Slow Flow of a Viscoelastic Fluid Past a Circular Cylinder

by

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Abstract

Flow around a cylinder is important in the motion of microorganisms found in biological viscoelastic fluids that propel themselves by flagella. Flow around a cylinder experiments are difficult to perform because of the influence of the walls and ends. An approach was developed to measure the drag on a cylinder by correcting for wall and end effects. Cylinders were vertically dipped into fluids in an annular shaped tank, which was rotated to generate a flow. The force acting on a cylinder was measured using a custom force transducer. This method was used for a Newtonian fluid and two Boger fluids. The drag of the Boger fluids was several times that of an equivalent Newtonian fluid. A cavity was observed to develop behind the cylinders once the flow surpassed a critical velocity. Streakline images taken during the experiment confirmed the presence of a wake region behind the cylinders.
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Chapter 1
Introduction

Flow around a circular cylinder is a fundamental flow in fluid mechanics. For this type of flow, a fluid flowing steadily at a uniform velocity approaches an immersed cylinder in the transverse direction. The flow is freestream in that it is not influenced by boundaries or obstructions other than the cylinder. The length of the cylinder is much greater than its diameter, so the cylinder can be represented as a two dimensional circle, as shown in Figure 1.1. Here, $U$ is the uniform upstream velocity, and $D$ is the diameter.

![Figure 1.1- Steady and uniform flow approaching a circular cylinder.](image)

The focus of the present study is flow around a circular cylinder in the absence of inertia. The drag at low Reynolds numbers ($Re < 0.01$) has been given little attention in the past because, experimentally, low Reynolds number flows are difficult to achieve and control. In reality and especially at low Reynolds numbers, it is difficult to make boundaries that are sufficiently far
from a cylinder that their influence cannot be felt, so wall effects are an ever present problem. Additionally, the flow cannot be made fully two dimensional because the ends of the cylinder always contribute to the drag. Interest has grown in the subject due to recent advances in micro- and nano-scale technology in biological applications such as microscale swimming, where low Reynolds number flows dominate.

The drag on a cylinder at high Reynolds numbers, on the other hand, is well known. Numerous investigators have experimentally verified the drag on cylinders at high Reynolds numbers [1] due to its many applications in aerodynamics and hydrodynamics. Drag values for low Reynolds number flows are not as well refined because drag measurements from past investigations are few. Even fewer drag values exist for viscoelastic fluids.

An analogous problem is the drag on a sphere. Like flow around a cylinder, the drag on a sphere is also experimentally difficult to measure due to the influence of the walls and the small forces involved. However, unlike flow around a cylinder, there are no ends to contend with for a sphere. Moreover, the Newtonian drag in the complete absence of inertia is solvable and is given by [2]:

\[
F_{\text{sphere}} = 3\pi \eta UD
\]  

(1.1)

where \(\eta\) is the fluid dynamic viscosity, \(U\) is the upstream uniform velocity and \(D\) is the diameter of the sphere.

Although researchers have devised approximate predictions using various approaches, no such exact solution exists for flow around cylinders. Since the drag is not positively known, experimental results for flow around cylinders are important. A novel approach for determining the drag at low Reynolds numbers was used in the present study: one that accounts for scale effects and allows for a wide range of Reynolds numbers.
1.1 Motivation

Two dimensional slow flow around a circular cylinder is a seemingly simple flow, but the existence of only inexact solutions and the lack of experimental work for both Newtonian fluids and viscoelastic fluids owe to the fact that inertia is always a factor, even at the lower Reynolds numbers. The unexplored nature of the problem alone makes flow around a cylinder worthy of investigation.

Besides the purely scientific motivations, results from the present study can be applied to studies of the propulsive mechanism of a flagellum. In micro-organisms such as spermatozoa, movement is achieved by the waving of a tail-like projection called the flagellum. The flagella of different microorganisms have been observed to generate either planar sinusoidal waves or helical rotations [3]. The resulting relative motion between the flagellum and the surrounding fluid generates normal and tangential force components that propel the flagellum. A representation of a flagellum and the forces acting on it are shown in Figure 1.2.

![Flagellum propulsion mechanism](image)

Figure 1.2 - Flagellum propulsion mechanism. The motion of a flagellum generates normal and tangential force components for propulsion.
The mechanism by which a flagellum generates propulsion is different from propulsion on the macro-scale. Many macro-scale swimmers rely on inertia and a reciprocating motion to generate propulsion. On the micro-scale, where Reynolds numbers are typically less than 0.01, reciprocating motion is not a viable form of propulsion because viscous forces dominate inertial forces. With a lack of inertia, reciprocating motion produces no net movement for microorganisms.

G.I. Taylor showed that Reynolds numbers associated with flagellum motion are of the order of $10^{-6}$, based on the frequency of oscillation [4]. Purcell suggested that typical Reynolds numbers for flagellum motion are of the order of $10^{-4}$ to $10^{-5}$ [5]. Gittleson compiled a list of flagellum Reynolds numbers for various organisms [6], but his Reynolds numbers were on the order of $10^{-2}$ to $10^{3}$. The difference can be attributed to Gittleson’s use of the flagellum length in the Reynolds number while Taylor and Purcell used the flagellum diameter. Because the flagellum is basically a two-dimensional cylinder, Taylor’s definition for the Reynolds number makes more sense.

Taylor theoretically investigated the mechanism by which the flagellum of a microorganism generates propulsion [4]. He demonstrated that movement can be achieved by rotating a spring-like coil which produced spiral waves. The model was shown to propel itself in glycerine at low Reynolds numbers [7].

Flagellum-based micro-organisms are found in biological media. Cervical mucus is one such fluid, where the presence of biopolymers makes it viscoelastic. Because of fluid elasticity, studies performed in Newtonian fluids may not accurately portray the propulsive mechanism of a flagellum.
For the results of the present study to be comparable to flow around a flagellum, not only do the Reynolds numbers have to be on a similar order of magnitude but also the strength of elasticity.

A common measure of fluid elastic strength is the dimensionless Deborah number $De$, which is discussed in detail in the Conceptual Background section. Typically, a Deborah number greater than 1 indicates that fluid elasticity is a factor. For sperm travelling in a fluid like cervical mucus, Deborah numbers have been reported to be anywhere from $O(10)$ to $O(10^3)$ [3] [8] [9]. In this range, elastic effects are expected to be significant and cannot be ignored. Researchers have attempted to model viscoelastic behaviour of the flow around a flagellum analytically and numerically. Analytical methods typically require many simplifications, often to the point that too much of the original problem is lost. For example, even in Taylor’s 1951 Newtonian study, he used small amplitude waves, treated the flagellum as a two-dimensional sheet with zero width and made a number of other mathematical simplifications [4]. In a later study in 1952, he included a small diameter assumption and was able to predict the behaviour of his working flagellum model [7]. Numerical methods typically require less simplification, but in the past, computing power has always been a limiting factor in the accuracy of the models.

Studies of flagellum in viscoelastic fluids require more simplifications and typically show less agreement with reality. Fulford et al. performed a similar analytical study, but used a simple viscoelastic fluid model (the Maxwell model) and included the drag on the spherical head of a spermatozoon [10]. Their results indicated that a flagellum could produce higher propulsive speeds in a viscoelastic fluid than in an equally viscous Newtonian fluid. Their study, like Taylor’s, used several mathematical simplifications that can change the conditions of the motions of flagellum. The use of the Maxwell model for cervical mucus is questionable because the model does not account for shear thinning effects. Fu et al. performed a similar analysis using more sophisticated fluid models and found that speeds were slower in such fluids, even
generating negative velocities under certain conditions [11]. Lauga also attempted to analytically determine the motion of flagella and found that the swimming velocity and efficiency decreased in viscoelastic fluids [3].

In addition to the assumptions already mentioned, many of the above studies assumed that the hydrodynamic force for slow flow travelling perpendicular to the cylinder is twice that for flow in the axial direction. Although this assumption can be valid under Newtonian flow conditions, it does not likely apply when the fluid is elastic as well as viscous.

Because the diameter is typically much smaller than the length, the flagellum can be treated as a long cylinder. By measuring the drag on cylinders in viscoelastic fluids at appropriate Reynolds numbers and Deborah numbers, the behaviour of flagella on the microscale can be determined. The drag, as the normal component of the propulsive force, produces the greatest effect on forward motion, so it is the key hydrodynamic component. Hence, the present research focused on the drag acting on a circular cylinder.

1.2 Conceptual Background

1.2.1 Drag

In fluid mechanics, drag is defined as the force exerted on an immersed object in reaction to a flow. The object disturbs the flow, altering the flow behaviour and producing a reaction force.
Two mechanisms are generally responsible for drag: inertial forces and viscous forces. In general, inertial forces arise when a fluid imparts its lost momentum to an object. Viscous forces arise from the sheared flow of a fluid flowing around an object. The resulting shear stress is responsible for the viscous component of drag.

Depending on the strength of the flow and the size of the object, either the inertial forces or the viscous forces will be the major contributor to the drag. A measure of the strength of the inertial forces to viscous forces is given by the Reynolds number, defined by:

\[ Re = \frac{\rho UD}{\eta} \]  

(1.2)

where \( \rho \) is the density, \( U \) is the velocity, \( D \) is the length scale, and \( \eta \) is the viscosity.

The present study deals with cases in which Reynolds numbers are so low that inertial effects are considered negligible.

1.2.2 Shear Flow

Shear flow is a basic flow and is used in rheometers to measure the flow properties of fluids. Shear flow is typically a result of the fluid coming into contact with solid boundaries and inducing the no-slip condition at the surfaces.

The most basic case of shear flow is simple shear. The rheometers used in the present study, the AR2000 and ARES, use a flow similar to simple shear flow to determine the viscosity and elasticity of fluids. The operation of these rheometers is further explained in subsection 1.2.5. In simple shear flow, a fluid is located between two parallel surfaces in 2D where the bottom
surface is fixed and the top surface moves at a velocity $U$. The no-slip condition requires that the fluid stick to both surfaces. The fluid then follows a linear velocity profile, as shown in Figure 1.3.

![Figure 1.3 - Simple shear flow. A fluid between two parallel plates is sheared as the top plate moves at a velocity $U$.](image)

The velocity gradient $\dot{\gamma}$, also known as the shear rate, is given by:

$$\dot{\gamma} = \frac{du(y)}{dy} \quad (1.3)$$

where $y$ is the distance from the stationary wall and $u(y)$ is velocity as a function of the $y$.

When a shear rate is induced in the fluid, a shear stress is produced in the direction of shear. The constant of proportionality between the shear stress and shear rate is called viscosity and is defined by:

$$\eta = \frac{\tau_{xy}}{\dot{\gamma}_{xy}} \quad (1.4)$$

where $\tau_{xy}$ is the induced shear stress and $\dot{\gamma}_{xy}$ is the shear rate.
For Newtonian fluids, the viscosity does not vary with shear rate. In non-Newtonian fluids, however, the viscosity is generally a function of the shear rate. Typically, the viscosity decreases with shear rate – a property known as shear thinning. Examples include paint, ketchup, and blood. The present study involves non-Newtonian fluids for which the viscosity varies little with shear rate.

1.2.3 Viscoelastic Fluids

Viscoelastic fluids are a type of non-Newtonian fluid that exhibit elastic behaviour. These fluids typically contain long polymer chains that are responsible for shear thinning and elasticity. Examples of viscoelastic fluids include synovial fluid, which lubricates animal joints; latex paint, whose shear thinning and elastic properties prevent dripping; and polymer melts, the liquid form of plastics used in injection moulding. For each of these fluids, viscoelasticity play an important role in their function and/or behaviour.

When not under stress, a single polymer chain in solution forms a random coil, as shown on the left side of Figure 1.4. When the fluid is sheared, the polymer chain begins to stretch out in the direction of shear, as shown on the right of Figure 1.4, reducing the frontal profile of the polymer chains. This reduces the resistance to shear flow, decreasing the shear stress. As a result, the viscosity of the fluid decreases as the shear rate increases.
The stretching of the polymer chains also produces normal stresses resulting from the tendency of the polymer chains to return to equilibrium. The two main normal stresses operating under shear flow are $\sigma_{xx}$, which acts in the direction of shear and $\sigma_{yy}$, which acts perpendicular to the direction of shear. The 1\textsuperscript{st} normal stress difference $N_I$ is the difference between the two stress components and is given by:

$$N_I = \sigma_{xx} - \sigma_{yy}$$

$N_I$ arises when a viscoelastic fluid undergoes shear. The magnitude of $N_I$ increases with shear rate and is approximately proportional to $\dot{\gamma}^2$. For Newtonian fluids, which do not contain long polymer chains, $N_I$ is always zero, regardless of shear rate.

A common measure of the elasticity of a viscoelastic fluid is its relaxation time. This property relates to the time required for polymer molecules in a fluid to return to their unstretched state after flow. Several methods exist for calculating the relaxation time of a fluid; each method partially depends on the rheological model used, and is discussed in section 4.1.
The strength of elastic effects with a given fluid depends on the fluid and a number of flow factors. The Deborah number is a convenient dimensionless number that measures the strength of the elastic effect by comparing the relaxation time of the fluid $\lambda$ with a characteristic time of the flow $t_p$, as given by [12]:

$$ De = \frac{\lambda}{t_p} \quad (1.6) $$

For $De \gg 1$, the flow process happens too quickly for the fluid to return to equilibrium and produces strong elastic effects. A low Deborah numbers, typically $De \ll 1$, indicates that the fluid is close to the equilibrium state during the flow process and behaves like a Newtonian fluid.

### 1.2.4 Boger Fluids

Boger fluids are viscoelastic fluids that have nearly constant viscosities. Like all viscoelastic fluids, Boger fluids are shear thinning, i.e., the viscosity decreases with shear rate. However, Boger fluids are so weakly shear thinning that their viscosity can be considered constant.

The use of Boger fluids in experiments allows viscous effects to be separated from elastic effects [13]. This can be achieved by conducting experiments with both a Newtonian fluid and a Boger fluid. For a given Reynolds number, the viscous effect on the results should be the same for both types of fluids. Any difference is due to elasticity.

James [13] indicated that the Oldroyd-B model with a single relaxation time provides a suitable constitutive equation for Boger fluids. The Oldroyd-B equation is given by [14]:
\[ \tau + \lambda_1 \dot{\tau} = \eta (\dot{\gamma} + \lambda_2 \ddot{\gamma}) \] (1.7)

where \( \tau \) is the stress tensor, \( \dot{\gamma} \) is the rate of strain tensor, \( \eta \) is the shear viscosity, \( \lambda_1 \) is the fluid relaxation time, \( \lambda_2 \) is the fluid retardation time, \( \dot{\tau} \) is the upper convected time derivative of the stress tensor and \( \ddot{\gamma} \) is the upper convected time derivative of the strain rate tensor.

With this model, the viscosity is constant and the polymer viscosity and solvent viscosity are separate contributions to the total viscosity. Other constitutive models, like the Giesekus, Bird-Aguiar, FENE-P models and models with multiple relaxation times, have been shown to better fit Boger fluids. However, the Oldroyd-B model with a single relaxation time was selected to represent the Boger fluids in the present study because of its relative simplicity.

1.2.5 Rheometry

The flow properties of the present fluids, such as viscosity and \( N_j \), must be characterised before drag measurements can be made. These properties can be measured using rheometers. A cone and plate rheometer is a common type of rheometer that holds a test fluid between a rotating cone and a fixed plate. The advantage of a cone and plate rheometer is that it produces a uniform shear rate everywhere in the fluid, thus allowing for more accurate measurements of the viscosity. Two such rheometers were used in the present study: an AR2000 from Rheometric Scientific and an ARES from TA Instruments. The rheometric measurements were made with a 40-mm cone with an angle of 2° on the AR2000 and with a 50-mm cone with an angle of 2.29° on the ARES. A drawing of a cone and plate rheometer is given in Figure 1.5.
Figure 1.5 - Cone and plate rheometer. The cone angle $\alpha$ is normally between $0.5^\circ$ and $4^\circ$.

The fluid is located between a fixed bottom plate and a shallow angle cone. The cone rotates and the bottom plate is fixed on the AR2000, while the bottom plate rotates and the cone is fixed on the ARES. Both rheometers are capable of precisely controlling the speed of rotation and measuring the torque acting on the cone. At a fixed rotational speed, the fluid between the cone and plate reacts by producing a torque on the cone, which the rheometer measures. The viscosity of the fluid $\eta$ can then be determined using the following equation:

$$\eta = \frac{\tau}{\dot{\gamma}} = \frac{3T\alpha}{2\pi\omega R^3} \quad (1.8)$$

where $\tau$ is the shear stress, $\dot{\gamma}$ is the shear rate, $T$ is the torque acting on the cone, $\alpha$ is the cone angle, $\omega$ is the angular velocity, and $R$ is the cone radius [12].

The rheometers are also capable of determining the first normal stress difference, $N_1$, by measuring the normal force on the top cone under steady shear. The relation between $N_1$ and the normal force $F$ is given by [12]:

$$N_1 = \frac{2F}{\pi R^2} \quad (1.9)$$

where $F$ is the normal force measured by the rheometers.
Small amplitude oscillatory tests can also be performed by rotating the upper cone in a sinusoidal motion at small amplitudes to determine two fluid properties: the storage modulus $G'$ and loss modulus $G''$. Oscillatory measurements are useful for determining the rheological properties of a fluid when it is not possible to perform tests at steady shear rates. $G'$ and $G''$ measurements can be used to determine the relaxation times and polymer viscosities of viscoelastic fluids. The calculation of these properties is further discussed in section 4.1.

The storage modulus $G'$ is the measure of the solidity or elasticity of a viscoelastic fluid determined by measuring the component of the shear stress in phase with the shear strain $\gamma(t)$ of the fluid. For a Newtonian fluid, the storage modulus is zero. The loss modulus $G''$ is the measure of the viscosity of the fluid determined by gauging the component of shear stress in phase with the shear rate $\dot{\gamma}(t)$ or out of phase with the shear strain $\gamma(t)$. For a Newtonian fluid, the loss modulus is given by [12]:

$$G'' = \eta \omega_f$$  \hspace{1cm} (1.10)

where $\omega_f$ is the angular velocity.

To determine $G'$ and $G''$, a rheometer records the oscillatory shear stress $\tau(t)$ as a function of time. Based on the phase lag and magnitude of the shear stress, $G'$ and $G''$ are determined using the following equation [12]:

$$\frac{\tau(t)}{\gamma_0} = G'' \cos(\omega t) + G' \sin(\omega t)$$  \hspace{1cm} (1.11)

By properly characterizing a viscoelastic fluid using the methods mentioned, its relaxation time and the strength of elasticity during a given flow process can be determined. The results of the characterization of two Boger fluids are applied in section 4.1.
1.3  Prior Research: Newtonian

Investigations into the drag on a cylinder typically involve three possible approaches: analytical, numerical and experimental approaches. Analytical approaches involve solving simplified forms of the Navier-Stokes equations as well as other applicable equations to determine the drag. Analytical approaches are typically limited either by mathematical complexity or over-simplification. Numerical approaches typically involve directly solving the Navier-Stokes equations and other relevant equations using computational methods. Because numerical approaches must be performed by computers, their accuracy is limited by computational power. Experimental approaches involve physically measuring the drag, but are often the most difficult to perform because they are limited by measurement precision, influence from undesired effects, and scaling issues.

Analytically, inertialess flow around circular cylinders for Newtonian fluids is difficult because the boundary conditions at the surface of the cylinder and those of the far-field cannot both be satisfied. This is known as Stoke’s Paradox [2]. C.W. Oseen devised a method to get around the paradox, using a matched asymptotic expansion approach to match the near-field and far-field solutions. Lamb provided a solution for the flow around a circular cylinder using the Oseen approximation [15]. His solution, however, is not accurate near the surface of the cylinder, so the resulting drag deviates from reality. Kaplun developed an alternate solution for the flow, using a singular perturbation method [16] [17]. This solution is considered to be more accurate than Oseen’s solution and was, therefore, used in the present study. Various other solutions exist, such as those of Sasic [18], Proudman and Pearson [19], Tomotika and Aoi [20], and Skinner [21], but they are essentially derivatives of Lamb’s and Kaplun’s solutions and provide only marginal
improvements. Numerical solutions have also been attempted by Tamada et al. [22], and Kropinski et al. [23].

Experimental measurements of cylinder drag at low Reynolds numbers have also been performed. The approach in these experiments has been to drop a cylinder into a fluid of known viscosity and density and measure the rate of fall in order to determine the drag. This technique was used by Jayaweera and Mason [24], White [25], and Jones and Knudsen [26]. Their approach was limited by the fixed velocities achievable in their experiment; the densities of the fluids and cylinders they used allowed for only set combinations of cylinder fall velocities. As will be shown later, the boundaries of the experiment can have a significant impact on the drag. Controlling orientation during the fall also presents a challenge for a non-spherical body like a cylinder. Other experimental results include those of Tritton [1], which are considered the most reliable and most often cited results, and Finn [27], who measured drag on a wire in an air flow. These results, along with the analytical solutions above, are plotted in Figure 1.6 using the viscous drag coefficient, defined by the following equation [28]:

\[ q = \frac{F}{L} = 4\pi \eta C_\eta U \]  \hspace{1cm} (1.12)

where \( L \) is the length of the cylinder, \( C_\eta \) is the viscous drag coefficient and \( U \) is the freestream flow velocity.
The results of White, and Jones and Knudsen are not included in Figure 1.6 because they were under the influence of walls. It will be shown later that the influence of the walls can have a significant effect on the drag. The results of Finn agreed well with Lamb’s solution down to Reynolds numbers as low as 0.01 and are not plotted in Figure 1.6. The viscous drag coefficient was used in favour of the more common definition of the drag coefficient, $C_D$ because inertial effects do not come into play in the present study.

None of the above experiments other than White’s were performed at Reynolds numbers below 0.01. As indicated earlier, the Reynolds numbers associated with flagellum motion are approximately two to four orders of magnitude lower, so these past experiments are not applicable to flagellum motion. White was able to achieve Reynolds numbers as low as 0.0001, but his drag data were highly influenced by the walls of his tank, which must be eliminated to make a proper comparison. One of the key focuses of the present study was to account for the effect of the walls.
1.4 Prior Research: Viscoelastic Fluids

As for viscoelastic fluids, few investigations have been performed for flow past a single circular cylinder at low Reynolds number. James and Acosta measured the drag for extremely dilute polymer solutions but only for \( \text{Re} > 0.1 \) [29]. McKinley et al. investigated elastic effects for flow around a confined (not an isolated) cylinder, but did not measure the drag [30]. Numerical studies have also been carried out for viscoelastic flows. Baaijens et al. investigated the flow and stress field around a confined cylinder using a variety of viscoelastic models [31], both experimentally and numerically. They found that certain constitutive models can sometimes predict experimental flow behaviour but tend to fall short in terms of accuracy. The drag was not found despite their exhaustive collection of stress data.

Chilcott and Rallison numerically calculated the drag on a circular cylinder using the Finite Extensible Nonlinear Elastic (FENE-P) dumbbell model, a viscoelastic model that treats the molecules of a viscoelastic fluid as beads connected by non-linear springs. They found that the region of highest stress resides downstream of the rear stagnation point and that the drag was generally higher for viscoelastic fluids than an equivalent Newtonian counterpart. Their study, however, was limited by computational power, reducing the allowable Deborah numbers and control volume size.

These past investigations have shortcomings such that they are not directly comparable to the motion of flagellum. They were either operated at excessively high Reynolds numbers that the flow can no longer be considered inertialess or were operated at such low Deborah numbers that elastic effects are negligible. Additionally, many of the above viscoelastic studies do not determine the exact effect viscoelasticity has on flow around a circular cylinder. A fresh experimental approach is called for that addresses these shortcomings.
The main objective of the present study was to experimentally measure the drag on a circular cylinder in viscoelastic fluids at low Reynolds numbers (Re < 0.01) to determine the effect of elasticity on the drag. A secondary objective was to observe the flow pattern of viscoelastic flow around a cylinder to help understand the viscoelastic behaviour of the flow.

2.1 Conceptual Design

2.1.1 Flow Field

To meet the requirements of the experimental objectives, the oncoming flow needs to be steady and well controlled. Two-dimensional uniform flow below a Reynolds number of 0.01 is required so that inertial effects are small enough, compared with viscous effects, that they can be neglected. It is virtually impossible to experimentally create a uniform flow field but it can be approximated by producing a rotational flow that is nearly uniform locally, on the scale of the cylinder rather than using a linear flow apparatus.

A controllable large-diameter industrial turntable was available for use. A fluid tank was also available, constructed by attaching two hollow circular cylinders concentrically to a base, forming an annular shaped tank that can be attached to the turntable. Solid-body rotational flow
can be produced by rotating the tank at a steady velocity. A test cylinder can be vertically dipped in the fluid midway between the inner and outer walls to create a flow around a test cylinder. The cylinder diameter needs to be small so that the flow approaching it is nearly uniform and so that wall effects are minimal. Also, because the ends of the cylinder will contribute to the drag, the length needs to be much longer than the diameter.

To make the flow upstream of the cylinder nearly uniform, the diameter of the cylinder of the tank must be much smaller than the distance between the tank walls. The flow then becomes nearly identical to a flow around a cylinder inside a straight channel. This setup is shown in Figure 2.1.

![Figure 2.1 — Steady-state rotational flow around a cylinder in an annular tank. The diameter of the cylinder needs to be much smaller than the diameter of the tank, by at least 2 orders of magnitudes. In this range, the upstream flow is nearly uniform, as shown on the right.](image)

Using this approach, a steady and controllable flow can be generated without requiring a very large volume of fluid.
2.1.2 Drag Measurement

To measure the drag on the test cylinder, a force transducer is required. The measurement range in typical force transducers is usually limited to 2 or 3 decades. In the present study, because various fluid viscosities and cylinder diameters are to be used, the expected range of forces may be too wide for a commercial transducer to measure accurately. A simple home-made transducer for this primarily exploratory study was considered more appropriate. Moreover, such a transducer can be created, for an adjustable force range and fine measurement resolution.

An optical measurement system is best suited because it does not require physical contact. To create a measurable quantity, a cylinder can be attached to a horizontal cantilever beam. When fluid flows around the cylinder, the resulting drag causes the free end of the beam to deflect. The deflection at the free end of the beam can be measured optically using a camera, and by calibration, the deflection can be related to the drag.

Because a wide range of forces is expected in the present study, the measurement range can be varied by making the length of the beam and the point of attachment of test cylinder adjustable. The immersed length of the cylinder can be made adjustable to correct for end effects. The immersion depth will be further discussed in section 2.2. The design of drag the measurement system is shown in Figure 2.2.
Figure 2.2 – Design of the drag measurement system. The annular tank containing the test fluid rotates to produce a force on the vertical test cylinder connected to a horizontal flexible cantilever beam.

2.1.3 Fluids

The fluids and speeds must produce sufficiently-low Reynolds numbers and also, in the case of viscoelastic fluids, Deborah numbers greater than unity. The viscosity of the fluid must also produce a force that falls within the measurement range of the force transducer. A Newtonian fluid must be tested first and compared against known drag results to verify the accuracy of the experimental approach.

Then viscoelastic fluids can be tested. Shear thinning effects, as described in sub-section 1.2.3, can potentially be problematic with viscoelastic fluids. The viscosity of a shear-thinning fluid would not be uniform everywhere in the flow and would vary with the velocity. Not knowing the fluid viscosity would make drag measurements meaningless. With Boger fluids, viscoelastic fluids with nearly constant viscosities, shear-thinning effects can be eliminated. Because Boger fluids do not shear thin, a single viscosity can be used in calculations.
For the Boger fluids, both the elasticity and viscosity must be such that that the drag falls within the measurement range of the force transducer. The elasticity will likely increase the drag, but the degree of increase is unknown. Therefore, a wide range of forces should be expected.

The Deborah number, a measure of elastic strength as described in section 1.2.3, is also a design criterion for the experiments involving viscoelastic fluids. One of the key components of the Deborah number is the relaxation time of a fluid, which represents the time required for the fluid to return to equilibrium after shear and is a measure of the fluid’s elasticity. To produce the desired range of Deborah numbers, a Boger fluid with appropriate relaxation times must be chosen. The Deborah number allows fluids of different viscosities to be compared so that the strength of the elastic effect on drag can be determined.

2.2 Experimental Design Considerations

Before the experimental apparatus described in the conceptual design section can be assembled, a number of design considerations must first be worked out. In this section, the important considerations are discussed in detail.

2.2.1 Velocity Range

The velocity range of the flow must be able to produce the desired Reynolds numbers. The motorized turntable in the Measurement Laboratory (of the Department of Mechanical and
Industrial Engineering at the University of Toronto) is capable of turning at an angular velocity from 0.5 rpm to 60 rpm. By placing an annular cylindrical tank filled with liquid on the turntable, steady flow velocities can be generated. The available plexiglass tank has an outer wall diameter of 49.5 cm and an inner diameter of 19 cm. At a point midway between the inner and outer walls of the tank, the velocity ranges from 0.009 m/s to 1 m/s. As the velocity increases, however, the angle of the fluid surface due to centripetal acceleration also increases. A change in the surface angle could affect the surface behaviour and immersion depth. At the low drag anticipated in the present study, the error could be significant. To minimize this effect, the angular velocity was capped at 20 rpm so that the centripetal acceleration was no more than 10% of the gravitational acceleration.

2.2.2 Test Cylinder Diameter Range

The test cylinders are subject to bending, so their diameters must be chosen to minimize bending but must also be much smaller than the distance between the inner and outer diameter of the tank in order to minimize wall effects. The wall effects are especially significant at very low Reynolds numbers, i.e., in the absence of inertial effects, and will be fully discussed in section 2.3.

Ideally, the cylinder diameters are so small that wall effects can be ignored and the cylinders are so long that end effects can be ignored. However, the strength of the cylinder material and drag measurement resolution limits the minimum allowable diameter. The minimum cylinder diameter can be determined based a maximum allowable beam deflection angle and expected force range. The deflection of a circular cylindrical beam $\theta$ is given by [32]:
where $q$ is the applied force per unit length, $E$ is the elastic modulus, $I$ is the area moment of inertia and $L$ is the length of the cylinder.

Here, it is assumed that the cylinder is entirely immersed in the fluid. Experimentally, part of the cylinder will always be above the surface of the fluid. However, the bending angle is always smaller when the cylinder is partially immersed than when it is fully immersed and need not be considered when calculating the maximum cylinder bending angle.

The quantity $q$ can be estimated from Kaplun’s solution [16] for low-Re drag on a circular cylinder, which is given by:

$$C_\eta = \frac{1}{S} - \frac{0.87}{S^3}$$  \hspace{1cm} (2.2)

Here, $C_\eta$ is the viscous drag coefficient, defined by equation (1.12), and $S$ is:

$$S = 0.5 - 0.57721 - \ln\left(\frac{Re}{8}\right)$$  \hspace{1cm} (2.3)

The uniformly applied load, $q$, is then given by:

$$q = \frac{F}{L} = 4\pi \eta C_\eta U$$  \hspace{1cm} (2.4)
Under a maximum imposed cylinder deflection angle of 10°, and for a steel cylinder with an elastic modulus \( E \) of 200 GPa, a length \( L \) of 30 mm, immersed in fluid of viscosity \( \eta \) of 10 Pa.s and flowing up to a velocity of 1 m/s, the cylinder diameter must be greater than 0.5 mm. Hence, cylinders no smaller than 0.5 mm were used in the present study.

2.2.3 Bending Beam Selection

The dimensions and material of the bending beam must be selected to strike a balance between measurement resolution and range. The drag on a test cylinder can be determined by attaching the test cylinder to a calibrated cantilever beam at a distance \( a \) from its base and optically measuring the deflection at the free end. The general setup is shown below in Figure 2.3.

![Figure 2.3 - Cantilever beam, bending in a horizontal plane in the experiment. Here, \( a \) is the distance from the fixed end to the attached cylinder, \( l \) is total length of the beam, \( F_d \) is the drag acting on the cylinder and \( \delta \) is the deflection at the free end of the beam.](image)

Aluminum was chosen for the cantilever beam material for its low Young’s modulus of 69 GPa and its high yield stress. An aluminum beam allows for high deflections before the onset of plastic deformation.
The cantilever beam had a rectangular cross section with a small thickness for large deflections in the horizontal plane, and with a large depth as shown in Figure 2.4 to prevent the beam from bending vertically. For a rectangular cross section, the area moment of inertia for deflection in the horizontal plane is given by [32]:

\[
I_y = \frac{hb^3}{12}
\]  \hspace{1cm} (2.5)

Figure 2.4 - Rectangular cantilever beam cross-section. Small \( b \) and large \( h \) values allow for deflection in the horizontal plane but not in the vertical plane.

The beam has cross-sectional dimensions of 1.59 mm by 12.7 mm, giving an area moment of inertia of \( 4.23 \times 10^{-12} \text{ mm}^4 \).

According to Gere [32], the deflection of the end of the beam is:

\[
\delta = \frac{F_d a^2}{6EI_y} (3l - a)
\]  \hspace{1cm} (2.6)

where \( EI_y \) is the flexural rigidity of the beam, and the angle of deflection \( \theta_B \) is:

\[
\theta_B = \frac{F_d a^2}{2EI_y}
\]  \hspace{1cm} (2.7)
Equation (2.6) is valid up to a deflection angle of $10^\circ$. Based on equations (2.2) and (2.4), and assuming a cylinder diameter of 3 mm, a cylinder length of 30 mm, a fluid viscosity of 10 Pa.s, and a maximum velocity of 2 m/s, the drag apparatus must be able to measure a maximum drag of 1 N. According to equation (2.7), the distance $a$ needs to be approximately 0.30 m.

It is important to maximize drag measurement resolution without increasing measurement error. A force measurement resolution of about 0.1 mN was desired so that minute differences in drag measurements could be detected. For a desired beam deflection resolution 0.01 mm, the length of the beam was set at 0.70 m in accordance with equation (2.6).

Both the beam length, $l$ of 0.70 m and the distance $a$ of 0.30 m were initial estimates used to carry out the first runs of the apparatus. They were later adjusted, based on measurement results, to better balance resolution and range, and to accommodate various fluid viscosities. Table 2-1 summarizes the material and dimensions of the cantilever beam, as determined in this section.

**Table 2-1 - Beam selection summary**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus, $E$</td>
<td>$69 \times 10^9$ Pa</td>
</tr>
<tr>
<td>Width, $b$</td>
<td>1.59 mm</td>
</tr>
<tr>
<td>Height, $h$</td>
<td>12.7 mm</td>
</tr>
<tr>
<td>Area moment of inertia, $I$</td>
<td>$4.23 \times 10^{-12}$ mm</td>
</tr>
<tr>
<td>Length, $l$</td>
<td>0.70 m*</td>
</tr>
<tr>
<td>Distance to load, $a$</td>
<td>0.30 m*</td>
</tr>
</tbody>
</table>

* These quantities are initial estimates. They were changed as required.
2.2.4  Beam Deflection Measurement

The human eye is not sensitive enough to measure the beam deflection and so, a Sony DSC-T10 camera was used to record measurements. It was placed directly above the pointer end of the horizontal beam, and pictures were taken vertically using the camera’s magnification mode. To measure the magnitude of the deflection, a scale was placed within 0.5 mm of the pointer end, as shown in Figure 2.5. The top surface of the pointer and scale were arranged to be flush so that both were within the same focal plane of the camera. To reduce vibrations, the camera was fixed to the scale. Pictures were taken with a 10 second timer, giving vibrations enough time to dampen out.

![Figure 2.5 – The cantilever beam deflects when drag acts on the attached test cylinder. The deflection at the end was measured using a scale.](image)

At the beginning of each set of deflection measurements, the zero load deflection was recorded while the fluid was still. Then when the fluid velocity was steady, the deflection was measured. Each time, the camera was moved directly above the pointer to eliminate angular optical errors.
2.2.5 Calibration

To accurately measure the drag, the cantilever beam must be calibrated. Although Young’s modulus $E$ and the area moment of inertia $I$ of the beam can be calculated, the actual value of the product $EI$, the flexural rigidity, was found by calibration. A string was tied to the test cylinder and connected to known weights via a pulley, applying known loads to the beam as shown in Figure 2.6 and Figure 2.7. The beam deflected linearly with the applied load. The constant of proportionality between the load and the measured beam deflection is given by equation (2.6).

![Figure 2.6 - Cantilever beam calibration, where F is a known weight – view looking horizontally.](image)

![Figure 2.7 - Cantilever beam calibration - top view.](image)
The deflection results for the initial calibration, with $a = 0.30$ m and $l = 0.70$ m, are shown in Figure 2.8. By fitting a straight line to the data and using equation (2.6), the flexural rigidity, $EI$ was determined to be $0.266$ Nm$^2$. Theoretically, $EI$ should be $0.29$ Nm$^2$ if the elastic modulus is assumed to be $69$ GPa and the area moment of inertia is $4.23 \times 10^{-12}$ mm. Because the experimentally determined flexural rigidity is the true value, it was used for the drag measurements. To ensure accurate measurements, the flexural rigidity was determined each time the beam lengths $a$ and $l$, were adjusted. The measurement error was approximately $\pm 0.05$ mm.

![Figure 2.8 - Sample beam calibration. A straight line was fitted to the load vs. deflection data to determine the flexural rigidity.](image)

### 2.2.6 Choice of Fluids

The viscosity of all test fluids and the elasticity of viscoelastic fluids play an important role in the drag and must be selected to produce appropriate drag ranges. For the Newtonian component of the experiment, a silicone oil with a viscosity of $0.98$ Pa.s and a density of $1008.7$ kg/m$^3$ was available for use. The viscosity of the oil was measured using the AR2000 rheometer with a 40-
mm diameter 2° angle cone and plate fixture. With this oil, the Reynolds number ranged from 0.005 to 0.5, an appropriate range for the experiment.

For the viscoelastic component of the experiment, two Boger fluids were available, termed B1 and B4. These fluids were developed by Dr. Ronnie Yip at the University of Toronto [33]. The two have similar compositions, being composed of polyisobutylene, polybutene, and kerosene. The exact composition of each fluid is shown below in Table 2-2.

**Table 2-2 - Composition of B1 and B4 Boger fluids.**

<table>
<thead>
<tr>
<th></th>
<th>Polymer</th>
<th>Solvent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B1</strong></td>
<td>0.20% PIB</td>
<td>92.8% PB, M.W. 910</td>
</tr>
<tr>
<td></td>
<td>M.W. 2.8x10^6</td>
<td>7% Kerosene</td>
</tr>
<tr>
<td><strong>B4</strong></td>
<td>0.20% PIB</td>
<td>92.8% PB, M.W. 635</td>
</tr>
<tr>
<td></td>
<td>M.W. 4.7x10^6</td>
<td>7% Kerosene</td>
</tr>
</tbody>
</table>

B1 had a steady shear viscosity of 26.2 Pa.s and B4 had a steady shear viscosity of 6.6 Pa.s. The two viscosities allowed the Reynolds numbers to range from 0.0001 to 0.01 for the viscoelastic component of the experiment.

**2.2.7 Final Design**

The final design of the experimental apparatus is shown in Figure 2.9. The force measurement apparatus is attached to an overhead frame (the “fixture”) that is decoupled from the rotating turntable. The only physical point of contact between the two is the test cylinder. The aluminum bending beam is fixed to the main fixture by vises so that its length is adjustable. The cylinder
fixture can be slid along the bending beam and can be fixed at a desired position. The cylinder fixture also allows the cylinder to be moved up and down, allowing for variations in cylinder depths. A vertical scale was imprinted onto the vertical slider of the cylinder fixture to determine the exact depth of the cylinder. The cylinder fixture is shown more clearly in Figure 2.10.

Figure 2.9 - Final design of experimental apparatus.
Figure 2.10 - Cylinder fixture attached to the cantilever beam. The cylinder is attached to the fixture by a clamp. A vertical scale is visible on the slider to determine the immersion depth of the cylinder.

The deflection measurement scale is attached to the support beams, as shown in Figure 2.11. The camera which recorded the beam deflection is supported by an aluminum block held on the scale. The calibration fixture is attached to one of the support beams where it is decoupled from the bending beam.

Figure 2.11 - The cantilever beam has a sharp pointer at the free end to precisely measure the deflection.
2.3 Overcoming Non-Idealities

Some classes of experimental studies require ideal conditions to test theories and hypotheses, and these conditions require that certain factors be eliminated. Often, however, these factors are nearly impossible to remove.

In the present study, the ideal case consists of a uniform flow across a circular cylinder. The flow is assumed to be properly two-dimensional and the necessary bounding walls are assumed to have no influence on the flow. Experimentally however, neither of these requirements can be met. No matter how long the cylinder is compared to its diameter, the flow at the ends will always differ from the flow around the cylindrical surface. Furthermore, no matter how far the walls are from the cylinder, the flow field is always affected when the Reynolds number is less than unity. In this study, end and wall effects were accounted for such that the resulting drag closely matches that of the ideal case.

2.3.1 End Effects

For three-dimensional cylindrical bodies whose lengths are much greater than their widths or depths, drag results are typically treated as if the bodies were two-dimensional so that the length factor can be eliminated in calculations. That is, the drag caused by the ends can be ignored because it is negligible compared to the cylinder drag. In the present experiment, however, the cylinder length/diameter ratio varied between 5 and 40. In this range, end drag is not negligible.

For an immersed cylinder, end drag is expected to be independent of the immersion depth, while the cylinder drag is expected to vary linearly with depth. This allows for a scheme in which the
end drag can be accounted for by varying the depth of the cylinder in a systematic way. The drag at the free end, $F_e$, and the drag induced by surface effects at the liquid-air interface, $F_s$, can be accounted for using this scheme. The impact of each on the total drag will be discussed later in this section. The total drag will then be the sum of the end drags and the two-dimensional cylinder drag, $F_D$, as shown in Figure 2.12.

![Figure 2.12 - Eliminating end effects from measurements. The cylinder drag $F_D$ can be separated from end effects, $F_e$ and surface effects, $F_s$.](image)

By measuring the drag at different depths, plotting the drag versus depth, $L$ and extrapolating the data to $L = 0$, the cylinder drag alone is zero and the difference is the end drag. An example of this approach is shown in Figure 2.13 for five depths, obtained with a 0.50 mm diameter cylinder in a 0.98 Pa.s Newtonian silicone oil at a speed of 0.25 m/s. A linear regression curve-fit, shown by a solid line in Figure 2.13, was used to extrapolate the length/diameter ratio to zero. This yields an intercept of 0.0016 N, which is due to end effects. A similar plot for the same cylinder in the B4 Boger fluid is shown in Figure 2.14.
Figure 2.13 - Example: extrapolating the drag to a length/diameter ratio of zero.

Figure 2.14 - Example: extrapolating drag to a length/diameter ratio of zero for B4 Boger fluid.
This scheme can be used to determine the end effects for both Newtonian and viscoelastic fluids at all Reynolds numbers, and was used to determine the end effects for all measurements in the present study. However, the error associated with this scheme can be very large. Because the lengths at which the drag measurements were performed were far removed from the end effect extrapolation length of \( L = 0 \), measurement errors are more intensified at \( L = 0 \).

Non-dimensionalized end drags, as given by equation (2.8), are plotted in Figure 2.15, for cylinders with diameters of 0.50 mm, 0.97 mm and 1.18 mm in a 0.98 Pa.s silicone oil.

\[
C_{\text{end}} = \frac{F_{\text{end}}}{\eta UD} \tag{2.8}
\]

where \( F_{\text{end}} \) is the drag and \( D \) is the spherical diameter.

![Figure 2.15 - Experimentally-determined end drag in a 0.98 Pa.s Newtonian fluid.](image-url)
The error bars in Figure 2.15 give the standard error of the end effect extrapolation as defined by [34]:

\[
\sigma_{xy} = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n} (F_i - \bar{F})^2 - \left( \frac{\sum_{i=1}^{n} (L_i - \bar{L}) (F_i - \bar{F})}{\sum_{i=1}^{n} (L_i - \bar{L})^2} \right)^2} \tag{2.9}
\]

where \( F_i \) is the individual drag measurement and \( L_i \) is the corresponding cylinder length, \( \bar{L} \) is the average cylinder length, \( \bar{F} \) is the average drag and \( n \) is total number of measurements.

Figure 2.15 shows that end effect error is very significant compared to the calculated end effect values. For the worst case scenario, this error can be 5 times greater than the end drag itself. However, the error is likely due to the smaller drag at the lower Reynolds numbers, which is near the resolution of the measurement system of approximately \( 5 \times 10^{-4} \) N.

To determine whether the measured end effect drag is reasonable with respect to theory, the end drag was estimated theoretically. In slow flows, \( Re \ll 1 \), the dependence on body shape is weak. As a result, the end drag is expected to be close to that of half a sphere, whatever the actual shape. Therefore, the end can be approximated as a half-sphere. At low Reynolds numbers, the drag of a sphere is given by [2]:

\[
F_{sphere} = 3\pi \eta UD \tag{2.10}
\]

The drag of the submerged end should then be approximately given by:

\[
F_{end} = \frac{3\pi}{2} \eta UD \tag{2.11}
\]
To test whether this assumption is valid, Figure 2.16 compares the end drag data with the theoretical half-sphere drag.

![Graph showing end drag data compared to theoretical half-sphere drag](image)

Figure 2.16 – Experimentally-determined end drag, and the prediction based on a half-sphere.

The theoretical half-sphere drag is on the same order of magnitude as the experimentally determined end drag. The end drag was lower than the half-sphere drag at low Reynolds numbers, which can be attributed to poor measurement accuracy and instrument zeroing due to smaller forces. The end drag appears to increase systematically beyond the half sphere drag, however, at higher Reynolds numbers. The higher than expected results are indicative of other effects.

These other effects, like surface tension effects and bottom-wall effects should be accounted for with the above end drag correction scheme. Under static conditions, surface tension acts evenly around the circumference of a cylinder at its liquid-air interface. Thus, it produces a net-zero
force on the cylinder. However, the surface equilibrium can be upset by a flow, in which case, a net force can be produced on the cylinder as shown in Figure 2.17y.

![Figure 2.17 - Imbalance of surface forces on a circular cylinder under flow.](image)

Assuming that surface tension is capable of producing a net force on the cylinder, the maximum force that can be produced by surface effects is given by [35]:

\[ F_y = \gamma D \]  \hspace{1cm} (2.12)

where \( \gamma \) is the surface tension coefficient and \( D \) is the Diameter of the cylinder.

The surface tension coefficient of the 0.98 Pa.s silicone oil was measured with a ring tensiometer by Fisher Scientific and was 0.024 Nm. Using equation (2.12), the maximum non-dimensionalized end force due to surface effects was on the order of \( 10^{-5} \) N. The excess end drag at the higher Reynolds in Figure 2.16, however, were on the order of \( 10^{-3} \) N. Therefore, surface tension effects are not the sole cause of the higher than expected end drag at higher Reynolds numbers.

Whether or not the net surface force has any measurable contribution to the drag should not affect the present study because its dependence on velocity should be weak and its dependence
on the cylinder immersion depth should be zero. Therefore, surface tension effects can be factored out by the end effect correction scheme along with the free-end drag.

The contribution of the tank bottom wall to the drag varies with the immersion depth of the cylinder. As will be shown in subsection 2.3.2, the flow profile and the drag are highly influenced by the presence of walls at low Reynolds numbers. As the immersion depth changes, the distance between the free end and the bottom wall also changes. Because of this dependence on depth, the end effect correction scheme cannot completely factor out the end effect.

However, this dependence on depth is expected to be small. The 2D cylinder drag depends proportionally on the immersion depth, while the contribution of the bottom wall effect decays as the end of the cylinder moves away from the bottom wall. Typical immersion depths range from 10 mm to 30 mm. The total depth of the fluid is typically about 100 mm. If the cylinder immersion depth were to be increased from 10 mm to 20 mm, the 2D cylinder component of the drag would be increased by 100%. For a 0.5 mm diameter cylinder, the change would represent a decrease in the distance between the free end of the cylinder and the bottom wall from 180 to 160 diameters. The Stokes flow velocity profile across a sphere perpendicular to the flow direction is given by [2]:

\[
\frac{u_r}{u_0} = 1 - \frac{a^3}{4r^3} - \frac{3a}{4r}
\]  

(2.13)

where \(a\) is the radius of the sphere and \(r\) is distance from the centre of the sphere perpendicular to the flow direction.

At 180 diameters from the sphere, the velocity is 0.9958 of the freestream velocity and drops to 0.9953 of the freestream velocity at 160 diameters from the sphere. The velocity drop is only 0.05%, which is negligible compared to the 100% increase in the 2D drag component and is
likely to be overwhelmed by instrument noise. The bottom wall effect can then be assumed to be independent of depth, and thus, can be separated from the cylinder drag with the end effect correction scheme. Additionally, at 160 to 180 diameters from the sphere, the velocity deviates by only 0.5% from the freestream velocity. The bottom walls are therefore unlikely to be significant enough to cause the rise in end drag seen in Figure 2.16.

Neither surface effects nor bottom wall effects are large enough to cause the dependence of the end drag on the Reynolds number. The cause of the dependence remains a mystery and should be investigated in a future study. In the present study, the dependence should have no bearing on the experiment because the end drag can be eliminated with the end effect correction scheme.

2.3.2 Wall-effects at low Reynolds numbers

At high Reynolds numbers, the disturbance to flow by an immersed body virtually vanishes within several body lengths. For example, when air flows at 10 m/s around a 0.1 m diameter flagpole, the Reynolds number is $6.8 \times 10^4$ and the velocity increases by only 1% at a distance of 1 m from the flagpole in the direction perpendicular to the flow [2]. The flagpole, then, is effectively immersed in an infinite medium.

At low Reynolds numbers, however, i.e., at $\text{Re} < 0.1$, the disturbance to the flow does not die out so rapidly. In essence, the entire flowfield around the cylinder can be considered a boundary layer flow. The increase in drag was evident in C.M. White’s experimental investigation, where he demonstrated that, at a Reynolds number of 0.0013, the drag on a falling cylinder increased by 2.8 times in a tank where the walls were 50 diameters away from the cylinder [25].
To appreciate the influence of walls in the present experiment, a numerical simulation of two-dimensional flow was carried out using the computational fluid dynamics software Fluent. A straight channel was first used. The numerical mesh produced in Fluent is shown in Figure 2.18, using a width of 0.15 m, the distance between the inner and outer walls of the experimental tank.

![Fluent mesh of flow around a circular cylinder in a straight channel.](image)

The channel length was 2 m because the channel needed to be long enough that upstream and downstream boundary errors could be eliminated. A cylinder diameter of 0.5 mm was chosen to examine the flow disturbance at the highest ratio of tank width to cylinder diameter, namely 300. The walls of the channel were set to “slide” with the same velocity as the oncoming uniform flow. This is analogous to the experimental conditions, but with a straight rather than an annular channel.

Two simulations were performed in Fluent using two different viscosities, 5 Pa.s and 0.0001 Pa.s to produce respective Reynolds numbers of 0.001 and 50. The geometry, velocity and fluid
density were the same in the two simulations. The resulting velocity profiles across the cylinder perpendicular to the flow are shown in Figure 2.19.

![Velocity Profile](image)

**Figure 2.19 – Dimensionless velocity profiles across a circular cylinder in a sliding channel at Re = 0.001 and Re = 50. The flowfield upstream has a uniform velocity $U_0$.**

At a Reynolds number of 50, the influence of the cylinder vanishes at about 10 diameters from the cylinder and the flow returns to the uniform upstream velocity. That is, the influence of the walls is negligible. At a Reynolds number of 0.001, though, the influence of the cylinder is present throughout the entire flowfield. The velocity stays below the uniform upstream velocity until 40 diameters away from the cylinder and then overshoots it. The velocity does not return to the upstream uniform velocity until the walls where it is forced to do so because of the no-slip condition. This difference in the flowfield at low Reynolds numbers can force the drag to depend significantly on the walls.
The experimental situation pertains to an annular channel rather than a straight one. Because the outer wall moves at a higher velocity than the inner wall, the upstream velocity distribution is not uniform but varies linearly with distance from the centre of the circular walls. This distribution produces a variation of only 2% in the upstream velocity over a distance of 10 diameters. The velocity distribution across the cylinder and that on the opposite side of the channel, as determined by Fluent, are shown in Figure 2.20.

![Figure 2.20 – Velocity profile across a circular cylinder in an annular channel](image)

The velocity across the cylinder appears to be similar to that in the straight channel; that is, the velocity is zero at the cylinder due to the no-slip condition, gradually rises above the upstream velocity, and then, matches the wall velocity. However, the flow opposite the cylinder does not follow a linear velocity profile as expected of solid body rotation, even though the circumferential distance along the centre of the channel, of 21400 cylinder diameters, was initially expected to be large enough that the influence of the cylinder would have disappeared.
The non-linear velocity profile suggests that the circumferential distance along the centre of the channel was not large enough. Although this effect cannot be eliminated from the experiment, the velocity deficit of 6% opposite the cylinder should have a small effect on the flow approaching the cylinder.

To further understand the effect of walls, Fluent was used to determine how the drag depends on the wall width to cylinder diameter ratio. A conceptual representation of the numerical model is presented in Figure 2.21.

![Figure 2.21 - Flow around a cylinder between two parallel sliding walls. The wall velocity is equal to the inlet velocity. The h/D ratio was varied from 5 to 5000.](image)

The cylinder diameter of 1 mm, compatible with the expected range of 0.5 mm to 3 mm, was used. To match the properties of the B1 Boger fluid, the viscosity and density of the simulation fluid were set to 26 Pa.s and 867 kg/m³, respectively. The inlet velocity and the no-slip wall velocity were set to 0.085 m/s, a typical experimental velocity. The wall width, h was varied between 5 mm and 5000 mm to yield width/diameter ratios ranging from 5 to 5000. The Reynolds number was 0.0028 and fixed, i.e., to be independent of the width/diameter ratio. The
drag per unit length from the numerical simulations is plotted in Figure 2.22 along with Kaplun’s freestream cylinder drag solution.

![Figure 2.22](image_url)

Figure 2.22 - Drag per unit length from numerical simulations of flow around a cylinder between two parallel sliding walls and Kaplun’s low-Re prediction. The channel width, h was varied to determine the wall effect.

The figure indicates that, as the h/D approaches infinity, the drag approaches Kaplun’s drag solution, as expected. This comparison validates both the numerical method and Kaplun’s solution. The figure demonstrates that the walls have a very significant effect on the drag, increasing it by a factor of two even for a width/diameter ratio of 100 and by three for a width/diameter ratio of 10. Figure 2.22 also suggests that at a Reynolds number of 0.0028, the wall effect does not fully disappear even at a width/diameter ratio of 5000. The minimum and maximum experimental cylinders diameters give width/diameter ratios of only 300 and 50, respectively. Achieving a width/diameter ratio as high as 5000 is not experimentally feasible.

To remove the wall effect from the measured drag, the numerical solution is used to correct experimental data using the following equation:
\[ F_{\text{corrected}} = F_{\text{experimental}} \frac{F_{Kaplun}}{F_{\text{numerical}}} \]  \hspace{1cm} (2.14)

In order to use this method as it stands, a numerical simulation must be run for each experimental case, which is time-consuming because each simulation can take up to 30 minutes. Consequently, a more efficient scheme was used, based on Faxens work. He provided an analytical solution to the problem of a cylinder moving between two parallel plane walls [36] [28]. His solution is given by:

\[ \frac{F_D}{L} = \frac{4\pi \eta U}{\ln \left( \frac{h}{D} \right) - 0.9157 + 1.7244 \left( \frac{D}{h} \right)^2 - 1.7302 \left( \frac{D}{h} \right)^4} \]  \hspace{1cm} (2.15)

Figure 2.23 adds equation (2.15) to Figure 2.22.

![Graph showing analytical solution and experimental cylinder diameter range added to previous figure.](image)

Figure 2.23 – Faxens’ analytical solution and experimental cylinder diameter range added to previous figure.
Despite some small discrepancies, likely due to convergence errors, the numerical results appear to line up very well with the Faxens’ analytical solution, at least up to a width/diameter ratio of 1000.

Faxens’ solution, as provided by Happel and Brenner [28], does not converge to a constant value at high width/diameter ratios. In fact, it crosses Kaplun’s solution, which gives the drag in a freestream flow. The likely cause is that his solution only extends to the power of \((D/h)^4\), viz, it is accurate to \(O(10^4)\) at best. The use of Faxens’ solution must, therefore be limited to width/diameter ratios less than 1000 where it agrees with the numerical solution.

Because the minimum and maximum experimental width/diameter ratios of 50 and 300 are below 1000, Faxen’s solution can be used in place of the numerical solution to correct for errors in experimental results to save time. The corrected drag is then given by:

\[
F_{\text{corrected}} = F_{\text{experimental}} \frac{F_{\text{Kaplun}}}{F_{\text{Faxens}}} \quad (2.16)
\]
Chapter 3
Newtonian Results

As indicated in the conceptual design section, the experimental approach was tested by starting with a Newtonian fluid. After applying end and wall effect corrections, the resulting drag was compared with known Kaplun’s Newtonian drag solution to learn whether the experimental approach could be used for viscoelastic fluids.

The cantilever beam was calibrated with known weights ranging from 0.2 g to 10 g. The length of the beam \( l \) was set at 0.70 m and the distance \( a \) was lengthened to 0.45 m, based on the results of trial runs, to match the expected range of loads.

The drag was measured on cylinders having diameters of 0.50 mm, 0.97 mm, and 1.18 mm. Three different diameters were used to verify the consistency of the results. The three diameters gave a tank-width-to-diameter ratio of 129 to 309. In this range, the experimental drag is expected to be only two to three times the freestream drag, according to Figure 2.22 of section 2.3.

Each cylinder was immersed in the Newtonian fluid and held at a given depth. The velocity was ramped up and the drag was measured at each velocity setting. The cylinder depth was changed, and the process was repeated to obtain data for different depths for end corrections. The uncorrected drag results for the 0.97 mm diameter cylinder are plotted in Figure 3.1.
Figure 3.1 shows that the drag increased with the Reynolds number and with cylinder diameter, as expected. However, it is more useful to look at dimensionless drag data. These results, in terms of the viscous drag coefficient $C_\eta = \frac{F_D}{L} = \frac{F_D}{4\pi \eta U}$ (defined in section 2.2) are plotted with Kaplun’s solution in Figure 3.2.
In Figure 3.2, the uncorrected results for different diameters did not agree with each other and did not agree with Kaplun’s solution. This was expected because wall effects and end effects were not yet corrected for.

### 3.1 Applying End Effect Correction

The end correction technique, described in sub-section 2.3.1, was used to remove end effects from the raw Newtonian drag measurements. The corrected results, for the same data as Figure 3.2, are shown in Figure 3.3.

---

**Figure 3.3 - Data of Figure 3.2 corrected for end effects.**
A comparison of Figure 3.2 and Figure 3.3 shows that contribution of the end effect was small. At low Reynolds numbers, the two plots appear to be identical, while at high Reynolds numbers, the corrected drag and Kaplun’s solution agree better.

The difference between the measured drag and Kaplun’s solution at low Reynolds numbers is consistent with the results obtained by C.M. White [25]. He dropped small, horizontally positioned rods through Newtonian fluids of known viscosities and measured their terminal velocities to determine the drag. White used an end effect correction scheme that was similar to the present one, in which he varied the lengths of his test cylinders and extrapolated the drag to a length of zero. Wall effects were also present in White’s study, but were not corrected for. A comparison of our results to his is shown in Figure 3.4. The 0.97 mm diameter cylinder used in the present study corresponds to $h/D = 157$. The error bars attached to the present data give the standard error of the drag measurements.

Figure 3.4 - Comparison of the end-corrected results with White’s wall influenced results but not Kaplun’s freestream drag solution. Errors bars give standard deviations.
Our data agree well with White’s results, despite the different experimental setup. However, both results were influenced by the proximity of the walls and do not reflect the freestream drag. As a result, neither our data nor White’s results agree with Kaplun’s freestream drag solution. To make up for the discrepancy with Kaplun’s freestream drag solution, the influence of the walls must be accounted for.

### 3.2 Applying Wall Effect Correction

Wall effects were corrected for using the method given in section 2.3.2. The solution for drag on a cylinder moving between two parallel walls, provided by Faxens [36], was used to factor out the wall effect, so that the corrected drag applies to freestream flows. The corrected results for the 0.97 mm cylinder from Figure 3.4 are given in Figure 3.5. The figure shows that the drag data, corrected for both end and wall effects, agree well with Kaplun’s solution. Thus, the correction methods used in the present study appear to be accurate for Newtonian fluids and may be applied for viscoelastic fluids.
Figure 3.5 - Fully corrected drag coefficient for the 0.97 mm diameter cylinder. The uncorrected data are presented in the previous plot.

Using the same correction technique, the results for the 0.50 mm and 1.18 mm cylinders, with standard error bars, are shown in Figure 3.6.

Figure 3.6 - Fully corrected drag for 0.50 mm diameter cylinder (left) and for 1.18 mm diameter cylinder (right).
In these last two figures, the agreement between theory and experiments is not as good as that in Figure 3.5; for both cylinders, the data are above the theoretical curve. There are a number of possible explanations for the discrepancy. For the smaller 0.50 mm diameter cylinder, the drag was smaller. The measurement noise was higher in comparison, so the force transducer could not measure the drag as precisely. As for the 1.18 mm diameter cylinder, it was shorter than the other two cylinders, the maximum L/D ratio being 19.5. The corresponding maximum ratios for the 0.50 mm and 0.97 mm cylinders were 50 and 57.2, respectively. Hence, a significant percentage of the measured drag on the 1.18 mm diameter cylinder could have been due to end effects for lower immersion depths.

For all cylinders, the error associated with the end effect correction could contribute up to 40% of the total drag at the lowest immersion depths, as shown by the error bars in Figure 3.5 for the 0.98 mm diameter cylinder. The end effect correction errors were no greater than 15% of the total drag for the 0.50 mm and 1.18 mm diameter cylinders. These errors arose because the corrections were based on linear extrapolations of the experimental measurements, which were prone to noise and variation. Thus, the cylinder drag results in Figure 3.5 and 3.6 were heavily dependent on the precision of the end correction.

Despite the discrepancies, the experimental approach of the present study produces drag measurements that agree reasonably well with Kaplun’s drag solution after corrections for end and wall effects. The results are encouraging enough that the correction methods can be used to determine the drag in viscoelastic fluids.
Chapter 4
Viscoelastic Results

The previous chapter showed that end effects and wall effects were major sources of error, but could be accounted for in determining the cylinder drag. This same approach was applied to drag measurements in viscoelastic fluids.

It was expected that viscoelastic behaviour does not change the end drag when the immersion depth is varied, as with Newtonian fluids. As such, the end drag can be eliminated with the technique described in section 2.3. The wall effect, however, may be of concern. The correction factor used in the previous section may not apply because the relation between wall effects and elasticity is not known. Without such information, it was assumed that the Newtonian wall effect correction factor can be used for the viscoelastic fluids.

4.1 Viscoelastic Fluid Characterization

In this section, the characteristics of the two viscoelastic fluids in the present study, B1 and B4, are examined. The key properties involved in the drag of viscoelastic fluids are the viscosity and relaxation time. The measurement of these two parameters is discussed below.
4.1.1 Viscosity

The viscosities of the two Boger fluids were measured using steady shear mode of the ARES and AR2000 rheometers as described in section 1.2.5. Their viscosities with respect to shear rate are shown in Figure 4.1.

![Figure 4.1 – Viscosities of B1 and B4 Boger fluids. The temperatures at which the viscosities were measured correspond to the average ambient temperatures during which the drag measurements were taken.](image)

B1 had an average steady shear viscosity of 26.2 Pa.s at 23°C, and B4 of 6.6 Pa.s at 21°C. The difference in viscosity between the two fluids allowed a wider range of Reynolds numbers than with just one fluid. The viscosities were measured at different temperatures because the room temperature varied during the period in which the drag for each fluid was measured. However, at a temperature difference of only 2°C, the variation in viscosity was expected to be inconsequential.

The Oldroyd-B viscoelastic model, as given by the constitutive equation (1.7), was used to represent B1 and B4. Under the Oldroyd-B model, the zero-shear viscosity is the sum of the...
solvent viscosity and the polymer viscosity. The solvent component of viscosity can be determined from measurements of $G''$ and the following equation [14]:

$$G'' = \eta_s \omega + \frac{\eta_p \omega}{1 + (\lambda \omega)^2}$$ \hspace{1cm} (4.1)

where $\eta_s$ is the solvent viscosity, $\eta_p$ is the polymer viscosity, $G''$ is the loss modulus, $\omega$ is the angular frequency of oscillation and $\lambda$ is the relaxation time.

When the angular frequency approaches infinity, equation (4.1) can be simplified to give the solvent viscosity $\eta_s$:

$$\eta_s = \frac{G''}{\omega} , \omega \to \infty$$ \hspace{1cm} (4.2)

The loss modulus $G''$ and storage modulus $G'$ was determined for both fluids by performing a series of frequency sweep measurements. Equation (1.11) was used to extract $G''$ and $G'$ from the measured oscillatory shear stress. The average value of $G'' / \omega$ at high frequencies ($\omega > 10 \text{ rad/s}$) was used to determine the solvent viscosity $\eta_s$. Curve fittings of $G''$ for B1 and B4, using equation (4.2) are shown in Figure 4.2 for each fluid. The solvent viscosities of B1 and B4 were found to be 19 Pa.s and 4.2 Pa.s, respectively.
The polymer viscosity $\eta_p$, caused by the dissolved polymer, can be determined from:

$$\eta_p = \eta_0 - \eta_s \quad (4.3)$$

where $\eta_p$ is the polymer viscosity and $\eta_0$ is the solution viscosity.

The polymer viscosities of B1 and B4 were found to be 7.2 Pa.s and 2.4 Pa.s, respectively.

### 4.1.2 Relaxation Time

The relaxation time of a viscoelastic fluid is a measure of its elasticity and was described in section 1.2.3. Several methods based on the Oldroyd-B viscoelastic model were used to determine the relaxation time for each fluid. The methods and validity of the resulting relaxation times are discussed below.
Low $G'$

The Oldroyd-B constitutive equation, given in equation (1.7), yields the storage modulus $G'$ [14]:

$$G' = \frac{\eta_p \lambda \omega^2}{1 + (\lambda \omega)^2} \quad (4.4)$$

where $\lambda$ is the relaxation time.

When the frequency of oscillation approaches zero, equation (4.4) can be rearranged to give the relaxation time [14]:

$$\lambda_{G'} = \frac{G'}{\eta_p \omega^2}, \omega \to 0 \quad (4.5)$$

$G'$ data were taken at the lowest measurable angular frequencies ($\omega \sim 0.1$ rad/s) and curve fitted using equation (4.5) to obtain relaxation times of 2.7 s and 2.1 s for B1 and B4, respectively. The curve fittings of equation (4.5) to the measured $G'$ data are shown in Figure 4.3.
Figure 4.3 - Determining the relaxation time from the storage modulus $G'$.

For B4, the slope of equation (4.5) matches the slope of the $G'$ data at low frequencies, suggesting that the curve fit used was valid and gave an appropriate relaxation time. For B1, the agreement was poor, indicating that the frequency was not low enough for the attained relaxation time to be considered accurate. $G'$ and $G''$ were not measured at angular frequencies less than 0.1 rad/s because the rheometers could not measure the oscillatory shear stress precisely at such a low range and required prohibitively longer amounts of time to complete each measurement as frequencies decreased.

**Steady Shear $N_1$**

The relaxation time can also be determined using the first normal stress difference $N_1$. Under steady shear, the Oldroyd-B constitutive equation yields $N_1$ [13]:

$$N_1 = \sigma_{xx} - \sigma_{yy} = 2\eta_p\lambda N_1 \dot{y}_{xy}^2$$

(4.6)
The AR2000 and ARES rheometers, in conjunction with equation (1.9), were used to determine the $N_1$ under steady shear. Thus equation (4.6) can be used to calculate the relaxation time based on measured $N_1$ data.

The relaxation time was determined by curve fitting the plot of $N_1$ vs. $\dot{\gamma}_{xy}$ to a quadratic equation of the form $y = ax^2$ and solving for $\lambda_{N_1} = \frac{a}{2\eta_p}$. Using this method, the relaxation times were found to be 1.7 s for B1 and, interestingly, also 1.7 s for B4. The 1st normal stress difference measurements results are plotted in Figure 4.4 along with equation (4.6) using the determined relaxation times.

![Figure 4.4 - Determining relaxation time from measured 1st normal stress difference data.](image)

The curve fits of $N_1$ agree with the data for both fluids in the shear rate range of 1-10 s$^{-1}$, indicating that the Oldroyd-B model is a good fit for these two Boger fluids.
Step Shear Strain, Step Shear Rate and Cessation of Shearing

This approach to find \( \lambda \) involves shearing the fluid, then stopping the motion and measuring the residual stress over time. The stress vs. time results can then be fitted to an exponential decay curve:

\[
\tau(t) = \tau_0 e^{-\frac{t}{\lambda}} \tag{4.7}
\]

where \( \tau \) is the shear stress and \( \tau_0 \) is the steady state shear stress.

It was found that the residual stress over time could not be fitted with a single relaxation time. Instead, several relaxation times were necessary. The shear stress from stress relaxation measurements can be given by the following equation:

\[
\tau(t) = \tau_1 e^{-\frac{t}{\lambda_1}} + \tau_2 e^{-\frac{t}{\lambda_2}} + \cdots + \tau_n e^{-\frac{t}{\lambda_n}} \tag{4.8}
\]

where \( \lambda_1, \lambda_2, \lambda_n \) are the theoretical 1st, 2nd, \( n \)th relaxation times of the fluid and \( \tau_1, \tau_2, \tau_n \) are the theoretical constants of proportionality.

The procedure for determining the relaxation time, by curve fitting equation (4.8) to the results of a cessation of shearing experiment, is shown in Appendix A. As will be shown later, when a Boger fluid flows around a cylinder, the molecules of the Boger fluid do not return to equilibrium for several seconds – a similar order of magnitude to the longest relaxation times determined using this approach. To condense the number of relaxation times required to fit the relaxation profile to one, only the longest relaxation time was considered. The representative relaxation times of B1 and B4 were found to be 7.8 s for B1 and 8.0 s for B4.
4.1.3 Comparison OF B1 and B4

The properties of B1 and B4 determined in this section are summarized in Table 4-1. The zero shear viscosity of B1 was four times higher than that of B4. The agreement between the two fluids was expected as the solvent used in B1, Indopol Polybutene H-100, was about four times more viscous than Indopol Polybutene H-25, the solvent of B4. As such, the viscosity of B1 was expected to be four times greater than that of B4. For both fluids, the solvent viscosity and polymer viscosity were approximately 0.7 and 0.3 times the zero shear viscosity, respectively.

Table 4-1 - Summary of properties of B1 and B4 Boger fluid

<table>
<thead>
<tr>
<th></th>
<th>B1</th>
<th>B4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-shear viscosity, $\eta_0$</td>
<td>26.2 Pa.s</td>
<td>6.6 Pa.s</td>
</tr>
<tr>
<td>Solvent viscosity, $\eta_s$</td>
<td>19 Pa.s</td>
<td>4.2 Pa.s</td>
</tr>
<tr>
<td>Polymer viscosity, $\eta_p$</td>
<td>7.2 Pa.s</td>
<td>2.4 Pa.s</td>
</tr>
<tr>
<td>Density, $\rho$</td>
<td>867 kg/m$^3$</td>
<td>873 kg/m$^3$</td>
</tr>
<tr>
<td>$G'$ relaxation time, $\lambda_{G'}$</td>
<td>2.7 s</td>
<td>2.1 s</td>
</tr>
<tr>
<td>$N_1$ relaxation time, $\lambda_{N_1}$</td>
<td>1.7 s</td>
<td>1.7 s</td>
</tr>
<tr>
<td>Stress relaxation time, $\lambda_\tau$</td>
<td>7.8 s</td>
<td>8.0 s</td>
</tr>
</tbody>
</table>

As Table 4-1 shows, the relaxation times between the two fluids were similar for all the methods used. According to Table 2-2, the polyisobutylene in B1 and B4 had molecular weights of $2.8 \times 10^6$ and $4.7 \times 10^6$, respectively. B4 was expected to be more elastic and have a longer relaxation time due to its higher molecular weight of polyisobutylene, but this was not the case.
4.1.4 Comparison of Relaxation Times

Of the three relaxation times, one was picked as the representative for each fluid. The $G'$ data were obtained using small amplitude oscillations at low frequencies. However, the flow around a cylinder is continuous and steady. As such, the behaviour of the fluid at small amplitudes is not appropriate.

The $N_1$ relaxation time was determined from steady shear measurements. The flow conditions were then more similar to flow around a cylinder. However, $N_1$ is more representative of an upward force on a cylinder rather than across it, which is the direction the drag acts upon. Therefore, the $N_1$ relaxation time may not be the most suitable parameter.

The relaxation determined from the stress relaxation experiments seemed most related to drag on a cylinder. A fluid is sheared as it flows around the cylinder, and then relaxes as it passes by. The drag is a result of the shear stress developed during shearing, and so, is similar to a step shear experiments. Although the stress relaxation times are different from the $G'$ and $N_1$ relaxation times, the stress history is the closest to that of flow around a cylinder. Therefore, the stress relaxation time was used as the representative relaxation time of the Boger fluids.
4.2 Drag Results

4.2.1 Drag Coefficient

The drag on four circular cylinders was measured for both Boger fluids using the experimental apparatus described in chapter 2. The test cylinders had diameters of 0.50 mm, 0.97 mm, 1.59 mm, and 3.18 mm. Typical drag data for a single cylinder and one Boger fluid, after end effect and wall effect corrections, are plotted in Figure 4.5 in terms of $C_\eta$ as defined by equation (1.12).

![Figure 4.5 - Drag coefficient for the 0.97 mm diameter cylinder in the B4 Boger fluid.](image)

Figure 4.5 indicates that the drag with B4 was higher than Kaplun’s Newtonian drag, being about three times higher at low Reynolds numbers and up to six times higher at the higher Reynolds numbers. The increase in drag was due entirely to elasticity. To see what effect viscosity has on
the drag, the results for B1, which has a higher viscosity but similar relaxation time to B4, are plotted with the results for B4 for the same cylinder, in Figure 4.6.

![Figure 4.6 - Drag coefficient for the 0.97 mm diameter cylinder for both Boger fluids.](image)

The drag coefficients appear to agree between the two fluids, despite the difference in viscosities. The agreement was expected because the relaxation times, which dictate the magnitude of elasticity, were similar between the two fluids. The effect of viscosity was factored out by the Reynolds number.

Figure 4.7 presents the data for B4 and all four cylinders. Figure 4.8 presents the corresponding data for B1. The Reynolds number ranged from 0.0005 to 0.050 for B4 and 0.0001 to 0.020 for B1. In both plots, it is evident that the elastic effect depends on the diameter; as the cylinder diameter increased, the drag coefficient decreased.
Figure 4.7 - Experimental drag coefficient for B4 for the four cylinders.

Figure 4.8 - Experimental drag coefficient for B1 for the four cylinders.
The results for both Boger fluids are combined in Figure 4.9.

Figure 4.9 - Drag coefficient for all four cylinder diameters for B1 and B4.

As with Figure 4.6, Figure 4.9 shows that the results between the two Boger fluids tended to agree with each other for the same cylinder. In general, the elastic component of the drag increases with the Reynolds number. However, there was no agreement between different cylinders. Neither the drag coefficient nor the Reynolds number correlated with the elastic effect. In order to account for the elasticity, another dimensionless quantity is required.

The Deborah number, a dimensionless number described in sub-section 1.2.3, is a common measure of the strength of elastic effects. By plotting the drag versus the Deborah number, the elastic component of the drag can be collapsed. In the present study, the time scale of observation $t_p$ is the time required for the fluid to flow around the cylinder, i.e.,
\[ t_p = \frac{D}{U} \]  

(4.9)

where \( U \) is the uniform upstream velocity, and \( D \) is the diameter of the cylinder [12].

The Deborah number is then:

\[ De = \frac{\lambda r U}{D} \]  

(4.10)

To find the effect of elasticity, the normalized drag, defined as the ratio of the Boger fluid drag to the Newtonian drag, was plotted versus with the Deborah number. The normalized drag was used to eliminate the other factors in the drag, leaving only the elasticity. The drag for Boger fluids can then be compared to that of an equivalent Newtonian fluid. The results are plotted in Figure 4.10 and Figure 4.11 for B4 and B1, respectively.

Figure 4.10 - Normalized drag vs. Deborah number for B4 Boger Fluid.
For both fluids, the normalized drag increased with Deborah number, which was expected because elastic strength typically increases with Deborah number [12]. Data could not be collected at Deborah numbers as low as 1 because lower turntable speeds could not be achieved. Figure 4.10 and Figure 4.11, however, do suggest that the Boger fluid drag approaches the Newtonian drag at around De = 1. At this point, a Boger fluid is just able to return to equilibrium upon deformation.

In Figure 4.10, the results for different cylinder diameters appear to collapse for B4. This result shows that the cylinder diameter dependence of the drag can be factored out using the Deborah number. A similar trend can be seen in B1 in Figure 4.11, although the results for B1 appear more scattered. The scatter was perhaps because of stronger viscous stresses compared to elastic stresses, which adds more noise to the results.
4.3 Flow Visualization

The dramatic increase in drag for the Boger fluids indicates that the flow departs from Newtonian flow. As a result, a change in the flow path from Newtonian flow was also expected. In this section, the shape of the streamlines was observed to help explain the increase in drag.

4.3.1 Low Speed Flow Visualization

Streakline images were produced by seeding the B4 Boger fluid with reflective particles that have a similar refractive index and density as B4. Under standard room lighting, the particles appear invisible in the fluid. However, when a light sheet produced by a laser is passed through the fluid, a plane of particles light up. When photographed under a long exposure time, they form streaklines that show flow paths. Figure 4.12 shows a streakline image for flow around a cylinder in B4, along with a numerically predicted Fluent solution for Newtonian flow for the same Reynolds number of 0.00075.

Figure 4.12 - Comparison of Numerical Newtonian Streamline Prediction (left) with Experimental B4 laser streakline imaging (right) for Re = 0.00075.
The Fleunt prediction was symmetric about the x and y axis, which was expected for Stokes flow. This was not the case for the B4 Boger fluid. As the flow approached the cylinder, the streaklines diverged away from the cylinder more than in the Newtonian case. The boundary layer around the cylinder was larger in B4, as the velocity disturbance extended further away from the cylinder. The most noticeable difference was the presence of a wake region behind the cylinder. In this region, the flow was substantially slower than the rest of the flow, having velocities nearing zero. Wake regions are common in high Reynolds number flows, where the wake is inertially driven. This was not the case in Figure 4.12, as inertial effects are negligible at such low Reynolds numbers. Since the wake only showed up for the Boger fluid at this Reynolds number, it must have been induced by the elasticity of the fluid.

In section 4.2, it was shown that the drag was related to the elastic effect and that both increased with the velocity. The flow paths were expected to also change with increasing elastic effect. A series of streakline images are shown in Figure 4.13 to Figure 4.15, in order of increasing velocity.

![Figure 4.13](image)

*Figure 4.13 – Streakline image for the 0.50 mm cylinder in B4 at Re = 0.000357, De = 86. The streaklines are nearly symmetric in the direction of flow and the wake is small.*
Figure 4.14 - Streakline image for the 0.50 mm cylinder in B4 at Re = 0.00114, De = 276. The streaklines become skewed downstream of the cylinder and the wake becomes larger. The velocity also appears to decrease just upstream of the cylinder.

Figure 4.15 - Streakline image for the 0.50 mm cylinder in B4 at Re = 0.00237, De = 571. The velocity of the fluid inside the wake becomes much lower than the upstream velocity. Near the surface of the cylinder and towards the inner wall of the tank, the flow appears to reverse directions and travel around the other side of the cylinder.

As the velocity increased, the size of the wake region also gradually increased. Because the upstream flow was not perfectly uniform and followed a rotational path, some curvature in the streamlines was evident. The curvature became more pronounced at higher velocities, especially at the rear on the cylinder where the streamlines “curled” towards the inner walls of the tank. Evidently, this was an elastic effect, in which the streamlines were drawn in the direction of polymer elongation rather than in the direction of curvature of rotation. The magnitude that the “curling” effect has on the drag could not be determined in the present study.
4.3.2 High Speed Flow Visualization

Once the velocity was increased to a critical value, an air cavity was observed to form behind the cylinder. The cavity first depressed the free surface and then, over time, the depression slowly propagated down the length of the cylinder until it reached the free end. Photographs of the air cavity are shown in Figure 4.16 and Figure 4.17.

Figure 4.16 – Above-surface view of an air cavity behind cylinder in B4 for the 0.97 mm diameter cylinder. Re = 0.005 and De = 270.

Figure 4.17 – The air cavity from Figure 4.16 viewed from the side.
The cavity did not form in the Newtonian oil, and was therefore an indication of elasticity in the fluid. Figure 4.18 shows a top view of the air cavity overlaid with an outline of its shape. The air cavity appears to displace approximately 30% of the fluid from the surface of the cylinder. The significant size of the cavity was also expected to produce a significant effect on the drag.

Figure 4.18 - Top view of an air cavity with an outline showing its shape for the 3.18 mm diameter cylinder. Re = 0.016 and De = 80.

4.3.3 Impact of Air Cavity on the Drag

To test the effect the cavity has on the drag, drag measurements were made at steady flow velocities before the cavity formed and then recorded as the cavity developed. However, the velocity of the turntable could not be held perfectly constant and varied over time, typically by ±15%. Small changes in the drag, caused by variations in the velocity, could be mistakenly attributed to the growth of the air cavity. To eliminate the velocity dependence, the velocity was monitored over time, along with the drag, by optically recording the position of the turntable and calculating the angular velocity at any given time during the experiment. The drag data vs. time and velocity vs. time are shown in Figure 4.19 for the 1.58 mm cylinder in B4 and in Figure 4.20
for the 0.50 mm cylinder in B4. The approximate depth of the cavity is given as the percentage of the cylinder immersion depths in both figures.

Figure 4.19 - Drag and velocity run chart as an air cavity developed behind a 1.59 mm diameter cylinder. The approximate depth of the cavity is given as the percentage of the cylinder immersion depths. The cavity developed approximately 300 seconds after the start of the experiment.

Figure 4.20 - Drag and velocity run chart as an air cavity developed behind 0.50 mm diameter cylinder. The approximate depth of the cavity is given as the percentage of the cylinder immersion depths. The cavity developed approximately 150 seconds after the start of the experiment.
Comparing the drag and velocity in both figures, there appears to be a strong correlation between the two. The velocity dependence of the drag was factored out using the correlation between the drag and velocity.

Without the velocity dependence of the drag, the remaining variation in the drag must be due to the growth of the cavity. The drag results minus the velocity dependence for the 1.58 mm and 0.50 mm diameter cylinders are shown in Figure 4.21 and Figure 4.22, respectively.

Figure 4.21 - Corrected drag run chart as an air cavity developed behind a 1.59 mm diameter cylinder.
In Figure 4.21 and Figure 4.22, the corrected drag changed little over time, despite the increase in the size of the air cavity. The lack of change in drag indicates that the air cavity did not have a measurable effect on the drag. This finding suggests that the force acting on the upstream side of the cylinder contributes more to the drag than the force acting on the downstream side of the cylinder. The pressure at the downstream side of the cylinder is likely atmospheric once elastic stresses are sufficiently high, independent of the formation of the cavity there.

4.3.4 Critical Onset Velocity for Cavity Development

As mentioned earlier, the cavity started to form once a critical velocity was reached. The critical velocity was pinpointed by slowly ramping up the velocity of the turntable until a noticeable cavity formed. Due to the hysteretic nature of the air cavity in which it continued to grow even if the velocity was decreased, the turntable speed was reset if it appeared that the critical velocity was overshot. The process was repeated for each of the four cylinders used in the present study.
and two additional large diameter cylinders: a 3.18 mm and a 5.50 mm diameter cylinder. The critical velocities are plotted in Figure 4.23.

![Graph](image)

*Figure 4.23 – Critical velocity for the onset of cavity development for various cylinder diameters.*

The critical velocities were roughly the same, varying between 0.030 m/s to 0.038 m/s, over cylinder diameters which varied by 10 to 1. Figure 4.23, however, suggests a possible trend. Because of large errors for the 0.50 mm diameter cylinder, the critical velocity may actually increase with diameter, although only weakly. An explanation for the increase in critical velocity could be that as the cylinder diameter increases, the width of the air cavity also increases. To maintain the wider cavity, higher stresses are required, which can be induced with higher velocities.
4.3.5 Cavity Size

The dependence of the air cavity size on the velocity and cylinder diameter was also studied. The distance from the rear of the cylinder to the tip of the air cavity, defined as the horizontal length of the air cavity, is a suitable representative dimension for the cavity size. The horizontal cavity length was measured by taking pictures of the cavity from the side, then comparing the horizontal length to the known immersion depth. The side view of an air cavity behind a 1.58 mm diameter cylinder and the method by which the horizontal length is determined is illustrated in Figure 4.24, where $C_{\text{min}}$ and $C_{\text{max}}$ are the minimum and maximum horizontal lengths of an air cavity, respectively.

![Figure 4.24 - Determining the air cavity length for the 1.58 mm diameter cylinder at Re = 0.008 and De 160.](image)

The maximum and minimum horizontal cavity lengths were recorded because the horizontal length varied along the length of the cylinder. The range of the horizontal lengths is plotted in Figure 4.25 for both the 1.58 mm and 0.50 mm diameter cylinders in the B4 Boger fluid. The maximum and minimum horizontal cavity lengths, at each velocity, were represented by the top
and bottom of the vertical lines, respectively. The length of the vertical lines then represents the difference between the maximum and minimum horizontal length at each velocity.

As expected, both the diameter and velocity affected the horizontal length of the air cavity; the horizontal cavity length increased as both the diameter and velocity increased. Also, as the velocity increased, so too did the range between the maximum and minimum horizontal length. As shown in Figure 4.24, the cavity was close to uniform over the length of the cylinder at lower velocities, but became less uniform at higher velocities. At the higher velocities, the cavity was longer at the top and bottom of the cylinders, and shorter near the middle. This finding suggests that the pressure at lower end of the cylinder was lower than the pressure near the centre of a cylinder.
Based on the growth and shape of the cavity, an explanation for its presence in the Boger fluids is that the polymer chains of the fluids elongate as they flow around a cylinder. Once the polymer chains move past the cylinder, they don’t relax quickly enough to easily flow behind it, creating a pressure deficit behind the cylinder. Because the top surface of the fluid is exposed to the atmosphere, air fills the region behind the cylinder where the pressure is the lowest.

For future studies involving viscoelastic flows around an object, investigators should be aware of the significant pressure drop behind an object and the potential for cavity development. Specifically, for flow around a cylinder, the characterization of the critical cavity onset velocity, in the present study, can be used to help investigators estimate the pressure drop behind a cylinder. The characterization of the range of horizontal cavity lengths at a given velocity can help investigators estimate the pressure drop at the ends of a cylinder in viscoelastic fluids.
Chapter 5
Discussion

There were errors associated with the experimental technique and correction method. The noise in the drag measurements were largely due to poor speed control of the turntable, as was evident in section 4.3. The noise was not easy to remove because several components of the turntable must be replaced to better control its speed. The small impact on the drag did not warrant the cost of replacing the components. The depths to which the cylinders were immersed in the fluids were accurate to within ±0.5 mm, which represented no more than 5% of the total depth of the cylinder. Since the measured force is proportional to the depth, a 5% error in the cylinder immersion depth corresponds to a 5% error in the drag.

The use of the wall effect correction for the Boger fluids was somewhat questionable. The correction factor for the wall effect was determined from the ratio of Faxens’ confined flow solution [28] and Kaplun’s infinite medium solution. Both solutions only apply to Newtonian fluids, and the wall effect correction factor proved to be accurate. However, the correction factor for viscoelastic fluids, specifically for B1 and B4, is not known. There is no method at present by which an appropriate correction factor can be determined. Because the elastic effect on the drag did not scale with the Reynolds number, a correction factor could not be determined by varying the cylinder diameter to extrapolate the drag in a freestream flow. Therefore, the correction factor for Newtonian fluids was used as a first approximation. For future investigations, a more accurate correction factor could be determined by varying the inner and outer diameters of the annular tank. The measured drag could then be extrapolated to determine the drag in an infinitely
large channel for each cylinder diameter. The extrapolation will not be linear so at least three annular tanks are needed.

The air cavities that formed behind the cylinders did not appear to have any effect on the drag. This result contrasts that for Newtonian fluids where the flow at the rear of a cylinder contributes half or more of the total drag. The streakline images indicate that a wake was present behind the cylinder for the Boger fluids. However, drag measurements, recorded as a cavity developed, indicate that the elastic component of the drag acted strongly at the front of the cylinder and made up the majority of the total drag. This result is consistent with the results of Chilcott and Rallison [37], who indicated that regions of highest polymer extension occur just upstream of and at the sides of a cylinder. They also observed a region of relaxation just past the sides of the cylinder, towards the rear stagnation point. The higher stress upstream of the cylinder and the region of relaxation towards the rear of the cylinder may explain the small contribution to the drag at the rear of the cylinder.

The wake region observed in the present study was also predicted by Chilcott and Rallison. In their study, the velocity downstream of the cylinder was lower than the expected Stokes flow velocity but was relatively higher than the corresponding velocity in the present study. However, the Deborah numbers in their study were closer to unity, which may explain the weaker impact on the downstream velocity. Chilcott’s and Rallison’s study, however, does not explain the presence of the air cavity observed in the present study. In their work, the cavity observed in the present study would have occurred in a region of maximum polymer extension. Whether such a region can induce the flow to separate and allow for an air cavity to develop is not known.

For future work, the air cavity can be eliminated from the experiment by fully immersing the test cylinders into the fluid, preventing air from entering behind the cylinder. A numerical study,
similar to Chilcott’s and Rallison’s, could be performed under the same conditions as the present study, to explain the presence of the air cavity and wake region.

For microorganisms travelling in a viscoelastic fluid, the results of the present study suggest that fluid elasticity assists flagellum propulsion. The drag on a cylinder in the transverse direction is analogous to the normal component of the propulsive force of a flagellum, which is key to forward motion. Because the transverse drag has been shown to increase with elasticity in the present study, it is expected that elasticity increases the propulsive force of flagellum in microorganisms. However, the tangential component of propulsion, which likely opposes forward motion, was not studied. It is possible that elasticity can have a significant effect on the tangential component of the propulsive force. In addition, the drag was measured in a continuous and steady flow whereas a flagellum moves in either planar sinusoidal or helical rotational motion. It is unclear what effect elasticity has on propulsion in these types of motions.

For future work on the propulsion of flagellum in viscoelastic fluids, a drag experiment similar to the one in the present study can be conducted for flow tangent to the length of a cylinder. The effect of elasticity on the tangential drag, determined from such an experiment, and the deviation of the ratio of the tangential to transverse drag from Newtonian can contribute to the understanding of the propulsion of flagellum.

Also, because the elastic strength on drag was found to depend on the cylinder diameters, future analytical and numerical studies should avoid treating the flagellum as a zero-width waving sheet or zero-diameter filament. The diameter of the flagellum should be taken into consideration when dealing with viscoelastic fluid mediums.
Chapter 6
Conclusion

In the present study, the drag on a circular cylinder was determined experimentally by flowing a Newtonian fluid and two high-viscosity Boger fluids around test cylinders at low Reynolds numbers. Because wall and end effects could not be eliminated from the experiment, they were corrected by using known drag solutions. The drag for the Newtonian fluid matched known analytical solutions, verifying the experimental approach. The drag of the two viscoelastic Boger fluids was found to be three to six times higher than that for an equivalent Newtonian fluid. The rise in drag increased with both the Reynolds number and Deborah number.

During the drag experiments, an air cavity was found to develop behind the cylinder once a critical velocity was reached, and aspects of the cavity were characterized. The presence of the cavity, and its growth along cylinders had no effect on the drag. The size of the cavity increased with increasing velocity. Along with the distorted and asymmetric streaklines observed around the cylinder, it was evident that the disturbance of the flow by the cylinder resulted in high polymer extension just upstream and on the sides of the cylinder.

Based on the results of the present study, it is expected that fluid elasticity contributes to the propulsion of flagellum in microorganisms. The rise in drag with elasticity, seen in the present study, suggests that a cylindrical filament experiences higher forces in a viscoelastic fluid than in a Newtonian fluid, and thus, produces higher propulsive forces.
Bibliography


Appendix A

In this section, the procedure for determining the fluid relaxation time from a stress relaxation experiment is shown.

The stress response of a viscoelastic fluid can vary depending on the applied shear rate and the shear strain. Two special cases arise from the combination of shear rates and shear strains. When the shear rate becomes high enough to be considered infinite, the test becomes a step strain experiment, in which the test fixture rotates a given amount instantaneously. When the shear strain becomes high enough to be considered infinite, the test becomes a cessation of steady shearing. Both of these cases, along with intermediate cases, were studied to determine the fluid relaxation time.

The data for a cessation of shearing test for B4 at a shear rate of $5s^{-1}$ was curve fitted using equation (4.8). The process is shown in Figure A.1.
Figure A.1 - Example of relaxation time curve fit for B4 for cessation of shear at 5 s⁻¹.

Once the relaxation times were obtained, the cessation of shearing data was fitted using equation (4.8). The result is shown in Figure A.2.
Figure A.2 – Curve fitting of Shear stress after cessation of shearing at 5s-1 fit with 4 relaxation times.

The curve fit agreed well with the cessation of shearing data. The portion of data during time, \( t < 0.1 \) s was not curve fitted because the change in stress was likely due to machine stopping time. The fixture could not stop instantaneously after shearing and must decelerate to a stop, inducing a shear stress profile over a period of 0.1 s. The resulting shear stress follows a stress decay similar to the one seen in Figure A.1 for time \( t < 0.1 \) s. The stress profile also agrees with the Oldroyd-B model which suggests that the solvent viscosity of the fluid was responsible for the sudden drop in shear stress at \( t < 0.1 \) s. Under the Oldroyd-B model, the solvent viscosity component only induces a shear stress when the fluid is under shear. As soon as the fixture stops, the shear stress due to the solvent viscosity should also go to zero. Therefore, the shear stress in this region was not included in the curve fit.

Although the curve fitting with four relaxation times fit the data well, the true relaxation times for Boger fluids are more likely to follow a continuous relaxation spectrum rather than discrete times, according to Bird et al.[13]. However, the discrete relaxation times determined using the
method in this study should be close to or on the same order of magnitude as the strongest points along the continuous spectrum and should be representative of the relaxation profile of the fluid.

The longest relaxation time is expected to have the greatest contribution to the drag with a Boger fluid. As will be shown later, the molecules of the Boger fluids become highly disturbed after flowing around the test cylinder. It was observed that the fluids did not return to equilibrium for several seconds – a similar order of magnitude to the longest relaxation times. To condense the number of relaxation times required to fit the relaxation profile to one, only the longest relaxation time was considered. The longest relaxation times for a range of shear rates and shear times are listed in Table A-1 for B1 and Table A-2 for B4. It is interesting to note that different shear rates and shearing times do not appear to correlate with the longest relaxation time. At a shear rate of 1 s\(^{-1}\), there was variation in the relaxation times for both fluids, likely because of the low stress magnitude. At higher shear rates, the longest relaxation times were scattered, indicating that neither the shear rate nor shearing time affected the longest relaxation time. Due to the scatter, the average of the longest relaxation times was used as the representative relaxation time of each fluid. Some relaxation times were not considered in the average because they were either outliers or simply unreliable. The longest relaxation time was found to be 7.8 s for B1 and 8.0 s for B4.
Table A-1 - Summary of longest relaxation times from stress relaxation data for B1

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<td>1</td>
<td>2</td>
<td>5</td>
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<td>20</td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>1</td>
<td>14.93*</td>
<td>72.99*</td>
<td>10.00*</td>
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<td>9.52</td>
<td>7.82</td>
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* shear rate too low to accurately determine a relaxation time

** no data

Table A-2 - Summary of longest relaxation times from stress relaxation data for B4

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<td>7.63</td>
<td>7.60</td>
<td>10.54</td>
</tr>
</tbody>
</table>

* result are not consistent with the other results at shear rate = 1 s⁻¹

** longest relaxation time was not measurable.