Utilizing Managerial Cash Flow Estimates for Applied Real Options Analysis

by

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This thesis submitted in conformity with the requirements for the degree of Master’s of Applied Science.

Graduate Department of Chemical Engineering and Applied Chemistry

University of Toronto

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Abstract

Real options analysis has been recommended to identify and quantify opportunities where managerial flexibility can influence worth. However, real options models in the literature have become increasingly sophisticated, and managers have cited their reluctance to use such models due to their level of complexity and lack of transparency. Presented in this thesis is a real options model that can be easily incorporated into the current project selection methodology of a firm; the model uses managerial cash flow estimates to price real options on tangible investment opportunities in a financially consistent manner. Next, to demonstrate the application of real options analysis in practice, five real options models, including the proposed model, are applied to value a medical device project. The models all price the real option differently, due to the differences in their underlying assumptions, but they all yield the same investment conclusion: the medical device project has value.
Acknowledgements

- To my supervisor Yuri Lawryshyn. I am grateful for your guidance and leadership. You have made this processes enjoyable, insightful, and deeply rewarding.

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- To my family. I am forever indebted to you for your love, understanding, and encouragement.

- Most importantly, to Tyler. I dedicate this thesis to you. Thank you for your continual love and support. As I complete this journey I wish you the best of luck on yours.

-Kelsey
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Part I

Thesis Introduction

1 Research Introduction

The limitations of traditional valuation tools, such as payback period, return on investment, or net present value (NPV), are widely known (see Dean, 1951, Hayes and Abernathy, 1980, or Hayes and Garvin, 1982), yet most managers are reluctant to adopt new strategies for corporate budgeting. The current financial climate is one where excessive risk-taking is scrutinized; therefore, conservatism and the use of standard metrics prevails. Critics of these traditional techniques realize that these tools tend to undervalue opportunities and neglect strategic considerations. A reliance on these methods leads to shortsighted decision-making, underinvestment, and a loss in long-term competitive advantages (Schwartz and Trigeorgis, 2001). To address the issues with NPV analysis, an increasing community of academics and corporate practitioners have recommended real options analysis to identify and to quantify opportunities where managerial flexibility can influence worth. Real options analysis is a favoured tool for valuating projects when future uncertainties are high, if there are competitive interactions, or if there are interdependent projects; all of which are typical of real-world investments. Using real options can provide firms with sustainable competitive advantages realized from patents, proprietary technologies, ownership of natural resources, and the development of human capital (Trigeorgis, 1996).

The study of real options analysis spans several academic disciplines. It combines the mathematics of financial option pricing with the foresight of strategic planning and decision analysis to yield an insightful tool for investment decision-making. However, to capture the realities of tangible investments real options models in literature have become increasingly mathematically complex. Unfortunately, managers have cited their reluctance to use such models due to their level of sophistication and lack of transparency (see Block, 2007). Some academics have noticed this growing disparity between academia and practice. Trigeorgis (1996) outlines several areas
for future theoretical and practical real options analysis research\(^1\) in the attempt to bridge this divide. Some of the necessary contributions to real options literature, relevant to this thesis, include:

- developing generic options-based user-friendly software packages with simulation capabilities that can handle multiple real options as a practical aid to corporate planners;
- analyzing more actual case applications, and tackling real-life implementation issues and problems in more practical detail;
- doing more field, survey, or empirical studies to test the conformity of theoretical real-option valuation and its implications with management’s intuition and experience, as well as with actual market data when available; and
- applying real options to the valuation of flexibility in other related areas - e.g., competitive bidding, information technology or other platform investments, energy and R&D problems, or international finance options.

Furthermore, to strengthen its appeal and gain its acceptance amongst practitioners, Copeland and Antikarov (2005) outline seven criteria for real options methodologies. An acceptable model should:

- intuitively dominate other valuation methods;
- capture the reality of the real world situation;
- rule out the possibility of mispricing by eliminating arbitrage;
- use mathematics that managers can understand;
- be empirically testable;
- appropriately incorporate risk; and
- use as much market information as possible.

\(^1\)Ten important directions are listed, refer to Trigeorgis (1996), page 375.
1.1 Thesis Objectives

The suggestions of Copeland and Antikarov (2005) and Trigeorgis (1996) motivate this thesis; this work is focused on translating the theory and mathematics of real options analysis into a model that is understandable for managers. The two objectives of this work are to:

1. develop a user-friendly model which integrates real options analysis into the current project decision-making framework of managers, and to

2. analyze the applicability of the proposed real options model through real-world case studies.

It is thought that real options analysis will gain popularity if it is easily integrated into the preexisting project valuation framework of the corporation. With the help of industry partners and resources available to the Centre for Management of Technology and Entrepreneurship (CMTE) the proposed model will be applied to value real-world projects. Successful adoption of the proposed framework by our partners will likely lead to expanded adoption of the framework by other academics and practitioners.

1.2 Thesis Outline

The remainder of this thesis is organized as follows. Part II is a review of literature that is relevant to the topics presented in this thesis. Part III contains the paper Integrating Real Options into Managerial Cash Flow Estimates coauthored by Yuri Lawryshyn. This article was submitted and accepted for publication in the Engineering Economist Journal in May, 2011. Presented is a real options model that fits managerial cash flow estimates (optimistic, likely, and pessimistic projections) to a continuous GBM cash flow process that is correlated to a traded asset, so the real option is priced under the risk-neutral measure with a closed-form solution. The preliminary work for this paper was presented at the 2010 Real Options Conference. Based on the comments and feedback provided by the conference attendees this work was revised and the analysis has been extended to value sequential compound real options for investments with multiple cash outlays. In line with the second research objective, the paper A Review of Real Options Approaches: Applying Models to Value a Medical Device Project
coauthored by Yuri Lawryshyn is included in Part IV. This paper will be submitted to The Review of Financial Studies. In this paper, five real options approaches designed for managers are compared, contrasted, and then used to value a medical device project. Last, Part V concludes this thesis and makes recommendations for future real options research.
Part II

Literature Review

The study of real options analysis has developed to address the inadequacies of traditional valuation methods. The primary issues with NPV analysis is that it fails to account for the impact of flexibility and it inaccurately accounts for risk. These limitations are discussed in Section 1. Real options, like financial options, derive their value based upon risky investments, namely real-world projects, and provide managers an innovative way of formulating strategic investment decisions. This review introduces the key concepts underlying real options analysis. The quantitative origins of real options analysis are based upon the mathematics of financial options, so the basics of financial option pricing are also reviewed. Real options are typically categorized by the types of flexibilities they introduce to managers, so fundamental option types are explained. Last, a survey of the literature on the use of real options analysis in industry is included to reiterate the need for this practical research.

1 Traditional Project Valuation Tools

Capital budgeting is the process of identifying and valuing long-term investments that require significant expenditures. Examples of long-term projects include replacement projects to maintain the business or for cost reduction, expansion projects, R&D initiatives, or mandatory projects such as those required by a governmental or regulatory agency. For each investment decision every project undergoes a valuation to determine which alternative should be pursued.

Today NPV analysis is the most widely used tool for valuing corporate investments, yet this was not always the case. The use of NPV analysis arose from even more rudimentary methods and was aided by the development of pocket calculators and personal computers. In 1959 only 19% of firms reported using NPV analysis (Klammer, 1972), whereas in 2002 96% of the Fortune 1000 firms were using this tool (Ryan and Ryan, 2002). A NPV analysis compares the cost of an investment to the future after tax cash flows that the investment is expected to generate. Mathematically, the NPV is expressed as the sum of the discounted future cash flows $f_i$ that occur at times $T_i$ with $k = 1, 2, \ldots, n$, 

$$\text{NPV} = \sum_{i=1}^{n} \frac{f_i}{(1+r)^{T_i}}$$
\[ NPV = \sum_{t=1}^{n} \frac{f_t}{(1 + k)^t}. \]  
\[ (1.1) \]

A positive cash flow \( f_t > 0 \) is a cash inflow to a project, and an negative cash \( f_t < 0 \) is a cash outflow. Cash flows are discounted to account for the time value of money: a sum of cash today is not equivalent to its value sometime in the future. The discount rate \( k \) captures the declining value of future monies received; or alternatively viewed, the discount rate may be understood as the opportunity cost of investing in the considered project and choosing to forgo another alternative.

When considering several alternative projects, the most common discount rate employed is the weighted average cost of capital (WACC). Projects should only be undertaken if the NPV is positive; these projects will earn sufficient cash flows to pay debt holders and will add value to the organization’s shareholders. However, using the WACC to discount all projects oversimplifies the analysis as it assumes that all investments have similar risk characteristics to the firm itself, and that the riskiness of the project is constant over the project life. In fact, the uncertainty in a project will vary over time, and as new information arrives the riskiness of a project will diminish. The WACC is the weighted average of the after tax marginal cost of equity \( k_E \) and cost of debt \( k_D \), and it is weighted by the the market value of the firm’s debt \( D \) and equity \( E \),

\[ WACC = \frac{E}{E + D} k_E + \frac{D}{E + D} k_D (1 - T), \]
\[ (1.2) \]

where \( T \) is the marginal tax rate imposed upon the firm. The cost of debt \( k_D \) is usually the average interest cost of the firm’s outstanding debt, and the cost of equity \( k_E \) is estimated by
the capital asset pricing model (CAPM)\textsuperscript{2}. The CAPM, introduced by Treynor (1962), Sharpe (1964), and Lintner (1965), states that the return of a traded asset is the risk-free interest rate \( r \), the yield on a short-term government treasury note, plus an added premium to the investor for bearing market risk. An equity investor should expect a return equal to

\[
k_E = r + \beta(r_M - r) \tag{1.3}
\]

where \( r_M \) is the expected return of the market portfolio, and

\[
\beta = \frac{\rho \sigma}{\sigma_M} . \tag{1.4}
\]

The volatility of the stock and the market portfolio are \( \sigma \) and \( \sigma_M \), respectively, and \( \rho \) is the correlation of returns. The term \( \beta \) measures the level of systematic risk between the market portfolio and the traded equity. If \( \beta = 1 \), the asset is said to be \textit{perfectly correlated} to the market, and that than the variation in the returns of the asset mimic the upward and downward movements of the market. This type of risk is termed \textit{systemic} or market risk. If the stock is \textit{uncorrelated} to the market \( \beta = 0 \) and the expected return on the asset is \( k_E = r \). This relationship implies that the uncertainty in the returns of the asset is dominated by \textit{idiosyncratic} or private risks. If the asset is uncorrelated to the market, yet risky (\( \sigma \) is large) investors only expect a risk-free return; there is no premium to bear additional risk as idiosyncratic risks may be diversified away by investors if they hold a balanced portfolio of investments (Luenberger, 1998).

Forecasting the investment profit stream accurately is another impediment to NPV analy-

\textsuperscript{2}The assumptions of the CAPM, from Booth and Cleary (2006) are:

1. All investors have identical expectations about expect returns, standard deviations, and correlation coefficients for all securities.
2. All investors have the same one-period investment time horizon.
3. All investors can borrow and lend at the risk-free rate.
4. There are no transaction costs.
5. There are no personal income taxes so investors are indifferent to capital gains over dividends.
6. Investors are price-takers, i.e., there are many investors and no one single investor can affect the price of a stock through her buying decisions.
7. Capital markets are in equilibrium.
sis. If a project is undertaken, the realization of cash flows usually differs from a manager’s initial projections, but this method assumes that cash flows are rigid and inflexible. NPV analysis assumes that a firm follows a fixed scenario in which a project is started, executed, and completed without any contingencies (Dixit and Pindyck, 1994) In the real-world, management has the capability of influencing the cash flow path in response to uncertainty and competitive interactions.

In addition to NPV analysis, which calculates the value of a project, risk management tools are used to assess the perceived riskiness of a project by weighting the probability and the financial impact of possible project outcomes. Common techniques employed by managers include sensitivity analysis, scenario analysis, and Monte Carlo simulation. Sensitivity analysis allows managers to assess the impact of important variables on the return of a project, and scenario analysis measures the combined affect of different variables (or operating scenarios) on the outcomes of a project. In sensitivity analysis, the NPV is calculated as single variables, such as revenues, operating costs, or corporate tax rates, are manipulated, holding all else constant. Scenario analysis considers the sensitivity of the project value to changes in one or more input variables as well as the likelihood that various outcomes will occur. In both techniques an expected cash flow is compared to a pessimistic and an optimistic case. Although scenario analysis is readily used in industry, the subjective and heuristic nature of this technique is debated.

Monte Carlo simulation relies on repeated sampling from a predefined probability distributions to find solutions to problems where there are many uncertainties present. Monte Carlo simulation was first used to solve option valuation problems by Boyle (1977). Random price paths are generated and the option payoff for each path is calculated. The average payoff is the price of the option. Monte Carlo simulation is a flexible methodology that can solve problems with multiple state variables, but it is limited by a low solution speed, high computation requirements, and the difficulty in finding solutions for options with early exercise (Areal, Rodrigues, and Armada, 2008). Commercially available software, such as @Risk or Crystal Ball, which can be used within a Microsoft Excel spreadsheet can aid managers when using these techniques.
2 Introducing Options Analysis

Real options analysis and reasoning is grounded in financial option theory (Myers, 1977), so financial options are first introduced. An option is a contract that gives the owner the right, but not the obligation, to purchase or sell an underlying risky asset at a future date (the maturity) and at a specified price (the strike). Options are financial instruments, traded on both exchanges and over-the-counter markets, that derive their value from an underlying asset. Underlying financial assets may include: shares of a public firm, foreign currencies, index prices, commodities, or futures (Higham, 2004). A common feature is that the current price of these assets is known with certainty, but the price is liable to change in the future. Financial options are purchased for several reasons, but are predominantly used for hedging or speculation. Hedging is a strategy of mitigating risk should the underlying asset yield an unfavourable outcome, while speculating is the act of taking a position in the market; betting that the price of the asset will increase, or betting that it will decrease (Hull, 2009).

Two parties are involved in any option contract. The investor who purchases the option is said to take the long position while the investor who sells an option (or writes the option) takes the short position in the contract. Two main option types exists. Call options provide the holder the opportunity to purchase the underlying asset at the strike price, whereas put options provide the holder the right to sell the underlying asset at the strike. The holder is not obliged to exercise the option at maturity, but will do so only if it is economically favourable. If the right to exercise the option is fixed to a certain date, the option is a European option, or if it may be exercised before or on the exercise date it is an American option. Option payoffs are written as piecewise linear functions. The payoff of a call option is

\[ c(S_t,t) = \max(S_t - K, 0) \]

(2.1)

and the payoff of a put option is

\[ p(S_t,t) = \max(K - S_t, 0) \]

(2.2)

where \( S_t \) is the underlying stock price at time \( t \) and \( K \) is the strike price. The form of the
payoff functions reveal the desirability of options: these contracts limit the downside potential of a risky investment, yet the benefits remain unrestricted. The asymmetric payoffs of put and call options are presented in Figure 2.1a and Figure 2.1b.

![Payoff Diagrams](image)

**Figure 2.1** – The payoff diagrams for a) call option and b) put option. The payoff of the underlying stock is also plotted.

In their seminal work, Black and Scholes (1973) derive the following partial differential equation (PDE) which represents the changing value of the call option with respect to the changing value of the stock price over time,

\[
rc_t = \frac{\partial c_t}{\partial t} + rS_t \frac{\partial c_t}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 c_t}{\partial S_t^2} \tag{2.3}
\]

where \(\sigma\) is the volatility of the underlying asset. By applying the boundary conditions in Equation 2.1 or Equation 2.2, Black and Scholes (1973) find closed-form solutions to value the options on a traded stock\(^3\). The value of a European call is

\[
c(S_t, t) = e^{-r(T-t)} [S \Phi(d_1) - K \Phi(d_2)], \tag{2.4}
\]

\(^3\)The assumptions underlying the Black-Scholes formula, taken from Hull (2009), are as follows:
1. the stock price follows a geometric Brownian motion with constant growth \(\mu\) and volatility \(\sigma\);
2. the short selling of securities is permitted;
3. there are no transaction costs or taxes;
4. there are no dividends paid over the duration of the option;
5. there are no arbitrage opportunities;
6. security trading is continuous; and
7. the risk-free rate is constant and the same for all maturities.
and the value of the put option is

$$p(S_t, t) = e^{-r(T-t)}[K\Phi(-d_2) - S\Phi(-d_1)],$$

(2.5)

with

$$d_1 = \frac{\ln(S_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}.$$  

(2.6)

and

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$  

(2.7)

In this formula $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, and $T - t$ represents the time until the option expires. The value of an option is influenced by six factors: the underlying asset price, the strike price, the volatility of the underlying asset, the time to maturity, the risk-free interest rate, and the dividend yield (if the underlying asset pays regular dividends). Table 2.1, adapted from Hull (2009), shows the sensitivity of the option value to an increase in each parameter.

**Table 2.1** – Call and put option sensitivities, and the affect on the option value if the underlying variable is increased, holding all else constant.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sensitivity</th>
<th>Call option value</th>
<th>Put option value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying asset price</td>
<td>Delta, $\Delta = \frac{\partial c}{\partial S}$</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>Volatility</td>
<td>Vega, $\nu = \frac{\partial c}{\partial \sigma}$</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Strike price</td>
<td>Xi $\Xi = \frac{\partial c}{\partial K}$</td>
<td>↓</td>
<td>↑</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>Theta, $\Theta = \frac{\partial c}{\partial (T-t)}$</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>Rho, $\rho_t = \frac{\partial c_t}{\partial S_t}$</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>Dividends</td>
<td>-</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>
3 Linking Financial Options to Real Options

An analogy is made between financial options and real options because of the similarities in their risk structure and payoffs (Vollert, 2003). Real options provide the holder, managers, the opportunity to make a decision on a tangible investment. Akin to financial options, this added managerial flexibility can help to limit the potential losses while exploiting potential gains in projects. Table 3.1, adapted from Trigeorgis (1996), highlights some of these similarities.

Table 3.1 – A comparison between the variables influencing the value of financial and real options.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Financial Option</th>
<th>Real Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying asset price</td>
<td>Stock price</td>
<td>Present value of the future cash flows of the project</td>
</tr>
<tr>
<td>Volatility</td>
<td>Variation in the returns of the stock</td>
<td>Variation in the returns of the project</td>
</tr>
<tr>
<td>Strike price</td>
<td>Price to purchase the stock</td>
<td>Investment cost</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>Time until the option expires</td>
<td>Time until the opportunity expires</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>Risk-free interest rate</td>
<td>Risk-free interest rate</td>
</tr>
<tr>
<td>Dividends</td>
<td>Cash flows from stock</td>
<td>Cash flows from project</td>
</tr>
</tbody>
</table>

According to Trigeorgis (1996), the added value to the project gained by viewing projects as opportunities is the expanded NPV: this is the sum of the value project without flexibility, the static NPV, and the value from considering optionality in the project,

\[
expanded \ NPV = static \ NPV + real \ option \ value . \quad (3.1)
\]

Options are always non-negative, so even if the static NPV suggests the project is invaluable the embedded optionality in a project may result in a situation where the expanded NPV is positive and it is advantageous to undertake the venture\(^4\). Trigeorgis (1996) represents this added optionality by as skewness or asymmetry in a project’s NPV probability distribution. The result of managerial flexibility is shown in Figure 3.1.

\(^4\)Refer to Trigeorgis (1996) for an example. The author provides and example of a pharmaceutical project. The static NPV is negative, but when the optionality in the project is considered the expanded NPV is positive.
The literature classifies real options by the types of flexibilities they introduce to management. Literature has modeled and priced deferral options, time-to-build options, options to alter the operational scale of projects, abandonment options, switching options, and multiple interacting options, among others. In more detail:\(^5\):

- In a **deferral option**, management holds the opportunity to invest in a project, but the investment date is flexible. Conditional on the current market and technological conditions, the opportunity cost of waiting (foregone cash flows) might be outweighed by the benefits of deferring immediate investment (Vollert, 2003). This type of option is important in natural resource investments, such as mining or oil and gas, where the value of the project is highly dependant upon traded commodity prices.

- Staging investment, with a **time-to-build option**, allows management to adjust to changes in the technology or in market conditions, or to abandon the project altogether. It also allows management to find sources of financing as the product is developed. This type of option is present in all R&D projects.

- Management may make the decision to increase or decrease the scale of operations when exercising an **option to alter the operational scale**. The option to expand is viewed as a call option on additional capacity, whereas the choice to downsize is viewed as a put.

\(^5\)Refer to Trigeorgis (1996, pg, 2-14) for a complete description of the real option, potential applications, and supporting references in literature.
option (Vollert, 2003). This option is important for investments in cyclic industries, such as commercial real estate, consumer goods, automotive, or manufacturing, so managers can adapt their business changing market conditions.

- Management has the option to abandon production and instead recuperate any salvage value that remains in the project. This option is important to capital intensive industries such as airlines or railroads.

- Flexible operations have the option to switch inputs or outputs in response to external changes in the market. Flexibility may be present in the product or process. Process flexibility may add value to feedstock-dependant facilities, where product flexibility would enable to the firm to alter outputs depending upon demand.

- Every project has combinations of multiple interacting options. One important type to multistage R&D investments is compound options. Compound options are options on underlying options; a series of expenditures are made only if the previous stage is successful.

This thesis is not focused on the pricing of a given type of option, but instead attempts to provide managers with a framework where they can value individual projects. European options and compound options will be analyzed.

4 The Use of Real Options Analysis in Industry

The use of real options analysis in industry is attributed to several surveys. Bain & Company (2000) found that in 2000 only 9% of respondents were using real options analysis, whereas in 2002, Ryan and Ryan (2002) found from a sample of Fortune 1000 firms (205 respondents) that 11.4% use real options. To contrast, 96% of firms in this population use NPV analysis, and 85.1% of firms use sensitivity analysis to estimate project risks. Block (2007) later surveyed the same population (279 respondents) and only 14.3% use real options in their budgeting process. Recently, Bennouna, Meredith, and Merchant (2010) conducted a survey of large Canadian corporations and found in this population the adoption rate was lower: only 8.1% of firms are using real options analysis. Overall, the use of real options analysis in industry is stagnant.
Integrating Real Options with Managerial Cash Flow Estimates

Abstract

This paper presents a real options model that fits managerial cash flow estimates (optimistic, likely, and pessimistic projections) to a continuous GBM cash flow process with changing growth and volatility parameters. The cash flows and the value of a project are correlated to a traded asset, so the real option is priced under the risk-neutral measure with a closed-form solution. The analysis is extended to a sequential compound call option for investments with multiple cash outlays. If the project is correlated to the market then some of the risk may be mitigated by a delta-hedging strategy. A numerical example shows that the affect of the correlated asset on the real option value is significant, and the relationship between the volatility of the project and the real option value is not analogous to the typical relationship found in financial option pricing. Integrating the expertise and industry knowledge of management, this approach makes possible a more rigorous estimation of model inputs for real option pricing.

Keywords: Real options, project valuation, investment under uncertainty, compound options, managerial information
1 Introduction

Business and academic literature have recommended real options as a technique to analyze real-world investments where uncertainty is present as traditional valuation methods, namely net present value (NPV) analysis, fail to account for the impact of managerial flexibility on the value of the project. However, industry surveys have demonstrated that there is a limited adoption of real options analysis within an applied setting (Ryan and Ryan, 2002, Teach, 2003, Hartmann and Hassan, 2006, Block, 2007, Bennouna, Meredith, and Merchant, 2010). Complaints of real options analysis are due to the complexity of analytical models, the restrictive assumptions required, and the overall lack of intuition in the solution procedures (Block, 2007). Furthermore, it can be argued that many of the models proposed by academics can not be simply integrated into the quantitative estimates typically provided by managers.

This paper addresses these cited issues. Presented is a model that is financially sound yet tractable and correctly accounts for the sensitivity of the real option value to risk. In this model, the discrete cash flows of a project, extracted from managerial optimistic, likely, and pessimistic estimates, are correlated to a traded index. It is shown that the volatility and growth of the cash flows are identical to that of the project value process, and that a real option on the project value can be determined by a closed-form solution under the risk-neutral measure. If the real option exhibits some correlation to the market then a hedging strategy can mitigate some of the market exposure. Two issues regarding risk and real option value are addressed: the procedure to estimate volatility, and the sensitivity of the real option value to the underlying volatility. In contrast to approaches in the literature (Copeland and Antikarov, 2001, Brandão, Dyer and Hahn, 2005a) in which historical estimates are combined through simulation, the proposed model relies on the expertise and experience of managers to predict forward looking cash flows from which volatility is deduced. Furthermore, this model allows management to model the impact of market volatility on the project value by acknowledging the influence of a correlated asset. Implicit is the assumption that management understands the firm and its investment opportunities best and is capable of making adept predictions.

The remainder of this paper is organized as follows. Section 2 reviews applicable real options literature. Section 3 develops the proposed model, details a hedging strategy, and shows how
managerial estimates can be integrated into the valuation procedure. The model is then extended to a compound real option. In Section 4, a project previously valued by Datar and Mathews (2005) is revalued by the proposed model. Results of the example and aspects of the model are discussed. Section 5 addresses the relationship between real option value and volatility, and conclusions are provided in Section 6.

2 Related Literature

Real options analysis is an auxiliary tool when compared to NPV analysis, and there has been little change in the usage of real options analysis by practitioners in the last decade (see Ryan and Ryan, 2002, Hartmann and Hassan, 2006, Block, 2007, Bennouna, Meredith, and Merchant, 2010). The reluctance to use real options analysis stems from a lack of management support, but is also due to sophisticated mathematics required from many academic models, the notion that real options encourages excessive risk taking, and the overall lack of intuition in the solution procedures (Block, 2007).

Borison (2005) provides a review of five real options models in the literature that are intended for managers, and gives a detailed analysis of the differences in the assumptions, applicability, and solution mechanics of the approaches. The classical approach (Brennan and Schwartz, 1985) assumes that capital markets are complete, and that the dynamics of a project can be replicated by a traded asset. This assumption allows for a simple solution procedure; the project is valued using the Black-Scholes equation. In this model the historical volatility of the asset is taken to be the best estimate for the future volatility of the project. However, the risks affecting the portfolio of assets are markedly different than those influencing the real-world project. Copeland and Antikarov (2001), proponents of the market asset disclaimer (MAD) approach, make this distinction: “the volatility of gold is not the same as the volatility of the gold mine”. The subjective approach (Leuhrmann, 1998a, 1998b) eliminates the need to find an equivalent portfolio and instead uses industry standards or estimates as inputs into the Black-Scholes model. Although this procedure is straightforward, the accuracy and the financial rigour are compromised as an inconsistent combination of no-arbitrage assumptions and subjective estimates are utilized (Borison, 2005). The revised classical approach (Dixit and
Pindyck, 1994) is merely an extension of the classical approach and decision tree analysis (DTA): if the project is dominated by market uncertainties, then the classical approach is employed, but if technological risks prevail DTA is applied. The integrated approach (Smith and Nau, 1995) assumes that markets are partially complete, and it mixes market and probabilistic lattices. The assumption underlying the MAD approach is a contrarian view when compared against the classical approach: Copeland and Antikarov (2001) assume that markets are incomplete, so it is futile to search for a replicating portfolio. Instead, the value of the project without flexibility calculated at the risk-adjusted discount rate is the best estimate for the underlying asset. The MAD approach uses minimal market information, so there is a challenge in estimating a reasonable volatility for the underlying project value. The method Copeland and Antikarov (2001) suggest to estimate project volatility has been shown to over-predict the volatility of the project cash flows, and modifications have since been recommended (see Smith, 2005, Brandão, Dyer, and Hahn, 2005b).

Researchers (Brandão, Dyer, and Hahn, 2005a, Schneider et al., 2008) have suggested refinements to the model of Copeland and Antikarov (2001). Brandão, Dyer, and Hahn (2005a) adopt the same assumption underlying the MAD approach, but suggest a binomial decision tree as opposed to a recombining binomial lattice to model the underlying GBM process. This allows for management to visualize scenarios with multiple uncertainties, complex options, and projects with heteroskedasticity. Schneider et al. (2008) extend the work of Smith and Nau (1995) and Copeland and Antikarov (2001) by proposing an multidimensional integrated tree. Multidimensionality allows for managers to model the value of exercising defer or switching options, but because of its complexity and computational burden is unlikely to be as intuitive to managers as other solution procedures.

As an alternative to lattice techniques, Datar and Mathews (2004) propose a method (the DM approach) that relies on Monte Carlo simulation to produce a distribution of project values based on triangular cash flow probability distribution functions (see also Datar and Mathews, 2007 and Mathews, 2009). Collan, Fullr, and Mezei (2009) extend the work of Datar and Mathews (2004) with the use of fuzzy numbers. The main contribution of these strategies is that it includes managers predictions of cash flow estimates (an optimistic, a likely, and a pessimistic case) to price the real option. Scenario analysis is commonly used by managers to
analyze project risks (Bennouna, Meredith, and Merchant, 2010). The DM approach makes an important link in the attempt to integrate real options analysis into the current valuation framework used by managers. However, the work of Datar and Mathews and Collan, Fullr, and Menzi (2009) assumes that the market is complete and fails to differentiate between market and technological risks. Additionally, the underlying process driving the real option value does not follow a true GBM; this assumption is not financially consistent.

Sequential cash flow outlays are typical of R&D investing; investments will occur only as the technology is proven through the research stages and as financing sources are secured. Managers can use compound option pricing models, as derived from Geske’s compound option model (Geske, 1977), to value R&D projects. Geske’s model (Geske, 1977) has been extended to compute the value of sequential put or call options with time dependent volatility and interest rate parameters (see Chen, 2002, Lajeri-Chahelri, 2002, Carr, 1988, Chen, 2003, Thomassen and VanWouve, 2001, and Lee, Yeh, and Chen, 2008). Compound real options have been applied to several R&D applications in the literature including the valuation of a new drug discovery (Casimon et al., 2004), technology adoption (Kauffman and Li, 2005), and capital investment (Herath and Park, 2002), but there is little evidence that practitioners are comfortable with this tool in industry. Hence, a tool for managers to practically value compound real options will be proposed.

Ultimately, the dissimilarities among the real options models designed for practitioners result in stark differences in valuations and lead to contradictory recommendations (Borison, 2005). For real options analysis to be implemented by managers, the proposed tools must be easily integrated into the current forecasting and valuation framework of the corporation, yet models must be transparent and understandable for managers. Considering the budgeting procedures in place, the proposed real option model will build on the current practice of cash flow forecasting and scenario analysis. As it will be discussed, the advantage of the proposed work is that it is practical, financially correct, and accurately accounts for the sensitivity of the real option value to risk.
3 Model Development

The development of this model is carried out in three sections. Section 3.1 unifies the work of Berk, Green and Naik (2004) and Sick and Gamba (2005). The relationship between the dynamics of the cash flows and the project value, as found by Berk, Green, and Naik (2004), is presented. The key result is that the growth and volatility of the cash flow process is identical to that of the project value process. Next, the risk-neutral PDE and the analytical solutions for European call and put real options, originally derived by Sick and Gamba (2005), are restated. In Section 3.2 the cash flow process is changed to discrete cash flow process to match managerial estimates. A closed-form solution for the real option on the value of this process is derived. Last, in Section 3.3, an analytical solution, derived from the work of Lee, Chen and Yen (2008), for a European compound real option is presented.

3.1 PDE for the Valuation of Real Options

The asset underlying the value of a real option is typically the project value. This is is the case with the proposed model; however, an understanding of the dynamics of the cash flows of the project and their relationship to the dynamics of the project value is required in subsequent sections to utilize managerial cash flow estimates. The cash flows $g_t$ are assumed to follow a GBM process and are subject to stochastic variations. In this formulation under the real-world measure $\mathbb{P}$,

$$g_t = \nu g_t dt + \eta g_t dZ_t,$$

where $g_t$ is the incremental change in the cash flow over the interval $dt$; $\nu$ is the growth rate of the cash flows; $\eta$ is the volatility of the cash flows; and $Z_t$ is a standard Wiener process. Neither the cash flows nor the project value are traded, so the market is incomplete. Consequently, a traded index $S_t$ is introduced, and the success of the project is assumed to be strongly correlated to the returns of the index. This traded index also follows a GBM process with

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$
The growth rate and volatility of the index are $\mu$ and $\sigma$, and $W_t$ is a second Wiener process. The process driving the cash flow variations $Z_t$ can be decomposed into two motions: one correlated to the index $W_t$ and one independent of market fluctuations $W_t^\perp$.

$$dZ_t = \rho dW_t + \sqrt{1 - \rho^2} dW_t^\perp.$$  \hspace{1cm} (3.3)

Here $\rho$ represents the level of instantaneous correlation between the cash flows and the index, and $-1 \leq \rho \leq 1$. Greater correlation between the cash flows and the index implies that the success of the cash flows is strongly tied to the success of the traded index. The cash flow process under the risk-neutral measure $Q$ is

$$dg_t = \tilde{\nu} g_t dt + \eta g_t (\rho d\tilde{W}_t + \sqrt{1 - \rho^2} d\tilde{W}_t^\perp),$$  \hspace{1cm} (3.4)

with

$$\tilde{\nu} = \nu - \frac{\rho \eta}{\sigma} (\mu - r),$$  \hspace{1cm} (3.5)

where $\tilde{W}_t$ and $\tilde{W}_t^\perp$ are uncorrelated Brownian motions under this measure; $\tilde{\nu}$ is the risk-adjusted growth rate; and $r$ is the risk-free interest rate. In the model presented it is assumed that $r$, $\sigma$, $\mu$, $\eta$, $\nu$, $\tilde{\nu}$ and $\rho$ are constant over the duration of the project. The growth rate of the cash flows under the real-world measure $\nu$ is reduced by the market price of risk to yield $\tilde{\nu}$. As historical information about the market variable, the traded index, is available the market price of risk is estimated using the capital asset pricing model (Hull, 2009). The expected value of the of the cash flow process is equal to,

$$\mathbb{E}_Q^Q [g_s | \mathcal{F}_t] = g_t e^{\tilde{\nu}(s-t)}, \ s > t$$  \hspace{1cm} (3.6)

where $\mathcal{F}_t$ denotes the filtration at time $t$ and $\mathbb{E}_Q^Q[\cdot]$ denotes the expectation under the risk-neutral measure. The project value is the sum of all future cash flows discounted at the risk free rate,

$$V_t = \mathbb{E}_Q^Q \left[ \int_t^\infty e^{-r(s-t)} g_s ds | \mathcal{F}_t \right].$$  \hspace{1cm} (3.7)

Simplifying, a relationship between the project value and cash flow is found,
\[ V_t = \frac{g_t}{r - \bar{\nu}}. \]  

For the project to have finite value \( r > \bar{\nu} \). The form of Equation 3.8 is akin to the continuous-time form of the Gordon growth model used to price a dividend paying stock (Gordon, 1959), and if the project is held indefinitely then the cash flows will act as a perpetuity of payouts to the investor. A similar relationship was found by Berk, Green, and Naik (2004) when valuing a staged R&D investment. Substituting Equation 3.8 into Equation 3.4, it is realized that the volatility and growth of the project value are identical to the volatility and growth of the cash flows, i.e.

\[
dV_t = \bar{\nu}V_t dt + \eta V_t (\rho d\tilde{W}_t + \sqrt{1 - \rho^2}d\tilde{W}_t^\perp). \tag{3.9}
\]

Although the value of the underlying real option, which is the cash flow stream, is not traded it is still possible to mitigate some of the market risk with a delta-hedging strategy. To replicate the payoff of the option the self-financing hedge portfolio should hold

\[
\Delta_t^{RO} = \rho \eta V_t \frac{\partial RO_t}{\partial V_t} \tag{3.10}
\]

units of the index and the balance in the money market account. Trading in this manner leads to the derivation of the following partial differential equation (PDE),

\[
rRO_t = \frac{\partial RO_t}{\partial t} + \bar{\nu}V_t \frac{\partial RO_t}{\partial V_t} + \frac{1}{2} \eta^2 V_t^2 \frac{\partial^2 RO_t}{\partial V_t^2}. \tag{3.11}
\]

This PDE resembles the Black-Scholes PDE; a second order, linear parabolic equation. This result was detailed in Sick and Gamba (2005), and for completeness the analytical solution for European call and put options presented by the authors is included. The value the call option at time \( T > t \) is

\[
RO_{\text{call}}(V, t) = e^{-r(T-t)}[Ve^{\bar{\nu}(T-t)}\Phi(d_1) - K\Phi(d_2)], \tag{3.12}
\]
and the value of the put option is

\[ R^\text{put}(V,t) = e^{-r(T-t)}[K\Phi(-d_2) - Ve^{\tilde{\nu}(T-t)}\Phi(-d_1)], \]

(3.13)

with

\[ d_1 = \frac{\ln(V/K) + (\tilde{\nu} + \frac{1}{2}\eta^2)(T-t)}{\eta\sqrt{T-t}}, \]

(3.14)

and

\[ d_2 = d_1 - \eta\sqrt{T-t}. \]

(3.15)

Here \( K \) is the strike and \( \Phi(\cdot) \) denotes the standard normal cumulative distribution function.

3.2 Valuing an Option on a Cash Flow Stream

To utilize managerial optimistic, likely, and pessimistic cash flow estimates the underlying GBM process from Section 3.1 is modified. A new process \( f_t \) is introduced with discontinuous parameters \( \nu_i \) and \( \eta_i \) representing the discrete cash flow that would occur at given time \( t \). Here \( f_t \) is no longer a cash flow rate, as in Section 3.1. The parameters \( \nu_i \) and \( \eta_i \) are chosen so that the density of \( f_{t_i} \) matches closely the optimistic, likely, and pessimistic managerial cash flow estimates supplied at \( t_i \), with \( i = 1, 2, \ldots, n \). It follows that

\[ f_i \overset{d}{=} f_{0e^{(\nu_1-\frac{1}{2}\eta^2)t_1+\ldots+(\nu_i-\frac{1}{2}\eta^2)(t_i-t_{i-1})+\sqrt{\eta^2t_1+\ldots+\eta^2(t_{i-1}-t_{i-1})}Z}, \]

(3.16)

where \( Z \sim \mathcal{N}(0,1) \) and \( \overset{d}{=} \) denotes that both sides of Equation 3.16 are equal in distribution.

This is a GBM process, so lognormal probability density functions (PDFs) describe the periodic cash flows. An example of three cash flows estimates is given in Table 3.1 and the results of the fitting the lognormal PDFs to these estimates is shown in Figure 3.1.
Table 3.1 – Sample optimistic, likely, and pessimistic cash flow estimates provided by managers ($)

<table>
<thead>
<tr>
<th>Time</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>...</th>
<th>$T_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>10</td>
<td>12</td>
<td>18</td>
<td>...</td>
<td>$x_{n,\text{optimistic}}$</td>
</tr>
<tr>
<td>Most likely</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>...</td>
<td>$x_{n,\text{likely}}$</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>...</td>
<td>$x_{n,\text{pessimistic}}$</td>
</tr>
</tbody>
</table>

Figure 3.1 – Lognormal probability distribution functions fit optimistic, likely, and pessimistic cash flow estimates provided by managers.

Given three managerial cash flow estimates, a system of equations is used to solve for the periodic growth, volatility, and the initial cash flow parameters $\nu_1$, $\eta_1$, and $f_0$, respectively. The most likely estimate is the expected value of the process, whereas the optimistic and pessimistic estimates are upper and lower bounds of the cash flow PDF. For example, in the first year the three equations are

$$x_{1,\text{optimistic}} = f_0 e^{(\nu_1 - \frac{1}{2} \eta_1^2)T_1 + \sqrt{\eta_1^2 T_1} Z},$$

$$x_{1,\text{likely}} = E[f_1] = f_0 e^{\nu_1 T_1},$$

$$x_{1,\text{pessimistic}} = f_0 e^{(\nu_1 - \frac{1}{2} \eta_1^2)T_1 + \sqrt{\eta_1^2 T_1} Z},$$

(3.17)

where $x_{1,\text{likely}}$, $x_{1,\text{optimistic}}$, and $x_{1,\text{pessimistic}}$ are the most likely, the optimistic and pessimistic estimates provided by management. Here $E[\cdot]$ denotes the expectation under the real-world measure.

At this point several comments are necessary. As the time intervals are defined, the cash flow growth in the first period $\nu_1$ will be zero. Second, $f_0$ is not a realized cash flow but an
arbitrary initial value from which cash flow growth is realized in subsequent periods. Last, if a future cash flow is known with certainty then the deterministic value should replace the range of potential outcomes. This might be the case where operating costs or revenues are guaranteed by futures or forward contracts.

The fitting procedure is continued for each successive cash flow. In general, if cash flows occur at times \( t_1 \ldots t_i \) the parameters for \( \nu_i \) and \( \eta_i \) are found from the three equations

\[
x_{i,\text{optimistic}} = f_0 e^{(\nu_1 - \frac{1}{2} \eta_1^2)t_1 + \ldots + (\nu_i - \frac{1}{2} \eta_i^2)(t_i - t_{i-1}) + \sqrt{\eta_i^2(t_1 + \ldots + \eta_i^2(t_i - t_{i-1})} Z, \\
x_{i,\text{likely}} = \mathbb{E}[f_i] = f_0 e^{\nu_1 t_1 + \ldots + \nu_i (t_i - t_{i-1})}, \\
x_{i,\text{pessimistic}} = f_0 e^{(\nu_1 - \frac{1}{2} \eta_1^2)t_1 + \ldots + (\nu_i - \frac{1}{2} \eta_i^2)(t_i - t_{i-1}) + \sqrt{\eta_i^2(t_1 - t_i) + \ldots + \eta_i^2(t_i - t_{i-1})} Z.
\] (3.18)

When solving for these parameters, appropriate weightings to each estimate may be applied. Stronger emphasis on the expected value and the pessimistic outcome for will produce a distribution weighted more towards lower expectations. Conversely, if the outlook is positive managers may apply greater weighting to the upper tail. Ultimately, this fitting method serves as a framework for projects when uncertainty is present, but if additional information resolves future uncertainty then the model should undergo appropriate revision.

Once parameters for the real-world cash flow process have been determined, the process under the risk-neutral measure \( \mathbb{Q} \) can be written as

\[
\tilde{f}_i = \frac{d}{f_0 e^{(\tilde{\nu}_1 - \frac{1}{2} \tilde{\eta}_1^2)t_1 + \ldots + (\tilde{\nu}_i - \frac{1}{2} \tilde{\eta}_i^2)(t_i - t_{i-1}) + \sqrt{\tilde{\eta}_i^2(t_1 - t_i) + \ldots + \tilde{\eta}_i^2(t_i - t_{i-1})} Z},
\] (3.19)

with \( Z \sim \mathcal{N}(0,1) \), and the value of the \( i \)-th cash flow that occurs at time \( t_i \) is

\[
V_i = \mathbb{E}_\mathbb{Q}\left[ \tilde{f}_i | \mathcal{F}_t \right] = \tilde{f}_i e^{\tilde{\nu}_1 t_1 + \ldots + \tilde{\nu}_i (t_i - t_{i-1}) - rt(t_i - t_{i-1})}
\] (3.20)

The value of the project is the sum of discounted discrete cash flows,

\[
V_t = \sum_{i=1}^{n} V_i \\
= \tilde{f}_t (e^{\tilde{\nu}_1 t_1 - rt_1} + e^{\tilde{\nu}_1 t_1 + \tilde{\nu}_2 (t_2 - t_1) - rt(t_2 - t_1)} + \ldots + e^{\tilde{\nu}_1 t_1 + \tilde{\nu}_2 (t_2 - t_1) + \ldots + \tilde{\nu}_i (t_i - t_{i-1}) - rt(t_i - t_{i-1}))
\] (3.21)
Prior to the investment at time $t_K \leq t_1$ the cash flow and value processes of the project are

\[ \hat{f}_t = f_0 e^{(\tilde{\nu}_1 - \frac{1}{2} \eta_1^2) t + \eta_1 \sqrt{t} Z}, \tag{3.22} \]

and

\[ V_t = V_0 e^{(r - \frac{1}{2} \eta_1^2) t + \eta_1 \sqrt{t} Z}, \tag{3.23} \]

so an expression for the present value of the future cash flows at time $t = 0$ is found,

\[ V_0 = f_0 \left( e^{\tilde{\nu}_1 t_1 - r t_1} + e^{\tilde{\nu}_1 t_1 + \tilde{\nu}_2 (t_2 - t_1) - r t_2} + \ldots + e^{\tilde{\nu}_1 t_1 + \tilde{\nu}_2 (t_2 - t_1) + \ldots + \tilde{\nu}_m (t_m - t_{m-1}) - r t_m} \right). \tag{3.24} \]

The value of the opportunity to purchase the present value of the future cash flows is found with risk-neutral valuation. The value of the real option at time $t = 0$ with $t_K \leq t_1$ is

\[ RO_{\text{call}}(V_0, t_0) = V_0 \Phi(d_1) - K e^{-r t_K} \Phi(d_2), \tag{3.25} \]

with

\[ d_1 = \frac{\ln \left( \frac{V_0}{K} \right) + (r + \frac{1}{2} \eta_1^2) t_K}{\eta_1 \sqrt{t_K}}, \tag{3.26} \]

and

\[ d_2 = d_1 - \eta_1 \sqrt{t_K}. \tag{3.27} \]

### 3.3 Valuing an Option on a Project with Multiple Investments

The closed-form solution on a compound real option is adapted from the work of Lee, Yeh and Chen (2008). A typical cash flow diagram for a project that requires $m$ stages of investment is presented in Figure 3.2. An investment of $K_j$ is made at time $t_{K_j}$ with $j = 1, 2, \ldots, m$ before cash flows $f_i$, found by the previously outlined fitting procedure, are received at discrete times $t_i$, with $i = 1, 2, \ldots, n$. Here $0 \leq t_{K_1} < t_{K_2} \ldots < t_{K_m} \leq t_1 < t_2 \ldots < t_n$. $RO_m(t_0)$ is the $m$-fold compound option that matures at time $t_{K_1}$ with strike $K_1$. It derives its value from
$RO_{m-1}(t_{K_2})$, which is the $m - 1$ compound option that matures at $t_{K_2}$. The first fold option $RO_1(t_{K_m})$ is a call option on the underlying present value of the future cash flows $V_0$, where $V_0$ is given by Equation 3.24.

Figure 3.2 – The notation for the $m$-fold compound real option.

The value of the $m$-fold compound real option at $t_0$ is

$$RO_m(V_0, t_0) = V_0 N_m(a_1, a_2, \ldots, a_m; A^m) - \sum_{j=1}^{m} K_j e^{-rt_{K_j}} N_j(b_1, b_2, \ldots, b_j; A^j), \quad (3.28)$$

with

$$b_m = \ln\left(\frac{V_0}{V_m}\right) + \left(r + \frac{1}{2}\eta_1^2\right)(t_{K_m} - t_{K_{m-1}})$$

and

$$a_m = b_m + \eta_1 \sqrt{t_{K_m} - t_{K_{m-1}}}, \quad (3.29)$$

where $N_m(a_1, a_2, \ldots, a_m; A^m)$ is the $m$-variate cumulative normal distribution function. $V_m$ is the solution of $RO_{m-j}(t_{K_j}) = K_j$. The upper limit to this distribution is $a_m$ and $A^m$ as the correlation matrix with

$$A^m = (a_{ij}^m)_{i,j=1,\ldots,m} \begin{cases} a_{ii} = 1 \\ a_{ij} = a_{ji} = \rho_{ij} \end{cases} \quad (3.30)$$

and
$\rho_{ij} = \sqrt{\frac{t_{K_i}}{t_{K_j}}}$ \hspace{1cm} (3.32)

4 Valuation of an UAV Project

In this section a numerical example demonstrates how managers can utilize cash flow projections to quantify the value of the real option on the project and to examine the real option value when its success is influenced by a traded asset. Within this example positive market correlation is examined for $0 \leq \rho \leq 1$, but in practice managers would be expected to estimate this parameter based upon historical information, related project successes, or industry experience.

An unmanned aerial vehicle (UAV) project, previously valued by Datar and Mathews (2007), is examined. The manager must invest $K=325$ million at $t_K=2$ years to the receive cash flows presented in Table 4.1. Using standard NPV analysis, at risk-adjusted discount rate of 15% (used by Datar and Mathews, 2007) the likely cash flow stream has a present value of $242$ million, whereas the discounted value of the investment is $246$ million. Using this standard approach, a manager would not invest in the UAV project. To contrast, using their method Datar and Mathews (2007) calculate the option on the cash flows in Table 4.1 to be worth $23$ million.

Table 4.1 – Optimistic, likely, and pessimistic cash flow estimates for the UAV project ($\text{\$ million}$) (Datar and Mathews, 2007).

<table>
<thead>
<tr>
<th>Year</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>80</td>
<td>116</td>
<td>153</td>
<td>177</td>
<td>223</td>
<td>268</td>
<td>314</td>
</tr>
<tr>
<td>Most likely</td>
<td>52</td>
<td>62</td>
<td>74</td>
<td>77</td>
<td>89</td>
<td>104</td>
<td>122</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>20</td>
<td>23</td>
<td>24</td>
<td>18</td>
<td>20</td>
<td>29</td>
<td>22</td>
</tr>
</tbody>
</table>

Two indicies are considered to compare the influence of the correlated asset on the real option value. The first index selected is the PowerShares Aerospace and Defence Portfolio, an exchange traded fund (ETF) that tracks the SPADE Defence Index and is listed on the New York Stock Exchange (NYSE: PPA). This ETF includes companies involved in the development, manufacturing, and support of U.S. defence, homeland security, and aerospace operations (Invesco,
2010). The second index is the S&P 500, which is a broader representation of the market. The historical growth and volatility of the indices are presented in Table 4.3. The risk-free rate is 3%. Figure 4.2 plots the real option value of the UAV project. A clear trend emerges: the real option value decreases when the correlation to the indices increases.

Table 4.3 – The historical growth and volatility of the PPA index and the S&P 500.

<table>
<thead>
<tr>
<th>Index</th>
<th>Growth (%)</th>
<th>Volatility (%)</th>
<th>Current Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPA</td>
<td>9.2</td>
<td>17.2</td>
<td>14.95</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>7.0</td>
<td>14.0</td>
<td>1202.08</td>
</tr>
</tbody>
</table>

If the success of the project is partially correlated to one of the traded indices the manager can hedge some of the exposure by trading positions of the index prior to when the investment is made. To hedge the market exposure, ten thousand (10,000) sample paths of the indices and the corresponding project values are simulated, and the holdings of the hedge portfolio are rebalanced twice weekly. The effectiveness of the hedging strategy is revealed by the discounted profit and loss (P&L) histograms in Figure 4.1. As expected, the delta-hedging strategy is more effective when the correlation to marker is stronger.
To illustrate the proposed model for a project with multiple investments, the original investment ($325 million) is broken into staged cash outlays prior to commercialization in year 3 so that $K_1$=$50$ million at $t_{K_1}=0.5$ years, $K_2$=$50$ million at $t_{K_2}=1$ year, $K_3$=$75$ million at $t_{K_3}=1.5$ years, and $K_4$=$150$ million at $t_{K_4}=2$ years. Figure 4.2 shows the compound real option value. Again, the compound real option value is a decreasing function of the market correlation.
Figure 4.2 – The real option value and compound real option value on the UAV project for with a changing level of correlation to the PPA and S&P 500 indices.

The UAV example demonstrates that the level of correlation between the project and the traded index and choice of the correlated index have a significant impact on the real option value. Figure 4.2 shows that when the cash flows are uncorrelated to the indices the real option values for both the compound and single investment case are identical. This is to be expected; the only uncertainty is the uncertainty in the cash flows. However, as the correlation to the indices increase a greater portion of the cash flow growth is adjusted for risk related to the traded index, so \( \hat{\nu} \) decreases and the annual cash flows are less. The parameters of the correlated index also affect the real option value. At any level of correlation, all else being equal, a larger volatility or a smaller growth rate in the traded index would lead to a larger risk-adjusted growth rate \( \hat{\nu} \) and a higher option value. However, as observed in Table 4.3, the PPA index has a higher growth and volatility than the S&P 500; the combined affect of these two parameters, for any level of \( \rho \), leads to a risk-adjusted growth rate that is less than if the project were correlated to the S&P 500. The real option value when the project is correlated to the PPA index is less valuable than if it was correlated to the S&P 500 as noticed in Figure 4.2.

A delta-hedging strategy to mitigate market exposure was outlined previously, and if \( \rho = 1 \) a perfect hedge can be obtained. However, the UAV example shows that hedging the real option in practice may be difficult. This may be attributed to the difficulty in estimating a project volatility (with the outlined fitting procedure or by any other method) before the investment is made. If the project is unique and unrelated to the market, typical of biotechnology
or pharmaceutical innovations, hedging the market risk is difficult, if not impossible. Other implications such as transaction costs and taxes were ignored in the simulations, so these and other realistic factors may make this strategy impractical.

A final comment is made with respect to the correct rate to use when discounting the project cash flows. A standard NPV analysis uses a risk-adjusted rate, but this discount rate is often set prohibitively high, and projects with superior technologies may be disregarded. This is the case with the UAV example. An NPV analysis with a discount rate of 15% undervalues the UAV project when compared to the valuation provided by the proposed model. Here, an advantage of this work is revealed: it removes the need for managers to arbitrarily assign a risk-adjusted rate to each project and removes the inconsistent use of risk-adjusted and risk-free discount rates observed in previously recommended real options approaches (Datar and Mathews, 2004, Copeland and Antikarov, 2001, and Brandão, Dyer, and Hahn, 2005). Instead, the cash flows of a project are adjusted for their risk relative to the traded index and are discounted at the risk-free rate. This allows for the real option to be priced under the risk-adjusted measure in a financially sound manner.

5 Sensitivity of the Real Option to Risk

As mentioned, a key contribution of this work is that this method correctly accounts for the affect of uncertainty on the real option value. For real options models that use a traded asset to replicate the value of the project payoffs, the MAD or classical approaches, real option value is an increasing function of the underlying asset volatility. This is akin to the relationship found in financial option pricing and is a consequence of the asymmetric payoff of options: the potential benefits increase with additional risk but the potential losses are limited. However, this relationship is not translated to real option pricing as the underlying asset, the present value of the future cash flows, is not traded.

A simplified example supports this claim. Consider a scenario where an investment in one year is required to receive the rights to the value of a project. The sensitivity of the real option (calculated by Equation 3.25) to risk is contrasted to the sensitivity of the real option calculated by the MAD approach in Figure 5.1a. The plot shows that uncertainty of the underlying project
value (by increasing the range of values between the periodic optimistic and pessimistic cash flow forecasts) does not necessarily increase the option value. This is because the option value is calculated under the risk-neutral measure $Q$, not the real-world measure, and is contingent on the present value of the future cash flows, a value capture by risk-neutral expectations.

The real option is also sensitive to the cost of investing. For a given level of correlation to the index at low strike prices the real option value is a decreasing function of the project value uncertainty. This trend is noticed in Figure 5.1b for when the strike $K=0$. This is because as the value of the project is dominated by the expectation of future cash flows. For the case when $K=0$, it is clearly understood, as by standard NPV analysis, that as risk increases the project value decreases. However, when the uncertainty (and the potential upside) increases with the cost of investing the benefit of added optionality is higher; managers should choose to invest in the option under these conditions. Thus, for certain strike levels, this lower project value induces a lower option value, however, as the investment costs increases, the increased risk (and therefore upside potential) takes over the decrease in the project value, i.e. optionality wins out over the project value itself.

![Figure 5.1](attachment:Figure_5.1.png)

**Figure 5.1** – The affect of a) the cash flow volatility and b) the strike on the real option value. Cash flows are correlated to the S&P 500 index with $V = 100$, $\mu = 7\%$, $\sigma = 14\%$, $\nu = 10\%$, and $r = 3\%$. 

33
6 Conclusion

This paper addresses the issues with real option models commonly cited by managers. The proposed model is financial sound yet tractable, so managers may easily integrate real options analysis into their current capital budgeting framework, and the model correctly accounts for the sensitivity of the real option value to risk. The discrete cash flows of a project, extracted from managerial cash flow estimates, are correlated to a traded index so that the real option is priced under the risk-neutral measure. This results in a closed-form solution similar to the Black-Scholes equation. The model is extended to value compound real options, which is typical of R&D investing. The valuation of the UAV project, previously examined by Datar and Mathews (2007), shows that the affect of the correlated index on the real option value is significant, but if there is some correlation a portion of the market risk may be mitigated by a delta-hedging strategy. Unlike financial option valuation, it is shown that the value of the real option is not necessarily an increasing function of volatility as the real option value is contingent on the present value of the future cash flows, a value capture by risk-neutral expectations. Integrating the expertise and knowledge of managers, this approach makes possible a more rigorous estimation of model inputs for real options valuation, leading to heightened accuracy and sophistication when valuing projects.
A Review of Real Options Approaches: Applying Models to Value a Medical Device Project

Abstract

A number of practical real options approaches in literature, some of which have been embraced by industry, lack financial rigor while many theoretical approaches are not practically implementable. Industry surveys reveal that managers are reluctant to apply some of the more theoretical real options approaches due to their lack of transparency, their complexity, and because of the lack of firm acceptance and top management support. In this paper we compare and contrast five real options approaches designed for managers and apply each to value a medical device product development project. Models proposed by Amram and Kulatilaka (1999), Copeland and Antikarov (2001), Datar and Mathews (2004), Collan, Fullér, and Menzei (2009), Barton and Lawryshyn (2011), and Jaimungal and Lawryshyn (2011) are used to value the real option. The medical device project is modeled as a compound call option, and the results are contrasted to a traditional NPV calculation. Issues of volatility estimation and the possibility of arbitrage opportunities in the pricing procedures are examined.

Keywords: Real options, sequential compound options, managerial information, medical devices, volatility estimation, market asset disclaimer (MAD), DM Method
1 Introduction

Medical devices form a broad set of instruments, machines, implants, and in vitro reagents that are used for the purpose of medical diagnosis, therapy, or surgery. Examples of common medical devices include: stethoscopes, adhesive bandages, stints, and pacemakers. Unlike pharmaceuticals, which exert a biochemical effect, medical devices affect the body by physical means. As compared to pharmaceuticals, regulatory approval for medical devices typically occurs more quickly and at a lower cost. The market for medical devices is large and is growing rapidly due to an aging population and an increasing demand for health care in emerging markets. The medical device industry is expected to have sales of $273.3 billion USD this year, and are projected to be over $348.6 billion USD by 2016 (Crofts, 2011). More and more, medical devices are becoming an integral part of modern health care systems.

To ensure continual growth and prosperity, it is essential that managers of medical device firms have adequate tools to select the best projects and technologies. Traditional valuation tools, like net present value (NPV) analyses, fail to capture the value of these new innovations. Project revenues, expected in the distant future, are heavily discounted and come up short of the staged investments necessary to commercialize the product. Critics realize that discounted cash flow analysis tends to undervalue opportunities and neglect strategic considerations; this leads to leading to shortsighted decision-making, underinvestment, and an overall loss in long-term competitive advantage (Schwartz and Trigeorgis, 2001). Instead, an increasing community of academics and corporate practitioners have recommended real options analysis to identify opportunities where managerial flexibility can influence worth and to quantify the ability to adapt to favourable or undesirable outcomes. However, to capture the realities of the tangible investments the real options models in the literature have become increasingly more sophisticated and complex.

As will be discussed, a number of practical and theoretical approaches for real option valuation have been proposed in the literature, but industry surveys have revealed that there is a limited adoption of this methodology within an applied setting (Ryan and Ryan, 2002, Teach, 2003, Hartmann and Hassan, 2006, and Block 2007). A number of leading practical approaches (see for instance Borison, 2005 for a review of such models), some of which have been embraced
by industry, lack financial rigor while many theoretical approaches are not practically implementable. Managers are reluctant to apply some of the more theoretical real options approaches due to their lack of transparency, their complexity, and because of the lack of firm acceptance (Block, 2007, Hartman and Hassan, 2006). Hartman and Hassan (2006) suggest that this disconnect stems from a failure of academia to demonstrate the use of real options strategies through real-world case studies and to highlight the realistic advantages and disadvantages of the models in literature.

Considering the comments of Hartman and Hassan (2006), in this work we present a review of five real options approaches designed for managers, use the reviewed techniques to value a medical device project, and discuss the limitations and advantages of each model. Two ideas are addressed throughout this paper: the procedure to estimate volatility, and whether or not the real option is priced using a financially consistent methodology. The remainder of this paper is arranged as follows. In Section 2 we review literature that applies real options to value pharmaceutical and medical devices. In Section 3 we introduce the medical device project and show a standard valuation with a NPV calculation. Then we review five real options approaches designed for managers, and we use these models to value a medical device project. The real options approaches we review are the classical approach (Amram and Kulatilaka, 1999), the market asset disclaimer (MAD) approach (Copeland and Antikarov, 2001), the DM Method proposed by Datar and Mathews (2004), the Fuzzy Pay-off Method (Collan, Fullér, and Menzei, 2009), and models proposed by Barton and Lawryshyn (2011) and Jaimungal and Lawryshyn (2011). In Section 4 we compare and discuss the medical device project valuations and provide conclusions.

2 Literature Review

Johal, Oliver, and Williams (2008) calculate the real option value of a noninvasive blood glucose monitor by the MAD approach of Copeland and Antikarov (2001). This is the only study to apply real options analysis to value a medical device project, but the literature contains many applications of real options analysis to value R&D and pharmaceutical projects. For example, Merck and Co. apply the Black-Scholes formula to value biotechnology investments (Nichols,
1994). The authors take the present value of the projected cash flows as the underlying asset, and the volatility is calculated from historical returns of similar biotechnology stocks. Kellogg and Charnes (2000) use real options to value an antiviral drug produced by Agouron Pharmaceuticals. The authors use both decision tree analysis (DTA) and binomial lattice techniques to compute the value of the firm and compare it to actual historical stock prices. Loch and Bode-Gruel (2001) compute the value of compound growth options of three pharmaceutical research projects using decision trees. Berk, Green, and Naik (2004) outline a valuation model of R&D projects that includes a parameter to capture the correlation between the cash flow process and a traded index. Analytical solutions are not available for the derived partial differential equations (PDEs), but the real option value is found numerically. Schwartz (2004) values R&D projects and patents. Uncertainty is modeled in the cash flows generated by the project, the investment costs, and the possibility of catastrophic events (such as technological failures). Schwartz (2004) applies the least-squares Monte Carlo (LSMC) algorithm of Longstaff and Schwartz (2001) to value a pharmaceutical project with average industry cost, timing, and return data.

Cassimon et al. (2004) use a closed-form solution to price a 6-fold sequential compound call option on a new drug application. The six stages of investment correspond to the pre-clinical test phase, three clinical test phases, the FDA approval phase, and the commercialization phase. Similar to Schwartz (2004), average industry cost figures and timing data are used for the analysis. Cassimon et al. (2004) performing a sensitivity analysis and find that, in line with standard financial option theory, the real option value increases with increasing volatility. This relationship is consistent with the assumption underlying the classical (Amram and Kulatilaka, 1999) and MAD approaches (Copeland and Antikarov, 2011) (see details in Section 3), but researchers have challenged this observation and show that real option value is not necessarily an increasing function of project volatility (Barton and Lawryshyn, 2011, Jaimungal and Lawryshyn, 2011).

Schwartz and Hsu (2008) are the first to include a variable for pharmaceutical efficacy into a real options model. The authors study the value of HIV/AIDS vaccines and examine the effectiveness of research sponsorship programs. Willigers and Hansen (2008) value a pharmaceutical project with stochastic cash flows, project costs, and technical project failures. The work of Willigers and Hansen (2008) is similar to our work in that it compares the real option value calculated using the LSMC algorithm to values calculated by the MAD approach (Copeland and
Antikarov, 2001) and standard NPV and expected net present value (ENPV) analyses.

Hartman and Hassan (2006) reveal that managers of pharmaceutical companies tend to rely on expert opinions or sensitivity and scenario analysis to estimate volatility, and that they prefer modeling real options with binomial lattices, Monte Carlo simulations, and the Black Scholes formula (Black and Scholes, 1973). The models reviewed in this work reflect these preferences, whereas much of the academic literature to date uses more complex solution algorithms, like the LSMC approach (for instance Schwartz (2004), Schwartz and Hsu (2008), and Willigers and Hansen (2008)), and uses the historical equity returns of related pharmaceutical companies or indicies to estimate volatility (for instance, Nichols, 1994, Schwartz, 2004, Cassimon et al., 2004, Willigers and Hassan, 2008, Schwartz and Hsu, 2008). The models reviewed in this paper address the limitations of NPV analysis but also reflect the stated modelling preferences of managers. There is an inherent trade off between the metrics of managerial application and numerical tractability; this is an issue that will be highlighted in Section 4.

3 Assessing the Value of a Medical Device Project

In this section we introduce the medical device project and show a standard valuation with a NPV calculation. Then five real options approaches are used to calculate the compound real option value of the medical device project. The framework of the classical real options approach, popularized by Amram and Kulatilaka (1999) (see also Brennan and Schwartz, 1985), is extended to value compound options using the mathematics of Geske (1979) and Lee, Yeh, and Chen (2008). Then, the market asset disclaimer (MAD) approach of Copeland and Antikarov (2001), the DM Method of Datar and Mathews (2004), the fuzzy pay-off method (FPOM) of Collan Fullér, and Menzei (2009), and the approaches of Barton and Lawryshyn (2011) and Jaimungal and Lawryshyn (2011) are outlined. In each subsection the assumptions underlying the given approach are described and key equations are provided. The procedure to estimate the project value volatility, arguably the most difficult parameter for managers to determine reliably\(^6\), is also discussed. Then, each model is used to value the medical device project as a

\(^6\)However, the widely used discounted cash flow analysis based on the weighted average cost of capital (the WACC) inherently makes the assumption that the project cash flows have the same beta, a function of the cash flow volatility and the level of correlation to the market, as the average project in the company.
sequential compound real option.

3.1 Introducing the Medical Device Project

A partner firm is analyzing a potential investment in a medical device project that shows promise for the treatment of dry eye syndrome. Based on managers expectations of future market share, patient adoption, and profit margins and optimistic, most likely, and pessimistic cash flow scenario have been constructed. The three cash flow scenarios are shown in Table 3.1 and a complete cash flow analysis is given in Appendix B.

The project requires three staged investments $K_j$ that are made at fixed times $t_K$ with $j = 1, 2, 3$. These investments correspond to the funds necessary to develop a prototype, rework the device before testing, and then fund the product through clinical trials. After each phase managers may choose to continue development or to abandon the venture. If all three investments are made, then the cash flows $f_n$, with $n = 1, 2, \ldots, 10$, are received at times $t_i$, with $i = 4, 5, \ldots, 13$.

**Table 3.1** – The managerial cash flow estimates for the medical device project ($\$ million$).

<table>
<thead>
<tr>
<th>Phase</th>
<th>Development</th>
<th>Commercialization</th>
<th>NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Optimistic</td>
<td></td>
<td>2.0</td>
<td>10.3</td>
</tr>
<tr>
<td>Most Likely</td>
<td></td>
<td>0.6</td>
<td>1.5</td>
</tr>
<tr>
<td>Pessimistic</td>
<td></td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The managers of the medical device firm would typically value the company’s potential R&D projects using a standard NPV analysis. The risk-adjusted discount rate for this project has been set at a high rate of 75% due to the perceived level of uncertainty in the venture. The standard NPV of the optimistic, most likely, and pessimistic cash flow scenarios are given in Table 3.1. The limitations of NPV analysis, especially when used to value R&D investments, have been cited (Trigeorgis, 1996).
3.2 Classical Approach (Amram and Kulatilaka, 1999)

The classical approach (Amram and Kulatilaka, 1999) applied the mathematics of financial option pricing, namely the Black Scholes formula, to value real options. The solution procedure is transparent for managers, assuming that managers are familiar with the Black Scholes equation, yet several simplifying assumptions are required. The classical approach of Amran and Kulatilaka (1999) assumes that the underlying asset, the value of the project, follows a geometric Brownian motion (GBM) process with a constant drift and volatility. The capital markets are assumed to be complete so the dynamics of a project are perfectly replicated by a traded asset \( (\beta = 1) \). To value the medical device project, which has staged investments, we extend the the framework developed by Amram and Kulatilaka (1999) to compound options. Simple financial compound options (a European call on a European call) were first valued with a closed-form solution by Geske (1977), and his work has been extended to price multiple put or call options with time dependent volatility or interest rate parameters (Lee, Yeh, and Chen, 2008).

Let \( RO_m(V, t) \) be the \( m \)-fold sequential compound call option value that matures at time \( t_{K_1} \) with strike \( K_1 \). It derives its value from \( RO_{m-1}(t_{K_2}) \), which is the \( m - 1 \) compound call option that matures at \( t_{K_2} \). The first fold option \( RO_1(t_{K_m}) \) is a call option on the underlying value of the cash flows \( V \). The general form of the \( m \)-fold compound real option, adapted from Lee, Yeh, and Chen (2008) is

\[
RO_m(V, t) = VN_m(a_1, a_2, \ldots, a_m; A^m) - \sum_{j=1}^{m} K_j e^{-rt_j} N_j(b_1, b_2, \ldots, b_j; A^j),
\]

with

\[
b_j = \ln\left(\frac{V}{V_m}\right) + (r + \frac{1}{2}\sigma^2)(t_{K_m} - t_{K_{m-1}}) \over \sigma \sqrt{t_{K_m} - t_{K_{m-1}}}, \tag{3.2}
\]

and

\[
a_j = b_j + \sigma \sqrt{t_{K_m} - t_{K_{m-1}}}, \tag{3.3}
\]

where \( N_m(a_1, a_2, \ldots, a_m; A^m) \) is the \( m \)-variate cumulative normal distribution function. \( V_m \) is the solution of \( RO_{m-j}(t_{K_j}) = K_j \). The upper limit to this distribution is \( a_m \) and \( A^m \) as the
correlation matrix with

$$A^m = (a_{ij}^m)_{i,j=1,...,m} \begin{cases} 
  a_{ii} = 1 \\
  a_{ij} = a_{ji} = \rho_{ij}
\end{cases}$$

(3.4)

and

$$\rho_{ij} = \sqrt{\frac{t_{K_i}}{t_{K_j}}}.$$  

(3.5)

The risk-free interest rate $r$ is assumed to be constant over the duration of the project.

As mentioned above, in the application of the classical approach to value compound call options, the volatility of the project value, $\sigma$, is assumed to be the same as a comparable traded asset. In this analysis we chose the common shares of Medtronic, Inc. (NYSE: MDT) to represent the returns of the medical device project. MDT is a medical technology company that is involved in the R&D, manufacture, and sale of medical devices. The current share price of MDT is $32.95, the market capitalization is $40.61 billion and the historical drift and volatility of MDT are 8.2% and 22.0%, respectively. An estimate for the spot price, the current value of the project, is found by financial statement multiples. The price to operating cash flow (P/CF) ratio for MDT in fiscal 2011 was 10.9, and the expected cash flow for the medical device project in the first year of commercialization is $600,000, so the spot price for the project is $6.54 million. In summary, the inputs to the classical real option approach are:

- Current value of the underlying asset: $V = $6.54 million
- Strike prices: $K_1 = $100,000, $K_2 = $200,000, and $K_3 = $400,000
- Strike date: $t_{K_1} = 1$ year, $t_{K_2} = 2$ years, and $t_{K_3} = 3$ years
- Risk-free rate: $r = 3.0\%$
- Volatility: $\sigma = 22.0\%$

The real option value calculated by the classical approach is $5.89 million.

---

7All market data for MDT was taken at the time of analysis: July, 2011. The historical drift and volatility are annualized values taken from daily closing prices over the past 2 years.
The classical real options approach, when applied to compound options for R&D projects, is intuitive for managers. The analogy between financial and real options is easy to grasp, so similarities between the pricing formulas are also understood. Under the assumptions of the classical model, arbitrage opportunities are limited as the asset itself may be traded in the market. Should the manager choose to do so, a delta or delta-gamma hedging strategy may be employed to mitigate project risk. The interested reader may refer to Lee, Yeh, and Chen (2008) for the sensitivity formulas.

A Microsoft Excel spreadsheet or pocket calculator is capable of computing the real option value for a single investment; however, when the analysis is extended to $m$-fold compound options, the computation of the multivariate integral must be done numerically and can be easily handled by mathematical packages such as MATLAB or Maple. The need for added software may be viewed as a disadvantage when compared to other real options models. Another limitation is the assumption that there exists a perfectly comparable asset that can be identified. In particular, for R&D or medical device projects, the success of the venture is partially driven by the underlying technological development. Technological (or idiosyncratic) risks cannot be replicated in the market, so these types of projects are ill-suited to be analyzed using this approach. Last, this approach uses minimal managerial information; it does not incorporate the optimistic, likely, and pessimistic cash flow estimates provided by managers in its procedure.

### 3.3 Market Asset Disclaimer Approach (Copeland and Antikarov, 2001)

Copeland and Antikarov (2001) (see also Copeland and Tufano, 2004 and Copeland and Antikarov, 2005) assume that markets are incomplete, so the dynamics of the project value cannot be replicated by a traded asset. Instead, the value of the project without flexibility, calculated at the weighted average cost of capital (WACC) is argued to be the best estimate for the underlying asset; this is the market asset disclaimer (MAD) assumption. Since there is no historical price information for the underlying asset, a proxy for the project volatility must be estimated. Copeland and Antikarov (2001) recommend a Monte Carlo simulation approach. The key drivers of cash flow uncertainty, any quantity known to alter the project value including sales quantity, selling price, fixed costs, or variable costs, are simulated in each period. The cash flows are then discounted at the WACC to produce a distribution of project values. The logarithmic returns
of the project in the first period \( z \) are monitored through the simulation, i.e.,

\[
z = \ln\left(\frac{\tilde{V}_1}{V_0}\right).
\] (3.6)

Here \( V_0 \) is the deterministic present value calculated at the WACC and \( \tilde{V}_1 \) is the present value of the simulated cash flows in the first period\(^8\). The consolidated volatility of the project \( \sigma_{MAD} \) is the standard deviation of \( z \), and the return for the period \( \mu_{MAD} \) is the mean of \( z \).

Copeland and Antikarov (2001) recommend lattices to price the option. The estimated volatility is assumed to be constant over the duration of the project, so \( \sigma_{MAD} \) is used to construct the project value binomial lattice with the standard parametrization of Cox, Ross, and Rubinstein (1979). The multiplicative up and down factors are

\[
u_{MAD} = e^{\sigma_{MAD}\sqrt{\Delta t}},
\] (3.7)

and

\[
d_{MAD} = e^{-\sigma_{MAD}\sqrt{\Delta t}}.
\] (3.8)

The value of the real option on the underlying project value is found by applying the boundary condition (or option payoff equation) to the terminal project value nodes at maturity. The value of all previous nodes is found by a dynamic programming procedure. The risk-neutral probability of an up movement is

\[
p_{MAD} = \frac{e^{r\Delta t} - d_{MAD}}{u_{MAD} - d_{MAD}}
\] (3.9)

and the risk-neutral probability of a down movement is

\[1 - p_{MAD}.
\] (3.10)

To value a sequential compound option, layers of option lattices are created on underlying option

\(^8\)It has been show that the volatility of the project value is equal to the volatility of the project cash flows, (see Berk, Green, and Naik, 2003, Barton and Lawryshyn, 2011, or Hahn, Brandão, and Dyer, 2011), so it is appropriate to manipulate the cash flows.
lattices. The lattices are solved in reverse chronological order so that the final compound option, 

\[ RO_m(V, t) \]

which matures at time \( t_{K_1} \) with strike \( K_1 \), is the real option value of the project.

The application of the MAD approach to value the medical device proceeds as follows. First a proxy for the project volatility is estimated. For each of the variables influencing the medical device project cash flows (market share, product adoption, price per treatment, and royalty from the partner distributor) high, likely, and low values are matched to triangular distributions in @Risk with Microsoft Excel. The annual scenarios are related year-over-year by correlations of 0.9 as per the suggestions of Copeland and Antikarov (2001). Ten thousand samples create a distribution of logarithmic returns \( z \). The mean return \( \mu_{MAD} \) of the distribution is 24.9%, and the standard deviation \( \sigma_{MAD} \) is 26.0%. In summary, the inputs to the MAD approach lattices are:

- Present value of the most likely cash flow scenario: \( V_0 = \$1.40 \) million
- Strike prices: \( K_1 = \$100,000, K_2 = \$200,000, \) and \( K_3 = \$400,000 \)
- Strike dates: \( t_{K_1} = 1 \) year, \( t_{K_2} = 2 \) years, and \( t_{K_3} = 3 \) years
- Risk-free rate: \( r = 3.0\% \)
- Volatility: \( \sigma_{MAD} = 26.0\% \)

First, the underlying value lattice, shown in Figure 3.1a, is constructed with multiplicative up and down jumps of \( u_{MAD} = 1.67 \) and \( d_{MAD} = 0.60 \). Then, the real options are valued in sequential order. \( RO_1 \) (in Figure 3.1b) is the option on the underlying project value. For example, the uppermost terminal node in this lattice is found by the call option payoff equation \( \max(6.45 - 0.4, 0) = \$6.05 \) million. This boundary condition is applied to all of the terminal nodes, and the remainder of the \( RO_1 \) lattice values are found by a dynamic programming procedure using the risk-neutral probabilities of \( p_{MAD} = 0.40 \) and \( 1 - p_{MAD} = 0.60 \). This procedure is repeated for \( RO_2 \), the option on \( RO_1 \), and \( RO_3 \), the option on \( RO_2 \). The real option value of the medical device project using the MAD approach is \$76,000. A screen shot of the Microsoft Excel worksheet is available in Appendix C.
Yet, the MAD approach has its shortcomings. Two primary limitations, the volatility estimation procedure and possibility of arbitrage opportunities, are a consequence of the MAD assumption. Minimal market information is used in the solution procedure, so a proxy for the project risk must be calculated. The proposed volatility estimation procedure has been shown to overestimate volatility (Smith, 2005), so researchers (Herath and Park, 2002, Brandão, Dyer, and Hahn, 2005, Godhino, 2006, Haahtela, 2010, and Hahn, Brandão, and Dyer, 2011) had since provided modifications to this procedure. As pointed out by Haahtela (2011), another disadvantage of the MAD volatility is that the value is not based on market prices, so the project risk cannot be hedged. Furthermore, arbitrage opportunities could exist between the project and traded assets if similar traded assets can be identified in the market to replicated project returns (Borison, 2005).

### 3.4 Datar Mathews Method (Datar and Mathews, 2004)

Datar and Mathews (2004) (see also Datar, Mathews, and Johnson, 2007, and Mathews, 2009) make an important contribution in the attempt to translate the theory of real options analysis
into a usable tool for managers. The proposed model (henceforth the DM Method), utilizes managerial cash flow estimates (an optimistic, a pessimistic, and most likely scenario) to price the real-world investment. In their model Datar and Mathews (2004) fit triangular cash flow probability density functions (PDFs) to the periodic estimates; this procedure is shown in Figure 3.2.

![Figure 3.2](image)

**Figure 3.2** – Triangular distributions are used to fit optimistic, most likely, and pessimistic cash flow estimates provided by managers.

Annual PDFs are related year-to-year using rank-order correlations with risk analysis software. Monte Carlo simulation produces random draws of the cash flows in each year, and the NPV distribution is calculated at the WACC. Datar and Mathews (2004) argue that the investment is risk-less and will occur only in favourable cash flow outcomes, so the investment is discounted at the risk-free rate. Simulated project values $\hat{V}_0$, which is the discounted sum of the simulated cash flows $\hat{f}_n$,

$$\hat{V}_0 = \sum_{n=1}^{N} \frac{\hat{f}_n}{(1 + WACC)^n},$$  \hspace{1cm} (3.11)

in excess of the strike $K$ are deemed successful, so the real option value is the mean of the truncated real option distribution,

$$RO = mean[(\hat{V}_0 - K)_+].$$ \hspace{1cm} (3.12)

Applying the DM Method to value the medical device project, the input parameters are:

- Optimistic, likely, and pessimistic cash flow estimates (Table 3.1)
• Strike prices: $K_1 = 100,000, K_2 = 200,000, and K_3 = 400,000$

• Strike dates: $t_{K_1} = 1$ year, $t_{K_2} = 2$ years, and $t_{K_3} = 3$ years

• Risk-free rate: $r = 3.0\%$

• Cash flow correlations: $0.9^9$

The present value of the simulated cash flows at the risk-adjusted discount rate of 75% is $2.33$ million. Accounting for the compounding nature of the three staged investments, the real option value calculated using the DM Method is $1.48$ million.

The DM method is the first work in real options literature to utilize managerial cash flow estimates in its solution procedure; this is its key contribution. However, the DM Method links cash flows from one year to the next in an ad hoc manner. This approach does not explicitly estimate a volatility parameter, so the uncertainty in the project cannot be hedged in the market, nor does the model differentiate between market and private risks.

3.5 **Fuzzy Pay-off Method (Collan, Fullér, and Menzei, 2009a)**

Collan, Fullér, and Menzei (2009a) use possibility distributions and fuzzy set theory (Zadeh, 1965, Zadeh, 1978) in the fuzzy pay-off method (FPOM) to represent uncertain and flexible information. Triangular fuzzy numbers to represent the future possible set of NPV outcomes. Each possible NPV value is weighted from 0 to 1; this is the degree of membership of the fuzzy set. A triangular fuzzy number is defined by the three points: $a$, the expected value or peak, $\alpha$, the left width, and $\beta$, the right width. Figure 3.3 shows a fuzzy NPV distribution. Making use of managerial cash flow estimates, the points $a - \alpha$, $a$, and $a + \beta$ correspond to the NPVs of the pessimistic, most likely, and optimistic cash flow estimates.

---

9In Datar, Mathews, and Johnson (2007), the authors use correlations of 0.7 to link the annual cash flows of a unmanned aerial vehicle project. However, for this example the correlation is chosen to be 0.9 to be consistent with the MAD approach.
Figure 3.3 – The triangular fuzzy NPV number $A$ is defined by the peak $a$, with membership 1, the leftmost number $a - \alpha$ and rightmost number $a + \beta$. The real option value is calculated by the ratio of the positive area, shown in white, to the entire area multiplied by the by fuzzy mean value $E[A+]$.

Carlsson and Fullér (2001) derive the fuzzy mean value $E[A+]$ for the triangular fuzzy number $A$ given four cases,

$$
E[A+] = \begin{cases} 
\frac{a + \beta - \alpha}{6}, & 0 < a - \alpha, \\
\frac{(\alpha - a)^3}{6\alpha^3} + \frac{\beta - \alpha}{6}, & a - \alpha < 0 < a, \\
\frac{(\alpha - \beta)^3}{6\beta^3}, & a < 0 < a + \beta, \\
0, & a + \beta < 0.
\end{cases}
$$

(3.13)

The real option value $RO$ is calculated as the fuzzy mean weighted by the ratio of the positive area to the total area of the triangular fuzzy NPV. That is,

$$
RO = \frac{\int_{0}^{\infty} A(x)dx}{\int_{-\infty}^{\infty} A(x)dx} E[A+],
$$

(3.14)

where $\int_{-\infty}^{\infty} A(x)dx$ is the area below the fuzzy NPV $A$, and $\int_{0}^{\infty} A(x)dx$ is the area below the positive fuzzy NPV $A$. The inputs to the FPOM for the valuation of the medical device project are:

- Optimistic, likely, and pessimistic NPVs (Table 3.1).

For the medical device project the values $a$, $\alpha$, and $\beta$, are $1.40$ million, $1.23$ million, and $3.50$ million, respectively, so that the peak, lower, and upper bounds of the triangle correspond to the NPV of the most likely, pessimistic, and optimistic scenario. This case corresponds to
the situation where the whole fuzzy number is larger than zero, so the real option value \( RO \) simplifies to the fuzzy mean value \( \mathbb{E}[A_+] \). The real option value is $1.28 million.

The FPOM is easily implemented within a Microsoft Excel spreadsheet; this assists in its potential adoption by real options practitioners. However, the calculation underlying the FPOM is still a discounted cash flow analysis; therefore, the FPOM suffers from the same pitfalls as a standard discounted cash flow analysis. The process linking the cash flows from one year to the next is not explicitly defined, nor is the volatility for this process estimated so the the project value cannot be hedged in the market. If the project cannot be replicated by the market than it cannot be accurately priced in a financially consistent manner.

### 3.6 Distribution Fitting Approach (Barton and Lawryshyn, 2011)

The model proposed by Barton and Lawryshyn (2011) (henceforth the distribution fitting approach) assumes that the cash flows and the value of a project follow a GBM process. The dynamics of the process are found by fitting the discrete managerial cash flow estimates to lognormal PDFs so that discontinuous drift \( \nu_i \) and volatility \( \eta_i \) parameters are found over each period \( t_{i-1} \) to \( t_i \). For each period the most likely cash flow estimate is the expected value of the process, whereas the optimistic and pessimistic estimates are upper and lower bounds of the cash flow PDF. Figure 3.4 shows this fitting procedure.

The cash flows are assumed to be correlated to a traded index \( I_t \) which also follows a GBM, with drift \( \mu \) and volatility \( \sigma \). The cash flow process \( df_t \) under the risk-neutral measure is

\[
\hat{\nu}_t = \nu_t - \frac{\rho \eta_t}{\sigma} (\mu - r),
\]

(3.15)

where \( \rho \) is the instantaneous correlation between the cash flows and the index.
Figure 3.4 – Lognormal distributions are used to fit optimistic, most likely, and pessimistic cash flow estimates provided by managers.

Since the drift and volatility of the project value is identical to the drift and volatility of the cash flow process, the authors show that the present value of the future cash flows is

\[V_0 = f_0\left(e^{\nu_1 t_1 - r t_1} + e^{\nu_1 t_1 + \nu_2 (t_2 - t_1) - r t_2} + \ldots + e^{\nu_1 t_1 + \nu_2 (t_i - t_1) + \ldots + \nu_i (t_i - t_{i-1}) - r t_i}\right), \quad (3.16)\]

and the value of the \(m\)-fold compound real option is

\[RO_m(V_0) = V_0 N_m(a_1, a_2, \ldots, a_m; A^m) - \sum_{j=1}^{m} K_j e^{-r t_j} N_j(b_1, b_2, \ldots, b_j; A^j), \quad (3.17)\]

where

\[b_m = \frac{\ln(V_m)}{V_m} + (r + \frac{1}{2} \eta_1^2)(t_{K_m} - t_{K_{m-1}}) \quad \eta_1 \sqrt{t_{K_m} - t_{K_{m-1}}}, \quad (3.18)\]

and

\[a_m = b_m + \eta_1 \sqrt{t_{K_m} - t_{K_{m-1}}}, \quad (3.19)\]

where \(N_m(a_1, a_2, \ldots, a_m; A^m)\) is the \(m\)-variate cumulative normal distribution function. \(V_m\) is the solution of \(RO_{m-j}(T_{K_j}) = K_j\). The upper limit to this distribution is \(a_m\) and \(A^m\) as the correlation matrix with
\[ A^m = (a^m_{ij})_{i,j=1,...,m} \begin{cases} 
  a_{ii} = 1 \\
  a_{ij} = a_{ji} = \rho_{ij} 
\end{cases} \] (3.20)

and

\[
\rho_{ij} = \sqrt{\frac{t_{K_i}}{t_{K_j}}}.
\] (3.21)

In the classical approach the returns of a single project, Medtronic, Inc. (NYSE:MDT), are used to represent the possible returns of the project. Instead of a single firm, in the distribution fitting process a traded index chosen should better represent the returns of the related market. It is assumed that the cash flows are correlated to the iShares Dow Jones U.S. Medical Devices Index Fund (NYSE: IHI). This exchange traded fund (ETF) tracks the performance of companies that manufacture and distribute medical devices. (BlackRock, 2011). The historical drift and volatility of IHI is 22.18% and 16.96%, respectively\(^{10}\). Within this example, positive market correlation is examined for \(0 \leq \rho \leq 1\), but in practice managers are expected to estimate a tighter range for this parameter based upon historical information, related project successes, and industry experience.

In summary, the inputs to the distribution fitting model are:

- Optimistic, likely, and pessimistic cash flow estimates (Table 3.1)
- IHI drift: \(\mu=22.2\%\)
- IHI volatility: \(\sigma=17.0\%\)
- Strike prices: \(K_1=\$100,000, K_2=\$200,000, \) and \(K_3=\$400,000\)
- Strike dates: \(t_{K_1}=1\) year, \(t_{K_2}=2\) years, and \(t_{K_3}=3\) years
- Risk-free rate: \(r=3.0\%\)
- Market correlation: \(0 \leq \rho \leq 1\)

\(^{10}\)All market data for IHI was taken at the time of analysis: July, 2011. The historical drift and volatility are annualized values taken from daily closing prices over the past 2 years.
The value of the medical device project is shown in Figure 3.5. Real option value is a decreasing function of market correlation.

![Figure 3.5](image)

**Figure 3.5** – The real option value of the medical device project with changing correlation of the cash flows to the traded index IHI.

The real options approach of Barton and Lawryshyn (2011) accurately incorporates varying proportions of market and private uncertainties by introducing the correlated index to risk-adjust the dynamics of the project value. If there exists some level of correlation to the index, then the value of the project may be replicated by trading proportions of the correlated index. The distribution fitting model does not require managers to assign a risk-adjusted discount rate for each project, so it removes the inconsistent use of risk-adjusted and risk-free discount rates observed in previously recommended real options approaches (such as the DM method of MAD approach). Instead, the cash flows of a project are adjusted for their risk relative to a traded index and are discounted at the risk-free rate. This allows for the real option to be priced under the risk-adjusted measure and by a financially sound methodology. The ambiguity in estimating the project volatility is also removed; volatility is derived from management’s cash flow expectations. Although, the underlying mathematics are more involved than the FPOM or the MAD approach, the model may still be implemented with the help of macros or functions written in VBA or MATLAB. A closed-form solution, similar to the classical approach, aids in the applicability of this model.
3.7 Matching Approach (Jaimungal and Lawryshyn, 2011)

Jaimungal and Lawryshyn (2011) present another methodology, the matching approach, that utilizes managerial estimates to price real-world investments. Unlike other real option approaches that assume the project value follows a GBM process, the authors allow an observable but non-traded stochastic process to influence the project value. The process is termed a market sector indicator $S_t$. While the process is not restricted to GBM, the authors present their work for the case where it is assumed to be so. The essence of the methodology requires that this process is correlated to a traded index index by $\rho$, and the process drives the cash flows of the project by matching the probability associated with the sector index value to managerial supplied cash flow estimates. The market sector indicator may be any factor including: the market size, the market share, or the demand for the investment. Unlike the FPOM, the MAD approach, and the DM Method the matching approach, like the fitting approach, does not rely on an artificial method to correlate cash flows between periods, since the cash flows are driven by a stochastic process. When the market sector indicator is large the value of discrete cash flows $f_i$ is high and the resulting project value is also high; the converse relationship is also true. Ultimately, this natural correlation of cash flows between periods ensures a project and real option valuation that is consistent with hedging and financial theory.

The authors show that under the assumption of a GBM process for both the market sector indicator, with drift $\nu$ and volatility $\eta$, and the traded index, with drift $\mu$ and volatility $\sigma$, the present value of the project cash flows at time $t_0$ is the sum of the discounted value of a stream of uncertain cash flows $f_i$, made at time $t_i$, with $t_i > t_0$, that is,

$$ V(S, t_0) = \sum_{i=1}^{n} e^{-r(t_i-t_0)} \int_{0}^{\infty} (1 - \hat{P}_{f_i|S}(f)) dv. \quad (3.22) $$

where the risk-neutral distribution function conditioned on $S_t$ at time $t$, where $t_i > t$, for the $i$-th cash flow is given as

$$ \hat{P}_{f_i|S_t}(f) = \Phi(\sqrt{\frac{t_{Km}}{t_{Km}-t}} \Phi^{-1}(F_i^*(f)) - \frac{1}{\eta \sqrt{t_{Km}-t}} \ln(\frac{S}{S_0}) - \frac{\mu - \frac{1}{2} \eta^2}{\eta} \sqrt{t_{Km} - t} - \frac{\nu - \frac{1}{2} \eta^2}{\eta} \sqrt{t_{Km} - t}), \quad (3.23) $$
where $S_0$ is the initial indicator value, is the manager specified distribution - in this case, a triangular distribution (see Appendix A for details on the triangular distribution) is used, $\Phi(\cdot)$ denotes the standard normal cumulative distribution function (CDF), $\Phi^{-1}(\cdot)$ denotes the inverse of standard normal CDF, and the risk-adjusted drift of the market sector indicator $\tilde{\nu}$ is

$$
\tilde{\nu} = \nu - \frac{\rho \eta}{\sigma}(\mu - r)
$$  \hspace{1cm} (3.24)

Under the risk-neutral measure the value of the first real option on the underlying project value is

$$
RO_t(S) = e^{-r(T_{Km} - t)}\mathbb{E}^Q[(V_{T_{Km}}(S_{T_{Km}}) - K)_+ | S_t = S]
$$

$$
= e^{-r(T_0 - t)} \int_{-\infty}^{\infty} (V_{T_0} (Se^{\tilde{\nu} - \frac{1}{2}\eta^2(T_{Km} - t) + \eta \sqrt{T_{Km} - t} x} - K_{m})_+ \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx,
$$  \hspace{1cm} (3.25)

and the value of the compound real option is

$$
RO_{m}(V, S) = e^{-r(t_{K1} - t_0)}\mathbb{E}^Q[(RO_{m-1}(t_{K1}) - K_1)_+ | \mathcal{F}_0].
$$  \hspace{1cm} (3.26)

The value of the real option may be easily solved numerically - through numerical integration, lattice methods or the finite difference method. Managers emphasize that the market share captured by the medical device project is the primary influence on its cash flows, so it is chosen to be the market sector indicator. Again, IHI is used as a traded index, so the market share of the medical device project is assumed to be correlated to this index. The inputs to the matching approach are:

- Optimistic, likely, and pessimistic cash flow estimates (Table 3.1)
- IHI drift: $\mu = 22.2\%$
- IHI volatility: $\sigma = 17.0\%$
- Market share drift: $\nu = 10.0\%$
- Market share volatility: $\eta = 20.0\%$
• Initial value for the market sector indicator: \( S_0 = 4.0\% \)

• Strike prices: \( K_1 = \$100,000, \ K_2 = \$200,000, \) and \( K_3 = \$400,000 \)

• Strike dates: \( t_{K_1} = 1 \) year, \( t_{K_2} = 2 \) years, and \( t_{K_3} = 3 \) years

• Risk-free rate: \( r = 3.0\% \)

• Market correlation: \( 0 \leq \rho \leq 1 \)

The real option value calculated with the matching approach is shown in Figure 3.6. Real option value is a decreasing function of the level of correlation between the market share of the medical device project and the IHI index.

The matching method utilizes both cash flow estimates, which managers are comfortable predicting, and a market sector indicator, which is a variable that managers will know intuitively, to price the real option. By introducing the market sector indicator, Jaimungal and Lawryshyn (2011) remove the requirement that the project value or the project cash flows follow a standard GBM process. This is a departure from most real options approaches, and generalizes the model to more real-world investment scenarios. By correlating the market sector indicator to a traded index, the matching approach accounts for both market and private risks and the
option is correctly priced under the risk-neutral measure. If there is some level of correlation between the market sector indicator and the index, the payouts of the project may be replicated in the market, thereby negating arbitrage opportunities. Similar to the model of Barton and Lawryshyn (2011), volatility is inferred from the subjective managerial estimates provided, so the difficulty in estimating this parameter seen in other approaches is also removed. However, this model, although robust, is the most mathematically intensive of the reviewed real options approaches. It would be viewed as a “black box” to managers which would hinder its adoption within industrial settings. Ultimately, the valuation of compound options is admittedly more complex than standard European options, so a higher level of sophistication in the models should be expected by managers looking to value R&D projects.

4 Discussion and Concluding Remarks

Similar to the conclusion made by Borison (2005), it is observed that all of the real options approaches, even with their methodological differences, are intended to provide managers with a more sophisticated tool to select projects. In turn, better projects will enhance firm profitability and contribute to shareholder value. The medical device project is valued favourably by all of the real options approaches. Table 4.1 summarizes the valuations for a range of assumptions. The risk-adjusted discount rate used by the managers of this project, 75%, is set high due to the level of uncertainty in the project, so for comparison, the real option valuations were then performed using rates of 15%, 25%, and 35%; these parameters are more in line with those used in industry. Overall, the magnitude of the valuations by all of the real options approaches is of the same order.
Table 4.1 – Comparison of medical device project valuations

<table>
<thead>
<tr>
<th>Real Options Approach</th>
<th>Real Option Value ($ million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV (most likely case)</td>
<td>40.2 (k=15%) 19.8 (k=25%) 10.5 (k=35%) 1.40 (k=75%)</td>
</tr>
<tr>
<td>Classical approach</td>
<td>5.9 5.9 5.9 5.9</td>
</tr>
<tr>
<td>MAD approach</td>
<td>41.4(k=15%) 19.17(k=25%) 9.86(k=35%) 0.78 (k=75%)</td>
</tr>
<tr>
<td>DM method</td>
<td>46.3 (k=15%) 21.2 (k=25%) 11.6(k=35%) 1.4 (k=75%)</td>
</tr>
<tr>
<td>Fuzzy pay-off method</td>
<td>44.9 (k=15%) 21.3 (k=25%) 10.9 (k=35%) 1.3 (k=75%)</td>
</tr>
<tr>
<td>Distribution fitting</td>
<td>111.4 (ρ =0) 13.3 (ρ =0.5) 7.4 (ρ =0.8) 3.7 (ρ =0.9)</td>
</tr>
<tr>
<td>Matching approach</td>
<td>98.2 (ρ =0) 59.4 (ρ =0.5) 48.5 (ρ =0.8) 42.8(ρ =0.9)</td>
</tr>
</tbody>
</table>

All of the real options models reviewed in this paper are designed to address the shortcomings of NPV analysis, but the models differ in several ways. A triangle diagram helps to visualize and compare the qualitative differences among the reviewed real options approaches. In one corner are models that are more strongly grounded in financial option pricing theory (the classical approach), in another are those that are more inclined to managerial relevance (the DM Method, the FPOM or the MAD approach), and in the third are those that are more numerically tractable. Models such as the distribution fitting approach and the matching approach fall somewhere in between regions. Figure 4.1 illustrates this idea.
Figure 4.1 – A qualitative comparison of real options approaches is presented. Some approaches are more strongly grounded in financial options theory, some are easier to implement and understand numerically, and some use managerial estimates and the available data more readily.

The advantages (and disadvantages) of the models correspond to where they fall within this chart. The mathematics of the classical real options approach are easily transferred from financial option pricing, yet the applicability of this model is limited. It is unlikely that managers will find a traded security to replicate the returns of an R&D project. On the other hand, the DM method and the FPOM, models that incorporate managerial cash flow estimates, are appealing to managers but lack financial rigor, and, in some cases, are inconsistent with financial theory. Likewise, the volatility simulation procedure of Copeland and Antikarov (2001) and the use of the market asset disclaimer assumption is debatable, but the MAD approach is tractable due to its implementation with binomial trees. The distribution fitting and matching approaches attempt to bridge the divide between the three extremes, albeit they are more computationally intensive. Although the matching approach may be generalized to many types of projects, the distribution fitting approach has a closed-form solution, which aids in the implementation of the model for managers. The matching approach is still a “black box”.

In the reviewed real options approaches volatility is estimated in one of three ways. Volatility is either calculated from the historical returns of a comparable traded asset (the classical approach), derived through a Monte Carlo simulation of cash flows (the MAD approach), or de-
duced from managerial cash flow estimates (the distribution fitting and matching approaches). Models that do not estimate project volatility explicitly (the DM method and the FPOM) miss an important ingredient in the real options paradigm. Similar to financial options, the value of the real option is sensitive to changes in the underlying asset’s volatility. It is necessary for managers to be able to accurately estimate the risk inherent to a venture in order to compare one project to its alternatives or so the risk may be minimized by hedging procedures. The classical, distribution fitting, and matching approaches allow for managers to identify risk and hedge this risk in the capital markets should they choose to do so, whereas the DM method, MAD approach, and FPOM do not.

To conclude, in this paper we have applied five real options approaches (the classical, the MAD, the FPOM, the DM Method, the distribution fitting, and the matching) to value a medical device project. The project is modeled as a compound call option with three staged investments before annual cash flows are received. The real options models vary in their underlying assumptions, procedures to estimate volatility, and mathematics, but all of the models value the medical device project favourably.

5 Appendix A - Triangular Distributions

In the matching method Jaimungal and Lawryshyn (2011) allow managers to specify the distribution that is matched to managerial cash flow estimates \( F^*_i(f) \). The simplest distribution, the triangular distribution, is used in the solution procedure to match the cash flow estimates. A random number \( A \) has the triangular density function

\[
f_A(y) = \begin{cases} 
\frac{y - y_+}{a y_0 - y_-}, & y_- < y \leq y_0, \\
\frac{y_+ - y}{a y_0 - y_-}, & y_- < y \leq y_+, \\
0, & \text{otherwise.} 
\end{cases}
\]  

(5.1)

Using the normalizing constant \( a = \frac{2}{y_+ - y_-} \). The corresponding cumulative distribution function is
\begin{equation}
F_A(y) = \begin{cases}
0, & y \leq y_-, \\
\frac{a(y - y_-)^2}{2(y_0 - y_-)}, & y_- < y \leq y_0, \\
1 - \frac{a(y_+ - y)^2}{2(y_+ - y_0)}, & y_0 < y \leq y_+, \\
1, & y > y_+,
\end{cases}
\end{equation}

where $y_-$, $y_0$, and $y_+$ are the low, likely, and high points of the distribution.

6 Appendix B - Medical Device Project Cash Flows
Table 6.1 – Cash flow spreadsheet for the medical device project

<table>
<thead>
<tr>
<th>Phase</th>
<th>Prototype</th>
<th>Preclinical</th>
<th>Clinical</th>
<th>Commercialization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Market size (millions)</td>
<td>High</td>
<td>30.0</td>
<td>30.6</td>
<td>31.2</td>
</tr>
<tr>
<td></td>
<td>Expected</td>
<td>5</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>3</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>Market share (%)</td>
<td>High</td>
<td>48</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Expected</td>
<td>4</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>3</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>Patient adoption (%)</td>
<td>High</td>
<td>7.5</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Expected</td>
<td>7.5</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>5</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Price per treatment ($)</td>
<td>High</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>Expected</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Annual treatments per patient</td>
<td>High</td>
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<td></td>
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<td>Profit margin (%)</td>
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7 Appendix C - Solution Screen Shots

Screen shots of the medical device project solutions in Microsoft Excel for the MAD approach and the FPOM are show in Figure 7.1 and Figure 7.2.
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<th>MAD Method</th>
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<td>Up Movement ($u$)</td>
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<td>Probability of Down Movement ($1-p$)</td>
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<td>29.84</td>
<td>49.69</td>
<td>82.74</td>
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<td>381.99</td>
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<th>RO$_1$ ($million$)</th>
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<th>6.46</th>
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<td>.84</td>
<td>1.40</td>
<td>.51</td>
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| RO$_3$ ($million$)  | 1.40 | 2.33 | .84  |    |

Figure 7.1 – Screen shot of the MAD approach
<table>
<thead>
<tr>
<th>Fuzzy Pay-Off Method</th>
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<td>Discounted Cash Flows ($)</td>
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<td>Optimistic</td>
<td>163,213</td>
<td>475,648</td>
<td>1,386,175</td>
<td>807,942</td>
<td>470,915</td>
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<td>127,984</td>
<td>74,597</td>
<td>43,479</td>
<td>25,342</td>
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<td>Likely</td>
<td>48,964</td>
<td>71,347</td>
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<td>169,529</td>
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<td>Cumulative Present Value ($)</td>
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</tr>
<tr>
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<td>(57,143)</td>
<td>(122,449)</td>
<td>(178,868)</td>
<td>(15,656)</td>
<td>459,993</td>
<td>1,846,168</td>
<td>2,654,110</td>
<td>3,125,028</td>
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<td>(122,449)</td>
<td>(178,868)</td>
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<td>(178,868)</td>
<td>(170,163)</td>
<td>(146,487)</td>
<td>(72,557)</td>
<td>3,281</td>
<td>47,485</td>
<td>73,249</td>
<td>84,511</td>
<td>91,076</td>
<td>94,902</td>
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</table>

| a | 983,762 |
| alpha | 886,630 |
| beta | 2,687,140 |
| Fuzzy Mean Value E[A] ($) | 1,283,847 |
| Fuzzy Real Option Value ($) | 1,283,847 |

Figure 7.2 – Screen shot of FPOM
Part V

Conclusions and Recommendations for Future Work

This thesis addresses some of the issues with real options analysis, as cited by managers. Reflecting on the criteria outlined by Copeland and Antikarov (2005), the proposed real options model in Part III fits several of the necessary requirements to be easily adopted in practice. The model accurately incorporates risks with the introduction of a correlated traded index; therefore too uses market data. Unlike other real options approaches, the ambiguity in estimating the project volatility is removed; volatility is derived from management expectations. Albeit the mathematics of the model are more sophisticated than some real options approaches in the literature, a closed-form solution allows for managers to implement the framework within Microsoft Excel or MATLAB; this ensures that the model remains mathematically transparent and computationally efficient.

Trigeorgis (1996) raises the need for real options analysis to be applied to real-world case studies and for academics to more deeply understand when models are useful to managers in their decision-making and when they are not. In Part IV, five real options models, including the model proposed in Part III, are applied to value a real-world project currently considered by a CMTE industry partner. The classical approach (Amram and Kulatilaka, 1999), the market asset disclaimer (MAD) approach (Copeland and Antikarov, 2001), the DM Method (Datar and Mathews, 2004), the Fuzzy Pay-off Method (Collan, Fullér, and Menzei, 2009), the distribution fitting approach (Barton and Lawryshyn, 2011) and the matching method (Jaimungal and Lawryshyn, 2011) price the compound real option differently, but unlike the work of Borison (2005) all of the models yield the same investment conclusion: the medical device project has value. Ultimately, each real options model reviewed has its advantages and disadvantages (these are summarized in Figure 4.1), but all models provide a manager a more complete view of the medical device project than if she uses NPV analysis in isolation.

In short, the contributions of this thesis are
1. a new financially correct real options model that integrates managerial cash flow estimates (Part III), and

2. five real options models are compared, contrasted, and used to value a medical device project (Part III).

This work fulfills the previously stated research objectives and will contribute significantly to real options literature.

1 Recommendations for Future Work

This research may be extended in several directions, and the following suggestions are broken into two categories: extensions to the model proposed in Part III, and areas for further work in the field of applied real options analysis. The model suggested in Part III can be developed to further suit the specific needs of managers. Opportunities for improvement to the model include:

- **Modeling more than one stochastic process.** Often the cash flows of a project are driven by more than one uncertainty. The cash flows of these projects may be decomposed into a stochastic revenue and a stochastic cost process. Both processes can incorporate managerial estimates to yield two GBMs partially correlated to a traded index and partially correlated to each other. The discrete periodic cash flow would be the revenue less the cost in each period, and the project value would be the sum of the discounted discrete cash flows. A closed-form solution for the option value would not exist as there is no analytical expression for the sum of lognormal distributions, but a Monte Carlo simulation would provide a range of possible cash flow and present value outcomes. Furthermore, the investment cost and investment timing may be allowed to vary stochastically. Researchers have studied (see for instance Dixit and Pindyck, 1994 or Berk et al., 2004) the affect of varying investment costs on investment timing and real option value. In the proposed model, a deterministic strike was used for both European and compound real option cases, but, instead, managers may wish to estimate optimistic, likely, and pessimistic investment costs. For compound real option valuation in particular, it may be an interesting exercise.
to see how the uncertainty in the staged investment costs would affect the real option value.

- **Modeling stochastic processes other than GBMs.** The assumption of GBM is used commonly in real options literature because of its tractability. However, cash flows may follow processes other than GBMs; this may be the case for projects that are highly dependent upon commodity prices, which tend to follow mean reverting (Ornstein-Uhlenbeck) processes,

\[
dX(t) = \theta(\bar{\mu} - \mu_t)X_t dt + \sigma dW_t, \tag{1.1}
\]

where \( \mu_t \) is the growth of \( X_t \) but reverts to the long-term mean \( \bar{\mu} \). Here \( \theta \) is a parameter that depicts the speed of reversion. The model proposed in Part III is fit to a process with discontinuous growth and volatility parameters, but this framework could be extended to match the long term mean \( \bar{\mu} \) and the reversion speed parameter \( \theta \) too.

- **Modeling the effect of managerial risk-aversion.** Managers will interpret numerical valuations differently depending upon their appetite for risk. Three broad categories of decision-makers exist: an agent is either risk-neutral, risk-averse, or risk-seeking. When faced with two projects with the same expected value a risk-neutral manager will feel indifferent to either choice, but she will always prefer more wealth to less. A risk-averse manager behaves conservatively; she will choose the less risky project, whereas the risk-seeking manager will prefer the riskier alternative. Risk aversion is typically modeled through utility functions; utility is a measure of these varying degrees of satisfaction, and the utility functions of risk-neutral, risk-averse, and risk-seeking managers are linear, concave, and convex, respectively. Henderson (2007), Hugonnier and Morellec (2007), and Miao and Wang (2007) study the effect of risk-aversion within the context of real options. The authors apply methods that penalize option values for varying levels of risk-aversion. For example, a manager that is more strongly risk-averse will view idiosyncratic risks more negatively; therefore, projects that are less correlated to the market will be penalized more heavily than those more strongly correlated to the market. However, the models of Henderson (2007), Hugonnier and Morellec (2007), and Miao and Wang (2007) are mathematically complex and are inaccessible to managers for practical situations.
None of the real options models in literature which are relevant to managers incorporate a managers utility function into the analysis; this would be an interesting extension to this thesis.

- **Incorporating stochastic interest rates.** In most applied real options literature the risk-free interest rate is constant. In reality, variable interest rates may affect the viability of highly capital- intensive and long-term investments. Schulmerich (2008) outlines the importance of interest rate modeling for real options and demonstrates that even a binomial lattice may accommodate a stochastic term structure; this allows for practitioners to model investments more accurately. The reality of stochastic interest rates may be integrated into this work, providing it has consequential influence on the numerical results and overall investment recommendations.

- **Implementing the model.** A graphical user interface (GUI) could simplify real options analysis and the use of the proposed model for managers. Figure 1.1 presents a screen shot of a sample GUI to implementation the real options model proposed in Part III. To date, this GUI has been designed but the detailed coding and debugging still needs to occur, so this remains as future work for the interested reader.

By introducing the preceding elements in the model managers would have a more comprehensive analysis of individually investments. Next two ideas are proposed for future work in the field of applied real options analysis.

- **Using real options as a solution to the principal–agent problem.** The principal-agent problem arises in business when a principal (shareholders) elect an agent (the firm manager) to act on their behalf. In most firms there are likely to be agency problems (a lack of managerial effort, tendency to make short-term decisions, or empire building) and information asymmetries (managers are better suited than shareholders to estimate project cash flows) (Grenadier and Wang, 2004). In traditional real options frameworks it is assumed that there are no agency conflicts, so, from the perspective of the shareholder, managers will always make optimal investment decisions. Grenadier and Wang (2004) investigate the design of financial incentives for managers in a real options context, but
Figure 1.1 – A graphical user interface can provide the end user with an efficient and easy-to-use platform to price the real option.

this model is limited to highly specific situations and could be generalized to a broader set of applications.

- **Analyzing portfolios of real options.** It is important for managers to have tools to analyze real-world opportunities case-by-case, but managers typically have more than one investment opportunity in which to allocate a company’s finite resources. To select a portfolio of real options, a naive approach would be to directly apply financial portfolio optimization techniques, such as Modern Portfolio Theory (MPT) (Markowitz, 1952), to the set of potential projects. Yet, analogous to the differences between financial and real option valuation, there are several practical considerations that limit the translation from financial portfolio optimization to real option portfolio optimization. Projects are typically all-or-nothing propositions; a manager cannot short sell or trade multiple units of project. Unlike the shares of a traded company, projects have finite durations and varying time horizons. As noted by Solak et al. (2010), the concepts of correlation and interdependency between projects are different than those of financial assets. In MPT the
return of a financial portfolio is the weighted sum of the returns of individual assets, but
synergies or dis-synergies of portfolio returns may result from interacting projects. The
correlation between projects alters how funds are allocated among investments, whereas
in MPT correlation is independent of how funds are distributed. Most significantly, MPT
has been criticized because of its sensitivity to input parameters, and as discussed in Part
III, the volatility and growth parameters for real option are difficult to estimate reliably.
Clearly there are limitations to selecting real-world projects with methods designed for
financial assets. A interested extension of this work would be to develop a practical
portfolio selection algorithm for managers.

In summary, this thesis may be extended in several directions. The model proposed in Part
III may be improved upon by the inclusion of more than one stochastic process, by modelling
processes other than GBMs, by including a parameter for risk-aversion, and by implementing the
work into a user-friendly GUI. Furthermore, two areas for future work in the field of practical
real options analysis include using real options analysis as a solution to the principal agent
problem, and to analyze portfolios of real options.

2 References

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