MODELING OF DIELECTRIC BARRIER DISCHARGE PLASMA ACTUATORS FOR FLOW CONTROL SIMULATIONS

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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Abstract

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Single-dielectric-barrier-discharge (SDBD) plasma actuators have shown much promise as an actuator for active flow control. Proper design and optimization of plasma actuators requires a model capable of accurately predicting the induced flow for a range of geometrical and excitation parameters. A number of models have been proposed in the literature, but have primarily been developed in isolation on independent geometries, frequencies and voltages. This study presents a comparison of four popular plasma actuator models over a range of actuation parameters for three different actuator geometries typical of actuators used in the literature. The results show that the hybrid model of Lemire & Vo (2011) is the only model capable of predicting the appropriate trends of the induced velocity for different geometries. Additionally, several modifications of this model have been integrated into a new proposed model for the plasma actuator, introducing a number of improvements.
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Chapter 1

Introduction

1.1 Background

The application of active flow control is of huge importance in the aeronautical community. Efficient flow control systems are capable of manipulating the flow to achieve desired effects such as drag reduction, mitigating noise pollution, and increasing reducing stall on an airfoil. These results typically involve techniques such as separation control and laminar-to-turbulent transition suppression.

Popular flow control methods involve the use of mechanical flaps, suction and blowing techniques, piezoelectric actuators, synthetic jets, as well as MEMS devices. Recently, the introduction of plasma actuators in the field of aerodynamics has demonstrated much promise. Plasma-based devices exploit the momentum coupling between the surrounding gas and plasma to manipulate the flow. Unlike other flow control techniques such as suction and mechanical flaps, plasma actuators require low power consumption [20], involve no moving mechanical parts, and have a very high frequency response allowing real-time control. For these reasons, the plasma actuator has become a very attractive device in the flow control community.

Plasma actuators can be sub-categorized into two major families: the corona discharge, and the dielectric-barrier-discharge. Actuation due to corona discharge actuators is typically implemented through the ignition of a plasma in atmospheric pressure through the partial
ionization of the surrounding air.

This actuator involves the placement of two electrodes, which are both exposed to the air, on the surface of a dielectric separated by a fixed distance. Typical corona discharge plasma actuator geometries can be seen in Figure 1.1. The application of a DC high voltage potential across the electrodes ignites a weakly ionized plasma that is capable of inducing flows up to 5 m/s [17]. Geometry 1.1(a) represents one of the earliest depictions of plasma actuators, in which it was demonstrated that an electric field could affect the transition point on a flat plate [28]. Unlike the other geometries, this case had no grounded electrode. The major disadvantage of the corona discharge actuators is the glow-to-arc transition that occurs at large potential differences. This transition introduces a large surge of current towards the anode, effectively creating a short circuit.

Figure 1.1: Various geometries for the corona discharge plasma actuator. HV represents high voltage source in the above figure. Figure from [17].
Figure 1.2: Single-Dielectric-Barrier-Discharge Plasma actuator geometrical configuration. $\omega$, $g$, and $h$ represent the electrode width, gap width spacing, and dielectric height, respectively.

Recently, the most commonly used plasma actuator has become the single-dielectric-barrier-discharge (SDBD). A typical configuration of the SDBD is shown in Figure 1.2, indicating the geometrical parameters of interest for its operation. Two electrodes are typically separated by a dielectric barrier – usually glass, Kapton or teflon. When a high voltage AC signal of sufficient amplitude ($5-40\text{kV}_{pp}$) and frequency (1-20 kHz) is applied between the electrodes, the intense electric field partially ionizes the surrounding air producing a non-thermal plasma on the dielectric surface. The collisions between the neutral particles and accelerated ions generate a net body force on the surrounding fluid leading to the formation of a so called “ionic wind”. The body force can be used to impart the desired flow control outcome on a given fluid system.

For the SDBD configuration shown in Figure 1.2, the momentum-coupling of the plasma and fluid induces an initial vortex that propagates downstream. This can be observed through the PIV measurements of Post et al. [21] shown in Figure 1.3. Further PIV measurements by Balcon et al. [1], shown in Figure 1.4, also confirm that the plasma actuator eventually evolves into a wall-jet directed in the downstream direction.

The major difference between the SDBD and the surface corona discharge is the presence of a dielectric barrier separating the anode and cathode in the former configuration. The dielectric barrier introduces a region of large electric breakdown strength, allowing for the application of larger potential differences and thus larger electric field intensities in the plasma region. In addition, the presence of the dielectric barrier increases the stability of
the plasma, preventing a glow-to-arc transition at typical potentials for which this would occur on the surface corona discharge.

Promising results of plasma actuators have been observed in a wide range of applications in the aeronautics community. Hanson et al. [8] were able to effectively control transient growth instabilities through the injection of a counter-disturbance into the boundary layer using plasma actuators. Separation control has also been successfully achieved for a wide range of bodies including a NACA 0015 airfoil, a cylinder and a hump in the presence of a turbulent boundary layer [2]. Other applications include the control of a rotor blade wake [13], increasing the lift on a UAV [7], and noise reduction [27].

1.2 Objectives

Due to the large number of variables involved in the design of the plasma actuator (electrode width, applied voltage, driving frequency, etc), there is a considerable level of complexity that is involved. An accurate model that is capable of predicting the output of the actuator under variations in geometry, voltage, and frequency can alleviate the current trial-and-error design approach that is typically implemented. Furthermore, optimizing the performance of the plasma actuator is most easily achieved when the underlying physics is understood through an accurate model.

Recently, there has been a flurry of models that have been proposed in the literature. Typically, however, none of these models share similar geometry, voltage, and frequency values and are difficult to compare. Additionally, limited comparisons to experiments have been performed and thus it is difficult to validate many of these models.

The work presented in this study provides a systematic study between four popular plasma actuator models. Weaknesses and strengths are evaluated of each model, and the dependence on geometrical variation is also explored. Additionally, a number of corrective measures are investigated to provide a new framework for more sophisticated model formulations.

The main objective of this work is to identify a model for the plasma actuator that
is capable of predicting an accurate flow response given a set of input parameters on the actuator geometry and operating conditions. Consequently, variations of voltage and specific geometrical parameters are investigated in terms of their effect on the maximum velocity output by the plasma actuator.

Finally, if no existing model is able to accurately predict the flow response, then modifications must be applied to the existing work in an attempt to develop a new model capable of integrating into a control system. To improve upon existing models, weaknesses must be identified and improved upon accordingly. Furthermore, validation of the new model must be performed in a similar manner as previous comparisons to evaluate the performance of the new model.
Figure 1.3: PIV measurements of ensemble-averaged velocity vectors illustrating the initial startup vortex induced by the plasma actuator. Figure from [21].
Figure 1.4: PIV measurements of velocity vectors illustrating the steady-state wall-jet evolution of the induced flow by the plasma actuator. Figure from [1].
Chapter 2

Literature Review

Currently, the design of plasma actuators has primarily been focused towards the optimization of the momentum-air coupling for maximizing the induced flow. This has been accomplished through a variety of techniques from geometrical modifications to waveform optimization, incorporating a large set of variables that contribute to the performance of the plasma actuator. These variables are primarily divided into two categories: the geometrical parameters and the electrical operating conditions.

2.1 Geometrical Parameters

The geometrical parameters define the placement of the electrodes around the dielectric barrier. They include the electrode gap, electrode thickness, electrode width and spanwise length, as well as the dielectric thickness. In a typical application, these parameters are fixed and are not typically controlled in real-time. Therefore, they are generally prescribed to optimize the actuator for the maximum possible thrust while maintaining a stable plasma discharge. Many parametric studies have been dedicated to investigating the properties of the plasma actuator’s geometrical parameters.

One of the key parameters that has a significant influence on the induced flow is the thickness of the exposed electrode. Enloe et al. [5] was the first to investigate the properties of this geometrical parameter. In their work, Enloe et al. used two sets of actuators that
differ in exposed electrode shape. The first set used a 0.08 mm copper foil tape as the exposed electrode. The thickness of the electrode was varied from 0.08 mm to 0.64 mm using additional layers of the foil tape. The second set employed steel music wire of varying diameter ranging from 0.36 to 0.98 mm for the exposed electrode. In each case, the encapsulated electrode was 0.08 mm thick and 1.27 mm wide while the Kapton polyimide dielectric barrier was 0.3 mm thick.

Enloe et al. found that despite the two different exposed electrode geometries, the gross structure of the plasma remained unchanged in all cases. Additionally, both electrode geometries exhibited a power law relationship for the dissipated power as a function of the voltage amplitude in the form of approximately $P \propto V^{7/2}$.

However, Enloe et al. further observed that the efficiency of the plasma actuator is strongly dependent on the thickness and diameter of the exposed electrode. Figure 2.1 depicts the relationship observed of the measured thrust by varying the thickness and diameter of the exposed electrode. Regardless of geometry for the exposed electrode, the thrust of the plasma actuator is shown to increase linearly as the thickness or diameter of the exposed electrode decreases.

Figure 2.1: Momentum transfer to air depends strongly on characteristic dimension of exposed electrode. Figure from [5].
Hoskinson et al. [10] further investigated the effect of the electrode thickness of the plasma actuator using a stagnation probe. For this work, Hoskinson et al. varied the electrode thickness to a minimum value of approximately 0.02 mm, a value 4x smaller than the minimum used by Enloe et al.. In addition to this, three different materials were investigated for the exposed electrode: copper (Cu), tungsten (W) and stainless steel (SS). Figure 2.2 illustrates the results of Hoskinson et al. for the induced thrust of the plasma actuator as a function of exposed electrode thickness.

![Figure 2.2: Time-averaged linear force densities parallel to the dielectric surface for single barrier actuators, inferred from stagnation probe measurements. Figure from [10].](image)

In contrast with the observed results of Enloe et al., Hoskinson et al. found an exponential relationship between the induced thrust and the exposed electrode thickness. Furthermore, the thrust was completely independent of the material used for the exposed electrode. This has implications to the chemistry-based plasma models since they are dependent on the secondary electron emission coefficient of a material which describes the induced emission of secondary particles due to collisions of incoming particles and the dielectric. The results of [5] and [10] thus suggest that changing the secondary electron emission coefficient has little effect on the induced force for the exposed electrode. To confirm the stagnation probe measurements, results were also reproduced using a force balance.
Another important geometrical parameter of the plasma actuator is the chordwise width of the encapsulated electrode. As observed by Enloe et al. [5], and Orlov et al. [18], the width of the encapsulated electrode governs the maximum extent of the generated plasma. In their investigation, Enloe et al. used a photomultiplier tube to measure the light emissions from the plasma actuator. The light emissions were then taken as a representation of the electron and ion density that is present in the plasma. Holding the operating conditions and actuator parameters constant, they varied the width of the encapsulated electrode and measured the maximum induced velocity using PIV for two voltage amplitudes. For this experiment, the width of the exposed electrode was 12.7 mm long and a 0.15 mm Kapton polyimide dielectric was used. The results of the investigation are shown in Figure 2.3. From these results, Enloe et al. observed that the extent of the plasma was limited by the width of the encapsulated electrode and would not generate more than a few millimetres past the trailing edge. Furthermore, the performance of the actuator appears to depend greatly on the extent of the plasma and, thus, the width of the encapsulated electrode.

Figure 2.3: Maximum velocity induced by the plasma actuator, for two different driving voltages, as a function of width of lower (insulated) electrode. Figure obtained from [5].
Similar results were also observed by Forte et al. using a different plasma actuator configuration. In their work, Forte et al. used a 2 mm-thick Plexiglas plate for the dielectric and a 5 mm-wide exposed electrode. The driving frequency was also 700 Hz and the voltage amplitude was 20 kV for their setup. As shown by Figure 2.4, the same monotonically decreasing relationship is illustrated as in Figure 2.3 confirming the result obtained by Enloe et al. [5].

![Figure 2.4: Maximum velocity induced by the plasma actuator as observed by Forte et al. [6]. A monotonically decreasing trend is also observed confirming the results of Enloe et al. [5]. Figure obtained from [6].](image)

Finally, the electrode separation gap also plays a significant role in the optimization of the plasma actuator. This parameter governs the chordwise distance between the trailing edge of the exposed electrode and leading edge of the encapsulated electrode. Forte et al. [6], studied the effects of the electrode separation gap on the maximum induced velocity. In their experiment, they again used a 2 mm thick Plexiglass plate for the dielectric barrier, and 5 mm-wide exposed and encapsulated electrodes. A voltage amplitude of 20 kV was driven at a frequency of 700 Hz. The electrode separation gap was varied between -5 to 15 mm to investigate the effects on the maximum velocity.

The experimental results on the electrode separation gap performed by Forte et al. can be seen in Figure 2.5. A clear maximum exists in the empirical trend at the electrode gap width of 5 mm. These results indicate that there is a fine balance between maximizing the
electric field intensity with a small gap width and creating tangentially oriented force vectors with a larger gap width. However, there is no reason to believe that this result is universal to all actuator geometries and operating conditions. Thus, more studies must be performed on the effects of actuator gap on the performance of the actuator across multiple configurations.

![Figure 2.5: Maximum induced velocity as a function of the electrode separation gap. A clear maximum can be observed at g = 5 mm. Figure obtained from [6].](image)

### 2.2 Electrical Operating Parameters

The electrical parameters relating to plasma actuators are the most important when it comes to real-time control. These parameters include the applied voltage, applied waveform and frequency. The applied voltage is directly responsible for the strength of the electric field and thus, the magnitude of the induced flow. Since the charges are constantly rearranging to cancel out the electric field, a monotonically increasing voltage is required to maintain plasma ignition. This is the primary reason why sawtooth and ramp voltage waveforms are much more effective than sinusoidal and square. Thus the type of waveform used has a significant impact on the performance of the actuator. Finally, the driving frequency controls how many times the plasma is ignited within a specific time period.

Investigations into the effect of the applied voltage have been performed by various studies
including Enloe et al. \[5\], and Forte et al. \[6\], and Thomas et al. \[26\]. The former two works investigated the effect of the applied voltage on the maximum induced velocity, while the latter work focuses on the performance of the actuator thrust vs the applied voltage.

Enloe et al. \[5\] used a 0.08 mm thick, and 1.27 cm wide copper foil electrodes in combination with a 0.3 mm thick Kapton polyimide dielectric barrier. A square and triangle waveform at 5 kHz were used to study how the waveform affects the empirical trends. The voltage range used for both cases ranged from $8\text{kV}_{pp}$ to $16\text{kV}_{pp}$, and the results were obtained via PIV measurements.

The experimental findings of Enloe et al. are depicted in Figure 2.6 for the square wave, and Figure 2.7 for the triangle wave. From both figures a power law relationship can be observed for the maximum induced velocity as the applied voltage is varied in both cases. Furthermore, it is obvious that the triangular waveform does produce a slightly larger maximum velocity output compared to the square wave. However, the functional relationship in both experiments follows the same $U_{\text{max}} \propto V^{7/2}$ power law curve, concluding that the dependence on the applied voltage is independent of the voltage waveform.

![Figure 2.6: Maximum induced velocity as a function of the applied voltage for a square wave driven at 5 kHz. Figure obtained from \[5\].](image)

Corke et al. \[2\] found a similar relationship in their experimental study. Instead of measuring the maximum velocity, the induced thrust was measured using a spring force balance. The voltage range used for this study varied between 10 kV-rms to 19 kV-rms.
Figure 2.7: Maximum induced velocity as a function of the applied voltage for a triangle wave driven at 5 kHz. Figure obtained from [5].

The results of their experiment indicate that the induced thrust varies with a $7/2$ power law relationship to the applied voltage. Although the dependent variable in this case is different than the one used by Enloe et al., the empirical trends found by both authors are in agreement.

Unlike the previous two studies, Forte et al. [6] found a contradictory result through their experiments. In their work, the maximum induced velocity was measured using a glass total pressure probe. Two actuator geometries were used in this study varying only in the dielectric material: glass, and PMMA. The dielectric thickness was 2 mm in both cases and a 5 mm width was used for both the exposed and encapsulated electrodes. The excitation frequency was fixed at 1 kHz while the applied voltage was varied between 8 kVpp to 25 kVpp. The empirical relationships for both of these cases are shown in Figure 2.9.

As shown, the results of Forte et al. [6] contradict the findings of Enloe et al. [5], and Corke et al. [2]. Instead of the $7/2$ power law relationship previously observed, Forte et al. found a monotonically decreasing function that approaches an asymptotic limit as voltage is increased. This is an interesting result and could be related to a number of causes. Firstly, the actuator geometry and measurement method is different than the one used by Enloe et al. in their study, especially the dielectric thickness which is nearly 7 times thicker in the
Secondly, the voltage range used in the former study reaches a maximum of 25 kV_{pp}, nearly twice as large as the value used by the latter two. This could reveal hidden features that may not have been captured in the lower range experiments. Finally, if the plasma reached the maximum extent in the experiment of Forte et al., it would explain the asymptotic limit since the efficiency of the actuator would drop dramatically, as discussed in Section 2.1.1.

PIV measurements were also performed in our laboratory over a voltage range of 3-14 kVpp. The results are shown in Figure 2.10 for an actuator geometry consisting of a 0.2 mm glass dielectric, 1.27 cm wide electrodes, and a 0 gap width. All cases were driven by a 2 kHz sinusoidal waveform. This figure demonstrates a linear trend and resembles a similar pattern as the experiment conducted by Forte et al. within the range of 8-14 kVpp for a glass dielectric.
2.3 Modeling

Ideally, the optimization and design of plasma actuators can be performed using a reliable model, removing the tedious trial and error procedure commonly used in applications. A model capable of predicting the induced flow must be able to reproduce the properties discussed in Sections 2.1, and 2.2. The most common way of modeling plasma actuators is to incorporate the Lorentz Force into the Navier-Stokes equations as a body force term. This procedure allows the calculation of the Lorentz Force to be independent of the Navier-Stokes equations, thus decoupling the two solutions. The primary advantage of this approach is that the calculation of the Lorentz force can be performed without adhering to the time-step restrictions that may be required to ensure a stable flow solution. Additionally, Lorentz force distributions may be independently acquired for direct comparison with thrust measurements.

Plasma actuator models can be generally classified to belong to one of two families, defined by the method in which the charge density is calculated. The first consists of chemistry based models that attempt to spatially resolve the plasma phenomena directly, and the second are algebraic models that are approximated based on the solution of Poisson’s equation. The significant difference between the two families is that the latter generally require assumptions on the behaviour of either the charge density or electric field intensity produced.
Figure 2.10: Time-averaged PIV measurements of the maximum velocity response due to voltage variations.

by the actuator.

The chemistry based family typically consists of drift-diffusion type models such as [14], [24], and [12]. These models track the chemical species present in the plasma, such as electrons and ions, using a set of transport equations. Using empirically found relationships throughout the plasma physics literature, essential features such as ionization, recombination, and streamer propagation are all modeled. Generally speaking, these models are capable of accurately resolving and predicting the plasma phenomena that occur with plasma actuator operation.

The most sophisticated chemistry model that applies to the plasma actuator is the model proposed by Likhanskii et al. [14]. In this model, three chemical species are modeled: electrons, positive ions, and negative ions. Since the number of neutral particles is typically nearly 5 orders of magnitude larger than any other chemical species, it is assumed to be constant throughout this process. The solution consists of a set of drift-diffusion based transport equations to describe the time-evolution of each species. Additionally, many of the coefficients and parameters that are found in this model are also dependent on the electric field intensity through experimentally observed relationships. These relations describe the behaviour of plasma ionization, electron recombination, drift speeds, diffusion coefficients and other parameters in the presence of a varying electric field intensity. The governing
equations are described by,

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_{ej} = \alpha |\Gamma_e| - \beta n_e n_+ + k_d n n_- - \nu_a n_e, \tag{2.1}
\]

\[
\frac{\partial n_+}{\partial t} + \nabla \cdot \Gamma_{+j} = \alpha |\Gamma_e| - \beta n_e n_+ - \beta_{ii} n_+ n_-, \tag{2.2}
\]

\[
\frac{\partial n_-}{\partial t} + \nabla \cdot \Gamma_{-j} = - \beta_{ii} n_+ n_- + k_d n n_- + \nu_a n_e, \tag{2.3}
\]

where

\[
\Gamma_{ej} = -\mu_j E_j n_e - D_e \nabla_j n_e - D_e n_e \frac{\nabla_j T_e}{T_e}, \tag{2.4}
\]

\[
\Gamma_{+j} = \mu_j E_j n_+ - D_i \nabla_j n_+, \tag{2.5}
\]

\[
\Gamma_{-j} = -\mu_j E_j n_- - D_i \nabla_j n_- \tag{2.6}
\]

The electric field is calculated on every time iteration through the solution of Poisson’s equation for the electric potential,

\[
\nabla (\epsilon \nabla \phi) = e (n_e + n_+ - n_-) \tag{2.7}
\]

\[
E_j = -\nabla_j \phi \tag{2.8}
\]

The variables \(n_e\), \(n_+\), and \(n_-\) represent the spatially-dependent electron, positive ion and negative ion concentration. Further parameters and coefficients that are required are listed in Table 2.1. The complexity of this model is further emphasized in the fact that these rates are also non-constant and depend on the electric field intensity.

Table 2.1: Definitions for the parameters used in the Likhanskii et al. model \[14\].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Rate of ionization</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Electron-ion recombination rate</td>
</tr>
<tr>
<td>(\beta_{ii})</td>
<td>Ion-ion recombination rate</td>
</tr>
<tr>
<td>(k_d)</td>
<td>Electron detachment rate</td>
</tr>
<tr>
<td>(\nu_a)</td>
<td>Electron reattachment rate</td>
</tr>
<tr>
<td>(D_j)</td>
<td>Diffusion coefficients for species j</td>
</tr>
<tr>
<td>(\mu_j)</td>
<td>Mobility for species j</td>
</tr>
</tbody>
</table>
Voltages of up to 9 kV\textsubscript{pp} for nanosecond pulses were investigated in the numerical study conducted by Likhanskii \textit{et al.} using the chemistry model defined above. The essential physics was resolved and demonstrated through the production of features such as streamer propagation and plasma quenching. Likhanskii \textit{et al.} further utilized this model to also optimize for the applied voltage waveform, concluding that nanosecond pulses with a positive dc-bias showed significant improvement in the actuator performance.

Although the chemistry model proposed by Likhanskii \textit{et al.} \cite{14} has the potential to resolve the plasma phenomena directly, the computational cost presents a significant limitation in its implementation. Requiring a spatial resolution of 25x25 \(\mu\text{m}\) for a relatively low voltage amplitude of 9 kV\textsubscript{pp}, a dramatically finer resolution is required for larger voltages. Additionally, no investigation has been performed into the solution of this model over commonly used kHz frequencies which would require an extremely large number of iterations to complete at the nanosecond time steps required for numerical stability. Finally, although the optimal waveform was found to be nanosecond pulses, this presents a high demand for expensive high voltage amplifiers and function generators that are capable of handling such large slew rates.

The above computational limits are common amongst many of the chemistry models proposed throughout the literature. The requirement for very small spatial resolution and the significant restriction on the computational time step prohibits the application of these models for typically used plasma actuator geometries and operating conditions. Because of this, the chemistry based family is not typically feasible for the design and optimization of plasma actuators and is not particularly useful for simulations of flow control.

For the above-mentioned reasons, the focus of the present study is on algebraic models, specifically the models proposed by \cite{23}, \cite{25}, \cite{19}, \cite{15}, and \cite{13}. For convenience, they will be referred to as the SJA02, SH05, OC06, MC09, and LV11 models, respectively, after the initials of each author and the date of publication. Each one provides a unique method for calculating the generated Lorentz force by the plasma, alleviating the use of very fine spatial resolutions and time steps. A significant difference between these low-order algebraic models and the drift-diffusion chemistry based models is that the former depends on assumptions on
the behaviour of either the electric field or plasma charge density. Many of these models rely on the solution of commonly used electrostatic equations such as Poisson’s equations, with an imposed modification to include transient elements. The assumptions of the algebraic models are generally also followed by the presence of one or more parameters that are calibrated to fit experimental results. We present below a brief description of the key aspects for the 4 primary models used in this work in the following sections. Further details about the models can be found in the original papers.

2.3.1 SJA02 Model

One of the first suggested plasma actuator models was the SJA02 model proposed by Shyy et al. [23]. In this study, the maximum generated electric field was assumed to be equivalent to that of a parallel plate capacitor with an equivalent potential difference across the exposed and encapsulated electrodes. A further assumption was imposed suggesting that the electric field decays linearly in the horizontal and vertical directions from its prescribed maximum value located at the trailing edge of the exposed electrode. The electric field intensity is diminished at the trailing edge of the encapsulated electrode, and at a prescribed plasma height presenting a triangular region of plasma generation as shown in Figure 2.11. Equations (2.9)-(2.12) depict the governing equations for the SJA02 model with the parameters defined in Table 2.2.

\[ E_o = \Delta V/g \Rightarrow |E| = E_o - k_1x - k_2y \quad (2.9) \]
\[ k_1 = \frac{E_o - E_b}{b}, k_2 = \frac{E_o - E_b}{a} \quad (2.10) \]
\[ E_x = \frac{E_k_1}{\sqrt{k_1^2 + k_2^2}}, E_y = \frac{E_k_2}{\sqrt{k_1^2 + k_2^2}} \quad (2.11) \]
\[ \vec{F} = \nu \rho_c e_c \Delta t \delta \vec{E} \quad (2.12) \]

There are several disadvantages of the SJA02 model that restrict its generality across all actuator geometries. First, there exists no accurate calculation of the spatial charge density
Table 2.2: Definitions for the parameters used in the SJA02 model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta V$</td>
<td>Potential difference across electrodes</td>
</tr>
<tr>
<td>$E_o$</td>
<td>Maximum Electric Field</td>
</tr>
<tr>
<td>$E_b$</td>
<td>Electric field breakdown strength</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Driving Frequency</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Charge Density</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Plasma Actuation Time</td>
</tr>
<tr>
<td>$\vec{E}$</td>
<td>Electric Field</td>
</tr>
</tbody>
</table>

Figure 2.11: The electric field intensity and plasma generation is restricted to the above triangular domain for the SJA02 model. Outside of this region, no electric field is assumed to exist. Figure from [23].

and is maintained at a constant value of $\rho_c = 10^{11}/m^3$. The plasma charge density is an important indicator of force intensity and is highly dependent on the applied electric field. Therefore, neglecting its spatial variance can have a dramatic affect on the overall force distribution and accuracy of the model. Secondly, consideration of the dielectric height is not present in the governing equations. The inclusion of this parameter is critical since it plays a significant role in the performance of plasma actuators as it governs the electric field strength, thereby affecting the ignition of the plasma. Finally, there is no temporal variation in the SJA02 model which prevents the use of alternate signal waveforms.

Despite these shortcomings, the SJA02 model provides an attractive method for calculating the Lorentz force at extremely minimal computational cost. If the parameters of the plasma actuator are not subject to variation, the SJA02 model can be combined with an appropriate scaling factor to provide a reasonable flow prediction.
2.3.2 SH05 Model

A slightly more sophisticated approach was proposed in the SH05 model by Suzen et al. [25] that involved fewer parameters while at the same time capturing more of the essential plasma physics than the SJA02 model. The model is focused on applying Gauss’ Law and solving for the electric potential via Laplace’s equation. The governing equations and parameter definitions for the SH05 model are detailed in (2.13)-(2.16) and Table 2.2, as follows

\[ \nabla \cdot (\varepsilon_r \nabla \phi) = 0, \]  
\[ (2.13) \]
\[ \nabla \cdot (\varepsilon_r \nabla \rho_c) = \frac{\rho_c}{\lambda_D^2}, \]  
\[ (2.14) \]
\[ \rho_c^{BC} = \rho_c^{max} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right] f(t), \]  
\[ (2.15) \]
\[ \vec{f}_B = \rho_c \vec{E} = -\rho_c \nabla \phi. \]  
\[ (2.16) \]

Table 2.3: Definitions for the parameters used in the SH05 model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>Electric potential</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>Charge density</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>Dielectric constant</td>
</tr>
<tr>
<td>( \lambda_D )</td>
<td>Debye length of plasma</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Shape factor for half-gaussian boundary condition</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Position of trailing edge for exposed electrode</td>
</tr>
<tr>
<td>( f(t) )</td>
<td>Excitation signal</td>
</tr>
</tbody>
</table>

The boundary conditions for the Suzen et al. model are also shown in Figure 2.12. The calculation of (2.14) requires an explicit definition of a boundary condition for the charge density. Suzen et al. thus assumed that the charge density on the dielectric surface follows a half-gaussian distribution with the form given in (2.15). The maximum value of the half-gaussian distribution is specified by \( \rho_c^{max} \) which was calibrated to \( 8 \cdot 10^{-4} \) C/m\(^3\) to match the experimental flow results of [11]. Since this value was defined for a specific actuator geometry and is thus constant, the \( \rho_c^{max} \) parameter is not able to account for geometrical
changes or voltage and frequency variation. The generality of this model when extrapolating to new geometries is thus in question.

\[ \rho = 0 \]
\[ \frac{\partial \rho}{\partial x_n} = 0 \]
\[ \frac{\partial \phi}{\partial x_n} = 0 \]
\[ \phi = V_p \sin(\omega t) \]
\[ \phi = 0 \]
\[ \rho_{BC} \]

Figure 2.12: Boundary Conditions for the SH05 model.

### 2.3.3 Enloe Circuit Model

The concept of modeling the plasma actuator as an electric circuit was first proposed by Enloe et al. [5]. The model that was suggested in this work is depicted in Figure 2.13. In this circuit the capacitor \( C_3 \) represents the constant capacitance between the exposed and encapsulated electrode. The plasma is represented by the parallel element containing the capacitive elements \( C_1 \) and \( C_2 \), and the resistive element \( R_1 \).

A key underlying feature of this model is the existence of a virtual electrode on the dielectric surface. This virtual electrode depicts the accumulation of charges on the dielectric that are prevented from reaching the encapsulated grounded electrode. The capacitance of the virtual electrode to the exposed and encapsulated electrodes is represented by the \( C_1 \) and \( C_2 \) elements and are time-dependent. Additionally, the resistance of the plasma is accounted for in the element \( R_1 \) which is also time-dependent.

The difficulty of defining the elements \( C_1 \), \( C_2 \), and \( R_1 \), however, prevents extensive application of this model. Although previous attempts at explaining the behaviour of these elements has been provided by Pons et al. [20], there has been no functional relationship defined for these values.
2.3.4 OC06 Model

The OC06 model, proposed by Orlov et al. \cite{19}, extends the implementation of the Enloe circuit model and provides closure coefficients for the capacitive and resistive elements. As shown in Figure 2.14, the OC06 model discretizes the virtual electrode into an analog circuit consisting of a series of RC sub-circuit elements which represent the time-dependent capacitance and resistance of the plasma as it extends along the dielectric surface. The circuit is governed by a set of linear ODE’s which describe the voltage on each n\textsuperscript{th} element, viz

\[ C_{an} = \frac{\varepsilon_o A_n}{l_n}, \quad (2.17) \]
\[ C_{dn} = \frac{\varepsilon_o \varepsilon_r A_d}{l_d}, \quad (2.18) \]
\[ R_n = \frac{\rho l_n}{A_n}, \quad (2.19) \]
\[ I_{pn}(t) = \frac{1}{R_n} [V_{app}(t) - V_n(t)], \quad (2.20) \]
\[ \frac{dV_n(t)}{dt} = \frac{dV_{app}(t)}{dt} \left( \frac{C_{an}}{C_{an} + C_{dn}} \right) + k_n \frac{I_{pn}}{C_{an} + C_{dn}}. \quad (2.21) \]

When the following equation is satisfied, representing an electric field intensity greater than the critical breakdown threshold,

\[ \left| \frac{V_{app} - V_n}{l_n} \right| \geq E_b, \quad (2.22) \]
Figure 2.14: The electric circuit proposed by Orlov et al. Each sub-circuit element is activated once the electric field intensity reaches the critical breakdown threshold.

The sub-circuit element is activated via the zener diode at the element $n$ and $k_n = 1$. The solution of (2.21) thus provides the voltages on the dielectric surface, $V_n$, as a function of time for each sub-circuit element. These voltages act as an additional boundary condition for the electric potential, as shown in Figure 2.15. Incorporating the new boundary condition, the electric potential is then solved through the application of Poisson’s equation in the form of

$$\nabla (\epsilon_r \nabla \phi) = \phi / \lambda_D^2$$  \hspace{1cm} (2.23)

$$\vec{f}_B = \left( \frac{\epsilon_o}{\lambda_D^2} \right) \phi \nabla \phi.$$  \hspace{1cm} (2.24)

Thus, unlike the SH05 model, the plasma charge density in the OC06 model is directly proportional to the electric potential as follows,

$$\rho_c = - \left( \frac{\epsilon_o}{\lambda_D^2} \right) \phi.$$ \hspace{1cm} (2.25)

Therefore, no a priori information such as boundary conditions about the charge density is required to provide completeness.

Orlov et al. further tailored the parameters in the circuit model such that the results would agree with experimentally observed plasma behaviour. Specifically, the critical break-
down threshold, plasma resistivity and plasma height represent empirical parameters that were defined to fit experimental results of plasma maximum extent and sweep out velocity performed by [5], and [18].

2.3.5 MC09 Model

Evaluating the OC06 model, Mertz et al. performed flow comparisons [16] on the model and discovered that the induced flow was directed in the upstream direction, contradicting observed behaviour from PIV measurements of [21], [11]. An updated version of the model was suggested by [15] by reversing the sign of the electric potential on the dielectric surface. Although this modification continues to produce a significant portion of the flow in the upstream direction, a downstream directed flow is also generated. This upstream flow behaviour, however, is again not present in the previous PIV results, and appears to be an artifact generated by the MC09 and OC06 models. Nevertheless, the MC09 model proposed by Mertz et al. presents the most up-to-date version of the electric-circuit model.

2.3.6 LV11 Model

Finally, Lemire & Vo [13] combined the SH05 and OC06 models into a single hybrid model, labeled LV11 for this study. The virtual electrode concept from the OC06 model is employed...
by the LV11 model and the definitions of the plasma height, and resistivity parameters are also retained. Calculation of the charge density, electric potential, and Lorentz force all follow directly from the SH05 model, however. The half-gaussian boundary condition represented by (2.15) in the SH05 model is replaced with the following relationship that is obtained through (2.20) from the OC06 model,

$$\rho_{BC} = \frac{I_p n \Delta t}{\Delta V_n},$$

where $\Delta V_n$ is the volume of the $n^{th}$ virtual electrode element. Figure 2.16 further illustrates the boundary conditions that are implied for the electric potential and plasma charge density in the LV11 model.

A common feature amongst each of the above low-order algebraic models is the need for empirical calibration. For example, the SJA02 model defines a charge density that is spatially constant and acts as a scaling parameter to the electric field, while the SH05 model has a similiar dependence on the maximum charge density and Debye length parameters which are specified to match the simulated flow with experiments. The OC06, MC09, and LV11 models also all share the same empirical parameters in the form of plasma height, resistivity, and critical breakdown. In these cases, the values have been prescribed to match the plasma extent and sweep out velocities measured by [5], and [18] using a photomultiplier tube. A
significant concern arises in the implementation of each of these models when extrapolating to new geometries that are different from the ones in which the calibration was performed. Since the parameters of each model has been specified based on experimental results obtained on a single actuator geometry only, the generality of these parameters is unclear.

An additional feature that has large implications on the behaviour of each model is the orientation and distribution of the body force vectors. Vectors that are oriented to be parallel to the dielectric surface impart a larger momentum to the induced wall-jet, and therefore produce larger streamwise velocities. Furthermore, vectors that possess an upstream-directed force can result in an incorrectly directed flow that is not witnessed experimentally, as observed in the OC06 model by Mertz et al. [16].

Previous comparisons of plasma actuator models has been performed by Mertz et al. [15] to identify models which are effective in an iterative design approach. Four different models were investigated that were each derived from the electric circuit model originally proposed by Orlov et al. [19]. The study focused on various integrations of the circuit model into the calculation of the electrostatic potential. The four integration approaches are classified as the one-phi with negative voltage boundary condition, one-phi with current-H boundary condition, one-phi with current-h boundary condition, and the two-phi with current-H boundary condition models.

The one-phi with negative voltage boundary condition model represents the MC09 model and applies the negative of the voltage output from the circuit model as the boundary condition for the virtual electrode. The one-phi with current-h and current-H boundary condition models both exploit dimensional analysis to produce an expression for the electric potential boundary condition from the current output of the electric circuit model. This expression is described as follows,

$$
\phi_{BC} = -\frac{I_{ps}(t)\Delta t \lambda_D^2}{\Delta V_n^2 \epsilon_o},
$$

where each parameter is described in Table 2.4. The variation of the current-h and current-H models is the difference in which the $\Delta V$ volume parameter is calculated. In the former
technique, a variable plasma height is calculated for each element through
\[ h_n = \frac{H}{n_{\text{max}}} \],
while in the latter technique a constant plasma height, \( H \), is assumed for all virtual electrode elements.

Finally, unlike the one-phi models in which the charge density is assumed to follow the following expression
\[ \rho = -\frac{\epsilon_o \lambda D}{\phi} \],
\[ \nabla \cdot (\epsilon_r \nabla \phi) = 0 \],
the two-phi model incorporates an additional calculation of the charge density through a diffusive-based equation as follows,
\[ \rho = -\frac{\epsilon_o \lambda D}{\psi} \],
\[ \nabla \cdot (\epsilon_r \nabla \psi) = \frac{\psi}{\lambda^2} \]
\[ \nabla \cdot (\epsilon_r \nabla \phi) = 0 \].

### Table 2.4: Definitions for the parameters used in equation (2.14)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{BC} )</td>
<td>Boundary condition for electric potential</td>
</tr>
<tr>
<td>( I_{pn}(t) )</td>
<td>Time-dependent current output for element ( n ) from electric circuit model</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>Computational time-step used for electric-circuit model calculation</td>
</tr>
<tr>
<td>( \lambda_D )</td>
<td>Debye length of plasma</td>
</tr>
<tr>
<td>( \Delta V )</td>
<td>Volume of virtual electrode element ( n )</td>
</tr>
<tr>
<td>( \epsilon_o )</td>
<td>Permittivity of free space</td>
</tr>
</tbody>
</table>

Comparisons on the above four models were performed through a qualitative analysis of the orientation of the body force distribution as well as examining the power law behaviour of the RMS-force over voltage variation. The significant findings of the comparison from [15] can be depicted in Figure 2.17. The results indicate that the spatially averaged RMS force from the one-phi with negative voltage boundary condition model (MC09) most accurately represents the power law \( U_{\text{max}} \propto V^{7/2} \) relationship commonly observed in the literature.
Figure 2.17: Spatially-averaged RMS body force magnitudes as a function of voltage. Figure from [15].
Chapter 3

Plasma Actuator Model

Implementation

3.1 Numerical Details

3.1.1 Lorentz Force Calculation

The calculation of the SH05, MC09, and LV11 models all involve the numerical solution of Poisson’s equation. The following discretization and iterative approach is thus used to solve the governing equations for the electrostatic potential. To represent all variations of Poisson’s equation that are used, the solution of a general form for Poisson’s equation is described. The extension to the specific forms used in each model is straight-forward.

The general form of Poisson’s equation that will be used is,

\[ \nabla \cdot (\epsilon(\vec{x})\nabla \phi(\vec{x})) = \alpha(\vec{x}) + \delta \phi(\vec{x}), \]

where \( \phi \) is a general scalar quantity, \( \epsilon \) is the spatial permittivity, \( \alpha \) is a general scalar quantity independent of \( \phi \) and \( \delta \) is 1 only when \( \phi \) is included in the charge density, otherwise 0.

For simplicity, the dependence on \( \vec{x} \) is assumed for all further calculations and is thus
dropped in the notation. Expanding the derivatives, (3.1) can also be written as

\[ \frac{\partial}{\partial x} \left( \epsilon \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \epsilon \frac{\partial \phi}{\partial y} \right) = \alpha + \delta \phi \]  

(3.2)

\[ \Rightarrow \frac{\partial \epsilon}{\partial x} \frac{\partial \phi}{\partial x} + \epsilon \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \epsilon}{\partial y} \frac{\partial \phi}{\partial y} + \epsilon \frac{\partial^2 \phi}{\partial y^2} = \alpha + \delta \phi. \]  

(3.3)

Implementing a grid transformation from \( x \) to \( \xi \) and from \( y \) to \( \eta \) such that \( x = f(\xi) \) and \( y = g(\eta) \), the equations in the new coordinates become

\[ \left( \frac{\partial \xi}{\partial x} \right)^2 \frac{\partial \epsilon}{\partial x} \frac{\partial \phi}{\partial x} + \epsilon \left( \frac{\partial^2 \xi}{\partial x^2} \right) \frac{\partial \phi}{\partial x} + \epsilon \left( \frac{\partial \xi}{\partial x} \right)^2 \frac{\partial^2 \phi}{\partial \xi^2} + \epsilon \left( \frac{\partial \eta}{\partial y} \right)^2 \frac{\partial \epsilon}{\partial \eta} \frac{\partial \phi}{\partial \eta} + \epsilon \left( \frac{\partial \eta}{\partial y} \right)^2 \frac{\partial^2 \phi}{\partial \eta^2} = \alpha + \delta \phi. \]

A central difference discretization scheme is used for the spatial derivatives and rearranged into an iterative form as shown by

\[ \epsilon_{i,j} \left( \frac{\partial \xi}{\partial x} \right)^2 \left[ \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta \xi} \right] + \epsilon_{i,j} \left( \frac{\partial^2 \xi}{\partial x^2} \right) \left[ \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta \xi} \right] + \epsilon_{i,j} \left( \frac{\partial \eta}{\partial y} \right)^2 \left[ \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2\Delta \eta} \right] = \alpha_{i,j} + \delta \phi_{i,j}. \]

The above equation can also be rewritten in a more readable form as follows

\[ \Rightarrow A \phi_{i+1,j} + B \phi_{i-1,j} + C \phi_{i,j+1} + D \phi_{i,j-1} + E \phi_{i,j} = R_{i,j}, \]  

(3.4)
where

\[
A = \frac{1}{2\Delta \xi} \left( \frac{\partial \xi}{\partial x} \right)^2 \left[ \frac{\epsilon_{i+1,j} - \epsilon_{i-1,j}}{2\Delta \xi} \right] + \frac{\epsilon_{i,j}}{2\Delta \xi} \left( \frac{\partial^2 \xi}{\partial x^2} \right) + \frac{\epsilon_{i,j}}{\Delta \xi^2} \left( \frac{\partial \xi}{\partial x} \right)^2, \tag{3.5}
\]

\[
B = -\frac{1}{2\Delta \xi} \left( \frac{\partial \xi}{\partial x} \right)^2 \left[ \frac{\epsilon_{i+1,j} - \epsilon_{i-1,j}}{2\Delta \xi} \right] - \frac{\epsilon_{i,j}}{2\Delta \xi} \left( \frac{\partial^2 \xi}{\partial x^2} \right) + \frac{\epsilon_{i,j}}{\Delta \xi^2} \left( \frac{\partial \xi}{\partial x} \right)^2, \tag{3.6}
\]

\[
C = \frac{1}{2\Delta \eta} \left( \frac{\partial \eta}{\partial y} \right)^2 \left[ \frac{\epsilon_{i,j+1} - \epsilon_{i,j-1}}{2\Delta \eta} \right] + \frac{\epsilon_{i,j}}{2\Delta \eta} \left( \frac{\partial^2 \eta}{\partial y^2} \right) + \frac{\epsilon_{i,j}}{\Delta \eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2, \tag{3.7}
\]

\[
D = -\frac{1}{2\Delta \eta} \left( \frac{\partial \eta}{\partial y} \right)^2 \left[ \frac{\epsilon_{i,j+1} - \epsilon_{i,j-1}}{2\Delta \eta} \right] - \frac{\epsilon_{i,j}}{2\Delta \eta} \left( \frac{\partial^2 \eta}{\partial y^2} \right) + \frac{\epsilon_{i,j}}{\Delta \eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2, \tag{3.8}
\]

\[
E = -\frac{2\epsilon_{i,j}}{\Delta \xi^2} \left( \frac{\partial \xi}{\partial x} \right)^2 - \frac{2\epsilon_{i,j}}{\Delta \eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2 - \delta, \tag{3.9}
\]

\[
R = \alpha_{i,j}. \tag{3.10}
\]

Finally, the solution is obtained via (3.4)-(3.10) using the Gauss-Seidel relaxation approach, via

\[
\phi_{i,j}^{n+1} = \frac{R_{i,j} - B\phi_{i-1,j}^{n+1} - C\phi_{i,j+1}^{n+1} - D\phi_{i,j-1}^{n+1} - A_{i+1,j}}{E}. \tag{3.11}
\]

The solution is considered converged when the following condition is obtained for a specified error tolerance, typically \(10^{-12}\),

\[
\phi_{i,j}^{n+1} - \phi_{i,j}^n \leq \text{err}_{\text{converg}}. \tag{3.12}
\]

A stretched spatial grid is employed for the solution of Poisson’s equation and is shown in Figure 3.1. The computational domain in both directions spans from -4 cm to 4 cm. The minimum cell width is 52 \(\mu m\) in the x-direction and 8 \(\mu m\) in the y-direction. A refinement study was performed on this grid and is depicted in Figure 3.2. For this study, an exact solution was assumed to exist on a grid with 315,000 cells which was used for the error.

The governing equations of the OC06 and MC09 models also further involve the solution of the electric circuit model, described by (2.13). For this set of equations, the solution is
Figure 3.1: Spatial grid used for the solution of Poisson’s equation for the electric potential. Plasma actuator is symmetrically centered about the origin.

integrated using the 4th order Runge-Kutta method as follows,

\[
\frac{dV_n(t)}{dt} = \frac{dV_{app}(t)}{dt} \left( \frac{C_{an}}{C_{an} + C_{dn}} \right) + k_n \frac{I_{pn}}{C_{an} + C_{dn}},
\]

\[
\beta_1 = \Delta t \frac{dV_n}{dt} (t^n, V_n^n),
\]

\[
\beta_2 = \Delta t \frac{dV_n}{dt} (t^n + 1/2 \Delta t, V_n^n + 1/2 \beta_1),
\]

\[
\beta_3 = \Delta t \frac{dV_n}{dt} (t^n + 1/2 \Delta t, V_n^n + 1/2 \beta_2),
\]

\[
\beta_4 = \Delta t \frac{dV_n}{dt} (t^n + \Delta t, V_n^n + \beta_3),
\]

\[
V_{n+1} = V_n^n + \frac{1}{6} (\beta_1 + 2\beta_2 + 2\beta_3 + \beta_4).
\]

The timestep used for the electric circuit solution in both models is \( \Delta t = 8 \cdot 10^{-7} \) s, as originally defined in [18].

### 3.1.2 CFD Implementation

The solution of the Navier-Stokes equations is obtained using the OpenFOAM CFD software suite. A transient, incompressible solver is employed for a laminar flow with no turbulence.
Chapter 3. Plasma Actuator Model Implementation

Figure 3.2: Spatial refinement study on the computational domain for the Lorentz force calculation.

models. The spatial derivatives are discretized using a second order centered difference, and the time marching scheme was implemented using an implicit second order backwards-difference.

The Lorentz force from section 3.1.1 is incorporated directly into the Navier-Stokes equations as a time-dependent volumetric body force, viz

\[
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \mu \nabla^2 \vec{u} + \vec{f}_b. \tag{3.18}
\]

Since the force distribution was solved on a different grid than the one used for the CFD simulations, it is interpolated onto the CFD grid using bilinear interpolation.

The grid used by the OpenFOAM solver is a stretched quadrilateral unstructured grid to accommodate large velocity gradients at the location of the plasma actuator. The computational domain spans from -10 to 50 cm in the x-direction, and 0 to 10 cm in the y-direction. A total of 200 points are used for the y-axis, and 250 points are used on the x-axis, for a total of 50000 cells in the domain. The minimum cell size used is 152 \times 43 \mu m, located at
the trailing edge of the exposed electrode.

The plasma actuator is centered at the origin of the grid, located 10 cm downstream from the left boundary condition. Fifty points along the x-axis are placed upstream of the actuator, and 200 are placed downstream from the origin. Since the region below the exposed electrode is filled with dielectric material, no points are used to resolve the flow in this area.

To assess the accuracy of the grid that is used for the CFD calculation, a spatial refinement study was conducted. The results are displayed in Figure 3.3. For each grid, the total momentum was integrated over the entire domain. The error was obtained by taking the difference of the integrated momentum between each grid with a refined grid consisting of 172,800 cells which was treated as exact. As the number of cells increases, we can observe a quadratic decrease in the error representing the second-order scheme which was used. A slight divergence from a quadratic can be noted most likely due to the bilinear interpolation of the Lorentz force onto the CFD grid.

![Figure 3.3: Spatial refinement study on the computational domain for the CFD calculation. Typical momentum magnitude is $O(10^{-6})$.](image)
3.2 Experimental Details

3.2.1 Experimental Setup

To fully explore the behaviour of each model across varying conditions, four separate comparisons were conducted. Among these comparisons, 4 different actuator geometries were used. The details of the geometrical setup used for each actuator is described in Table 3.1. Geometries A, B, and C are used to provide comparisons of measured vertical velocity profiles with simulations in Section 4.5. They provide actuator geometries that are similar to those used in the literature with modifications to observe the trends that arise between each geometry. Variations of voltage to the maximum induced velocity by the plasma actuator are investigated over geometry D in Section 4.6. A range of 3-20 kV$_{pp}$ is used to represent similar ranges used by Enloe et al. [5] and Forte et al. [6]. Finally, geometry E was used by Forte et al. [6] to study variations in electrode gap width which is also investigated in Section 4.7.

Table 3.1: Plasma Actuator Configurations

<table>
<thead>
<tr>
<th>Geometry</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrode Width (mm)</td>
<td>6.35</td>
<td>12.7</td>
<td>5</td>
<td>12.7</td>
<td>5</td>
</tr>
<tr>
<td>Electrode Gap (mm)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dielectric Height (mm)</td>
<td>0.19</td>
<td>0.57</td>
<td>0.18</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>Dielectric Constant</td>
<td>2.9</td>
<td>2.9</td>
<td>2.9</td>
<td>6.1</td>
<td>2.6</td>
</tr>
<tr>
<td>Applied Voltage (kV$_{pp}$)</td>
<td>12</td>
<td>15</td>
<td>10</td>
<td>3-20</td>
<td>40</td>
</tr>
<tr>
<td>Frequency (kHz)</td>
<td>3</td>
<td>3</td>
<td>2.75</td>
<td>2</td>
<td>0.7</td>
</tr>
<tr>
<td>Downstream Location (mm)</td>
<td>6</td>
<td>10</td>
<td>10</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Measurements were obtained using a 0.45 mm diameter glass pitot tube in quiescent air for geometries A, B, and C. The probe was directed to measure the streamwise velocity and mounted to a vertical traverse with a resolution of 3 $\mu$m. Each velocity data point was averaged over a sample measured at 5 kHz for an interval of 30 seconds. The sample population was recorded using a NI-USB5239 data acquisition device connected to a PC.

The downstream location of the measurement relative to the trailing edge of the exposed electrode for each actuator is specified in Table 3.1. Each excitation signal was sinusoidal
and was delivered using a Rigol DG1011 waveform generator amplified by a Trek 20/20C High Voltage Amplifier. For each case, the dielectric material that was used was Kapton film tape that contained a total thickness of 90 $\mu m$ for each layer including the adhesive. Copper tape was used for the electrodes and was 74$\mu m$ thick including the adhesive.
Chapter 4

Results and Discussion

The results in this section present a systematic comparison between four previously discussed plasma actuator models: SJA02, SH05, MC09 and LV11. We present an analysis across identical actuator geometries and operating conditions to investigate the performance of each model against experimental measurements and trends observed in the literature. Furthermore, the parameters that have been inherently calibrated for each model remain unchanged and only parameters relating to voltage, frequency and geometrical specifications are adjusted for each case. Therefore, this study further investigates the generality of these inherent parameters when applied to alternate geometries.

In this section, comparisons are made to test the performance of each model across varying conditions. The purpose of these comparisons is to evaluate the generality of each model, regardless of operating condition. Since there is a large variation in actuator configurations used in the original proposals for many of the models, the comparisons presented here are evaluated on similar geometries and operating conditions. Additionally, several features of the implementation on a few models are slightly adjusted to allow for their use on any actuator geometry.

The first direct comparison is an evaluation of the spatial force distribution of each model in Sections 4.1-4.4. From these observations, insight can be obtained into the behaviour of the fluid flow, such as an upstream flow, given the overall distribution of the Lorentz force. Furthermore, force magnitudes can be evaluated amongst each model to provide a comparison
on similar operating conditions and to identify weaknesses of certain models.

Measured vertical velocity profiles are then compared against simulations for three different plasma actuators in Section 4.5. The intended purpose of this comparison is not only to assess predictions of velocity magnitudes, but also to evaluate the capability of each model to reproduce the experimental variations when geometry is adjusted. Therefore, if rates of change can be captured correctly, then straight-forward scalar multiplication can correct for deviations in magnitude. The ability to predict an accurate response when any of the geometrical or operating parameters has been modified is a vital component when assessing a model’s generality.

Empirical trends of velocity output and force magnitudes are also evaluated in Section 4.6 over a range of voltages. Experimentally, the velocity output has been observed to follow a power law relationship of the form $U_{max} \propto V^{7/2}$ in the literature. The recovery of this trend is paramount in the successful implementation of a plasma actuator model for active flow control.

Finally, the parametric study of Forte et al.\cite{6} investigating the velocity response to electrode gap width is also reproduced in Section 4.7. Figure 2.5 illustrates the result that was found for a specific actuator geometry when electrode gap width was varied. This actuator geometry is replicated for each plasma actuator model to discover whether any models are able to recover the maximum gap width that was observed in experiments.

## 4.1 SJA02 Model

The implementation of the SJA02 model is based on the algebraic governing equations of (2.9)-(2.12). However, to allow for actuator geometries with zero gap width, the calculation of $E_o$ was modified to $E_o = \Delta V / \sqrt{g^2 + d^2}$ to avoid undefined results. Typical values for the gap width are typically one or two orders of magnitude larger than the dielectric height. Thus, for geometries which include a non-zero gap width, our modification only includes a slight change from the original formulation since $g^2 + d^2 \approx g^2$. For actuator geometries with zero gap width the above modification incorporates the dielectric height into the model to
A contour plot of the Lorentz force on geometry A for the SJA02 model with the above modification is shown in Figure 4.1. Since there is no transient dependence inherent in the model, this force distribution is constant throughout the entire actuation period. From the figure, we notice that the induced force is limited to the triangular region restricted by the encapsulated electrode width, and a prescribed maximum plasma height of 1.5 mm. The maximum force is located at the minimum distance between the exposed and encapsulated electrode and imparts a force of $180 \text{ N/m}^3$ to the surrounding fluid.

Additionally, a vector plot of the Lorentz force distribution from the SJA02 model can also be seen in Figure 4.2 for the same actuator geometry. The body force vectors are primarily oriented in the wall-tangential direction with significantly smaller component in the wall-normal direction. Since this distribution is not time-dependent, the orientation of the Lorentz force restricts the induced flow in the downstream direction, as observed in PIV flow field measurements of Balcon et al\textsuperscript{[1]} and Post et al \textsuperscript{[21]}.

Finally, it is important to note that the electric field intensity for this model is completely based on a parallel plate formulation. The maximum electric field intensity is located at the
trailing edge of the exposed electrode and approximated to equal the value of a parallel plate configuration with equivalent potential difference and gap separation. However, since the actuator geometry is highly asymmetric, the electric field intensity is not required to follow a parallel plate geometry. Furthermore, the decay of the electric field in both the x- and y-directions has been shown throughout experiments \cite{18 \cite{5} to vary exponentially and not linearly as assumed in the SJA02 model.

The flow behaviour of the SJA02 model can be observed in Figure 4.3 for the same actuator geometry and operating conditions as Figure 4.2. As can be seen, the flow replicates the behaviour that is witnessed in PIV measurements with the downstream propagation of an initial starting vortex. In this case, the strength of the Lorentz force that is predicted by the SJA02 model places the fully evolved initial vortex approximately 20 mm downstream of the plasma actuator.

While this model seems to provide several basic assumptions that may potentially provide significant inaccuracies, it is particularly attractive since it does not depend on the solution of any partial different equations. This model has been successfully implemented in several applications including \cite{3} as well as \cite{22} in which a slightly modified version was derived. As will be shown, the other models present a much higher degree of complexity in their formulations leading to a significantly higher cost of computation, requiring the iterative
Chapter 4. Results and Discussion

4.2 SH05 Model

Unlike the SJA02 model, the SH05 model employs two decoupled partial differential equations to solve for the electrostatic potential, and the plasma charge density as shown in (2.13)-(2.16). Since no time derivatives are present in the formulation of the SH05 model, the governing equations are only solved once for an applied voltage of 1. The final time-dependent solution is then scaled by $V_{app}f(t)$ for the electric potential, and $\rho_c^{max}f(t)$ for the charge density.

The maximum Lorentz force distribution for the SH05 model is shown in Figure 4.4 which corresponds to a time of $t = T/4$. Since the force is solved for an initial voltage of 1 Volt, and is scaled by $V_{app}\rho_c^{max}f^2(t)$, we only show the distribution for a single time. Comparing
these plots with the distribution obtained for the SJA02 model in Figure 4.1, we immediately notice that the magnitude of the force distribution is 55 times larger.

![Figure 4.4: Maximum Lorentz force distribution for the SH05 model on geometry A.](image)

Instead of a predetermined region of plasma activity, the SH05 model depends on the charge density, \( \rho_c \), as a spatial indicator of plasma formation. Thus, we can see that the Lorentz force distributions in Figure 4.4 depend strongly on the distribution of the charge density depicted in Figure 4.5. Therefore, a vital property of the SH05 model depends on the accurate prediction of the plasma charge density and its empirical parameters. Most notably, the maximum charge density, \( \rho_c^{\text{max}} \), must be appropriately prescribed to account for all variations in geometry and voltage.

A vector distribution of the Lorentz force for the SH05 model can also be seen in Figure 4.6. Unlike the SJA02 model, we can see that the Lorentz force is defined everywhere in space and decays very rapidly away from the trailing edge of the exposed electrode. Additionally, we see a clear direction away from the exposed electrode and towards the grounded electrode resembling the negative orientation of the electric field distribution.

The flow behaviour of the SH05 model, shown in Figure 4.7, is also similar to results witnessed in previously discussed PIV measurements throughout the literature. Although
for the SH05 model, the propagation of the initial vortex is much slower and does not extend downstream as far as the SJA02 model. This is a surprising result since the force was significantly larger in the SH05 model. The underlying cause of this result is that the force vectors for the SH05 model are primarily oriented in the wall-normal direction whereas the distribution in the SJA02 model was almost exclusively wall-tangential. The result here is important and demonstrates that a strong wall-tangential Lorentz force distribution is required to fully develop the induced wall-jet.

### 4.3 MC09 Model

Unlike the previous models, the MC09 model presents a unique modeling approach, treating the plasma actuator as an analog circuit. The solution of this model is split into two parts: the first solves the voltages inherent in the electric circuit, and the second applies those voltages as boundary conditions for the electric potential. This model depends on several equations in its formulation including a set of linear ordinary differential equations governing the behaviour of the electric circuit, and the typical electric potential solution approach
applying Poisson’s equation to calculate the Lorentz force. In addition, the electric circuit model requires several assumptions to provide a closed system for calculation. These assumptions include the definition of the capacitive elements as well as the plasma resistivity, plasma height, and electric breakdown parameters.

The solution of the governing ODEs for the electric circuit provides the voltages on the dielectric surface due to charge accumulation. Using the same geometry as in the previous two models, these voltage waveforms are depicted in Figure 4.8 for the first 5 sub-elements. Each curve represents the time-dependent voltage at the \( n^{th} \) sub-circuit element and clearly demonstrates a monotonic decrease as \( n \) increases. Using these waveforms, it is possible to calculate the time-dependent extent of the plasma using the prescribed electric breakdown, and thus restrict the generation of plasma within this boundary.

The Lorentz force is calculated in a similar approach to the SH05 model through the solution of Poisson’s equation for the electric potential. In this case, however, the charge density is defined as

\[
\rho_c = -\frac{\varepsilon_o}{\lambda_D^2} \phi
\]

within the domain \( x_{ex} < x < x_p \), where \( x_{ex} \) is the trailing edge of the exposed electrode,
and \( x_p \) is the plasma extent, and zero outside of this region. Additionally, the voltage waveforms of Figure 4.8 are implemented as boundary conditions for the virtual electrodes, and represent a spatial and temporal variation across the dielectric surface.

The logarithmic Lorentz force distributions of the MC09 model can be seen for four distinct times of a single actuation period in Figure 4.9. The extent of the plasma is clearly visible at each time presenting the domain for which the distribution is restricted to. Since the sign of the voltage on the dielectric surface is reversed, a visible barrier in the contour can also be seen indicating where the potential difference is zero. Furthermore, the magnitude of the force for this model is extremely large when compared against the SH05 and SJA02 models, presenting a force that is nearly 5 orders of magnitude larger than the SH05 model.

The time-averaged Lorentz force distribution can also be seen in Figure 4.10 using the same electrode relation as Figure 4.9. We can see that, apart from the first few tenths of a millimetre, the force distribution is primarily wall-normal, directing momentum towards the wall and the grounded electrode. The primary source of the upstream-directed flow is the
large wall-tangential force components that exist at the very edge of the exposed electrode, located at $x = -0.4$ mm.

The underlying cause of this difference in magnitude is the prescribed value for the Debye length parameter, $\lambda_D$. Since the plasma charge density is defined by (4.1), the Lorentz force can be calculated via

$$\vec{f}_b = \frac{\epsilon_o}{\lambda_D^2} \phi \nabla \phi.$$

Therefore it is obvious that the coefficient $\epsilon_o/\lambda_D^2$ has a large effect on the magnitude of the force distribution. The value used for the Debye length parameter in the MC09 model is $\lambda_D = 0.19$ mm which leads to a value of $3.06 \cdot 10^{-4}$ F/m$^3$ for the above coefficient. Incorporating this value into (4.1) and calculating the charge density at $t = 0.25$ T we obtain a maximum charge density of $0.7$ C/m$^3$, a value 875 times larger than the SH05 model. In combination with the large gradients experienced by the electric potential due to the presence of the virtual electrode, this leads to a very large and unrealistic magnitude for the Lorentz force. Therefore, to evaluate the performance of the model under geometric variation it is scaled by a constant factor of 0.25 in this study to produce velocities that are within the same order of magnitude as experiments.

Figure 4.8: Voltage waveforms for the first 5 sub-circuit elements.
However, even with the reduction in magnitude by 75% we still remain with a very large force distribution which leads to a powerful wall-jet evolution as seen in Figure 4.11. We realize that within the first 10 milliseconds, the initial starting vortex displaces a distance of approximately 8 mm, a value that took the SH05 and SJA02 models nearly 400ms and 100ms, respectively, to achieve. The steady-state evolution of the wall-jet is also reached fairly quickly in the MC09 model at approximately 100 ms. Finally, an upstream-directed flow can also be observed in the time-series evolution which has not been observed in any of the previous PIV measurements by Balcon et al. [1] and Post et al. [21].

Another significant concern of the MC09 model is that the sub-circuit elements continuously switch on and off when the model is applied for actuator geometry A. This switching
effect can be observed in Figure 4.8, where a succession of steps are visible in the waveforms due to the activation and deactivation of the capacitive and resistive sub-circuit elements. Figure 4.12 illustrates the plasma current waveforms for 4 different geometries that only differ in dielectric height. The scale on each subfigure has been adjusted to emphasize the spiking that occurs rather than the difference in current magnitude. Figure 4.12 (b) represents the initial geometry that the MC09 electric circuit model was designed on and will reproduce the results found by Mertz et al. [15]. These results demonstrate that when the dielectric height is increased, a significant amount of on-and-off switching occurs and can be seen as a series of current spikes. It should be noted that current spikes have been observed in electrical measurements of the plasma actuator which correspond to a series of micro discharges [4] [20]. However, the phenomena observed in Figure 4.12 is a result from the governing equations for the electric circuit model and does not represent any essential physics of the plasma actuator. While these spikes do not seem to affect the behaviour of the induced flow, they present an unrealistic phenomenon and are considered an erroneously generated artifact in the MC09 model.
Figure 4.11: Time-series of the velocity distribution for the MC09 model on geometry A.

4.4 LV11 Model

The foundation of the LV11 model is a combination of the electric circuit concept that was originally proposed by Orlov & Corke [19] and the electric potential and plasma charge density decoupling that is present in the SH05 model. Therefore, the LV11 model shares some of the concerns that have been previously discussed for the SH05 and MC09 models.

The major difference between the LV11 and MC09 models is the calculation of the plasma charge density. Unlike the MC09 model, LV11 applies Poisson’s equation to solve for the plasma charge density similarly to the SH05 model. Instead of the half-gaussian distribution that was imposed by the SH05 model, however, the boundary condition is calculated proportionally to the time-dependent plasma current, $I_p$, from Figure 4.12. Therefore, when the plasma is switched off ($k_n = 0$) the charge density is set to zero at that specific virtual electrode element, introducing a very discontinuous charge distribution due to the spikes in
Figure 4.12: Plasma current waveforms for 4 different dielectric height geometries. Subcircuit on-and-off switching is highly dependent on the dielectric height.

The resulting force distributions using this hybrid approach are shown for \( t = 0.25 \, T, 0.5 \, T, 0.75 \, T, \) and \( T \) in Figure 4.13. Inspecting the distributions, it is obvious that the constant sub-circuit on-and-off switching that occurs in the electric circuit model introduces a very patchy distribution for the Lorentz force. Despite the discontinuity present on the boundary, however, the overall distribution is not forced to exist in a fixed region like the SJA02 and MC09 models. Additionally, the magnitude of the induced force is smaller than the SH05 model directly indicating that the charge density on the dielectric surface is also smaller in magnitude.
Chapter 4. Results and Discussion

The time-averaged Lorentz force for the LV11 model is illustrated in Figure 4.14 for geometry A. The distribution demonstrates that all of the force vectors are directed to produce a downstream-directed flow. There is some variation on the $y$-component of the distribution which is primarily caused by the patches of isolated charge density that can be observed in Figure 4.13.

A time-series of the velocity distribution for the LV11 model can be observed in Figure 4.13. A clear evolution of the wall-jet can be observed, reaching a steady-state at approximately 400 ms. At this time, the initial vortex has displaced a downstream distance of approximately 35 mm.
4.5 Vertical Velocity Profile Comparisons

Changes in actuator geometry can have a dramatic effect on the output of the plasma actuator. Geometrical parameters such as the dielectric height have a significant role in the strength of the electric field intensity and is a major contributor to the ignition or inhibition of plasma generation. For example, a dielectric barrier that is too thick can strongly diminish the strength of the electric field and prevent significant plasma ionization. In this case, the plasma actuator acts no differently than a capacitive load. On the other hand, too thin of a dielectric barrier can cause arcing across the barrier and create a short circuit, resulting in the destruction of the actuator.

Additionally, the effect of other geometrical parameters such as the width of the encapsulated electrode, and the electrode separation gap can also present significant challenges in actuator design. The ability to predict the output of the actuator given the geometrical specifications can yield accurate and optimized actuator designs which can replace the current trial and error approach.

To evaluate the effect of the geometrical variations on the present models, vertical velocity profiles are obtained for three different plasma actuator geometries. These geometries are listed in Table 4.1 alongside the operating conditions and downstream measurement location of the measurement. Geometries were chosen to provide cases on which specific geometrical
trends could be highlighted. For example, compared to geometry A, B has a dielectric height nearly 3x in thickness as well as a larger applied voltage. Since many of the actuator models rely upon the electric field intensity and applied voltage in their calculation of the Lorentz force, this case provides an interesting test on the importance of the dielectric height in the model’s formulation. Similarly, geometry C consists of a zero electrode gap width and is an interesting test case when compared against geometry A since it highlights the importance of the electrode gap in the performance of the actuator.

Table 4.1: Plasma Actuator Configurations

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrode Width (mm)</td>
<td>6.35</td>
<td>12.7</td>
<td>5</td>
</tr>
<tr>
<td>Electrode Gap (mm)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Dielectric Height (mm)</td>
<td>0.19</td>
<td>0.57</td>
<td>0.18</td>
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<td>Dielectric Constant</td>
<td>2.9</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Applied Voltage (kVpp)</td>
<td>12</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Frequency (kHz)</td>
<td>3</td>
<td>3</td>
<td>2.75</td>
</tr>
<tr>
<td>Downstream Location (mm)</td>
<td>6</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 4.16: Comparison of numerical and experimental vertical velocity profiles on actuator geometry A. Legend: -〇- Experiment -◇- SJA02 -□- MC09 -▽- SH05 -∗- LV11

Figure 4.16 shows the vertical profiles of streamwise velocity obtained on actuator geometry A. In order to facilitate comparison, each profile is normalised by its maximum velocity, which is listed in Table 4.2. From these results, it is clear the models have difficulty replicating not only the profile shape, but also the maximum induced velocity. From the figure, the most accurate model seems to be the MC09 model since the profile shapes are nearly identical below $y = 0.5\text{mm}$. However, the maximum velocity of the MC09 model is over 4x larger than the experimental velocity. This is a clear indication that a very large force is required to generate the profile shape that is delivered by the plasma actuator.

It can also be seen that the SJA02 and SH05 models both dramatically underestimate the maximum induced velocity. In these cases, the predicted velocities for SJA02 and SH05 have an error of 63% and 89%, respectively. Finally, the only model that seems to be within an acceptable tolerance to the experimental result is the LV11 model with an error of 21%. This is a surprising result since both the MC09 and SH05 models were unable to calculate the induced velocity, yet the hybrid combination of both as seen in LV11 seems to produce a more accurate value.

The vertical velocity profiles of streamwise velocity for geometry B can be seen in Figure
Table 4.2: Maximum Velocities on Each Configuration

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
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<tbody>
<tr>
<td>Experiment (m/s)</td>
<td>1.948</td>
<td>1.277</td>
<td>1.518</td>
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<tr>
<td>SJA02 (m/s)</td>
<td>0.727</td>
<td>0.838</td>
<td>0.304</td>
</tr>
<tr>
<td>SH05 (m/s)</td>
<td>0.211</td>
<td>0.249</td>
<td>0.212</td>
</tr>
<tr>
<td>MC09 (m/s)</td>
<td>8.931</td>
<td>7.367</td>
<td>3.977</td>
</tr>
<tr>
<td>LV11 (m/s)</td>
<td>1.534</td>
<td>0.293</td>
<td>1.238</td>
</tr>
</tbody>
</table>

4.17. This geometry consists of a dielectric barrier that is 3x thicker than geometry A, an electrode width that is twice as large, and a slightly larger applied voltage. Since the dielectric thickness and applied voltage have an inverse effect on the induced force, increasing both quantities provides information on which models depend on which quantity more heavily.

From Figure 4.17, a similar shape of the profile is shown as in geometry A. The large magnitude of the force from MC09 is apparent since it is the only model capable of reproducing the narrow profile shape of the experimental measurement. The results from Table 4.2 present a different conclusion, however. Other than the MC09 model, every model dramatically under predicts the magnitude of the induced flow for this geometry.

Figure 4.17: Comparison of numerical and experimental vertical velocity profiles on actuator geometry B. Legend: -○- Experiment -◊- SJA02 -□- MC09 -▽- SH05 -∗- LV11
Chapter 4. Results and Discussion

The SJA02 model presents the most accurate prediction on geometry B with an error of 35%. However, inspecting the difference in measured velocity magnitudes between geometry A and geometry B, there is a decreasing trend due to the thicker dielectric. Despite possessing the most accurate prediction, the SJA02 model is unable to capture this decreasing trend and actually predicts an increasing velocity. This behaviour is a direct result of no strong dependence on the dielectric height in the SJA02 model.

A similar behaviour is also seen with the SH05 model when inspecting the trend from A to B. An increase in velocity is predicted by the model, yet the measurements indicate the opposite effect. Unlike the SJA02 model, the dielectric height is in fact incorporated into the formulation of the SH05 model from solving Laplace’s equation for the electric potential. However, the primary quantity that is responsible for the performance of the SH05 model is the plasma charge density, which is prescribed independent of actuator geometry.

The third geometry used for the comparison of vertical velocity profiles is listed in Table 4.1 as geometry C. The profile results are displayed in Figure 4.18, once again normalized by their respective maximum velocity. The geometrical changes of this case include a dielectric barrier similar in thickness to geometry A, and an electrode width and applied voltage that is smaller than both of the previous geometries, but no gap width between electrodes. The experimental result indicates an induced velocity that is larger in magnitude than geometry B, but less than geometry A.

A similar behaviour to the previous two geometries is once again witnessed for the profile comparison in Figure 4.11. To better understand the effect of these geometrical changes, the maximum velocities must be inspected from Table 4.1. From these results, we can see good agreement between the LV11 model and the experiment with a slight underestimation of 18%, and a large difference from the other models. The MC09 model presents an overestimation of 162%, while the SJA02 and SH05 models underestimate the measured value by 80% and 86%, respectively.

The most important behaviour, however, is the capture of the geometrical trends observed through the experiments. For geometry C, we see a larger induced velocity from geometry B, yet a smaller velocity from geometry A. The only model that is capable of reproducing this
Figure 4.18: Comparison of numerical and experimental vertical velocity profiles on actuator geometry C. Legend: -〇- Experiment -◇- SJA02 -□- MC09 -▽- SH05 -☆- LV11

trend is the LV11 model. In addition, the LV11 is also able to maintain a nearly identical velocity ratio from geometry A to geometry C with a value of 1.24 compared to a value of 1.28 from the experiment. However, a dramatic underestimation is witnessed in geometry B due to the large dielectric thickness.

While some of the geometrical trends are captured by the other models, they are unable to correctly predict the changes for all geometries. The largest velocity for the MC09 model was obtained with geometry A, and the lowest with geometry C. The maximum velocities for the SJA02 model reflect the applied voltage with the largest velocity for geometry B and smallest velocity obtained by geometry C. Finally, the SH05 model predicts a similar velocity magnitude across all three geometries, with a slight increase for geometry B where the largest potential difference was applied.

From the above analysis, results clearly show that the LV11 model appears to be the most accurate model in reproducing the trends observed by modifying the actuator geometries. However, there is still much room for improvement in the LV11 model since it was unable to predict comparable velocity magnitudes for all three plasma actuator geometries used in this study. Chapter 5 discusses modifications that can be applied to provide a more accurate description of the plasma actuator properties.
4.6 Voltage scaling

An important property of the plasma actuator is the velocity dependence on the applied voltage. This relationship has been previously studied by Enloe et al. [5], and found to follow a power law relationship in the form of $U \propto V^{7/2}$. This section investigates the capability of each model to reproduce the power-law behaviour as seen in the literature when the voltage is varied.

The range of voltages used for this study are between 3 $kV_{pp}$ to 20 $kV_{pp}$. The minimum value of 3 $kV_{pp}$ was chosen since the majority of plasma actuators presented no plasma ignition at voltages below this value. Also, the maximum voltage was limited to 20 $kV_{pp}$ since it represented a value that was similar to the maximum value used in the literature [5].

The geometry chosen for this study was widely used within our laboratory in various applications due to the high resilience of glass as a dielectric barrier. It consists of two 1.27 cm wide electrodes separated by a zero gap width, and a 0.2 mm thick glass dielectric with a dielectric constant of 6.1. The frequency was maintained at 2 kHz for the entire voltage range and driven using a sinusoidal waveform.

The induced body force dependence on the applied voltage is shown in Figure 4.19. Results for Figure 4.19 are obtained through a time-averaged integration of total force for a single cycle of the applied waveform. To facilitate comparison, the results are normalized by their respective force magnitudes at a voltage of 20 $kV_{pp}$.

The voltage scaling relationships seen for both of the SJA02 and SH05 models are linear. This is an expected result since in the formulation of these models, the voltage acts as a direct multiplicative scaling to the body force distribution.

The results for the LV11 and MC09 models, however, follow a power law relationship of the form $F \propto V^{7/2}$. A similar relationship was also seen in the study by Mertz et al. [15] using a spatially averaged and time-RMS calculation in place of the spatially integrated and time-average approach used here. An interesting result is that both the LV11 and MC09 models share nearly identical scaling relationships. Since both models directly depend on the results of the electric circuit model, this suggests that the circuit model is primarily
Figure 4.19: Time-averaged integrated force comparisons. Legend: -♦- SJA02 -□- MC09 -▽- SH05 -∗- LV11

It is difficult to predict how the flow will behave to the various force distributions. Since the Navier-Stokes equations follow a highly non-linear, coupled set of second-order partial differential equations, it is not expected that the trends for the force distributions carry directly for velocity. Therefore, a required comparison is the variation in induced velocity with respect to voltage.

To provide further comparison, Figure 4.20 shows the voltage scaling for the maximum induced velocity. Once again, results are normalized by the maximum velocity obtained at a voltage of 20 kV$_{pp}$. From the figure, the variation in the predicted velocity for MC09 and LV11 with voltage show a significant deviation from the power-law observed for the induced body force. In the case of MC09, the velocity produced by the plasma actuator follows approximately a linear relationship. This appears to be a consequence of the actual direction of the body force vector calculated by each model (not only the force magnitude).

From Figure 4.20, the only model that demonstrates a slightly non-linear behaviour is the LV11 model with a power law relationship of exponent 1.5. This behaviour is most closely related to the experimental results obtained by Enloe et al. [5] where a 7/2 power law relationship was observed, although there is still a large difference in the two exponential relationships.
The above comparisons provide several insightful points of interest into the behaviour of the plasma actuator models. Firstly, none of the models are able to capture the non-linear power law relationship that has been measured when the voltage is scaled across a range of $0 \to 20\text{kV}_{pp}$. This indicates that the key assumptions made in the development of each model may be incorrect. Furthermore, it has been observed that the time-averaged integrated force cannot be used as an indicator of the induced flow output. The response of the flow is therefore dependent not only on the magnitude of the body force, but also the spatial variation and distribution.

### 4.7 Electrode gap scaling

To further supplement comparison, each model was evaluated on its ability to reproduce the behaviour measured by Forte et al. [6], shown in Figure 2.5. The geometry used for the models in this section is replicated from [6] and consisted of two electrodes of 5mm width separated by a 2mm plexiglass dielectric. The electrical operating conditions consisted of 40 $\text{kV}_{pp}$ driven at a 700 Hz sinusoidal wave. The gap width representing the horizontal displacement between the two electrodes was varied between -5 mm, and 15 mm.
As previously discussed in Chapter 2, Forte et al. measured a peak maximum with a gap width spacing of approximately 5mm. This suggests that there is an optimal balance between electric field intensity and the orientation of the force vectors to achieve a peak maximum in velocity output. To replicate this behaviour, the above geometry was used to calculate the Lorentz force distribution for each of the plasma actuator models that has been the focus of this study. The results of the scaling are illustrated in Figure 4.21, compared against the experimental measurements performed by Forte et al. There is a large range of different behaviours depending on which model is used for the calculation.

The SH05 model has a peak maximum value located at a zero gap width. A zero gap width corresponds to a actuator geometry in which the electrode separation distance is minimized, while the length of the surface charge boundary condition is maximized. Thus, by minimizing the distance between both electrodes we maximize the electric field intensity and produce a larger body force. As we decrease the gap width from zero, we greatly reduce the length of our surface charge boundary condition, dramatically decreasing the body force. Conversely, increasing the gap width from zero decreases the electric field intensity according to the inverse square law, \( E \propto \frac{1}{r^2} \), as demonstrated in the figure.

Alternatively, the SJA02 model predicts a peak maximum at a gap width of 2.5 mm which is closest to the measured value of 5 mm. A condition that must hold for plasma ignition in the SJA02 model is

\[
E_o = \frac{\Delta V}{\sqrt{g^2 + d^2}},
\]

\[
E_b = 30,000\text{V/cm},
\]

\[
E_o \geq E_b.
\]

For values of gap width, \( g \), greater than 5 mm this condition no longer holds and no Lorentz force distribution is calculated which is the cause of a zero velocity prediction for \( g > 5 \text{ mm} \).

It appears that the presence of the virtual electrode is primarily responsible for the output response when electrode gap is varied in the LV11 and MC09 models. This can be demonstrated through the fact that both models predict similar behaviours, including a peak
maximum at $g = 12.5$ mm. As the electrode gap is increased, the Lorentz force vectors are oriented to become slightly more wall-tangential in order to point towards the encapsulated electrode. Simultaneously, however, the virtual electrode length decreases as the elements farthest away from the exposed electrode are no longer able to meet the ignition condition.

![Graph showing comparison of electrode gap width scaling vs experimental measurements performed by Forte et al. [6]]. Legend: -○- Experiment -◇- SJA02 -□- MC09 -▽- SH05 -∗- LV11

Overall, no model is capable of predicting the maximum velocity peak that was observed by Forte et al. [6] when electrode gap width was varied. The electrode gap width parameter is difficult to take into account when formulating low-order plasma actuator models since it is unknown how the plasma charge density behaves spatially. Therefore, unlike other parameters such as the applied voltage, the gap width is more subtle in its overall affect.
Chapter 5

Proposition of a New Plasma Actuator Model

Although much progress has been demonstrated in developing low-order plasma actuator models, many of the models continue to display difficulty when extrapolating to new geometries. This is a direct cause of the inherent assumptions that are required to provide a closed set of governing equations for parameters that are difficult to predict. In many cases, the most common assumption is made on the generation and distribution of the plasma charge density. Calculations of the electric field, on the other hand, can generally be performed through the application of Gauss’ Law. Specifically, Poisson’s equation for an electric potential is employed for many of these models. Attempts at describing the plasma distribution have involved various methods including the presumption of a spatially constant charge density, a half-gaussian surface density distribution, as well as modeling the plasma actuator as an electric circuit to derive the surface charge density.

From previous comparisons we have observed that the LV11 model, despite its drawbacks, most accurately captures the plasma physics when compared to the other 3 models. For these reasons, it is clear that the combination of the electric circuit model and a diffusive equation for the plasma charge density seems to provide the most accurate approach for low-order modeling of the plasma actuator.

This present chapter introduces a new low-order model for the plasma actuator inspired
by the work of Lemire et al. [13] and the LV11 model. The new model, here-in referred to as PL11, maintains the hybrid approach between the MC09 and SH05 models with a number of corrective approaches. Unlike the LV11 model, the virtual electrode is not included as an additional boundary condition to generate a more desirable force distribution. Furthermore, several of the parameters are adjusted to eliminate the on-and-off switching that is observed with the previous circuit models and to provide a much more continuous charge distribution along the sub-circuit elements. Previous implementations of the electric circuit model also included assumptions on the behaviour of the capacitive elements that took the form of ideal parallel plate capacitors. A new approach is taken in which the actual capacitance is calculated using a numerical approach. Finally, the calculation of the plasma charge density has been reformulated to include a much more rigorous integral approach.

5.1 Algorithm & Results

Similar to the LV11 and MC09 models, the proposed model employs the use of the electric circuit model for its evaluation of the surface charge density. Modifications to the approach, which can be more rigorously justified, were made to the electric circuit model. Given new insight into the problem, we build upon the previous electric circuit that has been established and used in many models. Included in these modifications is a complete reevaluation for the calculation of the capacitive elements, an integrative approach to calculating the surface charge density from plasma current, as well as deriving a new constant value for the electric field breakdown strength.

Previous implementations of the electric circuit model, such as those seen in the LV11, MC09 and OC06 models all consist of an electric breakdown condition for the activation of each sub-circuit element. This condition follows an expression of the form

$$\left| \frac{V_{app} - V_n}{l_d} \right| \geq E_{crit}. \quad (5.1)$$

The left side of this expression represents the uniform electric field that is generated
between two parallel plates with a potential difference of \( V_{\text{app}} - V_n \), and separation of \( l_d \). While the geometry of the actuator with the virtual electrode differs highly from a parallel plate capacitor, this condition provides a reasonable first-order approximation to the electric field intensity at the location \( l_d \). The calculation of the circuit model must be performed before the electric potential is solved, and it is difficult as well as computationally expensive to calculate the electric field intensity using Laplace’s equation for this condition.

For the reasons stated above, (5.1) is employed as a conditional requirement for the activation of the sub-circuit element, \( n \), located at \( l_d \). However, since the actuator geometry does not resemble a parallel plate capacitor, we do not expect the parameter \( E_{\text{crit}} \) to fully correspond with the electrical breakdown of air, which is approximately 3 MV/m. Therefore, a one-time calibration must be performed on \( E_{\text{crit}} \) to provide a result that is comparable to experiments under the assumption of (5.1). To avoid confusion with the electric breakdown strength of air, we redefine the \( E_{\text{crit}} \) parameter to \( \tilde{E}_{\text{crit}} \) in the present model.

The approach that is used to calibrate \( \tilde{E}_{\text{crit}} \) is a comparison with the maximum extent measurements performed with a photo-multiplier tube by Enloe et al. \cite{Enloe2007}. From these results, a maximum extent of 5 mm was found corresponding to an applied voltage of 10 kV_{pp}. The actuator geometry used for this experiment consisted of 1.27 cm wide copper electrodes separated by a 0.3 mm Kapton polyimide dielectric with zero gap separation.

Replicating the actuator geometry used by Enloe et al., it is possible to calibrate the value of \( \tilde{E}_{\text{crit}} \) analytically. When the sub-circuit elements are not activated, \( k_n = 0 \), and thus the governing equations become

\[
\frac{dV_n(t)}{dt} = \frac{dV_{\text{app}}}{dt} \left( \frac{C_{an}}{C_{an} + C_{dn}} \right). \tag{5.2}
\]

Directly integrating the above, we obtain

\[
V_n(t) = V_{\text{app}} \left( \frac{C_{an}}{C_{an} + C_{dn}} \right). \tag{5.3}
\]
Thus, the electric breakdown condition can be transformed as follows,

\[
\frac{V_{\text{app}}(t)}{l_d} \left[ 1 - \frac{C_{an}}{C_{an} + C_{dn}} \right] \geq \tilde{E}_{\text{crit}}.
\]  

(5.4)

The maximum possible value for \( V_{\text{app}}(t) \) is \( V_p = 5000 \) V, which is substituted into the above condition. Since the maximum extent of the plasma is 1.27 cm (the width of the encapsulated electrode) and using 100 sub-circuit elements, the value of \( l_d = 5 \) mm corresponds with the sub-circuit element of \( n = 39 \). Substituting these values for the above parameters, and using the new method to calculate the capacitance as will be described below, the breakdown condition becomes

\[
\frac{5000}{0.005} \left[ 1 - \frac{C_{a39}}{C_{a39} + C_{d39}} \right] \frac{1}{E_{\text{crit}}} \geq 1.
\]  

(5.5)

Therefore, the value for the parameter \( E_{\text{crit}} \) is found by replacing the above inequality with an equality and solving. This leads to the value of

\[
\tilde{E}_{\text{crit}} = 430 \text{ kV/m},
\]  

(5.6)

which is the new value for the ignition of plasma for the PL11 model, which follows the conditional statement

\[
\left| \frac{V_{\text{app}} - V_n}{l_d} \right| \geq \tilde{E}_{\text{crit}}.
\]  

(5.7)

The implementation of the capacitive elements between the virtual electrodes to the exposed and encapsulated electrodes is also modified in the PL11 model. Previous models assumed that these capacitors followed an ideal parallel plate formulation, and are implemented using the following well-known expression

\[
C_{pp} = \varepsilon A/d.
\]  

(5.8)
However, the parallel plate capacitor is only applicable in situations for which the cross-sectional area of the electrode is much larger than the electrode separation distance, \( A \gg d \).

The implementation of the dielectric capacitive element in (2.10) would require that the width of the virtual electrode element be much greater than the dielectric height. For a typical electrode size of 1 cm, and for \( n_{\text{max}} = 100 \) sub-circuit elements, the dielectric height must be smaller than 0.127 mm for this to be true. Inspecting the configurations in Table 4.1 which correspond to typical actuator geometries, it is clear that the virtual electrode widths are on the same order of magnitude as the dielectric heights. Thus, we cannot conclude with confidence that the parallel plate assumption is valid for the dielectric capacitive elements.

A similar analysis can be done for the air capacitive element in (2.9). In this case, the electrode separation distance is the distance between the virtual electrode element and the edge of the exposed electrode. The cross-sectional area of the parallel plate formulation also corresponds to the plasma height. Except for the first few virtual electrode elements, which are near the same order of magnitude as the plasma height, the separation distance becomes much larger than the height of the plasma. Based on these results, it is clear that the parallel plate capacitor assumption for the air capacitive element is not justified since the fringing effects in this case cannot be neglected.

To correct for this behaviour, an alternative method is implemented to numerically calculate the capacitive elements for the circuit model. In the circuit model for the PL11 model, the parameter \( C_{an} \) represents the capacitance between the exposed electrode and the virtual electrode sub-circuit element \( n \), and is represented by solving Gauss’s Law to calculate the capacitance.

The capacitance calculation is accomplished by employing Laplace’s equation for each sub-circuit element, viz.

\[
\nabla (\varepsilon_r \nabla \phi) = 0 \quad (5.9) \\
\vec{E} = -\nabla \phi. \quad (5.10)
\]

This equation is solved for each virtual electrode in a set of 10 elements evenly spaced
across the dielectric surface to accurately resolve the relationship of capacitance vs electrode separation width. Figure 5.1 illustrates 3 electrode arrangements that progress in separation width and are representative of the geometries that are inclusive in this set.

Since capacitance is solely dependent on geometry and not the potential difference, the values for the electrode boundary conditions are 1 Volt and 0 Volts for the exposed and virtual electrodes, respectively. A zero gradient boundary condition is further imposed on the outer domain.

Applying a Gaussian surface enclosing each electrode, as shown in Figure 5.2, the total charge on each electrode can be calculated using the integral form of Gauss’ Law, as follows,

\[ \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon}, \]  

\[ \Rightarrow \int_{y=y_B}^{y=y_T} E_x(x = x_R, y)dy - \int_{y=y_B}^{y=y_T} E_x(x = x_L, y)dy \]

\[ + \int_{x=x_L}^{x=x_R} E_y(x, y = y_T)dy - \int_{x=x_L}^{x=x_R} E_y(x, y = y_B)dy = \frac{Q_{\text{enc}}}{\epsilon}. \]

Then, the capacitance of the electrode arrangement can be obtained using the following
formula,

\[ C = \frac{Q_{enc}}{\Delta V}. \]  

(5.14)

For each virtual electrode element, a new capacitance value is calculated and the results are tabulated to form a relationship for the capacitance as a function of the distance from the exposed electrode’s edge, \( l_n \). This relationship is then used for the air capacitor parameter, \( C_{an} \), in the electric circuit model equations.

A comparison can be made between the original value of the air capacitor and the new modified version used in the PL11 model. Figure 5.3 depicts the capacitance of \( C_{an} \) as a function of the distance from the exposed electrode, \( l_n \), for both the original parallel plate formulation, and the modified numerical calculation.

Inspecting the results from Figure 5.3, we see a significant difference in the rate of change between the new modified air capacitor and the original version. For the first few virtual electrode elements, the capacitance is approximately equal in magnitude for both the numerical calculation and the original formulation. This is an indication that the parallel plate formulation is valid for the first few elements. However, after approximately 1 cm downstream, the two capacitance curves diverge. The rate of decrease is much greater in the original parallel plate formulation, and could potentially become a cause of error when
calculating the virtual electrode voltages and the maximum plasma extent.

Finally, including the above modifications, the governing equations for the electric circuit model for the PL11 model are given in (5.15) - (5.19). The parameter \( C_{fit} \) in (5.15) represents an interpolated curve representing the function of Capacitance to electrode gap separation, as shown in Figure 5.3.

\[
\begin{align*}
C_{an} &= C_{fit} \quad (5.15) \\
C_{dn} &= \frac{\epsilon_0 \epsilon_r A_d}{l_d} \quad (5.16) \\
R_n &= \frac{\rho l_n}{A_n} \quad (5.17) \\
I_{pn}(t) &= \frac{1}{R_n} [V_{app}(t) - V_n(t)] \quad (5.18) \\
\frac{dV_n(t)}{dt} &= \frac{dV_{app}(t)}{dt} \left( \frac{C_{an}}{C_{an} + C_{dn}} \right) + k_n \frac{I_{pn}}{C_{an} + C_{dn}} \quad (5.19)
\end{align*}
\]

To calculate the plasma charge density distribution, a boundary condition must be specified on the dielectric surface representing the charge accumulation. The SH05 model maintains a constant half-gaussian distribution that is directly scaled according to the function
of the applied waveform. However, as seen from the previous comparison study, this inhibits
the universality of the model. Alternatively, the LV11 model obtains the boundary condition
by relating the plasma current from the electric circuit model to the plasma charge density via

$$\rho_n^{BC} = \frac{I_{pn} \Delta t}{\Delta V_n}. \quad (5.20)$$

However, a significant disadvantage of the LV11 model is the incorporation of the time
step parameter, $\Delta t$, in the calculation of the charge density. This parameter corresponds to
the computational time step used to solve the linear ODE equations corresponding to the
electric circuit model, (2.13). The computational time step is used to ensure the numerical
stability and accuracy of the implemented time marching scheme, and is subject to change
for varying conditions such as geometry and applied voltage. As this value is modified,
however, the plasma current, $I_{pn}$, and the volume of the element, $\Delta V_n$ remain constant.
Therefore, the plasma charge density is directly dependent on the computational time step
and adjusting the time step will directly scale the plasma charge density proportionally.

In order to correct for this behaviour, an alternative approach was used to calculate
the charge density boundary condition in the PL11 model. Instead of relying on the com-
putational time step for the appropriate time scale, an integral method is implemented as
follows,

$$I_{pn} = \frac{dQ_{pn}}{dt}, \quad (5.21)$$

$$\int_{t_o}^{t_f} I_{pn} dt = \int_{t_o}^{t_f} \frac{dQ_{pn}}{dt} dt, \quad (5.22)$$

$$Q_{pn}(t = t_f) - Q_{pn}(t = t_o) = \int_{t_o}^{t_f} I_{pn} dt. \quad (5.23)$$
Using a value of $t_o = 0$ and $Q(t = 0) = 0$, then

$$Q_{pm}(t) = \int_0^t I_{pm} dt. \quad (5.24)$$

Thus, the value of the total current charge on the $n^{th}$ virtual electrode element is the sum of the transferred charge from initialization to the specified time. The above approach presents a method that is independent of the computational time step. This allows the time step to be adjusted according to the stability criteria of the numerical scheme and the resulting charge density will remain unchanged.

To further evaluate the implications of the new calculation method, a comparison is made between the two charge density calculations of (5.20) and (5.24) in Figure 5.4. The figure presents a comparison of the surface charge density calculated via (5.20) in Figures 5.4 (b) and (d) to the calculation method of (5.24) in Figures 5.4 (a) and (c). For these results, geometry A from Table 3.1 was used with a voltage of $12 \text{kV}_{pp}$ and a frequency of $3 \text{ kHz}$.

There are three key results that can be observed from Figure 5.5. The most significant change is the difference in the order of magnitude for the plasma charge density values. In subplot (b), the maximum value of the charge density can be seen to be nearly $6 \mu\text{C/m}^3$. Incorporating the new calculation of (5.24) into the circuit-model presents a charge density peak maximum of approximately $500 \mu\text{C/m}^3$, a value nearly two orders of magnitude larger.

The second observation that can be made is the $90^\circ$ phase change between the two waveforms in subplots (a) and (b). The applied waveform for all the cases within this study has been a sinusoidal function. Due to the capacitive elements that exist throughout the circuit, the plasma current has consistently been $90^\circ$ out of phase with the applied voltage waveform. Mathematically, any current may be expressed as the time rate of change of the charge at that location, specified by

$$I_{pn} = \frac{dQ_{pm}}{dt}. \quad (5.25)$$

Experimentally, when a sinusoidal waveform is driven the current has been shown to be out
of phase with the applied voltage [20]. Thus, from (5.25) the charge density must also be out of phase with the current and therefore in phase with the applied waveform. The only method that meets this criteria is the calculation of (5.24) in subplots (a) and (c).

Additionally, the variation of the computational time step is further analyzed for both calculation methods. Subplots (a) and (c) depict the same calculation using (5.24) with a computational time step that varies by a factor of 2. Similarly, subplots (b) and (d) employ (5.20) with the same variation in computational time step. Comparing these results, we notice that the surface charge density of the LV11 model using (5.20) is directly proportional to the computational time step, while the new approach using (5.24) in the PL11 model is independent of this variation.

Figure 5.4: Surface charge density for the LV11 and PL11 models for two different computational time steps using identical actuator geometries.
The force distribution for the PL11 model can be seen in Figure 5.5 for geometry A in Table 1. Comparing this result with the distributions of the MC09 and LV11 models from Figures 4.6 and 4.8, we notice that the PL11 model provides a much more continuous and uniform force distribution. The patchy distribution seen in the LV11 model is no longer present due to the new reformulation of the plasma charge density.

Furthermore, we notice that the maximum force distributions on the PL11 model exists at times $t=0.25 \, T$, and $t=0.75 \, T$. These times represent the maximum voltage value that is delivered on the sinusoidal signal, corresponding to $+V_p$ and $-V_p$, respectively. This distribution is also out of phase with the distribution calculated from the LV11 model. This can be attributed to the difference in methods for the calculation of the charge density which was previously shown be out of phase. Finally, the magnitude of the Lorentz force in the PL11 model is an order of magnitude greater than the LV11 model. Unfortunately, since

Figure 5.5: Filled contour field of the logarithmic force field at 0.25 T time intervals of a single period for the PL11 model.
limited information exists on the force distribution response caused by plasma actuators, direct comparison with experiments is difficult in this case.

A time series of the induced flow can be observed in Figure 5.6. A qualitative analysis on these results shows the expected propagation of the initial vortex in the downstream direction, as compared to the PIV results shown in section 2 by [21] and [1]. The wall-jet becomes steady-state after approximately 400 ms. Additionally, there is no generation of an up-stream directed flow, adhering to the PIV observations previously mentioned.

![Figure 5.6: Flow comparisons of the PL11 model with two vertical velocity profile measurements.](image)

Nevertheless, the PL11 model provides a framework that is similar to the LV11 model with several improvements. The major advantage of the PL11 model is a more robust formulation that is devoid of the on-and-off switching that is present in previous adaptations of the electric circuit models seen in the MC09 and the LV11 models. Additionally, the model incorporates the exact values for its capacitive elements rather than relying on parallel plate assumptions that have been demonstrated to inaccurately represent the capacitive elements present in previous electric circuit adaptations.
5.2 Experimental Validation

The validation of the PL11 model is accomplished through a similar approach as in Chapter 4 for the previously discussed actuator models. Vertical velocity comparisons using the experimentally measured profiles for geometries A, B, and C from Section 4.5. Additionally, validation of the velocity response as a function of voltage is also performed against relationships found within the literature. Finally, the electrode gap scaling results of Forte et al. [6] are reproduced in a similar fashion in as Section 4.7.

Figure 5.7 depicts the vertical velocity profiles of the PL11 model compared against the measured values for geometries A, B and C in Table 3.1. From these results, we see an underestimation of the measured profiles in all cases. The profile shape of the flow response is also much wider than the experimental case, however, they are similar to the results for the LV11 model in Figures 4.16-4.18. The PL11 model is able to produce a stronger force for geometry B than the LV11 model, however it under predicts the result in geometry C where as the LV11 model was able to capture this response. The important decreasing trend from geometry A to geometry B is preserved, as well as the trend from geometry A to geometry C. However, the PL11 model is unable to account for changes in geometry from B to C.

The electrode gap scaling of the PL11 model is evaluated on the actuator geometry used in Forte et al. [6]. Results are shown in Figure 5.8 for the PL11 model and the experimental measurements. In this case, the PL11 model was able to capture the peak maximum velocity observed by the experiment, unlike any of the previously discussed models. Additionally, there is a dramatic change from the results of the LV11 model in Figure 4.21, where the peak maximum was observed at a gap width location of 12 mm. The PL11 model also produces a decreasing trend from the maximum value which is also witnessed in the experiment. Despite the flow response mismatches in Figure 5.7 over the three set geometries, the PL11 model is able to predict a very accurate result over electrode gap variations.

The integrated force and velocity response for the PL11 model over voltage variations is demonstrated in Figures 5.9 and 5.10, respectively. The integrated force is calculated by spatially integrating the force and time-averaging for one cycle. Its response exhibits
Figure 5.7: Flow comparisons of the PL11 model with two vertical velocity profile measurements.

a slightly non-linear behaviour with the functional relationship of $F \propto V^{3/2}$. Observing the velocity response in Figure 5.10, we immediately notice a linear output indicating that the force response does not directly carry over. Thus, we can induce that the spatial force behaviour has a significant impact on the flow result.

The results in this section demonstrate that the performance of the PL11 model introduces several improvements in certain aspects of the plasma actuator. Specifically, the instability of the surface charge previously seen in the LV11 model has been eliminated and replaced with a much more rigorous approach. This new approach has been demonstrated to present stable results and has also been shown to be independent of the computational time step chosen.

The electrode gap variation of the PL11 model when compared against experimental results of Forte et al. [6] has shown very promising results. The location of the maximum velocity was matched, and similar increasing and decreasing trends were also reproduced. This shows significant improvement from the previous results in Section 4.7, in which none
Figure 5.8: Electrode gap scaling comparison of the PL11 model to experimental results performed by Forte et al. [6].

of the models were able to accurately predict the maximum velocity location.

Finally, there are some limitations to the PL11 model in that the correct velocity magnitudes were not reproduced when compared against our set of actuator geometries. In general, the PL11 model demonstrated an under prediction of the velocity, and was unable to capture the increase observed in experiments from geometry B to geometry C. Further modifications to the plasma resistivity and the spatial charge density calculation could lead to significant improvements to these limitations.
Figure 5.9: Response of integrated force over a voltage range of 3-20 kVpp. Actuator geometry is 0.2 mm glass dielectric, 1.27 cm wide electrodes and driven at 2 kHz sinusoidal waveform.

Figure 5.10: Response of maximum velocity over a voltage range of 3-20 kVpp. Actuator geometry is 0.2 mm glass dielectric, 1.27 cm wide electrodes and driven at 2 kHz sinusoidal waveform.
Chapter 6

Conclusions and Future Work

6.1 Conclusions

Plasma actuators have become an increasingly popular and very promising device in the fields of active flow control. The implementation of these devices have been demonstrated to yield very powerful results in applications of separation control [2], bypass transition control [9], noise mitigation [27], as well as preventing stall on an UAV [7]. To effectively develop a feedback control system integrating plasma actuators, an accurate model of the induced output flow must first be acquired.

A comparison of four popular plasma actuator models was presented over a set of five different actuator geometries in this study. The models used were labeled SJA02, SH05, MC09 and the LV11 after their original authors in [23], [25], [15], and [13], respectively. Together, they formed a wide variation of modeling techniques including electric circuit analysis, algebraic approximation, and diffusive approximations. The major challenge in all of these models is effectively capturing the behaviour of the plasma charge density without extensive modeling of the fundamental chemical reactions that occur. As a consequence of this, an inherent property of each of these models is the existence of experimental parameters that require calibration.

Several methods have been employed in the comparison of the above four models. Vertical velocity profiles were measured for a set of three actuator geometries and compared against
CFD results for each model. Additionally, the variation of voltage and electrode gap on the maximum induced velocity was investigated which also included experimental observations from the literature. The results of the comparison demonstrated that none of the models is able to predict flow results that match experimental behaviour over the entire set of geometries that were applied.

Finally, a new plasma actuator model, labeled PL11, has been proposed which relies upon several significant modifications on the existing electric circuit model. These modifications include a complete reformulation of the capacitive elements in the electric circuit model, a new integrative approach to calculating the surface charge density on the dielectric barrier, and a new value for the electric field strength required for plasma ignition.

The validation of the PL11 model against experimental vertical velocity profiles reveals that the model under predicts the magnitude of the induced maximum velocity. However, the PL11 model eliminates the instabilities that are seen in the LV11 and MC09 models at large dielectric heights. Furthermore, when varying the electrode gap, results of the PL11 model indicate that the correct velocity response is predicted when compared against experimental findings.

6.2 Future Work

Although the PL11 model showed significant promise in its results, there are still a large number of cases that must be validated before its implementation in a feedback control system. Most importantly, frequency variations must be investigated since these two parameters represent the input of the control system.

Additionally, validation of the PL11 model must be performed on a set of other geometrical parameters such as the dielectric height and electrode width. Unfortunately, since no collection of parametric studies for all of the plasma actuator’s parameters exists, it is difficult to perform these validations without a large database of supplemented experimental measurements. A database of experimental measurements could also be used to identify strengths of the previously discussed models, which may be further implemented in future
revisions of the PL11 model.

Finally, the under prediction of the velocity magnitude must be addressed in future revisions of the PL11 model. Possible causes of this under prediction could be the volume approximation of each sub-circuit element, as well as an incorrect specification for many of the inherent properties of the model such as the Debye length, plasma resistivity, and the critical electric field breakdown.
Bibliography


