AN INVESTIGATION INTO THE FLEXURAL BEHAVIOUR
OF GFRP REINFORCED CONCRETE BEAMS

by

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A thesis submitted in conformity with the requirements
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Graduate Department of Civil Engineering
University of Toronto

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ABSTRACT

Non-corroding materials, such as Fibre-Reinforced Polymer (FRP) bars, are now being used as reinforcement for reinforced concrete structures in order to extend their lifetime and minimize maintenance costs. Because of the softer and brittle behaviour of GFRP bars, behaviour of structural members reinforced with this material is different than that of steel-reinforced members. In this study, 16 GFRP reinforced beams were constructed and tested under flexure and shear loads to failure. Effects of different variables, such as amount of longitudinal and lateral reinforcements, type of bars and concrete strength, were investigated for their effects. The flexural provisions of design codes, namely CSA S806-12, CSA S6-06, and report ACI 440.1R-06, were evaluated against the test data. The main provisions investigated are failure modes, ultimate strength, moment-curvature response, deflection, crack widths, and deformability.
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NOTATION

\( A_{ct} = \) average effective tension area of concrete per longitudinal bar

\( A_d = \) relative area under load-deflection curve (at ultimate or service)

\( A_f = \) area of longitudinal tensile FRP reinforcement

\( a = \) height of equivalent rectangular stress block above neutral axis

\( b = \) beam width

\( c = \) neutral axis depth

\( D_d = \) deflection deformability factor

\( DF = \) deformability factor

\( d = \) effective depth (distance from top of beam to centroid of reinforcement)

\( d_c = \) thickness of cover from tension face to center of closest bar

\( E_c = \) modulus of elasticity of concrete

\( E_f = \) modulus of elasticity of FRP

\( E_s = \) modulus of elasticity of steel

\( EFS = \) energy factor of safety

\( f_c' = \) concrete compressive strength

\( f_f = \) longitudinal FRP reinforcement stress

\( f_t = \) concrete tensile strength

\( f_r = \) concrete rupture strength

\( f_{ult} = \) longitudinal FRP reinforcement ultimate stress

\( f_y = \) steel yield stress

\( h = \) overall beam height

\( h_1 = \) distance from the neutral axis to the centroid of the tensile reinforcement
\( h_z \) = distance from the neutral axis to the bottom surface of the beam

\( I_{cr} \) = cracked moment of inertia

\( I_e \) = effective moment of inertia

\( I_e' \) = equivalent moment of inertia

\( I_g \) = gross moment of inertia

\( I_m \) = moment of inertia proposed by Faza and GangaRao (1992)

\( J \) = deformability factor

\( k_b \) = FRP bond factor to account for surface profile

\( L \) = center-to-center span length

\( L_g \) = uncracked beam length

\( l_n \) = clear span length

\( M_a \) = applied moment

\( M_c \) = applied moment when top concrete strain equals \(-1.0 \times 10^{-3}\)

\( M_{cr} \) = cracking moment

\( M_{exp} \) = experimental ultimate moment

\( M_{pred} \) = predicted ultimate moment

\( M_{ult} \) = ultimate moment resistance

\( m \) = moment of inertia transition coefficient

\( P \) = total applied load

\( P_{cr} \) = cracking load

\( S_{exp} \) = experiment load-deflection stiffness

\( S_{ACI} \) = predicted load-deflection stiffness (from ACI 440.1R-06)
\( s \) = stirrup spacing

\( w \) = maximum crack width

\( \alpha \) = stiffness reduction factor (from Benmokrane and Masmoudi)

\( \alpha_1 \) = equivalent stress block factor

\( \beta \) = ratio of \( h_2 \) to \( h_1 \)

\( \beta_d \) = uncracked stiffness reduction factor

\( \beta_s \) = service coefficient for deflection deformability factor

\( \beta_u \) = ultimate coefficient for deflection deformability factor

\( \beta_1 \) = equivalent stress block factor

\( \Delta \) = midspan deflection

\( \Delta_{exp} \) = experimental midspan deflection

\( \Delta_{pred} \) = predicted midspan deflection

\( \varepsilon_{cu} \) = ultimate concrete strain

\( \varepsilon_t \) = strain at the extreme top surface of the beam

\( \varepsilon_{ult} \) = ultimate FRP strain

\( \varepsilon_c' \) = peak concrete strain

\( \gamma \) = integration factor to account for stiffness variation along the beam length

\( \gamma_d \) = cracked stiffness reduction factor

\( \varphi_c \) = curvature corresponding to a top concrete strain of \(-1.0 \times 10^{-3}\)

\( \varphi_{ult} \) = ultimate curvature

\( \eta \) = factor to account for ratio of cracked to gross moments of inertia

\( \rho_b \) = balanced longitudinal reinforcement ratio
\[ \rho_f \quad = \quad \text{longitudinal reinforcement ratio} \]
\[ \chi \quad = \quad \text{shrinkage restraint factor to account for a low cracking moment} \]
1. **INTRODUCTION**

1.1 **Background**

One of the biggest challenges facing engineers today is the problem of aging infrastructure, particularly with respect to reinforced concrete. In Ontario alone, it has been estimated that the replacement cost of deficient bridges and highways is 57 billion dollars (MTO, 2009). The major cause of deterioration among reinforced concrete structures is corrosion of the reinforcing steel. Among others, one viable option is to reinforce concrete with glass fiber-reinforced polymer (GFRP) bars, a non-corrosive material.

GFRP reinforcing bars are made primarily of glass fibers. Being non-corrosive, GFRP bars can help extend the lifecycle of reinforced concrete structures substantially, as well as reduce their maintenance, repair, and replacement costs. While GFRP is becoming a viable reinforcement alternative, it presents design challenges which are different than those in the design of conventional steel reinforced concrete. One important challenge is consideration of a brittle failure mode in GFRP-reinforced members.

The difference in failure modes of steel and GFRP drastically changes the design principle for members subjected to flexure, such as beams. The CSA concrete design code (CSA A23.3-04) specifies that bottom steel reinforcement must yield before top concrete crushes, ensuring a ductile failure. GFRP rupturing is considered more brittle than concrete crushing, so beams need to be preferably designed as over-reinforced to ensure top concrete crushes before bottom reinforcement ruptures (CSA S806-02, CSA S806-12). The research presented herein aims to investigate these failure modes, specifically in the areas of predictability, post-failure behaviour, and ductility.

Other than the brittle failure mode, the major shortcoming of GFRP reinforcing bars is their relatively low stiffness, when compared to steel. This reduced stiffness, combined with other factors such as different bond behaviour and lower tension stiffening, results in deflections that are larger than conventional steel-reinforced beams, at any load stage. Because of these large deflections, designs may often be governed by
deflection limitations. As such, it is critical that load-deflection behaviour can be accurately predicted. The majority of research done to date on deflection involves modifying equations currently employed for use in steel reinforced beams, such as Branson’s equation for an effective modulus of inertia (Branson, 1968). This research aims to determine if such methods are accurate, and suggest a revision if they are deemed inaccurate.

Another difficulty that arises when designing with GFRP bars is the concept of ductility. Traditional steel reinforced concrete beams’ ductility can be defined by the displacement ductility factor. The displacement ductility factor is the ratio of ultimate displacement to yielding displacement (Sheikh and Yeh, 1986, Sheikh and Khoury, 1993). In general, ductility factors are the ratio between a property, such as displacement, curvature, rotation, or strain, at ultimate and that same property at yielding.

\[
Ductility \ Factor = \frac{Deflection \ (or \ curvature, \ or \ strain) \ at \ ultimate}{Deflection \ (or \ curvature, \ or \ strain) \ at \ yield} \tag{1.1}
\]

Ductility factors provide an opportunity for forces to redistribute in an indeterminate structure resulting in a better factor of safety against failure. With a sufficient ductility factor, visibly excessive deformations will be observed before failure, allowing occupants to evacuate the structure. Because GFRP reinforcement does not yield, Equation 1.1 is not applicable and thus conventional ductility factors cannot be applied. Therefore, new concepts which use the energy absorption capacity of beams have been developed in order to quantify deformability of a structure. These methods, known as deformability factors, involve calculating the ratio of the energy absorbed by the beam at ultimate loads to the energy absorbed at approximately service loads, and are discussed in depth in Chapter 5.

1.2 Objectives and Scope

The main objective of this research is to investigate flexural behaviour of GFRP reinforced concrete beams, with a focus on critically evaluating current design code provisions relating to design with GFRP. The design codes discussed are CSA S806-12 (Design and Construction of Building Structures with Fibre-Reinforced Polymers), CSA
S6-06 (Canadian Highway Bridge Design Code), and ACI 440.1R-06 (Guide for the Design and Construction of Structural Concreted Reinforced with FRP Bars). This investigation was carried out in two distinct phases, an experimental program and accompanying analytical program.

The experimental program consisted of constructing and testing sixteen GFRP reinforced concrete beams. The major variables among specimens in this test series were types of GFRP bars, longitudinal reinforcement ratio, transverse reinforcement ratio, and concrete strength. Bars made by three GFRP manufacturers were used: Hughes Brothers, Pultrall, and Schöck. The mechanical properties of their bars, along with the concrete, are described in detail in Chapter 2 and Appendix A.

This test series investigated the behaviour of concrete beams reinforced with GFRP bars only; no steel reinforcement was used. The beams were all 200 mm wide by 325 mm high by 3620 mm long, herein known as small beams. Size effect was established by comparing results between similar beams (GS) in this test series and large beam (JS) series from another test program carried out at the University of Toronto (Johnson and Sheikh, 2012). The beams in JS test series were 400 mm wide by 650 mm high and 3625 mm long.

The analytical portion of the program consisted of the evaluation of various code provisions relating to flexure, such as deflections and crack widths. Most GFRP designs are largely limited by these service conditions and as such they require accurate determination. The deflection and crack width equations currently in use by CHBDC S6-06, CSA S806-12 and ACI 440.1R-06 are compared with the experimental results, with the goal of determining which, if any, code is more applicable in design.

Regarding the performance at ultimate limit state, the linear-elastic behaviour of GFRP bars can present significant design challenges. While the CSA S806-12 explicitly prescribes failure by concrete crushing, CSA S6-06 and ACI 440.1R-06 do not. These codes, however, discourage the use of under-reinforced beams in their own way: CSA S6-06 prescribes the J factor, and ACI 440.1R-06 reduces the FRP material safety factor. These provisions are described and discussed in detail in Chapter 6.
1.3 Organization

Chapter 2 covers a literature review of previously performed studies on the flexural behaviour of GFRP-reinforced beams, with a summary of the models currently employed by design codes. Chapter 3 details the experimental program consisting of sixteen flexurally-critical GFRP reinforced concrete beams and Chapter 4 presents the experimental results from this test program. Chapter 5 describes the computer program Zeppelin, created as a tool to analyze data collected from a three-dimensional positioning camera. Chapter 6 discusses results from the experimental work, including moment-curvature response, load-deflection behaviour, deformability, and crack widths. Chapter 7 concludes with a summary of results, conclusions, and recommendations for future work.

Appendices contain experimental material properties, specimen details, load-deflection results, moment-curvature results, crack maps, and results from this study.
2. LITERATURE REVIEW

2.1 Introduction

The first reported development of FRP reinforcing bars occurred shortly after World War II. Their potential for use in reinforced concrete applications, however, was not fully recognized until the 1960’s. Some of the first serious academic studies on GFRP-reinforced members were carried out by Nawy and Neuwerth in 1971 and 1977. In the early 1990’s, GFRP manufacturing processes had improved enough to, combined with a widespread deteriorating infrastructure, make GFRP bars serious competitors with other reinforcing materials. Consequently, research in GFRP reinforced concrete members began to be performed worldwide, specifically in areas such as flexural behaviour, shear behaviour, bond performance, and column behaviour.

This section focuses on past research performed on the flexural behaviour of GFRP reinforced concrete beams. Early studies attempted to modify existing steel reinforced concrete equations, with mixed results. In general, this method provided accurate strength predictions, but significantly under predicted corresponding deflections. Modern studies have attempted to derive FRP reinforced concrete equations based on first principles. A chronological overview of this research is presented below.

2.1.1 Nawy and Neuwerth (1971, 1977)

Some of the original investigations into the behaviour of concrete beams reinforced with GFRP bars were performed by Nawy and Neuwerth at Rutgers University in the 1970’s. The authors recognized that, because of its high tensile strength and corrosive resistance, GFRP could be advantageous for certain structural applications. To further investigate the flexural behaviour of structural elements reinforced with this material, two test programs consisting of 36 steel- and GFRP-reinforced beams and 12 GFRP-reinforced slabs were conducted. The majority of the beams were designed as under-reinforced, as to best compare the results of GFRP and steel bars. Many of these beams had no shear reinforcement, and thus shear failures were the dominant failure mode in most specimens.
The authors attempted to predict the deflection behaviour of GFRP-reinforced beams by solving Branson’s equation (Equation 2.1) to calculate a suitable moment of inertia, and applying this value in classical mechanics equations (such as Equation 2.2). This method can reasonably predict behaviour of steel-reinforced beams by simultaneously accounting for change in beam stiffness due to cracking and increase in stiffness due to tension stiffening; however, the experimental results presented in these studies suggested that Branson’s equation underestimated deflection of GFRP-reinforced beams at all load levels. These predictions are slightly more accurate for beams with high reinforcement ratios. The stiffness and deflection of the beams were calculated using Equations 2.1 and 2.2

\[
I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1.0 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g
\]

where:

- \(I_e\) = effective moment of inertia (mm\(^4\))
- \(I_g\) = gross moment of inertia (mm\(^4\))
- \(I_{cr}\) = cracked moment of inertia (mm\(^4\))
- \(M_{cr}\) = cracking moment (Nmm)
- \(M_a\) = applied moment (Nmm)

Upon determination of the effective moment of inertia, it is possible to calculate deflection by using classical mechanics equations, such as Equation 2.2 for four-point loading.

\[
\Delta = \frac{P a}{48E_c I_e} (3l^2 - 4a^2)
\]

where:

- \(P\) = total applied load (N)
- \(E_c\) = modulus of elasticity of concrete (MPa)
The authors concluded with results that are echoed in modern GFRP research. Deflections in GFRP beams are in the order of three times that of their steel counterparts, at any specific load level. GFRP beams exhibit large amounts of flexural cracking, and deformation serviceability limits may govern design. These limits result in large amounts of reserve strength in the GFRP bars at service levels and large strength factors of safety. The load-deflection behaviour can accurately be described by a bilinear curve (pre-cracking and post-cracking). Ultimate moment resistances can be accurately predicted by applying well understood steel-reinforced concrete flexural equations, and there is potential for this material in structural applications.

2.1.2 Faza and GangaRao (1992)

The authors present a comprehensive study of 25 concrete beams reinforced with both steel and GFRP bars. The major variables between specimens are type and size of reinforcing bars, type of shear reinforcement, reinforcement distribution, and concrete compressive strength. The goal of this study was to develop a suitable model to predict pre- and post-cracking behaviour of GFRP reinforced beams, in terms of load and deflection. The authors recognize that Branson’s equation (Equation 2.1) is accurate for conventional steel reinforced beams but must be modified for use in GFRP reinforced concrete applications.

Experimental results from the 25 beams indicate that Branson’s equation is accurate for steel reinforced beams, but significantly under-predicts deflection for GFRP reinforced beams at service level. For the load setup being used in this test series (that is, four point bending with two concentrated loads applied at the third points), the authors propose the following model:

\[
I_m = \frac{23I_{cr}I_e}{8I_{cr} + 15I_e}
\]

\[2.3\]
This model agrees with the test results at service conditions. Two similar models are proposed, one for one concentrated load (three-point bending) and one for a uniformly distributed load. These models are not confirmed by experiments.

2.1.3 Jaeger, Tadros and Mufti (1995)

This study by Jaeger, Tadros and Mufti, performed at the Technical University of Nova Scotia, was an early investigation into performance factors, such as deformability factors, for use in FRP-reinforced concrete members. The authors present a design procedure for steel- and FRP-reinforced concrete beams, including a calculation process to determine the full moment-curvature response of a beam. They suggest that any performance factor must account for curvature response, as well as moment resistance, because high values of curvature are beneficial to beam performance as long as moment resistance increases, due to the potential for load redistribution in indeterminant structures at high curvatures.

Two performance factors are presented. One factor involves comparing ultimate behaviour to nominal behaviour (with nominal behaviour being defined as when the load is 77% of ultimate). The authors suggest that this nominal approximation may not be suitable for design purposes, and propose another, more suitable, factor. This second factor compares ultimate behaviour to behaviour at the onset of non-linearity (defined as when the top concrete of the beam reaches a strain of -1.0 x 10^{-3}). The authors mathematically define this deformability factor with the following equation:

\[ J = \frac{M_{ult} \phi_{ult}}{M_c \phi_c} > 4 \]  \hspace{1cm} 2.4

where:

\( M_{ult} \) = ultimate moment resistance

\( \phi_{ult} \) = ultimate curvature

\( M_c \) = moment resistance at \( \varepsilon_t = -1.0 \times 10^{-3} \)

\( \phi_c \) = curvature at \( \varepsilon_t = -1.0 \times 10^{-3} \)
This equation has been adopted by the Canadian Highway Bridge Design Code (CSA S6-06) in clause 16.8.2.1. The limiting factor of 4 is chosen for rectangular beams. For T-beams, this limiting value was increased to 6. This design principal inherently favours designing beams which fail by concrete crushing instead of GFRP rupturing, a desirable philosophy as the former failure mode is considered more ductile than that latter.

2.1.4 Alsayed (1997)

In this study, the author reported results from twelve beams; four test series of three specimens each. Beams in one series, the control series, were reinforced with conventional steel reinforcement and designed as under-reinforced. The remaining three test series consisted of beams reinforced with GFRP bars and designed as over-reinforced. The goal of this study was to construct a computer program model that can accurately predict the load-deflection behaviour of both steel and GFRP-reinforced beams, and to validate the existing equations. The authors evaluated two existing models: the ACI model (Equation 2.1) and the model proposed by Faza and GangaRao (Equation 2.3).

The authors conclude that Branson’s equation under-predicts service deflection by up to 70% at service conditions, and is thus unsuitable for application in GFRP reinforced concrete design codes. Replacing this model with Faza and GangaRao’s model (1992) improves the accuracy from 70% error to 15% error; however, more tests must be conducted to confirm these results. The proposed computer model is shown to be significantly more accurate than both numerical models, and can account for the properties of GFRP or steel reinforced beams at all load stages. The ultimate moment capacity of the beam can be accurately predicted using the ultimate design method used for steel reinforced beams.

2.1.5 Benmokrane and Masmoudi (1996, 1998)

Two studies were conducted at the Université de Sherbrooke to further the understanding of flexural behaviour of GFRP-reinforced concrete beams. The first study, conducted in 1996, involved testing eight beams: four beams reinforced with GFRP bars,
and four companion beams reinforced with steel bars. A major goal of this test program was to investigate deflection behavior for beams which fail by both compression and tension failures. Both failure modes were observed in the GFRP-reinforced beams. The authors, similar to Alsayed (1997), recorded beam deflections which were significantly higher than those predicted by Branson’s equation. Based on this study, the authors proposed the following deflection model based on a modified Branson’s equation:

\[
I_e = \frac{1}{\beta} \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \alpha \left[ 1.0 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \tag{2.5}
\]

where \( \alpha \) and \( \beta \) are reduction factors, equal to 0.84 and 7, respectively. In addition, the authors propose a material safety factor for GFRP bars of \( \Phi_f = 0.75 \).

A follow up study by the authors was conducted in 1998. This study included a test series of eight GFRP reinforced beams and two steel reinforced beams. Again, a major goal of this study was to determine a model which can accurately predict deflections. The authors propose two models. One of these models, a modification of the authors’ previously proposed model (Equation 2.5), can be seen in Equation 2.6:

\[
I_e = \beta \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1.0 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \tag{2.6}
\]

where \( \beta \) is equal to 0.6. Differences between this equation and the authors’ previous equation are: the cracking reduction factor \( \alpha \) is increased from 0.84 to 1.0, and the pre-cracking stiffness reduction factor \( \beta \) is increased from 0.143 to 0.6.

Equation 2.6 is directly derived from Faza and GangaRao’s work (1992). Their original equation (Equation 2.3) is suitable for predicting deflections for beams loaded in a specific manner (by two concentrated loads at third points). Benmokrane and Masmoudi, using the same mathematical approach, generalized the equation for any beam under four-point bending. This model can be seen in Equation 2.7:

\[
\Delta = \frac{Pa}{E_c} \left( \frac{4a(L/2 - a) + (L/2 - a)^2}{8l_{cr}} + \frac{a^2}{3l_e} \right) \tag{2.7}
\]
The authors report that both deflection models (Equations 2.6 and 2.7) can accurately predict deflections of GFRP reinforced beams.

2.1.6 Toutanji and Saafi (2000)

In this study, six GFRP reinforced concrete beams were constructed and tested in flexure; two balanced-condition beams and four over-reinforced beams. The authors investigated load-deflection behaviour and crack widths, and compared the experimental results with results from existing models. They acknowledge that Branson’s equation over-estimates flexural stiffness for GFRP reinforced concrete beams, and that this effect is more pronounced as longitudinal reinforcement ratio decreases. Results from beams tested in other investigations (Benmokrane et al., 1995, and Masmoudi et al., 1998) confirm this effect. A model is proposed which modifies Branson’s equation to account for, among other things, longitudinal reinforcement ratio (Equation 2.8).

\[
I_e = \left(\frac{M_{cr}}{M_0}\right)^m I_g + \left[1.0 - \left(\frac{M_{cr}}{M_0}\right)^m\right] I_{cr} \leq I_g
\]

\[m = 6 - 10 \rho f \frac{E_f}{E_s} \geq 3\]  \hspace{1cm} 2.8

\[
I_e = \left(\frac{M_{cr}}{M_0}\right)^m I_g + \left[1.0 - \left(\frac{M_{cr}}{M_0}\right)^m\right] I_{cr} \leq I_g
\]

\[m = 6 - 10 \rho f \frac{E_f}{E_s} \geq 3\]  \hspace{1cm} 2.9

The authors also proposed an equation to predict crack widths, developed using a regression analysis on experimental results which is a modified Gergely-Lutz equation, with \(k_g\) being modified from a constant to a factor which depends on reinforcement ratio and reinforcement modulus of elasticity, as seen in Equation 2.10.

\[
w = 10^{-6} \left(\frac{2000}{E_f}\right) \rho f^{-0.5} f_f h_2^3/d_c A_{ct}\]  \hspace{1cm} 2.10

This equation is further discussed in Chapter 6.

2.1.7 Razaqpur et al. (2000)

Razaqpur et al., at Carleton University, propose a deflection model derived independently of Branson’s equation. The authors stated that applying empirically derived modification factors to Branson’s equation may not be suitable for GFRP
reinforced beams, as these factors depend upon numerous variables, such as loading arrangement, GFRP properties, and support conditions. The model proposed by the authors is based on first principles, and is independent of support or loading conditions.

In developing this model, the authors make the following assumptions: the depth of the neutral axis after cracking is constant, regardless of the applied load; tension-stiffening of concrete is negligible; shear deflections are negligible; and the moment-curvature curve of the beam is trilinear, as seen in Figure 2.1. By applying direct integration methods of virtual work (Equation 2.11), it is possible to determine the deflection of any beam.

\[ \Delta_A = \int_0^L \frac{mM}{EI} \, dx \]

Figure 2.1: Idealized trilinear moment-curvature behaviour used by Razaqqur et. al

The authors apply this method to common arrangements of load and support conditions for beams. For example, deflection of a beam under three-point loading:

\[ \Delta = \frac{PL^3}{48E_c I_{cr}} \left[ 1 - 8\eta \left( \frac{L_g}{L} \right)^3 \right] \]

and for four-point bending:

\[ \Delta = \frac{PL^3}{48E_c I_{cr}} \left[ 3 \left( \frac{a}{L} \right)^3 - 4 \left( \frac{a}{L} \right)^3 - 8\eta \left( \frac{L_g}{L} \right)^3 \right] \]
where $L_g$ is the uncracked length, and:

$$\eta = \left(1 - \frac{l_{cr}}{L_g}\right)$$  \hspace{1cm} 2.14

A comprehensive experimental verification was provided by the authors, which applied this model to 31 beams from various studies performed at multiple institutions. The authors concluded that, for these 31 beams, their model was 95% accurate. This deflection model has since been adopted by the CSA S806-12 (clause 8.3.2.4). For deflection equations for load and support arrangements not listed here, refer to Table 7 of CSA S806-12.

2.1.8 Vijay and GangaRao (2001)

The authors of this study compiled a database of 64 GFRP reinforced concrete beams from 14 various studies with the goal of understanding the flexural behaviour of this type of beam. They concluded that the ultimate moment capacities of GFRP beams could be well predicted using existing mathematical models. Members designed to fail by compressive crushing of the concrete fail more gradually and show better performance than their under-reinforced counterparts, as proven by experimental and theoretical results. An initial depth estimate of around $d = l/13$ should be suitable to control deflection and crack width criteria.

The authors also propose the following deformability factor equation, based on energy absorption (or, the area under the moment-curvature curve).

$$DF = \frac{\text{Area under moment – curvature curve at ultimate}}{\text{Area under moment – curvature curve at } \varphi = 0.005/d}$$  \hspace{1cm} 2.15

This equation defines deformability as the ratio of the area under the moment-curvature curve at the ultimate condition to the same area at a limiting condition. The limiting condition corresponds to a curvature $\varphi = 0.005/d$, which has been empirically determined to satisfy deflection and crack width criteria. The authors stated that beams that failed by tension failures were observed to have a DF in the range of 5.80 to 6.78, and beams that failed by concrete compression had a DF in the range of 6.70 to 13.90.
These results agree with the generally accepted theory that over-reinforced beams show better performance than under-reinforced beams.

2.1.9 Yost and Gross (2002)

Yost and Gross, at Villanova University, attempted to define a suitable GFRP design methodology by modifying existing working stress design methods. The authors acknowledge that three levels of structural energy exist at any given load: member-level energy (area under a load-deflection curve), section-level energy (area under a moment-curvature curve), and material-level energy (area under the stress-strain curves of the constitutive materials). It is recognized that material-level energy (also known as strain energy density) may be the most useful for design purposes, as it can provide a design approach consistent with existing design methodologies. An energy factor of safety (EFS) is defined as the ratio of the energy absorbed at ultimate loads to the energy at absorbed at service loads. The authors also evaluated EFS at the member-level and material-level for a range of conventional steel-reinforced beams, and show that these methods yield similar results. It was observed that a properly designed steel-reinforced beam has an EFS greater than 25; thus, a GFRP beam must have the same limit to be considered safe.

The authors, for the steel analysis, define service levels as when the stress in the concrete is less than $0.45f'_c$, and the stress in the steel is less than $0.4f_y$. For a GFRP beam to have an EFS similar to a steel beam (that is, greater than 25), the authors show that the concrete stress limit must be reduced to $0.35f'_c$. The reinforcement service stress limit for GFRP does not need to exist, as long as $\rho_f > 1.33\rho_b$. Using this stress limit results in a typical service load of only 20% of the ultimate load; however, this is suitable as it reflects the necessity to avoid catastrophic failures associated with GFRP rupturing.

2.1.10 Habeeb and Ashour (2008)

Habeeb and Ashour investigated the behaviour of continuous GFRP-reinforced concrete beams by testing six specimens. Four beams were supported at three points and loaded at the two midspans, and two beams were simply supported and loaded at its midspan. One conclusion drawn by the authors was that the modified Branson’s equation greatly under-predicted deflections at all load stages, for continuous beams. To mitigate
this error, the authors introduced a post-cracking stiffness reduction factor, $\gamma_d$, into Branson’s equation, in the following form:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \beta_d I_g + \left[1.0 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \gamma_d I_{cr} \leq I_g$$  \hspace{1cm} 2.16

They suggested that, for continuous beams, experimental values indicated that $\gamma_d = 0.6$ is a suitable value. The results from the two simply supported beams show that $\gamma_d = 1.0$ can accurately predict service level deflections.

2.1.11 Bischoff et al. (2009)

Bischoff et. al carried out an intensive study on the currently used deflection equations, specifically those which modify Branson’s equation. The authors concluded that Branson’s equation had several deficiencies, particularly due to requirements of empirical calibrations (which can be inaccurate) and tension stiffening (which is not incorporated in Branson’s equation). It is recognized that Branson’s equation is inaccurate for beams with low reinforcement ratios because it was calibrated for moderately reinforced beams ($\rho > 1\%, l_g/I_{cr} < 3$). Beams outside of this range are greatly affected by the low tension stiffening effect of GFRP. The ACI 440 code equation is accurate for beams with $\beta_d l_g/l_{cr} < 3$ and reinforcing bars with $E_f/f_a$ close to 60, because it was calibrated using these values. The authors expected beams reinforced with the new generation of bars, which are both stiffer and stronger, to show higher deflection than that predicted by the current ACI 440.1 equation.

By working with first principles of equilibrium, compatibility, and tension stiffening, the authors proposed a new model to predict deflection, which is briefly described below through Equation 2.17.

$$I_e' = \frac{I_{cr}}{1 - \gamma \beta \chi \eta (M_{cr}/M_a)}$$  \hspace{1cm} 2.17

where:

$I_e' = \text{equivalent moment of inertia (mm}^4\text{)}$
\[ \beta = \text{tension stiffening factor for the cross section (equal to } \frac{M_{cr}}{M_a}) \]

\[ \chi = \text{shrinkage restraint factor to account for a low cracking moment (equal to 0.64)} \]

\[ \gamma = \text{integration factor to account for variation in stiffness along the member length} \]

\[ \eta = 1 - \frac{l_{cr}}{l_g} \]

This approach requires integration, depending on loading and support conditions. The authors provided closed form solutions for common arrangements. Equations for \( \gamma \) are listed below for three point bending and four point bending (the loading conditions used in the test series presented).

For three point bending:

\[ \gamma = 3 - 2\left(\frac{M_{cr}}{M_a}\right) \quad 2.18 \]

For four point bending:

\[ \gamma = \frac{3(a/L) - 16(M_{cr}/M_a)(a/L)^3 + 12(a/L)^3}{3(a/L) - 4(a/L)^3} \quad 2.19 \]

The authors suggested using Equation 2.17 for deflection of critical members, as it accounts for the greatest number of factors. For members where deflection is not critical, the authors provided simpler, conservative equations which ignore tension stiffening and assume the beam is cracked throughout.

2.2 Discussion

2.2.1 Deflection

At the earliest stages of research into the behaviour of GFRP reinforced concrete beams, it was recognized that deflection is a critical parameter and must be predicted accurately due to the low stiffness of such beams. One common approach to calculate deflections is to use classical mechanics equations, such as Equation 2.20 (for three-point bending) or Equation 2.21 (for four point bending), with a modified effective modulus of inertia \( (I_e) \) to account for the unique properties of GFRP reinforced beams.
These equations are, for three point bending:

\[ \Delta = \frac{PL^3}{48EI_e} \]  \hspace{1cm} (2.20)

and for four point bending:

\[ \Delta = \frac{Pa}{48EI_e}(3L^2 - 4a^2) \]  \hspace{1cm} (2.21)

The earliest attempts to determine a model which accurately predicts the effective modulus of inertia involved modifying Branson’s equation (Equation 2.1), which is currently considered to be acceptable for steel reinforced beams. To be applicable for use in GFRP reinforced beams, this equation must include modification factors to account for the low stiffness of GFRP, variation in bond properties and tension stiffening effect of GFRP. Much research has been done to determine accurate modification factors, using empirical data. The ACI 440R-06 has adopted one such equation, first suggested by Benmokrane et. al (1998).

\[ I_e = \beta_d \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1.0 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \]  \hspace{1cm} (2.22)

The ACI committee, upon analyzing empirical data gathered from 88 tests in 10 test series, recommended that the reduction factor \( \beta_d \) be determined in the following manner:

\[ \beta_d = \frac{1}{5} \left( \frac{\rho_f}{\rho_b} \right) \]  \hspace{1cm} (2.23)

where:

\( \rho_f = \) longitudinal reinforcement ratio

\( \rho_b = \) balanced longitudinal reinforcement ratio

A new deflection equation has been proposed to the ACI committee, and, at the time of writing, is in the ballot phase. This equation, proposed by Bischoff et. al (2010),
uses first principles to determine deflection, unlike the modified Branson’s equation above. Theoretically, this equation could work accurately for both steel and GFRP reinforced beams. It accounts for factors such as tension stiffening, loading conditions, and support conditions and is seen in Equation 2.25.

\[
I_e = \frac{I_{cr}}{1 - \gamma \beta \chi \eta (M_{cr}/M_a)}
\]  

The current CSA code, CSA S806-12, does not use an effective modulus of inertia. Instead, it includes a table with closed form solutions to solve for deflection directly. Table 7 in this code includes equations for common loading and support conditions. These equations were derived from integrating theoretical moment curvature diagrams, assuming a tri-linear moment curvature response. The deflection equations, from Table 7 of CSA S806-12 are:

for three point loading:

\[
\Delta = \frac{PL^3}{48E_cI_{cr}} \left[ 1 - 8\eta \left(\frac{L_g}{L}\right)^3 \right]
\]  

and for four point loading:

\[
\Delta = \frac{PL^3}{48E_cI_{cr}} \left[ 3 \left(\frac{a}{L}\right) - 4 \left(\frac{a}{L}\right)^3 - 8\eta \left(\frac{L_g}{L}\right)^3 \right]
\]  

2.2.2 Deformability

As described in chapter 1.1, deformability is a measure of performance of a flexural-critical structural element, which indicates a factor of safety. If adequate deformability is provided in the design by the engineer, pre-emptive warning (such as excessive deflections) will become apparent significantly before failure, allowing evacuation of the structure. Sufficient deformability will also allow load redistribution in indeterminate structures to prevent catastrophic failures. This is critical in elements, such as GFRP reinforced concrete beams, which fail in a brittle and abrupt manner. There are currently three methods for quantifying deformability: the J factor, Deformability Factor (DF), and Energy Factor of Safety (EFS). These methods use a ratio of energy absorption
of the beam at ultimate load levels to energy absorption at a limiting state which approximates service load levels.

The first deformability model, proposed by Jaeger, Tadros, and Mufti in 1995, is known as the J factor. This factor is currently the only code-adopted approach for determining deformability (CSA S6-06). This approach approximates the energy absorption ratio by multiplying the moment resistance by the corresponding curvature at two points (ultimate conditions and limiting conditions), as shown in Equation 2.4. The limiting conditions ($M_c$ and $\varphi_c$) are calculated when the concrete strain at the top surface of the beam equals $-1.0 \times 10^{-3}$. The authors, as well as CSA S6-06, recommend that J be greater than 4 for a rectangular beam to have sufficient deformability. Vijay and GangaRao (2001) proposed an alternative mode, the Deformability Factor (DF); see Equation 2.15.

This method differs slightly from the J factor in two ways. The DF method requires the exact energy absorption of a beam, which involves integrating the moment-curvature diagram at two points. This requires knowledge of the entire moment-curvature response, which is significantly more calculation intensive than determining two points, as for the J factor method. Secondly, the limiting condition is set at a curvature equal to $0.005/d$ rads/mm, which the authors suggest approximates service conditions based on empirical data.

Both J and DF utilize the moment-curvature behaviour of a beam to define deformability. Because moment-curvature is a sectional response, these factors are defined as sectional-level energy approaches. Yost and Gross (2002) proposed a deformability approach that utilize material-level energy, known as strain energy density (or the area under the stress-strain graph of the respective constitutive materials), to numerically determine deformability. This factor, Energy Factor of Safety (EFS) is defined in Equation 2.27.

$$EFS = \frac{\sum f^u_0 \sigma d\varepsilon}{\sum f^S_0 \sigma d\varepsilon}$$ 2.27
In the case of GFRP-reinforced beams, the summations in Equation 2.27 must be the addition of the concrete strain energy density and the GFRP strain energy density. This is accomplished by assuming the concrete stress-strain behaviour is parabolic, and the GFRP stress-strain behaviour is linear-elastic. The service condition is defined as when either the concrete stress is 35% of its ultimate, or the GFRP stress is 40% of its ultimate.
2.2.3 Test Programs

Before embarking on the current test program, a data base was created based on the existing related reported work. This resulted in 64 beams from 9 studies, with the specific goal of investigating load-deflection behaviour. This database is presented below.

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<th>h (mm)</th>
<th>d (mm)</th>
<th>ρ (%)</th>
<th>f'_c (MPa)</th>
<th>E_f (MPa)</th>
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3. EXPERIMENTAL PROGRAM

To accomplish the goal of investigating flexural behaviour of GFRP reinforced concrete beams, a test program consisting of 16 beams was undertaken. These beams were reinforced completely with GFRP. No steel reinforcement was used. All of the beams possessed enough transverse reinforcement to prevent shear failure; thus the desired failure mode was consistently flexural (either top concrete crushing or bottom reinforcement rupturing). The major variables in this program were concrete strength, type of GFRP bars and hence the manufacturer, and longitudinal reinforcement ratio. The specimens were designed in such a manner that two or more specimens could be compared to investigate only one variable between them. A number of such comparisons could be made for each variable. The details of the GFRP bars and concrete material properties, individual test specimens, and test setup are presented in this chapter.

3.1 Material Properties

The material properties of the three components of the beams are summarized here: longitudinal reinforcement, transverse reinforcement, and concrete. Detailed results of all the material tests can be found in Appendix A (Material Properties) including the strength vs. age relationship for the two types of concrete.

3.1.1 Longitudinal GFRP Reinforcement

The mechanical properties of GFRP bars have been well studied and are well understood. This material has approximately one third to one-quarter of the stiffness of steel (~50 000 to 60 000 MPa vs. ~200 000 MPa), three times the ultimate strength of steel (~1200 MPa vs. ~400 MPa), and approximately ten times the elastic strain of steel (~20x10^{-3} vs. ~2.0x10^{-3}). GFRP bars exhibit no post-elastic behaviour; upon reaching its ultimate stress, it ruptures in a brittle manner. Typical stress-strain curves for steel and GFRP bar are shown in Figure 3.1. This brittle behaviour is the main reason for the differences between steel-reinforced and GFRP-reinforced members and for detailed provisions of structural design codes, such as CSA S806-12 and ACI 440.1R-06.
For this test series, three GFRP manufacturers provided reinforcing bars: Hughes Bros., Pultrall, and Schöck. For the purposes of this document, they will herein be referred to as ‘A’ (Hughes Bros.), ‘B’ (Pultrall), and ‘C’ (Schöck).

Each manufacturer provided two sizes of longitudinal reinforcing bars. Bar types A and B were provided with nominal diameters of $\frac{1}{2}$” (#4) and $\frac{3}{8}$” (#5). Bars C had nominal diameters of 12 mm (12M) and 16 mm (16M). Because the mechanical properties, such as ultimate strength, modulus of elasticity, bond strength, and surface treatment, differ substantially between manufacturers and bar sizes, direct tensile coupon tests were performed on these bars to accurately determine these properties. These bars can be seen in Figure 3.2. As can be seen, bars A and B are sand-coated while bars C are helically grooved to improve bond with concrete. Bars A also use helical-wrapping to develop bond.
The test setup used to perform the direct tensile tests consisted of coupling a bar sample with hollow steel pipe, using expansive mortar. A displacement controlled 1000 kN testing machine was used to grip the coupling pipes and apply uniaxial tension until failure, as seen in Figure 3.3. A standard clip gauge was used to measure the change in length and calculate strain and modulus of elasticity. The clip gauge was removed between 25% and 50% of the ultimate load to prevent damage to the instrument. A linear-relationship was assumed until failure. For each type of bar, three such tests were performed. Further details of these tests can be found in Appendix A (Material Properties).
Determination of the cross-sectional areas of GFRP bars is a significant issue that must be addressed in design codes, such as in Annex A of CSA S806-12. Manufacturers of GFRP bars provide nominal dimensions, which can be seen in Table 3.1 below. In this table, the elongation was determined directly from experimental results. The ultimate strength was calculated by dividing the ultimate tensile load (in N) by the nominal area (in mm²). The modulus of elasticity corresponds to the ultimate strength divided by the elongation. Both the ultimate strength and modulus of elasticity depend on the cross-sectional bar area.
Table 3.1: Experimental Longitudinal GFRP Reinforcement Coupon Test Results
Calculated Using Nominal Diameters

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<td>16</td>
<td>201.0</td>
<td>1228</td>
<td>60700</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Uncertainties of properties may arise because of the difference in surface treatment of the bars. Sand coating on Bars A and B increases their cross-sectional area without contributing to ultimate strength. Bar C uses helical deformations. As stated above, both the ultimate strength and modulus of elasticity of these bars depend on their cross-sectional area. To allow comparison between these bars, their interior diameters (without surface treatments) were measured. The mechanical properties were calculated using the same experimental results from the tensile coupon tests (Table 3.1) and measured diameters, and are presented in Table 3.2 below.
Table 3.2: Experimental Longitudinal GFRP Reinforcement Coupon Test Results
Calculated Using Measured Diameters

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Bar Desig.</th>
<th>Measured Diameter (mm)</th>
<th>Calculated Area (mm²)</th>
<th>Actual Ult. Strength (MPa)</th>
<th>Actual Mod. of Elasticity (MPa)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar A</td>
<td>#4</td>
<td>12.22</td>
<td>117.3</td>
<td>819</td>
<td>54498</td>
<td>1.50</td>
</tr>
<tr>
<td>Bar A</td>
<td>#5</td>
<td>15.78</td>
<td>195.6</td>
<td>751</td>
<td>50092</td>
<td>1.50</td>
</tr>
<tr>
<td>Bar B</td>
<td>#4</td>
<td>12.69</td>
<td>126.5</td>
<td>1056</td>
<td>57785</td>
<td>1.83</td>
</tr>
<tr>
<td>Bar B</td>
<td>#5</td>
<td>15.86</td>
<td>197.6</td>
<td>1005</td>
<td>58084</td>
<td>1.73</td>
</tr>
<tr>
<td>Bar C</td>
<td>12M</td>
<td>12.45</td>
<td>121.7</td>
<td>1221</td>
<td>59088</td>
<td>2.07</td>
</tr>
<tr>
<td>Bar C</td>
<td>16M</td>
<td>16.35</td>
<td>210.0</td>
<td>1176</td>
<td>58100</td>
<td>2.02</td>
</tr>
</tbody>
</table>

The measured properties in Table 3.2 are used in all analysis calculations presented herein, unless otherwise specified.

3.1.2 Transverse GFRP Reinforcement

As a goal of this test series was to reinforce the beams with only GFRP bars, the transverse reinforcement was required to be bent GFRP bar. The size of the beams in this test series was relatively small, which constrained the use of GFRP reinforcement significantly. GFRP bars cannot be bent post-production, thus stirrups must be ordered directly from the manufacturer. Different manufacturers use different manufacturing processes to create bent bars, and these processes impose a strict requirement on minimum bend radius and ultimate strength.

Because of the limits on bend radius, stirrups made from bar A did not meet the required width dimension; thus, only bars B and C were used as transverse reinforcement, in the form of closed hoop stirrups. Bar C stirrups were made of 12M bars, which is consistent with the large beam test series. Smaller bars allow for a smaller bend radius. Because of this, bar B stirrups were made out of #3 (½”) bars. The specified dimensions can be seen in Figure 3.4. The stirrups were designed to provide minimum concrete cover, as prescribed in CSA S806-12 (1.6 $d_b$). Figure 3.5 shows the final stirrups. Note
that the stirrup hooks differ between samples due to differences in manufacturing processes.

Figure 3.4: Specified stirrup dimensions for Bar B and Bar C

Figure 3.5: Transverse GFRP Reinforcing Stirrups

The bending process significantly reduces the ultimate strength of GFRP bars. For example, the manufacturer of bar B recommends that, for a 90° bend, the design tensile strength of a bent bar be 40% that of a straight bar. The small size of the stirrups used in this test series made determination of the mechanical properties by direct tensile tests difficult; however, direct tensile tests were performed on similar stirrups used in the
large beam test series. Both test series had beams reinforced with 12 mm bar C stirrups. The large test series had beams reinforced with #4 bar B stirrups, similar to the #3 bar B stirrups used in this test series. The results of these tests are shown below in Table 3.3. Similar to the longitudinal reinforcement, Table 3.3 presents the properties measured at the University of Toronto. Note that these tests were performed on the straight portion of the stirrups, and the bent portion may be weaker.

Table 3.3: Measured Properties of Transverse Reinforcement

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Bar Desig.</th>
<th>Measured Diameter (mm)</th>
<th>Measured Area (mm²)</th>
<th>Ult. Strength (MPa)</th>
<th>Modulus of Elasticity (MPa)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bent Bar B (Straight Portion)</td>
<td>#4</td>
<td>11.9</td>
<td>111.2</td>
<td>655</td>
<td>41533</td>
<td>1.57</td>
</tr>
<tr>
<td>Bent Bar C (Straight Portion)</td>
<td>12M</td>
<td>13.0</td>
<td>132.7</td>
<td>912</td>
<td>57533</td>
<td>1.61</td>
</tr>
</tbody>
</table>

3.1.3 Concrete

Two different concrete strengths were used in this test series; normal strength concrete (NSC) specified at 40 MPa and high strength self consolidated concrete (HSC) specified at 80 MPa. The difference between concrete strengths allowed comparison between the failure modes of normal strength concrete and high strength concrete. HSC may become common in design practice, due to its capacity to utilize the high tensile strength of GFRP bars (Faza and Ganga Rao, 1992). Cylinder tests and modulus of rupture tests were performed on these two concretes with the results shown in Table 3.4. The cylinder and modulus of rupture test setup is shown in Figure 3.6.

Table 3.4: Concrete properties at 90 days

<table>
<thead>
<tr>
<th>Concrete Type</th>
<th>f'c (MPa)</th>
<th>f_r (MPa)</th>
<th>E_c (MPa)</th>
<th>ε_c (mm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Strength</td>
<td>41.4</td>
<td>3.56</td>
<td>30800</td>
<td>2.08</td>
</tr>
<tr>
<td>High Strength</td>
<td>80.9</td>
<td>5.55</td>
<td>37100</td>
<td>2.39</td>
</tr>
</tbody>
</table>
3.2 Test Specimen

The test program consisted of sixteen beams, split into two main series; eight beams made of normal strength concrete and eight beams made of high strength concrete. Among these test series’, the main variables investigated are longitudinal reinforcement ratio and type of GFRP bars. Shear reinforcement ratio and shear spacing were also changed between specimens in order to ensure adequate shear resistance. Beams in each test series were cast in one concrete pour to ensure consistent concrete properties. The formwork used for these pours and the cast specimens can be seen in Figure 3.7.

Figure 3.6: Test setups for cylinder tests and modulus of rupture tests

Figure 3.7: Formwork and cast beams

a) Formwork design  b) Formwork with beams cast
Three different tensile longitudinal reinforcement ratios were studied in this test program: approximately 0.5%, 1%, and 2%. These ratios roughly correspond to the predicted failure mode being, respectively, tension reinforcement rupturing, balanced failure, and concrete crushing. All beams were rectangular in cross-section; 325 mm high, 200 mm wide, and 3620 mm long. Figure 3.8 shows the cages for three beams, reinforced at 0.5%, 1%, and 2% longitudinal reinforcement ratios.

![Figure 3.8: Reinforcing cages for beams reinforced at a) 0.5%, b) 1%, and c) 2%](image)

The longitudinal reinforcement ratios and beam dimensions were chosen to allow size effect to be determined between the small beam test series (presented here) and the large beam test series (Johnson and Sheikh, 2012). All beams in the large test series had a longitudinal reinforcement ratio of 1% (as with seven small beams) and cross-sectional dimensions of 650 mm high by 500 mm wide (twice that of the small beams). The span of 3360 mm was kept constant for both test programs.

Upon determination of the beam dimensions and longitudinal reinforcement ratios, the moment resistance can be calculated. To ensure shear failures did not occur, the stirrups were spaced in such a way that the nominal shear resistance would always be greater than 150% of nominal applied shear at beam failure. This shear resistance was calculated as prescribed in CSA S806-02.
The specimen naming scheme presented herein is as follow: XX-Y-Z.Z, where XX is the specified concrete strength (40 or 80 MPa), Y is the longitudinal bar manufacturer (A, B, or C), and Z.Z is the approximate longitudinal reinforcement ratio (0.5%, 1.0%, or 2.0%, that would represent failure mode as stated above). For example, beam 40-B-2.0 was comprised of normal strength (40 MPa) concrete and reinforced with bar B at a ratio of 2%, which corresponds to an over-reinforced beam. A summary of the test specimens with reinforcement details can be seen in Table 3.5. Note that a subscript 3 in beam designation (XX-Y-Z.Z3) indicates that this beam was tested under three-point loading. A beam tested under four-point loading has no subscript. The effective depth, d, is the distance from top of the beam section to the centroid of the tension longitudinal reinforcement.

Table 3.5: Test Specimen

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Reinf. Condition</th>
<th>90-Day Concrete Strength (MPa)</th>
<th>GFRP Bars</th>
<th>Reinf. Bar Quantity</th>
<th>Long. Reinf. Ratio (%)</th>
<th>Trans. Reinf. Ratio (%)</th>
<th>Effective Depth, d (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-A-0.5</td>
<td>Under</td>
<td>41.4</td>
<td>A</td>
<td>2 #4</td>
<td>0.45</td>
<td>0.48</td>
<td>282</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>Over</td>
<td>41.4</td>
<td>A</td>
<td>5 #4</td>
<td>1.17</td>
<td>0.71</td>
<td>270</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>Over</td>
<td>41.4</td>
<td>B</td>
<td>5 #4</td>
<td>1.17</td>
<td>0.71</td>
<td>270</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>Over</td>
<td>41.4</td>
<td>B</td>
<td>5 #5</td>
<td>1.89</td>
<td>0.95</td>
<td>263</td>
</tr>
<tr>
<td>40-C-1.03</td>
<td>Over</td>
<td>41.4</td>
<td>C</td>
<td>5 12M</td>
<td>1.12</td>
<td>0.57</td>
<td>272</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>Over</td>
<td>41.4</td>
<td>C</td>
<td>5 12M</td>
<td>1.12</td>
<td>0.57</td>
<td>272</td>
</tr>
<tr>
<td>40-C-2.03</td>
<td>Over</td>
<td>41.4</td>
<td>C</td>
<td>5 16M</td>
<td>1.9</td>
<td>0.57</td>
<td>265</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>Over</td>
<td>41.4</td>
<td>C</td>
<td>5 16M</td>
<td>1.9</td>
<td>0.57</td>
<td>265</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>Under</td>
<td>80.9</td>
<td>A</td>
<td>2 #4</td>
<td>0.45</td>
<td>0.48</td>
<td>282</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>Balanced</td>
<td>80.9</td>
<td>A</td>
<td>5 #4</td>
<td>1.17</td>
<td>0.71</td>
<td>270</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>Over</td>
<td>80.9</td>
<td>A</td>
<td>5 #5</td>
<td>1.89</td>
<td>0.95</td>
<td>263</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>Balanced</td>
<td>80.9</td>
<td>B</td>
<td>5 #4</td>
<td>1.17</td>
<td>0.71</td>
<td>270</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>Over</td>
<td>80.9</td>
<td>B</td>
<td>5 #5</td>
<td>1.89</td>
<td>0.95</td>
<td>263</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>Under</td>
<td>80.9</td>
<td>C</td>
<td>2 12M</td>
<td>0.4</td>
<td>0.57</td>
<td>282</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>Balanced</td>
<td>80.9</td>
<td>C</td>
<td>5 12M</td>
<td>1.04</td>
<td>0.75</td>
<td>272</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>Over</td>
<td>80.9</td>
<td>C</td>
<td>5 16M</td>
<td>1.9</td>
<td>0.75</td>
<td>265</td>
</tr>
</tbody>
</table>

*Note: Subscript 3 (as in XX-Y-Z.Z3) indicates beam tested under three-point loading
Further details of these specimens can be found in Appendix B (Specimen Details).

### 3.3 Test Procedure

#### 3.3.1 Test Setup

Testing was performed in two phases. The first phase consisted of testing two beams (40-C-1.03 and 40-C-2.03) under three point loading, and two identical beams (40-C-1.0 and 40-C-2.0) under four point loading, as shown in Figure 3.9 and Figure 3.10. After performing analysis on these four tests, it was found that the flexural behavior of the beam was not significantly affected by the presence of shear. It was thus decided that the remaining twelve beams be tested under four-point loading due to the advantages of analyzing flexural behaviour of a section within a pure flexure zone.

![Figure 3.9: Three-point test setup](image)

![Figure 3.10: Four-point test setup](image)
The supports for the beam consisted of 150 mm wide steel plates resting on free rollers, allowing the support conditions to be idealized as simply-supported. The center-to-center span between supports was 3360 mm, with a clear span of 3210 mm. To ensure a solid contact surface with the supports and loading plates, plaster bags were placed between the concrete beam and steel plates. For the three point tests, the load was applied from the loading machine onto a 150 mm wide steel plate resting on a plaster bag, and onto the concrete beam. For the four-point tests, the load was applied onto a steel spreader beam, which applied the load onto two 150 mm wide rollers resting on plaster bags, and then onto the beam. The two rollers were offset 450 mm from midspan, resulting in a shear span of 1230 mm. This distance was chosen to reduce local effects in the pure flexure zone at loading points.

Testing was performed in multiple load stages. Each test was paused at certain stages to measure crack widths and locations. The first load stage was taken at initial cracking, followed by load stages corresponding to the strain at the top surface of the concrete being \(-0.5 \times 10^{-3}\), \(-1.0 \times 10^{-3}\), and, in the case of over-reinforced beams, \(-2.0 \times 10^{-3}\). In the case of under-reinforced beams, the test was not paused at a concrete strain of \(-2.0 \times 10^{-3}\) as it was deemed unsafe due to the possibility of sudden collapse as a result of bar rupture.

The load was applied using a 2700 kN MTS displacement controlled testing machine as seen in Figure 3.11. The loading rate varied from 0.008 mm/s (prior to the first load stage) up to 0.032 mm/s (after the final load stage pause until completion of the test.)
3.3.2 Instrumentation

Data was measured during testing using three main instrument types: strain gauges, linear potentiometers, and three-dimensional LED targets. The location of these instruments can be seen in Figure 3.12.

Each beam was instrumented with eight strain gauges: six 5 mm internal gauges placed on the longitudinal GFRP reinforcement and two 60 mm external gauges placed on the top surface of the concrete.

Of the six internal strain gauges, three were placed on a top longitudinal reinforcing bar and three were placed in the identical locations on a bottom reinforcing bar. One of the three strain gauges was located at midspan, with the other two being
located 600 mm from midspan on either side. These locations can be seen in Figure 3.10. The internal strain gauges used in this series were rated for use in high strain applications, such as those seen when GFRP ruptures.

The two external strain gauges were located on the top surface of the concrete beam, at 225 mm offset from midspan. Both of these gauges were located inside the pure flexure zone in the four-point test setup. The surface gauges were 60 mm long.

Seven potentiometers were used during testing. Four of these were located at the supports (one on each corner of the beam) to measure support displacements. The remaining three were located at the quarter points of the span to measure vertical displacements.

The data from the strain gauges, potentiometers, and MTS machine was collected using a data acquisition system sampling at a rate of 2 Hz.

In addition to the above-mentioned instrumentation, a three-dimensional LED targeting system was used. This system uses three cameras mounted on a tripod (Figure 3.13) and a grid of LED’s (Figure 3.14) to accurately determine the change of position of certain points on the beam. By positioning the LEDs in a regular square grid, it is possible to determine axial strains, shear strains, curvature, and various other properties of interest. For this test series, the LEDs were placed in a 150 mm by 150 mm grid.
Figure 3.13: Three-dimensional LED scanning camera

Figure 3.14: LED grid used for three-dimensional positioning
3.4 Summary

Details of the experimental program are presented in this chapter in which sixteen GFRP reinforced concrete beams were constructed in order to investigate their flexural behaviour. All beams were of a constant size (200 mm wide by 325 mm high by 3620 mm long) with the major variables being the type of GFRP bars (Bars A, B, or C), longitudinal reinforcement ratio (−0.5%, 1%, or 2%) and concrete strength (~40 MPa or 80 MPa). Fourteen beams were tested under four-point loading with a 1230 mm shear span, while the remaining two beams were tested under three-point loading. Details of instrumentation including internal and surface strain gauges, potentiometers, and a three-dimensional positioning camera are also provided in this chapter.
4. EXPERIMENTAL RESULTS

This chapter presents the experimental results from the sixteen beams, and is split into two main sections, failure modes and experimental discussion. The failure mode section discusses the behaviour and characteristics of the two main failure modes experienced during testing (tension failure in under-reinforced beams and concrete crushing in over-reinforced beams). The experimental results section discusses and compares the results between different beams. This chapter also discusses general results and comparisons between tests. For details of the individual tests, refer to Appendix B (Specimen Details).

4.1 Failure Modes

GFRP reinforced beams, as with steel reinforced beams, can be designed as under-reinforced, balanced, or over-reinforced. Traditionally, concrete beams have been designed as under-reinforced to ensure that the tensile longitudinal steel reinforcement yields and allows its beneficial plastic behaviour to govern the failure mode of the beam, resulting in a ductile response. Because GFRP bars do not exhibit any such plastic behaviour, typical design methodology for beams reinforced with this material has shifted from under-reinforced to over-reinforced. Tests have shown that the compression failure mode of over-reinforced beams (concrete crushing) is preferable over the more brittle tension failure mode of under-reinforced beams (GFRP rupturing).

In this test series, the behaviour of both under-reinforced and over-reinforced beams was investigated. Three beams (40-A-0.5, 80-A-0.5, and 80-C-0.5) were designed as under-reinforced, with the remaining thirteen beams being over-reinforced. The failure modes are discussed herein.

4.1.1 Balanced Failure Mode

A balanced failure is defined as one in which the top surface of the concrete and the bottom longitudinal reinforcement both reach their ultimate strains simultaneously, causing the concrete to crush at the same instant as the GFRP ruptures. This failure mode represents the limit between under-reinforced and over-reinforced beams. A beam with a reinforcement ratio lower than balanced will fail by GFRP rupture, while a beam with a
higher reinforcement ratio will fail by concrete crushing. Balanced conditions are a useful
design limit; however, it is difficult to design an actual beam in this manner due to
material inconsistencies, discrete reinforcing bar sizes, errors in construction, and other
factors. The mechanics of a balanced failure are seen in Figure 4.1.

To design a balanced beam, one of two methods must be used. The first of these
methods is to balance the forces of the concrete above the neutral axis with the forces in
the FRP, and design the FRP area respectively. This method is seen in Equation 4.1.

\[
\rho_b = \frac{0.85 \beta_1 f'_c}{f_{ult}} \frac{\varepsilon_{cu} E_f}{f_{ult} + \varepsilon_{cu} E_f}
\]

where:

\( \beta_1 = \text{equivalent stress block factor} \)

\( f'_c = \text{concrete compressive strength (MPa)} \)

\( f_{ult} = \text{FRP reinforcement ultimate stress (MPa)} \)

\( \varepsilon_{cu} = \text{concrete ultimate strain (mm/mm)} \)

\( E_f = \text{FRP reinforcement modulus of elasticity (MPa)} \)

The second method is derived from the strain profile found in Figure 4.1. By
using similar triangles, Equation 4.2 can be derived.
While it is difficult to achieve a perfect balanced failure mode in most cases, one beam in this test series exhibited a failure mode close to balanced. This beam (80-A-1.0) was reinforced slightly above balanced ($\frac{\rho_f}{\rho_b} = 1.01$). The experimental load-deflection behaviour can be seen below, in Figure 4.2. Note that in this figure, as with the following load-deflection figures, deflection was calculated by subtracting the average support displacement from the total midspan displacement (collected using either potentiometers or LED targets).

![Figure 4.2: Load vs. midspan deflection of beam 80-A-1.0 indicating a balanced failure mode](image)

Observations during testing confirm that at peak load of 200.5 kN, concrete crushing occurred on the top surface. Immediately after crushing, the longitudinal reinforcement ruptured, accompanied by popping noises. This happened at a load of about 155 kN which is indicated by a small peak in Figure 4.2. These observations confirm this beam failed in a balanced manner. Note that this beam was the closest to
being reinforced at a balanced condition in the entire test series. The failure mechanism of this beam is seen below, in Figure 4.3.

![Image of beam failure](image_url)

Figure 4.3: Crushing and tensile (balanced) failure of beam 80-A-1.0

### 4.1.2 Under-Reinforced Failure Mode

As stated above, under-reinforced beams fail by tensile failure of the bottom longitudinal reinforcement; in the case of GFRP reinforced beams, the failure mode is GFRP rupture.

As noted by Nanni (1993) and Vijay and GangaRao (1997), concrete compression failure is preferable to tension rupture failure as it provides a slightly more ductile response. This design philosophy is directly incorporated into CSA S806-12 by stating that failure in the section must be initiated by crushing of the concrete in the compression zone. ACI 440.1R-06 allows FRP beams to be designed as under-reinforced, but requires such beams to contain extra reserve strength by decreasing the material reduction safety factor from 0.65 for beams with $\rho_f > 1.4\rho_b$ to 0.55 for beams with $\rho_f < \rho_b$, and scaling linearly between these two points. Similarly, CSA S6-06 allows, but punishes, under-reinforced beams by incorporating the J factor, discussed in Section 5.4 (Deformability).
Figure 4.4 below shows a typical stress and strain profiles for an under-reinforced beam.

![Diagram showing under-reinforced strain and stress conditions](image)

Figure 4.4: Under-reinforced strain and stress conditions

Figure 4.4 shows that, because the FRP reinforcement strain is at ultimate and the concrete top surface strain is less than ultimate, the \( \frac{c}{d} \) ratio of an under-reinforced beam must be smaller than that of a balanced beam, as expressed in Equation 4.3.

\[
\frac{c}{d} < \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ult}} = \left( \frac{c}{d} \right)_b
\]

Equation 4.3

Figure 4.5 shows the load-deflection curves of one of the three under-reinforced beams in this test series (40-A-0.5).
Figure 4.5: Load vs. midspan deflection of under-reinforced beam 40-A-0.5

The characteristic feature of under-reinforced beams that is critical in design consideration is the complete and sudden failure of the beam upon reaching its ultimate load. There is generally no post-peak response. It is apparent that, if designing an under-reinforced beam, large factors of safety and careful consideration must be applied by the designer.

Figure 4.6 and Figure 4.7 show the overall beam failure and GFRP reinforcement failure, respectively, of an under-reinforced beam (80-C-0.5).
Figure 4.6: Failure of under-reinforced beam 80-C-0.5

Figure 4.7: Tensile failure of GFRP longitudinal reinforcement in beam 80-C-0.5

4.1.1 Over-Reinforced Failure Modes

**Initial Failure (First Peak)**

Beams with more reinforcement than balanced ($\rho_f > \rho_b$) are defined as over-reinforced and fail by crushing of concrete at the top surface. The stress and strain states of a typical over-reinforced beam can be seen in Figure 4.8.
EXPERIMENTAL RESULTS

Because the majority of beams designed in practice are over-reinforced, thirteen of the sixteen beams presented here were designed in such a manner. The design process for an over-reinforced beam involves either supplying more reinforcement than that calculated by Equation 4.1, or using Equation 4.4 below. As seen in Figure 4.8, the $c/d$ ratio of an over-reinforced beam is greater than that of a balanced beam.

\[
\frac{c}{d} > \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ult}} = \left(\frac{c}{d}\right)_b
\]

Figure 4.9 shows an over-reinforced beam (80-A-2.0) that has failed by concrete crushing in the pure flexure zone.
Analyzing the load-deflection response of over-reinforced beams can reveal information about their characteristic behaviour. One such figure, for beam 40-B-1.0, is shown below in Figure 4.10.

![Load vs. midspan deflection of over-reinforced beam 40-B-1.0](image)

Several characteristic patterns can be observed by investigation of the above load-deflection figure. Firstly, this beam exhibited typical pre-cracked and cracked response. Initially, the beam is stiff due to the tensile resistance of the uncracked concrete. At a load of approximately 20 kN, the beam behaviour transitions from uncracked to cracked as the concrete on the bottom face begins to crack, which results in a significant decrease in stiffness. After cracking, the stiffness remains approximately constant until ultimate load. Over-reinforced beams are typically stiffer than under-reinforced beams.

Secondly, the over-reinforced beams in this test series always show characteristic post-peak behaviour. In Figure 4.10, the load decreases significantly after the initial peak and then it begins to increase again. The initial peak load corresponds to crushing of the top concrete surface, as seen in Figure 4.9. Initial failure is defined by this crushing, and this point is defined as the first peak load; however, only the concrete that is outside of the stirrups (the concrete cover) crushes. As test progresses and neutral axis depth...
increases, the concrete below the top surface experiences higher strain and the moment is reduced primarily due to reduced lever arm. Inside of the stirrups exists a core which is partially confined by stirrups and thus can undergo greater strains than the unconfined cover without losing strength. This confinement effect allows beams which have displayed cover concrete crushing to experience a second peak load. Confinement is discussed further below.

Confined Failure (Second Peak)

As discussed above, over-reinforced sections fail initially by crushing of the top concrete surface as it reaches its ultimate strain ($\varepsilon_{cu}$). However, every over-reinforced beam in this test series experienced an increase in strength directly subsequent to the drop in load due to concrete crushing. A typical such beam (80-A-2.0) response is shown below in Figure 4.11. At a deflection of 84.1 mm, the load dropped from 234.6 kN to 189.0 kN but the beam did not lose its capacity completely. Continuing the test with further downward displacement resulted in the beam showing positive stiffness and reaching a second peak before the final failure. This phenomenon is most likely due to concrete confinement provided by GFRP stirrups.

![Figure 4.11: Load vs. Midspan Deflection of Beam 80-A-2.0](image)
It is understood that rectangular ties in columns improve the compressive behaviour of the concrete inside the ties, in terms of both strength and ductility (Sheikh and Uzumeri, 1982). The ties and longitudinal bars restrain the core concrete from dilating, improving the mechanical properties, as seen in Figure 4.12. In this diagram, the behaviour of typical unconfined concrete is represented by the dotted line and the behaviour of confined concrete is represented by the solid line.

Concrete in the compressed core of a beam, restrained by the closed hoop stirrups and top longitudinal reinforcement, behaves similar to the concrete in the core of a column, restrained by rectangular ties and longitudinal reinforcement. Thus, upon crushing of the top surface of the concrete and subsequent spalling of the cover in the compression zone, confinement effects allow the core concrete inside of the stirrups to continue sustaining compressive load. The FRP at this stage had not reached its ultimate strain, and thus continues to resist the tensile load. This behaviour can be seen in Figure 4.13.
Upon initiation of the confinement effect, the beam behaves similar to a conventional beam (with improved concrete properties) except that part of the section is lost due to spalling of concrete. As load increases from the initial confinement point, the bottom reinforcement experiences increases in tensile strain, and the top of the core concrete experiences increases in compressive strain. As load increases further, one of two failure modes must occur: confined concrete crushing, or tensile reinforcement rupturing.

The confined-tensile rupture failure mode is similar in appearance to the balanced failure mode; however, confined-tensile failures are characterized by an increase in load after initial crushing, as seen in Figure 4.14. The failure mode associated with the second peak was tensile rupturing. Beams which exhibit this failure mode do not contain enough longitudinal reinforcement to allow the core concrete to reach its ultimate strain. The overall beam failure is seen in Figure 4.15.
Figure 4.14: Load vs. midspan deflection of beam 80-B-1.0, indicating a confined-tensile rupture failure

Figure 4.15: Confined-tensile rupture failure mode of beam 80-B-1.0

The other confined failure mode, confined-crushing, occurs in beams with sufficient longitudinal reinforcement to allow the confined core concrete to reach its ultimate strain. This failure mode is characterized by complete failure of the beam, accompanied by complete crushing of core concrete, stirrups rupturing, or top reinforcing bar buckling, as seen in Figure 4.16.
Mechanical properties of confined concrete are dependent on several variables, including: properties of unconfined concrete and reinforcement, spacing of ties, and beam dimensions. The mechanism of confined concrete is beyond the scope of this work and is not detailed here.

4.2 Discussion of Experimental Results

This section discusses the experimental results with emphasis on the comparison of beam behaviour. The effects of longitudinal reinforcement ratio, concrete compressive strength, and GFRP manufacturer on load-deflection behaviour are discussed.

4.2.1 Load Deflection Results

The load deflection responses of the sixteen beams presented here are shown in Figure 4.17 to Figure 4.21. These figures are grouped in series’ with similar concrete strength and reinforcement ratio (YY-X-0.5, 40-X-1.0, 40-X-2.0, 80-X-1.0, and 80-X-2.0). Note that Figure 4.17 presents the data from the three under-reinforced beams, despite having two different concrete strengths.
Figure 4.17: Load vs. midspan displacement of under-reinforced beams

Figure 4.18: Load vs. midspan deflection of normal strength beams with 1% longitudinal reinforcement
Figure 4.19: Load vs. midspan deflection of normal strength beams with 2% longitudinal reinforcement

Figure 4.20: Load vs. midspan deflection of high strength beams with 1% longitudinal reinforcement
Figure 4.21: Load vs. midspan deflection of high strength beams with 2% longitudinal reinforcement

Table 4.1 below presents this data in tabular form, including the failure mode, loads, and deflections for the first peak (unconfined) and second peak (confined) failures.
Table 4.1: Experimental failure modes, loads and midspan deflections

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Failure Mode</th>
<th>First Peak</th>
<th>Second Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Load</td>
<td>Deflection</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(kN)</td>
<td>(mm)</td>
</tr>
<tr>
<td>40-A-0.5</td>
<td>Tensile</td>
<td>88.6</td>
<td>91.2</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>Confined-Crushing</td>
<td>140.3</td>
<td>77.0</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>Confined-Crushing</td>
<td>155.8</td>
<td>75.1</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>Confined-Crushing</td>
<td>180.2</td>
<td>58.6</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>Confined-Crushing</td>
<td>110.8</td>
<td>69.0</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>Confined-Crushing</td>
<td>152.0</td>
<td>73.7</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>Confined-Crushing</td>
<td>132.4</td>
<td>45.3</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>Tensile</td>
<td>94.0</td>
<td>106.0</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>Balanced</td>
<td>200.5</td>
<td>109.0</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>Confined-Crushing</td>
<td>234.8</td>
<td>88.8</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>Confined-Tensile</td>
<td>204.2</td>
<td>96.1</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>Confined-Crushing</td>
<td>251.2</td>
<td>81.8</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>Tensile</td>
<td>122.0</td>
<td>131.2</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>Confined-Crushing</td>
<td>193.7</td>
<td>99.7</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>Confined-Crushing</td>
<td>256.3</td>
<td>79.8</td>
</tr>
</tbody>
</table>

4.2.2 Effect of Concrete Strength and Longitudinal Ratio

The effect of modifying concrete strength is intrinsically related to the longitudinal reinforcement ratio. For example, in an under-reinforced beam, increasing the concrete strength may not improve ultimate strength substantially due to overall beam strength being governed by reinforcement strength instead of concrete strength. However, for an over-reinforced beam, improving the concrete strength could greatly improve performance. For this reason, it is advantageous to study these two variables and their effects simultaneously. The following analysis discusses the first peak load and deflection only; confinement effects are ignored in this section.
Figure 4.22a, b, and c show the effect of concrete strength and longitudinal reinforcement ratio on the first peak (unconfined) moment resistance for beams reinforced with Bars A, B and C, respectively. Longitudinal reinforcement ratio is plotted on the horizontal axis, concrete compressive strength on the vertical axis, and first peak moment plotted as a colour contour graph. Blue colour indicates low peak moment resistance (relatively weak beams, ~50 kNm) and red indicates high peak moment resistance (relatively strong beams, ~160 kNm). Note that the experimental results are displayed as black dots, with the interior points being linearly interpolated between these results. For beams with duplicate specimens (40-C-1.0 and 40-C-2.0), the average experimental moment resistance is shown.

Figure 4.22: Effect of longitudinal reinforcement ratio and concrete strength on first peak load
Investigation of Figure 4.22b reveals the effect concrete strength has on the performance of under-reinforced beams, specifically by comparing the results of 40-A-0.5 and 80-A-0.5. These beams are represented as the left-most points. At the lowest reinforcement ratio (0.5%), concrete strength does not affect overall strength. This is represented by the constant blue colour at this reinforcement ratio. Doubling the concrete strength of this beam, from 41.4 MPa to 80.9 MPa, resulted in an increase in moment capacity of 1.06 times (88.6 kN to 94.0 kN). This relatively small increase in moment is due to the fact that the strength of under-reinforced beams is mainly governed by the reinforcement strength and not concrete strength.

As the reinforcement ratio increases, the contour colour bars in Figure 4.22b begin to bend (i.e. at a reinforcement ratio of ~0.5%, moment resistance is approximately constant for all concrete strengths, while at higher reinforcement ratios moment resistance varies with concrete strength). This pattern reflects that increasing concrete compressive strengths is most effective at high reinforcement ratios. The converse is also true: increasing reinforcement ratio is most effective at high concrete strengths. This effect can also be seen in Table 4.2.

Table 4.2: Effect of concrete strength on first peak load of over-reinforced beams

<table>
<thead>
<tr>
<th>Specimen</th>
<th>40 MPa</th>
<th>80 MPa</th>
<th>( \frac{P_{HSC}}{P_{NSC}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX-A-1.0</td>
<td>140.3</td>
<td>200.5</td>
<td>1.43</td>
</tr>
<tr>
<td>XX-B-1.0</td>
<td>155.8</td>
<td>204.2</td>
<td>1.31</td>
</tr>
<tr>
<td>XX-B-2.0</td>
<td>180.2</td>
<td>251.2</td>
<td>1.39</td>
</tr>
<tr>
<td>XX-C-1.0</td>
<td>152.0</td>
<td>193.7</td>
<td>1.27</td>
</tr>
<tr>
<td>XX-C-2.0</td>
<td>159.3</td>
<td>256.3</td>
<td>1.61</td>
</tr>
</tbody>
</table>

On average, the peak load of high strength concrete over-reinforced beams is 1.40 times that of their normal strength counterparts. The corresponding ratio for under-reinforced beams (40-A-0.5 and 80-A-0.5) is 1.06. This is expected because the concrete strength governs failure for over-reinforced beams.

The consequences of these results to a designer are as follows. If designing an under-reinforced beam with a given cross-section and GFRP bar type, increasing beam strength can only be achieved efficiently by increasing reinforcement ratio. For over-
reinforced beams, the most effective sections involve balancing the reinforcement ratio and concrete strength. It is not efficient to use excessive amounts of reinforcement if the concrete strength cannot balance the tensile forces. Alternatively, high strength concrete is not useful without also increasing reinforcement ratio.

4.2.3 Effect of GFRP Bar Type

Table 4.3 lists the first peak load and deflection for the specimen, organized in such a way to allow comparison between similar specimen in which the only variable is the bar type.

Table 4.3: Effect of GFRP bar type on first peak load and first peak deflection

<table>
<thead>
<tr>
<th>Specimen</th>
<th>First Peak Load (kN)</th>
<th>First Peak Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>40-X-0.5</td>
<td>88.6</td>
<td>-</td>
</tr>
<tr>
<td>40-X-1.0</td>
<td>140.3</td>
<td>155.8</td>
</tr>
<tr>
<td>40-X-2.0</td>
<td>-</td>
<td>180.2</td>
</tr>
<tr>
<td>80-X-0.5</td>
<td>94.0</td>
<td>-</td>
</tr>
<tr>
<td>80-X-1.0</td>
<td>200.5</td>
<td>204.2</td>
</tr>
<tr>
<td>80-X-2.0</td>
<td>234.8</td>
<td>251.2</td>
</tr>
</tbody>
</table>

This data is presented graphically below.

Figure 4.23: Effect of GFRP manufacturer on first peak load
By comparing the results from 80-A-0.5 and 80-C-0.5, the effect of reinforcement properties on overall strength in under-reinforced beams can be investigated. The failure load of 80-C-0.5 was 1.30 times that of 80-A-0.5 (122.0 kN and 94.0 kN, respectively). This roughly corresponds to the failure strain of Bar C being 1.38 times that of Bar A (2.07% and 1.50%, respectively). It should be noted that Bars C had about 15% to 20% higher stiffness than Bars A. This was reflected by an increased stiffness of the beam containing Bars C. 80-C-0.5 also exhibited 1.24 times the ultimate deflection of 80-A-0.5 (131.2 mm vs. 106.0 mm). For visual comparisons, refer to Figure 4.17. These results indicate that Bar C performs better than Bar A in under-reinforced conditions.

Modifying the mechanical properties of the reinforcement does not affect over-reinforced beams as significantly as under-reinforced beams. As stated above, under-reinforced beams reinforced with Bar C can sustain loads up to 1.30 times greater than a similar beam reinforced with Bar A (80-C-0.5 vs. 80-A-0.5). For similar over-reinforced beams 80-C-2.0 and 80-A-2.0, however, this ratio drops to 1.09 (256.3 kN to 234.8 kN respectively). Comparing these beams at 1% reinforcement ratio shows this strength ratio to be 0.97 (193.7 kN to 200.5 kN). Over-reinforced beams reinforced with Bar C showed less deflection than their Bar A counterpart (reinforced at 1%: 99.7 mm vs. 109.0 mm respectively, and at 2%: 79.8 mm vs. 88 mm).
Comparisons between beams reinforced with Bar B and Bar C reveal similar performance, in general. For normal strength concrete, beams with 1% reinforcement ratio using Bars B and C displayed almost equal strength but at 2% reinforcement ratio, beam with Bar B showed about 13% higher strength than that with Bar C. Strengths for high strength concrete beams are very similar for both types of bars. The deflection values at first peak also show similar results for the two types of bars.

**Confinement**

It is understood that columns with closely spaced ties result in improved confinement performance than similar columns with large spacing between ties. Based on this premise, it is reasonable to expect beams with closely spaced stirrups to show a significantly higher confinement effect compared to beams with large stirrup spacing. The impact stirrup spacing has on confinement can be seen in Figure 4.25. This figure shows three similarly reinforced beams (80-A-2.0, 80-B-2.0, and 80-C-2.0), with the exception of stirrup spacing. Stirrups in 80-A-2.0 and 80-B-2.0 were spaced at 75 mm, and 80-C-2.0 had stirrups spaced at 150 mm.

![Figure 4.25: Load vs. displacement graph, illustrating effect of stirrup spacing on confinement](Image)
Figure 4.25 shows that the pre-peak behaviour is similar within this group except the higher stiffness displayed by beams containing higher stiffness bars. Their post-peak behaviour, however, is significantly different. Specimen 80-C-2.0 showed a larger drop in load and a smaller increase in load associated with confinement than either 80-A-2.0 or 80-B-2.0 due to its much larger stirrup spacing (150 mm vs. 75 mm). Obviously, smaller stirrup spacing is beneficial to post-first peak performance of the beams.

Since the top longitudinal reinforcing bars were instrumented with strain gauges, it is possible to compare the confined concrete crushing strains of certain beams. These results, for the eleven beams that experienced confinement of concrete, are listed in Table 4.4. Note that, due to imperfect bond between reinforcing bars and concrete, the crushing strain of the confined concrete may be less than the strain in the top reinforcement at the time of concrete crushing.

Table 4.4: Top reinforcing bar longitudinal strain at confined concrete crushing

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Stirrup Spacing (mm)</th>
<th>Transverse Reinforcement Ratio (%)</th>
<th>FRP Strain (mm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-A-1.0</td>
<td>100</td>
<td>0.713</td>
<td>-7.5</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>100</td>
<td>0.713</td>
<td>-9.0</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>75</td>
<td>0.951</td>
<td>-11.0</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>200</td>
<td>0.565</td>
<td>-8.9</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>200</td>
<td>0.565</td>
<td>-12.0</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>200</td>
<td>0.565</td>
<td>-7.6</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>200</td>
<td>0.565</td>
<td>-15.0</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>200</td>
<td>0.565</td>
<td>-11.0</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>75</td>
<td>0.951</td>
<td>-10.0</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>75</td>
<td>0.951</td>
<td>-6.0</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>150</td>
<td>0.753</td>
<td>-7.2</td>
</tr>
</tbody>
</table>

The results in Table 4.4 do not indicate a strong trend. It is important to note, however, that the crushing strain in all the cases is significantly greater than that assumed by concrete codes for unconfined concrete (such as -3.5 mm/m for CSA S806-12)
4.3 Size Effect

Coinciding with this test series was another designed to investigate the shear behaviour of GFRP reinforced concrete beams, known as the large beam test series or JS series. The JS series involved testing 24 large beams which were 400 mm wide, 650 mm high and 3620 mm long. Note that these beams were twice as wide and twice as deep, but had the same length as those presented here (referred to in this section as the small beam test series).

A goal of designing these two beam sizes was to determine the size effect, if any, on flexural behavior of beams. It is possible to determine this size effect by comparing the two most similar beams (one from each series). The properties of these two beams can be seen in Table 4.5. The two beams have similar properties outside of their width and height which makes them good candidates for determination of size effect.

Table 4.5: Properties of small beam 40-C-1.03 and similar large beam DJC40-50B

<table>
<thead>
<tr>
<th>Test Series:</th>
<th>Small</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Designation:</td>
<td>40-C-1.03</td>
<td>JSC40-50B</td>
</tr>
<tr>
<td>Width (mm):</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>Height (mm):</td>
<td>325</td>
<td>650</td>
</tr>
<tr>
<td>Length (mm):</td>
<td>3620</td>
<td>3620</td>
</tr>
<tr>
<td>Shear Span (mm):</td>
<td>1680</td>
<td>1680</td>
</tr>
<tr>
<td>Reinforcement Ratio:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal:</td>
<td>1.12%</td>
<td>1.00%</td>
</tr>
<tr>
<td>Transverse:</td>
<td>0.57%</td>
<td>0.50%</td>
</tr>
<tr>
<td>GFRP Bars:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal:</td>
<td>ComBAR 12mm</td>
<td>ComBAR 12mm</td>
</tr>
<tr>
<td>Transverse:</td>
<td>ComBAR 12mm</td>
<td>ComBAR 12mm</td>
</tr>
<tr>
<td>Stirrup Spacing (mm):</td>
<td>200</td>
<td>112.5</td>
</tr>
<tr>
<td>Concrete Compressive Strength (MPa):</td>
<td>41.4</td>
<td>40.0</td>
</tr>
</tbody>
</table>

By comparing gross moments of inertia, the large beam series should be approximately 16 times stiffer than an equivalent small beam ($\frac{l_{\text{large}}}{l_{\text{small}}} \approx 16$). This can be experimentally verified by comparing the slope of the linear portions of their moment-curvature responses in Figure 4.26 which shows these responses, normalized in terms of first peak moment.
In Figure 4.26, \( M_{\text{Peak}} \), the moment corresponding to unconfined crushing, equals 91.4 kNm for the small beam and 717.0 kNm for the large beam. The slope of the linear portion equals 1.682 MNm\(^2\)/rad for the small beam, and 26.973 MNm\(^2\)/rad for the large beam. The ratios of moment capacities and stiffness’s between the large and small beam are 7.85 and 16.04. This indicates that there is no size effect on the flexural strength and stiffness of the beams.

A probable effect from scaling sectional size on beam performance is related to confinement. As discussed above, concrete confined inside of the stirrups has capacity to experience greater strains and stresses than the unconfined cover. Assuming an equal concrete cover size (as with these two test series), the concrete core in the large series beams will be proportionally large to the overall area, compared to the small series beams. This can be seen in Figure 4.27 where for the same cover and stirrup size, the core area changes from approximately 44% to 72% of the overall beam area for the small beam and large beam, respectively.
Due to this change in ratio, it is reasonable to expect the larger beam to show better post-first peak response due to the fact that a smaller portion of section would be lost due to concrete spalling and a more significant confined effect would be present. By normalizing the load-deflection response to the first peak (unconfined) load and deflection, the confined response can be visually compared in Figure 4.28. The load-deflection behaviour is presented, as opposed to moment-curvature behaviour, due to difficulty determining curvature values after the first peak failure (due to damaged strain gauges and lost LED targets).
The two beam sizes show similar behaviour up to initial crushing. After this point, however, the two curves diverge. For the small beam, the second peak load does not reach the unconfined load. In addition to losing a smaller portion of the section due to cover spalling, confinement in the large beam allowed the second peak load to exceed the first peak load substantially. The ratio between first peak load and second peak load is 0.84 for the small beam and 1.24 for the large beam. The first to second peak deflection ratios are 1.46 for the small beam and 2.17 for the large beam. These results agree with the expectation that having a larger core to total area ratio would increase the benefit gained from confinement effects. Therefore, it is reasonable to conclude that confinement is influenced by size.
5. PROGRAM ZEPPELIN

5.1 Introduction

The quality of test results is greatly influenced by the accuracy of the instrumentation used to measure these test results. Strain gauges placed on reinforcing bars and concrete surfaces, potentiometers and LVDT’s placed at various locations around the specimen, and DEMEC mechanical gauges have given good quantitative data to numerically analyze test results; however, these devices have their shortcomings. Strain gauges can be damaged during construction, and can reach their maximum strain before failure. Potentiometers can be bent, misaligned, or incorrectly calibrated. DEMEC gauges can only be measured at pauses in tests and are susceptible to human error. For these reasons, a three-dimensional positioning camera was used in conjunction with mechanical instrumentation in this test series.

The three-dimensional positioning camera measures the real time three dimensional positions of multiple LED targets. By placing these targets on the surface of a beam in a regular grid, as seen in Figure 3.14, it is possible to measure their relative displacements throughout a test. Analysis can be performed on these displacements to determine a full strain state on each individual element at any time during a test. This strain state can reveal significant information, including: strain in the longitudinal and transverse directions, shear strains, principle strains, and principle angle. By assuming a concrete stress-strain relationship (such as Hognestad’s parabola), a theoretical stress-state can be calculated.

The three dimensional camera takes three positional measurements (x, y, and z) per LED target at a rate of 16 Hz. Because of the inherent length of tests, a large volume of data is generated which requires post-processing. This post-processing lends itself well to being performed by computer programs.

The positioning camera measures the position of the LED targets in local coordinates, relative to the camera. The first stage of post-processing involves rotating and translating these local positions to a global coordinate system. For beams, it is convenient to define this coordinate system in the following manner: the x-axis
corresponding to the longitudinal direction, the y-axis corresponding to the transverse
direction, and the z-axis corresponding to the out-of-plane direction. A computer program
was written in the programming language Python, named RotTrans, which performs the
required matrix rotations and translations. This program also reduces the number of
readings from sixteen per second to one per second to reduce the quantity of data and
outputs the results in a text file.

A second program, named Zeppelin, was written to further analyze this data and
provide user-friendly graphical output, as well as numerical output. Zeppelin was written
in Python, using the following libraries: numpy, wxPython, and matplotlib. By using
Zeppelin, a user can visually investigate the behaviour of any individual element within
an LED grid, and determine if any LED’s were obstructed or otherwise deemed unusable.

Zeppelin requires input from a comma delimited text file (*.csv), such as the
output file of RotTrans. This file must have $3 + 3n$ columns, where $n$ is the number of
LED’s. The first three columns correspond to time, load, and displacement, respectively;
the next $3n$ columns correspond to the global x, y, and z coordinates of all LED’s. The
LED coordinates must be placed in the correct order, with the upper left LED being
number 1, and the numbers incrementing downwards first, and then to the right, as seen
in Figure 5.1. For this reason, it is recommended that Zeppelin only be used for beam
tests, although it may be possible to revise and utilize this program with other structural
elements (such as columns or panel elements).

![Figure 5.1: Required LED Numerical Designation for Zeppelin](image)

Zeppelin, using the LED target coordinates, calculates and visually displays
various properties of the concrete elements within the LED grid.
5.2 Interface

Upon running Zeppelin, the user is prompted to input the following data: number of LED’s in the x direction (7, in the example shown in Figure 5.1), the number of LED’s in the y direction (3, in the example), concrete strength (in MPa), and concrete peak strain (in millistrain). This prompt screen is shown in Figure 5.2.

![Initial Prompt Screen for Zeppelin](image)

Figure 5.2: Initial Prompt Screen for Zeppelin

After receiving the above input data from the user, the program calculates output data, stores it in a separate comma delimited text file (OutputData.csv), and opens the main graphical user interface window. This window can be seen in Figure 5.3.
The above window displays information about the concrete element at any given load stage. The output can be divided into 7 main areas, described below.

**Load Stage Slider**

A slider bar is visible on the left side of the screen. This bar, known as the load stage slider, controls the load stage which the other visual elements display. The increment of load stage depends on the test setup; because this test series used a displacement-controlled actuator, the load stage slider increments displacement, from 0 mm to ultimate displacement. The current applied load and applied displacement is displayed in the lower right corner of the screen.

**Visual Output Buttons**

The top of the screen displays 13 buttons, seen in Figure 5.4.
These buttons modify which property is currently being displayed on the contour graph and numerical output grid. The properties which can be selected are (from left to right): element top longitudinal strain \( \varepsilon_x^{\text{top}} \), element middle longitudinal strain \( \varepsilon_x^{\text{mid}} \), element bottom longitudinal strain \( \varepsilon_x^{\text{bot}} \), element average transverse strain \( \varepsilon_y \), curvature \( \varphi \), average horizontal displacement \( \Delta_x \), average vertical displacement \( \Delta_y \), shear strain \( \gamma_{xy} \), principle tensile strain \( \varepsilon_1 \), principle compressive strain \( \varepsilon_2 \), principle angle \( \theta \), principle tensile stress \( f_1 \), and principle compressive stress \( f_2 \). The calculations behind these properties are described in Chapter 5.3.

**Contour and Numerical Element Output**

Figure 5.5 below shows the contour and numerical element output of the requested property.

Displaying element top horizontal strain (m/m)

Contour Graph

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2812e-03</td>
<td>3.5175e-03</td>
<td>5.6789e-03</td>
<td>5.0008e-03</td>
<td>6.2277e-03</td>
<td>2.6032e-03</td>
</tr>
<tr>
<td>-2.0325e-03</td>
<td>-1.8828e-03</td>
<td>-1.4610e-03</td>
<td>-1.1408e-03</td>
<td>-1.7357e-03</td>
<td>-1.4826e-03</td>
</tr>
</tbody>
</table>

Figure 5.5: Zeppelin Contour and Numerical Element Output
These visual elements display, for the chosen load stage, the requested property of every element. In Figure 5.5, the property being displayed is element top strain. Using the contour graph, it is easy to see that the top row of elements is experiencing negative strain while the bottom row of elements is under positive strain. This indicates the top elements are under compression, while the bottom elements are in tension, as expected in a beam. This contour graph easily allows relative comparison between elements. For exact results, the user can view the numerical output results, directly below the contour graph.

**LED Positioning Graph**

Figure 5.6 shows the LED Position Graph. This graph shows the two-dimensional position of the LED’s at the given load stage in red, with the original position of the LED’s in blue for comparison.

For this example, Figure 5.6 indicates that this beam was loaded symmetrically about the center line of LED’s, as the center line of LED’s displaced downwards further than the outer lines. This figure also displays when LED’s lose line of sight with the camera.

**Mohr’s Circle of Strain and Stress**

Figure 5.7 below shows the Mohr’s circle of strain for any individual element at the given load stage.
The user can choose which element is being represented by the displayed Mohr’s circle by using the dropdown box above the Mohr’s circle of strain. For this example, the user is looking at element 5 out of 12, which represents a top element (see Figure 5.8). The blue line represents the strain state in the longitudinal and transverse direction. This element, because the blue line is close to the horizontal axis, indicates axial strains close to the principle direction with low shear strains. This is expected in this example because this element is in the pure flexure zone of the four-point bending set-up. The Mohr’s circle of stress indicates that this element is undergoing 11 MPa of compression and almost 1 MPa of tension. The equations to determine the Mohr’s circles are explained in Section 5.3.

Figure 5.8: Element numerical order
Load-Deflection Plot

The load-deflection plot for the beam being examined is shown in Figure 5.9, with load along the y-axis and machine deflection along the x-axis. The state of the current load stage is indicated by the red circle.

Strain and Stress Profiles

Figure 5.10 shows strain and stress profiles for a given column of elements, which is selected by using the dropdown box. This example indicates that plain sections remain plane, due to the linear strain profile. The stress profile exhibits parabolic behaviour, because the current model for the concrete stress-strain relationship is Hognestad’s parabola. A further explanation of these equations can be seen in Section 5.3.
5.3 **Equations**

The following equations were used to calculate the properties discussed above. In this section, the equations will be used for a sample element with the following corner coordinates: top left, \((x_a, y_a)\); bottom left, \((x_b, y_b)\); top right, \((x_c, y_c)\); bottom right \((x_d, y_d)\). Any value with a subscript 1 indicates the original (baseline) value, while a subscript 2 indicates the value at the desired load stage. This can be seen in Figure 5.11.

![Figure 5.11: Node coordinate system used in Zeppelin equations](image)

**Top horizontal strain:**

\[
\varepsilon_{x,\text{top}} = \frac{\Delta L_x}{L_x} = \frac{(x_c - x_a)_2 - (x_c - x_a)_1}{(x_c - x_a)_1} \tag{5.1}
\]

The middle, bottom and transverse strains were calculated in a similar fashion.

**Curvature:**

\[
\varphi = \frac{\varepsilon_{x,\text{top}} - \varepsilon_{x,\text{bottom}}}{L_y} = \frac{\varepsilon_{x,\text{top}} - \varepsilon_{x,\text{bottom}}}{(y_a - y_b + y_c - y_d)_1/2} \tag{5.2}
\]

**Average x and y deflections:**

\[
\Delta_x = \frac{(x_a + x_b + x_c + x_d)_2 - (x_a + x_b + x_c + x_d)_1}{4} \tag{5.3}
\]

\[
\Delta_y = \frac{(y_a + y_b + y_c + y_d)_2 - (y_a + y_b + y_c + y_d)_1}{4} \tag{5.4}
\]
Shear strain is calculated by taking the average of the change in angle of two opposite corners of an element. This average is done to account for the effects of curvature on the element. For one angle (the angle at B):

\[ \gamma_{xy} = \alpha_2 - \alpha_1 \]  
\[ \alpha = \frac{\pi}{2} - \tan^{-1}\left(\frac{x_a - x_b}{y_a - y_b}\right) - \tan^{-1}\left(\frac{y_a - y_b}{x_a - x_b}\right) \]

Upon determination of the axial strains and shear strains, it is possible to construct Mohr’s circle of strain, using the following equations.

\[ c = \frac{\varepsilon_x + \varepsilon_y}{2} \]  
\[ r = \sqrt{\left(\frac{\gamma_{xy}}{2}\right)^2 + (\varepsilon_x - c)^2} \]  
\[ \varepsilon_1 = c + r \]  
\[ \varepsilon_2 = c - r \]  
\[ 2\theta = \tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right) \]

Because the complete state of strain is known, it is possible to determine the complete state of stress by assuming a constitutive stress-strain relationship. In this case, the Modified Compression Field Theory (Collins and Vecchio, 1986) is utilized.

\[ f_1 = \frac{0.33\sqrt{f'_{c}}}{1 + \sqrt{500\varepsilon_1}} \]  
\[ f_{2,max} = \frac{f'_{c}}{0.8 + 170\varepsilon_1} \leq f'_{c} \]
5.4 Discussion

High tensile environments, such as those experienced when GFRP reaches its ultimate strain, can interfere with the quality of data collected by strain gauges. In this test series, several strain gauges including high-elongation gauges stopped recording data when strain reached a value greater than $20.0 \times 10^{-3}$. In these circumstances, the LED camera data is useful. The test series presented here was instrumented with LED targets approximately 10 mm below the level of the strain gauge on the bottom longitudinal reinforcing bar. By analyzing the longitudinal strain recorded by these two instruments, it is possible to investigate their accuracy. Figure 5.12 shows these different longitudinal strain readings at the level of the bottom reinforcement plotted against machine displacement.

![Figure 5.12: Comparison of Strain Gauge and LED Strain Measurements for 80-B-1.0](image)
Figure 5.12 indicates both measurement instruments record similar strains, particularly at moderate load levels. At high loads, the LED data shows slightly higher strains than the strain gauge data at the same displacement. This effect may be caused by local cracking or strain gauge simply is not working very well. At high loads, the beam is significantly cracked with large crack widths. Strain gauges measure local effects well. If a crack opens away from the location of the strain gauge, it will measure lower strain readings than average due to the stiffening effect of concrete. Similarly, if a crack opens directly on a strain gauge, it will measure higher strains than average. This effect, known as tension stiffening, is illustrated in Figure 5.13. In contrast, the LED camera system can measure global strains along a certain length accurately. The cracking behaviour also explains how, in Figure 5.5, the bottom elements exhibit varied tensile strains. When a crack opens between LED targets, that element is shown to have higher strain than un-cracked elements.

![Diagram of concrete beam with reinforcement and cracks]

Figure 5.13: Idealized variation of strain along reinforcement in cracked concrete

For this test series, Zeppelin was utilized primarily to construct moment-curvature diagrams for each beam. This was done for four-point tests by plotting the average curvature of all of the elements against to the moment. The results of these calculations, along with the moment-curvature diagram constructed using strain gauge data, for beam
Figure 5.14: Comparison of Zeppelin and Strain Gauge Moment-Curvature Diagrams for 80-B-1.0

Figure 5.14 indicates that the Zeppelin results show a slightly softer response than the strain gauge data. This can be attributed to the false local stiffening effect of the strain gauges described above. Because, in this case, the strain gauge measures lower strains (due to not being at a crack location), the resulting curvature is slightly lower. It is difficult to determine which instrument is more accurate without checking the location of cracks at each step. For the analysis performed on the moment curvature behaviour, described in Chapter 6, the data collected by the LED camera is used.

Zeppelin can also be used to analyze local effects. Beam 80-A-0.5 will be used as an example, shown in Figure 5.15.
This beam is under-reinforced, and showed significant cracking. A primary flexural crack can be seen slightly to the right of the second column of LED targets (highlighted in red). Upon inspection, it can be seen that this crack crosses the LED grid line and actually forms slightly to the left of the second column of targets. This means that, theoretically, the bottom left element (element 2) and its neighbouring element (element 4) will exhibit interesting local effects. The strain profile for these two element columns is shown below, at a load of 74.3 kN.
Figure 5.16a shows a typical strain profile; that is, plane sections remain plane. Figure 5.16b, however, shows a quite opposite response, due to the effect of the crack crossing LED gridlines. The middle layer of LED’s in element column 2 has a significant flexural crack running through it, so it displays high tensile strains. The bottom layer of the same column of LEDs does not have such a crack. This un-cracked element, therefore, shows relatively low tensile strains at its bottom face.

Similarly, the rightmost column of elements in Figure 5.15 (elements 7 and 8) shows a significant flexural crack (highlighted green). This crack eventually becomes the primary failure crack. By looking at the bottom element strain contour graph displayed in Zeppelin at the ultimate load stage (Figure 5.17), it is possible to see high tensile strain and the crack.

5.5 Future Improvements

At this stage in development, Zeppelin is optimized for beam tests; however, minor modifications could be made to enable its use for analysis of specimen such as columns and panel elements. Out-of-plane effects, for elements such as shell element tests, would require significant modifications.

Currently, Zeppelin calculates properties such as principle strains and principle angle using the strain at the middle of an element. Consequently, principle strains and stresses will be lower than at the extreme concrete surfaces. This could be mediated by using methods such as shape functions. The usability of the program could be
significantly improved by implementing Save and Open features, reducing or removing the flicker visible during load stage changes, implementing more sophisticated models, particularly with respect to concrete stress-strain behaviour, incorporating cracking behaviour, and incorporating reinforcement conditions.
6. RESULTS AND DISCUSSION

The following discussion focuses on comparing the experimental results presented in Chapter 4 with the behaviour predicted by using current design codes and guidelines. The documents discussed are design codes CSA S806-12 and CSA S6-06, and the report ACI 440.1R-06, with focus being placed on the moment-curvature, load-deflection, deformability, and cracking behaviour.

6.1 Moment-Curvature

CSA S806-12 has provisions to compute the moment-curvature behaviour of a beam similar to the steel reinforced concrete code, CSA A23.3-04. Both codes utilize a sectional approach, equivalent rectangular stress blocks, and the following assumptions:

a) Plane sections remain plane

b) A perfect bond exists between the concrete and the reinforcing bars

c) Failure of concrete occurs at a strain of $\varepsilon_c = -3.5 \times 10^{-3}$

The following iterative procedure was used to calculate the ultimate moment resistance and corresponding curvature of every beam.

**Over-Reinforced Beams**

i) Assume the beam is over-reinforced (beam will fail by crushing at top concrete surface)

ii) Set the top concrete strain equal to $\varepsilon_t = -3.5 \times 10^{-3}$ (failure strain)

iii) Calculate equivalent rectangular stress block factors

\[
\alpha_1 = 0.85 - 0.0015f'_c \\
\beta_1 = 0.97 - 0.0025f'_c
\]

iv) Estimate stress in tensile GFRP reinforcement, $f_f$ ($f_f < f_{ult}$)
v) From equilibrium ($C = T$), calculate the depth of neutral axis, $c$

$$c = \frac{A_f f_f}{\alpha_1 \beta_1 \beta f c'}$$  \hspace{1cm} (6.3)

vi) Using similar triangles of strain, calculate strain in tensile longitudinal reinforcement

$$\varepsilon_f = \varepsilon_t \times \left(\frac{c - d}{c}\right)$$  \hspace{1cm} (6.4)

vii) Calculate stress in tensile GFRP reinforcement

$$f_f = \varepsilon_f E_f$$  \hspace{1cm} (6.5)

viii) Check if calculated $f_f$ equals assumed $f_f$ from step iv. If stresses are equal, continue to step ix. If stresses are not equal, repeat process from step iv with a new assumed $f_f$ (Assume $f_f = \left(\frac{2f_{f,calc} + f_{f,est}}{3}\right)$)

ix) Check assumption that beam is over-reinforced by ensuring stress in GFRP is below ultimate ($f_f < f_{ult}$). If assumption is correct, continue to step x. If not, skip remaining steps and redo calculations using under-reinforced procedure (see below).

x) Calculate ultimate moment resistance and ultimate curvature

$$M_r = T \left(d - \frac{\beta_1 c}{2}\right) = A_f f_f \left(d - \frac{\beta_1 c}{2}\right)$$  \hspace{1cm} (6.6)

$$\varphi_{ult} = \varepsilon_t / c$$  \hspace{1cm} (6.7)

**Under-Reinforced Beams**

If the procedure above resulted in the GFRP reinforcement reaching its ultimate stress before concrete crushing ($f_f > f_{ult}$), the beam is defined as under-reinforced. The ultimate moment resistance of an under-reinforced beam is calculated using the following procedure:
RESULTS AND DISCUSSION

i) Set the strain in the GFRP tensile reinforcement equal to its ultimate strain 
\( \varepsilon_f = \varepsilon_{ult}, \quad f_f = f_{ult} \)

ii) Estimate top concrete strain \( (\varepsilon_t < -3.5 \times 10^{-3}) \)

iii) Calculate equivalent rectangular stress block factors (Collins and Mitchell, 1997)

\[
\beta_1 = \frac{4 - \frac{\varepsilon_t}{\varepsilon_c}}{6 - 2 \frac{\varepsilon_t}{\varepsilon_c}}
\]

\[
\alpha_1 = \frac{\frac{\varepsilon_t}{\varepsilon_c^2} - \frac{1}{3(\varepsilon_c^2)^2}}{\beta_1}
\]

iv) From equilibrium \( (C = T) \), calculate the depth of neutral axis, \( c \)

\[
c = \frac{A_f f_{ult}}{\alpha_1 \beta_1 b f_c}
\]

v) Using similar triangles of strain, calculate top concrete strain

\[
\varepsilon_t = \varepsilon_{ult} \times \left( \frac{c}{c - d} \right)
\]

vi) Check that calculated strain, \( \varepsilon_t \), from step v equals estimated strain from step ii. If it does, continue to step vii. If it does not, re-estimate \( \varepsilon_t \) and go back to step ii.

vii) Calculate ultimate moment resistance and curvature

\[
M_r = T \left( d - \frac{\beta_1 c}{2} \right) = A_f f_{ult} \left( d - \frac{\beta_1 c}{2} \right)
\]

\[
\varphi_{ult} = \varepsilon_t / c
\]

The moment resistance and curvature at points other than at ultimate (such as at a top concrete strain of \(-1.0 \times 10^{-3}\)) can be calculated by setting the top concrete strain to a
certain value, estimating the GFRP tensile reinforcement strain \((\varepsilon_f < \varepsilon_{ult})\), and using the under-reinforced equations above 6.8 to 6.13.

Table 6.1 compares the predicted results of the moment resistance found using the above procedure and the experimental data. Note that these results exclude any increase in ultimate moment resistance due to confinement effects.

Table 6.1: Ultimate Moment Resistance Predicted and Experimental Results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Predicted (kNm)</th>
<th>Experimental (kNm)</th>
<th>(\frac{M_{exp}}{M_{pred}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-A-0.5</td>
<td>58.0</td>
<td>54.5</td>
<td>0.940</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>90.9</td>
<td>86.3</td>
<td>0.949</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>91.9</td>
<td>95.8</td>
<td>1.042</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>101.9</td>
<td>110.8</td>
<td>1.087</td>
</tr>
<tr>
<td>40-C-1.0(_2)</td>
<td>95.0</td>
<td>93.1</td>
<td>0.980</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>95.0</td>
<td>93.5</td>
<td>0.984</td>
</tr>
<tr>
<td>40-C-2.0(_2)</td>
<td>107.7</td>
<td>111.2</td>
<td>1.033</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>107.7</td>
<td>98.0</td>
<td>0.910</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>57.8</td>
<td>57.8</td>
<td>0.999</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>122.4</td>
<td>123.3</td>
<td>1.007</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>127.6</td>
<td>144.4</td>
<td>1.132</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>123.9</td>
<td>125.6</td>
<td>1.014</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>139.4</td>
<td>154.5</td>
<td>1.108</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>87.0</td>
<td>75.0</td>
<td>0.862</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>128.3</td>
<td>119.1</td>
<td>0.928</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>147.9</td>
<td>157.6</td>
<td>1.066</td>
</tr>
</tbody>
</table>

Table 6.1 indicates that flexural resistance can be accurately predicted using simplified equations, for both over-reinforced and under-reinforced beams (average \(\frac{M_{exp}}{M_{pred}} = 1.003\) and standard deviation = 0.07). Concrete strength does not seem to greatly affect the accuracy of the predictions. These results agree with results from other authors, which suggest steel-reinforced concrete flexural theories can accurately predict ultimate moment resistance of GFRP-reinforced concrete beams. The accuracy of predictions can be visually verified below, in Figure 6.1. These figures show the experimental contour
maps (repeated from Chapter 3) with the theoretical contour maps beside them for comparison purposes. Note that the black line on the predicted graphs represents balanced condition. Any point to the left of this line represents an under-reinforced beam, and any point to the right of this line represents an over-reinforced beam.
RESULTS AND DISCUSSION

Figure 6.1: Moment resistance as a function of concrete strength and reinforcement ratio

- **a)** Colour scale
  - i) Experimental
  - ii) Predicted

- **b)** Bar A
  - i) Experimental
  - ii) Predicted

- **c)** Bar B
  - i) Experimental
  - ii) Predicted

- **d)** Bar C
  - i) Experimental
  - ii) Predicted
Figure 6.1 indicates that strength can accurately be predicted using code-adopted equations, at all reinforcement ratios and concrete strengths. The trends which are visible in the experimental response are also apparent in the predicted response (as seen by the similar shapes of contour lines).

6.2 Deflection

6.2.1 CSA S608-12 Procedure

The CSA S608-12 code deflection procedure states that deflection shall be computed by methods based on the integration of curvature at sections along the span. This can be a calculation intensive process subject to human error, so the code also provides closed form equations for common the loading and support conditions, found in Table 7 of S806-12. The equations applicable to the test setup presented here are listed below. For three point loading:

$$\Delta = \frac{PL^3}{48E_cI_{cr}} \left[ 1 - 8\eta \left( \frac{L_g}{L} \right)^3 \right]$$  \hspace{1cm} 6.14

and for four point loading:

$$\Delta = \frac{PL^3}{48E_cI_{cr}} \left[ 3 \left( \frac{a}{L} \right) - 4 \left( \frac{a}{L} \right)^3 - 8\eta \left( \frac{L_g}{L} \right)^3 \right]$$  \hspace{1cm} 6.15

where:

\(P\) = total applied load (N)

\(L\) = span length (mm)

\(L_g\) = uncracked length in half of the beam (mm)

\(E_c\) = modulus of elasticity of concrete (MPa)

\(a\) = shear span (mm)

\(I_g\) = gross moment of inertia (mm\(^4\))

\(I_{cr}\) = cracked moment of inertia (mm\(^4\))
The procedure described above was first proposed by Razaqpur et al. (2000). For a detailed description of the derivation, refer to Section 2.1 (Literature Review).

6.2.2 ACI 440.1R-06 Procedure

The ACI 440.1R-06 approach to deflection assumes the beam has a constant moment of inertia along the entire span, known as the effective moment of inertia ($l_e$). This assumption is obviously an approximation because the beam will be cracked near the center of the span and uncracked near the supports; however, this assumption has sufficed for predictions of deflections for steel-reinforced beams. The value of the effective moment of inertia is between the gross moment of inertia, $l_g$, and the cracked moment of inertia, $l_{cr}$, depending on how much of the beam has cracked. If the applied moment is lower than the cracking moment of the beam, the effective moment of inertia equals the gross moment of inertia. As the applied moment increases beyond the cracking moment to the ultimate moment, the effective moment of inertia decreases from the gross moment of inertia to the cracked moment of inertia. The mathematical procedure is as follows:

$$l_e = \left(\frac{M_{cr}}{M_a}\right)^3 \beta_d l_g + \left[1.0 - \left(\frac{M_{cr}}{M_a}\right)^3\right] l_{cr} \leq l_g$$

where:

- $M_{cr} =$ cracking moment
- $M_a =$ applied moment
- $\beta_d = \frac{1}{5} \left(\frac{\rho_f}{\rho_b}\right) \leq 1$

$\rho_f =$ longitudinal reinforcement ratio

$\rho_b =$ balanced reinforcement ratio
Equation 6.18 above describes what is known as the precracked stiffness reduction factor ($\beta_d$). This value accounts for the presumed low bond strength and tension stiffening effects of GFRP reinforcement, compared to steel reinforcement. Empirical data shows that, at higher reinforcement ratios, these differences become less apparent; thus, this reduction factor is proportional to reinforcement ratio. The balanced reinforcement ratio can be calculated using Equation 6.19.

$$\rho_b = 0.85\beta_1 \frac{f'_c E_f \epsilon_{cu}}{f_{ult} E_f \epsilon_{cu} + f_{ult}}$$

$f_{ult} = $ ultimate stress of GFRP

$\epsilon_{cu} = $ ultimate strain of concrete

It is important to note that for loads below cracking ($M_a < M_{cr}$), the first term of Equation 6.17 dominates the equation; thus the effective moment of inertia is approximately calculated as the gross moment of inertia ($I_e \approx I_g$). At high loads ($M_a \ll M_{cr}$), the second term dominates, and the equation approximates the beam as almost entirely cracked ($I_e \approx I_{cr}$). Note that the first term includes the stiffness reduction factor ($\beta_d$) discussed above, while the second term, which accounts for post-cracked stiffness, does not include such a factor.

Once the effective moment of inertia is calculated, midspan deflection can be calculated using solid mechanics formulae:

for three point bending,

$$\Delta = \frac{PL^3}{48E_c I_e}$$ 6.20

and for four point bending,

$$\Delta = \frac{Pa}{48E_c I_e} (3l^2 - 4a^2)$$ 6.21
6.2.3 ACI 440.1 Ballot Procedure

Recently, a novel approach to calculate deflection has been proposed by Bischoff et al. (2010) and is, at the time of this writing, in the ballot phase for the ACI 440.1. This procedure attempts to define a single moment of inertia to describe the beam (similar to ACI 440.1R-06 described above) based on integration of curvature along the member (similar to CSA S806-12). This new moment of inertia is known as the equivalent moment of inertia \( I_e' \). The authors provide closed form solutions for common arrangements of loads and supports so direct integration is not required. This procedure is described below.

\[
I_e' = \frac{I_{cr}}{1 - \gamma \beta \chi \eta (M_{cr}/M_a)} \tag{6.22}
\]

where:

\( I_e' \) = equivalent modulus of inertia (mm\(^4\))

\( \beta \) = tension stiffening factor for the cross section (equal to \( M_{cr}/M_a \))

\( \chi \) = shrinkage restraint factor to account for a low cracking moment (equal to 0.64)

\( \gamma \) = integration factor to account for variation in stiffness along the member length

\( \eta = 1 - I_{cr}/I_g \)

\( \gamma \) is listed below for three point bending and four point bending (the loading conditions used in this test series). For other arrangements of loads and supports, refer to Bischoff et al. (2010).

For three point bending:

\[
\gamma = 3 - 2(M_{cr}/M_a) \tag{6.23}
\]

and for four point bending:

\[
\gamma = \frac{3(a/L) - 16(M_{cr}/M_a)(a/L)^3 + 12(a/L)^3}{3(a/L) - 4(a/L)^3} \tag{6.24}
\]
6.2.4 Deflection Results

The deflection procedures listed above do not incorporate failure criteria. The calculations yield a certain deflection corresponding to a certain load. Ultimate deflection is thus calculated based on ultimate moment calculations. If the ultimate moment calculation is inaccurate, ultimate deflection will also be inaccurate. Due to this, it will not be appropriate to compare the experimental deflection to the predicted deflection calculated with predicted loads. Table 6.2 below compares the ultimate experimental deflection of the sixteen specimens with the analytical deflection corresponding to the experimental ultimate loads, using the three procedures described above.

Table 6.2: Comparison of Analytical and Experimental Deflections

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experimental Results</th>
<th>Analytical Results</th>
<th>ACI Ballot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta_{\text{exp}} ) (mm)</td>
<td>( \Delta_{\text{pred}} ) (mm)</td>
<td>( \Delta_{\text{pred}} ) (mm)</td>
</tr>
<tr>
<td>40-A-0.5</td>
<td>91.2</td>
<td>65.8</td>
<td>0.722</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>77.0</td>
<td>50.8</td>
<td>0.659</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>75.1</td>
<td>55.0</td>
<td>0.732</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>58.6</td>
<td>47.4</td>
<td>0.809</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>69.0</td>
<td>40.5</td>
<td>0.588</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>73.7</td>
<td>50.2</td>
<td>0.682</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>45.3</td>
<td>34.4</td>
<td>0.759</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>49.8</td>
<td>37.3</td>
<td>0.759</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>106.0</td>
<td>68.0</td>
<td>0.642</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>109.0</td>
<td>70.8</td>
<td>0.650</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>88.8</td>
<td>67.2</td>
<td>0.756</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>96.1</td>
<td>70.3</td>
<td>0.732</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>81.8</td>
<td>64.2</td>
<td>0.785</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>131.2</td>
<td>82.1</td>
<td>0.626</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>99.7</td>
<td>62.4</td>
<td>0.626</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>79.8</td>
<td>58.2</td>
<td>0.729</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td><strong>0.703</strong></td>
<td><strong>0.697</strong></td>
</tr>
</tbody>
</table>

The results above indicate that the described deflection prediction methods do not accurately predict ultimate deflection for this test series. All the three methods predict an ultimate deflection of approximately 70% of the experimental results. The standard deviations are 0.065, 0.066, and 0.072, respectively for the CSA S806-12, ACI440.1R-
06, and ACI Ballot approaches. There is also little difference between the results of the three methods, suggesting that there are factors or variables that have not been addressed.

6.2.5 Cracked Stiffness Reduction Factor

Table 6.2 shows that, for all sixteen beams, the current code provisions under predict ultimate midspan deflection. Under prediction of deflection is undesirable, particularly at service levels. The cause of this discrepancy can be seen in Figure 6.2 and Figure 6.3, which compare the predicted load-deflection behaviour to the experimental behaviour for two typical beams, 40-A-0.5 and 80-B-1.0. For comparison between the predicted and experimental load-deflection behaviour from the entire test series, refer to Appendix C.

![Figure 6.2: Experimental and predicted load-deflection behaviour of 40-A-0.5](image-url)
The above figures indicate that, beyond cracking, the three methods do not provide significantly different results. The figure also shows that the major cause of discrepancy between the predicted and experimental results can be attributed to an over-prediction of cracked stiffness (or the difference in slopes post-cracking).

Figure 6.4 shows a graphical representation of Branson’s original equation, without reduction factor. It can be seen that, at low loads, the gross moment of inertia controls beam stiffness. At higher loads, however, the beam behaviour is controlled by the cracked moment of inertia.
RESULTS AND DISCUSSION

If the cause of the under-prediction of ultimate deflection is due to the over-prediction of cracked stiffness, it is logical to implement a cracked stiffness reduction factor, similar to the pre-cracked stiffness reduction factor already implemented ($\beta_d$) in the ACI equations. The pre-cracked stiffness reduction factor is related to the reduced tension stiffening exhibited by FRP-reinforced members, as stated in ACI 440.1R-06. However, tension stiffening is a phenomena associated with cracked concrete; thus, a reduction factor implemented to account for low tension stiffening effects should be applied to the cracked stiffness term, or $I_{cr}$.

A cracked stiffness reduction factor ($\gamma_d$) has been proposed by Habeeb and Ashour (2008) for use in the ACI440-1R-06 effective moment of inertia equation (Equation 6.17). This is expressed in Equation 6.25:

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 \beta_d I_g + \left[ 1.0 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \gamma_d I_{cr} \leq I_g$$  \hspace{1cm} 6.25

where:

$M_{cr} = $ cracking moment

$M_a = $ applied moment

Figure 6.4: Contribution of $I_g$ and $I_{cr}$ to $I_e$ in Branson’s equation
RESULTS AND DISCUSSION

\[ \beta_d = \text{pre-cracked stiffness reduction factor} \]

\[ l_g = \text{gross moment of inertia} \]

\[ \gamma_d = \text{cracked stiffness reduction factor} \]

\[ l_{cr} = \text{cracked moment of inertia} \]

Habeeb and Ashour originally proposed using the \( \gamma_d \) factor for use in continuous beams, as they discovered the ACI provisions over-predicted post-cracked stiffness. While the beams presented here are simply supported, the same results were seen. By evaluating Equation 6.25 for the beams this test series, with \( \gamma_d = 0.6 \) (the value proposed by Habeeb and Ashour), it was found to over predict deflection. This means that, for this test series, \( \gamma_d \) should be between 0.6 and 1.0.

The individual cracked stiffness reduction factor, theoretically, is the ratio of the experimental cracked stiffness to the predicted cracked stiffness (as seen in Equation 6.26). The numerical results of the post-cracked stiffness, of either the experimental or predicted response, is the slope of the linear portion of the load-deflection graphs, as seen in Figure 6.2 and Figure 6.3 above, with units of kN/mm.

\[ \gamma_d = \frac{S_{exp}}{S_{ACI}} \quad 6.26 \]

where:

\[ S_{exp} = \text{post-cracked stiffness of the experimental results (kN/mm)} \]

\[ S_{ACI} = \text{post-cracked stiffness of the ACI predicted response (kN/mm)} \]

The results from Equation 6.26 for all the specimens are shown in Table 6.3.
Table 6.3: Experimental post-cracked stiffness reduction factors

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{exp}$</th>
<th>$S_{ACI}$</th>
<th>$\gamma_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-A-0.5</td>
<td>0.89</td>
<td>1.17</td>
<td>0.756</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>1.78</td>
<td>2.52</td>
<td>0.707</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>1.98</td>
<td>2.55</td>
<td>0.775</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>2.99</td>
<td>3.46</td>
<td>0.863</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>1.60</td>
<td>2.42</td>
<td>0.663</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>2.04</td>
<td>2.68</td>
<td>0.759</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>2.84</td>
<td>3.48</td>
<td>0.817</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>3.11</td>
<td>3.79</td>
<td>0.821</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>0.76</td>
<td>1.18</td>
<td>0.642</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>1.73</td>
<td>2.64</td>
<td>0.656</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>2.64</td>
<td>3.28</td>
<td>0.804</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>2.03</td>
<td>2.63</td>
<td>0.772</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>2.99</td>
<td>3.61</td>
<td>0.826</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>0.82</td>
<td>1.24</td>
<td>0.666</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>1.88</td>
<td>2.74</td>
<td>0.686</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>3.15</td>
<td>3.97</td>
<td>0.794</td>
</tr>
</tbody>
</table>

The average of $\gamma_d$ for the sixteen beams is 0.75, with a standard deviation of 0.071. This value is in the range of the literature values of 0.6 (Habeed and Ashour, 2008), 0.84 (Benmokrane et. al, 1996) and the current ACI value of 1.0. By plotting $\gamma_d$ against $\rho_f/\rho_{cr}$, it is possible to determine if the longitudinal reinforcement ratio of a beam has an influence on the post-cracking stiffness reduction value, as is the case with the pre-cracking stiffness reduction value. This plot is shown in Figure 6.5.
Figure 6.5: Cracked Stiffness Reduction Factor vs. Reinforcement Ratio

Figure 6.5 indicates that the cracked stiffness reduction factor increases as longitudinal reinforcement ratio increases, similar to the pre-cracking stiffness reduction factor. This agrees with the literature consensus that current deflection equations are more accurate for higher reinforced beams. By evaluating the line of best fit, the theoretical cracked reduction factor can be calculated using Equation 6.27.

\[ \gamma_d = 0.07 \ln \left( \frac{\rho_f}{\rho_b} \right) + 0.7 \]  

6.27

Figure 6.6 and Figure 6.7 below show the results of the same two beams from Figure 6.2 and Figure 6.3 above using the modified ACI 440.1R-06 equations (Equations 6.25 and 6.27). The original ACI 440.1R-06 prediction is shown as the dotted line for comparison purposes.
Figure 6.6: Experimental and Modified ACI Calculated Load-Deflection Behaviour of 40-A-0.5

Figure 6.7: Experimental and Modified ACI Calculated Load-Deflection Behaviour of 80-B-1.0
The above figures show that, at all load stages, the proposed modification more accurately estimated deflection than the current ACI provisions. The CSA S806-12 equations can be similarly modified, by multiplying the cracked moment of inertia \((I_{cr})\) by the same stiffness reduction factor \((\gamma_d)\) from Equation 6.27, as seen below.

\[
\Delta = \frac{PL^3}{48E_c\gamma_d I_{cr}} \left[ 3 \left( \frac{a}{L} \right) - 4 \left( \frac{a}{L} \right)^3 - 8 \eta \left( \frac{L_d}{L} \right)^3 \right] \quad 6.28
\]

\[
\eta = 1 - \frac{\gamma_d I_{cr}}{I_g} \quad 6.29
\]

In order to investigate this proposed modification beyond the test series presented here, a database of 80 beams (64 beams from other test series’ plus the 16 beams presented here) was investigated. The details of these beams can be found in Section 1.3. Ultimate midspan deflection was calculated using the five procedures described above (CSA S806-12, ACI 440.1R-06, ACI Ballot, proposed Modified ACI, and proposed Modified CSA). The results from this comparison can be found in Table 6.4. Note that, for this analysis, an un-conservative prediction is considered one in which \(\frac{\Delta_{pred}}{\Delta_{exp}} < 0.95\).

Table 6.4: Comparison of deflection prediction procedures with experimental results

<table>
<thead>
<tr>
<th>Procedure</th>
<th>(\Delta_{pred}/\Delta_{exp})</th>
<th>Standard Deviation</th>
<th>Unconservative Predictions</th>
<th>Percent Unconservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSA S806-12</td>
<td>0.789</td>
<td>0.178</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>ACI 440.1R-06</td>
<td>0.757</td>
<td>0.169</td>
<td>66</td>
<td>82.5</td>
</tr>
<tr>
<td>ACI Ballot</td>
<td>0.801</td>
<td>0.190</td>
<td>59</td>
<td>73.75</td>
</tr>
<tr>
<td>Proposed ACI Modification</td>
<td>1.063</td>
<td>0.248</td>
<td>27</td>
<td>33.75</td>
</tr>
<tr>
<td>Proposed CSA Modification</td>
<td>1.091</td>
<td>0.283</td>
<td>26</td>
<td>32.5</td>
</tr>
</tbody>
</table>

* Proposed modifications calculated with \(\gamma_d = 0.07 \ln \left( \frac{P_f}{P_b} \right) + 0.7\)

Table 6.4 above indicates that the proposed modification to both the ACI and CSA code significantly improves ultimate deflection predictions over the current procedures. The percent accuracy (defined as the ratio of predicted deflection to experimental deflection) improves from, on average, 80% to 108%. The number of unconservative predictions decreases from 62 to 27.
RESULTS AND DISCUSSION

Note that the above calibration was performed using the sixteen presented beams. Regression analysis performed on the database of 80 beams yielded an average value for the post-cracking stiffness reduction factor of 0.8. This value may be more suitable for design at this stage.

\[ \gamma_d = 0.8 \]

The results of applying this reduction factor to the proposed modified equations are shown below, in Table 6.5.

### Table 6.5: Comparison of experimental deflections to predicted results, using \( \gamma_d = 0.8 \)

<table>
<thead>
<tr>
<th>Procedure</th>
<th>( \frac{\Delta_{pred}}{\Delta_{exp}} )</th>
<th>Standard Deviation</th>
<th>Unconservative Predictions</th>
<th>Percent Unconservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSA S806-12</td>
<td>0.789</td>
<td>0.178</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>ACI 440.1R-06</td>
<td>0.757</td>
<td>0.169</td>
<td>66</td>
<td>82.5</td>
</tr>
<tr>
<td>ACI Ballot</td>
<td>0.801</td>
<td>0.190</td>
<td>59</td>
<td>73.75</td>
</tr>
<tr>
<td>Proposed ACI Modification</td>
<td>0.980</td>
<td>0.218</td>
<td>37</td>
<td>46.3</td>
</tr>
<tr>
<td>Proposed CSA Modification</td>
<td>1.000</td>
<td>0.238</td>
<td>46</td>
<td>46.3</td>
</tr>
</tbody>
</table>

* Proposed modifications calculated with \( \gamma_d = 0.8 \)

In conclusion, in deflection calculations for GFRP-reinforced beams, applying a post-cracked reduction factor equal to 0.8 to the cracked moment of inertia improves the accuracy of ultimate deflection predictions.

### 6.2.6 Response-2000 Analysis

Response-2000 is an analytical computer program developed at the University of Toronto (Bentz, 2000). This program has the capacity to perform two dimensional, member-level analysis on beams with custom material properties. These features make it an ideal tool to predict the load-deflection behaviour of GFRP-reinforced beams studied here.

**Input Parameters**

Response-2000 allows users to input custom stress-strain models for the constitutive materials. In addition, the program includes default models and values.
For concrete, the default model provided uses Popovics/Thorenfeldt/Collins (from Collins and Mitchell, 1997) for the base curve, Vecchio-Collins (1986) for compression softening, and Bentz (2000) for tension stiffening. This default model was used in the presented analysis. The required input values for the model are concrete cylinder strength, tensile strength, peak strain, aggregate size, and tension stiff factor. These values were experimentally determined with cylinders and prisms, as seen in Chapter 3, and presented in Table 6.6. Note that tensile strength was calculated via rupture strength (found using modulus of rupture prisms) and Equation 6.31.

\[ f_t = \frac{0.33}{0.6} \times f_r \]

Table 6.6: Concrete properties used in Response-2000 models

<table>
<thead>
<tr>
<th>Normal Strength</th>
<th>High Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder Strength (MPa)</td>
<td>41.4</td>
</tr>
<tr>
<td>Tension Strength (MPa)</td>
<td>1.96</td>
</tr>
<tr>
<td>Peak Strain (mm/m)</td>
<td>2.08</td>
</tr>
<tr>
<td>Max Aggregate Size (mm)</td>
<td>14</td>
</tr>
<tr>
<td>Tension Stiff Factor</td>
<td>1</td>
</tr>
</tbody>
</table>

The results of these models can be seen in Figure 6.8. Also shown in this figure is the experimental cylinder data for comparison purposes.

Figure 6.8: Concrete compressive stress-strain models used in Response-2000 analysis
Similarly, Response-2000 allows the user to input custom reinforcement stress-strain properties. To simulate the properties of GFRP bars, it is assumed this material behaves linearly elastic until failure. Thus, the required input values are ultimate strength, modulus of elasticity, and ultimate strain. Due to a known bug in Response-2000 (Bentz, 2000), FRP bars must be modeled with its strain hardening strain equal to ultimate strain, and its rupture strain twice that of ultimate strain. The input parameters used for analysis are presented in Table 3.2. Figure 6.9 shows the stress-strain relationship used for longitudinal and transverse reinforcement of type Bar C.

![Stress-strain relationship](image)

**Figure 6.9: Longitudinal and transverse Bar C stress-strain relationship used in Response-2000 models**

Beam geometry and reinforcement details were modeled as detailed in Chapter 3. An example of the cross-section properties (for beam 40-B-2.0) can be seen in Figure 6.10.
RESULTS AND DISCUSSION

Figure 6.10: Cross-sectional geometry for analyzing 40-B-2.0 in Response-2000

Using the material properties and geometry described above, Response-2000 performs layered sectional analysis and numerical integration to calculate load-deflection behaviour.

Results

The following three figures show the Response-2000 predicted load-deflection curves for the two beams discussed in the previous section (40-A-0.5 and 80-B-1.0), as well as 40-C-2.0. For the load-deflection results from all 16 beams, refer to Appendix C.
Figure 6.12: Experimental and Response-2000 prediction load-deflection behaviour for 80-B-1.0

The above figures show Response-2000 predicts load-deflection values similar to those calculated using the code-adopted approaches described above. Table 6.7 below shows the predicted ultimate load resistance compared to the experimental results.
Table 6.7: Ultimate load predictions found using Response-2000

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experimental Results</th>
<th>Response-2000 Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{\text{exp}}$ (kN)</td>
<td>$P_{\text{pred}}$ (kN)</td>
</tr>
<tr>
<td>40-A-0.5</td>
<td>88.6</td>
<td>86.1</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>140.3</td>
<td>147.4</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>155.8</td>
<td>154.4</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>180.2</td>
<td>170.5</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>110.8</td>
<td>115.9</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>159.3</td>
<td>184.6</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>94.0</td>
<td>91.6</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>200.5</td>
<td>189.7</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>234.8</td>
<td>216.7</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>204.2</td>
<td>203.5</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>251.2</td>
<td>228.9</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>122.0</td>
<td>139.7</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>193.7</td>
<td>208.1</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>256.3</td>
<td>247.1</td>
</tr>
</tbody>
</table>

Average: $1.01$
Standard Deviation: $0.07$

Table 6.7 indicates that Response-2000 can accurately predict ultimate loads for GFRP reinforced concrete beams, with an average error of 1% and standard deviation of 7%.

The ultimate deflection predictions, compared with the experimental results, can be found below in Table 6.8. Note that these values incorporate the predicted ultimate load, so it is possible that errors propagate (i.e. if Response-2000 over-predicted strength, it is probable that it will over-predict deflection).
Table 6.8: Ultimate deflection predictions found using Response-2000

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experimental Results</th>
<th>Response-2000 Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δ_exp (mm)</td>
<td>Δ_pred (mm)</td>
</tr>
<tr>
<td>40-A-0.5</td>
<td>91.2</td>
<td>72.8</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>77.0</td>
<td>69.7</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>75.1</td>
<td>64.7</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>58.6</td>
<td>53.8</td>
</tr>
<tr>
<td>40-C-1.03</td>
<td>69.0</td>
<td>50.8</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>73.7</td>
<td>64.8</td>
</tr>
<tr>
<td>40-C-2.03</td>
<td>45.3</td>
<td>40.9</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>49.8</td>
<td>53.1</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>106.0</td>
<td>71.1</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>109.0</td>
<td>79.1</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>88.8</td>
<td>67.1</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>96.1</td>
<td>74.8</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>81.8</td>
<td>61.9</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>131.2</td>
<td>98.5</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>99.7</td>
<td>73.8</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>79.8</td>
<td>60.8</td>
</tr>
</tbody>
</table>

Average: 0.81  
Standard Deviation: 0.10

The above table indicates that Response-2000 predictions echo the results from the code-adopted predictions. In general, Response-2000 under predicts deflection. It is, however, more accurate than the other prediction methods (Δpred/Δexp = 0.81 for Response-2000, vs. 0.701, 0.697, or 0.684 for the other methods). A study of Table 6.8 shows that deflection predictions are more accurate for beams with high reinforcement ratio, and less accurate for under-reinforced beams. In addition, deflection predictions are more accurate for normal strength concrete beams.

In conclusion, Response-2000 is a powerful tool which can predict ultimate loads for GFRP-reinforced concrete beams accurately. Caution must be used if using this program to predict deflections; however, it is more accurate than the code-adopted deflection prediction procedures.
6.3 Deformability

6.3.1 Deformability Factors

Deformability factors are numerical values that attempt to quantify the safety of a GFRP-reinforced beam, similar to ductility factors in steel-reinforced beams. Conventional design dictates a minimum level of reserve strength, or a strength factor of safety. This factor, however, does not incorporate the advantageous post-yield behaviour of steel-reinforced beams. According to Jaeger, Tadros, and Mufti (1995), unlimited increase of curvature is more beneficial if the moment of resistance also goes up. Thus, an accurate quantification of safety must account for load resistances (moments, forces, or stresses) as well as member deformations (curvature, displacement, or strains). Deformability factors are a proposed solution to this problem. A beam, properly designed with adequate deformability, can meet serviceability requirements, but still have enough reserve strength and deflection to allow pre-emptive warning of failure.

Deformability factors account for this effect by comparing energy absorption at two different load levels. Energy absorption can defined in three ways: member-level (area under the load-deflection graph); sectional-level (area under the moment-curvature graph); and material-level (area under the stress-strain curve). In general, deformability factors compare the energy absorbed at ultimate with the energy absorbed at a specific limiting state (such as service level). Two deformability factors are discussed herein: J factor and Deformability Factor (DF).

J Factor

First proposed by Jaeger, Tadros, and Mufti (1995), the J factor was an initial attempt at defining deformability. The authors investigated sectional behaviour of GFRP and steel reinforced beams, and proposed a method to evaluate moment-curvature response. The result of their work was the J factor, a sectional-level deformability factor.

The authors recognized that performance factors must include curvature and suggested that a suitable safety factor, instead of the ratio of strength at ultimate to strength at service, is in fact the ratio of energy absorbed at ultimate to energy absorbed at service. To simplify calculations, the authors proposed multiplying the moment
resistance by its corresponding curvature. The J factor equation can be seen in Equation 6.32, with a graphical representation in Figure 6.14. Subscript ‘c’ indicates the moment and curvature at the limiting state, which the authors define as top concrete strain equaling -0.001, a value which approximately corresponds to the onset of non-linear behaviour of normal concrete.

\[ J = \frac{M_{\text{ult}}\varphi_{\text{ult}}}{M_c\varphi_c} > 4 \]  

In Figure 6.14, J equals the total shaded area divided by the dark shaded area.

**Deformability Factor**

The Deformability Factor (DF), first proposed in 2001 by Vijay and GangaRao, is similar to the J factor in several ways. Both factors are sectional deformability factors, which rely on the moment-curvature behaviour of a beam. There are several differences between how to determine these factors.

Firstly, DF compares the true energy absorbed by the beam section, unlike J. Thus, the area under the moment-curvature graph must be determined. This required more intensive calculations, including pre-cracking behaviour and full knowledge of the moment-curvature response.
Secondly, the DF defines its limiting state at a curvature of $\varphi = 0.005/d$, instead of a value of top strain equal to -0.001. This curvature was empirically determined to satisfy serviceability limits, including deflection and crack widths.

$$DF = \frac{\text{Area under moment} - \text{curvature curve at ultimate}}{\text{Area under moment} - \text{curvature curve at } \varphi = 0.005/d}$$  \hspace{1cm} 6.33$$

DF can be calculated by dividing the total shaded area in Figure 6.15 by the dark shaded area.

6.3.2 Deformability Calculations

The J and DF factors, being sectional-level methods, require knowledge of the full moment-curvature behaviour. To calculate this behaviour, the procedure described in Section 6.1 was used in combination with a computer program written in Python. This computer program logic can be seen in the flowchart in Figure 6.16. It calculates full moment-curvature response and deformability factors of 60 beams, with given material properties and geometry ($E_f$, $f_{ult}$, $f_c'$, $b$, $h$, $d$ and $E_c$), and a varying reinforcement ratio. The reinforcement ratio varies from $0.1\rho_b < \rho_f < 6.0\rho_b$ in increments of $0.1\rho_b$. 

Figure 6.15: Graphical definition of DF
Figure 6.16: Logic flowchart to calculate deformability factors

Using this program, it becomes simple to isolate controllable variables available to a design engineer which may influence deformability, such as beam depth, GFRP properties, or concrete strength. The results are discussed in the following section.

Experimental J and DF factors were calculated using the moment-curvature response determined from Zeppelin and strain gauges. The DF factor requires integration of the moment-curvature response. This was calculated by using trapezoidal approximation.
6.3.3 Deformability Results

Experimental results for the two deformability factors, calculated using LED data, are shown plotted against relative reinforcement ratio in Figure 6.17 and Figure 6.18. Relative reinforcement ratio \( (\rho_f/\rho_b) \) is plotted on the horizontal axis as it has a large effect on beam performance, and it is convenient when comparing the behaviour of under-reinforced and over-reinforced beams.

![Figure 6.17: Experimental J Results](image)

It can be seen in Figure 6.17 that all of the beams in this test series satisfied the J limit of 4. Also, in general, the high strength beams had significantly higher J values than the normal strength beams. While the under-reinforced beams tended to have lower J values than the over-reinforced beams, this trend is not obvious.
The DF experimental results indicate a stronger trend than the J results, as seen in Figure 6.18. As expected, beams with a reinforcement ratio less than balanced have a low DF. DF peaks at $\rho_f/\rho_b = 1$. This is explained by the fact that, for all of these beams, the limiting curvature of $0.005/d$ is close to constant. The ultimate curvature, however, is maximum for beams reinforced with a balanced reinforcement ratio because, at this condition, both the concrete and GFRP are at their maximum strains.

The data presented above indicates that deformability factors can have large deviation, and may be difficult to predict. For this reason, the computer program described in Section 6.3.2 was used to isolate which design parameters have an effect on deformability. First, a standard set of data was created. This was done by using properties similar to beam 40-B-1.0: $E_f = 57230$ MPa, $f_{ult} = 1000$ MPa, $f'_c = 40$ MPa, $\varepsilon'_c = -0.002$, $\varepsilon_{cu} = -0.0035$, $b = 200$ mm, $h = 325$ mm, and $d = 270$ mm. Reinforcement ratio varied from $0.1\rho_b < \rho_f < 6.0\rho_b$. Herein, these properties are referred to as standard, and any
variation from standard is described. The standard results are shown in Figure 6.19 with the term J plotted on the left vertical axis and DF plotted on the right vertical axis in order to keep a reasonable scale.

![Figure 6.19: Theoretical deformability factors of standard beam](image)

Figure 6.19 shows expected and desirable results for under-reinforced beams; extremely under-reinforced beams have low deformability factors, and as the reinforcement ratio approaches balanced, the beams exhibit more deformability. Beams with a reinforcement ratio greater than balanced display almost a constant J. In contrast, the DF for over-reinforced beams sharply drops for highly reinforced beams. A standard beam reinforced with greater than three times the amount of GFRP than that required by balanced conditions has a DF lower than a beam with 10% of the balanced reinforcement ratio. This drop is also seen in the experimental data (Figure 6.18)

The over-reinforced plateau seen for J can be explained by the linear-elastic behaviour of GFRP. The limiting stage for this factor in over-reinforced beams is defined as a constant fraction of peak concrete strain (~50%). Because ultimate concrete strain is
constant and the behaviour of GFRP is linear-elastic, the energy ratio between the ultimate conditions and limiting conditions used to calculate $J$ will also be close to constant, around 5.8.

The decline seen in the DF curve for over-reinforced beams can be explained by two factors:

i) Over-reinforced beams have a lower ultimate curvature than balanced condition beams, while the limiting curvature is kept constant at $\varphi_s = 0.005/d$. This reduces the $\varphi_{ult}/\varphi_s$ ratio.

ii) The corresponding moment at the limiting curvature is greater for over-reinforced beams than for balanced conditions. This also reduces the $M_{ult}/M_s$ ratio.

An analysis is presented below which compares the deformability curves discussed above with curves created by modifying the standard properties. The standard deformability curves are always displayed as solid lines, with the modified curves always displayed as dotted or dashed lines.

**Effect of Concrete Compressive Strength**

Figure 6.20 shows the theoretical deformability curves resulting from changing the concrete cylinder strength from 40 MPa to 80 MPa. Note that no other concrete properties, such as peak strain or ultimate strain, were modified. While this is not realistic, it is useful for purely theoretical examination. Results from inputting the experimental properties for concrete used in this test series are discussed further in this document.
Increasing concrete strength is shown to slightly improve DF, but does not have any effect on J. Note that this chart compares deformability factors at the same $\rho_f/\rho_b$. Increasing concrete strength also increases $\rho_b$.

**Effect of Peak Concrete Strain**

High strength concrete, in addition to its increased compressive strength, tends to display higher peak strains. The effect of modifying the peak concrete strain, such as seen in Figure 6.21, is shown in Figure 6.22. This analysis was performed for concrete with a compressive strength of 40 MPa. Note that $\varepsilon'_c/\varepsilon_{cu}$ was kept at a constant ratio of 2/3.5.
It can be seen in Figure 6.22 that varying the peak concrete strain significantly impacts J and DF.
When $\epsilon'_c = -0.001$, the J limit of greater than 4 is never satisfied, meaning it is impossible to design a deformable beam under this definition. This is due to the fact that, at limiting condition of top concrete strain equaling -0.001, the concrete has reached its peak stress. The strain in the concrete and the moment resistance are more than halfway to ultimate, which results in a J factor of less than 2. In contrast, increasing the peak concrete strain from $\epsilon'_c = -0.002$ to $\epsilon'_c = -0.003$ doubles the theoretical J factor, from ~6 to ~12.

DF can be said to have similar, albeit not as extreme, results. Decreasing from $\epsilon'_c = -0.002$ to $\epsilon'_c = -0.001$ reduced DF to around 75% of its original value. Increasing $\epsilon'_c$ to -0.003 increased DF to around 125% of its base value.

**Effect of Ultimate Concrete Strain**

Ultimate concrete strain is an unpredictable and variable property inherent in the nature of concrete production; however, ultimate strain also is a governing property in design codes. This can be seen by comparing the prescribed ultimate strain in CSA S806-12 ($\epsilon_{cu} = -0.0035$) and ACI 440.1-R06 ($\epsilon_{cu} = -0.003$). In this test series, the ultimate concrete strain varied, in NSC, from -0.0023 to -0.0037, and in HSC, from -0.0034 to -0.0043 (as measured by the surface strain gauges placed on the top surface of the beam during testing). Because ultimate strain has an effect on ultimate curvature and ultimate moment resistance, its effect on deformability was investigated. The results can be seen below, in Figure 6.23.
Reducing the ultimate strain from $\varepsilon_{cu} = -0.0035$ to $\varepsilon_{cu} = -0.0022$ causes a significant reduction in both deformability factors at all reinforcement ratios. DF is reduced to approximately 80% of its original value, while the J values are reduced by up to about 38% of their standard values.

The reduction to 62% of standard value in J corresponds to the ultimate strain being reduced to 62% (from -0.0035 to -0.0022). This is expected, because, for this factor, the limiting condition remains unchanged, but the ultimate curvature and moment resistance are both reduced.

The DF factor is also lowered from its standard value but not to the same extent. Again, the limiting condition of $\varphi_s = 0.005/d$ is unaffected. The ultimate condition, however, is reduced.
Effect of Normal Strength Concrete vs. High Strength Concrete

Upon analyzing the effect that individual parameters of the concrete stress-strain relationship have on deformability, it is possible to understand the results of simulating realistic concrete properties. For this analysis, two concrete stress-strain relationships are used, corresponding to the NSC and HSC used in this test series. These properties are, for NSC: $f'_c = 41.4$ MPa, $\varepsilon'_c = -0.00208$, $\varepsilon_{cu} = -0.003$; and for HSC: $f'_c = 80.9$ MPa, $\varepsilon'_c = -0.0023$, $\varepsilon_{cu} = -0.0038$.

Figure 6.24: Theoretical J and DF factors using realistic concrete properties

Figure 6.24 indicates that high strength concrete beams exhibit higher deformability than low strength beams. This effect can be associated with the higher peak strains and ultimate strains in HSC ($\varepsilon'_c = 2.39 \times 10^{-3}$ for HSC vs. $2.08 \times 10^{-3}$ for NSC, $\varepsilon_{cu} = -3.8 \times 10^{-3}$ for HSC vs. $3.0 \times 10^{-3}$ for NSC). This effect can also be seen in Figure 6.21 and 6.22.
Effect of GFRP Ultimate Strength

It was shown above that concrete properties can significantly impact deformability. Similarly, it can be shown that the mechanical properties of the longitudinal GFRP reinforcement can affect a beam’s performance and its deformability.

In Figure 6.25, it is possible to see the effect of modifying the ultimate strength of longitudinal GFRP reinforcement from 1000 MPa to 2000 MPa (dashed lines) or 500 MPa (dotted lines) on J factor. Note that the balanced reinforcement ratio is largely affected by FRP ultimate strength. It is important to note that balanced reinforcement ratio is much lower for 2000 MPa bars than 500 MPa bars (0.12% vs. 1.56%).

The effect of GFRP bar strength on J is negligible. Considering balanced conditions, ultimate curvature is much greater for high strength bars (due to higher ultimate strains). However, the limiting curvature is also significantly higher due to the lower balanced reinforcement ratio (and less bar area). The increase in curvature at
ultimate and limiting conditions is proportional, and the ratio stays constant. In a similar fashion, the moment resistances are decreased for beams reinforced with high strength bars, due to less bar area. This effect is also proportional at ultimate and limiting cases. Thus, the J factor stays constant.

In contrast, DF factor is proportional to ultimate strength to a power of greater than 1; if ultimate strength is doubled, DF increases approximately threefold. In the DF equation, limiting curvature stays constant. The ultimate curvature, however, is significantly larger for high strength GFRP bars than low strength (due to higher ultimate strain). This effect increases DF as ultimate strength increases.

**Effect of GFRP Stiffness**

The modulus of elasticity of GFRP reinforcing bars, as with the stiffness of concrete, can impact deformability, as shown in Figure 6.26 below.

![Figure 6.26: Theoretical effect of GFRP modulus of elasticity on deformability](image-url)
Similar to the effect of GFRP ultimate strength on deformability, GFRP stiffness does not impact J significantly. DF, however, is inversely proportional to GFRP stiffness. Reducing the GFRP stiffness from ~60000 MPa to 30000 MPa almost triples DF. The opposite is true for increasing the modulus of elasticity. This is due to the GFRP ultimate strain being inversely proportional to modulus of elasticity.

While it has been shown that individual GFRP mechanical properties have significant effects on DF, these mechanical properties are related. For the GFRP bars used in this test series, stiffness is proportional to ultimate strength. Thus, it is logical to compare realistic bar types. Figure 6.27 shows the deformability results found by using properties from the three manufacturers’ smaller bars (#4 or 12M) used in this test series. These properties, determined experimentally, can be found in Table 3.2.

![Figure 6.27: Theoretical J and DF factors using realistic GFRP bar properties](image)

It can be seen in Figure 6.27 that J is theoretically not greatly influenced by bar type. Beams reinforced with Bar C show higher DF than those reinforced by Bar B; and Bar B shows higher DF than Bar A. This effect is proportional to the bars’ $f_{ult}/E_f$ ratio,
or ultimate strain. Bars which can sustain higher strains before failure theoretically perform better than bars with lower ultimate strains, according to the DF factor.

With the understanding of the effect each individual mechanical property of the constituent materials, it is possible to analyze the experimental results. These results are shown in Table 6.9.

Table 6.9: Predicted and experimental deformability factors

<table>
<thead>
<tr>
<th>Specimen</th>
<th>J</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J_{pred}</td>
<td>J_{exp}</td>
</tr>
<tr>
<td>40-A-0.5</td>
<td>3.98</td>
<td>6.95</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>5.39</td>
<td>7.75</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>5.38</td>
<td>11.44</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>5.35</td>
<td>8.81</td>
</tr>
<tr>
<td>40-C-1.03</td>
<td>5.40</td>
<td>7.32</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>5.40</td>
<td>7.77</td>
</tr>
<tr>
<td>40-C-2.03</td>
<td>5.35</td>
<td>4.12</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>5.35</td>
<td>5.09</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>2.21</td>
<td>4.36</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>6.13</td>
<td>15.99</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>7.59</td>
<td>14.42</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>7.60</td>
<td>13.53</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>7.57</td>
<td>15.42</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>4.15</td>
<td>14.78</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>7.58</td>
<td>10.14</td>
</tr>
<tr>
<td>80-C-2.0</td>
<td>7.54</td>
<td>10.93</td>
</tr>
<tr>
<td><strong>Average:</strong> &amp; 0.645 &amp; 1.111</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard Deviation:</strong> &amp; 0.248 &amp; 0.478</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.9 indicates that the J factor equations under-predicted the experimental values, while DF factor is over predicted. Both factors show large standard deviations (0.248 for J, 0.478 for DF). Note that data from beam 40-C-2.03 is an outlier data point for the J and DF factors. This may be due to inaccurate experimental data. The LED data for this beam was considered unusable for moment-curvature analysis, and only strain
gauge data was used to calculate deformability (the remainder of the beams used the average deformability between the strain gauge data and LED data). Ignoring this data point causes the average $DF_{\text{pred}}/DF_{\text{exp}}$ to be 1.001, with a standard deviation of 0.189. $J_{\text{pred}}/J_{\text{exp}}$ decreases to 0.601 with a standard deviation of 0.182. These ratios of predicted to experimental results are shown graphically below, plotted against reinforcement ratio.

Figure 6.28: Accuracy of deformability predictions vs. reinforcement ratio

Figure 6.28 shows that DF predictions are more accurate than the J predictions, as seen by DF being scattered around an accuracy ratio of ~1, and J being scattered around an accuracy ratio of ~0.6. It can also be seen from this figure that as reinforcement ratio increases, predictions for the J value tend to become more accurate.

Beam geometry (width, height, and effective depth) does not impact deformability factors.
6.3.4 Concluding Remarks on Deformability

**J Factor**

J factor, as expected, is largely affected by reinforcement ratio while under-reinforced. However, J factor reaches a maximum at balanced reinforcement ratio and stays almost constant in over-reinforced beams ($\rho_f \geq \rho_b$). Concrete strength, GFRP strength, GFRP stiffness, and beam geometry do not significantly impact J; however, it is greatly impacted by the concrete stress-strain curve.

The major advantage J offers over DF is the ease of calculation. Only two points on the moment-curvature curve need to be calculated, as opposed to the entire moment-curvature curve as for DF.

J factor, however, has several disadvantages when compared to DF. It does not reflect the effects of different variables. If a beam is over-reinforced, and the concrete stress-strain behaviour as described by CSA S806-12 is assumed, the beam will have a J of ~5.8. This suggests this factor is unnecessary in design of over-reinforced sections. The experimental results do not show a strong agreement with theoretical results, as seen in Table 6.9 and Figure 6.28.

**Deformability Factor (DF)**

DF factor is related to reinforcement ratio. As with J, DF yields low deformability results for under-reinforced beams, increasing with reinforcement ratio up to balanced. The analysis above, however, indicates that DF is maximized at balanced ratio, and above this ratio, DF is inversely proportional to reinforcement ratio. This effect penalizes highly reinforced beams. DF is influenced by concrete stress-strain properties (similar to J); however, DF is strongly influenced by GFRP mechanical properties, unlike the J Factor.

The decline of DF in over-reinforced beams is due to the assumption that service curvature equals 0.005/d rads/mm. This curvature is assumed to satisfy serviceability limits for crack widths and deflection. For a heavily over-reinforced beam, member strength may govern over serviceability. Over-reinforced beams have lower ultimate curvatures, which is the cause of the lower DF. However, the service curvature may also be lower than 0.005/d for these beams and it is not considered in DF calculations.
To calculate DF, the full moment-curvature diagram must be determined and integrated. This is calculation-intensive exercise which can be prone to error in design and may not be an acceptable approach for design codes. The original authors did not suggest a lower limit which provides acceptable deformability.

Theoretical DF values show better agreement with experimental values compared to J, as seen in Table 6.9 and Figure 6.28.

### 6.3.5 Deflection Deformability Factor

It is apparent from the shortcomings listed above for J and DF that there is a need for an accurate, easily implementable and simple method to determine deformability. Due to advances in prediction of load-deflection behaviour, it is possible to predict a member-level deformability factor. Such a deformability factor is defined as the ratio of the area under the load-deflection curve at ultimate to the same area at a limiting condition.

Calculating the area under the load-deflection curve involves integrating the assumed load-deflection response over the given load range. In this analysis, the area under the curve is defined as $A_d$. Thus, the deflection deformability factor ($D_d$) is given by Equation 6.34.

$$D_d = \frac{A_{d,u}}{A_{d,limit}} = \frac{\int_0^{\Delta_u} Pd\Delta}{\int_0^{\Delta_{limit}} Pd\Delta}$$

where:

- $A_{d,u} = \int_0^{\Delta_u} Pd\Delta = \text{area under load-deflection curve at ultimate conditions}$
- $A_{d,limit} = \int_0^{\Delta_{limit}} Pd\Delta = \text{area under load-deflection curve at limiting conditions}$
- $P = \text{applied load (N)}$
- $d\Delta = \text{integration increment of deflection (mm)}$

At this stage of analysis, assumptions must be made, specifically regarding the limiting load stage and the load-deflection relationship.
It is convenient to define the limiting load stage as the service condition for two reasons. Firstly, it is simple for the design engineer to calculate the service conditions, as they should already be known. Secondly, GFRP reinforced concrete beams are often governed by deflection criteria. Thus, any beam that is not governed by deflection will likely have higher deformability than a beam that is. Setting the limiting condition to service represents the expected conditions experienced by that beam.

For this analysis, the assumed load-deflection relationship is the proposed Modified CSA procedure. For more details on this relationship, refer to Section 6.2 (Deflection). This assumption offers several advantages. The resulting deformability factor can be easily integrated into the existing code, and yields simple, closed form solutions for the existing support conditions, found in Table 7 of CSA S806-12. An example is given in the following on the determination of $D_d$ for three-point loading. The proposed Modified CSA equation for the deflection of a three-point loaded beam is given by Equation 6.35.

$$\Delta = \frac{PL^3}{48E_c\gamma_d I_{cr}} \left[ 1 - 8\eta \left( \frac{L_g}{L} \right)^3 \right]$$

where:

$P = \text{total applied load (N)}$

$L = \text{span length (mm)}$

$E_c = \text{modulus of elasticity of concrete (MPa)}$

$I_{cr} = \text{cracked moment of inertia (mm}^4\text{)}$

$\gamma_d = \text{post-cracked stiffness reduction factor}$

$\eta = 1 - \gamma_d I_{cr} / I_g$

$L_g = \text{uncracked length in half of beam (equal to } L/2 \text{ for uncracked beam)}$

In order to integrate Equation 6.35, the discontinuity at cracking loads must be accounted for. It can be seen that, prior to cracking, the uncracked length $L_g$ equals half of
the total beam length. Post-cracking, the uncracked length is somewhere between zero and half of the beam length, dependent on total load. That is:

\[ M_a < M_{cr}, \quad L_g = \frac{L}{2} \]

\[ M_{cr} < M_a < M_{ult}, \quad L_g = \frac{M_{cr} L}{M_a} \times \frac{1}{2} \]

where:

\( M_a = \) applied moment (kNm)

\( M_{cr} = \) cracking moment (kNm)

\( M_{ult} = \) ultimate moment (kNm)

Note that the ratio between cracking moment and applied moment is the same as the ratio between cracking load and applied load \( \frac{M_{cr}}{M_a} = \frac{F_{cr}}{F_a} \).

Equation 6.34 is thus changed to Equation 6.36, assuming that, at service loads, the beam is cracked.

\[ D_d = \frac{A_{d,u}}{A_{d,s}} = \frac{\int_0^{c_r} P \Delta d + \int_{c_r}^{u} P \Delta d}{\int_0^{c_r} P \Delta d + \int_{c_r}^{s} P \Delta d} \]

where:

\( A_{d,s} = \) area under load-deflection curve at service conditions

\( \int_0^{c_r} P \Delta d = \) area under load-deflection curve at cracking

\( \int_{c_r}^{u} P \Delta d = \) area under load-deflection curve between cracking and ultimate

\( \int_{c_r}^{s} P \Delta d = \) area under load-deflection curve between cracking and service

Evaluating the numerator integral \( (A_{d,u}) \) yields Equation 6.37.
In Equation 6.37, the term outside the brackets is constant for any given beam. Thus, dividing the area under the load deflection curve at ultimate to the area at service (Equation 6.36) cancels this constant term. This, combined with defining \( \frac{M_{cr}}{M_u} = \beta_u \), changes Equation 6.37 into Equation 6.38.

\[
A_{d,u} = \frac{1}{\eta \beta_u^2 - 4 \beta_u + 3} \quad 6.38
\]

Thus, the deflection deformability factor for a beam under three-point loading can be evaluated using Equation 6.39.

\[
D_a = \frac{A_{d,u}}{A_{d,s}} = \frac{1}{\frac{\eta \beta_u^2 - 4 \beta_u + 3}{\eta \beta_s^2 - 4 \beta_s + 3}} \quad 6.39
\]

The above procedure was used for the beams presented in Table 7 of CSA S806-12, with the results shown in Table 6.10.
RESULTS AND DISCUSSION

Table 6.10: Closed form deflection deformability factors

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>$A_d$ (for $A_{d,u}$ use $\beta_u$, $A_{d,s}$ use $\beta_s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>$A_d = \frac{1}{\eta \beta^2} - 4 \beta + 3$</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>$A_d = \frac{1}{\eta \beta^2} \left[ 3 \left( \frac{L}{a} \right)^2 - 4 \right] - 32 \beta + 16 + \left( \frac{L}{a} \right)^3$</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>$A_d = \frac{48 \left( \frac{L}{L} \right)^4 - 64 \left( \frac{L}{L} \right)^3 + 8 \left( \frac{L}{L} \right)}{\beta^2} + \frac{5}{\eta \beta^2} - \frac{2 \sqrt{1 - \beta}}{\beta} - 3 \ln \left[ (1 + \sqrt{1 - \beta})^2 \right] - 1.5$</td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram 4" /></td>
<td>$A_d = \frac{1}{\eta \beta^2} - 4 \beta + 3$</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 5" /></td>
<td>$A_d = \frac{1}{\eta \beta^2} - 1 - 2 \ln \beta$</td>
</tr>
</tbody>
</table>

$p = \frac{M_{cr}}{M_u}$ or $\frac{p_{cr}}{p_u}$ or $q \frac{q_{cr}}{q_u}$ 

$\beta_u = \frac{M_{cr}}{M_u}$ or $\frac{p_{cr}}{p_u}$ or $q \frac{q_{cr}}{q_u}$ 

$\beta_s = \frac{M_{cr}}{M_s}$ or $\frac{p_{cr}}{p_s}$ or $q \frac{q_{cr}}{q_s}$ 

$\eta = 1 - \frac{\gamma a l_{cr}}{L_g}$ 

$L_g = \frac{1 - \sqrt{1 - \beta}}{2}$

It is apparent that, from Table 6.10, the deflection deformability factor is simple to calculate, with the exception of the simply-supported uniformly distributed load case. In this case, it is reasonable to make the assumption that $L_g \approx \frac{M_{cr}}{4M_s}$, assuming $M_a > 1.5M_{cr}$ where $M_a$ is the applied moment. The resulting integration constant is relatively simpler, as seen in Table 6.11.
Table 6.11: Closed form deflection deformability factors suitable for design

**Deflection Deformability Factor**

\[ D_d = \frac{A_{d,u}}{A_{d,s}} \]

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>( A_d ) (for ( A_{d,u} ) use ( \beta_u ), ( A_{d,s} ) use ( \beta_s ))</th>
</tr>
</thead>
</table>
| \( \begin{array}{c} 
\begin{array}{c}
\uparrow \quad P \\
L/2 \\
L/2 \\
\end{array} \\
\end{array} \) | \( A_d = \frac{1}{\eta \beta^2} - 4\beta + 3 \) |
| \( \begin{array}{c} 
\begin{array}{c}
\uparrow \quad P \\
\uparrow \quad P \\
L \\
\end{array} \\
\end{array} \) | \( A_d = \frac{1}{\eta \beta^2} \left[ 3 \left( \frac{L}{a} \right)^2 - 4 \right] - 32\beta + 16 + \left( \frac{L}{a} \right)^3 \) |
| \( \begin{array}{c} 
\begin{array}{c}
\uparrow \quad \text{q} \\
L \\
\end{array} \\
\end{array} \) | \( A_d = \frac{1}{\eta \beta^2} \left( \frac{4\beta}{5} - \frac{9\beta^2}{80} + \frac{109}{80} \right) \) |
| \( \begin{array}{c} 
\begin{array}{c} \\
\uparrow \quad P \\
L \\
\end{array} \\
\end{array} \) | \( A_d = \frac{1}{\eta \beta^2} - 4\beta + 3 \) |
| \( \begin{array}{c} 
\begin{array}{c}
\uparrow \quad \text{q} \\
L \\
\end{array} \\
\end{array} \) | \( A_d = \frac{1}{\eta \beta^2} - 1 - 2 \ln \beta \) |

\[ \beta_u = \frac{M_{cr}}{M_u} \text{ or } \frac{P_{cr}}{P_u} \text{ or } \frac{q_{cr}}{q_u} \quad \beta_s = \frac{M_{cr}}{M_s} \text{ or } \frac{P_{cr}}{P_s} \text{ or } \frac{q_{cr}}{q_s} \quad \eta = 1 - \frac{Y_d L_{cr}}{I_g} \]

Note that Table 6.11 can only be used when the beam is cracked at service conditions.
6.3.6 Discussion on Deflection Deformability

The deflection deformability factor presented above allows a design engineer to easily check the deformability of a beam with beam properties that have already been determined. No extra calculations are required. The loading and support conditions above are the same as those already listed in Table 7 of CSA S806-12. For loading conditions composed of a combination of the above loads, the design engineer can use their judgment to determine the worst case scenario and design accordingly.

To compare the predicted and experimental results, a limiting service condition must be assumed. For the comparison discussed herein, it is assumed that the beams presented are all deflection-critical, meaning their service conditions are governed by deflection. If the beams are assumed to be supporting or attached to non-structural elements not likely to be damaged by large deflections, the allowable deflection (from CSA S806-12) would be equal to $l_n/240$, or 14 mm for this test series.

Table 6.12 compares the experimental and predicted results of the proposed deflection deformability factor. The predicted results were calculated using the equations found in Table 6.11. The experimental results were calculated by using trapezoidal integration of the experimental load-midspan deflection results at ultimate load (corresponding to the first peak failure) and service load (corresponding to $\Delta = 14$ mm), found in Appendix C.
### Table 6.12: Experimental and predicted results of deflection deformability

<table>
<thead>
<tr>
<th>Beam</th>
<th>Experimental</th>
<th>Predicted</th>
<th>( \frac{D_{d,pred}}{D_{d,exp}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-C-1.0</td>
<td>17.14</td>
<td>14.37</td>
<td>0.839</td>
</tr>
<tr>
<td>40-C-1.0</td>
<td>7.91</td>
<td>7.40</td>
<td>0.935</td>
</tr>
<tr>
<td>40-C-2.0</td>
<td>9.51</td>
<td>8.61</td>
<td>0.905</td>
</tr>
<tr>
<td>40-A-0.5</td>
<td>21.91</td>
<td>15.66</td>
<td>0.715</td>
</tr>
<tr>
<td>40-A-1.0</td>
<td>18.22</td>
<td>15.09</td>
<td>0.829</td>
</tr>
<tr>
<td>40-B-1.0</td>
<td>17.26</td>
<td>17.08</td>
<td>0.990</td>
</tr>
<tr>
<td>40-B-2.0</td>
<td>12.79</td>
<td>14.10</td>
<td>1.102</td>
</tr>
<tr>
<td>80-C-0.5</td>
<td>35.57</td>
<td>16.81</td>
<td>0.473</td>
</tr>
<tr>
<td>80-C-1.0</td>
<td>23.92</td>
<td>17.85</td>
<td>0.746</td>
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<td>80-C-2.0</td>
<td>22.91</td>
<td>18.45</td>
<td>0.805</td>
</tr>
<tr>
<td>80-A-0.5</td>
<td>20.60</td>
<td>11.80</td>
<td>0.573</td>
</tr>
<tr>
<td>80-A-1.0</td>
<td>28.82</td>
<td>22.66</td>
<td>0.786</td>
</tr>
<tr>
<td>80-A-2.0</td>
<td>26.52</td>
<td>23.34</td>
<td>0.880</td>
</tr>
<tr>
<td>80-B-1.0</td>
<td>25.40</td>
<td>21.96</td>
<td>0.864</td>
</tr>
<tr>
<td>80-B-2.0</td>
<td>23.33</td>
<td>22.02</td>
<td>0.944</td>
</tr>
</tbody>
</table>

**Average:** 0.819  
**Standard Deviation:** 0.155

![Figure 6.29: Experimental and prediction deflection deformability factors vs. reinforcement ratio](image_url)
Figure 6.29 shows that, at reinforcement ratios above balanced, the deflection deformability factor can be predicted reasonably accurately using the equations found in Table 6.11. At low reinforcement ratios, deformability is significantly under-predicted. While this is conservative, it is not desirable. The source of this discrepancy may be that the assumed service limit of 14 mm is close to the cracking deflection for under-reinforced beams.

The results above show that as reinforcement ratio increases, deformability tends to decrease, similar to the DF factor. This is due both of these factors having a limiting condition controlled by serviceability. This pattern may not be typical of beams controlled by strength (such as service moment being a constant ratio of ultimate moment). It should be noted that beams with high reinforcement ratios have less ultimate deflection than beams with low reinforcement ratios; thus, 14 mm is closer to ultimate conditions than in beams with lower reinforcement ratios. There is an advantage in setting the deformability limit to service conditions, which represents the expected load conditions that will be placed on the beam.

In order for this deflection deformability to be implemented into design codes, there must be a limit which satisfactorily designed beams will pass, but unsafely designed beams will fail. It is recommended that GFRP reinforced concrete beams achieve at least the same deformability as an adequately designed steel reinforced beam. This may be a unique value dependent on multiple variables, such as service conditions. More research, both analytical and experimental, must be completed in this area before a deflection deformability limit is proposed.

6.4 Cracks

6.4.1 Crack Widths

Predicting crack widths is critical for GFRP reinforced concrete beams because, as a serviceability limit, they can often govern design. Crack widths were measured during testing. This section compares the experimental crack widths with the ACI 440.1R-06 predicted crack widths.
RESULTS AND DISCUSSION

For all 16 beams, the tests were paused at four stages to measure crack widths, using a crack width gauge, and locations. These stages correspond to: a) initial cracking; b) top concrete strain = -0.5x10^{-3}; c) top concrete strain = -1.0x10^{-3}; and d) top concrete strain = -2.0x10^{-3}. These values were chosen to simulate service level, with focus being placed on the load stage when top concrete strain = -1.0x10^{-3}. The first load stage was used to attempt to accurately determine the cracking load of the beams, and always corresponded to crack widths of 0.05 mm. The cracking behaviour of this load stage is not discussed herein due to the small width of these cracks (0.05 mm was the smallest possible measurable crack width). Figure 6.30 shows beam 40-B-2.0 at the load stage corresponding to top strain equal to -1.0 x 10^{-3} with the marked cracks.

![Figure 6.30: Marked Cracks with Crack Width Tags](image)

Figure 6.30 shows a typical beam with cracks marked at a load stage corresponding to top concrete strain = -1.0x10^{-3}, and is typical of all results. For the data from the other load stages, see Appendix E. Figure 6.31 shows that, as expected, crack widths decrease as longitudinal reinforcement ratio increases. Two models for calculating crack widths will be discussed and compared herein: ACI 440.1R-06 equations and Tountanji’s model (2000).
Figure 6.31: Maximum Crack Width vs. Reinforcement Ratio (Top Strain = -1.0x10^{-3})

**ACI Code Equations**

The ACI 440.1 R-06 code prescribes the following equations for calculating maximum crack width at any given load:

\[
\begin{align*}
    w &= 2 \frac{f_f}{E_f} \beta k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \\
    \beta &= \frac{h-c}{d-c} \\
    d_c &= \text{thickness of cover from tension face to center of closest bar (mm)} \\
    s &= \text{bar spacing (mm)}
\end{align*}
\]

Where:

- \(f_f\) = FRP longitudinal reinforcement stress (MPa)
- \(E_f\) = FRP modulus of elasticity (MPa)
- \(\beta\) = 
- \(d_c\) = thickness of cover from tension face to center of closest bar (mm)
- \(s\) = bar spacing (mm)
\( k_b = \text{FRP bond factor (taken as 1.0)} \)

Equation 6.40 is based on a physical, strain-based model, which assumes the crack locations are at stirrup locations. The \( k_b \) factor is an empirically derived bond factor to account for the difference between steel bond and GFRP bond. A factor of 1.0 corresponds to a bond equivalent to that of steel; a factor less than 1.0 indicates a superior bond to steel and a factor greater than 1.0 indicates a weaker bond than steel. For the purposes of this investigation, \( k_b \) was assumed to be 1.0 to be conservative. ACI 440.1R-06 specifies an experimental range of \( k_b \) from 0.60 to 1.72, with a mean of 1.10. For design purposes, this code recommends a value of 1.4.

![Figure 6.32: Beam 80-A-2.0 indicating cracks initially form at stirrup locations](image)

Figure 6.32 shows that the assumption that cracks are initiated at stirrup locations is reasonable. The initial cracks (here marked in red, blue and green thin markers) mostly correspond with stirrups.

**Toutanji’s Model**

Toutanji and Saafi (2000) proposed the following modified Gergely-Lutz equation for predicting maximum crack widths in GFRP reinforced concrete beams.
where $A_{ct}$ is defined as the average effective tension area of concrete per bar (Equation 6.42).

\[
A_{ct} = \frac{2(h - d)b}{\text{number of bars}}
\]

Comparing these two models, for this test series, indicates that Toutanji’s model is slightly more accurate overall than the current ACI provision (on average for all beams, $w_{exp}/w_{ACI} = 0.64$, and $w_{exp}/w_{Toutanji} = 0.79$). These results can be visually compared in Figure 6.33 below:

![Figure 6.33: Experimental and Analytical Crack Width Comparison](image)

Figure 6.33 indicates that, for beams that are over-reinforced ($\rho_f/\rho_b > 1$), Toutanji’s model predicts maximum crack width more accurately than the current ACI code provisions. The ACI code is slightly more accurate than Toutanji’s model for under-reinforced beams in this test series. It should be noted that most GFRP reinforced concrete beams are over-reinforced.
This analysis does not take into account long-term effects such as creep, which may be significant in GFRP reinforced beams. Further research is needed in the area of cracks in order to fully understand the crack development.
7. CONCLUSIONS AND RECOMMENDATIONS

7.1 General

At the earliest stages of research into glass-fibre reinforced polymer reinforcing bars, it was recognized that this material has large potential for use in concrete structures, but also has significant disadvantages when compared to steel. To be suitable as a reinforcing material, design equations must account for such properties as low stiffness and brittle failure mode.

As early as 1971, it was understood that the flexural behaviour of GFRP reinforced concrete beams could be predicted using equations developed for steel-reinforced beams, particularly with respect to moment resistance. Since that era, both the mechanical properties of GFRP bars and the understanding of their behaviour have significantly improved. It is recognized, for instance, that the use of GFRP bars results in significantly less tension stiffening effects than steel bars. Due to this, and other phenomena, modifications to the steel-reinforced concrete equations must be made in order to accurately predict the unique behaviour of GFRP-reinforced concrete in flexure.

The research presented here investigated the flexural behaviour of sixteen GFRP-reinforced concrete beams, varying in concrete strength (41.4 MPa or 80.9 MPa), GFRP bar type (Manufacturer: Hughes Bros., Pultrall, or Schöck), and longitudinal reinforcement ratio (0.5%, 1%, or 2%). The goal of this research was to understand flexural properties such as failure modes, ultimate moment resistance, load-deflection behaviour, deformability, crack widths, and confinement. This chapter provides a summary of the findings from this research, as well as recommendations for future work on flexural behaviour of GFRP reinforced concrete beams.
7.2 Conclusions

The following conclusions can be drawn based on the current experimental work:

1. Data collected using a three-dimensional positioning camera paired with an LED target grid located on the surface of a beam provided valuable experimental data, validated by the data collected by strain gauges and potentiometers. These instruments can, for example, accurately measure global behaviour in the pure flexure zone of a four-point load setup, as opposed to the local effects collected by strain gauges. A computer program, Zeppelin, was developed for the analysis of test data and proved to be a valuable tool.

2. Primary failure modes and failure loads can be accurately predicted using conventional sectional analysis techniques and assumptions, such as plane sections remain plane and equivalent rectangular stress blocks. Failure load can also be accurately predicted by modeling beams in the computer program Response-2000. Both under-reinforced and over-reinforced failure modes were investigated in this test series. Over-reinforced beams appeared to fail in a more favourable manner than under-reinforced beams as indicated by an increase in load following initial crushing and drop in load.

3. The confining effect of the closed hoop stirrups allowed beams that initially fail by concrete crushing to experience a second peak load. This effect is attributed to the favourable change in mechanical properties of confined concrete, with its magnitude depending on such factors as unconfined concrete response, mechanical properties, configuration and spacing of lateral reinforcement, and beam dimensions.

4. Cracks associated with flexure tend to initiate at stirrup locations. The maximum crack width can be slightly more accurately predicted using the model proposed by Toutanji (2000), as compared to the current ACI 440.1R-06 equations. Both models, however, yield reasonably accurate results. Maximum crack widths decreased with an increase in longitudinal reinforcement ratio.

5. Investigation of the load-deflection behaviour suggests that current code provisions over-estimate the stiffness of GFRP reinforced beams at loads above cracking. While there is currently an uncracked stiffness reduction factor to
account for the relatively low tension-stiffening effect of GFRP (\(\beta_d L_g\) in ACI 440.1R-06), no such factor exists for cracked stiffness. Because the tension-stiffening phenomena occurs only at loads beyond cracking, it is logical that a stiffness reduction factor which accounts for low tension-stiffening must be applied to the post-cracked stiffness (\(\gamma_d L_{cr}\)). From the analysis of 80 specimens in 10 test series reported by various investigators (including the one presented here), \(\gamma_d = 0.8\) is proposed as an initial suitable value for this reduction factor. Applying this factor to the current ACI 440.1 deflection equations resulted in average prediction accuracy of the 80 beams improving from 79% to 98%, with the percentage of unconservative estimates decreasing from 75% to 46%. Applying this factor to the current CSA S806-12 deflection equations resulted in average prediction accuracy improving from 80% to 100%, with the percentage of unconservative estimates decreasing from 74% to 46%.

6. While deformability attempts to define an energy safety factor for GFRP reinforced beams, it is shown that the predicted results using available models do not reflect the experimental results well. The two available deformability factors, J-factor and Deformability Factor DF, are based on the section properties as displayed by moment-curvature response. Designers do not have a large amount of control over deformability, beyond selecting reinforcement ratio. The stress-strain relationship of concrete has the largest effect of deformability, which may be difficult to control in practice. In addition, some of these predictions can be calculation intensive and cumbersome in practice.

7. A beam deflection deformability factor \(D_d\) is proposed. This member deformability factor is defined as the area under the load-deflection graph at ultimate to the same area at service. Closed form solutions are presented which allow a design engineer to quickly and easily check deformability of a beam. By placing the limiting condition at service, instead of assuming a constant limit (such as for J, or DF), the unique conditions expected for each individual beam can be accounted for. The predicted results agree reasonably well with the experimental results, provided service load moment is at least equal to 1.5 times the moment at cracking \(M_s > 1.5M_{cr}\).
7.3 Recommendations for Future Work

This research was performed on beams of a constant size tested under short-term monotonically applied loads. Larger size beams with a larger concrete core tested in a parallel series, would provide data to validate the conclusions presented here in particular related to deflection and concrete confinement in compression zone of the beams. Creep and shrinkage may be significant problems associated with GFRP reinforced concrete members, and need to be addressed.

The proposed deflection modification factor ($\gamma_d$) should be validated further. While this factor was calibrated with a database of 80 beams, most of these beams were of a similar size (~300 mm by ~200 mm). Further work could investigate the effect of other variables, such as manufacturer, surface treatment, or loading conditions.

The deflection deformability factor ($D_d$) also requires further validation. In addition, for this factor to be usable in design purposes, a limit must be created. Such a limit could state whether a beam is suitable for its intended purpose, or if it is safely designed.
8. References


REFERENCES


**Codes and Standards**

American Concrete Institute (2006), “Guide for the design and construction of concrete reinforced with FRP bars”, ACI 440.1R-06, American Concrete Institute, Farmington Hills, Mi., 2006


APPENDIX A: MATERIAL PROPERTIES

The following appendix reports the mechanical properties of the structural material used in this test series: longitudinal GFRP reinforcement, transverse GFRP reinforcement, and concrete. All tests were performed at the University of Toronto.

The mechanical properties (ultimate tensile strength, ultimate tensile strain, modulus of elasticity) of the GFRP reinforcement were determined via direct tension tests in a 1000 kN displacement controlled actuator.

Concrete cylinder tests provided the required data to determine peak concrete compressive strength, modulus of elasticity, and peak concrete compressive strain. Modulus of rupture tests were performed to determine tensile strength of concrete.

The test results are presented in the following pages, with longitudinal reinforcement properties presented first, followed by transverse reinforcement properties, and concluding with concrete properties.
Bar A (#4) Coupon Test Summary

<table>
<thead>
<tr>
<th>Bar Type:</th>
<th>Asian (US #4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>Hughes Brothers Inc.</td>
</tr>
<tr>
<td>True Diameter:</td>
<td>12.22mm</td>
</tr>
<tr>
<td>No. of Tests:</td>
<td>3</td>
</tr>
<tr>
<td>Test Machine:</td>
<td>MTS 793 1000 kN Test Frame (Huggins Lab)</td>
</tr>
<tr>
<td>Date of Tests:</td>
<td>August 18th, 2011</td>
</tr>
</tbody>
</table>

Table A.1: Experimental Results of 3 #4 Bar A Specimen

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Tester</th>
<th>Modulus (MPa)</th>
<th>Ult. Stress (MPa)</th>
<th>Ult. Strain (10^-6)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal (x10^-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>230.3</td>
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<td>3</td>
<td>JM</td>
<td>47976</td>
<td>761.6</td>
<td>15874</td>
<td>220.7</td>
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<td>Average</td>
<td></td>
<td>50447</td>
<td>759</td>
<td>15061</td>
<td>197.3</td>
<td>3498.0</td>
</tr>
</tbody>
</table>

Figure A.2: Failed #4 Bar A
Figure A.3: Close-up of failure
Figure A.4: Bar A (#4) Test #1

Figure A.5: Bar A (#4) Test #2

Figure A.6: Bar A (#4) Test #3

Material Properties
Bar A (#5) Coupon Test Summary

<table>
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<th>Manufacturer:</th>
<th>Hughes Brothers Inc.</th>
</tr>
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<tbody>
<tr>
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<td>15.77mm</td>
<td>Grade:</td>
<td>I</td>
</tr>
<tr>
<td>No. of Tests:</td>
<td>3</td>
<td>Date of Tests:</td>
<td>August 18th, 2011</td>
</tr>
<tr>
<td>Test Machine:</td>
<td>MTS 793 1000 kN Test Frame (Huggins Lab)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.2: Experimental Results of 3 #5 Bar A Specimen

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Tester</th>
<th>Modulus (MPa)</th>
<th>Ult. Stress (MPa)</th>
<th>Ult. Strain (10^-6)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal (x10^-6)</th>
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<tbody>
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Figure A.7: Average Stress-Strain Curve of #5 Bar A

Figure A.8: Failed #5 Bar A (1)

Figure A.9: Failed #5 Bar A (2)
Figure A.10: Bar A (#5) Test #1

Figure A.11: Bar A (#5) Test #2
### Bar B (#4) Coupon Test Summary

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</thead>
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### Coupon Tests

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<th>Ult. Stress (MPa)</th>
<th>Ult. Strain (10^-6)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal (x10^-6)</th>
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<td>18280</td>
<td>480.3</td>
<td>8335.0</td>
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Table A.3: Experimental Results of 3 #4 Bar B Specimen

**Figure A.12:** Average Stress-Strain Curve of #4 Bar B

**Figure A.13:** Two #4 Bar B Failed Specimen
Figure A.14: Bar B (#4) Test #1

Figure A.15: Bar B (#4) Test #2

Figure A.16: Bar B (#4) Test #3
### Bar B (#5) Coupon Test Summary

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<th>VROD (US #5)</th>
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</tr>
<tr>
<td>No. of Tests:</td>
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<tr>
<td>Test Machine:</td>
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</tr>
<tr>
<td>Date of Tests:</td>
<td>September 8th, 2011</td>
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</table>

### Coupon Tests

<table>
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<tr>
<th>Test ID</th>
<th>Tester</th>
<th>Modulus (F)</th>
<th>Ult. Stress (MPa)</th>
<th>Ult. Strain ($10^{-6}$)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal ($x10^{-6}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JM</td>
<td>62129</td>
<td>1006.4</td>
<td>16199</td>
<td>202.2</td>
<td>3356</td>
</tr>
<tr>
<td>2</td>
<td>JM</td>
<td>56120</td>
<td>1003.5</td>
<td>17881</td>
<td>500.5</td>
<td>9031</td>
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<tr>
<td>3</td>
<td>JM</td>
<td>55703</td>
<td>999.4</td>
<td>17942</td>
<td>602.1</td>
<td>10949</td>
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<td>Average</td>
<td></td>
<td>57984</td>
<td>1003</td>
<td>17340</td>
<td>434.9</td>
<td>7778.7</td>
</tr>
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</table>

![Average Stress-Strain Curve of #5 Bar B](image1.png)

**Figure A.17:** Average Stress-Strain Curve of #5 Bar B

![#5 Bar B Tensile Failure](image2.png)

**Figure A.18:** #5 Bar B Tensile Failure
Material Properties

Figure A.19: Bar B (#5) Test #1

Figure A.20: Bar B (#5) Test #2

Figure A.21: Bar B (#5) Test #3
**Bar C (12M) Coupon Test Summary**

<table>
<thead>
<tr>
<th>Bar Type:</th>
<th>ComBar (12M)</th>
<th>Manufacturer:</th>
<th>Schöck Bauteille GmbH</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Diameter:</td>
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<td>Grade:</td>
<td>II</td>
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<tr>
<td>No. of Tests:</td>
<td>3</td>
<td>Date of Tests:</td>
<td>September 20th, 2011</td>
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<tr>
<td>Test Machine:</td>
<td>MTS 793 1000 kN Test Frame (Huggins Lab)</td>
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**Coupon Tests**

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Tester</th>
<th>Modulus (MPa)</th>
<th>Ult. Stress (MPa)</th>
<th>Ult. Strain (10^-6)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal (x10^-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JM</td>
<td>62191</td>
<td>1378.9</td>
<td>22171</td>
<td>411.9</td>
<td>6360</td>
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<td>2</td>
<td>JM</td>
<td>64070</td>
<td>1345.7</td>
<td>21003</td>
<td>355.2</td>
<td>5700</td>
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<td>3</td>
<td>JM</td>
<td>64711</td>
<td>1220.2</td>
<td>18856</td>
<td>356.1</td>
<td>5585</td>
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<td><strong>Average</strong></td>
<td></td>
<td>63657</td>
<td>1315</td>
<td>20759</td>
<td>374.4</td>
<td>5881.7</td>
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**Figure A.22:** Average Stress-Strain Curve of 12M Bar C

**Figure A.23:** 12M Bar C Failure

**Figure A.24:** 12M Bar C Close-Up
Figure A.25: Bar C (12M) Test #1

Figure A.26: Bar C (12M) Test #2

Figure A.27: Bar C (12M) Test #3
Table A.6: Experimental Results of 3 Bent #4 Bar B Specimen

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Tester</th>
<th>Modulus (MPa)</th>
<th>Ult. Stress (MPa)</th>
<th>Ult. Strain (10^-6)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal (x10^-6)</th>
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<tbody>
<tr>
<td>1</td>
<td>JM</td>
<td>41400</td>
<td>656.2</td>
<td>15806</td>
<td>318.4</td>
<td>7646</td>
</tr>
<tr>
<td>2</td>
<td>JM</td>
<td>41800</td>
<td>658.1</td>
<td>15669</td>
<td>316.6</td>
<td>7500</td>
</tr>
<tr>
<td>3</td>
<td>JM</td>
<td>41400</td>
<td>652</td>
<td>15690</td>
<td>317.1</td>
<td>7599</td>
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<td>Average</td>
<td></td>
<td>41533.3</td>
<td>655.4</td>
<td>15721.7</td>
<td>317.4</td>
<td>7581.7</td>
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Figure A.28: Average Stress-Strain Curve of Bent #4 Bar B

Figure A.29: Close up of bent #4 Bar B failure
Figure A.30: Bent Bar B (#4) Test #1

Figure A.31: Bent Bar B (#4) Test #2

Figure A.32: Bent Bar B (#4) Test #3
### Bar C-Bent (12M) Coupon Test Summary

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>ComBar-Bent (12M)</th>
<th>Manufacturer</th>
<th>Schöck Bauteille GmbH</th>
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<tr>
<td>True Diameter</td>
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<td>II</td>
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<td>No. of Tests</td>
<td>3</td>
<td>Date of Tests</td>
<td>August 22nd, 2011</td>
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<td>Test Machine</td>
<td>MTS 793 1000 kN Test Frame (Huggins Lab)</td>
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Table A.7: Experimental Results of 3 12M Bent Bar C Specimen

<table>
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<th>Test ID</th>
<th>Tester</th>
<th>Modulus (MPa)</th>
<th>Ult. Stress (MPa)</th>
<th>Ult. Strain (10^6)</th>
<th>Stress at Gage Removal (MPa)</th>
<th>Strain at Gage Removal (\times 10^6)</th>
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<tbody>
<tr>
<td>1</td>
<td>JM</td>
<td>62900</td>
<td>1018.1</td>
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<td>357.1</td>
<td>5595</td>
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<td>2</td>
<td>JM</td>
<td>49400</td>
<td>989</td>
<td>20043</td>
<td>346.3</td>
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<td>JM</td>
<td>60300</td>
<td>728.9</td>
<td>12104</td>
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<td>912.0</td>
<td>16082.0</td>
<td>352.3</td>
<td>6170.0</td>
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Note: Test No.3, Specimen Slip

---

![Average Stress-Strain Curve of Bent 12M Bar C](image)

Figure A.33: Average Stress-Strain Curve of Bent 12M Bar C

![12M Bent Bar C during testing](image)

Figure A.34: 12M Bent Bar C during testing
Material Properties

Figure A.35: Bent Bar C (12M) Test #1

Figure A.36: Bent Bar C (12M) Test #2

Figure A.37: Bent Bar C (12M) Test #3
Concrete Tests

<table>
<thead>
<tr>
<th>Specified Strength:</th>
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<td>Dufferin</td>
</tr>
<tr>
<td>Aggregate Size:</td>
<td>14 mm</td>
</tr>
<tr>
<td>Date of Pour:</td>
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<tr>
<td>Number of Tests:</td>
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Table A.8: Cylinder Test Results of NSC (MPa)

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Tester</th>
<th>3 Day Strength</th>
<th>7 Day Strength</th>
<th>14 Day Strength</th>
<th>21 Day Strength</th>
<th>28 Day Strength</th>
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<tbody>
<tr>
<td>1</td>
<td>XS</td>
<td>13.69</td>
<td>26.81</td>
<td>31.32</td>
<td>30.76</td>
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<td>XS</td>
<td>13.69</td>
<td>25.88</td>
<td>31.78</td>
<td>36.32</td>
<td>36.87</td>
<td>40.00</td>
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<tr>
<td>3</td>
<td>XS</td>
<td>13.69</td>
<td>25.07</td>
<td>31.88</td>
<td>35.70</td>
<td>35.80</td>
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<td>25.92</td>
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<td>34.26</td>
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Figure A.38: NSC Cylinder Compressive Strength vs. Time
Concrete Tests

Normal Strength Concrete Test Properties

Date of Tests: November 1, 2011

Specified Strength: 40 MPa

Aggregate Size: 14 mm

Table A.9: Test Day Properties of NSC

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Modulus ($E_c$) (GPa)</th>
<th>Ult. Stress ($f'_c$) (MPa)</th>
<th>Ult. Strain ($\varepsilon'_c$) ($10^{-3}$)</th>
<th>Rupture Strength ($f_r$) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.80</td>
<td>41.43</td>
<td>2.08</td>
<td>3.69</td>
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<td>2</td>
<td>29.06</td>
<td>42.86</td>
<td>2.13</td>
<td>3.72</td>
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<tr>
<td>3</td>
<td>29.35</td>
<td>42.92</td>
<td>2.12</td>
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<td>Average</td>
<td>29.74</td>
<td>42.40</td>
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Figure A.39: Test Day NSC Cylinder Stress-Strain Results
Concrete Tests

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<th>Specified Strength:</th>
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<td>Innocon</td>
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<td>Aggregate Size:</td>
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<td>Date of Pour:</td>
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<tr>
<td>Number of Tests:</td>
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</table>

Table A.10: Cylinder Test Results of HSC (MPa)

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<tr>
<th>Test ID</th>
<th>Tester</th>
<th>3 Day Strength</th>
<th>9 Day Strength</th>
<th>14 Day Strength</th>
<th>21 Day Strength</th>
<th>28 Day Strength</th>
<th>Ultimate Strength</th>
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<tr>
<td>1</td>
<td>XS</td>
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<td>2</td>
<td>XS</td>
<td>47.52</td>
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<td>76.99</td>
<td>80.25</td>
<td>80.66</td>
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<tr>
<td>3</td>
<td>XS</td>
<td>48.62</td>
<td>67.63</td>
<td>72.05</td>
<td>75.59</td>
<td>78.11</td>
<td>81.64</td>
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<td>48.12</td>
<td>67.16</td>
<td>73.30</td>
<td>74.13</td>
<td>78.55</td>
<td>80.87</td>
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Figure A.40: HSC Cylinder Compressive Strength vs. Time
Concrete Tests | High Strength Concrete Test Properties

Date of Tests: November 27, 2011

Specified Strength: 80 MPa

Aggregate Size: 14 mm

Table A.11: Test Day Properties of HSC

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Modulus ($E_c$) (GPa)</th>
<th>Ult. Stress ($f_c'$) (MPa)</th>
<th>Ult. Strain ($\varepsilon_c'$) (10^-3)</th>
<th>Rupture Strength ($f_r$) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.11</td>
<td>80.87</td>
<td>2.39</td>
<td>6.06</td>
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<tr>
<td>2</td>
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<td>80.66</td>
<td>2.40</td>
<td>5.13</td>
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<tr>
<td>3</td>
<td>35.75</td>
<td>81.64</td>
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<td>5.48</td>
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<tr>
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<td>37.14</td>
<td>81.06</td>
<td>2.39</td>
<td>5.56</td>
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</table>

Figure A.41: Test Day HSC Cylinder Stress-Strain Results
APPENDIX B: SPECIMEN DETAILS

The following appendix details the individual specimen and test results. Important numerical data is reported, along with a qualitative description of the test.

Note that experimental results are shown for two points: unconfined and confined peak load. Unconfined peak load corresponds to the first failure mode (concrete cover crushing or tensile reinforcement rupturing), and confined peak load corresponds to the failure of confined core concrete, in the case of over-reinforced beams. Beams which fail by tensile rupturing do not experience a confined peak load.

The strains reported correspond to the peak tensile strain in the reinforcement and the peak compressive strain in the concrete, at the failure mode indicated, as recorded by strain gauges. The confined peak concrete strain was measured by the strain gauge located on the top reinforcing bar, and may not represent the exact concrete strain.

All displacements reported in this appendix are machine displacements. Thus, for four-point tests, the midspan displacement is greater than that reported here.
# Beam 40-C-1.0₃

## Specimen Details

<table>
<thead>
<tr>
<th>Concrete Type:</th>
<th>Normal Strength</th>
<th>Design Failure Mode:</th>
<th>Crushing</th>
</tr>
</thead>
</table>

### Longitudinal Reinforcement Properties

<table>
<thead>
<tr>
<th>Bar Type:</th>
<th>Bar C 12M</th>
<th>No. of Bars:</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Depth:</td>
<td>271.8 mm</td>
<td>Reinforcement Ratio:</td>
<td>1.12%</td>
</tr>
<tr>
<td>Reinforcement Area:</td>
<td>608.5 mm²</td>
<td>Balanced Reinforcement Ratio</td>
<td>0.33%</td>
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</tbody>
</table>

### Transverse Reinforcement Properties

<table>
<thead>
<tr>
<th>Bar Type:</th>
<th>Bar C 12M</th>
<th>Stirrup Spacing:</th>
<th>200 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stirrup Area:</td>
<td>226 mm²</td>
<td>Transverse Reinforcement Ratio:</td>
<td>0.57%</td>
</tr>
</tbody>
</table>

## Test Results

<table>
<thead>
<tr>
<th>Unconfined Peak Load:</th>
<th>111.4 kN</th>
<th>Unconfined Peak Deflection:</th>
<th>69.0 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Peak GFRP Strain:</td>
<td>14172 με</td>
<td>Unconfined Peak Concrete Strain:</td>
<td>-2905 με</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Confined Peak Load:</th>
<th>120.2 kN</th>
<th>Confined Peak Deflection:</th>
<th>127.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confined Peak GFRP Strain:</td>
<td>17349 με</td>
<td>Confined Peak Concrete Strain:</td>
<td>-7748 με</td>
</tr>
</tbody>
</table>

![Figure B.1: Load vs. MTS-Deflection for Beam 40-C-1.0₃](image-url)
Test Date: September 27, 2011

Test Description:

Second three-point loaded test. First load stage performed at initial cracking (P = 12.1 kN). Second load stage at P = 25.5 kN. Third load stage at P = 41.8 kN. Fourth load stage at P = 79.0 kN.

Initial failure mode was crushing immediately to the south of the loading plate. (P = 111.4 kN, Δ = 69.0 mm). Confinement effects allowed the beam to regain load, up to the same crushing load. At this point, the north side top concrete crushed (P=110.8 kN, Δ = 86.7 mm). Confinement effects again allow the beam to regain load. Large deformations occurred, and it was decided to remove potentiometers to minimize damage. Some load was removed for safety. Upon reloading, the beam failed in a diagonal failure mode before reaching the prior load.

![Figure B.2: Diagonal Failure Mode](image1)
![Figure B.3: Compression Reinforcement Crushing](image2)
![Figure B.4: 40-C-1.03 Reinforcing Cage](image3)
![Figure B.5: Three-Point Test Setup](image4)

![Figure B.6: Load vs. Midspan Deflection of 40-C-1.03](image5)
Beam 40-C-1.0

Specimen Details

Concrete Type: Normal Strength  Design Failure Mode: Crushing

Longitudinal Reinforcement Properties

Bar Type: Bar C 12M  No. of Bars: 5
Effective Depth: 271.8 mm  Reinforcement Ratio: 1.12%
Reinforcement Area: 608.5 mm²  Balanced Reinforcement Ratio: 0.33%

Transverse Reinforcement Properties

Bar Type: Bar C 12M  Stirrup Spacing: 200 mm
Stirrup Area: 226 mm²  Transverse Reinforcement Ratio: 0.57%

Test Results

Unconfined Peak Load: 151.4 kN  Unconfined Peak Deflection: 73.7 mm
Unconfined Peak GFRP Strain: 15040 µε  Unconfined Peak Concrete Strain: -3161 µε

Confined Peak Load: 126.7 kN  Confined Peak Deflection: 91.8 mm
Confined Peak GFRP Strain: 20285 µε  Confined Peak Concrete Strain: -14273 µε

Figure B.7: Load vs. MTS-Displacement for Beam 40-C-1.0
Test Date: October 3, 2011

Test Description:

First four-point loaded test. First load stage performed at initial cracking (P = 10.0 kN). Second load stage at P = 29.5 kN. Third load stage at P = 58.5 kN. Fourth load stage at P = 109.4 kN.

Initial failure mode was concrete crushing near the south side of the pure flexure zone. After initial crushing, some load was removed to the potentiometers could be safely removed. This corresponds with the backwards jog seen in the load displacement curve. The second peak corresponds to the north side of the pure flexure zone crushing, upon which confinement allows the beam to regain load. Finally, confined crushing failure occurred in the pure flexure zone.

Figure B.8: Confined Crushing Failure Mode
Figure B.9: Compression Reinforcement Buckling
Figure B.10: Four-point load test setup
Figure B.11: 40-C-1.0 Reinforcing Cage
Figure B.12: Load vs. Midspan Deflection of 40-C-1.0
Beam 40-C-2.0₃

**Specimen Details**

Concrete Type: Normal Strength  
Design Failure Mode: Crushing

**Longitudinal Reinforcement Properties**

Bar Type: Bar C 16M  
No. of Bars: 5

Effective Depth: 264.2 mm  
Reinforcement Ratio: 2.00%

Reinforcement Area: 1056 mm²  
Balanced Reinforcement Ratio 0.32%

**Transverse Reinforcement Properties**

Bar Type: Bar C 12M  
Stirrup Spacing: 200 mm

Stirrup Area: 226 mm²  
Transverse Reinforcement Ratio: 0.57%

**Test Results**

Unconfined Peak Load: 132.4 kN  
Unconfined Peak Deflection: 45.4 mm

Unconfined Peak GFRP Strain: 7955 με  
Unconfined Peak Concrete Strain: -2667 με

Confined Peak Load: 122.3 kN  
Confined Peak Deflection: 67.7 mm

Confined Peak GFRP Strain: 8781 με  
Confined Peak Concrete Strain: -6074 με

Figure B.13: Load vs. MTS-Displacement for Beam 40-C-2.0₃
Test Date: September 22, 2011

Test Description:

First test. First load stage performed at initial cracking (P = 11.9 kN). Second load stage at P = 34.7 kN. Third load stage at P=60.3. Fourth load stage at P = 110.0 kN.

At a load of 132.4 kN, concrete crushing occurred on the north side of the loading plate, causing the load to drop. Confinement effects allowed the beam to regain load, up to 122.3 kN. At this point, confined crushing occurred and the top reinforcement bars buckled.
Beam 40-C-2.0

Specimen Details

Concrete Type: Normal Strength
Design Failure Mode: Crushing

**Longitudinal Reinforcement Properties**

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<thead>
<tr>
<th>Bar Type</th>
<th>Bar C 16M</th>
<th>No. of Bars:</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Depth</td>
<td>264.2 mm</td>
<td>Reinforcement Ratio:</td>
<td>2.00%</td>
</tr>
<tr>
<td>Reinforcement Area</td>
<td>1056 mm²</td>
<td>Balanced Reinforcement Ratio</td>
<td>0.32%</td>
</tr>
</tbody>
</table>

**Transverse Reinforcement Properties**

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Bar C 12M</th>
<th>Stirrup Spacing:</th>
<th>200 mm</th>
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</thead>
<tbody>
<tr>
<td>Stirrup Area</td>
<td>226 mm²</td>
<td>Transverse Reinforcement Ratio:</td>
<td>0.57%</td>
</tr>
</tbody>
</table>

**Test Results**

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<tr>
<th></th>
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<tbody>
<tr>
<td>Unconfined Peak Load:</td>
<td>159.3 kN</td>
<td>Unconfined Peak Deflection:</td>
</tr>
<tr>
<td>Unconfined Peak GFRP Strain:</td>
<td>8921 με</td>
<td>Unconfined Peak Concrete Strain:</td>
</tr>
<tr>
<td>Confined Peak Load:</td>
<td>137.5 kN</td>
<td>Confined Peak Deflection:</td>
</tr>
<tr>
<td>Confined Peak GFRP Strain:</td>
<td>11222 με</td>
<td>Confined Peak Concrete Strain:</td>
</tr>
</tbody>
</table>

Figure B.19: Load vs. MTS-Displacement for Beam 40-C-2.0
SPECIMEN DETAILS

Test Date: October 6, 2011

Test Description:

Second three-point loaded test. First load stage performed at initial cracking (P = 13.6 kN). Second load stage at P = 34.8 kN. Third load stage at P = 71.1 kN. Fourth load stage at P = 127.4 kN.

At 159.3 kN, crushing occurred on north side (inside the pure flexure zone). Typically, confinement effects increased load, back up to 137.5 kN. The south-quarter potentiometer was not installed correctly, and should not be used in analysis.

Figure B.20: Confined crushing failure mode
Figure B.21: Buckled Compression Reinforcement
Figure B.22: 40-C-2.0 Reinforcing Cage
Figure B.23: First four beams after failure

Figure B.24: Load vs. Midspan Deflection of 40-C-2.0
Beam 40-A-0.5

Specimen Details

Concrete Type: Normal Strength  Design Failure Mode: Rupture

Longitudinal Reinforcement Properties

Bar Type: Bar A #4  No. of Bars: 2
Effective Depth: 282.5 mm  Reinforcement Ratio: 0.45%
Reinforcement Area: 254 mm\(^2\)  Balanced Reinforcement Ratio 0.68%

Transverse Reinforcement Properties

Bar Type: Bar B #3  Stirrup Spacing: 150 mm
Stirrup Area: 142.6 mm\(^2\)  Transverse Reinforcement Ratio: 0.48%

Test Results

Unconfined Peak Load: 88.6 kN  Unconfined Peak Deflection: 86.6 mm
Unconfined Peak GFRP Strain: SATURATED  Unconfined Peak Concrete Strain: -3000 με
Confined Peak Load: X  Confined Peak Deflection: X
Confined Peak GFRP Strain: X  Confined Peak Concrete Strain: X

Figure B.25: Load vs. MTS-Displacement for Beam 40-A-0.5
Test Date: October 19, 2011

Test Description:

First load stage performed at initial cracking ($P = 11.1 \text{ kN}$). Second load stage at $P = 13.8 \text{ kN}$. Third load stage at $P = 34.1 \text{ kN}$. Fourth load stage at $P = 50.7 \text{ kN}$ (at top strain of $-1500 \mu \varepsilon$).

First test of a beam designed to fail by tensile rupturing. Low stiffness caused the load displacement curve to show sawtooth behaviour. (Cracking causes significant drop in load). LED #3 moved out of field of camera vision due to elongation of bottom cord of beam. Failure of reinforcement cause beam to fail completely, no confinement effects. Rollers no longer touched loading spreader beam.

Figure B.26: Large deflection of 40-A-0.5
Figure B.27: 40-A-0.5 immediately after failure

Figure B.28: 40-A-0.5 Reinforcing Cage
Figure B.29: Close-up of ruptured reinforcement

Figure B.30: Load vs. Midspan Deflection for 40-A-0.5
Beam 40-A-1.0

**Specimen Details**

Concrete Type: Normal Strength  
Design Failure Mode: Crushing

**Longitudinal Reinforcement Properties**

Bar Type: Bar A #4  
No. of Bars: 5  
Effective Depth: 269.9 mm  
Reinforcement Ratio: 1.18%  
Reinforcement Area: 635 mm²  
Balanced Reinforcement Ratio: 0.67%

**Transverse Reinforcement Properties**

Bar Type: Bar B #3  
Stirrup Spacing: 100 mm  
Stirrup Area: 142.6 mm²  
Transverse Reinforcement Ratio: 0.71%

**Test Results**

Unconfined Peak Load: 140.3 kN  
Unconfined Peak Deflection: 71.2 mm  
Unconfined Peak GFRP Strain: 16754 με  
Unconfined Peak Concrete Strain: -4064 με

Confined Peak Load: 144.3 kN  
Confined Peak Deflection: 96.1 mm  
Confined Peak GFRP Strain: 21554 με  
Confined Peak Concrete Strain: -6692 με

**Figure B.31: Load vs. MTS-Displacement for Beam 40-A-1.0**
Test Date: October 28, 2011

Test Description:

First load stage performed at initial cracking ($P = 12.6$ kN). Second load stage at $P = 23.3$ kN. Third load stage at $P = 46.8$ kN. Fourth load stage at $P = 91.9$ kN.

Top bar north and bottom bar south strain gauges not working, probably damaged during construction. Quarter-point potentiometers were removed immediately after crushing. Confined failure mode primarily top reinforcement buckling (due to two confined load peaks). Confined crushing not experienced, beam still relatively elastic. Test deemed finished due to roller almost at max displacement.

![Figure B.32: Top concrete cover crushed](image)

![Figure B.33: No confined crushing](image)

![Figure B.34: 40-A-1.0 failure (elastic behaviour)](image)

![Figure B.35: Three-Point Test Setup](image)

![Figure B.36: Load vs. Midspan Deflection of 40-A-1.0](chart)
**Beam 40-B-1.0**

**Specimen Details**

Concrete Type: Normal Strength  
Design Failure Mode: Crushing

**Longitudinal Reinforcement Properties**

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>No. of Bars</th>
<th>Effective Depth</th>
<th>Reinforcement Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar B #4</td>
<td>5</td>
<td>269.9 mm</td>
<td>1.18%</td>
</tr>
</tbody>
</table>

Reinforcement Area: 635 mm²  
Balanced Reinforcement Ratio: 0.46%

**Transverse Reinforcement Properties**

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Stirrup Spacing</th>
<th>Stirrup Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar B #3</td>
<td>100 mm</td>
<td>142.6 mm²</td>
</tr>
</tbody>
</table>

Transverse Reinforcement Ratio: 0.71%

**Test Results**

Unconfined Peak Load: 155.8 kN  
Unconfined Peak Deflection: 68.5 mm

Unconfined Peak GFRP Strain: SATURATED  
Unconfined Peak Concrete Strain: -3290 με

Confined Peak Load: 168.2 kN  
Confined Peak Deflection: 100.6 mm

Confined Peak GFRP Strain: SATURATED  
Confined Peak Concrete Strain: -8933 με

Figure B.37: Load vs. MTS-Displacement for Beam 40-B-1.0
Test Date: November 1, 2011

Test Description:

First load stage performed at initial cracking ($P = 14.4$ kN). Second load stage at $P = 26.8$ kN. Third load stage at $P = 51.3$ kN. Fourth load stage at $P = 102.0$ kN.

Bottom bar south strain gauge not working, probably damaged during construction. Bottom bar strain gauges maxed out during test. Similar to last test, confined failure mode was not confined crushing failure. The beam started to lose load, possibly due to internal buckling of bars. Note that the spacing of these beams was very small, allowing significant confinement effects.
Beam 40-B-2.0

Specimen Details
Concrete Type: Normal Strength  Design Failure Mode: Crushing

Longitudinal Reinforcement Properties
Bar Type: Bar B #5  No. of Bars: 5
Effective Depth: 261.9 mm  Reinforcement Ratio: 1.90%
Reinforcement Area: 995 mm²  Balanced Reinforcement Ratio: 0.51%

Transverse Reinforcement Properties
Bar Type: Bar B #3  Stirrup Spacing: 75 mm
Stirrup Area: 142.6 mm²  Transverse Reinforcement Ratio: 0.95%

Test Results
Unconfined Peak Load: 180 kN  Unconfined Peak Deflection: 54.4 mm
Unconfined Peak GFRP Strain: 9500 με  Unconfined Peak Concrete Strain: 2789 με
Confined Peak Load: 217.4 kN  Confined Peak Deflection: 108 mm
Confined Peak GFRP Strain: 14389 με  Confined Peak Concrete Strain: -11194 με

Figure B.43: Load vs. Displacement for Beam 40-B-2.0
Test Date: November 4, 2011

Test Description:

Second three-point loaded test. First load stage performed at initial cracking (P = 13.1 kN). Second load stage at P = 30.8 kN. Third load stage at P = 60.6 kN. Fourth load stage at P = 124.6 kN.

Very desirable behaviour for an over-reinforced beam. Initial crushing at P = 180.0 kN, following by increase in load due to confinement. Confined failure mode was confined crushing. Strain measurement in top reinforcing bar at confined crushing was -11194 με. Explosive failure, stirrups rupture, and reinforcing bar buckling. Complete failure. Small spacing of stirrups allowed for significant confinement effect, (confined deflection roughly equaled unconfined deflection). Beam still exhibited some elastic behaviour.

Figure B.44: Initial crushing failure mode
Figure B.45: Confined crushing failure mode
Figure B.46: Top reinforcing bars crushed
Figure B.47: 40-B-2.0 after failure

Figure B.48: Load vs. Midspan Deflection of 40-B-2.0
**Beam 80-C-0.5**

**Specimen Details**

Concrete Type: High Strength  
Design Failure Mode: Rupture

**Longitudinal Reinforcement Properties**

Bar Type: Bar C 12M  
No. of Bars: 2  
Effective Depth: 271.8 mm  
Reinforcement Ratio: 0.43%  
Reinforcement Area: 608.5 mm$^2$  
Balanced Reinforcement Ratio: 0.58%

**Transverse Reinforcement Properties**

Bar Type: Bar C 12M  
Stirrup Spacing: 200 mm  
Stirrup Area: 226 mm$^2$  
Transverse Reinforcement Ratio: 0.57%

**Test Results**

Unconfined Peak Load: 121.8 kN  
Unconfined Peak Deflection: 119.4 mm  
Unconfined Peak GFRP Strain: SATURATED  
Unconfined Peak Concrete Strain: -3775 με

Confined Peak Load: X  
Confined Peak Deflection: X  
Confined Peak GFRP Strain: X  
Confined Peak Concrete Strain: X

Figure B.49: Load vs. MTS-Deflection for Beam 80-C-0.5
Test Date: November 9, 2011

Test Description:

First high strength concrete test. First load stage performed at initial cracking (P = 18.2 kN). Second load stage at P = 24.2 kN. Third load stage at P = 41.4 kN. Fourth load stage at P = 78.8 kN.

Test was designed to fail by tensile rupturing, and did. Saw-tooth behaviour explained by cracking. Middle potentiometer removed for safety. Initial failure associated with crack opening directly below north loading point. Popping noises heard. Beam gained strength, but still heard popping noises. Eventually, bottom reinforcement failed and beam fell down. No elastic behaviour after removing load.
Beam 80-C-1.0

**Specimen Details**

Concrete Type: High Strength  
Design Failure Mode: Crushing

**Longitudinal Reinforcement Properties**

- **Bar Type:** Bar C 12M  
- **No. of Bars:** 5  
- **Effective Depth:** 271.8 mm  
- **Reinforcement Ratio:** 1.12%  
- **Reinforcement Area:** 608.5 mm²  
- **Balanced Reinforcement Ratio:** 0.58%

**Transverse Reinforcement Properties**

- **Bar Type:** Bar C 12M  
- **Stirrup Spacing:** 150 mm  
- **Stirrup Area:** 226 mm²  
- **Transverse Reinforcement Ratio:** 0.75%

**Test Results**

- **Unconfined Peak Load:** 193.6 kN  
- **Unconfined Peak Deflection:** 91.4 mm  
- **Unconfined Peak GFRP Strain:** 18821 με  
- **Unconfined Peak Concrete Strain:** -3506 με

- **Confined Peak Load:** 190.9 kN  
- **Confined Peak Deflection:** 111.2 mm  
- **Confined Peak GFRP Strain:** SATURATED  
- **Confined Peak Concrete Strain:** -6553 με

**Figure B.55: Load vs. MTS-Displacement for Beam 80-C-1.0**
Test Date: November 15, 2011

Test Description:

First load stage performed at initial cracking (P = 15.8 kN). Second load stage at P = 31.4 kN. Third load stage at P = 62.1 kN. Fourth load stage at P = 119.7 kN.

At P = 193.6 kN, concrete crushed on north side of loading plate (outside pure flexure zone). Beam regained load and crushed on the south side of loading plate (again, outside of pure flexure zone). A horizontal crack appeared in the pure flexure zone, at the level of top reinforcement. A load pop was heard, and the test was stopped before confined crushing. Concrete cover peeled off in one layer is slabs. Beam possessed significant elastic behaviour.
Beam 80-C-2.0

Specimen Details

Concrete Type: High Strength  Design Failure Mode: Crushing

Longitudinal Reinforcement Properties

Bar Type: Bar C 16M  No. of Bars: 5
Effective Depth: 264.2 mm  Reinforcement Ratio: 2.00%
Reinforcement Area: 1056 mm²  Balanced Reinforcement Ratio 0.56%

Transverse Reinforcement Properties

Bar Type: Bar C 12M  Stirrup Spacing: 150
Stirrup Area: 226 mm²  Transverse Reinforcement Ratio: 0.75%

Test Results

Unconfined Peak Load: 256.2 kN  Unconfined Peak Deflection: 73.8 mm
Unconfined Peak GFRP Strain: 13286 με  Unconfined Peak Concrete Strain: 3542 με

Confined Peak Load: 185.6 kN  Confined Peak Deflection: 90.7 mm
Confined Peak GFRP Strain: 14996 με  Confined Peak Concrete Strain: -7155 με

Figure B.61: Load vs. MTS-Deflection for Beam 80-C-2.0
Test Date: November 17, 2011

Test Description:

First load stage performed at initial cracking (P = 16.0 kN). Second load stage at P = 39.3 kN. Third load stage at P = 81.6 kN. Fourth load stage at P = 159.6 kN.

Initial failure mode was crushing in the center of the pure flexure zone, as expected (P = 256.2 kN). Brittle crushing, compared to normal strength concrete. No warning and airborne concrete. Core still intact, and confinement effects kicked in. Eventually, top bars crushed due to confinement. Core concrete not crushed.

![Figure B.62: Initial crushing in pure flexure zone](image1)
![Figure B.63: Final failure of concrete](image2)

![Figure B.64: Close-up of top bar failure](image3)
![Figure B.65: 80-C-2.0 after removal of cover](image4)

![Figure B.66: Load vs. Midspan Deflection of 80-C-2.0](image5)
Beam 80-A-0.5

Specimen Details

Concrete Type: High Strength  Design Failure Mode: Rupture

Longitudinal Reinforcement Properties

Bar Type: Bar A #4  No. of Bars: 2
Effective Depth: 282.5 mm  Reinforcement Ratio: 0.45%
Reinforcement Area: 254 mm²  Balanced Reinforcement Ratio: 1.16%

Transverse Reinforcement Properties

Bar Type: Bar B #3  Stirrup Spacing: 150 mm
Stirrup Area: 142.6 mm²  Transverse Reinforcement Ratio: 0.48%

Test Results

Unconfined Peak Load: 93.9 kN  Unconfined Peak Deflection: 98.4 mm
Unconfined Peak GFRP Strain: 20515 με  Unconfined Peak Concrete Strain: -2687 με
Confined Peak Load: X  Confined Peak Deflection: X
Confined Peak GFRP Strain: X  Confined Peak Concrete Strain: X

Figure B.67: Load vs. Deflection for Beam 80-A-0.5
Test Date: November 22, 2011

Test Description:

First load stage performed at initial cracking (P = 19.5 kN). Second load stage at P = 23.3 kN. Third load stage at P = 39.1 kN. Fourth load stage at P = 73.5 kN (at top strain of -1500 με).

Again, load-deflection curve showed significant sawtooth behaviour, as with the other under-reinforced beams. This may be due to the fact that the tensile strength contributes a significant portion of the beams stiffness. Primary failure crack opened directly in middle of beam. No post-peak behaviour, no elastic behaviour. Beam pretty much split in two pieces.
Beam 80-A-1.0

Specimen Details

Concrete Type: High Strength  Design Failure Mode: Crushing

Longitudinal Reinforcement Properties

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>No. of Bars</th>
<th>Effective Depth</th>
<th>Reinforcement Ratio</th>
</tr>
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<tbody>
<tr>
<td>Bar A #4</td>
<td>5</td>
<td>269.9 mm</td>
<td>1.18%</td>
</tr>
</tbody>
</table>

Reinforcement Area: 635 mm²  Balanced Reinforcement Ratio: 1.16%

Transverse Reinforcement Properties

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Stirrup Spacing</th>
<th>Stirrup Area</th>
<th>Transverse Reinforcement Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar B #3</td>
<td>100 mm</td>
<td>142.6 mm²</td>
<td>0.71%</td>
</tr>
</tbody>
</table>

Test Results

<table>
<thead>
<tr>
<th></th>
<th>Unconfined Peak Load: 200.4 kN</th>
<th>Unconfined Peak Deflection: 100 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Peak GFRP Strain</td>
<td>20848 με</td>
<td>Unconfined Peak Concrete Strain: -4470 με</td>
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</table>

Confined Peak Load: 154.6 kN  Confined Peak Deflection: 102.6 mm

Confined Peak GFRP Strain: SATURATED  Confined Peak Concrete Strain: -6619 με

Figure B.73: Load vs. MTS-Displacement for Beam 80-A-1.0
SPECIMEN DETAILS

Test Date: November 24, 2011

Test Description:

First load stage performed at initial cracking ($P = 17.5$ kN). Second load stage at $P = 28.6$ kN. Third load stage at $P = 44.8$ kN. Fourth load stage at $P = 103.8$ kN.

This test was very close to balanced condition. Initial failure mode was top concrete crushing at $P = 200.4$ kN. Following this failure, the load dropped substantially. The test was paused here for approximately one minute. During this minute, popping noises were heard until the beam finally failed. This is defined as a confined balanced failure mode.

![Initial compression failure](image1)

![Final tensile rupture failure mode](image2)

![Failure with loose concrete removed](image3)

![Overall beam failure](image4)

![Load vs. Midspan Deflection of 80-A-1.0](image5)
Beam 80-A-2.0

Specimen Details

Concrete Type: High Strength  Design Failure Mode: Crushing

Longitudinal Reinforcement Properties

Bar Type: Bar A #5  No. of Bars: 5
Effective Depth: 261.9 mm  Reinforcement Ratio: 1.90%
Reinforcement Area: 995 mm²  Balanced Reinforcement Ratio: 1.34%

Transverse Reinforcement Properties

Bar Type: Bar B #3  Stirrup Spacing: 75 mm
Stirrup Area: 142.6 mm²  Transverse Reinforcement Ratio: 0.95%

Test Results

Unconfined Peak Load: 234.6 kN  Unconfined Peak Deflection: 81.2 mm
Unconfined Peak GFRP Strain: SATURATED  Unconfined Peak Concrete Strain: -4425 με
Confined Peak Load: 220.6 kN  Confined Peak Deflection: 113.2 mm
Confined Peak GFRP Strain: SATURATED  Confined Peak Concrete Strain: -10226 με

Figure B.79: Load vs. MTS Deflection for Beam 80-A-2.0
Test Date: November 28, 2011

Test Description:

First load stage performed at initial cracking (P = 15.9 kN). Second load stage at P = 30.5 kN. Third load stage at P = 64.6 kN. Fourth load stage at P = 132.6 kN.

Initial crushing failure occurred at P = 239.5 kN. Again, this failure mode is more brittle than corresponding beams made of NSC. Confinement effects allowed beam to increase in load. Closely spaced stirrups allowed this confinement to be significant. Eventually, confined concrete crushed. The failure mode appeared brittle, however, inspection of the load-deflection graph indicated the opposite. The load slowly dropped off, in a curved manner.

Figure B.80: Initial crushing failure mode
Figure B.81: Confined crushing failure mode
Figure B.82: Reinforcing cage after confined failure
Figure B.83: Three-Point Test Setup
Figure B.84: Load vs. Midspan Deflection of 80-A-2.0
Beam 80-B-1.0

Specimen Details

Concrete Type: High Strength  
Design Failure Mode: Crushing

Longitudinal Reinforcement Properties

Bar Type: Bar B #4  
No. of Bars: 5  
Effective Depth: 269.9 mm  
Reinforcement Ratio: 1.18%  
Reinforcement Area: 635 mm²  
Balanced Reinforcement Ratio: 0.80%

Transverse Reinforcement Properties

Bar Type: Bar B #3  
Stirrup Spacing: 100 mm  
Stirrup Area: 142.6 mm²  
Transverse Reinforcement Ratio: 0.71%

Test Results

Unconfined Peak Load: 204.1 kN  
Unconfined Peak Deflection: 88.1 mm  
Unconfined Peak GFRP Strain: 21249 με  
Unconfined Peak Concrete Strain: -4301 με

Confined Peak Load: 175.3 kN  
Confined Peak Deflection: 98 mm  
Confined Peak GFRP Strain: SATURATED  
Confined Peak Concrete Strain: -5532 με

Figure B.85: Load vs. MTS-Displacement for Beam 80-B-1.0
Test Date: November 30, 2011

Test Description:

Second three-point loaded test. First load stage performed at initial cracking ($P = 14.8 \text{ kN}$). Second load stage at $P = 30.0 \text{ kN}$. Third load stage at $P = 45.7 \text{ kN}$. Fourth load stage at $P = 102.7 \text{ kN}$.

Similar to all the over-reinforced HSC beams, the top concrete crushed in a brittle manner. This occurred in the pure flexure zone at a load of 204.1 kN. Following this crushing, the bottom middle bar strain gauge maxed out. Confinement effects kicked in, up to tensile rupturing. Similar failure mode to 80-A-1.0, with a more visible confined response.

Figure B.86: Initial crushing failure mode

Figure B.87: Final failure mode after rupture

Figure B.88: Close-up of failure crack

Figure B.89: Close-up of ruptured bars

Figure B.90: Load vs. Midspan Deflection of 80-B-1.0
Beam 80-B-2.0

Specimen Details

Concrete Type: High Strength  Design Failure Mode: Crushing

Longitudinal Reinforcement Properties

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Bar B #5</th>
<th>No. of Bars: 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Depth:</td>
<td>261.9 mm</td>
<td>Reinforcement Ratio: 1.90%</td>
</tr>
<tr>
<td>Reinforcement Area:</td>
<td>995 mm²</td>
<td>Balanced Reinforcement Ratio 0.87%</td>
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</tbody>
</table>

Transverse Reinforcement Properties

<table>
<thead>
<tr>
<th>Bar Type</th>
<th>Bar B #3</th>
<th>Stirrup Spacing: 75 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stirrup Area:</td>
<td>142.6 mm²</td>
<td>Transverse Reinforcement Ratio: 0.95%</td>
</tr>
</tbody>
</table>

Test Results

- Unconfined Peak Load: 251.1 kN
- Unconfined Peak Deflection: 75.3 mm
- Unconfined Peak GFRP Strain: 15134 με
- Unconfined Peak Concrete Strain: -3755 με
- Confined Peak Load: 234.2 kN
- Confined Peak Deflection: 111.5 mm
- Confined Peak GFRP Strain: SATURATED
- Confined Peak Concrete Strain: -10072 με

Figure B.91: Load vs. Displacement for Beam 80-B-2.0
Test Date: December 2, 2011

Test Description:

Final test. First load stage performed at initial cracking (P = 17.6 kN). Second load stage at P = 30.1 kN. Third load stage at P = 64.9 kN. Fourth load stage at P = 140.9 kN.

Typical over-reinforced response, similar to 40-A-2.0. Initial crushing occurred in the pure flexure zone, followed by confinement effects. This beam had closely spaced stirrups, which allowed for a large post-peak response. Ultimate failure mode was confined core crushing, in a brittle manner.

Figure B.92: Beam before failure
Figure B.93: Beam after initial crushing failure
Figure B.94: Confined crushing failure mode
Figure B.95: Close-up of crushed core

Figure B.96: Load vs. Midspan Deflection of 80-B-2.0
APPENDIX C:  LOAD DEFLECTION RESULTS

The following appendix is split into two main sections. Both sections display the experimental load vs. midspan deflection of the sixteen beams and compare them with predicted results. Midspan deflection is calculated by subtracting the support displacements from the net midspan displacement.

This first section compares the experimental results with the current prediction methods (CSA S806-12, ACI 440.1R-06, and ACI Ballot) and predictions determined from the analysis program Response-2000. The second section shows the experimental results and prediction results using the proposed Modified ACI method, with the original ACI 440.1R-06 shown as a dashed line.
Figure C.1: Load vs. Midspan Deflection for Beam 40-A-0.5

Figure C.2: Load vs. Midspan Deflection for Beam 40-A-1.0
Figure C.3: Load vs. Midspan Deflection for Beam 40-B-1.0

Figure C.4: Load vs. Midspan Deflection for Beam 40-B-2.0
Figure C.5: Load vs. Midspan Deflection for Beam 40-C-1.03

Figure C.6: Load vs. Midspan Deflection for Beam 40-C-1.0
Figure C.7: Load vs. Midspan Deflection for Beam 40-C-2.0₃

Figure C.8: Load vs. Midspan Deflection for Beam 40-C-2.0
Figure C.9: Load vs. Midspan Deflection for Beam 80-A-0.5

Figure C.10: Load vs. Midspan Deflection for Beam 80-A-1.0
Figure C.11: Load vs. Midspan Deflection for Beam 80-A-2.0

Figure C.12: Load vs. Midspan Deflection for Beam 80-B-1.0
Figure C.13: Load vs. Midspan Deflection for Beam 80-B-2.0

Figure C.14: Load vs. Midspan Deflection for Beam 80-C-0.5
Figure C.15: Load vs. Midspan Deflection for Beam 80-C-1.0

Figure C.16: Load vs. Midspan Deflection for Beam 80-C-2.0
Figure C.17: Load vs. Midspan Deflection for Beam 40-A-0.5

- **Experimental**
- **Modified ACI Prediction**
- **Original ACI Prediction**

Figure C.18: Load vs. Midspan Deflection for Beam 40-A-1.0

- **Experimental**
- **Modified ACI Prediction**
- **Original ACI Prediction**
Figure C.19: Load vs. Midspan Deflection for Beam 40-B-1.0

Figure C.20: Load vs. Midspan Deflection for Beam 40-B-2.0
Figure C.21: Load vs. Midspan Deflection for Beam 40-C-1.03

- Experimental
- Modified ACI Prediction
- Original ACI Prediction

Figure C.22: Load vs. Midspan Deflection for Beam 40-C-1.0

- Experimental
- Modified ACI Prediction
- Original ACI Prediction
Figure C.23: Load vs. Midspan Deflection for Beam 40-C-2.0₃

Figure C.24: Load vs. Midspan Deflection for Beam 40-C-2.0
Figure C.25: Load vs. Midspan Deflection for Beam 80-A-0.5

Figure C.26: Load vs. Midspan Deflection for Beam 80-A-1.0
Figure C.27: Load vs. Midspan Deflection for Beam 80-A-2.0

Figure C.28: Load vs. Midspan Deflection for Beam 80-B-1.0
Figure C.29: Load vs. Midspan Deflection for Beam 80-B-2.0

Figure C.30: Load vs. Midspan Deflection for Beam 80-C-0.5
Figure C.31: Load vs. Midspan Deflection for Beam 80-C-1.0

Figure C.32: Load vs. Midspan Deflection for Beam 80-C-2.0
APPENDIX D: MOMENT CURVATURE RESULTS

The following appendix presents the moment-curvature results of the sixteen specimens.

The charts for beams loaded under four-point loading contain two sets of experimental data: one calculated using strain gauge data, and one calculated using LED data. The charts for beams loaded under three-point loading only contain the strain gauge data, due to difficulty calculated the LED moment-curvature response for this test set-up.

The predicted moment-curvature behaviour was calculated using the computer program described in Chapter 6 in Figure 6.16.
Figure D.1: 40-A-0.5 Moment vs. Curvature

Figure D.2: 40-A-1.0 Moment vs. Curvature
Figure D.3: 40-B-1.0 Moment vs. Curvature

Figure D.4: 40-B-2.0 Moment vs. Curvature
Moment Curvature Results

Figure D.5: 40-C-1.0\textsubscript{3} Moment vs. Curvature

Figure D.6: 40-C-1.0 Moment vs. Curvature

Theoretical

Strain Gauge

LED Data

Predicted
Moment Curvature Results

Figure D.7: 40-C-2.0 Moment vs. Curvature

Figure D.8: 40-C-2.0 Moment vs. Curvature
Figure D.9: 80-A-0.5 Moment vs. Curvature

Figure D.10: 80-A-1.0 Moment vs. Curvature
Figure D.11: 80-A-2.0 Moment vs. Curvature

LED Data
Strain Gauge
Theoretical

Figure D.12: 80-B-1.0 Moment vs. Curvature

LED Data
Strain Gauge
Theoretical
Figure D.13: 80-B-2.0 Moment vs. Curvature

Figure D.14: 80-C-0.5 Moment vs. Curvature
Figure D.15: 80-C-1.0 Moment vs. Curvature

Figure D.16: 80-C-2.0 Moment vs. Curvature
APPENDIX E: CRACKS

The following appendix presents data on the cracking behaviour of the beams in this test series. The first section displays crack maps recorded during load stage pauses, along with the maximum crack width at these stages. Each stage has two crack maps: west side and east side. The west side crack map is presented first (distinguished by the LED targets displayed on the map).

The second section presents the crack width data in chart form. It is presented in three charts corresponding to three load stage pauses (top concrete strain = -0.5 x 10^{-3}, -1.0 x 10^{-3}, or -2.0 x 10^{-3})
**Beam 40-A-0.5**

Load Stage: Cracking

- **Load Stage: \(-0.5 \times 10^{-3}\)**
  - \(w = 0.05\text{mm}\)
  - \(w = 0.05\text{mm}\)

- **Load Stage: \(-1.0 \times 10^{-3}\)**
  - \(w = 0.25\text{mm}\)
  - \(w = 0.3\text{mm}\)

- **Load Stage: \(-1.5 \times 10^{-3}\)**
  - \(w = 0.8\text{mm}\)
  - \(w = 1.8\text{mm}\)

**Load Stage: \(-2.0 \times 10^{-3}\)**

- **Load Stage: \(-2.0 \times 10^{-3}\)**
  - \(w = 1.6\text{mm}\)
  - \(w = 2.0\text{mm}\)

**Notes:** The final load stage was reduced from \(-2.0 \times 10^{-3}\) to \(-1.5 \times 10^{-3}\) for safety reasons. This beam was under-reinforced, so concrete strain could not indicate proximity to failure.
**Beam 40-A-1.0**

Load Stage: Cracking

- Load Stage: \(-0.5 \times 10^{-3}\)
  - \(w = 0.05\,\text{mm}\)

- Load Stage: \(-1.0 \times 10^{-3}\)
  - \(w = 0.2\,\text{mm}\)
  - \(w = 0.15\,\text{mm}\)

- Load Stage: \(-2.0 \times 10^{-3}\)
  - \(w = 0.35\,\text{mm}\)
  - \(w = 0.45\,\text{mm}\)
  - \(w = 0.6\,\text{mm}\)

Notes:
**Beam 40-B-1.0**

Load Stage: Cracking

Load Stage: $-0.5 \times 10^{-3}$

Load Stage: $-1.0 \times 10^{-3}$

Load Stage: $-2.0 \times 10^{-3}$

Notes:
**Beam 40-B-2.0**

Load Stage: Cracking

Load Stage: -$0.5 \times 10^{-3}$

Load Stage: -$1.0 \times 10^{-3}$

Load Stage: -$2.0 \times 10^{-3}$

Notes:
Beam 40-C-1.0

Load Stage: Cracking

- $w = 0.05\text{mm}$
- $w = 0.05\text{mm}$

Load Stage: $-0.5 \times 10^{-3}$

- Not Recorded
- Not Recorded

Load Stage: $-1.0 \times 10^{-3}$

- $w = 0.35\text{mm}$
- $w = 0.3\text{mm}$

Load Stage: $-2.0 \times 10^{-3}$

- Not Recorded
- Not Recorded

Notes:
**Beam 40-C-2.0**

Load Stage: Cracking

Load Stage: $-0.5 \times 10^{-3}$

Load Stage: $-1.0 \times 10^{-3}$

Load Stage: $-2.0 \times 10^{-3}$

**Notes:**
Note:
**Beam 80-A-0.5**

Load Stage: Cracking

Load Stage: \(-0.5 \times 10^{-3}\)

Load Stage: \(-1.0 \times 10^{-3}\)

Load Stage: \(-1.5 \times 10^{-3}\)

Notes:
**Beam 80-A-1.0**

Load Stage: Cracking

- **Load Stage: -0.5 \times 10^{-3}**
  - $w = 0.05\text{mm}$
  - $w = 0.05\text{mm}$

- **Load Stage: -1.0 \times 10^{-3}**
  - $w = 0.2\text{mm}$
  - $w = 0.3\text{mm}$

- **Load Stage: -2.0 \times 10^{-3}**
  - $w = 0.55\text{mm}$
  - $w = 0.5\text{mm}$
  - $w = 1.0\text{mm}$
  - $w = 1.4\text{mm}$

Notes:
**Beam 80-A-2.0**

Load Stage: Cracking

- Load Stage: $-0.5 \times 10^{-3}$
  - $w = 0.05\text{mm}$
  - $w = 0.05\text{mm}$

- Load Stage: $-1.0 \times 10^{-3}$
  - $w = 0.15\text{mm}$
  - $w = 0.15\text{mm}$

- Load Stage: $-2.0 \times 10^{-3}$
  - $w = 0.35\text{mm}$
  - $w = 0.4\text{mm}$
  - $w = 0.55\text{mm}$
  - $w = 0.6\text{mm}$

**Notes:**
Beam 80-B-1.0

Load Stage: Cracking

Load Stage: \(-0.5 \times 10^{-3}\)

Load Stage: \(-1.0 \times 10^{-3}\)

Load Stage: \(-2.0 \times 10^{-3}\)

Notes:
**Beam 80-B-2.0**

Load Stage: Cracking

- $w = 0.05\text{mm}$
- $w = 0.05\text{mm}$

Load Stage: $-0.5 \times 10^{-3}$

- $w = 0.1\text{mm}$
- $w = 0.15\text{mm}$

Load Stage: $-1.0 \times 10^{-3}$

- $w = 0.3\text{mm}$
- $w = 0.4\text{mm}$

Load Stage: $-2.0 \times 10^{-3}$

- $w = 0.7\text{mm}$
- $w = 1.1\text{mm}$

Notes:
**Beam 80-C-0.5**

Load Stage: Cracking

- Load Stage: $-0.5 \times 10^{-3}$
  - $w = 0.05\text{mm}$
  - $w = 0.8\text{mm}$

- Load Stage: $-1.0 \times 10^{-3}$
  - $w = 2.0\text{mm}$
  - $w = 1.6\text{mm}$

- Load Stage: $-2.0 \times 10^{-3}$
  - $w = 3.5\text{mm}$
  - $w = 3.5\text{mm}$

**Notes:**
Beam 80-C-1.0

Load Stage: Cracking

Load Stage: \(-0.5 \times 10^{-3}\)

Load Stage: \(-1.0 \times 10^{-3}\)

Load Stage: \(-2.0 \times 10^{-3}\)

Notes:
Beam 80-C-2.0

Load Stage: Cracking

Load Stage: -0.5 x 10^{-3}

Load Stage: -1.0 x 10^{-3}

Load Stage: -2.0 x 10^{-3}

Notes:
Maximum Crack Widths

Figure E.1: Crack Widths at Top Strain = -0.5x10^{-3}

Figure E.2: Crack Widths at Top Strain = -1.0x10^{-3}
Figure E.3: Crack Widths at Top Strain = -2.0x10^{-3}

- ACI Prediction
- Experimental