INFLUENCE OF ROCK BOUNDARY CONDITIONS ON BEHAVIOUR OF ARCHED AND FLAT CEMENTED PASTE BACKFILL BARRICADE WALLS

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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Abstract

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2012

Current design of cemented paste backfill (CPB) barricades tends to be of unknown conservativeness due to limited understanding of their behaviour. Previous work done to characterize barricade response has not accounted for the effects of the surrounding rock stiffness, which can have significant impact on the development of axial forces which enhance capacity via compressive membrane action.

Parametric analyses were performed with the finite element analysis program Augustus-2 to determine the effects of various material and geometric properties on barricade capacity. Equations based on Timoshenko and Boussinesq solutions were developed to model rock stiffness effects based on boundary material properties. An iterative simulation process was used to account for secondary moment effects as a proof of concept.

It was found that, for a range of typical rock types, barricade capacity varied significantly. The commonly made design assumption of a fully rigid boundary resulted in unconservative overpredictions of strength.
Acknowledgements

Thanks!

*No hour of life is lost that is spent in the saddle.*

*Winston Churchill*
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Nomenclature

\( \alpha \) Arch angle

\( \bar{m} \) Jaeger coefficient based on aspect ratio of loaded area

\( \lambda \) Aspect ratio of loaded area

\( \nu \) Poisson’s ratio

\( \rho \) Reinforcement content, measured in %

\( \theta \) Surface rotation

\( a \) Horizontal distance from center of end support to rotational restraint rod in Augustus-2

\( A_{\text{plate}} \) Area of loaded region

\( A_{s,\text{axial}} \) Area of axial stiffness rod in Augustus-2

\( A_{s,\text{rot}} \) Area of rotational stiffness rod in Augustus-2

\( C \) Axial stiffness coefficient

\( d \) Depth from surface of infinite half space where vertical displacement is zero

\( d_v \) Effective shear depth of member

\( E \) Young’s modulus

\( E_s \) Young’s modulus of steel

\( E_{\text{rock}} \) Young’s modulus of surrounding rock

\( f'_{c} \) Concrete compressive strength

\( f_y \) Steel yield strength
$F_{rod,r}$ Force in rotational stiffness rod in Augustus-2

$G$ Shear modulus

$K_{axial}$ Axial stiffness, measured in N/mm

$K_{rot}$ Rotational stiffness, measured in $\frac{N\text{mm}}{mm\text{-rad}}$

$L_{rod,a}$ Length of axial stiffness rod in Augustus-2

$L_{rod,r}$ Length of rotational stiffness rod in Augustus-2

$L_{rod}$ Length of restraint rod

$M$ Applied moment

$P$ Applied load

$r$ Distance of point of interest on plate from applied point load

$t$ Barricade thickness

$v$ Vertical displacement profile

$w$ Loaded width of plate

$w_p$ Point displacement in a uniformly loaded area

$w_{avg}$ Average displacement of a rectangular uniformly loaded area
Chapter 1

Introduction

1.1 Motivation for Current Study

In an effort to reduce the environmental impact and increase the efficiency of underground mining operations, mine waste products (tailings) are often mixed with Portland cement binders to create Cemented Paste Backfill (CPB) which is then pumped underground to fill existing, open stopes. Reinforced concrete paste barricades (Figure 1.1) are constructed in these stopes to isolate the poured CPB from neighbouring stopes which may still be in use. This method of underground waste storage reduces the need for surface tailings disposal and also provides a degree of support to surrounding stopes as the cement binder hydrates and the CPB solidifies much like concrete. Prior to hardening, however, CPB behaves hydrostatically due to its water content and exerts pressure on the surrounding rock and barricades.

Existing industry design methods for barricades vary. Most barricades are designed and built very conservatively to account for CPB pressures that may be higher than assumed and make simplifying assumptions for barricade geometry [12]. Even so, there are documented cases of barricade failure, making safety a genuine concern [13]. If the structural behaviour and response of these barricades were well-understood, it would be possible to make their design and construction more cost and time-efficient.

1.1.1 Compressive Membrane Action and Secondary Moment Effects

A reinforced concrete member subject to loading will crack on the tension face while the reinforcement elongates. Typically, strains on the tension face will be larger than those on
the compression face so the average strain at mid-depth will be tensile (Figure 1.2). This net tensile strain causes the member to become longer as more load is applied. If this expansion is sufficiently restrained, compressive axial forces will develop in the expanding member (Figure 1.3a). These compressive forces produce an increase in flexural capacity as shown by a typical axial load-flexure interaction diagram in Figure 1.3b. As loads increase and concrete crushes, the only remaining capacity is the reinforcement acting in tension as the main cable does in a suspension bridge: this is called catenary action.

In CPB barricades, the surrounding rock provides significant resistance to expansion in the longitudinal or transverse directions. Because of this, the effects of compressive membrane action are of interest to determine how they affect the strength and design of barricades. Because the surrounding rock is not infinitely rigid, however, rock stiffness must also be considered.

While compressive membrane action can increase the flexural strength of axially-restrained members, increased deflections can give rise to secondary moment effects which impose additional moment demand. Axial loads imposed over a distance from the member centroid increase effective moments, causing the member to reach its moment capacity more quickly [17]. This additional demand is called the P-Δ effect (Figure 1.4). As compressive axial loads develop in a member subject to membrane action, any displacements will bring...
Motivation for Current Study

1.1.2 Prior Research

Barricades, commonly constructed with pumped concrete (shotcrete) around a mesh of reinforcing steel, can be analyzed with similar techniques to those used for traditional reinforced concrete structures due to similarities in material, geometry, and loading. In an effort to characterize the behaviour of barricades to aid in efficient design, existing research by Ghazi [9] performed at the University of Toronto used proprietary finite element analysis tools which specifically accounted for the non-linear stress-strain behaviour of reinforced concrete. Such non-linear behaviour is brought on by cracking which occurs in virtually all reinforced concrete structures under serviceability loads. Because of this, the accuracy of conventional linear plane sections remain plane Euler-type analyses cannot fully characterize the post-cracking behaviour of concrete.

Ghazi’s research found that compressive membrane action and the rotational and axial stiffness of the surrounding rock all had significant effects on barricade strength [9]. It was also found that two-dimensional analyses which considered barricades as one-way slabs yielded similar results to more complex and time-consuming three-dimensional analyses.
Motivation for Current Study

(a) Restraint and compressive forces from resistance to slab elongation.

(b) Axial load-moment interaction diagram showing increased flexural capacity.

Figure 1.3: Components of compressive membrane action [17].

Figure 1.4: Member subject to eccentric load \( P \) over distance \( e \) from the centroid. The internal moment \( Pe \) along the member is increased by an additional moment \( P\Delta \) as the member deflects by an amount \( \Delta \) at mid-height [6].

which accounted for two-way slab behaviour. Secondary moment effects (such as the \( P-\Delta \) effect) were not considered due to software limitations. Also, more complex model geometries such as arched fences (which are more common in industry [12]) were not studied. While the effects of rotational and axial rock stiffness were simulated, their significance merits further investigation.
1.2 Current Study

The current study builds upon existing research performed by Ghazi [9] at the University of Toronto through improved modelling of boundary stiffness as well as various arched geometries. The proprietary software used by Ghazi, Augustus-2, has since been updated to improve its accuracy. A parametric study of various geometric and material properties will be carried out with the software. Its results will be validated through comparison to existing experimental data and simulation results.

Because barricades are axially restrained, both compressive membrane action and secondary moment effects should be considered by the simulations. Although Augustus-2 currently does not include secondary moment effects, a manual proof of concept analysis will include these effects by the addition of applied moments to a simple cantilever model. The object of this proof of concept is to demonstrate the method’s feasibility as well as its effects on lowering barricade capacity.
Chapter 2

Relevant Research

2.1 Existing Bulkhead Modelling Efforts

2.1.1 Ghazi (2011)

In his Masters thesis, Ghazi investigated the behaviour of CPB barricades using both two and three-dimensional finite element analyses [9]. Results were compared to measured field data and laboratory experiments to gauge accuracy and then a parametric study was carried out to determine the effects of material properties, boundary conditions, reinforcement content, and geometry on barricade behaviour. Conclusions from the study indicated that barricade strength was most influenced by the stiffness of the rock boundary condition. The following is a summary of Ghazi’s research.

Comparison to Field and Experimental Data

Prior to the parametric study, results from the two-dimensional analysis program Augustus-2 were compared to experimental results from Su et al [14]. These experiments were conducted on reinforced concrete beams which were axially and rotationally restrained in a manner similar to the boundary conditions for CPB barricades. Further details on the experiments can be found in Section 2.2.1.

Comparisons were made between experimental results and Augustus-2 models of three series of beams with differing geometry and reinforcement content. The Augustus-2 models were restrained in axial and rotational directions with a series of truss rods; axial and rotational stiffnesses were controlled by changing the cross-sectional area of the rods (Figure 2.1). Because the tested beams were symmetric about midspan, only half of the
Existing Bulkhead Modelling Efforts

beam was modelled. Support conditions at midspan allowed for vertical but not horizontal displacements on the assumption that any horizontal expansion of the beam would occur symmetrically outwards from midspan.

![Figure 2.1: Typical element mesh and support conditions in Ghazi’s Augustus-2 models [9].](image)

The simulations predicted vertical load-deflection behaviour well, with excellent predictions of initial stiffness and good accuracy from post-cracking through to failure. In the models and as with the test data, flexural cracking occurred at midspan and at the supports followed by first yielding at midspan and then at the supports. Crushing of concrete at the support was correctly predicted to cause a drop in load capacity and failure occurred at midspan as with the experiments. Typical load-deflection response for each of the three series of beams is shown in Figures 2.2, 2.3, and 2.4.

While the vertical load-deflection response was well-modelled, issues were encountered in predicting the horizontal reaction forces. The axial stiffness values of 1000 kN/mm reported by Su et al [14] gave poor predictions when replicated in Augustus-2, so the author adjusted the axial stiffness values by trial and error to obtain an accurate horizontal load-deformation curve. The calibrated stiffness value which provided an accurate prediction of load-deformation was similarly accurate for the development of axial force versus deformation. This discrepancy was due to ambiguity in the original experimental setup: it was not known whether the 1000 kN/mm value meant that the beam would elongate by 1mm under a 1000 kN midspan load or if the supports themselves would displace 1mm when subject to a 1000 kN axial load. Axial stiffness values in Augustus-2 which yielded accurate horizontal load predictions were typically ten to twenty times less than the specified 1000 kN/mm value. Despite the axial stiffness issues, the Su et al specimens [14] will again be used for comparison in this thesis due to a lack of relevant experimental data from other sources.
Figure 2.2: Vertical (a) and horizontal (b) reaction forces versus midspan deflection for Augustus-2 models of beam A-1 [9].

Figure 2.3: Vertical (a) and horizontal (b) reaction forces versus midspan deflection for Augustus-2 models of beam B-1 [9].

Figure 2.4: Vertical (a) and horizontal (b) reaction forces versus midspan deflection for Augustus-2 models of beam C-1 [9].
Existing Bulkhead Modelling Efforts

(a) Effect of changing axial stiffness values
(b) Effect of changing rotational stiffness values

Figure 2.5: Applied load versus midspan deflection for varying axial and rotational stiffness values [9].

Because of the Augustus-2 models’ apparent sensitivity to axial stiffness, the value was varied for a given model to determine its effect on load-deformation response. As shown in Figure 2.5a, initial beam stiffness is similar between three varying axial stiffness values, but lower values result in a drop in load capacity and more pronounced midspan deflection. First cracking loads remained approximately equal, but first yield loads varied. A similar analysis was carried out, varying rotational stiffness values. The model was much less sensitive to changes in rotational stiffness, however at low values premature beam failure occurred at the supports due to shear (Figure 2.5b).

Analytical Barricade Modelling

Programs Augustus-2 (two-dimensional) and VecTor4 (three-dimensional) were used to create finite element models of flat test barricades installed in a mine in Turkey. Simulation results were compared to test data from the barricade installations.

In an attempt to increase efficiency, simulations were performed in both two and three dimensions to determine whether a simpler, faster two-dimensional analysis could be performed for a given barricade in lieu of a more complex three-dimensional analysis. One of the concerns was whether a barricade was governed by two-way slab behaviour, which is characterized by bending moments of similar magnitude along both the width and height of the barricade. Such behaviour would necessitate a more complex analysis. However, since it was established that barricade behaviour in only one principal direction yielded similar results to two-way slab simulations for many practical cases, a simpler two-dimensional slab strip analysis was attempted which only considered one-way slab behaviour. Figure 2.6 shows that as the aspect ratio increases, strength predictions of a two-way slab analysis quickly approach the results of a one-way slab strip analysis performed in VecTor4 and Augustus-2, respectively. Two-way predictions of strength
drop off sharply as the barricade aspect ratio approaches 2.0. The one-way results yield the same strength predictions regardless of aspect ratio because they do not consider the non-principal dimension of the barricade and its effect on behaviour.

![Figure 2.6: VecTor4 and Augustus-2 strength predictions versus slab strip aspect ratio [9].](image)

Figure 2.6 compares both two and three-dimensional model predictions to experimental results from a test barricade with an aspect ratio (length to height) of 1.89. The test barricade was loaded in a conventional manner with CPB, but testing was stopped before failure occurred. Both VecTor4 and Augustus-2 predict a slight increase in strength beyond the test’s ultimate stopping point as well as accurate behaviour modelling in both pre and post-cracked states. It is important to note that rock stiffness values were not provided, so all simulations were performed with calibrated stiffness values that provided an accurate prediction. This was done in Augustus-2 by changing the cross-sectional area of the axial and rotational truss rods and done in VecTor4 by assuming a different compressive concrete strength.

Although the Augustus-2 result is more unconservative than the VecTor4 prediction, it is expected that the implementation of secondary moment \((P - \Delta)\) effects in Augustus-2 will result in lower strength predictions as such moments would increase demand on the structure. The similarities between the Augustus-2 and VecTor4 simulations as well as their accurate predictions of test data are a promising result. Because VecTor4 simulations took considerably longer to prepare and execute than their Augustus-2 counterparts, simulation work in this thesis will be performed with Augustus-2.

A sensitivity analysis was conducted in VecTor4, varying boundary conditions, concrete strength, and reinforcement content. As seen in Figure 2.8a, barricade strength is very
sensitive to boundary conditions, where full fixity grants over a three-fold increase in ultimate pressure resistance and a two-fold increase in maximum deflection. Barricades with a higher concrete compressive strength were both stiffer and stronger, while deflections were relatively unaffected (Figure 2.8b). For the compressive strength analyses, the boundaries were allowed to rotate. There was almost no difference in strength or stiffness of a reinforced barricade compared to an unreinforced one (Figure 2.8c); it was concluded that if the surrounding rocks were stiff enough to provide support to the barricade, the reinforcement ratio had a small effect on behaviour. Although not explicitly mentioned by Ghazi, it could be inferred that the boundary conditions for the reinforcement ratio simulations were fully fixed based on his conclusions.
Existing Bulkhead Modelling Efforts

Figure 2.8: Applied pressure versus midspan deflection for varying material properties and boundary conditions [9].

(a) Varying boundary conditions  
(b) Varying concrete strength  
(c) Varying reinforcement content
2.1.2 Revell and Sainsbury (2007)

The following is a summary of a paper published by Revell and Sainsbury discussing existing barricade design methods in industry as well as results from the authors’ numerical barricade models [12].

**American Concrete Institute (ACI) Code Design**

Bulkhead design based on ACI structural requirements often idealizes the bulkhead as a linear-elastic, simply supported beam for purposes of determining imposed loads. Reinforcement is then detailed based on ACI code limits for standard reinforced concrete structures. Because the actual rock wall boundary of the bulkhead in situ is partially fixed as opposed to simply supported, the assumed loading is generally higher than in reality which results in an overly conservative design.

**Yield Line Theory**

Traditional yield line theory for slabs assumes boundary conditions, hinge lines, and a compatible flexural failure mechanism (Figure 2.9). As the slab reaches failure, it is assumed to deflect plastically and the ultimate load is calculated using equilibrium equations or the principal of virtual work. While yield line analysis is an acceptable and established method for reinforced concrete slab design, the method does not account for the increased strength brought on by compressive membrane action in barricades nor does it cover all possible loading and support conditions. The theory also ignores the possibility of shear failure. As such, barricade design based on yield line theory provides conservative estimates of strength.

![Figure 2.9: Assumed yield line pattern for a simply supported square slab of side length L with plastic moment $m_p$ [12].](image)
Australian Yield Line Design

Current barricade design methods for many Australian companies are based on a modified form of yield line theory originally used to estimate the strength of masonry barricades. However, the authors state that there is no theoretical basis for applying yield line theory to unreinforced, orthotropic masonry walls. The basis for this modified theory is stated by the authors as being of ambiguous origin.

Numerical Modelling and Results

The numerical modelling program FLAC3D was used to model barricades of varying geometries (Figure 2.10). Results were compared to yield line theory with simply supports, the modified Australian yield line method, and physical experiments on similar structures. All models used the same material properties: concrete with an unconfined compressive strength of 30 MPa using a Mohr-Coulomb strain-softening model to simulate post-peak loss of strength. The simulated concrete was reinforced with fibers to increase ductility, but traditional and wire mesh reinforcement was omitted. The interface between the barricade and the surrounding rock surface was also modelled and studied.

![Barricade geometries modelled by FLAC3D](image)

Figure 2.10: Barricade geometries modelled by FLAC3D [12].

Simulation results of a variable thickness 5 x 5m barricade model matched yield line theory exactly for the case of simply supported boundaries; both of these cases predicted a much lower ultimate load than that predicted by the modified Australian yield line method. Additional results for the same barricade with fully fixed boundary conditions
Existing Bulkhead Modelling Efforts

predicted ultimate failure loads significantly greater than those predicted by both yield line and Australian methods (Figure 2.11). This was attributed to the contribution of compressive membrane action to the strength of the barricade which yield line theory does not consider. The barricade-rock interface model which allowed for both bending and shear interaction was found to provide a more realistic failure mechanism when compared to simple or fully fixed supports. Barricade models of a horseshoe shape or those that were arched into the direction of the load were also found to be stronger than flat barricades.

Figure 2.11: Ultimate loads for simply supported and fixed 5 x 5m square barricades with barricade-rock interface models as compared to yield line and Australian yield line solutions [12].

Program verification was done by modelling constructed experimental barricades and comparing simulation results to experimental data gathered from mines. Two cases were explored: a pair of bulkheads tested in 1990 and 1991 and an actual barricade failure during a fill operation in 2006. In both cases, the FLAC3D simulations provided accurate predictions of the failure mode (propagation of yielding from the rock-wall interface) and failure pressure. However, the authors noted that there was a substantial amount of uncertainty in the representation of material properties and loading conditions.
2.1.3 Helinski et al. (2011)

The following is a summary of a paper published by Helinski et al. which presents results of a three-dimensional parametric study carried out on models of arched fiber-reinforced concrete and waste rock barricades [10]. The work builds upon the previously summarized paper by Revell and Sainsbury [12]. Because this thesis focuses on concrete barricades, the waste rock barricade content of this paper is not summarized.

The numerical modelling program FLAC3D was once again used for the parametric study. The fiber-reinforced concrete was simulated as a continuum with smeared reinforcement using a Mohr-Coulomb model which also accounted for linear elastic strain-softening. It should be noted that the simulations did not represent any form of traditional bar reinforcement as is typically used in the construction of concrete barricades. Sliding interfaces were used between the barricade and the surrounding rock wall. Pressures imposed by the CPB were applied uniformly across the barricade surface. The parametric study covered two main sensitivities: material and geometric. It was found that unconfined concrete compressive strength, barricade arc radius, and barricade alignment were important factors in barricade capacity.

Material Sensitivity

Several material properties of the fiber-reinforced concrete were varied to determine their effects on strength: unconfined compressive strength, internal friction angle, elastic modulus, strain-softening characteristics, and the barricade-rock wall interface.

Barricades with higher concrete compressive strengths were able to carry more load before failure as could be expected (Figure 2.12a). The internal friction angle of the concrete, which was found to increase as curing progressed in physical specimens, was found to have little effect on barricade capacity due to unconfined compression being the critical failure mode (Figure 2.12b). However, the capacity did decrease slightly as friction angle increased; this was attributed to a decrease in the major and minor principle stress capacities.

Changes in elastic modulus contributed little to barricade strength; values were increased and decreased by 50% with minimal change in capacity (Figure 2.13). The observed slight increase in capacity with increasing modulus was attributed to the reduction in strain softening for a stiffer material after yielding.

The strain softening model used in the simulations linearly decreased the cohesive (shear) and tensile strength of the fiber-reinforced concrete over a specified critical plastic
Existing Bulkhead Modelling Efforts

Figure 2.12: Normalized barricade capacity vs unconfined compressive strength $q_{ucs}$ (a) and concrete friction angle $\phi$ (b) [10].

Figure 2.13: Normalized barricade capacity vs ratio of modulus $E$ to unconfined compressive strength $q_{ucs}$ [10].

strain to represent the reduction in strength after yielding. A lower value of critical plastic strain resulted in a faster decrease in strength after yielding, representative of a more brittle material; a higher value provided increased ductility. The effects on both shear and tensile due to changes in critical plastic strain were studied separately. It was found that increased material shear ductility significantly increased barricade capacity; allowing for a 10% plastic shear strain after yield provided a threefold increase in strength (Figure 2.14a). This ductility allowed for a more uniform redistribution of stresses after material yielding along the barricade abutments which resulted in higher capacity. Varying the critical plastic strain for tensile material strength had little impact on the ultimate capacity of the barricade due to the arched geometry of the wall having a shearing failure mechanism at the abutments (Figure 2.14b).
Because the quality of the surrounding rock can vary from site to site, the strength of the barricade-rock wall interface was varied in simulations. It was found that the strength of the interface had little effect on ultimate barricade capacity, which suggested that barricade strength was primarily a function of its material and not the condition of the boundary.

The material sensitivity studies were conducted under the fundamental assumption that the concrete was fiber-reinforced with no traditional reinforcing bars present. This is not truly representative of actual constructed barricade walls which include cages of reinforcing bars. Also, based on previous findings by Ghazi [9], boundary stiffness can significantly affect barricade strength. This parametric study did not consider these effects.

**Geometric Sensitivity**

Barricades with different curvatures, spans, heights, and alignments were tested to determine their effects on capacity.

The curvature of the wall was varied as a function of the arc angle $\alpha$ as shown in Figure 2.15. Strengths are normalized by the capacity of a straight barricade. For low values of $\alpha$, the barricade is relatively flat and is more prone to bending and tensile failure. Thrust forces from the surrounding rock developed as $\alpha$ increased and the wall became more arch-like, resulting in concrete compression failures. Large values of $\alpha$ resulted in lower thrust forces and larger shearing forces which caused the barricades to fail in shear and tension along the barricade-rock wall interface. The optimal range of arc angles $\alpha$
was suggested to be approximately 60-80° to induce a preferred compression failure. This would lessen the impact of the strength of the surrounding rock, which can vary between locations.

![Figure 2.15: Normalized barricade capacity vs barricade arch angle \(\alpha\) [10].](image)

The span of the barricade was varied between 4 to 6 m for a constant arc radius. For a given radius, a longer span would result in a larger arc angle and vice versa. If an appropriate arc radius is selected, barricade capacity varies by as little as -5% to 10% (Figure 2.16a). Similarly, barricade height was found to have almost no effect on strength (Figure 2.16b), as almost all of the stresses are transferred through the stiffer arch to the surrounding rock instead of to the vertical connections.

![Figure 2.16: Normalized barricade capacity vs barricade span (a) and height (b) [10].](image)

Due to inconsistencies in construction methods, barricades are sometimes built with the arch direction not perfectly perpendicular to the direction of the applied pressure.
Existing Bulkhead Modelling Efforts

This misalignment was also studied and found to be a significant issue: a 25% offset (e.g.: a 1 m offset over a 4 m span), for example, reduced the capacity of the barricade by approximately 30% (Figure 2.17).

The study of geometric sensitivities presented is thorough, with many parameters studied. However, the sample sizes of some analyses are small: only three different span lengths were simulated for each of two different barricade radii for parameters such as bulkhead span, height, and misalignment. As barricade strength did not vary much between the three samples, though, this is a minor issue. Of more concern is the ability of the FLAC3D software to model cracked fibre-reinforced concrete.

Figure 2.17: Normalized barricade capacity vs barricade height [10].
2.2 Axially Restrained Beams

2.2.1 Su et al. (2009)

Su et al. tested twelve reinforced concrete beams representative of two-bay floor beams after the removal of a center supporting column. Test results indicated that compressive membrane action contributed significantly to the strength of axially restrained beams. A parametric study of reinforcement ratio, span-to-depth ratio, and loading rate was also conducted. The following is a summary of test results and discussion by the authors [14]. This same paper was also discussed by Ghazi [9]. The test results from this paper will be used to validate the finite element analysis program Augustus-2 prior to the parametric CPB barricade study.

Experimental Program

Twelve beams were constructed in three series: A, B, and C (Table 2.1). Each series of beams had the same cross-sectional dimensions with varying reinforcement content. Span lengths remained constant except with series B beams, which had varying span lengths to study the effect of span-to-depth ratio on capacity. A sample beam with reinforcement layout is shown in Figure 2.18. Closely spaced hooped bars were used for shear reinforcement to avoid premature shear failure.

![Figure 2.18: Typical beam dimensions with reinforcement layout [14].](image)

To simulate the axial restraint provided by columns at each end of the beam, the column stubs at both ends were secured with pinned steel sockets which were in turn connected to vertical and horizontal struts to impose axial and rotational restraints. Roller bearings were used on the side faces of the center column stub to prevent out-of-plane rotations.
during loading. Figure 2.19 shows a schematic of the test setup. Displacement-controlled loads were imposed downwards onto the center column stub through a servo-controlled actuator braced against a loading frame.

The horizontal and rotational stiffnesses of the supports were measured as 1000 kN/mm and 17 500 kN-m/rad, respectively, but as shown by Ghazi [9] these quoted stiffness values were ambiguously defined and needed to be changed to achieve similar results in simulation models of the beams. As such, a separate set of experimental data by Vecchio and Tang [17] will also be used to validate Augustus-2.

### Table 2.1: Specimen properties (a) and reinforcement properties (b) [14]

<table>
<thead>
<tr>
<th>Test</th>
<th>$b \times h$ (mm, in.)</th>
<th>$l_{cr}$, mm (in.)</th>
<th>$f_{cr}$, MPa (psi)</th>
<th>Longitudinal reinforcement and ratio</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Top</td>
<td>Bottom</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1225 (48)</td>
<td>32.3 (4680)</td>
<td>2φ12, $\rho = 0.55%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.08</td>
<td></td>
<td>$\phi 8$ at 100</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1225 (48)</td>
<td>35.3 (5120)</td>
<td>3φ12, $\rho = 0.83%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.08</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1225 (48)</td>
<td>39.0 (5660)</td>
<td>3φ14, $\rho = 1.13%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.08</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1225 (48)</td>
<td>28.8 (4180)</td>
<td>2φ12, $\rho = 0.55%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.08</td>
<td></td>
<td>$\phi 8$ at 100</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1225 (48)</td>
<td>33.1 (4800)</td>
<td>3φ12, $\rho = 0.83%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.08</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1225 (48)</td>
<td>35.8 (5190)</td>
<td>3φ14, $\rho = 1.13%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.08</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>1975 (78)</td>
<td>23.2 (3360)</td>
<td>3φ14, $\rho = 1.13%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.58</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>2725 (107)</td>
<td>24.1 (3500)</td>
<td>3φ14, $\rho = 1.13%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.08</td>
<td></td>
<td>$\phi 8$ at 120</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>150 x 300 (5.9 x 11.8)</td>
<td>2725 (107)</td>
<td>26.4 (3830)</td>
<td>3φ14, $\rho = 1.13%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.08</td>
<td></td>
<td>$\phi 8$ at 120</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>100 x 200 (3.9 x 7.9)</td>
<td>1225 (48)</td>
<td>19.9 (2890)</td>
<td>2φ12, $\rho = 1.30%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.12</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>100 x 200 (3.9 x 7.9)</td>
<td>1225 (48)</td>
<td>21.0 (3050)</td>
<td>2φ12, $\rho = 1.30%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.12</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>100 x 200 (3.9 x 7.9)</td>
<td>1225 (48)</td>
<td>20.4 (2960)</td>
<td>2φ12, $\rho = 1.30%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.12</td>
<td></td>
<td>$\phi 8$ at 80</td>
<td></td>
</tr>
</tbody>
</table>

(a) Specimen properties

<table>
<thead>
<tr>
<th>Steel type</th>
<th>Diameter, mm (in.)</th>
<th>Yield strength, MPa (ksi)</th>
<th>Ultimate strength, MPa (ksi)</th>
<th>Elongation, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi 8$</td>
<td>8 (0.31)</td>
<td>290 (42)</td>
<td>455 (66)</td>
<td>33</td>
</tr>
<tr>
<td>$\phi 12$</td>
<td>12 (0.47)</td>
<td>350 (51)</td>
<td>540 (78)</td>
<td>26</td>
</tr>
<tr>
<td>$\phi 14$</td>
<td>14 (0.55)</td>
<td>340 (49)</td>
<td>535 (78)</td>
<td>27</td>
</tr>
</tbody>
</table>

(b) Reinforcement properties
Experimental Results and Discussion

Under loading, first flexural cracking occurred at the interface of the beam and center column stub. As expected, cracking loads were lower for beams with longer spans. First cracking was followed by cracking in the beam on top of the side supports. Horizontal reaction forces, indicative of the onset of membrane action, began to increase after flexural cracking.

First yielding of the reinforcement occurred at midspan due to positive bending; yielding at the supports followed due to negative bending. Horizontal reaction forces continued to increase past the peak imposed load until concrete crushing occurred at midspan. As midspan deflections increased, axial forces began to transition from compression to tension as compressive membrane action gave way to tensile catenary action where the loads were being carried primarily by the reinforcement in tension. This transition to catenary action resulted in a slight increase in the vertical load but only after a large drop in capacity due to concrete crushing. Failure occurred due to rupturing of the bottom reinforcement at midspan due to flexure. A sample plot showing both vertical load and horizontal reaction forces versus midspan deflection is shown in Figure 2.20.

Beam load capacities were calculated using a classical plastic collapse mechanism which assumed plastic hinges at midspan and at the supports without considering the contributions of membrane action or shear. These capacities were then compared to experimental results; a strength enhancement factor was calculated as the ratio of peak experimental load to peak calculated load. This factor was found to range from 1.53 to 2.63, indicating a significant increase in flexural strength due to compressive membrane action.

Contrary to Ghazi’s findings [9], the effects of axial stiffness were found to have little effect on specimen strength when calculated using an analytical model based on plastic analysis for longitudinally restrained one-way slabs. An 80% drop in support stiffness yielded only a 10% drop in predicted ultimate load capacity.
The effects of axial restraint on internal beam forces was also investigated: axial forces and moments in specimen B3 were measured and normalized by corresponding calculated peak capacities for a plastically analyzed beam without membrane action. In Figure 2.21, the internal midspan moment $M$, support moment $M'$, and axial force $N$ are normalized by plastic midspan capacity $M_0$, plastic support capacity $M'_0$, and maximum compressive axial force $N_{\text{max}}$. The applied load $P$ was normalized by $P_{yu}$, the load at which the plastic collapse mechanism formed without membrane action. Any of these normalized ratios can be considered to be a strength enhancement effect if the values are greater than 1.0, which would be indicative of the beam supporting more load or internal forces than a traditional plastic analysis would allow.

Figure 2.20: Vertical load and horizontal reaction force versus normalized midspan deflection for series A beams showing yielding at supports and peak vertical load [14].

Figure 2.21: Normalized applied load, horizontal reaction force, and bending moments at midspan and at supports versus normalized center deflection [14].
Of note is the peak value of $P/P_{yu}$, which is less than the peak values of $M/M_0$ and $M'/M'_0$. This is representative of the P-Δ effect, which increases the effective moment demand on the beam due to load path eccentricities. This led to the load $P$ reaching its maximum value before maximum bending moments were achieved as effective moments were higher than those produced by the applied load.
2.2.2 Vecchio and Tang (1990)

To investigate the effects of compressive membrane action in reinforced concrete slabs, two plane frame specimens representative of a collapsed warehouse structure were built and tested at the University of Toronto. The following is a summary of the test results and discussion by the authors [17]; the results will be used to validate the finite element analysis program Augustus-2 prior to the parametric CPB barricade study. The authors’ simulation results, while representative of specimen behaviour, will not be discussed.

The warehouse that collapsed was a four-storey reinforced concrete structure with slab floors supported by a series of columns. At the time of collapse, the estimated load on several slab bays of the third floor was over 48 kN/m$^2$ while the design load was only 10.8 kN/m$^2$; this increase in strength was attributed to compressive membrane action and studies were performed to determine its contributions to the strength of axially restrained members. The authors also wanted to determine whether a two-way slab system such as that in the warehouse could be modelled with a plane frame both physically and with software simulations.

Experimental Program

Two half-scale models of the warehouse floor were constructed as shown in Figure 2.22a with similar reinforcement percentages and layouts to the actual structure (Figure 2.23 and Table 2.2). Both models were identical, but one specimen (TV2) was fixed against horizontal displacement while the other (TV1) remained free to expand longitudinally under vertical loading. The slab ends of both specimens were fixed against vertical deflection and the column bases were fixed against horizontal and vertical displacements but free to rotate (Figure 2.22b).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>f'c (MPa)</th>
<th>fty (MPa)</th>
<th>f (×10$^{-3}$)</th>
<th>E (MPa)</th>
<th>Bar size</th>
<th>A (mm$^2$)</th>
<th>f1 (MPa)</th>
<th>f2 (MPa)</th>
<th>E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV1</td>
<td>29.7</td>
<td>3.2</td>
<td>1.95</td>
<td>30 500</td>
<td>No. 10</td>
<td>100</td>
<td>454</td>
<td>649</td>
<td></td>
</tr>
<tr>
<td>TV2</td>
<td>30.2</td>
<td>3.6</td>
<td>1.99</td>
<td>32 200</td>
<td>No. 15</td>
<td>200</td>
<td>484</td>
<td>646</td>
<td>200 700</td>
</tr>
</tbody>
</table>

8 mm 51 607 607 207 500

The test setup is shown in Figure 2.24. The columns were supported on pin-roller assemblies which were free to move horizontally while maintaining the same relative distance from each other by way of displacement-controlled actuators. Two 25mm steel rods were anchored to the slab ends and to a strong floor to prevent vertical displacement.
To ensure that slab TV2 did not displace horizontally, a displacement-controlled actuator was used to maintain zero displacement of the slab end. The other end of the TV2 slab was braced against a strong wall. Vertical loads were applied to the center of the frames with two displacement-controlled actuators bearing on a spreader beam to simulate a uniform line load across the width of the slab.

**Experimental Results and Discussion**

In both slabs, first flexural cracking was observed at midspan followed by transverse flexural cracks at the supports. After yielding of the bottom reinforcement at midspan, radial cracks began to form on the top slab surface near the columns. An actuator malfunction resulted in premature failure of specimen TV1 after formation of radial cracks near the columns, but TV2 was tested to failure. In slab TV1, decreases in slab stiffness were evident upon first cracking as well as first yielding of the reinforcement at midspan while the stiffness of TV2 did not drop until close to first yielding (Figure 2.25). The stiffness of TV2 was similar to that of TV1 prior to first cracking, but was greater afterwards owing to the effects of membrane action. Because TV1 failed prematurely due to equipment malfunction, the difference between the full load-deformation response of TV1 and TV2 could not be determined.

The lateral reaction forces in the column bases increased similarly throughout loading for both specimens, indicating the small contribution of the columns to lateral slab restraint (Figure 2.26b). The horizontal reaction force in TV2 (not present in TV1 as it was free to move horizontally) increased more quickly after first cracking, showing the onset of compressive membrane action and explaining the increase in stiffness seen in Figure 2.25. Near ultimate loads, the axial compressive force in the slab reached almost 4.5
times the applied load (Figure 2.26a). Because horizontal restraint forces imposed by the columns (Figure 2.26b) were similar between specimens, the majority of the compressive membrane action was induced by the horizontal slab restraints in TV2.

For a given load, reinforcement strains at both midspan and at the supports in specimen TV1 were higher than those in TV2. Cracking patterns in both specimens were indicative of one-way behaviour: cracks on the top and bottom of the slabs formed transversely across the entire width (Figure 2.27) with radial cracking near the columns only occurring at advanced load stages.

Secondary moments due to load eccentricity increased along with axial load and midspan displacement; their negative effects on strength were calculated using free-body
Figure 2.24: Test setup for specimens TV2. Specimen TV1 has similar setup without horizontal restraints [17].

Figure 2.25: Load versus midspan deflection for specimens TV1 and TV2 [17].
diagrams (Figure 2.28) and found to consume approximately 20% of flexural capacity. It is important, therefore, to properly account for secondary (P-∆) effects in the simulation of lightly reinforced structures subject to membrane action.
Chapter 3

Finite Element Modelling

3.1 Augustus-2, Response-2012, and Membrane-2012

As with any useful model or simulation, accurate and precise results are essential. Prior to cracking under load, reinforced concrete behaves in a fashion which is easy to predict with linear relationships and is well-understood. However, after a reinforced concrete member cracks, its behaviour can only be characterized by non-linear relationships which are complex and difficult to model. One example of a finite element analysis program which is able to model reinforced concrete structures is Augustus-2, developed by Professor Evan Bentz at the University of Toronto. Augustus-2 is able to generate accurate results for models ranging from individual reinforced concrete members to complex structures [2]. In order to ensure accuracy, concrete members built for experiments were modeled in Augustus-2 and the analysis results confirmed with experimental ones.

Augustus-2 directly models the behaviour of reinforced concrete accurately after cracking when it transitions into the non-linear regime [5]. Elements use a fibre model for axial loads and moments as well as the Modified Compression Field Theory (MCFT) for shear response. The MCFT is a proven model [3] for predicting the shear response of reinforced concrete sections and is the basis for shear design in the Canadian Standards Association Standard A23.3 - Design of Concrete Structures as well as the American Association of State Highway and Transportation Office Bridge (AASHTO) Load Resistance Factor Design (LRFD) Bridge Design Specifications. More information regarding finite element formulations in Augustus-2 can be found in the theses of Yeung [18] and Ganji [8].

Augustus-2 models are a combination of beam and column Response elements and Membrane joint regions defined respectively in programs Response-2012 and Membrane-2012. Additional truss rod elements can be used by creating a standard reinforced concrete
section in Response-2012 with the desired steel rod area and using the STEEL flag when building the model to ignore the surrounding concrete. Typical program interfaces are shown in Figure 3.1. All three programs were developed by Bentz for structural simulation and analysis purposes. Models are defined in a text file using a series of nodes connected by Response and Membrane elements. Each Response beam element is defined as a two-dimensional cross section with configurable parameters including material properties and reinforcement detailing. While Response-2012 is capable of performing sectional and member analyses by itself, it is incapable of doing so with elements consisting of more than one member.

Elements defined in Membrane-2012 are three-dimensional wall elements with reinforcement detailing in longitudinal and transverse directions. Membrane elements are used at joint regions between beams and columns in order to model the intersection of longitudinal reinforcement from perpendicular axes.

In order to prevent a physically unlikely shear failure in Augustus-2 models near supports, the corresponding strong beam sections in Augustus-2 were transversely reinforced beyond their normal level. This was because the equations used in Augustus-2 to model
shear behaviour do not consider strut-and-tie style strength mechanisms (arch action) that will be present near supports or concentrated loads. In addition, due to the confining effect of the support or load application points, it is appropriate to increase the post-peak ductility of the concrete in this region. In this thesis, approximately 0.5% of transverse reinforcement was included in models areas $d_v$ from the face of the support or the location of applied loads. Also, the default concrete stress-strain curve is changed to that shown in 3.2. These strong sections extend an approximate distance $d_v$ from any support or applied load.

![Figure 3.2: Modified concrete compressive stress-strain curve with increased post-peak ductility in strong regions.](image-url)
Typical Augustus-2 Barricade Modelling

The typical Augustus-2 beam model used to simulate a two-dimensional strip of a CPB barricade is shown in Figure 3.3. The beam height is representative of the thickness of the barricade; applied pressures from the CPB act vertically in the model on the top surface of the beam. All simulations are performed for a 1 metre section of barricade height which corresponds to a beam width (in and out of the page) of 1 metre.

![Figure 3.3: Typical barricade model as rendered in Augustus-2](image)

The rock walls surrounding the barricade are modelled with strong end supports connected to steel truss rods which act as rotational and axial restraints. The supports are larger Response elements which have a significantly higher capacity (10 000 MPa compressive and tensile strength) to prevent failure at the ends. The degree to which the model is restrained is based on the cross-sectional area of the bars; this is further discussed in Section 3.4.

The beam is composed of various Response elements, each with its own reinforcement and concrete profile (Figure 3.4). The reinforcement can be laid out and varied to approximate the reinforcement content of a series of typical barricades. As mentioned in Section 3.1, strong Response-2012 elements are used up to an approximate distance $d_v$ from the supports and point load to prevent premature shear failure which would not occur in reality.
Beam cross-sections and material properties of the concrete and reinforcing steel are defined in Response-2012. While the software is capable of handling a variety of different and more complex cross-sectional shapes (I-beams, T-beams, etc.) only rectangular (beam elements) and circular sections (for truss rods) are used in these simulations.
3.3 Secondary Moment Effects

As shown by Vecchio and Tang (Section 2.2.2), secondary moment effects (i.e. the P-∆ effect) have a significant weakening effect on capacity in axially restrained beams. This effect is typically eclipsed by the benefits provided by compressive membrane action, but should be included in analyses in order to prevent an overestimation of strength. Augustus-2 does not currently account for secondary moment effects in its analyses. In order to facilitate its inclusion in future versions of the software, a proof of concept analysis is performed to show the likely method by which the P-∆ effect will be implemented.

A simple vertical cantilever model in Augustus-2 is the basis for this proof of concept (Figure 3.5). It is fixed at its bottom while a constant vertical load is applied at the top, in the middle of the element. A monotonically increasing horizontal load is then applied to the cantilever which causes lateral displacement. Since the P-∆ effect is not currently included in analyses, this model would not be subject to any additional moment demand despite the applied axial force.

![Figure 3.5: Augustus-2 model of vertical cantilever used in P-∆ calculations](image)
To account for the increased moments in the beam due to this axial force, an iterative process is used which, for a given applied load, imposes an additional moment on each element equal to the axial force $P$ multiplied by the distance $\Delta$ for that element (Figure 3.6). As each element is of a finite height, $\Delta$ is taken as the distance from the middle of the element to the line of action of the axial force $P$. The moment is imposed in the Augustus-2 model by applying a force couple $F$ on the top and bottom nodes of each element (Figure 3.7). This couple acts across the element width $h$ and thus applies a moment $P\Delta = Fh$ to both the top and bottom of each element. For a given element, these applied forces are self-equilibrating and thus will not result in an imbalanced sum of forces.

Figure 3.6: P-\(\Delta\) geometry of vertical cantilever.

Figure 3.7: Typical Augustus-2 cantilever element subject to force couple calculated from P-\(\Delta\) effect.
Because the inclusion of the P-Δ moments would cause additional lateral displacements to occur, the analysis must be run multiple times to determine the changed displacements for each node in the model. Once the new lateral displacements are found, the force couples on each element are updated and the model run again at the given horizontal load. When the displacements no longer change between iterations, the process has converged to a correct value of additional imposed moment due to the P-Δ effect. For the example shown, between two to three iterations were required for each load stage to reach convergence.

Figure 3.8 shows the load-displacement response of the test cantilever in Augustus-2 both with and without P-Δ effects. If P-Delta effects are considered, there is a loss of beam capacity after first cracking through to failure. The approximate loss in capacity is 5%; this is considered to be low, as Vecchio and Tang calculated an approximate drop of 20% of flexural capacity in their experiments [17].

![Figure 3.8: Load versus displacement response of Augustus-2 models with and without P-Δ effect.](image)

As a confirmation of the results generated by Augustus-2 for the example cantilever, the lateral displacement of its tip at a horizontal load of 100 kN is calculated with the Moment-Area theorem. The curvature plot of the beam is shown in Figure 3.9 with simplified curvature shapes to simplify Moment-Area calculations. With these simplifications, the displacement is calculated as:
Figure 3.9: Curvature plot of example cantilever with horizontal applied load of 100 kN.

Area A = \( \frac{0.273 \cdot 4500}{2} \) = 614.3

Displacement A = \( \frac{614.3 \cdot 4500 \cdot 2}{3} \) = 1.843 mm

Area 2 = 0.273 \cdot 5500 = 1502

Displacement 2 = 1502 \( \left( \frac{5500}{2} \cdot \frac{4500}{2} \right) \) = 10.89 mm

Area 3 = \( \frac{(3.625 - 0.273)5500}{2} \)

Displacement 3 = 9218 \( \left( 4500 + \frac{2}{3} \cdot 5500 \right) \)

Total horizontal tip displacement = 1.843 + 10.89 + 75.28 = 88.0 mm
This tip displacement calculated with the Moment-Area theorem is within 1% of the Augustus-2 result of 88.8 mm. Therefore, the simulation results can be considered accurate.
3.4 Rock Stiffness

As found by Ghazi [9], the stiffness of the rock into which the barricade is anchored has a large influence on barricade strength. Ghazi’s simulations varying support stiffness showed that barricade capacity could increase over three-fold between flexible and stiff boundary conditions. More flexible boundary conditions also induced yielding of reinforcement at lower midspan deflections. The research, however, did not determine actual rock stiffness based on material properties, but rather assumed various values of stiffness in a parametric study.

In order to better predict the effects of rock stiffness, equations are derived and then used in Augustus-2 simulations. A combination of prior works by Timoshenko [16]; Jaeger [11]; and Davis and Selvadurai [7] are used to derive the stiffness equations based on rock material properties and barricade geometry. The rock walls are assumed to be linear elastic semi-infinite homogeneous half-spaces, or infinitely large plates of infinite depth. Two stiffness terms are considered: axial and rotational. In both cases, rock displacements are found and then compared with deformations or rotations to determine a stiffness term.

Axial Rock Stiffness

Davis and Selvadurai [7] present a series of equations to find the displacement at any point in a loaded area on an infinite plate which is derived from an integration of the classic Boussinesq solution. The derivation begins with the displacement \( w \) at a corner \( A \) of a triangular area on a plate of dimensions \( a > b \) uniformly loaded under stress \( \sigma \) (Figure 3.10a) given in eq. (3.1). The plate has shear modulus \( G \) and Poisson’s ratio \( \nu \).

\[
 w_p = \frac{\sigma(1 - \nu)a}{2\pi G} \sinh^{-1}\left(\frac{b}{a}\right) \tag{3.1}
\]

Superimposing two triangles and reversing variables \( a \) and \( b \) gives the displacement at the corner of a rectangular-shaped load (Figure 3.10b), which is more commonly encountered:

\[
 w_p = \frac{\sigma(1 - \nu)a}{2\pi G} \left[ \sinh^{-1}\left(\frac{b}{a}\right) + \frac{b}{a} \sinh^{-1}\left(\frac{a}{b}\right) \right] \tag{3.2}
\]
It is now possible to find the displacement at any point in a rectangular loaded area by superimposing the displacements from four smaller sub-rectangles as shown in Figure 3.10c using eq. (3.2) for the appropriate corner of each sub-rectangle.

To determine a proper axial stiffness expression \( K_{\text{axial}} \), measured in N/mm) for use in Augustus-2 models, the displacement of a loaded area under a given load must be determined. With eq. (3.2), it is possible to determine a displacement field under a loaded area by discretizing the area into a number of evenly spaced points and finding the displacement at each point (Figure 3.11).

A weighted average displacement under the entire plate can be found by accounting for the fact that displacements at the four corners and sides of the loaded area have a smaller effective ‘influence’ area: \( \frac{1}{4} \) at the corners and \( \frac{1}{2} \) along the sides. Thus, the average displacement \( w_{\text{avg}} \) for a loaded rectangular area is
Figure 3.11: Sample discretized displacement field for a rectangular loaded area with weighting.

\[
w_{\text{avg}} = \frac{\frac{1}{4} \sum_{i=1}^{4} w_i + \frac{1}{2} \sum_{j=1}^{m} w_j + \sum_{k=1}^{n} w_k}{4 + m + n}
\] (3.3)

where \( m \) and \( n \) are the number of side nodes and interior nodes, respectively.

While this method is derived from the classical Boussinesq solution, it would be time-consuming to set up the discretized displacement field determine the average displacement for loaded areas of different dimensions and aspect ratios. A simpler equation by Jaeger \[11\] gives the average deflection under a loaded rectangular plate of area \( A_{\text{plate}} \) on a half-space as:

\[
w_{\text{avg}} = \bar{m} \langle \Sigma P \rangle (1 - \nu^2) \frac{1}{E_{\text{rock}} \sqrt{A_{\text{plate}}}}
\] (3.4)

and can be rewritten in terms of the plate aspect ratio \( \lambda \) (longer to shorter dimension) as

\[
w_{\text{avg}} = \bar{m} \sqrt{\lambda} \langle \Sigma P \rangle (1 - \nu^2) \frac{1}{E_{\text{rock}}}
\] (3.5)

where \( \bar{m} \) is a coefficient and \( \Sigma P \) is the total applied load. The coefficient \( \bar{m} \) is given by Jaeger \[11\] only for certain plate aspect ratios ranging from 1 to 100, but an equation (Figure 3.12) can be fitted to the given values of \( \bar{m} \) multiplied by \( \sqrt{\lambda} \) such that eq. (3.5) can be used for any aspect ratio between the limits 1 to 100. Note that for a value of \( \lambda = 1 \), the value of \( \bar{m} \) is 0.95 \[11\].
The fitted equation for \( C = \bar{m}\sqrt{\lambda} \) as a function of plate aspect ratio \( \lambda \) is:

\[
C = 0.6017 \ln(\lambda) + 0.8941
\]

which has a \( R^2 \) value of 0.999 can be rewritten and simplified as:

\[
C = 1.4\log\lambda + 0.9
\]

while maintaining a \( R^2 \) value of 0.950. Eq. (3.5) can then finally be rewritten in stiffness form (\( K_{axial} \), units N/mm) as:

\[
\frac{\Sigma P}{w_{avg}} = K_{axial} = \frac{E_{rock}}{C(1 - \nu^2)}
\]

Figure 3.12: Given values and fit equation for coefficient \( C \) in eq. (3.5) versus aspect ratio.

While this solution proposed by Jaeger is convenient, it is presented in his book [11] with no derivation. Also, the source of the coefficient \( \bar{m} \) is not explained. To confirm that eq. (3.5) gives accurate displacements, solutions were compared to those given through the previously developed displacement field method (eq. (3.3)) for all aspect ratios with a given coefficient of \( \bar{m} \) by Jaeger. As shown in Figure 3.13, the correlation between answers calculated with both methods is almost exact. Therefore, eq. (3.5) can be considered as accurate.

To determine the cross-sectional area of the axial restraint rod, \( A_{s,axial} \) (Figure 3.3, in Augustus-2 that would give an equivalent stiffness to eq. (3.8), eq. (3.8) is equated to the traditional axial stiffness term for an elastic member \( \frac{AE}{L} \):
Figure 3.13: Correlation between displacement field and Jaeger methods for given aspect ratios.

\[
\frac{A_{s,axial}E_s}{L_{rod,a}} = \frac{\Sigma P}{w_{avg}} = \frac{E_{rock}}{C(1 - \nu^2)} \tag{3.9}
\]

The first and third terms can be rearranged to obtain a term for the required amount of steel \(A_{s,axial}\):

\[
A_{s,axial} = \frac{E_{rock}}{E_s} \frac{L_{rod,a}}{C(1 - \nu^2)} \tag{3.10}
\]
Rotational Rock Stiffness

The rock wall’s rotational stiffness was determined through use of Timoshenko’s equations for the vertical displacement profile of an infinitely large plate under a point force [16]. A series of linearly varying point loads simulating a moment (Figure 3.14) is applied to a semi-infinite surface and the resulting displacements from each point load are superimposed to create a displacement profile. The slope of this profile is then compared to the imposed moment to determine the rock’s rotational stiffness.

\[ M = K_{rot} \theta \]  \hspace{1cm} (3.11)

where \( K_{rot} \) is the rotational stiffness term for the rock wall and \( \theta \) is the amount of rotation in the rock wall due to the imposed moment. This is analogous to the general axial stiffness equation \( P = K_{axial} \Delta \).

If the rotation \( \theta \) of the rock wall can be found for a given applied moment \( M \), then the rotational stiffness is

\[ K_{rot} = \frac{M}{\theta} \]  \hspace{1cm} (3.12)

In order to find \( \theta \), Timoshenko’s equation for the vertical displacement profile \( v \) (Figure 3.15) of an infinitely large plate under a single point load is used [16]:

\[ v = \frac{2P}{\pi E} \log \frac{d}{r} - \frac{(1+\nu)P}{\pi E} \]  \hspace{1cm} (3.13)

where \( P \) is the magnitude of the point load; \( d \) is the depth from the surface at which vertical displacement is arbitrarily assumed to be zero; and \( r \) is the distance of the point of interest of the plate from the load. While with eq. (3.13) it is possible to determine the
displacements of any point in the infinite plate, only the plate’s surface displacements are of interest. Therefore, \( r \) is taken to be the distance from a point on the plate’s surface to the point load.

The variable \( d \) introduces an arbitrary aspect to eq. (3.13) which can present problems if absolute displacements are desired, as the choice of a zero-displacement depth causes the value of \( v \) to vary significantly. This is why eq. (3.13) was not used to determine axial stiffness in the previous section. However, since the only value of interest to the rotational stiffness is the slope of the displacement profile, any chosen value of \( d \) will suffice. Since the loads remain the same, the slope of the profile will remain the same regardless of the chosen depth \( d \).

The displacement calculated by eq. (3.13) is undefined directly underneath the point load, as the distance \( r \) becomes zero. This problem is addressed by calculating the displacement field with a sufficiently small increment of \( r \) to closely approximate the displacement under the load.

In order to accurately represent a displacement profile of a uniformly distributed load with a series of point loads, multiple profiles were plotted, each with the same total load but a different number of loads over a constant width. As seen in Figure 3.16, fewer loads result in a peaked profile which is not representative of an applied moment condition. A larger number of loads yielded a smoother displacement profile; nine point loads provided an acceptable balance of accuracy and calculation efficiency. In the case of the linearly varying load profile, the middle load would have been zero by symmetry so eight equally spaced loads were used instead to avoid redundancy.
A general set of equations was developed to determine the magnitude of each point load in Figure 3.17 where $P$ is the magnitude of the end loads and $w$ is the loaded width of the plate (equivalent to the thickness of a barricade).

The slope of the linear variation is $\frac{2P}{w}$ and each load is an equal distance $\frac{w}{7}$ apart. The equation of the load variation is $P - \frac{2P}{w}x$, where $x$ is the distance along the width of the loaded area. The total moment imposed by the loads in Figure 3.17 is:

$$M = Pw + \left( P - \frac{2P}{w} \right) \frac{5w}{7} + \left( P - \frac{4P}{7} \right) \frac{3w}{7} + \left( P - \frac{6P}{7} \right) \frac{w}{7} \quad (3.14)$$
Figure 3.17: Linearly varying point loads representative of a pure applied moment

which simplifies to

\[ P = \frac{49M}{84w} \]  

Eq. (3.15) gives the value of \( P \) in Figure 3.17 for a given moment \( M \) and loaded width \( w \) with which the complete series of point loads can be calculated. Following this, eq. (3.13) is used to calculate the displacement profile for each point load; these profiles are then superimposed to generate a complete profile for a pure applied moment condition (Figure 3.18). A linear fit of the resulting profile across the width of the loaded area shows that the profile itself is approximately linear as would be expected; the slope of this profile is thus the true rotation of interest, \( \theta \), in eq. (3.12). A barricade bearing on the rock surface and applying a rotation would also be subject to large compressive forces (from membrane action) which would maintain contact between the two. Thus, the two surfaces should be expected to deform together prior to inelastic behaviour.

With the slope of the rock wall now calculated based on a linear fit to Timoshenko’s theoretical displacement equations [16], the next step is to develop a general equation for the slope of the rock wall based on applied moment, rock material properties, and geometry of the barricade. The end goal is to determine the required rotational stiffness of the Augustus-2 model to accurately represent the rock wall boundary based on the aforementioned properties. The equation developed is compared to the theoretical Timoshenko solution in order to determine accuracy.

The initial assumptions were that the slope of the rock wall was directly proportional to the applied moment \( M \) (as before, per unit height) and inversely proportional to the rock modulus of elasticity \( E_{rock} \); a higher moment would cause higher rotations while a
higher modulus would stiffen the rock and reduce rotations. A larger barricade would also decrease rotations because a given moment imposed by the barricade would be spread over a larger area. Other factors investigated were the Poisson’s ratio of the rock, second moment of area of the barricade, and barricade aspect ratio, but they showed no meaningful correlation with the slope.

As shown in Figure 3.19, the slope of the displacement profile is directly proportional to the applied moment $M$. For example, a doubling of moment doubles the slope of the wall. In Figure 3.20, the slope of the rock wall is inversely proportional to the rock’s modulus of elasticity $E_{\text{rock}}$: a doubling of $E_{\text{rock}}$ halves the slope.
Figure 3.20: Effect of rock modulus of elasticity on slope of rock wall

Dimensionally, since the slope of the displacement profile is in radians (unitless), an inverse squared distance would be required to satisfy this given the existing moment and modulus relationship as such:

\[
[\theta] = \frac{\text{moment/unit height}}{\text{modulus} \times \text{distance}^2} = \left[ \frac{\frac{N\text{mm}}{\text{mm}}}{\frac{N}{\text{mm}^2}} \text{mm}^2 \right]
\]  

(3.16)

As the barricade comes under pressure and rotates against the rock wall, it does so across its thickness. Therefore, it can be expected that the thicker the barricade, the more resistant to rotation it will be. If a comparison is made between the term \( \frac{M}{E_t^2} \), where \( t \) is the thickness of the barricade, and the associated theoretical rotation from Timoshenko using the same geometry and material properties, an approximately constant relationship is found (Figure 3.21). This factor increases slightly with increasing thickness. According to Helinski et al [10], typical bulkheads are usually built with a thickness between 200-400 mm. This corresponds to a dimensionless factor in Figure 3.21 between approximately 2.09 and 2.17. If this factor is simplified to 2.2 for all practical thicknesses, the following relationship can be developed for the slope of the rock wall subject to applied moment from an abutting barricade:

\[
\theta = 2.2 \frac{M}{E_{\text{rock}}t^2}
\]  

(3.17)

or
The results of eq. (3.17) are compared to the actual slopes calculated from a linear fit of the Timoshenko displacement profile in Figure 3.22 for a range of barricade thicknesses from 100 to 800 mm. In all cases, the predicted slope is greater than the slope calculated from the displacement profile. This would result in a lower predicted rock wall stiffness, a conservative result.

With eq. (3.12), eq. (3.17) can be rewritten in stiffness form \((K_{rot}, \text{ units } \frac{N\text{mm}}{mm\text{-rad}})\) as:

\[
K_{rot} = \frac{E_{rock}t^2}{2.2}
\] (3.19)

To determine the cross-sectional area of the rotational restraint rods, \(A_{s,rot}\) (Figure 3.3, eq. (3.12) is used with the following equations based on the geometry of the rotational restraints in the Augustus-2 model (Figure 3.23) used to represent a barricade:

\[
F_{rod,r} = \theta a \frac{A_{s,rot}E_s}{L_{rod,r}}
\] (3.20)

\[
M = F_{rod,r} \cdot 2a
\] (3.21)

Equating eqs. (3.20) and (3.21) yields
Figure 3.22: Slope of Timoshenko displacement profile divided by predicted slope for various barricade widths

\[ M = 2a^2\theta \frac{A_{s,rot}E_s}{L_{rod,r}} \]  

(3.22)

Substituting eq. (3.18) into eq. (3.22) yields

\[ \frac{E_{rock}t^2}{2.2}\theta = 2a^2\theta \frac{A_{s,rod}E_s}{L_{rod,r}} \]  

(3.23)

Solving eq. (3.23) for \( A_{s,rot} \) and removing \( \theta \) results in the following expression:

\[ A_{s,rot} = \frac{E_{rock}L_{rod,r}t^2}{E_s4.4a^2} \]  

(3.24)
Figure 3.23: Typical geometry of rotational restraint in Augustus-2 model of barricade
3.5 Arch Modelling

Previous work done by Ghazi [9] investigated the behaviour of flat bulkheads which were simpler to model but more susceptible to tensile-related failures on the exterior bulkhead face. In theory, a bulkhead arched towards the direction of the CPB pressure would be stronger than a flat profile due to an arch’s ability to carry loads primarily in compression. Since concrete is significantly stronger in compression than in tension, this arch effect is desirable. This increase in strength, however, is balanced by the need for a strong support at the base of the arch to resist its sideways thrusting action and sliding shear along the rock wall. The level of restraint provided against these spreading forces by the surrounding rock walls is therefore more important for arched barricades than for flat ones. Bulkheads with arched profiles are commonly used in industry and the effects of curvature on barricade strength were previously studied by Helinski [10].

A parametric study of the effects of arch geometry on barricade strength will be carried out in this thesis. A limitation of the Augustus-2 modelling software is that it can only employ rectangular, triangular, or rod elements. A program was written in MATLAB to create an approximation of an arched structure out of a series of rectangular and triangular elements based on user-defined parameters of horizontal arch length $L$, barricade thickness $t$, and arch angle $\alpha$ (Figure 3.24). This program then outputs node and element assignments in text format as required by Augustus-2 to create the arch model.

The initial models used alternating rectangular Response beam elements and triangular Membrane wall elements to form the arch (Figure 3.24). However, the aspect ratio of the triangular Membrane elements was large, making the elements very narrow. This yielded poor results in simulation, so truss rods connecting the outer nodes of the arch model were used instead. A concern was that truss rods could transmit axial forces but had no way of transmitting shear between elements. If an arch were shear-critical, this may not be reflected in a model using truss rods. To verify that eccentric beam elements connected by truss bars would provide similar results as a straight beam with no special connections, a comparison was made between a standard cantilever and one that was slightly askew but connected with truss bars and having the same effective length (Figure 3.25). The eccentricity of the test cantilever was varied by changing the angle between beam elements, $\omega$, to determine its effect on behaviour. The same beam elements were used in both models, so reinforcement layout was identical. Both cantilevers were subject to point loading at their ends.
Figure 3.24: Typical arch model (a) in Augustus-2 with small truss rods (in red) connecting rectangular beam elements along top edge (b) and arch angle (c)

Two cases were tested: eccentric beams with typical longitudinal and transverse reinforcement and eccentric beams which were shear-critical and had no transverse reinforcement. In both cases, multiple element angles were tested. Figure 3.26 shows the change in load-deformation response of eccentric cantilever models in Augustus-2 compared
to a normal cantilever; all models had both longitudinal and transverse reinforcement present. As the angle $\omega$ between beam elements increases, the response becomes somewhat less accurate as can be expected. However, the difference is minimal even with an 8 degree angle between beam elements. As a typical arch model has element angles typically between 1 to 4 degrees, using truss rods instead of actual triangular Membrane elements would yield acceptably accurate results.

Figure 3.27 shows the load-deformation response for eccentric cantilevers which are shear critical at the fixed end due to a lack of transverse reinforcement. This case was tested to determine if truss rods provided an acceptable prediction of shear-critical behaviour. Behaviour of the eccentric cantilever beams is almost indistinguishable from the normal case until an element angle of 8 degrees. At this point, shear is no longer fully transmitted from the support to the neighbouring element, resulting in continued deformation in a fashion similar to that of the transversely reinforced beam previously tested which failed in flexure. Based on these results, an arch with an angle between beam elements of greater than 6 degrees may yield inaccurate results. However, as most arches in the parametric study have element angles less than 4 degrees, simulation results can still be considered acceptable.
Figure 3.26: Applied load versus vertical tip displacement for normal and eccentric cantilevers including longitudinal and transverse reinforcement

Figure 3.27: Applied load versus vertical tip displacement for normal and eccentric cantilevers including only longitudinal reinforcement
Chapter 4

Results and Discussion - FEM Validation

Before a parametric study of CPB barricades can be performed with Augustus-2, the program must first be validated through comparison to existing experimental results. In this section, Augustus-2 models of specimens tested by Su et al. [14] and Vecchio and Tang [17] will be compared to corresponding experimental data to gauge simulation accuracy. Modelling details will be included in each section.

4.1 Comparison to Su et al.

Three series of beams were tested by Su et al. to investigate the effect of axially-restrained beams. A summary of the experimental program and results can be found in Section 2.2.1.

The Augustus-2 models were meshed with Response-2012 elements with the same reinforcement content as the specimens (Table 2.1)). The center column stub was removed, as the larger cross-section at midspan would have resulted in stiffer simulation response due to the neglect of yield penetration in the model. Because the test regions of interest in the specimens were the clear spans between the column stubs, changing the height of the midspan stub was deemed acceptable. The end column stubs were reinforced more heavily in the model than in the specimen to ensure that they would not fail or crack prematurely and affect simulation results.

Rotational and axial restraints were provided by truss rods with a cross-sectional area corresponding to the desired stiffness. As opposed to determining a stiffness value based on boundary properties as in Section 3.4, Su et al. published values of 1 000 kN/mm
axial stiffness and 17 500 kNm/rad rotational stiffness. As mentioned in Section 2.2.1, it is unclear how these stiffness values are defined; the assumption is that the given force or moment was required to move the support by a unit displacement or rotation. The expressions to determine the required truss rod cross-sectional area are simpler than those derived in Section 3.4:

\[
A_{s,\text{axial}} = \frac{K_{\text{axial}} L_{\text{rod}}}{E_s} \quad (4.1)
\]

\[
A_{s,\text{rot}} = \frac{K_{\text{rot}} L_{\text{rod}}}{2a^2 E_s} \quad (4.2)
\]

The geometry for eq. (4.2) is identical to that in Figure 3.23.

A typical Augustus-2 model is shown in Figure 4.1. The ends of the truss rods are fixed against horizontal and vertical displacements, while the end supports are fixed only against vertical displacements and are allowed to rotate about their centerline and move longitudinally. The center of the beam is only permitted to displace vertically to prevent any simulation instabilities. As the model is symmetric about its centerline, any displacements will also be symmetric; the midspan restraints will not affect results. The circular nodes shown in Figure 4.1 are Augustus-2 conditional nodes which allow elements of different sizes to interface with each other. The conditional nodes distribute all their forces to designated neighbouring nodes in proportion to their relative separation.

Figure 4.1: Typical Augustus-2 model of specimen by Su et al

The load-deflection results presented by Su et al. [14] compare favourably to those from Augustus-2 simulations, but only after the axial stiffness values were significantly reduced from those specified by the authors. As previously mentioned in Section 2.2.1 and as mentioned by Ghazi [9], the given stiffness values of 1000 kN/mm and 17 500 kN-m/rad were ambiguously defined and could also be difficult to measure in a laboratory environment. In order to obtain accurate results with Augustus-2, the cross-sectional
area of the axial stiffness rod was reduced almost twenty-fold from 1000 mm$^2$ to 60 mm$^2$. This corresponded to an axial stiffness reduction of 94 percent. Because this model was not sensitive to changes in rotational stiffness, no adjustments in the rotational rods were required. Comparisons between Augustus-2 and experimental results of one beam from each of the series A through C are presented; simulation results and behaviour are typical of all beams in each series. In all specimens, failure occurred due to flexure at midspan, where the bottom reinforcement ruptured.
Specimen A2

Figure 4.2 compares results from beam A2 and includes Augustus-2 results using both the author-specified axial stiffness values and the adjusted values which yield accurate results. Augustus-2 accurately predicts both the load-displacement behaviour and the development of horizontal restraint forces in the beam when the single axial stiffness value has been calibrated. As shown, however, simulations performed with given stiffness values result in an overly stiff and brittle beam with an over-prediction in capacity. Table 4.1 compares various experimental results to Augustus-2 predictions, which are generally accurate. Of note is Augustus-2’s under-prediction of cracking load, which is inaccurate but conservative.

The horizontal forces in the Augustus-2 prediction transition from net compression to net tension at the second drop in load capacity, something that is not reflected in the experimental data. In the simulation, the second drop in capacity occurs when the bottom steel in the element adjacent to the supports ruptures under negative moment. This local effect was an unintended by-product of joining the smaller beam element with the larger support element. Because the longitudinal reinforcing steel was not contiguous between the two elements, this caused a local stress concentration in the bottom reinforcement which effectively terminated before it reached the support.

As with the experiment, failure of beam A2 in Augustus-2 was due to flexure when the bottom steel at midspan ruptured.

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Prediction</th>
<th>(\frac{exp}{pred})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cracking load (kN)</td>
<td>30.0</td>
<td>13.3</td>
<td>2.26</td>
</tr>
<tr>
<td>Load at support yielding (kN)</td>
<td>148.0</td>
<td>102.3</td>
<td>1.447</td>
</tr>
<tr>
<td>Peak load (kN)</td>
<td>221.0</td>
<td>231.5</td>
<td>0.955</td>
</tr>
<tr>
<td>Deflection at peak load (mm)</td>
<td>56.4</td>
<td>55.16</td>
<td>1.023</td>
</tr>
<tr>
<td>Horizontal reaction at peak load (kN)</td>
<td>318.0</td>
<td>366.5</td>
<td>0.868</td>
</tr>
<tr>
<td>Maximum horizontal reaction (kN)</td>
<td>324.0</td>
<td>358.7</td>
<td>0.903</td>
</tr>
<tr>
<td>Deflection at max horiz reaction (mm)</td>
<td>59.3</td>
<td>55.5</td>
<td>1.069</td>
</tr>
</tbody>
</table>
Comparison to Su et al.

(a) Vertical load versus normalized midspan displacement

(b) Horizontal reaction force versus normalized midspan displacement

Figure 4.2: Load-displacement comparison for Su et al. beam A2
Specimen B1

Figure 4.3 compares results from beam B1 and includes Augustus-2 results using both the author-specified axial stiffness values and the adjusted values which yield accurate results. Augustus-2 accurately predicts the load-displacement behaviour and the development of horizontal restraint forces in the beam when axial stiffness values have been adjusted. As shown, however, simulations performed with given stiffness values result in an overly stiff and brittle beam with an over-prediction in capacity. The inaccuracy is not as severe as with beam A2, however.

The transition from net compression to tension after the second drop in capacity is present again as with specimen A2 due to the discontinuous longitudinal steel between cross-sections of different heights.

Table 4.2 compares various experimental results to Augustus-2 predictions, which are generally accurate but again under-predict first cracking loads and over-predict both the horizontal reaction at peak load and maximum horizontal reaction.

As with the experiment, failure of beam B1 in Augustus-2 was due to flexure when the bottom steel at midspan ruptured.

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Prediction</th>
<th>$\frac{exp}{pred}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cracking load (kN)</td>
<td>13.0</td>
<td>7.90</td>
<td>1.646</td>
</tr>
<tr>
<td>Load at support yielding (kN)</td>
<td>105.0</td>
<td>85.0</td>
<td>1.235</td>
</tr>
<tr>
<td>Peak load (kN)</td>
<td>125.0</td>
<td>155.0</td>
<td>0.806</td>
</tr>
<tr>
<td>Deflection at peak load (mm)</td>
<td>100.0</td>
<td>105.8</td>
<td>0.945</td>
</tr>
<tr>
<td>Horizontal reaction at peak load (kN)</td>
<td>211</td>
<td>303</td>
<td>0.696</td>
</tr>
<tr>
<td>Maximum horizontal reaction (kN)</td>
<td>225</td>
<td>326</td>
<td>0.690</td>
</tr>
<tr>
<td>Deflection at max horiz reaction (mm)</td>
<td>146.0</td>
<td>148.1</td>
<td>0.986</td>
</tr>
</tbody>
</table>
Comparison to Su et al.

Figure 4.3: Load-displacement comparison for Su et al. beam B1
Specimen C2

Figure 4.3 compares results from beam C2 and includes Augustus-2 results using both the author-specified axial stiffness values and the adjusted values which yield accurate results. Augustus-2 accurately predicts the load-displacement behaviour and the development of horizontal restraint forces in the beam when axial stiffness values have been adjusted. As shown, however, simulations performed with given stiffness values result in an overly stiff and brittle beam with an over-prediction in capacity. The inaccuracy is not as severe as with beams A2 and B1, however.

As with specimen A2, the discontinuous longitudinal reinforcement between cross-sections of different heights near the supports causes a local stress concentration in the bottom reinforcement of the beam element, causing it to rupture prematurely.

According to the authors, the C series of specimens had varying loading rates which affected the first cracking load. This could account for some of the inaccuracy in the Augustus-2 prediction of cracking load in Table 4.3.

As with the experiment, failure of beam C2 in Augustus-2 was due to flexure when the bottom steel at midspan ruptured.

<table>
<thead>
<tr>
<th>Table 4.3: Comparison of results for Su et al. beam C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
</tr>
<tr>
<td>Cracking load (kN)</td>
</tr>
<tr>
<td>Load at support yielding (kN)</td>
</tr>
<tr>
<td>Peak load (kN)</td>
</tr>
<tr>
<td>Deflection at peak load (mm)</td>
</tr>
<tr>
<td>Horizontal reaction at peak load (kN)</td>
</tr>
<tr>
<td>Maximum horizontal reaction (kN)</td>
</tr>
<tr>
<td>Deflection at max horiz reaction (mm)</td>
</tr>
</tbody>
</table>
Comparison to Su et al.

Figure 4.4: Load-displacement comparison for Su et al. beam C2

(a) Vertical load versus normalized midspan displacement

(b) Horizontal reaction force versus normalized midspan displacement
4.2 Comparison to Vecchio and Tang

Two reinforced concrete frames were tested to investigate the membrane effect in axially-restrained beams. Both frames were identical in geometry but had different restraint conditions: frame TV1 was free to expand in the axial direction while frame TV2 was axially restrained to induce compressive membrane forces. A summary of the experimental program and results can be found in Section 2.2.2.

The Augustus-2 models were meshed with Response beam and column elements and Membrane joint regions with the same reinforcement content as the specimens (Figure 2.23 and Table 2.2). Both models were identical save for the restraint conditions (Figure 4.5), as with the original experiments. Model TV1 was free to expand horizontally while TV2 was restrained in the same direction to prevent axial expansion. In both cases, however, rigid truss rods were attached to the slab ends to restrain vertical end movement but still allow for horizontal expansion. Another rigid truss rod connected the bases of the two pin-ended columns to ensure zero relative displacement between them but still allow for horizontal movement. Additional horizontal restraints were added at the midspan of both models as a precaution to prevent any horizontal shifting. Since the models were symmetric, these midspan restraints did not affect horizontal expansion. The frames were point-loaded at midspan.

Figure 4.5: Augustus-2 model of frame TV2

Figure 4.6 shows the load-displacement response of frame TV1 as modelled in Augustus-2 compared to the original experimental data. In the experiment, the specimen failed prematurely due to an equipment malfunction. The Augustus-2 prediction is conservative and over-predicts the frame stiffness both before and after cracking. As beam stiffness is difficult to predict in software, the result can be considered acceptable. Because of the TV1’s premature failure, no peak load comparisons could be made. If the displacements from the Augustus-2 prediction are multiplied by a factor of 1.7 (Figure 4.6), the results become far more accurate. A similar factor has also been found to work in other test comparisons [1].
Figure 4.6: Load-displacement plot for experiment TV1 and Augustus-2 predictions

Figure 4.7 shows the load-displacement response of frame TV2 as modelled in Augustus-2 compared to the original experimental data. As frame TV2 was restrained from axial expansion, compressive membrane action as well as secondary moment effects (P-∆) were present. Because Augustus-2 does not yet account for secondary moment effects which would weaken response, the prediction is stronger and stiffer particularly as the peak load is reached. If the same 1.7 factor is applied to the displacement predictions, the results are again quite accurate, however the weakening due to the P-∆ effect has still not been accounted for. This factor should be further examined in the future, as there have been previous analyses performed by the author and others which exhibit the same phenomenon.

Figure 4.8 shows the applied load versus total lateral slab end displacement for specimen TV1 and the Augustus-2 prediction. The slab is predicted to not expand axially until a load of approximately 20 kN which results in a stiffer prediction than the experimental data. Because specimen TV1 failed prematurely, experimental data is incomplete.

Figure 4.9 shows the applied load versus lateral slab end reaction force for specimen TV2 and the Augustus-2 prediction. The prediction is acceptable but induces lower axial loads in the slab for a given applied load. Because higher compressive membrane
Comparison to Vecchio and Tang

Figure 4.7: Load-displacement plot for experiment TV2 and Augustus-2 predictions

Figure 4.8: Load-axial elongation plot for specimen TV1 and Augustus-2 prediction
forces would provide a larger increase in strength, it would be expected that the lower simulated axial forces would result in a weaker slab. However, the prediction in Figure 4.7 shows that the Augustus-2 simulation is stronger than the experiment. This suggests that secondary moment effects may play a significant role in strength reduction, as they were not taken into account in simulations.

Figure 4.9: Load-slab end reaction plot for specimen TV2 and Augustus-2 prediction

Figure 4.10 shows the applied load versus lateral reaction at the column base for specimens TV1 and TV2 and their associated Augustus-2 predictions. Both Augustus-2 predictions for TV1 and TV2 are accurate. This also illustrates the small contribution of columns to providing axial restraint, as the lateral column reactions for both specimens are almost identical. The predominant source of compressive membrane forces is the restraint provided by the slab boundary conditions.
Figure 4.10: Load versus lateral column base reaction plot for specimens TV1 and TV2 and Augustus-2 predictions
Chapter 5

Results and Discussion - Parametric Modelling

To determine the sensitivity of a typical barricade to a variety of geometric and material properties, a parametric analysis was carried out in Augustus-2. Reinforcement content; clear cover; concrete compressive strength; barricade thickness and length; Young’s modulus of the surrounding rock; and arched walls of various curvatures were tested. The reference model was similar in geometry and reinforcement content to a previously constructed barricade at the Cayeli Bakir mine in Turkey which was studied by Thompson et al. [15]. In each test case, only one parameter was varied while the others remained identical to the reference model. The parameter ranges are shown in Table 5.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barricade thickness</td>
<td>mm</td>
<td>100 200 300 400 500 600 700 800 900 1000</td>
</tr>
<tr>
<td>Barricade length</td>
<td>m</td>
<td>2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>Barricade reinforcement, $\rho$</td>
<td>%</td>
<td>0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 2.0</td>
</tr>
<tr>
<td>Depth to centroid of reinforcement</td>
<td>mm</td>
<td>50 100 150 200 250</td>
</tr>
<tr>
<td>Concrete comp. strength, $f'_c$</td>
<td>MPa</td>
<td>5 10 15 20 25 30 35 45 55 65</td>
</tr>
<tr>
<td>Young’s modulus of rock, $E_{rock}$</td>
<td>GPa</td>
<td>25 30 35 40 45 50 55 60 65 70</td>
</tr>
<tr>
<td>Arch angle, $\alpha$</td>
<td>°</td>
<td>-20 0 20 40 60 80 120 140 160</td>
</tr>
<tr>
<td>Steel yield strength, $f_y$</td>
<td>MPa</td>
<td>400</td>
</tr>
</tbody>
</table>

All simulations were performed on a 1 metre-wide strip taken along the shorter (and weaker) dimension of the barricade. The applied pressure, therefore, can be calculated as $\frac{P}{L}$ where $P$ is the total applied load and $L$ is the length of the barricade. Loads were uniformly distributed across the length of the model. Only reinforcement in the
longitudinal, or horizontal, direction could be included in the simulations, as Response-2012 elements are only able to model longitudinal and transverse steel. The vertical reinforcement was not modelled, but would have had little impact on behaviour, which is predominantly similar to that of a one-way slab as shown by Ghazi [9].

5.0.1 Reference Model

The reference model for the parametric study is as described in Table 5.1. First cracking occurred at an applied pressure of 12.25 kPa while first yielding occurred at midspan at a pressure of 49.0 kPa. The reference rock stiffness value was not high enough to allow for sufficient development of negative moment at the supports, so no yielding occurred there prior to failure. Failure occurred due to concrete crushing at the midspan element. Plots of applied pressure and the development of axial compression in the barricade against midspan displacement are shown in Figure 5.1. Various plots from Augustus-2 showing the internal forces and stresses in the critical element at failure are shown in Figure 5.2. Of note is the longitudinal concrete stress plot, which shows the top portion of the cross-section having reached a concrete compressive strength of 35 MPa. As this limit is reached, flexural crushing will occur which will bring about failure of the barricade.

The magnified displaced shapes and average crack directions are shown in Figure 5.3. As applied pressure increases, cracks propagate from midspan towards the supports. First cracking at the supports due to negative moment occurred at approximately 73.5 kPa, well after first yielding of the midspan reinforcement.

The shear force and bending moment diagrams at failure are shown in Figure 5.4. The moment diagram is drawn on the tension side of the barricade.
(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement

Figure 5.1: Response of reference model
Figure 5.2: Augustus-2 plots showing internal forces and stresses of critical midspan element at failure.
(a) First cracking, 12.25 kPa

(b) First yielding, 49 kPa

(c) Ultimate capacity, 80.9 kPa

Figure 5.3: Reference barricade displaced shapes (magnified 10x) and average crack directions in red
Figure 5.4: Shear and bending moment diagrams at failure

(a) Shear force diagram

(b) Bending moment diagram
5.1 Barricade Reinforcement Content, $\rho$

The longitudinal reinforcement content of the barricade, $\rho$, was varied from zero to two percent. The zero reinforcement case was analysed for completeness and should not be used in practice. The required area of steel, $A_s$, in the cross-section was calculated using the formula $\rho = \frac{A_s}{b_w d}$ where $b_w$ is the width of the section (1 metre in all cases) and $d$ is the effective depth of the member taken as the depth to the centroid of the layer of steel (150 mm in all cases).

Figure 5.5 shows the predicted pressures for first cracking, yielding, and failure, in all cases at midspan. The percentages shown on the figure are the change in ultimate strength compared to the reference case. As $\rho$ increases, the barricade is capable of withstanding more pressure first yielding and failure. This result is expected, as a larger area of reinforcement would require more force to reach yield and also increases flexural capacity. First cracking occurred on the tensile face at the same pressure regardless of steel content, as reinforcement content has no effect on the tensile strength of the concrete. In the case of a barricade with no reinforcement, failure occurred at the onset of first cracking because there was no steel to bear the tensile forces in the barricade. The $\rho = 0.2\%$ case was below the minimum amount of reinforcement of 0.296\% (specified by CSA A23.3 [4] based on concrete compressive strength and steel yield strength), so simulations yielded poor results. In all other cases, failure occurred due to flexural crushing at midspan.

There is a significant increase in strength between $\rho = 0.2\%$ and 0.4\%: for $\rho = 0.2\%$, failure was considered to have occurred at yielding of the reinforcement when the crack depth reached the rebar at mid-depth. The minimal area of steel was insufficient to resist the tensile forces in the barricade. For poorly-reinforced barricades, a smaller amount of steel results in higher steel stresses (and therefore, strains) for a given applied pressure. These larger steel strains correspond to higher overall curvatures in the cross-section which lead to an increased rate of crack formation after the first one develops. For this reason, it is recommended that a minimum amount of reinforcement be required in the construction of CPB barricades to avoid premature failure.

While a 2.0\% reinforcement ratio gives almost a doubling in capacity compared to the minimally reinforced 0.4\% case, such amounts of reinforcement would not be recommended due to potential construction issues, particularly with shotcrete penetration through the rebar cage.

Figure 5.6 shows the applied pressure and axial force versus the midspan displacement for each analysed case. Barricade capacity increases with higher values of $\rho$, while
peak barricade displacement remains fairly similar until $\rho = 1.0\%$ then decreases with increasing $\rho$. Larger amounts of steel also result in stiffer post-cracking barricade response, with more pressure being required to cause the same amount of midspan displacement. With the exception of minimally-reinforced barricades which failed at low pressures, the development of axial forces in the barricade decreases with increasing amounts of reinforcement. This is likely due to smaller net strains on account of higher $\rho$ which result in less axial elongation. The development of compressive axial forces in these barricades is wholly dependent on a net tensile axial strain which is restrained by the boundary conditions. More steel would result in less strain for a given tensile force, resulting in a lower net strain distribution in the cross-section. Although this reduction in axial force would correspond to a smaller strengthening contribution from compressive membrane action, the increased amount of steel still raises flexural capacity and produces a stronger barricade.

The lower peak midspan displacements at higher values of $\rho$ are due to the larger steel area, which induces correspondingly higher compression forces in the concrete. This causes a drop in ductility despite an increase in capacity. Yielding occurs at larger displacements with increasing $\rho$ possibly due to the larger forces (and thus displacements) required to yield a larger area of steel.
As an example, if a barricade of the simulated dimensions were designed for 60 kPa capacity (without considering safety factors), the predicted peak midspan displacement would vary from approximately 2 to 12 mm as $\rho$ is varied.
Figure 5.6: Effect of varying reinforcement content, $\rho$

(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement
5.2 Depth to Centroid of Reinforcement

The depth of the reinforcement in the barricade was changed to address possible variability in construction. Flexural capacity would likely decrease as the distance between the barricade’s tension face and the reinforcement increases. For example, if concrete was to be sprayed to such a thickness that the reinforcement was located further towards the CPB face of the barricade (the flexural compression side at midspan), then its contributions to strength would be lessened. To simulate these effects, the depth to the centroid of the reinforcement in Response-2012 beam elements was varied from 50 mm (reinforcement near the midspan tension face) to 250 mm (reinforcement near the midspan compression face) over a constant barricade thickness of 300 mm.

Figure 5.7 shows the predicted pressures for first cracking, yielding, and failure, in all cases at midspan. As cover increases and the steel moves from the tension to the compression face, the barricade is capable of withstanding less pressure before the reinforcement yields. First cracking occurred at similar pressures regardless of steel depth; since cracking occurs on the tension face, steel height had no effect on the cracking strength. In all cases, failure occurred due to flexural crushing at midspan. Because the amount of reinforcement was constant, the change in applied pressure to cause yield and failure was only a function of rebar height within the cross-section. If the steel were closer to the flexural compression side, the moment arm between the concrete compressive zone and the steel centroid would be smaller, resulting in higher steel and concrete stresses for a given applied pressure and thus lower flexural capacity.

If the barricades were flexure-critical at the supports, where tension occurs on the side facing the CPB, then steel placed close to the paste side would increase strength considerably. This behaviour, however, did not arise in simulations because negative moments at the supports were not high enough to cause failure (Figure 5.4). Nevertheless, during construction it would be prudent to place steel on the tension face at all locations in the barricade: towards the paste near the supports and away from the paste elsewhere.

Figure 5.8 shows the applied pressure and axial force versus the midspan displacement for each analysed case. As the steel moves from the tension to the compression face, barricade capacity decreases while the peak displacement is similar in all cases. This is primarily due to the shorter lever arm between the steel in tension and the concrete compressive zone which decreases moment capacity. The development of axial forces remains similar as well with a slight increase as the steel moves further towards the compression face. This increase can be considered minor enough to conclude that the
position of the steel within the barricade has a minimal effect on the development of axial forces, and thus compressive membrane action.

A barricade of the simulated dimensions designed for 50 kPa capacity (without safety factors) would have a peak midspan displacement between approximately 2 to 11 mm depending on the location of reinforcement within the barricade depth.
Figure 5.8: Effect of varying bottom clear cover

(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement
5.3 Barricade Thickness

Barricade thickness was varied from 100 to 1000 mm to determine the strengthening effect of increasing concrete content. If it were possible to significantly strengthen a barricade by adding slightly more concrete instead of more steel, construction costs could potentially be lowered.

Figure 5.9 shows the predicted pressures for first cracking, yielding, and failure. The percentages shown above the extreme values are the change in ultimate strength compared to the reference case. Thicker barricades can support higher pressures before first cracking and yielding. Cracking resistance increased on account of a larger area of concrete supporting a given amount of tension when subject to flexure. However, at a thickness of 900 mm and greater, shear failure at the supports occurs prior to the reinforcement yielding. As the barricade grows thicker, so does the moment arm between the steel and the compressive concrete zone. This larger moment arm reduces stresses in the steel, resulting in shear failure in the concrete prior to yielding. Because the barricade is uniformly loaded across its span and is partially fixed at both ends, maximum shear would occur near the supports, where failures occurred in thicker models. All other cases failed due to flexural crushing at midspan with the 700 and 800 mm thickness specimens being close to shear failure.

![Figure 5.9: Pressures causing first cracking, yielding, and failure versus barricade thickness](image-url)
Figure 5.10 shows the applied pressure and axial force versus the midspan displacement for each analysed case. As thickness increases, there is an increase in capacity but a decrease in midspan deflection beyond 400 mm. This decrease in ductility is on account of the increased cross-sectional area of concrete which requires a larger force to produce a given curvature. As thickness increases, higher forces are needed to deflect the barricade by the same amount.

The peak axial force increases until a thickness of 400 mm and then drops significantly as thickness increases. Because the development of axial compression in the barricade is dependent on strains caused by flexure, a stiffer model will develop smaller strains and thus less axial force. Since flexural concerns are minimized with increasing thickness, as the cross-section thickens, shear demand begins to dominate due again to higher stiffnesses.

Based on these results, varying barricade thickness has a significant effect: doubling the thickness can more than double the capacity. It is suggested, however, that barricade thicknesses not exceed 400 mm to prevent brittle behaviour and to allow as much deflection as possible to act as a warning prior to failure.
Figure 5.10: Effect of varying barricade thickness

(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement
5.4 Barricade Length

The length of a barricade is dictated by the cross-sectional dimensions of the stope, and so cannot be controlled. However, if it is determined that increased length dramatically reduces capacity, additional measures can be taken to strengthen the barricade to avoid a premature failure. For simulations varying barricade length, the reinforcement ratio $\rho$ was varied by a factor $(L_{\text{barricade}}/L_{\text{reference}})^2$ to account for the increasing moments (which, for a uniformly loaded beam, vary by the square of the loaded length) for beams of a different length. The reinforcement content varied from 0.03% for the 1 m long case to 3.1% for the 10 m long case. It should be noted that both 1 m and 2 m lengths had a $\rho$ value below the minimum required by CSA standards.

Figure 5.11 shows the predicted pressures for first cracking, yielding, and failure. Cracks occurred at lower pressures in longer barricades due to larger midspan moments, while yielding of steel only occurred in test barricades of length 6 m or less. Because longer barricades support significantly more moment, the concrete reached crushing at a faster rate and failed prior to sufficient stress developing in the reinforcing steel. If the steel were located closer to the tensile face of the barricade, it is possible that it would have yielded prior to crushing even in the longer cases. For the shorter barricades in which the reinforcement yielded, the scaling of reinforcement content $\rho$ with the square of the length resulted in a similar yield pressure independent of length. In all cases, failure was due to flexural crushing at midspan despite the lack of yielding in barricades longer than 6 m. Although the 1 m and 2 m long barricades had less than the minimum amount of reinforcement required by CSA standards, they still developed significant strength. This is possibly due to their short spans which reduced bending moments.

Figure 5.12 shows the applied pressure and axial force versus the midspan displacement for each analysed case. Barricade capacity drops with increased length while peak axial forces grow with decreasing length until a value of 3 m, after which axial forces decrease dramatically. Longer barricades are slightly more ductile: in general, an increasing span to depth ratio results in more ductile behaviour as flexure demand overtakes shear demand.

As length increases, the rate of development of axial forces decreases. This is due to a lower curvature for a given displacement as barricade length increases. Longitudinal strains are dependent on curvature, so there would be lower axial forces as a result.

Because there are significant decreases in capacity with increasing barricade length, it is recommended that longer barricades be designed more conservatively, possibly by
Figure 5.11: Pressures causing first cracking, yielding, and failure versus barricade length

increasing thickness and using more reinforcement and ensuring that it is placed near tensile faces.
Figure 5.12: Effect of varying barricade length

(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement
5.5 Concrete Compressive Strength, $f'_c$

Concrete compressive strength, $f'_c$, was varied between 5 to 65 MPa while maintaining constant geometry and boundary conditions.

Figure 5.13 shows the predicted pressures for first cracking, yielding, and failure. Concretes with higher compressive strengths also have higher tensile strengths, so cracking pressures increased with $f'_c$. Yielding pressures remained similar across different values of $f'_c$, as concrete strength had no effect on steel behaviour. As expected, failure in all cases was due to flexural crushing at midspan, with higher strength concretes failing at higher pressures due to increased compressive capacity. Interestingly, however, there was little increase in capacity for barricades using concrete with $f'_c$ values between 35 to 65 MPa. One possible explanation is that, prior to failure, the depth of concrete still capable of carrying compression is so small that any increase in $f'_c$ beyond a certain point provides a negligible increase in the length of the lever arm between tensile forces in the steel and net compressive forces in the concrete, thus maintaining compressive capacity.

Figure 5.13: Pressures causing first cracking, yielding, and failure versus concrete comp. strength $f'_c$

Figure 5.14 shows the applied pressure and axial force versus the midspan displacement for each analysed case. As expected, lower concrete strengths result in a weaker barricade with lower stiffness and peak midspan displacement. Values of $f'_c$ above 35 MPa, however, provide no appreciable increase in strength or stiffness as previously explained. Similar behaviour is seen in the development of axial forces in the barricade: lower concrete
Concrete Compressive Strength, $f'_c$

strengths result in less axial compression. Compression forces stop increasing at values of $f'_c$ higher than 35 MPa. Based on these simulation results, any reasonable concrete compressive strength would have little strengthening effect in a barricade. However, severely deficient concrete would result in a significant loss of strength and displacement capacity.
Concrete Compressive Strength, $f'_c$

Figure 5.14: Effect of varying compressive concrete strength $f'_c$

(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement
5.6 Young’s Modulus of Rock Wall, $E_{rock}$

As determined by Ghazi [9], barricade strength is very sensitive to the stiffness of the surrounding rock due to the onset of compressive membrane action. Because even a slight increase in axial compressive force due to a stiffer boundary can cause a significant increase in flexural capacity, it is important to properly account for these effects. Using the methods discussed in Section 3.4, an equivalent axial and rotational stiffness value was derived for a given value of $E_{rock}$. Expressions were also developed to determine the required area of steel $A_{s,axial}$ and $A_{s,rot}$ for the truss rods in the Augustus-2 model based on the calculated stiffness values.

The values of $E_{rock}$ cover a range from 0 to 70 GPa. These results were also compared to the case of a barricade with fully fixed boundaries and one which was free to expand longitudinally (effectively simply supported).

Figure 5.15 shows the predicted pressures for first cracking, yielding, and failure. Cracking pressure remains unchanged and yielding occurs at slightly higher pressures as $E_{rock}$ increases. Any increase in yield strength would likely be brought on by a reduction in curvature (and thus steel stresses) due to a more rigid boundary, but the effect is minimal. The more pronounced, and valuable, result is the significant difference in ultimate barricade capacity afforded by a stiffer boundary. A higher rock stiffness would provide more resistance to axial expansion, resulting in the accelerated development of compressive membrane forces which enhance strength. A barricade constructed in rock with a modulus of 25 GPa has only half the capacity of the same barricade in rock with a modulus of 70 GPa.

In the fully fixed case, the pressure to cause first cracking is doubled compared to the simply supported and partially restrained cases. The simply supported barricade has lowest yielding pressure and ultimate capacity of all simulations. Failure for all models was by flexural crushing at midspan with the exception of the fully fixed case, which failed in shear at the barricade supports. Figure 5.15 also shows the capacity of the fully fixed case, which is at least 1.75 times stronger than any of the other simulated barricades. This illustrates the danger in assuming fully rigid boundary conditions, which could easily give a twofold overestimate of strength and a severe underestimate of ductility. It should be noted that the response of the fully fixed case is unlike that modelled by Ghazi (Figure 2.8a). Ghazi’s fully fixed model was able to deflect over 60 mm at midspan prior to failure compared to a deflection of 6.3 mm in this parametric study. Despite this difference, the ultimate pressures between both cases were similar.
Figure 5.16 shows the applied pressure and axial force versus the midspan displacement for each analysed case. Barricade capacity increases with boundary stiffness while peak midspan displacement remains fairly constant prior to failure. This suggests that while the rock wall modulus has a large effect on ultimate capacity, peak displacements would remain similar for a given barricade design regardless of the boundary conditions. This result would likely change with the inclusion of secondary moment effects in analysis which would result larger displacements for the same applied pressure. Axial forces reached a higher peak and developed more quickly with increasing rock stiffness. The simply supported model reached a peak midspan deflection almost twice that of the other models, while the fully fixed case deflected minimally before shear failure. The capacity of the fully fixed barricade was approximately 4.5 times that of the simply supported case and 1.75 times that of the strongest partially restrained case.

In practice, situations may arise where the quality of the rock wall is poor and its load-bearing capacity may not be as high as if it were intact. In this case, it may be possible to apply a damage factor $D_f$ less than 1.0 to reduce the effective modulus ($E_{rock,eff} = D_f E_{rock} < E_{rock}$). Such application of a damage factor would be to the discretion of the supervising engineer and should be used conservatively.
Figure 5.16: Effect of varying the rock wall Young’s modulus, $E_{\text{rock}}$
5.7 Arch Angle, $\alpha$

The effect of arched geometry on barricade strength was studied by varying the arch angle (Figure 5.17). An arched barricade should provide a significant increase in strength over a flat one due to it carrying imposed loads primarily in compression. Rectangular Response-2012 elements were laid out in a segment of a circle and connected with truss rods to form an approximation of an arch for simulation. The span of the arch was kept constant at 4 m while the arch angle was increased from 0 to 160 degrees. Because each arch is laid out in a circular shape, it is expected that there will be more significant tensile stresses than in arch shapes such as the parabola or the catenary, which are more efficient because they carry more of the applied load in compression. To investigate the expected loss in capacity for a reversed arch which thrusts away from the direction of loading, a -20 degree case was also simulated. Such a situation could arise with improper construction techniques.

As discussed in Section 3.5, concerns about proper transfer of forces between eccentrically connected elements (such as those in an arch) arise when the angle between neighbouring elements approaches six degrees as shear behaviour is not properly simulated at the transition points. It should be noted that for an arch angle of 160°, the angle between elements reaches this six degree limit which could provide inaccurate results. One potential inaccuracy would be the inability to predict a shear failure.

![Figure 5.17: Arch angle, $\alpha$](image)

Figure 5.17 shows the predicted pressures for first cracking, yielding, and failure. Cracking pressures remained fairly similar to the flat reference case for all arch angles; first cracking always occurred at midspan for every model. As arch angle increased, however, first yielding and failure pressures increased until an arch angle of 100°, where simultaneous yielding and crushing led to failure. At arch angles larger than 100°, failure
Arch Angle, $\alpha$

occurred due to concrete crushing before the reinforcement approached yielding due to higher compressive forces. As the arch angle increases, more of the applied pressure is carried in direct compression, which places higher demand on the concrete. After an arch angle of 100°, the compression stress in the concrete increases significantly, resulting in crushing failure prior to yielding. Along with this, more shear demand was placed on the area near the supports, but not enough to cause a shear failure at that location. Because it is safer to have reinforcing steel yield before crushing (thus allowing for more warning prior to failure), it is suggested that a more moderate arch angle be adopted in construction. Additionally, barricades with larger arch angles are more prone to being shear critical, which may result in sudden failures with little to no warning.

The simulated arch with negative curvature had a lower ultimate capacity than the reference flat barricade due to nearly the entire cross-section being placed in direct tension when pressured were applied. If a barricade were constructed poorly with the incorrect direction of curvature, capacity would decrease; this deficiency could be mitigated by the use of sufficient reinforcing steel which could act as a tensile net. Despite this, reversed curvatures must be avoided in practice.

![Figure 5.18: Pressures causing first cracking, yielding, and failure versus arch angle, $\theta$](image)

Figure 5.18 shows the applied pressure and axial force versus the midspan displacement for each analysed case. Larger arch angles result in a stronger and stiffer barricade with
less midspan deflection before failure. As previously mentioned, larger arch angles result in higher compression stresses in the concrete for a given applied force, resulting in more brittle behaviour. A peculiar, contrary result is the response of the 120° model, which is more ductile than the 100° model and develops less axial forces for a given midspan displacement. It is possible that because the 100° model places more demand on the reinforcement (which has a higher Young’s modulus than concrete), its response is stiffer than that of the 120° model which experiences lower steel stresses.

Axial forces develop at a faster rate with increasing arch angle but reach a peak at an angle of 100°. Because the cross-section is the same between all models, it would require the same compressive force to cause concrete crushing. Thus, the peak axial force for all models with an arch angle greater than 100° is approximately equal.

The strongest arch, the 160° model, has a 4.5-fold strength increase over the base flat barricade with an approximate halving of peak midspan displacement.

The results for arches of angles greater than 80° differ from those found by Helinski et al. [10]; in their simulations, these arches failed in shear and tensile separation at the rock supports. While the Augustus-2 models did not fail in shear, models with large arch angles did have more shear demand. Due to the simplifying assumptions made in modelling arched barricades in Augustus-2 (namely, truss rods connecting beam elements and strong Membrane support elements), further more detailed investigation of high arch angles is warranted.
Figure 5.19: Effect of varying barricade arch angle, $\theta$

(a) Applied pressure versus midspan displacement

(b) Barricade axial force versus midspan displacement
Chapter 6

Conclusions

Based on the parametric analysis, CPB barricade strength can vary by as much as 30% depending on the Young’s modulus of the surrounding rock. Although other parameters may have had a higher effect on capacity, the type of rock in a stope cannot be changed. Barricade designs must therefore be scaled appropriately to account for boundary conditions. Arched barricades are significantly stronger than flat ones: more than a doubling in strength can be achieved with a moderate curvature. Because the arched barricades were modelled in Augustus-2 did not use curved elements, some inaccuracy should be expected with the corresponding results.

Of the material and geometric barricade properties, thickness and length have the highest effects on capacity and displacement. Care should also be taken to properly position the reinforcement within the concrete to support tensile stresses. Factors such as reinforcement ratio and concrete compressive strength contribute little to capacity so long as reasonable values are used.

Secondary moment (P-Δ) effects can consume a significant amount of flexural capacity. An iterative proof of concept simulation was performed in Augustus-2 to show both the negative effects of P-Δ and a potential implementation method for future versions of the software.

The equations derived for rock stiffness calculate a stiffness value based on the Young’s modulus of the surrounding rock using theoretical solutions provided by Timoshenko and Boussinesq for loaded areas on an infinite half space. Although the equations were developed for simulating boundary conditions in Augustus-2, they can be applied in practical situations where theoretical rock boundary displacements are desired.
Chapter 7

Recommendations

The following recommendations are made for future research:

**Implement secondary moment effects in Augustus-2**

Currently, Augustus-2 does not account for secondary moment effects (i.e. the P-Δ effect) in its simulations. Because this could significantly decrease the capacity of the modelled barricades, it should be included to increase accuracy and minimize unconservative overpredictions of strength. Such functionality would also improve simulations results of other models subject to eccentric axial compressive loads.

**Improve arch modelling capabilities in Augustus-2**

Because Augustus-2 is not currently capable of modelling curved elements, truss rods were used instead to arrange rectangular elements in a segmented circular layout. Although simulations were performed to verify that the truss rod connections were still able to properly simulate the behaviour of a typical beam, the model geometry was still not representative of reality. If Membrane elements with high aspect ratios could be used stably in simulations, this could improve accuracy.

**Conduct experimental barricade tests**

Experiments with actual barricade walls under CPB loading conditions would provide an opportunity to not only verify Augustus-2 simulation results but to also determine the in situ effects of rock wall stiffness. The impact of concrete shrinkage could also be considered.

**Investigate the effect of varying pressure distributions**

All simulations in this thesis considered only uniformly distributed CPB pressures. Because the paste is often poured in stages, it would be beneficial to determine the
effects of partial uniform pressure applied to barricades. Additionally, the profile of the CPB pressure may not truly be uniform, which could affect behaviour.

**Develop standard design methods**
Currently, there are many different design methods for CPB barricades. After experimental work is carried out, it is recommended that a design methodology be established which accounts for various geometry and material properties. This would minimize uncertainty and result in more efficient use of time and materials.

**Determine interaction between parameters**
The parametric study in this thesis only considered the variation of one parameter at a time. To further characterize barricade behaviour, the effects of simultaneously varying multiple parameters should be studied as their effects may not be merely superimposed.

**Investigate the effects of concrete shrinkage**
Augustus-2 does not currently account for concrete shrinkage effects which could affect barricade behaviour. Shrinkage cracking could adversely affect barricade strength, particularly at boundary conditions where shear failures are of more concern.
Bibliography


Appendices
Appendix A

Typical Augustus-2 Input Files
Element Input File V2.0

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**Notes:**
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- Table 2: Data are presented as mean ± standard error (SE).
- Table 3: Data are presented as median (IQR).
- Table 4: Data are presented as count (percentage).
- Table 5: Data are presented as percentage (95% confidence interval).
Appendix C

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Appendix D

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**TV2 Lateral rxn at slab end taken as axial force in slab exp Data from Graph**

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*NOTE: COMPRESSION IS POSITIVE FOR PLOTTING PURPOSES*
Lateral reaction at column base TV2

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### Controls

- **Title:** Control Chart
- **x-axis:** x-axis
- **y-axis:** y-axis

#### Data from Graph

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#### Lateral reaction at column base TV2

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**TV 1**

- **Load RXN**
- **Data from Graph**
- **Label:** Title: Control Chart

**TV 2**

- **Load RXN**
- **Data from Graph**
- **Label:** Title: Control Chart

---

**Description:**

- The table presents data related to the lateral reaction at column base TV2, including TV1 and TV2 values, load, and control chart information.
- The dataset includes various numerical values indicating the reaction conditions and outcomes.
- Controls are noted with specific titles for the x and y axes.
- The table format aids in visualizing the relationship between TV1 and TV2 with load data.

---

**Observations:**

- The data suggests a complex reaction occurring under controlled conditions, indicated by the labeled control charts.
- The load values range from 0.0 to 28.7, reflecting the reaction intensity.
- The TV1 and TV2 values range from 0.1159 to 15.63, highlighting the reaction's variability.
- The dataset is designed to provide insights into the reaction's behavior, possibly under different experimental conditions.

---

**Analysis:**

- The reaction's complexity is reflected by the extensive range of values for TV1 and TV2, indicating a multidimensional approach to the reaction study.
- The control charts offer a means to validate the reaction's stability and potential outliers.
- Further analysis, such as correlation coefficients and statistical tests, could provide deeper insights into the reaction dynamics.
Appendix E

P-Delta Simulation Results
Appendix F

Element Angle Simulation Results