Ptolemy's Planetary Theory: An English Translation of Book One, Part A of the *Planetary Hypotheses* with Introduction and Commentary

by

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Abstract

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This study comprises a translation and commentary of Book I of the *Planetary Hypotheses* by the second century A.D. Greco-Roman astronomer, Claudius Ptolemy. It closely examines the *Planetary Hypotheses* on its own and in relation to Ptolemy's other writings. Where necessary I rely on astronomical, philosophical, and technological works by other writers in order to better situate Ptolemy's ideas into the context of Greco-Roman science.

The dissertation is organized into three sections. Section I consists of an extended introduction to the *Planetary Hypotheses*. I offer a synopsis of the *Planetary Hypotheses* and a history of the text in Sections I.1 and I.2. Section I.3 consists of a brief introduction to notation and sexagesimal numbers while Section I.4 analyzes the aim and function of Ptolemy's planetary models.

Section II is a translation of the existing Greek text of the *Planetary Hypotheses*, namely Book I Part A, and a précis of Book I, Part B. The translation is made from J.L. Heiberg's edited Greek text and the précis relies on the English translation by Bernard Goldstein, the French translation by Regis Morelon, and the Arabic Manuscripts found in the British Library (Arabic-A) and the Library at the University of Leiden (Arabic-B).

The footnotes include variant readings from the different Greek and Arabic Manuscripts. A list of all existing manuscripts of the *Planetary Hypotheses* can be found in Section I.2

Section III is a commentary of the entirety of Book I (Parts A and B). This section is arranged so that it loosely follows the order of topics found in the *Planetary Hypotheses*. Section III.1 examines the *Planetary Hypotheses* in terms of instrument-making. Section III.2 discusses the geometric models that Ptolemy presents along with a discussion of the changes that he makes. I give an overview of the period relations and mean motions presented in the *Planetary Hypotheses* in Section III.3 and III.4 and the new frame of reference in Section III.5. Section III.6 briefly examines Book II of the *Planetary Hypotheses* and Section III.7 addresses the relationship of Book I and Book II and contextualizes this work in the history of Greco-Roman science. Finally, Section III.8 examines the role the *Planetary Hypotheses* played in developments within Medieval Islamic astronomy.

While I focus on the changes that Ptolemy made to the models in the *Planetary Hypotheses* from his theories in the *Canobic Inscription*, *Handy Tables*, and the *Almagest*, this work aims to explore the motivations behind these changes. Additionally, I contextualize the *Planetary Hypotheses* within Greco-Roman and Islamic astronomy and technology. What emerges from this dissertation is a consideration of Ptolemy's ideas about the practice of science and an analysis of how he modeled astronomical observations.

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Section I:

Introduction

The second-century A.D. natural philosopher Claudius Ptolemy is arguably the most important author whose works exist on Greco-Roman science. His works on science encapsulated and exceeded the work of his predecessors.[1] The wide variety of subjects on which Ptolemy wrote includes astronomy, astrology, optics, harmonics, spherical geometry, and geography. While he owes much of his success to his predecessors, it is Ptolemy's own contributions – namely his ingenuity, his thoroughness, and his ability to coalesce theories – that made him an authority in so many fields.[2]

While several of Ptolemy's works have survived, little is known about the man himself. There are a small number of references to Ptolemy by late Greco-Roman and Byzantine authors, such as Olympiodorus and Theodore Meliteniotes, but the little that historians know about Ptolemy's life comes primarily from his surviving texts.[3] Ptolemy lived and worked in Alexandria, which was part of the Roman Empire. His name is representative of both his geographical location and nationality: his first name, Claudius, indicates that he was a Roman citizen and his last name, Ptolemaeus, suggests that he

[1] For a description of Ptolemy's works see the article in the *New Dictionary of Scientific Biography*, s.v. "Ptolemy."
[2] *Dictionary of Scientific Biography*, s.v. "Ptolemy."
[3] Toomer 1987, 186.

lived in Egypt and had Greek ancestors.[4] Alexandria was an intellectual and trading center; it was highly influenced by Greek culture and philosophy and possessed one of the most comprehensive libraries in the ancient world.

Although the precise years of Ptolemy's birth and death are unknown, his dates can be deduced from his recorded astronomical observations. The earliest observation documented by Ptolemy and recorded in his astronomical work, the *Almagest*, is dated March 26, 127 A.D. and the latest is dated February 2, 141 A.D.[5] Since Ptolemy is known to have been active between A.D. 127 and A.D. 141, and given that most of his works were completed after the *Almagest*, it is reasonable to suggest that Ptolemy lived somewhere between A.D. 100 and A.D. 175.

This study aims to examine Book I of the *Planetary Hypotheses* in detail.[6] The *Planetary Hypotheses* is related to and directly builds upon many of Ptolemy's other works, including: the *Canobic Inscription*,[7] *Handy Tables*,[8] *Optics*,[9] *Harmonics*,[10] and the *Almagest*.[11] The *Planetary Hypotheses* is an astronomical work, which was written after Ptolemy's comprehensive and influential astronomical, the *Almagest*. Similar to the *Almagest*, the *Planetary Hypotheses* provides a geocentric, mathematical account of the heavenly motions. However, while the *Almagest* consists of thirteen detailed books and many tables, which can used to calculate the exact positions of the heavenly bodies, the *Planetary Hypotheses* is a much briefer account. Additionally, there are many changes to

4 Toomer 1987, 187.
5 Toomer 1998, 450, 525.
6 Heiberg 1907, 70-143.
7 Jones 2005a, 53-98. Heiberg 1907, 148-155.
8 Heiberg 1907, 159-185.
9 Lejeune 1989.
10 Düring 1930.
11 Heiberg 1898.

the *Planetary Hypotheses* from Ptolemy's other astronomical works, such as the *Canobic Inscription* and *Handy Tables* – the former believed to have been a public inscription that was copied and has been preserved in a manuscript tradition[12] and the latter consists of an introduction and a series of astronomical tables that have been revised from the *Almagest*. Finally, there are many unique ideas that Ptolemy only addresses in the *Planetary Hypotheses*, such as the distances of the planets, and constructing an instrument that displays the motion of the cosmos.

While there are have been important contributions to the study of the *Planetary Hypotheses*, namely by Dennis Duke,[13] Bernard Goldstein,[14] Willy Hartner,[15] Regis Morelon,[16] Andrea Murschel,[17] Otto Neugebauer,[18] Noel Swerdlow,[19] and others, this is the first systematic study of Book I of the *Planetary Hypotheses*. The methodology used in this dissertation can most accurately be described as a textual study. I examine the *Planetary Hypotheses* both internally and in relation to Ptolemy's other works and surviving scientific texts by other authors.

Section I of this dissertation consists of a brief introduction to Ptolemaic astronomy and in particular the *Planetary Hypotheses*. The introduction is broken up into several parts, where I offer a description of the *Planetary Hypotheses* (Section I.1), a history of the manuscripts and scholarship (Section I.2), an overview of notation (Section

[12] Jones 2005a, 3.
[13] Duke 2009, 635-654.
[14] Goldstein 1967.
[15] Hartner. 1964.
[16] Morelon 1993.
[17] Murschel 1995.
[18] Neugebauer. 1975.
[19] Swerdlow 1968, 2005.

I.3), and an examination of the aim and function of the *Planetary Hypotheses* (Section I.4).

Section II offers a translation of Book I, Part A made from Heiberg's Greek text and a summary of Book I, Part B made from the Arabic text, Morelon's French translation, and Goldstein's English translation.[20] Variant readings from the Greek and Arabic manuscripts are included.

Finally, the third section is a commentary of Book I where I analyze the changes Ptolemy makes to his periodicities, parameters, and models. Like the introduction, the commentary consists of multiple sections, where I examine *Sphairopoiia* (Section III.1), Ptolemy's geometrical models (Section III.2), simple and complex mean motion periods (Section III.3-4), point of reference (Section III.5), Book I, Part B (Section III.6), the *Planetary Hypotheses* as a complete work (Section III.7), and finally the *Planetary Hypotheses* in Medieval Islamic astronomy (Section III.8).

This study engages with existing scholarship in the fields of history of astronomy, history of technology, philosophy of science, and history of mathematics. I examine the purpose of the *Planetary Hypotheses* and its place in the context of Greco-Roman science and technology. Additionally, I focus on the changes Ptolemy made in the *Planetary Hypotheses* from his earlier astronomical theories, such as the *Almagest*, *Canobic Inscription*, and *Handy Tables*. The aim is not to catalog Ptolemy's changes, but rather to explore the motivations for these changes. As such, this study offers insight into what Ptolemy thought about the practice of science and how he modeled the world around him.

[20] Heiberg 1907. Morelon 1993. Goldstein 1967.

I.1 Description of the *Planetary Hypotheses*

The *Planetary Hypotheses* is divided into two books. In Book I Ptolemy generated a comprehensive model of the universe including the distances and relative sizes of the planets. At the beginning of Book I, he states that he would present the models from the *Almagest*, but in a more succinct fashion so as to make them more comprehensible to astronomers and instrument-makers. Ptolemy wanted the craftsman to build a device that would represent the movements of the celestial bodies, including their various anomalies, using a mechanical motion, or in a more "naked" way by moving the parts of the device by hand. Additionally, each model of a celestial body could be presented as a separate device so that there would be one model for Venus, for example, or so that all of the models could be combined to make one large device.[21]

The theory of nesting spheres was only discussed by Ptolemy in the *Planetary Hypotheses*. This idea – that the spheres of the planets are situated so that a planet's greatest distance from the Earth is equal to the closet distance of the planet above it, and that there is no empty space – was utilized by Islamic and Medieval European astronomers.[22] Ptolemy used the ratios of the deferent and epicycle circles along with his estimates of the distance of the Sun and Moon to determine the distances of the other planets by assuming that there is no empty space in the universe.[23] Making this assumption was not a problem for Ptolemy since a void free universe is in line with ideas on the natural world put forth by Aristotle and the Stoic philosophers, which had a significant influence on Ptolemy's works. However, this theory caused Ptolemy problems

[21] See Section III.1 below.
[22] Rosen 1978. Duncan 1981. Neugebauer 1975, 918.
[23] Goldstein 1967, 7-9. See Section III.7 below.

when calculating the distances of Mercury and Venus. His calculated distances of the Moon and the Sun do not correspond exactly with the ratios of eccentric circles to epicycles for Mercury and Venus; consequently, Ptolemy is left with a void of 81 Earth radii between the outermost sphere of Venus and the inner most sphere of the Sun.[24]

In Book II, which is different in its scope and objective than Book I, Ptolemy examined the physical cosmology of the heavens and the laws by which the heavens are governed. Each body in Ptolemy's cosmology, both the celestial and terrestrial, acts according to its own natural motion; a terrestrial body can be moved by a violent force, but a celestial body is not under the influence of any other body.[25] Ptolemy believed that the cosmos is governed by a different set of laws than the ones that govern the terrestrial world and that we cannot understand the way the heavens work by examining the sublunary sphere. Although Ptolemy was arguing against Aristotle and Peripatetic natural philosophers, he often used terrestrial analogies to describe the heavenly motions.[26]

When discussing the spheres in the heavens, Ptolemy criticized Aristotle's unwinding spheres because the counteracting spheres would take up space, but serve no purpose.[27] He rejected the view that the motion of the outer spheres is transmitted to the inner spheres. Instead, Ptolemy said that each celestial body is driven by its own soul, which moves the sphere containing it. Consequently, each sphere acts according to its own natural motion, so that the motion of the epicycle sphere and the motion of the deferent sphere are both caused by the planet, but manifest in different types of motion.[28]

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[24] Goldstein 1967, 7-9. Murschel 1995, 36. See Section III.7 below.
[25] Murschel 1995, 38.
[26] For example, he compared the movement of the spheres to the movement of synchronized dancers. Heiberg 1907, 119. Murschel 1995, 38, n. 34.
[27] Heiberg 1907, 112-115. Aristotle, *Metaphysics*, 1073a14-1074b14.
[28] Murschel 1995, 39.

Ptolemy disagreed with the idea that the pole of one of the heavenly spheres was attached to the surface of the sphere that contains it. He contended that the ethereal spheres move uniformly since they are in their natural place and they did not require a mechanism, such as an axis, to move them.[29]

Finally, in Book II Ptolemy stated that entire spheres were not necessary to exhibit the motion of the planets and all of the heavenly spheres, except for the sphere of the fixed stars, could be replaced with *manshūrāt,* i.e. sawn-off spheres similar to rings or tambourines. Instead of concentric spheres nested within each other, the cosmos would consist of sawn-off spheres. Ptolemy wanted to use the smallest number of spheres possible to represent the heavenly motions; he said 34 spheres are needed with the full sphere model, and that 22 are needed with the sawn-off model. Although Ptolemy maintained that there was no difference between the model with the full spheres and the model with the sawn-off spheres, he preferred the sawn-off model.[30] Starting with the geometric models of the *Almagest*, Ptolemy provided a physical, three-dimensional description of the cosmos in the *Planetary Hypotheses* that adheres to the foundations he sets out in the *Almagest*.[31] Book II ends with a series of tables, which unfortunately have not survived.[32]

[29] Heiberg 1907, 113-115. Murschel 1995, 39.
[30] Heiberg 1907, 122-125. Murschel 1995, 50-52.
[31] Murschel 1995, 51-55.
[32] Heiberg 1907, 113-115.

I.2 History of the Text

Like most of Ptolemy's works, the *Planetary Hypotheses* is an undated text. The only Ptolemaic text that has a date is the *Canobic Inscription*, which is believed to be Ptolemy's earliest work. It was erected in the city of Canopus in A.D. 146 or 147.[33] The *Canobic Inscription* is dated at least five years after the latest observation recorded in the *Almagest*, but N.T. Hamilton, N.M. Swerdlow, and G.J. Toomer have established that it was written before the *Almagest*.[34] Since the *Canobic Inscription* was written after A.D. 147, then Ptolemy must have completed the *Almagest* after that, perhaps as early as A.D. 150. Ptolemy states in the *Planetary Hypotheses* that he will present a revision of the models of astronomical motion from the *Almagest*, which indicates that he had already completed the *Almagest* when he wrote the *Planetary Hypotheses*. We can then safely assume that Ptolemy wrote the *Planetary Hypotheses* at some point between A.D. 150 and A.D. 175.[35]

The *Planetary Hypotheses* has a complicated history. The first part of Book I of the *Planetary Hypotheses* has survived in the original Greek; the remainder of the work exists in translation only. The entirety of the *Planetary Hypotheses* was translated from Greek into Arabic during the Middle Ages and a Hebrew translation was made from one of the Arabic translations.

[33] Jones 2005a, 53.
[34] Hamilton, Swerdlow, Toomer 1987, 55-72.
[35] The *Almagest* is mentioned in the introductions of the *Tetrabiblos*, *Handy Tables*, and *Planetary Hypotheses*. Toomer 1998, 1.

I.2.1. Manuscripts

The following is a list of manuscripts of the *Planetary Hypotheses*. I will use Heiberg's system of referencing unless otherwise noted.[36] Since several Arabic and Greek manuscripts have the same letter assigned to them, the language of the manuscript is designated in addition to the letter. Manuscripts not utilized by Heiberg are referenced using Morelon's system of reference.[37]

Greek Manuscripts

Greek-A
Vaticanus Gr. 208 (a fourteenth-century paper manuscript)

Greek-B
Vaticanus Gr. 1594 (a ninth-century parchment manuscript)

Arabic Manuscripts

Arabic-A.
British Library Manuscript (complete)
British Library, MS. Arab. 426 (Add. 7473), fol. 81b-102b.[38]
This manuscript is dated 1242, but the author is unknown. Heiberg refers to it as manuscript A, Goldstein refers to it as manuscript BM, and Morelon refers to it as manuscript B.

Arabic-B.
Leiden Manuscript (complete)
Leiden, MS. Arab. 1155 (cod. 180 Gol.), fol. 1a-44a.[39]
The Leiden Manuscript is undated; it was translated by Thābit ibn Qurra, who died in 901 A.D., which means that was probably written in the ninth-century or tenth-century. It is located in the Leiden University Library. Heiberg refers to this manuscript as manuscript B; Goldstein and Morelon refer to it as manuscript L.

[36] Heiberg 1907, vi-ix. Goldstein 1967, 5. Tihon 1978, 47-49.
[37] Morelon 1993, 9-10.
[38] Goldstein 1967, 5. Heiberg 1907, ix. Morelon 1993, 9-10.
[39] Goldstein 1967, 5. Heiberg 1907, ix. Morelon 1993, 9-10.

Arabic-Q.
Cairo Manuscript (incomplete)
Dār al-kutub, Taymūr, Riyadiyyāt 238, pp. 1-5.[40]
This is a partial Arabic manuscript located in Cairo. It was made from the British Library manuscript. Morelon refers to this manuscript as manuscript Q; Heiberg and Goldstein do not utilize it.

Hebrew Manuscripts

Hebrew-H.
Paris Manuscript (complete)
Paris, MS. Hebr. 1028 (*ancien fonds* 470), fol. 54b-87a.[41]
Kalonymos b. Kalonymos translated this manuscript and it is dated 1342. Made from the Arabic manuscript in the British Library, it is located in the Bibliothèque Nationale de France in Paris. Heiberg does not use this manuscript. Goldstein refers to it as manuscript Hebrew and Morelon refers to it as manuscript H.

[40] Morelon 1993, 9-10. Neugebauer 1975, 900.
[41] Goldstein 1967, 5. Morelon 1993, 10.

I.2.2 Translations and Scholarship

The *Planetary Hypotheses* has been translated into several languages from the Greek and Arabic manuscripts. The first part of Book I was first translated from Greek into Latin in 1620 by John Bainbridge, professor of astronomy at Oxford University. Bainbridge also made corrections to the parameters found in the text, noting the new latitude theory presented by Ptolemy.[42] In 1820 N. Halma published a translation of Ptolemy's *Planetary Hypotheses* and Proclus Diadochus's (fifth-century A.D) *Hypotyposis.*[43]

J.L. Heiberg included a translation by Ludwig Nix of the *Planetary Hypotheses*, from Arabic into German, in *Opera Astronomica Minora,* published in 1907.[44] L. Nix conducted the translation of the *Planetary Hypotheses* from Arabic into German for Heiberg's edition. Nix died before completing his translation, which was then completed by F. Buhl and P. Heegaard. Unbeknownst to Buhl and Heegard, Nix had not completed the entirety of the translation of Book I; the two translators began their translation at the beginning of Book II. The translation was thus published with a substantial section at the end of Book I missing. After examining passages in al-Bīrūnī's (d. c. 1050) works, *India* (*Kitāb Taʾrīkh al-Hind*) and *Canon Masudicus* (*Kitāb al-Qanūn al-Masʿūdī fī 'l-hayʾa wa-'l-nujūm*) that discussed the distances of the planets attributing the ideas to Ptolemy, Willy Hartner began to suspect that the Arabic references to the sizes and distances of the planets stemmed from a work by Ptolemy.[45] These passages attributed the theory of the sizes and distances of the planets to Ptolemy's *Kitāb al-Manshūrāt.* C.A. Nallino had

[42] Bainbridge 1620.
[43] Halma 1820.
[44] Heiberg 1907.
[45] Sachau 1887 and 1910. Haydarabad, ed. 1954-1956.

previously hypothesized that al-Bīrūnī's reference to *Kitāb al-Manshūrāt* was in fact a reference to Proclus's *Hypotyposis* and that al-Bīrūnī had mistakenly attributed this work to Ptolemy instead of its proper author, Proclus.[46] Hartner argued that *Kitāb al-Manshūrāt* was in fact the Arabic name for Ptolemy's work the *Planetary Hypotheses.*[47] Hartner correctly predicted that Ptolemy discussed the sizes and distances of the celestial bodies in the text, but concluded that this portion of the text was lost. Bernard Goldstein examined the Hebrew manuscript of the *Planetary Hypotheses*, which was not consulted for Heiberg's edition and found that the portion on the sizes and distances of the planets was indeed found in the manuscript. The Arabic manuscripts confirmed that these ideas were found in Book I. Goldstein published an English translation of the previously unpublished portion of the *Planetary Hypotheses*, along with the Arabic text (Arabic-A) and variant readings.

In 1993 Regis Morelon published a French translation of Book I from the Arabic. His translation utilizes all three Arabic manuscripts as well as the Hebrew manuscript. Morelon included an edited version of the Leiden manuscript (*Arabic-B*) along with an apparatus of variant readings. Additionally, Eulalia Pérez Sedeño published a translation of Book I of the *Planetary Hypotheses* into Spanish along with a brief commentary of both Book I and Book II.[48]

Much of the considerable scholarship on Ptolemy is devoted to the *Almagest.* The translation of the *Almagest* by Gerald Toomer is an excellent English translation; it includes a detailed introduction and comprehensive footnotes. Both Otto Neugebauer and

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[46] Proclus calculates the sizes and distances of the celestial bodies in his work *Hypotyposis Astronomicarum Positionum* and *Commentary on Plato's Timaeus*. Manitus 1909; 142, 220-225. Schneider 1847, 624. Swerdlow 1968, 15. Nallino 1899-1907, 2888-2889.
[47] Hartner 1964. Swerdlow 1968, 17.
[48] Sedeño 1987.

Olaf Pedersen provide indispensable and extensive commentary of the *Almagest* in *A History of Ancient Mathematical Astronomy* and *A Survey of the Almagest*, respectively.[49] Full or partial translations exist of most of Ptolemy's major works, including the *Tetrabiblos*,[50] *Canobic Inscription*,[51] *Harmonics*,[52] *Geography*,[53] *Criterion of Truth*,[54] *Optics,*[55] and *Planisphaerium*.[56] While Ptolemy makes connections between different branches of science – for example, in the *Harmonics* he compares the three main musical intervals with the three parts of the Platonic soul – unfortunately there are only a few works that examine Ptolemy as a natural philosopher across disciplines or in relation to Greco-Roman philosophical schools.[57]

Important scholarship on the *Planetary Hypotheses* includes Noel Swerdlow's unpublished Ph.D. dissertation, *Ptolemy's Theory of the Distance and Sizes of the Planets: A Study of the Scientific Foundations of Medieval Cosmology.* Here Swerdlow examines the size of the cosmos in Ptolemy's work, including an examination of Book I, Part B of the *Planetary Hypotheses*.[58] Additionally, Albert Van Helden provides a discussion of cosmic distances in *Measuring the Universe: Cosmic Dimensions from Aristarchus to Halley*. While Van Helden discusses the *Planetary Hypotheses* minimally,

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[49] Pedersen. 1974. Neugebauer 1975.
[50] Robbins 1980.
[51] Jones 2005a. Hamilton, Swerdlow, Toomer 1987.
[52] Barker 2000 and 1991. Solomon 1999.
[53] Berggren and Jones 2000.
[54] Long 1989.
[55] Smith 1988, 1996.
[56] Anagnostakis 1985. Berggren and Sidoli 2007.
[57] Hübner 2000. For discussions of Ptolemy in relation to philosophical schools see Jones 2003, Taub 1993 and Feke 2009.
[58] Swerdlow 1968.

this work provides an informative synopsis of the various theories about the dimensions of the cosmos throughout the history of science.[59]

Otto Neugebauer's discussion of the *Planetary Hypotheses* in *A History of Ancient Mathematical Astronomy* provides a critical analysis of the period relations found in Book I.[60] Concerning Book II, Neugebauer says: "The second Book of the 'Planetary Hypotheses' is a rather sad affair. Looked at superficially it seems to suggest some mechanism which connects the motions of the planets within a larger cosmic system. In fact nothing of this kind is achieved."[61] While Neugebauer discusses many of the problems with Book II of the *Planetary Hypotheses*, his overall analysis is limited.

Finally, the article by Andrea Murschel titled, "The Structure and Function of Ptolemy's Physical Hypotheses of Planetary Motion" is the most recent and comprehensive work on the *Planetary Hypotheses*.[62] Murschel provides an insightful overview of both books, though the thrust of the article deals mainly with the physical models described in Book II; it is essential to any scholarly consideration of the *Planetary Hypotheses*.

[59] Van Helden 1985.
[60] Neugebauer 1975. A period relation is some phenomenon, such as distance traveled by a planet in degrees or returns in position that is made relative to a second phenomenon, such as days, months, or years. See Section III.3.5 below.
[61] Neugebauer 1975, 922-926.
[62] Murschel 1993, 33-61.

I.3 Notations and Sexagesimals

1.3.1 The Zodiac

The division of the zodiac into twelve signs originates in Babylonian astronomy. These twelve signs provide an ideal background upon which to measure the motion of the Sun, Moon, and planets. Ptolemy used Aries 0º as the vernal equinoctial point and he used a tropical frame of reference, not a sidereal one, for the motion of the stars. Below are the symbols, names, and longitudinal intervals for the twelve zodiacal signs.

♈	Aries	0°-30°
♉	Taurus	30°-60°
♊	Gemini	60°-90°
♋	Cancer	90°-120°
♌	Leo	120°-150°
♍	Virgo	150°-180°
♎	Libra	180°-210°
♏	Scorpio	210°-240°
♐	Sagittarius	240°-270°
♑	Capricorn	270°-300°
♒	Aquarius	300°-330°
♓	Pisces	330°-360°

I.3.2 Egyptian Months

The Egyptian year consisted of twelve months, each thirty days in length, plus five extra days. The five extra days, epagomenal days, came at the end of the year and are referred to as ‘first epagomenal day’, ‘second epagomenal day’, etc. The names of the Egyptian months are provided here in chronological order. The names are English transliterations of Greek transliterations of the Egyptian names.[63]

[63] Toomer 1998, 9.

I	Thoth
II	Phaophi
III	Athyr
IV	Choiak
V	Tybi
VI	Mechir
VII	Phamenoth
VIII	Pharmouthi
IX	Pachon
X	Payni
XI	Epiphi
XII	Mesore

I.3.3 Sexagesimal Place Value System

Records show that the sexagesimal place value system came into use as early as 2000 B.C.; however, early evidence of this system in use is rare since calculations were either done mentally or the calculation itself was not recorded, only the outcome.[64] The sexagesimal system is useful because 60 is a number that can be easily divided by multiple numbers; for example, sixty is divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30.

In ancient Greek astronomy, the twenty-seven letters of the archaic Greek alphabet were used to denote numbers, so that α represented the number one, β represented the number two, γ represented the number three, and so forth. The number ten was represented by the tenth letter ι, and the number twenty was represented by the letter κ. The length of the tropical year would have been written as τξε ιδ μη. This represents the value $365+\frac{14}{60}+\frac{48}{3600}$. Relying on Neugebauer's notation, I write this number as 365;14,48, so that a semicolon is used to separate whole numbers from fractions. In Greek astronomy the sexagesimal place value system was used for fractions,

[64] Robson 2008, 5, 77-78.

but not for whole numbers. For example, the value for the tropical year appears as 365;14,48 instead of appearing as 6,5;14,48 as it would in a strictly sexagesimal notation.

In the *Arabic-A* manuscript of the *Planetary Hypotheses*, which is dated 1242, the numbers are presented in three different ways; a) spelled out as words (e.g. واحد); b) presented using *abjad* numerals, (e.g. ا ب ج); and finally in at least a few instances, c) presented as Hindu-Arabic digits (e.g. ١ ٢ ٣).[65] The *Arabic-B* manuscript, which is a ninth-century or tenth-century copy, presents numbers either spelled out as words (a); or using *abjad* numerals (b). Similar to the system used in the Greek manuscripts, in the *abjad* notation each of the twenty-eight letters of the Arabic alphabet denotes a numerical value; however, the order in which letters are assigned does not follow the order of the alphabet. The first letter of the Arabic alphabet, ا (alif), represents the number one. The second letter of the alphabet, ب (bā'), represents the number two, but the number three is assigned to, jīm(ج) which is the fifth letter of the Arabic alphabet and the number four is assigned to د (dāl), which is the eighth letter of the Arabic alphabet. The Hindu-Arabic numbers represent numbers using nine ciphers, or digits. Zero was considered to be a placeholder and it represented the lack of a number. The Hindus used this system to represent whole numbers as early as 520 A.D. and the Muslims were the first to use it to represent fractions.[66] *The Book of Addition and Subtraction According to the Hindu Calculation* is the earliest existing explanation of the Hindu number system; it was written probably early in the ninth-century by Muḥammad ibn Mūsā al-Khwārizmī. The Hindu-Arabic number system was not yet extensively used in the ninth and tenth centuries, which explains why it does not appear in the *Arabic-B* manuscript, but it was

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[65] For example see Goldstein 1967; 16. Berggren 2003, 41.
[66] Berggren 2003, 29-31.

much more widely known by the thirteenth-century, which accounts for its appearance in the *Arabic-A* manuscript.

I.3.4 Direction of Celestial Motion

In the *Planetary Hypotheses*, Ptolemy references the direction that a celestial body moves relative to the daily motion of the sphere of the fixed stars. According to Ptolemy, the sphere of the fixed stars rotates on its axis once a day, from east to west. When Ptolemy refers to the motion of another body, he does so relative to the directional motion of the sphere of the fixed stars. For example, he says εἰς τὰ προηγούμενα τοῦ κόσμου to mean the same direction of the sphere of the fixed stars. I have chosen to translate this as “in the in the forward direction of the cosmos”. Alternatively, when referring to the opposite direction that the sphere of the fixed stars travels Ptolemy says εἰς τὰ ἑπόμενα τοῦ κόσμου, which I have chosen to translate as “in the trailing direction of the cosmos”. While these phrases may seem strange to a modern reader, I think they stay true to the Greek while still conveying the meaning. Throughout the commentary, I refer to the forward motion and the rearward motion of the cosmos, by which I mean εἰς τὰ προηγούμενα τοῦ κόσμου and εἰς τὰ ἑπόμενα τοῦ κόσμου, respectively.

I.4 The Aim and Function of Ptolemy's Models

While it is clear that Ptolemy's models endeavor to predict the motions of the heavens – after all they come with detailed tables and clear methods of computation so that the location of any of the heavenly bodies can be calculated for any given date – it is unclear how Ptolemy viewed his own models, specifically how his models relate to the true motions in the heavens and whether he believed that it was possible to know the true motions. Pierre Duhem famously addressed the question of whether Ptolemy's models were meant to be predictive tools, or realistic theories in his 1908 publication, *To Save the Phenomena*.[67] Using a philosophy of science framework, Duhem represented Greek astronomical models as instrumentalist models. He cited the works of Proclus, Aristotle, Ptolemy and others to emphasize the different objectives of the astronomer, concluding that ancient Greek astronomers thought of astronomical models as predictive instruments rather than realistic descriptions. While Duhem's conjecture that ancient astronomers were uninterested in constructing instrumentalist models[68] and not realistic models[69] has been met with criticism, the question at the heart of this debate – what did Ptolemy think were the aims and functions of his models? – is a particularly interesting subject. In later publications Duhem modified his ideas, but he maintained his earlier views. Scholars

[67] Duhem 1969, 20.
[68] Instrumentalism is the view that theories are instruments or tools that can be used to make predictions about observable phenomena. Instrumentalist models do not begin from first principles or deduced reasoning, nor do they include a theory of causation. Chakravartty 2007, 8-13.
[69] Scientific realism is a position in the philosophy of science that says that the world is mind-independent and that our scientific theories can be true (or, at least, approximately true). When faced with a decision between two theories that are both able to account for the phenomena, the realist will rely on pragmatic reasoning. There are many different types of realism, such as entity realism or property realism, as well as different degrees of commitment to any given theory. The various types of realism and ways to be a realist are written about extensively in the philosophy of science. Worrall. 2000, 353.

such as G.E.R. Lloyd, and Jamil Ragep have criticized Duhem's position.[70] Their arguments focus on clarifying the objectives of Greek and Islamic astronomy and demonstrating that ancient astronomers were interested in more than producing predictive models.

Ptolemy had a detailed philosophy concerning one's ability to construct a theory describing natural phenomena, the limits of theories, and how one can achieve certain knowledge. Technically, he falls under the category of scientific realist, however, applying a label to him does not explain what he thought were the aims, purposes, and limits of his astronomical models. His opinions about models are complex and he does not state his views on models in any one place. Instead, he touches on topics concerning the objectives of his models, the criteria used to construct models, and the limits of his models throughout his works and in several different places in his texts.

Before we can examine what Ptolemy says about models, we first need to understand the vocabulary that he uses. Throughout the *Almagest* and *Planetary Hypotheses* Ptolemy uses the Greek word, ὑποθέσις. The English word *hypothesis* is derived from the Greek word ὑποθέσις, but the English word *model* is a more accurate translation of what Ptolemy means by ὑποθέσις. A *hypothesis* implies an unproven explanation made from limited evidence. For example, we often think of testing a hypothesis. When Ptolemy says ὑποθέσις he means a proposed theory based on the careful analysis of data. Ptolemy often refers to the "ὑποθέσις that he has *demonstrated* to be true". Furthermore, Ptolemy uses the word ὑποθέσις to refer to both abstract, geometrical models, namely imagined, mental models, and tangible, material models.

[70] Lloyd 1978. Ragep 1990.

I.4.1 Physical Foundations

The physical foundations upon which Ptolemy bases his astronomical system – that celestial bodies exhibit uniform, circular motion, the heavens are finite, and the Earth is at the center – are present in the work of his predecessors, such as Plato, Eudoxus, Aristotle, and Hipparchus. Ptolemy builds on the work of his predecessors and his astronomical theory is not revolutionary; however, it is more thorough, extensive and precise then anything contemplated before him.

While not intuitive, Ptolemy's concept of uniform motion is fundamental to reconciling his models with Aristotelian principles of motions in the heavens. According to Ptolemy, uniform motion means that a body travels uniformly around a point. However, he does not insist that this point needs to be the Earth. By making a point other than the Earth the center of uniform motion, Ptolemy constructs a system where uniform motion occurs but it does not appear uniform when viewed from the Earth. Ptolemy calls this new point the center of uniform motion; Latin Medieval astronomers referred to it as the equant. In the *Almagest*, Ptolemy does not give credit to anyone for the discovery that the Earth and the center of uniform motion are different for the celestial bodies. Nor does Ptolemy take credit for this discovery himself. While Ptolemy's use of the equant in the *Almagest* is the earliest known use of such a model, Dennis Duke argues that the data necessary to bisect the equant was available as early as Hipparchus, and possibly even earlier. Furthermore, Duke argues that Ptolemy's use of the equant is the not the first appearance of the bisected equant in ancient astronomy. Duke also suggests that Indian astronomers may have known of the equant.[71]

[71] Some of the Indian astronomical texts dated from about 400-500 A.D. were influenced by Greek astronomers that predate the *Almagest*. This is known because the astronomy is clearly Greco-Roman, yet

In Book I of the *Almagest* Ptolemy lays out most of his physical assumptions. He begins by distinguishing practical philosophy from theoretical philosophy,[72] and relying on Aristotle, Ptolemy divides theoretical philosophy into theology, physics, and mathematics.[73] Ptolemy says:

> that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously.[74]

Mathematics is able to assist in better understanding both physics and theology. For Ptolemy, it is important to study astronomy since the heavens are ordered and uniform. One should devote time studying things that are eternal so that one will become familiar with certain characteristics – such as order, consistency and calmness – and eventually emulate these characteristics.[75] The only way to produce certain knowledge is through mathematics and since astronomy is the study of things that are eternal and unchanging, astronomy can provide us with knowledge that neither physics nor theology can. Additionally, the heavenly bodies are not only eternal but they are also perceptible. The study of astronomy relies on evidence gained through the senses, which makes the knowledge that one obtains from astronomy more secure than the knowledge gained from theology.

less developed than that presented in the *Almagest*. However, this is an understudied topic and there is much that remains unknown about Indian astronomy. Duke 2005.

[72] Ptolemy stated that practical philosophy does not necessarily require instruction and that a person can lead a relatively virtuous life without ever being taught about such a life. Conversely, theoretical philosophy requires instruction. Ptolemy asserted that one cannot gain a theoretical understanding of the heavens without guidance. Consequently, he believed that it is important to strive to live a virtuous life, but that one should devote one's efforts to intellectual matters. Toomer 1998, 35.

[73] Aristotle, *Metaphysics*, 1026a13-23.

[74] Toomer 1998, 36.

[75] Toomer 1998, 36-37.

In Aristotle's writings, mathematics exists between theology and physics. The latter two are like guesswork; theology because its subject matter is ungraspable; physics because its subject matter is constantly changing.[76] Mathematics is an attribute of all things, both mortal things that are perpetually changing and immortal things that are eternal and have an ethereal nature.[77] No real conclusions can be made from theology or physics alone and it is only through mathematics, as long as it is approached rigorously, that one can reach "unshakeable knowledge".[78]

Ptolemy's belief that the heavens are eternal and unchanging comes from patterns that were observed, such as uniform, circular motion, and then made into principles. The only type of motion that can account for the motions of the heavens, according to Ptolemy, is spherical motion. In Book I.3 of the *Almagest,* Ptolemy explains that early astronomers observed that the sun, Moon, and stars are carried from east to west in parallel circles; they rise, set, vanish and then rise again.[79] He says:

> To sum up, if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upwards, must vary, wherever and however one supposes the earth itself to be situated. Hence the sizes and mutual distances of the stars must appear to vary for the same observers during the course of each revolution, since at one time they

[76] Aristotle, *Metaphysics*, 1026a13-23.
[77] The idea that the heavens are eternal and have uniform motion goes back to Plato and Aristotle. In the *Timaeus*, Plato presents a model for the heavens that consists of two spheres; the Same and the Different. The outer sphere is the sphere of the Same and it moves the fixed stars uniformly. The inner sphere is the sphere of the Different; it accounts for the motion of the wondering bodies, such as the Sun, the Moon and the planets. While Plato says that this motion is "different", it is still uniform. Aristotle discusses ether, which is the fifth element and is qualitatively different from the corruptible sublunary substances, in the *Physics*. Toomer 1998, 36. Plato, *Timaeus*, 35a-40b.
[78] Toomer 1998, 36. The universe is limited in space, but eternal in time. This inconsistency was never completely addressed in antiquity. Hübner 2000, 62-63.
[79] Ptolemy argues against the Epicurean idea that stars move in a straight line toward infinity. Furthermore, he argues against the idea supported by Heraclitus, Xenophanes, and Epicurus that the stars are kindled as they rise and extinguished when they set. Toomer 1998, 38-39, n. 22, 23.

> must be at a greater distance, at another at a lesser. Yet we see that no such variation occurs.[80]

Since parallax of the fixed stars cannot be observed, Ptolemy claims that the fixed stars must always be the same distance from the Earth; the only type of motion that would allow for the fixed stars to remain equidistant from the Earth throughout their revolution is spherical motion.[81] Ptolemy goes on to argue that not only do the heavens move spherically, but that they are also spherical in shape:

> The following considerations also lead us to the concept of the sphericity of the heavens. No other hypothesis but this can explain how sundial constructions produce correct results; furthermore, the motion of the heavenly bodies is the most unhampered and free of all motions, and freest motion belongs among plane figures to the circle and among solid shapes to the sphere; similarly, since of different shapes having an equal boundary those with more angles are greater [in area or volume], the circle is greater than [all other] surfaces, and the sphere greater than [all other] solids; [likewise] the heavens are greater than all other bodies.[82]

The motion in the heavens is the freest motion and since the freest motion for three-dimensional objects belongs to sphere, then the heavens must be spherical. The spherical shape of the heavens is not a necessary consequence of spherical motion; nonetheless, Ptolemy argues that the heavens must be spherical because of their spherical motion. Furthermore, he states that since the sphere has the greatest surface area among solids, and the heavens are spherical in shape, the heavens are greater than all bodies.

Ptolemy hastily justifies his shift from spherical motion in the heavens to a spherical shape; however, he offers one final reason for the sphericity of the heavens. Using ether, Ptolemy argues:

> Furthermore, one can reach this kind of notion from certain physical considerations. E.g., the aether is, of all bodies, the one with constituent parts

[80] Toomer 1998, 39.
[81] Parallax of the stars was not observed until 1838 by Friedrich Wilhelm Bessel. Hübner 2000, 63.
[82] Toomer 1998, 39-40.

> which are finest and most like each other; now bodies with parts like each other have surfaces with parts like each other; but the only surfaces with parts like each other are the circular, among planes, and the spherical, among three-dimensional surfaces. And since the aether is not plane, but three-dimensional, it follows that it is spherical in shape. Similarly, nature formed all earthly and corruptible bodies out of shapes which are round but of unlike parts, but all aethereal and divine bodies out of shapes which are like parts and spherical. For if they were flat or shaped like discus they would not always display a circular shape to all those observing them simultaneously from different places on earth. For this reason it is plausible that the aether surrounding them, too, being of the same nature, is spherical, and because of the likeness of its parts moves in a circular and uniform fashion.[83]

The heavens contain ether and ether consists of constituent parts that are homogeneous. Since these parts are homogeneous, Ptolemy contends that they must be spherical. The ethereal constituents could not be circular shaped discs because they would not appear spherical when viewed from different locations. Since the visible heavenly bodies always appear circular despite the location from where they are viewed, they must be made from spherical ethereal parts. Ptolemy asserts that because all ethereal parts are alike, the ether everywhere must be spherical and move in uniform, circular motion.

Ptolemy believed that the heavens exhibit uniform, circular motion. This conjecture is based on preliminary observations, which he deems principles and his theory is constructed so that it adheres to these principles. According to Ptolemy's assumptions about the heavens, all celestial bodies are spherical and move in uniform, circular motion; however, observations indicate that some bodies appear to have anomalistic motion. In the midst of discussing his solar theory in Book III.3 of the *Almagest*, Ptolemy explains:

> Our next task is to demonstrate the apparent anomaly of the sun. But first we must make the general point that the rearward displacements of the planets with respect to the heavens are, in every case, just like the motion of the universe in advance, by nature uniform and circular. That is to say, if we imagine the bodies or their

[83] Toomer 1998, 40.

> circles being carried around by straight lines, in absolutely every case the straight line in question describes equal angles at the centre of its revolution in equal times. The apparent irregularity [anomaly] in their motions is the result of the position and order of those circles in the sphere of each by means of which they carry out their movements, and in reality there is in essence nothing alien to their eternal nature in the 'disorder' which the phenomena are supposed to exhibit.[84]

The celestial bodies move in uniform circles, but their center of motion is not necessarily the Earth. When viewed from the Earth, the Sun, Moon and planets seem to exhibit anomalistic motion. Ptolemy would say that this irregularity is not real, but only apparent, since the angular motion of the body is perfectly uniform when judged in relation to the equant point. Ptolemy reiterates this point when he discusses the theory of the planets in Book IX.1 of the *Almagest*, saying that all of the apparent anomalies can be accounted for using uniform, circular motion.[85] The Earth is not the center of all circular motion; however, the Earth maintains its position at the center of the universe.

According to Ptolemy, the Earth, like the heavens, is spherical. Observations demonstrate that the Sun, Moon and stars rise at different times depending on one's location on the Earth. For example, he says that lunar eclipses, which occur at the same moment everywhere, are not observed to take place at the same time relative to noon.[86] If the Earth's surface were two-dimensional, concave, triangular, or cubical, then astronomical events would not appear to occur at different times.[87] If the Earth were concave then stars would be observed to rise by people further west before those in the east. If the Earth were a plane then the stars would appear to rise/set at the same time

[84] Toomer 1998, 141.
[85] Toomer 1998, 420.
[86] While Ptolemy explains that the Earth can be deduced to be spherical because eclipses are observed to take place at different times by observers in different locations, this observation is actually made after the conclusion that the Earth is spherical. Very few eclipse observations were made simultaneously from different locations, and none were made before the Greeks had reached the conclusion that the Earth was round. I must thank James Evans for bringing this point to my attention.
[87] Toomer 1998, 40.

everywhere; this would also be the situation of the sides of the Earth if it were a polygon. However, none of these examples match the phenomena. Furthermore, the sphericity of the Earth can be observed when sailing towards mountains, since the mountains appear to gradually rise from the water and grow in size.[88] For all of these reasons, Ptolemy asserts that the Earth must be spherical.

Like the majority of ancient astronomers and natural philosophers, Ptolemy believed that the Earth is motionless and at the center of the heavens.[89] Ptolemy argued that the Earth is at the center of the cosmos for multiple reasons. He eliminated any location for the Earth other than at the center of the cosmos since an equinox could not occur if the Earth was not at the center. This would only occur if the Earth were equidistant from each pole and on the axis between the two poles. Furthermore, Ptolemy contended that the Earth is motionless at the center and that all heavy things move toward their natural place, the center of the cosmos. The Earth is unable to rotate since it is a heavy body and a heavy body has a violent motion, which cannot persist.[90] The heavens, however, are composed of ethereal matter, which is rare and light and easily movable.

All of the physical foundations of Ptolemaic astronomy were subject to the concept of simplicity. Ptolemy believed that the heavens would not be more complicated than they need to be, so that when faced with two theories that explained the same phenomena, Ptolemy picked the simpler theory. When discussing the period of the solar year in the *Almagest* Ptolemy said:

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[88] Toomer 1998, 41.
[89] In addition to geocentric theories, there were also heliocentric theories such as the one proposed by Aristarchus. *Dictionary of Scientific Biography*, s.v. "Aristarchus of Samos."
[90] Toomer 1998, 45.

> And in general, we consider it a good principle to explain the phenomena by the simplest hypotheses possible, in so far as there is nothing in the observations to provide a significant objection to such a procedure.[91]

When discussing his lunar theory in Book IV.9 of the *Almagest*, Ptolemy commented about correcting a theory. After presenting a preliminary lunar model that achieves excellent results for eclipses, Ptolemy replaced this model with a more complicated model that gave better agreement with the Moon's latitudinal positions. He said:

> For those who approach this science in a true spirit of enquiry and love of truth ought to use any new methods they discover, which give more accurate results, to correct not merely the ancient theories, but their own too, if they need it. They should not think it disgraceful, when the goal they profess to pursue is so great and divine, even if their theories are corrected and made more accurate by others beside themselves.[92]

A lover of truth should account for the phenomena by using the simplest theory, but if that does not accurately account for the phenomena, then he should put forth a theory that does. While the observations should be explained using the simplest theory, Ptolemy says that the theories should not disagree with the observed phenomena. For Ptolemy, an accurate theory is more important than a simple theory.

Ptolemy makes some insightful comments about his objectives in his study of astronomy and what kinds of models he hopes to achieve in Book XIII of the *Almagest*. He says:

> Now let no one, considering the complicated nature of our devices, judge such hypotheses to be over-elaborated. For it is not appropriate to compare human [constructions] with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what [could one compare] more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself? Rather, one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions, but if this does not succeed, [one should apply hypotheses] which do fit. For provided that each of the phenomena is duly saved by the

[91] Toomer 1998, 136.
[92] Toomer 1998, 206.

> hypotheses, why should anyone think it strange that such complications can characterise the motions of the heavens when their nature is such as to afford no hindrance, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another?[93]

He goes on to say a few lines later:

> Rather, we should not judge 'simplicity' in heavenly things from what appears to be simple on earth, especially when the same thing is not equally simple for all even here. For if we were to judge by those criteria, nothing that occurs in the heavens would appear simple, not even the unchanging nature of the first motion, since this very quality of eternal unchangingness is for us not [merely] difficult, but completely impossible. Instead [we should judge 'simplicity'] from the unchangingness of the nature of things in the heaven and their motions. In this way all [motions] will appear simple, and more so than what is thought 'simple' on earth, since one can conceive of no labour or difficulty attached to their revolutions.[94]

What is simple in the heavens is not the same as what is simple on Earth. Simplicity equals the unchanging nature of things in the heavens. This is simpler than what one would call simple on Earth, since there is no labor or hindrance. Ptolemy asserts that celestial motions should not be judged in the same way that sublunary motions are judged. The astronomer should try to account for the phenomena by using the simplest theory, but if that does not accurately account for the phenomena, then he should put forth a theory that does.

As long as the astronomer records careful observations and looks for the simplest theory that accurately represents the phenomena, then the astronomer can attain the true, or approximately true, theory. When discussing the length of the year in the Book III of the *Almagest*, Ptolemy says:

> And in general, we consider it a good principle to explain the phenomena by the simplest hypotheses possible, in so far as there is nothing in the observations to provide a significant objection to such a procedure.[95]

[93] Toomer 1998, 600-601.
[94] Toomer 1998, 600-601.
[95] Toomer 1998, 136.

Ptolemy states that the astronomer should pick the simplest explanation that accounts for the phenomena, but he does not explain why the heavens are simple, nor does he justify why simplicity is a route to truth.

Ptolemy believes that through careful observation, and with an eye for uniform, circular motion, astronomers can create a model that, for the most part, accurately accounts for the motions in the cosmos.[96] However, when determining parameters, Ptolemy makes it clear that the greater the time interval between two observations, the more accurate the final value will be. Ptolemy argues that in order to arrive at the most accurate value for the length of the tropical year, for example, it is best to use observations that are separated by large intervals of time. He says in Book III of the *Almagest*:

> For the error due to the inaccuracy inherent in even carefully performed observations is, to the senses of the observer, small and approximately the same at any [two] observations, whether these are taken at a large or small interval. However, this same error, when distributed over a smaller number of years, makes the inaccuracy in the yearly motion [comparatively] greater (and [hence increases] the error accumulated over a longer period of time), but when distributed over a larger number of years makes the inaccuracy [comparatively] less.[97]

The period relations that Ptolemy presents are only as accurate as the observations will allow. An accumulation of observations spanning many centuries is needed to construct a more accurate theory. Ptolemy is unique in his opinion about the accuracy of mean motions; many of his successors did not echo this view.[98] As a result, Ptolemy asserts that

[96] Toomer 1998, 600-601.
[97] Toomer 1998, 137.
[98] Toomer 1998, 137, n. 18.

astronomers cannot consider period motions valid for eternity, or even for a length of time that is longer than the time between observations.[99]

In sum, for Ptolemy an astronomical model must meet a clear set of criteria. First, it must consist of uniform, circular motion. Although the motion in the heavens may appear anomalistic from Earth, according to Ptolemy they move with uniform, circular motion only. Additionally, the simplest model must be presented, but the accuracy of the model cannot be sacrificed. If two models are capable of representing a phenomena, but one model accounts for the phenomena better, than that is the model that should be used, even if it is not the simpler of the two. Finally, while astronomers can construct models, the models are only as good as the data. With more data and more time between observations, calculations for the length of an astronomical event, e.g. the length of the year, will be more precise.

1.4.2 Decisions Ptolemy makes when constructing astronomical models

In addition to what Ptolemy says about models, we can gain insight into his understanding of the role of models by examining his construction of astronomical models. The decision that Ptolemy makes when faced with two physically different, but mathematically equivalent, models reveal the criteria Ptolemy emphasizes in his astronomical theory. Mathematically equivalent models are understood as two models that situate a heavenly body in exactly the same location in space at all times.

In three different places in the *Almagest* and the *Planetary Hypotheses* Ptolemy is forced to choose between two mathematically equivalent models. In the first example, when determining his solar model in Book III of the *Almagest,* Ptolemy demonstrates that

[99] Ptolemy is possibly referring to the 'Aeon tables' with his reference to eternity. Toomer 1998, 422, n. 12.

both the epicycle hypothesis and the eccentric hypothesis will account for the Sun's motion and he says that either model could be used. In the second example, Ptolemy argues in Book II of the *Planetary Hypotheses* that the physical make-up of the models does not necessarily have to consist of nested spheres, an idea found in Plato and Aristotle, but that the heavens could be made up of a series of nested rings. Finally, in the third example Ptolemy discusses in both Book IX of the *Almagest* and Book I of the *Planetary Hypotheses* the location of the Sun relative to the planets; he debates whether the Sun is located below all of the planets, or between Venus and Mars, or above all of the planets. By examining these examples closely, we can gain insight into Ptolemy's criteria for his models and his commitment to his models by studying the places where Ptolemy must choose between models.

In the first example, when discussing the solar anomaly, Ptolemy demonstrates the equivalence of the epicycle and eccentric models in Book III.3 of the *Almagest*.[100] In the epicycle model, the Sun travels uniformly around the epicycle circle (small circle HDGC), while the epicycle travels uniformly around circle ABCD, in the opposite direction. (See Figure 1)

[100] The epicycle and eccentric models will be discussed further in Section III.2 below.

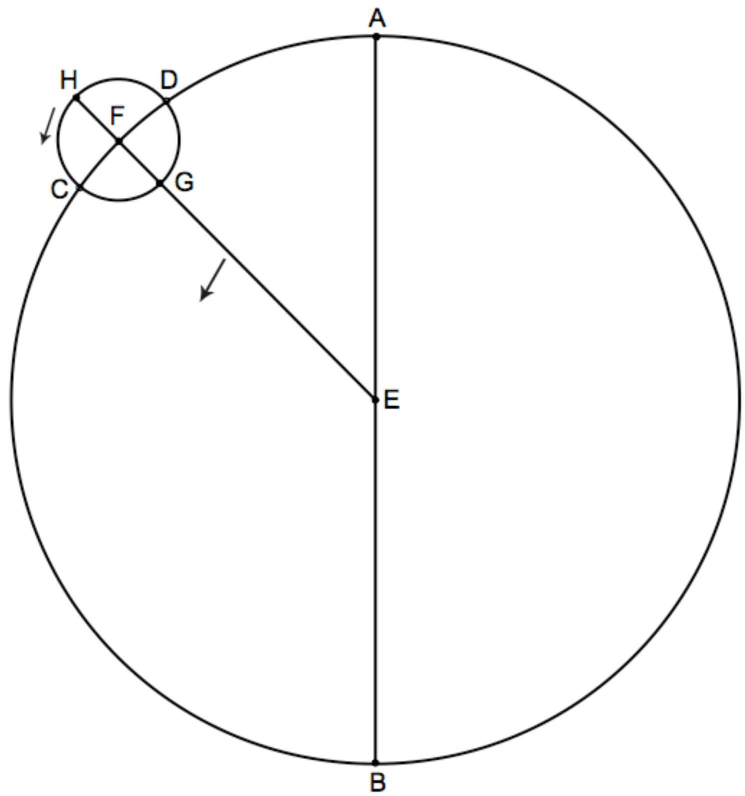

Figure 1: Epicycle Model

In the eccentric model, the Sun travels around circle ABC centered on point F, and the observer is at point E. (See Figure 2)

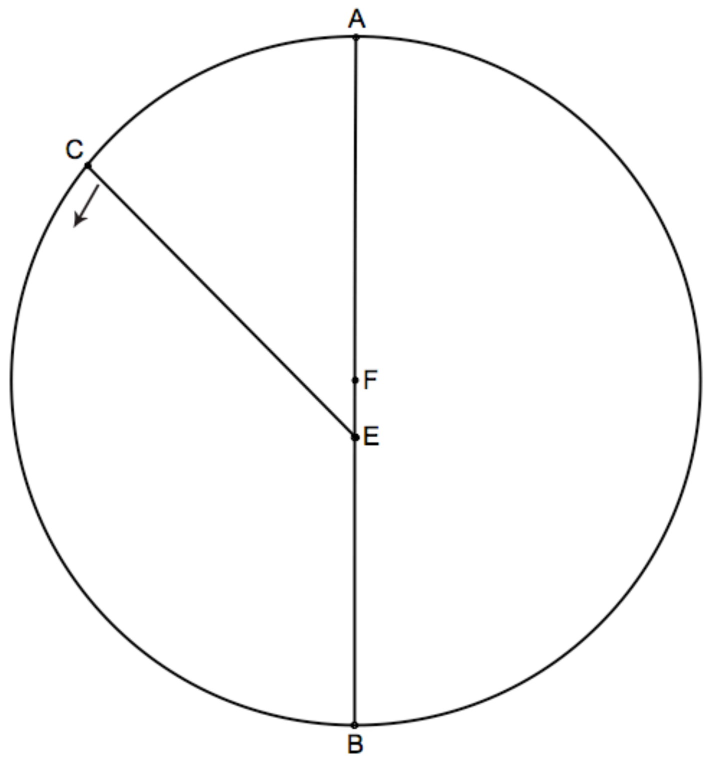

Figure 2: Eccentric Model

Both the models are equivalent in that they place the Sun in the same position in space and time – in each case the Sun moves in a circle and its motion appears anomalistic when viewed from point E. (See Figure 2) Ptolemy states that *either* model can be used to represent the solar anomaly; however, Ptolemy ultimately chooses between these models. For the solar theory he chooses the eccentric model to represent the motion of the Sun. He argues that the eccentric model is simpler and requires one motion instead of two. In Book III.4 of the *Almagest* Ptolemy says:

> Now this could be represented by either of the hypotheses described above, though in case of the epicyclic hypothesis the motion of the sun on the apogee arc of the epicycle would have to be in advance. However, it would seem more

> reasonable to associate it with the eccentric hypothesis, since that is simpler and is performed by means of one motion instead of two.[101]

Ptolemy does not imply that one theory is mathematically simpler than the other. Instead he appeals to the physical motion of the two hypotheses; the eccentric model has one physical motion while the epicyclic model has two physical motions.[102] Both models account for the observations and both models adhere to uniform, circular motion, but Ptolemy thinks that one, namely the eccentric model, is physically simpler than the other. Ptolemy chooses this model for pragmatic reasons, relying on criteria of simplicity.

In the second example, in Book II of the *Planetary Hypotheses* Ptolemy discusses the physical attributes of the models that he presents in the *Almagest.* He goes into great detail in describing the models in the *Almagest* in three-dimensional terms. He argues against Aristotle's idea that there are unwinding spheres. Ptolemy claims that an entire sphere is not necessary and all of the heavenly spheres, except for the sphere of the fixed stars, could be replaced with *manshūrāt,* which means sawn-off pieces in Arabic. Instead of concentric spheres nested within each other, the cosmos would consist of sawn-off spheres. Ptolemy says that there are two types of spheres; a) those which are hollow and enclose other spheres or enclose the Earth, and, b) those which are solid and enclose nothing, like the epicycles.

[101] Toomer 1998, 153.
[102] Toomer 1998, 153.

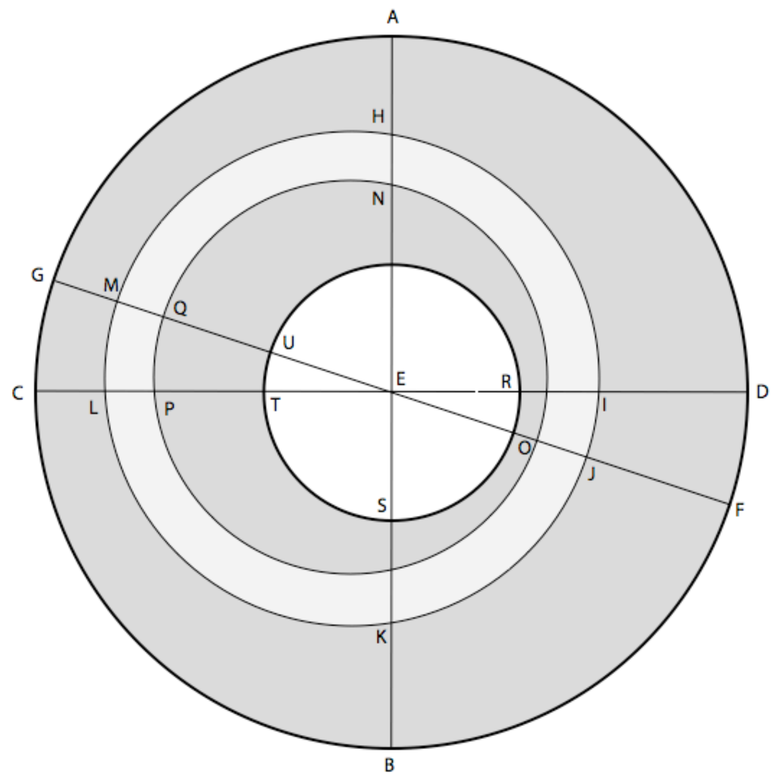

Figure 3: Solid Sphere Planetary Model

Figure 3 is a cross-section of the model for one of the planets other than Mercury. Point E represents the Earth, line CD represents the celestial equator, and line GF represents the path of the planet along the ecliptic. The epicycle travels within the light grey section between sphere HIJKLM and sphere NOPQ; its width is equal to the diameter of the epicycle. The dark grey space between sphere ADFBCG and sphere HIJKLM along with the space between sphere NOPQ and sphere RSTU is called the paraecliptic. The epicycle travels in the light grey area with the planet embedded on the sphere of the epicycle. The white space inside circle RSTU would contain the model for the planets below this planet, so that the spheres nest within each other.

The sawn-off pieces hypothesis consists of two types of cut-off spheres; a) those which are hollow (like a hollow ring) and enclose other rings, and b) those which are solid (like a solid tambourine) and enclose nothing.[103]

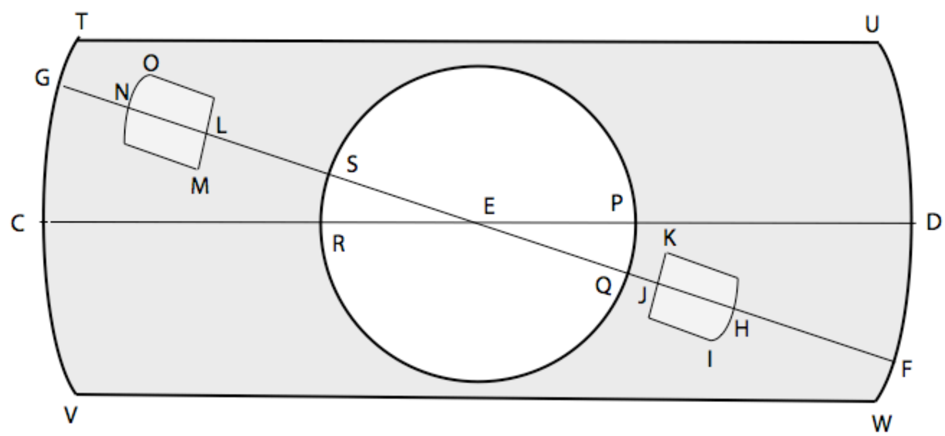

Figure 4: Sawn-off Sphere Planetary Model

Figure 4 is a cross-section of the sawn-off pieces model for any of the planets, other than Mercury. As in the previous figure, Point E represents the Earth, line CD represents the celestial equator, and line GF represents the path of the planet along the ecliptic. The ring is seen in sections HIJK and LMNO and the epicycle travels within this ring. In this model, the paraecliptic surrounds the ring (seen in sections HIJK and LMNO) in which the epicycle travels. The dark grey space in the ring, the space between TUVW and QRS,

[103] Heiberg 1907, 116-118.

is the paraecliptic. In the full sphere model the paraecliptic consists of two different components; however, in the sawn-off model it consists of one. The epicycle travels in the light grey area with the planet embedded on it. The white space inside circle PQRS would contain the model for the planets below this planet, so that there is a series of nested rings.

Ptolemy says that both hypotheses are equivalent from a mathematical point of view. While he believes that either hypothesis will work mathematically, Ptolemy thinks that the sawn-off sphere hypothesis is a more probable theory because nature would not render anything useless and this hypothesis is more economical.[104] Furthermore, Ptolemy wants to use the smallest number of components possible; he says 34 spheres are needed with the full sphere model, and 22 are needed with the sawn-off model.[105] Although Ptolemy maintains that there is no difference between the model with the full spheres and the model with the sawn-off spheres, he prefers the sawn-off model.[106] Starting with the geometric models of the *Almagest*, Ptolemy provides a physical, three-dimensional description of the cosmos in the *Planetary Hypotheses* that adheres to the foundations he sets out in the *Almagest*.[107] Once again, when faced with two mathematically equivalent, but physically different theories, Ptolemy makes a decision by appealing to simplicity. Ptolemy considers the model consisting of sawn-off spheres to be simpler since it would require fewer bodies. Furthermore, an entire sphere is unnecessary in the sawn-off model, whereas a series of nested rings would be more economical.[108]

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[104] Heiberg 1907, 117.
[105] The sawn-off model would have fewer parts since the paraecliptic surrounds the path of the epicycle; however, in the sphere model, the paraelciptic is separated into two spheres with the path of the epicycle dividing it. In Figures 1.3 and 1.4 the paraecliptic is denoted by the grey shaded areas.
[106] Heiberg 1907, 122-125. Murschel 1995, 50-52.
[107] Murschel 1995, 51-55.
[108] Heiberg 1907, 117.

In the third example, when discussing the order of the planets, Ptolemy advances an order, but he says that astronomers cannot know this matter with certainty. In Book IX.1 of the *Almagest* he states:

> But concerning the spheres of Venus and Mercury, we see that they are placed below the sun's by the more ancient astronomers, but by some of their successors these too are placed above [the sun's], for the reason that the sun has never been obscured by them [Venus and Mercury] either. To us, however, such a criterion seems to have an element of uncertainty, since it is possible that some planets might indeed be below the sun, but nevertheless not always be in one of the planes through the sun and our viewpoint, but in another [plane], and hence might not be seen passing in front of it, just as in the case of the moon, when it passes below [the sun] at conjunction, no obscuration results in most cases.[109]

Ptolemy is debating where to place the Sun, either before Mercury and Venus, or after them. Ptolemy goes on to say that we cannot be certain about the order of the heavenly bodies since we are not able to observe any parallax and, consequently, we cannot make progress in our theory concerning the order. He argues that the order of Mercury, Venus, the Sun, Mars, Jupiter, Saturn, which is the order supported by many other ancient astronomers, is more plausible because it separates the planets that are always near the Sun from those that are not. Since Ptolemy handles each planetary theory separately, the order of the planets does not affect his predictive models; he could leave the matter unresolved and still develop a theory for each planet. However, Ptolemy does put forth an order, and while he admits that we cannot know with certainty if it is correct, he says that this separation is "more in accordance with nature."[110] In Book IX.1 of the *Almagest* he states:

> For, by putting the sun in the middle, it is more in accordance with the nature [of the bodies] in thus separating those which reach all possible distances from the sun and those which do not do so, but always move in its vicinity; provided only that it does not remove the latter close enough to the earth that there can result a parallax

[109] Toomer 1998, 419.
[110] Toomer 1998, 419.

> of any size.[111]

This is not a simplicity argument. Here Ptolemy is suggesting that similar structures belong near each other. If a given model is not simpler than any other, as in the case of the planetary order, then Ptolemy uses organization as his criteria. The heavens are better organized if like objects are near each other, and given the choice, Ptolemy adheres to this principle. In the *Planetary Hypotheses* Ptolemy returns to this problem:

> But with respect to the Sun, there are three possibilities; either all five planetary spheres lie above the sphere of the Sun just as they all lie above the sphere of the Moon; they all lie below the sphere of the Sun; or some lie above, and some below the sphere of the Sun.[112]

Here he seems even less sure of the location of the Sun with respect to the planets. Additionally, in Book I, Part B Ptolemy explains that the spheres closet to the air, Mercury and the Moon, move with many motions since they resemble the turbulent elements near them. He ultimately offers the same order that he presents in the *Almagest*, namely: Moon, Mercury, Venus, Sun, Mars, Jupiter, and Saturn. In total, Ptolemy suggests three different options for the place of the Sun amongst the planets:

[111] Toomer 1998, 419-420.
[112] Goldstein 1967, 6.

Table 1: Order of the Planets

Moon	Moon	Moon
Mercury	**Sun**	Mercury
Venus	Mercury	Venus
Sun	Venus	Mars
Mars	Mars	Jupiter
Jupiter	Jupiter	Saturn
Saturn	Saturn	**Sun**

All three of the orders that Ptolemy suggests meet the criteria of uniform, circular motion and adhere to the observations. He does not argue that one order is simpler than the other, but instead he picks the order that neatly organizes the planets so that those that stay near the Sun, namely Mercury and Venus, are separate from those that are capable of reaching opposition with the Sun, namely Mars, Jupiter, and Saturn.

In both the *Almagest* and the *Planetary Hypotheses*, Ptolemy forms a philosophy of science by incorporating empirical observations, along with arguments of simplicity and order, to construct a realistic model of the natural world. According to Ptolemy, an astronomical theory begins with general observations and these observations provide insight into the nature of the heavenly motions. While Ptolemy discusses the nature of the cosmos and uses simplicity and order as criteria, there are a few places in his theory

where he is unable to make a decision with certainty. His reliance on natural philosophy, as well as his attempts to unify different scientific theories shows that he is concerned with more than merely the predictive capabilities of his models. When faced with a decision, Ptolemy says that one cannot know with certainty, but he does not imply that it makes no difference which model is used. In every instance that he is faced with two mathematically equivalent models, he advances one hypothesis over the other based on simplicity or order.

Ptolemy is very clear about how the study of mathematics leads to "unshakeable knowledge" and addresses this both in the *Almagest* and *Harmonics*. He thinks that it is only through mathematics that we can know anything about the eternal and unchanging and that mathematics is a useful tool for the study of physics and theology. He does not think that the astronomer, or the lover of music, is imposing order on the natural world, but instead recognizing order that is already present.

Ptolemy regards empirically based arguments as the strongest criteria, resorting to kinds of dialectic arguments concerning order or simplicity only where empirical evidence is lacking. Ptolemy uses these types of arguments when faced with two competing theories, but he is ambivalent about how one can know this type of argument will lead to certain knowledge. Since his theory is a comprehensive theory with model systems and tables, he does not have the choice of remaining undecided. The beauty of Ptolemy's astronomy is that you can predict the location of any given body for any given date, but in order to accomplish this Ptolemy cannot have any holes in his theory.

Ultimately, Ptolemy's different degrees of commitment to different aspects of his models are directly connected to whether the model was formed using empirical data or

physical foundations, such as order and simplicity. This goes to show how consistent Ptolemy is throughout his works concerning the mathematics ability and to achieve certain knowledge.

Section II:

Translation of the *Planetary Hypotheses*

II.1 Translation of Book I, Part A[1]

H70 <A>[2]

[1] We have worked out, Syrus,[3] the models (ὑποθέσεις) of heavenly motions through the books of the *Mathematical Syntaxis* (μαθηματικῆς συντάξεως),[4] demonstrating by arguments, concerning each example, both the logicality and agreement everywhere with the phenomena, with a view to a presentation of uniform and circular motion (τῆς ὁμαλῆς καὶ ἐγκυκλίου κινήσεως), which necessarily was to arise in things taking part in (κεκοινωνηκόσι) eternal and orderly motion and that are not capable to undergo increase or decrease in any way.[5] Here we have taken on the task to set out the

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[1] I have chosen to translate the Greek word ὑποθέσεις as *models* everywhere except in the title. While the English word *hypothesis* is derived from the Greek word ὑποθέσις, the word *model* is a more accurate translation for this text. See Section I.4 above. Toomer 1998, 23-24. Greek-A: ὑποθέσεων. Greek-B: περὶ ὑποθέσεων. Heiberg 1907, 70.
[2] Greek-A and Greek-B: α omitted. Heiberg 1907, 70. I have only included significant variations between manuscripts in the footnotes; for a complete list of variations see Heiberg 1907, 70-106.
[3] Ptolemy dedicated several of his works to Syrus, including the *Almagest*. Syrus was probably a benefactor and unfortunately this is all that is known about him. Toomer 1998, 35 n. 5.
[4] Ptolemy refers here to the *Almagest*.
[5] Ptolemy reveals his criteria for constructing a model of the heavens; first, the model must account for the observed motion of the heavenly bodies; second, it must be made with an effort to explain the phenomena using uniform and circular motion. The third criterion, which Ptolemy discusses in the *Almagest*, is that the

thing itself (αὐτὸ μόνον) briefly, so that it can be more readily comprehended by both ourselves and by those choosing to arrange the models[6] in an instrument (ὀργανοποιίαν), either doing this in a more "naked" way by restoring each of the motions to its respective epoch by hand, or through a mechanical approach, combining the models[7] to one another and to the motion of the whole. Indeed, this is not the accustomed manner of *sphairopoiia* (σφαιροποιεῖν);[8] for this [sort of manner], apart from failing to represent the models, presents the phenomenon only, and not the underlying [reality], so that the craftsmanship, and not the models, becomes the exhibit; but rather [the manner] according to which the
H72 arrangement together with the different motions under our view, along with the anomalies that are apparent to observers, are subject to uniform and circular courses, even if it is not possible to intertwine them all in a way that is worthy of the aforementioned, but having to exhibit separately each [model] in this way.

[2] We shall make the exposition, so far as the general assumptions are concerned, in agreement with the things delineated in the *Syntaxis*, so far as the details are concerned, following the corrections we have produced in many places on the basis of more continuous observations, either corrections to the models themselves, or corrections to the spatial ratios, or corrections to the periods of restitutions. Adhering to the presentation of the models themselves, that is, where it is necessary, dividing and fitting together the uniform motions in the *Syntaxis*,[9]

model should be made using the simplest representation that meets and the first two criteria. See Section III.1 below.

[6] Taking αὐτὰ to refer back to ὑποθέσεις.

[7] Taking αὐτὰς to refer to ὑποθέσεις.

[8] See Section III.1 below for more on the meaning and use of this word in Greek astronomy.

[9] Taking τὰς ἐκείνῃ to refer back to Συντάξει.

in order that the definitions (ἀφορισμοὺς) of the models[10] are tied to (ἀναδεδομένας) the parts of the zodiac and the starting points, since this is useful for calculations, so that here the individuality of each course is made visible, even if there are many motions in a combination of efforts (συντελῶνται). Concerning the positions and arrangement of the circles causing the anomalies, we will apply the simpler version in respect to the method of instrument-making (ὀργανοποιίας), even if some small variations will follow, and moreover we fit the motions to the circles themselves, as if they are freed from the spheres that contain them (ὡς ἀπολελυμένοις τῶν περιεχουσῶν αὐτοὺς σφαιρῶν),[11] so that we can gaze upon the visual impact of the models bare and unconcealed. We shall
H74 begin with the motions of the whole, because they lead all motions and contain the other motions, and they would give us an example with respect to many things,[12] since the most wondrous nature imparts similar properties to similar things, as will become clear from the things that we shall demonstrate.

[3] Let there be imagined (νοείσθω)[13] a stationary great circle that is centered on the center of the sphere of the cosmos and let it be called the "equator" (ἰσημερινός), and dividing the circumference of it into 360 equal parts, let us call each part a "time-degree" (χρόνοι).[14] Next, let another concentric circle to it be carried in the same plane, and with the same center, with a uniform speed from east to west, and let it be called the "carrier" (φέρων). Let it carry another great circle inclined to it in a fixed position and with the same center, and let it be

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[10] τοῦ.
[11] Taking αὐτοὺς to refer back to τὰς κινήσεις. Greek-A: αὐτά(ς).
[12] Ptolemy is referring to *Almagest* I.1 here.
[13] Ptolemy is using the language of Greek geometry. See Section III.2 below.
[14] I have chosen to translate χρόνοι as "time-degree" and μοῖραι as "degree".

called the “zodiac” (ζῳδιακός). Let the inclination of these planes contain an angle of 23;51,20, such that one right angle is 90; dividing the circumference of the zodiac into 360 equal parts, let these individual divisions be called “degrees” (μοῖραι). And let the points where the carrier and the zodiac cut one another be called “equinoctial”; and let the points that are a quadrant away from them on each side be called the “solsticial”; and of these, let the point that is inclined toward the north be called “summer” and “northern limit”, and the point lying opposite “winter” and “southern limit”. Similarly, of the equinoctial points, let the one in advance of the summer solsticial point, according to the established direction of revolution, be called “vernal”, and let the one in advance of the winter solsticial point be called “autumnal”.

[4] One revolution of the cosmos occurs whenever some point of the carrier begins moving away from some point of the points[15] of the stationary
H76
equator and is restored to the same point for the first time; and clearly such a restitution contains 360 time-degrees. But since the restitutions of the revolutions of the cosmos are not completed in a visible way, whereas the restitutions of the *nychthemera* are visible from the Sun, on this account we measure the other motions by the first restitutions. A *nychthemeron* (νυχθήμερον)[16] is the time in which the Sun, relative to the stationary equator, makes one circuit as a consequent of the revolution of the cosmos. Clearly, if the Sun did not move around the zodiac, then a *nychthemeron* would be the same as a revolution of the

[15] Taking τῶν to refers back to points.
[16] The Greek word νυχθήμερον refers to a 24-hour period encompassing both day and night. I have chosen to use a transliteration of the Greek instead of the word *day*, since the word *day* can imply the period of time between sunrise and sunset.

cosmos, but since the Sun is assumed as moving toward the east, the *nychthemeron* is longer in time than a revolution of the cosmos, and one *nychthemeron* contains one revolution of the cosmos, that is 360 time-degrees, plus so great a part of the equator as the Sun travels along the zodiac in one *nychthemeron*, assuming the courses are uniform.

[5] Now that these have been sketched out, next let us look at the models of the planets, first setting out their simple and unmixed periods (τὰς ἁπλᾶς καὶ ἀμιγεῖς αὐτῶν περιόδους), from which particular, complex ones arise (αἱ κατὰ μέρος καὶ ποικίλαι συνίστανται). These periods were obtained by us as approximations from the restitutions calculated by the corrections (διορθώσεως).

In 300 Egyptian years and 74 *nychthemera*, let it be assumed (ὑποκείσθω)[17] that the Sun makes 300 circuits taken relative to the solsticial and equinoctial points of the zodiac; and let it be assumed that the sphere of the fixed stars and the apogees of the 5 planets make one one-hundred-and-twentieth part of a single similar circuit, that is 3 time-degrees, such that a circle is 360. So that in
H78 36,000 of the aforementioned solar years, which is 36,024 Egyptian years and 120 *nychthemera*, one circuit of the sphere of the fixed stars is completed, and 35,999 overtakings by the Sun relative to it, and the same number of revolutions of the cosmos as the number of *nychthemera* contained in the previously established time plus the circuits of the Sun in the same time.

[17] ὑποκείσθω is a standard term in Greek geometry, which is often translated as "it is given". Toomer 1998, 23-25.

[6] In 8,523[18] solar years taken relative to the solsticial and equinoctial points, which is 8,528 Egyptian years and 277;20,24[19] *nychthemera,* let the Moon make 105,416[20] overtakings of the Sun, i.e. complete months; and again in 3,277 complete months let the Moon make 3,512 restitutions in anomaly; and in 5,458 complete months let the Moon make 5,923 restitutions in latitude.

[7] Likewise, in 993[21] solar years taken relative to the apogees and the sphere of the fixed stars, which is 993 Egyptian years and 255;0,54,46,51[22] *nychthemera* approximately, let the star of Mercury (Ἑρμοῦ) make 3,130[23] restitutions in anomaly. And in 964 of the same kind of solar years, which is 964 Egyptian years and 247;34,2,45,23,40,28[24] *nychthemera* approximately, let the star of Venus (Ἀφροδίτης) make 603 restitutions in anomaly. And in 1,010 of the same kind of solar years, which is 1,010 Egyptian years and 259;22,50,56,16,27,50 *nychthemera* approximately, let the star of Mars (Ἄρεως) make 473 restitutions in anomaly. And in 771[25] of the same kind of solar years, which is 771 Egyptian years and 198;0,9,18,0,26,57 *nychthemera* approximately, let the star of Jupiter (Διὸς) make 706 restitutions in anomaly. And in 324[26] of the same kind of solar years, which is 324 Egyptian years and 83;12,26,19,14,25,48

[18] The Greek text says ηφκη with a subscript of σιν. The subscript is the dative case and clarifies that the number is referring to solar years. The Greek says 8,528 and the Arabic says 8,523. Heiberg 1907 does not make a note of this.
[19] Arabic-B: 257;20,24. See Heiberg 1907, 79.
[20] Heiberg 1907: 109,416. Arabic: 106,416. Corrected by Bainbridge 1620.
[21] The Greek text in Heiberg 1907 says ϡϙγ with a subscript of σιν. The subscript is the dative case and clarifies that the number is referring to solar years.
[22] Arabic-A: 255;54,0,4,46,51. Arabic-B: 255;0,54,0,4,40,51.
[23] The Greek text in Heiberg 1907 says 3150. Corrected by Dennis Duke.
[24] Arabic-A: 247;34,2,45,23,57,28. Arabic-B: 247;34,2,45,23,48,28. Heiberg 1907: 247;33,2,45,23,40,28.
[25] Greek-B and Arabic: 770.
[26] The Greek number in Heiberg 1907 has a subscript of σιν. The subscript is the dative case and clarifies that the number is referring to solar years, which is also in the dative case.

nychthemera approximately, let the star of Saturn (Κρόνου) make 313 restitutions in anomaly.

[8] Concerning the solar sphere, let there be imagined in the plane of the zodiac an eccentric circle situated so that the radius (τὴν μὲν ἐκ τοῦ κέντρου αὐτοῦ)[27] has the same ratio to the line between the centers of it and of the zodiac, which is 60 to $2\frac{1}{2}$; and the straight line produced through the centers and apogee of the eccentric always cuts off an arc of the zodiac of $65\frac{1}{2}$ degrees away from the vernal equinoctial point in the trailing direction of the cosmos. Let the center of the Sun move along the aforementioned eccentric circle from west to east around its center with uniform speed, so that in 37 complete *nychthemera* plus 150 Egyptian years it makes 150 restitutions taken relative to the apogee of the eccentric. And let the sphere of the fixed stars move relative to the center of the zodiac and its poles towards the east $1\frac{1}{2}$ degrees, such that the zodiac is 360, with uniform speed in the established time.

But in the first year from[28] the death of Alexander the Founder,[29] Thoth 1 according to the Egyptians, at noon in Alexandria, the Sun was 162 and 10 sixtieths degrees away from the apogee of the eccentric in the trailing direction of the cosmos. Likewise, the star on the heart of the lion was 117 degrees and 54

[27] The phrase τὴν ἐκ τοῦ κέντρου αὐτοῦ is a common phrase used in Greek geometry used to refer to the radius of a circle.
[28] Greek-A and Greek-B: ἀπό τῆς. This error occurs for this same phrase throughout the text. Corrected by Bainbridge 1620.
[29] Alexander the Great.

sixtieths away from the vernal equinoctial point in the trailing direction of the cosmos along the zodiac.[30]

[9] Concerning the lunar sphere, let there be imagined again a circle concentric with the zodiac circle moving in its plane and around the same center with uniform speed (ἰσοταχῶς) from east to west, [and with its motion being] the excess by which the course in latitude projected on the zodiac exceeds the motion of both the Sun and the elongations in equal time, so that in 88 complete
H82 *nychthemera* plus 37 Egyptian years, it makes approximately two restitutions of the zodiac, for it takes up 1 sixtieth of a degree more in precise computation. Let this circle carry another circle inclined to it with a fixed position around the same center, with the inclination containing an angle of 5 of such parts, such that one right angle is 90. In the aforementioned plane of the aslant circle let there be imagined an eccentric circle, so that its radius has the same ratio to the [line] between the centers of it and of the zodiac, which is 60 to $12\frac{1}{2}$; and let the center of the eccentric move around the center of the zodiac with uniform speed from east to west, with its motion from the northern limit being the excess by which double the course of the mean elongation of the Sun exceeds the course in latitude projected on the zodiac circle in equal time, so that in 348 complete *nychthemera* plus 17 Egyptian years, it makes approximately 203 restitutions relative to the aslant circle; for it falls short 2 sixtieths of a degree in precise computation. And let the center of the epicycle move from west to east, away from the apogee of the eccentricity, always having its position on the eccenter, being double the mean

[30] Ptolemy gives Regulus a location at the epoch, just as he does for the planets. He is treating the sphere of the fixed stars as having a motion relative to the equinoctial and solsticial points.

elongation itself, i.e. the sum of the aforementioned motions, so that in 300 complete *nychthemera* plus 19 Egyptian years, it makes approximately 490 restitutions relative to the eccentricity, for it takes up 4 sixtieths of a degree more in precise computation. Finally, around the aforementioned center of the epicycle that is in the plane of the aslant circle and having the straight line through both centers of it and the zodiac, which it moves around with uniform speed always cutting off the same points of the little circle that we call the apogee and perigee, so that the radius of the eccentric relative to the radius of the epicycle has the same ratio of 60 to $6\frac{1}{3}$. Let the center of the Moon be assumed to travel with uniform speed towards the west, with its motion from the apogee section being the very course of the anomaly, so that in 99 complete *nychthemera* plus 26 Egyptian years it makes approximately 348 restitutions relative to the epicycle, for it falls short 1 sixtieth of a degree in precise computation.

In the same first year from the death of Alexander, Thoth 1 according to the Egyptians, at noon in Alexandria, the northern limit of the aslant circle was 230 degrees and 13[31] sixtieths from the vernal equinoctial point in the advance direction of the cosmos;[32] and the center of the epicycle was 261[33] degrees and 32[34] sixtieths from the apogee of the eccentricity in the trailing direction of the

[31] Greek-F and Arabic-B: 230;19,13.
[32] Bainbridge 1620 adds: τὸ δὲ ἀπόγειον τοῦ ἐκκέντρου ἀπὸ βορείου πέρατος ὡς εἰς τὰ προηγούμενα τοῦ κόσμου μοίρας σιβ καὶ ξ̄. “and the apogee of the eccentre 212;20 degrees from the northern limit towards
H84 the leading parts of the cosmos.” The Arabic adds that the elongation of the apogee of the eccentric circle from the northern limit was 82;40 degrees to the west.
[33] Arabic: 260.
[34] Arabic: 40.

cosmos; and the center of the Moon was 85 degrees and 36[35] sixtieths from the apogee of the epicycle in the advance direction of the cosmos.

[10] Concerning the sphere of Mercury, let there be imagined a concentric circle with the zodiac circle traveling in its plane and around the same center from west to east, so that [its motion is the same as that of] the sphere of the fixed stars. H86 Let this circle carry another circle around the same center and inclined to it in a fixed position, [so that] the inclination of the planes contain an angle of one sixth of a degree, such that one right angle is 90. In the plane of the aslant circle let there be assumed a diameter through the northern and southern limit; and around this between the center of the zodiac and the southern limit, let there be taken two points relative to the center of the zodiac, and around the one of these that is further away from the Earth, let the center of the eccentric circle move with uniform speed away from the apogee of eccentricity in the advance direction of the cosmos, [and with its motion being] the excess by which the course of the Sun exceeds the course of the fixed stars in equal time, so that in 37 complete *nychthemera* plus 144 Egyptian years, it makes approximately 144 restitutions, for it takes up 2 sixtieths of a degree more in precise computation. Concerning the [point] that is closer to the Earth, let the center of the epicycle always move away from the apogee of the eccentricity in the trailing direction of the cosmos, always having its position on the eccenter [and] being a course equal to the aforementioned course, so that in 37 complete *nychthemera* plus 144 Egyptian years, it makes approximately 144 restitutions relative to the eccentricity, for it takes up 2 sixtieths of a degree more in precise computation. Let there be assumed

[35] Arabic: 19 or 17.

that such as the radius of the eccenter is 60, the [line] between the center of the zodiac and one of the two points that is closer to the Earth is 3; and the [line] between the center of the zodiac and one of the two points that is further away from the Earth is $5\frac{1}{2}$; and the [line] between the point closer to the Earth and the center of the eccenter is $2\frac{1}{2}$. Again, let there be imagined a little circle around the center of the epicycle sphere in the plane of the aslant circle, [so that] the straight line through both the centers of it and the point closer to the Earth of the two points, which it moves around with uniform speed, always cutting away the same points of the little circle that we call apogee and the perigee. [And let there be imagined] another little circle concentric with it traveling in the same plane and around the same center with uniform speed, such that the separation relative to the
H88 apogee is completed in the same direction as the revolution of the cosmos, with a course equal to the aforementioned [course] of the center of the eccenter or the epicycle; let this little circle carry another [circle] around the same center and inclined to it in a fixed position, with the inclination containing an angle of $6\frac{1}{2}$, such that one right angle is 90; the radius of the eccenter relative to the radius of the little circle has the ratio of 60 to $22\frac{1}{4}$; and on this little circle let the star move[36] around the center of it with uniform speed, such that a change in position relative to the apogee is completed in the opposite direction of the revolution of the cosmos, [making] an equal course to the sum of the radius of the eccenters, or [the radius] of the epicycle, and [the radius] of the anomaly of the star, so that in

[36] Greek-A, Greek-B, and Arabic: "let the star be imagined". Corrected by Bainbridge 1620.

174[37] complete *nychthemera* plus 208[38] Egyptian years it makes approximately 865 restitutions relative to the inclined circle, for it takes up 4 sixtieths of a degree more in precise computation.

Again, in the first year from the death of Alexander, Thoth 1 according to the Egyptians, at noon in Alexandria the apogee of the eccentricity was 185
H90 degrees and 24 sixtieths away from the vernal equinoctial [point] in the trailing direction of the cosmos; likewise, the northern limit was 5 degrees and 24[39] sixtieths; the center of the eccenter was 52[40] degrees and 16 sixtieths away from the apogee of the eccentricity in the advance direction of the cosmos; the center of the epicycle was 52[41] degrees and 16 sixtieths away from the apogee of the eccentricity in the trailing direction of the cosmos; and again the northern limit of the aslant little circle was 132 degrees and 16 sixtieths away from the apogee of the epicycle in the advance direction of the cosmos. The star was 346 degrees and 41 sixtieths away from the northern limit of the aslant little circle in the trailing direction of the cosmos.

[11] Concerning the star of Venus, again let there be imagined a circle concentric with the zodiac circle traveling in its plane and around the same center with uniform speed, from west to east, so that [its motion is the same as that of] the sphere of the fixed stars. Let this circle carry another circle around the same center and inclined to it in a fixed position, [so that] the inclination of the planes contain an angle of one sixth of a degree, such that one right angle is 90. In the

[37] Arabic-A: 194.
[38] Arabic: 250.
[39] Omitted in Greek-A.
[40] Arabic: 42.
[41] Arabic: 42.

plane of the aslant circle, let there be assumed a diameter through the northern
H92 and southern limit, and on this [diameter] between the center of the zodiac and the northern limit, [let there be assumed] two points cutting off a straight line equal to the [line][42] between the center of the zodiac and one of the two points towards it; and around the point that is closer to the Earth [let there be assumed] an eccentric circle in a fixed position, and the radius of it relative to the line between the centers of it and the zodiac has the ratio of 60 to 1;15;[43] [let] the center of the epicycle [be assumed] to move around the point that is further from the Earth with uniform speed having its position on the eccentric circle always around the aforementioned diameter in the trailing direction of the cosmos, [and with its motion being] the excess by which the course of the Sun exceeds the course of the fixed stars in equal time. Again, let there be imagined a little circle in the sphere of the epicycle around its center and in the plane of the aslant circle, [and having] the straight line through both of the centers of it and one of the two aforementioned points that is further from the Earth, which it moves around with uniform speed always cutting off the same points of the little circle that we call the apogee and the perigee. [And let there be imagined] another little circle concentric with it traveling in the same plane and around the same center with uniform speed, such that the separation relative to the apogee is completed in the same direction as the revolution of the cosmos, with a course equal to the aforementioned [course] of the center of the epicycle; let this little circle carry another [circle] around the same center and inclined to it in a fixed position, with

[42] The reading should be τῇ instead of τὴν. This error occurs for this same phrase throughout the text. Corrected by Jones.
[43] Greek-C: 1;15. Greek-A and Greek-B: 0;15. Arabic: 1.

the inclination containing an angle of $3\frac{1}{2}$, such that one right angle is 90; the radius of the eccenter relative to the radius of the little circle has the ratio of 60 to 43 and $\frac{1}{6}$; and on this little circle let the star move around its center with uniform speed, such that a change in position relative to the apogee is completed in the opposite direction of the revolution of the cosmos, [making] a course equal to the sum of [the motion of] the epicycle and the star, so that in 33 complete *nychthemera* plus 35 Egyptian years it makes approximately 57 restitutions, for it takes up 1 sixtieth more in precise computation.

In the first year [from] the death of Alexander, Thoth 1, according to the Egyptians, at noon in Alexandria, the apogee of the eccentricity was 50 degrees and 24 sixtieths away from the vernal equinoctial [point] in the trailing direction
H94 of the cosmos; and the northern limit [was] the same [number of degrees away from the vernal equinoctial point in the trailing direction of the cosmos]; the center of the epicycle was 177 degrees and 12[44] sixtieths away from the apogee of the eccentricity in the trailing direction of the cosmos; and again the northern limit of the aslant little circle was 87 degrees and 16 sixtieths away from the apogee of the epicycle in the advance direction of the cosmos. The star was 186 degrees and 35 sixtieths away from the northern limit of the aslant little circle in the trailing direction of the cosmos.

[12] Concerning the sphere of Mars, let there be imagined a circle concentric with the zodiac [circle] traveling in its plane and around the same center with uniform speed, from west to east, so that [its motion is the same as

[44] Arabic: 16.

H96 that of] the sphere of the fixed stars. Let this circle carry another circle around the same center and inclined to it in a fixed position, [so that] the inclination of the planes contains an angle of $1\frac{1}{2}$ and $\frac{1}{3}$ degrees,[45] such that one right angle is 90. In the plane of the aslant circle let there be assumed a diameter through the northern and southern limit and on this [diameter] between the center of the zodiac and the northern limit [let there be assumed] two points cutting off a straight line equal to the [line] between the center of the zodiac and one of the two points towards it; and around the point that is closer to the Earth [let there be assumed] an eccentric circle in a fixed position, and the radius is relative to the [line] between the centers of it and the zodiac has the ratio of 60 to 6; [let] the center of the epicycle [be assumed] to move around the point that is further from the Earth with uniform speed, having its position on the eccentric circle always around the aforementioned diameter in the trailing direction of the cosmos, [and with its motion being] the excess by which the course of the Sun exceeds the course both of the fixed stars and of the planet in equal time, so that in 361 complete *nychthemera* plus 95 Egyptian years it makes approximately 51 restitutions, for it falls short 3 sixtieths of a degree in precise computation. Again, let there be imagined a little circle in the sphere of the epicycle around its center and in the plane of the aslant circle, [and having] the straight line through both of the centers of it and one of two aforementioned [points] that is further from the Earth, which it moves around with uniform speed always cutting off the same points of the little circle that we call apogee and perigee. [And let there be imagined] another

[45] Arabic says 4;50. Neugebauer 1975, 908.

little circle concentric with it traveling in the same plane and around the same center with uniform speed, such that the change in position relative to the apogee is completed in the opposite[46] direction as the revolution of the cosmos, with a course equal to the aforementioned [course] of the center of the epicycle; let this little circle carry another [circle] around the same center and inclined to it in a fixed position, with the inclination containing an angle of $1\frac{1}{2}$ and $\frac{1}{3}$ degrees,[47] such that one right angle is 90. The radius of the eccentric relative to the radius of the little circle has the ratio of 60 to $39\frac{1}{2}$; and on this little circle let the star move around the center of it with uniform speed, such that a change in position relative to the apogee is completed in the opposite direction of the revolution of the cosmos, [making] a course equal to the sum of [the motion of] the epicycle and the star, [and with its motion being] the excess by which the course of the Sun exceeds the course of the fixed stars in equal time.

In the first year [from] the death of Alexander, Thoth 1 according to the Egyptians, at noon in Alexandria, the apogee of the eccentricity was 110 degrees and 44[48] sixtieths away from the vernal equinoctial [point] in the trailing direction of the cosmos; and the northern limit [was] the same [number of degrees away from the vernal equinoctial point in the trailing direction of the cosmos]; the center of the epicycle was 356 degrees and 20[49] sixtieths away from the apogee of the eccentricity in the trailing direction of the cosmos; and again the northern

[46] Heiberg 1907 says "opposite" (ἰσταχῶς). The Greek-B and the Arabic have "same". Corrected by Bainbridge 1620.
[47] Arabic: 4;50.
[48] Arabic: 54.
[49] Arabic: 7.

H98 limit of the aslant little circle was 176 degrees and 20 sixtieths away from the apogee of the epicycle in the advance direction of the cosmos. The star was 296[50] degrees and 46 sixtieths away from the northern limit of the aslant little circle in the trailing direction of the cosmos.

[13] Concerning the sphere of Jupiter, let there be imagined a circle concentric with the zodiac [circle] traveling in its plane and around the same center with uniform speed, from west to east, so that [its motion is the same speed as that of] the sphere of the fixed stars. Let this circle carry another circle around the same center and inclined to it in a fixed position, [so that] the inclination of the planes contain an angle of $1\frac{1}{2}$, such that one right angle is 90. In the plane of the aslant circle let there be assumed a straight line from the center of the zodiac to [a point] 20 degrees in advance of the northern limit, and let there be assumed on it two points cutting off a line equal to the [line] between the center of the zodiac and one of the two points towards it; and around the point that is closer to the Earth of the two points [let there be assumed] an eccentric circle in a fixed position, and the radius of it relative to the [line] between the centers of it and the zodiac has the ratio of 60 to $2\,\frac{1}{2}$ and $\frac{1}{4}$; let the center of the epicycle move around the point that is further from the Earth with uniform speed, having its position on the aforementioned eccentric [circle] always, in the trailing direction of the cosmos around the aforementioned diameter, [and with its motion being] the excess by which the course of the Sun exceeds the sum of the course of the

[50] Arabic A and B and Bainbridge 1620: 296. Greek-A and Greek-B: 96.

[sphere] of the fixed stars and of the star in equal time, so that in 238[51] complete *nychthemera* plus 213 Egyptian years it makes approximately 18 restitutions, for
H100 it takes up 1 sixtieth of a degree more in precise computation. Again, in the epicycle sphere, let there be imagined a little circle around its center in the plane of the aslant circle, [so that] the straight line through both the centers of it and the point further away from the Earth of the two aforementioned points, which it moves around with uniform speed, always cutting away the same points of the little circle that we call apogee and perigee. [And let there be assumed] another little circle concentric with it traveling in the same plane and around the same center with uniform speed, such that a change in position relative to the apogee is completed in the same direction as the revolution of the cosmos, with a course equal to the aforementioned [course] of the center of the epicycle; let this little circle carry another [circle] around the same center and inclined to it in a fixed position, with the inclination containing an angle of $1\frac{1}{2}$, such that one right angle is 90. The radius of the eccenters relative to the radius of the little circle has the ratio of 60 to $11\frac{1}{2}$; and on this little circle, let the star move around the center of it with uniform speed, such that a change in position relative to the apogee is completed in the opposite direction as the revolution of the cosmos, [making] an equal course to the sum of [the motion of] the epicycle and the star, again [with its motion being] the excess by which the course of the Sun exceeds the course of the fixed stars in equal time.

[51] Greek-A, Greek-B, and Arabic A and B: 240. Bainbridge 1620: 238.

In the first year from the death of Alexander, Thoth 1 according to the
H102 Egyptians, at noon in Alexandria, the apogee of the eccentricity was 156 degrees and 24 sixtieths from the vernal equinoctial [point] in the trailing direction of the cosmos;[52] the center of the epicycle was 292 degrees and 43[53] sixtieths away from the apogee of the eccentricity in the trailing direction of the cosmos; and again the northern limit of the aslant circle was 92 degrees 43 sixtieths away from the apogee in the forward[54] direction of the cosmos. The star was 231[55] degrees and 31[56] sixtieths away from the northern limit of the aslant little circle in the trailing direction of the cosmos.

[14] Concerning the sphere of Saturn, let there be imagined a circle concentric with the zodiac traveling in its plane and around the same center with uniform speed, from west to east, so that [its motion is the same as that of] the sphere of the fixed stars. Let this circle carry another circle around the same center and inclined to it in a fixed position, [so that] the inclination of the planes contain an angle of $2\frac{1}{2}$, such that one right angle is 90. In the plane of the aslant circle let there be assumed a straight line from the center of the zodiac to the point
H104 40 degrees behind the northern limit, and let there be assumed on it two points cutting off a line equal to the [line] between the center of the zodiac and one of the two points towards it; and around the point that is closer to the Earth of the two points [let there be assumed] an eccentric circle in a fixed position, and the

[52] The Arabic adds: the elongation of the northern limit from this point was 176;24 degrees.
[53] Arabic A and B: 23.
[54] Heiberg 1907 says "trailing" (ἑπόμενα). Arabic: forward direction. Corrected by Bainbridge 1620. Heiberg 1907 says "trailing" (ἑπόμενα).
[55] Greek-A and Greek-B: 31. Arabic: 231.
[56] Arabic: 16.

radius of it relative to the [line] between the centers of it and the zodiac has the ratio of 60 to $3\frac{1}{3}$;[57] let the center of the epicycle move around the point that is further from the Earth with uniform speed, having its position on the aforementioned eccentric [circle] always, in the trailing direction of the cosmos, [and with its motion being] the excess by which the course of the Sun exceeds the sum of [the motion of] the course of the sphere of the fixed stars and the star in equal time, so that in 330 complete *nychthemera* plus 117 Egyptian years it makes approximately 4 restitutions, for it takes up 1 sixtieth of a degree more in precise computation. Again, in the epicycle sphere, let there be imagined a little circle around its center in the plane of the aslant circle, [so that] the straight line through both the centers of it and the point further away from the Earth of the two aforementioned [points], which it moves around with uniform speed, always cutting away the same points of the little circle that we call apogee and perigee. [And let there be assumed] another little circle concentric with it traveling in the same plane and around the same center with uniform speed, such that a change in position relative to the apogee is completed in the same direction as the revolution of the cosmos, with a course equal to the [course] of the center of the epicycle; let this little circle carry another [circle] around the same center and inclined to it in a fixed position, again with the inclination containing an angle of $2\frac{1}{2}$, such that one right angle is 90. The radius of the eccenter relative to the radius of the little circle

[57] Arabic says $3\frac{1}{3}$ and $\frac{1}{12}$ (i.e. 3;25). This is the same value given in the *Almagest*. The Greek is missing the $\frac{1}{12}$.

has the ratio of 60 to $6\frac{1}{2}$;[58] and on this little circle, let the star move around the center of it with uniform speed, such that a change in position relative to the apogee is completed in the opposite direction as the revolution of the cosmos, [making] an equal course to the sum of [the motion of] the epicycle and the star, again [with its motion being] the excess by which the course of the Sun exceeds the course of the sphere of the fixed stars in equal time.

In the first year [from] the death of Alexander, Thoth 1 according to the Egyptians, at noon in Alexandria, the apogee of the eccentricity was 228 degrees
H106 and 24 sixtieths from the vernal equinoctial [point] in the trailing direction of the cosmos; the center of the epicycle was 210 degrees and 38 sixtieths away from the apogee of the eccentricity in the trailing direction of the cosmos; and again the northern limit of the aslant circle was 70 degrees and 38 sixtieths away from the apogee in the trailing direction of the cosmos. The star was 219 degrees and 16[59] sixtieths away from the northern limit of the aslant little circle in the in the forward[60] direction of the cosmos.

[58] The text in Greek-B ends here. The rest of the text continues in Greek-A, but instead of numbers there are blanks. The remainder of the text in Greek-A was probably copied from similar section for one of the other planets, such as Jupiter. This would explain why the numbers do not appear in the Greek. Heiberg 1907 includes numbers, which have been taken from the Arabic text.
[59] Omitted in Arabic-A.
[60] Heiberg 1907 says “trailing” (ἑπόμενα). Arabic: forward. Corrected by Bainbridge 1620.

II.2 Précis of Book I, Part B[61]

[1] Ptolemy begins Part B of Book I by stating that according to what has been previously said, the sphere of the fixed stars has a motion that is close to the universal movement.[62] The planets, all of which lie below this movement,[63] move from east to west with it, but they each have a movement in the opposite direction from west to east. They also move forward and backward and from north to south, all of which are local motions. The local motion is the primary of all movements and things whose nature is eternal have only this kind of motion. The changes with respect to quality or quantity exist in things that are not eternal are changes to the thing itself and its substance.[64]

Ptolemy summarizes the motions of the Sun, Moon, and five planets. He says that in the case of the Sun there is only one anomaly in its motion in the ecliptic, because there is nothing stronger than it from which it receives another anomaly. For the planets there are two kinds of anomaly: one that is similar to the motion just mentioned and the other depends on the return to the Sun. Each planet has a voluntary motion and a constrained motion.[65] For the Moon there are two sorts: the one just mentioned and another which corresponds to its inclination on the ecliptic in its inclined orbit. For the five planets there are three sorts: two were

[61] I have constructed this précis using Goldstein, Morelon, and two Arabic manuscripts (Arabic-A and Arabic-B). Significant variances between these translations are noted in the footnotes.
[62] Goldstein translates this as: "…and containing no contrarity at all". Goldstein 1967, 5. Morelon translates this as: "…et sans que l'on puisse du tout y accepter des situations contradictorires". Morelon 1993, 56.
[63] Goldstein translates *hādhā al-ḥaraka* as "the (prime) mover" and Morelon translates it as "ce mouvement". Goldstein 1967, 5, 26. Morelon 1993, 56-57.
[64] Goldstein and Morelon's translations vary here. Goldstein 1967, 5. Morelon 1993, 16.
[65] Goldstein translates the text as "Each of the planets has one free motion, the other is determined of necessity." Goldstein 1967, 6. Morelon translates it as "un mouvement volontaire et un mouvement auquel il est contraint". Morelon 1993, 56-57.

just mentioned and the third occurs due to the orbits that revolve around the Earth inclined on the epicycle. The latter do not surround the Earth and it is because of this reason we have come to think that the inclined orbits move in opposite orientation and that their movement produces parallel planes.

The descriptions of the solsticial and equinoctial points are given along with a description of latitudinal motion. Ptolemy explains how to move from imagining one type of inclination to another type of inclination and ends with a discussion of latitudinal motion.[66]

[2] The arrangement of the spheres, Ptolemy explains, has been debated for some time. He contends that the Moon is certainly the closest to the Earth, followed by Mercury, Venus, Mars, Jupiter, Saturn, and the sphere of the fixed stars. Concerning the position of the Sun, there are three options: all five planets lie above the sphere of the Sun, they all lie below, or some lie above and others lie below. We cannot decide this matter with certainty. He explains that it is more difficult to determine the distance of the five planets than the distance of the two luminaries, since the distance of the two luminaries were determined relying on eclipses. While a planet has never been observed passing in front of the Sun, this does not mean that this phenomenon has never occurred. Consequently, one cannot be certain about the position of the Sun relative to the planets.

[3] Turning to the distances of the planets, Ptolemy determines the ratio for each planet of least distance to greatest distance. He assumes the order of the planets as: Moon, Mercury, Venus, Sun, Mars, Jupiter, and Saturn. Relying on the

[66] While Ptolemy seems to be referring to latitudinal motion, the meaning of this sentence is ambiguous. Both Morelon 1993 and Goldstein 1967 translate it differently; while Morelon's translation is closer to the Arabic text, it is not clear what precisely Ptolemy is referencing. See Section III.6.1 below.

calculations from the *Almagest*, he assumes the least distance of the Moon to be 33 Earth radii and the greatest distance to be 64 Earth radii. Additionally, he assumes the least distance of the Sun to be 1,160 Earth radii and the greatest distance to be 1,260 Earth radii. The ratios of least distance to greatest distance for Mercury is 34:88 and for Venus 16:104, making the least absolute distance for Mercury 64 Earth radii and the greatest 166 Earth radii and for Venus 166 and 1,079 Earth radii, respectively. Since the least distance of the Sun is 1,160 Earth radii, there is a discrepancy in the distances. The gap is not large enough for Mars, which has a ratio of least distance to greatest distance of 7:1, to fit between Venus and the Sun. Ptolemy says that when the distance of the Moon is decreased then we must also decrease the distance of the Sun and vice versa. Consequently, if the distance of the Moon is increased then the distance of the Sun will increase, so that the greatest absolute distance of Venus will correspond with the least absolute distance of the Sun.

The order of the planets given, Ptolemy contends, is not based on distances alone, but also on the types of anomalies the planets exhibit. He argues that the spheres closest to the air move with many motions, since they resemble the nature of the elements near them and the spheres closest to the universal motion move with a simple motion. This is why the Moon and Mercury exhibit a complex motion, having two perigees in a revolution and the sphere of the fixed stars move with a simple motion that is unchanging.

For the remaining planets, the ratios of least distance to greatest distance are as follows; for Mars 7:1, for Jupiter 23:37[67], and for Saturn 5:7. The ratios of greatest distance to least distance are 1,1260 Earth radii to 8,820 Earth radii for Mars, 8,820 to 14,187 for Jupiter, and 14,187 to 19,865 for Saturn. The greatest distance for Saturn is the distance of the sphere of the fixed stars. Ptolemy reiterates that the unit used is the radius of the Earth and water and he summarizes the greatest distances for each planet.

[4] Having already given the distances of the planets in Earth radii, Ptolemy then gives their distances in myriad stades. He assigns the Earth a radius of 2;52 myriad stades and a circumference of 18 myriad stades. The boundary between the terrestrial and the celestial is 94;36[68] myriad stades. The boundary separating the lunar sphere from the sphere of Mercury is 183;28[69] myriad stades, Mercury from Venus is 475;52 myriad stades, Venus from the Sun is 3,093;8 myriad stades, the Sun from Mars is 2 myriad myriad 3,612 myriad stades, Mars from Jupiter is 5,284, Jupiter from Saturn is 4 myriad myriad and 4,769;22[70] myriad stades, and Saturn from the sphere of the fixed stars is 5 myriad myriad and 6,946;20 myriad stades.

Ptolemy states that if the universe is constructed in the way that he describes, then there is no void between the greatest distance of a sphere and the least distance of the sphere that follows it. He claims that this arrangement is most plausible, since it is not conceivable that there would be a vacuum or anything

[67] Hebrew reads 23:38. Goldstein 1967, 7.
[68] Arabic-A says 74 instead of 94. Goldstein 1967, 8.
[69] Arabic-A says 133 instead of 183. Goldstein 1967, 8.
[70] Arabic-A says 4,999 instead of 4,769. This number is corrupt see Goldstein 1967, 8, 11.

that is useless or without meaning in nature.[71] If there is indeed emptiness between the spheres then it is clear that the distances given are the least possible distances.

[5] Following the calculations for the distances of the planets, Ptolemy calculates the sizes of the planets. To determine the sizes he utilizes the apparent diameters, the models, and the aforementioned distances. Hipparchus said, according to Ptolemy, that the Sun is 30 times as big as the smallest star and the apparent diameter of the Sun, which appears to be the largest star, is approximately $\frac{1}{10}$ the apparent diameter of the Sun. These values were determined assuming the Earth is a point. Ptolemy finds the apparent diameter of Venus to be $\frac{1}{10}$ that of the Sun, the diameter of Jupiter is $\frac{1}{12}$ that of the Sun, Mercury $\frac{1}{15}$ of the Sun, Saturn $\frac{1}{18}$ of the Sun, and Mars and the first magnitude stars $\frac{1}{20}$ the diameter of the Sun. The Moon at mean distance is $1\frac{1}{3}$ the diameter of the Sun.

If all of the planets subtended equal apparent angles at mean distance, then the ratio of their diameters would be equal to the ratio of their distances. Ptolemy begins by determining the mean distance of each planet by finding the product of the mean distance and the apparent diameter. For the Moon he determines the mean distance to be 48 Earth radii, 115 Earth radii for Mercury, $622\frac{1}{2}$ for Venus, 5,040 for Mars, 11,504 for Jupiter, 17,026 for Saturn, and 20,000 for the first magnitude stars.

[71] Goldstein and Morelon's translations vary throughout this section. Morelon 1993, 72. Goldstein 1967, 8.

Ptolemy explains that in the *Almagest* the solar diameter was found to be $5\frac{1}{2}$ when the diameter of the Earth is 1. He says that $5\frac{1}{2}$ is to 1,210 as 1 part is to 220 and using this ratio he finds that the diameter of the Moon is $\frac{1}{4}$ and $\frac{1}{24}$, the diameter of Mercury is $\frac{1}{27}$, the diameter of Venus is $\frac{1}{4}+\frac{1}{20}$, the diameter of Mars is $1\frac{1}{7}$, the diameter of Jupiter is $4\frac{1}{3}+\frac{1}{40}$, the diameter of Saturn is $4\frac{1}{4}+\frac{1}{20}$, and the diameter of the first magnitude fixed stars is at least $4\frac{1}{2}+\frac{1}{20}$. If the volume of the Earth is 1, then the volume of the Moon is $\frac{1}{40}$, the volume of Mercury is $\frac{1}{19683}$,[72] the volume of Venus is $\frac{1}{44}$, the volume of the Sun is $166\frac{1}{3}$, the volume of Mars is $1\frac{1}{2}$, the volume of Jupiter is $82\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$, the volume of Saturn is $79\frac{1}{2}$, and the volume of the first magnitude stars are at least $94\frac{1}{6}+\frac{1}{8}$. The Sun has the greatest volume, followed by the fixed stars of the first magnitude, Jupiter, Saturn, Mars, Venus, the Moon, and lastly Mercury.

If the distances are correct, Ptolemy repeats, then the volumes will also be correct. However, if the distances are greater, then the volumes presented will be the minimum sizes. If these distances are correct, then Mercury, Venus, and Mars should display some parallax. The ratio of each of them to the lunar and solar parallax is equal to the ratio of their distances to those of the Sun and Moon.

[6] The first appearance and disappearance of a star relative to the Sun takes place when the star is near the horizon, either rising or setting, and the Sun is near the horizon. The *arcus visionis* is measured on the circle through the center

[72] Hebrew says $\frac{1}{19663}$. Goldstein 1967, 9.

of the Sun and the zenith. For the first magnitude stars, it is about 15 degrees, 13 degrees for Saturn, 9 degrees for Jupiter, $14\frac{1}{2}$ for Mars, 7 degrees for Venus at morning setting and evening rising and 5 degrees at evening setting and morning rising, and 12 degrees for Mercury. For acronychal morning risings of the outer planets, the Sun must be below the horizon by half the aforementioned values.

[7] Ptolemy examines why our imagination ascribes magnitudes to the celestial bodies, which are not in the same ratio as their distance. He contends that this effect is an optical illusion due to the difference in outlook and that this difference is apparent in everything that is seen at great distances. The planets appear closer to us than they are because of the degeneration of sight and the eye has a visual disability to estimate distances.[73]

[73] Goldstein's translation varies from the Arabic text throughout this section. Goldstein 1967, 9.

Section III:

Commentary of Book I of the *Planetary Hypotheses*

III.1 *Sphairopoiia*

Ptolemy relied on various instruments when taking observations. Among them, a meridian ring, a meridian plaque, an armillary sphere, and a parallactic instrument are all discussed in the *Almagest*.[1] In most instances Ptolemy provided a brief description of the instrument, an explanation of how it works, and some details about how to construct such a device, such as necessary materials or dimensions.[2] From Ptolemy's brief discussions of the instruments introduced in the *Almagest*, it appears that Ptolemy had some experience handling each of the instruments he describes. Unlike the instruments he describes in the *Almagest*, Ptolemy's discussion of the instrument he references in the first paragraph of the *Planetary Hypotheses* is imprecise and ambiguous. While Ptolemy refers to an instrument (ὀργανοποιίαν) in the *Planetary Hypotheses*, he does not imply an instrument

[1] Toomer 1998, 61-63, 217-219, 244-247.

[2] For example, Ptolemy offers a detailed description of the armillary sphere in Book V of the *Almagest*. Ptolemy explains the overall appearance of the instrument and how to construct it. He discusses the details, such as how he joined two rings together at diametrically opposite points so that the rings form right angles. He also explains how the poles of the ecliptic are marked and how a peg is placed through the rings so that a third ring can be attached and can freely pivot. In his description of the parallactic instrument he says: "We made two rods, rectangular [in cross-section], no less than 4 cubits long….". Toomer 1998, 217, 244.

that could be used for observations. Instead, he discusses a device that would exhibit the motions of the celestial bodies.[3]

Ptolemy lays out his objectives, establishes his intended audience, and briefly discusses this device in the first paragraph of the *Planetary Hypotheses.* He writes:

> We have worked out, Syrus, the models (ὑποθέσεις) of heavenly motions through the books of the *Mathematical Syntaxis* (μαθηματικῆς συντάξεως), demonstrating by arguments, concerning each example, both the logicality and agreement everywhere with the phenomena, with a view to a presentation of uniform and circular motion (τῆς ὁμαλῆς καὶ ἐγκυκλίου κινήσεως), which necessarily was to arise in things taking part in (κεκοινωνηκόσι) eternal and orderly motion and that are not capable to undergo increase or decrease in any way. Here we have taken on the task to set out the thing itself (αὐτὸ μόνον) briefly, so that it can be more readily comprehended by both ourselves and by those choosing to arrange the models in an instrument (ὀργανοποιίαν), either doing this in a more "naked" way by restoring each of the motions to its respective epoch by hand, or through a mechanical approach, combining the models to one another and to the motion of the whole. Indeed, this is not the accustomed manner of *sphairopoiia* (σφαιροποιεῖν); for this [sort of manner], apart from failing to represent the models, presents the phenomenon only, and not the underlying [reality], so that the craftsmanship, and not the models, becomes the exhibit; but rather [the manner] according to which the arrangement together with the different motions under our view, along with the anomalies that are apparent to observers, are subject to uniform and circular courses, even if it is not possible to intertwine them all in a way that is worthy of the aforementioned, but having to exhibit separately each [model] in this way.[4]

Everything that Ptolemy says about instrument-making in the *Planetary Hypotheses* can be found in the first paragraph of the *Planetary Hypotheses*. In the remainder of Book I Ptolemy does precisely what he stated he would do, namely describing the celestial models and the geometric arrangement of the cosmos, including the order, distances and relative volumes of the Sun, moon and planets. The description of the models, Ptolemy explains, will adhere to uniform, circular motion. He does not describe what qualifies as

[3] I will use the word "instrument" to refer to the object Ptolemy describes in the *Planetary Hypotheses*; however, it should be noted that this is not an instrument used for taking observations, but instead a tangible representation of the heavenly motions.
[4] Heiberg 1907, 71.

uniform motion, he simply states: “agreement everywhere with the phenomena, with a view to a presentation of uniform and circular motion, which necessarily was to arise in things taking part in eternal and orderly motion, and that are not capable to undergo increase or decrease in any way”(τὸ πανταχοῦ πρὸς τὰ φαινόμενα σύμφωνον πρὸς ἔνδειξιν τῆς ὁμαλῆς καὶ ἐγκυκλίου κινήσεως, ἣν ἀναγκαῖον ἦν ὑπάρχειν τοῖς τῆς ἀιδίου καὶ τεταγμένης κινήσεως κεκοινωνηκόσιν καὶ κατὰ μηδένα τρόπον τὸ μᾶλλον καὶ τὸ ἧττον ἐπιδέξασθαι δυναμένοις).[5] While Ptolemy endeavored to describe heavenly phenomena using uniform, circular motion, his priority is agreement between the models and the phenomena.

Relying on the models from the *Almagest*, Ptolemy aims to succinctly describe the models for ourselves and for instrument-builders. While Ptolemy does not explain specifically to whom he is referring when he says “ourselves”, he is likely referring to astronomers like himself, and Syrus, to whom the text is addressed. The instrument-makers that Ptolemy addresses are not merely craftsman, but builders who will use this text to arrange the planetary models he describes into a device that will illustrate his theory. The instructions that Ptolemy gives for the construction of the armillary sphere are far more informative and descriptive than what he gives in the *Planetary Hypotheses*. This is owing to the fact that he has seen and used the instruments he describes in the *Almagest*, while the instrument described in the *Planetary Hypotheses* had probably never been previously constructed. Ptolemy provides little detail in the *Planetary Hypotheses* about the instrument he anticipates the instrument-maker building and there is no evidence that the instrument he describes in the *Planetary Hypotheses* was ever built.

[5] Heiberg 1907, 71.

Ptolemy wanted this instrument to represent the motions of the celestial bodies, including the various anomalies, in one of two ways. First, he claimed it could be done in "a more 'naked' way by restoring each of the motions to its respective epoch by hand" (ἐάν τε γυμνότερον διὰ χειρὸς ἑκάστης τῶν κινήσεων ἐπὶ τὰς οἰκείας ἐποχὰς ἀποκαθισταμένης τοῦτο δρῶσιν).[6] While Ptolemy does not elaborate on this "more naked way", I believe he means that each celestial body would be moved manually to its appropriate position for a given date. This instrument would not necessarily be gear powered, for example the different parts of the model could be moved into position by simply moving the body with one's hand.

The second way that this instrument could be constructed, Ptolemy suggests, is "through a mechanical approach, combining the models to one another and to the motion of the whole" (ἐάν τε διὰ τῶν μηχανικῶν ἐφόδων συνάπτωσιν αὐτὰς ἀλλήλαις τε καὶ τῇ τῶν ὅλων).[7] The models would be connected to one another mechanically, so that certain parts of the model move other parts, possibly though a series of gears, although this is not specified. The motion of the whole, which is the daily motion of the sphere of the fixed stars, would also be displayed. Ptolemy does not want the celestial bodies to move against a stationary background of the stars, like the front dial of the Antikythera Mechanism; he wants the motion of the fixed stars to be exhibited.

Ptolemy says that this instrument should not be constructed in the accustomed manner of *sphairopoiia* (σφαιροποιεῖν).[8] James Evans and J. Lennart Berggren demonstrate that the Greek word *sphairopoiia*, which literally translates to sphere-making, has an interesting range of meanings. Strabo (c. 64 B.C. – c. 25 A.D.) and

[6] Heiberg 1907, 71.
[7] Heiberg 1907, 71.
[8] Heiberg 1907, 71.

Plutarch (first-century A.D.) both used *sphairopoiia* to describe making something into a ball or sphere. Archimedes (d. 212 B.C.) wrote a text called *sphairopoiia*, which is now lost. Geminos, who lived in the first-century B.C., used *sphairopoiia* as an astronomical term to mean: (1) a branch of mechanics; (2) a particular astronomical device; (3) a spherical theory of the world; or (4) a spherical system that actually exists in nature.[9] Theon of Smyrna, who flourished around 100 A.D., uses *sphairopoiia* to mean a particular instrument and he says that he made a *sphairopoiia* based on Plato's spindle whorls.[10] Ptolemy indicates that there was a tradition of *sphairopoiia* in the second-century A.D. and that within this tradition there was an accustomed manner. According to Ptolemy, in this accustomed manner of *sphairopoiia* the craftsmanship, not the astronomical models, is the object of display. Furthermore, the accustomed manner does not display the underlying reality of the models, but the phenomena only. This means that the instrument would not display the true movement of each body, but instead it would display the apparent motion or the mean motion. It does not appear that Ptolemy was working closely with a particular instrument-maker, since he addressed it to "those choosing to arrange the models in an instrument." The *Planetary Hypotheses* was written so that one choosing to construct an instrument could consult this work. Ptolemy makes no claims that he intends to build this instrument himself; he does not state that he is working with an instrument-maker, and it is possible he has not even conferred with one to discuss the logistics of building the instrument he discusses.

[9] Geminos says "there is a certain spherical construction (*sphairopoiia*) proper for each planet". Evans and Berggren 2006, 52-53.
[10] Dupuis 1892, 16. Evans and Berggren 2006, 52.

III.1.1 Textual Evidence of Instrument-making

The instrument that Ptolemy anticipates the craftsman constructing, while technically complex, is not without precedent. There are significant textual references, as well as archaeological remains, that suggest that many mechanical astronomical models had been built in the classical world. While few technological books from antiquity survive, the ones that do show evidence of heavy gear work as early as the third-century B.C. In the pseudo-Aristotelian text, *The Mechanical Problems*, the author discusses wheels engaging in order to transfer motion; however, it is unclear whether these wheels have gears, or if they are smooth and transfer their motion through frictional contact.[11] Gear work is also present in the text of the first-century B.C. Roman architect-engineer, Vitruvius. In *On Architecture,* Vitruvius explicitly discusses gear work in relation to the construction of a water mill.[12] Finally, gear work is present in Hero of Alexandria's *A Treatise on Pneumatics*, in which he discusses a lamp that uses geared wheels to distribute the wick of the lamp and an automaton using gears. Additionally, Hero discusses an odometer that can be connected to a chariot in the *Dioptra*.[13] Clearly gears were used in the Greco-Roman world long before Ptolemy's time.

In addition to textual references of heavy gear work, there is also evidence of finer, mathematical gear work. Several ancient textual references alluded to mechanical astronomical devices, with the most significant coming from Cicero, the Roman lawyer and philosopher who lived in the first-century B.C. Cicero often used the Latin word *sphaera* to refer to these devices. English translations of Cicero's text translate *sphaera*

[11] Aristotle, *Mechanics*, 855a30-857b8. Drachmann 1963, 200-203.
[12] Ganger 1931, 305-307.
[13] Woodcroft 1851, 52-53, 109-110.

as *globe* or *orrery*, neither of which capture the meaning of the word that Cicero uses, since *sphaera* is a technical term that Cicero is deliberately borrowing from the Greek.[14]

In the following quotation from *De Re Publica*, Cicero is at the home of Marcus Marcellus. Gallus, who is also there, brings out a model of the heavens. He says:

> Though I had heard this globe (*sphaerae*) mentioned quite frequently on account of the fame of Archimedes, when I actually saw it I did not particularly admire it; for that other celestial globe, also constructed by Archimedes, which the same Marcellus placed in the temple of Virtue, is more beautiful as well as more widely known among the people. But when Gallus began to give a very learned explanation of the device, I concluded that the famous Sicilian had been endowed with greater genius than one would imagine it possible for a human being to possess. For Gallus told us that the other kind of celestial globe (*sphaerae*), which was solid and contained no hollow space, was a very early invention, the first one of that kind having been constructed by Thales of Miletus, and later marked by Eudoxus of Cnidus (a disciple of Plato, it was claimed) with the constellations and stars which are fixed in the sky. He also said that many years later Aratus, borrowing this whole arrangement and plan from Eudoxus, had described it in verse, with out any knowledge of astronomy, but with considerable poetic talent. But this newer kind of globe (*sphaerae*), he said, on which were delineated the motions of the sun and moon and of those five stars which are called wanderers, or, as we might say, rovers, contained more than could be shown on the solid globe (*sphaera solida*), and the invention of Archimedes deserved special admiration because he had thought out a way to represent accurately by a single device for turning the globe those various and divergent movements with their different rates of speed (*quem ad modum in dissimillimis motibus inaequabiles et varios cursus servaret una conversio*). And when Gallus moved the globe (*sphaeram*), it was actually true that the moon was always as many revolutions behind the sun on the bronze contrivance as would agree with the number of days it was behind it in the sky. Thus the same eclipse of the sun happened on the globe (*sphaera)* as it would actually happen, and the moon came to the point where the shadow of the earth was at the very time when the sun… out of the region…[15]

In this passage Cicero discusses two different *sphaera*. The first he describes as a *sphaera solida*, which seems to be a solid sphere of some kind. However, none of the existing texts by Arastus mention this instrument. Cicero does not explain how the

[14] I have decided to leave the English translations of Cicero as they are, but I have included the Latin word or words in parenthesis whenever an astronomical device is discussed.
[15] At this point two and half pages of the text appear to have been lost. Keyes [1928] 1966, 41-43.

sphaera solida worked nor does he provide any other details about it. In Book VII of the *Almagest*, Ptolemy mentions Hipparchus's στερεὰ σφαῖρα, which literally means a 'solid sphere'. Given the context, it appears that Ptolemy is referring to a celestial globe.[16] Cicero may be using the Greek technical term for a celestial globe and translating it into the Latin.

According to Cicero, Archimedes constructed the second type of *sphaera* described in *De Re publica,* which displayed the motions of the Sun, Moon, and five planets. It displayed the different speeds of the celestial bodies accurately and one motion drove the device. Archimedes' *sphaera* was calibrated so that the instrument displayed the heavenly phenomena as it appeared in the cosmos. Cicero clearly admired this device and Archimedes's ability to build such an instrument. The *sphaera*, Cicero explains, came from Syracuse and was taken by the Romans during the capture of the city during the Second Punic War, 214-212 B.C.[17] It is not clear that Cicero himself saw the device he discusses. Cicero's dialogue was written over one hundred years after the siege of Syracuse, but the dialogue is set around 129 B.C. and the particular event in the life of Sulpicius Gallus that is recalled took place even earlier, possibly around 166 B.C.

Cicero discusses a celestial device again in *Tuscalan Disputations*. Similar to the previous passage, here he says that a single motion drives the *sphaera's movement.* All of the other celestial bodies move at their own speed, but they have one driving force. Cicero says:

> For when Archimedes fastened on a globe (*sphaeram*) the movements of moon, sun and five wandering stars, he, just like Plato's God who built the world in the *Timaeus*, made one revolution of the sphere control several movements utterly unlike in slowness and speed (*ut tarditate et celeritate dissimillimos motus una*

[16] Toomer 1998, 327 n. 48.
[17] King 1927, 74 n. 1.

> *regeret conversio*). Now if in this world of ours phenomena cannot take place without the act of God, neither could Archimedes have reproduced the same movements upon a globe (*sphaera*) without divine genius.[18]

It is likely that Cicero is referring to the same *sphaera* built by Archimedes both in *Tuscalan Disputations* and *De Re publica.* Cicero explains that the motion of the Moon, Sun and five planets is accomplished due to one revolution. It appears that turning the *sphaera* so that it completes a revolution results in the Moon, Sun and five planets moving each at their respective rate, so that the all of the celestial bodies are driven by one motion. Unfortunately, Cicero does not provide more details on how the device turns.

Finally, Cicero mentions a *sphaera* in *De Natura Deorum.* When discussing a creator of the world, he says:

> When you see a statue or painting, you recognize the exercise of art; when you observe from a distance the course of a ship, you do not hesitate to assume that its motion is guided by reason and by art; when you look at a sun-dial or a water-clock, you infer that it tells the time by art and not by chance; how then can it be consistent to suppose that the world, which includes both the works of art in question, the craftsmen who made them, and everything else besides, can be devoid of purpose and of reason? Suppose a traveler to carry into Scythia or Britain the orrery (*sphaeram)* recently constructed by our friend Posidonius, which at each revolution reproduces the same motions of the sun, the moon and the five planets that take place in the heavens every twenty-four hours, would any single native doubt that this orrery (*sphaera*) was the work of a rational being? These thinkers however raise doubts about the world itself from which all things arise and have their being, and debate whether it is the product of chance or necessity of some sort, or of divine reason and intelligence; they think more highly of the achievement of Archimedes in making a model of the revolutions of the firmament than of that of nature in creating them, although the perfection of the original shows a craftsmanship many times as great as does that counterfeit.[19]

In this passage Cicero makes the argument for the existence of God based on design. According to Cicero, Posidonius, a stoic philosopher and astronomer who was active during the first and second centuries B.C, constructed this *sphaera*. Posidonius's *sphaera*

[18] King 1927, 73-75.
[19] Rackham 1967, 206-209.

is described similar to the way Cicero describes the instrument constructed by Archimedes. Both instruments reproduce the movements of the Sun, Moon, and five planets.

In total, Cicero refers to three or four different devices, depending on whether the *sphaera* he mentioned in *De Re publica* and *Tuscalan Disputations* were indeed the same device. The devices that Cicero referenced are: an early *sphaera solida*, the *sphaera* constructed by Archimedes, and the *sphaera* constructed by Posidonius. Thales of Miletus lived during the sixth and seventh centuries B.C., so if the *sphaera solida* was indeed constructed by Thales, it would have been very old in Cicero's time. However, Cicero does not explicitly say that Thales constructed this *sphaera*, but instead says that it was constructed using the same design. Additionally, it is not clear that Cicero ever saw the device.

If the *sphaera* Cicero discussed were indeed constructed by Archimedes, then we recognize that the instrument-maker had both astronomical and mechanical knowledge. Archimedes wrote texts on mechanics and astronomy, such as *On Balances and Levers* and *The Sand Reckoner*. He would have understood the astronomical models of the time, and he would have had at least some of the skills necessary to implement these models into a device that exhibited their motions. Like Archimedes, Posidonius had astronomical expertise. If Cicero's descriptions can be trusted, we learn that the instrument-makers had astronomical knowledge and in the case of Archimedes, knowledge of mechanics. If Archimedes did in fact build the device that Cicero describes, it did not display Ptolemaic or Hipparchian astronomy, since Archimedes predates both Ptolemy and Hipparchus.

Without knowing the details of the *sphaera* it is difficult to discern whether the mechanical capabilities and limitations had any impact on the astronomical models, i.e. if what the instrument-maker had the ability to build influenced the models that astronomers used to describe heavenly motion. In the *Planetary Hypotheses*, Ptolemy does not appear to be working with an instrument-maker closely, but some of the changes he makes to his models may have been done with the instrument-maker in mind, in particular the changes he makes to the latitude theory.[20] There is a lot that remains unknown about third-century B.C. astronomy, so it is difficult to deduce the exact types of models that would have been displayed by the device attributed to Archimedes or Posidonius; however, it is likely that the lunar and planetary models displayed were simpler than Ptolemy's models. The lunar model, for example, most likely would have displayed one anomaly. It is even more difficult to speculate about what the planetary models would have looked like. It is interesting to note that in the passage in *De Re publica* Cicero points out that the instrument displayed solar eclipses as they actually happened, which seems to mean that the device was designed so that it could corresponded with the heavenly phenomena. The Antikythera Mechanism displayed eclipse calendars on its back panels, which may suggest that Cicero was referencing a device similar in design to that one.

In addition to the passages by Cicero, textual evidence of astronomical instruments exists in a few other places. Ovid, the first-century A.D. Roman poet, provides a description of an astronomical device in his work, *Fasti*. He says:

> They say that Rome had forty times celebrated the Parilia when the goddess, Guardian of Fire, was received in her temple; it was the work of that peaceful king, than whom no man of more god-fearing temper was ever born in Sabine

[20] See Section III.2.6 below.

> land. The buildings which now you see roofed with bronze you might then have seen roofed with thatch, and the walls were woven of tough osiers. This little spot, which now supports the Hall of Vesta, was then the great palace of unshorn Numa. Yet the shape of the temple, as it now exists, is said to have been its shape of old, and it is based on a sound reason. Vesta is the same as the Earth; under both of them is a perpetual fire; the earth and the hearth are symbols of the home. The earth is like a ball, resting on no prop; so great a weight hangs on the air beneath it. Its own power of rotation keeps its orb (*orbem*) balanced; it has no angle which could press on any part; and since it is placed in the middle of the world and touches no side more or less, if it were not convex, it would be nearer to some part than to another, and the universe would not have the earth as its central weight. There stands a globe (*globus*) hung by Syracusan art in closed air, a small image of the vast vault of heaven, and the earth is equally distant from the top and bottom. That is brought about by its round shape. The form of the temple is similar: there is no projecting angle in it; a dome protects it from the showers of rain.[21]

The Hall of Vesta was dome shaped and inside was the globe, placed equally distant from the edge of the dome. Ovid does not discuss what the globe does nor does he say whether the globe consisted of moving parts. While Ovid does not specify who constructed the instrument, he says that the globe is hung by Syracusan art. Since Archimedes was from Syracuse, it is probable that Ovid is making a reference to Archimedes.

In his work, *The Divine Institutes*, Lactantius, who wrote during the fourth-century A.D., references an instrument constructed by Archimedes. He writes:

> Could Archimedes the Sicilian have devised from hollowed metal a likeness and figure of the world, in which he so arranged the sun and moon that they should effect unequal motions and those like to the celestial changes for each day, as it were, and display or exhibit, not only the risings and settings of the sun and the waxings and wanings of the moon, but even the unequal courses of revolutions and the wanderings of the stars as that sphere turned, and yet God Himself be unable to fashion and accomplish what the skill of a man could simulate by imitation? Which answer, therefore, would a Stoic give if he had seen the forms of stars painted and reproduced in that sphere? Would he say that they were moved by their own purpose or would he not rather say by the skill of the designer?[22]

[21] Frazer 1959, 339-341.
[22] McDonald 1964, 114.

Lactantius says Archimedes' device was made from hollow metal. The device had the capability to describe the unequal motion of the Sun and Moon, the waxing and waning of the Moon, the risings and settings of the Sun, and unequal course of the wanderings of the stars. This instrument sounds very similar to the instrument Cicero attributes to Archimedes. Lactantius lived several centuries after Cicero and does not explicitly state that he had seen the instrument, making it possible that he had only heard or read about such a device.

Finally, in *Shorter Poems*, Claudian references a sphere constructed by Archimedes. He says:

> *Archimedes' sphere* (*sphaeram*). When Jove looked down and saw the heavens figured in a sphere of glass he laughed and said to the other gods: "Has the power of mortal effort gone so far? Is my handiwork now mimicked in a fragile globe (*orbe*)? An old man of Syracuse has imitated on earth the laws of the heavens, the order of nature, and the ordinances of the gods. Some hidden influence within the sphere directs the various courses of the stars and actuates the lifelike mass with definite motions. A false zodiac runs through a year of its own, and a toy moon waxes and wanes month by month. Now bold invention rejoices to make its own heaven revolve and sets the stars in motion by human wit. Why should I take umbrage at harmless Salmoneus and his mock thunder? Here the feeble hand of man has proved Nature's rival".[23]

The glass instrument Claudian described had the capability to direct the courses of the stars and zodiac and represents the waning and waxing of the Moon. Claudian attributes the device to Archimedes. Claudian, who was active around 400 A.D., does not say that he had seen the device he attributes to Archimedes; like Lactantius he seems to be describing something he has only heard or read about. Given the gulf of time separating Archimedes and Claudian it is probable that he is describing something he has never seen.

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[23] Platnauer 1922, 278-281.

While it is possible that each of the authors discussed, observed, or handled the devices they mention, the span of years between Cicero and the later authors, Lactantius and Claudian, suggests that the references were dependent on Cicero. Neither Lactantius nor Claudian claim to have seen the device personally. Ovid, on the other hand, does not reference Archimedes in his passage concerning an astronomical device. The device Ovid describes is unlike the devices described by either Cicero, Lactantius, or Claudian.

Unfortunately all of the passages where *sphaera* are discussed contain few details about the device. Fortunately archeological evidence, specifically the Antikythera Mechanism, can help to illuminate the textual evidence on the history of instrument making in the ancient world.

III.1.2 Archeological Evidence of Instrument-making

The Antikythera Mechanism is the primary example of a mechanical astronomical model in the ancient world. It was discovered, in pieces, on a wrecked merchant ship sunk 42 meters below the surface of the water, near the island of Antikythera, by a group of sponge divers in the Mediterranean Sea in 1900. Over time the 82 corroded fragments of the Antikythera Mechanism were taken to the National Archaeological Museum in Athens.

The historian Derek De Solla Price closely studied the fragments using x-rays. In 1974 he published *Gears for the Greeks: The Antikythera Mechanism – A calendar computer from ca. 80 B.C.*, in which he proposed that the mechanism was an astronomical calendar with a front dial displaying the location of the Sun and the Moon in the zodiac, and a back dial displaying solar-lunar cycles with over 30 gears inside the

mechanism.[24] Price's important analysis laid the groundwork for additional studies of the Antikythera Mechanism. Michael Wright, former curator of the Science Museum in London, used X-rays and linear tomography in his study of the mechanism to advance scholar's understanding of how the device worked and how it was constructed.[25]

Building on the work of Price and Wright, an international group of historians and scientists studied the device using three-dimensional X-ray computer tomography and high-resolution surface imaging, culminating in the 2006 publication of the article "Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism".[26] Tony Freeth, Alexander Jones, John M. Steele and Yanis Bitsakis revealed previously unreadable inscriptions, unveiling calendar month names and a four-year Olympiad dial, initially believed to be 76-year Callippic dial.[27] The discovery of the Olympiad dial demonstrates that the Antikythera Mechanism related astronomical phenomena to social customs. Due to the collaborative research on this device, scholars now know that the Antikythera Mechanism was used to exhibit and calculate the anomalistic motion of the Sun, the Moon and possibly the planets, as well lunar-solar cycles.[28] It is more technologically advanced than any known gear instrument in the first millennium A.D.[29]

The front dial of the Antikythera Mechanism exhibited the position of the Sun and the Moon in the zodiac with pointers; the Moon pointer also displayed the phases of the Moon. On the back dial were two spirals displaying solar-lunar calendars. The upper

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[24] Price [1974] 1975.
[25] Wright, Bromley, and Magou 1995. Wright 2002. Wright 2003. Wright 2005a. Wright 2005b. Wright 2005c. Wright 2006.
[26] Freeth, et al. 2006.
[27] Freeth, et al. 2008.
[28] Freeth, et al. 2006, 587-591.
[29] Freeth, et al. 2006, 587.

back dial represents the Metonic cycle, which is a 19-year calendar. The upper back dial has a subsidiary dial consists a four-year cycle used to determine the Pan-Hellenic Games, such as the Olympics. The bottom back dial represents the Saros cycle, which is an eclipse predication cycle. The bottom dial also has a subsidiary dial, which represents an Exeligmos cycle.[30] More recently, James Evans, Christián C. Carman, and Alan S. Thorndike have found that in addition to displaying the variable motion of the Moon, the Antikythera Mechanism also displayed the solar anomaly. They argue that the two circular scales representing the zodiac circle and the Egyptian calendar were divided differently. The zodiac scale was divided non-uniformly, accounting for the solar anomaly. The solar pointer displayed the position of the mean Sun relative to the Egyptian calendar and it also displayed the position of the true Sun, relative to the zodiac scale.[31]

Other than one gear with 63 teeth and no assigned role in current reconstructions of the Antikythera Mechanism, no gear works from the planets survive. Wright proposed that the Antikythera Mechanism displayed the position of the planets relative to the zodiac by means of pointers, similar to the way that the Sun and Moon are displayed.[32] Evans, Carman, and Thorndike propose that the planets were represented on small dials, which displayed the events in the respective planet's synodic cycle. For Venus, for example, the events displayed would include first and last appearance, conjunctions with the Sun, the commencement and conclusion of retrograde motion, and the greatest elongations for the Sun.[33]

[30] Freeth, et al. 2008, 614-617.
[31] Evans, Carman, Thorndike 2010, 5.
[32] Wright 2002, 169-173.
[33] Evans, Carman, Thorndike 2010, 24.

The Antikythera Mechanism was likely constructed in the second century B.C. and the ship that it was on sank in the first half of the first-century B.C. It is possible that Cicero, who lived in the first-century B.C., may have been referencing the Antikythera Mechanism in his works. He claims that either Archimedes or Posidonius constructed the devices he discusses; the Antikythera Mechanism is not old enough to have been constructed by Archimedes, but the dates when the Antikythera Mechanism was constructed correspond with Posidonius's life. It is unlikely that the Antikythera Mechanism was the first of its kind to be built.[34] Since bronze was a valuable metal, the other assumed mechanisms were may have been melted down, or simply lost.

In addition to the Antikythera Mechanism there are other, later, examples of mechanical astronomical devices that have survived. Housed in the Science Museum of London is a device known as the Byzantine Sundial-Calendar. It dates from approximately the fourth to the seventh centuries A.D. This device exists in four fragments. Exactly how it was acquired is unknown. A reconstruction of this instrument, built by Michael Wright, includes a portable sundial with a geared calendar that shows the shape of the Moon, its age in days and the position of the Sun and Moon in the zodiac. The number of teeth on several of the different gears corresponds with parts of a mechanical calendar described in a medieval Islamic manuscript, written around 1000 A.D. The author of the manuscript, al-Bīrūnī, describes an instrument called "The Box of the Moon".[35] D.H. Hill argues that al-Bīrūnī uncharacteristically does not credit the

[34] Merchant 2006, 536-537.
[35] Al-Bīrūnī's description appears in a manuscript housed in the University Library, Leiden. Field and Wright 1985, 200-201. In addition to the text by al-Bīrūnī, there a several Arabic books describing different mechanical inventions, such as the ninth century *Book of Ingenious Devices* by three brothers, Muḥammad, Aḥmad, and al-Ḥasan, known collectively as Banū Mūsā, and al-Jazarī's *The Book of Knowledge of Mechanical Devices* written in 1206.

inventor of this device, possibly because it was a well-known instrument in the Hellenistic or Byzantine world.[36]

An astrolabe dated 1221-1222 A.D. also seems to have connections to al-Bīrūnī's "Box of the Moon". Constructed by Muḥammad ibn Abī Bakr al-Ibarī, it is the oldest existing complete gear train, containing five gears. Like the Byzantine sundial-calendar, and al-Bīrūnī's "Box of the Moon", this device also gives the shape of the Moon, its age in days, and positions of the Sun and Moon in the zodiac.[37] There is a small window on the top of the back of the astrolabe that shows the phases of the Moon. Also, on the back, there is a window that gives the age of the Moon, and finally there are two rings that give the position of the Sun and Moon in the zodiac. An axle attached to the front of the astrolabe powers the device.[38]

In the Byzantine sundial-calendar, al-Bīrūnī's "Box of the Moon" description, and al-Ibarī's astrolabe, we have three similar devices or references all separated by at least two centuries. Despite the lack of existing mechanical astronomical devices, it seems Byzantine and Islamic scholars were writing and building such devices. Even some of the earliest clocks have connections to mechanical astronomical devices, such as the clock built by Richard of Wallingford and the clock built by Giovanni de' Dondi, both of which were constructed in the first part of the fourteenth-century. Historians, such as Field and

[36] Hill 1985, 140-141. The best copy of the manuscript, which consists of 88 folios, can be found in the University Library, Leiden (Leiden Or 591). Two other copies of the manuscript exist, namely: Staatsbibliothek Berlin; Catalogue W. Ahlwardt 5794-5796 (Sprenger 1896); and British Library OR 5593.
[37] Field and Wright 1985, 132-136.
[38] Field and Wright 1985, 200-201. This unique astrolabe is housed in the Museum of History of Science in Oxford. The Museum of History of Science, University of Oxford website contains a detailed database of all of its astrolabes. See www.mhs.ox.ac.uk. A later example of an astrolabe with gear work is the French astrolabe from the fourteenth century in the Science Museum in London. The front of the dial has gears and pointers that display the relative position of the Sun and Moon. Turner 1987, 30.

Wright, have suggested that these early clocks are more closely related to the tradition of mechanical astronomical models than to the early tradition of clockwork.[39]

Richard of Wallingford's astronomical clock was constructed in approximately 1327-1330. While there are several references to Richard's clock, the information about the device itself is limited. Richard did not write a text to accompany the clock like he did for the instrument he constructed called an Albion, which was essentially an equatorium.[40] Silvio A. Bedini and Francis R. Maddison write that Richard of Wallingford's astronomical clock and Giovanni de' Dondi's astrarium were mechanically similar; however, in their details they are very different.[41] Giovanni's astrarium was driven by a balance wheel and displayed the motions of the Sun, Moon, and planets according the Ptolemaic theory. The device is seven sided and each heavenly body was displayed on its own dial. A calendar wheel drove the planetary dials. The lower part of the frame consisted of a 24-hour dial, a dial indicating cultural events, namely religious feasts, and a dial indicating the lunar nodes.[42] It is an ingenious design and although no originals survive, several reconstructions have been built, such as the one found in the Science Museum in London.[43]

A unique instrument displaying Mercury's nesting spheres, referred to as a *theorica orbium,* was constructed by Hieronymus Vulparius in 1582.[44] The device, which has similarities to an armillary sphere, displays the Ptolemaic model for Mercury, complete with eccentric spheres and epicycles. While not all of the spheres in the

[39] Field and Wright 1985, 202.
[40] Bedini and Maddison 1966, 6.
[41] Bedini and Maddison 1966, 8.
[42] Bedini and Maddison 1966, 15.
[43] Turner 1987, 24-25.
[44] A nice photograph of this instrument can be found in Stephenson, Bolt, and Friedman 2000, 114-115. Gingerich 1977, 39. The device is preserved at the Adler Planetarium and History of Astronomy Museum in Chicago.

theorica orbium are concentric with the Earth, both the outer and inner spheres are. Similar devices are represented in Islamic manuscripts as early as the fourteenth-century. Vulparius's device was constructed relying on George Peurbach's *Theoricae Novae Planetarum* first published in 1473 and Johann Schöner's *Opera Mathematica* published in 1551.[45] This device displayed the model for Mercury, accounting for the eccentricity. An instrument similar to this, but which displays the Sun and Moon can be found in the Národní Tecknické Museum in Prague. While this instrument is unsigned, the instrument-maker was most likely also Vulparius.[46]

By examining examples like this we can gain insight into how Byzantine, Islamic, and Medieval European instrument-makers attempted to display Ptolemaic astronomy in the form of a mechanical device. While they can show us how later instrument-makers handled the same problem, they do not provide us with information about how an instrument-maker in the second-century A.D. would have constructed the device Ptolemy describes.

Returning to the device discussed in the *Planetary Hypotheses*, Ptolemy says that the instrument builder should not construct the device in the conventional manner of sphere-making, since this manner does not show the reality of the models. In this statement Ptolemy implies there was a tradition of *sphairopoiia* in the second-century A.D. and that within this tradition there was an "accustomed manner". According to Ptolemy, the accustomed manner does not display the underlying reality of the models, but the phenomena only. This seems to suggest that the instrument would not display the true movement of each body, but instead it would display the apparent motion or the

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[45] Gingerich 1977, 39.
[46] Turner 1987, 34-35.

mean motion. The Antikythera Mechanism is possibly an example of what Ptolemy intends when he says “traditional manner of sphere-making”. On the front dial of the Antikythera Mechanism the apparent motion of the Sun and Moon are shown, but not the actual motion. The Antikythera Mechanism is a calculation tool, rather than an instrument that displays the true motion of a celestial body (namely the motion of the body around its eccentric circle and or epicyclic circle). The several motions that come together to comprise the motion of the Sun and the Moon, such as the daily rotation, the motion of the eccenters and, in the case of the Moon, the motion around the epicycle, would not be visible. Hidden inside the Antikythera Mechanism were the gears that produced these motions so that only the final collective motion was exhibited through the use of a pointer. In order to demonstrate the underlying reality of the models, Ptolemy’s mechanism would have represented the motion of the Sun, the Moon and the planets along the zodiac, possibly like the front dial of the Antikythera Mechanism; however, the various individual motions would also be exhibited. In this way, his instrument would display, for the outer planets for example, the motion of the planet around the epicycle and the motion of the epicycle around the deferent circle.

Ptolemy says that he will use the simpler version of the positions and the arrangements of the circles that will cause anomalies for the sake of instrument-making. He states:

> Concerning the positions and arrangement of the circles causing the anomalies, we will apply the simpler version in respect to the methods of instrument-making (ὀργανοποιὰς), even if some small variations will follow, and moreover we fit the motions to the circles themselves, as if they are freed from the spheres that contain them, so that we can gaze upon the visual impact of the models bare and unconcealed.[47]

[47] Heiberg 1907, 70-72.

This is an interesting point, since Ptolemy implies that he wants simplicity for the sake of instrument-making and not because the heavens are simple, which is what he says in the *Almagest.* He uses the simpler arrangements and positions, even if this means there will be small variations. This statement seems at odds with his emphasis on accuracy earlier in the passage. However, here he seems to make allowances given the difficulty of instrument-making. While Ptolemy makes changes to some of the parameters in the *Planetary Hypotheses*, the only place where he makes a change to model is the simplified latitude theory. This is the only place where the changes Ptolemy makes could have been done "for the sake of instrument-making".[48] If Ptolemy's justification for choosing a model includes the ease with which the model can be built, then the relationship between instrument-making and astronomy is much closer than has previously been suggested.

It is not clear whether the instrument Ptolemy discussed would have consisted of a two-dimensional representation or a three-dimensional representation. Ptolemy's model could have been constructed so that it displayed a cross-section of the heavens through the ecliptic, similar to the front dial on the Antikythera Mechanism. Alternatively, it could have been constructed as a series of nested spheres, similar to an armillary sphere or a *theorica orbium*. Obviously it would be easier to construct a model that displayed a cross section of the cosmos along the ecliptic, so that the path of the celestial bodies would be displayed as circles. Constructing a three-dimensional representation, where the path of the celestial bodies was displayed using spheres would not be possible since Ptolemy's use of an equant would mean that the model would need to display a sphere traveling uniformly around a point that is not on the sphere's axis.

[48] I will discuss this point in more detail in Section III.2 below.

The formerly lost section of Book I of the *Planetary Hypotheses*, Book I, Part B, includes a discussion of the relative distances of the cosmos. In the *Almagest* Ptolemy calculated the distances of the Sun and Moon. Using these distances, along with the ratios of the minimum and maximum distance of each celestial body, Ptolemy calculated the relative distance of each planet. Ptolemy says that at its closest, the Moon is 33 Earth radii away from the Earth. Alternatively, at its furthest, Saturn is 19,856 Earth radii away.[49] It is unlikely that the instrument Ptolemy discusses could have been built to scale. For example, if the instrument-maker were to use a scale of one centimeter to ten Earth radii, then the minimum distance of the Moon from the Earth would be 3.3 centimeters, while the maximum distance of Saturn from the Earth would be almost 20 meters. Consequently, the device would have to have been built out of proportion, or it would have to have been built as separate pieces so that each celestial body used a different scale. Ptolemy gives this second option as a feasible approach since he says that the instrument-maker could build one instrument with all of the different motions intertwined, or alternatively, exhibit each of the motions separately. If the instrument-maker built a separate model for each celestial body, then different scales could be used for the Sun, Moon and planets, therefore maintaining the ratios Ptolemy gives in the second part of Book II.

While there is no evidence that the type of instrument Ptolemy describes was ever built, we can use the existing archeological evidence to examine whether it would have been possible to build such an instrument. Ptolemy does not insist that the instrument be mechanically operated. A hand-operated instrument, where the epicycles and celestial bodies could be moved to the appropriate position manually, would present a smaller

[49] Goldstein 1967, 9-11.

challenge for the craftsman to construct. This type of device would require that the different parts of the model be moved on a slider, or something equivalent, so that the epicycle and planet could be moved into place. Incorporating the latitudinal motion would be more complicated, but Ptolemy presents a simplified latitude theory in the *Planetary Hypotheses* that could be more easily translated to an instrument than the one he presents in the *Almagest*. A mechanical instrument, specifically one that is gear operated, would present a larger challenge, since Ptolemy's astronomical system is more complicated than the system exhibited in the Antikythera Mechanism.

Ptolemy's instrument would need to display the planet's motion around the epicycle, which is uniform relative to the center of the epicycle. It would also need to display the epicycle's motion around the deferent circle, which is uniform relative to the equant point. The theory for the Sun, outer planets, and Venus would be challenging to build, but it would be possible if we assume that the technical knowledge we see displayed in the Antikythera Mechanism was known in the second-century A.D. The models for Mercury and the Moon would be more of a challenge to construct, since their models are more complicated. For example, Mercury's model has a second eccentric point, which moves. This may not have been too great a challenge for the instrument-maker, but it would have presented problems.

In conclusion, it appears Ptolemy is providing general guidelines for the instrument-maker in the *Planetary Hypotheses,* ambiguous though those guidelines may be. He does not provide a description of the instrument. Instead he gives a list of criteria and options of feasible approaches that could be used to build such an instrument. It is possible that the text of *Planetary Hypotheses* is part of the same tradition of

astronomical instruments that started with Archimedes and includes the Antikythera Mechanism. While we have no evidence that a device based on the *Planetary Hypotheses* was ever built, this does not mean that it could not have been built. Although the descriptions in the *Planetary Hypotheses* are not detailed, the information that we have, viewed in relation to the textual references and archeological evidence, suggests that ancient craftsmen probably had the necessary technological expertise to build a hand operated astronomical instrument. Finally, Ptolemy's brief discussion of "an accustomed manner of *sphairopoiia*" means that Ptolemy was aware of other *sphairopoiia*. This adds to our knowledge of astronomical instrument-making in the ancient world by confirming that this tradition was, to some extent, alive in the second-century A.D.

III.2 Ptolemy's Geometrical Models

One of the distinguishing features of Greek astronomy is the use of geometric models. This type of model was not a feature of Babylonian astronomy, which often relied on arithmetic models.[50] The models used in Greek astronomy are simple, yet sophisticated tools to account for the motion in the heavens. Ptolemy constructed models that allowed the precise location of a celestial body to be calculated for a given date and time. While the models for each body are different, there are two mechanisms that Ptolemy relies on to describe celestial motion: the epicycle model and the eccentric model. In this section I will discusses Ptolemy's objectives in laying out his models, introduce the epicyclic and eccentric models, outline the basics of observable astronomy, and briefly explain the models Ptolemy presents in the *Almagest*. The models in the *Planetary Hypotheses* are based on the models Ptolemy describes in the *Almagest*, with a few important modifications.

In the *Planetary Hypotheses* Ptolemy says that the general assumptions are in agreement with those in the *Almagest*. The details, however, have been amended. Ptolemy says:

> We shall make the exposition, so far as the general assumptions are concerned, in agreement with the things delineated in the *Syntaxis*, so far as the details are concerned, following the corrections we have produced in many places on the basis of more continuous observations, either corrections to the models themselves, or corrections to the spatial ratios, or corrections to the periods of restitutions.[51]

[50] An example of this is System A and System B, which are both arithmetic models that describe monthly solar and lunar progress. System A used a step function and System B used a zig-zag function. Both systems were used in Babylon and Uruk simultaneously. Aaboe 2001, 57.
[51] Heiberg 1907; 72.

The corrections that Ptolemy makes in the *Planetary Hypotheses* are due to more continuous observations, which implies observations taken after those included in the *Almagest*. Alternatively, this could be more broadly taken to mean further study of the data already available.

Ptolemy tells the reader that the presentation of the models will vary from that in the *Almagest*. In Book III of the *Almagest,* Ptolemy says:

> …we think that the mathematician's task and goal ought to be to show all the heavenly phenomena being reproduced by uniform circular motions, and that the tabular form most appropriate and suited to this task is one which separates the individual uniform motions from the non-uniform [anomalistic] motion which [only] seems to take place, and is [in fact] due to the circular models; the apparent places of the bodies are then displayed by the combination of these two motions into one.[52]

While in the *Almagest* Ptolemy's objective is to demonstrate that the heavenly phenomena is produced due to uniform, circular motion and therefore separates the uniform motion from the so-called anomalistic motion, in the *Planetary Hypotheses* Ptolemy says:

> Adhering to the presentation of the models themselves, that is, where it is necessary, dividing and fitting together the uniform motions in the *Syntaxis*, in order that the definitions (ἀφορισμούς) of the models are tied to (ἀναδεδομένας) the parts of the zodiac and the starting points, since this is useful for calculations, so that here the individuality of each course is made visible, even if there are many motions in a combination of efforts (συντελῶνται).[53]

Ptolemy will present particular motions as a combined motion in the *Planetary Hypotheses*. For example, when presenting the complex mean motions of the outer planets, Ptolemy gives one period relation that represents the motion of the epicycle around the eccentric circle and the motion of the apogee. Additionally, Ptolemy will also

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[52] Toomer 1998, 140.
[53] Heiberg 1907, 72.

present divide motions, for instance, he presents the anomalistic motion of each individual planet.

Relying on the passive imperative in Greek, Ptolemy begins his construction of the models by saying, “Let there be imagined a stationary great circle that is centered on the center of the sphere of the cosmos…” (νοείσθω μέγιστος κύκλος περὶ τὸ κέντρον τῆς τοῦ κόσμου σφαίρας...).[54] Ptolemy is using the same language found in Euclid’s *Elements*. In Euclid’s geometrical works most propositions are composed of several parts: enunciation, definition, setting-out, construction, proof, and finally the conclusion.[55] Each component has a different role, as Sir Thomas L. Heath explains:

> Now of these the *enunciation* states what is given and what is that which is sought, the perfect *enunciation* consisting of both of these parts. The *setting-out* marks off what is given, by itself, and adapts it beforehand for use in the investigation. The *definition* or *specification* states separately and makes clear what the particular thing is which is sought. The *construction* or *machinery* adds what is wanting to the datum for the purpose of finding what is sought. The *proof* draws the required inference by reasoning scientifically from acknowledged facts. The *conclusion* reverts again to the *enunciation*, confirming what has been demonstrated.[56]

While not every proposition in the *Elements* contains all of these components, they comprise a general formula that is used consistently throughout Euclid’s geometry, making the proofs standardized and structured.

The term *given* has a different meaning in the pure geometry of Euclid than it does in applied mathematics of Ptolemy. Nathan Sidoli explains this:

> Thus, when Euclid says that a triangle is *given in form* he means that we can construct angles that are equal to the angles of the given triangle and we can set out a series of three lines that have to one another the same ratios as the sides of the given triangle. On the other hand, if Ptolemy were to speak of a triangle *given in*

[54] Heiberg 1907; 74.
[55] For a description of Euclid’s works see the article in the *New Dictionary of Scientific Biography,* s.v. “Euclid.”
[56] Heath 1926, 129.

> *form* he would mean one in which we can name what part each of its angles forms of a whole circle and can state the lengths of each of its sides in terms of some assumed unit. Although Euclid's concepts and theorems lay the foundation upon which later trigonometric methods were developed, the path that led from the one to the other was not direct.[57]

When a figure is *given in form* in pure geometry then it can be geometrically constructed. For example, Book One, Proposition 1 of Euclid's *Elements* states: "On a given finite straight line to construct an equilateral triangle." The finite straight line is *given*, since the definitions and postulates of Book One explain how a finite straight line can be constructed and the reader thus has the knowledge of how to construct such a line. Alternatively, in applied mathematics, *given in form* means that the mathematician or astronomer can utilize the figure to calculate the length of its sides and the value of its angles. For instance, in Book I.10 of the *Almagest*, when constructing the table of chords Ptolemy states: "I say, that if we join AG, that [chord] too will be given." Ptolemy implies that since the lengths of certain chords have already been calculated, relying on this information the length of another, specific chord (AG in particular) can be computed.[58] The length of a particular chord is *given*, since Ptolemy has the ability to calculates its length based on the previous work. While the meaning of *given* has to do with its construction in Euclidean geometry, for Hipparchus and Ptolemy the meaning of *given*, for the length of a chord for example, concerns having the ability to calculate its the value relative to an established unit.[59]

In addition to the mathematician, Ptolemy's clear description of the basic construction of the models is useful to an instrument-builder intending to construct a

[57] Sidoli 2004, 60.
[58] Toomer 1998, 53.
[59] Sidoli argues that the concept of *given in form* changed between Euclid and Hipparchus. Sidoli 2004, 63-67.

tangible representation. Not only did Ptolemy utilize his geometrical figures to perform calculations, but also in constructing these figures he described his astronomical system in a way that is useful beyond calculating purposes.

III.2.1 The Epicyclic and Eccentric Models

Ptolemy first introduced the epicyclic and eccentric models when discussing the Sun in Book III.3 of the *Almagest*. The epicyclic and eccentric models are capable of describing anomalistic motion using uniform motion. The ability to assign sizes for the different components of the epicyclic and eccentric models, such as the size of the epicycle or the size of the eccentricity, provides the flexibility needed for Ptolemy to describe the unique orbit of each body.

For the eccentric hypothesis, Ptolemy constructs a circle eccentric to the ecliptic. The body travels on this circle uniformly, with respect to the center of the circle. The observer is not at the center of the circle, but slightly off center.

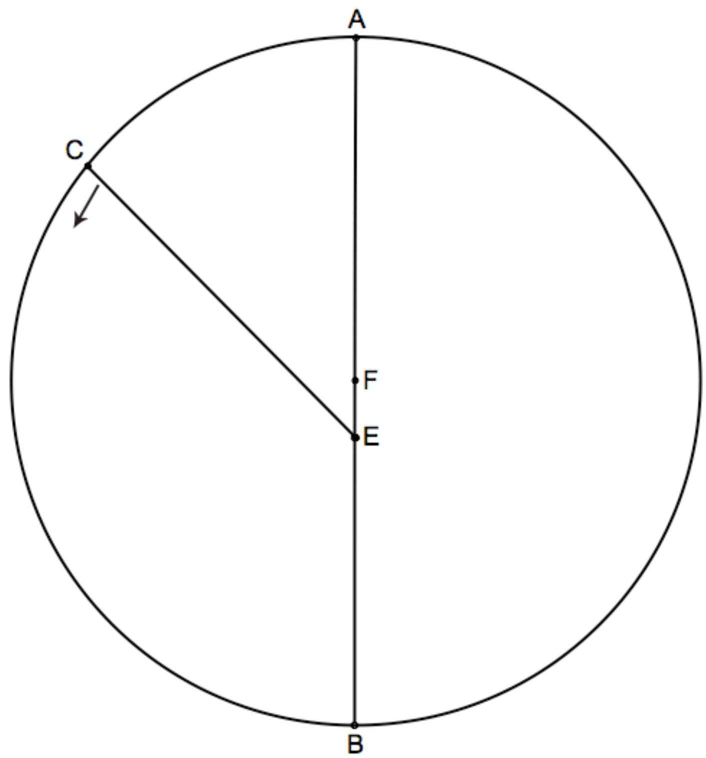

Figure 5: Eccentric Model

The eccentric model is displayed in Figure 5. The body travels with uniform motion, relative to point F, around the circle ABC, which is centered on point F. The observer is represented by point E. Consequently, the body travels uniformly around a circle, but the observer is not located at the center of the circle, but slightly off center (at point E). Point A is the apogee and point B is the perigee. The body has the greatest apparent speed at the perigee and its least apparent speed at the apogee. The distance between the center of the deferent circle and the eccentric circle can be adjusted to represent different orbits.

For the epicycle model, Ptolemy constructs a circle concentric with the ecliptic. A smaller circle, called the epicycle, travels around the center of the larger circle with

uniform speed. Finally, the body travels around the epicycle with uniform speed relative to the center of the epicycle, point F.[60]

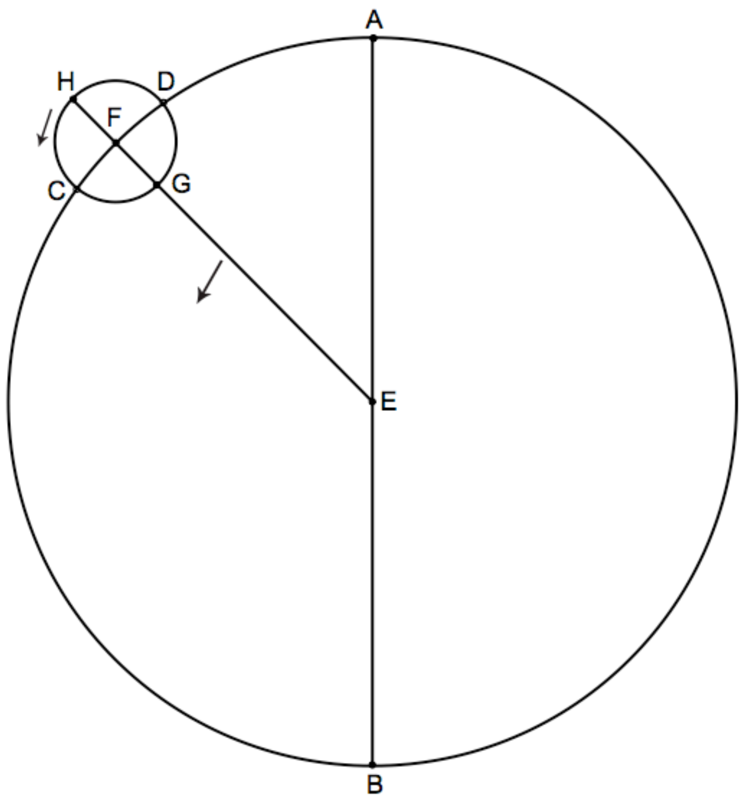

Figure 6: Epicyclic Model

The epicycle model is displayed in Figure 6. The circle ABCD is concentric with the ecliptic. It is centered on point E. The body moves uniformly around the epicycle CHDG, which is centered on point F. The center of the epicycle, point F, moves uniformly around circle ABCD.

The epicycle hypotheses can work in two different ways; in the first, the body and the center of the epicycle move in the same direction, namely from H to C and from A to D respectively. Under these circumstances, the greatest apparent speed occurs at the

[60] Toomer 1998, 144.

apogee and the least apparent speed occurs at the perigee. In the second way, the body and the center of the epicycle move in opposite directions, so that the epicycle moves from A to D and the body moves from H to D. In this scenario, the least apparent speed occurs at the apogee and the greatest apparent speed occurs at the perigee.[61] Additionally, the apsidal line, line AB, can be made to revolve uniformly in both the epicyclic and eccentric models.

Since both of the above hypotheses are equivalent, if a body displays one anomaly, then either of the above models will suffice to describe it, given that the correct ratios are used.[62] But, if a body displays two anomalies, then the eccentric and epicyclic hypotheses can be combined to describe the motion.[63] Ptolemy does this to describe the motions of the moon and planets.

III.2.2 The Fixed Stars

In the *Almagest*, Ptolemy explains that heavens move like a sphere in Book I.3, that the Earth is spherical in Book I.4, and the Earth is at the center in Book I.5.[64] He supports these statements with observations, such as the stars appear to travel in circles about one center, and the Earth's surface is evenly curved, and the horizon appears to bisect the zodiac, respectively. Ptolemy observes that the stars always remain in the same position relative to one another and the Sun, Moon and planets slowly move eastwardly in relation to the stars.

[61] Toomer 1998, 144-145.
[62] Ptolemy demonstrates the equivalence of the eccentric and epicyclic hypotheses in Book III.3 of the *Almagest*. Toomer 1998, 146-153.
[63] Toomer 1998, 145.
[64] Toomer 1998, 38-43.

The Sun, according to Ptolemy, can be observed to travel from west to east approximately one degree a day. The Sun takes one year to complete a revolution; however, the length of the year is different depending on whether the Sun's motion is measured relative to one of the fixed stars, or to one of the solsticial or equinoctial points. A tropical year, which is shorter than a sidereal year, is measured relative to one of the solsticial or equinoctial points. The difference in the length of the sidereal and tropical year is due to the slow eastwardly motion of the fixed stars relative to the solsticial and equinoctial points.[65] This motion is called precession.

In the *Planetary Hypotheses*, Ptolemy describes a stationary great circle that is centered on the center of the sphere of the cosmos. This is the "equator" (ἰσημερινός), and Ptolemy divides it into 360 parts, each of which he calls a "time-degree" (χρόνοι). Next, Ptolemy constructs a circle that is also centered on the center of the sphere of the cosmos, which moves from east to west.[66] He calls this circle the "carrier" (φέρων). The carrier, which is appropriately named, carries another great circle with the same center, called the zodiac (ζῳδιακός).[67] These two circles, the carrier and the zodiac, are inclined to each other in a fixed position at an angle of 23;51,20. This value represents the obliquity of the ecliptic. The seven celestial bodies – Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn – travel along the zodiac.

Ptolemy divides the zodiac into 360 parts, and he calls each division a degree (μοῖραι). Since the equator and the zodiac are inclined to each other, the points where the two circles intersect are called the equinoctial points. The points that are exactly a

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[65] Toomer 1998, 321 n. 2.
[66] Ptolemy refers to east and west as rotational directions, not linear.
[67] Heiberg 1907, 74. In the *Almagest* Ptolemy does not call this circle the zodiac, but instead he regularly refers to it as 'the inclined (circle) through the middle of the zodiacal signs' (ὁ λόξος καὶ διὰ μέσων τῶν ζῳδίων κύκλος). Toomer 1998, 20.

quadrant away are called the tropical points. The northernmost point on the zodiac is called the summer tropical point, or the summer solstice. Likewise, the southern most point is called the winter tropical point, or the winter solstice. The order of the four points from west to east is as follows: vernal equinoctial point, summer solsticial point, autumnal equinoctial point, and winter solsticial point. Ptolemy not only describes the arrangement of the spheres, he also provides the names of the various parts.

Ptolemy tells the reader that a revolution of the cosmos occurs when the carrier completes a full rotation, or 360 degrees, with reference to the equator. This restitution is not visible and, as a result, we measure motions in *nychthemera* since this type of motion is in fact visible. Ptolemy defines a *nychthemeron* (νυχθήμερον) in the following way:

> A *nychthemeron* is the time in which the sun, relative to the stationary equator, makes one circuit as a consequent of the revolution of the cosmos. Clearly, if the sun did not move around the zodiac, then a *nychthemeron* would be the same as a revolution of the cosmos, but since the sun is assumed as moving toward the east, the *nychthemeron* is longer in time than a revolution of the cosmos, and one *nychthemeron* contains one revolution of the cosmos, that is 360 time-degrees, plus so great a part of the equator as the sun travels along the zodiac in one *nychthemeron*, assuming the courses are uniform.[68]

The motion of the Sun is not in fact uniform, so a mean *nychthemeron* is not observable. Nonetheless, a *nychthemeron* is the time it takes the Sun to complete one revolution with respect to a point on the stationary equator, plus a little extra, since the Sun has its own motion. One *nychthemeron* represents more than the time of daylight, but a full mean day and night.

[68] Heiberg 1907, 76-77.

III.2.3 The Solar Model

Ptolemy discusses his solar model in Book III.4 of the *Almagest*.[69] He attributes this model to Hipparchus. In Ptolemy's model the path of the Sun around the Earth is a great circle called the ecliptic.[70] The ecliptic and the celestial equator form an angle; Ptolemy reports this angle as: 23;51,20º. The two points where the equator and the ecliptic intersect are the equinoctial points; these points are located at zero degrees Aries and zero degrees Libra. For Ptolemy, an equinox occurs when the Sun is at one of these points and a summer solstice occurs when the Sun is at the northernmost point in its path around the ecliptic and a winter solstice occurs when the Sun reaches the southernmost point of its path around the ecliptic.

The stars that lie along the ecliptic are broken into twelve constellations, collectively referred to as the zodiac.[71] The twelve signs are: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, and Pisces. The angular speed of the Sun varies as it travels through the zodiac. This variation is called the solar anomaly. Ptolemy reports the interval from the summer solstice to autumnal equinox is $92\frac{1}{2}$ days. The interval from the autumnal equinox to winter solstice consists of $88\frac{1}{8}$ days and the interval from the winter solstice to spring equinox consists of $90\frac{1}{8}$ days. Finally, the interval from the spring equinox to summer solstice comprises $94\frac{1}{2}$ days.[72] Since the periods between solstices and equinoxes appears to be similar, and

[69] Pedersen provides a detailed description of Ptolemy's solar theory. Pedersen 1974, 122-158.
[70] A great circle is a circle that cuts through the center of the sphere.
[71] By the Greco-Roman period the zodiac referred to 30-degree sections.
[72] Toomer 1998, 154.

Ptolemy concludes that the Sun's apogee maintains the same position relative to solsticial and equinoctial points.[73]

While either the eccentric or epicyclic model could be used to represent the Sun's motion, Ptolemy says; "However, it would seem more reasonable to associate it with the eccentric hypothesis, since that is simpler and is performed by means of one motion instead of two."[74] In Ptolemy's solar model, the Sun moves uniformly, in the opposite direction of the daily motion, namely from west to east around a point that is eccentric to the Earth. Ptolemy uses simplicity as a criterion when choosing between the eccentric and epicyclic models, and here he argues that the simpler theory is more likely to represent the true motion of the Sun.

[73] Unlike the apogees of the five planets, the Sun's apogee does not move due to precession. Toomer 1998, 154 n. 46.
[74] Toomer 1998, 153.

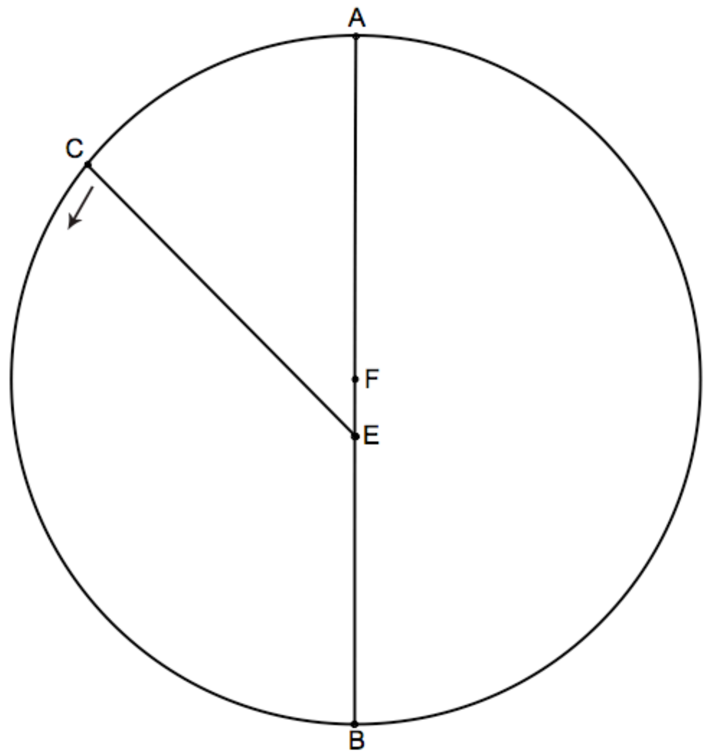

Figure 7: Solar Model

Ptolemy's solar model is displayed in Figure 7. The Sun, point C, travels uniformly in the trailing direction with respect to the heavens around the circle ABC, which is centered on point F. The Sun's motion is not observed from the center of the circle that it travels along, but from an eccentric point, point E. The Sun is furthest from the Earth and its apparent speed is the slowest at its apogee, point A. Conversely, the Sun is closest to the Earth and has the greatest apparent speed at its perigee, point B.

After giving the arrangement of the equator, carrier, and zodiac and discussing the daily rotation, and the simple and unmixed period relations in the *Planetary Hypotheses*, Ptolemy moves to a description of the models for the celestial bodies. For the solar model an eccentric circle lies in the plane of the zodiac with an eccentricity of 60 to 2;30. The

same value is given in the *Almagest* and the *Canobic Inscription*.[75] The direction of the Sun's apogee is also the same in both the *Planetary Hypotheses* and the *Almagest*; however, Ptolemy uses different points of reference in each text when describing the location of the apogee.[76] In the *Planetary Hypotheses*, he says that the apogee is 65;30 degrees from the vernal equinoctial point in the trailing direction of the cosmos. In the *Almagest* he says that the apogee is 24;30 degrees in advance of the summer solsticial point. This is the same point, but Ptolemy uses the summer solstice as the reference point in the *Almagest* and the vernal equinox as the reference point in the *Planetary Hypotheses*. While there is no clear advantage for using one point over another, Ptolemy converts his parameters from the *Almagest* so that the vernal equinox demarcates the reference point.[77] The apogee and the eccentricity for the Sun, as well as the static model itself, are the same in the *Planetary Hypotheses* as they are in the *Almagest,* and *Canobic Inscription*.

III.2.4 The Lunar Model

Ptolemy begins his treatment of the lunar theory in Book IV with a discussion about observations. He explains that he only uses observation records made during lunar eclipses, since the Moon's apparent position in longitude and true position in longitude when viewed from the Earth are the same during an eclipse.[78] At other points in the Moon's orbit the true position of the Moon is difficult to observe due to parallax. An

[75] In the *Almagest*, Ptolemy shows that the distance between the Earth and the eccentric point is 2;29,30, when the radius of the eccentric circle is 60. He then rounds this to 60 to 2;30, or 24 to 1. Toomer 1998, 154-155.
[76] The Sun's apogee, unlike the apogees of the five planets, is tropically fixed. Toomer 1998, 153.
[77] I discuss reasons for Ptolemy's change of reference points in Section III.5.
[78] Toomer 1998, 173. The Moon displays no parallax in longitude near mid-eclipse.

eclipse occurs at full Moon only, which means that Ptolemy relied on specific types of observations to construct a theory that was able to account for the Moon at any point. In constructing this theory, Ptolemy used period relations passed down to him and observations spanning over 900 years.[79] For the first lunar model, Ptolemy utilized 15 eclipse observations.[80]

Ptolemy explains that the Moon's motion is more complicated than the Sun's motion and unlike the Sun, the Moon can exhibit its maximum velocity at any point along its path. Additionally, the Moon's maximum northern or southern latitude can occur at any point along its path, which means the points of intersection between the ecliptic and the Moon's orbit are not fixed.[81] These two factors make the lunar phenomena more complicated to model than the solar model phenomena.

Ptolemy presents two lunar models in the *Almagest.* The first model accounts for a single lunar anomaly and the second model accounts for two lunar anomalies. In Book IV Ptolemy explains:

> Our next task is to demonstrate the type and size of the moon's anomaly. For the time being we shall treat this as if it were single and invariant. It is apparent that this anomaly, namely the one with a period corresponding to the above period of return, is the only one which our predecessors (just about all of them) have hit upon. Later, however, we shall show that the moon also has a second anomaly, linked to its distance from the sun; this [second anomaly] reaches a maximum round about both [waxing and waning] half-moons, and goes through its period of return twice a month, [being zero] precisely at conjunction and opposition.[82]

In the first lunar model, Ptolemy accounts for one lunar anomaly using the epicycle model. The first anomaly is dependent on the Moon's position in the ecliptic, while the second anomaly concerns the relative positions of the Moon and Sun, so that it is greatest

[79] Pedersen 1974, 161, 169-170.
[80] Pedersen 1974, 169-170.
[81] Pedersen 1974, 175.
[82] Toomer 1998, 180-181.

at quadrature.[83] In the second lunar model, Ptolemy accounts for the first lunar anomaly using the epicycle model and he accounts for the second lunar anomaly using the eccentric model.[84] He explains that either model, the epicycle or eccentric, could be used to explain the lunar anomalies. Ptolemy chooses the eccentric model over the epicyclic model since he finds the eccentric model "more suitable to represent the second anomaly" when the two anomalies are combined in one model.[85]

When describing the first lunar model, Ptolemy constructs a circle in the sphere of the Moon with a second circle inclined to it at an angle that corresponds to the maximum inclination in latitude. This circle travels in advance with respect to the heavens. The epicycle moves on this circle in the trailing direction with respect to the heavens and the Moon moves on the epicycle in advance with respect to the heavens. The Moon's orbit is inclined approximately five degrees to the ecliptic; the two places where the Moon's orbit and the ecliptic intersect are called the nodes. Like the Sun, the Moon's apparent motion varies; however, unlike the Sun, the apsidal line of the Moon has a slow motion in advance with respect to the heavens.

[83] Pedersen 1974, 165.
[84] For a discussion of the first lunar model see Neugebauer 1975, 68-84.
[85] Toomer 1998, 181.

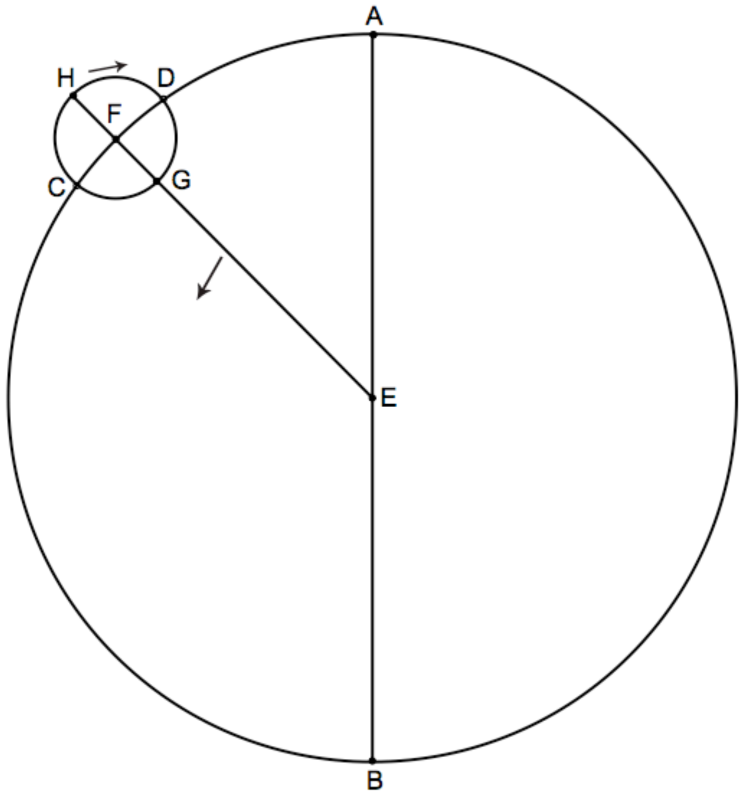

Figure 8: First Lunar Model

Figure 8 displays Ptolemy's first lunar model, ignoring the advance motion of the latitude and the inclination of the Moon's orbit.[86] Ptolemy also ignores these parameters in his discussion of the Moon's first anomaly "since such a small inclination has no noticeable effect on the position in longitude". In Figure 8, epicycle CHDG moves uniformly around point E in the trailing direction with respect to the heavens. The Moon travels on the epicycle, circle CHDG, in advance with respect to the heavens. The Moon completes a return in longitude sooner that it completes a return in anomaly.

The plane of the Moon's deferent circle (circle AFB above) is inclined to the ecliptic at an angle of 5 degrees. The points where the deferent circle and ecliptic

[86] Toomer 1998, 191.

intersect are called the nodal points and the line connecting these points is the nodal line. These points are not fixed, but instead rotate in the advance direction of the heavens. The epicycle (circle CHDG) is inclined to the deferent circle, namely circle AFB.

In Book V of the *Almagest* Ptolemy explains that the first lunar model is not sufficient when the Moon is at quadrature. Ptolemy says “We were led to awareness of and belief in this [second anomaly] by the observations of lunar positions recorded by Hipparchus, and also by our own observations, which were made by means of an instrument which we constructed for this purpose.”[87] Ptolemy provides a description of the instrument he used. While he refers to the instrument as an astrolabe, he describes what a modern reader would call an armillary sphere.[88] Ptolemy finds that while the first lunar model is able to account for the lunar phenomena when the Moon is at conjunction or opposition, it does not account for the phenomena when the Moon is at quadrature. Consequently, Ptolemy constructs a second, more complex lunar model.

In Ptolemy’s second lunar model, the Moon travels along the epicycle in advance through the heavens and the epicycle travels on the deferent circle in the opposite direction, namely in the trailing direction of the heavens.[89] The center of the deferent circle rotates around the Earth with a constant angular velocity in advance through the heavens, so that the angular width of the epicycle would be larger at quadrature than at the syzygies. Since the second lunar anomaly is due to the Moon’s elongation from the Sun, in the second lunar model the center of the deferent circle is coupled to the motion of the Sun, so that the second lunar anomaly is greatest at quadrature and it is zero at conjunction and opposition. Consequently, the second lunar anomaly relies on

[87] Toomer 1998, 217.
[88] Toomer 1998, 217 n. 1.
[89] For a discussion of the second lunar model see Neugebauer 1975, 84-101.

observations that do not occur at eclipses. The center of uniform motion is not the center of the circle.[90] This is the first time Ptolemy defines uniform motion this way, but he does not draw attention to this new definition. In the planetary models Ptolemy also describes uniform motion around a circle relative to point that is not the center of the circle.

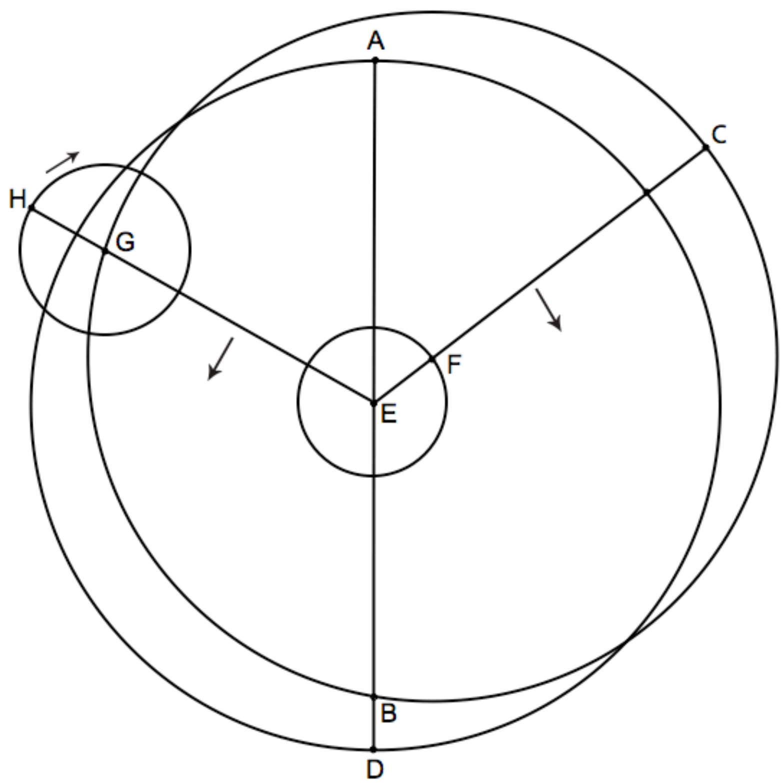

Figure 9: Second Lunar Model

The lunar model is displayed in Figure 9. The Moon, point H, travels along the epicycle, the circle centered on G with uniform motion. The epicycle travels on the deferent circle, circle CBG centered on point F. The center of the deferent circle, circle CBG, rotates uniformly on a circle centered on E with the radius of EF. Both the lines EGH and EFC

[90] Neugebauer 1975, 86, 1229.

move with a constant angular velocity around point E, but they move in opposite directions.

The eccentric circle of the Moon and the ecliptic circle are inclined to each other at an angle of five degrees. The two points of intersection are called the nodes. When both the Sun and Moon are at one of the nodes an eclipse will occur (a solar eclipse occurs when the Sun and Moon are at the same node and a lunar eclipse occurs when they are at opposite nodes). The points of intersection are not fixed, so that the nodes move through the ecliptic in the reverse direction with respect to the heavens.

For the lunar model in the *Planetary Hypotheses*, Ptolemy describes a circle concentric to the zodiac circle, moving in the same plane with uniform speed (ἰσοταχῶς) in advance with respect to the heavens.[91] This rotation accounts for the movement of the lunar nodes. This circle carries another circle that is inclined to it at an angle of five degrees and the point where these two circles intersect represent the lunar nodes. In this inclined circle there is an eccentric circle, which is eccentric with a ratio of 60 to 12;30. The center of the eccentric circle moves around the center of the zodiac, which is also the same point as the center of the aslant circle, in advance with respect to the heavens. The center of the epicycle moves along the eccentric circle towards the rear with respect to the heavens. The plane of the epicycle is inclined to the plane of the eccentric circle and the radius of the epicycle circle relative to the eccentric circle is 60 to 6;20. The Moon travels along the epicycle circle with equal speed in advance with respect to the heavens.

The construction that Ptolemy describes for the lunar model in the *Planetary Hypotheses* and the values that he gives for the parameters, are the same as those described in the *Almagest*. However, the values for the radius of the epicycle and the

[91] Heiberg 1907, 80.

eccentricity, 6;20 and 12;30 respectively, appear differently in the *Almagest*. In the *Almagest* Ptolemy assigns 60 to the distance between center of the Moon's epicycle and center of the eccentric circle, plus the distance between the center of the eccentric circle and the center of the zodiac. In the *Planetary Hypotheses* and *Canobic Inscription*, Ptolemy assigns 60 to the distance between the center of the epicycle and the center of the eccentric circle only. Consequently, the value for the eccentricity in the *Almagest*, 10;19, is equal to the value found in the *Planetary Hypotheses* and *Canobic Inscription*, but it is simply measured differently.[92] Likewise, the value for the radius of the epicycle are equivalent in Ptolemy's works, but presented as 5;15 in the *Almagest* and 6;20 in the *Planetary Hypotheses* and *Canobic Inscription*.[93]

III.2.5 Planetary Models

There are two types of planets: those that have a maximum elongation from the Sun, called the inner planets; and those that have unlimited elongations, called outer planets. According to the order Ptolemy gives in Book IX of the *Almagest*, the inner planets, Mercury and Venus, lie between the Moon and Sun, and the outer planets, Mars, Jupiter, and Saturn, lie above the Sun.[94] The inner planets never reach opposition with the Sun. The outer planets have a different observable phenomena: they travel along the ecliptic eastward, come to a stop, retrograde, come to a stop again, and then continue traveling eastwards. The speed at which an outer planet's epicycle moves around the ecliptic and the speed the planet moves around the epicycle varies for each planet.

[92] Neugebauer 1975, 903.
[93] I will discuss this change in more detail in Section III.3.5.
[94] Ptolemy debates whether the Sun is located below the planets, above the planets, or in between the planets in Book IX of the *Almagest*. He settles on the order presented here, but he brings up the topic again in the *Planetary Hypotheses*. See Section I.4.2 above.

Ptolemy discusses the planetary theories in Books IX-XI of the *Almagest*. While Ptolemy credits Hipparchus with the advancement of the solar and lunar theory, he states that the planetary theory is his own contribution. Although there were probably advancements to the planetary theories between Hipparchus and Ptolemy, not much is known about planetary modeling during this time period. Ptolemy gives two reasons as to why none of his predecessors developed satisfactory planetary models: they did not have enough accumulated observations to develop a theory, and the planets each have two anomalies, which are difficult to distinguish from one another.[95] Ptolemy is not reprimanding earlier astronomers, but instead he appears to offering an explanation as to why proposed planetary models were either erroneous or provided inadequate methods of demonstration.

In his model, the planet travels uniformly around an epicycle. The epicycle travels on a deferent circle and moves uniformly relative to the center or uniform motion, called the equant point. The observer is neither located at the center of the deferent circle nor at the equant, but at a third point. The plane of the eccentric circle is inclined to the plane of the ecliptic and the plane of the epicycle is inclined to the epicycle.[96]

[95] Toomer 1998, 420. The Keskintos Inscription provides evidence of planetary theories that preceded the *Almagest.* Neugebauer 1975, 698-705. Jones 2006, 3-41.
[96] The latitudinal theories in the *Almagest* and *Planetary Hypotheses* are different; I discuss this in Section III.2.6 below.

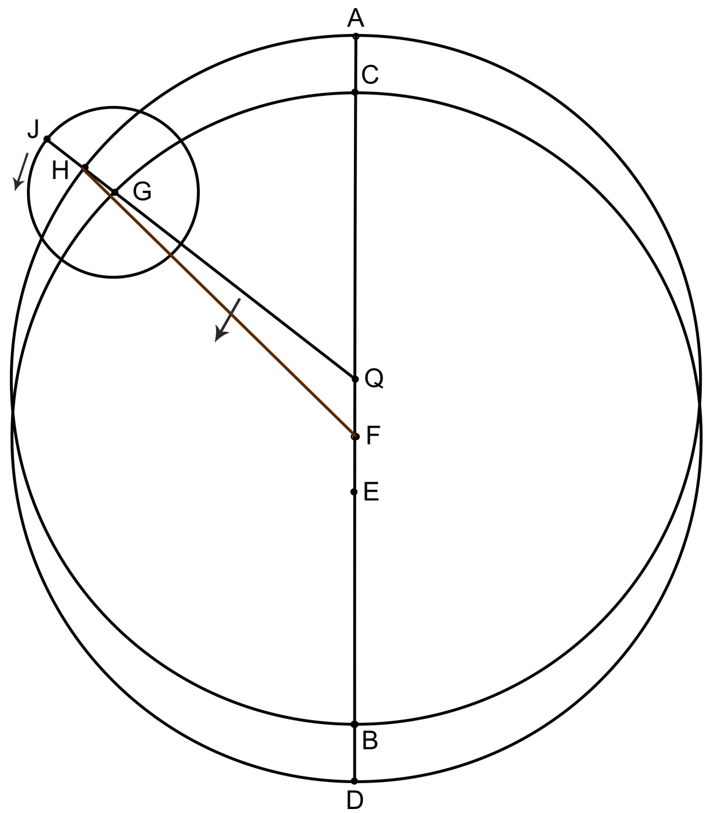

Figure 10: Planetary Model[97]

Figure 10 displays Ptolemy's planetary model for all of the planets except for Mercury. The planet J travels uniformly around the epicycle, the circle centered on G. The epicycle travels uniformly around point Q; however, it does not travel on the circle centered on Q, namely circle ABH, but instead it travels around the circle centered on F, namely circle CDG. The planet and the epicycle both move towards the rear with respect to the heavens. The observer is located at point E. Point F bisects line QE.

Both Mercury and Venus have a mean motion in longitude that is equal to the Sun's mean motion; in terms of Figure 10, each epicycle travels around circle CDG with a mean speed that is equivalent to the Sun's mean speed. Mercury requires its own model

[97] Toomer 1998, 444.

since it has two perigees. Venus's model is the same as the model for the outer planets, except for one important difference. For the outer planets the line between the center of the epicycle and the planet (line GJ in Figure 9) remains parallel to the line between the Earth and the mean Sun.[98] For Venus, the line between the equant and the center of the epicycle (line QG in Figure 9) remains parallel to the line between the Earth and the mean Sun.[99]

III.2.6 Mercury's Model

Mercury's model is based on 16 observations. When determining the perigee of Mercury, Ptolemy finds that it is not located opposite the apogee, but 120 degrees from the apogee. In hopes of finding symmetry, Ptolemy checks the observations for 120 degrees from the apogee in the other direction and determined that Mercury has two perigees.[100] Mercury's apogee and two perigees are all separated by 120º. Furthermore, Ptolemy calculated that Mercury's apogee travels approximately four degrees in 400 years in advance through the heavens. This is the same rate that the fixed stars travel relative to the solsticial and equinoctial points. From this, Ptolemy concluded that the Mercury's apogee, as well as the apogees of all of the other planets, must be sidereally fixed.[101] The apogee of all of the planets moves westward at the rate of one degree every one hundred years.

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[98] The mean Sun is the longitude the Sun would occupy if the Sun moved uniformly around the Earth at the average speed of the real Sun.
[99] This phenomenon is due to the fact that Ptolemy is describing a heliocentric system instead of a geocentric system. Pedersen 1974; 303. Neugebauer [1969] 1993, 123.
[100] Pedersen 1974, 314.
[101] Pedersen 1974, 303. See Section III.3.4 for a discussion of the planetary apogees.

Like the other planetary models, in Ptolemy's model of Mercury the planet revolves around its epicycle towards the rear with respect to the heavens. The epicycle rotates around its deferent circle from west to east and the epicycle rotates uniformly around the equant point. However, unlike the other planetary models, the center of the deferent circle rotates uniformly around a point.

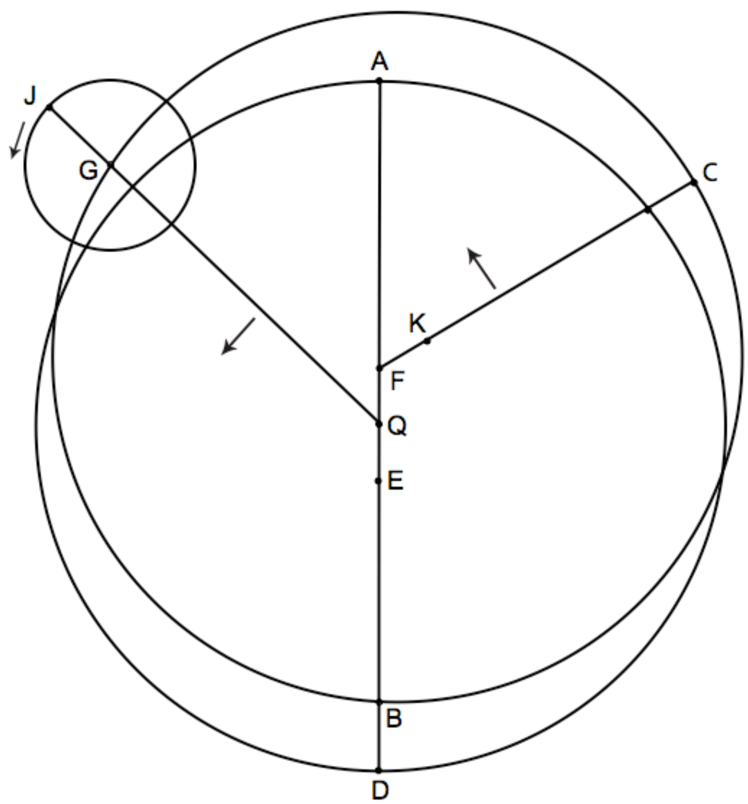

Figure 11: Mercury Model

Mercury's model is shown in Figure 11. The planet, point J, travels uniformly around its epicycle, the circle centered on G, towards the rear with respect to the heavens. The epicycle travels uniformly towards the rear with respect to the equant, point Q. The epicycle travels on the deferent circle, circle CBG, centered on point K. Point K travels uniformly in a circle around point F in advance with respect to the heavens. The lines

QGJ and FKC both move at the same angular velocity, but in opposite directions. Point F and point E are equidistant from point Q. Furthermore, Point K and point Q are equidistant from point F.

The structures of the models for the planets are the same in the *Planetary Hypotheses* as they are in the *Almagest*; however, there are some changes to the parameters, specifically for Mercury. In fact, Ptolemy presents slightly different parameters for Mercury in the *Canobic Inscription*, *Almagest*, and *Planetary Hypotheses*. In the *Planetary Hypotheses* Ptolemy discusses the motion of the center of the epicycle around the equant point similar to the way he does so in the *Almagest*. When discussing Venus in the *Planetary Hypotheses*, Ptolemy says:

> let the center of the epicycle be assumed to move around the point that is further from the earth with uniform speed having its position on the eccenteric circle always around the aforementioned diameter in the trailing direction cosmos…[102]

The explanation of uniform, circular motion around a point that is not at the center of the circle found in both the *Almagest* and *Planetary Hypotheses* would present a problem for an instrument-maker. As in the rest of the *Planetary Hypotheses*, Ptolemy does not offer guidance to the instrument-maker; however, he also does not make any changes in the model in order to simplify it for the purposes of creating a mechanical representation.

In the *Almagest*, Ptolemy gives Mercury a unique model that contains two perigees that are each 120 degrees from the apogee. Similar to the lunar model, Mercury's model also has a crank mechanism, which moves the center of the eccentric circle. The parameters that Ptolemy gives Mercury in the *Almagest* are 22;30 for the radius of the epicycle, and 3;0 for the both eccentricities. In the *Canobic Inscription*, which is an earlier work than the *Almagest*, Ptolemy gives the same parameter for the

[102] Heiberg 1907, 86.

radius of the epicycle as he does in the *Almagest*, namely 22;30. He gives the eccentricity value as 2;30, but he does not indicate that the model for Mercury is different than the models for the other planets, which raises question the structure of Mercury's model in the *Canobic Inscription*.[103]

In the *Planetary Hypotheses* Ptolemy changes the radius of the epicycle of Mercury to 22;15 and the eccentricity of Mercury to 2;30. The distance from the equant to the Earth remains 3;0. As Willy Hartner shows, the perigees for Mercury according to the *Almagest* model are not at ±120º, but at ±120;28,20º.[104] Additionally, the changes that Ptolemy makes to Mercury's eccentricities in the *Planetary Hypotheses* affect the positions of the perigees so that they are at ±124;6,5º, approximately four degrees further away from where they should be. Swerdlow contends that Ptolemy decided that the position of Mercury's two perigees were at ±120º, not because he calculated this to be the *exact* location, but because his calculations were *close* to ±120 degrees and the symmetry appealed to Ptolemy.[105] By changing the radius of the epicycle from 22;30 in the *Almagest* to 22;15 in the *Planetary Hypotheses* and changing the eccentricity from 3;0 to 2;30, Ptolemy does not make a noticeable change to the model. The elegance of having both the value of the eccentricity and the value for the distance between the equant and

[103] Hamilton, Swerdlow, and Toomer state that it is possible that Mercury's model in the *Canobic Inscription* may be the same as the models for the other planets; however, they contend that there is not enough evidence to confirm this. Hamilton, Swerdlow, and Toomer 1987, 65. Jones examines this question more thoroughly, discussing evidence that supports the idea that the model for Mercury in the *Canobic Inscription* is the same as the model in the *Almagest*. Jones suggests that the models are the same in both works, but he concludes that we cannot be absolutely sure about the structure of Mercury's model in the *Canobic Inscription*. Jones 2005a, 64-67.
[104] Willy Hartner first pointed out that the perigee of Mercury is not exactly 120 degrees from the apogee in his 1955 publication. He gives the calculated value for the position of the perigee of Mercury, namely ± 120;28,20º, in his 1974 publication. Hartner 1955, 117. Hartner 1974, 7.
[105] Swerdlow 1989, 53.

the Earth equal in the *Almagest* might be what led Ptolemy to choose these values.[106] However, this does not explain why Ptolemy changed the values in the *Planetary Hypotheses*. The new parameters offer no perceptible change and they are no longer mathematically aesthetic. In the *Almagest*, the maximum elongation of Mercury could be increased but the parameters used in the *Planetary Hypotheses* actually decrease the maximum elongation and do not significantly change the minimum elongation. Consequently, the changes that Ptolemy makes to the *Almagest* parameters do not result in parameters, which more accurately account for Mercury's maximum and minimum elongation in the *Planetary Hypotheses*.[107]

The parameters for the eccentricity and radius of the epicycle for the models of Venus, Mars, Jupiter, and Saturn are exactly the same in the *Planetary Hypotheses* as they are in the *Almagest*.[108] Additionally, the structures of the models are also unchanged.

III.2.7 Latitude Theory for the Planets

In order to account for the varying latitudes of the planets, Ptolemy presents a latitude theory in Book XIII of the *Almagest*. The latitude theory for the planets is complex compared to the Sun and the Moon; the former does not require a latitude theory and the latter has a latitude theory that consists of a plane inclined to the ecliptic circle at

[106] Hartner shows that the parameters in the *Planetary Hypotheses* do not alter the model so that it better accounts for the observations. Hartner 1964, 267 n. 24. Swerdlow examines Ptolemy's steps in calculating the values for Mercury and Venus. Swerdlow says, "Ptolemy was empirical up to a point, but where observation was inadequate, and in precisely locating the maximum sums of elongations it doubtless was, he had no alternative but to turn to theory, adopting what seemed to him the most simple, reasonable or elegant hypothesis (whatever such criteria may mean) that made sense out of less than certain observations." Swerdlow 1989, 53-54.
[107] Hartner 1964, 267 n. 24.
[108] In addition to changes to Mercury's parameters between the *Canobic Inscription* and the *Almagest*, Ptolemy also makes changes to the parameters for the Moon and Saturn. Hamilton, Swerdlow, Toomer 1987, 64-65.

fixed angle of 5 degrees. Constructing the planetary latitude system was no easy task since the data used to construct Ptolemy's theory consisted of observations of small angles.[109] Since the inclinations in latitude for the planets are small and do not affect the longitude of the planet, Ptolemy ignored the inclinations when he laid out the longitude theory in Books IX-XII.[110]

According to his latitude theory, for all of the planets the plane of the eccentric circle is inclined to the plane of the ecliptic and the plane of the epicycle is inclined to the plane of the eccentric circle. While these respective inclinations are fixed for the outer planets – Mars, Jupiter, and Saturn – for Mercury and Venus the inclination of the plane of the epicycle oscillates relative to the plane of the deferent. Ptolemy explains how the models are situated in Book XIII. He says:

> Now [first], just as each [planet] appears to perform a twofold anomaly in longitude, each exhibits a twofold difference in latitude, one [varying] with respect to the parts of the ecliptic, and due to the eccentre, the other with respect to [its elongation from] the sun, and due to the epicycle. Therefore in every case we suppose that the eccentre is inclined to the plane of the ecliptic, and that the epicycle is inclined to the plane of the eccentre. However, as we said [IX 6, p. 433], no noticeable difference occurs in the longitudinal position or the demonstrations of the anomalies on account of such small inclinations, as we shall show later. [Secondly,] from individual observations of every planet, [we see that] the planets appear exactly in the plane of the ecliptic when the corrected longitude is approximately a quadrant from the northern or southern limit of the eccentre, and at the same time the corrected anomaly is approximately a quadrant from its own apogee. So we suppose the inclinations of the eccentres to take place at the centre of the ecliptic (just as for the moon), and with respect to the diameters through the northern and southern limits; and [we suppose] that the inclinations of the epicycles take place with respect to that diameter of the epicycle which points towards the centre of the ecliptic, on which its apparent apogee and perigee are observed.[111]

[109] In our modern astronomical theory the plane that a planet travels on does not pass through the Earth, but instead passes through the Sun, which makes constructing an accurate geocentric latitude theory complicated. Neugebauer 1975, 209.
[110] Toomer 1998, 443.
[111] Toomer 1998, 597.

Just as each planet has two longitudinal anomalies, each planet also has two latitudinal anomalies. The inclination of the eccentric circle occurs at the center of the ecliptic and the inclination of the epicycles occurs with respect to the diameter of the epicycle, which is the line where apparent apogees and perigees can be seen. The inclination of the epicycle only occurs along the apogee-perigee line. Relying on observations, Ptolemy concludes that for Mars, Jupiter, and Saturn if the eccenter is north of the ecliptic, then the perigee of the epicycle is north of the eccenteric circle, and if the former is south then the latter is also south.[112] However, for Mercury and Venus the situation is different, since each planet experiences two inclinations along the diameter of the epicycle.

> [We also conclude that] the epicycle brings about two variations [in latitude]: it produces the greatest inclination of the diameter through the apparent apogee at the nodes of the eccentre, and the greatest 'slant' (let us use this term to distinguish this kind of angular variation) of the diameter at right angles to the former at the apogee and perigee of the eccentre.[113]

One inclination for the inner planets occurs along the diameter of the epicycle when the epicycle is at apogee or perigee of the eccentric circle. The second inclination, called the slant, occurs along the diameter of the epicycle when the epicycle is a quadrant away from the apogee or perigee on the eccentric circle.

> But in the case of the 3 planets Saturn, Jupiter and Mars the eccentre has a fixed inclination, so that diametrically opposite positions of the epicycle have opposite directions in latitude, whereas in the case of Venus and Mercury the eccentre moves together with the epicycle in the same latitudinal direction, for Venus always to the north, for Mercury always to the south. [114]

For all five of the planets the eccentric circle is inclined to the ecliptic circle through the center of the ecliptic. For the outer planets – Mars, Jupiter, and Saturn – the inclination of the eccentric circle to the epicycle occur in the same directions "so that diametrically

[112] Toomer 1998, 597-598 n. 7.
[113] Toomer 1998, 599.
[114] Toomer 1998, 599.

opposite positions have opposite directions in latitude"; however, for Mercury and Venus, the inclination of the epicycle and eccentric circle occur in the same latitudinal direction.[115] The inclination of the epicycle is controlled by small vertical circles that are perpendicular to the plane of the ecliptic and that move with the same longitudinal motion as the epicycle. Ptolemy explains:

> The epicycle [for all 5 planets] has the diameter though its apparent apogee moved from a starting-point in the plane of the eccentre, by a small circle which we may suppose attached to the end [of the diameter] nearer the earth. This circle is of a size corresponding to the appropriate [maximum] deviation in latitude, is perpendicular to and centred in the plane of the eccentre, and revolves with uniform motion, with a period equal to that of the motion in longitude, from one end of the intersection of its own plane and the plane of the epicycle towards the north (by hypothesis), carrying with it the plane of the epicycle: in its revolution through the first quadrant it carries the epicycle's plane, obviously, to the northern limit, in the second back to the plane of the eccentre, in the third to the southern limit, and in its return to [the end of] the remaining quadrant back to the original plane.[116]

These circles, which provide a mechanical explanation of the latitudinal motion, are responsible for the movement of the diameter of the epicycle. Ptolemy provides minimal detail concerning how these circles move.[117]

Since Ptolemy treats the outer planets and inner planets differently, I will first examine the latitude theory of the outer planets and the then the latitude theory of the inner planets. For the outer planets, Ptolemy gives a value of the angle between the ecliptic and plane of the eccentric (i_1), and a second value for the angle between the eccentric plane and the epicycle plane ($i_1 + i_2$).[118]

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[115] Toomer 1998, 599. See Figure III.8 and Figure III.9 below.
[116] Toomer 1998, 599.
[117] I discuss these small circles below.
[118] N.M. Swerdlow provides a thorough and detailed discussion of Ptolemy's planetary latitude theories. I have decided to use Swerdlow's configuration and notation. Unlike Neugebauer, Swerdlow divides the epicycle's inclination into two angles: angle i_1 is fixed, and angle i_2 varies. Swerdlow 2005, 42-43. Neugebauer 1975, 908-909.

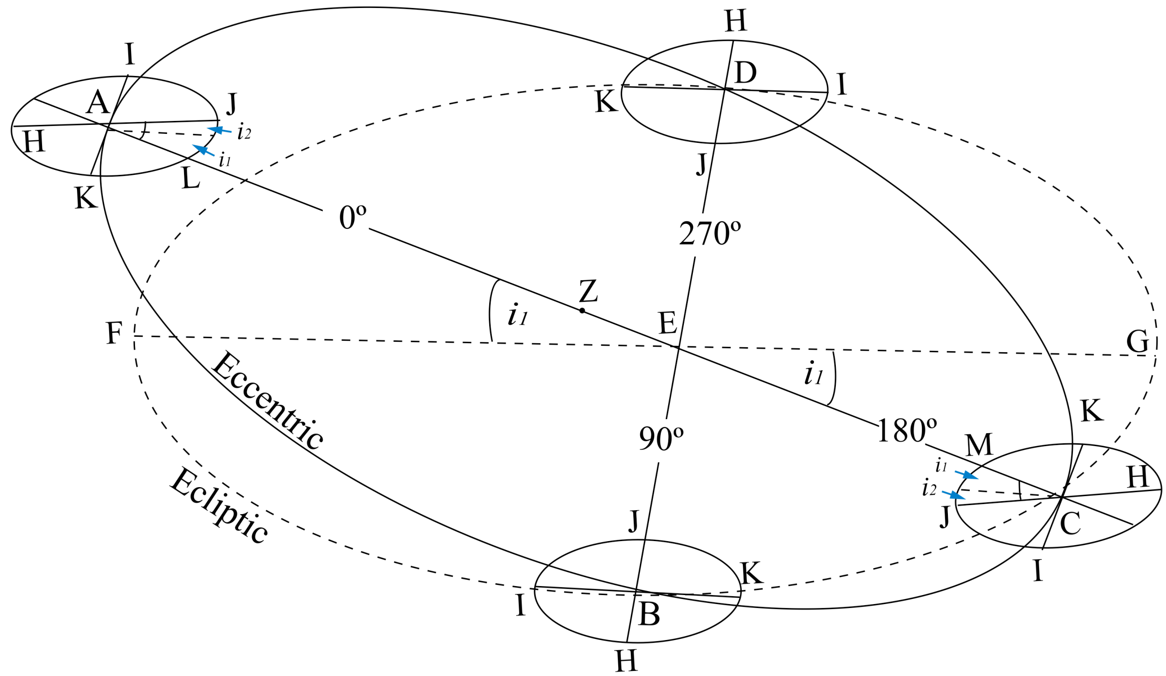

Figure 12: Latitude Model for the Outer Planets

Figure 12 represents the latitude model for the outer planets (the inclinations are exaggerated). Point E represents the Earth and point Z represents the center of the eccentric circle. The epicycle is shown at four different locations in its orbit around the Earth. The plane of the epicycle is inclined to the eccentric circle so that when the center of the epicycle is at point A the apogee of the epicycle, point H, is angled to the south and the perigee of the epicycle, point J, is angled to the north. When the center of the epicycle is at point C, the apogee of the epicycle, point H, is angled to the north and the perigee of the epicycle, point J, is angled to the south. When the center of the epicycle is a point B or D, the epicycle lies in the same plane as the ecliptic. The inclination of i_2 varies as the epicycle rotates around the Earth, hitting its maximum and minimum inclinations ($+i_2$

and $-i_2$) at the apogee and perigee of the eccentric circle. When the center of the epicycle is one of its quadratures, namely point B or D, then angle i_2 diminishes to zero.

Calculating the apparent inclination using Ptolemy's latitude tables is difficult since the inclination of the plane of the epicycle will appear smaller or greater depending on the epicycle's distance from the Earth. For example, the inclination of $i_2 + i_2$ will appear greater when the center of the epicycle is at C compared to when the center of the epicycle is at A.[119] As N.M. Swerdlow shows, the difficulty in accounting for the latitude of the planets in the Ptolemaic systems lies with the variation in the epicycle's inclination. In a heliocentric system, the epicycle's path represents the Earth's motion around the Sun for the outer planets. Consequently, the epicycle of the outer planets should not have a large variation in latitude because the Earth is always in the plane of the ecliptic.[120] In order to construct the latitudinal theory, Ptolemy required observations of the outer planet at opposition or near conjunction when the epicycle is at one of its limits. The necessary conditions to record these observations occur rarely, making it difficult to acquire accurate data. Not surprisingly, the observations that Ptolemy used were inaccurate.[121] Furthermore, the method of calculation that Ptolemy used to determine the inclinations was very sensitive, so that errors or rounding had a strong impact on the outcome.

Similar to the latitude models for the outer planets, the eccentric plane for the inner planets is inclined to the plane of the ecliptic in a fixed position (i_0). However, the models for Mercury and Venus are more complicated than those of the outer planets

[119] See Swerdlow 2005, 42-58.
[120] Swerdlow 2005, 46.
[121] For Saturn, an opposition or conjunction occurs near each limit approximately every thirty years, for Jupiter approximately every twelve years, and for Mars approximately every fifteen or seventeen years. Swerdlow 2005, 47.

because Ptolemy gives Mercury and Venus two separate inclinations between the epicycle and the eccentric circle (i_1 and i_2).

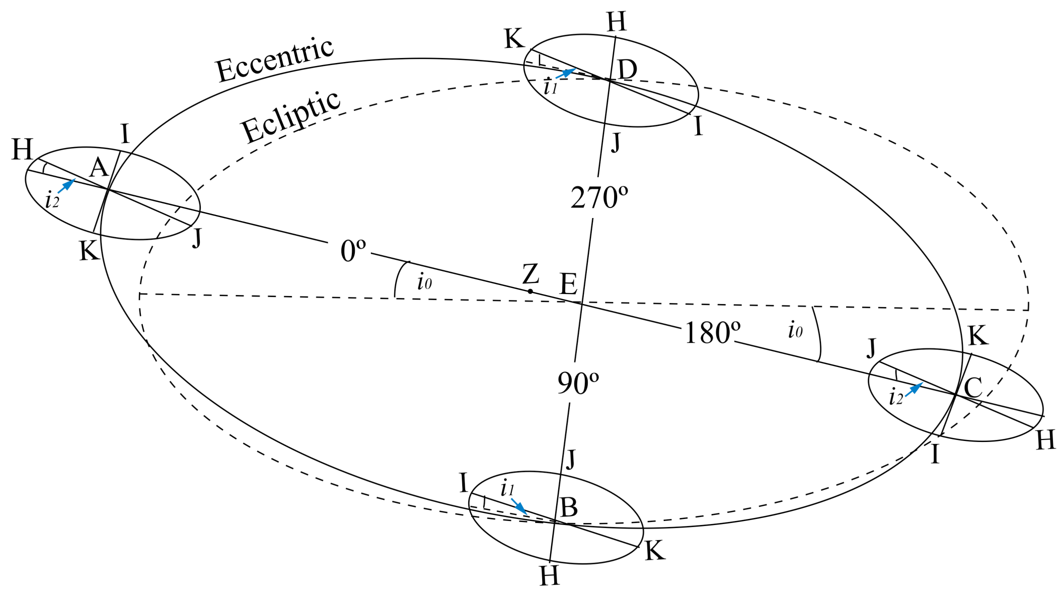

Figure 13: Latitude Model for the Inner Planets

In Figure 13 the latitude model from the inner planets is shown (once again, the inclinations are exaggerated). Point E represents the Earth, point Z represents the center of the eccentric circle, and the epicycle is shown in four different places. The ecliptic circle is inclined to the eccentric circle in a fixed position (in the chart below this inclination is called i_0). For the inner planets the apsidal lines and nodal lines coincide.[122] The epicycle experiences two different inclinations in the time it takes the center of the epicycle to complete one complete rotation around the Earth. One inclination (i_2) occurs about the line connecting the apogee and perigee of the epicycle, point H and J in the

[122] Toomer 1998, 598.

diagram. A second inclination (i_1) occurs a quadrant away from the first, about the line that connects point K and I. Over the course of one solar year i_2 oscillates between its extremes, so that point H hits its maximum height ($+i_2$) when the center of the epicycle is at apogee, zero degrees longitude measured from the apogee of the eccentric circle. Angle i_2 hits its minimum height ($-i_2$) when the center of the epicycle is at 180 degrees longitude measured from the apogee of the eccentric circle. Angle i_1 behaves similar to angle i_2, reaching its extremes over the course of one solar year; however, these extremes ($\pm i_1$) occur at 90 degrees and 270 degrees longitude, measured from the apogee of the eccentric circle. When the epicycle is at zero degrees longitude, angle i_2 is at its maximum inclination. As the epicycle moves to 90 degrees longitude, angle i_2 decreases to zero and angle i_1 increases to its maximum inclination, so that when i_2 is at maximum inclination, i_1 is at zero and the reverse is true. In order to determine the latitude of the inner planets, three inclinations – instead of two with the outer planets – must be calculated separately and then combined.

As with the inclinations of the outer planets in the *Almagest*, the problems with the parameters for the inner planets can be traced back to the observations that Ptolemy used. The latitude of the inner planets when at the apogee or perigee of the epicycle (i.e. superior or inferior conjunction) cannot be observed and must therefore be estimated.[123] This proves to be difficult and the observations that Ptolemy used are problematic, which provides inaccurate inclination values particularly for angle i_1 for Venus.[124]

[123] Swerdlow 2005, 55.
[124] Swerdlow, Pedersen, and Neugebauer provide discussions of how Ptolemy computed the inclinations for the inner planets.

The latitude values for the outer planets are corrupt in the *Canobic Inscription*, making it difficult to say anything about changes Ptolemy made with certainty.[125] However, since Ptolemy provided an angle of inclination between the ecliptic plane and eccentric plane (i_1) and an angle of inclination between the eccentric plane and the epicyclic plane (i_1+i_2), it seems that there is no major change in the model between the *Almagest* and *Canobic Inscription*. The angles of inclination are the same in the *Almagest* and *Canobic Inscription* for Mars, but Ptolemy made changes to the parameters for the other planets.[126] The angles of inclination for the *Almagest*, *Canobic Inscription*, *Handy Tables*, and *Planetary Hypotheses* are compared in the table below:

Table 2: Angles of Inclination for the Outer Planets

	Almagest		Canobic Inscription		Handy Tables		PH	
	i_1	i_1+i_2	i_1	i_1+i_2	i_1	i_1+i_2	i_1	i_1+i_2
Mars	1;0	2;15	1;0	2;15	1;0	2;15	1;50	1;50
Jupiter	1;30	2;30	1;30	1;0	1;30	2;30	1;30	1;30
Saturn	2;30	4;30	0;0[?]	9;5,0[?]	2;30	4;30	2;30	2;30

In the *Handy Tables* Ptolemy made changes from the latitude theory found in the *Almagest*.[127] The inclination between the epicycle and the eccentric for the outer planets is fixed in the *Handy Tables*; however, this is not an improvement to the model since the epicycle would be inclined to the ecliptic when the center of the epicycle is at one of the nodes of the eccentric circle.[128]

[125] Hamilton, Swerdlow, Toomer 1987, 68.
[126] Jones 2005a, 70-73. Neugebauer 1975, 909.
[127] Neugebauer reconstructs the latitudinal models for the *Handy Tables*. His solution is based on the work of B.L. Van der Waerden. Neugebauer 1975, 1006-1016. Van der Waerden 1953, 265-270. Van der Waerden 1958, 54-78.
[128] Swerdlow 2005, 61.

As with the outer planets, the corruption of the text makes it difficult to come to any conclusions about the parameters, but it is safe to say that Ptolemy did not make any substantial changes to the models for the inner planets.[129] The table below contains the latitudinal values for the *Almagest*, *Canobic Inscription*, *Handy Tables*, and *Planetary Hypotheses*:

Table 3: Angles of Inclination for the Inner Planets

	Almagest			Canobic Inscription			Handy Tables		PH	
	i_1	i_2	i_0	i_1	i_2	i_0	i_1	i_0	i_1	i_0
Mercury	6;15	7;0	–0;45	7;0	2;30	[–?]0;40	6;30	0;10	6;30	0;10
Venus	2;30	3;30	+0;10	2;30	2;30	[+?]0;15	3;30	0;10	3;30	0;10

In the *Handy Tables*, Ptolemy changed the inclination between the epicycle and the eccentric so that this value is fixed, which means there is only one value for i_1 and i_2 (we will call this value i_1). There are no longer two angles of inclination (i_1 *and* i_2) between the eccentric circle and the epicycle as there were in the *Almagest*. Additionally, Ptolemy adjusted the angle of inclination between the ecliptic and the eccentric circle (i_0), so that the angle is fixed. For both Mercury and Venus this angle is 0;10˚; however, for Venus the northern limit is at apogee, and for Mercury the northern limit is at perigee. The values in the *Planetary Hypotheses* are the same as those in the *Handy Tables* for the inner planets.

Ptolemy's final attempt to construct a latitude model for the outer planets that accounts for the phenomena comes in the *Planetary Hypotheses*. Ptolemy made several important changes from the latitude theory found in the *Almagest,* to the latitude theory found in the *Planetary Hypotheses*. He presented a different set of parameters from his

[129] Jones 2005a, 70-73. Hamilton, Swerdlow, Toomer 1987, 68.

previous work, giving the same values for the angle between the ecliptic and plane of the eccentric (i_1), and the angle between the plane of the eccentric and the plane of the epicycle (i_1+i_2).[130] In other words, in the *Planetary Hypotheses* angle i_2 equals zero. The plane of the epicycle is consequently parallel to the plane of the ecliptic and when the center of the epicycle is at one of the two nodes of the eccentric circle the planet exhibits the same latitude as the ecliptic no matter where the planet is on the epicycle.

Additionally, in the *Planetary Hypotheses* Ptolemy simplified the theory so that the inclination of the epicycle is fixed. In doing this, Ptolemy eliminated the need for the small circles that control the latitude of the planets, which he introduced in Book XIII of the *Almagest*.

One of the most important changes Ptolemy made between the latitude models he presented in the *Almagest* and the ones presented in the *Planetary Hypotheses* is a change in the point of reference. In the *Almagest* the direction of the inclinations of the epicycle is given as seen from the observer. For example, in Figure 13 when the center of the epicycle is at 0 degrees (point A), the epicycle has an inclination along the apogee-perigee line of the epicycle, so that the point H is higher than point J. After the epicycle has traveled 90 degrees, so that the center of the epicycle is at point B, then the epicycle no longer has an inclination along the apogee-perigee line, but instead it has an inclination along the diameter of the epicycle perpendicular to the apogee-perigee line, namely line KI. In both cases Ptolemy judged the inclination of the epicycle radially, so that the direction of the inclination is relative to the radius of the eccentric circle. In the *Planetary Hypotheses*, Ptolemy changed the point of reference, so that the inclination of the epicycle is given relative to its absolute direction. In the *Planetary Hypotheses* the

[130] Neugebauer 1975, 908-909.

location of the observer in relation to the plane being measured is irrelevant in determining the inclination of the plane.

The latitude theory that Ptolemy presents in the *Planetary Hypotheses* bears many similarities to latitudinal theories seen in Indian astronomy. Although Greek astronomy had a significant impact on Indian astronomy, there appears to have been virtually no transmission of Ptolemaic astronomy into India until the seventeenth-century.[131] Because of this, careful analysis of Indian astronomy can provide insight into the state of pre-Ptolemaic Greek astronomy.[132] Between the second and fifth-century A.D. a branch of non-Ptolemaic astronomy was transmitted to India.[133] During this period five schools of astronomy came about, called the five pakṣas. These five pakṣas were: Brāhmapakṣa, Āryapakṣa, Ārdharātrikapakṣa, Saurapakṣa, and Gaṇeśapakṣa.[134] Brāhmapakṣa astronomy, which originated in western and northwestern India about 400 A.D., possessed latitude theories with a similar structure to those found in the *Planetary Hypotheses*. For example, the *Paitāmahasiddhānta* (often referred to as the *Paitāmaha*), the main text of Brāhmapakṣa, contains a latitude theory where an absolute perspective is used instead of a geocentric perspective as in the *Almagest*.[135] In the *Paitāmaha* each

[131] The Greek texts that were transmitted consisted of different astronomical theories, including planetary theories from Mesopotamia and Hellenistic astronomical theories. There is no evidence of Ptolemaic astronomy within Indian astronomy until a much later date and it is widely believed by historians of science that the works that were transmitted to India during the third to fifth centuries were pre-Ptolemaic theories. For a detailed discussion of the transmission of Greek astronomy to India see Neugebauer 1955; Pingree 1976 and 1978; and *Dictionary of Scientific Biography*, supplement, "History of Mathematical Astronomy in India."
[132] This task is not easy due to corruptions in early Sanskrit texts and modifications to theories made by Indian astronomers. *Dictionary of Scientific Biography*, supplement, "History of Mathematical Astronomy in India." Kim Plofker provides a detailed and balanced discussion of this topic. Plofker 2009, 113-120.
[133] Pingree 1976, 110.
[134] *Dictionary of Scientific Biography*, supplement, "History of Mathematical Astronomy in India.": 533-534.
[135] This text was written around 450 A.D. and survives in a later compilation of texts called *Viṣṇudharmottarapuāṇa*. *Dictionary of Scientific Biography*, supplement, "History of Mathematical Astronomy in India.": 555, 560.

planet has two epicycles: a manda epicycle and a sīghra epicycle. The plane of the sīghra epicycle is tilted to the plane of the ecliptic so that each planet has a given orbital inclination.[136]

While the details of the *Paitāmaha* planetary theories clearly differ from the planetary theories found in the *Planetary Hypotheses*, the latitude theory is comparable in structure. If this theory originates from pre-Ptolemaic Greek astronomical theories, then the latitude theory found in the *Planetary Hypotheses* would represent conventional latitudinal theories. Consequently, Ptolemy's latitude theory in the *Almagest* would have been a new theory where a radial perspective was used so as to maintain a geocentric point of reference. This would explain the defensive passage in the *Almagest* about simplicity in the heavens. In Book XIII Ptolemy followed his introduction to the latitude theory with a discussion of simplicity in the heavens versus simplicity on Earth. Following a brief explanation of how the small vertical circles move, Ptolemy provides a defense of his model. He says:

> Now let no one, considering the complicated nature of our devices, judge such hypotheses to be over-elaborated. For it is not appropriate to compare human [constructions] with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what [could one compare] more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself? Rather, one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions, but if this does not succeed, [one should apply hypotheses] which do fit. For provided that each of the phenomena is duly saved by the hypotheses, why should anyone think it strange that such complications can characterise the motions of the heavens when their nature is such as to afford no hindrance, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another?[137]

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[136] *Dictionary of Scientific Biography*, supplement, "History of Mathematical Astronomy in India.": 560.
[137] Toomer 1998, 600-601. See Section I.4.1 above.

Ptolemy makes a concerted effort to defend and explain the apparent complexity of his model. He argues that the simplest hypothesis should be used; however, if it does not describe the phenomena then a hypothesis that does should be used. He defends the complexity of his latitudes models, yet he does not give a similar defense after presenting other complex models, such as Mercury's model and there is no other passage in the *Almagest* that has this same tone.

Ptolemy remains silent concerning his precise reasons for changing his latitude theory in the *Planetary Hypotheses*. It is possible that Ptolemy changed the models for reasons that he does not explicitly state. For example, he may have merely wanted to simplify his latitude theory (for reasons unconnected to instrument-making) or he may have wanted to return to the type of latitude theory found in India astronomy, which presumably represents pre-Ptolemaic latitude theories. While Ptolemy does not directly state in the *Planetary Hypotheses* why he changed his latitude theory, there are two passages in the *Planetary Hypotheses* that offer possible motivations, both toward the beginning of Book I, Part A. Firstly, Ptolemy states that the models may be due to more continuous observations. He says:

> We shall make the exposition, so far as the general assumptions are concerned, in agreement with the things delineated in the *Syntaxis*, so far as the details are concerned, following the corrections we have produced in many places on the basis of more continuous observations, either corrections to the models themselves, or corrections to the spatial ratios, or corrections to the periods of restitutions.[138]

Secondly, Ptolemy also states that the models may be simplified for the purposes of constructing an instrument. He says:

[138] Heiberg 1907, 72.

> Concerning the positions and arrangement of the circles causing the anomalies, we will apply the simpler version in respect to the methods of instrument-making (ὀργανοποιίας), even if some small variations will follow…[139]

These two passages offer reasons, namely due to more continuous observations or simplifying the models for the purposes of constructing an instrument, as to why Ptolemy decided to change his latitude theory.

Concerning the first, since Ptolemy does not provide observations in the *Planetary Hypotheses,* and since the tables that were meant to accompany the text have not survived, we cannot determine if the alternations were in fact due to further observations. Since the conditions needed to take observations of one of the outer planets when its epicycle is at apogee or perigee and the planet is simultaneously at opposition or near conjunction occur rarely, it would have been difficult for Ptolemy to gather these observations himself in the period of time between when the *Almagest* and the *Planetary Hypotheses* were written. Given the difficulty of acquiring the observations required to improve or reconstruct his latitude theory and the rarity of the outer planets reaching their maximum and minimum latitudes, the changes that Ptolemy made to his latitude models were not motivated by new observations. Swerdlow proposes an explanation when he says: "Since it was not possible in the years of his observations to observe all the planets under the special conditions used to derive the inclinations in the *Almagest*, for the superior planets at opposition and near conjunction at the limits, he must have derived the inclinations from other sorts of observations."[140] It is entirely possible that Ptolemy had

[139] Heiberg 1907, 72-74.
[140] Swerdlow 2005, 67. Swerdlow proposes an example of the type of data that Ptolemy could have used. He says: "One possibility is a series of oppositions over several years with a large although not maximum latitude, showing by computation both the inclination i_1 and, from $i_1+i_2\approx i_1$, that $i_2=0°$, that the epicycle remains parallel to the ecliptic, which would require two oppositions for each planet. In this way he could correct both the *Almagest* and the *Handy Tables*. But on such things one can only speculate, fully aware of the difficulty of finding these parameters even from accurate observations." Swerdlow 2005, 67.

access to some types of observations that he was able to use to improve his theory, but we can only conjecture about the type of observations Ptolemy would have used to make the changes we see to the latitude theory in the *Planetary Hypotheses*.

Concerning the second motivation, namely simplifying the latitude theory for the purpose of instrument-making, this seems to be a more probable possibility since there is no place in the *Planetary Hypotheses* where the models were deliberately simplified compared to the *Almagest* models. Ptolemy made changes to parameters in the *Planetary Hypotheses* in several places, but the only place where he altered a model was for his latitude theory. Setting aside some of the details, the orientation of the inclination of the epicycle remains the same in the latitude theory in the *Almagest* and the *Planetary Hypotheses*, but the models differ in the perspective (either relying on radial or absolute perspective) and how the one refers to the inclinations. Consequently, shifting from the model used in the *Almagest* to the model used in the *Planetary Hypotheses* significantly simplifies how the planetary latitude theories are presented and calculated. Additionally, the positions and motions of the eccentric, epicyclic, and ecliptic planes are less complicated than in the *Almagest*, which results in a model that could more easily be represented in an instrument.

III.3 Mean Motions: Simple Periods

When presenting the mean motion period relations in the *Planetary Hypotheses*, Ptolemy states the number of revolutions that each body makes in both Egyptian years and solar years.[141] Ptolemy uses two different types of solar years – sidereal years and tropical years – and as a result, he uses in total three different lengths of the year: the Egyptian year, the tropical year and the sidereal year.

III.3.1 The Egyptian Year

The Egyptian year comes from the Egyptian calendar and it consists of 365 days. For Ptolemy's purposes, this means 365 mean *nychthemera.* The Egyptian year has a constant length; it is always made up of 12 months, each consisting of 30 days, plus an extra 5 days called epagomenal days.[142] Most ancient calendrical systems, such as the luni-solar calendar, resulted in an underlying parameter for the length of the year that was not constant. The Egyptian calendar was one of the few ancient time reckoning systems with an unvarying length and this made it well situated for computation. Ptolemy was not the first to use the Egyptian year for astronomical purposes and it continued to be used for calculating purposes in astronomy into the Renaissance.[143]

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[141] I should note that some lunar periods in the *Planetary Hypotheses* are presented in synodic months.
[142] A list of the names of Egyptian months can be found the *Almagest*. The calendar that Ptolemy uses was not the civil calendar in use in Egypt at the time. Toomer 1998, 9.
[143] Neugebauer 1975, 559-560.

III.3.2 The Solar Year

The solar year is described as the time it takes for the Sun to complete one revolution of the ecliptic. In the *Almagest* Ptolemy says that the ancients were in disagreement about the length of the solar year because:

> …when one examines the apparent returns [of the sun] to [the same] equinox or solstice, one finds that the length of the year exceeds 365 days by less than ¼-day, but when ones examines its return to [one of] the fixed stars it is greater [than 365¼ days].[144]

As a result, there are two types of solar years: tropical years and sidereal years. A tropical year is the length of time it takes the Sun to complete one revolution relative to one of the solsticial or equinoctial points. A sidereal year is the length of time it takes the Sun to complete one revolution relative to one of the fixed stars on the ecliptic. The discrepancy in the length of the solar year is what led Hipparchus to suspect that the sphere of the fixed stars had a slow motion in the trailing direction of the heavens, i.e. precession.[145]

There was not universal agreement among ancient astronomers that the length of the year was constant. For instance, Hipparchus had doubts as to whether the tropical year was constant, which Ptolemy discusses with great detail in Book 3.1 of the *Almagest*. Ptolemy believed that the length of the tropical year was invariable when measured relative to the same reference point, and he conjectures that Hipparchus's suspicion that the length of the tropical year is inconsistent is due to erroneous observations that he used.[146] After listing the observations Hipparchus used, Ptolemy

[144] Toomer 1998, 131.
[145] Toomer 1998, 131.
[146] Toomer 1998, 131-133.

states that Hipparchus himself did not believe that the observations provided enough support to validate his suspicion that the length of the year varied.[147]

Ptolemy calculates the precise length of the tropical year in Book III.1 of the *Almagest*. By comparing several observations of equinoxes and solstices, Ptolemy confirms that the length of the tropical year is constant. Ptolemy says that he will abandon all crudely made observations and that he will only use observations that Hipparchus believes were done accurately or observations that Ptolemy himself made on a meridian ring or a meridian plaque; however, Ptolemy in fact uses the early summer solstice observation made by Meton and Euktemon.[148]

Ptolemy determines the length of the tropical year using three sets of observations.[149] For all three examples, Ptolemy reaches the same value for the length of the tropical year. He concludes that the length of the tropical year is $365\frac{1}{4}$ days minus $\frac{1}{300}$ of a day. In Book III.1 he says:

> Thus I think it appears plainly from the agreement of present-day [observations] with earlier ones, that all phenomena observed up to the present time having to do with the length of the solar year accord with the abovementioned figure for the return to solstices or equinoxes. This being so, if we distribute the one day over

[147] Toomer 1998, 135.
[148] Toomer 1998, 137. Ptolemy describes both a meridian ring and a meridian plaque in Book I.12 of the *Almagest*. Toomer 1998, 61-63.
[149] Ptolemy compares the autumnal equinox that he observed in the 463rd year after the death of Alexander, on Athyr 9 (September 26, 139 A.D.), one hour after sunrise with the autumnal equinox observed by Hipparchus in the 178th year after the death of Alexander, on the third/fourth epagomenal (September 26/27, 147 B.C.), at midnight. Secondly, Ptolemy compares the spring equinox he observed in 463rd year after the death of Alexander, Mechir 27 (March 22, 140 A.D.), at dawn with the spring equinox Hipparchus observed in the 178th year after the death of Alexander, Pachon 7 (March 14, 146 B.C.), approximately one hour after noon. Finally, Ptolemy uses an early observation, even though it is reported to have been made somewhat inaccurately; he compares the summer solstice he observed in 463rd year from the death of Alexander, on Mesore 11/12 (June, 24/25, 140 A.D.), approximately two hours after midnight with the summer solstice Meton and Euktemon observed in the year Apseudes was archon at Athens, on Phamenoth 21 (June 27, 432 B.C.), at dawn. Ptolemy almost certainly uses Meton and Euktemon's summer solstice observation because it is an old observation relative to the others that he uses. Ptolemy says that using observations that have a large interval of time between them produces more accurate parameters because any error will be distributed over many years, making the inaccuracy relatively small.

> the 300 years, every year gets 12 seconds of a day. Subtracting these from the 365;15^{d} of the $\frac{1}{4}$-day increment, we get the required length of the year as 365;14,48^{d}. Such, then, is the closest possible approximation which we can derive from the available data.[150]

Relying on three different examples, Ptolemy arrives at the same value for the length of the tropical year as Hipparchus. [151] According to Ptolemy, this value for the length of the tropical year is the best estimate he can calculate, since he is limited by the observations available to him.

Ptolemy uses the length of the sidereal year when converting solar years into Egyptian years for the planets in the *Planetary Hypotheses*. While Ptolemy does not provide the length of the sidereal year in any of his works, this number can be derived through calculation. In the *Planetary Hypotheses* Ptolemy lists certain periods in both sidereal solar years and Egyptian years. Dividing the converted Egyptian years for each planet by the corresponding solar years produces the length of the sidereal year that Ptolemy uses for all of the planets. This value is exactly: 365;15,24,31,32,27,7 days[152] While Ptolemy uses the length of the sidereal year when converting solar years into Egyptian years for the planets, he uses the length of tropical year when converting solar years into Egyptian years for the Sun, Moon, and sphere of the fixed stars.

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[150] Toomer 1998, 139-140.
[151] It has long been suspected that many of the observations given by Ptolemy have been manipulated or fabricated. In his 1817 work, J. B. J. Delambre suggests that Ptolemy's solar data may have been calculated and not observed. Delambre [1817] 1965, xxvi. Delambre [1819] 1965, lxvii–lxx. Robert R. Newton argues that if Ptolemy had indeed used the instruments he describes in Book 1.12 of the *Almagest* when observing the equinoxes and solstices, and if he took careful observations, then his observations would not be as erroneous as they are. Newton suggests that Ptolemy started with the result he wanted and calculated backwards, fabricating his observations. Newton 1977, 87-94. John P. Britton provides a close analysis of many of Ptolemy's observations, including his solar observations. He concludes that the error in Ptolemy's data is not due to systematic observing and that Ptolemy altered his data. Britton 1992, 12–37. A thorough discussion of Ptolemy's calculation for the length of the tropical year can be found in Jones 2005b, 18–27.
[152] This number is found by dividing the number of days equivalent to the number of given Egyptian years by the number of sidereal years. There is a typo in the value that Neugebauer gives in *HAMA*, which is: 365;15,24,31,22,27,7. Neugebauer 1975, 901.

In Book III.1 of the *Almagest* Ptolemy states his preference for the tropical year over the sidereal year. He justifies his definition of the length of the year as the tropical year with the following:

> We must define the length of the year as the time the sun takes to travel from some fixed point on this circle back again to the same point. The only points which we can consider proper starting-points for the sun's revolution are those defined by the equinoxes and solstices on that circle. For if we consider the subject from a mathematical viewpoint, we will find no more appropriate way to define a 'revolution' than that which returns the sun to the same relative position, both in place and in time, whether one relates it to the [local] horizon, to the meridian, or to the length of the day and night; and the only starting-points on the ecliptic which we can find are those which happen to be defined by the equinoxes and solstices. And if, instead, we consider what is appropriate from a physical point of view, we will not find anything which could more reasonably be considered a 'revolution' than that which returns the sun to a similar atmospheric condition and the same season; and the only starting-points one could find [for this revolution] are those which are the principal means of marking off the seasons from one another [i.e. solsticial and equinoctial points]. One might add that it seems unnatural to define the sun's revolution by its return to [one of] the fixed stars, especially since the sphere of the fixed stars is observed to have a regular motion of its own towards the rear with respect to the [daily] motion of the heavens. For, this being the case, it would be equally appropriate to say that the length of the solar year is the time it takes the sun to go from one conjunction with Saturn, let us say, (or any other of the planets) to the next. In this way many different 'years' could be generated. For the above reasons we think it appropriate to define the solar year as the time from one equinox or solstice to the next of the same kind, as determined by observations taken at the greatest possible interval.[153]

Ptolemy gives both mathematical and physical reasons for using the equinoctial or solsticial points as reference points.[154] From a mathematical point of view, he says it is reasonable to define a revolution as the Sun returning to its same relative position in place and time. In addition, from a physical viewpoint, Ptolemy argues that a revolution

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[153] Toomer 1998, 132.

[154] I use "mathematics" and "physics" in the sense Ptolemy gives in Book I.1 of the *Almagest* where he says: "The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'." Toomer 1998, 36.

should be defined as the Sun returning to "a similar atmospheric condition and the same season".[155] Consequently, for both mathematical and physical reasons Ptolemy argues that the equinoctial or solsticial points should be used to as reference points for measuring the solar year. Furthermore, Ptolemy contends that it would be unnatural to use a star as a starting point when measuring a revolution of the Sun since the sphere of the fixed stars has its own motion.

In Book VII.1 of the *Almagest*, Ptolemy explains that the stars are fixed, so that they do not move relative to one another and they maintain their formations. The sphere of the fixed stars, however, has its own motion from west to east. Ptolemy says:

> But the sphere of the fixed stars also performs a motion of its own in the opposite direction to the revolution of the universe, that is, [the motion of] the great circle through both poles, that of the equator and that of the ecliptic. We can see this mainly from the fact that the same stars do not maintain the same distances with respect to the solsticial and equinoctial points in our times as they had in former times: rather, the distance [of a given star] towards the rear with respect to [one of] those points is found to be greater in proportion as the time [of observation] is later.[156]

Ptolemy says Hipparchus discusses this phenomenon in his work *On the Displacement of the Solsticial and Equinoctial Points*.[157] Unfortunately, Hipparchus's work is lost. In the *Almagest*, Ptolemy demonstrates the motion of the sphere of the fixed stars using only one example, in which he compares two observations. He shows that Regulus moved $2\frac{2}{3}$ degrees in the trailing direction of the cosmos in approximately 265 years and he concludes that in approximately 100 years the sphere of the fixed stars moved one-degree

[155] Toomer 1998, 132. This is not a straightforward criterion, since Ptolemy accepted the traditional belief that the rising and settings of the fixed stars influenced the weather. Riley 1995, 242. Neugebauer 1975, 926-931.
[156] Toomer 1998, 327.
[157] Toomer 1998, 327.

rearward.[158] Hipparchus, Ptolemy says, reached this same conclusion. Additionally, in Book VII.1-3 Ptolemy compares Hipparchus's stellar observations with his own, concluding that the sphere of the fixed stars has its own motion in the opposite direction of the daily rotation.[159]

Ptolemy describes the sphere of the fixed stars moving relative to the stationary tropical points, and not vice versa. He takes the solsticial and equinoctial points to be his reference points.[160] Hipparchus, on the other hand, seems to have attributed the motion to the tropical points, and not the sphere of the fixed stars. This is apparent from Hipparchus's quote from his work *On the Length of the Year*, which Ptolemy gives us in Book VII.2. Hipparchus says:

> For if the solstices and equinoxes were moving, from that cause, not less than $\frac{1}{100}$th of a degree in advance [i.e. in the reverse order] of the signs, in the 300 years they should have moved not less than 3°.[161]

Meton's observation of the solstice occurred in 432 B.C. and the time interval between Meton's observation and Hipparchus's time, is approximately 300 years.[162] Ptolemy's deduction, namely that the sphere of the fixed stars moves in the trailing direction relative to the tropical points at a rate of about one degree every 100 years, is in agreement with Hipparchus's findings.

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[158] The first observation that Ptolemy uses is of Regulus in the second year of Antoninus, on Pharmouthi 9 (February 23, 139 A.D.), at six equinoctial hours after noon. Ptolemy shows that here Regulus had a distance from the summer solstice of $32\frac{1}{2}$ degrees. Ptolemy compares this observation with an observation made by Hipparchus in the 50th year of the Third Kallippic Cycle (129/8 B.C.), where Regulus had a distance to the rear of the summer solstice of $29\frac{5}{6}$ degrees. Toomer 1998, 328.

[159] Toomer 1998, 321-329.

[160] Toomer 1998, 321 n. 2.

[161] Toomer 1998, 328.

[162] Toomer 1998, 328 n. 53.

III.3.3 Precession

In the *Almagest*, Ptolemy does not specify whether he is referring to 100 tropical years, 100 sidereal years or 100 calendar years when he assigns the rate of motion for the sphere of the fixed stars one degree every 100 years. Ptolemy offers only rounded values for precession and he does not provide a table of precession. Nonetheless, we can determine the type of year that Ptolemy uses since he calculates the interval of time between Hipparchus's observation of Regulus and his own to be 265 Egyptian years. Ptolemy says that Regulus moved $2\frac{2}{3}$ degrees in this time period, so that the precession of the fixed stars is one degree in approximately 100 Egyptian years.

Ptolemy presents the values for precession differently in the *Planetary Hypotheses* and the *Canobic Inscription*. In the *Canobic Inscription* Ptolemy gives a precise value for the mean daily motion of the sphere of the fixed stars, which is: $0;0,0,5,55,4,7^{\circ/d}$ [163] From this value we can determine the type of year Ptolemy uses for precession by retracing his steps. Since the sphere of the fixed stars moves one degree every one hundred years, if we divide one degree by the number of days that occur in 100 Egyptian years, we get the following value: $0;0,0,5,55,4,6,34\ldots^{\circ/d}$. Rounding this value we get: $0;0,0,5,55,4,7^{\circ/d}$, which is the same value Ptolemy gives us in the *Canobic Inscription*. Consequently, in the *Canobic Inscription* Ptolemy uses the length of the Egyptian year when determining precession, so that the sphere of the fixed stars moves one degree every 100 Egyptian years. This is the same rate of motion he gives in the *Almagest*.

[163] Jones 2005a, 70-71. Bullialdus corrected the last two digits from 37 to 4,6 and Heiberg corrected it to 4,7. Heiberg 1907, 150.

The value for the sidereal year that Ptolemy uses in the *Planetary Hypotheses*, namely 365;15,24,31,32,27,7 days, is precise enough to determine the type of year Ptolemy uses for precession.[164] By finding the daily mean motion of the Sun for the sidereal year and then subtracting it from the daily mean motion of the Sun for the tropical year, we can determine the daily mean motion of the sphere of the fixed stars. The Sun completes one revolution, i.e. 360 degrees, relative to the solsticial and equinoctial points in 365;14,48 days. Consequently, the mean daily motion of the Sun is: 0;59,8,17,13,12,30…$^{\circ/d}$. By dividing 360 degrees by the length of the sidereal year, namely 365;15,24,31,32,27,7 days, we can find the mean daily motion of the Sun for the sidereal year. We get the following value: 0;59,8,11,18,22,47,40…$^{\circ/d}$. Subtracting the daily mean motion of the sidereal year from the daily mean motion of the tropical year, gives us the daily mean motion of the sphere of the fixed stars, which is: 0;0,0,5,54,49,43,18,53…$^{\circ/d}$. This value, which is different from the value Ptolemy gives in the *Canobic Inscription,* represents the mean daily motion of the sphere of the fixed stars in the *Planetary Hypotheses*. We can find how Ptolemy obtained this value by using the length of the tropical year for the time it takes the sphere of the fixed stars to move. Since the sphere of the fixed stars moves one degree every one hundred years, if we divide one degree by the number of days that occur in 100 tropical years, we get the following value:

$$\frac{1^\circ}{(100)(365;14,48^{d})} = 0;0,0,5,54,49,43,19,15\ldots^{\circ/d}$$

[164] This value is discussed above in Section III.2.2.

This number is very similar to the value that we get by subtracting the mean daily length of the tropical year from the mean daily length of the sidereal year; the difference is 0;0,0,0,0,0,0,0,22$^{o/d}$. The discrepancy is due to rounding.

In the *Almagest* and *Canobic Inscription* Ptolemy assigns the sphere of the fixed stars a rate of motion of one degree every 100 Egyptian years; however, in the *Planetary Hypotheses*, Ptolemy calculates the rate of motion for the sphere of the fixed stars as one degree every 100 tropical years. Ptolemy does not discuss why he uses a different type of year nor does he point out this modification. We should, however, keep in mind that Ptolemy gives the value as a rounded number in the *Almagest*, after all he says the rearward motion of one degree occurs in "approximately 100 years".[165] Additionally, the difference between the precessional value given in the *Almagest* and *Canobic Inscription*, versus the one given in the *Planetary Hypotheses* is extremely small. It is indeed too small for Ptolemy to compute from the observations he was using. Ptolemy did not justify the modifications he made to the rate of precessional motion, either dialectically or numerically. The change in the rate of motion may have been an oversight, either because Ptolemy realized that the difference was too small to observe, or he simply forgot which type of year he used to make the calculations for precession. Alternatively, since the difference would have been imperceptible, Ptolemy may have simply made the change without drawing attention to it.

[165] Toomer 1998, 328.

III.3.4 Apogees of the Planets

Ptolemy states that the apogees of the five planets exhibit the same motion as the sphere of the fixed stars, each moving about one degree every 100 years.[166] He bases his theory of precessional motion of the planets' apogees on observations of Mercury. In Book IX.7 of the *Almagest*, he determines the location of the apsidal line for Mercury. Ptolemy uses two pairs of observations of greatest elongations of Mercury as morning star and evening star, both separated by an equal interval of time. The apogee of the eccentricity can then be identified as the point halfway between these points. From these observations, he calculates that, in his own time, the apsidal line of Mercury is located at 10 degrees Aries or Libra.[167] Since Ptolemy had access to six ancient observations for Mercury, all from the third-century B.C., he was able to use these observations to determine the previous location of Mercury's apsidal line. From these observations, Ptolemy finds that the apsidal line traveled to six degrees Libra.[168] Accordingly, in the past four hundred years, Mercury's apsidal line has traveled four degrees rearward at approximately one degree a century. Ptolemy says:

> From the above, and also because the phenomena associated with the other planets individually fit [the assumption], we find it consistent [with the facts to assume] that the diameters through the apogees and perigees of the five planets shift about the centre of the ecliptic towards the rear through the signs, and that this shift has the same speed as that of the sphere of the fixed stars. For the latter moves about 1° in 100 years, as we demonstrated; and here too the interval from the ancient observations, in which the apogee of Mercury was in about the 6th degree [of the signs in question], to the time of our observations, during which it has moved about 4° (since it [now] occupies the 10th degree), is found to comprise approximately 400 years.[169]

[166] Toomer 1998, 328.
[167] Neugebauer 1975, 159.
[168] Toomer 1998, 449-453.
[169] Toomer 1998, 453.

When Ptolemy's observations and calculations are examined closely, it appears that Ptolemy altered his data since some observations are misdated by at least one day and the calculations are imprecise.[170] Without further demonstration, Ptolemy concludes that the apsidal line for Mercury and the apsidal lines for the other four planets have a slow shift that corresponds to the precessional motion of the fixed stars.[171] Using the most complex of all of the planets, Mercury, Ptolemy freely applies his results for Mercury's apsidal line to the other planets. As Alexander Jones says: "It is one thing to fiddle with one's data, or the analysis of it, so as to make a plausible result come out more cleanly and unambiguously. It is quite another to fiddle with a second analysis of a second set of data so as to demonstrate a small, slow effect that could never be detected from the unmanipulated data."[172] Jones also argues that Ptolemy is being conservative since the Babylonian planetary theories assumed that the planets' zodiacal anomaly is sidereally fixed. While Ptolemy is committed to constructing models that best represent the phenomena, he is, however, willing to alter data and manipulate calculations in order to achieve round numbers and mathematically unambiguous results.

[170] For a discussion of Ptolemy's methodology in determining the position of the apsidal line for the planets see Sawyer 1977, 169-181. Swerdlow suggests that Ptolemy preformed preliminary calculations for Mercury's model and then selected observations and constructed a demonstration so that his results confirm what he had found in his preliminary assessment. Swerdlow 1989, 29-60. A nice discussion of Ptolemy's calculation for the apsidal line of Mercury can be found Jones 2005b, 27-30.

[171] For the outer planets, Ptolemy uses one observation per planet from the third-century B.C. in refining his values for the mean motions of the outer planets. Neugebauer 1975, 160.

[172] Jones 2005b, 27.

III.3.5 Simple and Unmixed Periods

After discussing the layout of the spheres and the definition of a *nychthemeron*, Ptolemy presented the mean motion parameters for the Sun, Moon, planets, and fixed stars. He states:

> Now that these have been sketched out, next let us look at the models of the planets, first setting out their simple and unmixed periods (τὰς ἁπλᾶς καὶ ἀμιγεῖς αὐτῶν περιόδους), from which particular, complex ones arise (αἱ κατὰ μέρος καὶ ποικίλαι συνίστανται). These periods were obtained by us as approximations from the restitutions calculated by the corrections (διορθώσεως).[173]

The "simple and unmixed periods" (τὰς ἁπλᾶς καὶ ἀμιγεῖς αὐτῶν περιόδους) refer to the period relations of: the fixed stars and planetary apogees, the Sun, the Moon, and the planets with respect to the Sun. These periods come from what Ptolemy called the "approximations from the restitutions calculated by the corrections". The "corrections" are small adjustments based on observations that have been made to the period relations, resulting in allegedly more accurate period relations.[174]

Ptolemy presented the mean motion of a celestial body in two different ways throughout his astronomical works. The first was as period relations, so that some phenomenon, such as distance traveled in degrees or returns in position is made relative to a second phenomenon, such as days, months, or years.[175] The second way that Ptolemy expressed mean motion was as a rate of mean motion, so that the number of degrees traveled is made relative to a specific quantity of time, for example, one *nychthemeron*, one month, or one tropical year.

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[173] Heiberg 1907, 76.
[174] Duke 2009, 636. James Evans provides a brief, but clear discussion of how period relations can be constructed. Evans 1998, 304-305.
[175] Period relations were used regularly in Babylonian astronomy and they can be found in MUL.APIN. Britton 2002, 21-78

In the *Planetary Hypotheses* Ptolemy presented all mean motion periods as period relations. The numbers of years and the number of restitutions (ἀποκαταστάσεις) were not necessarily integers; for example, Saturn makes 313 restitutions in anomaly (ἀνωμαλίας ἀποκαταστάσεις) in 324 Egyptian years and 83;12,26,19,14,25,48 *nychthemera*.[176] In the *Almagest* Ptolemy initially relies on period relations that were passed down; however, the final mean motion periods that he calculated were presented as a rate of mean motion. For example, in the *Almagest* Saturn's rate of mean motion in anomaly is 0;57,7,43,41,43,40 degrees per day.

It is easier to compare calculated values for periods when they are presented as rates of mean motions per day. For instance, a reader can quickly determine that Saturn's mean motion in anomaly, 0;57,7,43,41,43,40 degrees per day, is more rapid than Jupiter's, which is 0;54,9,2,46,26,0 degrees per day.[177] This information does not come across as clearly when comparing period relations, such as 324 Egyptian years and 83;12,26,19,14,25,48 *nychthemera* to 313 restitutions in anomaly for Saturn and 771 Egyptian years and 198;0,9,18,0,26,57 *nychthemera* to 706 restitutions in anomaly for Jupiter.[178] Additionally, acquiring a period relation that is precise can be challenging, since it requires, for an outer planet for example, two observations that are taken years apart where the planet is observed to be at nearly the same longitude and in the same location with relation to the Sun, e.g. at opposition. The period relation is therefore the ratio between the number of years and revolutions made during this interval of time. Corrections can be made to an existing period relation by combining period relations or by using continuous fraction expansions, so that a new period relation is acquired through

[176] Heiberg 1907, 102-104.
[177] Toomer 1998, 424.
[178] Heiberg 1907, 98-104.

calculations. This seems to be the case for the period relations found in the *Planetary Hypotheses*.

Moving from period relations, like the ones found in the *Planetary Hypotheses*, to rates of mean motion for a set amount of time, such as the ones found in the *Almagest*, can be a simple task. In the *Planetary Hypotheses,* however, Ptolemy claimed he made the reverse conversion, starting with the rates of mean motion for a given period calculated in the *Almagest* and converting those values into period relations for the *Planetary Hypotheses*. Although Ptolemy explained that the parameters he presented in the *Planetary Hypotheses* were based on the parameters found in the *Almagest,* it is not evident how Ptolemy would have made these calculations. There is, in fact, a straightforward way of moving from a rate of mean motion in a specific period of time to a period relation by converting the sexagesimal number into a fraction, so that the numerator and denominator represent the period relation. While this produces a period relation that is exactly equivalent to the rate of mean motion per day, it usually results in numbers that are too large to be useful. For example, the period relation for Saturn's mean motion in anomaly based on parameters in the *Almagest* would result with the following period relation: 2,221,167,911 degrees to 2,332,800,000 days.[179] The period relations in the *Planetary Hypotheses*, which consists of relatively small values, were not found using this method. How then, did Ptolemy find good approximate representations as period relations consisting of smaller numbers?

[179] In the *Almagest* Ptolemy gives the mean motion in anomaly parameter as 0;57,7,43,41,43,40 degrees per day, which can be expressed as, 44,423,358,220 degrees to 46,656,000,000 days and then reduced. Converting these values to revolutions and years respectively would further reduce the numbers. Toomer 1998, 424.

The origins of the period relations that Ptolemy gives in the *Planetary Hypotheses* are not apparent; however, relying on scholarship by Lis Brack-Bernsen, John P. Britton, and Bernard Goldstein, Dennis Duke has found that Ptolemy worked from older known period relations, combining them along with small adjustments.[180] Duke shows that rates of mean motions can be used to construct period relations that represent good approximations of the original mean motion rates. Assuming that Ptolemy started with the rates of mean motion from the *Almagest*, which is what he said he did, then Ptolemy was trying to find a period relation that best represented the given rates of mean motion. This can be done using; (1) a rate of mean motion; and (2) two period relations, one which results in a rate of mean motion that is larger than (1), i.e. (2a), and the other which results in a rate of mean motion that is smaller than (1), i.e. (2b). In the equation below, $\frac{a}{b}$ represents the upper period relation (2a) (2b), and ω represents the corrected rate of mean motion (1), so that:[181]

$$\frac{a}{b} < \omega < \frac{c}{d}$$

A good period relation approximation can be found for ω using the following equation:

$$\frac{a}{b} < \frac{ma + nc}{mb + nd} < \frac{c}{d}$$

[180] Lis Brack-Bernsen demonstrates that Babylonian values for column Φ in System A could have been constructed from Babylonian observations (1993), and how lunar observations were used to predict specific time intervals, known as the "Lunar Six", around lunar opposition and conjunction (1999). Brack-Bernsen. 1993 and 1999. John P. Britton examines the lunar theory in System A, demonstrating how period relations can be used to construct other period relations. Britton 1999. Relying on the work of Britton and Brack-Bernsen, Goldstein provides a detailed discussion of how the Babylonians may have constructed accurate period relations for lunar motions. Goldstein. 2002. Duke uses a similar methodology to the above works when examining the period relations presented in the *Planetary Hypotheses*. Duke 2009, 635-654.
[181] I use Duke's notation here and below.

The period relations $\frac{a}{b}$ and $\frac{c}{d}$ can be converted into a rate of mean motion. The difference between *ω* and each period relation as a rate of mean motion can then be used to make an approximate period relation.[182] Another option is to approximate using continued fractions, a method that stems from the Euclid's algorithm presented in Book VII and Book X of the *Elements*.

While it is possible to reconstruct the combinations of period relations used in the *Planetary Hypotheses*, not all of the changes that Ptolemy makes are for clear reasons nor or all of them improvements over the parameters in the *Almagest*, as I will discuss in more detail below. I will mostly follow what Duke has demonstrated, adding further analysis and hypothesizing on the origin of the mean motion periods Ptolemy presents in the *Planetary Hypotheses*.

III.3.6 Solar and Lunar Period Relations

The period relations for the Sun in the *Planetary Hypotheses* agree with the values found in the *Almagest*. Ptolemy uses the length of the tropical year to convert solar years into Egyptian years. In the *Planetary Hypotheses* he says that in 300 Egyptian years and 74 days the Sun makes 300 revolutions. This means that the Sun makes 300 revolutions in 109,574 Egyptian days, i.e. one revolution in 365;14,48 Egyptian days, which is the length of the tropical year Ptolemy gives in the *Almagest*. This is not the smallest number of revolutions that gives a whole number of days, since 150 revolutions would occur in 54,787 days. Ptolemy most likely arrived at this period relation by

[182] For an example see Duke 2009, 638-640.

multiplying the length of the tropical year, namely $365\frac{1}{4}$ minus $\frac{1}{300}$ days, by 300. In doing so, he gets rid of the fractions and ends with a period relation consisting of two integers.

The apogees of the five planets and the sphere of the fixed stars move at the same rate: $\frac{1}{120}$ of one revolution in 300 tropical years, which is equal to one degree in 100 tropical years. This is different from the precession value Ptolemy gives in the *Canobic Inscription*, which is one degree every 100 Egyptian years.[183] The sphere of the fixed stars and the apogees of the five planets each complete one revolution in 36,000 tropical years. This is equal to 36,024 Egyptian years and 120 days, which is also equal to 35,999 sidereal years. For the solar period relations Ptolemy does not make any adjustments to these values when compared to the parameters given in the *Almagest*.

For the Moon, Ptolemy discusses three different types of periodicities: restitutions in longitude, restitution in anomaly, and restitutions in latitude. Since it is necessary to know the period of anomaly in order to determine the other periods of the Moon, Ptolemy says that the ancient astronomers tried to find a period for which the Moon's motion in longitude would be constant. They believed that only such a period could produce a return in anomaly. Ptolemy noted the estimate made by "the even more ancient [astronomers]" of a period of 223 restitutions in longitude, 239 restitutions in anomaly, and 242 returns in latitude in $6585\frac{1}{3}$.[184] Ptolemy calls this interval a 'periodic'. In modern scholarship it is often referred to as a 'saros cycle'.[185]

According to Ptolemy, Hipparchus proved that the saros cycle estimates were erroneous and consequently Hipparchus gave different parameters for the smallest

[183] See Section III.2 above.
[184] Toomer 1998, 175.
[185] Neugebauer shows that the name saros cycle is not ancient, but instead a misnomer that originated with Edmond Halley in 1691. Neugebauer 1975, 497 n. 2.

interval of time in which a whole number, or nearly a whole number, of synodic months, anomalistic months and longitudinal revolutions are completed. The interval he found consisted of 4,267 synodic months, 4,573 returns in anomaly and 4,612 revolutions on the ecliptic less about $7\frac{1}{2}$ degrees, in 126,007 days and 1 equinoctial hour. Ptolemy stated that the ratio of synodic months to anomalistic months could be reduced to 251 to 269. The above interval does not contain returns in latitude; however, Hipparchus found the period relation of 5,458 mean synodic months to 5,923 draconitic months.[186] Ptolemy reported the length of one mean synodic month as 29;31,50,8,20 days. Ptolemy said that Hipparchus derived this number by dividing 126,007 days and 1 equinoctial hour by 4,267 synodic months; however, performing this calculation produces the number 29;31,50,8,9,20,12. The length of the synodic month that Ptolemy reports is a Babylonian value with its roots in System B.[187] Subsequently, Ptolemy made corrections to the parameters for the mean motion in latitude and the mean motion in anomaly in Book IV.3.[188]

In the *Planetary Hypotheses* Ptolemy said that in 8,523 tropical years, or in 8,528 Egyptian years and 277;20,24 days, the Moon completes 105,416 complete months, also known as synodic months. A synodic month represents the average amount of time between one conjunction, or opposition, with the Sun and a conjunction of a syzygy of the same type, i.e. conjunction to conjunction, or opposition to opposition. From the

[186] Toomer 1998, 175-179. Kugler was the first to point out that these values come from Babylonian cuneiform tablets. Kugler 1900, 4-46. Asger Aaboe shows that Hipparchus manipulated these values, which have origins in the zigzag function representing lunar velocity, through calculations to produce an eclipse period. Aaboe 1955. G.J. Toomer provides a clear explanation of Hipparchus' procedure and rationale. Toomer 1980, 97-109.
[187] al-Ḥajjāj b. Maṭar noticed that the value Ptolemy gives for the length of the synodic month (29;31,50,8,20 days) and the quotient of the calculation (29;31,50,8,9,20,12 days) were different; he included the corrected parameter (29;31,50,8,9,20,12 days) in his Arabic translation of the *Almagest*. Saliba 2007, 79-80 n. 12. Copernicus also noticed the discrepancy in the lunar value. Aaboe 1955, 123.
[188] Toomer 1998, 179.

parameters above, we can determine the length of one mean synodic month in the *Planetary Hypotheses,* which is 29;31,50,8,48,6,7… ^{d}. This exact value does not appear anywhere else.[189]

Duke shows that the period relation of 105,416 synodic months to 8,523 tropical years found in the *Planetary Hypotheses* may have been built from known period relations, namely the relationship of 235 synodic months to 19 years and the period relation of 136 synodic months to 11 tropical years.[190] It can be broken in the following way:

$$\frac{105416}{8523} = \frac{(448)(235) + 136}{(448)(19) + 11}$$

If Ptolemy found the period relation of 105,416 synodic months to 8,523 tropical years by starting with the mean length of the synodic month, then he used neither the value for the synodic month that can be calculated from the saros cycle, nor the value he gives in the *Almagest*, namely 29;31,50,8,9^{d} and 29;31,50,20^{d} respectively. The value Ptolemy used ranged between 29;31,50,8,37^{d} to 29;31,50,52^{d}.[191]

The second type of lunar period that Ptolemy discusses is restitution in anomaly, also known as an anomalistic month. An anomalistic month refers to the interval of time it takes the Moon to complete one revolution around its epicycle. In the *Planetary Hypotheses* Ptolemy says that in 3,277 synodic months the Moon makes 3,512 restitutions in anomaly. This period relation is unattested elsewhere.[192] Expressing this period relation as one value gives the following:

189 Neugebauer 1975, 902.
190 Duke 2009, 638.
191 Duke 2009, 638-640.
192 Neugebauer 1975, 902.

1 anomalistic month = 0;55,59,6,41,49,20… synodic months

In the *Almagest* Ptolemy provides the period relation from Hipparchus, which is:

251 synodic months = 269 anomalistic months
or
1 anomalistic month = 0;55,59,6,28,6,14,43… synodic months

The period relation of 251 synodic months to 269 anomalistic months is not the final ratio Ptolemy gives in the *Almagest*. By multiplying the number of anomalistic months by 360 and dividing by the number of synodic months multiplied by the length of the synodic month, Ptolemy determined the mean daily motion in anomaly for the Moon.[193] He calculated the following:

$$\frac{(269)(360^\circ)}{(251)(29;31,50,8,20^{d})} = 13;3,53,56,29,38,38^{\circ/d}$$

Ptolemy found this number to be unsatisfactory. In Book IV.7 he corrected this value before constructing the mean motion tables in Book IV.[194] Ptolemy obtained the value of $13;3,53,56,17,51,59^{\circ/d}$ for the Moon's daily mean motion in anomaly.

Using the same procedure Ptolemy employed, but with the parameters from the *Planetary Hypotheses*, we can determine the Moon's mean motion in anomaly:

$$\frac{(3512)(360^\circ)}{(3277)(29;31,50,8,48,6,7^{d})} = 13;3,53,53,5,7,46,36,14\ldots^{\circ/d}$$

[193] Toomer 1998, 179.
[194] Toomer 1998, 204.

Between the corrected value from the *Almagest*, and the value in the *Planetary Hypotheses*, there is a difference of 0;0,0,3,12,44,12,23,46$^{\circ/d}$.[195] The difference between the uncorrected value in the *Almagest,* namely the daily mean motion in anomaly of the Moon found from the period relation of 269 anomalistic months to 251 synodic months, and the daily mean motion in anomaly of the Moon in the *Planetary Hypotheses* is 0;0,0,0,11,46,39$^{\circ/d}$. The period relation of 3,277 synodic months to 3,512 restitutions in anomaly can be broken down in the following way:

$$\frac{3512}{3277} = \frac{(13)(269) + 15}{(13)(251) + 14}$$

Ptolemy is starting with the uncorrected ratio of synodic months to restitutions in anomaly, namely 269 to 251. The period relation of 3,512 synodic months to 3,277 restitutions in anomaly that Ptolemy used is a worse approximation to the value given in the *Almagest* than the period relation 269 to 251 (the uncorrected period relation Ptolemy built from in the *Almagest*).[196] This suggests that Ptolemy was not working from the parameters found in the *Almagest*.

The final type of lunar period that Ptolemy referred to was a restitution in latitude, i.e. a draconitic month. A draconitic month refers to the interval of time between the Moon's return to the same node. In the *Planetary Hypotheses* Ptolemy said that in 5,458 synodic months the Moon makes 5,923 restitutions in latitude. By multiplying the returns in latitude by 360 degrees and dividing this value by the number of days in 5,458 synodic months, we get the mean daily motion of the Moon in latitude, which is:

[195] In the *Canobic Inscription* Ptolemy gives the value for the anomalistic month, which is: 13;13,53,56,17,51,59$^{\circ/d}$. This value is larger than the value Ptolemy gives in either the *Almagest* or the *Planetary Hypotheses*. Heiberg 1907, 151.
[196] Duke 2009, 640-641.

$$\frac{(5923)(360°)}{(5458)(29;31,50,8,20^{d})} = 13;13,45,39,40,17,19^{°/d}$$

However, using “more elegant methods”, Ptolemy corrected this value in Book IV.9 of the *Almagest*. Hamilton, Swerdlow, and Toomer have demonstrated that Ptolemy’s use of “more elegant methods” is a reference to an erroneous procedure he used in the *Canobic Inscription.* Ptolemy is making corrections to both Hipparchus’s work and his own work in this passage.[197] Ptolemy showed that Hipparchus’s solution for the Moon’s motion in latitude is in error. [198] Consequently, he gave the corrected mean daily motion of the Moon in latitude as: 13;13,45,39,48,56,37°. Ptolemy’s mean motion tables are based on this new, corrected value. Using Ptolemy’s method for determining the daily mean motion of the Moon in latitude, we can find this parameter using the data from the *Planetary Hypotheses*. Since the period relation is identical, this value can be found by replacing the value for the mean synodic month found in the *Almagest*, with the one found in the *Planetary Hypotheses*:

$$\frac{(5923)(360°)}{(5458)(\ 29;31,50,8,48,6,7^{d})} = 13;13,45,39,27,41,57,7,26,47,50^{°/d}$$

The difference between the values for the mean daily motion of the Moon in latitude in the *Almagest* and *Planetary Hypotheses* is: 0;0,0,0,21,14,39,52,33,12,10.[199] The ratio of 5,923 synodic months to 5,458 restitutions in anomaly can be broken down in the following way:

$$\frac{5923}{5458} = \frac{(7)(777) + (2)(242)}{(7)(716) + (2)(223)}$$

[197] Hamilton, Swerdlow, Toomer 1987, 57-60.
[198] Toomer 1998, 205.
[199] In addition to correcting this value in the *Almagest*, Ptolemy also corrects it in the *Canobic Inscription*. In the *Canobic Inscription* Ptolemy gives the value of 0;3,0,41,48,20,51 for the “node of the moon towards the leading signs”.

Ptolemy began with the ratio of 242 draconitic months to 223 synodic months. The eclipses studied by Hipparchus imply the ratio of 777 to 716.[200] Unlike what he did for the restitutions in longitude and restitutions in anomaly, Ptolemy built from the numbers found in the *Almagest* for the restitutions in latitude; however, he did not include the corrections he made in the *Almagest*.

III.3.7 Planetary Period Relations

Before looking at the period relations in the *Planetary Hypotheses*, let us examine the mean motion periods in the *Almagest*. In the *Almagest*, Ptolemy provided two different methods for how he obtained the period relations; (1) by comparing an ancient observation with an observation that Ptolemy made himself;[201] and, (2) from well-known Babylonian period relations and corrections.[202] For the first method, the observations that Ptolemy compares do not result in the rates of mean motion that he presents in Book IX.3.[203] Dennis Duke and Alexander Jones demonstrate in "Ptolemy's Planetary Mean Motions Revisited" that Ptolemy does indeed use the second method and that his mean motions are derived from the corrected Babylonian period relations that he presents in

[200] Toomer 1998, 309.
[201] In Book IX.10 of the *Almagest* Ptolemy compares two observations for Mercury that are over 400 years apart. He does the same for Venus in Book X.4, Mars in Book X.9, Jupiter in Book XI.3, and finally Saturn in Book XI.7. Toomer 1998, 461-467, 474-479, 502-504, 522-525.
[202] While it seems that Ptolemy describes these period relations as parameters computed by Hipparchus, they are in fact well-known period relations that appear in the Babylonian Goal-Year Texts. Toomer 1998, 423-424 n. 19. Neugebauer 1975, 554.
[203] Toomer 1998, 423-424. Neugebauer and Newton independently point out this discrepancy. Neugebauer acknowledges the difference, while Newton suggests that the mean motion periods that Ptolemy presents in Book IX.3 were from an unknown source and not derived from the observations Ptolemy presents. Neugebauer 1975, 151, 157, 167-168, 182. Newton 1977, 320-321, 325-327. Toomer also discusses this discrepancy in Appendix C of his translation of the *Almagest*, concluding that Ptolemy's mean motions were based on period relations unknown to us. Toomer 1998, 669-672. Rawlins contends that Ptolemy's period relations originate with period relations determined by his predecessors. Rawlins 1987, 237-239.

Book IX.3. Ptolemy claims that these corrections came from observation comparisons, and while this may be true for some cases, it is not true for Mercury.[204]

Turning to the *Planetary Hypotheses*, when presenting the mean motion period relations Ptolemy used two different types of solar years: tropical solar year's and sidereal solar years. He used tropical years for the Sun and Moon and sidereal years for the planets. Conversely, in the *Almagest* Ptolemy only used tropical years for all of the celestial bodies. Ptolemy gave the number of restitutions in anomaly for each planet in both sidereal solar years and Egyptian years. These values represent the anomalistic motion of the planet only; the motion of the epicycle around the eccentric point for each planet is given in conjunction with the anomalistic motion when Ptolemy discusses the complex periods of each planet. By dividing the total number of degrees by the number of Egyptian days,[205] we can find the daily mean motion in anomaly for each planet. These values are listed below:

Table 4: Daily Mean Motion in Anomaly

	Days	Degrees	Mean Daily Motion
Mercury	362700;0,54,0,4,46,51^{d}	1126800°	3;6,24,7,7,7,8,12,56,50…$^{o/d}$
Venus	352107;34,2,45,23,40,28^{d}	217080°	0;36,59,27,29,14,8…$^{o/d}$
Mars	368909;22,50,56,16,27,50^{d}	170280°	0;27,41,40,34,55,28…$^{o/d}$
Jupiter	281613;0,9,18,0,26,57^{d}	254160°	0;54,9,3,16,17,8…$^{o/d}$
Saturn	118343;12,26,19,14,25,48^{d}	112680°	0;57,7,43,30,54,14…$^{o/d}$

These parameters do not appear in this form in any other source.[206] However, they are similar to the values found in the *Almagest* and *Canobic Inscription*.[207] The values for the

[204] Jones and Duke show that Ptolemy's mean daily motions were derived from the period relations, but for reasons that are unclear, the method of derivation was different for Mercury, Venus, and Saturn than it was for Mars and Jupiter. Jones and Duke 2005, 229-231.
[205] The number of days has been calculated by multiplying Ptolemy's value for the length of the sidereal year in days by the number of sidereal years.
[206] Neugebauer 1975, 906.

daily mean motion for the *Planetary Hypotheses*, *Almagest,* and *Canobic Inscription* are as follows:

Table 5: Comparison of the Daily Mean Motion in Anomaly

	Planetary Hypotheses	*Almagest* and *Canobic Inscription*
Mercury	3;6,24,7,7,7,8,12,56,50…$^{\circ/d}$	3;6,24,6,59,35,50$^{\circ/d}$
Venus	0;36,59,27,29,14,8…$^{\circ/d}$	0;36,59,25,53,11,28$^{\circ/d}$
Mars	0;27,41,40,34,55,28…$^{\circ/d}$	0;27,41,40,19,20,58$^{\circ/d}$
Jupiter	0;54,9,3,16,17,8…$^{\circ/d}$	0;54,9,2,46,26,0$^{\circ/d}$
Saturn	0;57,7,43,30,54,14…$^{\circ/d}$	0;57,7,43,41,43,40$^{\circ/d}$

Although the values we see in the *Planetary Hypotheses* and those presented in the *Almagest* and *Canobic Inscription* appear similar, Duke has shown that Ptolemy did not use the values in the *Almagest* as a starting point for calculating the period relations given in the *Planetary Hypotheses*.[208] The period relations that Ptolemy gives in the *Planetary Hypotheses* can be broken down into following well-known period relations:

Table 6: Period Relations in the *Planetary Hypotheses*

	Period Relation	Breakdown
Saturn	$\frac{313}{324}$	$\frac{(5)(57)+28}{(5)(59)+29}$
Jupiter	$\frac{706}{771}$	$\frac{391+315}{427+344}$
Mars	$\frac{473}{1010}$	$\frac{303+170}{647+363}$
Venus	$\frac{603}{964}$	$\frac{(3)(152)+(29)(5)+2}{(3)(243)+(29)(8)+3}$ or $\frac{(4)(152)-5}{(4)(243)-8}$
Mercury	$\frac{3130}{993}$	$\frac{(2)(1223)+684}{(2)(388)+217}$

[207] Toomer 1998, 424-425. Heiberg 1907, 150-151.
[208] Duke 2009, 640-643.

The period relations Ptolemy gives in the *Planetary Hypotheses* are sound approximations to the mean motion values he gives in the *Almagest* for Saturn and Mercury only. For Venus, Mars, and Jupiter, there exist other period relations that would better represent the values in the *Almagest* (namely, 152/243; 303/647; and 391/427, respectively).[209]

III.3.8 Origins of the Period Relations in the *Planetary Hypotheses*

Ptolemy's values for the Sun and sphere of the fixed stars appear to come directly from the values found in the *Almagest*; however, the simple period relations that Ptolemy gives in the *Planetary Hypotheses* for the Moon and Venus, Mars and Jupiter do not incorporate the corrections that Ptolemy made in the *Almagest*.[210] The Babylonian period relations that are well known to us may have been starting points for Ptolemy, but if he was working from period relations to period relations, then either he was using period relations or a method unknown to us. Furthermore, if Ptolemy was working form rates of mean motions to period relations, what is the benefit of constructing a period relation? Moving from a rate of mean motion to a period relation would only result in an approximate value for the original mean motion rate (assuming relatively small numbers, like the ones found in the *Planetary Hypotheses*, were the objective).

Yet, Ptolemy may have had reasons for using period relations in the *Planetary Hypotheses*. Period relations were used consistently throughout Babylonian astronomy and were the conventional way of expressing relationships between two different phenomena in ancient astronomy. Consequently, Ptolemy may have been conforming to

[209] Duke 2009, 640-643.
[210] Duke 2009, 642-643.

the standard methods of representing periodic parameters. Moreover, Ptolemy may have been motivated by mechanical reasons to use period relations, since the period relations could have been used as a basis to construct ratios for mechanical gear work.[211]

[211] See Section III.1 above.

III.4 Mean Motion: Complex Periods

Ptolemy says that he will first set out the “simple and unmixed periods, from which particular complex ones will arise”. The simple periods, which I discussed in the pervious section, refer to the period relations that Ptolemy gives early in Book I, Part A where he gives period relations consisting of the years and returns in anomaly. The complex periods, on the other hand, refer to the period relations Ptolemy gives in the remainder of Book I, Part A when working through each particular model in more detail.

The particular complex periods offer approximate linear combinations of the simple period relations. For the moon, the complex periods refer to the rate of mean motion of the lunar node and the rate of mean motion of the lunar eccentric circle. The complex motions represent, for the outer planets, the mean motion of the epicycle around the eccentric circle. Alternatively, for the inner planets, they represent the mean motion of the planet itself around the epicycle.

Beginning with the Sun, Ptolemy examined the models of each celestial body with closer detail. Ptolemy stated that the Sun made 150 restitutions in anomaly relative to the equinox in 150 Egyptian years and 37 days, so that the Sun made one restitution in anomaly every 365;14,48 Egyptian days. The ratio of 150 Egyptian years and 37 days to 150 restitutions in anomaly is the same ratio of days to restitutions in anomaly that Ptolemy used when he discussed the simple periods; however, here it appears in its reduced form. Ptolemy previously stated the ratio as doubled (i.e. 300 Egyptian years and 74 days to 300 restitutions in anomaly).[212]

[212] Heiberg 1907, 76-78.

For the Moon, Ptolemy presents simple, unmixed lunar period relations from which he constructs the complex periods. The simple period relations are: 105,416 restitutions in elongation in 8,523 tropical years, 3,512 restitutions in anomaly in 3,277 synodic months, and 5,923 restitutions in latitude in 5,458 synodic months. The complex lunar motions that Ptolemy gives are:

Table 7: Lunar Complex Period Relations

	Egyptian Years	Returns
Lunar Node	37 years and 88 days	2 returns plus $\frac{1}{60}$°
Center of the Lunar Eccenter	17 years and 348 days	203 returns minus $\frac{2}{60}$°
Lunar Elongation	19 years and 300 days	490 returns plus $\frac{4}{60}$°
Lunar Anomaly	26 years and 99 days	348 returns minus $\frac{1}{60}$°

Ptolemy explains which simple motions will be combined in order to construct the complex periods for the Moon. In reference to the lunar node Ptolemy says:

> Concerning the lunar sphere, let there be imagined again a circle concentric with the zodiac circle moving in its plane and around the same center with uniform speed (ἰσοταχῶς) from east to west, [and with its motion being] the excess by which the course in latitude projected on the zodiac exceeds the motion of both the Sun and the elongations in equal time, so that in 88 complete *nychthemera* plus 37 Egyptian years, it makes approximately two restitutions of the zodiac, for it takes up 1 sixtieth of a degree more in precise computation.[213]

Relying on these instructions along with the simple, unmixed periods, namely the lunar mean motion in elongation with respect to the equinox (η), the lunar mean motion in anomaly (α), the rate of change of the lunar argument of latitude (ω'), and the solar mean motion with respect to the equinox (ω_s), the mean motion of the lunar node (ω_n) can be reconstructed according to the following equation:

$$\omega_s + \eta - \omega' = \omega_n$$

[213] Heiberg 1907, 80-82.

This motion represents the mean motion of the lunar node with respect to the equinox, so that the lunar node makes 2 returns plus $\frac{1}{60}$° in 37 tropical years and 88 days in the rearward direction of the cosmos.

For the motion for the center of the lunar eccenter, Ptolemy says the following:

> …and let the center of the eccentric move around the center of the zodiac with uniform speed from east to west, with its motion from the northern limit being the excess by which double the course of the mean elongation of the Sun exceeds the course in latitude projected on the zodiac circle in equal time, so that in 348 complete *nychthemera* plus 17 Egyptian years, it makes approximately 203 restitutions relative to the aslant circle; for it falls short 2 sixtieths of a degree in precise computation.[214]

The rate of mean motion of the center of the lunar eccenter (ω'_m) can be reconstructed as follows:

$$2\eta - \omega' = \omega'_m$$

The center of the lunar eccenter (ω'_m) completes 203 returns minus $\frac{2}{60}$° in 17 years and 348 days in the rearward direction of the cosmos. Additionally, the center of the lunar epicycle's motion is double the mean elongation (2η) so that in 19 Egyptian years and 300 days it completes 490 restitutions plus $\frac{4}{60}$°. Finally, the Moon completes 348 restitutions plus $\frac{1}{60}$° on its epicycle in 26 Egyptian years and 99 days (α).

For the outer planets, Ptolemy presents the complex periods and he explains what these periods represent. For example, with regard to Mars, Ptolemy says:

> …[let] the center of the epicycle [be assumed] to move around the point that is further from the Earth with uniform speed, having its position on the eccentric circle always around the aforementioned diameter in the trailing direction of the cosmos, [and with its motion being] the excess by which the course of the Sun exceeds the course both of the fixed stars and of the planet in equal time, so that

[214] Heiberg 1907, 82-84.

> in 361 complete *nychthemera* plus 95 Egyptian years it makes approximately 51 restitutions, for it falls short 3 sixtieths of a degree in precise computation.[215]

Ptolemy explains that for the complex planetary periods he will combine: the solar motion with respect to the equinox (ω_s), the motion of the apogee i.e. three degrees in 300 years (ω_π), and the anomalistic motion (ω_α). These parameters are combined in the following way:

$$\omega_s - \omega_\pi - \omega_\alpha = \omega_p$$ [216]

The motion to which Ptolemy refers, namely ω_p, is shown in the following diagram:

[215] Heiberg 1907, 94-96.
[216] Ptolemy calculates $\omega_s - \omega_\pi$ separately and then subtracts ω_α for the outer planets. Duke uses the following equation $\omega_s + \omega_\pi - \omega_\alpha = \omega_p$, where ω_π is a negative value.

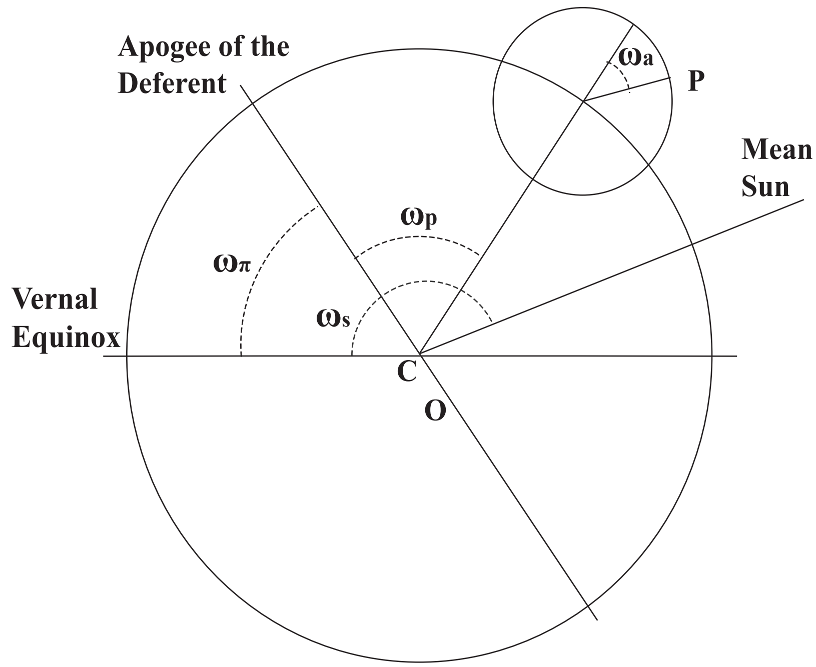

Figure 14: Complex Mean Motion of Outer Planets

The result, namely the rate of complex motion for the outer planets (ω_p), represents the motion of the center of the epicycle along the eccentric circle with reference to the apogee of the eccentric.

Regarding the inner planets, Ptolemy provides the complex period relation and explains what this period relation represents. For example, concerning Venus he says:

> …and on this little circle let the star move around its center with uniform speed, such that a change in position relative to the apogee is completed in the opposite direction of the revolution of the cosmos, [making] a course equal to the sum of [the motion of] the epicycle and the star, so that in 33 complete *nychthemera* plus

> 35 Egyptian years it makes approximately 57 restitutions, for it takes up 1 sixtieth more in precise computation.[217]

For the inner planets, the complex motions are found according the following combination:

$$\omega_s - \omega_\pi + \omega_\alpha = \omega_p{}^{218}$$

The motion to which Ptolemy refers, namely ω_p, is shown in the following diagram:

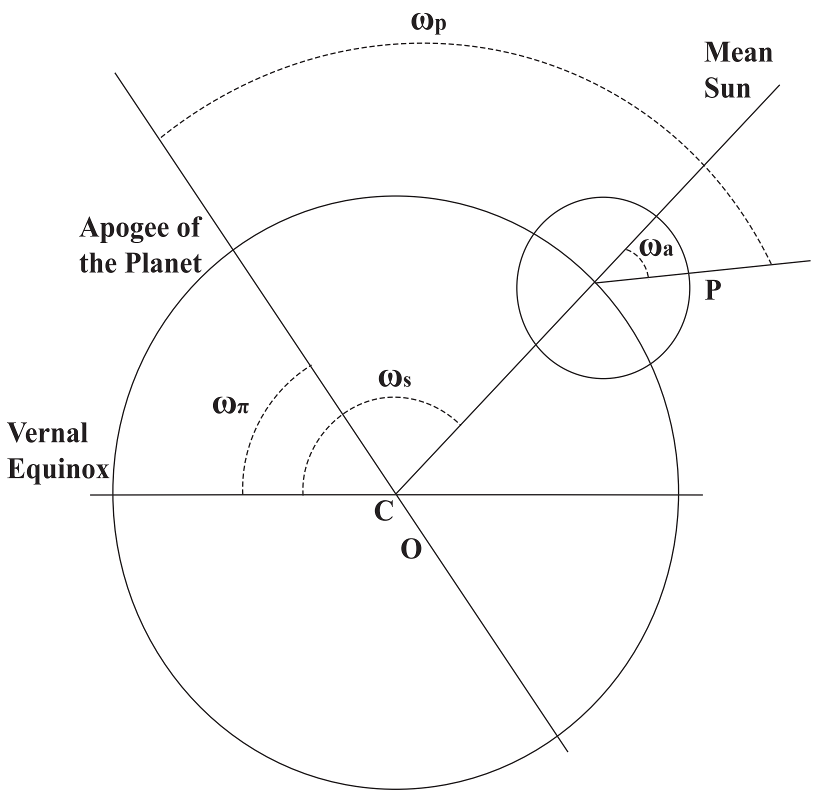

Figure 15: Complex Mean Motion of Inner Planets

[217] Heiberg 1907, 90-92.
[218] Duke uses the following equation $\omega_s + \omega_\pi + \omega_\alpha = \omega_p$, where again ω_π is a negative value.

The complex motion (ω_p) for the inner planets represents the motion of the planet along the epicycle with reference to the apogee of the eccentric circle.[219] The values that Ptolemy provides for the complex periods are as follows:

Table 8: Planetary Complex Period Relations

	Egyptian Years	Returns with respect to the apogee
Mercury	208 years 174 days	865 returns plus $\frac{4}{60}°$
Venus	35 years 33 days	57 returns plus $\frac{1}{60}°$
Mars	95 years 361 days	51 returns minus $\frac{3}{60}°$
Jupiter	213 years 240 days	18 returns plus $\frac{1}{60}°$
Saturn	117 years 330 days	4 returns plus $\frac{1}{60}°$

Neugebauer suggested that the small remainders were insignificant; however, Duke has shown them to be important to understanding the origins of Ptolemy's complex motions.[220] Ptolemy states that these values were found from the simple period relations.[221] While Ptolemy did not walk through the calculations of the complex periods, by starting with the simple periods and relying on the formulas above, the method can be replicated. Once again, the simple mean motion periods are as follows:[222]

[219] Duke 2009, 643-644.
[220] Neugebauer says that the corrections are too small to have much of an effect on the values. Since these adjustments are found in both the Greek and Arabic texts, Neugebauer suggests that the Greek text has been tampered with and that these adjustments were a later addition. Neugebauer 1975, 903 n. 11. Duke 2009, 644.
[221] See Section III.3 above.
[222] These mean motion periods are calculated by multiplying Ptolemy's length for the sidereal year, 365;15,24,31,32,27,7 days, by the number of sidereal years. See Section III.3 above for a discussion of the simple periods.

Table 9: Planetary Simple Period Relations

	Sidereal Years	Returns in Anomaly with Respect to the Apogee	Daily Mean Motion
Mercury	993	3,130	$3;6,24,7,7,7,8,12,56,50\ldots^{\circ/d}$
Venus	964	603	$0;36,59,27,29,14,8\ldots^{\circ/d}$
Mars	1,010	473	$0;27,41,40,34,55,28\ldots^{\circ/d}$
Jupiter	771	706	$0;54,9,3,16,17,8\ldots^{\circ/d}$
Saturn	324	313	$0;57,7,43,30,54,14\ldots^{\circ/d}$

Using daily mean motion rates and the formulas above, the complex mean motions rates can be reconstructed. For instance, for the inner planets we can combine the solar mean motion with respect to the equinox (ω_s), the anomalistic mean motion (ω_a), and subtract the mean motion of the apogees (ω_π), in order to get the motion of the planet along the epicycle with reference to the apogee of the eccentric circle (ω_p).[223] In the *Planetary Hypotheses* the rounded solar mean motion relative to the equinox is $0;59,8,17,13,12,31^{\circ/d}$ and the rounded mean motion of the apogee of the planets is the same as the mean motion of the sphere of the fixed stars, namely $0;0,0,5,54,49,43,19^{\circ/d}$.[224] Accordingly, the complex mean motion rates for the planets can be reconstructed from the simpler periods resulting in the following mean motion rates:

[223] In the case of the outer planets we subtract the anomalistic motion.

[224] This is the same solar mean motion rate that Ptolemy gives in the *Almagest*. Toomer 1998, 140. See Section III.2.3 above for a discussion of the mean motion rate for the sphere of the fixed stars and planetary apogees.

Table 10: Complex Daily Mean Motions Reconstructed from the Simple Periods

	Complex Mean Motion
Mercury	4;5,32,18,25… $^{o/d}$
Venus	1;36,7,38,47… $^{o/d}$
Mars	0;31,26,30,43… $^{o/d}$
Jupiter	0;4,59,8,2,5,39… $^{o/d}$
Saturn	0;2,0,27,47,28… $^{o/d}$

From these complex mean motion rates, Ptolemy can construct period relations. Ptolemy wanted a period relation where each planet completes an integral number of rotations in an integral number of days, with very small remainders.[225] All of the complex period relations that Ptolemy provided had remainders of less than $\frac{4}{60}$ degrees. Ptolemy contends that the complex period relations he presents are based on the simple period relations presented at the beginning of Book I, Part A and Duke confirms this by comparing the small fractional remainders for the complex mean motion period in the *Almagest* and simple periods in the *Planetary Hypotheses*.[226]

Ptolemy does not explicitly state why he makes these particular combinations; however, the complex mean motions certainly would have been useful for the tables mentioned at the end of Book II, which unfortunately have been lost. The complex period relations represent particular, combined motions, which would have been valuable for calculating the position of celestial bodies. Each complex period relation incorporates more than one motion; for example, the complex period relations for the outer planets

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[225] Duke demonstrates how these values can be constructed into period relations using Mercury as an example. Duke 2009, 644-645. Additionally, see Section III.3.5 above for a brief discussion of methods used in converting mean motion rates to period relations.

[226] Duke argues that Ptolemy started with the simple periods found in the *Planetary Hypotheses,* or something close to those, and not the corrected or uncorrected periods relations found in the *Almagest*. Duke 2009, 646.

represent the motion of the epicycle around the eccentric circle *and* the motion of apsidal line. As such, these periods would have been effective for determining the positions of celestial bodies, since the astronomer would have been required to make fewer calculations without sacrificing accuracy.

III.4.1 Regulus

Ptolemy gave the position of the star Regulus relative to the vernal equinox. In the *Handy Tables*, which predates the *Planetary Hypotheses*, Ptolemy treated Regulus similarly to the way he treated a planet. He ascribes a mean motion to Regulus, which is equal to precession. In doing this he asserted that the solsticial and equinoctial points are stationary and everything else exhibits motion, including the sphere of the fixed stars. Since Regulus is only a fraction of a degree from the ecliptic, it is a good choice to represent the motion of the sphere of the fixed stars and in the *Handy Tables* Ptolemy assigns Regulus the latitude of +10 minutes.

In the *Planetary Hypotheses* Ptolemy said that the heart of the lion, i.e. Regulus, was 117;54 degrees rearward from the vernal equinoctial point. Comparing this to the *Almagest*, Ptolemy said that in the second year of Antoninus, on Pharmouthi 9 in the Egyptian calendar, or February 23, 139 in the Julian calendar, Regulus was 32;30 degrees rearward from the summer solstice, which is equivalent to 122;30 degrees rearward from the vernal equinox.[227] The difference between these two dates is 462 Egyptian years. At one degreeevery 100 tropical years, precession would be approximately 4;37 degrees in 462 years. Subtracting the amount of precession from the position of Regulus on the date

[227] Toomer 1998, 328.

of the *Almagest* epoch gives us the location of Regulus on the date of the *Planetary Hypotheses* epoch:

$$122;30° - 4;37° = 117;53°$$

The position of Regulus is 117;53 degrees according the *Almagest*. This is nearly 117;54 degrees, which is the position that Ptolemy states in the *Planetary Hypotheses*.[228] For Regulus, Ptolemy seems to be using his epoch positions from the *Almagest* and adding the appropriate mean motion value for the given time in order to achieve the position of the star for Year 1 of the Era of Phillip.

III.4.2 Epochs

In the *Planetary Hypotheses*, Ptolemy used a different date for the epoch than he does in the *Almagest*. In the *Planetary Hypotheses* he said, "But in the first year from the death of Alexander the Founder, Thoth 1 according to the Egyptians, at noon in Alexandria, the Sun was 162 and 10 sixtieths degrees away from the apogee of the eccentric in the trailing direction of the cosmos."[229] In the *Almagest* he said, "we find for the epoch in mean motion in the first year of Nabonassar, Thoth 1 in the Egyptian calendar, noon, that the Sun's distance in mean motion is 265;15 degrees to the rear of the apogee."[230] The difference between the Year 1 of the Era of Nabonassar and Year 1 of the Era of Phillip is approximately 426 years. According to the mean motion tables from the *Almagest*, the Sun would travel 256;55 degrees in 426 years. The position of the Sun in the *Almagest* is 265;15 degrees rearward (east) from the apogee. Adding the

[228] The *Canobic Inscription* can be used instead of the *Almagest* to compare the position of Regulus as Neugebauer shows. Neugebauer 1975, 903.
[229] Heiberg 1907, 80.
[230] Toomer 1998, 169.

position of the Sun at its epoch during the Era of Nabonassar to the value the Sun would travel in 426 years produces the following:

$$265;15° + 256;55° = 522;10° \text{ (or } 162;10°)$$

In Year 1 of the Era of Phillip the Sun was 162;10 degrees rearward from the apogee. This is the exact location that is given for the epoch in the *Planetary Hypotheses*.[231] Ptolemy did not recalculate the position of the Sun, but instead used the parameters from the *Almagest,* along with his mean motion tables, to determine the location of the Sun at the new epoch. The epoch in the *Planetary Hypotheses*, which also happens to be the same epoch for the *Handy Tables*, was the Era of Phillip at noon. Year 1 of the Era of Phillip is the same year of the death of Alexander the Great. The epoch in the *Almagest* is the Era of Nabonassar. The Julian dates for these epochs are:

Year 1 of the Era of Nabonassar:	February 26, 747
Year 1 of the Era of Phillip:	November 12, 324

Nabonassar is the first ruler presented in Ptolemy's Royal Canon, a chronological list of Assyrian, Babylonian, Chaldaean, Ptolemaic, Macedonian, and Persian rulers often included with manuscripts of Ptolemy's *Handy Tables*.[232] While the Canon was constructed for astronomical purposes, it is instrumental in determining chronology in the ancient world.[233]

The years in the Canon are recorded in Egyptian years and they are all integers. The Egyptian year is consistent in length and consequently a better choice for such a list than

[231] Neugebauer 1975, 902.
[232] A reconstructed Royal Canon, also known as Canon Basileon, can be found in the introduction to Toomer's translation of the *Almagest*. Toomer 1998, 11.
[233] Consisting of three columns, the Canon begins with Era of Nabonassar and ends with the Era of Cleopatra VII, covering a millennium in total. In addition to the names of the rulers (first column), the Canon contains the length of each ruler's reign (second column), and the total number of years that have passed (third column). Depuydt 1995, 99.

a lunar-solar calendar, where each month has 29 or 30 days and each year has 12 or 13 months. Although a new Egyptian day began in the morning, Ptolemy considered the astronomical epoch for the day to be noon since the period of time from noon to noon on any two consecutive days is no greater than the time variation due to the anomaly.[234] For this reason, all of the Eras begin at noon.

The Canon begins with Nabonassar, but in the year of Alexander the Great's death, or the first year of the Era of Phillip, the years are counted in the third column from the Era of Phillip instead of the Era of Nabonassar.[235] Consequently, the Era of Nabonassar and the Era of Phillip denote epochs on the list that are easy to calculate from. Ptolemy explains in Book III.7 of the *Almagest* that he uses the Era of Nabonassar as his epoch because the earliest observations he uses date back to the eighth-century B.C.[236] Since the *Planetary Hypotheses* does not include observation dates, Ptolemy can use a later epoch. The Era of Phillip is an obvious choice because the Canon is constructed in such a way that counting back to the Era of Phillip is simple. While Ptolemy uses observations from different Eras – for example, when determining the apogee of Mercury, Ptolemy uses observations from the "Era of Dionysius" and the "Chaldean Era" – he only uses the Era of Nabonassar, the Era Augustus, and the Era of Phillip as epochs.[237]

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[234] The period from noon to noon is not fixed due to the equation of time. Ptolemy used a noon epoch since the time difference at noon does not vary as much as the time difference at sunrise or sunset (where the times can vary depending on latitude). Depuydt 1995, 106.
[235] Depuydt 1995, 99-101.
[236] Toomer 1998, 9-14, 166 n. 59.
[237] Neugebauer 1975, 159.

III.5 Point of Reference

In the *Almagest* Ptolemy stated that while one can measure the length of the year relative to a sidereal point, such as a star, or relative to a tropical point, such as a solstice or equinox, he clearly stated that he preferred to use a tropical point. Throughout the *Almagest*, when measuring a planet's longitudinal motion and anomalistic motion, Ptolemy used the solsticial and equinoctial points.[238] Likewise, for the longitudinal motion of the Sun and the anomalistic motion of the Moon, Ptolemy measured the mean motion relative to the solsticial and equinoctial points.[239]

III.5.1 Type of Year and Reference Point

Likewise, in the *Planetary Hypotheses* Ptolemy measured the motion of the Sun and Moon using tropical years, relying on the equinoctial and solsticial points as reference points. For example, Ptolemy writes:

> In 8,523 solar years taken relative to the solsticial and equinoctial points, which is 8,528 Egyptian years and 277;20,24 *nychthemera,* let the moon make 105,416 overtakings of the sun, i.e. complete months; and again in 3,277 complete months let the moon make 3,512 restitutions in anomaly; and in 5,458 complete months let the moon make 5,923 restitutions in latitude.[240]

Ptolemy used the solsticial and equinoctial points for the Sun, as we can see above. Similarly, he measured the lunar motion using this same point of reference. Ptolemy says:

> In 8,523 solar years taken relative to the solsticial and equinoctial points, which is 8,528 Egyptian years and 277;20,24 *nychthemera,* let the moon make 105,416 overtakings of the sun, i.e. complete months; and again in 3,277 complete months let the moon make 3,512 restitutions in anomaly; and in 5,458 complete months let the moon make 5,923 restitutions in latitude.[241]

[238] Toomer 1998, 423-424.
[239] Toomer 1998, 132.
[240] Heiberg 1907, 78.
[241] Heiberg 1907, 78.

When presenting the locations of the Sun and Moon at the epoch, Ptolemy uses the apogees as a point of reference.[242] Consequently, for the Sun and Moon Ptolemy uses tropical years as a unit of measurement and he uses tropically fixed points as reference points.

Alternatively, for the planets Ptolemy used a different point of reference than he did for the Sun and Moon. When presenting the anomalistic motion for each planet in the *Planetary Hypotheses*, Ptolemy says:

> Likewise, in 993 solar years taken relative to the apogees and the sphere of the fixed stars, which is 993 Egyptian years and 255;0,54,46,51 *nychthemera* approximately, let the star of Mercury make 3,130 restitutions in anomaly. And in 964 of the same kind of solar years, which is 964 Egyptian years and 247;33,2,45,23,40,28 *nychthemera* approximately, let the star of Venus make 603 restitutions in anomaly.[243]

Not only did Ptolemy use the apogees and the sphere of the fixed stars as reference points for Mercury and Venus, but he also used these points for the remaining planets. This means that the solar years that he refers to are not in fact tropical years, as they are for the Sun and Moon, but sidereal years. Additionally, when describing the location of each planet at the epoch, Ptolemy gives the positions in relation to the apogee, which is sidereally fixed. For example, when presenting the location of Venus, Ptolemy says:

> In the first year [from] the death of Alexander, Thoth 1, according to the Egyptians, at noon in Alexandria, the apogee of the eccentricity was 50 degrees and 24 sixtieths away from the vernal equinoctial [point] in the trailing [direction] of the cosmos; and the northern limit was the same [number of degrees away from the vernal equinoctial point in the trailing direction of the cosmos]; the center of the epicycle was 177 degrees and 12 sixtieths away from the apogee of the eccentricity in the trailing [direction] of the cosmos; and again the northern limit of the aslant little circle was 87 degrees and 16 sixtieths away from the apogee of the epicycle in the forward [direction] of the cosmos. The star was 186 degrees

[242] Only the apogees of the planets are sidereally fixed. Heiberg 1907, 76-78.
[243] Heiberg 1907, 78.

> and 35 sixtieths away from the northern limit of the aslant little circle in the trailing [direction] of the cosmos.[244]

Egyptian years are used as the unit of measurement and the apogee of the planet is used as the longitudinal point of reference. The apogee is sidereally fixed. The sphere of the fixed stars has a slow motion relative to the tropical points, as Ptolemy assigns a mean motion for the sphere of the fixed stars and an epoch location for Regulus relative to the vernal equinox.

III.5.2 Reasons for a New Point of Reference

Ptolemy's use of the sidereal year and sidereal reference points in the *Planetary Hypotheses* is not consistent throughout. Since he only makes these changes for the planets, and not the Sun, Moon, or sphere of the fixed stars, Ptolemy uses two different points of reference. The changes that Ptolemy makes to the *Planetary Hypotheses* concerning how motion is measured and judged do not affect the accuracy of his theory. This change is particularly strange given that Ptolemy weighs the benefits of using one method over the other in his discussion of the solar year in Book III of the *Almagest* and ultimately decides that there are more benefits to using the tropical year. In particular, Ptolemy argued that equinoctial or solsticial points should be used over sidereal points because it is more sensible to define a revolution by when the Sun reaches the same relative position in place and time. Moreover, Ptolemy argues that the Sun should be defined as completing a revolution when it returns to the same season and atmospheric condition.[245] While the second reason does not apply to planetary periods, the atmospheric changes we see on Earth do not neatly correspond to the planetary periods,

[244] Heiberg 1907, 90-94.
[245] Toomer 1998, 132.

the first reason does apply. Ptolemy argues that the most reasonable way to judge a revolution is when the body returns to the same position in place and time. Since the sphere of the fixed stars has its own motion, if a planet returns to the same position relative to the sphere of the fixed stars it has not completed a revolution relative to the fixed solsticial and equinoctial points.

Finally, Ptolemy assigned the sphere of the fixed stars a mean motion relative to the tropical points in the *Planetary Hypotheses*. In the *Canobic Inscription* he assigned Regulus a mean motion, implying the tropical points are fixed and the stars move relative to the fixed solsticial and equinoctial points.[246] Since Ptolemy assigned the sphere of the fixed stars a mean motion, then he measures the motion of the planets against a background that is moving. But, he also gives the background, namely the sphere of the fixed stars, a motion. Additionally, it makes sense to express the simple and unmixed planetary period relations in terms of sidereal year since from these periods Ptolemy derived the complex periods relative to the apogees.

[246] Jones 2005a, 70-71. In the *Almagest*, Ptolemy describes the solsticial and equinoctial points moving relative to the sphere of the fixed stars. In the *Planetary Hypotheses* and *Canobic Inscription*, he changes his point of reference. Section III.2-3.

III.6 Book I, Part B

Book I, Part B comprises the formerly missing section of the *Planetary Hypotheses*, the section that was left out of Heiberg's 1907 edition. This intriguing section of the *Planetary Hypotheses* was consequently unknown to modern scholars for the first half of the twentieth-century.[247] Several interesting ideas are discussed in this section, including the distances, diameters, and volumes of the planets. It was only in this section that Ptolemy discussed what was later known as the "Ptolemaic System", namely the idea that the spheres in the heavens are nested and that there is no empty space between them. Additionally, this was the only place where Ptolemy calculated the distances of the celestial bodies. His calculations for these values influenced Medieval Arabic and European estimates of the sizes and distances of the planets.

III.6.1 Beginning of Book I, Part B

At the beginning of Book I, Part B[248] Ptolemy refers back to Book I, Part A informing the reader that the models laid out there are the models describing planetary motion.[249] He distinguishes between the planetary spheres and the spheres of the fixed stars, since the former have more anomalies and the latter are simple and unmixed. The sphere of the fixed stars, Ptolemy explains, is close to that of the universal movement, which is simple by nature and not mixed with anything.[250] The planets all lie below this

[247] See Section I.2 above.
[248] Throughout this section I rely on the Arabic-A and Arabic-B published manuscripts, Morelon 1993 and Goldstein 1967. There are several differences between the translations by Goldstein and Morelon; the former takes several liberties and the latter, in general, more closely matches the Arabic text.
[249] Goldstein 1967, 5, 26. Morelon 1993, 55. The Arabic word that he uses for models, *hay'a,* has multiple meanings in Arabic astronomy. See Section III.7.4 for a discussion of *ʿilm al-hayʾa* texts in Arabic astronomy.
[250] Goldstein 1967, 5, 26. Morelon 1993, 56-57.

movement and move from east to west with it, but they also move from west to east.[251] Ptolemy does not imply that a mover, or prime mover, is moving the planets, but instead that the planets move with the universal movement.

Ptolemy says that the planets move from east to west, from north to south, and forward and backward; however, all of these motions are local motions.[252] Ptolemy relates types of celestial motion to Aristotle's discussion of motion in the *Physics*. In the *Physics*, Aristotle contends that there are three types of motion: local, qualitative, and quantitative. He says that local motion is the primary motion and "Everything that is in locomotion is moved either by itself or by something else."[253] According to Ptolemy, the type of motion that the planets undergo is local motion, since they move in respect to place only and not in respect to quality or quantity, which are changes of substance.[254]

Ptolemy then turns to a discussion of the motions of each celestial body. The Sun, he states, has only one anomaly because there is nothing stronger from which it could receive another anomaly.[255] While the Sun only has one type of anomaly, the planets have two anomalies. The first motion, which Ptolemy says he has already mentioned, concerns the planet's place along the ecliptic and the second concerns the planet's return to the Sun. According to Ptolemy each planet has a voluntary motion and a constrained motion. Ptolemy does not clearly explain from where the Sun receives its one anomaly, nor does he elaborate on why one of the motions of the planets is constrained and the other is voluntary.

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[251] Goldstein 1967, 5, 26. Morelon 1993, 56-57.
[252] Goldstein 1967, 5. Morelon 1993, 57.
[253] Aristotle, *Physics*, 243a.
[254] Goldstein 1967, 5, 25. Morelon 1993, 56-57.
[255] Goldstein 1967, 6. Morelon 1993, 56. Goldstein 1967 says "there is nothing stronger than it to give it another anomaly in its motion". Morelon 1993 is closer to the Arabic: "il n'y a rien qui soit plus fort lui de sorte qu'il pourrait en recevoir une autre anomalie dans sa progression".

Ptolemy's treatment of the motion of the celestial bodies continues with a discussion about the northern and southern motions of the planets. The Sun has a northern and southern motion due to the inclination of the ecliptic to the equator. The Moon has two inclinations: the aforementioned and the inclination of the Moon's orbit to the ecliptic. The planets each have three inclinations: the two aforementioned and the inclination of the epicycle to the planet's orbit around the Earth (the eccentric circle). While he described the layout of the models briefly earlier in Book I, Ptolemy repeats this in Book I, Part B. He explains how the ecliptic and the equator are inclined to each other in a fixed position and describes the location of the solsticial and equinoctial points. Ptolemy says that one can imagine a difference in the motions since the first two occur around the Earth and the third, namely the epicyclic motion, does not surround the Earth. The first two types of inclinations result in northern and southern motion, while the third occurs on planes parallel to which they have a fixed inclination.[256] He explains that the inclination of the epicycle to the eccentric circle and the inclination of the eccentric circle to the equator are fixed.[257] With respect to the third type of inclination – the one that is not centered on the Earth – Ptolemy says that inclination of the epicycle sphere is the correspondent to the ecliptic. For the spheres around the Earth, a return is relative to that which moves it, whether it is the Sun, or the center of the epicycle, or the Moon, or a planet. For the epicycles, he says, a return is measured by the center of the epicycle sphere and not by that which moves it. It is not clear precisely what Ptolemy means in this passage. Goldstein translates this sentence as: "The epicycles return with the return

[256] Goldstein 1967, 6, 26-27. Morelon 1993, 58-59.
[257] Goldstein 1967, 6, 26-27. Morelon 1993, 56-60. In the *Almagest* the inclination between the epicycle and eccentric circle is not fixed. See Section III.2 above for a discussion of the models in the *Planetary Hypotheses* and *Almagest.*

of the centers of the epicyclic spheres, not with the return of the planet that moves on them – this is the condition for each of one of the spheres".[258] Morelon's translation, which is closer to the Arabic, says: "Quant à ceux qui sont des orbes epicycles, leur retour se produit avec celui du centre des orbes epicycles, non avec celui de ce qui se meut sur eux".[259] Ptolemy seems to be referring to latitude motion of the epicycle and how that motion should be measured, but the meaning of the passage is ambiguous and it is not entirely clear what the motion should be measured against.

III.6.2 Order of the Planets

The arrangement of the planets that Ptolemy uses – Moon, Mercury, Venus, Sun, Mars, Jupiter, and Saturn – is not the only order of the planets discussed by astronomers and philosophers in the ancient world. In the third-century A.D. the Christian bishop, Hippolytus, reports that Archimedes presented two orders for the arrangements of the planets and that he gave estimates of the distances of the planets.[260] The two orders of the planets attributed to Archimedes are presented as: (A) Earth, Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, zodiac; and (B) Earth, Moon, Venus, Mercury, Sun, Mars, Jupiter, Saturn, zodiac. Due to corruptions in the text, it is difficult to analyze the content; however, Catherine Osborne offers a reconstruction of the data and argues that distances assigned are based on harmonics.[261]

[258] Goldstein 1967, 6.
[259] Morelon 1993, 60-61.
[260] Van Helden 1985, 5-10.
[261] Osborne 1983, 234-235. See Section III.6.3 below.

Aristotle discussed the arrangement of the planets briefly in *On the Heavens* and offered a theory that accounted for the order of the planets and their respective periods. In Book II of *On the Heavens* Aristotle says:

> It is established that the outermost revolution of the heavens is a simple movement and the swiftest of all, and that the movement of all other bodies is composite and relatively slow, for the reason that each is moving on its own circle with the reverse motion to that of the heavens. This at once makes it reasonable that the body which is nearest to that first simple revolution should take the longest time to complete its circle, and that which is farthest from it the shortest, the others taking a longer time the nearer they are and a shorter time the farther away they are. For it is the nearest body which is most strongly influenced, and the most remote, by reason of its distance, which is least affected, the influence on the intermediate bodies varying, as the mathematicians show, with their distance.[262]

Aristotle proposes that the planets closest to the sphere of the fixed stars take the longest time to complete a period and those furthest from the sphere of the fixed stars take the shortest amount of time. The arrangement of the planets and their periods are consequently related for Aristotle.

In the *Almagest* Ptolemy also introduces two orders for the planets, though he admits that the order of the planets is a matter of debate. He argued that since astronomers are not able to observe any parallax of the planets, then this matter cannot be judged with certainty. In Book IX.1 of the *Almagest* he says:

> But concerning the spheres of Venus and Mercury, we see that they are placed below the sun's by the more ancient astronomers, but by some of their successors these too are placed above [the sun's], for the reason that the sun has never been obscured by them [Venus and Mercury] either. To us, however, such a criterion seems to have an element of uncertainty, since it is possible that some planets might indeed be below the sun, but nevertheless not always be in one of the planes through the sun and our viewpoint, but in another [plane], and hence might not be seen passing in front of it, just as in the case of the moon, when it passes below [the sun] at conjunction, no obscuration results in most cases.[263]

[262] Aristotle, *On the Heavens*, 291a-292-291b.
[263] Toomer 1998, 419.

In spite of the uncertainty surrounding the order of the planets, Ptolemy chooses to use the order attributed to the more ancient astronomers, namely Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn. He says:

> For, by putting the sun in the middle, it is more in accordance with the nature [of the bodies] in thus separating those which reach all possible distances from the sun and those which do not do so, but always move in its vicinity; provided only that it does not remove the latter close enough to the earth that there can result a parallax of any size.[264]

Ptolemy chose an order based on organization, so that like bodies were near like bodies.[265] Bodies that can reach opposition with the Sun, such as Mars, Jupiter, and Saturn, are separated from those that are always near the Sun, namely Mercury and Venus.

In the *Planetary Hypotheses* Ptolemy addresses this topic again. This time, however, he puts forth three possibilities for the order of the celestial bodies. He says:

> But with respect to the Sun, there are three possibilities: either all five planetary spheres lie above the sphere of the Sun just as they all lie above the sphere of the Moon; or they all lie below the sphere of the Sun; or some lie above, and some below the sphere of the Sun, and we cannot decide this matter with certainty.[266]

Ptolemy considered the Moon to lie closest to the Earth and his only debate concerned the location of the Sun and whether it should be located directly above the Moon, somewhere between the planets, or above all of the planets. The three options Ptolemy stated were not the only options, since the celestial bodies could have paths that overlap one another; however, Ptolemy does not discuss this idea. In the end, when calculating the distances of the celestial bodies Ptolemy maintained the order of the planets that he presented in the *Almagest*: Moon, Mercury, Venus, Sun, Mars, Jupiter, and Saturn.

[264] Toomer 1998, 419-420.
[265] See Section I.4.2 above.
[266] Goldstein 1967, 6.

Calculating the distances of the planets, Ptolemy states, is a difficult task. For while the distances of the Sun and Moon can be calculated relying on eclipses, the distances of the planets cannot be determined using the same means since their parallax cannot be established.[267] Furthermore, Ptolemy states that no astronomer has observed a planet passing in front of the Sun, raising the possibility that all the planets lay beyond the sphere of the Sun. However, Ptolemy attributed the difficulty of making such an observation to the brightness of the Sun and the rarity at which such an event could be observed.[268] Ptolemy concluded that one cannot assume the arrangement of the planets with certainty. While the difficulties in determining the arrangement of the celestial bodies were discussed more thoroughly in the *Planetary Hypotheses* than in the *Almagest*, Ptolemy reached the same conclusions in both works.

III.6.3 The Distances of the Planets

The idea that the distances of the planets correspond to musical ratios or produce tones was discussed in multiple ancient texts.[269] Plato discussed this idea in the *Republic*. He says:

> The spindle itself turned on the lap of Necessity. And up above on each of the rims of the circles stood a Siren, who accompanied its revolution, uttering a single sound, one single note. And the concord of the eight notes produced a single harmony.[270]

While Plato briefly described the music of the heavenly spheres, or more appropriately the music of the Sirens who accompany the spheres, he did not discuss the distance of

[267] Goldstein 1967, 6. In the *Almagest* Ptolemy states that none of the stars, by which he means both the fixed stars and the planets, has an observable parallax. Toomer 1998, 419.
[268] Goldstein 1967, 6-7.
[269] Swerdlow 1968, 96.
[270] Plato, *The Republic*, 616e-d.

each sphere nor whether the distance of the sphere has any relation to the note each Siren sings.

In *On the Heavens* Aristotle argued against the idea that the heavens produce sound when they move. He says that while others argue that the motions of something as large as the heavenly spheres must produce a sound, that this is not necessarily the case. Building on this he says, “Starting from this argument and from the observation that their speeds, as measured by their distances, are in the same ratios as musical concordances, they assert that the sound given forth by the circular movement of the stars is a harmony.”[271] While Aristotle did not list the distances of the stars and planets nor describe the musical ratios, he was clearly reacting to the estimates by other thinkers for these values. Aristotle did not name the person or people whose ideas he was engaging with nor did he explain how the estimates for the distances of the celestial bodies were calculated.

Since the distances of the planets cannot be calculated by determining the parallax of each planet, Ptolemy used a method to calculate the distances that relies on the ratios of least distance to greatest distance. Ptolemy did not calculate the distances of the planets in any of his other works. His calculations for the distances of the planets are not the only instances in Greek astronomy that we see the distances determined using this method; the distances of the planets are discussed by both Proclus and an anonymous scholia to the *Almagest*.

Ptolemy arranges the spheres so that the furthest distance from the Earth that one celestial body can reach is equal to the least distance the celestial body beyond it can

[271] Aristotle, *On the Heavens*, 290b.

reach.[272] This assumes that there is no empty space in the cosmos. While Ptolemy does not expand on the idea that the heavens contain no empty space, he is surely building on Aristotle's idea that nature abhors a void.[273] Ptolemy does not want an empty space in the heavens, but he does not seem to consider the idea of a gap between celestial bodies that could be filled with ether.

Despite not possessing the data needed to calculate the parallax of the planets, Ptolemy was able to estimate the distance of each planet relying on ratios for the planet's model. In the *Almagest*, each planetary model was constructed using a deferent circle with a radius of 60 units. This value was used for all of the models, regardless of the absolute radius of the deferent circle, which Ptolemy considered to be unknown. Ptolemy constructed a ratio for the least distance and the maximum distance of each planet relying on the value of eccentricity and epicycle size. Using this information, along with the order of the planets, the absolute distance for at least one celestial body, and the assumption that there is no empty space in the heavens, Ptolemy was able to calculate the true distance for each celestial body.

In both the *Canobic Inscription* and the *Harmonics* Ptolemy discussed the tones of the cosmos. The scales that Ptolemy described in the *Canobic Inscription* do not correspond to the distances in the *Planetary Hypotheses*. Unfortunately, the part of the *Harmonics* where Ptolemy discusses the tones of the planets, Book III 14-16, has been lost.[274] In the *Canobic Inscription* the distance of the Moon is given as 64 Earth radii and the distance of the Sun is 729 Earth radii. Using the method to find the solar distance that Ptolemy uses in Book V.15 of the *Almagest*, but using the diameter of the shadow that

[272] Goldstein 1967, 7.
[273] Aristotle, *Physics*, 212a-217b.
[274] Swerdlow 1968, 97 n. 2.

Ptolemy uses in the *Canobic Inscription*, the distance of the Sun is, with some adjustment, approximately 729 Earth radii.[275] In the *Almagest* Ptolemy found that the least distance for the Moon, which was at quadrature, was 33;33 Earth radii. The maximum distance, found at syzygy, was 64;10 Earth radii.[276] In the *Planetary Hypotheses* Ptolemy rounds these values to 33 and 64, respectively. For the Sun, in the *Almagest* Ptolemy finds the Sun's distance to be 1,210 Earth radii.[277] He does not specify whether this represents the minimum, mean, or maximum distance. In the *Planetary Hypotheses* Ptolemy gives the Sun's minimum and maximum distance as 1,160 Earth radii and 1,260 Earth radii respectively.[278] This means that the distance he gives in the *Almagest* is in fact the Sun's mean distance; however, this must have been a retrospective decision since in the *Almagest* Ptolemy ignored the variation in solar distance when he calculated the absolute distance to the Sun.[279]

Since Ptolemy assumes that the order of the planets is the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn, he states that the Moon's maximum distance is equal to Mercury's minimum distance, Venus's minimum distance is equal to Mercury's maximum distance, and so forth. With the value for the Moon's distance in Earth radii and the ratios for each planet, Ptolemy calculates the minimum and maximum distance for each planet in Earth radii. His values are:

[275] Swerdlow 1968, 74. Hamilton, Swerdlow, Toomer 1987, 68-69.
[276] Toomer 1998, 251.
[277] Toomer 1998, 257.
[278] Goldstein 1967, 7.
[279] Toomer 1998, 255-257.

Table 11: Distances of the Planets in Earth Radii

	Ratio of minimum to maximum distance	True distance in Earth radii
Moon	–	33 : 64
Mercury	34 : 88	64 : 166
Venus	16 : 104	166 : 1,079
Sun	–	1,160 : 1,260
Mars	7 : 1	1,260 : 8,820
Jupiter	23 : 37	8,820 : 14,187
Saturn	5 : 7	14,187 : 19,865

Table 11 shows a gap of 81 Earth radii between the maximum distance of Venus and the minimum distance of the Sun. Since Ptolemy uses the true distances for both the Moon and Sun, he is unable to get Mercury and Venus to fit between the Sun and Moon without a gap. Ptolemy acknowledges this discrepancy and says that he cannot account for it.[280] Mars, which has a ratio of 7:1, cannot fit between Venus and the Sun and so Ptolemy places Mars after the Sun. This also supports the arrangement of the planets that Ptolemy gives in the *Almagest* and the *Planetary Hypotheses*. Ptolemy justifies the empty distance between Venus and the Sun by reminding the reader that when the distance of the Moon is increased, the distance of the Sun decreases.[281] Consequently, Ptolemy states that the lunar distance can be increased, which will cause the solar distance to decrease rectifying the gap between Venus and the Sun.[282] Ptolemy does not specify the value by which the Moon's distance needs to be increased. His instructions here are vague, and he seems to suggest that toying with the distances can solve the problem. By assigning the exact same distance for the maximum distance of a celestial body and the minimum distance of the

[280] Goldstein 1967, 7. The ratio of 34 to 88 for Mercury is not due to a translation error in the Arabic, since Proclus confirms this value for Mercury in the *Planetary Hypotheses* in his work *Hypotyposis*. Hartner 1964, 286. Manitus 1909, 142, 220-225.
[281] This comes from the property of Aristarchus's eclipse diagram.
[282] Goldstein 1967, 7.

body that is beyond it, the possibility arises that these two bodies could potentially collide. This problem is not addressed in any of Ptolemy's works.

The ratios of the planets that Ptolemy presents come from rounding the values for the eccentricities and epicycle radii. While these parameters are the same for most of the models in the *Almagest* and *Planetary Hypotheses*, Ptolemy makes changes to his model for Mercury in the *Planetary Hypotheses*. Specifically he changes the radius of the epicycle from 20;30 to 20;15 and the radius of the circle on which the deferent moves from 3 to 2;30. In the *Planetary Hypotheses*, Ptolemy gives 34 to 88 as the ratio of minimum distance to maximum distance for Mercury; however, this ratio does not correspond with the parameters Ptolemy gives in Book I.[283] Given the changes that Ptolemy makes to Mercury's parameters, the ratio for Mercury should actually be 34 to 90;15. The ratio that Ptolemy gives, 34 to 88, does not match the *Planetary Hypotheses* values for the models or the *Almagest* values. Using the parameters from the *Almagest*, the ratio should be 33;4 to 91;30.[284] Christián C. Carman has demonstrated that had Ptolemy used the correct ratio for Mercury, but rounded the numbers to 33 and 92 then that ratio along with the ratio for Venus, would have resulted in a solar minimum distance of 1,159.76 Earth radii, which could then be rounded to 1,160 Earth radii.[285] This is the precise minimum distance that Ptolemy calculates in the *Planetary Hypotheses*. This discovery makes Ptolemy's error particularly intriguing. Carman suggests that Ptolemy's

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[283] Hartner suggests that the error in Mercury's ratio is due to a mistake and instead of equating the minimum distance of Mercury to the maximum distance of the Moon, the minimum distance of Mercury was taken to be 60 Earth radii instead of 64;10 Earth radii. Hartner argues that this is how the maximum distance of 166 was found for Mercury and the ratio of 34 to 88 was deduced from these distances. Once this mistake was made it was not corrected and consequently the incorrect ratio for Mercury appears in Arabic sources. al-Bīrūnī tried to correct Ptolemy's error of 88 for Mercury, but he makes his own mistakes. Hartner 1964, 267-269, 276-77.
[284] Goldstein 1967, 10.
[285] Carman 2009, 226.

initial calculations for the Earth-Sun distance were inconclusive since the calculation method used for finding the distance of the Sun in the *Almagest* is very sensitive. The calculation relies on eclipse observations, with different observations providing different results for the distance of the Sun.[286] Carman argues that Ptolemy determined which eclipse observations to use based on his calculations for the distance of the Sun using the apogee-perigee proportions and the distance of the Moon.[287] Ptolemy then used his preliminary calculations when giving the distance of the Sun in the *Almagest*; however, when he presented the ratios in the *Planetary Hypotheses* he mistakenly uses an incorrect proportion for Mercury and did not realize his mistake since his results were close to what he expected to attain based on his preliminary calculations.

Proclus's discussion of the distance of the planets in *Hypotyposis* gives the essence of the Ptolemy's argument from Book I, Part B. Similar to Ptolemy, Proclus assigned the Sun the minimum distance of 1,160 Earth radii and the maximum distance of 1,210 Earth radii.[288] In the *Almagest* Ptolemy calculated the distance of the Sun to be 1,210 Earth radii, but did not specify if this is the minimum, maximum, or mean distance. Proclus takes it to be the maximum distance.[289] The Moon has a maximum distance of 64;10 Earth radii, which is equivalent to Mercury's minimum distance. Since the ratio of Mercury's minimum and maximum distance is 33;15 to 91; 30, then Mercury's maximum distance would be 177;33 Earth radii. Venus has a maximum and minimum ratio of 15;35 to 104;25 and its minimum and maximum distance in Earth radii would be 177;33 and 1,190. Since the maximum distance that Venus reaches from the Earth,

[286] Carman 2009, 216-225.
[287] Carman 2009, 238.
[288] Manitus 1909, 222.
[289] Toomer 1998, 257.

namely 1,190 Earth radii, is greater than the minimum distance that the Sun reaches, namely 1,160 Earth radii, Venus and the Sun would overlap. Proclus says that Venus's maximum distance was very close to the Sun's minimum distance.[290] The values that Proclus gives in parts for the minimum and maximum distances are not the rounded values that Ptolemy provides in the *Planetary Hypotheses*, but values derived from the *Almagest*.

In addition to Proclus's work, there is an anonymous scholion in the *Almagest* B and C manuscripts, which discusses the distances of the Moon, Mercury, Venus, and the Sun.[291] The values for the minimum and maximum distances of planets found in the scholion are not the rounded values that appear in the *Planetary Hypotheses*. They are instead taken directly from the *Almagest*. The values found in the scholion and Proclus's *Hypotyposis* are compared in the table below:

Table 12: Ratios of the Distances of the Planets in Proclus's *Hypotyposis* and the Anonymous Scholion

	Proclus's *Hypotyposis*		Anonymous Scholion	
	Ratio of minimum to maximum distance	True distance in Earth radii	Ratio of minimum to maximum distance	True distance in Earth radii
Moon	–	64;10 (max.)	–	64;10 (max.)
Mercury	33;15 : 91;30	64;10 : 177;33	33;4 : 91;30[292]	64;10 : 177;33
Venus	15;35 : 104;25	177;33 :1,190	15;31 : 104;25	177;33 : 1,190
Sun	–	1,160 : 1,210	–	1,210

[290] Manitus 1909, 220-225.
[291] I would like to thank Alexander Jones for bringing this scholion to my attention, and for providing me with a translation.
[292] The maximum distance in parts for Mercury appears as 91;30 in the text of the scholion, but as 94;30 in the table that follows the text.

Both Proclus and the author of the scholion give different values for the minimum distances of Mercury and Venus, but these do not affect the values for the absolute distances of the planets. The author of the scholion follows the same method as Proclus and Ptolemy, except that the distance of the Sun was always assumed to be 1,210 Earth radii. The text says that this is because the difference between its greatest and least distance is imperceptible to the senses, an argument which neither Ptolemy nor Proclus make. The author of the scholion may have realized that the difference between the minimum distance and maximum distance for the Sun found in the *Hypotyposis* and *Planetary Hypotheses* are insignificant. Like Ptolemy, the author of the scholion says that there should be no void, yet makes no comment on the gap between maximum distance of Venus, namely 1,190 Earth radii, and the distance of the Sun, namely 1,210 Earth radii.

After determining the distances of the planets in Earth radii, Ptolemy calculates the distance of each planet in myriad stades. He begins by stating that the circumference of the Earth is 180,000 stades and therefore the radius of the Earth is 2;52 myriad stades or 28,666;40 stades.[293] The value for the circumference of the Earth is the same one he gives in the *Geography*.[294] Ptolemy presents the following minimum and maximum distances for the Sun, Moon, and planets in myriad stades:[295]

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[293] A stade is a unit of measurement used regularly in the Greek world. It originates with the length of a racetrack (στάδιον). There were stades of several different lengths in use and the precise distance of the stade that Ptolemy uses is unknown. Evans 1998, 48. A myriad stades is 10,000 stades. Goldstein 1967, 7.
[294] Berggren and Jones 2000. Aristotle reports that mathematicians calculate the circumference of the Earth to be 400,000 stades. Aristotle, *On the Heavens*, 289a15-20. Aristotle's report is the earliest recorded calculation for the circumference of the Earth. Eratosthenes estimated the Earth's circumference to be either 250,000 or 252,000 stades. Evans 1998, 65. Archimedes's work, the *Sand Reckoner*, examined how many grains of sand were needed to fill the universe. In this work, Archimedes assigns the fixed stars a distance of 100,000,000 Earth radii. Van Helden 1985, 5-10. Swerdlow 1968, 213-214.
[295] Goldstein 1967, 7-8.

Table 13: Distances of the Planets in Myriad Stades

	Minimum Distance	Maximum Distance
Moon	94;36[296]	183;28
Mercury	183;28[297]	475;52
Venus	475;52	3,093;8
Sun	3,093;8	3,612
Mars	3,612	25,284
Jupiter	25,284	44,769;22
Saturn	44,769;22[298]	56,946;2

If we assume that one stade is $\frac{1}{8}$ of a Roman mile, then Ptolemy's estimate for the maximum distance of Saturn, which is equal to the distance of the sphere of the fixed stars, was over 50 million miles.[299] After giving the absolute distances of the celestial bodies in myriad stades, Ptolemy returns to the idea that there is no empty space in the universe. He says:

> If (the universe is constructed) according to our description of it, there is no space between the greatest and least distances (of adjacent spheres), and the sizes of the surfaces that separate one sphere from another do not differ from the amounts we mentioned. This arrangement is most plausible, for it is not conceivable that there be in Nature a vacuum, or any meaningless and useless thing. The distances of the spheres that we have mentioned are in agreement with our hypotheses.[300]

In the *Almagest*, Ptolemy elaborated on the idea that there was nothing superfluous in nature. When deciding between mathematically equivalent, but physically different, models Ptolemy's theory that there is no empty space in the cosmos guided his conclusions.[301]

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[296] *Arabic-A* reads 74 instead of 94. Goldstein 1967, 8 n. 8.
[297] *Arabic-A* reads 133. Goldstein 1967, 8 n. 9.
[298] *Arabic-A* reads 4,999 instead of 4,769. This number is corrupt see Goldstein 1967, 8, 11.
[299] Van Helden 1985, 5, 24.
[300] Goldstein 1967, 8.
[301] Swerdlow 1968, 98. Other than the method Ptolemy used to determine the distances of the planets, there is only one other method we see used. This method is discussed by al-Bīrūnī in *India*. In this method, the velocity of each planet was considered to be the same, so the speed of the celestial bodies are equal to the distance they cover is different depending on the size of the sphere they traverse.

III.6.4 Diameters and Volumes of the Planets

Following his discussion of the relative and absolute distances of the planets, Ptolemy calculated the apparent and relative diameters of the Sun, Moon, and planets. Additionally, he calculated the volume for each celestial body. The size of a planet can be determined from the distance and apparent size. Therefore, Ptolemy outlined the apparent diameter of each planet relative to the Sun. Building off of the observations of Hipparchus, Ptolemy gave the apparent diameter of each planet for the planet at mean distance. His observations corresponded with the observations that Hipparchus made for Venus, the first magnitude stars, and the Sun.[302] Ptolemy did not specify how the estimates of planetary width were made, but with reference to the Keskintos Inscription, Alexander Jones suggests that the study of the sizes of the planets may have relied on small units of arc other than degrees. Jones says: "Thus investigations of cosmic sizes and distances could give rise to a metrology based on small units of arc independent of degrees. Interestingly, two passages of the Sanskrit *Pañcasiddhāntikā* of Varāhamihira, likely derived from Greek sources, specify a division of the Moon's disk into 15 units."[303] This explanation accounts for Ptolemy's comparison of the apparent size of each planet to that of the Sun's.

Ptolemy explained that if the diameters of each planet subtended the same angle, "the ratio of one diameter to another would equal the ratio of their distances, because the ratio of the circumferences of circles, as well as of similar arcs, one to another, is equal to

[302] Ptolemy does not report Hipparchus's observations for the remaining bodies. Goldstein 1967, 8. Morelon 1993, 74-75. The angular diameters that Ptolemy gives in the *Canobic Inscription* may have been taken from Hipparchus. Swerdlow 1968, 76.
[303] Jones 2006, 15-16.

the ratio of their radii."[304] However, Ptolemy explains that from the Earth the diameter of each celestial body subtends a different size angle, so Euclid's proposition cannot be utilized in this case. Consequently, Ptolemy determines the diameter of each planet relative to the Sun, assuming the Sun has a distance of 1,210 Earth radii. He does this by multiplying the apparent size of each body by its mean distance in Earth radii.[305] For example, for the Moon Ptolemy multiplies the apparent diameter, $1\frac{1}{3}$ Earth radii, by the mean distance, namely 48 Earth radii.[306] The result, 64, is the diameter of the Moon relative to the Sun's diameter when the Sun's diameter is 1,210 Earth radii.

Table 14: Sizes of the Planets

	Apparent size relative to the Sun	Mean distance in Earth radii	Diameter relative to the Sun when the Sun's diameter is 1,210
Moon	$1\frac{1}{3}$	48[307]	64
Mercury	$\frac{1}{15}$	115	8
Venus	$\frac{1}{10}$	$622\frac{1}{2}$	62
Sun	1	1,210	1,210
Mars	$\frac{1}{20}$	5,040	252
Jupiter	$\frac{1}{12}$	11,504[308]	959
Saturn	$\frac{1}{18}$	17,026	946
1st magnitude stars	$\frac{1}{20}$	19,865 or 20,000	1,000

304 Goldstein 1967, 8. Morelon 1993, 74-75. Ptolemy relies on Euclid, proposition VI.33, which states: "In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences". Heath 1926, 273-274.

305 Goldstein 1967, 8. Morelon 1993, 74-75.

306 For a discussion about the different diameters Ptolemy gives for the Sun and Moon see Carman 2009, 238 n. 37.

307 This value should in fact be 48.5, since it is supposed to be the average of 33 and 64.

308 This value should in fact be 11,503.5, since it is supposed to be the average of 8,820 and 14,187.

Ptolemy reminds the reader that in the *Almagest* he reported that when the diameter of the Earth is 1, the diameter of the Sun is $5\frac{1}{2}$. Using this ratio, Ptolemy calculated the sizes of all of the celestial bodies relative to the Earth. He then calculated the volume of each body, relative to the volume of the Earth and then ranked the sizes of the celestial bodies from largest to smallest.[309] His results are as follows:

Table 15: Sizes and Volumes of the Planets

	Diameter in Earth radii	Volume compared to the Earth	Order of the sizes from largest to smallest
Moon	$\frac{1}{4}+\frac{1}{24}$	$\frac{1}{40}$	8
Mercury	$\frac{1}{27}$	$\frac{1}{19683}$	9
Venus	$\frac{1}{4}+\frac{1}{20}$	$\frac{1}{44}$	7
Sun	$5\frac{1}{2}$	$166\frac{1}{3}$	1
Mars	$1\frac{1}{7}$	$1\frac{1}{2}$	5
Jupiter	$4\frac{1}{3}+\frac{1}{40}$	$82\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$	3
Saturn	$4\frac{1}{4}+\frac{1}{20}$	$79\frac{1}{2}$	4
1st magnitude stars	$4\frac{1}{2}+\frac{1}{20}$	$94\frac{1}{16}+\frac{1}{8}$	2
Earth	1	1	6

Ptolemy determined the volume of each celestial sphere by cubing the diameter of the sphere.[310] Ptolemy explained that if the distances that he presented for the planets were indeed correct, then the sizes were also correct; however, if the true distances of the planets were greater than his calculations, then the sizes of the planets would also be greater.[311] Given the arrangement of the planets, Ptolemy argued that Mercury, Venus, and Mars should all display some parallax, specifically that the ratio of Mercury's

[309] Goldstein 1967, 8-9. Morelon 1993, 74-76.
[310] Goldstein 1967, 9, 12, 32. In the third column of the table the Moon has a larger volume than Venus, but is ranked as being smaller. Morelon 1993, 80-83
[311] Goldstein 1967, 9, 32. Morelon 1993, 80-83

parallax to the Moon's parallax is equal to the ratio of their distances. He says that the same is true for Venus and Mars.[312] Consequently, the parallax of Mars at its perigee should be equal to the parallax of the Sun at its apogee and so forth. If the parallax of the planets could in fact be observed, then the order of the planets could be determined with more certainty. For example, the celestial body that follows the Moon should, at its closest distance, display the same amount of parallax that the Moon does at its furthest distance. This alone would have allowed Ptolemy to conclude whether the Sun or Mercury followed the Moon, something he debated in the *Almagest* and *Planetary Hypotheses*.[313] Moreover, since Ptolemy contended that Mars should display parallax, then the Sun's position could be determined based on if its parallax at its minimum distance matched the Moon's parallax at its furthest distance (placing the Sun above the Moon) or Venus's parallax at its furthest distance (placing the Sun above Venus). If neither were the case, then Ptolemy could have concluded that the Sun followed Saturn, the third option for the position of the Sun that he presents in the *Almagest*.

Although he did not estimate the sizes of the planets, in the *Almagest* Ptolemy calculated the sizes of the Sun and Moon in Book V.16. Ptolemy explained that if the Earth's radius is equal to 1, then the Moon's radius is equal to 0;17,33 and the Sun's radius is approximately $5\frac{1}{2}$ times the radius of the Earth's.[314] In the *Planetary Hypotheses* Ptolemy achieved nearly the same results as he did in the *Almagest*. He lists the Moon's diameter as $\frac{1}{4}+\frac{1}{24}$ of the Earth's, which equals 0;17,30 and he lists the Sun's diameter as

[312] Goldstein 1967, 9, 32. Morelon 1993, 80-83
[313] Toomer 1998, 419. Goldstein 1967, 9, 32. Morelon 1993, 80-83
[314] Ptolemy also gives the diameters of the Earth and Sun in terms of the Moon's diameter: the Earth's diameter is $3\frac{2}{5}$ times the Moon's diameter and the Sun's diameter is $18\frac{4}{5}$ times the Moon's diameter. Toomer 1998, 257.

$5\frac{1}{2}$ times the Earth's. In the *Almagest* Ptolemy concluded that if the Moon's volume is 1 (i.e. the diameter cubed, so 1^3), then the Earth's volume is $39\frac{1}{4}$ (i.e. $3\frac{2}{5}$^3), and the Sun's volume is $6644\frac{1}{2}$ (i.e. $18\frac{4}{5}$^3). In the *Planetary Hypotheses*, Ptolemy assigned the Earth, instead of the Moon, the diameter of 1. His result for the size of the Moon was still approximately $\frac{1}{3}$ the size of the Earth and his result for the size of the Sun was approximately $5\frac{1}{2}$ times the size of the Earth; however, there was a problem in the calculation of Venus's volume. Ptolemy gave the diameter of Venus as $\frac{1}{4}+\frac{1}{20}$, which equals 0;18. With this diameter, the volume of Venus should be 0;5,24, but instead it is $\frac{1}{44}$ (0;1,22). Goldstein hypothesizes that this discrepancy was due to a transcription error and that the diameter of Venus should be $\frac{1}{4}+\frac{1}{30}$, which results in a volume of approximately $\frac{1}{44}$.[315] After giving the volume of each celestial body, Ptolemy ranked them in size from largest to smallest. The order that he gives is: Sun, first magnitude stars, Jupiter, Saturn, Mars, Earth, Venus, Moon, and Mercury. This ranking does not match the volumes that he calculated since the volume Ptolemy determined for Venus, which was $\frac{1}{44}$ the size of the Earth, was smaller than the volume that he determined for the Moon, $\frac{1}{40}$ the size of the Earth, but he ranked Venus as being larger than the Earth.[316] Either the text is corrupt here or Ptolemy was inconsistent with his calculations.[317]

While determining the volumes of the celestial bodies is a somewhat futile exercise, it was also done regularly in Greek astronomy. Hipparchus is reported to have

[315] Goldstein 1967, 9; 12, 32. Morelon 1993, 80-83
[316] Goldstein 1967, 8-9, 32. Morelon 1993, 80-83
[317] Carman hypothesizes that the diameter and the ranking that Ptolemy gives for Venus are correct and the error occurs in the value for Venus's volume. Carman 2009, 229-232.

calculated the volume of the Sun and Moon. According to Theon of Smyrna, Hipparchus calculated the Sun to be 1,180 times the volume of the Earth and he calculated the volume of the Earth to be 27 times that of the Moon.[318] Additionally, Cleomedes reports that Hipparchus demonstrated the Sun to be 1,050 times the size of the Earth.[319] Ptolemy's estimate of the Sun's volume relative to the Earth's, $6644\frac{1}{2}:1$, is larger than estimate of the Sun's volume by either Hipparchus or Cleomedes. The difference in the volumes was due to the discrepancy in the lunar and solar distances calculated by Hipparchus, namely $67\frac{1}{3}$ Earth radii and 490 Earth radii respectively. Additionally, when calculating the distance of the Sun and Moon in the *Almagest* Ptolemy did not use the same method that Hipparchus used.[320]

The *arcus visionis* refers to the distance the Sun must be below the horizon in order for a planet to be visible. Each celestial body has a different *arcus visionis*, which, as Ptolemy explains in the *Almagest*, is due to three factors: "the first of these is due to the fact that they are of unequal size, the second due to the variation of the inclination of the ecliptic to the horizon, and the third due to their positions in latitude."[321] In the *Canobic Inscription* the values for first and last appearance are similar to those in the *Almages*t, except in the case of Mercury.[322] The parameters Ptolemy presents in the

[318] Hiller 1878, 197. Bowen and Todd 2004, 118. A discussion of Hipparchus estimates for the sizes and distances of the Sun and Moon can be found in Toomer 1974, 126-142.
[319] Toomer 1974, 140. Ziegler 1891, 152.
[320] Ptolemy criticizes the method Hipparchus used, but Toomer shows that Hipparchus used a more sophisticated procedure than Ptolemy gives him credit for. Toomer 1974, 128-129.
[321] Toomer 1998, 636-647. For a discussion of *arcus visionis* see Neugebauer 1975, 234-238.
[322] The value for Mercury in the *Almagest* is 10º while in the *Canobic Inscription* it is 10;31º or 10;34º. Manuscript A and C say 10;31º. Manuscript B says 10;34º. Bullialdus corrects this to 10;30º. Jones 2005a, 74-75. Hamilton, Swerdlow, Toomer 1987, 68.

Planetary Hypotheses and the *Handy Tables* are the same, although Ptolemy gives a value for first magnitude stars in the *Planetary Hypotheses*.[323]

Table 16: Comparison of *Arcus Visionis*

	Almagest and *Canobic Inscription*	*Planetary Hypotheses* and *Handy Tables*
Mercury	10;30	12;0
Venus (morning setting and evening rising)	5;0	7;0
Venus (evening setting and morning rising)	5;0	5;0
Mars	11;30	14;30
Jupiter	10;0	9;0
Saturn	11;0	13;0

While the preface of the *Handy Tables* has come down to us, the tables themselves only exist as reported by Theon of Alexandria. The *Handy Tables* significantly deviate from the *Almagest* concerning the planetary latitude theory and the visibility of the planets. However, B.L Van Der Waerden has shown that the planetary latitudes agree with the preface of the *Handy Tables* and Asger Aaboe has shown that the planetary visibilities agree with the *Almagest*. This led Aaboe to conclude that the tables reported by Theon of Alexandria are indeed Ptolemy's tables.[324] While the values presented in the *Almagest* and the *Handy Tables* are similar, two sets of values are different: the values for the arcs, and the calculations for planetary latitude. As for the former, Aaboe contends that values for *arcus visionis* presented in the *Handy Tables* (and therefore in the *Planetary*

[323] The *Planetary Hypotheses* gives the *arcus visionis* for first magnitude stars as 15;0.
[324] Van der Waerden 1953, 261-272. Aaboe 1960.

Hypotheses as well since they are the same) are more conservative estimates than those found in the *Almagest*; with regards to the latter, Aaboe argues that the differences are due to simplifications in the method of calculation, specifically ignoring the eccentricity and identifying first and last appearance with conjunction.[325] Aaboe says: "The reason for Ptolemy's crude latitude computations in the visibility section of the Handy Tables, as well as for his summary treatment of this question in the Almagest, may be that this topic no longer was of great interest to him and that he included it at all only out of deference for the central position which it used to hold in mathematical astronomy."[326] When writing the *Planetary Hypotheses* Ptolemy relied on the values for *arcus visionis* from the *Handy Tables* despite the fact that he introduces a new latitude theory for the planets in the *Planetary Hypotheses*. Since the changes to the *arcus visionis* values would have affected the new latitude theory minimally, it is unlikely that Ptolemy found it necessary to recalculate the values.

III.6.5 Optical theory and the *Planetary Hypotheses*

Following his calculations for the distances and sizes of the Moon, Sun, and planets, Ptolemy discussed the difficulties the eye encounters when determining great distances. He explained that an optical illusion occurs due to a difference in the view and this difference is apparent in everything that is seen at great distances. He claims that the planets appear closer to us than they are because of the degeneration of sight, suggesting that the eye has a visual disability to estimate distances.[327] Ptolemy is referring to the sizes of the planets that he has calculated and why these sizes do not match observations.

[325] Aaboe 1960, 16-18.
[326] Aaboe 1960, 17-18.
[327] Goldstein 1967, 9, 32-35. Morelon 1993, 80-85.

Both Ptolemy and Euclid present visual theories and in each distance is perceived by interpreting the angles of objects. For example, definition four of Euclid's *Optics* states: "those things seen within a larger angle appear larger, and those seen within a smaller angle appear smaller, and those seen within equal angles appear to be of the same size".[328] Visual rays are emitted from the eye in the shape of a cone. When these rays meet an object the size of the object can be determined by the size of the angle that subtends the object. Unlike Euclid, Ptolemy takes into account distance when explaining perception. In the *Optics* Ptolemy explains that the eye emits visual flux and "the nature of visual radiation is perforce continuous rather than discrete".[329] Sight is similar to tangible contact and when the visual flux encounters an object it senses it through touch.[330] The size of an object is judged by taking into account angles, slant, and distance. In Book II of the *Optics* Ptolemy says:

> The visual faculty also discerns the place of bodies and apprehends it by reference to the location of its own source-points [i.e., the vertices of the visual cones], which we have already discussed, as well as by the arrangements of the visual rays falling from the eye upon those bodies. That is, longitudinal distance [is determined] by how far the rays extend outward from the vertex of the cone, whereas breadth and height [are determined] by the symmetrical displacement of the rays away from the visual axis. That is how differences in location are determined, for whatever is seen with a longer ray appears farther away, as long as the increase in [the ray's] length is sensible.[331]

Ptolemy contends that an object is judged to be close to the eye when the ray that makes contact with the center of the object is short and an object is judged to be further away when the center visual ray is longer.[332] The visual ray that touches the center of the object is vital in determining the distance of the object. The power of the visual flux differs from

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[328] Burton 1945, 357.
[329] Smith 1996, 91.
[330] Smith 1996, 75.
[331] Smith 1996, 81-82.
[332] Smith 1996, 92.

person to person; while some people can see objects at great distances, others must hold objects close to their eyes to see since the visual flux is weak in them.[333] Consequently, the eye's ability to perceive distances depends on the distance between the eye and the object and the power of the observer's visual flux.

Ptolemy refers to optical illusions when discussing the perception of distance in the *Planetary Hypotheses*. In both the *Optics* and the *Almagest* Ptolemy discusses optical illusions and the difficulties that are present when observing the celestial realm. In Book III of the *Optics*, Ptolemy says:

> Generally speaking, in fact, when a visual ray falls upon visible objects in a way other than is inherent to it by nature and custom, it perceives less clearly all the characteristics belonging to them. So too, its perception of the distances it apprehends will be diminished. This seems to be the reason why, among celestial objects that subtend equal visual angles, those that lie near the zenith appear smaller, whereas those that lie near the horizon are seen in another way that accords with custom. Things that are high up seem smaller than usual and are seen with difficulty.[334]

The distance of a celestial object near the zenith is difficult to establish because the eye must gaze up to see the object and the visual ray falls upon the object in a way in which the visual ray is not accustomed to doing. When the same celestial object is viewed near the horizon then the visual ray makes contact with the object in a way that is "inherent to it by nature and custom".[335] The difficulty in viewing celestial objects near the zenith does not concern the distance of the object (although that can play a role), but the manner in which the eye views the object.

Ptolemy accounts for the Moon illusion – the phenomenon that the Moon's angular diameter appears greater when the Moon is at the horizon than when it is at its

[333] Smith 1996, 107. Smith 1999, 42.
[334] Smith 1996, 151.
[335] Smith 1996, 151.

zenith – in the *Optics* and the *Almagest*. In Book V of the *Optics* and in Book I.3 of the *Almagest* Ptolemy attributes the difference in the lunar diameter to atmospheric vapors and refraction.[336] In the *Optics* Ptolemy says:

> It has thus been demonstrated how stellar observation must be affected by the refraction of the visual ray. It would also be possible for us not only to examine the degree of such refractions, but also to analyze such refraction in the case of certain [celestial] bodies whose distance is given – e.g., the sun and the moon – and to determine the degrees [of refraction] toward the horizon as well as the amount by which the refraction of the visual ray shifts the apparent position upward if the distance of the interface between the two media [i.e., air and ether] were known. But, although this distance lies nearer than the earth to the lunar sphere, where the ether stops, it is not known whether the [refractive] interface lies at the same distance as the aforesaid surface, or whether it lies nearer the earth, or whether it lies farther from the [aforesaid] surface. Therefore, it is impossible to provide a method for determining the size of the angles of deviation that occur in this sort of refraction.[337]

Ptolemy explains that when viewing a celestial object the visual ray must pass through two mediums: air and ether. In Book V of the *Optics* Ptolemy explains that when the visual ray passes through two transparent mediums the visual ray can be broken. At the beginning of Book V Ptolemy says:

> There are two ways in which the visual ray is broken. One involves rebound and is caused by reflection from bodies that block the [visual ray's] passage and that are included under the heading of "mirrors." The other way, however, involves penetration and is caused by a deflection in media that do not [completely] block the visual ray's passage, and those media are included under the single heading of "transparent".[338]

According to Ptolemy's theory, when the visual ray passes through two different mediums, it should be refracted. This refraction would affect celestial observations, such as the apparent positions or sizes of celestial bodies. Since the difference in the medium

[336] Toomer 1998, 39.
[337] Smith 1996, 241-242.
[338] Smith 1996, 229.

of the air and ether is not known and since it is unclear where ether begins and air ends, Ptolemy argues that it is impossible to determine the refraction that occurs.

In the *Optics* Ptolemy discusses the topic of illusion in relation to celestial bodies and how the mind can misinterpret information. In Book II Ptolemy says:

> At this juncture, though, we ought to point out that everything we have said about illusion applies not only to an illusion due to the sense of sight itself but also to the perception that arises from it. And since we are deceived in several cases when the visual sense impinges on visible objects according to its nature and habit, while the mind, remaining in continuous apperceptual touch with such objects, judges them to be abnormal, we must reiterate that this is an illusion involving mental inference.[339]

Ptolemy points out that illusions can be to the result of the mind's flawed deductions. The passages about visual perception in Ptolemy's astronomical theory combined with his optical theory provide insight into the relationship between what can be observed in the heavens and how these observations relate to reality. According to Ptolemy, optical illusions can affect the perception of objects. Ptolemy's explanation of the Moon illusion, which says that refraction causes the Moon to appear bigger when near the horizon than when in the zenith, allow the astronomer guidance in determining which observations of the Moon are affected by refraction in a significant way. While in the *Almagest* Ptolemy argues that observations taken near the horizon are particularly problematic, in the *Optics* he discusses the difficulties in taking observations near the zenith. The problems with observations near the zenith lie with the unaccustomed way that the visual ray falls an object near the zenith. The visual ray is better able to examine objects when the line of sight is horizontal than when it is vertical; observations of celestial bodies directly overhead are more difficult to see and are usually perceived as being smaller than they

[339] Smith 1996, 126.

really are. Consequently, observations taken both near the zenith and near the horizon can be problematic.

In the *Planetary Hypotheses* Ptolemy does not refer to the same problems in astronomical observation that he mentions in the *Almagest* and *Optics*. Instead, he refers to the problem of determining the distance of a celestial object as being due to the eye's difficulty in estimating large distances. Ptolemy says that these difficulties are in line with the problems laid out according to the principles of optics. While he surely must mean his ideas on optics presented in other works, his explanations of the difficulties of astronomical observations do not match those found in his other works.

In the *Almagest*, *Optics*, and *Planetary Hypotheses* Ptolemy discusses the various difficulties that exist when observing the heavens. He does not, however, explicitly examine what this means for the accuracy of the observations that astronomers record. While he offers an explanation for the discrepancy in the angular measurements of the Moon when it is taken near the horizon and near the zenith, he does not give an account of how the astronomer can overcome difficulties due to the distance of the celestial body or the relative magnitudes.

III.7 The *Planetary Hypotheses* as a whole

The content and style of Book I and Book II of the *Planetary Hypotheses* vary considerably. While Book I provides a succinct description of the astronomical models and parameters to be utilized by astronomers and instrument-makers, Book II consists of a discussion and description of the physical components of Ptolemy's models. Book II follows Book I in sequence only, for the substance of the Books is widely divergent.

III.7.1 The Unity of Book I and Book II

While there are important differences between the content of Book I and Book II, there is a natural progression to the text. Ptolemy began Book II by briefly reviewing what he covered in Book I. He tells the reader that in Book I he described the arrangement of the spheres and their motions and that he will now turn to the nature of ethereal unchanging bodies.[340] The material that Ptolemy undertook in Book II has similarities to the material he addresses in Book I of the *Almagest*. In the *Almagest*, Ptolemy began with a discussion of the principles that guide his astronomical theories, such as the spherical shape of the Earth and cosmos, the location of the Earth at the center of the heavens, the uniform movement of the heavens, and the finite structure of the celestial realm. These principles are based on observations and Ptolemy used them as tenets when constructing his theories. Other concepts that guide Ptolemy's practice of science, such as the idea that the heavens are simple, appear elsewhere in the *Almagest*. After laying out these ideas, Ptolemy plunges into the mathematical details of this theory, beginning with the construction of a table of chords in Book II.

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[340] Heiberg 1907, 111. Goldstein 1967, 36.

In Book I of the *Almagest*, Ptolemy tells the reader that mathematics leads to certain knowledge and that it is through mathematics that one can gain knowledge of physics and theology.[341] If we investigate the unity of Book I and Book II of the *Planetary Hypotheses* in light of Book I of the *Almagest*, then Ptolemy's approach to understanding the physical aspects of celestial realm is consistent with his philosophical ideas about how to acquire knowledge. The order and content of the *Planetary Hypotheses* seems to follow the principles laid out in Book I of the *Almagest*: after a detailed and mathematical discussion of the celestial motions in Book I, Ptolemy discusses the physical attributes of the heavens, including the material, position, size and cause of motions. Because of the mathematical content of Book I, Ptolemy can expand the boundaries of his investigation and discuss the physics of the celestial realm.

III.7.2 The Lost Tables at the End of Book II

Ptolemy explains that Book II will conclude with a set of tables. Unfortunately for historians these tables have been lost. The *Planetary Hypotheses* tables are not mentioned until the end of Book II. Ptolemy says in Book II that the tables are a set of mean motion tables that contain one column for the Sun and four columns for the other bodies.[342] Although these tables are lost, Dennis Duke has suggested that the tables can be reconstructed from the information found in Book I.[343] Duke examines the values that Ptolemy included in the *Planetary Hypotheses* tables and compares them to the values

[341] Feke 2009, 39-64, 219-220. Feke explores Ptolemy's division of practical philosophy into physics, mathematics, and religion.
[342] Heiberg 1907, 143.
[343] Duke 2009, 650-653.

found in the *Almagest*. He concludes that Ptolemy made changes to the parameters for Saturn, Jupiter, and Mercury from the *Almagest*, leaving Mars and Venus the same.[344]

Since tables would have been included in the original text, we can draw some conclusions about for whom the *Planetary Hypotheses* was written. The tables probably would have been tools for astronomers to use in predicting the motions of the celestial bodies. Similar to the tables found in the *Handy Tables*, the tables in the *Planetary Hypotheses* would have offered astronomers a way of calculating the positions of the celestial bodies, without working through the details of the theories found in the *Almagest.* The descriptions that Ptolemy gives of his astronomical models are comprehensive, yet succinct. This implies that they were not necessarily written for a specialist astronomer, but instead an astronomer who was not familiar with the *Almagest* or someone new to the study of astronomy. The tables, however, mean that the *Planetary Hypotheses* is not simply an introduction to Ptolemaic astronomy, but that it is also intended for someone who wants to calculate the position of celestial bodies.

III.7.3 The Context of the *Planetary Hypotheses* in Greco-Roman Astronomy

Given the scope and content of the *Planetary Hypotheses*, this work is unique compared to other Greco-Roman astronomical works. This is the only work in Ptolemy's corpus in which he discusses building a comprehensive model describing celestial motion, the physical components of his astronomical theory, or examines in detail the nature of ethereal bodies; however, the *Planetary Hypotheses* was not an isolated text, since it complements Ptolemy's other astronomical works and addresses contemporary philosophical discussion on cosmology. In Book I, Ptolemy clearly specified for whom

[344] Duke 2009, 653.

his work is intended: instrument-makers and those like himself, namely astronomers. Conversely, in Book II, Ptolemy appears to address a wider audience. In addition to astronomers and instrument-makers, his discussion addresses philosophical and physical ideas concerning the composition and driving forces of the cosmos. Consequently, Ptolemy's work engages with several different traditions.

For the astronomer, the description of his astronomical models, parameters, volumes, and distances provides a succinct overview of the major components of Ptolemy's theory. Additionally, the instrument-maker could find the concise descriptions and the parameters useful in attempting to construct a mechanical or hand-operated model of the heavens. As I discussed in Section III.1, Ptolemy assumed that his readers were familiar with the traditional type of instrument-making. Precisely whom Ptolemy is addressing is unclear, since few references to *sphairopoiia* by predecessors or contemporaries exist. While Ptolemy is clearly addressing instrument-makers, historians unfortunately know very little about this constituency.[345]

The *Planetary Hypotheses* complements Ptolemy's technical astronomical works in many ways, in particular the *Canobic Inscription*, *Almagest*, and *Handy Tables*. The text begins by summarizing the *Almagest*, but it does not go into the level of detail the *Almagest* does, thus appealing to a wider audience. Moreover, the *Planetary Hypotheses* addresses the physical mechanics of Ptolemy's astronomical models and discusses how those models work. These ideas are conspicuously left out of the *Almagest* and are not addressed in the *Canobic Inscription* or *Handy Tables*; however, these concepts underscore questions about whether Ptolemy's theory is a realistic description of the celestial realm. Finally, the *Planetary Hypotheses* is, according to Ptolemy, a resource for

[345] See Section III.1 above.

the instrument-maker. As I discussed in Section III.1, this infers that there is a tradition of astronomical instrument-making. Since it is only in the *Planetary Hypotheses* that Ptolemy mentions this tradition, Ptolemy implies that the *Planetary Hypotheses* serves this group better than his other works.

The discussions about cosmology in the ancient world included astronomers and philosophers alike. In the *Planetary Hypotheses* Ptolemy engaged with both technical astronomical traditions and philosophical cosmological debates. In Book II of the *Planetary Hypotheses*, Ptolemy mentions Plato's whorls when describing the arrangement of the spheres. In Book II, Ptolemy says:

> The first of them is to assign a whole (*tâmm*) sphere to each motion, either hollow like the spheres that surround each other or the earth, or solid and not hollow like those which do not contain anything other than the thing [itself], namely those that set the stars in motion and are called epicyclic orbs. The other way is that we set aside for each one of the motions not a whole sphere but only a section (*qiṭᶜah*) of a sphere. This section lies on the two sides of the largest circle which is in that sphere, namely that from which the motion is longitude [is taken]. That which this section encloses from the two sides is [equal to] the amount of latitude. Thus the shape (*shakl*) of this section, when [taken] from an epicyclic orb, is similar to a tambourine (*duff*). When taken from the hollow sphere, it is similar to a belt (*niṭâq*), an armband (*siwâr*), or a whorl (*fulkah*), as Plato said. Mathematical investigation shows that there is no difference between these two ways that we have described.[346]

The different components of the celestial realm that are responsible for the movements of the bodies can be, Ptolemy suggests, solid or hollow spheres. Alternatively, the components could consist of pieces that are shaped like tambourines or belts. Both options are equivalent mathematically and the only difference is the physical shape of these sections. In describing the set up consisting of solid and hollow spheres, Ptolemy

[346] Langermann 1990, 19. For the Arabic see Goldstein 1967, 37. For a description of Ibn al-Haytham's works see the article in the *New Dictionary of Scientific Biography,* s.v. "Ibn al-Haytham, Abū ʿAlī al-Ḥasan ibn al-Ḥasan."

makes a comparison to the whorls Plato describes. In the *Republic*, Plato describes the cosmos as a series of eight whorls, where each whorl is nested inside another:

> The nature of the whorl was this: Its shape was like that of an ordinary whorl, but, from what Er said, we must understand its structure as follows. It was as if one big whorl had been made hollow by being thoroughly scooped out, with another smaller whorl closely fitted into it, like nested boxes, and there was a third whorl inside the second, and so on, making eight whorls altogether, lying inside one another, with their rims appearing as circles from above, while from the back they formed one continuous whorl around the spindle, which was driven through the center of the eighth.[347]

Ptolemy does not address Plato's theory in any detail; the only reason that Ptolemy mentions this theory at all seems to be as a reference point for those who are familiar with Plato's *Republic*.

Additionally, in Book II Ptolemy refers to Aristotle's unwinding spheres, a device Aristotle discusses in the *Metaphysics*.[348] Aristotle tells the reader that Eudoxus's system was comprised of three spheres for each of the Sun and Moon and four spheres for each of the planets. Additionally, he says that Callippus added two more spheres to the Sun and Moon and one more sphere to each of Mercury, Venus, and Mars. For Eudoxus and Callippus, the combined motion of multiple concentric spheres drove each celestial body. The poles of each sphere were connected to the inside of the sphere above it, so that the motion of the outermost sphere was transferred to the spheres inside of it. Callippus added more spheres to Eudoxus's model and Aristotle added more spheres to Callippus's model. In particular, Aristotle added a series of unwinding spheres. Aristotle says:

> But it is necessary, if all the spheres combined are to explain the phenomena, that for each of the planets there should be other spheres (one fewer than those hitherto assigned) which counteract those already mentioned and bring back to the same position the first sphere of the star which in each case is situated below the

[347] Plato, *The Republic*, 616b-616e.
[348] Aristotle, *Metaphysics*, 1073a14-1074b14.

> star in question; for only thus can all the forces at work produce the motion of the planets. Since, then, the spheres by which the planets themselves are moved are eight and twenty-five, and of these only those by which the lowest-situated planet is moved need not be counteracted, the spheres which counteract those of the first two planets will be six in number, and the spheres which counteract those of the next four planets will be sixteen, and the number of all the spheres – those which move the planets and those which counteract these – will be fifty-five. And if one were not to add to the moon and to the sun the movements we mentioned, all the spheres will be forty-nine in number.[349]

Aristotle assigned three unwinding spheres for each of Saturn and Jupiter, four unwinding spheres for each of Mars, Venus, and Mercury, and four unwinding spheres for the Sun. The Moon, being the innermost celestial body, did not require unwinding spheres. Aristotle's total number of spheres for the seven celestial bodies came to fifty-five. Moving at the same speed, but in the opposite direction, the unwinding spheres cancelled out the movement of the spheres above so that motion of five spheres responsible for the motion of Saturn, for example, would not be passed on to the spheres responsible for the motion of Jupiter.

Ptolemy argued that the arrangement proposed by Aristotle would not be physically feasible and he produced reasons as to why the concentric spheres proposed by Eudoxus and Aristotle did not provide a realistic description of the cosmos. Ptolemy argues that the spheres can be the cause of their own motion and thus do not need to be driven by something else. Furthermore he states that a stationary object, such as a pole, can be the cause of motion. Finally, he questions how the poles of the sphere would be bound the inside surface of the sphere surrounding it. He argues that if the place where

[349] Aristotle, *Metaphysics*, 1074a1-1074a14.

the pole and sphere are fixed together were denser than the surrounding ether then the denser objects would sink towards the center of the cosmos.[350]

In Book II Ptolemy says that the heavens have an animate power and he refers to the heavens as a celestial animal. Ptolemy says:

> For this reason we must construe the notion (*ʹamr*) of the celestial animal (*al-ḥayawân al-falakiy*). We hold that each one of the stars in its [individual] ranking possesses an animate (*nafsâniy*) power. It moves by itself, bestows motion upon the bodies naturally connected to it beginning with that which is close to it, and passes it on to that which is adjacent to it. For example, it bestows motion first on the epicyclic orb, then on the eccentric orb, and then on the orb whose center is the center of the world. This motion which it bestows is different in many places. For the motion of reason in us is not exactly like the motion of impulse (*ʹinbiᶜâth*), nor is this motion like the motion of the nerves, nor is this motion like the motion of the foot. Rather they have some difference in their outward disposition (*fî maylihâ ʹilâ khârij*).[351]

The celestial motion is different for the distinctive pieces of the heavens and not all parts move in the same way. Ptolemy makes a comparison to the human body, saying that the different components of the body exhibit their motion in different ways. Ptolemy is not alone in describing the heavens anthropomorphically. In Plato's *Timaeus*, the heavens are described as a living thing, complete with a soul and intelligence.

> Accordingly, the god reasoned and concluded that in the realm of things naturally visible no unintelligent thing could as a whole be better than anything which does possess intelligence as a whole, and he further concluded that it is impossible for anything to come to possess intelligence apart from soul. Guided by this reasoning, he put intelligence in soul, and soul in body, and so he constructed the universe. He wanted to produce a piece of work that would be as excellent and supreme as its nature would allow. This, then, in keeping with our likely account, is how we must say divine providence brought our world into being as a truly living thing, endowed with soul and intelligence.[352]

[350] Goldstein 1967, 39. Murschel 1993, 40.
[351] Langermann 1990, 20. For the Arabic see Goldstein 1967, 41.
[352] Plato, *Timaeus*, 30b-c.

Ptolemy does not go into much detail about the heavens as a celestial animal and there is much more he could have said, especially if he introduces this analogy as a way to account for the cause of celestial motion.

Ptolemy clearly states at the beginning of Book II that he will not recount the theories of his predecessors, nor correct their theories. He says that those who want to read the hypotheses written by them can do so since the texts are readily available; however, those who are interested in learning about the celestial realm as it truly exists should read his work. Ptolemy has no intention of engaging with the work of his predecessors, such as Plato and Aristotle, and he does not systematically work through their theories. Instead, he is interested in examining how the heavens move from a physical point of view. As I have shown in Section I.4.2, there are places in both the *Almagest* and *Planetary Hypotheses* where Ptolemy is not fully committed to one theory over another and he consequently presents more than one possible explanation for the phenomena he describes. While in these places he always makes a decision as to which theory he thinks is more plausible, he acknowledges that more than one theory could work. With the exception of whether the spheres are comprised of hollow/solid spheres or tambourines/belts, Ptolemy presents his ideas as findings that he has established to be true. Ptolemy concludes Book II by saying that his arrangements and descriptions of the celestial motions are simpler than those given by his predecessors, and that this can be demonstrated if his work his compared with the work of others.[353] This comment follows Ptolemy's conclusion that his model consisting of whole spheres would require 34 spheres and his model consisting of sawn-off pieces would require 22 pieces. Since these number are substantially lower than the 55 spheres that Aristotle needs, Ptolemy seems to

[353] Heiberg 1907, 143.

be claiming that his model is simpler than the theory Aristotle presents in the *Metaphysics*.

The *Planetary Hypotheses* engages with several different mathematical, astronomical, and philosophical works including Plato's *Timaeus,* Plato's *Republic*, Aristotle's *Metaphysics*, and works by Ptolemy's predecessors, such as Hipparchus and Euclid. There are, however, no existing Greco-Roman texts that are similar in scope and content to the *Planetary Hypotheses*. The *Planetary Hypotheses* engages with metaphysical explanations of celestial motion, providing an account of the motion that is distinct from the ideas proposed by Ptolemy's predecessors.

III.8 The *Planetary Hypotheses* in Islamic Astronomy

Astronomers in the Islamic world were particularly interested in astronomical models in physical terms. The term *ʿilm al-hayʾa* is often used to describe the tradition of works that primarily examined the physical reality of models.[354] The *Planetary Hypotheses* addresses many of the topics often discussed in *ʿilm al-hayʾa* texts. While Book I of the *Almagest* and Aristotle's *On the Heavens* and *Metaphysics* were the focus of these texts, astronomers in the Islamic world indeed referred to the ideas in the *Planetary Hypotheses*.[355] A prominent example is Ibn al-Haytham (d. 1039), who refers extensively to both Book I and Book II of the *Planetary Hypotheses* in his work *Doubts Concerning Ptolemy* (*al-Shukūk ʿalā Baṭlamyūs*).[356] Ibn al-Haytham examines several aspects of Ptolemy's astronomical theory.[357] He discusses the ways in which Ptolemaic astronomy deviates from Aristotelian cosmology. One of his fundamental arguments, which can be found in Essay VIII in *Doubts Concerning Ptolemy,* centers on Ptolemy's account of uniform motion. Since Ptolemy uses a center of uniform motion that is not the Earth, Ibn al-Haytham argues that Ptolemy has not in fact constructed a system that adheres to uniform motion.[358] In Essay XI, Ibn al-Haytham discusses the model for the outer planets, which exhibit uniform motion around a point that is not at the center of the circle (equant point). He says:

> It is not the case that if a man were to postulate a line in his imagination and to move it in his imagination that a corresponding line in the heavens should move that [same] motion. Nor is it the case that if the man were to imagine a circle in

[354] As I mentioned earlier, the term *ʿilm al-hayʾa* has multiple interpretations and can also refer to "the science of the configurations [of the celestial spheres]", i.e. astronomy. Saliba 1999, 134-138.
[355] Langermann 1997, 186-189.
[356] Langermann 1997, 13.
[357] The first person to provide an overview of this text in Western scholarship was Pines 1964, 547-550.
[358] Voss 1985, 44-47.

> the sky and to imagine a planet moving on the circle, that the planet would move on the imaginary circle. This being the case, the models which Ptolemy imagined for the five (5) planets are false models. He established them knowing they were false because he could not do otherwise. There exists a correct model for the motions of the planets in existing bodies which Ptolemy did not understand and did not obtain. It is not correct to find perceptible, eternal motion preserving an order and a structure unless it has a correct model in existing bodies.[359]

Ibn al-Haytham argues that Ptolemy constructed a geometrical model, but failed to construct a model that adheres to the principles of heavenly motion, namely uniform, circular motion.[360] For these reasons, Ibn al-Haytham contends that Ptolemy's models are not physically possible and should be rejected.[361]

In addition to these studies of Ptolemy's astronomical theory, Ibn al-Haytham investigated specific aspects of the *Planetary Hypotheses*, such as changes Ptolemy made to the latitude theory in Book I and the description of the movement of the spheres in Book II.[362] In both cases he argued that the theories that Ptolemy proposed in the *Planetary Hypotheses* did not match the theories presented in the *Almagest* and since the theories in the *Almagest* were constructed using "observations and reasoning" the models in the *Almagest* better accounted for the phenomena. Additionally, in the case of the small vertical circles Ptolemy described in his latitude theory in Book XIII of the *Almagest*, but did not include in his latitude theory in the *Planetary Hypotheses*, Ibn al-Haytham argues that Ptolemy either neglected the circles in the *Planetary Hypotheses* or imagined them in the *Almagest*.[363] Ibn al-Haytham does not seem to have considered that Ptolemy may have, for any number of reasons, made improvements to his latitude model, which would account for the changes.

[359] Voss 1985, 60.
[360] Voss 1985, 52.
[361] Dallal 2010, 70.
[362] Voss 1985, 55-63, 69-77.
[363] Voss 1985, 68-78.

While astronomers in the Islamic world were primarily concerned with the *Almagest*, they also paid attention to the *Planetary Hypotheses*, consistently examining several of the topics that Ptolemy addressed in Book II, primarily the function and physical reality of his models. Astronomers, such as Ibn Bāja (d. c. 1138), Ibn Rushd (d. 1198), and al-Biṭrūjī (d. c. 1200), attempted to eliminate epicycles and eccentrics and reinstate Aristotle's homocentric spheres. Additionally, astronomers such as al-Ṭūsī (d. 1274), Ibn al-Shāṭir (d. c.1375), and al-Qūshdjī (d. 1474) attempted to solve the problems of Ptolemaic astronomy, such as the use of the equant, by proposing new, technical models that combined uniform, circular motion.[364] For example, al-Ṭūsī offered a new and ingenious model, known today as the Ṭūsī couple. This model consisted of two internally tangent circles, one inside of the other with the smaller circle having a diameter half the size of the larger. The two circles travel in opposite directions so that the motion of the smaller circle is twice that of the larger circle and the smaller circle rolls along the inside circumference of the larger circle. The combination of the two results in the oscillation of a point carried by the smaller circle, tracing a straight line.[365] Consequently, linear motion can be created from uniform circular motion and al-Ṭūsī used this device as part of his planetary theory to account for latitudinal motion.[366]

Although the specifics of the *Planetary Hypotheses* are not extensively discussed in works other than Ibn al-Haytham's *Doubts Concerning Ptolemy*, F. Jamil Ragep proposes that the *Planetary Hypotheses* is a model for works such as Naṣīr al-Dīn al-Ṭūsī's *Memoir on Astronomy* (*al-Tadhkira fī ʿilm al-hayʾa*).[367] While the two texts appear

[364] Dallal 2010, 64-93.
[365] Ragep 1993, 194-197.
[366] Saliba 2007, 109-110.
[367] Ragep 1993, 28.

very different in scope and layout they both discuss astronomical models, aiming to provide a physical depiction. For example, in Book I.2 al-Ṭūsī's says:

> Nothing having the principle of circular motion can undergo any rectilinear motion at all, and conversely, except by compulsion. Thus the celestial bodies neither tear nor mend, grow nor diminish, expand nor contract; neither does their motion intensify nor weaken. They do not reverse direction, turn, stop, depart from their confines, nor undergo any change of state except for their circular motion, which is uniform at all times.[368]

In Book I al-Ṭūsī clearly states what he means by uniform motion and introduces his work with an overview of geometrical terms, such as point, line, sphere, circular motion.[369] The emphasis on the physical properties of the geometrical figures that al-Ṭūsī describes seems, at least in part, motivated by Ibn al-Haytham's critiques of Ptolemy's models from the *Almagest* and *Planetary Hypotheses*. Not only does al-Ṭūsī aim to present a theory that matches the phenomena, but he also proposes new models that eliminate the equant.

The calculations of the sizes and distances of the planets and the idea that the spheres are nested so that there is no empty space appear in the works of many of Ptolemy's counterparts in the Islamic world. While al-Bīrūnī attributes the distances of the planets to the *Planetary Hypotheses* in his work *India*, others who rely on these values do not credit Ptolemy. There are several mediaeval scholars who reference the theory of distances and sizes, the roots of which can be traced back to Ptolemy's *Planetary Hypotheses*. These authors include: al-Farghānī (d. c. 861),

[368] Ragep 1993, 100-101.
[369] Ragep 1993, 92-101.

Thābit ibn Qurra (d. 901), al-Battānī (d. 929), al-Qazwīnī (d. c.1283), Bar Hebraeus (d. 1286), Dante Alighieri (d. c. 1321), and Roger Bacon (d. c. 1292).[370] None of these scholars credit Ptolemy for providing the original calculations for the sizes and distances of the planets and most probably did not know that they were relying on his work.

The ideas expressed in the *Planetary Hypotheses* permeated astronomical and cosmological works throughout the Middle Ages and into the Renaissance. Many astronomers in the Islamic world both built on Greco-Roman theories and reacted to the problems and inconsistencies within these theories. The *Planetary Hypotheses* not only deepens our understanding of astronomy and cosmology, but it also provides insight into Greco-Roman and Islamic views of the natural world and the practice of science.

[370] For example, in *De Hiis que indigent expositione antequam legatur Almagesti* (*Tashīl al-Majisṭī*), which survives in both Arabic and Latin, Thābit ibn Qurra gives values for the minimum and maximum distances of the planets that match those found in the *Planetary Hypotheses*, with the exception of the maximum distance of Venus. For the maximum distance of Venus Thābit ibn Qurra provides the same value as the minimum distance of the sun, namely 1,079 Earth radii, consequently eliminating the 81 Earth radii void found in the *Planetary Hypotheses*. The values given by other authors such as al-Farghānī, al-Battānī, and Bar Hebraeus, vary from those presented by Ptolemy. Carmody 1960, 130-139. For a discussion of the use of the *Planetary Hypotheses* values by these authors see Swerdlow 1968, 130-165 and Carmody 1960, 122-130.

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