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Mathematics learning in groups: Analysing equity in two cooperative activity structures

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RUNNING HEAD: ANALYSING EQUITY

Abstract

Many mathematics classrooms use cooperative learning to support equitable learning environments for all students. Past research in the field has focused primarily on increasing achievement, rather than contexts that support equitable interactions. This year-long study in three secondary mathematics classes compares two activity structures – a group quiz and a presentation – by examining group interaction within the two activities. The analysis shows that groups constructed a range of work practices, including a practice focused on collaboration, one focused on individual work, and one focused on ‘helping.’ In addition, students adopted a variety of positions, including expert, novice, in-between and facilitator. In this data corpus, experts tended to dominate interactions during group quizzes, whereas presentation preparations were more equitable, particularly when a student was positioned as a facilitator. Based on the analysis, suggestions are provided for structuring more equitable mathematics group work.

Mathematics Learning in Groups: Analysing Equity in two Cooperative Activity Structures

Many mathematics teachers have adopted cooperative group work as a daily classroom practice, along with standards-based mathematics curricula and pedagogies based on constructivist views of learning (Antil, Jenkins, Wayne, & Vadasy, 1998). Although there have been no recent studies quantifying the prevalence of cooperative group work, several widely used mathematics curricula for middle and high school are based on group work (including the Connected Mathematics Project, the College Preparatory Mathematics program, and the Interactive Mathematics Program). In addition, the National Council of Teachers of Mathematics (NCTM), serving both the U.S. and Canada, recommends the use of cooperative learning at least part of the time (National Council of Teachers of Mathematics, 2000).

Proponents of cooperative learning argue that working together provides students with more opportunities to talk about mathematics, to learn from others, and to learn through teaching (see, for example, Brown & Palincsar, 1989; Farivar & Webb, 1994; Nattiv, 1994; Stevens & Slavin, 1995). Still, when not implemented carefully, using group work may exacerbate equity issues in the classroom by supporting students who are already successful, while leaving less successful students behind (Cohen, Lotan, Scarloss, & Arellano, 1999; Weissglass, 2000).

The equitable implementation of cooperative learning in mathematics classrooms depends not just on what teachers do; students' learning depends on how they interact with one another. How do students go about learning together in groups? How can we encourage more productive and equitable interactions? As I will show, we cannot answer these questions simply by outlining how a teacher might structure the perfectly equitable

activity. A single cooperative activity structure may be taken up in multiple ways in the classroom, and issues of equity are complex enough that some aspects of an activity might support equity, while other aspects detract from it. Therefore, in order to better understand issues of equity, we must examine this uptake process – within particular activities, how do students interact and what are the consequences for their learning? To this end, I present an analysis of two different activity structures adopted in high school mathematics classes. The focus of this paper is how particular activities, set in classroom contexts, afford particular kinds of engagement from cooperative groups. I study groups' uptake of the two activity structures and how this uptake affected the distribution of opportunities to learn in the group.

In this paper I will not directly address issues of race, class, and gender, although these terms are central to debates and discussions of equity in schools, and in society more broadly. Rather than beginning with definitions of equity that draw on these terms, I begin with definitions that highlight participation in classroom practices, focusing on how people choose to participate, how they shape the participation of others, and how both are constrained by activity structures within particular practices. Although positioning in relation to racialized, gendered, and socioeconomic categories may often be associated with positioning in relation to mathematical competence, these forms of positioning are not always made visible. Because positioning in terms of mathematical competence is often visible in classroom interaction, this type of positioning has formed the basis for the analysis of positioning in this paper.

Equity and Opportunities to Learn in Mathematical Group Work

In this paper, the goal is to explore the conditions that support equity by fostering adequate opportunities to learn for all students, and to understand how two different classroom activities affect these opportunities to learn. Although equity has not been a central focus for most prior research on group work in mathematics (with research on ‘Complex Instruction’ a notable exception, see, e.g., Boaler, 2008; Cohen & Lotan, 1997), a careful consideration of these findings can still inform our analysis of equity in schools. In the classroom context, I define equity as the *fair distribution of opportunities to learn*, where situated and sociocultural theories of learning are brought to bear on the question of what constitutes an opportunity to learn.

The term ‘fair’ here refers to a qualitative understanding of justice, rather than a strict equality (i.e., in terms of counting the number of questions asked, or the number of utterances for each student) (Secada, 1989). What we consider to be fair is ultimately a political question as much as an empirical one; after reviewing theoretical and empirical research on learning in groups, I will return to the question of how I will define fair in the context of this paper.

Turning now to opportunities to learn, it is important to define first what learning is, and the conditions under which it occurs. In mathematics classes, learning encompasses more than socializing students into particular mathematical practices; learning also includes changes to who they become. Thus, when analyzing opportunities to learn, we should consider not only students’ *access to mathematical content and discourse practices*, but also their *access to (positional) identities* as knowers and doers of mathematics (Gresalfi & Cobb, 2006). Of course, participation in mathematical discourse practices may influence one’s identity in the classroom, and one’s identity may

reciprocally influence how one participates in the group. These two dimensions of opportunities to learn, and the relationships between them, are illustrated in Figure 1 and will be discussed further below.

Figure 1. Opportunities to learn includes access to a) mathematical content and discourse practices, and b) positional identities

To elaborate on these two branches of OTL, and the fair distribution of OTL in cooperative learning, I will highlight these issues as they arise in the following vignette.

Tony, Sarah, Mustafa, and Kendra are working together on a mathematics problem. From the teacher's vantage point at the front of the room, they look extraordinarily productive; their heads are bent together, they are engaged in animated discussion and Sarah's hands gesture towards her own paper as well as Kendra's. As the teacher circulates around the room, she pauses to observe and listen closely to the talk. Tony and Mustafa are both writing, heads down, in their own notebooks. She hears Sarah explain her strategy for solving the problem, as she gestures toward her notebook and the diagrams she has written there. She hears Kendra tell Sarah, "Oh... I get it. But I did it a different way. What do you think? I chose-" and

then notices Sarah's gaze drift back to her own paper as she begins working on the next question.

In this vignette, opportunities to make sense of mathematical ideas and to participate in mathematical discourse practices are not evenly distributed across the group. Recent research drawing on sociocultural theories of learning has emphasized the importance of having students “construct knowledge, negotiate meanings, and participate in mathematical communication” (Moschkovich, 2002, p. 190). Learning is conceptualized as happening not just ‘in the head,’ but mediated by community practices, available cultural forms, language and representations (Saxe, 1999). In cooperative groups, mathematics learning has been shown to be associated with specific types of interactions, including asking questions (King, 1991), discussing problem-solving strategies (Chizhik, 2001), observing someone else's problem-solving strategies (Azmitia, 1988), explaining one's thinking (Nattiv, 1994; Webb, 1991), and maintaining joint attention (Barron, 2000, 2003). In this example, Sarah does the explaining and Kendra listens, while Kendra does not have a chance to explain, nor Sarah to listen to an alternative strategy. The group as a whole does not maintain joint attention, as evidenced by Tony and Mustafa's independent work, and Sarah's dismissal of Kendra's attempted explanation.

This is not to say that students in the group are not learning or will not learn from this interaction. Mustafa and Tony may be engaged in important sense-making processes, and individual work may at times be a productive part of cooperative learning. An emphasis on participation in discourse practices should not be confused with an emphasis on talk alone. The point here is that for a group's interactions to be equitable, they should

explicitly attend to the meaning-making processes of each of the group members, not just the few who are considered ‘smart’ or most competent.

Access to positioning as competent mathematicians is also distributed unevenly in this vignette of group interaction. Sarah is positioned as competent through Kendra’s uptake of her idea, while Kendra’s ideas are ignored. I understand identity through the ways that students position themselves and the way they position others (Davies & Harré, 1990), rather than understanding identity as a personal or individual trait that a person carries with them from place to place. Students are positioned in multiple ways in interaction, including positioning with respect to mathematical competence, behavioral norms, friendship, and socially constructed norms of race, gender, socioeconomic status, and a host of other social categories.

When I refer to ‘access to identities,’ I refer to students’ opportunities to develop positive positional identities, that place them as authoritative and competent members of the classroom community. This access occurs through the moment-by-moment practice of positioning, and longer term trajectories of positioning (Wortham, 2004), though the relationship of one moment of interaction to a person’s longer term trajectory of identity development is not always a simple one (Nasir & Saxe, 2003).

In mathematics education research, issues of positioning, identity and identification are gaining currency. One particularly relevant study focuses on what Boaler calls ‘relational equity’ – when students of varying cultural backgrounds, genders, and prior achievement treat one another’s classroom contributions with respect (Boaler, 2006). Based on the pedagogical approach Complex Instruction (Cohen & Lotan, 1997), the teachers in Boaler’s study used complex, ‘group-worthy’ problems, emphasized the wide variety of skills necessary for success in the class, and publicly recognized the

important contributions of low-status students in the classroom. Over time, students began to treat one another with respect and to consider everyone's contributions to be worthy of serious consideration. In effect, the students in Boaler's study demonstrated how classroom positioning can shift over time, in her case so that more students were positioned as competent. Relational equity is encompassed in the broader construct of opportunities to learn, as one aspect of access to positional identities as knowers and doers of mathematics.

Analysing Equity as Opportunities to Learn

The foregoing research allows me to elaborate more on the meaning of 'fair' as it relates to opportunities to learn in mathematics classrooms. To analyse group members' access to mathematical content and discourse practices, it is important to look at what I will call the group's *work practices*: interactional patterns that groups construct as they get work done together. Although the work of negotiation may at times be invisible, work practices are always negotiated between group members, in the sense that no one student can determine the group's patterns of interaction, without the cooperation of others. In my analysis of two different group activity structures, I will try to characterize the different work practices that groups construct together, and discuss the implications for equity. The key analytic question will be: Based on the group's work practices, did all group members have access to mathematical content and participation in mathematical discourse practices?

To analyse access to mathematical identities, I will investigate whether group members' positions allow them access to the mathematical ideas at play in the group (thus promoting mathematical learning), as well as whether group members' positions allow them to be perceived (at the moment, or in the future) as mathematically competent

and confident. The key analytical question is: How do various acts of positioning influence access to mathematical content, mathematical practices, and mathematical identities?

In this paper I consider two kinds of classroom activities in detail, in an effort to understand how the structure of an activity might influence work practices and positioning, and to consider whether some activities are likely to foster more equitable work practices and positioning than others. The group's work practices and positioning are, of course, related, although the relationship between the two is by no means simple or clear. For this study, I made no assumptions about the relationship between work practices and positional identities, except the assumption that both work practices and positioning were constantly negotiated and could shift in interaction.

Below, I describe data collection and selection methods, and provide examples of how work practices and positioning were identified in the data corpus. I then discuss how these practices and positioning affected the distribution of opportunities to learn in the groups, and close by considering implications for teachers and classrooms.

Methods

The research was conducted in three different high school mathematics classes, all taught by the same mathematics teacher, Ms. Delack. (I will sometimes refer to the classes as Period 1, Period 4, and Period 6.) The three classes were all housed on the campus of a large, diverse urban high school, Bay Area High School (BAHS). They were diverse with respect to race, gender, prior achievement, and grade level.

Ms. Delack used the Interactive Mathematics Program (IMP) curriculum in all of her classes. This curriculum is a strong reform curriculum with an emphasis on

conceptual understanding and open-ended problems; the curriculum is explicitly designed for use with cooperative groups. In IMP, a day's work typically consists of one to three deep, rich problems, often with multiple solution paths or several 'correct' answers. The three classes that participated in this study were taking the Year 2 course, known as IMP2. Ms. Delack followed the IMP2 curriculum during most of the year, occasionally supplementing with her own activities and materials.

Major Data Sources

The study was designed so as to obtain rich ethnographic data of cooperative group interactions and mathematics learning over an academic year, with a secondary goal of observing cooperative learning in more than one classroom. Video recordings and ethnographic methods were essential to capture the details of talk, gesture and body positioning in interaction. Approximately 150 hours of video were collected in all, though only a small sample of the data is reported here. Supplementary data in the form of student work, classroom artifacts, interviews, and questionnaires were collected to support interpretation of the video data.

In order to facilitate a comparison of group activity across classrooms and within a single classroom over the course of the year, two contrasting activity structures were selected to be the central focus of analysis.

In Ms. Delack's classroom, students worked in groups on almost a daily basis. In these classes, 'group work' did not mean just one type of activity. In a single 55-minute period, there were typically 3-4 different activities, including group discussion, presentations, whole-class discussions, quizzes, 'classwork,' and 'homework.' Therefore,

to analyse issues of equity and opportunities to learn in cooperative work, selected two activities to compare.

Two Activity Structures

Group quizzes were paper-and-pencil tests in which students worked in groups, and received two grades. One grade was for the mathematical correctness of their papers – the teacher graded only one paper per group, and the whole group got that grade. The second part of the grade was for group participation – a rubric was posted and students were evaluated based on whether they included everyone in the group, worked together, asked questions, and explained their thinking. In the terms used in the research literature, group quizzes had a group reward (a grade) and students were individually accountable to the group for correct answers and 'showing their work' on the quiz paper. I therefore selected group quizzes as one important task structure to investigate more closely.

I selected a second activity structure to contrast with the first. I wanted the second activity structure to be one that also seemed to have a high level of interaction (though no other activity seemed to come close to the group quiz in terms of the palpable intensity of group interaction). I also wanted the second activity to contrast with the group quiz in terms of group rewards or individual accountability, as Slavin's (1994) work suggests that these factors affect learning outcomes.

I therefore chose *presentation preparation* as the second activity structure. In the three classes, students presented problem solutions in their class at least once a week. Ms. Delack required all students in the group to be prepared to present. Sometimes the group would choose the presenter, and sometimes Ms. Delack would randomly choose just prior to the presentation.

There was no official reward for this activity, and students were somewhat interdependent in the sense that they had to produce a single product to represent the group. When the task was implemented in the classroom, students seemed to interact more than during an unstructured classwork activity.

From informal observations, these two activity structures seemed to prompt high levels of interaction within groups. However, without close analysis I could not determine the nature of these interactions; whether students focused on answers, procedures, or conceptual explanations, whether all students had opportunities to explain and ask questions, or whether the interactions were dominated by one or more students. A sample of group quizzes and presentation preparations was selected for analysis.

Selecting a Sample of Group Quizzes and Presentation Preparation Periods

Over the academic year, students participated in five thematic units. In four of these units, the teacher administered a group quiz, which took place a few days before the individual assessment for that unit. For the three class periods in which I collected data, I had videotape of 11 group quizzes. For data-analytic purposes, 6 group quizzes were selected for close analysis.

All four of the Period 4 group quizzes were included for analysis. I selected an additional two group quizzes for analysis; one from Period 1 and one from Period 6. I selected the first group quiz of the year, because I had good video recordings of all three class periods. Thus, in selecting the six group quizzes for analysis, I was able to contrast the participation of three different groups on one quiz (the first quiz of the year), and still include four different quizzes that occurred at different points in the year and covered different mathematical concepts.

In the data corpus, there were seven examples of the presentation preparation from Period 1, six from Period 4, and five from Period 6. Since for group quizzes I had selected four examples from the Period 4 class, ranging across the academic year, I did the same for presentation preparations, and chose a comparable presentation from each of the Period 1 and Period 6 classes. The goal was to get a sample that was large enough to expect some variability (with variability built in because of differences in mathematical task and drawing from different classes) but that was small enough to analyse in detail.

Identifying Work Practices

As a way to identify differing patterns of interaction in the group, each video sample was chunked into segments, where each segment corresponded to a unique topic of conversation and/or physical grouping (Erickson, 2006). A sample of this coding system is included in Appendix A. Within each segment, a very brief description of the major task(s) is included in the left-most column, along with a description of which students were involved in the task(s). In the right hand column, the following set of questions are listed, with brief descriptive answers.

How are ideas solicited? Some groups read the materials together and students offered input, whereas in other groups, students worked individually and responded to direct questions when asked. If there was a particular ‘division of labor’ within the group, this would be recorded here. For example, in some groups students divided up the work and each answered different questions. This was an example of a more formal division of labor. In other groups, there did not appear to be an explicit division of labor, but one student took on more of the explaining and monitored other students’ progress. I noted the participation structures within the group – roughly, who was included or excluded in activity, and how (Goffman, 1981).

How is correctness determined? Once mathematical ideas were ‘out on the floor,’ groups had to have some way to decide if those ideas were correct or not. In some cases, mathematical argumentation was used, while in others, correctness seemed to be largely attributed to individuals (as in, ‘smart’ students were assumed to be correct).

Were there multiple correct answers/strategies? This question was relatively straightforward to code, although brief descriptions were usually included in addition to a yes/no answer.

How are students positioned? Although some aspects of positioning could be inferred from previous questions in this list, here I explicitly recorded how students were positioned with respect to classroom authority, including but not limited to mathematical competence.

Based on repeated viewings of the tapes, and iterative cycles of coding based on the questions listed above, several themes emerged in the groups’ work practices and positioning. I was able to loosely group work practices into three basic types of work practice that were present in this data corpus: each segment of interaction could be coded as exemplifying an ‘individualistic,’ ‘collaborative,’ or ‘helping’ work practice. Here, I report briefly on these themes. Although the existence of these three basic types of interaction represent a finding for this study, I will describe them briefly here to help the reader get a broad overview of the coding system. I will provide many illustrative examples of these three types of work practices in the results sections of this paper.

Individualistic Work Practice. Many groups went through periods of individual work during the activities. The distinguishing characteristics for interactions that I coded as individualistic were a propensity for working individually before consulting one another, for not asking for help when needed, and for denying help to group members’

who expressed confusion or requested assistance. Groups were *not* coded for an individualistic work practice if, for example, group members tended to work individually to carry out strategies that had previously been discussed in the group.

Evidence for an individualistic work practice would include a) seeing a group member ask for help and be refused (especially if the group had previously helped someone else, or had displayed somehow that they were capable of helping), b) a group member visibly struggling with the work without asking their peers for assistance, or c) groups working alone with no discussion.

Collaborative Work Practice. I coded group work practices as collaborative when group members put their ideas together, worked together, and seemed to act as ‘critical friends’ when considering one another’s ideas – rather than quickly accepting or rejecting one another’s ideas, collaborative groups discussed and critiqued ideas put out onto the public floor.

In collaborative interactions in which groups displayed confidence about a correct answer, several students would contribute to the correct answer, or several different formulations or strategies were considered to be correct. Interactions in which groups faced greater levels of uncertainty or disagreement were considered collaborative if group members put more than one idea forward for discussion, or if several people jointly constructed a single strategy.

Helping Work Practice. The ‘helping’ work practice was used to distinguish groups in which mathematical talk was asymmetrically organized, in which one or more students instructed other students about what to do. In contrast with the collaborative work practice, a helping work practice was characterized by the uncritical uptake of ideas, and an insistence on just one correct idea or solution, while ignoring other

possibilities (usually denying contributions from other group members once a correct answer had been found).

Although it might seem counter-intuitive, the helping work practice could still occur in groups in which there were no expert students to provide the help. Groups were coded as ‘helping’ if group members oriented their actions towards obtaining an answer from a more expert other, or if they gave up when there was no such expert available.

Identifying Positioning

As I will show throughout the analyses, differences in positioning for individuals in the group made a difference in opportunities to learn for all group members. For positioning, the major differences appeared to be associated with levels of authority in the group (Engle & Conant, 2002). There were two types of authority that clearly influenced the nature of group interactions and affected opportunities to learn: whether or not a group positioned at least one group member as expert and one as novice, as well as whether or not a group positioned at least one member with the authority to organize participation (e.g., to get the group started, to encourage particular kinds of participation, etc.).

Experts, Novices and In-betweens. In many of the data examples analysed here, students positioned themselves with respect to mathematical competence. Students could position themselves as more or less competent than a peer (e.g., “I know less than you”), or more or less competent at a particular task (e.g., “this is easy”).

To describe the ways that students positioned themselves and one another with respect to mathematical competence, I have appropriated the terms experts and novices but use them slightly differently from their typical use in cognitive science research. Traditionally, research from this approach takes as a given that there are experts and

novices, and that they can be distinguished by their respective levels of content knowledge, or their experience in a particular field (see, e.g., Chi, Feltovich, & Glaser, 1981; Hatano & Inagaki, 1986; Hinds, Patterson, & Pfeffer, 2001). I am concerned instead with understanding how *in interaction*, students *position* one another as novices or experts.

There are, of course, many ways of enacting expertise, and I use the term to characterize one particular type of expert that appeared frequently in these groups: the unchallengeable expert. I used the code 'expert' for a group member who was frequently deferred to (mathematically), and who was often granted authority to decide whether their own and other students' work was correct. In order to be coded as an expert a student must have positioned her or himself as such, and must also have been positioned as an expert by peers. For a group to position one member as an expert, there must also be a student positioned as novice – with no novice, there is no one to defer to the expert, and to take up their ideas.

When coding a particular interaction, I use the term 'novice' to refer to a student who deferred to an expert (positioning themselves as less competent), and whose opinion was frequently passed over in discussions of mathematical controversies (positioned by others as less competent). Novices were often instructed by others, and often accepted these instructions, though they sometimes challenged or questioned the expert's advice. However, disagreements between expert and novice were resolved quickly, usually by a simple assertion by the expert.

In some interactions, group members could be positioned as neither expert nor novice¹, and students who were coded as expert in one situation may not be coded as

such in another. In some interactions there was one student positioned as expert, in others there was more than one, and some interactions had no experts.

Facilitators. The second type of positioning that will be considered here is that of the facilitator. I use the term to describe students who orchestrated the group activity, and fostered broad participation from group members. In an interaction, a student positioned as facilitator made sure that all or most group members participated in group discussion in some way. This could take the form of making sure that group members asked questions when they needed help, of assigning different tasks to different group members, or (in some of the most productive cases) actively encouraging group members to contribute to joint problem-solving.

Results

The goal of this paper is to examine recurrent patterns of interaction within two cooperative activities, and to understand how variations in interaction would affect equity in opportunities to learn. A summary of the 6 group quizzes and 6 presentation preparations, along with their codes for work practices and positioning, is presented in Appendix B. The combinations of negotiated work practices and positioning found in this data sample are listed in the appendix with descriptions of each of the data samples. I will consider the group quiz and presentation preparation separately, and discuss the kinds of interactions that were found in these two activity structures, showing how the work practices and positioning influenced group members' opportunities to learn.

Equity and Interaction in the Group Quiz

Figure 2 displays the negotiated work practices and positioning for 6 different group quizzes. The figure shows that in these six activities, students negotiated primarily helping and collaborative work practices, and very rarely constructed an individualistic

work practice. Further, though in several interactions, group members were positioned as novices or experts, no group interactions included students positioned as facilitators. The fact that students adopted a variety of work practices and positions is interesting, but may have little impact on equity in the classroom unless it can be shown to have affected the distribution of opportunities to learn within the group. What these differences mean for equity and opportunities to learn, is not clear from a cursory examination of the table.

Figure 2. Positioning and negotiated work practices for 6 group quizzes

		POSITIONING			
GROUP QUIZ		Expert and facilitator	Expert, no facilitator	Facilitator, no expert	No facilitator, no expert
WORK PRACTICES	Asymmetric: Helping		3 group quizzes 'Expert tells'		2 group quizzes Stuck - ask teacher for help
	Symmetric: Collaborating		1 group quiz Expert shares ideas with group		3 group quizzes Peers collaborate
	Individualistic				1 group quiz Stuck - individual asks teacher to explain

In an effort to maintain a broad level view of how issues of equity played out in group quiz interactions, while also providing detailed accounts of these interactions in an effort to help the reader get a sense of what these group interactions looked like, I will first summarize the general findings about how interaction in the group quiz shaped

opportunities to learn, and then ‘zoom in’ to present detailed descriptions of the data that corroborate those findings.

The group quiz had been selected for analysis because of the palpable feeling of intense engagement that came over the classroom during this activity. As the analysis revealed, students *were* likely to work on the task, and to actively participate in discussions with their group, without relying on any student facilitators to keep everyone involved. In this activity, getting students to work together was not the problem – but there *was* a problem in how some groups accomplished their joint work.

In the six examples of group quizzes that were analysed here, three groups adopted a work practice focused on helping. In all cases, the group interaction resulted in inequitable learning opportunities. When interactions in which group members were positioned as expert and novice, the expert typically did most of the work, told the novice exactly what to write down, and did not offer conceptual explanations. In the interactions in which no student was positioned as expert, the students did not engage in mathematical discussions at all, as they waited for the teacher to help them. This interactional pattern was of no benefit to students who did not understand how to do the problems.

The collaborative groups tended to be much more equitable, in that multiple students had opportunities to contribute to joint problem-solving. Although the collaborators did not always provide conceptual explanations for their ideas, in order to participate in these discussions and to make contributions, group members had to ‘stay on the same page’ (Barron, 2000).

This finding – that collaborative work practices were more equitable, and helping practices less so – is not simply an artifact of the coding scheme itself. It would be possible, for example, for a group to engage in primarily ‘helping’ interactions that did

not privilege only the more expert students. Experts sometimes provided written help by writing on the novice's test paper and solving problems for them (although this was considered cheating by the teacher). When experts gave oral help, it was most often a description of a set of steps to follow. This allowed experts to make their own thinking visible, while masking the thinking of the novice – so the novice's prior understanding of the problem was not taken into account when help was given. If the expert had begun by asking the novice what they did understand, and taken this into account when crafting an explanation, the interaction would have been much more equitable because both students would have the opportunity to express their thoughts and to make connections. Instead, in the group quiz activity, the conversations between students seemed geared towards transmitting information from one person to another.

In all cases, the groups did not seem focused on explaining their thinking. Explanations that were offered were brief, and students rarely asked for clarification of *why* a particular strategy was being used. As a result, the mathematical discussions were relatively superficial. This may be related to the nature of the task; because of the pressure of the assessment, perhaps students were focused more on correct answers than on extensive explanations.

In summary, the group quiz had some strengths and weaknesses. As a strength, this task encouraged students to work together and in many cases, to persevere despite difficulty. However, the mathematical discussions tended to focus on correct answers, and multiple strategies were not encouraged, so some students were silenced. The more equitable group interactions tended to focus on collaborating. However, the group quiz interactions could have been more equitable, even within the helping work practice, if expert students had allowed the novice students to work through their mathematical

problems on their own. Instead of showing the novices how to do the problems, and then stepping back to let them try it, the experts tended to do most of the work for their peers.

I will now present and discuss several examples. These examples were selected to highlight patterns of interaction from the various cells of Figure 2, and to exemplify (and in some cases, complicate) the generalized findings reported above.

Collaborative work practice, with no expert and no facilitator. The first example is taken from a group whose interactions were all coded as collaborative, with no expert and no facilitator. This excerpt highlights a collaborative work practice in a group that did not initially know how to proceed to solve a problem. Dawn, May and Namaya were working on a group quiz, and the problem they discuss in the excerpt was about finding the x -intercepts of quadratic functions like $y = 49 - (x - 3)^2$. (Each student had a slightly different problem on her quiz.)

Transcript conventions include the following: numbers in subscript correspond to and index the onset of an action or gesture, the boundaries of overlapping talk are shown with square brackets at the beginning ([) and end (]) of the overlap, and elongated pronunciation is shown with a series of colons.

Excerpt 1. Dawn, May and Namaya find the x -intercept of a parabola, 05/24/05, 4th period

1	10:05	Dawn	₁ I don't know how to find [x -intercepts. 1. Gaze down towards her quiz paper, head in hands
2	10:07	May	₂ [like? Yeah. Neither do I. ₃ Um. 2. Gaze towards her quiz paper, pencil in hand poised above the problem. 3. Sits closer to her desk, head still down.

3	10:10	Namaya	<p>⁴Y'all do it- [whe::re₅</p> <p>4. <i>Takes pencil out of her pencil case and zips it up</i></p> <p>5. <i>Looks up at May, then back down and starts to erase from her quiz paper</i></p>
4	10:11	May	<p>[Well this has to be zero, right?₆ (3s)</p> <p>[So</p> <p>6. <i>Wiggles pencil slightly at a point on her quiz</i></p>
5	10:16	Namaya	<p>⁷[Don't y have to be ₈zero for the x-intercept?</p> <p>7. <i>Finishes erasing, starts brushing debris off her paper</i></p> <p>8. <i>May's gaze comes up towards Namaya</i></p>
6	10:18	May	<p>The whole-</p> <p>Er this has to equal forty-nine</p> <p>Wait well₉</p> <p>9. <i>Leans closer, looks at Dawn's quiz paper</i></p>
7	10:23	Namaya	<p>Where are (you at?)₁₀</p> <p>10. <i>Gaze moves to her paper, pencil pointing to a problem</i></p>
8	10:24	May	<p>¹¹Yours has –</p> <p>This whole thing has to equal sixty-four right?</p> <p>So that₁₂ y can equal zero.</p> <p>11. <i>Points with right hand and pencil to a spot on Dawn's paper</i></p> <p>12. <i>Namaya turns her gaze towards Dawn's paper</i></p>
9	10:29	Dawn	<p>Oh yeah.₁₃ (2s)</p> <p>¹⁴So make that equal sixty four?</p> <p>13. <i>Nods slightly</i></p> <p>14. <i>Points to a spot on her paper</i></p>
10	10:34	Namaya	[Yeah.
11	10:34	May	[Yeah.
12		Dawn	<i>Clears throat, opens calculator</i>
			<i>All three students bent over their own papers, writing and using calculators, with 13s of no talking</i>

13	10:51	?May	Hmmm
			<i>All continue working, 22s of no talking</i>
14	11:13	May	Man! ¹⁵ <i>15. Straightens up partially, head comes up, then bends back down and punches numbers into calculator</i>
15	11:31	May	Right? ¹⁶ <i>16. Straightens</i>
16	11:39	Dawn	¹⁷ You lookin for what ¹⁸ this would be to equal forty-nine? [Te:n. ¹⁹ <i>17. Looks over to May's paper, begins moving left hand towards it</i> <i>18. Points to May's paper</i> <i>19. Brings hand back to own desk</i>
17	11:41	May	[Yeah. Ten minus- (1s) three. Really?
18	11:46	Dawn	Cuz you ²⁰ gotta do- no you don't do it like that ((<i>inaudible</i>)) <i>20. Reaching over and pointing to May's paper</i>

In this excerpt, Dawn began with a generalized plea for help – stating that she did not know how to find x-intercepts. Danielle's quadratic function was $y = 64 - (x - 3)^2$, and May's was $y = 49 - (x - 3)^2$. Although May admitted that she, too, didn't know how to find the x-intercepts, she soon suggested a strategy: "this has to be zero, right?" (turn 4).

In turn 5, Namaya chimed in with the suggestion that y has to be zero for the x -intercept. (It is not clear whether she was agreeing with or correcting May – since in May's utterance, the referent for 'this' is not clear.) In turn 6, May suggested that 'this' would have to equal 49 if the whole right side of the equation were to equal zero, while gesturing to her paper - probably pointing to $(x - 3)^2$ in her equation.

In turn 7, Namaya asked May where she was at in the quiz – perhaps because she needed to know what May's equation was. The fact that each student had slightly different equations written on their quizzes added complexity to their talk – when students spoke about their own specific problems, as May had in turn 6, the other students either had to generalize to their own equations or watch one another's gestures carefully. May did not verbally respond to Namaya's question, but leaned closer to observe Dawn's paper, and suggested that the $(x - 3)^2$ in Dawn's equation would have to equal 64 'so that y can equal zero' (turn 8). Dawn repeated these instructions, which were confirmed by both Namaya and May with their simultaneous 'yeahs.'

The segment so far was coded as collaborative because both Namaya and May had made substantive contributions to the discussion about how to find the x-intercept. Their ideas built on and echoed one another. Then all three students began a period of individual work, carrying out this strategy. In this coding system, this period was still considered 'collaborative' because they were carrying out results from their joint conversation, and because they checked in with one another at the end of this individual work.

During this quiet period in the group, May displayed that she was encountering difficulty; she made this display through sighs, and small comments to herself, as well as shifts in posture. Dawn, apparently having finished her own x -intercept problem, leaned over and asked May if she needed help, then suggested that the number May was looking for was 10 (turn 16). May then took up Dawn's suggestion and plugged it into her calculator to check that it worked. Thus, although Dawn was initially only a recipient of the help of others on this problem, she built on May's suggestion and was able to make a concrete suggestion of her own. The group's work on this problem was co-constructed,

with each group member contributing comments that built on one another's suggestions. In order to do so, the group maintained joint attention on their set of quiz papers, looking at their own and at one another's work, and listened closely to one another's talk. They finished each other's sentences, and built on one another's ideas. Each student in that group was involved in making their thinking visible, and each student was afforded opportunities to engage with the mathematical concepts at play. There was therefore a relatively equitable distribution of opportunities to learn in this type of interaction.

No one student in the group was positioned as any more or less expert than the others, and no student made overt facilitative moves designed to open space for participation for other group members. The fact that no group member was positioned as expert is interesting, especially in light of the fact that earlier in this group interaction, before Namaya arrived, Dawn had expressed dismay that Namaya was not there, saying "she's smart!" Despite this earlier positioning of Namaya as 'smart' and necessary for the group, during the group's problem-solving activities with the quiz, all group members were positioned as equally smart and valuable.

This example also nicely illustrates that participation is not synonymous with vocal interaction. Until the very end of the excerpt, Dawn does not articulate her own understanding of the problem of finding the x -intercept. In turn 9, she repeats the instruction given to her by the other group members, but she had not had the opportunity to explain her thinking about the problem. However, she did successfully solve her own problem, and then succeeded in helping May solve the problem as well. In this collaborative interaction, in which (primarily) May and Namaya contributed ideas about a solution method, the three students maintained joint attention, as is evident from the links between adjacent turns at talk. This joint attention, May and Namaya's explanations of a

strategy and not just an *answer*, and the independent work time while each student had the opportunity to test their understanding, allowed Dawn to benefit from the interaction, although her vocal contributions were minimal.

Collaborative work practice, with expert, no facilitator. In the group quiz, there was only one example of a group interaction that was coded as collaborative but in which one student was still positioned as expert. In that example, one student Sarah had been positioned as expert throughout the group quiz, and she had been the one to give instructions and advice to her peers. Another group member, Tony, was positioned as ‘in-between’ – he did most of his work by himself and was trusted to do so, but generally deferred to Sarah’s instructions. When, towards the end of the group quiz, Sarah found what she believed to be a mistake in her paper, she enlisted Tony’s help (and only Tony’s, although a third group member was available) to figure out how to fix the error. They both contributed ideas and built off of one another’s contributions, but Sarah consistently positioned herself as more expert, even while she was asking Tony’s advice.

In this case, two group members were struggling to make sense of a mathematical idea together, and listened closely to one another’s ideas, considered alternatives, and created space for open dialogue. However, the discussion only took place between Tony and Sarah, and Candie, the third student, who had been positioned as a novice throughout the quiz interactions, was excluded. Thus, the conversation was relatively equitable for those who were involved – but inequitable for the group as a whole because only students positioned as relatively competent were a part of the joint problem-solving endeavour. The novice student was completely excluded from this critical discussion.

Helping work practice, with expert, no facilitator. I now turn to several examples of group interactions that were coded as ‘helping.’ The first example I will give occurred

in Sarah, Candie and Tony's group, and was much more typical of interactions in that group. Just prior to the beginning of this excerpt, all three students in the group had been working silently and individually on their quiz papers. In this problem, students were graphing a series of inequalities and trying to locate the region of a 2-dimensional coordinate plane that corresponded to all of the inequalities – the 'feasible region.'

Excerpt 2. Sarah helps Candie find the feasible region, 03/18/05, 4th period

1	25:29	Sarah	<i>(gaze on her quiz paper, writing)</i> Okay (2s) So the::n= =The feasible region is right here ₁ 2Cuz less than is this side (5s) So the feasible region is this area. 1. <i>(gestures to her paper with pencil)</i> 2. <i>(begins to color in a section of her graph, Tony and Candie have eyes on their own papers)</i>
	25:42	Tony	<i>(reaches forward and takes a colored pencil from Sarah's desk)</i>
		Sarah	<i>(leans forward, looks at Candie's paper)</i> <i>(inaudible)</i>
2	25:52	Candie	This is the feasible region? ₃ This? ₄ 3. <i>(drops hand to point to section of her paper)</i> 4. <i>(shading in a part of her graph)</i>
3	25:55	Sarah	No. Color the ₅ whole, like ₆ thing like this? 5. <i>(holds palm flat, waves it twice over Candie's paper)</i> 6. <i>(points to her own graph, fingers gesture downwards in a line)</i>
4	25:58	Candie	₇ Mm hmm 7. <i>(shades in part of her graph, gaze towards her paper)</i>

5	25:59	Sarah	₈ We have to color the (<i>inaudible</i>) Like (<i>inaudible talk</i>) 8. (<i>gaze still on Candie's paper</i>)
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This interaction was coded as ‘helping’ because Sarah gave instructions to Candie without waiting to see if they were needed – in fact, when Sarah began her instructions in turn 1, Candie was in the midst of working on something else. In addition, Sarah’s ideas were the only ones put forth, and Candie carried out the instructions without offering her own opinion, or seeming to critically consider Sarah’s advice. This distinguishes the helping work practice from a more collaborative work practice.

Although both students were actively engaged in the discussion, they were both focused on clarifying *Sarah’s* point of view, without acknowledging or discussing Candie’s prior (or even current) understanding of the problem. This type of discussion was the hallmark of ‘helping’ work practices in this data corpus – the focus squarely on broadcasting the helper’s understanding to other group members.

This interaction was coded as having an expert present, because both Sarah and Candie frequently positioned Sarah as competent and expert, and Candie as a novice. Tony was positioned as in-between novice and expert. He was positioned as competent enough to work mainly on his own while Sarah helped Candie, but Sarah did watch over his work and several times offered unsolicited advice about how to proceed.

This style of interaction was common during the group quiz. While Sarah, the expert, watched over and gave advice to Candie, this helping interaction did not allow spaces for the novice student to express her own ideas or to explicitly make connections between her own mathematical thinking and that of her peers. Tony, as a student who

was positioned as competent enough to work on his own, also did not benefit from group discussion of the responses. The opportunities to learn in this type of interaction were skewed heavily towards the expert student, who had opportunities to explain her thinking and get feedback from peers as they tried to carry out her advice.

Helping work practice, with no expert, no facilitator, and individualistic work practice with no expert, no facilitator. Interactions that were coded as helping, but with no expert, occurred when group members became stuck and tried to call on an expert, although there was no student in the group positioned as expert. Sometimes groups in this position tried to recruit the teacher's help, although in a group quiz situation the teacher refused. Other potential 'experts' included textbooks, course notes (both of which were allowed by the teacher), and other groups – though this last was considered cheating because each group was supposed to solve the problems on their own. These interactions typically did not allow group members access to mathematical practices or content, because the groups waited for an expert to tell them what to do, rather than asking group members to contribute what they already did know.

One such group, composed of Kameko, Cecilia and Kim, got stuck on a problem about computing the chi-squared statistic. They asked the teacher to help, they recruited other groups, all to no avail. Coincidentally, earlier in the day, Kameko had worked with the teacher to solve a very similar problem. After exhausting other avenues for finding help, Kameko decided to try to reconstruct what she had done with the teacher (thus relying on the teacher's expertise). Cecilia and Kim waited silently for her to finish, so that she could instruct them in how to do it. This stretch of interaction was coded as *individualistic, with no expert and no novice*, because Kameko struggled alone to complete the problem. Kameko was not positioned as expert because she never did solve

the problem, and repeatedly positioned herself as not knowledgeable enough to solve the problem. This type of interaction was ultimately unproductive because the students could not make progress without an expert, and were not able to develop a sense of their own authority to solve mathematical problems.

As these detailed analyses of group interaction have shown, the group's work practices and the students' positional identities made a difference for individual group members' opportunities to learn. These analyses also demonstrated that the type of work practice and the types of positional identities made available to students were only loosely coupled – for example, helping interactions could take place with or without experts. In helping interactions with an expert present, the expert did much of the explaining. In helping interactions without an expert, the group expended their energy trying to locate an expert who could help, or waiting for a group member to step into the role of expert.

Equity and Interaction in the Presentation Preparation

Figure 3 displays the negotiated work practices and positioning for the 6 examples of presentation preparations. Comparing figures 2 and 3, the presentation preparation activity structure seems to allow for a wider variety of work practices as well as of classroom positions. The individualistic work practice occurred frequently here, though the helping and collaborative work practices were also represented several times. In addition, some interactions positioned one or more students as facilitator, whereas no group quiz interactions did so.

Figure 3. Positioning and negotiated work practices for 6 presentation preparations

PRESENTATION PREPARATION		POSITIONING			
		Expert and facilitator	Expert, no facilitator	Facilitator, no expert	No facilitator, no expert
WORK PRACTICES	Asymmetric: Helping		2 presentations 'Expert tells'	1 presentation Need outside help, facilitator organizes	2 presentations Stuck - ask teacher for help
	Symmetric: Collaborating	2 presentations Everyone contributes			1 presentation Peers collaborate
	Individualistic	1 presentation Little talk, one person writes	3 presentations Little talk, expert writes		2 presentations Stuck - individual asks teacher to explain

For the presentation preparation analysis, I will again begin with a general summary of how group interactions influenced equity and opportunities to learn, and then illustrate with examples of interaction segments that fell into the various cells in Figure 3.

The strength of the presentation preparation is that its structure encouraged multiple people to get involved. For each presentation, multiple students in the group had to be involved either in preparing a transparency or poster to present, or in rehearsing an explanation. Further, since the group typically did not know who would present, or knew that several presenters would be needed, it was possible for multiple students to rehearse explanations and to benefit from participating in this practice.

The weakness of this activity was that group members were able to opt out of discussing their ideas, listening to peers, or participating in writing solutions. Although all group members were typically offered the chance to participate, many declined. In some cases, group members even declined to help their peers when asked. It appeared that students did not always hold themselves accountable to one another.

In this activity, the most equitable group collaborated to come up with multiple ways to present a solution (and were coded as collaborative, expert present, facilitator present). One student acted as a facilitator (and also expert) and was able to encourage most students to rehearse presentations. However, this could have been improved if the group focused more on connections between the varied ideas, rather than considering them one by one. The mathematical discussion also seemed to pass through the expert student, as rather than speaking to one another, group members spoke to the expert.

The individualistic groups were much less equitable, in that students did not get the help they needed, they turned to the teacher for help rather than one another, and students opted out of participation. This may be because the presentation preparation was comparatively low-stakes for students. Though the group knew that one or more of their group members would have to present, they were not graded, and there was less interdependence built in to this activity. There were two reasons for diminished interdependence. First of all, presentations in these three classes often involved a student standing at the front and reading aloud what had been written on a prepared transparency or poster. This did not require a deep understanding of the material. Secondly, when questions were posed to the presenter, sometimes other group members would step in to explain. Thus, the presenter did not have to master the material they were presenting. The structure of the activity, as well as classroom work practices for presentations, allowed

for a more individualistic approach to preparing presentations. As long as the transparency or poster was prepared, the group could consider their task done.

Another interesting pattern that appeared in group interactions during the presentation preparation was that many groups constructed several different kinds of work practices, for different phases of activity. It appeared that for many groups, the work of preparing the transparency comprised one task, and the work of rehearsing presentations was another. Some groups did not rehearse; in these groups, opportunities to learn were skewed heavily towards the person who prepared the transparency and any students who helped or collaborated with that task.

I now turn to examples of group interaction to add detail and complexity to the general characterization given above. Because there was such a wide variety of interactional styles represented in Figure 3, I will not try to give examples of each, as I did with the group quiz. Instead, I focus on several examples that again highlight the diversity within this particular activity structure, but also provide some contrast with the examples discussed for the group quiz.

Individualistic work practice, with expert, no facilitator. Because the prevalence of the ‘individualistic’ work practices was much higher here than in the group quiz, I provide several examples to highlight how this work practice played out during interactions with and without experts and facilitators. In the first example, we see a student attempt to position himself as facilitator but fail because his peers do not respond to his overtures; he is, however, acknowledged as the group’s expert.³

Example 1. Riley prepares the transparency for his group

Riley pulled the transparency closer to him and offered to carry out Ms.

Delack's instructions. While he wrote on the transparency, Shayenne and

Dawn chatted with one another about other matters, and Ayodele sat in

silence. At times, Riley quietly verbalized what he was writing out loud.

When he did so, he did not look at the other students, and they did not

visibly react. His tone indicated that he was talking mainly to himself.

When he had finished writing a section of the transparency, he turned to

Ayodele and asked him if he wanted to write a section. Ayodele said no,

giving his bad handwriting as a reason. (02/01/05)

In this example, the group allowed one student to do the work without consultation from the rest of the group. It was coded as individualistic because Riley did the work without checking with other group members, and because when he invited Ayodele to participate, Ayodele refused to do so. The mathematical ideas on the public floor were constructed by Riley, with no contributions from other group members. Other group members did not participate in this construction, even through listening to or reading Riley's explanations.

I consider Riley's attempt to engage other students in preparing the transparency as a bid to facilitate the group; since Ayodele refused (as the other two group members did at times during this group's joint work), Riley was not coded as facilitator. He was coded as expert, in this excerpt and throughout this group's interactions, because he consistently was positioned as knowing what the group should do mathematically, and he was responsible for making sure that the answers on the transparency were correct. This interaction was inequitable because the student who was positioned as most expert ended up doing all of the work, with no involvement from the other students, who were positioned as less competent and who may have needed to learn the material.

Individualistic work practice, with expert, facilitator. In the interaction coded as individualistic but with a group expert and a facilitator, a group of four students had to prepare a transparency with answers to 4 different homework problems. A student positioned as facilitator suggested that they split up the work, so that each student was responsible for a particular piece of it (each of those students then being positioned as expert on their portion). This interaction was coded as individualistic because then each group member wrote their own portion of the transparency, with no input from others. This interaction was equitable in the sense that each student was positioned as expert, and therefore as mathematically competent and able to represent the group. However, the group interaction could have taken greater advantage of their mathematical competence by discussing the problems to ensure that each student had opportunities to learn from one another.

Individualistic work practice, with no expert, no facilitator. In a later interaction, this same group was coded as individualistic, with no expert and no facilitator, because one of the group members became confused and asked for help on his problem. No group member agreed to help. In fact, they all positioned themselves as incompetent to help, and did not display any sense of being accountable to help their group member. None of the students were positioned as expert for that small stretch of interaction. This interaction was inequitable in that a student who asked for help was denied any opportunity to engage in talk that might have supported the development of mathematical understanding.

Helping interactions, with expert, no facilitator. Turning to examples of interaction that were coded as helping, there were two basic kinds: helping interactions in which one student was positioned as expert, and interactions in which there was no

expert. Helping interactions that included an expert tended to be quite similar to those shown in the group quiz. In these interactions, the expert did most of the talking, with very little input from students positioned as less expert. Although it is, of course, possible for the non-expert students to learn from such interactions, the expert student had no way of knowing what they were learning, because so little floor-time was given to the non-experts.

Helping interactions, with expert, with or without facilitator. In helping interactions with no expert, groups had to find help from some other source. They invariably turned to the teacher, who was usually able to provide the information that the group needed to continue to make progress. The teacher's help was often provided in a more equitable fashion than the help of expert students, because the teacher usually asked questions of the students to find out what they had already done, and to encourage them to make conjectures about how to solve the problem. If the teacher was not available, these groups tended to sit and wait for her.

Two examples of collaborative work practice, with expert, facilitator. Finally, I present two contrasting examples of a collaborative work practice. In Excerpt 3, we see two students collaborating to construct an explanation for a problem that has already been solved. The problem involved trying to find the number of computations a computer could do in 30 seconds, if it could do one computation in 5×10^{-7} seconds.

Excerpt 3. Candie rehearses her explanation, with help from Christa, 05/03/2005

1	18:29	Christa	Do you know how to explain this? ₁ 1.(points to the transparency)
2	18:30	Tony	₂ Do you get it? 2. (turns head towards Candie, then back to the front,

			<i>looking down at his calculator)</i>
3	18:31	Candie	(1s) ₃ Big numbers? ₄ 3. <i>(reaches for transparency)</i> 4. <i>(takes the transparency that Christa hands to her)</i>
4	18:32	Christa	Yeah
5	18:33	Candie	Okay (1s) It's point zero ₅ zero zerozero <u>fi::ve</u> <i>(clicks tongue)</i> And then five times ten to the negative seven equals 5. <i>(Tony turns head towards the transparency in Candie's hand)</i>
6	18:40	Christa	<i>(points to the transparency)</i> ₆ This number right here ₇ 6. <i>(points to a spot on the transparency)</i> 7. <i>(leans back in her seat)</i>

This interaction was coded as collaborative for two reasons: because Christa invited Candie to participate even after a correct solution had been accepted by the group, and also because Christa and Candie jointly produced an explanation. (Even though the explanation was guided heavily by the transparency, Candie took the lead in producing it.) In this excerpt, in turn 1, Christa opened up space for collaboration by inviting Candie to participate. Tony echoed this invitation. Although this group had already prepared a transparency and could be considered finished with their task, Christa and Tony purposely invited Candie to co-construct an explanation to prepare for the upcoming presentation. In turn 5 Candie began an explanation, which Christa helped her with in turn 6.

Christa and Candie co-constructed the explanation, focusing mainly on explaining which arithmetical operations were used, and how the different lines of the written

solution corresponded to one another. Still, Christa was clearly positioned as expert – she had written the transparency and she was charged with decoding it for others – and Candie, the novice. The explanation focused at the procedural level, rather than addressing what the problem was about and how to marshal mathematical reasoning to solve it. As this example shows, a collaborative work practice does not necessarily mean that students were engaged in particularly meaningful or intense mathematical discussion. The key difference between a discussion of this nature that was coded as collaborative, and one that was not, is that in collaborative discussions multiple students had the opportunity to voice their developing understandings of the mathematics at play. Of particular interest, though, was the way that discussions tended to pass through and be dominated by an expert student – even if that expert was opening up opportunities for others.

Finally, one group interaction in the data corpus was particularly interesting because students seemed to spontaneously construct multiple ways of displaying competence. They distinguished between *doing* a mathematical task, *understanding* a mathematical concept and *explaining* the concept. Students were able to position themselves as competent at some parts of the task (usually, understanding), and less competent at another part of the task (usually, explaining). As in Complex Instruction where the recognition of multiple abilities allows more students to be positioned as high status, the distinction between doing, understanding and explaining allowed group members to be positioned as authorities while still expressing some uncertainty and confusion.

For example, in this group when one student told her group, “I know what I’m doing a little bit? I just can’t explain it, I guess,” she was asked how she might explain it.

She then attempted to give an explanation while the whole group listened, and was given feedback by the most expert student in the group. Rather than being marginalized and positioned as incompetent, she was offered an opportunity to voice her developing explanation for finding the feasible region.

These detailed examples have shown once again that there were a number of different ways that students could take up the presentation preparation, and that the varying work practices and types of positioning influenced the opportunities to learn that were made available to group members.

Discussion

The goals of this paper were to consider two mathematical classroom activities – the group quiz and the presentation preparation – in an effort to understand how their structures influenced groups' work practices and positioning, and to consider whether one activity was likely to foster more equitable work practices and positioning than the other. While an activity structure might encourage specific forms of engagement, all participants are constantly improvising within (and sometimes crossing the boundaries of) that structure, so some variation was expected (Erickson, 1982). The examples given here illustrated a variety of ways that different groups could take up a single activity, and demonstrated that these variations affected the group's distribution of opportunities to learn.

In both the group quiz and the presentation preparation, the most equitable groups tended to work collaboratively rather than individualistically. In the group quiz, these collaborative interactions only occurred when group interaction did not position any students as experts or novices. In the presentation preparation, there was more variety in

the collaborative interactions, and some groups managed to position one or more students as experts, yet still were very collaborative as they prepared the presentation.

The results of this study are both specific to mathematical learning and more general. While the facilitator positional identity might occur generally in cooperative groups, the particular characteristics of the ‘expert’ and ‘novice’ positional identities are intimately connected to students’ perceptions of mathematics learning. Mathematics is often presented as a cumulative field of study, so students who were high achievers in the past may be assumed to be better prepared for that day’s activities – and thus more likely to be positioned as experts within an interaction. Mathematics is also often characterized as fairly procedural and well-defined, with a single correct answer. The ‘helping’ interactions appeared to be oriented towards finding the single correct answer, and quickly. In some classroom communities, mathematical ‘smartness’ is displayed through ‘finding the answer’ quickly, and not necessarily through other mathematical practices such as coming up with conjectures, justifying one’s thinking, or creating a representation (Boaler, 2008). Although the teacher in these classrooms explicitly tried to move students away from these conceptions (and was successful to some degree), these beliefs about mathematics seem to underlie the interactions in which some students were positioned as expert, some as novice. Other aspects of the work practices and positional identities that were identified in this study may be more generally a result of the school context, and not specific to mathematics. School contexts typically support a concern for grades, for following instructions, for displaying one’s smartness, and for working independently.

The findings here resonate with much prior research in the field of cooperative learning, and extend and deepen the discussion of equity in cooperative classroom contexts. For example, this study echoes and adds complexity to the finding that activity

structures with interdependence and individual accountability can sometimes increase student achievement. This study goes beyond prior work in this area because of the careful analysis of the relationship between the activity structure and student interactions. The group quiz included in this study was associated with the active engagement of all group members, and an absence of individualistic group work practices. When working on a group quiz, no group required a facilitator to make sure that all students did a share of the work. However, a close look at work practices for participation in the group quiz revealed that some groups were content with relatively superficial mathematical discussions that may have prevented novice students from gaining a deeper conceptual understanding of the material. Similar concerns about this type of activity structure have been voiced in the past (e.g., Damon, 1984); the current study provides further empirical evidence for this concern. Thus, although the group quiz might increase student achievement *if all students write down correct answers on their quiz paper*, it is questionable whether all students actually understood the material or had the opportunity to genuinely learn from their peers.

A second finding from prior research in this area is that those who seem to have greater knowledge prior to a group interaction, often tend to benefit the most from those interactions (Cohen et al., 1999; Fantuzzo, King, & Heller, 1992; Webb, 1991). This study suggests why this may be the case. In this study, in groups with experts and no facilitator, the expert students were likely to take on the bulk of the work, focus much of the interaction on explaining their own ideas, and neglecting the understanding of their peers.

This study also complicates and lends nuance to a large body of research on the benefits of interaction with a ‘more competent other’ for learning. For example, many

educators draw on Vygotsky's work to argue that working with more knowledgeable peers or adults can help students move into their 'zone of proximal development,' thus enabling them to develop more sophisticated mathematical understandings (Kozulin, Gindis, Ageyev, & Miller, 2003; Wertsch, 1984). For example, Wertsch (1985) provides a detailed analysis of adult-child interactions and demonstrates that adults are often quite sensitive to the children's level of understanding, and work to develop a common language with which to communicate.

By contrast, in this study, helping interactions in which one student was positioned as more expert than the others were sometimes found to be detrimental to learning. The experts in this study did not, for the most part, act like the adults in Wertsch's study, or like their own teacher. They did not actively try to understand and build from the novice's perspective, which was a technique that their teacher usually used when working with groups. Many of the expert-novice interactions provided a fairly narrow window on mathematical content, with the expert student focused mainly on conveying their own mathematical ideas without considering the ideas of the novice students. Some novice students actually set aside their prior understandings and displayed less mathematical competence at this task than they had prior to their interaction with the expert (Esmonde, 2006).

Of course, the most pressing questions for educators will be how to marshal the evidence presented here to improve equity in cooperative learning in their classrooms. We must exercise caution when applying the results of an ethnographic study, deeply rooted in a particular classroom community, to more broad contexts in education. However, this careful examination of a particular classroom context does provide clues as to how to structure cooperative learning more equitably.

As I noted above, in both the group quiz and the presentation preparation, the most equitable groups constructed work practices focused on collaborating, rather than helping or working individually. Groups without experts who worked collaboratively still made progress on mathematical understanding, and groups with facilitators were able to take advantage of an expert's understanding, without compromising opportunities to learn for those positioned as less expert. So, finding ways to support more collaborative interactions between students should be a priority for mathematics educators.

Still, it would be a misinterpretation of the analysis presented here to suggest that there is one best model of group interaction, or one type of positioning that all students should take up. There are at least two reasons why we should not assume that what I have characterized as 'collaboration' is always the best work practice. First of all, a number of studies have found that different styles of interaction may be appropriate to help students learn different kinds of material (Chizhik, Alexander, Chizhik, & Goodman, 2003; Cohen, 1994; Damon, 1984). For more procedural types of learning, a helping interaction may be perfectly adequate – though other studies have found that helping interactions are more helpful when the recipient of help has the opportunity to state their own ideas, and to carry out independent problem-solving after being helped (Webb & Mastergeorge, 2003). And of course, students might benefit from periods of individualistic learning as they test out new ideas or try to consolidate ideas that they were introduced to in group collaboration.

Secondly, students' preferences for and interpretations of particular work practices may be related to their repertoires of practice, developed through participation in communities inside and outside of school (Gutierrez & Rogoff, 2003). A study in a Dutch middle school found that immigrant students were more likely to construct (what I

would call) helping interactions, while Dutch-born students were more likely to construct (what I would call) collaborative interactions. Further, each group tended to consider the work practices of the other group to be disrespectful (de Haan & Elbers, 2005). Thus, cultural background, communities of practice, and probably age, subject matter, and experience in the classroom, all affect how students work together. It would be a mistake, then to try to impose some a priori set of work practices and classroom positions on students, or to think that only one way of learning together could be productive. Instead, we must find ways to capitalize on and benefit from the diversity of students' approaches to learning (Nasir, Rosebery, Warren, & Lee, 2006).

The analysis presented in this paper suggests two major directions for future research on cooperative learning in mathematics. One direction would pursue the idea of shifting interactional patterns, and explore how teachers might intervene, through changing the structure of activities, the mathematical content, or explicitly positioning students. The findings of this study do suggest some lessons for teachers attempting to structure equitable cooperative learning experiences for their students.

For example, one could consider the relationship between an activity's structure, and the work practices groups use within that structure. In the group quiz, for example, students were graded and for a number of reasons, the quiz was relatively high stakes. This structure seems to have encouraged students to look to experts to make sure their answers were correct, rather than encouraging the group to explore multiple solution paths, or to consider each student's perspective. One could shift the structure, and hope to shift the group's work practices as well. The year after this study was conducted, Ms. Delack decided to continue with group quizzes, but to grade students *only* on group participation, and not on mathematical correctness. She hoped to avoid students' focus

only on correct answers. Another change to the structure would be to alter the nature of the questions on the quiz, requiring students to present multiple perspectives.

Consider the presentation preparation. Groups that were the most equitable took advantage of this activity to allow students to practice giving explanations. Multiple correct explanations were sometimes encouraged, providing group members with a window into one another's thinking. However, some groups did not rehearse any explanations at all, satisfying themselves with only a written representation of a solution, prepared by a single group member. One could alter the structure of this activity by requiring that each student rehearse an explanation, and requiring the group to give feedback to one another on these rehearsals.

Another possible way to foster more equitable cooperative interactions, discussed in depth in Boaler and her colleagues' recent papers (Boaler, 2008; Boaler et al., 2006; Boaler & Staples, In press), is to change the way expertise gets assigned and defined in the classroom. As I discussed with respect to the presentation preparation, in one group students were able to position themselves as experts in one domain (*understanding* how to find the feasible region) and as less expert in another (*explaining* how to find the feasible region). This assignment of competence appeared to open up possibilities for interaction that were not often seen in the data corpus. The crucial difference seemed to be that students who expressed uncertainty about explaining, but who said they understood, were asked to give a practice explanation. They were then supported in constructing an explanation. This contrasts with the work practices in many other groups in the data corpus, in which students who expressed uncertainty were often positioned as 'not understanding' (as opposed to 'understanding but not explaining well') and were

often given more directed explanations, explanations which often did not shed much light on the mathematical phenomenon in question.

Of course, changes to the activities would require further research, as classroom interactions are complex, and we cannot always predict how groups will react to the shifts in structure. As I have tried to show here, one can set up a cooperative activity, but what happens within that activity depends on the choices that are made by participants.

A second direction for future research would focus more explicitly on issues such as race, gender, (dis)ability, language, socioeconomic status. Although I do not do so in this paper, it should be a priority to build on this approach to consider how race, gender, socioeconomic status, and other social categories, play out in terms of participation. One could investigate, for example, whether particular ways of participating (specifically, those associated with white, middle-class values and norms for communication) are more valued than others (specifically, those associated with communities of color, and poor and working-class communities). I leave this important analysis for the future, as more complex analytic tools would have to be brought to bear.

The inequities that we see in the world outside of school are often replicated within schools. This study suggests several important questions about how this process works: How do work practices get formulated, and what is the basis for student positioning? Can it be shown that patterns of interaction within small group work systematically privilege or marginalize students from certain groups? Is positioning with respect to mathematical competence related to positioning of other kinds – with respect to race, gender, and other socially constructed identities? The approach presented in this paper discourages essentialization of marginalized groups and instead offers a way to

analyse these systemic inequities through participation patterns, by focusing on micro-interactions in classrooms that are consequential for opportunities to learn.

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Appendix A. Sample Coding for a Group Quiz

Coding, Group Quiz Participants: Sarah, Candie, Tony	
Phases of participation	
I: Whole group reads the problem together, identifies key information	How are ideas solicited? Group reads together, all group members make suggestions of interpretation Sarah tells others what to write down
	How is correctness determined? Students record ideas proposed, little controversy
	Multiple correct answers/strategies? No – Sarah instructs others in what to write down
	Positioning? All group members participate, Sarah positions self and is positioned as more competent than others
II: Tony works on his own, Candie and Sarah work together	How are ideas solicited? Tony works on his own, asks group members occasional questions Sarah monitors Candie's progress – offers instruction before being asked
	How is correctness determined? Sarah asserts correctness, with minimal justification
	Multiple correct answers/strategies? No – Sarah treats differences in Candie's work as problems to resolve, not potential correct answers
	Positioning? Sarah positioned as group expert – unchallenged by others Sarah positions Candie as novice by monitoring her work and giving direct instruction Tony positioned as in-between because he is not monitored in same way as Candie is, but not positioned as Sarah's equal
III: Tony and Sarah discuss a suspected error, check one another's work. Candie not directly involved	How are ideas solicited? Sarah notes an error in her paper, enlists Tony's help in correcting it
	How is correctness determined? Although Tony makes several suggestions, Sarah overrules them, again, with minimal justification
	Multiple correct answers/strategies? No
	Positioning? Although Sarah requests help from Tony, most of their talk still positions her as more competent Candie marginalized – not positioned as competent enough to help

Appendix B. Summary of Coding

Name of Group Quiz	Description	Code
1 st period, ‘Bees’	Phase 1 (0:00 – 6:03) Darrell and Adriano work somewhat independently, occasionally verbalizing their work or comparing ideas. (Coded as collaborative because independent work is carrying out ideas from joint discussion)	Collaborative, no expert, no facilitator
	Phase 2 (6:03 – 27:00) Chidima arrives and starts working on her test. They work independently, consulting one another for help when needed. Towards the end of this phase, there is a lengthy discussion of one problem, which they believe they have solved incorrectly.	Collaborative, no expert, no facilitator
	Phase 3: (27:00 – 45:00) Adriano says that he can help the group and has solved the difficult trig problem. He tells his solution process to the others, who challenge it.	Collaborative, no expert, no facilitator
4 th period, ‘Bees’	Phase 1 (0:00 – 1:45) Solving the volume and surface area problems	Helping, no expert, no facilitator
	Phase 2 (1:45 – 16:30) Working individually and in pairs (Coded as helping because group members frequently ask questions of one another, always get help when they ask)	Helping, expert, no facilitator
	Phase 3: (16:30 – 30:12) Sarah finds a mistake in their work, persists in arguing that it is right	Helping, no expert, no facilitator
6 th period,	Phase 1 (10:00 – 32:40)	Collaborative, no

'Bees'	Joint work among the whole group on all problems.	expert, no facilitator
4 th period, 'Difference'	Phase 1 (8:25 – 19:40) Cecilia helps Kim and Kameko get the answers for the first few problems	Helping, expert, no facilitator
	Phase 2 (19:40 – 45:45) Trying to solve difficult problem; Kameko works on her own, others wait for her to solve and explain.	Helping, expert, no facilitator Then: Individualistic, no expert, no facilitator
	Phase 3: (45:45 – 62:45) Kameko has figured out the answer. The students decide to switch papers so Kameko can do the expected numbers on each, Cecilia can do the chi-squared, and Kim can complete any calculations already written down.	Helping, expert, no facilitator
4 th period, 'Cookies'	Phase 1 (5:23 - 10:00) Whole group reading problem and identifying key information	Collaborative, expert, no facilitator
	Phase 2 (10:00 – 38:44) Tony works mainly on his own, Sarah and Candie work together	Helping, expert, no facilitator
	Phase 3 (38:44 – 51:00) After Sarah discovers a suspected error on her paper, she and Tony work to resolve it	Collaborative, expert, no facilitator
4 th period, 'Fireworks'	Phase 1 (6:55 – 8:50) Dawn and May working together, Namaya absent.	Collaborative, no expert, no

		facilitator
	<p>Phase 2 (8:50 – 38:40)</p> <p>Namaya is present. All continue to work on their quizzes, with periods of joint work, and periods of individual work. (Coded as collaborative because independent work is carrying out ideas from joint discussion)</p>	<p>Collaborative, no expert, no facilitator</p>

Name of Presentation Preparation	Description	Code
4 th period, feasible region Jigsaw	<p>Phase 1 (26:00 – 41:20)</p> <p>Riley facilitates the group and asks people for their ideas about how to find the feasible region. Multiple ideas are expressed.</p>	Collaborative, expert, facilitator
6 th period, maximum profit Jigsaw	<p>Phase 1 (32:21 – 35:50)</p> <p>Noreen, Elly and Lisbet discuss how to find the maximum profit.</p>	Helping, no expert, no facilitator
	<p>Phase 2 (35:50-41:45)</p> <p>When they are satisfied they can go no further, they lapse into off-task talk. Daniel tries to get their help in explaining, and they refuse.</p>	Off-task Individualistic, no expert, no facilitator
	<p>Phase 3: (41:45-43:20)</p> <p>Ms. Johnson comes and explains how parallel profit lines help one to find the point of maximum profit.</p>	Helping, no expert, no facilitator
4 th period, 'How many	<p>Phase 1 (0:20 – 10:30)</p> <p>Preparing the transparency. Riley facilitates, trying to get someone to do the writing. Shayenne agrees to do</p>	Helping, expert, no facilitator

of each kind?’	it and carries out his instructions while Riley explains to Dawn what she had missed the previous day.	
	Phase 2 (10:30 – 13:50) The transparency is finished. All group members wait for next task. Ms. Johnson comes and tells them to write more detail.	Off-task
	Phase 3 (13:50 – 23:15) Riley starts to carry out Ms. Johnson’s instructions, other group members refuse but eventually agree when the teacher insists. Riley finishes it off, then checks in with group members to see if they understand it.	Individualistic, expert, no facilitator
4 th period, ‘Picturing cookies’	Phase 1 (40:10 – 44:40) Shayenne and Dawn chat for a bit about people they know. Ms. Johnson comes by and gets them started on the task. Shayenne and Dawn start working on the graph. When they run into trouble, they stop working.	Off-task and Helping, no expert, no facilitator
	Phase 2 (44:40 – 57:00) Researcher helps Dawn and Shayenne. They continue working. For next step of problem, Shayenne asks teacher to explain. To graph points, they each take turns, facilitated by Dawn. Ayodele participates in graphing when asked.	Helping, no expert, facilitator
1 st period, ‘Big numbers’	Phase 1 (6:44 – 16:55) Beginning of the class period. Individual work on the class ‘warmup,’ along with social talk. Amir has done the homework, and Cherish and Darrell ask questions about it.	Off-task and Helping, expert, no facilitator, Then: Individualistic, no expert, no facilitator
	Phase 2 (16:55 – 30:30) Brandon joins the group. They decide which	Individualistic, expert, facilitator (when they split

	presentation to do, and then divide the labor of preparing the transparency. Everyone does their own part of it, and when Amir needs help on his part, no one helps him.	up the work) Then: Individualistic, expert, no facilitator
	Phase 3 (30:30 – 42:40) Darrell helps Amir. Cherish and Brandon continue to talk socially. Ms. Johnson comes over one last time and helps Amir see that his answer is correct. He then writes out the transparency while the others talk socially.	Collaborative, no expert, no facilitator and off-task
4 th period, 'Big numbers'	Phase 1 (1:53 – 9:30) Christa and Tony work, primarily independently, on the warmup. Christa prepares the transparency on her own.	Individualistic, expert, no facilitator
	Phase 2 (9:30 – 26:10) Candie comes in – late. When Christa finishes making the transparency, she explains it, and then Candie practices explaining the transparency. Researcher comes over to talk to the whole group, help them resolve some mathematical issues on the transparency.	Collaborative, expert, facilitator

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Footnotes

1. I sometimes refer to these students as ‘in-betweens’ in reference to Eckert’s (1989) description of Jocks, Burnouts, and In-Betweens in her high school ethnography.
2. An x -intercept is a point where a graph intersects with the x -axis. It can be found by substituting $y = 0$ into the equation and then solving for x .
3. This example is presented in prose form rather than through a transcript, because large portions of the interaction occurred in silence.