Vertical and horizontal structure in cross-shore flows: An example of undertow and wave set-up on a barred beach

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ABSTRACT


Eulerian measurements of the horizontal, cross-shore velocity field in the lowermost meter of the water column, in association with measurements of waves and the mean elevation of the water surface, across a nontidal, low relief, barred surf-zone reveal: (a) spatially coherent patterns of the time-averaged first, second and third moments of the velocity field, which vary temporally in direct response to the incident waves; (b) large asymmetries in the flow field, with offshore-directed mean flows up to \( \sim 0.20 \) m s\(^{-1}\) and onshore-directed velocity skewness (the third moment normalized by the standard deviation) up to \( \sim +0.60\); (c) mean flows decrease and oscillatory flows (measured by the standard deviation of the velocity field) increase towards the water surface; (d) a fourth-order polynomial provides the best fit to the cross-shore set-up, reflecting the influence of variable energy dissipation due to topographic controls on wave breaking; (e) significant linear correlations exist between the cross-shore mean velocities and the maximum set-up of the still-water surface near to the shoreline (up to 82\% of the observed variability in the former can be accounted for by linear regressions); (f) set-up is highly correlated with the incident wave height (86\% of the observed variability in the former can be accounted for by a linear regression); (g) gradients in the mean water surface are significantly less than those predicted by a linear function of beach slope as originally proposed by Bowen et al.\(^1\) even using an appropriate breaking criterion (0.4–0.6) for the spilling breakers recorded.

Cross-shore circulation over the low relief, barred nearshore slope is predominantly two-dimensional over the outer bar in this case study. The near-bed mean flow is an undertow responding to the balance between the wave-generated, depth-dependent momentum flux directed onshore, the stress induced by the onshore mass transport under waves and wind, and the hydraulic pressure gradient induced by set-up of the mean water-level.

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\(^1\)Bowen et al. (1968).
INTRODUCTION

Recently, considerable attention has been directed at a re-evaluation of Gilbert's (1890, p. 30) classic two-dimensional circulation in the surf zone known as "undertow". As originally conceived, the undertow represented a mass conservation response to the associated landward drift of water under the crests of the breaking incident waves (see also Johnson, 1919; and Evans, 1938). Subsequently, as a result of work by Shepard and LaFond (1939), Shepard et al. (1941), a conceptual shift occurred towards a conservation of mass maintained primarily by horizontal circulations rather than vertically stratified flows (see for example Shepard and Inman, 1950; McKenzie, 1958; Bowen, 1969; Bowen and Inman, 1969; Cook, 1970a, b; Sonu, 1972; Hino et al., 1974). The seminal work of Longuet-Higgins (1953), Longuet-Higgins and Stewart (1962, 1963, 1964) and Lundgren (1963) however, provided the theoretical basis for an explanation of vertically stratified flows based on momentum conservation (Dyhr-Nielsen and Sorensen, 1970; Longuet-Higgins, 1983). A considerable body of research has now been directed at modelling the vertically stratified mean flows which occur normal to a shoreline under shoaling and breaking waves (Dally, 1980; Stive, 1981; Wang and Yang, 1981; Borecki, 1982; Wang et al., 1982; Dally and Dean, 1984; Svendsen, 1984a; Hansen and Svendsen, 1985; Stive and Battjes, 1985; Stive and Wind, 1986; Svendsen, 1986; Kim et al., 1987; Stive and DeVriend, 1987; Svendsen et al., 1987; Hedegaard and Deigaard, 1989; Okayasu et al., 1989; Svendsen and Hansen, 1989; Svendsen and Lorenz, 1989) and the driving mechanism, the pressure gradient due to set-up (and set-down) of the mean water-surface above (and below) the still-water level (LeMehaute, 1962; Bowen et al., 1968; Battjes, 1972, 1974; Battjes and Janssen, 1979; Brevik, 1979; Dally, 1980; Borecki, 1982; Stive and Wind, 1982; Longuet-Higgins, 1983; Svendsen, 1984b, c; Dally et al., 1985; Stive and Battjes, 1985; Kim et al., 1987; Svendsen and Hansen, 1987; Svendsen et al., 1987a, b; Lo, 1989; Okayasu et al., 1989).

To a first approximation undertow is conceived as a current driven by vertical differences within the water column between the depth-dependent radiation stresses and the uniform pressure gradient force due to wave-induced set-up at the shoreline; the mass flux induced by the propagating waves is also important and decreases with depth as do the radiation stresses. In two dimensions the undertow is part of a simple stress balance (Phillips, 1977; Svendsen et al., 1987a; see Fig. 1):

\[
\frac{dS_{xx}}{dx} + \rho \frac{d}{dx} \left( \frac{q_{f}^2}{h + \eta} \right) = -\rho g (h + \eta) \frac{d\eta}{dx} - \bar{\tau}_b \tag{1}
\]

where \( S_{xx} \) is the onshore component of the onshore momentum flux, \( q_{f} \) is the
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Fig. 1. Schematic of the stress balance constraints on undertow (after Svendsen, 1984a; Svendsen et al., 1987a). $\eta$ is the water surface elevation, SWL is the still-water level and the subscripts $cr$ and $tr$ refer respectively to the wave crest and trough. The other symbols are defined in the text (Eq. 1).

total mean volume flux of water due to waves and currents, $\eta$ is the water surface elevation, $h$ is the still-water depth, $\rho$ is mass density, $g$ is the gravitational constant and $\tau_b$ is the turbulent shear stresses induced by the undertow current; $x$ is the shore-normal coordinate and the overbar indicates time-averaged values. Other stresses, such as wind stress, which is largest at the water surface, also contribute to the set-up in a natural environment, while a small contribution to the undertow shear stress will result from streaming within the turbulent wave boundary layer (Longuet-Higgins, 1953). Unfortunately, the lack of a suitable theory for breaking waves (Longuet-Higgins, 1981; Peregrine, 1983) has meant use of either a simple linear theory (Borecki, 1982; Dally, 1980) or empirical solutions (Svendsen, 1984a, b) to model the wave stresses, resulting in considerable debate as to the applicability of existing models (Dally and Dean, 1986; Svendsen, 1986). Furthermore, the choice of appropriate boundary conditions dramatically affects the predicted mean flows, producing profiles which may indicate flows of opposite direction near the bed (contrast Svendsen, 1984a, and Dally, 1980 with Stive and Wind, 1986, and Okayasu et al., 1989, for example). Clearly such model differences have major implications for sediment transport predictions.
Early laboratory observations (Caligny, 1887; Bagnold, 1947; Russell and Osorio, 1953; Scott, 1954) provided some support for the existence of two-dimensional stratification of flow under waves, and experimental documentation of the possible driving mechanism, wave-induced set-up, was provided by Savage (1957), Saville (1961), Fairchild (1958), Bowen et al. (1968) and Van Dorn (1976). Bowen et al. showed that on a planar beach with monochromatic waves the wave set-up was linearly proportional to the beach slope and the square of the ratio of wave height to water depth:

$$\frac{d\eta}{dx} = \frac{-\tan \beta}{[1 + 2.66 \gamma^2]}$$

where $\tan \beta$ is the beach slope and $\gamma$ is the wave height-to-water depth ratio. More recently a large number of laboratory experiments have provided some confirmation of these early findings and allowed tests of explicit models for set-up generation and the structure of the depth-varying mean flow (e.g., Stive, 1981; Longuet-Higgins, 1983; Svendsen, 1984a, b; Stive and Wind, 1982, 1986; Hansen and Svendsen, 1987; Oelerich and Dette, 1989; Okayasu et al., 1989; Ramsden and Nath, 1989).

Traditionally, in the geological, geomorphological and engineering literature, undertow (often in association with steep plunging breakers) was thought to be a major control on shoreface erosion and the development and maintenance of nearshore bars (see for example, Johnson (1909), Keulegan (1948), Shepard (1950), Herbich (1970), King (1972), Miller (1976), Dally (1980) and Dally and Dean (1984)). More recent theory for cross-shore sediment transport under waves (e.g., Bowen, 1980; Bailard, 1981) are especially sensitive to the presence of flow asymmetries such as mean flows, no matter how small these may be; indeed several authors have modelled bar development on the basis of undertow-induced cross-shore transport (e.g. Dally and Dean, 1984; Roelvink and Stive, 1988). However, there is a dearth of good prototype observations with which to examine the assumptions underlying existing theory and to assess the role that undertow plays in nearshore hydrodynamics and sediment transport.

Strong cross-shore mean flows on a natural planar beach, not directly associated with rip currents, were first conclusively demonstrated from the extensive instrument arrays associated with the Nearshore Sediment Transport Study in the U.S.A. (Guza and Thornton, 1985; Seymour, 1989), although both Inman and Quinn (1951) and Schiffman (1965) had reported broad offshore flows – undertow – in natural surf zones. Measurements by Wright et al. (1979), Mizuguchi et al. (1980), Wright et al. (1982) and Wright et al. (1986) revealed the presence of strong cross-shore mean flows on both non-barred and barred slopes under dissipative wave conditions, and which on occasions indicated a landward directed current at the surface and an off-
shore current at depth. This was interpreted as an undertow. Greenwood and Sherman (1984) measured mean flows up to 0.26 m s\(^{-1}\) across a barred slope unassociated with rip currents and subsequently illustrated a strong vertical stratification, with, in one instance, near-bed flows being directed landward while flows above were offshore (Greenwood and Sherman, 1985). Sallenger et al. (1983, 1985), Leont’ev (1985), Davidson-Arnott and McDonald (1989) and Greenwood and Osborne (1990) have all recorded strong cross-shore mean flows of considerable spatial extent on barred beaches.

Dorrestein (1961) was one of the first to document field evidence for set-up at the shoreline on a barred beach. Sonu (1972) documented sea-level measurements using a portable low-pass hydraulic filter, which suggested, in contrast to theory, that maximum set-up occurred seaward of the breaker line and that the lowest mean level was at the shoreline. More recently, however, Guza and Thornton (1981) using a capacitance-type run-up sensor in the swash zone demonstrated a linear relationship between wave set-up and deepwater wave height, with set-up clearly increasing landwards on a planar beach. An analysis of time series of swash excursions determined photographically led Holman and Sallenger (1985) to the conclusion that set-up was strongly correlated linearly with a surf zone similarity parameter (the Iribarren Number). Neilsen (1988), using an array of manometer tubes (Davis and Neilsen, 1989), has shown that there is a difference between the set-up gradients in the area from the still-water shoreline to the mean waterline and that experienced offshore of the still-water shoreline, owing to the effect of the water table.

However, none of the field studies to date have documented the spatial (horizontal and vertical) and temporal structure of the cross-shore mean flow, nor has its direct relationship to a set up of the still-water level induced by wave breaking been demonstrated. In this paper we present Eulerian measurements of the horizontal velocity field, including, locally, its vertical structure, across a natural barred nearshore slope under “surf” conditions. An undertow is identified and shown to respond in a coherent manner to measurements of a wave-induced set-up of the still-water level.

LOCATION OF STUDY

The experimental site was at Wymbolwood Beach, Nottawasaga Bay, southern Georgian Bay, Ontario, Canada (Fig. 2), a tideless, fetch-limited coastal environment. Water level shifts occur in response to the seasonal hydrological cycle and longer term climatic shifts. Higher frequency fluctuations occur in response to seiche effects in the coastal boundary layer and wind and wave set-up. During the experiment, two bars were present (Fig. 3) on an average nearshore slope of 0.015 in medium-to-fine sands; mean grain diameter varied across shore from a maximum of 0.71 mm at the step to 0.13
mm, 300 m offshore. The slope is that of a least-squares linear fit to the survey points from the beach step (intersection of still-water line with the beach face) offshore to a distance of 230 m. Bar crest heights, and therefore crest positions, were defined as the maximum distance perpendicular to a line of mean beach slope tangential to the bar troughs. The inner and outer bars had crest heights of 0.55 and 0.45 m, respectively, and were located at approximately 30 and 90 m offshore. The inner bar had relatively steep local slopes (landward = 0.083; lakeward = 0.047 to 0.031). The outer bar in contrast consisted of a laterally extensive, very gently sloping landward slope (0.005) from 55 to 75 m offshore and a lakeward slope approximately equal to that of the mean beach slope; the profile in this area was therefore intermediate in form between a step-type profile and a well-developed barred profile. It is worth noting that neither the inner nor outer landward slopes were close to the angle
of repose during this experiment, though such slopes do occur in the inner bar system in this area at those times when the inner bar migrates onshore and/or alongshore prior to merging with the beach face in a broad shore terrace. The beach face slope varied between 0.17 and 0.20. Figure 3 illustrates the alongshore variability of the nearshore profiles prior to the experiment; the nearshore slope was essentially two-dimensional, with two sub-linear bars and an absence of rip channels or other irregularities.

EXPERIMENTAL DESIGN

The aims of this experiment required that measurements be made with enough spatial (vertical and horizontal) and temporal resolution to identify
an undertow and the associated forcing (wave set-up); a further aim was to
relate both the set-up and the undertow to the bar topography and dynamics.
The former demand was satisfied by sensors deployed to cover as much of the
surf zone as possible; the latter demand was satisfied by detailed monitoring
of hydrodynamics in the critical area of the landward slope of a bar (the outer
bar in this case), where offshore transport of sediment might be expected to
be driven by undertow during bar growth, according to theory. However, in-
strument deployment (Fig. 4) was also constrained by the limited number of
sensors available, especially current meters.

**Monitoring waves and currents**

Incident wave conditions and mean water-surface elevations were recorded
with a shore-normal array of 15 (1.5–3 m long), surface piercing, continuous
resistance wave staffs (modified after a design by Truxillo, 1970; see Green-
wood and Sherman, 1984). The lakewardmost staff was ~130 m offshore in
~2.5 m water depth while the shorewardmost staff was ~15 m offshore in
~0.4 m water depth (Fig. 4). The wave staffs were individually calibrated in
the field and provide a linear calibration except for the lowermost 0.25 m.
Local wind speed and direction, important for wave generation in this fetch-
limited environment and in assisting water level set-up, was measured rela-
tive to the shore-normal and recorded at the beach face using a Belfort ane-
mometer and wind vane.

The horizontal components of flow in the shore-normal and shore parallel
directions were recorded using biaxial electromagnetic current meters (Marsh-
McBirney Inc., Models OEM 512, 711 and 551), with time constants of 0.2 s.

![Instrument deployment across-shore. Current meters 1–12 (subsequently referred to as CM1-CM12) are Marsh-McBirney OEM 512 Models; FM2 is a Marsh-McBirney Model 711 and EM1 is a Marsh-McBirney Model 551. [Note: current meter FM2 (50 m offshore) and wave wire WW05 (75 m offshore) proved unreliable and data from these sensors are not considered in this paper. Wave wire WW12 (30 m offshore) gave only partial records as it sustained damage during the experiment.]

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**Fig. 4. Instrument deployment across-shore. Current meters 1–12 (subsequently referred to as CM1-CM12) are Marsh-McBirney OEM 512 Models; FM2 is a Marsh-McBirney Model 711 and EM1 is a Marsh-McBirney Model 551. [Note: current meter FM2 (50 m offshore) and wave wire WW05 (75 m offshore) proved unreliable and data from these sensors are not considered in this paper. Wave wire WW12 (30 m offshore) gave only partial records as it sustained damage during the experiment.]**
Nine OEM 512's were deployed in a tri-level, shore-normal array across the landward slope of the outer bar (Fig. 4, CM1–CM9); sensors in each vertical array were placed at 0.1, 0.5 and 1.0 m above the pre-storm bed, and the arrays separated by 5 m at 60, 65 and 70 m offshore. Single current meters were deployed on the lakeward slope of this bar (~110 m offshore, EM1 in Fig. 4), on the lakeward slope of the inner bar (~50 m offshore, FM2 in Fig. 4) and landward of the inner bar, in the trough (~5 m from the beach step, CM9 in Fig. 4). Unfortunately FM2 proved unreliable and is not considered further. The current meters were calibrated individually in a large towing tank at the Canadian National Calibration Facility (De Zeeuw, 1972); they exhibited a linear gain, and calibration coefficients were determined from least-squares linear regression. Considerable concern has been voiced recently over the accuracy of flow measurement with the Marsh-McBirney sensors (Aubrey et al., 1984; Aubrey and Trowbridge, 1985, 1988; Hamblin et al., 1987) especially with respect to measured mean flows and asymmetries (see also the comments of Guza, 1988 and Aubrey and Trowbridge, 1988). However, it is now clear that these sensors can be used with confidence to measure mean flows greater than ~0.04 m s\(^{-1}\) in an oscillatory flow field and that the gain error is not more than 10% (Doering and Bowen, 1987a, b; Guza et al., 1988).

**Data acquisition**

All sensors were hardwired to a shore-based power supply and computer-controlled data acquisition system (Greenwood, 1983; Greenwood and Sherman, 1983). Sensors were scanned every 0.42 s for twenty-one minutes during each sample period, to give data records of approximately 3000 values.

**Data analysis**

Owing to the stochastic nature of the surf zone processes, driven by wave breaking in random wave fields, it is necessary to adopt a statistical approach when analyzing field data. In this paper the waves and currents are described by their time-averaged values and functional relationships between related variables are sought using regression procedures.

The normalized moments of the surface elevation and cross-shore velocity field were computed:

\[
\eta = \frac{1}{n} \sum \eta_{(x,t)} \quad \text{or} \quad \bar{u} = \frac{1}{n} \sum u_{(x,t)}
\]

\[
\eta_s = \left[ \frac{1}{n-1} \sum (\eta_{(x,t)} - \bar{\eta})^2 \right]^{1/2} \quad \text{or} \quad \bar{u}_s = \left[ \frac{1}{n-1} \sum (u_{(x,t)} - \bar{u})^2 \right]^{1/2}
\]
\[ \eta_{sk} = \frac{1}{(n-1)} \sum (\eta_{(x,t)} - \bar{\eta})^3/\eta_s^3 \quad \text{or} \quad u_{sk} = \frac{1}{(n-1)} \sum (u_{(x,t)} - \bar{u})^3/u_s^3 \]

where \( \eta_{(x,t)} \) and \( u_{(x,t)} \) are the instantaneous surface elevation and shore-normal velocity, respectively, at location \( x \), and time \( t \); \( n \) is the sample size (2925 points), the overbar indicates time-averaged mean values and the subscripts “s” and “sk” refer to the standard deviation and skewness, respectively. It should be noted that the skewness statistic computed here represents the normalized time-averaged third moment about the mean. It is therefore not identical to Bowen’s (1980) or Bailard’s (1981) time-averaged third moment of velocity, \( \langle u^3 \rangle \). Further, the skewness of the surface elevation incorporates the effects of asymmetry about both the horizontal and vertical axes (see Doering, 1988).

Variance density spectra for the water surface elevation (\( \eta \)) and for both cross-shore (\( u \)) and alongshore (\( v \)) components of the horizontal velocity field were computed using the BMDP2T Power Spectrum Analysis (Dixon, 1971; Dixon and Brown, 1979). Peak wave period (\( T_{pk} \)) and the peak periodicity in the cross-shore current (\( u_{pk} \)) were determined from the maximum variance spectral densities; significant wave heights were estimated from the total record standard deviations (\( H_s = 4 \times \eta_s \)).

INCIDENT WAVE CYCLE: JUNE 16-18, 1986

The high-energy wave cycle documented in this paper was forced by a meteorological depression tracking northeastward across Georgian Bay, June 16-18, 1986, which generated waves propagating into the experimental site for a period of 49 hours. Further detail on the storm event is presented in Greenwood and Osborne (1990) and Osborne (1987) and will not be re-iterated here. It is sufficient to note that the storm was of a magnitude frequently encountered in this location (estimated recurrence interval of 0.2 years) and caused considerable sediment re-activation over the whole of the nearshore profile out to a depth relative to still water of at least 2.5 m, and at a distance offshore of 130 m.

The temporal variation in wind speed and direction measured at the beach face, and the significant wave height and peak period, measured at the outermost wave staff (130 m offshore) are presented in Fig. 5. It is evident that maxima in wind speed and wave height were nearly in phase during this particular storm (peak wave height lags peak wind speed by only 0.5 h) owing to the coincidence of the greatest wind speeds with the maximum effective fetch (WNW). The relatively rapid reduction of wave height following the storm peak at 1800 h can be attributed to both a reduction in wind speed as well as a shift in wind direction to the N and NW away from the maximum fetch.
The primary wave conditions were therefore wind-forced with no propagation of swell from long distances.

The general pattern of growth and decay of the incident wave spectrum is illustrated by Fig. 6a. The shift in peak period from 2.8 s at 1300 h to 4.7 s at 1800 h as the storm grew, and its maintenance through the decay period (2300 h) is clearly visible. At 1800 h the first harmonic of the incident period (2.4 s) is also evident, indicative of the nonlinearities that develop as waves shoal prior to breaking (Thornton et al., 1976). The spectrum for the storm peak (1800 h) is relatively narrow-banded centred around a frequency of 0.2 Hz. Incident significant wave heights reached 1.5 m, with peak periods of 4–6 s. A series of spectra (Fig. 6b) from the storm peak (1800 h) shows that the absolute levels of energy decrease progressively across the surf zone as dissipation occurs through breaking and friction. The dominant energy peak across the whole surf zone was that of the incident frequency (0.2 Hz, 5 s). The relative proportion of low frequency energy (<0.10 Hz) increased progressively shorewards from 5% of the total energy at WW01 (130 m offshore) to 7% at WW06 (70 m offshore), to 9%, 20% and 29%, respectively, at WW10.
(50 m offshore), WW13 (20 m offshore) and WW15 (15 m offshore). This spatial shift (confirmed by all other wave staff records) to increasing proportions of the energy at lower frequencies in a landward direction is characteristic of this nearshore system during storms (Greenwood and Sherman, 1984) and suggests the presence of long waves, such as trapped (edge) waves or leaky mode waves; the former have been clearly identified at this location by
Bauer and Greenwood (1990). Either of such wave modes could theoretically induce offshore-directed drift velocities close to the bed (Bowen and Inman, 1971; Carter et al., 1974). However, as more than 70% (inner part of surf zone) and 90% (outer part of the surf zone) of the total energy occurs at frequencies between 0.10 and 0.5 Hz, it is unlikely that these low frequency waves could be the dominant mechanism responsible for inducing strong mean flows during the storm.

Wave angles determined from collocated wave staffs and current meters 65 m offshore, using the procedure of Sherman and Greenwood (1986), varied from 9° at 1600 h to 6° at the storm peak in direct response to the wind forcing as the latter shifted to a more shore-normal orientation (6° from the shore-normal). At the end of the storm peak (2330 h) waves were at an angle of 4° although the wind had shifted to 45° from the shore-normal; this resulted from the refraction of the longer period waves as the direct forcing by wind was dramatically reduced with the smaller speeds and reduced fetch length. Short-crested spilling breakers were the dominant wave form in the outer part of the surf zone with occasional plunging breakers present on the inner bar crest. Longer-crested, spilling breakers developed in the decay period. If a breaker height-to-water depth ratio of 0.78 (Galvin, 1972; Hardisty and Levin, 1989) is assumed, then depth-controlled breaking occurred up to 130 m from the shoreline with the 1.5-m high waves. This distance coincides with the location of the outermost wave staff. Visual observations suggest this estimate is conservative and a more appropriate breaking criterion is in the 0.4–0.6 range, which corresponds with the breaking criterion suggested by many other authors for random wave fields on gently sloping beaches (Huntley and Bowen, 1975; Thornton and Guza, 1982; Sallenger and Holman, 1985). This criterion would set the outer limit of the surf zone at the storm peak some 180–200 m offshore. Certainly, a surf zone dominated by spilling breakers, extended well beyond the outer bar crest for a period of at least 11 hours and breaking was present on the outer bar crest and the area landward for a significantly longer period. Although wave activity lasted for 49 hours, the most intense part of this activity was concentrated between 1600 h and 2330 h on June 16 (Fig. 5) and the following analysis of hydrodynamics focusses on this period of time.

**SURF ZONE HYDRODYNAMICS**

In an attempt to understand the nature of the cross-shore fluid dynamics and kinematics associated with this barred surf zone, both the nearshore velocity and wave fields need to be examined.
Nearshore velocity field

Variability in the time-averaged, cross-shore velocity moments throughout the storm are shown in Fig. 7; data from the flowmeters at the 0.1 m elevation only are plotted and illustrate a number of salient points:

1. The magnitudes of the wave-induced, oscillatory currents ($u_0$) were both spatially and temporally coherent, as might be expected from the continuous propagation of spilling breakers across the surf zone. Near-identical values were recorded from the closely spaced flowmeters on the outer bar at both the beginning and end of the storm peak, when waves would be shoaling rather than breaking across the gentle landward slope. The latter gives confidence to

![Fig. 7. Variation of the near-bed velocity parameters throughout the storm period. Values from the mid-surf zone (60, 65 and 70 m offshore) are shown on the left-hand side and values from the inner (15 m offshore) and outer (110 m offshore) parts of the surf zone are shown on the right-hand side. [Note: $u_0$ is the standard deviation of the time-varying, cross-shore velocity and is a measure of the magnitude of the wave-induced oscillatory currents; $\bar{u}$ is the time-averaged, mean cross-shore velocity; $u_{sk}$ is the skewness (normalized third moment) of the time-varying, cross-shore velocity. The current meter notation is the same as in Fig. 4 and positive values indicate the onshore direction.]
instrument performance and credibility to the variation in measured values during more energetic conditions. An increase in gain in the current meter at 65 m does appear on June 17 and although the trends are coherent with the other sensors close by, there will be some overestimate of current speeds for this location.

(2) $u_c$ values at all stations were well in excess of the threshold for motion for the local sand throughout the storm and values of $u_m$ were well above the threshold for sheet flow at times. This was corroborated by the considerable depths of sediment re-activation (0.06–0.42 m) recorded (see Greenwood and Osborne, 1990).

(3) During the storm peak at 1800 h, cross-shore velocities were at a maximum at the outermost station on the landward slope of the outer bar (70 m), and decreased landward. The velocity on the lakeward slope (110 m) was almost the same as that at 70 m.

(4) From 1300 h on June 16 until 2330 h on the same day, a distinct offshore mean flow ($\vec{u}$) was superimposed on the oscillatory motion at all measurement stations. The maximum value of $\vec{u}$ was 0.20 m s$^{-1}$ at the innermost station (60 m offshore) on the landward slope of the outer bar. This cross-shore mean flow was not, therefore, a local anomaly, but was coherent spatially and temporally.

(5) However, the near-bed mean flow ($\bar{u}$) was neither temporally nor spatially constant. At 1600 h for example, a lakeward decrease in this flow was present across the landward slope of the outer bar, with currents dropping from 0.20 m s$^{-1}$ to 0.12 m s$^{-1}$ over a distance of 10 m; at 1830 h in contrast, a lakeward increase in velocity from 0.12 m s$^{-1}$ to 0.18 m s$^{-1}$ was observed. After 2330 h on June 16 the mean flows had been reduced to less than 0.04 m s$^{-1}$; if we accept a conservative estimate of flowmeter accuracy (Aubrey et al., 1984; Doering and Bowen, 1987a, b; Guza et al., 1988) it would suggest that at this time mean flows were essentially zero. It is significant to note that the spatial pattern of mean flow during the early part of the storm, when waves built from the SW for a short time, showed an increase in mean velocity from the innermost trough (15 m offshore) to the innermost flowmeter on the landward slope of the outer bar (60 m offshore); lakeward of this point mean velocities decrease such that the flow was detectable but small (0.04 m s$^{-1}$) 40 m further lakeward. During the storm peak, however, the mean flow on the lakeward slope at 110 m was of the same order as that on the landward slope of the outer bar.

(6) Coincident in time with the strong offshore mean flow there was a strong positive (onshore-directed) skewness within the cross-shore velocity field. There does not appear to be a clear spatial pattern in the values of the normalized third moment, but rather a marked uniformity for a statistic which is highly susceptible to sampling variability. Certainly velocity skewness increased dramatically after 1600 h but returned to near zero by 2330 h. This
period of time coincides well with the period of time when spilling breakers were propagating through the tri-level sensor array, and this asymmetry is undoubtedly due to the nonlinear nature of the surface waves at this time.

A more detailed analysis of the local structure of the cross-shore flow reveals a distinct vertical structure. Vertical variations in the cross-shore mean flow ($\bar{u}$), the standard deviation of the cross-shore velocity ($u_s$) and cross-shore velocity skewness ($u_{sk}$) recorded by the tri-level array of flowmeters at 60–70 m offshore, during the storm peak (1700–2030 h) are shown in Fig. 8. A number of points can be noted:

1) The offshore-directed mean flows (Figs. 8a, b, c) increased and then decreased at all elevations over this time period, and exhibited a distinct stratification during the most intense part of the storm.

2) Of considerable importance is the fact that the offshore mean flow was a maximum at the 0.1 m elevation and decreased in magnitude with elevation above the bed; at times the mean flow actually exhibited a reversal, with mean flows at the 1 m elevation being directed onshore, though of small magnitude. This contrasts markedly with the wave-induced oscillatory motion ($u_o$) (Figs. 8d, e, f) which increases in magnitude vertically away from the bed. The time-averaged, shore-parallel velocities have also been shown to increase away from the bed and in fact vary in a log-linear relationship with elevation (Greenwood and Osborne, 1987), in a manner similar to that of a quasi-steady open channel flow.

3) Velocity skewness ($u_{sk}$; Figs. 8g, h, i) exhibited considerable variation with elevation, as might be expected from a parameter that magnifies all sampling and instrument errors. It was, however, almost always positive (directed onshore) and increased and decreased with wave energy at all elevations; this simply reflects the strong nonlinearity in wave shape about both horizontal and vertical axes (Doering, 1988).

The pattern of cross-shore mean flow described above is in marked contrast with that expected from simple rip currents; available measurements of rips indicate a marked increase in mean velocities away from the bed towards the water surface (Sonu, 1972; Wright et al., 1986). Even though a significant quasi-steady flow can be clearly demonstrated at Wymbolwood Beach, the time-dependent cross-shore flows are still dominated by oscillatory motion. Figure 9 illustrates histograms of the cross-shore velocities for records from the lowermost current meters (0.1 m elevation) at the vertical arrays throughout the storm. Even when the offshore mean flow was at a maximum (1800 h), near-bed currents were still predominantly oscillatory, with large percentages of onshore and offshore flows induced by the propagation of spilling breakers (compare this signature to that of an active rip recorded by Bowman et al., 1988a, b). Furthermore, the magnitudes of the mean flows observed at Wymbolwood are distinctly smaller than those normally associated with rips, where velocities are commonly between 0.5 and 2.0 m s$^{-1}$ (see
Fig. 8. Vertical variation in the horizontal velocity field through the storm peak (1700–2030 h) at 60, 65 70 m offshore. The time-averaged, cross-shore, mean velocity ($\bar{u}$) is shown in (a), (b) and (c); the magnitude of the time-averaged, cross-shore orbital velocity ($u_m = 2.8 \bar{u}$) is shown in (d), (e) and (f); the time-averaged, cross-shore velocity skewness ($u_{sk}$) is shown in (g), (h) and (i). Note: positive values indicate the onshore direction and both the absolute elevations and elevations relative to the still-water level are plotted on the ordinates.

Shepard et al., 1941; Draper and Dobson, 1965; Cook, 1970a, b; Greenwood and Davidson-Arnott, 1975; Basco, 1982, etc.). As noted earlier, there was no evidence in the nearshore topography for a channel which could induce
topographic forcing of this flow and suggest a rip-type circulation. Indeed there was no evidence from direct visual observation of the presence of rip currents at a distance of up to 110 m offshore. The longevity of the cross-shore quasi-steady flow would indicate that it does not represent unconfined transient rips such as those described by Vos (1976), which tend to have life spans of the order of minutes not hours.

In summary, therefore, it should be noted that the velocity field across the surf zone during an extended period of propagation of spilling breakers was predominantly oscillatory, but with a well-defined offshore mean flow superimposed. The velocities exhibited a marked landward-directed skewness and
both these asymmetries increased with increases in incident wave height. The spatial and temporal coherence of the mean flow documented above, indicates an extensive lakeward flow; this flow with its maximum near the bed has the characteristics of an undertow and it remains therefore to examine its relationship to the major forcing agents of nearshore flows.

Nearshore wave field

The variation in significant wave height across the nearshore zone for the storm buildup (1300–1700 h) and the storm decay period (2030–2330 h) is shown in Figs. 10a and c, while Fig. 10b illustrates that for the most intense part of the storm (1800–1930 h). With the exception of the records from 1300 h and 2330 h, wave height reduction was occurring across the whole of the nearshore slope, with a concomitant dissipation of energy, from at least 130 m offshore to the shoreline.

To the first order, the spatial decay in wave height is described well by simple linear functions of offshore distance (Figs. 10a, b, c), which account for between 87 and 90% of the variability in wave height during the period of highest waves. On the assumption that topographic variation should constrain wave breaking (especially the intensity), high order polynomials (such as a fourth order) might be expected to provide better descriptions of the spatial variation in wave height. Although these higher order functions do provide an increase in the amount of variability accounted for, the increases are not statistically significant. During the storm buildup (1300 h) and during the latter part of the decay period (2330 h), however, there is a marked reduction in the variability accounted for by the linear functions, with \( R^2 \) values as low as 0.46. Statistically significant fourth-order polynomials account for 70 and 80% of the observed variability, respectively, at these times. This suggests, therefore, that with such low relief, gentle slopes, and spilling breakers initiated some distance lakeward of the bars at the storm peak, a linear decay of wave height similar to that proposed for a planar beach (Longuet-Higgins, 1982) may provide an adequate approximation for certain purposes (see also Greenwood and Sherman, 1987).

There is strong evidence, nevertheless, that local variation in topography does indeed introduce variability in the wave height pattern through enhanced breaking, shoaling and/or frictional effects. Lakeward of the outer bar crest a significant break of slope is encountered by the propagating waves between 80 and 90 m offshore; a rapid decay in wave height locally is associated with more intense wave breaking between this break of slope and the bar crest at approximately 75 m offshore (note the negative residuals from the regression lines in this area; see Fig. 10). Locally increased breaking on the inner bar crest is again marked by a steep gradient in wave height decay (Fig. 10a), although lack of a complete record from wave wire 12 (30 m offshore)
Fig. 10. Shore-normal variation in significant wave height ($H_s$). (a) During storm-wave growth (1330–1700 h); (b) during the storm-wave peak (1800–1930 h); (c) during the storm-wave decay (2030–2330 h).

prevents this being seen throughout the storm cycle. Field data cited in Watanabe and Dibajnia (1989) illustrate just such large local gradients in wave height decay across both barred and step-type profiles. Wave height decay also occurs across the lakeward slope of the inner bar but is less rapid and locally may be reversed (for example between 55 and 65 m offshore; Fig. 10), because of the offsetting effects of shoaling over this steep slope. A small, but
temporally consistent, increase in wave height near the shoreline (note the consistent positive residuals at this location; see Fig. 10) is associated with the wave steepening prior to the final break at the shoreline.

Set-up and set-down of the still water surface

Since set-up and set-down are displacements of the mean water level above or below the still-water level, their determination under surf zone conditions requires measurements of possibly small changes in the time-averaged surface elevation in the presence of very large absolute shifts in water surface elevation operating at high frequency. Furthermore, it is necessary to determine the still-water level with some confidence. The accuracy of the resistance wire wave staffs with respect to these determinations is therefore of some concern. To assess wave staff performance, several tests of the output signals were performed.

During the immediate post-storm period, burst samples at ~2 Hz of 3-min duration were recorded from each staff under low waves (June 18) and near flat calm conditions (June 19). The intersection of the still-water surface with the beach face was also surveyed accurately on June 19 and the still-water levels on the staffs recorded. Table 1 documents the mean calibrated signal for each wave staff, together with the standard error of estimate for the means measured on June 19, when the lake surface was at its calmest. Taking a confidence band of three standard errors of estimate (observed at wave wire 01), it is clear that the maximum error in estimation of the mean sensor signal (with 95% confidence) for a single record is < ±0.03 m. As the shifts in the means recorded during the storm were an order of magnitude larger than this, such errors can be considered negligible. It is worth noting that a collocated pressure sensor at wave wire 09 (50 m offshore) revealed a very similar standard error of estimate (0.002). It is obvious from Table 1 that the mean water surface was still somewhat elevated on June 18 in the early morning with levels falling through the day to 1600 h. The calibrated mean signals continued to drop through to June 19; however, with the exception of wave wire 05, which had a difference from the previous day of almost twice that of any other sensor, the changes were small and eminently plausible. Examination of the spectral signature of the still-water signal from wave wire 05 (see Fig. 4) revealed a significant difference from white noise so that this sensor was eliminated from consideration. As a further check on sensor stability the mean signals from 1300 h on June 16 during the very beginning of wave activity are also given in Table 1. It is clear that the mean signals at the beginning of the storm and those from the end of the storm differ by less than 10%. Unfortunately the connecting cable to wave wire 12 was damaged (and subsequently replaced) during the storm; as it did not give a continuous time series which could be confidently calibrated, it was only used as a confirmatory indicator.
TABLE 1

Comparison of the mean water level values from the calibrated wave wire signals

<table>
<thead>
<tr>
<th>Wave staff number</th>
<th>June 13</th>
<th>June 18</th>
<th>June 19</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{\eta}$ (1300 h)</td>
<td>$\bar{\eta}$ (0100 h)</td>
<td>Diff $\bar{\eta}$ (1600 h)</td>
</tr>
<tr>
<td>15</td>
<td>0.92</td>
<td>0.94</td>
<td>0.02</td>
</tr>
<tr>
<td>14</td>
<td>1.07</td>
<td>1.03</td>
<td>0.04</td>
</tr>
<tr>
<td>13</td>
<td>0.91</td>
<td>0.86</td>
<td>0.05</td>
</tr>
<tr>
<td>12</td>
<td>0.86</td>
<td>m.f.</td>
<td>m.f.</td>
</tr>
<tr>
<td>11</td>
<td>0.97</td>
<td>1.02</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>1.28</td>
<td>1.21</td>
<td>0.07</td>
</tr>
<tr>
<td>09</td>
<td>1.36</td>
<td>1.32</td>
<td>0.04</td>
</tr>
<tr>
<td>08</td>
<td>1.13</td>
<td>1.09</td>
<td>0.04</td>
</tr>
<tr>
<td>07</td>
<td>1.16</td>
<td>1.15</td>
<td>0.01</td>
</tr>
<tr>
<td>06</td>
<td>1.82</td>
<td>1.82</td>
<td>0.00</td>
</tr>
<tr>
<td>05</td>
<td>1.34</td>
<td>1.81</td>
<td>0.47</td>
</tr>
<tr>
<td>04</td>
<td>1.07</td>
<td>1.07</td>
<td>0.00</td>
</tr>
<tr>
<td>03</td>
<td>1.54</td>
<td>1.54</td>
<td>0.00</td>
</tr>
<tr>
<td>02</td>
<td>1.57</td>
<td>1.64</td>
<td>0.07</td>
</tr>
<tr>
<td>01</td>
<td>1.78</td>
<td>1.78</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\bar{\eta}$ = time-averaged mean.
$\bar{\eta}_{s.e.}$ = standard error of estimate for the mean, June 19.
Diff = difference from preceding record.
m.f. = instrument malfunction.

of trends in set-up revealed by the other sensors. It appears therefore that with the exception of wave wires 05 and 12, the wave sensors performed with errors in mean signal significantly less than 10%.

Set-up (set-down) was measured as the difference between the time-averaged water-surface elevation determined from each wave staff at any point in time and the time-averaged still-water elevation determined on June 19. Since it is clear from the earlier appraisal of wave staff performance that there will be some error in the measured values, the best descriptor of the spatial variation in set-up was determined statistically using least-squares regression. Furthermore, the two bars were present, and since set-up could be expected to reflect this topography, then nonlinear functional relationships between set-up and offshore distance might reasonably be expected (e.g., Battjes and Janssen, 1978; Dally et al., 1985). Figure 11 illustrates the set-up ($\bar{\eta}$), for each time period plotted against the offshore distance ($x$); also shown are the best-fit fourth-order polynomials for each time period. Table 2 documents the $R^2$ values and Table 3 lists the partial $F$ coefficients for an analysis of variance of each additional term in the polynomial regression. From this it is clear that, while a linear function is statistically significant, a fourth-order polynomial is the best estimator of the observed variability in set-up. The values
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Fig. 11. Shore-normal variations in the mean water surface set-up during the storm peak, June 16, 1300–2330 h. Fourth-order polynomial approximations to the mean water surface set-up are shown as is the topography along the instrument transect and the locations of the bar crests (C) and troughs (T) of the two-bar system. Note: the offshore distance used in the least-squares computation is that relative to a baseline which was 0.5 m landward of the pre-storm still water shoreline.

of the coefficient of determination ($R^2$) for the latter vary between 0.66 and 0.71 for the period of the storm peak (1700–2130 h); even at the time of wave buildup and decay this polynomial accounts, respectively, for 46 and 63% of the observed variability. Standardized residuals from the regressions
TABLE 2

$R^2$ values for the least squares polynomial regressions between mean water surface elevation and offshore distance

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Order of polynomial</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300</td>
<td></td>
<td>0.23 n.s.</td>
<td>0.41 n.s.</td>
<td>0.45 n.s.</td>
<td>0.46 n.s.</td>
<td>0.49 n.s.</td>
</tr>
<tr>
<td>1600</td>
<td></td>
<td>0.17 n.s.</td>
<td>0.55 *</td>
<td>0.56 *</td>
<td>0.63 *</td>
<td>0.68 n.s.</td>
</tr>
<tr>
<td>1700</td>
<td></td>
<td>0.27 n.s.</td>
<td>0.62 *</td>
<td>0.63 *</td>
<td>0.66 *</td>
<td>0.70 *</td>
</tr>
<tr>
<td>1800</td>
<td></td>
<td>0.29 n.s.</td>
<td>0.63 *</td>
<td>0.65 *</td>
<td>0.71 *</td>
<td>0.74 *</td>
</tr>
<tr>
<td>1830</td>
<td></td>
<td>0.19 n.s.</td>
<td>0.54 *</td>
<td>0.55 n.s.</td>
<td>0.71 *</td>
<td>0.72 n.s.</td>
</tr>
<tr>
<td>1930</td>
<td></td>
<td>0.15 n.s.</td>
<td>0.47 *</td>
<td>0.50 n.s.</td>
<td>0.69 *</td>
<td>0.72 n.s.</td>
</tr>
<tr>
<td>2030</td>
<td></td>
<td>0.13 n.s.</td>
<td>0.48 *</td>
<td>0.43 n.s.</td>
<td>0.67 *</td>
<td>0.68 n.s.</td>
</tr>
<tr>
<td>2330</td>
<td></td>
<td>0.02 n.s.</td>
<td>0.30 n.s.</td>
<td>0.31 n.s.</td>
<td>0.63*</td>
<td>0.65 n.s.</td>
</tr>
</tbody>
</table>

*Significant at 0.05 level.

n.s. = not significant at 0.05 level.

Significance based upon an “Analysis of variance” and F-test.

TABLE 3

Partial $F$ values for the contribution to polynomial regression of each order up to eighth order

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Order of polynomial</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300</td>
<td></td>
<td>0.040</td>
<td>0.852</td>
<td>2.163</td>
<td>3.371</td>
<td>4.292</td>
<td>4.933</td>
<td>5.358</td>
<td>5.630</td>
</tr>
<tr>
<td>1700</td>
<td></td>
<td>1.174</td>
<td>1.808</td>
<td>2.257</td>
<td>2.468</td>
<td>2.527</td>
<td>2.506</td>
<td>2.448</td>
<td>2.377</td>
</tr>
<tr>
<td>1800</td>
<td></td>
<td>0.748</td>
<td>1.063</td>
<td>1.211</td>
<td>1.139</td>
<td>1.139</td>
<td>1.047</td>
<td>0.957</td>
<td>0.878</td>
</tr>
<tr>
<td>1830</td>
<td></td>
<td>0.480</td>
<td>0.764</td>
<td>0.906</td>
<td>0.919</td>
<td>0.871</td>
<td>0.803</td>
<td>0.733</td>
<td>0.671</td>
</tr>
<tr>
<td>1930</td>
<td></td>
<td>0.239</td>
<td>0.420</td>
<td>0.484</td>
<td>0.457</td>
<td>0.397</td>
<td>0.344</td>
<td>0.279</td>
<td>0.234</td>
</tr>
<tr>
<td>2030</td>
<td></td>
<td>0.326</td>
<td>0.505</td>
<td>0.549</td>
<td>0.500</td>
<td>0.421</td>
<td>0.434</td>
<td>0.279</td>
<td>0.277</td>
</tr>
<tr>
<td>2130</td>
<td></td>
<td>0.180</td>
<td>0.312</td>
<td>0.338</td>
<td>0.297</td>
<td>0.238</td>
<td>0.183</td>
<td>0.138</td>
<td>0.105</td>
</tr>
<tr>
<td>2330</td>
<td></td>
<td>0.524</td>
<td>0.847</td>
<td>0.955</td>
<td>0.914</td>
<td>0.818</td>
<td>0.713</td>
<td>0.618</td>
<td>0.539</td>
</tr>
</tbody>
</table>

indicate normally-distributed variables with equal variances; the Durbin–Watson statistic (Durbin and Watson, 1951a, b, 1971) computed for the residuals reveal an absence of spatial autocorrelation. The fourth-order polynomial functions are thus statistically robust predictors.

Several observations can be made with respect to the set-up (set-down) pattern (Fig. 11), which compares favourably, at least in a qualitative sense, with the patterns predicted by Battjes and Janssen (1978).

1) In general set-up increased from offshore towards the shoreline as expected and there was a general raising and lowering of the water surface across the whole surf zone through time as the storm increased and then decreased in intensity. Only at the earliest and latest stages of the storm cycle (1300 h
and 2330 h) was the mean surf-zone water level close to the still-water value. This suggests, therefore, that spatial variation in set-up was superimposed on a general super-elevation of the still-water level over the whole nearshore, which was wind-forced. Table 4 documents the measured winds at the beach face during this time period and shows that the shore-normal wind vector increased up to 1800 h and then decreased to 2330 h, coincident with the overall raising and lowering of the mean water surface.

(2) There was set-up landward of the inner bar, with a maximum in the trough and a distinct decrease towards the shoreline. The inner trough experienced the maximum set-up (0.41 m at 1800 h) and the deepest part of the trough always exhibited the maximum set-up at any time. This set-up was associated with the observed intensification of breaking (frequently with plunging breakers) over the crest and upper lakeward slope of the inner bar. Unfortunately, identification of the set-down over the inner bar crest was limited owing to the failure of wave wire 12 (in fact the damage incurred by this sensor resulted from its position within the zone of intense breaking). Furthermore, an absolute set-down might not be expected owing to both wind stress effects and set-up induced by wave breaking over the outer bar. Measurements from wave wire 12 prior to its failure (see Fig. 11) indicate that indeed a relative reduction in the set-up was occurring at this location. The decrease in set-up from the inner trough towards the shoreline is again to be expected, since a zone of more intense breaking occurred at the shoreline with an associated relative set-down. No detailed measurements of the water surface across the beach face were possible in this experiment to confirm the

### TABLE 4

Wind parameters measured at the beach face and wave angles measured 65 m offshore, Wymbolwood Beach, June 16, 1986

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Wind speed (m s⁻¹)</th>
<th>Direction (°)</th>
<th>Shore-normal wind speed (m s⁻¹)</th>
<th>Wave angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300</td>
<td>3.78</td>
<td>110</td>
<td>1.29</td>
<td>n.a.</td>
</tr>
<tr>
<td>1500</td>
<td>4.32</td>
<td>145</td>
<td>3.54</td>
<td>n.a.</td>
</tr>
<tr>
<td>1600</td>
<td>6.48</td>
<td>33</td>
<td>5.43</td>
<td>9</td>
</tr>
<tr>
<td>1700</td>
<td>8.10</td>
<td>15 (WNW)</td>
<td>7.82</td>
<td>2</td>
</tr>
<tr>
<td>1800</td>
<td>9.18</td>
<td>6 (WNW)</td>
<td>9.12</td>
<td>6</td>
</tr>
<tr>
<td>1830</td>
<td>8.10</td>
<td>18 (WNW)</td>
<td>7.70</td>
<td>6</td>
</tr>
<tr>
<td>1930</td>
<td>8.10</td>
<td>23 (WNW)</td>
<td>7.46</td>
<td>6</td>
</tr>
<tr>
<td>2030</td>
<td>8.10</td>
<td>65 (N)</td>
<td>3.42</td>
<td>n.a.</td>
</tr>
<tr>
<td>2130</td>
<td>4.86</td>
<td>45 (NNW)</td>
<td>3.44</td>
<td>n.a.</td>
</tr>
<tr>
<td>2330</td>
<td>4.86</td>
<td>45 (NNW)</td>
<td>3.44</td>
<td>4</td>
</tr>
</tbody>
</table>

n.a. = not available.
expected set-up due to this shoreline break, but supporting visual observations were made of increases and decreases in the elevation of run-up.

(3) Lakeward of the inner bar, the mean water surface exhibited a relative set-down associated with wave breaking across a broad area of the outer bar crest and lakeward slope; the outermost wave staffs at 110 m and 130 m show little difference in the mean water level at any one time and reflect essentially the super-elevation due to wind stress.

(4) It is worth noting that there was a small but detectable increase in the set-up gradient (determined from difference in maximum set-up between the outer and inner parts of the surf zone) as incident wave energy increased (1300–1800 h), and a subsequent decrease as wave energy decreased (1800–2030 h).

Topography does, therefore, exert a strong influence upon set-up, as both theory and laboratory experiments would suggest (Battjes and Jansen, 1978; Izumiya and Horikawa, 1984; Dally et al., 1985); however, the relative spatial gradients in set-up are significantly smaller than those associated with the wave height decay. The latter, at least in part, accounts for the successful fit to the set-up achieved by the smoothing of the fourth-order polynomial. This strong influence of the bars upon set-up would appear to contrast with the recent measurements of Nielsen (1988). However, measureable differences also exist with some of the models proposed. For example, Dally et al. (1985) applied their model of set-up to a large scale two-barred profile developed experimentally by Saville (1975); predicted variation in the mean water surface elevation was of the order of 100% across the “offshore” slope of the inner bar, within a horizontal distance of only ~7% of the surf zone width (see their fig. 12). The maximum local variation in set-up measured at Wymbolwood Beach across the offshore slope of the inner bar was also of the order of 100%, but this occurred over a distance of >30% of the surf zone width over which the set-up was measured.

Since the absolute set-up consists of that resulting from both wind and wave stresses, it is useful to consider the set-up differentials as well as the absolute elevations. Table 5 summarizes the maximum set-up and set-up gradient for each time period determined using the fourth-order polynomial and its first two roots. Values for the absolute set-up range between 0.12 m and 0.36 m, while the gradients range from 0.0027 to 0.0037. Also given in Table 5 are the ratios of maximum set-up to incident wave height (significant breaking wave height, $H_{sb}$, measured at wave wire 03 (85 m offshore) and the estimated deep water significant wave height ($H_{so}$); the latter was determined using simple linear shoaling transformations of the wave height measured at the outermost wave staff (130 m offshore). These observations indicate that:

\[
0.15 < \frac{d\bar{h}}{H_{so}} < 0.23
\]  

(3)
and:

\[ 0.18 < \frac{d\bar{\eta}}{H_{sb}} < 0.32 \]  

These ratios are lower than the maximum set-up at the shoreline determined for deep-water incident wave spectra by Battjes (1974) and Battjes and Janssen (1978). At Wymbolwood Beach the average value of \( d\bar{\eta}/H_{so} = 0.19 \) and the average value of \( d\bar{\eta}/H_{sb} = 0.25 \) (see Table 5). The latter value lies well below the lower limit of the range proposed by Battjes (1974) for breaking monochromatic waves, but his values were shoreline values over a planar slope. The maximum gradients in set-up measured at Wymbolwood Beach are significantly smaller than those predicted using an average nearshore slope value and the empirical expression of Bowen et al. (1968), even when the solitary wave breaking criterion is used; using a more appropriate breaking criterion for this beach, the predicted gradient is three times the magnitude of that observed. This difference results partly from the irregularity in the wave field, since Dean (1974) has shown that irregular waves at breaking generate significantly less radiation stress than monochromatic waves of the same significant height. Another reason that may explain this deviation is that the radiation stress gradients in the outer region of a surf zone may be relatively constant, even with a marked reduction in wave height (see Battjes, 1974; Battjes and Janssen, 1978) owing to the mode of energy dissipation in wave breaking (Svendsen, 1984). Thus, with an average nearshore slope of

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>( \bar{\eta}_m ) (m)</th>
<th>( \frac{d\bar{\eta}}{dx} )</th>
<th>( \bar{\eta}<em>m/H</em>{so} )</th>
<th>( \bar{\eta}<em>m/H</em>{sb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700</td>
<td>0.28</td>
<td>0.0027</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>1800</td>
<td>0.36</td>
<td>0.0029</td>
<td>0.23</td>
<td>0.32</td>
</tr>
<tr>
<td>1830</td>
<td>0.35</td>
<td>0.0037</td>
<td>0.23</td>
<td>0.31</td>
</tr>
<tr>
<td>1930</td>
<td>0.24</td>
<td>0.0036</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td>2030</td>
<td>0.22</td>
<td>0.0035</td>
<td>0.16</td>
<td>0.23</td>
</tr>
<tr>
<td>2130</td>
<td>0.19</td>
<td>0.0034</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td>2330</td>
<td>0.12</td>
<td>0.0027</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>Average</td>
<td>0.25</td>
<td>0.0032</td>
<td>0.19</td>
<td>0.25</td>
</tr>
</tbody>
</table>

\( d\bar{\eta}/dx \) = the gradient determined from the maximum set-up difference resolved from the roots of the polynomial predictor.

\( H_{so} \) = deep water significant height determined by linear shoaling from wave wire 01.

\( H_{sb} \) = significant breaker height measured at wave wire 03.
Fig. 12. Functional relationship between the time-averaged, cross-shore mean currents (under-tow) over the outer bar and: (1) the time-averaged maximum set-up in the inner surf zone (computed using the best-fit fourth-order polynomial; a, b, c); (2) the incident significant wave height measured at 110 m offshore (d, e, f). Current records for the outer bar are plotted for the 0.1 m and 0.5 m elevations on the landward slope (CM2 and CM12 at 70 m, CM6 and CM5 at 65 m and CM10 and CM8 at 60 m; see Fig. 4) and from the 0.25 m elevation on the lakeward slope (EM1; see Fig. 4). The best-fit linear least-squares regression lines and the $R^2$ values for the regressions are also shown.
TABLE 6

Linear correlation coefficients ($R$) and $R^2$ values for correlations between cross-shore mean flow ($\bar{u}$) and incident significant wave height ($H_o$), breaking wave height ($H_{sb}$) and predicted maximum set-up ($\bar{\eta}_m$).

<table>
<thead>
<tr>
<th>Location of mean flow</th>
<th>$H_o$</th>
<th>$H_{sb}$</th>
<th>$\bar{\eta}_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$R^2$</td>
<td>$R$</td>
</tr>
<tr>
<td>20 m</td>
<td>0.226</td>
<td>0.051</td>
<td>0.131</td>
</tr>
<tr>
<td>60 m</td>
<td>0.580</td>
<td>0.337</td>
<td>0.517</td>
</tr>
<tr>
<td>65 m</td>
<td>0.905</td>
<td>0.820*</td>
<td>0.911</td>
</tr>
<tr>
<td>70 m</td>
<td>0.887</td>
<td>0.786*</td>
<td>0.910</td>
</tr>
<tr>
<td>70 m</td>
<td>0.949</td>
<td>0.900</td>
<td>0.954</td>
</tr>
<tr>
<td>110 m</td>
<td>0.482</td>
<td>0.337</td>
<td>0.921</td>
</tr>
<tr>
<td>0.10 m</td>
<td>0.517</td>
<td>0.267</td>
<td>0.921</td>
</tr>
<tr>
<td>0.50 m</td>
<td>0.620</td>
<td>0.384</td>
<td>0.608</td>
</tr>
<tr>
<td>0.50 m</td>
<td>0.845</td>
<td>0.718*</td>
<td>0.858</td>
</tr>
<tr>
<td>0.50 m</td>
<td>0.787</td>
<td>0.619*</td>
<td>0.823</td>
</tr>
<tr>
<td>110 m</td>
<td>0.886</td>
<td>0.784*</td>
<td>0.898</td>
</tr>
</tbody>
</table>

*Significant at the 0.1 level.

Fig. 13. Functional relationship between maximum set-up in the inner surf zone (computed using the best-fit fourth-order polynomial) and the deep-water significant wave height. The best-fit linear least-squares regression line and the $R^2$ value for the regression is also shown.

0.015 and local slope across the lakeward slope of the inner bar of 0.044, it is evident that the average set-up gradient is less than 20% of the beach slope.
Cross-shore flows and wave set-up

Significant linear correlations exist between the cross-shore mean flows (undertow) across the outer bar and both set-up and incident wave height. Figure 12 illustrates linear least-squares fits between the undertow velocities in the lower part of the water column (0.1 m and 0.5 m) and both the maximum set-up in the inner surf zone estimated from the polynomial predictor (Fig. 12a, b, c) and the incident wave height (Fig. 12d, e, f). The undertow at all four measurement locations is strongly correlated (Table 6) with set-up (average $R^2=0.64$). Only at the 60 m station is the degree of explanation provided by the linear function relatively poor ($R^2$ values of 0.21 and 0.32 for the 0.1 m and 0.5 m flows, respectively). Almost identical levels of explanation of the variability in the current records is achieved by a linear correlation with the set-up values and incident wave height (on average 64%); this is to be expected of course if the currents were set-up driven and the set-up was primarily wave-forced. Figure 13 illustrates the correlation between the maximum set-up and the incident significant wave height. Eighty-six percent of the variability in the observed maximum set-up can be explained by a linear function of incident wave height. The above analysis would therefore appear to indicate unequivocally that the cross-shore mean flows were set-up induced and that the set-up itself was forced primarily by the incident wave field. Although some wind stress effect must have been present it clearly was not the dominant force, inducing mean water-level changes.

DISCUSSION AND CONCLUSIONS

Measurements from eleven strategically located current meters, together with measurements of water surface elevation from fifteen wave staffs, provide a preliminary description of the nearshore hydrodynamics controlling the cross-shore flow field on a barred beach. Asymmetry in the shore-normal velocity distribution was evident in large nonzero mean values and large positive skewness values, both of which increased up to the storm peak and decreased as the storm decayed. Mean flows in the lowermost 0.5 m of the water column were directed offshore and decreased in magnitude away from the bed. Velocity skewness was always positive (i.e., velocity vectors skewed in the onshore direction) and also increased with the height of the breaking waves. Vertical profiles of the wave-induced oscillatory velocity ($u_o$) and mean velocity ($\bar{u}$) indicated radically different patterns. Oscillatory velocities decreased slightly with depth, apparently influenced by boundary effects, while the mean of the velocity distribution varied significantly with depth independently of the oscillatory current magnitude. The magnitude of the mean flows was most often largest near the bed and in some cases, reversals of mean flows (in the vertical) were present in the upper part of the water column. These observations reveal a separate and distinct mean flow and imply the existence of a current superimposed on the oscillatory motion.
The cross-shore velocity vectors during the storm peak were clearly influenced by the existence of shoaling and breaking waves; nonlinearity in the surface waves, for example, would have contributed to asymmetry in the oscillatory currents. Asymmetry of wave shape about the horizontal axis would induce skewness to the velocity distribution; this alone may have been strong enough to offset the mean from zero, although not necessarily in the same direction as the observed skewness. Velocity skewness exhibited less stratification than the mean, but increasing positive skewness was associated with increasing elevation away from the bed. This suggests either a high surface stress due to breaking waves (Haines, 1984) or incident waves with a significant translatory component. The shoreward mass and momentum flux, which is theoretically greater near the surface and decreases with depth (Fig. 1), is indicated in the prototype by larger positive (onshore) values of velocity skewness as well as slightly higher oscillatory velocity values near the water surface. A balance is achieved by a lakeward mass and momentum flux near the bed, resulting from the larger offshore mean currents in the lower water column. The significant landward mass and momentum fluxes associated with the breaking wave at the surface also served to reduce the strength of the offshore mean flow in the upper water column, even to the point of inducing a reversal in direction of flow near the surface. This hypothesis is substantiated by many of the measured vertical profiles from the storm peak and early decay.

Low-frequency energy levels in the wave spectra did increase landwards across the surf zone, but were small relative to the energy levels at the incident frequency; this was especially the case over the outer bar system, where mean flows were large. Therefore, it is unlikely that drift velocities from a low frequency edge wave or leaky wave were responsible for the observed asymmetry in cross-shore velocity distribution in the outer parts of the surf zone. A more detailed examination of periodicity in the mean flow and the phase relationships with water surface elevation is necessary to confirm this speculation.

Recent work on the vertical structure of the cross-shore velocity field in the surf zone has renewed interest in "undertow", especially where longshore currents and rip cells are weak or absent. Undertow is clearly illustrated by the vertical structure of the offshore mean flows observed under breaking waves in this study. Vertical stratification in the magnitudes of the oscillatory currents and velocity skewness support this contention. A large set-up of the mean water level in the inner surf zone and significant set-up gradients were measured during the storm peak. Although the gradients were significantly smaller than those suggested by existing model studies, they were large enough to provide a pressure gradient capable of driving large near-bed lakeward currents. Longuet-Higgins (1983) suggested that the reverse bottom current or undertow in the surf zone may be expected to be strongest when the beach is fairly steep and the surf zone is narrow. The correlations between the set-up gradient and the steepest part of the beach (lakeward slope of the inner bar),
and between the set-up and the mean cross-shore flow, are by no means conclusive, but are at least encouraging.

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