CONTROLLER DESIGN OF MULTIVARIABLE LTI UNKNOWN SYSTEMS

by

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A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
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Abstract

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2011

This thesis deals with the design of multivariable controllers for stable linear time-invariant multi-input multi-output systems, with an unknown mathematical model, subject to constant reference/disturbance signals and actuator saturation constraints. A new controller parameter optimization approach, which can be carried out experimentally with no knowledge of the plant model nor of the order of the system, is proposed. The approach has the advantage that controllers can be optimized by perturbing only the initial conditions of the servocompensator, and that the order of the resulting controller obtained can be specified by the designer. Implementation of the proposed controller design approach is described, and an experimental application study of the proposed method applied to a multivariable system with industrial sensor/actuator components is presented to illustrate the feasibility of the design method in an industrial environment.

Acknowledgements

It is my extreme honour and pleasure to have Professor E.J. Davison as my thesis supervisor. My gratitude for his continuous patience, guidance, and insights on “real world” problems is everlasting.

I would also like to extend my appreciation to Bartek Roszak and Simon Lam for their helping hand in the setup of the MARTS system. Thanks also go to all the members of the Systems Control Group for making my graduate years here an exciting, rewarding, and unforgettable experience that I will forever treasure.

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Chapter 1

Introduction

Many modern control techniques such as Model Predictive Control (MPC), Linear Quadratic Gaussian (LQG) control, and H-infinity control are model-based control methods. If the dynamic model of the process is initially unknown, one can obtain the model by way of system identification. System identification deals with the problem of obtaining a “good” mathematical model of a system, based solely on using the input-output data of a system. The system identification procedure can be broken down into four main steps [1]:

1. *Identification tests.* This stage of the procedure involves the design of input test signals that would make the measured data informative about system properties.
2. *Model order/structure selection.* The types of model that are useful for the application at hand depends on a number of factors such as the desired level of model accuracy, the operating conditions of the process, the type of noise in the environment, and any available *a priori* knowledge about the process structure and dominant time constant.
3. *Parameter estimation.* Once the experimental data is obtained and model structure is chosen, the parameters of the model can be estimated by using a variety of methods via optimization.
4. *Model validation.* At this stage, the model property is first compared against the process *a priori* knowledge. Other methods such as residual analysis and simulations can then be used to check if the model fits well with the test data.

The above procedure is an iterative process as it is possible for a model to fail the validation stage which may require re-identification. In general, system identification is a *time consuming and expensive process*, especially in the process industry where processes are typically large scale systems with complex dynamics, e.g., an industrial chemical plant may have a huge number of interacting subsystems. The following quotes from reference [1] suggest some of the downsides of system identification:

“... , after a long step test, an equally long time is needed to perform identification and hence it is often defined as a separate project phase. This is caused by two things. First, many disturbances occur during the long step test. Data slicing is used to cut the bad portion out, which is a tedious process. Secondly, there is no systematic approach to model order selection (or, settling time determination) and model validation/selection. Hence much time is spent using a trial-and-error approach.”

“Commonly used methods of model validation are simulation using estimation data or fresh data, whiteness tests residuals and the tests on the independence between the residuals and past MV's¹. These methods only tell how well the model agrees with the test data. They can neither quantify the model quality with respect to the purpose of closed-loop control, nor can they give good advice for re-identification. The trial-and-error approach in this step makes industrial identification a very expensive practice.”

The difficulties in obtaining a process model has motivated the interest and effort made towards developing controller design methods which are non model-based. The Ziegler-Nichols, Extremum Seeking, and Iterative Feedback Tuning are methods that fall under the non model-based controller design category and are reviewed next.

1.1 Ziegler-Nichols (for Single-Input Single-Output Systems)

1.1.1 Description of Ziegler-Nichols

Ziegler-Nichols (ZN) [2] is a popular heuristic method for tuning the parameters of a Proportional-Integral-Derivative (PID) controller of the form

$$u(t) = K \left(e(t) + \frac{1}{T_i} \int_0^t e(s) ds + T_d \frac{de(t)}{dt} \right). \quad (1.1)$$

¹MV stands for Manipulated Variable (or process input)

The method does not require knowledge of the plant's mathematical model and the controller gains can be computed based on a few simple experiments. There are two versions of the ZN tuning rules: the step-response method and the ultimate-sensitivity method [3].

The Step Response Method

In this method the unit step response of the open-loop system is performed. The tangent to the step response that has the steepest slope of slope R is drawn. The intersection of the tangent with the horizontal coordinate axis gives the apparent deadtime L of the system. See Figure 1.1. The PID controller parameters are then given by Table 1.1.

Figure 1.1: Determination of R and L parameters for the Ziegler-Nichols step response method [3].

Controller	K	T_i	T_d
P	$1/(RL)$		
PI	$0.9/(RL)$	$3L$	
PID	$1.2/(RL)$	$2L$	$0.5L$

Table 1.1: PID parameters obtained from Ziegler-Nichols step-response method [3].

The Ultimate-Sensitivity Method

In this method a proportional controller is connected to the plant under a unity

feedback configuration. The gain of the controller is slowly increased until the closed-loop system exhibits an undamped, oscillatory output response. The gain at which this occurs is known as the ultimate gain (K_u) and the period of oscillation is called the ultimate period (T_u). The PID controller parameters are then given by Table 1.2.

Controller	K	T_i	T_d
P	$0.5K_u$		
PI	$0.45K_u$	$T_u/1.2$	
PID	$0.6K_u$	$T_u/2$	$T_u/8$

Table 1.2: PID parameters obtained from Ziegler-Nichols ultimate-sensitivity method [3].

Even though the ZN method is very straight forward to use and often produces acceptable closed-loop performance, there are still a number of disadvantages listed as follows.

- The method is applicable only to *single-input single-output (SISO) systems* that are open-loop asymptotically stable.
- The method is designed for noise-free conditions and does not take into account actuator saturation constraints.
- The tuning rules do not apply to other controller structures such as integral or proportional-derivative control.
- The ultimate-sensitivity method of the ZN tuning rules requires the system to be brought to its stability limits , which could cause the system to become out of control.
- The tuning rules were originally designed for disturbance rejection performance and may give poor tracking performance.
- The step response method of the ZN tuning rules assumes that the system can be described as a first order time delay process of the form

$$G(s) = \frac{Ke^{-sT}}{(a + s)}. \quad (1.2)$$

For systems with a low time delay/time constant ratio the R and L parameters extracted from the unit step response will have small values. This will cause both the proportional and integral gains to become very large [4].

There are a number of methods that can be found in the literature that are similar to the ZN method. The Kappa-Tau tuning rules [5] characterizes the process dynamics using three parameters instead of two to give better closed-loop performance. The phase and amplitude margins design method [6] provides an alternative way of finding the ultimate gain and ultimate period without pushing the system to its stability limits. A description of other single-loop PID tuning techniques can be found in a survey presented by Åström et al. [7].

1.1.2 Extension to Multi-Input and Multi-Output Systems

One possible approach for extending the application of the ZN method to multi-input and multi-output (MIMO) asymptotically stable systems is by using a multi-loop PID scheme, where PID controllers are tuned specifically for each pair of inputs and outputs via the ZN method. The one-to-one pairing of the inputs and outputs can be selected based on a matrix known as the Relative Gain Array (RGA) [8]. RGA provides an indication of the level of interaction which occurs between input-output pairs and can be obtained experimentally as follows. Assume that we have a two input and two output system

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} g_1(s) & g_2(s) \\ g_3(s) & g_4(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (1.3)$$

and that we wish to control the system using the decentralized controller

$$u_1 = K_1(s)y_1 \quad (1.4)$$

$$u_2 = K_2(s)y_2. \quad (1.5)$$

Then to measure the level of interaction existing between these two controllers, we perform the following two sets of experiments:

For $u_1 = K_1(s)y_1$:

First set $u_2 = 0$ and measure the process gain:

$$u_2 = 0 : \quad \text{process gain } G_1 = \frac{y_1(\infty)}{u_1(\infty)} = g_1(0). \quad (1.6)$$

Next, regulate output y_2 to be 0 and measure the process gain:

$$y_2 = 0 : \quad \text{process gain } G_2 = \frac{y_1(\infty)}{u_1(\infty)} = g_1(0) - \frac{g_3(0)g_2(0)}{g_4(0)}. \quad (1.7)$$

The corresponding RGA measure for controller (1.4) is

$$\lambda_{11} = \frac{G_1}{G_2}. \quad (1.8)$$

For $u_2 = K_2(s)y_2$:

First set $u_1 = 0$ and measure the process gain:

$$u_1 = 0 : \quad \text{process gain } G_3 = \frac{y_2(\infty)}{u_2(\infty)} = g_4(0). \quad (1.9)$$

Next, regulate output y_1 to be 0 and measure the process gain:

$$y_1 = 0 : \quad \text{process gain } G_4 = \frac{y_2(\infty)}{u_2(\infty)} = g_4(0) - \frac{g_3(0)g_2(0)}{g_1(0)}. \quad (1.10)$$

The corresponding RGA measure for controller (1.5) is

$$\lambda_{22} = \frac{G_3}{G_4}. \quad (1.11)$$

According to Bristol [8], if λ_{ii} is positive and has a value of 1, this means that the control of output y_i using input u_i is unaffected by other control loops for $i = 1, 2$. Values that are much larger than 1 implies that independent control of each output is difficult to achieve, and input-output pairs that give rise to negative λ values should be avoided as the system can become unstable. It is to be noted that counter-examples can be given to these “rules” [9].

1.2 Extremum Seeking (for Single-Input Single-Output Systems)

1.2.1 Description of Extremum Seeking

Extremum seeking (ES) [10] is a nonmodel-based method that can be used either for seeking an unknown set point to achieve an optimal value of an unknown output function, or for tuning parameters of a feedback law. In general, it is mainly applied to SISO systems. Figure 1.2 shows a block diagram of the general sinusoidal perturbation-based

extremum seeking scheme. To illustrate how ES works, consider the search for only one unknown parameter initially. As described in [10], the input θ of an output objective function $f(\theta)$ is perturbed by a “slow” sinusoidal signal thereby creating a periodic response at the output. The high pass filter extracts the variation of the objective function in response to the perturbation signal or dither signal. The gradient ∇f is estimated by demodulating the filtered signal with a sinusoid of the same frequency as the dither signal. The gradient estimate is then integrated, yielding a new parameter estimate $\hat{\theta}$, which converges to the optimal parameters θ^* . A channel is composed of a high pass filter, a demodulating signal, an integrator and a dither signal. If there are now multiple unknown parameters to tune, a matching number of channels will be required, each with a different dither frequency since all channels share the same output $f(\theta)$. The overall feedback system operates on three different time scales [11]:

- fastest – the plant with the stabilizing controller,
- medium – the periodic perturbation,
- slowest – the filters in the extremum seeking scheme.

If the perturbation signal is slow, then the plant can be treated as a static map and its dynamics do not interfere with the extremum seeking scheme. In general, the plant is assumed to be open-loop asymptotically stable.

1.2.2 Application of Extremum Seeking for Controller Tuning

The extremum seeking scheme described in Section 1.2.1 can be applied to tune parameters of a controller using the setup shown in Figure 1.3. The step-response experiment, which is contained within the upper dash box in Figure 1.3, is carried out iteratively. The cost $J(\theta(k))$ is calculated during the step-response experiment with the controller parameters set as $\theta(k)$ at iteration k . The discrete version of the ES algorithm uses the value $J(\theta(k))$ to generate the next set of controller parameters $\theta(k+1)$ in the algorithm. Another step response experiment is then performed with the new parameters, and the process continues iteratively until the optimal parameters are found. This optimization approach has the advantage that no knowledge of the mathematical model of the plant is required. The time scale separation between the plant and the ES optimizer can be achieved by activating the ES optimizer only at instances when the step-response experiment reaches steady state. This is equivalent to decelerating the dynamics of the

Figure 1.2: Multiparameter extremum seeking with $i = 1, 2, \dots, m$. Each unknown parameter of interest θ_i requires 1 channel for tuning [10].

optimizer [12]. According to [13], the discrete ES algorithm can be implemented using the following equations, which are equivalent to the system representation shown in the lower dash box in Figure 1.3,

$$\zeta(k) = -h\zeta(k-1) + J(\theta(k-1)) \quad (1.12)$$

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) - \gamma_i \alpha_i \cos(\omega_i k) [J(\theta(k)) - (1+h)\zeta(k)] \quad (1.13)$$

$$\theta_i(k+1) = \hat{\theta}_i(k+1) + \alpha_i \cos(\omega_i(k+1)) \quad (1.14)$$

where the subscript i indicates the i^{th} entry of a vector, $\zeta(k)$ is a scalar, γ_i is the adaptation gain, α_i is the perturbation amplitude, and ω_i is the dither frequency. ω_i is chosen such that $\omega_i = a^i \pi$ with $0 < a < 1$. Additionally, the high pass filter is designed with $0 < h < 1$ and a cutoff frequency well below ω_i .

Setting up an extremum seeking controller tuning scheme for an unknown system is in general a non-trivial task since the achievable domain of attraction and convergence rate of the ES scheme *depends on careful selections of $\gamma, \alpha, \omega, h$, and the initial conditions of $\zeta(0)$* . The convergence speed of the overall system is affected by the magnitude of $\gamma\alpha^2$. The larger $\gamma\alpha^2$ is, the faster the convergence becomes, but now the domain of attraction becomes smaller [14]. Large γ and ω speeds up the convergence rate, but causes the closed-loop system to be sensitive to noise and can lead to steady state tracking error between the estimated optimal controller parameters $\hat{\theta}$ and the true optimal parameters

Figure 1.3: Overall extremum seeking controller tuning scheme [13].

θ^* [15]. The perturbation or dither amplitude α should be chosen sufficiently large to overcome the noise effect, and at the same time sufficiently small to obtain convergence to a smaller neighborhood of the optimal values and to prevent the closed-loop system from becoming unstable [16]. By examining equations 1.12 to 1.14 it appears that the choice of the initial condition $\zeta(0)$ also has an impact on the convergence rate and stability of the system. The extremum seeking scheme shown in Figure 1.2 assumes that the design of m channels for the identification/tracking of m parameters is required. As stated in [10], there are however *no systematic means of multivariable design for unknown systems, using this approach*.

1.3 Iterative Feedback Tuning (for Multivariable Controllers)

Another gradient-based, model-free tuning method is the iterative feedback tuning (IFT) [17]. In the IFT scheme the following cost function is adopted:

$$J(\rho) = \frac{1}{2N} E \left[\sum_{k=1}^N (L_y \tilde{y}_k(\rho))^2 + \lambda \sum_{k=1}^N (L_u u_k(\rho))^2 \right] \quad (1.15)$$

where ρ is a vector that contains the controller parameters, L_y and L_u are frequency weighting filters, $\tilde{y}_k$ denotes the error between the achieved and desired output response, u_k is the control input, and the subscript k represents discrete time instants. IFT attempts to minimize the cost function $J(\rho)$ by iteratively estimating the gradient of the cost function $\frac{\partial J}{\partial \rho}(\rho_i)$ with respect to the current controller parameters ρ_i .

At each step of the iterative design for a SISO plant, IFT uses only input-output data from three closed-loop experiments to estimate the gradient of the cost function. The first and third experiments involve a standard servomechanism setup where a reference signal is given and the input-output data is collected. The second experiment is different. It involves subtracting the output signal from the first experiment from the reference, and using this error signal as the new reference signal. The controller parameters are then updated by using a stochastic approximation algorithm of the form

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \frac{\partial J}{\partial \rho}(\rho_i) \quad (1.16)$$

where R_i is the Hessian of the cost function J and γ_i is the step size. Estimations of R_i and $\frac{\partial J}{\partial \rho}(\rho_i)$ can be computed using the data obtained from the three experiments.

One of the drawbacks of IFT, as discussed in [17], is that when feeding back the output at the reference input which is measured during the first experiment, noise that was captured with the output signal will be reinjected into the system during the second experiment and may cause unacceptable system behavior to occur. Another disadvantage of IFT is the *increase in complexity when applying to MIMO systems*. It is shown in [18] that if a plant has r outputs and m control inputs, $r \times m$ additional experiments are required to be able to compute an estimate of the gradient.

To overcome the disadvantages of the various tuning methods discussed, in this thesis we propose an alternative solution to the design of practical controllers for process control type problems. The proposed approach provides a systematic method of multi-variable design for MIMO unknown systems, copes with stability and actuator saturation constraints, and requires essentially no *a priori* knowledge of the plant. In process industries, the plants or processes are generally assumed to be open-loop asymptotically stable and subject to slow time-varying reference inputs and unmeasurable disturbances that can be approximated as constant signals [19]. The material presented in this thesis is developed within the discrete time framework, as computer-based automation systems are often used within the process industries.

1.4 Thesis Outline

In Chapter 2, the properties of a tuning regulator and a multivariable two-term controller that solve the robust servomechanism problem for unknown multi-input and multi-output systems are first discussed, and then an online controller parameter optimization method for designing a two-term controller is proposed. The focus of Chapter 2 then shifts to a discussion on the implementation of the proposed controller design approach, and in Chapter 3, the mechanism and properties of a chosen optimization algorithm that is used to solve the optimization problem is presented. The proposed design method is then illustrated through a number of simulation examples: a small-scale minimum phase system (Chapter 4), a large-scale minimum phase system (Chapter 5), a medium-scale nonminimum phase system (Chapter 6), and a single-input and single-output time delayed system (Chapter 7). Finally, an experimental application study of the proposed controller design approach is presented in Chapter 8.

Chapter 2

An Online Optimization Approach to the Robust Servomechanism Problem

A method for designing multivariable controllers that solves the robust servomechanism problem for mathematically unknown systems is proposed. The method involves performing real-time online parameter optimization on the system subject to stability and saturation constraints.

2.1 Problem Statement

Assume a sampled system to be controlled, described by the following linear shift-invariant model:

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k + E\omega \\y_k &= Cx_k + F\omega \\e_k &= y_k - y_{ref}\end{aligned}\tag{2.1}$$

where $x_k \in \mathbb{R}^n$ is the state of the system, $u_k \in \mathbb{R}^m$ is the input, $y_k \in \mathbb{R}^r$ is the output, $\omega \in \mathbb{R}^\Omega$ is a constant input disturbance, $y_{ref} \in \mathbb{R}^r$ is a constant reference input, $e_k \in \mathbb{R}^r$ is the error in the system, and there is no knowledge of the plant parameters (A, B, C, E, F) nor of the order n . It is assumed, however, that A is asymptotically stable¹, the control

¹i.e., the eigenvalues of A are all contained in the unit disk centered about the origin of the complex plane.

input u_k can be excited, the outputs y_k can be measured, and that the plant is sampled with a sampling period $h > 0$. The objective is to find a controller to solve the *robust servomechanism problem* (RSP), as described in [20], for this system, i.e., to find a linear, shift-invariant controller so that

1. the closed-loop system is asymptotically stable,
2. $\lim_{k \rightarrow \infty} e_k = 0$ for all $x_0 \in \mathbb{R}^n, u_0 \in \mathbb{R}^m, \omega \in \mathbb{R}^\Omega, y_{ref} \in \mathbb{R}^r$, and
3. property 2 holds for all perturbations in the plant model which do not cause the perturbed closed-loop system to become unstable.

Lemma 2.1 ([20]). The necessary and sufficient conditions for the existence of a solution to the RSP are that the following conditions should all hold:

1. (C, A, B) is stabilizable and detectable,
2. $m \geq r$, and
3. (C, A, B) has no transmission zeros at 1, i.e., $\text{rank} \left(\begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix} \right) = n + r$.

Definition 2.1. Define the *steady-state tracking gain matrix* of the system (2.1) to be

$$\Gamma = C(I - A)^{-1}B. \quad (2.2)$$

It is shown in [21] that Γ can be measured experimentally by carrying out a finite number of steady-state experiments on the system.

Since the system has been assumed to be open-loop asymptotically stable, the conditions of Lemma 2.1 are equivalent to requiring the steady-state tracking matrix Γ to have full row rank [21], i.e., $\text{rank}(\Gamma) = r$. It has been shown in [22] that if $\text{rank}(\Gamma) < r$, then there exists constant ω and y_{ref} signals so that for every control law u_k , either x_k will be unbounded or error regulation will not occur.

In reality, actuator saturation constraints are always present in every “real world” problem; thus, an additional necessary condition for the existence of a solution to RSP is that the control inputs of the plant should be sized so that the steady-state value of the i^{th} control input $u_\infty(i)$ is within the saturation bounds of the i^{th} actuator $u_{Sat}(i)$, $i = 1, 2, \dots, m$; otherwise, error regulation cannot be achieved [23]. In particular, it is assumed that the tracking and disturbance signals applied to the plant have the property that $|u_\infty(i)| \leq |u_{Sat}(i)|, i = 1, 2, \dots, m$.

2.2 Controller Structure

Assuming that $\text{rank}(\Gamma) = r$, then the following robust servomechanism controller for the plant (2.1) is now considered [22]

$$u_k = K_0 \hat{x}_k + K_1 \eta_k \quad (2.3)$$

where η_k is the output of a servocompensator, and where $\hat{x}_k$ is the output of a stabilizing compensator. For the case of a constant disturbance and tracking signal, the servocompensator is a controller defined as

$$\eta_{k+1} = \eta_k + h e_k \quad (2.4)$$

with input e_k , output η_k , and h is the sampling period. A stabilizing compensator has the following general structure:

$$\begin{aligned} \xi_{k+1} &= \Lambda_0 \xi_k + \Lambda_1 y_k + \Lambda_2 \eta_k + \Lambda_3 y_{ref} \\ \hat{x}_k &= \Lambda_5 \xi_k + \Lambda_6 y_k + \Lambda_7 y_{ref} \end{aligned} \quad (2.5)$$

with inputs $y_k, \eta_k, y_{ref}, u_k$ and output $\hat{x}_k$. As an example of a controller which satisfies (2.3) and (2.4), assume that $m = r$ and consider the multivariable two-term controller of the form

$$u_k = -S(K_P y_k + K_I \eta_k) \quad (2.6)$$

where $S = \Gamma^{-1}$ is a decoupling matrix, and K_P, K_I are scalar parameters of the controller. It is to be noted that if $m > r$, we replace the inverse of the Γ matrix by its pseudoinverse, i.e., $S = \Gamma^+$ where $(\cdot)^+ = (\cdot)^\top [(\cdot)(\cdot)^\top]^{-1}$.

2.2.1 Motivation

To give motivation for choosing controller (2.6), we will use the results of the following lemma. The proof of this lemma follows the approach taken in [24], which assumes a sample time of $h = 1$.

Lemma 2.2. Consider the system

$$\begin{aligned} x_{k+1} &= A x_k + B u_k \\ y_k &= C x_k \end{aligned} \quad (2.7)$$

subject to the following assumptions:

1. A is asymptotically stable,
2. Γ^{-1} exists, and
3. K is chosen so that ΓK has eigenvalues contained in the open left complex plane, e.g., $K = -\epsilon\Gamma^{-1}$ where $0 < \epsilon < 1$,

and let $Q \in \mathbb{R}^{m \times n}$ be a constant matrix; then there exists $\theta^* > 0$ such that, for all $\theta \in (0, \theta^*]$,

$$\mathcal{A} \triangleq \begin{bmatrix} A + \theta BQ & \theta BK \\ hC & I \end{bmatrix} \quad (2.8)$$

is asymptotically stable.

Proof. Let $\Phi \triangleq [C(I - A)^{-1}B]K$. There exists an $\epsilon^* > 0$ such that, for all $\theta \in (0, \epsilon^*]$, $I + \theta h\Phi$ has eigenvalues within the unit disk and is asymptotically stable. Since A and $I + \theta h\Phi$ are asymptotically stable, there exists unique positive definite matrices M, N that satisfy the discrete Lyapunov equations

$$\begin{aligned} A^\top M A - M &= -I \\ (I + \theta h\Phi)^\top N (I + \theta h\Phi) - N &= -I. \end{aligned} \quad (2.9)$$

Apply the coordinate transformation $\mathcal{A} \mapsto T^{-1}\mathcal{A}T$:

$$\begin{aligned} \Psi &= T^{-1}\mathcal{A}T \\ &= \begin{bmatrix} I & -(I - A)^{-1}BK\theta \\ 0 & \theta I \end{bmatrix} \begin{bmatrix} A + \theta BQ & \theta BK \\ hC & I \end{bmatrix} \begin{bmatrix} I & (I - A)^{-1}BK \\ 0 & \frac{I}{\theta} \end{bmatrix} \\ &= \begin{bmatrix} A + \theta\Delta_1(\theta) & \theta\Delta_3(\theta) \\ \theta\Delta_2(\theta) & I + \theta\Delta_4(\theta) \end{bmatrix} \end{aligned} \quad (2.10)$$

where

$$\begin{aligned} \Delta_1(\theta) &= BQ - h(I - A)^{-1}BKC \\ \Delta_2(\theta) &= hC \\ \Delta_3(\theta) &= -h(I - A)^{-1}BK\Phi + BQ(I - A)^{-1}BK \\ \Delta_4(\theta) &= h\Phi. \end{aligned}$$

Since eigenvalues are invariant under coordinate transformations, $\mathcal{A}$ is stable if and only if Ψ stable. Consider the system $\xi_{k+1} = \Psi\xi_k$ and define a tentative positive definite

Lyapunov function $V(\xi_k)$:

$$V(\xi_k) = \xi_k^\top \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix} \xi_k \quad (2.11)$$

where $M > 0$ and $N > 0$, then it remains to show that $\Delta V(\xi) \triangleq V(\xi_{k+1}) - V(\xi_k)$ is negative definite for Ψ to be asymptotically stable.

$$\begin{aligned} \Delta V(\xi) &= \xi_k^\top \left(\Psi^\top \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix} \Psi - \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix} \right) \xi_k \\ &\triangleq \xi_k^\top \tilde{\mathcal{S}} \xi_k \end{aligned} \quad (2.12)$$

where

$$\tilde{\mathcal{S}} = \left[\begin{array}{c|c} \begin{array}{c} A^\top M A - M \\ +\theta(A^\top M \Delta_1 + \Delta_1^\top M A) \\ +\theta^2(\Delta_1^\top M \Delta_1 + \Delta_2^\top N \Delta_2) \end{array} & \begin{array}{c} \theta(A^\top + \theta \Delta_1^\top) M \Delta_3 \\ +\theta \Delta_2^\top N (I + \theta \Delta_4) \end{array} \\ \hline \begin{array}{c} \theta \Delta_3^\top M (A + \theta \Delta_1) \\ +\theta(I + \theta \Delta_4^\top) N \Delta_2 \end{array} & \begin{array}{c} (I + \theta \Delta_4)^\top N (I + \theta \Delta_4) - N \\ +\theta^3(\Delta_4^\top N \Delta_5 + \Delta_5^\top N \Delta_4) \\ +\theta^2(N \Delta_5 + \Delta_5^\top N) + \theta^2 \Delta_3^\top M \Delta_3 \end{array} \end{array} \right].$$

Since $\tilde{\mathcal{S}}$ is a symmetric matrix, the principal minors can be used to determine if $\tilde{\mathcal{S}}$ is negative definite. Notice that for sufficiently small values of θ the main diagonal elements of $\tilde{\mathcal{S}}$ dominate. From (2.9) it is found that the diagonal elements reduce to $-I$ as $\theta \rightarrow 0$; thus, by inspection, there exists a $\theta^* > 0$ such that, for all $\theta \in (0, \theta^*]$, $\tilde{\mathcal{S}} < 0$, which implies $\Delta V(\xi \neq 0) < 0$. $\square$

On applying the controller (2.6) to the plant (2.1) we obtain the following closed-loop augmented system:

$$\begin{bmatrix} x_{k+1} \\ \eta_{k+1} \end{bmatrix} = \begin{bmatrix} A - BSK_P C & -BSK_I \\ hC & I \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + \begin{bmatrix} E - BSK_P F & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}. \quad (2.13)$$

Since A is stable, there exists a $K_P^* > 0$ so that $A - BSK_P C$ is stable for all $K_P \in [0, K_P^*]$. A stable state matrix $A - BSK_P C$ gives rise to a well defined steady-state tracking gain matrix $C(I - (A - BSK_P C))^{-1}B$. Recall that $C(I - A)^{-1}B$ has full rank from the assumption that there exists a solution to the servomechanism problem, then by continuity, $(C(I - (A - BSK_P C))^{-1}B)^{-1}$ exists. Since (see Appendix A.1)

$$(C(I - (A - BSK_P C))^{-1}B)^{-1} = (C(I - A)^{-1}B)^{-1} + SK_P, \quad (2.14)$$

then

$$-\Gamma(C(I - (A - BSK_PC))^{-1}B)^{-1} = -(1 + K_P)I \quad (2.15)$$

has its eigenvalues contained in the open left complex plane, which implies that its inverse $-(C(I - (A - BSK_PC))^{-1}B)S$ also has eigenvalues in the open left complex plane. Thus from Lemma 2.2, for a given $K_P \in [0, K_P^*]$, there exists a $K_I^* > 0$ such that, for all $K_I \in (0, K_I^*]$,

$$\begin{bmatrix} A - BSK_PC & -BSK_I \\ hC & I \end{bmatrix}$$

is asymptotically stable. In other words, sufficiently small K_P and K_I parameters of controller (2.6) guarantee closed-loop stability of the augmented plant (2.13), and it is shown in [25] that any compensator that stabilizes the augmented plant solves the RSP. Furthermore, the controller has a parameter space that is independent of the system order, and it can be applied to plants without a knowledge of their mathematical models, since the S matrix (2.6) can be determined experimentally.

2.2.2 Experimental Calculation of Steady-State Tracking and Disturbance Gain Matrices

Recall system (2.1)

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ew \\ y_k &= Cx_k + Fw \\ e_k &= y_k - y_{ref} \end{aligned}$$

where $x_k \in \mathbb{R}^n$ is the state of the system, $u_k \in \mathbb{R}^m$ is the input, $y_k \in \mathbb{R}^r$ is the output, $\omega \in \mathbb{R}^\Omega$ is the constant input disturbance, and $y_{ref} \in \mathbb{R}^r$ is a constant reference input, and assume that the system is open-loop asymptotically stable. Then at steady state, on denoting $\lim_{k \rightarrow \infty} x_k$, $\lim_{k \rightarrow \infty} y_k$, $\lim_{k \rightarrow \infty} u_k$ by x_∞ , y_∞ , u_∞ respectively, the system equations become

$$\begin{aligned} x_\infty &= Ax_\infty + Bu_\infty + E\omega \\ \Rightarrow x_\infty &= (I - A)^{-1}Bu_\infty + (I - A)^{-1}E\omega \\ y_\infty &= Cx_\infty + F\omega \\ &= (C(I - A)^{-1}B)u_\infty + (F + C(I - A)^{-1}E)\omega \end{aligned} \quad (2.16)$$

where $\Gamma = C(I - A)^{-1}B$ is the *steady-state tracking gain matrix*, and $\Lambda = (F + C(I - A)^{-1}E)$ is the *steady-state disturbance gain matrix*. Γ can be experimentally found using the following procedure [21, 26]:

Experimental Determination of Γ

1. Assume the output y_k can be measured, the inputs u_k can be excited and no disturbances ω are acting on the plant.
2. For $i = 1, 2, \dots, m$, apply a constant input vector $u_k = [0 \dots 0 \alpha_i 0 \dots 0]^\top$ to the plant (2.1), with $\alpha_i \neq 0$, and measure the corresponding steady-state output to obtain the vector $y_k = [\beta_i^1 \ \beta_i^2 \ \dots \ \beta_i^r]^\top$.
3. Solve the equation for Γ :

$$\Gamma \begin{bmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ & & \ddots & \\ 0 & 0 & \dots & \alpha_m \end{bmatrix} = \begin{bmatrix} \beta_1^1 & \beta_2^1 & \dots & \beta_m^1 \\ \beta_1^2 & \beta_2^2 & \dots & \beta_m^2 \\ & & \ddots & \\ \beta_1^r & \beta_2^r & \dots & \beta_m^r \end{bmatrix}.$$

Experimental Determination of Λ

Similarly, Λ can be experimentally found using the following procedure:

1. Assume the output y_k can be measured, the disturbances ω can be measured and be excited, and no inputs u_k are applied to the plant.
2. For $i = 1, 2, \dots, \Omega$, apply a constant disturbance vector $\omega = [0 \dots 0 \gamma_i 0 \dots 0]^\top$ to the plant (2.1), with $\gamma_i \neq 0$, and measure the corresponding steady-state output to obtain the vector $y = [\rho_i^1 \ \rho_i^2 \ \dots \ \rho_i^r]^\top$.
3. Solve the equation for Λ :

$$\Lambda \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ & & \ddots & \\ 0 & 0 & \dots & \gamma_\Omega \end{bmatrix} = \begin{bmatrix} \rho_1^1 & \rho_2^1 & \dots & \rho_\Omega^1 \\ \rho_1^2 & \rho_2^2 & \dots & \rho_\Omega^2 \\ & & \ddots & \\ \rho_1^r & \rho_2^r & \dots & \rho_\Omega^r \end{bmatrix}.$$

2.3 Tuning Regulator

If K_P in (2.6) is set to 0, the resulting controller is of the form of a *tuning regulator* [21]:

$$\begin{aligned}\eta_{k+1} &= \eta_k + h e_k \\ u_k &= -\epsilon S \eta_k\end{aligned}\tag{2.17}$$

where h is the sampling period, $S = \Gamma^{-1}$ and $\epsilon > 0$. The tuning regulator possesses three appealing properties:

Property 2.1. The tuning regulator (2.17) has the properties that

1. it solves the robust servomechanism problem.
2. it can be implemented without knowledge of the plant model.
3. There exists an $\epsilon^* > 0$ such that, for all $\epsilon \in (0, \epsilon^*]$ and $t \geq 0$, $\|u(t)\| \leq u_{sat}$, i.e., the control input actuators will not saturate at all times.

Properties 1 and 2 were already discussed in Sections 2.2.1 and 2.2.2, respectively. We will prove property 3 under the continuous time domain setup. If property 3 holds for the continuous controller

$$\begin{aligned}\dot{\eta} &= e \\ u &= -\epsilon S \eta,\end{aligned}\tag{2.18}$$

then by continuity, an implementation of the continuous controller using a “small enough sampling period” will yield the same results.

To prove the non-saturating property, we will consider an equivalent form of controller (2.18) in terms of the control input u perspective, i.e.,

$$\begin{aligned}\dot{\eta} &= \epsilon e \\ u &= -S \eta\end{aligned}\tag{2.19}$$

as it is in a form that will allow us to formulate the problem within a singular perturbation framework. The following theorem presents the singular perturbation result of interest. The following definition will be used in Theorem 2.1.

Definition 2.2 ([27]). $\delta_1(\epsilon) = O(\delta_2(\epsilon))$ if there exist positive constants k and ϵ^* such that, for all $\epsilon \in [0, \epsilon^*)$,

$$|\delta_1(\epsilon)| \leq k |\delta_2(\epsilon)|.\tag{2.20}$$

Theorem 2.1 ([27]). Consider the following system where $\epsilon > 0$ is a scalar

$$\begin{bmatrix} \frac{dx(t)}{dt} \\ \epsilon \frac{dz(t)}{dt} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}, \quad \begin{bmatrix} x(t_0) \\ z(t_0) \end{bmatrix} = \begin{bmatrix} x_0 \\ z_0 \end{bmatrix} \quad (2.21)$$

where A_4 is asymptotically stable, and assume $\epsilon \rightarrow 0$. Define the reduced system to be

$$\frac{d\bar{x}(t)}{dt} = A_1\bar{x}(t) + A_2\bar{z}(t), \quad \bar{x}(t_0) = x_0 \quad (2.22)$$

where

$$\bar{z}(t) = -A_4^{-1}A_3\bar{x}(t) \quad (2.23)$$

is the quasi-steady-state of z . Furthermore, define the boundary-layer system to be

$$\frac{dp(\tau)}{d\tau} = A_4p(\tau), \quad p(0) = z_0 - \bar{z}(t_0). \quad (2.24)$$

Assume that the following conditions are satisfied:

1. The reduced system (2.22) is exponentially stable.
2. The boundary-layer system (2.24) is exponentially stable.

Then there is a positive constant ϵ^* such that for all $t_0 \geq 0$, $\bar{x}(t_0)$, $p(0)$, and $0 < \epsilon < \epsilon^*$, the singular perturbation problem of (2.21) has a unique solution $x(t, \epsilon)$, $z(t, \epsilon)$ on $[t_0, \infty)$, and

$$\begin{aligned} x(t, \epsilon) - \bar{x}(t) &= O(\epsilon) \\ z(t, \epsilon) - \bar{z}(t) - p(t/\epsilon) &= O(\epsilon) \end{aligned} \quad (2.25)$$

hold uniformly for $t \in [t_0, \infty)$, where $\bar{x}(t)$ and $p(\tau)$ are the solutions of the reduced and boundary-layer problems (2.22) and (2.24).

Now we proceed to the proof of property 2.1, result 3. A similar proof can be found in [28] where the tuning regulator is applied to positive LTI systems.

Proof. Consider the continuous time LTI plant

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + E\omega \\ y(t) &= Cx(t) + F\omega \\ e(t) &= y(t) - y_{ref} \end{aligned} \quad (2.26)$$

and the tuning regulator

$$\begin{aligned}\dot{\eta}(t) &= \epsilon e(t) \\ u(t) &= -K\eta(t)\end{aligned}\tag{2.27}$$

where $K = (-CA^{-1}B)^{-1}$, $\epsilon > 0$, and A is assumed to be asymptotically stable. Applying controller (2.27) to plant (2.26) gives rise to the following closed-loop augmented plant:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} A & -BK \\ \epsilon C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} E & 0 \\ \epsilon F & -\epsilon \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ \eta(0) \end{bmatrix} \triangleq \begin{bmatrix} x_0 \\ \eta_0 \end{bmatrix}.\tag{2.28}$$

Define

$$\begin{bmatrix} z(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} - \begin{bmatrix} x(\infty) \\ \eta(\infty) \end{bmatrix},$$

system (2.28) becomes

$$\begin{bmatrix} \dot{z}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} A & -BK \\ \epsilon C & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ q(t) \end{bmatrix}, \quad \begin{bmatrix} z(0) \\ q(0) \end{bmatrix} = \begin{bmatrix} x_0 - x(\infty) \\ \eta_0 - \eta(\infty) \end{bmatrix} \triangleq \begin{bmatrix} z_0 \\ q_0 \end{bmatrix}.\tag{2.29}$$

(2.29) can be re-written as

$$\begin{aligned}\begin{bmatrix} \dot{q}(t) \\ \dot{z}(t) \end{bmatrix} &= \begin{bmatrix} 0 & \epsilon C \\ -BK & A \end{bmatrix} \begin{bmatrix} q(t) \\ z(t) \end{bmatrix} \\ \Rightarrow \begin{bmatrix} \frac{1}{\epsilon}\dot{q}(t) \\ \dot{z}(t) \end{bmatrix} &= \begin{bmatrix} 0 & C \\ -BK & A \end{bmatrix} \begin{bmatrix} q(t) \\ z(t) \end{bmatrix}.\end{aligned}\tag{2.30}$$

Let $\frac{1}{\epsilon}\frac{d(\cdot)}{dt} = \frac{d(\cdot)}{d\tau} = (\cdot)^\circ$, so that the system is finally expressed as

$$\begin{bmatrix} \dot{q}^\circ(t) \\ \epsilon \dot{z}^\circ(t) \end{bmatrix} = \begin{bmatrix} 0 & C \\ -BK & A \end{bmatrix} \begin{bmatrix} q(t) \\ z(t) \end{bmatrix}.\tag{2.31}$$

Our model has now been transformed into that of the singular perturbation model. We need to show that the reduced and boundary-layer models are exponentially stable. By setting $\epsilon = 0$ we obtain

$$\begin{aligned}0 &= -BKq(t) + Az(t) \\ \Rightarrow z(t) &= h(t, q(t)) = A^{-1}BKq(t) \\ \dot{q}^\circ(t) &= CA^{-1}BKq(t) \\ &= (CA^{-1}B)(-CA^{-1}B)^{-1}q(t) \\ &= -q(t).\end{aligned}\tag{2.32}$$

The expression for $q(t)$ can be obtained through integration:

$$\begin{aligned}
& \frac{dq(t)}{d\tau} = -q(t) \\
\Rightarrow & \int_{q_0}^{q(t)} \frac{dq(t)}{q(t)} = - \int_0^\tau d\tau \\
\Rightarrow & \ln q(t) = -\tau + \ln q_0,
\end{aligned} \tag{2.33}$$

which implies that

$$\begin{aligned}
q(t) &= e^{-\epsilon t} q_0 \\
z(t) &= A^{-1} B K e^{-\epsilon t} q_0.
\end{aligned} \tag{2.34}$$

Let the solutions of $q(t)$ and $z(t)$ obtained above in (2.34) be denoted as $\bar{q}(t)$ and $\bar{z}(t)$, respectively. These are the solutions to the reduced model, which is obtained on setting ϵ to 0. This model is exponentially stable. To obtain the boundary-layer model, we perform the change of variables $p(t) = z(t) - h(t, q(t))$ that shifts the quasi-steady-state of z to the origin. In the new variables $(q(t), p(t))$, the full problem is

$$\begin{aligned}
\dot{\bar{q}}(t) &= -DKq(t) + Cz(t) \\
&= -DKq(t) + Cp(t) + Ch(t, q(t)) \\
\epsilon \dot{\bar{p}}(t) &= \epsilon \dot{\bar{z}}(t) - \epsilon \dot{h}(t, q(t)).
\end{aligned} \tag{2.35}$$

Recall

$$\epsilon \frac{d(\cdot)}{d\tau} = \frac{d(\cdot)}{dt} = (\dot{\cdot}),$$

then

$$\begin{aligned}
\frac{dp(t)}{dt} &= \frac{dz(t)}{dt} - \frac{dh(t, q(t))}{dt} \\
&= \dot{z}(t) - A^{-1} B K \dot{q}(t) \\
&= -BKq(t) + Az(t) - A^{-1} B K \dot{q}(t) \\
&= -BKq(t) + Az(t) - A^{-1} B K (-\epsilon DKq(t) + \epsilon Cz(t)) \\
\dot{p}(t)|_{\epsilon=0} &= -BKq(t) + Ap(t) + BKq(t) + A^{-1} B K (\epsilon DKq(t) - \epsilon Cz(t))|_{\epsilon=0} \\
&= Ap(t).
\end{aligned} \tag{2.37}$$

Since A is assumed to be asymptotically stable, the origin $p(t) = 0$ of the boundary-layer model is exponentially stable. The solution of (2.37) is

$$\hat{p}(t) = e^{At} p(0) = e^{At} (z_0 - A^{-1} B K q_0). \tag{2.38}$$

According to Theorem 2.1 we can conclude that there exists a time $t_{O(\epsilon)}$ such that, for all $t \geq t_{O(\epsilon)}$,

$$\begin{aligned} q(t) - \bar{q}(t) &= O(\epsilon) \\ \Rightarrow q(t) &= e^{-\epsilon t} q_0 + O(\epsilon). \end{aligned} \quad (2.39)$$

Substituting back $q(t) = \eta(t) - \eta(\infty)$, we obtain

$$\eta(t) = \eta(\infty) + e^{-\epsilon t} (\eta_0 - \eta(\infty)) + O(\epsilon). \quad (2.40)$$

Similarly, for all $t \geq t_{O(\epsilon)}$,

$$\begin{aligned} z(t) - \bar{z}(t) - \hat{p}(t) &= O(\epsilon) \\ \Rightarrow z(t) &= \bar{z}(t) + \hat{p}(t) + O(\epsilon) \\ &= A^{-1}BK e^{-\epsilon t} q_0 + e^{At} (z_0 - A^{-1}BK q_0) + O(\epsilon). \end{aligned} \quad (2.41)$$

Substituting back $z(t) = x(t) - x(\infty)$, we obtain

$$\begin{aligned} x(t) &= x(\infty) + A^{-1}BK e^{-\epsilon t} (\eta_0 - \eta(\infty)) \\ &\quad + e^{At} \left((x_0 - x(\infty)) - A^{-1}BK (\eta_0 - \eta(\infty)) \right) + O(\epsilon). \end{aligned} \quad (2.42)$$

The $O(\epsilon)$ terms in equations (2.40) and (2.42) capture the transient peaking and/or overshoot portions of the $\eta(t)$ and $x(t)$ signals. This means that the amount of peaking or overshoot diminishes as we make ϵ smaller and smaller. In other words, $[x(t) \quad \eta(t)]^\top$ can be made to approach $[x(\infty) \quad \eta(\infty)]^\top$ monotonically, with no overshoot, but at the cost of a slower system response. It should be noted that the dependency on x_0 for $\eta(t)$ is encapsulated in the $O(\epsilon)$ term in (2.40). Since $u(t) = K\eta(t)$, if $[x_0 \quad \eta_0 \quad \omega \quad y_{ref}]^\top$ is chosen such that $\|u(\infty)\| \leq u_{sat}$ and a sufficiently small ϵ is used, saturation will not occur for all $t \geq 0$. $\square$

2.4 Controller Parameter Selection

Even though small values of K_P and K_I guarantee closed-loop stability, the system response can be excessively slow. Motivated by the *optimal transient response shaping* results in [29, 30], we propose that K_P and K_I are to be chosen by solving one of the two parameter optimization problems presented below, which will be referred to as *optimization problem A* and *optimization problem B*.

2.4.1 Optimization Problem A

Consider cascading the servocompensator (2.4) with the plant (2.1) to form an augmented plant given by

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ \eta_{k+1} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ hC & I \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} E & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix} \\ z_k &= \begin{bmatrix} \theta C & I \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} \end{aligned} \quad (2.43)$$

where θ is a non-negative scalar value. Now, let

$$\Delta x_k \triangleq x_k - x_{k-1}, \quad \Delta \eta_k \triangleq \eta_k - \eta_{k-1}, \quad \Delta u_k \triangleq u_k - u_{k-1}, \quad \Delta z_k \triangleq z_k - z_{k-1}. \quad (2.44)$$

Then, on substituting (2.44) into (2.43), we obtain the following equivalent representation of (2.43):

$$\begin{bmatrix} \Delta x_{k+1} \\ \Delta \eta_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ hC & I \end{bmatrix} \begin{bmatrix} \Delta x_k \\ \Delta \eta_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u_k \quad (2.45)$$

$$\Delta z_k = \begin{bmatrix} \theta C & I \end{bmatrix} \begin{bmatrix} \Delta x_k \\ \Delta \eta_k \end{bmatrix}. \quad (2.46)$$

The K_P and K_I parameters will be chosen so as to minimize the performance index

$$J_A \triangleq \frac{1}{h} \sum_{k=1}^{\infty} (\Delta z_k)^\top (\Delta z_k) + \mu (\Delta u_k)^\top (\Delta u_k), \quad \mu > 0, \quad (2.47)$$

subject to (2.45) and (2.46). Substituting the controller (2.6) into (2.45) yields the closed-loop equations

$$\begin{aligned} \tilde{x}_{k+1} &= \tilde{\mathcal{A}} \tilde{x}_k \\ \Delta z_k &= \tilde{\mathcal{C}}_z \tilde{x}_k \\ \Delta u_k &= \tilde{\mathcal{C}}_u \tilde{x}_k \end{aligned} \quad (2.48)$$

where

$$\begin{aligned} \tilde{\mathcal{A}} &= \begin{bmatrix} A - BSK_PC & -BSK_I \\ hC & I \end{bmatrix} \\ \tilde{\mathcal{C}}_z &= \begin{bmatrix} \theta C & I \end{bmatrix} \\ \tilde{\mathcal{C}}_u &= \begin{bmatrix} -SK_PC & -SK_I \end{bmatrix} \\ \tilde{x}_k &:= \begin{bmatrix} \Delta x_k \\ \Delta \eta_k \end{bmatrix} = \begin{bmatrix} x_k - x_{k-1} \\ \eta_k - \eta_{k-1} \end{bmatrix}. \end{aligned} \quad (2.49)$$

With the sampling index k starting from 1, Δz_k and Δu_k in (2.48) can be expressed as

$$\begin{aligned}\Delta z_k &= \tilde{\mathcal{C}}_z \tilde{\mathcal{A}}^{k-1} \tilde{x}_1 \\ \Delta u_k &= \tilde{\mathcal{C}}_u \tilde{\mathcal{A}}^{k-1} \tilde{x}_1.\end{aligned}\tag{2.50}$$

Next, on substituting the Δz_k and Δu_k from (2.50) into performance index (2.47), we obtain

$$J_A = \frac{1}{h} \tilde{x}_1^\top \left[\sum_{k=1}^{\infty} (\tilde{\mathcal{A}}^\top)^{k-1} (\tilde{\mathcal{C}}_z^\top \tilde{\mathcal{C}}_z + \mu \tilde{\mathcal{C}}_u^\top \tilde{\mathcal{C}}_u) (\tilde{\mathcal{A}})^{k-1} \right] \tilde{x}_1.\tag{2.51}$$

If we apply the change of variables $i = k - 1$, (2.51) becomes

$$J_A = \frac{1}{h} \tilde{x}_0^\top \left[\sum_{i=0}^{\infty} (\tilde{\mathcal{A}}^\top)^i (\tilde{\mathcal{C}}_z^\top \tilde{\mathcal{C}}_z + \mu \tilde{\mathcal{C}}_u^\top \tilde{\mathcal{C}}_u) (\tilde{\mathcal{A}})^i \right] \tilde{x}_0\tag{2.52}$$

or

$$J_A = \frac{1}{h} \tilde{x}_0^\top \tilde{\mathcal{P}} \tilde{x}_0\tag{2.53}$$

where $\tilde{\mathcal{P}} > 0$ is the solution of the discrete Lyapunov equation

$$\tilde{\mathcal{A}}^\top \tilde{\mathcal{P}} \tilde{\mathcal{A}} - \tilde{\mathcal{P}} + (\tilde{\mathcal{C}}_z^\top \tilde{\mathcal{C}}_z + \mu \tilde{\mathcal{C}}_u^\top \tilde{\mathcal{C}}_u) = 0.\tag{2.54}$$

The initial condition in (2.53) can be expressed as

$$\tilde{x}_0 = \begin{bmatrix} (A - BSK_P C) - I & -BSK_I \\ hC & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ \eta_0 \end{bmatrix} + \begin{bmatrix} E - BSK_P F & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}.\tag{2.55}$$

From (2.53), we can define the following performance indices:

- Assume $(y_0 \neq 0, \eta_0 = 0, \omega = 0, y_{ref} = 0)$:

$$J_{Ay_0} = \frac{1}{h} \times \text{trace} \left(\begin{bmatrix} ((A - BSK_P C) - I)^\top & hC^\top \end{bmatrix} \tilde{\mathcal{P}} \begin{bmatrix} (A - BSK_P C) - I \\ hC \end{bmatrix} \right).\tag{2.56}$$

This performance index measures the “average cost” of (2.47) over all initial conditions y_0 uniformly distributed on a unit disk ball [31]. Likewise, we can define the following performance indices:

- Assume $(x_0 = 0, \eta_0 \neq 0, \omega = 0, y_{ref} = 0)$:

$$J_{A\eta_0} = \frac{1}{h} \times \text{trace} \left(\begin{bmatrix} (-BSK_I)^\top & 0 \end{bmatrix} \tilde{\mathcal{P}} \begin{bmatrix} -BSK_I \\ 0 \end{bmatrix} \right).\tag{2.57}$$

- Assume $(x_0 = 0, \eta_0 = 0, \omega \neq 0, y_{ref} = 0)$:

$$J_{A\omega} = \frac{1}{h} \times \text{trace} \left(\begin{bmatrix} (E - BSK_P F)^\top & hF^\top \end{bmatrix} \tilde{\mathcal{P}} \begin{bmatrix} (E - BSK_P F) \\ hF \end{bmatrix} \right). \quad (2.58)$$

- Assume $(x_0 = 0, \eta_0 = 0, \omega = 0, y_{ref} \neq 0)$:

$$\begin{aligned} J_{Ay_{ref}} &= \frac{1}{h} \times \text{trace} \left(\begin{bmatrix} 0 & -h \end{bmatrix} \tilde{\mathcal{P}} \begin{bmatrix} 0 \\ -h \end{bmatrix} \right) \\ &= h \times \text{trace} \left(\begin{bmatrix} 0 & I \end{bmatrix} \tilde{\mathcal{P}} \begin{bmatrix} 0 \\ I \end{bmatrix} \right). \end{aligned} \quad (2.59)$$

The motivation for choosing such performance indices can be understood by first substituting the $\Delta\eta_{k+1} = hC\Delta x_k + \Delta\eta_k$ equation from (2.45) into (2.46) to give

$$\Delta z_k = \frac{\theta}{h} (\Delta\eta_{k+1} - \Delta\eta_k) + \Delta\eta_k. \quad (2.60)$$

On noting that $\Delta\eta_k = he_{k-1}$, (2.60) can then be re-written as

$$\begin{aligned} \Delta z_k &= \theta(e_k - e_{k-1}) + he_{k-1} \\ &= \theta\Delta e_k + he_{k-1}. \end{aligned} \quad (2.61)$$

If $\Delta z_k = 0$, this implies

$$\begin{aligned} e_k &= \left(1 - \frac{h}{\theta}\right) e_{k-1} \\ \Rightarrow e_k &= \left(1 - \frac{h}{\theta}\right)^k e_0, \end{aligned} \quad (2.62)$$

i.e., the error response has a smooth, noninteracting decaying exponential type of behavior whose time constant is set by the θ value. Note that (2.62) can also be obtained by sampling $e(t) = \exp(-\theta^{-1}t)e(0)$ at $t = kh$ and using the exponential series approximation of $\exp(-\theta^{-1}t) \approx I - h\theta^{-1}$ [29].

In addition to having the desired transient response characteristics embedded within it, the performance index (2.47) has the property that if a stabilizing compensator is applied to a minimum phase plant as to minimize (2.47), the eigenvalues of the controlled system will either approach the origin or the transmission zeros of the plant as $\mu \rightarrow 0$. This may not be true for other performance indices such as

$$J = \sum_{k=1}^{\infty} (\Delta e_k)^\top (\Delta e_k) + \mu (\Delta u_k)^\top (\Delta u_k) \quad (2.63)$$

since the system

$$\left\{ \begin{bmatrix} C & 0 \end{bmatrix}, \begin{bmatrix} A & 0 \\ hC & I \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix}, 0 \right\}$$

is nonminimum phase with transmission zeros at $z = 1$. As $\mu \rightarrow 0$, some closed-loop eigenvalues will approach $z = 1$, which means the system becomes closer to becoming a marginally stable system [25].

The reason for having a $1/h$ factor in the performance index (2.47) can be understood by substituting the output expression Δz_k from (2.61) into (2.47). If we assume θ and μ to be 0, the performance index simplifies to

$$\begin{aligned} J_A &= \frac{1}{h} \sum_{k=1}^{\infty} (he_{k-1})^\top (he_{k-1}) \\ &= h \sum_{k=1}^{\infty} (e_{k-1})^\top (e_{k-1}). \end{aligned} \quad (2.64)$$

Hence, we obtain the Riemann sum approximation of

$$J = \int_0^\infty e^\top e \, d\tau. \quad (2.65)$$

2.4.2 Optimization Problem B

Reconsider cascading now the servocompensator (2.4) with the plant (2.1) to form an augmented plant given by

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ \eta_{k+1} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ hC & I \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} E & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix} \\ z_k &= \begin{bmatrix} \theta C & I \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} \end{aligned} \quad (2.66)$$

where θ is a non-negative scalar value. Now, let

$$\Delta x_k \triangleq x_k - x_\infty, \quad \Delta \eta_k \triangleq \eta_k - \eta_\infty, \quad \Delta u_k \triangleq u_k - u_\infty, \quad \Delta z_k \triangleq z_k - z_\infty. \quad (2.67)$$

Then, on substituting (2.67) into (2.66), we obtain the following equivalent representation of (2.66):

$$\begin{bmatrix} \Delta x_{k+1} \\ \Delta \eta_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ hC & I \end{bmatrix} \begin{bmatrix} \Delta x_k \\ \Delta \eta_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u_k \quad (2.68)$$

$$\Delta z_k = \begin{bmatrix} \theta C & I \end{bmatrix} \begin{bmatrix} \Delta x_k \\ \Delta \eta_k \end{bmatrix}. \quad (2.69)$$

The K_P and K_I parameters will be chosen so as to minimize the performance index

$$J_B \triangleq h \sum_{k=0}^{\infty} (\Delta z_k)^\top (\Delta z_k) + \mu (\Delta u_k)^\top (\Delta u_k), \quad \mu > 0, \quad (2.70)$$

subject to (2.68) and (2.69). On substituting the controller (2.6) into (2.68) we obtain the closed-loop equations

$$\begin{aligned} \bar{x}_{k+1} &= \bar{\mathcal{A}} \bar{x}_k \\ \Delta z_k &= \bar{\mathcal{C}}_z \bar{x}_k \\ \Delta u_k &= \bar{\mathcal{C}}_u \bar{x}_k \end{aligned} \quad (2.71)$$

where

$$\begin{aligned} \bar{\mathcal{A}} &= \begin{bmatrix} A - BSK_PC & -BSK_I \\ hC & I \end{bmatrix} \\ \bar{\mathcal{C}}_z &= \begin{bmatrix} \theta C & I \end{bmatrix} \\ \bar{\mathcal{C}}_u &= \begin{bmatrix} -SK_PC & -SK_I \end{bmatrix} \\ \bar{x}_k &:= \begin{bmatrix} \Delta x_k \\ \Delta \eta_k \end{bmatrix} = \begin{bmatrix} x_0 - x_\infty \\ \eta_0 - \eta_\infty \end{bmatrix}. \end{aligned} \quad (2.72)$$

With the sampling index k starting from 0, Δz_k and Δu_k in (2.71) can be expressed as

$$\begin{aligned} \Delta z_k &= \bar{\mathcal{C}}_z \bar{\mathcal{A}}^k \bar{x}_0 \\ \Delta u_k &= \bar{\mathcal{C}}_u \bar{\mathcal{A}}^k \bar{x}_0 \end{aligned} \quad (2.73)$$

and on substituting Δz_k and Δu_k from (2.73) into performance index (2.70), yields

$$J_B = h \bar{x}_0^\top \left[\sum_{k=0}^{\infty} (\bar{\mathcal{A}}^\top)^k (\bar{\mathcal{C}}_z^\top \bar{\mathcal{C}}_z + \mu \bar{\mathcal{C}}_u^\top \bar{\mathcal{C}}_u) (\bar{\mathcal{A}})^k \right] \bar{x}_0 \quad (2.74)$$

or

$$J_B = h \bar{x}_0^\top \bar{\mathcal{P}} \bar{x}_0 \quad (2.75)$$

where $\bar{\mathcal{P}} > 0$ is the solution of the discrete Lyapunov equation

$$\bar{\mathcal{A}}^\top \bar{\mathcal{P}} \bar{\mathcal{A}} - \bar{\mathcal{P}} + (\bar{\mathcal{C}}_z^\top \bar{\mathcal{C}}_z + \mu \bar{\mathcal{C}}_u^\top \bar{\mathcal{C}}_u) = 0. \quad (2.76)$$

The initial condition in (2.75) can be expressed as (see Appendix A.2 for derivation)

$$\bar{x}_0 = \begin{bmatrix} x_0 \\ \eta_0 \end{bmatrix} - \begin{bmatrix} I - (A - BSK_PC) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} E - BSK_PF & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}. \quad (2.77)$$

From (2.75), we can now define the following performance indices:

- Assume $(y_0 \neq 0, \eta_0 = 0, \omega = 0, y_{ref} = 0)$:

$$J_{B_{y_0}} = h \times \text{trace} \left(\begin{bmatrix} I & 0 \end{bmatrix} \bar{\mathcal{P}} \begin{bmatrix} I \\ 0 \end{bmatrix} \right). \quad (2.78)$$

- Assume $(x_0 = 0, \eta_0 \neq 0, \omega = 0, y_{ref} = 0)$:

$$J_{B_{\eta_0}} = h \times \text{trace} \left(\begin{bmatrix} 0 & I \end{bmatrix} \bar{\mathcal{P}} \begin{bmatrix} 0 \\ I \end{bmatrix} \right). \quad (2.79)$$

- Assume $(x_0 = 0, \eta_0 = 0, \omega \neq 0, y_{ref} = 0)$:

$$J_{B_{\omega}} = h \times \text{trace} \left(\left(\begin{bmatrix} I - (A - BSK_P C) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} E - BSK_P F \\ hF \end{bmatrix} \right)^{\top} \times \right. \\ \left. \bar{\mathcal{P}} \times \left(\begin{bmatrix} I - (A - BSK_P C) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} E - BSK_P F \\ hF \end{bmatrix} \right) \right). \quad (2.80)$$

- Assume $(x_0 = 0, \eta_0 = 0, \omega = 0, y_{ref} \neq 0)$:

$$J_{B_{y_{ref}}} = h \times \text{trace} \left(\left(\begin{bmatrix} I - (A - BSK_P C) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -h \end{bmatrix} \right)^{\top} \times \right. \\ \left. \bar{\mathcal{P}} \times \left(\begin{bmatrix} I - (A - BSK_P C) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -h \end{bmatrix} \right) \right). \quad (2.81)$$

The motivation for choosing such a performance index and output Δz_k can be understood by first substituting the $\Delta \eta_{k+1} = hC\Delta x_k + \Delta \eta_k$ equation from (2.68) into (2.69) to give

$$\Delta z_k = \frac{\theta}{h} (\Delta \eta_{k+1} - \Delta \eta_k) + \Delta \eta_k. \quad (2.82)$$

Notice that

$$\begin{aligned} \eta_{k+1} - \eta_k &= he_k \\ \Rightarrow (\eta_{k+1} - \eta_{\infty}) - (\eta_k - \eta_{\infty}) &= he_k \\ \Rightarrow \Delta \eta_{k+1} - \Delta \eta_k &= he_k. \end{aligned} \quad (2.83)$$

Thus, output (2.82) can also be expressed as

$$\Delta z_k = \theta e_k + \Delta \eta_k. \quad (2.84)$$

If $\Delta z_k = 0$, this means that (2.82) becomes

$$\Delta\eta_{k+1} = (1 - \frac{h}{\theta})\Delta\eta_k \quad (2.85)$$

$$\Rightarrow \Delta\eta_k = (1 - \frac{h}{\theta})^k \Delta\eta_0. \quad (2.86)$$

Substituting (2.86) into (2.84) yields

$$e_k = -\frac{1}{\theta}(1 - \frac{h}{\theta})^k \Delta\eta_0. \quad (2.87)$$

This means that the ideal error response has a smooth, noninteracting decaying exponential type of behavior whose time constant is set by the θ value. Note that (2.87) can be obtained by sampling $e(t) = -\theta^{-1} \exp(-\theta^{-1}t) \Delta\eta(0)$ at $t = kh$ and using the approximation of $\exp(-\theta^{-1}t) \approx I - h\theta^{-1}$. The desirable properties of performance index (2.47) discussed in Section 2.4.1 hold for (2.70) as well.

2.5 Proposed Controller Tuning Approach

2.5.1 Tuning for Tracking Performance

Since the closed-loop matrices $(\tilde{\mathcal{A}}, \tilde{\mathcal{C}}_z, \tilde{\mathcal{C}}_y)$ in (2.49) are identical to the closed-loop matrices $(\bar{\mathcal{A}}, \bar{\mathcal{C}}_z, \bar{\mathcal{C}}_y)$ in (2.72) for given fixed values of K_P, K_I, θ and μ , solving the Lyapunov equation under either optimization problem A (2.54) or B (2.76) yields the same solution. This implies that minimizing the performance index $J_{A_{y_{ref}}}$ (2.59) will yield the same optimal controller as minimizing $J_{B_{\eta_0}}$ (2.79). In other words, we can indirectly tune the controller for optimal tracking performance by perturbing only the states of the servocompensator η_0 while minimizing performance index J_B (2.70). This approach is advantageous as the tuning process can then be carried out without perturbing the plant's initial states or changing the plant's tracking set-points.

2.5.2 Tuning for Disturbance Rejection Performance

If the disturbance signal ω can be excited, one can tune the controller for optimal disturbance rejection performance by minimizing the performance index $J_{A\omega}$ (2.58) or $J_{B\omega}$ (2.80). However, for many systems or processes ω can not be measured nor excited. For these systems, we propose to first tune the controller for tracking performance and have no disturbances acting upon the system during the optimization process, then apply this

same controller for disturbance rejection. It can be shown that a controller which is tuned for tracking will also perform well for disturbance rejection. For example, consider the following scalar 2 input and 2 output system:

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k \\y_k &= Cx_k \\e_k &= Cx_k - y_{ref}\end{aligned}\tag{2.88}$$

where disturbance $\omega = 0$ and $y_{ref} \neq 0$. As previously mentioned, the parameters of the two-term controller (2.6) for this system can be tuned by minimizing the performance index

$$J_A = \frac{1}{h} \sum_{k=1}^{\infty} (he_{k-1})^\top (he_{k-1}) + \mu(u_k - u_{k-1})^\top (u_k - u_{k-1})\tag{2.89}$$

over a representative class of y_{ref} signals distributed on the unit disk, e.g., $y_{ref} = [1 \ 0]^\top$, $y_{ref} = [0 \ 1]^\top$. The designed controller in this case

$$u_k = -S(K_{Popt}y_k + K_{Iopt}\eta_k)\tag{2.90}$$

will perform well for the tracking signals $y_{ref} = [1 \ 0]^\top$, $y_{ref} = [0 \ 1]^\top$, and the linear combination $y_{ref} = a[1 \ 0]^\top + b[0 \ 1]^\top$ for all $a, b \in \mathbb{R}$. Now, consider the system

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k \\y_k &= Cx_k + F\omega \\e_k &= Cx_k + F\omega\end{aligned}\tag{2.91}$$

where $\omega \neq 0$, $y_{ref} = 0$, and assume the disturbance term $F\omega \neq 0$. In this case, $F\omega$ is a 2×1 column vector that can be expressed as a linear combination of the vectors $[1 \ 0]^\top$ and $[0 \ 1]^\top$, i.e., the error e_k in (2.88) and (2.91) have the same form; thus, the controller (2.90) obtained from minimizing the error signal will also perform well for disturbance rejection. If the system has a state disturbance term, i.e., the system is described by

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k + E\omega \\y_k &= Cx_k \\e_k &= Cx_k\end{aligned}\tag{2.92}$$

where $\omega \neq 0$ and $y_{ref} = 0$, we can apply the following transformations to recast (2.92) into the form of (2.91): Let $x_k = \bar{x} + \Delta x_k$ and substitute it into (2.92) to obtain

$$\begin{aligned}\Delta x_{k+1} &= A\Delta x_k + Bu_k + E\omega + (A - I)\bar{x} \\ y_k &= C\Delta x_k + C\bar{x} \\ e_k &= C\Delta x_k + C\bar{x}.\end{aligned}\tag{2.93}$$

Furthermore, let $\bar{x} = (I - A)^{-1}E\omega$ and we obtain

$$\begin{aligned}\Delta x_{k+1} &= A\Delta x_k + Bu_k \\ y_k &= C\Delta x_k + C(I - A)^{-1}E\omega \\ e_k &= C\Delta x_k + C(I - A)^{-1}E\omega\end{aligned}\tag{2.94}$$

where $C(I - A)^{-1}E$ is the new F matrix. Systems (2.91) and (2.94) are now in the same form; thus, a controller which is tuned for tracking will also have good disturbance rejection performance for these classes of systems.

2.5.3 Using a Non-Noise Sensitive Performance Index

Choosing the optimization problem B method over A has the additional advantage that performance index J_B (2.70) is less sensitive to random error (i.e., noise) that can arise as a result of quantization effects, or unpredictable and unrepeatable fluctuations from experiments. This can be illustrated by the following analysis on the structure of performance index J_A (2.47) where we reconsider the performance index of optimization problem A given by

$$J_A = \frac{1}{h} \sum_{k=1}^{\infty} (\Delta z_k)^\top (\Delta z_k) + \mu (\Delta u_k)^\top (\Delta u_k)$$

where $\Delta z_k = \theta(e_k - e_{k-1}) + he_{k-1}$ and $\Delta u_k = u_k - u_{k-1}$. When noise is present, large values for the disturbance signal ω and tracking reference signal y_{ref} may be required to obtain good signal-to-noise ratio of the measurable signals for computing J_A due to the structure of Δu_k and Δz_k . This can be illustrated by first assuming a simple controller of the form is used:

$$u_k = K(y_k + W_k)\tag{2.95}$$

where W_k is a white Gaussian noise with zero mean and finite variance. Then

$$\Delta u_k = u_k - u_{k-1} = K(y_k - y_{k-1}) + K(W_k - W_{k-1})\tag{2.96}$$

where W_k and W_{k-1} are independent and identical distributed random variables. According to the Bienaymé equality [32], which states that the variance of a sum of independent random variables is equal to the sum of their variances, this implies that $W_k - W_{k-1}$ is a white Gaussian noise with twice the variance of W_k and the noise is further amplified by a factor of K . A similar analysis can be done to show that the $W_k - W_{k-1}$ term is also present in the expression for Δz_k and plays a role if $\theta \neq 0$. This implies that it becomes more difficult to obtain reliable cost values as the controller gains increase when solving optimization problem A.

2.6 Controller Comparison

If we choose our servomechanism controller to be

$$u_k = -S(K_P e_k + K_I \eta_k) \quad (2.97)$$

instead of the controller (2.6)

$$u_k = -S(K_P y_k + K_I \eta_k), \quad (2.98)$$

the performance index $J_{Ay_{ref}}$ (2.59) can be reexpressed as

$$\overline{J_{Ay_{ref}}} := J_A = \frac{1}{h} \times \text{trace} \left(\begin{bmatrix} (BSK_P)^\top & -h \end{bmatrix} \tilde{\mathcal{P}} \begin{bmatrix} BSK_P \\ -h \end{bmatrix} \right) \quad (2.99)$$

while the performance index $J_{B\eta_0}$ (2.79) remains to be the same

$$\overline{J_{B\eta_0}} = J_{B\eta_0} := J_B = h \times \text{trace} \left(\begin{bmatrix} 0 & I \end{bmatrix} \bar{\mathcal{P}} \begin{bmatrix} 0 \\ I \end{bmatrix} \right). \quad (2.100)$$

Minimizing $\overline{J_{Ay_{ref}}}$ (2.99) and $\overline{J_{B\eta_0}}$ (2.100) gives rise to different optimal controllers even if the $\tilde{\mathcal{P}}$ and $\bar{\mathcal{P}}$ matrices are the same. This implies that we no longer have the advantageous property of being able to tune the controller for tracking performance by perturbing only the initial conditions of the servocompensator η_0 while minimizing the performance index J_B (2.70), as already described in Section 2.5.1. This is another reason for choosing controller (2.6) in addition to the properties of the controller discussed in Section 2.2.1.

2.7 Experimental Implementation of Proposed Controller Design

The evaluation of the performance index $J_{B\eta_0}$ (2.79)

$$J_{B\eta_0} = h \times \text{trace} \left(\begin{bmatrix} 0 & I \end{bmatrix} \bar{\mathcal{P}} \begin{bmatrix} 0 \\ I \end{bmatrix} \right)$$

where $\bar{\mathcal{P}}$ is the solution to the discrete Lyapunov equation in (2.76), requires either a knowledge of the plant model or an infinite number of experiments to be performed in order to measure the “average cost” over all initial conditions η_0 uniformly distributed on a unit disk ball. Instead, we propose to take the approach of evaluating the performance index J_B (2.70)

$$J_B = h \sum_{k=0}^{\infty} (\Delta z_k)^\top (\Delta z_k) + \mu (\Delta u_k)^\top (\Delta u_k)$$

where $\Delta z_k = \theta e_k + (\eta_k - \eta_\infty)$ and $\Delta u_k = u_k - u_\infty$, using only the measurable signals $e_k, \eta_k, u_k, \eta_\infty$, and u_∞ for a finite number of points uniformly distributed on the unit disk ball. Each point on the unit ball corresponds to carrying out an experiment. This approach provides an approximation to the performance index $J_{B\eta_0}$ and the approximation improves as we increase the number of points on the unit ball. It should be noted that we need to consider only half of the unit disk ball when choosing the finite number of points due to symmetry. We will denote the chosen number of points on the unit ball by the variable q and Figure 2.1 illustrates the approach with $q = 5$. For example, assume the controller is $u_k = -2y_k - 0.5\eta_k$ and carry out an experiment with $\eta_0 = [0 \ 1]^\top$ to obtain the first cost value J_1 . We then repeat the experiment with $\eta_0 = [1/\sqrt{2} \ 1/\sqrt{2}]^\top$ to obtain the second cost J_2 . A total of 5 experiments is performed (each time with a different η_0) and an average value of the respective costs is then obtained. This average value represents the final cost for this specific controller (with gains $K_P = 2$ and $K_I = 0.5$) and is used as one data point in the controller parameter optimization. For convenience, we will denote the points chosen on the unit ball by a set denoted by $\mathcal{Z}$. For the example presented above we have

$$\mathcal{Z}_{\eta_0} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\} \quad (2.101)$$

where there are q number of elements and the subscript η_0 indicates the variable of interest. The experimental approach described can also be adopted for measuring the

average cost over y_{ref} or ω signals that are uniformly distributed on the unit ball. In general, x_0 can be assumed to be 0 if one lets the system settle to its equilibrium point before carrying out the experiments.

Figure 2.1: A selection of η_0 values for calculating the average cost.

2.7.1 Performance Index Modification

To assure that the resultant closed-loop system remains stable during the experimental parameter optimization, the following modified performance indices are used:

$$J_A = \begin{cases} \frac{1}{h} \sum_{k=1}^T (\Delta z_k)^\top (\Delta z_k) + \mu (\Delta u_k)^\top (\Delta u_k), & \text{if actuators do not saturate} \\ 10^{20}, & \text{otherwise} \end{cases} \quad (2.102)$$

where J_A is given by either (2.56), (2.57), (2.58), (2.59), and where $\Delta z_k = \theta(e_k - e_{k-1}) + h e_{k-1}$ and $\Delta u_k = u_k - u_{k-1}$. Likewise,

$$J_B = \begin{cases} h \sum_{k=0}^T (\Delta z_k)^\top (\Delta z_k) + \mu (\Delta u_k)^\top (\Delta u_k), & \text{if actuators do not saturate} \\ 10^{20}, & \text{otherwise} \end{cases} \quad (2.103)$$

where J_B is given by either (2.78), (2.79), (2.80), (2.81), and where $\Delta z_k = \theta e_k + (\eta_k - \eta_\infty)$ and $\Delta u_k = u_k - u_\infty$. Here the measure of instability used is that if the control signal saturates it is assumed that the closed-loop system is unstable. The duration of each experiment is chosen to be long enough such that both transient and steady state responses

are captured, i.e., T is a sufficiently large number to approximate ∞ . When actuator saturation occurs, due to system instability, a large value such as 10^{20} can be assigned to the performance index value J . This will force the optimization algorithm to converge to a set of controller gains that will maintain closed-loop stability and will not cause the actuators to saturate. This implies that the parameter optimization algorithm must not fail on discontinuous or non-differentiable functions (see Section 3.1 for the choice of algorithm). It is also desired to avoid actuators from saturating since a non-linear phenomenon known as *reset windup* [5] will occur if a controller with integral action is used which saturates. When this happens the actuator will remain at its limit independently of the plant's output and the error signal will continue to be integrated. This means that the integral term may become very large and will require the error signal to have an opposite sign for a long period to “undo” the integration. The consequence is that large transients may arise when the actuator saturates.

2.7.2 Controller Implementation

When implementing the discrete controller (2.6), the sampling period h must be chosen to be greater than the time h_c required to execute all of the following instructions during each sampling period:

1. read input port,
2. compute control signal,
3. set output port, and
4. update controller variables.

In view of the real-time scheduling constraints, the controller (2.6) can be implemented as

$$\begin{aligned} u_t &= -S(K_P y_{t-h_c} + K_I \eta_{t-h_c}) \\ \eta_t &= \eta_{t-h_c} + h e_{t-h_c} \\ &= \eta_{t-h_c} + h(y_{t-h_c} - y_{ref}) \quad , \end{aligned} \tag{2.104}$$

which is an “approximation” to the continuous controller

$$\begin{aligned} u(t) &= -S(K_P y(t) + K_I \eta(t)) \\ \dot{\eta}(t) &= e(t). \end{aligned} \tag{2.105}$$

The time sequence for controller (2.104) is shown in Figure 2.2.

Figure 2.2: The time sequence for the two-term controller.

2.8 Optimization Procedure

The controller parameter optimization process can be summarized into the following steps:

1. Choose a performance index either (2.56), (2.57), (2.58), (2.59), (2.78), (2.79), (2.80), or (2.81) depending on the nature of the controller design specifications.
2. Measure the steady-state tracking matrix Γ to determine if a controller exists that solves the robust servomechanism problem.
3. Select a set of y_0, η_0, ω or y_{ref} signals that are uniformly distributed on a scaled unit disk ball such that the steady-state value of the i^{th} control input is within the saturation limits of the i^{th} actuator, i.e., $|u_\infty(i)| \leq |u_{sat}(i)|$, $i = 1, 2, \dots, m$. According to (2.16), u_∞ can be expressed as $u_\infty = \Gamma^{-1}y_{ref} - \Gamma^{-1}\Lambda\omega$. Both the Γ and Λ matrices can be obtained experimentally using the procedure outlined in Section 2.2.2 to achieve this.

4. Precondition the plant with the tuning regulator $u_k = -\epsilon S\eta_k$, where ϵ is sufficiently small.
5. Carry out experiments to minimize the performance index J_A (2.102) or J_B (2.103). Start with a large control penalty coefficient μ in the performance index and choose sufficiently small K_P and K_I values as feasible starting gains for the optimization problem.
6. Decrease the μ value and repeat the optimization problem, using the optimal controller gains obtained in step 5 as a new starting point.
7. If a desired closed-loop response is achieved, stop.
If a desired closed-loop response is not achieved, this may or may not be due to the plant having nonminimum phase effects. Alternately, it may be due to the simplicity of the two-term controller being used, and so one could repeat the design process using a more complex controller.

Pre-conditioning the system with the tuning regulator in step 3 guarantees a non-saturating solution to the robust servomechanism problem. Recall from Section 2.3 that the tuning regulator has the property of achieving closed-loop stability without violating the saturation constraints. The addition of the controller $u_k = -S(K_P y_k + K_I \eta_k)$ is solely to improve the system response. If μ is large, the performance index or cost function is dominated by the control effort u_k . In the case of a stable plant, solving the optimization problem with $\mu \rightarrow \infty$ will result in $(K_P, K_I) \rightarrow (0, 0)$. Thus, a non-saturating solution to RSP is obtained, which is just the tuning regulator itself.

We will now apply the proposed controller design approach to 4 different classes of systems:

- Open-loop stable and minimum phase MIMO small-scale system (see Chapter 4).
- Open-loop stable and minimum phase MIMO large-scale system (see Chapter 5).
- Open-loop stable and nonminimum phase MIMO medium-scale system (see Chapter 6).
- Open-loop stable SISO system with time delay (see Chapter 7).

Chapter 3

Optimization Algorithm and Simulation Procedure

In this chapter, the mechanics and properties of the optimization algorithm that we chose to solve the two optimization problems A and B, are discussed. The properties of the algorithm are further elaborated on through comparisons made against gradient-based algorithms and other popular direct search methods. A procedure for carrying out the proposed controller design method in simulation is also provided.

3.1 The Chosen Algorithm

The optimization problems A and B will be solved using the *Nelder-Mead* algorithm [33], which is a direct search method for the minimization of a function of n variables. The algorithm operates by first comparing the function values at the $(n + 1)$ vertices of a simplex in a n dimensional parameter space, followed by the replacement of the vertex with the highest function value by another point obtained via one of the four operations at each iteration — reflection, contraction, expansion and shrinkage.

Assume a n dimensional simplex with vertices $v_0, v_1, \dots, v_n$, and let f_i be the function value at v_i . Further, define v_l to be the vertex with lowest function value f_l ; v_h to be the vertex with highest function value f_h ; and $\bar{v}$ to be the centroid of the remaining n vertices. The reflection of v_h over $\bar{v}$ along the line joining v_h and $\bar{v}$ is a new vertex v^* , whose coordinates are defined by the relation

$$v^* = \bar{v} + \alpha(\bar{v} - v_h) \quad (3.1)$$

where $\alpha > 0$ is the positive reflection coefficient. If $y_h > y^* > y_l$, then a new working simplex is formed by replacing v_h with v^* . If $y^* < y_l$, then the simplex is further expanded along the reflection line to a new vertex v^{**} by the relation

$$v^{**} = \bar{v} + \gamma(v^* - \bar{v}) \quad (3.2)$$

where $\gamma > 1$ is the expansion coefficient. If $y^{**} < y_l$, v_h is replaced by v^{**} ; otherwise, v_h is replaced by v^* .

If the initial reflection was unsuccessful, i.e., $y^* > y_i$ for all $i \neq h$, then v_h remains unchanged or is set to $v_h = v^*$ depending on which has a lower function value. A contracted point v^{**} is then formed by the relation

$$v^{**} = \bar{v} + \beta(v_h - \bar{v}) \quad (3.3)$$

where $0 < \beta < 1$ is the contraction coefficient and set $v_h = v^{**}$. However, if the contraction fails to give rise to a vertex with a lower function value than y_h , the current simplex is shrunk by replacing all the v_i by $(v_i + v_l)/2$.

In setting up the algorithm, we choose to set $\alpha = 1, \beta = 0.5, \gamma = 2$ and let the iterations continue until the length of the longest edge of the simplex becomes smaller than some tolerance value. The mechanics of the Nelder-Mead algorithm described above can be visualized as seen in Figures 3.1 and 3.2.

The Nelder-Mead method has some advantages and disadvantages:

Advantages

- The method does not involve approximating derivatives and thus is “robust,” i.e., it does not fail on discontinuous or non-differentiable functions. This allows us to impose stability and saturation constraints for solving the controller parameter optimization problem by modifying the function to take large positive values for controller gains that result in closed-loop instability or actuator saturation.
- The method has the ability to adapt itself to “local landscape” such that every trespassing by the simplex over constraint borders will be followed by contraction moves which force the simplex to remain in the feasible region [33].
- The method tends to give significant improvements in reducing the function value in the first few iterations. This is appealing in many practical problems such as process control, where each function evaluation can be time consuming and expensive.

Figure 3.1: Possible transformations of the simplex: (a) reflection, (b) expansion, (c) contraction, and (d) shrinkage [34].

Figure 3.2: The sequence of triangles $\{T_k\}$ converging to the optimal point for the Nelder-Mead algorithm [34].

Disadvantages

- The method can quickly approach the vicinity of the optimal point, but may be slow in locating the minimum itself, thus, requiring many more function evaluations as compared to gradient based methods.
- There are essentially no theoretical results or proofs on the convergence properties of the algorithm, except for the case of a one dimensional parameter space [35]. It may only give local solutions. As a result, one may need to try different starting points, different sizes or orientations of the initial simplex and/or repeat the algorithm multiple times before reaching the global minimizer.

The first motivation for using the Nelder-Mead method was to recognize that the performance index to be minimized may not be differentiable, which implies that gradient-based methods such as Quasi-Newton or Conjugate Gradient methods may have numerical problems. This situation can arise when a penalty function is used when the closed-loop eigenvalues are unstable. For example, penalty functions in the form of

$$F(x) = \begin{cases} f(x) & \text{if } x \in \Omega \\ +\infty & \text{otherwise} \end{cases} \quad (3.4)$$

or

$$F(x) = f(x) + \sum_{i=1}^m \omega_i \max(0, c_i(x)) \quad (3.5)$$

where $f(x)$ is the objective function, $c(x)$ is the constraint function, Ω is the feasible set and the weights $\omega_i > 0$ are chosen to be sufficiently large, clearly leads to a non-differentiable performance index.

The issues associated with gradient-based methods naturally lead to the investigation of heuristic methods or so called Direct Search methods. Some of the popular algorithms in this category includes Nelder-Mead, Pattern Search, and the Genetic Algorithm. Pattern search, in short, is a method where the the points at which the function is evaluated form a grid of points in $\mathbb{R}^n$. A grid is formed around the current iterate by selecting points that radiate outwards from this center point according to a set of specified directional vectors. In general, the grid is composed of at least $2n$ points, and the emphasis is on keeping the points close to the current iterate as to aquire a detailed local picture of the objective function before making a decision about where to move [36]. This means that all points in the grid are evaluated at each iteration. If the initial starting point is

far from the optimal point, much time is wasted on function evaluations that may not be necessary, where as Nelder-Mead performs only one or two function evaluations per iteration regardless of the size of n , except in shrink transformations, which are rare in practice [37].

The Genetic Algorithm is a method that simulates biological evolution to solve optimization problems. The algorithm starts out with a random initial population and creates a sequence of new generations, through random biological selections, crossovers, and mutations at each iteration, that evolve towards an optimal solution [38]. Similar to the concerns for Pattern Search, Genetic Algorithm involves a large number of function evaluations per iteration and can be computationally intensive. As a result of all the discussions thus far, we have chosen the Nelder-Mead algorithm for our applications.

3.2 Illustration Example

Consider an unstable SISO plant given by

$$G(s) = \frac{s^2 + 3s + 2}{s^2 - 8s + 15} \quad (3.6)$$

where the poles are located at $s = 3$ and $s = 5$. A proportional controller is to be designed to stabilize the plant as shown in Figure 3.3.

Figure 3.3: Block diagram of the feedback control system.

From the root locus plot shown in Figure 3.4, the plant can indeed be stabilized by some controller gain K . We propose that K is to be chosen by solving the following unconstrained optimization problem:

$$\underset{K}{\text{minimize}} \max(\Re\{\sigma(G(s))\}) \quad (3.7)$$

where $\sigma(\cdot)$ is the pole spectrum of the argument. The objective function (3.7) can be interpreted as minimizing the real component of the largest closed-loop pole, and is *non-differentiable* at where the break-in and breakaway occurs in the root locus diagram.

According to the root locus diagram, the optimal gain is $K = 116$ and the largest pole will be located at $s = -1.45$.

Figure 3.4: Root locus of the feedback control system.

By applying the Nelder-Mead algorithm, the following optimization output is obtained for the initial starting point of $K = 10^{-3}$:

Iteration	Func-count	min f(x)	Procedure
0	1	4.97891	
1	2	4.97786	initial simplex
2	4	4.97574	expand
3	6	4.97149	expand
4	8	4.96298	expand
5	10	4.94588	expand
6	12	4.91129	expand
7	14	4.84045	expand
8	16	4.69032	expand
9	18	4.32516	expand
10	20	3.72739	expand
11	22	3.48482	expand
12	24	3.06148	expand
13	26	2.39919	expand

14	28	1.52173	expand
15	30	0.583846	expand
16	32	-0.214277	expand
17	34	-0.771962	expand
18	36	-1.11015	expand
19	38	-1.29791	expand
20	40	-1.39706	expand
21	42	-1.44804	expand
22	44	-1.44804	contract inside
23	46	-1.44804	contract inside
24	48	-1.44804	contract inside
25	50	-1.45107	reflect
26	52	-1.45107	contract inside
27	54	-1.45246	reflect
28	56	-1.45246	contract inside
29	58	-1.45246	contract inside
30	60	-1.45279	reflect
31	62	-1.45279	contract inside
32	64	-1.45296	reflect
33	66	-1.45296	contract inside
34	68	-1.45296	contract inside
35	70	-1.45296	contract inside
36	72	-1.45296	contract inside
37	74	-1.45297	reflect
38	76	-1.45297	contract inside
39	78	-1.45297	reflect
40	80	-1.45297	contract inside
41	82	-1.45298	reflect
42	84	-1.45298	contract inside
43	86	-1.45298	contract inside
44	88	-1.45298	reflect
45	90	-1.45298	contract inside
46	92	-1.45298	reflect
47	94	-1.45298	contract inside
48	96	-1.45298	reflect
49	98	-1.45298	contract inside
50	100	-1.45298	contract inside

Optimization terminated:

the current x satisfies the termination criteria using
 OPTIONS.TolX of 1.000000e-004 and F(X) satisfies the
 convergence criteria using OPTIONS.TolFun of 1.000000e-004

optimal gain is $K = 115.9654$

real part of largest pole is -1.4530

The results show that the expected optimal gain and pole locations are obtained, but it also shows that the algorithm is slow in locating the minimum itself.

3.3 Description of the Simulation Procedure

The controller design process can be carried out in simulation, using a combination of MATLAB and Simulink, where the Simulink model simulates the closed-loop system response for generating input and output data and the MATLAB environment executes the Nelder-Mead algorithm. The process consists of the following steps:

1. Run the code for the Nelder-Mead algorithm in the MATLAB environment and provide a starting point for the algorithm.
2. Simulate the closed-loop response in Simulink whenever the performance index of a controller with specified gains needs to be evaluated. Meanwhile, the Nelder-Mead algorithm is paused in the background.
3. Calculate the performance index value or cost using the input and output data signals that are logged into the MATLAB workspace at the end of the current simulation run. If control input saturation occurs, the cost is assigned a value of 10^{20} .
4. Store the cost and resume the execution of the algorithm to determine a new set of controller gains to evaluate.
5. Repeat steps 2 to 4 until the algorithm converges.

Chapter 4

MARTS Simulation Example (Minimum Phase System)

4.1 Description of MARTS

The MARTS (Multivariable Apparatus for Real Time Control Studies) in [39] is a two vessel hydraulic system model with 2 inputs, 2 outputs, and 2 disturbance input channels. A schematic of MARTS is shown in Figure 4.1 with the inputs u and outputs y measured in volts. A $1V$ increase/decrease at the output represents approximately $2.4L$ of water rise/drop or a $30cm$ change in water level height. A $1V$ change in the input to control valve 1 and 2 corresponds to a change in water flow rate of approximately $0.0072L/s$ and $0.0108L/s$ respectively.

Figure 4.1: Schematic of MARTS setup.

By applying the identification methods given in [39] for the case when the interconnecting valve 1 is set to $\theta \approx 30^\circ$ and the disturbance signals ω disabled, the following approximate continuous linear time-invariant model is obtained:

$$\begin{aligned}\dot{x}(t) &= \mathcal{A}x(t) + \mathcal{B}u(t) \\ y(t) &= \mathcal{C}x(t) + \mathcal{D}u(t)\end{aligned}\tag{4.1}$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are given by

$$\begin{aligned}\mathcal{A} &= \begin{bmatrix} -6.18 \times 10^{-2} & 1.35 \times 10^{-2} \\ 1.35 \times 10^{-2} & -8.00 \times 10^{-2} \end{bmatrix} \\ \mathcal{B} &= \begin{bmatrix} 3.00 \times 10^{-3} & 0 \\ 0 & 4.50 \times 10^{-3} \end{bmatrix} \\ \mathcal{C} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \mathcal{D} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.\end{aligned}$$

The eigenvalues of this continuous system are given by

$$\{-8.7181 \times 10^{-2}, -5.4619 \times 10^{-2}\}$$

and there are no transmission zeros. If we sample the system with $h = 0.1$, the resultant sampled system is open-loop stable and minimum phase with no transmission zeros, and with eigenvalues given by

$$\{9.9945 \times 10^{-1}, 9.9913 \times 10^{-1}\}.$$

It is to be noted that the mathematical model of the plant (4.1) is used in this chapter only to carry out simulations, but *it is not used for controller design*.

The operating range of the control signals is given by

$$\begin{aligned}-5V &\leq u_k(1) \leq 5V \\ -5V &\leq u_k(2) \leq 5V.\end{aligned}$$

Open-loop responses of this system are given in Figure 4.2. The steady-state tracking matrix Γ (2.2) is experimentally obtained to be:

$$\Gamma = \begin{bmatrix} 5.04 \times 10^{-2} & 1.28 \times 10^{-2} \\ 8.51 \times 10^{-3} & 5.84 \times 10^{-2} \end{bmatrix}.$$

Figure 4.2: MARTS Example, Open-Loop Response.

4.2 Control Design 1 for MARTS

In this study, it is desired to design a controller to provide tracking for constant tracking signals $y_{ref}(1)$, $y_{ref}(2)$. The tuning regulator (2.17) with $\epsilon = 10^{-3}$ is first applied to the plant and the two-term controller (2.6) is then obtained by minimizing the performance index $J_{Ay_{ref}}$ (2.59) using $\theta = 0, \mu = 10^3$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \eta_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \mathcal{Z}_{y_{ref}} = \left\{ \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \begin{bmatrix} \frac{0.2}{\sqrt{2}} \\ \frac{0.2}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{0.2}{\sqrt{2}} \\ -\frac{0.2}{\sqrt{2}} \end{bmatrix} \right\}.$$

The above settings correspond to approximating the performance index $J_{Ay_{ref}}$ (2.59); in this case the optimal control gains K_P and K_I are found to be

$$K_P = 3.4475 \times 10^{-2}, \quad K_I = 9.3686 \times 10^{-4}. \quad (4.2)$$

Simulation of the tracking reference input is given in Figures 4.3-1 to 4.3-4 for the resulting closed-loop system.

Remark 4.1. Although the states x_k can not be measured, we allow the system to settle to its equilibrium point before performing each experiment; hence, x_0 can be assumed to be 0.

Remark 4.2. The y_{ref} signals are chosen from a disk of radius 0.2. A simple calculation of $u_\infty = \Gamma^{-1}y_{ref}$ shows that the steady state control input is within the actuator saturation bounds for the chosen set of tracking signals y_{ref} .

Remark 4.3. With the chosen y_{ref} signals, the steady-state control input $u_\infty = 4.12V$ is close to the saturation bounds $|u| < 5V$. It is seen that the closed-loop system using the tuning regulator is *unsaturated*, where the control input u_k monotonically approaches to u_∞ with no overshoot. A large control penalty coefficient μ is chosen to ensure that the addition of the tuned two-term controller does not cause saturation during the optimization process.

Figure 4.3-1: MARTS Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
Input $y_{ref} = [0.2 \ 0]^T$
Design 1.

Figure 4.3-2: MARTS Example, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $y_{ref} = [0 \ 0.2]^\top$
 Design 1.

Figure 4.3-3: MARTS Example, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.2)
 Input $y_{ref} = [0.2 \ 0]^T$
 Design 1.

Figure 4.3-4: MARTS Example, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.2)
 Input $y_{ref} = [0 \ 0.2]^T$
 Design 1.

Design 1:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	1000	14000
Iteration	Func-count	min f(x)	Procedure	
0	4	20.1123		
1	12	20.0702	initial simplex	
2	20	20.0204	expand	
3	28	19.9511	expand	
4	36	19.8197	expand	
5	44	19.6224	expand	
6	52	19.2823	expand	
7	60	18.7713	expand	
8	68	17.9822	expand	
9	76	16.9163	expand	
10	84	15.6248	expand	
11	92	14.497	expand	
12	100	14.2989	expand	
13	108	14.2989	contract	
14	116	14.198	contract	
15	120	14.198	reflect	
16	136	14.198	shrink	
17	144	14.1929	contract	
18	148	14.1929	reflect	
19	156	14.1927	contract	
20	164	14.1926	contract	
21	172	14.1923	contract	
22	176	14.1923	reflect	
23	192	14.1923	shrink	
24	200	14.1923	contract	
25	208	14.1923	contract	
26	216	14.1923	contract	
27	220	14.1923	reflect	
28	228	14.1923	contract	
29	244	14.1923	shrink	
30	252	14.1923	contract	
31	260	14.1923	contract	
32	268	14.1923	expand	
33	272	14.1923	reflect	
34	280	14.1923	expand	
35	288	14.1923	contract	
36	296	14.1923	expand	
37	304	14.1923	expand	
38	312	14.1923	expand	
39	316	14.1923	reflect	
40	324	14.1923	expand	
41	332	14.1923	expand	
42	340	14.1923	expand	
43	348	14.1923	expand	
44	352	14.1923	reflect	

45	360	14.1923	expand
46	368	14.1923	expand
47	376	14.1923	reflect
48	384	14.1923	contract
49	388	14.1923	reflect
50	396	14.1923	contract
51	404	14.1923	contract
52	412	14.1923	contract
53	420	14.1923	contract
54	428	14.1923	contract
55	432	14.1923	reflect
56	440	14.1923	contract
57	448	14.1923	contract
58	456	14.1923	contract
59	464	14.1923	contract
60	472	14.1923	contract
61	480	14.1923	contract
62	488	14.1923	contract
63	496	14.1923	reflect
64	504	14.1923	reflect
65	512	14.1923	contract
66	520	14.1923	contract
67	536	14.1923	shrink
68	544	14.1923	contract
69	552	14.1923	contract
70	560	14.1923	contract
71	568	14.1923	contract
72	576	14.1923	contract
73	584	14.1923	contract

Optimal KP Optimal KI Optimal cost J
0.0344746 0.000936862 14.1923

Algorithm run # 2

Starting KP Starting KI theta mu Duration T (sec)
0.0344746 0.000936862 0 1000 14000

Iteration	Func-count	min f(x)	Procedure
0	4	14.1923	
1	12	14.1923	initial simplex
2	20	14.1923	contract
3	36	14.1923	shrink
4	52	14.1923	shrink
5	68	14.1923	shrink
6	84	14.1923	shrink
7	100	14.1923	shrink
8	116	14.1923	shrink
9	132	14.1923	shrink
10	148	14.1923	shrink
11	164	14.1923	shrink
12	180	14.1923	shrink
13	196	14.1923	shrink
14	212	14.1923	shrink

15	228	14.1923	shrink
Optimal KP 0.0344746	Optimal KI 0.000936862	Optimal cost J 14.1923	

4.3 Control Design 2 for MARTS

The tuning regulator (2.17) with $\epsilon = 10^{-3}$ is first applied to the plant and the two-term controller (2.6) is then obtained by minimizing the performance index $J_{B\eta_0}$ (2.79) using $\theta = 0, \mu = 10^3$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, y_{ref} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \mathcal{Z}_{\eta_0} = \left\{ \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \begin{bmatrix} \frac{0.2}{\sqrt{2}} \\ \frac{0.2}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{0.2}{\sqrt{2}} \\ -\frac{0.2}{\sqrt{2}} \end{bmatrix} \right\}.$$

The above settings correspond to approximating the performance index $J_{B\eta_0}$ (2.79); in this case the optimal control gains K_P and K_I are found to be

$$K_P = 3.4475 \times 10^{-2}, \quad K_I = 9.3686 \times 10^{-4}. \quad (4.3)$$

Simulation of plant's initial condition input is given in Figures 4.4-1 to 4.4-4 for the resulting closed-loop system.

Remark 4.4. *The controller found by minimizing $J_{B\eta_0}$ (2.79) is identical to that obtained by minimizing $J_{Ay_{ref}}$ (2.59). The approach used in Control Design 2 has the advantage of being able to tune controllers without perturbing the plant's initial states, changing tracking set-points, or injecting disturbances into the system.*

Figure 4.4-1: MARTS Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
Input $\eta_0 = [0.2 \ 0]^\top$
Design 2.

Figure 4.4-2: MARTS Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
Input $\eta_0 = [0 \ 0.2]^\top$
Design 2.

Figure 4.4-3: MARTS Example, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.3)
 Input $\eta_0 = [0.2 \ 0]^T$
 Design 2.

Figure 4.4-4: MARTS Example, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.3)
 Input $\eta_0 = [0 \ 0.2]^T$
 Design 2.

Design 2:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	1000	14000
Iteration	Func-count	min f(x)	Procedure	
0	4	20.1122		
1	12	20.0701	initial simplex	
2	20	20.0203	expand	
3	28	19.951	expand	
4	36	19.8197	expand	
5	44	19.6224	expand	
6	52	19.2823	expand	
7	60	18.7713	expand	
8	68	17.9822	expand	
9	76	16.9163	expand	
10	84	15.6248	expand	
11	92	14.497	expand	
12	100	14.2989	expand	
13	108	14.2989	contract	
14	116	14.198	contract	
15	120	14.198	reflect	
16	136	14.198	shrink	
17	144	14.1929	contract	
18	148	14.1929	reflect	
19	156	14.1927	contract	
20	164	14.1926	contract	
21	172	14.1923	contract	
22	176	14.1923	reflect	
23	192	14.1923	shrink	
24	200	14.1923	contract	
25	208	14.1923	contract	
26	216	14.1923	contract	
27	220	14.1923	reflect	
28	228	14.1923	contract	
29	244	14.1923	shrink	
30	252	14.1923	contract	
31	260	14.1923	contract	
32	268	14.1923	expand	
33	272	14.1923	reflect	
34	280	14.1923	expand	
35	288	14.1923	contract	
36	296	14.1923	expand	
37	304	14.1923	expand	
38	312	14.1923	expand	
39	316	14.1923	reflect	
40	324	14.1923	expand	
41	332	14.1923	expand	
42	340	14.1923	expand	
43	348	14.1923	expand	
44	352	14.1923	reflect	

45	360	14.1923	expand
46	368	14.1923	expand
47	376	14.1923	reflect
48	384	14.1923	contract
49	388	14.1923	reflect
50	396	14.1923	contract
51	404	14.1923	contract
52	412	14.1923	contract
53	420	14.1923	contract
54	428	14.1923	contract
55	432	14.1923	reflect
56	440	14.1923	contract
57	448	14.1923	contract
58	456	14.1923	contract
59	464	14.1923	contract
60	472	14.1923	contract
61	480	14.1923	contract
62	488	14.1923	contract
63	492	14.1923	reflect
64	500	14.1923	reflect
65	508	14.1923	contract
66	516	14.1923	contract
67	524	14.1923	contract
68	532	14.1923	contract
69	540	14.1923	contract
70	548	14.1923	contract
71	556	14.1923	reflect
72	564	14.1923	contract
73	572	14.1923	contract
74	580	14.1923	contract
75	588	14.1923	contract

Optimal KP Optimal KI Optimal cost J
0.034475 0.000936863 14.1923

Algorithm run # 2

Starting KP Starting KI theta mu Duration T (sec)
0.034475 0.000936863 0 1000 14000

Iteration	Func-count	min f(x)	Procedure
0	4	14.1923	
1	12	14.1923	initial simplex
2	20	14.1923	contract
3	36	14.1923	shrink
4	52	14.1923	shrink
5	68	14.1923	shrink
6	84	14.1923	shrink
7	100	14.1923	shrink
8	116	14.1923	shrink
9	132	14.1923	shrink
10	148	14.1923	shrink
11	164	14.1923	shrink
12	180	14.1923	shrink

13	196	14.1923	shrink
14	212	14.1923	shrink
15	228	14.1923	shrink
Optimal KP	Optimal KI	Optimal cost J	
0.034475	0.000936863	14.1923	

4.4 Control Design 1 for MARTS Repeated under the Influence of Noise

The effectiveness of the design approach under the influence of noise in the outputs of the system can be briefly investigated by introducing an additive white Gaussian noise (AWGN) channel with 0 mean and a variance of 5×10^{-6} at the output of the plant. The two-term controller (2.6) is obtained by minimizing the performance index $J_{Ay_{ref}}$ (2.59) using $\theta = 0, \mu = 10^3$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \eta_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \mathcal{Z}_{y_{ref}} = \left\{ \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \begin{bmatrix} \frac{0.2}{\sqrt{2}} \\ \frac{0.2}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{0.2}{\sqrt{2}} \\ -\frac{0.2}{\sqrt{2}} \end{bmatrix} \right\};$$

in this case the optimal control gains K_P and K_I are found to be

$$K_P = 4.6517 \times 10^{-5}, \quad K_I = 8.5770 \times 10^{-4}. \quad (4.4)$$

Simulation of tracking reference input is given in Figures 4.5-1 and 4.5-2 for the resulting closed-loop system.

Remark 4.5. The mean and variance values of the white noise signal are obtained from experimental measurements made on MARTS.

Remark 4.6. The initial Nelder-Mead simplex is enlarged so that the performance of the various controllers being compared can be distinguished from each other.

Remark 4.7. The optimal control gains are significantly smaller than those obtained when noise was absent. As previously discussed, the structure of performance index J_A (2.102) is sensitive to noise with the result that Nelder-Mead converges prematurely to a non-stationary point.

Figure 4.5-1: MARTS Example with Noise, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.4)
 Input $y_{ref} = [0.2 \ 0]^T$
 Design 1 with Noise.

Figure 4.5-2: MARTS Example with Noise, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.4)
 Input $y_{ref} = [0 \ 0.2]^T$
 Design 1 with Noise.

Design 1 with Noise:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	1000	14000
Iteration	Func-count	min f(x)	Procedure	
0	4	22.9474		
1	12	22.9474	initial simplex	
2	20	22.5605	reflect	
3	28	22.5561	contract	
4	36	22.2176	reflect	
5	44	22.1868	reflect	
6	52	21.8889	reflect	
7	60	21.8297	reflect	
8	68	21.584	reflect	
9	76	21.504	reflect	
10	84	21.2949	reflect	
11	92	21.1885	reflect	
12	100	21.0137	reflect	
13	108	20.8959	reflect	
14	116	20.7519	reflect	
15	124	20.599	reflect	
16	132	20.5038	reflect	
17	140	20.3408	reflect	
18	148	20.2697	reflect	
19	156	20.0829	reflect	
20	164	20.0565	reflect	
21	172	19.8427	reflect	
22	176	19.8427	reflect	
23	184	19.6156	reflect	
24	192	17.0916	contract	
25	200	16.8835	reflect	
26	216	16.8835	shrink	
27	232	16.8835	shrink	
28	248	16.8835	shrink	
29	256	16.8835	contract	
30	264	16.8701	expand	
31	272	16.8111	expand	
32	280	16.7909	expand	
33	288	16.6242	expand	
34	292	16.6242	reflect	
35	300	16.3284	expand	
36	304	16.3284	reflect	
37	312	15.8196	expand	
38	316	15.8196	reflect	
39	324	15.0599	expand	
40	328	15.0599	reflect	
41	336	14.5441	expand	
42	344	14.3687	expand	
43	352	14.3687	contract	
44	360	14.3687	contract	

45	364	14.3687	reflect
46	368	14.3687	reflect
47	376	14.3675	contract
48	384	14.3632	contract
49	392	14.3537	contract
50	396	14.3537	reflect
51	412	14.3537	shrink
52	420	14.3527	contract
53	428	14.3527	contract
54	436	14.3513	contract
55	452	14.3513	shrink
56	456	14.3513	reflect
57	464	14.3513	contract
58	472	14.3506	reflect
59	488	14.3506	shrink
60	504	14.3506	shrink
61	508	14.3506	reflect
62	524	14.3506	shrink

Optimal KP	Optimal KI	Optimal cost J
4.13171e-005	0.000871634	14.3506

Algorithm run # 2

Starting KP	Starting KI	theta	mu	Duration T (sec)
4.13171e-005	0.000871634	0	1000	5000

Iteration	Func-count	min f(x)	Procedure
0	4	14.2495	
1	12	14.2495	initial simplex
2	28	14.2495	shrink
3	44	14.2495	shrink
4	52	14.2495	contract
5	60	14.2495	contract
6	76	14.2495	shrink
7	84	14.2495	contract
8	92	14.2495	contract
9	100	14.2481	contract
10	116	14.2481	shrink
11	132	14.2481	shrink
12	148	14.2481	shrink
13	152	14.2481	reflect
14	156	14.2481	reflect
15	172	14.2481	shrink
16	188	14.2481	shrink

Optimal KP	Optimal KI	Optimal cost J
4.6517e-005	0.000857696	14.2481

4.5 Control Design 2 for MARTS Repeated under the Influence of Noise

The AWGN channel is again introduced at the output of the plant. The two-term controller (2.6) is obtained by minimizing the performance index $J_{B\eta_0}$ (2.79) using $\theta = 0, \mu = 10^3$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, y_{ref} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \mathcal{Z}_{\eta_0} = \left\{ \begin{bmatrix} 0 \\ 100 \end{bmatrix}, \begin{bmatrix} \frac{100}{\sqrt{2}} \\ \frac{100}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 100 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{100}{\sqrt{2}} \\ -\frac{100}{\sqrt{2}} \end{bmatrix} \right\};$$

in this case the optimal control gains K_P and K_I are found to be

$$K_P = 2.8193 \times 10^{-2}, \quad K_I = 9.4791 \times 10^{-4}. \quad (4.5)$$

Simulation of servocompensator's initial condition input is given in Figures 4.6-1 to 4.6-4 for the resulting closed-loop system.

Remark 4.8. The large η_0 values are required to prevent the plant output from being completely masked by noise. It is seen that the tuning regulator with a small ϵ ensures that the closed-loop system will be *unsaturated* when η_0 is large. This time, the optimal control gains are approximately the same as those obtained when noise was absent. These results suggest that performance index J_B (2.103) should be used when designing controllers for experimental systems if no filters are used.

Remark 4.9. When evaluating a controller's performance for a specific η_0 setting, the experiment is performed only once, as opposed to repeating the same experiment multiple times to find an accurate measure. The motivation is to approach to the vicinity of the optimal point using as few function evaluations as possible.

Remark 4.10. When the Nelder-Mead algorithm converges to a point that is closer to the minimizer, one can restart the algorithm from this point and use a shorter fixed time duration for all subsequent experiments (corresponds to reducing T in the performance index) as the closed-loop system response becomes faster. In this case, a shorter time duration has the benefit of reducing the impact of noise on the cost value at steady state. If the duration of each experiment is much longer than the system's transient response time, then the cost does not well represent the performance of the controller as it is corrupted mostly by the accumulated noise at steady state.

Figure 4.6-1: MARTS Example with Noise, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $\eta_0 = [100 \ 0]^\top$
 Design 2 with Noise.

Figure 4.6-2: MARTS Example with Noise, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $\eta_0 = [0 \ 100]^\top$
 Design 2 with Noise.

Figure 4.6-3: MARTS Example with Noise, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.5)
 Input $\eta_0 = [100 \ 0]^\top$
 Design 2 with Noise.

Figure 4.6-4: MARTS Example with Noise, Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.5)
 Input $\eta_0 = [0 \ 100]^\top$
 Design 2 with Noise.

Design 2 with Noise:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	1000	14000
Iteration	Func-count	min f(x)	Procedure	
0	4	5.02799e+006		
1	12	4.89226e+006	initial simplex	
2	20	4.77073e+006	expand	
3	28	4.55873e+006	expand	
4	36	4.29557e+006	expand	
5	44	3.94579e+006	expand	
6	52	3.64872e+006	expand	
7	60	3.56664e+006	expand	
8	68	3.56664e+006	contract	
9	76	3.55465e+006	contract	
10	84	3.55428e+006	reflect	
11	92	3.54923e+006	contract	
12	100	3.54857e+006	reflect	
13	116	3.54764e+006	shrink	
14	124	3.54764e+006	contract	
15	140	3.54764e+006	shrink	
16	144	3.54764e+006	reflect	
17	152	3.54623e+006	contract	
18	168	3.54623e+006	shrink	
19	176	3.54623e+006	contract	
20	192	3.54623e+006	shrink	
21	196	3.54623e+006	reflect	
22	204	3.54623e+006	contract	
23	220	3.54623e+006	shrink	
24	236	3.54623e+006	shrink	
25	240	3.54623e+006	reflect	
26	248	3.54613e+006	contract	
27	264	3.54613e+006	shrink	
28	272	3.54613e+006	contract	
29	288	3.54613e+006	shrink	
30	292	3.54613e+006	reflect	
31	308	3.54613e+006	shrink	
Optimal KP	Optimal KI	Optimal cost J		
0.021107	0.000931408	3.54613e+006		

Algorithm run # 2

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.021107	0.000931408	0	1000	5000
Iteration	Func-count	min f(x)	Procedure	
0	4	3.54784e+006		
1	12	3.54784e+006	initial simplex	
2	20	3.54784e+006	contract	

3	28	3.54784e+006	contract
4	36	3.54784e+006	contract
5	44	3.54784e+006	contract
6	52	3.54706e+006	contract
7	60	3.54578e+006	contract
8	76	3.54578e+006	shrink
9	92	3.54578e+006	shrink
10	100	3.54578e+006	contract
11	116	3.54578e+006	shrink
12	132	3.54547e+006	shrink
13	148	3.54547e+006	shrink
14	152	3.54547e+006	reflect
15	156	3.54547e+006	reflect
16	172	3.54547e+006	shrink
17	176	3.54547e+006	reflect
18	184	3.54547e+006	contract
19	200	3.54547e+006	shrink
20	216	3.54547e+006	shrink
21	232	3.54547e+006	shrink
22	236	3.54547e+006	reflect
23	244	3.54547e+006	contract
24	252	3.54547e+006	contract
25	260	3.54547e+006	contract
26	276	3.54547e+006	shrink
27	292	3.54547e+006	shrink
Optimal KP	Optimal KI	Optimal cost J	
0.0281928	0.000947905	3.54547e+006	

4.6 Control Design 3 for MARTS

This study shows that “high performance control” can be achieved by decreasing the μ parameter in the performance index (J_B 2.103).

The tuning regulator (2.17) with $\epsilon = 10^{-3}$ is first applied to the plant and the two-term controller (2.6) is then obtained by minimizing the performance index J_{By_0} (2.78) using $\theta = 0, \mu = 1$. The optimization is carried out under the system settings of:

$$y_0 = \begin{bmatrix} -0.1 \\ -0.15 \end{bmatrix}, \quad \eta_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad y_{ref} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

in this case the optimal control gains K_P and K_I are found to be

$$K_P = 8.0229 \times 10^{-1}, \quad K_I = 6.3518 \times 10^{-2}. \quad (4.6)$$

Simulation of plant’s initial condition input and tracking reference input is given in Figures 4.7-1 to 4.7-8 for the resulting closed-loop system.

Remark 4.11. For the tuning regulator, the initial peak value of u_k decreases as the gain ϵ is made smaller.

Remark 4.12. In this example, the controller with gains (4.6) is optimal only for the initial condition $y_0 = [-0.1 \quad -0.15]^\top$. The “averaging” technique described in Section 2.7 was not applied.

Figure 4.7-1: MARTS Example, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $y_0 = [-0.1 \ 0]^T$
 Design 3.

Figure 4.7-2: MARTS Example, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $y_0 = [0 \ -0.15]^\top$
 Design 3.

Figure 4.7-3: MARTS Example , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.6)
 Input $y_0 = [-0.1 \ 0]^T$
 Design 3.

Figure 4.7-4: MARTS Example , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.6)
 Input $y_0 = [0 \ -0.15]^T$
 Design 3.

Figure 4.7-5: MARTS Example , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.6)
 Input $y_0 = [-0.1 \quad -0.15]^T$
 Design 3.

Figure 4.7-6: MARTS Example , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.6)
 Input $y_{ref} = [0.1 \ 0]^T$
 Design 3.

Figure 4.7-7: MARTS Example , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.6)
 Input $y_{ref} = [0 \ 0.15]^T$
 Design 3.

Figure 4.7-8: MARTS Example , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (4.6)
 Input $y_{ref} = [0.1 \ 0.15]^T$
 Design 3.

Design 3:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	1	14000
Iteration	Func-count	min f(x)	Procedure	
0	1	4681.85		
1	3	4671.31	initial simplex	
2	5	4657.62	expand	
3	7	4641.18	expand	
4	9	4606.12	expand	
5	11	4556.95	expand	
6	13	4466.23	expand	
7	15	4332.2	expand	
8	17	4110.83	expand	
9	19	3795.87	expand	
10	21	3346.87	expand	
11	23	2794.83	expand	
12	25	2169.21	expand	
13	27	1566.05	expand	
14	29	1045.47	expand	
15	31	653.869	expand	
16	33	385.277	expand	
17	35	222.244	expand	
18	37	138.48	expand	
19	39	115.815	expand	
20	40	115.815	reflect	
21	42	115.815	contract	
22	46	115.8	shrink	
23	50	115.679	shrink	
24	52	115.679	contract	
25	54	115.679	contract	
26	58	115.679	shrink	
27	62	115.679	shrink	
28	64	115.679	contract	
29	66	115.676	expand	
30	67	115.676	reflect	
31	69	115.672	expand	
32	70	115.672	reflect	
33	72	115.669	expand	
34	74	115.657	expand	
35	76	115.655	expand	
36	78	115.62	expand	
37	79	115.62	reflect	
38	81	115.554	expand	
39	82	115.554	reflect	
40	84	115.426	expand	
41	86	115.37	expand	
42	88	115.063	expand	
43	90	114.911	expand	
44	92	114.196	expand	

45	93	114.196	reflect
46	95	112.932	expand
47	97	112.161	expand
48	99	109.779	expand
49	101	107.325	expand
50	103	104.162	expand
51	105	103.705	expand
52	107	103.705	contract
53	109	103	contract
54	110	103	reflect
55	114	102.965	shrink
56	116	102.954	contract
57	118	102.95	contract
58	120	102.944	contract
59	122	102.944	contract
60	124	102.942	contract
61	126	102.941	contract
62	128	102.941	contract
63	130	102.941	contract
64	132	102.941	contract
65	134	102.941	contract
66	136	102.941	contract
67	138	102.941	contract
68	139	102.941	reflect
69	140	102.941	reflect
70	142	102.941	contract
71	144	102.941	contract
72	146	102.941	contract
73	150	102.941	shrink
74	152	102.941	contract
75	154	102.941	contract
76	156	102.941	contract
77	158	102.941	contract
78	160	102.941	contract
79	162	102.941	contract
80	164	102.941	contract
81	166	102.941	contract
82	168	102.941	contract
83	170	102.941	contract
84	172	102.941	contract
85	174	102.941	contract
86	176	102.941	contract
87	178	102.941	contract
88	180	102.941	contract
89	182	102.941	contract

Optimal KP	Optimal KI	Optimal cost J
0.802289	0.0635183	102.941

Algorithm run # 2

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.802289	0.0635183	0	1	14000

Iteration	Func-count	min $f(x)$	Procedure
0	1	102.941	
1	3	102.941	initial simplex
2	4	102.941	reflect
3	6	102.941	contract
4	8	102.941	contract
5	10	102.941	contract
6	12	102.941	contract
7	14	102.941	contract
8	15	102.941	reflect
9	17	102.941	contract
10	19	102.941	contract
11	21	102.941	contract
12	22	102.941	reflect
13	24	102.941	contract
14	26	102.941	contract
15	28	102.941	contract
16	30	102.941	contract
17	32	102.941	contract
18	34	102.941	contract
19	36	102.941	contract
20	38	102.941	contract
21	40	102.941	contract
22	42	102.941	contract
23	44	102.941	contract
24	46	102.941	contract
25	48	102.941	contract
26	50	102.941	contract
27	52	102.941	contract
28	54	102.941	contract
29	56	102.941	contract
30	58	102.941	contract
31	60	102.941	contract
32	62	102.941	contract
33	64	102.941	contract
34	66	102.941	contract
35	67	102.941	reflect
36	69	102.941	contract
37	71	102.941	contract
38	73	102.941	contract
39	74	102.941	reflect
40	76	102.941	contract
41	78	102.941	contract
Optimal KP 0.802289	Optimal KI 0.0635183	Optimal cost J 102.941	

4.7 Summary

The proposed controller design approach is applied to a small-scale system that is open-loop stable and minimum phase. A tuning regulator with a sufficiently small gain ϵ ensures that the control input u_k monotonically approaches to the steady state value u_∞ with no overshoot. Furthermore, as ϵ decreases, “peaking” in the control input u_k diminishes. Solving either optimization problem A or B gives rise to the same optimal controller, but *the performance index J_B (2.103) is less sensitive to noise and can be minimized without disturbing any of the plant’s settings.*

Chapter 5

Williams and Otto Chemical Plant Example (Minimum Phase System)

5.1 Description of the Williams and Otto Chemical Plant

The Williams and Otto chemical plant in [40] is described by a 41st order model with 4 inputs, 4 outputs, and 3 disturbance input channels. The continuous plant model is given in Appendix B. The eigenvalues and transmission zeros of this continuous system are given by

Eigenvalues =	Transmission zeros =
-3.5000e+002	-1.7695e+002
-2.0640e+001	-3.5000e+002
-1.6420e+001	-3.5000e+002
-3.5000e+002	-3.5000e+002
-8.2720e+001	-3.5000e+002
-4.5510e+001	-3.5000e+002
-1.8915e+001 +1.0789e+001i	-8.2392e+001 +5.0909e-001i
-1.8915e+001 -1.0789e+001i	-8.2392e+001 -5.0909e-001i
-8.1561e-001	-8.3120e+001
-6.2046e+000	-8.2720e+001
-6.1025e+002	-8.2606e+001
-1.9544e+002	-5.8963e+001
-1.7695e+002	-4.5510e+001
-3.5000e+002	-4.5303e+001
-3.5000e+002	-4.0598e+001 +2.1082e+000i
-3.5000e+002	-4.0598e+001 -2.1082e+000i
-3.5000e+002	-4.2363e+001
-8.2223e+001	-2.0599e+001 +1.1060e+001i

-8.3146e+001		-2.0599e+001	-1.1060e+001i
-8.2607e+001		-1.8885e+001	+1.1275e+001i
-8.2688e+001		-1.8885e+001	-1.1275e+001i
-5.8594e+001		-1.8915e+001	+1.0789e+001i
-4.6270e+001		-1.8915e+001	-1.0789e+001i
-4.5125e+001		-1.9842e+001	
-4.1266e+001		-1.5695e+001	
-3.5820e+001		-9.5391e+000	+3.8361e-001i
-3.5197e+001		-9.5391e+000	-3.8361e-001i
-1.8873e+001	+1.0987e+001i	-6.2046e+000	
-1.8873e+001	-1.0987e+001i	-5.1118e+000	
-2.1160e+001	+9.9877e+000i	-1.7432e+000	+1.9494e+000i
-2.1160e+001	-9.9877e+000i	-1.7432e+000	-1.9494e+000i
-2.0584e+001		-4.2012e-001	
-1.5720e+001		-9.5316e-001	
-9.7921e+000		-8.1561e-001	
-7.6450e+000		-2.0640e+001	
-6.7738e+000		-3.5000e+002	
-4.9051e+000			
-3.3063e-001			
-6.0560e-001			
-9.9564e-001	+3.9048e-002i		
-9.9564e-001	-3.9048e-002i		

If we sample the system with $h = 0.01$, the resultant sampled system is open-loop stable and minimum phase with eigenvalues and transmission zeros given by

Eigenvalues =	Transmission zeros =
3.0197e-002	-8.0444e-002
8.1351e-001	1.7041e-001
8.4857e-001	4.3872e-001 +2.3065e-003i
4.3727e-001	4.3872e-001 -2.3065e-003i
2.2372e-003	4.3553e-001
6.3438e-001	4.3727e-001
1.4165e-001	4.3777e-001
1.7041e-001	5.5448e-001
8.2285e-001 +8.9127e-002i	6.3438e-001
8.2285e-001 -8.9127e-002i	6.3571e-001
9.3984e-001	6.5496e-001
9.9188e-001	6.6605e-001 +1.4500e-002i
9.9670e-001	6.6605e-001 -1.4500e-002i
9.9396e-001	8.0890e-001 +8.9829e-002i
9.9009e-001 +3.8661e-004i	8.0890e-001 -8.9829e-002i
9.9009e-001 -3.8661e-004i	8.2266e-001 +9.3155e-002i
9.5213e-001	8.2266e-001 -9.3155e-002i
9.3450e-001	8.2285e-001 +8.9127e-002i
9.2640e-001	8.2285e-001 -8.9127e-002i
9.0672e-001	9.8253e-001 +1.9155e-002i
8.2301e-001 +9.0787e-002i	9.8253e-001 -1.9155e-002i
8.2301e-001 -9.0787e-002i	9.9051e-001
8.0525e-001 +8.0695e-002i	9.9188e-001
8.0525e-001 -8.0695e-002i	9.9581e-001
8.5453e-001	9.5017e-001

8.1396e-001	9.3984e-001
5.5658e-001	9.0901e-001 +3.4893e-003i
7.0330e-001	9.0901e-001 -3.4893e-003i
6.9893e-001	8.5474e-001
6.6189e-001	8.2003e-001
6.2958e-001	8.1351e-001
6.3683e-001	3.0197e-002
4.3945e-001	3.0197e-002
4.3541e-001	3.0197e-002
4.3777e-001	3.0197e-002
4.3741e-001	3.0197e-002
3.0197e-002	3.0197e-002
3.0197e-002	
3.0197e-002	
3.0197e-002	
3.0197e-002	

It is to be noted that the mathematical model of the plant (B.1) is used in this chapter only to carry out simulations, but *it is not used for controller design*.

Since actuator constraints were not given in [40], we assume that the operating range of the control signals is given by

$$-5 \leq u_k(1) \leq 5$$

$$-5 \leq u_k(2) \leq 5$$

$$-5 \leq u_k(3) \leq 5$$

$$-5 \leq u_k(4) \leq 5.$$

Open-loop responses of this system are given in Figures 5.1-1 to 5.1-4. The steady-state tracking matrix Γ (2.2) is experimentally obtained to be:

$$\Gamma = \begin{bmatrix} 9.38 \times 10^{-2} & 0 & 0 & -7.54 \times 10^{-4} \\ 6.95 \times 10^{-2} & 2.59 \times 10^{-1} & 0 & -5.58 \times 10^{-4} \\ 6.95 \times 10^{-2} & 2.59 \times 10^{-1} & 6.09 \times 10^{-4} & -5.58 \times 10^{-4} \\ 0 & 0 & 0 & -1.00 \times 10^{-5} \end{bmatrix}.$$

Figure 5.1-1: Williams and Otto Chemical Plant Example, Open-Loop Response.

Figure 5.1-2: Williams and Otto Chemical Plant Example, Open-Loop Response.

Figure 5.1-3: Williams and Otto Chemical Plant Example, Open-Loop Response.

Figure 5.1-4: Williams and Otto Chemical Plant Example, Open-Loop Response.

5.2 Control Design for Williams and Otto Chemical Plant

In this study, it is desired to design a controller to provide disturbance rejection for the case of constant disturbances in $\omega(1)$, $\omega(2)$, $\omega(3)$. The tuning regulator (2.17) with $\epsilon = 10^{-2}$ is first applied to the plant and the two-term controller (2.6) is then obtained by minimizing the performance index $J_{A\omega}$ (2.58) using $\theta = 0, \mu = 10^{-6}$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{41 \times 1}, \quad \eta_0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{4 \times 1}, \quad y_{ref} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{4 \times 1}, \quad \text{and}$$

$$\mathcal{Z}_\omega = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right\}.$$

The above settings correspond to approximating the performance index J_{Aw} (2.58); in this case the optimal control gains K_P and K_I are found to be

$$\text{Case } \theta = 0 : \quad K_P = 8.7621 \times 10^{-2}, \quad K_I = 6.1933 \times 10^{-1}. \quad (5.1)$$

The two-term controller (2.6) is then redesigned using $\theta = 20$ for performance index J_{Aw} (2.58), while holding all other settings to be the same. In this case, the new optimal control gains obtained are

$$\text{Case } \theta = 20 : \quad K_P = 1.1653, \quad K_I = 7.7464 \times 10^{-2}. \quad (5.2)$$

Simulation of various disturbance inputs is given in Figures 5.2-1 to 5.2-9 for the resulting closed-loop system. A summary of the optimization results is given in Tables 5.1 and 5.2, and the detailed optimization output can be found in Appendix C.1.

Remark 5.1. It is seen that the controller designed with $\theta = 20$ produces significantly less transient oscillations as compared to the case when $\theta = 0$, but is slower (as expected due to the structure of the performance index when $\theta \neq 0$). It is to be noted that we *must be able to excite the disturbances in order to carry out this optimization*, and the order of the resulting controller obtained is low (4^{th} order).

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-4}	1.00×10^{-5}	8.76×10^{-2}	6.19×10^{-1}	6.58×10^{-6}	1.97×10^{-7}	1683

Table 5.1: Summary of controller parameter optimization for the Williams and Otto chemical plant with $\theta = 0$ and $\mu = 10^{-6}$.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-4}	1.00×10^{-5}	1.17	7.75×10^{-2}	8.28×10^{-4}	6.16×10^{-4}	1800

Table 5.2: Summary of controller parameter optimization for the Williams and Otto chemical plant with $\theta = 20$ and $\mu = 10^{-6}$.

Figure 5.2-1: Williams and Otto Chemical Plant Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-2}$
Input $\omega = [1 \ 0 \ 0]^T$.

Figure 5.2-2: Williams and Otto Chemical Plant Example, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-2}$
 Input $\omega = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.

Figure 5.2-3: Williams and Otto Chemical Plant Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-2}$
Input $\omega = [0 \ 0 \ 1]^T$.

Figure 5.2-4: Williams and Otto Chemical Plant Example, Closed-Loop Response
Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (5.1)
Input $\omega = [1 \ 0 \ 0]^T$.

Figure 5.2-5: Williams and Otto Chemical Plant Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 20$ with Controller Gains (5.2)
 Input $\omega = [1 \ 0 \ 0]^T$.

Figure 5.2-6: Williams and Otto Chemical Plant Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (5.1)
 Input $\omega = [0 \ 1 \ 0]^T$.

Figure 5.2-7: Williams and Otto Chemical Plant Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 20$ with Controller Gains (5.2)
 Input $\omega = [0 \ 1 \ 0]^T$.

Figure 5.2-8: Williams and Otto Chemical Plant Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (5.1)
 Input $\omega = [0 \ 0 \ 1]^T$.

Figure 5.2-9: Williams and Otto Chemical Plant Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 20$ with Controller Gains (5.2)
 Input $\omega = [0 \ 0 \ 1]^T$.

5.3 Summary

The proposed controller design approach is applied to a large-scale system that is open-loop stable and minimum phase. The purpose is to experimentally find a controller that minimizes the average cost of performance index J_A (2.47) over all disturbance signals uniformly distributed on a unit ball. The resulting controller obtained is 4th order for this 41st order system and the closed-loop output response is fast. Setting $\theta \neq 0$ in the performance index J_A (2.47) produces less transient oscillations as compared to the case when $\theta = 0$.

Chapter 6

Turbo-Generator Example (Nonminimum Phase System)

6.1 Description of the Turbo-Generator

The Koeberg turbo-generator in [41] is a 6th order system with 2 inputs and 2 outputs. The system is described by the following continuous linear time-invariant model:

$$\begin{aligned}\dot{x}(t) &= \mathcal{A}x(t) + \mathcal{B}u(t) \\ y(t) &= \mathcal{C}x(t) + \mathcal{D}u(t)\end{aligned}\tag{6.1}$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are given by

$$\mathcal{A} = \begin{bmatrix} -18.446 & 4.2263 & -2.2830 & 0.22600 & 0.42200 & -0.0951 \\ -4.0977 & -6.0706 & 5.6825 & -0.69660 & -1.2246 & 0.2873 \\ 1.4449 & 1.4336 & -2.6477 & 0.60920 & 0.8979 & -0.2300 \\ -0.0093 & 0.23020 & -0.5002 & -0.17640 & -6.3152 & 0.1350 \\ -0.0464 & -0.3489 & 0.7238 & 6.3117 & -0.6886 & 0.3645 \\ -0.0602 & -0.23610 & 0.2300 & 0.0915 & -0.3214 & -0.2087 \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} -0.2748 & 3.1463 \\ -0.0501 & -9.3737 \\ -0.1550 & 7.4296 \\ 0.0716 & -4.9176 \\ -0.0814 & -10.265 \\ 0.0244 & 13.794 \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} 0.5971 & -0.7697 & 4.8850 & 4.8608 & -9.8177 & -8.8610 \\ 3.1013 & 9.3422 & -5.6000 & -0.7490 & 2.9974 & 10.572 \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The eigenvalues of this continuous system are

$$\{-15.873, -10.387, -0.34925 \pm 6.3444i, -1.0444, -0.23455\}$$

and the transmission zeros are given by

$$\{2170.6, 22.623, -33.628 \pm 7.2463i\}.$$

If we sample the system with $h = 0.1$, the resultant sampled system is open-loop stable and nonminimum phase with eigenvalues at

$$\{0.20448, 0.35391, 0.77776 \pm 0.57238i, 0.90083, 0.97682\}$$

and with transmission zeros given by

$$\{20.076, 0.042514, -0.15968, -1.0262\}.$$

It is to be noted that the mathematical model of the plant (6.1) is used in this chapter only to carry out simulations, but *it is not used for controller design*.

Since actuator constraints were not given in [41], we assume that the operating range of the control signals is given by

$$-5 \leq u_k(1) \leq 5$$

$$-5 \leq u_k(2) \leq 5.$$

Open-loop responses of this system are given in Figure 6.1. The steady-state tracking matrix Γ (2.2) is obtained experimentally to be:

$$\Gamma = \begin{bmatrix} -1.46 & -585 \\ 0.557 & 664 \end{bmatrix}.$$

Figure 6.1: Turbo-Generator Example, Open-Loop Response.

6.2 Control Design 1 for Turbo-Generator

In this study, it is desired to design a controller for the case of tracking constant set points. The tuning regulator (2.17) with $\epsilon = 10^{-3}$ is first applied to the plant and the two-term controller (2.6) is then obtained by minimizing the performance index $J_{Ay_{ref}}$ (2.59) using $\theta = 0, \mu = 10^{-4}$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \eta_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \mathcal{Z}_{y_{ref}} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right\};$$

in this case the optimal control gains K_P and K_I are found to be

$$\text{Case } \theta = 0 : \quad K_P = 7.1299 \times 10^{-1}, \quad K_I = 2.0062. \quad (6.2)$$

The two-term controller (2.6) is then redesigned using $\theta = 10$ for performance index $J_{Ay_{ref}}$ (2.59), while holding all other settings to be the same. In this case, the new optimal control gains are

$$\text{Better Controller Case } \theta = 10 : \quad K_P = 4.5141 \times 10^{-1}, \quad K_I = 1.3764 \times 10^{-1}. \quad (6.3)$$

Simulation of the tracking reference input is given in Figures 6.2-1 to 6.2-6 for the resulting closed-loop system. A summary of the optimization results is given in Tables 6.1 and 6.2, and the detailed optimization output can be found in Appendix C.2.

Remark 6.1. When the Nelder-Mead algorithm converged for the first time, the controller gains $K_P = 6.6778 \times 10^{-1}$, and $K_I = 2.1259 \times 10^{-2}$ were obtained and the cost was $J = 38.3182$ (see Appendix C.2). With this set of controller gains, the closed-loop system is actually unstable with the output y_k slowly growing unbounded. The cost, however, was not assigned a value of 10^{20} since the control input u_k never reached the actuator saturation bounds before the experiment was terminated after $T = 10000$ seconds. This implies that a longer experimental duration will be required for the cost value to better reflect system instability. Nevertheless, restarting the Nelder-Mead algorithm from this unstable point in the controller parameter space gave rise to a set of optimal gains $K_P = 7.1299 \times 10^{-1}$ and $K_I = 2.0062$ that ensured closed-loop stability.

Remark 6.2. It is seen that the controller found with $\theta = 10$ gives a more desirable transient response compared to the case when $\theta = 0$, which is highly oscillatory. The oscillation arises from the physics of the system where the turbo-generator has a resonance

frequency at 6.35 rad/sec with a damping factor of only 0.05 [41]. The controller design approach is shown to be effective for this open-loop stable, non-minimum phase system.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-4}	1.00×10^{-5}	7.13×10^{-1}	2.01	496.69	1.59	1108

Table 6.1: Summary of controller parameter optimization A for the Turbo-Generator with $\theta = 0$ and $\mu = 10^{-4}$.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-4}	1.00×10^{-5}	4.51×10^{-1}	1.38×10^{-1}	486.74	1.20	876

Table 6.2: Summary of controller parameter optimization A for the Turbo-Generator with $\theta = 10$ and $\mu = 10^{-4}$.

Figure 6.2-1: Turbo-Generator Example, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $y_{ref} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$
 Design 1.

Figure 6.2-2: Turbo-Generator Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
Input $y_{ref} = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$
Design 1.

Figure 6.2-3: Turbo-Generator Example, Closed-Loop Response
Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (6.2)

Input $y_{ref} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$
Design 1.

Figure 6.2-4: Turbo-Generator Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 10$ with Controller Gains (6.3) (Better Controller)
 Input $y_{ref} = [1 \ 0]^T$
 Design 1.

Figure 6.2-5: Turbo-Generator Example, Closed-Loop Response
Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (6.2)

Input $y_{ref} = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$
Design 1.

Figure 6.2-6: Turbo-Generator Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 10$ with Controller Gains (6.3) (Better Controller)
 Input $y_{ref} = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$
 Design 1.

6.3 Control Design 2 for Turbo-Generator

The tuning regulator (2.17) with $\epsilon = 10^{-3}$ is first applied to the plant and the two-term controller (2.6) is then obtained by minimizing the performance index $J_{B\eta_0}$ (2.79) using $\theta = 0, \mu = 10^{-4}$. The optimization is carried out under the system settings of:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, y_{ref} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } \mathcal{Z}_{\eta_0} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right\};$$

in this case the optimal control gains K_P and K_I are found to be

$$\text{Case } \theta = 0 : \quad K_P = 7.1299 \times 10^{-1}, \quad K_I = 2.0062. \quad (6.4)$$

The two-term controller (2.6) is then redesigned using $\theta = 10$ for performance index $J_{B\eta_0}$ (2.79), while holding all other settings to be the same. In this case, the new optimal control gains are

$$\text{Better Controller Case } \theta = 10 : \quad K_P = 4.5141 \times 10^{-1}, \quad K_I = 1.3764 \times 10^{-1}. \quad (6.5)$$

Simulation of servocompensator's initial condition input is given in Figures 6.3-1 to 6.3-6 for the resulting closed-loop system. A summary of the optimization results is given in Tables 6.3 and 6.4, and the detailed optimization output can be found in Appendix C.2.

Remark 6.3. *It is seen that the controllers found by minimizing $J_{B\eta_0}$ (2.79) are identical to those obtained by minimizing $J_{Ay_{ref}}$ (2.59). The approach used in Control Design 2 provides an alternative way of tuning which requires only an offset in the initial conditions of the servocompensator.*

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-4}	1.00×10^{-5}	7.13×10^{-1}	2.01	496.69	1.59	1264

Table 6.3: Summary of controller parameter optimization B for the Turbo-Generator with $\theta = 0$ and $\mu = 10^{-4}$.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-4}	1.00×10^{-5}	4.51×10^{-1}	1.38×10^{-1}	486.74	1.20	876

Table 6.4: Summary of controller parameter optimization B for the Turbo-Generator with $\theta = 10$ and $\mu = 10^{-4}$.

Figure 6.3-1: Turbo-Generator Example, Closed-Loop Response
Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
Input $\eta_0 = [1 \ 0]^T$
Design 2.

Figure 6.3-2: Turbo-Generator Example, Closed-Loop Response
 Tuning Regulator (2.17) with $\epsilon = 10^{-3}$
 Input $\eta_0 = [0 \ 1]^\top$
 Design 2.

Figure 6.3-3: Turbo-Generator Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (6.4)
 Input $\eta_0 = [1 \ 0]^\top$
 Design 2.

Figure 6.3-4: Turbo-Generator Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 10$ with Controller Gains (6.5) (Better Controller)
 Input $\eta_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$
 Design 2.

Figure 6.3-5: Turbo-Generator Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 0$ with Controller Gains (6.4)
 Input $\eta_0 = [0 \ 1]^\top$
 Design 2.

Figure 6.3-6: Turbo-Generator Example, Closed-Loop Response
 Two-Term Controller (2.6), $\theta = 10$ with Controller Gains (6.5) (Better Controller)
 Input $\eta_0 = [0 \ 1]^\top$
 Design 2.

6.4 Summary

The proposed controller design approach is applied to a medium-scale system that is open-loop stable and nonminimum phase. During the design phase, the plant model is assumed to be unknown and there is no knowledge that the plant is nonminimum phase nor that it has a high resonance frequency with a low damping factor. By setting θ to be non-zero and μ to be sufficiently small in the performance indices J_A (2.102) or J_B (2.103), a relatively fast and smooth closed-loop output response is obtained for a given tracking reference input.

Chapter 7

Comparison Example

In this example, we will compare our proposed controller design approach with the extremum seeking (ES) controller tuning scheme using one of the simulation examples presented in [13] with some modifications.

7.1 Description of the Original Problem

In [13], the objective is to tune the parameters of a PID controller of the form

$$u(s) = Ke(s) + \frac{K}{T_i s}e(s) + KT_dsy(s) \quad (7.1)$$

for the plant

$$G(s) = \frac{1}{1 + 20s}e^{-5s} \quad (7.2)$$

using the performance index of the form

$$J(\theta) \triangleq \frac{1}{T - t_0} \int_{t_0}^T e^2(t, \theta) dt \quad (7.3)$$

where $\theta \triangleq [K \quad T_i \quad T_d]^\top$ and the cost $J(\theta)$ is calculated for each step-response experiment performed. The open-loop response of this system is given in Figure 7.1. *The PID controller parameters given by the Ziegler-Nichols (ZN) tuning rules [3] are used as a starting point for the ES tuning, i.e., $\theta(0) = [4.06 \quad 9.25 \quad 2.31]^\top$ in [13].* The cost function spans from $t_0 = 10s$ to $T = 100s$, where t_0 is approximately the time at which the step response of the closed-loop system reaches the first peak under the controller gains given by ZN. This essentially puts zero weighting on the initial transient response.

The ES tuning scheme is configured with adaptation gains $\gamma = [200 \ 1200 \ 200]^\top$, perturbation amplitudes $\alpha = [0.1 \ 1 \ 0.1]^\top$, dither frequencies $\omega = [0.8\pi \ 0.8^2\pi \ 0.8^3\pi]^\top$, and filter parameter $h = 0.5$ [13].

Figure 7.1: Comparison Example, Open-Loop Response.

7.2 Modifications to the Original Problem

In order to be able to compare results obtained using ES tuning against those obtained using our proposed design approach with a two-term controller, we make the following modifications to the ES setup for the example presented in [13]:

- The PID controller is simplified to a PI controller of the form

$$u(s) = Ke(s) + \frac{K}{T_i s}e(s). \quad (7.4)$$

- The starting controller parameters for ES tuning are found to be $\theta(0) = [2.8 \quad 14.4]^\top$ using the ZN method.
- The cost function now spans from $t_0 = 18s$ to $T = 100s$, where t_0 is still approximately the time at which the step response of the closed-loop system reaches the first peak under the controller gains given by ZN.
- The initial conditions of $\hat{\theta}(0)$ and $\zeta(0)$ in Equations (1.12) to (1.14) are set to be $\hat{\theta}(0) = \theta(0)$ and $\zeta(0) = J(\theta(0))$, as the values for $\hat{\theta}(0), \zeta(0)$ were not given in the original example [13].

7.3 Control Design using Extremum Seeking

The PI controller (7.4) is obtained by minimizing the performance index (7.3). The optimization is carried out under the ES parameter settings of:

$$\gamma = \begin{bmatrix} 200 \\ 1200 \end{bmatrix}, \alpha = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, \omega = \begin{bmatrix} 0.8\pi \\ 0.64\pi \end{bmatrix}, \text{ and } h = 0.5;$$

in this case the optimal control parameters $\theta = [K \quad T_i]^\top$ are found to be

$$K = 1.9351, \quad T_i = 21.0554. \quad (7.5)$$

Simulation of the tracking reference input is given in Figure 7.2 for the resulting closed-loop system. Figures 7.3-1 to 7.3-4 illustrate the impact of the choice of ES parameters on the stability and convergence rate of the ES tuning scheme.

Remark 7.1. It is seen that ES converges at different rates depending on the choice of γ, α and ω values. There are also cases where ES diverges and causes the overall closed-loop system to become unstable. For example, setting $\zeta(0) = 0.5J(\theta(0))$ or $\alpha = [0.4 \quad 4]^\top$ destabilizes the system. Furthermore, if the starting controller parameters $\theta(0) = [2.8 \quad 14.4]^\top$ are changed to $\theta(0) = [0.28 \quad 14.4]^\top$, the ES tuning scheme does not converge. The domain of attraction can be increased by decreasing γ at the cost of a slower convergence rate.

Remark 7.2. In the simulation results presented, ES converges after several hundreds of function evaluations or more. This amounts to a long period of time to simulate the ES tuning process using the simulation procedure described in Chapter 3.3. In order to demonstrate the need of increasing the perturbation amplitude α when the plant is subjected to noise, the ES tuning example presented would need to be repeated starting with smaller α values. However, this also means that the convergence rate is much slower and much more time is required for simulation. As a result, noise sensitivity analysis via Simulink simulations was not performed.

Figure 7.2: Comparison Example, Closed-Loop Response
PI Controller
Input $y_{ref} = 1$.

Figure 7.3-1: Comparison Example, Evolution of PI Parameters.

Figure 7.3-2: Comparison Example, Evolution of PI Parameters.

Figure 7.3-3: Comparison Example, Evolution of PI Parameters.

Figure 7.3-4: Comparison Example, Evolution of PI Parameters.

7.4 Control Design using Proposed Approach

In order to be able to compare the proportional and integral gains obtained using our proposed design approach against those obtained using the ES tuning, we modify our two-term controller (2.6) to become

$$u_k = -S(K_P e_k + K_I \eta_k). \quad (7.6)$$

To design the controller (7.6), we first sample $G(s)$ with a sampling period of $h = 0.1$ seconds and implement the time delay by using a 3rd order Padé approximation. The sampling period is chosen to be sufficiently small so that our discrete domain results can be compared with the continuous domain results obtained in Section 7.3 using extremum seeking. The steady-state tracking matrix Γ (2.2) for plant $G(s)$ (7.2) is obtained experimentally to be $\Gamma = 1$. The two-term controller is then obtained by minimizing performance index $J_{Ay_{ref}}$ given in (2.59) using $\theta = 0, \mu = 10^{-8}$. The optimization is carried out under the system settings of:

$$x_0 = 0, \eta_0 = 0, \text{ and } y_{ref} = 1;$$

in this case the optimal controller gains K_P and K_I are found to be

$$\text{Case } t_0 = 0s : \quad K_P = 3.2519, \quad K_I = 1.0581 \times 10^{-1}, \quad (7.7)$$

or equivalently,

$$\text{Case } t_0 = 0s : \quad K = K_P = 3.2519, \quad T_i = \frac{K_P}{K_I} = 3.0732 \times 10^1. \quad (7.8)$$

If we modify the summation boundaries of the performance index J_A (2.102) such that it corresponds to a time span from $t_0 = 10s$ to $T \approx \infty$, we obtain

$$\text{Case } t_0 = 10s : \quad K = 2.6821, \quad T_i = 2.5596 \times 10^1. \quad (7.9)$$

If the cost function is further modified to span from $t_0 = 18s$ to $T \approx \infty$, the optimal gains are found to be

$$\underline{\text{Best Controller}} \quad \text{Case } t_0 = 18s : \quad K = 1.9489, \quad T_i = 2.1151 \times 10^1. \quad (7.10)$$

Simulation of the tracking reference input is given in Figure 7.4 for the resulting closed-loop system. A summary of the optimization results is given in Tables 7.1 to 7.3, and the detailed optimization output can be found in Appendix C.3.

Remark 7.3. There is a slight discrepancy between the optimal gains obtained using our proposed method and those obtained using the sinusoidal perturbation-based extremum seeking scheme. Under extremum seeking, the controller gains locally exponentially converge to an $O(\alpha)$ neighborhood of the true optimal gains, where α is the perturbation amplitude [10]. We will, however, treat the optimal gains obtained using the two different methods to be identical.

Remark 7.4. Even though a “truncated” cost function, which places zero weighting on the initial transient response, gives rise to a faster system response with smaller overshoot, choosing a good value for the starting time t_0 is not straightforward. Unlike the ES tuning method, changing the time span of the performance index or performing the optimization from a new set of initial controller gains did not require any changes in our setup. We obtained the same optimal controller gains and closed-loop response as those obtained using ES tuning but with a much simpler setup.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-3}	1.00×10^{-3}	3.25	1.06×10^{-1}	510.06	7.53	126

Table 7.1: Summary of controller parameter optimization for the SISO time-delayed system with the cost function spanning from $t_0 = 0s$ to $T \approx \infty$.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
5.00×10^{-3}	1.00×10^{-3}	2.68	1.05×10^{-1}	500.07	2.40×10^{-1}	118

Table 7.2: Summary of controller parameter optimization for the SISO time-delayed system with the cost function spanning from $t_0 = 10s$ to $T \approx \infty$.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
1.00×10^{-4}	1.00×10^{-3}	1.95	9.21×10^{-2}	494.59	4.49×10^{-3}	175

Table 7.3: Summary of controller parameter optimization for the SISO time-delayed system with the cost function spanning from $t_0 = 18s$ to $T \approx \infty$.

Figure 7.4: Comparison Example, Closed-Loop Response
PI Controller
Input $y_{ref} = 1$.

7.5 Summary

The proposed controller design approach is applied to a single-input single-output system with time delay. During the design phase, no knowledge of the plant model or presence of time delay was required. Even though the “Window” cost function used in ES tuning gives rise to a response with the smallest overshoot and fastest settling time, there are no guarantees for bounded peaking of the state trajectories or the stability of the closed-loop system. If a different plant, performance index or starting controller gains is of interest, one would have to maintain the stability of ES through the careful selection of the ES parameters. Unlike ES, our proposed method is more “robust” in the sense that stability is maintained for every non-negative θ and positive μ values in our performance indices.

Chapter 8

MARTS Experiment

The purpose of this chapter is to validate, experimentally, the effectiveness of the proposed controller design approach via the use of an experimental test bed known as MARTS. A description of MARTS and the details of the experimental setup are initially provided. The process of developing linear control valve characteristics, measuring steady-state tracking matrix and setting up the optimization problem on MARTS are also described. Thereafter, we concentrate on the experimental results.

8.1 Description of MARTS

The experimental test bed is called MARTS(Multivariable Apparatus for Real Time Systems) [42] and is composed of two interconnected vessels, a network of pipes and valves, and a variety of industrial commercial process-control sensors and actuators that monitor/control a hydraulic dynamic process. The setup which we are concerned with in this thesis is shown in Figure 8.1. The objective in using this configuration is to find a controller to control the position of the water level in both vessels by using only the two control valves. The controller design process can be challenging due to nonlinear effects of the plant, time delays, actuator valve dynamics, as well as actuator saturation and sizing effects, and interaction effects when the interconnecting valve 1 linking the two vessels is “widely” open.

In Figure 8.1, the control inputs to the plant are valve control voltages for the two control valves ($-5V$ corresponds to fully closed and $5V$ corresponds to fully opened) that affect the amount of water inflow into the two vessels. The outputs of the plant are the water height measurements in the two water vessels via sensors which provide

voltage levels ($1V$ corresponds to near empty and $5V$ corresponds to near full, with $1V$ increase/decrease representing approximately $2.4L$ of water rise/drop or a $30cm$ change in water level height). There are two disturbance inputs to the plant that provide unmeasurable water inflow disturbances via digital on/off control inputs. To perform real-time control, a Windows-based PC running MATLAB Version 7.2.0.232(*R2006a*) with the Real-Time Windows Target software is inserted within the control loop. Furthermore, the online optimization process is carried out using the combination of MATLAB and Simulink as described in Section 3.3, where the two-term controller (2.6) is implemented in Simulink and the Nelder-Mead algorithm is executed in MATLAB. A photograph of the experimental setup is shown in Figure 8.2, and a full list of components used within the experiment can be found in [43].

Figure 8.1: Schematic of MARTS setup.

Figure 8.2: MARTS experimental setup.

8.2 Developing Linear Control Valve Characteristics

The control valves on MARTS exhibit the non-linear characteristics as shown in Figure 8.3-1. In an attempt to “remove” the nonlinear valve characteristics, a square root function of the form

$$u = \alpha\sqrt{(\hat{u} + 5)} - 5$$

is applied, where $\hat{u}$ is the output of a controller and u is the valve control voltage. α is chosen to be 2.8 for control valve 2, and no compensation was done for control valve 1 since a better linear fit can only be obtained, using the square root function, at the cost of reducing the maximum flow rate by 50%. The linearized valve characteristics are shown in Figure 8.3-2. A 1V change in the control input now corresponds to a change of approximately 0.0072L/s in flow rate for control valve 1 and 0.0108L/s for control valve 2.

Figure 8.3-1: Nonlinear valve characteristics obtained experimentally.

Figure 8.3-2: Linearized valve characteristics obtained experimentally.

8.3 Unexpected Observation

Figure 8.4 shows the plant output over a 12 hours period with control valve inputs fixed at $0V$. The water inflow and outflow rates were initially balanced by manually adjusting the drainage and by-pass valves. Due to the fact that the system is not open-loop asymptotically stable (but is only stable), one would expect the water level in the vessels to remain at constant values or possibly to “drift” somewhat over a 12 hours period; however, this was not the case as shown in Figure 8.4 with the appearance of non-stationary saw-tooth like waveforms. The behavior observed in the two water vessels are different from each other, and we suspect that it may be due to various aging effects in the pumps and/or control valves; thus it was observed that the amount of control effort required to maintain the water level at a specific height changes overtime. Combined with the high frequency signal noise that arises as a result of water splashes and quantization noise, the saw-tooth like waveform poses difficulties in accurately comparing costs of different controllers. As a work around solution, we can

1. enlarge the initial Nelder-Mead simplex so that the performance of the various controllers being compared can be distinguished from each other, and
2. re-adjust the by-pass and drainage valves such that the water inflow and outflow rates are balanced at the desired water level height upon each restart of the Nelder-Mead algorithm.

Figure 8.4: A 12 hour record of water level measurements with zero input flow applied to the two vessels.

8.4 Measuring the Steady-State Tracking Matrix

MARTS is an open-loop stable plant (but not asymptotically stable) under the configuration shown in Figure 8.1. Thus to stabilize the system, we precondition the system with a stabilizing output feedback $u_k(1) = -10y_k(1)$ and $u_k(2) = -10y_k(2)$ so that steady-state measurements can be made. Recall that the outputs $y_k(1), y_k(2)$ of the vessels are measured in *volts* (V) and that the control valve inputs $u_k(1), u_k(2)$ are also measured in V . With MARTS sampled at $h = 0.1$ seconds, the steady-state tracking matrix Γ was measured via the following procedure:

1. With the interconnecting valve between the water vessels set as $\theta = 30^\circ$ and with the control valves set as half-opened, the by-pass and drainage valves were manually adjusted till the water level settled roughly at the chosen equilibrium heights in the water vessels.
2. A step input of magnitude $-1V$ was applied to control valve 1 and the corresponding steady-state outputs were measured.
3. The water level was then restored back to the same equilibrium height chosen in step 1, followed by a step input of magnitude $-1V$ applied to control valve 2.
4. Γ was obtained experimentally to be:

$$\Gamma = \begin{bmatrix} 8.5 \times 10^{-2} & 1.0 \times 10^{-2} \\ 1.0 \times 10^{-2} & 7.5 \times 10^{-2} \end{bmatrix}.$$

Open-loop responses of the system are given in Figures 8.5-1 and 8.5-2.

Figure 8.5-1: MARTS experiment, open-loop response.

Figure 8.5-2: MARTS experiment, open-loop response.

8.5 Control Design for MARTS

The two-term controller (2.6) is obtained by minimizing the performance index J_{By_0} (2.78) using $\theta = 0, \mu = 1$. The optimization is carried out under the system settings of:

$$y_0 = \begin{bmatrix} -0.1 \\ -0.15 \end{bmatrix}, \quad \eta_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad y_{ref} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad \omega = \begin{bmatrix} \text{off} \\ \text{off} \end{bmatrix}.$$

The physical setup that corresponds to the above system settings is given as follows: On MARTS, with the interconnecting valve between the water vessels set as $\theta = 30^\circ$ and with the control valves set as half-opened, the equilibrium water level was adjusted to be at 4.0V for water vessel 1 and 4.2V for vessel 2 by adjusting the by-pass and drainage valves manually. Control valve 1 was set at 0V and control valve 2 at 1.26V so that the valves were only half-opened when trying to achieve the desired equilibrium water level. Control valve 2 does not start at 0V due to the linearization of the control valve characteristics. Before the start of each experiment, the water level was regulated to be at 3.9V for vessel 1 and 4.05V for vessel 2 to achieve the required setting of $y_0 = [-0.1 \quad -0.15]^\top$ ¹.

By carrying out the optimization process, the optimal control gains K_P and K_I are found to be

$$K_P = 6.4262 \times 10^{-1}, \quad K_I = 7.7555 \times 10^{-2}. \quad (8.1)$$

The measured closed-loop system waveforms are given in Figures 8.6-1 to 8.6-7. It is seen that the response of the system is fast and non-oscillatory. The controller gains and system responses obtained experimentally are very similar to those obtained in simulation (under the “Control Design 3 for MARTS” simulation example in Section 4.6). Furthermore, the peak output values apparent in Figures 8.6-6 and 8.6-7 are much smaller than the open-loop steady-state outputs for the input disturbances shown in Figure 8.5-2. It should be emphasized that at no time during the design phase, was system identification performed. The optimization process took a total of 10.5 hours to complete, and the results show that the proposed design approach works well on experimental systems. It is to be noted that the choice of carrying out the experiments by perturbing y_0 was made,

¹The two-term controller (2.6) with a fixed K_P and K_I was used to ensure that the water level is at 3.9V and 4.05V before the start of each experiment. When an experiment begins, the controller gains are switched to the K_P and K_I values that we would like to compute a cost value for. The switch results in a discontinuous jump in the control input. Since y_0 is set to be relatively small in value, the jump in u_k is also small in magnitude; thus, the amount of inrush water added over that small period of time is little and the effects on the calculation of the performance index is negligible compared to those from the measurement noise. Furthermore, this conclusion drawn was re-confirmed by turning on and off the pumps in rapid intermittent fashion with no change in the output waveform observed.

to show that it is not essential to have actually apply physical tracking signals or disturbance signals to design a controller for a plant. A summary of the optimization results is given in Table 8.1, and the detailed optimization output can be found in Appendix C.4.

Initial K_P	Initial K_I	Final K_P	Final K_I	Initial cost J	Final cost J	Number of function evaluations
1.00×10^{-2}	5.00×10^{-3}	6.43×10^{-1}	7.76×10^{-2}	1068.18	63.28	58

Table 8.1: Summary of controller parameter optimization for the experimental MARTS system with $\theta = 0$ and $\mu = 1$.

Figure 8.6-1: MARTS Experiment , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (8.1)
 Input $y_0 = [-0.1 \ 0]^T$.

Figure 8.6-2: MARTS Experiment , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (8.1)
 Input $y_0 = [0 \quad -0.15]^T$.

Figure 8.6-3: MARTS Experiment , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (8.1)
 Input $y_0 = [-0.1 \quad -0.15]^T$.

Figure 8.6-4: MARTS Experiment , Closed-Loop Response
Two-Term Controller (2.6) with Controller Gains (8.1)
Input $y_{ref} = [0.1 \ 0]^T$.

Figure 8.6-5: MARTS Experiment , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (8.1)
 Input $y_{ref} = [0 \ 0.15]^T$.

Figure 8.6-6: MARTS Experiment , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (8.1)
 Input $\omega(1) = \text{on}$, and $\omega(2) = \text{off}$.

Figure 8.6-7: MARTS Experiment , Closed-Loop Response
 Two-Term Controller (2.6) with Controller Gains (8.1)
 Input $\omega(1) = \text{off}$, and $\omega(2) = \text{on}$.

Chapter 9

Conclusion

In this chapter, the main results in this thesis are summarized, and possible future work is discussed.

9.1 Summary of Results

In this thesis, a new method of designing multivariable robust servomechanism controllers for MIMO open-loop stable systems, subject to constant references and disturbances, has been proposed and successfully applied to a real physical plant. The method requires no knowledge of the plant model nor of the order of the system, but assumes the inputs can be excited, and the outputs can be measured. The design process is carried out within a closed-loop setting. The controller parameters are optimized by minimizing the average cost of a performance index over the plant initial conditions, or disturbance/tracking signals uniformly distributed on a unit disk ball. The performance indices are expressed in a form where the cost can be directly computed via measured data. Once the cost value is computed, it is used as a single data point in the controller parameter optimization and an algorithm such as the Nelder-Mead algorithm is then employed to find the local minimizer. An important advantage that the method offers is that optimizing for tracking performance can be achieved by perturbing only the initial conditions of the servocompensator while minimizing the noise insensitive performance index. The method is well suited for applications that exhibit actuator saturation, and the order of the resulting controller obtained is low and is independent of the system order. An experimental application study of the proposed controller design approach was then performed on a multivariable hydraulic system (MARTS), which had industrial

actuators and sensors, and the results obtained indicate that the design method is feasible to use in an industrial environment.

9.2 Future Work

A natural extension of this work is to consider ways of applying more general controller structures that are optimal in some sense for different classes of systems or if more *a priori* knowledge of the plant is known. For example, one can consider three-term controllers or state based controllers to replace the PI controllers used in the thesis. Once the controller structure is determined, one can proceed to find the optimal controller parameters using the same method presented in this thesis. Finally, it would be of interest to extend the theory and application of the proposed method to the design of decentralized controllers.

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Appendix A

Mathematical Derivations

This appendix contains the derivation of some equations that were used in the chapters of the thesis document.

A.1 Equivalent Steady-State Control Input Expressions

The relationship of $(C(I - (A - BSK_PC))^{-1}B)^{-1} = (C(I - A)^{-1}B)^{-1} + SK_P$ in Section 2.2.1 can be derived as follows. Consider the system

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k \\y_k &= Cx_k \\u_k &= Ky_k + \hat{u}_k\end{aligned}\tag{A.1}$$

where $K = -SK_P$. There are two approaches in determining the steady-state relationship between $\hat{u}_k$ and y_k :

Approach 1

$$\begin{aligned}x_\infty &= Ax_\infty + BKCx_\infty + B\hat{u}_\infty \\&= (A + BKC)x_\infty + B\hat{u}_\infty \\ \Rightarrow x_\infty &= (I - (A - BSK_PC))^{-1}B\hat{u}_\infty \\y_\infty &= C(I - (A - BSK_PC))^{-1}B\hat{u}_\infty \\ \hat{u}_\infty &= (C(I - (A - BSK_PC))^{-1}B)^{-1}y_\infty.\end{aligned}\tag{A.2}$$

Approach 2

$$\begin{aligned}
x_\infty &= Ax_\infty + Bu_\infty \\
\Rightarrow x_\infty &= (I - A)^{-1}Bu_\infty \\
&= (I - A)^{-1}B(Ky_\infty + \hat{u}_\infty) \\
&= -(I - A)^{-1}BSK_P y_\infty + (I - A)^{-1}B\hat{u}_\infty \\
y_\infty &= -C(I - A)^{-1}BSK_P y_\infty + C(I - A)^{-1}B\hat{u}_\infty \\
\hat{u}_\infty &= (C(I - A)^{-1}B)^{-1}(I + C(I - A)^{-1}BSK_P)y_\infty \\
&= ((C(I - A)^{-1}B)^{-1} + SK_P)y_\infty.
\end{aligned} \tag{A.3}$$

Therefore, $(C(I - (A - BKC))^{-1}B)^{-1} = (C(I - A)^{-1}B)^{-1} + SK_P$.

A.2 Initial Conditions for Optimization Problem B

The expression for $[\Delta x_0 \quad \Delta \eta_0]^\top = [x_0 - x_\infty \quad \eta_0 - \eta_\infty]^\top$ given in Section 2.4.2 can be derived as follows. On applying the controller (2.6) to the plant (2.1) we obtain the following closed-loop augmented system:

$$\begin{bmatrix} x_{k+1} \\ \eta_{k+1} \end{bmatrix} = \begin{bmatrix} A - BSK_PC & -BSK_I \\ hC & I \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + \begin{bmatrix} E - BSK_PF & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}. \tag{A.4}$$

At steady state, the system becomes

$$\begin{aligned}
\begin{bmatrix} x_\infty \\ \eta_\infty \end{bmatrix} &= \begin{bmatrix} A - BSK_PC & -BSK_I \\ hC & I \end{bmatrix} \begin{bmatrix} x_\infty \\ \eta_\infty \end{bmatrix} + \begin{bmatrix} E - BSK_PF & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix} \\
\Rightarrow \begin{bmatrix} x_\infty \\ \eta_\infty \end{bmatrix} &= \begin{bmatrix} I - (A - BSK_PC) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} E - BSK_PF & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}.
\end{aligned} \tag{A.5}$$

Therefore, we obtain

$$\begin{bmatrix} \Delta x_0 \\ \Delta \eta_0 \end{bmatrix} = \begin{bmatrix} x_0 \\ \eta_0 \end{bmatrix} - \begin{bmatrix} I - (A - BSK_PC) & BSK_I \\ -hC & 0 \end{bmatrix}^{-1} \begin{bmatrix} E - BSK_PF & 0 \\ hF & -h \end{bmatrix} \begin{bmatrix} \omega \\ y_{ref} \end{bmatrix}. \tag{A.6}$$

Appendix B

Model of Williams and Otto Chemical Plant

The following continuous state-space model is used in the “Williams and Otto Chemical Plant Example” in Chapter 5. In the example, the discrete model used was generated by sampling the continuous-time model with a sampling period $h = 0.01$ seconds.

$$\begin{aligned}\dot{x}(t) &= \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{E}\omega \\ y(t) &= \mathcal{C}x(t) + \mathcal{D}u(t) + \mathcal{F}\omega\end{aligned}\tag{B.1}$$

where

$\mathcal{A} =$

Columns 1 through 5

-4.0740e+001	-6.3600e+000	0	0	0
-2.0100e+001	-3.1470e+001	-6.8400e+001	0	0
4.0200e+001	3.7800e+000	-1.8244e+002	0	-7.3500e+000
0	8.9400e+000	1.3680e+002	-2.0640e+001	0
0	4.4700e+000	5.5900e+001	0	-2.4315e+001
0	0	3.7500e+001	0	1.1025e+001
3.0300e+003	1.4495e+003	8.1680e+003	0	1.9060e+002
0	0	0	0	0
0	0	0	0	0
3.5000e+002	0	0	0	0
0	3.5000e+002	0	0	0
0	0	3.5000e+002	0	0
0	0	0	3.5000e+002	0
0	0	0	0	3.5000e+002
0	0	0	0	0

	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
3.6400e+000	0	0	0	0	0
	0	1.5600e+001	0	0	0
	0	0	1.5600e+001	0	0
	0	0	0	1.5600e+001	0
	0	0	0	0	1.5600e+001
-4.6000e+001	0	0	0	0	0
	0	-5.3950e+001	0	0	0
	0	0	-5.3950e+001	0	0
	0	0	0	-5.3950e+001	0
	0	0	0	0	-5.3950e+001
3.5650e+001	0	0	0	0	0
	0	3.5650e+001	0	0	0
	0	0	3.5650e+001	0	0
	0	0	0	3.5650e+001	0
	0	0	0	0	3.5650e+001
	0	0	0	0	0

Columns 36 through 40

	0	1.0750e+001	0	0	0
	0	0	1.0750e+001	0	0
	0	0	0	1.0750e+001	0
	0	0	0	0	1.0750e+001
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
1.5600e+001	0	0	0	0	0

0	1.8300e+001	0	0	0
0	0	1.8300e+001	0	0
0	0	0	1.8300e+001	0
0	0	0	0	1.8300e+001
-5.3950e+001	0	0	0	0
0	-3.5650e+001	0	0	0
0	0	-3.5650e+001	0	0
0	0	0	-3.5650e+001	0
0	0	0	0	-3.5650e+001
3.5650e+001	0	0	0	0

Column 41

	0
	0
	0
	0
1.0750e+001	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
	0
1.8300e+001	0
	0
	0
	0
-3.5650e+001	0

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

$\mathcal{F} =$

0	0	0
0	0	0
0	0	0
0	0	0

Appendix C

Detailed Optimization Outputs

This appendix contains the MATLAB optimization outputs for the control design examples presented in Chapters 4 to 8.

C.1 Control Design for Chemical Plant

The following optimization outputs were obtained when designing the two-term controller (2.6) for the Williams and Otto chemical plant in Section 5.2.

Design of Controller for the Case $\theta = 0$:

Algorithm run # 1				
Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	1e-006	1000
Iteration	Func-count	min f(x)	Procedure	
0	9	6.57684e-006		
1	27	6.57502e-006	initial simplex	
2	45	6.57192e-006	expand	
3	63	6.56965e-006	expand	
4	81	6.56233e-006	expand	
5	99	6.55416e-006	expand	
6	117	6.53553e-006	expand	
7	135	6.51007e-006	expand	
8	153	6.46075e-006	expand	
9	171	6.38686e-006	expand	
10	189	6.25634e-006	expand	
11	207	6.05628e-006	expand	
12	225	5.73105e-006	expand	
13	243	5.25628e-006	expand	
14	261	4.58099e-006	expand	
15	279	3.74244e-006	expand	

16	297	2.8289e-006	expand
17	315	2.06782e-006	expand
18	333	1.80004e-006	expand
19	342	1.80004e-006	reflect
20	360	1.79615e-006	contract
21	396	1.78695e-006	shrink
22	414	1.78695e-006	contract
23	432	1.78695e-006	contract
24	468	1.78679e-006	shrink
25	504	1.78678e-006	shrink
26	522	1.7867e-006	contract
27	540	1.7867e-006	contract
28	558	1.78652e-006	expand
29	567	1.78652e-006	reflect
30	585	1.78625e-006	expand
31	603	1.78602e-006	expand
32	621	1.78526e-006	expand
33	639	1.78474e-006	expand
34	657	1.78262e-006	expand
35	675	1.78211e-006	expand
36	693	1.77708e-006	expand
37	702	1.77708e-006	reflect
38	720	1.77176e-006	expand
39	738	1.76241e-006	expand
40	756	1.75227e-006	expand
41	774	1.723e-006	expand
42	792	1.71363e-006	expand
43	810	1.64124e-006	expand
44	819	1.64124e-006	reflect
45	837	1.57286e-006	expand
46	855	1.45823e-006	expand
47	873	1.38373e-006	expand
48	891	1.11937e-006	expand
49	900	1.11937e-006	reflect
50	918	9.02927e-007	expand
51	936	8.4313e-007	expand
52	954	4.96447e-007	expand
53	963	4.96447e-007	reflect
54	981	3.3263e-007	expand
55	999	3.3263e-007	contract
56	1017	2.21986e-007	expand
57	1026	2.21986e-007	reflect
58	1044	2.13742e-007	expand
59	1062	2.10425e-007	contract
60	1080	2.03483e-007	contract
61	1098	1.99425e-007	contract
62	1116	1.99425e-007	contract
63	1134	1.97915e-007	reflect
64	1152	1.97469e-007	contract
65	1170	1.97169e-007	contract
66	1188	1.97026e-007	contract
67	1206	1.96974e-007	reflect
68	1224	1.96879e-007	contract
69	1242	1.96819e-007	contract

70	1260	1.96819e-007	contract
71	1278	1.96819e-007	contract
72	1296	1.96815e-007	contract
73	1314	1.9681e-007	contract
74	1323	1.9681e-007	reflect
75	1341	1.9681e-007	contract
76	1359	1.9681e-007	contract
77	1377	1.96809e-007	contract
78	1395	1.96809e-007	contract
79	1413	1.96809e-007	contract
80	1431	1.96809e-007	contract
81	1440	1.96809e-007	reflect
82	1458	1.96809e-007	contract
83	1476	1.96809e-007	contract
84	1485	1.96809e-007	reflect
85	1503	1.96809e-007	contract
86	1521	1.96809e-007	contract
87	1539	1.96809e-007	contract
88	1557	1.96809e-007	contract
89	1575	1.96809e-007	contract
90	1593	1.96809e-007	contract
91	1611	1.96809e-007	contract
92	1629	1.96809e-007	contract
93	1647	1.96809e-007	contract
94	1665	1.96809e-007	contract
95	1683	1.96809e-007	contract

Optimal KP Optimal KI Optimal cost J
0.0876208 0.619331 1.96809e-007

Algorithm run # 2

Starting KP Starting KI theta mu Duration T (sec)
0.0876208 0.619331 0 1e-006 200

Iteration	Func-count	min f(x)	Procedure
0	9	1.96809e-007	
1	27	1.96809e-007	initial simplex
2	36	1.96809e-007	reflect
3	54	1.96809e-007	contract
4	90	1.96809e-007	shrink
5	126	1.96809e-007	shrink
6	162	1.96809e-007	shrink
7	198	1.96809e-007	shrink
8	234	1.96809e-007	shrink
9	270	1.96809e-007	shrink
10	306	1.96809e-007	shrink
11	342	1.96809e-007	shrink
12	378	1.96809e-007	shrink
13	414	1.96809e-007	shrink
14	450	1.96809e-007	shrink
15	468	1.96809e-007	contract
16	486	1.96809e-007	contract
17	504	1.96809e-007	contract

Optimal KP	Optimal KI	Optimal cost J
0.0876208	0.619331	1.96809e-007

Design of Controller for the Case $\theta = 20$:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	20	1e-006	1000

Iteration	Func-count	min f(x)	Procedure
0	9	0.000827682	
1	27	0.000827632	initial simplex
2	45	0.000827568	expand
3	63	0.000827488	expand
4	81	0.00082732	expand
5	99	0.000827077	expand
6	117	0.000826621	expand
7	135	0.00082591	expand
8	153	0.00082465	expand
9	171	0.00082262	expand
10	189	0.000819151	expand
11	207	0.000813532	expand
12	225	0.000804274	expand
13	243	0.0007897	expand
14	261	0.000767263	expand
15	279	0.000735047	expand
16	297	0.000692522	expand
17	315	0.000644765	expand
18	333	0.000619763	expand
19	369	0.000619763	shrink
20	405	0.000617309	shrink
21	423	0.000617086	contract
22	459	0.00061708	shrink
23	495	0.000617052	shrink
24	513	0.000617052	contract
25	531	0.000617052	contract
26	549	0.000617052	contract
27	567	0.000617052	contract
28	603	0.000617052	shrink
29	639	0.000617052	shrink
30	675	0.000617052	shrink
31	711	0.000617052	shrink
32	729	0.000617052	contract
33	747	0.000617052	contract
34	765	0.000617052	expand
35	774	0.000617052	reflect
36	792	0.000617052	expand
37	810	0.000617052	expand
38	828	0.000617052	expand
39	846	0.000617052	expand
40	864	0.000617052	expand

41	882	0.000617052	expand
42	900	0.000617051	expand
43	909	0.000617051	reflect
44	927	0.000617051	expand
45	945	0.000617051	expand
46	963	0.000617051	expand
47	981	0.00061705	expand
48	999	0.000617049	expand
49	1017	0.000617048	expand
50	1035	0.000617045	expand
51	1053	0.000617041	expand
52	1071	0.000617035	expand
53	1089	0.000617023	expand
54	1107	0.000617013	expand
55	1125	0.000616986	expand
56	1134	0.000616986	reflect
57	1152	0.000616967	expand
58	1170	0.000616925	expand
59	1179	0.000616925	reflect
60	1197	0.000616859	expand
61	1215	0.000616847	expand
62	1233	0.000616701	expand
63	1242	0.000616701	reflect
64	1260	0.000616643	expand
65	1278	0.000616496	expand
66	1287	0.000616496	reflect
67	1305	0.000616466	expand
68	1323	0.000616442	contract
69	1341	0.00061643	reflect
70	1359	0.000616422	contract
71	1377	0.000616418	contract
72	1395	0.000616414	contract
73	1413	0.000616414	contract
74	1431	0.000616414	contract
75	1449	0.000616414	reflect
76	1467	0.000616414	contract
77	1485	0.000616414	contract
78	1503	0.000616414	contract
79	1521	0.000616414	contract
80	1539	0.000616414	contract
81	1557	0.000616414	reflect
82	1575	0.000616414	contract
83	1593	0.000616414	contract
84	1611	0.000616414	contract
85	1629	0.000616414	contract
86	1647	0.000616414	contract
87	1683	0.000616414	shrink
88	1701	0.000616414	contract
89	1719	0.000616414	contract
90	1728	0.000616414	reflect
91	1746	0.000616414	contract
92	1764	0.000616414	contract
93	1782	0.000616414	contract
94	1800	0.000616414	contract

Optimal KP	Optimal KI	Optimal cost J		
1.16531	0.0774639	0.000616414		
Algorithm run # 2				
Starting KP	Starting KI	theta	mu	Duration T (sec)
1.16531	0.0774639	20	1e-006	300
Iteration	Func-count	min f(x)	Procedure	
0	9	0.000616414		
1	27	0.000616414	initial simplex	
2	36	0.000616414	reflect	
3	54	0.000616414	contract	
4	90	0.000616414	shrink	
5	126	0.000616414	shrink	
6	162	0.000616414	shrink	
7	198	0.000616414	shrink	
8	234	0.000616414	shrink	
9	270	0.000616414	shrink	
10	306	0.000616414	shrink	
11	342	0.000616414	shrink	
12	378	0.000616414	shrink	
13	414	0.000616414	shrink	
14	450	0.000616414	shrink	
15	486	0.000616414	shrink	
16	522	0.000616414	shrink	
17	558	0.000616414	shrink	
Optimal KP	Optimal KI	Optimal cost J		
1.16531	0.0774639	0.000616414		

C.2 Control Design for Turbo-Generator

The following optimization outputs were obtained when designing the two-term controller (2.6) for the Turbo-Generator in Sections 6.2 and 6.3 .

Design 1 with $\theta = 0$:

Algorithm run # 1				
Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	0.0001	10000
Iteration	Func-count	min f(x)	Procedure	
0	4	496.693		
1	12	495.633	initial simplex	
2	20	494.376	expand	

3	28	492.622	expand
4	36	489.283	expand
5	44	484.231	expand
6	52	475.415	expand
7	60	461.875	expand
8	68	440.139	expand
9	76	408.556	expand
10	84	364.228	expand
11	92	308.999	expand
12	100	246.97	expand
13	108	186.435	expand
14	116	134.291	expand
15	124	94.1337	expand
16	132	65.8417	expand
17	140	46.9734	expand
18	148	41.3391	reflect
19	164	41.3391	shrink
20	180	41.3391	shrink
21	188	39.5219	reflect
22	196	38.5712	reflect
23	212	38.5712	shrink
24	228	38.5712	shrink
25	244	38.5712	shrink
26	252	38.3787	reflect
27	268	38.3787	shrink
28	276	38.3238	reflect
29	292	38.3238	shrink
30	308	38.3238	shrink
31	324	38.3238	shrink
32	332	38.3192	reflect
33	348	38.3192	shrink
34	364	38.3183	shrink
35	372	38.3183	contract
36	388	38.3182	shrink
37	404	38.3182	shrink
38	420	38.3182	shrink
39	428	38.3182	contract
Optimal KP Optimal KI Optimal cost J			
0.667782 0.0212588 38.3182			
Algorithm run # 2			
Starting KP Starting KI theta mu Duration T (sec)			
0.667782 0.0212588 0 0.0001 1000			
Iteration	Func-count	min f(x)	Procedure
0	4	38.3134	
1	12	38.3134	initial simplex
2	28	38.3134	shrink
3	44	38.3134	shrink
4	52	36.354	expand
5	56	36.354	reflect
6	64	34.6553	expand

7	72	31.329	expand
8	80	26.4811	expand
9	88	19.9677	expand
10	96	10.7447	expand
11	104	9.2205	reflect
12	120	9.2205	shrink
13	136	9.2205	shrink
14	152	9.2205	shrink
15	168	9.2205	shrink
16	176	9.20384	contract
17	184	9.16041	expand
18	188	9.16041	reflect
19	196	9.10263	expand
20	204	9.07892	expand
21	212	8.88061	expand
22	216	8.88061	reflect
23	224	8.56901	expand
24	232	8.50489	expand
25	240	7.84742	expand
26	244	7.84742	reflect
27	252	7.11033	expand
28	260	6.55647	expand
29	268	5.67518	expand
30	276	4.8346	expand
31	284	4.17743	expand
32	292	3.40633	expand
33	300	3.40633	contract
34	308	3.0921	expand
35	316	2.78411	reflect
36	324	2.3917	expand
37	332	2.3917	contract
38	336	2.3917	reflect
39	344	2.22491	expand
40	352	1.90301	expand
41	356	1.90301	reflect
42	364	1.72055	reflect
43	372	1.72055	contract
44	376	1.72055	reflect
45	384	1.64056	expand
46	392	1.64056	contract
47	400	1.60404	reflect
48	408	1.60404	contract
49	416	1.59117	contract
50	424	1.59117	contract
51	432	1.59117	contract
52	440	1.59117	contract
53	444	1.59117	reflect
54	448	1.59117	reflect
55	456	1.59074	contract
56	464	1.59068	contract
57	472	1.59065	contract
58	480	1.59056	contract
59	488	1.59056	contract
60	496	1.59056	contract

61	504	1.59055	contract
62	512	1.59055	contract
63	520	1.59055	contract
64	528	1.59055	contract
65	532	1.59055	reflect
66	540	1.59055	contract
67	548	1.59055	contract
68	556	1.59055	contract
69	564	1.59055	contract
70	572	1.59055	contract
71	580	1.59055	contract
72	588	1.59055	contract
73	596	1.59055	contract
74	604	1.59055	contract
75	612	1.59055	contract
76	620	1.59055	contract
77	628	1.59055	contract
78	636	1.59055	contract
79	644	1.59055	contract
80	652	1.59055	contract
81	660	1.59055	contract
82	664	1.59055	reflect
83	672	1.59055	contract
84	680	1.59055	contract

Optimal KP	Optimal KI	Optimal cost J
0.712985	2.00619	1.59055

Algorithm run # 3

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.712985	2.00619	0	0.0001	300

Iteration	Func-count	min f(x)	Procedure
0	4	1.59055	
1	12	1.59055	initial simplex
2	16	1.59055	reflect
3	24	1.59055	contract
4	28	1.59055	reflect
5	44	1.59055	shrink
6	60	1.59055	shrink
7	76	1.59055	shrink
8	92	1.59055	shrink
9	108	1.59055	shrink
10	124	1.59055	shrink
11	140	1.59055	shrink
12	156	1.59055	shrink
13	172	1.59055	shrink
14	188	1.59055	shrink
15	204	1.59055	shrink
16	220	1.59055	shrink
17	236	1.59055	shrink
18	252	1.59055	shrink
19	268	1.59055	shrink

Optimal KP	Optimal KI	Optimal cost J
0.712985	2.00619	1.59055

Design 1 with $\theta = 10$:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	10	0.0001	10000
Iteration	Func-count	min f(x)	Procedure	
0	4	486.743		
1	12	485.683	initial simplex	
2	20	484.426	expand	
3	28	482.672	expand	
4	36	479.333	expand	
5	44	474.282	expand	
6	52	465.467	expand	
7	60	451.929	expand	
8	68	430.196	expand	
9	76	398.617	expand	
10	84	354.296	expand	
11	92	299.08	expand	
12	100	237.071	expand	
13	108	176.569	expand	
14	116	124.478	expand	
15	124	84.4005	expand	
16	132	56.2252	expand	
17	140	37.5146	expand	
18	148	31.9591	reflect	
19	164	31.9591	shrink	
20	180	31.9591	shrink	
21	188	30.1804	reflect	
22	196	29.3101	reflect	
23	212	29.3101	shrink	
24	228	29.3101	shrink	
25	244	29.3101	shrink	
26	248	29.3101	reflect	
27	256	29.3101	contract	
28	264	29.265	contract	
29	268	29.265	reflect	
30	284	29.265	shrink	
31	300	29.265	shrink	
32	308	29.2649	reflect	
33	316	29.2641	reflect	
34	324	29.2637	contract	
35	332	29.2635	reflect	
36	340	29.2635	contract	
37	348	29.2628	expand	
38	352	29.2628	reflect	
39	360	29.2615	expand	
40	364	29.2615	reflect	

41	372	29.261	expand
42	380	29.258	expand
43	384	29.258	reflect
44	392	29.2519	expand
45	396	29.2519	reflect
46	404	29.2412	expand
47	412	29.2343	expand
48	420	29.2075	expand
49	428	29.1872	expand
50	436	29.1157	expand
51	444	29.1008	expand
52	452	28.9211	expand
53	460	28.9211	contract
54	468	28.774	expand
55	476	28.4249	expand
56	484	28.0376	expand
57	492	27.1478	expand
58	500	26.1245	expand
59	508	24.1048	expand
60	516	21.6709	expand
61	524	17.8847	expand
62	532	13.6857	expand
63	540	8.92275	expand
64	548	4.82966	expand
65	556	1.96297	expand
66	564	1.32873	expand
67	572	1.32873	contract
68	580	1.25181	contract
69	584	1.25181	reflect
70	592	1.2166	contract
71	596	1.2166	reflect
72	604	1.21398	contract
73	612	1.21398	contract
74	620	1.21312	reflect
75	628	1.2107	contract
76	636	1.20909	expand
77	644	1.20909	contract
78	652	1.20613	expand
79	656	1.20613	reflect
80	664	1.20096	expand
81	668	1.20096	reflect
82	676	1.19688	expand
83	684	1.19642	reflect
84	700	1.19617	shrink
85	708	1.19617	contract
86	716	1.19617	contract
87	724	1.19614	contract
88	732	1.19612	contract
89	740	1.19612	contract
90	748	1.19612	contract
91	756	1.19611	contract
92	764	1.19611	contract
93	772	1.19611	contract
94	776	1.19611	reflect

95	784	1.19611	contract
96	792	1.19611	contract
97	800	1.19611	contract
98	808	1.19611	contract
99	816	1.19611	contract
100	824	1.19611	contract
101	832	1.19611	contract
102	840	1.19611	contract
103	848	1.19611	contract
104	856	1.19611	contract
105	860	1.19611	reflect
106	868	1.19611	contract
107	876	1.19611	contract

Optimal KP Optimal KI Optimal cost J
0.451414 0.137636 1.19611

Algorithm run # 2

Starting KP Starting KI theta mu Duration T (sec)
0.451414 0.137636 10 0.0001 1000

Iteration	Func-count	min f(x)	Procedure
0	4	1.19611	
1	12	1.19611	initial simplex
2	20	1.19611	contract
3	28	1.19611	contract
4	36	1.19611	contract
5	40	1.19611	reflect
6	48	1.19611	contract
7	56	1.19611	contract
8	64	1.19611	contract
9	68	1.19611	reflect
10	76	1.19611	contract
11	84	1.19611	contract
12	92	1.19611	contract
13	100	1.19611	contract
14	108	1.19611	contract
15	116	1.19611	contract
16	124	1.19611	contract
17	132	1.19611	contract
18	140	1.19611	contract
19	148	1.19611	contract
20	156	1.19611	contract
21	164	1.19611	contract
22	172	1.19611	contract
23	180	1.19611	contract
24	188	1.19611	contract
25	196	1.19611	contract
26	204	1.19611	contract
27	212	1.19611	contract
28	216	1.19611	reflect
29	224	1.19611	contract
30	232	1.19611	contract

31	240	1.19611	contract
Optimal KP	Optimal KI	Optimal cost J	
0.451414	0.137636	1.19611	

Design 2 with $\theta = 0$:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	0	0.0001	15000

Iteration	Func-count	min f(x)	Procedure
0	4	496.693	
1	12	495.633	initial simplex
2	20	494.376	expand
3	28	492.621	expand
4	36	489.282	expand
5	44	484.231	expand
6	52	475.415	expand
7	60	461.875	expand
8	68	440.139	expand
9	76	408.556	expand
10	84	364.228	expand
11	92	308.999	expand
12	100	246.97	expand
13	108	186.435	expand
14	116	134.291	expand
15	124	94.1337	expand
16	132	65.8417	expand
17	140	46.9734	expand
18	148	41.3391	reflect
19	164	41.3391	shrink
20	180	41.3391	shrink
21	188	39.5219	reflect
22	196	38.5712	reflect
23	212	38.5712	shrink
24	228	38.5712	shrink
25	244	38.5712	shrink
26	252	38.3787	reflect
27	268	38.3787	shrink
28	276	38.324	reflect
29	292	38.324	shrink
30	308	38.324	shrink
31	324	38.324	shrink
32	340	38.324	shrink
33	348	38.3214	reflect
34	364	38.3214	shrink
35	372	38.3214	contract
36	380	38.3214	contract
37	388	38.3213	contract
38	396	38.3213	contract
39	400	38.3213	reflect

40	408	38.3213	contract
41	416	38.3213	contract
Optimal KP 0.667647	Optimal KI 0.0212544	Optimal cost J 38.3213	
Algorithm run # 2			
Starting KP 0.667647	Starting KI 0.0212544	theta 0	mu 0.0001
			Duration T (sec) 15000
Iteration	Func-count	min f(x)	Procedure
0	4	38.3213	
1	12	38.3213	initial simplex
2	28	38.3213	shrink
3	44	38.3213	shrink
4	60	38.3213	shrink
5	76	38.3213	shrink
6	92	38.3213	shrink
7	108	38.3213	shrink
8	116	38.1922	expand
9	120	38.1922	reflect
10	128	38.0885	expand
11	136	37.8421	expand
12	144	37.5124	expand
13	152	36.8687	expand
14	160	35.9034	expand
15	168	34.2218	expand
16	176	31.6075	expand
17	184	27.5019	expand
18	192	21.4043	expand
19	200	13.4983	expand
20	208	10.1282	reflect
21	224	10.1282	shrink
22	232	9.4035	reflect
23	248	9.37931	shrink
24	264	9.25342	shrink
25	268	9.25342	reflect
26	276	9.25342	contract
27	284	9.25342	contract
28	292	9.25342	contract
29	300	9.25342	contract
30	308	9.25342	contract
31	316	9.24775	expand
32	320	9.24775	reflect
33	328	9.24068	expand
34	336	9.23092	expand
35	344	9.21939	expand
36	352	9.18595	expand
37	360	9.17889	expand
38	368	9.09544	expand
39	372	9.09544	reflect
40	380	8.99826	expand
41	388	8.86286	expand

42	396	8.64943	expand
43	404	8.28432	expand
44	412	7.88871	expand
45	420	7.15808	expand
46	428	6.66465	expand
47	436	5.73331	expand
48	444	5.39812	expand
49	452	4.9825	reflect
50	460	4.9825	contract
51	468	4.72609	expand
52	476	4.48985	reflect
53	484	3.99258	expand
54	492	3.5858	expand
55	500	3.13197	expand
56	508	2.50572	expand
57	512	2.50572	reflect
58	520	1.94473	expand
59	528	1.94473	contract
60	536	1.8023	expand
61	552	1.78907	shrink
62	560	1.64784	reflect
63	568	1.64784	contract
64	576	1.60724	expand
65	592	1.60724	shrink
66	596	1.60724	reflect
67	604	1.59696	reflect
68	612	1.59093	contract
69	620	1.59093	contract
70	628	1.59093	contract
71	636	1.59093	contract
72	644	1.59067	contract
73	652	1.59061	contract
74	660	1.59061	contract
75	668	1.59058	contract
76	676	1.59055	contract
77	684	1.59055	contract
78	688	1.59055	reflect
79	696	1.59055	contract
80	704	1.59055	contract
81	712	1.59055	contract
82	720	1.59055	contract
83	728	1.59055	contract
84	736	1.59055	contract
85	744	1.59055	contract
86	752	1.59055	contract
87	760	1.59055	contract
88	768	1.59055	contract
89	776	1.59055	contract
90	784	1.59055	contract
91	792	1.59055	contract
92	800	1.59055	contract
93	808	1.59055	contract
94	816	1.59055	contract
95	824	1.59055	contract

96	832	1.59055	contract
97	840	1.59055	contract
98	848	1.59055	contract
Optimal KP	Optimal KI	Optimal cost J	
0.712984	2.00618	1.59055	
Algorithm run # 3			
Starting KP	Starting KI	theta	mu
0.712984	2.00618	0	0.0001
		Duration T (sec)	500
Iteration	Func-count	min f(x)	Procedure
0	4	1.59055	
1	12	1.59055	initial simplex
2	16	1.59055	reflect
3	24	1.59055	contract
4	28	1.59055	reflect
5	44	1.59055	shrink
6	60	1.59055	shrink
7	76	1.59055	shrink
8	92	1.59055	shrink
9	108	1.59055	shrink
10	124	1.59055	shrink
11	140	1.59055	shrink
12	156	1.59055	shrink
13	172	1.59055	shrink
14	188	1.59055	shrink
15	204	1.59055	shrink
16	220	1.59055	shrink
17	236	1.59055	shrink
18	252	1.59055	shrink
19	260	1.59055	contract
20	264	1.59055	reflect
21	272	1.59055	reflect
22	280	1.59055	contract
23	288	1.59055	contract
24	296	1.59055	contract
Optimal KP	Optimal KI	Optimal cost J	
0.712985	2.00619	1.59055	

Design 2 with $\theta = 10$:

Algorithm run # 1				
Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0005	1e-005	10	0.0001	15000
Iteration	Func-count	min f(x)	Procedure	
0	4	486.743		
1	12	485.683	initial simplex	
2	20	484.426	expand	

3	28	482.671	expand
4	36	479.333	expand
5	44	474.282	expand
6	52	465.467	expand
7	60	451.929	expand
8	68	430.196	expand
9	76	398.617	expand
10	84	354.296	expand
11	92	299.08	expand
12	100	237.071	expand
13	108	176.569	expand
14	116	124.478	expand
15	124	84.4005	expand
16	132	56.2252	expand
17	140	37.5146	expand
18	148	31.9591	reflect
19	164	31.9591	shrink
20	180	31.9591	shrink
21	188	30.1804	reflect
22	196	29.3101	reflect
23	212	29.3101	shrink
24	228	29.3101	shrink
25	244	29.3101	shrink
26	248	29.3101	reflect
27	256	29.3101	contract
28	264	29.265	contract
29	268	29.265	reflect
30	284	29.265	shrink
31	300	29.265	shrink
32	308	29.2649	reflect
33	316	29.2641	reflect
34	324	29.2637	contract
35	332	29.2635	reflect
36	340	29.2635	contract
37	348	29.2628	expand
38	352	29.2628	reflect
39	360	29.2615	expand
40	364	29.2615	reflect
41	372	29.261	expand
42	380	29.258	expand
43	384	29.258	reflect
44	392	29.2519	expand
45	396	29.2519	reflect
46	404	29.2412	expand
47	412	29.2343	expand
48	420	29.2075	expand
49	428	29.1872	expand
50	436	29.1157	expand
51	444	29.1008	expand
52	452	28.9211	expand
53	460	28.9211	contract
54	468	28.774	expand
55	476	28.4249	expand
56	484	28.0376	expand

57	492	27.1478	expand
58	500	26.1245	expand
59	508	24.1048	expand
60	516	21.6709	expand
61	524	17.8847	expand
62	532	13.6857	expand
63	540	8.92275	expand
64	548	4.82966	expand
65	556	1.96297	expand
66	564	1.32873	expand
67	572	1.32873	contract
68	580	1.25181	contract
69	584	1.25181	reflect
70	592	1.2166	contract
71	596	1.2166	reflect
72	604	1.21398	contract
73	612	1.21398	contract
74	620	1.21312	reflect
75	628	1.2107	contract
76	636	1.20909	expand
77	644	1.20909	contract
78	652	1.20613	expand
79	656	1.20613	reflect
80	664	1.20096	expand
81	668	1.20096	reflect
82	676	1.19688	expand
83	684	1.19642	reflect
84	700	1.19617	shrink
85	708	1.19617	contract
86	716	1.19617	contract
87	724	1.19614	contract
88	732	1.19612	contract
89	740	1.19612	contract
90	748	1.19612	contract
91	756	1.19611	contract
92	764	1.19611	contract
93	772	1.19611	contract
94	776	1.19611	reflect
95	784	1.19611	contract
96	792	1.19611	contract
97	800	1.19611	contract
98	808	1.19611	contract
99	816	1.19611	contract
100	824	1.19611	contract
101	832	1.19611	contract
102	840	1.19611	contract
103	848	1.19611	contract
104	856	1.19611	contract
105	860	1.19611	reflect
106	868	1.19611	contract
107	876	1.19611	contract
Optimal KP	Optimal KI	Optimal cost J	
0.451414	0.137636	1.19611	

Algorithm run # 2

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.451414	0.137636	10	0.0001	300

Iteration	Func-count	min f(x)	Procedure
0	4	1.19611	
1	12	1.19611	initial simplex
2	20	1.19611	contract
3	28	1.19611	contract
4	36	1.19611	contract
5	40	1.19611	reflect
6	48	1.19611	contract
7	56	1.19611	contract
8	64	1.19611	contract
9	68	1.19611	reflect
10	76	1.19611	contract
11	84	1.19611	contract
12	92	1.19611	contract
13	100	1.19611	contract
14	108	1.19611	contract
15	116	1.19611	contract
16	124	1.19611	contract
17	132	1.19611	contract
18	140	1.19611	contract
19	148	1.19611	contract
20	156	1.19611	contract
21	164	1.19611	contract
22	172	1.19611	contract
23	180	1.19611	contract
24	188	1.19611	contract
25	196	1.19611	contract
26	204	1.19611	contract
27	212	1.19611	contract
28	216	1.19611	reflect
29	224	1.19611	contract
30	232	1.19611	contract
31	240	1.19611	contract

Optimal KP	Optimal KI	Optimal cost J
0.451414	0.137636	1.19611

C.3 Control Design for Time-Delayed System

The following optimization output was obtained when designing the two-term controller (7.6) for the SISO time-delayed system in Section 7.4.

Design of Controller for the Case $t_0 = 0s$:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.005	0.001	0	1e-008	10000

Iteration	Func-count	min f(x)	Procedure
0	1	510.057	
1	3	419.381	initial simplex
2	5	343.394	expand
3	7	279.258	expand
4	9	202.944	expand
5	11	146.48	expand
6	13	98.432	expand
7	15	67.3507	expand
8	17	45.5836	expand
9	19	32.6306	expand
10	21	24.5119	expand
11	23	20.3691	expand
12	25	18.8239	expand
13	26	18.8239	reflect
14	28	18.8239	contract
15	30	18.8239	contract
16	32	18.8239	contract
17	34	18.8239	contract
18	36	18.7215	expand
19	37	18.7215	reflect
20	39	18.5328	expand
21	41	18.4331	expand
22	43	17.9814	expand
23	45	17.7027	expand
24	47	16.6433	expand
25	49	16.2956	expand
26	51	14.1993	expand
27	52	14.1993	reflect
28	54	12.1035	expand
29	56	10.6963	expand
30	58	8.43731	expand
31	60	8.15482	expand
32	62	7.99759	reflect
33	64	7.79621	contract
34	66	7.56982	reflect
35	68	7.56982	contract
36	70	7.56982	contract
37	72	7.54407	reflect
38	74	7.54407	contract

39	76	7.538	contract
40	78	7.53593	contract
41	80	7.53265	contract
42	82	7.53265	contract
43	83	7.53265	reflect
44	85	7.53176	contract
45	87	7.53176	contract
46	89	7.53176	contract
47	91	7.53174	contract
48	93	7.53172	contract
49	95	7.53167	contract
50	96	7.53167	reflect
51	98	7.53167	contract
52	99	7.53167	reflect
53	100	7.53167	reflect
54	102	7.53167	contract
55	104	7.53167	contract
56	106	7.53167	contract
57	108	7.53166	contract
58	110	7.53166	contract
59	112	7.53166	reflect
60	114	7.53166	contract
61	116	7.53166	contract
62	118	7.53166	contract
63	120	7.53166	contract
64	122	7.53166	contract
65	124	7.53166	contract
66	126	7.53166	contract

Optimal KP Optimal KI Optimal cost J
3.25181 0.105815 7.53166

Algorithm run # 2

Starting KP Starting KI theta mu Duration T (sec)
3.25181 0.105815 0 1e-008 10000

Iteration	Func-count	min f(x)	Procedure
0	1	7.53166	
1	3	7.53166	initial simplex
2	4	7.53166	reflect
3	6	7.53166	contract
4	8	7.53166	contract
5	10	7.53166	contract
6	12	7.53166	contract
7	14	7.53166	contract
8	16	7.53166	contract
9	18	7.53166	contract
10	20	7.53166	contract
11	22	7.53166	contract
12	24	7.53166	contract
13	26	7.53166	contract
14	28	7.53166	contract
15	30	7.53166	contract

16	32	7.53166	contract
17	34	7.53166	contract
18	36	7.53166	contract
19	38	7.53166	contract
20	40	7.53166	contract
21	42	7.53166	contract
22	44	7.53166	contract
23	46	7.53166	contract
24	47	7.53166	reflect
25	49	7.53166	contract
26	50	7.53166	reflect
27	52	7.53166	contract
28	56	7.53166	shrink
Optimal KP	Optimal KI	Optimal cost J	
3.25188	0.105814	7.53166	

Design of Controller for the Case $t_0 = 10s$:

Algorithm run # 1

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.005	0.001	0	1e-008	10000
Iteration	Func-count	min f(x)	Procedure	
0	1	500.065		
1	3	409.39	initial simplex	
2	5	333.405	expand	
3	7	269.27	expand	
4	9	192.961	expand	
5	11	136.501	expand	
6	13	88.4665	expand	
7	15	57.3994	expand	
8	17	35.6658	expand	
9	19	22.7576	expand	
10	21	14.7269	expand	
11	23	10.7147	expand	
12	25	9.40087	expand	
13	26	9.40087	reflect	
14	28	9.40087	contract	
15	32	9.40087	shrink	
16	34	9.40087	contract	
17	36	9.31978	expand	
18	38	9.31978	contract	
19	40	9.31395	expand	
20	42	9.16123	expand	
21	43	9.16123	reflect	
22	45	9.08315	expand	
23	47	8.74462	expand	
24	49	8.43401	expand	
25	51	7.67938	expand	
26	53	6.826	expand	
27	55	5.40933	expand	

28	57	3.79578	expand
29	59	2.24792	expand
30	61	0.684738	expand
31	62	0.684738	reflect
32	64	0.386672	reflect
33	65	0.386672	reflect
34	67	0.309162	contract
35	69	0.24295	contract
36	71	0.24295	contract
37	72	0.24295	reflect
38	74	0.24295	contract
39	76	0.24295	contract
40	78	0.24295	contract
41	80	0.241603	contract
42	82	0.240745	contract
43	84	0.240127	contract
44	86	0.240127	contract
45	88	0.240127	contract
46	90	0.240055	reflect
47	92	0.240037	contract
48	94	0.240023	contract
49	96	0.24002	contract
50	98	0.240016	contract
51	100	0.240014	contract
52	102	0.240013	contract
53	104	0.240012	contract
54	106	0.240012	contract
55	108	0.240012	contract
56	110	0.240012	contract
57	112	0.240012	contract
58	114	0.240012	contract
59	116	0.240012	contract
60	118	0.240012	contract

Optimal KP Optimal KI Optimal cost J
2.68214 0.104779 0.240012

Algorithm run # 2

Starting KP Starting KI theta mu Duration T (sec)
2.68214 0.104779 0 1e-008 10000

Iteration	Func-count	min f(x)	Procedure
0	1	0.240012	
1	3	0.240012	initial simplex
2	4	0.240012	reflect
3	6	0.240012	contract
4	8	0.240012	contract
5	10	0.240012	contract
6	12	0.240012	contract
7	14	0.240012	contract
8	16	0.240012	contract
9	18	0.240012	contract
10	20	0.240012	contract

11	22	0.240012	contract
12	24	0.240012	contract
13	26	0.240012	contract
14	28	0.240012	contract
15	30	0.240012	contract
16	32	0.240012	contract
17	34	0.240012	contract
18	36	0.240012	contract
19	38	0.240012	contract
20	40	0.240012	contract
21	41	0.240012	reflect
22	43	0.240012	contract
23	45	0.240012	contract
24	47	0.240012	contract
25	48	0.240012	reflect
26	50	0.240012	contract
27	52	0.240012	contract
Optimal KP Optimal KI Optimal cost J			
2.68205 0.104782 0.240012			

Design of Controller for the Case $t_0 = 18s$:

Algorithm run # 1				
Starting KP	Starting KI	theta	mu	Duration T (sec)
0.0001	0.001	0	1e-008	10000
Iteration	Func-count	min f(x)	Procedure	
0	1	494.588		
1	3	403.703	initial simplex	
2	5	327.965	expand	
3	7	263.331	expand	
4	9	187.25	expand	
5	11	130.436	expand	
6	13	82.666	expand	
7	15	51.5358	expand	
8	17	30.2733	expand	
9	19	17.8395	expand	
10	21	11.1073	expand	
11	23	9.50261	expand	
12	24	9.50261	reflect	
13	26	9.42662	contract	
14	30	9.4024	shrink	
15	34	9.39781	shrink	
16	38	9.39609	shrink	
17	42	9.39592	shrink	
18	46	9.39575	shrink	
19	48	9.39575	contract	
20	50	9.39573	contract	
21	52	9.39573	contract	
22	54	9.39564	expand	
23	56	9.39564	contract	

Optimal KP	Optimal KI	Optimal cost J
0.00379458	0.0597978	9.39564

Algorithm run # 2

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.00379458	0.0597978	0	1e-008	10000

Iteration	Func-count	min f(x)	Procedure
0	1	9.39564	
1	3	9.39564	initial simplex
2	5	9.39111	contract
3	9	9.39111	shrink
4	11	9.38525	contract
5	13	9.37172	expand
6	15	9.37172	contract
7	17	9.3555	expand
8	19	9.34006	expand
9	21	9.29058	expand
10	22	9.29058	reflect
11	24	9.19216	expand
12	26	9.15074	expand
13	28	8.91525	expand
14	30	8.79194	expand
15	32	8.23681	expand
16	34	8.04392	expand
17	36	6.98654	expand
18	37	6.98654	reflect
19	39	5.82043	expand
20	41	4.76903	expand
21	43	3.84793	expand
22	45	2.3836	expand
23	46	2.3836	reflect
24	48	1.64297	expand
25	50	0.742881	expand
26	51	0.742881	reflect
27	53	0.149588	expand
28	55	0.130187	reflect
29	57	0.0883087	contract
30	59	0.0313307	contract
31	61	0.0313307	contract
32	63	0.0313307	contract
33	65	0.0223514	expand
34	67	0.016719	contract
35	69	0.0121824	contract
36	71	0.00470792	contract
37	73	0.00470792	contract
38	75	0.00470792	contract
39	76	0.00470792	reflect
40	78	0.00470792	contract
41	80	0.00470792	contract
42	82	0.00452537	contract
43	84	0.00452166	contract

44	86	0.00451874	contract
45	88	0.00449695	contract
46	90	0.00449695	contract
47	92	0.0044959	contract
48	94	0.00449428	reflect
49	96	0.00449235	contract
50	98	0.00449235	contract
51	100	0.00449235	contract
52	102	0.00449218	contract
53	104	0.00449217	contract
54	106	0.00449212	contract
55	108	0.00449206	contract
56	109	0.00449206	reflect
57	111	0.00449206	contract
58	113	0.00449206	contract
59	115	0.00449206	contract
60	117	0.00449206	contract
61	119	0.00449206	contract

Optimal KP Optimal KI Optimal cost J
1.94885 0.0921382 0.00449206

Algorithm run # 3

Starting KP Starting KI theta mu Duration T (sec)
1.94885 0.0921382 0 1e-008 10000

Iteration	Func-count	min f(x)	Procedure
0	1	0.00449206	
1	3	0.00449206	initial simplex
2	5	0.00449206	contract
3	7	0.00449206	contract
4	8	0.00449206	reflect
5	10	0.00449206	contract
6	12	0.00449206	contract
7	14	0.00449206	contract
8	16	0.00449206	contract
9	17	0.00449206	reflect
10	19	0.00449206	contract
11	21	0.00449206	contract
12	23	0.00449206	contract
13	25	0.00449206	contract
14	27	0.00449206	contract
15	29	0.00449206	contract
16	31	0.00449206	contract
17	33	0.00449206	contract
18	35	0.00449206	contract
19	37	0.00449206	contract
20	39	0.00449206	contract
21	41	0.00449206	contract
22	43	0.00449206	contract
23	45	0.00449206	contract
24	47	0.00449206	contract
25	49	0.00449206	contract

26	51	0.00449206	contract
27	53	0.00449206	contract
28	54	0.00449206	reflect
Optimal KP	Optimal KI	Optimal cost J	
1.94885	0.0921382	0.00449206	

C.4 Control Design for Experimental MARTS

The following optimization output was obtained when designing the two-term controller (2.6) for the experimental MARTS system in Section 8.5.

Algorithm run # 1				
Starting KP	Starting KI	theta	mu	Duration T (sec)
0.01	0.005	0	1	500
Iteration	Func-count	min f(x)	Procedure	
0	1	1068.18		
1	3	246.505	initial simplex	
2	5	123.664	expand	
3	7	87.3302	expand	
4	9	79.3822	expand	
5	11	79.3822	contract	
6	13	79.3822	contract	
7	15	79.1632	reflect	
8	19	78.0586	shrink	
9	21	76.8838	expand	
10	23	76.8838	contract	
11	25	73.5444	expand	
12	27	72.0638	expand	
13	29	70.1961	expand	
14	30	70.1961	reflect	
15	32	68.231	expand	
16	34	65.8539	expand	
17	35	65.8539	reflect	
18	36	65.8539	reflect	
19	38	64.1645	contract	
20	40	64.1645	contract	
21	42	64.1607	reflect	
22	44	63.7266	contract	
23	48	63.7266	shrink	
24	50	63.282	reflect	
25	52	63.282	contract	
26	56	63.282	shrink	
27	58	63.282	contract	
Optimal KP	Optimal KI	Optimal cost J		

0.642618 0.0775545 63.282

Algorithm run # 2

Starting KP	Starting KI	theta	mu	Duration T (sec)
0.642618	0.0775545	0	1	240

Iteration	Func-count	min f(x)	Procedure
0	1	61.0768	
1	3	61.0768	initial simplex
2	5	61.0768	contract
3	7	61.0768	contract
4	8	61.0768	reflect
5	10	61.0768	contract
6	12	61.0768	contract
7	13	61.0768	reflect
8	15	61.0768	contract
9	17	61.0768	contract
10	19	61.0768	contract
11	23	61.0768	shrink
12	27	61.0768	shrink
13	29	61.0768	contract
14	30	61.0768	reflect
15	34	61.0768	shrink
16	38	61.0768	shrink

Optimal KP	Optimal KI	Optimal cost J
0.642618	0.0775545	61.0768

Appendix D

Program Listings

This appendix contains MATLAB programs and Simulink models that were used to generate the simulation and experimental results presented in the thesis document.

D.1 Valve Linearization

The following two programs were used to obtain the square-root functions for MARTS control valve linearization.

```
%-----  
%File name: LinearizeValve1.m  
%By: William Szu-Wei Wang 994792079  
  
%This program attempts to find the appropriate square-root function  
%to linearize MARTS control valve 1 based on experimental data  
%-----  
  
%Input voltage to control valve in 0.5V increments  
Voltage = [5;4.5;4;3.5;3;2.5;2;1.5;1;0.5;0;-0.5;-1;-1.5;-2;  
          -2.5;-3;-3.5;-4;-4.5;-5];  
  
%Rate of change of measured water height  
Rate = [ 0.0291;0.0282;0.0278;0.0277; 0.0273;0.0260; 0.0256;  
         0.0238; 0.0219;0.0192; 0.0151; 0.0118; 0.0106;0.0088;  
         0.0071;0.0047; 0.0034;0.0022; 0.0010;5.6966e-004; 0];  
  
%Perform curve fitting on the original experimental data  
%to obtain a poly equation  
N = 4; %fourth order polynomial seems to be sufficient  
Z = polyfit(Voltage,Rate,N);  
  
u = [-5:0.01:5]; %control valve input range
```

```

for alpha=0.1:0.1:3 %plot a curve for each alpha
new_u.to.output = ((alpha*sqrt((u+5)))-5); %apply square root fcn
h = polyval(Z, new_u.to.output); %obtain corresponding rates
plot(u,h), hold on;
end
plot(Voltage, Rate, 'r'); %impose the experimental data
hold off;
ylabel('Rate of water height increase in Vessel 1 (V/s)')
xlabel('Control input voltage (V)')
%Note: the plot shows that no alpha value gives rise to a better
%linear characteristic than the experimental data itself without
%sacrificing maximum flow rate.

```

```

%-----
%File name: LinearizeValve2.m
%By: William Szu-Wei Wang 994792079

%This program attempts to find the appropriate square-root function
%to linearize MARTS control valve 2 based on experimental data
%-----

%Input voltage to control valve in 0.5V increments
Voltage = [5;4.5;4;3.5;3;2.5;2;1.5;1;0.5;0;-0.5;-1;-1.5;-2;
          -2.5;-3;-3.5;-4;-4.5;-5];

%Rate of change of measured water height
Rate = [0.047;0.046;0.046;0.042;0.041;0.036;0.032;0.029;0.026;
        0.021;0.017;0.013;0.011;0.007;0.006;0.004;0.003;0.002;
        0.001;0.001;0.001];

%Perform curve fitting on the original experimental data
%to obtain a poly equation
N = 4; %fourth order polynomial seems to be sufficient
Z = polyfit(Voltage,Rate,N);

alpha = 2.8; %scale constant of square root function

u = [-5:0.01:5]; %control valve input range
new_u.to.output = ((alpha*sqrt((u+5)))-5); %apply square root fcn
h = polyval(Z, new_u.to.output); %obtain corresponding rates

%The plot gives an idea of what to expect when the square root
%function is applied on MARTS during an experiment
plot(u,h)
title('Linearized valve 2 based on curve fitting process')
ylabel('Rate of water height increase in Vessel 2 (V/s)')
xlabel('Control input voltage (V)')

```

D.2 Proportional Controller Design

The following program was used to design the proportional controller in Section 3.2.

```
%-----
%File name: RLOptimization.m
%By: William Szu-Wei Wang 994792079

%This program solves the illustration example
%(design of proportional controller) presented in the "Optimization
%Algorithm" chapter by calling the Nelder-Mead algorithm
%-----

clear
clc

%Initial condition for gain K , which is denoted by variable x
x0 = 1e-3;

options = optimset('FunValCheck','on','Display','iter',...
                  'MaxFunEvals', 1000, 'MaxIter',1000);

x = x0;

display('fminsearch results')
pause

display('first iteration: ')
[x,fval,exitflag,output] = fminsearch(@fnc,x,options);
display('optimal gain is:')
x
display('real part of largest pole is:')
fval

pause

display('2nd iteration: ')
[x,fval,exitflag,output] = fminsearch(@fnc,x,options);
display('optimal gain is:')
x
display('real part of largest pole is:')
fval
```

```

%-----
%File name: fnc.m
%By: William Szu-Wei Wang 994792079

%This file contains the performance index for the
%illustration example and is called from within RLOptimization.m
%-----

function f = fnc(x)

%Denominator of Close-loop transfer function
den = [(x(1)+1) (3*x(1)-8) (15+2*x(1))];
CLSpoles = roots(den);

f = max(real(CLSpoles));

end

```

D.3 Extremum Seeking

The following program and Simulink model were used to carry out the extremum seeking controller design example in Section 7.

```

%-----
%File name: ESControllerTuning.m
%By: William Szu-Wei Wang 994792079

%This program simulates and solves the extremum seeking
%controller design problem presented in the
%"Comparison Example" chapter
%-----

clear
clc

%3rd order pade approximation of a time delay of 5 sec
[num,den] = pade(5,3);

%starting parameters found via Ziegler-Nichols method
KES = 2.8;
TiES = 14.4;
thetaES = [KES;TiES]; %controller parameters (theta)

%parameters of ES
aES = 0.8; %perturbation frequency parameter (a)
hES = 0.5; %filter parameter (h)
alphaES = [0.1;1]; %perturbation amplitude (alpha)

```

```

gammaES = [200;1200];    %adaptation gain (gamma)
omegaES = [aES^1*pi;aES^2*pi]; %perturbation frequency (omega)

%parameters for cost function
T = 100;    %Tfinal
t0 = 18;    %Tpeak

%define arrays to keep track of gains and costs
thetaESarray= []; %to keep track of theta
JESarray = []; %to keep track of cost J
xiESarray = []; % to keep track of the variable xi
theta_hESarray = []; %to keep track of theta_hat
theta_hESarray(:,1) = thetaES;    %theta_hat(0) = theta(0)
indexK = 0;
TotalNumEvaluation = 2000; %terminate tuning process after this
                           %number of function evaluations

ESSimCommands; %Simulate for k = 0, call simulation commands
thetaESarray(:,1) = thetaES    %record gains
JESarray(:,1) = J    %record cost

for i = 1:TotalNumEvaluation
    if indexK == 0

        xiESarray(:,1) = J;    % xi(0) = J(0)
    else
        xiESarray(:,indexK+1) = -hES*xiESarray(:,indexK)...
                                +JESarray(:,indexK);
    end

    for j = 1:length(thetaES)
        temp1=cos(omegaES*indexK);
        theta_hESarray(j,indexK+2)=theta_hESarray(j,indexK+1)...
        -gammaES(j,1)*alphaES(j,1)*temp1(j,1)*...
        (JESarray(:,indexK+1)-(1+hES)*xiESarray(:,indexK+1));
    end

    for v = 1:length(thetaES)
        temp1=cos(omegaES*(indexK+1));
        thetaESarray(v,indexK+2)=theta_hESarray(v,indexK+2)...
        +alphaES(v,1)*temp1(v,1);
    end

    %upload the next set of controller parameters to simulate
    thetaES = thetaESarray(:,indexK+2)
    ESSimCommands;
    JESarray(:,indexK+2)=J
    indexK = indexK+1;
end

%plot results
temp = [1:1:length(thetaESarray)];
plot(temp, thetaESarray)

```

```

xlabel('Number of function evaluations, k')
ylabel('Controller parameters \theta')
legend('K', 'T_i')
title(['\alpha=[0.1 1]^T \gamma=[200 1200]^T'...
      '\omega=[0.8\pi 0.64\pi]^T '])

```

```

%-----
%File name: ESSimCommands.m
%By: William Szu-Wei Wang 994792079

%This file contains commands that control the execution of the
%Simulink model and is called from within ESControllerTuning.m
%-----
set_param(bdroot, 'StopTime', 'T'); %set simulation duration
pause(0.5);
set_param(bdroot, 'SimulationCommand', 'connect'); %connect model
pause(0.5);
set_param(bdroot, 'SimulationCommand', 'start'); %start

%check if simulation stopped
while strcmp(get_param(bdroot, 'SimulationStatus'), 'stopped') ~= 1
    pause(0.5)
end
pause(0.5);

%retrieve the logged performance index value
J = CostValueScopeData.signals(1).values;

```


Figure D.1: Simulink model for extremum seeking controller tuning.

D.4 Optimization Problem A and B

The following program and Simulink model were used to solve optimization problems A and B for the simulation examples presented in Chapters 4 to 7.

```
%-----
%File name: RSPsetup.m
%By: William Szu-Wei Wang 994792079

%This is the main file to run. It calls PlantAndIC.m at
%start-up to define the plant model and initial conditions,
%then calls for the Nelder-Mead algorithm in NelderAlgorithm.m
%-----

%Define plant model, saturation bounds and initial conditions
PlantAndIC

%Seed variables for generating sensor noise
seed1 = 0;
seed2 = 0;

%Performance index parameters
mu = 1e3;
theta = 0;

%Tuning regulator parameters
%flipped signs cause of implementation form
epsilon_tuningReg = -1e-3;
K_tuningReg = S;

%Two-term controller starting PI parameters
%flipped signs cause of implementation form
initial_KP = -5e-4 ;
initial_KI = -1e-5 ;

%Nelder initial simplex parameters
simplex_lower_bound = 0.9;
simplex_upper_bound = 1.1;

%Control Saturation bounds
ControlUpperBound = Usat;
ControlLowerBound = -Usat;

%Time after which the performance index value
%is reset to 0. This essentially defines the cost function's
%sum or integral lower bound
TimeToResetJ = 0;
%Extract the last x seconds of steady state
SStimeForExtraction = 100;
%Total simulation time in seconds
TotalSimTime = 14000;
```

```

%data management counter
savecounter = 1;

%%%%%%%%%%Execution of algorithm %%%%%%%%%%
totalAlgoRepeats = 1;
simoption = 2;      %1 = Test single set of gain in simulink
                   %2 = complete nelder optimization
simmode = 1;        %1 = Solve Optimization Problem A
                   %2 = Solve Optimization Problem B

if simoption == 1
    nextSetOfGains = [initial_KP;initial_KI];
    NelderSimulationScript %call simulation commands
    J_value
else
    disp(' ')
    disp(['Algorithm run # ' num2str(1)])
    %starting point of algorithm
    FirstSetOfGains=(1/simplex_lower_bound)*[initial_KP;initial_KI];
    NelderAlgorithm %call Nelder Mead algorithm
    pause
    for numOfRepeats = 1: totalAlgoRepeats
        disp(' ')
        disp(['Algorithm run # ' num2str(numOfRepeats+1)])
        FirstSetOfGains = (1/simplex_lower_bound)*xopt;
        NelderAlgorithm
        pause
    end
end
end

```

```

%-----
%File name: PlantAndIC.m
%By: William Szu-Wei Wang 994792079

%This file defines the plant model, saturation bounds and initial
%conditions for the controller parameter optimization setup.
%This file is called from within RSPsetup.m
%-----

clear
format short e
clc

% MARTS continuous model
%Parameters before pre-compensating with an output feedback
a_pre = [-0.0318 0.0135; 0.0135 -0.0350];
b_pre = [0.003 0; 0 0.0045];
c_pre = eye(2,2);
d_pre = zeros(2,2);
e_pre = zeros(2,2);
ff_pre = zeros(2,2);
Kpre = 10*eye(2,2); %gain of stabilizing output feedback

```

```

%Parameters after compensation
a = a_pre+b_pre*-Kpre;
b = b_pre;
c = c_pre;
d = d_pre;
e = e_pre;
ff = ff_pre;
Scont = inv(d-c*inv(a)*b);

%dimensions of matrices
[nn,mm]=size(b);
[r,nn]=size(c);
[nn,g]=size(e);

%Discretize the plant
temp_a = a;
temp_b = [b e];
samp_period=1e-1;
[temp_ad, temp_bd]=c2d(temp_a,temp_b,samp_period);
a_discrete = temp_ad;
b_discrete = temp_bd(:, (1:mm));
e_discrete = temp_bd(:, ((mm+1):(mm+g)));
c_discrete = c;
d_discrete = d;
ff_discrete = ff;

S=inv(d_discrete+c_discrete*inv(eye(nn,nn)-a_discrete)*b_discrete);

Usat = [5;5]; %Saturation bounds

%Pick initial conditions x0, eta0, yref and disturbance w from
%unit disk ball and put into the matrices XList, EtaList,
%YrefList, and WDisturbList respectively. Each row of the lists
%represents 1 initial condition. For example,the ith row of
%YrefList goes with ith row of WDisturbList and the number of
%rows of Yref must match the number of rows of WDisturbList.
X_scaleCoeff = 1;
Eta_scaleCoeff = 1;
Yref_scaleCoeff = 0.2;
W_scaleCoeff = 1;
XList = X_scaleCoeff*[0 0; 0 0; 0 0; 0 0];
EtaList = Eta_scaleCoeff*[0 0; 0 0; 0 0; 0 0];
YrefList = Yref_scaleCoeff*[0 1; 1/sqrt(2) 1/sqrt(2); ...
                            1 0; 1/sqrt(2) -1/sqrt(2)];
WDisturbList = W_scaleCoeff*[0 0; 0 0; 0 0; 0 0];

[numOfDiffSetsOfIC, junk] = size(YrefList);

```

```

%-----
%File name: NelderSimulationScript.m
%By: William Szu-Wei Wang 994792079

%This file contains commands to control the execution of the
%Simulink model and finds the average cost of the experiments.
%The value of performance index J_A for each experiment is
%computed during each simulation run "on" the Simulink model.
%The value of index J_B is computed in the MATLAB workspace using
%logged signals after each simulation run due to the need
%of finding the steady-state values of the u_k and eta_k signals.
%This file is called from within NelderAlgorithm.m
%-----

output_form_sumJ = 0;
output_form_avgJ = 0;

for ICi = 1:numOfDiffSetsOfIC

    %the initial conditions/inputs to be simulated in current run
    x_not = XList(ICi,:)';
    eta_not = EtaList(ICi,:)';
    yref = YrefList(ICi,:)';
    disturb_w = WDisturbList(ICi,:)';

    %random sensor noise
    seed1 = randseed;
    seed2 = randseed;

    %simulate
    set_param(bdroot, 'StopTime', 'TotalSimTime');
    pause(0.5);
    set_param(bdroot, 'SimulationCommand', 'connect');
    pause(0.5);
    set_param(bdroot, 'SimulationCommand', 'start');

    while strcmp(get_param(bdroot, 'SimulationStatus'), 'stopped') ~= 1
        pause(0.5)
    end
    pause(0.5);
    if sum((USatData.signals(1).values) ~= 0) > 0 %saturation occurred
        output_form_tempJ = 1e20;
    else

        if simmode == 1 % Performance Index J_A value

            %extract logged data
            output_form_tempJ = outputJData.signals(1).values;

        else %Performance Index J_B value

            %extract eta_infty and form eta_k - eta_infty

```

```

%convert seconds to number of data points
SSpointForExtraction = SStimeForExtraction / samp_period;
%extract all of eta signal, total of r of them ;
%matrix of values where each column is 1 channel
tempetaNData1 = etaNData.signals(1).values;
[TotalrowOftempetaNData1,TotalcolOftempetaNData1] = ...
    size(tempetaNData1);
%extract the last x seconds of data for all r channels
tempetaNData2 = tempetaNData1((TotalrowOftempetaNData1-...
    SSpointForExtraction):TotalrowOftempetaNData1,:);
%returns a row vector where each element is
%the average of each column vector
eta_infty = mean(tempetaNData2);
%predfine a matrix called etaDifferenceForm which represents
%eta_k- eta_infty, where each column represents an ith
%channel of the r number of channels
tempetaDifferenceForm = zeros(TotalrowOftempetaNData1, ...
    TotalcolOftempetaNData1);
for i = 1:TotalcolOftempetaNData1
    tempetaDifferenceForm(:,i)=tempetaNData1(:,i)-eta_infty(1,i);
end
%extract only those within the time span of cost function
IndexToResetJ = (TimeToResetJ/samp_period)+1;
etaDifferenceForm = ...
    tempetaDifferenceForm(IndexToResetJ:TotalrowOftempetaNData1,:);

%Extract u_infty and form u_k - u_infty

%Extract all of u signal, total of m of them ;
%matrix of values where each column is 1 channel
tempcontrolUData1 = controlUData.signals(1).values;
[TotalrowOftempcontrolUData1,TotalcolOftempcontrolUData1]=...
    size(tempcontrolUData1);
%extract the last x seconds of data for all m channels
tempcontrolUData2 = tempcontrolUData1...
    ((TotalrowOftempcontrolUData1-SSpointForExtraction)...
    :TotalrowOftempcontrolUData1,:);
%returns a row vector where each element is
%the average of each column vector
u_infty = mean(tempcontrolUData2);
%predfine a matrix called uDifferenceForm which represents
%u_k -u_infty, where each column represents an ith
%channel of the m number of channels
tempuDifferenceForm = zeros(TotalrowOftempcontrolUData1, ...
    TotalcolOftempcontrolUData1);
for i = 1:TotalcolOftempcontrolUData1
    tempuDifferenceForm(:,i)=tempcontrolUData1(:,i)-u_infty(1,i);
end
uDifferenceForm = ...
tempuDifferenceForm(IndexToResetJ:TotalrowOftempcontrolUData1,:);

%Extract error
temperrorEData1 = errorEData.signals(1).values; % e

```

```

tempererrorData2 = theta*tempererrorData1;    % theta*e
tempererrorData3 = ...
    tempererrorData2(IndexToResetJ:TotalrowOfTempcontrolUData1,:);
zDifferenceForm = tempererrorData3 + etaDifferenceForm;

%compute cost
output_form_tempJ=samp_period*(sum(sum(zDifferenceForm.^2))...
    + mu*sum(sum(uDifferenceForm.^2)) );

end
end

output_form_sumJ = output_form_sumJ+output_form_tempJ;
end

%find the average cost
output_form_avgJ = (1/numOfDiffSetsOfIC )*output_form_sumJ;
J_value = output_form_avgJ; %final average cost value used as
    %a single data point in optimization

```

```

%-----
%File name: NelderAlgorithm.m
%By: William Szu-Wei Wang 994792079

%This file contains the Nelder-Mead algorithm. The original code
%was obtained from MathWork:
% C. Moler, 8-19-86
% Copyright (c) 1987 by the MathWorks, Inc.
% Ref: D. J. Woods, Report 85-5, Dept. Math. Sciences, Rice Univ.,
% May, 1985.
%Various modifications were made. This file is called
%from within RSPsetup.m
%-----

x = FirstSetOfGains;
tol = 1e-6;
prnt = 0; %for displaying details of each iteration 0 = disable
header = ' Iteration  Func-count      min f(x)      Procedure';
header2 = 'Starting KP Starting KI theta mu Duration T (sec)';
header3 = ' Optimal KP      Optimal KI      Optimal cost J';
how = '';

[n,m] = size(x);
if m > n
    x = x';
    n = m;
end
cnt = 0; %counter for number of iterations

%Set up a simplex near the initial guess.
v = simplex_lower_bound*x;
nextSetOfGains = v;
gain_array=nextSetOfGains; %saving data in to a record array

```

```

NelderSimulationScript %simulate by calling simulation commands
J_array = J_value; %saving data into a record array
f = J_value;

%Print out starting parameters
%Note: gains are displayed as positive values to match notation
%      used in thesis document.
disp(' ')
disp(header2)
disp(sprintf('%10.6g %10.6g %5.0f %10.6g %5.0f ', ...
            -v(1), -v(2), theta, mu, TotalSimTime));

%Print out initial f(x) as 0th iteration
disp(' ')
disp(header)
disp(sprintf('%5.0f %5.0f %12.6g %s', cnt, ...
            length(J_array)*numOfDiffSetsOfIC, f(1), how));

for j = 1:n
    y = x;
    if y(j) ~= 0
        y(j) =simplex_upper_bound*y(j);
    else
        y(j) = 0.1;
    end
    v = [v y];
    nextSetOfGains=y;
    gain_array=[gain_array nextSetOfGains];
    NelderSimulationScript
    J_array = [J_array J_value];
    f = [f J_value];
end
% sort so v(1,:) has the lowest function value
[f, j] = sort(f);
v = v(:, j);

how = 'initial simplex';
cnt = cnt + 1;
disp(sprintf(' %5.0f %5.0f %12.6g %s', cnt, ...
            length(J_array)*numOfDiffSetsOfIC, f(1), how));

if prnt
    clc
    format compact
    format short e
    home
    cnt
    disp('initial ')
    disp(' ')
    v
    f
end

% Iterate until the diameter of the simplex is less than tol.

```



```

while 1
    test = 0;
    for j = 2:n+1, test = max(test,norm(v(:,j)-v(:,1),1)); end
    if test ≤ tol, break, end

    %coefficients for reflection, expansion, contraction and
    %shrinkage. These selection guarantee convergence to
    %minimizer in 1D strictly convex cases
    alpha = 1; beta = 1/2; gamma = 2;

    [n,np1] = size(v);
    vbar = mean(v(:,1:n))';
    vr = (1 + alpha)*vbar - alpha*v(:,n+1);
    nextSetOfGains=vr;
    gain_array=[gain_array nextSetOfGains];
    NelderSimulationScript
    J_array = [J_array J_value];
    fr = J_value;
    vk = vr; fk = fr; how = 'reflect ';
    if fr < f(n)
        if fr < f(1)
            ve = gamma*vr + (1-gamma)*vbar;
            nextSetOfGains=ve;
            gain_array=[gain_array nextSetOfGains];
            NelderSimulationScript
            J_array = [J_array J_value];
            fe =J_value;
            if fe < f(1)
                vk = ve; fk = fe;
                how = 'expand ';
            end
        end
    else
        vt = v(:,n+1); ft = f(n+1);
        if fr < ft
            vt = vr; ft = fr;
        end
        vc = beta*vt + (1-beta)*vbar;
        nextSetOfGains=vc;
        gain_array=[gain_array nextSetOfGains];
        NelderSimulationScript
        J_array = [J_array J_value];
        fc = J_value;
        if fc < f(n)
            vk = vc; fk = fc;
            how = 'contract';
        else
            for j = 2:n
                v(:,j) = (v(:,1) + v(:,j))/2;
                nextSetOfGains= v(:,j);
                gain_array=[gain_array nextSetOfGains];
                NelderSimulationScript
                J_array = [J_array J_value];
                f(j) = J_value;
            end
        end
    end
end

```

```

    end
    vk = (v(:,1) + v(:,n+1))/2;
    nextSetOfGains=vk;
    gain_array=[gain_array nextSetOfGains];
    NelderSimulationScript
    J_array = [J_array J_value];
    fk = J_value;
    how = 'shrink ';
end
end
v(:,n+1) = vk;
f(n+1) = fk;
[f,j] = sort(f);
v = v(:,j);

    cnt = cnt + 1;
    disp(sprintf(' %5.0f %5.0f %12.6g %s', cnt,...
        length(J_array)*numOfDiffSetsOfIC, f(1), how))
if prnt
    home
    cnt
    disp(how)
    disp(' ')
    v
    f
    test
end
end
x = v(:,1);
xopt = x;

for p=1:length(gain_array)
    if gain_array(:,p)==xopt
        fopt = J_array(1,p);
    end
end
fopt;

% Print out optimal parameters
disp(' ')
disp(' ')
disp(header3)
disp(sprintf(' %10.6g %10.6g %10.6g ',...
    -xopt(1), -xopt(2), fopt));

if prnt, format, end

```

Figure D.2-7: Simulink model for optimization problems A and B.

D.5 MARTS Experiment

The following MATLAB files and Simulink model were used to carry out optimization problem B on the experimental test bed MARTS.

```
%-----
%File name: ExperimentSetup.m
%By: William Szu-Wei Wang 994792079

%This file defines the controller gains and various parameters for
%the MARTS Experiment. This is the first file to run.
%-----

clear
clc

%Stabilizing output feedback
Kpre = 10*eye(2,2);
%Sampling period
h = 0.1;
%use experimentally obtained steady-state tracking matrix
measuredGamma = [0.085 0.011; 0.012 0.075];
S = inv(measuredGamma);
%Valve compensation constant
scale = 2.8;
%Allocated time for restoring initial conditions
restore_time = 100;
%Measured equilibrium water level in volts
equilib=[4.0520;4.2670];
%Saturation bounds
uSatBound = 5;
%Disturbance enabling signal
disturbance_on = 0; %0 = disabled, 1 = enable
%Saturation check enabling signal
saturation_on = 1; % 0 = disabled, 1 = enabled
%Initial conditions
y0=[-0.1; -0.15];
eta0 = [0;0];
yref = [0;0];
%Experiment duration
TotalSimTime = 600;
%Control penalty
mu=1;
theta=0;

%Nelder initial simplex parameters
simplex_lower_bound = 0.5;
simplex_upper_bound = 2;

%First set of controller gains to evaluate
initial_gainKP = (1/simplex_lower_bound)*-1e-2;
initial_gainKI = (1/simplex_lower_bound)*5e-3;
```

```

current_gainKP = 0; %just for initialization
current_gainKI = 0; %just for initialization
%Fixed controller gains for restoring plant initial conditions
%upon the start of each experiment
restore_gainKP= -1.2289;
restore_gainKI= 0.11726;

%counter used in saving workspace
savecounter=1;

```

```

%-----
%File name: ExperimentScript.m
%By: William Szu-Wei Wang 994792079

%This file is controls the real-time execution of Simulink through
%MATLAB command line and calculates the performance index from
%logged data. This file is invoked by ExperimentNelderAlgorithm.m
%-----

%Upload parameters of interest
current_gainKP = nextSetOfGains(1);
current_gainKI = nextSetOfGains(2);

%Use command line to control the execution of Simulink
set_param(bdroot, 'StopTime', 'TotalSimTime');
pause(1);
set_param(bdroot, 'SimulationCommand', 'connect');
pause(1);
set_param(bdroot, 'SimulationCommand', 'start');

%pause the algorithm code until Simulink Simulation finishes
while strcmp(get_param(bdroot, 'SimulationStatus'), 'stopped') ~=1
    pause(10) %check if simulation terminated every 10 seconds
end

pause(1); %give a delay before accessing logged data

%check if saturated, if so assign arbitrarily large cost
if sum((StopScopeData.signals(1).values) ~= 0) > 0
    J_value=1e10;
else
    %calculate cost based on logged data
    %Extract eta
    tempEta = EtaData.signals(1).values;
    %ignore restore initial condition stage
    StartTime = (restore_time / h)+1;
    EndTime = size(tempEta,1);
    Eta1 = tempEta(StartTime:EndTime,1);
    Eta2 = tempEta(StartTime:EndTime,2);

    %Extract control input
    tempPIu = PIuData.signals(1).values;
    PIu1 = tempPIu(StartTime:EndTime,1);

```

```

PIu2 = tempPIu(StartTime:EndTime,2);

%Extract error signal
tempError = ErrorData.signals(1).values;
Error1 = tempError(StartTime:EndTime,1);
Error2 = tempError(StartTime:EndTime,2);

%Form output Z
outputZ1 = theta*Error1 + Eta1;
outputZ2 = theta*Error2 + Eta2;

%Cost value
integrand = (outputZ1.^2 + outputZ2.^2)+mu*(PIu1.^2+PIu2.^2);
J_value = trapz(integrand)*h;
end
%save workspace in case system crashes
name = strcat('run_', int2str(savecounter));
save(name);
pause(5)
savecounter=savecounter+1;

```

```

%-----
%File name: NelderAlgorithm.m
%By: William Szu-Wei Wang 994792079

%This file contains the Nelder-Mead algorithm. The original code
%was obtained from MathWork:
% C. Moler, 8-19-86
% Copyright (c) 1987 by the MathWorks, Inc.
% Ref: D. J. Woods, Report 85-5, Dept. Math. Sciences, Rice Univ.,
% May, 1985.
%Various modifications were made. This file is executed after
%the ExperimentSetup.m and it automatically invokes
%ExperimentScript.m
%-----

x = [initial_gainKP;initial_gainKI];
tol = 1e-2;
prnt = 0; %for displaying details of each iteration 0 = disable
header = ' Iteration   Func-count       min f(x)    Procedure';
header2 = 'Starting KP Starting KI theta mu Duration T (sec)';
header3 = ' Optimal KP    Optimal KI    Optimal cost J';
how = '';

[n,m] = size(x);
if m > n
    x = x';
    n = m;
end
cnt = 0; %counter for number of iterations

% Set up a simplex near the initial guess.
v = simplex_lower_bound*x;

```

```

nextSetOfGains = v;
gain_array=nextSetOfGains; %saving data in to a record array
ExperimentScript %simulate by calling simulation commands
J_array = J_value; %saving data into a record array
f = J_value;

% Print out starting parameters
disp(' ')
disp(header2)
disp(sprintf('%10.6g %10.6g %5.0f %10.6g %5.0f ', ...
            -v(1), v(2), theta, mu, TotalSimTime));

% Print out initial f(x) as 0th iteration
disp(' ')
disp(header)
disp(sprintf('%5.0f %5.0f %12.6g %s',cnt, ...
            length(J_array), f(1), how));

for j = 1:n
    y = x;
    if y(j) ~= 0
        y(j) =simplex_upper_bound*y(j);
    else
        y(j) = 0.1;
    end
    v = [v y];
    nextSetOfGains=y;
    gain_array=[gain_array nextSetOfGains];
    ExperimentScript
    J_array = [J_array J_value];
    f = [f J_value];
end
% sort so v(1,:) has the lowest function value
[f,j] = sort(f);
v = v(:,j);

how = 'initial simplex';
cnt = cnt + 1;
disp(sprintf(' %5.0f %5.0f %12.6g %s', cnt,...
            length(J_array), f(1), how));

if prnt
    clc
    format compact
    format short e
    home
    cnt
    disp('initial ')
    disp(' ')
    v
    f
end

% Iterate until the diameter of the simplex is less than tol.

```

```

while 1
    test = 0;
    for j = 2:n+1, test = max(test,norm(v(:,j)-v(:,1),1)); end
    if test ≤ tol, break, end

    %coefficients for reflection, expansion, contraction and
    %shrinkage. These selection guarantee convergence to
    %minimizer in 1D strictly convex cases
    alpha = 1;  beta = 1/2;  gamma = 2;

    [n,np1] = size(v);
    vbar = mean(v(:,1:n))';
    vr = (1 + alpha)*vbar - alpha*v(:,n+1);
    nextSetOfGains=vr;
    gain_array=[gain_array nextSetOfGains];
    ExperimentScript
    J_array = [J_array J_value];
    fr = J_value;
    vk = vr;  fk = fr; how = 'reflect ';
    if fr < f(n)
        if fr < f(1)
            ve = gamma*vr + (1-gamma)*vbar;
            nextSetOfGains=ve;
            gain_array=[gain_array nextSetOfGains];
            ExperimentScript
            J_array = [J_array J_value];
            fe =J_value;
            if fe < f(1)
                vk = ve; fk = fe;
                how = 'expand ';
            end
        end
    else
        vt = v(:,n+1); ft = f(n+1);
        if fr < ft
            vt = vr;  ft = fr;
        end
        vc = beta*vt + (1-beta)*vbar;
        nextSetOfGains=vc;
        gain_array=[gain_array nextSetOfGains];
        ExperimentScript
        J_array = [J_array J_value];
        fc = J_value;
        if fc < f(n)
            vk = vc; fk = fc;
            how = 'contract';
        else
            for j = 2:n
                v(:,j) = (v(:,1) + v(:,j))/2;
                nextSetOfGains= v(:,j);
                gain_array=[gain_array nextSetOfGains];
                ExperimentScript
                J_array = [J_array J_value];
                f(j) = J_value;
            end
        end
    end
end

```



```

        end
        vk = (v(:,1) + v(:,n+1))/2;
        nextSetOfGains=vk;
        gain_array=[gain_array nextSetOfGains];
        ExperimentScript
        J_array = [J_array J_value];
        fk = J_value;
        how = 'shrink ';
    end
end
v(:,n+1) = vk;
f(n+1) = fk;
[f,j] = sort(f);
v = v(:,j);

cnt = cnt + 1;
disp(sprintf(' %5.0f %5.0f %12.6g %s', cnt,...
    length(J_array), f(1), how))

if prnt
    home
    cnt
    disp(how)
    disp(' ')
    v
    f
    test
end
end
x = v(:,1);
xopt = x;

for p=1:length(gain_array)
    if gain_array(:,p)==xopt
        fopt = J_array(1,p);
    end
end
fopt;

% Print out optimal parameters
disp(' ')
disp(' ')
disp(header3)
disp(sprintf(' %10.6g %10.6g %10.6g ',...
    -xopt(1), xopt(2), fopt));

if prnt, format, end

```

Figure D.3-1: Simulink model for MARTS experiment.

Figure D.3-2: “Controller” subsystem block.

Figure D.3-3: “Valve Linearization” subsystem block.

Figure D.3-4: “Spike Filter” subsystem block.

Figure D.3-5: “Saturation check” subsystem block.

Figure D.3-6: “Unit Step Response Test” subsystem block.