Slow Flow of Viscoelastic Fluids Through Fibrous Porous Media

by

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Department of Mechanical & Industrial Engineering
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ABSTRACT

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This thesis reports on an experimental study of slow viscoelastic flow through models of fibrous porous media. The models were square arrays of parallel cylinders, with solid volume fractions or ‘solidities’ of 2.5%, 5.0%, and 10%. An initial study using a Newtonian fluid provided a baseline for comparison with results for two Boger fluids, so that the effects of fluid elasticity could be determined. Boger fluids are elastic fluids that have near constant viscosities and can be used in experiments without having to account for shear-thinning effects. The experimental approach involved measurements of pressure loss through the three arrays and interior velocity measurements using particle image velocimetry (PIV).

For the Newtonian flows, pressure loss measurements were in good agreement with the analytical predictions of Sangani and Acrivos (1982). PIV measurements showed velocity profiles which were symmetrical and independent of flow rate.

Pressure loss measurements for the Boger fluid flows revealed that the onset of elastic effects occurred at a Deborah number of approximately 0.5, for both fluids and the three arrays.
Flow resistance data collapsed for the two Boger fluids, and increased with solidity. For all three models, the flow resistance increased monotonically with Deborah number, reaching values up to four times the Newtonian resistance for the 10% model.

PIV measurements showed that the transverse velocity profiles for the Newtonian and Boger fluids were the same at Deborah numbers below the elastic onset. Above onset, the profiles became skewed. The skewness, like the flow resistance, was observed to increase with both Deborah number and solidity.

In the wake regions between cylinders in a column, periodic flow structures formed in the spanwise direction. The structures were staggered from column to column, consistent with the skewing. As either Deborah number or solidity increased, the flow structures became increasingly three-dimensional, and the stagger became more symmetric.

An analysis of fluid stresses reveals that the elastic flow resistance is attributed to additional normal stresses caused by shearing, and not by extension.
DECLARATION

No part of this thesis has been previously published in any journal by any person except where due reference has been made in the text. To the best of the author’s knowledge, this thesis contains no material previously written towards any degree in any university.
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NOMENCLATURE

\(a\) \hspace{1cm} \text{cylinder radius [mm]}

\(C(x,y)\) \hspace{1cm} \text{cross-correlation function at coordinates (x, y)}

\(D\) \hspace{1cm} \text{distance between the centres of adjacent cylinders [mm]}

\(De\) \hspace{1cm} \text{Deborah number}

\(De_c\) \hspace{1cm} \text{onset Deborah number}

\(d_p\) \hspace{1cm} \text{particle diameter [\(\mu\)m]}

\(f\) \hspace{1cm} \text{friction factor}

\(f \Re\) \hspace{1cm} \text{flow resistance parameter}

\(f \Re_N\) \hspace{1cm} \text{Newtonian flow resistance parameter}

\(F\) \hspace{1cm} \text{thrust on the plate [N]}

\(g\) \hspace{1cm} \text{gravitational acceleration [m/s}\(^2\)]

\(G'\) \hspace{1cm} \text{dynamic storage modulus [Pa]}

\(G''\) \hspace{1cm} \text{dynamic loss modulus [Pa]}

\(G''/\omega\) \hspace{1cm} \text{dynamic viscosity [Pa s]}

\(h\) \hspace{1cm} \text{height of the fluid in collection tank [m]}

\(I(x,y)\) \hspace{1cm} \text{light intensity function of frame 1 at coordinates (x, y)}

\(I'(x,y)\) \hspace{1cm} \text{light intensity function of frame 2 at coordinates (x, y)}

\(k\) \hspace{1cm} \text{permeability of porous medium [m}\(^2\)]

\(l\) \hspace{1cm} \text{length of porous medium [m]}

\(M\) \hspace{1cm} \text{applied torque [N m]}

\(M_W\) \hspace{1cm} \text{molecular weight [g/mol]}

\(N_1\) \hspace{1cm} \text{first normal stress difference [Pa]}

\(Q_{x/D}\) \hspace{1cm} \text{two-dimensional flow rate in a unit cell a fixed coordinate of x/D [mm}\(^2\)/s\)]
\( R \)  
gas constant [\( J/(\text{mol K}) \)]

\( r \)  
cone radius [\( \text{m} \)]

\( \text{Re} \)  
Reynolds number

\( \text{Re}_p \)  
particle Reynolds number

\( S_{\Delta p} \)  
relative standard deviation of the pressure loss measurement

\( S_h \)  
relative standard deviation of the height measurement

\( S_t \)  
relative standard deviation of the time measurement

\( S_{u_A} \)  
relative standard deviation of the velocity measurement within an interrogation area

\( T \)  
absolute temperature [\( \text{Kelvin} \)]

\( t \)  
time [\( \text{s} \)]

\( t \)  
time required for fluid collection [\( \text{s} \)]

\( T_D \)  
characteristic time of flow [\( \text{s} \)]

\( Tr \)  
Trouton ratio

\( U \)  
superficial velocity or bulk velocity [\( \text{mm/s} \)]

\( u \)  
velocity in the \( x \)-direction [\( \text{mm/s} \)]

\( u_x \)  
velocity within fluid filament in the \( x \)-direction [\( \text{m/s} \)]

\( u_y \)  
velocity within fluid filament in the \( y \)-direction [\( \text{m/s} \)]

\( u_z \)  
velocity within fluid filament in the \( z \)-direction [\( \text{m/s} \)]

\( u_{IA} \)  
mean velocity within an interrogation area [\( \text{mm/s} \)]

\( V \)  
velocity of upper plate [\( \text{m/s} \)]

\( v \)  
velocity vector [\( \text{m/s} \)]

\( v_s \)  
particle settling velocity [\( \text{mm/s} \)]

**Greek**

\( \Delta d \)  
particle displacement [\( \text{mm} \)]
\( \Delta_h \) height measurement subdivision [m]

\( \Delta p \) pressure loss [Pa]

\( \Delta t \) frame separation time [s]

\( \Delta_t \) time measurement subdivisions [s]

\( \varepsilon \) Hencky strain

\( \dot{\varepsilon} \) extensional rate \([s^{-1}]\)

\( \gamma_0 \) amplitude of oscillatory input strain

\( \gamma_{xy} \) oscillatory input strain

\( \dot{\gamma} \) shear rate \([s^{-1}]\)

rate of strain tensor \([s^{-1}]\)

\( \dot{\gamma} \) upper convected time derivative of the rate of strain tensor \( \dot{\gamma} \) \([Pa \ s^{-1}]\)

\( \dot{\gamma}_{xy} \) shear rate in parallel plates \([s^{-1}]\)

\( \eta \) shear viscosity [Pa s]

\( \eta_0 \) low-shear rate shear viscosity [Pa s]

\( \eta_E \) extensional viscosity [Pa s]

\( \eta_p \) polymer contribution to the shear viscosity [Pa s]

\( \eta_s \) solvent shear viscosity [Pa s]

\([\eta]_0 \) intrinsic viscosity at zero shear [Pa s]

\( \lambda \) fluid relaxation time [s]

\( \lambda_1 \) fluid relaxation time for the Oldroyd-B model [s]

\( \lambda_2 \) fluid retardation time for the Oldroyd-B model [s]

\( \lambda_p \) fluid relaxation time for the Oldroyd-B model [s]

\( \nabla v \) velocity gradient tensor \([s^{-1}]\)

\((\nabla v)^T\) transpose of the tensor \( \nabla v \) \([s^{-1}]\)
$\omega$ \hspace{1cm} oscillation frequency \([s^{-1}]\)

$\Omega$ \hspace{1cm} angular velocity of the cone \([\text{rad/s}]\)

$\phi$ \hspace{1cm} solid volume fraction or solidity

$\Psi_1$ \hspace{1cm} first normal stress coefficient \([\text{Pa s}^2]\)

$\rho$ \hspace{1cm} fluid density \([\text{kg/m}^3]\)

$\rho_p$ \hspace{1cm} particle density \([\text{kg/m}^3]\)

$\sigma$ \hspace{1cm} surface tension \([\text{N/m}]\)

$\sigma_{xx}$ \hspace{1cm} normal stress in the \(x\)-direction acting on a fluid element surface whose normal is in the \(x\)-direction \([\text{Pa}]\)

$\sigma_{yy}$ \hspace{1cm} normal stress in the \(y\)-direction acting on a fluid element surface whose normal is in the \(y\)-direction \([\text{Pa}]\)

$\sigma_{zz}$ \hspace{1cm} normal stress in the \(z\)-direction acting on a fluid element surface whose normal is in the \(z\)-direction \([\text{Pa}]\)

$\tau$ \hspace{1cm} shear stress in cone-and-plate \([\text{Pa}]\)

$\nabla \tau$ \hspace{1cm} upper convected time derivative of the tensor \(\tau\) \([\text{Pa s}^{-1}]\)

$\tau_p$ \hspace{1cm} polymer contribution to the stress tensor \(\tau\) \([\text{Pa}]\)

$\nabla \tau_p$ \hspace{1cm} upper convected time derivative of the tensor \(\tau_p\) \([\text{Pa s}^{-1}]\)

$\tau_s$ \hspace{1cm} solvent contribution to the stress tensor \(\tau\) \([\text{Pa}]\)

$\tau_{xy}$ \hspace{1cm} shear stress in the \(x\)-direction on a fluid element surface whose normal is in the \(y\)-direction \([\text{Pa}]\)

$\tau_R$ \hspace{1cm} particle response time \([\text{s}]\)

$\theta$ \hspace{1cm} cone angle \([\text{radians}]\)
CHAPTER 1: INTRODUCTION

A porous medium consists of a solid matrix permeated by a network of pores. A physical property commonly used to characterize the porous medium is the solid volume fraction, defined as the ratio of the volume occupied by the solid to the total volume. ‘Solidity’ will be used in the current study to denote solid volume fraction; the term is the complement of ‘porosity’.

Porous media made of grains are normally compact with solidities of approximately 65%. Soil, sand layers, limestone, aquifers are a few examples of granular porous media. Porous media made of fibres, on the other hand, are not compact, and have a wider range of solidities, which can be as low as 1%. Examples of fibrous porous media include paper, biological tissue, textiles, polymer membranes, and filters.

Slow elastic flow through fibrous porous media is a topic of interest because of its relevance to several engineering applications. In manufacturing polymer composites, a common technique is resin transfer molding, as shown in Figure 1.1. In this process, resin mixed with a hardener is slowly injected into a preformed bed of fibres encased in a mold. The preform mat resistance and the desired mold filling time are controlled by the injection pressure. The design of the flow process is an important factor in the quality of the finished part. For instance, in order to avoid air entrapment, an accurate knowledge of the flow’s front position and its advancement in time is necessary.
Another method of manufacturing composites utilizes an autoclave process. In this technique, a laminate, formed from layers of carbon-reinforced epoxy preimpregnate, is placed onto a tool surface. The prepreg layer consists of aligned carbon fibres enveloped by partially cured resin, as shown in Figure 1.2. Absorbent layers are placed on top of the prepreg laminate, and the entire setup is enclosed in a vacuum seal. As the sealed layers are subjected to temperature and pressure, the resin in the prepreg layers first softens and then flows out of the laminate into the absorbent layers. The resin, which initially exhibits Newtonian behaviour, becomes highly non-Newtonian as the temperature drops and the resin gel point is approached. To improve this type of manufacturing process, which is
normally developed on a costly trial-and-error basis, a better understanding of resin flow within a fibrous medium is required.

![Figure 1.2 Autoclave process, adapted from Skartsis et al. (1992).](image)

In the textile industry, waterproofing breathable fabric is achieved by coating a natural or synthetic fibrous substrate with a polymeric substance, such as a silicone elastomer or polyurethane (PU). Depending on end product requirements, different types of fibrous substrate may be used. Common examples include nylon, rayon, polyester, and polypropylene. One type of coating process known as 'reverse roll coating' is illustrated in Figure 1.3. In this procedure, by setting the proper gap between the metering roller and the
application roller, a precise amount of coating material, e.g. a silicone elastomer, is measured. This liquid is applied onto the fibrous substrate and removed from the application roller as the substrate passes between the support roller and the application roller. In order to optimize coating adhesion strength, fabric tear strength, thermal resistivity, and breathability, an understanding of the flow dynamics of the coating material through the fibrous substrate is required.

**Figure 1.3** Reverse roll coating.

Several approaches have been taken by different authors to assess slow viscoelastic flow through fibrous porous media. The prior investigations, along with related rheological concepts, are introduced in the next chapter.
CHAPTER 2: BACKGROUND

2.1 Rheological Concepts

2.1.1 Shear Flow

Shear flows are found in every manufacturing and industrial process involving a fluid. The basic flow is a fluid confined between two parallel plates moving relative to each other, as illustrated in Figure 2.1. In the diagram, the flow is bounded by a lower stationary plate, and an upper plate moving at a constant velocity $V$.

![Figure 2.1 Shear flow between parallel plates.](image)

The velocity gradient in the fluid is the shear rate $\dot{\gamma}_{xy}$, equal to $V$ divided by the gap distance $l$. The shear viscosity $\eta$ is defined as:

$$\eta = \frac{\tau_{xy}}{\dot{\gamma}_{xy}},$$

(2.1)
where \( \tau_{xy} \) is the shear stress in the \( x \)-direction on a fluid element surface whose normal is in the \( y \)-direction. The shear viscosity is independent of the shear rate for Newtonian fluids but generally varies for non-Newtonian fluids. Most non-Newtonian fluids are shear-thinning, meaning the viscosity decreases as the shear rate increases; examples include paint, blood, nail polish, shampoo, and cosmetic lotions.

Non-Newtonian fluids under shear experience normal stresses in addition to shear stresses. Given the fluid deformation in Figure 2.1, the first normal stress difference \( N_1 \) is defined as:

\[
N_1 = \sigma_{xx} - \sigma_{yy},
\]

where \( \sigma_{xx} \) is the normal stress in the \( x \)-direction acting on a fluid element surface whose normal is in the \( x \)-direction, and \( \sigma_{yy} \) is similarly defined for the \( y \)-direction. The first normal stress difference is a measure of fluid elasticity in shear.

To characterize the viscoelasticity of a non-Newtonian fluid, it is instructive to measure the response of the fluid under oscillatory shear. For a fluid that is sheared sinusoidally at a frequency \( \omega \) and with a small amplitude \( \gamma_0 \), the stress response is also sinusoidal, but is generally out-of-phase. If the strain input is:

\[
\gamma_{xy}(t) = \gamma_0 \sin(\omega t),
\]

the oscillating stress response can be decomposed into in-phase and out-of-phase components:

\[
\tau_{xy}(t) / \gamma_0 = G'(\omega) \sin(\omega t) + G''(\omega) \cos(\omega t),
\]
where $\tau_{xy}$ is the output shear stress, $G'$ is termed the dynamic storage modulus, and $G''$ the dynamic loss modulus. For a Newtonian fluid, $\tau_{xy}$ is $\pi/2$ radians out of phase with $\gamma_{xy}$; therefore, $G'$ is identically zero, and $G'' = \eta \omega$. For a viscoelastic fluid, $G'$ and $G''$ reflect the fluid’s solid-like and liquid-like responses, respectively. In other words, $G'$ is a measure of elasticity, while $G''$ is a measure of viscosity.

### 2.1.2 Rheometry in Shear

A shear rheometer is used to characterize fluids in response to an applied shear deformation. Suspensions, emulsions, and polymer solutions are a few examples of materials typically tested. Although the device can measure a variety of fluid properties, it is most commonly used to measure viscosity as a function of shear rate. For the current study, a rheometer with a cone-and-plate fixture was used to characterize the test fluids.

![Figure 2.2 Cone-and-plate fixture.](image-url)
For a rheometer using a cone-and-plate fixture, as shown in Figure 2.2, a sample of fluid is placed between a horizontal plate and a shallow-angled cone, typically about 2 to 4 degrees.

As the cone rotates about the shaft axis, the shear rate in the fluid sample is constant everywhere. For a cone angle of $\theta$, and an angular velocity of $\Omega$, the shear rate of the fluid $\dot{\gamma}$ is given by:

$$\dot{\gamma} = \frac{\Omega}{\theta}.$$  

(2.5)

The shear stress within the fluid $\tau$ is related to the applied torque $M$ by the following equation (Macosko, 1994):

$$\tau = \frac{3M}{2\pi r^3},$$  

(2.6)

where $r$ is the cone radius. By applying Eq. 2.1, Eq. 2.6 becomes:

$$\eta = \frac{3M \theta}{2\pi r^4 \Omega}.$$  

(2.7)

Given the geometry of the cone, the rheometer measures the applied torque $M$ to determine the shear viscosity $\eta$ as a function of the shear rate $\dot{\gamma}$. The first normal stress difference $N_1$ can also be determined by measuring the total thrust $F$ on the plate (Tanner, 2000):

$$N_1 = \frac{2F}{\pi r^2}.$$  

(2.8)

By applying an oscillatory strain, the rheometer can obtain the dynamic storage modulus $G'$ and the dynamic loss modulus $G''$ by measuring the amplitude of torque response and its phase shift with the applied strain (Collyer and Clegg, 1998).
2.1.3 Extensional Flow and Extensional Viscosity

In many industrial applications, e.g. paper coating and fibre spinning, extensional flow plays an important role in the operation. In processes where there is fluid acceleration, such as a flow through a contraction or an orifice, a significant proportion of the flow can be extensional, leading to flow characteristics that cannot be predicted by shear viscosity alone. The most commonly encountered extensional motion is uniaxial extension.

In uniaxial extension, a fluid element is stretched along the $x$-axis at an extensional rate $\dot{\varepsilon}$, as illustrated in Figure 2.4. The velocity field of this fluid element, for a constant extensional rate, is:

\begin{align}
    u_x &= \dot{\varepsilon} x , \quad (2.9) \\
    u_y &= -\dot{\varepsilon} y / 2 , \quad (2.10) \\
    u_z &= -\dot{\varepsilon} z / 2 , \quad (2.11)
\end{align}

where $u_x$, $u_y$, $u_z$ are components of the velocity components in the $x$, $y$, and $z$ directions, respectively.

![Figure 2.3 Uniaxial extensional flow.](image-url)
The stresses acting on the fluid element can be expressed as:

\[
\sigma_{xx} - \sigma_{yy} = \dot{\varepsilon} \eta_E (\dot{\varepsilon}), 
\]

(2.12)

\[
\sigma_{xx} - \sigma_{zz} = \dot{\varepsilon} \eta_E (\dot{\varepsilon}), 
\]

(2.13)

\[
\tau_{xy} = \tau_{xz} = \tau_{yz} = 0, 
\]

(2.14)

where \( \eta_E \) is the extensional viscosity, a measure of fluid elasticity. This quantity is generally a function of both the strain and the extensional rate \( \dot{\varepsilon} \), and is unrelated to the shear viscosity. For Newtonian fluids, however, it can be shown that \( \eta_E \) is identically three times the value of its shear viscosity \( \eta \) (Tirtaatmadja and Sridhar, 1993).

### 2.1.4 Rheometry in Extension

Extensional viscosity is ideally determined under constant extensional rates, analogous to determining shear viscosity at constant shear rates. Various instruments to measure \( \eta_E \), including opposed jet devices, two-roll and four-roll mills, and fibre spinning rheometers, were reviewed by James and Walters (1993). Their review of these different techniques showed that measurements of the extensional viscosity varied greatly.

Creating experimental conditions in which the extensional viscosity can be measured properly is not easy. The main challenge lies in generating a flow that is free of shear and in which the extensional rate remains constant. The filament-stretching rheometer (FSR) (Tirtaatmadja and Sridhar, 1993) is the only instrument which creates these flow conditions.
Although the CaBER device (McKinley and Tripathi, 2000) generates a purely extensional flow, it does not measure extensional viscosity directly. This instrument operates by rapidly stretching a fluid filament and holding the strain; the decrease in its diameter is measured as function of time to obtain the fluid relaxation time.

The FSR, on the other hand, continuously stretches a column of fluid between two coaxial end plates. By separating them such that the length of the fluid element grows exponentially in time, a constant extensional rate is produced. The simultaneous measurements of the extensional stress and the filament diameter lead to the evaluation of the transient extensional viscosity $\eta_E(t)$. An inter-laboratory comparison validated this technique (Anna et al., 2001).

A commonly used dimensionless parameter describing fluid elasticity in extension is the Trouton ratio $Tr$. This parameter is defined as the ratio of the extensional viscosity to the shear viscosity:

$$Tr = \frac{\eta_E(\dot{\varepsilon}, t)}{\eta(\dot{\gamma})}.$$ (2.15)

Any increase in the Trouton ratio from the Newtonian value of three can be attributed to the fluid elasticity. Elastic fluids can reach Trouton ratios up to $10^4$. For the current work, a filament-stretching rheometer was used to measure the Trouton ratios of the viscoelastic test fluids.
2.1.5 Polymer Solution

Polymer solutions are an important class of non-Newtonian fluids that has been studied extensively (e.g., Larson, 1998). These solutions typically consist of linear long-chain molecules dissolved in a solvent. Depending on the amount of overlap between individual chains, the polymer solution may be classified as concentrated, semi-dilute, or dilute.

For a Newtonian fluid, neighbouring molecules rapidly change their positions through Brownian motion. The average lifespan of adjacent molecular pairs, or the local structural memory, is very short, being on the order of $10^{-10}$ s. Any applied deformation increases the intermolecular potential energy. However, the additional energy is quickly dissipated, as the fluid relaxes back to its equilibrium state almost instantaneously. This type of response to applied deformations is entirely viscous. In contrast, a Hookean solid has a long structural memory. The increase in intermolecular potential energy from deformation is preserved, and the relaxation of local structures requires long periods, without a definitive time limit. The response to an applied deformation for a Hookean solid is entirely elastic.

Polymer solutions are viscoelastic, as they exhibit the characteristics of both a Newtonian fluid and a Hookean solid. After a solution has been deformed, it takes some time to relax to its equilibrium state and there is a wide range of relaxation times among fluids. As the polymer coils unravel and align in the direction of flow during shear or extension, normal stresses are generated. The additional stresses are caused by an increase in flow resistance as entropic forces of the unraveling polymer coils compete against
hydrodynamic forces of the solvent. Upon cessation of flow, Brownian motion of the solvent molecules acts to rearrange the stretched polymer coils back to their equilibrium conformations. This rearrangement is known as elastic recovery.

The fluid’s relaxation time $\lambda$ is a measure of its elasticity. It is the characteristic time required for a fluid to relax after an applied stress. For a polymer solution, $\lambda$ is related to the time required for the deformed polymer molecules to return to their equilibrium conformations.

The Deborah number $De$ is a dimensionless number which is a measure of elastic effects in a specific flow:

$$De = \frac{\lambda}{T_D},$$  \hspace{1cm} (2.16)

where $T_D$ is a characteristic time of flow. A high $De$ corresponds to a flow that is greatly influenced by elasticity, whereas a low $De$ describes more Newtonian-like flow behaviour. In the present investigation, $De$ is used to determine the relative importance of elastic effects for the viscoelastic flows through porous media.

2.1.6 The Oldroyd-B Constitutive Model

The Oldroyd-B constitutive equation is a commonly used model in simulations of viscoelastic flows. This model is the simplest constitutive equation which most successfully predicts the behaviour of a special class of polymeric fluids known as ‘Boger
fluids’ (James, 2009). Boger fluids are highly elastic with minimal shear-thinning, and are used as test fluids in the current study. A more detailed description of Boger fluids is provided in a Section 3.2.

The Oldroyd-B model will be used to characterize the relaxation times of the Boger fluids used in the current study. The constitutive equation for the model is given by (Bird et al., 1987, p. 345):

\[
\tau + \dot{\lambda}_1 \tau = \eta \left( \dot{\gamma} + \dot{\lambda}_2 \dot{\gamma} \right),
\]

(2.17)

where \(\tau\) is the stress tensor, \(\dot{\gamma}\) is the rate of strain tensor, \(\eta\) is the shear viscosity, \(\lambda_1\) is the fluid relaxation time, and \(\lambda_2\) is fluid retardation time. The terms \(\dot{\tau}\) and \(\dot{\gamma}\) are the upper convected time derivatives of the stress and strain rate tensors, respectively. This derivative is defined as (Bird et al., 1987, p. 296):

\[
\dot{\tau} = \frac{\partial \tau}{\partial t} + v \cdot \nabla \tau - (\nabla v)^T \cdot \tau - \tau \cdot \nabla v,
\]

(2.18)

where \(v\) is the fluid velocity vector, \(\nabla v\) is the fluid velocity gradient tensor, and \((\nabla v)^T\) is the transpose of that tensor.

The Oldroyd-B model can be interpreted at the molecular level as a dilute concentration of polymer chains suspended in a Newtonian solvent, where the polymer chains are made up of a series of dumbbells connected by Hookean springs (Larson, 1998, p. 142). The stress tensor \(\tau\) found in Eq. 2.17 can be expressed as a sum of the stress contributions from the polymer \(\tau_p\) and the solvent \(\tau_s\):

\[
\tau = \tau_p + \tau_s.
\]

(2.19)
By separating these two stresses, Eq. 2.17 can be considered the sum of two separate constitutive equations.

The constitutive equation describing the contribution of the polymer can be expressed as:

\[ \tau_p + \dot{\lambda}_p \tau_p = \eta_p \dot{\gamma}, \]  

(2.20)

where \( \tau_p \) is the polymer contribution to the stress tensor, \( \dot{\lambda}_p \) is the upper convected time derivative of \( \tau_p \), \( \eta_p \) is the polymer contribution to the shear viscosity, \( \lambda_p \) is the relaxation time.

The constitutive relation for the Newtonian solvent is:

\[ \tau_s = \eta_s \dot{\gamma}, \]  

(2.21)

where \( \tau_s \) is the solvent contribution to the stress, and \( \eta_s \) is the solvent viscosity. When Eqs. 2.20 and 2.21 are combined appropriately, the result is Eq. 2.17.

The parameters \( \lambda_p, \eta_p, \) and \( \eta_s \) can be expressed in terms of \( \lambda_1, \lambda_2, \) and \( \eta \) used in Eq. 2.17 as follows:

\[ \dot{\lambda}_p = \dot{\lambda}_1, \]  

(2.22)

\[ \eta_p = \frac{\eta(\lambda_1 - \lambda_2)}{\lambda_1}, \]  

(2.23)

\[ \eta_s = \frac{\eta \dot{\lambda}_2}{\dot{\lambda}_1}. \]  

(2.24)

Using Eqs. 2.22 to 2.24, the total viscosity of shear viscosity \( \eta \) can be expressed as:
\[ \eta = \eta_p + \eta_s. \]  

Eq. 2.25 indicates that the total shear viscosity can be considered as a sum of the viscosity contributions from the polymer and the solvent.

For steady shearing, the Oldroyd-B model predicts that the first normal stress difference varies quadratically with shear rate (Bird et al., 1987, p. 347):

\[ N_1 = 2\lambda_p \eta_p \dot{\gamma}^2. \]  

(2.26)

For small amplitude oscillatory shear, the dynamic properties of the Oldroyd-B model are given by (Bird et al., 1987, p. 347):

\[ \frac{G''}{\omega} = \eta_s + \frac{\eta_p}{1 + (\lambda_p \omega)^2}, \]  

(2.27)

\[ \frac{G'}{\omega} = \frac{\eta_p \lambda_p \omega}{1 + (\lambda_p \omega)^2}. \]  

(2.28)

The quantity \( G''/\omega \) is also known as the ‘dynamic viscosity’ and is often denoted as \( \eta' \).

Although Eq. 2.26 predicts a constant value of \( N_1 / 2\dot{\gamma}^2\eta_p \), experimental data for dilute polymer solutions show that this quantity varies with shear rate, and asymptotes to a constant value at low shear. Therefore, to obtain the relaxation time using experimental data of \( N_1 \), the low-shear rate limit of \( N_1 / 2\dot{\gamma}^2\eta_p \) must be calculated:

\[ \lambda_p = \lim_{\dot{\gamma} \to 0} \left( \frac{N_1}{\dot{\gamma}^2\eta_p} \right). \]  

(2.29)

Alternatively, Eq. 2.28 can be used to obtain the relaxation time by taking the following limit:
\[
\lambda_p = \lim_{\omega \to 0} \left( \frac{G'}{\omega^2 \eta_p} \right).
\] (2.30)

The Oldroyd-B constitutive equation predicts the properties of dilute polymer solutions with reasonable accuracy (Larson, 1998). The model exhibits a constant shear viscosity, a zero second normal stress difference, and a fluid stress relaxation.

## 2.2 Prior Studies

Parallel arrays of cylinders offer the simplest and best-defined structures of fibrous porous media for analysis. For this reason, several previous authors have studied slow flow through cylinder arrays. Prior experimental and numerical analyses of viscoelastic flow through fibrous porous media of cylinder arrays are reviewed in the following sections.

### 2.2.1 Experimental Viscoelastic Flow Through Cylinder Arrays

Studies of viscoelastic flows past cylinders in uniform arrays of various configurations have consistently revealed an increase in the flow resistance, relative to Newtonian flow, above a critical Deborah number. Vossoughi and Seyer (1974) performed pressure drop measurements for slow flow through a regular hexagonal array of closely spaced cylinders (solidity of 70.2%). Using a Deborah number based on the time required for the flow to travel between two adjacent rows of cylinders and a shear-rate-dependent relaxation time estimated from normal stress data, they found that the increase in pressure drop started at \( De = 0.08 \).
Similar to the data of Vossoughi and Seyer, Skartsis *et al.* (1992) found a pressure drop increase, greater than that of the Newtonian flow, for viscoelastic flows through cylinder arrays arranged in either a square (solidity of 55%) or a staggered (solidities of 32% and 56.6%) configuration. The authors found that the onset of elastic effects occurred at $De = 0.01$, where the Deborah number was based on the shear-rate-dependent relaxation time and the velocity through the cylinder gap.

The effects of fluid elasticity for flows through square and hexagonal cylinder arrays (solidity of 30%) were investigated by Chmielewski and Jayaraman (1992, 1993). For three Boger fluids with different relaxation times, the authors found that the increase in flow resistance, beyond that of the Newtonian flow, occurred at $De = 1$ for both models. At high Deborah numbers ($De > 3$), the flow resistance plateaued and became independent of $De$. The Deborah number used by the authors was based on the zero-shear-rate relaxation time and the bulk velocity. Pressure drop fluctuations, signifying the presence of a flow instability, were observed in either array configuration at a critical Deborah number. For the square array, the transition from steady to unsteady, three-dimensional flow occurred at $De = 1.5$; for the hexagonal array, the change occurred at $De = 0.5$. Using laser Doppler anemometry (LDA), the authors measured the velocity distribution for in the streamwise direction for a *shear-thinning* fluid, along the centreline between rows of cylinders. The data revealed that, prior to the onset of elastic effects, where a *reduced* pressure drop was observed, the velocities increased by up to 50% due to shear-thinning effects. No velocity measurements were made in the elastic regime.
Khomami and Moreno (1997) experimentally investigated slow flow of two Boger fluids through square arrays of cylinders with solidities of 14% and 55%. For both models, the flow was steady and two-dimensional below a critical Deborah number. Their $De$ was based on the zero-shear-rate polymer relaxation time and the average flow rate across the minimum gap. Pressure fluctuations appeared above Deborah numbers of 2.95 for the 55% solidity model and 0.95 for the 14% solidity model. The authors used streak photography to visualize the flow at high Deborah numbers. In the 55% solidity model, a transition from a steady, two-dimensional flow to an unsteady, three-dimensional flow occurred above the critical Deborah number. In the low solidity model, however, the transition was from steady, two-dimensional flow to steady, three-dimensional flow. In particular, the authors found steady "periodic cellular structures" in the wake regions between the first and second rows of cylinders. Beyond the second row, another transition occurred as the flow became transient and three-dimensional.

Koshiba et al. (1998) examined slow flow past periodic cylinder arrays of square and hexagonal configurations. For each arrangement, two different solidities were studied (66% and 44%). In their experiments, shear-thinning polyacrylamide (PAA) solutions were used. They found the onset of increased pressure drop due to fluid elasticity occurred in the Deborah number range of 0.1 to 0.2 for a 0.1 wt% PAA solution. For a more elastic 1.0% wt% PAA solution, an increased pressure drop occurred at a Deborah number ranging from 0.3 to 0.5. The $De$ used by the authors was based on a shear-rate-dependent relaxation time estimated from normal stress data, and the average stretch rate.
Moss and Rothstein (2010) used particle image velocimetry (PIV) to produce velocity vector maps for the flow of a shear-thinning wormlike micellar solution through a porous medium. Their model consisted of six cylinders installed in a channel, in a 3x2 array arranged broadside to the flow. Flow asymmetry was observed, which they attributed to a flow instability caused by fluid elasticity.

These previous experiments of viscoelastic flow through cylinder arrays have shown that flow resistance increases above the Newtonian value once a critical Deborah number is reached. Several authors have reported flow asymmetry and fluctuations in pressure measurements at just beyond the elastic onset, which was explained by an apparent flow instability.

A related experimental investigation was undertaken by McKinley et al. (1993), who examined the elastic instability of a Boger fluid flow around a cylinder confined in a rectangular channel. The cylinder diameter to channel width ratio $\beta$ varied from 0.17 to 0.50. The authors used laser Doppler velocimetry to measure the streamwise velocity in the centre plane. They found that, upstream of the cylinder, the dimensionless velocity component did not vary with increasing $De$ when compared with Newtonian results. However, the size of the wake increased with $De$. The authors observed a flow instability beyond an initial critical Deborah number, where the flow transitioned from steady and two-dimensional to steady and three-dimensional, consisting of spatially periodic structures. As a second critical $De$ was reached, another flow transition took place, in which the flow in the wake became time dependent.
2.2.2 Numerical Analyses of Viscoelastic Flow Through Cylinder Arrays

Steady, two-dimensional viscoelastic flow through cylinder arrays in square and staggered configurations has been investigated by Talwar and Khomami (1992), Souvaliotis and Beris (1992), Khomami et al. (1994), and Talwar et al. (1994). Solutions using the Oldroyd-B and the upper convected Maxwell (UCM) constitutive equations were computed. While an increase in flow resistance above a critical Deborah number was shown in the experimental results, the computational results based on steady-state simulations did not exhibit the same increase.

Talwar and Khomami (1995) compared the Oldroyd-B and UCM flow resistance data with results obtained using more complex constitutive equations, such as the Phan-Thien-Tanner (PTP) and the Giesekus models. However, no significant improvement in terms of predicting viscoelastic flow resistance was shown by using the more complex models.

Hua and Schieber (1998) used a numerical method, known as CONNFFESSIT, which is a finite element technique combined with stochastic simulations, to solve viscoelastic flows through an infinite array of square-arrayed cylinders. Like the previous authors, their results showed no dramatic increase in flow resistance with increasing fluid elasticity.

The possible explanation for the qualitative change in the behaviour of flow resistance with increasing $De$ may be that the flow exhibits a transition to another flow structure which substantially increases the flow resistance. Skartsis et al. (1992), Talwar
and Khomami (1992), Chmielewski and Jayaraman (1993), and Talwar and Khomami (1995), among others, have suggested that the experimentally observed increase in flow resistance is due to a transition to an unstable flow structure, which was not predicted by any numerical simulations.

### 2.2 Research Objectives

The general objective of this study is to examine the effects of fluid elasticity on slow flow through cylinder arrays of low solidity. There has been no study of viscoelastic flows through arrays having solidities below 14%. This range is important because numerous fibrous media have such low solidities; also, fundamental knowledge about viscoelastic flows may be gained when cylinders are widely spaced.

More specifically, flows through square arrays of cylinders with solidities of 2.5%, 5%, and 10% are to be examined, as they adequately represent fibrous media in the low solidity range. Square arrays of cylinders are chosen because of their simple and well-defined structures. The goals of this research are then:

- To measure the flow resistance and velocity fields for the flow of a Newtonian fluid through square arrays of cylinders, to establish an inelastic baseline.
- To make the same measurements for several model elastic fluids.
- To characterize the flow properties of the fluids in shear and extension to determine which properties are important in generating flow resistance.
CHAPTER 3: TEST FLUIDS

To explore the effects of fluid elasticity, several test fluids were prepared. A Newtonian fluid was used first to establish a baseline, the results of which were compared with data obtained from viscoelastic fluids. In order to isolate the contribution of elasticity, special viscoelastic fluids were made. Boger fluids, having a high elasticity and an almost constant viscosity, served that purpose. This chapter discusses the selection of the test fluids, and describes their rheological characterizations.

3.1 Glycerol

A Newtonian fluid was used for the initial phase of the experimental investigation because it has a constant viscosity and is inelastic. Because creeping flow is the main interest in the current study, the effects of inertia were marginalized by keeping the Reynolds number low. Therefore, a fluid with a relatively high viscosity was desired. Glycerol obtained from EMD Chemicals was chosen for this reason. Using a TA Instruments AR2000 rheometer, the fluid was found to have a viscosity of 0.85 Pa.s at 24°C (the nominal temperature of the test fluid). This viscosity was high enough to ensure that Reynolds numbers were less than 0.1 in the test apparatus.

An advantage of using glycerol is that it is water soluble, making it easy to clean from the test apparatus. However, because glycerol is hygroscopic, the water content continuously increases due to absorption of moisture from the environment. Because the
viscosity of the glycerol is sensitive to the water content, the viscosity and the fluid temperature were measured during each experimental run.

3.2 Boger Fluids

While polymeric solutions are often used to investigate the effects of fluid elasticity, a common concern is that most polymeric solutions are shear-thinning, in addition to being elastic. To address this problem, Boger developed an elastic fluid that has a nearly constant shear viscosity (Boger, 1977). ‘Boger fluids’ can be used in experiments without having to account for shear-thinning effects. These dilute solutions are obtained by dissolving a small concentration of a high molecular weight polymer in a highly viscous Newtonian solvent (James, 2009).

Various Boger fluids have been used in the past. Early Boger fluids were aqueous solutions of polyacrylamide (PAA) in corn syrup (Boger, 1977), or polyacrylamide in glucose and water (Chhabra et al. 1980). Prilutski et al. (1983) created the first organic Boger fluid, which was composed of a dilute concentration of polyisobutylene (PIB) in a mixture of polybutene (PB) and kerosene. This type of Boger fluid has since become the one which is most widely used (Chmielewski and Jayaraman, 1992, 1993; Khomami and Moreno, 1997; Shiang et al., 1997; Verhoef et al., 1999; Arora et al., 2002).

Both aqueous and organic Boger fluids are difficult to handle. Aqueous solutions require a long mixing process at low shear rates to achieve a homogenous state; also the sugar based solutions (corn syrup and glucose) have a tendency to form a skin on the
exposed surface upon due to evaporation (Binnington and Boger, 1986). Organic solutions, on the other hand, require large amounts of solvent for cleaning (James, 2009). Nevertheless, organic PIB-PB based Boger fluids offer several advantages. Compared with aqueous solutions, PIB-PB solutions are free from degradation over long periods of storage, are less sensitive to evaporation, and have a viscosity which is less dependent on temperature (Binnington and Boger, 1986). Moreover, because this type of Boger fluid is transparent, it is suitable for use in optical velocimetry experiments. Based on these considerations, PIB-PB based Boger fluids were chosen for this study.

3.2.1 Boger Fluid Preparation

Based on the known characteristics of the above PIB-PB based Boger fluids, two PIB-PB compositions were chosen for the present experiments, both consisting of 0.2 wt% polyisobutylene in a solvent mixture of 7 wt% kerosene and 92.8 wt% polybutene. The first Boger fluid (B1) solvent was polybutene with a molecular weight of 910 g/mol (Brenntag Indopol H-100) and the PIB had a molecular weight of 4.7 million g/mol (Scientific Polymer CAT# 040E). For the second Boger fluid (B2), the same polyisobutylene was used, but the polybutene had a lower molecular weight of 635 g/mol (Brenntag Indopol H-25). Table 3.1 summarizes the compositions of the two Boger fluids.
### Table 3.1 Compositions of the two Boger fluids. Molecular weight is denoted as $M_W$.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.2 wt% PIB</td>
</tr>
<tr>
<td></td>
<td>$M_W = 4.7 \times 10^6$ g/mol</td>
</tr>
<tr>
<td></td>
<td>$M_W = 910$ g/mol</td>
</tr>
<tr>
<td>B2</td>
<td>0.2 wt% PIB</td>
</tr>
<tr>
<td></td>
<td>$M_W = 4.7 \times 10^6$ g/mol</td>
</tr>
<tr>
<td></td>
<td>$M_W = 635$ g/mol</td>
</tr>
</tbody>
</table>

Both fluids were prepared in a similar fashion. Solid blocks of polyisobutylene were initially dissolved in kerosene in carefully measured proportions. The mixture was sealed in an Erlenmeyer flask and agitated with a magnetic stirrer for approximately 3 days. The mixture was also heated to help dissolve the polyisobutylene. Once the mixture was free of clumps, it was poured into a drum of liquid polybutene of known volume. This fluid was then stirred for a period of 5 to 6 days with a slow mixer (Cole-Parmer Stir Pak) until the fluid became homogenous in its consistency. By this method, approximately 50 litres of each Boger fluid was prepared.

### 3.2.2 Boger Fluid Characterization in Shear

The rheological properties of the Boger fluids were measured with a TA Instrument AR2000 rheometer. Steady shear and small amplitude oscillatory shear measurements were obtained with a cone-and-plate fixture (60 mm diameter, with a cone angle of 4°). Figures 3.1a and 3.1b present the steady shear viscosity $\eta$ and the dynamic viscosity $G'/\omega$. The shear rates tested in the rheometer cover the ranges of shear rates found in the
three cylinder arrays. The plots show that the low-shear rate viscosities $\eta_0$ for B1 and B2 were 34 Pa.s and 4.0 Pa.s, respectively. These fluids exhibit slight shear-thinning behaviour: the B1 viscosity decreased by about 8% at the highest available steady shear rate of 2.5 s$^{-1}$; and a 10% decrease was shown for the B2 fluid at a steady shear rate of 4 s$^{-1}$. Figures 3.1a and 3.1b also show that at low oscillation frequencies, the dynamic viscosity of each Boger fluid approaches its respective low-shear rate viscosity. The predictions of the Oldroyd-B model for dynamic viscosities (Eq. 2.27), using the relaxation times calculated from Eq. 2.30, are also plotted on the figures. The method of determining relaxation times will be discussed in detail later in this section. Figures 3.1a and 3.1b show that the Oldroyd-B constitutive equations accurately predict the dynamic viscosity only in the high (greater than 10 s$^{-1}$) and low (less than 0.1 s$^{-1}$) frequency ranges. The deviation of the predictions at the mid-frequency ranges is due to the limitation of the single-mode Oldroyd-B model. Because only one relaxation time is required for the Deborah number, the single-mode model is adequate for the current study.

Figure 3.2 presents the first normal stress difference $N_1$ of the two Boger fluids. The figure shows that, for each fluid, $N_1$ increases with shear rate to the power of 2.2; this is close to the value of 2 predicted by the Oldroyd-B model.

Measurements for the storage modulus $G'$ and the loss modulus $G''$, along with their Oldroyd-B predictions (Eqs. 2.27 and 2.28), are presented in Figure 3.3a for fluid B1, and Figure 3.3b for fluid B2. While $G''$ is accurately predicted by the Oldroyd-B model for all frequencies measured, the predictions of $G'$ is only accurate at the low frequency range of less than 0.1 s$^{-1}$. 
Figure 3.1a Viscous properties of fluid B1 at 24°C.

Figure 3.1b Viscous properties of fluid B2 at 24°C.
Figure 3.2 First normal stress difference \( N_1 \) of fluids B1 and B2 at 24°C.

In order to determine the fluid relaxation time \( \lambda \) based on the Oldroyd-B model, the storage modulus and the first normal stress difference are replotted as \( G'/\omega^2 \) and \( \Psi'_1/2 \), respectively, where \( \Psi'_1 \) is the first normal stress coefficient defined as:

\[
\Psi'_1 = N_1/\dot{\gamma}^2.
\]

These quantities for the two Boger fluids are presented in Figures 3.4a and 3.4b. To calculate the Oldroyd-B relaxation times for each fluid using Eqs. 2.29, and 2.30:
Figure 3.3a Storage modulus $G'$ and loss modulus $G''$ of fluid B1 at 24°C.

Figure 3.3b Storage modulus $G'$ and loss modulus $G''$ of fluid B2 at 24°C.
\[
\lambda = \frac{1}{2} \lim_{\gamma \to 0} \left( \frac{N_1}{\gamma^2 (\eta_0 - \eta_s)} \right),
\]
\[
\hat{\lambda} = \lim_{\omega \to 0} \left( \frac{G'}{\omega^2 (\eta_0 - \eta_s)} \right),
\]

it is necessary to determine the limits of $\Psi_1 / 2$ and $G' / \omega^2$ as the shear rate and the oscillation frequency approach zero. However, it can be seen in Figures 3.4a and 3.4b that the two lower limits do not really coincide. This shortcoming arises because the AR2000 rheometer cannot measure $N_1$ accurately at low shear rates for these fluids. Therefore, the more definitive lower limit of the storage modulus data, $G' / \omega^2$, was used to determine the relaxation time for each fluid. By this method, the relaxation time for B1 was found to be 8.2 s at 24°C, and 3.9 s at 24°C for B2. The relaxation times, along with other fluid properties, are summarized in Table 3.2.

### 3.2.3 Boger Fluid Characterization in Extension

A filament-stretching rheometer (FSR), as described in Section 2.1.4, was used to characterize fluid B1 in extension. This device stretches a filament at a constant extensional rate, so that the ‘flow’ is almost completely free of shear. Appendix A shows the modifications to the FSR control software to ensure that a ‘true’ constant extensional rate was achieved, by creating conditions in which the filament diameter decreased exponentially in time.

By measuring the extensional viscosity as a function of time, the elasticity of the fluid in extension can be quantified (cf. $N_1$ quantifies the elasticity of the fluid in shear).
Figure 3.4a $G'/\omega^2$ and $\Psi_1/2$ of fluid B1 at 24°C.

Figure 3.4b $G'/\omega^2$ and $\Psi_1/2$ of fluid B2 at 24°C.
The extensional viscosity of B1 was measured at extensional rates of 1.0 s\(^{-1}\), 1.5 s\(^{-1}\), and 2.0 s\(^{-1}\); this range adequately covers the stretch rates that the fluid experiences in the arrays. The results, shown in Figure 3.5, are presented in dimensionless form: the Trouton ratio \(Tr\) as a function of the Hencky strain \(\varepsilon\). The Trouton ratio was previously defined in Eq. 2.15 as:

\[
Tr(\dot{\varepsilon}, t) = \frac{\eta_{E}(\dot{\varepsilon}, t)}{\eta} ,
\]

and \(\varepsilon\) is defined as:

\[
\varepsilon = \dot{\varepsilon} t ,
\]

where \(\eta_{E}\) is the extensional viscosity, \(\eta\) is the shear viscosity, \(\dot{\varepsilon}\) is the extensional rate, and \(t\) is the time of stretch.

![Figure 3.5](image-url)  

**Figure 3.5** Trouton ratio of Boger fluid B1 at three different extensional rates.
The results reveal that the Trouton ratio data collapse into a single curve for different extensional rates when the time variable is made dimensionless. At low Hencky strains \((0 < \varepsilon < 2)\), B1 behaves like a Newtonian fluid; and as \( \varepsilon \) increases above 5, the Trouton ratio plateaus to a value of approximately 10,000. The measurements shown in Figure 3.5 are similar to the data obtained by Tirtaatmadja and Sridhar (1993) and Anna et al. (2001). These data do not follow the Oldroyd-B model, which predicts that extensional viscosity reaches an infinite limit at a finite elongation rate.

Characterization for fluid B2 in extension was not performed because a suitably sensitive force transducer was not available. However, it can be surmised that at Hencky strains of less than 2, B2 would exhibit a Trouton ratio of 3, as elastic effects are not important in that range; this behaviour has been found for other PIB-PB Boger fluids (Anna et al., 2001).

### 3.2.4 Additional Boger Fluid Properties

The measurements of surface tension were required for the calculation of the extensional viscosity. Surface tension measurements of the two Boger fluids were performed on a Fisher Scientific DuNouy ring tensiometer. These results, along with the other previously-stated fluid properties, are summarized in Table 3.2.

It should be noted that, as the kerosene slowly evaporated, the properties of the Boger fluids changed. More specifically, the shear viscosity and relaxation time drifted to
slightly higher values. It is estimated that these values increased by approximately 5\% for every three months of usage.

The solvent viscosity $\eta_s$, needed for the calculation of the fluid relaxation time, was approximated using the dynamic properties of the Oldroyd-B model (Eq. 2.27). The expression for $\eta_s$ is found by:

$$
\eta_s = \lim_{\omega \to \infty} \frac{G''}{\omega}.
$$

(3.3)

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density, $\rho$</th>
<th>Surface Tension, $\sigma$</th>
<th>Low-Shear Rate Viscosity, $\eta_0$</th>
<th>Solvent Viscosity, $\eta_s$</th>
<th>Relaxation Time, $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>880 kg/m$^3$</td>
<td>3.4x10$^{-2}$ N/m</td>
<td>34 Pa.s</td>
<td>21 Pa.s</td>
<td>8.2 s</td>
</tr>
<tr>
<td>B2</td>
<td>854 kg/m$^3$</td>
<td>3.2x10$^{-2}$ N/m</td>
<td>4.0 Pa.s</td>
<td>2.1 Pa.s</td>
<td>3.9 s</td>
</tr>
</tbody>
</table>

**Table 3.2** Fluid properties of Boger fluids B1 and B2.
CHAPTER 4: PRINCIPLES OF PIV

4.1 Introduction

Particle image velocimetry (PIV) is a flow visualization technique used to obtain instantaneous velocity vector measurements in a cross section of a flow. Other related field properties, such as vorticity and rate of deformation, can also be determined. The PIV technique has been used to analyze a wide variety of problems, such as flow in turbulent jets (Tandalam et al., 2010; Gutmark et al. 2011), airfoil dynamics (Tinar and Cetiner, 2006; Ragni et al., 2009; Lee, 2011), and blood flow around prosthetic heart valves (Kaminsky et al., 2007; Akutsu and Matsumoto, 2010; Hutchison et al., 2011).

Figure 4.1 shows a typical arrangement for a PIV flow measurement system. It consists of a fluid seeded with tracer particles, a light source, an optical arrangement to expand the light source into a thin sheet (the ‘lens’ in the figure), a charged couple device (CCD) sensor, a frame grabber, a synchronizer, and a CPU with image acquisition and raw image analysis software.

The fluid is initially seeded with tracer particles that follow the flow without exhibiting any dynamic effects. A sheet of light, generated by either a continuous wave or pulsed laser beam, illuminates the particles in a plane of interest within the flow. Scattered light from the bright particles are then captured by the CCD sensor to record their positions. Timing between the laser pulses and image captures is coordinated by the synchronizer, which acts as an external trigger.
Images of the particles are captured at set time intervals. A set of two consecutive images, identifying the locations of the particles at two instances in time, are analyzed by software to determine particle displacements, from which the velocity field is obtained.

Because of its optical nature, the technique does not require intrusive measurement probes. When properly chosen, the tracer particles cause negligible flow disturbances. Unlike other methods of flow measurement, such as laser Doppler anemometry (LDA) or hotwire anemometry, PIV is capable of measuring an entire cross section of the flow field instantaneously. With the advent of powerful CPUs capable of high volume data
processing, a large number of image pairs may be captured and analyzed, allowing for an almost continuous stream of vector plots to be obtained.

A detailed discussion of the components of the PIV setup used for the current investigation is provided in the following sections.

### 4.2 Seeding Particles

The tracer particles used in seeding the flow must follow the fluid faithfully, since the PIV technique infers fluid motion by determining particle displacements. The choice of particles is largely dependent on the type of fluid being observed. Typical seeding particles used in previous studies include glass beads (Sakakibara et al., 1996; Hardalupas and Horender, 2003), polystyrene beads (Akselli et al., 2009; Chamarthy et al., 2009), aluminum flakes (Ohyama and Kaneko, 1997; Shin et al., 2000), and oil droplets (Baert and Klaassen, 2010).

In order for the tracer particles to follow the flow, the particle settling velocity must be several orders below the fluid velocities. To determine the velocity at which the particles settle in the fluid, Stokes’ law for the drag force on a sphere may be used. This velocity, which arises from the difference in the particle and fluid densities, is expressed as (Mei et al., 1991):

\[
    v_s = \frac{(\rho_p - \rho)}{18\eta} gd_p^2,
\]

(4.1)
where \( v_s \) is the settling velocity, \( d_p \) is the particle diameter, \( \rho_p \) is the particle density, \( \rho \) is the fluid density, \( \eta \) is the viscosity of the fluid, and \( g \) is the gravitational acceleration. For slow settling, particles with a small diameter should be chosen; also, they should have a density close to that of the fluid.

For the particles to accurately follow the fluid flow, they should be small enough that the particle response time is of several orders less than the characteristic flow time. The particle response time \( \tau_R \) is given by (Westerweel et al., 1996):

\[
\tau_R = \frac{\rho_p d_p^2}{18 \eta}.
\]  

The expressions for settling velocity and the particle response time are only applicable in the Stokes’ regime, where the Reynolds number is less than one. The particle Reynolds number \( \text{Re}_p \) is given by:

\[
\text{Re}_p = \frac{\rho_p v_s d_p}{\eta}.
\]  

The tracer particles used for the current investigation were silver-coated hollow glass spheres (Conduct-O-Fil, Potters Industries). This product was chosen because of its high refractive index (2.6), low density (1650 kg/m\(^3\)), and small size (a mean particle diameter \( d_p \) of 14 µm).

The settling velocity, the response time, and the Reynolds number of the particles in the three test fluids are given in Table 4.1. The settling velocities in all test fluids are six
orders less than the velocities found in the arrays. Moreover, the short response times ensure that fluid motion is properly tracked by the particles. Because the flow is in the low Reynolds number regime (Re < 0.1), the particles respond almost instantaneously to any changes in fluid velocity.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Particle Settling Velocity, ( v_s )</th>
<th>Response Time, ( \tau_s )</th>
<th>Particle Reynolds Number, ( Re_p )</th>
<th>Maximum Velocity in Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glycerol/Water</td>
<td>( 5.2 \times 10^{-3} ) mm/s</td>
<td>( 2.1 \times 10^{-8} ) s</td>
<td>( 1.4 \times 10^{-9} )</td>
<td>54 mm/s</td>
</tr>
<tr>
<td>B1</td>
<td>( 0.24 \times 10^{-5} ) mm/s</td>
<td>( 5.3 \times 10^{-10} ) s</td>
<td>( 1.6 \times 10^{-12} )</td>
<td>8.2 mm/s</td>
</tr>
<tr>
<td>B2</td>
<td>( 2.1 \times 10^{-5} ) mm/s</td>
<td>( 4.5 \times 10^{-9} ) s</td>
<td>( 1.2 \times 10^{-10} )</td>
<td>16 mm/s</td>
</tr>
</tbody>
</table>

Table 4.1 PIV particle characteristics in the three test fluids.

### 4.3 Light Source

A laser beam is converted to a light sheet as it passes through a cylindrical lens. This sheet of light can be used to illuminate tracer particles in a plane. Depending on the types of flow, different lasers may be used. For high speed flows, a double-pulsed Nd:YAG (neodymium-doped yttrium aluminum garnet) laser is typically used. The pulsed action of the high powered laser is required to freeze the fast motion of the particles. For slower flows, a continuous wave (CW) laser, such as Copper Vapor, Helium Neon, Argon Ion, or laser diodes, may be used.
When the flow is traveling at very low velocities, it is possible to use a CW laser without severing the beam into pulses. The requirement of the light source is to provide enough illumination so that the CCD camera can capture images of resolved particles. If the flow is slow enough, the electronic shutter of the camera can be set to a lower speed and still capture sharp particle images. Therefore, a low power CW laser can be integrated into a PIV system without the need of an external synchronizer. In this scenario, frame separation is solely controlled by the acquisition frame rate of the camera.

In the current apparatus, two CW diode-pumped solid-state (DPSS) lasers were used to measure creeping flow within the three test sections used in the current study. These two green lasers, with 532 nm wavelengths, were purchased from Intelite Inc. Each laser was paired with a cylindrical glass lens to expand the light beam into a sheet of approximately 1 mm in thickness; this is sufficiently thin for velocity measurements in which the flows are expected to be essentially two-dimensional. The two light sheets were subsequently made to overlap one another to increase the overall luminance within the flow area of interest and to minimize ‘shadows’ (to be shown in Figure 5.5).

4.4 Image Acquisition

Light scattered by tracer particles can be captured on an analogue film or by a CCD sensor. To perform a PIV analysis, two exposures, separated in time, are required to determine particle displacements. Because early analogue cameras were not capable of
Chapter 4: Principles of PIV

capturing multiple frames at high speeds, consecutive exposures were often captured on a single frame. Analysis of the double exposure on a single frame was performed via a process called auto-correlation. Modern digital cameras with the ability to capture successive frames at high speeds allow exposures to be isolated on their own frames. These separate frames are analyzed with the more accurate cross-correlation technique (Raffel et al., 1998), which was used for the present PIV study.

For the current PIV setup, a digital camera (JAI TM-6740CL) capable of capturing up to 200 frames per second was used. The progressive scan CCD sensor has a diagonal dimension of 1/3 inch, and an aspect ratio of 4:3. The output resolution is 640 horizontal pixels by 480 vertical pixels, with each square pixel having a side dimension of 7.4 µm. The adjustable electronic shutter can set exposure speeds up to a maximum of 1/6400 second.

During an experiment, a PCI-bus frame grabber (Euresys Grablink Value) captured the digital frames from the high speed stream of images produced by the CCD sensor. These still images were initially stored on the random access memory (RAM) of a personal computer, and were later transferred to the system’s hard disk. The control of the shutter speed, image storage allocation, and frame rate adjustments were performed by video recording software called Norpix StreamPix 4. This program, by default, lacked the ability to adjust the camera frame rate; however, a script was developed which modified the software to include this important feature. This script is included in Appendix B. Once the camera frame rate was set, the frame separation time $\Delta t$ is simply given by:
$$\Delta t = \frac{1}{\text{frame rate}}.$$ \hspace{1cm} (4.4)

Because flow velocities were low compared with the camera frame rate, no external trigger was required to synchronize the light source emission with the timing of the image captured.

### 4.5 PIV Analysis

The images captured by the camera were divided into a square grid of sub-sections known as ‘interrogation areas’. These were small enough that all particles within were assumed to move at uniform displacements during the frame separation time. Hence, each interrogation area yielded one mean particle displacement vector.

Two different types of image correlations can be used for PIV analysis. Auto-correlation is used when a single image frame, containing two exposures separated by $\Delta t$, is captured. This method of analysis yields a result which contains three different correlation peaks: a central self-correlation peak, and two displacement peaks. There exist two possibilities for the mean particle displacement, which can be directed from the self-correlation peak to either one of the displacement peaks. The directional ambiguity of the auto-correlation technique is the main disadvantage when analyzing a single frame with double exposures.
For consecutive frames of single isolated exposures, cross-correlation is used. Unlike auto-correlation, this method provides a distinct displacement vector for each interrogation area because the order of the two images is known. The cross-correlation function is applied to every pixel within the interrogation area to find the mean spatial shift. In discretized form, this function is expressed as follows (Willert and Gharib, 1991):

\[
C(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(i, j)I'(i + x, j + y),
\]

where \( C(x, y) \) denotes the cross-correlation function for the pixel located at coordinates \((x, y)\), \( I(x, y) \) and \( I'(x, y) \) are the pixel light intensities for the two frames separated by \( \Delta t \). The infinite limits found in the double summation of Eq. 4.5 can be replaced by the boundary coordinates. Once the entire interrogation area is scanned, the coordinates of the highest correlation can be determined, and this represents the most likely particle displacement in that sub-section.

A PIV software developed by AEA Tech, called VISIFLOW was used in our experimental setup. During PIV analysis of individual interrogation areas, instead of directly applying Eq. 4.5, the software performs a fast Fourier transform (FFT) to the two light intensity functions, \( I(x, y) \) and \( I'(x, y) \). The transformed functions are then subjected to a complex conjugate multiplication. Taking the inverse Fourier transform of the resulting product yields the desired cross-correlation function \( C(x, y) \). Having determined the mean particle displacement from the correlation peak location, the velocity at the sub-section can be calculated by dividing by \( \Delta t \). This process is repeated for all interrogation
areas to obtain a full vector field. A diagram summarizing the software cross-correlation algorithm is shown in Figure 4.2. In the current PIV analysis, the transverse and wake region velocity distributions in the cylinder arrays were measured for the Newtonian fluid and the two Boger fluids. Any differences in the flow fields found between the Newtonian and Boger fluids can then be attributed to the effects of fluid elasticity.
Figure 4.2 Cross-correlation analysis of image pairs.
CHAPTER 5: EXPERIMENTAL METHODOLOGY

5.1 Flow Apparatus

A diagram of the flow apparatus is shown in Figure 5.1. The apparatus contains a large overhead reservoir connected to a vertical entry section, below which the test section is located. Below the exit section is a 3-inch diameter ball valve to control the flow rate, and underneath is a collection tank. Collected fluid is returned to the upper reservoir by hand to avoid using a mechanical pump which can degrade a polymeric fluid. The large cross section of the reservoir (500 mm x 500 mm) ensures that the driving head remains relatively constant during test runs.

The main component of the flow apparatus is the test section, a square array of cylinders at solidities of 2.5%, 5.0%, and 10%, which covers a range of low solidities which have previously not been investigated. The front and side views of the three test sections are shown in Figures 5.2 to 5.4. Each test section consists of multiple columns and rows of cylindrical rods mounted in a rectangular channel having a cross section of 178 mm x 100 mm. The numbers of columns and rows for each test section are summarized in Table 5.1.
Figure 5.1 Experimental apparatus.
Figure 5.2 Front and side views of the 2.5% test section.
Figure 5.3 Front and side views of the 5.0% test section.
Figure 5.4 Front and side views of the 10% test section.
Table 5.1 Number of columns and rows of cylinders in the three test sections.

<table>
<thead>
<tr>
<th>Solidity of Test Section</th>
<th>Columns</th>
<th>Rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>5.0%</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>10%</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Each rod had a uniform diameter of 3.18 mm and an aspect ratio of more than 30. Flows were driven by gravity and so the flow direction was vertically downward. The small rod diameter, along with high fluid viscosities, ensured that flows remained in the Stokes regime, with Reynolds numbers less than 0.1. The test section was designed with long rods and many columns to minimize the effect of shearing caused by the walls. However, even with the large number of columns and a rod aspect ratio of more than 30, it was found that wall effects had to be accounted for in each test section, the calculations for which are described in Zhong et al. (2006). Theses authors determined the flow rate with shearing at the walls by applying Brinkman’s equation, which is a combination of Darcy’s Law and Stokes equation. The decrease in the flow rate due to wall effects for the three test sections is listed in Table 5.2.
Table 5.2 Decrease in flow rate due to wall effects. (Zhong et al., 2006).

<table>
<thead>
<tr>
<th>Solidity of Test Section</th>
<th>Decrease in Flow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>13.7%</td>
</tr>
<tr>
<td>5.0%</td>
<td>8.6%</td>
</tr>
<tr>
<td>10%</td>
<td>5.2%</td>
</tr>
</tbody>
</table>

In mounting the cylinders, the holes in one wall were drilled through while blind holes were drilled in the other wall. The blind holes minimized leakage on the optical side of the test section, i.e., on the side through which the PIV images were captured. The walls and rods were fabricated of acrylic plastic to provide clear optical paths throughout. As illustrated in Figures 5.2 to 5.4, pressure taps were installed along the centre of each test section for the measurement of pressure drop. The various test sections were the same size so that they were interchangeable in the experimental apparatus.

### 5.2 Pressure Drop Measurements

Flow rates through the apparatus were controlled using calibrated openings of the ball valve. The pressure drop across the test section was monitored by a differential pressure transducer connected to a pair of well-separated pressure taps. These pressure taps were located between the first and second rows of cylinders (high pressure port), and
between the second last and last rows of cylinders (low pressure port). By using these taps, the maximum pressure drop within an array could be measured.

Depending on the magnitude of the pressure drop, either a low differential transducer (0 to 5” water, Omega PX154-005DI) or a high differential transducer (0 to 25” water, PX154-025DI) was used. Real-time signals from the transducer were displayed on a digital meter (Omega DPi8-C24), while a personal computer connected to the meter acquired pressure differential data as a function of time. The real-time digital displays provided instant feedback, enabling precise control of the flow rate. The pressure loss was calculated by taking the average pressure drop obtained while the valve was set to a desired opening.

5.3 Bulk Velocity Measurements

The flow rate was determined by dividing the measured volume of collected fluid by the time of collection. This flow rate was then adjusted to account for the retarded flow near the walls; for example, the decrease in flow rate due to wall effects is 5.2% for the 10% test section (Table 5.2). Therefore, the ratio of the measured flow rate to the adjusted flow rate is 0.948, which holds for all flow rates. The bulk (or superficial) velocity was determined by dividing the adjusted flow rate by the total cross sectional area of the test section.
5.4 PIV Setup

The individual PIV components were described in detail in Chapter 4. A brief summary of the entire system is summarized in this paragraph for convenience. Two diode-pumped solid state (DPSS) lasers were utilized in the PIV system to illuminate the flow field in a plane. Each laser beam had a wavelength of 532 nm and each passed through a cylindrical lens to create a light sheet of approximately 1 mm in thickness. Pulsed lasers were not necessary because velocities were low, of order 1 mm/s. Images within an area of interest were captured with a CCD camera (JAI 6740CL) connected to a PCI-bus frame grabber (Euresys Grablink Express). The latter was controlled by software (Norpix StreamPix 4) which could set acquisition rates up to 200 frames per second. Consecutive images were processed to increase the brightness and to sharpen the particles. Enhanced image pairs were cross-correlated with a software package produced by AEA Technology, called VISIFLOW.

The seeding particles were silver-coated hollow glass spheres, with a diameter of 14 μm and a density of 1650 kg/m³ (Conduc-O-Fil, Potters Industries). Dispersing the particles in the glycerol/water mixture required the use of a slow mixer (Stir Pak, Cole-Parmer) over a period of 12 hours. For the Boger fluids, because of their elasticity and high viscosity, dispersing the particles required running the seeded fluid through the flow apparatus several times, until streaks formed by clumps of the particles disappeared. Particles were added in small amounts to the fluids until the interrogation areas of 32 x 32
Chapter 5: Experimental Methodology

Pixels had a particle density of about 10 to 15 per area, which is considered optimal for image pair cross-correlation algorithms (Raffel et al., 1998).

5.5 PIV Procedures

PIV measurements were carried out in two orthogonal planes, one plane perpendicular to the cylinder axes, as indicated by Figure 5.5, and the other through the centres of cylinders in a column, as illustrated in Figure 5.6. Although Figures 5.5 and 5.6 show the 10% solidity model, the same orthogonal planes were used for the two other test sections.

In the first PIV setup, the two laser sheets were aligned to create a single x-y vertical plane. Figure 5.5b shows the locations of the lasers, lenses and test section for this configuration. To minimize shadows caused by the cylinders, the light sheets were oriented at different angles to illuminate an interior region as much as possible. The optimal configuration, found by trial and error, was a vertical separation of 8 to 10 cm. The CCD camera focused on a ‘unit cell’, a set of four cylinders near the middle of the array, as illustrated in Figure 5.5a. Because of the wide range of velocities within a unit cell, the frame rate was chosen such that the average distance travelled by the particles between frames was about 25% of the distance across an interrogation area (Raffel et al., 1998). For the velocities in this study, the frame rates varied from 10 to 200 frames per second.
Figure 5.5 PIV setup for velocity measurements in an $x$-$y$ plane within a unit cell. 5.5a shows the origin of the $x$-$y$ coordinates and 5.5b shows the positions of the two lasers and lenses outside the test section.
The second PIV setup enabled observations of the flow in the wake regions between cylinders, i.e., in vertical $x$-$z$ planes passing through the cylinder centres, as illustrated by Figure 5.6. With two lasers available, one laser sheet illuminated the plane through one column of cylinders and the other through an adjacent column, as Figure 5.6a shows, with one column designated as the left side and the other as the right side. During a run, images were initially gathered with one laser on to illuminate one column; then it was shut off and immediately afterward the other laser was turned on to illuminate the adjacent column. In this way, virtually simultaneous velocity fields were obtained since the flows were time independent. Since the strongest illumination was near the wall where the light entered, measurements were obtained near the walls as well as within the array, as will be seen in the results.
Figure 5.6 PIV setup for measurement in $x$-$z$ planes. The dashed lines in 5.6a designate the locations of light sheets for illuminating flow in the wake regions of two adjacent columns of rods. 5.6b shows the positions of the laser and lens to illuminate an $x$-$z$ plane.
CHAPTER 6: RESULTS AND DISCUSSIONS

Results for the flow resistance and PIV measurements are presented and discussed in this chapter. Where applicable, comparisons with prior experimental and analytical results are made.

6.1 Flow Resistance Measurements

6.1.1 Parameters and Equations for Porous Media Flow

Measurements of pressure drop through the three cylinder arrays were first made with the Newtonian fluid. These results will be used to draw comparisons with data obtained from the Boger fluids. To ensure accuracy of the pressure drop measurements, the Newtonian data are compared with known analytical results. To present the data, the relevant parameters and equations for porous media flow are now introduced.

Darcy’s law is commonly used in studies of slow flow of Newtonian fluids through a porous medium where the superficial velocity is uniform. It relates the pressure gradient to the flow rate, fluid viscosity, and the permeability of the medium. The law has the following form:

\[
\frac{\Delta p}{l} = \frac{n}{k} U, \tag{6.1}
\]
where $\Delta p$ is the pressure loss over a length $l$, $\eta$ is the viscosity, $k$ is the permeability and $U$ is the superficial velocity or bulk velocity. Darcy’s law pertains to Newtonian fluid flows at Reynolds number less than 10 (de Marsily, 1986), where inertial effects are negligible. Darcy’s law cannot account for velocity gradients due to shear, such as those near a solid boundary.

In order to quantify the flow resistance across the cylinder arrays, two dimensionless parameters are introduced. The friction factor $f$ is defined as:

$$f = \frac{\Delta p}{l \rho U^2},$$

and the Reynolds number $Re$ is defined as:

$$Re = \frac{\rho U (2a)}{\eta},$$

where $a$ is the cylinder radius, and $\rho$ is the fluid density. Darcy’s law can be expressed in terms of $f$ and $Re$ in the following form:

$$f Re = \frac{4a^2}{k}.$$ 

The group $f Re$ is the preferred measure of flow resistance because the product depends only on the porous medium, i.e., it is independent of the flow rate and fluid properties.
Several predictions have been made for the permeability of square arrays of widely-spaced cylinders. Although the analytical techniques differ, the solutions of Hasimoto (1959), Sangani and Acrivos (1982), and Drummond and Tahir (1984) yield essentially the same formulas. The previous experimental work of Skartsis et al. (1992), Chmielewski and Jayaraman (1993), and Zhong et al. (2006) confirmed that flow resistance measurements and the three analytical solutions are in good agreement. The solution of Sangani and Acrivos – the most precise of the three – is used as the basis of comparison for the current study.

Sangani and Acrivos solved the Stokes equations for transverse flow by using a Fourier series method applied to a ‘unit cell’, a set of four cylinders, to determine the permeability:

\[
k = \frac{a^2}{4\phi} \left[ \frac{1}{2} \ln(1/\phi) - 0.738 + \phi - 0.887\phi^2 + 2.038\phi^3 + O(\phi^4) \right],
\]

where \( \phi \) is the solidity. The flow resistance parameter \( f \, Re \) for a square array can then be obtained by combining Eqs. 6.4 and 6.5, resulting in the following expression:

\[
f \, Re = \frac{16\phi}{\frac{1}{2} \ln(1/\phi) - 0.738 + \phi - 0.887\phi^2 + 2.038\phi^3 + O(\phi^4)}.
\]

For the three test sections, where the cylinder radius \( a \) is 3.18 mm, the values for \( f \, Re \) and permeability \( k \) are summarized in Table 6.1. These theoretical values will be compared with the experimental data.
Table 6.1 Analytical solutions for $k$ and $fRe$ for the three test sections according to Sangani and Acrivos (1982).

### Table 6.1

<table>
<thead>
<tr>
<th>Solidity $\phi$</th>
<th>Permeability $k$ (m$^2$)</th>
<th>Flow Resistance Parameter $fRe$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>2.86x10$^{-5}$</td>
<td>0.354</td>
</tr>
<tr>
<td>0.050</td>
<td>1.02x10$^{-4}$</td>
<td>0.990</td>
</tr>
<tr>
<td>0.10</td>
<td>3.20x10$^{-6}$</td>
<td>3.16</td>
</tr>
</tbody>
</table>

6.1.2 Flow Resistance Error Analysis

The flow resistance measurements to be presented contain error bars. The method of calculating these error bars is discussed in this section. By combining the expressions for the friction factor $f$ and the Reynolds number $Re$ as given in Eq. 6.2 and Eq. 6.3, respectively, the experimental flow resistance parameter is given by:

$$f Re = \frac{\Delta p}{\rho} \frac{4a^2}{l} \frac{U}{\eta U}. \quad (6.7)$$

To determine the accuracy of $f Re$, an error analysis which accounts for all measured quantities in Eq. 6.7 is necessary. Uncertainty in a measurement generally comes from two sources, namely, bias and precision (Coleman and Steele, 1995). In the case of the flow resistance parameter, bias errors are corrected with proper instrument calibrations. On the other hand, precision errors are determined from standard deviations. Values of
Chapter 6: Results and Discussions

The flow resistance $f \, \text{Re}$ are dependent on three measured quantities: the pressure loss $\Delta p$, the bulk velocity $U$, and the fluid shear viscosity $\eta$.

Measurements of pressure loss $\Delta p$ were obtained by taking the average of the output from the pressure differential transducer while the valve was open. The relative standard deviation of the $\Delta p$ data can be expressed as (Coleman and Steele, 1995):

$$S_{\Delta p} = \frac{1}{\Delta p} \left[ \frac{1}{N} \sum_{i=1}^{N} (\Delta p_i - \Delta p)^2 \right]^{\frac{1}{2}},$$

where the $\Delta p$ is calculated from:

$$\Delta p = \frac{1}{N} \sum_{i=1}^{N} \Delta p_i,$$

and $N$ is the total number of measured discrete pressure loss signals $\Delta p_i$.

The bulk velocity $U$ was calculated by dividing the bulk flow rate by the cross sectional area of the test section. To determine the flow rate, the collection tank was used to measure the volume of fluid collected during a run of the experiment, the duration of which was recorded by a stopwatch. Therefore, the sources of the error derive from the difference in height $h$ of the fluid in the collection tank, and the time $t$ required for fluid collection, of which the relative standard deviations are, respectively:

$$S_{h} = \frac{\Delta h}{h},$$

and

$$S_{t} = \frac{\Delta t}{t}.$$
\[ S_t = \frac{\Delta_h}{t}, \]  

(6.11)

where \( \Delta_h \) and \( \Delta_t \) are the subdivisions in the two measurements.

With the relative standard deviation of viscosity measurements from a calibrated rheometer approximated at 2\%, the total precision uncertainty of \( f \text{Re} \) is expressed as:

\[ S_{f_{\text{Re}}} = \left[ (S_{hp})^2 + (S_h)^2 + (S_t)^2 + 0.02^2 \right]^{\frac{1}{2}}. \]  

(6.12)

Calculated this way, the total precision uncertainty of \( f \text{Re} \) was found to vary from 5\% to 20\% for the Newtonian flows.

### 6.1.3 Newtonian Flow Resistance Data

Flow resistance data for the glycerol, in terms of \( f \text{Re} \), and predictions of \( f \text{Re} \) are presented in three graphs. Figures 6.1, 6.2, and 6.3 show the measurements obtained for the 2.5\%, 5.0\%, and 10\% solidity models, respectively. The data indicate that \( f \text{Re} \) is independent of the Reynolds number, confirming that inertial effects were negligible. Furthermore, the three graphs show good agreement between the data and the predictions of Sangani and Acrivos (1982). Wall effects were necessarily accounted for in the bulk flow rate measurement, as described in Section 5.3. These corrections were needed despite the large aspect ratio of the cylinders and the large number (ten) of columns normal to the
flow (Zhong et al., 2006). The good agreements validate the experimental technique. The data for the three solidity models are presented on a single graph in Figure 6.4.

The Newtonian results provide the baselines for comparison with results obtained with Boger fluids. Because elasticity causes increased flow resistance, the data for a Boger fluid will lie above the baseline. Any increase in $f \text{Re}$ is due solely to elasticity, as viscous effects are accounted for in the Newtonian results.

![Graph showing measurements of glycerol flow resistance for the 2.5% solidity model. The solid line indicates the prediction by Sangani and Acrivos (1982).](image)

**Figure 6.1** Measurements of glycerol flow resistance for the 2.5% solidity model. The solid line indicates the prediction by Sangani and Acrivos (1982).
Figure 6.2 Measurements of glycerol flow resistance for the 5.0% solidity model. The solid line indicates the prediction by Sangani and Acrivos (1982).

Figure 6.3 Measurements of glycerol flow resistance for the 10.0% solidity model. The solid line indicates the prediction by Sangani and Acrivos (1982).
Figure 6.4 Measurements of glycerol flow resistance for the three solidity models. The solid lines indicate the predictions by Sangani and Acrivos (1982).

6.1.4 Flow Resistance Data with the Boger Fluids

For Boger-fluid flows, elastic effects are expressed here in terms of the Deborah number, which is the ratio of the fluid’s characteristic relaxation time \( \lambda \) to the characteristic time in the flow (refer to Eq. 2.16). Although the Weissenberg number has been used in previous work (Khomami and Moreno, 1997), the Deborah number seems more appropriate here because of fluid acceleration. A straightforward flow time in a square array is that for the fluid to flow from one row of cylinders to the next, i.e., \( D/U \), where \( D \) is the distance between the centres of adjacent cylinders, and \( U \) is the bulk
velocity. However, to account for the effect of varying solidities in the three test sections, a more precise flow time is \((1 - \phi)D/U\). This flow time was also used in the experimental work of Skartsis et al. (1992). Hence, the Deborah number in this work is defined as:

\[
De = \frac{\lambda U}{(1 - \phi)D}.
\]  

(6.13)

The flow resistance data for the two Boger fluids are presented in Figures 6.5, 6.6, and 6.7 for the 2.5%, 5.0%, 10.0% solidity models, respectively. In all cases, the Reynolds numbers remained below 0.1 so that inertial effects were negligible. To ensure reproducibility, the data were gathered over a range of flow rates and the process was repeated for several days. The plots reveal that the onset of elastic effects consistently occurred at a Deborah number of approximately 0.5, regardless of test section solidity or the relaxation time of the Boger fluid. Beyond the onset Deborah number \(De_c\), \(f \text{Re}\) increases monotonically. Each graph shows excellent agreement between the two Boger fluids.

Figure 6.8 presents the flow resistance data, in which \(f \text{Re}\) is normalized with the respective Newtonian value \(f \text{Re}_N\). The three sets of data were fitted with trend lines beyond the onset Deborah number of 0.5 using the least squares method. Linear equations were used to fit the 2.5% and 5.0% solidity model data, while an exponential regression was found to best fit the 10% solidity model. Plotted this way, the data clearly show that \(f \text{Re}\) increases with solidity, as expected.
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Figure 6.5 Flow resistance data of the two Boger fluids for the 2.5% solidity model. The solid line indicates the Newtonian analytical solution of Sangani and Acrivos (1982).

Figure 6.6 Flow resistance data of the two Boger fluids for the 5.0% solidity model. The solid line indicates the Newtonian analytical solution of Sangani and Acrivos (1982).
Figure 6.7 Flow resistance data of the two Boger fluids for the 10% solidity model. The solid line indicates the Newtonian analytical solution of Sangani and Acrivos (1982).

Figure 6.8 Normalized flow resistance data of the two Boger fluids for the 2.5%, 5.0%, and 10% solidity model.
Considering the factors discussed above – the consistent onset Deborah number and the similar $f \text{Re}$ values for the two Boger fluids – it appears that the definition of Deborah number in this work was appropriately chosen. This is the first experimental investigation, working with more than one Boger fluid and with test sections of various solidities, to show Deborah number consistency of flow resistance results. Table 6.2 provides a summary of the Boger-fluids flow resistance data and the curve fits.

<table>
<thead>
<tr>
<th>Solidity $\phi$</th>
<th>Boger Fluid</th>
<th>Onset Deborah Number $De_c$</th>
<th>Maximum Attainable Deborah Number</th>
<th>Curve Fitting of $f \text{Re}/f \text{Re}_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>B1</td>
<td>0.55 ± 0.15</td>
<td>3.8</td>
<td>0.53($De - 0.5$) + 1</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>0.50 ± 0.15</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>B1</td>
<td>0.45 ± 0.15</td>
<td>3.0</td>
<td>0.63($De - 0.5$) + 1</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>0.50 ± 0.15</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>B1</td>
<td>0.55 ± 0.20</td>
<td>2.5</td>
<td>exp[0.53($De - 0.5$)]</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>0.55 ± 0.20</td>
<td>3.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2 Summary of the flow resistance data for the two Boger fluids.

Figure 6.9 presents the viscoelastic flow resistance data from all previous authors who investigated flows through square arrays, along with the present data. The solidities of the earlier models varied from 30% to 66%, and the test fluids were either Boger fluids or shear-thinning elastic fluids. The flow resistance and $De$ values used by previous authors were converted to the forms given in Eq. 6.7 and Eq. 6.13. However, the methods of obtaining fluid relaxation times varied among the authors.
Skartsis et al. (1992) applied Rouse’s expression (Rouse, 1953) to estimate the relaxation time of their shear-thinning corn syrup/polyacrylamide solution:

\[
\lambda = \frac{6\eta_s [\eta]_0 M_w}{\pi^2 RT}, \tag{6.14}
\]

where \(\eta_s\) is the solvent viscosity, \([\eta]_0\) is the intrinsic viscosity at zero shear, \(M_w\) is the polymer molecular weight, \(R\) is the gas constant, and \(T\) is the absolute temperature in
degrees Kelvin. For the shear thinning fluids of Chmielewski and Jayaraman (1992) and of Koshiba et al. (1998), a shear-rate \( \dot{\gamma} \) dependent relaxation time was employed:

\[
\lambda(\dot{\gamma}) = \frac{N_1(\dot{\gamma})}{2\eta(\dot{\gamma})\dot{\gamma}^2}.
\]  

(6.15)

Khomami and Moreno (1997) and Chmielewski and Jayaraman (1992) estimated the relaxation times of their Boger fluids using the Oldroyd-B model. Khomami and Moreno used the low-shear limit of \( N_1 \) to obtain \( \lambda \) (Eq. 2.29), while Chmielewski and Jayaraman used the low-shear limit of \( G' \) (Eq. 2.30).

The different methods of determining the relaxation time may be the cause of the wide range of onset Deborah numbers, which vary by three orders of magnitude. An exact determination of relaxation time is difficult (Solomon and Muller, 1996), and depending on the method of calculation, the relaxation time may vary by a factor of two (Khomami and Moreno, 1997). The additional shearing due to wall effects, as discussed in Section 5.3, may be another aspect which may contribute to the variation in onset Deborah number. No other authors have considered wall effects in their flow resistance calculations. Because Skartsis et al. and Koshiba et al. used only shear-thinning elastic fluids, it is difficult to separate shear-thinning effects from elastic effects when analyzing their flow resistance data.

With the current apparatus, the maximum attainable Deborah number varied from 2.5 to 4.1, depending on the solidity, and Figure 6.8 shows that \( f \text{Re} \) was three to four times the Newtonian value. Chmielewski and Jayaraman obtained \( f \text{Re} \) values ten times
the Newtonian value, but their data were obtained for an array with a much higher solidity, 30%, in which elastic effects are expected to be higher. Also, their data reached an asymptotic limit for each of their three Boger fluids, in contrast to the present monotonic data.

Khomami and Moreno also showed that $f \text{Re}$ increased monotonically in their 55% solidity model. At the highest Deborah number of 0.11, they obtained $f \text{Re}$ values five times the Newtonian value. Their experimental results showed an increasing pressure drop per unit length ($\Delta p/l$) along the flow direction. That is, downstream of the array entrance, there was a greater increase in $f \text{Re}$ with $De$, compared with the flow closest to the entrance.

Observations of increasing pressure drop along the flow direction were not found in the present study. This was confirmed by pressure drop measurements made at several different locations from the entrance within the arrays, which showed that $\Delta p/l$ for all three test sections remained the same, regardless of the pressure tap locations chosen for measurements.

### 6.2 PIV Analysis

Changes in the flow field should aid in understanding the role of fluid elasticity. Chmielewski and Jayaraman (1993) are the only prior authors who measured a velocity distribution within a square array. They showed that normalized velocities along the
centrel ine were higher than those corresponding to Newtonian flows. In the current work, the velocity field *throughout* the array is examined and these results are presented in this section.

### 6.2.1 PIV Error Analysis

The sources of bias errors for the velocity data to be presented derive from the frame separation time $\Delta t$, the particle displacement $\Delta d$, and the particle response time $\tau_g$. However, these errors are small when compared with the precision errors stemming from local mean velocity measurements. For each interrogation area, the local mean velocity $u_{IA}$ is taken as the average of $N$ measurements of $u_{IA,i}$. Hence, the total error of the PIV measurements is approximated as the relative standard deviation of $u_{IA}$:

$$S_{u_{IA}} = \frac{1}{u_{IA}} \left[ \frac{1}{N} \sum_{i=1}^{N} (u_{IA,i} - u_{IA})^2 \right]^\frac{1}{2}. \quad (6.16)$$

$S_{u_{IA}}$ was found to be larger in the regions of low velocity close to the cylinders. The values of the relative standard deviations were as high as 15% near the cylinders and 5% in the regions away from rods. A typical set of error values are shown in Figure 6.10 for the 2.5% model. The error values were similar for the three test sections, regardless of the test fluids used.
6.2.2 Newtonian Velocity Distributions

The vector plots in the $x$-$y$ plane obtained with PIV for the Newtonian fluid are presented in Figures 6.10, 6.11, and 6.12 for the 2.5%, 5.0%, and 10% solidity models, respectively. The Reynolds number remained less than 0.1, ensuring that inertia effects were negligible. The $x$-$y$ coordinate system, introduced in Figure 5.5, has the origin located at the centre of the unit cell. The positive $x$-axis is downward, in the flow direction. In the three plots, the arrays are drawn to scale. To minimize wall and entrance effects, the selected unit cell was about half way down the array and about halfway between the walls. The velocity profiles, normalized by the bulk velocity $U$, are symmetrical at the different streamwise locations for all three solidity models. Flow symmetry was found within other unit cells away from the walls, and the same normalized velocity profiles were obtained at other Reynolds numbers. That is, the velocity distributions were dependent only on the solidity. Measurements of velocity fields in surrounding unit cells were also obtained to check for the influence of wall effects in the centre region of the test section. Wall effects were found to be negligible, as long as PIV measurements were made at least two unit cells away from the walls.

The accuracy of the PIV data was verified by calculating the flow rates at different locations within the unit cell to determine if continuity was satisfied. At a fixed coordinate of $x/D$, the two-dimensional flow rate $Q_{x/D}$ is:

$$Q_{x/D} = \int_{-D/2}^{D/2} u\left(\frac{x}{D}, \frac{y}{D}\right) dy$$  \hspace{1cm} (6.17)
Figure 6.10 Dimensionless velocity distributions for the Newtonian fluid within a unit cell of the 2.5% solidity model. Measurements were made in an x-y plane at a Reynolds number of 0.08. The coordinate system is defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity. Error bars are shown for the velocity profile at $x/D = 0$. 
Figure 6.11 Dimensionless velocity distributions for the Newtonian fluid within a unit cell of the 5.0% solidity model. Measurements were made in an x-y plane at a Reynolds number of 0.09. The coordinate system is defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.12 Dimensionless velocity distributions for the Newtonian fluid within a unit cell of the 10% solidity model. Measurements were made in an x-y plane at a Reynolds number of 0.1. The coordinate system is defined in Figure 5.5, D is the distance between cylinder centres, and U is the superficial velocity.
Figure 6.13 shows the flow rate calculations for the 2.5% solidity model, which is a typical set of flow rate data for the three test sections. The values of $Q_{x/D}$, calculated from the data obtained in Figure 6.10, shows a variation of approximately 3%. These results, along with the standard deviation calculations shown in Figure 6.10, indicate the accuracy of the velocity measurements.

![Graph showing flow rate calculations for the 2.5% solidity model.](image)

**Figure 6.13** Two-dimensional flow rate at different locations within a unit cell $Q_{x/D}$ for the 2.5% solidity model.

### 6.2.3 Boger Fluids Velocity Distributions in the $x$-$y$ Plane

With the two Boger fluids, dimensionless velocity distributions were measured for the three test sections at several Deborah numbers. The unit cells selected for the Newtonian flow measurements were again used for the Boger fluid flows. These velocity
distributions are presented in Figures 6.14 to 6.23 for fluid B1, and Figures 6.24 to 6.33 for fluid B2.

For Boger fluid B1, Figure 6.14 shows the velocity profiles of the 2.5% solidity model prior to the onset of elastic effects, at $De = 0.2$, below the critical Deborah number of approximately 0.5. As expected, these profiles match those of the Newtonian fluid in the previous plots, found in Figure 6.10. Low $De$ velocity profiles matching those of the Newtonian counterparts were also observed for the 5.0% and 10% solidity models. A direct comparison of the Newtonian velocity profiles (Figure 6.12) with those at a low Deborah number (Figure 6.20) is shown in Figure 6.33.

Figure 6.15 presents the velocity profiles at $De = 1.6$ for the 2.5% solidity model, where the flow was well into the elastic regime. The profiles have slightly higher maxima and are close to symmetrical but no longer perfectly so. At the maximum obtainable Deborah number of 3.9 for the 2.5 solidity model, the velocity profiles are asymmetrical, as shown in Figure 6.16. By comparing the velocity measurements obtained at different times during an experimental run lasting several minutes, flows were observed to remain steady for the full range of Deborah numbers.

Comparing Figures 6.17 to 6.19 (5% solidity model) and Figures 6.20 to 6.22 (10% solidity model) with Figures 6.14 to 6.16 (2.5% solidity model), similar trends to the ones described above are observed as elastic effects become increasingly important. Moreover,
Figure 6.14 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 2.5% solidity model. Measurements were made in the x-y plane at a Deborah number of 0.2. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.15 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 2.5% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 1.6. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.16 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 2.5% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 3.9. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.17 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 5.0\% solidity model. Measurements were made in the x-y plane at a Deborah number of 0.3. The coordinate system was defined in Figure 5.5, \(D\) is the distance between cylinder centres, and \(U\) is the superficial velocity.
Figure 6.18 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 5.0% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 2.1. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.19 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 5.0% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 2.7. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.20 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 10% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 0.3. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.21 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 10% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 1.4. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.22 Dimensionless velocity distributions for the fluid B1 within the unit cell of the 10% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 2.5. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.23 Vector map corresponding to the flow field of Figure 6.22 (Fluid B1, 10% solidity, $De = 2.5$).
for approximately the same Deborah number, the asymmetry of the velocity profiles increase with solidity. Hence, skewness was observed to increase with both Deborah number and solidity, analogous to the increase in flow resistance with $De$ and $\phi$.

Figure 6.23 is the vector map corresponding to Figure 6.22, where $De$ is 2.5 for the 10% solidity model. This plot shows that the flow is more disturbed than the velocity profiles indicate. Some vectors on the right side of the unit cell have a strong lateral component, indicating that the flow curves into and out of the wake region on that side. This pattern is different from the small curvature found on the left side. Investigation at other locations revealed that the flow may skew to either side in any unit cell.

Comparing Figures 6.14 to 6.22 (fluid B1) with Figures 6.24 to 6.32 (fluid B2) shows that, for approximately the same Deborah numbers at the same solidity, the velocity distributions are the same for the two Boger fluids. A selected comparison is shown in Figure 6.34 for the 2.5% solidity model, where the velocity profiles of fluid B1 at $De = 3.9$ (Figure 6.16) are plotted with those of fluid B2 at $De = 3.6$ (Figure 6.26). The two sets of data show that the flow fields are virtually identical for the two Boger fluids at similar Deborah numbers. This observation, along with the flow resistance measurements of Figures 6.5 to 6.7, confirms that the Deborah number was appropriate, and that the relaxation times were measured accurately.
Figure 6.24 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 2.5% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 0.2. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.25 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 2.5% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 1.8. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.26 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 2.5% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 3.6. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.27 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 5.0% solidity model. Measurements were made in the x-y plane at a Deborah number of 0.3. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.28 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 5.0% solidity model. Measurements were made in the x-y plane at a Deborah number of 2.1. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.29 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 5.0% solidity model. Measurements were made in the \(x\)-\(y\) plane at a Deborah number of 2.9. The coordinate system was defined in Figure 5.5, \(D\) is the distance between cylinder centres, and \(U\) is the superficial velocity.
Figure 6.30 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 10% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 0.3. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.31 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 10% solidity model. Measurements were made in the x-y plane at a Deborah number of 2.9. The coordinate system was defined in Figure 5.5, D is the distance between cylinder centres, and U is the superficial velocity.
Figure 6.32 Dimensionless velocity distributions for the fluid B2 within the unit cell of the 10% solidity model. Measurements were made in the $x$-$y$ plane at a Deborah number of 3.4. The coordinate system was defined in Figure 5.5, $D$ is the distance between cylinder centres, and $U$ is the superficial velocity.
Figure 6.33 Comparison of velocity profiles, for the 10% solidity model, between the Newtonian flow and the low Deborah number flow. The smaller solid symbols represent the Newtonian profiles in Figure 6.12. The larger open symbols are the velocity profiles for fluid B1, at $De = 0.3$, replotted here from Figure 6.20.
Figure 6.34 Comparison of velocity profiles between the two Boger fluid flows at similar Deborah numbers for the 2.5% solidity model. The smaller solid symbols represent the velocity profiles of fluid B1 at $De = 3.9$, from Figure 6.16. The larger open symbols are the velocity profiles for fluid B2 at $De = 3.6$, replotted here from Figure 6.26.
6.2.4 Boger Fluids Velocity Distributions in the x-z Plane

As described in Section 5.5, PIV data were obtained in the orthogonal direction, in x-z planes of the test sections, to further study flow modifications due to fluid elasticity. As indicated in Figure 5.6, these planes bisect the cylinders in a column and the measurements were made in adjacent columns in the centre of an array.

Vector plots in consecutive wake regions of the cylinder arrays are presented in Figures 6.35 to 6.39. The upper plot in these figures pertains to the right column as shown in Figure 5.6 and the lower plot to the adjacent left column. The flow is downward, in the x-direction, and the white regions indicate cylinder positions. The side wall is at the left-hand edge, at \( z = 0 \), providing the same reference location for the two plots. Velocity measurements in the x-z planes revealed that the flows remained steady for the full range of Deborah numbers tested; this is in agreement with the same finding from the measurements in the x-y plane.

Figure 6.35 shows the vector plots for the 10% solidity model for fluid B1 at \( De = 1.4 \), where elastic effects are visible. The flow is not uniform in the x-direction and some vectors have a lateral component. The non-uniform flow is in contrast to flows below the onset of elastic effects, where the vectors were found to be in the x-direction only, indicating a two-dimensional flow field throughout the array. Figure 6.35 does not show any obvious flow pattern in the wake region. Spurious vectors found in both the upper and lower plots are present because of the low velocities in these regions, leading to weak cross-correlations between image pairs where the lighting was dim.
Figure 6.35 Vector plots for Boger fluid B1 in the wake regions between cylinders of the 10% solidity model, at $De = 1.4$, obtained with the setup shown in Figure 5.6. The measurements were made in $x$-$z$ planes cutting through the centres of four rods in two adjacent columns, with the top figure pertaining to the right column in Figure 5.6 and the bottom figure to the adjacent left column. The white regions in each plot indicate cylinder positions.
Figure 6.36 Vector plots for Boger fluid B1 in the wake regions between cylinders of the 10% solidity model, at $De = 2.4$, obtained with the setup shown in Figure 5.6. The measurements were made in $x$-$z$ planes cutting through the centres of four rods in two adjacent columns, with the top figure pertaining to the right column in Figure 5.6 and the bottom figure to the adjacent left column. The white regions in each plot indicate cylinder positions.
As $De$ increased and elastic effects became more important, a regular three-dimensional flow pattern emerged, as shown in Figure 6.36 for the same test section at $De = 2.4$. At this flow rate, more vectors have a significant lateral component compared with those at $De = 1.4$. Figure 6.36 reveals that the flow was modified in a regular way in the spanwise direction. Each of these regularly-spaced structures appears to be symmetric, with high-speed fluid bursting outward from a central path.

Within the same column, as shown in the upper plot of Figure 6.36, the locations of these structures remained unchanged in the consecutive wake regions below. Similarly, in the adjacent column (the bottom plot), the locations of the structures were unchanged in the consecutive wake regions. However, a closer examination reveals that these structures have ‘shifted’ between the two adjacent columns. The spacing of the structures will be discussed in more detail later in this section.

Figure 6.37 presents the flow pattern for the 2.5% solidity model for fluid B2 at $De = 2.6$, comparable to $De = 2.4$ for fluid B1 in Figure 6.36. Although regularly-spaced structures are again found in two adjacent wake regions, as shown in the top and bottom plots, these structures appear less pronounced when compared with the ones found in the 10% solidity model.
Figure 6.37 Vector plots for Boger fluid B2 in the wake regions between cylinders of the 2.5% solidity model, at $De = 2.6$, obtained with the setup shown in Figure 5.6. The measurements were made in $x$-$z$ planes cutting through the centres of three rods in two adjacent columns, with the top figure pertaining to the right column in Figure 5.6 and the bottom figure to the adjacent left column. The white regions in each plot indicate cylinder positions.
In the highly elastic regime shown in Figure 6.38, at $De = 3.6$, the structures are very well defined, with strong lateral bursting velocities. The same shift in alignment can be clearly seen by comparing the top and bottom plots.

At all solidities, the three-dimensional periodic structures in one column appear to have shifted sideways from those in the adjacent column. Figure 6.39 is Figure 6.36 with lines added to indicate locations of the structures. In the upper plot (the right side in Figure 5.6), the centre of each structure is denoted by a thick dashed line, making it easier to see that the structures are regularly spaced and formed at the same distances from $z = 0$. In the lower plot (the left side in Figure 5.6), the structures are again spaced about every two or four cylinder diameters, with the same spacing in consecutive wake regions. The centres in the lower plot are indicated by **solid** lines, showing that these structures are also periodically spaced and aligned the same in consecutive wake regions. Dashed lines from the upper plot are superposed, revealing that the two sets of structures are offset and almost symmetrically so. Hence, in the open space between columns, it seems that the structures alternate in dovetail fashion to fit into the available space. Observations of many such vector plots over the full $De$ range for the three test sections show that, as either $De$ or solidity increases, the flow structures become more pronounced, i.e., increasingly three-dimensional. Each one is more symmetrical, the spacing becomes more regular, and the offset becomes more symmetric.

A modified flow field like the above was seen by Khomami and Moreno (1997). Using streak photography, they observed steady periodic “cellular structures” in the elastic
Figure 6.38 Vector plots for Boger fluid B2 in the wake regions between cylinders of the 10% solidity model, at $De = 3.4$, obtained with the setup shown in Figure 5.6. The measurements were made in $x$-$z$ planes cutting through the centres of three rods in two adjacent columns, with the top figure pertaining to the right column in Figure 5.6 and the bottom figure to the adjacent left column. The white regions in each plot indicate cylinder positions.
Figure 6.39 Vector plots for Boger fluid B1 in the wake regions between cylinders, at $De = 2.4$. The top figure pertains to the right column in Figure 5.6 and the bottom figure pertains to the adjacent left column. In the right column (top figure), the centres of the structures are marked by dashed lines. In the left column (bottom figure), the centres are marked by solid lines, and the dashed lines of the top figure have been superposed to the bottom figure to show the relative locations of the structures.
flow region in their 14% solidity model. Their images imply circulation or secondary flow, but the current vector plots clearly show that secondary motion does not occur in the flow structures. Moreover, their structures appeared only in the spaces between the first and second rows of cylinders, and not between pairs further downstream, as we found. Downstream of their second row, their “streak patterns” became less periodic and after the third row, were “no longer periodic and … highly time dependent”. They describe the flow as three-dimensional and transient. In our models, the structures appeared to be periodic and steady throughout the entire array.

Figure 6.40 presents ideal boundaries of the periodic structures as viewed from above, i.e., in the y-z plane. For the flow to satisfy continuity, the boundaries must be hexagonal so that the structures can dovetail. Because the spacing of the structures is close to the rod spacing $D$, the sides of the hexagons are close to the same length.

Figure 6.40 Ideally staggered periodic structures for the 10% solidity model in the y-z plane. The hexagons show the theoretical boundaries of the periodic structures. The grey regions indicate rod locations.
6.3 Increase in *f* Re Due to Fluid Elasticity

To investigate the cause of the flow resistance increase at high Deborah numbers, as high as 4 times the Newtonian resistance, the differences between the Newtonian and viscoelastic flow fields are examined. Extensional effects will first be examined by using the PIV data to determine the Hencky strains $\varepsilon$ within the unit cells, which are related to elastic stresses in extension. As the Trouton ratio measurements in Figure 3.5 indicate, at Hencky strains of less than a value of two, Boger fluids behave like inelastic Newtonian fluids. Therefore, values of $\varepsilon$ found within the test sections will determine if the increase in flow resistance may be caused by extensional effects.

There are two possible locations where the maximum stretching can occur. One is along the centreline of the unit cell, and the other is in the wake region between stagnation points. To obtain the Hencky strain along the centreline, the average extensional rate is first determined between the origin of the unit cell ($\frac{x}{D} = 0, \frac{y}{D} = 0$) and the point on the centreline intersecting the lower boundary ($\frac{x}{D} = 0.5, \frac{y}{D} = 0$). This quantity is then multiplied by the ‘stretching time’ to determine the Hencky strain, which is the length between the two points, $D/2$, divided the bulk velocity $U$. A similar calculation is used to determine the Hencky strain in the wake region.

Along the centreline ($y/D = 0$), the average extensional rate is:

$$\dot{\varepsilon}_{y/D=0} = \frac{u(\frac{x}{D} = 0.5, \frac{y}{D} = 0) - u(\frac{x}{D} = 0, \frac{y}{D} = 0)}{D/2},$$

(6.18)
and the corresponding Hencky strain is:

\[ \varepsilon_{y/D=0} = \dot{\varepsilon}_{y/D=0} \left( \frac{D/2}{U} \right). \]  

(6.19)

Similarly, for the wake region \( y/D = -0.5 \), the average extensional rate is:

\[ \dot{\varepsilon}_{y/D=-0.5} = \frac{u(\frac{y}{D} = 0, \frac{y}{D} = -0.5)}{D/2-a}. \]  

(6.20)

and the corresponding Hencky strain is:

\[ \varepsilon_{y/D=-0.5} = \dot{\varepsilon}_{y/D=-0.5} \left( \frac{D/2-a}{U} \right). \]  

(6.21)

The results of these calculations for the three test sections, using the velocity plots of Figures 6.14 to 6.22 (fluid B1) and Figures 6.24 to 6.32 (fluid B2), are presented in Figures 6.41 and 6.42 for \( \varepsilon_{y/D=0} \) and \( \varepsilon_{y/D=-0.5} \), respectively.

Figure 6.41 shows that along the unit cell centreline, \( \varepsilon_{y/D=0} \) increased with \( De \) and solidity, as would be expected. Similar to the flow resistance data, the Hencky strains for the two Boger fluids collapse for the different test sections. In the wake regions, Figure 6.42 indicates that Hencky strains for all three solidity models decreased to approximately 0.25 and become independent of \( De \), after the onset of elastic effects. The figures demonstrate that strains are larger in the central region.
Figure 6.41 Hencky strains along the centreline of a unit cell ($y/D = 0$) for the two Boger fluids and three test sections. The strains were calculated using data obtained from Figures 6.14 to 6.22 for fluid B1, and Figures 6.24 to 6.32 for fluid B2.

Figure 6.42 Hencky strains in the wake region of a unit cell ($y/D = -0.5$) for both fluids and three test sections. The strains were calculated using data obtained from Figures 6.14 to 6.22 for fluid B1, and Figures 6.24 to 6.32 for fluid B2.
The highest Hencky strain found in our data was 0.95 for the 10% solidity model at $De = 3.4$, much less than the value of 2 necessary for extensional effects. This finding indicates that extensional effects were unimportant in our test sections over the range of $De$ tested. Evans et al., (1994) studied the flow of dilute PIB solutions in low solidity random fibrous media. Using birefringence, they showed that significant polymer extension occurred within a fibre bed of 2.47% solidity. Evans et al. suggested that the increase in pressure loss was correlated with a large conformation change of the polymers. The current study is in contrast to that study. The difference may be caused by larger polymer stretching for flow through a disordered fibre bed, compared with flow through a regular array of cylinders.

The other aspect to consider is effects caused by shearing. An examination of the velocity profiles of Figure 6.14 to 6.31 shows that the maximum shear rates are at or near the cylinder surfaces at $x/D = \pm 0.5$. A comparison of the Boger fluid profiles with Newtonian ones shows an increase in shear rate at these locations, and therefore an increase in shear stress, but the increase is clearly not enough to cause the large rises in flow resistance.

In addition to shear stress, the first normal stress difference $N_1$ also increases with shear rate. For the cylinder array, $N_1$ is largest at $x/D = \pm 0.5$, where the maximum shear rate occurs. To determine if the flow resistance increases are related to normal stresses due to shear, a comparison between the excess pressure loss and $N_1$, as functions of shear rate, is examined. To obtain the additional pressure loss due to elasticity, the difference between
\[ \frac{\Delta p}{l} \text{ of the Boger flow and that of the Newtonian flow is calculated. Pressure loss per length of the Boger flow is determined from Eq. 6.7:} \]

\[
\left[ \frac{\Delta p}{l} \right]_{\text{Boger}} = \frac{\eta}{4a^2} (f \text{Re})(U). \tag{6.22}
\]

For Newtonian flow, pressure loss per length is determined from Darcy’s law:

\[
\left[ \frac{\Delta p}{l} \right]_{\text{Newtonian}} = \frac{\eta}{k} U. \tag{6.1}
\]

The additional pressure loss is calculated from the difference of the losses:

\[
\left[ \frac{\Delta p}{l} \right]_{\text{Due to Elasticity}} = \eta U \left( \frac{f \text{Re}}{4a^2} - \frac{1}{k} \right), \tag{6.23}
\]

where \( f \text{Re} \) and \( U \) are functions of \( De \) (refer to Table 6.2 and Eq. 6.13).

As the flow undergoes maximum shear at each row of cylinders, the first normal stress difference \( N_1 \) can act to increase the pressure loss. Therefore, the pressure loss of Eq. 6.23 is scaled for each rod spacing \( D \), and compared with \( N_1 \) at equivalent shear rates. Assuming parabolic velocity profiles at \( x/D = \pm0.5 \), the maximum shear rate (on the cylinder surface) as a function of \( De \) can be determined.

Figure 6.43 shows a comparison between the \( N_1 \) measurements from Figure 3.2 and the pressure loss due to elasticity per rod spacing (Eq. 6.23) for the three solidity models and two Boger fluids. The pressure loss is plotted as a function of the maximum shear rate (at \( x/D = 0.5 \), \( y/D = 0.5 \)), \( y/D = 0.5 - a/D \). The figure shows that the first normal stress difference is
approximately six times the value of the excess pressure loss per rod spacing. Therefore, normal stresses due to shear can easily account for the large rises in flow resistance. The ‘mean’ shear rate may be more appropriate, accounting for the direction of shear and the gradient of the velocity field.

![Graph showing the relationship between shear rate and pressure loss](image)

**Figure 6.43** $N_1$ measurements obtained from Figure 3.2 and the excess pressure loss due to elasticity per rod spacing $D$ for the two Boger fluids and three solidity models.

As discussed in Section 6.2.3, PIV data in the $x$-$z$ plane illustrate the transition from two-dimensional to three-dimensional flow, with periodic structures appearing in the wake regions throughout an entire array. These structures are similar to the ones observed by
Khomami and Moreno (1997) in the spaces between the first and second rows of cylinders in their 14% solidity model. A comparison of flow patterns for the 10% solidity model as shown in Figure 6.35 for \( De = 1.4 \) and Figure 6.38 for \( De = 3.4 \) reveals the dramatic change in flow modification in the highly elastic regime. The structures formed at \( De > 2.5 \) (Figures 6.36 to 6.39) are spaced very regularly and are staggered from column to column. Because PIV analysis in the \( x-y \) plane measures the flow field at a fixed \( z \)-coordinate, the observed stagger in the two adjacent \( x-z \) planes, as shown in Figure 6.39, is consistent with the skewed velocity profiles in Figure 6.22 and with the asymmetry in the vector streamlines of Figure 6.33. Fluid elasticity causes a less streamlined flow, requiring more energy to form the well-defined periodic structures at high Deborah numbers. However, the periodic structures alone cannot account for the large rises in flow resistance. As shown above, the pressure loss increases are more likely related to normal stresses due to shearing.

Chmielewski and Jayaraman (1992, 1993) and Khomami and Moreno (1997) argue that their increases in flow resistance were caused by a “purely elastic instability.” Above the onset Deborah number, Chmielewski and Jayaraman found fluctuations in pressure drop measurements, along with unsteady flow patterns in their 30% solidity model. Similarly, Khomami and Moreno discovered a highly time-dependent three-dimensional flow for their 14% and 55% solidity models. In current study, where the test sections had a maximum solidity of 10%, the increase in flow resistance was clearly not due to a flow instability because our flows remained steady for all three models throughout the entire arrays, even for flows in the highly elastic regime.
CHAPTER 7: CONCLUDING REMARKS

7.1 Summary

In the present work, the effects of fluid elasticity on slow flows through cylinder arrays of low solidities (2.5%, 5.0%, and 10%) were examined. A Newtonian flow provided a baseline study so that the effects of elasticity can be determined from an investigation with Boger fluids.

From pressure loss measurements, it was found that for all cases the onset of elastic effects occurred at a Deborah number of 0.5. Normalized flow resistance data show that $f \frac{Re}{Re_f}$ increased with solidity. The consistent onset Deborah number, and the collapse of flow resistance data of the two Boger fluids with $De$, shows that the Deborah number used in the current investigation was appropriately chosen and scaled the effects of fluid elasticity properly.

A PIV system was used to map the velocity fields within the arrays. To minimize the shadows caused by the cylinder arrays, overlapping light sheets were used to illuminate a unit cell within an array. Captured image pairs were post-processed to enhance the contrast and sharpness of the particle locations. A consideration of the appropriate frame rate at which to capture images was important to ensure that particle displacements between pulse separations were well within the defined length of the interrogation areas.
With the Newtonian fluid, the lateral velocity profiles are found to be symmetric about $x/D$ and $y/D$ axes. When scaled with the bulk velocity, the profiles are independent of the flow rate, confirming that the experiment was carried out in the Stokes flow regime. PIV measurements of the Boger fluid flow showed that, at low Deborah numbers prior to the onset of elastic effects, the velocity profiles were the same as those with the Newtonian fluid. As elastic effects become increasingly important, the velocity profiles start to become pinched in the middle, yielding higher maxima than their Newtonian counterparts. At Deborah numbers greater than 2.5, velocity profiles begin to skew, showing a flow asymmetry. The skewness of the velocity profiles increases with Deborah number and solidity. PIV measurements in the wake regions of the cylinder arrays revealed the formation of steady periodic structures along the lengths of the cylinders. These structures appeared to be perfectly staggered at adjacent columns.

An analysis of the velocity data shows that the increase in flow resistance is not due to additional stresses caused by fluid extension, as suggested by previous authors. Flow modifications in the wake region, which become stronger and increasing well-defined with $De$, also cannot account for the large rises in pressure loss. Rather, the rise in $fRe$ appears to be caused by additional normal stresses due to shear. A comparison of $N_1$ measurements and $\Delta p/D$ for the three solidity models shows that the first normal stress difference is approximately six times the value of the excess pressure loss per rod spacing. Therefore, normal stresses due to shear can account for the large rises in flow resistance.
7.2 Conclusions

- The onset Deborah number $D_{Ec}$ at which elastic effects start to occur is 0.5 for all three solidity models, for the two Boger fluids, where $De$ is defined as:

\[
De = \frac{\lambda}{(1-\phi)} \frac{U}{D},
\]  

(6.13)

- Normalized $f \text{Re}$ data collapse for the two Boger fluids, and increase with solidity, using the same definition of $De$.

- Periodic flow structures form in the wake regions, which are staggered from column to column, at $De > 0.5$.

- As either $De$ or solidity increases, the flow structures become more pronounced, i.e., increasingly three-dimensional; the structures become more symmetrical, their spacing becomes more regular, and the stagger becomes more symmetric.

- Elasticity causes the velocity profiles to skew at $De > 0.5$, with the asymmetry increasing with $De$. The asymmetry in the vector streamlines is related to the staggered periodic structures found in the wake regions.

- Flows remained steady for all three models throughout the entire arrays, even at the maximum attainable Deborah numbers, in contrast to the findings of Khomami and Moreno (1997), who observed unsteady flow in their 14% and 55% solidity models. Chmielewski and Jayaraman (1992, 1993) also found unsteady flow in their 30% solidity model.
• Flow resistance increase is caused the additional normal stresses due to the shearing of an elastic fluid.

• First normal stress difference $N_1$ is approximately six times the value of the excess pressure loss per rod spacing $\Delta p / D$, for all three solidity models and both Boger fluids.

• Flow resistance increase is not attributed to additional stresses caused by extension.

• Flow resistance increase is not attributed to the flow modification in wake regions.
APPENDIX A: VELOCITY COMPENSATION OF THE FSR

To ensure that the filament-stretching rheometer (FSR) provides a constant stretch rate $\dot{\varepsilon}$ during uniaxial extension of a fluid element, it is necessary to create a velocity control of the lower end plate such that the mid-point filament diameter decreases as a function of time as follows (Kolte et al., 1997; Huang, 1999):

$$D(t) = D_0 \exp(-\frac{1}{2} \dot{\varepsilon} t) . \quad (A.1)$$

Nominally, the FSR applies the velocity profile:

$$v(t) = \dot{\varepsilon} L_0 \exp(\dot{\varepsilon} t) , \quad (A.2)$$

so that the filament length increases as a function of time as follows:

$$L(t) = L_0 \exp(\dot{\varepsilon} t) . \quad (A.3)$$

By stretching the filament based on the prescribed velocity of Eq. A.2, the mid-point diameter will decrease in the form of Eq. A.1, if the filament remains perfectly cylindrical. However, due to non-homogeneous effects induced by the end plates, the sample is exposed to shear, as well as extension. As a result, the filament undergoes significant necking, and the effective stretch rate is higher than the one imposed. Therefore, the velocity profile $v(t)$ needs to be adjusted so that the mid-point diameter follows Eq. A.1. An existing control program was modified to allow for the required
velocity adjustments. The Visual Basic script containing the modified code is included at the end of Appendix A.

The procedure for obtaining the required velocity control function is outlined as follows:

1. Using the nominal velocity control, given by Eq. A.2, run tests to obtain length and diameter plots as functions of time.
2. Plot the measured length and diameter as $L(t)/L_0$ vs. $D_0/D(t)$.
3. Curve fit the plot to obtain:

$$\frac{L(t)}{L_0} = f\left[\frac{D_0}{D(t)}\right].$$

(A.4)

Use up to three continuously differentiable 6-power polynomial functions for the curve fits, such that for $0 \leq t < t_1$:

$$\frac{L(t)}{L_0} = A_1\left[\frac{D_0}{D(t)}\right]^6 + A_2\left[\frac{D_0}{D(t)}\right]^5 + \ldots + A_7.$$

(A.5)

For $t_1 \leq t < t_2$,

$$\frac{L(t)}{L_0} = B_1\left[\frac{D_0}{D(t)}\right]^6 + B_2\left[\frac{D_0}{D(t)}\right]^5 + \ldots + B_7.$$

(A.6)

For $t \geq t_2$, ...
Appendix A: Velocity Compensation of the FSR

\[
\frac{L(t)}{L_0} = C_1 \left[ \frac{D_0}{D(t)} \right]^6 + C_2 \left[ \frac{D_0}{D(t)} \right]^5 + \ldots + C_7. \quad (A.7)
\]

4. Substitute the ideal diameter from Eq. A.1 into Eq. A.4. Differentiate the resulting equation to obtain:

\[
v(t) = \frac{dL(t)}{dt} = L_0 \frac{d}{dt} f \left[ \frac{D_0}{D(t)} \right] = L_0 \frac{d}{dt} f[\exp(\frac{1}{2} \dot{e}t)]. \quad (A.8)
\]

Therefore, for \(0 \leq t < t_1\):

\[
v(t) = \dot{e} L_0 [3A_1 \exp(3\dot{e}t) + 2.5A_2 \exp(2.5\dot{e}t) + \ldots + 0.5A_2 \exp(0.5\dot{e}t)]. \quad (A.9)
\]

For \(t_1 \leq t < t_2\):

\[
v(t) = \dot{e} L_0 [3B_1 \exp(3\dot{e}t) + 2.5B_2 \exp(2.5\dot{e}t) + \ldots + 0.5B_2 \exp(0.5\dot{e}t)]. \quad (A.10)
\]

For \(t \geq t_2\):

\[
v(t) = \dot{e} L_0 [3C_1 \exp(3\dot{e}t) + 2.5C_2 \exp(2.5\dot{e}t) + \ldots + 0.5C_2 \exp(0.5\dot{e}t)]. \quad (A.11)
\]

By applying the velocity compensation technique outlined above, the desired diameter decrease as a function of time (Eq. A.1) can be obtained. In addition to the compensated velocity profile, the modified program allows for two other types of velocity control by selecting the appropriate ‘ProfileFlag’, as summarized in Table A.1. When the compensated velocity control is selected (ProfileFlag=2), 23 input parameters are required,
these constants are obtained from the coefficients of Eqs. A.5 to A.7, and the two time constants $t_1$ and $t_2$. 

<table>
<thead>
<tr>
<th>ProfileFlag</th>
<th>Velocity Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Nominal velocity control based on Eq. A.2</td>
</tr>
<tr>
<td>1</td>
<td>Velocity control based on: $v(t) = \dot{L}_c \exp(\dot{\xi} t) - \dot{\xi} L_0$</td>
</tr>
<tr>
<td>2</td>
<td>Compensated velocity control based on Eqs. A.8, A.9, and A.10</td>
</tr>
</tbody>
</table>

_table A.1_ The three types of velocity profiles used in the modified control program.
Sub CalculateVelocityProfile()
  ' R. Yip modified this code to allow for velocity compensation, as described in Huang [1999].
  ' L(t)/L0 = f(D0/D(t)) Fitted with three continuously differentiable 6-power polynomial functions.
  ' Modified sections are indicated by: '''R. Yip (Begin Insert) / R. Yip (End Insert)'''
  ' This procedure calculates the velocity profile which is
  ' downloaded to the DAP to control the actuator velocity.
  ' The velocity profile is based on ideal extensional behaviour
  ' i.e., no end effects and therefore a simple exponential
  ' velocity profile describes the motion.
  ' No correction is included to maintain the extensional
  ' rate constant over the entire stroke.
  ' Any required velocity correction to maintain constant extensional
  ' rate is performed in the DAP code and all calculations
  ' related to the correction are performed by the DAP.
  ' The profile ends with a simple linear deceleration to zero
  ' velocity. This deceleration occurs over a fixed time set by the
  ' variable DecelerationTime.
  ' The procedure utilizes the current value of ExtensionalRate
  ' and Ls (the initial filament length + the pre-stretch length) in
  ' the calculation.
  ' The results of the calculation are stored in the dynamic
  ' array VelocityProfile().
  ' A user selectable pause time is added before the exponential motion
  ' begins. During the pause the actuator is held at zero speed and
  
  Global A1 As Single ' Curve 1, 6th Power Coefficient
  Global A2 As Single ' Curve 1, 5th Power Coefficient
  Global A3 As Single ' Curve 1, 4th Power Coefficient
  Global A4 As Single ' Curve 1, 3rd Power Coefficient
  Global A5 As Single ' Curve 1, 2nd Power Coefficient
  Global A6 As Single ' Curve 1, 1st Power Coefficient
  Global A7 As Single ' Curve 1, 0th Power Coefficient
  
  Global B1 As Single ' Curve 2, 6th Power Coefficient
  Global B2 As Single ' Curve 2, 5th Power Coefficient
  Global B3 As Single ' Curve 2, 4th Power Coefficient
  Global B4 As Single ' Curve 2, 3rd Power Coefficient
  Global B5 As Single ' Curve 2, 2nd Power Coefficient
  Global B6 As Single ' Curve 2, 1st Power Coefficient
  Global B7 As Single ' Curve 2, 0th Power Coefficient
  
  Global C1 As Single ' Curve 3, 6th Power Coefficient
  Global C2 As Single ' Curve 3, 5th Power Coefficient
  Global C3 As Single ' Curve 3, 4th Power Coefficient
  Global C4 As Single ' Curve 3, 3rd Power Coefficient
  Global C5 As Single ' Curve 3, 2nd Power Coefficient
  Global C6 As Single ' Curve 3, 1st Power Coefficient
  Global C7 As Single ' Curve 3, 0th Power Coefficient

  Global Time1 As Single
  Global Time2 As Single
  ''' R. Yip (End Insert)
A user selectable pre-stretch value can be specified. If this value is greater than 0 mm then the profile will include a trapezoidal velocity profile after the pause time and before the exponential profile. The purpose of this trapezoidal profile is to move the actuator a distance equal to the pre-stretch value. The velocity is held at zero after the pre-stretch for a time which is also set by the user.

Begin:

' Declare local variables.
' Integer
Dim i As Integer                   ' loop counter.
Dim j As Integer                   ' loop counter.
Dim NptsTotal As Integer           ' total number of points in velocity profile array.
Dim NptsStart As Integer           ' Number of points for the 0 velocity start of the profile array.
Dim NptsPrestretch As Integer      ' Number of points in prestretch profile.
Dim NptsPrestretchTime As Integer  ' Number of points in prestretch pause.

'... R. Yip (Begin Insert)

Dim Npts1 As Integer               ' Number of points in the first segment (Curve1) of the velocity profile.
Dim Npts2 As Integer               ' Number of points in the second segment (Curve2) of the velocity profile.
Dim Npts3 As Integer               ' Number of points in the third segment (Curve3) of the velocity profile.

'... R. Yip (End Insert)

' Real
Dim LowerBoundTotalTime As Single  ' Lower bound on total test time.
Dim UpperBoundTotalTime As Single  ' Upper bound on total test time.
Dim LowerBoundAcceleration As Single  ' Time for acceleration portion of profile.
Dim Tol As Single                  ' Tolerance on calculation of test time.
Dim Stroke As Single               ' Stroke.
Dim StrokeError As Single          ' Difference between MaxStroke and Stroke.

'... R. Yip (Begin Insert)

Dim Lslope                         ' Slope of the Stroke function.
Dim Vslope                         ' Slope of the Velocity function.

'... R. Yip (End Insert)

Dim T1 As Single                   ' Time for acceleration portion of trapezoidal move profile [s].
Dim Vmax As Single                 ' Maximum velocity of actuator during trapezoidal move profile [mm/s].

' String
Dim Msg1 As String                 ' String variable for line 1 of message box message.
Dim Msg2 As String                 ' String variable for line 2 of message box message.
Dim Msg3 As String                 ' String variable for complete message box message.
Dim MsgTitle As String             ' String variable for message box title.

' Global variables used by this subroutine.
' TimeArray() As Single            ' Array for time values of velocity profile.
' VelocityProfile() As Single      ' Array for velocity profile data.
' ExtensionalRate As Single        ' Extensional rate for a test.
' L0 As Single                     ' Initial length of fluid filament.
' MaxStroke As Single              ' Stroke length of actuator [mm].
' MaxVelocity As Single            ' Maximum velocity of actuator [mm/s].
' TotalTime As Single              ' Total time for test at given Ext Rate [s].
' TotalStroke As Single            ' Total stroke for test at given Ext Rate [mm].
' FinalVelocity As Single          ' Final velocity attained by actuator at given Ext Rate [mm/s].
' DeltaTime As Single              ' time between successive updates of velocity [s].
' Npts As Integer                  ' number of points in velocity profile array.
' NptsDeceleration As Integer      ' Npts in deceleration portion of velocity profile.
' ControlTime As Single            ' time between output values to DAP [usec].
' SampleTime As Single             ' time between input values from DAP [usec].

' Subroutine Start
' Calculate DeltaTime (time between ouput values) from Control Time.
DeltaTime = ControlTime / 10000000  ' convert [usec] to [s]
' Calculate the test time, stroke length and final velocity for the given extensional rate.
The test will end either when the velocity exceeds MaxVelocity or when the stroke exceeds MaxStroke. Set tolerance on convergence of calculation. Tol = 0.001

Calculate the total test time based on MaxStroke. If ProfileFlag = 0 then use simple length formula (no linear term containing time).
\[ L(t) = L_s \cdot e^{(ExtensionalRate \cdot t)} \]
If ProfileFlag = 1 then use more complex length formula.
\[ L(t) = L_s \cdot e^{(ExtensionalRate \cdot t)} - (L_s \cdot ExtensionalRate \cdot t) \]
Set L(t) = MaxStroke and solve for t.
When ProfileFlag = 0 this equation can be solved directly. Otherwise solve the equation by trial and error, the lower bound for t can be found by ignoring the linear term as follows.
\[ t = \ln \left( \frac{MaxStroke}{L_s} \right) / ExtensionalRate \]
Once the lower bound is determined the error in the stroke can be used to generate an error term and then convergence can be achieved using a bisection iterative procedure.
Calculate Ls, starting length = L0 + InitialStretch.
\[ L_s = L_0 + InitialStretch \]
First calculate lower bound on total time.
\[ LowerBoundTotalTime = \left( \frac{\log(MaxStroke/L_s)}{ExtensionalRate} \right) \]
If ProfileFlag = 0 Then
\[ TotalTime = LowerBoundTotalTime \]
ElseIf ProfileFlag = 1 Then

\[ \text{' R. Yip (Begin Insert) ...} \]
\[ \text{Stroke} = L_s \cdot (\exp(ExtensionalRate \cdot LowerBoundTotalTime)) \]
ElseIf ProfileFlag = 1 Then

\[ \text{' R. Yip (End Insert)} \]

Else (edited out by R. Yip)

\[ \text{' Calculate stroke when } t = \text{LowerBoundTotalTime.} \]
\[ \text{Stroke} = L_s \cdot (\exp(ExtensionalRate \cdot LowerBoundTotalTime) - ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ \text{' Use the Stroke Error to estimate the upper bound of total test time.} \]
\[ \text{' Set upper bound of test time = LowerBoundTotalTime + Stroke Error/MaxStroke} \]
\[ \text{UpperBoundTotalTime = LowerBoundTotalTime + \left( \frac{\text{Stroke Error}}{\text{MaxStroke}} \cdot \text{LowerBoundTotalTime} \right)} \]
\[ \text{' Set total time to be UpperBoundTotalTime.} \]
\[ \text{NewTotalTime = UpperBoundTotalTime} \]
\[ \text{' Iterate to get test time within tolerance.} \]
\[ \text{Do While Stroke Error / MaxStroke} > \text{Tol} \]
\[ \text{' Recalculate Stroke to see if stroke is within tolerance of MaxStroke.} \]
\[ \text{Stroke = Ls \cdot (exp(ExtensionalRate \cdot NewTotalTime) - ExtensionalRate \cdot NewTotalTime)} \]
\[ \text{Stroke Error = MaxStroke - Stroke} \]
\[ \text{' If Stroke Error} > 0 \text{' Then} \]
\[ \text{Reset lower bound.} \]
\[ \text{LowerBoundTotalTime = NewTotalTime} \]
\[ \text{' Reset upper bound.} \]
\[ \text{UpperBoundTotalTime = NewTotalTime} \]
\[ \text{' End If} \]
\[ \text{' Calculate new total time.} \]
\[ \text{NewTotalTime = LowerBoundTotalTime + (UpperBoundTotalTime - LowerBoundTotalTime) / 2} \]
\[ \text{Loop} \]
\[ \text{' Set the test total time.} \]
\[ \text{TotalTime = NewTotalTime} \]
\[ \text{' R. Yip (Begin Insert)} \]

Else
\[ \text{' Calculate stroke when } t = \text{LowerBoundTotalTime, based on 7-parameter curve fit.} \]
\[ \text{Stroke} = L_s \cdot \left( C1 \cdot \exp(3.5 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \right) \]
\[ + C2 \cdot \exp(2.5 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ + C3 \cdot \exp(2.0 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ + C4 \cdot \exp(1.5 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ + C5 \cdot \exp(1.0 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ + C6 \cdot \exp(0.5 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ + C7 \right) \]
\[ \text{Lslope = Ls \cdot ExtensionalRate \cdot (3.5 \cdot C1 \cdot \exp(3.5 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \right) \]
\[ + 2.5 \cdot C2 \cdot \exp(2.5 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
\[ + 2.0 \cdot C3 \cdot \exp(2.0 \cdot ExtensionalRate \cdot LowerBoundTotalTime) \]
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\[ + \frac{1.5 \times C_4 \times \exp(1.5 \times \text{ExtensialRate} \times \text{LowerBoundTotalTime})}{2} + \frac{1 \times C_5 \times \exp(1 \times \text{ExtensialRate} \times \text{LowerBoundTotalTime})}{2} + 0.5 \times C_6 \times \exp(0.5 \times \text{ExtensialRate} \times \text{LowerBoundTotalTime}) \]

\[ \text{StrokeError} = \text{Abs}((\text{MaxStroke} - \text{Stroke}) \times \text{Abs}((Lslope) \times \text{Tol} \times \text{Stroke}) \times \text{MaxStroke}) \leq \text{Tol} \text{ And } (\text{StrokeError} / \text{MaxStroke}) \leq \text{Tol} \text{ Then} \]

\[ \text{TotalTime} = \text{LowerBoundTotalTime} \]

ElseIf (\text{Stroke} < \text{MaxStroke} \text{ And } \text{Lslope} > 0) \text{ Then} \]

\[ \text{TotalTime} = \text{LowerBoundTotalTime} \]

Do Until (\text{Stroke} > \text{MaxStroke} \text{ Or } \text{Lslope} < 0)

\[ \text{TotalTime} = \text{TotalTime} + 0.004 \]

\[ \text{Stroke} = \text{Ls} \times (C_1 \times \exp(3 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_2 \times \exp(2.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_3 \times \exp(2 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_4 \times \exp(1.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_5 \times \exp(1 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_6 \times \exp(0.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_7) \]

\[ \text{Lslope} = \text{Ls} \times \text{ExtensialRate} \times (3 \times C_1 \times \exp(3 \times \text{ExtensialRate} \times \text{TotalTime}) \times 2 \times C_2 \times \exp(2.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times 2 \times C_3 \times \exp(2 \times \text{ExtensialRate} \times \text{TotalTime}) \times 1.5 \times C_4 \times \exp(1.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times 1 \times C_5 \times \exp(1 \times \text{ExtensialRate} \times \text{TotalTime}) \times 0.5 \times C_6 \times \exp(0.5 \times \text{ExtensialRate} \times \text{TotalTime}) \]

Loop

Else

\[ \text{TotalTime} = \text{LowerBoundTotalTime} \]

Do Until (\text{Stroke} < \text{MaxStroke} \text{ And } \text{Lslope} > 0)

\[ \text{TotalTime} = \text{TotalTime} - 0.004 \]

\[ \text{Stroke} = \text{Ls} \times (C_1 \times \exp(3 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_2 \times \exp(2.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_3 \times \exp(2 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_4 \times \exp(1.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_5 \times \exp(1 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_6 \times \exp(0.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times C_7) \]

\[ \text{Lslope} = \text{Ls} \times \text{ExtensialRate} \times (3 \times C_1 \times \exp(3 \times \text{ExtensialRate} \times \text{TotalTime}) \times 2 \times C_2 \times \exp(2.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times 2 \times C_3 \times \exp(2 \times \text{ExtensialRate} \times \text{TotalTime}) \times 1.5 \times C_4 \times \exp(1.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times 1 \times C_5 \times \exp(1 \times \text{ExtensialRate} \times \text{TotalTime}) \times 0.5 \times C_6 \times \exp(0.5 \times \text{ExtensialRate} \times \text{TotalTime}) \]

Loop

End If

\[ \text{FinalVelocity} = \text{Ls} \times \text{ExtensialRate} \times \exp(\text{ExtensialRate} \times \text{TotalTime}) \]

\[ \text{FinalVelocity} = \text{FinalVelocity} - \text{Ls} \times \text{ExtensialRate} \]

Else

\[ \text{FinalVelocity} = \text{Lslope} \]

\[ \text{Vslope} = \text{Ls} \times \text{ExtensialRate} \times 2 \times (9 \times C_1 \times \exp(9 \times \text{ExtensialRate} \times \text{TotalTime}) \times 6.25 \times C_2 \times \exp(2.5 \times \text{ExtensialRate} \times \text{TotalTime}) \times \text{TotalTime}) \]

End If
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\[ + 4\# * C_3 * \exp(2\# * \text{ExtensionalRate} * \text{TotalTime}) \]
\[ + 2.25 * C_4 * \exp(1.5 * \text{ExtensionalRate} * \text{TotalTime}) \]
\[ + 1\# * C_5 * \exp(1\# * \text{ExtensionalRate} * \text{TotalTime}) \]
\[ + 0.25 * C_6 * \exp(0.5 * \text{ExtensionalRate} * \text{TotalTime}) \]

End If

... R. Yip (End Insert)

' Check to see if final velocity > MaxVelocity.

... R. Yip (Begin Insert)

If ProfileFlag = 0 Or ProfileFlag = 1 Then

... R. Yip (End Insert)

If FinalVelocity > MaxVelocity Then

' Set FinalVelocity = MaxVelocity

FinalVelocity = MaxVelocity
' Calculate the time when the MaxVelocity is reached.
If ProfileFlag = 0 Then

TotalTime = \(\log\left(\frac{\text{MaxVelocity}}{\text{Ls} \times \text{ExtensionalRate}}\right) / \text{ExtensionalRate} \)
' Re-calculate the total stroke length.
TotalStroke = Ls * \(\exp(\text{ExtensionalRate} \times \text{TotalTime}) \)
Else

' \( t = \ln\left(\frac{\text{MaxVelocity}}{(\text{Ls} \times \text{ExtensionalRate}) + 1}\right) / \text{ExtensionalRate} \).

TotalTime = \(\log(\text{MaxVelocity} / (\text{Ls} \times \text{ExtensionalRate})) + 1\) / \text{ExtensionalRate} \)
' Re-calculate the total stroke length.
TotalStroke = Ls * \(\exp(\text{ExtensionalRate} \times \text{TotalTime}) - \text{ExtensionalRate} \times \text{TotalTime} \)

End If

End If

... R. Yip (Begin Insert)

Else

Do Until (FinalVelocity <= MaxVelocity And Vslope >= 0)

TotalTime = TotalTime - 0.004

FinalVelocity = Ls * \(\exp(3\# * \text{ExtensionalRate} * \text{TotalTime}) \)
\[ + 2.5 \times C_2 \times \exp(2.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 2\# \times C_3 \times \exp(2\# \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 1.5 \times C_4 \times \exp(1.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 1\# \times C_5 \times \exp(1\# \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 0.5 \times C_6 \times \exp(0.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]

Vslope = Ls * \(\text{ExtensionalRate} \times 2 \times (9\# \times C_1 \times \exp(3\# \times \text{ExtensionalRate} \times \text{TotalTime}) \)
\[ + 6.25 \times C_2 \times \exp(2.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 4\# \times C_3 \times \exp(2\# \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 2.25 \times C_4 \times \exp(1.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 1\# \times C_5 \times \exp(1\# \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + 0.25 \times C_6 \times \exp(0.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]

Loop

TotalStroke = \(Ls \times (C_1 \times \exp(3\# \times \text{ExtensionalRate} \times \text{TotalTime}) \)
\[ + C_2 \times \exp(2.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + C_3 \times \exp(2\# \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + C_4 \times \exp(1.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + C_5 \times \exp(1\# \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + C_6 \times \exp(0.5 \times \text{ExtensionalRate} \times \text{TotalTime}) \]
\[ + C_7 \]

End If

... R. Yip (End Insert)

' Calculate the time required to move the actuator the pre-stretch distance

' InitialStretch [mm]. Note: the move profile is trapezoidal. Acceleration and
deceleration are set by \(A_{max} = \text{TrapezoidalAcceleration}\) and the assumption is
made that the acceleration, constant velocity and deceleration
portions of the trapezoidal profile all take the same time.
' \(\text{TrapezoidalAcceleration}\) is set in the declarations section of the RHEO.BAS.
' Define \(t_1\) = time at end of acceleration, \(t_2\) time at end of constant velocity, and
\(t_3\) = time at end of move.
' Given above assumption then \(t_2 = 2^*t_1\) and \(t_3 = 3^*t_1\).
' Also (1) \(t_1 = V_{max} / A_{max}\) and (2) Pre-stretch = \(2^*t_1^*V_{max}\).
' Solve for \(V_{max}\) in (1), substitute into (2) and solve for \(t_1\).
' \(t_1 = \sqrt{\text{Pre-stretch} / (2)^*A_{max}}\).
' \(V_{max} = t_1^*A_{max}\) and \(\text{Pre-stretchTime} = 3^*t_1\).
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' Check to see if InitialStretch = 0.0 mm, if so then T1 = 0 s.
If InitialStretch > 0 Then
' Calculate T1, Vmax and PreStretchTime.
    T1 = Sqr(InitialStretch / (2 * TrapezoidalAcceleration))
    Vmax = T1 * TrapezoidalAcceleration
Else
    T1 = 0
End If

' Ensure that the VelocityProfile array is empty
Erase TimeArray
Erase VelocityProfile
' Redimension the dynamic VelocityProfile array and TimeArray.

' The complete velocity profile consists of 5 segments which are:
' 1) pause before test starts (PauseTime),
' 2) trapezoidal velocity profile to move actuator by pre-stretch distance (InitialStretch),
' 3) pause after pre-stretch (StretchTime),
' 4) exponential velocity profile,
' 5) deceleration profile.
' Calculate the number of points in each section of the complete velocity profile.

' 1) PauseTime.
    NptsStart = Int(PauseTime / DeltaTime)
' 2) Trapezoidal velocity profile.
    NptsPreStretch = 3 * Int(T1 / DeltaTime)
' 3) Pre-stretch pause time.
    NptsStretchTime = Int(StretchTime / DeltaTime)
' 4) Exponential velocity profile = Npts. (Also valid of velocity-compensated profile, R. Yip)

''' R. Yip (Begin Insert)'''

If ProfileFlag = 2 Then
    If (Time1 <= 0 Or Time1 >= Time2 Or Time1 >= TotalTime) Then
        A1 = 0
        A2 = 0
        A3 = 0
        A4 = 0
        A5 = 0
        A6 = 0
        A7 = 0
        Time1 = 0
    End If
    If (Time2 <= 0 Or Time2 >= TotalTime) Then
        A1 = 0
        A2 = 0
        A3 = 0
        A4 = 0
        A5 = 0
        A6 = 0
        A7 = 0
        Time2 = 0
    End If
    If Time1 <> 0 And Time2 <> 0 Then
        Npts1 = Int(Time1 / DeltaTime)
        Npts2 = Int((Time2 - Time1) / DeltaTime)
        Npts3 = Int((TotalTime - Time2) / DeltaTime)
        Npts = Npts1 + Npts2 + Npts3
    ElseIf Time1 = 0 And Time2 <> 0 Then
        Npts2 = Int(Time2 / DeltaTime)
        Npts3 = Int((TotalTime - Time2) / DeltaTime)
        Npts = Npts2 + Npts3
    ElseIf Time1 <> 0 And Time2 = 0 Then
        Npts1 = Int(Time1 / DeltaTime)
        Npts = Npts1 + Npts2 + Npts3
    End If
End If
Else
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Npts = Int(TotalTime / DeltaTime)
End If

' 5) Deceleration profile = NptsDeceleration
NptsDeceleration = Int(DecelerationTime / DeltaTime)

' Calculate the total number of points in the complete velocity profile.
NptsTotal = NptsStart + NptsPreStretch + NptsStretchTime + Npts + NptsDeceleration

' Check to see if NptsTotal is greater than 10000, if so then warn user and reset
' the ControlTime to a longer time.
If NptsTotal > 10000 Then

' Warn user
MsgTitle = "WARNING"
Msg1 = "Control Time Interval is set too low for chosen Extensional Rate"
Msg2 = "Control Time Interval will be reset to 10.0 msec"
MsgBox Msg1 & Chr(13) & Chr(13) & Msg2
MsgTitle = 0, MsgTitle

' Reset ControlTime
ControlTime = 1000#

' Update values in Hardware Parameters
frmHardwareParameters.inpControl.Text = ControlTime / 1000#

GoTo Begin
End If

' Redimension and then fill arrays.
ReDim VelocityProfile(NptsTotal)
ReDim TimeArray(NptsTotal)

' Calculate the pause time portion of the velocity profile array and start to
' fill the arrays.
For i = 0 To NptsStart
    TimeArray(i) = i * DeltaTime
    VelocityProfile(i) = 0
Next i

' Calculate the exponential acceleration portion of the velocity profile array
' and fill the arrays.
' *** R. Yip (Begin Insert)
'
' *** R. Yip (End Insert)
For i = 0 To Npts
    j = i + NptsStart + NptsPreStretch + NptsStretchTime
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * Exp(ExtensionalRate * (i * DeltaTime))
End For

If ProfileFlag = 1 Then
    VelocityProfile(i) = VelocityProfile(i) - Ls * ExtensionalRate
End If

ElseIf Time1 <> 0 And Time2 <> 0 Then
For i = 0 To Npts1
    j = i + NptsStart + NptsPreStretch + NptsStretchTime
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * (3# * A1 * Exp(3# * ExtensionalRate * (i * DeltaTime))
                      + 2.5 * A2 * Exp(2.5 * ExtensionalRate * (i * DeltaTime))
                      + 1.5 * A4 * Exp(1.5 * ExtensionalRate * (i * DeltaTime))
                      + 1# * A5 * Exp(1# * ExtensionalRate * (i * DeltaTime))
                      + 0.5 * A6 * Exp(0.5 * ExtensionalRate * (i * DeltaTime)))
Next i
For i = 0 To Npts2
    j = i + NptsStart + NptsPreStretch + NptsStretchTime + Npts1
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * (3# * B1 * Exp(3# * ExtensionalRate * ((i + Npts1) * DeltaTime))
                      + 2.5 * B2 * Exp(2.5 * ExtensionalRate * ((i + Npts1) * DeltaTime))
                      + 1.5 * B4 * Exp(1.5 * ExtensionalRate * ((i + Npts1) * DeltaTime))
                      + 1# * B5 * Exp(1# * ExtensionalRate * ((i + Npts1) * DeltaTime))
                      + 0.5 * B6 * Exp(0.5 * ExtensionalRate * ((i + Npts1) * DeltaTime)))
Next i
For i = 0 To Npts3
    j = i + NptsStart + NptsPreStretch + NptsStretchTime + Npts1 + Npts2
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * (3# * C1 * Exp(3# * ExtensionalRate * ((i + Npts1 + Npts2) * DeltaTime))
                      + 2.5 * C2 * Exp(2.5 * ExtensionalRate * ((i + Npts1 + Npts2) * DeltaTime))
                      + 1.5 * C4 * Exp(1.5 * ExtensionalRate * ((i + Npts1 + Npts2) * DeltaTime))
                      + 1# * C5 * Exp(1# * ExtensionalRate * ((i + Npts1 + Npts2) * DeltaTime))
                      + 0.5 * C6 * Exp(0.5 * ExtensionalRate * ((i + Npts1 + Npts2) * DeltaTime)))
Next i
ElseIf Time1 = 0 And Time2 <> 0 Then
For i = 0 To Npts2
    j = i + NptsStart + NptsPreStretch + NptsStretchTime
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * (3# * B1 * Exp(3# * ExtensionalRate * (i * DeltaTime))
                      + 2.5 * B2 * Exp(2.5 * ExtensionalRate * (i * DeltaTime))
                      + 1.5 * B4 * Exp(1.5 * ExtensionalRate * (i * DeltaTime))
                      + 1# * B5 * Exp(1# * ExtensionalRate * (i * DeltaTime))
                      + 0.5 * B6 * Exp(0.5 * ExtensionalRate * (i * DeltaTime)))
Next i
For i = 0 To Npts3
    j = i + NptsStart + NptsPreStretch + NptsStretchTime + Npts2
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * (3# * C1 * Exp(3# * ExtensionalRate * ((i + Npts2) * DeltaTime))
                      + 2.5 * C2 * Exp(2.5 * ExtensionalRate * ((i + Npts2) * DeltaTime))
                      + 1.5 * C4 * Exp(1.5 * ExtensionalRate * ((i + Npts2) * DeltaTime))
                      + 1# * C5 * Exp(1# * ExtensionalRate * ((i + Npts2) * DeltaTime))
                      + 0.5 * C6 * Exp(0.5 * ExtensionalRate * ((i + Npts2) * DeltaTime)))
Next i
Else
For i = 0 To Npts
    j = i + NptsStart + NptsPreStretch + NptsStretchTime
    TimeArray(j) = j * DeltaTime
    VelocityProfile(j) = Ls * ExtensionalRate * (3# * C1 * Exp(3# * ExtensionalRate * (i * DeltaTime))
                      + 2.5 * C2 * Exp(2.5 * ExtensionalRate * (i * DeltaTime))
                      + 1.5 * C4 * Exp(1.5 * ExtensionalRate * (i * DeltaTime))
                      + 1# * C5 * Exp(1# * ExtensionalRate * (i * DeltaTime))
                      + 0.5 * C6 * Exp(0.5 * ExtensionalRate * (i * DeltaTime))
End For
End If
Appendix A: Velocity Compensation of the FSR

\[ + 2.5 \cdot C2 \cdot \exp(2.5 \cdot \text{ExtensionalRate} \cdot (i \cdot \Delta \text{Time})) \]
\[ + 2\# \cdot C3 \cdot \exp(2\# \cdot \text{ExtensionalRate} \cdot (i \cdot \Delta \text{Time})) \]
\[ + 1.5 \cdot C4 \cdot \exp(1.5 \cdot \text{ExtensionalRate} \cdot (i \cdot \Delta \text{Time})) \]
\[ + 1\# \cdot C5 \cdot \exp(1\# \cdot \text{ExtensionalRate} \cdot (i \cdot \Delta \text{Time})) \]
\[ + 0.5 \cdot C6 \cdot \exp(0.5 \cdot \text{ExtensionalRate} \cdot (i \cdot \Delta \text{Time})) \]

Next i

End If

End If

End Sub

Sub defaults()

A1 = 0                        ' Curve 1
A2 = 0                        ' Curve 1
A3 = 0                        ' Curve 1
A4 = 0                        ' Curve 1
A5 = 0                        ' Curve 1
A6 = 0                        ' Curve 1
A7 = 0                        ' Curve 1
B1 = 0                        ' Curve 2
B2 = 0                        ' Curve 2
B3 = 0                        ' Curve 2
B4 = 0                        ' Curve 2
B5 = 0                        ' Curve 2
B6 = 0                        ' Curve 2
B7 = 0                        ' Curve 2
C1 = 0                        ' Curve 3
C2 = 0                        ' Curve 3
C3 = 0                        ' Curve 3
C4 = 0                        ' Curve 3
C5 = 1                        ' Curve 3 (2nd Power Coefficient)
C6 = 0                        ' Curve 3
C7 = 0                        ' Curve 3

Time1 = 0                     ' Time at intersection of Curve 1 and Curve 2.
Time2 = 0                     ' Time at intersection of Curve 2 and Curve 3.

...
APPENDIX B: CAMERA FRAME RATE CONTROL

The digital camera (JAI TM-6740CL) used in the PIV system captures images at an acquisition rate of 200 frames per second. The images are recorded onto the random access memory (RAM) of the personal computer, before being transferred to the hard disk. The data management is handled by a software called Norpix StreamPix 4. Although the acquisition rate of the camera cannot be changed, it is possible to instruct StreamPix to record every $n$-th frame onto the hard disk, thereby effectively reducing the frame rate as follows:

$$\text{frame rate} = \frac{200}{n}. \quad \text{(B.1)}$$

The script used by StreamPix to control the camera frame rate is provide at the end of Appendix A. To use the script, change the parameter $n$ (from TargetCount="n") to number of frames StreamPix is to skip recording. Once the file is saved, select the ‘Recording Manager’ option within the ‘StreamPix Settings’ menu to load the script. The effective frame rate is now modified according to equation Eq. B.1.
Script to Modified Digital Camera Frame Rate.

Load this script in StreamPix 4

<?xml version="1.0" ?>
<Streampix4RecScript>
  <STEP NAME="Main">
    <COMMANDS>
      <GRAB_FRAME />
      <CALL_STEP>
        <STEP NAME="skipframes">
          <COMMANDS>
            <SKIP_FRAME />
          </COMMANDS>
          <LOOPCONTROL CurrentControl="0" HH="0" MM="0" SS="0" MS="0" FrameCount="0" />
          <CONDITIONS>
            <DO_X_TIMES Interrupt="1" TargetCount="n" />
          </CONDITIONS>
        </STEP>
      </CALL_STEP>
    </COMMANDS>
    <LOOPCONTROL CurrentControl="0" HH="0" MM="0" SS="0" MS="0" FrameCount="0" />
    <CONDITIONS />
  </STEP>
</Streampix4RecScript>
REFERENCES


References


