Dynamic Analysis of Whiplash

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Abstract

This study is concerned with whiplash injuries resulting from the sudden acceleration and deceleration of the head relative to the torso in vehicle collisions. Whiplash is the most common automobile injury, yet it is poorly understood. The objective of this thesis is to develop a representative rigid linkage lumped parameter model using Lagrangian mechanics to capture the relative motion of the head and cervical spine. Joint locations corresponding to the intervertebral centers of rotation are used to simulate the normal spinal movements and an inverse analysis is applied to determine the viscoelastic parameters for the spine, based on cadaver test results. The model is further validated using ANSYS dynamic finite element analysis and experimentally validated using a newly designed and fully instrumented whiplash test fixture. Our findings reveal the effectiveness of the simplified model which can be easily scaled to accommodate differences in collision severity, posture, gender, and occupant size.
Acknowledgments

I extend my appreciation and gratitude to Dr. S.A. Meguid for his expert advice, technical guidance and financial assistance throughout the course of my research. I also wish to acknowledge the assistance of Professors J. Zu and C. Simmons for their kind advice as well as Ben Cornwell-Mott for his experimental contributions. I would also like to pay special thanks to the members of the Mechanics and Aerospace Design Laboratory for their friendship and help during this research. Furthermore, the financial support of the University of Toronto is gratefully acknowledged.
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Notation

$C_1 - C_7$      cervical vertebrae
$T_1$            first thoracic vertebra
$m_{head}$        head mass
$I_{zz,head}$    moment of inertia of the head about the center of mass in the sagittal plane
$L_i$            rigid linkage length
$\theta_i, \dot{\theta}_i$  rigid linkage angle, angular velocity
$\theta_{i,initial}$  initial rigid linkage angle
$\dot{\theta}_i$    rigid linkage angular velocity
$k_i$            joint stiffness
$c_i$            joint damping
$m_i$            joint mass (note: $m_8 = m_{head}$)
$T$              total kinetic energy
$V$              total potential energy
$v_i$            net joint velocity
$x_i, \dot{x}_i$  x component of joint position, velocity
$y_i, \dot{y}_i$  y component of joint position, velocity
$Q_i$            generalized force
$[M]$            mass matrix
$[V]$            gyroscopic matrix
$[C]$            damping matrix
\[K\] stiffness matrix

\[\{Q_e\}\] generalized force vector

\[\{\theta\}_i, \{\dot{\theta}\}_i, \{\ddot{\theta}\}_i\] generalized coordinate vector for joint angle, angular velocity, and angular acceleration

\(m_{i-h}\) sum of joint masses \(m_i\) to \(m_{head}\)

\(\{\ddot{\theta}_i\}, \{\dot{\theta}_i\}\) row vector of angular acceleration and angular velocity

\(\{\dot{\theta}_i\}, \{\theta_i\}\) row vector of angular velocity and joint angle

fit fit for overall head and neck motion

extfit fit for neck motion

headfit fit for head motion

headposfit fit for head position

headrotfit fit for head rotation

\(x_{exp}(t)\) experimental head position in x direction

\(y_{exp}(t)\) experimental head position in y direction

\(x_{head}(t)\) model head position in x direction

\(y_{head}(t)\) model head position in y direction

\(\theta_{exp}(t)\) experimental head rotation

\(\theta_{head}(t)\) model head rotation

\(\Delta_i(t)\) model intervertebral extension

\(\Delta_{exp,i}(t)\) experimental intervertebral extension

\(tsteps\) total number of time steps
\( \theta_{\text{seg}} \) segment rotation

\( M \) joint moment

\( M_{\text{bend}} \) beam bending moment

\( E \) modulus of elasticity

\( I \) second moment of area

\( EI \) bending stiffness

\( \frac{d\theta_{\text{beam}}}{dl} \) beam curvature

\( k_{\text{spring}} \) spring stiffness

\( m_{\text{sled}} \) sled mass

\( \sigma \) beam stress

\( \gamma \) distance from neutral axis

\( \alpha \) smoothing factor

\( \text{smoothed}_{i+1} \) smoothed data point at new time

\( \text{smoothed}_i \) smoothed data point at previous time

\( x_{i+1} \) original data point at new time

\( x_i \) original data point at previous time
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abduction</td>
<td>The movement of a limb away from the midline or axis of the body</td>
</tr>
<tr>
<td>Acute symptoms</td>
<td>Symptoms of short duration but typically severe</td>
</tr>
<tr>
<td>Adduction</td>
<td>The movement of a limb toward the midline or axis of the body</td>
</tr>
<tr>
<td>Anthropometry</td>
<td>The scientific study of the measurements and proportions of the human body</td>
</tr>
<tr>
<td>Biofidelity</td>
<td>The quality of being lifelike in appearance or responses and often refers to dummies used in safety investigations of motor vehicles</td>
</tr>
<tr>
<td>Cervical spine</td>
<td>Of or pertaining to the neck. Cervical vertebrae C1-C8</td>
</tr>
<tr>
<td>Chronic symptoms</td>
<td>Symptoms that are ongoing. Pain that extends beyond the expected period of healing</td>
</tr>
<tr>
<td>Extension</td>
<td>Act of stretching or straightening out a flexed limb</td>
</tr>
<tr>
<td>Facet joints</td>
<td>Any of the four projections that link one vertebra of the spine to an adjacent vertebra. Facet joints (also known as zygodiphyseal joints) are the small joints that connect vertebral bodies to each other.</td>
</tr>
<tr>
<td>Flexion</td>
<td>Act of bending a joint; especially a joint between the bones of a limb so that the angle between them is decreased</td>
</tr>
<tr>
<td>In vitro</td>
<td>A biological process made to occur in a laboratory vessel or other controlled experiment rather than within a living organism or natural setting.</td>
</tr>
<tr>
<td>In vivo</td>
<td>A biological process or experiment occurring in a living body</td>
</tr>
<tr>
<td>Instantaneous axis of rotation (IAR)</td>
<td>See instantaneous center of rotation</td>
</tr>
<tr>
<td>Instantaneous center of rotation (ICR)</td>
<td>Defines the point about which the object rotates. When an object moves it is subject to a combination of rotation and translation. A point may be determined about which the object is only subject to a rotation. This point, called the instantaneous center of rotation or instantaneous axis of rotation, will be defined at a specific point for each movement the object makes.</td>
</tr>
<tr>
<td>Interindidual</td>
<td>Between different individuals</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Intervertebral discs</td>
<td>Lie between each adjacent vertebrae in the spine. Forms a cartilaginous joint to allow movement between vertebrae.</td>
</tr>
<tr>
<td>Kyphosis</td>
<td>Exaggerated curvature of the thoracic spine. Also called a ‘hunchback’.</td>
</tr>
<tr>
<td>Ligament</td>
<td>Fibrous tissue that connects bones to other bones</td>
</tr>
<tr>
<td>Lordosis</td>
<td>Exaggerated lumbar curvature of the spine</td>
</tr>
<tr>
<td>Morphometry</td>
<td>The measurement of the shape of objects. Morphometry includes a large range of measurements including numbers, length, surface area, volume, angles, and curvature.</td>
</tr>
<tr>
<td>Muscle</td>
<td>Is a contractile tissue whose purpose is to produce force and motion</td>
</tr>
<tr>
<td>Nervous system</td>
<td>An organ containing a network of specialized cells called neurons that coordinate the actions and transmit signals between different parts of the body. The central nervous system contains the brain, spinal cord, and retina.</td>
</tr>
<tr>
<td>Parestesias</td>
<td>A sensation of tingling, pricking, or numbness of a person’s skin with no apparent long-term physical effect. More generally known as the feeling of ‘pins and needles’</td>
</tr>
<tr>
<td>Pathophysiology</td>
<td>The study of changes of normal mechanical, physical, and biochemical functions, either caused by disease, or resulting from an abnormal syndrome. Pathophysiology emphasizes quantifiable measurements</td>
</tr>
<tr>
<td>Revolute joint</td>
<td>Allows only relative rotation between two bodies (also called a pin or hinge joint)</td>
</tr>
<tr>
<td>Sagittal plate</td>
<td>A vertical plane passing through the standing body from front to back. The mid-sagittal, or median, plane splits the body into left and right halves.</td>
</tr>
<tr>
<td>Soft tissue</td>
<td>Refers to tissues that connect, support, or surround other structures and organs of the body, not including bone. Soft tissue includes tendons, ligaments, fascia, skin, fibrous tissues, fat, synovial membranes, muscles, nerves, and blood vessels.</td>
</tr>
<tr>
<td>Spinal cord</td>
<td>A long, thin, tubular bundle of nervous tissue and support cells that extends from the brain. The brain and spinal cord together make up the central nervous system. The spinal cord has three main functions including communicating motor information, communicating senses, and coordinating reflexes</td>
</tr>
<tr>
<td><strong>Vertebrae</strong> (vertebra singular)</td>
<td>Any of the bones or segments composing the spinal column, consisting typically of a cylindrical body and an arch with various processes, and forming a foramen, or opening, through which the spinal cord passes.</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Viscoelastic</strong></td>
<td>A property of materials that exhibit both viscous and elastic characteristics when undergoing deformation.</td>
</tr>
</tbody>
</table>
Chapter 1
Introduction and Justification

In this chapter, we define the problem and justify the undertaking of the study. The method of approach to achieve the stated objectives is outlined, followed by a summary of the layout of the thesis.

1.1 Problem Statement

Whiplash is defined by the Quebec Task Force as, “an acceleration-deceleration mechanism of energy transferred to the neck” [1]. During a rear-end automotive collision the torso is accelerated forward as the energy from the collision is transferred to the human body. For unrestrained motion the head is approximately stationary as the torso is initially accelerated forward. This leads to an S-shaped curvature of the cervical spine (see Appendix Figure A.1) with the upper segments flexed and the lower segments extended as shown in Figure 1.1. After this the head begins to rotate rearward until the upper and lower segments of the cervical spine are extended. This is followed by the head rebounding forward into a flexed configuration. Although whiplash injuries are still poorly understood research has suggested they may be associated with abnormal motions in the lower cervical vertebrae early in the collision sequence [2] and the differential motion between the head and the torso [3].

![Figure 1.1: Typical cadaver whiplash motion for 8.5 g rear-end collision acceleration [4]](image-url)

Whiplash injuries are the most common type automobile injury [5], with 85% of all whiplash injuries occurring during rear-end collisions [6]. More than 65% of all whiplash injuries occur at
speeds below 30 km/h [7] with some injuries even occurring where there is no vehicle damage at all [8]. This high prevalence, coupled with human vulnerability, leads to more than 1 million cases of whiplash reported annually in the U.S. alone [7]. These injuries lead to significant costs, painful symptoms, and reduced quality of life for individuals with chronic symptoms. There are many acute and chronic symptoms associated with whiplash as shown in Table 1.1, including neck, back, and shoulder pain and headaches which can range from mild to severe [9]. These injuries are difficult to assess and treat, owing to the complex behaviour of soft tissue and the nervous system in the head and neck and the inability of modern X-Ray, MRI, and CT scan techniques to detect injury [6, 10, 11]. These factors have led to costs of whiplash associated disorders estimated at 10 billion dollars/euros annually in the U.S. and Europe [12-14]. In the U.S. these figures refer to the costs of healthcare and insurance alone. The addition of lost work time due to whiplash associated disorders would make this figure even larger.

Table 1.1: Symptoms of Whiplash [15]

<table>
<thead>
<tr>
<th>Symptom</th>
<th>Acute Prevalence (%)</th>
<th>Chronic ≥3 months</th>
<th>Prevalence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neck Pain</td>
<td>94</td>
<td>Neck Pain</td>
<td>100</td>
</tr>
<tr>
<td>Neck Stiffness</td>
<td>96</td>
<td>Shoulder Pain</td>
<td>88</td>
</tr>
<tr>
<td>Headache</td>
<td>44</td>
<td>Neck Stiffness</td>
<td>83</td>
</tr>
<tr>
<td>Interscapular Pain</td>
<td>35</td>
<td>Headache</td>
<td>68</td>
</tr>
<tr>
<td>Sleeping Problems</td>
<td>35</td>
<td>Back Pain</td>
<td>64</td>
</tr>
<tr>
<td>Intrusion/Avoidance</td>
<td>30</td>
<td>Dizziness</td>
<td>43</td>
</tr>
<tr>
<td>Numbness/Paresthesias</td>
<td>22</td>
<td>Numbness</td>
<td>40</td>
</tr>
<tr>
<td>Dizziness</td>
<td>15</td>
<td>Sleeping Problems</td>
<td>34</td>
</tr>
<tr>
<td>Visual Symptoms</td>
<td>12</td>
<td>Concentration Problems</td>
<td>34</td>
</tr>
<tr>
<td>Auditory Symptoms</td>
<td>13</td>
<td>Memory Problems</td>
<td>25</td>
</tr>
<tr>
<td>Memory Problems</td>
<td>15</td>
<td>Auditory Symptoms</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Visual Symptoms</td>
<td>14</td>
</tr>
</tbody>
</table>

With these statistics, much research has focused on studying the many facets of whiplash to understand, treat, and prevent it with improved vehicle safety. Unfortunately, improvements in vehicle safety have only resulted in nominal gains in whiplash prevention. The addition of the
first vehicle head restraints in the late 1960’s only resulted in a 14-18% reduction in whiplash injuries [16, 17]. This was likely due to improper head restraint adjustment and excessive backset distance between the head restraint and the individual. About 75% of all head restraints were found to be left in the down position in an early study [18], which is still a common trend today. Current global Research Council for Automotive Repairs (RCAR) standards require that the head restraint is positioned close to the back of the head (backset) and top of the head (topset) as shown in Figure 1.2, which has been correlated with reduced injuries in rear-end collisions [19, 20]. Even properly adjusted passive head restraints have been found to only provide a 24% reduction in the incidence of neck pain in rear-end collisions [6]. Many different design strategies have been proposed to reduce whiplash injury. The strategies for reducing whiplash injury in the event of a collision can be lumped into 3 main categories: (i) reduce energy transferred to the occupant, (ii) restrict occupant head movement, and (iii) alert the occupant to look forward and engage neck muscles as shown in Figure 1.3. The novel ideas proposed by the author are italicized in this figure. Opportunities for reducing the energy transferred to the occupant are limited because of high speed collision safety requirements and space restrictions within the vehicle. Volvo has designed a system that allows the seat to swivel backwards when the torso is pushed back into the seat during a rear-end collision [21-23]. This absorbs some of the energy of the collision and encourages a flexed cervical spine during the collision. This design is considered an ‘active’ system because it is activated during a collision to protect the occupant from injury. Other ‘active’ systems have focused on restricting the occupant head movement to prevent injury. For these systems, the head restraint will move forward to close the gap between the head and the head restraint to provide support early in the collision. It should be noted that the head restraint cannot be designed to have no initial gap with the passengers head because it must not interfere with passengers with different driving postures and during movements to check mirrors etc. The Saab Active Head Restraint (SAHR) and Mercedes Neck-Pro are two examples of ‘active’ head restraint systems [24, 25]. These types of active systems have been found to reduce whiplash claims by 31-75% compared with typical designs [26-28]. There is potential for further gains as well. These active head restraint designs are only 'semi-active' in that they are only activated once in the collision sequence (moving forward to close the gap between the passenger and the head). A truly active system, as listed in Figure 1.3, would allow the head restraint movement prior to and during the collision sequence to produce the optimal response for
passenger safety. During normal driving the conditions the active head restraint would move to maintain the desired topset and backset between the head and head restraint without interfering with passenger motions. Upon sensing a collision, the active head restraint would activate to close gap between head restraint and head and then move in the optimal way to minimize injury.

Figure 1.2: RCAR head restraint position requirements [29]
Further safety improvements to provide the optimal head restraint characteristics (for standard and ‘semi-active’ designs) and response (for ‘active’ designs) have been limited by the development of robust human dynamic simulation models and the understanding of whiplash injury. Most computer simulation models provide poor accuracy for cervical and head motions during whiplash due to the difficulty of capturing the motion and properties of the intervertebral discs, facet joints, ligaments, muscles, etc. throughout the dynamic whiplash motion. Complex finite element models have been able to approximate the intervertebral motion at great computational expense [30]. Efficient and accurate head/neck models are required to determine the optimal head restraint design for the given collision severity, passenger size, gender, and posture as shown in Figure 1.4. An efficient model with fast solution time is beneficial to allow for head restraint optimization over many iterations and design parameters. This model must also provide an accurate response for both the head and intervertebral motions to enable an accurate assessment of injury given our current understanding of whiplash trauma.
1.2 Objectives

The objective of this thesis is to develop a representative rigid linkage lumped parameter model using Lagrangian mechanics to capture the relative motion of the head and cervical spine during whiplash. Joint locations corresponding to intervertebral centers of rotation are used to simulate the normal spinal movements and an inverse analysis is applied to determine the viscoelastic parameters for the spine based on cadaver test results. The model is further validated using high resolution dynamic finite element analysis (ANSYS) and experimental results.

1.3 Method of Approach

Figure 1.5 shows the schematic of the method of approach adopted for this work. Two types of dynamic models (analytical and finite element) are developed to capture the whiplash motion during rear-end collisions. These models attempt to capture the dynamic whiplash response for a healthy 50th percentile male looking forward in normal seated posture.
1.4 Layout of Thesis

The remainder of the thesis is organized as follows. Chapter 2 reviews the state of the available literature for whiplash. It includes the background information for whiplash; whiplash injury assessment criteria; dynamic models for whiplash; and human, cadaver, and test dummy whiplash results. Chapter 3 describes the development of the analytical rigid linkage model for whiplash. It covers the anthropometric and mass values for the head and neck, the modeling approach for the intervertebral motion of the cervical spine, the Lagrange method for determining the equations of motion, and the inverse method for determining the viscoelastic parameters of the head and neck. Chapter 4 describes the two different dynamic finite element models used to model whiplash. Chapter 5 discusses the experimental results for whiplash including the development of a whiplash test fixture, simulated head and neck, and the dynamic response of the system. Chapter 6 demonstrates the performance of the rigid linkage and finite element models and their validation with experimental results. The strengths and weaknesses of the different approaches are discussed as well as a review of the important factors influencing whiplash. Chapter 7 presents the conclusions of the work and future work for improving the model.
Experimental data currently missing in the field of whiplash to date is discussed as well. Following the main body of the thesis are the list of references and appendices. Glossary terms are placed in the front of the thesis.
Chapter 2
Literature Review

This thesis is concerned with the dynamic analysis of whiplash. As such, the literature review will cover the tissue properties of the head and neck, the causes and pathophysiology of whiplash, injury criteria for whiplash, dynamic models for whiplash, experimental results for whiplash testing, and opportunities for model improvements. These sections are included to provide the reader with a brief summary of these topics as they relate to this thesis.

2.1 Background

There are many factors that influence the dynamic response during whiplash. The main collision factors are summarized in Table 2.1. Rear-end collisions are of the greatest interest in the study of whiplash because they lead to 85% of all whiplash injuries [6]. During a rear-end collision the bullet vehicle collides with the target vehicle, causing it to accelerate forward. The crash severity corresponds to the net change in the velocity of the target vehicle in such a collision. This is related to the energy transferred during the collision and, not surprisingly, to the risk of whiplash injury [31]. Based on the crash severity, vehicle, and seat characteristics, a given acceleration of the target passenger is produced. Stiffer vehicles, with less energy absorbing bumpers, produce shorter collision profiles with higher peak accelerations. This peak acceleration is found to be an indicator for whiplash injury as well as crash severity [32, 33]. In some cases, individuals can be injured even in low-speed collisions where there is no vehicle damage [8]. The seat inclination and stiffness also determines the net acceleration profile and direction of the occupant. For typical seat inclinations, the occupant is accelerated upwards as well as forward, leading to an axial compression of the cervical spine during the whiplash motion [34]. The properties of the seat also affect the acceleration of the occupant and the relative head and torso motion associated with injury [35, 36].
Table 2.1: Collision factors for whiplash injury

<table>
<thead>
<tr>
<th>Collision Factors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collision Type</td>
<td>Front-End, Rear-End, Side-Impact</td>
</tr>
<tr>
<td>Crash Dynamics</td>
<td>Crash severity (ΔV), Acceleration, Deceleration</td>
</tr>
<tr>
<td>Vehicle Characteristics</td>
<td>Mass, Stiffness, Energy Absorption, Coefficient of Restitution</td>
</tr>
<tr>
<td>Seat</td>
<td>Profile, Inclination, Stiffness, Damping</td>
</tr>
<tr>
<td>Head Restraint</td>
<td>Standard, Semi-Active, Active, Topset, Backset, Stiffness, Damping</td>
</tr>
</tbody>
</table>

In addition to the collision factors there are many human factors that affect the dynamic response and influence the outcome of whiplash injury. All of the human factors listed in Table 2.2 play a role in the whiplash response. Individuals looking to the side to check their mirrors or vehicle blind spots are particularly susceptible to chronic whiplash injury in the case of a collision. In this head-turned posture the facet joints are in a vulnerable position with the potential for excessive facet capsular ligament strains in the event of a rear-end collision [37]. In the most common scenario the passenger is looking forward in the vehicle, leading to a 2-dimensional whiplash motion in the sagittal plane (see Figure 2.1) during a rear-end collision. Variations in initial posture affect the dynamics of the whiplash motion, with kyphotic spine curvatures more susceptible to injury due to greater extension of the lower cervical vertebrae during normal posture [34, 38]. Variations in height and weight have an effect on the forces and motions sustained during the whiplash motion, with greater mass leading to greater loading [39]. Gender has also been suggested as a risk factor for whiplash [40, 41]. Females may be more vulnerable to whiplash injury due to anthropometric and strength differences in the head and neck [42]. It has also been suggested that children and the elderly may be more susceptible to whiplash injury [6, 43, 44].
Most people in rear-end collisions are unaware of the impending collision and are not bracing for the impact. In this scenario, very little muscle activation is required to hold the head upright, and the neck provides minimal resistance to the collision acceleration. Even if the muscles are not highly activated initially, they will become activated during a rear-end collision as the natural response to resist the whiplash motion. The sternocleidomastoid, splenius capitis, and trapezius muscles are all activated during whiplash with the greatest muscle force coming from the sternocleidomastoid muscle group. These muscles are typically activated late in the collision sequence, with muscle activation beginning approximately 120 ms after impact and peak activation occurring at approximately 205-888 ms, 225-682 ms, and 416-1635 ms for the sternocleidomastoid, splenius capitis, and trapezius muscles, respectively [7]. The awareness of an individual to the impending collision can lead to increased muscle activation prior to the collision, which can alter the dynamics and loading of the cervical spine. Subject awareness and bracing for impact can alter the time to peak muscle contraction and peak muscle contraction levels during whiplash [7]. This was found to reduce the peak head acceleration by 40%, as well [46]. It is still unclear whether this altered loading and dynamic response would reduce, exacerbate, or create new mechanisms of whiplash injury.

The human head and neck weigh approximately 4.4 ± 0.6 kg and 1.6 ± 1.4 kg respectively [47]. The vertebrae, intervertebral discs, facet joints, ligaments, and muscles of the head and neck all work together to support this load and provide excellent range of motion in the sagittal, transverse, and frontal planes. The general arrangement of the vertebrae, intervertebral discs, and
ligaments is shown in Figure 2.2. The vertebrae are stiff bony structures with a complex three-dimensional structure. They protect the spinal cord and provide connection points for the intervertebral discs, facet joints, and ligaments of the spine. There are seven vertebrae of the cervical spine (C₁-C₇). The upper two vertebrae (C₁ atlas and C₂ axis) have a unique geometry that allow for rotation of the head about the vertical axis (‘no’ movement). The lower vertebrae, from C₂ down to C₇, are separated by intervertebral discs and facet joints that allow them to slide and rotate past one another during flexion/extension, abduction/adduction, and axial rotations as shown in Figure 2.3.

<table>
<thead>
<tr>
<th>Human Factors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head Position</td>
<td>Looking Forward, Rotated Left / Right, Inclined Up / Down</td>
</tr>
<tr>
<td>Posture</td>
<td>Normal, Slouching, Head Fore / Aft, Cervical Lordosis / Kyphosis</td>
</tr>
<tr>
<td>Anthropometric and Weight Variations</td>
<td>Height, Size, Weight</td>
</tr>
<tr>
<td>Gender</td>
<td>Male, Female</td>
</tr>
<tr>
<td>Age</td>
<td>Child, Adult, Elderly</td>
</tr>
<tr>
<td>Awareness</td>
<td>Unaware / Aware of Collision (Initial Muscle Activation)</td>
</tr>
<tr>
<td>Vertebrae</td>
<td>Complex Geometry, Stiffness, Mass</td>
</tr>
<tr>
<td>Intervertebral Discs</td>
<td>Viscoelastic, Non-Homogeneous, Non-Isotropic, Mass</td>
</tr>
<tr>
<td>Ligaments</td>
<td>Non-linear Stiffness, Path, Attachment, Mass</td>
</tr>
<tr>
<td>Muscles</td>
<td>Viscoelastic, Activation, Morphometry, Path, Attachment, Mass</td>
</tr>
<tr>
<td>Spinal Cord</td>
<td>Viscoelastic, Attachment, Mass</td>
</tr>
<tr>
<td>Cervical Motion</td>
<td>Intervertebral Motion during Whiplash</td>
</tr>
<tr>
<td>Tissue Variation</td>
<td>Interindividual Tissue Stiffness and Damping</td>
</tr>
<tr>
<td>Body Function</td>
<td>Fractures, Injuries, Disease</td>
</tr>
</tbody>
</table>
Figure 2.2: Vertebrae, intervertebral discs, and ligaments of the spinal unit [48]
The intervertebral discs support the large compressive loads transferred through the vertebrae while producing very little resistance to shear. The intervertebral discs are composed of a nucleus pulposis gel with a high water content (70-90%) [48], surrounded by a fibrous annulus fibrosis tissue around the outside of the disc. This structure gives the intervertebral discs their anisotropic and viscoelastic properties. The facet (or zygapophyseal) joints also act to maintain spacing between the vertebrae to facilitate motion and maintain stability in the spine. Two facet joints are located between each cervical vertebra below C2, dorsal to the intervertebral disc and symmetric about the sagittal plane. The facet joint achieves large shear motions by the unfolding of the fibroadipose meniscoid between the inferior and superior articular processes of each adjacent
vertebra. The ligaments of the spine connect between the vertebrae to support the spine. Ligaments display a highly non-linear stress-strain relationship with very low stiffness for low strain levels. They also display viscoelastic properties due to their high (~2/3) water content [45]. Higher stress values will occur for higher strain rates due to this viscoelasticity. As is the case with most biomechanical tissue, significant variations in ligament stiffness are observed between individuals and age groups [50]. The muscles of the spine provide support and facilitate movement. During normal seated posture, a small level of muscle activation is required to hold the head upright. In its passive state muscle stretched beyond its resting length will display a non-linear increase in load with deflection. In its contracted state the muscle will be shortened, thereby producing a greater increase in load with deflection. As a result, a subject bracing for a collision will contract the muscles of the neck, increasing their resistance to loading and decreasing overall head motion during whiplash [51]. At the same time, the loads applied to the cervical spine will be increased because of higher muscle forces. The spinal cord is protected by the vertebrae and acts to communicate motor, sensory, and reflex signals to the body. During the flexion and extension motions of the spine, the length and cross sectional area of the spinal cord is forced to change as the vertebrae slide and rotate past one another. The spinal cord accounts for these deflections by maintaining very low resistance to deformation for strain values up to 5%. The spinal fluid within the spinal cord gives it a highly viscoelastic response while making it susceptible to developing pressure waves under rapid deformations [3]. For a more detailed description of these tissues the reader is referred to the appendix and the work of Ethier [45], White [48], and Gray [52].

In addition to understanding the mechanical properties of the individual structures of the spine, some studies have been performed on the properties of the spinal unit in vivo and in vitro. Table 2.3 shows the resistance to spinal rotation for flexion and extension motions of cadaver samples. These properties provide a rough estimate of the net stiffness of the spine to resist whiplash motion and demonstrate the vulnerability of the cervical spine to external loading due to its lower stiffness. Similar studies have been performed to quantify the flexibility of the spine and intervertebral range of motion [48, 53, 54]. In order to understand the relative motion of the vertebrae during flexion/extension and whiplash motion, a number of studies have examined the locations of the instantaneous axes of rotation at each vertebral level [2, 55, 56]. Using x-ray cinematography, the vertebrae can be mapped to understand the motion of each vertebra relative
to the vertebra inferior to it. The instantaneous axis of rotation defines the axis perpendicular to the sagittal plane about which the superior vertebra has rotated from one time step to the next. Penning [53] found these centers of rotations to be repeatable for voluntary flexion/extension motions across multiple individuals and over the entire voluntary range of motion. This approach simplifies the complex intervertebral motion of the spine considerably, allowing the motion of the spine to be approximated by one angular degree of freedom at each vertebral level instead of three degrees of freedom for shear, compression, and rotation.

**Table 2.3: Spinal unit stiffness coefficients [48]**

<table>
<thead>
<tr>
<th>Spinal Unit</th>
<th>Stiffness (Nm/deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flexion</td>
</tr>
<tr>
<td>Cervical</td>
<td>0.43</td>
</tr>
<tr>
<td>Thoracic</td>
<td>2.22</td>
</tr>
</tbody>
</table>

### 2.2 Injury Criteria

Many injury criteria are used to assess whiplash trauma. The most established injury criteria are summarized in Table 2.4. The presence of multiple injury criteria speaks to the complex involvement of the different soft tissues in the cervical spinal unit associated with whiplash. In order to capture the injury risk due to multiple potential injury modes, more than one injury criterion is required. For example, the development of Boström’s Neck Injury Criterion (NIC) ensures that the pressure effects in the spinal canal do not exceed threshold levels associated with injury to the dorsal root ganglion [3]. The Intervertebral Neck Injury Criterion (IV-NIC) developed by Panjabi et al. [57] analyses the extension at each of the intervertebral joints throughout the whiplash motion. Injury is assessed based on the ratio of the intervertebral extensions to their physiologic values. This injury criterion provides an assessment of soft-tissue injury to the facet joints and intervertebral discs by capturing the motion beyond the physiologic limit of the spine. Facet joint injury has also been linked to the S-shaped curvature of spine occurring early in the collision sequence [2]. The normalized Neck Injury Criterion (N_{ij}) and Neck Protection Criterion (N_{km}) consider the forces and moments during the whiplash motion to assess the injury severity. The 

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scale, level 2 injury (AIS2) in frontal impacts and is applicable in high-speed rear-end collisions [58, 59]. The $N_{km}$ criterion requirement has been proposed to assess injuries in rear impacts by reducing injurious loads and moments acting on the spine. The intercept values for these $N_{ij}$ and $N_{km}$ criteria are based on the BioRIDII test dummy to assess the whiplash response in high-speed rear-end collision scenarios. The Neck Displacement Criterion (NDC) developed by Viano et al. [60] ensures the head extension, posterior displacement of the head (at the occipital condyles) relative to the torso, and axial compression of the cervical spine from T1 to the occipital condyles are below threshold values.

Based on these criteria it is clear that an accurate model of whiplash must capture not only the realistic motions of the head relative to the torso but also the individual intervertebral rotations throughout the whiplash motion.
### Table 2.4: Whiplash injury criteria

<table>
<thead>
<tr>
<th>Injury Criteria</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neck Injury Criterion (NIC) [3]</td>
<td>$NIC = 0.2a_{rel} + v_{rel}^2$</td>
</tr>
<tr>
<td>$a_{rel} = \text{relative acceleration } T_1/C_1$</td>
<td>$NIC &lt; 14.4m^2/s^2$</td>
</tr>
<tr>
<td>$v_{rel} = \text{relative velocity } T_1/C_1$</td>
<td></td>
</tr>
<tr>
<td>Normalized Neck Injury Criterion (N_{ij}) [58, 59]</td>
<td>$N_{ij} = \frac{F_z}{F_{int}} + \frac{M_y}{M_{int}}$</td>
</tr>
<tr>
<td>$F_z = \text{axial force}$</td>
<td>$N_{ij} &lt; 0.09$</td>
</tr>
<tr>
<td>$F_{int} = +6806N/-6160N$</td>
<td></td>
</tr>
<tr>
<td>$M_z = \text{Ext bending moment}$</td>
<td></td>
</tr>
<tr>
<td>$M_{int} = 125Nm$</td>
<td></td>
</tr>
<tr>
<td>Neck Protection Criterion (N_{km}) [58, 59]</td>
<td>$N_{km} = \frac{F_z}{F_{int}} + \frac{M_y}{M_{int}}$</td>
</tr>
<tr>
<td>$F_z = \text{shear force}$</td>
<td>$N_{km} &lt; 0.33$</td>
</tr>
<tr>
<td>$F_{int} = +845N/-845N$</td>
<td></td>
</tr>
<tr>
<td>$M_z = \text{Ext bending moment}$</td>
<td></td>
</tr>
<tr>
<td>$M_{int} = 47.5Nm$</td>
<td></td>
</tr>
<tr>
<td>Neck Displacement Criterion (NDC) [60]</td>
<td>$\cdot \text{Peak Head/T1 extension}$</td>
</tr>
<tr>
<td>$\cdot \text{Posterior shear displacement}$</td>
<td>Minimum excellent rating: \text{Extension} &lt; 25°</td>
</tr>
<tr>
<td>$\cdot \text{Axial compression displacement}$</td>
<td>Extension &lt; 25° $\quad \text{Posterior Shear} &lt; 3.5cm$</td>
</tr>
<tr>
<td>$\quad \text{Axial Compression} &lt; 1.5cm$</td>
<td></td>
</tr>
<tr>
<td>Intervertebral Neck-Injury Criterion (IV-NIC) [57]</td>
<td>$IV - NIC = \frac{\theta_{\text{whiplash}}}{\theta_{\text{physiological}}}$</td>
</tr>
<tr>
<td>$\quad Head / C_1 &lt; 1.1$</td>
<td></td>
</tr>
<tr>
<td>$\quad C_3 / C_4 &lt; 1.1$</td>
<td></td>
</tr>
<tr>
<td>$\quad C_4 / C_5 &lt; 2.1$</td>
<td></td>
</tr>
<tr>
<td>$\quad C_5 / C_6 &lt; 1.5$</td>
<td></td>
</tr>
<tr>
<td>$\quad C_6 / C_7 &lt; 1.8$</td>
<td></td>
</tr>
<tr>
<td>$\quad C_7 / T_1 &lt; 2.9$</td>
<td></td>
</tr>
</tbody>
</table>
2.3 Dynamic Models

A great variety of computational dynamic models have been developed using multi-body dynamics and finite element approaches to date. Both full-body and head and neck models have been developed, although most recent models have focused on the head and neck alone in an effort to capture the complex dynamic response in this area. The simplest multi-body models use rigid linkages with joints and lumped parameters to capture the net effect of the muscles, ligaments, intervertebral discs etc. These models have been developed with varying levels of success [62-64]. The major limitations of these models have been the difficulty to capture the complex vertebral geometry and intervertebral movements and the difficulty in obtaining realistic lumped parameter values. In some scenarios these simplified models have been found to accurately determine the human head response during rear-end whiplash loading with very low solution times [62]. More complex multi-body models have been developed to capture the geometry of the individual vertebrae and head and model each intervertebral disc, facet joint, ligament, and muscle. The most proven multi-body model is probably the De Jager model used in the MADYMO software package [65]. This model has been studied in a number of different scenarios and has been refined over time [65-68]. Most recently Lopik et al. [69] presented a head and neck multi-body model with very good head and intervertebral response over a range of acceleration profiles. This model used the visualNastran4d software package in combination with Matlab to model the muscle response. In this model, intervertebral discs were modeled by non-linear viscoelastic constraints connecting adjacent vertebrae, ligaments were modeled by non-linear viscoelastic elements, and facet joints were modeled by frictionless contact. Muscles are typically modeled with passive and active components using Hill muscle elements [70]. These models display relatively fast computation times (~20 minutes) [69] but are still limited in their ability to accurately capture intervertebral rotations and motions. This is due to the lack of understanding of the viscoelastic properties of the intervertebral disc and facet joints to characterize the intervertebral response and a lack of data for the kinematic response of the cervical spine during whiplash.

A number of finite element (FE) models of the head and neck have also been developed in an effort to assess the dynamic response and susceptibility for injury during rear-end collisions. These finite element models attempt to accurately model the geometry and material properties of
the head and neck. This approach tends to be computationally intensive, with Halliden et al. [71] developing a FE model with run times of up to 45 hours to solve [69]. The finite element approach is also limited by the application of accurate material properties for all of the soft tissue of the neck and head. Recently, Fice et al. [30] demonstrated the use of a FE model to capture the overall dynamic whiplash response and assess the potential for injury to the facet joints during whiplash loading. Other FE models have been presented as well [72-75]. Compared to the multi-body modeling approach, the FE approach has the ability to produce a more complete picture of the stresses and strains observed during the whiplash motion [76], but is still limited by the accurate material properties required to capture the motion of the intervertebral joints.

2.4 Testing

The experimental testing of whiplash can be broken into three major groups: in vivo testing of human subjects during low-speed rear-end collisions; in vitro testing of cadaver specimens (head/neck or head/neck/torso); and testing using anthropometric test dummies. Human testing provides the most realistic response to the whiplash motion but it is limited to low-speed testing to avoid chronic injury to the test subjects. In vitro cadaver testing is performed at all impact velocities, but muscle tissue properties tend to be stiffer than in vivo test subjects [77] and muscle activation is typically ignored with this type of testing. Anthropomorphic test dummies (ATDs) are full size physical specimens made to match the geometry and mass of individual body parts. These dummies have been validated against in vivo, in vitro and computational models with limited biofidelity in low-speed rear-end collisions [78]. A hybrid ATD (HUMON) with a cadaver head and neck on an ATD body has been developed as well [79]. In the motor vehicle industry, ATDs are used for vehicle validation of static and dynamic whiplash criteria [29, 80]. The dynamic acceleration profile specified by the RCAR seat/head restraint evaluation standard is shown in Figure 2.4. This is a global standard adopted by the Insurance Institute of Highway Safety (IIHS) and others to assess injury using the BioRID ATD. This acceleration profile is based on typical collision characteristics [8] and is commonly used in assessing the dynamic response during whiplash.
Another critical aspect in the dynamic analysis of whiplash is the determination of the anthropometric values, mass, and inertia for the neck and head. These parameters will be dependent on subject size, age, gender, and ethnicity. The anthropometric data for the 50th percentile male (average height and mass of North American male) in normal seated posture is shown in Figure 2.5. A more detailed three-dimensional model of a 50th percentile male has also been presented by Vasavada using the OpenSim software for visualization [42, 49]. Various other compilations of anthropometric data exist to define mass, inertia, and geometry values for male and female subjects with a summary of these values shown in Table 2.5.

Measurements of the range of motion for the cervical spine have also been completed in vivo and in vitro to understand the limitations of the cervical spine [54, 57]. These measurements have led to an improved understanding of the range of motion for the cervical spine and have led to the development of the Intervertebral Neck Injury Criterion (IV-NIC) [57].
Figure 2.5: 50th percentile male in seated posture [81]

Table 2.5: Human adult male head and neck inertial properties

<table>
<thead>
<tr>
<th>Source</th>
<th>Adult Male Size (%)</th>
<th>Body Mass (kg)</th>
<th>Head Mass (kg)</th>
<th>Neck Mass (kg)</th>
<th>Head Inertia (about center of mass) (kg-m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>lxx (forward)</td>
</tr>
<tr>
<td>Grosso et al. [82]</td>
<td>50th</td>
<td>-</td>
<td>6.5</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>NASA 1 [83]</td>
<td>50th</td>
<td>82.0</td>
<td>5.0</td>
<td>1.8</td>
<td>0.017</td>
</tr>
<tr>
<td>NASA 2 [82]</td>
<td>50th</td>
<td>81.5</td>
<td>-</td>
<td>-</td>
<td>0.020</td>
</tr>
<tr>
<td>Walker et al. [47]</td>
<td>-</td>
<td>-</td>
<td>4.4 ± 0.6</td>
<td>1.6 ± 1.4</td>
<td>-</td>
</tr>
</tbody>
</table>

2.4.1 In Vivo Human Testing

A variety of *in vivo* human low-speed rear-end collision experiments have been performed over the past 30 years. They have been performed using subjects in both vehicle and sled test scenarios [41, 84]. Testing has been performed on male and female subjects, with and without head restraints, but the test results performed without head restraints are of the greatest interest to this research in order to understand the uninhibited motion and mechanical properties of the head and
neck during whiplash. Most studies involve subjects looking forward with normal initial posture during the collision, but head-turned postures have also been examined [37]. Human testing has focused on both the kinematic and electromyographic (EMG) response during whiplash. To determine the kinematic response, accelerometers and reflective targets are commonly used to track the locations of interest on the head and neck. In most cases, the position, velocity, acceleration, and rotation of the head center of gravity and torso (T1) are reported [32, 39, 51, 85]. While this information is useful, the intervertebral rotations, vertebral instantaneous axes of rotation, and vertebral motions are required for assessing facet joint injury and for understanding the complex motion of the spine during whiplash. Unfortunately, this information is limited. Kaneoka et al. [2] performed a groundbreaking test on male volunteers using high-speed x-ray cinematography to track the motion of the individual vertebra during low-speed whiplash testing. This study found an abnormal shift in the instantaneous axis of rotation of the C5 vertebra associated with facet joint injury during the S-shaped curvature of the spine. However, the motions of the instantaneous axis of rotation at each vertebral level through the entire whiplash motion were not reported. EMG signals have been used to measure the level of muscle activation during whiplash to understand the level and time of muscle activation during whiplash [2, 7, 46]. This testing has demonstrated the lack of muscle activation early in the collision, believed to be associated with the time of whiplash injury.

X-ray cinematography has also been used to capture the motion of the cervical spine during voluntary flexion/extension motions in the sagittal plane. Figure 2.6A shows the concept of the instantaneous axis of rotation applied to flexion/extension movements of the cervical spine. In this figure, two x-ray images are superimposed from two different flexion/extension positions. By overlaying the x-rays such that the vertebra at a given level is overlaid, the motion of the superior vertebra relative to this vertebra can be observed. The point corresponding to the axis of rotation for this vertebra and time step can then be defined. This analysis can be carried out for each vertebra, time step, and individual to determine the variability in this approach. These results are shown in by the dots in Figure 2.6b provided by Amevo et al. [55]. The localized positions of these dots show that this approach provides a reliable means for interpreting the complex motion of the cervical spine during flexion and extension movements. It should be noted that the lack of dots on the C2 vertebrae is due to the altered geometry of the C1 and C2 neck region allowing the head to rotate about the occipital condyles and rotation to occur about the C1 vertebra. These
locations are also for voluntary flexion/extension movements which may differ from that for the dynamic whiplash response.

Figure 2.6: Instantaneous centers of rotation for the cervical spine

(A) ICR locations [53] (B) variability [55]

2.4.2 In Vitro Cadaver Testing

In vitro dynamic testing allows cadavers to be tested in rear-end collisions at acceleration levels associated with whiplash injury. This provides the most realistic human response to whiplash available at high levels of acceleration (>6.5 g). A number of different in vitro studies have been performed to assess the dynamic response during rear-end collisions including Stemper [68, 86], Panjabi [57, 87], Grauer [4], and others [61, 88-90]. These tests have been performed on both full-body and head-neck complexes. Testing on head-neck complexes is most commonly performed by applying a uniaxial acceleration to the upper thoracic vertebrae while observing the response of the head and neck. This testing ignores the effects of axial spine loading that occurs due to the upward force on the occupant as a result of seat inclination. Muscle simulation is another area of concern with in vitro testing due to its increased stiffness compared with in vivo properties [77] and the difficulty to simulate muscle activation to hold the head in normal posture. Two approaches have been used to deal with this limitation: (i) removing muscle tissue and supporting the head in normal initial posture prior to the collision, and (ii) using a muscle
force replication system to capture the muscle properties of the neck and hold the head upright prior to the collision. The first approach, used by Grauer et al. [4], fails to capture the passive stiffness of the muscles of the neck, producing what should be a slightly more flexible specimen. The second approach, used by Panjabi et al. [91], uses a muscle force replication system to support the head and capture the passive muscle properties in the neck. This system benefits from a more complete test specimen, with the potential for the most realistic whiplash response, but adds the uncertainty of the overall performance of the muscle replication system to accurately capture the muscle response. Both of these types of cadaver tests have been able to duplicate the typical S-shaped curvature characteristic of the *in vivo* whiplash motion, which supports their validity [4, 91].

![Figure 2.7: In vitro experimental test apparatus [4]](image)

The test setup used by Grauer et al. is shown in Figure 2.7. For this test the T1 vertebra was fixed to the test sled at a 20 degree angle corresponding to that of normal posture. The head was replaced by a surrogate head with a mass of 5.5 kg and moment of inertia of 0.035 kg·m² to represent a 50th percentile human head. The surrogate head was supported by a magnetic piston holding the foramen magnum (base of skull) at a horizontal orientation corresponding to an individual looking straight ahead while driving. The magnet was used to rapidly remove the support at the start of the experiment, allowing the head and neck to move freely during the collision. Muscle and external soft tissue was removed and motion-monitoring flags were secured to each vertebra to track their motion and rotation. This allowed for the motion capture of the
vertebrae in addition to that of the head, which is very important for understanding the complex intervertebral motion of the neck. A horizontal acceleration was applied to the trauma sled to perform the test. The results of this testing are shown in Figures 2.8 and 2.9 for an 8.5 g acceleration profile. Head rotation is seen to increase sharply after approximately 50 ms with the horizontal motion of the neck starting approximately 25 ms before this. This is characteristic of the S-shaped curvature and motion of the neck during whiplash, where the head initially moves backwards relative to the torso (S-shaped curvature) before rotating backwards and rebounding forward as shown in Figure 1.1. Figure 2.9 also demonstrates the S-shaped curvature response with C₀-C₁ and C₁-C₂ intervertebral flexion occurring while the lower intervertebral joints are extended around 50-75 ms. After this time the upper intervertebral joints transition from flexion to extension as the head rotates backwards. As such, these results provide a realistic response of the head and neck during whiplash injury.

Figure 2.8: In vitro head response relative to the T₁ vertebra [4]
2.4.3 Testing of Anthropomorphic Test Dummies

Anthropomorphic test dummies (ATDs) are full-body human replicas that can be used to simulate the whiplash response during rear-end collisions. HybridIII-TRID, RID2, and BioRID2 are three such ATDs that have been specifically designed for rear-end collisions. By capturing segment masses and geometry representing 50th percentile human properties, these dummies attempt to duplicate the human response during whiplash. Although ATDs have been shown to approximate the head motion and loading during whiplash, the biofidelity of the neck is limited [78, 92]. This is likely due to the pin joints and other attachments used to simulate the complex motion of the vertebrae and soft tissue. This limits the potential for ATDs to assess injury due to the IV-NIC and makes them a poor choice in supporting the development of a fundamental dynamic model for whiplash. There are still many applications for ATDs and they have been used to validate vehicle safety improvements for over 20 years using the NIC and other injury criterion for whiplash [32, 93-95].
2.5 Opportunities for Model Improvements

In order to facilitate injury assessment during whiplash, dynamic models must accurately capture both the head motion relative to the T1 vertebra and the intervertebral rotations of the cervical spine [3, 57]. Current lumped-parameter models have been limited in their ability to capture the dynamic intervertebral response of the cervical spine due to:

(i) the complex sliding and rotation motion of the vertebrae relative to one another and

(ii) the difficulty in determining accurate lumped-parameter values to include the net effects of the intervertebral discs, facet joints, ligaments, muscles, and other soft tissue.

By addressing these areas of concern, a simplified lumped parameter model capable of capturing the complex whiplash motion in the cervical spine and head may be achieved.
This chapter describes the development of a rigid linkage dynamic model of whiplash. It covers the development of realistic joint locations, the Lagrange method for developing the equations of motion for the system, the fitting method for determining the lumped parameter values for the system, and the results for the rigid linkage model.

Figure 3.1: Rigid linkage model for whiplash
The rigid linkage model of whiplash is shown in Figure 3.1. The segment lengths are defined by \(L_1\) to \(L_8\) for the T1 to C7 segment up to the uppermost segment ending at the center of gravity of the head. The segment angles corresponding to initial posture are given by \(\theta_1\) to \(\theta_8\), starting from the bottom to top locations shown in Figure 3.1. These angles are measured in the counterclockwise direction from the vertical. There are 8 degrees of freedom for the system corresponding to each of the segments angles \(\theta_1\) to \(\theta_8\), which are measured in the CCW direction and allowed to vary over time. The rotational stiffness at each of the joints is given by \(k_1\) to \(k_8\) and the rotational damping is given by \(c_1\) to \(c_8\). Segment masses, starting at the second intervertebral joint, are given by the values \(m_1\) up to \(m_{\text{head}}\). At this top \(m_{\text{head}}\) location the moment of inertia of the head \(I_{\text{head}}\) is also defined at the center of gravity of the head.

Whiplash injury is characterized by the motion of the head and cervical spine [3, 57, 61], so this model will focus on the response in this region. The important characteristics for modeling whiplash are shown in Table 3.1. The rigid linkage model is developed to model the most common whiplash scenario. For example, the model will assume the subject is looking forward with normal posture during the rear-end collision, leading to a 2-dimensional whiplash response in the sagittal plane. The model mass and linkage values correspond to that of the average North American male. Muscle activation is ignored as most individuals are unaware of an impending collision and involuntary muscle activation is believed to occur after whiplash injury is sustained [2, 7, 46]. Passive muscle stiffness is ignored in accordance with the experimental results of Grauer et al. [4]. Lumped parameters are used to capture the net effects of the vertebrae, intervertebral discs, facet joints, ligaments, and spinal cord in accordance with Grauer et al.’s [4] testing on healthy cadaver subjects. In order to achieve an acceptable level of accuracy, the model is formulated to handle the large deflections which occur during whiplash.
Table 3.1: Important characteristics for modeling whiplash

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Description</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head Position</td>
<td>Looking Forward, Left, Right, Up / Down</td>
<td>Looking Forward</td>
</tr>
<tr>
<td>Posture</td>
<td>Normal, Slouching, Head Fore / Aft, Cervical Lordosis / Kyphosis</td>
<td>Normal Seated Posture</td>
</tr>
<tr>
<td>Anthropometric and Weight Variations</td>
<td>Height, Size, Weight</td>
<td>50th Percentile</td>
</tr>
<tr>
<td>Gender</td>
<td>Male, Female</td>
<td>Male</td>
</tr>
<tr>
<td>Age</td>
<td>Child, Adult, Elderly</td>
<td>Adult</td>
</tr>
<tr>
<td>Awareness</td>
<td>Unaware / Aware of Collision</td>
<td>Unaware</td>
</tr>
<tr>
<td>Vertebrae</td>
<td>Complex Geometry, Stiffness, Mass</td>
<td>Lumped Parameters for Mass, Linear Stiffness and Damping</td>
</tr>
<tr>
<td>Intervertebral Discs</td>
<td>Viscoelastic, Non-Homogeneous, Non-Isotropic, Mass</td>
<td></td>
</tr>
<tr>
<td>Ligaments</td>
<td>Non-linear Stiffness, Path, Attachment, Mass</td>
<td></td>
</tr>
<tr>
<td>Spinal Cord</td>
<td>Viscoelastic, Attachment, Mass</td>
<td></td>
</tr>
<tr>
<td>Muscles</td>
<td>Viscoelastic, Activation, Morphometry, Path, Attachment, Mass</td>
<td>None</td>
</tr>
<tr>
<td>Cervical Motion</td>
<td>Intervertebral Motion during Whiplash</td>
<td>Center of Rotation</td>
</tr>
<tr>
<td>Tissue Variation</td>
<td>Interindividual Tissue Stiffness and Damping</td>
<td>Experimental Tissue Properties</td>
</tr>
<tr>
<td>Body Function</td>
<td>Fractures, Injuries, Disease</td>
<td>Healthy</td>
</tr>
<tr>
<td>Large Deformations</td>
<td>Head and Cervical Vertebrae Motions during Whiplash</td>
<td>Large Deformations</td>
</tr>
</tbody>
</table>

3.1 Joint Locations for Intervertebral Motion

For individuals looking forward during a rear-end collision the motion of the head and vertebrae can be approximated by a 2-dimensional motion in the sagittal plane. Assuming no additional constraints are applied, this allows for 3 degrees of freedom (x position, y position, and rotation) for the motion of each vertebra. The relative motion of each vertebra can be simplified further by understanding the motion of the spine during normal flexion/extension movements. During these movements the vertebrae slide and rotate past one another in such a way that their motion can be approximated by a rotation of each superior vertebra about a specific location on each adjacent inferior vertebra. As discussed in section 2.4.1, this approach has been found to provide a good
approximation for voluntary flexion/extension motions through the entire cervical range of motion and across multiple individuals [53, 55]. The mean locations of the centers of rotation are a function of vertebra size, allowing them to be defined for a range of individual sizes [55]. For the 50th percentile North American male, the size and location of each vertebra can be defined using the anthropometric data shown in Figure 2.5 [81]. The resulting locations for the IAR’s are shown in Table 3.2. The skull center of gravity (CoG) does not correspond to an instantaneous axis of rotation but is included for completeness. The skull rotation IAR location corresponds to the rotation center for the skull as it moves about the occipital condyles of the atlas. The C₁/C₂ IAR location captures the rotation of the atlas about the axis and the following locations define the IAR’s for C₃ through to T₁.

Table 3.2: Instantaneous axis of rotation locations for a 50th percentile male

<table>
<thead>
<tr>
<th>IAR Location</th>
<th>X Position (mm)</th>
<th>Y Position (mm)</th>
<th>Mass (kg)</th>
<th>Inertia (kg·m²)</th>
<th>Linkage</th>
<th>Length (mm)</th>
<th>Initial Angle (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull CoG</td>
<td>13.0</td>
<td>182.0</td>
<td>5.5</td>
<td>0.035</td>
<td>L₆ (top)</td>
<td>51.4</td>
<td>-37.2</td>
</tr>
<tr>
<td>Skull Rotation</td>
<td>-18.2</td>
<td>141.1</td>
<td>0</td>
<td>0</td>
<td>L₇</td>
<td>21.4</td>
<td>40.6</td>
</tr>
<tr>
<td>C₁/C₂</td>
<td>-4.2</td>
<td>124.8</td>
<td>0.27</td>
<td>0</td>
<td>L₆</td>
<td>31.8</td>
<td>6.3</td>
</tr>
<tr>
<td>C₃</td>
<td>-0.8</td>
<td>93.2</td>
<td>0.25</td>
<td>0</td>
<td>L₅</td>
<td>16.8</td>
<td>23.4</td>
</tr>
<tr>
<td>C₄</td>
<td>5.9</td>
<td>77.8</td>
<td>0.32</td>
<td>0</td>
<td>L₄</td>
<td>20.4</td>
<td>7.1</td>
</tr>
<tr>
<td>C₅</td>
<td>8.4</td>
<td>57.6</td>
<td>0.37</td>
<td>0</td>
<td>L₃</td>
<td>16.8</td>
<td>3.2</td>
</tr>
<tr>
<td>C₆</td>
<td>9.4</td>
<td>40.8</td>
<td>0.30</td>
<td>0</td>
<td>L₂</td>
<td>19.0</td>
<td>-10.6</td>
</tr>
<tr>
<td>C₇</td>
<td>5.9</td>
<td>22.1</td>
<td>0.29</td>
<td>0</td>
<td>L₁</td>
<td>22.9</td>
<td>-14.9</td>
</tr>
<tr>
<td>T₁</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>Total</td>
<td>200.5</td>
<td></td>
</tr>
</tbody>
</table>

The concept of using IAR’s to define the movements of the vertebrae is analogous to that of a rigid linkage structure with revolute joints at the IAR locations. In both cases, the distance between the centers of rotation for adjacent vertebrae is conserved and rotations are only permitted about the IAR locations. The resulting revolute joints and rigid linkages are shown in Figure 3.1, overlaid on the figure for the 50th percentile male. Revolute joint locations are shown by ‘+’ signs with dashed lines indicating linkages between these joints. Linkage values for the length and initial angle (corresponding to initial posture) are provided in Table 3.2. Table 3.2 also shows the mass and inertia values defined at each of the joint locations. The mass and inertia values for the head were chosen to match those of Grauer et al. [4] and approximate those of a
50th percentile male as shown in Table 2.5. The mass values at the other joint locations were calculated based on the percentage of the inferior vertical segment length to the total vertical neck length. The total neck mass was taken to be 1.8 kg, corresponding to that of a 50th percentile male as shown in Table 2.5. The inertia of the neck at each of the neck joints is neglected because the soft tissue of the neck does not tend to rotate during intervertebral rotations, but instead translates in a more linear fashion during this motion.

Although the location of IAR’s are well established for voluntary motions, their application in the dynamic whiplash response is still largely unknown. To the authors knowledge, only one experimental study has been performed involving IAR analysis at the C₅/C₆ level during whiplash. This research found a shift in the C₅/C₆ IAR location during whiplash, believed to be associated with facet joint injury [2]. The motions of the IAR locations at all of the vertebral locations throughout the whiplash motion were not reported, however. In order to assess the validity of the IAR approximation during whiplash, the experimental intervertebral rotations of Grauer et al. [4] can be applied to the rigid linkage model to compare the resulting head response with that found experimentally. The Grauer experimental results capture both the intervertebral extensions of the neck and the relative motion of the head to the T₁ vertebra, so if the IAR approximation is reasonable it should provide a link between these two sets of results. It should be noted that the IAR locations of cadaver samples cannot be expected to be the same as those of the rigid linkage model because of differences in height, size, etc. The comparison between the rigid linkage model (shown in Figure 3.1), using IAR locations for a 50th percentile male, and the experimental results of Grauer et al. [4] are shown in Figure 3.2 below. The results show a very good fit for both head rotation and head position using this IAR rigid linkage approach. This supports the validity of using IAR joint locations to approximate the cervical spine motion during whiplash.
Figure 3.2: Dynamic rigid linkage model IAR assessment

(A) head rotation (B) head displacement relative to T₁ vertebra

3.2 Constitutive Equations for Rigid Linkage Model

The rigid linkage dynamic model of the head and neck is shown in Figure 3.1. The model consists of 8 rigid linkages running from the T₁ vertebra up to the center of gravity of the head. Revolute joints with mass, stiffness, and damping lumped parameters are defined at each of the joint locations from the T₁ joint upwards. Joint rotational stiffness and damping values are applied in a Voigt element as shown in Figure 3.3. The inertia of the head is defined at the center of gravity of the head, as well. The input acceleration, applied at the T₁ vertebrae, in both the x (forward) and y (upward) directions will define the response of the system over time.

For this rigid linkage configuration, the kinetic energy $T$ of the system can be defined as a function of the joint mass $m_i$ and velocity $v_i$, as shown in equation 3.1. This summation starts at the first joint above the T₁ joint and continues up to the center of mass of the head according to
Figure 3.1. The velocity vector, which lies in the sagittal plane, can be further decomposed into \( x_i \) and \( y_i \) components as provided in equation 3.2.

\[
T = \frac{1}{2} I_{\text{head}} \dot{\theta}_s^2 + \frac{1}{2} \sum_{i=1}^{8} m_i v_i^2 \tag{3.1}
\]

\[
T = \frac{1}{2} I_{\text{head}} \dot{\theta}_s^2 + \frac{1}{2} \sum_{i=1}^{8} m_i \left( \dot{x}_i^2 + \dot{y}_i^2 \right) \tag{3.2}
\]

The joint mass locations in the forward \( x_i \) and upward \( y_i \) directions are a function of the linkage lengths \( L_i \) and angles \( \theta_i \) (measured in the CCW direction from the vertical) as shown in equations 3.3 and 3.4. Taking the derivative of equations 3.3 and 3.4 with respect to time gives the \( \dot{x}_i \) and \( \dot{y}_i \) joint velocities in equations 3.5 and 3.6. These velocities are a function of the linkage lengths \( L_i \), angles \( \theta_i \), and angular velocities \( \dot{\theta}_i \) for fixed segment lengths \( L_i \). Equations 3.2, 3.5, and 3.6 can be combined to define the kinetic energy of the rigid linkage system in terms of the generalized coordinates \( \theta_i \) and \( \dot{\theta}_i \).

\[
x_i = -\sum_{j=1}^{i} L_j \sin(\theta_j) \tag{3.3}
\]
\[ y_i = \sum_{j=1}^{i} L_j \cos(\theta_j) \]  
(3.4)

\[ \dot{x}_i = -\sum_{j=1}^{i} L_j \dot{\theta}_j \cos(\theta_j) \]  
(3.5)

\[ \dot{y}_i = -\sum_{j=1}^{i} L_j \dot{\theta}_j \sin(\theta_j) \]  
(3.6)

The potential energy \( V \) for the system is a function of the stored spring energy at each of the joints, ignoring the effects of gravity. Equation 3.7 shows the potential energy as a function of the rotational spring stiffness \( k_i \) and linkage angle \( \theta_i \), measured from the vertical in the counterclockwise direction.

\[ V = \frac{1}{2} k_i \theta_i + \sum_{i=2}^{8} k_i (\theta_i - \theta_{i-1}) \]  
(3.7)

Once the kinetic and potential energy have been defined in terms of the generalized coordinates \( \theta_i \) and \( \dot{\theta}_i \), the Lagrange method can be used to determine the equations of motion for the system. Lagrange's equations are given by equation 3.8 where the generalized force \( Q_i \) is shown in equation 3.9. The generalized force is a function of the joint mass \( m_i \), horizontal external acceleration \( a_{x\text{ref}} \), vertical external acceleration \( a_{y\text{ref}} \), partial derivative of the joint horizontal position with respect to the generalized coordinate \( \frac{\partial x_k}{\partial \theta_i} \), and partial derivative of the joint vertical position with respect to the generalized coordinate \( \frac{\partial y_k}{\partial \theta_i} \).

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}_i} \right) - \left( \frac{\partial T}{\partial \theta_i} \right) + \frac{\partial V}{\partial \theta_i} = Q_i \]  
(3.8)
\[ Q_i = \sum_{k=1}^{8} \left( -m_k a_{xref} \frac{\partial x_k}{\partial \theta_i} - m_k a_{yref} \frac{\partial y_k}{\partial \theta_i} \right) \]  

(3.9)

There are 8 degrees of freedom for the rigid linkage system corresponding to the angles of the 8 segments in the model. Equation 3.8 defines Lagrange’s 8 equations of motion for this system which can be arranged using matrix algebra into the format of equation 3.10. In this equation the 8 by 8 mass matrix \( [M] \) is multiplied by the 8 by 1 angular acceleration row vector of segment angular accelerations \( \ddot{\theta}_i \) down to \( \ddot{\theta}_8 \). The \( [V] \) matrix characterizes the damping in the system due to coriolis forces and the \( [C] \) matrix defines the damping in the system as a result of the joint damping. In most cases, the \( [V] \) and \( [C] \) matrices are combined but they have been separated here for clarity. These matrices are multiplied by the segment angular velocity row vector \( \{\dot{\theta}\} \). The stiffness matrix \( [K] \) arises from the joint stiffness parameters \( k_i \) and defines the moment produced for a given change in the generalized coordinates \( \{\theta\} \). The generalized force vector \( \{Q_e\} \) defines the generalized external forces acting on the system due to the external accelerations applied at the T1 vertebra. Equation 3.10 can be modified to that of equation 3.11 where the rigid linkage model is given an initial configuration \( \{\theta_{initial}\} \) about which it will resist movement. This initial configuration corresponds to the normal seated posture of the average 50th percentile male.

\[
[M] \{\ddot{\theta}\} + [V] \{\dot{\theta}\} + [C] \{\dot{\theta}\} + [K] \{\theta\} = \{Q_e\}
\]

(3.10)

\[
[M] \{\ddot{\theta}\} + [V] \{\dot{\theta}\} + [C] \{\dot{\theta}\} + [K] \{\theta - \theta_{initial}\} = \{Q_e\}
\]

(3.11)

Each of the matrices defined in equation 3.11 are shown explicitly in equations 3.12 to 3.15. The mass matrix in equation 3.12 is a function of the joint masses, segment lengths, and segment angles. The notation \( m_{1-b} \) defines the addition of the joint masses \( m_1, m_2, m_3 \) up to the head mass \( m_h \). The coriolis matrix in equation 3.13 is a function of the joint masses and segment lengths, angles, and angular velocities. The damping matrix in equation 3.14 is a function of each
of the joint damping values corresponding to the rotational Voigt elements. The stiffness matrix in equation 3.15 is a function of the joint stiffness values corresponding to the rotational Voigt elements. In equation 3.16 the generalized force vector is a function of the joint masses, segment lengths, and input acceleration for the system.

\[
[M]_{(8.1-4)} = \begin{bmatrix}
L_1^2 m_{1-h} & L_1 L_2 \cos(\theta_1 - \theta_2) m_{2-h} & L_1 L_3 \cos(\theta_1 - \theta_3) m_{3-h} & L_1 L_4 \cos(\theta_1 - \theta_4) m_{4-h} \\
L_1 L_2 \cos(\theta_2 - \theta_3) m_{2-h} & L_2 L_3 \cos(\theta_2 - \theta_3) m_{3-h} & L_2 L_4 \cos(\theta_2 - \theta_4) m_{4-h} & L_2 \int_3^4 m_{4-h} \\
L_1 L_3 \cos(\theta_3 - \theta_4) m_{3-h} & L_1 L_4 \cos(\theta_3 - \theta_4) m_{4-h} & L_1 \int_3^4 m_{4-h} & L_1 \int_3^4 m_{4-h} \\
L_1 L_4 \cos(\theta_4 - \theta_5) m_{4-h} & L_1 \int_3^4 m_{4-h} & L_1 \int_3^4 m_{4-h} & L_1 \int_3^4 m_{4-h} \\
\end{bmatrix}
\]

(3.12)

\[
[M]_{(8.5-8)} = \begin{bmatrix}
L_1 L_2 \cos(\theta_1 - \theta_2) m_{2-h} & L_1 L_3 \cos(\theta_1 - \theta_3) m_{3-h} & L_1 L_4 \cos(\theta_1 - \theta_4) m_{4-h} & L_1 L_5 \cos(\theta_1 - \theta_5) m_{5-h} \\
L_2 L_3 \cos(\theta_2 - \theta_3) m_{2-h} & L_2 L_4 \cos(\theta_2 - \theta_4) m_{3-h} & L_2 L_5 \cos(\theta_2 - \theta_5) m_{4-h} & L_2 \int_3^5 m_{5-h} \\
L_3 L_4 \cos(\theta_3 - \theta_4) m_{3-h} & L_3 L_5 \cos(\theta_3 - \theta_5) m_{4-h} & L_3 \int_3^5 m_{5-h} & L_3 \int_3^5 m_{5-h} \\
L_4 L_5 \cos(\theta_4 - \theta_5) m_{4-h} & L_4 \int_3^5 m_{5-h} & L_4 \int_5 \int_{5-h} m_{5-h} \\
\end{bmatrix}
\]
(3.13)

\[
[v]_{(a1-a4)} = \begin{bmatrix}
0 & L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} \\
-L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & 0 & L_2 L_3 \beta_3 \sin (\theta_3 - \theta_4) m_{3-n} & L_2 L_3 \beta_3 \sin (\theta_3 - \theta_4) m_{3-n} & L_2 L_3 \beta_3 \sin (\theta_3 - \theta_4) m_{3-n} \\
-L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & -L_2 L_3 \beta_3 \sin (\theta_3 - \theta_4) m_{3-n} & 0 & L_3 L_4 \beta_4 \sin (\theta_4 - \theta_4) m_{4-n} & L_3 L_4 \beta_4 \sin (\theta_4 - \theta_4) m_{4-n} \\
-L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & -L_2 L_3 \beta_3 \sin (\theta_3 - \theta_4) m_{3-n} & -L_3 L_4 \beta_4 \sin (\theta_4 - \theta_4) m_{4-n} & 0 & L_4 L_5 \beta_5 \sin (\theta_5 - \theta_5) m_{5-n} \\
-L_1 L_2 \beta_2 \sin (\theta_2 - \theta_4) m_{2-n} & -L_2 L_3 \beta_3 \sin (\theta_3 - \theta_4) m_{3-n} & -L_3 L_4 \beta_4 \sin (\theta_4 - \theta_4) m_{4-n} & -L_4 L_5 \beta_5 \sin (\theta_5 - \theta_5) m_{5-n} & 0
\end{bmatrix}
\]

(3.14)

\[
[C] = \begin{bmatrix}
C_1 + C_2 & -C_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-C_2 & C_2 + C_3 & -C_3 & 0 & 0 & 0 & 0 & 0 \\
0 & -C_3 & C_3 + C_4 & -C_4 & 0 & 0 & 0 & 0 \\
0 & 0 & -C_4 & C_4 + C_5 & -C_5 & 0 & 0 & 0 \\
0 & 0 & 0 & -C_5 & C_5 + C_6 & -C_6 & 0 & 0 \\
0 & 0 & 0 & 0 & -C_6 & C_6 + C_7 & -C_7 & 0 \\
0 & 0 & 0 & 0 & 0 & -C_7 & C_7 + C_8 & -C_8 \\
0 & 0 & 0 & 0 & 0 & 0 & -C_8 & C_8
\end{bmatrix}
\]

(3.15)

\[
[K] = \begin{bmatrix}
K_1 + K_2 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-K_2 & K_2 + K_3 & -K_3 & 0 & 0 & 0 & 0 & 0 \\
0 & -K_3 & K_3 + K_4 & -K_4 & 0 & 0 & 0 & 0 \\
0 & 0 & -K_4 & K_4 + K_5 & -K_5 & 0 & 0 & 0 \\
0 & 0 & 0 & -K_5 & K_5 + K_6 & -K_6 & 0 & 0 \\
0 & 0 & 0 & 0 & -K_6 & K_6 + K_7 & -K_7 & 0 \\
0 & 0 & 0 & 0 & 0 & -K_7 & K_7 + K_8 & -K_8 \\
0 & 0 & 0 & 0 & 0 & 0 & -K_8 & K_8
\end{bmatrix}
\]
Substituting the joint and linkage values for each of the matrix variables above produces a matrix equation which is a function of the angular position, velocity, and acceleration of each segment according to equation 3.11. Equation 3.11 can then be rearranged to obtain the angular acceleration in terms of the angular velocity and position as shown by equation 3.17. At time $t$ equal to zero the initial angular position (given in Table 3.2) and initial velocity (zero initial angular velocity) are known, so for an external acceleration shown in Figure 3.4, the angular acceleration of each of the segments can be found. At this point, the segment angular acceleration and angular velocity vectors can be combined to form the 16 by 1 column vector $\{\dot{\theta}_i\}$ shown on the left hand side of equation 3.18. This column vector is then numerically integrated over time to obtain the segment angular velocity and position vector $\{\dot{\theta}_i, \theta_i\}$ shown on the right hand side of equation 3.18, which is a function of time. The Runge-Kutta numerical integration method (ODE45 in Matlab) is used to obtain the response of the rigid linkage model for a time varying input acceleration for the models without joint damping. For the numerical integration using fully-defined viscoelastic parameters, ODE15s (designed for numerical integration of stiff systems in Matlab) was found to produce faster solution results.

$$\{\dot{\theta}\} = [M]^{-1}(\{Q_e\} - [V]\{\dot{\theta}\} - [C]\{\dot{\theta}\} - [K]\{\theta - \theta_{initial}\})$$  (3.17)
\[
\begin{align*}
\{\ddot{\theta}_i\} &= \frac{d}{dt} \{\dot{\theta}_i\} \\
\{\dot{\theta}_i\} &= \frac{d}{dt} \{\theta_i\}
\end{align*}
\] (3.18)

Figure 3.4: Rigid linkage 8.5 g whiplash acceleration applied at the T1 vertebra

3.3 Determining Viscoelastic Joint Parameters

Based on the previous formulation, the dynamic response of the rigid linkage model is determined for a given stiffness and damping configuration and acceleration profile. The goal at this stage is to determine the lumped parameter stiffness and damping values which will provide the most realistic head and neck response during whiplash. Although the tissue properties of the intervertebral discs, facet joints, ligaments, etc. are generally well understood, their contribution to the lumped parameter stiffness and damping joint values of the rigid linkage model is not trivial. The cadaver test results of Grauer et al. [4] provide the realistic whiplash response for both the head and neck under a known collision acceleration. By applying the same collision acceleration to the rigid linkage model, the model response and experimental response are compared to assess the performance of the rigid linkage model. A numerical fit quantifying how closely the model response matches the experimental response is then defined, where an improved model response corresponds to a lower fit value. At this point, an optimization routine is implemented to determine the stiffness and damping values which will minimize the fit value.
and attempt to capture the experimental cadaver whiplash response. This procedure is illustrated in Figure 3.5.

![Diagram](image_url)

**Figure 3.5: Inverse method to determine viscoelastic parameters**

The fit value, mentioned previously, must capture not only the head response (displacement and rotation) but also the neck response (intervertebral extensions) for it to accurately assess the whiplash motion. As discussed in section 2.2, it is believed that whiplash injury depends on both the head movement relative to the torso [3] and also intervertebral neck extensions [57], so it is important that the model attempt to accurately capture all of these motions. The fit value for head displacement $headposfit$ is a summation of the square of the difference between the experimental head position $x_{\text{exp}}(t), y_{\text{exp}}(t)$ and model head position $x_{\text{head}}(t), y_{\text{head}}(t)$ at each time step as shown in equation 3.19. Position values are in millimeters and $tsteps$ corresponds to the total number of time steps for the response. The experimental results of Grauer et al. [4] were available from 0 to 0.170 s over 0.005 s increments so there were 35 time steps $tsteps$ for the summation.

$$headposfit = \sum_{t=1}^{tsteps}\left(\left(x_{\text{exp}}(t) - x_{\text{head}}(t)\right)^2 + \left(y_{\text{exp}}(t) - y_{\text{head}}(t)\right)^2\right)$$  (3.19)

In order to assess the head rotation fit $headrotfit$ the sum of the squares of the difference between the experimental head rotation $\theta_{\text{exp}}(t)$ and the model head rotation $\theta_{\text{head}}(t)$ are added at each time step according to equation 3.20.
$$\text{headrotfit} = \sum_{t=1}^{\text{step}} (\theta_{\text{exp}}(t) - \theta_{\text{head}}(t))^2$$  \hspace{1cm} (3.20)$$

By adding the sum of \textit{headposfit} and \textit{headrotfit}, an assessment of the level of the fit for the complete head response is achieved.

$$\text{headfit} = \text{headposfit} + \text{headrotfit}$$  \hspace{1cm} (3.21)$$

The intervertebral extension of the neck is a measure of the rotation of each vertebra relative to its adjacent inferior vertebra as shown in Figure 3.6. In the case where the T$_1$ vertebra is fixed, the C$_7$-T$_1$ intervertebral extension $\Delta_i(t)$ is equal to the rotation of C$_7$ vertebra $\theta_i(t)$ beyond its initial orientation $\theta_{i,\text{initial}}(t)$ as given by equation 3.22. At the higher vertebral levels, both the inferior and superior vertebra can rotate. In this scenario the intervertebral extension $\Delta_i(t)$ is equal to the difference in the rotation of the superior and inferior vertebral rotations relative to their initial configurations as shown in equation 3.23 and Figure 3.6. The rotation of the superior vertebra is equal to the difference in the net rotation $\theta_i(t)$ from its initial orientation $\theta_{i,\text{initial}}(t)$.

The rotation of the inferior vertebra is equal to the difference in the net rotation $\theta_{i-1}(t)$ from its initial orientation $\theta_{i-1,\text{initial}}(t)$.

$$\Delta_i(t) = \theta_i(t) - \theta_{i,\text{initial}}$$  \hspace{1cm} (3.22)$$

$$\Delta_i(t) = \theta_i(t) - \theta_{i-1}(t) - \theta_{i,\text{initial}} + \theta_{i-1,\text{initial}} \text{ where } i > 1$$  \hspace{1cm} (3.23)$$
The fit for the neck intervertebral extension $extfit$ is equal to the sum of the squares of the difference between the experimental intervertebral extension $\Delta_{exp,t}(t)$ and model intervertebral extension $\Delta_{i}(t)$ over the 8 intervertebral levels and time. This is given by equation 3.24. As mentioned previously, the value for $tsteps$ is 35, based on the number of experimental data points obtained.

$$
 extfit = \sum_{t=1}^{tsteps} \sum_{i=1}^{8} (\Delta_{exp,i}(t) - \Delta_{i}(t))^2 
$$  \hspace{1cm} (3.24)

The total fit $fit$ is equal to the sum of the fit contribution from the head motion $headfit$ and the neck intervertebral rotations $extfit$ as shown in equation 3.25. This fit value gives a measure of how well the model response replicates the experimental whiplash motion of cadaver subjects. The lowest fit value is desired, corresponding to best overall approximation of the whiplash motion.

$$
 fit = headfit + extfit 
$$  \hspace{1cm} (3.25)
In order to determine the stiffness and damping parameters to provide the most realistic neck response, a constrained optimization routine was performed in Matlab. Four different models of increasing complexity were used to determine the parameters which would provide the best whiplash response as shown in Table 3.3. For the uniform joint stiffness no damping (UJSND) model, a common stiffness value is defined at each of the intervertebral joints. For the joint stiffness no damping model (JSND), the stiffness values at each of the joints are assumed to be independent of one another. For the joint damping no stiffness model (JDNS), 8 independent damping variables define the damping at each of the joints. In the joint stiffness joint damping model (JSJD), independent variables for stiffness and damping are defined at each of the intervertebral levels.

Table 3.3: Rigid linkage whiplash models and input parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>Name</th>
<th>Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>UJSND</td>
<td>Uniform Joint Stiffness, No Damping</td>
<td>$K_{\text{uniform}}$</td>
</tr>
<tr>
<td>JSND</td>
<td>Joint Stiffness, No Damping</td>
<td>$K_{1-8}$</td>
</tr>
<tr>
<td>JDNS</td>
<td>Joint Damping, No Stiffness</td>
<td>$C_{1-8}$</td>
</tr>
<tr>
<td>JSJD</td>
<td>Joint Stiffness, Joint Damping</td>
<td>$K_{1-8}, C_{1-8}$</td>
</tr>
</tbody>
</table>

The optimization criteria for determining the model parameters and response are shown in Table 3.4. Upper and lower bounds were set within the optimization to constrain the optimization variables within realistic limits. The lower bound was set to zero and the upper bound for each of the models is given in Table 3.4. Many optimization routines were performed over the optimization space using the parameter values shown in Table 3.4 as the starting points for the optimization routine. For example, with the JSND model there were 8 stiffness parameters with two different starting values used for each parameter. This led to $2^8$ optimizations starting from different initial conditions. The parameters from the best optimization run (lowest overall fit value) were chosen as the most realistic configuration for that model.
The optimized parameters for the best fit optimized models are shown in Table 3.5. For the simplest model the joint stiffness is found to be 366 Nm/rad at the 8 intervertebral joints of the spine. This corresponds to a total neck stiffness of 48.5 Nm/rad (0.85 Nm/deg) which provides very good agreement with 0.73 Nm/deg cervical stiffness in extension given by White et al. [48]. The joint stiffness no joint damping model has high stiffness in the lower segments and low stiffness in the upper segments. This restricts the motion in the lower segments during whiplash which limits its ability to achieve the realistic S-shaped whiplash motion as shown in Figure 1.1. The joint stiffness joint damping model displays a range of stiffness values from nearly zero up to 256 Nm/rad. At the joints where the stiffness is very close to zero, the associated Voigt damping parameter is seen to be non-zero. As a result, the damper will resist the motion at these locations. This suggests that the energy in the neck is primarily stored in the joints 2, 3, 6, 7, and 8 with joints 1, 3, 4, 5, and 6 acting to dissipate the energy of the collision. It should be noted this analysis has focused on capturing the whiplash response early in the collision sequence as it is this region that is believed to be associated with injury [2, 34]. For this reason the fit values and optimization parameters are based on the initial whiplash response as shown in Figures 3.7 to 3.10. This may lead to model and joint parameters may not capture the accurate whiplash response beyond this time window.

### Table 3.4: Optimization criteria for determining viscoelastic parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>Upper Bound</th>
<th>Initial Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>UJSND</td>
<td>5 000 Nm/rad</td>
<td>Entire range</td>
</tr>
<tr>
<td>JSND</td>
<td>100 000 Nm/rad</td>
<td>5 Nm/rad, 10 000 Nm/rad</td>
</tr>
<tr>
<td>JDNS</td>
<td>100 000 Nms/rad</td>
<td>5,500 Nm/rad, 10 000 Nm/rad</td>
</tr>
<tr>
<td>JSJD</td>
<td>100 000 Nm/rad</td>
<td>5 Nm/rad</td>
</tr>
<tr>
<td></td>
<td>100 000 Nms/rad</td>
<td>5 Nms/rad, 500 Nms/rad</td>
</tr>
</tbody>
</table>

#### 3.4 Dynamic Response for Rigid Linkage Model

The optimized parameters for the best fit optimized models are shown in Table 3.5. For the simplest model the joint stiffness is found to be 366 Nm/rad at the 8 intervertebral joints of the spine. This corresponds to a total neck stiffness of 48.5 Nm/rad (0.85 Nm/deg) which provides very good agreement with 0.73 Nm/deg cervical stiffness in extension given by White et al. [48]. The joint stiffness no joint damping model has high stiffness in the lower segments and low stiffness in the upper segments. This restricts the motion in the lower segments during whiplash which limits its ability to achieve the realistic S-shaped whiplash motion as shown in Figure 1.1. The joint stiffness joint damping model displays a range of stiffness values from nearly zero up to 256 Nm/rad. At the joints where the stiffness is very close to zero, the associated Voigt damping parameter is seen to be non-zero. As a result, the damper will resist the motion at these locations. This suggests that the energy in the neck is primarily stored in the joints 2, 3, 6, 7, and 8 with joints 1, 3, 4, 5, and 6 acting to dissipate the energy of the collision. It should be noted this analysis has focused on capturing the whiplash response early in the collision sequence as it is this region that is believed to be associated with injury [2, 34]. For this reason the fit values and optimization parameters are based on the initial whiplash response as shown in Figures 3.7 to 3.10. This may lead to model and joint parameters may not capture the accurate whiplash response beyond this time window.
Table 3.5: Optimized rigid linkage lumped parameter values

<table>
<thead>
<tr>
<th>Model</th>
<th>Rotational Stiffness (Nm/rad)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
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<tr>
<td></td>
<td>Joint 1 C7-T1</td>
<td>Joint 2 C6-C7</td>
<td>Joint 3 C5-C6</td>
<td>Joint 4 C4-C5</td>
<td>Joint 5 C3-C4</td>
<td>Joint 6 C2-C3</td>
<td>Joint 7 C1-C2</td>
<td>Joint 8 Skull rot</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UJSND</td>
<td>366</td>
<td>366</td>
<td>366</td>
<td>366</td>
<td>366</td>
<td>366</td>
<td>366</td>
<td>366</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JSND</td>
<td>9864</td>
<td>358</td>
<td>295</td>
<td>9994</td>
<td>10000*</td>
<td>181</td>
<td>50</td>
<td>10000*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JDNS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JSJD</td>
<td>0</td>
<td>256</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>29</td>
<td>55</td>
<td>246</td>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Rotational Damping (Nms/rad)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Joint 1</td>
<td>Joint 2</td>
<td>Joint 3</td>
<td>Joint 4</td>
<td>Joint 5</td>
<td>Joint 6</td>
<td>Joint 7</td>
<td>Joint 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UJSND</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JSND</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
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</tr>
<tr>
<td>JDNS</td>
<td>1483</td>
<td>336</td>
<td>27</td>
<td>14</td>
<td>20319</td>
<td>32</td>
<td>4</td>
<td>6748</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JSJD</td>
<td>906</td>
<td>0</td>
<td>569</td>
<td>11</td>
<td>530</td>
<td>645</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* upper bound

The response for each of the optimized models is shown in Figures 3.7-3.10. The relative displacement of the head to the T1 vertebra in the horizontal direction is shown in Figure 3.7. The solid line in the figure demonstrates the cadaver head response observed by Grauer et al. [4] during experimental testing. The other 4 lines demonstrate the responses for the 4 other rigid linkage models: uniform joint stiffness no joint damping (UJSND), joint stiffness no joint damping (JSND), joint damping no joint stiffness (JDNS), and joint damping joint stiffness (JDJS). During whiplash, the head is initially forced backward relative to the T1 vertebra before rebounding forward as shown by the negative relative displacement in Figure 3.7. Apart from the joint damping (JDNS) model, all models show a reasonable approximation of the head peak x displacement, with the uniform joint stiffness model (UJSND) slightly over predicting the deflection and the joint stiffness (JSND) and joint stiffness joint damping (JSJD) models slightly under predicting the response. All 3 of these models display peak relative head displacement at approximately 130 ms while the cadaver response reaches a peak at 90 ms after the initial collision. After reaching the peak deflection, the viscoelastic joint stiffness joint damping model produces the best response with a more gradual decrease in deflection over time, similar to that of the cadaver results.
Figure 3.7: Rigid linkage model comparison of relative head to T₁ vertebra horizontal displacement during whiplash

The head displacement relative to the T₁ vertebra for the rigid linkage models and cadaver response is shown in Figure 3.8. It should be noted that the scale is greatly reduced from that of the x displacement shown in Figure 3.7. The models show similar trends to that of the head x displacement, with the UJSND, JSND, and JSJD models reaching a peak value 20 ms after the cadaver. As before, the uniform joint stiffness model slightly exceeds the peak y relative displacement from the cadaver testing and the JSND and JSJD models underestimate the peak deflection. After reaching its peak value, the joint stiffness joint damping model shows a good fit with the cadaver response over time.
The overall model performance for head rotation over time is shown in Figure 3.9. The joint damping model demonstrates a very poor response compared to the cadaver experimental results. The other 3 models produce a good approximation of the peak head rotation with the peak head rotation occurring at 100 ms, 125 ms, and 135 ms for the cadaver, joint stiffness joint damping, and uniform joint stiffness models. Beyond this peak, the joint stiffness joint damping model produces the most realistic response over time. Although both the model and cadaver responses are similar, the cadaver head rotation is seen to increase more rapidly than the rigid linkage models after around 50 ms. The cadaver response also displays a concave downward decrease in the rotation from about 100 to 150 ms. The models, by contrast, exhibit a smooth transition before and after reaching their peak rotation levels.
In addition to the head response of whiplash, the motion of the neck during whiplash is also important for assessing whiplash injury [57]. Figure 3.10 demonstrates the intervertebral response during whiplash for the four different rigid linkage models and cadaver experimental results. Looking at the cadaver response in Figure 3.10A, there are two main characteristics to note. The first is the flexion of the cervical spine at the ‘skull to C1’ and ‘C1 to C2’ levels occurring around 40 to 80 ms after the collision. At this time, the upper spine segments are flexed while the other segments are extended, corresponding to the S-shaped curvature shown in Figure 1.1. It is important to note that this does not imply that the segments are actually rotating forward in global coordinates but only that they have rotated forward relative to their adjacent inferior segments. The other characteristic of note is the large extension of the upper cervical segments as the head rotates backward during whiplash. The JSND, JDNS and JSJD models all exhibit large intervertebral extensions at the ‘C1 to C2’ joint similar to the cadaver results. Of these models, only the joint stiffness joint damping model exhibits the s-shaped flexion/extension characteristic of the upper and lower cervical neck segments during whiplash. The JSJD model also displays an increase in the ‘C4 to C5’ joint extension over time which is not seen in the cadaver results.
Figure 3.10: Rigid linkage model comparison of intervertebral extensions during whiplash
Figure 3.10D shows the intervertebral extensions over time for the viscoelastic JSJD model. From this figure it is evident that the ‘Skull-C1’, ‘C1-C2’, ‘C4-C5’, and ‘C6-C7’ joints allow motion while the other segments are virtually ‘locked’. Referring to Table 3.5 these flexible locations correspond to joints 2, 4, 7, and 8 which have low rotational damping values of 0 Nms/rad, 11 Nms/rad, 0 Nms/rad, and 0 Nms/rad. By contrast, the rigid joints 1, 3, 5, and 6 have damping values of 906 Nms/rad, 569 Nms/rad, 530 Nms/rad, and 645 Nms/rad, with little or no associated Voigt stiffness. This demonstrates the ability of the Voigt damper to resist motion during the rapid whiplash motion.

The resulting fit values for each model configuration is shown in Figure 3.11. The joint stiffness joint damping model provides the best overall fit for head (headfit) and neck (extfit) motion. This is not surprising considering the viscoelastic nature of the soft tissue of the neck [45, 48]. Both the uniform joint stiffness and joint stiffness joint damping models show a good overall fit for the intervertebral extensions of the neck, but the uniform joint stiffness does not capture the realistic head motion as well as the JSJD model. This is demonstrated in Figures 3.8 and 3.9 where the UJSND model produces excessive head deflection relative to the T1 vertebra in the horizontal and vertical directions. The joint damping model (JDNS) is seen to produce the worst overall whiplash response due to its inability to spring backward and forward during whiplash. This demonstrates the importance of the stiffness of the neck to produce the forward motion of the head in the later stages of the whiplash motion. The simplest uniform joint stiffness model has the best (lowest) extfit value corresponding to the best approximation of the neck motion during whiplash. As discussed previously, this model does not capture the important s-shaped curvature of the neck during whiplash, which is important in assessing whiplash injury. With this in mind the joint stiffness joint damping model is seen to provide the most realistic assessment of both the head and neck motion during whiplash.
Figure 3.11: Rigid linkage model performance summary for cadaver fit results
In this chapter, we use the finite element method to develop 2 dynamic models of whiplash. The first makes use of rigid linkages to simulate the intervertebral motions and the second treats the problem using beam elements. The purpose of this work is threefold: (i) to enable the verification of the lumped mass model developed in chapter 3, (ii) to investigate the application of a continuous beam structure to model whiplash, and (iii) to support the experimental investigations.

4.1 Overview of Finite Element Method

The finite element method is a powerful numerical tool for solving engineering and biomedical problems. All finite element methods involve dividing the physical system into small subsections known as elements. For each element the degrees of freedom and response to external conditions can be applied. Then, by using large numbers of elements, the features of the overall system can be captured. One of the attractions of the finite element method is the ease with which it can be applied to real biomedical engineering problems involving complex geometrical features. The drawback of using multiple elements comes with the amount of numerical computations required to solve the resulting sets of simultaneous algebraic equations. In the following sections of this chapter, we intend to discuss the finite element details of the developed models. The rigid linkage model defines rigid linkage, viscoelastic joints, and lumped masses in a multi-body dynamics assessment of the whiplash response. The beam model, on the other hand, follows the same profile using many beam elements to define the model. Both formulations use non-linear transient analysis to capture the motion over time. The non-linear approach is required to take into account the changes in the mass matrix and applied loading which occur during the large deformations of the whiplash response. Transient analysis is required to determine the response of the system over each time step due to the applied loads. It is worth noting that the models were developed in the commercial software ANSYS. A 2-dimensional analysis of the whiplash response in the sagittal plane is performed using this software.
4.2 Rigid Linkage Model

In this model, the motion of head and intervertebral discs is captured using rigid linkages and viscoelastic joints. Figure 4.1 shows the approximated geometry of the head and neck using the discretized finite element rigid linkage model. Eight rigid beam elements (known in ANSYS as MPC184) and eight viscoelastic revolute joints (known in ANSYS as MPC184) were used to define the structure of the neck in the same manner as the rigid linkage model developed in chapter 3. Joint locations and mass values were defined according to Table 3.2. Joint mass values were defined at each of the locations shown in Figure 4.1 with a 5.5 kg mass and 0.035 kg\cdot m^2 inertia defined at the location of the center of gravity of the head, corresponding to that of a 50th percentile male. Each of the viscoelastic joints in the model were defined with unique linear rotational stiffness and damping values. These values were defined according to the optimized joint stiffness joint damping values shown in Table 3.5.

![Rigid Linkage FE model with revolute joints and rigid beam elements](image)

**Figure 4.1: Rigid linkage FE model with revolute joints and rigid beam elements**

(R – revolute joint, M – point mass, I – inertia)

Typical whiplash acceleration is used to load the system at the base of the neck at node 1. As Figure 4.2 indicates, the acceleration profile contains a linear increase in the horizontal acceleration up to 8.5 g’s at 0.0525 s and decreases linearly back to zero at 0.105 s. Zero
acceleration is applied in the forward direction beyond this time. Zero acceleration is applied in the upward (y) direction throughout as the effects of gravity are ignored.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Acceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>8.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.2: Rigid linkage FE 8.5g whiplash acceleration applied at the T₁ vertebra

Full transient analysis with large deflections is used in this analysis. Four load steps are used to define the input acceleration for the system over time: (i) zero initial acceleration, (ii) ramped acceleration up to 8.5 g at 0.0525 s, (iii) ramped acceleration down to zero at 0.105 s, and (iv) zero acceleration up to 0.225 s. The initial conditions for the system were zero initial position and velocity at each of the nodes. At this point, the response over each load step was solved and a stable and converged solution was achieved.

The results for the whiplash head response are shown in Figure 4.3. The head displacement relative to the T₁ vertebra is shown on the left axis and the head rotation is shown on the right axis. The head x displacement is approximately 8 cm relative to the T₁ vertebra at its maximum and the relative y displacement is approximately 2 cm, also in the negative direction. The peak head rotation is approximately 45 degrees. This supports the earlier claim for the importance of performing non-linear analysis to capture the effects of large displacements.
Figure 4.3: Rigid linkage FE time variation of head response during whiplash

Figure 4.4 shows the relative acceleration between the head and the T\textsubscript{1} vertebra. The left ordinate shows the horizontal (x) and vertical (y) acceleration and the right ordinate shows the angular acceleration of the head. This figure shows the head initially accelerates backwards before rebounding and accelerating forward. At the same time, the angular acceleration of the head is initially positive, causing the head to rotate counterclockwise (when viewed from the right-hand side as per Figure 4.1) before reaching a maximum velocity and accelerating in the opposite direction.
Figure 4.4: Rigid linkage FE time response of head acceleration during whiplash

The intervertebral extension of the rigid linkage finite element model during whiplash is shown in Figure 4.5. Similar to the rigid linkage response of chapter 3, extensions are observed in the ‘C₆-C₇’, ‘C₄-C₅’, ‘C₁-C₂’, and ‘Skull-C₁’ joints during the whiplash motions. The initial flexion in the upper segments around 40 to 90 ms is not evident in the response for the finite element rigid linkage model and extensions are lower than those of Grauer et al. [4] and the viscoelastic rigid linkage model developed in chapter 3. These findings are discussed further in Chapter 6.
The present work was further extended by modeling the motion of the head and intervertebral discs using beam elements. Beam elements provide 2 useful benefits in addition to those of the rigid linkage model: (i) the capture of a continuous simulated neck deformation profile resulting from whiplash loading and (ii) ease of application for experimental validation of the dynamic whiplash response.

Figure 4.6 demonstrates the finite element beam model used to approximate the response of the cervical spine and head during whiplash. The initial posture of the model is equivalent to that of the rigid linkage models developed previously, with the beam elements forming straight line segments between the instantaneous axis of rotation locations shown in Table 3.2. Each beam segment is divided into 10 elements, leading to 80 total beam elements (known in ANSYS as Beam3 elements) for the model as shown in Figure 4.7. These elements are capable of supporting both axial and compression loads as well as transverse loads and bending moments. The beam elements of the model are given a density of 2.5 kg/m$^3$ to produce a total neck mass of 0.5 kg. This mass is chosen to be roughly equivalent to the mass of the physical neck samples used in the experimental validation. Additional mass and inertia values are defined at the center of gravity of the head using an ANSYS Mass21 element.
Figure 4.6: Beam FE model with continuous beam mass, head mass, and inertia

Figure 4.7 demonstrates the approach used to determine the bending stiffness to approximate the stiffness of the rigid linkage model. In the case of the rigid linkage structure, the rotations in each segment $\theta_{seg}$ are a function of the net moment acting on the joint $M$ and the rotational joint stiffness $k_i$ according to equation 4.1.

$$\theta_{seg} = \frac{M}{k} \quad (4.1)$$

For the beam element, the deformation is continuous over its length. A pure moment applied to the end of a beam segment will lead to a constant internal moment and curvature along its length as shown in Figure 4.7. The bending moment $M_{bend}$ in the beam is equal to the product of the modulus of elasticity of the material $E$, the second moment of area $I$, and the curvature of the beam $\frac{d\theta}{dl}$ as shown in equation 4.2.

$$M_{bend} = EI \frac{d\theta_{bend}}{dl} \quad (4.2)$$
Rearranging this equation and integrating over the segment length gives the beam angle $\theta$ as a function of the bending moment $M_{\text{bend}}$, bending stiffness $EI$, and segment length $L_i$ as per equation 4.3.

$$\theta_{\text{bend}} = \frac{M_{\text{bend}}L_i}{EI} \quad (4.3)$$

Equating equations 4.1 and 4.3 gives the bending stiffness $EI$ as a function of the segment length $L_i$ and rigid linkage rotational joint stiffness $k_i$ for the equivalent beam model. This is shown in equation 4.4.

$$EI = k_i L_i \quad (4.4)$$

Using equation 4.4, the equivalent bending stiffness can be determined for each of the segment lengths shown in Figure 4.6 and given in Table 3.2. For the uniform joint stiffness joint damping rigid linkage model, the stiffness at each of the joints was found to be 366 Nm/rad to provide the most realistic response. With this stiffness value, the average equivalent bending stiffness is found to be equal to 9.2 Nm². Note that this approach seeks to match the equivalent rotations at the nodal locations of interest but that the deflections, and therefore nodal displacements, will differ. The parameters specified to define the beam element to achieve this equivalent bending stiffness value are shown in Table 4.1.
Figure 4.7: Equivalent beam element

Table 4.1: Finite element beam model parameters

<table>
<thead>
<tr>
<th>Beam Properties</th>
<th>Head Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass</td>
<td>0.5kg</td>
</tr>
<tr>
<td>Mass</td>
<td>5.5kg</td>
</tr>
<tr>
<td>Density</td>
<td>2.5kg/m³</td>
</tr>
<tr>
<td>Inertia</td>
<td>0.035kg·m²</td>
</tr>
<tr>
<td>Thickness</td>
<td>1.6002x10⁻³m</td>
</tr>
<tr>
<td>Depth</td>
<td>0.1314m</td>
</tr>
<tr>
<td>Izz</td>
<td>4.5x10⁻¹¹m⁴</td>
</tr>
<tr>
<td>E</td>
<td>205GPa</td>
</tr>
<tr>
<td>EI</td>
<td>9.2Nm²</td>
</tr>
</tbody>
</table>

Typical whiplash acceleration was used to load the system at the location of the T₁ vertebra. As Figure 4.8 indicates, the acceleration profile contains a sinusoidal profile, which decays to a small level corresponding to the appropriate components of the gravitational acceleration along the x and y axes. This acceleration profile was used to model the acceleration profile for an experimental test fixture inclined at 10 degrees (global x axis inclined 10 degrees from the horizontal). In order to determine the initial posture for the model under the application of gravitational loads, a static analysis was performed to determine the static deformation of the
neck under gravitational forces. These nodal deformations were then used to define the initial configuration for the model in the transient analysis.

![Graph showing T1 acceleration profile](image)

**Figure 4.8: Beam FE whiplash acceleration applied at the T₁ vertebra**

In the following we provide only a sample of the output to demonstrate the response of the beam model during whiplash. Detailed analysis of the results, including the analytical model and the experimental investigations, will be discussed in chapter 6.

Figure 4.9 shows the time variation of the displacement components of the head relative to the T₁ vertebra in the x and y directions. The figure shows that for the given 8.5 g acceleration profile, the relative head displacement in the x direction increases dramatically and reaches a peak value of -0.105 m at 130 ms. The relative head displacement in the y direction also reaches its much smaller peak value of -0.030 m at this time. The head rotation reaches its maximum value of 57.8 degrees slightly before this at 120 ms.
Figure 4.9: Beam FE time variation of head response during whiplash

The head acceleration relative to the T₁ vertebra is shown in Figure 4.10 for the x and y rectilinear coordinates. During the initial stage of whiplash (0 to 60 ms), the head is accelerated in the negative x direction relative to the T₁ vertebra. After the head has reached its maximum relative velocity in the backward direction, it sustains a larger forward acceleration as the neck resists the posterior motion of the head. The forward horizontal acceleration of the head reaches a peak of 5.2 g at 145 ms. The vertical acceleration of the head displays a smaller and delayed initial negative acceleration as the head approaches its maximum rotation. This is followed by a strong increase in the vertical acceleration up to a maximum value of 4.0 g at a time of 135 ms. The angular acceleration of the head increases steadily up to a maximum of 589 rad/s² at a time of 102 ms. The positive angular acceleration of the head corresponds to the counterclockwise rotation of the head in the sagittal plane when viewed from the right-hand side of the individual (x in forward direction, y in upward direction relative to the seated individual). After the head reaches a maximum angular velocity in the counterclockwise direction, the acceleration decreases to a minimum value of -6 g at 100 ms as the neck resists the CCW rotation of the head. These
large rectilinear accelerations and velocities (area under each acceleration curve) have been linked to injury due to their link with damaging pressure effects in the neck during whiplash [3].

**Figure 4.10: Beam FE time variation of head acceleration during whiplash**

Due to the continuous deformation of the beam finite element structure, the intervertebral extension of the neck can be defined in two different ways to assess the whiplash response. The first method, shown in Figure 4.11A, compares the nodal rotations at each of the instantaneous axis of rotation locations. This gives a measure of the relative extensions at the joints similar to that of the rigid linkage model. While this method is analogous to that of the rigid linkage model, it is challenging to measure these rotation values during physical testing due to their small size. Figure 4.11B shows the alternative method for determining the intervertebral extension of the neck by tracking the locations of the instantaneous axes of rotation and assuming linear segments between these points. Once these IAR locations are determined and the segments defined, the intervertebral extensions can be determined using the same approach demonstrated in Figure 3.6. Figure 4.11 compares the results for these two methods. The response for both methods are similar with a few minor differences. The extension values for the ‘joint position’ method in Figure 4.11B are reduced compared to the ‘joint rotation’ method in Figure 4.11A. This is due to the linear segment approximation for the ‘joint position’ extension calculation, leading to the
underestimation of the rotations at the IAR nodal locations. This ‘joint position’ approach also affected the relative magnitude of the extension values observed at each of the joints, with the largest changes occurring at the C7-T1 joint as shown in Figure 4.11.

![Graph showing intervertebral extension during whiplash](image)

**Figure 4.11: Beam FE time variation of intervertebral extension during whiplash**

(A) joint rotations (B) joint positions
In this chapter, we outline the details of the fixture design and experiments performed to test the whiplash models developed in this thesis. The design contains a linear sled mounted on a steel frame with precision rails and guides as well as a simulated head and neck. The fixture is fully instrumented with strain gauges, accelerometers, and reflectors for image capture and image analysis. The test rig is designed to be capable of producing accelerations up to 8.5 g according to that of the dynamic RCAR head restraint standards [29].

5.1 Test Fixture Design

The design of the inclined sled is characterized by its ability to provide impact loads to examine the effect of the acceleration upon the motion of the head relative to the torso. The exploded view of the complete whiplash test fixture design is shown in Figure 5.2. The end supports act to secure two 2” diameter and two ½” diameter precision steel rails at 10 degrees to the horizontal. The lower end support shown in Figure 5.2 is bolted to the floor to avoid any unwanted motion of the fixture itself. In order to avoid binding issues, the rails are fixed parallel to one another to within 1/1000 of an inch. The parallelism of the ½” diameter rails are less of an issue due to their flexibility over their 92” span. The other important components of the fixture design are the test sled to simulate the typical acceleration during whiplash, compression springs to provide the desired overall sled acceleration, and a release latch to ensure repeatability between whiplash test trials. The location of the release latch controls how far the test sled travels under the acceleration due to gravity prior to contacting the fixture springs. This determines the impact velocity of the sled into the springs and the peak acceleration of the sled. The spring stiffness $k_{spring}$ and sled mass $m_{sled}$ determine the approximate collision time for the experiment based on the natural frequency of vibration for a spring and mass system according to equation 5.1. This equation
ignores the effects of head oscillation, energy losses, and non-linear spring stiffness on the response.

\[
t_{\text{collision}} = \pi \sqrt{\frac{m_{\text{sled}}}{k_{\text{spring}}}}
\]  

(5.1)

Four springs (2 on each ½” rail) are used in order to provide the desired stiffness and load capacity for the system. The spring stiffness and sled mass for the fixture is given in Table 5.1. Using these values the total collision time is calculated to be 0.083s which is slightly lower than the 0.105 s collision time typically used for whiplash testing [29]. In order to reduce high frequency vibrations from the fixture to the test sled, dampers were added on both ends of the springs.

Figure 5.1: Experimental whiplash test fixture
### Table 5.1: Whiplash test fixture parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>(+/-)</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sled</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>16.6</td>
<td>0.8</td>
<td>kg</td>
</tr>
<tr>
<td><strong>Springs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>24000</td>
<td>3600</td>
<td>N/m</td>
</tr>
<tr>
<td><strong>Neck</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wide instrumented mass</td>
<td>0.64</td>
<td>0.01</td>
<td>kg</td>
</tr>
<tr>
<td>Nominal instrumented mass</td>
<td>0.53</td>
<td>0.01</td>
<td>kg</td>
</tr>
<tr>
<td>Narrow instrumented mass</td>
<td>0.40</td>
<td>0.01</td>
<td>kg</td>
</tr>
<tr>
<td><strong>Head</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>5.56</td>
<td>0.01</td>
<td>kg</td>
</tr>
<tr>
<td>Instrumented mass</td>
<td>5.57</td>
<td>0.01</td>
<td>kg</td>
</tr>
<tr>
<td>Center of gravity location (measured radially from CAD CoG)</td>
<td>2</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>0.035</td>
<td>0.002</td>
<td>kg·m²</td>
</tr>
</tbody>
</table>

The sled design with the simulated head and neck is shown in Figure 5.2. The sled consists of an aluminum plate and block to provide a mounting surface for the linear bearings and simulated head. Four low-friction 2” diameter linear bearings with pillow blocks are used to support the sled loads and allow the sled to slide down the 8” precision rails. Two ½” diameter linear bearings with pillow blocks are used to track along the ½” precision rails and transfer the loads from the fixture springs to the test sled. Dowel pins are inserted on the back side of these pillow blocks to provide additional support during loading. The neck mounting block is firmly secured to the sled base using four M8 fasteners. The simulated neck is secured to the sled by two fasteners sandwiching the neck between the neck block and neck plate. This ensures accurate alignment of the neck sample while restricting any undesirable deflection in the region below the T₁ vertebra location.
The simulated head and neck design used to capture the whiplash response are shown in Figures 5.3 and 5.5. These designs capture the head properties of the 50th percentile male and the stiffness of the neck.

The neck design chosen for modeling whiplash is based on the beam finite element model discussed previously. A bending stiffness of 9.2 Nm² is chosen to be roughly equivalent to the rigid linkage joint stiffness found for the simplest uniform joint stiffness joint damping model. As a result, this experimental testing can serve to validate both methodologies developed in chapters 3 and 4. The bending stiffness \( EI \) is equal to the product of the modulus of elasticity for the material \( E \) and the second moment of area \( I \) for a given beam cross section. A range of
materials and cross-section designs can be used to achieve this desired beam flexibility. It is also important, however, that the beam withstand the large deflections and loads during whiplash without yielding. The stress $\sigma$ in a beam subjected to bending is given in equation 5.2, as a function of the moment about the neutral axis $M$, the perpendicular distance from the neutral axis $y$, and the second moment of area about the neutral axis $I$.

$$\sigma = \frac{My}{I}$$

(5.2)

For a given material, a certain second moment of area is required to achieve the required 9.2 Nm$^2$ bending stiffness. In this case, both the applied moment (based on the whiplash loading) and second moment of area are constrained in equation 5.2. In order to reduce the peak stress in the sample, the perpendicular distance from the neutral axis must be limited by making the sample as thin as possible. The final design parameters using AISI4130 steel to satisfy these requirements are shown in Tables 5.2 and 5.3. This material is selected because of its low cost and high strength and ductility after heat treatment.

**Table 5.2: Experimental neck sample mass and heat treatment summary**

<table>
<thead>
<tr>
<th>Neck Sample</th>
<th>Mass [+/- 0.005] (kg)</th>
<th>Mass* [+/- 0.005] (kg)</th>
<th>Heat Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrow</td>
<td>0.375</td>
<td>0.400</td>
<td>HRC 34-38 (HT-1) Outsourced</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.500</td>
<td>0.530</td>
<td>HRC 34-38 (HT-1) Outsourced</td>
</tr>
<tr>
<td>Wide</td>
<td>0.610</td>
<td>0.640</td>
<td>HRC 34-38 (HT-2) In-house</td>
</tr>
</tbody>
</table>

* Instrumented with strain gauge

**Table 5.3: Experimental neck sample geometry and mechanical properties**

<table>
<thead>
<tr>
<th>Neck Sample</th>
<th>Width [+/- 0.05] (mm)</th>
<th>Thickness [+/- 0.01] (mm)</th>
<th>E (GPa)</th>
<th>I (x10$^{11}$ m$^4$)</th>
<th>EI (N·m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value [+/-]</td>
<td>Value [+/-]</td>
<td>Value [+/-]</td>
<td>Value [+/-]</td>
<td></td>
</tr>
<tr>
<td>Narrow</td>
<td>98.55</td>
<td>1.60</td>
<td>203</td>
<td>38</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>0.06</td>
<td>7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Nominal</td>
<td>131.40</td>
<td>1.59</td>
<td>203</td>
<td>38</td>
<td>4.38</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>0.08</td>
<td>9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Wide</td>
<td>164.25</td>
<td>1.62</td>
<td>203</td>
<td>38</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>0.1</td>
<td>12</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
The neck samples are formed out of 2 plates of steel by cutting them to the desired length and bending them to the profile shown in Figure 5.3. A template is used to ensure that each sample is made to the profile specified on the drawing. Each of the bend locations on the beam sample correspond to the locations of the instantaneous axes of rotation for the 50th percentile male as provided in Table 3.2. Four neck samples and 3 tensile samples are made from the 4130 plate material for testing and material validation. The heat treatment and mass for each of the neck samples is given in Table 5.2. The wide sample was heat treated ‘in-house’ using a conventional oven to heat the sample to 855 degrees Celsius followed by a water quench and 480 degree Celsius temper. This caused significant carbon buildup and warpage of the sample so the other two test samples were outsourced to obtain their heat treatment in a more controlled environment.

<table>
<thead>
<tr>
<th>LOC</th>
<th>X-POS (MM)</th>
<th>Y-POS (MM)</th>
<th>LINKAGE</th>
<th>LENGTH (MM)</th>
<th>ANGLE (DEG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKULL C1</td>
<td>13.0</td>
<td>102.0</td>
<td>0.0</td>
<td>51.4</td>
<td>-37.2</td>
</tr>
<tr>
<td>SKULL C2</td>
<td>-15.2</td>
<td>141.1</td>
<td>-0.7</td>
<td>21.4</td>
<td>40.6</td>
</tr>
<tr>
<td>C1/JC2</td>
<td>-4.0</td>
<td>124.5</td>
<td>-0.6</td>
<td>31.0</td>
<td>6.3</td>
</tr>
<tr>
<td>C3</td>
<td>-9.0</td>
<td>93.2</td>
<td>-0.5</td>
<td>14.6</td>
<td>23.4</td>
</tr>
<tr>
<td>C4</td>
<td>5.9</td>
<td>77.5</td>
<td>-0.4</td>
<td>20.4</td>
<td>7.1</td>
</tr>
<tr>
<td>C5</td>
<td>6.4</td>
<td>74.6</td>
<td>-0.3</td>
<td>14.6</td>
<td>3.2</td>
</tr>
<tr>
<td>C6</td>
<td>9.4</td>
<td>40.6</td>
<td>-0.2</td>
<td>19.0</td>
<td>10.6</td>
</tr>
<tr>
<td>C7</td>
<td>5.9</td>
<td>22.3</td>
<td>0.1</td>
<td>22.9</td>
<td>-14.9</td>
</tr>
<tr>
<td>T1</td>
<td>0.0</td>
<td>0.0</td>
<td>TOTAL</td>
<td>200.5</td>
<td></td>
</tr>
</tbody>
</table>

SAMPLE 1: 98.85MM WIDTH, HEAT TREAT TO HRC 34-38
SAMPLE 1: 134.4MM WIDTH, NO HEAT TREATMENT
SAMPLE 2: 124.2MM WIDTH, HEAT TREAT TO HRC 34-38
SAMPLE 4: 131.4MM WIDTH, HEAT TREAT TO HRC 34-38

**Figure 5.3: Heat treated 4130 steel neck sample design**

The resulting material properties for each of the heat treatments and the base 4130 steel material are shown in Figure 5.4. For the nominal width sample, ANSYS predicted a peak stress of 898 MPa at the base of the neck for the 8.5 g collision. In order to avoid yielding for this aggressive loading, the yield point for the material should be above this value. From Figure 5.4, the yield stress is 222 MPa, 747 MPa, and 1080 MPa for the virgin (no heat treatment), HT-2, and HT-1 heat treatments. The yield stress for the nominal and narrow neck samples is well above the
expected maximum stress during whiplash, so the material should behave elastically during the response. For the wide neck sample with HT-2 heat treatment, the maximum stress is approximately equal to the maximum stress expected for the nominal sample. This sample is designed to be 25\% stiffer than the nominal sample, so lower deformations and maximum stress values are expected. This is confirmed by the experimental results for head deflections during whiplash and the absence of observable yielding in the neck sample after testing. The resulting bending stiffness for each of the neck samples tested is shown in Table 5.3.

![Stress vs Strain Graph](image)

**Figure 5.4: Neck material properties.**

The head is designed to capture the mass and inertia of the 50th percentile male. In accordance with the work of Grauer et al. [4] and matching that of the analysis in chapters 3 and 4, the head is designed to be 5.5 kg with a moment of inertia of 0.035 kg·m². The head center of gravity is designed to be located at the center of the slot where the neck attaches to the head. This ensures the center of mass of the head is positioned at the location corresponding to the tip of the neck sample. A robust attachment, using 4 set screws, ensures that the head and neck are securely fastened to one another. The measured values for the final head specimen are shown in Table 5.1. These values show excellent agreement with the design values in the computer aided design software SolidWorks. The moment of inertia of the head about the center of mass is found using a bifilar pendulum to suspend the head sample and observe its period of oscillation [96, 97].
Accelerometers, strain gauges, and high speed photography are used to capture the motion of the sled, head, and neck during whiplash. Accelerometers are placed on the sled, along the axis of motion (x axis for analysis) and on the head in the local x and y directions (aligned with the skull-C1 neck segment). During the whiplash motion, the rotation of the head causes these local axes to rotate relative to the global x and y coordinates, so the orientation of the head at a given time is used to resolve these accelerations into their components in the global x and y directions. A strain gauge is placed at the base of the neck to monitor the peak stress observed during the experiments. The strain gauge is connected to a Wheatstone bridge and an amplifier circuit to amplify the output signal. A ‘measurement computing’ analog to digital converter (USB-1608FS) is used to transfer the data to the local computer. Two different accelerometers are used for the
measurements and data acquisition: (i) Kistler 8632C50 and (ii) Measurement Specialties ACH-01. Both accelerometers require different electrical configurations with the Kistler accelerometers requiring a constant current source and the ACH-01 accelerometer requiring a constant voltage source and amplifier for the output signal. The circuit diagram for each of the accelerometers is included in appendix B. Figure 5.6 demonstrates the whiplash test fixture setup including the Fastec Imaging TSHRMS high-speed camera and Lowel 250 W adjustable light source.

![Image of whiplash test fixture](image)

**Figure 5.6: Motion capture instrumentation for whiplash test fixture**

Figure 5.7 demonstrates the reflective targets applied to the sled, head, and neck to accurately assess the whiplash motion. Two reflective targets on the sled are used to ensure the camera frame is aligned with the direction of the sled motion. The reflective targets on the sled (x direction) and vertical sled block (y direction) are spaced apart by 200 mm to provide accurate scaling of the motion capture images. The distance from the camera to the x-axis sled points, y-axis sled points, head and neck are used to scale the motions observed using the motion capture software. Three points on the head are used to define the location of the center of gravity of the head (middle reflective target on head) and the rotation of the head at each time step. Additional
reflective targets are placed on the instantaneous axes of rotation at the C₇-T₁ joint up to the Skull-C₁ joint.

Figure 5.7: Experimental reflective targets for high-speed motion capture

5.4 Motion Capture Software

Motion capture software, developed by Ben Cornwell-Mott in the Mechanics and Aerospace Design Lab, is used to record the location of each of the reflective targets over time. The software works by analyzing each frame to determine the location of each of the reflective locations and assigns a number to each location. The numeric center of a cluster of pixels above a certain brightness range is chosen for that location. A typical frame during the whiplash motion is shown in Figure 5.8. For each successive frame, the software attempts to maintain the numbering structure for the points that did not move beyond a given range between frames. If the point moves beyond this zone between frames, a new unused number is assigned to that location. During the analysis, multiple numbers corresponding to the same reflective target are stitched together to provide the response of that target over the entire test run. This methodology is found to provide a very good overall assessment of the whiplash motion over time. In some cases, points are found to disappear or shift as interfering reflective areas influence the reflective center location. Frame corruption is also an occasional issue where individual frames are unreadable. These issues posed no difficulty when analyzing the whiplash positions over time but are an issue when the velocities and accelerations of the associated motion are considered. Digital smoothing
of the data is used to overcome this issue and points associated with discontinuities in the position data have been removed as required.

![Figure 5.8: Typical motion tracking frame for head, neck, and sled during whiplash](image)

### 5.5 Typical Dynamic Results

The typical dynamic results for the whiplash motion are shown for the wide sample in the following figures. For both the accelerometer and motion capture data, smoothing is used to digitally filter out the experimental noise. The smoothing is applied according to equation 5.3, where the smoothed value $\text{smoothed}_{i+1}$ is a function of the smoothing factor $\alpha$, unsmoothed value $x_{i+1}$, and previous smoothed value $\text{smoothed}_i$. For the motion capture data, the position data is smoothed using a smoothing factor of 0.18 for the wide and nominal neck samples (using a camera frame rate of 250 frames per second) and smoothing factor of 0.02 for the narrow neck sample for a camera frame rate of 1000 frames per second. The accelerometer data is smoothed using a 0.18 smoothing factor. Note that this is the digital equivalent of applying a low pass filter to the accelerometer output signal.

$$
\text{smoothed}_{i+1} = \alpha x_{i+1} + (1-\alpha)\text{smoothed}_i
$$  \hspace{1cm} (5.3)

The 8.5 g experimental sled acceleration is shown in Figure 5.9. The initial collision time is taken as the time corresponding to the maximum sled velocity prior to the collision. The collision time is 170 ms for the collision based on the motion capture data. The peak sled acceleration is 5 g for the collision. Both accelerometers are not found to provide a good assessment of the collision...
time and peak acceleration during this test trial due to fluctuations and high frequency oscillations in their signals. These oscillations are found to be greatly reduced for lower peak acceleration experiments.

![Graph showing typical experimental sled acceleration for the wide neck](image)

**Figure 5.9: Typical experimental sled acceleration for the wide neck**

The experimental head response to whiplash is shown in Figure 5.10 for the wide neck sample using the motion capture data. The motion capture data shows peak x and y head displacements relative to the T1 vertebra of -0.094 m and -0.022 m at times of 148 ms and 144 ms. The peak head rotation of 47.9 degrees occurs slightly before this, at a time of 132 ms.
The segment angles for the simulated neck sample are shown in Figure 5.11. These initial angles correspond well with the initial segment angles provided for the posture of the 50th percentile male in Table 3.2. The segment angles all increase over time initially with the onset of the collision.

Figure 5.10: Experimental head response for the wide neck during whiplash

Figure 5.11: Experimental segment angles for the wide neck during whiplash
The intervertebral extension for the wide experimental neck sample is shown in Figure 5.12. The extension profile at each of the joints follows approximately the same path, reaching their peak values at around 140 ms. The intervertebral extension for the C6-C7 joint reaches its maximum value sooner, at approximately 90 ms.

![Graph showing intervertebral extension](image)

**Figure 5.12: Experimental intervertebral extension for the wide neck during whiplash**

The relative head accelerations in rectilinear and angular coordinates are shown in Figure 5.13 based on the smoothed motion capture data. The data shows an initial head acceleration relative to the T1 vertebra, followed by the positive acceleration later in the collision sequence. The results are reversed for the angular acceleration, with an initial positive angular acceleration followed by a smooth transition to negative acceleration values.
Figure 5.13: Experimental relative head acceleration for the wide neck during whiplash
Chapter 6
Results and Discussion

This chapter assesses the whiplash response observed for the rigid linkage, finite element, and experimental models developed in chapters 3 to 5. These results are compared to those established in literature and the interesting characteristics of the models and analytical findings are discussed. The following sections deal with the effect of neck stiffness, gravity, acceleration profile, and acceleration magnitude on the whiplash response, using the previously discussed models.

6.1 Outline of Results and Discussion

The results of this work are concerned with the dynamic response of the head and the intervertebral response to dynamic loads to simulate the kinematics experienced by humans in whiplash trauma. The whiplash motion, velocities, and accelerations of the head and neck are closely linked with whiplash injury [3, 34, 57]. Accordingly, the results are concerned with the whiplash motion of the following: (i) head movement relative to the T₁ vertebra (x and y), (ii) head rotation, (iii) intervertebral accelerations, and (iv) head acceleration relative to the T₁ vertebra.

In our discussions, we intend to account for the rigid linkage model, finite element models, and experimental results to elucidate the influence of the pertinent parameters that dictate whiplash associated disorders. This comparison will also provide some form of quality assurance between the different techniques adopted for the treatment of the problem in this thesis.

6.2 Comparison of Developed Models

The input acceleration applied at the T₁ vertebra is shown in Figure 6.1 for the viscoelastic rigid linkage model, finite element models, experimental testing completed by the author, and for an established computational model [69] and experimental cadaver test results from literature [4].
The Van Lopik et al. [69] model is a three-dimensional multi-body dynamics formulation that has been shown to provide good results for the whiplash response by capturing the net effect of the muscles, ligaments, intervertebral discs, etc. The Grauer et al. [4] cadaver results present the cadaver response for whiplash loading with muscle tissue removed and other soft tissue intact. These cadaver whiplash results provide the closest link to the human whiplash response available for full-speed whiplash testing expected to produce injury. As shown in Figure 6.1, most computational models and dynamic responses are based on the response from a triangular acceleration profile, increasing linearly up to 8.5 g at 0.0525 s and back down to zero at 0.105 s. The ‘FE-beam’ line in Figure 6.1 corresponds to the acceleration in the sled frame of reference for the expected load profile with the experimental whiplash test fixture including the effects of gravity. The experimental test results (given by the ‘Exp’ line) show a similar overall trend for the collision, but the peak acceleration is delayed by close to 50 ms as the acceleration increases up to its peak value.

![Figure 6.1: Comparison of acceleration profiles for rigid linkage (RL), finite element rigid linkage (FE-RL), finite element beam (FE-beam), experiments (exp), Van Lopik et al. model (model) [69], and Grauer et al. cadaver testing [4]](image)
The associated head response for each of these models is shown Figure 6.2 below. For the relative head displacement to the T1 vertebra in the x direction, all of the models provide peak relative displacements in the range of -7.9 to -11.4 cm. The whiplash experimental results are significantly delayed relative to the cadaver test results and beam test results. This is believed to be due to the delayed increase in the input acceleration for the experimental model as shown in Figure 6.1. If this was shifted backwards 50 ms according to that required to equate the acceleration input profile, the experimental model would closely align with that of the finite element beam model. All of the models display a slower increase in the initial x displacement with the Van Lopik et al. model producing the fastest increase of all of the models. This may be due to a number of factors, including intervertebral joint freedom, additional muscle stiffness, and non-linear ligament stiffness in the multi-body dynamics model. An additional point of interest is the reduction in the peak displacement following 100 ms in Figure 6.2. In this region, the viscoelastic models and cadaver results align well, while the non-viscoelastic results for the experiment and finite element beam show a sharp decrease in the displacement from their peak values. This is believed to be due to the damping effects opposing the ‘spring-back’ response of the neck during whiplash. For the relative y head displacement, the scale is greatly reduced compared to that of the x displacements. All of the models show the same general trend of a decrease in the relative height of the head as it is forced backwards during the whiplash motion. For this displacement, the Lopik model and cadaver test results display peak displacements of -5 cm and the other models develop peak values of -3 cm. As before, the experimental response is seen to lag behind that of the rigid linkage viscoelastic model and that of the finite element beam model.
Figure 6.2: Relative head to T1 vertebra comparison between rigid linkage (RL), finite element rigid linkage (FE-RL), finite element beam (FE-beam), experiments (exp), Van Lopik et al. model (model) [69], and Grauer et al. cadaver testing [4] during whiplash.

Figure 6.3 shows the head rotation over time for all of the results. The finite element model is seen to poorly model the ramped increase for the head rotation observed with the other results. This may be due to the course time scale used for the rigid linkage analysis and also that the ANSYS gyroscopic matrix (associated with angular rotations for the system) does not support large deflection effects [98]. All of the models show the general trend of an increase in the head rotation beginning at 50 ms followed by a reduction in the peak rotation as the head begins to rebound forward. The peak head rotations for both the finite element and experimental beam models show excellent agreement with one another, exhibiting peak rotations of 57.8 degrees for the finite element beam model and 57.0 degrees for the experimental beam model. The rigid linkage model closely matches the cadaver test results for both peak rotation and the decrease in the head rotation beyond 100 ms. Between 100 to 175 ms the cadaver head rotation exhibits a slight oscillation in the rotation values as the rotation continues back to zero. This effect is not captured by any of the models shown and is not well understood. This type of response could be
the result of the activation of multiple modes of vibration of the neck during the whiplash sequence.

Figure 6.3: Head rotation comparison between rigid linkage (RL), finite element rigid linkage (FE-RL), finite element beam (FE-beam), experiments (exp), Van Lopik et al. model (model) [69], and Grauer et al. cadaver testing [4] during whiplash

In addition to the head response during whiplash, the intervertebral response of the neck is important for assessing injury to the facet joints during whiplash [2, 30, 57]. While many models have been developed to capture the realistic motion of the head, few have addressed the additional challenge of capturing the accurate neck motions during whiplash. Figure 6.4 shows the intervertebral response for each of the whiplash models. Figures 6.4A and B show the intervertebral response for the rigid linkage model using analytical and finite element approaches. Both of these models show similar results with intervertebral extensions localized to the ‘Skull-C₁’, ‘C₁-C₂’, ‘C₄-C₅’, and ‘C₆-C₇’ joints. The viscoelastic rigid linkage JSJD model shown in Figure 6.4A also exhibits slight flexion in the upper neck segments and extension in the lower segments, which corresponds to the characteristic S-shaped curvature of the spine associated with whiplash [2, 4, 34].

Figures 6.4C and D show the intervertebral response for the finite element and experimental beam models. The finite element model also demonstrates the initial flexion at the ‘Skull-C₁’,
with the oscillations in the ‘skull-C1’ extension suggesting that multiple modes of vibration are activated during the input acceleration. It should be noted that these beam models are not viscoelastic (stiffness only), enabling these oscillations to be more easily observed. Although the presence of high frequency modes of vibration in the experimental response were readily observed with the naked eye, oscillations of the ‘Skull-C1’ joint according to Figure 6.4C were not recorded during experiments. This may be due to the attachment of the head on the neck sample, which clamps the head to the neck along the length of the upper segment. This attachment resists deformations in this area, contrary to that of the finite element model where a point mass and inertia is specified at the end of the segment.

The intervertebral response for the Van Lopik complete multi-body dynamics model [69] is shown in Figure 6.4D. This model also demonstrates the S-shaped flexion/extension whiplash profile early in the collision sequence. It predicts the maximum intervertebral extension in the C1-C2 joint similar to that for the viscoelastic rigid linkage model. The rigid linkage model, Van Lopik model, and cadaver intervertebral extensions all differ slightly with respect to the details of the response. The rigid linkage model predicts an increase in the ‘Skull-C1’, ‘C1-C2’, ‘C6-C7’ similar to that of the cadaver results of Figure 6.4F, however, it also includes an increase in the ‘C4-C5’ extension over time dissimilar to the Van Lopik model and cadaver results. The Van Lopik model displays a smooth extension profile over time, with larger extensions for the ‘Skull-C1’ and ‘C1-C2’ joints. The other joints in this model converge to positive extension values in the range of 5-7 degrees after 100 ms. In contrast, the experimental cadaver results of Grauer et al. [4] show a jagged profile over time. These results display a peak extension of 30 degrees at the ‘C1-C2’ joint beyond 100 ms, with the other joint extensions contained within a window of approximately -12 to 12 degrees. More experimental cadaver testing, with well defined input accelerations, is required to gain a better understanding of the extent to which the trends observed in the intervertebral response are repeatable.

There are a number of potential sources of error in the rigid linkage model, including the locations for the ‘instantaneous axes of rotation’, the viscoelastic stiffness and damping parameters defining the model, the segment and head mass and inertia values, and the limitations of the material model used to define the response. For the rigid linkage model, stiffness and damping parameters were assumed to be linear and only acting between adjacent segments. In
reality, muscles, ligaments, and other soft tissue extend along the length of the cervical spine and attach at multiple locations. As a result, stiffness and damping parameters are expected to be highly coupled to multiple segments of the spine. These tissue properties are known to be highly non-linear as well. Even with these approximations the model provides a design tool that can be used to determine the pertinent parameters that influence the dynamic response of occupants to a given collision acceleration profile.
Figure 6.4: Intervertebral extension comparison between rigid linkage (RL), finite element rigid linkage (FE-RL), finite element beam (FE-beam), experiments (exp), Van Lopik et al. model (model) [69], and Grauer et al. cadaver testing [4] during whiplash.
6.3 Effect of Neck Stiffness on Whiplash Response

The effect of neck stiffness on the overall whiplash response is shown in Figure 6.5. For these experiments, steel beam samples were used to capture the elastic deformation of the neck with very low internal damping. Narrow, nominal, and wide neck samples were used to assess the whiplash response. These samples displayed bending stiffness values of $7 \pm 1 \text{ Nm}^2$, $9 \pm 2 \text{ Nm}^2$, and $12 \pm 2 \text{ Nm}^2$ as shown in Table 5.3. The acceleration profile for each of the test trials is shown in Figure 6.5A. The peak acceleration applied to each of the neck samples is roughly equivalent, with slight variations in the peak acceleration times. The resulting head rotation during whiplash is shown in Figure 6.5B. As expected, the narrow sample with lower bending stiffness displays greater peak head rotation for the same input acceleration. The wide sample displays the least head motion, as the stiffer neck is better able to resist the loads applied during whiplash. Figures 6.5C and D display similar trends, with a decrease in neck stiffness corresponding to an increase in peak head deflections in the x and y directions. The period of oscillation for the head vibration is found to decrease with an increase in neck stiffness, as well. This trend is expected as the natural frequency of the system will decrease as the stiffness of the system is reduced.

These findings demonstrate the importance of the stiffness of the soft tissue of the cervical spine to determine the whiplash response, namely the peak deformations achieved and the time scale for the whiplash response.
Figure 6.5: Experimental neck stiffness assessment during whiplash

6.4 Effect of Gravity on Whiplash Response

The overall effect of gravity on the whiplash response is shown in Figure 6.6. The analysis is performed using the ANSYS beam model developed in chapter 4. Three different analyses of the dynamic whiplash response are developed using the three different acceleration profiles as illustrated in Figure 6.6A. The first acceleration profile, labeled Tri X and Tri Y, ignores the effects of gravity and applies a linear triangular acceleration pulse to the T1 vertebra. In the second scenario, the triangular acceleration pulse is maintained in the horizontal direction, with an additional gravitational acceleration in the negative y direction. The third acceleration profile applies a triangular acceleration that includes the effects of gravity for a reference frame inclined at 10 degrees to the vertical. This corresponds to the scenario for the experimental whiplash test fixture discussed in chapter 5. The whiplash response for the head rotation and displacement
relative to the T₁ vertebra in each of the gravitational scenarios is shown in Figures 6.6B-D. These results show very little effect of gravity on the overall head motion. This supports the assumption that gravitational effects can be ignored in the development of the rigid linkage models of chapters 2 and 3.

![Graphs showing head motion and acceleration](image)

**Figure 6.6: FE beam gravity assessment during 8.5g whiplash**

### 6.5 Effect of Acceleration Profile on Whiplash Response

Using the finite element beam model of chapter 4, the effect of different acceleration profiles on the overall whiplash response is assessed. Figure 6.7A shows the accelerations applied to the T₁ vertebra in the horizontal direction. Four main acceleration criteria are believed to play a role in determining the injury during a collision: (i) collision time, (ii) collision severity, (iii) collision profile, and (iv) peak acceleration [31-33]. The collision time corresponds to the total time where
a non-zero acceleration is applied to the system and the collision severity corresponds to the total change in velocity during the collision. This is equal to the area under the acceleration curve.

Five different acceleration profiles were chosen to capture the common acceleration profiles used in the computational and experimental analysis of whiplash. The baseline acceleration profile was taken to be the 8.5 g triangular acceleration profile, used in the RCAR specification for seat and head restraint validation globally [29]. This acceleration profile rises up from zero to its peak value at 0.0525 s and then decreases back to zero at 0.105 s. A square acceleration pulse was also applied, similar to that used by Stemper et al. [68] in experiments and computer simulations. This peak acceleration for the square profile was set to 8.5 g and the width was set to 0.0525 s to match the collision severity of the triangular pulse. Three different sinusoidal acceleration profiles were investigated: (i) matching the triangular pulse peak acceleration and collision time (sin-at); (ii) matching the peak acceleration and collision severity (sin-vt); and (iii) matching the collision severity and collision time.

The whiplash response, shown in Figures 6.7B-D, shows a number of interesting results for the different input accelerations. The square acceleration pulse appears to activate multiple modes of vibration in the response, leading to fluctuations in the head rotation curve shown in Figure 6.7B. Both the ‘Sin-at’ and ‘Sin-av’ acceleration profiles produce an altered whiplash response relative to the triangular pulse, as well. Only the ‘Sin-vt’ acceleration profile, which matched the collision time and severity, produced a virtually identical response to the triangular pulse. In this case a 6.7 g sinusoidal acceleration pulse is found to be nearly equivalent to that of an 8.5 g triangular pulse as shown in Figure 6.7A. This supports the experimental approach used in chapter 5, where a 6.7 g peak sinusoidal acceleration pulse was used to elucidate the whiplash response. These findings also support the claims that all four of these factors will influence the whiplash response. However, the application of the sinusoidal acceleration to capture the effect of the triangular pulse shows that the collision time and collision severity are the most important factors influencing the whiplash response.
The effect of the acceleration magnitude on the whiplash response is shown in Figure 6.8 for the four triangular pulses applied to the finite element beam model. The response shows a roughly linear increase in the rotations and relative head displacements with an increase in the peak collision acceleration. Interestingly, the time for the head to return back to zero was unchanged for all of the collision accelerations. This is analogous to the simple spring and mass system where the period of oscillation is unaffected by the amplitude of oscillation for the system. In this case, the neck stiffness, head mass and head inertia will determine the natural frequency of vibrations as discussed in section 6.3.

Figure 6.7: FE beam acceleration profile assessment during whiplash

6.6 Effect of Acceleration Magnitude on Whiplash Response

The effect of the acceleration magnitude on the whiplash response is shown in Figure 6.8 for the four triangular pulses applied to the finite element beam model. The response shows a roughly linear increase in the rotations and relative head displacements with an increase in the peak collision acceleration. Interestingly, the time for the head to return back to zero was unchanged for all of the collision accelerations. This is analogous to the simple spring and mass system where the period of oscillation is unaffected by the amplitude of oscillation for the system. In this case, the neck stiffness, head mass and head inertia will determine the natural frequency of vibrations as discussed in section 6.3.
Figure 6.8: FE beam acceleration magnitude assessment during whiplash
Chapter 7
Conclusions and Future Work

7.1 Conclusions

Whiplash is a major health concern with many associated acute and chronic symptoms [15]. This common injury is difficult to treat and accounts for more than 10 billion dollars annually in healthcare and insurance related costs in the United States alone [14]. Much work has been done in an effort to develop improved ‘active’ head restraints and safety systems to minimize injury during whiplash but this work has been limited by the development of an accurate and efficient computational model to capture the head and neck response to assess whiplash injury and facilitate the optimization of said systems. It is with this in mind that the current investigation was conducted. The objective of this research was to develop a rigid linkage lumped parameter model to capture the relative motion of the head and cervical spine during whiplash.

To this end, a viscoelastic model capturing the bulk properties of a healthy 50th percentile male was developed with good overall performance and solution time for modeling whiplash. This model was found to capture both the dynamics of the head, including head rotations and relative rectilinear displacements relative to the T1 vertebra, and the intervertebral rotations of the cervical spine. With respect to the individual intervertebral motions of the spine, this model was able to capture the S-shaped curvature of the upper and lower spinal segments characteristic of the whiplash response.

These results of the present work demonstrate a number of important features including:

(i) the application of an inverse method to determine the viscoelastic lumped parameters for the dynamic rigid linkage model based on an experimental whiplash response,
(ii) the application of joint locations at the ‘instantaneous axes of rotation’ locations to capture the relative motions of the intervertebral discs of the cervical spine and simplify the number of degrees of freedom for the system,

(iii) the model allowed us to determine the overall whiplash response using the lumped parameter approach,

(iv) the model produced solution times for the dynamic response on the order of 2.5 s,

(v) the effect of pertinent parameters upon the dynamic response of the head-neck response during whiplash,

(vi) the finite element models were compared with the rigid linkage model and the results are in general agreement,

(vii) to the author’s knowledge this work presents the first application of a dynamic multi-body model utilizing realistic IAR joint locations and non-trivial viscoelastic parameters [62],

(viii) and a number of other findings were made during the extensive computational and experimental trials including:

   a. the reduction in the peak head deflection and rotation with an increase in neck stiffness,

   b. the decrease in the period of the whiplash response with an increase in neck stiffness,

   c. the equivalence of collision time and collision severity matched triangular and sinusoidal input acceleration profiles,

   d. and the increase of rectilinear peak head displacement and rotation with peak input acceleration
7.2 Future Work

This research provides a good first-step into developing a simplified model of whiplash but there are a number of potential areas for future improvements based on the present research. The following areas are worthy of future research in order to further validate the viscoelastic rigid linkage model and improve the current understanding of the whiplash response and pathophysiology during whiplash trauma:

(i) experimentally determine the intervertebral motions of the cervical spine (translations, rotations, and IAR locations) using multiple subjects subjected to the standard 8.5 g triangular acceleration profile to obtain a clear understanding of the average response and variability associated with whiplash,

(ii) study the effect of coupled viscoelastic parameters on the response of the rigid linkage model (off-diagonal coefficients in stiffness and damping matrices),

(iii) study the effect of non-linear stiffness on the response of the rigid linkage model,

(iv) study the effect of alternative material models to define joint parameters (Kelvin body etc.),

(v) improve the experimental apparatus including shortening the collision time, synchronizing the motion capture and accelerometer start times, and improving overall accelerometer circuitry for improved signal performance,

(vi) and the development of a complete multiphysics model of the cervical spine to model the complete head and neck complex to capture the overall relationship between the pressure effects, strains, and kinematics during whiplash.
References


63. Huston, *Multibody dynamics including translation between the bodies with application to head-neck systems*, in *Department of Engineering Science*. 1978, University of Cincinatti: Cincinatti, Ohio, USA.


93. Bostrom, O.R., M; Aldman, B; et al., et al., *Comparison of car seats in low speed rear-end impacts using the BioRID dummy and the new neck injury criterion (NIC)*. Accident Analysis & Prevention, **32**: pp. 321-328, 2000.


98. Reference, A.C., *NLGEOM*. 2011, ANSYS.


Appendix A

Tissue and Vertebrae Characteristics for the Head and Neck

The soft tissue and bones in the head and neck provide the head with an excellent range of motion. The ligaments, intervertebral discs, facet joints, and vertebrae in the cervical spine work in combination to provide a flexible and stable support structure for the head as well as protection for the spinal cord. Muscles provide the input forces to support the head and provide mobility. All of these systems have unique properties which make them particularly well adapted for their particular roles during normal movements, however, the flexibility of the spine makes it vulnerable to the dynamic loading present during rear-end automobile collisions.

A.1 Vertebrae

The cervical vertebrae (C₁-C₇) and thoracic vertebrae (T₁-T₁₂) form the bones of the cervical and thoracic spine as shown in Figure A.1. The C₁ vertebra (atlas) contains facets for supporting the occipital condyles of the head and allows for the flexion/extension movement (‘yes’ head motion) of the head relative to C₁. The C₂ vertebra (axis) allows for the axial rotation of the head (‘no’ head motion) and C₁ relative to C₂. The inferior aspect of C₂ down to C₇ contains facets and a vertebral body for the facet joints and intervertebral discs as shown by Figures A.2 and A.3.

Figure A.1 Human spine segments [99]
The vertebrae are composed of cortical and trabecular bone, which gives it high strength and toughness [48]. The compression strength of the cervical vertebrae is on the order of 1500-2000 N [100] and the modulus of elasticity of vertebral cancellous and trabecular bone is found to be 11.5-17.0 GPa and 291 +/- 113 MPa [101, 102]. The characteristic load deformation curve for bone corresponds to an initial linear region followed by a plateau of plastic collapse of the trabecular bone and a final increase in the load corresponding to densification [103]. With these properties, the cervical vertebrae will have negligible deflection during whiplash loading.

![Facet](image)

**Figure A.2: Typical cervical (C₄) vertebra [104]**

### A.2 Intervertebral Discs and Facet Joints

The intervertebral discs and facet joints connect between the vertebrae (inferior to C₂) to allow the vertebrae to move relative to one another as shown in Figure A.3. The complex geometry of the vertebral body and facets permit significant rotation in both the transverse plane (‘no’ head movement) and sagittal plane (‘yes’ head movement) between vertebrae. In the sagittal plane, the vertebrae slide and rotate past one another due to the angle of the vertebral facets relative to the vertebral body. The motions at each of the cervical joints all contribute to the net motion of the head.
The intervertebral discs are composed of the nucleus pulposus, annulus fibrosis, and cartilaginous end-plate as shown in Figure A.4. The nucleus pulposis consists of a mucoprotein gel with fine fibrous strands. The water content in this gel ranges from 70-90% which gives the intervertebral discs their viscous properties [48]. The annulus fibrosis forms the outside of the disc and is characterized by fibrous tissue aligned at approximately 30 degrees from the disc body. Each layer of the annulus is aligned in the opposite direction (120 degrees), giving the disc very high resistance to compression loading (0.5 MN/m) [105] and an anisotropic behaviour [51]. During normal motions, the intervertebral discs provide low resistance to shear motions (0.06 MN/m) [105], facilitated by the nucleus pulposis gel and surrounding angled fibrous tissue. Cartilaginous end plates adhere to the vertebra to form the bond between the vertebrae and the disc. The intervertebral discs are susceptible to injury due to fatigue and excessive dynamic and static loading [48].

The cervical facet (or zygapophyseal) joints connect between the adjacent superior articular process and inferior articular processes of adjacent vertebrae. The facet joints are located dorsal to the intervertebral disc and lateral to the spinal cord. There are two facet joints between each adjacent vertebra, starting below the C₂ vertebra. Facet joints resist out of plane movements and provide very little resistance to shear motions, similar to that of intervertebral discs. Due to the angle of the facet joints in the sagittal plane they provide some resistance to both axial and forward loading of the vertebrae while facilitating axial rotations and sliding motions in the sagittal and coronal planes. During the sliding motion the facet joint the fibroadipose meniscoid
unfolds, connecting between both articular processes of the adjacent vertebrae [106]. This allows the joint to move with very little resistance within its normal range of motion.

Figure A.4: Cervical vertebra and intervertebral disc [107]

A.3 Ligaments

Ligaments in the spine provide bone to bone connection to support the normal motion of the spine. The ligaments of the spine include the anterior longitudinal ligament, posterior longitudinal ligament, intertransverse ligaments, capsular ligaments, ligament flava, interspinous ligaments, and supraspinous ligaments as shown in Figure A.5 [48]. Ligaments are composed of collagen, elastin fibers, and proteoglycans with most collagen fibers aligned along the length of the ligament [45]. Ligaments exhibit a highly non-linear stress-strain relationship in tension and buckle easily in compression. Roughly two thirds of the ligament weight is water which gives it viscoelastic properties as well. The typical load-deflection curve for a ligament consists of an initial concave up increase in the load with increasing deformation. In this low deflection region internal fibers of the ligament follow a wavy path, providing very little resistance to deflection. As the load increases the fibers become aligned and the load increases approximately linearly. This is followed by the failure of individual fibers leading to a reduced increase in the load with deflection. As the deflection is increased, further collagen fibers will continue to fail, leading to the eventual failure of the ligament [108]. Significant variations in ligament stiffness are observed between individuals and across age groups. Nachemson [50] measured a tensile modulus of 20MPa and 98MPa for the ligamentum flavum of young and old humans. In addition
to its non-linearity and variability, ligaments are strain-rate dependent owing to their viscoelasticity. For higher deformation rates ligament loads will increase faster with deformation. This makes the ligament response sensitive to the dynamic loading applied during whiplash.

Figure A.5: Ligaments of the spine [48]

A.4 Muscles

The muscles of the spine provide stability and facilitate movement. A complex arrangement of muscles in the anterior and posterior regions of the neck allow for flexion, extension, abduction, adduction, and rotations of the head. The major intermediate and superficial muscles of the neck are shown in Figure A.6 with the levator scapulae, sternocleidomastoid, trapezius, semispinalis, and splenius capitus muscles activated during whiplash. The reader is referred to Gray [52] for an in-depth treatment of the muscles of the neck. Muscle behaves in a markedly different manner depending on its activation level. For passive extension, the muscle load is seen to increase in a non-linear manner for extension beyond its resting length. For activated muscle, this non-linear increase with extension occurs sooner and with a steeper increase as shown in Figure A.7. This muscle activation can apply loading to produce motion or provide support. The muscles of the head and neck are required to hold the head upright for example.
During whiplash, the sternocleidomastoid muscles produce the greatest muscle activation followed by the splenius capitis and trapezius muscles. Muscle activation for rear-end impacts typically occurs in the range of 120ms after impact with activation ramping up over time [109, 110]. Activation of each muscle group typically occurs at different times with activation occurring sooner for greater collision accelerations [7].

**Figure A.6: Muscles of the neck. Adapted from Gray [52]**

**Figure A.7: Active and passive muscle force characteristics [48, 111, 112]**
A.5 Spinal Cord

The spinal cord extends from the brain down the spine to communicate the motions, senses and reflexes for the body. During flexion and extension motions of the spine, the length and cross sectional area of the spinal cord changes [48]. The spinal cord is protected by the surrounding pia mater, dentate ligaments, spinal fluid, and dura mater. The spinal cord and pia mater exhibit marked non-linear elasticity in tension with very little resistance up to approximately 5% strain and dramatically increased resistance to loading beyond this point [113-115]. The spinal cord also shows pronounced viscoelastic behavior owing to its fluid content. These properties will affect the response of the neck during whiplash and make the neck susceptible to injury due to pressure affects [3].
Appendix B
Experimental Circuit Diagrams

Figure B.1: ACH001 accelerometer circuit diagram

Figure B.2: Kistler 8632C50 accelerometer circuit diagram
Figure B.3: Wheatstone bridge circuit diagram

Figure B.4: Instrumentation amplifier