ESSAYS ON DYNAMIC CONTRACTS:
MICROFOUNDATION AND MACROECONOMIC IMPLICATIONS

by

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Graduate Department of Economics
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Abstract

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Doctor of Philosophy
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This thesis consists of three chapters pertaining to issues of long-term relationships in labour markets. In Chapter 1, I analyze a model of a two-period advice game. The decision maker chooses to retain or replace the advisor after the first period depending on the first period events. Even though the decision maker and the advisor have identical preferences, this potential replacement creates incentive for the advisor to avoid telling the truth. I show the condition under which the decision maker can find a random retention rule that induces a truthful report from the advisor, and I characterize an optimal retention rule that maximizes the decision maker’s expected payoff.

In Chapter 2, I propose a search theoretic model of optimal employment contract under repeated moral hazard. The model integrates two important attributes of the labour market: workers’ work incentive on the job and their mobility in the labour market. Even though all workers and firms are ex ante homogeneous, these two factors jointly generate (1) wages and productivity that increase with worker’s tenure and (2) endogenous dynamic heterogeneity of the labour productivity of the match. The interaction of these factors provides novel implications for wage dispersion, labour mobility, and the business cycle behaviour of macroeconomic variables.

Lastly, in Chapter 3, I quantitatively assess wage dispersion and business cycle implications of the model developed in Chapter 2. In terms of wage dispersion, the model with on-the-job search with wage-tenure contracts seems to accommodate sizable frictional wage dispersion. The
model, however, generates very small productivity difference among workers, and shows weak evidence that the productivity difference generated by the endogenous variations in incentives is responsible for frictional wage dispersion. In terms of business cycle implications, workers' endogenous effort choice first amplifies the effect of productivity shock on unemployment rate. Second, responses of workers to productivity shocks generate marked difference between the effects of temporary productivity shock and that of permanent shock. Third, the analysis shows the importance of the distributional effect on macroeconomic variables during the transitory periods after a shock.
To my late father Masayuki Tsuyuhara
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Chapter 1

An Advice Game with Reputational and Career Concerns
1.1 Introduction

Consider a simple case of information transmission where the decision maker seeks advice from an expert. If the expert’s preferences are perfectly aligned with the preferences of the decision maker and if both know this fact, then the decision maker can efficiently use the available information to make the statistically optimal decision. However, if there are uncertainties about the expert’s preferences when hiring, and if the decision maker thinks that the expert may be in favor of one particular action, then the decision maker will discount the expert’s advice when making a statistical inference. The latter situation results in decision making that is informationally inefficient even if the advisor is truly unbiased and provides truthful advice. If the decision maker faces a recurrent decision making process, she will want to replace the advisor with a new expert if she is suspicious of the current advisor based on the advice history. Given this potential replacement, an unbiased advisor does not want to be seen as biased type, and he may lie when his specialist information supports the decision that the biased advisor prefers. The uncertainty of the decision maker about the advisor’s preferences generates the advisor’s reputational concern, and the potential replacement generates the career concern. I investigate how these reputational and career concerns influence the incentive to truthfully report information\textsuperscript{12}.

The current paper augments Morris’ (2001) two-period advice game in the following way. Consider an economist who is advising an uninformed policy maker for a fixed two-period term. Suppose a new government regulation of a certain industry is an issue. The economist assesses the pros and cons of the regulation to the best of his ability and advises

\textsuperscript{1}The basic model of strategic information transmission was first formulated by Crawford and Sobel (1982). Most of the cheap talk models following Crawford and Sobel’s (1982) original contribution were static variations: Farrell and Gibbons (1986) on situations where there is more than one receiver; Austen-Smith (1993) and Krishna and Morgan (2001) on situations where there is more than one sender; Battaglini (2002) on situations where uncertainty and the policy space are multidimensional.

\textsuperscript{2}Sobel (1985) and Morris (2001) study the repeated version of cheap talk game with reputational concern studied in this paper.
the policy maker to either support or oppose the new regulation. If the policy maker knows that the economist does not have any conflicts of interest with the industry at issue, she can incorporate the advice of the economist without reservation into her decision. If, however, she thinks that the economist might be biased in favor of the industry, the economist’s advice on the regulation will be discounted. If the economist does not want to be thought to be biased, he has a reputational incentive to lie. The economist will have reputational concerns simply because he wants his valuable and unbiased advice to be fully considered in future decision making. This is the source of informational inefficiency in a repeated version of advice game in Morris (2001). Now, suppose his appointment is a two-period term, but the policy maker has a right to replace him with a new advisor after the first period. In this case, the policy maker’s retention/replacement decision has a strategic influence on the economist’s advice, which was absent in Morris’ original argument. I will analyze how this additional element of the model influences the equilibrium of the game and welfare of the decision maker. I address several possible questions in this model, including 1) Is the economist more or less likely to lie? 2) How does the policy maker choose to retain or replace her advisor based on the first period outcome? 3) Is there any optimal retention/replacement strategy in terms of welfare of the decision maker?

This paper studies a two-period cheap talk game in which the decision maker chooses at the end of the first period whether to retain her advisor or fire him and hire a new advisor from the outside market. A decision maker has a two-period decision problem. In each period, her optimal decision depends on unknown state of the world. She has no information about the state in both periods. In the first period, a decision maker hires an expert as an advisor from the market for advisors. When hiring an expert, the decision maker is aware that there are two types of experts in the market. The preferences of an unbiased expert are perfectly aligned with the decision maker in each period, while a biased advisor always wants to lead the decision maker towards one particular decision.
The advisor gives his report to the decision maker based on his specialist information. Based on the report by the advisor, the decision maker infers the state of the world and makes a decision conditional on the uncertainties about the type of the advisor. After the decision is made, the true state of the world is publicly observed and respective payoffs are realized. At the end of the first period, the decision maker updates her belief about the type of the advisor. Depending on this inference, the decision maker either replaces or retains the first-period advisor for the second period. The rest of the game in the second period follows exactly the same way as in the first period. The game ends after the second period.

In this paper, to focus on the incentive problem faced by an unbiased advisor, I assume that the biased advisor is a behavioural type, i.e., he always gives one particular report independent of the signal. Hence, the unbiased advisor can always differentiate himself from biased advisors by giving a different report than what the biased type will report. However, the unbiased advisor faces the following problem. In the first period, if the signal supports the action, which the biased type does not prefer, then truthfully reporting the signal achieves the statistically optimal decision in the current period as well as secures the second period appointment by establishing his credential as an unbiased type. However, if the signal supports the action, which the biased advisor prefers, then truthfully reporting the signal can lead the decision maker towards the statistically optimal action, but confuses the decision maker about the advisor’s type. The advisor may be either an unbiased type reporting the signal truthfully or a biased type reporting independent of a signal. It may cause replacement after the first period. On the other hand, sending a false report will secure his second period appointment, but will lead to statistically worse action by the decision maker. His tradeoff depends on how he values the current action by the decision maker and how he values the career in the second period.

In the benchmark model, I assume that the decision maker cannot commit to any
retention/replacement rule. In this case, the decision maker replaces the advisor whenever she receives the report that a biased advisor sends. The decision maker understands that the report may be sent by an unbiased advisor who is truthfully reporting the signal. Yet, her uncertainty about the advisor’s type after receiving the report increases, and she finds it optimal to replace the first period advisor. Given this retention/replacement strategy by the decision maker, there is always an equilibrium in which an unbiased advisor separates himself from biased advisors by reporting what biased experts never report (politically correct equilibrium). If career concerns are sufficiently large for the unbiased advisor, this is the unique equilibrium of the game. He has an incentive to lie so that his valuable and unbiased report will be considered for the next period decision and also so that he will be retained for the second period. In this equilibrium, the decision maker cannot learn anything about the current state, but she can be certain about the type of the first-period advisor. However, if the level of career concerns is sufficiently small, there is another equilibrium in which the unbiased advisor truthfully reports the observed information in the first period (truthful equilibrium). In this equilibrium, the decision maker can partially learn about the type of the first-period advisor as well as the state of the world. In this sense, it is the most informative equilibrium to the decision maker, and the decision maker strictly prefers the truthful equilibrium when there are multiple equilibria.

In the last section, I augment the model with the decision maker’s commitment to a retention/replacement rule. When the politically correct equilibrium is the unique equilibrium outcome and socially valuable information is lost, I ask whether the decision maker can recover the possibility of truthful report by offering a rewarding retention rule. In this case, the decision maker makes a trade-off between the benefit from extracting the useful information from the unbiased advisor in the first period and the cost of retaining the biased advisor in the second period. I show the upper bound of the level of career concern under which the decision maker can find a retention rule that recovers the
truthful report and characterizes an optimal retention rule that maximizes the decision maker’s expected payoff. In addition, I show that the decision maker’s expected payoff decreases as the advisor’s level of career concern increases.

This paper builds on the work by Sobel (1985) and Morris (2001) in analyzing reputational concerns of the expert in the repeated cheap talk model. Despite similarities, several important differences exist between these models. First, Sobel’s (1985) analysis assumed that an unbiased advisor (friend) always tells the truth to focus on a biased advisor’s (enemy) behaviour. An enemy will sometimes tell the truth to invest in reputation and sometimes lie to exploit that reputation. In my model, a biased advisor always lies, and an unbiased advisor will sometimes avoid telling the truth to invest in reputation. Morris (2001) endogenizes the behaviour of both an unbiased (good) advisor and a biased (bad) advisor. Second, both Sobel (1985) and Morris (2001) do not analyze the decision maker’s retention/replacement strategy. In Sobel (1985), the advisor can potentially be replaced at any time, but it will not happen in equilibrium. In Morris (2001), the relationship between the decision maker and the advisor is fixed over two periods. On the other hand, in my model, replacement happens in equilibrium. Moreover, the decision maker’s retention/replacement decision plays an important role in inducing the truth-telling when the unbiased advisor chooses to lie in equilibrium.

The reputation effect was first formulated in the game-theoretic models of repeated interactions under imperfect information (Kreps, Milgrom, Roberts, and Wilson, 1982; Kreps and Wilson, 1982; Milgrom and Roberts, 1982). Hörner (2002), Mailath and Samuelson (2001), and others examine the reputational concerns in a market framework where firms and consumers repeatedly interact and the consumers infer the firms’ effort levels from the product quality. The general implication of these studies is that the reputational concerns of agents provide implicit commitment device and allow them to achieve efficient outcomes. However, Ely and Välimäki (2003) point out that it could work the other way around; that is, the reputational concern of the agent to look good in
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the current period may result in the loss of socially valuable information. The literature calls this negative implication bad reputation.

The above studies examine the impacts of reputational concerns on the payoff-relevant actions under asymmetric information. The reputational concerns, however, naturally arise in cheap talk games, where the message is costless and does not directly affect the agents’ payoffs, if the communication occurs in multiple periods (Kim, 1996; Ottaviani and Sørensen, 2006a, 2006b; Park, 2005). Morris’ (2001) contribution is to formalize the notion of bad reputation in the context of cheap-talk games; a “good” advisor may lie in an attempt to be more influential in the future.

The models of career concerns identify the incentive issues in the presence of variable outside options—market values—for the agent. The way I model the agent’s career concerns is closely related to Prat (2005). He analyzes a reduced form of a two-period career concerns model in which the principal can either retain the first period agent or hire new agent in the second period. In the model by Prat (2005), the informed agent takes an action and the principal faces a moral hazard problem.

The final part of this paper on the game with partial commitment is related to the mechanism design approach to eliciting private information. The standard mechanism design problems often consider the situation in which an informed agent takes an action and the principal designs a mechanism to achieve a desirable outcome. There are few studies analyzing the issue of strategic information transmission from the mechanism design approach (Melumad and Shibano, 1991). Recently, Li (2007) analyzes the issue of designing optimal reporting protocols in a multistage cheap talk environment. In my model, however, the decision maker cannot commit to the decision rule regarding an

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4See Dewatripont, Jewitt, and Tirole (1999), Gibbons and Murphy (1992), and Holmstrom (1999).
5See Hurwicz and Reiter (2006) for an introduction to the literature.
6Alonso, Dessein, and Matouschek (2008) study the effect of organizational structure on communication. Instead of designing the decision rule as a function of message, the principal commits to the allocation of decision rights. See Alonso and Matouschek (2008) and Dessein (2002) for related work.
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action in each period, but can commit only to the retention rule. In this sense, the approach taken in this paper differs from these previous contributions.

1.2 An Advice Game

A decision maker faces a two-period decision problem. A period is indexed by \( t = 1, 2 \). She takes an action, \( a_t \in [0, 1] \), in each period, but her optimal decision depends on the state of the world, \( \omega_t \in \{0, 1\} \). Each state occurs with equal probability, and its probability distribution is independent over time. She has no information about the state in both periods, so she hires an advisor. The advisor, on the other hand, observes a signal about the state, \( s_t \in \{0, 1\} \), which equals the true state with probability \( \gamma \). I assume that \( \gamma \in (\frac{1}{2}, 1) \) so that the signal is informative, but not perfectly so. However, when hiring an advisor, the decision maker is aware that there are two types of advisors in the market: unbiased and biased. \( \lambda_1 \in (0, 1) \) is a fraction of unbiased advisors in the market, so the decision maker’s prior belief that her advisor is an unbiased type is \( \lambda_1 \).

In the following, I call the belief (prior or posterior) that the advisor is an unbiased type reputation and use these terms interchangeably depending on the context.

The advisor gives his report \( r_t \in \{0, 1\} \) to the decision maker after observing a signal, and then the decision maker takes an action. After an action is taken, the true state of the world is publicly observed and respective payoffs are realized. After the first period, the decision maker updates her belief about the type of the advisor based on the report and the realized state. Depending on these inferences, the decision maker either replaces or retains the first-period advisor for the second period. The second period of the game follows the exact same procedure as the first period. The distribution of the state in the second period is independent of the action taken in the first period. The game ends after the second period.

The decision maker’s preferences are given by the quadratic loss function which de-
Chapter 1. Advice Game with Reputational and Career Concerns

Figure 1.1: Timeline of the first period game

pends on the state \( \omega_t \) and the action \( a_t \). I assume no discounting over time for the decision maker. Thus her life-time utility is given by

\[-(a_1 - \omega_1)^2 - (a_2 - \omega_2)^2.\]

With this form of preferences, the statistically optimal decision for \( a_t \) when \( \omega_t \) is uncertain is to choose \( a_t \) that equals to her expectation that \( \omega_t = 1 \).

The preferences of the unbiased advisor are perfectly aligned with those of the decision maker in each period. However, the second period utility is conditional on being retained as the advisor. Let \( \mathbb{I}_{(r_1, \omega_1)} \) be the indicator function, which takes 1 if he is retained in the second period and 0 if otherwise. The decision maker’s retention decision depends on publicly available information: \( r_1 \) and \( \omega \). The unbiased advisor’s life-time utility is given by

\[1 - (a_1 - \omega_1)^2 + \mathbb{I}_{(r_1, \omega_1)} \cdot \delta \left[1 - (a_2 - \omega_2)^2\right].\]

This implies that the advisor’s reservation payoff in the second period is zero. Since the shape of the indicator function is the decision maker’s strategy, which is determined in equilibrium, the advisor takes it as given. The coefficient \( \delta \in \mathbb{R}_+ \) is a level of career concern and measures how much the advisor cares about the second period payoff relative to that of the first period. I take an affine transformation over the decision maker’s period utility function so that zero utility when unemployed is the minimum utility, instead of the maximum.
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The biased advisor has preferences in favor of $a_t = 1$ independent of the underlying state. Especially, I assume that the biased advisor is a behavioural type; i.e., he has a myopic preferences $-(a_t - 1)^2$ whenever hired and does not consider the influence of his report on future outcome.

1.3 Analysis of the Game

It is worth recapitulating the problem that an unbiased advisor faces. In the first period, if the signal supports the action that the biased type does not prefer, then truthfully reporting the signal results in the statistically optimal decision in the current period, as well as secures the second period appointment by establishing his credential as an unbiased type. However, if the signal supports the action, which the biased prefers, then truthfully reporting the signal can lead closer to the statistically optimal action by the decision maker, but confuses her about his type. He may be either an unbiased type reporting the signal truthfully or a biased type reporting independent of a signal. It may cause replacement after the first period. On the other hand, sending a false report will secure his second period appointment but will lead to a statistically worse action by the decision maker. His tradeoff depends on how he values the current action by the decision maker and how he values the career in the second period.

I will characterize the sequential equilibrium of the game\footnote{The footnote in page 14 explains the situation where the consistency of the assessment is necessary for this analysis.}, and the analysis is shown by backward induction. First, I will characterize the equilibrium behaviour in the second period. It characterizes the continuation values for the second period. The continuation values depend on the reputation of the advisor in the second period, and it creates the reputational concern of the advisor in the first period. Given these continuation values, I will characterize the equilibrium of the game in which both the decision maker and the advisor maximize their respective \textit{ex-ante} payoffs.
1.3.1 Equilibrium in the Second-Period Game

This is the final period and the advisor does not have concerns about the future. He simply maximizes his current expected utility. That is, given the perfect alliance of the preferences, the unbiased advisor is willing to share the private information with the decision maker so that the decision maker can make a statistically optimal decision. Since the signal is informative, the unbiased advisor has a strict incentive to truthfully report his signal. The biased type reports 1 independent of signal.

Suppose the decision maker enters the second period with an advisor whose reputation is $\lambda_2$. $\lambda_2$ is equal to $\lambda_1$ if the decision maker has replaced the advisor in the first period, or $\lambda_2$ is her updated belief about the advisor if she retained the advisor after the first period. The decision maker’s statistically optimal decision is to match the action with her posterior belief that the state is 1. Let $\Gamma$ denote such a belief. The decision maker calculates $\Gamma$ based on the advisor’s report by Bayes’ rule$^8$. Given the advisor’s optimal strategy, if the decision maker receives the report 0, she can be certain that the unbiased advisor is truthfully reporting the signal 0. In this case, she updates the belief so that $\Gamma(r = 0) = 1 - \gamma$ and she chooses $a = 1 - \gamma$, independent of $\lambda_2$. On the other hand, if she receives the report 1, there are two possibilities: an unbiased type is truthfully reporting the signal or a biased type is reporting independently of his signal. Taking these cases into consideration, the decision maker updates her belief so that $\Gamma(r = 1) = \frac{1 - \lambda_2 + \lambda_2 \gamma}{2 - \lambda_2}$ and chooses $a$ accordingly.

Given these equilibrium strategies of the advisor and the decision maker, I can calculate the expected payoff in the second-period for each player as functions of $\lambda_2$. First, let $\Pr(r, \omega)$ be the probability that the decision maker receives report $r$ when the underlying

\footnote{I provide a formula of this posterior belief that incorporate the advisor’s general strategy in Appendix A.1.}
state is $\omega$. With the specified strategy, $\Pr(r, \omega)$ is given by

$$\Pr(r, \omega) = \begin{cases} 
\frac{1}{2}(\lambda_2 \gamma + (1 - \lambda_2)) & \text{if } r = 1, \, \omega = 1 \\
\frac{1}{2}(\lambda_2(1 - \gamma) + (1 - \lambda_2)) & \text{if } r = 1, \, \omega = 0 \\
\frac{1}{2}\lambda_2(1 - \gamma) & \text{if } r = 0, \, \omega = 1 \\
\frac{1}{2}\lambda_2 \gamma & \text{if } r = 0, \, \omega = 0 
\end{cases}$$

Using this probability function, the decision maker’s expected payoff of the second period is

$$V(\lambda_2) = -\sum_{\omega} \sum_{r} \Pr(r, \omega) (\Gamma(r) - \omega)^2.$$ 

Substituting $\Pr(r, \omega)$ and $\Gamma(r)$ and rearranging the resulting equation yield

$$V(\lambda_2) = -\frac{1}{2} \left( (1 - \lambda_2 + \lambda_2 \gamma) \left( \frac{1 - \lambda_2 \gamma}{2 - \lambda_2} \right)^2 + (1 - \lambda_2 \gamma) \left( \frac{1 - \lambda_2 + \lambda_2 \gamma}{2 - \lambda_2} \right)^2 + \lambda_2 \gamma (1 - \gamma) \right).$$

On the other hand, let $\Pr(\omega|s)$ be the posterior belief that the state is $\omega$ when he observes the signal $s$. By the assumption of informative signal,

$$\Pr(\omega|s) = \begin{cases} 
\gamma & \text{if } \omega = s \\
1 - \gamma & \text{if otherwise.} 
\end{cases}$$

Since the unbiased advisor truthfully reports the signal ($r = s$), he uses this probability to calculate the expected payoff in the second period. The expected payoff of an unbiased advisor is

$$v(\lambda_2) = 1 - \frac{1}{2} \sum_{\omega} \sum_{s} \Pr(\omega|s)(\Gamma(s) - \omega)^2.$$ 

Substituting $\Pr(s|\omega)$ and $\Gamma(r)$ and rearranging the resulting equation yield

$$v(\lambda_2) = 1 - \frac{1}{2} \left( \gamma \left( \frac{1 - \lambda_2 \gamma}{2 - \lambda_2} \right)^2 + (1 - \gamma) \left( \frac{1 - \lambda_2 + \lambda_2 \gamma}{2 - \lambda_2} \right)^2 + \gamma (1 - \gamma) \right).$$

These expected payoffs in the second period game characterize the continuation values for each player and are represented as continuous functions of the advisor’s reputation.
It can be shown that both functions are increasing in $\lambda_2$. With higher reputation, the decision maker trusts the advisor more and can choose a statistically better action based on the advisor’s report. Therefore, the decision maker is better off entering the second period with the advisor with higher reputation. It implies the following lemma\(^9\).

**Lemma 1.1.** In any equilibrium, the decision maker replaces the advisor in the first period if and only if his posterior reputation is lower than the market average, $\lambda_1$.

Because it is always possible to hire a new advisor with the reputation $\lambda_1$ from the outside market, it is optimal for the decision maker to fire the current advisor if his updated reputation is lower than this average reputation of the market.

### 1.3.2 Equilibrium of the Game

Given the values of the second period described in the previous section, the equilibrium of the game is defined by the advisor’s reporting strategy and the decision maker’s strategies in terms of the choice of action and retention/replacement decision.

The unbiased advisor’s reporting strategy in the first period is characterized by his reputational and career concerns. Due to the presence of biased advisors in the market, his report influences his reputation. The reputation, in turn, affects his position as well as his value in the second period. Since biased advisors never report 0, the unbiased advisor can perfectly separate himself from the biased type if he reports 0. Therefore, if he receives a signal 0, reporting 0 can not only lead to the statistically optimal decision but also improve his reputation as the unbiased advisor. However, if he receives a signal 1, he faces a nontrivial trade-off. Reporting 1 can make the action by the decision maker closer to statistically optimal decision compared to falsely reporting 0, but it will negatively affect his reputation and thus his continuation value. On the other hand, if he reports 0 instead, the current action will be further away from a statistically optimal decision, but

\(^9\)Proofs for all the lemmas and propositions are given in Appendix A.2.
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14

<table>
<thead>
<tr>
<th>S = 0</th>
<th>S = 1</th>
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<tr>
<td></td>
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<tr>
<td>Unbiased advisor reports</td>
<td>0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Biased advisor reports</td>
<td>1</td>
</tr>
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Table 1.1: Advisor’s reporting strategy when $s = 1$

he can separate himself from biased advisors and will enjoy a higher payoffs in the second period. Given these tradeoffs, it is without loss to restrict attention to strategies where the unbiased advisor truthfully reports the signal if it is 0, but if it is 1, he reports 1 with probability $\eta$ and 0 with probability $1 - \eta$. The pure strategy of reporting 0 corresponds to $\eta = 0$ and reporting 1 to $\eta = 1$. If $\eta$ is strictly between 0 and 1, the unbiased advisor uses a mixed strategy. Table 1.1 summarizes such a strategy.

As Lemma 1.1 indicates, the decision maker’s retention/replacement strategy depends on her updated reputation at the end of the first period. Let $\Lambda$ be such a belief. The decision maker calculates $\Lambda$ based on the advisor’s report and the realized state in the first period. Therefore, using the notation in the previous section, $\lambda_2 = \Lambda(r, \omega)$. Since the biased advisors always report 1 independent of signal, receiving 0 will make the decision maker certain about the type of advisor$^{10}$. Therefore, $\Lambda(0, \omega_1) = 1$ for $\omega_1 = 0, 1$. However, if the decision maker receives the report 1, she updates her belief based on Bayes’ rule, and

$$
\Lambda(1, \omega) = \begin{cases} 
\frac{\lambda_1 \gamma \eta}{\lambda_1 \gamma \eta + (1 - \lambda_1)} & \text{if } \omega = 1 \\
\frac{\lambda_1 (1 - \gamma) \eta}{\lambda_1 (1 - \gamma) \eta + (1 - \lambda_1)} & \text{if } \omega = 0.
\end{cases}
$$

Note that this inference depends on the advisor’s strategy $\eta$. The decision maker not only considers the report she receives, but also the strategy that sends the report. It is

$^{10}$If an unbiased advisor also reports 1 independent of the signal, Bayes’ rule does not apply. In this case, the decision maker’s belief is arbitrary, and I can construct a belief system where the unbiased advisor’s strategy—reporting 1 regardless of the signal—is indeed optimal. However, the consistency requirement for the sequential equilibrium excludes this assessment because any small probability that the unbiased advisor report 0 when signal is 0 makes $\Lambda(0, \omega_1) = 1$ (Osborne and Rubinstein, 1994).
easy to show that $\Lambda(1, \omega_1) < \lambda_1$ for both $\omega_1 = 0, 1$ and for any $\eta$ and $\gamma$. This inference rule, together with Lemma 1.1, implies the following proposition.

**Proposition 1.1.** In equilibrium, the decision maker replaces the advisor in the first-period if and only if she receives the report 1, independent of the realized state.

The advisor’s reputation is reduced when the decision maker receives a report 1, even if the realized outcome turned out to be 1. Since the decision maker’s continuation value is an increasing function of the advisor’s reputation, there is no loss in replacing the advisor after the first period, and the decision maker can be better off by doing so.

The decision maker’s optimal choice of action follows the similar argument as in the previous section. Again, let $\Gamma$ denote her posterior belief that the state is 1. She calculates $\Gamma$ based on the advisor’s report and her belief about the type of the advisor by Bayes’ rule, and

$$\Gamma(r, \eta) = \begin{cases} 
\frac{1-\gamma\eta}{2-\eta} & \text{if } r = 0 \\
\frac{1-\lambda_1+\lambda_1\gamma\eta}{2(1-\lambda_1)+\lambda_1\eta} & \text{if } r = 1.
\end{cases}$$

(1.1)

As in the second period game, these updated beliefs are identified as statistically optimal decisions given the report, 0 and 1, respectively. This inference also depends not only on the report she actually receives but on the strategy through which the report is delivered to her.

The above arguments imply that the reporting strategy of the unbiased advisor affects his expected payoff of the game for two reasons: one through the effect on the current action taken by the decision maker, and the other through the effect on his own reputation. The advisor’s equilibrium strategy maximizes his expected lifetime payoff given the decision maker’s strategy. Because the advisor has an unambiguous optimal report when the signal is 0, I focus on the case when the signal is 1 in the following argument. Let $E(r, \eta)$ denote his expected lifetime payoff from reporting $r$ when $s = 1$. I make it
explicit that the advisor’s expected payoff depends on his strategy $\eta$. In general,

$$E(r, \eta) = 1 - \sum_{\omega} \Pr(\omega|1)(\Gamma(r, \eta) - \omega)^2 + \mathbb{I}_r \delta v(1).$$

The last term on the right hand side is the expected payoff for the advisor in the second period. From Proposition 1.1, the advisor’s reputation becomes 1 if he is retained in the second period. Also from Proposition 1.1, the indicator function depends only on $r$. I substitute the functions from the previous discussion to explicitly calculate $E$. If the decision maker receives the report 0, the advisor will obtain

$$E(0, \eta) = 1 - \left( \gamma \left( \frac{1 - \gamma \eta}{2 - \eta} - 1 \right) \right)^2 + (1 - \gamma) \left( \frac{1 - \gamma \eta}{2 - \eta} \right)^2 + \delta v(1).$$

In this case, the advisor’s second period reputation becomes 1 and is retained in the second period to obtain the continuation value of $v(1)$. On the other hand, if the decision maker receives the report 1, the advisor will obtain

$$E(1, \eta) = 1 - \left( \gamma \left( \frac{1 - \lambda_1 + \lambda_1 \gamma \eta}{2(1 - \lambda_1) + \lambda_1 \eta} - 1 \right) \right)^2 + (1 - \gamma) \left( \frac{1 - \lambda_1 + \lambda_1 \gamma \eta}{2(1 - \lambda_1) + \lambda_1 \eta} \right)^2.$$

In this case, the advisor will be replaced after the first period, so the continuation value is zero since $\mathbb{I}_1 = 0$. The dependence of these payoffs on the advisor’s strategy $\eta$ is characterized by the following lemma.

**Lemma 1.2.** $E(0, \eta)$ is strictly decreasing in $\eta$ and $E(1, \eta)$ is strictly increasing in $\eta$.

It is worth emphasizing that I am focusing on the case when the signal is indeed 1 and that higher $\eta$ implies a more truthful report with respect to the signal. If the decision maker expects a high $\eta$, she expects that the advisor is truthfully reporting the signal and puts heavier weight on the report. If she receives the report 0, she takes an action in favor of state 0, even though the signal suggests the true state is more likely to be 1. The higher $\eta$, the more her decision is biased towards 0. That is, the bias of the action taken by the decision maker relative to the statistically optimal action is increasing in $\eta$ when
she receives the report 0. Therefore, the advisor’s expected payoff will be lower in this case. On the other hand, if she indeed receives the report 1, higher \( \eta \) makes it possible for the decision maker to take an action closer to the statistically optimal action, so the advisor’s expected payoff is increasing in \( \eta \).

This lemma implies three possible cases: (1) \( E(0, 1) > E(1, 1) \) in which case \( E(0, \eta) > E(1, \eta) \) for all \( \eta \), (2) \( E(0, 0) > E(1, 0) \) in which case \( E(1, \eta) > E(0, \eta) \) for all \( \eta \), and (3) there is a unique \( \eta \in (0, 1) \) to make \( E(0, \eta) \) and \( E(1, \eta) \) equal. The relative value of \( E(0, \eta) \) depends on the measure of career concern \( \delta \) of the advisor. If the unbiased advisor evaluates the second period value highly enough, the benefit from the statistically better action in the first period by truthfully reporting is outweighed by the benefit of securing the second period position by falsely reporting, independent of the decision maker’s expectation about the advisor’s strategy. In this case, \( \eta = 0 \) is indeed the dominant strategy by the advisor. However, as \( \delta \) decreases, the benefit from securing the second period position becomes lower for any value of \( \eta \). In particular, as the decision maker expects that the advisor is truthfully reporting the signal (higher \( \eta \)), the benefit from the statistically better action in the first period dominates the value of securing the position for the second period. Using these properties of expected payoffs, the equilibrium of the game is characterized as follows.

**Proposition 1.2.** There exists a level of career concern, \( \bar{\delta} > 0 \), such that

1. for any \( \delta > \bar{\delta} \), \( \eta = 0 \) is the unique equilibrium, and

2. for any \( \delta < \bar{\delta} \), there are three equilibria, namely \( \eta = 0 \), \( \eta = 1 \), and \( \eta^m \in (0, 1) \).

The proof in the appendix gives the explicit form for \( \bar{\delta} \). Figure 1.2 shows each case in the proposition. I call the equilibrium with \( \eta = 1 \) truthful equilibrium, and call the equilibrium with \( \eta = 0 \) politically correct equilibrium. The left panel of the figure shows the case where there is a unique politically correct equilibrium. In this equilibrium, the
Figure 1.2: Possibility of unique and multiple equilibria

decision maker ignores the report by the advisor for her choice of action. If I substitute $\eta = 0$ in equation (1.1), $\Gamma(0)$ becomes $\frac{1}{2}$. That is, she takes an action as if no information is available. She then only uses the report from the advisor to ensure that he is an unbiased type. In this case, socially valuable information is wasted. However, it is important that there are strictly positive levels of career concern with which the unbiased advisor provides a truthful report with positive probability even though it will cause termination of his career. The right panel of the figure shows this case. If the equilibrium with $\eta = 1$ is realized, the information is utilized statistically efficiently, and I call the truthful equilibrium informationally efficient.

1.4 The Game with Partial Commitment

In the above benchmark model, I assumed that the decision maker cannot commit to any specific retention/replacement policy, i.e., she cannot write any contract that specifies the conditions for retention based on the report and the realized state. She bases her retention/replacement decision solely on the reputation of her advisor. Therefore, the unbiased advisor has a stronger incentive to misreport a true signal than the case where
his reputation only affects the action taken in the second period as in Morris (2001). In this section, I relax this assumption and suppose she can commit to the following mechanism.

**Definition 1.1.** A retention rule determines the probability that the advisor is retained after the first period as a function of the report that the decision maker received and the realized state.

In the real-life situations of advisory relationship, which lasts for several distinct periods, it does not seem that advisors are replaced after some term because of a particular report independent of the realized state (as in the above equilibrium replacement) or that advisors are retained with certainty (as in Morris’ (2001) original case). The real situations, of course, involve many different variables than this simple theory can capture. However, a stochastic component of the relationship should be included in the model, whose outcome depends on the report as well as the realized state. The definition of the probabilistic retention rule is intended to capture such a stochastic aspect of the long-term relationship.

### 1.4.1 Existence and Characterization of the Optimal Retention Rule

As the above analysis shows, if an extent of the advisor’s career concern is sufficiently large, i.e., $\delta > \bar{\delta}$, the unique equilibrium of the game is the politically correct equilibrium. Truthfully reporting the signal is too costly for the unbiased advisor because it will result in replacement after the first period. Given this property of the game without commitment, I ask whether the decision maker can find any retention rule in which the game has a truthful equilibrium. However, it indicates that the decision maker will also need to keep a biased advisor for the second period with positive probability. Therefore, inducing truthful report by an unbiased advisor incurs cost to the decision maker. These
arguments motivate the following definition.

**Definition 1.2.**

1. A retention rule recovers the truthful equilibrium if there is an equilibrium with $\eta = 1$.

2. A retention rule profitably recovers the truthful equilibrium if the decision maker is better off in the truthful equilibrium with the retention rule than in the politically correct equilibrium without commitment.

3. The optimal retention rule maximizes the decision maker’s expected payoff.

Given the equilibrium properties of the benchmark problem, I can focus on the following simplified rule. If the decision maker receives a report 0, she is certain that the advisor is an unbiased type. Therefore, the advisor is retained with probability one independent of the realized state. On the other hand, if she receives a report 1, the retention probability depends on the realized state: the advisor is retained with probability $\phi_0$ if the realized state is 0 and retained with probability $\phi_1$ if the realized state is 1. Let $\Phi = \{(\phi_0, \phi_1) \in [0, 1]^2\}$. I call the retention rule of this type a $\phi$-rule, for short. For any $\phi$-rule and the advisor’s strategy $\eta$, let $\Pi(\phi, \eta)$ denote the decision maker’s life-time expected payoff. Using these notations, a $\phi$-rule recovers the truthful equilibrium if

$$E(0, 1) \leq E(1, 1) + \delta [\phi_1 \gamma v(\Lambda(1, 1)|_{\eta=1}) + \phi_0 (1 - \gamma) v(\Lambda(1, 0)|_{\eta=1})]$$

(1.2)

holds, and it profitably recovers if

$$\Pi(0, 0) \leq \Pi(\phi, 1)$$

(1.3)

is satisfied. The left hand side of the condition (1.2) is the unbiased advisor’s expected payoff

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111 Explicit form of this function is given in page 23.
Figure 1.3: $\phi$-rule that recovers the truthful equilibrium

payoff in the politically correct equilibrium, and the first term of the right hand side is his expected payoff in the truthful equilibrium without commitment. Inside the bracket is the expected value of the second period under the $\phi$-rule. This condition is the incentive compatible constraint for this problem; that is, the decision maker needs to provide sufficient incentive for the advisor to truthfully report when he receives the signal $s = 1$. On the other hand, the left hand side of the condition (1.3) is the decision maker’s expected payoff in the politically correct equilibrium, and the right hand side is that in the recovered truthful equilibrium with $\phi$-rule. Therefore, the condition (1.3) requires that, if a $\phi$-rule profitably recovers the truthful equilibrium, the decision maker’s expected payoff must be at least as high as in the politically correct equilibrium. This is the individual rationality constraint for the decision maker. Therefore, if this condition is not satisfied, the decision maker does not benefit from the possible truthful report by an unbiased advisor.

$^{12}\phi = (0, 0)$ corresponds to the equilibrium retention rule that is characterized in the previous section.
Define the following subsets of \( \phi \)-rules.

\[
\Phi_R \equiv \{ \phi : E(0, 1) \leq E(1, 1) + \delta[\phi_1 \gamma v(\Lambda(1, 1)|_{\eta=1}) + \phi_0(1 - \gamma)v(\Lambda(1, 0)|_{\eta=1})] \}
\]

and

\[
\Phi_{PR} \equiv \{ \phi : \Pi((0, 0), 0) \leq \Pi(\phi, 1) \}.
\]

That is, \( \Phi_R \) and \( \Phi_{PR} \) are subsets of \( \phi \) that satisfy conditions (1.2) and (1.3), respectively. The existence of the optimal retention rule that profitably recovers the truthful equilibrium is equivalent to that the intersection of \( \Phi_R \) and \( \Phi_{PR} \) is nonempty. Let \( \Omega \equiv \Phi_R \cap \Phi_{PR} \).

Note that \( \Phi_{PR} \) depends on the fraction of unbiased advisors in the market \( \lambda_1 \) and the precision of signal \( \gamma \) while \( \Phi_R \) also depends on the extent of career concern \( \delta \). Therefore, the existence of the optimal retention rule as well as its characterization depends on these three parameters.

First, I characterize the set \( \Phi_R \). Rearranging the condition (1.2) gives

\[
3\gamma(1 - \gamma) + \delta[v(1) - \phi_1 \gamma v(\Lambda(1, 1)|_{\eta=1}) - \phi_0(1 - \gamma)v(\Lambda(1, 0)|_{\eta=1})] \leq E(1, 1).
\]

The left hand side is decreasing in \( \phi_0 \) and \( \phi_1 \), so the set is the upper contour set of the negatively sloped line on the \((\phi_0, \phi_1)\) plane that is given by (1.2) with equality (the left
panel of Figure 1.4). Let \( \bar{\phi}_1 \) denote the minimum of 1 and the intercept of the line on \( \phi_1 \)-axis, and \( \bar{\phi}_0 \) denote the maximum of 0 and the \( \phi_0 \)-coordinate of the line at \( \phi_1 = 1 \). I denote this pair by \( \bar{\phi} \). By definition, solving (1.2) with equality for \( \phi_1 \) at \( \phi_0 = 0 \) gives

\[
\bar{\phi}_1 \equiv \min \left\{ \frac{E(0, 1) - E(1, 1)}{\delta \gamma v(\Lambda(1, 1)_{\eta=1})}, 1 \right\} .
\]

Similarly, solving (1.2) with equality for \( \phi_0 \) at \( \phi_1 = 1 \) gives

\[
\bar{\phi}_0 \equiv \max \left\{ 0, \frac{E(0, 1) - E(1, 1) - \delta \gamma v(\Lambda(1, 1)_{\eta=1})}{\delta(1 - \gamma) v(\Lambda(1, 0)_{\eta=1})} \right\} .
\]

These two thresholds have an important property.

**Lemma 1.3.** \( \bar{\phi}_0 \) is positive if and only if \( \bar{\phi}_1 \geq 1 \).

Together with this lemma, the following lemma proves useful.

**Lemma 1.4.** There exists an extent of career concern \( \delta' \) such that, for any \( \delta \in (\bar{\delta}, \delta'] \), \( \bar{\phi}_1 \leq 1 \). This threshold value is given by

\[
\delta' \equiv \frac{E(1, 1) - 3\gamma(1 - \gamma)}{v(1) - \gamma v(\Lambda(1, 1)_{\eta=1})}.
\]

The threshold value is characterized by the ratio between the net benefit in the first period from truthfully reporting the signal—due to the statistically optimal action—and the net benefit in the second period from misreporting the signal—due to the enhanced reputation under the \( \phi \)-rule. This lemma implies that as long as the advisor’s career concern is less than \( \delta' \), the decision maker does not need to offer a positive retention probability when the report turns out to be incorrect.

Next, I characterize the set \( \Phi_{PR} \). The general form for \( \Pi(\phi, \eta) \) is given by

\[
\Pi(\phi, \eta) = \Pr(1, 1) \left( - (\Gamma(1) - 1)^2 + \phi_1 \cdot V(\Lambda(1, 1)) + (1 - \phi_1) \cdot V(\lambda_1) \right) \\
+ \Pr(1, 0) \left( -\Gamma(1)^2 + \phi_0 \cdot V(\Lambda(1, 0)) + (1 - \phi_0) \cdot V(\lambda_1) \right) \\
+ \Pr(0, 1) \left( - (\Gamma(0) - 1)^2 + V(1) \right) \\
+ \Pr(0, 0) \left( -\Gamma(0)^2 + V(1) \right) .
\]
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Since \( \Lambda(1, j) < \lambda_1 \) for \( j = 0, 1 \) and \( V \) is increasing, \( \Pi(\phi, \eta) \) is a decreasing function in \( \phi \) for a given \( \eta \). Thus, the set \( \Phi_{PR} \) is the lower contour set of the line on \((\phi_0, \phi_1)\) plane that is given by \( \Pi(\phi, 1) = \Pi(0, 0) \) (the right panel of Figure 1.4). By evaluating \( \eta = 1 \), I find

\[
\Pi(\phi, 1) = -\frac{1}{2} \phi_1 (1 - \lambda_1 + \lambda_1 \gamma) [V(\lambda_1) - V(\Lambda(1, 1))] - \frac{1}{2} \phi_0 (1 - \lambda_1 \gamma) [V(\lambda_1) - V(\Lambda(1, 0))] + (1 + \frac{1}{2} (2 - \lambda_1)) V(\lambda_1) + \frac{\lambda_1}{2} V(1). \quad (1.4)
\]

Similarly, \( \Pi(0, 0) \) is given by

\[
\Pi(0, 0) = -\frac{1}{4} + (1 - \lambda_1) V(\lambda_1) + \lambda_1 V(1).
\]

Similarly to \( \Phi_R \), let \( \bar{\phi}_1 \) denote the minimum of 1 and the intercept of the line on \( \phi_1 \)-axis, and \( \bar{\phi}_0 \) denote the maximum of the \( \phi_0 \)-coordinate of the line at \( \phi_1 = 1 \) and 0. By definition, \( \bar{\phi}_1 \) solves \( \Pi((0, \phi_1), 1) = \Pi(0, 0) \), and thus

\[
\bar{\phi}_1 \equiv \min \left\{ \frac{\frac{1}{2} + (2 + \lambda_1) V(\lambda_1) - \lambda_1 V(1)}{(1 - \lambda_1 + \lambda_1 \gamma) [V(\lambda_1) - V(\Lambda(1, 1)|\eta=1)]}, 1 \right\}.
\]

Moreover, if \( \bar{\phi}_1 \geq 1 \), \( \bar{\phi}_0 \) solves \( \Pi((\phi_0, 1), 1) = \Pi(0, 0) \), and thus

\[
\bar{\phi}_0 \equiv \max \left\{ 0, \frac{\frac{1}{2} + (1 - \lambda_1 + \lambda_1 \gamma) V(\Lambda(1, 1)) + (1 + 2 \lambda_1 - \lambda_1 \gamma) V(\lambda_1) - \lambda_1 V(1)}{(1 - \lambda_1 \gamma) [V(\lambda_1) - V(\Lambda(1, 0)|\eta=1)]} \right\}.
\]

I call the straight lines characterizing sets \( \Phi_R \) and \( \Phi_{PR} \) boundaries of the sets.

**Lemma 1.5.** The boundary of \( \Phi_{PR} \) is steeper than that of \( \Phi_R \).

Having established these properties, I provide the first characterization of the optimal retention rule when it exists.

**Proposition 1.3.** The condition (1.2) holds with equality with the optimal retention rule. Moreover, the optimal retention rule must satisfy \( \phi_0 = 0 \) unless \( \phi_1 = 1 \).

The first statement is simply due to the fact that the decision maker’s payoff is decreasing in both \( \phi_0 \) and \( \phi_1 \). She raises \( \phi \) just as high to recover the truthful equilibrium.
Moreover, Lemma 1.3 suggests that $\phi$-rule is not maximizing the decision maker’s payoff if $\phi_0 > 0$ when $\phi_1 < 1$. This implies that, when there are two instruments for rewarding a truthful report, the decision maker first rewards the advisor whose report turns out to be correct. She rewards the advisor when his report turns out to be incorrect only if setting $\phi_1 = 1$ is not attractive enough for the advisor to report truthfully. Immediate observation from this result is the following.

**Corollary 1.1.** When $\Omega$ is nonempty, the optimal retention rule in $\Omega$ is the $\bar{\phi}$-rule.

Finally, I establish the condition for the existence of the optimal $\phi$-rule, i.e., nonemptiness of $\Omega$.

**Lemma 1.6.** There exists an upper bound of career concern, $\delta''$, such that, for any $\delta > \delta''$, $\Omega$ is empty. This threshold value is given by

$$
\delta'' \equiv \frac{E(1, 1) - 3\gamma(1 - \gamma)}{v(1) - \gamma v(\Lambda(1, 1)|_{\eta=1}) - (1 - \gamma)v(\Lambda(1, 0)|_{\eta=1})}.
$$

Similar to the threshold value $\delta'$, $\delta''$ is characterized by the ratio between the net benefit in the first period from truthfully reporting the signal and the net benefit in the second period from misreporting the signal under the $\phi$-rule. The intuition behind this lemma is that if the unbiased advisor’s career concern is too large, it dwarfs his consideration about the effect of misreport on the current decision making. Therefore, no retention rule, even a perfect job security, can induce a truthful report by the advisor. However, if the advisor’s career concern is sufficiently small, i.e., $\delta < \delta''$, I have the following further characterization.

**Proposition 1.4.** For any $(\lambda, \gamma)$, there exists a $\delta_{\lambda, \gamma} \in (\bar{\delta}, \delta'']$ such that, for any $\delta \in (\bar{\delta}, \delta_{\lambda, \gamma}]$, $\Omega$ is nonempty. Moreover, if $\delta_{\lambda, \gamma} < \delta''$, then $\delta_{\lambda, \gamma}$ is increasing in both $\lambda$ and $\gamma$.

This proposition shows that for any $(\lambda, \gamma)$, there is at least some range of career concerns for which the optimal retention rule that profitably recovers the truthful equilibrium exists. Intuitively, it is derived from the comparison between the maximum
retention probability that the decision maker is willing to offer—the willingness to offer—and the minimum retention probability that the advisor is willing to accept for the truthful report—the willingness to accept. As mentioned earlier, the advisor’s willingness to accept is increasing in $\delta$. Then, as long as $\delta$ falls into that region, the decision maker’s willingness to offer is at least as large as the advisor’s willingness to accept. In those situations, the decision maker can profitably recover the truthful equilibrium by offering an appropriate retention probability to the advisor. I can now summarize the argument in the following proposition.

**Proposition 1.5.** Suppose the decision maker can commit to a retention rule.

1. For any $\delta \in (\bar{\delta}, \delta_{\lambda, \gamma}]$, $\bar{\phi}$-rule is the optimal retention rule.

2. For $\delta > \delta_{\lambda, \gamma}$, $\phi = (0, 0)$ is the optimal retention rule.

### 1.4.2 Effects on the decision maker’s welfare

Without commitment to a retention rule discussed in the previous section, there is no truthful equilibrium if $\delta > \bar{\delta}$. Therefore, the decision maker’s expected payoff drops discontinuously at $\bar{\delta}$ from $\Pi(0, 1)$, payoff in the truthful equilibrium, to $\Pi(0, 0)$, payoff in the politically correct equilibrium. However, if $\delta \leq \delta_{\lambda, \gamma}$, the decision maker can achieve a higher payoff by offering the optimal retention rule to obtain a truthful report compared to that in the politically correct equilibrium.

The optimal $\bar{\phi}$-rule is shown to be an increasing function of the advisor’s career concern. That is, the necessary retention probability increases as the advisor’s career concern increases. Since the decision maker’s payoff is decreasing in the necessary retention probability, her expected payoff also decreases as the advisor’s career concern increases.

The optimal retention rule, therefore, bridges the gap between $\Pi(0, 1)$ and $\Pi(0, 0)$ for a value $\delta \in [\bar{\delta}, \delta_{\lambda, \gamma}]$. Let $\phi^*(\delta)$ denote the optimal $\phi$-rule given the level of career concern
\[ \omega \lambda, \gamma \bar{\omega} \omega' \bar{\omega} \omega' \]

\[ \Pi(\omega^*(\delta), 1) \]

\[ \Pi(0, 1) \]

\[ \Pi(0, 0) \]

\[ \Pi(1, 0) \]

\[ \delta \]

\[ \delta_{\lambda, \gamma} \]

\[ \delta'' \]

\[ \Pi(0, 1) \]

\[ \Pi(\phi^*(\delta), 1) \]

\[ \Pi(0, 0) \]

\[ \Pi(1, 0) \]

\[ \Pi(0, 0) \]

\[ \Pi(1, 0) \]

\[ \delta \]

\[ \delta'' \]

\[ \text{Figure 1.5: Welfare effects of the optimal retention rule} \]

\[ \delta. \] By construction, \( \Pi(0, 1) = \Pi(\phi^*(\delta), 1) \) at \( \delta = \bar{\delta} \), and \( \Pi(1, 0) = \Pi(\phi^*(\delta), 1) \) at \( \delta = \delta'' \).

There are two possible cases. One is the case where \( \delta_{\lambda, \gamma} < \delta'' \) and, \( \Pi(0, 0) = \Pi(\phi^*(\delta), 1) \).

In this case, when \( \delta > \delta_{\lambda, \gamma} \), the decision maker could potentially recover the truthful report by raising the retention probability, but she chooses not to do so because it is too costly to her. Here, \( \Pi(0, 1) \) and \( \Pi(0, 0) \) is connected with the optimal retention rules, and the decision maker’s expected payoff is continuous with respect to \( \delta \). This case is shown on the left panel of the Figure 1.5. The other is the case where \( \delta_{\lambda, \gamma} = \delta'' \). In this case, the decision maker finds it optimal to raise the retention probability according to \( \delta \) as long as \( \delta \leq \delta'' \), at which point there is no feasible offer she can make for recovering the truthful report. Here, the decision maker’s expected payoff is discontinuous at \( \delta'' \). This case is shown on the right panel of the Figure 1.5.

\subsection{1.5 Conclusion}

This paper characterized the equilibrium of a two-period advice game when the decision maker can choose to retain or replace the advisor after the first period, and the advisor has reputational and career concerns. If the extent of career concern is small, there is an equilibrium in which the unbiased advisor truthfully reports the signal, while if it is sufficiently large there is a unique equilibrium in which the unbiased advisor gives the
politically correct report regardless of the observed signal.

When the politically correct equilibrium is the unique outcome of the benchmark model, the decision maker can still be better off by committing to a retention rule that potentially retains a biased advisor for the second period. The decision maker makes a trade-off between the benefit from extracting the useful information from the unbiased advisor in the first period and the cost of retaining the biased advisor in the second period. I showed the upper bound of the level of career concern under which the decision maker can find a retention rule that recovers the truthful report and constructed an optimal retention rule that maximizes the decision maker’s expected payoff. Moreover, I showed that the decision maker’s expected payoff is decreasing in the advisor’s level of career concern.
Chapter 2

Repeated Moral Hazard with Worker Mobility via Directed On-the-Job Search
2.1 Introduction

Firms usually do not hire new workers each period and many jobs are characterized by long-term relationships between firms and workers. Yet, workers typically change jobs many times in their lifetimes. Even if already working for a firm, they can look for better employer in the labour market\(^1\). When workers are mobile, their incentives at their current jobs depend not only on the current contract, but also on the labour market environment, such as a likelihood of obtaining an alternative and potentially more desirable employment opportunities within the labour market. The firm needs to take these complex incentives into account when offering a long-term wage contracts. On the other hand, a worker’s outside options in the market are the contracts offered by other firms that will face similar complexity. Therefore, the contracts offered by an individual firm and the worker’s outside options influence each other through their mobility and should be determined jointly in equilibrium\(^2\).

This paper proposes a search theoretic model of optimal employment contracts that integrates these two important attributes of the labour market—workers’ mobility in the labour market and their work incentive on the job—into a unified framework. Job search literature studies allocation of workers in a frictional labour market and the subsequent effects on macroeconomic variables, such as wage inequality or labour productivity (Pissarides, 1985; Mortensen and Pissarides, 1994). In particular, the models with on-the-job search explicitly study the workers’ job-to-job movement (Pissarides, 1994; Burdett and Coles, 2003; and Shi, 2009). However, they often do not explore the strategic interaction between workers and employers that results from the private information\(^3\). On the other

\(^1\)In the US, job-to-job transitions constitute 49% of all separation from employers over the past decade (Nagypál, 2008).

\(^2\)The literature on CEO compensation studies the interaction between the market and incentive contracts (Axelson and Bond, 2009; Cao and Wang, 2010). Cao and Wang (2010) also incorporate a dynamic search model and provide empirical support for the significance of this interaction. Manoli and Sannikov (2005) also study a dynamic contracting problem in a competitive equilibrium framework.

\(^3\)An important exception is Guerrieri, Shimer, and Wright (2010). They analyze a static model of search with adverse selection.
hand, dynamic contracting literature studies the strategic interactions between a worker and an employer over a long period of time (Rogerson, 1985; Holmstrom and Milgrom, 1987; Spear and Srivastava, 1987). However, those studies often assume that the outside environment is exogenously given to the contracting party and their relationship is fixed over the course of relationship. Therefore, despite its dynamic nature, dynamic contracting literature does not address the interaction between contracts and dynamic labour market environment through the worker mobility. This paper argues that the strategic interaction within a firm and worker mobility within the labour market are not mutually independent and must be considered together to fully elucidate the mechanism of the labour market. The framework will shed new light on wage inequality, business cycle fluctuation, and unemployment issues.

To analyze the interaction between work incentives and labour mobility within an equilibrium framework, I integrate a repeated moral hazard model à la Spear and Srivastava (1987) into an equilibrium search environment. Shi (2008) suggests the potentials of this type of integration of contract theory and search theory. However, analyzing a dynamic contracting problem in a standard random search framework is not an easy task. The difficulty comes from the fact that workers’ mobility, which is a critical variable for the incentive provision, is endogenously determined in equilibrium. In a random search framework, the mobility is determined by the distribution of workers in the labour market. Therefore the distribution, which is an infinite dimensional object, is a state variable for the optimal contracting problem, and it usually leads to an intractable model. The useful framework introduced by Shi (2009) and Menzio and Shi (2010) makes it possible to overcome this problem. By modeling search as a directed process, they introduce the class of equilibria in which the workers’ optimal decisions and the firms’ optimal contracts are independent of the distribution of workers over different contracts.\footnote{Two of the earliest formulations of directed search are introduced by Peters (1991) and Montgomery (1991). Acemoglu and Shimer (1999a, 1999b) and Moen (1997) are applications of the directed search in the labour market, and Delacroix and Shi (2006) is the first to study the directed on-the-job search.}
independence property, called block recursivity, is the key to finding an equilibrium in this framework\(^5\).

In my model, firms are risk-neutral and workers are risk-averse, and both are ex ante homogeneous. The firms competitively enter the labour market and offer long-term wage contracts to attract workers. A contract specifies a wage profile that depends on the worker’s tenure in the contract. Workers, both employed and unemployed, observe these offers and choose which contract to try to obtain. Once a firm and a worker form a match and sign a contract, they will engage in a series of projects. In each period, the firm pays the promised wage at the beginning, and the worker provides unobservable and costly effort in the project. A project results in either success or failure, and the probability of success increases as the worker’s effort increases. The match continues as long as its projects keep succeeding. Failure, however, physically destroys the match, and the worker is displaced into unemployment and needs to search for a new job in the market.

There are two types of frictions in this economy: search frictions and informational frictions due to the worker’s unobservable effort. On the one hand, the worker’s mobility is restricted by search frictions. The current value of contract subsequently becomes a reservation value for the on-the-job search, and it in turn determines the value of outside option–market value–for the worker. On the other hand, given moral hazard, labour productivity and job destruction probability are determined by the incentive structure. Therefore, the contract influences both how likely it is that the project succeeds and how likely it is that the worker leaves for another job through on-the-job search.

In equilibrium, there is a non-degenerate distribution of workers over different contract offers in the market. Even though the workers and firms are ex-ante homogeneous, search frictions and workers’ on-the-job search generate ex-post heterogeneity with respect to

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\(^{5}\)My modeling of the labour market is closely related to the latter.

\(^{5}\)Other applications of the block recursivity include Gonzalez and Shi (2010) and Menzio, Shi, and Sun (2009).
the values of contract. Despite the potential complication arising from this endogenous heterogeneity, the aforementioned block recursivity enables me to obtain a very simple characterization of the optimal contract. First, because of the firm’s incentives to induce effort as well as to retain the worker, the optimal contract exhibits an increasing wage-tenure profile; that is, the wage increases with the worker’s tenure on the job. In addition, the optimal incentive compatible effort also increases with tenure. Since the match productivity is positively correlated with the worker’s effort provision, the increasing effort process implies that the match productivity is also increasing over time. Therefore, the model provides an alternative theory to explain why a worker’s wage and productivity both increase with tenure. The key to this alternative theory is the evolution of the worker’s incentive in a market environment.

The model illustrates how the worker’s incentives endogenously evolve over time in a dynamic environment. The work incentives on the job come from two sources: one is the explicit incentives that are directly affected by the current wage contract, and the other is the implicit incentives that are affected by the worker’s outside option through on-the-job search. Firms use contracts in which the value to the worker increases with tenure by means of an increasing wage. The increasing value of the current job, in turn, increases the worker’s explicit incentive with tenure. However, as the value of the current job increases, it becomes more difficult for the worker to find a job that offers a higher value. It decreases the marginal value of the outside option, and therefore the market driven implicit incentive decreases with tenure. Firm’s optimal contract controls the flow of these incentives and optimizes total incentives. The result suggests that the

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6To the best of my knowledge, Lazear (1981) and Lazear and Moore (1984) are the first to study the incentive aspect of the age-earnings profiles. They show that steepening the age-earnings profile reduces the worker’s incentive to shirk and thereby increases his/her productivity. These studies, however, do not explicitly model the labour market as in the current paper and do not analyze the interaction between the work incentive and labour mobility.

7This type of incentive is often called the worker’s career concern. See Gibbons and Murphy (1992), Dewatripont, Jewitt, and Tirole (1999), Holmstrom (1999) for other models of optimal incentive contracts with career concerns. Gibbons and Murphy (1992) provide empirical support for the following prediction of the model.
total incentives increase with tenure, and the workers increase their work effort with tenure. This is why a worker's productivity increases with tenure. The mechanism predicts wage-productivity dynamics caused by worker mobility, which contrast with the predictions from other standard theories, including learning-by-doing, accumulation of human capital, and learning of worker's ability.

The incentive theory of productivity in this framework predicts an empirically sound endogenous job-destruction process. The increasing effort provision implies that the worker is more likely to succeed in the project as tenure increases and less likely to be displaced into unemployment. Moreover, workers are less likely to leave for another job the longer they stay on a job. Therefore, the rate of job-destruction—voluntary or involuntary—decreases with the tenure on the job. In addition, the increasing wage and productivity profiles provide plausible macroeconomic implications. First, the model's endogenous distribution of workers over different contracts generates dispersion in effective productivity among ex ante homogeneous workers as an equilibrium outcome. It is a natural consequence of the joint forces of endogenous productivities through the incentive mechanism and of the workers' on-the-job search in a frictional labour market. Moreover, the productivity dispersion that is endogenously generated in equilibrium can be used for the business cycle research. The model enables us to analyze how the workers' incentives affect the business cycle through their productivity on the job as well as their mobility within the labour market. For example, the model shows how a temporary reduction in workers' productivity on the job propagates in equilibrium and yields persistent effects on the economy's aggregate productivity through the distribution of workers.
2.2 Labour Market with Search Friction and Moral Hazard

2.2.1 Physical Environment

I consider a labour market with a continuum of infinitely lived workers with measure 1 and a continuum of firms whose measure is determined by competitive entry. All workers and firms are ex ante homogeneous. Time is discrete, continues forever, and is indexed by \( t = 1, 2, \ldots \). Each worker has a utility function \( u(w) \) where \( w \) is income in a period. I assume that workers cannot save nor borrow against their future income. The utility function \( u : \mathbb{R} \to \mathbb{R} \) is twice continuously differentiable, strictly increasing, and weakly concave. I further assume that the first derivative is bounded, i.e., \( u'(w) \in [\underline{u}', \underline{u}'] \) for all \( w \). When employed, each worker exerts costly effort, \( e \in \mathbb{R}_+ \), for the project of the firm in each period. The worker’s effort is unobservable to the employer. I assume that the cost of effort by a worker is given by a function \( c : \mathbb{R} \to \mathbb{R} \) that is twice continuously differentiable, strictly increasing, and weakly convex. Each worker maximizes the expected lifetime sum of utilities of income minus disutility of effort discounted at the rate \( \beta \in (0, 1) \).

Each firm is endowed with a series of projects. One project is executed in each period while the firm is employing a worker. It results in one of two possible outcomes: \( \{0, y\} \); when outcome is \( y \) it is called a “success,” and when 0 a “failure.” A failure physically destroys the project, and the employment relation breaks up. The probability of success in each period depends on an effort level by the worker employed in the firm and is given by \( r(e) \). I assume that \( r : \mathbb{R} \to \mathbb{R} \) is twice continuously differentiable, strictly increasing

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8This is a standard assumption in the literature. Also, it plays a key role in my model to isolate the effects of long-term contracts and the outside market on the workers’ incentive on the job.

9The following analyses are qualitatively unchanged if destruction after failure occurs with exogenous probability or if I assume that the firm fires the worker with some predetermined probability as in the efficiency wage models. I assume this particular separation rule for expositional simplicity.
and weakly concave in \( e \). I also assume that \( r'(0) < \infty \) and \( \lim_{e \to \infty} r(e) = 1 \). Each firm maximizes the expected sum of profits discounted at the rate \( \beta \).

### 2.2.2 Contractual Environment

I assume that firms commit to a long-term contract. An employment contract specifies the worker’s wage profile as a function of his tenure in the firm conditional on the employment relationship. In addition, since the worker’s effort levels are unobservable, the firm needs to determine how much effort by the worker to induce given the proposed wage offer. Therefore, a contract specifies a wage profile \( \{w_t\}_{t \geq 0} \) and a recommended effort profile \( \{e_t\}_{t \geq 0} \). Such wage and recommended effort profiles jointly deliver an expected lifetime utility to the workers that is referred to as a value of contract and denoted by \( x \).

Firms offer contracts with different values to attract workers, so the labour market consists of a continuum of submarkets indexed by \( x \). I assume that \( x \in X \), where \( X \) is a large enough closed interval \([\bar{x}, \bar{x}]\). The ratio of vacant firms to searching workers in submarket \( x \) is denoted by \( \theta(x) \) and is referred to as the tightness of submarket \( x \). Let \( G \) be the cumulative distribution of workers over \( X \) and \( u \) be the fraction of unemployed workers.

### 2.2.3 Workers’ Job Search

In each period, there are three stages: separation stage, search and matching stage, and production stage. In the separation stage, an employed worker loses her job if the project in the previous period failed. If a worker loses the job in the separation stage, she must stay unemployed for a period and cannot immediately search in the following stage.

In the search and matching stage, firms post a vacancy at a flow cost \( k > 0 \) and offer a contract to recruit a worker. Both employed and unemployed workers search for a new job. An employed worker who did not separate from her job in the previous stage finds an opportunity of searching for a new job with probability \( \lambda_e \in [0, 1] \). An
unemployed worker who lost her job in the previous period finds an opportunity of searching with probability \( \lambda_u \in [0,1] \). Given the opportunity to search, all workers observe all the available contract offers in the labour market and choose which submarket to enter. I assume a standard matching technology as in the job-search literature. If a worker chooses to visit submarket \( x \), she meets a vacant firm with probability \( p(\theta(x)) \), where \( p : \mathbb{R}_+ \rightarrow [0,1] \) is twice continuously differentiable, strictly increasing and strictly concave, and satisfies \( p(0) = 0, p'(0) < \infty \). On the other hand, if a vacant firm enters a submarket \( x \), it finds a worker with probability \( q(\theta(x)) \), where \( q : \mathbb{R}_+ \rightarrow [0,1] \) is twice continuously differentiable, strictly decreasing and strictly convex, and satisfies \( \theta^{-1}p(\theta) = q(\theta), q(0) = 1 \). In addition, the matching technology is assumed to satisfy the condition that \( p(q^{-1}(\cdot)) \) is concave. If an employed worker matches with a firm and accepts the offer, she must leave her previous employment position before entering the production stage with a new firm\(^{10}\). If she rejects the offer, she enters the production stage with her current employer.

### 2.2.4 Workers’ Productive Behaviour

In the production stage, each unemployed worker receives and consumes unemployment benefit \( b \). An employed worker receives the current period wage \( w_t \) as specified in the contract and exerts effort \( e \) for the current period project. At the end of this stage, the project outcome, which stochastically depends on the worker’s effort and technology \( r(\cdot) \), is publicly realized. It is important to note that the timing of events implies that the wage in the current period has no effect on the worker’s effort choice in the current period. I will precisely explain the worker’s choice of effort in the following section.

\(^{10}\)As shown in the following, the optimal contract takes into account the probability of losing the worker for another firm and maximizes the flow value of the job. Therefore, it is without loss of generality to assume that the current employer does not make a counteroffer in response to the employee’s on-the-job search.
2.3 Equilibrium Conditions

In the following, I describe the equilibrium conditions for this economy. First, I describe a market tightness, which is consistent with the firm’s free entry condition. Second, taking the market tightness function as given, I describe the worker’s problems in terms of optimal job search and optimal effort choice when employed. The value of the unemployed worker is also defined. Then, following the approach taken by Spear and Srivastava (1987), I describe the optimal contracting problem as a recursive problem, which takes the value of contract as the state variable. Lastly, I will define the equilibrium of the economy that constitutes these equilibrium conditions.

2.3.1 Market Tightness and Free Entry Condition

During the search stage, firms choose how many vacancies to create and where to locate them. Let $J(x)$ denote a firm’s value of employing a worker in submarket $x$. Then, the firm’s expected benefit of creating a vacancy in submarket $x$ is given by $q(\theta(x))J(x)$, the product of the probability and the value of meeting a worker in the submarket. Given the market tightness function $\theta(x)$, if the cost $k$ of creating a vacancy is strictly greater than the expected benefit, then firms do not create any vacancies in submarket $x$. If $k$ is strictly smaller than the expected benefit, then firms create infinitely many vacancy in $x$. When they are equal, the expected profits of the firm are zero and are independent from the number of vacancies created by an individual firm in submarket $x$. Therefore, in any $x$, $\theta(x)$ is consistent with firms’ profit maximization if

$$q(\theta(x))J(x) - k \leq 0,$$  \hspace{1cm} (2.1)

and $\theta(x) \geq 0$, with complementary slackness.
2.3.2 Worker’s Problems

Optimal Search of the Worker

Consider a worker in the search stage. For an employed worker, the worker can compute the value of the remaining contract that has been offered by the firm. For an unemployed worker, the remaining value is the value of unemployment. Suppose the remaining value of the contract for a worker is $W$. Given this value, the worker’s search problem is as follows. If the worker receives an opportunity to search for a new contract and visits submarket $x$, he finds an employer with probability $p(\theta(x))$ and obtains the additional value of $x - W$. The worker chooses which submarket to visit to maximize the expected value of search. I denote the worker’s value of search given $W$ as

$$D(W) = \max_{x \in \mathbb{R}} p(\theta(x))(x - W).$$ (2.2)

I denote with $m(W)$ the worker’s optimal search policy of this problem and define the composite function $\hat{p}(W) = p(\theta(m(W)))$. That is, $\hat{p}(W)$ is the probability that a worker with remaining value $W$ successfully finds an employer in the optimally chosen submarket.

Optimal Effort Choice of the Worker

Consider an employed worker in the production stage. He chooses how much effort to provide for the current project. The trade-off is between the benefit of staying employed and the cost of current effort. When the worker exerts effort $e$, the project succeeds with probability $r(e)$. If the project succeeds, the worker can stay employed, and the current contract will provide a continuation value to the worker for the next period $W$. In the next search stage, if he receives the opportunity to search with probability $\lambda_e$, he will obtain the expected value of $W + D(W)$ through the optimal search; that is, the continuation value of the current contract plus an expected value of optimal on-the-job search with $W$. If he does not receive the opportunity to search, he keeps the reservation value $W$. On the other hand, if the project fails, the worker will lose the job and spend
as unemployed during the next period, receiving the value of unemployed $U$, which is explained below. Hence, the net expected value of success with the effort level $e$ is calculated as

$$r(e)(\lambda_e(W + D(W)) + (1 - \lambda_e)W) + (1 - r(e))U.$$  

The cost of effort level $e$ is $c(e)$. As mentioned earlier, given the structure of the contract and the timing of events, the wage in the current period has no impact on the worker’s choice of effort. Therefore, given the continuation value $W$, the worker will choose his effort level to solve the following problem.

$$\max_{e \in \mathbb{R}} \left(- c(e) + \beta \left(r(e)(W + \lambda_e D(W)) + (1 - r(e))U\right)\right).$$

Note that the expected benefit from the effort is given in the next period and the worker discount its value at $\beta$. The optimal choice of effort is given as a function of the continuation value $W$ and denoted by $e(W)$.

**Worker’s Value of Unemployment**

Finally, consider an unemployed worker who fails to find a job during the current search stage. If $U$ denotes the value of unemployment, it must satisfy the following functional equation.

$$U = u(b) + \beta \left(U + \lambda u D(U)\right). \quad (2.3)$$

First, the worker obtains $u(b)$ from the unemployment benefit. In the next period, if he receives an opportunity to search for a job, he will obtain the expected benefit of $U + D(U)$ through the optimal search. If he does not receive an opportunity to search, he will stay unemployed and receive $U$ again. Simplifying the expression with discount factor $\beta$ gives the right hand side of the equation.
2.3.3 The Firm’s Optimal Contracting Problem

Consider a firm that promises to provide a continuation value $V$. Let $J(V)$ denote the value of a contract for the firm. Following the recursive contract approach, the firm’s problem is expressed as the choice of three objectives: i) $w$, how much wage to pay in this period, ii) $e$, how much effort to induce, and iii) $W$, how much continuation value to promise to the worker in the next period conditional on success of the project. I augment the problem by allowing for a randomization over these choices; that is, the firm offers two sets of subcontract and a probability distribution, $\{\pi_i\}_{i=1,2}$, over these subcontracts\textsuperscript{11}. Let $\xi = (\{w_i, e_i, W_i, \pi_i\}_{i=1,2})$ denote the contract offered by the firm at the beginning of a period. The firm’s optimal contracting problem is given by

$$J(V) = \max_{\xi \in \Xi} \sum_{i=1,2} \pi_i \{r(e_i)y - w_i + \beta r(e_i)(1 - \lambda e\delta(W_i))J(W_i)\}$$

subject to

$$\Xi = \{w_i, e_i, W_i, \pi_i\}_{i=1,2} : W_i \in X \text{ for } i = 1, 2; \pi_1 + \pi_2 = 1, \pi_i \in [0,1] \text{ for } i = 1, 2$$

$$V = \sum_{i=1,2} \pi_i \{u(w_i) - c(e_i) + \beta [r(e_i)(W_i + \lambda eD(W_i)) + (1 - r(e_i))U]\}$$

$$e_i \in \arg \max_{e \in \mathbb{R}} \left(-c(e) + \beta (r(e)(W + \lambda eD(W)) + (1 - r(e))U)\right), \text{ for } i = 1, 2.$$ 

By inducing an effort level $e$ and paying $w$, the firm’s expected current period payoff is $r(e)y - w$. In the next period, by offering continuation value $W$, the current worker will stay in the current firm with probability $r(e)(1 - \lambda e\delta(W))$: product between a probability the project succeeds and a probability the worker will not leave for the outside option, and the firm will enjoy the value $J(W)$ of remaining contract.

In designing the contract, the firm faces three constraints. The first is the promise-keeping constraint; that is, the contract has to provide the worker with the promised

\textsuperscript{11}The purpose of introducing the lottery is the potential nonconvexity of the constraint set and resulting nonconcavity of the firm’s objective function; if the firm faces this problem, the value can be improved by randomizing over the control variables.
value $V$. Since the offer of the contract is evaluated ex ante, only the expected value of subcontracts must provide the promised continuation value; either subcontract may fail to provide the promised value. The other constraint is that the effort that the firm wants to induce must be incentive compatible; that is, based on the contract offered, the worker voluntarily chooses to exert that level of effort. Since realization of a subcontract occurs before the worker chooses his effort level, both subcontracts the firm prepares need to meet the incentive compatibility constraint. Finally, when the firm randomizes the contracts, the probabilities assigned to each subcontract must sum to one. I denote with $\xi(V)$ the optimal policy functions given $V$ associated with this contracting problem.

### 2.3.4 Block Recursive Equilibrium

Given the above description, a market equilibrium is defined as follows.

**Definition 2.1.** A Block Recursive Equilibrium is a set of functions \{{J^*, \theta^*, D^*, m^*, U^*, \xi^*}\} and a sequence of distribution of workers \{\{(G^*_t, u^*_t)\}_{t \geq 0}\) such that

1. A market tightness $\theta^*$ satisfies condition (2.1),

2. A value of job search $D^*$ and an optimal job search policy $m^*$ satisfy the equation (2.2),

3. A value of unemployment $U^*$ satisfies the equation (2.3),

4. A value of firm $J^*$ and a optimal contract policy $\xi^*$ satisfy the equation (2.4), and

5. $\{(G^*_t, u^*_t)\}_{t \geq 0}$ is consistent with $m^*$ and $\xi^*$.

Since the definition is not restricted to the stationary equilibrium, the distribution of workers over the value of the contract evolves over time depending on workers’ transition across jobs in the market, as well as within the firm. The last condition, therefore, requires that the evolution of distributions over time is consistent with the optimal job search policy, as well as optimal contract policy.
Note that the functions \( \{J^*, \theta^*, D^*, m^*, U^*, \xi^*\} \) are independent of the distribution of workers \((G_t^*, u_t^*)\) in any period \( t \geq 0 \). For each individual’s decision making, the market tightness of each submarket is the sole relevant variable. For example, a searching worker only cares about the tradeoff between the employment probability and the value of contract offered in the submarket in which she will visit, and not about how many workers and firms are distributed in other submarket. This class of equilibrium makes the analysis of the model very tractable. First of all, we can solve the model as if there is a representative worker and a representative firm without referring to the distribution of workers over a set of contracts. Secondly, since the individual worker’s and firm’s behaviour can be characterized outside of the steady-state, we can trace the evolution of the distribution of workers outside of the steady-state. It enables us to analyze the macroeconomic variables, such as unemployment rate and aggregate productivity, on the transition phase over the business cycle. I will discuss its implication in more detail in the final section.

### 2.4 General Properties and Existence of an Equilibrium

In this section, I first describe general properties of equilibrium objects. I will then establish the existence of an equilibrium with these properties. Proofs of each lemma are minor modifications of Menzio and Shi (2009) due to the worker’s effort choice. These necessary modifications are included in Appendix B.1.

I start with specifying a set of functions with certain properties. Then, I take an arbitrary function from the set as a firm’s value function and characterize the equilibrium objects given the properties of functions. With all these results, I construct an operator defined over the set of functions and show that any fixed point of the operator is a Block Recursive Equilibrium.
2.4.1 Market Tightness and Free Entry Condition

First, I define the set $\mathcal{J}(X)$ of functions $J : X \to \mathbb{R}$ such that:

(i) $J(V)$ is strictly decreasing and Lipschitz continuous with respect to $V$,

(ii) $J(V)$ is bounded both from below and above, and

(iii) $J(V)$ is concave.

I can show that the set $\mathcal{J}(X)$ is non-empty, bounded, closed, and a convex subset of the space of bounded, continuous functions on $X$ with the sup norm.

Then, I take an arbitrary firm’s value function $J \in \mathcal{J}(X)$ and solve the equilibrium condition (2.1) with respect to the market tightness function $\theta$. I get

$$
\theta(x) = \begin{cases} 
q^{-1}(k/J(x)) & \text{if } J(x) \geq k \\
0 & \text{otherwise.}
\end{cases}
$$

(2.5)

Because $q$ is a probability, $q^{-1}$ is defined only on $[0, 1]$, that is $\theta$ is well defined only when $J(x) \geq k$. Since $J(x)$ is strictly decreasing, there exists a unique $\tilde{x} \in \mathbb{R}$ such that $J(x) > k$ for all $x < \tilde{x}$ and $J(x) < k$ for all $x > \tilde{x}$. That is, offering any contract with the value more than $\tilde{x}$ provides negative expected profits even if the firm can hire a worker with probability one. Therefore, no firm will enter submarkets with $x > \tilde{x}$, and the market tightness takes nonnegative value only if $x \leq \tilde{x}$. I call this threshold value an upper bound of the competitive entry given $J$. Since $J$ is bounded from above, $\theta$ is also bounded from above. Now I have the following properties of market tightness functions.

**Lemma 2.1.** If $x < \tilde{x}$, the market tightness function $\theta(x)$ is strictly positive and strictly decreasing, and Lipschitz continuous. $\theta(x) = 0$ for all $x \geq \tilde{x}$.

2.4.2 Worker’s Problem

Optimal Search of the Worker

I have the following properties of worker’s optimal search problem.
Lemma 2.2. Given the market tightness $\theta(x)$ defined in (2.5),

1. For all $W \in X$, the worker’s objective function $f(x; W) = p(\theta(x))(x - W)$ is strictly concave with respect to $x$.

2. The worker’s optimal search strategy $m(W) \in \arg \max p(\theta(x))(x - W)$ is unique, weakly increasing and Lipschitz continuous.

3. If $W < \tilde{x}$, the worker’s value of searching $D(W)$ is strictly positive and weakly decreasing and Lipschitz continuous. If $W \geq \tilde{x}$, $D(W) = 0$.

The strict concavity of the problem implies that, given a continuation value of the current contract, the worker will find a unique submarket that she optimally visits to find a new job. Moreover, the worker’s optimal search policy is monotone. A worker with higher continuation value $W$ searches in a submarket that offers a higher value of the contract, and a worker with lower continuation value searches in a submarket that offers a lower value of the contract, i.e., $m(W_2) \geq m(W_1)$ for $W_2 > W_1$. Finally, as long as a worker’s current continuation value is less than $\tilde{x}$, there are submarkets that offer higher values and they provide positive expected value of search. However, once his continuation value reaches the bound of the market, there are no outside firms offering better offers and the value of the search becomes zero; $D(W) = 0$ since $p(0) = 0$. Combining these properties results in the following lemma.

Lemma 2.3. $\hat{p}(W)$ is weakly decreasing and Lipschitz continuous. Moreover $\hat{p}(\tilde{x}) = 0$.

Remember that the composite function $\hat{p}(W)$ is a probability that a worker meets a firm in the optimally selected submarket when a continuation value of the current contract is $W$. Since a worker with higher $W$ applies to a submarket with higher tightness, this property implies that a worker with higher continuation value is less likely to find a favorable outside option.
Optimal Effort Choice of the Worker

I now turn to a worker’s optimal effort choice problem:

\[ \max_{e \in \mathbb{R}} \left( -c(e) + \beta(r(e)(W + \lambda eD(W)) + (1 - r(e))U) \right). \]

Under the assumption on \(c(\cdot)\) and \(r(\cdot)\), it is a concave problem with respect to \(e\). Therefore, the first-order condition sufficiently characterizes the optimal effort.

Lemma 2.4. Given a continuation value \(W\), there is a unique level of optimal effort. Moreover, the worker’s optimal effort function \(e(W)\) is increasing in the continuation value of contract, \(W\).

2.4.3 Existence of a Block Recursive Equilibrium

So far, I have examined properties of equilibrium objects given an arbitrary firm’s value function \(J \in \mathcal{J}(X)\). In this subsection, I establish that a Block Recursive Equilibrium exists.

Proposition 2.1. A Block Recursive Equilibrium exists.

Detailed proof is given in the appendix. I will only outline the argument in the proof. First, by inserting all the previous equilibrium objects, given an arbitrary value function \(J \in \mathcal{J}(X)\), into a firm’s optimal contracting problem, I construct the following operator that maps from \(\mathcal{J}(X)\) to some space of functions:

\[ (TJ)(V) = \max_{\xi \in \Xi} \sum_{i=1,2} \pi_i \{ r(e_i)y - w_i + \beta r(e_i)(1 - \lambda \hat{p}(W_i))J(W_i) \} . \]

I show that \(T\) is a self-map; \(T\) maps from \(\mathcal{J}(X)\) to itself (Lemma B.8). Then, I show that \(T\) is a continuous map (Lemma B.9). These suffice to show that the operator \(T\) satisfies the assumptions of Schauder Fixed Point Theorem (Stokey and Lucas with Prescott, 1989, Theorem 17.4); that is, (i) \(T\) is continuous, (ii) the family of functions \(T(\mathcal{J})\) is equicontinuous, and (iii) \(T\) maps the set \(\mathcal{J}(X)\) into itself. Therefore, there exists a firm’s
value function $J^*(V) \in \mathcal{J}(X)$ such that $TJ^* = J^*$. Let $\{\theta^*, D^*, m^*, U^*, \xi^*\}$ denote the respective functions associated to $J^*$. By construction, the functions $\{J^*, \theta^*, D^*, m^*, U^*, \xi^*\}$ satisfy conditions in the definition of a Recursive Equilibrium and do not depend on the distribution of workers. Therefore, they constitute a Block Recursive Equilibrium.

### 2.5 Characterization of the Optimal Long-Term Contracts

I now characterize the contract that solves a firm’s optimal contracting problem in Section 2.3.3. I first state the following result.

**Lemma 2.5.** Under the optimal contract, the current period wage is independent of the realization of lottery.

This proposition implies that the lottery does not randomize the current wage. However, it does not imply that the lottery is not used in equilibrium. In principle, a lottery randomizes over subcontracts $\{w_i, e_i, W_i\}_{i=1,2}$. As Lemma 2.4 implies, the choice of $W_i$ uniquely determines $e_i$, so the lottery technically randomizes over $\{w_i, W_i\}_{i=1,2}$. As I mentioned earlier, the lottery is used to randomize over the control variables and to improve the firm’s value when the firm’s objective function is not concave. This proposition implies that randomizing over the continuation values of contract is sufficient for this matter.

Using this result, the firm’s optimal contracting problem takes the following reduced form.

$$
J(V) = \max_{\phi} \left\{ -w(V, \phi) + \sum_i \pi_i r(e(W_i)) \left[ y + \beta (1 - \lambda e \hat{p}(W_i)) J(W_i) \right] \right\} 
$$

where

$$
\phi \in \left\{ \{W_i, \pi_i\}_{i=1,2} : W_i \in X, \; \pi_i \in [0,1] \; \text{for} \; i = 1,2 \; \text{and} \; \pi_1 + \pi_2 = 1 \right\}.
$$
Here $w(V, \phi)$ is a wage that is consistent with the current value of contract $V$ and the randomized continuation values.\(^\text{12}\)

I am interested in the shape of long-term wage-tenure profile in this environment, and this problem is simple enough that I can characterize the path of optimal wages in a recursive manner. But, first, I establish the following lemma.

**Lemma 2.6.** Under the optimal contract, a firm offers a contract with nondecreasing continuation values over the tenure, independent of the realization of lottery, i.e., $V \leq W_i$ for $i = 1, 2$.

This lemma characterizes the dynamics of continuation values of the contract. Once the value profile is determined, the optimal wage profile is recursively characterized as follows.

**Proposition 2.2.** Under the optimal contract, a firm offers a contract with a nondecreasing wage profile, independent of the realization of the lottery. The wage dynamics on the contract is characterized by:

$$\frac{1}{w'(w(W_i))} - \frac{1}{w'(w(V))} = -\frac{\lambda e J(W_i) \hat{p}'(W_i)}{1 - \lambda e \hat{p}(W_i)} + \frac{r'(e(W_i))}{r(e(W_i))} \left( \frac{y}{\beta} + (1 - \lambda e \hat{p}(W_i)) J(W_i) \right) g'(\Omega(W_i))$$

(2.7)

where $w(W_i)$ is the next period wage when the continuation value given the current period lottery is $W_i$.

In addition to the optimal wage-tenure profile in the above proposition, Lemma 2.6 implies the following property of an equilibrium effort provision by the workers.

**Proposition 2.3.** Under the optimal contract, the worker’s optimal effort levels are nondecreasing with tenure.

Propositions 2.2 and 2.3 completely characterize the optimal contract. The next section discusses the key features of the optimal contract and the implied wage dynamics, as well as the productivity dynamics in the labour market.

\(^{\text{12}}\)See proof of Lemma 2.6 in Appendix B.1 for the explicit expression.
2.6 Empirical Implications and Related Literature

This section discusses key features of the optimal contract characterized in the previous section and economic insights based on these key features. I will compare the results of this paper to previous studies to highlight the novelty of this unified framework.

2.6.1 The Optimal Incentive Contract

The essence of Propositions 2.2 and 2.3 is that the interaction between workers’ work incentive on the job and their mobility in the labour market jointly induce both wages and productivity to increase over a worker’s tenure in a firm. There is an extensive amount of literature that examines the properties of long-term wage contracts. Harris and Holmstrom (1982) and Holmstrom (1983) characterize the equilibrium long term labour contracts in a frictionless labour market, and Burdett and Coles (2003), Stevens (2004), and Shi (2009) characterize the optimal wage-tenure contract in search and matching framework. With or without friction in the labour market, these studies both show that wages never decline with age or tenure at the firm. In the former, wages increase in response to an increased market value of the worker as a result of revelation of the worker’s private information, and in the latter wages increase to prevent the worker from leaving for another firm. These are plausible reasons for an increasing wage-tenure profile, but these do not speak to the question of how wages and productivity on the job are correlated.

Lazear and Moore (1984) examine the relationship between age-earnings profiles and worker incentives and show that most of the slope in age-earning profiles is accounted for by the firm’s desire to provide incentives, rather than by the effects of human capital accumulation through on-the-job training. This approach, however, does not speak to the question of why workers’ job-to-job transitions usually result in wage increase.

These economic insights are provided by (2.7), which describes how wages evolve
optimally with tenure. If the worker’s productivity is exogenously given, then $r' \equiv 0$ and the wage dynamics will be characterized only by the first term of the right hand side in (2.7). This term regulates how the current promised value affects the probability that the worker finds an alternative job. The firm faces a trade-off between the cost and benefit of retaining the worker conditional on the current value of contract. In this case, if the worker mobility is exogenously given, i.e., $\hat{p}(\cdot)$ is fixed at some nonnegative constant (e.g., fixed at 0 for no on-the-job search and 1 for frictionless economy), then the firm has no incentive to raise the wage, and a constant wage offer would result.

However, if the productivity is endogenously determined by the workers’ unobservable effort choice, the firm still needs to provide incentives. As Lazear and Moore (1984) show, it is optimal for the firm to offer increasing wages with tenure at the firm, which is characterized by the second term on the right hand side in (2.7). This model provides a simple framework to analyze the wage dynamics that is consistent with these two mechanisms.

The above analysis implies that the labour market is divided into a segment of competitive markets and a segment of monopsonistic markets. As long as the promised value of a contract $x$ is below $\tilde{x}$, $J(x) \geq k$ and new firms enter the markets to find a worker. Here, workers move across firms depending on their search opportunity. In this competitive segment, wages and productivity increase due both to the increasing continuation value within the firm and to the move across jobs. Once the continuation value reaches $\tilde{x}$, no outside firm offers a higher value to recruit him, i.e., $J(x) \leq k$. If there is no incentive problem for the effort provision, the firm that has been employing the worker up to this value does not have an incentive to offer any higher value to the worker. However, in this model, the firm still wants to induce a higher effort by offering a higher continuation value. According to (2.7), this process continues until either an additional

\footnote{In the previous studies on on-the-job search, this point determines the upper bound of the labour market.}
effort by the worker will not provide a higher probability of success, i.e., \( r' = 0 \) or an additional value to the worker will not generate higher effort by the worker, i.e., \( g' = 0 \). Therefore, even though there is no search opportunity for the worker, the firm’s optimal contract still exhibits an increasing wage profile. In this range, the current employer is the sole buyer of the worker’s labour, and this range of promised value constitutes a monopsonistic segment of the labour market.

### 2.6.2 Worker Mobility and Wage-Productivity Dynamics

Search and matching models are useful framework to analyze the individual earning process as an outcome of the worker mobility. Kambourov and Manovskii (2009 ab) study the workers’ occupational mobility in the frictional labour market and analyze the resulting individual wage dynamics and wage inequality. They assume that, based on empirical findings, the workers accumulate human capital that is occupation specific, so changes in occupation naturally result in earnings changes. On the other hand, Postel-Vinay and Turon (2010) analyze the model with on-the-job search where workers face idiosyncratic i.i.d. productivity shocks. They show the combined assumptions of on-the-job search and wage renegotiation act as a propagation mechanism of the shocks, which generates the individual earnings process.

The current model has none of these potential sources of wage dynamics, but the simple structure of the optimal contract provides explicit prediction of the earning processes due to worker mobility within the firm and the labour market. In this model, an unemployed worker searches for a job with a low value but a high probability of employment. A low value contract offers a wage profile with low initial wage. However, her wages increase with tenure if the projects keep succeeding. If she finds an opportunity to look for the outside option, she will search for a contract with a higher value offering a wage profile with higher initial wage. Therefore, as long as she stays employed, her wages keep increasing with tenure, not only on the job, but also on the labour market. On the
other hand, if her project fails (and it inevitably will), she becomes unemployed. Once displaced, she searches for a job with a low value and begins the process all over again. Therefore, a worker faces significant wage decrease if she goes through an unemployment spell. These wage dynamics based on workers’ transitions across employment statuses is consistent with empirical observations.

In addition, a novel prediction of this model is the productivity dynamics. As above, an unemployed worker initially finds a low value contract, and it offers a low continuation value to the worker. A low continuation value provides a low work incentive, and as a result his productivity is initially low. However, the continuation values increase with tenure if he stays employed, so his productivity increases with tenure on the job. Moreover, if he moves to a new job, it offers an even higher continuation value than the original contract would provide. Therefore, his productivity further increases if he moves across jobs without a period of unemployment. On the other hand, if a worker loses the job due to project failure, he goes through a period of unemployment and will again start with a low incentive job. Therefore, his productivity drops lower than the previous job. This is a novel, but plausible explanation why unemployment reduces workers’ productivity.

The implications of this incentive theory of wage and productivity dynamics qualitatively contrast with the other potential mechanisms. First, the human capital approach does not explain why a period of unemployment reduces productivity and thus wages, even though the worker stays in the same industry or occupation. The literature often assumes that human capital depreciates during the period of unemployment (e.g. Shimer and Werning 2006). Though this might be a reasonable explanation, the incentive approach also provides natural explanation without an additional assumption. Second, the match-specific productivity approach does not explain upward wage changes as a result of job-to-job transition without further assumptions of ex ante heterogeneity and sorting in the market.\footnote{See Bagger and Lentz (2008), Lise, Meghir, and Robin (2008), and Lapes de Melo (2008) for this}
simpler explanation.

2.6.3 Endogenous Job Destruction

In the standard search and matching models without on-the-job search, job destruction occurs when the productivity of a job falls below a certain threshold value, either because of the arrival of idiosyncratic productivity shocks or because of the arrival of a general shock that affects many firms (Mortensen and Pissarides 1994, Pissarides 2000). In this scenario, job destruction displaces the worker into unemployment. In addition, several studies incorporate workers’ on-the-job search to analyze the endogenous job destruction, which results in the worker’s job-to-job transitions (Pissarides 1994, Menzio and Shi 2008).

There are two sources of job destruction in this model. However, unlike previous models, there is no job destruction that is caused by exogenous productivity shocks. On the one hand, the labour productivity is determined by the worker’s effort, which in turn is determined by the firm’s incentive provision. Therefore, the worker’s involuntary separation is still endogenously determined. On the other hand, the worker’s voluntary separation is determined by the on-the-job search whose likelihood is determined in the market. Therefore, this model provides a simple unified framework to capture both voluntary and involuntary job destruction that is determined in equilibrium.

Not only does this model provide potential mechanisms of voluntary and involuntary job destruction, this incentive approach provides a plausible explanation as to why the rate of job destruction, both voluntary and involuntary, often decreases with tenure on the job. That is, in this model, probability of losing the job is closely related to the worker’s effort provision. A higher effort implies higher probability of project success and thus, higher probability of keeping the job. The above characterization of the optimal contract implies that the worker’s effort provision increases with tenure on the job. This in turn approach.
implies that probability of losing the job (i.e., involuntary job destruction) decreases with tenure on the job. Moreover, as in the previous studies evaluating on-the-job search, the probability of finding a better job offer decreases with tenure ($\hat{p}(\cdot)$ is a decreasing function). Therefore, the probability that a worker voluntarily leaves the job for a better job also decreases with tenure on the job.

### 2.6.4 Wage-Productivity Dispersion

The job-search models have often been analyzed in the context of wage dispersion (Burdett and Mortensen 1998, Burdett and Coles 2003, Shi 2009). While these models provide potential mechanisms of wage dispersion, it is generally understood that the standard models generate only a small amount of frictional wage dispersion (Hornstein et al, 2010). Due to the aforementioned wage dynamics in a frictional labour market, this model also generates wage dispersion. Whether the model provides a quantitatively significant contribution to the literature is left for further study, and Chapter 3 of this thesis investigates this potentially important contribution.

This model, however, has its important implications with respect to productivity dispersion. Acemoglu and Shimer (2000) study a search model with the firm’s endogenous technology choice. The same forces causing wage dispersion also generate endogenous technology dispersion, and thus dispersion of the productivity of labour. The current model, on the other hand, endogenizes the productivity of labour through workers’ effort choice under moral hazard. Unlike firms’ technology choices, workers’ effort choices change over time depending on the incentives, even if the worker stays in one firm. This leads to dynamic heterogeneity among identical workers and thus to a productivity dispersion. This interaction of work incentives and labour mobility is a new mechanism, and it has further useful business-cycle implications as I discuss in detail in the next section.
2.6.5 Business-Cycle Implication

The job-search models have natural business cycle implications, thanks to their explicit description of the labour market—how workers are allocated and how jobs are created and destructed (Andolfatto 1996, Shimer 2005, 2010). The closest work to my model is Menzio and Shi (2008) in that they model the search as a directed process and allow on-the-job search. On the other hand, early efficiency wage models developed by Shapiro and Stiglitz (1984) analyze workers’ effort choices and resulting unemployment as a source for the business cycle fluctuation. Recently, Alexopoulos (2004) presents a model without search friction, but with a moral hazard in a dynamic general equilibrium framework and studies the business cycle behaviour.

The present paper incorporates those two approaches to provide further implications for the business cycle research. In particular, this model predicts a mechanism through which a temporary productivity shock to the economy propagates with persistent effects on the aggregate productivity over the business cycle. If a negative shock to aggregate productivity destroys some matches despite workers’ high effort on the job, newly displaced workers will search for jobs. As discussed above, these workers start with a low incentive, low productivity job. Therefore, the average productivity immediately drops due to the initial shock. The initial shock not only affects the current productivity on the job, but also affects worker distribution over the values of the contract\textsuperscript{15}, i.e., more workers are employed at low incentive, low productivity jobs and fewer workers are employed at high incentive, high productivity jobs than would occur in the steady state. Over time, each worker’s productivities increase with their tenure in the labour market, and the worker distribution gradually moves back toward the steady state condition that existed before the shock. Due to these distributional effects, overall productivity slowly returns to previous levels. This interaction between worker’s incentives on-the-job and

\textsuperscript{15}See Appendix B.3 for detailed argument.
their mobility in the labour market may be the keystone mechanism for the delayed recovery response after an economic shock.

2.7 Conclusion

The purpose of this paper was to explore how workers’ incentives inside a firm interact with their mobility in the labor market. To this end, I have developed a search theoretic model of employment contracts with repeated moral hazard. I found that in equilibrium the optimal long-term contract is characterized by an increasing wage-tenure profile. The optimal incentive compatible effort level also increases with tenure. Even though all workers and firms are ex ante homogeneous, these two outcomes jointly generate time-varying endogenous heterogeneity of the wages and labor productivities of the match.

The contribution of this paper is primarily theoretical. Nevertheless, these results provide novel implications for the workers’ wage and productivity profiles, and process of job destruction. Moreover, the theory makes important predictions about how the microstructures of the economy interact with business cycle behavior. I discussed some of those key features to distinguish my approach from the previous studies.
Chapter 3

Work Incentives and Labour Mobility: Quantitative Assessment of Macroeconomic Implications
3.1 Introduction

Despite their sound qualitative features and analytical tractability, standard search and matching models of equilibrium unemployment share two undesirable quantitative properties: one in the cross-section, and the other in time-series. First, even though the models provide potential mechanisms of observed wage differential, once properly calibrated, they can generate only a small amount of frictional wage dispersion (Hornstein et al., 2010). Second, even though a search equilibrium can be consistent with a number of business-cycle facts, standard models cannot generate empirically reasonable labour market volatility over the business cycle (Shimer, 2005). This paper addresses these difficulties from the viewpoint of the worker’s incentives and mobility.

Standard search theoretic labour market models naturally generate wage differentials among identical workers and/or firms (Diamond, 1982; Mortensen, 1982; and Pissarides, 1985). A matched worker and firm jointly draw a productivity, which determines flow surplus of the relationship, and they determine the wage rate in order to share the surplus. Different realizations of productivity result in different wages, and the frictional labour market sustains the wage differential in equilibrium. In this sense, stochastically created productivity differences drive wage differentials. This mechanism, however, clearly does not account for the wage’s role of incentive provision and thus the productivities of workers. Identical workers will yield different productivities if one worker works harder than the other. These effort choices are determined by the incentives provided by the contracts\(^1\). Despite its potentially important roles for productivity differences among workers, this incentive mechanism has not been fully investigated in macroeconomic contexts\(^2\).

Modeling work incentives on the job within a macroeconomic framework, however, in-

---

2Acemoglu and Shimer (2000) analyze the endogenous productivity differences due to firms’ technology choices. They focus on the firm’s side of incentives but do not analyze the worker’s side.
troduces a nontrivial challenge. Due to workers’ job-to-job transition, the labour market environment naturally influences these incentives on the job. If we want to incorporate the incentive perspective within the search theoretic framework, we need to model the interaction between work incentives within a firm and labour mobility within the labour market. However, modeling this interaction in turn enables us to analyze how workers’ productive behaviour reacts to the allocation and to changes in economic environment over the business cycle. In other words, not only do different contracts yield different levels of productivity at a time, the interaction generates the endogenous dynamic heterogeneity among ex ante identical workers, as a result of evolving labour market environment.

The objective of this paper is to assess quantitative implications of this mechanism for the wage dispersion as well as labour market behaviour over the business cycle. I use a model developed in Chapter 2. The model builds on the framework developed by Shi (2009), but will integrate workers’ moral hazard problem on the job. That is, how hard the worker works, which determines the firm’s labour productivity, is not observable to the employer. The firm, therefore, designs a contract to induce a desired level of effort by the worker as well as to attract the worker to stay in the firm. In this sense, the model combines the efficiency wage argument of the wage contracts as in Lazear (1981) and Shapiro and Stiglitz (1984) and the wage-tenure contracts as in Burdett and Coles (2003) and Shi (2009).

The model has several desirable properties for this analysis. First, the equilibrium is characterized by the block recursive property; that is, the workers’ optimal decisions and the firms’ optimal contracts are independent of the distribution of workers over different contracts\(^3\). This property enables me to compute the behavior of the model outside the

\(^3\)Directed search by workers and firms’ competitive entry into the labour market are key to establish the existence of the block recursivity. See Shi (2009) and Menzio and Shi (2010) for the original contributions. Directed search was originally introduced by Peters (1991) and Montgomery (1991). Acemoglu and Shimer (1999a, 1999b) and Moen (1997) are applications of the directed search in the labour market, and Delacroix and Shi (2006) is the first to study the directed on-the-job search.
steady state of the economy\(^4\). Second, the model endogenously generates both voluntary and involuntary job destruction processes in a unified framework. The workers’ on-the-job search behaviour determines the voluntary job destruction process while endogenous productivities due to work incentives on the job determine the involuntary process. These two properties—block recursivity and endogenous job destruction processes—provide an ideal framework for analyzing the labour market behaviour over the business cycle.

The model introduces two additional elements into the standard model: one is the endogenous productivity as a function of the worker’s effort, and the other is the worker’s cost of effort. The worker’s effort is unobservable to the employer. Therefore, the employer needs to induce a desirable level of effort in the presence of asymmetric information. These two elements directly influence the workers’ flow from employed to unemployed status and indirectly influence the aggregate unemployment rate. I use these two statistics to pin down these elements in the calibration of the model.

I first investigate the implication for the wage dispersion. Hornstein et al. (2010) use the mean-min ratio \(Mm\), i.e., the ratio between the average wage and the lowest wage paid in the labour market to an employed worker, as a measure of the frictional wage dispersion. They calculate its empirical statistics using three different data sources: the November 2000 survey from the Occupational Employment Statistics (OES) program, the 1967-1996 wages of the Panel Study of Income Dynamics (PSID), and the 5 % Integrated Public Use Microdata Series (IPUMS) sample of the 1990 U.S. Census. Their estimates of the mean-min ratio are 1.67 from the OES data, 1.46 from the PSID data, and 1.98 from the Census data. Hornstein et al. report that, based on the plausibly calibrated canonical search model, \(Mm = 1.036\); that is, the model predicts only 3.6 percent differential between the average wage and the lowest wage in the labour market. On the other hand, the calibrated model of the current paper yields \(Mm = 1.153\).

\(^4\)See Gonzalez and Shi (2010) and Menzio, Sun, and Shi (2009) for other applications of the block recursivity.
Though it is still lower than the empirical counterpart, it is marked improvement over the standard model. I compare the models with and without the on-the-job search and with and without the endogenous effort choice, and the analysis indicates that the key source of this improvement of the $Mm$-measure may well be the worker’s on-the-job search, but the endogenous effort choice significantly increases the variety of available contracts in the labour market.

The key implication of the model for the business cycle research is its new propagation mechanism through the incentives. The mechanism of endogenous effort choice amplifies the effect of productivity shock on unemployment rate. That is, a slight temporary idiosyncratic productivity shock, which results in 4.5% decrease in the aggregate productivity, increases the unemployment rate from the steady state level of 5.6% to an astonishingly high 10% following the shock. This result starkly contrasts with Shimer’s (2005) finding that, in the standard search models, fluctuations in labour productivity have little impact on the unemployment rate. However, if the same magnitude of the idiosyncratic productivity shock becomes permanent, the economy settles into its new steady state with an 8.5% unemployment rate, instead of 10%. When the productivity shock becomes permanent, workers respond to the shock by increasing their effort provision to avoid displacement. Due to this workers’ response, the aggregate productivity decreases only by 2.9%. The comparison between a temporary shock and a permanent shock illustrates the novel implication of this model. In addition, the model provides interesting implications for the transitory dynamics of wage dispersion, as well as the unemployment rate following a temporary idiosyncratic productivity shock.

3.2 Calibration of the Model

In this section, I will numerically compute the equilibrium of the model in Chapter 2 using the following specific functional forms. I use the standard CRRA utility function
\( u(w) = \frac{w^{1-\sigma}}{1-\sigma} \). To my knowledge, there is no standard method to compute the worker’s production under the moral hazard. In this paper, the production technology is summarized by the probability of success \( r(e) = \exp(-\xi e) \) and the cost of effort \( c(e) = \frac{1}{2} \eta e^2 \) as functions of workers’ level of effort. Furthermore, I assume that the matching technology is summarized by \( p(\theta) = \theta(\psi + \theta)^{-1} \). In addition to these functional parameters, I need to specify other environmental parameters. In total, I have ten parameters in the model to be specified.

I specified these parameters and calibrated the model as follows. I set the model period to be one quarter. I set the discount factor \( \beta \) equal to 0.988, so that the annual interest rate in the model is 5 percent. I set the CRRA coefficient \( \sigma \) equal to 2, a common value in the literature\(^5\). The production level of successful project \( y \) is normalized to 1. Also, I normalized search probability \( \lambda_u = 1 \), so that unemployed workers can search with probability one. Following Shimer’s (2005) baseline calibration methodology, I set \( b = 0.34 \) so that the value of non-market time is 40 percent of the average wage, which is the upper end of the range of income replacement rates in the US\(^6\). I set the vacancy cost \( k = 0.135 \) so that the flow cost of open vacancy equals 14 percent of quarterly average wage\(^7\). I set the search probability of employed worker \( \lambda_e \), the parameter in the matching technology \( \psi \), and the idiosyncratic labour productivity parameter \( \rho \) so that the computed average EE, UE, and EU transition rates match the respective empirical moments\(^8\). Lastly, I set the parameter for the cost of effort \( \eta \) so that the unemployment

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\(^5\)Hornstein et al. (2010) show that higher \( \sigma \) by itself induce significant wage dispersion in a standard search model.

\(^6\)This baseline measure does not take into account direct search costs, or the psychological cost of unemployment (Hornstein et al., 2010).

\(^7\)Silva and Toledo (2007) report that recruiting costs are 14 percent of quarterly pay per hire. Hall and Milgrom use the similar measurement of the flow cost of open vacancy.

\(^8\)I computed the empirical quarterly transition rates as follows. First, Menzio and Shi (2009) report that the average monthly UE rate in US between 1951 and 2006 was 45%. Aggregating it to a quarterly rate yields 83.4%. To find an estimate of quarterly EU rates, I use the characterization of the steady state unemployment rate to find \( EU = \frac{u}{1-u} UE \) where \( u \) is the unemployment rate. If we use \( u = 0.056 \) and \( UE = 0.834 \), we get \( EU = 0.049 \). Hornstein et al. (2010) use the monthly total separation rate of 4%, which implies the quarterly rate of 11.5%. This implies the quarterly EE transition rate to be 6.5%.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ - Discount factor</td>
<td>0.988</td>
<td>Annual interest rate</td>
</tr>
<tr>
<td>$y$ - Output level</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$k$ - Vacancy creation cost</td>
<td>0.135</td>
<td>$k$/Average quarterly wage</td>
</tr>
<tr>
<td>$b$ - Unemployment benefit</td>
<td>0.34</td>
<td>Income replacement rate</td>
</tr>
<tr>
<td>$\lambda_u$ - Probability of search (unemployed)</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\lambda_e$ - Probability of search (employed)</td>
<td>1</td>
<td>EE transition rate</td>
</tr>
<tr>
<td>$\sigma$ - CRRA</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>$\psi$ - Matching technology</td>
<td>0.32</td>
<td>UE transition rate</td>
</tr>
<tr>
<td>$\rho$ - Labour productivity</td>
<td>0.09</td>
<td>EU transition rate</td>
</tr>
<tr>
<td>$\eta$ - Cost of effort</td>
<td>0.038</td>
<td>Unemployment rate</td>
</tr>
</tbody>
</table>

Table 3.1: Calibration of parameters

rate equals 5.6%. Table 3.1 summarizes the results.

It is important to note that the model does not generate enough job-to-job flow of workers and the simulated EE transition is 0.34 even with $\lambda_e = 1$. There are two potential reasons. First, the worker typically gets promoted very quickly to the highest value of contract that no outside offer can match. Second, because of such a promotion within a firm, the worker typically searches for a job that is offering so high a value that the search is not often successful. These both result in quite a low EE transition rate in the model.
3.3 Preliminary Comparison with the Previous Models

The current model differs from the previous models in two dimension; one in terms of worker’s on-the-job search, and the other in terms of worker’s endogenous choice of effort and moral hazard. Before addressing their macroeconomic implications, it will be useful to discuss some of the basic implications these additional features add to the previous models.

First of all, in terms of market structure, the current model divides the labour market into two segments; one characterized by firms’ competitive entry and the other characterized by the firm’s monopsony power. This is the outcome of the worker’s endogenous choice of effort. For example, Shi (2009) analyzes the similar on-the-job search model with dynamic contracts, but there, the productivity of the match is exogenously given. Therefore, once the value of contract reaches the level that no outside offer can match, the firm has no incentive to increase the continuation value any further. This is essentially the upper bound of the labour market (characterized by the competitive entry). However, if the firm can induce a higher level of effort and thus yield higher productivity by offering still increasing continuation value, the firm would do so optimally. In such a case, the current firm is a sole buyer of the labour service of the worker.

Second, the workers’ on-the-job search generates endogenous job-to-job flow of the workers, which is absent from the canonical search model. In Shi (2009), this is the driving force for characterizing the increasing wage-tenure profile and the resulting wage dispersion among identical workers. In the current model, the workers’ endogenous effort choice also generates increasing wage-tenure profile, due to a mechanism similar to Lazear and Moore’s (1984) back-loaded wages. Quantitatively, the current model combines these two mechanisms of back-loaded wages into a single model. The result provides a rather interesting implication. Comparing the equilibrium wage profile with on-the-job search
and without it, the wage profile with on-the-job search shows a distinct kink at a point (Figure 3.1). This is the point where the market is divided into two segments. To the left of the point, the wage increases with the continuation values due to those combined forces. However, to the right of the point, the force due to the competition is absent and the wage profile becomes flatter. It still increases for inducing further effort by the worker.

As the previous argument suggests, if I exclude the on-the-job search mechanism and keep the worker’s effort choice, the employed workers still face an upward sloping wage offer and get promoted to a higher wage as long as the project keeps succeeding. Because the workers find jobs at different times and some workers stay on the job longer than the others, the steady state is still characterized to some extent by wage dispersion. However, if I exclude both the mechanisms, i.e., the labour productivity is also exogenously given, firms offer fixed wage contracts and there is a unique optimal contract that all the firms offer. Therefore, there is no wage dispersion in this environment.
3.4 Macroeconomic Implications

3.4.1 Wage Dispersion

In this section I investigate the frictional wage dispersion predicted by the calibrated model. I evaluate the results by comparing with empirical measures of frictional wage dispersion presented in Hornstein et al. (2010).

Hornstein et al. (2010) use a particular measure of frictional wage dispersion: the mean-min ratio \( (Mm) \), which is the ratio between the average wage and the lowest wage paid in the labour market to an employed worker. This measure has a convenient property; in canonical models of equilibrium unemployment, it can be expressed as a function only of a small set of structural parameters, independent of the shape of the whole wage distribution. Based on plausibly chosen parameters, the canonical search models predict \( Mm = 1.036 \); that is, the model can only generate a 3.6% differential between the average wage and the lowest wage paid in the labour market. To evaluate the model’s performance, they report the empirical counterpart of the model’s mean-min ratio using three different data sources: the November 2000 survey from the Occupational Employment Statistics (OES) program, the 1967-1996 wages of the Panel Study of Income Dynamics (PSID), and the 5 \% Integrated Public Use Microdata Series (IPUMS) sample of the 1990 U.S. Census\(^9\). Their estimates of the mean-min ratio are 1.67 from the OES data, 1.46 from the PSID data, and 1.98 from the Census data.

Unlike the standard models discussed in Hornstein et al. (2010), I cannot obtain an explicit expression for the mean-min ratio of my model. Therefore, I compute the model’s simulated statistic based on the calibrated parameters. The calibrated model predicts \( Mm = 1.157 \), i.e., the model now generates a 15.7% differential between the average wage and the lowest wage in the labour market. Though it is still lower than

\(^9\)To accommodate to the issue of measurement in each data set, they use 1st, 5th, 10th percentile of the respective wage distribution as appropriate estimates of minimum wage in the labour market.
the empirical counterpart, it is significant improvement over the canonical search models discussed above.

As in the previous preliminary comparison, I will discuss the implication of the current model by comparing it with the model with exogenous productivity and the model without on-the-job search\textsuperscript{10}. To compare with the model with exogenous productivity, I fix the probability of success so that EU rate matches the empirical moment, and re-calibrate the other model parameters. One immediate observation from this restricted model is that the equilibrium does not generate much variation in contract offers, and subsequently in wages, in the whole labour market. Specifically, the equilibrium is characterized by a two-point distribution of submarkets: the one in which unemployed workers find a job and the other to which all the employed workers get promoted. Since all the employed workers get promoted in one step to the value that no outside offers can match, employed workers do not search on the job in equilibrium. With this distribution, I find $Mm = 1.158$, which is very close to the full model. Even though the full model with workers’ effort choice exhibits marked variation in contract offers, this difference in variation in wages is not reflected in the $Mm$-measure. Yet, if I calculate the variances of respective wage distribution, the full model has a slightly larger variance than the restricted model. Based on the current parametrization and calibration of the model, these two models can generate similar wage dispersion, but the current model generates richer wage variety.

To compare with the model without on-the-job search, I recalibrate the model with restriction $\lambda_e = 0$. Restricting on-the-job search lowers workers’ opportunity cost of being unemployed and the workers find it optimal to search for a contract that offers a somehow higher wage than they would accept with an on-the-job search. Therefore, theoretically, this restriction raises the lower bound of the wage distribution and thus

\textsuperscript{10}As I mentioned before, if I restrict both the endogenous effort choice and on-the-job search, the model does not generate any wage dispersion as there is a unique fixed contract offer.
Chapter 3. Work Incentives and Labour Mobility

This prediction is confirmed with the calibrated model, and here I find $M_m = 1.046$, which turns out to be similar to what Hornstein et al. (2010) find with the canonical models. The equilibrium, however, is characterized by larger number of contracts offered in the labour market.

This analysis implies that it is the interaction between on-the-job search and the effort choice—incentives on-the-job—that generates the much larger wage dispersion in my model than in the canonical model.

3.4.2 Business Cycle Implications

In this section, I investigate how an idiosyncratic productivity shock affects the economy over the business cycle.

In order to address this question, I compute the Block Recursive Equilibrium with stationary distribution. Then, I compute the response of the equilibrium a negative shock to the productivity by changing a technology parameter $\rho$ in $r(e)$ function. That is, the probability of success becomes lower for the same level of effort. It is important to note that the productivity shock specified as above is not the same as the so called “aggregate productivity shock” in the usual business cycle analyses. The aggregate shock would correspond to a shock to $y$ in the current model, which decreases the output of each employed worker. This in turn decreases the aggregate output per employed worker. However, the productivity shock specified here does not decrease the output if the project is successful, but rather decreases the probability of success. This also results in a decrease in the aggregate output per employed worker. With respect to aggregate output per worker, these two types of shock feature a similar outcome, but the underlying mechanisms are very different.

First, I analyze a temporary shock to $\rho$ that lasts only one period so that the optimal contract and workers’ behaviour cannot respond to the shock and are fixed as before. To make it comparable with the current economic climate, I choose a shock to $\rho$ so that
the shock raises the unemployment rate temporarily to 10%. This requires that ρ be set to 0.175. Note that a larger ρ implies lower productivity for a given level of effort. This temporary shock causes about 4.5% decrease in the average productivity. In other words, only 4.5% decrease in the realized aggregate productivity reduction generates this magnitude of unemployment. To assess this finding, I repeat the same analysis with restrictions as I mentioned earlier. First, if I decrease the aggregate productivity by 4.5% in the model with exogenous labour productivity, it generates a slightly lower unemployment rate (9.5%). Second, in the model with endogenous productivity, but without on-the-job search, setting ρ = 0.175 temporarily generates an unemployment rate of 10% as in the full model. Whether this 0.5% point difference in the simulated unemployment rate is significant is not clear, but the comparison implies the difference may well come from the underlying difference in the determination of the productivity.

Next, I analyze the steady state of the economy when ρ = 0.175, while all the other parameters are set as before; that is, I perform a comparative statics with respect to ρ. In this environment, the firms and workers adjust their behaviour optimally, and the result of this analysis suggests the direction of the evolution of the economy if the previous technology shock becomes permanent. I found that the simulated unemployment rate is 8.5%, which is substantially lower than the case of temporary shock. The result clearly shows the significance of the incentive mechanism and the resulting worker behaviour. As the previous analysis shows, when a technology shock hits the economy, the firms and workers cannot immediately respond to the shock and a high rate of unemployment results. However, if the shock is permanent and the lower level of technology becomes the norm, the workers exert a higher level of effort at each value of contract to avoid replacement after a project failure. As a result, the average productivity in the steady state here is 0.92, which is only a 2.9% decrease in the aggregate productivity. Combining this finding and the previous finding with a temporary shock, if I trace the time-series of average productivity after the realization of a permanent technology shock, I would
expect a sudden drop of the productivity followed by a gradual recovery toward a new steady state. I cannot perform a similar comparative statics with the model of exogenous productivity. Yet, if I directly and permanently decrease the level of average productivity by 2.9%, the model predicts 8.5% unemployment. This result implies that these two models are consistent in the long run, and the worker’s endogenous effort choice provides a compelling microfoundation of the productivity of the match. It also provides reasonable predictions of dynamics of the model that the exogenous productivity models cannot provide.

The model also predicts how a temporary shock to the idiosyncratic productivity affects wage dispersion and unemployment rate during transitory period through its effect on the distribution of workers. Figure 3.2 shows its effects on the $Mm$ statistic and unemployment rate over eight periods after the shock. In this model, a temporary shock does not affect the equilibrium functions, including the search behaviour of unemployed workers, so the minimum wage in the labour market does not change. The effects on the $Mm$, therefore, reflect the effects on the average wage over periods after the shock. The top panel of the figure shows an interesting behaviour of the $Mm$, i.e., the bottom of the trough comes two period after the shock. This is due to distributional effects on the average wage. After a shock, newly displaced workers start from the lowest wage job, and this initially decreases the average wage. Not all those newly displaced workers can find a job right after the shock, so still more workers start from the lowest wage job in the following period than would in the steady state. Even though the workers who found a job in the previous period have moved to higher wage jobs through promotion or on-the-job search, the negative impact on the average wage caused by the second wave of new employment at the lowest wage job dominates the effect of wage increases within the market.

On the other hand, unemployment rate does not show this sort of delayed peak after the shock. The initial productivity shock causes a larger displacement than in
the steady state. However, this excess mass of unemployed workers keeps finding a job at a higher rate than the rate at which currently employed workers lose their jobs and become unemployed. This lack of delayed peak of unemployment rate, however, is not as general a result as it may seem to be. If there were a large productivity differences among low wage jobs and high wage jobs, the rate at which workers employed at low wage jobs lose their job may be higher than the rate that unemployed workers find a job for some time after the initial shock. It may further increase unemployment rate. As the above argument suggests, the delayed bottom of the trough is caused by a sufficiently large difference between the initial average wage and the minimum wage in the market. If there were not such dispersion, the distributional impact of the second wave of new employment would be smaller, and this model does not predict such a large productivity dispersion to create a large enough distribution effect.
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3.5 Conclusion

The paper investigated the quantitative implications of the interaction between work incentives and labour mobility. Endogenous determination of work incentive within the frictional labour market framework plays an important role in generating heterogeneous productivity among ex ante homogeneous workers and firms. I analyzed whether this new mechanism enhances further understanding of frictional wage inequality and examined how it contributes to the business cycle behaviour of labour market outcomes.

In terms of frictional wage inequality, the result supports the finding of Hornstein et al. (2010) that models with on-the-job search with firms posting wage-tenure contract seem more easily to accommodate sizable frictional wage dispersion. I found weak evidence that the endogenous productivity difference due to work incentive is responsible for frictional wage dispersion. Moreover, the model predicts very small productivity difference. Yet, the full model exhibits a richer market structure than the previous models. The firm has incentive to keep raising wage offers to induce further effort by the worker, even when the wage has become so high that there is no market competition for workers and there is no risk of losing the workers to another firm. At this point, the firm has a monopsony power.

The model also provides novel business cycle implications. First, the mechanism of endogenous effort choice amplifies the effect of productivity shock on unemployment rate. Second, endogenous productivity mechanism enables me to illustrate an important difference between temporary and permanent productivity shock, which comes from the workers’ response to the shock. Third, the model shows the importance of the distributional effect on macroeconomic variables during the transitory periods after a shock. These are important properties for further investigation into the business cycle behaviour.
Appendix A

Supplementary Materials for
Chapter 1

A.1 Formal mathematical expressions and inference rules

First, define the following notations. Let $U$ denote the unbiased type and $B$ the biased type, and $I \in \{U, B\}$.

- $\sigma_I(s)$: Mixed strategy of report of type $I$ advisor, i.e., probability that $r = 1$ given the signal $s$. I assume that the biased advisor is a behavioral type—he reports 1 independent of the signal. Therefore, $\sigma_B(s) = 1$ for $s = 0, 1$.

- $\Theta_I(r|\omega)$: Probability that advisor of type $I$ reports $r$ conditional on the state $\omega$.

\[
\Theta_I(1|\omega) = \gamma \sigma_I(\omega) + (1 - \gamma) \sigma_I(1 - \omega)
\]

and $\Theta_I(0|\omega) = 1 - \Theta_I(1|\omega)$.

- $\Pr(r, \omega)$: Probability that the decision maker receives the report $r$ when the real
state is $\omega$.

$$\Pr(r, \omega) = \lambda \Theta_U(r|\omega) + (1 - \lambda) \Theta_B(r|\omega).$$

The inferences given in the text are formally given by the following Bayes’ rule with above definitions.

- $\Lambda(r, \omega)$: The decision maker’s posterior belief that the advisor in the first period is of an unbiased type when she receives the report $r$ and state $\omega$ is realized.

$$\Lambda(r, \omega) = \frac{\lambda \Theta_U(r|\omega)}{\lambda \Theta_U(r|\omega) + (1 - \lambda) \Theta_B(r|\omega)}$$

- $\Gamma(r)$: The decision maker’s posterior belief that $\omega_t = 1$ if the advisor reports $r$.

$$\Gamma(r) = \frac{\lambda \Theta_U(r|1) + (1 - \lambda) \Theta_B(r|1)}{\lambda(\Theta_U(r|1) + \Theta_U(r|0)) + (1 - \lambda)(\Theta_B(r|1) + \Theta_B(r|0))}$$

### A.2 Omitted Proofs

**Proof of Lemma 1.1:** Obvious from two facts that $V$ is increasing in reputation and that the reputation decreases when $r = 1$.

**Proof of Lemma 1.2:** First, it is clear that $\Gamma(0)$ is decreasing in $\eta$ and $\Gamma(1)$ is increasing in $\eta$. Then, differentiating each expression with respect to $\eta$ gives the desired results. □

**Proof of Proposition 1.2:** First, evaluating $E(0, \eta)$ and $E(1, \eta)$ at $\eta = 0$ gives

$$E(0, 0) = \frac{3}{4} + \delta (1 - \gamma (1 - \gamma)), \quad \text{and} \quad E(1, 0) = \frac{3}{4}.$$  

Obviously, $E(0, 0) \geq E(1, 0)$ if $\delta \geq 0$. This implies that reporting $r = 0$ is the best response to the decision maker’s inference about the state if she expects $\eta$ is sufficiently small.

Lemma 1.2 implies that $E(0, \eta) > E(1, \eta)$ for all $\eta \in [0, 1]$ if and only if $E(0, 1) > E(1, 1)$. Evaluating $E(0, \eta)$ and $E(1, \eta)$ at $\eta = 1$ gives

$$E(0, 1) = 3\gamma (1 - \gamma) + \delta (1 - \gamma (1 - \gamma))$$
\[ E(1, 1) = 1 - \left( \frac{1 - \lambda_1 + \lambda_1 \gamma}{2 - \lambda_1} \right)^2 + 2\gamma \left( \frac{1 - \lambda_1 + \lambda_1 \gamma}{2 - \lambda_1} \right) - \gamma. \]

From these expressions, \( E(0, 1) > E(1, 1) \) holds if and only if \( \delta > \bar{\delta} \) where

\[
\bar{\delta} \equiv \frac{E(1, 1) - 3\gamma(1 - \gamma)}{1 - \gamma(1 - \gamma)} = \frac{1}{1 - \gamma(1 - \gamma)} \left\{ 1 - \left( \frac{1 - \lambda_1 + \lambda_1 \gamma}{2 - \lambda_1} \right)^2 + 2\gamma \left( \frac{1 - \lambda_1 + \lambda_1 \gamma}{2 - \lambda_1} \right) - \gamma - 3\gamma(1 - \gamma) \right\}.
\]

That is, if \( \delta > \bar{\delta} \) holds, there is a unique politically correct equilibrium.

On the other hand, if \( \delta < \bar{\delta} \), then \( E(0, 1) < E(1, 1) \). Therefore, reporting \( r = 1 \) is the best response to the decision maker’s inference about the state if she expects \( \eta \) is sufficiently large. That is, there is a truthful equilibrium if the advisor’s career concern is less than \( \bar{\delta} \).

Moreover, if \( \delta < \bar{\delta} \), \( E(0, \eta) \) and \( E(1, \eta) \) cross only once in \((0, 1)\). That is, there exists unique \( \eta^m \in (0, 1) \) such that \( E(0, \eta^m) = E(1, \eta^m) \). At \( \eta^m \), the advisor is indifferent between reporting \( r = 0 \) and \( r = 1 \), and thus randomizing between \( r = 0, 1 \) is a mixed strategy equilibrium.

To show that \( \bar{\delta} > 0 \) is indeed the case, it suffices to show that \( E(1, 1) - 3\gamma(1 - \gamma) > 0 \). Note that \( \bar{\delta} = 0 \) at \( \gamma = \frac{1}{2} \). I can show that \( E(1, 1) \) is increasing in \( \gamma \in (\frac{1}{2}, 1) \). This together with that \( 3\gamma(1 - \gamma) \) being decreasing in \( \gamma \in (\frac{1}{2}, 1) \) implies the desired result. □

**Proof of Lemma 1.4:** From the definition of \( \bar{\phi}_1 \), solving (1.2) with equality for \( \phi_1 \) at \( \phi_0 = 0 \) gives

\[
\bar{\phi}_1 \equiv \frac{E(0, 1) - E(1, 1)}{\delta \gamma v(\Lambda(1, 1)|_{\eta=1})} = \frac{3\gamma(1 - \gamma) + \delta v(1) - E(1, 1)}{\delta \gamma v(\Lambda(1, 1)|_{\eta=1})}.
\]

Since \( v(1) > \gamma v(\Lambda(1, 1)|_{\eta=1}) \), this expression is increasing in \( \delta \). Therefore \( \phi_1 \leq 1 \) if and only if \( \delta \leq \delta' \) where

\[
\delta' \equiv \frac{E(1, 1) - 3\gamma(1 - \gamma)}{v(1) - \gamma v(\Lambda(1, 1)|_{\eta=1})}.
\]

This proves the statement. □
Appendix A. Supplementary Materials for Chapter 1

Proof of Lemma 1.5: The slope of the boundary of $\Phi_R$ is $-(1-\gamma)\frac{v(\Lambda(1,0)|_{\eta=1})}{\gamma v(\Lambda(1,1)|_{\eta=1})}$, and the slope of the boundary of $\Phi_{PR}$ is $-(1-\lambda_1 \gamma)\frac{v(\Lambda(1,1)|_{\eta=1})}{v(\Lambda(1,0)|_{\eta=1})}$. Rearranging terms shows that the boundary of $\Phi_{PR}$ is steeper than that of $\Phi_R$ if and only if
\[
(1-\gamma)(1-\lambda_1 + \lambda_1 \gamma)v(\Lambda(1,0)|_{\eta=1})[V(\lambda_1) - V(\Lambda(1,1))] \\
< \gamma(1-\lambda_1 \gamma)v(\Lambda(1,1)|_{\eta=1})[v(\Lambda(1,0) - V(\Lambda(1,0))] 
\]
But since $(1-\gamma)(1-\lambda_1 + \lambda_1 \gamma) < \gamma(1-\lambda_1 \gamma)$ for $\gamma > \frac{1}{2}$, the right hand side is clearly larger. This gives the desired result.

Proof of Proposition 1.3: Since $\Pi(\phi, \cdot)$ is decreasing both in $\phi_0$ and $\phi_1$, the optimal retention rule must be on the boundary of $\Phi_R$. Using this result, I will show that $\Pi(\phi, \cdot)$ is decreasing in $\phi_0$ on the boundary. If it is the case, then the decision maker is better off by reducing $\phi_0$ while increasing $\phi_1$ along the boundary. This implies the statement of the proposition.

The relation of $\phi_0$ and $\phi_1$ on the boundary is given by
\[
\phi_1 = -\frac{(1-\gamma)v(\Lambda(1,0)|_{\eta=1})}{\gamma v(\Lambda(1,1)|_{\eta=1})} \phi_0 + \frac{E(0,1) - E(1,1)}{\delta v(\Lambda(1,1)|_{\eta=1})}.
\] (A.1)

Substituting (A.1) into (1.4) and differentiating with respect to $\phi_0$ give
\[
\frac{\partial \Pi(\phi_0, 1)}{\partial \phi_0} = \frac{1}{2} \left[ \frac{(1-\gamma)v(\Lambda(1,0)|_{\eta=1})}{\gamma v(\Lambda(1,1)|_{\eta=1})} (1-\lambda_1 + \lambda_1 \gamma)[V(\lambda_1) - V(\Lambda(1,1))] \\
- (1-\lambda_1 \gamma)[V(\Lambda(1,0) - V(\Lambda(1,1))] \right]
\]
Since the term $\gamma v(\Lambda(1,1)|_{\eta=1})$ is positive, I have
\[
\text{sign} \frac{\partial \Pi(\phi_0, 1)}{\partial \phi_0} = \text{sign} \left\{ (1-\gamma)(1-\lambda_1 + \lambda_1 \gamma)v(\Lambda(1,0)|_{\eta=1})[V(\lambda_1) - V(\Lambda(1,1))] \\
- \gamma(1-\lambda_1 \gamma)v(\Lambda(1,1)|_{\eta=1})[V(\lambda_1) - V(\Lambda(1,0))] \right\}
\]
As shown in the proof of lemma 1.5, the right hand side is negative, which implies the desired result.
**Proof of Lemma 1.6**: The condition (1.2) can be rewritten as

\[ \delta \{ v(1) - [\phi_1 \gamma v(\Lambda(1,1)|_{\eta=1}) + \phi_0 (1 - \gamma) v(\Lambda(1,0)|_{\eta=1})] \} \leq E(1,1) - 3\gamma(1 - \gamma) \]

The inside the square bracket of the left hand side is the difference between the advisors continuation value when he enters the second period with reputation 1 and the expected continuation value when he reports truthfully under the \( \phi \)-rule. On the other hand, the right hand side is the difference between his expected payoffs of the first period when reports truthfully and when he misreport the signal.

Note that inside the square bracket of the left hand side is positive. Therefore, for each \( \phi \), there exists \( \delta_\phi \) such that for any \( \delta \leq \delta_\phi \) the inequality holds, and for any \( \delta > \delta_\phi \) the inequality is reversed. That is, the inequality is monotonic with respect to \( \delta \). Since \( \phi \)'s are probabilities, \( \phi_j \leq 1 \) for \( j = 0, 1 \), and by monotonicity of the inequality, there is a unique \( \delta \) to make the condition (1.2) hold when \( \phi = (1,1) \). The threshold value of \( \delta \) is given by

\[ \delta(1,1) = \frac{E(1,1) - 3\gamma(1 - \gamma)}{v(1) - \gamma v(\Lambda(1,1)|_{\eta=1}) - (1 - \gamma) v(\Lambda(1,0)|_{\eta=1})}. \]

Clearly, if \( \delta > \delta(1,1) \), no \( \phi \) satisfies this inequality. That is, if \( \delta > \delta(1,1) \), \( \Phi_R \) is empty. Defining \( \delta'' \equiv \delta(1,1) \) proves the statement.

**Proof of Proposition 1.4**: First, I need to show the condition where \( \bar{\phi}_1 \geq \bar{\phi}_1 \). If it does not hold for any \( \delta \), then \( \Omega \) is clearly empty. On the other hand, if \( \bar{\phi}_1 \geq \bar{\phi}_1 > 1 \), then I further need to show \( \bar{\phi}_0 \geq \bar{\phi}_0 \).

Now, \( \bar{\phi}_1 \geq \bar{\phi}_1 \) is expressed as

\[ \bar{\phi}_1 \geq \frac{E(0,1) - E(1,1)}{\delta \gamma v(\Lambda(1,1)|_{\eta=1})}. \]

Note that only \( E(0,1) \) depends on \( \delta \). Rearranging terms shows that the inequality holds if and only if \( \Delta_1(\delta, \lambda, \gamma) > 0 \) where

\[ \Delta_1(\delta, \lambda, \gamma) \equiv \delta \left[ \bar{\phi}_1 \gamma v(\Lambda(1,1)|_{\eta=1}) - v(1) \right] + E(1,1) - 3\gamma(1 - \gamma) \quad (A.2) \]
For the other inequality, $\bar{\phi}_0 \geq \bar{\phi}_0$, similar rearrangement of terms shows that the inequality holds if and only if $\Delta_0(\delta, \lambda, \gamma) > 0$ where

$$\Delta_0(\delta, \lambda, \gamma) \equiv \delta \left[ \bar{\phi}_0(1 - \gamma)v(\Lambda(1, 0)|_{\eta=1}) + \gamma v(\Lambda(1, 1)|_{\eta=1}) - v(1) \right] + E(1, 1) - 3\gamma(1 - \gamma)$$

(A.3)

I consider the cases $\delta \in (\bar{\delta}, \bar{\delta}')$ and $\delta \in (\delta', \delta'')$ separately. In the former, $\Delta_1 > 0$ is necessary and sufficient for the nonemptiness of $\Omega$ while in the latter, I also need to have $\Delta_0 > 0$.

First, when $\delta \in (\bar{\delta}, \delta')$, if $\bar{\phi}_1 \geq 1$ then I am done and can move on to the latter case. On the other hand, if $\bar{\phi}_1 < 1$, then inside the bracket of (A.2) is negative and $\Delta_1$ is decreasing in $\delta$. I can show that $\Delta_1(\bar{\delta}, \cdot, \cdot) > 0$ for any $(\lambda, \gamma)$. Then, there is a $\delta_{\lambda, \gamma} \in (\bar{\delta}, \delta')$ such that $\Delta_1 > 0$ for $\delta \in (\bar{\delta}, \delta_{\lambda, \gamma}]$ and $\Delta_1 < 0$ for $\delta > \delta_{\lambda, \gamma}$. That is, $\Omega$ is nonempty if and only if $\delta \in (\bar{\delta}, \delta_{\lambda, \gamma}]$.

Second, when $\delta \in (\delta', \delta'')$, if $\bar{\phi}_0 \geq 1$ then I am done and can set $\delta_{\lambda, \gamma} = \delta''$. On the other hand, if $\bar{\phi}_0 \in (0, 1)$, then inside the bracket of (A.3) is negative and $\Delta_0$ is decreasing in $\delta$. Note that if $\bar{\phi}_0 > 0$ then $\bar{\phi}_1 \geq 1$. I can show that $\Delta_0(\delta', \cdot, \cdot) > 0$ for any $(\lambda, \gamma)$. Then, there is a $\delta_{\lambda, \gamma} \in (\delta', \delta'')$ such that $\Delta_0 > 0$ for $\delta \in (\bar{\delta}, \delta_{\lambda, \gamma}]$ and $\Delta_0 < 0$ for $\delta > \delta_{\lambda, \gamma}$. That is, $\Omega$ is nonempty if and only if $\delta \in (\bar{\delta}, \delta_{\lambda, \gamma}]$.

Finally, when $\delta_{\lambda, \gamma} < \delta''$, partially differentiating $\Delta_1$ and $\Delta_0$ with respect to $\lambda$ and $\gamma$ gives positive derivatives. They imply that the threshold $\delta_{\lambda, \gamma}$ is increasing in respective variables.

$\square$
Appendix B

Supplementary Materials for
Chapter 2

B.1 Omitted Proofs

Proof of Lemma 2.4: First, for a given continuation value $W$, the worker’s optimal effort is implicitly characterized by

$$-c'(e(W)) + \beta r'(e(W))(W + \lambda e D(W) - U) = 0.$$ 

Define $\Omega(W) = W + \lambda e D(W) - U$. Then I can write $\frac{c'(e(W))}{\beta r'(e(W))} = \Omega(W)$. Under the assumptions ($c(\cdot):$ convex, continuous, $r(\cdot):$ concave continuous, and $r' > 0$ everywhere), $\frac{c'}{\beta r'}$ is continuous and monotonically increasing, and thus invertible. Therefore, there exists a unique level of $e$ that satisfies the equality. Moreover the inverse function is continuous. Hence, if write $e(W) = g(\Omega(W))$ where $g$ is the inverse function of $\frac{c'}{\beta r'}$. 

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differentiating the composite function yields

\[
\frac{\partial e(W)}{\partial W} = g'(\cdot)\Omega'(W) = \left( \left( \frac{c'}{\beta r'} \right)^{-1} \right)'(1 - \lambda e\tilde{\phi}(W)) = \left( \frac{c''r' - c'r''}{\beta r'^2} \right)^{-1}(1 - \lambda e\tilde{\phi}(W)) = \frac{\beta r'^2}{c''r' - c'r''}(1 - \lambda e\tilde{\phi}(W)).
\]

The first equality is by the chain rule and the third by the inverse function theorem. Since the denominator of the right hand side is strictly positive by assumption, the derivative is positive. Therefore, the worker’s optimal effort is increasing in \(W\). \(\square\)

**Proof of Lemma 2.5:** Let \(\eta(V)\) be the Lagrange multiplier for the promise-keeping constraint. The first order condition of the maximization problem with respect to \(w_i\) implies

\[
\eta(V) = \frac{1}{u'(w_i)} \text{ for } i = 1, 2.
\]

This implies that \(w_1 = w_2\). Therefore, the current wage, \(w\), does not depend on the realization of the lottery. \(\square\)

**Proof of Lemma 2.6:** First, I derive the equation (2.6). Using Lemma 2.4, substituting \(e(W_i)\) into both the objective function and the promise-keeping constraint eliminates the incentive compatibility constraint. Then, using the result that \(w_1 = w_2\) from Lemma 2.5, the promise-keeping constraint becomes

\[
V = u(w) + \beta U + \sum_i \pi_i \left[ -c(e(W_i)) + \beta r(e(W_i))[W_i + \lambda D(W_i) - U] \right].
\]

Since \(u\) is strictly concave and thus invertible, this constraint can be solved for \(w\). Let \(w(V, \phi)\) be the solution. Then,

\[
w(V, \phi) = u^{-1} \left( V - \beta U - \sum_i \pi_i \left[ -c(e(W_i)) + \beta r(e(W_i))[W_i + \lambda eD(W_i) - U] \right] \right).
\]
Substituting this expression into the objective function gives the desired form of optimization problem.

To characterize the optimal contract given the simplified problem, first I focus on the interior solution by ignoring the feasibility constraints for \( W_i \) and \( \pi_i \). The first order condition of the reduced form of the problem (2.6) with respect to \( W_i \) is

\[
\frac{1}{u'(w(V))} \pi_i \beta r'(\varepsilon(W_i))(1 - \lambda_e \hat{p}(W_i)) \\
+ \pi_i r'(\varepsilon(W_i)) \left[ y + \beta (1 - \lambda_e \hat{p}(W_i)) J(W_i) \right] g'(\Omega(W_i)) \left( 1 - \lambda_e \hat{p}(W_i) \right) \\
+ \pi_i r'(\varepsilon(W_i)) \beta \left[ (1 - \lambda_e \hat{p}(W_i)) J'(W_i) - \lambda_e J(W_i) \hat{p}'(W_i) \right] = 0.
\]

I used the fact that \( \frac{\partial \Omega(W)}{\partial W} = 1 - \lambda_e \hat{p}(W) \) and \( e'(W) = g'(\Omega(W_i))\Omega'(W) \). Then, dividing through by \( \beta, \pi_i, r(e), \) and \( (1 - \lambda_e \hat{p}(W_i)) \) gives

\[
\frac{1}{u'(w(V))} + \frac{r'(\varepsilon(W_i))}{r(\varepsilon(W_i))} \left( \frac{y}{\beta} + (1 - \lambda_e \hat{p}(W_i)) J(W_i) \right) g'(\Omega(W_i)) + J'(W_i) - \lambda_e J(W_i) \hat{p}'(W_i) = 0.
\]

Now, from the first order condition of the constrained problem, I have \( \eta(V) = \frac{1}{u'(w(V))} \). The theorem of Lagrange multiplier implies that \( J'(V) = -\eta(V) \). Thus

\[
J'(V) = -\frac{1}{u'(w(V))}.
\]

Substituting it into the previous equation and rearranging the terms yield

\[
J'(V) - J'(W_i) = -\frac{\lambda_e J(W_i) \hat{p}'(W_i)}{1 - \lambda_e \hat{p}(W_i)} + \frac{r'(\varepsilon(W_i))}{r(\varepsilon(W_i))} \left( \frac{y}{\beta} + (1 - \lambda_e \hat{p}(W_i)) J(W_i) \right) g'(\Omega(W_i)).
\]

(B.2)

The right hand side is positive since \( \hat{p}'(W_i) \) is non-positive. Therefore, I have \( J'(V) - J'(W_i) > 0 \). Then, concavity of \( J \) implies \( W_i \geq V \) for \( i = 1, 2 \). This characterizes the dynamics of wages on the contract.

\( \square \)

**Proof of Proposition 2.2**: Shifting (B.1) one period forward gives \( J'(W_i) = -\frac{1}{u'(w(W_i))} \) where \( w(W_i) \) is the wage in the next period when the realization of the lottery is \( i \). Substituting it into (B.2), together with (B.1) shows

\[
\frac{1}{u'(w(W_i))} - \frac{1}{u'(w(V))} > 0.
\]
Since \( u \) is concave function, this implies \( w(W_i) \geq w(V) \) for \( i = 1, 2 \). Hence, the next period wage is at least as high as the current wage.

\[ \Box \]

**Proof of Proposition 2.3**: As shown in lemma 2.4, the optimal effort function is increasing in the continuation value of the contract. Since the continuation value of the contract is increasing with tenure, the worker will provide higher effort next period than the current period effort if the worker stays on the contract.

\[ \Box \]

### B.2 Existence of a Block Recursive Equilibrium

The following is the general properties of the equilibrium as well as lemmas for proof of the existence.

#### B.2.1 Other Properties of Equilibrium

Note that all the equilibrium objects in the characterizations depend on a specific \( J \in \mathcal{J}(X) \). To show the existence of an equilibrium, I need to find a fixed point of the operator \( TJ(V) \). The proof of existence of a fixed point requires the continuity of each object with respect to \( J \in \mathcal{J}(X) \). The following lemmas prove this property. Proofs are given in Menzio and Shi (2010) and are omitted here.

**Free Entry Condition and Market Tightness**

**Lemma B.1.** Consider \( J_m, J_n \in \mathcal{J}(X) \). Let \( \theta_j(x) \) be the market tightness function implied by \( J_j \) for \( j = m, n \). If \( ||J_m - J_n|| < \rho \), then \( ||\theta_m - \theta_n|| < \varepsilon_\theta \rho \).

**Worker’s Search Problem**

**Lemma B.2.** Consider \( J_m, J_n \in \mathcal{J}(X) \). Let \( D_j(V) \) be the worker’s value of searching implied by \( J_j \) for \( j = m, n \). If \( ||J_m - J_n|| < \rho \), then \( ||D_m - D_n|| < \varepsilon_D \rho \).
Lemma B.3. Consider $J_m, J_n \in \mathcal{J}(X)$. Let $m_j(V)$ be the optimal search strategy implied by $J_j$ for $j = m, n$. If $||J_m - J_n|| < \rho$, then $||m_m - m_n|| < \varepsilon_m \rho$.

Lemma B.4. Consider $J_m, J_n \in \mathcal{J}(X)$. Let $\hat{p}_j(V) = p(\theta_j(m_j(V)))$ be the composite function implied by $J_j$ for $j = m, n$. If $||J_m - J_n|| < \rho$, then $||\hat{p}_m - \hat{p}_n|| < \varepsilon_p \rho$.

Worker’s Value of Unemployment

Lemma B.5. Consider $J_m, J_n \in \mathcal{J}(X)$. Let $U_j$ be the worker’s unemployment value implied by $J_j$ for $j = m, n$. If $||J_m - J_n|| < \rho$, then $||U_m - U_n|| < \varepsilon_U \rho$.

Worker’s Optimal Effort

Lemma B.6. Consider $J_m, J_n \in \mathcal{J}(X)$. Let $\Omega_j(W) = W + \lambda D_j(W) - U_j$ be the function implied by $J_j$ for $j = m, n$. If $||J_m - J_n|| < \rho$, then $||\Omega_m - \Omega_n|| < \varepsilon_\Omega \rho$.

Proof of Lemma B.6:
\[
|\Omega_m(W) - \Omega_n(W)| \\
= |\lambda(D_m(W) - D_n(W)) - (U_m - U_n)| \\
\leq |\lambda \varepsilon_D - \varepsilon_U| \rho.
\]

Lemma B.7. Consider $J_m, J_n \in \mathcal{J}(X)$. Let $e_j(W) = g(\Omega_i(W))$ be the worker’s optimal effort function implied by $J_j$ for $j = m, n$. If $||J_m - J_n|| < \rho$, then $||e_m - e_n|| < \varepsilon_e \rho$.

Proof of Lemma B.7: Let $\bar{g}' = \sup g'(\cdot)$. Given the assumptions about $c(\cdot)$ and $r(\cdot)$, $\bar{g}' < \infty$. Then,
\[
|e_m(W) - e_n(W)| = |g(\Omega_m(W)) - g(\Omega_n(W))| \\
\leq \bar{g}' |\Omega_m(W) - \Omega_n(W)| \\
\leq \bar{g}' \varepsilon_\Omega \rho.
\]
B.2.2 Existence of a Block Recursive Equilibrium

These following lemmas extend the existence result for the Block Recursive Equilibrium given by Menzio and Shi (2010) in the presence of a moral hazard problem on the job. Let \( \hat{J}(V) \) be a firm’s updated value function by the operator \( T \), i.e., \( \hat{J}(V) = (TJ)(V) \).

**Lemma B.8.** The firm’s value function \( \hat{J}(V) \) belongs to the set \( J(X) \). That is,

(i) \( \hat{J}(V) \) is strictly decreasing, and Lipschitz continuous with respect to \( V \).

(ii) \( \hat{J}(V) \) is bounded both from below and above.

(iii) \( \hat{J}(V) \) is concave.

**Proof of Lemma B.8:**

(i) From the characterization result, let \( F \) be the objective function of the reduced problem, i.e.,

\[
F(V, \phi) = \left\{ -u^{-1} \left( V - \beta U - \sum_i \pi_i \left[ -c(e(W_i)) + \beta r(e(W_i))[W_i + \lambda D(W_i) - U] \right] \right) = w(V, \phi) \right. \\
+ \left. \sum_i \pi_i r(e(W_i)) \left[ y + \beta (1 - \lambda \hat{p}(W_i) J(W_i)) \right] \right\}
\]

By the Inverse Function Theorem,

\[
F'(V, \phi) \equiv \frac{\partial F(V, \phi)}{\partial V} = -\frac{1}{u'(w)} \in \left[-\frac{1}{u'}, -\frac{1}{u'}\right]
\]

Now, for any \( V_a, V_b \in X \), such that \( V_a \leq V_b \), I have

\[
|\hat{J}(V_b) - \hat{J}(V_b)| \leq \max_{\phi} |F(V_b, \gamma) - F(V_a, \gamma)|
\]

\[
= \max_{\phi} \left| \int_{V_a}^{V_b} F'(V, \gamma) dV \right|
\]

\[
\leq \max_{\phi} \int_{V_a}^{V_b} |F'(V, \gamma)| dV
\]

\[
\leq \frac{1}{u'} |V_b - V_a|
\]
Therefore $\hat{J}(V)$ is Lipschitz continuous in $V$. From this result, $\hat{J}(V)$ is absolutely continuous and thus almost everywhere differentiable (Folland, 1999). Moreover, at any point of differentiability, I have $\hat{J}'(V) = F'(V, \phi(V))$ where $\phi(V)$ is the optimal contract given $V$ (Milgrom and Segal, 2002). Then,

$$\hat{J}(V_b) - \hat{J}(V_a) = \int_{V_a}^{V_b} F'(V, \phi(V)) dV \in \left[ -\frac{1}{u'}(V_b - V_a), -\frac{1}{u'}(V_b - V_a) \right]$$

Hence, $\hat{J}(V)$ is strictly decreasing and the difference is bounded.

(ii) Next, I estimate the bounds of $\hat{J}(V)$. Let $w$ be the lowest possible wage under the feasible contract. That is

$$w = \min_{\phi} u^{-1}\left(V - \sum_{i=1,2} \pi_i(-c(e(W_i)) + \beta[r(e_i)(W_i + \lambda D(W_i)) + (1 - r(e_i))U])\right)$$

Since $u'$ is increasing function and the expected continuation value for the worker is bounded by $\bar{x} = \sup_{x \in \hat{\mathcal{J}}} \hat{x}$ and $\underline{x} = \frac{b}{1-\beta}$, which is finite, I have $w \geq u^{-1}\left(\bar{x} + c(\bar{e}) - \beta\bar{x}\right)$. Using the fact that $\hat{J}(V)$ is strictly decreasing in $V$, I have

$$\hat{J}(V) < \hat{J}(\bar{x})$$

$$\leq r(\bar{e})y - u^{-1}(\bar{x} + c(\bar{e}) - \beta\bar{x}) + \beta \bar{J} \equiv \bar{J}.$$

Here, $\bar{e}$ and $\bar{e}$ are the lower bound and the upper bound of the level of effort, which the firm can induce with $\bar{x}$ and $\underline{x}$, respectively. Then, $\hat{J}(V) \leq \bar{J} = \frac{y - u^{-1}(\bar{x} + c(\bar{e}) - \beta\bar{x})}{1-\beta}$. Similarly to the previous argument,

$$\hat{J}(V) > \hat{J}(\bar{x})$$

$$\geq r(e)y - u^{-1}(\bar{x} + c(\bar{e}) - \beta\bar{x}) + \beta \bar{J} \equiv J.$$
Lemma B.9. (Continuity of the operator) Consider $J_m, J_n \in \mathcal{J}(X)$. Let $\hat{J}_j(W)$ be the firm’s value implied by $J_j$ for $j = m, n$. If $||J_m - J_n|| < \rho$, then $||\hat{J}_m - \hat{J}_n|| < \varepsilon T \rho$.

Proof of Lemma B.9: Let $F_j$ be the objective function of the firms optimal contracting problem implied by $J_j$: $F_j : \Gamma \times X \rightarrow \mathbb{R}$. Consider $J_m, J_n \in \mathcal{J}(X)$ such that $||J_m - J_n|| < \rho$. Take $V \in X$ such that $\hat{J}_m(V) - \hat{J}_n(V) > 0$. Let $\phi_j$ be the maximizer of $F_j$ and $w_j(\phi)$ be the wage function given by $J_j$. Then, I have

$$0 \leq |\hat{J}_m(V) - \hat{J}_n(V)|$$

$$= |F_m(V, \phi_m) - F_n(V, \phi_n)|$$

$$\leq |F_m(V, \phi_m) - F_n(V, \phi_m)|$$

$$\leq |-w_m(\phi_m) + \sum_i \pi_i,m r(e_m(W_{i,m})) [y + \beta(1 - \lambda \hat{p}_m(W_{i,m})) J_m(W_{i,m})]$$

$$+ w_n(\phi_m) - \sum_i \pi_i,m r(e_n(W_{i,m})) [y + \beta(1 - \lambda \hat{p}_n(W_{i,m})) J_n(W_{i,m})]|$$

$$\leq |w_m(\phi_m) - w_n(\phi_m)|$$

$$+ \sum_i \pi_i,m |r(e_m(W_{i,m})) [y + \beta(1 - \lambda \hat{p}_m(W_{i,m})) J_m(W_{i,m})]$$

$$- r(e_n(W_{i,m})) [y + \beta(1 - \lambda \hat{p}_n(W_{i,m})) J_n(W_{i,m})]|.$$  \hspace{1cm} (B.3)

The objective here is to estimate a bound for $|\hat{J}_m(V) - \hat{J}_n(V)|$. I will consider a bound for each part of the last expression separately as follows.

1. Consider $|w_m(\phi_m) - w_n(\phi_m)|$ first. Since $u$ is concave function, for any $w_1$ and $w_2$, $|w_1 - w_2|u' < |u(w_1) - u(w_2)|$. Also, by definition

$$u(w_m(\phi_m)) = V - \beta U_m - \sum_i \pi_i,m [-c(e_m(W_{i,m})) + \beta r(e_m(W_{i,m})) \Omega_m(W_{i,m})],$$

and

$$u(w_n(\phi_m)) = V - \beta U_n - \sum_i \pi_i,m [-c(e_n(W_{i,m})) + \beta r(e_n(W_{i,m})) \Omega_n(W_{i,m})].$$
Therefore, I can express the distance as

\[
|u(w_m(\phi_m)) - u(w_n(\phi_m))| \\
\leq \beta |U_m - U_n| + \sum_i \pi_{i,m} \left\{ |c(e_m(W_{i,m})) - c(e_m(W_{i,m}))| \\
+ \beta |r(e_m(W_{i,m}))\Omega_m(W_{i,m}) - r(e_n(W_{i,m}))\Omega_n(W_{i,m})| \right\}. 
\]

|U_m - U_n| is bounded by \( \varepsilon_U \). The last term of the previous expression is bounded as

\[
|r(e_m(W_{i,m}))\Omega_m(W_{i,m}) - r(e_n(W_{i,m}))\Omega_n(W_{i,m})| \\
\leq |r(e_m(W_{i,m}))\Omega_m(W_{i,m}) - r(e_n(W_{i,m}))\Omega_m(W_{i,m})| \\
+ |r(e_n(W_{i,m}))\Omega_m(W_{i,m}) - r(e_n(W_{i,m}))\Omega_n(W_{i,m})| \\
= |r(e_m(W_{i,m})) - r(e_n(W_{i,m}))|\bar{\varepsilon} + r(e_n(W_{i,m}))|\Omega_m(W_{i,m}) - \Omega_n(W_{i,m})| \\
\leq r'(\varepsilon)|e_m(W_{i,m}) - e_n(W_{i,m})|\bar{\varepsilon} + |\Omega_m(W_{i,m}) - \Omega_n(W_{i,m})| \\
\leq (r'(\varepsilon)\varepsilon + \varepsilon)\rho. 
\]

I use the fact that \( \Omega(\cdot) \) is bounded by \( \bar{\varepsilon} \). Collecting these bounds together, I have

\[
|u(w_m(\phi_m)) - u(w_n(\phi_m))| \\
\leq (\beta\varepsilon_U + c'(\varepsilon)\varepsilon + \beta r'(\varepsilon)\varepsilon + \varepsilon)\rho. 
\]

Hence, from the property of concave function I mentioned earlier, the first term of (B.3) is bounded as

\[
|w_m(\phi_m) - w_n(\phi_m)| \leq (u')^{-1} \cdot (\beta\varepsilon_U + c'(\varepsilon)\varepsilon + \beta r'(\varepsilon)\varepsilon + \varepsilon)\rho. 
\]

2. Next, consider the following term in (B.3):

\[
\sum_i \pi_{i,m} \left| r(e_m(W_{i,m})) [y + \beta(1 - \lambda\hat{p}_m(W_{i,m}))J_m(W_{i,m})] \\
- r(e_n(W_{i,m})) [y + \beta(1 - \lambda\hat{p}_n(W_{i,m}))J_n(W_{i,m})] \right|. 
\]
This expression can still be divided into subcomponents after expanding the brackets and collecting terms. Similarly to the above estimate of the bound, the bound for each subcomponent can be found as follows.

(i) \[ |r(e_m(W_{i,m}))J_m(W_{i,m}) - r(e_n(W_{i,m}))J_n(W_{i,m})| : \]

\[ |r(e_m(W_{i,m}))J_m(W_{i,m}) - r(e_n(W_{i,m}))J_n(W_{i,m})| \]
\[ \leq |r(e_m(W_{i,m}))J_m(W_{i,m}) - r(e_n(W_{i,m}))J_m(W_{i,m})| \]
\[ + |r(e_n(W_{i,m}))J_m(W_{i,m}) - r(e_n(W_{i,m}))J_n(W_{i,m})| \]
\[ = |r(e_m(W_{i,m})) - r(e_n(W_{i,m}))|J_m(W_{i,m}) \]
\[ + r(e_n(W_{i,m}))|J_m(W_{i,m}) - J_n(W_{i,m})| \]
\[ \leq r'(\varepsilon)|e_m(W_{i,m}) - e_n(W_{i,m})| \bar{J} \]
\[ + |J_m(W_{i,m}) - J_n(W_{i,m})| \]
\[ \leq (r'(\varepsilon)e_{\bar{J}} + 1)\rho. \]

(ii) \[ |\hat{p}_m(W_{i,m})J_m(W_{i,m}) - \hat{p}_n(W_{i,m})J_n(W_{i,m})| : \]

\[ |\hat{p}_m(W_{i,m})J_m(W_{i,m}) - \hat{p}_n(W_{i,m})J_n(W_{i,m})| \]
\[ \leq |\hat{p}_m(W_{i,m})J_m(W_{i,m}) - \hat{p}_n(W_{i,m})J_m(W_{i,m})| \]
\[ + |\hat{p}_n(W_{i,m})J_m(W_{i,m}) - \hat{p}_n(W_{i,m})J_n(W_{i,m})| \]
\[ = |\hat{p}_m(W_{i,m}) - \hat{p}_n(W_{i,m})|J_m(W_{i,m}) \]
\[ + \hat{p}_n(W_{i,m})|J_m(W_{i,m}) - J_n(W_{i,m})| \]
\[ \leq (\varepsilon_p\hat{J} + 1)\rho. \]
Finally, using these estimates of bounds for respective terms, the inequality (B.3) becomes:

\[
| r(e_m(W_{i,m}))\hat{p}_m(W_{i,m})J_m(W_{i,m}) - r(e_n(W_{i,m}))\hat{p}_n(W_{i,m})J_n(W_{i,m}) | \\
\leq | r(e_m(W_{i,m}))\hat{p}_m(W_{i,m})J_m(W_{i,m}) - r(e_n(W_{i,m}))\hat{p}_m(W_{i,m})J_m(W_{i,m}) | \\
+ | r(e_n(W_{i,m}))\hat{p}_m(W_{i,m})J_m(W_{i,m}) - r(e_n(W_{i,m}))\hat{p}_n(W_{i,m})J_n(W_{i,m}) | \\
= | r(e_m(W_{i,m})) - r(e_n(W_{i,m}))|\hat{p}_m(W_{i,m})J_m(W_{i,m}) \\
+ r(e_n(W_{i,m}))|\hat{p}_m(W_{i,m})J_m(W_{i,m}) - \hat{p}_n(W_{i,m})J_n(W_{i,m}) | \\
\leq (|r'(\varepsilon)\varepsilon_e\bar{J} + 1)|\bar{J} + (\varepsilon_p\bar{J} + 1))\rho.
\]

Therefore,

\[
\sum_i \pi_{i,m} | r(e_m(W_{i,m})) [ y + \beta(1 - \lambda\hat{p}_m(W_{i,m}))J_m(W_{i,m})] \\
- r(e_n(W_{i,m})) [ y + \beta(1 - \lambda\hat{p}_n(W_{i,m}))J_n(W_{i,m}) ] | \\
\leq \{ (y + \beta + \beta\lambda\bar{J})(|r'(\varepsilon)\varepsilon_e\bar{J} + 1) + r(\bar{\varepsilon})(\varepsilon_p\bar{J} + 1) \} \rho.
\]

Finally, using these estimates of bounds for respective terms, the inequality (B.3) becomes

\[
| \hat{J}_m(V) - \hat{J}_n(V) | \\
\leq u^{(t-1)} \cdot (\beta\varepsilon_U + \bar{c}'(\bar{\varepsilon})\varepsilon_e + \beta(r'(\varepsilon)\varepsilon_e\bar{x} + \varepsilon_{\Omega})) \rho \\
+ \{ (y + \beta + \beta\lambda\bar{J})(|r'(\varepsilon)\varepsilon_e\bar{J} + 1) + r(\bar{\varepsilon})(\varepsilon_p\bar{J} + 1) \} \rho \\
\equiv \varepsilon_T \rho.
\]

Hence, the operator $T$ is continuous.

\[\square\]

**Proof of Proposition 2.1:** Given Lemma B.8 and B.9, I can show that the operator $T$ satisfies the assumptions of Schauder's Fixed Point Theorem (Stokey and Lucas with Prescott, 1989, Theorem 17.4); that is, (i) $T$ is continuous, (ii) the family of functions $T(\mathcal{J})$ is equicontinuous, and (iii) $T$ maps the set $\mathcal{J}(X)$ into itself. Therefore, there exists a firm's value function $J^*(V) \in \mathcal{J}(X)$ such that $TJ^* = J^*$. Denote with
\{\theta^*, D^*, m^*, U^*, e^*\} the respective functions associated to \( J^* \). By construction, the functions \( \{ J^*, \theta^*, D^*, m^*, U^*, e^* \} \) satisfy conditions in the definition of a Recursive Equilibrium and do not depend on the distribution of workers. Therefore, they constitute a Block Recursive Equilibrium. 

\[ \square \]

**B.3 Distributional Effects on the Average Productivity**

This section describes how the worker distribution evolves outside the steady state, and how the temporary shock to the economy affects its dynamics and thus the average productivity of the economy. I will use the probability that project succeeds for a given level effort as a measure of productivity. Therefore, the average productivity of this economy is an integration of success probabilities with respect to the distribution of workers over the values of contracts, i.e., \( \int r(\cdot)dG \).

At any time \( t \) and for any \( V \in X \), \( G_t(V) \) is the fraction of worker having a value of contract less than \( V \), including unemployed worker. Given the optimal contract, the distribution of workers evolves as follows:

\[
G_{t+1}(V) = G_t(V) - \left[ \int_{m^{-1}(H^{-1}(V))}^{H^{-1}(V)} r(e(H(x)))\lambda e\hat{p}(x)dG_t(x) + \int_{H^{-1}(V)}^{V} r(e(H(x)))dG_t(x) \right] \\
+ \int_{V}^{\bar{x}} (1 - r(e(H(x))))dG_t(x).
\]

Here \( H \) is a function that specifies the continuation value \( W \) under the current contract and is implicitly defined by the equation (2.7). Note that, given the current value \( x \), the current period effort is determined by the continuation value \( W = H(x) \).

The whole bracket in the first line is the outflow from \( G_t(V) \). Workers in the interval \([H^{-1}(V), V]\) will be promoted to a value higher than \( V \) after a success of their current project with probability \( r(e(H(x))) \). Workers in the interval \([m^{-1}(H^{-1}(V)), H^{-1}(V)]\), though not be promoted as much, will search for new contracts that offer more than \( V \).
after the success, and they will successfully find a firm with probability $r(e(H(x)))\lambda\hat{p}(x)$.

On the other hand, workers whose current value is above $V$ fails in their current project with probability $(1 - r(e(H(x))))$, and they will have the value of unemployed. This is the inflow into $G_t(V)$. A steady state is where the inflow equals the outflow.

Consider, at any steady state of distribution $G^*$, a temporary (one period) productivity shock that decreases the probability of a project success for a given level of effort. To clarify the point, suppose the shock is private to the worker and the firms do not modify the contract. This decrease in productivity decreases the outflow and increases the inflow due to higher probability of displacement after a failure. This implies that $G_{t+1}(V) > G_t^*(V)$ for all $V$, and thus, the next period distribution is first-order stochastically dominated by the current steady state distribution. It is equivalent to $\int_x r(e(H(x)))dG_{t+1}(x) < \int_x r(e(H(x)))dG_t^*(x)$. The property of Block recursivity that the optimal contract is independent of the distribution of workers enables me to calculate these integrals. The inequality implies that the average productivity is lower after the technology shock. Since the distribution of workers does not return to the stationary distribution immediately and follows the above law of motion, a temporary shock to the productivity will yield a persistent effect on the average productivity.

**B.4 The Case of No On-the-Job Search**

This section examines the case where there is no on-the-job search. This corresponds to the case where $\lambda_e = 0$. Then, the optimal contracting problem becomes:

$$J(V) = \max_\xi \{r(e)y - w_i + \beta r(e)J(W)\}$$ (B.4)
subject to

$$
\xi \in \Xi'' = \left\{ (w, e, W)_{i=1,2} : W \in X \right\} \\
V = u(w) - c(e) + \beta \left( r(e)W + (1 - r(e))U \right) - c'(e) + \beta r'(e)(W - U) = 0
$$

where I simplify it by ignoring the lottery to clarify the point. By reducing the problem following the method in the paper, it becomes:

$$
J(V) = \max_W \left\{ r(g((W - U))(y + \beta J(W)) - w) \right\}
$$

where

$$
w = u^{-1} \left( V + c(g((W - U))) - \beta r(g((W - U)))(W - U) - \beta U \right).
$$

Taking the first order condition with respect to $W$ and rearranging term give:

$$
\frac{1}{u'(w(W_i))} - \frac{1}{u'(w)} = \frac{r'(e(W))}{r(e(W))} \left( \frac{y}{\beta} + J(W) \right) g'(W - U).
$$

The right hand side is clearly positive. Therefore, we have

$$
\frac{1}{u'(w(W))} - \frac{1}{u'(w)} > 0.
$$

Therefore, $w(W) \geq w$ by the concavity of $u$. This implies that, even though there is no on-the-job search, it is still optimal for the firm to pay increasing wages over tenure for inducing worker’s effort.
Bibliography


