MULTIDISCIPLINARY DESIGN OPTIMIZATION OF A HIGHLY FLEXIBLE AEROServoELASTIC WING

by

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Abstract

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A multidisciplinary design optimization framework is developed that integrates control system design with aerostructural design for a highly-deformable wing. The objective of this framework is to surpass the existing aircraft endurance limits through the use of an active load alleviation system designed concurrently with the rest of the aircraft. The novelty of this work is two fold. First, a unified dynamics framework is developed to represent the full six-degree-of-freedom rigid-body along with the structural dynamics. It allows for an integrated control design to account for both manoeuvrability (flying quality) and aeroelasticity criteria simultaneously. Secondly, by synthesizing the aircraft control system along with the structural sizing and aerodynamic shape design, the final design has the potential to exploit synergies among the three disciplines and yield higher performing aircraft. A co-rotational structural framework featuring Euler–Bernoulli beam elements is developed to capture the wing’s nonlinear deformations under the effect of aerodynamic and inertial loadings. In this work, a three-dimensional aerodynamic panel code, capable of calculating both steady and unsteady loadings is used. Two different control methods, a model predictive controller (MPC) and a 2-DOF mixed-norm robust controller, are considered in this work to control a highly flexible aircraft. Both control techniques offer unique advantages that make them promising for controlling a highly flexible aircraft. The control system works towards executing time-dependent manoeuvres along with performing gust/manoeuvre load alleviation. The developed framework is investigated for demonstration in two design cases: one in which the control system simply worked towards achieving or maintaining a target altitude, and another where the control system is also performing load alleviation. The use of the active load alleviation system results in a significant
improvement in the aircraft performance relative to the optimum result without load alleviation. The results show that the inclusion of control system discipline along with other disciplines at early stages of aircraft design improves aircraft performance. It is also shown that structural stresses due to gust excitations can be better controlled by the use of active structural control systems which can improve the fatigue life of the structure.
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**Nomenclature**

\( d \) structural deformation vector \([m]\)

\( f_e \) generalized aeroelastic forces

\( g \) gravitational acceleration vector \([m/s^2]\)

\( r \) rigid-body mass distribution vector \([m]\)

\( v \) structural deformation rate vector \([m]\)

\( x_k \) state vector at time-step \(k\)

\( u_\Delta \) the uncertainty block output signal

\( y_\Delta \) input signal to the uncertainty block

\( z_{\text{flexible}} \) structural outputs

\( z_{\text{rigid}} \) rigid-body outputs

\( dv \) infinitesimal volume

\( m \) aircraft mass \([kg]\)

\( C_{bi} \) transfer matrix from the inertial-frame to the body-frame

\( C_{ee} \) modal damping matrix of the aircraft structure

\( C_{pi} \) panel coefficient of pressure

\( D \) transfer matrix from the Euler angle rates to the body-frame angular velocities
$H_{rs}(k)$  $r \times s$ generalized Hankel matrix

$J$  aircraft moment of inertia matrix [kgm$^2$]

$K_{ee}$  The aircraft structure modal stiffness matrix

$K_{fb}$  feedback gain

$K_{ff}$  feedforward gain

$K_l$  local material stiffness matrix of a beam element

$K_{t\sigma}$  global geometrical stiffness matrix of a beam element

$K_{tl}$  global material stiffness matrix of a beam element

$L$  aircraft’s Lagrangian

$M_\infty$  free stream Mach number

$M_{ee}$  The aircraft structure modal mass matrix

$N_p$  prediction horizon

$N_u$  control horizon

$Q$  State weighting matrix

$Q_f$  Terminal weighting matrix

$R$  Input weighting matrix

$SU$  matrix that relates the nodal displacement to von Mises stress

$T$  aircraft’s kinetic energy

$W_e, W_u, W_s$  frequency-domain weighting functions

$\Delta A_i$  panel area [m$^2$]

$\mathbf{F}$  rigid-body aerodynamic and propulsive forces [N]
$M$          rigid-body aerodynamic and propulsive moments [Nm]

$R_c$       the body-frame origin absolute position [m]

$U$         future input sequence vector

$V_c$       the body-frame origin velocity vector [m/s]

$X$         future output sequence vector

$V_{rel}$   relative velocity of a point on surface due to control deflections [m/s]

$V_s$       total surface velocity of point on aircraft [m/s]

$\eta$      structural modal deformation rate vector

$\xi$       structural modal deformation vector

$\omega$    the body-frame rotational velocity vector [rad/s]

$\theta$    Euler angles vector [rad]

$\mu_i$     panel vorticity strength

$\mu_{wi}$  wake panel vorticity strength

$\phi_r$    modal deflection at location $r$

$\sigma_i$  panel source strength

$\sigma_i$  von Mises stress in the $i$th finite element [Pa]

$\zeta$     aerostructural solver adaptive relaxation factor

$\Phi$      complete shape function matrix for all nodes on the aircraft structure

$F$         Rayleigh’s dissipation term for the complete aircraft

$L$         matrix of differential stiffness operators

$U$         Strain energy of the complete aircraft
\( \varphi \) flow potential

\( \hat{\cdot} \) volumetric density

\( \vec{\cdot} \) cross-product operator

\( \gamma_i \) jig-shape twist angle of the \( i^{\text{th}} \) spanwise panel [deg]

\( \rho \) Kreisselmeier–Steinhauser (KS) function parameter

\( AR \) wing aspect ratio

\( CD \) aircraft drag coefficient

\( CL \) aircraft lift coefficient

\( Cl \) section lift coefficient

\( d_i \) spar diameter at the \( i^{\text{th}} \) spanwise section [m]

\( h_{\text{err}} \) steady-state altitude error

\( S_{\text{ref}} \) wing reference area [m\(^2\)]

\( t_i \) spar thickness at the \( i^{\text{th}} \) section [m]

\( V_s \) stall speed [m/s]
Chapter 1

Introduction

During the past decade, high-altitude long-endurance (HALE) unmanned aircraft have gained considerable attention from aircraft designers. Airborne intelligence, surveillance, and reconnaissance (ISR); airborne communication relays; and atmospheric research platforms are among the many applications of HALE aircraft that have drawn the attention of government agencies and aircraft designers.

NASA’s Environmental Research Aircraft and Sensor Technology (ERAST) program was among the first few attempts to develop cost-effective, slow-flying UAVs that can perform long-duration science missions at altitudes above 60,000 ft [48]. The NASA’s Pathfinder and Helios aircraft were among a series of solar and fuel cell powered UAVs that AeroVironment, Inc. developed under the ERAST program before it’s termination in 2003. Pathfinder and Helios were essentially flying wings with solar photovoltaic cells mounted on the top of the wing to produce electricity powering the aircraft’s electric-driven propellers.

In 2008, The Defence Advanced Research Projects Agency (DARPA) has selected Aurora Flight Sciences, Boeing and Lockheed Martin as contractors for the first phase of the Vulture program [99]. The Vulture program envisions a system carrying a 1,000-pound payload that is able to stay airborne for an uninterrupted period of at least five years while remaining in the required mission airspace 99 percent of the time.

Recently, the Vulture program has entered the second phase. Under the Vulture II agreement, Boeing’s Phantom Works division will develop a full-scale demonstrator called the ‘So-
larEagle’ that is scheduled to make its first demonstration flight in 2014 \cite{29}. An artistic drawing of the proposed ‘SolarEagle’ is shown in Figure \ref{fig:1.1}. The aircraft will have highly efficient electric motors and propellers. The high aspect ratio 400-foot wing provides increased solar power and aerodynamic performance. During testing, the SolarEagle demonstrator will remain in the upper atmosphere for 30 days, harvesting solar energy during the day that will be stored in fuel cells and used to provide power through the night. However, solar power is not the only energy source being looked harvested to keep aircraft in the skies for extended periods. Boeing’s Phantom Works division is also working on a hydrogen-powered demonstrator called the Phantom Eye, a High Altitude Long Endurance (HALE) aircraft designed to stay aloft for up to four days. In June 2010 Boeing unveiled its hydrogen-powered Phantom Eye unmanned airborne system. The demonstrator, which will stay aloft at 65,000 ft for up to four days, is powered by two 2-litre, four-cylinder engines that provide 150 horsepower each. It has a 150-foot wingspan, will cruise at approximately 150 knots and can carry up to a 450-pound payload.

To fulfil the desire for an aircraft that can stay airborne for extended periods and act as a pseudo-satellite, a number of ongoing projects considering solar-powered aircraft are initiated in recent years. The Solar Impulse recently flew through the night passing another milestone on its way to an attempt to fly around the world non-stop in 2012.
1.1 Literature Review

In all attempts for designing HALE UAVs, the requirement for both lightweight structures and superior aerodynamic performance have resulted in high aspect ratio aircraft that are highly flexible. However, wing flexibility in aircraft is typically undesirable. The more flexible a wing is, the more prone it is to exhibit undesirable aeroelastic phenomena, such as flutter and divergence. Inertial and gust induced loads are traditionally addressed through passive aeroelastic tailoring and this usually results in an over designed wing for the cruise condition. NASA’s active aeroelastic wing (AAW) initiative aimed to perform aircraft roll control by twisting a flexible wing on a full-size aircraft. The AAW research (1995–2005) was performed on a modified F/A-18A \cite{67}. Through the use of multiple control surfaces, AAW technology can also be exploited to reduce the wing critical loads due to manoeuvring and atmospheric turbulence \cite{100,91,62,61,25}. The application of active control techniques to large aspect ratio transport aircraft have recently become more prevalent. Boeing has designed an outboard aileron modal suppression (OAMS) system to dampen out the limit cycle oscillation (LCO) that was observed during flutter testing of 747-8 aircraft \cite{65}.

With the advent of multidisciplinary design optimization (MDO) techniques, designers have started to integrate various disciplines in the aircraft design process. The strong coupling between aerodynamics and structures has motivated aircraft designers to apply MDO techniques to aerostructural design problems. Previous work has focused on minimizing the total drag or maximizing the aircraft range by simultaneously optimizing the structural and aerodynamic design variables \cite{28,53,54}. In order to improve aircraft efficiency, researchers have also investigated nonplanar lifting surface configurations, where aerostructural optimization was performed to find optimal configurations, such as C-wings, joined wings, and winglets \cite{63,96}. In all these efforts, the aircraft structure was designed to have optimum cruise performance while withstanding the critical loads corresponding to other flight conditions.

More than 25 years of extensive research in the field of aeroservoelasticity has shown that aeroservoelastic analysis and design should be considered as early as possible in the aircraft design process \cite{51,50}. Previous work on aeroservoelastic synthesis has focused either on
achieving the required maneuverability or avoiding catastrophes such as flutter through the use of active control systems. However, aeroservoelasticity can also be used to optimize the overall performance of an aircraft by minimizing an objective function, such as endurance, range, or fuel consumption. The new high aspect ratio structures of HALE aircraft have lower natural frequencies that makes the study of the rigid-elastic interaction an integral part of the design process. The true characteristics of these aircraft cannot be revealed without considering the interaction between the two dynamics and any design procedure that is based on traditional design schemes will suffer from lack of reliability [82, 72, 32].

One of the earliest works in the field of aeroservoelastic optimization was performed by Suzuki [91], who minimized wing structural weight by designing the structure and control system simultaneously. This optimization was subject to control stability and stress constraints while flying through an atmospheric gust. Aerodynamic design, however, was not considered in this work and the aircraft planform was kept unchanged. An aeroservoelastic design framework was also presented by Idan et al. [35], who considered the interaction between the aircraft structure and control system. First, a preliminary structural and control optimization was performed separately. The resulting structure and control system were then optimized considering closed-loop control margins and flutter. The same framework was later used to perform simultaneous structural and control optimization in the presence of parameter uncertainty [60].

Zink et al. [105] addressed the design of structural parameters and gear ratios of a lightweight fighter performing steady symmetric and antisymmetric (rolling) maneuvers. The optimization formulation was based on static aeroelastic equations. The integrated approach results were compared to those obtained using a sequential approach. The former was shown to be more effective and converged to a lower structural weight. The same authors also considered maneuver load inaccuracies and their effects on the optimum design [106].

The design variables used in previous work have mostly been limited to structural and control system parameters, while the aerodynamic shape has been kept constant during optimization. Nam et al. [62] conducted the only investigation that considered integrated planform and control system design, where parameters such as wing area, aspect ratio, taper ratio, wing twist distribution were considered in the design optimization procedure.
Since simultaneous optimization yields better results than those obtained through performing sequential optimization \cite{26,105,13}, the real potential of including aeroservoelastic synthesis in the aircraft design process can only be realized with a simultaneous optimization approach. In a traditional design process, the aircraft configuration is first designed, followed by the design of the control system design. This prevents the flight control system from affecting the aircraft configuration. It has been shown that by designing the aircraft configuration and the control system concurrently, trade-offs between these two design subspaces can improve the aircraft performance \cite{3,68,31}.

The development of a detailed mathematical model that can integrate flight dynamics with structures and aerodynamics is essential in the context of highly flexible wing design optimization. Some of the early work that addressed the active aeroelastic wing design adopted quasi-steady flight models, where structural deflections are assumed to have reached their steady-state conditions \cite{67,105,78}. The more flexible an aircraft is, the stronger the interaction between the rigid-body and the structural dynamics becomes. This interaction between the aerodynamic forces and the inertial forces can adversely affect the aircraft dynamics and destabilize the aircraft. Therefore, the exclusion of this coupled dynamics results in significant model inaccuracies. Mathematical formulations of flexible aircraft flight dynamics are mainly based on one of two approaches: the mean-axes method, or the body-fixed axes method. Waszak and Schmidt \cite{96} and Schmidt and Raney \cite{80} developed the equations of motion for a flexible aircraft using the mean-axes method. The use of simple aerodynamic strip theory and the small structural deflections assumption are the main limitations of this formulation. The derivation of aeroservoelastic formulations based on body-fixed axes system have also been considered by aeroelastic researchers \cite{58,66,81,89,5}. Shearer and Cesnik \cite{81}, for example, derived the non-linear equations of motion for a highly flexible aircraft and performed time-domain simulations to characterize the behaviour of a general HALE aircraft. The dynamic behaviour of an elastically deformable aircraft was compared to a statically deformed one to highlight the effect of structural flexibility on aircraft dynamics. The comparison concluded that the behaviour of an elastically deformed aircraft cannot be accurately captured using a statically deformed model. The derived formulation was then used to develop a control system to perform time-dependent
manoeuvre [82].

In flexible aircraft design optimization, it is of particular importance to consider the design of control systems along side the more commonly considered disciplines such as aerodynamics and structures. Among the technical recommendations included in the investigation that followed the crash of NASA’s Helios, the development of more advanced multidisciplinary approaches that include control systems and time-domain analysis methods appropriate to flexible aircraft was emphasized [64]. Such approach has the potential to exploit synergies between the three disciplines and yield higher performing aircraft than is possible without this multidisciplinary approach. Also, highly flexible aircraft can deform in unexpected ways under varying atmospheric conditions (e.g. Helios aircraft incident) from which the operational flight condition can only be recovered if active control systems are used.

The high level of interaction between the rigid-body dynamics and structural dynamics poses a significant challenge to a successful control system design. Aircraft structures are required to sustain critical loading conditions during manoeuvre or when flying through atmospheric turbulence. In addition to the various formulations that have been used to model flexible aircraft dynamics, researchers have used a variety of control techniques, ranging from classical frequency-based controllers to modern time-domain methods for designing active gust and manoeuvre load alleviation systems. The performance of the closed-loop system depends not only on the accuracy of the model used to represent the plant but also on the control architecture used to control the system. Hence, the selection of an appropriate control method, warrants a through review of the reported control applications in the related literature.

In a seminal paper, Karpel [41] effectively applied a simple output feedback controller to a simple wing section in order to perform gust load alleviation and flutter suppression. In that paper, state-space system representation was used and the feedback matrix coefficients were solved to move the closed-loop poles of the system to the desirable locations. McLean et al. [56] used a linear quadratic (LQ) optimal controller to reduce the wing bending moment of the C-5A Galaxy aircraft. The deformable equations of motion were developed by augmenting the short period equations with a series of second-order differential equations representing the structural dynamics. Elevators and ailerons (applied symmetrically) were used as control
surfaces and the final result was an improved damping of most structural modes. It was also shown through numerical simulation that the application of the designed control system resulted in less oscillatory motion in the wing bending moment that can assist in reducing the accumulation of fatigue cycles in the aircraft wing.

Aouf et al. [4] studied the application of optimal and robust control methods to flexible aircraft gust load alleviation. Optimal controllers were applied to the nominal model (which excludes the plant uncertainties) of the flexible B-52 aircraft. The controllers were designed to reduce the transient peak value of structural loadings caused by the Dryden gust spectrum using both ailerons and horizontal canards. However, it was shown that when plant uncertainties were considered, the designed controllers failed to satisfy the robust performance criterion (i.e. the structured singular value ($\mu$) < unity). On the other hand, a controller designed based on the $\mu$–synthesis using a $D - K$ iteration procedure was shown to be capable of successfully providing the robust performance.

Obtaining reliable aeroservoelastic models has always been a challenge for aeroelasticians, especially when the rational approximation of the unsteady aerodynamic loads has to be considered. Silva et al. [86] performed experimental analysis on the SensorCraft wind-tunnel model at NASA Langley transonic dynamics tunnel. The obtained unsteady measurements were used to extract the impulse responses of the aircraft to control inputs and gust excitations, which itself was transformed into time-domain state-space matrices. The obtained model was used to develop a simple gust load alleviation controller using the generalized predictive control method. Simulated open- and closed-loop results indicated that the developed controller had significantly lowered the gust response.

Moulin et al. [61] investigated the effectiveness of different control surfaces for large transport aircraft gust load alleviation. A baseline model with aileron control was compared with models equipped with under wing and wing-tip control surfaces. A simple low-pass controller was designed and the control parameters were tuned for each case. The three models were excited with a $(1 - \cos)$ discrete gust profile. It was shown that the new proposed surfaces were more effective in relaxing the wing structure. More specifically, ailerons lose their effectiveness at higher speeds, whereas the other two control surfaces become more effective.
The application of adaptive feed-forward controllers for load alleviation purposes has also been reported by some researchers. Zeng et al. \cite{102} proposed an adaptive feed-forward controller for the suppression of aircraft structural vibrations induced by gusts. Wildschek et al. \cite{97, 98} combined an $H_\infty$ based inner feedback controller with an adaptive feed-forward controller. The system was tuned to improve the ride comfort and to reduce the wing fatigue using ailerons, elevators, and direct lift control flaps \cite{98}. However, the main challenge for a feed-forward controller is the availability of a reliable gust induced angle of attack measurement.

Recently, Dillsaver et al. \cite{18} designed a gust load alleviation controller for the highly flexible flying wing X-HALE \cite{11}. A linear quadratic Gaussian (LQG) controller was designed and applied to the linear reduced-order model representing the aircraft dynamics. The controller was shown to reduce the peak wing curvatures by an average of 47\% and RMS curvatures by an average of 83.7\%.

The traditional aircraft control design philosophy is based on separating the rigid-body control from the structural-control and has been successfully used by control designers in conventional aircraft. In a highly flexible aircraft control, the rigid-body control, if performed separately from the structural control, can induce structural oscillations that makes the structural control and relaxation more challenging. Therefore, this approach is not effective when there is significant coupling between the rigid and deformable dynamics in highly flexible aircraft. Therefore, an integrated control system has to be designed in order to execute time-dependent manoeuvres, perform structural load alleviation, and suppress catastrophic aeroelastic phenomena such as flutter.

1.2 Thesis Contributions

In the present work, an MDO framework is developed that integrates control system design with aerostructural design of a highly-deformable aircraft at the conceptual design level considering time dependent manoeuvres and gust excitations. The main goal of this work is to exploit the synergies between the design of the control system, aerodynamics, and structures. The objective of this framework is to surpass the aircraft endurance limits through the use of
an active load alleviation system that is designed concurrently with the aerodynamic shape and structural sizing. The novelty of this framework is two fold: first, a unified dynamics framework is developed to represent the full six-degree-of-freedom rigid-body along with the structural dynamics. The derived nonlinear equations allow for an integrated control design to account for both manoeuvrability (which is related to flying qualities) and aeroelasticity criteria simultaneously. Second, by synthesizing the aircraft control system along with the structural sizing and aerodynamic shape design, the final design has the potential to exploit synergies between the three disciplines and yield a higher performing aircraft.

Due to the high flexibility of aircraft structure, the aircraft wing deforms beyond the range of validity of linear finite element methods. Therefore, a co-rotational structural framework featuring Euler–Bernoulli beam elements is developed to capture the wing nonlinear deformations under the effect of aerodynamic and inertial loadings. In this work, a three-dimensional panel code, capable of calculating both steady and unsteady loadings is used. The lifting surfaces are discretized using source and doublet panels and the wake elements are represented by doublet panels. In order to reduce the computational cost associated with the unsteady aerodynamic calculations, the eigensystem realization algorithm (ERA) is used to construct reduced-order models (ROMs), which can reduce the computational time dramatically.

The more flexible an aircraft is, the stronger the potential for interaction between the rigid-body and the structural dynamics becomes. This interaction, which is caused by the aerodynamic forces and the inertial forces, can adversely affect the aircraft dynamics and destabilize the aircraft. As a result, the aeroelastic analysis and the control design should be based on the actual trimmed shape of the aircraft. As shown by Haghighat et al. [30], the design of a control system based on the undeformed linearized model leads to instabilities. In this work, a comparison is presented that reveals the detrimental effect of the interaction between the two dynamics that cannot be captured by using the simple models that are usually used in previous work [56, 4, 61, 102]. The results of this comparison reveals the significance of the inertial interaction in the flexible aircraft modelling.

Two different control methods are considered in this work: a model predictive controller (MPC) and a 2-degree-of-freedom mixed-norm robust controller. Both control techniques offer
unique advantages that make them appealing for controlling a highly flexible aircraft. The MPC technique offers a systematic approach to handle input and state constraints directly and can be effectively used to control nonlinear plants. An MPC framework is proposed to address the gust load alleviation for highly flexible aircraft. Model predictive controllers are capable of handling model nonlinearities and constraints on states and inputs. Predictive controllers are generally sensitive to the accuracy of the future output prediction and the performance can be adversely affected by inaccurate predictions. In the current work, a technique is proposed to increase the prediction accuracy of the MPC, which improves the overall controller performance. On the other hand, the multi-objective robust control methodology uses frequency-domain weighting functions to bound the inputs and outputs and provides a direct approach to address uncertainties due to unmodelled dynamics, parameter uncertainties, and model nonlinearities. The control system was developed to execute time-dependent manoeuvres along with performing gust/manoeuvre load alleviation.

The developed framework is used to design a HALE aircraft wing and its control system at multiple manoeuvring conditions. Two different design scenarios are compared in this work: one for which the control system simply worked towards achieving maneuverability, and another where the control system was also performing load alleviation and structural control.
Chapter 2

The Nonlinear Equations for Motion of a Highly Flexible Aircraft

Early efforts in aeroservoelasticity have derived the flexible aircraft equations of motion by simply adding a series of second-order differential equations, where each set of equations represents one structural mode \[56, 4\]. Such an approach, may work for a mildly flexible aircraft, but cannot capture the correct interaction between the rigid-body and the structural dynamics. Therefore, this type of formulation should only be used where the frequency gap between the the fastest rigid-body dynamics and the first structural mode is considerably large to avoid any significant interaction between the two dynamics \[20\].

More detailed mathematical formulations of flexible aircraft flight dynamics have been developed in recent years. Schmidt et al. \[96, 80\] have developed the equations of motion for a flexible aircraft using the mean-axes method. Several assumptions were made to develop the formulation. The application of small structural deflections is the main drawback of that formulation, which makes it less suitable for modelling a highly flexible aircraft. The development of more rigorous formulations using a body-fixed axes system have also been reported \[58, 66, 81, 5, 94\].

The development of a mathematical model that can be used to integrate flight dynamics with other disciplines, such as structures and aerodynamics is the key to developing a framework for
design optimization of a highly flexible aircraft. In this work, the nonlinear equations of motion of a highly flexible aircraft are derived using a body-fixed axes system. Since the coordinate system that is used is not an inertial coordinate system, the original form of the Lagrange’s equations cannot be used. Hence, the derivation begins with the Lagrange’s equations for quasi-coordinates \[59\],

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{V}_c} \right) + \omega \frac{\partial L}{\partial \omega} - C_{bi} \frac{\partial L}{\partial \dot{R}} = F, \tag{2.1}
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \omega} \right) + \dot{V}_c \frac{\partial L}{\partial \dot{V}_c} + \omega \frac{\partial L}{\partial \omega} - D^{-T} \frac{\partial L}{\partial \theta} = M, \tag{2.2}
\]

\[
\frac{d}{dt} \left( \frac{\partial \hat{L}}{\partial \dot{v}} \right) - \frac{\partial \hat{T}}{\partial \dot{d}} + \frac{\partial \hat{F}}{\partial \dot{v}} + \hat{L} d = \hat{U}, \tag{2.3}
\]

where \(L\) and \(T\) are the system’s Lagrangian and kinetic energy, respectively. \(F\) represents the Rayleigh’s dissipation term, \(U\) represents the distributed forces acting on the aircraft structure, \((\cdot)\) indicates the volume density of the energy terms and \((\tilde{\cdot})\) is the cross-product operator representing a skew-symmetric matrix with the following form,

\[
\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T \Rightarrow \tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{2.4}
\]

The two transformation matrices, \(C_{bi}\) and \(D\), transform the inertial translational velocities and the inertial rotational velocities into the corresponding velocity vectors in the body-fixed coordinate system, respectively. The derivation of this modified Lagrange’s equations can be found in the related literature \[57\]. For the reader’s convenience a short derivation is presented in
Figure 2.1 shows a deformable aircraft with its original and deformed wing, as well as the associated axes systems. $F_I$ represents the inertial frame and $F_B$ the body-fixed frame, not necessarily connected to the centre of mass. The absolute position and velocity of an infinitesimal mass element $dm$ on the aircraft is given by,

$$R = R_c + r + d,$$

$$V = \dot{R} = V_c + \omega \times (r + d) + v.$$ (2.5)

The location vector of the body-axis $R_c$, the mass element rigid distribution vector $r$, and the displacement of the mass element $d$ are shown in Figure 2.1. The deformational velocity of the mass element is represented by $v$. The structural deformations can be expressed in a discrete finite-element formulation (nodal deformations) or as a linear combination of a complete set of mode shapes (basis functions). The required characteristics of these mode shapes can be found in the related literature [34]. A common choice for these mode shapes are the eigenvectors of the structural system. In this work, using the modal approximation method, the structural displacement and velocity vectors are expressed as follows,

$$d = \phi_r \xi \Rightarrow v = \phi_r \eta,$$ (2.6)

where $\xi$ is a vector of modal amplitudes, $\eta$ is the rate of change of the modal amplitudes, and $\phi_r$ represents the modal deflection at location $r$, which has the following form,

$$\phi_r = \begin{bmatrix} x_{r,1} & x_{r,2} & \cdots & x_{r,n} \\ y_{r,1} & y_{r,2} & \cdots & y_{r,n} \\ z_{r,1} & z_{r,2} & \cdots & z_{r,n} \end{bmatrix},$$ (2.7)

where $x_{r,i}$ is the deformation at location $r$ in the $x$ direction due to the $i^{th}$ structural mode. Although the application of the modal representation in dynamic response and stability analysis is widely practiced, static aeroelastic analysis is usually based on discrete finite-element methods. This is mainly due to the effect of concentrated forces, such as the weight of external stores, which the modal representation does not capture adequately, unless the structural
modes are calculated using large fictitious masses at the locations of the lumped forces [42]. In this work, the modal representation is used for both static and dynamic analysis. Note that the adoption of modal representation for structural deformations does not impose any restriction on modelling large nonlinear structural deformations. The details of the structural formulation are reported in the next chapter. Here, the gravitational force is modelled as a distributed force and therefore, the following terms in Equations (2.1) and (2.2) vanish, i.e.,

\[
\frac{\partial L}{\partial R} = 0, \quad \frac{\partial L}{\partial \theta} = 0.
\] (2.8)

By using the structural modal approximation, Equation (2.3) can be re-arranged and adapted to the modal form as follows,

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \eta} \right) - \frac{\partial T}{\partial \xi} + \frac{\partial U}{\partial \xi} + \frac{\partial F}{\partial \eta} = f_e,
\] (2.9)

where \( U \) and \( F \) represent the strain energy and Rayleigh's dissipation for the complete aircraft structure. Also, \( f_e \) represents the modal forces that act on the aircraft structure (generalized forces).

### 2.1 Kinetic Energy

The kinetic energy of a deformable aircraft can be formulated as follows:

\[
T = \frac{1}{2} \int_v \rho \mathbf{V}^T \mathbf{V} dv \\
= \frac{1}{2} m \mathbf{V}_c^T \mathbf{V}_c + \mathbf{V}_c^T \left[ \int_v \rho \left( \tilde{r} + \tilde{\phi} r \xi \right) d v \right] \omega + \mathbf{V}_c^T \left[ \int_v \rho \phi r dv \right] \eta \\
+ \omega^T \left[ \int_v \rho \left( \tilde{r} + \tilde{\phi} r \xi \right) \phi r dv \right] \eta + \frac{1}{2} \omega^T \left[ \int_v \rho \left( \tilde{r} + \tilde{\phi} r \xi \right) \left( \tilde{r} + \tilde{\phi} r \xi \right)^T dv \right] \omega \\
+ \frac{1}{2} \eta^T \left[ \int_v \rho \phi r^T \phi r dv \right] \eta.
\] (2.10)

If the origin of \( \mathbf{F}_B \) is placed at the undeformed aircraft center of gravity, the term \( \int_v \rho \tilde{r}^T dv \) is zero.
2.2 Potential Energy

The strain energy of an elastic system can be written as,

\[ U = \frac{1}{2} U^T K_{GG} U = \frac{1}{2} \xi^T \Phi^T K_{GG} \Phi \xi = \frac{1}{2} \xi^T K_{ee} \xi, \quad (2.11) \]

where \( \Phi \) is the matrix of the structural modes for all DOFs on the aircraft structure and has the following form,

\[ \Phi^T = \begin{bmatrix} \phi_{r_1}^T & \phi_{r_2}^T & \ldots & \phi_{r_m}^T \end{bmatrix}^T. \quad (2.12) \]

2.3 Nonlinear Equations of Motion

In order to derive the nonlinear equations of motion the partial derivatives \( \partial T / \partial V_c \), \( \partial T / \partial \omega \), \( \partial T / \partial \xi \), and \( \partial T / \partial \eta \) have to be derived first. These partial derivatives are presented here:

\[ \frac{\partial T}{\partial V_c} = m V_c + \left( \int_v \rho \dot{\phi}_r \xi dv \right)^T \omega + \left( \int_v \rho \phi_r dv \right) \eta, \quad (2.13) \]

\[ \frac{\partial T}{\partial \omega} = \left[ \int_v \rho \phi_r \xi dv \right] V_c + J \omega + \left( \int_v \rho \dot{\phi}_r \phi_r dv \right) + \left( \int_v \rho \dot{\phi}_r \xi \phi_r dv \right) \eta, \quad (2.14) \]

\[ \frac{\partial T}{\partial \xi} = \left[ \int_v \rho \phi_r \phi_r dv \right]^T \dot{\omega} V_c - \left[ \int_v \rho \phi_r \phi_r \dot{r} dv \right] \omega - \left[ \int_v \rho \phi_r \phi_r \dot{r} \phi_r dv \right] \xi - \left[ \int_v \rho \phi_r \phi_r \phi_r dv \right] \eta, \quad (2.15) \]

\[ \frac{\partial T}{\partial \eta} = \left[ \int_v \rho \phi_r \phi_r dv \right] V_c + \left[ \int_v \rho \phi_r \phi_r \dot{r} dv + \int_v \rho \phi_r \phi_r \xi dv \right] \omega + M_{ee} \eta. \quad (2.16) \]

Calculating the time derivative of the above terms and substituting the kinetic and potential energy partials and their time derivatives into Equations (2.1, 2.2 and 2.9) and rearranging
them, the equations of motion of a deformable aircraft become

\[
\begin{bmatrix}
    m & X_1^T & S_1 \\
    X_1 & J & S_2 + X_2 \\
    S_1^T & S_2^T + X_2^T & M_{ee}
\end{bmatrix}
\begin{bmatrix}
    \dot{V}_c \\
    \dot{\omega} \\
    \dot{\eta}
\end{bmatrix} =
\begin{bmatrix}
    m\ddot{\omega} & \tilde{\omega}X_1^T & 2\ddot{\omega}S_1 \\
    \tilde{\omega}X_1 & \tilde{\omega}J + \tilde{V}_c X_1^T & \ddot{\omega}(S_2 + X_2) \\
    -S_1^T\ddot{\omega} & \int_v \rho \phi_T^T \ddot{\omega}Tdv & 2\int_v \rho \phi_T^T \ddot{\omega}\phi_Tdv + C_{ee}
\end{bmatrix}
\begin{bmatrix}
    V_c \\
    \omega \\
    \eta
\end{bmatrix}
\]

\[
\begin{bmatrix}
    O_{3\times 3} & O_{3\times 3} & O_{3\times n} \\
    O_{3\times 3} & O_{3\times 3} & O_{3\times n} \\
    O_{n\times 3} & O_{n\times 3} & \int_v \rho \phi_T^T \ddot{\omega}\phi_Tdv + K_{ee}
\end{bmatrix}
\begin{bmatrix}
    R_c \\
    \theta \\
    \xi
\end{bmatrix} +
\begin{bmatrix}
    mC_{bi} \\
    X_1C_{bi} \\
    S_1^T C_{bi}
\end{bmatrix}
\begin{bmatrix}
    g \\
    \{\text{f}e\}
\end{bmatrix}
\]

\[
(2.17)
\]

where, \( \text{F} \) and \( \text{M} \) are the total forces and moments about the origin of the body-frame, including propulsive and aerodynamic forces. The \( C_{ee} \) matrix represents the modal damping of the aircraft structure, \( \text{g} \) is the vector of gravitational acceleration, and \( \{\text{f}e\} \) represents the generalized forces due to elastic deformations and is calculated as,

\[
\{\text{f}e\} = \int_s \phi_T^T p\hat{n}ds,
\]

\[
(2.18)
\]

where \( p \) is the pressure acting on the surface of the aircraft and \( \hat{n} \) is the surface normal vector. Also, \( S_1, S_2, X_1 \) and \( X_2 \) are the rigid-elastic interaction terms and are defined as follows,

\[
\begin{align*}
S_1 &= \int_v \rho \phi_T dv, \\
S_2 &= \int_v \rho \tilde{\phi}_T dv, \\
X_1 &= \int_v \rho \tilde{\phi}_T \xi dv, \\
X_2 &= \int_v \rho \tilde{\phi}_T \phi_T dv,
\end{align*}
\]

\[
(2.19)
\]

The integrals in Equation (2.19) are all volumetric integrals whereas the integral in Equation (2.18) is a surface integral. As can be seen, the terms \( S_1 \) and \( S_2 \) are constant and they
depend on the selected structural modes and the rigid-body mass distribution. On the other hand, $X_1$ and $X_2$ are time-dependent and vary as the aircraft structure deforms. Therefore, the inertia matrix of a highly flexible aircraft is time-varying and has to be updated at each time instant. As aircraft become more flexible, the relative magnitudes of the terms $X_1$ and $X_2$ with respect to the other terms in Equation (2.17) become larger, which results in a stronger interaction between the two types of dynamics. In addition to the above terms, the term $\int_v \rho \phi_T \tilde{\omega}^2 \phi_r dv$ that is added to the modal stiffness matrix in the nonlinear equations is the geometric stiffening that accounts for the coupling between the transverse and the axial deformations and the rigid-body rotational velocity. It is worth mentioning that the derived equations of motion are independent of the structural representation, so they can be used with the designer’s preferred tools.

2.4 Simplified Equations of Motion

When the aircraft structure is not highly flexible, the cross product term $\omega \times \mathbf{d}$ compared to other terms present in Equation (2.5) can be ignored. Dropping this term simplifies the equations of motion significantly. The equations of motion for a slightly flexible aircraft are presented here:

\[
\begin{bmatrix}
  mL_{3x3} & O_{3x3} & S_1 \\
  O_{3x3} & J & S_2 \\
  S_1^T & S_2^T & M_{ee}
\end{bmatrix}
\begin{bmatrix}
  \dot{V}_c \\
  \dot{\omega} \\
  \dot{\eta}
\end{bmatrix}
= -\begin{bmatrix}
  m\ddot{\omega} & O_{3x3} & \tilde{\omega}S_1 \\
  O_{3x3} & \tilde{\omega}J & \tilde{V}_c + \tilde{\omega}S_2 \\
  O_{n\times3} & O_{n\times3} & C_{ee}
\end{bmatrix}
\begin{bmatrix}
  V_c \\
  \omega \\
  \eta
\end{bmatrix}
- \begin{bmatrix}
  O_{3x3} & O_{3x3} & O_{3xn} \\
  O_{3x3} & O_{3x3} & O_{3xn} \\
  O_{n\times3} & O_{n\times3} & K_{ee}
\end{bmatrix}
\begin{bmatrix}
  R_c \\
  \theta \\
  \xi
\end{bmatrix}
+ \begin{bmatrix}
  mC_{bi} \\
  X_1C_{bi} \\
  S_{T}C_{bi}
\end{bmatrix}
\begin{bmatrix}
  g \\
  \mathbf{M} \\
  \mathbf{f}_e
\end{bmatrix} \tag{2.20}
\]

The above equations are significantly simpler than the complete form of equations (Equation 2.17). Unlike the complete form of the equations, the inertia matrix of the simplified equations is a constant matrix. In addition to that, terms such as $\tilde{J}$, $2\int_v \rho \phi_T \tilde{\omega}^2 \phi_r dv$, and $\int_v \rho \phi_T \tilde{\omega}^2 \phi_r dv$ (geometric stiffening) are not present in the simplified equations. For a high-
altitude long-endurance unmanned aircraft, such as Helios, that does not have a fuselage and for which the aircraft wing deforms significantly, the variation in the center of gravity and the moment of inertia cannot be ignored and the original form of Equation (2.17) must be used. However, for a conventional transport aircraft that has its mass concentrated in the fuselage and does not experience high rotational velocities, the simplified equations (Equation (2.20)) would suffice. As the design optimization of a highly flexible aircraft is of interest in this work, the complete form of the equations of motion (Equation 2.17) are used throughout the rest of this research.

2.5 Summary

In this chapter, the equations of motion for a highly flexible aircraft were derived. The derivation started from the quasi-coordinate Lagrange’s equations. The developed equations clearly revealed the coupling between the rigid-body and structural dynamics. In addition to the general form of the equations, a simplified form of the equations of motion which can be used for mildly flexible aircraft was presented. In the next chapter, the aerodynamic model and the structural representation are presented and then the iterative algorithm used to solve the aerostructural problem is explained.
Chapter 3

Aerostructural modelling

Over the past few decades, the aeroservoelasticity has been extensively researched and researchers have used a wide range aerodynamic and structural representations to perform aerostructural modelling. The fidelity of the aerodynamic model depends on basic flow parameters, such as Mach number or Reynolds number. A wide variety of aerodynamic models ranging from the lifting line theory to nonlinear CFD modelling have been considered by researchers. The structural representation used to design aeroservoelastic systems are commonly based on beam elements. Nonlinear beam models capable of handling geometric nonlinearities can be employed when structure experiences very large deformations \cite{33, 90}. In this chapter, the aerodynamic model and the structural representation are first presented and then the iterative algorithm used to find the steady trim condition is explained and sample results are shown.

3.1 Aerodynamic modelling

In this work a steady aerodynamic representation is used to perform steady trim calculations that is required to evaluate the performance of the aircraft in cruising flight. In addition to steady aerodynamics, an unsteady (or quasi-steady) representation of forces and moments are needed when time-dependent manoeuvres or flying through gusts are considered. A threedimensional panel code capable of handling both steady and unsteady flight conditions is used to obtain aerodynamic loads. The fluid surrounding the body is assumed to be inviscid, irro-
tational, and incompressible over the entire flow field. As a result, the flow solution can be represented by a velocity potential that satisfies the Laplace equation.

\[ \nabla^2 \phi = 0 \] (3.1)

Two boundary conditions are used in the procedure of finding the solution of this equation. The first boundary condition requires zero normal velocity across the body’s solid surface.

\[ (\nabla \phi + \mathbf{V}_s) \cdot \hat{n} = 0 \] (3.2)

where \( \mathbf{V}_s \) is the surface velocity which is represented as,

\[ \mathbf{V}_s = - (\mathbf{V}_c + \omega \times (\mathbf{r} + \Phi \xi) + \Phi \eta) \] (3.3)

The vector \( \hat{n} \) is a unit vector normal to the surface of the body. The second boundary condition states that the flow disturbances should diminish far from the body which is represented as follows,

\[ \lim_{\| \mathbf{R} - \mathbf{R}_0 \| \to \infty} \nabla \phi = 0. \] (3.4)

Using the superposition principle, the general solution to the Laplace equation (3.1) can be constructed by summing the elementary flow solutions of sources (\( \sigma_i \)) and doublets (\( \mu_i \)) over the body surface and wakes. Therefore, the lifting surfaces are discretized using quadrilateral source and doublet panels and the wake elements are represented by quadrilateral doublet panels, as shown in Figure 3.1. Equation (3.5) expresses the flow potential of point \((x, y, z)\) as a function of source and doublet distribution.

\[ \phi(x, y, z) = \frac{1}{4\pi} \sum_{\text{body+wake}} \mu_i \hat{n}_i \cdot \nabla \left( \frac{1}{r} \right) \Delta A_i - \frac{1}{4\pi} \sum_{\text{body}} \sigma_i \left( \frac{1}{r} \right) \Delta A_i \] (3.5)

This solution automatically satisfies the far-field boundary condition as the disturbance induced by any elementary solution vanishes at far distances. The zero normal flow boundary condition can be satisfied by differentiating the above equation with respect to the body coordinates. When the body surface is discretized, the zero normal velocity condition is satisfied at collocation points that are located at the centre of each quadrilateral panel. Numerous combinations of sources and doublets can be found to satisfy the zero normal velocity boundary
Chapter 3. Aerostructural modelling

One common approach to overcome this problem is to select the value of the source elements equal to the normal local kinematic velocity, i.e.,

\[ \sigma_i = -\hat{n}_i \cdot (V_c + \omega \times (r + \Phi \xi) + \Phi \eta) \]  

(3.6)

The wake panels are force-free and have to satisfy the Kutta–Joukowsi theorem,

\[ \mathbf{V} \times \mu_w = 0 \]  

(3.7)

which can be interpreted as forcing the total velocity to be parallel to the wake circulation vector. Using the above equations, the doublets vorticities can be found from the following set of equations,

\[ \sum_{k=1}^{N_{\text{panel}}} D_{ik} \mu_k + \sum_{l=1}^{N_{\text{wake}}} D_{il} \mu_{wl} + \sum_{k=1}^{N_{\text{panel}}} S_{ik} \sigma_k = -V_s \cdot \hat{n}_i ; \quad i : 1 \rightarrow N_{\text{panel}} \]  

(3.8)

where \(D_{ik}\) and \(S_{ik}\) are induced velocity coefficients representing the induced velocity at collocation point \(i\) by panel \(k\) for doublets and sources, respectively. The derivation of these coefficients for quadrilateral panels can be found in related references [43]. The Kutta condition requires that the vorticity at the trailing edge to remain zero. In order to satisfy the Kutta condition,
the vorticity of a wake panel is related to the vorticity of the trailing edge panels as follows,

$$\mu_w = \mu_{\text{upper}} - \mu_{\text{lower}}$$  \hspace{0.5cm} (3.9)

Using the above equations, the equations can be represented as a set of linear equations in \( \mu_i \)'s and is used to find the panel vorticities. The obtained doublet vorticities and source strengths, are then used to compute the induced velocity and the local pressure coefficients at each collocation point using the following relations.

$$V_k = V_{sk} + V_{\text{ind}_k}$$

$$C_{pk} = 1 - \frac{V_k^2}{V_{\infty}^2}$$  \hspace{0.5cm} (3.10)

Using the pressure at each panel, the rigid-body forces and moments and elastic generalized aerodynamic forces can be computed as follows,

$$F_{\text{aero}} = \sum_j p_j \hat{n}_j \Delta A_j,$$

$$M_{\text{aero}} = \sum_j \hat{r} (p_j \hat{n}_j) \Delta A_j,$$

$$f_{e_i} = \sum_j \phi_{i|j} p_j \hat{n}_j \Delta A_j.$$  \hspace{0.5cm} (3.11)

### Compressibility Correction

The above formulation can successfully represent an incompressible flow around any configuration where there is no separation. However, the range of validity of the formulation can be extended by introducing compressibility correction factors. The Prandtl–Glauert rule is the first correction factor that was first introduced in 1922 [38]. Since then, more accurate correction factors have been proposed. In this work, the Karman–Tsien compressibility correction factor is used to take into account the flow compressibility [38].

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_{\infty}^2} + \frac{M_{\infty}^2}{1 + \sqrt{1 - M_{\infty}^2}} \left( \frac{C_{p0}}{2} \right)}$$  \hspace{0.5cm} (3.12)

In the above equation, \( C_{p0} \) represents the incompressible pressure coefficient, \( M_{\infty} \) is the free stream Mach number, and \( C_p \) is the pressure coefficient of the compressible flow. Unlike the Prandtl–Glauert rule, which is based on the linear theory, the Karman–Tsien transformation
is a nonlinear correction factor to find the pressure coefficient of a compressible, inviscid flow. Note that this correction factor tends to slightly overestimate the magnitude of the fluid’s pressure. More detailed information can be found in related literature [39].

Validation

In order to validate the results of the developed panel method, ONERA M6 wing is analyzed at different mach numbers and different angles of attack and the obtained results are compared with the results of a CFD code. The CFD solver used to generate these results is SUmb [95], a structured multi-block flow solver developed at Stanford University under the sponsorship of the Department of Energy. SUmb uses a second order finite volume discretization and the equations are solved using a preconditioned matrix-free Newton–Krylov approach. The obtained results (Figures 3.2 and 3.3) show a very close match between the panel method results and the CFD results.
3.1.1 Unsteady Aerodynamics

When unsteady flight conditions are of interest, some modifications have to be made to the above procedure. However, the basics of the algorithm remain unchanged. Unlike the steady aerodynamic calculation, the doublet vorticities and source strengths are now time-dependent. The zero normal force boundary condition has to be enforced at each time-step as the aircraft manoeuvres or when the structure undergoes deformation. In addition to that, a new series of wakes are shed at each time-step. It is assumed that once a wake vortex is shed, the vorticity in it stays constant over time. The newest wake panel is bounded at one end by the lifting surface trailing edge and the other end is placed in the interval covered by the trailing edge during the latest time-step. It is usually placed closer to the trailing edge within 0.2–0.3 of the travelled distance [43].

The wake panels are considered to be force-free and therefore they travel according to the local flow velocity. This local flow velocity is the result of the kinematic motion of the aircraft body and the velocity induced by doublet and source panels. The flow velocities at each wake panels corner points are calculated and used to displace the corner points to their new location.
Other than these changes, the other steps mentioned in the steady aerodynamic calculations remain unchanged. The pressure can be computed using the Bernoulli equation,

\[ C_{p_k} = 1 - \frac{V_k^T V_k}{V_{\infty}^T V_{\infty}} - \frac{2}{V_{\infty}^T V_{\infty}} \frac{\partial \varphi}{\partial t}. \] (3.13)

The aerodynamic forces, moments and generalized forces can be computed using Equation (3.11).

3.1.2 Unsteady Aerodynamic for Aircraft Flight Simulation

The efficient and accurate calculation of generalized aerodynamic forces (GAFs) have always posed a major challenge to conducting aeroservoelastic analysis and optimization. Although, the approach explained in the previous section can be used to compute unsteady aerodynamic loads to perform time-domain aircraft simulation and control, the computational burden of this method has made it less attractive to aeroservoelastic practitioners.

Different methods have been used by researchers to efficiently calculate and represent the unsteady aerodynamic forces. The early efforts were mainly based on the application of Theodorsen theory of unsteady 2D flat plate aerodynamics [92]. However, the direct application of Theodorsen theory is only limited to the frequency domain analysis and undamped periodic motion, which makes this approach less applicable to control system design and time-domain simulation.

Finite-state unsteady models are unsteady aerodynamic techniques that offers more practicality in calculating the unsteady forces and moments [81, 89]. Finite state modelling allows one to cast the aerodynamics in the same state-space context as the structural dynamics and controls. This allows the application of advanced time-domain control design techniques to aeroservoelastic design. Peters et al. [69] developed a 2D finite state theory that has been the basis for calculating the unsteady aerodynamic force by many researchers. The theory calculates aerodynamic loads on a thin-airfoil section undergoing large motions in an incompressible inviscid subsonic flow as a function of kinematic parameters, such as \( \alpha \) and \( \dot{\alpha} \), and a set of inflow states. Theoretically, an infinite number of inflow states are required to correctly calculate
airfoil loads. However, the desired accuracy can be obtained by using 4 to 8 inflow states. Note that a finite span correction factor has to be applied to account for accurate spanwise force distribution.

Another traditional approach for constructing linearized GAFs includes performing time-domain aerodynamic calculations (using CFD or a panel method) to obtain individual modal responses to sine wave modal excitations of different aeroelastic modes at different frequencies. In order to obtain the individual modal responses for \( n \) modes at \( r \) frequencies, \( n \times r \) separate code evaluations are required. Each of these evaluations has to be performed over a long time interval for the transient response to damp out.

A more efficient approach involves the time-domain excitation of the aerodynamic system using exponential pulse signals. An exponential pulse signal excites a range of frequencies that can be altered for each individual mode. Since such a signal excites only a specific range of frequencies, a single analysis per mode has to be performed, which is then used to construct a GAF influence coefficient matrix. This time-domain influence coefficient matrix can be transformed into the frequency domain using FFT techniques.

With the advancement of state-space control techniques and the availability of powerful time-domain methods, the frequency-domain GAFs are usually required to be transformed back into the time-domain using rational function approximation (RFA) techniques. In such a case, the time-domain aerodynamic information obtained from the exponential pulse excitation is first transformed into the frequency-domain just to reconstruct the proper time-domain representation ready to be integrated into a state-space control system.

In order to overcome the above challenges, reduced-order models (ROMs) based on the Volterra theory of nonlinear systems have gained significant attention. The Volterra theory was first used to model unsteady aerodynamic systems by Silva [83], and has since been developed further by other researchers [84, 74, 85, 52]. Volterra-based ROMs are based on the creation
of linearized and nonlinear unsteady aerodynamic impulse/step responses that are then used in the Volterra series to provide the linearized and nonlinear responses of the system to arbitrary inputs. Guendel and Cesnik [27] applied this method to the PMARC aerodynamic panel code to perform rapid linear and nonlinear aeroelastic analysis of a high-altitude long-endurance aircraft. The combination of this method with a system identification technique such as eigen-system realization algorithm (ERA) [40] can be used to construct state-space matrices \((A, B, \text{and } C)\) that represent the generalized aerodynamic forces.

In the present work, the computation of pulse responses for each mode of an aeroelastic system is first performed using the developed 3D panel code for which the developed aerodynamic code has to be modified to perform pulse simulations. As reported by Raveh et al. [74], the inclusion of the second order kernel terms does not improve the accuracy of the results significantly. Therefore, based on the findings of Raveh, and the fact that panel code is a linear method at its core, only a first order kernel (the obtained pulse responses) is used in this work, and the Volterra theory is not applied to compute higher-order kernels. The recorded pulse responses, obtained from the excitations of the panel code, are the Markov parameters \((Y_k = CA^{k-1}B)\) that are required by the ERA technique to reconstruct the state matrices. The algorithm is based on forming the \(r \cdot p \times s \cdot m\) generalized Hankel matrix as,

\[
H_{rs}(k - 1) = \begin{bmatrix}
Y_k & Y_{k+t_1} & \cdots & Y_{k+s-1} \\
Y_{k+j_1} & Y_{k+j_1+t_1} & \cdots & Y_{k+j_1+s-1} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{k+j_{r-1}} & Y_{k+j_{r-1}+t_1} & \cdots & Y_{k+j_{r-1}+s-1}
\end{bmatrix}
\]  

(3.14)

where \(j_i (i : 1 \rightarrow r - 1)\) and \(t_i (i : 1 \rightarrow s - 1)\) are arbitrary integers and can be chosen to avoid using corrupted recorded outputs [40]. Using singular value decomposition (SVD), \(H_{rs}(0)\) can be decomposed into the following form,

\[
H_{rs}(0) = U \Sigma V^T,
\]

(3.15)

where \(\Sigma\) is a diagonal matrix containing the singular values of \(H_{rs}(0)\). The size of the reduced order model can be determined using the rank of \(H_{rs}(0)\) (number of singular values, \(\Sigma\), that are
greater than the desired accuracy). According to ERA, a minimum realization of the dynamic system of interest can be constructed using the triple \[ \Sigma^{-1/2} U^T H_{rs} (1) V \Sigma^{-1/2}, \quad \Sigma^{1/2} V E_m, \quad E_p^T U \Sigma^{1/2} \]

where \( E_m^T = \begin{bmatrix} I_m & O_m & \cdots & O_m \end{bmatrix} \) and \( E_p^T = \begin{bmatrix} I_p & O_p & \cdots & O_p \end{bmatrix} \).

As aircraft conceptual design is the main focus of this work and the flutter calculation is beyond the scope of this thesis (reserved for future work), the dynamic gust response analysis is performed using the quasi-steady aerodynamic loads. Fidkowski et al. [22] showed that, for a transport aircraft, gust encounters at typical flight conditions can be reasonably modelled using quasi-steady aerodynamics assumption. The use of quasi-steady aerodynamic loads is expected to reduce the computational cost and speed up the optimization process significantly.

### 3.2 Structural modelling

In aeroelastic modelling, beam elements have traditionally been used to represent the wing structure. Depending on the aspect ratio and other parameters, such as airfoil thickness-to-chord ratio, different beam elements such as Euler–Bernoulli beam, Timoshenko beam [9], and nonlinear geometrically-exact beam [88] representations have been used by researchers. The aerostructural model used in this work is shown in Figure 3.4. The wing spar is modelled as a hollow tubular beam. The beam diameter is determined by the wing cross-section and its thickness is specified by the optimizer. As shown in Figure 3.4, in order to simplify the load and displacement transfer between the structural model and aerodynamic panels, the same span-wise discretization is used for the structural beams and the aerodynamic panels. The recently studied high altitude long endurance (HALE) planes have highly flexible wing structures that deform beyond the range of validity of simple linear finite element representations. The development of a beam element that can efficiently model large displacement has been an area of study for many researchers [33, 90]. The efforts for developing finite element frameworks to capture large deformations fall into three categories: total Lagrangian, updated Lagrangian, and co-rotational formulation.

In nonlinear structural analysis, the forces are applied in an incremental manner where the
Chapter 3. Aerostructural modelling

Figure 3.4: Wing structural representation

position and state of the structure is computed at the end of each increment. The nature of
these increments is depicted in Figure 3.5 where the shown solutions are referred to as accept-
able solutions or incremental solutions. One common feature of all nonlinear finite element
formulations is that the program does not converge from one solution $C_i$ to the next incremen-
tal solution $C_{i+1}$ in one step, as would be the case in a linear analysis. Instead most solution
methods iterate through several intermediate configurations. To verify whether a solution is
an acceptable solution, it must satisfy the governing equations. To verify this, the state of the
structure (forces, displacements, strains, etc) must be expressed with respect to a ‘reference
frame’. The selection of the this reference frame is the source of differences between nonlinear
structural solvers [21]. In the total Lagrangian formulation, the FEM equations are for-
mulated with respect to a fixed reference frame which is not changed throughout the analysis.
Commonly, the frame attached to the initial configuration $C_0$ is used for this purpose. The
updated Lagrangian relies on expressing the FEM equations in the reference frame associated
with the last accepted solution ($C_n$). This reference frame is kept fixed as the solution method
iterates over the intermediate configurations to find the next acceptable configuration $C_{n+1}$.
The co-rotational formulation is the most recent of the the three aforementioned approaches.
This approach is based on decomposing the motion of an element into rigid-body motion and
pure straining deformation. In co-rotational framework two sets of reference frames are em-
employed. One is a fixed or base configuration such as the one used in the total Lagrangian formulation. The other one is a local (co-rotated) frame that continuously translates and rotates with each element, which is used to separate the rigid body motion from the strain producing deformations. While the fixed system is the same for all elements, each element is associated with a local frame (Figure 3.6). This local frame is used as a reference frame to measure the local nodal variables that represent the element’s straining deformations. By choosing the appropriate local coordinate system, the straining deformations are always small relative to the local axes, and simple linear elements can be used to discretize the structure. Furthermore, these straining deformations are related to the local internal forces via element tangent stiffness matrix. In this approach the geometric nonlinearities are captured through the rigid-body motion of the local frame and are shown in form of transformation matrices \[14, 16, 49\]. Two main approaches have been used in developing co-rotational formulations. In the first approach, the co-rotational aspects and the local element behaviour were considered in deriving the formulation. The second approach, which was pioneered by Rankin and Brogan \[73\], is based on separating the local element behaviour from the co-rotational computations. This element-independent formulation has great practical importance as the element library of an existing (linear or materially nonlinear) finite element program can be used without a significant
modifications. The latter methodology is adopted in this work. Among many advantages of an element-independent co-rotational formulation, the following are most relevant to the context of this work; (1) Effective treatment of problems with large-rotation but small strains. This covers important problems in engineering because many structural materials can experience only fairly small strains in service. This approach is well suited to the treatment of structural elements with rotational degrees of freedom such as beams, plates, and shells commonly used in modelling aerospace structures. (2) Takes advantage of existing small-strain FEM libraries. (3) Decouples small-strain material nonlinearities from geometric nonlinearities \cite{21}.

The local coordinate system, \( \mathbf{E} = \left[ \hat{e}_1 \hat{e}_2 \hat{e}_3 \right] \), is shown in Figure 3.6. The first axis, \( \hat{e}_1 \), is placed along the line that connects the first node to the second node, yielding

\[
\hat{e}_1 = \frac{X_2 + d_2 - (X_1 + d_1)}{l_n} \tag{3.16}
\]

Other axes are selected in a manner to form a right handed orthogonal axes system. Different approaches have been presented by researchers to obtain \( \hat{e}_2 \) and \( \hat{e}_3 \). In this work, the construction of these two axes is based on the approach that is reported by Crisfield \cite{15}. The other two axes systems shown in Figure 3.6 are \( \mathbf{T} = \left[ \hat{t}_1 \hat{t}_2 \hat{t}_3 \right] \) and \( \mathbf{U} = \left[ \hat{u}_1 \hat{u}_2 \hat{u}_3 \right] \).
are the nodal coordinates that are connected to the elements end nodes and rotate as the element deforms. The nodal variables are used to define the deformations of each individual beam element. The local nodal variables defining the deformations of a two-node 3D beam are as follows:

\[ \mathbf{p}_l^T = \left\{ u_l \quad \theta_{l1}^T \quad \theta_{l2}^T \right\} \]  

(3.17)

where \( u_l \) represents the element elongation and is defined as:

\[ u_l = \frac{l_n^2 - l_o^2}{l_n + l_o} \]  

(3.18)

\( \theta_{l1} \) and \( \theta_{l2} \) are pseudo-vectors that represent the rotation transformation from the element coordinate, \( E \), to the nodal coordinates \( T \) and \( U \), respectively. The local rotation vectors can be found as follows:

\[ 2 \sin(\theta_{l1,1}) = -\hat{\mathbf{t}}_3^T \hat{\mathbf{e}}_2 + \hat{\mathbf{t}}_2^T \hat{\mathbf{e}}_3 \]  

(3.19)

\[ 2 \sin(\theta_{l1,2}) = -\hat{\mathbf{t}}_2^T \hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2^T \hat{\mathbf{t}}_1 \]  

(3.20)

\[ 2 \sin(\theta_{l1,3}) = -\hat{\mathbf{t}}_3^T \hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_3^T \hat{\mathbf{t}}_1 \]  

(3.21)

\[ 2 \sin(\theta_{l2,1}) = -\hat{\mathbf{u}}_3^T \hat{\mathbf{e}}_2 + \hat{\mathbf{u}}_2^T \hat{\mathbf{e}}_3 \]  

(3.22)

\[ 2 \sin(\theta_{l2,2}) = -\hat{\mathbf{u}}_2^T \hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2^T \hat{\mathbf{u}}_1 \]  

(3.23)

\[ 2 \sin(\theta_{l2,3}) = -\hat{\mathbf{u}}_3^T \hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_3^T \hat{\mathbf{u}}_1 \]  

(3.24)

The global nodal variables that are presented in a vector form are:

\[ \mathbf{p}^T = \left\{ \mathbf{d}_1^T \quad \alpha^T \quad \mathbf{d}_2^T \quad \beta^T \right\} \]  

(3.25)

where \( \alpha^T \) and \( \beta^T \) are pseudo-vectors that represent the rotation transformation from the inertial frame to the nodal coordinates \( T \) and \( U \), respectively. The two vectors \( \mathbf{d}_1 \) and \( \mathbf{d}_2 \) represent the global displacement of the beam nodes measured from the inertial reference frame. To form a local right handed axes system and obtaining \( \hat{\mathbf{e}}_2 \) and \( \hat{\mathbf{e}}_3 \), first an intermediate frame between \( T \) and \( U \) has to be constructed.

\[ R_{av} \approx R \left( \frac{\beta + \alpha}{2} \right) \]

\[ R_{av} = R \left( \frac{\beta - \alpha}{2} \right) T(\alpha) \text{ where } R(\beta - \alpha) = UT^T \]  

(3.26)
Note that pseudo-vectors are not additive, however, $\frac{\beta - \alpha}{2}$ does not become very large and above approximation can be used to obtain an intermediate frame. While $R_{av} = \begin{bmatrix} \hat{r}_1 & \hat{r}_2 & \hat{r}_3 \end{bmatrix}$ is an intermediate frame, the vector $\hat{r}_1$ is not aligned with $\hat{e}_1$. Therefore, the intermediate frame must be rotated on to $\hat{e}_1$. Following the rotation the vectors $\hat{e}_2$ and $\hat{e}_3$ can be constructed as follows:

$$\hat{e}_2 = \hat{r}_2 - \frac{\hat{r}_2^T \hat{e}_1}{2} (\hat{e}_1 + \hat{r}_1) \quad (3.27)$$

$$\hat{e}_3 = \hat{r}_3 - \frac{\hat{r}_3^T \hat{e}_1}{2} (\hat{e}_1 + \hat{r}_1) \quad (3.28)$$

To derive the tangent stiffness matrix, the variation of the local nodal variables have to be related to the global nodal variable variation. The variation of the local nodal variables can be related to the global variables as follows,

$$\delta p_l = F \delta p \quad (3.29)$$

where $F^T = \begin{bmatrix} f_1 & f_2 & \ldots & f_7 \end{bmatrix}$. The derivation of $f_i$s are left to the reader and the final results are only presented in here. The reader is referred to Crisfield [15] for more details.

$$f_1^T = \begin{bmatrix} -\hat{e}_1^T & 0_{3 \times 1} & -\hat{e}_1^T & 0_{3 \times 1} \end{bmatrix} \quad (3.30)$$

$$2 \cos (\theta_{11,1}) f_2 = L (\hat{r}_3) \hat{t}_2 - L (\hat{r}_2) \hat{t}_3 + h_1 \quad (3.31)$$

$$2 \cos (\theta_{11,2}) f_3 = L (\hat{r}_2) \hat{t}_1 + h_2 \quad (3.32)$$

$$2 \cos (\theta_{11,3}) f_4 = L (\hat{r}_3) \hat{t}_1 + h_3 \quad (3.33)$$

$$2 \cos (\theta_{12,1}) f_5 = L (\hat{r}_3) \hat{u}_2 - L (\hat{r}_2) \hat{u}_3 + h_4 \quad (3.34)$$

$$2 \cos (\theta_{12,2}) f_6 = L (\hat{r}_2) \hat{u}_1 - h_5 \quad (3.35)$$

$$2 \cos (\theta_{12,3}) f_7 = L (\hat{r}_3) \hat{u}_1 - h_6 \quad (3.36)$$

While $f_1$ relates $\delta u_l$ to the changes in the global nodal displacements, the remaining rows of matrix $F$ can be obtained by differentiating the Equations (3.19)–(3.24). The matrix $L$ and the vectors $h_i$ can be computed as follows:

$$L^T = \begin{bmatrix} L_1^T & L_2^T & -L_1^T & L_2^T \end{bmatrix} \quad \text{where} \quad \begin{cases} L_1 (\hat{r}_i) = \frac{\hat{r}_i^T \hat{e}_1}{2} A + (1/2) A \hat{r}_i (\hat{e}_1 + \hat{r}_1)^T \\ L_2 (\hat{r}_i) = \frac{\hat{r}_i}{2} - \frac{\hat{r}_i^T \hat{e}_1}{4} \hat{r}_1 + (1/4) \hat{r}_1 (\hat{e}_1 + \hat{r}_1)^T \end{cases} \quad (3.37)$$
\[
\mathbf{h}_1^T = \begin{bmatrix} 0_{3\times1} & (\tilde{t}_3 \hat{\mathbf{e}}_2 + \tilde{t}_2 \hat{\mathbf{e}}_3)^T \end{bmatrix} \begin{bmatrix} 0_{3\times1} & 0_{3\times1} \end{bmatrix}
\]
\[
\mathbf{h}_2^T = \begin{bmatrix} (A\tilde{\mathbf{t}}_2)^T & (\tilde{t}_2 \hat{\mathbf{e}}_1 + \tilde{t}_1 \hat{\mathbf{e}}_2)^T \end{bmatrix} \begin{bmatrix} 0_{3\times1} & 0_{3\times1} \end{bmatrix}
\]
\[
\mathbf{h}_3^T = \begin{bmatrix} (A\tilde{\mathbf{t}}_3)^T & (\tilde{t}_3 \hat{\mathbf{e}}_1 + \tilde{t}_1 \hat{\mathbf{e}}_3)^T \end{bmatrix} \begin{bmatrix} 0_{3\times1} & 0_{3\times1} \end{bmatrix}
\]
\[
\mathbf{h}_4^T = \begin{bmatrix} 0_{3\times1} & 0_{3\times1} & 0_{3\times1} \end{bmatrix} \begin{bmatrix} (\tilde{u}_3 \hat{\mathbf{e}}_2 + \tilde{u}_2 \hat{\mathbf{e}}_3)^T \end{bmatrix}
\]
\[
\mathbf{h}_5^T = \begin{bmatrix} (A\hat{\mathbf{u}}_2)^T & 0_{3\times1} & - (A\hat{\mathbf{u}}_2)^T \end{bmatrix} \begin{bmatrix} (\tilde{u}_2 \hat{\mathbf{e}}_1 + \tilde{u}_1 \hat{\mathbf{e}}_2)^T \end{bmatrix}
\]
\[
\mathbf{h}_6^T = \begin{bmatrix} (A\hat{\mathbf{u}}_3)^T & 0_{3\times1} & - (A\hat{\mathbf{u}}_3)^T \end{bmatrix} \begin{bmatrix} (\tilde{u}_3 \hat{\mathbf{e}}_1 + \tilde{u}_1 \hat{\mathbf{e}}_3)^T \end{bmatrix}
\]

where \( A = \frac{1}{\ln} \begin{bmatrix} I - \hat{\mathbf{e}}_1 \hat{\mathbf{e}}_1^T \end{bmatrix} \).

The virtual work performed by the internal forces in the both local and global frames are the same. Therefore, the global internal forces can be related to the local ones by,

\[
\mathbf{q}_i = F^T \mathbf{q}_{il}
\]

At this stage of the development, the linear element behaviour can be used to relate the local internal forces to the local displacements through the element stiffness matrix, i.e.,

\[
\delta \mathbf{q}_{il} = \mathbf{K}_l \delta \mathbf{p}_l
\]

Differentiating the force equation (3.44) and rearranging it, the tangent stiffness matrix can be found as follows,

\[
\delta \mathbf{q}_i = F^T \delta \mathbf{q}_{il} + \delta F^T \mathbf{q}_{il} = (\mathbf{K}_l + \mathbf{K}_\sigma) \delta \mathbf{p}
\]

where \( \mathbf{K}_l = F^T \mathbf{K}_l F \) is the global material stiffness matrix and \( \mathbf{K}_\sigma \delta \mathbf{p} = \sum_j \mathbf{q}_{il} (j) \delta \mathbf{f}_j \) is the stiffness matrix due to geometric nonlinearities that depend on the local internal force vector.

### 3.2.1 Load control algorithm for co-rotational beam analysis

Due to the nonlinear nature of the problem and the dependency of the stiffness matrix on the internal forces, iterative solution techniques have to be used to find the correct structural deformation under the acting aerodynamic loads. A load control approach in which the acting
load is applied to the structure over a finite number of increments is used in this work. The deformed configuration and the state of the structure (such as internal forces) are calculated at each increment. At each increment the state of the structure has to be acceptable in a residual sense. Therefore, a Newton–Raphson iteration technique is used to obtain equilibrium at each incremental step. The load control algorithm is depicted in Algorithm 1.

### 3.3 Aerostructural trim calculation

In order to calculate the trim condition, the steady-state equations have to be derived from the general form of the equations of motion for a flexible aircraft. The derivation of the equations for rectilinear flight starts from the equations of motion \(2.17\) and all derivatives and \(\omega\) are set to zero, yielding,

\[
F_{\text{aero}} + F_{\text{prop}} + mC_{b\theta}g = 0 \tag{3.47}
\]

\[
M_{\text{aero}} + M_{\text{prop}} + X_1C_{b\theta}g = 0 \tag{3.48}
\]

\[
-K_{ee}(\xi)\xi + (S_1)^T C_{b\theta}g + f_e = 0 \tag{3.49}
\]

The above equations are coupled to each other through aerodynamic forces and moments, and can be used to solve the trim parameters \((C_L, C_D, \alpha, \delta_a)\) and the steady-state wing deformation \((\xi)\). An iterative approach with successive-over-relaxation (SOR) is employed to find the steady-state flight conditions and deformations \([6]\). This iteration is depicted in Figure 3.7. First, the angle of attack and elevator deflection are calculated to trim the aircraft while keeping the wing deformation constant. A Newton-Raphson nonlinear solver is employed to find the roots of Equations \(3.47\) and \(3.48\). The generalized aerodynamic forces \((f_e)\) along with the gravitational forces are then used to calculated the structural deformations using the algorithm shown in Algorithm \([1]\). The calculated wing deformation is used to warp the aerodynamic panels and collocation points. Transferring the exact calculated deflections can destabilize the iterative algorithm or can induce numerical oscillations and therefore, the relaxed structural response has to considered. The optimal value of this relaxation factor, \(\zeta\) depends on the problem of interest and has a significant effect on the stability of the algorithm and the convergence rate. The optimal value of \(\zeta\) at each step can be calculated by performing a line search at every single
iteration which itself requires some additional inner-iterations. In this work, the adaptation of
the relaxation factor is performed using the procedure suggested by Barcelos et al. [6].

\[
\zeta^n = \zeta^{n-1} \left( 1 - \frac{(\Delta \xi^n - \Delta \xi^{n-1})^T \Delta \xi^n}{(\Delta \xi^n - \Delta \xi^{n-1})^T (\Delta \xi^n - \Delta \xi^{n-1})} \right)
\]  
(3.50)

where \(\Delta \xi^n\) is the difference between the actual calculated deformation and previous step de-
formation.

\[
\Delta \xi^n = \xi^n_{\text{actual}} - \xi^{n-1}
\]  
(3.51)

Finally, the relaxed transferable deformation can be found as follows,

\[
\xi^{n-1} = \xi^{n-1} + \zeta^n \Delta \xi^n
\]  
(3.52)

![Figure 3.7: Aerostructural iterative solver](image)

As shown in Figure 3.7, an XDSM (eXtended Design Structure Matrix) diagram is used to
visualize the aerostructural solver process. A numbering system is introduced in the diagram to
define the order in which the software is run. The aerostructural iteration starts at the number
zero and proceeds sequentially. When a component block is entered at number \(k\), the computer
code that corresponds to the component is executed and the process exits at number \(k + 1\) unless the process involves a loop which is denoted by \(k \rightarrow j\). In this diagram the inputs to
components are placed in the same column and outputs are placed in the same row. External
inputs and outputs are placed on the outer edges of the system [46]. The angle of attack (\(\alpha_{\text{trim}}\)
and the aileron deflection angle ($\delta_{\text{trim}}$) are adjusted to trim the aircraft for level flight. Lift and drag coefficients at cruise are used to calculate the aircraft endurance and the wing deflections ($\xi_{\text{trim}}$) are used to evaluate the structural stresses present in the constraints. The outputs of the aerostructural solver are used to linearize the equations of motion needed in the control synthesis.

### 3.4 Summary

In this chapter, the theory used to develop the panel code was first reported. The developed panel code can be used to model both steady and un-steady flows. In order to validate the developed panel code, the ONERA M6 wing was analyzed and the obtained results were compared to CFD results. In order to capture nonlinear structural deformations, a co-rotational framework was developed and an iterative load control algorithm is used to calculated the structural deformations. The developed aerodynamic and structural models were used to develop an aerostructural solver that can be used to solve the steady trimmed shape of the aircraft.
Algorithm 1 Load control algorithm

1: Set $\lambda \leftarrow 1/n_{\text{iter}}$

2: while $F_{n+1} - F \leq \varepsilon 1$ do

3: Set the incremental force vector $\delta F \leftarrow \lambda F$

4: Update the global stiffness matrix ($K$) based on the current values of the local elemental properties

5: Update the global nodal displacement vector: $p_{n+1} \leftarrow p_{n} + \delta p$ where $\delta p = K^{-1}\delta F$

6: Update the global nodal force vector: $F_{n+1} \leftarrow F_{n} + \delta F$

7: Compute the internal global force vector $F_{n+1}^{\text{int}}$ using the local nodal displacement ($p_{n+1}^{\text{local}}$)

8: Calculate the residual norm: $R \leftarrow \sqrt{(F_{n+1}^{\text{int}} - F_{n+1})^T (F_{n+1}^{\text{int}} - F_{n+1})}$

9: while $R \geq \varepsilon$ do

10: Set $k \leftarrow 0$ and $\delta p \leftarrow 0$

11: Compute the new global stiffness matrix ($K$) based on the current values of the local elemental properties

12: Compute the correction to $p_{n+1}$: $\delta p_{k+1}^{\text{update}} \leftarrow \delta p_{k}^{\text{update}} - K^{-1}(F_{n+1}^{\text{int}} - F_{n+1})$

13: Update the element data: $p_{\text{current}} \leftarrow p_{n+1} + \delta p_{k+1}^{\text{update}}$

14: Compute the new residual to account for $p_{\text{current}}$

15: end while

16: Update variables to their final value for the current increment

17: end while
Chapter 4

Control System Design for a Highly Flexible Aircraft

While active control systems have been successfully used to address aeroelastic phenomena for a wide range of research aircraft, the application of these systems to control highly flexible aircraft is not prevalent. Rigid-elastic interaction and high sensitivity of structural deflections to flight parameters pose significant challenges to control design procedures.

Over the past three decades, active control systems have been successfully developed to suppress undesirable aeroelastic phenomena such as flutter and LCO \cite{2}. In addition to flutter suppression, active control systems have also been utilized to perform gust and manoeuvre load alleviation whereby ailerons are mainly used to reduce the stress level in the wing structure by counteracting the induced aerodynamics and inertial forces \cite{4, 61, 18, 100}.

One major comprehensive attempt was reported by Shearer and Cesnik \cite{82}, who developed a heuristic approach based on pilot behaviour that separates the control design problem into a fast inner loop and a slower outer loop. The faster loop commands pitch rate, pitch angle, roll rate, and sideslip angle. The slower loop, on the other hand, controls altitude and rate of climb. At the inner loop level, the aircraft dynamics was separated into a longitudinal and a lateral motion where the lateral dynamics was controlled by a linear quadratic controller and
the longitudinal control was accomplished by the use of a dynamic inversion controller. Finally, a nonlinear PID controller is used to control the outer loop. The control system was designed for a heavyweight configuration and was tested with both heavyweight (including the fuel weight) and lightweight (excluding the fuel weight) configurations to perform a climbing and a turning flight. The developed controller managed to perform the desired time-dependent maneuvering, however, structural control and manoeuvre load alleviation were not addressed.

Raghavan and Patil [71] designed a flight control system to perform trajectory following for a flexible, high aspect-ratio wing. A multi-loop dynamic inversion technique is applied to a 6-degree-of-freedom statically deformed model of aircraft. The elastic deformations were updated at each time step in the simulation model to account for the static deformation and the aerodynamic loadings due to the wing deformation. The developed multi-loop controller combined with a nonlinear guidance law was used to perform trajectory following.

Unlike these approaches, the approach proposed in this work performs the trajectory control and the gust/maneuver load alleviation simultaneously. The main novelty in the proposed control design approach is the unified dynamics framework that is developed for the full 6-DOF rigid-body along with deformation dynamics. This allows for an integrated control design to account for both the rigid-body dynamic performance and aeroelasticity simultaneously, therefore potentially leading to overall improvement in control of a highly flexible aircraft.

Two different control methods are developed: a Model Predictive Controller (MPC) and a 2-DOF mixed-norm robust controller. Both control techniques offer unique advantages that make them promising for controlling a highly flexible aircraft. The MPC method offers a systematic approach to address state and input constraints and due to its nature, that the control input is calculated at each time-step, it can be used to handle model nonlinearities. On the other hand the mixed-norm formulation can be used to address opposing objective functions in a control design problem and is well suited to address model uncertainties. In this chapter both controllers are discussed in details. Because the latter approach is much faster than the MPC
method, it is considered in the next chapter for design optimization.

4.1 Linearized State-Space Representation

In order to obtain a linear plant representation, which is needed in designing a linear control system, the equations of motion are first linearized. For straight rectilinear flight the linearization can be easily performed by substituting the perturbed flight parameters into Equation (2.17) and ignoring the terms of the higher orders. For the more general case, however, the linearization process is not tractable and therefore it has to be performed numerically. The linearized equations of motion for a straight level flight are,

\[
\begin{bmatrix}
\delta \dot{V}_c \\
\delta \dot{\omega} \\
\delta \dot{\eta}
\end{bmatrix} = M^{-1} \begin{bmatrix}
\mathcal{O}_{3\times3} & m \mathcal{V}_c & \mathcal{O}_{3\times n} \\
\mathcal{O}_{3\times3} & X_{10}^T \mathcal{V}_c - \mathcal{V}_c X_{10} & \mathcal{O}_{3\times n} \\
\mathcal{O}_{3\times3} & S^T \mathcal{V}_c & -C_{ee}
\end{bmatrix} \begin{bmatrix}
\delta V_c \\
\delta \omega \\
\delta \eta
\end{bmatrix} + \begin{bmatrix}
\mathcal{O}_{3\times3} & m G & \mathcal{O}_{3\times n} \\
\mathcal{O}_{3\times3} & X_{10} G & \mathcal{O}_{3\times n} \\
\mathcal{O}_{3\times3} & S^T G & -K_{ee}
\end{bmatrix} \begin{bmatrix}
\delta R_c \\
\delta \theta \\
\delta \xi
\end{bmatrix} + \begin{bmatrix}
\delta F \\
\delta M \\
\delta f_e
\end{bmatrix},
\]

where \( M \) and \( G \) matrices are given by

\[
M = \begin{bmatrix}
m & X_{10}^T & S_1 \\
X_{10}^T & J_0 & S_2 + X_{20} \\
S_1^T & S_2^T + X_{20}^T & M_{ee}
\end{bmatrix}, \quad G = \begin{bmatrix}
0 & -g \cos(\theta_0) & 0 \\
-g \cos(\theta_0) & 0 & 0 \\
0 & g \sin(\theta_0) & 0
\end{bmatrix}.
\]

The linearized inertia matrix \( M \) includes off-diagonal terms that indicates the coupling between the two dynamics. Note that, \( S_1 \) and \( S_2 \) are constant and \( X_{10} \) and \( X_{20} \) are shape dependent and have to be computed based on the trimmed shape of the aircraft. These equations are driven by the perturbed aerodynamic forces and moments which can be represented using the obtained ROM for the aerodynamic forces and moments. Elevator and aileron deflections are the two major control inputs used here. The left and the right aileron deflect differentially for roll control, but they can also be commanded independently to perform load alleviation for the left or the right wing. The total control surface deflection for each aileron is calculated by adding these two components. Equation (4.1), when augmented with the obtained aerodynamic
ROM, can be expressed in a state-space form. The state vector and other system matrices are

\[
\begin{aligned}
    x_T &= \begin{bmatrix} V \\ \omega \\ \xi \\ \eta \\ \theta \\ R_c \end{bmatrix}, \\
    \dot{x}_{\text{ac}} &= Ax_{\text{ac}} + Bu + B_\omega \omega \\
    y &= Cx_{\text{ac}}
\end{aligned}
\]

(4.3)

where \( \theta \) is the vector of Euler angles ([\( \phi \ \theta \ \psi \)]^T), \( \eta \) represents structural deflection rates (\( \eta = \dot{\xi} \)), and \( x_{\text{aero}} \) is the state vector form the aerodynamic reduced order model (Section 3.1.2). The \( B_\omega \) matrix is used to model effect of disturbances, such as gusts, on the aircraft dynamics.

### 4.2 Linearized dynamics

As mentioned previously, the more flexible an aircraft is, the stronger the interaction between the rigid-body and the structural dynamics becomes. In this section, in order to reveal the significance of this interaction, the open-loop dynamics (eigenvalues and eigenvectors) of a flexible HALE aircraft are computed for three cases. The geometry of the aircraft is shown in Figure 4.1 and the geometric parameters are listed in Table 4.1. More detailed information about this UAV can be found in Shearer and Cesnik [81, 82].

![Figure 4.1: 3D geometry of a generic high aspect ratio UAV](image)

Our unified dynamics framework allows for integrated design and control because it reveals strong inertial interactions, which are especially critical for very flexible aircraft. To illustrate this point, we give a frequency analysis by comparing the frequency characteristics of three models: 1) a statically deformed aircraft model (deformed model); 2) a statically deformed
Table 4.1: Aircraft geometric, structural, and Inertial properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuselage length</td>
<td>26.4 m</td>
</tr>
<tr>
<td>Wing span</td>
<td>58.6 m</td>
</tr>
<tr>
<td>Wing area</td>
<td>196 m²</td>
</tr>
<tr>
<td>Wing taper</td>
<td>0.48</td>
</tr>
<tr>
<td>Horizontal tail span</td>
<td>18 m</td>
</tr>
<tr>
<td>Horizontal tail area</td>
<td>53.5 m²</td>
</tr>
<tr>
<td>Horizontal tail taper</td>
<td>0.7</td>
</tr>
<tr>
<td>Vertical tail span</td>
<td>4 m</td>
</tr>
<tr>
<td>Vertical tail area</td>
<td>8.9 m²</td>
</tr>
<tr>
<td>Vertical tail taper</td>
<td>0.81</td>
</tr>
<tr>
<td>Weight</td>
<td>528887 N</td>
</tr>
<tr>
<td>Fuel weight</td>
<td>32000 N</td>
</tr>
<tr>
<td>E.A. position</td>
<td>0.45c</td>
</tr>
<tr>
<td>Spar thickness at root</td>
<td>10 cm</td>
</tr>
<tr>
<td>Spar thickness at tip</td>
<td>1 cm</td>
</tr>
</tbody>
</table>

The aircraft model augmented by a series of elastic modes (augmented model); and 3) a fully developed model under a unified framework (unified model). Figure 4.2 shows the Bode diagrams that represent the transfer functions relating the vertical gust excitation to the load factor.
Figure 4.2 shows that: 1) the statically deformed model and the simplified model have similar DC-gain values whereas the DC-gain value of the proposed model is considerably lower. 2) The short period dynamics of the statically deformed aircraft is captured by two stable real poles at 8.9222 rad/s and 17.1779 rad/s. In the other two models, the first dominant dynamics at 6.1 rad/s is dominated by the first structural mode that has a natural frequency of 4.849 rad/s; this dynamics is not captured by the statically deformed model. 3) At higher frequencies, the gain and the phase diagram for the augmented model show similar behaviour to the rigid-body dynamics captured by the statically deformed model. The two spikes at 139.44 rad/s and 320.49 rad/s correspond to two modes that are heavily dominated by the first and second torsional dynamics. Because of the aerodynamic interaction, the torsional modes can significantly affect the rigid-body dynamics even when the inertial interaction is not considered. In contrast, the unified model is significantly different from the other two models, and at frequencies higher than the short period frequency, the transfer function gain is not attenuated.

As it is expected, the provided comparison clearly demonstrates that the exclusion of the
inertial interaction makes a significant difference to the model accuracy at both low and high frequencies. Therefore, for the successful control of a very flexible aircraft the control design should be based on Eq. (4.1) that captures both the inertial and aerodynamic interactions. In addition, it shows that the linearized equations based on the developed model can capture the interaction between the two dynamics and this interaction is not lost in the linearization process.

4.3 Challenges

In traditional aircraft control design, the rigid-body tracking is performed separately from structural control, and the interaction of the structural and rigid-body dynamics is avoided by the use of notch filters. When aircraft is highly flexible however, there is a strong interaction between the rigid-body dynamics and the structural dynamics that makes the design of a control system challenging. There are major challenges in designing a control system for a highly flexible aircraft; (1) a unified controller needs to address both rigid-body motion and the aircraft flexibility. (2) In addition, the high flexibility of aircraft structure results in a time delay between the control input deflections and the rigid-body motion of the aircraft. This time delay results in a non-minimum phase system that is challenging to control. (3) In a highly flexible aircraft, the high frequency structural modes are very lightly damped. This condition makes the design of a state observer challenging, as the transient response of the filter can adversely affect the closed-loop stability and performance. (4) Finally, as seen in Figure 4.2 the short period dynamics at 6.1 rad/s, is dominated by the first wing bending dynamics. Using the forward flying speed and the characteristics length, the short period frequency corresponds to a non-dimensional reduced frequency of 0.145729 which indicates the presence of unsteady aerodynamic behaviour. Therefore, reduced-order models capable of capturing unsteady behaviours must be used for control design and simulation purposes. The first and second above-mentioned challenges are addressed in the next section. However, the selection of appropriate sensors and the design of an observer filter is not addressed in the present work. As a result, the availability of full state information for feedback control is
assumed in this work. The design of an appropriate observer shall be considered for future developments.

4.4 Model Predictive Control

Model predictive control (MPC), also known as receding horizon control (RHC), is a discrete method that that is well known in optimal control [45]. Using this control technique, the control signal is calculated by performing a constrained optimization over a finite control horizon (and in some cases a constraint horizon), denoted by the number of future control steps $N_u$, at each sampling time. MPC is specially advantageous when a plant is nonlinear or time-varying and also when states or inputs have to be constrained [45, 55]. As it was described before, the dynamics of a highly flexible aircraft is highly nonlinear and the inertia and damping matrices are time-varying. Therefore the MPC approach has a significant potential in controlling these type of aircraft.

Due to the high computational cost of MPC, it has traditionally been applied to systems with slow time constants. However, the increased capability of computer hardware and the advent of fast and reliable quadratic programming techniques have made it possible to apply MPC to problems with very fast dynamics, such as path planning and aircraft dynamics [1, 75]. Although MPC provides the control designer with a systematic approach to handle plant nonlinearities and hard state/input constraints, the use of a finite horizon which is required to make the numerical optimization tractable, may destabilize the closed-loop system. The stability of model predictive finite horizon control problems have been extensively studied and techniques to ensure the stability of these systems can be found in the literature [45, 55]. Since MPC is a discrete control method, the discretized state-space representation of the plant has to be used.

In this work, the MPC is applied to provide gust load alleviation for the aircraft wing structure. The desirable performance is achieved by minimizing the following quadratic performance function. Mini-maximizing of quadratic performance function with disturbance effects have also
been considered by other researchers, where linear matrix inequality (LMI) techniques are used to solve $H_\infty$ model predictive control problems \cite{12}.

$$ V(x_k, U, N_u, N_p) = \sum_{i=1}^{N_p-1} \left( (x_{k+i} - x_{ref,k+i})^T Q (x_{k+i} - x_{ref,k+i}) \right) + \sum_{i=0}^{N_u-1} (u_{k+i}^T R u_{k+i}) $$

\hspace{1cm} + \left( x_{k+N_p} - x_{ref,k+N_p} \right)^T Q_f \left( x_{k+N_p} - x_{ref,k+N_p} \right), \quad (4.4)$$

where $U$ is the control signal sequence over the control horizon $N_u$ and $N_p$ represents the prediction horizon which can be longer than the control horizon. Although the generalized inertial and damping matrices in the nonlinear equations of motion are time-varying, the linearized equations is time-independent. In order to address the time-dependent nonlinear nature of the aircraft dynamics, successive linearizations are performed at every time step and the matrices of the obtained linearized system are used to perform future output prediction. The predicted output sequence $X$ can be expressed, in a compact form, as a function of the future input sequence $U$ and the current state $x_0$, i.e.,

$$ X = HU + Gx_k, \quad (4.5)$$

where $HU$ represents the forced output, and $Gx_k$ is the free output due to the initial condition $x_k$. In the absence of all state information, the estimated values of the current state ($\hat{x}_k$) must be used. In this work, however, we assume that the exact values of the current state vector is available for feedback.

$$ H = \begin{bmatrix} B_d \\
A_d B_d & B_d \\
\vdots & \ddots \\
A_d^{N_u-1} B_d & A_d^{N_u-2} B_d & \ldots & B_d \\
A_d^{N_u} B_d & A_d^{N_u-1} B_d & \ldots & A_d B_d \\
\vdots & \ddots & \vdots & \vdots \\
A_d^{N_p-1} B_d & A_d^{N_p-2} B_d & \ldots & \sum_{i=0}^{N_p-N_u} A_d^i B_d \end{bmatrix} \quad , \quad G = \begin{bmatrix} A_d \\
A_d^2 \\
\vdots \\
A_d^{N_p} \end{bmatrix} \quad (4.6)$$

To avoid unrealistic control actions, saturation of control surface deflection and their rates are considered. Therefore, the optimization problem becomes a constrained optimization problem,
i.e.,

$$\min \ V(x_k, U, N_u, N_p)$$ \hspace{1cm} (4.7) \hspace{1cm} \text{w.r.t } U,$$

s.t. \hspace{1cm} $$u_{\text{min}} \leq u_i \leq u_{\text{max}},$$

$$\Delta u_{\text{min}} \leq \Delta u_i \leq \Delta u_{\text{max}}.$$ 

The above constraints can also be expressed in a compact form as $$FU \leq c$$ where the matrix $$F$$ and the vector $$c$$ are formed as follows,

$$F = \begin{bmatrix} I_{N_u \times m} \\ -I_{N_u \times m} \\ M \\ -M \end{bmatrix}, \hspace{1cm} c = \begin{bmatrix} U_{\text{max}} \\ -U_{\text{min}} \\ \Delta U_{\text{max}} + b \\ -\Delta U_{\text{max}} - b \end{bmatrix},$$ \hspace{1cm} \text{with} \hspace{1cm} (4.8)

$$M = \begin{bmatrix} I \\ -I \\ \ddots \\ -I \end{bmatrix} \hspace{1cm} \text{and} \hspace{1cm} b = \begin{bmatrix} u_{k-1} \\ 0 \\ \vdots \end{bmatrix}.$$

Using the provided compact formulation for the future output prediction and the constraints, the performance index and constraints are re-arranged in the following form,

$$\min \ V(\hat{x}_k, U, N_u, N_p) = U^T \left(H^T \hat{Q}H + \hat{R}\right) U + 2 \left(\hat{x}_k^T G^T - X_{\text{ref}}^T\right) \hat{Q}HU,$$ \hspace{1cm} (4.9) \hspace{1cm} \text{w.r.t } U,

s.t. \hspace{1cm} $$F \cdot U \leq c,$$

where $$\hat{Q}, \hat{R}$$ are block diagonal matrices that can be written as

$$\hat{Q} = \text{diag} \begin{bmatrix} Q & Q & \cdots & Q_f \end{bmatrix},$$

$$\hat{R} = \text{diag} \begin{bmatrix} R & R & \cdots & R \end{bmatrix}.$$ \hspace{1cm} (4.10)

The above problem is a quadratic programming (QP) problem. For a positive definite weighting matrix $$Q$$, it can be shown that $$H^T \hat{Q}H$$ is also positive definite and the optimization problem becomes a convex quadratic programming (CQP) problem for which the solution for the future input sequence $$U$$ is unique. These type of problems can be solved efficiently using
fast cone programming algorithms \cite{8}. A schematic representation of the control system is shown in Figure 4.3.

![Figure 4.3: Model predictive control system block diagram](image)

### 4.4.1 Stability of the MP Controller

It is well known that if a system is stabilizable and detectable, then a standard linear quadratic infinite horizon optimal control problem yields an optimal stabilizing controller \cite{10}. However, a linear finite horizon model predictive controller with a quadratic performance index is not asymptotically stabilizing in general and the use of a finite horizon, which is required to make the numerical optimization tractable, may destabilize the closed-loop system. It can be shown that an LQ model predictive controller is stable if and only if the pair \((A, B)\) is stabilizable, the pair \((A, Q^{1/2})\) is observable, and the cost monotonicity condition, \(V(x_k, U^*, N + 1) \leq V(x_k, U^*, N)\), is satisfied.

The cost monotonicity of a constrained MPC is guaranteed if any of the following terminal conditions is applied: (1) terminal equality constraints, (2) terminal set constraints, or (3) terminal penalty function. The first two approaches are very restrictive, especially the terminal equality constraint, which may reduce the feasibility region. Richards and How \cite{76} developed an analytical method for the performance prediction of a constrained model predictive control. In that work, a terminal inequality constraint is used along with a recursive tightening scheme that guarantees the stability of the system in presence of a disturbance. The application of the recursive terminal constraint tightening allows the control designer to chose an initially wide terminal constraint, which improves the feasibility of the design. In the present work, the terminal penalty approach is used. The cost monotonicity condition with the application of
terminal weighting can be guaranteed if the terminal penalty function satisfies the following equation [15]:
\[
Q_f \geq Q + K^T R K + (A_d - B_d K)^T Q_f (A_d - B_d K)
\]
for some \( K \in \mathbb{R}^{m \times n} \).

In this work, the \( K \) matrix is chosen as the gain of an LQR controller with the same \( Q \) and \( R \) weighting matrices.

### 4.4.2 MP Controller with Prediction Enhancement

The performance of a model predictive controller is highly dependent on its ability to predict the future state values. Poor state prediction may result in performance degradation and also system instability. The discrepancies between the predicted values and the actual state values stem from un-modelled dynamics, system nonlinearities, parameter mismatching, and disturbances. The proposed modification here, is based on the addition of a feedback loop to the traditional formulation to improve the accuracy of the predictive controller. The added feedback path is used to update the prediction at each sampling time, based on the difference between the previous step prediction and the actual estimated value of the state vector using the following formulation,
\[
x_{p,k+i} = A_d \hat{x}_k + B_d u_k + L (\hat{x}_k - x_{p,k}), \quad i = 1, \ldots
\]

where \( L \) is a transition matrix that is used to represent the effect of the previous step error on the future output prediction. The above series of equations can be expressed in the following compact form,
\[
X_p = H U + G \hat{x}_k + G_o L e_k,
\]

with \( G_o = \begin{bmatrix} I \\ A_d \\ A_d^2 \\ \vdots \end{bmatrix} \)

Using this equation for predicting the future states, the cost function becomes,
\[
V(x_k, U, N_u, N_p) = U^T \left( H^T \hat{Q} H + \hat{R} \right) U + 2 (\hat{x}_k^T G^T + e_k^T L^T G_o^T - X_{ref}) \hat{Q} H U.
\]
If $L$ is set to zero, then the cost function is the same as the cost function in the traditional formulation. In absence of detailed information about the disturbance and the dynamics that is not modelled, an identity matrix of proper size can be used for $L$. When the variation of the plant matrices from the nominal values are available, a better representative transition matrix can be constructed using the available information. A diagram of this MP controller with prediction enhancement (PE) is shown in Figure 4.4. A similar approach was utilized by Richards and How [77] to improve performance and guaranty controller stability in absence of perfect measurement. In that work the estimation error is used to modify the prediction performance to improve the closed-loop stability and performance. In this work, the availability of perfect full state information is assumed and the proposed approach is used to improve the performance of in presence of modelling errors and model uncertainties.

![Model Predictive Control System Block Diagram](image)

**Figure 4.4: Model predictive control system block diagram**

### 4.4.3 MP Controller Applied to a Non-minimum Phase System

One of the difficulties of controlling a non-minimum phase system is the internal instability problem. This problem arises when a high performance level that is of interest forces the controller to contain an implicit approximate inverse of the system model. When the controller dynamics contains the inverse of a system model with unstable zeros, the cancellation of unstable poles and zeros causes the internal instability. García-Gabin *et al.* [24] showed how the application of a multivariable model predictive controller to a non-minimum phase plant can lead to internal instabilities. It can be shown that if no control and state constraints are considered, the optimal control sequence can be calculated as follows,

$$U = \left( H^T \hat{Q} H + \hat{R} \right)^{-1} H^T \hat{Q} (G\hat{x}_k - X_{ref})$$  \hspace{5cm} (4.15)
If the control effort is not penalized ($R = O$), and the prediction and the control horizon are equal ($H$ is a square matrix) then the optimal control sequence becomes,

$$U^* = H^{-1}(G\dot{x}_k - X_{\text{ref}}) \quad (4.16)$$

Substituting the optimal control sequence into Equation (4.5), there is a cancellation of unstable poles with unstable zeros that causes the internal instability. Penalizing the control effort and also the application of a longer prediction horizon than the control horizon (tall $H$ matrix) have been reported to avoid the internal instability [23]. In this work, control penalization is used to avoid internal instabilities.

### 4.5 Mixed-Norm 2-DOF Robust Controller

In addition to the model predictive controller that was described in the previous section, a mixed-norm 2-DOF controller is also considered for controlling a highly flexible aircraft. The MPC technique offers a systematic approach to handle input and state constraints directly and can be effectively used to control nonlinear plants. However, due to the requirement of solving an optimization problem at each sampling time, MPC may not be applicable to systems with fast dynamics. On the other hand, the multiobjective robust control methodology uses frequency-domain weighting functions to bound the inputs and outputs, and requires a certain level of knowledge about the plant and experience in the selection of the weighting functions for a successful control design. However, multi-objective robust control provides the designers with freedom to use different system norms to design the control system. It also offers a systematic approach to address uncertainties due to un-modelled dynamics, parameter uncertainties, and model nonlinearities. In this section, a short description of the multiobjective robust control technique is provided, and different system norms that are useful in controlling a highly flexible aircraft are explained. Finally, the linear matrix inequalities (LMI) technique is used to design a mixed-norm controller.
4.5.1 Proposed 2-DOF Architecture

Traditional feedback-only structure (1-DOF controller) uses the difference between the desired reference signal and the measured output to calculate the new control command. On the other hand, a 2-DOF controller uses both feedback and feedforward loops to act on the measured output and the desirable reference signal independently. This type of control structure offers a higher degree of flexibility, which is highly advantageous when opposing performance specifications, such as low tracking error and sensor noise rejections are considered simultaneously (multiobjective control design). The proposed 2-DOF control architecture is shown in Figure 4.5.

Figure 4.5: The proposed 2-DOF control architecture

where $\tilde{e}$ is the weighted tracking error signal, $\tilde{s}$ represents the weighted structural deformations and $\tilde{u}$ is the weighted control effort. The weighting function $W_e$ is used to guarantee desirable tracking (regulation) performance with respect to the design specifications that are represented by $W_{ref}$ weighting function. The weighting function $W_u$ is used to avoid excessive actuator displacement and control saturation. Finally, $W_s$ is employed to shape the structural deformations and minimize the interaction between the rigid-body dynamics and structural dynamics.
These weighting functions can be written as,

\[
W_e = \frac{s/M_e + \omega_b e}{s + \omega_b e} 
\]  
(4.17)  
\[
W_u = \frac{s + \omega_b c u / M_u}{\epsilon_u s + \omega_b c u} 
\]  
(4.18)  
\[
W_s = \frac{s/M_s + \omega_b s}{s + \omega_b s} 
\]  
(4.19)

where frequencies $\omega_{bcu}, \omega_{be}$ and $\omega_{bs}$ indicate the roll-off frequencies for the actuator deflections, rigid-body tracking error, and structural deformations, respectively. Finally, $\epsilon_s$ is used to regulate the low frequency structural deformation and $M_u$ is an indicator of the control surface saturation limit. The reference signal weighting function $W_{\text{ref}}$ can be chosen based on the time-domain tracking specifications such as overshoot and settling time. In this work, static feedback and feedforward gains are used to control the system. The control system can be represented in linear fractional transformation (LFT) form (Figure 4.6), where $P$ includes all the frequency-domain weightings. The system dynamics can be represented as follows,

\[
\dot{x}_p = A_p x_p + B_u u + B_w w + B_{\Delta u} \Delta
\]  
\[
z_{\text{rigid}} = C_r x_p + D_{ru} u
\]  
\[
z_{\text{flexible}} = C_f x_p + D_{fu} u
\]  
(4.20)  
\[
y_{\Delta} = C_{\Delta} x + D_{\Delta u} u + D_{\Delta w} w
\]  
\[
y = C_y x_p
\]

where $u_{\Delta}$ and $y_{\Delta}$ are related to each other through the uncertainty block $\Delta$. Detailed information about the construction of the uncertainty model is provided in Section 4.5.3.
The feedback and feedforward controllers are combined. The gain matrix for both controllers can be written as,

\[ K = \begin{bmatrix} K_{fb} & K_{ff} \end{bmatrix} \] (4.21)

### 4.5.2 Multiobjective Control

Multiobjective control problems are commonly referred to synthesis problems with mixed time- and frequency-domain specifications. These specifications can range from \( H_\infty \) and \( H_2 \) performance to pole placement, and asymptotic tracking and regulation. While mixed \( H_2/H_\infty \) control design problems have been extensively studied by the control designers [44, 104, 19], general multiobjective control designs are difficult to solve and rarely tackled. Recent advancements in convex programming have made LMIs a useful tool for solving a wide variety of control problems especially for multiobjective problems [7]. Scherer et al. [79] reported a thorough overview of the application of the LMI approach to multiobjective control design. In this section, a brief description of some performance measures is first provided and then the proposed application of the LMI methodology to multiobjective control synthesis is presented.
\( \mathcal{H}_\infty \) Performance

The \( \infty \)-norm of a system is the largest singular value across frequency. The \( \mathcal{H}_\infty \) optimization minimizes the system output energy to unknown but bounded energy inputs. The \( \mathcal{H}_\infty \) performance is therefore convenient to design robust controllers in the presence of model uncertainties where according to small gain theorem it can guarantee the stability of a system in the presence of an uncertainty block, \( \Delta \), that has an \( \infty \)-gain smaller than \( 1/\|G\|_\infty \). It can also be used to address frequency domain specifications, such as bandwidth and roll-off frequencies. It can be shown that \( \|G\|_\infty < \gamma \) if and only if a symmetric \( P \) exists matrix such that [79]:

\[
\begin{pmatrix}
A^T P + PA & PB & C_z^T \\
* & -\gamma I & D_z^T \\
* & * & -\gamma I
\end{pmatrix} < 0,
\]

for \( P < 0 \).

\( \mathcal{H}_2 \) Performance

The traditional \( \mathcal{H}_2 \) approach attempts to minimize the 2-norm of a system defined as,

\[
\|G\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \text{Tr} \left( G(j\omega) ^H G(j\omega) \right) d\omega \tag{4.23}
\]

where \( \text{Tr}(.) \) represents the trace of a matrix. The 2-norm of a system can be interpreted as the root-mean-square (RMS) value of the performance output driven by unit impulse or unit intensity white noise. The result is a controller adept at handling noise, but potentially weak in tracking performance, and vulnerable to model uncertainties. The 2-norm can be computed as [103],

\[
\|G\|_2^2 = \text{Tr} \left( C_z S C_z^T \right) \tag{4.24}
\]

where \( S \) is the solution of the Lyapunov function \( AS + SA^T + BB^T = 0 \).
Using the above equation, it can be shown that \( \|G\|_2^2 < \alpha \) if and only if symmetric \( P \) and \( Q \) matrices exist such that \[79\]:

\[
\begin{pmatrix}
A^T P + PA & PB \\
* & -I
\end{pmatrix} < 0,
\]

\[
\begin{pmatrix}
P & C^T_z \\
* & Q
\end{pmatrix} > 0,
\]

\[\text{Tr}(Q) < \alpha, D = 0.\] \hspace{1cm} (4.25)

**Generalized \( \mathcal{H}_2 \) Performance**

The \( \infty \)-norm represents the system gain when both input and output signals are measured in energy or \( L_2 \) norm. However, when control saturation avoidance or maximum output level are considered, it is more desirable to consider a system gain that represents the peak amplitude of the output when the input signal is still measured by its energy. Such a system gain can be represented by the generalized 2-norm of the system that is defined as,

\[
\|G\|_g := \sup \left\{ \|z(T)\| : x(0) = 0, T \geq 0, \int_0^T |\omega(t)|^2 dt \leq 1 \right\}
\] \hspace{1cm} (4.26)

The generalized 2-norm can be represented in an LMI form as follows \[79\], \( \|G\|_g^2 < \alpha \) if and only if a symmetric \( P \) matrix exists such that:

\[
\begin{pmatrix}
A^T P + PA & PB \\
* & -I
\end{pmatrix} < 0,
\]

\[
\begin{pmatrix}
P & C^T_z \\
* & \alpha I
\end{pmatrix} > 0,
\]

\[D = 0.\] \hspace{1cm} (4.27)

In addition to the above mentioned performance, the *peak-to-peak* system gain that represents the peak amplitude of the output signal over all bounded input signals can also be employed to satisfy some performance measures. Although it is possible to calculate an upper bound for this norm, the calculation of this norm calls for an iterative procedure that increases the computational cost. In addition to that, the LMI representation of an upper bound for
the peak-to-peak system norm is fairly conservative and can result in performance degradation. Therefore, in this work, the peak-to-peak system norm is not considered in the control design procedure.

**Multiobjective Control Synthesis**

While the $H_\infty$ performance can be used to guarantee system stability in presence of uncertainties, the $H_2$ and the $H_\infty$ performance measures are more suitable to address temporal performance.

The system output is divided into two categories. First, the performance outputs $z_{\text{rigid}}$ and $z_{\text{flexible}}$ and second, the robustness output $y_\Delta$. As a performance output, $z_{\text{rigid}}$ represents the tracking (regulating) error and the control surface deflections. In order to obtain good temporal performance, the following linear quadratic cost function is used for minimization,

$$J = \int_0^\infty \left( \hat{e}^T Q \hat{e} + \hat{u}^T R \hat{u} \right) dt = \|z\|^2_2$$

(4.28)

This cost function can be minimized by minimizing $\|Tzw\|_2$, which is a classical $H_2$ control design problem. In addition to the rigid-body output, $z_{\text{flexible}}$ is a representation of structural deformation and as well control deformations. In order to maintain the stress level below the desirable level and avoid control saturation, the generalized 2-norm of the transfer function from the exogenous input to $z_{\text{flexible}}$ is considered. Finally, according to small gain theorem, in order to ensure the robust stability of the aircraft in presence of uncertainties, $\|Ty_\Delta u_\Delta\|_\infty$ should be smaller than $1/\|\Delta\|_\infty$ [103]. Therefore, a mixed-norm multiobjective controller can be designed to satisfy both the stability robustness and the temporal performance requirements.

Having all elements for control synthesis in place, the design problem can be stated as,

$$\min_{w.r.t. K} \|Ty_\Delta u_\Delta\|_\infty + \sum_i \rho_i \|Tz_i w\|_{\alpha_i}$$

(4.29)

The goal in a multiobjective control synthesis is to find a single controller $K$ that stabilizes the plant and satisfies different design specifications of interest for different signals. The different sets of LMIs, presented in this chapter, can be used to satisfy different performance specifications ($S_i$), if there exists a positive-definite Lyapunov matrix $P_i$. When combining the
LMI representations for different performance specifications, $\mathcal{S}_i$, a single Lyapunov matrix $\mathcal{P}$ is used to enforce all the design requirements (i.e. $\mathcal{P} = \mathcal{P}_1 = \cdots = \mathcal{P}_N$). The enforcement of this restriction on the Lyapunov matrix introduces conservatism into the design. However, it makes the problem of finding the controller gains more numerically tractable. Since expressions such as $A^T\mathcal{P} + \mathcal{P}A$ involves the product of the Lyapunov matrix ($\mathcal{P}$) and the controller gain ($K$), the inequality conditions become nonlinear (bi-linear matrix inequalities —BMIs) and cannot be solved using convex optimization methods. Therefore, a congruence transformation is required to transform the bi-linear matrix inequalities that represent these problems back into LMI forms. For the $\mathcal{H}_\infty$ problem, the congruence transformation is performed using $\text{diag} \left[ \mathcal{X}, \mathcal{I}, \mathcal{I} \right]$ and congruence transformation for the $\mathcal{H}_2$ and $\mathcal{H}_g$ is performed using $\text{diag} \left[ \mathcal{X}, \mathcal{I} \right]$ on both inequalities, where $\mathcal{X} = \mathcal{P}^{-1}$. The resulting LMI representation for the $\mathcal{H}_\infty$ problem is

\[
\begin{pmatrix}
(AX + BNC_v) + (AX + BuNC_y)^T & B_\Delta & (C_\Delta X + D_\Delta u NC_y)^T \\
* & -\gamma I & 0 \\
* & * & -\gamma I
\end{pmatrix} < 0, \tag{4.30}
\]

\[
X > 0, \tag{4.31}
\]

\[
MC_v = C_v X. \tag{4.32}
\]

The LMI formulation of the $\mathcal{H}_g$ problem is

\[
\begin{pmatrix}
(AX + BNC_y) + (AX + BNC_y)^T & B_w \\
* & -I
\end{pmatrix} < 0, \tag{4.33}
\]

\[
\begin{pmatrix}
X & (C_f X + D_f u NC_v)^T \\
* & \alpha I
\end{pmatrix} > 0, \tag{4.34}
\]

\[
MC_y = C_y X. \tag{4.35}
\]
And finally the LMI formulation of the $\mathcal{H}_2$ problem is
\[
\begin{bmatrix}
(AX + BNC_y) + (AX + BNC_y)^T & B_w \\
* & -I
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
X & (C_rX + D_{ru}NC_v)^T \\
* & Q
\end{bmatrix} > 0,
\]
\[
Tr(Q) < \alpha,
\]
\[
MC_v = C_vX.
\]

By using the above-mentioned LMI problems, the control design problem represented in the Equation (4.29) can be cast as follows,
\[
\min. \quad \gamma + \sum_i \rho_i \alpha_i
\]
\[
\text{w.r.t.} \quad X, N
\]
\[
\text{s.t.} \quad \text{Equations (4.30) - (4.39)}
\]

While $X$ and $N$ matrices are found by solving the above problem, the $M$ matrix can be found by solving Equation (4.32). Finally, the controller gain has the following form,
\[
K = NM^{-1}.
\]

The above LMI problems are cast in semi-definite programming form and CVXOPT [17] is used to solve the problems to find the appropriate matrices and the controller gains. More detailed discussion on the LMI representation of $\mathcal{H}_\infty$ and $\mathcal{H}_2$ problem can be found in the robust and optimal control literature [7, 79, 47].

### 4.5.3 Model Uncertainty

In this work, a control system is designed to stabilize the plant and improve the performance robustness in the presence of a $\pm 20\%$ flight speed variation. The aerodynamic forces and moments that affect the cruise condition are quadratic functions of the cruise speed. In addition to those, the stability and control derivatives show quadratic dependency on the flight speed. Therefore, a quadratic uncertainty model is used for the control design procedure. The state
and control matrices can be represented as a quadratic function of speed variation $\delta v$,

$$A = A_o + A_1 \delta v + A_2 \delta^2 v$$

$$B = B_o + B_1 \delta v + B_2 \delta^2 v$$

(4.42)

where $\delta V = \frac{V - V_o}{0.2 V_o}$ and $\delta V \in [-1, 1]$. In order to obtain an uncertainty model, the linearized aircraft dynamics is evaluated at two different flight conditions ($V = 1.1 V_o$ and $V = 1.2 V_o$) in addition to the nominal one. The linear and quadratic uncertainty terms can be found from the following set of equations,

$$\begin{align*}
A_1 + A_2 &= A|_{V_o+20\%V_o} - A|_{V_o} \\
0.5A_1 + 0.25A_2 &= A|_{V_o+10\%V_o} - A|_{V_o}
\end{align*}$$

(4.43)

A similar set of equations can be used to find $B_1$, $B_2$, $B_{d1}$, and $B_{d2}$. Based on the above definition, the actual plant can be represented as follows,

$$\begin{align*}
\dot{x}_{ac} &= \left( A + \sum_{i=1}^{2} \delta_i^i A_i \right) x_{ac} + \left( B + \sum_{i=1}^{2} \delta_i^i B_i \right) u + \left( B_{\omega} + \sum_{i=1}^{2} \delta_i^i B_{\omega i} \right) w \\
y &= C x_{ac}
\end{align*}$$

(4.44)

State-space equation can be re-arranged in the following form.

$$\begin{align*}
\dot{x}_{ac} &= Ax_{ac} + Bu + B_{\omega} w + \sum_{i=1}^{2} \delta_i^i \left[ \begin{array}{ccc}
A_i & B_i & B_{\omega i}
\end{array} \right] \begin{bmatrix}
x_{ac} \\ u \\ w
\end{bmatrix}
\end{align*}$$

(4.45)

Using singular value decomposition (SVD) the above equation can be re-arranged as follows:

$$\begin{align*}
\dot{x}_{ac} &= A_{\Delta} x_{ac} + B u + B_{\Delta \omega} w + \delta v \left[ \begin{array}{ccc}
\delta v & X_{r1} \\ \delta^2 v & X_{r2}
\end{array} \right] \begin{bmatrix}
C_{\Delta 1} & D_{\Delta u1} & D_{\Delta w1} \\ C_{\Delta 2} & D_{\Delta u2} & D_{\Delta w2}
\end{bmatrix} \begin{bmatrix}
x_{ac} \\ u \\ w
\end{bmatrix}
\end{align*}$$

(4.46)

where $r_i = \text{rank} \left( \begin{array}{ccc}
A_i & B_i & B_{\omega i}
\end{array} \right)$. 


4.6 Gust Load Alleviation

The stress levels are calculated based on the modal deflections as follows [42],

\[ \sigma = SU\Phi \xi. \]  \hspace{1cm} (4.47)

where \( SU \) is a constant matrix that relates the von Mises stress to the nodal displacement and can be formed when the structural mesh is generated. As it is clear from the above equation, the stress level in the wing is directly proportional to the amplitude of structural modes (\( \xi \)). Therefore, by regulating the modal amplitudes to zero the stress can be regulated to the steady-state level. The developed state-space representation is used to design a control system that performs rigid-body tracking and minimizes structural deformations in order to lower the stress at the critical flight conditions. In this work, we only consider the longitudinal dynamics of the aircraft and therefore the ailerons are applied symmetrically.

4.6.1 Gust Profile

As mentioned previously, both discrete and continuous gust profiles are used in this work. For discrete gust evaluation, a \((1 - \cos)\) gust profile of the following form is used:

\[ w_g = \bar{w}_g \left( 1 - \cos \frac{2\pi Vt}{L_g} \right), \]  \hspace{1cm} (4.48)

where \( \bar{w}_g \) is the maximum gust velocity, and \( L_g \) is the gust length, nondimensionalized with respect to the time for a point in the aircraft to travel across the gust field.

For continuous gust modeling, the Dryden model is used to create stochastic gust excitations. In this approach the gust is modeled as a stationary, random, Gaussian process with the prescribed power spectral density. For a continuous gust where the gust vector is generated using the Dryden gust model, a gust filter is used to generate the numerical values of gust [107], i.e.,

\[ G_w (s) = \bar{w}_g \sqrt{\frac{L}{\pi V}} \frac{1 + \sqrt{3} L_s}{(1 + \frac{L}{\pi V} s)^2}, \quad G_q (s) = \frac{\bar{\varphi}}{(1 + (\frac{4b}{\pi V}) s)} G_w (s) \]  \hspace{1cm} (4.49)

where \( G_w (s) \) and \( G_q (s) \) are the gust filters representing the vertical and rotational gusts. The parameter \( L \) represents the gust scale and \( V \) represents the aircraft flying speed. In this work,
\( \bar{w} \) is set to 5 m/s for both continuous and discrete profiles.

### 4.6.2 Simulation Results

In this section, the simulation results for the two proposed controllers are presented. The proposed MPC controller is applied to the baseline configuration of the HALE UAV presented in Chapter 3 while cruising at 70 m/s. The obtained results highlight the potential improvement that can be gained using a load alleviation system. In addition, the performance of the proposed improved MPC formulation are compared to the traditional MPC and to the LQR controller. The 2-DOF mixed-norm controller was applied to the same aircraft, cruising at 80 m/s, to perform climb manoeuvres and gust load alleviations at different flight conditions. The obtained results show the superiority of the mixed-norm controller to a traditional \( H_2 \) controller.

#### Model Predictive Control Results

The controller parameters used in the numerical analysis are presented in Table 4.2. The performance of MPC is dependent of the weighting matrices and the control and prediction horizons length. As mentioned before, in this work the same the control and prediction horizons are considered. The control horizon is selected through a trial and error process. The longer the control horizon, the better the results become. However, a very long control horizon slows down the control signal calculation. The weighting matrices can be chosen using performance

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling time</td>
<td>( \Delta t )</td>
<td>2.5e-4 s</td>
</tr>
<tr>
<td>Control horizon</td>
<td>( N_u )</td>
<td>20</td>
</tr>
<tr>
<td>Prediction horizon</td>
<td>( N_p )</td>
<td>20</td>
</tr>
<tr>
<td>Max surface deflection</td>
<td>max(( \delta_e )) or max(( \delta_a ))</td>
<td>10 deg</td>
</tr>
<tr>
<td>Min surface deflection</td>
<td>min(( \delta_e )) or min(( \delta_a ))</td>
<td>-10 deg</td>
</tr>
<tr>
<td>Max surface deflection rate</td>
<td>max(( \Delta \delta_e / \Delta t )) or max(( \Delta \delta_a / \Delta t ))</td>
<td>60 deg/s</td>
</tr>
</tbody>
</table>
optimization. The focus of this work is to employ the proposed MPC framework to perform
gust load alleviation and compare its performance to a traditional MPC and an LQR controller.
Therefore, the weighting matrices are found through trial-and-error. In order to find appropriate
weighting matrices, first the \( R \) matrix is chosen. The \( R \) matrix is \( \text{diag} \left[ 10 \ 10 \right] \). The \( Q \) matrix
is represented by a diagonal matrix. First the diagonal entries corresponding to rigid-body
dynamics are chosen and then the structural weightings are selected. The following structure
is proposed for the \( Q \) matrix,
\[
Q = \text{diag} \left[ q_u \ q_w \ q_q \ q_{\xi} \ldots q_{\xi} \ q_{\theta} \right]
\]
where \( n \) is the number of structure modes used to construct the plant model.

Figures 4.7–4.15 show the results of a linear simulation subjected to discrete gust excita-
tions with three gust lengths of 1 s, 0.5 s and 0.25 s, respectively. The plotted results compare
the rigid-body regulation and gust load alleviation (GLA) performance of the proposed MPC,
a traditional MPC, and an LQR control. The simulation results for a case without a GLA
system is also plotted as a reference. In these figures, the maximum gust-induced stress level
at the wing root and mid-span locations are significantly reduced by the application of GLA.
In addition to an improvement in the stress level, the inclusion of the load alleviation system
in the controller has significantly lowered the maximum deviation of the rigid-body parameters
from their steady-state values. The reduction percentage for the maximum gust-induced stress
using the three controllers are provided in Table 4.3. The results show that the proposed MPC
outperforms the traditional MPC and the LQR controller in reducing the induced stress. When
the aircraft is flying through a wide gust (\( L_g =1s \)), all three controllers offer similar stress re-
duction capabilities. The rigid-body regulation performance of the proposed MPC controller,
as shown in Figure 4.7, is also comparable to the other controllers for a wide gust with \( L_g \) of 1s.
As the gust becomes sharper, the GLA performance is lowered. While the performance of the
proposed controller is only reduced by 2.5%, the GLA performance of the traditional MPC and
LQR controllers are dropped significantly. This performance degradation is also notable when
the rigid-body time-history of the three controllers are compared to each other (Figures 4.11
Table 4.3: Different controllers stress-level reduction

<table>
<thead>
<tr>
<th>$L_g$ (s)</th>
<th>1.0</th>
<th>0.5</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPC with P.E.</td>
<td>45.4%</td>
<td>43.9%</td>
<td>42.9%</td>
</tr>
<tr>
<td>MPC</td>
<td>41.6%</td>
<td>37.7%</td>
<td>29.3%</td>
</tr>
<tr>
<td>LQR</td>
<td>44.5%</td>
<td>37.9%</td>
<td>26.7%</td>
</tr>
</tbody>
</table>

and [4.14]). To further compare the performance of the three employed controllers, Table 4.3 shows the results of these controllers applied to the aircraft while it flies through continuous gusts generated using the Dryden gust filter. To perform the comparison, the gust filter is used to generate 10 gust profiles each 10 seconds long. The stochastic simulation is first performed without using the GLA system. Thereafter, the three controllers are used to stabilize the aircraft and perform load alleviation. It should be noted that the stochastic gust simulations begin from rest and $t = 0$ s marks the entrance of the aircraft into the turbulent atmosphere. Therefore the aircraft response to gust excitation starts from zero and expands as aircraft continues flying in the turbulent region. The average RMS values and the average peak values of the gust-induced parameters such as load factor and maximum stress for the 10 study cases are presented in Table 4.4. The time response of the three controllers to the applied stochastic excitations are plotted in Figure 4.16. Unlike the discrete gust study, the traditional MPC and LQR controllers do not offer any performance improvement. Surprisingly, the exclusion of the GLA system resulted in lower average RMS and average peak values for both the rigid-body parameters and stress levels. However, when GLA system is excluded, the average RMS and peak values for elevator deflection are considerably higher than the respective values for the traditional MPC and LQR controllers. This makes the aircraft prone to control saturation when more severe gusts are considered.

Contrary to the traditional MPC and the LQR controller, the obtained results for the proposed MPC with prediction enhancement, shows a significantly lower RMS and peak values for both rigid-body and structural parameters. The average RMS value of the load factor is
Table 4.4: Average RMS and peak values of rigid-body and structural parameters in response to Dryden gust

<table>
<thead>
<tr>
<th>Load factor</th>
<th>$w$ (m/s)</th>
<th>$\sigma_{\text{root}}$ (MPa)</th>
<th>$\sigma_{\text{mid}}$ (MPa)</th>
<th>$\delta_e$ (deg)</th>
<th>$\delta_a$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average RMS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC with P.E.</td>
<td>0.4162</td>
<td>1.2222</td>
<td>2.5867e+06</td>
<td>2.3782e+05</td>
<td>2.00002</td>
</tr>
<tr>
<td>MPC</td>
<td>0.5724</td>
<td>2.3311</td>
<td>5.0891e+06</td>
<td>4.5781e+05</td>
<td>1.2768</td>
</tr>
<tr>
<td>LQR</td>
<td>0.5658</td>
<td>2.2951</td>
<td>4.9946e+06</td>
<td>4.5038e+05</td>
<td>1.2852</td>
</tr>
<tr>
<td>Without GLA</td>
<td>0.4394</td>
<td>1.6595</td>
<td>4.7404e+06</td>
<td>4.4014e+05</td>
<td>2.6167</td>
</tr>
<tr>
<td>Average peak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC with P.E.</td>
<td>1.0426</td>
<td>2.3748</td>
<td>5.1686e+06</td>
<td>5.1574e+05</td>
<td>4.3849</td>
</tr>
<tr>
<td>MPC</td>
<td>1.5687</td>
<td>4.5455</td>
<td>1.0166e+07</td>
<td>1.0208e+06</td>
<td>2.9316</td>
</tr>
<tr>
<td>LQR</td>
<td>1.5317</td>
<td>4.4586</td>
<td>9.9287e+06</td>
<td>9.9498e+05</td>
<td>2.9467</td>
</tr>
<tr>
<td>Without GLA</td>
<td>1.1521</td>
<td>3.1054</td>
<td>1.0802e+07</td>
<td>1.0826e+06</td>
<td>5.9604</td>
</tr>
</tbody>
</table>

reduced by 5.2% compared to the results without GLA system, and the average peak value of the load factor is reduced by 9.5%. However, the greatest improvement can be seen in the stress-level reduction. The average RMS value of the root maximum stress is lowered by 45.4% and the average peak value is lowered by 52.1% when the prediction enhancement is introduced. In addition to these improvements, the average RMS and peak values for the elevator deflection is also lowered compared with the reference condition without the load alleviation.

When a system is inherently unstable, inaccuracies in the control effectiveness matrix ($B$) or the loss of control surface effectiveness can adversely affect the closed-loop performance. In this work, the inaccuracies of the control surface effectiveness is considered for both ailerons and elevators. The three controllers are applied to the same aircraft with 50% loss of control surface effectiveness. The simulation results for the crippled aircraft flying through two discrete gust excitations, $L_g=0.25$ and $L_g=0.5$, are presented in Figures 4.17–4.22. Note that, in the absence of GLA, the aircraft experienced control surface saturation and the controller failed to stabilize the plant when it was flying through a discrete gust with $L_g$ of 0.5s. Both traditional MPC and LQR controllers suffered from loss of control surface effectiveness. When the aircraft is flying through a sharp discrete gust ($L_g = 0.25$ s) the maximum stress at both root and tip show high frequency oscillations that can cause premature structural fatigue. The rigid-body tracking
Table 4.5: Average RMS and peak values of rigid-body and structural parameters in response to Dryden gust

<table>
<thead>
<tr>
<th></th>
<th>Load factor</th>
<th>$w$ (m/s)</th>
<th>$\sigma_{\text{root}}$ (MPa)</th>
<th>$\sigma_{\text{mid}}$ (MPa)</th>
<th>$\delta_e$ (deg)</th>
<th>$\delta_a$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average RMS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC with P.E.</td>
<td>0.7128</td>
<td>2.0928</td>
<td>4.9537e+06</td>
<td>4.3503e+05</td>
<td>3.4353</td>
<td>1.4580</td>
</tr>
<tr>
<td>LQR</td>
<td>0.9821</td>
<td>3.7354</td>
<td>8.9246e+06</td>
<td>7.4617e+05</td>
<td>2.5772</td>
<td>3.3027</td>
</tr>
<tr>
<td><strong>Average peak</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC with P.E.</td>
<td>1.9363</td>
<td>4.2413</td>
<td>1.0473e+07</td>
<td>9.8308e+05</td>
<td>7.6642</td>
<td>3.6368</td>
</tr>
<tr>
<td>LQR</td>
<td>2.6036</td>
<td>7.9605</td>
<td>2.0225e+07</td>
<td>1.7985e+06</td>
<td>6.1842</td>
<td>7.6825</td>
</tr>
</tbody>
</table>

performance of these controllers are as well degraded when handling a crippled aircraft. On the contrary, the proposed MPC effectively controlled the damaged aircraft without experiencing high frequency oscillations. While the modified predictive controller managed to damp out the excitation, the traditional MPC and the LQR controllers failed to reject the excitation in the simulation time.

In addition to discrete $(1 - \cos)$ gust excitations, stochastic continuous gusts are also considered to analyze the controllers performance in the presence of control effectiveness inaccuracies. The results of this study are presented in Table 4.5. When the load alleviation was not considered and when it was performed using a traditional MPC, control input saturation destabilized the system. On the other hand, the proposed MPC method and the LQR controller managed to successfully stabilize the aircraft. The response of these two controllers to the excitations are presented in Figure 4.23. The results presented in Table 4.5 show that the average RMS and peak values corresponding to the proposed MPC are significantly lower than the LQR results. The same as before, when the proposed control method is employed higher elevator deflections are commanded.
Figure 4.7: Load factor and rigid-body parameters ($L_g = 1.0s$)

Figure 4.8: Structural modes and wing root and mid stresses ($L_g = 1.0s$)
Figure 4.9: Control surface deflections ($L_g = 1.0s$)

Figure 4.10: Load factor and rigid-body parameters ($L_g = 0.5s$)
Figure 4.11: Structural modes and wing root and mid stresses ($L_g = 0.5s$)

Figure 4.12: Control surface deflections ($L_g = 0.5s$)
Figure 4.13: Load factor and rigid-body parameters ($L_g = 0.25s$)

Figure 4.14: Structural modes and wing root and mid stresses ($L_g = 0.25s$)
Figure 4.15: Control surface deflections ($L_g = 0.25s$)

Figure 4.16: The response of the aircraft GLA system to continuous stochastic gusts
Figure 4.17: Load factor and rigid-body parameters for the crippled aircraft ($L_g = 0.25s$)

Figure 4.18: Structural modes and wing root and mid stresses for the crippled aircraft ($L_g = 0.25s$)
Figure 4.19: Control surface deflections for the crippled aircraft \((L_g = 0.25s)\)

Figure 4.20: Load factor and rigid-body parameters for the crippled aircraft \((L_g = 0.5s)\)
Figure 4.21: Structural modes and wing root and mid stresses for the crippled aircraft ($L_g = 0.5s$)

Figure 4.22: Control surface deflections for the crippled aircraft ($L_g = 0.5s$)
2DOF Mixed-Norm Controller Results

Figures 4.24–4.26 show the performance of the designed controllers in performing the altitude change manoeuvre. The red line in Figure 4.24 represents the reference altitude input. Both controllers show a zero steady error. However, as expected the $H_2$ controller has a better performance in tracking the input signal and has a shorter settling time. On the other hand, due to a slower rate of climb, the mixed-norm controller shows a significantly lower maximum stress (27%). In addition, the manoeuvring load factor is lower when the mixed-norm controller is used.

Two different gust lengths are considered and numerical simulations are performed to study the performance of the two controllers. Figures 4.27–4.29 show the simulation results for a gust length of 20 m and Figures 4.30–4.32 represent the obtained results for an 80 m gust length. Both controllers have effectively regulated the altitude and the wing root bending moment. The $H_2$ controller has a better regulating performance and has a slightly lower maximum stress in both gust simulations. By comparing the results, it is clear that a shorter gust length causes
more oscillations and therefore it is selected as a more critical condition for the robustness analysis.

The results show that when performing an altitude manoeuvre, the controller becomes more vulnerable to oscillations and instabilities as the flight speed increases. Figures 4.33–4.38 show the simulation results for two flight speeds of 90 m/s and 96 m/s. As can be observed from the altitude oscillations in Figures 4.33 and 4.34, altitude tracking performance of the $H_2$ controller degrades at higher flight speeds. In contrast, the mixed-norm controller does not suffer from oscillations and manages to maintain the same tracking performance at different flight conditions. In addition to poor altitude tracking performance of the $H_2$ controller, significant oscillations are present in the load factor and the root bending moment time history. While the maximum manoeuvring stress has increased for both controllers, the difference between the stress level of the two controllers is increased showing that the load alleviation capability of the $H_2$ controller decreases as the velocity of aircraft increases.

At a lower flight speed, the same gust excitation results in a larger gust-induced angle of attack. Therefore, aircraft dynamics show higher sensitivity to gust excitations at lower flight speeds. Figures 4.39–4.44 show the simulation results for two flight speeds: 70 m/s and 68 m/s. Both controllers show a higher number of oscillations compared to the nominal flight condition (80 m/s). While the $H_2$ controller has a better altitude regulation performance, the oscillations take double the time to damp out when flying at 70 m/s. At the flight speed of 68 m/s, the $H_2$ controller becomes unstable, while the mixed-norm controller manages to stabilize the aircraft. The control surface deflections, for both controllers, increase as the flight speed decreases.

The obtained results show that, while the $H_2$ controller has a better input tracking and disturbance rejection performance at the nominal flight condition, its performance deteriorates at neighbouring flight conditions. As the deviation in the flight condition becomes more significant, the $H_2$ control starts to suffer from marginal stability (or lack of stability). On the other hand, the proposed mixed-norm controller manages to maintain the same level of performance
even at significantly different flight conditions.

As discussed previously, a nonlinear parameter varying (NLPV) model is used to represent the actual model and to construct the uncertainty model. A simple linear parameter varying (LPV) representation, however, has a lower computational cost. A simple numerical simulation was performed to compare the controller performance for the two different models. The analysis is limited to a gust load alleviation study. The two controllers are evaluated at a non-nominal flight speed of 70 m/s and the results of this simulation are shown in Figures 4.45 and 4.46. While the two controllers have successfully stabilized the plants, the controller that is designed based on a nonlinear parameter varying model shows fewer oscillations and has a better disturbance rejection performance. The disturbance rejection performance is closely tied to the $H_2$ norm of the system. It can be concluded that the use of a less accurate uncertainty model (LPV model) in the control synthesis leads to a controller that has a worse tracking and regulating performance.

![Figure 4.24: Load factor and altitude time history of a maneuvering flight ($V = 80$ m/s)](image-url)
Figure 4.25: Wing bending moment and maximum stress due to a maneuvering flight ($V = 80$ m/s)

Figure 4.26: Control effectors deflection for a maneuvering flight ($V = 80$ m/s)
Figure 4.27: Load factor and altitude time history due to a discrete gust ($V = 80$ m/s, $L_g = 20$ m)

Figure 4.28: Wing bending moment and maximum stress due to a discrete gust ($V = 80$ m/s, $L_g = 20$ m)
Figure 4.29: Control effectors deflection due to a discrete gust ($V = 80$ m/s, $L_g = 20$ m)

Figure 4.30: Load factor and altitude time history due to a discrete gust ($V = 80$ m/s, $L_g = 80$ m)
Figure 4.31: Wing bending moment and maximum stress due to a discrete gust \((V = 80 \text{ m/s}, \quad L_g = 80 \text{ m})\)

Figure 4.32: Control effectors deflection due to a discrete gust \((V = 80 \text{ m/s}, \quad L_g = 80 \text{ m})\)
Figure 4.33: Load factor and altitude time history of a maneuvering flight ($V = 90 \text{ m/s}$)

Figure 4.34: Wing bending moment and maximum stress due to a maneuvering flight ($V = 90 \text{ m/s}$)
Figure 4.35: Control effectors deflection for a maneuvering flight ($V = 90$ m/s)

Figure 4.36: Load factor and altitude time history of a maneuvering flight ($V = 96$ m/s)
Figure 4.37: Wing bending moment and maximum stress due to a maneuvering flight ($V = 96$ m/s)

Figure 4.38: Control effectors deflection for a maneuvering flight ($V = 96$ m/s)
Figure 4.39: Load factor and altitude time history due to a discrete gust ($V = 70$ m/s, $L_g = 20$ m)

Figure 4.40: Wing bending moment and maximum stress due to a discrete gust ($V = 70$ m/s, $L_g = 20$ m)
Figure 4.41: Control effectors deflection due to a discrete gust ($V = 70 \text{ m/s}, L_g = 20 \text{ m}$)

Figure 4.42: Load factor and altitude time history due to a discrete gust ($V = 68 \text{ m/s}, L_g = 20 \text{ m}$)
Figure 4.43: Wing bending moment and maximum stress due to a discrete gust ($V = 68 \text{ m/s}$, $L_g = 20 \text{ m}$)

Figure 4.44: Control effectors deflection due to a discrete gust ($V = 68 \text{ m/s}$, $L_g = 20 \text{ m}$)
Figure 4.45: Load factor and altitude time history due to a discrete gust ($V = 70 \text{ m/s}$, $L_g = 20 \text{ m}$)

Figure 4.46: Wing bending moment and maximum stress due to a discrete gust ($V = 70 \text{ m/s}$, $L_g = 20 \text{ m}$)
4.6.3 Nonlinear Simulation Results

The results presented in Section 4.6.2 represent a linear controller applied to a linearized plant. However, due to the geometric nonlinearities and the rigid-elastic coupling, the equations of motion for a highly flexible aircraft (2.17) are highly nonlinear. Therefore, the performance of a controller designed based on the linearized plant must be verified by performing nonlinear simulations. In this Section, an LQR controller is applied to the nonlinear plant. The simulation results are limited to a discrete gust excitation. To be comparable with the presented linear results (Section 4.6.2) a sharp gust ($L_g = 0.25$ s) which is more challenging to control is considered. The obtained results are given in Figures 4.47 – 4.49. The gust amplitude is set to $0.3048$ m/s (1 fps). As seen from the results, the nonlinear plant follows a trend similar to the linear model. However, both the rigid-body dynamics and the structural dynamics show greater deviation from the steady condition. The fact that this greater deviation is generated by a small gust excitation reveals the importance of the nonlinear simulation. The obtained results show that a controller design based on the linearized plant can stabilize the aircraft. To improve the performance of the controller, the control parameters (weighting functions) must be further optimized.

Figure 4.47: Load factor and rigid-body parameters for the nonlinear aircraft ($L_g = 0.25$ s)
As this work is focused on the conceptual design of a highly flexible aircraft, a linearized model is used to evaluate the aircraft performance and design control systems. However, nonlinear models must be considered for future detailed design purposes.

4.7 Summary

In this chapter, the derived nonlinear equations of motion were first linearized. The linearized equations were then used to perform eigenvalue analysis to reveal the rigid-elastic interaction in a highly flexible aircraft. An MPC controller and a 2-DOF mixed-norm controller were proposed.
and developed to control a highly flexible aircraft. Finally, both controllers were applied to a HALE UAV to perform gust and manoeuvre load alleviation. The integration of the proposed feedback loop with the traditional MPC architecture improved the controller performance, especially when modelling errors were considered. The comparison of the developed mixed-norm controller to the traditional $H_2$ controller revealed the superiority of the proposed controller in rigid-body tracking and structural control in the presence of flight condition deviation.
Chapter 5

Design optimization and case studies

The objective of this thesis is to surpass current aircraft endurance limits through the use of an active load alleviation system that is designed concurrently with the rest of the aircraft. In the optimization problem solved here, design variables from all three disciplines are included. The aerodynamic design variables may include the aspect ratio \((AR)\), wing area \((S_{\text{ref}})\), and the wing spanwise twist distribution \((\gamma_i)\). The structural sizing is represented by the spar wall thicknesses \((t_i)\). Finally, the weighting parameters \(\rho_{\text{rigid}}\) and \(\rho_{\text{elastic}}\), are the control discipline design variables. In this thesis, two different case studies are considered for optimization. First, a jet propelled HALE UAV is considered; HALE UAVs are the main focus of this thesis. In order to show the applicability of the developed integrated optimization approach to other aircraft configurations, an electric, propeller-driven, high aspect ratio flying wing is selected for the second case study optimization. The lack of vertical stabilizers makes the control of flying wings very challenging. In addition, when active load alleviation is also of interest, the lack of control effector redundancy may degrade the overall controller performance which makes this case an interesting target for integrated design optimization.
5.1 Objective Function

Aircraft endurance is considered as the objective function in this work. Two different propulsion systems are considered in this work. For a jet propelled aircraft the endurance is,

$$ E = \frac{1}{c_T C_D} \frac{C_L}{C_D} \ln \frac{W_{initial}}{W_{empty}} $$

(5.1)

where $c_T$ represents the engine specific fuel consumption and $C_L$ and $C_D$ are aircraft lift and drag coefficients, respectively. The aircraft empty weight ($W_{empty}$) is the total weight of the aircraft’s structure, subsystems, and payload and the initial weight ($W_{initial}$) is the summation of empty weight and the fuel weight. Since the selection of the propulsion system is not the focus of this work, a representative endurance measure is selected, as opposed to the actual endurance value.

$$ \frac{C_L}{C_D} \ln \frac{W_{initial}}{W_{empty}} $$

(5.2)

The derivation of endurance and range formulas for an electric aircraft powered by fuel cells were recently reported by Traub [93] who derived the following expression,

$$ E = R_t^{1-n} \left[ \frac{\eta_{total} \nu C}{qS(C_{D_o} + C_{D_i})} \right]^n $$

(5.3)

where $R_t$ is the battery hour rating (in hours), $n$ is the discharge parameter dependent on battery type and temperature, $\nu$ and $C$ represent battery voltage and capacity, respectively. The term $\eta_{total}$ is the total efficiency of the propulsion system. The denominator of the above equation represents the required power for a steady level-flight. Minimizing the required power maximizes the aircraft endurance.

5.2 Flight Conditions

The cruise condition is considered in order to compute the endurance. The cruise performance depends on aerodynamic variables that dictates the wing configuration and the structural weight. Two other flight conditions are also analyzed here: an altitude change maneuver and a flight through a sharp vertical gust. Both conditions are selected to study the effect of inertial and aerodynamic loads on the aircraft structure and the design optimization. The performance
of aircraft in these flight conditions depends heavily on the controller gains and the structural sizing. It should be noted that both study cases represent longitudinal flight conditions. In the first maneuver, the aircraft is commanded to gain 1,000 m in 40 s. Unfortunately, there are no published performance requirements for HALE highly flexible aircraft. However, Shearer and Cesnik [82] have considered a highly flexible aircraft as a large transport type aircraft (class III-L) and used a maximum climb rate of 10.16 m/s at sea level as a guideline for control design.

In this work, a fast maneuver is performed that results in large wing loadings. For the second flight condition, a \((1 - \cos)\) discrete gust profile is considered. The gust profile is generated using the following definitions [101].

\[
\begin{align*}
    w_g &= \frac{\bar{w}_g}{2} \left(1 - \cos \frac{2\pi t}{L_g}\right), \\
    p_g &= \frac{\bar{p}_g}{2} \left(1 - \cos \frac{2\pi t}{L_g}\right),
\end{align*}
\]

where \(\bar{w}_g\) and \(\bar{p}_g\) are the maximum gust velocities, and \(L_g\) is the gust length. The gust scale factor \((L_g)\) is selected so that maximum stress is achieved.

5.2.1 Constraints

As previously mentioned, two different flight conditions are analyzed: an altitude change manoeuvre and a gust. Two constraints are enforced: one guarantees the structural integrity by constraining the maximum stress in the structure for the duration of the manoeuvre or gust, while the other ensures manoeuvring capability by limiting the altitude error at 40 s after the start of the manoeuvre. The addition of the altitude constraint at 40 s ensures that the optimizer achieves load alleviation capability without sacrificing the aircraft manoeuvrability.

In order to reduce the number of structural constraints, they are aggregated using the Kreisselmeier–Steinhauser (KS) function [87]. For a vector of constraints \(g_i \leq 0 \quad (i = 1, \ldots, n)\) the KS function is a differentiable function that has the form,

\[
KS(g_i) = \frac{1}{\rho} \ln \sum_{i=1}^{n} e^{\rho g_i},
\]

where \(\rho\) is a parameter that controls how close the function is to the maximum of the constraints,
as we can infer from the inequality property,
\[ g_{\text{max}} \leq KS(g_i) \leq g_{\text{max}} + \frac{\ln(n)}{\rho}. \] (5.6)

Based on this property, all stress constraints can be enforced by the single constraint,
\[ KS(g(\sigma_i)) < 0, \] (5.7)

where,
\[ g_i(\sigma_i) = \frac{\sigma_i}{\sigma_{\text{yield}}} - 1. \] (5.8)

A more detailed analysis of the properties of KS function and its application to optimization problems can be found in previous work [87, 70].

In our optimization, the taper ratio is fixed. Therefore, increasing the aspect ratio may decrease the wing chord, resulting in high sectional lift coefficients near the wing tips. In order to avoid tip stall, we require the stall speed of the designed wing to be smaller than that of the baseline wing. In order to satisfy this condition, all sectional lift coefficients must be lower than \( C_{l_{\text{max}}} \), the maximum sectional lift coefficient of the baseline airfoil.

\[ V_s \leq V_{s_{\text{ref}}} \Rightarrow C_{l_i|V_{s_{\text{ref}}}} \leq C_{l_{\text{max}}} \] (5.9)

The KS function is also used for aggregating the stall constraints,
\[ KS(g(C_{l_i})) \leq 0, \] (5.10)

where,
\[ g(C_{l_i}) = \frac{C_{l_i}}{C_{l_{\text{max}}}} - 1. \] (5.11)

In this design problem, the thickness-to-chord ratio is fixed for the whole wing, and therefore, the spar diameter depends on the local chord. Ultimately, this means that the spar diameter decreases as the aspect ratio increases. The spar wall thickness is bounded below by the minimum gauge thickness and above by the spar radius. Spar thicknesses and wing twist angles are varied at six locations along the span and the intermediate values are linearly interpolated. The XDSM diagram proposed by Lambe and Martins [46] is used to visualize the aeroservoelastic optimization flow (Figure 5.1).
Due the presence of multiple local minima, a gradient-free optimizer is chosen over a gradient-based optimizer. An Augmented Lagrange Multiplier Particle Swarm Optimizer (ALPSO) is used to solve the proposed optimization problem. This gradient-free algorithm tends to find global optima and this particular version can solve constrained problems [37].

5.3 HALE Aircraft Design Optimization

A HALE UAV with a large aspect ratio is considered as a target for design optimization. The geometry of the baseline aircraft is shown in Chapter 4. The chord-wise location of the spar is set to 45% of the chord. The design cruise and stall speeds are set to 80 m/s and 60 m/s, respectively. A maximum airfoil lift coefficient of 1.6 is used for the stall calculations. The initial and final altitudes for the altitude gain maneuver are 2,000 m and 3,000 m, respectively.

The complete optimization problem is as follows:

$$\min_{AR,S_{ref},t_i,\gamma_i,\rho_{rigid},\rho_{elastic}} \left( -\frac{C_L}{C_D} \ln \frac{W_{initial}}{W_{empty}} \right)$$

subject to

$$KS \left( g(\sigma_i) \right) \leq 0$$
$$KS \left( g(C_{li}) \right) \leq 0$$
$$h_{err} \leq 0.05$$
$$t_i - \frac{d_i}{2} \leq 0$$
$$-10 \leq \gamma_i \leq 10$$
$$145 \leq S_{ref} \leq 245$$
$$10 \leq AR \leq 20$$
$$10 \leq \rho_{rigid} \leq 100$$
$$500 \leq \rho_{elastic} \leq 1500$$

where \((i = 1, \ldots, n)\). (5.12)

Since the wing area \((S_{ref})\) is a design variable, the viscous drag must be considered in the endurance calculation. The drag coefficient used in this work takes into account both the induced drag and the viscous drag. The induced drag is calculated using the previously described vortex-lattice panel code and the viscous drag is estimated using an empirical formulation based on the wet-surface area.
While the structural and aerodynamic parameters directly affect the endurance through weight and aerodynamic performance, the control parameters have an indirect effect on the objective function through load alleviation. A simple linear control theory can be used to explain the effect of the control parameters on the final design. A system’s steady-state error is dependent on the choice of gain matrix, \( K \). Therefore, different values of control weighting parameters result in different gain matrices that can affect the altitude constraint. However, maximum stress can be related to the maximum overshoot of the structural modes amplitudes, which is related to the eigenvalues of the system. The system’s eigenvalues depend on the gain matrix, which itself depends on the weighting parameters. The limits for \( \rho_{\text{rigid}} \) and \( \rho_{\text{flexible}} \) are selected through a trial-and-error process. Although the \( R \) matrix is selected to penalize the control deflections and avoid controls saturation, large values of \( \rho_{\text{rigid}} \) and \( \rho_{\text{flexible}} \) can result in controls saturation over a long period of time, which can destabilize the aircraft. Therefore, an upper limit is used to avoid prolonged controls saturation. However, as this type of aircraft is marginally stable, a sharp atmospheric gust can result in excessive oscillations. As a result, a lower bound is set in place for \( \rho_{\text{rigid}} \) and \( \rho_{\text{flexible}} \) to avoid low gain controllers that cannot damp out the gust-induced oscillations effectively.

Two design optimization cases are considered: one with load alleviation and the other without. The optimized endurance and design variable values are summarized in Table 5.1 and Figs. 5.2 and 5.3. The constraint results are shown in Figs. 5.4 and 5.5 where the horizontal lines represent the stall and the stress constraint values, respectively.

The optimization results presented in Table 5.1 show that the optimization of the load alleviation system in the design procedure results in a wing structure that is 41.5% lighter. The optimization results plotted in Fig. 5.2 show that the spar thickness decreases monotonically when load alleviation is used. However, the optimal thickness distribution is different when load alleviation is not employed: a significant amount of material is added near the tip. When the active load alleviation system is not used, the optimizer is forced to explore passive options. Increasing the wing tip mass can help stabilize the aircraft and lower the maximum stress when encountering an atmospheric turbulence. The increase of the wing tip thickness effectively adds mass to the wing tip.
Table 5.1: Optimization results with and without load alleviation system.

<table>
<thead>
<tr>
<th>Load alleviation</th>
<th>Off</th>
<th>On</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\text{ref}}$ ($m^2$)</td>
<td>219.18</td>
<td>191.47</td>
</tr>
<tr>
<td>$AR$</td>
<td>13.98</td>
<td>14.03</td>
</tr>
<tr>
<td>$L/D$</td>
<td>34.29</td>
<td>34.37</td>
</tr>
<tr>
<td>$q_{\text{elastic}}$</td>
<td>1499.95</td>
<td>1499.88</td>
</tr>
<tr>
<td>$q_{\text{rigid}}$</td>
<td>90.63</td>
<td>75.71</td>
</tr>
<tr>
<td>Wing mass (kg)</td>
<td>13,378</td>
<td>7,817</td>
</tr>
<tr>
<td>Endurance factor</td>
<td>31.90</td>
<td>38.83</td>
</tr>
</tbody>
</table>

It can be shown that a lower tip mass would result in control effector saturation and aircraft instability. In this work, only an upward wind gust is considered. However, increasing the wing tip mass would also be helpful in reducing the maximum stress of a flexible wing passing through a downward gust. In order to satisfy the stress constraints without using the added mass at the tip, a much higher thickness near the wing root would be required, resulting in a far heavier aircraft. Note that mounting the engines or external stores on the wing could also help reduce the accumulated dead mass at the tip of the wing. In this work, however, the baseline model did not have engines mounted on the wing.

The converged values of control weighting parameters listed in Table 5.1 and the altitude and load factor time history (Fig. 5.5) show that both the cases achieve comparable performance in the climb manoeuvre.

Fig. 5.4 shows the lift coefficient distribution at the reference stall speed of 60 m/s. The stall constraint is satisfied in both cases. However, when load alleviation is not employed, the wing area is 14.5% higher than the wing area when load alleviation is considered. This is mainly due to the weight of the wing structure, which is 41.5% higher than the weight of the wing that uses the load alleviation system.

Finally, the results show that the aircraft endurance is increased by 21.7% when an active load alleviation system is used and optimized concurrently with the wing aerodynamics and structural sizing.
Figure 5.2: Spanwise distribution of spar wall thickness and wing twist

Figure 5.3: Spanwise distribution of vertical displacement, stress, and lift at the cruise condition
Figure 5.4: Spanwise $C_l$ distribution at the minimum allowable speed

Figure 5.5: Load factor, altitude and maximum stress time history (stress and maneuver constraints)
5.4 Flying Wing Design Optimization

In addition to the jet propelled HALE UAV, a fuel cell powered flying wing is also considered.
While in a jet propelled study case, the aircraft weight had a direct effect on the optimization objective function. For an electric aircraft where the weight does not change during the flight, the aircraft weight affects the performance by indirectly affecting the drag. A large aspect ratio flying wing is considered for optimization. Similar configurations have been studied by other researchers over the last decade [66, 71, 11]. The baseline geometry is shown in Figure 5.6 and the structural and geometric parameters are listed in Table 5.2.

Figure 5.6: 3D geometry of a generic high aspect flying wing
Table 5.2: Aircraft geometric properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing span</td>
<td>60 m</td>
</tr>
<tr>
<td>Wing area</td>
<td>150 m²</td>
</tr>
<tr>
<td>Wing taper ratio</td>
<td>1</td>
</tr>
<tr>
<td>Wing airfoil</td>
<td>EMX07</td>
</tr>
<tr>
<td>Wing tip dihedral</td>
<td>5 deg</td>
</tr>
<tr>
<td>Aileron chord</td>
<td>0.2 c</td>
</tr>
<tr>
<td>Position of wing tip dihedral</td>
<td>10 m from tip</td>
</tr>
<tr>
<td>Elastic axis position</td>
<td>25% chord</td>
</tr>
<tr>
<td>Sectional center of gravity</td>
<td>25% chord</td>
</tr>
<tr>
<td>Horizontal stabilizer span</td>
<td>20 m</td>
</tr>
<tr>
<td>Horizontal stabilizer area</td>
<td>30 m</td>
</tr>
<tr>
<td>Horizontal stabilizer airfoil</td>
<td>NACA0012</td>
</tr>
</tbody>
</table>

Although the configuration shown in Figure 5.6 has horizontal stabilizer, a reflex airfoil is considered for the wing section to avoid high pitching moments and make the trim procedure more simple. The EMX07 airfoil is selected for this purpose which is the same airfoil used in the design of X-HALE [11]. As seen in Figure 5.6, a positive wing folding is considered to improve the lateral controllability of the flying wing. This wing folding angle ($\Gamma$) is considered as a design variable for this optimization study. The wing aspect ratio ($AR$) is also considered as a design variable. However, unlike the previous optimization problem, the wing area ($S_{\text{ref}}$) and the wing incidence angle are kept constant in the course of optimization. Similar to the previous study case, the spar thickness is a design variable and it is varied at three locations along the span (the root, the wing folding location, and the tip) and the intermediate values are linearly interpolated between each two adjacent design sections. Since the operational condition optimization is not addressed in this work and the wing area is constant, the minimum power
condition (5.3) can be obtained by minimizing the drag. The complete system-level optimization is as follows:

$$\min \ P_{\text{req}} \propto (C_{D_o} + C_{D_i})$$

w.r.t \ $AR, \ \Gamma, \ t_i, \ \rho_i$

s.t. \ $\begin{cases}
KS (g (\sigma_i)) < 0 \\
t_i - \frac{D_i}{2} \leq 0 \\
|\delta_{\text{ailer}}| \leq 10^\circ
\end{cases}$

(5.13)

As mentioned before, the spar diameter is determined by the airfoil geometry. During the course of the optimization, thickness-to-chord ratio is kept constant. This means that a variation in wing planform, changes the airfoil thickness and therefore the spar diameter along the span. The stress constraint is evaluated by numerically simulating the aircraft encountering a $(1 - \cos)$ vertical gust and rotational gust (Equation (5.4)).

To highlight the importance of the geometric nonlinearities in modelling the behaviour of a highly flexible aircraft, the calculation of the trim condition and the deformed shape of the wing is performed; one using a linear finite element representation and another using the developed co-rotational framework. The wing deformation and the values for the angle of attack and the elevator deflection are presented in Figure 5.7 and Table 5.3, respectively.

Figure 5.7: Wing deformation under cruise loadings using linear and co-rotational structural representation
Table 5.3: The steady cruise condition using linear and co-rotational structural representation

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$ (deg)</th>
<th>$\delta_e$ (deg)</th>
<th>Lift (N)</th>
<th>Drag (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-rotational approach</td>
<td>2.227</td>
<td>-1.244</td>
<td>5875.37</td>
<td>965.66</td>
</tr>
<tr>
<td>Linear FE</td>
<td>2.719</td>
<td>-1.743</td>
<td>5760.29</td>
<td>979.81</td>
</tr>
</tbody>
</table>

As it can be seen from Figure 5.7, when a linear structural representation is used lower tip deflection is obtained. In addition, the zero axial shrinkage that can be seen in the linearly deformed wing along with the bending deformation, increases the wing surface area and leads to inaccurate aerodynamic calculations. The exclusion of the geometric nonlinearities from the structural calculation resulted in 18.09% and 28.6% error in the angle of attack and elevator deflection calculation, respectively. Therefore a controller designed based on the linearly obtained trim condition, adversely affects the aircraft performance and can destabilize the aircraft.

In this section, the results of the application of the proposed controller to the baseline configuration flying through a longitudinal and a lateral gust is presented. The obtained results are presented in Figures 5.8–5.11. For each gust excitation, the presented results compare the open-loop response of the aircraft with the closed-loop results. The two closed-loop simulations represent two different controllers: one designed to emphasize rigid-body regulation, and another that targets the structural dynamics. The load alleviation system manages to lower the magnitude of the structural modes for both longitudinal and lateral excitations. However, as it can be seen in Figures 5.9, 5.11, the load alleviation system is more efficient in regulating the first bending mode. This shortcoming can be addressed by introducing multiple control effectors along the span. In our case, however, only one control effector is used along span, which is not very effective in regulating structural modes that have more than one anti-node.
Chapter 5. Design optimization and case studies

Figure 5.8: Longitudinal gust induced rigid-body excitation

Figure 5.9: Longitudinal gust induced structural excitation
The converged optimized values for the design parameters are presented in Table 5.4. The
converged thickness and stress distributions for both design cases are shown in Figure 5.12. When the load alleviation system is included in the design process, the optimized cruise drag is 2.4% lower than when this system is not used. The structural weight in the presence of the load alleviation system is also lowered by 17.4%. Comparing the aspect ratio of the two optimized configurations reveals that in the absence of the load alleviation system, the traditional high aspect ratio configuration is used to improve the aerodynamic efficiency. However, the integration of this system has driven the optimizer in a different direction to achieve the desired performance through lowering the structural weight. In addition to the inferior performance, the significantly higher structural weight of the wing in the absence of the active control system can lead to premature wingtip stall. It is worth mentioning that greater performance improvements similar to the previous case results can be achieved by designing the wing area and twist distribution at the same time with the other design variables.

<table>
<thead>
<tr>
<th>Load alleviation</th>
<th>Off</th>
<th>On</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>23.84</td>
<td>20.84</td>
</tr>
<tr>
<td>Folding angle (deg)</td>
<td>7.89</td>
<td>4.32</td>
</tr>
<tr>
<td>Drag (N)</td>
<td>1034.25</td>
<td>1009.04</td>
</tr>
<tr>
<td>Structural weight (N)</td>
<td>6069.68</td>
<td>5010.63</td>
</tr>
</tbody>
</table>
Figure 5.12: Optimized thickness and stress distribution
Figure 5.1: Aeroservoelastic optimization flow diagram
Chapter 6

Conclusions and Future Work

In this work, the quasi-coordinate Lagrange’s equations were used to derive the nonlinear equations of motion of a highly flexible aircraft (Chapter 2). The derivation of the equations was based on a body-fixed axes system. The derived equations, which allow for large rigid-body motions along with large structural deformations, can be used to perform time-dependent analysis and can also be linearized to perform control system design.

In Chapter 3 a three-dimensional panel-code was developed and modified to obtain the aerodynamic responses to pulse excitations, which are used to construct a linearized state-space representation of the unsteady aerodynamics. A co-rotational framework was developed to capture large structural deformations that are experienced by the aircraft structure. The developed framework is independent of the beam element used for discretizing the structure providing the designer with more freedom in modelling a wider range of wing structures.

The linearized equations of motion were used to design control systems to perform manoeuvre/gust load alleviations. An eigenvalue analysis of the linearized equations revealed that the dynamics of a highly flexible aircraft cannot be accurately modelled when the rigid-elastic interactions are not considered. Two different control methodologies were considered in this work.

A model predictive controller was developed to perform gust load alleviation for an aircraft encountering discrete and continuous atmospheric disturbances. In order to improve the performance and robustness of the traditional model predictive formulation, a feedback loop
was introduced to improve the prediction performance of the controller. To demonstrate the effectiveness of the controller in gust load alleviation, the proposed predictive framework was applied to a highly flexible aircraft. The introduced feedback loop improved the controller performance in handling rigid-body dynamics while maintaining the same level of structural relaxation. In addition to better performance, the addition of the feedback loop made the controller less sensitive to the loss of control effector effectiveness. In a comparison with the linear quadratic controller, the proposed predictive controller performed better in regulating the maximum stress and the rigid-body parameters. The proposed framework integrates aerostructural design with active control, which fully exploits the potential of an active aeroservoelastic wing, and making it possible to perform design optimization of high aspect ratio, flexible wings.

A 2-DOF mixed-norm control architecture that is capable of performing rigid-body tracking and load alleviation was also presented in this work. The control structure was specifically designed to improve the stability robustness of a very flexible aircraft when subjected to trim condition variations. The uncertainty model is obtained by trimming the aircraft at different cruise speeds and calculating the variation in the aircraft state-space matrices. The optimal control problem was cast in LMI form that can be solved efficiently using convex optimization techniques. The performance of the designed mixed-norm controller and an $\mathcal{H}_2$ controller were evaluated by performing gust load alleviation and climb/descent manoeuvre simulations. While both controllers achieved desirable performance with the nominal plant, the $\mathcal{H}_2$ controller destabilized the aircraft when it was not trimmed at the nominal cruise speed. On the other hand, the designed $\mathcal{H}_2/\mathcal{H}_\infty$ controller managed to stabilize the aircraft at different flight speeds and achieve desirable performance.

Finally, the analysis framework was used to compare two design cases: one for which the control system simply worked towards achieving or maintaining a target altitude, and another where the control system was also performing load alleviation. The design framework was use to optimize the wing of a HALE UAV, a large aspect ratio flying wing, and a futuristic transport aircraft wing. The use of the active load alleviation system resulted in a 21.7% improvement in the endurance of the HALE UAV relative to the optimum result without load alleviation. The inclusion of the load alleviation system in flying wing design optimization reduced power by
2.4%. In addition to that, the structural weight in the presence of the load alleviation system is lowered by 17.4%.

The results clearly show that the inclusion of control system discipline along with other disciplines at the early stages of aircraft design improves aircraft performance. It was also shown that structural stresses due to gust excitations can be better controlled by the use of active structural control systems.

6.1 Future Work

Although the general form of the developed equations of motion allows for detailed structural representations, a simple hollow tubular spar was used in this work. While the use of a simple spar representation was satisfactory for the purpose of this study, a more detailed structural representation should be used at the detailed design level. The availability of a detailed structural representation, along with unsteady aerodynamic loadings, can be used to perform flutter analysis. Flutter analysis is an integral part of aircraft design procedure. The presented results indicate that the application of the proposed methodology in aircraft design results in flexible wings that are prone to flutter failure. In order to reveal the full potential of the proposed integrated optimization framework, flutter analysis has to be included in the design process. In addition to that, the developed nonlinear equations along with the co-rotational beam framework should also be used to evaluate limit cycle oscillations and design controllers to damp out the vibration.

In order to avoid unrealistic control designs, control input saturation was considered in this work. To better address the control design problem, actuator dynamics should also be considered in the design problem. In addition, the availability of full state information was assumed in this work. Future designs can benefit from the inclusion of state observers which makes the problem more realistic. Finally, a simple parametric uncertainty model based on the aircraft cruise speed was represented in this work. A more detailed uncertainty model that better represents aerodynamic model inaccuracies is necessary.
Appendix A

The Lagrange’s Equations for Quasi-Coordinates

In this section the Lagrange’s equations for quasi-coordinates are derived from the original form of the Lagrange’s equations. The derivation begins from the matrix form of the Lagrange’s equations,

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = Q \tag{A.1}
\]

where \( q \) is the vector of the true (inertial) coordinates. In aircraft flight dynamics, the equations of motion are commonly expressed in a non-inertial body-fixed coordinate system. In the remainder of this section, \( w \)’s are the quasi-coordinates rates that can be represented as a linear combination of \( \dot{q} \)’s as follows,

\[
w = \sum_{r=1}^{n} \alpha_{rs} \dot{q}_r \tag{A.2}
\]

where \( n \) represents the number of degrees of freedom and \( \alpha_{rs} \) are known functions of the true coordinates \( q_i \). While the rate of change of the true coordinates \( \dot{q}_i \) can be integrated over time to calculate the corresponding true coordinates, the quasi coordinates cannot be found by integrating the rate of quasi-coordinates. As a result the original form of kinetic energy \( T(q, \dot{q}) \) is replaced with \( \bar{T}(q, w) \).

In order to drive the modified Lagrange’s equations, each term in Equation \( \text{[A.1]} \) have to be
Appendix A. The Lagrange’s Equations for Quasi-Coordinates

expressed in terms of the true coordinates and rates of the quasi-coordinates.

\[
\frac{\partial T}{\partial \dot{q}_k} = \sum_{i=1}^{n} \frac{\partial T}{\partial w_i} \frac{\partial w_i}{\partial \dot{q}_k} = \sum_{i=1}^{n} \alpha_{ki} \frac{\partial T}{\partial q_i} \Rightarrow \frac{\partial T}{\partial \dot{q}} = \alpha \frac{\partial T}{\partial \dot{w}} \tag{A.3}
\]

Using the above equation, the first term on the left hand-side of the Lagrange’s equation can be expressed as follows,

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) = \alpha \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{w}} \right) + \dot{\alpha} \frac{\partial T}{\partial \dot{w}} \tag{A.4}
\]

As mentioned before \(\alpha_{ij}\) are dependent of the true coordinates and \(\dot{\alpha}\) can be represented in the following form

\[
\dot{\alpha}_{ij} = \sum_{r=1}^{n} \frac{\partial \alpha_{ij}}{\partial q_r} \dot{q}_r = \dot{q}^T \left\{ \frac{\partial \alpha_{ij}}{\partial q} \right\} = w^T \beta^T \left\{ \frac{\partial \alpha_{ij}}{\partial q} \right\} \tag{A.5}
\]

It is worth mentioning that the above triple matrix product does not involve summation over the indices of \(\alpha_{ij}\) and the \(\dot{\alpha}\) term can be expressed in a matrix form as,

\[
\dot{\alpha} = \left[ w^T \beta^T \left\{ \frac{\partial \alpha}{\partial q} \right\} \right]. \tag{A.6}
\]

Using the same technique the second term of Equation \(A.1\) can also be expressed as

\[
\frac{\partial T}{\partial q_k} = \sum_{i=1}^{n} \frac{\partial T}{\partial w_i} \frac{\partial w_i}{\partial q_k} + \sum_{i=1}^{n} \frac{\partial T}{\partial \dot{w}_i} \left( \sum_{j=i}^{n} \frac{\partial \alpha_{ji}}{\partial q_k} \dot{q}_j \right) + \frac{\partial T}{\partial q_k} = \dot{q}^T \left[ \frac{\partial \alpha}{\partial q_k} \right] \frac{\partial \dot{w}}{\partial w} + \frac{\partial T}{\partial q_k} \tag{A.7}
\]

and can be represented in a matrix form of

\[
\frac{\partial T}{\partial q_k} = \left[ w^T \beta^T \left[ \frac{\partial \alpha}{\partial q_k} \right] \right] \frac{\partial \dot{w}}{\partial w} + \frac{\partial T}{\partial q_k} \tag{A.8}
\]

where the triple matrix products \(w^T \beta^T \left[ \frac{\partial \alpha}{\partial q_k} \right]\) results is \(n\) row matrices, each corresponding to an index of \(\partial q\). These \(n\) row matrices are then stacked up to form a square matrix \(\left[ w^T \beta^T \left[ \frac{\partial \alpha}{\partial q_k} \right] \right]\).

Using the Equations \(A.4\), \(A.6\) and \(A.8\) the original form of the Lagrange’s equations can be re-arranged into the following form,

\[
\alpha \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{w}} \right) + \left[ w^T \beta^T \left[ \frac{\partial \alpha}{\partial q} \right] \right] \left( \frac{\partial \dot{T}}{\partial \dot{w}} \right) + \frac{\partial T}{\partial \dot{w}} - \left[ w^T \beta^T \left[ \frac{\partial \alpha}{\partial q} \right] \right] \left( \frac{\partial \dot{T}}{\partial \dot{w}} \right) = Q \tag{A.9}
\]

Pre-multiplication by \(\beta^T\), the equations become,

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{w}} \right) + \beta^T \left[ \left[ w^T \beta^T \left[ \frac{\partial \alpha}{\partial q} \right] \right] - \left[ w^T \beta^T \left[ \frac{\partial \alpha}{\partial q} \right] \right] \right) \left( \frac{\partial \dot{T}}{\partial \dot{w}} \right) + \frac{\partial T}{\partial \dot{q}} = \beta^T Q \tag{A.10}
\]
In aircraft flight dynamics, the rate of the true coordinates and the quasi-velocity vector are

\[
\dot{q} = \begin{bmatrix}
\dot{R}_x & \dot{R}_y & \dot{R}_z & \dot{\theta}_x & \dot{\theta}_y & \dot{\theta}_z \\
\end{bmatrix}^T,
\]

\[
w = \begin{bmatrix}
V_x & V_y & V_z & \omega_x & \omega_y & \omega_z \\
\end{bmatrix}^T.
\]

And the corresponding coordinate transformation matrices have the following form,

\[
\alpha^T = \begin{bmatrix}
C_{bi} & 0 \\
O & D \\
\end{bmatrix}, \quad \beta^T = \alpha^{-1} = \begin{bmatrix}
C_{bi} & 0 \\
O & (D^T)^{-1} \\
\end{bmatrix}.
\]

It can be shown that,

\[
\beta^T E = \begin{bmatrix}
\tilde{\omega} & 0 \\
\tilde{V}_c & \tilde{\omega} \\
\end{bmatrix}.
\]

Substituting the above equation into Equation (A.10) the Lagrange’s equations for quasi-coordinates (Equations 2.1 and 2.2 as represented in Chapter 2) are derived.

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{V}_c} \right) + \tilde{\omega} \frac{\partial L}{\partial \dot{V}_c} - C_{bi} \frac{\partial L}{\partial R} = F,
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \tilde{\omega}} \right) + \tilde{V}_c \frac{\partial L}{\partial \dot{V}_c} + \tilde{\omega} \frac{\partial L}{\partial \omega} - D^T \frac{\partial L}{\partial \theta} = M.
\]
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