Early Fault Detection for Gear Shaft and Planetary Gear
Based on Wavelet and Hidden Markov Modeling

by

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Abstract

Fault detection and diagnosis of gear transmission systems have attracted considerable attention in recent years, due to the need to decrease the downtime on production machinery and to reduce the extent of the secondary damage caused by failures. However, little research has been done to develop gear shaft and planetary gear crack detection methods based on vibration signal analysis. In this thesis, an approach to gear shaft and planetary gear fault detection based on the application of the wavelet transform to both the time synchronously averaged (TSA) signal and residual signal is presented. Wavelet approaches themselves are sometimes inefficient for picking up the fault signal characteristic under the presence of strong noise. In this thesis, the autocovariance of maximal energy wavelet coefficients is first proposed to evaluate the gear shaft and planetary gear fault advancement quantitatively. For a comparison, the advantages and disadvantages of some approaches such as using variance, kurtosis, the application of the Kolmogorov-Smirnov test (K-S test), root mean square (RMS), and crest factor as fault indicators with continuous wavelet transform (CWT) and discrete wavelet transform (DWT) for
residual signal, are discussed. It is demonstrated using real vibration data that the early faults in gear shafts and planetary gear can be detected and identified successfully using wavelet transforms combined with the approaches mentioned above.

In the second part of the thesis, the planetary gear deterioration process from the new condition to failure is modeled as a continuous time homogeneous Markov process with three states: good, warning, and breakdown. The observation process is represented by two characteristics: variance and RMS based on the analysis of autocovariance of DWT applied to the TSA signal obtained from planetary gear vibration data. The hidden Markov model parameters are estimated by maximizing the pseudo likelihood function using the EM iterative algorithm. Then, a multivariate Bayesian control chart is applied for fault detection. It can be seen from the numerical results that the Bayesian chart performs better than the traditional Chi-square chart.
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Chapter 1

Introduction

1.1 A Brief Overview

The gearbox is a vital part on most types of machinery for changing the shaft speed, the torque and the power. The concept of gear transmission has been used thousands of years ago. Gear trains are considered to be among the earliest machine elements. In modern days, the existence of the gearbox is believed to have control over the economies of the industries and in fact nations. Besides, in some types of machinery such as the helicopter or aircrafts in general, the existence of the gearbox is extremely important and its failure may lead to catastrophic results such as loss of assets and life. Statistics from 1999 show that out of the 192 turbine helicopter accidents which occurred, 28 were directly due to mechanical failures with the most common failure occurring on the drive train of the gearboxes [1]. Over the past 30 years, many researchers have extensively studied and focused on failure and damage detection techniques in mechanical equipments. However, faults can still take place at any time on rotating machinery which will lead to harmful results or delays in production. It is important to detect any problems at an early stage to prevent unexpected breakdown. There are several techniques available for the early detection of failure, and one of the most useful is vibration analysis. A popular technique of vibration analysis during the last decade has been spectral analysis, in which the amplitude spectrum of the measured vibration time signal is calculated and displayed. Spectral analysis is a particularly powerful technique because the different elements of a mechanical system in general produce vibration at different frequencies. But this identification can be difficult in very complex systems such as helicopter gearboxes, which may have as many as 30 gears and 50 bearings. The great number of spectral lines can make it difficult to detect changes in the spectrum. An alternative technique of vibration analysis which is becoming more common for the early detection of failure in gears is time domain averaging. This technique is particularly useful for complex systems such as gearboxes as it eliminates the vibration from other system
elements, thus reducing the problem to the study of a simpler system. The early applications of these gear faults detecting methods in particular began with Forrester's work [2-4]. He applied the Wigner-Ville distribution (WVD) to the averaged gear vibration signals and showed that different faults such as a tooth crack and pitting can be detected in the WVD plot. Later, McFadden and Wang applied the normal WVD and a weighted version of the WVD to gear failure to improve the detection capability of the method [5-6]. Staszewski and Tomlinson applied the wavelet transformation (WT) and the Wigner-Ville distribution to a spur gear to detect a broken tooth. They also used statistical and neural networks for the classification of fault conditions [7]. More recent work conducted by Yesilyurt applied the smoothed instantaneous power spectrum distribution to gear faults and identified incipient faults in the gears [8]. A more general study was carried out by Paya, Badi and Esat on gears and bearing faults. They used the wavelet as a processor to classify the types of fault [9].

Figure 1.1 Classification of Vibration-Based Analysis Techniques and Parameters

Figure 1.1 shows the vibration-based analysis that may be used for gear fault detection...
There are several feature functions such as Kurtosis and RMS that can be calculated at different preprocessing stages.

In recent years, condition-based maintenance (CBM) modeling has gained significant attention and recognition in many areas. Diagnostics and prognostics are two important aspects in a CBM program. Diagnostics deal with fault detection, and prognostics deal with fault prediction. The main goal of these techniques is to minimize the cost and time in machine repairs as well as to enhance the ways of detecting failures.

The gear deterioration process is considered to be a continuous time homogeneous Markov process which is generally unobservable. There are examples of research carried out which extensively studied and focused on partially observable systems. For example, Ohnishi et al. [12] applied a Markov decision process model for a discrete-time deterioration system in order to find the optimal replacement policy. Hontelez et al. [13] formulated the decision problem as a discrete Markov decision problem based on a continuous deterioration process to find the optimum maintenance policy with respect to cost. Kumar and Westberg [14] proposed a reliability-based approach for estimating the optimal maintenance time interval as well as the optimal threshold of the maintenance policy to minimize the total cost per unit time. Makis and Jiang [15] presented a framework for CBM optimization based on a continuous-discrete stochastic model. The evolution of the hidden machine state was described by a continuous-time Markov process, and the condition monitoring process was described by a discrete-time observation stochastic process which depends on the hidden machine state. The optimal replacement policy was found minimizing the expected cost per unit time in the long run using the optimal stopping theory. Makis [16-17] has proved that the optimal control policy can be represented by a Bayesian control chart.

1.2 Organization of the Thesis

This thesis consists of five chapters. Chapter 1 is the introduction. In Chapter 2, the autocovariance of maximal energy coefficients combined with wavelet transform approaches is proposed for gear shaft crack detection. The TSA and residual signal are used as the source
signals, and wavelet transform approaches such as CWT and DWT are considered. Measures such as standard deviation, kurtosis and the K-S test are used as fault indicators. The TSA method for the planetary gearbox is introduced and wavelet transform approaches are combined with the autocovariance of maximal energy coefficients to a planetary gearbox vibration signal in Chapter 3. Chapter 4 presents a CBM model parameter estimation problem. A partially observable hidden Markov model with unobservable system state (except the failure state) was used to describe the relationship between the observations and the three states of a gear deterioration process. The EM algorithm was used to determine the maximum likelihood estimates of the state and observation model parameters. The final chapter, Chapter 5, presents the conclusions and recommendations for future work.

The main body of this thesis consists of three chapters: Chapter 2, Chapter 3 and Chapter 4 which will be described below in detail.

**Chapter 2: Wavelet Analysis — Based Gear Shaft Fault Detection**

In this chapter, a parallel gear transmission system is first introduced. Then, the time synchronous averaging (TSA) technique and the residual signal based on time synchronous averaging are briefly described. We can extract different TSA signals based on the different gears and their shafts. In this chapter, only the signals from the pinion gear shaft were extracted by the TSA method. A quick overview of wavelet transforms such as CWT and DWT are provided. When WT is used, the wavelet coefficients will highlight the changes in signals, which often indicate the occurrence of a fault. Hence, the wavelet coefficients-based features are suitable for early fault detection. However, it is not easy to detect a fault using only the wavelet coefficients. Thus, a method consisting of combining correlation analysis with wavelet transform for the purpose of denoising is proposed. Each data file was collected from the Mechanical Diagnostics Test Bed of the Applied Research Laboratory at Pennsylvania State University in a 10s window which covers 200000 sampling points in total. The time interval between every two adjacent data files is 30 minutes. The sampling frequency is 20 kHz. The gear shaft ran from the healthy state to the completely broken state. The Daubechies wavelet with order 4 is considered in this chapter. Fault indicators such as
standard deviation, kurtosis and the value of the K-S test statistic with continuous wavelet transform (CWT) and discrete wavelet transform (DWT) for residual signal are discussed.

Chapter 3: Wavelet Analysis with Time-synchronous Averaging of Planetary Gearbox Vibration Data

In this chapter, a planetary gear transmission system is introduced. A planetary gearbox has a number of planet gears which all mesh with a sun and ring gear. The planetary gears are mounted onto the planetary carrier and contained within an internal toothed ring gear. Drive is provided via the sun gear. The ring gear is stationary and the axes of the planet gears are connected to a carrier which rotates in relation to both the sun gear and ring gear. The planet carrier provides the output of the planetary gear train. Signal averaging has proved to be the most useful vibration analysis tool for detecting faults in gears in Chapter 2. The implementation of a TSA algorithm for fixed-axis gears is relatively straightforward. The technique for the extraction of the time averaged vibration signals associated with the individual planet gears and sun gear in a planetary transmission is quite complex. In this chapter, a mathematical derivation of the method by proportionally dividing the vibration data among the individual gear meshes is introduced. Vibration data were collected during 300s for each file. For the files from rtf_1st_run to rtf_3rd_run, the sampling frequency is 10000Hz, which covers 3000000 sampling points. For the files from rtf_4th_run to rtf_13th_run, the sampling frequency is 5000Hz, which covers 1500000 sampling points. Firstly, the original vibration signals measured from the gearbox were synchronously averaged per one rotation of the gear of interest, and then the residual signals were obtained based on the TSA. After that, Daubechies wavelet transform with order 4 based on the TSA and the residual vibration data were investigated. Frequencies with original signals, TSA signals, residual signals, CWT signals, DWT signals and autocorrelation for these signals were analyzed and compared. Data files running from the healthy state to the failure state (File 1 to File 13) were investigated. Also, some fault feature indicators such as variance, kurtosis, RMS, and Crest factor for planetary gearbox vibration signals were extracted and investigated with the autocovariance of maximal energy wavelet coefficients.
Chapter 4: Multivariate Bayesian Control Chart for CBM

In this chapter, data were collected from the new condition to the failure. The system can be in one of three states \( \{1, 2, 3\} \). State 1 represents a healthy state in which the system is running under normal conditions and State 2 represents an unhealthy state in which the system is undergoing deterioration. Both States 1 and 2 are unobservable as is the case in many real-life situations. State 3 then represents the failure state, which is observable. The gear deterioration process is considered to be a continuous time homogeneous Markov process with three states: good, warning, and breakdown. The observation process is represented by 2 characteristics: variance and RMS based on the analysis of autocovariance for DWT applied to the TSA signal obtained from the planetary gear vibration data in the previous chapter. Unobservable as well as observable conditions are both involved in the deterioration process of the gear transmission system. A partially observable hidden Markov model is used to model the process. The model parameters are estimated by maximizing the pseudo likelihood function using the EM iterative algorithm. Numerical results are then applied to estimate the parameters and generate a set of reasonable parameter estimates in comparison to the results obtained from judgment. Then, a multivariate Bayesian control chart for condition-based maintenance applications is considered by using the control limit policy structure and involving an observable failure state.

1.3 Research Contributions

This thesis investigates the fault detection and the CBM of the gear shaft and planetary gear using vibration data, and the contributions of the research are summarized below.

1. Reviews of the literature indicate that there has not been much study on the gear shaft and the planetary gear fault detection schemes. This research has developed methodologies for fault detection on the gear shaft and the planetary gear using wavelet transform.

2. Fault detection schemes for planetary gear defects as well as HMM-based CBM models utilizing health index processes for the assessment of the system state have been developed.
A multivariate Bayesian control model for early fault detection of the gear transmission system has been developed. The schemes and the models have been tested using Syncrude data.

3. The research can be extended to other areas such as vibration monitoring and CBM of automobiles, aircraft engines, heavy machinery and marine vehicles. The developed fault detection schemes and CBM models will be applicable in real situations and will contribute to substantial cost savings.
Chapter 2

Wavelet Analysis-Based Gear Shaft Fault Detection

2.1 Introduction

Fault detection and diagnosis of gear transmission systems have attracted considerable attention in recent years, due to the need to decrease the downtime on production machinery and to reduce the extent of the secondary damage caused by failures. Much work has been done using vibration data analysis and modeling to detect and diagnose tooth faults in the gears, especially fatigue crack due to cyclic loading, but there are very few papers dealing with the early detection of shaft cracks, which can also cause catastrophic breakdown. Vibration behaviour induced by shaft cracks is different from that induced by tooth cracks. Hence, fault indicators used for the detection of tooth faults may not be effective in detecting shaft cracks.

There has been extensive research on the vibration behaviour of cracked shafts and crack identification in rotating shafts [18-20]. However, all the papers have focused on the crack identification in a non-gear shaft, specifically in a rotor shaft. As summarized by Hamidi et al. [21], several publications have proposed a number of techniques such as the use of natural frequencies, mode shapes and frequency response functions for the damage detection of rotor shafts. However, the method of crack detection in a gear shaft is different from that in a non-gear shaft. Little research has been done to develop gear shaft crack detection methods based on vibration signal analysis. An autoregressive model-based technique to detect the occurrence and advancement of gear shaft cracks was proposed by Wang and Makis [22].

Recently, wavelet transform (WT), which is capable of providing both the time-domain and frequency-domain information simultaneously, has been successfully used in non-stationary vibration signal processing and fault diagnosis [23-26]. Wavelet approaches themselves are sometimes inefficient for picking up the fault signal characteristic under the presence of strong noise. In this chapter, the autocovariance of maximal energy coefficients combined with wavelet transform approaches is firstly proposed for gear shaft crack detection. The results reveal that the method can enhance the capability of feature extraction and fault diagnosis for gear shaft. The time synchronous averaging (TSA) and residual signal are used as the source signal, and some
wavelet transform approaches such as continuous wavelet transform (CWT) and discrete wavelet transform (DWT) are considered. Measures such as standard deviation, kurtosis and the K-S test are used as fault indicators.

The remainder of the chapter is organized as follows. In section 2.2, a parallel gear transmission system is briefly introduced. Section 2.3 briefly describes residual signal based on time synchronous averaging. Section 2.4 provides a quick overview of wavelet transforms such as CWT and DWT. Fault indicators based on wavelet transform are considered in section 2.5. In section 2.6, we describe briefly the experimental gear test rig, and summarize the data-processing techniques used in this study. The results are presented in section 2.7, followed by the conclusions in section 2.8.

### 2.2 The Parallel Gear Transmission System

The scheme of a gear transmission system is shown in Figure 2.1. The system consists of a pinion gear and a driven gear. The pinion gear has a smaller number of teeth than the driven gear. Normally, a gear transmission system is designed to reduce the angular velocity in order to increase the output torque. In such a speed reduction gear transmission system, the pinion is connected with an input shaft, and the driven gear is connected with an output shaft.

![Figure 2.1 The Parallel Gear Transmission System](image)
2.3 Residual Signal Based on Time Synchronous Averaging (TSA)

TSA technique [27] is widely accepted as a powerful tool in the fault detection and diagnosis of rotating systems. The technique attempts to isolate the raw vibration signal from the gearbox by reducing the effects of noises. Noises can be from the external environment or be from other gears in the same gearbox. Suppose that there is a discrete time series \( x(n) \) \( n = 0, 1, \ldots, N-1 \), which covers a number of revolutions of the gear. Then, the TSA signal is calculated using the following formula:

\[
y(n) = \frac{1}{L} \sum_{i=0}^{L-1} x(n - iK), n = (L-1)K, (L-1)K+1, \ldots, N-1
\]

(2.1)

where \( y(n) \) is the TSA signal, \( L \) is the number of revolutions to be averaged, and \( K \) is the number of sampling points per revolution.

We can extract different TSA signals based on the different values of \( K \), which are dependent on the different gears and their shafts. In this chapter, only the signals from the pinion gear shaft were extracted by the TSA method.

Residual signal is obtained by eliminating from the FFT spectrum of the TSA signal the fundamental and harmonics of the tooth-meshing frequency, subsequently applying the inverse Fourier transform and then reconstructing the remaining signal in the time-domain. The residual signal can hence be expressed as:

\[
z(n) = y(n) - g(n)
\]

(2.2)

where \( z(n) \) is the residual signal, and \( g(n) \) is the signal composed of the eliminated components.

2.4 Wavelet Methodology [28-29]

Wavelet transform is a powerful method for studying how the frequency content changes with time and consequently is able to detect and localize short-duration phenomena. The basic idea is to choose a wavelet function whose shape is similar to the vibration signal caused by the mechanical fault. When a fault occurs, the vibration signal of the machine which includes periodic impulses is well described by the wavelet behaviour, which has been widely used in
fault diagnosis. In this section, we first give a brief introduction of the CWT and then summarize the theory of DWT.

### 2.4.1 Continuous Wavelet Transform (CWT)

The wavelet transform is a linear transform which uses a series of oscillating functions with different frequencies as window functions, \( \psi_{a,b}(t) \), to scan, and translate the signal \( x(t) \), where \( \alpha \) is the dilation parameter for changing the oscillating frequency and \( \beta \) is the translation parameter. At high frequencies, the wavelet reaches a high time resolution but a low frequency resolution, whereas at low frequencies a low time resolution and a high frequency resolution is achieved, which makes these transformations suitable for non-stationary signal analysis. The basis function for the wavelet transform is given in terms of a translation parameter \( \beta \) and a dilation parameter \( \alpha \) with the mother wavelet represented as:

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{\alpha}} \psi\left(\frac{t-\beta}{\alpha}\right)
\]

Suppose that all signals \( x(t) \) satisfy the condition

\[
\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty
\]

Which implies that \( x(t) \) decays to zero.

The wavelet transform, \( CWT(\alpha, \beta) \), of a time signal \( x(t) \) can be defined as

\[
CWT(\alpha, \beta) = \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} x(t) \psi^\ast\left(\frac{t-\beta}{\alpha}\right) dt
\]

Where \( \psi(t) \) is an analyzing wavelet and \( \psi^\ast(t) \) is the complex conjugate of \( \psi(t) \).

There are a number of different real and complex valued functions that can be used as analyzing wavelets.
2.4.2 Discrete Wavelet Transform (DWT)

In practice, calculating wavelet coefficients at every possible scale is a fair amount of work and generates a lot of redundant data. It turns out that, if we limit the choice of $\alpha$ and $\beta$ to discrete numbers, then our analysis will be sufficiently accurate.

By choosing fixed values $\alpha = \alpha_0^j$ and $\beta = k\beta_0\alpha_0^j$, and $j, k = 0, \pm 1, \pm 2, \cdots$, we obtain for the DWT

$$DWT(j, k) = \alpha_0^{-j/2} \int_{-\infty}^{\infty} x(t)\psi^*\left(\alpha_0^{-j/2} t - k\beta_0\right)dt$$

(2.6)

In particular, if $\alpha$ and $\beta$ are replaced by $2^j$ and $2^j \beta_0$, then the DWT is given by

$$DWT(j, k) = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t - 2^j \beta_0}{2^j}\right)dt$$

(2.7)

Generally speaking, the original signal, $S$, passes through two complementary filters and emerges as a low frequency signal [approximations (A)] and a high frequency signal [details (D)]. The decomposition process can be iterated, so that a signal can be broken down into many lower-resolution components. The decomposition process can be seen in Figure 2.2 below.

![Figure 2.2 The Discrete Wavelet Decomposition Process at Three Levels](image)
2.5 Fault Feature Extraction

2.5.1 Selection of Maximal Energy Coefficients for Fault Detection

The signal processed by wavelet transform can be the raw vibration signal, the TSA signal or the gear motion residual signal. In this chapter, the residual signal is used as the source signal to which the wavelet transform is applied.

The effectiveness of the fault diagnosis and prognosis techniques depends very much on the quality of the selected fault features. When WT is used, the wavelet coefficients will highlight the changes in signals, which often indicate the occurrence of a fault. Hence, the wavelet coefficient-based features are suitable for early fault detection. However, it is not easy to detect a fault using only wavelet coefficients. Hence, a method consisting of combining correlation analysis with wavelet transform for the purpose of denoising is proposed. The procedure can be described as follows.

**Computing Threshold Wavelet Coefficients**

The threshold used here is determined by calculating the mean and the variance values of $W(\alpha, \beta)$ for different scales, where $W(\alpha, \beta)$ are the coefficients of WT. This value was chosen to effectively remove the noise. After determining the threshold, denoising and threshold wavelet coefficients are then computed. These are denoted as:

$$M(\alpha_i) = \frac{1}{K} \sum_{j=1}^{K} W(\alpha_i, \beta_j)$$

$$\sigma(\alpha_i) = \left[ \frac{1}{K} \sum_{j=1}^{K} [W(\alpha_i, \beta_j) - M(\alpha_i)]^2 \right]^{1/2}$$

$$Thr(\alpha_i) = M(\alpha_i) + \sigma(\alpha_i)$$

$$\hat{W}(\alpha_i, \beta_j) = \begin{cases} 
\text{sign}(W(\alpha_i, \beta_j)) \times \left[ W(\alpha_i, \beta_j) - Thr(\alpha_i) \right] & \text{if } |W(\alpha_i, \beta_j)| \geq Thr(\alpha_i) \\
0 & \text{if } |W(\alpha_i, \beta_j)| < Thr(\alpha_i) 
\end{cases}$$

(2.8)
where $K$ is the number of sampling points, $M(\alpha_i)$ is the mean, $\sigma(\alpha_i)$ is the variance, $\hat{W}(\alpha_i, \beta_j)$ represent coefficients after denoising, sign is the signum function where $\text{signum}(x)$ is -1 when $x$ is negative, 0 when $x$ is 0, and 1 when $x$ is positive.

### Computing Autocovariance of Maximal Energy Coefficients

First, we find the maximal energy coefficients at a special scale $\alpha_{\text{max}}$ defined below, and then autocovariance of the special scale series using the following formulas:

$$
\hat{E}(\alpha_i) = \sum_{j=1}^{K} \left( \hat{W}(\alpha_i, \beta_j) \right)^2
$$

$$
\left[ \hat{E}(\alpha_{\text{max}}) \right] = \max_{\alpha_i \rightarrow \sigma_i} \left[ \hat{E}(\alpha_i) \right]
$$

$$
\hat{R}_{\parallel\parallel} (m) = \sum_{j=1}^{M-m} \hat{W}(\alpha_{\text{max}}, \beta_j) \hat{W}(\alpha_{\text{max}}, \beta_{j+m})
$$

$$
P(i) = \left( \hat{R}_{\parallel\parallel}(i) \right)^2 \quad (2.9)
$$

Finally, these can be used as fault indicators combined with some statistical measures such as kurtosis, standard deviation (std) and the K-S test.

#### 2.5.2 K-S Test and Some Measures for Wavelet Transform

In statistics, the K-S test is used to determine whether two underlying probability distributions differ, or whether an underlying probability distribution differs from a hypothesized distribution [30]. Recently, the K-S test has been found to be an extremely powerful tool in the condition monitoring of rotating machinery [31]. The K-S test - based signal processing technique compares two signals and tests the hypothesis that the two signals have the same probability distribution. Using this technique, it is possible to determine whether the two signals are similar or not. Note that the application of this test for condition monitoring assumes that the fault is strong enough to cause a change in the cumulative distribution function of the original vibration signature, which is the case of fatigue cracks and many other gear mechanical faults.
More specifically, the K-S test considers the null hypothesis that the cumulative distribution function (CDF) of the target distribution, denoted by \( F(x) \), is the same as the cumulative distribution function of a reference distribution, \( R(x) \). The K-S statistic \( K \) is then the maximum difference between the two distribution functions, which can be used as the fault indicator.

In this chapter, the coefficients of wavelet transform in the healthy state are chosen to represent the reference distribution, and the K-S test is performed to compare coefficients of the wavelet transform of other data files with the reference file.

In order to compare the effectiveness to indicate fault occurrence, other statistical measures such as kurtosis which is a statistical parameter commonly used to assess the peakedness of a signal, and standard deviation are also considered in the following sections.

2.6 Experimental Set-up

Typically, vibration data are collected from accelerometers located on the transmission housing. The vibration data used in this chapter were obtained from the mechanical diagnostics test-bed (MDTB) in the Applied Research Laboratory at Pennsylvania State University [32-33]. It is functionally a motor-drive train-generator test stand. The gearbox is driven at a set input speed using a 22.38kW, 1750 rpm AC drive motor, and the torque is applied by a 55.95kW, 1750 rpm AC absorption motor. The MDTB is highly efficient because the electrical power generated by the absorber is fed back to the driver motor. The gearboxes are nominally in the 3.73~14.92kW range with ratios from about 1.2:1 to 6:1. The system can be seen in Figure 2.3.
Each data file was collected in a 10s window which covers 200000 sampling points in total. The time interval between every two adjacent data files is 30 minutes. The sampling frequency is 20 kHz. The signals of the MDTB accelerometers are all converted to digital data format with the highest resolution. Among all accelerometers located in the MDTB, the single axis shear piezoelectric accelerometer data A03 for axial direction presents the best quality data for the state diagnosis of the gearbox. Therefore, the data recorded by this accelerometer is selected in this study. In this chapter, we have only extracted and analyzed the signals of the input gear shaft with period \( K = 686 \) (sampling frequency*60/gear speed = 20000*60/1750 = 686).

### 2.7 Results and Discussion

Several data files (194–195,197,199–206,208–212,214, 217–218, 223, 225, and 228–231) of A03 from the test run #13 have been randomly selected to investigate the gear shaft (21 teeth pinion gear) fault. The gear shaft ran from the healthy state to the completely broken state (See Figure 2.4) at 300% output torque (188.14 Nm). The duration of the whole experiment was 15.5 hours. The gear shaft states were unknown during the running period when the data files were collected. The shaft was inspected after completing the experiment. There are a number of different real and complex valued functions that can be used in analyzing wavelets. After a thorough investigation and analysis of the results, we have found that the Daubechies wavelet with order 4 is most effective for processing the vibration data considered in this chapter.

![Figure 2.4 Broken Gear Shaft in Test Run #13](image-url)
2.7.1 CWT for Gear Shaft Crack Detection

2.7.1.1 CWT Based on the TSA Signal and Residual Signal

CWT is often graphically represented in a time-scale plane. However, using the relationship between frequency and scale, and by transforming the time of one wheel revolution to 360 degrees of wheel angular location, the results of CWT amplitude maps can be displayed in the angle-frequency plane.

We have investigated all selected files of Test Run #13 with CWT applied to the corresponding TSA and residual signals.

![Figure 2.5](image)

Figure 2.5 CWT Based on TSA Signal (a) and Residual Signal (b) for File 194
Figure 2.6 CWT Based on TSA Signal (a) and Residual Signal (b) for File 214

Figure 2.7 CWT Based on TSA Signal (a) and Residual Signal (b) for File 217
Figure 2.8 CWT Based on TSA Signal (a) and Residual Signal (b) for File 218

Figure 2.9 CWT Based on TSA Signal (a) and Residual Signal (b) for File 229
The following results can be observed from the plots:

(1) In the healthy state of the gear shaft (Figures 2.5-2.7), the TSA signatures vary very regularly, oscillating along the center line. There are 21 signature periods in one revolution, corresponding to 21 teeth.

(2) In the broken state of the gear shaft (Figure 2.10), there is a very large variation in the whole waveform of the TSA signal. The TSA signal fluctuation induced by the gear shaft crack does not show a sharp impulse, but a hump in the shape of the waveform. Also, the curve deviates far from the center line which is induced by the shaft eccentricity due to shaft crack. However, as there is no peak impulse in the curve, which is often induced by tooth fault, we can identify the fault as a gear shaft fault rather than a gear tooth fault or some other fault.

(3) In the corresponding CWT plots, the waveform of the mean amplitude of the CWT based on TSA signal behaves just like the waveform of the TSA amplitude. In the healthy states, there is little fluctuation in the waveform, which is expected and it is due to small imperfections in the gear shaft. However, there are evident amplitude fluctuations in the gear faulty states (Figure 2.10), and this can be explained by the bigger impact caused by faulty states (broken shaft).

(4) In the residual signal, most of the vibration energy generated by the gear meshing action has been removed, so the amplitude values of residual signal are relatively small. The residual
signals and CWT based on residual signals appear unorderly, and no special symptoms can be found in the maps of healthy states and small fault states. Thus, these maps cannot be used to indicate and to prognosticate the gear shaft fault advancement quantitatively.

2.7.1.2 CWT Based on Residual Signal for Gear Shaft Crack Detection

In order to detect the gear shaft fault, some fault feature extracting indicators such as standard deviation, kurtosis and the K-S test using the wavelet coefficients of residual signal are computed and investigated in this section. At first, a detailed procedure was presented in Table 2.1 for getting standard deviation from CWT with autocovariance for some data of residual signal. Then, The plots based on CWT and on the \( \{P(i)\} \) values (Equation (2.9)) for residual signal can be seen in Figures 2.11-2.13.
Table 2.1 An Example (i is 12, j is 8)

\[
W(\alpha_i, \beta_j) = \begin{bmatrix}
49.1787 & 15.9872 & -6.0738 & 12.2833 & 29.5146 & 30.8915 & 49.5288 & 47.2169 & 65.0466 \\
37.5669 & 47.3882 & 51.5916 & 38.3324 & 17.2763 & -13.2949 & -0.5372 & 41.1172 & -55.4765 \\
51.6651 & 79.7728 & 88.5339 & 77.9586 & 46.1954 & 1.9042 & -38.8215 & -71.0918 \\
84.2580 & 100.3494 & 101.5039 & 78.6083 & 33.6688 & -14.0771 & -53.7140 & -83.4656 \\
109.0655 & 123.2023 & 122.0819 & 95.3317 & 50.0899 & -0.5372 & -43.7681 & -77.7305 \\
\end{bmatrix}
\]

\[
M(\alpha_i) = \frac{1}{M} \sum_{j=1}^{M} W(\alpha_i, \beta_j)
\]

\[
= \begin{bmatrix}
3.0977 & 4.6521 & 1.2047 & 2.7638 & 0.2212 & 10.2833 \\
12.2833 & 29.5146 & 30.8915 & 49.5288 & 47.2169 & 65.0466 \\
\end{bmatrix}
\]

\[
\hat{\sigma}(\alpha_i) = \left[ \frac{1}{M} \sum_{j=1}^{M} [W(\alpha_i, \beta_j) - M(\alpha_i)]^2 \right]^{\frac{1}{2}}
\]

\[
= \begin{bmatrix}
57.0689 & 59.3761 & 72.9000 & 68.5578 & 79.0097 & 70.9893 \\
\end{bmatrix}
\]

\[
Thr(\alpha_i) = \hat{M}(\alpha_i) + \hat{\sigma}(\alpha_i)
\]

\[
= \begin{bmatrix}
69.3522 & 88.8907 & 103.7915 & 118.0866 & 126.2266 & 136.0359 \\
\end{bmatrix}
\]
\[
W(\alpha_i, \beta_j) = \begin{cases} 
\text{sign}(W(\alpha_i, \beta_j)\ast(W(\alpha_i, \beta_j)-\text{Thr}(\alpha_i))) & |W(\alpha_i, \beta_j)| \geq \text{Thr}(\alpha_i) \\
0 & |W(\alpha_i, \beta_j)| < \text{Thr}(\alpha_i)
\end{cases}
\]

\[
\begin{bmatrix}
6.67840 & -1.9015 & 0 & 0 & 0 & 0.0000 & 0.0000 & 0.0000 \\
23.0760 & 0.0000 & 0 & 0 & 0 & 0.0000 & 0.0000 & 0.0000 \\
21.7375 & 0.0000 & 0 & 0 & 0 & -2.0756 & 0.0000 & 0.0000 \\
22.3028 & 0.0000 & 0 & 0 & 0 & -0.5520 & 0.0000 & 0.0000 \\
7.83640 & 0.0000 & 0 & 0 & 0 & 0.0000 & -3.8610 & -2.1067 \\
0.0000 & 0.0000 & 0 & 0 & 0 & 0.0000 & 0.0000 & -3.4705 \\
0.0000 & 0.0000 & 0 & 0 & 0 & 0.0000 & 0.0000 & -4.6011 \\
0.0000 & 0.0000 & 0 & 0 & 0 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0 & 0 & 0 & 0.0000 & 0.0000 & 0.0000 \\
\end{bmatrix}
\]

\[
\hat{E}(\alpha_i) = \sum_{j=1}^{M} \left( \tilde{W}(\alpha_i, \beta_j) \right)^2
\]

\[
= \begin{bmatrix}
48.2165 & 532.5031 & 476.8285 & 497.7214 & 80.7539 & 12.0441 \\
21.1704 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\left[ \hat{E}(\alpha_{\text{max}}) \right] = \max_{a_i \rightarrow a_k} \left[ \hat{E}(\alpha_i) \right]. \text{When } i = 2, \text{we can get the maximum value of } \hat{E}(\alpha_i) = \sum_{j=1}^{M} \left( \tilde{W}(\alpha_i, \beta_j) \right)^2, \text{the value of } \hat{E}(\alpha_2) = \sum_{j=1}^{M} \left( \tilde{W}(\alpha_2, \beta_j) \right)^2 \text{is } 532.5031
\]

\[
\hat{R}_{\hat{W}}(m) = \sum_{j=1}^{M-m} \tilde{W}(\alpha_{\text{max}}, \beta_j) \tilde{W}(\alpha_{\text{max}}, \beta_{j+m})
\]

\[
= \sum_{j=1}^{M-m} \tilde{W}(\alpha_2, \beta_j) \tilde{W}(\alpha_2, \beta_{j+m}) = \begin{bmatrix}
532.5031 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
P(i) = \hat{R}_{\hat{W}}^2(i) = 1.0e+005 \ast \begin{bmatrix}
2.8356 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The result of Std based on \( P(i) \) is \( 1.0025e+005 \)
Figure 2.11 Std of CWT (a) and Std of \( \{P(i)\} \) Values (b) for Residual Signal

Figure 2.12 Kurtosis of CWT (a) and Kurtosis Based on \( \{P(i)\} \) Values (b) for Residual Signal
Figure 2.13 K of CWT (a) and K Based on \{P(i)\} Values (b) for Residual Signal

The behavior of the standard deviation, kurtosis and K-S test of the amplitude of the CWT maps over the gear shaft’s full lifetime are shown in Figures 2.11-2.13, respectively. The following results can be drawn from the plots:

(1) Both plots in Figure 2.11 based on the std present the same trend, but there are evident differences. First, the values of standard deviation based on \{P(i)\} are far larger than those of CWT, since \{P(i)\} calculations are carried out using the square of a sum of squares. Also, starting from Data File 194 to File 217 (Figure 2.11a), the values of std oscillate with a decreasing trend, followed by a dramatic increase for file 218 which is caused by an early gear shaft fault (small crack). After Data File 218, the values show a gradual increase with fluctuation. The values of std in Figure 2.11b remain constant with little fluctuation between Data Files 194 and 217, but a sudden increase occurred in Data File 218, indicating the first stage of the fault development. After that, the values tend to decrease, then increase again until the occurrence of the catastrophic fault when gear shaft is broken (data files 230 to 231). We can conclude that std based on \{P(i)\} values is a better indicator of fault presence than the std of CWT.

(2) The values of kurtosis based on both CWT and \{P(i)\} values over the full gearbox lifetime are plotted in Figure 12a and 12b, respectively. Kurtosis is used in engineering for the detection of fault symptoms because it is sensitive to impulses in signals. Obviously, the sharper the impulse in a signal, the greater the value of the kurtosis. However, from Figure 2.12, we can
observe that the values of kurtosis oscillate irregularly. The kurtosis value of the residual signal is not proportional to the advancement of the gear fault, particularly when the gear shaft is involved in a fault. Thus, kurtosis values based on residual signal are unable to diagnose early gear shaft fault.

(3) For a comparison, the K-S test is also considered for the gear shaft fault detection. Data File 194 was used as the reference signal. The results are shown in Figure 2.13. Although the values have an increasing trend, the K-S test applied to CWT based on residual signal cannot diagnose early gear fault. There is no obvious jump or sudden increase of K value.

2.7.2 DWT for Gear Shaft Crack Detection

2.7.2.1 DWT Based on TSA and Residual Signal

In discrete wavelet analysis, the details which give the identity of the signal are the low-scale, high-frequency components, and the approximations which indicate the overall behavior are the high-scale, low-frequency components. Since the process is iterative, it can be continued indefinitely in theory. In practice, we select a suitable number of levels based on the nature of the signal. In this chapter, we consider three levels of decomposition. The DWT coefficients for some typical files based on the TSA and residual signals are shown in Figures 2.14-2.19 considering the lowest level of DWT decomposition.

Figure 2.14 DWT Based on TSA Signal (a) and Residual Signal (b) for File 194
Figure 2.15 DWT Based on TSA Signal (a) and Residual Signal (b) for File 214

Figure 2.16 DWT Based on TSA Signal (a) and Residual Signal (b) for File 217
Figure 2.17 DWT Based on TSA Signal (a) and Residual Signal (b) for File 218

Figure 2.18 DWT Based on TSA Signal (a) and Residual Signal (b) for File 229
From Figures 2.14-2.19, we obtain the following results:

(1) The amplitude values of the coefficients of DWT for the healthy state are little smaller than those for the unhealthy state, but there is no evident difference for different data files.

(2) Like CWT, these maps of DWT cannot be used to identify the early gear shaft fault. They must be combined with fault feature extracting indicators such as standard deviation, kurtosis and the K-S test.

2.7.2.2 DWT Based on Residual Signal for Gear Shaft Crack Detection

In this section, we perform similar analysis as in section 2.7.1.2 using DWT. The results are summarized in Figures 2.20-2.22.
Figure 2.20 Std of DWT (a) and Std of \( \{P(i)\} \) Values (b) for Residual Signal

Figure 2.21 Kurtosis of DWT (a) and Kurtosis Based on \( \{P(i)\} \) Values (b) for Residual Signal
The following results can be obtained from Figures 2.20-2.22:

(1) The values of the std of DWT (Figure 2.20a) oscillate with a slightly decreasing trend before Data File 218, but a sharp jump occurs in File 218, revealing that the gear shaft crack may have occurred at that time. After Data File 218, the values increase gradually, then fluctuate with a slightly decreasing trend. However, the trend of Figure 2.20 is different from that of Figure 2.11, which shows the development of the gear shaft crack until the shaft is broken, since DWT requires a far smaller amount of work compared to CWT. Nevertheless, by using std for DWT based on residual signal, it can still be detected that an early fault occurred in the gear shaft starting with Data File 218. The values of std in Figure 2.20b remain almost constant with limited fluctuation between Data Files 194 and 217, then an abrupt change occurs in Data File 218, and after that, the process shows the same behaviour as the process in Figure 2.20a.

(2) Using the residual signal, the values and waveform of kurtosis show obvious differences for DWT when compared to CWT, but the same conclusion can still be obtained that kurtosis based on residual signal is unable to diagnose early gear fault.

(3) We can observe that the trend of the K-S test value K (Figure 2.22) is similar to that for the std (Figure 2.20), which is a clear indication of the fault presence. Therefore, we can conclude that the K-S test applied to DWT based on residual signal can indicate the occurrence of an early fault in the gear shaft.
2.8 Conclusions

In this chapter, the approach using the autocovariance of the maximal energy coefficients combined with wavelet transform has been proposed for gear shaft fault detection using gear shaft vibration signal data. Several indicators such as std, kurtosis and the value of the K-S test statistic have been calculated and analyzed in detail. The main results can be summarized as follows:

(1) For both CWT and DWT, the statistical measure kurtosis is unable to reveal the occurrence and advancement of gear shaft cracks.

(2) The standard deviation of the residual signal as an indicator over the full gear shaft lifetime is able to diagnose early gear fault and fault advancement. We have also found that the std based on \( \{P(i)\} \) values is a good indicator of the presence of faults. Considering the amount of work required for CWT, DWT also proves to be an efficient method.

(3) With the DWT based on residual signals, the K-S test statistic K is able to detect the gear shaft crack occurrence, its advancement, and the faulty state effectively. However, it has been shown that for the CWT based on the residual signal, the K value is incapable of revealing the occurrence of the gear shaft crack clearly.

(4) It can be concluded from the analysis that the gear shaft was in a healthy state during Data Files 194 to 217, there is an indication of a crack occurrence in Data File 218, and the gear shaft can be diagnosed as being in the faulty state after Data File 218. The diagnosis indicates that the impending fault using the method presented in this chapter can be identified earlier than the inspection performed at the actual shutdown time of gearbox due to shaft cracks estimating fault occurrence between Data Files 230 and 231.

In this chapter, we have employed the feature extraction approach based on the application of the autocovariance of maximal energy coefficients combined with wavelet analysis to gear shaft fault detection. It has been demonstrated using real vibration data that the faults in gear shafts can be early detected and identified successfully using this approach.
3.1 Introduction

Gearboxes are components that transmit rotating motion and play critical roles in the stable operation of rotating machinery. Gearboxes typically use fixed-axis gear sets or planetary gear sets.

Fault detection and diagnostics require the development of effective methods for analyzing vibration data. A literature research has shown that while there is a body of literature on vibration analysis techniques for gearboxes with fixed-axis gear sets, techniques for gearboxes with planetary gear sets have been explored to a lesser extent. Planetary gear sets are more complex mechanical systems than fixed-axis gear sets. Firstly, there are multiple tooth contacts, with each planet being simultaneously in mesh with both the sun and ring gear, and secondly, the axis of the planets move with respect to both the sun and ring gear. They are used in applications such as helicopters [34], automobiles [35], aircraft engines [36], heavy machinery, and marine vehicles. These gear sets have comparative advantages such as compactness, large torque-to-weight ratio, reduced noise and vibration, diminished loads on shafts bearings, improved reliability, more efficient power transfer, and reduced maintenance costs.

Signal averaging has proved to be the most useful vibration analysis tool for detecting faults in gears. The implementation of a time synchronous averaging (TSA) algorithm for fixed-axis gears is relatively straightforward. However, the technique for the extraction of the time averaged vibration signals associated with the individual planet gears and sun gear in a planetary transmission is quite complex. A planetary gearbox has a number of planet gears which all mesh with a sun and ring gear. An earlier method of performing selective signal averaging on planetary gearboxes was developed and tested at the Aeronautical and Maritime
Research Laboratory Defense Science and Technology Organization, Melbourne, Australia [38]. That method was useful in detecting faults on individual planet gears. However, it was tedious to implement, requiring an excessively long time to perform even a small number of averages as well as a selective chopping up of the time signal which proved to introduce discontinuities in the signal average. Another method attempted to extract a representative signal average for each planet by taking a narrow window or snap-shot of the vibration signal each time a planet came past the transducer. These small packets of vibration were then assembled into appropriate buckets in a stored signal average. Typically each packet of vibration would represent a single tooth mesh and only one of these could be collected for a particular planet for each revolution of the planet carrier. To create one complete ensemble representing a single revolution of a planet, required \( N \), revolutions of the planet carrier, where \( N_p \) is the number of teeth on a planet gear, and \( p \) is the planet gear. This technique was first introduced by McFadden [37]. After that, another TSA method of extracting representative signal averages for each planet by incorporating a selective (continuous) time filter into the signal averaging process was proposed by Forrester [38].

In this chapter, the TSA method of Forrester is introduced and wavelet transform approaches are combined with the autocovariance of maximal energy coefficients to a planetary gearbox vibration signal. The results reveal that the method can enhance the capability of feature extraction and fault diagnosis for the gearbox. According to the method, the TSA and residual signals are used as the source signals, and some wavelet transform approaches such as continuous wavelet transform (CWT) and discrete wavelet transform (DWT) are considered. Measures such as variance, kurtosis, root mean square (RMS) and crest factor are used as fault indicators.

The remainder of the chapter is organized as follows. In Section 3.2, planetary gear transmission systems are briefly introduced. The TSA technique applied to the residual signal is described shortly in Section 3.3. In Section 3.4, we describe briefly the experimental gear test rig, and summarize the data-processing techniques used in this study. The results are presented in Section 3.5, followed by the conclusions in Section 3.6.
3.2 Planetary Gear Transmission Systems [36]

Planetary gearboxes are typically used in applications requiring a large reduction in speed at high loads, such as the final reduction in the main rotor gearbox of a helicopter. A typical planetary reduction gearbox has three or more planet gears each meshing with a sun and a ring gear. The planetary gears are mounted onto the planetary carrier and contained within an internal toothed ring gear. Drive is provided via the sun gear, the ring gear is stationary and the axes of the planet gears are connected to a carrier which rotates in relation to both the sun gear and ring gear. The planet carrier provides the output of the planetary gear train. Figure 3.1 is an illustration of a single stage planetary gearset which consists of a sun gear, a ring gear, several planets and a carrier.

![Figure 3.1 One Stage of a Planetary Gear Diagram](image)

3.3 Time Synchronous Averaging and Residual Signal

3.3.1 Time Synchronous Averaging (TSA) of a Planetary Gearbox Vibration Data [38]

The implementation of a TSA algorithm for fixed-axis gears is relatively straightforward. However, the technique for the extraction of the time averaged vibration signals associated with the individual planet gears and sun gear in a planetary transmission is quite complex.

In this chapter, a mathematical derivation of the method is provided which shows that the averaging can be performed by proportionally dividing the vibration data amongst the individual gear meshes.
Figure 3.2 shows a diagram of a simple planetary gearbox with the number of teeth on the planet, sun and annulus gears $N_p$, $N_s$, and $N_a$, respectively, while the rotation frequencies of the sun gear, planet gear and the planet carrier are $f_s$, $f_p$ and $f_c$, respectively.

![Figure 3.2 Simple Planetary Gearbox](image)

From the theory of planetary gear transmission, the tooth meshing frequency $f_m$ is given by

$$f_m = N_s f_c = N_p (f_p + f_c) = N_s (f_s - f_c)$$  \hspace{1cm} (3.1)

The relative frequencies, $f_p + f_c$ of the planet to the carrier and $f_s - f_c$ of the sun to the carrier are:

$$f_p + f_c = f_m / N_p = f_c (N_a / N_p)$$  \hspace{1cm} (3.2)

$$f_s - f_c = f_m / N_s = f_c (N_a / N_s)$$  \hspace{1cm} (3.3)

The only place where it is normally feasible to locate a transducer to monitor the vibration of a planetary gear train is on the outside of the ring gear. This gives rise to planet pass modulation due to the relative motion of the planet gears to the transducer location.

As each planet approaches the location of the transducer, an increase in the amplitude of the vibration will be seen, reaching a peak when the planet is adjacent to the transducer then receding as the planet passes and moves away from the transducer. For a planetary gear train with $P$ planets, this will occur $P$ times per revolution of the planet carrier, resulting in an apparent amplitude modulation of the signal at frequency $Pf_c$.

The expected planet gear vibration signal recorded at a transducer mounted on the ring gear of a planetary gear train will be the sum of the individual planet gear vibrations multiplied by the planet pass modulations.
\[ x(t) = \sum_{p=1}^{P} \alpha_p(t)\nu_p(t) \quad (3.4) \]

where:

- \( \alpha_p(t) \) is the amplitude modulation due to planet number \( p \), and
- \( \nu_p(t) \) is the tooth meshing vibration for planet \( p \).
- \( p \) identifies the individual planet gear
- \( P \) is the number of planet gears
- \( t \) is time

The amplitude modulation function \( \alpha_p(t) \) has the same form for all planets, differing only by a time delay, and will repeat with the planet carrier rotation period \( 1/f_c \).

\[ \alpha_p(t) = a(t + \frac{P}{f_c}) = \sum_{n=0}^{\infty} A(n) \cos(2\pi nf_c t + \frac{2\pi np}{P}) \quad (3.5) \]

where \( \alpha(t) \) is the planet pass modulation function and \( A(n) \) is its Fourier Series. Eq.(3.4) can be rewritten in terms of the common planet pass modulation function giving

\[ x(t) = \sum_{p=1}^{P} a(t + \frac{P}{f_c})\nu_p(t) \quad (3.6) \]

For each planet signal average \( \bar{\nu}_p(t) \), the time window, \( b(t) \), is centred at the point at which the planet is adjacent to the transducer. Signal averaging of the filtered vibration signal is performed with a period equal to the relative planet rotation \( 1/(f_p+f_c) \) giving, where \( N \) is the number of averages,

\[ \bar{\nu}_p(t) = \frac{1}{N} \sum_{l=1}^{N-1} b(t + \frac{P}{f_c}) \left( x(t + \frac{l}{f_p+f_c}) - x(t + \frac{l}{f_p+f_c}) \right) \quad (3.7) \]

where \( b(t) \) is real valued and periodic with the planet carrier rotation \( 1/f_c \). It is a Fourier series with less than \( P \) terms.

\[ b(t) = \sum_{m=0}^{P-1} B(m) \cos(2\pi mf_c t) \quad (3.8) \]
One separation window which has been found to perform well is a cosine window raised to the power of \( P - 1 \).

\[
b(t) = (1 + \cos(2\pi f_c t))^{P-1}
\]  

(3.9)

which is a tapered function, with maximum value when the planet is adjacent to the transducer and a value of 0 when the planet is furthest from the transducer.

Assuming that all vibration which is not synchronous with the relative planet rotation will tend toward zero with the signal averaging process, the same filtered signal average Eq.(3.7) using the time window \( b(t) \) taken over \( N \) periods of the relative planet rotation, \( 1/(f_p + f_c) \), can be expressed as

\[
\bar{z}_p(t) = \frac{1}{N} \sum_{i=0}^{N-1} b(t + \frac{p}{f_c} + \frac{l}{f_p + f_c}) x(t + \frac{l}{f_p + f_c})
\]

\[
= \frac{1}{N} \sum_{i=0}^{N-1} b(t + \frac{p}{f_c} + \frac{1}{f_c N_r}) \sum_{k=1}^{P} a(t + \frac{k}{f_c N_r}) \psi_k(t + \frac{l}{f_p + f_c})
\]

\[
= \sum_{k=1}^{P} \bar{\psi}_k(t) \frac{1}{N} \sum_{i=0}^{N-1} b(t + \frac{p}{f_c} + \frac{1}{f_c N_r}) a(t + \frac{k}{f_c N_r})
\]

(3.10)

where \( \bar{\psi}_k(t) \) is the mean vibration value for planet \( k \), which repeats with period \( 1/(f_p + f_c) \).

The signal averaging is performed over the relative planet rotation period \( 1/(f_p + f_c) \) and the number of averages is an integer multiple of the number of teeth on the ring gear \( N_r \). The time filtered signal average for planet \( p \), Eq. (3.10) reduces to

\[
\bar{z}_p(t) = \sum_{k=1}^{P} \bar{\psi}_k(t) \psi(p, k, t) = \sum_{k=1}^{P} \bar{\psi}_k(t) c(p - k)
\]

(3.11)

where

\[
\psi(p, k, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B(m) A(n) \frac{1}{2N} \sum_{i=0}^{N-1} \left[ \cos(2\pi(m+n)f_c t + \frac{2\pi(mp+nk)}{P} + \frac{2\pi l(m+n)N_p}{N_r}) + \cos(2\pi(m-n)f_c t + \frac{2\pi(mp+nk)}{P} + \frac{2\pi l(m-n)N_p}{N_r}) \right]
\]

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\[
= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B(m) A(n) \frac{1}{2N} \sum_{l=0}^{N-1} \left[ \cos(2\pi(m + n) f_c t + \frac{2\pi(mp + nk)}{P}) \cos\left(\frac{2\pi l(m + n) N_r}{N_p}\right) - \right.
\]
\[\sin(2\pi(m + n) f_c t + \frac{2\pi(mp + nk)}{P}) \sin\left(\frac{2\pi l(m + n) N_r}{N_p}\right) + \]
\[\cos(2\pi(m - n) f_c t + \frac{2\pi(mp - nk)}{P}) \cos\left(\frac{2\pi l(m - n) N_r}{N_p}\right) - \]
\[\sin(2\pi(m - n) f_c t + \frac{2\pi(mp - nk)}{P}) \sin\left(\frac{2\pi l(m - n) N_r}{N_p}\right) \right]. \tag{3.12}
\]

If the number of averages \( N \) is an integer multiple of the number of teeth on the ring gear, \( N_r \), then
\[
\frac{1}{iN_r} \sum_{l=0}^{N-1} \cos\left(\frac{2\pi l(m \pm n) N_p}{N_r}\right) = \delta(m \pm n)
\]
and
\[
\frac{1}{iN_r} \sum_{l=0}^{N-1} \sin\left(\frac{2\pi l(m \pm n) N_p}{N_r}\right) = 0 \tag{3.13}
\]

We have
\[
\psi(p,k,t) = \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B(m) A(n) \left[ \cos(2\pi(m + n) f_c t + \frac{2\pi(mp + nk)}{P}) \delta(m + n) + \right.
\]
\[\cos(2\pi(m - n) f_c t + \frac{2\pi(mp - nk)}{P}) \delta(m - n) \right]
\[= \frac{1}{2} \left[ \sum_{m=0}^{\infty} B(m) A(-m) \cos\left(\frac{2\pi m(p - k)}{P}\right) + B(m) A(m) \cos\left(\frac{2\pi m(p - k)}{P}\right) \right] \tag{3.14}
\]
\[= \frac{1}{2} \left[ B(0) A(0) + \sum_{m=0}^{\infty} B(m) A(m) \cos\left(\frac{2\pi m(p - k)}{P}\right) \right]
\]
\[c(p-k) \text{ is defined by both the applied time window } b(t) \text{ with less than } P \text{ terms and the planet pass modulation function } \alpha(t), \]
\[c(p-k) = \frac{1}{2} \left[ B(0) A(0) + \sum_{m=0}^{P-1} B(m) A(m) \cos\left(\frac{2\pi m(p - k)}{P}\right) \right] \tag{3.15}
\]
The method of separating the individual planet vibration signatures in a planetary gearbox has
now been developed, and has significant advantages over previous methods. One advantage is a significant reduction in the time required to perform sufficient separation of the planet signatures for diagnostic purposes, which makes implementation of the new method feasible in operational aircraft.

3.3.2 Residual Signal Based on TSA

Residual signal is obtained by eliminating from the FFT spectrum of the TSA signal the fundamental and harmonics of the tooth-meshing frequency, subsequently applying inverse Fourier transform and then reconstructing the remaining signal in the time-domain. The residual signal can thus be expressed as:

\[ x(t) = \mathcal{T}_p(t) - g(t) \]  

(3.16)

where \( x(t) \) is the residual signal and \( g(t) \) is the signal composed of the eliminated components.

3.4 Experimental Set-up

3.4.1 Experimental Set-up for our test rig

The vibration data were obtained from our test bed (see the transmission diagram of the test bed in Figure 3.3). The teeth number and the reduction ratio of each stage can be seen in Table 3.1.

![Figure 3.3 The Transmission Diagram of Our Test Bed](image)
Table 3.1 Teeth and Speed Information for our test Gearbox

<table>
<thead>
<tr>
<th>Stage</th>
<th>Sun/Input Speed (rpm)</th>
<th>Reduction ratio</th>
<th>Carrier Speed (rpm)</th>
<th>Number of teeth of Sun/Shaf</th>
<th>Number of teeth of Planet/Bevel</th>
<th>Number of teeth of Ring</th>
<th>Gear Meshing Freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Stage</td>
<td>1800</td>
<td>10</td>
<td>180</td>
<td>12</td>
<td>48</td>
<td>108</td>
<td>324</td>
</tr>
<tr>
<td>2nd Stage</td>
<td>180.0</td>
<td>7</td>
<td>25.7143</td>
<td>18</td>
<td>45</td>
<td>108</td>
<td>46.2857</td>
</tr>
<tr>
<td>3rd Stage</td>
<td>25.7143</td>
<td>5.5</td>
<td>4.6753</td>
<td>24</td>
<td>42</td>
<td>108</td>
<td>8.4156</td>
</tr>
</tbody>
</table>

The data file was collected in a 10mins window which covers 600000 sampling points in total. The sampling frequency is 1000 Hz. The signals of the test bed accelerometers are all converted to digital data format. Because no failure data were obtained from our test bed, the vibration data obtained from Syncrude which represented the complete the time from “new” to “failure” will be investigated in the next section.

3.4.2 Experimental Set-up for Syncrude Gearbox Rig

The experiment uses vibration data obtained from a mining company [39]. The transmission diagram of the gearbox is described in Figure 3.4. It contains three stages. All the gears in the gearbox are straight-toothed gear. The tooth number, speed and the reduction ratio of each stage are listed in Table 3.2, and the detailed calculation can be seen in Appendix A.

Vibration data were collected during 300s for each file. For the files from rtf_1st_run to rtf_3rd_run, the sampling frequency is 10000Hz, which covers 3000000 sampling points. For the files from rtf_4th_run to rtf_13th_run, the sampling frequency is 5000Hz, which covers 1500000 sampling points.

Figure 3.4 Transmission Diagram of the Gearbox from Syncrude
### Table 3.2 Tooth and Speed Information for Syncrude Gearbox

<table>
<thead>
<tr>
<th>Stage</th>
<th>Sun/Input Speed (rpm)</th>
<th>Reduction ratio</th>
<th>Carrier Speed (rpm)</th>
<th>Number of teeth of Sun/Shaft</th>
<th>Number of teeth of Planet/Bevel</th>
<th>Number of teeth of Ring</th>
<th>Gear Meshing Freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s</td>
<td>1200</td>
<td>4.0</td>
<td></td>
<td>18</td>
<td>72</td>
<td></td>
<td>360</td>
</tr>
<tr>
<td>2s</td>
<td>300.0</td>
<td>6.429</td>
<td>46.667</td>
<td>28</td>
<td>62</td>
<td>152</td>
<td>118.2222</td>
</tr>
<tr>
<td>3s</td>
<td>46.667</td>
<td>5.263</td>
<td>8.867</td>
<td>19</td>
<td>31</td>
<td>81</td>
<td>11.9700</td>
</tr>
</tbody>
</table>

#### 3.5 Results and Discussion

The vibration data include 13 files from rtf_1st_run to rtf_13th_run, each file having five groups: Input-h, Planetary1-v, Planetary2-v, HS Planetary1-v, and HS Planetary2-v (See Figure 3.5). Based on some research, we have found that the data for Planetary2-v better reflect their meshing frequencies. Thus, Planetary2-v for each file is chosen to be investigated.

![Run-to Failure Sensor Orientation](image)

Figure 3.5 Run-to Failure Sensor Orientation

#### 3.5.1 Signals and Their FFT

First, the original vibration signals measured from the gearbox were synchronously averaged per one rotation of the gear of interest, and then residual signals were obtained based on the TSA. After that, Daubechies wavelet transform with order 4 based on TSA and residual vibration data were investigated. Frequencies with original signals, TSA signals, residual
signals, CWT signals, DWT signals and autocovariance for these signals are analyzed and compared. All files with data which run from the healthy state to the failure state between File 1 and File 13 are investigated. Here, only the results of the three typical states: healthy state, warning state and failure state are shown in the following plots.
DWT Signal based on TSA Signal and its FFT

CWT Signal based on TSA Signal and its FFT for Autocovariance

DWT Signal based on TSA Signal and its FFT for Autocovariance
Residual Signal and its FFT

CWT Signal based on Residual Signal and its FFT

DWT Signal based on Residual Signal and its FFT
The following results can be drawn from the plots:

(1) No special symptoms can be seen in the waveform of the signals from the healthy state. Thus, these maps cannot be used to indicate and prognosticate the gearbox.

(2) There are evident meshing frequencies of stage 1 in the FFT plot with the original signal and stage 2 in the FFT plot with the CWT signal. We cannot clearly distinguish between meshing frequencies and noise frequencies when FFT was applied to the TSA signal and DWT signal.

(3) There are no strong frequencies in the FFT plot with TSA, CWT, and DWT signals. In the corresponding FFT plots of DWT, plots of FFT for DWT with TSA signal behave just like the plot of FFT for TSA.

(4) Meshing frequencies of CWT were relatively easier to distinguish than those of DWT.
(5) The vibration energy generated by gear meshing action has been removed by the residual signal. However, there are evident meshing frequencies and their harmonic from FFT based CWT can still be observed clearly.
DWT Signal based on TSA Signal and its FFT

CWT Signal based on TSA Signal and its FFT for Autocovariance

DWT Signal based on TSA Signal and its FFT for Autocovariance
Residual Signal and its FFT

CWT Signal based on Residual Signal and its FFT

DWT Signal based on Residual Signal and its FFT
The following results can be drawn from the plots:

(1) No special symptoms can be seen in the waveform of the signals from the warning state. Compared to the healthy state, the waveform of the signals for the two states shows no obvious difference. Hence, these maps cannot be used to indicate and prognosticate the gearbox.

(2) There are two evident frequencies in the FFT plot with original signal and residual signal, which are the meshing frequencies of stage 1 and its harmonic. Also, meshing frequencies of stage 1 and stage 2 were observed in the FFT plot with CWT signal, and were relatively easier to distinguish than those of the TSA signal and DWT signal.

(3) Although we can find the meshing frequencies of stages 1, 2, 3 and their harmonics in the FFT plots of CWT and DWT applied to TSA and the residual signal with autocovariance, it is
not easy to distinguish between the meshing frequencies and the noise frequencies.
DWT Signal based on TSA Signal and its FFT

CWT based on TSA and its FFT for autocovariance

DWT based on TSA and its FFT for autocovariance
Residual Signal and its FFT

CWT Signal based on Residual Signal and its FFT

DWT Signal based on Residual Signal and its FFT
The following results can be drawn from the plots:

(1) No special symptoms can be seen in the waveform of the signals obtained in the failure state. Compared to the healthy and unhealthy states, the waveforms of the signals for the three states have no obvious differences. Therefore, these maps cannot be used to indicate and prognosticate the gearbox.

(2) There are some evident frequencies in the FFT plot with the original signal, which are the meshing frequencies of stage 1, its harmonic and noise frequencies. Also, the meshing frequencies of stage 1 and stage 2 were observed in the FFT plot with CWT signal, and were relatively easier to distinguish than those of the TSA signal and DWT signal.

(3) It is difficult to distinguish between meshing frequencies and noise frequencies in the FFT
plots of CWT and DWT applied to TSA and the residual signal with autocovariance, and there was a large variation in the whole waveform of the FFT due to noise.

The main results can be obtained from analysis of the signals and their FFT below:

(1) In the healthy state, there are evident meshing frequencies of stages 1, 2, 3 and some harmonics in the FFT plots of CWT and DWT applied to TSA and residual signal with autocovariance.

(2) Whether the state is healthy or not, we cannot clearly distinguish between the meshing frequencies and noise frequencies when FFT was applied to the original, TSA, CWT and DWT based on TSA and residual signals.

(3) It is not easy to distinguish between the meshing frequencies and noise frequencies in the FFT plots of CWT and DWT applied to TSA and the residual signal with autocovariance from Rtf_9th_run. Thus, we can conclude that the deterioration stage occurred from the Rtf_9th_run on.

(4) It is difficult to distinguish between the meshing frequencies and noise frequencies in the FFT plots of CWT and DWT applied to TSA and the residual signal with autocovariance from Rtf_13th_run, and there was a large variation in the whole waveform of FFT due to noise. Hence, we can conclude that the failure stage occurred from the Rtf_13th_run on.

Although FFT plots were intended to be used for identifying healthy, deterioration and failure states, the meshing frequencies and their harmonics were not very easy to distinguish. Thus, the behaviors of selected indicators with different signals are further discussed below.

3.5.2 Indicator Analysis of Signals

In order to detect the planetary gear fault, some fault feature indicators such as variance, kurtosis, RMS, and crest factor for signals are computed and investigated in this section. A comparison for different signals with some indicators can be seen in the following figures, and tables corresponding to these figures can be seen in Appendix B.
From Figure 3.9, there are no observable patterns in the values of variance, kurtosis, RMS and crest factor for the original signal, hence the selected indicators are not suitable for purposes of feature extraction for the original signal.
The variance value undergoes a gradual increase from 1 to 3 then a decrease for 4, after that it follows the same patterns for later files. The RMS value shows a similar trend as the variance value. There are no observable patterns in the kurtosis and crest factor values. We can conclude that the selected indicators are not suitable for purposes of feature extraction for TSA signal.
The variance value undergoes an increase from 1 to 2 then a decrease from 3 to 5, after then it follows a similar pattern for later files. The RMS value shows a similar trend to the variance value. There are no observable patterns in the kurtosis and crest factor values. Hence, a similar result can be obtained that the selected indicators are not suitable for purposes of feature extraction for residual signal.
Both plots for variance and RMS present a similar trend, and the values have a gradual increase from 1 to 3 then a decrease for 4, after then they have a steady trend, and this is a counter-intuitive truth. There are also no observable patterns in the kurtosis and crest factor values. We can conclude that the selected indicators are not suitable for purposes of feature extraction for CWT applied to TSA signal.
These plots for variance, RMS and crest factor present a similar trend, and the values have an increase from 1 to 3 then a decrease for 4, after then they have a steady trend. We cannot see an observable pattern in the kurtosis values. Hence, the selected indicators are not suitable for purposes of feature extraction for CWT applied to TSA signal with autocovariance.
The results for the CWT applied to residual signal are similar to those for the CWT applied to TSA signal, and the conclusion can be made that the selected indicators are not suitable for purposes of feature extraction for CWT applied to residual signal.
The result for CWT applied to residual signal with autocovariance is similar to that for CWT applied to TSA signal with autocovariance. Hence, the selected indicators are not suitable for purposes of feature extraction for CWT applied to residual signal with autocovariance.
The plots in variance and RMS are comparable. The values of variance and RMS for all files except Files 3 and 9 show a steady increasing trend. Although we cannot identify which file begins to deteriorate, we can see that File 13 presents a failure state with a large value for variance and RMS. Also, we can get that kurtosis and crest factor are not suitable for purposes of feature extraction for DWT applied to TSA signal.

Figure 3.16 Indicator Analysis of DWT Applied to TSA Signal
Figure 3.17 Indicator Analysis of Autocovariance for DWT Applied to TSA Signal

From Figure 3.17, we can see that the values of variance show a steady trend with a sudden increase in Data File 9, indicating the occurrence of deterioration, after which there is a dramatic increase for Data File 13 which is caused by failure. A similar result is applied to RMS, although there are little differences between variance and RMS. From the plot of crest factor, we can find that File 13 presents a failure state with a large value. Finally, we can conclude that variance and RMS are suitable indicators for DWT applied to TSA signal with autocovariance.
Both plots in variance and RMS present the same trend, and the values show a gradual increase with fluctuation, then there is a dramatic increase for File 13 which is caused by the failure state. A similar result is applied to crest factor, although there are some differences. We cannot see observable patterns in the kurtosis values. Thus, the selected indicators are not suitable for purposes of feature extraction for DWT applied to residual signal.
The values of variance remain constant with little fluctuation between Data Files 1 to 12 with a sudden increase in Data File 13, indicating the occurrence of failure. A similar result is applied to crest factor, although there are little differences between the variance and the crest factor. Also, we can seen that the values of RMS show a steady trend with a sudden increase in Data File 9, indicating the occurrence of deterioration, after which there is a dramatic increase for Data File 13 which is caused by failure. Finally, we can conclude that variance and RMS are better indicators for DWT applied to residual signal with autocovariance.

The main results can be obtained from indicator analysis of signals below:

(1) Variance, kurtosis, RMS and crest factor are not suitable for purposes of feature extraction for original, TSA, residual, CWT applied to TSA and residual, CWT applied to TSA and residual signals with autocovariance.

(2) Variance and RMS can be used to detect the failure state for DWT applied to TSA and residual signals.

(3) Variance and RMS are suitable for purposes of feature extraction for DWT applied to TSA
and residual signals with autocovariance.

(4) Finally, we can conclude that deterioration stage occurred from the Rtf_9th_run data on and that the failure stage occurred from the Rtf_13th_run data on.

Below are some pictures for the healthy, deterioration and failure state from Syncrude, which is a proof for our analysis. In order to compare them, we have chosen the same planet gear (planet 2) for different times.

![Figure 3.20 Pictures of Planet 2 Taken by Digital Camera](image-url)
In the healthy state (Figure 3.20(a)), possible plastic deformation and heavy polishing occurred on the outboard tips of all the sun gear meshing faces. Slight wear occurred on the outboard tips of all the ring gear meshing faces. For the warning state (Figure 3.20(b)), flattening of the pitch line on the sun gear mesh face is quite evident. The flat spot is widest at the output end, and includes a 'divot' at the input end. Once the gear enters the failure state (Figure 3.20(c)), the gear flattening along the pitch line on the sun gear mesh face is evident. A 'divot' was observed at the input end. More pits were observed and distributed along the pitch line of the sun gear mesh face.

3.6 Conclusion

In this chapter, using the TSA of a planetary gearbox and residual signals, FFT has been performed and wavelet analysis with the autocovariance of maximal energy coefficients applied. Behavior of the variance, kurtosis, RMS, and crest factor value has been calculated and analyzed. The major conclusions can be summarized as follows:

(1) Whether it is in a healthy state or not, we can see the meshing frequency of stage 1, 2 and 3, but we cannot clearly distinguish between the meshing frequencies and the noise frequencies. There was no apparent difference in the whole waveform of FFT when it was applied to the original, TSA, CWT and DWT based on TSA signals.

(3) For the healthy and unhealthy states, the meshing frequencies and noise frequencies could not be distinguished in the FFT plot of DWT applied to TSA signal with autocovariance, but obvious differences in FFT plots can be seen. The noise strength of the failure state exceeds that of the healthy state.

(4) Plots of FFT for the residual signal behave just like the plot of FFT for DWT applied to the signal, but not for CWT applied to the residual signal. Although the meshing frequencies of stage 2 and stage 3 were observed, FFT applied to these signals exhibited a wide spectrum of other frequencies as well. We also cannot clearly distinguish between the meshing frequencies and noise frequencies.
(5) FFT plots of CWT and DWT applied to residual signal with autocovariance were submerged in noise, which makes it difficult to distinguish between the meshing frequencies and noise frequencies. However, in the FFT analysis for which the noise strength of the failure state exceeds that of the healthy state, noticeable differences can be seen.

(6) Variance, kurtosis, RMS, and crest factor are not suitable for the purposes of feature extraction for the original, TSA, residual, CWT applied to TSA and residual, and CWT applied to TSA and residual signals with autocovariance.

(7) Variance and RMS can be used to detect failure state for DWT applied to TSA and residual signals.

(8) Variance and RMS are suitable for purposes of feature extraction for DWT applied to TSA and residual signals with autocovariance.

(9) Finally, we can conclude that deterioration stage occurred from Rtf_9th_run on and that the failure stage occurred from Rtf_13th_run on.

Vibration signals collected from a planetary gearbox contain unwanted noises, which make it difficult to find the early symptoms of potential failures. In this chapter, we have employed the feature extraction approach based on the application of the autocovariance of maximal energy coefficients combined with wavelet analysis to planet gear fault detection. It has been demonstrated that DWT applied to TSA signals with autocovariance is the most effective for fault detection and CBM purposes, which are further discussed in the next chapter.
Chapter 4
Multivariate Bayesian Control Chart for CBM

4.1 Introduction

In recent years, condition-based maintenance (CBM) modeling has gained significant attention and recognition in many regions for its abilities to improve the performance of the system, gain economic benefits, and decrease the system’s downtime. The CBM approach evaluates the need for maintenance by considering the estimate of the actual condition of the equipment or system, and in order to determine this estimate, samples are collected from the system at regular epochs. In a real situation, the system condition can only be monitored by sampling and inspection. For example, sensors such as accelerometers may be installed to gather vibration data so as to evaluate indirectly the condition of the gearbox. Clearly, the real state of the gearbox can only be assessed and determined after an examination by technicians with the machine shut down. The observation process is stochastically related to the unobservable deterioration process, which then defines the system’s condition. As failures may occur and are also observable in most real systems, an evident condition of failure would be included in CBM optimization models. In many situations, the inspection is costly, and optimization of the performing inspection schedule, as well as preventive maintenance, can be allowed by the ability to model the deterioration of the gear transmission system with a hidden Markov model. Some CBM optimization models which consider a failure condition can be found in the literature [15, 40-42]. Makis [16-17, 43] has proved that the optimal control policy for both short and long production runs is a control limit policy which can be represented by a Bayesian control chart. The statistic plotted is the posterior probability that the system is in the warning state. The lower control limit of the Bayesian chart is zero. An alarm occurs when the value of the statistic exceeds the upper control limit. Such a maintenance policy is optimal and easy to implement in real industrial systems with condition monitoring already in place.

Hidden Markov models have been studied considerably in the literature and have been successfully applied in several areas. These models have been used with much success in the area of speech recognition [44] and later also in the area of system diagnostics and fault detection [45-51]. Partially observable hidden Markov models have been employed also in various maintenance applications. In Jiang & Makis [15], a hidden Markov model was
proposed to describe a partially observable deteriorating system. The hidden Markov model’s condition develops as a continuous-time homogeneous Markov process with a finite state space. The system states are unobservable except the breakdown state. The condition monitoring data can nevertheless be used in estimating these unobservable states. A general recursive filter for the partially observable process model with continuous range observations was given in Lin & Makis [52-53] as a filtering and parameter estimation problem for a partially observable hidden Markov process with discrete range observations. In addition, two on-line parameter estimation algorithms, a recursive prediction error algorithm, along with a recursive maximum likelihood algorithm were presented in the paper [54]. Furthermore, explicit formulas for the partially observable hidden Markov model parameter estimation were developed by Kim & Makis [55] maximizing a pseudo likelihood function by using the expectation-maximization (EM) iterative algorithm. The EM algorithm was established in the literature first by Dempster et al.[56]. After that, researchers including McLachlan and Krishnan [57] and Laporice [58] have studied the numerical advantages as well as the algorithm’s optimal convergence properties.

In this chapter, the planetary gear deterioration process from the new condition to failure is modeled as a continuous time homogeneous Markov process with three states: good, warning, and breakdown. The observation process is represented by two characteristics: variance and RMS based on the analysis of the autocovariance of discrete wavelet transform (DWT) applied to the time synchronous averaging (TSA) signal obtained from planetary gear vibration data. A partially observable hidden Markov model is used to model the process, and the hidden Markov model parameters are estimated by maximizing the pseudo likelihood function using the EM (expectation-maximization) iterative algorithm. Then, a multivariate Bayesian control chart is applied for fault detection.

The rest of the chapter is organized as follows. In Section 4.2, a quick summary of the model formulation of the gear deterioration process is provided. Section 4.3 describes the parameter estimation for a partially observable hidden Markov model. EM iterative algorithm is introduced and then applied to estimate the numerical parameters. The multivariate Bayesian control chart of the gear deteriorating process is studied in Section 4.4., followed by the conclusions in Section 4.5.
4.2 Model Formulation of Gear Deterioration Process

Consider a gear deterioration process system which can be in one of three states \( \{1, 2, 3\} \), where states 1 and 2 are unobservable operational conditions (state 1 is in no worse condition than state 2) and state 3 represents the observable failure state. Here, state 1 can also be referred to as the normal state and state 2 as the warning state. The state process is a continuous-time homogeneous Markov chain with state space \( \{1, 2, 3\} \). In order to avoid costly failures, prevention replacement can be performed. The model assumptions made for the deterioration process of a gear transmission system are as follows:

1) The condition of the deterioration process cannot be enhanced with the exceptions of replacement or repair.

2) The sojourn time, or time spent in each state, is denoted by the random variable \( \tau_i \), which follows an exponential distribution with mean \( 1/\nu_i \), where \( \nu_1 = \lambda_{12} + \lambda_{13} \) and \( \nu_2 = \lambda_{23} \).

3) The failure replacement cost \( s_3 \) is higher than the preventive maintenance cost \( s_2 \) in the warning state.

4.2.1 Stochastic Relationship between Observation Process and Operational States

The system is assumed to be renewed to the normal state after replacement. The system is monitored at times \( h, 2h, \ldots, mh, \ldots \) to facilitate the preventive replacement decision, and \( \{Y_{1h}, Y_{2h}, \ldots, Y_{mh}, \ldots\} \) represents the vector observation process, where

\[ Y_{mh} = (y_{1}^m, y_{2}^m, \ldots, y_{j}^m, \ldots y_{n}^m) \]

represents the observation matrix with a sample of size \( n \) collected at time \( mh \) for \( m = 1, 2, \ldots \), and \( y_{j}^m = (y_{j1}^m, y_{j2}^m, \ldots, y_{jq}^m)^T \) denotes the \( q \)-dimensional column observation vector for the \( j^{th} \) sample unit for \( j = 1, 2, \ldots, n \).

If the gear deterioration process is in the normal state \( s_1 \), \( y_{j}^m \) follows a multivariate normal distribution with mean vector \( \mu_1 \) and covariance matrix \( \Sigma_1 \)

\[
f_{Y_{j}^m \mid X_{mh} = s_1} = \frac{1}{(2\pi)^{q/2} |\Sigma_1|^{1/2}} \exp \left[ -\frac{1}{2} (y_{j}^m - \mu_1)^T \Sigma_1^{-1} (y_{j}^m - \mu_1) \right]
\]

where
\[ \mathbf{\mu}_j = \mathbb{E}(y_j^m) = \left[ \mathbb{E}(y_{1j}^m) \quad \mathbb{E}(y_{2j}^m) \quad \cdots \quad \mathbb{E}(y_{nj}^m) \right]^T \]

\[ \Sigma_j = \text{cov}(y_j^m) = \mathbb{E} \left[ (y_j^m - \mathbf{\mu}_j)(y_j^m - \mathbf{\mu}_j)^T \right] \] is a symmetric matrix, and \( T \) denotes the transpose of the matrix.

If the gear deterioration process proceeds into the warning state \( s_2 \), \( y_j^m \) will then follow a multivariate normal distribution with mean vector \( \mathbf{\mu}_2 \) and covariance matrix \( \Sigma_2 \)

\[ f_{y_j^m|x_{mh}}(y_j^m | X_{mh} = s_2) = \frac{1}{(2\pi)^{\frac{m_j}{2}}} \Sigma_2^{-\frac{1}{2}} \exp \left( -\frac{1}{2} (y_j^m - \mathbf{\mu}_2)^T \Sigma_2^{-1} (y_j^m - \mathbf{\mu}_2) \right) \]

The probability density functions of the observations depend on the state of the deterioration process at time \( mh \).

However, as the operational states are unobservable, a partially observable hidden Markov model needs to be constructed in order to make inferences with regard to the probability of the operational states at time \( mh \) from the observations.

### 4.2.2 Continuous-Time Markov Model for Gear Deterioration Process

Based on the model assumptions, the gear deterioration process \( \{X_t : t \in \mathbb{R}_+\} \) develops as a continuous-time homogeneous Markov process with state space \( S_X = \{s_i\}, 1 \leq i \leq 3 \). The transition probability matrix is denoted as

\[ R(t) = \begin{pmatrix} r_{11}(t) & r_{12}(t) & 1 - r_{11}(t) - r_{12}(t) \\ r_{22}(t) & 1 - r_{22}(t) \\ 0 & 0 & 1 \end{pmatrix} \] (4.1)

where \( r_{ij}(t) = \Pr \left\{ X_{(m-1)h+t} = s_j \left| X_{(m-1)h} = s_i \right. \right\} \) for \( m = 1, 2, \ldots \),

The length of time for which the system remains in an operational state before entering a different state is exponentially distributed. Let \( \lambda_{12} \) represent the transition rate from state 1 to state 2, \( \lambda_{23} \) the transition rate from state 2 to state 3, and \( \lambda_{13} \) the transition rate from state 1 to state 3.

Using \( \lambda = (\lambda_{12}, \lambda_{13}, \lambda_{23}) \) to express transition rate parameters, the instantaneous transition rate
matrix of the continuous-time Markov chain can be denoted as

\[
Q = \{q_{ij}\} = \begin{pmatrix}
-\lambda_{12} + \lambda_{13} & \lambda_{12} & \lambda_{13} \\
0 & -\lambda_{23} & \lambda_{23} \\
0 & 0 & 0
\end{pmatrix}, \ s_i, s_j \in S_X
\] (4.2)

where

\[
q_{ij}(t) = \lim_{t \to 0} \frac{r_{ij}(t)}{t} = \lim_{t \to 0} \frac{\Pr\{X_{(m-1)k+t} = s_j | X_{(m-1)k} = s_i, s_j \neq s_j \in S_X\}}{t}
\]

\[
q_{ij} = -\sum_{j \neq i} q_{ij}
\]

The transition probability matrix of the embedded process is given as:

\[
P = \{p_{ij}\} = \begin{pmatrix}
0 & \lambda_{12} & \lambda_{13} \\
\lambda_{12} & \lambda_{12} + \lambda_{13} & \lambda_{12} + \lambda_{13} \\
0 & 0 & 1
\end{pmatrix}
\] (4.3)

where \( p_{ij}(t) \) is the probability that the process moves to state \( j \) after leaving state \( i \).

The embedded transition matrix shows that all \( p_{ij} = 0 \) when \( i > j \). This satisfies the assumption that the gear deterioration process cannot transfer into a better state from a worse state without applying maintenance.

By solving the Kolmogorov’s backward differential equations, the gear states transition probability matrix for the continuous-time Markov chain can be expressed, with respect to the transition parameters \( \lambda_{12}, \lambda_{13} \) and \( \lambda_{23} \), as:

\[
R(t) = \{r_{ij}(t)\} = \begin{pmatrix}
e^{-\lambda_{12}t} - e^{-\lambda_{12}t} & \frac{\lambda_{12}e^{-\lambda_{12}t} - e^{-\lambda_{12}t} - e^{-\lambda_{12}t}}{\lambda_{12} + \lambda_{13} - \lambda_{23}} & 1 - e^{-\lambda_{12}t} - e^{-\lambda_{12}t} \\
e^{-\lambda_{12}t} - e^{-\lambda_{12}t} & \frac{\lambda_{12}e^{-\lambda_{12}t} - e^{-\lambda_{12}t} - e^{-\lambda_{12}t}}{\lambda_{12} + \lambda_{13} - \lambda_{23}} & 1 - e^{-\lambda_{12}t} - e^{-\lambda_{12}t} \\
0 & 1 & 0
\end{pmatrix}
\] (4.4)

4.2.3 Partially Observable Hidden Markov Model

Three components are used to describe a partially observable hidden Markov model for the gear deterioration process. This can be defined as
\[ \Omega = \{ R(\lambda), F(\psi), \pi(1, 0, 0) \} \]  

(4.5)

where:

\[ \lambda = (\lambda_{12}, \lambda_{13}, \lambda_{23}) \]

\[ \psi = (\mu_1, \Sigma_1, \mu_2, \Sigma_2) \]

\[ \pi = \{ \pi_i \} \] is the initial distribution. Since the deterioration process starts in a good state at time \( t_0 = 0 \), the probabilities of \( s_i \) being the initial state are

\[ \pi_1 = \Pr \{ X(t_0) = s_1 \} = 1 \]
\[ \pi_2 = \Pr \{ X(t_0) = s_2 \} = 0 \]
\[ \pi_3 = \Pr \{ X(t_0) = s_3 \} = 0 \]

Thus, the partially observable hidden Markov model can also be denoted as a function with respect to these two sets of parameters

\[ \Omega = f_{\Omega}(\lambda, \psi) \]

Using the maximum likelihood method and the EM algorithm (a general computational method providing an iterative procedure for computing MLEs), the observation and state model parameters estimates can then be established.

### 4.3 Parameter Estimation for Partially Observable Hidden Markov Model

The observation process is represented by two characteristics: variance and root mean square (RMS) based on the analysis of the autocovariance of DWT applied to the TSA signal obtained from planetary gear vibration data. The state and observation model parameters are estimated using the EM algorithm.

#### 4.3.1 The EM Algorithm

Let \( y \) represent all the historical observable data with likelihood function \( L(\lambda, \psi \mid y) \). However, computing and maximizing \( L(\lambda, \psi \mid y) \) directly is quite difficult. Hence, let \( \overline{y} \) represent the observable data \( y \) with the sample path information of the state process. Then, the maximum likelihood estimation becomes straightforward with an explicit likelihood of \( L(\lambda, \psi \mid \overline{y}) \).
The EM algorithm is introduced by the concept of the pseudo likelihood function. Let \( \hat{\lambda}, \hat{\psi} \) be some initial values of the unknown parameters. The computation of the pseudo likelihood function is required in the ‘E-step’ of the EM algorithm.

**E-step** Compute the pseudo likelihood function \( Q(\lambda, \psi | \hat{\lambda}, \hat{\psi}) \)

\[
Q(\lambda, \psi | \hat{\lambda}, \hat{\psi}) := E_{\hat{\lambda}, \hat{\psi}} (L(\lambda, \psi | \bar{y}) | y)
\]  
(4.6)

The ‘M-step’ of the EM algorithm requires the maximization of \( Q(\lambda, \psi | \hat{\lambda}, \hat{\psi}) \) with respect to \( \lambda, \psi \)

**M-step** Choose \( \lambda^*, \psi^* \) such that

\[
Q(\lambda^*, \psi^* | \hat{\lambda}, \hat{\psi}) \geq Q(\lambda, \psi | \hat{\lambda}, \hat{\psi}) \quad \text{for all } \lambda, \psi
\]  
(4.7)

The updates \( \lambda^*, \psi^* \) are then treated as the initial values \( \hat{\lambda}, \hat{\psi} \) as the E- and M-steps (4.6) and (4.7) are repeatedly executed until reaching the \( \epsilon \)-convergence of the pseudo likelihood function.

**4.3.2 Analysis of Failure Histories**

Consider the case with observations \( y = (y_{1h}, \ldots, y_{Th}) \). The system is known to have failed at time \( \zeta = t \), where \( Th < t < (T+1)h \). The pseudo log-likelihood function then has the following decomposition with the distributional properties of the sojourn time \( \tau_0 \) and failure time \( \zeta \).

This section is not only an application of reference [55]. We have made some new developments which have been applied to planetary gear fault detection for the first time in this thesis. Some challenges were encountered which are summarized below:

1. Conducting a pre-processing of planetary gear vibration data and extracting important feature values using planetary gear vibration data. It has been done in Chapter 3.

Previously, these research results have been used only for oil data analysis or simple fixed-axis gearset data analysis. However, planetary gearbox vibration data is extremely complicated and the number of data is huge. If the research results are directly applied to these vibration data, it will take a long time and perhaps no solution will ever be reached.
Thus, we must conduct a pre-processing of these vibration data. Here, we extract important feature values of indicators for the input data.

(2) Developing the EM formulae to be applied to planetary gear vibration data.

Research results from reference [55] are used for many histories including suspension and failure. However, planetary gear vibration data have only failure histories. Hence, we had to modify and develop these EM formulae for the planetary gearbox.

Detailed formulae are presented below.

\[
Q_f \left( \psi, \lambda, \hat{\psi}, \hat{\lambda} \right) = E_{\psi, \hat{\lambda}} \left( \ln \left( f_{N \psi, \lambda, \hat{\lambda}} (\hat{y}, t, t_0) \right) \right) \mid \bar{Y} = \bar{y}, \bar{\xi} = t \\
= \int \ln \left( f_{N \psi, \lambda, \hat{\lambda}} (\hat{y}, t, s) \right) f_{N \tau_{0 \lambda}} (s \mid t) ds + \ln \left( f_{N \psi, \lambda, \hat{\lambda}} (\hat{y}, t, t) \right) f_{N \tau_{0 \lambda}} (t \mid t) m_{\tau_{0 \lambda}} (t \mid t) \\
= Q_{f, \text{state}} (\lambda \mid \hat{\psi}, \hat{\lambda}) + Q_{f, \text{rel}} (\psi \mid \hat{\psi}, \hat{\lambda}) \tag{4.8}
\]

where

\[
f_\xi (t) = p_{12} \frac{v_1 v_2}{v_1 - v_2} \left( e^{-v_2 t} - e^{-v_1 t} \right) + p_{13} v_1 e^{-v_1 t} \quad t \geq 0
\]

\[
f_{\tau_{0 \lambda}} (s \mid t) = \frac{p_{12} v_2 e^{-v_2 s} e^{-(v_1 - v_2) s}}{p_{12} \frac{v_2}{v_1 - v_2} \left( e^{-v_2 t} - e^{-v_1 t} \right) + p_{13} e^{-v_1 t}} \quad 0 \leq s < t
\]

\[
P(\tau_0 = t \mid \bar{\xi} = t) := m_{\tau_{0 \lambda}} (t \mid t) = \frac{p_{13} e^{-v_1 t}}{p_{12} \frac{v_2}{v_1 - v_2} \left( e^{-v_2 t} - e^{-v_1 t} \right) + p_{13} e^{-v_1 t}} \quad t \geq 0
\]

\[
f_{\psi \lambda} (\hat{y}, t, kh) = (2\pi)^{-kd/2} \det^{-k/2} (\Sigma_1) \det^{-t/2} (\Sigma_2) \exp \left\{ -\frac{1}{2} \sum_{m=1}^{k} (y_{ah} - \mu_i)^T \Sigma_1^{-1} (y_{ah} - \mu_i) \right\}
\]

\[
f_{\psi \lambda} (\hat{y}, t, t) = (2\pi)^{-kd/2} \det^{-t/2} (\Sigma_1) \exp \left\{ -\frac{1}{2} \sum_{m=1}^{t} (y_{ah} - \mu_i)^T \Sigma_1^{-1} (y_{ah} - \mu_i) \right\}
\]

The first term of (4.8) is a function only of the state parameters \( \lambda = (\hat{\lambda}_{12}, \hat{\lambda}_{13}, \hat{\lambda}_{23}) \) given by
\[ Q_{\text{state}}(\lambda | \psi, \hat{\lambda}) \]
\[ = \int \ln \left( f_{\eta|\zeta}^{\psi}(s | t) f_{\zeta|\eta}^\lambda (s | t) d s + \ln \left( f_{\eta|\zeta}^{\psi}(t | t) f_{\zeta|\eta}^\lambda (t | t) \right) \right) \]
\[ = \hat{d} \left( \ln (\lambda_{i2}) \hat{b}_{i2} + \ln (\lambda_{i3}) \hat{b}_{i3} + \ln (\lambda_{i23}) \hat{b}_{i23} + \lambda_{i2} \hat{c}_{i2} + \lambda_{i3} \hat{c}_{i3} + \lambda_{i23} \hat{c}_{i23} \right) \]

with
\[ \hat{b}_{i2} = \hat{b}_{i2} = \hat{p}_{i2} \hat{b}_2 e^{d_{ij}} \left( \sum f_{\psi|\zeta}^\psi (\psi | t, kh) \left( e^{-(\hat{\lambda}_{i2} - \hat{\lambda}_{i2}) d_{ij}} - e^{-(\hat{\lambda}_{i2} - \hat{\lambda}_{i2}) d_{ij}} \right) + f_{\psi|\zeta}^\psi (\psi | t, t) \left( e^{-(\hat{\lambda}_{i2} - \hat{\lambda}_{i2}) d_{ij}} - e^{-(\hat{\lambda}_{i2} - \hat{\lambda}_{i2}) d_{ij}} \right) \right) \]
\[ \hat{b}_{i3} = \hat{b}_{i3} = \hat{p}_{i3} \hat{b}_3 e^{d_{ij}} \left( \sum f_{\psi|\zeta}^\psi (\psi | t, kh) \left( e^{-(\hat{\lambda}_{i3} - \hat{\lambda}_{i3}) d_{ij}} - e^{-(\hat{\lambda}_{i3} - \hat{\lambda}_{i3}) d_{ij}} \right) + f_{\psi|\zeta}^\psi (\psi | t, t) \left( e^{-(\hat{\lambda}_{i3} - \hat{\lambda}_{i3}) d_{ij}} - e^{-(\hat{\lambda}_{i3} - \hat{\lambda}_{i3}) d_{ij}} \right) \right) \]

and the second term of (4.8) is a function only of the observation parameters 
\[ \psi = (\mu_1, \mu_2, \Sigma_1, \Sigma_2) \] given by
\[ Q_{f,\text{aux}}(\psi, \hat{\psi}, \hat{\lambda}) \]

\[ = \int \ln \left( f^{\psi}_{Y|t,s}(Y|t,s) \right) f^{\psi}_{Y,\xi,t_0}(Y|t,t) f^{\hat{\psi}}_{\xi,\tau_{\xi},t_0}(s|t)ds + \ln \left( f^{\psi}_{Y|t,t}(Y|t,t) \right) f^{\psi}_{Y,\xi,t_0}(Y|t,t)m^{\xi}_{\tau_{\xi}}(t|t) \]

\[ \int f^{\psi}_{Y,\xi,t_0}(Y|t,u) f^{\hat{\psi}}_{\xi,\tau_{\xi}}(u|t)du + f^{\psi}_{Y,\xi,t_0}(Y|t,t)m^{\xi}_{\tau_{\xi}}(t|t) \]

\[ = \sum_{k=1}^{T} \ln \left( f^{\psi}_{Y,\xi,t_0}(Y|t,kh) \right) \left( f^{\hat{\psi}}_{\xi,\tau_{\xi}}(Y|t,kh) \int f^{\hat{\psi}}_{\xi,\tau_{\xi}}(s|t)ds \right) \]

\[ + \ln \left( f^{\psi}_{Y,\xi,t_0}(Y|t,t) \right) \left( \int f^{\hat{\psi}}_{\xi,\tau_{\xi}}(s|t)ds + m^{\xi}_{\tau_{\xi}}(t|t) \right) \]

\[ (4.10) \]

\[ = \sum_{k=1}^{T} \hat{\alpha}_k \ln \left( f^{\psi}_{Y,\xi,t_0}(Y|t,kh) \right) + \hat{\alpha}_t \ln \left( f^{\psi}_{Y,\xi,t_0}(Y|t,t) \right) \]

with

\[ \hat{\alpha}_k = \hat{d}f^{\psi}_{Y,\xi,t_0}(Y|t,kh) \left( \frac{\hat{\psi}_{t_2} \hat{\psi}_{t_2} e^{-\phi_2} (e^{-(t_1-t_2)kh} - e^{-\phi_2})}{\hat{\psi}_{t_2} \hat{\psi}_{t_2} (e^{-\phi_2} - e^{-\phi_2}) + (\hat{\psi}_1 - \hat{\psi}_2) \hat{\psi}_1 e^{-\phi_2}} \right), k = 1, \ldots, T \]

\[ \hat{\alpha}_t = \hat{d}f^{\psi}_{Y,\xi,t_0}(Y|t,t) \left( \frac{\hat{\psi}_{t_2} \hat{\psi}_{t_2} e^{-\phi_2} (e^{-(t_1-t_2)\xi} - e^{-(t_1-t_2)\psi})}{\hat{\psi}_{t_2} \hat{\psi}_{t_2} (e^{-\phi_2} - e^{-\phi_2}) + (\hat{\psi}_1 - \hat{\psi}_2) \hat{\psi}_1 e^{-\phi_2}} + \frac{p_{12} e^{-\psi t}}{p_{12} - p_{12} e^{-\psi t}} \right) \]

This means that for the state and observation parameters, the M-step (4.7) can be carried out separately. This then further helps the process of the computations.

### 4.3.3 Explicit Formulas for Failure Histories

Suppose we have observed \( N_f \in \mathbb{N} \) independent failure histories which all follow the statistical model of section 4.2. That is, the information in the form \( Y^l = (y^l_{1h}, \ldots, y^l_{(T_t + 1)h}) \) and \( \xi^l = t_l \), where \( T_th < t_l < (T_t + 1)h \) and \( l = 1, \ldots, N_f \) are considered. By independence of the histories, it follows that the pseudo log-likelihood function is given by

\[ Q \left( \psi, \lambda | \hat{\psi}, \hat{\lambda} \right) = \sum_{l=1}^{N_f} Q_{f} \left( \psi, \lambda | \hat{\psi}, \hat{\lambda} \right) \]

then
\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \lambda_{12}} = \sum_{l=1}^{N_r} \hat{d}(l) \left( \frac{\hat{b}_{12}(l)}{\lambda_{12}} + \hat{c}_{12}(l) \right) = 0
\]
\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \lambda_{13}} = \sum_{l=1}^{N_r} \hat{d}(l) \left( \frac{\hat{b}_{13}(l)}{\lambda_{13}} + \hat{c}_{13}(l) \right) = 0
\]
\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \lambda_{23}} = \sum_{l=1}^{N_r} \hat{d}(l) \left( \frac{\hat{b}_{23}(l)}{\lambda_{23}} + \hat{c}_{23}(l) \right) = 0
\]

(4.11)

and the following optimal updates can be obtained for the transition rate parameters

\[
\lambda_{12}^* = -\frac{\sum_{l=1}^{N_r} \hat{d}(l) \hat{b}_{12}(l)}{\sum_{l=1}^{N_r} \hat{d}(l) \hat{c}_{12}(l)}
\]
\[
\lambda_{13}^* = -\frac{\sum_{l=1}^{N_r} \hat{d}(l) \hat{b}_{13}(l)}{\sum_{l=1}^{N_r} \hat{d}(l) \hat{c}_{13}(l)}
\]
\[
\lambda_{23}^* = -\frac{\sum_{l=1}^{N_r} \hat{d}(l) \hat{b}_{23}(l)}{\sum_{l=1}^{N_r} \hat{d}(l) \hat{c}_{23}(l)}
\]

(4.12)

Similarly, the observation parameters are computed by solving

\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \mu_1} = \sum_{l=1}^{N_r} \sum_{k=1}^{T_l} \sum_{m=1}^{T_k} \hat{a}_k(l) \left( y_{m_h} - \mu_1 \right) + \hat{a}_y(l) \sum_{m=1}^{T_r} \left( y_{m_h} - \mu_1 \right) = 0
\]
\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \mu_2} = \sum_{l=1}^{N_r} \sum_{k=1}^{T_l} \sum_{m=1}^{T_k} \hat{a}_k(l) \sum_{m=1}^{T_r} \left( y_{m_h} - \mu_2 \right) = 0 \]

(4.13)

\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \Sigma_1^{-1}} = \sum_{l=1}^{N_r} \left\{ \sum_{k=1}^{T_l} \hat{a}_k(l) \left( \frac{k}{2} \Sigma_1 - \frac{1}{2} \sum_{m=1}^{T_k} (y_{m_h} - \mu_1)(y_{m_h} - \mu_1)^T \right) \right\} = 0
\]
\[
\frac{\partial Q(\psi, \lambda | \hat{\psi}, \hat{\lambda})}{\partial \Sigma_2^{-1}} = \sum_{l=1}^{N_r} \sum_{k=1}^{T_l} \hat{a}_k(l) \left( \frac{T_l - k + 1}{2} \Sigma_2 - \frac{1}{2} \sum_{m=1}^{T_k} (y_{m_h} - \mu_2)(y_{m_h} - \mu_2)^T \right) = 0
\]

and the following optimal updates for the observation parameters are obtained:
### 4.3.4 Numerical Results

In this section, the observations are defined by two characteristics: variance and RMS based on analysis of autocovariance for DWT applied to TSA signal from planetary gear vibration data in Chapter 3. Considering the run-in period, files Rtf_1st to Rft_3rd were discarded and only files Rtf_4th to Rtf_13th were used in the analysis.

The extracted observations for the partially observable hidden Markov model parameter estimation can be seen in Table 4.1.

<table>
<thead>
<tr>
<th>File</th>
<th>Variance ($y_1$)</th>
<th>RMS ($y_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rtf_4th</td>
<td>7.4748e-005</td>
<td>0.7021</td>
</tr>
<tr>
<td>Rtf_5th</td>
<td>9.0243e-005</td>
<td>0.8384</td>
</tr>
<tr>
<td>Rtf_6th</td>
<td>3.7882e-004</td>
<td>1.6576</td>
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<td>Rtf_7th</td>
<td>3.8035e-004</td>
<td>1.8446</td>
</tr>
<tr>
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<td>4.2011e-004</td>
<td>1.4313</td>
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<tr>
<td>Rtf_9th</td>
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<td>2.8564</td>
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<tr>
<td>Rtf_10th</td>
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</tr>
<tr>
<td>Rtf_11th</td>
<td>12.000e-004</td>
<td>4.1973</td>
</tr>
</tbody>
</table>
The corresponding plots to Table 4.1 can be seen below:

Figure 4.1 Observations Collected by Variance and RMS

In building a hidden Markov model and designing a Bayesian fault detection scheme, the independence and normality of the bivariate indicator (variance and RMS) are first tested. The p-values for the Portmanteau Independence Test and the Henze-Zirkler Multivariate Test from Technical Report [59] are given in the following table:

Table 4.2 p-values of the Independence and Normality Tests for Variance and RMS

<table>
<thead>
<tr>
<th></th>
<th>Healthy data set</th>
<th>Unhealthy data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independence (Portmanteau)</td>
<td>0.6350</td>
<td>0.5338</td>
</tr>
<tr>
<td>Normality (Henze-Zirkler)</td>
<td>0.3325</td>
<td>0.3469</td>
</tr>
</tbody>
</table>

Table 4.2 shows no statistical evidence to reject the null hypotheses which state that the variance and RMS signals are independent and follow a multivariate normal distribution.

The observation can be represented by

\[
y = \begin{bmatrix} y(t_4) & y(t_5) & \cdots & y(t_{13}) \end{bmatrix} = \begin{bmatrix} y_1(t_4) & y_1(t_5) & \cdots & y_1(t_{13}) \\ y_2(t_4) & y_2(t_5) & \cdots & y_2(t_{13}) \end{bmatrix} \tag{4.15}
\]

10 files were processed for the testing with the data collection interval of 5 minutes. All the data files were separated into two groups. Files 4-8 were used as healthy state files with the consideration that the gear system is in the good state \(s_1\), while Files 9-12 were used as unhealthy state files considering the system to be in the warning state \(s_2\). The intervals between samples varied and the following procedure is presented for an illustration only.
The time interval for each sample is $\Delta h = 5$ mins. Assuming the process proceeds into the warning state from the good state in the middle of the data collection interval for files 8 and 9, the sojourn time of the good state is $t_{s_i} = 5.5\Delta h$. The time spent in the warning state is $t_{w} = 4.5\Delta h$ and the gearbox failure occurred at $t_f = 10\Delta h$. The relation between the partially observable hidden Markov model parameters can be obtained, by subjective judgment, as:

$$p_{13} = 0 \Rightarrow \frac{\lambda_{13}}{\lambda_{12} + \lambda_{13}} = 0 \Rightarrow \lambda_{13} = 0$$

The sojourn time of the good state is then

$$\nu_1 = \frac{1}{t_{s_1}} = \frac{1}{5.5 \times 5 \text{(min)}} = 0.03636$$

$$\lambda_{12} = \nu_1 - \lambda_{13} = 0.03636 - 0 = 0.03636$$

The sojourn time of the warning state is estimated as

$$\nu_2 = \frac{1}{t_{s_2}} = \frac{1}{4.5 \times 5 \text{(min)}} = 0.0435$$

$$\lambda_{23} = \nu_2 = 0.0435$$

When the gearbox is in the good state, the mean vector is calculated by

$$\mu_1 = \begin{bmatrix} 2.6885 \times 10^{-4} \\ 1.2948 \end{bmatrix}$$

and the covariance matrix is

$$\Sigma_1 = \begin{bmatrix} 2.9245 \times 10^{-8} & 0.7960 \times 10^{-4} \\ 0.7960 \times 10^{-4} & 0.2530 \end{bmatrix}$$

When the gearbox is in the warning state, the mean vector is

$$\mu_2 = \begin{bmatrix} 12.1574 \times 10^{-4} \\ 3.9102 \end{bmatrix}$$

and the covariance matrix is

$$\Sigma_2 = \begin{bmatrix} 16.9868 \times 10^{-8} & 3.4993 \times 10^{-4} \\ 3.4993 \times 10^{-4} & 0.8110 \end{bmatrix}$$

Below are the estimated partially observable hidden Markov model parameters using the EM
iteration algorithm.

Because the EM iteration algorithm displays local convergence, the iteration results will then depend on its initial values. If the initial values chosen is far from the real values, the estimated parameter values obtained from these primary initial values cannot be guaranteed to be ideal. Hence in this case, the EM iteration algorithm is firstly ran with initial values as given by common logic. Then, as the iteration values converge to constant values which are seen as unideal, these constant values then act as the initial values as the EM iteration algorithm is used a second time to finally obtain the estimated partially observable hidden Markov model parameters.

The first initial values are chosen as

\[
\begin{align*}
\mathbf{\mu}_1 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\
\mathbf{\mu}_2 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\
\mathbf{\Sigma}_1 &= \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, \\
\mathbf{\Sigma}_2 &= \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, \\
\mathbf{\lambda} &= \begin{bmatrix} \lambda_{12} \\ \lambda_{13} \\ \lambda_{23} \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix}
\end{align*}
\]

The number of iterations for the EM algorithm is selected as 30 because the values of the parameters stop changing at that point. The EM estimation values are shown in the following tables, and plots corresponding to the tables are also given. Due to the difference between the first initial values and their updated values, these plots do not include the first initial values.

The EM estimations of the mean vectors for the first initial values are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Table 4.3 EM Algorithm Iteration Results for the First Initial Values: Mean Vectors $\mathbf{\mu}_1$ and $\mathbf{\mu}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>The First Initial Value</td>
</tr>
<tr>
<td>Update 1</td>
</tr>
<tr>
<td>Update 2</td>
</tr>
<tr>
<td>Update 3</td>
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<tr>
<td>Update 28</td>
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<tr>
<td>Update 29</td>
</tr>
<tr>
<td>Update 30</td>
</tr>
</tbody>
</table>

The plots corresponding to Table 4.3 are given below:
Figure 4.2 EM algorithm Estimated Mean Vectors $\mu_1$ and $\mu_2$ for the First Initial Values

The EM estimations of Covariance Matrix $\Sigma_i$ for the first initial values are shown in Table 4.4.

Table 4.4 EM Algorithm Iteration Results for the First Initial Values: Covariance Matrix $\Sigma_i$

<table>
<thead>
<tr>
<th></th>
<th>$\Sigma_i(1,1)$</th>
<th>$\Sigma_i(1,2) = \Sigma_i(2,1)$</th>
<th>$\Sigma_i(2,2)$</th>
</tr>
</thead>
<tbody>
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<td>3.0000E+01</td>
<td>0.0000E+00</td>
<td>3.0000E+01</td>
</tr>
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<td>4.9338E-04</td>
<td>2.9572E+00</td>
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<td>3.8510E-01</td>
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<td>3.0487E-01</td>
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<td>4.2994E-01</td>
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</table>
The EM estimations of Covariance Matrix $\Sigma_2$ for the first initial values are shown in Table 4.5.

<table>
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<th>$\Sigma_2 (1,2)$</th>
<th>$\Sigma_2 (2,2)$</th>
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</thead>
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Table 4.5 EM Algorithm Iteration Results for the First Initial Values: Covariance Matrix $\Sigma_2$
<table>
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<th>Σ(1,1)</th>
<th>Estimate Covariance</th>
<th>Σ(1,1)</th>
<th>Estimate Covariance</th>
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</tr>
</tbody>
</table>

The plots corresponding to Tables 4.4-4.5 are shown:
Figure 4.3 EM Algorithm Estimated Covariance Parameters $\Sigma_1$ and $\Sigma_2$ for the First Initial Values

Table 4.6 shows the EM estimation results of transition rate parameters $\lambda = (\lambda_{12}, \lambda_{13}, \lambda_{23})$.

Table 4.6 EM Algorithm Iteration Results for the First Initial Values:

<table>
<thead>
<tr>
<th>States Transition Parameters $\lambda$</th>
<th>$\lambda_{12}$</th>
<th>$\lambda_{13}$</th>
<th>$\lambda_{23}$</th>
</tr>
</thead>
<tbody>
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<td>5.0000E-02</td>
<td>5.0000E-02</td>
</tr>
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<tr>
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<tr>
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<td>Update 22</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5082E-02</td>
</tr>
<tr>
<td>Update 23</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5083E-02</td>
</tr>
<tr>
<td>Update 24</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5083E-02</td>
</tr>
<tr>
<td>Update 25</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5083E-02</td>
</tr>
<tr>
<td>Update 26</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5084E-02</td>
</tr>
<tr>
<td>Update 27</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5084E-02</td>
</tr>
<tr>
<td>Update 28</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5084E-02</td>
</tr>
<tr>
<td>Update 29</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5084E-02</td>
</tr>
<tr>
<td>Update 30</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5084E-02</td>
</tr>
</tbody>
</table>

The plots corresponding to Table 4.6 are as follows:
Table 4.7 is a summary on the iteration results of the parameters for the first initial values and their percentage difference. From this table, it can be seen that the percentage difference is quite large for each parameter despite the fact that the iteration values converge to constant values. Therefore, it is not reasonable to depend only on their convergence in identifying the feasibility of EM algorithm iteration results. By judgment, the EM algorithm iteration is again needed.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Subjective Judgment</th>
<th>Iteration Results (First initial values)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$ of $y_1$</td>
<td>2.6885E-04</td>
<td>2.1890E-04</td>
<td>20.4818</td>
</tr>
<tr>
<td>$\mu_2$ of $y_1$</td>
<td>12.1574E-04</td>
<td>11.8100E-04</td>
<td>2.8989</td>
</tr>
<tr>
<td>$\mu_1$ of $y_2$</td>
<td>1.2948</td>
<td>1.3978</td>
<td>7.6506</td>
</tr>
<tr>
<td>$\mu_2$ of $y_2$</td>
<td>3.9102</td>
<td>3.9647</td>
<td>1.3841</td>
</tr>
<tr>
<td>$\Sigma_1(1,1)$</td>
<td>2.9245E-08</td>
<td>6.2046E-08</td>
<td>71.8603</td>
</tr>
<tr>
<td>$\Sigma_1(1,2)$</td>
<td>0.7960E-04</td>
<td>1.3149E-04</td>
<td>49.1639</td>
</tr>
<tr>
<td>$\Sigma_1(2,2)$</td>
<td>0.2530</td>
<td>0.48708</td>
<td>63.2580</td>
</tr>
<tr>
<td>$\Sigma_2(1,1)$</td>
<td>16.9868E-08</td>
<td>11.8120E-08</td>
<td>35.9376</td>
</tr>
<tr>
<td>$\Sigma_2(1,2)$</td>
<td>3.4993E-04</td>
<td>2.4488E-04</td>
<td>35.3222</td>
</tr>
<tr>
<td>$\Sigma_2(2,2)$</td>
<td>0.8110</td>
<td>0.57291</td>
<td>34.4083</td>
</tr>
<tr>
<td>$\lambda_{12}$</td>
<td>3.636E-02</td>
<td>2.3142E-02</td>
<td>44.4288</td>
</tr>
<tr>
<td>$\lambda_{13}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{23}$</td>
<td>4.3500E-02</td>
<td>4.5084E-02</td>
<td>3.5763</td>
</tr>
</tbody>
</table>
Based on the updated results for the first initial values, the EM iteration algorithm for the second initial values, which are the values from Update 30 in the above tables, are as follows:

\[
\mu_1 = \begin{bmatrix} 2.1890 \times 10^{-4} \\ 1.3978 \end{bmatrix}
\]

\[
\mu_2 = \begin{bmatrix} 11.8100 \times 10^{-4} \\ 3.9647 \end{bmatrix}
\]

\[
\Sigma_1 = \begin{bmatrix} 6.2046 \times 10^{-8} & 1.3149 \times 10^{-4} \\ 1.3149 \times 10^{-4} & 0.48708 \end{bmatrix}
\]

\[
\Sigma_2 = \begin{bmatrix} 11.8120 \times 10^{-8} & 2.4488 \times 10^{-4} \\ 2.4488 \times 10^{-4} & 0.57291 \end{bmatrix}
\]

\[
\lambda = \begin{bmatrix} \hat{\lambda}_{12} \\ \hat{\lambda}_{13} \\ \hat{\lambda}_{21} \end{bmatrix} = \begin{bmatrix} 2.3142 \times 10^{-2} \\ 0 \\ 4.5084 \times 10^{-2} \end{bmatrix}
\]

In this run, the number of iterations for the EM algorithm is selected as 10 as the values again stop changing at that point. The EM estimation values are shown in the following tables, and plots corresponding to these tables are also given. Due to the fact that the second initial values and their updated values do not have a huge difference, these plots include the second initial values.

The EM estimations of mean vectors for the second initial values are shown in Table 4.8.

### Table 4.8 EM Algorithm Iteration Results for the Second Initial Values:

<table>
<thead>
<tr>
<th>Mean Vectors $\mu_1$ and $\mu_2$</th>
<th>$y_1$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The Second Initial Value</strong></td>
<td>$\hat{\mu}_1$</td>
<td>$\hat{\mu}_2$</td>
</tr>
<tr>
<td>Update 1</td>
<td>2.1890E-04</td>
<td>1.1891E-03</td>
</tr>
<tr>
<td>Update 2</td>
<td>2.9989E-04</td>
<td>1.2270E-03</td>
</tr>
<tr>
<td>Update 3</td>
<td>2.7024E-04</td>
<td>1.2161E-03</td>
</tr>
<tr>
<td>Update 4</td>
<td>2.6959E-04</td>
<td>1.2174E-03</td>
</tr>
<tr>
<td></td>
<td>2.6976E-04</td>
<td>1.2173E-03</td>
</tr>
</tbody>
</table>
Update 5  2.6977E-04  1.2173E-03  1.2995E+00  3.9165E+00
Update 6  2.6978E-04  1.2173E-03  1.2995E+00  3.9165E+00
Update 7  2.6979E-04  1.2173E-03  1.2995E+00  3.9165E+00
Update 8  2.6979E-04  1.2173E-03  1.2995E+00  3.9165E+00
Update 9  2.6979E-04  1.2173E-03  1.2995E+00  3.9165E+00
Update 10 2.6979E-04  1.2173E-03  1.2995E+00  3.9165E+00

The plots corresponding to Table 4.8 are given below

Figure 4.5 EM algorithm Estimated Mean Vectors $\mu_1$ and $\mu_2$ for the Second Initial Values

The EM estimations of Covariance Matrix $\Sigma_i$ for the second initial values are shown in Table 4.9.

Table 4.9 EM Algorithm Iteration Results for the Second Initial Values:

<table>
<thead>
<tr>
<th>The Second Initial Value</th>
<th>$\Sigma_i(1,1)$</th>
<th>$\Sigma_i(1,2) = \Sigma_i(2,1)$</th>
<th>$\Sigma_i(2,2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Second Initial Value</td>
<td>6.2046E-08</td>
<td>1.3149E-04</td>
<td>4.8708E-01</td>
</tr>
<tr>
<td>Update 1</td>
<td>3.3504E-08</td>
<td>1.1176E-04</td>
<td>3.0686E-01</td>
</tr>
<tr>
<td>Update 2</td>
<td>3.2182E-08</td>
<td>8.2827E-05</td>
<td>2.4871E-01</td>
</tr>
</tbody>
</table>
The EM estimations of Covariance Matrix $\Sigma_2$ for the second initial values are shown in Table 4.10.

Table 4.10 EM Algorithm Iteration Results for the Second Initial Values:

<table>
<thead>
<tr>
<th>Update</th>
<th>$\Sigma_2(1,1)$</th>
<th>$\Sigma_2(1,2) = \Sigma_2(2,1)$</th>
<th>$\Sigma_2(2,2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Second Initial Value</td>
<td>1.1812E-07</td>
<td>2.4488E-04</td>
<td>5.7291E-01</td>
</tr>
<tr>
<td>Update 1</td>
<td>1.6702E-07</td>
<td>3.4626E-04</td>
<td>7.8603E-01</td>
</tr>
<tr>
<td>Update 2</td>
<td>1.7008E-07</td>
<td>3.4974E-04</td>
<td>8.0534E-01</td>
</tr>
<tr>
<td>Update 3</td>
<td>1.6928E-07</td>
<td>3.4954E-04</td>
<td>8.1145E-01</td>
</tr>
<tr>
<td>Update 4</td>
<td>1.6934E-07</td>
<td>3.4938E-04</td>
<td>8.1038E-01</td>
</tr>
<tr>
<td>Update 5</td>
<td>1.6931E-07</td>
<td>3.4930E-04</td>
<td>8.1022E-01</td>
</tr>
<tr>
<td>Update 6</td>
<td>1.6931E-07</td>
<td>3.4930E-04</td>
<td>8.1021E-01</td>
</tr>
<tr>
<td>Update 7</td>
<td>1.6931E-07</td>
<td>3.4930E-04</td>
<td>8.1020E-01</td>
</tr>
<tr>
<td>Update 8</td>
<td>1.6931E-07</td>
<td>3.4930E-04</td>
<td>8.1020E-01</td>
</tr>
<tr>
<td>Update 9</td>
<td>1.6931E-07</td>
<td>3.4930E-04</td>
<td>8.1020E-01</td>
</tr>
<tr>
<td>Update 10</td>
<td>1.6931E-07</td>
<td>3.4930E-04</td>
<td>8.1020E-01</td>
</tr>
</tbody>
</table>

The plots corresponding to Tables 4.9-4.10 are given.
Figure 4.6 EM Algorithm Estimated Covariance Parameters $\Sigma_1$ and $\Sigma_2$ for the Second Initial Values

Table 4.11 show EM estimation results of transition rate parameters $\lambda = (\lambda_{12}, \lambda_{13}, \lambda_{23})$ for the second initial values.

<table>
<thead>
<tr>
<th>States Transition Parameters $\lambda$</th>
<th>$\lambda_{12}$</th>
<th>$\lambda_{13}$</th>
<th>$\lambda_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Second Initial Value</td>
<td>2.3142E-02</td>
<td>0.0000E+00</td>
<td>4.5084E-02</td>
</tr>
<tr>
<td>Update 1</td>
<td>3.5747E-02</td>
<td>0.0000E+00</td>
<td>4.5083E-02</td>
</tr>
<tr>
<td>Update 2</td>
<td>3.6317E-02</td>
<td>0.0000E+00</td>
<td>4.3472E-02</td>
</tr>
<tr>
<td>Update 3</td>
<td>3.6260E-02</td>
<td>0.0000E+00</td>
<td>4.3696E-02</td>
</tr>
<tr>
<td>Update 4</td>
<td>3.6266E-02</td>
<td>0.0000E+00</td>
<td>4.3682E-02</td>
</tr>
<tr>
<td>Update 5</td>
<td>3.6265E-02</td>
<td>0.0000E+00</td>
<td>4.3686E-02</td>
</tr>
<tr>
<td>Update 6</td>
<td>3.6264E-02</td>
<td>0.0000E+00</td>
<td>4.3687E-02</td>
</tr>
<tr>
<td>Update 7</td>
<td>3.6264E-02</td>
<td>0.0000E+00</td>
<td>4.3687E-02</td>
</tr>
<tr>
<td>Update 8</td>
<td>3.6264E-02</td>
<td>0.0000E+00</td>
<td>4.3687E-02</td>
</tr>
<tr>
<td>Update 9</td>
<td>3.6264E-02</td>
<td>0.0000E+00</td>
<td>4.3687E-02</td>
</tr>
<tr>
<td>Update 10</td>
<td>3.6264E-02</td>
<td>0.0000E+00</td>
<td>4.3687E-02</td>
</tr>
</tbody>
</table>
The plots corresponding to Table 4.11 are as follows:

![Plots](image.png)

Figure 4.7 EM Algorithm Estimated States Transition Parameters $\lambda$ for the Second Initial Values

Table 4.12 is a summary on the iteration results of parameters for the second initial values and their percentage difference. From this table, it can be seen that the percentage difference is quite small. By judgment, EM algorithm iteration is again not needed.

Table 4.12 EM Algorithm Iteration Results for the Second Initial Values and Percentage Difference

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Subjective Judgment</th>
<th>Iteration Results (Second initial values)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$ of $y_1$</td>
<td>2.6885E-04</td>
<td>2.6979E-04</td>
<td>0.3490</td>
</tr>
<tr>
<td>$\mu_2$ of $y_1$</td>
<td>12.1574E-04</td>
<td>12.173E-04</td>
<td>0.1282</td>
</tr>
<tr>
<td>$\mu_1$ of $y_2$</td>
<td>1.2948 E+00</td>
<td>1.2995E+00</td>
<td>0.3623</td>
</tr>
<tr>
<td>$\mu_2$ of $y_2$</td>
<td>3.9102 E+00</td>
<td>3.9166E+00</td>
<td>0.1635</td>
</tr>
<tr>
<td>$\Sigma_1(1,1)$</td>
<td>2.9245E-08</td>
<td>2.9392E-08</td>
<td>0.5014</td>
</tr>
<tr>
<td>$\Sigma_1(1,2)$</td>
<td>7.9600E-05</td>
<td>7.9769E-05</td>
<td>0.2121</td>
</tr>
<tr>
<td>$\Sigma_1(2,2)$</td>
<td>2.5300 E-01</td>
<td>2.5244E-01</td>
<td>0.2216</td>
</tr>
<tr>
<td></td>
<td>( \Sigma_2(1,1) )</td>
<td>( \Sigma_2(1,2) )</td>
<td>( \Sigma_2(2,2) )</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
</tbody>
</table>
|        | 1.69868E-07    | 3.4993E-04      | 8.1100 E-01    | 0.3290  
|        | 1.6931E-07    | 3.4930E-04      | 8.1020E-01    | 0.1802  
|        | 0.169868E-07  | 0.34993E-04     | 0.081100 E-01 | 0.0987  

The EM iteration results for the estimated final values are given here:

\[
\mu_1 = \begin{bmatrix}
2.6979 \times 10^{-4} \\
1.2995
\end{bmatrix}
\]

\[
\mu_2 = \begin{bmatrix}
12.1730 \times 10^{-4} \\
3.9166
\end{bmatrix}
\]

\[
\Sigma_1 = \begin{bmatrix}
2.9392 \times 10^{-8} & 0.79769 \times 10^{-4} \\
0.79769 \times 10^{-4} & 0.25244
\end{bmatrix}
\]

\[
\Sigma_2 = \begin{bmatrix}
16.9310 \times 10^{-8} & 3.4930 \times 10^{-4} \\
3.4930 \times 10^{-4} & 0.81020
\end{bmatrix}
\]

\[
\lambda = \begin{bmatrix}
\lambda_{12} \\
\lambda_{13} \\
\lambda_{23}
\end{bmatrix} = \begin{bmatrix}
3.6360E-02 \\
0 \\
4.3500E-02
\end{bmatrix}
\]

Based on the above analysis, all parameters, estimated using the EM algorithm, converge in the end to the values which were calculated using subjective judgments (with some small discrepancies). It has thus been found that the procedure is computationally efficient.

### 4.4 The Multivariate Bayesian Control Chart for CBM Application

The sample size, control limit parameters, and sampling interval must be specified for a multivariate Bayesian control chart.

Based on the results from the above section, if the gear state transforms from the good state \( s_1 \) to the warning state \( s_2 \), the mean vector and covariance matrix of the deterioration process will then increase. It is assumed in this section that as the gear state changes from \( s_1 \) into \( s_2 \), the gear deterioration process mean vector shifts from \( \mu_1 \) to \( \mu_2 \) with unchanged covariance matrix i.e. \( \Sigma_1 = \Sigma_2 = \Sigma \).
4.4.1 Multivariate Bayesian Procedure for Posterior Probability of Gear Deteriorating Process in Warning State [60]

Let $\xi$ denote the observable failure time of the system for $t \geq 0$ and $P_i(t)$ denote the probability that the process is in a state $i$ at time $t$ given the observations up to time $t$, which can be expressed as:

$$P_i(t) = P(X_t = i \mid Y_h, \ldots, Y_{[t/h]h}, I_{[t/h]}).$$

For $m = 1, 2, \ldots$, the probability $P_{mh}(2)$ that the system is in the warning state at time $mh$ given the system history up to time $mh$ is plotted on the multivariate Bayesian control chart to facilitate maintenance decision-making. Let $\overline{P}_i(t) = P(X_t = i \mid Y_h, \ldots, Y_{[t/h]h}, I_{[t/h]}h)$ be the probability that the process is in state $i$ at time $t$ given the observable information up to time $[t/h]h$.

Clearly, at sampling time $mh$, $P_{mh}(i) = \overline{P}_{mh}(i)$. The formula to update $P_{mh}(2)$ is denoted

$$P_{mh}(2) = \frac{\overline{P}_{mh}(2)}{\exp\left[\frac{1}{2}(nd^2 + Z_m)\right] \cdot \overline{P}_{mh}(1) + \overline{P}_{mh}(2)} \quad (4.16)$$

where

$$\overline{P}_{mh}(1) = e^{-\left(\lambda_2 + \lambda_3\right)h} \cdot \left[1 - P_{(m-1)h}(2)\right]$$

$$\overline{P}_{mh}(2) = \frac{\hat{\lambda}_{12} e^{-\hat{\lambda}_{12}h} - e^{-\left(\hat{\lambda}_{12} + \lambda_3\right)h}}{\lambda_{12} + \lambda_{13} - \lambda_{23}} \cdot \left[1 - P_{(m-1)h}(2)\right] + e^{-\hat{\lambda}_{12}h} \cdot P_{(m-1)h}(2).$$

$$Z_m = 2 \sum_{j=1}^{n} (y_j^m - \mu_1)^T \Sigma^{-1} (\mu_1 - \mu_2),$$

and follows

$$Z_m \sim N(0, 4nd^2)$$

when the system is in state 1, and

$$Z_m \sim N(-2nd^2, 4nd^2)$$

when it is in state 2.

$$d = [(\mu_2 - \mu_1)^T \Sigma^{-1} (\mu_2 - \mu_1)]^{1/2}$$
4.4.2 Multivariate Bayesian Control Chart of Gear Deteriorating Process [60]

It is important to choose the optimal Bayesian control chart parameters which will minimize the expected average cost per unit time. The Bayesian control chart parameters include sampling interval $h$, sample size $n$, and upper control limit $P^*$. As sample size, $n$, is equal to one, the optimization of $n$ will not be considered.

The optimization of the multivariate Bayesian control chart to be applied to CBM can be formulated as

$$
\min z(h, P^*) = \frac{E(\text{Cycle Cost})}{E(\text{CL})}
\text{ s.t. } 0 \leq P^* \leq 1
\quad h > 0
$$

As the upper control limit for the posterior probabilities, $P^*$ is required to be between 0 and 1. The following policies are applied in order to make a decision on performing inspection:

a) If $P_{mh}(2) \leq P^*$ no inspection is needed

b) When $P_{mh}(2) > P^*$, inspect the system immediately

In the following sections, the formulae for the expected cycle length and expected cycle cost will be introduced to construct the objective function.

Cost structure

Below is a list of the notations used:

- $C_1$ = the operation and regular maintenance cost per unit time in the normal state.
- $C_2$ = the operation and regular maintenance cost per unit time in the warning state.
- $C_{fa}$ = the inspection cost per alarm.
- $C_p$ = the preventive maintenance cost.
- $C_f$ = the failure maintenance cost (includes the system installation cost and the system downtime cost).
- $C_s$ = the sampling cost.
- $T_{fi}$ = expected full inspection time.
\( T_{pm} \) = expected time for preventive maintenance.

\( T_f \) = expected time to restore the system after the occurrence of system failure.

\( E(\text{Cycle Cost}) \) = expected cycle cost.

\( E(\text{CL}) \) = expected cycle length.

It is assumed that \( T_f + T_{pm} < T_f \). Similarly, it is also assumed that \( C_{fa} + C_p < C_f \).

**Expected cycle length**

The expected cycle length is determined as follows.
\[ E(\text{CL}) = \hbar J \cdot [I - P(N)]^{-2} v - (1 - e^{-i\epsilon^h}) \hbar J \cdot [I - P(N)]^{-2} [I - e^{-i\epsilon^h} P(N)]^{-1} v \\
- (1 - e^{-i\epsilon^h}) e^{-i\epsilon^h} [I - P(N)]^{-2} [I - e^{-i\epsilon^h} P(N)]^{-1} P(N)v \\
+ p_{12} \frac{1}{\nu_2} [1 - J \cdot [I - P(N)]^{-1} v] + p_{12} \frac{1}{\nu_2} (1 - e^{-i\epsilon^h}) J \cdot [I - e^{-i\epsilon^h} P(N)]^{-1} [I - P(N)]^{-1} v \\
+ \hbar \frac{1}{\nu_1} (1 - e^{-i\epsilon^h}) e^{-i\epsilon^h} \left[ 1 - J \cdot [I - P(N)]^{-1} v \right] + J \cdot [I - e^{-i\epsilon^h} P(N)]^{-2} P(N)[I - P(N)]^{-1} v \\
+ \frac{1}{\nu_1} (1 - e^{-i\epsilon^h}) + (1 - e^{-i\epsilon^h}) h - h \right] J \cdot [I - e^{-i\epsilon^h} P(N)]^{-1} [I - P(N)]^{-1} v \\
- \frac{\nu_1}{\nu_1 - \nu_2} p_{12} e^{-i\epsilon^h} (e^{-(v_1 - v_2 \epsilon^h)} - 1) \\
\begin{align*}
&\left[ [I - P(W)]^{-2} \right] \\
&\left[ (1 - e^{-i\epsilon^h}) [I - P(W)]^{-2} [I - e^{-i\epsilon^h} P(W)]^{-1} P(W) \right] \cdot w \\
&+ e^{-i\epsilon^h} h J \cdot [I - e^{-i\epsilon^h} P(N)]^{-2} P(N)[I - P(W)]^{-1} w \\
&- (1 - e^{-i\epsilon^h}) J \cdot [I - e^{-i\epsilon^h} P(N)]^{-1} [I - P(W)]^{-1} w \\
&+ \frac{1}{\nu_2} \left( 1 + 2h \right) (1 - e^{-i\epsilon^h}) - h \right] J \cdot [I - e^{-i\epsilon^h} P(N)]^{-1} [I - e^{-i\epsilon^h} P(W)]^{-1} P(W)[I - P(W)]^{-1} w \\
&+ e^{-i\epsilon^h} h (1 - e^{-i\epsilon^h}) J \cdot [I - e^{-i\epsilon^h} P(N)]^{-2} P(N)[I - e^{-i\epsilon^h} P(W)]^{-1} P(W)[I - P(W)]^{-1} w \\
&- e^{-i\epsilon^h} h J \cdot [I - e^{-i\epsilon^h} P(N)]^{-2} P(N)[I - P(W)]^{-1} w \\
&+ e^{-i\epsilon^h} h (1 - e^{-i\epsilon^h}) J \cdot [I - e^{-i\epsilon^h} P(N)]^{-1} [I - e^{-i\epsilon^h} P(W)]^{-2} P^2(W)[I - P(W)]^{-1} w \\
&+ (1 - e^{-i\epsilon^h}) (T_{pm} + T_{T} - T_{j}) J \cdot [I - e^{-i\epsilon^h} P(N)]^{-1} \left( 1 - e^{-i\epsilon^h} \right)^{-1} I \left[ -[I - e^{-i\epsilon^h} P(W)]^{-1} P(W) \right] [I - P(W)]^{-1} w \\
&\left( T_{j} - T_{j} \right) J \cdot \left[(1 - e^{-i\epsilon^h})^{-1} I - [I - e^{-i\epsilon^h} P(N)]^{-1} \right] [I - P(N)]^{-1} v + (1 - e^{-i\epsilon^h})^{-1} T_{j} \\
&+ \frac{(1 - e^{-i\epsilon^h}) + p_{12} e^{-i\epsilon^h} \left( e^{-(v_1 - v_2 \epsilon^h)} - 1 \right) }{\nu_1 - \nu_2} P_{12} e^{-i\epsilon^h} (e^{-(v_1 - v_2 \epsilon^h)} - 1) \\
\end{align*}
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\[
P_j(W) = \Phi \left( \{2 \ln[(b_j / L_j - b_j) / a_j] + nd^2\} / (2d \sqrt{n}) \right) 
- \Phi \left( \{2 \ln[(b_j / U_j - b_j) / a_j] + nd^2\} / (2d \sqrt{n}) \right) 
\text{for } 0 \leq i \leq l, 0 < j \leq l
\]

\[
P_{0i}(W) = 1 - \Phi \left( \{2 \ln[(b_j / U_0 - b_j) / a_j] + nd^2\} / (2d \sqrt{n}) \right) 
\text{for } j = 0
\]

where \( U_j = (j + 0.5)P^* / l \) and \( L_j = (j - 0.5)P^* / l \) for \( 1 \leq j < l \) and \( U_0 = 0, \ U_j = P^* \).

\[
a_i = \bar{P}_{nh} (1) \approx e^{-(\lambda_{12} + \lambda_{34})h} (1 - \frac{iP^*}{l})
\]

\[
b_i = \bar{P}_{nh} (2) = \frac{\lambda_{12}(e^{-\lambda_{34}h} - e^{-(\lambda_{12} + \lambda_{34})h})}{\lambda_{12} + \lambda_{13} - \lambda_{23}} (1 - \frac{iP^*}{l}) + e^{-\lambda_{23}h} \frac{iP^*}{l}
\]

\[
v = (I - P(N))1
\]

\[
w = (I - P(W))1
\]

Define \( \mathbf{1} = (1, \ldots, 1)^T \) and let \( 1 \times (l + 1) \) row vector \( \mathbf{J}(i) \) have a 1 in the place corresponding to the starting state \( i \) and zeros in the other places for \( i = 1, \ldots, l \).

**Expected cycle cost**

The expected cycle cost can be deduced as follows:

\[
E(\text{Cycle Cost}) = \text{Cost 1} + \text{Cost 2} + \text{Cost 3} + \text{Cost 4}
\]

\[
+ \frac{\nu_1}{\nu_1 - \nu_2} p_{12} e^{-\nu_2 h} (e^{-(\nu_1 - \nu_2)h} - 1)(\text{Cost 51} + \text{Cost 52} + \text{Cost 53} + \text{Cost 54} + \text{Cost 55})
\]

where

Cost 1

\[
= (C_s + hC_c) \left[ \frac{(1 - e^{-\nu_2 h})}{\nu_1 - \nu_2} p_{12} e^{-\nu_2 h} (e^{-(\nu_1 - \nu_2)h} - 1) \right] \cdot \mathbf{J} \left\{ [I - P(N)]^{-2} [1 - e^{-\nu_2 h}]^{-1} I - [I - e^{-\nu_2 h} P(N)]^{-2} P(N) \right\} v
\]

Cost 2

\[
= C_{s2} \left[ \frac{(1 - e^{-\nu_2 h})}{\nu_1 - \nu_2} p_{12} e^{-\nu_2 h} (e^{-(\nu_1 - \nu_2)h} - 1) \right] \cdot \mathbf{J} \left\{ (1 - e^{-\nu_2 h})^{-1} I - [I - e^{-\nu_2 h} P(N)]^{-1} [I - P(N)]^{-1} \right\} v
\]
\begin{align*}
\text{Cost 3} & \quad [1-e^{-\alpha h}]C_f + \left[ \frac{1}{\nu_2} (1-e^{-\alpha h}) - h \right] C_1 + \frac{1}{\nu_2} p_{12} (1-e^{-\alpha h})C_2 \\
& = + \frac{\nu_1}{\nu_1 - \nu_2} p_{12} e^{-\alpha h} \left\{ (e^{-(\alpha - \beta \nu_2 h)} - 1)(C_f + \frac{1}{\nu_2} C_2) - \left[ \frac{1}{\nu_1 - \nu_2} (e^{-(\alpha - \beta \nu_2 h)} - 1) + h \right] (C_2 - C_1) \right\} \\
& \frac{1-e^{-\alpha h}}{h} C_1 + \frac{\nu_1}{\nu_1 - \nu_2} p_{12} e^{-\alpha h} (e^{-(\alpha - \beta \nu_2 h)} - 1) \\
\cdot \left[ (1-e^{-\alpha h})^{-1} \right] - J \cdot \left[ (1-e^{-\alpha h})^{-1} I - \left[ I - e^{-\alpha h} P(N) \right]^{-1} \right] [I - P(N)]^{-1} \cdot v \\
\text{Cost 4} & \quad [1-e^{-\alpha h}]C_s + \frac{\nu_1}{\nu_1 - \nu_2} p_{12} e^{-\alpha h} (e^{-(\alpha - \beta \nu_2 h)} - 1)C_s \\
& = \left\{ [1-e^{-\alpha h}] + \frac{\nu_1}{\nu_1 - \nu_2} p_{12} e^{-\alpha h} (e^{-(\alpha - \beta \nu_2 h)} - 1) \right\} C_s h \\
& \cdot e^{-\alpha h} \left[ (1-e^{-\alpha h})^{-2} \right] - J \cdot \left[ (1-e^{-\alpha h})^{-2} I - \left[ I - e^{-\alpha h} P(N) \right]^{-2} P(N) \right] [I - P(N)]^{-1} \cdot v \\
\text{Cost 51} & \quad - (C_s + hC_s) \cdot J \left\{ [I - P(N)]^{-2} \left[ (1-e^{-\alpha h})^{-1} I - \left[ I - e^{-\alpha h} P(N) \right]^{-1} \right] \cdot v \\
& - e^{-\alpha h} \left[ [I - P(N)]^{-1} P(N)[I - P(W)]^{-2} P(N) \right. \\
& \left. + e^{-\alpha h} \left[ [I - e^{-\alpha h} P(N)]^{-1} P(N)[I - P(W)]^{-2} \right] \right\} \cdot w \\
& - C_s \cdot J \cdot \left( (1-e^{-\alpha h})^{-1} I - \left[ I - e^{-\alpha h} P(N) \right]^{-1} \right) [I - P(N)]^{-1} \cdot v \\
& - (C_s + C_p) \cdot J \cdot \left[ [I - e^{-\alpha h} P(N)]^{-1} [I - P(W)]^{-1} \right] w \\
\text{Cost 52} & \quad \left\{ (C_s + hC_s) \cdot J \cdot [I - e^{-\alpha h} P(N)]^{-1} \cdot \left[ [I - P(W)]^{-2} \left[ I - e^{-\alpha h} P(W) \right]^{-1} P(W) \right) \right. \\
& + [I - P(W)]^{-1} \left[ I - e^{-\alpha h} P(W) \right]^{-2} P(W) \right\} \\
& \cdot \left( (1-e^{-\alpha h}) \right) + hC_s e^{-\alpha h} \cdot J \cdot [I - e^{-\alpha h} P(N)]^{-2} P(N) \cdot \left[ I - e^{-\alpha h} P(W) \right]^{-1} P(W) \cdot [I - P(W)]^{-1} \cdot w \\
& + (C_s + C_p) \cdot J \cdot \left[ [I - e^{-\alpha h} P(N)]^{-1} [I - e^{-\alpha h} P(W)]^{-1} P(W) [I - P(W)]^{-1} \right] \cdot w \\
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4.4.3 Numerical Results

In this section, the previously provided partially observable hidden Markov model parameters (found in 4.3.4) are considered in applying the Bayesian process control model to optimize the gear system inspection policy. In [60], it is assumed that the covariance matrix does not change ($\Sigma_1 = \Sigma_2 = \Sigma$). Nevertheless, there is an obvious large difference between $\Sigma_1$ and $\Sigma_2$ during the gear deterioration process. Thus, three situations: $\Sigma_1 = \Sigma_2$, $\Sigma_1 = \Sigma_2 = \Sigma$, and $\Sigma = \Sigma_2$ are used in the application of the Bayesian process control model. In the process of the application, all parameters are kept the same with the exceptions of $P^*$, which is found in Technical Report [61], and $\Sigma$. The unchanged parameters are shown below:

$$
\text{Cost 53}
= e^{-v \cdot h} (1 - e^{-v \cdot h})^{-2} [1 - J \cdot [I - P(N)]^{-1} v] (-C_s - hC_2)
+ (1 - e^{-v \cdot h})^{-1} [1 - J \cdot [I - P(N)]^{-1} v] \left\{ (C_s + C_f) - C_2 \left[ \frac{1}{v_1} (1 - e^{-v \cdot h})^{-1} + 2h - h(1 - e^{-v \cdot h})^{-1} \right] \right\}
+ e^{-v \cdot h} \left\{ J [I - e^{-v \cdot h} P(N)]^{-2} P(N) \left[ [I - P(N)]^{-1} v - [I - P(W)]^{-1} w \right] \right\} (-C_s - hC_2)
+ J \cdot [I - e^{-v \cdot h} P(N)]^{-1} \left\{ [I - P(N)]^{-1} v - [I - P(W)]^{-1} w \right\}
\cdot \left\{ (C_s + C_f) - C_2 \left[ \frac{1}{v_2} (1 - e^{-v \cdot h})^{-1} + 2h - h(1 - e^{-v \cdot h})^{-1} \right] \right\}
$$

$$
\text{Cost 54}
= -C_2 (1 - e^{-v \cdot h}) h e^{-v \cdot h} \left\{ J [I - e^{-v \cdot h} P(N)]^{-1} P(N) [I - e^{-v \cdot h} P(W)]^{-1} P(W) [I - P(W)]^{-1} w \right\}
- \left\{ J [I - e^{-v \cdot h} P(N)]^{-1} [I - e^{-v \cdot h} P(W)]^{-1} P(W) [I - P(W)]^{-1} w \right\}
\cdot \left\{ C_2 \left[ \frac{1}{v_2} + 2h \right] (1 - e^{-v \cdot h}) - h \right\} + (1 - e^{-v \cdot h}) (C_s + C_f) \right\}
\cdot \left\{ (1 - e^{-v \cdot h})^{-1} - J \cdot \left\{ \left( 1 - e^{-v \cdot h} \right)^{-1} J - [I - e^{-v \cdot h} P(N)]^{-1} v - J \cdot [I - e^{-v \cdot h} P(N)]^{-1} [I - P(W)]^{-1} w \right\}
- e^{-v \cdot h} (1 - e^{-v \cdot h}) (C_s + hC_2) J \cdot [I - e^{-v \cdot h} P(N)]^{-1} [I - e^{-v \cdot h} P(W)]^{-2} P^2(W) [I - P(W)]^{-1} w
- (C_1 - C_2) e^{-v \cdot h} h \left\{ (1 - e^{-v \cdot h})^{-2} - J \cdot \left\{ (1 - e^{-v \cdot h})^{-2} J - [I - e^{-v \cdot h} P(N)]^{-2} P(N) [I - P(N)]^{-1} v \right\}
- (C_1 - C_2) \left[ \frac{1}{v_1 - v_2} + h e^{-v \cdot h} \frac{e^{-v \cdot h} - 1}{v_2} \right] \right\}
\cdot \left\{ (1 - e^{-v \cdot h})^{-1} - J \cdot \left\{ (1 - e^{-v \cdot h})^{-1} J - [I - e^{-v \cdot h} P(N)]^{-1} [I - P(N)]^{-1} v \right\}
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\[ \mu_1 = \begin{bmatrix} 2.6979 \times 10^{-4} \\ 1.2995 \end{bmatrix} \]
\[ \mu_2 = \begin{bmatrix} 12.1730 \times 10^{-4} \\ 3.9166 \end{bmatrix} \]
\[ \lambda = \begin{bmatrix} \lambda_{12} \\ \lambda_{13} \\ \lambda_{23} \end{bmatrix} = \begin{bmatrix} 3.6264 \times 10^{-2} \\ 0 \\ 4.3687 \times 10^{-2} \end{bmatrix} \]
\[ v_1 = \lambda_{12} + \lambda_{13} = 3.6264 \times 10^{-2} \]
\[ v_2 = \lambda_{23} = 4.3687 \times 10^{-2} \]

Using Equation 4.17, the control limit for the multivariate Bayesian control chart can be calculated. \( C_1 \) and \( C_2 \) are assumed to be zero. The other cost parameters include \( C_{fi} = 20, \)
\( C_p = 30, \) \( C_f = 150, \) \( C_s = 5 \) and \( T_{fi} = T_{pm} = T_f = 0. \)

(a) \( \Sigma = \Sigma_1 = \begin{bmatrix} 2.9392 \times 10^{-8} & 0.79769 \times 10^{-4} \\ 0.79769 \times 10^{-4} & 0.25244 \end{bmatrix} \) with \( P^* = 0.38 \) for the multivariate Bayesian control chart

A detailed calculation of \( P_{1h} (P_{mhr}, \text{the multivariate Bayesian procedure for posterior probability, where } m = 1) \) is presented below. The sample size \( n = 1 \) and a sample is taken every one time unit.

Assume that \( P_0 (2) = 0, P_0 (1) = 1 \) and \( y_1 = [7.4748 \times 10^{-5}, 0.7021], \overline{P}_{1h} (1), \overline{P}_{1h} (2), Z_1, d \) and \( P_{1h} (2) \) are then calculated as follows:

\[ \overline{P}_{1h} (1) = e^{-(\lambda_{12} + \lambda_{13})h} \cdot [1 - P_0 (2)] = e^{-(3.6264 \times 10^{-2})h} \cdot (1 - 0) = 9.6439 \times 10^{-1} \]
\[ \overline{P}_{1h} (2) = \frac{\lambda_{12} (e^{-\lambda_{12} h} - e^{-(\lambda_{12} + \lambda_{13})h})}{\lambda_{12} + \lambda_{13} - \lambda_{23}} \cdot [1 - P_0 (2)] + e^{-\lambda_{23} h} \cdot P_0 (2) = \frac{3.6264 \times 10^{-2} \cdot (e^{-3.6264 \times 10^{-2}h} - e^{-3.6264 \times 10^{-2}h})}{3.6264 \times 10^{-2} - 3.6264 \times 10^{-2}} = 3.4843 \times 10^{-2} \]
\[ Z_1 = 2 \frac{2}{(y_1 - \mu_1)^T \Sigma^{-1} (\mu_1 - \mu_2)} \]
\[ = 2 \left( \begin{bmatrix} 7.4748 \times 10^{-5} & 2.6979 \times 10^{-4} \\ 0.7021 & 1.2995 \end{bmatrix} - \begin{bmatrix} 2.6979 \times 10^{-4} \\ 1.2995 \end{bmatrix} \right) \left( \begin{bmatrix} 2.9392 \times 10^{-8} & 0.79769 \times 10^{-4} \\ 0.79769 \times 10^{-4} & 0.25244 \end{bmatrix} \right)^{-1} \]
\[ = 12.7480 \]
\[ d^2 = (\mu_2 - \mu_1)^T \Sigma^{-1} (\mu_2 - \mu_1) \]
\[ = \left( \begin{bmatrix} 12.1730 \times 10^{-4} \\ 3.9166 \end{bmatrix} - \begin{bmatrix} 2.6979 \times 10^{-4} \\ 1.2995 \end{bmatrix} \right) \left( \begin{bmatrix} 2.9392 \times 10^{-8} & 0.79769 \times 10^{-4} \\ 0.79769 \times 10^{-4} & 0.25244 \end{bmatrix} \right)^{-1} \]
\[ = 30.6030 \]

Finally, it is obtained that
\[ P_{1h} (2) = \frac{1}{\exp \left[ \frac{1}{2} (d^2 + Z) \right] \cdot P_{1h} (1) + P_{1h} (2)} \]
\[ = 3.4843 \times 10^{-2} \]
\[ \exp \left[ \frac{1}{2} (30.6030 + 12.7480) \right] \]
\[ = 1.3946 \times 10^{-11} \]

For the purpose of comparison with the Bayesian control chart, the statistic values of the
Chi-square chart are calculated using Equation (4.20) as shown below:
\[ \chi_{mh} = \sum_{j=1}^{n} (y_j - \mu_1)^T \Sigma^{-1} (y_j - \mu_1) \] (4.20)

The control limit for the Chi-square chart is 10.6 with the probability of type I error \( \alpha = 0.005 \).

A detailed \( \chi_{1h} \) calculation is obtained below:
\[ \chi_{1h} = (y_1 - \mu_1)^T \Sigma^{-1} (y_1 - \mu_1) \]
\[ = \left( \begin{bmatrix} 7.4748 \times 10^{-5} \\ 0.7021 \end{bmatrix} - \begin{bmatrix} 2.6979 \times 10^{-4} \\ 1.2995 \end{bmatrix} \right) \left( \begin{bmatrix} 2.9392 \times 10^{-8} & 0.79769 \times 10^{-4} \\ 0.79769 \times 10^{-4} & 0.25244 \end{bmatrix} \right)^{-1} \]
\[ = 1.4231 \times 10^{-1} \]

Following the above procedure, the complete computational results for the performance
comparison between the Bayesian control chart and traditional Chi-square control chart are
given in the following Table 4.13.

Table 4.13 Observation and Control Chart Statistics for $\Sigma = \Sigma_1$

<table>
<thead>
<tr>
<th>File</th>
<th>Variance ($y_1$)</th>
<th>RMS ($y_2$)</th>
<th>$P_{mh}$ (2)</th>
<th>$\chi_{mh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rtf_4th</td>
<td>7.4748e-005</td>
<td>0.7021</td>
<td>1.3946E-11</td>
<td>1.4231E+00</td>
</tr>
<tr>
<td>Rtf_5th</td>
<td>9.0243e-005</td>
<td>0.8384</td>
<td>2.5898E-11</td>
<td>1.1159E+00</td>
</tr>
<tr>
<td>Rtf_6th</td>
<td>3.7882e-004</td>
<td>1.6576</td>
<td>2.9736E-07</td>
<td>5.1205E-01</td>
</tr>
<tr>
<td>Rtf_7th</td>
<td>3.8035e-004</td>
<td>1.8446</td>
<td>3.9392E-07</td>
<td>2.0862E+00</td>
</tr>
<tr>
<td>Rtf_8th</td>
<td>4.2011e-004</td>
<td>1.4313</td>
<td>7.3286E-07</td>
<td>2.8903E+00</td>
</tr>
<tr>
<td>Rtf_9th</td>
<td>8.7496e-004</td>
<td>2.8564</td>
<td>6.8535E-01</td>
<td>1.2664E+01</td>
</tr>
<tr>
<td>Rtf_10th</td>
<td>9.8800e-004</td>
<td>3.6058</td>
<td>9.9989E-01</td>
<td>2.1097E+01</td>
</tr>
<tr>
<td>Rtf_11th</td>
<td>12.000e-004</td>
<td>4.1973</td>
<td>1.0000E+00</td>
<td>3.3315E+01</td>
</tr>
<tr>
<td>Rtf_12th</td>
<td>18.000e-004</td>
<td>4.9814</td>
<td>1.0000E+00</td>
<td>8.5838E+01</td>
</tr>
<tr>
<td>Rtf_13th</td>
<td>83.000e-004</td>
<td>10.7413</td>
<td>1.0000E+00</td>
<td>6.4380E+03</td>
</tr>
</tbody>
</table>

Figure 4.8 is the plots corresponding to Table 4.13.

It can be seen from Figure 4.8 that the statistic on the Bayesian control chart increases
dramatically after File 8. The File 9 statistic value falls outside the control limit, which
indicates that the system has moved from State 1 to State 2 between Files 8 and 9, and the
system fault occurs after File 8. Although the Chi-square chart follows a similar trend and
also indicates that the system made the transition between States 1 and 2 between Files 8 and
9, the statistic value difference on the Chi-square chart for the two files is not large and distinct. In many cases, the deterioration of the system cannot be displayed clearly enough by the Chi-square chart (see Figures 4.9-4.10 below).

\[
\begin{bmatrix}
9.9351 \times 10^{-8} & 2.1453 \times 10^{-4} \\
2.1453 \times 10^{-4} & 0.53132
\end{bmatrix}
\]

with \( P^* = 0.053 \) for multivariate Bayesian control chart

Following the same procedure as (a), we can obtain Table 4.14 below.

<table>
<thead>
<tr>
<th>File</th>
<th>Variance ( (y_1) )</th>
<th>RMS ( (y_2) )</th>
<th>( P_{mh} ) (2)</th>
<th>( \chi_{mh} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rtf_4th</td>
<td>7.4748e-005</td>
<td>0.7021</td>
<td>1.2741E-06</td>
<td>8.3922E-01</td>
</tr>
<tr>
<td>Rtf_5th</td>
<td>9.0243e-005</td>
<td>0.8384</td>
<td>3.4998E-06</td>
<td>4.0362E-01</td>
</tr>
<tr>
<td>Rtf_6th</td>
<td>3.7882e-004</td>
<td>1.6576</td>
<td>2.8420E-04</td>
<td>3.4072E-01</td>
</tr>
<tr>
<td>Rtf_7th</td>
<td>3.8035e-004</td>
<td>1.8446</td>
<td>1.3559E-03</td>
<td>1.5020E+00</td>
</tr>
<tr>
<td>Rtf_8th</td>
<td>4.2011e-004</td>
<td>1.4313</td>
<td>3.0980E-05</td>
<td>7.7355E-01</td>
</tr>
<tr>
<td>Rtf_9th</td>
<td>8.7496e-004</td>
<td>2.8564</td>
<td>8.5983E-02</td>
<td>4.6054E+00</td>
</tr>
<tr>
<td>Rtf_10th</td>
<td>9.8800e-004</td>
<td>3.6058</td>
<td>9.8566E-01</td>
<td>1.3576E+01</td>
</tr>
<tr>
<td>Rtf_11th</td>
<td>12.0000e-004</td>
<td>4.1973</td>
<td>1.0000E+00</td>
<td>2.0325E+01</td>
</tr>
<tr>
<td>Rtf_12th</td>
<td>18.0000e-004</td>
<td>4.9814</td>
<td>1.0000E+00</td>
<td>2.5664E+01</td>
</tr>
<tr>
<td>Rtf_13th</td>
<td>83.0000e-004</td>
<td>10.7413</td>
<td>1.0000E+00</td>
<td>1.5656E+03</td>
</tr>
</tbody>
</table>

The corresponding plots to Table 4.14 can be seen in Figure 4.9 below.

Figure 4.9 The Multivariate Bayesian and Chi-square Control Chart for \( \Sigma = (\Sigma_1 + \Sigma_2)/2 \)
From Figure 4.9, it can be seen that the File 9 statistic value falls outside the control limit which indicates that the system made a transition between State 1 and State 2 between Files 8 and 9. Although the value difference between File 8 and File 9 is not large in this case, the occurrence of the system fault can still be identified. However, the deterioration of the system cannot be demonstrated clearly only by the Chi-square chart, since File 9 on the Chi-square chart is below the control limit. This case shows that while the Bayesian chart can definitely prevent the costly system failure, the Chi-square chart may not always provide a signal when the system is in the warning state.

\[(c) \quad \Sigma = \begin{bmatrix}
16.9310*10^{-8} & 3.4930*10^{-4} \\
3.4930*10^{-4} & 0.81020
\end{bmatrix} \text{ with } P^* = 0.018 \text{ for multivariate Bayesian control chart}
\]

Also, Table 4.15 and the plots corresponding to Table 4.15 can be obtained following the same procedure as (a).

<table>
<thead>
<tr>
<th>File</th>
<th>Variance ((v_1))</th>
<th>RMS ((v_2))</th>
<th>(P_{mh} (2))</th>
<th>(\chi_{mh})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rtf_4th</td>
<td>7.4748e-005</td>
<td>0.7021</td>
<td>1.7560E-05</td>
<td>6.4929E-01</td>
</tr>
<tr>
<td>Rtf_5th</td>
<td>9.0243e-005</td>
<td>0.8384</td>
<td>4.1261E-05</td>
<td>2.8221E-01</td>
</tr>
<tr>
<td>Rtf_6th</td>
<td>3.7882e-004</td>
<td>1.6576</td>
<td>1.0854E-03</td>
<td>2.6819E-01</td>
</tr>
<tr>
<td>Rtf_7th</td>
<td>3.8035e-004</td>
<td>1.8446</td>
<td>4.3709E-03</td>
<td>1.1942E+00</td>
</tr>
<tr>
<td>Rtf_8th</td>
<td>4.2011e-004</td>
<td>1.4313</td>
<td>1.5311E-04</td>
<td>4.8849E-01</td>
</tr>
<tr>
<td>Rtf_9th</td>
<td>8.7496e-004</td>
<td>2.8564</td>
<td>6.0122E-02</td>
<td>3.2249E+00</td>
</tr>
<tr>
<td>Rtf_10th</td>
<td>9.8800e-004</td>
<td>3.6058</td>
<td>9.3764E-01</td>
<td>1.0638E+01</td>
</tr>
<tr>
<td>Rtf_11th</td>
<td>12.000e-004</td>
<td>4.1973</td>
<td>9.9996E-01</td>
<td>1.5805E+01</td>
</tr>
<tr>
<td>Rtf_12th</td>
<td>18.000e-004</td>
<td>4.9814</td>
<td>1.0000E+00</td>
<td>1.6907E+01</td>
</tr>
<tr>
<td>Rtf_13th</td>
<td>83.000e-004</td>
<td>10.7413</td>
<td>9.8933E-01</td>
<td>9.4768E+02</td>
</tr>
</tbody>
</table>

Figure 4.10 is the plots corresponding to Table 4.15.
A similar result to (b) can be obtained.

Thus, Figures 4.8-4.10 indicate that the Bayesian chart has a much better statistical performance than the Chi-square chart.

4.5 Conclusions

In this chapter, a CBM model parameter estimation problem was examined. A partially observable hidden Markov model with unobservable system state (except the failure state) was used to depict the relationship between the observations and the three states of a gear deterioration process. The observation process was modeled in the hidden Markov framework while the EM algorithm was used to determine the maximum likelihood estimates of the state and observation model parameters. It can be seen from the numerical results that the numerical iteration procedure has generated a set of reasonable parameter estimates in comparison to the results from judgment. A multivariate Bayesian control chart for CBM applications, considering the control limit policy structure and including an observable failure state, is then introduced. The Bayesian chart is observed to perform better than the traditional Chi-square chart from the numerical results.
Chapter 5

Conclusions and Recommendations for Future Research

This chapter presents a summary of the major results obtained during the course of study documented here, followed by some recommendations for future research.

5.1 Summary of Major Conclusions

Major results are summarized as follows on a chapter basis.

Chapter 2: Wavelet Analysis-Based Gear Shaft Fault Detection

An approach to gear shaft fault detection based on the application of the wavelet transform to both the time synchronously averaged (TSA) signal and residual signal is presented. The autocovariance of maximal energy coefficients based on the wavelet transform is first proposed to evaluate the gear shaft fault advancement quantitatively. For a comparison, the advantages and disadvantages of some approaches such as using standard deviation, kurtosis, and the value of the K-S test statistic as fault indicators with continuous wavelet transform (CWT) and discrete wavelet transform (DWT) for residual signal, are discussed. The main results can be summarized as follows:

(1) The statistical measure kurtosis is unable to reveal the occurrence and advancement of gear shaft cracks for both CWT and DWT.

(2) The standard deviation based on \( \{P(i)\} \) for the residual signal as an indicator over full the gear shaft lifetime is able to diagnose early gear fault and fault advancement. It is a good indicator.

(3) The K-S test statistic \( K \) can effectively detect the gear shaft crack occurrence, its
advancement, and the faulty state with DWT based on residual signals.

(4) The conclusion was obtained that the gear shaft was in a healthy state during data files 194 to 217, a state with small crack occurrence in data file 218, and completely broken state between data files 230 and 231.

It was demonstrated using real vibration data that the early faults in gear shafts can be detected and identified successfully using wavelet transforms combined with the approaches mentioned in Chapter 2.

Chapter 3: Wavelet Analysis with Time-synchronous Averaging of Planetary Gearbox Vibration Data

In Chapter 3, planetary gearbox fault detection based on the application of wavelet transform with the autocovariance of maximal energy coefficients to both the TSA signal and the residual signal has been investigated. Some fault indicators such as variance, kurtosis, root mean square (RMS), and crest factor with CWT and DWT for the original signal, TSA signal, and CWT and DWT applied to the TSA and residual signals are analyzed and discussed. Based on these investigations, the following conclusions may be given:

(1) Whether it is in a healthy state or not, we cannot clearly distinguish between meshing frequencies and noise frequencies, when FFT was applied to the original, TSA, CWT and DWT applied to TSA signals, and CWT and DWT applied to the residual signal with autocovariance.

(2) There is evident meshing frequency of stage 1 and some harmonics of stage 2 in the FFT plot of only CWT applied to the TSA signal with autocovariance. However, this result is not obtained for DWT applied to the TSA signal with autocovariance. The noise strength of the failure state exceeds that of the healthy state for both CWT and DWT.

(3) The variance, kurtosis, RMS and crest factor are not suitable for purposes of feature extraction for the original, TSA, residual, CWT applied to TSA and residual, and CWT applied to
TSA and residual signals with autocovariance.

(4) Variance and RMS are suitable for purposes of feature extraction for DWT applied to the TSA and residual signals with or without autocovariance, but the result for TSA is better.

(5) Finally, we can conclude that the deterioration stage occurred from the Rtf_9th_run data on and that the failure stage occurred from the Rtf_13th_run data on.

The results reveal that the method can enhance the capability of feature extraction, fault diagnosis, and CBM purposes for the planetary gearbox.

Chapter 4: Multivariate Bayesian Control Chart for CBM

In this chapter, a CBM model parameter estimation problem was examined. A partially observable hidden Markov model with unobservable system state (except the failure state) was used to depict the relationship between the observations and the three states of a gear deterioration process. The observation process was modeled in the hidden Markov framework while the EM algorithm was used to determine the maximum likelihood estimates of the state and observation model parameters. The results are shown in the table below:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Subjective Judgment</th>
<th>Iteration Results</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$ of $y_1$</td>
<td>2.6885E-04</td>
<td>2.6979E-04</td>
<td>0.3490</td>
</tr>
<tr>
<td>$\mu_2$ of $y_1$</td>
<td>12.1574E-04</td>
<td>1.2173E-03</td>
<td>0.1282</td>
</tr>
<tr>
<td>$\mu_1$ of $y_2$</td>
<td>1.2948E+00</td>
<td>1.2995E+00</td>
<td>0.3623</td>
</tr>
<tr>
<td>$\mu_2$ of $y_2$</td>
<td>3.9102E+00</td>
<td>3.9166E+00</td>
<td>0.1635</td>
</tr>
<tr>
<td>$\Sigma_1(1,1)$</td>
<td>2.9245E-08</td>
<td>2.9392E-08</td>
<td>0.5014</td>
</tr>
<tr>
<td>$\Sigma_1(1,2)$</td>
<td>7.9600E-05</td>
<td>7.9769E-05</td>
<td>0.2121</td>
</tr>
<tr>
<td>$\Sigma_1(2,2)$</td>
<td>2.5300E-01</td>
<td>2.5244E-01</td>
<td>0.2216</td>
</tr>
<tr>
<td>$\Sigma_2(1,1)$</td>
<td>1.69868E-07</td>
<td>1.6931E-07</td>
<td>0.3290</td>
</tr>
<tr>
<td>$\Sigma_2(1,2)$</td>
<td>3.4993E-04</td>
<td>3.4930E-04</td>
<td>0.1802</td>
</tr>
<tr>
<td>$\Sigma_2(2,2)$</td>
<td>8.1100 E-01</td>
<td>8.1020E-01</td>
<td>0.0987</td>
</tr>
<tr>
<td>$\lambda_{01}$</td>
<td>3.6360E-02</td>
<td>3.6264E-02</td>
<td>0.2644</td>
</tr>
<tr>
<td>$\lambda_{02}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{12}$</td>
<td>4.3500E-02</td>
<td>4.3687E-02</td>
<td>0.4290</td>
</tr>
</tbody>
</table>

It can be seen from the numerical results that the numerical iteration procedure has generated a set of reasonable parameter estimates when compared to the results obtained from simple judgment.

Then, a multivariate Bayesian control chart for condition-based maintenance applications is considered by using the control limit policy structure and involving an observable failure state. The following results have been obtained.

1. The system moved from State 1 to State 2 between Files 8 and 9, and the system fault occurs after File 8.

2. The Bayesian chart can definitely prevent the costly system failure in three situations: $\Sigma = \Sigma_i$, $\Sigma = (\Sigma_i + \Sigma_2)/2$, and $\Sigma = \Sigma_2$, which are used in the application of the Bayesian process control model as approximations.

3. The Chi-square chart may not always provide a signal when the system is in the warning state. Although the Chi-square chart indicates that the system made the transition between States 1 and 2 between Files 8 and 9 for the case $\Sigma = \Sigma_i$, the deterioration of the system cannot be displayed clearly enough by the Chi-square chart in other cases.

4. The Bayesian chart has a much better statistical performance than the Chi-square chart.
5.2 Recommendations for Future Research

The following recommendations can be made for future research.

5.2.1 Extending the Research to Planetary Gear Bearings Using Vibration Data

Bearings are one of the most important and frequently encountered components in rotating machinery. Failure of the bearings can cause catastrophic breakdown. Fault identification of rolling bearings using condition monitoring techniques has been the subject of extensive research.

Two types of bearings are used to support a shaft: sliding and rolling bearings. With small friction, high rotation flexibility and easy maintenance, rolling bearings are widely used standard components of mechanical equipment. The structure of rolling bearings consists of four components: inner race—tightly mounted on the shaft and rotating with it; outer race—usually mounted on the bearing’s bracket hole and fastened, but for planetary gear bearings, rotating together with a carrier; rolling elements—located between the inner and outer rings which include balls, columns, and cones; retainer—used to isolate the rolling element. The structure of rolling bearings is shown in Figure 5.1.

![Figure 5.1 The Structure of Rolling Bearings](image)

During the bearing operation, wide band impulses are generated when rollers pass over the defect at a frequency determined by shaft speed, bearing geometry, and defect location (inner-race, roller, or outer-race).
Therefore, bearing monitoring poses important challenges to the machine maintenance and the automation processes.

5.2.2 Extending the Research to Planetary Gear Carrier Using Vibration Data

The carrier plate is a critical component of the transmission, aiding to transmit mechanical power from the engines to the main rotor blades. Much work has been done using vibration analysis to detect and diagnose tooth faults in the gears, but failure in the carrier can also cause catastrophic event such as aircraft accidents, which may lead to the loss of aircraft and the lives of people on board. The carrier plate crack, illustrated in Figure 5.2, developed on the root of one of the five planet gear mounting posts of the planetary carrier plate, which is a region of high stress.

![Figure 5.2 Planetary Carrier Plate with a Crack](image)

Note that the fault on the planetary gear is a crack in the carrier plate, and it is different from the usual tooth crack or breakage. It is possible that the resonance frequency of the planetary gear that may indicate the plate crack fault are averaged out, since TSA tends to average out external disturbances and noises that are not synchronized with the carrier rotation. Even some meshing harmonics and their sidebands may also be averaged out or reduced if their initial phases at the start of each carrier rotation are different.

It is highly desirable to develop a simple, cost-effective test capable of diagnosing this fault
based on vibration data analysis techniques.

5.2.3 Fault Detection and CBM of Planetary Gear Bearings and Carrier Using Vibration Data

A major problem with detecting and diagnosing faults in the bearings and carrier in gearboxes is that their vibration signals are strongly masked by gear signals, because the vibration signals collected by the sensors tend to be composites of vibrations associated with all of the transmission components, such as the meshing gears, shafts, bearings, and other parts. This is more complicated in the planetary drive, where signals from the planet gear bearings and carrier must be transmitted via a time-varying path through the ring gear to externally mounted accelerometers, meaning that mixing between the bearing and carrier signals and the gear signals cannot be avoided.

The future research should focus on the diagnosis of single and multiple faults using wavelet transform applied to planetary gear bearings and carrier vibration data as well as on the development of CBM models using health indexes obtained from the wavelet transform data as observation processes. The work should investigate the fault detection and CBM of planetary gear bearings and carrier using vibration data, and the objectives of the study should be:

a) Planetary gear bearings and carrier vibration data analysis using wavelet approaches.
b) Identification and modeling of health indexes.
c) Development of a 3 state CBM model with partial information for fault detection and maintenance decision-making.
d) Development of effective estimation procedures for simultaneous estimation of parameters of a 3 state hidden Markov model describing the evolution of the unknown system state as well as the model parameters of the observation process.
e) Model testing using lab and real data.
Bibliography


[32] MDTB Data (Data CDs: Test-Runs #9, #7, #5 and #13), Condition-Based Maintenance Department, Applied Research Laboratory, The Pennsylvania State University, 1998.


[39] Vibration Data (Baseline Data: Syncrude Gearbox Rig-Commissioning Data), Syncrude Canada Ltd., 2010.


Appendix A

Summaries of Values towards Each Stage

<table>
<thead>
<tr>
<th>Notation</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; Stage</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; Stage</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun /Shaft Gear Teeth</td>
<td>$N_s$</td>
<td>18</td>
<td>28</td>
</tr>
<tr>
<td>Planet/Bevel Gear Teeth</td>
<td>$N_p$</td>
<td>72</td>
<td>62</td>
</tr>
<tr>
<td>Annulus Gear Teeth</td>
<td>$N_a$</td>
<td>152</td>
<td>81</td>
</tr>
<tr>
<td>Stage Ratio</td>
<td>$R$</td>
<td>4.0</td>
<td>6.429</td>
</tr>
<tr>
<td>Ring (rpm)</td>
<td>$R_s$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| Sun Gear (rpm)           | $f_s$                 | 1200                  | 300                   | 46.667                |
| Carrier (rpm)            | $f_c$                 |                       | 46.667                | 8.867                 |
| Planet (rpm)             | $f_p$                 |                       | 67.737                | 14.301                |
|                          | $f_{rp}$              |                       | 114.401               | 23.167                |
| Gear Meshing             | $f_m$                 |                       |                       |                       |
| Freq. (Hz)               | $f_{rs}$              | 1200                  | 253.333               | 37.8                  |
|                          | $=N_s f_c=N_p f_{rp}$ | 18*1200/60            | 28*253.333/60         | 19*37.8/60            |

The calculation method is divided into two steps: carrier speed and planet speed

**Step 1: Carrier Speed (rpm)**

The carrier speed of each gearbox can be calculated:

$$R = \frac{N_a + N_s}{N_s} ; f_c = \frac{f_s}{R}$$

The first stage:

$$R_1 = \frac{N_{p1}}{N_{s1}} = \frac{72}{18} = 4$$
\[
f_{c1} = \frac{f_{s1}}{R_1} = \frac{1200}{4.0} = 300
\]

The second stage:

\[
R_2 = \frac{N_{a2} + N_{s2}}{N_{s2}} = \frac{152 + 28}{28} = 6.429
\]

\[
f_{c2} = \frac{f_{s2}}{R_2} = \frac{f_{c1}}{R_2} = \frac{1200}{4 \times 6.429} = 46.667
\]

The third stage:

\[
R_3 = \frac{N_{a3} + N_{s3}}{N_{s3}} = \frac{811 + 19}{19} = 5.263
\]

\[
f_{c3} = \frac{f_{s3}}{R_3} = \frac{f_{c2}}{R_3} = \frac{f_{c1}}{R_1 R_2 R_3} = \frac{1200}{4 \times 6.429 \times 5.263} = 8.867
\]

**Step 2: Planet Speed (rpm)**

\[f_{p} = f_c \times \frac{N_a}{N_p}, \text{ where } f_p \text{ is planet relative rotation}\]

\[f_p = -f_c + f_{p} = -f_c + f_c \times \frac{N_a}{N_p} = f_c \left( \frac{N_a - N_p}{N_p} \right), \text{ where } f_p \text{ is planet absolute rotation}\]

The second stage:

\[
f_{p2} = f_{c2} \times \frac{N_{a2}}{N_{p2}} = \frac{f_{s1}}{R_1 R_2} \times \frac{N_{a2}}{N_{p2}} = \frac{1200}{4 \times 6.429} \times 152 = 114.401
\]

\[
f_{p2} = -f_{c2} = f_{p2} = f_{c2} \left( \frac{N_{a2} - N_{p2}}{N_{p2}} \right) = \frac{f_{s1}}{R_1 R_2} \left( \frac{N_{a2} - N_{p2}}{N_{p2}} \right) = \frac{1200}{4 \times 6.429} \times \frac{152 - 62}{62} = 67.737
\]
The third stage:

\[ f_{p3} = f_{c3} \times \frac{N_{a3}}{N_{p3}} = \frac{f_{s1}}{R_1R_2R_3} \times \frac{N_{a3}}{N_{p3}} = \frac{1200}{4 \times 6.429 \times 5.263} \times \frac{81}{31} = 23.167 \]

\[ f_{p3} = -f_{c3} + f_{p3} = f_{c3} \left( \frac{N_{a3} - N_{p3}}{N_{p3}} \right) = \frac{f_{s1}}{R_1R_2R_3} \left( \frac{N_{a3} - N_{p3}}{N_{p3}} \right) = \frac{1200}{4 \times 6.429 \times 5.263} \times \frac{81 - 31}{31} = 14.301 \]
## Appendix B

### Results of Indicators Analysis

Table B.1 Indicators Analysis of Original Signal

<table>
<thead>
<tr>
<th></th>
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<th>Crest Factor</th>
</tr>
</thead>
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Table B.2 Indicators Analysis of TSA Signal

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<td>RMS</td>
<td>Crest Factor</td>
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<th>Crest Factor</th>
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Table B.5 Indicators Analysis of Autocovariance with CWT Applied to TSA Signal

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Table B.6 Indicators Analysis of CWT Applied to Residual Signal

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### Table B.7 Indicators Analysis of Autocovariance for CWT Applied to Residual Signal

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### Table B.8 Indicators Analysis of DWT Applied to TSA Signal

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<th>Crest Factor</th>
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### Table B.9 Indicators Analysis Autocovariance for DWT Applied to TSA Signal

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### Table B.10 Indicators Analysis of DWT Applied to Residual Signal

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