Abstract

Essays on Banking, Institutions, and Macroeconomic Activity

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This thesis investigates the role of institutions in shaping macroeconomic phenomena. The first two chapters focus on financial institutions, formalizing interactions between information and competition in frictional credit markets and providing novel predictions for output and efficiency. The third chapter then presents a new approach for empirically assessing the relationship between political institutions and growth.

In Chapter 1, I construct a credit-based model of production to analyze how learning through lending relationships affects the monetary transmission mechanism. I examine how monetary policy changes the incentives of borrowers and lenders to engage in relationship lending and how these changes then shape the response of aggregate output. A central finding is that relationship lending induces a smoother steady state output profile and a less volatile response to certain monetary shocks. This result provides a theoretical basis for cross-country transmission differences via a relationship lending channel.

In Chapter 2, I investigate financial sector inefficiency when banks divide resources between attracting clients and learning about them via screening. I show that banks do not fully internalize the effects that their allocation decisions have on the beliefs and outside options of other lenders. These externalities result in an inefficiently high amount of low-quality credit and thus motivate a tax on activities designed to attract rather than screen borrowers. Steady state results suggest that production exhibits a hump-shaped response to increases in this tax and the model’s dynamics indicate that a mild tax can also attenuate business cycle fluctuations.
Chapter 3 then turns to the interaction between political institutions and economic outcomes. In collaboration with Gordon Anderson, I use a notion of distributional dominance to evaluate intertemporal dependence between polity and growth without hindrance from the mix of discrete and continuous variables in our data set. We also use this notion to measure the joint contribution of polity and growth to wellbeing. The results support the view that institutions promote growth more than growth promotes institutions. They also suggest that polity has dominated growth in determining the evolution of wellbeing over the past few decades.
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To my parents and my sister.
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Chapter 1

Relationship Lending and the Transmission of Monetary Policy

1.1 Introduction

Macroeconomists have shown how intermediation costs can propagate shocks when lenders are imperfectly informed about their borrowers. Commonly omitted from the analysis though is the potential for lenders to learn about their clients through repeated interactions. This omission is problematic because recent empirical evidence suggests that there is a link between relationship lending and the transmission of monetary shocks. As defined in Boot (2000), relationship lending is the provision of credit by intermediaries that acquire proprietary information about their borrowers over time or across products. Among the major European economies, Ehrmann et al (2001) establish that relationship lending is much more prevalent in Germany and Italy than in Spain and France. Incidentally, they also find that the quantity of bank loans responds less severely to a monetary contraction in the first two countries. Borio and Fritz (1995) find a similar pattern on the pricing side, with the pass-through from higher policy rates to higher loan rates occurring more slowly in Germany and Italy than in Spain.\footnote{Additional support for the impact of relationship lending is provided for Italy by Gambacorta (2004) and for Germany by Weth (2002) and Iacoviello and Minetti (2008). Moreover, based on U.S. survey data, Berger and Udell (1995) conclude that American borrowers with larger banking relationships tend to pay lower interest rates and are less likely to pledge collateral.} A correlation with spending is also visible as Mojon and Peersman (2003) demonstrate that the peak decline in investment following a monetary contraction is smaller in Germany and Italy than in Spain and France.
In this paper, I establish a theory of how relationship lending affects the macroeconomic response to monetary policy. I begin by constructing a credit-based model of production where lenders can uncover private information about their borrowers’ abilities over time. While such learning decreases the informational asymmetry between a lender and his borrower, it introduces one between the lender and competing financiers so the extent and effect of relationship lending must be endogenously determined. I undertake this determination and analyze how monetary policy changes the incentives of borrowers and lenders to engage in such relationships by changing the cost of funds on the interbank market. I then examine how the response of aggregate output depends on the response of relationship lending. In contrast to many models of financial acceleration, I show that relationship lending prevails in equilibrium and dampens the canonical credit channel. The prevalence of such relationships depends not only on the policy rate but also on institutional parameters so, given differences in these parameters, cross-country differences in relationship lending and monetary transmission are supported.

The analysis uncovers two mechanisms through which relationship lending affects monetary transmission. The first mechanism operates during the credit relationship while the second operates beforehand. As lenders acquire information over the course of their relationships, they retain only sufficiently good borrowers. The presence of other lenders limits monopoly power and, in order to induce higher repayment rates, it is optimal for informed lenders to concede positive surplus to some of their borrowers. I demonstrate that this concession includes offering policy-invariant credit terms over intermediate ranges of the policy rate, giving rise to a first mechanism. To be sure, informed lenders do not concede the entire surplus from relationship lending and, in anticipation of future relationship profits, unmatched lenders compete more intensely for new borrowers. A second mechanism arises because this competition lowers loan rates for any given policy, alleviating some of the tightness that information frictions may impart on first-time borrowers without actually changing these frictions in the first period. At an aggregate level, the two mechanisms combine to produce a smoother steady state output profile and a less volatile response to
certain monetary shocks.

The importance of financial intermediation for real activity has been emphasized in the macroeconomics literature.\(^2\) However, in analyzing how credit markets transmit shocks to the real economy, macroeconomics has essentially discounted the propensity of agents in these markets to engage in relationship lending: Williamson (1987), Bernanke and Gertler (1989), and Kiyotaki and Moore (1997) abstract from multi-period credit relationships while Gertler (1992), Khan and Ravikumar (2001), and Smith and Wang (2006) abstract from the learning benefits of such relationships. In contrast, the key feature of multi-period lending relationships in my model is learning and, in particular, the informational advantage of an inside lender over all other lenders. Indeed, I assume that agents cannot commit ex ante to long-term contracts, making multi-period lending relationships a sequence of one-period arrangements whose benefits are derived solely from the possibility of lender learning.\(^3\) To the extent that I emphasize relationship lending, this paper is also related to the banking literature and, in particular, work by Schmeits (2005) and Van Tassel (2002) on the properties of these relationships. However, neither study investigates how policy rates affect the resulting contracts or how these contracts then transmit shocks to the macroeconomy, two questions which are key components of my analysis.\(^4\)

The rest of the paper proceeds as follows: Section 1.2 describes the environment in more detail; Section 1.3 characterizes the optimal credit decisions; Section 1.4 determines the resulting output functions; Sections 1.5 and 1.6 discuss the output implications of relationship


\(^3\)Multiple periods are important here both because they permit learning and because learning has long-term implications. This contrasts with the growth model of Bose and Cothren (1997) where lenders invest in learning about borrowers but the information acquired cannot be used in future contracts since agents are two-period-lived overlapping generations who only enter into credit contracts in their first period.

\(^4\)To this end, I also extend the oft-used two-type banking environment to a continuum of borrower types, permitting non-degenerate lender beliefs and continuous output functions. Moreover, the first period credit market in my model is competitive, borrowers can choose a different project each period, and all lenders can condition their second period loan rates on first period default history. While some of the last three features are present in either Schmeits (2005) or Van Tassel (2002), neither paper contains all three. Combining these elements allows me to explore more avenues through which relationship lending can influence real activity.
lending; and Section 1.7 concludes. Figures are presented at the end of the chapter and all proofs and derivations are collected in Appendix A.

1.2 Environment

Time is discrete. All agents are infinitely-lived, risk neutral, and have discount factor \( \beta \in (0, 1) \). There is a continuum of firm types, denoted by \( \omega \) and distributed over the interval \([0, 1]\) according to a non-degenerate probability density function \( f(\cdot) \). All firms have access to the same production technologies: an investment project called \( P1 \) and a speculative project called \( P2 \). Types are private information and high-\( \omega \) firms are better in the sense that they are more likely to operate the investment project successfully. In particular, a type \( \omega \) who operates \( P1 \) is able to produce \( \theta_1 \) units of output with probability \( p(\omega) \) and zero units with probability \( 1 - p(\omega) \), where \( p : [0, 1] \rightarrow [0, 1] \) is a continuously differentiable and strictly increasing function. In contrast, the outcome of the speculative project \( P2 \) is independent of firm type, yielding \( \theta_2 \) with probability \( q \) and zero with probability \( 1 - q \). Assume \( \theta_2 > \theta_1 \) and \( q\theta_2 = p(0)\theta_1 \) so that the speculative project is riskier in the sense that it is second order stochastically dominated by the investment project.\(^5\) The presence of a speculative outside option allows the credit contracts described below to affect real activity by changing the relative attractiveness of safe projects.

To undertake either project, firms need one unit of capital. Project output is not storable so this capital must be borrowed from a measure of ex ante identical lenders that also populates the economy. Lenders cannot operate the production technologies but they have access to an interbank market for capital. The interest rate in the interbank market is denoted by \( r \). It is affected by central bank monetary policy and enters the model as the lenders’ cost of funds.\(^6\) To simplify the analysis, I assume that \( r \) is exogenous and refer to it directly as the policy rate.

Lenders also have an ability to learn about the borrowers they lend to. In particular, a

\(^5\) \( p(0)\theta_1 > q\theta_2 \) requires more algebra but yields similar conclusions.

\(^6\) Essentially, lenders who need more capital can obtain it at the interbank rate while lenders who have enough capital interpret the rate as an opportunity cost.
lender who has provided credit to a borrower in the past knows more about that borrower than do all the other lenders. Label the informed or relationship lender an insider and the other lenders outsiders. I abstract from the process through which insiders acquire information, summarizing it instead by a positive probability of type discovery. Outsiders are not privy to the information gathered by insiders. To avoid situations where insiders design credit contracts to distort the beliefs of outsiders, assume that outsiders are also not privy to an insider’s offer when they make their own offers. They do, however, find out if a borrower defaulted on a past loan and can revise their beliefs about the borrower’s type conditional on this information.\footnote{This is the only cost of default in the model. If the borrower is also forced to wait a few periods before his next contract, the marginal type that chooses the risky project may fall but the qualitative conclusions of the model are unlikely to change.}

Firms borrow from one lender at a time while lenders can take on more than one borrower. Agents cannot commit to long-term contracts, inducing a sequence of one-period credit arrangements. This limits the scope for intertemporal incentives à la Townsend (1982) and keeps the focus on the learning property of relationship lending.\footnote{Note that the scope is limited but not necessarily eliminated. In the environment presented in Appendix A.2, for example, first period defaulters end up with higher loan rates the next time around so there is effectively an intertemporal punishment for choosing the riskier project. However, the punishment is not complete since uncommitted borrowers can switch to another lender and this new lender may not find it optimal to punish default as much as the original lender would have liked.} Contracts are characterized by a loan rate so there is no quantity rationing in the model.

There are two notions of time: \textit{dates} \(t = 1, 2, \ldots\) denote calendar time in the general economy while \textit{periods} \(k = 1, 2, \ldots\) denote time spent by a borrower in the credit market. As will be described at the end of this section, borrowers can be exogenously separated from the credit market so the two notions of time are distinct.

At any date \(t\), first-time (i.e., \(k = 1\)) borrowers enter the market with private information about their types. All lenders have the same priors about a new borrower so this borrower chooses randomly among perfectly competitive lenders. Based on his type and the loan rate charged, the borrower then chooses which project to undertake. At the end of date \(t\), project outcomes are realized and debts are settled. Capital is not destroyed in the production
process so the borrowed unit is always recovered by the lender. Interest payments, however, can only be made by borrowers with successful projects. Lenders cannot observe the exact output of a project but can detect whether their borrowers have positive consumption so borrowers settle interest payments if and only if their projects are successful. Default occurs when a borrower cannot pay interest. Since this happens when his project yields no output, the probability of default is just the probability of project failure.

Also at the end of date $t$, a lender learns the type of his first-time borrower with probability $\phi \in (0, 1]$. To simplify the exposition, I consider $\phi = 1$ here. All other lenders learn whether or not the borrower defaulted on his first period loan. The borrower then moves to date $t + 1$ and becomes a second-time (i.e., $k = 2$) borrower. Conditional on their cost of funds and their beliefs about the borrower’s type, lenders set their second period loan rates simultaneously.\(^9\) Since insiders and outsiders have different information sets, the borrower no longer chooses among perfectly competitive lenders. After receiving all offers, the borrower decides which contract to accept and which project to then undertake. Once again, project outcomes are realized and debts are settled.

At the end of date $t + 1$, the types of all second-time borrowers are made public. This is done to avoid carrying credit history throughout the model and, therefore, to keep the state space finite. Also starting at the end of the borrower’s second period is a positive probability, $\mu \in (0, 1)$, of exogenous separation from the credit market. This separation eliminates all information about the borrower and requires that he draw a new type and re-enter the market as a first-time borrower in $t + 2$. In contrast, borrowers that are not separated become third-time borrowers in $t + 2$. As before, loan rates are determined, lenders and projects chosen, outcomes realized, and debts settled. Borrowers who do not survive separation at the end of their third period must start anew while borrowers who do survive it become fourth-time borrowers in $t + 3$ and face the same environment they did in $t + 2$. The market continues in this way and, even though information is revealed after

\(^9\)As we will see in Section 1.3, the type-independent nature of $P^2$ allows for a pure strategy equilibrium in the simultaneous game between second period lenders.
two periods, the possibility of exogenous separation ensures that there are always first-time, second-time, and advanced (i.e., $k \geq 3$) borrowers at any date $t$.

The three borrower classes can be interpreted as an approximation of the borrower life cycle. When a borrower first enters the credit market, little is known about him so $k = 1$ reflects the market for new borrowers. In contrast, after sufficiently many realizations of the borrower’s credit history, all lenders can form reasonably precise beliefs about his type so $k \geq 3$ approximates the market for established borrowers. $k = 2$ captures the intermediate market: after enough time has elapsed for lending relationships to inform insiders but before enough time has elapsed for credit history to inform outsiders.

### 1.3 Optimal Decisions

I now characterize the optimal decisions for each period of the credit market. In what follows, $J_k (r|\Omega)$ denotes the value of a lender with cost of funds $r$ and information set $\Omega$ about a period $k$ borrower. The optimal loan rate offer is then denoted by $R^*_k (r|\Omega)$. It will be convenient to study the optimization problems recursively, starting with the market for advanced borrowers.

#### 1.3.1 Lenders with Advanced Borrowers

Since the types of advanced borrowers are public, the problem for $k \geq 3$ is one of perfect information. Project choice does not affect future outcomes as borrowers either start anew with exogenous probability $\mu$ or continue to period $k + 1$ with exogenous probability $1 - \mu$. Consequently, each borrower will choose the project that yields him a higher expected return in the current period. A tradeoff arises, however, since $P1$ generates more expected revenue but also increases the likelihood of interest payments. At high loan rates then, the borrower may have an incentive to choose the riskier project. Formally, the borrower’s optimal strategy is characterized relative to a threshold loan rate. For a type $\omega$ borrower, the one-period return to $P1$ is $p(\omega)[\theta_1 - R]$ and the one-period return to $P2$ is $q[\theta_2 - R]$. The loan rate that makes him indifferent between the two projects is:
\[
\overline{R}(\omega) = \frac{p(\omega) \theta_1 - q \theta_2}{p(\omega) - q}
\]  

(1.1)

where \( \overline{R}(0) = 0 \) and \( \overline{R}'(\omega) > 0 \). Type \( \omega \) borrowers thus choose \( P1 \) if charged \( R \leq \overline{R}(\omega) \) and \( P2 \) otherwise. I summarize this strategy as follows:

\[
\pi(R|\omega) = \begin{cases} 
  p(\omega) & \text{if } R \leq \overline{R}(\omega) \\
  q & \text{if } R > \overline{R}(\omega) 
\end{cases}
\]  

(1.2)

Given the borrower’s project strategy, lenders choose the loan rate. With probability \( \mu \), lenders in \( k \geq 3 \) are separated from their borrowers and must start the next period with a first-timer. All lenders have the same information set in \( k = 1 \) so competition forces expected profits there to zero. Symmetric information in periods \( k \geq 3 \) also means zero profits. Therefore, the expected revenue of an informed lender who charges his type \( \omega \) borrower \( R \) is \( \pi(R|\omega) R \). Proposition 1 establishes the optimal loan rate offer:

\textbf{Proposition 1} Define the following critical type:

\[
\tilde{\omega}(r) \equiv \arg \min_{\omega \in [0,1]} |p(\omega) \overline{R}(\omega) - r|
\]  

(1.3)

In the competition for \( k \geq 3 \) borrowers, lenders offer:

\[
R_{k \geq 3}(r|\omega) = \begin{cases} 
  r/q & \text{if } \omega \in [0,\tilde{\omega}(r)) \\
  r/p(\omega) & \text{if } \omega \in [\tilde{\omega}(r), 1] 
\end{cases}
\]  

(1.4)

Since \( p(\cdot) \overline{R}(\cdot) \) is monotonically increasing, \( \tilde{\omega}(r) \) is unique and increasing in \( r \). Moreover, \( p'(\cdot) > 0 \) and \( p(\cdot) > q \) imply that lower types are charged higher loan rates at any given policy rate, consistent with their more costly nature.

\textsuperscript{10}Since an advanced borrower operates in an environment of perfect information and homogeneous separation rates, he attracts the same offer from every lender and is thus indifferent among them. Without loss of generality, I complete the borrower’s strategy by assuming that he stays with his second period lender for all \( k \geq 3 \).
1.3.2 Lenders with Intermediate Borrowers

In the second period, first period credit histories are public. Denote default by \( d = D \) and non-default by \( d = N \). While \( d \) is the only new piece of information observed by outside lenders in \( k = 2 \), an insider also learns his borrower’s type before making an offer. This type is revealed to outsiders at the end of the second period though so the borrower’s future outcomes are independent of current project choice and the optimal strategy of a type \( \omega \) borrower is still given by \( \pi(R|\omega) \).

As discussed in Section 1.2, the game between second period lenders is simultaneous. Consider first an insider who has discovered that his borrower is type \( \omega \). With competition driving future profits to zero, the insider’s expected revenue is again \( \pi(R|\omega) R \). Now, however, the insider has an informational advantage over all other lenders so competition will not necessarily eliminate current profits. Noting that the insider can charge above the outsider rate and lose the borrower, his value is:

\[
J_2(r|\omega, d) = \max \left\{ 0, \max_{R} \pi(R|\omega) R - r \quad \text{s.t.} \quad R \leq R^*_2(r|d) \right\}
\]  

(1.5)

Below \( \bar{R}(\omega) \), the insider’s revenue is strictly increasing in \( R \). Therefore, if \( R^*_2(r|d) \leq \bar{R}(\omega) \), equation (1.5) reduces to \( J_2(r|\omega, d) = \max \left\{ 0, \pi(\omega) R^*_2(r|d) - r \right\} \). Now suppose \( R^*_2(r|d) > \bar{R}(\omega) \). The insider will either offer \( R = \bar{R}(\omega) \) and induce the selection of \( P1 \), offer \( R = R^*_2(r|d) \) and induce the selection of \( P2 \), or offer \( R > R^*_2(r|d) \) and lose the borrower. Without loss of generality, I assume that insiders only keep borrowers who net them positive expected profit.\(^{11}\) Proposition 2 establishes that the second period credit market splits neatly between insiders and outsiders. In particular, lending relationships are formed with better borrowers, consistent with the empirical prediction of Memmel et al (2007) that high quality firms are more likely to choose relationship lenders.

\(^{11}\)This assumption simplifies the exposition aimed at here but is innocuous. Assuming instead that the insider keeps types for which he is indifferent yields the same loan rates and output functions derived below but the split between borrowers and project choice may occur within the insider rather than across insiders and outsiders.
Proposition 2 For a given credit history $d$, outsiders attract second period borrowers with $\omega \in [0, c_d(r)]$ and insiders retain those with $\omega \in (c_d(r), 1]$, where $c_d(r)$ is a threshold borrower type that satisfies $J_2(r|c_d(r), d) = 0$.

Consider now an outsider who has attracted a $k = 2$ borrower with history $d$. Represent his beliefs about the borrower’s type by a cumulative distribution function, $\hat{F}_d(\cdot)$, defined over the interval $[0, c_d(r)]$ according to Bayes’ rule. Whatever the borrower’s type, it will be known to everyone next period so future profits will be competed away. The expected profit of an outsider who charges his group $d$ borrower $R_d$ is thus:

$$f_0^{c_d(r)} \overline{T}(R_d|x) R_d d\hat{F}_d(x) - r$$

In equilibrium, the outsider’s beliefs will depend on $R_d$. Moreover, all outsiders must have the same beliefs so competition also drives (1.6) down to zero. Proposition 3 summarizes the outcome of the game between inside and outside lenders vying for second period borrowers:

Proposition 3 In the competition for $k = 2$ borrowers, outsiders offer $R^*_2(r|d) = r/q$ and get $\omega \in [0, \tilde{\omega}(r)]$. In contrast, insiders keep $\omega \in (\tilde{\omega}(r), 1]$ by offering:

$$R^*_2(r|\omega, d) = \begin{cases} R(\omega) & \text{if } \omega \in (\tilde{\omega}(r), \tilde{\omega}(r)) \\ r/q & \text{if } \omega \in [\tilde{\omega}(r), 1] \end{cases}$$

(1.7)

where $\tilde{\omega}(r)$ is as defined in Proposition 1 and $\tilde{\omega}(r) \equiv \arg \min_{\omega \in [0,1]} |qR(\omega) - r|$.

Proposition 3 establishes two important results. First, when the insider discovers his borrower’s type with certainty (i.e., $\phi = 1$), default history is irrelevant. Second, instead of the monotonically increasing function of $r$ that would arise in a pooled equilibrium, relationship lenders charge their borrowers a flat rate over certain ranges of the policy rate.

The adverse selection problem faced by competitive outsiders is key for the first result. Insiders want to keep the best types and, since outsiders cannot observe insider offers before
making their own, no inferences about type can be made based on current loan rates. For each default group then, outsiders know that they will attract the bottom of the distribution so they offer \( r/q \), the maximum competitive rate. This rate is above \( \bar{R}(\omega) \) for \( \omega < \tilde{\omega}(r) \) so, instead of matching the outsider and inducing \( P_2 \), the insider can offer these borrowers \( \bar{R}(\omega) \) and induce \( P_1 \). Given equation (1.3), \( \tilde{\omega}(r) \) is the lowest type for which undercutting the outsider is profitable and the market splits according to it.\(^\text{12}\)

The second result from Proposition 3 is illustrated in Figure 1(a). Details on the construction of the figure are provided in Appendix A.1. Consider type \( \omega_A \) as shown on the vertical axis. If \( r > r_B \), then \( \omega_A \) falls below \( \tilde{\omega}(r) \) and the borrower moves to an outsider. If \( r \leq r_B \), then \( \omega_A \) stays with his insider and, for policy rates between \( r_A \) and \( r_B \), he is charged his reservation loan rate. For policy rates below \( r_A \), the insider charges him \( r/q \) but \( \omega_A \) is sufficiently high that this rate is less than \( \bar{R}(\omega_A) \). Notice the role of relationship lending here: by revealing type to the insider, it allows him to gauge how much can be extracted from the borrower without inducing the risky project. In equilibrium, policy rates between \( r_A \) and \( r_B \) are such that the insider profits from using information generated by his lending relationship with \( \omega_A \) to undercut the outsiders. The shaded region in Figure 1(a) illustrates that the length of the policy rate interval over which a borrower is charged his reservation rate is increasing in the borrower’s type (i.e., the horizontal distance between \( \tilde{\omega}(r) \) and \( \tilde{\omega}(r) \) rises). Figure 1(a) also demonstrates that the proportion of types charged their reservation rate as a result of relationship lending exhibits a hump-shaped response to increases in the policy rate (i.e., the vertical distance between \( \tilde{\omega}(r) \) and \( \tilde{\omega}(r) \) rises then falls). As \( r \) increases, the marginal type on which an insider breaks even rises so the fraction of borrowers admitted into lending relationships falls. Within the group of relationship borrowers, however, the insider wants to increase the marginal type that he undercuts on. Initially, the second effect dominates the first and the vertical distance rises but, eventually, the first effect dominates the second and the vertical distance falls.

\(^{12}\)Note, however, that \( d \) would not be irrelevant if \( \phi < 1 \). Instead, \( \phi < 1 \) implies a positive probability that no one is informed about the borrower’s type in \( k = 2 \), making credit history the only piece of information available to the market and mitigating the adverse selection problem.
Before proceeding to \( k = 1 \), let us elaborate on the role of outsiders in these results. The free entry of other lenders forces the insider to solve a constrained optimization problem and, with monopoly rents precluded, insiders choose to tailor contracts around reservation rates in the manner discussed above.\(^\text{13}\) Note, however, that the absence of monopoly rents does not mean the absence of all rents: equation (1.3) and \( p (\omega) > q \) imply \( R_2^* (r|\omega, d) > r/p (\omega) \), where \( r/p (\omega) \) is the zero-profit perfect information loan rate offered to \( \omega > \tilde{\omega} (r) \) in Subsection 1.3.1. It is also interesting to note that insiders just match the outsider rate for types above \( \tilde{\omega} (r) \). This is consistent with the empirical finding of Bharath et al (2009) that prices of relationship and non-relationship loans are indistinguishable for borrowers at the top of the asset size distribution.\(^\text{14}\) Here though, \( \tilde{\omega} ' (r) > 0 \) so the fraction of borrowers for which insider and outsider prices are indistinguishable declines with the policy rate.

### 1.3.3 Lenders with New Borrowers

Recall from Proposition 3 that the second period equilibrium does not depend on default history when \( \phi = 1 \). Therefore, the reservation rate of a first-time borrower is still \( \overline{R} (\omega) \) and his project strategy is once again \( \gamma (R|\omega) \) as defined in equation (1.2). Moreover, the \( d's \) drop out of (1.5) and the insider’s valuation of a second period contract with a type \( \omega \) borrower can be denoted by \( J_2 (r|\omega) \). Assuming for now that the future policy rate is expected to be the same as the current one, first period lenders obtain the following expected profit from charging their borrowers \( R_1 \):

\[
\int_0^{1} \gamma (R_1|x) R_1 dF (x) + \beta \int_{\tilde{\omega}(r)}^{1} J_2 (r|x) dF (x) - r
\]

Let \( R_1 (r) \) denote the equilibrium first period loan rate and define type \( \xi (r) \) such that

\(^{13}\)In other words, competition prevents the borrower from being informationally captured and, as in Schmeits (2005), mitigates the hold-up problem. The result that competition can help sustain a mutually beneficial second period credit contract contrasts somewhat with the Petersen and Rajan (1995) argument that concentration increases the value of lending relationships. Therefore, consistent with Cao and Shi (2001), the treatment of information appears critical in analyzing interactions between credit market structure and credit market outcomes.

\(^{14}\)To the extent that ability and assets are positively correlated, the distribution of assets can be viewed as one approximation of the distribution of types.
\( R(\xi(r)) = R_1(r) \). By this definition, all types above \( \xi(r) \) choose \( P1 \) and all types below it choose \( P2 \). Since competition between identically uninformed lenders drives (1.8) down to zero, \( \xi(r) \) is characterized by:

\[
\xi(r) \equiv \arg \min_{\omega \in [0,1]} \left[ \int_0^\omega q dF(x) + \int_\omega^1 p(x) dF(x) \right] - R(\omega) + \beta \int_\omega^1 J_2(r|x) dF(x) - r \tag{1.9}
\]

where (1.5) and Proposition 3 can be used to substitute out \( J_2(r|x) \). Further discussion of \( \xi(r) \) is deferred until Section 1.4. For now though, note that a continuous function over a compact set has at least one \( \text{argmin} \) and, as demonstrated in Appendix A.1, the \( \text{argmin} \) that defines \( \xi(r) \) is unique and non-decreasing under the following regulatory conditions:

**Assumption 1** \( f(\cdot) \) is well-behaved with \( f(1) \leq 1 \)

**Assumption 2** \( p(0) \geq q + \frac{|p(1) - q|^3}{qp'(1) + |p(1) - q|} \)

**Assumption 3** \( p''(\cdot) \) is sufficiently low

**Assumption 4** \( p(1) \leq 2q \)

Assumption 1 precludes the economy from having a disproportionately large group of high types. It is a relatively innocuous assumption, satisfied by both uniform and truncated normal distributions over the unit interval. In Section 1.2, we imposed \( p(0) > q \) so Assumption 2 just refines the margin by which \( p(0) \) exceeds \( q \). Assumption 3 then says that \( p(\cdot) \) is either a concave, linear, or mildly convex function. In other words, while the probability of succeeding in the investment project increases with firm type, it does not increase exponentially. Finally, Assumption 4 regulates the speculative option by putting a lower bound on \( q \).

### 1.4 Output Functions

From the preceding analysis, there are two channels through which relationship lending can affect economic activity. First and as demonstrated in Subsection 1.3.2, insiders use

\footnotetext[15]{If \( R_1(r) > R(1) \), then \( \xi(r) \) is corner at 1.}
the information they acquire over the course of their relationships to tailor loan rates non-
monotonically. Second and as visible in Subsection 1.3.3, the loan rate for new borrowers
depends on expectations of future relationship profits. Having determined the effect of
relationship lending on credit terms in the previous section, let us now formalize its effect
on the output produced under these terms. As will become apparent below, the key variables
for the output calculation are the cutoff types \( \omega(r) \) and \( \xi(r) \).

1.4.1 Advanced Borrowers

Proposition 1 established that \( R_{k \geq 3}^e(r|\omega) = r/p(\omega) \) for all \( \omega \geq \omega(r) \) and, according to the
borrower strategy in equation (1.2), this loan rate will induce investment in \( P1 \) as long
as \( r/p(\omega) \leq \bar{R}(\omega) \). Given (1.3), the latter condition is guaranteed by \( \omega \geq \omega(r) \) so we
can conclude that all advanced borrowers above \( \omega(r) \) choose \( P1 \). Consider now \( \omega \leq \omega(r) \).
Proposition 1 also established that \( R_{k \geq 3}^e(r|\omega) = r/q \) for borrowers below \( \omega(r) \) so they will
choose \( P2 \) as long as \( r/q > \bar{R}(\omega) \). A sufficient condition for this inequality is \( r > p(\omega) \bar{R}(\omega) \),
which is guaranteed by \( \omega < \omega(r) \). Normalized by the relevant population then, the total
output of advanced borrowers is:

\[
Y_{k \geq 3}(r) = \int_{0}^{\omega(r)} q\theta_2 dF(x) + \int_{\omega(r)}^{\bar{R}(\omega)} p(x) \theta_1 dF(x) \tag{1.10}
\]

With \( \omega'(r) > 0 \) and \( p(x) \theta_1 > q\theta_2 \) for \( x \in (0, 1] \), equation (1.10) defines an output function
that is decreasing in the policy rate.

1.4.2 Intermediate Borrowers

Proposition 3 established that intermediate borrowers below \( \omega(r) \) are financed by outsiders
at loan rate \( r/q \). From Subsection 1.4.1, we know that \( r/q \) induces types below \( \omega(r) \) to
choose the speculative project so we can conclude that all intermediate borrowers below
\( \omega(r) \) undertake \( P2 \). Proposition 3 also established that intermediate borrowers between
\( \omega(r) \) and \( \omega(r) \) are financed by insiders at loan rate \( \bar{R}(\omega) \) so, given (1.2), they clearly
choose $P1$. Consider now intermediate borrowers above $\tilde{\omega}(r)$. These types are financed by insiders at loan rate $r/q$ so they will also choose $P1$ as long as $r/q \leq \overline{R}(\omega)$. With $\tilde{\omega}(r)$ as defined in Proposition 3, the latter condition is guaranteed by $\omega \geq \tilde{\omega}(r)$ and we can conclude that all intermediate borrowers above $\tilde{\omega}(r)$ choose $P1$. In other words, $Y_2(r)$ is also given by equation (1.10).

1.4.3 New Borrowers

As discussed in Subsection 1.3.3, the loan rate for first-time borrowers induces all types above $\xi(r)$ to choose $P1$ and all types below it to choose $P2$. Therefore, the total output of new borrowers is:

$$Y_1(r) = \int_0^{\xi(r)} q\theta_2 dF(x) + \int_{\xi(r)}^1 p(x) \theta_1 dF(x) \quad (1.11)$$

With $\xi'(r) > 0$, equation (1.11) also defines an output function that is decreasing in the policy rate.

1.4.4 Aggregation

There are new, intermediate, and advanced borrowers at any date $t$ so we must now determine the distribution of borrowers across periods in order to calculate aggregate output. Without loss of generality, set the population size to one and let $\Psi_{1,t}$, $\Psi_{2,t}$, and $\Psi_{k \geq 3,t}$ denote the proportions of period 1, 2, and $k \geq 3$ borrowers at date $t$. Aggregate output can then be written as:

$$Y(r,t) = \Psi_{1,t}Y_1(r) + \Psi_{2,t}Y_2(r) + \Psi_{k \geq 3,t}Y_{k \geq 3}(r)$$

where $Y_1(r)$ and $Y_2(r) = Y_{k \geq 3}(r)$ are given by (1.11) and (1.10) respectively. With the possibility of exogenous separation beginning at the end of the second period, the distribution evolves according to:
\[
\begin{align*}
\Psi_{1,t+1} &= \mu (\Psi_{2,t} + \Psi_{k\geq 3,t}) \\
\Psi_{2,t+1} &= \Psi_{1,t} \\
\Psi_{k\geq 3,t+1} &= 1 - \Psi_{1,t+1} - \Psi_{2,t+1}
\end{align*}
\]

Substituting \( \Psi_{k\geq 3,t} \) into the expression for \( \Psi_{1,t+1} \), the evolution of \( \Psi_{1} \) is determined by a one-dimensional difference equation and, with \( \mu \in (0, 1) \), the entire system is asymptotically stable. Therefore, starting from any initial distribution, the proportions converge to \( \Psi_{1} = \frac{\mu}{1+\mu} \) and \( \Psi_{k\geq 3} = \frac{1-\mu}{1+\mu} \). Steady state aggregate output is thus:

\[
Y (r) = \frac{\mu}{1+\mu} Y_{1} (r) + \frac{1}{1+\mu} Y_{k\geq 3} (r)
\]

Moreover, the extent of relationship lending is captured by \( (1 - \tilde{\omega} (r))/ (1 + \mu) \) and is decreasing in the policy rate and the rate of exogenous separation.

### 1.4.5 Benchmark for Comparison

To better appreciate the macroeconomic effects of relationship lending, it will be instructive to compare the results of my model to those of a standard credit channel model where exogenous separation occurs with certainty every period and private information is never revealed. In this context, a representative lender’s expected profit from charging \( R_{S} \) is \( \int_{0}^{1} \gamma (R_{S}|x) R_{S} dF (x) - r \). Competition drives this expression down to zero and yields an equilibrium loan rate denoted by \( R_{S} (r) \). Defining type \( \eta (r) \) such that \( \overline{R} (\eta (r)) = R_{S} (r) \), the solution to the standard model and the resulting output function are characterized by (1.13) and (1.14) respectively:

\[
\eta (r) \equiv \arg \min_{\omega \in [0,1]} \left| \int_{0}^{\omega}qdF (x) + \int_{\omega}^{1}p (x) dF (x) \right| \overline{R} (\omega) - r \tag{1.13}
\]

\[
Y_{S} (r) = \int_{0}^{\eta (r)} q \theta_{2} dF (x) + \int_{\eta (r)}^{1} p (x) \theta_{1} dF (x) \tag{1.14}
\]
1.5 Output Implications by Borrower Class

Given the structure of the output functions derived above, the effect of relationship lending on per-period output can be gauged by comparing $\tilde{\omega} (r)$, $\xi (r)$, and $\eta (r)$. The key properties of these cutoffs are derived in Appendix A.1 and illustrated in Figure 1(b). The associated output functions are then shown in Figure 2(a).\textsuperscript{16}

1.5.1 Output of Advanced and Intermediate Borrowers

Begin by comparing $\eta (r)$ and $\tilde{\omega} (r)$. Aside from the corners, $\eta (r)$ only intersects $\tilde{\omega} (r)$ once. Moreover, $\eta (r)$ approaches this intersection from below $\tilde{\omega} (r)$. Given the output functions in equations (1.10) and (1.14), we can then conclude that $Y_S (r)$ is greater than $Y_{k \geq 3} (r)$ for low policy rates but less than $Y_{k \geq 3} (r)$ otherwise. The difference between $Y_{k \geq 3} (r)$ and $Y_S (r)$ is intuitive. At low policy rates, informed lenders can grant favourable credit terms (i.e., loan rates low enough to induce $P1$) to more types without suffering a loss. The same is true for uninformed lenders in the standard model but, since they can only offer a pooled rate, some of the lower types who would not otherwise receive favourable terms now do. In contrast, when the cost of funds is sufficiently high, this mechanism has the opposite effect, yielding $\eta (r) > \tilde{\omega} (r)$ and $Y_S (r) < Y_{k \geq 3} (r)$. As can be gleaned from Subsection 1.3.2, relationship lending results in credit terms which push $Y_2 (r)$ towards $Y_{k \geq 3} (r)$ so the first macroeconomic impact of these relationships is a less severe second period output profile relative to the standard model.

1.5.2 Output of New Borrowers

Consider now $\xi (r)$ and $\eta (r)$. Aside from the corners, $\eta (r)$ does not intersect $\xi (r)$. Instead, $\xi (r) < \eta (r)$ which implies $Y_1 (r) > Y_S (r)$. Therefore, even though the first period of my model is characterized by the same information frictions as the standard model, the former generates higher output. To see why, note that first period lenders compete more fiercely for

\textsuperscript{16}Since all output functions were normalized by the relevant population, they are directly comparable. Moreover, to ease notation in Figure 2, I use $\mathcal{p}$ to denote $\int_0^1 p(x) dF(x)$. 
borrowers in anticipation of the second period insider profits afforded by relationship lending. As a result, the first period loan rate is driven down further, a greater number of types opt for \( P1 \), and the second macroeconomic impact of relationship lending is an improvement in first period output relative to the standard model. This effect is most pronounced over moderate policy rates since very high values of \( r \) are associated with few lending relationships while very low values of \( r \) provide only limited scope for further reductions in the loan rate.

1.6 Implications for Aggregate Output

How do these effects roll up into economy-wide output? In this section, I establish that relationship lending leads to a smoother steady state aggregate output function and a less dramatic response to certain monetary shocks.

1.6.1 Steady State

Aggregate steady state results are illustrated in Figure 2(b). With a higher probability of exogenous separation, there is a greater mass of first-time borrowers and the benefits of relationship lending are more visible at moderate policy rates. In contrast, lower separation probabilities push \( Y (r) \) towards \( Y_{k≥3} (r) \) and magnify the benefits of relationship lending at higher policy rates. For any value of \( \mu \) though, \( Y (r) \) is better than \( Y_S (r) \) in two respects. First, \( Y (r) \) is smoother over the interval \( r ∈ [0, qR (1)] \) and, second, \( Y (r) > qθ_2 = Y_S (r) \) over the interval \( r ∈ [qR (1), p (1) R (1)] \). Proposition 4 establishes the smoothness result more formally. To simplify the exposition both here and in the next subsection, I strengthen Assumptions 1 and 3 so that \( f (\cdot) = 1 \) and \( p'' (\cdot) = 0 \).

**Proposition 4** Let smoothness be a notion of curvature and define the smoothness of \( y (r) \) over \( r ∈ [a, b] \) by \( S \equiv \int_a^b |y'' (r)| \, dr \). It can be shown that \( \int_0^{qR (1)} |Y'' (r)| \, dr < \int_0^{qR (1)} |Y''_S (r)| \, dr \), implying that \( Y (r) \) is smoother than \( Y_S (r) \). The same conclusion holds if smoothness is taken to be a notion of dependence and defined by \( \tilde{S} \equiv \int_a^b |y' (r)| \, dr \).
In words, Proposition 4 says that relationship lending smooths the aggregate output function both by decreasing the average curvature of this function and by decreasing the average dependence of steady state output on policy rates.

To see that the smoothing effect is not just a by-product of the timing of exogenous separation or the fact that information is eventually revealed to all lenders, suppose that separation occurs with probability $\mu$ at the end of the first period and probability 1 at the end of the second. The $k = 2$ problem is unchanged so $Y_2 (r)$ is still given by (1.10). The $k = 1$ problem is slightly different since the second term in (1.8) must now be multiplied by $(1 - \mu)$. Once again though, higher values of $\mu$ will imply that the economy is able to sustain fewer lending relationships, shifting weight from $Y_2 (r) = Y_{k \geq 3} (r)$ to $Y_1 (r)$ and steepening the aggregate output function. Moreover, with exogenous separation beginning at the end of period 1, higher values of $\mu$ will lead first period lenders to view future relationship profits as less likely, pushing $\xi (r)$ closer to $\eta (r)$, $Y_1 (r)$ closer to $Y_S (r)$, and hastening the fall in output. Therefore, the institutional parameters that affect an economy’s ability to sustain lending relationships affect its real response to policy. Here, $\mu$ can be interpreted as the firm death rate and, as suggested by Adachi and Aidis (2007), its magnitude will be influenced by the regulatory environment (i.e., enforcement of property rights, anti-trust laws, hiring and firing restrictions, predatory tax practices, inspection agencies, etc.).

1.6.2 Dynamics

Let us now investigate whether relationship lending also fosters smoothness in a stochastic environment. In particular, if the policy rate follows an AR(1) process, does the presence of relationship lending generate a less volatile output response to temporary shocks? As proven in Appendix A.1 and summarized in the following proposition, the answer is unambiguously yes for small shocks in an already contractionary environment.\(^\text{18}\) That is, when liquidity

\(^{17}\)In a more general version of the model, both $\mu$ and $\phi$ would qualify as institutional parameters with $\phi$ representing average lender quality.

\(^{18}\)The effects in an expansionary environment are not as clear. For example, if $r_{ss}$ as defined in Proposition 5 is very low and the shock is small enough, then the output of new borrowers will exhibit less volatility while that of intermediate and advanced borrowers will exhibit more.
is low, relationship lending does indeed dampen the dynamic response to monetary policy and, for that matter, the propagation of other liquidity shocks.

**Proposition 5** Suppose the policy rate evolves according to \( r_t = r_{ss} + \alpha (r_{ss} - r_{t-1}) + \varepsilon_t \), where \( r_{ss} \) is its steady state value, \( \alpha \) captures the speed of mean reversion, and \( \varepsilon_t \) is an IID shock with \( E(\varepsilon_t) = 0 \). If \( r_{ss} \) is high, then a small temporary shock – that is, a one-time shock that keeps the policy rate between 0 and \( qR(1) \) – induces a less volatile transition path for aggregate output when relationship lending is present.

Consider now a permanent shock. As long as the policy rate is expected to stay at its post-shock level, the total output of each borrower class – new, intermediate, and advanced – adjusts immediately to its new steady state. Recalling the laws of motion presented in Subsection 1.4.4, the policy rate does not affect the distribution of borrowers across these classes so aggregate output also adjusts immediately after a permanent shock. In light of Proposition 4, the size of the adjustment will often be smaller in the presence of relationship lending so there is still a sense in which the transition is less pronounced.

This, however, raises the question of how distributional dynamics would affect the aggregate output response when credit markets feature relationship lending. To gain some insight into this issue, I consider the propagation of a permanent shock when the current model is extended to allow for different separation rates. In particular, the extension assumes that borrowers who stay with their insiders have a lower probability of exogenous separation. The results so far suggest that this assumption is not unreasonable. As demonstrated above, relationship lending makes loans more affordable for new borrowers and sometimes induces lenders with intermediate borrowers not to increase loan rates. Since better credit terms may help borrowers overcome idiosyncratic events that would have otherwise put them out of business, relationship lending may indeed foster lower firm exit rates.

The difference in separation rates has two important implications. First and as noted above, it allows us to consider distributional dynamics. We have already seen that the policy rate affects which borrowers enter into lending relationships so, when separation rates differ
between insiders and outsiders, the policy rate will also affect the distribution of borrowers across periods. Second, having different separation rates gives insiders more bargaining power over high types. A separated borrower must draw a new type and re-enter the credit market as a first-timer so separation is very costly for high-$\omega$ firms. By supporting a lower exit rate, insiders can now charge slightly above the outsider offer without losing these borrowers. Details of the extended model are provided in Appendix A.2 and, although the introduction of more bargaining power complicates the analysis, numerical results suggest that relationship lending induces distributional dynamics which then foster a more gradual output response to certain permanent shocks.

1.7 Conclusion

This paper has constructed a credit-based model of production to examine how relationship lending affects the monetary transmission mechanism. I analyzed how monetary policy changes the incentives of borrowers and lenders to engage in relationship lending and how these changes then shape the response of aggregate output. I find that sufficiently good borrowers enter into lending relationships and, over intermediate ranges of the policy rate, their loan rates are policy-invariant and preferable to the terms offered by uninformed lenders. In addition, competition among lenders for future relationship profits alleviates some of the tightness that could otherwise arise in the market for new borrowers. On average then, the informational properties of relationship lending lead to improved credit terms and economies that can sustain these relationships have a smoother steady state aggregate output profile and a less dramatic response to certain monetary shocks. These results provide a theoretical basis for cross-country transmission differences via a relationship lending channel so future work will be directed at calibrations to quantify the effect.
Figure 1: Critical Types

(a) Graphical representation of Proposition 3

(b) Comparison of critical types
Figure 2: Steady State Output

(a) Per-period output functions

(b) Aggregate output functions
Chapter 2

Screening, Lending Intensity, and the Aggregate Response to a Bank Tax

2.1 Introduction

Troubled assets have led to huge losses over the past few years, prompting many to explain the Great Recession as a miscalculation of risk by banks. In theory, however, the raison d’être of banks is that they are good at intermediation, providing risk sharing in incomplete markets and screening under asymmetric information. How, then, could they have gotten it so wrong? This paper contributes to a growing literature on financial sector inefficiency by investigating an as-yet unexplored margin: inefficiencies arising from the allocation of bank resources across intermediation activities.

I focus on two such activities. First, because of competition among lenders, banks devote some resources to attracting clients. Second, because of asymmetric information, they also devote some resources to screening those clients once attracted. No economic agent has unlimited resources though so a tradeoff between quantity and quality arises. The relevance of this tradeoff provides one interpretation of the recent rise and fall of mortgage-backed securities. In particular, the proliferation of these instruments fostered a credit boom but, ex post, it is clear that not enough information underlay the ratings. When viewed in this way, the evolution of mortgage-backed securities may symptomize a more fundamental problem of inefficiently low screening by financial intermediaries.

To understand why and when such inefficiencies can arise, I build a model that formal-
izes the allocation decision of banks. The economy features a continuum of heterogeneous borrowers differing in production ability. Each borrower needs one unit of capital to produce but this capital can only be intermediated by a mass of ex ante identical lenders. As described above, the intermediation process consists of attracting borrowers (i.e., by creating and/or advertising financial products) and screening them. Matches are necessary because credit is needed for production. At the same time though, screening is necessary because low quality borrowers are more likely to destroy capital by running unprofitable projects. Although lenders may want to undertake both matching and screening, the allocation of resources across these activities will be non-trivial if it is either too costly or too time-consuming to undertake each activity until its marginal return is zero. To ease the exposition, I capture this restriction as a unit resource constraint whereby lenders do not have enough resources to make both activities succeed with probability 1.

After analyzing an individual lender’s optimal division of resources between attracting and screening borrowers, I establish the existence of a unique steady state and calibrate the model to U.S. data in order to investigate efficiency in the decentralized equilibrium. I find that the equilibrium is not efficient and identify two key externalities behind the inefficiency. The first externality operates through the distribution of available borrowers when matches can be preserved over time. Since attracting a borrower today limits the need for matching resources tomorrow, lenders who carry their clients over can devote more of tomorrow’s resources towards screening if today’s screening efforts are unsuccessful. The eventual rejection of unprofitable borrowers then worsens the pool that currently unmatched lenders will draw from should they try to attract someone later on. The result is an "attract now, screen later" motive that drives the matching activity above the efficient level. This inefficiency is exacerbated by a second externality which arises because unmatched lenders do not take into account that their value function is the outside option of a matched lender. By allocating resources to maximize this value, unmatched lenders increase the opportunity cost of being matched and prompt informed lenders to be more selective in the types they retain. All else constant then, borrowers that a social planner would have deemed good
enough to finance are let go in the decentralized market as informed lenders pursue the potential of higher profits. To decrease the prospect of re-matching and thus decrease the endogenous destruction of informed financing, the efficient allocation would prescribe a lower matching intensity.

A corollary of these results is that bank taxes which limit the drive to attract borrowers (for example, regulations on certain aspects of the financial innovation process) can improve social welfare. I investigate a simple version of this policy, namely a proportional tax on the matching activity, and find that steady state production exhibits a hump-shaped response to increases in the tax. I also find that a mild tax can attenuate business cycle fluctuations.

To the extent that it emphasizes financial non-neutrality, my paper is related to the macroeconomic literature on credit channels. It is also related to a more recent branch of this literature which builds on the asset price propagation mechanism of Kiyotaki and Moore (1997) to investigate financial sector inefficiency. In Lorenzoni (2008) and Korinek (2009), for example, fire sales of collateralizable assets impart pecuniary externalities which can culminate in a financial crisis. In my model, however, inefficiency arises even if credit constraints are decoupled from asset prices, thus providing a new justification for regulatory intervention. Since the problem I propose exists at the level of bank decision-making, my paper is also related to previous work on the microfoundations of banking. Particularly relevant are Direr (2008) and Cao and Shi (2001) who examine screening externalities, Parlour and Rajan (2001) who examine competition externalities with strategic default, and Becsi et al (2009) who examine search frictions in the credit market matching process. None of these studies, however, takes into account the tradeoffs that can arise when lenders engage in both screening and matching so implications at both the bank and aggregate level have yet to be investigated.

The rest of the paper proceeds as follows: Section 2.2 describes the environment in more detail; Section 2.3 analyzes individual decisions in the decentralized market; Section

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19 See, for example, Gurley and Shaw (1955), Williamson (1987), Bernanke and Gertler (1989), and Kiyotaki and Moore (1997).

20 See Freixas and Rochet (1997) for an overview of such models.
2.4 establishes the existence of a unique steady state and calibrates the model; Section 2.5 compares the market equilibrium with the efficient allocation and discusses the externalities; Section 2.6 proposes a simple corrective tax and presents its effect on both steady state activity and recovery from a crisis; and Section 2.7 concludes. Figures appear at the end of the chapter and all proofs and derivations are collected in Appendix B.

2.2 Environment

All agents are risk neutral and endowed with a unit of effort each period. There is a continuum of firm types, \( \omega \in [0,1] \), with symmetric density function \( f(\cdot) \). Types are private information. Each firm has access to a risky production project that requires one unit of external financing (i.e., capital) to operate. A firm that obtains the necessary capital and exerts effort \( e \) runs a successful project with probability \( (1 + z) e \), where \( z \in (-\varepsilon, \varepsilon) \) is an unanticipated mean-zero aggregate productivity shock that is IID over time. \( z \) is not contractible and all decisions are made before its realization. An unsuccessful project yields zero while a successful one yields \( \theta(\omega) \), where \( \theta(\cdot) \) is an increasing and concave function of firm type. Project yields include the original capital input so unsuccessful projects effectively destroy capital. The firm’s cost of exerting effort \( e \) is \( -c \ln(1 - e) \), where \( c > 0 \) is a constant.

Firms cannot store project output and they do not have direct access to capital so they must borrow from a measure of ex ante identical lenders that also populates the economy. Lenders cannot operate the production project but, in addition to capital, they have access to two technologies that allow them to emerge as intermediaries. First, lenders can create and/or advertise financial products to match firms with capital. The greater the number of matches, the greater the lending intensity. Second, lenders can screen firms to determine whether facilitating such matches is indeed profitable. Although lenders may want to undertake both activities, it is either too costly or too time-consuming to undertake each one until its marginal return is zero. This restriction is captured by a unit resource constraint. In particular, a lender who devotes fraction \( \pi \) of his effort endowment to matching gets a borrower with probability \( \pi \) and discovers that borrower’s type with probability \( 1 - \pi \) im-
mediately thereafter. Lenders cannot support more than one match at a time and cannot search "on-the-contract" so the matching technology is only available to unmatched lenders. In contrast, screening can be undertaken by all lenders.

To understand the implications of the resource allocation decision, let us examine how lenders evolve over time. Begin with a lender who is unmatched as of date $t$. At the beginning of $t$, the lender chooses $\pi$. If he fails to attract a borrower, then he stays unmatched and must try again in $t + 1$. If, however, he succeeds in forming a match, then he exerts screening effort $1 - \pi$ right after getting that match. Successful screening means that the lender’s information set contains the borrower’s true type whereas unsuccessful screening means that it only contains his beliefs about the pool of borrowers from which he drew the match. To keep the analysis tractable, I assume that these beliefs cannot be conditioned on credit history if screening fails. Given his information set, the newly matched lender must make two more decisions at the beginning of date $t$. First, he must decide whether to finance the borrower he just attracted or let him go and try again in $t + 1$. Information is clearly important here because only lenders who have successfully screened will be able to gauge how profitable the borrower really is. In contrast, lenders who must rely on their beliefs about the borrower pool can only gauge average profitability across types. In what follows, I denote the retention strategy of a matched and informed lender by $a(\omega)$, where $a(\omega)$ will be an indicator that equals 1 if and only if the lender accepts to finance a type $\omega$ borrower.

Conditional on him keeping the borrower, the lender’s second decision is what contract terms to offer. I assume no intertemporal commitment so each contract is defined by a one-period loan rate. This rate includes the borrowed unit of capital and must be paid to the lender if the project succeeds. Lenders cannot observe the exact result of a project but

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21 To some extent, the lender’s constraint can be thought of as a budget constraint. However, using budgets would complicate the long-run analysis by endogenizing the right-hand side of the constraint without really changing the idea that lenders face a tradeoff between the two activities.

22 That is, discovering credit history requires screening to be at least partly successful.

23 To simplify the analysis, assume that creating and/or advertising financial products is enforceable in that a lender who has exerted $\pi > 0$ in the current period and attracted a match cannot reject that match unless he can prove that the applicant’s $\omega$ is too low (i.e., can only discriminate based on $\omega$).
can detect the presence of positive output so borrowers repay if and only if their projects are successful. The information on which the lender conditions his loan rate is again important. Since the same rate can induce different $\omega$’s to exert different production effort, the lender’s offer affects whether the borrower’s project will fail and, thus, whether capital will be destroyed.

Once retention decisions have been made and loan rates set, matched borrowers undertake production. The output of a successful project is then split so that, given loan rate $R$, the borrower gets $\theta(\omega) - R$ and the lender gets $R$. Borrowers consume their entire cut. In contrast, lenders save $(1 - \delta)R$ as capital for future financing and deplete the rest, $\delta R$ where $\delta \in (0, 1)$, as operating expenses and/or consumption. In what follows, I assume the existence of an interbank market and denote its market clearing cost of capital by $r$. Lenders who do not have enough capital in their reserves must borrow at $r$ while lenders who do have enough interpret $r$ as an opportunity cost. Therefore, a lender’s gross cost of funds is $1 + r$, where 1 represents the loan made to the borrower. Since each borrower needs only one unit of capital and lenders can finance only one borrower at a time, I can now focus on aggregate rather than individual capital accumulation.

At the end of date $t$, matches are subject to an exogenous separation probability $\mu \in (0, 1)$. Separation implies that the lender starts $t + 1$ unmatched. Non-separation implies that he carries his match into $t + 1$ and, therefore, cannot operate the matching technology that period. Since screening is still available to all lenders and an agent’s effort endowment is not transferable through time, it then follows that any matched lender who enters $t + 1$ without full information about his borrower’s type will undertake complete screening. As a result, uninformedness lasts for at most one period and within-lender credit history is rendered irrelevant. The lender’s problem is now the same as that of a matched lender who enters $t + 1$ with full information: at the beginning of the period, he also decides whether to finance the borrower again and, if he accepts to finance, then he also chooses a one-period loan rate. If he rejects, then he enters $t + 2$ unmatched.

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$^{24}$This ensures that the steady state features both informed and uninformed lending.


2.3 Optimal Decisions

2.3.1 Borrowers

Consider a type $\omega$ borrower who has obtained financing at loan rate $R$. With credit history irrelevant and intertemporal incentives precluded, the borrower’s problem is a static one. In particular, given $R$, he chooses how much effort to put into the production project so as to maximize his one-period expected utility. Recall that a type $\omega$ who exerts production effort $e$ succeeds with probability $(1 + z)e$, in which case the project yields output $\theta(\omega)$. Since $z$ is IID with mean 0, expected output is $e\theta(\omega)$ and the borrower’s expected utility from consumption is $e[\theta(\omega) - R]$. Taking into account the disutility of effort, the borrower thus solves the following problem:

$$\max_{e \in [0, 1]} \{ e[\theta(\omega) - R] + c \ln (1 - e) \}$$

Conditional on $R$, his optimal strategy is:

$$e(\omega, R) = \begin{cases} 
0 & \text{if } R > \theta(\omega) - c \\
1 - \frac{c}{\theta(\omega) - R} & \text{if } R \leq \theta(\omega) - c
\end{cases} \quad (2.1)$$

If the loan rate is higher than the choke rate $\theta(\omega) - c$, then the project will fail with certainty because the borrower has no incentive to exert production effort. On the other hand, if the loan rate is lower than the choke rate, then the borrower’s effort is positive but strictly decreasing in $R$. Note that since $\theta(\omega)$ is an increasing function of the borrower’s type, a better borrower is more likely to exert positive effort and his effort in this case will be higher for any given loan rate $R$.

2.3.2 Lenders

As described in Section 2.2, a lender’s problem depends on whether he is matched or unmatched and, if matched, it also depends on whether he is informed or uninformed about
his borrower’s type. Since a lender’s choices affect how he evolves over time, I formulate
the problem using dynamic programming. In what follows, the aggregate state is summa-
ized by $S \equiv \{K, V(\cdot), V_u(\cdot), \lambda_{-1}(\cdot), \phi_{-1}(\cdot)\}$, where $K$ is the beginning-of-period stock of
financing capital, $V(\omega)$ is the value of type $\omega$ under informed financing, $V_u(\omega)$ is the value
of $\omega$ if unmatched, $\lambda_{-1}(\omega)$ is the proportion of $\omega$’s financed by informed lenders last period,
and $\phi_{-1}(\omega)$ is the proportion financed by uninformed lenders.

2.3.2.1 Informed Lenders Consider an informed lender matched with a type $\omega$ bor-
rower. The lender takes as given $S$ and his individual state $\{\omega, v\}$, where $v$ is the value
attained by his borrower. In turn, he must choose whether to keep the borrower ($a$), what
loan rate to charge if he does keep him ($R$), and what continuation value to offer ($v_{+1}$).
Since the borrower has the option of turning down the contract and hoping for a new lender
next period, the continuation value must satisfy the borrower’s participation constraint (i.e.,
the present discounted value of staying cannot be less than $\beta V_{u,+1}(\omega)$). Letting $J$
the value function of an informed lender and $U$ the value function of an unmatched lender,
the informed problem can be written as:

$$J(\omega, v, S) = \max_{a, R, v_{+1}} \left\{ \begin{array} {l}
(1-a) \beta U(S_{+1}, \psi_{+1}) \\
+ a \left[ \left( 1 - \frac{c}{\theta(\omega)-R} \right) R - (1+r(S)) \right. \\
\left. + \beta \left[ (1-\mu) J(\omega, v_{+1}, S_{+1}) + \mu U(S_{+1}, \psi_{+1}) \right] \right] \end{array} \right\}$$

subject to

$$a \in [0, 1], \ R \in [0, \theta(\omega) - c]$$

$$v = \theta(\omega) - R - c + c \ln \left( \frac{c}{\theta(\omega)-R} \right) + \beta \left[ (1-\mu) v_{+1} + \mu V_{u,+1}(\omega) \right] \geq \beta V_{u,+1}(\omega)$$

$$S_{+1} = \Gamma(S), \ \psi_{+1} = \mathcal{G}(S_{+1})$$

Let us now work through equation (2.2). If the lender rejects the borrower, then he
gets the discounted value of being unmatched next period ($\psi$, the individual state of an
unmatched lender, will be discussed in the next subsection). If he accepts the borrower,
then his current period payoff depends on the borrower’s strategy. With $e(\omega, R)$ as in (2.1),
the lender will never want to charge above \( \theta (\omega) - c \). Moreover, although higher values of \( R \) increase conditional revenue, they also decrease the probability of repayment so the lender would not want to monopolize the borrower irrespective of the participation constraint. Expected revenue is thus \( e (\omega, R) R \) and the lender’s gross cost of funds is \( 1 + r (S) \), where the market clearing cost of capital depends on the aggregate state. The lender’s future value is then \( J (\omega, v_{+1}, S_{+1}) \) if the match is not exogenously destroyed and \( U (S_{+1}, \psi_{+1}) \) otherwise. To complete the problem, the lender’s beliefs about the evolution of \( S \) and \( \psi \) are governed by laws of motion which, as will be discussed in Section 2.4, must be consistent with aggregate behaviour.

### 2.3.2.2 Unmatched Lenders

Consider now an unmatched lender. As discussed earlier, this lender has to choose how to divide resources between getting matches and screening applicants. A lender who devotes \( \pi \) units to attracting a borrower becomes matched and uninformed with probability \( \pi^2 \), matched and informed with probability \( \pi (1 - \pi) \), and stays unmatched with probability \( 1 - \pi \). Recall from Section 2.2 that uninformedness lasts for at most one period so I now define the one-period revenue function of an uninformed lender. Since uninformed lenders cannot discriminate among borrowers, they can only offer a pooled rate \( \overline{R} \) which, if below \( \theta (\omega) - c \), will induce a type \( \omega \) to exert effort. Letting \( \psi (\omega) \) denote the lender’s beliefs about the share of type \( \omega \)’s in the pool from which he drew, his maximized expected revenue is:

\[
X (S, \psi) = \max_{\overline{R}} \int_{\eta(\overline{R})}^{1} \left( 1 - \frac{c}{\theta (\omega) - \overline{R}} \right) \overline{R} \psi (\omega) \, d\omega \\
subject to \quad \overline{R} \in [0, \theta (1) - c] \\
\eta (\overline{R}) = \arg \min_{w \in [0, 1]} \left| \theta (w) - c - \overline{R} \right|
\]

A newly matched lender who has not discovered his borrower’s type gets \( X (S, \psi) - (1 + r (S)) \) in the current period. If the match is exogenously destroyed at the end of the
period, then his future value is \( U(S_{+1}, \psi_{+1}) \). If it is not destroyed, then his future value is \( J(\omega, V_{+1}(\omega), S_{+1}) \) weighted by \( \psi(\omega) \) since \( \omega \) is not known at the time of the match. We can now write the value function of an unmatched lender:

\[
U(S, \psi) = \max_\pi \left\{ \pi^2 \left[ X(S, \psi) - (1 + r(S)) + \beta \mu U(S_{+1}, \psi_{+1}) \right] + \beta (1 - \mu) \int_0^1 J(\omega, V_{+1}(\omega), S_{+1}) \psi(\omega) \, d\omega + \pi (1 - \pi) \int_0^1 J(\omega, V(\omega), S) \psi(\omega) \, d\omega + (1 - \pi) \beta U(S_{+1}, \psi_{+1}) \right\}
\]

subject to

\[
\pi \in [0, 1], \ S_{+1} = \Gamma(S), \ \psi_{+1} = \mathcal{G}(S_{+1})
\]

### 2.3.3 Optimal Resource Allocation

The key allocation decision is the unmatched lender’s choice of \( \pi \). The choice clearly depends on the distributions \( \lambda, \phi \), and \( \psi \) so we must now establish them.

#### 2.3.3.1 Distributions

Recall that there are two classes of financing - informed and uninformed. The proportion of type \( \omega \)'s with informed financing is \( \lambda(\omega) \) and the proportion with uninformed financing is \( \phi(\omega) \). Let \( \Pi \) denote the aggregate lending intensity of unmatched lenders and \( A(\omega) \) the aggregate retention strategy of informed lenders. The law of motion for \( \lambda(\omega) \) is then:

\[
\lambda(\omega) = A(\omega) \left[ (1 - \mu) [\lambda_{-1}(\omega) + \phi_{-1}(\omega)] + [1 - (1 - \mu) [\lambda_{-1}(\omega) + \phi_{-1}(\omega)]] \Pi (1 - \Pi) \right]
\]

As long as \( A(\omega) = 1 \), borrowers who were financed by informed lenders last period and who are still around this period again obtain informed financing. The same is true for borrowers who were financed by uninformed lenders last period and who are still around.
this period. These two statements explain \( A(\omega)(1 - \mu) \left[ \lambda_{-1}(\omega) + \phi_{-1}(\omega) \right] \). To see where
\( A(\omega) \left[ 1 - (1 - \mu) \left[ \lambda_{-1}(\omega) + \phi_{-1}(\omega) \right] \right] \Pi(1 - \Pi) \) comes from, note that some borrowers obtain informed financing from a new lender. This group is drawn both from borrowers who were unmatched last period and from borrowers who would have stayed with their last period lender had they not been exogenously separated.

Exogenously separated borrowers are also relevant for \( \phi(\omega) \). In particular, some of the borrowers who were exogenously separated last period and who match again this period are not discovered. Moreover, some of the borrowers who were discovered last period (or are sure to be discovered at the start of this period) are not retained by informed lenders yet, if they match again, they may be able to obtain uninformed financing. The same is of course true for previously unmatched borrowers so the law of motion for \( \phi(\omega) \) is:

\[
\phi(\omega) = \left[ 1 - A(\omega)(1 - \mu) \left[ \lambda_{-1}(\omega) + \phi_{-1}(\omega) \right] \right] \Pi^2 \tag{2.6}
\]

Indeed, \( 1 - A(\omega)(1 - \mu) \left[ \lambda_{-1}(\omega) + \phi_{-1}(\omega) \right] \) is the proportion of type \( \omega \) borrowers looking for new lenders at the beginning of the current period so, in equilibrium, beliefs about the composition of available borrowers must satisfy:

\[
\psi(\omega) = \frac{\left[ 1 - A(\omega)(1 - \mu) \left[ \lambda_{-1}(\omega) + \phi_{-1}(\omega) \right] \right] f(\omega)}{\int_{\xi}^{J} \left[ 1 - A(j)(1 - \mu) \left[ \lambda_{-1}(j) + \phi_{-1}(j) \right] \right] dF(j)} \tag{2.7}
\]

### 2.3.3.2 Best Response Function
With the distributions in hand, we can examine the optimal choice of \( \pi \). The following proposition simplifies the analysis by reducing the informed retention strategy \( A(\cdot) \) from a function to a scalar:

**Proposition 6** Suppose informed lenders are not bound by the borrower participation constraint. The informed retention strategy can be summarized by a cutoff type \( \xi \), where \( A(\omega) = 1 \) if and only if \( \omega \geq \xi \).
Proposition 7 now establishes the best response of one unmatched lender to the actions of other (symmetric) unmatched lenders, holding the informed side of the market constant. To fix ideas, I focus on steady state.

Proposition 7 Let $\pi(\Pi)$ denote the steady state best response of $\pi$ to $\Pi$ with $\xi$ held fixed. There exists a $\hat{\xi}$ such that:

(i) $\pi(\Pi) = 1$ with $\pi'(\Pi) = 0$ for $\xi < \hat{\xi}$;

(ii) $\pi(\Pi) \in (0, 1)$ with $\pi'(\Pi) < 0$ for $\xi \in (\hat{\xi}, 1)$;

(iii) $\pi(\Pi) = 0$ with $\pi'(\Pi) = 0$ for $\xi = 1$.

A lender has two incentives to learn his borrower’s type. First, the information will enable him to reject unprofitable applicants and, second, it will also enable him to extract some rents from the profitable ones. Very high values of $\xi$ mean that only a small group of borrowers are profitable so the desire to identify them drives lending intensity down and, in the extreme case, we observe $(iii)$. On the other hand, very low values of $\xi$ mean that almost all types are profitable so the first incentive is diminished. Moreover, for $\xi$ sufficiently low, the risk of not forming a match this period outweighs the second incentive and $\pi$ tends to 1 as per $(i)$.

The interesting case is $\xi \in (\hat{\xi}, 1)$. If $\Pi = 0$, then any unmatched lender who successfully expends $\pi > 0$ will have drawn from the initial distribution of types. As long as this distribution yields profitable expectations (which it must in order for the credit market to get off the ground), the lender will indeed choose $\pi > 0$. However, he will not go as far as $\pi = 1$ since the symmetry of $f(\cdot)$ across good and bad types also makes screening desirable. What happens if $\Pi$ is slightly positive? Although other lenders only get a few borrowers, they screen them so intensely that at least some good types are pulled off the market while almost all the bad types remain. The average quality of available borrowers thus decreases, increasing any individual lender’s incentive to screen and decreasing the choice of $\pi$. Consider now a very high value of $\Pi$. A lot of matches are being formed but immediate type discovery is not common among other lenders so both good and bad
borrowers are pulled off the market. If uninformed matches were to stay uninformed, beliefs would move back towards the initial distribution, \( \pi \) would increase, and the best response function would be U-shaped. Recall, however, that uninformedness is eventually resolved when lenders can preserve their matches across periods. In turn, high values of \( \Pi \) will translate into worse beliefs about the future and the best response will decrease. As we will see in Section 2.5, this result underlies one of two important externalities.

### 2.4 General Equilibrium

#### 2.4.1 Market Clearing

Given the strategies and distributions derived above, I now discuss the evolution of the capital base. Suppose the beginning-of-period stock is \( KS \). To get the end-of-period stock \( KS_{+1} \), we must first subtract the amount of capital put into production during the current period. Since each loan transfers one unit of capital to the borrower, the amount of capital used up in financing equals the number of borrowers financed. This is essentially a measure of capital demand and it can be calculated as follows:

\[
KD = \int_0^1 \left( \lambda(\omega) + \phi(\omega) \right) dF(\omega)
\]

If production is unsuccessful, then the borrowed unit of capital cannot be recovered. In contrast, a successful project returns this unit plus some additional capital to the lender and, after depleting a fraction \( \delta \), the lender adds back to the base. The law of motion for capital is then:

\[
KS_{+1} = KS - KD + (1 - \delta) \left[ \int_0^1 e(\omega, R(\omega)) R(\omega) \lambda(\omega) dF(\omega) + \int_0^1 e(\omega, \overline{R}) \overline{R}\phi(\omega) dF(\omega) \right]
\]

Note that the cost of funds does not enter this equation directly. If \( r \) is interpreted as an opportunity cost that the lender must be compensated for, then it does not enter
into aggregate accounting. Alternatively, if $r$ is interpreted as a direct cost - namely the cost of borrowing the required unit from another lender on the interbank market - then it is subtracted from the revenues of the borrowing lender but added to the revenues of the lending lender, effectively washing out. The role of $r$ is thus indirect. In particular, it adjusts to yield choices of $A$ and $\Pi$ that produce $KD = KS$. With such market clearing, the capital accumulation equation becomes:

$$K_{t+1} = (1 - \delta) \left[ \int_0^1 e(\omega, R(\omega)) R(\omega) \lambda(\omega) dF(\omega) + \int_0^1 e(\omega, \bar{R}) \bar{R} \phi(\omega) dF(\omega) \right]$$  \hspace{1cm} (2.8)

### 2.4.2 Definition and Existence of Equilibrium

To complete the characterization, we need rules for the two remaining state variables: $V(\cdot)$ and $V_u(\cdot)$. Since an informed lender will never charge above his borrower’s choke rate, production effort is always positive under informed financing and the borrower’s value satisfies the following functional equation:

$$V(\omega) = \theta(\omega) - R(\omega) - c + c \ln \left( \frac{e}{\theta(\omega) - R(\omega)} \right) + \beta \left[ (1 - \mu) V_{t+1}(\omega) + \mu V_{u,t+1}(\omega) \right]$$  \hspace{1cm} (2.9)

In contrast, uninformed lenders can only offer a pooled rate which may or may not induce $e(\omega, R) = 0$ so this borrower’s value if unmatched is:

$$V_u(\omega) = \Pi^2 \left[ \max \left\{ \theta(\omega) - \bar{R} - c + c \ln \left( \frac{e}{\theta(\omega) - \bar{R}} \right), 0 \right\} + \beta \left[ (1 - \mu) V_{t+1}(\omega) + \mu V_{u,t+1}(\omega) \right] \right]$$

$$+ \Pi \left( 1 - \Pi \right) V(\omega) + \left( 1 - \Pi \right) \beta V_{u,t+1}(\omega)$$  \hspace{1cm} (2.10)

An equilibrium in this model is then a set of lender value functions $\{J, U\}$ and sequences of borrower continuation values $\{V, V_u\}$, individual decision rules $\{a, \pi, R, \bar{R}, v_{t+1}\}$, aggregate decision rules $\{A, \Pi\}$, distributions $\{\lambda, \phi\}$, financing capital $\{K_{t+1}\}$, costs of funds $\{r\}$, and

\[25\text{ As } V(\omega) \text{ and } V_u(\omega) \text{ are really only used to construct the participation constraint that an informed lender must satisfy in order to retain } \omega, \text{ I will just present the rules for types that the lender does indeed want to keep.}\]
beliefs \{\psi, \Gamma, \mathcal{G}\} satisfying:

1. Lender optimality as per the optimization problems in Section 2.3.

2. Symmetry (i.e., \(A = a, \Pi = \pi, \) and \(V = v\)).

3. Capital market clearing.

4. Laws of motion (2.5), (2.6), and (2.8).

5. Functional equations (2.9) and (2.10).

6. Consistency of beliefs and, in particular, \(\psi\) as given by (2.7).

The proof of Proposition 6 in Appendix B.1 established the existence of \(J\) and \(U\). Moreover, the Theorem of the Maximum implies that the first order condition for \(\pi\) defines a continuous mapping from \(\Pi \in [0, 1]\) to \(\pi \in [0, 1]\) so there exists at least one symmetric equilibrium in the game between unmatched lenders. Since all unmatched lenders are ex ante identical, restricting attention to symmetry in the lending intensity choice is not unreasonable. Moreover, since all informed lenders matched with a type \(\omega\) borrower are also ex ante identical, the same can be said for symmetry in their retention strategies and continuation offers. Proposition 8 now addresses the existence and uniqueness of a stationary symmetric equilibrium in the overall economy.\(^{26}\)

**Proposition 8** If \(\mu\) is sufficiently high, then there exists a unique non-trivial steady state in the class of symmetric equilibria with non-binding borrower participation constraints.

The remainder of this paper investigates efficiency in the decentralized steady state and the potential for corrective taxes. Since the many interactions between agents make it difficult to obtain closed-form expressions, the next subsection parameterizes the model in order to conduct the analysis.

\(^{26}\)I restrict attention to non-trivial steady states since even the standard capital accumulation model admits an equilibrium where the economy shuts down.
2.4.3 Parameterization

I calibrate the model’s steady state to match features of the U.S. credit market over the period 1995-2005. Although the Gramm-Leach-Bliley Act did not officially institute broad banking until 1999, the Fed began easing Glass-Steagall in the late 1980s, effectively expanding the range of activities that banks could engage in. As such, I calibrate the model under $\tau = 0$ (i.e., very little to no regulation) then consider policy experiments where $\tau \neq 0$.

Suppose that the initial distribution of firm types is uniform over the unit interval and restrict attention to production functions of the form $\theta(\omega) = y_0 + y_1 \omega^\alpha$. I normalize $y_0 = 1$ so that it represents the borrowed unit of capital (i.e., every successful project returns enough to cover its input). I also define the model period to be a quarter and set the discount factor $\beta$ to match an annualized risk-free interest rate of 4%.

The parameters left to be calibrated are: the exogenous separation probability $\mu$, the lender parameter $\delta$, the borrower disutility parameter $c$, and the production parameters $y_1$ and $\alpha$. I use the following targets:

1. Based on Bharath et al (2009), 71% of business loans come from lenders who recently provided the firm with another loan. The analogous measure in the model is the proportion of loans not in their first period.

2. Define capacity as the production that could be achieved if, all else constant, borrowers exerted effort $e(\omega, 0)$. I use the capacity utilization rate for manufacturing, roughly 0.78 in the FRED database, to target the model’s ratio of actual production to capacity.

3. I target $K/Y$ to match the ratio of net business loans to GDP. With net business loans defined as the difference between credit market debt owed by non-farm non-financial businesses and the credit market assets they hold, FRED data suggests a ratio of 0.57.

4. For the period under consideration, the value-added of the financial industry as a fraction of GDP is 0.075 (BEA Economic Accounts). Value-added sums compensation
to employees, production taxes, and gross operating surplus so I interpret \( \delta K \) as the model’s counterpart. The targeted item is then \( \delta K/Y \).

5. From the 1997 Census of Manufactures, Dziczek et al (2008) estimate that the difference between the log labour productivity of the 90th and 10th percentile manufacturing plants is 1.62. I use this figure to target the dispersion of production among successful borrowers.

The resulting parameters are: \( \mu = 0.14, c = 0.285, \delta = 0.13, \alpha = 0.5 \), and \( y_1 = 2.05 \).

2.5 Externalities

Consider now a steady state social planner who holds the entire capital base. He faces the same constraints and intermediation technologies as lenders in the decentralized economy. Subject to aggregate feasibility, the planner chooses lending intensity \( \Pi \), the informed cutoff \( \xi \), and transfers \( R \equiv \{ R(\cdot), \overline{R} \} \) to maximize total welfare. Total welfare is taken here to be total present discounted net output. A formal statement of the planner’s constrained efficiency problem is presented in Appendix B.2 and the results are summarized below.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>PLANNER</th>
<th>MARKET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lending Intensity (( \Pi ))</td>
<td>0.3637</td>
<td>0.4309</td>
</tr>
<tr>
<td>Informed Cutoff (( \xi ))</td>
<td>0.3770</td>
<td>0.4901</td>
</tr>
<tr>
<td>Amount of Informed Credit</td>
<td>0.4750</td>
<td>0.4044</td>
</tr>
<tr>
<td>Amount of Uninformed Credit</td>
<td>0.0753</td>
<td>0.1169</td>
</tr>
<tr>
<td>Average Type Financed</td>
<td>0.6432</td>
<td>0.6578</td>
</tr>
<tr>
<td>Average Delinquency Rate</td>
<td>0.3201</td>
<td>0.3426</td>
</tr>
<tr>
<td>Aggregate Welfare</td>
<td>80.551</td>
<td>75.100</td>
</tr>
</tbody>
</table>

Relative to the decentralized market, the planner would devote more resources to screening new matches but be less restrictive in his cutoff once informed. The outcome would be
a smaller market for uninformed financing, a lower overall delinquency rate, and higher aggregate welfare.

To understand why unmatched lenders do not achieve the efficient resource allocation, consider first how their choices impact other unmatched lenders. For the relevant values of $\xi$, recall from Proposition 7 that the best response function $\pi (II)$ is decreasing. As an unmatched lender increases lending intensity, he negatively impacts the beliefs of other unmatched lenders as discussed in Subsection 2.3.3.2. Unmatched lenders do not take this externality into account and the result is an "attract now, screen later" motive which prompts lending intensity in the decentralized market to be too high.

Consider now how the resource allocation decision affects matched lenders. When an informed lender decides whether or not to keep his borrower, he compares the expected value from that borrower to the value he could get as an unmatched lender. The latter, however, is not exogenous. Instead, unmatched lenders choose lending intensity to maximize that value, thereby increasing the opportunity cost of being matched and prompting informed lenders to be more selective in the types they retain. As a result, borrowers that a social planner would have deemed good enough to fund are let go in the decentralized market. All else constant then, the efficient allocation would prescribe a lower matching intensity to decrease the endogenous destruction of informed financing by decreasing the likelihood of re-matching.\footnote{Computing a variant of the decentralized economy where the informed lender’s outside option is appropriately exogenized confirms that the outside option externality exacerbates the beliefs externality in this direction. Moreover, computing a pseudo problem in which the planner cares about capital rather than net output reveals that any additional distortions stemming from the division of output between lenders and liquidity-constrained borrowers are negligible.}

Overall, unmatched lenders impart externalities on both the beliefs and the outside options of other lenders, leading to an inefficiently high amount of low-quality credit.

### 2.6 A Corrective Tax

The direction of the inefficiency identified in Section 2.5 motivates a tax on lending intensity. In this section, I consider a linear tax which makes activities designed to attract borrowers
more costly. The tax rate is denoted by $\tau$ and only affects unmatched lenders. In particular, the maximization problem on the right-hand side of equation (2.4) now includes the term $-\tau \pi$. The tax revenues are then added back to the capital base so that all other equations are unchanged.28

### 2.6.1 Effect on Steady State

Figure 3 illustrates how steady state equilibrium outcomes vary with the lending intensity tax. There are four noteworthy features.

First, higher values of $\tau$ lead to lower values of $\Pi$ and $\xi$. Since $\tau$ makes lending intensity more costly, the decrease in $\Pi$ is straightforward. The decrease in $\xi$ then follows from the fact that higher taxes and lower re-matching probabilities decrease the outside option of informed lenders, making them less restrictive in their retention of borrowers.

Second, market size (the measure of borrowers financed) exhibits a hump-shaped response to increases in $\tau$. There are two competing forces here. On one hand, the decline in lending intensity decreases match formation but, on the other, the decline in informed selectivity increases match preservation. The second effect dominates at low tax rates but is eventually overtaken by the first.

Third, higher values of $\tau$ increase the average quality of the credit market. Recall that a borrower’s default probability depends on both his type and the loan rate he is charged. One of the advantages of informed lenders is that they can give borrowers better incentives to run successful projects. Although the decline in $\xi$ lowers the average type financed, it (along with the increase in $1 - \Pi$) increases the proportion of informed financing and decreases the average delinquency rate.

Finally, at a macroeconomic level, production exhibits a hump-shaped response to increases in the tax. In particular, the shift towards informed financing has a positive effect

28Although alternative specifications of $\tau$ are certainly possible, I begin with the simple version described here in order to fix ideas, leaving extensions for future work. In what follows, $\tau$ can be interpreted as either a direct tax on the number of loans or a regulation which increases the cost of engaging in the matching activity.
as long as the frequency of new matches does not become too small. Furthermore, welfare increases as $\xi$ and $\Pi$ approach the efficient allocation.

### 2.6.2 Effect on Dynamics

The response to a temporary negative aggregate productivity shock is shown in Figure 4. I consider an unanticipated one-time shock where $z$ drops to $-0.01$ at $t = 1$ but reverts to its mean right after. Appendix B.3 describes the algorithm used to compute the response.

A negative shock to the probability of successful production (after $\Pi_1$ and $\xi_1$ have been decided) leads to an immediate drop in aggregate output and the end-of-period capital stock. The fall in capital implies a higher cost of funds in $t = 2$, reducing the incentive to lend and prompting a decline in lending intensity. The increase in $r_2$ also puts upward pressure on the informed cutoff. At the same time though, higher costs and a lower re-matching probability put downward pressure on this cutoff by deteriorating the informed lender’s outside option.

The presence of a small tax reinforces the downward pressure on $\xi_2$. Along with providing an additional drag on the value of being unmatched, a small tax prolongs the recovery path of $\Pi_t$ and, all else constant, deteriorates the outside option further into the future. The net result is a short-term decline in $\xi_t$ so that, by limiting the contraction of informed financing, $\tau = 0.005$ hastens the re-accumulation of capital and fosters a faster recovery in production.

### 2.7 Conclusion

This paper has examined the allocation of bank resources across two important intermediation activities: creating credit market matches and screening the borrowers in these matches. I constructed a model to disentangle the implications of this allocation decision in an environment with private information and competing lenders. I then showed that banks do not fully internalize the effects of their resource allocation decisions, leading to an inefficiently high amount of low-quality credit. There are two main externalities behind this result. The first operates through the distribution of available borrowers when matches can be preserved over time while the second arises because unmatched lenders do not take into
account that their value function is the outside option of a matched lender. From a policy perspective, these results contribute to the current debate on bank taxes. In particular, the inefficiencies identified by my model suggest that a tax on matching activities (for example, certain regulations on financial innovation) would be more effective than some of the general profit taxes recently tabled. Indeed, steady state results show that production exhibits a hump-shaped response to increases in a matching tax and the model’s dynamics indicate that a mild version of this tax can also attenuate business cycle fluctuations.
Figure 3: Steady State Results

- **Lending Intensity**
- **Informed Cutoff**
- **Market Size**
  - Total
  - Informed
  - Uninformed
- **Average Delinquency Rate**
- **Production**
- **Welfare**
Figure 4: Dynamics After a Temporary Productivity Shock ($z = -0.01$ in $t = 1$)
Chapter 3

Institutions and Economic Outcomes: A Dominance-Based Analysis
(joint with Gordon Anderson)

3.1 Introduction

The interaction between political institutions (polity) and economic outcomes is a key issue in both welfare and development economics. While welfarists are concerned with how these variables contribute to overall wellbeing, development economists have taken a more intertemporal approach and focused on issues of causality.\textsuperscript{29} Neither of these questions, however, has been fully settled. On the welfare front, it is generally agreed that polity and growth jointly promote wellbeing yet empirical assessments of the effects are limited. On the causality front, the problem is almost reversed. Theory suggests that causality can run in both directions and, despite much empirical work to disentangle these channels, a consensus has not emerged. In this paper, we analyze both multivariate welfare and intertemporal dependence using a notion of distributional dominance. Our approach provides a powerful tool for addressing the aforementioned issues, especially when the data set contains discrete (polity) and continuous (growth) variables.

While our reason for studying welfare is to fill a gap in the literature, our reason for studying dependence is to tackle a long-standing debate using new techniques. As such, it will be useful to review some of the scholarship that underlies this debate. On one

\textsuperscript{29}As will be discussed later, we interpret this as an evaluation of dependence rather than pure causality.
hand, prolonged economic failure may compel agents to demand better institutions and any growth that makes them richer or more educated may also provide extra bargaining power to make these demands credibly. On the other hand though, better institutions (for example, property rights, political freedoms, and government accountability) may provide better investment incentives and thus encourage economic activity. A key empirical study that finds causality from institutions to economic outcomes is Acemoglu et al (2001). The authors argue that institutions introduced by European colonizers varied according to settlement objectives and show that the persistence of these institutions to the present day has had important income per capita implications for the ex-colonies. Using a growth accounting framework, Hall and Jones (1999) also argue for the primacy of institutions, finding that differences in social infrastructure help explain the large differences in capital accumulation and productivity observed across countries. More recent work by Gwartney et al (2006) confirms the importance of such an institutions-investment channel while Dawson (2003) identifies freedoms related to international finance as those which affect growth through investment and freedoms related to political, civil, and economic liberties as those which affect growth directly. Consistent with Calderon and Chong (2000) and Kaufmann and Kraay (2002), however, both Dawson (2003) and Gwartney et al (2006) also find evidence of reverse causality when certain institutional measures are used. The importance of disaggregating institutions is further established by the Heckelman (2000) result that an average measure of freedom along with its monetary, capital, and property rights components precedes growth but that growth likely precedes the extent of government intervention. A consistent conclusion is reached by Alvarez and Vega (2003) who find clear evidence of causality from institutions to growth when institutions are measured as economic freedoms but confounded evidence when they are measured as political freedoms.

That the debate is far from settled is also reflected in several papers which have raised additional theoretical linkages are discussed in Sen (1999) and Friedman (2005). Empirical tests of causality have also been debated in the financial development literature. See, for example, Beck et al (2000), King and Levine (1993), Rajan and Zingales (1998), Morris et al (2001), and Calderon and Liu (2003).
questions about the econometric methods used to investigate the relationship between growth and institutions. Levine and Renelt (1992), for example, demonstrate that slight changes in the list of explanatory variables can overturn the results of many empirical growth studies while De Haan et al (2006) also criticize the specification of certain growth models used in the literature. Perhaps the most searing criticism though is provided by Glaeser et al (2004) who argue that traditional methods for testing the relationship between polity and growth are flawed and, once proper measures and valid instruments are employed, political institutions have only a second order effect on economic performance.

In light of the preceding discussion, we abstract from conventional regression methods and analyze the relationship between institutions and economic outcomes in the context of intertemporal dependence instead of just intertemporal correlation. We stress that what is being considered here is dependence rather than pure causality. Indeed, the development literature’s use of the term causality is much in the spirit of Granger (1969) and thus has more to do with dependence than causality as we currently understand it. Moreover, even without the appropriate counterfactuals for a causal analysis, we can measure degrees of dependence. Given theoretical support for both the "polity causes growth" and "growth causes polity" hypotheses, we argue that they should not be treated as alternatives and instead focus on identifying the dominant hypothesis by adapting the overlap index proposed by Anderson et al (2009, 2010) for use with a mixture of discrete and continuous variables. The basic premise is that the joint density of two independent variables overlaps the product of their marginal densities at every point of support so, if polity does indeed determine growth more than growth determines polity, the joint density of earlier polity and later growth should be systematically further away from independence than that of earlier growth and later polity. Using this approach, we can admit non-linear relationships non-parametrically and bypass the error-term constraints that plague regression methods.

Dominance-based techniques are also germane because they are what we use to examine the other important aspect of the polity-growth interaction: the effect on overall welfare. Changes in economic and political variables have not always been in the same direction,
making their net impact on wellbeing difficult to ascertain. Drawing from the multivariate stochastic dominance literature, however, we can compare the current distribution over growth-polity pairs to past distributions over these pairs and make qualitative statements about the progress of wellbeing.

The next section presents our methodology in more detail. Section 3.3 then discusses the data used while Section 3.4 reports our results. We find that economic growth had a positive impact on wellbeing from 1960 to 2000 yet declines in polity over the earlier part of this period were sufficient to produce a decline in overall welfare until the mid-1970s. Subsequent increases in polity then reversed the trend and, ultimately, welfare in 2000 was higher than that in 1960. We also find evidence that the causal effects of polity on growth dominate those of growth on polity, particularly when the data are population weighted.

3.2 Methodology

3.2.1 Multivariate Wellbeing

With some modification, the multivariate stochastic dominance techniques presented in Anderson (2008) and Duclos, Sahn, and Younger (2006) can be used to assess changes in overall wellbeing. Although these techniques do not provide a complete ordering of states, when they do provide a ranking, the ordering is unambiguous. Suppose societal wellbeing at date \( t \) can be written as \( U(y_t, x_t) \): a monotonic, non-decreasing function of the continuous variable economic wellbeing \( y_t \) and the discrete variable political freedoms \( x_t \). Further, let \( y_t \) and \( x_t \) be jointly distributed with potentially time-varying PDF \( g_t(y, x) \) and corresponding CDF \( G_t(y, x) \). Suppose \( \frac{\partial^2 U}{\partial x \partial y} \) is negative and define \( D = G_t(y, x) - G_{t-i}(y, x) \). If \( \frac{\partial^2 U}{\partial x \partial y} \) is positive, then \( D \) is defined in terms of the counter cumulative densities (see Anderson (2008)).\(^{32}\) When \( D \leq 0 \) for all pairs \((y, x)\) with strict inequality for at least some, then \( E(U(y_t, x_t)) \geq E(U(y_{t-i}, x_{t-i})) \). Moreover, based on Atkinson and Bourguignon (1982), the society at \( t \) can be considered a welfare improvement over the

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\(^{32}\)The signs of the cross-partialials reflect the substitutability (-ve) or complementarity (+ve) of \( x \) and \( y \) in the wellbeing calculus.
society at \( t - i \) for all wellbeing functions in the monotonic non-decreasing family. In fact, as long as \( D \) is significantly negative for some pairs \((y, x)\) and not significantly positive for all other pairs, \( E(U(y_t, x_t)) \geq E(U(y_{t-i}, x_{t-i})) \) can be established and an approximately first order welfare improvement obtains. When dominance is ascertained for a family of wellbeing indicators, there is no need to choose composite indicators from those families since all those choices become ordinally equivalent.

In order to make quantitative statements about \( D \), we use the Kolmogorov-Smirnov statistic for differences between distributions. The statistic is based on the maximum value of \( D \) over the support of the two distributions being compared and an estimate of this value can be obtained from sample-based estimates of the joint densities in two periods.\(^{33}\) The formula used for \( P(\sqrt{n}D < \lambda) \) is \( 1 - e^{-2\lambda^2} \) which is Rayleigh’s formula for the univariate statistic \((K = 1, \text{where } K \text{ is the number of variables the distribution describes})\). Although Kiefer and Wolfowitz (1958) establish the existence of a distribution function for \( D \) when \( K > 1 \), they find that it generally depends on \( G \). Later work by Kiefer (1961), however, suggests that the formula for the univariate case provides a conservative (i.e. larger) estimate of the true value when \( K > 1 \).

### 3.2.2 Dependence Dominance

The literature abounds with types of dependence. Lehmann (1966) outlines three: positive (negative) quadrant dependence, positive (negative) regression dependence, and positive (negative) likelihood ratio dependence.\(^{34}\) The problem for our analysis is that all of these notions deal with monotone relationships yet there should be no presumption that the polity-growth nexus is characterized by such monotonicity.\(^{35}\) Therefore, we employ a more general concept of "distance from independence" which admits both monotone and non-

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\(^{33}\) Although the joint distribution of polity and growth describes a mixture of discrete and continuous variables, this does not pose a problem since sample cumulants are easily calculated.

\(^{34}\) See also Bartolucci et al (2001), Pandit and Shetty (2003), and Denzit and Scaillet (2004).

\(^{35}\) While the case for positive dependence can be gleaned from some of the development literature discussed in Section 3.1, the argument that negative dependence cannot be ignored is supported by the post-WWII literature on wage moderation. Maier (1987), for instance, argues that the limited freedoms of labour unions in Germany, Italy, and Japan after the war helped foster growth-promoting compromise during this period.
monotone relationships. Letting $z$ be an $n$-dimensional vector and $f_a(z)$ and $f_b(z)$ be two continuous multivariate distributions, the extent to which $f_a(z)$ and $f_b(z)$ overlap can be measured as:

$$OV = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \min \{f_a(z), f_b(z)\} dz_1 \cdots dz_n$$

If $f_a(z)$ is the unrestricted joint PDF of $z$ and $f_b(z)$ is the joint distribution when the $z$’s are independent, then $OV \in [0, 1]$ is an index of independence and $1 - OV$ is a general index of dependence, be it monotone or not. In recent work, Anderson, Linton, and Whang (2009) show that the kernel estimator of $\theta = \int \min \{f_a(z), f_b(z)\} dz$ is distributed as follows:

$$\sqrt{n} \left( \hat{\theta} - \theta \right) - \alpha_n \longrightarrow N(0, v)$$

where

$$v = p_0\sigma_0^2 + p_a (1 - p_a) + p_b (1 - p_b)$$

$$p_0 = P(Z \in C_{ab}); \ C_{ab} = \{z \in \mathbb{R}^n : f_a(z) = f_b(z) > 0\}$$

$$p_a = P(Z \in C_a); \ C_a = \{z \in \mathbb{R}^n : f_a(z) < f_b(z)\}$$

$$p_b = P(Z \in C_b); \ C_b = \{z \in \mathbb{R}^n : f_a(z) > f_b(z)\}$$

$\alpha_n$ and $\sigma_0^2$ are bias correction factors.

The slight wrinkle for the polity-growth application, however, is that $z$ is a mixture of discrete and continuous variables. Denoting them by $z_d$ and $z_c$ respectively so that $z = (z_d, z_c)$, the appropriate overlap measure is:

$$OV_{mix} = \int_{-\infty}^{\infty} \sum_{z_d} \min \{f_a(z), f_b(z)\} dz_c$$

The discrete version of $OV$ has been developed in Anderson, Ge, and Leo (2010) so the properties of $OV_{mix}$ can be derived as a mixture of the two cases. Moreover, $OV_{mix}$ lends itself quite naturally to a test of dependence dominance. To see how, let $y_t$ be a vector of economic variables at date $t$ with joint distribution $f(y_t)$ and let $x_t$ be a vector of
institutional indices at date $t$ with joint density $p(x_t)$. The joint distribution of economic outcomes at date $j$ and institutions at date $k$ is denoted by $g(y_j, x_k)$. Under independence, $g(y_j, x_k) = f(y_j) p(x_k)$ and the following measure of their dependence can be constructed:

$$d(y_j, x_k) = 1 - \int \min \{ g(y_j, x_k), f(y_j) p(x_k) \} \, dy_j \in (0, 1)$$

A greater degree of dependence between $y_j$ and $x_k$ implies less overlap between $g(y_j, x_k)$ and $f(y_j) p(x_k)$, leading to higher values of $d(y_j, x_k)$. To test for dependence dominance, we thus focus on $d(y_{t-i}, x_t) - d(y_t, x_{t-i})$ for $i = 1, \ldots, l$. Consistently negative differences support the hypothesis that institutions promote growth more than growth promotes institutions while consistently positive differences support the reverse. Essentially, conditions like $d(y_{t-i}, x_t) - d(y_t, x_{t-i}) \geq 0$ for all $i$ or $d(y_{t-i}, x_t) - d(y_t, x_{t-i}) \leq 0$ for all $i$ (with strict inequality holding somewhere) are forms of dominance relationships and establishing them empirically would lend considerable support to one view or the other. Since these inequalities need to hold simultaneously, the simultaneous comparison techniques in Wolak (1989) and Stoline and Ury (1979) are appropriate.

### 3.3 Data

We consider a sample of 84 developed and developing countries over the period 1960 to 2000, drawing data on economic outcomes and political institutions at 5 year intervals.\footnote{Dictated largely by data availability, our sample includes the following countries: Algeria, Argentina, Australia, Austria, Belgium, Benin, Bolivia, Brazil, Burkina Faso, Cameroon, Canada, Central African Rep, Chad, Chile, China, Colombia, Congo Brazzaville, Congo Kinshasa, Costa Rica, Denmark, Dominican Rep, Ecuador, Egypt, El Salvador, Finland, France, Gabon, Ghana, Greece, Guatemala, Haiti, Honduras, Hungary, India, Indonesia, Iran, Ireland, Israel, Italy, Ivory Coast, Japan, Kenya, Liberia, Madagascar, Malawi, Malaysia, Mauritania, Mexico, Morocco, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Panama, Paraguay, Peru, Philippines, Portugal, Rwanda, Senegal, Sierra Leone, Singapore, South Africa, South Korea, Spain, Sri Lanka, Sudan, Sweden, Switzerland, Syria, Thailand, Togo, Trinidad, Tunisia, United Kingdom, United States, Uruguay, Venezuela, Zambia.}

Economic outcomes are measured using real GDP per capita from the World Bank Development Indicators database. For political institutions, Glaeser et al (2004) suggest that constraints on the executive is the most defensible of all the standard measures so
we use the corresponding variable from the frequently cited Polity IV project.\textsuperscript{37} The Polity project was initiated in the late 1960s by Ted Robert Gurr, then of Princeton University. The first wave of data was collected by a single coder who, for each country in the sample, used historical records to ordinally score the country’s authority traits (i.e., executive constraints, political participation, etc.). The data was eventually expanded into an annual format and vetted on several occasions to ensure consistency.\textsuperscript{38} Polity IV, currently maintained by the Center for Systemic Peace, tracks six main political traits for over 160 countries. The executive constraints indicator we use here captures the extent to which decision-makers are limited by accountability groups (i.e., democratic legislatures, independent judiciaries and, in coup-prone states, the military). The bindingness of such constraints is measured on a seven-point scale. Higher values reflect better institutions, with 1 representing unlimited authority and 7 representing executive parity. Most parliamentary democracies are coded as 7’s while countries where the executive controls the judiciary and/or routinely overrides the constitution are coded as 1’s.

At this point, it is useful to address the often overlooked issue of population weighting. If the polity-growth nexus is viewed as a latent technological relationship, then each country should be interpreted as a particular draw from that technology and given equal weight. If, on the other hand, we take a representative agent view, then country-level observations should be population weighted so as to give each individual in the world sample equal weight. In what follows, we present the results of both approaches as well as a representative agent version that excludes the two most populous countries – China and India. Our population data also come from the World Bank database.

Summary statistics are reported in Tables 1(a) and 1(b) at the end of this chapter. When unweighted, average GDP per capita exhibits sustained growth throughout the period.\textsuperscript{37} Citations include Hanson (2004), Hausmann, Pritchett, and Rodrik (2005), and Klomp and De Haan (2009). Acemoglu et al (2001) also use the Polity IV data in their robustness checks. For the project’s user guide, see Gurr et al (2010).\textsuperscript{38} One caveat to the annual frequency is that measures of executive constraints are not reported for transition years. This affected a few of our observations so we used the closest available data point – usually within one or two years of the missing one – to circumvent it.
Average polity, in contrast, declined over the first 15 years of our sample, returning to its initial level in the mid-1980s and rising to unprecedented levels thereafter. The 1980s also saw a reversal in the plight of the poorest nation as minimum GDP per capita transitioned from consistent improvements to substantial losses late in the decade. With regard to dispersion, polity and GDP per capita seem to be driven by very different processes. Polity, in particular, appears to be a convergent measure whereas GDP per capita appears to be a divergent one. The population weighted statistics tell a similar story for polity but not for GDP per capita which, when weighted, is characterized by greater dispersion and substantially lower means and medians.

3.4 Results

3.4.1 Multivariate Wellbeing

Tables 2(a) to 2(f) report the Kolmogorov-Smirnov first order stochastic dominance comparisons for all possible pairs of years in the sample. The joint densities have been estimated using cumulants of the Epanechnikov kernel in the continuous dimension and straightforward cumulation in the discrete dimension. An increase in overall wellbeing from year $B$ to year $A$ is declared if "$H_0$: Year $A$ dominates Year $B$" is accepted and "$H_0$: Year $B$ dominates Year $A$" is rejected. If both hypotheses are rejected or both hypotheses are accepted, then an indeterminate change in welfare is reported.

In this application, the unweighted results in Table 2(a) are the clearest. Under negative cross-partials, the unweighted sample yields only 5 indeterminacies out of the 36 possible year-to-year comparisons while the weighted samples with and without China and India yield 21 and 9 respectively. The unweighted results reflect the fact that declines in polity between 1960 and 1975 outweighed progress in incomes, leading to declines in overall wellbeing relative to initial conditions. By 1985, however, the drop in polity had been made up and further progress in such institutions meant that unambiguous increases in wellbeing were

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39 Per capita GDP is very close to log normally distributed, hence the use of the Epanechnikov kernel with the optimal bandwidth for normality following Silverman (1986).
sustained through to 2000. The population weighted results in Tables 2(b) and 2(c) tell a consistent story, particularly when China and India are excluded from the sample. In the latter case, the main difference relative to Table 2(a) is that a swifter recovery in polity under population weighting pulls the welfare declines in the earlier part of the observation period up to indeterminacy. The results under positive cross-partials are similar in most respects, however the weighted calculation in Table 2(e) is now characterized by less indeterminacy than the unweighted one in Table 2(d).

3.4.2 Dependence Dominance

To avoid difficulties with joint density estimation at points with too few observations, we amalgamate polity categories 1 and 2 and polity categories 3 and 4 to form a new five-point polity scale. For all lags examined – 0 to 40 years in 5 year intervals – the dependence of GDP on past polity and the dependence of polity on past GDP are readily established. With causality in both directions, we now turn to the question of interest: does one direction dominate in the sense that the degree of dependence is always at least as great in that direction at every lag? Tables 3(a), 3(b), and 3(c) report the dependence dominance results for the unweighted, population weighted, and population weighted excluding China and India samples. We have used a discrete-continuous specification for the joint densities, employing a Gaussian kernel for the continuous component and Silverman’s rule of thumb for the window width.\textsuperscript{40} Tables 4(a), 4(b), and 4(c) report the results when the joint densities are specified with smoothed estimates of the discrete component probabilities as per Li and Racine (2003) and Li et al (2006).\textsuperscript{41}

With respect to the unweighted results in Table 3(a), there is some indication that the dependence of growth on polity dominates the dependence of polity on growth at longer lags but the differences are largely insignificant. The smoothed cell probability results in Table

\textsuperscript{40}Our dependence dominance analysis uses the multivariate overlap measure software of Anderson, Linton, and Whang (2009) which employs a multivariate Gaussian kernel. The window width suggested by Silverman (1986) is $1.06\sigma n^{-1/(4+k)}$.

\textsuperscript{41}This approach to calculating the densities was suggested by a referee and software advice was kindly provided by Jeff Racine. For the smoothed cell calculation, we are able to use the original seven-point polity scale.
4(a) are much stronger in this direction, with dominance at longer lags accepted under all usual levels of significance. The weighted results in Tables 3(b) and 3(c) also exhibit a clearer pattern than those in 3(a). At all lags where $d(y_{t-i}, x_t) - d(y_t, x_{t-i}) < 0$, the hypothesis that polity promotes growth more than growth promotes polity is accepted under modest degrees of significance. In contrast, the reverse hypothesis is readily rejected at all lags where $d(y_{t-i}, x_t) - d(y_t, x_{t-i}) > 0$. Moreover, if we exclude China and India to avoid the extraordinary importance that population weighting assigns to their circumstances, the conclusion emerges more strongly in both the discrete-continuous and smoothed cell specifications.

### 3.5 Conclusion

There has been considerable debate over whether it is polity that causes growth or growth that causes polity and the discussion has been fomented, at least in part, by the fact that the two hypotheses are not mutually exclusive. In this paper, we argued that the relevant question is not which hypothesis is correct but, rather, which hypothesis dominates. Since conventional regression techniques have difficulty capturing non-linear dependence, especially when one of the variables is an index with limited variation, we proposed a dependence dominance test based on the overlap measure of Anderson et al (2009, 2010) to examine the polity-growth nexus. Taken together, our results suggest that institutions promote economic outcomes more than economic outcomes promote institutions. Another advantage of the dominance-based approach is its natural link to multivariate welfare comparisons. Our results on this front suggest that, while economic growth had a positive impact on wellbeing between 1960 and 2000, early declines in polity were sufficient to produce a decline in overall welfare until the mid-1970s. Subsequent increases in polity then reversed the trend and, ultimately, wellbeing in 2000 was higher than that in 1960.
### Table 1: Summary Statistics

#### (a) Unweighted Sample

<table>
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<tr>
<th>Year</th>
<th>Polity Index</th>
<th>GDP per Capita</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>1960</td>
<td>4.0595</td>
<td>3</td>
</tr>
<tr>
<td>1965</td>
<td>3.9048</td>
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<td>1970</td>
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<td>3</td>
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<tr>
<td>1975</td>
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<td>3</td>
</tr>
<tr>
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<td>6</td>
</tr>
<tr>
<td>2000</td>
<td>5.2381</td>
<td>6</td>
</tr>
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</table>

#### (b) Population Weighted Sample

<table>
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<tr>
<th>Year</th>
<th>Polity Index</th>
<th>GDP per Capita</th>
</tr>
</thead>
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<td></td>
<td>Mean</td>
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<td>1965</td>
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<td>5</td>
</tr>
<tr>
<td>2000</td>
<td>5.1097</td>
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</table>
**Table 2(a): Multivariate Dominance Tests (Negative Second Partial)**

**Unweighted**

<table>
<thead>
<tr>
<th>Comparison Years</th>
<th>Change in Wellbeing</th>
<th>$P(A$ dominates $B)$</th>
<th>$P(B$ dominates $A)$</th>
</tr>
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<tbody>
<tr>
<td>1960 1965</td>
<td>↓</td>
<td>0.0261</td>
<td>0.1185</td>
</tr>
<tr>
<td>1960 1970</td>
<td>n/d</td>
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<tr>
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<td>0.0083</td>
<td>0.4770</td>
</tr>
<tr>
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</tr>
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<td>0.0199</td>
</tr>
<tr>
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<td>0.0000</td>
</tr>
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<tr>
<td>1965 1970</td>
<td>↓</td>
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<td>0.2655</td>
</tr>
<tr>
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<td>0.4559</td>
</tr>
<tr>
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<td>↓</td>
<td>0.0444</td>
<td>0.3183</td>
</tr>
<tr>
<td>1965 1985</td>
<td>↑</td>
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<td>0.0000</td>
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<td>1965 2000</td>
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<td>0.0006</td>
</tr>
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<td>1970 1975</td>
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<td>0.0480</td>
<td>0.1301</td>
</tr>
<tr>
<td>1970 1980</td>
<td>n/d</td>
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</tr>
<tr>
<td>1970 1985</td>
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<td>0.3863</td>
<td>0.0180</td>
</tr>
<tr>
<td>1970 1990</td>
<td>↑</td>
<td>0.9027</td>
<td>0.0056</td>
</tr>
<tr>
<td>1970 1995</td>
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<td>0.9859</td>
<td>0.0000</td>
</tr>
<tr>
<td>1970 2000</td>
<td>↑</td>
<td>0.9968</td>
<td>0.0000</td>
</tr>
<tr>
<td>1975 1980</td>
<td>↑</td>
<td>0.1101</td>
<td>0.0000</td>
</tr>
<tr>
<td>1975 1985</td>
<td>↑</td>
<td>0.4667</td>
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### Table 2(e): Multivariate Dominance Tests (Positive Second Partials)
**Population Weighted**

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### Table 2(f): Multivariate Dominance Tests (Positive Second Partials)

**Weighted Excl. CHN and IND**

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Table 3: Dependence Dominance Tests

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(b) Population Weighted

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(c) Weighted Excl. CHN and IND

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Table 4: Dependence Dominance Tests (Smoothed Cell Probabilities)

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References


Appendix to Chapter 1

A.1 Proofs

This section proves Propositions 1 to 5 and derives the key features of Figure 1. The material is presented in the order it is referenced in Chapter 1.

Proof of Proposition 1

The highest sustainable loan rate is $\theta_2$: anything above $\theta_2$ and the borrower will not want to undertake either project. For loan rates less than or equal to $\theta_2$, the lender’s revenue function is:

$$\tau(R|\omega)R = \begin{cases} p(\omega)R & \text{if } R \leq \bar{R}(\omega) \\ qR & \text{if } R > \bar{R}(\omega) \end{cases}$$

In equilibrium, competition forces $\tau(R|\omega)R = r$ so the zero-profit loan rates are:

$$R = \begin{cases} r/p(\omega) & \text{if } r \leq p(\omega)\bar{R}(\omega) \\ r/q & \text{if } r > q\bar{R}(\omega) \end{cases}$$

In other words, the lender charges $r/p(\omega)$ if $r \in [0,q\bar{R}(\omega)]$, either $r/p(\omega)$ or $r/q$ if $r \in (q\bar{R}(\omega), p(\omega)\bar{R}(\omega)]$, and $r/q$ if $r \in (p(\omega)\bar{R}(\omega), \theta_2]$. I now show that there exists a profitable deviation from $r/q$ when $r \in (q\bar{R}(\omega), p(\omega)\bar{R}(\omega)]$, ruling it out as an equilibrium offer. In particular, suppose the lender offers $\bar{R}(\omega)$ instead of $r/q$. Since $r > q\bar{R}(\omega)$, the lender can indeed do this without losing the borrower. Moreover, at $\bar{R}(\omega)$, type $\omega$ chooses $P1$ so the lender’s expected profit is $p(\omega)\bar{R}(\omega) - r > 0$, confirming the profitable deviation.
Therefore, the equilibrium loan rate for \( k \geq 3 \) is:

\[
R_{k \geq 3}^*(r|\omega) = \begin{cases} 
  r/p(\omega) & \text{if } r \leq p(\omega) \bar{R}(\omega) \\
  r/q & \text{if } r > p(\omega) \bar{R}(\omega)
\end{cases}
\]

The monotonicity of \( p(\cdot) \bar{R}(\cdot) \) and the definition of \( \bar{\omega}(r) \) then yield the form in (1.4).

**Proof of Proposition 2**

Suppose types \( \omega_0 \) and \( \omega_0 + \delta \) have the same credit history \( d \). Since credit history is the only information that the outsider can condition on, he offers \( \omega_0 \) and \( \omega_0 + \delta \) the same loan rate \( R_2^*(r|d) \). Consider now an insider who finds it optimal to keep \( \omega_0 \). Retaining \( \omega_0 \) when exogenous separation rates are homogeneous implies that the insider must be charging \( R_2^*(r|\omega_0, d) \leq R_2^*(r|d) \). Moreover, the fact that the insider finds it optimal to keep \( \omega_0 \) means that he must be making positive profit on this borrower. With \( \delta > 0 \) and \( \bar{R}'(\omega) > 0 \), equation (1.2) establishes that \( \bar{\gamma}(R|\omega_0 + \delta) \geq \bar{\gamma}(R|\omega_0) \) for any \( R \). Therefore, the insider could offer \( \omega_0 + \delta \) loan rate \( R_2^*(r|\omega_0, d) \), keep him, and make at least as much as he is making on \( \omega_0 \). Since \( \omega_0 \) and \( \delta \) were chosen arbitrarily, Proposition 2 follows by induction.

**Proof of Proposition 3**

Let \( R_d(r) \) denote the as yet undetermined solution to the outsider’s problem and define type \( \omega_d(r) \) such that \( \bar{R}(\omega_d(r)) = R_d(r) \). By definition, all types above \( \omega_d(r) \) choose P1 and all types below it choose P2 so setting (1.6) to zero yields:

\[
R_d(r) = \begin{cases}
  r/\left[\int_0^{\omega_d(r)} q d\tilde{F}_d(x) + \int_{\omega_d(r)}^{c_d(r)} p(x) d\tilde{F}_d(x)\right] & \text{if } \omega_d(r) \leq c_d(r) \\
  r/q & \text{if } \omega_d(r) > c_d(r)
\end{cases}
\]  

(A.1)

The proof of Proposition 3 proceeds in three steps. First, I prove \( R_2^*(r|d) = r/q \). Second, I prove \( c_d(r) = \bar{\omega}(r) \). Third, I establish equation (1.7). The following lemma will be useful for the first step:
Lemma 1 \( \hat{F}_d(\cdot) \) is the CDF of a non-degenerate distribution.

Proof. Using Bayes’ Rule, \( \hat{F}_d(x) \) is given by:

\[
\hat{F}_d(x) \equiv \Pr(\omega \leq x|d) = \frac{\Pr(d|\omega \leq x) \Pr(\omega \leq x)}{\Pr(d)}
\]

All first-time borrowers advance to the second period so the unconditional type distribution is \( F(\cdot) \). Note, however, that \( \Pr(\omega \leq x) = F(x)/F(c_d(r)) \) since the outsider only gets \( \omega \in [0,c_d(r)] \). If \( c_d(r) \leq \xi(r) \), where \( \xi(r) \) is the lowest type that chose \( P1 \) in the first period, then the outsider’s beliefs are given by:

\[
\hat{F}_d(x) = \frac{F(x)}{F(c_d(r))} \text{ for } x \in [0,c_d(r)]
\]

On the other hand, if \( c_d(r) > \xi(r) \), then Bayesian updating yields:

\[
\hat{F}_N(x) = \begin{cases} 
\frac{q F(x)}{q F(\xi(r)) + \int_{\xi(r)}^{c_n(r)} p(z) dF(z)} & \text{if } x \in [0,\xi(r)) \\
\frac{q F(\xi(r)) + \int_{\xi(r)}^{c_d(r)} p(z) dF(z)}{q F(\xi(r)) + \int_{\xi(r)}^{c_n(r)} p(z) dF(z)} & \text{if } x \in [\xi(r),c_n(r)]
\end{cases}
\]

\[
\hat{F}_D(x) = \begin{cases} 
\frac{(1-q) F(x)}{(1-q) F(\xi(r)) + \int_{\xi(r)}^{c_d(r)} (1-p(z)) dF(z)} & \text{if } x \in [0,\xi(r)) \\
\frac{(1-q) F(\xi(r)) + \int_{\xi(r)}^{c_d(r)} (1-p(z)) dF(z)}{(1-q) F(\xi(r)) + \int_{\xi(r)}^{c_d(r)} (1-p(z)) dF(z)} & \text{if } x \in [\xi(r),c_D(r)]
\end{cases}
\]

With \( F(\cdot) \) well-behaved, \( \hat{F}_d(\cdot) \) is the CDF of a non-degenerate distribution. \( \square \)

We can now establish \( R^*_2(r|d) = r/q \). Given (A.1), it will be enough to prove that \( \omega_d(r) > c_d(r) \). The proof proceeds by contradiction. In particular, if \( \omega_d(r) \leq c_d(r) \), then a type \( c_d(r) \) borrower will choose \( P1 \) if offered loan rate \( R_d(r) \), permitting his insider an expected profit of \( \Pi = p(c_d(r)) R_d(r) - r \). From Lemma 1, we know that \( \hat{F}_d(\cdot) \) is not degenerate at \( c_d(r) \) so \( \omega_d(r) \leq c_d(r) \) in (A.1) implies \( R_d(r) > r/p(c_d(r)) \) and, therefore, \( \Pi > 0 \). From Proposition 2 though, \( J_2(r|c_d(r),d) = 0 \) so the fact that \( J_2(r|\cdot) \) is a maximum
value function implies $\Pi \leq 0$. By contradiction then, $\omega_d(r) > c_d(r)$ and $\overline{R}(c_d(r)) < R^*_2(r|d) = r/q$.

To determine the value of $c_d(r)$, note that an insider with a type $c_d(r)$ borrower can make an expected profit of $p(c_d(r))\overline{R}(c_d(r)) - r \leq J_2(r|c_d(r), d) = 0$ by charging him $\overline{R}(c_d(r))$. If the inequality is strict, then there exists a $\delta > 0$ such that $\overline{R}(c_d(r) + \delta) < r/q$ and $p(c_d(r) + \delta)\overline{R}(c_d(r) + \delta) - r < 0$. Therefore, the best an insider can do with a type $c_d(r) + \delta$ borrower is charge $R^*_2(r|d) = r/q$ and break even but, given that insiders only keep borrowers which yield them positive profit, this contradicts the fact that they keep all $\omega > c_d(r)$. In equilibrium then, $c_D(r)$ and $c_N(r)$ are implicitly defined by $p(\cdot)\overline{R}(\cdot) = r$. This is the same equation that defines $\tilde{\omega}(r)$, implying $c_D(r) = c_N(r) = \tilde{\omega}(r)$.

Since we now know that the insider only keeps $\omega > \tilde{\omega}(r)$, we can restrict attention to $r < p(\omega)\overline{R}(\omega)$ in order to prove (1.7). Consider first $r \in (q\overline{R}(\omega), p(\omega)\overline{R}(\omega))$. From the proof of Proposition 1, we know that an informed lender would rather charge $\overline{R}(\omega)$ than $r/q$ for these policy rates. Therefore, since the insider cannot charge above the outsider offer of $R^*_2(r|d) = r/q$ and keep the borrower, he will charge $R^*_2(r|\omega, d) = \overline{R}(\omega)$ and get $J_2(r|\omega, d) = p(\omega)\overline{R}(\omega) - r > 0$. Consider now $r \leq q\overline{R}(\omega)$. In this case, the outsider’s offer falls below the borrower’s reservation loan rate and the best the insider can do is match it, yielding $R^*_2(r|\omega, d) = r/q$ and $J_2(r|\omega, d) = (p(\omega) - q) r/q$. Recalling that $r \in (q\overline{R}(\omega), p(\omega)\overline{R}(\omega))$ corresponds to $\omega \in (\tilde{\omega}(r), \tilde{\omega}(r))$ and $r \leq q\overline{R}(\omega)$ corresponds to $\omega \geq \tilde{\omega}(r)$ produces $R^*_2(r|\omega, d)$ as in (1.7).

**Derivation of Figure 1**

(a) Consider first $\tilde{\omega}(r)$. At $r = 0$, we need $q\overline{R}(\tilde{\omega}(0)) = 0$ which occurs if and only if $\tilde{\omega}(0) = 0$. Since $\overline{R}(\cdot)$ is increasing, $\tilde{\omega}(r)$ is increasing for $r \in (0, q\overline{R}(1))$ and equal to 1 for $r \geq q\overline{R}(1)$. Consider now $\tilde{\omega}(r)$. Once again, $r = 0$ yields $\tilde{\omega}(0) = 0$. The fact that $p(\cdot)\overline{R}(\cdot)$ is increasing implies that $\tilde{\omega}(r)$ is increasing for $r \in (0, p(1)\overline{R}(1))$ and equal to 1 for $r \geq p(1)\overline{R}(1)$. At policy rate $r$, the set of types for which an insider charges reservation
rates has length \( |\tilde{\omega}(r) - \tilde{\omega}(r)| \) (i.e., the vertical distance between the \( \tilde{\omega}(r) \) and \( \tilde{\omega}(r) \) curves).

Since \( \omega \in (\tilde{\omega}(r), \hat{\omega}(r)) \) if and only if \( r \in (qR(\omega), p(\omega)R(\omega)) \), the range of policy rates over which an insider charges type \( \omega \) his reservation rate has length \( |p(\omega)R(\omega) - qR(\omega)| \) (i.e., the horizontal distance between the \( \tilde{\omega}(r) \) and \( \tilde{\omega}(r) \) curves). Having \( p(\cdot) > q \) establishes \( \tilde{\omega}(r) \leq \tilde{\omega}(r) \) and having \( p'(\cdot) > 0 \) establishes \( \frac{d}{dr} |p(\omega)R(\omega) - qR(\omega)| > 0 \).

(b) Consider \( \eta(r) \). As before, \( \eta(0) = 0 \) is immediate. Define:

\[
g(\omega) \equiv \left[ \int_0^\omega q dF(x) + \int_0^1 p(x) dF(x) \right] R(\omega)
\]

Note that \( q\theta_2 = p(0)\theta_1 \) allows us to write:

\[
g'(\omega) = - \left[ p(\omega) - p(0) \right] f(\omega)\theta_1 + \left( \frac{p'(\omega)(p(0) - q)}{p(\omega) - q} \right) g(\omega)
\]

Since \( g(0) = 0 \) and \( g(\omega) > 0 \) for all \( \omega \in (0, 1) \), it must be the case that \( g'(0) > 0 \). Lemma 2 establishes the sign of \( g'(\omega) \) over \( \omega \in (0, 1) \) under the assumptions in Subsection 1.3.3:

**Lemma 2** Under Assumptions 1 to 3, \( g(\omega) \) is monotonically increasing over \( \omega \in (0, 1) \).

**Proof.** If the claim is not true, then there must be at least one \( z \in (0, 1) \) such that \( g'(z) = 0 \). Using the expression for \( g'(\cdot) \) presented above, this \( z \) is implicitly defined by

\[
g(z) = \frac{[p(z) - q][p(z) - p(0)]^2 f(z)\theta_1}{p'(z)[p(0) - q]^2}.
\]

Denoting the right side of this equation by \( h(\cdot) \), we can say that \( g(\cdot) \) achieves a local optimum every time it intersects \( h(\cdot) \). In other words, \( g'(\cdot) \) switches signs at any \( z \) such that \( g(z) = h(z) \). Recalling \( g'(0) > 0 \), it follows that \( g(\cdot) \) begins to decrease after its first intersection with \( h(\cdot) \). Under the assumptions, \( h'(\cdot) > 0 \) and there can be no further intersections. Therefore, if there is indeed a \( z \in (0, 1) \) such that \( g(z) = h(z) \), then we will have \( g'(1) < 0 \). Taken together, \( g'(1) < 0 \) and \( f(1) \leq 1 \) imply \( p(0) < q + \frac{[p(1) - q]^3}{qp'(1) + [p(1) - q]^2} \), violating Assumption 2. As a result, there cannot be a \( z \in (0, 1) \) such that \( g(z) = h(z) \) and, by implication, \( g(\cdot) \) must be monotonically increasing over \( \omega \in (0, 1) \). \( \square \)

With \( g(\omega) \) increasing, the minimization problem in (1.13) yields only one \( \text{argmin} \). Moreover, \( \eta(r) \) is increasing for \( r \in (0, qR(1)) \) and equal to 1 for \( r \geq qR(1) \). Consider now...
all points such that \( \tilde{\omega}(r) = \eta(r) = z \). Using the definitions of \( \tilde{\omega}(r) \) and \( \eta(r) \), any such \( z \in (0, 1) \) must satisfy 
\[
p(z) = \left[ \int_0^z q dF(\omega) + \int_z^1 p(\omega) dF(\omega) \right] = u(z).
\]
Since \( u'(\cdot) < 0 \), \( p'(\cdot) > 0 \), \( u(0) > p(0) \), and \( u(1) < p(1) \), we know that \( p(\cdot) \) and \( u(\cdot) \) intersect only once. Therefore, there is only one point such that \( \eta(r) = \tilde{\omega}(r) \in (0, 1) \). Since both \( \eta(r) \) and \( \tilde{\omega}(r) \) are increasing, this implies that there is only one \( r \in (0, p(1) \overline{R}(1)) \) such that \( \eta(r) = \tilde{\omega}(r) \).

Moreover, \( \eta(0) = \tilde{\omega}(0) = 0 \) and \( \tilde{\omega}'(0) = \frac{1}{p(0)\overline{R}(0)} > \frac{1}{u(0)\overline{R}(0)} = \eta'(0) \) imply that \( \eta(r) \) approaches its intersection with \( \tilde{\omega}(r) \) from below \( \tilde{\omega}(r) \).

Consider now \( \xi(r) \). We can rewrite (1.9) as:

\[
\xi(r) = \arg \min_{\omega \in [0, 1]} \left| g(\omega) - \left[ r - \beta \int_{\tilde{\omega}(r)}^{1} J_2(r|x) dF(x) \right] \right|
\]

By Lemma 2, \( g'(\omega) > 0 \) so this \( \arg\min \) is unique and \( \xi(0) = 0 \). Furthermore, \( \eta(r) \) defined as the \( \arg\min \) of \( |g(\cdot) - r| \) and \( g(1) = \overline{qR}(1) \) imply \( \xi(r) < \eta(r) \) for all \( r \in (0, \overline{qR}(1)) \).

At \( r = p(1) \overline{R}(1) \) though, \( \tilde{\omega}(r) = 1 \) so \( \xi(r) = \eta(r) = 1 \). Therefore, \( \xi(r) \) reaches 1 for \( r \in (\overline{qR}(1), p(1) \overline{R}(1)) \). Now, using Proposition 3:

\[
\int_{\tilde{\omega}(r)}^{1} J_2(r|x) dF(x) = \int_{\tilde{\omega}(r)}^{1} \left[ p(x) \overline{R}(x) - r \right] dF(x) + \int_{\tilde{\omega}(r)}^{1} \left[ \frac{p(x)}{q} - 1 \right] rdF(x)
\]

Assumption 4 ensures that \( r - \beta \int_{\tilde{\omega}(r)}^{1} J_2(r|x) dF(x) \) is increasing in \( r \) so \( \xi(r) \) is also increasing until it reaches 1. Let us now examine all policy rates such that \( \xi(r) = \tilde{\omega}(r) \in (0, 1) \). Using the definitions of \( \tilde{\omega}(r) \) and \( \xi(r) \), any such \( r \) must satisfy:

\[
\frac{1}{\beta \int_{\tilde{\omega}(r)}^{1} q dF(x) + \int_{\tilde{\omega}(r)}^{1} p(x) dF(x)} + \beta \frac{1}{\int_{\tilde{\omega}(r)}^{1} J_2(r|x) dF(x)} = 1
\]

The first term on the left hand side is clearly decreasing in \( r \). The second term is also decreasing since \( \frac{d}{dr} \int_{\tilde{\omega}(r)}^{1} J_2(r|x) dF(x) = -\int_{\tilde{\omega}(r)}^{1} \frac{p(x) \overline{R}(x)}{r^2} dF(x) < 0 \). Therefore, the left hand side (LHS) is decreasing in \( r \) while the right hand side (RHS) is constant. Moreover, at \( r = 0 \), we have \( LHS = \infty > 1 = RHS \) and, at \( r = p(1) \overline{R}(1) \), we have \( LHS = \frac{q}{p(1)} < 1 = RHS \). Therefore, there is a unique \( r \in (0, p(1) \overline{R}(1)) \) such that \( \tilde{\omega}(r) = \xi(r) \). Moreover,
since $\xi (r) \leq \eta (r)$ and $\eta (r)$ is initially below $\omega (r)$, we know that $\xi (r)$ also approaches its intersection with $\omega (r)$ from below $\omega (r)$.

Note that Assumption 3 also identifies the curvature of $\omega (r)$. Consider $r \in (0, p (1) \bar{R} (1))$. Differentiating both sides of $p (\omega) \bar{R} (\omega) = r$ yields $\frac{d \omega}{dr} = \frac{p (\omega) - q}{p' (\omega) - q}$ and, therefore, $\frac{d^2 \omega}{dr^2} \propto 2 p' (\omega) q \left[ \frac{\theta_1 - \bar{R} (\omega)}{2 p (\omega) \theta_1 - q \theta_2 - r} \right] - p'' (\omega) [p (\omega) - q]$. It can be shown that $\bar{R} (\omega) < \theta_1$ so the first term in $\frac{d^2 \omega}{dr^2}$ is positive and, with $p'' (\cdot)$ sufficiently low, $\omega (r)$ is convex. A similar procedure establishes the convexity of $\omega (r)$ over $r \in (0, q \bar{R} (1))$. All other properties of $\omega (r)$ and $\omega (r)$ carry over from part (a).

Proof of Proposition 4

I start by establishing the convexity of $\eta (r)$ and $\xi (r)$ over policy rates where these cutoffs are interior (recall that the convexity of $\omega (r)$ was established in the previous paragraph).

With $\eta (r)$ defined by $\left[ q \eta + \int_{q}^{1} p (x) \, dx \right] \bar{R} (\eta) = r$, we have:

$$\frac{d\eta}{dr} = \frac{1}{- [p (\eta) - p (0)] \theta_1 + \left[ q \eta + \int_{q}^{1} p (x) \, dx \right] \bar{R}' (\eta)}$$

$$\frac{d^2 \eta}{dr^2} = \eta' (r)^3 \left( p' (\eta) \theta_1 + [p (\eta) - q] \bar{R}' (\eta) - \left[ q \eta + \int_{q}^{1} p (x) \, dx \right] \bar{R}'' (\eta) \right)$$

From the derivation of Figure 1, $\eta' (r) > 0$. Moreover, $\bar{R}'' (\omega) \propto p'' (\omega) [p (\omega) - q] - 2 p' (\omega)^2$ so Assumption 3 ensures $\bar{R}'' (\omega) < 0$ and, thus, $\eta'' (r) > 0$. Turn now to $\xi (r)$. Differentiating the equation which implicitly defines $\xi (r)$ yields:

$$\frac{d\xi}{dr} = \frac{1 - \beta A' (r)}{- [p (\xi) - q] \bar{R} (\xi) + \left[ q \xi + \int_{0}^{1} p (x) \, dx \right] \bar{R}' (\xi)}$$

where: $A (r) \equiv \int_{\omega (r)}^{\xi (r)} \left[ p (x) \bar{R} (x) - r \right] \, dx + \int_{\omega (r)}^{1} \left[ \frac{p (x)}{q} - 1 \right] r \, dx$

After some algebra, we get that the second derivative satisfies:

$$\frac{d^2 \xi}{dr^2} \propto - \beta A'' (r) + \left[ p' (\xi) \bar{R} (\xi) + 2 [p (\xi) - q] \bar{R}' (\xi) - \left[ q \xi + \int_{0}^{1} p (x) \, dx \right] \bar{R}'' (\xi) \right] \xi' (r)^2$$
Since $\tilde{R}''(\cdot) < 0$, a sufficient condition for $\xi''(r) > 0$ is $A''(r) < 0$. With $p(\cdot) > q$ and $A''(r) = \tilde{\omega}'(r) - \frac{p(\tilde{\omega}(r))}{q} \tilde{\omega}'(r)$, this sufficient condition is guaranteed by $\tilde{\omega}'(r) < \tilde{\omega}'(r)$. Note that $\tilde{\omega}'(r) < \tilde{\omega}'(r)$ is equivalent to $p'(\tilde{\omega}) \tilde{R}(\tilde{\omega}) + p(\tilde{\omega}) \tilde{R}'(\tilde{\omega}) > q \tilde{R}''(\tilde{\omega})$, which is certainly true since $\tilde{\omega} \leq \tilde{\omega}$ and $\tilde{R}''(\cdot) < 0$ imply that $\tilde{R}(\tilde{\omega}) > \tilde{R}'(\tilde{\omega})$. Thus, $A''(r) < 0$ and $\xi''(r) > 0$.

Turn now to the relevance of these results for steady state smoothness. That $\eta(r)$ is an increasing and convex function of $r$ yields:

$$Y_S'(r) = -[p(1) - p(0)] \theta_1 \eta(r) \eta'(r) < 0$$

$$Y_S''(r) = -[p(1) - p(0)] \theta_1 [\eta'(r)^2 + \eta(r) \eta''(r)] < 0$$

Moreover, since $\xi(r)$ and $\tilde{\omega}(r)$ are also increasing and convex, we have:

$$Y'(r) = -[p(1) - p(0)] \theta_1 \left[ \frac{\mu}{1+\mu} \xi(r) \xi'(r) + \frac{1}{1+\mu} \tilde{\omega}(r) \tilde{\omega}'(r) \right] < 0$$

$$Y''(r) = -[p(1) - p(0)] \theta_1 \left[ \frac{\mu}{1+\mu} [\xi'(r)^2 + \xi(r) \xi''(r)] + \frac{1}{1+\mu} [\tilde{\omega}'(r)^2 + \tilde{\omega}(r) \tilde{\omega}''(r)] \right] < 0$$

The concavity of $Y_S(r)$ and $Y(r)$ simplifies the smoothness measure. In particular, if $y(r)$ is a concave function, we can write:

$$S = -\int_a^b y''(r) \, dr = -[y'(b) - y'(a)]$$

The right-hand side limits of $Y'(0)$ and $Y_S'(0)$ are both zero so establishing the desired smoothness result amounts to establishing $Y'(q\tilde{R}(1)) > Y_S'(q\tilde{R}(1))$. Consider first $\mu = 0$. In this case, we want to establish $\tilde{\omega}(q\tilde{R}(1)) \tilde{\omega}'(q\tilde{R}(1)) < \eta(q\tilde{R}(1)) \eta'(q\tilde{R}(1))$. Using $\eta(q\tilde{R}(1)) = 1$ and the derivatives presented earlier, this amounts to showing:

$$\tilde{\omega}(q\tilde{R}(1)) \left[ \frac{p(1) - p(0)}{p(\tilde{\omega}) - q} \tilde{\omega} + p(\tilde{\omega}) \frac{|p(0) - q|}{|p(\tilde{\omega}) - q|^2} \right] < \frac{1}{1 + \frac{|p(0) - q|}{|p(1) - q|}}$$

A sufficient condition for the preceding to be true is $\frac{q}{|p(1) - q|^2} < \frac{p(\tilde{\omega})}{|p(\tilde{\omega}) - q|^2}$, which is guaranteed
by \( p(1) > p(\tilde{\omega}) > q \). Therefore, for \( \mu = 0 \), \( Y(r) \) is smoother than \( Y_S(r) \). Since \( \mu = 0 \) puts all the weight on \( Y_{k \geq 3}(r) \) in the calculation of \( Y(r) \), we can conclude that \( Y_{k \geq 3}(r) \) is smoother than \( Y_S(r) \). As \( \mu \) increases, weight shifts from \( Y_{k \geq 3}(r) \) to \( Y_1(r) \) so, to show that \( Y(r) \) is smoother than \( Y_S(r) \) for all \( \mu \), it will be enough to show that \( Y(r) \) is smoother than \( Y_S(r) \) for \( \mu = 1 \). In this case, what we want to establish is:

\[
\xi(qR(1)) \xi'(qR(1)) + \tilde{\omega}(qR(1)) \tilde{\omega}'(qR(1)) < 2\eta(qR(1)) \eta'(qR(1))
\]

Substituting in the derivatives (and suppressing the argument \( r \)), the above inequality becomes:

\[
\frac{\xi(1 + \beta[1 - \tilde{\omega}])}{[p(1) - p(0)] \theta_1 \left( -\xi + \left[q\xi + \int_{\xi}^1 p(x) \, dx \right] \frac{|p(0) - q|}{|p(1) - q|^2} \right)} + \frac{\tilde{\omega}}{p'(\tilde{\omega}) R(\tilde{\omega}) + p(\tilde{\omega}) \tilde{R}'(\tilde{\omega})} < \frac{2}{[p(1) - p(0)] \theta_1 \left( -1 + q \frac{|p(0) - q|}{|p(1) - q|^2} \right)}
\]

(A.2)

A sufficient condition can be established by noting the following:

\[
\frac{\xi(1 + \beta[1 - \tilde{\omega}])}{[p(1) - p(0)] \theta_1 \left( -\xi + \left[q\xi + \int_{\xi}^1 p(x) \, dx \right] \frac{|p(0) - q|}{|p(1) - q|^2} \right)} < \frac{2 - \tilde{\omega}}{[p(1) - p(0)] \theta_1 \left( -1 + q \frac{|p(0) - q|}{|p(1) - q|^2} \right)}
\]

In particular, substituting this upper bound into (A.2) and rearranging yields the sufficient condition below:

\[
[p(1) - p(0)] \theta_1 \left( -1 + q \frac{|p(0) - q|}{|p(1) - q|^2} \right) < p'(\tilde{\omega}) R(\tilde{\omega}) + p(\tilde{\omega}) \tilde{R}'(\tilde{\omega})
\]

Replacing \( R(\tilde{\omega}) \) and \( \tilde{R}'(\tilde{\omega}) \) with the appropriate expressions, the sufficient condition then becomes:

\[
q \frac{|p(0) - q|}{|p(1) - q|^2} < 1 + \frac{p(\tilde{\omega}) - p(0)}{p(\tilde{\omega}) - q} + p(\tilde{\omega}) \frac{|p(0) - q|}{|p(\tilde{\omega}) - q|^2}
\]

This inequality is certainly satisfied if \( \frac{q}{|p(1) - q|^2} < \frac{p(\tilde{\omega})}{|p(\tilde{\omega}) - q|^2} \), which is again guaranteed by
\( p(1) > p(\tilde{\omega}) > q. \) Therefore, \( Y(r) \) is smoother than \( Y_S(r) \) when smoothness is defined according to \( S. \)

Consider now \( \hat{S}. \) For a decreasing function \( y(r) \), we can write:

\[
\hat{S} = - \int_a^b y'(r) \, dr = - [y(b) - y(a)]
\]

Since \( Y(0) = Y_S(0) \), establishing that \( Y(r) \) is smoother than \( Y_S(r) \) when smoothness is defined according to \( \hat{S} \) amounts to establishing \( Y(q\bar{R}(1)) > Y_S(q\bar{R}(1)) \). A sufficient condition is \( \max \{ \tilde{\omega}(q\bar{R}(1)), \xi(q\bar{R}(1)) \} < \eta(q\bar{R}(1)) = 1 \), which was shown to be true in the derivation of Figure 1. \( \blacksquare \)

**Proof of Proposition 5**

Suppose an unexpected shock hits at \( t = 1 \) (i.e., \( \varepsilon_1 \neq 0 \) but \( \varepsilon_t = 0 \) for all \( t > 1 \)) and an additional \( T - 1 \) periods are required for the policy rate to return to its steady state value. Without relationship lending, output at time \( t \) is just \( Y_S(r_t) \), where \( Y_S(\cdot) \) is as defined in (1.14). The variance of the transition path relative to steady state in the standard credit model is thus:

\[
\sigma^2_S \equiv \frac{1}{T} \sum_{t=1}^{T} [Y_S(r_t) - Y_S(r_{ss})]^2
\]

The analogous measure for the model with relationship lending is:

\[
\sigma^2_R \equiv \frac{1}{T} \sum_{t=1}^{T} [Y_t(r_t) - Y(r_{ss})]^2
\]

Note that \( Y_t(r_t) \) is not necessarily equal to \( Y(r_t) \). In addition to the current cost of funds, first period output depends on expectations of future relationship profits and, therefore, expectations of future policy rates. With \( r_{t+1} = r_{ss} + \alpha(r_{ss} - r_t) \) for \( t \geq 1 \), the appropriate first period cutoff at time \( t \) is:

\[
\xi_t = \arg\min_{\omega \in [0, 1]} \left\{ q\omega + \int_0^1 p(x) \, dx \right\} \bar{R}(\omega) + \beta \int_{\omega(r_{t+1})}^{1} J_2(r_{t+1} | x) \, dx - r_t
\]
Expected relationship profits equal $J_{2}^{1}(r | x) dx$ and, at high policy rates, this expression is decreasing in $r$. If $\varepsilon_{1} > 0$, then $r_{t+1} < r_{t}$ for all $t \geq 1$ and, therefore, $J_{2}^{1}(r_{t+1} | x) dx > J_{2}^{1}(r_{t} | x) dx$. We know from Lemma 2 that $\left[ q_{\omega} + \int_{0}^{1} p(x) dx \right] \overline{R}(\omega)$ is increasing in $\omega$ so it must be the case that $\xi_{t} < \xi(\bar{r}_{t})$, where $\xi(\cdot)$ is as defined in (1.9). In other words, $Y_{t}(r_{t}) > Y(r_{t})$. Since $\varepsilon_{1} > 0$ yields policy rates that are always above $r_{ss}$ along the transition path, we can further conclude that $Y(r_{ss}) > Y_{t}(r_{t}) > Y(r_{t})$ and thus $\sigma_{R}^{2} < \frac{1}{T} \sum_{t=1}^{T} [Y(r_{t}) - Y(r_{ss})]^{2}$. If instead $\varepsilon_{1} < 0$, a similar argument establishes $Y(r_{ss}) < Y_{t}(r_{t}) < Y(r_{t})$ and, once again, $\sigma_{R}^{2} < \frac{1}{T} \sum_{t=1}^{T} [Y(r_{t}) - Y(r_{ss})]^{2}$.

A sufficient condition for $\sigma_{R}^{2} < \sigma_{S}^{2}$ is then $\sum_{t=1}^{T} [Y(r_{t}) - Y(r_{ss})]^{2} < \sum_{t=1}^{T} [Y_{S}(r_{t}) - Y_{S}(r_{ss})]^{2}$ and, since $r_{t} \in [\min \{r_{ss}, r_{1}\}, \max \{r_{ss}, r_{1}\}] \equiv \mathbf{r}$ along the transition path, having $|Y'(\cdot)| < |Y_{S}'(\cdot)|$ over this interval would guarantee it. Recalling the expressions for $Y'(r)$ and $Y_{S}'(r)$ from the proof of Proposition 4, what we would like to show is:

$$\frac{\mu}{1+\mu} \xi(r) \xi'(r) + \frac{1}{1+\rho} \bar{\omega}(r) \bar{\omega}'(r) < \eta(r) \eta'(r)$$

Consider first $\mu = 0$. Since $\bar{\omega}(r) < \eta(r)$ for large values of $r$, it will be enough to show that $\bar{\omega}'(r) < \eta'(r)$ or, equivalently:

$$-\eta + \left[ q_{\eta} + \int_{0}^{1} p(x) dx \right] \frac{[p(0) - q]}{[p(\eta) - q]} < \left[ \frac{p(1) - p(0)}{p(\bar{\omega}) - q} \right] \bar{\omega} + p(\bar{\omega}) \frac{[p(0) - q]}{[p(\bar{\omega}) - q]^{2}}$$

A sufficient condition for the above inequality is $q_{\eta} + \int_{0}^{1} p(x) dx < p(\bar{\omega})$. To see that this is indeed true, recall $\left[ q_{\eta} + \int_{0}^{1} p(x) dx \right] \overline{R}(\eta) = r = p(\bar{\omega}) \overline{R}(\bar{\omega})$ and $\overline{R}'(\cdot) > 0$. With $\bar{\omega} < \eta$, we have $\overline{R}(\bar{\omega}) < \overline{R}(\eta)$ and, thus, $q_{\eta} + \int_{0}^{1} p(x) dx < p(\bar{\omega})$. We can now conclude that $|Y'(r)| < |Y_{S}'(r)|$ for $r \in \mathbf{r}$ when $\mu = 0$.

Note that $\mu = 0$ puts all the weight on $Y_{k>3}(r)$ so what we have just shown is that $Y_{k>3}(r)$ is flatter than $Y_{S}(r)$ for $r \in \mathbf{r}$. As $\mu$ increases, weight shifts from $Y_{k>3}(r)$ to $Y_{1}(r)$. Since $Y_{k>3}(r)$ is assigned the smallest weight when $\mu = 1$, it will be enough to show that
Consider then $\mu = 1$. What we want to show now is $\xi(r) \xi'(r) + \overline{\omega}'(r) < 2\eta(r) \eta'(r)$. To do so, define $h(\omega) \equiv -\omega + \left[q \omega + \int_{\omega}^{1} p(x) \, dx\right] \frac{\left[p(0) - q\right]}{\left[p(\omega) - q\right]}$ and rewrite $\eta'(r)$ and $\xi'(r)$ as:

$$\eta'(r) = \frac{1}{\left[p(1) - p(0)\right] \theta_1 h(\eta)}$$

$$\xi'(r) = \frac{1 + \beta \left[1 - \overline{\omega}\right] - \beta \int_{\overline{\omega}}^{1} p(x) \, dx}{\left[p(1) - p(0)\right] \theta_1 h(\xi)} = \frac{2 - \overline{\omega}}{\left[p(1) - p(0)\right] \theta_1 h(\xi)}$$

Since $h'(\cdot) < 0$ and $\xi < \eta$, we know that $h(\xi) > h(\eta)$ and, therefore, $\xi'(r) < \frac{2 - \overline{\omega}}{\left[p(1) - p(0)\right] \theta_1 h(\eta)}$. A sufficient condition for what we currently want to show is thus $\overline{\omega}'(r) < \frac{\eta}{\left[p(1) - p(0)\right] \theta_1 h(\eta)}$.

Substituting in for $\overline{\omega}'(r)$, this sufficient condition becomes:

$$\frac{1}{\left[p(\overline{\omega}) - p(0)\right] \theta_1 h(\overline{\omega})} + p(\overline{\omega}) \frac{\left[p(0) - q\right]}{\left[p(\overline{\omega}) - q\right]} - \eta + \left[q \eta + \int_{\eta}^{1} p(x) \, dx\right] \frac{\left[p(0) - q\right]}{\left[p(\eta) - q\right]} \leq \frac{\eta}{\left[p(1) - p(0)\right] \theta_1 h(\eta)}$$

(A.3)

Recall that $p(\overline{\omega}) \overline{R}(\overline{\omega}) = \left[q \eta + \int_{\eta}^{1} p(x) \, dx\right] \overline{R}(\eta)$ and, under $p(\cdot)$ linear, $\overline{R}(\omega) = \frac{\left[p(1) - p(0)\right] \theta_1 h(\omega)}{p(\omega) - q}$. Combining these equations yields $q \eta + \int_{\eta}^{1} p(x) \, dx = p(\overline{\omega}) \frac{\overline{\omega} \left[p(\eta) - q\right]}{\eta \left[p(\overline{\omega}) - q\right]}$ and lets us write (A.3) as:

$$-1 + p(\overline{\omega}) \left[p(0) - q\right] \frac{\overline{\omega}}{\left[p(\overline{\omega}) - q\right]} \left[\frac{1}{\eta} - 1\right] - \frac{1}{\left[p(\overline{\omega}) - q\right]} < \frac{p(\overline{\omega}) - p(0)}{p(\overline{\omega}) - q}$$

With $0 < \overline{\omega} < \eta$, a sufficient condition for the preceding inequality is $p(\overline{\omega}) \left[\frac{1}{\eta} - 1\right] < 2p(\overline{\omega}) - p(0) - q$ or, equivalently, $\eta > \frac{p(\overline{\omega})}{3p(\overline{\omega}) - p(0) - q}$. In other words, (A.3) is guaranteed for $\eta$ sufficiently large. Since $\eta'(r) > 0$, it then follows that (A.3) is guaranteed for $r$ sufficiently large. 

### A.2 An Extension with Distributional Dynamics

The results of the main text suggest that, on average, the informational properties of relationship lending lead to improved credit terms. In reality though, borrowers with improved
terms may be better able to overcome adverse idiosyncratic events that would have otherwise put them out of business. Along with learning then, relationship lending may also foster lower firm exit rates. To understand the implications of this possibility, I extend the baseline model. In particular, as long as a borrower stays with his insider, he experiences exogenous separation with probability $\mu - \varepsilon$, where $\varepsilon > 0$. If or once he switches to an outsider, separation occurs with probability $\mu$.

As noted in Subsection 1.6.2, the difference in separation rates has two important implications. First, it gives insiders more bargaining power over high types so insiders can now charge slightly above the outsider offer without losing these borrowers. Second, it provides a new source of transition dynamics by making the policy rate affect the distribution of borrowers across periods. In this section, I set up the extended model more formally.

**Value Functions**

Outsiders still compete against each other and make zero expected profits so their $k \geq 3$ and $k = 2$ value functions are of the same form as those in Subsections 1.3.1 and 1.3.2. In contrast, the $k = 2$ value function of an insider is now:

$$J_{2,I}(r|\omega, d) = \max \left\{ 0, \left\{ \max_{R} \gamma(R|\omega) R - r + \beta (1 - \mu + \varepsilon) J_{k \geq 3,I}(r|\omega) \right\} \right\}$$

where $V_{2,I}(\omega|R)$ is the value of a second-time type $\omega$ borrower who stays with his insider and pays loan rate $R$ while $V_{2,U}(\omega|R_{2,U,d})$ is the value of this borrower should he move to an outsider charging $R_{2,U,d}$. The insider’s value function for any $k \geq 3$ is also given by the right hand side of (A.4) but with $V_{k \geq 3,I}(\omega|R) \geq V_{k \geq 3,U}(\omega|R_{k \geq 3,U,w})$ as the borrower’s participation constraint.

For the baseline model (i.e., $\varepsilon = 0$), it was proven that second-time borrowers are not divided according to default history. In the absence of an analytical solution for the extended model, there is no presumption that this is still the case. Therefore, second period loan rates are not restricted to be history-independent and the first period borrower strategy is now
denoted by \( \overline{\tau}_1 (R|\omega) \) to distinguish it from \( \overline{\tau} (R|\omega) \). The value function of a first period lender is then similar to equation (1.8) in the main text except that \( \overline{\tau}_1 (\cdot) \) is used instead of \( \overline{\tau} (\cdot) \) and expected future profits are determined using (A.4) and the market splitting that results.

Consider now the borrower side. For a first-time borrower facing loan rate \( R \), the expected payoffs associated with choosing \( P_1 \) and \( P_2 \) are given by (A.5) and (A.6) respectively:

\[
p(\omega) [\theta_1 - R + \beta V_2 (\omega|R_{2,I,N}, R_{2,U,N})] + (1 - p(\omega)) \beta V_2 (\omega|R_{2,I,D}, R_{2,U,D}) \tag{A.5}
\]

\[
q [\theta_2 - R + \beta V_2 (\omega|R_{2,I,N}, R_{2,U,N})] + (1 - q) \beta V_2 (\omega|R_{2,I,D}, R_{2,U,D}) \tag{A.6}
\]

where \( V_2 (\omega|R_{2,I,d}, R_{2,U,d}) = \max \{V_{2,I} (\omega|R_{2,I,d}), V_{2,U} (\omega|R_{2,U,d})\} \) is the borrower’s second period value function. His first period value, \( V_1 (\omega|R) \), is given by the maximum of (A.5) and (A.6) and his strategy is \( \overline{\tau}_1 (R|\omega) = p(\omega) \) if and only if (A.5) is greater than (A.6).

Finally, to determine \( V_2 (\omega|R_{2,I,d}, R_{2,U,d}) \), note that:

\[
V_{2,I} (\omega|R) = V_{k\geq 3,I} (\omega|R) = \max \{p(\omega) [\theta_1 - R], q[\theta_2 - R]\} + \beta (\mu - \varepsilon) \int_0^1 V_1 (x|R_1) dF(x) + \beta (1 - \mu + \varepsilon) \max \{V_{k\geq 3,I} (\omega|R_{k\geq 3,I,\omega}), V_{k\geq 3,U} (\omega|R_{k\geq 3,U,\omega})\}
\]

\[
V_{2,U} (\omega|R) = V_{k\geq 3,U} (\omega|R) = \max \{p(\omega) [\theta_1 - R], q[\theta_2 - R]\} + \beta \mu \int_0^1 V_1 (x|R_1) dF(x) + \beta (1 - \mu) V_{k\geq 3,U} (\omega|R_{k\geq 3,U,\omega})
\]

**Transition Dynamics**

Suppose an unanticipated permanent increase in \( r \) occurs at date \( t \). Lenders with advanced borrowers can adjust immediately to the new steady state but this may not be true for lenders with intermediate borrowers. Recall that the only piece of information available to a second period outsider is whether the borrower defaulted on his first period loan and, at date \( t \), this outcome depends on loan rates induced by the \( t - 1 \) policy rate. In the insider’s problem, however, expected future profits depend on loan rates induced by the \( t + 1 \) policy.
rate. Since an equilibrium is reached when each lender’s offer is a best response to the other’s, the second period loan rates that prevail at date $t$ depend on both pre-shock and post-shock policy rates. In contrast, the post-shock steady state is conditioned entirely on the post-shock policy rate so the $k = 2$ equilibrium at date $t$ may differ from the new $k = 2$ steady state.

To determine how long it takes to reach the new steady state, consider the market for new borrowers. If first period lenders at date $t$ expect a full adjustment by date $t + 1$, then they will adjust immediately. As a result, both outsider information and insider profits at date $t + 1$ will be conditioned on the new policy rate. This means that the $k = 2$ equilibrium will reach the new steady state by date $t + 1$, consistent with the time $t$ expectations of first period lenders. In what follows, I focus on this case. That is, all contracts adjust to the new steady state by date $t + 1$.\textsuperscript{42} Note, however, that even with a quick contract response, the effects of the policy rate shock continue to be propagated through the distribution. We can see this by formalizing the borrower flows. Let $I_{2,d,t}(\omega)$ be an indicator function that equals 1 if a second-time borrower with type $\omega$ and default history $d$ stays with his insider at date $t$. Similarly, define $I_{k \geq 3,t}(\omega)$ so that $I_{k \geq 3,t}(\omega) = 1$ if $V_{k \geq 3,t}(\omega | R_{k \geq 3,I,t}) \geq V_{k \geq 3,U}(\omega | R_{k \geq 3,U,t})$.

The mass of $k = 1$ borrowers at date $t + 1$ is now:

$$\Psi_{1,t+1} = (\mu - \varepsilon) \int_0^1 \left[ \psi_{2,I,t}(x) + \psi_{k \geq 3,I,t}(x) \right] dx + \mu \int_0^1 \left[ \psi_{2,U,t}(x) + \psi_{k \geq 3,U,t}(x) \right] dx$$

where $\psi_{2,I,t}(\cdot)$ is the distribution of borrower types across $k = 2$ insiders and $\psi_{2,U,t}(\cdot)$ is the distribution of borrower types across $k = 2$ outsiders. The corresponding distributions for $k \geq 3$ are denoted by $\psi_{k \geq 3,I,t}(\cdot)$ and $\psi_{k \geq 3,U,t}(\cdot)$. The laws of motion for these distributions are as follows:

\textsuperscript{42}Other assumptions about the time it takes for contracts to adjust would be ad hoc at this point. Issues of contract stickiness are thus left for future work.
\[\psi_{2,I,t+1}(\omega) = \Phi_{1,t} \left[ \tau_1 (R_{1,t}[\omega]) I_{2,N,t+1}(\omega) + [1 - \tau_1 (R_{1,t}[\omega])] I_{2,D,t+1}(\omega) \right] \]

\[\psi_{2,U,t+1}(\omega) = \Phi_{1,t} \left[ \tau_1 (R_{1,t}[\omega]) [1 - I_{2,N,t+1}(\omega)] + [1 - \tau_1 (R_{1,t}[\omega])] [1 - I_{2,D,t+1}(\omega)] \right] \]

\[\psi_{k \geq 3,I,t+1}(\omega) = (1 - \mu + \varepsilon) \left[ \psi_{2,I,t}(\omega) + \psi_{k \geq 3,I,t}(\omega) \right] I_{k \geq 3,t+1}(\omega) \]

\[\psi_{k \geq 3,U,t+1}(\omega) = \left[ (1 - \mu) \left[ \psi_{2,U,t}(\omega) + \psi_{k \geq 3,U,t}(\omega) \right] + (1 - \mu + \varepsilon) \left[ \psi_{2,I,t}(\omega) + \psi_{k \geq 3,I,t}(\omega) \right] [1 - I_{k \geq 3,t+1}(\omega)] \right] \]

Shocks to the policy rate affect the terms offered by various lenders and changes in these terms then affect which borrowers choose to stay with their insiders (i.e., \(I_{2,N}, I_{2,D}\), and \(I_{k \geq 3}\) respond). When \(\varepsilon\) is positive, types that stay with their insiders become more persistent so changes in \(r\) alter the distribution of borrower types in and across periods. As these distributions evolve to their new steady states, aggregate dynamics are observed well beyond time \(t\).

**Numerical Analysis**

To compute the equilibrium quantities, discretize the type space and the set of possible loan rates and initialize the loan rate functions and the value functions. Given the loan rates, I determine the borrowers’ strategies by iterating on their value functions then, based on these strategies, I iterate on the loan rates to find the optimal lender responses. The equilibrium is determined by iterating on the outer loop until the starting and ending loan rate functions converge.

To execute the iterations, I set \(\beta = 0.96\) and use a uniform distribution of types. Returns are \(\theta_1 = 5\) and \(\theta_2 = 6\) so that the speculative project yields 20 percent more than the investment project if successful. The probability of success for the investment project is assumed to be linear in borrower type, satisfying \(p(\omega) = p(0) + [p(1) - p(0)] \omega\). By definition, the
best type is very likely to succeed if he operates $P1$ so I set $p(1) = 0.9$. The success rate of the speculative project is much lower but, since it still needs to be a legitimate outside option, I consider $q = 0.65$. With values for $q, \theta_1, \text{and } \theta_2$, we can then use $p(0)\theta_1 = q\theta_2$ to pin down $p(0) = 0.78$. Unless otherwise specified, $\mu = 0.3$ and $\varepsilon = 0.065$.

**Steady State**

Figure S.1 illustrates steady state results for the extended model. The aggregate output profile resembles that in the main text but, by making better types more persistent, higher values of $\varepsilon$ shift the profile upwards. There are two additional differences relative to the baseline model. First and as shown in Figure S.1(a), the steady state measure of relationship lending exhibits a hump-shaped response to increases in the policy rate instead of a monotonic decline. Second and as shown in Figure S.1(b), credit history matters even though the insider still discovers his borrower’s type with certainty (i.e., $\phi = 1$).

A higher policy rate increases the cost of lending so, all else constant, the lowest type on which the insider breaks even rises. As before then, insiders become more selective in their retention of borrowers and fewer lending relationships are formed. Now, however, the additional bargaining power that $\varepsilon$ gives the insider over better types means that more of the necessary break even can be accommodated by increases in the loan rate, stemming the restriction of insider credit. The bargaining power effect plays out initially but is eventually dominated by the selectivity effect, leading to the hump-shaped response in relationship lending.

The bargaining and selectivity effects are also useful for understanding why credit history can matter with $\varepsilon > 0$. At higher policy rates, the increase in insider selectivity means that more types have to resort to outsider credit. This increases outsider uncertainty and makes credit history a natural screening mechanism. The informativeness of credit history, however, depends on the first period loan rate. In particular, a very high $R_1$ induces most types to choose $P2$ in the first period and implies high default probabilities across the board. The opposite is true when $R_1$ is very low. Therefore, by getting good firms to choose $P1$
and bad firms to choose $P_2$, moderate first period loan rates generate the most informative credit histories. For credit history to matter then, we need a relatively high value of $r$ but a relatively moderate value of $R_1$. This configuration can be achieved under $\varepsilon > 0$ since the bargaining power afforded to insiders over high types increases the expectation of future profits and, for any $r$, competitive first period lenders settle on an even lower value of $R_1$. To be sure, adverse selection still arises with $\varepsilon > 0$ but, for relatively high policy rates, the informativeness of credit history is such that $d$ cannot be completely crowded out.

**Dynamics**

Consider now the transition between steady states after an increase in the policy rate at date $t$. As shown in Figure S.2, the initial response of relationship lending tends to overshoot its new steady state value. To see why, define the extensive margin in period $k$ as the total number of borrowers in that period and the intensive margin as the proportion of these borrowers that enter into multi-period lending relationships. The extent of relationship lending in any given period is approximately equal to the product of its intensive and extensive margins.\(^{43}\) The extent of relationship lending at any given date is then equal to the sum of the extents for periods 2 and above. The right panel of Figure S.1(c) reveals that $k = 2$ is critical for the analysis. When $r$ increases from 0.5 to 0.75, the bargaining power effect drives up the second period intensive margin and we observe the immediate increase in relationship lending shown in Figure S.2(a). Over time though, more lending relationships mean fewer exogenous separations so the distribution of borrowers eventually shifts away from $k = 2$ and the extent of relationship lending declines along the transition path. Therefore, the increase in relationship lending overshoots its new steady state and the decrease in output undershoots. As set up in Subsection 1.4.5, the standard model adjusts to its new steady state immediately so the results presented here suggest that relationship lending leads to a more gradual transition after certain permanent shocks.\(^{44}\)

\(^{43}\)The result is an approximation for the second period since it aggregates across default histories.

\(^{44}\)This is not to say that traditional models do not generate dynamics. Instead, the transitions presented here should be interpreted as dynamics over and above those generated by a model that ignores relationship lending.
In comparison, Figure S.2(b) demonstrates that an increase in the policy rate from 0.75 to 1 causes the decrease in output to overshoot. For this range of $r$, the insider’s selectivity effect dominates, pushing the second period intensive margin back down. As the immediate decrease in relationship lending eventually increases the number of young borrowers (i.e., the pool of potential relationship borrowers), the initial declines in relationship lending and output are partially offset over time. Even at its trough, however, aggregate output in Figure S.2(b) exceeds the standard credit model’s new steady state of $Y_S(1) \approx 4.16$. 
Figure S.1: Steady State Results in the Extended Model

(a) Aggregate output and relationship lending

(b) Realized second period loan rates

(c) Intensive and extensive margins
Figure S.2: Dynamics in the Extended Model

(a) Transition between steady states, $r = 0.5$ to $r = 0.75$

(b) Transition between steady states, $r = 0.75$ to $r = 1$
Appendix to Chapter 2

B.1 Proofs

This section proves Propositions 6 to 8 as stated in Chapter 2.

Proof of Proposition 6

The proof amounts to showing that $J$, the informed value function, is increasing in $\omega$. I start by establishing the existence of $J$. Define indicator $i$ and value function $D$ such that:

$$TD(S, \omega, v, \psi, i) = i \cdot D(S, \omega, v, \psi, 1) + (1 - i) \cdot D(S, 0, 0, \psi, 0)$$

where

$$D(S, \omega, v, \psi, 1) = J(\omega, v, S)$$
$$D(S, 0, 0, \psi, 0) = U(S, \psi)$$

Now suppose $D$ exists in the set of bounded and continuous functions ($C$). By the Theorem of the Maximum, the right-hand side of equation (2.2) produces $D(\cdot, 1) \in C$ while the right-hand side of (2.4) produces $D(\cdot, 0) \in C$. Therefore, $TD \in C$. We can then show that Blackwell’s sufficient conditions for a contraction are satisfied so, by the Contraction Mapping Theorem, there does indeed exist a unique $D \in C$. By implication, $J$ and $U$ exist and are unique, bounded, and continuous. A similar contraction mapping argument can be used to show that $J$ lies in the set of increasing-in-$\omega$ functions, completing the proof. ■
Proof of Proposition 7

In what follows, \( J(\omega, v, S) \) and \( U(S, \psi) \) are shortened to \( J(\omega) \) and \( U \) while \( r \) is used in place of \( r(S) \). Let us first examine the value of an informed lender, ignoring the borrower’s participation constraint. If he keeps the borrower, then the optimal loan rate is \( R^*(\omega) = \theta(\omega) - \sqrt{c\theta(\omega)} \). Defining \( g(\omega) \equiv \left( \sqrt{\theta(\omega)} - \sqrt{c} \right)^2 \), we can then write \( J(\omega) \) as:

\[
J(\omega) = \begin{cases} 
\beta U & \text{if } \omega < \xi \\
\frac{\beta(1+r) + \mu U}{1 - \beta(1-\mu)} & \text{if } \omega \geq \xi 
\end{cases}
\]

where

\[
\xi = \arg \min_{x \in [0,1]} |g(x) - (1 + r) - \beta (1 - \beta) (1 - \mu) U|
\]

Turn now to the distribution of borrowers across financing class. In steady state, the expressions in Section 2.3 simplify to:

\[
\lambda(\omega) = \begin{cases} 
0 & \text{if } \omega < \xi \\
\frac{\Pi(1-\mu\Pi)}{\mu + (1-\mu)\Pi} & \text{if } \omega \geq \xi 
\end{cases}
\]

and \( \phi(\omega) = \begin{cases} 
\Pi^2 & \text{if } \omega < \xi \\
\frac{\mu\Pi^2}{\mu + (1-\mu)\Pi} & \text{if } \omega \geq \xi 
\end{cases} \]

Recall that the fraction of type \( \omega \)'s looking for a new lender is \( 1 - A(\omega) (1 - \mu) [\lambda(\omega) + \phi(\omega)] \). As a result, an unmatched lender will have the following beliefs about the borrower pool from which he may get a match:

\[
\hat{f}(\omega|\xi, \Pi) = \begin{cases} 
\frac{\mu + (1-\mu)\Pi}{\mu + (1-\mu)\Pi H(\xi)} & \text{if } \omega < \xi \\
\frac{\mu}{\mu + (1-\mu)\Pi H(\xi)} & \text{if } \omega \geq \xi 
\end{cases}
\]

To ease notation, let \( \hat{f}_L(\xi, \Pi) \) denote the first row of the above equation and let \( \hat{f}_H(\xi, \Pi) \) denote the second. Moreover, let \( X(\xi, \Pi) \) denote the maximized one-period revenue of an uninformed lender. The value of an unmatched lender pursuing optimal strategy \( \pi^* \) is then:
\[ U = \pi^* \left[ X(\xi, \Pi) - (1 + r) + \beta (1 - \mu) \int_{0}^{1} J(\omega) \, d\bar{F}(\omega|\xi, \Pi) + \beta \mu U \right] \\
+ \pi^*(1 - \pi^*) \int_{0}^{1} J(\omega) \, d\bar{F}(\omega|\xi, \Pi) + (1 - \pi^*) \beta U \]

Substituting in for \( J(\omega) \), we can rearrange the above expression to isolate \( U \). With \( \pi^* \) optimal, \( dU/d\pi^* = 0 \) so differentiating the isolated expression yields an implicit definition of \( \pi^* \). The definition can be simplified by combining terms to reconstitute \( U \) then using the definition of \( \xi \) to sub it out. After some algebra, we get the following expression for optimal lending intensity:

\[ \pi^* = \frac{1}{2 [1 - \beta (1 - \mu)]} \left( \frac{\int_{\xi}^{1} [g(\omega) - g(\xi)] \, dF(\omega)}{\int_{\xi}^{1} [g(\omega) - g(\xi)] \, dF(\omega) - \left( \frac{X(\xi, \Pi) - g(\xi)}{f_H(\xi, \Pi)} \right)} \right) \equiv s(\xi, \Pi) \quad (B.1) \]

Holding \( \xi \) fixed, equation (B.1) defines the best response function \( \pi(\Pi) \). If \( \xi = 0 \), then \( s(\xi, \Pi) > 1 \) so \( \pi(\Pi) = 1 \). If \( \xi = 1 \), then \( s(\xi, \Pi) = 0 \) so \( \pi(\Pi) = 0 \). For \( \xi < 1 \) sufficiently large though, \( \pi(\Pi) = s(\xi, \Pi) \in (0, 1) \). To determine the slope of \( \pi(\Pi) \) when \( \xi \) yields an interior solution, we need to write out the expression for \( X(\xi, \Pi) \). Defining \( h(\omega, \bar{R}) \equiv (1 - \frac{e}{\theta(\omega) - R}) \bar{R}^\circ \), it is:

\[ X(\xi, \Pi) = \tilde{f}_{L}(\xi, \Pi) \int_{\pi^*}^{\max{\pi^*, \xi}} h(\omega, \bar{R}^*) \, dF(\omega) + \tilde{f}_{H}(\xi, \Pi) \int_{\max{\pi^*, \xi}}^{1} h(\omega, \bar{R}^*) \, dF(\omega) \]

Applying the Envelope Theorem yields:

\[ \frac{\partial}{\partial \Pi} \left( \frac{X(\xi, \Pi) - g(\xi)}{f_H(\xi, \Pi)} \right) = \left( \frac{1 - \mu}{\mu} \right) \left[ \int_{\pi^*}^{\max{\pi^*, \xi}} h(\omega, \bar{R}^*) \, dF(\omega) - g(\xi) F(\xi) \right] \]

Note that \( g(\xi) \) must be greater than or equal to \( h(\xi, \bar{R}^*) \) since an informed lender can always charge type \( \xi \) an amount \( \bar{R}^\circ \). Combined with \( h'(\omega, \bar{R}^*) > 0 \), this implies that the denominator of equation (B.1) is increasing in \( \Pi \). Therefore, \( \pi'(\Pi) < 0 \) when \( \xi \in (\tilde{\xi}, 1) \).
Proof of Proposition 8

For \( \Pi > 0 \), the steady state market clearing equation can be written as:

\[
\Pi = \frac{1}{\mu} \left( \frac{\int_{\xi}^{1} [g(\omega) - \frac{1}{1-\delta}] dF(\omega)}{\int_{\xi}^{1} [g(\omega) - \frac{1}{1-\delta}] dF(\omega) - \left( \frac{X(\xi, \Pi) - \frac{1}{1-\delta}}{f_{H}(\xi, \Pi)} \right)} \right) \equiv m(\xi, \Pi) \tag{B.2}
\]

Proving that there exists a unique symmetric steady state amounts to proving that there is a unique combination of \( \xi \) and \( \Pi \) that satisfies \( \Pi = s(\xi, \Pi) \) and \( \Pi = m(\xi, \Pi) \), where \( s(\xi, \Pi) \) and \( m(\xi, \Pi) \) are given in equations (B.1) and (B.2) respectively. Let \( \Pi(\xi) \) denote the solution to \( \Pi = s(\xi, \Pi) \) and \( \Pi_{m}(\xi) \) denote the solution to \( \Pi = m(\xi, \Pi) \). That there exists one and only one point such that \( \Pi(\xi) = \Pi_{m}(\xi) \) is established in a series of steps.

Claim 1 Suppose the parameters are such that \( \int_{0}^{1} g(\omega) dF(\omega) < \frac{1}{1-\delta} \). Any non-trivial steady state must have \( \Pi \in (0, 1) \), allowing us to use the interior solution of equation (B.1).

Proof. Non-triviality rules out \( \Pi = 0 \). Consider now \( \Pi = 1 \). At unit lending intensity, the market clearing equation reduces to:

\[
(1 - \mu) \int_{\xi}^{1} g(\omega) dF(\omega) + [\mu + (1 - \mu) F(\xi)] X(\xi, 1) = \frac{1}{1-\delta}
\]

Since \( g(\omega) > 0 \), we have \( \int_{\xi}^{1} g(\omega) dF(\omega) < \int_{0}^{1} g(\omega) dF(\omega) \). Moreover, \( h(\omega, \overline{R}) \leq g(\omega) \), \( \overline{F} \geq 0 \), \( \widehat{f}_{L}(\xi, \Pi) \geq \widehat{f}_{H}(\xi, \Pi) \), and \( \widehat{f}_{H}(\xi, \Pi) \leq 1 \), imply \( X(\xi, \Pi) < \int_{0}^{1} g(\omega) dF(\omega) \). Therefore, under the condition assumed in Claim 1, \( \Pi = 1 \) cannot satisfy market clearing. \( \square \)

Claim 2 \( \Pi(\xi) \) and \( \Pi_{m}(\xi) \) intersect at least once.

Proof. Define \( \xi \) such that \( \int_{\xi}^{1} (g(\omega) - \frac{1}{1-\delta}) dF(\omega) = 0 \) and \( \overline{\xi} \) such that \( g(\overline{\xi}) = \frac{1}{1-\delta} \). With some algebra, we can show \( \Pi(\xi) > \Pi_{m}(\xi) \) and \( \Pi(\xi) < \Pi_{m}(\xi) \) for all \( \xi \geq \overline{\xi} \). \( \square \)

Claim 3 All intersections between \( \Pi(\xi) \) and \( \Pi_{m}(\xi) \) are associated with the same value of \( \Pi \), labelled \( \Pi_{0} \).
**Proof.** Rearrange $\Pi = s(\xi, \Pi)$ and $\Pi = m(\xi, \Pi)$ to get two expressions for \( \frac{X(\xi, \Pi)}{f_H(\xi, \Pi)} \). Equating these expressions and rearranging again yields a quadratic in $\Pi$, where the roots of this quadratic determine the values $\Pi$ can achieve at an intersection. Recall from the proof of Claim 2 that intersections require $\xi < \xi$ which, given $g'(\cdot) > 0$, is equivalent to $g(\xi) < \frac{1}{1-\delta}$. This fact combined with $\Pi > 0$ can be used to eliminate one of the roots, implying that any intersection must achieve the same value of $\Pi$. \( \square \)

**Claim 4** $\Pi'_s(\xi) < 0$ so there is only one value of $\xi$ such that $\Pi_s(\xi) = \Pi_0$.

**Proof.** Totally differentiate equation (B.1) under $\pi = \Pi$. Based on this expression, a sufficient condition for $\Pi'_s(\xi) < 0$ is $X(\xi, \Pi) \leq \frac{1}{1-F(\xi)} \int_0^1 g(\omega) dF(\omega)$. Since $h(\omega, \overline{R}) \leq g(\omega)$, $\overline{\Pi} > 0$, $\overline{F}_L(\xi, \Pi) \geq \overline{F}_H(\xi, \Pi)$, and $\overline{F}_H(\xi, \Pi) \leq 1$, this condition is satisfied. \( \square \)

Conditional on the participation constraint not binding, we can now conclude that the model has a unique non-trivial steady state. The following claim establishes that $\mu$ sufficiently large ensures non-bindingness, completing the proof of Proposition 8.

**Claim 5** If $\mu$ is sufficiently large, then $R(\omega) = \theta(\omega) - \sqrt{c\theta(\omega)}$ satisfies the borrower’s participation constraint.

**Proof.** After some algebra, the steady state participation constraint for a type $\omega \geq \xi$ borrower reduces to:

$$
\beta (1 - \mu) \Pi^2 \leq \frac{\Theta(\omega, R(\omega))}{\Theta(\omega, \overline{R}) - \Theta(\omega, R(\omega))}
$$

where

$$
\Theta(\omega, R) = \theta(\omega) - R - c + c \ln \left( \frac{c}{\theta(\omega) - R} \right)
$$

With a sufficiently high value of $\mu$, this equation will be satisfied. \( \square \)

From a computational perspective, Claim 5 can be interpreted as follows: if, for a given $\mu$, we find an equilibrium ignoring the participation constraint then confirm that this
equilibrium does indeed induce participation, we have found the unique equilibrium in the class of equilibria where the participation constraint does not bind.

## B.2 Planner’s Problem

The net output from financing type ω at rate R is:

\[
y(ω, R) = e(ω, R)θ(ω) + c \ln (1 - e(ω, R))
\]

Let \( W_I(ω, R) \) denote the present discounted value of putting ω in an informed match, \( W_U(ω, R) \) the value of putting him in an uninformed match, and \( W_N(ω, R) \) the value of keeping him unmatched. The steady state functional equations are:

\[
W_I(ω, R) = \begin{cases} 
βW_N(ω, R) & \text{if } ω < ξ \\
y(ω, R(ω)) + β(1 - μ)W_I(ω, R) + βμW_N(ω, R) & \text{if } ω ≥ ξ
\end{cases}
\]

\[
W_U(ω, R) = \begin{cases} 
y(ω, R) + βW_N(ω, R) & \text{if } ω < ξ \\
y(ω, R) + β(1 - μ)W_I(ω, R) + βμW_N(ω, R) & \text{if } ω ≥ ξ
\end{cases}
\]

\[
W_N(ω, R) = \begin{cases} 
Π^2W_U(ω, R) + (1 - Π^2)βW_N(ω, R) & \text{if } ω < ξ \\
Π^2W_U(ω, R) + Π(1 - Π)W_I(ω, R) + (1 - Π)βW_N(ω, R) & \text{if } ω ≥ ξ
\end{cases}
\]

Solving this system isolates \( W_I, W_U, \) and \( W_N \) as functions of only ω and the choice variables. Denote the isolated expressions by \( W^*_I(ω, R, ξ, Π), W^*_U(ω, R, ξ, Π), \) and \( W^*_N(ω, R, ξ, Π) \). In order to construct the aggregate welfare function, we now need the distribution of borrowers across informed financing, uninformed financing, and unmatchedness. For a given Π and ξ, the evolution of borrowers still follows equations (2.5) and (2.6), with the steady state versions as in Appendix B.1. To make the dependence of these distributions on the planner’s choices clear, I write \( λ(ω|ξ, Π) \) and \( φ(ω|ξ, Π) \). Measured at the beginning of the production
stage, the objective function is then:

\[
W(R, \xi, \Pi) = \int_0^1 W_T^\pi(w, R, \xi, \Pi) \lambda(\omega|\xi, \Pi) dF(\omega) + \int_0^1 W_U^\pi(w, R, \xi, \Pi) \phi(\omega|\xi, \Pi) dF(\omega) + \int_0^1 \beta W_N^\pi(w, R, \xi, \Pi) [1 - \lambda(\omega|\xi, \Pi) - \phi(\omega|\xi, \Pi)] dF(\omega)
\]

The planner’s problem is to choose \( \Pi, \xi, \bar{R} \), and \( R(\cdot) \) in order to maximize \( W(R, \xi, \Pi) \) subject to an aggregate feasibility constraint. The constraint requires that the amount of capital used to finance projects equals the amount of capital available to the planner each period. It is thus equivalent to the market clearing equation.

### B.3 Computational Algorithm for Dynamics

Suppose a one-time aggregate productivity shock hits at \( t = 1 \). Recall that \( z \) is realized after lending and production decisions have been made so all credit market variables are still in steady state at \( t = 1 \). The capital available for \( t = 2 \) is then:

\[
KS_2 = (1 - \delta) (1 + z) \left[ \int_{\xi_{ss}}^1 \left( 1 - \sqrt{\frac{e}{\theta(\omega)}} \right) R(\omega) \lambda_{ss}(\omega) dF(\omega) + \int_{\bar{R}_{ss}}^1 \left( 1 - \frac{c}{\theta(\omega)-R_{ss}} \right) \bar{R}_{ss}\phi_{ss}(\omega) dF(\omega) \right] < KS_{ss}
\]

Even though \( z \) returns to its expected value by \( t = 2 \), the effects of the \( t = 1 \) shock are propagated over time due to the change in the stock of financing capital. I start by computing the propagation in absence of the participation constraint. Let \( T +1 \) denote the date at which \( \xi_t \) returns to \( \xi_{ss} \) and let \( TT \) denote the date at which the economy returns to steady state. Note that \( T +1 < TT \) since the partition of the type space implied by the evolution of \( \xi_t \) must stabilize before the distribution over that space can stabilize. The rest of the transition path is computed in four steps:

1. For \( t = 2, \ldots, T \):
• Guess $\Pi_t$.

• Use $\Pi_t$, $\lambda_{t-1} (\cdot)$, and $\phi_{t-1} (\cdot)$ to get $\lambda_t (\cdot)$, $\phi_t (\cdot)$, $R_t$, and $K_S_{t+1}$.

• By bisection, find the $\xi_t$ that equates $KD_t$ to the previously determined $K_S_t$.

2. For $t = T + 1, \ldots, TT - 1$:

• Use $\lambda_{t-1} (\cdot)$, $\phi_{t-1} (\cdot)$, and $\xi_t = \xi_{ss}$ to get beliefs $\psi_t (\cdot)$.

• Use $J_{t+1} (\xi_{ss}, \cdot) = \beta U_{t+2} (\cdot)$ to get an expression for $r_t$.

• Recursive substitution of $J_{t+1} (\omega, \cdot)$ into $J_t (\omega, \cdot)$ yields:

$$J_t (\omega, \cdot) = \frac{g (\max \{\omega; \xi_{ss}\}) - g (\xi_{ss})}{1 - \beta (1 - \mu)} + \beta U_{t+1} (\cdot)$$

for all $t \in [T + 1, TT - 1]$

• Use $\psi_t (\cdot)$, $r_t$, and the expression for $J_t (\omega, \cdot)$ to get $U_t (\cdot)$.

• Based on the first order condition for $\pi_t$, get the optimal $\pi_t^*$.

3. For $t = TT - 1, \ldots, T + 1$:

• Recall that the value functions at date $TT$ are the steady state ones. Starting at $t = TT - 1$, use $\pi_t^*$ as computed in step 2 to get $U_t (\cdot)$ and $J_t (\cdot)$.

• Work back until $t = T + 1$.

4. For $t = T, \ldots, 2$

• From step 3, we know the date $T+1$ value functions. Starting at $t = T$, determine the optimal choice $\pi_t^*$ then the value functions $U_t (\cdot)$ and $J_t (\cdot)$.

• Work back until $t = 2$.

Symmetry requires $\Pi_t = \pi_t$ so compare the guess $\{\Pi_t\}_{t=2}^T$ with the result $\{\pi_t^*\}_{t=2}^T$. If the root mean squared error is not sufficiently small, then update the guess in the direction suggested by the result. Repeat until RMSE-convergence then verify that the unconstrained choice of $R (\omega)$ does indeed satisfy the borrower’s participation constraint.