person, comparability of individual utilities, and an aggregation function for utilities. In this regard, the first two requirements can readily be compiled from the existing mechanism, but a proper aggregation function is needed to be defined in order to deduce the SWF. Cowell (2004) points out two approaches (equal-ignorance assumption and PLUM principle) for defining the aggregation function. In future I plan to develop the aggregation function for our proposed clearing mechanism.

In any urban area, in addition to the owner-occupied housing market, the rental housing market also plays an important role in shaping up the distribution of population and built space. The households in the rental market are assumed in our work to be price-takers. Giroux-Cook (2010) has designed and implemented the rental market in ILUTE v1.0 using the price-taker
mechanism proposed by Farooq *et al.* (2010a). The current implementation of ILUTE runs the two markets completely independently and the transition between two tenures (owner, rental) is determined using ad hoc rules. In a future version of ILUTE, I intend to bring more interaction between the two markets and develop a transition model for tenure decisions.

By implementing a fully functional owner-occupied housing market within ILUTE and running a full population simulation for twenty years (1986–2006), the strength of the proposed mechanism has been ascertained in terms of its operationalization and ability to represent agent behaviour at a microsimulation scale. A rich variety of results are produced and a basic validation is performed with the historic data from census and other sources. In this regard, I plan to move forward and start a more systematic and rigorous validation process of ILUTE results in general and housing market in particular. It will also be worth an effort to test various scenarios and their implications on the agents’ behaviour and spatio-temporal distribution of the population and built space.

In the validation it was pointed out that the asking price forecasts by the current asking price model, that is implemented in ILUTE v1.0 are higher than expected. The clearing mechanism is highly sensitive to the asking prices that are representative of the seller’s perception of expected profit. The transaction price is searched in the vicinity of the asking price. If the search starts with higher asking prices, most of the time, it will end up forecasting higher than expected transaction prices as well. In the future I intend to work on calibration or re-estimation of the asking price model to have asking price estimates closer to the historic values.

The time intensive nature of the current ILUTE simulation in general and housing market in specific is a major obstacle to the large scale adoption of ILUTE v1.0 for planning and scenario testing. In this regard we have already tested prototypes to speed up the execution of the simulation by as large as 30 times using shared memory parallel processing architecture (Farooq *et al.*, 2010a, and Luu *et al.*, 2010). In future I intend to implement a full scale version of these speedups for the price-taker and–formation markets within ILUTE simulation framework.
PART III: DECISIONS
CHAPTER 7

BUILT SPACE VALUATION AND SUPPLY

7.1 Introduction

Part III of this dissertation is focused on the second important dimension of the built space evolution, Decisions. In particular, it addresses the decisions that are related to the valuation of built space and the supply of new built space. These decisions not only influence the activity levels in the built space markets, but also have an effect on each other in a lagged fashion.

Whenever owners decide to bring the built space they own into the active market, they have to decide on the value they expect from selling/leasing/renting the space—also referred to as asking price/rent. In the price formation markets (e.g. owner-occupied housing market), this value acts as the reference point for the bidding process. The resulting transaction price is a function of market conditions, buyers’ perception, and the asking price. In the case of price-taker markets (e.g. rental housing market) this value becomes the transaction rent at which the space is sold/rented out to the successful demander. The decision maker (owner of the built space) can be a single household selling its dwelling or a real estate company managing a number of apartment buildings for rent.

The valuation of a built space itself is a dynamic process that is affected by various factors, such as changes in market conditions, existing quality of the built space, accessibility levels, and changes in the features of the neighbourhood in which the built space is located. The valuation may vary for different categories of built space (residential, commercial, and industrial etc.). The type of decision makers (owners), value bearing attributes, their effects, location of the space, and methodology of valuation may also vary with the categories. For instance, in the case of residential built space, most of the time the decision maker is a household, while, in case of office space, the decision maker is usually the leasing company that may be managing/owning many office buildings. In terms of valuation of industrial built space, the proximity to a major highways intersection will have more importance than for a residential built space. The fluctuations in the value for residential space (at least in the case of a highly dynamic housing market like that of the GTA) may be bigger than in the industrial space market. It is due to these reasons that the valuation process for each category of built space is investigated separately in...
empirical studies. At the same time, the effects of different categories on each other should not be completely ignored in the empirical models.

Valuation of the residential space is widely investigated in the literature. On the other hand, there are very few studies found in the literature that are related to empirical modelling of the valuation for commercial and industrial space. In this part of the dissertation, this shortcoming is addressed by developing empirical models for the valuation of one specific type of commercial built space, i.e. office space. In the context of integrated land use and transportation modelling for the GTA, understanding the decision making processes in the office space market is extremely important. The Greater Toronto Area is the business centre of Canada and is the third largest financial centre in North America. Out of total employment in Toronto, 45% is office based. Thus, the GTA office space distribution is a major determinant of the overall travel patterns in the GTA.

The other important dimension of decision making in the context of built space evolution, addressed in this part of the dissertation, is supply decisions. Builders in the built space market respond to market conditions, availability of land and other resources (e.g. capital, construction material etc.), and regional economic conditions by adjusting the supply of different categories of built space. In case of brownfield redevelopment (De Sousa, 2002), builders transform an existing built space into a new space of the same category or some different category. In terms of the total inventory of built space in the urban area, due to these rebuild decisions by the builders, the stock of one category of built space may decrease and the stock of other’s may increase. Brownfield development usually requires changes in the zoning bylaws that are related to the land parcel.

In the outer limits of an urban area, land owners sell land to developers. In the context of the GTA, most of the time, developers will buy cheap agricultural land surrounding the city many years ahead of time and keep it until the local municipality designates it for development. Developers then help shape the final zoning of the land and develop it into parcels. Builders, based on zoning and the expected demand, build different categories of built space on these parcels. A developer and a builder can be the same or different agent. An analysis of the
InfoCanada\textsuperscript{14} dataset for businesses operating in the GTA for year 2006 shows that there are less than 500 builders operating the GTA. Buzzelli and Harris (2003) suggest that the number of active developers is even less than the number of builders. Developers usually work with the same builders and in very few cases the developer of land also builds the space. This supports the argument that the built space supply market in the GTA is a well-connected oligopoly.

The analysis of InfoCanada dataset on builders shows that in 2006 the sales volume of 13\% of the builders was less than 1 million dollars, 70\% of the builders had sales between 1 and 10 million, and 17\% of them had sales more than 10 million dollars. The building industry is thus dominated numerically by small- to medium-sized builders, but at the same time there is a significant presence of heavy-weight players in the industry. Buzzelli and Harris (2003) similarly report that the building industry in Ontario has a high number of small- to medium-sized builders. The total volume of the sales by the building industry in 2006 was approximately 5 billion dollars, with small to medium builders contributing 800 million dollars of this total. The large-sized builders contributed 4.2 billion dollars, which is more than five times of what was contributed by the small- and medium-sized (83\% numerically) builders. This shows that the large-sized builders play a dominating role in the building industry.

Another interesting fact about the building industry is that the number of employees for about 95\% of the builders is less than 25. This is because builders do not perform the construction job in-house. Instead, they heavily rely on contractors and sub-contractors to actually do the job for them and their employees are usually only managing the project. Buzzelli and Harris (2003) reported that this relation between the builders and contractors is spatially localized and long-term.

The building project has various identifiable stages (Somerville, 2001) (figure 7.1). In the first stage, a builder applies for a permit to construct a certain quantity of built space, seeks any required zoning changes, and acquires financial backing. Once approved, the builder may start the construction of the entire or some quantity of the built space it is permitted to build. The time to start the construction may vary, depending on market and regional economic conditions, but the latest time to start is dictated by the terms and conditions of the loans. The completion time

\textsuperscript{14} InfoCanada is a marketing consultant firm that conduct surveys related to business firms in various major cities of the world, including Toronto.
of the projects may also vary depending on these conditions. The introduction of space within the market may vary both in terms of time and quantity. Moreover, the whole project construction process may vary for different categories of built space.

![Construction Stages Diagram](image)

**Figure 7.1**
Various Stages of Construction

Supply of new built space is a very complex process in which there are markets (land, development, and building market) at various stages, different types/levels of finished product (land, developed parcels, and built space), and various types of decision making agents (land owners, developers, and builder) (figure 7.2). One approach to deal with this complexity is the grid cell concept (Waddell *et al.*, 2008; Hunt *et al.*, 2007). The urban area is divided into fixed grid cells that act as evolving cellular automata. The grid cell is a very rough abstraction of the developer and builder agents that maintains its own inventory and decides on the built space supply decisions (land is the decision maker). It seems to be an over simplification of the process. In my view, different markets, finished products and agents should be identified separately with inter- and intra-type interaction. Moreover, the parcels evolve by merging and splitting (especially in the case of brownfield redevelopment). The concept of a fixed dimension grid cell cannot represent this evolution. Martinez and Roy (2004) within the equilibrium framework modelled the supply process as a chain of market processes in which landowners, developers, and builders interact. Their modelling approach is a better representation of the various markets, finished products, and agents involved in the built space supply. But the strong
equilibrium assumption fails to capture the complex interactions occurring among the various agents within these markets.

Figure 7.2

Various Markets and Agents involved in the Built Space Supply

Realizing the highly complex nature of the supply process that results from the behaviour and interactions of the involved agents, in this dissertation, the focus is specifically on the decision making behaviour of one agent, the builder. In the context of new built space supply, a builder is faced with various types of decisions. It has to decide when, where, what type, and how much to build so that it can maximize its expected profit. It is assumed that the builder:
• Has access to financial and other resources required to build the amount of space it decides to build.
• Can acquire land parcels anywhere in the urban area.
• Can get a permit for the construction of the quantity it desires to build.
• Has a fixed time of construction.
• Introduces the entire quantity of built space at once to the built space market, at the end of the project.
• Is an expected profit maximizer, which it computes at the start of the project by speculating about the future revenues from sale, rent, and lease and various costs associated with the project.

Under these assumptions a novel approach is proposed that explicitly ties the time, location, quality, and quantity decisions for new built space supply together into a single dynamic framework based on expected profit maximization. As an application, a model for new office space development at a fairly disaggregate spatial resolution is then estimated. The problem of building new built space is treated as a situation in which a builder as a decision maker is faced with the decision of selection of a choice bundle and the associated quantities, while optimizing his expected profits. By doing so, the proposed framework is able to incorporate not only the relation between various decision dimensions, but also captures the behaviour of the decision maker (builder) and the influence of changing sub-market conditions and regional economy on the decision making.

The rest of the part III of this dissertation is organized as follows: this chapter first presents a brief review of the state of research in the real estate and integrated urban systems modelling literature in terms of modelling the valuation process and supply of new built space, in general, and office space in particular. In chapters 8 and 9 the formulation and application of the new built space supply model is presented. Chapter 10 then presents the formulation and application of a new hedonic model for valuation of the built space.
7.2 Literature Review

DiPasquale (1999) noted that while the demand-side of the housing market has been rigorously studied in the real estate literature, we do not have extensive empirical knowledge of the housing supply. The same is true for the integrated urban systems literature. In addition, the literature is even scarcer when it comes to commercial built space. This is despite the fact that commercial built spaces in an urban area are major activity centres and greatly influence general travel patterns, energy use, and population distributions. In previous dissertations completed here at the University of Toronto by Haider (1998, 2003) and Habib (2009), the current state of research related to housing demand, valuation, and supply has been extensively discussed. In this section the primary focus is on the state of literature related to one specific type of commercial space, i.e. office space.

7.2.1 Supply Modelling

McDonald (2002) concluded that despite the disastrous over construction of new office space in late 1980s in the United States, the literature on understanding and modelling the office space market is limited. In the office market literature, most of the focus seems to be on the valuation of the office space, absorption rates, and vacancy rates in the long term at the country or metropolitan area level. Another dimension that has been investigated in fair detail is the business cycles involved in the office market. The studies that have addressed the supply side have used a long-run equilibrium model in which supply is the result of a stock readjustment process in response to the lagged office sector employment growth and the difference in the effective rent level from the natural level. There is, however, rarely any work available on the location choice decisions for new space at submarket or lower aggregation levels within a metropolitan area. In the integrated urban systems modelling literature, the decisions are modelled at higher aggregation, but the models there are lacking in terms of completeness and sound economic foundations.

Rosen (1984) was the first to introduce a statistical modelling framework for the office space market. He reported that the existing methodology to model the office space market was only rate-based and deemed it ‘inadequate’ for investment and development decisions. The model forecasted vacancy rate, rent, and quantity of new floor space using separate linear regression equations. The model was estimated for San Francisco based on data from 1961 to
1983. Supply was assumed to be a function of vacancy rate, rent, interest rates, construction costs, and tax laws affecting commercial real estate. The model lacked any spatial representation and there was no differentiation in the quality of the office space. The model also lacked dynamics in that there was no investigation of lagged effects.

Howarth and Malizia (1998) proposed various strategies to improve our collective understanding of the office space market. It was recommended that more rigorous economic methods should be employed; the spatial variation should be incorporated in the model; and the long-term forecasting should incorporate the lag effects to capture the dynamics and the business and building cycles. It was also recommended to use changes in credit availability, trends in employment types, and that the regulatory environment should be taken into consideration. Product differentiation in terms of existing office space should be considered in decisions of differentiated new office space. The differences in the submarket dynamics should be incorporated in the location choice decision of new space. Risk and uncertainty faced by the builders should also be taken into account. This study provides a significant contribution in terms of exploration of the important explanatory measures of the office market dynamics. It didn’t, however, indicate the probable statistical and econometric modelling frameworks that could be used to model the office space market.

The Real Estate Econometric Forecast Model (REEFM), proposed by Viezer (1999) pooled the office space market data for fifty-one metropolitan areas in United States over the years 1985–1996. The REEFM framework consisted of six stochastic equations for forecasting occupancy, real rents, capitalization rates, market value per square foot, net change in stock, and real construction costs. Individual models were estimated using linear regression. The dynamics was captured using lagged variables. It was reported that inflation rates and interest rates were insignificant in explaining the variation. Due to the highly aggregate nature of the REEFM model and very few explanatory variables, it was limited in use for behavioural and spatially detailed analysis of the office market.

A comprehensive Walrasian Equilibrium based dynamic adjustment model of office market can be found in Hendershott et al. (1999). The relationship between supply and demand was used to link new construction, absorption, vacancies, and rent to employment growth and interest rates. Using structural equations, the model was estimated for the City of London for the
twenty year duration of 1977 to 1996. As reported in previous studies, the difference between actual and natural vacancy rates affected the rents. The construction responded to lagged changes in rent. Absorption rate was negatively related to rent while positively related to growth of financial services employment growth. While the proposed model was economically consistent and a complete model of office space market, it lacked submarket dynamics and spatial variation in the supply and rent levels.

Poterba's two-equation asset-market approach to modelling the housing market was adopted by Nanthakumaran et al. (2000) to model the office space market of the United Kingdom. This long-run equilibrium model treated supply as the “automatic stabiliser” to the changing rent level, resulting from changing demand. The parameter estimated for the capital value variable in the flow-supply equation was interpreted as price elasticity of supply. Other studies that used some variation of two-equation long-run equilibrium models for office space market include: Lentz and Tse (1977), McDonald (2000), Tse and Webb (2003), and Ho (2005).

Fürst (2006) used a three-stage simultaneous equation model to analyse the office space market of Manhattan. This study modelled the absorption rate, rent, and new supply of office space. Fuerst also investigated the existence of submarkets and their working as asynchronous autonomous economic units. This work is unique in the sense that it is the only study found in the real estate literature where the office market is analysed at a more disaggregate level than the generally-used metropolitan area level. The study divided Manhattan into 15 submarkets.

In UrbanSim, developed parcels or grid cells are used as the basic unit of built space (Waddell et al., 2003, 2008). The built space development is modelled as a discrete choice decision in which the land owner of a site (parcel/grid cell) decides on changing the state of site. This decision is modelled using a multinomial logit model. Land owners are faced with twenty-four choices that are a combination of built space type (residential, mixed use, commercial, industrial, government, vacant, developable, and undevelopable) and associated range of development intensity, in term of number of units or floor space (Waddell and Ulfarsson, 2003, 2004). In the simulation, these probabilities are used in combination with random draws to update the yearly built space stock. The simplicity of this approach makes it easier to operationalize in the urban simulation context, but at the same time, makes it very limited in terms of its ability to capture the underlying behaviour and structure of built space evolution. In
most of the cases, the development choices are rarely independent of one another. Haider and Miller (2004) reported the phenomena of spatial inertia in which the existence of one type attracted the intensification of the same type of development at a location. The associated quantity for each type of development is also overly simplified by limited categorization in term of choices. The quantity of development is a continuous dimension which is influenced by the market conditions, built capacity and various other factors. Moreover, the assumption of a homogenous decision maker is not behaviourally consistent. The landowner in the downtown will have a different perspective of choices than a landowner in the suburbs.

MUSSA (Martínez, 1996), like UrbanSim, assumes a homogenous builder, but its supply subsystem has a better economic foundation. The builder is faced with the decision of selecting the combination of built space type and location to maximize his profit (Martínez and Hurtubia, 2006). The probability of selection for each type and location combination is modelled using a multinomial logit model. MUSSA defines a linear profit function based on expected revenue and production cost function (Martínez and Henríquez, 2007). Using a strong equilibrium assumption, the share of each type of built space is determined from the total difference in demand and available stock and the probabilities of selection. This model is only implemented for residential built space. By imposing strong equilibrium and IIA assumptions, the solution for market clearing in the simulation becomes more tractable, but is not very representative of the actual built space markets.

PECAS simulates the evolution of spatial economics using a grid cell system under a strong equilibrium structure (Hunt and Abraham, 2003). The land use development for each year is determined in a multidimensional input-output table. These totals are then disaggregated using a set of logit models for each type of built space. Like MUSSA, PECAS also first determines the aggregate change in the built space from a strong equilibrium assumption, but unlike MUSSA, here the representation of different types of built space is more complete and there is some representation (at aggregate level) of the interplay between the share of each type of built space.

The current operational version of ILUTE (Miller et al., 2010), like the above mentioned urban simulations, first determines the total built space for a given year and then allocates it to individual locations. But it does not impose any equilibrium assumption on the market to determine the aggregate totals. Instead, the aggregate supply for each type of built space is
determined using a dynamic econometric model that represents the builders’ behaviour in different market conditions and the natural built cycles of the building industry (Farooq et al., 2008a). Separate logit models for each type of built space are estimated (Haider and Miller, 2004) that determine the probability of selection of a location by a type of built space. For each simulation year, the totals are computed and probabilities are updated. A Monte Carlo simulation is then used to assign location to the individual unit of built space (Farooq et al., 2008a). Currently, only the new housing supply model is operational. ILUTE’s land use evolution is economically more consistent as it incorporates various behavioural and market dimensions, including: risk taking attitude of the builders, spatial inertia, and lagged market conditions driving the new supply.

Here the discussion is focused on more recent, operational, and widely used urban systems model. A more detailed review of urban space evolution within various integrated urban systems modelling frameworks is discussed in Wegner (1995), Timmermans (2003), Hunt et al. (2005), and Miller (2006).

In general it can be concluded that there a lack of a single large-scale built space supply modelling framework that is spatially disaggregate, econometrically consistent, captures decision makers’ behaviour and associated heterogeneity, and is able to capture the interplay between various dimensions of decision making (where, when, what type, how much). Moreover, there is also a need of a modelling framework that moves away from the conventional equilibrium based aggregate approach and towards a dynamic disequilibrium microsimulation approach which is more representative of the actual built space markets.

7.2.2 Valuation

Valuation of the office space has been more extensively studied in the literature compared to the supply of new office space. As mentioned in the section 7.2.1, Rosen (1984) was the first to suggest a comprehensive model of the office space market. This study suggested that the change in demand and supply resulted in an adjustment of vacancy rates. In response to the change in vacancy rate, rent was found to change in a nonlinear fashion. The more rapidly the rent would
rise or fall, the further the actual vacancy rate moves away from the normal vacancy rate\textsuperscript{15}. This model was applied to the San Francisco office market and the inverse relationship between office rents and vacancy rate was confirmed. This highly aggregate model did not consider the spatial variations in rent or the effect of the quality of office space available on the transaction rent.

Shilling \textit{et al.} (1987) analyzed the changes in rental office space in 17 cities across the U.S. from 1969 to 1975. The study found that the normal vacancy rate varied across the cities as a function of the marginal cost of carrying inventory and the expected fluctuations in demand. A higher level of vacant space relative to the normal vacancy rate meant that landlords would lower their rents and hence reduce the difference between the desired and actual vacancies.

Mun and Hutchinson (1995) modelled the transaction rent for the old City of Toronto, parts of Mississauga, and parts of the North York region. Twenty five aggregate business nodes were defined and it was assumed that the office space within these nodes was homogenous. Short term equilibrium between the supply and demand of office space was used to determine the transaction rent. The inter-business-node travel times were used to study the influence of transportation on the office market, and the effect of office space quality was not considered. The study reported a high degree of agglomeration economies in the office sector and spatial variation in the transaction rent. From the dataset used in this study, it was found that there is a strong evidence of varying rents within the business nodes. In my view, the homogenous rent assumption by Mun and Hutchinson is a serious shortcoming in their study.

Individual building data from Greensboro, North Carolina were used by Frew and Jud (1988) to estimate a simulations equation based model for office space transaction rate. They argued that at any point in time, the supply is fixed; demand is a function of rent, and since rent is related to demand by the vacancy rate, this study effectively estimated the vacancy rate and rent level simultaneously. Linear, log-linear and square root specifications for the transaction rent were examined using the Box-Cox transformation. The hypothesis of a square root specification for both dependent and explanatory variables was accepted based on the higher maximum likelihood value. This study also explored the effects of distance from the CBD, the proximity to highways, and building characteristics on the transaction rents.

\textsuperscript{15} Normal (or natural) vacancy rate is defined as the level of vacancy rate that leaves the landlords indifferent in terms of vacant units that are held (Clapp 1993). This rate varies spatially and temporally and is a function of interest rates and expected rent levels.
The earliest hedonic modelling effort for office space rent can be found in Clapp (1980). Office space rent for buildings in Los Angeles metropolitan area was modelled as a function of land value, quality of space, neighbourhood features, pollution levels, distance to the nearest highway intersection, and average commuting time for employees. A double log function was used as the functional form.

Hedonic models of office space have also been estimated at the building level (Hough and Charles, 1983; Glascock et al., 1990; Sivitanides, 1997). The City of Chicago’s office space data was used in the hedonic analysis reported on by Hough and Kratz (1983). This study included distance from the CBD, proximity to train stations, and parking availability in the analysis to investigate the effect of accessibility to transport infrastructure on office rent. It was also reported that good new architecture resulted in a considerable rent premium.

Glascock et al. (1990) modelled rent as a function of building class, building location, and market conditions. The study used vacancy rate at the building level to incorporate the market conditions into the model. Rent was modelled both as linear and semi-log function of the explanatory variables, but the linear model was found to have better explanatory power. Sivitanides (1997) used the office space market data from 24 metropolitan areas in the U.S. between 1980-1988 to study temporal changes in office space rent. This study also explored the influence of lagged vacancy and absorption rates on office space rent.

Brennan et al. (1984) investigated the Box-Cox/Box-Tidwell type transformation to determine the functional form of the relationship between office space rent and explanatory variables. A log-linear form in which the natural logarithm of rent is a linear function of the explanatory variables was selected as the final model form. This was also the first study to use the office suite and not the whole building as the unit of analysis. Quality of office space, lease features, and distance from the CBD were found to be influential explanatory variables.

A hedonic regression model of asking rent for office space was developed by Dunse and Jones (1998) for the City of Glasgow. The office rent used here was at the individual suite level. This study defined the office rent as a function of the quality of the space, location, and tenure rights. The influence of quality was captured by age of the building, location of the suite within the building, and facilities in the suite and the building. A linear function was assumed between
the rent and explanatory variables. The resulting model pointed out age and location of the office space as the principal determinant of rents. The study could not include the influence of distance to the CBD or the variation in rent due to the availability of transportation infrastructure in the model.

Other important studies related to the office rent in the existing literatures are: Sivitanidou (1995), McGough et al. (2000), Ho (2005), and Stevenson and McGough (2006).

Overall, location, physical attributes, access to transport infrastructure, and market conditions have been used in past studies as principle determinants of office space rents. Most of these studies have used transaction rent as the dependent variable in the hedonic analysis. However, transaction rents should actually be the outcome of the market clearing process (Frew and Jud, 1988), and by using hedonic analysis, we are ignoring the effects of the dynamic interaction between supply and demand in the office space market. This part of the dissertation analysed the asking rent, which is an outcome of the owner’s perception of the rent, based on the quality, market conditions, location, and other utility bearing characteristics. The resulting rent model can then be used as a starting point in the supply and demand interaction that will result in the transaction rents. In all of the previous studies, the heterogeneity and other spatial phenomena that are important to the office space market are completely ignored in the analysis. The models developed in chapter 10 are unique in the sense that they explicitly investigate the effects of spatial heterogeneity and clustering on the asking rent of the office space in the GTA.
CHAPTER 8

MODEL OF NEW BUILT SPACE SUPPLY

8.1 Introduction

This chapter presents the theoretical formulation of the model for new built space supply that models the multidimensional decisions of when, where, what type, and how much to build, in a single consistent modelling framework. The underlying assumptions and the econometric foundations are discussed in detail. The estimation problem for this model is then defined. Various properties of the estimation problem are discussed and an estimation procedure is developed.

The decision makers here are a set of building construction firms (builders) that are active in the urban area at certain time \( t \). They are faced with the decision of choosing the quantity of different types of built space to be built, and the location where to build them. It is assumed that builders take these decisions so as to maximize their expected profits. Profit is determined by the difference of expected revenue and cost.

There are various large scale demand models available in the choice modelling literature that model the choice of a discrete bundle of goods (e.g. types of activities in which to engage in) and an associated continuous quantity (e.g. how much time to allocate to each activity) (Bhat, 2005, 2008; Habib et al., 2008; Habib and Miller, 2009, Kim et al. 2002). These models predominantly use the well-known random utility modelling (RUM) framework that assumes that the consumer is a utility maximizer. In terms of mathematical model formulation, the assumption of profit maximization by the producer (specifically, builders in the case at hand) is analogous to the utility maximization assumption in these large scale demand models. Moreover, profit from manufacturing a product is a more quantifiable concept than utility. Thus one can pose the problem of expected profit maximization in the same way as RUM does for the utility maximization of consumers faced with choice bundle selection and the associated quantities. This lets us use the same construct of optimization conditions (Kuhn-Tucker conditions) that in recent years has frequently been used in large scale demand models.
8.2 Theoretical Framework

The expected profit ($\Pi$) of a building construction firm, from $N$ differentiated products that it can decide to build at certain decision point $t$ can be represented by:

$$\Pi = \sum_{i=1}^{N} \frac{v_i}{a_i} \left\{ (f^r(X^r_i) - f^c(X^c_i))\theta \left( \left( \frac{q_i}{y_i} + 1 \right)^{\alpha_i} - 1 \right) + f^z(z) \right\}$$  \[8.1\]

Where:

$f^r(X^r_i)$ represents the expected unit revenue from selling product $i$

$X^r_i$ is a vector of variables related to product attributes, location features, and built space market conditions that influence the revenue

$f^c(X^c_i)$ represents the expected unit cost in building product $i$

$X^c_i$ is a vector of variables related to product attributes, location, state of regional economy, and conditions in various associated markets (labour, material etc.) that influence the cost

$q_i$ is the quantity of product $i$ that is decided to be built

The formulation here treats the share of profit from individual type of floor space $i$ in the same manner as Bhat (2005) and Kim et al. (2002) treat the share of individual choices in their utility function for large scale demand systems. The translation parameter $\gamma_i$ makes sure that there is a possibility of zero production of any given type of built space. Its value is greater or equal to zero ($\gamma_i \geq 0$), so as to make sure that the indifference curves touch the horizontal axis with a finite slope (Bhat, 2008). The parameter $\alpha_i$ is a scale parameter which adjusts the marginal profit with respect to the associated quantity of built space. $\theta$ here represents the risk behaviour of the builder and the structure of the space market in the region. In the simplest case $\theta$ could be a constant parameter, but in a more elaborate case it could be parameterized based on a combination of the builder’s and the market’s characteristics. The value of $\theta$ greater than one would mean that the builders inflate the expected revenue thus showing a more risk taker attitude, while a lower than one value would represent a risk avoiding attitude due to deflation of the expected revenue. Bhat (2008) performed a comprehensive analysis of the influence of different values of $\gamma_i$ and $\alpha_i$ on the indifference curves for the utility of the goods consumed. Similar analysis is needed for the formulation suggested in equation [8.1] for the profit.
8.2.1 Concept of Hicksian/Composite Product

As the builder has other options of investments besides the set of built space types that analysts are modelling, the concept of a Hicksian/composite product is introduced in the general formulation, \( f^z(z) \). If the conditions are extremely favourable, the builder would like to build as much as possible. The only limiting factor will be the technological or zonal constraints, as builders can only build to a certain extent with the current technology in a given time step. On the other hand, if the conditions are not highly favourable, the builder will carefully select the supply levels to an extent so that the profit is maximized and loss is avoided by overbuilding.

Profit from the Hicksian product in [8.1] represents the expected loss that is avoided at a given interest rate at the decision time by not building the built space that could have been built under the technological/zoning constraint. To represent this, a separate profit generation function similar to the one used by von Haefen and Phaneuf (2004), and Habib and Miller (2009), is used for the composite activity.

If one assumes that the revenue and cost functions are linear in parameters and the modeller’s inability to perfectly observe builder’s expected profit is represented by the error term \( \varepsilon_l \), then [8.1] can be rewritten as:

\[
\Pi = \sum_{i=1}^{N} \frac{y_i}{a_i} \left( (\beta^r_i X_i^r - \beta^c_i X_i^c)^\theta e^{\theta \varepsilon_i} \left( \left( \frac{q_i}{y_i} + 1 \right)^{\alpha_i} - 1 \right) \right) + \frac{1}{1-e^p} z^{(1-e^p)}
\]  

[8.2]

The form of the profit function for composite product here guarantees a positive profit from a nonzero composite product (Habib and Miller, 2009). Note that there is no error term associated with the profit from the Hicksian product. The rational here is that the error terms from rest of the products in [8.2] are the differences from the Hicksian part.

8.3. Estimation Problem

Using [8.2], our optimization problem can be defined as:

Maximize

\[
\Pi = \sum_{i=1}^{N} \frac{y_i}{a_i} \left( (\beta^r_i X_i^r - \beta^c_i X_i^c)^\theta e^{\theta \varepsilon_i} \left( \left( \frac{q_i}{y_i} + 1 \right)^{\alpha_i} - 1 \right) \right) + \frac{1}{1-e^p} z^{(1-e^p)}
\]  

[8.3a]

Subject to
\begin{align*}
\sum_{i=1}^{N} q_i + z &= K_T \tag{8.3b} \\
q_i &\geq 0 \quad i = 1, 2, \ldots, N \tag{8.3c} \\
z &> 0 \tag{8.3d}
\end{align*}

$K_T$ is the maximum possible space that could be built in the time interval under zoning and technological constraints.

The Lagrangian function for the problem in (8.3) becomes:

\begin{equation}
L = \left[ \sum_{i=1}^{N} \frac{\theta_i}{\gamma_i} \left( (\beta_i^r X_i^r - \beta_i^c X_i^c) e^{\theta_i \epsilon_i} \left( (\frac{q_i}{\gamma_i} + 1)^{\alpha_i} - 1 \right) \right) + \frac{1}{1-e^\rho} z^{(1-e^\rho)} \right] - \lambda \left[ \sum_{i=1}^{N} q_i + z - K_T \right] \tag{8.4}
\end{equation}

$\lambda$ = Lagrangian multiplier

The Khun-Tucker (KT) first order conditions for optimal allocations here will be:

\begin{align*}
\frac{\partial L}{\partial q_i} - \lambda &\leq 0 \quad \text{for } i = 1, 2, \ldots, n \tag{8.5a} \\
&\&
\frac{\partial L}{\partial z} - \lambda &\geq 0 \tag{8.5b}
\end{align*}

(8.5a) ensures that, for the selected levels of the product bundle, any further increase in the quantity of product $i$ will have no further positive effect on the total profit. (8.5b) ensures that the quantity of the composite product (not investing) is at the level where it has no negative effect on the profit.

From (8.5a) and (8.5b)

\begin{equation}
\frac{\partial L}{\partial q_i} \leq \frac{\partial L}{\partial z} \quad \text{for } i = 1, 2, \ldots, n
\end{equation}

\begin{equation}
(\beta_i^r X_i^r - \beta_i^c X_i^c) e^{\theta_i \epsilon_i} \left( \frac{q_i}{\gamma_i} + 1 \right)^{(\alpha_i-1)} \leq z^{-e^\rho} \tag{8.6}
\end{equation}

It can be shown that $\frac{\partial^2 L}{\partial q_i \partial \epsilon_i}$ is a $i \times i$ non-singular matrix and $\frac{\partial^2 L}{\partial z \partial \epsilon_i}$ is a zero valued vector.

Thus using explicit function theorem (Jittorntrum, 1978), one can express the error term as:

\begin{equation}
\epsilon_i \leq g_i(\beta_i^r, X_i^r, \beta_i^c, X_i^c, q_i, \theta, \gamma_i, \alpha_i, \rho)
\end{equation}
\[ \varepsilon_i \leq \frac{1}{\varrho} [(1 - \alpha_i) \log \left( \frac{q_i}{y_i} + 1 \right) - \beta_i \log (\beta_i X_i^r - \beta_i X_i^c) - e^\rho \log \left( \kappa T - \sum_{j=1}^n q_j \right)] \forall i \quad [8.7] \]

### 8.4 Econometric Model Structure

If the joint probability density function, \( f(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n) \) of the error terms is known then the probability associated with the quantities of a certain bundle of product that is selected by the building firm for construction is given by:

\[
P(Q) = \int_{-\infty}^{g_{m+1}} \cdots \int_{-\infty}^{g_n} f(g_1, g_2, \ldots, g_m, \varepsilon_{m+1}, \ldots, \varepsilon_n) \left| J \right| d\varepsilon_{m+1} \cdots d\varepsilon_n \quad [8.8]
\]

Where

\[ Q = [q_1^*, q_2^*, \ldots, q_m^*, 0, 0, \ldots, 0] \] is the vector of quantities of each type selected by the builder to build.

\( |J| \) is the determinant of \( m \times m \) Jacobian matrix, whose individual elements are defined by:

\[ \frac{\partial \varepsilon_i}{\partial q_i} \quad (\text{Kim et al., 2002; Bhat, 2005; and Habib and Miller, 2009}). \]

\[ |J| = \prod_{i=1}^N \frac{1}{\varrho} \left[ \frac{(1 - \alpha_i)}{(q_i + \gamma_i)} + \frac{e^\rho}{(\kappa_T + q_i)} \right] \]

Most of the discrete-continuous large scale demand models including (Bhat, 2005, 2008; Habib, 2008, 2009; Pinjari and Bhat, 2009), have assumed the error terms to be IID with Type I extreme value distribution. This assumption simplifies [8.8] and gives a closed form solution for the calculation of choice probabilities. The estimation of model parameters also becomes computationally manageable in cases where the size of choice-set is large.

However, in my view, this assumption is not valid in the case of new built space. Most of the time, the types of the space that are built by the builder are highly correlated to each other. Builders are localized in terms of their operations (Buzzelli and Harris, 2003). Moreover, builders and their associated contractors/sub-contractors typically specialize in building specific types of space. The builder that builds detached dwellings is more likely to build semi-detached and attached dwellings than high rise apartments. The location case is similar: A zone (business node) that primarily has Type-A office space will unlikely to get built an inferior, Type-C office space. A more appropriate assumption, therefore is that the error terms are jointly normally distributed with a mean of 0 and covariance matrix \( \Omega \). Hence:
\[ P(Q) = \frac{1}{(2\pi)^{n/2}|\Omega|^{1/2}} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \int_{-\infty}^{g_{m+1}} \cdots \int_{-\infty}^{g_n} \exp \left( -\frac{1}{2} \mathbf{E}' \Omega^{-1} \mathbf{E} \right) |\mathbf{f}| d\varepsilon_1 \cdots d\varepsilon_n \]  \[ 8.10 \]

Where \( \mathbf{E} = [\varepsilon_1, \ldots, \varepsilon_m, \varepsilon_{m+1} \ldots \varepsilon_n] \)

Equation [8.10] involves computing an \((n-m)\) dimensional integral of the function that will have a high computational cost associated for large choice sets. In the case of built space however, the builder is faced with very few choices (e.g. 3 in case of office space and 4 to 5 in the case of housing). Thus the evaluation of [8.10] remains computationally viable.

The resulting likelihood function from [8.10] for all the builders thus becomes:

\[
L(Q|\beta^T_1, \beta^C_1, \theta, \gamma_i, \alpha_i, \rho, \Omega) = \\
\prod_{b=1}^{B} \frac{1}{(2\pi)^{n/2}|\Omega|^{1/2}} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{g_{k+1}} \cdots \int_{-\infty}^{g_n} \exp \left( -\frac{1}{2} \mathbf{E}' \Omega^{-1} \mathbf{E} \right) |\mathbf{f}| d\varepsilon_1 \cdots d\varepsilon_n
\]

8.5 Parameter Estimation

The likelihood maximization based parameter estimation process involves two basic steps, the generation/evaluation of the candidate parameter values, and the evaluation of the likelihood function. In the logit-based conventional discrete choice models, the likelihood function has a closed form, so the evaluation of likelihood, gradient, and hessian of the function is trivial. Gradient based search methods like Newton-Raphson (NS), Broyden-Fletcher-Goldfarb-Shanno (BFGS), Berndt-Hall-Hall-Hausman (BHHH), David-Fletcher-Powell, Polak-Ribiere conjugate gradient, and simulated annealing (Ben-Akiva and Lerman, 1985; Washington et al., 2003; and Train, 2009) are used to estimate the parameters and their statistical properties.

In case of parameter estimation for probit models, the evaluation of the likelihood function becomes non-trivial, because of the involvement of the multi-dimensional integral. In case of the classic probit model the multidimensional integral involved in the likelihood function is approximated using one of several methods: numerical integration, tabulation, numerical approximation, and Monte Carlo simulation (Sheffi et al. 1982). MC Simulation is most widely used in which the likelihood function is evaluated using various simulation techniques like Accept–Reject (AR), smoothed AR, and Geweke–Hajivassiliou–Keane (GHK). The resulting approximate log-likelihood function is called a Simulated Log-Likelihood (SLL). The gradient of the function required for optimization problem can be approximated by dividing the change in SLL by the change in the parameter values (Train, 2009). Bolduc (1999) suggests a simulation
based procedure for the analytical solution of the gradient in the GHK-simulator. Another approach for the probit model parameter estimation is the Bayesian based Markov Chain Monte Carlo (MCMC) simulation technique that avoids the direct evaluation of the likelihood function. Instead it derives the posterior distributions from the prior belief and the data. The moments and other statistical properties are derived by sampling from the posterior distribution using simulation techniques like Metropolis-Hasting (M-H), Adaptive M-H, and Gibb’s sampler (Kim et al., 1999; Kim et al., 2002; and Train, 2009). Kim et al. (1999) used Markov chain Gibb’s sampler to draw directly from posterior distribution and performed finite sample likelihood inference.

Bhat (2001) used a quasi-random Monte Carlo simulation technique to estimate parameters for a mixed logit model. A Halton sequence for each dimension of the integral in the likelihood function was drawn by pairing k-sequences. The sequence ensured that the whole region under the integral is uniformly covered. The cyclic nature of the Halton sequence results in correlation issues. To avoid this problem, a scrambling technique was used, but this adds an exponential overhead with each dimension, so as to produce a “good” permutation (Hess and Polak, 2003). It is however not very clear what maximization criteria were used and how the approximate gradient/scores and hessian were calculated. It is also not very clear how the local maxima were avoided in the estimation process.

Train (2009) outlined a Bayesian based MCMC method for the parameter estimation in mixed logit models. Bayesian methods relax the constraint of maximizing the simulated-likelihood function, which could be complicated in cases where there might be various local maxima and thus might result in identification problems. In Bayesian methods, the prior distribution plays an important role and is assumed to be near the values that globally maximizes the likelihood function. Bayesian methods are also superior to standard simulated-likelihood maximization methods in terms of consistency and efficiency.

In the case of large-scale demand model estimation, Monte Carlo simulation, quasi-Monte Carlo simulation, and Markov Chain based Monte Carlo simulation methods are commonly used (Bhat, 2001, Kim et al., 2002, Habib, 2009). Bhat (2005) and Habib and Miller (2009) used the quasi-random Monte Carlo simulation procedure outlined in Bhat (2001) for the likelihood function that had extreme valued error terms and normally distributed parameters.
Kim et al. (2002), von Haefen and Phaneuf (2004), and Habib (2009), used Markov Chain Monte Carlo (MCMC) based on Metropolis-Hasting method to estimate their parameters from the likelihood function involving the normal distribution. The likelihood functions in the cases of von Haefen and Phaneuf (2004), and Habib (2009) had extreme valued error terms and normally distributed correlated parameters. Kim et al. on the other hand had a normally distributed correlated error terms as well. Kim et al. (2002) used GHK simulator to evaluate the multidimensional integral involved within the log likelihood function. The statistical properties of the estimated parameters were computed using Gibb’s sampling.

The likelihood function in equation [8.11] also involves correlated error terms that are normally distributed. In the process of estimation of parameters from this function, due to a relatively small sample size, it was decided to use Bayesian MCMC with Gibb’s sampling approach. For the evaluation of the multidimensional normal probability function involved in equation [8.11], a technique based on randomized lattice rules was used. This technique seeks to fill the hyper integration space evenly using a deterministic process. In principle, these lattice rules construct regular patterns, such that the projections of the integration points onto each axis produce an equidistant subdivision of the axis (Genz, & Bretz 2002, 2009). Robust integration error bounds are obtained by introducing additional shifts of the entire set of integration nodes in random directions. Since this additional randomization step is only performed to introduce a robust Monte Carlo error bound, 10 simulation runs are usually sufficient. This method was preferred against the more widely used Halton sequence based simulation procedure, because it has been numerically proven to outperform Halton or Sobel sequences in terms of efficiency and doesn’t suffer from correlation issues (Lai, 2009).

8.6 Estimation Procedure

The procedure that has been developed to estimate parameters in equation [8.11] is as follows:

Let the parameters in the likelihood function be represented as \( \bar{\zeta}_b = (\beta^b, \rho^b, \gamma, \rho) \)

1. Initialize \( \bar{\zeta}_b, \alpha, \bar{\rho}_b, \Omega_\zeta \)

2. Generate \( \{\bar{\zeta}_b, b = 1, ..., B\} \) from

98
\[ \psi(\zeta_b|q_{bt}, t = 1, ..., T), \alpha, \bar{\zeta}_b, \Omega_c] \propto det|\Omega_c|^{-\frac{1}{2}} \exp\left[-\frac{1}{2} (\zeta_b - \bar{\zeta}_b)^\prime \Omega_c^{-1} (\zeta_b - \bar{\zeta}_b)\right] \prod_t L_{bt} \]

Where

\( \psi \) is a \( N \times 1 \) vector representing all the alternatives

\( t \) represents the decision occasion

Generate a random number \( \tau_\psi \rightarrow N(0,0.0025) \), then the candidate value of \( \zeta_b \) for iteration \( k \) will be:

\[ \zeta_b^k = \zeta_b^{(k-1)} + \tau_\psi \]

Accept this new value with the probability:

\[
\min \left[ \frac{\exp\left[-\frac{1}{2} (\zeta_b^{(k)} - \bar{\zeta}^{(k)})\prime \Omega_c (\zeta_b^{(k)} - \bar{\zeta}^{(k)})\right] \prod_t L_{bt}^k}{\exp\left[-\frac{1}{2} (\zeta_b^{(k-1)} - \bar{\zeta}^{(k-1)})\prime \Omega_c (\zeta_b^{(k-1)} - \bar{\zeta}^{(k-1)})\right] \prod_t L_{bt}^{k-1}}, 1 \right]
\]

3. Generate \( \bar{\zeta}_b \) from

\[
\psi(\bar{\zeta}_b|\{\zeta_b, b = 1, ..., B\}, \Omega_c) = N\left(\frac{\sum_b \zeta_b}{B}, \frac{\Omega_c}{B}\right)
\]

4. Generate \( \Omega_c \) from

\[
\psi(\Omega_c|\{\zeta_b, b = 1, ..., B\}, \bar{\zeta}_b,) \propto \text{Inverted Wishart} \left( d_0 + B.D_0 + \sum_b (\zeta_b - \bar{\zeta}_b)\prime(\zeta_b - \bar{\zeta}_b) \right)
\]

Where \( d_0 \) is the prior degrees of freedom and \( D_0 \) is the sum of squares of \( \Omega_c \)

5. Generate \( \alpha \) from

\[
\psi(\alpha|q_{bt}, b = 1 ... B and t = 1, ..., T), \{\zeta_b, b = 1 ... B\}, \bar{\alpha}_0, \Sigma_0} \]

\[
\propto det|\Omega_0|^{-\frac{1}{2}} \exp\left[-\frac{1}{2} (\alpha - \bar{\alpha}_0)^\prime \Omega_0^{-1} (\alpha - \bar{\alpha}_0)\right] \prod_b \prod_t L_{bt}
\]

\( \bar{\alpha}_0 \) and \( \Omega_0 \) are the prior parameters

Generate a random number \( \tau_\alpha \rightarrow N(0,0.01) \), then the candidate value of \( \alpha \) for iteration \( k \) will be:

\[ \alpha^k = \alpha^{(k-1)} + \tau_\alpha \]
Accept this new value with the probability:

\[
\min \left[ \frac{\exp \left[ -\frac{1}{2} (\alpha^k - \bar{\alpha}_0) \Omega_0^{-1} (\alpha^k - \bar{\alpha}_0) \right] \prod_b^B \prod_t^T L_{bt}^k}{\exp \left[ -\frac{1}{2} (\alpha^{(k-1)} - \bar{\alpha}_0) \Omega_0^{-1} (\alpha^{(k-1)} - \bar{\alpha}_0) \right] \prod_b^B \prod_t^T L_{bt}^{(k-1)}}, 1 \right]
\]

6. Generate \( \vartheta \) from

\[
\psi(\vartheta | \{q_{bt}, b = 1 \ldots B \text{ and } t = 1, \ldots, T\}, \{\zeta_b, b = 1 \ldots B\}, \bar{\vartheta}_0, \Sigma_0)
\alpha \det(\Omega_0)^{-\frac{1}{2}} \exp \left[ -\frac{1}{2} (\vartheta - \bar{\vartheta}_0) \Omega_0^{-1} (\vartheta - \bar{\vartheta}_0) \right] \prod_b^B \prod_t^T L_{bt}
\]

\( \bar{\vartheta}_0 \) and \( \Omega_0 \) are the prior parameters.

Generate a random number \( \tau_\vartheta \rightarrow N(0,0.01) \), then the candidate value of \( \vartheta \) for iteration \( k \) will be:

\[
\vartheta^k = \vartheta^{(k-1)} + \tau_\vartheta
\]

Accept this new value with the probability:

\[
\min \left[ \frac{\exp \left[ -\frac{1}{2} (\vartheta^k - \bar{\vartheta}_0) \Omega_0^{-1} (\vartheta^k - \bar{\vartheta}_0) \right] \prod_b^B \prod_t^T L_{bt}^k}{\exp \left[ -\frac{1}{2} (\vartheta^{(k-1)} - \bar{\vartheta}_0) \Omega_0^{-1} (\vartheta^{(k-1)} - \bar{\vartheta}_0) \right] \prod_b^B \prod_t^T L_{bt}^{(k-1)}}, 1 \right]
\]

7. Iterate back to step 1

The simulation has to be run for a sufficient numbers of iterations before drawing inferences. It is suggested that around 25,000 iterations should be enough for the burn-in (Kim et al., 2002; von Haefen and Phaneuf, 2004; Train, 2009; and Habib, 2009). Gibb’s sampling is then done to construct the distributional summary statistics for \( \zeta_b, \bar{\zeta}_b, \Omega_\zeta, \alpha, \vartheta \). Gibb’s sampling induces a serial correlation in the parameters. To avoid this correlation, it is also suggested that every 10\(^{th}\) iteration be used in the simulation after warm up (von Haefen and Phaneuf, 2004; and Train, 2009).

### 8.7 Identification Problem

Parameter estimation from the data based on the underlying model structure is fundamentally an optimization problem that may have a non-unique solution set. The identification problem is the problem of determining what conclusions drawn from the data about a model parameters are
feasible (Manski, 1995; Train, 2009). Walker et al. (2007) defined the identification problem as the problem of determining the set of restrictions to impose in order to obtain a unique vector of consistent parameter estimates. These restrictions can be on the range in which the parameter values should exist, acceptable goodness-of-fit values for the estimated model, and definition of the regions in the search space in which to search.

In this context, it seems that the Bayesian method based on Markov Chain Monte Carlo simulation with Gibb’s sampling does a better job to properly identify the solution. Compared to the quasi-MC methods, the approach proposed in this chapter, gives more flexibility in defining the search space and guiding the search in the proper direction. This is because the search is based on a prior distribution. The prior distribution corresponds to the knowledge that we already have about the parameters and their correlations. It is rarely the case that we do not have any idea about the sign and/or scale of the parameter values. We can thus control the starting direction of search based on our prior beliefs about the solution. Secondly, the Metropolis-Hasting based search process itself is more controlled and directed. The MH search updates the value of the parameter distributions based on the increase in the likelihood from the new values. The statistical properties of the parameters in the solution are drawn from the posterior distribution of the parameters that are not just based on the likelihood values from the data, but also on the prior distribution and the search process.

Lastly, by introduction of the Hicksian good, it is ensured that the solution space for the problem is reduced to finding only those parameters that represent the set of quantities within the technological constraints. The parameters estimation also takes into account the fact that a builder will only invest in built space to an extent where it can maximize profit, but also have the option of investing elsewhere (or for that matter not investing at all).
CHAPTER 9

BUILT SPACE SUPPLY MODEL: APPLICATION TO OFFICE SPACE SUPPLY

9.1 Introduction

Using the model formulation proposed in chapter 8, this chapter estimates a model for the new office space supply evolution in the Greater Toronto Area (GTA). The office space market in the GTA is a vibrant and growing market. Greater Toronto Area is the third largest financial centre of North America, only after New York and Chicago. It is the centre of activity in Canada for various office-based employment sectors, including finance, information technology, banking, insurance, and legal consulting. The consistent demand of high quality office space with good accessibility and location arising from the office based employment sectors has driven the growth in office space market in the GTA.

Understanding the factors that affect office location decisions plays an extremely important role in our greater understanding of travel behaviour in urban areas. Since office location strongly influences the spatial distribution of morning and afternoon peak-period travel, where firms choose to locate their offices greatly influences short-term individual-level decisions such as mode of transportation, and long-term household-level decisions such as residential location. Conditions in the office space market affect firms’ location and relocation decisions, and hence influence the general travel patterns in an urban area. Moreover, modelling and understanding the office space market in general and office space supply in particular has high economic benefits. The large capital requirements and long development periods make office investment riskier than other types of built space (Tse and Webb, 2003). Using office space models for forecasting and understanding of the working of building industry could decrease these investment risks.

Many aggregate (country, municipality or CBD-suburb level) office space supply models can be found in the real estate and integrated land-use and transportation modelling literature. There are very few examples, however, of serious modelling efforts at the more disaggregate submarket level. This is in spite of the fact that there is strong evidence to suggest that submarkets within a metropolitan area are temporally asynchronous from each other in terms of growth and are characterised by a high level of agglomeration by industry type. The availability
of certain types of office floor space has an effect on firm location and relocation decisions. Literature on modelling the quality of the new office space supply is also relatively scarce.

Another important dimension in the modelling of office space market is that the location, quality, and quantity are interconnected decisions. At anytime, a location may have excess stock of one type, but is under-stocked in other type. Similarly, some locations are suitable for only a few specific types of built space while not suited for others. For instance, downtown Toronto has a high concentration of Type A\textsuperscript{16} and B office space, but rarely Type C space. The quantity that could be built at certain location is also influenced by the neighbourhood characteristics (zoning by-laws, technological constraints). In the real estate literature, quantity is mostly modelled at a very high level of aggregation. Operational integrated urban modelling frameworks model these decisions at a lower level of aggregation (census tracts, small grids), but do not treat them as related decisions within a single framework. Instead the individual dimensions are modelled separately, and then some kind of simulation or rule based allocation is used to simulate the built space evolution. In ILUTE for instance, Miller et al. (2010) used a separate model for the location choice probabilities for each type of dwelling and another model for the quantities to produce in the study area. These two models are then used in a Monte Carlo simulation to allocate the new stock to individual locations.

The building industry generally, and in the case of the GTA in particular, is an oligopoly with very few firms and these firms move in and out of the market very frequently with the boom and bust cycles of the built space market (Buzzelli and Harris, 2003). It is important to bring in the builders’ behaviour within the decision modelling framework. Their attitude towards risk taking and the expectation of profit might vary among individual builders. A modelling framework that can incorporate these issues is currently missing in the literature.

Farooq et al. (2010) reported a high spatial variation in the rent of office space. It was also reported that there were not only inter-cluster variation, but also intra-cluster variations. Under the profit maximizing assumption for the builder of space, this variation in rent will influence the decision to select the best location for the new office space.

\textsuperscript{16} Categories defined by Building Owners and Managers Association (BOMA). For details, please see section 9.2
Considering the above mentioned shortcomings in the existing literature, the estimation of a multidimensional decision model for office space that models the when, where, what, and how much decisions in a single framework, is a significant contribution towards the better understanding of the office space supply evolution. The remainder of this chapter is organized as follows: the next section discusses the dataset used in the model estimation, the model estimates are then presented, followed by a discussion of the estimation results and concluding remarks.

9.2 Data Description

The office space data collected by Coldwell Banker Richard Ellis (CBRE) consultants is used for the Greater Toronto Area from 1986 to 2005 to estimate the model. Office space is distributed across the study area in various identifiable clusters. Based on their geographic concentration in various regions, CBRE divided the study area into 36 visually identifiable business nodes (Figure 9.1) in the survey.

![Figure 9.1](image)

**Figure 9.1**

**Study area and approximate location of the business nodes**

The CBRE dataset that was used in this study is a quarterly account of the total inventory and market conditions, including new supply, vacancy rates, gross and net rent levels, and absorption rates in these business nodes for different types of office space. For the estimation of
the model in this chapter, the data is available from first quarter of 1986 to third quarter of 2005 is converted to yearly values.

The dataset classified office space into four standard types (A, B, C, and G), as defined by the Building Owners and Managers Association (BOMA). This is a subjective classification that uses a combination of factors including rent, building finishes, system standards and efficiency, building amenities, location/accessibility, and market perception (BOMA, 2009). Type A floor space has a high quality standard finish, state of the art systems in the building, exceptional accessibility and a definite market presence. Downtown Toronto and regional centres are dominated by Type A office space. Type A space has higher than average rents for the area. Type B office space has fair to good facilities and infrastructure, while Type C office space buildings are only providing a functional space at a lower rent level compared to the area average (BOMA, 2009; AtlusInSite, 2009). Type G office space are government owned buildings. Type G buildings don’t follow the general demand for office space in the market and are not included in this analysis.

Table 9.1 lists out the business nodes used in this study, their share of office space in year 2005, and the average number of floors in the buildings in these nodes. One can observe that there is a high concentration of office space in the downtown Toronto area followed by the regional centres of Mississauga, North York, and Airport area. The downtown is dominated by high rise buildings, while in the suburbs there is a high concentration of low rise buildings.

Figure 9.2 reports the yearly changes in the total supply of new office space in the GTA. During the analysis period, one can observe two boom cycles, 1986–1992 and 1999–2000, and one bust cycle, 1993–1998. The first boom cycle was the biggest and longest cycle in the history of the GTA. It started in early 80s and kept on rising until 1990. This boom cycle coincided with the building boom cycle in United States, when the office space building industry excessively overbuilt (McDonald, 2002). The cycle also coincides with the high growth in the population and employment of the GTA during this time period. With the recession in the Canadian economy in early 90s, the boom cycle also sharply subsided. Clearly the builders overbuilt in this boom cycle. In the bust cycle of 1993–1998, hardly any new office space was built. Even in the next boom cycle, by the time of which the Canadian economy had already recovered the intensity of new office space being built was not as high as the first boom.
<table>
<thead>
<tr>
<th>No.</th>
<th>Node</th>
<th>Office Space (million sqr. ft.)</th>
<th>% of Total Office Space in the Node</th>
<th>Avg. Number of Floors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Financial Core</td>
<td>32.92</td>
<td>20.07</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Downtown North</td>
<td>14.97</td>
<td>9.13</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Downtown West</td>
<td>10.86</td>
<td>6.62</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>Downtown East</td>
<td>3.15</td>
<td>1.92</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>Downtown South</td>
<td>2.38</td>
<td>1.45</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Bloor &amp; Yonge</td>
<td>8.84</td>
<td>5.39</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>St. Clair &amp; Yonge</td>
<td>3.05</td>
<td>1.86</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>Eglinton &amp; Yonge</td>
<td>4.84</td>
<td>2.95</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>North Yonge</td>
<td>8.95</td>
<td>5.46</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>Heartland</td>
<td>3.15</td>
<td>1.92</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>Yorkdale</td>
<td>2.57</td>
<td>1.57</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>Downsview</td>
<td>0.77</td>
<td>0.47</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>Dufferin and Finch</td>
<td>0.61</td>
<td>0.37</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>Vaughan</td>
<td>1.46</td>
<td>0.89</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>Bloor &amp; Islington</td>
<td>1.54</td>
<td>0.94</td>
<td>9</td>
</tr>
<tr>
<td>16</td>
<td>King &amp; Dufferin</td>
<td>2.88</td>
<td>1.76</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>Sheridan</td>
<td>1.11</td>
<td>0.68</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>Airport Dispersed</td>
<td>4.62</td>
<td>2.82</td>
<td>4</td>
</tr>
<tr>
<td>19</td>
<td>Highway 427 Corridor</td>
<td>2.51</td>
<td>1.53</td>
<td>6</td>
</tr>
<tr>
<td>20</td>
<td>Airport Corporate Centre</td>
<td>6.08</td>
<td>3.71</td>
<td>4</td>
</tr>
<tr>
<td>21</td>
<td>Etobicoke Dispersed</td>
<td>0.66</td>
<td>0.40</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>Mississauga Dispersed</td>
<td>0.82</td>
<td>0.50</td>
<td>4</td>
</tr>
<tr>
<td>23</td>
<td>Mississauga City Centre</td>
<td>3.41</td>
<td>2.08</td>
<td>9</td>
</tr>
<tr>
<td>24</td>
<td>Cooksville</td>
<td>0.74</td>
<td>0.45</td>
<td>6</td>
</tr>
<tr>
<td>25</td>
<td>Brampton</td>
<td>2.07</td>
<td>1.26</td>
<td>4</td>
</tr>
<tr>
<td>26</td>
<td>Meadowvale</td>
<td>4.05</td>
<td>2.47</td>
<td>3</td>
</tr>
<tr>
<td>27</td>
<td>Oakville</td>
<td>1.79</td>
<td>1.09</td>
<td>4</td>
</tr>
<tr>
<td>28</td>
<td>Burlington</td>
<td>3.28</td>
<td>2.00</td>
<td>3</td>
</tr>
<tr>
<td>29</td>
<td>Don Mills and Eglinton</td>
<td>4.23</td>
<td>2.58</td>
<td>6</td>
</tr>
<tr>
<td>30</td>
<td>Duncan Mill</td>
<td>1.87</td>
<td>1.14</td>
<td>6</td>
</tr>
<tr>
<td>31</td>
<td>Consumers Road</td>
<td>4.53</td>
<td>2.76</td>
<td>7</td>
</tr>
<tr>
<td>32</td>
<td>Scarborough</td>
<td>4.81</td>
<td>2.93</td>
<td>5</td>
</tr>
<tr>
<td>33</td>
<td>Markham &amp; Pickering</td>
<td>0.79</td>
<td>0.48</td>
<td>3</td>
</tr>
<tr>
<td>34</td>
<td>Highway 404 &amp; Highway 7</td>
<td>7.89</td>
<td>4.81</td>
<td>4</td>
</tr>
<tr>
<td>35</td>
<td>Highway 404 &amp; Steeles</td>
<td>5.53</td>
<td>3.37</td>
<td>4</td>
</tr>
<tr>
<td>36</td>
<td>Richmond Hill</td>
<td>0.26</td>
<td>0.16</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>164</td>
<td>100.00</td>
</tr>
</tbody>
</table>
By modelling the new office space supply for the time period from 1986 to 2005, the model is able to capture the builder’s behaviour in a few very different and equally interesting market scenarios. In the boom cycle when the economy is doing well and the demand of office space is high, the builders tend to over build and in recessions and low demands there are very few new development projects. The bust cycle 1993–1998 is a special case as it comes right after an exceptionally high boom cycle, right in the middle of a major recession.

Figure 9.3 breaks down the yearly new supply by the type of office space. The new supply seems to be dominated by the high quality Type A space, followed by Type B and C. It should also be noted that there is almost no new office space of Type B and C built after 1997 and all most all the new supply is of Type A. This reflects the demand of high quality office space by the office based employment sector in the GTA, which is dominated by banking, finance, IT, and consulting.
In the first boom cycle of the analysis period, it can be observed that the new office space building activity is going on, not only in the downtown Toronto, but also in all the suburban regions (Figure 9.4). However, in the last 10 years of the analysis period, one can observe that the construction activity is only concentrated in the suburbs and there are very few new office space construction projects in the downtown Toronto. This shows the ongoing trend in which the regional activity centres like downtown Mississauga, North York, and Oakville are experiencing a rapid growth rate.

Figure 9.5 shows the spatial distribution of the office space by different types. About 40% of the Type A space is located in the downtown, followed by West (Mississauga and Airport area) and then East (Scarborough and Markham area). Type B and C have a more even distribution among these regions.

Figure 9.6 shows a high concentration of high rise buildings in the downtown followed by the midtown and North region. The medium to low rise building are concentrated in the suburbs. The interaction between dimensions like zoning by-laws, availability of land, size of land parcels, and spatial distribution of demand for new space are the main causes of such distribution of the buildings in the GTA.
Figure 9.4
Yearly supply of floor space by regions in the GTA

Figure 9.5
Spatial distribution of floor space type (A, B, C) in the GTA
Figure 9.6
Distribution of buildings by number of floors
Statistics Canada was used as the main source for the data related to hourly wage rates of construction worker and number of construction workers in the labour force for each year in the study duration. Generally, it can be observed that the number of construction workers in the industry (figure 9.7) follows the trend of the bust and boom cycle of the construction industry (figure 9.2). However, contrary to the cycles in construction, a higher trend in number of construction workers in the industry is observed, during 1999–2006 rather than in the period of 1986–1991. This might indicate that there was a shortage in number of available construction workers in the first boom cycle of the analysis period.

![Figure 9.7: Number of Construction Workers](image)

**Figure 9.7**  
Number of Construction workers

Figure 9.8 shows the trend in the hourly wage rate for the construction workers. Using the consumer price index (CPI), all the values have been converted to 2001 Canadian dollars. One can observe a rise in the values around 1991 and then, with recession and a freeze on wage rates, a sharp decline in the rates can be observed. The rates rise again after the recession, peaking in 2002, and a downward trend can be seen again after 2002.
The construction cost estimates data for office space were collected from yearly per square foot estimates published by RSMean Inc. for office space construction in the Toronto region. These estimates are the major sources used by the builders to develop construction estimates for construction projects. Figure 9.9 shows the yearly trend in construction cost of one square foot of office space by the number of floors in the building. All the values are converted into 2001 Canadian Dollars. One can observe a bigger increase in the construction cost of low to medium rise than the high rise buildings. The trend shows that after 2000 it becomes cheaper to build a high rise than low or medium rise. This trend might be because of the fact that the infrastructure (cranes and heavy equipment) that was bought by the builders in the 80s and 90s required huge investments. The same infrastructure is now being reused and leased out for the construction of high rises. This brings the unit cost down for the high rises.

Figure 9.8
Wage Rates for Construction Workers (in 2001 CAN Dollars)
Low rise has seen the highest increase in cost followed by the medium rises. This might be due to the fact that the processes there are less automated and mechanized and thus construction requires more labour as compared to high rise buildings. Unfortunately, the cost data by type of office space in the building was not available. Computing that cost from RSMean unit cost forecasts will require knowing the details about the exact facilities offered in different types and using the base rates for them to compute total cost. We do not have any good estimate about the exact facilities provided in different types of office space, either. Because of this, in the estimated model, a single parameter and average cost for all the types were used.

Figure 9.9
Construction Cost for Square Foot of Office Space by Number of Floors in the Building (in 2001 CAN Dollars)
Yearly average gross rent rate and vacancy rates were generated from averaging the quarterly values in the CBRE dataset. Figure 9.10 shows the trends in the gross rent in the GTA. All the values are converted into 2001 Canadian Dollars. It can be observed that the rent fluctuates periodically in the range of about 22 to 30 dollars per square foot. The fluctuation in the rent represents the fluctuations in the interaction between demand and supply levels in the market during the period of analysis.

![Gross Rent per Sq. Ft. for Office Space (in 2001 CAN $)](chart)

**Figure 9.10**
Gross Rent per Sq. Ft. for Office Space (in 2001 CAN $)

Under the homogenous builder assumption, for the estimation of the model, 720 observations of types of space built and the associated quantity (i.e. from the combination of 36 nodes and 20 years) were created. Although the duration of construction varies by the type of project, for the sake of simplicity, it was assumed that the construction time of the project was 1 year. For the market indicator variables, a lag of 1 year was used. Table 9.2 provides the summary statistics and description of the dependent and explanatory variables used in the estimated model.
Table 9.2 Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean/Proportion</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supply_A</td>
<td>Supply of Type A (1000 sq. ft.)</td>
<td>76.57</td>
<td>226.78</td>
<td>0</td>
<td>2601.88</td>
</tr>
<tr>
<td>Supply_B</td>
<td>Supply of Type B (1000 sq. ft.)</td>
<td>11.92</td>
<td>44.99</td>
<td>0</td>
<td>525.00</td>
</tr>
<tr>
<td>Supply_C</td>
<td>Supply of Type C (1000 sq. ft.)</td>
<td>2.14</td>
<td>12.58</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td><strong>Independent Variable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Built_A</td>
<td>Already Built Type A in the Node (million sq. ft.)</td>
<td>1.833</td>
<td>3.815</td>
<td>0</td>
<td>24.667</td>
</tr>
<tr>
<td>Built_B</td>
<td>Already Built Type B in the Node (million sq. ft.)</td>
<td>1.307</td>
<td>1.448</td>
<td>0</td>
<td>5.987</td>
</tr>
<tr>
<td>Built_C</td>
<td>Already Built Type C in the Node (million sq. ft.)</td>
<td>0.368</td>
<td>0.438</td>
<td>0</td>
<td>1.640</td>
</tr>
<tr>
<td>Gr_Rtl_Rt_A</td>
<td>Average Rent of Type A in the Node (CAN $)</td>
<td>25.46</td>
<td>6.888</td>
<td>2.30</td>
<td>50.30</td>
</tr>
<tr>
<td>Gr_Rtl_Rt_B</td>
<td>Average Rent of Type B in the Node (CAN $)</td>
<td>20.82</td>
<td>5.525</td>
<td>1.77</td>
<td>40.89</td>
</tr>
<tr>
<td>Gr_Rtl_Rt_C</td>
<td>Average Rent of Type C in the Node (CAN $)</td>
<td>18.73</td>
<td>5.000</td>
<td>7.40</td>
<td>37.62</td>
</tr>
<tr>
<td>Vac_Rt_A</td>
<td>Vacancy Rate of Type A in the Node</td>
<td>0.139</td>
<td>0.077</td>
<td>0</td>
<td>0.442</td>
</tr>
<tr>
<td>Vac_Rt_B</td>
<td>Vacancy Rate of Type B in the Node</td>
<td>0.160</td>
<td>0.096</td>
<td>0</td>
<td>0.56</td>
</tr>
<tr>
<td>Vac_Rt_C</td>
<td>Vacancy Rate of Type C in the Node</td>
<td>0.158</td>
<td>0.175</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Con_Wrks</td>
<td>Number of Construction Workers in the GTA (×1000)</td>
<td>41.93</td>
<td>9.87</td>
<td>2.73</td>
<td>61.30</td>
</tr>
<tr>
<td>Wage_Rt</td>
<td>Hourly Wage Rate for Construction Workers</td>
<td>18.27</td>
<td>2.69</td>
<td>12.83</td>
<td>22.91</td>
</tr>
<tr>
<td>Con_Cost</td>
<td>Construction Cost per sq. ft.</td>
<td>85.15</td>
<td>13.75</td>
<td>70.5</td>
<td>117.55</td>
</tr>
</tbody>
</table>

Total observations: 720
9.3 Model Estimates

Using the dataset described in the previous section, the model of new office space supply for the Greater Toronto Area was estimated. The estimation process was implemented in R statistical language. This language was selected due to the open source nature of the language and availability of various support packages. The code is written in a very generic form and could be readily used for estimation of the models that are based on the framework proposed in chapter 8. The execution time for the estimation code is around 6 hours. The large computation time is due to the fact that the MCMC process has to run for a long time to warm up and then to avoid correlation among iterations, only 1 in 10 iterations is used to generate the distributions for the parameters. In future I intend to work on a faster implementation of the algorithm.

The dependent variable here is the probability of selection of a vector \((3 \times 1)\) of quantities for each type of office space to be built in a business node \(n\) at certain year \(t\) from 1986 to 2005. The explanatory variables used in the model represent the market conditions and land use characteristics of the business node and the state of regional economy at the time of decision to build. Parameter estimates and associated statistics are reported in Table 9.3. Table 9.4 reports the correlations between different types of office space.

The constant term for Type A space is the highest, followed by Type B. For Type C the constant term is negative. This suggests that builders in general prefer to build higher quality space. The office employment sector in the GTA is dominated by financial, accounting, law, and technology firms that generate the demand for high quality office space. Higher constants for Type A and B seems to be the response of builders to this demand and higher profit margins.

Haider and Miller (2004) reported the phenomena of spatial inertia in the new housing supply of the GTA. Same phenomena can be observed in the case of office space supply. The attractiveness which is captured by the amount of office space that is already available (Buit_\*) is the highest in case of Type A, while it is lowest in the case of Type C.
Table 9.3 Model Parameter Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Std. Error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const_A</td>
<td>29.71</td>
<td>0.182</td>
<td>163.29</td>
</tr>
<tr>
<td>Const_B</td>
<td>19.99</td>
<td>0.112</td>
<td>178.74</td>
</tr>
<tr>
<td>Const_C</td>
<td>-10.72</td>
<td>0.119</td>
<td>-89.75</td>
</tr>
<tr>
<td>Built_A</td>
<td>1.76</td>
<td>0.202</td>
<td>8.68</td>
</tr>
<tr>
<td>Built_B</td>
<td>0.68</td>
<td>0.057</td>
<td>11.94</td>
</tr>
<tr>
<td>Built_C</td>
<td>0.57</td>
<td>0.082</td>
<td>6.99</td>
</tr>
<tr>
<td>Gr_Rtl_Rt_A</td>
<td>0.69</td>
<td>0.219</td>
<td>3.13</td>
</tr>
<tr>
<td>Gr_Rtl_Rt_B</td>
<td>0.80</td>
<td>0.166</td>
<td>4.81</td>
</tr>
<tr>
<td>Gr_Rtl_Rt_C</td>
<td>1.11</td>
<td>0.108</td>
<td>10.31</td>
</tr>
<tr>
<td>Vac_Rt_A</td>
<td>-2.44</td>
<td>0.140</td>
<td>-17.44</td>
</tr>
<tr>
<td>Vac_Rt_B</td>
<td>-1.15</td>
<td>0.167</td>
<td>-6.91</td>
</tr>
<tr>
<td>Vac_Rt_C</td>
<td>-0.37</td>
<td>0.236</td>
<td>-1.58</td>
</tr>
<tr>
<td>Con_Wrks</td>
<td>0.84</td>
<td>0.072</td>
<td>11.73</td>
</tr>
<tr>
<td>Wage_Rt</td>
<td>-1.32</td>
<td>0.116</td>
<td>-11.37</td>
</tr>
<tr>
<td>Con_Cost</td>
<td>-0.64</td>
<td>0.135</td>
<td>-4.73</td>
</tr>
</tbody>
</table>

Model structure parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Std. Error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma_A</td>
<td>100.10</td>
<td>0.080</td>
<td>1246</td>
</tr>
<tr>
<td>Gamma_B</td>
<td>100.06</td>
<td>0.088</td>
<td>1135</td>
</tr>
<tr>
<td>Gamma_C</td>
<td>99.88</td>
<td>0.138</td>
<td>723.28</td>
</tr>
<tr>
<td>Rho</td>
<td>4.49</td>
<td>0.108</td>
<td>41.64</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.66</td>
<td>0.173</td>
<td>1.107</td>
</tr>
<tr>
<td>Theta</td>
<td>1.59</td>
<td>0.226</td>
<td>15.44</td>
</tr>
</tbody>
</table>

Table 9.4 Correlation Matrix between the error terms (significant with 95% confidence)

<table>
<thead>
<tr>
<th>Type of Office Space</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>1.00</td>
<td>-0.10</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

The rent per sq. ft. of the type office space at the time of decision was used as the indicator of market and the growth of office based employment. In general there is a positive effect of the supply decisions with the higher rents and this effect is highest in the case of Type C buildings. This result is unexpected as one would expect that the higher quality space will be
more sensitive to the increase in the rent. One reason for this might be the fact that in general there is a higher temporal variation found in the rent of Type C office space. While in case of Type A and B, the variation is both in terms of time and space.

Average vacancy rate in the node at time of decision was used as another indicator of the demand for office space. The model reports negative sensitivity of the supply decisions to the increase in vacancy rates. This effect is highest in case of Type A space. A higher project cost is associated with Type A space and at the same time the revenue (indicated by rents) from it is the highest as well. This explains the higher sensitivity to vacant space in case of Type A supply decisions.

The number of construction workers in the labour force at the time of decision is used as the indicator of building inertia and state of the regional economy. A positive effect is found on the supply decisions due to the increase in number of construction workers.

With the increase in the wage rates the cost of the project increases and thus effects the new office space supply decision in the negative fashion. Similar behaviour is evident in the case of increase in the construction cost. Unfortunately, due to the unavailability of the data, a model that reflects the effect of difference in cost by type on the probability of selection could not be estimated at this stage.

The greater than one theta variable that represents the behaviour of the builders towards risk shows that they are risk takers. Builders overbuild in the boom of construction cycles anticipating future revenues. This fact is evident from the discussion in the data description section. I would like to introduce more detailed behaviour of the builders and possible heterogeneity in the model by further parameterizing this variable using data on builders’ characteristics.

The translation parameters that make the corner solutions possible, are almost the same for all three types. The standard errors are also very small. Various starting values were tried that resulted in approximately the same values. It will be interesting to observe the effect of fixing the value of translation parameter to 100 and running the estimation again. This way, one will be able to control the bias that variable translation parameters may have introduce in the estimation
process. The scale parameter seems to be in the acceptable range. The rho parameter that is associated with the parameterization for the Hicksian good is also in the right range.

A limitation that arises due to the use of Bayesian-based MCMC estimation process is the inability to generate model level goodness-of-fit statistics. The goodness-of-fit test for these types of method is an evolving research topic. An alternate approach to test the goodness of fit for this model could be to use simulation forecasting and compare the results with the observed data. Once new data for the years after 2005 are available, I plan on performing the simulation tests.

9.4 Discussion and Concluding Remarks

This chapter presented a model of new office space supply for the Greater Toronto Area. The model is based on the novel multidimensional decision modelling framework for the supply of new built space, presented in detail in chapter 8. The modelling framework assumes that builders attempt to maximize expected profit. To our knowledge, this work is the first that models the where, when, how much, and what type of office space to supply in a single framework at a fairly disaggregate spatial zoning system.

The estimated model is dynamic in the sense that it captures the lagged effects of market conditions on the new supply. The phenomenon of spatial inertia was observed in terms of location choices for different types of office space. The behaviour of the builders in terms of risk is explicitly incorporated and estimated in the model. Builders are risk takers and tend to overbuild in the boom cycles. In future I intend to bring in more detail in the model in terms of builder’s behaviour and heterogeneity among builders. That will require specialized survey of builders and their characteristics and observations about their decision making behaviour in the context of new built space supply. The changes in the construction project’s expected cost on the builder’s decisions are also modelled. Depending on the data availability I intend to introduce more detailed representation of costs and its variation by type and location in the model.

The use of Markov Chain Monte Carlo simulation has a short-coming in the sense that the model fitness statistics cannot be estimated. On the estimation side, I intend to assess the goodness of fit for the model using simulation and comparison of simulation results with the observed data. Unfortunately the data for the new built space supply after 2005 is not currently
available. Another option for assessing the performance of the model would be to randomly pick some observations from the current dataset and not include them in the estimation process. One can then test the prediction power of the model on the excluded observations. This would provide some indication concerning how good the model is, however the size of the dataset would be reduced further (currently 720 observations), which might affect the quality of the estimation process.

In the future I also intend to develop an estimation procedure for the model using a maximum likelihood method. Such a method will require at least developing procedures to evaluate the first and second order derivative of the likelihood function that was developed in chapter 8. The advantage of using maximum likelihood based estimation is that one would be able to compute goodness-of-fit values from the estimation process. On the other hand, one will have to exercise extra caution so as to avoid getting stuck in the local maxima during the estimation process. Due to the higher degree from of the likelihood function and the problem’s dimensionality, it is not very clear if the global maximum exists. In any case, I want to make sure that the solution that the estimation process find, does make sense and is usable in planning and forecasting. For instance, if one sets the parameters value to infinity (with appropriate signs) that will give the maximum value for the likelihood function, but the resulting model estimates will be of no use to planners in terms of giving insight and its predictive power.

This model is part of the ongoing efforts towards operationalization of the office space market within Integrated Land Use Transportation and Environment (ILUTE) modelling framework, currently under development at the University of Toronto. In the general market-clearing framework\(^1\) of ILUTE, the asking rent model captures the role of accessibility, neighbourhood characteristics, quality of space, and market conditions to determine the asking price at each simulation year. The models for demand for office space in the Greater Toronto Area by small- and medium-sized firms have already been developed by Elgar et al. (2009). With the available demand and supply, these asking rents are then to be used in the market clearing module to match the space to the demander at a transaction rent that is endogenously determined. In next simulation year, the lagged transaction rents then influence the builders’ decisions of where, how much, and what type of office space to supply.

---

\(^{1}\) Detail discussion can be found in chapter 2.
Note that the model estimated in this chapter is based on 36 business nodes that are spatially quasi-independent sub-markets. While in the context of modelling the office space supply, it makes sense to use this spatial system, for the implementation of the model in ILUTE, another level of model will be needed that distribute the built space within a business node to the census tracts or dissemination areas that are marked as commercial by the zoning by-laws. This model can be a similar location choice model as the one estimated for the new housing supply by Haider (2003). This model will then be used in the similar fashion as we used Haider’s model for operationalizing housing supply in ILUTE v1.0.\(^\text{18}\)

Another important aspect to consider in terms of operationalization of this model in ILUTE is the evaluation of the multidimensional integral involved in the model. Evaluation of such an integral will have to take place many times in some sort of Monte Carlo simulation that will be required to implement this model within ILUTE. This will introduce a large computational overhead in the already time-intensive ILUTE simulation. Given the relatively modest correlations\(^\text{19}\) observed in Table 9.4, data limitations, and current assumptions, a possible option to be explored in future work would be to assume the error terms to be identically and independently Gumbel distributed and re-estimate the model. Such an assumption will have a close formed solution and will not require an integral evaluation, both in estimation and forecasting.

The Government of Ontario recently released a growth plan for Greater Golden Horseshoe Area that includes the Greater Toronto Area. This plan forecasts an increase of 3.7 million in the population and 1.8 million in the employment of the region by 2031. The long-range forecast suggests high levels of growth in employment sectors that are housed in office space. 25 nodes across the region are identified as high density employment centres and corridors linking them are given high priority for transit investment. The Province is also in the process of introducing a comprehensive transportation plan to support this growth. To quantitatively assess the implications of these policies and their effect on the urban growth pattern, demographics, and more specifically on the travel behaviour of the population, I intend to operationalize a comprehensive office market model within the Integrated Land-Use,

\(^{18}\) See chapter 6 for details on operationalization of housing supply in ILUTE v1.0

\(^{19}\) The probable reason for a low correlation is the assumption in the current model that all the builders are homogeneous and non-localized
Transportation, and Environment (ILUTE) modelling framework. This modelling effort is a major step forward in achieving this objective.
CHAPTER 10

HEDONIC ANALYSIS OF ASKING RENT FOR OFFICE SPACE

10.1 Introduction

This chapter presents analysis of another important decision that a supplier has to make in the built space market, i.e. setting up the asking price/rent for the active built space. In this context, the focus here is limited to analysis of the asking rent for the active office space in the Greater Toronto Area. The rent analysis here is primarily based on hedonic analysis theory.

A comprehensive study of the spatial and temporal distribution of population and space that results from the dynamics within Toronto’s office sector is thus a very important component of an integrated land use, transport, and environment framework like ILUTE (Integrated Land Use, Transportation, and Environment modeling framework). A previous study by Elgar et al. (2009) studied the behaviour of small office firms in the office market at a very disaggregate scale, and highlighted the importance of the office space market conditions in location and relocation decisions. Unfortunately, however, there is a paucity of literature available on the valuation and supply side of the office sector in general, and on the office space market in particular. This modelling effort is a first step towards our greater understanding of the role of asking rent model for office space evolution in the Greater Toronto Area (GTA). This chapter analyse the asking rent of available office space at a highly disaggregate spatial scale using hedonic analysis theory.

In hedonic analysis the price/rent of a differentiated product is assessed based on the utility bearing attributes of the product (Rosen, 1974). On one hand, the transaction rent is the outcome of the dynamic interaction between supply and demand in the office space market. On the other hand, asking rent is the rent demanded by the supplier-agent on the basis of his perception of the value offered by the product’s attributes and the existing market conditions. It is thus more appropriate to use hedonic analysis as a tool to study the asking rent, rather than the transaction rent.

In this chapter, a set of hedonic rent models are presented that analyse the effects of the accessibility, quality, location, and market conditions on the asking rent of office space in the GTA. Two important spatial phenomena, heterogeneity and clustering that are visible in the
study area, are tested and modelled using the Global Moran I test, the Anselin Local Moran I test, the Hierarchical Random Effect model, and the Semiparametric Regression model. Functional form of the hedonic relationship using the Box-Cox transformation was also explored.

The remainder of the chapter is organized as follows: the next section presents a detailed descriptive analysis of the dataset used in the analysis. The modelling structures that are explored in the analysis are then presented. This is followed by the estimated modelling results. Next, the chapter analyses and discusses the modelling results and their implications. Finally, conclusions are drawn with a summary of major contributions made by this analysis.

10.2 Data Description

For the estimation of hedonic asking rent models in this chapter, the office building data provided by Atlas InSite consulting is used. This dataset provides a snapshot of office space inventory for the Greater Toronto Area (GTA), in the second quarter of 2005. The dataset included all the buildings with an office space of 20,000 sq. ft. or more. Their exact locations were used to geocode the dataset for computing the accessibility, demographic, and land-use related variables. Asking rent rate, type of rent, building classification, total and available floor space, and the year built variables were also available in the dataset.

As discussed in the chapter 9, the office buildings are distributed across the study area in various identifiable clusters. Based on their geographic concentration in various regions, the study area was divided into 36 unique business nodes (Figure 9.1) in the survey that generated the dataset. Mun and Hutchison’s (1995) analysis of transaction rent was based on a subset of the study area used in this paper; the 25 unique business nodes they identified match 25 of the nodes used in this paper.

Table 10.1 lists the business nodes used in this study and their descriptive statistics. A high concentration of office space is located in the central business district and in the Toronto downtown area (40%). Major street and highway intersections and the airport have a high concentration of office space buildings. Regional centres in the suburbs such as Mississauga, Scarborough, Markham, Brampton, and Oakville also exhibit a concentration of office buildings.
Asking rent in the dataset is expressed as per square footage of office space, expressed in 2005 Canadian dollars. The average value of rent is recorded in the data for each building. The suite-level variation of office rent within the building based on orientation, floor, and other structural and architectural features is not considered. The average asking rent per square foot, in the study area was $13.48 with a standard deviation of $5. A large spatial variation was observed in the asking rent, with downtown Toronto having the highest variation. The rate of variation decreases as the distance from the central business district increases (Figure 10.1). A few peaks such as Mississauga City Centre and Oakville can also be observed. Downtown and regional centres are dominated by medium to high rise office buildings, while low rise buildings are located at the intersection of major highways and the airport.

Like the CBRE dataset used in chapter 9, this dataset also used Building Owners and Managers Association (BOMA) classification and classified office buildings into four standard types (A, B, C, and G).

![Figure 10.1](image.png)

**Figure 10.1**
Office space rent variation with reference to the distance from CBD
### Table 10.1 Business Nodes in the study area

<table>
<thead>
<tr>
<th>No.</th>
<th>Node</th>
<th>% of Total Office Area in the Node</th>
<th>Avg. Asking Rate (sqr. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Financial Core</td>
<td>20.07</td>
<td>18.76</td>
</tr>
<tr>
<td>2</td>
<td>Downtown North</td>
<td>9.13</td>
<td>15.16</td>
</tr>
<tr>
<td>3</td>
<td>Downtown West</td>
<td>6.62</td>
<td>15.45</td>
</tr>
<tr>
<td>4</td>
<td>Downtown East</td>
<td>1.92</td>
<td>14.68</td>
</tr>
<tr>
<td>5</td>
<td>Downtown South</td>
<td>1.45</td>
<td>17.25</td>
</tr>
<tr>
<td>6</td>
<td>Bloor &amp; Yonge</td>
<td>5.39</td>
<td>13.94</td>
</tr>
<tr>
<td>7</td>
<td>St. Clair &amp; Yonge</td>
<td>1.86</td>
<td>14.29</td>
</tr>
<tr>
<td>8</td>
<td>Eglinton &amp; Yonge</td>
<td>2.95</td>
<td>12.65</td>
</tr>
<tr>
<td>9</td>
<td>North Yonge</td>
<td>5.46</td>
<td>15.19</td>
</tr>
<tr>
<td>10</td>
<td>Heartland</td>
<td>1.92</td>
<td>12.20</td>
</tr>
<tr>
<td>11</td>
<td>Yorkdale</td>
<td>1.57</td>
<td>9.55</td>
</tr>
<tr>
<td>12</td>
<td>Downsview</td>
<td>0.47</td>
<td>11.30</td>
</tr>
<tr>
<td>13</td>
<td>Dufferin and Finch</td>
<td>0.37</td>
<td>11.66</td>
</tr>
<tr>
<td>14</td>
<td>Vaughan</td>
<td>0.89</td>
<td>12.81</td>
</tr>
<tr>
<td>15</td>
<td>Bloor &amp; Islington</td>
<td>0.94</td>
<td>14.21</td>
</tr>
<tr>
<td>16</td>
<td>King &amp; Dufferin</td>
<td>1.76</td>
<td>13.78</td>
</tr>
<tr>
<td>17</td>
<td>Sheridan</td>
<td>0.68</td>
<td>11.35</td>
</tr>
<tr>
<td>18</td>
<td>Airport Dispersed</td>
<td>2.82</td>
<td>11.65</td>
</tr>
<tr>
<td>19</td>
<td>Highway 427 Corridor</td>
<td>1.53</td>
<td>10.05</td>
</tr>
<tr>
<td>20</td>
<td>Airport Corporate Centre</td>
<td>3.71</td>
<td>14.35</td>
</tr>
<tr>
<td>21</td>
<td>Etobicoke Dispersed</td>
<td>0.40</td>
<td>9.72</td>
</tr>
<tr>
<td>22</td>
<td>Mississauga Dispersed</td>
<td>0.50</td>
<td>11.03</td>
</tr>
<tr>
<td>23</td>
<td>Mississauga City Centre</td>
<td>2.08</td>
<td>14.78</td>
</tr>
<tr>
<td>24</td>
<td>Cooksville</td>
<td>0.45</td>
<td>10.06</td>
</tr>
<tr>
<td>25</td>
<td>Brampton</td>
<td>1.26</td>
<td>12.07</td>
</tr>
<tr>
<td>26</td>
<td>Meadowvale</td>
<td>2.47</td>
<td>12.59</td>
</tr>
<tr>
<td>27</td>
<td>Oakville</td>
<td>1.09</td>
<td>14.49</td>
</tr>
<tr>
<td>28</td>
<td>Burlington</td>
<td>2.00</td>
<td>11.29</td>
</tr>
<tr>
<td>29</td>
<td>Don Mills and Eglinton</td>
<td>2.58</td>
<td>10.16</td>
</tr>
<tr>
<td>30</td>
<td>Duncan Mill</td>
<td>1.14</td>
<td>10.63</td>
</tr>
<tr>
<td>31</td>
<td>Consumers Road</td>
<td>2.76</td>
<td>10.02</td>
</tr>
<tr>
<td>32</td>
<td>Scarborough</td>
<td>2.93</td>
<td>11.00</td>
</tr>
<tr>
<td>33</td>
<td>Markham &amp; Pickering Dispersed</td>
<td>0.48</td>
<td>13.78</td>
</tr>
<tr>
<td>34</td>
<td>Highway 404 &amp; Highway 7</td>
<td>4.81</td>
<td>13.41</td>
</tr>
<tr>
<td>35</td>
<td>Highway 404 &amp; Steeles</td>
<td>3.37</td>
<td>11.36</td>
</tr>
<tr>
<td>36</td>
<td>Richmond Hill</td>
<td>0.16</td>
<td>13.00</td>
</tr>
</tbody>
</table>

|           | 100.00 | 13.48 |
In the second quarter of 2005, there were 1302 office buildings of 20,000 sq. ft. area or more in the study area. The total inventory of the office space was 164 million square feet, out of which 53.5% was Type A, 30.7% was Type B, 10.4% was Type C, and 5.4% was government office space (Figure 10.2). As already stated, Toronto is the commercial and financial capital of Canada with head offices of major national and international companies. A high percentage of Type A and Type B office space in the inventory is the result of high demand for such space by the types of office based firms operating in Toronto.

Table 10.2 Descriptive statistics for Asking Rent\(^{20}\) by Type of Building

<table>
<thead>
<tr>
<th>Type</th>
<th>Avg_Rent</th>
<th>St_Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15.95</td>
<td>4.66</td>
</tr>
<tr>
<td>B</td>
<td>12.35</td>
<td>3.83</td>
</tr>
<tr>
<td>C</td>
<td>12.22</td>
<td>4.44</td>
</tr>
</tbody>
</table>

Figure 10.2
Share of Office Space Type (A, B, C, G) in 2005

\(^{20}\) All the asking rent values used in this chapter are in terms of dollars per square foot of office space.
The average asking rent for Type A is highest followed by Type B and C (Figure 10.3). The difference between average asking rent for Type B and C is very small, but the standard deviation for Type C office space is higher than Type B (Table 10.2). Type A has the highest standard deviation. Type B buildings have the least variation in the rents among different types of office space.

A small fraction of office buildings (5%) in the study area offered the option of gross rent that included the utilities and taxes in the rent. No definitive pattern was found in the spatial distribution of such buildings.

Parcel level land use data for the year 2006 were aggregated at the Census Dissemination area (DA) level to compute the percentage of different types of land use. Census 2006 data for place of work at the DA level was used to compute employment data.

As evident from the figure 10.4, almost all the office space buildings are located in proximity to a subway station, regional transit station, major intersection, or highway ramp. The proximity also influenced the variation in the asking rents. Accessibility related data were prepared by overlaying the street network, regional transit, and subway system GIS maps over the office space data. Buffers of 0.5, 1, and 2kms were constructed around highway interchanges, regional transit stations, and subway stations to determine the building’s accessibility to these locations. Euclidean distances from the CBD and regional centres were computed using GIS operations.

75% of office buildings were found to be within 2km of highway interchanges or the intersection of highways or major roads. 65% of office buildings were within 2km distance of a regional transit station, and 50% of office buildings were within 1km of a subway station.

Figure 10.1 shows the variation of asking rent with respect to distance from the CBD. The rent drops rapidly with increasing distance at first, but then rises again at about the distance of the regional centres around the old City of Toronto. The first rise occurs at about 20km, corresponding to the Mississauga downtown, and a second smaller rise occurs at about 45km, which corresponds to the Oakville downtown.
It can be concluded from the descriptive analysis that there is strong evidence of spatial variation in the asking rent. The accessibility to transport infrastructure is highly desirable, while quality of the space, location attributes, and local market conditions also contribute to the variation in the asking rent. Table 10.3 presents the list of variables and their summary statistics that were explored in the next section.

Figure 10.3
Average Asking Rent by Type of Building
Figure 10.4
Spatial Description of Office Space and Transportation Network
### Table 10.3 Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean/Proportion</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ask_rate</td>
<td>Asking rent per sq. ft. (2005 CAD)</td>
<td>13.48</td>
<td>4.86</td>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td><strong>Office Space Attributes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RType_G</td>
<td>Dummy: Gross Rent</td>
<td>6%</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Vac_Rate</td>
<td>Vacancy rate of the building</td>
<td>0.136</td>
<td>0.189</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>BType_A</td>
<td>Dummy: Type A Building</td>
<td>33.5%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>BType_B</td>
<td>Dummy: Type B Building</td>
<td>37.5%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Business Node Attributes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FS_Node</td>
<td>Millions of sq. ft. in the node</td>
<td>6.669</td>
<td>7.816</td>
<td>0.156</td>
<td>29.437</td>
</tr>
<tr>
<td>Off_Emp</td>
<td>Total employees in office sector in the neighbourhood (DA)</td>
<td>415.21</td>
<td>476.71</td>
<td>0</td>
<td>3680</td>
</tr>
<tr>
<td><strong>Accessibility and Location Attributes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TTC_05K</td>
<td>Dummy: Office building within 500m of a subway station</td>
<td>38%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>GO_1K</td>
<td>Dummy: Office building within 1km of a regional transits station</td>
<td>35%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Sub_HWY_1K</td>
<td>Dummy: within 1km of a highway intersection in suburbs</td>
<td>29%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>CBD_Dist</td>
<td>Distance from Central Business District in kms</td>
<td>13.97</td>
<td>11.9</td>
<td>0.01</td>
<td>51.57</td>
</tr>
<tr>
<td>CBD_Dist_In</td>
<td>Log of distance from Central Business District</td>
<td>1.899</td>
<td>1.566</td>
<td>-4.605</td>
<td>3.943</td>
</tr>
<tr>
<td>Airport</td>
<td>Dummy: Office building near Toronto Airport</td>
<td>4%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mississauga</td>
<td>Dummy: Office building in Mississauga downtown</td>
<td>2%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Oakville</td>
<td>Dummy: Office building in Oakville</td>
<td>2.2%</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Neighbourhood’s Land Use Attributes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IND_AP</td>
<td>Percentage of industrial land use in the neighbourhood</td>
<td>0.375</td>
<td>0.31</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>RES_AP</td>
<td>Percentage of residential land use in the neighbourhood</td>
<td>0.283</td>
<td>0.31</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Total observations: 1302
10.3 Model Structure

The hedonic analysis formulates the value of a differentiated good as a function of its utility-bearing attributes as follows:

\[
g(r) = f(Z) + \epsilon, \quad [10.1]
\]

where

- \( r \) = value of the differentiated good
- \( g \) = functional form of value
- \( Z \) = vector of the explanatory variables
- \( f \) = functional form of explanatory variables
- \( \epsilon \) = error term.

However, the functions \( g \) and \( f \) in equation [10.1] are generally not defined. Thus to investigate the functional form of the relationship between asking rent and the explanatory variables, the Box-Cox transformation was used. Box-Cox is a form of power transformation designed to stabilize the variance and improve the correlation between the dependent and independent variables (Box and Cox, 1964). It has frequently been used by economists to investigate hedonic price models for housing and other differentiated goods (Brennan et al., 1984). The power function is assumed to be of the form:

\[
r^{(\theta)} = \beta Z^{(\lambda)} + \epsilon, \quad [10.2]
\]

where

- \( \beta \) = vector of parameters associated with explanatory variables

\[
r^{(\theta)} = \begin{cases} \frac{r^{\theta-1}}{\theta}, & \text{if } \theta \neq 0 \\ \log(r), & \text{if } \theta = 0 \end{cases}
\]

\[
Z^{(\lambda)} = \begin{cases} \frac{z^{\lambda-1}}{\lambda}, & \text{if } \lambda \neq 0 \\ \log(Z), & \text{if } \lambda = 0 \end{cases}
\]

& \( Z \) should be strictly positive.

The \( \beta, \theta, \text{and} \lambda \) are estimated by maximizing the likelihood function of form (Kim and Hill, 1993):
where
\[ n = \text{number of observations} \]
\[ \text{SSR} = \text{sum of square of residuals at } \beta', \theta', \lambda' \]

To investigate the spatial variation in the asking rent, a series of tests and models including the Global Moran I test, the Anselin Local Moran I test, Geographically Weighted Regression (GWR), the hierarchical random effect model, and the semiparametric regression model were used.

The Global Moran I test measures the existence of spatial autocorrelation in the spatial data. Spatial autocorrelation is the correlation of an object’s attribute(s) in space with the neighbouring objects (Cliff, 1973; Kelejian and Prucha, 2001). The Moran index is computed using (Griffith, 1987):

\[ I = \frac{n}{S_0} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} r_i r_j}{\sum_{i=1}^{n} r_i^2}, \quad [10.4] \]

where
\[ r_i = \text{deviation of an attribute of object } i \text{ from mean} \]
\[ w_{ij} = \text{spatial weight between object } i \text{ and } j. \text{ Typically, the inverse of Euclidian distance is used} \]
\[ S_0 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \quad [10.5] \]

The Z-score for the statistics is:
\[ z_I = \frac{I - E[I]}{\sqrt{V[I]}}, \quad [10.6] \]

and
\[ E[I] = -\frac{1}{(n-1)} \quad [10.7] \]
\[ V[I] = E[I^2] - E[I] \quad [10.8] \]

The expected value of \( I^2 \) is computed by a series of equations that relate \( w_{ij}, w_{ji} \), and the second and fourth power of \( r_i \). For details, see Griffith (1987).
The Anselin Local Moran I test is used to investigate autocorrelation at the local level. It is computed for individual spatial objects using (Anselin, 1995):

\[
I_i = \frac{r_i - \bar{R}}{S^2} \sum_{j=1,j\neq i}^{n} w_{ij}(r_i - \bar{R}) \tag{10.9}
\]

\[
S^2_i = \frac{\sum_{j=1,j\neq i}^{n} w_{ij}^2 - \bar{R}^2}{n-1} \tag{10.10}
\]

where

\[
\bar{R} = \text{mean rent}
\]

The Z-score is computed in a similar way as equation [10.6]. For details, see Anselin (1995).

The Hierarchical random effect model is a useful tool to analyse spatial datasets having clustering effects. Such models are able to incorporate not only the fixed effects due to the attributes of the spatial object, but also the random and/or fixed effects at the cluster level (Raudenbush and Bryk, 2002). The Hierarchical model is defined by two sets of equations, one within the spatial cluster and one between the clusters. Let us assume that there are \( m \) clusters, where there are \( n_j \) observations in a cluster \( j \), \( X_{ij} \) represents the vector of attributes of the buildings and are fixed; and \( Z_j \) represents the vector of attributes of the cluster and varies across clusters. Then, at the building level, the regression equation can be defined as (Grafarend, 2006):

\[
r_{ij} = \beta_j + \beta X_{ij} + \epsilon_{ij}, \tag{10.11}
\]

where \( \beta_j \) is a function of cluster \( j \)'s attributes \( Z_j \), and is defined as:

\[
\beta_j = YZ_j + \delta_j \tag{10.12}
\]

So, equation [10.11] becomes:

\[
r_{ij} = \beta X_{ij} + YZ_j + \delta_j + \epsilon_{ij} \tag{10.13}
\]

Here \( \beta \) is the fixed effect parameter vector and \( Y \) is the random effect parameter vector. \( \delta_j \) and \( \epsilon_{ij} \) are assumed to be independently and identically distributed and normally distributed with zero means, while \( Y \) is only assumed to be normally distributed with zero mean. \( \beta \) and the variance of \( Y \) are then estimated using the restricted maximum likelihood (REML) method.
REML is particularly good at estimating unbiased variance and covariance for random effect parameters (Liao and Lipsitz, 2002). Moreover, it outperforms the conventional maximum likelihood estimation in the case of smaller sample sizes.

Geographically Weighted Regression (GWR) is a form of weighted least square regression of form (Charlton and Fotheringham, 2009):

\[ r(u) = \beta(u)Z + \epsilon(u) \]  
\[ \text{And} \]
\[ \beta(u) = (Z^TW(u)Z)^{-1}Z^TW(u)r \]

Whereas \( W(u) \) is the square matrix of weights at location \( u \) in the study area.

\[ W(u) = \begin{bmatrix} w_1(u) & 0 & 0 \\ 0 & \ldots & 0 \\ 0 & 0 & w_i(u) \end{bmatrix} \]

\( w_i(u) \) is computed by generating a buffer around each observation and fitting a normal kernel on the observation and its neighbours. As a result, \( \beta \) vector is estimated for each observation. GWR is an interesting approach to deal with the spatial autocorrelation issue.

In the case of semiparametric specification, a partial linear model was used that was of the form (Ruppert et al., 2003; Robinson, 1988):

\[ r = \beta Z + h(X) + \epsilon, \]

where
\[ Z = \text{matrix of explanatory variables that are linear in parameter} \]
\[ X = \text{matrix of nonparametric explanatory variables.} \]

To estimate the linear part of equation [10.16], the nonparametric effects from \( r \) and \( Z \) were removed using the double residual method (Robinson, 1988). \( r \) and \( Z \) were regressed as a function of \( X \) to estimate the \( E(r|X) \) and \( E(Z|X) \). The resulting residuals were then used to estimate the \( \beta \) in equation 16 using OLS regression. To estimate the regressor function for
nonparametric effects due to $X$, a normal kernel smoother was used, as described by Yatchew (2003).

### 10.4 Model Estimates

In the first phase of the analysis, an ordinary least squares regression model was used to test various explanatory variables related to office space quality and location. I also introduced land use, demographics, regional office markets, and accessibility to the transportation system in the analysis. Model 1 in Table 10.4 shows the final specification of this analysis. The magnitude and signs of the explanatory variables were found to be as expected and similar to those reported in previous studies of office space asking rents for similar cities in North America.

The spatial effect of distance from the CBD, as shown in Figure 10.1, was incorporated into the model by including the distance from the CBD and dummy variables for regional centres as explanatory variables.

In the second phase of the analysis, the functional form of the relationship between asking rent and the explanatory variables was explored using the Box-Cox transformation. Transformation was analysed for the following three cases: dependent only (model 2, Table 3), both dependent and independent (model 3, Table 3), and independent only (model 4, Table 3). In model 2 and model 3, the transformation of the dependent variable resulted in a square root function that was found to be statistically different from a linear or logarithmic form. On the other hand, the transformation of the independent variables resulted in a logarithmic form, which was found to be statistically different from a linear form. Due to the strict positivity restriction of the variables in the Box-Cox transformation, the transformation was only applied to the distance from CBD variable.

On the basis of AIC and BIC values one can only compare the model 1 with model 4, since due to the change in functional form of the independent variable, it was not possible to compare these models with models 2 and 3. The models, at this phase were thus compared based on the improvement achieved in the models’ goodness-of-fit. The best model fit was achieved using the independent-only transformation in model 4, which was thus selected as our base model for further analysis. By using a logarithmic function for the distance from the CBD, the t-
statistics for its parameter improved. This functional form is also a better representation of the observed spatial variation of rent, as shown in figure 1c. The office based employment density in the neighbourhood became highly insignificant (t-stat = 0.38) in this phase. Past studies on the asking rent have not reported employment density as an influential factor in their models. Moreover, the market variables, vacancy rate and stock of office space in the model, have already been included. It was thus decided to exclude office employment from further analysis.

As shown in Figure 9.1, the office space in the study area is agglomerated into 36 distinguishable business nodes/clusters. The possibility of local and/or global spatial autocorrelation of the asking rent with respect to these nodes was investigated using the global Moran I and Anselin Local Moran I tests. The Moran Index value was 0.41 with a Z-score of 10.74 and p-value of 0.000. In the case of the Anselin Local Moran I tests, 20% of the individual observations were found to have very high autocorrelation (Z-score higher than 1.96) with the neighbouring observations. These tests revealed that there is not only a high degree of spatial autocorrelation among the asking rents in the study area, but also that autocorrelation exists in the asking rent within the cluster.

To investigate the spatial variation in the asking rate a combination of spatial variables (distance from the CBD and regional dummies) and the random effects were used within a hierarchical structure. Model 5 in Table 10.4 shows the final estimates of the hierarchical model that treated 36 business nodes as the upper level and the buildings within these nodes as the lower level in the hierarchy. The dummies for Mississauga and Oakville were left in the model to account for the additional constant fixed-effect that was observed in the descriptive analysis. The hierarchical model without these dummies (not reported here), had a higher standard deviation in the constant term.

The parameter values of the explanatory variables did not change significantly compared to Model 4, but variables that were previously insignificant at a 95% confidence level became significant. In an overall test of significance, the AIC and BIC values improved significantly. Random effects in asking rent at the lower level due to vacancy rate, industrial land use, and constant term were explored. The hierarchical model with random effects was found to be significantly different than the linear model with the same explanatory variables.
The spatial autocorrelation issue in the asking rent led me to also investigate a geographically weighted regression (GWR) based hedonic rent model. The GWR model, which is not reported here, resulted in higher adjusted $R^2$, AIC, and BIC values, but is not very useful in the context of integrated land use, transport, and environment microsimulation. The GWR estimates parameter values at the individual observation level by fitting a Gaussian kernel about that location, resulting in a fitted surface for each parameter in the study area. Operationalizing and using this surface for the forecasting in microsimulation frameworks like ILUTE will not be feasible and is thus not much of a use in policy scenario testing. Moreover, computing the t-stats for parameter surfaces is not computationally feasible and is still a subject of current research (Charlton and Fotheringham, 2009).

Finally, the analyses focused on the spatial variation in asking rent by assuming it to be a pure function of distance from the CBD without assuming any functional form of the relation. This was achieved by using a generalized additive model (GAM), in which all the parameters were assumed to be linear, except for the distance from the CBD parameter, upon which no functional form was imposed. Rather, it was only assumed to be a well behaved function that is first and second order differentiable. The nonparametric effects due to distance from the CBD were first removed using the double residual method discussed in (Yatchew, 2003). The ordinary least squares method was then used to estimate the linear parameters. A Gaussian kernel was fitted to estimate the nonparametric effects due to the distance from the CBD.

Model 6 shows the parameter values for the linear part of the model, while Figure 10.5 shows the variation in the asking rent due to distance from the CBD. The AIC and BIC values for Model 6 are significantly smaller than for Model 5. Except for one independent variable, there is not much of a difference in the significance, magnitude, or signs of the linear parameters. Model 6 is able to capture the centrality of downtown Toronto, the nonlinear decrease in the asking rent with respect to distance, and the rise in values at the suburban commercial centres. The value of the intercept in Model 6 decreases, as the nonparametric part now is able to explain the spatial variation more effectively.
### Table 10.4 Hedonic Analysis of Asking Rent

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Model 1 Linear OLS</th>
<th>Model 2 Box-Cox Transform (Dependent)</th>
<th>Model 3 Box-Cox Transform (Both)</th>
<th>Model 4 Box-Cox Transform (Explanatory)</th>
<th>Model 5 Hierarchical Random Effect</th>
<th>Model 6 Generalized Additive Semiparametric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>t-stat</td>
<td>P-value</td>
<td>Coef.</td>
<td>t-stat</td>
<td>P-value</td>
</tr>
<tr>
<td>Const</td>
<td>10.81</td>
<td>20.14</td>
<td>0.000</td>
<td>4.443</td>
<td>30.43</td>
<td>0.000</td>
</tr>
<tr>
<td>RType_G</td>
<td>8.460</td>
<td>18.25</td>
<td>0.000</td>
<td>2.179</td>
<td>17.27</td>
<td>0.000</td>
</tr>
<tr>
<td>Vac_Rate</td>
<td>-1.380</td>
<td>-2.39</td>
<td>0.017</td>
<td>-0.296</td>
<td>-1.89</td>
<td>0.060</td>
</tr>
<tr>
<td>BType_A</td>
<td>4.237</td>
<td>14.08</td>
<td>0.000</td>
<td>1.185</td>
<td>14.14</td>
<td>0.000</td>
</tr>
<tr>
<td>BType_B</td>
<td>0.396</td>
<td>1.51</td>
<td>0.132</td>
<td>0.152</td>
<td>2.02</td>
<td>0.044</td>
</tr>
<tr>
<td>FS_Node</td>
<td>0.118</td>
<td>6.56</td>
<td>0.000</td>
<td>0.028</td>
<td>1.89</td>
<td>0.006</td>
</tr>
<tr>
<td>Off_Emp</td>
<td>0.0002</td>
<td>0.86</td>
<td>0.390</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TTC_05K</td>
<td>1.986</td>
<td>5.63</td>
<td>0.000</td>
<td>0.566</td>
<td>5.90</td>
<td>0.000</td>
</tr>
<tr>
<td>GO_1K</td>
<td>0.860</td>
<td>3.40</td>
<td>0.001</td>
<td>0.225</td>
<td>3.26</td>
<td>0.001</td>
</tr>
<tr>
<td>Sub_HWY_1K</td>
<td>0.269</td>
<td>11.00</td>
<td>0.271</td>
<td>0.079</td>
<td>1.00</td>
<td>0.300</td>
</tr>
<tr>
<td>CBD_Dist</td>
<td>-0.040</td>
<td>-2.74</td>
<td>0.006</td>
<td>-0.010</td>
<td>-2.47</td>
<td>0.014</td>
</tr>
<tr>
<td>CBD_Dist_In</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Airport</td>
<td>0.922</td>
<td>1.55</td>
<td>0.121</td>
<td>0.295</td>
<td>1.84</td>
<td>0.068</td>
</tr>
<tr>
<td>Mississauga</td>
<td>2.865</td>
<td>3.42</td>
<td>0.001</td>
<td>0.833</td>
<td>3.65</td>
<td>0.000</td>
</tr>
<tr>
<td>Oakville</td>
<td>2.859</td>
<td>3.71</td>
<td>0.000</td>
<td>0.817</td>
<td>3.90</td>
<td>0.000</td>
</tr>
<tr>
<td>IND_AP</td>
<td>-1.096</td>
<td>-2.07</td>
<td>0.039</td>
<td>-0.261</td>
<td>-1.81</td>
<td>0.071</td>
</tr>
<tr>
<td>RES_AP</td>
<td>-1.245</td>
<td>-2.37</td>
<td>0.018</td>
<td>-0.287</td>
<td>-2.01</td>
<td>0.045</td>
</tr>
<tr>
<td>st_dev(const)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>st_dev(ind_ap)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>st_dev(Vac_Rt)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Θ</td>
<td>1</td>
<td>0.50</td>
<td>0.50</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>λ</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSR</td>
<td>15929.07</td>
<td>1873.443</td>
<td>1016.855</td>
<td>15594.10</td>
<td>13355.192</td>
<td>12129.084</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.457</td>
<td>0.439</td>
<td>0.450</td>
<td>0.468</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>AIC</td>
<td>3310.623</td>
<td>2277.893</td>
<td>1983.054</td>
<td>3302.368</td>
<td>3221.587</td>
<td>3175.122</td>
</tr>
<tr>
<td>BIC</td>
<td>1327.485</td>
<td>288.439</td>
<td>2.962</td>
<td>1320.531</td>
<td>1242.449</td>
<td>1189.893</td>
</tr>
<tr>
<td>χ²</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>697</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Prob &gt;= χ²</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The spatial analysis of the residuals revealed that for buildings in business nodes near the CBD (Downtown West, Downtown North, Bloor & Yonge, and St. Clair & Yonge), Model 5 resulted in higher values of residuals than Model 6. This is due to the fact that Model 6 was able to capture the premium associated with the proximity to CBD better than Model 5.

![Figure 10.5](image)

**Figure 10.5**  
Asking rate estimate as a function of distance from CBD

### 10.5 Discussion

#### 10.5.1 Effect of attributes of space on the asking price

Type-A buildings that are highest quality buildings have the highest asking price. Compared to Type-C buildings, floor space in Type-A buildings has an asking rent that is about CAN $4.5 higher.

Type-B buildings that are lower in quality as compared to Type-A, but are higher in quality than Type-C have the second highest asking price. Compared to Type-C building, floor space in Type-B buildings has an asking price that is about 60 cents higher.
Buildings that offer gross rents have CAN $8.5 higher asking rents compared to the buildings that don’t.

10.5.2 Effect of the market conditions on the asking price

The model suggests that the level of office space built-up in a business node has a positive impact on the asking rent. For every million square feet of office space in the business node, there is an increase of about 7 cents in the asking rate. Elgar, et al. (2009) have pointed out that the office based firms in the same study area tend to locate in areas with higher build-up of office space. The higher demand of office space in such areas thus results in the higher asking rents.

The vacancy rate of the building has a negative effect on the asking price for office space available in that building. For every 10% rise in the vacancy rate in a building, there is an average decrease of about 15 cents in the asking rent. It should also be noted that the standard deviation in the parameter due to the random effects is about the same order of magnitude. Our hypothesis is that this is due to the fact that there is no reference in the estimated models to the normal vacancy rate of the business nodes. As discussed above, Rosen (1984) pointed out that every office market has a normal vacancy rate and variation from it is not linear with respect to rent. Thus, it would have been more appropriate to use the deviation of the current vacancy rate of the building from the normal vacancy rate of its business node in the model. Unfortunately, due to the limited cross-sectional nature of the dataset used in the models, it was not possible to compute the natural vacancy rates for the business nodes in the study area.

10.5.3 Effect of land use on the asking rent

The asking price decreased with an increase in the percentage of residential land use in the neighbourhood (Census Dissemination Area, DA). For every 10% rise of residential area there is a 13 cents decrease in the asking rent of the floor space.

Similar to the effect of increase in residential area, the increase in the industrial area in the neighbourhood (DA) causes a decrease in the asking price. The rate of the mean decrease in the asking price with respect to a 10% increase in the industrial area in the DA is 13 cents. This effect varies across the business nodes with a standard deviation of 7 cents.
Commercial area was not used as an explanatory variable because it is found to be correlated with the total office floor space in the business node.

10.5.4 Effect of accessibility on the asking rent

Office buildings that are located within 500 meters of a subway station were found to have higher asking rents compared to other buildings. A premium of CAN $1 is added to asking rate for such office space. Office based firms prefer to locate near subway stations to offer higher accessibility to their employees (Vahaly, 1976; Ihlanfeldt and Raper, 1990; Shukla and Waddell, 1991). This preference results in higher demand in the market for office space located in the vicinity of the subway station and thus increases the asking rent.

Vicinity to regional transit stations also increased the asking price. Office buildings located within 1 km of a regional transit station have an asking rate that is higher by 63 cents.

In the suburbs, the vicinity to highway interchanges was found to result in increased asking rent. Proximity to highways has been found to be an important factor in the location decision of office firms (Elgar et al., Vahaly, 1976; Ihlanfeldt and Raper, 1990; Shukla and Waddell, 1991). This generates a higher demand for office buildings located near highways, thus resulting in a higher asking price.

With the increase in distance from the CBD, the asking price was found to generally decrease exponentially. This is consistent with Elgar, et al. (2009). Despite the rapid development of suburbs such as Mississauga, Oakville, Vaughan, Brampton, and Burlington, downtown Toronto remains highly in demand.

Mississauga downtown was found to have higher asking rates compared to locations like Scarborough that are approximately the same distance from the CBD. This increase was found to be CAD 2.86. Mississauga downtown has grown through a rapid development in the past 20 years and has attracted various types of businesses.

A similar variation was observed for Oakville. The asking price in Oakville was found to be CAD 2.62 higher than other locations situated at the same distance from CBD.
10.6 Concluding Remarks

The major contribution of this study towards the hedonic analysis and model structure for office space rent comes from its ability to incorporate the variation resulting from clustering and heterogeneity across space within the model structure. In terms of application, this study analysed the rent at a highly disaggregate level in comparison to the previous study done by Mun and Hutchinson (1995) of office space rent in the GTA. This chapter, not only analysed the effects of accessibility, but also looked into the effects of quality, sub-market conditions, and location characteristics on the office space rent.

Despite the sprawling nature of the study area and existence of rapidly developing regional centres, downtown Toronto remains a highly desirable location with the highest rents. Accessibility to transport infrastructure both downtown and in the suburbs has a high impact on asking rent. Office buildings that are located within 500 meters of a subway station were found to have premium of CAN $1 added to their asking rents. Office buildings located within 1km of a regional transit station have an asking rate that is higher by 63 cents. In the suburbs, the proximity to highway interchanges was found to result in increased asking rent. The effects of local market conditions were found to vary across the study area. Our analysis only explains about 50 percent of the variation in the price, which is comparable to similar past studies.

The analysis was able to include the market indicators as explanatory variables in the models, but was not able to bring in their lagged effects. The dataset that was used here is only a cross-sectional dataset for year 2005. I intend to explore the lagged effects when a temporal data of office space asking rent is available.

The models developed in which chapter are intended to be used in the operationalization of the office space market in ILUTE. The office space market will be based on the conceptual framework for built space markets presented in chapter 2. The asking rent model will be used to initiate the rents for active office space in the market. The clearing mechanism will then search for the transaction prices in the vicinity of the asking rent. The clearing mechanism will be similar to the one suggested in chapter 5. However, in case office space the clearing mechanism will have to take care of the amount of floor space in addition to the transaction price and leaser. Note that the suggested process here is based on the assumption that the office space market is
also a price formation market, just like the housing market. There is a lack of data at this point in time to verify this assumption, but the changes in new office space supply and the temporal changes in the rents (Figure 9.10) seems to suggest that this is, indeed, the case.
PART IV: CONCLUSION
CHAPTER 11
SUMMARY, CONCLUDING REMARKS, AND FUTURE RESEARCH DIRECTIONS

11.1 Summary

This dissertation contributes towards two very important, but rarely investigated dimensions of the built space evolution, i.e. *markets* and *decisions*—thus filling some very important gaps in the built space and integrated land use and transportation literature. In the first part of the dissertation, a generic built space modelling framework within the ILUTE microsimulation modelling system was proposed. The framework defines the role of supply and demand modules, their interaction with each other in the market and various other modules that are associated with the evolution of built space supply.

In the second part of the dissertation, an innovative market-disequilibrium based microsimulation modelling formulation is developed for the built space markets, in which price formation is completely endogenous. Using random utility and game theory, the theoretical formulation developed here not only provides an enriched framework to represent the market characteristics and detailed agents behaviour, but is also readily operationalizable in a microsimulation modelling system like ILUTE.

Sellers and buyers enter and leave the active market in which individual level clearing is continuously going on. Sellers while entering the market, set an asking price on the built space which represents their perception of the expected profit. Asking price, among other factors, is mainly influenced by sellers’ knowledge of the market. Buyers, who are getting active in the market, form their choice set based on their level of information about the market. Social network, work location, previous location, etc. may play key roles in formation of their choice sets. Sellers are interested in maximizing the profit by selling their built space in the market, while buyers are interested in maximizing the gain in their utility from the available choices. All the agents in the market are assumed to have limited information about the market, based on which sellers set the asking price for the built space, while buyers form their choice set. Agents are also assumed to be non-cooperative and interested in maximizing their individual profit/utility only.
With these characteristics of the market and behavioural representation of the agents, a micro-equilibrium condition is proposed that conforms to the existing consumer choice and production theory and guarantees a solution for the clearing ‘game’. Under the proposed condition, the clearing of individual built space is achieved by forcing the sum of probabilities of selection by the bidders (representing the demand) to unity (representing the supply). The price level that can achieve this condition determines the transaction price and the bidder with highest probability of selection becomes the buyer at this transaction price.

The mathematical representation of the resulting game becomes a polynomial root finding problem. The degree of this polynomial equation for a built space is the same as the number of bidders that are interested in it, representing the maximum each bidder is willing to pay if everything else remains the same. Asking price plays an important role here as it represents the seller’s expectations. The clearing process thus is guided, so that it only looks for the potential transaction prices in the vicinity of the asking price. For this purpose, a two-level directed and constrained root finding procedure is developed that searches for only those roots of the bidding polynomial that are within a certain range of values. The procedure may not return any root—in that case, the clearing attempt fails and the participating agents (both sellers and buyers) re-evaluate their options—including the option of leaving the market. If a single value is returned then that becomes the transaction price and the bidder, with highest probability of selection, becomes the successful buyer. In case of more than one values, a randomly selected value is chosen from the set.

As an application of the proposed framework, the owner-occupied housing market was developed and operationalized within the ILUTE microsimulation system. Based on the initial prototype developed by Salvini, a comprehensive software design and development effort was undertaken, resulting in a full scale operational version, i.e. ILUTE v1.0. Each simulation year, households decide on getting active in the market that may result in an increase in the active housing stock in ILUTE simulation. Builders decide on building new housing space that also results in increase of the active housing stock. The clearing mechanism works in a quasi-monthly time step. At the start of each time step, buyers and sellers may enter or exit the market. During the quasi-month, dwellings are individually cleared and based on the result of clearing, the market is updated. At the end of the last quasi-month of the year, unresolved dwellings and
households are rolled over to the next year. The full scale ILUTE simulation runs with the entire population of the GTA (4.2 million people in 1986) from 1986 to 2006. The results outputted by the simulation are validated using the historic data from various sources. The simulation may take 7 to 10 days and currently, it is highly demanding, both in terms of memory and computational power.

In the third part of the dissertation, the decisions aspect of the built space evolution was investigated. In particular, builders’ various decisions that are involved in the process of supply of new built space and the valuation decision were investigated. A multidimensional decision modelling approach for the when, where, what type, and how much decisions by the builders was proposed which models all these decisions in a single consistent framework. The proposed framework, not only is based on the builders’ behaviour in terms of risk and profit maximization, but also captures the interplay between the various dimensions of the decision making that is going on in the new built space supply.

The problem is posed as a case of modelling the selection of a choice bundle by the builder. Some key concepts were borrowed from random utility based large scale demand models. The econometric formulation uses hedonic functions for the expected revenue and cost so as to determine the profit function for the builders. The resulting mathematical formulation was reduced to a constrained optimization problem that maximizes profit for a builder. With the help of Lagrangian and Khun-Tucker (KT) first order conditions for optimal allocations, a function was developed for the error terms. The error terms were assumed to be jointly normally distributed and a likelihood function for the selection of a bundle of type of built spaces and the associated quantities was derived using this assumption.

The parameter estimation from the resulting likelihood function involved evaluation of a higher dimensional integral. A Markov Chain Monte Carlo procedure was developed that used Metropolis Hasting algorithm and Gibbs sampling, so as to estimate the distributions for the parameters. This estimation process was developed and implemented in R statistical language and has a very generic design. It can be used to estimate models that are based on the proposed framework, for built space evolution datasets of various types.
The proposed modelling framework was applied to estimate a model for new supply of office space in the GTA from 1986 to 2005. The model estimates indicated a high preference for Type A office space, followed by B and C. The phenomenon of spatial inertia was visible for Type A and Type B office space. The existing trend in the market rents and vacancy rates played an important role in all types of office space. Other important factors that influenced the supply were availability of construction workers, wage rates, and construction costs per square foot of the office space.

Another dimension of decision making that was investigated in this dissertation is the valuation of office space. A set of hedonic models were developed to model the asking rent for office space in the GTA. The form of hedonic function was estimated using the Box-Cox transformation. The phenomena of clustering and spatial heterogeneity that are usually ignored in existing literature were thoroughly investigated using hierarchical and semi-parametric models. A strong influence due to the quality of the office space in the building on the asking rent was observed. The estimated parameters indicated that the market conditions were important in determining the rent levels. The neighbourhood effects were also visible. Good accessibility to the highway intersections, subway stations, and regional public transit stations had a strong positive impact on the asking rents. The hierarchical model was able to capture the within and among clusters variation in vacancy rate on the asking rent. The semi-parametric model was to capture the spatial trends that were visible in the rent levels as the distance from the CBD increased.

11.2 Concluding Remarks

For the better understanding of the spatio-temporal evolution of population and built space within microsimulation based urban systems in general, and the housing market in particular, it is essential that we move away from oversimplifying, strong market level equilibrium based assumptions and focus our efforts on moving forward research that is based on disequilibrium. Disequilibrium based approaches can represent the markets in a more comprehensive manner; they capture the heterogeneity in the agents’ behaviour in a flexible and more detailed fashion; and can exploit the capabilities of the microsimulation approach better. In recent years there have been very few attempts that are based on disequilibrium, but at best these approaches can be
regarded as quasi-disequilibrium based clearing processes (e.g. market clearing in UrbanSim). The major weakness of such models is their inability to have a completely endogenous price formation mechanism. In these integrated modelling systems, the price formation mechanism is usually an exogenous hedonic process that is coupled to the clearing process.

The proposed clearing process is a significant step forward in terms of developing a truly disequilibrium based approach as: a) it represents the market characteristics and agents behaviour more comprehensively; b) it has a completely endogenous price formation mechanism that is the result of supply and demand interaction in the market; c) it is independent of the type of models used for location choice, mobility decisions, asking price, and supply of new built space; d) it is readily operationalizable for full scale urban systems microsimulation; and e) it is completely integrated with other modules, process, and aspects of urban systems.

The market clearing process is able to represent both buyer- and seller-driven markets. The price fluctuation based on the differences in demands in terms of time, space, and type of dwelling can be represented. The change in the choice set influenced by the shortage of certain type of dwellings or dwellings in certain neighbourhoods is also reflected in the clearing process. This dissertation was able to investigate some basic properties of the clearing game, but in future I intend to investigate these properties further and develop a formal mathematical argument. While a full population based 20 years historic microsimulation run is a significantly big step forward, due to the current computational time and memory constraints, this dissertation presented only a preliminarily validation and analysis of the owner-occupied housing market simulation result. This effort is first of its kind, but a more systematic validation of the simulation results, interactions of various modules, and recalibration of models used in the process is much needed.

For the size of population it simulates, the operationalized microsimulation is relatively fast, but there is a significant room to further speedup. Currently, the market clearing process takes approximately 70% of the processing time during an ILUTE microsimulation run. This is understandable, as each year the owner-occupied market in ILUTE simulation handles the clearing of around 50,000 dwellings. This may require more than one clearing attempt for each
dwelling, price search and adjustments, repeated probability evaluations, agent’s decision processing in market, choice-set updates, and handling of the transaction process.

The multidimensional decision modelling framework for new built space supply, proposed in this dissertation is unique in the sense that it models the when, where, what type, and how much decisions by the builder in a single consistent framework and also incorporates the interplay among these decision dimensions. The framework allows for any spatial and temporal scale. It can be used to estimate models for various different types of built spaces. The careful specification of the model’s structural parameters allows for corner solutions (zero quantity selected for a type of built space, at a location, at certain time). The behaviour of a builder as decision maker is explicitly incorporated in the model structure. The normally distributed error assumption ensures that phenomena like spatial inertia and spatial patterns of dominant built space are captured by the model structure. By using the hedonic formulation for the profit, this model avoids the necessity to assume any type of production and cost function for the builders. The specifications of such functions are first of all not very well defined for builders. Moreover, the function may vary with the type of the builders. To our knowledge this is the first modelling framework that models these decision dimensions in a single seamless framework.

The office space supply model estimated using the proposed modelling framework is a significant step in modelling the office space supply in the Greater Toronto Area. It gives a deeper insight into the dynamics of the office space evolution in the GTA. Business nodes are modelled as independent sub-market that may be influenced from each other. The model estimates suggests that the builders are risk takers. Though, there is much room for improvement, the model in its current state can be great tool for the planners and policy makers to understand the office space market in the GTA.

The office space asking rent model contributes to the valuation literature by modelling the rent at building level and explicitly incorporating the clustering and heterogeneity with respect to space, within the models. The model not only incorporates the quality and vacancy rate of the building in the model, but also captures the effects of location, accessibility, and market condition in the business node on the rent levels of space in an office building.
11.3 Future Research Directions

While this dissertation is an important step forward in our collective understanding of the built space evolution, it has also triggered many new unanswered research questions and exposed various short-comings in our understanding of built space evolution. On the positive side, it opens up abundant opportunities to extend and improve the state of research in the built space evolution. Various directions of extending and improving the research completed in this dissertation are as follows.

11.3.1 Markets

\textit{a. Agents’ Heterogeneity in the Market:} The behaviour of sellers and buyers in the market varies, based on their socio-economic characteristics and various other factors. Moreover, their level of information about the options and market conditions may vary. For instance, a builders’ attitude towards profit maximization from a batch of dwellings it is introducing in the market will be different from a household selling the dwelling it is currently living in. Or, a new immigrant looking to buy a dwelling in the market will be less informed than a household that has owned and sold many dwellings in the same urban area for past 20 years. Similarly, a household who has bought a new dwelling, but still has to sell or rent its current dwelling, will behave differently than one who has already sold the current dwelling and is still looking for a new dwelling to buy or rent. For the correct representation of the behaviour, it is very important to incorporate the heterogeneity of the agents in the market clearing models. To do so, I will need better datasets that directly or indirectly observe this heterogeneity.

\textit{b. Interaction between Rental and Owner-Occupied Markets:} In the existing implementation of the markets within ILUTE households initially decide on whether to participate in the rental or owner-occupied housing market. After that the two markets are cleared independently without any agents or information flow between them. In reality this is hardly the case. The two markets are much more interconnected. If a household cannot find a dwelling that it can afford to buy, it may start looking to rent a dwelling. Similarly, a household who was initially looking for a dwelling to rent may be encouraged by the buyer-driven trends in the owner-occupied market to consider buying a dwelling. A household may be active in both markets at the same time, looking for the most affordable option in both markets. To represent these interactions
between the two markets, we will have to run the two clearing processes in a parallel fashion, with information and agent flow between the two markets. The planned parallel computing on multi-core shared memory architecture will be perfect for the implementation of such a conceptualization.

c. Mathematical Properties of Market Formulation as a Game: While this dissertation has explored only some basic properties of the game theoretic formulation of the proposed price formation market, a formal and more detailed exploration and mathematical inference is needed. Such an effort will help us in further improving and extending the proposed market formulation.

d. Choice Set Generation: A correct representation of the choice set, which the active households consider in the market, is extremely important for capturing the behaviour of these households as buyers. In the current version of ILUTE microsimulation, a rule-based choice set generation process is implemented. These rules are derived from the findings reported in the retrospective survey of mobility decisions by the households in the GTA by Pushkar (1998).

A more sophisticated model, however, is needed that considers the heterogeneity among agents during the process of their choice set generation. For instance, a choice set of a newly married working couple will be different from a choice set of a household with two school-age children and a housewife. Such models can be based on more detailed rules or may have an econometric origin. This will require collection of better datasets that not only observe the dwellings that were considered by the buyer, but also the one on which it bid and failed. The information about the failed bids will help in improving the clearing process as well.

e. Asking Price Model: Asking price acts as the reference point in the clearing process. It is thus very important that the asking price model is forecasting the price correctly and capturing the behaviour of the decision maker (seller) properly. The current implementation of the model seems to overestimate the prices in general, and specifically in the case of single detached dwellings. This results in higher than expected transaction prices. A re-evaluation and calibration of the asking price model is needed so as to better forecast the prices.
f. Representation of Income and Mortgage: The existing models in the housing market of ILUTE simulation are essentially based on household total wages. While in general, wage gives a strong indication about the economic status of a household, it may or may not represent the total income and wealth of the household. Another short-coming of the existing market model is that there is no representation of mortgages in the models. Both these shortcomings are not due to inability to incorporate them in the models, but due to the unavailability of the data on total wealth and mortgage information. It is essential that future mobility survey efforts have at least some representation of these dimensions. Such an update will greatly improve our ability to analyse various policies targeted towards social equity and effects of various mortgage schemes offered by the banks.

g. Systematic Testing: The ILUTE modelling framework is a highly complex system and its complexity is exponentially increasing with the constant addition of new modules. The interactions between the various modules within ILUTE vary in terms of scale and lag, both spatially and temporally. The level of complexity and interactions make the testing and validation of ILUTE and its modules, a daunting task. The initial results reported in this dissertation and by Miller et al. (2010) are encouraging, but a more systematic testing and historic validation of the results is needed. By constraining the degrees of freedom and module by module testing, we can focus on the few major dimensions of the results. The testing should explore the effects of changes in one module on other modules, the associated lag, and scale during the simulation. Proper statistical designs and techniques (e.g. meta-model, response surface, compact model, and emulator) are needed for the sensitivity analysis of the ILUTE microsimulation. This may require a whole graduate level dissertation work focused only on the testing and validation.

h. Speedup of ILUTE Microsimulation: The current run-times and memory requirements for ILUTE and some of its components (including housing markets) are excessively high. It may take more than a week on a 2.4GHz processor, and 8GB of memory space to run a 20-year ILUTE simulation with full population of the GTA. A drastic decrease in the run-time and to a lesser extent in the memory usage is required to make it useful in planning and policy analysis.

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21 In recent past a complete Doctoral Dissertation was dedicated to validation and calibration of a travel demand model, MatSim.
The memory usage in ILUTE software is already optimized and with the availability of cheaper and larger memory hardware with time, the memory requirement can be overcome.

The run-time issue however, is more serious as it is related to the current design of the ILUTE software rather than availability of the processing power of existing hardware. These days, the focus in the processor development is on multi-core parallel shared memory systems, rather than on the speed of a single core. The ILUTE software on the other hand is a serial implementation requiring a very fast single core processor. To take full advantage of the available power in multi-core machines, the ILUTE software has to move towards parallelization.

The parallelization of decision making is trivial as it doesn’t require access to other objects in memory (100 households can decide in parallel if they want to get active in the housing market). The real challenge comes from the parallelization of the markets. In the markets, households are interacting with each other through the changes that are taking place in choice-sets and bidder-sets. Maintaining these interactions and changes in the memory in a synchronized manner requires sophisticated software design considerations. As a prototype we have developed and tested a parallel version of both the price-taker and price-formation markets (Farooq et al., 2010a; Lu, 2010) and have achieved a speedup of as high as 30 times. A full scale implementation of these speedups in ILUTE software is the next step forward.

11.3.2 Decisions

a. Better Representation of the Supply Process: Supply of a finished built space involves various decision making agents (land owner, land developer, builders, and contractors), various markets (land market, developed land market, and built space market), and steps (permits, starts, construction, finish). While a comprehensive representation of all these agents, markets, and steps within urban systems like ILUTE is necessary to capture the dynamics in the built space supply, the existing representation is less than adequate. Much effort is needed in terms of surveys and modelling to better understanding and represent of these dimensions of supply in urban systems modelling frameworks.
b. Builder’s Behaviour: The builder is the only agent that is most commonly considered as the decision maker in the context of supply modelling in integrated urban model systems. The model estimated for the supply of new office space in this dissertation assumed a homogenous building industry. It is a step forward in the sense that the risk behaviour of the builder is explicitly modelled, but the supply models need to consider the heterogeneity within builder agents. The data of builders in the GTA indicates the evidence of heterogeneity in terms of size among the builder. Large-sized builders may exhibit more dominating behaviour and may influence the market more. Similarly, the small- and medium-sized builders may exhibit different behaviour. It is essential that our models are able to capture these differences in the behaviour of builders. Better datasets will be needed for understanding and modelling the heterogeneity in the builder’s behaviour.

c. Error Structure and Operationalization: The properties that are assumed for the error structure used in the proposed built space supply model have good theoretical foundations, but more exploration in terms of mathematical inference is needed, especially the fact that it is assumed that the error terms in non-Hicksian goods are the differences from the error term of Hicksian good. The proposed equation for the profit function is nonlinear and will have consequence on this assumption.

In terms of operationalization, I will need to have a simpler assumption for the error structure (e.g. Gumbel distributed independent errors). This will give a closed form solution and avoid the computational overhead associated with evaluation of the integration term in existing model.

d. Estimation Process and Model’s Goodness-of-Fit: A Bayesian approach was developed to estimate the new supply model in this dissertation. Bayesian approach gives a better control of the parameter estimation process, in terms of identification problem, but is unable to give the goodness-of-fit estimates for the resulting model. In this regard, a simulation based test is required so as to determine the goodness-of-fit for the model estimated for supply of new office space. This will require a dataset of office space supply from the duration before or after the duration for which the model is estimated. Another option will be to develop a maximum likelihood based estimation process for the model estimation. For such a process, I will need to
adapt/develop a set of procedures that can evaluate the first few order of derivatives for the likelihood function.

e. Applications of Built Space Evolution Model: The proposed modelling framework is a general multidimensional decisions modelling framework for the builder’s behaviour. This dissertation applied the model only to the supply of new office space. In future I intend to apply the framework to estimate models for supply of other types of built spaces, especially the supply of new residential space.

f. Dynamic model of Asking Price: In the context of urban systems modelling, it is important that the dynamics in terms of time is captured in the models of built space. The asking price model of office space rent developed in this dissertation is dynamic in the sense that it uses current market conditions to explain the rent levels, but it is only based on a cross-sectional dataset. To capture the detailed dynamics in terms of lags, it is important to have a longitudinal dataset with a reasonably long duration. Such a dataset will also enable us to determine the natural vacancy rate of various clusters. That can then be used to determine the deviation in the vacancy rate from the natural rate to explain the rent levels.

g. Hedonic Model Incorporating Clustering and Heterogeneity: This dissertation used a hierarchical model to explain the clustering and a generalized additive semi-parametric model to explain the spatial heterogeneity in the asking rent of the office space in the GTA. A logical step forward in this regard was to merge the hierarchical model in the generalized additive modelling framework to have one model explaining both clustering and heterogeneity. While the mathematical formulation for such a model was successfully derived, this dissertation was not able to estimate a model that properly converges with the available dataset. A longitudinal dataset with more observations and variation will be needed to estimate such model.

h. Operationalization of office market model: The development of asking rent model and the model of new office space supply are two very important steps forward in operationalizing the office market in ILUTE modelling framework. The (re)location choice model and mobility model for small to medium office firms has already been developed by Elgar (2007). A small scale microsimulation was also developed to test the goodness-of-fit for the location choice decision of small to medium sized office firms in the GTA.
As a next step, a model for office space market clearing is needed. The price-taker market clearing process proposed by Farooq et al. (2010a) and price-formation market clearing process proposed in this dissertation can be adapted for the office space clearing mechanism. It is not very clear at this point, however, if firms are price-takers or not. Moreover, the clearing process in case of office space may not just involve the determination of rent, but it may also have to determine the quantity of office space (sq. ft) that is actually transacted. Another important aspect in terms of operationalization of a new office space model is that a separate distribution model for office space within a business node will be needed. This distribution model can be similar to the one that is used in the operationalization of housing supply in the current version of ILUTE.

11.4 Final Remarks

Analysis of urban transportation system is a complex process that involves understanding: the behaviour of various stakeholders, complexity of the system, and two-way interaction of transportation system with various other urban engineering systems (land use, environment, energy, etc.). Furthermore, transportation research requires building specialized tools that incorporate these dimensions so as to perform in-depth policy analysis, comprehensive scenario testing, and that have the ability to answer what-if questions rather than simply extending the trends. In this context, this dissertation adds some important components to the ILUTE modelling framework. The modelling approach taken here differs substantially from commonly used approaches in integrated land use transportation modelling and also advances the state-of-the-art by adding complexity and providing behaviourally richer specifications.

In recent years, focus of transportation planning in the Greater Golden Horseshoe Area (GGHA) has gradually shifted from local municipalities- to regional-level integrated approach. A regional transportation plan and accompanying smart growth plan are developed for maintaining a sustained urban growth and increasing the competitiveness of the region. In this context, it is anticipated that the sophisticated models and integrated microsimulation tool developed for the region, in this dissertation, will be highly useful for the policy and decision makers in formulating and scenario-testing sustainable integrated urban policies. A joint effort between academia and governmental organizations is needed for the large-scale adoption of ILUTE for policy analysis purposes.
REFERENCES


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163


APPENDICES
APPENDIX A

IMPORTANT ILUTE CLASSES

This appendix discusses major classes that are design and implemented in the software ILUTE v1.0.

A.1 Class Representation of Household, Family and Person

Census population agents in ILUTE are represented by the *Household*, *Family*, and *Person* classes. These classes implement the attributes and decision making behaviour of the respective census population agents. Figure A.1 shows the relationships that exist among these three classes, while Figures A.2, A.3, and A.4 show their relationships with other ILUTE classes. The existence of a *Person* class object is only associated with a *Family* class object or a *Household* class object. A *Family* class object can only exist if there are at least two or more associated *Person* objects. Similarly, a *Household* can only exist if there is at least one associated *Family* or *Person* object.

Figure A.1

*Household, Family, and Person* Class Diagram – Inter-Relationships
Figure A.2
*Household* Class Diagram – Relationships

Figure A.3
*Family* Class Diagram – Relationships
Figures A.5, A.6, and A.7 show the complete list of attributes and operations of household, family, and person classes, respectively. All the objects of these classes are uniquely identified by their identification number (ID) in the simulations. A list of the decision making operations implemented for these classes is presented in Table A.1.
Table A.1 List of Decisions Implemented in the Population Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>Decision List</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household</strong></td>
<td>- Mobility</td>
</tr>
<tr>
<td></td>
<td>- Residential search</td>
</tr>
<tr>
<td></td>
<td>- In migration</td>
</tr>
<tr>
<td></td>
<td>- Out migration</td>
</tr>
<tr>
<td></td>
<td>- Auto ownership</td>
</tr>
<tr>
<td><strong>Family</strong></td>
<td>- Divorce decision</td>
</tr>
<tr>
<td><strong>Person</strong></td>
<td>- Birth</td>
</tr>
<tr>
<td></td>
<td>- Marriage</td>
</tr>
<tr>
<td></td>
<td>- Education</td>
</tr>
<tr>
<td></td>
<td>- Change of Job/retirement</td>
</tr>
</tbody>
</table>

Household

- nextId : int
- myHouseholdType : EHouseholdType
- myHeadId : unsigned int
- myDwellingUnitId : unsigned int
- myActiveInHousingMarketFlag : bool
- timeInDwell : unsigned short
- myPersonIdList
- hhldSizeDesc : bool

+ processResidentialMobilityDecision(in d : SimulationDate) : void
+ performResidentialSearch(in availableDwellings, in choiceSet, in choiceSetSize : int) : void
+ computeNonPriceUtility(in did : int) : long double
+ computePriceUtility(in did : long) : long double
+ processOutMigrationDecision(in d : SimulationDate) : void
+ processDivorceDecision(in d : SimulationDate) : void
+ processAutoOwnershipDecision(in d : SimulationDate) : void
+ Setters...()
+ Getters...()
+ AddToLists...()
+ RemoveFromLists...()

Family

- nextId : int
- myHouseholdId : unsigned int
- myPersonIdList

+ Setters...()
+ Getters...()
+ AddToLists...()
+ RemoveFromLists...()
In the current version of ILUTE, space is represented by three levels of class hierarchy. The highest level of abstraction is defined by the \textit{SpatialObject} class. Every \textit{SpatialObject} class object has a shared aggregation with a \textit{Location} class object. \textit{Location} class objects can maintain coordinates in any coordinate system. \textit{Area} class is defined in the second level of the hierarchy. In the lowest level of the hierarchy, two types of area-based classes, the \textit{CensusZone} and the \textit{TTSZone} classes, are defined. Note that by maintaining these levels of spatial class hierarchy, in the design, it enables the space management system in ILUTE v1.0 to be highly flexible and scalable. Many other zoning systems, including: exact point locations, parcels, grid cells, etc and their mutual inter-operatability can be incorporated in the software.
The *CensusZone* class represents census tracts (CT), which is the lowest census zonal level at which ILUTE models are currently working\(^{22}\). The *TTSZone* class represents traffic analysis zones (TAZ) of the Toronto Tomorrow Survey (TTS), which again is the lowest TTS zonal level at which ILUTE models are currently working. These class definitions allow the results to be aggregated to any higher spatial level for both the census and TTS data. Spatial relationships (overlap) are also maintained between the *TTSZone* class and *CensusZone* class objects. This provides the flexibility to switch from one zoning system to other at run-time. Moreover, the models estimated in any of the two systems could be used in the software implementation.

Figure A.8 shows the relationships between the spatial classes, whereas Figures A.9, A.10, and A.1a show their relationships with other ILUTE classes. Figures A.12, A.13, A.14, A.15, and A.16 give complete lists of attributes and operations of these spatial classes.

\[^{22}\text{Incorporation of Census Dissemination Area (DA), which is a level lower in aggregation than Census Tract, is under development}\]
Figure A.9
SpatialObject Class Diagram – Relationships

Figure A.10
TTSZone Class Diagram – Relationships

Figure A.11
CensusZone Class Diagram – Relationships

Figure A.12
Location Class Diagram: Attributes and Operations

Figure A.13
SpatialObject Class Diagram: Attributes and Operations
### Figure A.14

*Area* Class Diagram: Attributes and Operations

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>myApproximateRadius</td>
<td>float</td>
</tr>
</tbody>
</table>

+ Area(in id : int)
+ ~Area()
+ Setters...()
+ Getters...()

---

### Figure A.15

*TTSZone* Class Diagram: Attributes and Operations

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>nextId</td>
<td>int</td>
</tr>
<tr>
<td>myTTSId</td>
<td>int</td>
</tr>
<tr>
<td>myPlanningDist</td>
<td>int</td>
</tr>
<tr>
<td>noVehHhld</td>
<td>int</td>
</tr>
<tr>
<td>noLicHhld</td>
<td>int</td>
</tr>
<tr>
<td>myCensusZones</td>
<td>int</td>
</tr>
</tbody>
</table>

+ Setters...()
+ Getters...()
+ AddToLists...()
+ RemoveFromLists()

---

### Figure A.16

*CensusZone* Class Diagram: Attributes and Operations

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>nextId</td>
<td>int</td>
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</table>

+ Setters...()
+ Getters...()
+ AddToLists...()
+ RemoveFromLists...()
+ Incrementers...()
A.3 Class Representation of Builder and Dwelling

In the ILUTE software, a dwelling is represented by the `DwellingUnit` class (figure A.17). Each year during the simulation, new dwellings are added to the housing stock. The `SpaceBuilder` class (figure A.18) represents the mechanism that adds new dwellings to the simulation. Figure A.19 shows the relationships between the `DwellingUnit` class and other classes of the ILUTE software.

![DwellingUnit Class Diagram](image1)

**Figure A.17**

*DwellingUnit* Class Diagram – Attributes and Operations

![SpaceBuilder Class Diagram](image2)

**Figure A.18**

*SpaceBuilder* Class Diagram – Attributes and Operations

185
A.4 Class Representation of the Owner-Occupied Housing Market

The *HousingMarket* class is the major class that represents the owner-occupied housing market clearing process in the ILUTE software. It is supported by two classes: *BidderSet* and *DwellingChoiceSet*, and two structures: *BidderInfo* and *DwellingChoiceInfo*. These classes and structures act as market related data holders and manipulators for the *Household* and *Dwelling* objects that are active in the housing market. Note that the supporting classes and structures only exist while their respective household and dwelling objects are active in the market. This design optimizes the use of the system memory space, as each year only a small percentage of the household and dwelling objects are active in the market. Figure A.20 shows the relationships that exist between the housing market classes and other classes of the ILUTE software. Figures A.21, A.22, and A.23 show the details related to the attributes, operations and relations for the housing market classes.

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23 In a recent development, a two level hierarchy (with an abstract parent class and two child classes: owner-occupied and rental housing market) is introduced by Giroux-Cook (2010) to seamlessly incorporate rental market within ILUTE. Here however, the discussion is only limited to owner-occupied housing market. The references *housing market* and *owner-occupied housing market* are used interchangeably in the discussion.
Figure A.20

*HousingMarket* Class Diagram – Relationships

Figure A.21

*BidderSet* Class Diagram – Attributes, Operations, and Relationships

Figure A.22

*DwellingChoiceSet* Class Diagram – Attributes, Operations, and Relationships
## HousingMarket

- myActiveHouseholdIdList
- myActiveDwellingIdList
- hhIdListAtSubStep
- dwellListAtSubStep
- myChoiceSetMap
- myBidderSetMap
- myChoiceSetSize : int
- myPriceIncrement : int
- maxHouseholdActiveTime : int
- maxDwellingActiveTime : int
- minPriceFactor : int
- maxPriceFactor : int

+ Init() : void
+ clear() : void
+ determineHouseholdChoiceSet(in hhIdList, in dwelIdList) : void
+ processTransaction(in currClDwell : DwellingUnit, in potPrices) : void
+ determinePotentialTransactionPrices(out probPrices, in dObj : DwellingUnit, in minPrice : long, in maxPrice : long) : void
+ findNearestPrice(in constC, in strtValue : long) : ulong
+ performHouseKeepingOfActiveLists(in dwelldList, in hhidldList) : void
+ performMarketHouseKeeping() : void
+ updateZonalAverages() : void
+ Setters...()
+ Getters...()
+ AddToLists...()
+ RemoveFromLists...()
+ handleHouseholdRemoval(in hid : int) : void
+ updateHhldsStatusInMarket(in hhldList) : void
+ updateDwellsStatusInMarket(in dwellList) : void

---

**Figure A.23**

*HousingMarket* Class Diagram – Attributes and Operations
APPENDIX B

IMPORTANT SEQUENCE FLOWS

This appendix describes important sequence flows that are implemented in the software ILUTE v1.0.

B.1 Simulating World Updating

Upon the instantiation of an Application class object, it first initialises the system configuration. Then, the object creates a new world by instantiating a World class object, and sets up the user interface for the system. The Application object acts as a master controller for the running and updating of the World object. It manages the main loop that sequentially executes the World object’s updates until the user-specified target simulation end date. Based on the loaded configuration, the individual updates and their sequence of execution are determined by the World object. Figure B.1 shows the system sequence diagram for the loading and execution of updates in simulated World.

Figure B.1
Simulated World – System Sequence Diagram
B.2 Updating the Housing Market

As described in the section A.4, the HousingMarket class is responsible for the clearing process of the housing market in the ILUTE simulation. At each time step, Household and DwellingUnit objects become active in the housing market. The HousingMarket Object associated with the World object maintains the list of Household and DwellingUnit objects that are active in the market. Figure B.2 shows the sequence in which HousingMarket operations are executed and other objects that are involved in the market clearing.

At each time step, once it has been decided which Household and DwellingUnit objects become active in the housing market, the HousingMarket object is called to be updated. The HousingMarket object first determines the asking price for the active DwellingUnit objects and the choice set for the Household objects. It then selects a DwellingUnit object one by one and determines its transaction price, which is based on the utility function of the Household objects.

Figure B.2
Housing Market Clearing – System Sequence Diagram
### B.3 Household’s States in Housing Market

At each time-step during an ILUTE simulation run, all Household objects decide on whether to get active in the market or not. Figure B.3 shows various states in which a Household object can exist once it decides to get active. If the Household object has an active association with a DwellingUnit object, then that object also becomes active in the housing market. If the Household object is a newly created object, it will have no association with any DwellingUnit object, in which case it will automatically enter the state of “Do not own a dwelling”. After becoming active, Household objects generate their choice set and assess them using their utility functions. During the search for association with a new DwellingUnit object, if the Household object already has an association with another DwellingUnit object, it may decide to go back to the inactive state and keeping the existing association. Household objects that do not have a current association with a DwellingUnit object remain active in the housing market until they find a suitable DwellingUnit object.

![State Diagram](image)

**Figure B.3**

*Household class – State Chart Diagram of Housing Market Interaction*
B.4 DwellingUnit's States in Housing Market

A DwellingUnit object can become active in the housing market if either its associated Household object becomes active in the market or if it has been newly created by the SpaceBuilder object. In the later case, the DwellingUnit object automatically enters the “Do not have an owner” state. Once it is active in the market, an asking price is set for the DwellingUnit object. If a buyer is found for a DwellingUnit object, its association is updated and it is removed from the housing market. In the case of DwellingUnit objects without any existing association with a Household object, the asking price is adjusted until a new association is found. In the case of DwellingUnit objects that already have an association with a Household object, if the Household object decides to back out from the market at any point during the market clearing process, that DwellingUnit object is also removed from the market. Figure B.4 shows the various states in which a DwellingUnit object can exist while it is active in the housing market.

![State Chart Diagram of Housing Market Interaction](image)

**Figure B.4**

DwellingUnit class – State Chart Diagram of Housing Market Interaction

B.5 Price Finding Process

The price finding process is a sub-process of the market clearing process. It generates a set of sale prices by iteratively increasing and re-evaluating the price from a given lower to an upper
range. It stops the search when the price reaches the maximum. Figure B.5 shows the steps involved in finding a set of prices for each dwelling unit that is active in the housing market.

Figure B.5
Price Finding – State Diagram of Housing Market Interaction

B.6 New Housing Supply

At the end of each year in the ILUTE simulation, the SpaceBuilder object creates new DwellingUnit objects that will be available in the housing market for the next year. Figure B.6 shows the sequence diagram of the supply process.

For each dwelling type, the SpaceBuilder object computes the total number of dwellings that are to be introduced in the simulation and the relative probabilities for the TTSZone objects. Using the total number of dwellings and the selection probabilities, the SpaceBuilder object creates an appropriate number of DwellingUnit objects by type for each TTSZone. The SpaceBuilder object then uses the selected TTSZone object to associate the newly created
DwellingUnit object with a CensusTract object. Finally, the SpaceBuilder object introduces the newly created DwellingUnit object to the housing market.

Figure B.6
New Housing Supply – System Sequence Diagram