Simple and Low-Cost Exposed -Layer Grain Drying Apparatus

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Abstract

Thin-layer drying apparatus was developed from standard "off-the- shelf" equipment: a fan convection laboratory oven and a weighing scale. Using this apparatus the thin-layer drying data for wheat under constant conditions were obtained for a range of drying air temperature from 30°C to 150°C and the initial moisture content from 0.20 to 0.42 (d.b.). The smooth drying characteristic curves were obtained and used to develop coefficients for three thin layer models namely: Grace (modified Henderson) model, Chen and Johnson, and the numerical diffusion models. The Grace and the Chen and Johnson models were the best at describing the drying curves giving a standard error of the moisture ratio less than 0.027. The drying apparatus enables monitoring of the sample weight during drying without opening the oven. The drying air properties (relative humidity and temperature) inside the
oven were kept constant throughout drying. The next special feature of it was that the sample kept on drying even at the time of recording its weight. No temperature change was observed after the oven was switched on after 30 seconds. This apparatus is relatively simple and can be used for drying of biological materials. However, several thin-layer grain drying researchers have obtained moisture loss by using fast sampling devices throughout the drying period. During the study a fast sampling device was not used because it was necessary to turn off the oven before taking measurements.

**Key words:** Drying apparatus, thin-layer grain drying, drying model

**Introduction**

Thin-layer grain drying is defined as a single kernel grain thickness exposed to a constant air temperature, flow rate and relative humidity. The drying characteristics of individual grain kernel are observed over a chosen period of time. The time required to attain the desired moisture content was found by Sokhansanj *et al* (1983) to be directly related to the temperature under which the mass transfer occurred. They found that to increase the moisture content of wheat from 12% to 22% at 2°C 46 hours of tempering was adequate, while tempering at 21°C reduced the tempering time to 32 hours. Understanding some of the most common problems in thin-layer drying apparatus is important to obtain drying data with a minimum error. The most important element is the accurate measurement of the loss of moisture. This is usually carried out by measuring the weight loss of the sample during drying. The apparatus must also be able to maintain constant, accurate and uniform drying conditions, especially air temperature and humidity. To control these parameters, the thin-layer drying apparatus in general consists of the following parts:

- an air conditioning unit
- a drying chamber
- a weighing system

Nellist and O'Callaghan (1971) used the modified version of the N.I.A.E. multi-unit drier, which was originally built to examine the effect of drying. In principle, the apparatus was based on the assumption that the drying air was blown upwards through the chamber at a constant air flow rate, which means the upward pressure force of the air on the sample tray was considered constant. Thus, the change in weight of the grain sample and the tray could be due to loss of the sample weight.
Bruce and Sykes (1983) developed an apparatus for determining mass transfer coefficients at temperatures up to 300°C and air speeds up to 1 m.s⁻¹ for thin-layer particulate crops. They solved the problem of the air pressure force on the perforated tray by introducing a bypass for the drying air during weight recording. The apparatus enables the measurement of the following parameters: weight and surface temperature of grain sample, and temperature, relative humidity and speed of the drying air. Data logging is controlled by a microcomputer that could perform a short scan or long scan. A short scan logs time, sample weight, sample temperature and dew point temperature. A long scan records the air pressure in a venturi. They suggested that the measurements of the surface temperature of the sample by infrared pyrometry are shown to be valid and are expected to be important in modelling crop quality.

Woods and Favier (1993) developed an apparatus for thin-layer drying and described the measurement techniques together with the analysis of error in drying. The apparatus consists of the air conditioning unit and the thin-layer drying chamber. They introduced the method to measure air humidity through the use of a differential psychrometer. Also, they showed a method for continuously monitoring the aerodynamic force on a sample by the means of a pressure differential, which they indicated to be effective in determining the true sample weight.

Siebenmorgan et al (1991) combined an IBM PC, a six-axis articulating robot, an analytical balance, and a temperature/relative humidity control unit to automatically weigh samples of grain during drying. In this case, the grain samples are sequentially removed from the chamber, weighed by placing on the balance and returned to the chamber by the use of the robotic arm. There may be weight loss due to dust particles falling from the sample during movement in this case.

Some of the most common problems in thin-layer drying apparatus are:

- air currents on analytical balance
- effect of heat on analytical balance
- aerodynamic effect on the sample weight measured in situ
- weight loss due to loss of dust particles if sample is moved, and
- accurate control of temperature and humidity.

Included among the factors that affect the accuracy of the thin-layer drying equations is the measurement of the weight loss of the drying sample manually by opening the drying apparatus, (which results in disturbing the drying air conditions). To control this problem, the literature shows that much effort has been spent developing an "in
situ" weighing systems to record the weight of the sample throughout the drying process; these avoid the need to open the drying apparatus and thus, help to maintain the required steady-state air conditions and reduce weight loss due to dust. The main shortcoming of this way of recording weight loss is the aerodynamic effect on the sample tray. In order to control the effect of the air pressure created by the fan on the sample weight, Bruce (1985), as presented previously, introduced a bypass for diverting air from the drying chamber at the time of recording. Henderson and Pabis (1961) controlled the buoyancy created around the sample tray by cutting off the air movement at the sample location at the time of recording weight loss. The other approach to control the aerodynamic effect on the weighing system was achieved by blowing the air at a constant flow-rate. Then, the upward pressure of air may be considered to be constant and the change in weight of the sample tray would be due to the loss of weight of the sample. Woods and Favier (1993) measured the pressure difference across the sample tray directly.

The objective of the study is to fit a laboratory weighing scales to a laboratory oven in such a way to enable the instantaneous recording of the loss in weight of the grain sample without opening the oven door. The standard equipment is used for both short construction time and accuracy of control and measurement during drying run where possible.

Material and Methods

Description of Thin-layer drying apparatus

Fig.1 shows a schematic diagram of the apparatus developed and used to obtain the thin-layer drying data. It was designed to enable drying sample weight loss to be recorded in situ without opening the drying chamber door. It was decided to use proven "off-the-shelf" equipment where possible. For this reason a standard fan convection laboratory oven was chosen as the drying chamber. A good oven can quickly achieve and maintain accurate uniform air temperature within the chamber. We also looked at the possibility of using a psychrometric chamber in which the air humidity could be controlled as well as the temperature. However, the cost of such a unit was beyond the budget for this project. Also, it was noted that air humidity only affects the drying rate indirectly through the equilibrium moisture content and therefore is much less important than air temperature.

As seen in the Fig. 1 the sample tray is suspended from the balance, which is, placed on top of the oven. To avoid the air currents disturbing the balance it was enclosed in a transparent container (D). Hot air coming out of the oven was diverted away from the balance along the air diversion channel (E) (see also Fig. 2). The oven has an in-
built thermocouple to control the air temperature but an extra thermocouple was placed near the sample tray to give accurate local measurements of temperature. The other apparatus consisted of a standard laboratory balance, suspension frame and sample tray.

The drying chamber used was the Gallenkamp Plus II 75 litre Oven. This oven was chosen because of its accurate temperature control and fast response time. It also had an air vent at the top of the oven through which the sample tray could be suspended and through which the grain samples could be inserted. The air in the oven was circulated with an in-built fan. A control thermostat controlled the temperature of the air in the drying zone. According to the manufacturer's specifications, this unit is capable of maintaining a dry-bulb temperature of accuracy: ±1.0°C up to 100°C and ±0.1°C in the range 100°C to 2000°C. This has been verified with the use of an extra thermocouple near the sample. The unit requires 15 minutes in order to reach a stable temperature. The recovery time required is 4 minutes and the circulation rate is 2500 litres/hour.

An electronic analytical and precision balance (Sartorius model LC 620P) with an accuracy of 1 milligram and response time of 1.5 seconds was used to monitor weight loss during drying. The balance is sensitive to air movement but not very sensitive to heat with a sensitivity drift of ±2 x 10^{-6} g.°C^{-1} in the temperature range +10 to +30°C. A special transparent plastic box was used to cover the balance during drying periods in order to minimise inaccuracies induced by air currents. Hot air from the oven vent, which would have otherwise affected the accuracy of the weighing process, was diverted away from the balance and its the enclosing box via the air diversion channel resting on top of the drying chamber. To minimise vibration during the weight recording, the whole unit was placed on a rigid concrete bench. Also, the movement of air in the oven caused the weight reading to fluctuate. This problem was minimised by switching off the oven for at least 15 seconds before taking weight reading.

The sample tray consisted of a wire screen (mesh size approximately 1.5 mm less than the smallest dimensions of the kernel of grain) of overall dimension 160 x 150 x 30 mm constructed in such a way that allows the drying air to circulate in any direction including from the sides and base in order to minimise error due to non uniformity of air distribution over the grain kernels. The sample tray was suspended in the chamber one quarter way down where the air flow pattern was reasonably uniform. Fig. 1 shows the airflow pattern inside the oven. The sample tray was bolted to the bottom of a stiff rod which passed up through the oven ventilation port and connected to the suspension frame, which transfers the weight without vibration to the top of the scales. The suspension frame is light but rigid so that the balance is not overloaded but at the same time the frame damps out any vibration in the system. This was essential to minimise shaking or vibration of the sample during weighing measurements. The
The overall weight of the suspension frame plus the sample tray and the rod was 502 grams. The initial weight of grain samples was about 20 grams. The maximum capacity of the balance is 620 grams. Therefore, there is a good factor of safety against exceeding the capacity of the balance.

**Sample preparation**

Re-wetted Alexandria spring wheat was used for the tests. A typical sample of wheat contains kernels of many different dimensions. For this reason it was decided to ensure that the drying samples had a uniform distribution of kernel dimensions. The most variable dimension in wheat grain is its length. Thus, the kernel selection was primarily based on kernel length to form the required drying sample of wheat. This was done by observations. A series of sieves were used to select grains of varying thickness.

To avoid having to wait for the harvesting season, it was decided to re-wet dry grain to the desired moisture content. Re-moistening was performed by adding the required amount of water to the grain before placing it in a double sealed container for about 40 hours. The container was shaken approximately every 6 hours in order to uniformly distribute the moisture throughout the grain. To create a microenvironment for mass transfer within the container, its contents were sealed. In this case, re-wetting was carried out at room temperature and a re-wetting time of about 40 hours was chosen. The procedure for re-wetting was: first, measure the weight of the grain sample; second, add the required amount of water, and close the container tightly and shake it well; and finally, periodically shake the water-grain mixture until the moisture is uniformly distributed.

The prepared, re-wetted grain was kept in the refrigerator until required for each drying test to avoid re-germinating of wheat grains and mould growth.

The total grain sample was divided into four groups, which were re-wetted to four different moisture contents varying between 20 and 42% (d.b.), in order to emulate moisture content at harvest which is typical of harvesting moisture contents in the wet part of the world. After re-wetting, these four sample groups were kept in a refrigerator until the tests for that particular group were completed.

All combinations of the following initial moisture content and air temperature were used:

- initial moisture content, $m_o$ (d.b.) kg kg$^{-1}$ dry grain
- group 1 $m_o = 0.22$
● group 2 $m_o = 0.28$
● group 3 $m_o = 0.36$
● group 4 $m_o = 0.42$
● drying air temperature °C: 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, and 150.

There were 52 distinct experimental runs with seven duplications. Moisture content determination was done according to the methods the standard analytical chemists (Official Method of the Association of Official Analytical Chemists, 1990). A ground grain method was employed.

Determination of the moisture contents were made in triplicates to statistically check the error that may occur. The mean value was used in subsequent calculations. The variations or accuracy of determined moisture content values were evaluated by using sample standard deviation.

**Thin-Layer grain drying under constant conditions**

A series of tests were conducted to establish if the relative humidity inside the oven increases while moisture from the grain sample is evaporating into the drying air inside the oven. This was carried out to check whether the humidity inside the drying chamber rises much during drying. The tests were conducted by heating the air to the desired temperatures both with and without the sample of grain in oven. A data logger with a temperature and relative humidity probe was used to record the temperature and the relative humidity of the drying air inside the oven. The tests with no grain sample in the oven, which was just heating the circulating and the in-going air, indicated that the relative humidity was approximately constant. The second test was conducted with a grain sample inserted into the hot oven through the ventilation port.

Prior to each drying test, all the sample group were held for approximately 1 hour at room temperature in a sealed container for the sample to reach ambient temperature. The grain sample for each drying run was randomly selected from the wet grain. In all of the drying runs about 20 grams of grain approximately 500 kernels were used. The remaining wet grain was immediately returned to the refrigerator to avoid mould development.

At the start of each test, the oven was switched on and the required temperature was set. The oven was allowed to stabilise to the pre-set temperature for 20 minutes if the drying temperature was less than 100°C and 30 minutes if the drying temperature was over 100°C. The sample tray, support rod and suspension frame were tared to give a zero weight reading. Without opening the drying chamber, the grain sample (at room temperature) was sprinkled
onto the sample tray through the ventilation port of the oven, then immediately the initial weight of the sample was recorded and the clock started. At the initial stage of the drying test, it was found to be important to take data points at short time intervals and as carefully as possible, since the maximum rate of weight loss occurs at this stage. The interval was increased as the drying rate decreased.

From the start of drying, the change in the sample weight was recorded at time intervals of 2 minutes for first 10 minutes, 5 minutes for up to 30 minutes, 10 minutes for up to 1 hour, and 15 minutes thereafter. The drying tests were terminated when the weight loss indicated 0.02 grams or less. At the end of each drying test, the grain sample was divided into two sub-samples; one for final moisture content analysis and the other for a 24-hour equilibrium moisture content determination. The final moisture content of each sample were required in order to calculate the moisture content at each weighing interval. The 24-hour equilibrium moisture contents were determined by the drying of the samples from each test in the same oven at the same temperature used in the drying test.

Moisture contents were calculated according to American Society of Agricultural Engineers Standard (1993) by the following equation:

\[
m = \frac{w_w - w_d}{w_d}
\]  

(1)

where

\(m\) = moisture content, dry basis, (decimal),
\(w_w\) = weight of the wet sample,
\(w_d\) = weight of the dry sample.

Equation (1) can be used to determine the moisture content at any time for which we have a weight recording if we know the weight of dry material. Because \(w_d\) is not measured for the whole sample, this value can be calculated from the final moisture content by reverting equation (1):
\[ w_d = \frac{w_f}{1 + m_f} \]  
\hspace{1cm} (2)

where \( m_f \) and \( w_f \) are the final weight and moisture content respectively at the end of each run.

Since the equilibrium moisture content models use the relative humidity of drying air, it was necessary to obtain the relative humidity of the drying air used in each drying run. The dry and wet bulb temperatures of the room air were measured. The absolute humidity of the moist air can be obtained from the dry and wet bulb temperatures using the psychrometric chart. The moisture added to the drying air from the drying sample was found to be negligible. Hence the absolute humidity and the vapour pressure of the drying air was assumed to be constant throughout each drying period. The vapour pressure (\( P \)) was calculated by the use of equation (3) from the absolute humidity. At its maximum water vapour holding capacity, the air is said to be fully saturated and the vapour pressure at this capacity is known as the saturated vapour pressure (\( P_s \)). The saturation pressure was obtained from the equation (4) using the oven temperature. Then, the relative humidity of the air in the oven was calculated using equation (5), since the vapour pressure in the oven was assumed to be equal to that in the room.

\[ P_v = \frac{P_{\text{abs}}H_a}{H_a + 0.62198} \]  
\hspace{1cm} (3)

The empirical equations relating the saturated vapour pressure and the absolute air temperature, \( T_{\text{abs}} \), for different ranges of air temperatures are expressed as:

\[ P_s = a \exp \left[ \frac{b + cT_{\text{abs}} + dT_{\text{abs}}^2 + eT_{\text{abs}}^3 + fT_{\text{abs}}^4}{gT_{\text{abs}} + T_{\text{abs}}^2} \right] \]  
\hspace{1cm} (4)

for \( 273.15 < T_{\text{abs}} < 533.15 \)
where

\[ a = 22105849.39 \]
\[ b = -27405.5258361 \]
\[ c = 97.54129373 \]
\[ d = -0.146244044 \]
\[ e = 0.1255753189 \times 10^{-3} \]
\[ f = -0.4850171032 \times 10^{-7} \]
\[ g = 4.349028978 \]
\[ h = -0.3938107141 \times 10^{-2} \]

So, for relative humidity (rh):

\[ rh = \frac{P_v}{P_s} \quad (5) \]

Finally, the equilibrium moisture content of the thin-layer grain sample for each drying test was calculated using the model proposed by Grace (1994) with the standard error of 0.0182:

\[ m_e = \frac{a_1 m_1 rh (1 - \ln(1 - rh))}{a_1 rh + 1} \quad (6) \]

where
Result and Discussions

Oven temperature and humidity tests Fig. 3 shows the variations of the relative humidity and the temperature with respect to time during drying. The result showed that the relative humidity rises for a few minutes and then decreases. The air temperature in the oven dropped for a few minutes as soon as the grain sample was inserted and became approximately constant thereafter. The reason for this is that heat was required to raise the temperature of the grain and to evaporate surface moisture. Assuming a constant air temperature seems to have significant effect only during the first few minutes of drying.

Using the relative humidity values given in the Fig. 3 the equilibrium moisture contents were calculated by the use of the Grace model (see Equation (6)). This was used to check the error that was induced by the variations in the relative humidity. Fig. 4 shows the calculated equilibrium moisture content as a function of the time. As can be seen in the Fig. 3 and 4, the relative humidity and the equilibrium moisture content follows the same pattern. The variations in the equilibrium moisture content values were found to be negligible. These tests therefore indicate that it is possible to assume the moisture added to the drying air from the thin-layer grain during drying to be negligible.

Thin-Layer Drying Models

The models presented in the table 1 were fitted to the experimental data for drying under constant conditions. The Stepwise regression analysis procedure (SAS, 1989) was used to pick out the important variables. The stepwise regression analysis did not pick any variables that did not meet 0.1500 significance level.

The REG regression analysis procedure (SAS, 1989) was then executed to check regression equations produced by
the stepwise and to try out other combinations. The principles pursued to look for the best relationships were:

- first, to find a model that maximises the coefficient of determination $R^2$ and minimise the standard error or root mean square error (root MSE),
- second, to find a model that has a capability of extrapolation beyond the experimental data range obtained in this study, which is important to model drying at low air temperatures, and
- third to find a relationship that is smooth and simple.

The final sub-coefficients for the models were found by combining the regression equations from this section with the parent models and by fitting them into the experimental data for all the runs together using the sub-coefficients as initial estimates for the optimisation program. The results of this analysis are shown in Table 1.

As an example, Figures 5 display the comparison of the calculated moisture ratio values with the experimental ones for drying wheat at 100°C with initial moisture content of 36%. The models best described the drying curve. These models have a higher number of coefficients. Again the comparison of these best fitting models showed that the Grace model came out best in describing the drying curve and there was no significant difference in standard error of moisture ratio. For the Grace and the Numerical diffusion models moisture dependent drying coefficients were possible.

As an example of the drying curve, Fig. 5 displays the comparison of the calculated moisture ratio values for the three best models with the experimental values for drying wheat at 100°C with initial moisture content of 35.9%. All three of these described the drying curve quite well.

This drying apparatus has many advantages. First, it enables the monitoring of the sample weight during drying without opening the oven door. As the result of this the drying air properties inside the oven were kept constant throughout drying. The second special feature of the drying apparatus was that during recording of the weight of the sample when the oven was turned off the sample continued to dry. This is because of the fact that even though the oven was turned off the air condition inside the drying chamber (oven) remains constant during the short time interval required for recording the sample weight. No temperature change was observed after the oven was switched on after 30 seconds. When measurement was taken the time recorded was that at which the measurement was taken rather than that at which the oven was turned off. Also the time required for recording the weight of the thin-layer grain sample was part of the drying time throughout this experimental work.
A continuous rise in relative humidity does not occur inside the oven during drying provided the ventilation port is open and the grain sample weight is small, which in this work, was about 20 grams. The variations in the equilibrium moisture content values were found to be less than 0.14 percentage points.

This apparatus is relatively simple. However, several thin-layer grain drying researchers have obtained moisture loss a data using a data logger collecting data at short time intervals throughout the drying period. In this case a data logger was not used because of the necessity to turn off the oven before taking measurements.

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Fig.1. Schematic arrangement of the thin-layer drying apparatus. Arrows show direction of air flow.

A = perforated sample tray
B = suspension frame
C = analytical balance/scales
D = draught excluding transparent box
E = hot air diversion channel
F = oven fan
G = balance stand
H = rigid support rod
J = oven chamber
Fig. 2. The two dimensional views of the hot air diversion channel.
Fig. 3. Oven temperature and relative humidity versus time for drying a wheat sample at 60°C.
**Fig. 4.** Equilibrium moisture content versus time for drying wheat at 60°C: to show the effect of fluctuation of the relative humidity on the equilibrium moisture content.
Figure 5. Moisture ratio versus drying time for thin-layer drying of wheat at 100°C and initial moisture content 35.9% (d.b.).
Table 1. Thin-layer mass transfer models, coefficients and standard error of moisture ratio.

<table>
<thead>
<tr>
<th>Model</th>
<th>coefficients</th>
<th>standard error of moisture ratio</th>
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<tbody>
<tr>
<td>Chen and Johnson model (Chen &amp; Johnson, 1969)</td>
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| \[
\frac{dm}{dt} = -k \left( m - m_e \right)^n \\
\]
| \[
k = \exp\left( a + b \ln(T_g) + cT_g + d \ln(m) \right) \\
\]
| \[
n = 1 + \exp\left( e + f \ln(T_g) + gT_g + hm \right) \\
\] | \[a = -16.44\]      | \[0.0223\]                        |
|                                                                       | \[b = 2.266\]        |                                  |
|                                                                       | \[c = -0.00483\]     |                                  |
|                                                                       | \[d = 0.3548\]       |                                  |
|                                                                       | \[e = -6.027\]       |                                  |
|                                                                       | \[f = 2.465\]        |                                  |
|                                                                       | \[g = -0.0428\]      |                                  |
|                                                                       | \[h = -15.88\]       |                                  |
| Grace model (Grace, 1994)                                             |                       |                                  |
| \[
\frac{dm_1}{dt} = -\frac{k_1'}{f_1} \left( m_1 - m_2 \right) \\
\]
| \[
\frac{dm_2}{dt} = \frac{k_1'(m_1 - m_2) - k_2'(m_2 - m_e)}{1 - f_1} \\
\] | \[a = -5.850\]      | \[0.0232\]                        |
|                                                                       | \[b = 0.0496\]       |                                  |
|                                                                       | \[c = 3.451\]        |                                  |
|                                                                       | \[d = -14.04\]       |                                  |
|                                                                       | \[e = 1.675\]        |                                  |
|                                                                       | \[f = 0.424\]        | \[0.654\]                        |
|                                                                       |                       |                                  |
|                                                                     |                       |                                  |
| Numerical diffusion model (Marchant, 1976)                            |                       |                                  |
| \[
\frac{\partial m}{\partial t} = D \left( \frac{\partial^2 m}{\partial r^2} + \frac{2 \partial m}{r \partial r} \right) + \left( \frac{\partial m}{\partial r} \right)^2 \frac{\partial D}{\partial m} \\
\]
| \[
D = \alpha \exp(\beta m) \\
\]
| \[
m = \exp(-\gamma t)(m_a - m_e) + m_e \\
\] | \[a = -32.555\]     | \[0.0268\]                        |
|                                                                       | \[b = 0.0521\]       |                                  |
|                                                                       | \[c = 2.90\]         |                                  |
|                                                                       | \[d = 3.3986\]       |                                  |
|                                                                       | \[e = 0.015022\]     |                                  |
|                                                                       | \[f = -1.0106 \times 10^{-4}\] |                                  |
|                                                                       | \[g = -0.95\]        |                                  |
|                                                                       | \[h = -9.3715\]      |                                  |
|                                                                       | \[i = -0.0033\]      |                                  |
\[ \alpha = \exp(a + bT_g + cm_o) \]
\[ \beta = \exp(d + eT_g^2 + f\ln(T_g) + gm_o) \]
\[ \gamma = \exp(h + iT_g) \]

*Optimum standard error in (brackets), if coefficients were modelled accurately.

**Source:** Chen and Johnson (1969a), Marchant (1976)