THEORETICAL MODEL OF THE LITTORAL DRIFT SYSTEM IN THE TORONTO WATERFRONT AREA, LAKE ONTARIO

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ABSTRACT. A computer model is described which simulates the interaction between shoreline geometry and shoaling waves in the Toronto waterfront area on the north shore of Lake Ontario, between Highland Creek in the east and the Credit River in the west, and establishes potential littoral drift patterns. Thirteen wave conditions, defined by height, period and direction of approach, characterising the annual hindcast spectrum for Toronto, are used in the determination of the shore-parallel components of wave energy flux. To identify the long term net effects of the distribution of wave energy flux which produce the dominant littoral drift pattern, a simple summation procedure is used whereby the individual effect of any wave condition at a shoreline point is weighted according to its frequency of occurrence. The long term average pattern of potential littoral drift based on zones of potential erosion (increasing alongshore wave energy flux), transport (constant alongshore wave energy flux) and deposition (decreasing alongshore wave energy flux) is established. Identification of nodal points (zero alongshore wave energy flux) and drift cells allows a test of the 'geomorphological sense' of the model by comparison with observed drift patterns.

INTRODUCTION

The coastal zone is characterized by a complex interactive system of air, water and land producing a dynamic environment highly sensitive to man-induced changes. In the Great Lakes many coastal areas, such as the Toronto Waterfront, are presently undergoing rapid development as a result of commercial, residential and recreational demands which frequently involve shoreline changes through the introduction of shore protection structures, reclamation and dredging activities. In each case the 'natural equilibrium regime' of the littoral drift system is altered and the system tends towards a new 'forced equilibrium regime' (Sonu, McCloy and McArthur 1966). This shift towards a 'forced equilibrium regime' may include severe changes in the littoral drift system resulting in deposition in some areas and serious erosion in others. In many cases planning for development in the nearshore zone is based on inadequate information concerning the processes to be affected by any such changes, while the physical collection of such data is a prodigious task. Environmental impact assessment is therefore either ignored or based on very

limited data. There is a clear need for the development of modelling techniques to aid in our understanding of coastal dynamics. The model must be flexible enough to allow testing of the response of the littoral system to alternate planning concepts and to a variety of natural stresses. Hard-ware hydraulic models can be used and although popular with engineers they are expensive, time consuming and subject to significant problems of scale. The most efficient models in this respect are numerical, utilizing the speed and flexibility of computers.

Little information is presently available, for example, on wave-generated longshore currents and sediment drift patterns in the Toronto Waterfront area although it is an area of considerable natural shoreline change and is presently undergoing extensive artificial alteration. The present study is, therefore, an attempt to develop a numerical simulation model of the potential time-averaged littoral drift system of the Toronto Waterfront and to evaluate the *geomorphological sense* of the model. It is hoped that the resulting model will provide not only an explanation of present shoreline changes but also a tool for use in environmental impact assessment studies.

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FIG. 1. Location of the study area.

LOCATION

The study area is located on the northwestern shore of Lake Ontario, between Highland Creek in the east and the Credit River in the west (Figure 1). It is the site for numerous large coastal structures proposed by the Metropolitan Toronto and Region Conservation Authority (1974). Some of these structures presently exist (for example, the Eastern Headland) and several others have yet to be constructed. However, the present study considers the Waterfront system without these structures present.

The bathymetric configuration of the waterfront is illustrated in Figure 2. In general the average nearshore slope is similar in the eastern and western sections (approximately 0.01) while the central section is marked by a sharp break in slope known as the Toronto Scarp (Lewis and Sly 1971) extending from 18 m to 55 m (10 to 30 fathoms). The slope of the scarp itself reaches a maximum of 0.17 while landward and lakeward slopes are 0.006 and 0.012 respectively.

LITTORAL DRIFT SYSTEMS

A littoral drift system may be defined as an integrated system of sediment dispersal in the near-shore zone controlled by wave activity; it may be

divided into a number of littoral drift cells, (Inman and Frautschy 1966; Pierce 1969; Stapor 1971, 1973 and 1974), each cell being characterized by one area of erosion, one area of deposition and a transport path between them (Tanner 1974b). Compartmentalization of a littoral drift system into cells is caused by specific wave and current conditions and therefore the division of the system into cells may occur frequently, infrequently or not at all.

Littoral drift reflects both shore-normal and shore-parallel transport processes and as such depends upon the orbital motion under shoaling waves and the wave energy flux at the shoreline upon wave breaking. At present sediment entrainment and rates of transport under oscillatory wave motion are poorly understood and in this study only the potential shore-parallel transport is modelled. While the results will thus be limited in this way it is nevertheless true that the dominant mechanism of sediment movement in the near-shore is that of longshore currents generated landward of wave breaking.

A conceptual model of a littoral drift system for the Toronto Waterfront area is illustrated in Figure 3 and relates the existing wind and wave climatology to time-averaged littoral drift patterns through a wave refraction procedure. The wave

FIG. 2. Bathymetry of the Toronto waterfront area.

climate and bathymetry are used to calculate a longshore component of wave energy flux and this information is then subjected to a simple summation procedure whereby the individual effect of each wave condition at a shoreline point is weighted according to its frequency of occurrence. The success of the operational method, therefore, rests heavily upon the theory and practice of calculating wave energy transformations over the zone of shoaling and the prediction of the rate of energy dissipation at the shoreline.

THEORETICAL BACKGROUND

Since the very early work of O'Brien (1942, 1947) and Munk and Traylor (1947), there has been an extended effort both to refine the basic theory concerning wave form and transformations and also to develop rapid numerical solutions to the equations using computers. Reviews of the former appear in Collins (1976) and Madsen (1976). A number of computer models for the prediction of wave shoaling and refraction have been developed, most notably those of: (1) the Hydraulics Research Station, Wallingford England (Abernethy and Gilbert 1975, Willis and Price 1975); (2) the

FIG. 3. A conceptual model of a littoral drift system for the Toronto waterfront.

Redsea Program of the Texas A and M University (McClenan *et al.* 1971, Worthington and Herbich 1971); (3) the Delaware Model (Birkmeier and Dalrymple 1975, 1976); (4) the Tetra Tech Near-shore Circulation Model (Noda 1972, Noda *et al.* 1974); (5) the Danish Programs (Skovgaard, Jonsson and Bertelsen 1975); (6) the Virginia Institute for Marine Science Programs (Goldsmith *et al.* 1974, Goldsmith 1976); (7) the Wave Energy Program of Florida State University (May and Tanner 1973, May 1974). May's (1974) program modified after Wilson (1966) was selected for this study because its computations include not only the construction of wave orthogonals but also the

wave height attenuation due to bottom friction and a measure of the longshore component of wave energy flux at the breaker point. Basically, it simulates the transformation of a deepwater wave across a shoaling bottom of known bathymetry to the point of breaking using linear wave theory. At the point of wave breaking (where the ratio of wave height to water depth is equal to or greater than 0.78) the refracted path of each selected wave ray stops and the following equations are used to compute wave energy flux toward the shoreline (Equation i), a longshore component of wave energy flux (Equation ii), both in joules per metre-second and a mean longshore current velocity in metres per second (Equation iii) after Komar and Inman (1970):

$$P_b = [E.C.n]_b \quad (i)$$

$$P_L = P_b \cdot \cos\alpha_b \cdot \sin\alpha_b \quad (ii)$$

$$V_L = 2.7 u_m \cos\alpha_b \cdot \sin\alpha_b \quad (iii)$$

where

P = wave energy flux per unit width of wave crest

E = wave energy density per unit width of wave crest

C = phase velocity

n = ratio between group and phase velocity

α = angle of wave approach

u_m = maximum orbital velocity at the breaking wave position

b = denotes the breaking condition

L = denotes a longshore component per unit length of shoreline.

The ability of computer simulation to model a littoral drift system rests upon: (1) the theoretical validity of the principles used to govern wave energy dissipation through shoaling, refraction and friction; and (2) the relationship between the longshore component of wave energy flux (P_L) and the longshore sediment transport rate. The use of P_L dates from early work at Scripps Institution of Oceanography (1947) and a recent restatement of the underlying theory has been presented by Galvin and Vitale (1976). Use of both the terms and equations defining the *longshore component of energy flux* or *longshore power* have been criticized by Longuet-Higgins (1972) as being physically inadequate and the term *lateral thrust* is preferred to describe the value of equation (ii). However, as a measure of the potential transporting power of longshore currents generated by breaking waves the longshore component of wave

energy flux has proven remarkably effective. Wave energy flux and the longshore sediment transport rate, expressed as an immersed weight (I_L) have been related successfully by Komar and Inman (1970) in the equation:

$$I_L = K.P_L \quad (iv)$$

where K is a constant of proportionality derived for instantaneous transport rates (Bagnold 1963, Inman and Bagnold 1963). For fully developed transport conditions on sandy beaches K has been determined by Komar and Inman (1970) as being equal to 0.77. However, it is not yet known how this parameter varies on mixed sand and gravel beaches such as those of the Toronto Waterfront. Thus while the value of K remains a questionable issue and although no real consideration of transport mechanics was involved in the derivation of Equation iv, the latter does suggest that P_L may serve as a coastal index for describing a littoral drift system, at least in a relative way. The spatial variability of the longshore components of wave energy flux at the shoreline can therefore be used to construct hypothetical littoral transport paths. Since the longshore component (or projection) of energy flux (P_L) represents the rate of doing work, or expending energy, parallel to the shoreline, it therefore must be associated closely with the work done in a littoral drift system (Bagnold 1963, Inman and Bagnold 1963, Tanner 1974b). Thus, the quantity of sediment transported (I_L) past any given point in a unit time must behave like P_L ; the only distinction will be that the value for these two parameters are numerically different. This approach is adopted in the theoretical model described in this paper.

COMPUTER MODEL

The computer model, therefore, provides a prediction of the longshore component of wave energy flux at breaking by computing the energy dissipation through shoaling, refraction and friction at the bed, for a number of points along the lake-shore. Basic input data used are the deep water wave height, period and direction of approach and the bathymetric characteristics of the Toronto waterfront.

WIND AND WAVE CLIMATOLOGY

Wind information collected over a 17 year period (1955-1972) at the Toronto Island Weather Station provides a representative indication of average annual and monthly trends in wind for the Toronto

TABLE 1. Wind data for Toronto Island Airport for the period 1955-1972.

PERIOD 1955-72				HEIGHT OF ANEMOMETER 11.8m									
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	YEAR
PERCENTAGE FREQUENCY													
N	7	8	7	6	5	5	5	5	7	6	6	7	6
NE	10	11	14	14	10	8	7	8	11	10	10	11	10
E	8	10	16	20	23	22	17	16	17	17	13	10	16
SE	3	3	2	3	5	5	5	6	7	5	5	4	4
S	4	5	6	8	9	12	14	14	11	9	7	5	9
SW	25	19	13	12	17	20	20	19	16	20	23	24	19
W	22	20	18	14	11	11	13	12	12	14	18	20	15
NW	21	24	24	22	19	16	8	19	18	18	18	19	20
Calm	*	*	*	1	1	1	1	1	1	1	*	*	1
AVERAGE WIND SPEED IN MILES PER HOUR													
N	9.5	9.7	8.9	9.0	9.0	8.7	6.6	7.1	7.3	7.5	9.8	9.4	8.5
NE	11.9	12.6	13.4	14.0	12.1	10.1	8.9	8.9	9.3	9.8	11.7	12.5	11.3
E	14.4	13.6	14.1	11.6	10.0	8.6	8.1	8.4	9.5	10.5	11.0	14.2	11.2
SE	11.5	9.8	7.6	7.7	7.1	6.9	7.3	7.5	7.8	8.2	9.6	9.9	8.4
S	12.8	11.7	9.4	9.9	8.7	8.7	8.7	8.7	8.4	8.8	10.6	11.8	9.9
SW	17.4	15.7	13.2	10.5	8.7	7.7	7.4	8.0	9.4	10.8	14.7	14.6	11.5
W	14.0	13.3	13.1	11.8	10.0	8.6	7.8	7.2	8.8	9.7	12.8	12.8	10.8
NW	12.1	12.2	11.9	11.8	11.1	10.1	9.4	9.1	9.4	10.2	11.3	11.3	10.8
All Directions	13.8	12.9	12.4	11.4	9.8	8.6	8.1	8.2	8.9	9.7	12.0	12.6	10.7

Source: Environment Canada, 1975, p. 128

waterfront area, (see Table 1). The prevailing winds are from the northwest and southwest but only winds from the southwest, south, southeast and east are important in the generation of waves reaching the study area. Winds from the east blow over the maximum fetch (257 km) and are responsible for the generation of the largest waves reaching the Toronto waterfront. Winds from the southeast and south are relatively lower in magnitude and frequency of occurrence and therefore play a secondary role in terms of wave activity. Southwesterlies are the prevailing onshore winds but owing to the limited fetch (maximum fetch of 34 km) they generate waves which have a high frequency of occurrence but are of low magnitude. Table 1 reveals the seasonality of winds in the Toronto area. Generally, the most severe winds occur during the late fall, winter and early spring months (November to April). Somewhat milder conditions prevail during the late spring, summer and early fall months (May to October). Ice cover is rare on Lake Ontario, therefore the greatest wave activity occurs during the winter months.

Only limited records of wave spectral character-

istics are available for the Toronto waterfront: some unpublished data collected by the Great Lakes Institute for 1972 and a single year's record of 1972-1973 collected by the Marine Environmental Data Service, Fisheries and Environment Canada (1975) are all that are available. Neither source provides directional information. To obtain an estimate of long term average conditions wave-hindcast techniques can be used. The most commonly used hindcast technique is the Sverdrup-Munk-Bretschneider Method (SMB) which has been employed in Lake Ontario by a number of researchers (Saville 1953, Baines and Lentheusser 1962, Brebner 1964a and 1964b, Fricbergs 1965, Coakley and Cho 1973). Other wave hindcasting techniques are available but comparison of actual to predicted wave heights for the north shore of Lake Ontario by Brebner (1964a) reveals that the SMB method provides the most reliable estimate for deep water waves.

At present the deep water wave climate of the Toronto area is best represented by the data provided by Fricbergs (1965). Using 16 years of hourly wind data collected at the Toronto Island Weather

TABLE 2. Toronto Wave Climate (1948-1963) hindcast using the SMB method.

OCTANTS	EAST					SOUTHEAST			SOUTH	SOUTHWEST			
SIGNIFICANT WAVE HEIGHT (H_S) (ft)	4-5.9	6-7.9	8-9.9	10-11.9	12-14	4-5.9	6-7.9	8-10	4-6.0	4-5.9	6-7.9	8-9.9	10-12
1948 (Aug-Dec)	—	—	—	—	—	14	13	—	—	4	—	—	—
1949	2	2	2	—	—	—	—	—	—	25	5	—	—
1950	—	21	15	7	6	20	—	—	—	16	7	—	—
1951	3	4	7	1	—	—	—	—	—	55	5	—	—
1952	13	23	12	5	—	—	—	—	—	5	—	—	—
1953	4	9	7	—	—	—	—	—	9	44	1	—	—
1954	8	13	6	4	—	—	—	1	—	17	2	—	—
1955	9	13	7	—	—	10	3	—	2	25	10	—	—
1956	8	2	1	2	—	3	2	—	5	23	9	—	—
1957	—	—	—	—	—	—	—	—	12	31	3	—	—
1958	—	—	—	—	—	—	—	—	—	9	—	—	—
1959	8	6	—	—	—	—	—	—	—	67	2	—	—
1960	12	21	9	5	—	4	5	2	37	—	—	—	—
1961	8	25	34	5	1	—	1	—	—	37	—	—	—
1962	11	18	24	4	—	—	—	—	—	96	—	—	—
1963	2	2	8	—	—	2	—	—	—	65	8	—	—
1964 (Jan-July)	20	15	17	7	—	—	—	—	—	27	21	7	1
TOTAL HOURS	108	174	149	40	7	53	23	3	28	581	73	7	1
HR/YR	6.7	10.9	9.3	2.5	0.4	3.3	1.5	0.2	1.7	36.3	4.6	0.4	0.1

Source: after Fricbergs, 1965; SMB hindcast method.

Station and employing the SMB method, Fricbergs determined the hourly frequency of occurrence of deep water waves and classified them by significant wave height (H_S) and direction, (see Table 2 and Figure 4). Thirteen wave classes were defined by Fricbergs with a lower limit on significant wave height of 4 feet (1.2 m). The wave data of the Marine Environmental Data Service (1975) indicated the likelihood of waves greater than four feet in height to be approximately 10 percent, but with waves less than one foot in height the likelihood was 30 percent. The limit set by the Fricbergs data is arbitrary and will therefore, restrict the value of the resulting model; however the large percentage of smaller waves will generate relatively lower velocity currents in the nearshore zone and may be expected to be less important in the overall drift pattern.

The computer simulation deals with one specific set of wave characteristics at a time, therefore 13 runs were required to examine the effect of the total wave climate on the littoral drift system. The deep water significant wave heights (H_S) were determined by taking the mid-point of each of Fricbergs' (1965) wave classes (Table 3). The significant periods (T_S) (not recorded in Fricbergs' study) were estimated for each corresponding H_S

FIG. 4. Significant wave roses for the Toronto waterfront area for both Winter and Summer seasons.

by re-examination of the SMB hindcast curves and noting the effective fetch for each wave propagation direction.

BATHYMETRY

Bathymetric data for the Toronto Waterfront are limited at present to those of the Canadian Hydro-

TABLE 3. Average hourly frequency per year of waves—by direction, significant height (H_s) and significant period (T_s)—for the Toronto waterfront area.

WAVES APPROACHING FROM	WAVE CLASS BY H_s (ft)	H_s (m)	T_s (sec)	AVERAGE HOURLY FREQUENCY PER YEAR
EAST	4 – 5.9	1.5	6.8	6.7
	6 – 7.9	2.1	7.4	10.9
	8 – 9.9	2.7	8.1	9.3
	10 – 11.9	3.4	8.7	2.5
	12 – 13.9	4.0	9.3	0.4
SOUTHEAST	4 – 5.9	1.5	5.5	3.3
	6 – 7.9	2.1	6.3	1.5
	8 – 9.9	2.7	6.8	0.2
SOUTH	4 – 5.9	1.5	5.5	1.7
SOUTHWEST	4 – 5.9	1.5	5.2	36.3
	6 – 7.9	2.1	6.0	4.6
	8 – 9.9	2.7	6.6	0.4
	10 – 11.9	3.4	7.3	0.1

Adapted from Fricbergs, 1965 — length of record: 16 years 1948 (Aug. – Dec.) – 1964 (Jan. – July)

graphic Charts (2062 and 2063; edition 34, August 1975) but were considered accurate enough for the large area covered. The depth values were entered into the program in the form of a rectangular matrix and were produced by laying a 1 cm square grid pattern over the hydrographic charts and noting depth values at the inter-section of each grid by interpolation. The distance between each grid line was 729 m and each data point represented 0.531 km².

The shore-normal dimension of the matrix (Y-axis) was dependent upon the offshore slope and the wave characteristics to be considered and was chosen to extend into depths greater than one-half the largest wave length, a depth of 78 m plus a two grid buffer zone. The shore-parallel dimension of the matrix (X-axis) was determined by the

length of the shoreline of interest (Highland Creek to the Credit River), plus the additional space required at each end of the matrix to allow oblique waves to reach the shoreline under examination, plus a two grid buffer at each end. The grid matrix used in this study was 113 by 30 and represented a study area of approximately 1802 km². The shoreline was positioned within the matrix by allocating negative values of depth to points on the land such that the actual shoreline received a depth value of zero.

THE LITTORAL DRIFT SYSTEM UNDER SPECIFIC WAVE CONDITIONS

Wave Refraction

To illustrate the spatial variation in wave energy

in the Toronto waterfront area, four wave refraction diagrams were drawn using the computer output for the largest significant waves from each direction (Figures 5 and 6). Waves of lower magnitude from each direction refract less but take on a very similar pattern to those shown; therefore the four diagrams selected are considered representative of waves approaching from the east, southeast, south and southwest.

In general, it appears that all waves are refracted very little owing to the relatively steep offshore and nearshore slopes and the short period waves generated in the lake; the differing orientation of shoreline segments appears to play the major role in controlling the effectiveness of wave energy reaching shore. Wave energy is relatively evenly dispersed along most of the shoreline; the zones of convergence are few in number (Figures 5 and 6) and indicate that very few areas receive a concentration of wave energy. Also illustrated on each of the diagrams is the direction of potential littoral drift represented by arrows and based on computed longshore currents. Waves from the east cause a southwesterly drift of material (Figure 5), while waves from south and southwest cause a northeasterly transport of material, (Figure 6). Waves from the southeast (Figure 5) appear to compartmentalize the littoral drift system into cells; the general direction of drift is not clear, potential transport appears to be southwesterly in some areas and northeasterly in others. These patterns, however, refer to specific wave conditions only and are controlled primarily by the breaker angle and shoreline orientation.

ALONGSHORE VARIATION IN THE LONGSHORE COMPONENT OF WAVE ENERGY FLUX

It has been shown by Tanner and his co-workers that it is the shore-parallel component of wave energy flux and its spatial variability (energy flux gradients) that theoretically correlate most successfully with zones of erosion, transport and deposition in an ideal littoral drift system. The shore-parallel component of wave energy flux (P_L) has been plotted against distance along the Toronto waterfront for each of the thirteen wave conditions. Figures 7, 8, 9 and 10 illustrate the resulting patterns. Note that the values of P_L are in joules per metre-second and are both negative and positive, with negative values indicating southwestward drift and positive values indicating northeasterly drift.

FIG. 5. Wave refraction diagrams for waves from the east and waves from the southeast.

FIG. 6. Wave refraction diagrams for waves from the south and waves from the southwest.

In general it appears that the Toronto waterfront littoral drift system is affected in different ways by waves of different magnitude and direction of approach. However, the overall trend in a number of instances is complicated by considerable 'noise': this noise results from the inability of the program to simulate accurately over very rapidly changing bed slope just prior to breaking. This may result in excessively high or low values of P_L ; where these occur the values are marked by a star

and were ignored in the construction of the smooth trend line between sample points. The line is drawn by visual approximation to best fit the predicted longshore power gradient.

Areas of potential erosion are indicated by the absolute values of P_L increasing in the direction of drift, i.e. more material would be transported out than would be transported into the area. Conversely, potential areas of deposition are denoted by the absolute values of P_L decreasing in the direction of drift, more material would be moved into the area than would be removed. A zone of transport is designated by constant values of P_L .

Wave From the East (Fig. 7)

Five graphs of the alongshore variation in P_L under specific deep water wave conditions show the effects of waves from the east on the littoral drift system in the Toronto waterfront area. While the general sense of drift appears to be consistently southwestward, the differing magnitude of waves from the east have dissimilar effects on the littoral drift system. The wave shadow in the area of Humber Bay should be noted on each graph. It is

expected that little or no material will be transported through the shadow area as a result of waves from the east. Thus, when the littoral drift system is under the influence of waves from the east, sediment drift in the area west of the shadow is likely derived from local sources.

Graph I ($H_S = 4.0$ m; $T_S = 9.3$ sec) indicates three zones of erosion, located: (1) along the eastern portion of Scarborough Bluffs; (2) along the western section of Cherry Beach and the eastern part of the Island Beaches and (3) in the area between New Toronto and Port Credit. Two areas of deposition are revealed: (1) along the Eastern Beaches and (2) in the area between Mimico Creek and New Toronto.

Graph II ($H_S = 3.4$ m; $T_S = 8.9$ sec) shows three zones of erosion located: (1) along the eastern section of Scarborough Bluffs; (2) along the Eastern Beaches, and (3) along the shoreline west of the Credit River. Three areas of deposition are indicated: (1) along the western portion of Scarborough Bluffs; (2) along Cherry Beach and the eastern Island beaches (which appear to receive drift from both directions) and (3) along the stretch of coast

FIG. 7. Longshore variation in P_L for waves from the east. Note: In this figure and Figures 8, 9 and 10 the location of the P_L values should be read vertically to the shoreline and the smoothed line is a visual approximation of the average trend.

between New Toronto and Port Credit.

Graph III ($H_S = 2.7$ m; $T_S = 8.1$ sec) reveals three areas of erosion: (1) along the eastern section of Scarborough Bluffs (which appears to lose material in both directions); (2) along the Eastern Beaches and the eastern part of Cherry Beach and (3) in the area of Cooksville Creek. Four zones of deposition are shown: (1) along the central stretch of Scarborough Bluffs; (2) along the western part of Cherry Beach and the eastern Island beaches; (3) along the coast between New Toronto and Long Branch, and (4) in the area of the Credit River.

Graph IV ($H_S = 2.1$ m; $T_S = 7.4$ sec) shows three zones of erosion: (1) along the eastern section of Scarborough Bluffs; (2) along the western portion of the Bluffs, and (3) along a small stretch of shoreline about 4 km west of the Credit River. Three areas of deposition are indicated: (1) along the central part of Scarborough Bluffs; (2) along Cherry Beach and the eastern Island beaches and (3) along the shore immediately west of the Credit River.

Graph V ($H_S = 1.5$ m; $T_S = 6.8$ sec) does not reveal any clear areas of erosion or deposition at this scale; however, a southwesterly drift is indicated. There appears to be some compartmentalization of the littoral drift system along the coast west of the Credit River.

Waves From the Southeast (Fig. 8)

Three graphs reveal the effect of waves from the southeast on the littoral drift system. The general direction of drift is complicated by the division of the system into littoral drift cells. The nodal points of each cell represent a reversal in the drift direction.

Graph I ($H_S = 2.7$ m; $T_S = 6.8$ sec) reveals the compartmentalization of the littoral drift system into 10 discrete cells. Eleven nodal points have been labelled: those identified by even numbers receive drift from both directions and are therefore depositional; those marked by odd numbers lose sediment to the northeast and southwest and are therefore erosional.

Graph II ($H_S = 2.1$ m; $T_S = 6.3$ sec) shows the division of the littoral drift system into 13 discrete cells with 14 nodal points. Again, the even numbers mark depositional nodes and the odd numbers denote erosional nodes.

It is interesting to draw comparisons between the nodal points on graphs I and II. Many of the depositional nodes in graph I are also depositional in graph II. This relationship also holds for the erosional nodes. An exception is shown in the area of Highland Creek; in graph I, nodal point '3' is depositional and in graph II it has switched to an erosion node ('4'). The lower magnitude wave, illustrated in graph II, appears to cause a greater

FIG. 8. Longshore variation in P_L for waves from the southeast. Note: The nodal points numbered from right to left indicate the zero position of the trend line.

FIG. 9. Longshore variation in P_L for waves from the south.

FIG. 10. Longshore variation in P_L for waves from the southwest.

compartmentalization of the littoral drift system.

Graph III ($H_s = 1.5$ m; $T_s = 5.5$ sec) indicates the division of the littoral drift system into approximately 17 cells; it is difficult, however, to determine exactly the location of nodal points. It is interesting to note, again, that a lower magnitude wave appears to cause greater compartmentalization of the system. A similar relationship was noted empirically by Shepard and Inman (1950) and McKenzie (1958) in their examination of rip currents which were considered as defining the

termini of small cells. The latter author specifically noted that rip-current spacing increased with wave energy. It appears that the littoral drift cells moving sediment northeasterly are extremely small as compared to the relatively larger cells transporting material southwesterly. The general sense of drift seems to be to the southwest as might be expected.

Waves From the South (Fig. 9)

Only one graph ($H_s = 1.5$ m; $T_s = 5.5$ sec) is

necessary to represent the limited wave activity from the south. The general sense of drift is toward the northeast, no areas of erosion or deposition can be delimited with any certainty at this scale. Two points should be noted: (1) the small cell indicated east of Highland Creek in the area of Duffin Creek and (2) the small cell revealed in the area of Gibraltar Point.

Waves From the Southwest (Fig. 10)

Due to the location of the Toronto waterfront and the shape of Lake Ontario, simulated waves from the southwest do not appear to have a significant effect on the littoral drift system west of the Toronto Islands. In general, all four graphs representing the effect of waves from the southwest indicate a drift of material to the northeast, as was expected from the refraction diagram.

Graphs I ($H_S = 3.4$ m; $T_S = 7.3$ sec), II ($H_S = 2.7$ m; $T_S = 6.6$ sec) and III ($H_S = 2.1$ m; $T_S = 6.0$ sec) indicate erosion of the island beaches and a transport of this material to the northeast. An erosional zone is also indicated east of Duffin Creek.

Graph IV ($H_S = 1.5$ m; $T_S = 5.2$ sec) reveals no clear zones of erosion and deposition at this scale; however, a northeasterly drift of material is indicated.

To summarize, it can be stated that waves from the east cause a drift of material to the southwest, while waves from the southeast tend to compartmentalize the system into littoral drift cells. Waves from both the south and southwest move material in a northeasterly direction along the shore. It is also apparent from Figures 7 to 10 that the predicted littoral drift patterns do vary with wave magnitude and approach direction. However, the long term effect of each wave condition must reflect the absolute effect of that condition and the frequency with which it occurs.

A MODEL OF THE LITTORAL DRIFT SYSTEM IN THE TORONTO WATERFRONT AREA

It has been demonstrated that the littoral drift system in the Toronto waterfront area responds differently to waves of different magnitude and approach direction. Each of the graphs in Figures 7 to 10 are, in fact, models of the littoral drift system under specific wave conditions. However, of more importance is the combined long term net effect of all significant wave conditions. For this

reason, it is necessary to estimate the net effect of both the magnitude and frequency of occurrence of all important waves. The technique involves the selection of a number of points along the shore (Figure 11) and at each point the value of P_L is determined from the graphs of the longshore variation of P_L , for each wave condition. Each value of P_L is then multiplied by the corresponding frequency of occurrence of the wave condition responsible for P_L (from Table 3) and these values, for each selected point, are summed to give an approximation of the net P_L along the shore. Thus:

$$\bar{P}_{Lj} = \sum_{i=1}^n (f_i \cdot P_{Lij}) \quad (v)$$

where

$\bar{P}_{Lj}$ is the net value of P_L in joules per metre-second, for all waves reaching the point of interest

f_i is the annual average frequency of occurrence of wave for each class

P_{Lij} is the longshore component of wave energy flux at point j produced under condition i .

The net values of $\bar{P}_{Lj}$ are then plotted along the shore in order to approximate the *time averaged* littoral drift condition of the waterfront. In this way, areas of erosion, transport and deposition due to the average wave climate are indicated. The combined effect of the magnitude and frequency of all 13 wave conditions is calculated for 35 selected points along the shoreline using equation (v). Plotting the values of the net longshore component of wave energy flux (P_L) against distance along the shore provides a model of the average long term trends in erosion, transport and deposition in the Toronto waterfront area (Figure 12).

The littoral drift model reveals that, in an average year, the stretch of shoreline between Highland Creek and the Credit River contains 5 discrete cells, each characterized by a zone of erosion, transport and deposition. Five nodal points mark the termini of each littoral drift cell and a reversal in the drift direction. Nodal point '5' is an exception; it appears to separate the littoral drift cell along the Scarborough Bluff shoreline into two subcells. The direction of drift in each subcell is southwesterly. Nodal points '2' and '4' are depositional nodes, receiving material from both directions and '1', '3' and '6' are erosional nodes, losing drift material in both directions.

FIG. 11. Sample points for the calculation of $\bar{P}_L$ —Toronto waterfront area.

GEOMORPHOLOGICAL JUSTIFICATION OF THE LITTORAL DRIFT MODEL

The model presented above indicates on purely theoretical grounds areas which are potentially sensitive to sediment erosion or deposition. While there are inherent shortcomings in theory and technique, the accuracy with which the prototype system is predicted is the only real test. The *geomorphological* sense of the model is therefore of paramount importance and a number of historical trends can be examined in the light of the model predictions:

(1) In a general sense the model reveals a progressive southwesterly transport of material eroded from Scarborough Bluffs toward a depositional site on Toronto Islands. From early maps and site descriptions (Bouchette 1793 and 1832, as quoted in Coleman 1937 and Fricbergs 1972a) it is clear that the islands formed a complex recurved spit formed by the deposition of large volumes of westerly moving material. Fleming as early as 1853 hypothesized a westerly growing spit fed by sediment eroded and transported from an ancient promontory some 3.2 km south of the present bluffs and more recently other workers have defined this general drift pattern (McConnell 1957, Coakley 1970). In the western section of Scarborough Bluffs, for example, and along the Eastern Beaches the accumulation of sedimentary materials on the eastern side of a number of structures clearly supports a southwesterly drift. A long groyne constructed in the early 1900's at the R. C. Harris Filtration Plant (located just west of the western limit of the bluffs) filled to capacity, depriving the Eastern Beaches to the west. In 1960, an

attached 'L' shaped riprap breakwater, 275 m long, was constructed in the central section of the Eastern Beaches. This breakwater trapped littoral material at an exceedingly fast rate and caused an accumulation of 1.4 hectares of beach material on the eastern side of the structure (Fricbergs 1972a).

(2) The model predicts greater erosion at the eastern end of the Scarborough Bluffs than further west, a pattern evident from recessional studies. Figure 13 illustrates the greater toe recession in the eastern bluffs section revealed by surveys carried out by the Toronto Harbour Commissioners and the Scarborough Township Engineers between the years 1922 and 1952 (Fricbergs 1972b).

(3) Three of the six nodal points indicated by the model can be verified, at least in part by other studies. Gibraltar Point is clearly identified by the model as an area sensitive to drift reversals (nodal point '1' in figure 12) and therefore of variable erosion at the distal end of the Toronto Islands spit. Although the recurved spit was in all probability built by a continual southwesterly drift, the instability of the area is clearly indicated by shoreline positional changes documented by Fricbergs (1972a) for the period 1879 to 1972 (Figure 14). In the period 1879 to 1915, for example, erosion to the west of the Point was complemented by deposition to the east. In the following time period (1915 to 1924), however, a reversal of this situation occurred. Furthermore, evidence for a reversal drift has been put forward by Coakley (1970) on the basis of natural tracer experiments which revealed that little material eroded from Scarborough Bluffs passes the distal end of the

FIG. 12. Model of the Toronto waterfront littoral drift system.

Toronto Islands spit.

The depositional nodal point '2' (Figure 12) indicated by the model at the location of the Eastern Gap has been partly verified by the necessity for extensive dredging activities to maintain the channel. Dredging data recorded from 1929 to 1965 revealed that the annual volume of dredged material varied from 10,500 to 68,000 m³ with an average of 27,000 m³ (Fricbergs 1970). However, it is likely that deposition was enhanced by the presence of jetties on the channel margins and historically the Eastern Gap was formed by erosion and breaching.

A littoral drift divide in the area of Highland Creek (nodal point '6' in figure 12) has been observed by Fricbergs (1976 personal communication).

Nodal points 3, 4 and 5 have not and cannot now be verified since each node is located at the site of existing man-made structures (Eastern Headland, Ashbridge's Bay and Bluffer's Park landfill sites). Nodal points 3 and 4 are in fact defined by one sampling point only and might be

considered erroneous. However, simulation of the littoral drift system at a finer scale in the central waterfront area reveals similar drift divides (McGillivray 1976).

DISCUSSION AND CONCLUSION

The results of this study uncover general trends in net erosion, transportation and deposition in the Toronto waterfront area. The littoral drift model is, however, imperfect and subject to a number of limitations.

(1) A fundamental weakness is that the computer simulation is based on linear wave theory and deals with monochromatic waves, both of which are extreme simplifications of wave conditions in nature. A closer approximation to reality would necessitate calculating the refraction of an entire directional wave spectrum (Karlsson 1969, Abernathy and Gilbert 1975) and using, possibly, higher order Stokes or cnoidal wave theory (McLennan *et al.* 1971, Chu 1975, Skovgaard and Petersen 1974, Walker 1976). However, Dean (1970) has illustrated the success of linear wave

SOURCE: BURT AND ROOZENBOOM (1952)
 DIAGRAM COMPILED BY: P. HALE

FIG. 13. Scarborough Bluffs recession—1922 to 1952.

Fricbergs, 1972a, DWG. NO. 15300 (THC)

FIG. 14. Gibraltar Point shoreline configuration—1879 to 1972.

theory in modelling surface profiles of gravity waves over a wide range of near breaking conditions and Komar and Gaughan (1972) have shown the value of linear theory in predicting breaker heights.

(2) The model assumes no reflection or diffraction of wave energy and clearly no consideration of non-linear effects, although their significance has been demonstrated by Whalin (1972) and may be worthy of serious consideration in future numerical models.

(3) The wave refraction procedure employed considers wave transformation only to the first point of breaking and does not consider energy dissipation beyond. Fundamental theory in this respect is lacking although work by Smith and Camfield (1972) illustrates the use of experimental data for prediction in this zone. In Lake Ontario where nearshore slopes are relatively steep and wave periods are short, extensive breaking, reformation and surf development is not likely to be very important in terms of refraction.

(4) The values of P_L were computed at the break point termini of each wave ray and these values were later plotted against distance along the shore to illustrate longshore variations. These were then interpreted to reveal trends in erosion, transportation and deposition. This procedure contains at least two limitations. Firstly, the value of P_L is dependent upon the ability of the program to predict wave energy dissipation through shoaling refraction and bottom friction and to estimate the breaker angle (α_b). The program does both within the accuracy of the theoretical constraints and the simulation of bottom topography, but it is not presently possible to test the accuracy in the prototype. Secondly, the program tends to generate 'noise' which causes erroneous values of P_L to be computed; thus all values of P_L are not of equal accuracy. The smoothing techniques used were limited to visual approximation rather than an objective best fit procedure. Nevertheless such a technique has proved equally successful in other work (May and Tanner 1973).

(5) The use of P_L as an index to the rates of sediment transport presents some theoretical problems in that no real consideration is given to the actual mechanisms of sediment transport or the availability of sediment. Attempts have been made to predict detailed water and sediment circulation patterns, actual rates of erosion and deposition and the resulting shoreline changes based on a full statement of the continuity equation (Price, Tomlinson and Willis 1972, Moytka and Willis 1974, Mogel

and Street 1974a and 1974b, Birkemeier and Dalrymple 1975, Wang, Dalrymple and Shian 1975, Skafel 1975a, Fleming and Hunt 1976, and Willis 1976). However, the problems associated with relaxing the constraints in this type of application are considerable. The use of the relationship in equation (iv) has been justified by a number of empirical studies (notably Komar and Inman 1970) but these were restricted to sandy beaches. At this stage it is not possible to compute actual rates of sediment transport based on P_L and the K value defined previously; nevertheless, the *relative* rates of longshore transport within the system are described by the values of P_L . It is not clear that significant gains will be made by using highly complex numerical equations for sediment transport rates particularly where the controlling inputs (wave climate and bathymetry) are themselves so poorly known. Further, the extension of computer time required for such simulation may quickly reach a point of diminishing returns (Birkemeier and Dalrymple 1975).

(6) A number of important factors which control the littoral drift system in the Toronto waterfront area were ignored: (a) the model was unable to deal with the possibility of waves from the north-east and west but these would have to be initiated in extremely shallow water and with a limited fetch would be extremely small; (b) annual and longer period lake level fluctuations clearly affect the availability of sediments as well as changing the depth matrix, however lake level changes could be incorporated as uniform shifts in bathymetry, (c) the lack of continuous records of wave spectral characteristics forced the use of wave hindcast data. Only relatively high magnitude, low frequency events were considered; waves less than 1.2 m in height were not accounted for at all. However, it must be pointed out that it is the high magnitude waves that have the most important effect on the littoral drift system and the great advantage of the hindcast data was that it yielded 16 years of record which provided a reasonable estimate of the average annual magnitude and frequency of occurrence of specific wave conditions.

In conclusion, it has been demonstrated that despite a number of imperfections, the erosional and depositional sensitivity of the Toronto waterfront area is revealed by the modelling technique. The *geomorphological sense* of the resulting model supports May's (1974) suggestion that the results of the simulation program are sufficiently accurate to warrant its use on real coasts. The combined

magnitude and frequency effects of wave conditions representing the wave climate of the area were shown to be extremely important in revealing the time-averaged trends in the littoral drift system. It is the generation of these long term models which are important in the examination of the effects of large man-made structures on a littoral drift system. It is thought that the modelling technique developed in this study may prove to be a valuable planning aid in present and future coastal development schemes.

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