Validation of Point and Pressure Loading Models for Simply Supported Composite Sandwich Beams

by

Bryan K Wright

A thesis submitted in conformity with the requirements for the degree of Masters of Applied Science Graduate Department of Aerospace Engineering University of Toronto

© Copyright 2012 by Bryan K Wright
Abstract

Validation of Point and Pressure Loading Models for Simply Supported Composite Sandwich Beams

Bryan K Wright
Masters of Applied Science
Graduate Department of Aerospace Engineering
University of Toronto
2012

Stiffness and strength models are derived for simply supported composite sandwich panels comprised of fibre-reinforced face sheets and polymer cores subject to symmetric four point bending and uniformly distributed loading. Optimal trajectories for minimum mass design are calculated using the models and situated on failure mechanism maps. A stiffness constraint is also derived to omit beam designs of excessive compliance. Analytical models were validated through an extensive series of experiments, considering beams comprised of GFRP face sheets with ROHACELL 51-IG and extruded polystyrene (EPS) polymer cores. An alternate loading fixture was used to simulate uniform pressure loads. In general, experiments were able to validate most analytical expressions for a range of experimental conditions. Though the predictions worked well with most beam cases, analytical models were noted to become unreliable for short or slender beams.
Acknowledgements

I first wish to express my esteemed gratitude to my thesis advisor Dr. Craig A. Steeves, for his support and guidance throughout the duration of this master’s project. It was a privilege to work with him and I wish him the best in his future pursuits. I also wish to thank my RAC Advisors, Dr. David W. Zingg, Dr. Philippe Lavoie, Dr. Prasanth B. Nair and Dr. Alis Ekmekci for their constructive feedback during the following project, and my second reader, Dr. Adam Steinberg, for taking the time to review my work. I also wish to thank Jennifer Higgs for helping edit my thesis.

To my family, a special thank you must go out to my parents for their never-ending love and support. They have always been my teachers, role models and friends. I cannot express how fortunate I am to be their son. I also wish to thank my sister Erin, for her support, wishing her the best in her teaching career, and to my Nana for her love and support.

I wish to thank my colleagues within the Multifunctional Structures Lab of University of Toronto Institute for Aerospace Studies: Dane Haystead, Richard Lee, Collins Ogumpide and John Chu, in addition to all my friends within UTIAS community. I also cannot forgot my friends within Shorinji Kan Jiu Jitsu for keeping me in the game all this time, even after the bruises.

I wish to thank Lilex Industries for machining the whippletree testing fixture for the following project.

Last, and certainly not least, I would like to thank my grandfather John "Jack" Keenan, my inspiration into pursuing a career in engineering.

Bryan Keenan Wright
University of Toronto Institute for Aerospace Studies
# Contents

1 Introduction 1  

1.1 Summary of Previous Work ................................. 1  

1.2 Project Scope ........................................ 2  

2 Background 3  

2.1 Review of Sandwich Beam Mechanics .......................... 3  

2.1.1 Stiffness ........................................... 4  

2.1.2 Strength ............................................ 6  

Face Yield and Microbuckling ................................ 6  

Core Shear ............................................... 7  

Core Crushing ........................................... 7  

Indentation ............................................... 7  

Face Wrinkling ........................................... 8  

Adhesion Failure/Buckling-Induced Delamination: .......... 8  

2.2 Modelling Techniques - Failure Mechanism Map ................. 9  

2.3 Modelling Techniques - Minimum Mass Trajectory Line Plots ... 14  

3 Analysis 17  

3.1 Case A: Simply Supported, Symmetric Four Point Bending .......... 17  

3.1.1 Stiffness ........................................... 18  

3.1.2 Strength ............................................ 20  

Failure Mechanism #1: Microbuckling ......................... 20  

Failure Mechanism #2: Core Shear ............................ 21  

Failure Mechanism #3: Indentation ............................ 22  

3.1.3 Failure Mechanism Mapping ............................ 24  

3.2 Case B: Uniformly Distributed Load Case ............... 31  

3.2.1 Stiffness ........................................... 31  

3.2.2 Strength ............................................ 32  

Failure Mechanism #1: Microbuckling ......................... 32  

Failure Mechanism #2: Core Shear ............................ 33  

Failure Mechanism #3: Core Crush ............................ 33  

Failure Mechanism #4: Indentation ............................ 33  

Failure Mechanism Mapping ................................. 36
4 Failure Mechanism Map Validation

4.1 Material Selection and Fabrication
   4.1.1 Face Sheet Material - GFRP
   4.1.2 Core Material - ROHACELL 51-IG, Extruded Polystyrene (EPS)
   4.1.3 Beam Fabrication

4.2 Data Acquisition
   4.2.1 Strain Gauge (A)
   4.2.2 Core Shear Strain Gauge (B)
   4.2.3 Laser Extensometry (C)-(D)
   4.2.4 Data Acquisition and Output

4.3 Test Outline
   4.3.1 Case A: Three Point Bending
   4.3.2 Case B: Four Point Bending
   4.3.3 Case C: Uniformly Distributed Loading
   Proposed Testing Method: Whippletree Loading Apparatus

4.4 Experiment Results
   4.4.1 Three Point Bending
   4.4.2 Four Point Bending
   4.4.3 Uniformly Distributed Loading

4.5 Stiffness Validation

4.6 Strength Validation
   4.6.1 Core Shear
   4.6.2 Indentation
   4.6.3 Microbuckling
   4.6.4 Adhesion Failure
   4.6.5 Results Summary - 3D Failure Mechanism Mapping

4.7 Summary of Experiments
   4.7.1 Stiffness Constraint Validation

5 Conclusion and Future Recommendations

Bibliography
### Nomenclature

#### Key Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Load</td>
<td>N</td>
</tr>
<tr>
<td>$w$</td>
<td>Pressure</td>
<td>kPa</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
<td>kg</td>
</tr>
<tr>
<td>$x$</td>
<td>Position along beam</td>
<td>mm</td>
</tr>
<tr>
<td>$y(x)$</td>
<td>Deflection</td>
<td>mm</td>
</tr>
<tr>
<td>$P/\delta, w/\delta$</td>
<td>Stiffness</td>
<td>N/mm, kPa/mm</td>
</tr>
</tbody>
</table>

#### Beam Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Face sheet thickness</td>
<td>mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Core thickness</td>
<td>mm</td>
</tr>
<tr>
<td>$L$</td>
<td>Beam length</td>
<td>mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Beam width</td>
<td>mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between face sheet centroids</td>
<td>(mm)</td>
</tr>
<tr>
<td>$H$</td>
<td>Beam overhang</td>
<td>mm</td>
</tr>
</tbody>
</table>

#### Material Properties

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_f$</td>
<td>Face sheet elastic modulus</td>
<td>GPa</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Face sheet compressive strength</td>
<td>MPa</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Face sheet density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Core elastic modulus</td>
<td>MPa</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Core compressive strength</td>
<td>kPa</td>
</tr>
<tr>
<td>$G_c$</td>
<td>Core shear modulus</td>
<td>MPa</td>
</tr>
<tr>
<td>$\tau_y$</td>
<td>Core shear yield stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>Core shear strength</td>
<td>kPa</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Core density</td>
<td>kg/m$^3$</td>
</tr>
</tbody>
</table>

#### Non-Dimensional Ratios

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{t} = \frac{t}{c}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{c} = \frac{c}{L}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{a} = \frac{a}{L}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{P} = \frac{P\phi}{\sigma_f b L}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{w} = \frac{w}{\sigma_f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{M} = \frac{M\psi}{\rho_f b L^2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\phi} = \frac{\phi}{\sigma_f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\psi} = \frac{\rho_f}{\rho_c}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{E}_f = \frac{E_f\phi}{\sigma_f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{E}_c = \frac{E_c\phi}{\sigma_f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{G} = \frac{G_c\phi}{\tau_c}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\tau} = \frac{\tau_c\phi}{\sigma_f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\sigma} = \frac{\sigma_c\phi}{\sigma_f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\rho} = \frac{\rho_c\psi}{\rho_f}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>E-Glass fibre-reinforced polymer</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon fibre-reinforced polymer</td>
</tr>
<tr>
<td>PMI</td>
<td>Polymethacrylimide foam</td>
</tr>
<tr>
<td>EPS</td>
<td>Extruded polystyrene insulation foam</td>
</tr>
<tr>
<td>FMM</td>
<td>Failure Mechanism Map</td>
</tr>
<tr>
<td>MMT</td>
<td>Minimum mass trajectory</td>
</tr>
<tr>
<td>MB</td>
<td>Microbuckling</td>
</tr>
<tr>
<td>CS</td>
<td>Core shear</td>
</tr>
<tr>
<td>CC</td>
<td>Core crush</td>
</tr>
<tr>
<td>A</td>
<td>Adhesion failure</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data acquisition</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Composite materials have become a dominant choice in lightweight structural design due to a high stiffness-to-weight ratio. One variation is the composite sandwich panel, a structure comprised of two thin stiff face sheets around a low density core material. Composite sandwich panels can be tailored to satisfy any strength requirement or weight limitation, based on the vast array of materials available. Such examples include metallic or fibre-reinforced face sheets and foam, truss or honeycomb cores.

Though composite materials in general have the potential to replace traditional structural materials (such as homogenous metallic beams or panels), predicting stiffness and strength behaviour using traditional beam theory is complicated by their complex geometry. As such, only a select number of material configurations have been investigated for an even smaller number of loading cases. Without providing valid analytical models that best represent the stiffness and strength behaviour for each face/core consideration, optimal structural configurations cannot be determined unless a prohibitive number of experiments are conducted.

1.1 Summary of Previous Work

Notable early work in sandwich beam theory includes Allen [1], Plantema [29] and Zenkert [43], each considering the simplified model of an arbitrary face sheet and core material deflecting under various applied loads. Subsequent work by Gibson [14] examines metal and polymer foam cores while Ashby et al. [3] focuses on metal foams.

Composite sandwich structures have been primarily studied for simply supported three point bending. Relevant works in three point bending include Steeves and Fleck [36, 37, 38], who determined analytical expressions for the stiffness and strength for simply supported composite sandwich beams comprised fibre-reinforced face sheets and polymer foam cores. Analytical models were validated through experiments then compared to finite element simulation results. Rizov [30, 31] compared 2D and 3D finite element models to experiments for beams comprised of GFRP face sheets and Divinycell foam core. Sadighi et al.[32], Gdoutos et al.[13], and Lim et al.[25] have investigated simply supported three point
bending for GFRP/PVC and GFRP/polyurethane beams, while Chiras et al. [8] investigated three point bending on metal truss cores.

Only recently have different load cases been investigated, including simply supported four point bending and concentrated punch loading. Works in four point bending include Gibson [15], who derived expressions for midspan deflection for beams comprised of CFRP and GFRP face sheets and polymethacrylimide (PMI) foam core. Other works include Chen et al. [7] for metal face sheets and cores, Casari et al. [6] for Novel K-Cor Structures and Madhi et al. [27] for GFRP/NOMEX honeycomb cores. Tagarielli and Fleck [39] validated strength expressions for concentrated punch loading for both simply supported and clamped beams.

Another case is uniformly distributed loading, whereby the structure is subject to a pressure, such as air or water. Though experiments have been completed for uniformly distributed loading, no attempt has been made to propose analytical models for strength and stiffness. From previous literature, the only noted testing method for pressure loads has been the implementation of airbags, shown by White [42] on wood pallets and Islam et al. [21] on composite beams. Introduced by Gibson and Ashby [14], failure mechanism mapping considers a design space of geometric variables that define a structure. For a fixed set of material properties, a design space of dimensionless structural parameters can be divided into regions, where in each region a single failure mechanism dominates. The result is a visual representation of all competing failure mechanisms for a structure. By comparing load to mass gradients, minimum mass trajectories for optimal beam design configurations are determined. Previous literature on failure mechanism mapping include Steeves and Fleck [36, 37, 38], Kazemahvazi et al. [24] for metallic core composite sandwich beams under shear loading and Mohan et al. [28] for CFRP/aluminum foam core beams subject to simply supported three point bending. Recent work by Steeves [35] accounts for mass and stiffness limits for simply supported three point bending.

1.2 Project Scope

The following master’s thesis investigates symmetric four point bending and uniform loading cases for simply supported composite sandwich beams comprised of fibre-reinforced face sheets and polymer foam cores. The study encompasses the following two primary objectives:

- To define and validate analytical models of stiffness and strength
- To determine the optimal beam design trajectories considering for minimum mass

Extensive research has already been completed for three point bending, and thus, is used as a reference model for sandwich beam theory. Analytical models for four point bending and uniformly distributed loading are validated through a series of experiments.
Chapter 2

Background

2.1 Review of Sandwich Beam Mechanics

Consider Figure 2.1, a simply supported composite sandwich beam comprised of two thin stiff face sheets and a low density core. Let $L$ be the beam length, and $H$ be the overhang. Remaining beam geometry includes face sheet thickness $t$, core thickness $c$, beam width $b$ and distance between face sheet centroids ($d = c + t$). Relevant material properties for the face sheets are density $\rho_f$, elastic modulus $E_f$ and compressive strength $\sigma_f$. Relevant material properties for the core are density $\rho_c$, elastic modulus $E_c$, compressive strength $\sigma_c$, shear modulus $G_c$, shear yield stress $\tau_y$ and shear strength $\tau_c$.

![Figure 2.1: Geometry of a simply supported composite sandwich beam](image)

For sandwich beam theory, two key design objectives must be addressed: stiffness and strength. Stiffness, or rigidity, is the measure of resistance a structure exhibits to deformation. Strength is the peak load a structure can withstand before failure. Multiple works have already validated stiffness and strength expressions for simply supported three point bending, considering different face and core materials [3, 36, 32, 39].
For three point bending, consider a transverse load $P$ that is applied to the midspan of the beam, seen in Figure 2.2.

![Figure 2.2: A simply supported composite sandwich beam subject to three point bending](image)

The corresponding shear force ($V$) and bending moment ($M$) diagrams for three point bending are shown in Figure 2.3.

![Figure 2.3: Shear and bending moment diagrams for a simply supported composite sandwich subject to three point bending](image)

### 2.1.1 Stiffness

Composite sandwich beam stiffness relates $P$, the applied load to $\delta$, the midspan deflection. An expression for beam stiffness is derived by evaluating both bending and shear effects of deflection. For three point bending,

$$\delta = \delta_{bending} + \delta_{shear} = \frac{PL^3}{48(EI)_{eq}} + \frac{PL}{4(AG)_{eq}}.$$  \hfill (2.1)

where $(EI)_{eq}$ and $(AG)_{eq}$ are the equivalent bending stiffness and shear stiffness terms for a beam. Therefore, the stiffness of a simply supported composite sandwich beam when subject to three point bending is

$$\frac{P}{\delta} = \left[\frac{48(EI)_{eq}(AG)_{eq}}{12(EI)_{eq}L + (AG)_{eq}L^3}\right].$$  \hfill (2.2)
For composite sandwich structures, \((EI)_{eq}\) is

\[
(EI)_{eq} = E_f \frac{bt^3}{6} + E_c \frac{bc^3}{12} + E_f \frac{btd^2}{2}.
\]  

(2.3)

From Equation (2.3), the first two terms define the individual contributions of the face sheets and core; the third term is a relationship for the bending axis of the face sheets with respect to the neutral axis of the beam.

The equivalent shear stiffness equation, \((AG)_{eq}\), is

\[
(AG)_{eq} = \frac{bG_c d^2}{c}.
\]  

(2.4)

Figure 2.4 compares the cross-sectional axial and shear stress profiles for a sandwich beam subject to bending.

In sandwich beam design, elastic and shear moduli of the core are small compared to the face sheet \((E_c << E_f, G_c << G_f)\). Therefore, it is assumed core shear stress is constant and core axial stress is negligible (seen in Figure 2.4(b)). Second, face sheets are very thin compared to the core, \(t << c\). Assuming \(d \approx c\), axial stress within each face is constant, and shear stress in the face sheet is linear (seen in Figure 2.4(c)). Using the above assumptions, the bending stiffness equation becomes

\[
(EI)_{eq} \approx E_f \frac{btd^2}{2},
\]  

(2.5)

and shear stiffness becomes

\[
(AG)_{eq} \approx bdG_c.
\]  

(2.6)
As thicker face sheets or stiffer cores are considered, the following simplifications cannot be used, for in those cases stiffness will be under-predicted. As such, the requirement of the full stiffness terms is case-specific and may only be determined through experiments.

2.1.2 Strength

Failure occurs when a critical stress in the core or face sheet is exceeded. In all cases, the corresponding peak load of the beam is a function of the component material properties and geometry. The weakest mechanism is considered the active failure mechanism for a given configuration. Figure 2.5 lists the failure mechanisms applicable to sandwich beams.

![Figure 2.5: Failure mechanisms for sandwich beams, (a) face yield, (b) core shear, (c) indentation, (d) core crushing, (e) face wrinkling, (f) adhesion failure](image)

Face Yield and Microbuckling

Face yield occurs when the tensile or compressive strength of the face material is exceeded. For fibre-reinforced face sheets, microbuckling is the associated compressive face failure, whereby the material fails through kink band formation [26, 5]. For fibre-reinforced polymer face sheets, the compressive strength is significantly lower than the tensile strength, hence only microbuckling is considered. Steeves and Fleck [36] express the microbuckling failure load equation for simply supported three point bending as

\[
P_{MB} = \frac{4htd\sigma_f}{L}. \tag{2.7}
\]

The use of the face sheet compressive strength to predict microbuckling failure requires caution. Though the face sheet failure is from an axial stress, the compressive strength only applies to a pure uniaxial
compressive load. Prior to microbuckling, the face sheet material is subject to a multiaxial stress state. As a result, the actual material strength may be higher than predicted [36].

Core Shear

Core shear failure occurs when the shear strength of the core material is exceeded. In response to core shear, the face sheet either develops plastic hinges (common for metallic face sheets) [1] or elastically deforms (for the case of polymers) [36]. Early models for core shear only consider the shear effects of core [1, 29],

\[ P_{CS} = 2bd\tau_c. \]  

(2.8)

More recent works consider the face sheet as an additional stiffening property [8]. The failure load expression is derived using Timoshenko beam theory, where for three point bending

\[ P'_{CS} = 2bd\tau_c + 8Efb\left(\frac{t}{L}\right)^3\delta. \]  

(2.9)

Noted in Tagarielli and Fleck, simply supported beams comprised of elastic-plastic cores are shown to yield a higher failure threshold if they are designed with considerable overhang [39]. To compensate, an overhang factor may be applied such that

\[ P''_{CS} = P_{CS}\left(\frac{(L+2H)}{L}\right). \]  

(2.10)

As the strengthening term of Equation (2.9) is negligible for cases of small beam deflections before failure, the following study considers the model in Equation (2.8) with the additional overhang factor for beams comprised of elastic-plastic cores.

Core Crushing

Core crushing occurs when the transverse applied pressure exceeds the compressive strength of the core,

\[ w_{CC} = \sigma_c. \]  

(2.11)

The simply supported three point bending case disregards core crush failure as it requires an applied pressure, compared to a point load.

Indentation

Indentation is an interaction failure between the face sheet and core. At a critical load, the face sheet collapses into the core material within a localized region of length 2\( \lambda \).
Figure 2.6 illustrates the occurrence of indentation failure for composite sandwich beams.

![Figure 2.6: Indentation failure region for a simply supported composite sandwich beam subject to three point bending](image)

For ductile face sheets, such as metals, indentation is caused by the formation of plastic hinges on the top face sheet [2, 39]. For elastic face sheets, such as polymers, the top face elastically buckles into the core at a critical load [16].

The reaction pressure of the core $q$ may be treated as: rigid-ideally plastic, elastic-ideally plastic and purely elastic. Rigid-ideally plastic cores collapse when the core compressive strength is exceeded [36, 33]. For an elastic-ideally plastic core, core indentation is initially assumed elastic until the load surpasses a yield point. The result is a two-phase indent of the core: elastic indentation surrounding an inner region of plastic collapse [36, 33, 13]. If the core is assumed purely elastic, the face sheet still buckles, but no plastic collapse is evident [16, 33]. For the simply supported three point bending expression derived by Steeves and Fleck, elastic face sheets and a rigid-ideally plastic core are considered [36, 37]:

$$P_t = bt \left( \frac{\pi^2 d E_f \sigma_c^2}{3L} \right)^{1/3}$$

(2.12)

**Face Wrinkling**

Face wrinkling is a material-specific failure mechanism whereby the face sheet buckles within the cell size of the core material. The following failure is common in honeycomb or lattice core composite sandwich materials but does not apply for polymer foams.

**Adhesion Failure/Buckling-Induced Delamination:**

Commonly referred to as debonding, decohesion or delamination, adhesion failure is the separation of the face sheet and core. Hutchinson and Suo detail various occurrences of adhesion failure and growth [22]. In addition to flat surfaces, interest has been also taken to curved substrates including Hutchinson [19], Clausen *et al.* with cornered composite materials [23] and Sorensen on spherical structures [34].

Buckling-induced delamination is the adhesion failure applicable to sandwich beams subject to bending, whereby the localized separation of the face and core is generally attributed to compliant face sheets and weak adhesive strength. Though delamination has been well documented in terms of a crack (or blister) growth [22, 40, 11, 12], few attempts have been made to predict initial buckling load.
However, adhesion failure rarely occurs as the strength of standard adhesives generally exceed the strength of the core material. If in the event adhesion failure occurs, it is more due to an underlying fabrication fault than material selection.

### 2.2 Modelling Techniques - Failure Mechanism Map

Introduced by Gibson and Ashby [14], the failure mechanism map considers a design space of geometric variables that define a structure. For a fixed set of material properties, the design space can be divided into regions, where in each region a single failure mechanism exists. The result is a visual representation of all competing failure mechanisms for a structure in terms of peak load and mass gradient plots. Steeves and Fleck [36, 37, 38], Tagarielli and Fleck [39], Chen et al. [7], Mohan et al. [28] and Kazemahvazi et al. [24] adapt this method to illustrate failure for various composite sandwich beams.

The following study compares two face sheet materials, carbon fibre-reinforced polymer (CFRP) and E-glass fibre-reinforced polymer (GFRP). Two different core materials are also considered: Evonik Industries ROHACELL Industrial Grade (IG) polymethacrylimide (PMI) foam and Foamular C-300 extruded polystyrene (EPS) insulation foam.

The associated material properties are listed in Tables 2.1 and 2.2. While technical data for GFRP, CFRP, ROHACELL 51-IG and 110-IG are provided [36, 9, 20], a complete set of material properties for EPS are not available through online sources. As such, compression and shear tests were conducted to determine material properties of EPS and are noted in Chapter 3.

#### Table 2.1: Face sheet material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho_f$ (kg/m$^3$)</th>
<th>Elastic Modulus, $E_f$ (GPa)</th>
<th>Compressive Strength, $\sigma_f$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Glass Fibre-Reinforced</td>
<td>1770</td>
<td>30</td>
<td>350</td>
</tr>
<tr>
<td>Polymer (GFRP)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon Fibre-Reinforced Polymer (CFRP)</td>
<td>1600</td>
<td>70</td>
<td>570</td>
</tr>
</tbody>
</table>

#### Table 2.2: Core material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho_c$ (kg/m$^3$)</th>
<th>Elastic Modulus, $E_c$ (MPa)</th>
<th>Compressive Strength, $\sigma_c$ (MPa)</th>
<th>Shear Modulus, $G_c$ (MPa)</th>
<th>Shear Yield, $\tau_y$ (MPa)</th>
<th>Shear Strength, $\tau_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS</td>
<td>28</td>
<td>13</td>
<td>0.422</td>
<td>7</td>
<td>0.282</td>
<td>0.357</td>
</tr>
<tr>
<td>ROHACELL 51-IG</td>
<td>52</td>
<td>70</td>
<td>0.9</td>
<td>19</td>
<td>-</td>
<td>0.8</td>
</tr>
<tr>
<td>ROHACELL 110-IG</td>
<td>110</td>
<td>160</td>
<td>3</td>
<td>50</td>
<td>-</td>
<td>2.4</td>
</tr>
</tbody>
</table>
Considering a design space of non-dimensional parameters \( \bar{t} = \frac{t}{c} \) and \( \bar{c} = \frac{c}{L} \), failure mechanism equations are expressed in non-dimensional terms using

\[
\hat{P} = \frac{P}{\sigma_F b L},
\]

(2.13)

while beam mass is expressed as

\[
\hat{M} = \frac{M}{\rho_f b L^2}.
\]

(2.14)

All material properties are expressed as:

\[
\tilde{E}_f = \frac{E_f \phi}{\sigma_f}, \quad \tilde{E}_c = \frac{E_c \phi}{\sigma_f}, \quad \tilde{G}_c = \frac{G_c \phi}{\sigma_f},
\]

\[
\bar{\sigma} = \frac{\sigma_c \phi}{\sigma_f}, \quad \bar{\tau} = \frac{\tau_y \phi}{\sigma_f}, \quad \bar{\tau} = \frac{\bar{\tau} \phi}{\sigma_f},
\]

\[
\bar{\rho} = \frac{\rho \psi}{\rho_f}, \quad \bar{\phi} = \frac{\phi}{\sigma_{GFRP}}, \quad \psi = \frac{\rho_f}{\rho_{GFRP}}.
\]

For comparative purposes, \( \psi \) and \( \phi \) are added to relate CFRP to GFRP. Converting the mass and peak load equations for three point bending to non-dimensional terms yields

\[
\hat{P}_{MB} = 4\bar{t}(\bar{t} + 1)\bar{c}^2 \phi,
\]

(2.15)

\[
\hat{P}_{CS} = 2\bar{\tau}(\bar{t} + 1)\bar{c},
\]

(2.16)

\[
\hat{P}_I = \left( \frac{\pi^2 \bar{\sigma}^2 \tilde{E}_c}{3} \right)^{1/3} \bar{t}(\bar{t} + 1)^{1/3} \bar{c}^{4/3},
\]

(2.17)

and

\[
\hat{M} = (2\bar{t} + \bar{\rho} \psi)\bar{c}.
\]

(2.18)

For the design space of \( 0 \leq \bar{c} \leq 0.3 \) and \( 0 \leq \bar{t} \leq 0.25 \), Figures 2.7-2.10 are the failure mechanism maps considering the following beam material combinations:

- GFRP and ROHACELL 51-IG
- GFRP and extruded polystyrene (EPS)
- GFRP and ROHACELL 110-IG
- CFRP and ROHACELL 51-IG
Figure 2.7: Failure mechanism map for a simply supported composite sandwich beam subject to three point bending, GFRP face sheets and ROHACELL 51-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red)

Figure 2.8: Failure mechanism map for a simply supported composite sandwich beam subject to three point bending, GFRP face sheets and extruded polystyrene (EPS) foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red)
Figure 2.9: Failure mechanism map for a simply supported composite sandwich beam subject to three point bending, GFRP face sheets and ROHACELL 110-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red)

Figure 2.10: Failure mechanism map for a simply supported composite sandwich beam subject to three point bending, CFRP face sheets and ROHACELL 51-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red)

For each face and core, the three competing failure mechanisms exist within a defined failure region. For simply supported three point bending, microbuckling failure is predicted to exist for long beams of thin cores, indentation for beams of thick cores and thin faces, and core shear for short beams of thick faces.

As ROHACELL is stronger than EPS foam, ROHACELL beams are less prone to core shear or indentation, and is represented by a larger microbuckling failure region. However, GFRP/EPS beams are more prone to indentation due to the core shear strength being relatively higher than the compressive strength.
As CFRP is stronger, stiffer and lighter than GFRP, CFRP-constructed beams are less prone to microbuckling and indentation, seen in Figures 2.7 and 2.10.

To define the boundaries between each failure mechanism, load index equations may be combined to form critical relationships between \( \bar{t} \) and \( \bar{c} \).

First, lines define the boundaries of two active failure mechanisms. For a known \( \bar{c} \), a corresponding \( \bar{t} \) may be determined using the relationships listed below:

i) Microbuckling/Core Shear

\[
\bar{t} = \frac{\bar{\tau}}{2\bar{c}\phi} \tag{2.19}
\]

ii) Core Shear/Indentation

\[
\frac{(\bar{t} + 1)^2}{\bar{t}^3} = \frac{\bar{E}_f \bar{\sigma}}{24 (\pi \bar{\sigma})^2} \bar{c} \tag{2.20}
\]

iii) Indentation/Microbuckling

\[
\bar{t} = \pi \bar{\sigma} \left( \frac{\bar{E}_f}{192 \phi^3} \right)^{1/2} - 1. \tag{2.21}
\]

A material-specific design point may also be defined for where all three failure mechanisms intersect:

iv) Microbuckling/Core Shear/Indentation

\[
\frac{\bar{t}}{\bar{t} + 1} = \frac{\bar{\tau}}{\pi \bar{\sigma}} \left( \frac{48 \phi}{\bar{E}_f} \right)^{1/2} \tag{2.22}
\]

\[
\phi \bar{c} (\phi \bar{c} + \bar{\tau}) = \left[ \frac{\pi^2 \bar{\sigma}^2 \bar{E}_f}{192 \phi} - \left( \frac{\bar{\tau}}{2} \right)^2 \right]. \tag{2.23}
\]

Denoted in red in Figures 2.7-2.10, the minimum mass trajectory (MMT) represents the optimal beam set for a given load and material composition, considering mass reduction for a given strength. Within each failure region, beams are considered optimal design points when both load and mass requirements are satisfied. With respect to the failure mechanism map, the minimum mass trajectory is defined as the set of design points where both load and mass gradients are parallel. All subsequent points may exist along failure region boundary lines. For three point bending, locus points exist in both microbuckling and indentation failure regions; for the remaining domain of \( \hat{P} \), the minimum mass trajectory exists along the microbuckling/indentation and core shear/indentation boundary lines.
2.3 Modelling Techniques - Minimum Mass Trajectory Line Plots

In addition to failure mechanism mapping, the minimum mass trajectory may be plotted with respect to the load index $\hat{P}$. The three design parameters considered are $\hat{M}$, $\bar{t}$ and $\bar{c}$. Figure 2.11 compares the optimal mass index to load index for the four considered beam designs. For the first figure, $\hat{M} - \hat{P}$ curves represent the optimal beams that exist within the design space of $(0 \leq \bar{c} \leq 0.3)$ and $(0 \leq \bar{t} \leq 0.25)$. An accompanying figure highlights the finer details of each minimum mass trajectory for a small $\hat{P}$.

Designing beams comprised of either stiffer face sheets or stronger cores allows for a significantly higher load capacity. Stronger materials increase beam strength, but often at the cost of a greater mass. As CFRP is stiffer, stronger and lighter than GFRP, its $\hat{M} - \hat{P}$ curve has a smaller minimum mass index for a larger domain of $\hat{P}$ to GFRP beams.
Alternatively, Figures 2.12-2.13 plot beam geometry ratios $\bar{t}$ and $\bar{c}$ with respect to load index $\hat{P}$.

**Figure 2.12**: Plot of minimum mass trajectory in terms of $\bar{t}$ to load index for simply supported composite beams subject to three point bending - within the chosen design space (left), for a small $\hat{P}$ (right)

**Figure 2.13**: Plot of minimum mass trajectory in terms of beam geometry to load index for simply supported composite beams subject to three point bending - within the chosen design space (left), for a small $\hat{P}$ (right)

For both microbuckling and indentation failure regions, optimal beam designs are defined by a critical $\bar{t}$. For microbuckling,

$$\bar{t}_{MMT} = \frac{\bar{\rho}}{2\psi(1 - \bar{\rho})};$$

therefore, the corresponding $\bar{c}$ and mass index are

$$\bar{c}_{MMT} = \frac{(1 - \bar{\rho})\hat{P}^{1/2}}{\sqrt{\phi\bar{\rho}(2 - \bar{\rho})}},$$

$$\hat{M}_{MMT} = \frac{\bar{\rho}(2 - \bar{\rho})^2 \hat{P}}{\phi(2\psi - \psi\bar{\rho} + \bar{\rho})}.$$
Similarly, for indentation

\[ \hat{t}_{MMT} = \frac{3\bar{\rho}}{2\psi(1 - \bar{\rho})}, \]  

(2.27)

where corresponding \( \bar{c} \) and mass index are

\[ \bar{c}_{MMT} = 2(1 - 2\bar{\rho}) \left[ \frac{\bar{\rho}\hat{P}^3}{9\bar{\rho}^3(2 - \bar{\rho})\pi^2\sigma^2E_f} \right]^{1/4}, \]  

(2.28)

\[ \hat{M}_{MMT} = \frac{4(2 - \bar{\rho})}{(\bar{\rho}(3 - 4\psi) + 2\psi)^{1/4}} \left[ \frac{\bar{\rho}\hat{P}^3}{9\pi^2\sigma^2E_f} \right]^{1/4}. \]  

(2.29)

For all remaining \( \hat{P} \), beam geometry ratios \( \hat{t} \) and \( \bar{c} \) are governed by the previously listed boundary line expressions. The resultant mass index for the indentation/microbuckling boundary line is

\[ \hat{M}_{MMT} = \frac{\pi\bar{\sigma}\bar{\rho}}{8\phi^3} \left( \frac{\bar{E}_f}{3} \right)^{1/2} + \frac{2(2\psi - \bar{\rho})}{\pi\bar{\sigma}} \left( \frac{3}{\bar{E}_f} \right)^{1/2} \hat{P}, \]  

(2.30)

while for the core shear/indentation boundary line,

\[ \hat{M}_{MMT} = \left( \frac{\bar{\sigma}^4(2\psi - \bar{\rho}) + (\pi\bar{\sigma})^2\bar{E}_f\bar{\rho}\hat{P}}{2(48^2)\phi^9} \right)(\pi\bar{\sigma})^2\bar{E}_f\hat{P}^2. \]  

(2.31)

In summary, using the simply supported three point bending case as reference, composite sandwich mechanics are reviewed. Expressions for both stiffness and strength are noted and, through the use of failure mechanism mapping, optimal beam designs are shown.
Chapter 3

Analysis

For the following section, two load cases are analyzed: four-point bending and uniformly distributed loading. Similar to the approach taken in Chapter 2, stiffness and strength expressions are derived with the end goal of plotting failure mechanism maps and determining optimal beam design trajectories.

3.1 Case A: Simply Supported, Symmetric Four Point Bending

Consider the simply supported composite sandwich beam in Figure 3.1: two loads of equal magnitude ($P/2$) are applied on the beam, at an equal distance $a$ apart from one another.

![Figure 3.1: A simply supported composite sandwich beam subject to four point bending](image)
Figure 3.2 compares shear ($V$) and bending moment ($M$) diagrams.

![Figure 3.2: Shear and bending moment diagrams for a simply supported composite sandwich subject to four point bending](image)

Although similar to three point bending, four point bending introduces a region of zero shear stress and constant bending moment between the applied point loads. While asymmetric variants of four point bending exist, only the symmetric case is considered here.

### 3.1.1 Stiffness

Recalling Chapter 2, deflection is predicted by combining bending and shear components:

$$\delta = \delta_b + \delta_s$$  \hspace{1cm} (3.1)

For the bending component of deflection, four point bending considers two regions:

- Outer region A: side support to the point load ($0 < x < (L - a)/2$)
- Inner region B: point load to the midspan ($(L - a)/2 < x < L/2$)

Seen in Figure 3.2, the maximum bending moment present is $M = P(L - a)/4$ at a distance $(L - a)/2$ from the edge. Considering Euler-Bernoulli beam theory,

$$-EI \frac{d^2 y(x)}{dx^2} = M = \frac{P(L - a)}{4};$$  \hspace{1cm} (3.2)

where $y(x)$ refers to the deflection. Similarly, the maximum shear force present on the outer regions of the beam is $P/2$, thus

$$-EI \frac{d^3 y(x)}{dx^3} = V = \frac{P}{2}. \hspace{1cm} (3.3)$$
By integrating Equation (3.2) and (3.3)

\[ y(x)|_A = -\frac{P}{12EI}x^3 + \frac{C_1}{2}x^2 + C_2x + C_3, \]  

(3.4)

where \( y(x)|_A \) is the deflection equation for the outer region, and for the inner region

\[ y(x)|_B = -\frac{P}{8EI} \left( x - \frac{(L-a)}{2} \right)^2 + D_1 \left( x - \frac{(L-a)}{2} \right) + D_2. \]  

(3.5)

To solve the above set of equations, the following boundary conditions are set:

- No deflection at the ends, \( y(0) = 0 \)
- Midspan slope is zero (symmetry), \( dy(L/2)/dx = 0 \)
- Bending moment at the ends are zero, \( d^2y(0)/dx^2 = 0 \)
- Slope is continuous at \( x = (L-a)/2, \quad dy/dx|_A = dy/dx|_B \)
- Deflection is continuous at \( x = (L-a)/2, \quad y|_A = y|_B \)

Solving for \( y(x), \)

\[ y(x)|_A = \frac{Px}{48EI} \left[ 3(L-a)(L+a) - 4x^2 \right] \]  

(3.6)

for the outer region and

\[ y(x)|_B = \frac{P(L-a)}{96EI} \left[ 12x(L-x) - (L-a)^2 \right] \]  

(3.7)

for the inner region respectively.

From here, the bending contribution of deflection may be estimated from two key nodes: at one of the two point loads

\[ \delta_b = y \bigg|_A \left( \frac{(L-a)}{2} \right) = \frac{P(L-a)^2(L+2a)}{48EI}, \]  

(3.8)

and at the midspan

\[ \delta_b = y \bigg|_B \left( \frac{L}{2} \right) = \frac{P(L-a)}{96EI} \left[ 3L^2 - (L-a)^2 \right]. \]  

(3.9)

In Figure 3.2, the maximum shear is \( P/2. \) At both the point load and midspan, the shear contribution to deflection is

\[ \delta_s = \frac{P(L-a)}{4AG}. \]  

(3.10)
Therefore, the deflection at the point loads is

\[ \delta_P = \frac{P(L-a)^2(L+2a)}{48EI} + \frac{P(L-a)}{4AG}, \]  

(3.11)

while for the midspan

\[ \delta = \frac{P(L-a)}{96EI} \left[ 3L^2 - (L-a)^2 \right] + \frac{P(L-a)}{4AG}. \]

(3.12)

Rewriting Equation (3.12), the stiffness of a simply supported composite sandwich beam subject to four point bending is

\[ \frac{P}{\delta} = \left[ \frac{96(EI)_{eq}(AG)_{eq}}{(L-a)(24(EI)_{eq} + (AG)_{eq}(3L^3 - (L-a)^2))} \right]. \]

(3.13)

Although deflection at the point loads may be required for certain experiment setups, only midspan deflection is considered beyond this point.

### 3.1.2 Strength

With four point bending, the same set of failure mechanisms seen in three point bending exist: microbuckling, core shear and indentation.

#### Failure Mechanism #1: Microbuckling

Figure 3.3 depicts the cross-section of a composite beam with neutral axis NA:

![Cross section of simply supported composite sandwich beam under buckling load F, with neutral axis NA](image)

*Figure 3.3: Cross section of simply supported composite sandwich beam under buckling load F, with neutral axis NA*

The applied buckling load \( F \) is derived from the bending moment stress equation

\[ \sigma_f = \frac{Mh}{I}, \]

(3.14)
where for a composite sandwich beam, the displacement $h$ from the neutral axis is $h = d/2$. In terms of the equivalent bending stiffness,

$$\sigma_f = \frac{Md}{2} \frac{E_f}{(EI)_{eq}},$$

(3.15)

where from Chapter 2, $(EI)_{eq} = E_f btd^2/2$. For $M = P/(L-a)$,

$$\sigma_f = \left(\frac{P(L-a)}{4}\right) \left(\frac{d}{2}\right) E_f \left(\frac{2}{E_f btd^2}\right).$$

(3.16)

Therefore, the load at microbuckling failure is

$$P_{MB} = \frac{4btd\sigma_f}{L-a}. $$

(3.17)

To recall, the microbuckling equation for three point bending is $P_{MB} = 4btd\sigma_f/L$. With the only difference being a reduction of bending moment arm from $L/2$ to $(L-a)/2$, four point bending is presumed to have a higher microbuckling failure load than three point bending.

**Failure Mechanism #2: Core Shear**

To solve for core shear, work done by Allen [1] is considered. For composite materials,

$$\tau = \frac{Vs}{(EI)_{eq}} \Sigma SE,$$

(3.18)

where $\tau$ is the shear stress, $s$ first moment of area, and $I$ and second moment of inertia. Considering only the core, the expression for $\Sigma SE$ is

$$\Sigma SE = \frac{E_f btd}{2} + \frac{E_c b}{2} \left(\frac{c}{2} - h\right) \left(\frac{c}{2} + h\right).$$

(3.19)

The resulting expression for $\tau$ becomes:

$$\tau = \frac{V}{(EI)_{eq}} \left[\frac{E_f btd}{2} + \frac{E_c}{2} \left(\frac{c^2}{4} - h^2\right)\right].$$

(3.20)

Assuming the core is compliant compared to the face sheets ($E_f >> E_c$) and replacing $(EI)_{eq}$ yields

$$\tau = \frac{V}{bd}.$$ 

(3.21)

Due to shear being independent of beam length, the shear force applied to the beam for both three and four point bending cases is $V = P/2$. 
Therefore, core shear failure for both three and four point bending is defined by

\[ P_{CS} = 2b\tau_c d, \]  

(3.22)

where overhang factor \((L + 2H)/L\) is added for beams comprised of elastic-plastic cores.

**Failure Mechanism #3: Indentation**

Following the three point bending model in Chapter 2, work by Hetenyi [16] is recalled considering an infinitely long elastic beam resting on a rigid-ideally plastic material, subject to an applied load \(P/2\) [16]. For a composite sandwich beam, the model describes the interaction between the top face sheet and core. Taking an infinitely small section of the top face sheet generates the free body diagram seen in Figure 3.4.

\[ dM + (V + dV) - qbdx^2 + F\frac{du}{dx} dx = 0, \]  

(3.24)

where the core foundation reaction pressure \(q = \sigma_c\).
Differentiating Equation (3.24) yields the following non-homogenous ordinary differential equation for $u(x)$:

$$\frac{d^4u}{dx^4} + \frac{F}{EI} \frac{d^2u}{dx^2} = -\frac{\sigma_c b}{EI}.$$  \hspace{1cm} (3.25)

Solving for $u(x)$ yields

$$u(x) = A_1 \cos(kx) + A_2 \sin(kx) + A_3 x + A_4 - \frac{\sigma_c b x^2}{2F},$$  \hspace{1cm} (3.26)

where

$$k = \sqrt{\frac{F}{EI}}.$$  \hspace{1cm} (3.27)

Considering the indentation has finite length $2\lambda$, the following boundary conditions are set:

- $du(0)/dx = 0$
- $d^3u/dx^3(0) = P/2EI$
- $u(\lambda) = 0$
- $du(\lambda)/dx = 0$
- $d^2u(\lambda)/dx^2 = 0$

Solving for the constants:

$$A_1 = \frac{2d}{(L-a)k} \left( \frac{1 - \cos\mu - \mu \sin\mu}{\sin\mu - \mu \cos\mu} \right),$$  \hspace{1cm} (3.28)

$$A_2 = \frac{-2d}{(L-a)k},$$  \hspace{1cm} (3.29)

$$A_3 = \frac{2d}{(L-a)},$$  \hspace{1cm} (3.30)

$$A_4 = \frac{2d}{(L-a)k} \left( \frac{1 - \cos\mu - \mu \sin\mu}{\sin\mu - \mu \cos\mu} \right) + \frac{d\mu^2}{(L-a)k} \left( \frac{1 + \cos\mu}{\sin\mu - \mu \cos\mu} \right),$$  \hspace{1cm} (3.31)

where $\mu = \lambda/k$. As $du(0)/dP = 0$,

$$P = bt \left[ \frac{4dE\sigma_c^2}{3(L-a)} \left( \frac{\sin\mu - \mu \cos\mu}{1 - \cos\mu} \right)^2 \right]^{1/3}.$$  \hspace{1cm} (3.32)

and

$$u(0) = \frac{4d}{(L-a)k} \left( \frac{1 - \cos\mu - \mu \sin\mu}{\sin\mu - \mu \cos\mu} \right) + \frac{d\mu^2}{(L-a)k} \left( \frac{1 + \cos\mu}{\sin\mu - \mu \cos\mu} \right).$$  \hspace{1cm} (3.33)
Assuming load P is maximum when \( \mu \approx \pi \),

\[
P_{\text{max}} = bt \left( \frac{\pi^2 d E_f \sigma_c^2}{3(L - a)} \right)^{1/3}.
\] (3.34)

Accounting for two separate instances of indentation on the beam, the failure expression becomes

\[
P_I = 2bt \left( \frac{\pi^2 d E_f \sigma_c^2}{3(L - a)} \right)^{1/3}.
\] (3.35)

### 3.1.3 Failure Mechanism Mapping

To transform the above failure load expressions into non-dimensional terms, the same relationships listed in Chapter 2 are recalled. To account for the load spacing term \( a \), dimensionless parameter \( \bar{a} = a/L \) is introduced. The resulting non-dimensional expressions for four point bending are

\[
\hat{P}_{MB} = \frac{4t(t+1)\bar{c}^2\phi}{(1 - \bar{a})},
\] (3.36)

\[
\hat{P}_{CS} = 2\bar{t}(t + 1)\bar{c},
\] (3.37)

\[
\hat{P}_I = 2 \left( \frac{\pi^2 \bar{\sigma}^2 \bar{E}}{3(1 - \bar{a})} \right)^{1/3} \bar{t}(\bar{t} + 1)^{1/3} \bar{c}^{4/3},
\] (3.38)

where the same expression for the mass index is used from Chapter 2 for three point bending:

\[
\hat{M} = \frac{M}{\rho_{GFRP} bL^2} = (2\bar{t} + \bar{\rho}\psi)\bar{c}.
\] (3.39)

Figures 3.5-3.8 are the various failure mechanism maps for four point bending, assuming the same four material combinations in Chapter 2. All plots assume a constant \( \bar{a} = a/L = 0.2 \).
Figure 3.5: Failure mechanism map for a simply supported composite sandwich beam subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red), $\bar{a} = 0.2$

Figure 3.6: Failure mechanism map for a simply supported composite sandwich beam subject to four point bending, GFRP face sheets and extruded polystyrene (EPS) foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red), $\bar{a} = 0.2$
Chapter 3. Analysis

Figure 3.7: Failure mechanism map for a simply supported composite sandwich beam subject to four point bending, GFRP face sheets and ROHACELL 110-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red), $\bar{a} = 0.2$

Figure 3.8: Failure mechanism map for a simply supported composite sandwich beam subject to four point bending, CFRP face sheets and ROHACELL 51-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red), $\bar{a} = 0.2$

The corresponding boundary lines between each failure region are defined by $(\bar{c}, \bar{t})$:

i) Microbuckling/Core Shear

\[ \bar{t} = \frac{\bar{\tau}(1 - \bar{a})}{2\bar{c}\phi} \]  

(3.40)
ii) Core Shear/Indentation

\[ \bar{t}^3 = \frac{\bar{E}_f \pi^2 \bar{\sigma}^2 \bar{c}}{3(1 - \bar{a}) \bar{t}^3} \]  \hspace{1cm} (3.41)

iii) Indentation/Microbuckling

\[ \bar{t}^* = \left( \frac{\bar{E}_f}{24\phi^3} \right)^{1/2} \left( \frac{\bar{\sigma} \pi (1 - \bar{a})}{\bar{c}} \right) - 1 \]  \hspace{1cm} (3.42)

iv) Microbuckling/Core Shear/Indentation

\[ \frac{\bar{t}}{(t + 1)} = \frac{\bar{\tau}(1 - \bar{a})}{\pi \bar{\sigma}} \left( \frac{6\phi}{\bar{E}_f} \right)^{1/2} \]  \hspace{1cm} (3.43)

\[ \phi \bar{c}(\phi \bar{c} + \bar{\tau}(1 - \bar{a})) = \left[ \frac{(\pi \bar{\sigma}(1 - \bar{a}))^2 \bar{E}_f}{24\phi} - \left( \frac{\bar{\tau}(1 - \bar{a})}{2} \right)^2 \right]. \]  \hspace{1cm} (3.44)

Compared to three point bending, an increase in the required load for both microbuckling and indentation forces core shear to become the more dominant failure mechanism. As was the case for three point bending, it is not possible for a minimum-mass beam to fail solely due to core shear. As a result, the minimum mass trajectory exists along the boundary lines, where the mass and load index gradients are not parallel.

Solving for the minimum mass trajectory, the optimal \( \bar{t} \) for the microbuckling and indentation failures regions is found to be the same as three point bending in Chapter 2 (Equations (2.24) and (2.27) respectively). Solving for \( \bar{c} \) and \( \bar{M} \) in the microbuckling failure region,

\[ \bar{c}_{MMT} = (1 - \bar{\rho}) \left[ \frac{(1 - \bar{a}) \bar{P}}{\phi \bar{\rho}(2 - \bar{\rho})} \right]^{1/2}, \]  \hspace{1cm} (3.45)

\[ \bar{M}_{MMT} = \sqrt{\frac{\bar{\rho}(2 - \bar{\rho})^2 \bar{P}(1 - \bar{a})}{(2\psi - \psi \bar{\rho} + \bar{\rho})\phi}}, \]  \hspace{1cm} (3.46)

while for indentation

\[ \bar{c}_{MMT} = 2(1 - 2\bar{\rho}) \left[ \frac{8\bar{\rho}(1 - \bar{a}) \bar{P}^3}{9\bar{\rho}^3(2 - \bar{\rho}) \pi^2 \bar{\sigma}^2 \bar{E}_f} \right]^{1/4}, \]  \hspace{1cm} (3.47)

\[ \bar{M}_{MMT} = \frac{4(2 - \bar{\rho})}{(\bar{\rho}(3 - 4\psi) + 2\psi)^{1/4}} \left[ \frac{8\bar{\rho}(1 - \bar{a}) \bar{P}^3}{9\pi^2 \bar{\sigma}^2 \bar{E}_f} \right]^{1/4}. \]  \hspace{1cm} (3.48)
For the boundary lines, mass index for indentation/microbuckling is

\[
\hat{M}_{MTM} = \frac{\pi \bar{\sigma}(1 - \bar{a})}{16 \phi^3} \left( \frac{\bar{E}_f}{6} \right)^{1/2} + \frac{4(2\psi - \bar{\rho})}{\pi \bar{\sigma}(1 - \bar{a})} \left( \frac{6}{\bar{E}_f} \right)^{1/2} \hat{P},
\]  

(3.49)

while for core shear/indentation

\[
\hat{M}_{MTM} = \left( \frac{\pi^4(1 - \bar{a})(2\psi - \bar{\rho}) + (\pi \bar{\sigma})^2 \bar{E}_f \bar{\rho} \hat{P}}{72(1 - \bar{a})^2 \pi^3} \right) \left( \pi \bar{\sigma} \right)^2 \bar{E}_f \hat{P}^2. 
\]  

(3.50)

Figures 3.9 and 3.10 plot the change in beam geometry variables \( \bar{t} \) and \( \bar{c} \) with respect to the predicted load index.

**Figure 3.9:** Plot of minimum mass trajectory in terms of \( \bar{t} \) to load index for simply supported composite beams subject to four point bending, \( \bar{a} = 0.2 \) - within the chosen design space (left), for a small \( \hat{P} \) (right)

**Figure 3.10:** Plot of minimum mass trajectory in terms of \( \bar{c} \) to load index for simply supported composite beams subject to four point bending, \( \bar{a} = 0.2 \) - within the chosen design space (left), for a small \( \hat{P} \) (right)
Figure 3.11 plots the mass index in terms of the load index below.

Comparing Figures 3.9-3.11, results are shown to be similar to three point bending. Aside from fewer optimal beam designs occurring within the indentation failure region, the main difference with four point bending is that the mass index for optimal beams is predicted to be smaller. Within the microbuckling and indentation failure regions, the same $\tilde{I}_{MT}$ is used for three point bending, so the drop in mass index is attributed to an increase in beam length. For the boundary lines, the drop in mass index may be attributed to changes in all three beam dimensions.
To consider the full range of possible $\bar{a}$, Figure 3.12 is a failure mechanism map using GFRP/ROHACELL 51-IG beams, comparing the failure region boundary lines for $\bar{a} = [0.05, 0.1, 0.2, 0.5, 0.75]$.

![Failure mechanism map for a simply supported composite sandwich beam subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core - Varied $\bar{a}$](image)

Figure 3.12: Failure mechanism map for a simply supported composite sandwich beam subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core - Varied $\bar{a}$

Figure 3.13 plots the $\hat{M} - \hat{P}$ curves for varying $\bar{a}$ below.

![Plot of minimum mass trajectory in terms of mass to load index for simply supported composite beams subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core - varied $\bar{a}$](image)

Figure 3.13: Plot of minimum mass trajectory in terms of mass to load index for simply supported composite beams subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core - varied $\bar{a}$

Seen above, as the load spacing becomes large in relation to the beam length, the decrease in mass index and $\bar{c}$ become more significant.

By adapting similar methods from three point bending, a new set of strength and stiffness equations are derived for four point bending. Though failure mechanism maps show similarities to three point bending, microbuckling and indentation failure regions are less dominant. By increasing load spacing factor $\bar{a}$, optimal beam designs of similar load index become more compliant, yielding a lower mass index.
### 3.2 Case B: Uniformly Distributed Load Case

For uniformly distributed loading, the applied load is replaced by a pressure \( w = \frac{P}{bL} \), applied over the beam span (excluding the overhang), seen in Figure 3.14.

\[
EIm \frac{d^4y}{dx^4} = wb. \tag{3.52}
\]

**Figure 3.14**: A simply supported composite sandwich beam subject to uniformly distributed loading

Figure 3.15 compares shear \((V)\) and bending moment \((M)\) diagrams for the following case.

**Figure 3.15**: Shear and bending moment diagrams for a simply supported composite sandwich subject to uniformly distributed loading

#### 3.2.1 Stiffness

Similar to the point loading cases, deflection is the sum of bending and shear effects:

\[
\delta = \delta_b + \delta_s. \tag{3.51}
\]
The following boundary conditions are used:

- Deflection at the end support is zero, \( y(0) = 0 \)
- Bending moment at the end support is zero, \( d^2 y(0)/dx^2 = 0 \)
- Midspan slope is zero, \( dy(L/2)/dx = 0 \)
- No shear at the midspan, \( d^3 y(L/2)/dx^3 = 0 \)

The resultant deflection equation from bending becomes

\[
y(x) = \frac{wbx}{24EI}(L^3 - 2Lx^2 + x^3),
\]

where at the midspan, the deflection is

\[
\delta_b = \frac{5wbL^4}{384EI}.
\]

Taking into account the shear component of deflection

\[
\delta_s = \left( \frac{wbL}{2} \right) \left( \frac{L}{2AG} \right) = \frac{wbL^2}{4AG},
\]

therefore, the total deflection for a simply supported sandwich composite beam is

\[
\delta = \frac{5wbL^4}{384EI} + \frac{wbL^2}{4AG},
\]

where in terms of stiffness

\[
\frac{w}{\delta} = \left[ \frac{384(EI)_{eq}(AG)_{eq}}{96(EI)_{eq}L^2 + 5(AG)_{eq}L^4} \right].
\]

### 3.2.2 Strength

For uniformly distributed loading, the predicted failure mechanisms are microbuckling, core shear, indentation and core crushing.

**Failure Mechanism #1: Microbuckling**

From Figure (3.14) above, the maximum bending moment \( M \) is \( wbL^2/8 \). Recalling the bending stress equation from Equation (3.14),

\[
\sigma_f = \frac{wL^2}{8td}.
\]
Therefore, the expression for microbuckling failure in terms of a uniformly distributed load is

\[ w_{MB} = \frac{8td\sigma_f}{L^2}. \] (3.59)

**Failure Mechanism #2: Core Shear**

From Figure 3.14, the maximum shear load is \( wbL/2 \). Using Equation (3.21), the core shear failure expression for uniformly distributed loading is

\[ w_{CS} = \frac{2\tau_c d}{L}. \] (3.60)

**Failure Mechanism #3: Core Crush**

Recalling work from Chapter 2, the core crushing peak pressure is equal to the compressive strength of the core

\[ w_{CC} = \sigma_c. \] (3.61)

**Failure Mechanism #4: Indentation**

Though indentation is generally considered as a point load failure mechanism, it is of interest to determine its feasibility for a uniformly distributed load. The following indentation model assumes the corresponding localized core collapse occurs at the midspan of the beam.

Recalling Hetenyi [16], consider an elastic beam of infinite length resting on a foundation material, now subject to a transverse pressure \( w \).

![Free body diagram for an infinitely small section of a simply supported elastic beam, subject to uniformly distributed load](image-url)
Chapter 3. Analysis

Taking moments from the left side of the element, equilibrium is

\[ M - (M + dM) + (V + dV)dx + (w - \sigma_c)bdx^2 + F \frac{du}{dx} dx = 0, \tag{3.62} \]

where:

\[ \frac{dV}{dx} = \frac{(q - w)b}{2}. \tag{3.63} \]

Both the rigid-ideally plastic and elastic-ideally plastic foundations are analyzed.

Rigid-Ideally Plastic Core

To reiterate, in the case of the rigid-ideally plastic core foundation, the reaction pressure \( q = \sigma_c \).

Differentiating the expression yields the non-homogenous expression

\[ \frac{d^4 u}{dx^4} + \frac{F}{EI} \frac{d^2 u}{dx^2} = \frac{w - \sigma_c}{2EI}. \tag{3.64} \]

Solving for the homogeneous left hand side of the expression yields the same \( k = \sqrt{F/\pi EI} \).

However, the particular solution of the expression is only true when

\[ u(x) = A_1 \cos(kx) + A_2 \sin(kx) + A_3x + A_4 - \frac{(w - \sigma_c)bx^2}{4F}. \tag{3.65} \]

To solve for constants \( A_1, A_2, A_3 \) and \( A_4 \), the following boundary conditions are set:

- Slope at the beam at the midspan is zero, \( u'(0) = 0 \)
- The shear component of \( u(x) \) is zero for pressure loading \( u''(0) = 0 \)
- \( u(\lambda) = 0 \)
- \( u'(\lambda) = 0 \)
- \( u''(\lambda) = 0 \)

With the shear component of \( u(0) \) at zero, constant \( A_2 \) is eliminated, thus simplifying the equation to

\[ u(x) = \frac{(w - \sigma_c)b}{2Fk^2} \left[ \frac{\cos(kx)}{\cos \mu} - 1 - \frac{\mu^2}{2} + \frac{x^2}{2} \right], \tag{3.66} \]

where at \( u(0) \) (assuming \( \mu \approx \pi \)),

\[ u(0) = u_{max} = \frac{(\sigma_c - w)b}{2Fk^2} \left[ 2 + \frac{\mu^2}{2} \right]. \tag{3.67} \]
Solving for the peak pressure,

\[ u'(\lambda) = \frac{(w - \sigma_c)b}{2Fk^2} \left( \frac{-k\sin\mu}{\cos\mu} + \lambda \right) = 0, \]  

(3.68)

where for \( \mu \approx \pi \),

\[ w_I = \sigma_c = w_{CC}. \]  

(3.69)

As Equation (3.69) is equivalent to core crush, a rigid-ideally plastic core cannot be considered.

**Elastic-Ideally Plastic Core**

For an initially elastic core, the reaction pressure is \( q = su(x) \), where \( s \) is foundation modulus of the core \( (s = E_c/c) \). When the indentation exceeds a critical point \( (u_c = \sigma_c/E_c) \), the indentation follows the traits of plastic collapse. For now, only the initial elastic indentation is of concern. Replacing the elastic expression for \( q \) for Equation 3.64 the expression for \( u(x) \) becomes:

\[ \frac{d^4u}{dx^4} + \frac{F}{EI} \frac{d^2u}{dx^2} + \frac{su(x)}{2EI} = \frac{w}{2EI} \]  

(3.70)

With the addition of a \( u(x) \) term to the homogenous portion of the following expression, the particular solution changes from above. Solving for \( u(x) \) yields

\[ u(x) = (B_1 e^{\beta x} + B_2 e^{-\beta x})\cos(\alpha x) + (B_3 e^{\beta x} + B_4 e^{-\beta x})\sin(\alpha x) + w/s, \]  

(3.71)

where

\[ \alpha^2 = \frac{F}{4EI} + \left( \frac{s}{8EI} \right)^{1/2}; \quad \beta^2 = -\frac{F}{4EI} + \left( \frac{s}{8EI} \right)^{1/2}. \]  

(3.72)

Assuming the same slope, bending moment and shear boundary conditions as from the rigid-ideally plastic core case, Equation (3.71) becomes

\[ u(x) = B\cos(\alpha x) + w/s, \]  

(3.73)

where \( B \) is an indeterminate constant. As core indentation is indeterminate within the elastic region, a plastic collapse can never be attained. Indentation is instead a purely elastic response of the core from a buckling of the top face sheet, given by

\[ F_{cr} = 2\sqrt{sb(EI)_{eq}} \]  

(3.74)

where the indent size is equal to the beam length.
Under the following conditions, the peak pressure before failure is

\[ w_I = \sqrt{\frac{128E_fE_c t^3 d}{cL^4}}. \]  

(3.75)

**Failure Mechanism Mapping**

Using the same non-dimensional relationships from Chapter 2, failure expressions for uniformly distributed loading are defined as:

\[ \hat{w}_{MB} = 8(\bar{t} + 1)\ddot{c}^2\phi, \]  

(3.76)

\[ \hat{w}_{CS} = 2\ddot{\tau}(\bar{t} + 1)\ddot{c}, \]  

(3.77)

\[ \hat{w}_{CC} = \ddot{\sigma}, \]  

(3.78)

and

\[ \hat{w}_I = \sqrt{128\bar{E}_f\bar{E}_c\bar{t}^3(\bar{t} + 1)^2\ddot{c}^4} \]  

(3.79)

where \( \hat{w} \) defines the peak pressure index.

Figures 3.17-3.20 plot the failure mechanism maps for simply supported beams subject to a uniformly distributed load for the same material selections as the previous two cases.
Figure 3.18: Failure mechanism map for a simply supported composite sandwich beam subject to uniformly distributed loading, GFRP face sheets and extruded polystyrene (EPS) foam core comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red)

Figure 3.19: Failure mechanism map for a simply supported composite sandwich beam subject to uniformly distributed loading, GFRP face sheets and ROHACELL 110-IG foam core, comparing non-dimensional contour plots of load and mass, dividing regions of active failure mechanism and predicting minimum mass trajectory (in red)
Solving for each design space, only two of the four competing failure mechanisms are present: microbuckling and core shear. Microbuckling occurs in cases of small $\bar{t}$, while core shear is the dominant failure region in cases of higher $\bar{t}$. A stronger face sheet compressive strength or weaker core shear strength results in a larger core shear failure region.

The boundary line equations are:

i) Microbuckling/Core Shear

$$\bar{t} = \frac{\bar{\tau}}{4\bar{c}\phi} \quad (3.80)$$

ii) Core Shear/Indentation

$$\bar{t} = \left[ \frac{3\bar{\sigma}^2}{32E_fE_c\bar{c}^2} \right]^{1/3} \quad (3.81)$$

iii) Indentation/Core Crush

$$\bar{t}^3(\bar{t} + 1)^2 = \frac{3\bar{\sigma}^2}{128E_fE_c\bar{c}^4} \quad (3.82)$$

iv) Microbuckling/Core Crush

$$\bar{t}(\bar{t} + 1) = \frac{\bar{\sigma}}{8\phi\bar{c}^2} \quad (3.83)$$
v) Indentation/Microbuckling

\[
\bar{t} = \frac{3\phi^2}{2E_fE_c} \tag{3.84}
\]

vi) Core Shear/Core Crush

\[
\bar{t} = \frac{\bar{\sigma}}{2\bar{\tau}c} - 1 \tag{3.85}
\]

vii) Microbuckling/Core Shear/Indentation

\[
\bar{t} = \frac{3\phi^2}{2E_fE_c} \tag{3.86}
\]

\[
\bar{c} = \frac{E_fE_c\bar{\tau}}{6\phi^3} \tag{3.87}
\]

viii) Core Shear/Indentation/Core Crush

\[
\frac{(\bar{t} + 1)^2}{\bar{t}^3} = \frac{8E_fE_c\bar{\sigma}^2}{3\bar{\tau}^4} \tag{3.88}
\]

\[
\left[ \frac{\bar{\sigma}}{2\bar{\tau}c} - 1 \right]^3 = \frac{3\bar{\tau}^2}{32E_fE_c\bar{c}^2} \tag{3.89}
\]

ix) Indentation/Core Crush/Microbuckling

\[
\bar{t} = \frac{3\phi^2}{2E_fE_c} \tag{3.90}
\]

\[
\bar{c} = \frac{(E_fE_c)^2\bar{\sigma}}{6\phi^3(3\phi^2 + 2E_fE_c)} \tag{3.91}
\]

x) Core Crush/Microbuckling/Core Shear

\[
\bar{t} = \frac{\bar{\tau}^2}{2\phi\bar{\sigma} - \bar{\tau}^2} \tag{3.92}
\]

\[
\bar{c} = \frac{2\bar{\sigma}\phi - \bar{\tau}^2}{4\phi\bar{\tau}}. \tag{3.93}
\]
Feasibility of Core Crush and Indentation Failures

As both indentation and core crush are not present in any of the above failure mechanism maps, it is of interest to determine if the failures can exist at all. For core crushing to exist,

\[
\frac{\sigma_c}{\tau_c} < 2(\bar{t} + 1)\bar{c},
\]  

(3.94)

whereby the compressive strength must be less than the required pressure for core shear failure. Considering the maximum limits of the design space (\(\bar{c} = 0.30, \bar{t} = 0.25\)), the compressive strength of the core must be less than 75% of its own shear strength for core crush to exist. Though core crushing is not considered beyond here, it is still feasible to exist if the selected core material has a significantly weaker compressive strength relative to its shear strength.

Considering the feasibility of indentation, the easiest failure mechanism to surpass is microbuckling, seen by

\[
\frac{2E_fE_c}{3\bar{c}^2} < \frac{c}{\bar{t}}.
\]  

(3.95)

For indentation to exist, the material properties parameter on the left must be less than the beam geometry parameter to the right. Even considering the largest design space dimensions, the required material properties must still be small. Unless face sheets with properties close to silicone are considered, indentation may be disregarded.

Minimum Mass Trajectory Analysis

For the minimum mass trajectories of uniformly distributed loading, optimal beams either lie within the microbuckling failure region for a set critical \(\bar{t}\), where

\[
\bar{t}_{MMT} = \frac{\bar{\rho}}{2(1 - \bar{\rho})},
\]  

(3.96)

\[
\bar{c}_{MMT} = \left(\frac{(1 - \bar{\rho})^2\bar{w}}{2\bar{\rho}(2 - \bar{\rho})}\right)^{1/2},
\]  

(3.97)

and

\[
\hat{\bar{M}} = \left(\frac{\bar{\rho}(2 - \bar{\rho})^2\bar{w}}{2\phi(2\psi - \psi\bar{\rho} + \bar{\rho})}\right)^{1/2},
\]  

(3.98)

or the optimal beam designs lie along the microbuckling/core shear boundary line, where

\[
\bar{M} = \frac{(2\psi - \bar{\rho})}{4\phi} + \frac{\bar{\rho}\bar{w}}{2\tau}.
\]  

(3.99)
Comparing the minimum mass trajectories from Figures 3.16-3.20, Figure 3.21 compares beam geometry variables \( \bar{t} \) to \( \bar{c} \) to the peak pressure index, while Figure 3.22 plots the resultant minimum mass index to peak pressure index.

Figure 3.21: Plot of minimum mass trajectory in terms of \( \bar{t} \) to peak pressure index for simply supported composite beams subject to uniform loading

Figure 3.22: Plot of minimum mass trajectory in terms of mass to peak pressure index for simply supported composite beams subject to uniform loading

As the dominant failure region for uniform loading is core shear, considering a stronger core material has a significant effect on the range of possible \( \bar{w} \). Though the proportionality of microbuckling and microbuckling-core shear optimal beam designs are subject to change, the difference that occurs in terms of peak pressure index is small.

In summary, a new set of strength and stiffness equations are derived for a simply supported composite beam subject to a uniform pressure. Though four failure mechanisms are considered, only two are likely to exist in general practice: microbuckling and core shear. The dominance of one failure mechanism over the other is dependant on shear strength of the core relative to the compressive strength of the face sheet.
3.3 Stiffness Constraint

In engineering practice, compliant beams are undesirable due to their excessive deflection. Utilizing stiffness equations, design points that fall below a target stiffness may be omitted from a selected design space if a maximum deflection is defined.

To note, the term "stiffness constraint" does not reflect a constraint used in structural optimization. The term "constraint" is used to differentiate between the set of beams omitted from the remaining design space, considering an arbitrary deflection limit.

3.3.1 Three Point Bending

Recalling Chapter 2, beam stiffness is defined by $P/\delta$. To impose a stiffness constraint on the design space, a fourth failure mechanism is derived by rewriting the beam stiffness equation for three point bending from Equation (2.2)

$$P = \frac{48EI_{eq}(AG)_{eq}}{12(EI)_{eq}L + (AG)_{eq}L^3} \delta.$$  \hspace{1cm} (3.100)

Converting the above expression into non-dimensional terms,

$$\hat{P} = \frac{48\hat{EI}\hat{AG}}{12\hat{EI} + \hat{AG}} \hat{\delta}.$$  \hspace{1cm} (3.101)

where

$$\hat{EI} = \frac{\bar{E}f(\bar{t} + 1)^2c^3}{2}$$  \hspace{1cm} (3.102)

and

$$\hat{AG} = \bar{G}_c(\bar{t} + 1)\bar{c}.$$  \hspace{1cm} (3.103)

Seen in Equation (3.101), the user-defined variable of the proposed stiffness constraint is $\hat{\delta}$, the ratio of predicted midspan deflection before failure over beam length ($\hat{\delta} = \delta/L$). Beams are considered to be undesirable if midspan deflection at the predicted peak load exceeds an arbitrary maximum allowable deflection. For now, the range of $\hat{\delta} = [0.015, 0.02, 0.025, 0.05, 0.1]$ or $\hat{\delta} = [1.5\%, 2.0\%, 2.5\%, 5.0\%, 10.0\%]$ is compared here, considering only GFRP/ROHACELL 51-IG beams.

Figure 3.23 plots the failure mechanism map for three point bending including the stiffness constraint. Only the boundary lines for each failure region are shown, as the mass and load index contour lines do not change from GFRP/ROHACELL 51-IG plot in Figure 2.7.
Below are the accompanying boundary lines related to stiffness for the three point bending model. The expressions are in terms of normalized equivalent bending and shear stiffness terms $\hat{E}I$ and $AG$:

i) Microbuckling/Stiffness

$$\bar{t}(\bar{t} + 1) = \left( \frac{12\hat{E}I\hat{A}G}{12\hat{E}I + AG} \right) \left( \frac{\hat{\delta}}{\bar{\varepsilon}^2\phi} \right)$$

(3.104)

ii) Core Shear/Stiffness

$$\bar{t} = \left( \frac{24\hat{E}I\hat{A}G}{12\hat{E}I + AG} \right) \frac{\hat{\delta}}{\bar{\tau}c} - 1$$

(3.105)

iii) Indentation/Stiffness

$$\bar{t}^3(\bar{t} + 1) = \left( \frac{144\hat{E}I\hat{A}G\hat{\delta}}{12\hat{E}I + AG} \right)^{1/3} \left( \frac{1}{\pi^2\sigma^2 E_f c^4} \right)$$

(3.106)

iv) Microbuckling/Core Shear/Stiffness

$$\frac{(\bar{t} + 1)}{t} = \left( \frac{48\hat{E}I\hat{A}G}{12\hat{E}I + AG} \right) \frac{\hat{\delta}}{\bar{\tau}^2}$$

$$\bar{c} = \left( \frac{48\hat{E}I\hat{A}G}{12\hat{E}I + AG} \right) \frac{\hat{\delta}}{2\bar{\tau}} - \frac{\bar{\tau}}{2\phi}$$

(3.107)
v) Core Shear/Indentation/Stiffness

\[
\frac{(t + 1)}{t} = \left[ \left( \frac{\pi^2 \bar{\sigma}^2 E_f}{48 \bar{c}} \right) \left( \frac{48 \bar{E} \bar{A} \delta}{12 \bar{E} I + \bar{A} G} \right) \right]^{1/3}
\]  
(3.109)

\[
\left[ \left( \frac{48 \bar{E} \bar{A} \delta}{12 \bar{E} I + \bar{A} G} \right) \frac{\delta}{2 \bar{p} \bar{c}} - 1 \right]^3 = \left( \frac{12 \bar{E} I + \bar{A} G}{48 \bar{E} I \bar{A} \delta} \right)^{2/3} \left( \frac{6 \bar{r}}{\pi^2 \bar{\sigma}^2 \bar{E} f \bar{c}} \right)
\]  
(3.110)

vi) Indentation/Microbuckling/Stiffness

\[
\frac{\bar{P}}{(t + 1)} = \left( \frac{48 \bar{E} I \bar{A} \delta}{12 \bar{E} I + \bar{A} G} \right) \left( \frac{3 \bar{E} f}{\pi \bar{\sigma}} \right) \left( \frac{4 \phi}{\pi \sigma} \right)^2
\]  
(3.111)

\[
\left[ \left( \frac{\pi^2 \bar{\sigma}^2 E_f}{192 \phi^3 \bar{c}^2} \right)^{1/2} - 1 \right]^2 = \left( \frac{12 \phi}{\pi^2 \bar{\sigma}^2 E_f \bar{c}^2} \right) \left( \frac{48 \bar{E} I \bar{A} \delta}{12 \bar{E} I + \bar{A} G} \right)^2
\]  
(3.112)

A point where all failure mechanisms boundaries intersect is a function of the material properties of the beam.

For larger deflection ratios (ie. 10%), the imposed stiffness constraint rules out a small fraction of the failure mechanism map design space. Every incremental decrease of the deflection ratio limit thereafter expands this territory. For the GFRP/ROHACELL 51-IG case, even with a 10% deflection ratio limit, beam designs that are predicted to fail by microbuckling are ruled out. Thus, the failure mechanisms that should realistically be considered for the following material case are core shear and indentation.

Figures 3.24 and 3.25 compare the constrained minimum mass trajectory from each deflection ratio limit through \( \bar{t} - \bar{P} \) and \( \bar{c} - \bar{P} \) plots respectively.

Figure 3.24: Plot of minimum mass trajectory in terms of \( \bar{t} \) to load index for simply supported composite beams subject to three point bending, GFRP face sheets and ROHACELL 51-IG foam core - varied stiffness constraint, defined by \( \delta, \bar{P} \leq 5 \times 10^{-5} \)
As the maximum allowable deflection is limited, optimal beams show a rise in $\bar{c}$ and drop in $\bar{l}$, meaning beams are recommended to be designed with a thicker cores relative to face sheet thickness and beam length.

Figure 3.26 compares the mass index of the constrained minimum mass trajectory, varying $\delta$.

Shown in Figure 3.26, as the deflection ratio limit decreases the predicted mass index rises.
3.3.2 Four Point Bending

Next, four point bending is considered, where:

\[
\frac{\dot{P}}{\delta} = \frac{96 \dot{E}I \dot{A}\dot{G}}{24(1 - \bar{a})\dot{E}I + (1 - \bar{a})(3 - (1 - \bar{a})^2)\dot{A}\dot{G}}
\]  

(3.113)

and the supporting boundary line equations are:

i) Microbuckling/Stiffness

\[
\bar{t}(\bar{t} + 1) = \left(\frac{24 \dot{E}I \dot{A}\dot{G}\delta}{(24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G})\left(\frac{1}{\phi\epsilon^2}\right)}\right)
\]  

(3.114)

ii) Core Shear/Stiffness

\[
\bar{t} = \left(\frac{48 \dot{E}I \dot{A}\dot{G}\delta}{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}\right)\left(\frac{1}{(1 - \bar{a})\bar{\tau}\epsilon^2}\right) - 1
\]  

(3.115)

iii) Indentation/Stiffness

\[
\bar{t}^3(\bar{t} + 1) = \left(\frac{48 \dot{E}I \dot{A}\dot{G}\delta}{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}\right)^3\left(\frac{3}{\bar{\sigma}\bar{\tau}(1 - \bar{a})\bar{\epsilon}^2}\right)
\]  

(3.116)

iv) Microbuckling/Core Shear/Stiffness

\[
\frac{(\bar{t} + 1)}{t} = \left(\frac{\phi\delta}{(1 - \bar{a})\bar{\tau}^2\epsilon^2}\right)\left[\frac{96 \dot{E}I \dot{A}\dot{G}}{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}\right]
\]  

(3.117)

\[
\bar{c} = \left[\frac{48 \dot{E}I \dot{A}\dot{G}}{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}\right]\left(\frac{\delta}{\bar{\tau}(1 - \bar{a})}\right) - \frac{\bar{\tau}(1 - \bar{a})}{2\phi}
\]  

(3.118)

v) Core Shear/Indentation/Stiffness

\[
\frac{(\bar{t} + 1)}{t} = \left[\left(\frac{2(1 - \bar{a})\bar{\pi}^2\bar{\sigma}^2E_f}{\bar{\tau}^4}\right)\left(\frac{96 \dot{E}I \dot{A}\dot{G}\delta}{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}\right)\right]^{1/3}
\]  

(3.119)

\[
\left[\left(\frac{96 \dot{E}I \dot{A}\dot{G}\delta}{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}\right)\left(\frac{1}{2\bar{\tau}(1 - \bar{a})\bar{c}} - 1\right)^3\right] = \left(\frac{24\dot{E}I + (3 - (1 - \bar{a})^2)\dot{A}\dot{G}}{96 \dot{E}I \dot{A}\dot{G}\delta}\right)^{2/3}\left(\frac{3(1 - \bar{a})\bar{\tau}}{4\pi^2\bar{\sigma}^2E_f\bar{c}^3}\right)
\]  

(3.120)
vi) Indentation/Microbuckling/Stiffness

\[
\bar{t} = \left( \frac{96 \hat{E} I \hat{A} G \hat{\delta}}{24 \hat{E} I + (3 - (1 - \bar{a})^2) \hat{A} G} \right) \left( \frac{24}{\hat{E} f} \right) \left( \frac{\phi (1 - \bar{a})}{\pi \sigma} \right)^2
\]  

(3.121)

\[
\left[ \left( \frac{\pi^2 \sigma^2 (1 - \bar{a})^2 \hat{E} f}{24 \phi^2 \bar{c}^2} \right)^{1/2} - 1 \right]^2 = \left( \frac{3\phi}{2\pi^2 \sigma^2 (1 - \bar{a})^2 \hat{E} f \bar{c}^2} \right) \left( \frac{96 \hat{E} I \hat{A} G \hat{\delta}}{24 \hat{E} I + (3 - (1 - \bar{a})^2) \hat{A} G} \right)^2
\]  

(3.122)

Figure 3.27 is the failure mechanism map for simply supported GFRP/ROHACELL 51-IG beams subject to four point bending, \( \bar{a} = 0.2 \).

Similarly to three point bending, microbuckling is not feasible. As the deflection ratio limit decreases, the minimum mass trajectory eventually becomes comprised of boundary line design points.
To compare, Figures 3.28 and 3.29 show the $t - \hat{P}$ and $c - \hat{P}$ plots with a varied stiffness constraint, while Figure 3.30 shows the resultant mass index to peak load plot.

Figure 3.28: Plot of minimum mass trajectory in terms of $t$ to load index for simply supported composite beams subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core, $a = 0.2$ - varied stiffness constraint, defined by $\delta$, $\hat{P} \leq 4 \times 10^{-5}$

Figure 3.29: Plot of minimum mass trajectory in terms of $c$ to load index for simply supported composite beams subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core, $a = 0.2$ - varied stiffness constraint, defined by $\delta$, $\hat{P} \leq 4 \times 10^{-5}$
As the design space is further constrained by stiffness in the indentation region, the minimum allowed core thickness increases. As stiffness further constrains the minimum mass trajectory along the core shear/indentation boundary line, $\bar{t}$ is suppressed to a smaller maximum value.

**Figure 3.30**: Plot of minimum mass trajectory in terms of mass index to load index for simply supported composite beams subject to four point bending, GFRP face sheets and ROHACELL 51-IG foam core, $\bar{a} = 0.2$ - varied stiffness constraint, defined by $\hat{\delta}$, $\hat{\bar{P}} \leq 1x10^{-4}$
3.3.3 Uniformly Distributed Loading

For the third and final case, the effect of stiffness is studied on uniformly distributed loading. For this case, the stiffness constraint, now $\hat{w}/\hat{\delta}$ is defined by

$$\frac{\hat{w}}{\hat{\delta}} = \frac{384 \hat{E} I \hat{A} G}{5 AG + 96 EI}.$$  \hspace{1cm} (3.123)

Below are the respective equations required for failure mechanism mapping, considering the addition of a stiffness constraint.

i) Stiffness/Microbuckling

$$\bar{t}(\bar{t} + 1) = \left( \frac{48 \hat{E} I \hat{A} G}{5 AG + 96 EI} \right) \frac{\hat{\delta}}{\phi \bar{c}^2}.$$  \hspace{1cm} (3.124)

ii) Core Shear/Stiffness

$$\bar{t} = \left( \frac{192 \hat{E} I \hat{A} G}{5 AG + 96 EI} \right) \frac{\hat{\delta}}{\bar{\tau} \bar{c}} - 1.$$  \hspace{1cm} (3.125)

v) Microbuckling/Core Shear/Stiffness

$$\frac{(\bar{t} + 1)}{\bar{t}} = \left( \frac{3072 \hat{E} I \hat{A} G}{5 AG + 96 EI} \right) \frac{\phi \hat{\delta}}{\bar{\tau}^2}.$$  \hspace{1cm} (3.126)

$$\bar{c} = \left( \frac{384 \hat{E} I \hat{A} G}{5 AG + 96 EI} \right) \frac{\hat{\delta}}{\bar{\tau}} - \frac{\bar{\tau}}{8 \phi}.$$  \hspace{1cm} (3.127)
Figure 3.31 is the modified failure mechanism map of uniformly distributed loading for simply supported GFRP/ROHACELL 51-IG beams, subject to an increasing stiffness constraints.

By imposing a large enough stiffness constraint, microbuckling may be ruled out for the uniform loading case as well. Figures 3.32-3.34 plot $\bar{t}$, $\bar{c}$ and mass index of the constrained minimum mass trajectories to pressure index.

*Figure 3.32: Plot of minimum mass trajectory in terms of $\bar{t}$ to load index for simply supported composite beams subject to uniformly distributed loading, GFRP face sheets and ROHACELL 51-IG foam core - varied stiffness constraint, defined by $\delta$*
In the event microbuckling is not feasible, the resultant minimum mass trajectory would be defined by the core shear/stiffness boundary line.

In general, when designing beams to be subject to pressure loading, the only plausible failure mechanism to exist is core shear. Though other failure mechanisms (microbuckling, indentation, core crush) may exist in extreme cases, core shear is the only active failure mechanism that both satisfies stiffness constraints and adheres to realistic material property assumptions.

Through failure mechanism identification and mapping, three point bending, four point bending and uniformly distributed loading are analyzed. To eliminate undesirable beam designs that fail under large deflections, a stiffness constraint is introduced. By omitting beam design points that exceed a set limit of compliance, is it determined that microbuckling is infeasible, considering realistic material properties for each load case.
Chapter 4

Failure Mechanism Map Validation

To validate the stiffness and strength expressions derived in Chapter 3, a series of experiments were conducted. Though the following tests may overlap other work, all three loading cases were considered for sake of continuity, allowing for a comparison of results afterward.

Experiments were performed on an MTS 550 Load Frame with a 100kN load cell. Various loading fixtures were installed to complete the required tensile, compressive and bending tests. Figure 4.1 is a summary of all loading fixtures used for three point bending, four point bending and uniformly distributed loading tests.

![Figure 4.1: Summary of testing apparatus: (a) three point bending, (b) four point bending, (c) uniform loading (d) base unit](image)

All three loading cases utilized the same base unit, Figure 4.1(d), a fully adjustable steel block with removable support arms of 55mm long contact rollers, 25mm in diameter. Governed by a track and set screws, the base unit spans 80-480mm in 20mm intervals. Load fixtures for three and four point bending are based on the same design as the base unit and were installed on the top cylinder of the load frame. As no testing standard existed for simply supported uniformly distributed loading testing, an alternate testing fixture was proposed (detailed later).
4.1 Material Selection and Fabrication

4.1.1 Face Sheet Material - GFRP

Glass fibre-reinforced polymer face sheets were constructed from 6oz. BGF plain weave fibreglass cloth [4], Huntsman Araldite LY8601 Resin and Aradur 8602 Hardener [18]. To ensure consistent face sheet quality, resin was applied on each glass fibre ply until saturation during the layering process. Face sheets were aligned in 0/90° orientation and cured at room temperature using a vacuum resin infusion unit. During the curing and consolidation process, GFRP sheet thicknesses were reduced by 15%. Therefore, the estimated post-cure thickness of GFRP, \( t \), was 85% of the total fabric material.

Tensile tests were conducted on 150mm x 15mm x 3 ply glass fibre ((0.85) x 0.6mm) strips, supported by four aluminum tabs of 5mm. Figure 4.2 is the tensile stress-strain plot from the 50mm x 15mm x 0.51mm GFRP specimen, performed at a strain rate of 0.005mm/s.

![Tensile stress-strain plot of GFRP face sheet material](image)

Though it would have been ideal determine the corresponding compressive strength through experiments, fibre-reinforced materials require specific testing fixtures for compressive testing (ie. a Celanese compression fixture). As no such testing fixture was available, estimated compressive strength was cited from other works [36].

Table 4.1 summarizes material properties for GFRP.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>E-Glass Fibre-Reinforced Polymer (GFRP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, ( E_f ) (GPa)</td>
<td>31</td>
</tr>
<tr>
<td>Compressive Strength, ( \sigma_c ) (kPa)</td>
<td>350 [37]</td>
</tr>
<tr>
<td>Density, ( \rho_f ) (kg/m(^3))</td>
<td>1592</td>
</tr>
</tbody>
</table>
4.1.2 Core Material - ROHACELL 51-IG, Extruded Polystyrene (EPS)

Of the three foams analyzed in previous chapters, two were used for the following experiments: Evonik Industries ROHACELL 51-IG (Industrial Grade) polymethacrylimide (PMI) foam, and Foamular C-300 extruded polystyrene (EPS) insulation foam [20, 10].

Figure 4.3 compares the compressive strength of the two foams using the MTS 550 load frame and compressive platens. Test specimens of dimensions 30mm x 30mm x 30mm were compressed at 0.01mm/s. The compressive strength was taken to be the stress associated with the long yield plateau.

![Figure 4.3: Compression stress-strain plot of core materials, EPS and ROHACELL 51-IG, plotted to 10% compression strain](image)

Figure 4.4 compares the shear stress for each core. Shear test specimens comprised of 120mm x 45mm x 30mm foam pieces were adhered to two 120mm x 45mm steel shear plates. Test specimens were compressed within the MTS 550 load frame at 0.005mm/s.

![Figure 4.4: Shear stress-strain plot of core materials, EPS and ROHACELL 51-IG](image)
The results of compression and shear testing for both ROHACELL 51-IG and EPS core materials are shown in Table 4.2.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Extruded Polystyrene (EPS)</th>
<th>ROHACELL 51-IG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, $E_c$ (MPa)</td>
<td>13</td>
<td>67</td>
</tr>
<tr>
<td>Compressive Strength (10% Strain), $\sigma_c$ (kPa)</td>
<td>422</td>
<td>1076</td>
</tr>
<tr>
<td>Shear Modulus, $G_c$ (MPa)</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>Shear Yield Stress, $\tau_y$ (kPa)</td>
<td>282</td>
<td>N/A</td>
</tr>
<tr>
<td>Shear Strength, $\tau_c$ (kPa)</td>
<td>357</td>
<td>792</td>
</tr>
<tr>
<td>Density, $\rho_f$ (kg/m$^3$)</td>
<td>28</td>
<td>50</td>
</tr>
</tbody>
</table>

Shear strength of the elastic-brittle ROHACELL 51-IG was 792kPa before fracture. For EPS, the foam underwent considerable plastic deformation before separating from the shear plates; therefore, shear failure was assumed at yielding using 0.2% strain offset ($\tau_y=282$ kPa).

### 4.1.3 Beam Fabrication

Face and core components were cut to the specified dimension requirements of each test, then bonded together using Huntsman Araldite 2011Y epoxy adhesive [17]; the curing time was 24 hours.

### 4.2 Data Acquisition

Figure 4.5 summarizes the data acquisition tools used during the following experiments.

![Figure 4.5: Placement summary of data acquisition instruments: (A) strain gauge, (B) core shear clip gauge, (C) core indentation, (D) midspan displacement](image-url)
4.2.1 Strain Gauge (A)

TML strain gauges of length 2mm (TML FLA-2-11, [41]) were installed on the top face sheet of the beam, 5mm from the load roller to track compressive axial strain.

4.2.2 Core Shear Strain Gauge (B)

A core shear clip gauge, comprised of an MTS displacement gauge and two 14-gauge aluminum wires, was adhered to top and bottom face sheets 40mm inside the side support position, seen in Figure 4.6. The bonding of the aluminum wires to the face sheets use the same Huntsman Araldite 2011Y epoxy adhesive [17] and were applied after initial beam construction.

![Figure 4.6: Test setup for clip gauge](image)

4.2.3 Laser Extensometry (C)-(D)

A LE-05 laser extensometer measured the linear displacement of two points marked by reflective tape, shown in Figure 4.7.

![Figure 4.7: Test setup for laser extensometer](image)

Core indentation (C) was measured from the displacement of the load roller to the bottom face sheet. Midspan displacement (D) was measured from the displacement of the bottom face sheet to a set marker on the base unit.
4.2.4 Data Acquisition and Output

During testing, MTS, laser extensometer, strain gauge and clip gauge test data were compiled through the use of a National Instruments 6325 signal conditioner and Labview 8.9 software. Each half-second iteration was the average of 100 data samples taken at 10kHz, sequentially extracted from each device.

4.3 Test Outline

For the following experiments, 84 design points were selected across the three possible loading cases and two material compositions: GFRP/ROHACELL 51-IG and GFRP/EPS. The goal each test set was to span as much of the design space as possible, given the limits of the testing fixtures, load frame and fabricated materials. Test sets were also strategically selected close to failure region boundary lines (if possible) to determine the true accuracy of the analytical models.

Composite sandwich beam test specimens were comprised of face sheets 5-ply (0.85mm) to 13-ply (2.26) thick, cores c=10mm to 30mm, spanning L=80-460mm long. The chosen beam width was b=35mm.

In Chapter 3, a stiffness constraint omits potential beam designs of undesired compliance. For the following study, tested beam designs were selected within a projected deflection ratio limit of $\hat{\delta} = 0.03 = 3\%$. It was predicted that any beams deflecting greater than this ratio had the potential of coming into contact with the base unit prior to failure.
4.3.1 Case A: Three Point Bending

Figures 4.8 and 4.9 plot the experiments on the corresponding failure mechanism maps, Figure 4.8 for GFRP/ROHACELL 51-IG beams and Figure 4.9 for GFRP/EPS.

![Failure Mechanism Map Validation](image)

Figure 4.8: Test layout for simply supported three point bending case with respect to failure mechanism map - GFRP face sheets and ROHACELL 51-IG foam core, $\delta = 3.0\%$

![Failure Mechanism Map Validation](image)

Figure 4.9: Test layout for simply supported three point bending case with respect to failure mechanism map - GFRP face sheets and extruded polystyrene (EPS) foam core, $\delta = 3.0\%$

To relate each design point of $(\bar{c}, \bar{t})$ from the failure mechanism maps, Tables 4.3 and 4.4 outline each test point in terms of chosen beam parameters. For both three and four point bending, beams were constructed with an overhang $H=10$mm. As the overhang was a fixed length rather than a set percentage of the beam length, failure mechanism maps for GFRP/EPS beams cannot account for the overhang factor in core shear failure prediction. Considering each test, the predicted failure in Table 4.4 was changed to indentation if the inclusion of the overhang factor resulted in indentation becoming the weakest failure mechanism.
Table 4.3: Test summary for simply supported three point bending - GFRP/ROHACELL

<table>
<thead>
<tr>
<th>Test ID</th>
<th>$\bar{c}$ (mm)</th>
<th>$\bar{r}$ (mm)</th>
<th>t (mm)</th>
<th>c (mm)</th>
<th>L (mm)</th>
<th>Predicted Failure</th>
<th>Predicted $\hat{\delta}$ (%)</th>
<th>DAQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRA1</td>
<td>0.023</td>
<td>0.085</td>
<td>0.85</td>
<td>10</td>
<td>440</td>
<td>I</td>
<td>2.16</td>
<td>Y</td>
</tr>
<tr>
<td>TRA2</td>
<td>0.023</td>
<td>0.119</td>
<td>1.19</td>
<td>10</td>
<td>440</td>
<td>I</td>
<td>2.45</td>
<td>Y</td>
</tr>
<tr>
<td>TRA3</td>
<td>0.023</td>
<td>0.170</td>
<td>1.7</td>
<td>10</td>
<td>440</td>
<td>CS</td>
<td>2.05</td>
<td>Y</td>
</tr>
<tr>
<td>TRB1</td>
<td>0.250</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>80</td>
<td>CS</td>
<td>1.04</td>
<td>Y</td>
</tr>
<tr>
<td>TRB2</td>
<td>0.100</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>200</td>
<td>CS</td>
<td>1.12</td>
<td>Y</td>
</tr>
<tr>
<td>TRB3</td>
<td>0.063</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>320</td>
<td>CS</td>
<td>1.26</td>
<td>Y</td>
</tr>
<tr>
<td>TRB4</td>
<td>0.045</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>440</td>
<td>CS</td>
<td>1.47</td>
<td>Y</td>
</tr>
<tr>
<td>TRC1</td>
<td>0.250</td>
<td>0.060</td>
<td>0.85</td>
<td>20</td>
<td>80</td>
<td>CS</td>
<td>1.05</td>
<td>Y</td>
</tr>
<tr>
<td>TRC2</td>
<td>0.100</td>
<td>0.060</td>
<td>0.85</td>
<td>20</td>
<td>200</td>
<td>I</td>
<td>0.98</td>
<td>Y</td>
</tr>
<tr>
<td>TRC3</td>
<td>0.063</td>
<td>0.060</td>
<td>0.85</td>
<td>20</td>
<td>320</td>
<td>I</td>
<td>1.03</td>
<td>Y</td>
</tr>
<tr>
<td>TRC4</td>
<td>0.045</td>
<td>0.060</td>
<td>0.85</td>
<td>20</td>
<td>440</td>
<td>I</td>
<td>1.21</td>
<td>Y</td>
</tr>
<tr>
<td>TRD1</td>
<td>0.375</td>
<td>0.028</td>
<td>0.85</td>
<td>30</td>
<td>80</td>
<td>I</td>
<td>0.65</td>
<td>Y</td>
</tr>
<tr>
<td>TRD2</td>
<td>0.150</td>
<td>0.028</td>
<td>0.85</td>
<td>30</td>
<td>200</td>
<td>I</td>
<td>0.54</td>
<td>Y</td>
</tr>
<tr>
<td>TRD3</td>
<td>0.094</td>
<td>0.028</td>
<td>0.85</td>
<td>30</td>
<td>320</td>
<td>I</td>
<td>0.56</td>
<td>Y</td>
</tr>
<tr>
<td>TRD4</td>
<td>0.068</td>
<td>0.028</td>
<td>0.85</td>
<td>30</td>
<td>440</td>
<td>I</td>
<td>0.66</td>
<td>Y</td>
</tr>
</tbody>
</table>

Legend: CS- Core Shear, I - Indentation,
(A) Strain Gauge, (B) Core Shear Clips, (C) Core Indentation, (D) Midspan Deflection
I* - Predicted to fail by indentation when overhang factor is applied

Table 4.4: Test summary for simply supported three point bending - GFRP/EPS

<table>
<thead>
<tr>
<th>Test ID</th>
<th>$\bar{c}$ (mm)</th>
<th>$\bar{r}$ (mm)</th>
<th>t (mm)</th>
<th>c (mm)</th>
<th>L (mm)</th>
<th>Predicted Failure</th>
<th>Predicted $\hat{\delta}$ (%)</th>
<th>DAQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEA1</td>
<td>0.023</td>
<td>0.100</td>
<td>0.85</td>
<td>10</td>
<td>440</td>
<td>I</td>
<td>1.88</td>
<td>Y</td>
</tr>
<tr>
<td>TEA2</td>
<td>0.023</td>
<td>0.140</td>
<td>1.19</td>
<td>10</td>
<td>440</td>
<td>CS</td>
<td>1.73</td>
<td>Y</td>
</tr>
<tr>
<td>TEA3</td>
<td>0.023</td>
<td>0.260</td>
<td>2.21</td>
<td>10</td>
<td>440</td>
<td>CS</td>
<td>1.38</td>
<td>Y</td>
</tr>
<tr>
<td>TEB1</td>
<td>0.250</td>
<td>0.120</td>
<td>2.04</td>
<td>20</td>
<td>80</td>
<td>CS</td>
<td>1.22</td>
<td>Y</td>
</tr>
<tr>
<td>TEB2</td>
<td>0.100</td>
<td>0.120</td>
<td>2.04</td>
<td>20</td>
<td>200</td>
<td>CS</td>
<td>1.17</td>
<td>Y</td>
</tr>
<tr>
<td>TEB3</td>
<td>0.063</td>
<td>0.120</td>
<td>2.04</td>
<td>20</td>
<td>320</td>
<td>CS</td>
<td>1.29</td>
<td>Y</td>
</tr>
<tr>
<td>TEB4</td>
<td>0.045</td>
<td>0.120</td>
<td>2.04</td>
<td>20</td>
<td>440</td>
<td>CS</td>
<td>1.25</td>
<td>Y</td>
</tr>
<tr>
<td>TEC1</td>
<td>0.250</td>
<td>0.050</td>
<td>0.85</td>
<td>20</td>
<td>80</td>
<td>I*</td>
<td>1.39</td>
<td>Y</td>
</tr>
<tr>
<td>TEC2</td>
<td>0.100</td>
<td>0.050</td>
<td>0.85</td>
<td>20</td>
<td>200</td>
<td>I</td>
<td>1.01</td>
<td>Y</td>
</tr>
<tr>
<td>TEC3</td>
<td>0.063</td>
<td>0.050</td>
<td>0.85</td>
<td>20</td>
<td>320</td>
<td>I</td>
<td>0.98</td>
<td>Y</td>
</tr>
<tr>
<td>TEC4</td>
<td>0.045</td>
<td>0.050</td>
<td>0.85</td>
<td>20</td>
<td>440</td>
<td>I</td>
<td>1.00</td>
<td>Y</td>
</tr>
<tr>
<td>TED1</td>
<td>0.375</td>
<td>0.067</td>
<td>1.7</td>
<td>30</td>
<td>80</td>
<td>CS</td>
<td>1.51</td>
<td>Y</td>
</tr>
<tr>
<td>TED2</td>
<td>0.150</td>
<td>0.067</td>
<td>1.7</td>
<td>30</td>
<td>200</td>
<td>CS</td>
<td>1.13</td>
<td>Y</td>
</tr>
<tr>
<td>TED3</td>
<td>0.094</td>
<td>0.067</td>
<td>1.7</td>
<td>30</td>
<td>320</td>
<td>I*</td>
<td>1.32</td>
<td>Y</td>
</tr>
<tr>
<td>TED4</td>
<td>0.068</td>
<td>0.067</td>
<td>1.7</td>
<td>30</td>
<td>440</td>
<td>I*</td>
<td>1.23</td>
<td>Y</td>
</tr>
</tbody>
</table>

Legend: CS- Core Shear, I - Indentation,
For the following experiments, four common beam cases were considered: long compliant beams of thin cores (Test Set A), beams comprised of thick face sheets (Test Set B), thin face sheets (Test Set C) and thick cores (Test Set D).

Strain gauges were installed on Test Set A beams as they are the mostly likely to promote microbuckling failure and core shear strain gauges were installed on all tests with the exception of short beams (ie. L=80mm) as the clips interfered with the load contacts. Although the MTS load frame provided crosshead displacement data, midspan deflection was measured by the laser extensometer.

To note, as the laser extensometer could only measure one displacement per test, a second test was performed of the same beam geometry to measure core indentation (if applicable).
4.3.2 Case B: Four Point Bending

For four point bending tests, the considered load roller spacing, \( a \), was 80mm. As the load spacing was fixed for a range of beam lengths, the experiments could not be plotted with respect to a single set of failure region boundary lines. Figures 4.10 and 4.11 uses the method seen in Figure 3.12, comparing boundary lines for a range of \( \bar{a} \). As reference, the \( \bar{a} \) of each test point is displayed in brackets.

![Figure 4.10: Test layout for simply supported four point bending case with respect to failure mechanism map - GFRP face sheets and ROHACELL 51-IG foam core, \( \hat{\delta} = 3.0\% \), varied \( \bar{a} \)](image)

![Figure 4.11: Test layout for simply supported four point bending case with respect to failure mechanism map - GFRP face sheets and extruded polystyrene (EPS) foam core, \( \hat{\delta} = 3.0\% \), varied \( \bar{a} \)](image)

Table 4.5 lists the tests for GFRP/ROHACELL 51-IG beams, Table 4.6 for GFRP/EPS beams. For the considered range of face sheet and core thicknesses, GFRP/EPS tests could not be selected within the predicted indentation failure region, only core shear.
### Table 4.5: Test summary for simply supported four point bending - GFRP/ROHACELL 51-IG

<table>
<thead>
<tr>
<th>Test ID</th>
<th>$\bar{c}$ (mm)</th>
<th>$\bar{t}$ (mm)</th>
<th>$c$ (mm)</th>
<th>$L$ (mm)</th>
<th>Predicted Failure</th>
<th>Predicted $\hat{\delta}$ (%)</th>
<th>DAQ</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRA1</td>
<td>0.033</td>
<td>0.119</td>
<td>0.19</td>
<td>10</td>
<td>300</td>
<td>CS</td>
<td>1.37</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRA2</td>
<td>0.033</td>
<td>0.170</td>
<td>1.7</td>
<td>10</td>
<td>300</td>
<td>CS</td>
<td>1.19</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRA3</td>
<td>0.033</td>
<td>0.204</td>
<td>2.04</td>
<td>10</td>
<td>300</td>
<td>CS</td>
<td>1.09</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRB1</td>
<td>0.200</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>100</td>
<td>CS</td>
<td>0.21</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRB2</td>
<td>0.077</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>260</td>
<td>CS</td>
<td>0.85</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRB3</td>
<td>0.056</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>360</td>
<td>CS</td>
<td>1.09</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRB4</td>
<td>0.043</td>
<td>0.102</td>
<td>2.04</td>
<td>20</td>
<td>460</td>
<td>CS</td>
<td>1.32</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRC1</td>
<td>0.200</td>
<td>0.043</td>
<td>0.85</td>
<td>10</td>
<td>100</td>
<td>CS</td>
<td>0.22</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRC2</td>
<td>0.077</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>260</td>
<td>CS</td>
<td>1.06</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRC3</td>
<td>0.056</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>360</td>
<td>CS</td>
<td>1.51</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRC4</td>
<td>0.043</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>460</td>
<td>I</td>
<td>2.05</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRD1</td>
<td>0.300</td>
<td>0.068</td>
<td>2.04</td>
<td>30</td>
<td>100</td>
<td>CS</td>
<td>0.22</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRD2</td>
<td>0.115</td>
<td>0.068</td>
<td>2.04</td>
<td>30</td>
<td>260</td>
<td>CS</td>
<td>0.81</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRD3</td>
<td>0.083</td>
<td>0.068</td>
<td>2.04</td>
<td>30</td>
<td>360</td>
<td>CS</td>
<td>0.99</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FRD4</td>
<td>0.065</td>
<td>0.068</td>
<td>2.04</td>
<td>30</td>
<td>460</td>
<td>CS</td>
<td>1.18</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: CS- Core Shear, I - Indentation, (A) Strain Gauge, (B) Core Shear Clips, (C) Core Indentation, (D) Midspan Deflection

### Table 4.6: Test summary for simply supported four point bending - GFRP/EPS

<table>
<thead>
<tr>
<th>Test ID</th>
<th>$\bar{c}$ (mm)</th>
<th>$\bar{t}$ (mm)</th>
<th>$c$ (mm)</th>
<th>$L$ (mm)</th>
<th>Predicted Failure</th>
<th>Predicted $\hat{\delta}$ (%)</th>
<th>DAQ</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA1</td>
<td>0.026</td>
<td>0.119</td>
<td>1.19</td>
<td>10</td>
<td>380</td>
<td>CS</td>
<td>1.26</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FEA2</td>
<td>0.026</td>
<td>0.170</td>
<td>1.7</td>
<td>10</td>
<td>380</td>
<td>CS</td>
<td>1.18</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA3</td>
<td>0.026</td>
<td>0.204</td>
<td>2.04</td>
<td>10</td>
<td>380</td>
<td>CS</td>
<td>1.07</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FEB1</td>
<td>0.200</td>
<td>0.104</td>
<td>2.04</td>
<td>20</td>
<td>100</td>
<td>CS</td>
<td>0.30</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEB2</td>
<td>0.083</td>
<td>0.104</td>
<td>2.04</td>
<td>20</td>
<td>240</td>
<td>CS</td>
<td>0.90</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEB3</td>
<td>0.059</td>
<td>0.104</td>
<td>2.04</td>
<td>20</td>
<td>340</td>
<td>CS</td>
<td>1.00</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEB4</td>
<td>0.045</td>
<td>0.104</td>
<td>2.04</td>
<td>20</td>
<td>440</td>
<td>CS</td>
<td>1.10</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEC1</td>
<td>0.200</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>100</td>
<td>CS</td>
<td>0.22</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEC2</td>
<td>0.083</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>240</td>
<td>CS</td>
<td>0.87</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEC3</td>
<td>0.059</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>340</td>
<td>CS</td>
<td>1.10</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEC4</td>
<td>0.045</td>
<td>0.043</td>
<td>0.85</td>
<td>20</td>
<td>440</td>
<td>CS</td>
<td>1.36</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FED1</td>
<td>0.300</td>
<td>0.057</td>
<td>1.7</td>
<td>30</td>
<td>100</td>
<td>CS</td>
<td>0.24</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FED2</td>
<td>0.125</td>
<td>0.057</td>
<td>1.7</td>
<td>30</td>
<td>240</td>
<td>CS</td>
<td>0.79</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FED3</td>
<td>0.088</td>
<td>0.057</td>
<td>1.7</td>
<td>30</td>
<td>340</td>
<td>CS</td>
<td>0.92</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FED4</td>
<td>0.068</td>
<td>0.057</td>
<td>1.7</td>
<td>30</td>
<td>440</td>
<td>CS</td>
<td>1.08</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: CS- Core Shear, I - Indentation, (A) Strain Gauge, (B) Core Shear Clips, (C) Core Indentation, (D) Midspan Deflection
4.3.3 Case C: Uniformly Distributed Loading

Seen in previous literature, a common strategy has been to use an airbag testing system [42, 21]. The pressure load is distributed via a flat plate compressing on a thick polymer airbag, lying across the entire length of the beam. As the beam defects, the airbag deforms to the curvature of the beam.

The following study an alternative approach was used, considering that a pressure load could be simulated by a finite number of point loads.

Proposed Testing Method: Whippletree Loading Apparatus

Figure 4.12 below shows the proposed testing fixture for simulating pressure loads. Based on a whippletree design, the point loads splay outward evenly as midspan deflection increases. Figure 4.12 compares the CAD drawing and final product of the proposed testing fixture, machined by Lilex Industries.

Ideally, an infinite number of roller contacts would best represent a uniform pressure. For the purpose of the following experiments, eight load rollers were considered acceptable for uniform pressure simulation. The proposed design in Figure 4.12 was comprised of three branch levels (1 primary, 2 secondary, and 4 tertiary), each allowed to freely pivot on lubricated pins. Eight contact rollers were installed to arms on the tertiary branches by set screws and springs. Parts were machined using machine steel with exception for the contact rollers made from stainless steel.

To stay within the size constraints of the MTS load frame, the total fixture length was 350mm, having each roller contact of radius 10mm a distance of 50mm apart. Though it would have been preferred to allow the test fixture to be length-adjustable, the complex design of the apparatus prevented the following design feature from being included.
Initial Composite Sandwich Beam Testing and Modification

A series of initial tests were conducted to determine what modifications, if any, were required to ensure the applied load simulated a uniform pressure. The tests also determined the ideal beam length that best coincided with stiffness and strength prediction. The following was assumed: if both analytical predictions and experiment data show similar results, both methods may represent the characteristics of a pressure load. An initial series of tests were conducted using 25mm wide beams, comprising of 1.19mm GFRP face sheets and 30mm ROHACELL 51-IG core material.

After the first set of tests, two challenges were noted. First, due to the significant surface roughness of the GFRP, the intended splay-out motion of the whippletree branches was frequently restricted. Once halted, the branches lifted up from the beam and any rollers still in contact became concentrated point loads. Secondly, all preliminary beams failed prematurely due to indentation occurring at the side supports.

To provide a friction-free surface, the first challenge was solved by inserting a thin strip of PVC between the top face sheet and roller contacts. The track and roller contacts were lubricated to minimize surface roughness. PVC was chosen over other materials due to its elastic modulus being significantly lower than the estimated GFRP stiffness. Figure 4.13 shows a simply supported composite sandwich beam subject to whippletree loading with the PVC track installed.

![Figure 4.13: Uniformly distributed loading test using the whippletree loading rig with PVC track, no side support tabs - subject to indentation](image)

To eliminate side support indentation, tabs comprised of two different materials (PVC and steel) were placed underneath the beams at the side supports. Varying beam length between 380mm and 400mm, Figures 4.14 compares the observed results of each tab material at the side supports. For continuity, overhang was held at $H = 10\text{mm}$.
Similar to the case of having no tab, adding PVC yielded no improvement in reducing side support indentation, however, it was prevented using the steel tabs. With initial performance issues resolved, the next objective was to determine the optimal beam length that best correlated both analytical and experiment results. To determine the best option, three beam lengths were considered: 360mm, 380mm and 400mm. The results of the following tests are summarized in Table 4.7 below:

In comparing the results of the three different beam lengths, both the 360mm and 380mm test beams performed close to their predicted values. Though either beam length could have been chosen from these results alone, prior to failure the whippletree roller contacts splayed beyond the side supports during the $L = 360\text{mm}$ test. Therefore, the only suitable beam length for testing was 380mm.

Seen in Figure 4.15, due to the chosen overhang being relatively short, all metal-tabbed beams incurred minor core crushing at the side support prior to core shear failure. By extending the overhang to 40mm, core crush at the side supports was eliminated and experiment results came within 6% error to both stiffness and strength predictions.

In summary, by providing a friction-free surface for the point loads, a larger surface area for side supports, and determining the ideal beam length and overhang for tests, similar results between predictions and experiments were observed. Thus, the following proposed whippletree loading fixture provided an adequate simulation of uniform pressure loading after initial testing.
Experiment Failure Mechanism Maps and Test Lists

Figures 4.15 and 4.16 display the tests for the uniform loading case in terms of failure mechanism maps, while Tables 4.8 and 4.9 list the beam geometry and data acquisition tools assigned.

Figure 4.15: Test layout for simply supported uniformly distributed load case with respect to failure mechanism map - GFRP face sheets and ROHACELL 51-IG foam core, $\hat{\delta} = 3.0\%$

Figure 4.16: Test layout for simply supported uniformly distributed load case with respect to failure mechanism map - GFRP face sheets and extruded polystyrene (EPS) foam core, $\hat{\delta} = 3.0\%$
Table 4.8: Test summary for simply supported uniformly distributed loading - GFRP/ROHACELL

<table>
<thead>
<tr>
<th>Test ID</th>
<th>$\bar{c}$</th>
<th>$\bar{t}$</th>
<th>$t$ (mm)</th>
<th>$c$ (mm)</th>
<th>$L$ (mm)</th>
<th>Predicted Failure</th>
<th>Predicted $\delta$ (%)</th>
<th>DAQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>URA1</td>
<td>0.039</td>
<td>0.073</td>
<td>1.19</td>
<td>15</td>
<td>380</td>
<td>CS</td>
<td>1.52</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>URA2</td>
<td>0.039</td>
<td>0.113</td>
<td>1.7</td>
<td>15</td>
<td>380</td>
<td>CS</td>
<td>1.35</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>URA3</td>
<td>0.039</td>
<td>0.147</td>
<td>2.21</td>
<td>15</td>
<td>380</td>
<td>CS</td>
<td>1.27</td>
<td>Y Y</td>
</tr>
<tr>
<td>URB1</td>
<td>0.053</td>
<td>0.060</td>
<td>1.19</td>
<td>20</td>
<td>380</td>
<td>CS</td>
<td>1.41</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>URB2</td>
<td>0.053</td>
<td>0.085</td>
<td>1.7</td>
<td>20</td>
<td>380</td>
<td>CS</td>
<td>1.28</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>URB3</td>
<td>0.053</td>
<td>0.111</td>
<td>2.21</td>
<td>20</td>
<td>380</td>
<td>CS</td>
<td>1.32</td>
<td>Y Y</td>
</tr>
<tr>
<td>URC1</td>
<td>0.066</td>
<td>0.048</td>
<td>1.19</td>
<td>25</td>
<td>380</td>
<td>CS</td>
<td>1.33</td>
<td>Y Y</td>
</tr>
<tr>
<td>URC2</td>
<td>0.066</td>
<td>0.068</td>
<td>1.7</td>
<td>25</td>
<td>380</td>
<td>CS</td>
<td>1.23</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>URC3</td>
<td>0.066</td>
<td>0.088</td>
<td>2.21</td>
<td>25</td>
<td>380</td>
<td>CS</td>
<td>1.18</td>
<td>Y Y</td>
</tr>
<tr>
<td>URD1</td>
<td>0.079</td>
<td>0.040</td>
<td>1.19</td>
<td>30</td>
<td>380</td>
<td>CS</td>
<td>1.29</td>
<td>Y Y</td>
</tr>
<tr>
<td>URD2</td>
<td>0.079</td>
<td>0.057</td>
<td>1.7</td>
<td>30</td>
<td>380</td>
<td>CS</td>
<td>1.20</td>
<td>Y Y</td>
</tr>
<tr>
<td>URD3</td>
<td>0.079</td>
<td>0.074</td>
<td>2.21</td>
<td>30</td>
<td>380</td>
<td>CS</td>
<td>1.16</td>
<td>Y Y</td>
</tr>
</tbody>
</table>

Legend: CS - Core Shear, I - Indentation, (A) Strain Gauge, (B) Core Shear Clips, (C) Core Indentation, (D) Midspan Deflection

Table 4.9: Test summary for simply supported uniformly distributed loading - GFRP/EPS

<table>
<thead>
<tr>
<th>Test ID</th>
<th>$\bar{c}$</th>
<th>$\bar{t}$</th>
<th>$t$ (mm)</th>
<th>$c$ (mm)</th>
<th>$L$ (mm)</th>
<th>Predicted Failure</th>
<th>Predicted $\delta$ (%)</th>
<th>DAQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>UEA1</td>
<td>0.039</td>
<td>0.073</td>
<td>1.19</td>
<td>15</td>
<td>380</td>
<td>CS</td>
<td>1.46</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>UEA2</td>
<td>0.039</td>
<td>0.113</td>
<td>1.7</td>
<td>15</td>
<td>380</td>
<td>CS</td>
<td>1.40</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>UEA3</td>
<td>0.039</td>
<td>0.147</td>
<td>2.21</td>
<td>15</td>
<td>380</td>
<td>CS</td>
<td>1.43</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>UEB1</td>
<td>0.053</td>
<td>0.060</td>
<td>1.19</td>
<td>20</td>
<td>380</td>
<td>CS</td>
<td>1.43</td>
<td>Y Y Y</td>
</tr>
<tr>
<td>UEB2</td>
<td>0.053</td>
<td>0.085</td>
<td>1.7</td>
<td>20</td>
<td>380</td>
<td>CS</td>
<td>1.36</td>
<td>Y Y</td>
</tr>
<tr>
<td>UEB3</td>
<td>0.053</td>
<td>0.111</td>
<td>2.21</td>
<td>20</td>
<td>380</td>
<td>CS</td>
<td>1.33</td>
<td>Y Y</td>
</tr>
<tr>
<td>UEC1</td>
<td>0.066</td>
<td>0.048</td>
<td>1.19</td>
<td>25</td>
<td>380</td>
<td>CS</td>
<td>1.41</td>
<td>Y Y</td>
</tr>
<tr>
<td>UEC2</td>
<td>0.066</td>
<td>0.068</td>
<td>1.7</td>
<td>25</td>
<td>380</td>
<td>CS</td>
<td>1.36</td>
<td>Y Y</td>
</tr>
<tr>
<td>UEC3</td>
<td>0.066</td>
<td>0.088</td>
<td>2.21</td>
<td>25</td>
<td>380</td>
<td>CS</td>
<td>1.31</td>
<td>Y Y</td>
</tr>
<tr>
<td>UED1</td>
<td>0.079</td>
<td>0.040</td>
<td>1.19</td>
<td>30</td>
<td>380</td>
<td>CS</td>
<td>1.38</td>
<td>Y Y</td>
</tr>
<tr>
<td>UED2</td>
<td>0.079</td>
<td>0.057</td>
<td>1.7</td>
<td>30</td>
<td>380</td>
<td>CS</td>
<td>1.33</td>
<td>Y Y</td>
</tr>
<tr>
<td>UED3</td>
<td>0.079</td>
<td>0.074</td>
<td>2.21</td>
<td>30</td>
<td>380</td>
<td>CS</td>
<td>1.33</td>
<td>Y Y</td>
</tr>
</tbody>
</table>

Legend: CS - Core Shear, I - Indentation, (A) Strain Gauge, (B) Core Shear Clips, (C) Core Indentation, (D) Midspan Deflection

To account for a fixed beam length of 380mm, each test set considered a core thickness range of 15-30mm, and face sheet thickness range of 1.19mm-2.21mm respectively. Under the following test and material conditions, only beams predicted to fail by core shear failure could be selected. Regardless, strain gauges were still applied to those tests near the predicted microbuckling failure region. As core indentation had been analytically disregarded, the laser extensometer was primarily used to measure midspan displacement. As the core shear clips had the potential to interfere with the PVC track and point load roller, two tests were conducted, with at least one beam having no core shear clips installed.
4.4 Experiment Results

Tables 4.10-4.15 list the results from all conducted experiments. The four parameters compared are: stiffness, observed failure mechanism, failure load, and observed deflection ratio. The observed deflection ratio is the predicted failure load over the observed stiffness. Tests that incorrectly predicted the active failure mechanism or exceeded the projected deflection ratio limit of $\delta = 3\%$ are highlighted in red. As multiple experiments of the same design point are conducted for the purpose of data acquisition, the result shown in each table is from the test measuring the midspan deflection. The corresponding statistical repeatability of the following tests are discussed in Section 4.7.

Stiffness results are analyzed in Section 4.5, followed by failure analysis in Section 4.6.
### 4.4.1 Three Point Bending

**Table 4.10: Summary of three point bending tests - GFRP/ROHACELL 51-IG**

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Observed Stiffness (N/mm)</th>
<th>Observed Failure Type</th>
<th>Observed Failure Load (N)</th>
<th>Stiffness Prediction/Observed</th>
<th>Failure Load Prediction/Observed</th>
<th>Observed Deflection Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRA1</td>
<td>32</td>
<td>I</td>
<td>429</td>
<td>1.45</td>
<td>0.87</td>
<td>3.12</td>
</tr>
<tr>
<td>TRA2</td>
<td>35</td>
<td>CS</td>
<td>579</td>
<td>1.60</td>
<td>1.05</td>
<td>3.92</td>
</tr>
<tr>
<td>TRA3</td>
<td>69</td>
<td>CS</td>
<td>733</td>
<td>1.05</td>
<td>0.88</td>
<td>2.15</td>
</tr>
<tr>
<td>TRB1</td>
<td>809</td>
<td>CS</td>
<td>1293</td>
<td>1.68</td>
<td>0.87</td>
<td>1.74</td>
</tr>
<tr>
<td>TRB2</td>
<td>426</td>
<td>CS</td>
<td>1207</td>
<td>1.22</td>
<td>0.97</td>
<td>1.37</td>
</tr>
<tr>
<td>TRB3</td>
<td>292</td>
<td>CS</td>
<td>1486</td>
<td>1.01</td>
<td>0.80</td>
<td>1.28</td>
</tr>
<tr>
<td>TRB4</td>
<td>203</td>
<td>I</td>
<td>1311</td>
<td>0.93</td>
<td>0.93</td>
<td>1.37</td>
</tr>
<tr>
<td>TRC1</td>
<td>406</td>
<td>CS</td>
<td>745</td>
<td>3.47</td>
<td>1.55</td>
<td>3.63</td>
</tr>
<tr>
<td>TRC2</td>
<td>386</td>
<td>CS</td>
<td>762</td>
<td>1.26</td>
<td>1.25</td>
<td>1.24</td>
</tr>
<tr>
<td>TRC3</td>
<td>305</td>
<td>I</td>
<td>858</td>
<td>0.78</td>
<td>0.91</td>
<td>0.80</td>
</tr>
<tr>
<td>TRC4</td>
<td>132</td>
<td>I</td>
<td>785</td>
<td>1.05</td>
<td>0.94</td>
<td>1.27</td>
</tr>
<tr>
<td>TRD1</td>
<td>1254</td>
<td>I</td>
<td>1219</td>
<td>1.63</td>
<td>0.87</td>
<td>1.06</td>
</tr>
<tr>
<td>TRD2</td>
<td>516</td>
<td>I</td>
<td>719</td>
<td>1.40</td>
<td>1.09</td>
<td>0.76</td>
</tr>
<tr>
<td>TRD3</td>
<td>330</td>
<td>I</td>
<td>681</td>
<td>1.13</td>
<td>0.99</td>
<td>0.64</td>
</tr>
<tr>
<td>TRD4</td>
<td>207</td>
<td>I</td>
<td>699</td>
<td>1.00</td>
<td>0.86</td>
<td>0.66</td>
</tr>
</tbody>
</table>

**Legend:** CS - Core Shear, I - Indentation

**Table 4.11: Summary of three point bending tests - GFRP/EPS**

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Observed Stiffness (N/mm)</th>
<th>Observed Failure Type</th>
<th>Observed Failure Load (N)</th>
<th>Stiffness Prediction/Observed</th>
<th>Failure Load Prediction/Observed</th>
<th>Observed Deflection Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEA1</td>
<td>24</td>
<td>I</td>
<td>203</td>
<td>1.14</td>
<td>1.12</td>
<td>2.14</td>
</tr>
<tr>
<td>TEA2</td>
<td>33</td>
<td>CS</td>
<td>270</td>
<td>0.94</td>
<td>0.87</td>
<td>1.63</td>
</tr>
<tr>
<td>TEA3</td>
<td>51</td>
<td>CS</td>
<td>313</td>
<td>0.80</td>
<td>0.79</td>
<td>1.10</td>
</tr>
<tr>
<td>TEB1</td>
<td>654</td>
<td>CS</td>
<td>590</td>
<td>0.79</td>
<td>0.86</td>
<td>0.97</td>
</tr>
<tr>
<td>TEB2</td>
<td>210</td>
<td>CS</td>
<td>488</td>
<td>0.92</td>
<td>0.92</td>
<td>1.07</td>
</tr>
<tr>
<td>TEB3</td>
<td>136</td>
<td>CS</td>
<td>504</td>
<td>0.88</td>
<td>0.98</td>
<td>1.14</td>
</tr>
<tr>
<td>TEB4</td>
<td>96</td>
<td>CS</td>
<td>424</td>
<td>0.86</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>TEC1</td>
<td>205</td>
<td>CS</td>
<td>465</td>
<td>2.36</td>
<td>1.08</td>
<td>3.05</td>
</tr>
<tr>
<td>TEC2</td>
<td>167</td>
<td>I</td>
<td>361</td>
<td>1.11</td>
<td>1.17</td>
<td>1.13</td>
</tr>
<tr>
<td>TEC3</td>
<td>91</td>
<td>I</td>
<td>301</td>
<td>1.07</td>
<td>1.10</td>
<td>1.05</td>
</tr>
<tr>
<td>TEC4</td>
<td>59</td>
<td>I</td>
<td>273</td>
<td>1.08</td>
<td>1.05</td>
<td>1.09</td>
</tr>
<tr>
<td>TED1</td>
<td>896</td>
<td>CS</td>
<td>745</td>
<td>0.82</td>
<td>1.20</td>
<td>1.24</td>
</tr>
<tr>
<td>TED2</td>
<td>297</td>
<td>CS</td>
<td>617</td>
<td>0.97</td>
<td>1.05</td>
<td>1.09</td>
</tr>
<tr>
<td>TED3</td>
<td>166</td>
<td>I</td>
<td>679</td>
<td>1.00</td>
<td>1.24</td>
<td>1.32</td>
</tr>
<tr>
<td>TED4</td>
<td>108</td>
<td>I</td>
<td>539</td>
<td>1.09</td>
<td>1.19</td>
<td>1.34</td>
</tr>
</tbody>
</table>

**Legend:** CS - Core Shear, I - Indentation
### 4.4.2 Four Point Bending

#### Table 4.12: Summary of four point bending tests - GFRP/ROHACELL 51-IG

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Observed Stiffness (N/mm)</th>
<th>Observed Failure Type</th>
<th>Observed Failure Load (N)</th>
<th>Stiffness Prediction/Observed</th>
<th>Failure Load Prediction/Observed</th>
<th>Observed Deflection Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRA1</td>
<td>88</td>
<td>CS</td>
<td>779</td>
<td>1.75</td>
<td>0.81</td>
<td>2.40</td>
</tr>
<tr>
<td>FRA2</td>
<td>99</td>
<td>CS</td>
<td>843</td>
<td>1.79</td>
<td>0.75</td>
<td>2.13</td>
</tr>
<tr>
<td>FRA3</td>
<td>177</td>
<td>CS</td>
<td>1050</td>
<td>1.16</td>
<td>0.65</td>
<td>1.27</td>
</tr>
<tr>
<td>FRB1</td>
<td>1534</td>
<td>CS</td>
<td>1857</td>
<td>3.84</td>
<td>0.67</td>
<td>0.81</td>
</tr>
<tr>
<td>FRB2</td>
<td>543</td>
<td>CS</td>
<td>1784</td>
<td>1.00</td>
<td>0.67</td>
<td>0.85</td>
</tr>
<tr>
<td>FRB3</td>
<td>286</td>
<td>CS</td>
<td>1486</td>
<td>1.06</td>
<td>0.79</td>
<td>1.15</td>
</tr>
<tr>
<td>FRB4</td>
<td>158</td>
<td>CS</td>
<td>1020</td>
<td>1.25</td>
<td>1.17</td>
<td>1.64</td>
</tr>
<tr>
<td>FRC1</td>
<td>2567</td>
<td>CS</td>
<td>1074</td>
<td>1.90</td>
<td>1.01</td>
<td>0.42</td>
</tr>
<tr>
<td>FRC2</td>
<td>240</td>
<td>CS</td>
<td>1023</td>
<td>1.72</td>
<td>1.11</td>
<td>1.82</td>
</tr>
<tr>
<td>FRC3</td>
<td>173</td>
<td>I</td>
<td>1012</td>
<td>1.16</td>
<td>1.08</td>
<td>1.75</td>
</tr>
<tr>
<td>FRC4</td>
<td>88</td>
<td>A*</td>
<td>664</td>
<td>1.31</td>
<td>1.54</td>
<td>2.70</td>
</tr>
<tr>
<td>FRD1</td>
<td>1283</td>
<td>CS</td>
<td>1617</td>
<td>6.27</td>
<td>0.99</td>
<td>1.31</td>
</tr>
<tr>
<td>FRD2</td>
<td>767</td>
<td>CS</td>
<td>1551</td>
<td>1.10</td>
<td>1.13</td>
<td>0.88</td>
</tr>
<tr>
<td>FRD3</td>
<td>478</td>
<td>CS</td>
<td>1850</td>
<td>1.02</td>
<td>0.94</td>
<td>1.01</td>
</tr>
<tr>
<td>FRD4</td>
<td>299</td>
<td>CS</td>
<td>1957</td>
<td>0.95</td>
<td>0.88</td>
<td>1.12</td>
</tr>
</tbody>
</table>

#### Table 4.13: Summary of four point bending tests - GFRP/EPS

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Observed Stiffness (N/mm)</th>
<th>Observed Failure Type</th>
<th>Observed Failure Load (N)</th>
<th>Stiffness Prediction/Observed</th>
<th>Failure Load Prediction/Observed</th>
<th>Observed Deflection Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA1</td>
<td>39</td>
<td>CS</td>
<td>264</td>
<td>1.29</td>
<td>0.90</td>
<td>1.62</td>
</tr>
<tr>
<td>FEA2</td>
<td>61</td>
<td>CS</td>
<td>278</td>
<td>0.84</td>
<td>0.83</td>
<td>0.99</td>
</tr>
<tr>
<td>FEA3</td>
<td>69</td>
<td>CS</td>
<td>325</td>
<td>0.88</td>
<td>0.76</td>
<td>0.95</td>
</tr>
<tr>
<td>FEB1</td>
<td>1792</td>
<td>CS</td>
<td>1077</td>
<td>1.02</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>FEB2</td>
<td>254</td>
<td>CS</td>
<td>571</td>
<td>0.97</td>
<td>0.93</td>
<td>0.87</td>
</tr>
<tr>
<td>FEB3</td>
<td>166</td>
<td>CS</td>
<td>560</td>
<td>0.78</td>
<td>0.79</td>
<td>0.78</td>
</tr>
<tr>
<td>FEB4</td>
<td>100</td>
<td>CS</td>
<td>565</td>
<td>0.98</td>
<td>0.85</td>
<td>1.08</td>
</tr>
<tr>
<td>FEC1</td>
<td>1472</td>
<td>CS</td>
<td>586</td>
<td>1.30</td>
<td>0.70</td>
<td>0.28</td>
</tr>
<tr>
<td>FEC2</td>
<td>200</td>
<td>CS</td>
<td>455</td>
<td>1.07</td>
<td>0.97</td>
<td>0.93</td>
</tr>
<tr>
<td>FEC3</td>
<td>100</td>
<td>CS</td>
<td>446</td>
<td>1.13</td>
<td>0.95</td>
<td>1.24</td>
</tr>
<tr>
<td>FEC4</td>
<td>70</td>
<td>CS</td>
<td>414</td>
<td>1.03</td>
<td>1.05</td>
<td>1.40</td>
</tr>
<tr>
<td>FED1</td>
<td>1558</td>
<td>CS</td>
<td>672</td>
<td>1.79</td>
<td>1.00</td>
<td>0.43</td>
</tr>
<tr>
<td>FED2</td>
<td>333</td>
<td>CS</td>
<td>720</td>
<td>1.02</td>
<td>0.89</td>
<td>0.80</td>
</tr>
<tr>
<td>FED3</td>
<td>167</td>
<td>CS</td>
<td>725</td>
<td>1.18</td>
<td>0.86</td>
<td>1.10</td>
</tr>
<tr>
<td>FED4</td>
<td>171</td>
<td>CS</td>
<td>615</td>
<td>0.78</td>
<td>1.03</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Legend: CS- Core Shear, I - Indentation, A* - Adhesion Failure
### 4.4.3 Uniformly Distributed Loading

*Table 4.14: Summary of uniformly distributed loading tests - GFRP/ROHACELL 51-IG*

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Observed Stiffness (kPa/mm)</th>
<th>Observed Failure Type</th>
<th>Observed Failure Pressure (kPa)</th>
<th>Stiffness Prediction/ Observed</th>
<th>Failure Pressure Prediction/ Observed</th>
<th>Observed Deflection Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>URA1</td>
<td>9.8</td>
<td>CS</td>
<td>75</td>
<td>1.19</td>
<td>0.89</td>
<td>1.81</td>
</tr>
<tr>
<td>URA2</td>
<td>15.0</td>
<td>CS</td>
<td>98</td>
<td>0.92</td>
<td>0.72</td>
<td>1.24</td>
</tr>
<tr>
<td>URA3</td>
<td>17.5</td>
<td>CS</td>
<td>109</td>
<td>0.86</td>
<td>0.67</td>
<td>1.09</td>
</tr>
<tr>
<td>URB1</td>
<td>13.5</td>
<td>CS</td>
<td>100</td>
<td>1.20</td>
<td>0.86</td>
<td>1.69</td>
</tr>
<tr>
<td>URB2</td>
<td>18.1</td>
<td>CS</td>
<td>93</td>
<td>1.03</td>
<td>0.97</td>
<td>1.32</td>
</tr>
<tr>
<td>URB3</td>
<td>26.1</td>
<td>CS</td>
<td>149</td>
<td>0.73</td>
<td>0.59</td>
<td>0.89</td>
</tr>
<tr>
<td>URC1</td>
<td>23.1</td>
<td>CS</td>
<td>136</td>
<td>0.94</td>
<td>0.80</td>
<td>1.25</td>
</tr>
<tr>
<td>URC2</td>
<td>26.5</td>
<td>CS</td>
<td>127</td>
<td>0.92</td>
<td>0.90</td>
<td>1.13</td>
</tr>
<tr>
<td>URC3</td>
<td>26.5</td>
<td>CS</td>
<td>108</td>
<td>1.09</td>
<td>1.04</td>
<td>1.28</td>
</tr>
<tr>
<td>URD1</td>
<td>31.7</td>
<td>CS</td>
<td>139</td>
<td>0.81</td>
<td>0.91</td>
<td>1.05</td>
</tr>
<tr>
<td>URD2</td>
<td>34.7</td>
<td>CS</td>
<td>140</td>
<td>0.84</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td>URD3</td>
<td>29.6</td>
<td>CS</td>
<td>130</td>
<td>0.98</td>
<td>0.99</td>
<td>1.14</td>
</tr>
</tbody>
</table>

*Table 4.15: Summary of uniformly distributed loading tests - GFRP/EPS*

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Observed Stiffness (kPa/mm)</th>
<th>Observed Failure Type</th>
<th>Observed Failure Pressure (kPa)</th>
<th>Stiffness Prediction/ Observed</th>
<th>Failure Pressure Prediction/ Observed</th>
<th>Observed Deflection Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UEA1</td>
<td>6.7</td>
<td>CS</td>
<td>32</td>
<td>0.80</td>
<td>0.94</td>
<td>1.18</td>
</tr>
<tr>
<td>UEA2</td>
<td>8.0</td>
<td>CS</td>
<td>38</td>
<td>0.70</td>
<td>0.78</td>
<td>0.98</td>
</tr>
<tr>
<td>UEA3</td>
<td>7.8</td>
<td>CS</td>
<td>30</td>
<td>0.77</td>
<td>1.08</td>
<td>1.09</td>
</tr>
<tr>
<td>UEB1</td>
<td>7.1</td>
<td>CS</td>
<td>42</td>
<td>0.98</td>
<td>0.91</td>
<td>1.40</td>
</tr>
<tr>
<td>UEB2</td>
<td>10.1</td>
<td>CS</td>
<td>42</td>
<td>0.74</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td>UEB3</td>
<td>8.7</td>
<td>CS</td>
<td>35</td>
<td>0.90</td>
<td>1.11</td>
<td>1.19</td>
</tr>
<tr>
<td>UEC1</td>
<td>9.1</td>
<td>CS</td>
<td>37</td>
<td>1.00</td>
<td>1.31</td>
<td>1.40</td>
</tr>
<tr>
<td>UEC2</td>
<td>13.3</td>
<td>CS</td>
<td>48</td>
<td>0.70</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>UEC3</td>
<td>14.3</td>
<td>CS</td>
<td>57</td>
<td>0.68</td>
<td>0.84</td>
<td>0.89</td>
</tr>
<tr>
<td>UED1</td>
<td>11.6</td>
<td>CS</td>
<td>48</td>
<td>0.90</td>
<td>1.15</td>
<td>1.25</td>
</tr>
<tr>
<td>UED2</td>
<td>13.0</td>
<td>CS</td>
<td>57</td>
<td>0.88</td>
<td>1.01</td>
<td>1.17</td>
</tr>
<tr>
<td>UED3</td>
<td>15.1</td>
<td>CS</td>
<td>61</td>
<td>0.75</td>
<td>0.93</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Legend: CS- Core Shear, I - Indentation
4.5 Stiffness Validation

Figures 4.17 and 4.18 are three dimensional surface plots of predicted stiffness index, with respect to the \([\bar{c}, \bar{t}]\) design space. Observed stiffness results from each test are converted to stiffness index terms and are shown as diamonds; the connecting vertical lines represent the corresponding prediction-to-observed error margins. The following plots serve as a visual representation for noting trends in stiffness prediction error. For quantitative results, refer back to Tables 4.10-4.15.

![Figure 4.17: 3D comparison of observed stiffness to analytical predictions for simply supported composite sandwich beams subject to three point bending with respect to design space \([\bar{c}, \bar{t}]\) - GFRP face sheets and ROHACELL 51-IG foam core, \(\hat{\delta} = 3.0\%\)](image1)

![Figure 4.18: 3D comparison of observed stiffness to analytical predictions for simply supported composite sandwich beams subject to three point bending with respect to design space \([\bar{c}, \bar{t}]\) - GFRP face sheets and extruded polystyrene (EPS) foam core, \(\hat{\delta} = 3.0\%\)](image2)
Considering the effect of $\bar{t}$, beams of thin face sheets (Test Set C) were well predicted for both material cases. As thicker faces were tested for GFRP/EPS beams (tests TEA3, TEB1-TEB4), stiffness predictions were shown to be smaller than experiment results. As noted in Chapter 2, equivalent bending and shear stiffness terms were originally simplified assuming the selected face sheets were relatively thin to the core material ($d \approx c$). For tests TEA3-TEB4, if the contribution of the facesheets were included, predictions would have been improved by an average of $\sim 8\%$. Comparatively, GFRP/ROHACELL 51-IG tests of similar face sheet thickness were well-predicted, so considering the contribution the face sheet was not required.

Considering the effect of $\bar{c}$, large errors were shown to occur when either compliant or short beams were tested. Recalling the shear force diagram for three point bending in Figure 2.3, the change in transmitted shear load at the midspan is assumed to occur over an infinitely small length. As shorter beams are tested, the change was much more apparent. However, as stubby beams were tested (in this case, $L=80\text{mm}$), the maximum intended shear load of $P/2$ was never attained, resulting in indentation at the side supports. As the added deflection did not represent either bending or shear effects, analytical models became unreliable.

For the case of long beams, tests TRA1 and TRA2 were shown to drastically over-predict stiffness. The following error is presumed to occur due to beam sliding, where beyond a given compliance, the applied load simply pushed the beam through the side supports. The resulting error in stiffness prediction was not solely attributed to the analytical expressions, but due a notable flaw that applies with simply supported beams.

Next, the stiffness results from all four point bending tests are detailed. Due to four point bending tests having a varied $\bar{a}$ term, predicted stiffness results could not be visually represented as a 3D mesh. Alternatively, Figures 4.19 and 4.20 compare stiffness index results in terms of prediction-to-observed ratios.

![Figure 4.19: 3D comparison of observed stiffness to analytical predictions for simply supported composite sandwich beams subject to four point bending with respect to design space $[\bar{c}, \bar{t}]$ - GFRP face sheets and ROHACELL 51-IG foam core, $\delta = 3.0\%$](image)
Figure 4.20: 3D comparison of observed stiffness to analytical predictions for simply supported composite sandwich beams subject to four point bending with respect to design space $[\bar{c}, \bar{t}]$ - GFRP face sheets and extruded polystyrene (EPS) foam core, $\hat{\delta} = 3.0\%$

Similar to three point bending, FRA1, FRA2 and FEA1 all had large errors in stiffness prediction due to the occurrence of beam sliding, short beams (for this case $L=100\text{mm}$) generally over-predicted stiffness, and beams comprised of thick face sheets and EPS cores under-predicted stiffness results due to simplifications made in equivalent bending and shear stiffness terms.

Finally, Figures 4.21 and 4.22 compare stiffness experiment results for uniformly distributed loading tests, simulated by the whippletree loading test fixture.

Figure 4.21: 3D comparison of observed stiffness to analytical predictions for simply supported composite sandwich beams subject to uniformly distributed loading with respect to design space $[\bar{c}, \bar{t}]$ - GFRP face sheets and ROHACELL 51-IG foam core, $\hat{\delta} = 3.0\%$
Despite being simulated by a whippletree load, pressure loading tests for both core considerations were well-predicted using pressure load equations. Any significant increase in error seen in Figures 4.22 and 4.23 was accounted to higher $\bar{t}$ tested (i.e. thicker face sheets). Though different core thicknesses were tested over the fixed beam length, the change in stiffness prediction error had no significant trend, as the cores were all relatively thick.

For three and four point bending tests, GFRP/ROHACELL 51-IG beams were generally well-predicted in stiffness as GFRP face sheets of 2.04mm were considered. Only during uniform loading tests were the first occurrences of stiffness under-prediction noted. As URA3 and URB3 were comprised of the thickest considered face sheets ($t=2.21$mm), it was determined that thicker faces were required to show a dominance in stiffness behaviour for beams comprised of rigid cores, compared to more compliant cores.

In summary, by comparing stiffness results of three point bending, four point bending and uniform loading tests (as simulated by whippletree loading), similar trends in accuracy were noted. Though the analytical models generally predicted stiffness within a $\sim 15\%$ error for most beam designs, they became unreliable as either short or compliant beams were considered. As well, when beams comprised thicker face sheets were tested, relative to the stiffness contribution of the core, results were under-predicted and the full terms were required.
4.6 Strength Validation

The following section analyzes failure prediction results for all loading cases. Core shear and indentation failures were identified using experiment results, then compared to predictions in terms of the failure mechanism map $[\bar{c} - \bar{t}]$ design space.

4.6.1 Core Shear

Identifying failure in terms of three point bending tests, Figure 4.23 compares ROHACELL 51-IG and EPS beams failing by core shear.

![Figure 4.23: Observed core shear failure from simply supported three point bending tests - GFRP/ROHACELL 51-IG (left), GFRP/EPS (right)](image)

Figure 4.24 compares stiffness and strength predictions for two beams failing by core shear, with respect to force-deflection curves.

![Figure 4.24: Force-deflection curves from simply supported three point bending tests, failing by core shear - results from TRA3 and TEB2](image)

For beams comprised of ROHACELL 51-IG, core shear failure was identified by an instantaneous fracture of the core. In Figure 4.24, failure was seen as an oblique crack across the core and (in some cases) possible separation from the face sheet.
For EPS cores, core shear was harder to identify as the beam continued to sustain load even after failure (seen by Figure 4.25). After EPS yielded in shear, the beam compressed, resembling the behaviour (and visual appearance) of a more compliant beam. As failure for EPS beams was defined as the shear yield stress, core shear failure was assumed to occur at the transition point between the elastic and plastic (or compliant) states. The considered 0.2% strain offset was the midspan deflection over total beam length.

Figure 4.25 and 4.26 shows the pressure-deflection curves for four point bending and uniformly distributed loading respectively, both yielding similar results.

As shown in Figure 4.24-4.26, beams comprised of GFRP/EPS greatly benefit from the use of the overhang factor term for core shear failure prediction. Comparing core shear failure results in Tables 4.10-4.15, most experiments over-predicted core shear failure. As most of these tests comprised of thick face sheets, however, prediction error could have been reduced if the face sheet stiffening property was considered from Equation (2.9). In terms of three point bending tests, EPS beams benefited most from the stiffening property, as predictions were improved by $\sim 2\%$. As ROHACELL was a stronger core material relative to the face sheet, the improvement was less significant ($< 1\%$). Similar results were observed from the other two loading cases.
Using results from core shear strain data, Figures 4.27 compares force-core shear strain plots for GFRP/ROHACELL 51-IG tests and GFRP/EPS tests respectively.

Noted earlier, core shear strain was measured 40mm from the side supports for each test. Though the results shown above are slightly varied per each loading case (based on different beam lengths used), core shear failure for ROHACELL 51-IG was observed at $\sim 5\%$ strain while EPS yielded at the 0.2% proof strain. As mentioned in the figure, interference between the track and core shear clips was observed during the initial movement of the whippletree point load rollers.

In summary, for composite sandwich beams, core shear failure were best predicted using core shear strength for ROHACELL 51-IG and core shear yield stress for extruded polystyrene (EPS).
4.6.2 Indentation

Figure 4.28 compares visual identification of indentation failure for both GFRP/ROHACELL 51-IG and GFRP/EPS beams, while Figures 4.29 and 4.30 compare the respective force-deflection curves for three and four point bending cases.

---

Figure 4.28: Observed indentation failure from simply supported three point bending tests - GFRP/ROHACELL 51-IG (left), GFRP/EPS (right)

---

Figure 4.29: Force-displacement curves from simply supported three point bending tests, failing by indentation - results from TRD3 and TEC4

---

Figure 4.30: Force-displacement curves from simply supported four point bending tests, failing by indentation - results from FRC3
For both core materials, drop-off in load-carrying capacity was associated with an observed plastic collapse of the core. Though multiple occurrences of indentation failure were observed during three point bending tests, only one indentation failure was observed during the four point bending test series: FRC3. Though FRC3 was predicted to fail by core shear, the difference between core shear and indentation failure loads was $\sim 2\%$. As the difference between failure load predictions was relatively small, it was determined that the active failure mechanism became exponentially harder to predict as beams were designed close to failure region boundary lines.

Figure 4.31 compares indentation failure between three and four point bending tests in terms of force-core indentation curves.

![Figure 4.31 Force-core indentation curve plots - TRC4, TEC4 and FRC3 failing by indentation](image)

From the above plot, core indentation results for long beams are compared ($320\text{mm} \leq L \leq 440$). Similar to results shown in core compression tests, both ROHACELL 51-IG and EPS cores plastically collapse at a critical plateau stress.

In summary, core indentation was observed as a drop-off in load-carrying capacity, associated with the plastic collapse of the core. By observing the results in Tables 4.10-4.15, experiments were well-predicted using the derived expressions for three and four point bending cases.
4.6.3 Microbuckling

Figure 4.32 compares force-axial strain plots for GFRP/ROHACELL 51-IG and GFRP/EPS beams respectively.

Though microbuckling did not occur in any test, it is still worth noting the effect of the face sheet in response core shear and indentation failures. In comparing several test results, face sheet axial strain did not exceed 1.7% strain in any test, as was assumed compressive and tensile failure strains were relatively similar in size [9]. After core shear failure for ROHACELL beams, the observed recovery in axial strain was instantaneous, while for EPS core shear, face sheet axial strain continued to increase until the test stopped. For indentation failure, the formation of a localized indentation region resulted in a decrease in compressive strain for TRA1. For TEA1 and FRA1, rapid progression in axial strain was noted just prior to their respective indentation and core shear failures. As the predicted microbuckling failure for TEA1 and FRA1 were both well beyond the observed failure load, the rise in axial strain was presumed to be an aftereffect of beams sliding, whereby the increase in beam length momentarily increased the compressive stress on the face sheet before failure occurred.
4.6.4 Adhesion Failure

During experiments, tests from design point FRC4 failed well below the predicted indentation failure load. Though multiple iterations of the same beam design were run, similar results were observed: a localized buckling and separation of the top face sheet from the core at the midspan of the beam. Although adhesion failure was disregarded during the early stages of the study, buckling-induced adhesion failure occurred during the conducted experiments. For FRC4, the required load for indentation failure was twice that to the three point bending case. As the beam was comprised of the thinnest considered face sheet thickness, any gaps evident between the face and core due to fabrication faults became a critical stress point for delamination, regardless of the adhesive strength considered. Possible reasoning for bond gaps are due to air bubbles being trapped in-between the face and core prior to curing. Therefore, as the demand for maximum load capacity increases for beams comprised of thin face sheets, it is important to not only consider a strong adhesive for beam fabrication, but to also ensure a consistent bond exists between the face and core components.

4.6.5 Results Summary - 3D Failure Mechanism Mapping

Using the failure mechanism map as the x-y plane, a three-dimensional visual representation of failure load results are plotted. As failure mechanism maps are plotted in terms of non-dimensional load and pressure indices in Chapters 2 and 3, results are converted to $\hat{P}$ and $\hat{w}$ terms respectively. For three point bending tests, Figures 4.33 shows results GFRP/ROHACELL 51-IG beams and Figure 4.34 refers to GFRP/EPS beams. Core shear predictions for EPS including overhang are denoted by squares.

![Figure 4.33: Comparison of observed failure load index to analytical predictions for simply supported composite sandwich beams subject to three point bending, represented by a 3D failure mechanism map - GFRP face sheets and ROHACELL 51-IG foam core, $\delta = 3.0\%$.](image-url)
For both core shear and indentation failures, the most significant errors were observed in all short beam tests. Recalling stiffness results, short beams experienced side support indentation due to a loss of shear force transmitted from the applied load. As such, short beam experiments generally under-performed against predicted failure loads. Though including the overhang factor for EPS beams greatly improved core shear prediction for longer beams, it was difficult to justify it as a contributing factor of error for any short beam test due to the observed side support indentation.

For core shear failure load predictions, most tests were well-predicted. As mentioned earlier, the stiffening property for thick face sheets improved the predictions of GFRP/EPS tests, however as ROHACELL was a stronger core relative to the contribution of the face sheet, the improvement was negligible.

In comparing all indentation failure predictions for three point bending, results show small error margins for all tests with the exception of short beams.

Though compliant beams TEA1, TRA1 and TRA2 under-performed in stiffness due to beam sliding, the corresponding observed failure loads were surprisingly close to their predictions. As core shear was neither dependent on deflection or beam length, failure prediction did not change on account of beam sliding. For indentation failure, it was noted that with a small increase in beam length, the change in predicted failure load was insignificant, so the increase in error was minimal.

For four point bending tests, Figure 4.35 plots failure load results for GFRP/ROHACELL 51-IG tests, while Figure 4.36 plots failure load results for GFRP/EPS tests. Similar to the stiffness results, failure load prediction results are in terms of prediction-observed ratios, due to a varied \( \hat{a} \) term.
Comparing four point bending test results in terms of failure load, similar observations were noted from the three point bending case. As most tests failed by core shear, a general under-prediction of core shear was due to not accounting for the stiffening property of thick face sheets. Short beams tests of \( L=100\text{mm} \) over-predicted experiment results as well.

In addition to each respective failure region, it is also important to note the accuracy of the predicted core shear/indentation failure region boundary for both three and four point bending cases. During experiments, three tests predicted to fail by indentation, failed by core shear (TRA2, TRC2 and TEC1) while two core shear tests failed by indentation (TRB4 and FRC3). Similar to the results of FRC3, the
average difference between predicted failure loads along the core shear/indentation boundary line was small (< 6%), except for short and compliant beams). As prediction error could have been due to a list of possible test specimen impurities, it was determined that the reality of predicting the active failure mechanism was difficult as design points existed close to a failure region boundary line.

For the final set of 3D failure region mechanism maps, Figures 4.37 and 4.38 compare failure pressure index results from the completed uniformly distributed loading tests.

Figure 4.37: Comparison of observed failure pressure index to analytical predictions for simply supported composite sandwich beams subject to uniformly distributed loading, represented by a 3D failure mechanism map - GFRP face sheets and ROHACELL 51-IG foam core, $\delta = 3.0\%$

Figure 4.38: Comparison of observed failure pressure index to analytical predictions for simply supported composite sandwich beams subject to uniformly distributed loading, represented by a 3D failure mechanism map - GFRP face sheets and extruded polystyrene (EPS) foam core, $\delta = 3.0\%$
After conducting whippletree loading tests for GFRP/ROHACELL 51-IG beams, a general under-prediction was observed in results compared to the point loading cases. Though the average error in simulated failure pressure prediction was $\sim 13.5\%$, select GFRP/ROHACELL 51-IG beams outperformed predictions in excess of $\sim 20 – 30\%$, a relatively large margin of error compared to point load tests of equal beam length.

Recalling the shear force diagram of pressure loading in Figure 3.15, the transmitted shear is predicted as a linear relationship. However, in terms of a constantly moving whippletree load (due to the splay-out motion of the point load rollers), the transmitted shear of whippletree loading is best represented as a piecewise function. At a given deflection and position on along the beam, the transmitted shear of the whippletree load may have been slightly above or below that of a uniform pressure. Noting that ROHACELL 51-IG fractures in shear within a localized region, the general under-prediction in core shear failure prediction was due to the corresponding difference in shear loading models. For the case of elastic-brittle cores, a higher load was required from the whippletree loading test fixture to promote core shear failure compared to what would have been observed from a uniform pressure.

However, as beams comprised of the elastic-plastic EPS distributed shear strain across the entire beam length, differences in transmitted shear load between the two loading cases became irrelevant. Therefore, core shear failures were better-predicted for EPS beams during whippletree loading tests using the core shear failure pressure equation for uniform pressures.

### 4.7 Summary of Experiments

To summarize core shear failure load prediction results, Figure 4.39 compares observed load indices to the predicted failure line for core shear, with respect of non-dimensional beam geometry term $(\bar{t} + 1)\bar{c}$, peak load index $\hat{P}$ and peak pressure index $\hat{w}$. Each test point is identified by its the respective loading and material case; the predicted failure for EPS with overhang is noted by the dot.

![Figure 4.39: Comparison of observed and predicted load index values for core shear failure in terms of non-dimensional beam geometry term related to the core shear failure equation](image)
Figure 4.40 shows all observed indentation failures with respect to the beam geometry term of the indentation failure equation and load index.

![Graph showing observed vs predicted load index values for indentation failure](image)

**Figure 4.40: Comparison of observed and predicted load index values for indentation failure in terms of non-dimensional beam geometry term related to indentation failure equations**

During three and four point bending experiments, the best prediction-to-observed stiffness and strength results were within a 5% error margin, and generally came from long beams of thin face sheets relative to the core (represented by small $t$ and $c$ beam geometry terms in Figures 4.39-4.40). When additional tests were conducted to measure core indentation, it was determined that the observed failure load only differed by, at most, by $\sim$ 5% from the initial test. All other tests were noted to either have a larger error margin or a larger difference in observed failure load during subsequent tests. Although beams of thicker cores were shown to under-predict both stiffness and strength results on average of 15% (any repeated tests for the sake of core indentation were within, at most, 10% from their initial test), the inclusion of full stiffness terms and a face sheet stiffening property for core shear prediction improved results to within $\sim$ 10% error. Therefore, as the prediction-to-observed results were shown to be similar to the three point bending case, the analytical models for four point bending were valid approximations for stiffness and strength, considering that error margins for most tests did not exceed $\sim$ 15%.

However, the following conclusions exclude both short and compliant beams as error margins were significantly large. Noting that the difference between repeated tests (for the sake of core indentation) were in excess of 20%, analytical predictions were unreliable for cases of short and overly compliant beams due to the added error from side support indentation and beam sliding respectively.

Though the core shear-indentation boundary lines could have been further examined during experiments, an expansion of tests was not carried out. As multiple occurrences of incorrect failure identification occurred during both three and four point bending tests, predicting failure within the close proximity of a boundary line was confirmed to be generally ineffective.

For the case of uniformly distributed loading, though stiffness predictions were shown to be (on average) within $\sim$ 15% difference to experiment results, it is worth repeating that the following experiments were an attempt to simulate pressure loading. The corresponding results for both stiffness and strength relate
to the whippletree test fixture, not an ideal pressure load. However, as EPS cores were observed to
distribute shear strain over the length of the core material (similar to the three and four point bend-
ing cases), the marginal differences between whippletree loading and uniform pressure models became
irrelevant. Based on the test results of the elastic-plastic EPS core material, test results suggest that
uniform pressure analytical models would predict with the same 15% error margin if an actual pressure
load was applied.

As a final mention, minimum mass trajectories for three point bending, four point bending and uniformly
distributed loading were investigated. By comparing the observed mass and load indices from all con-
ducted experiments to the minimum mass trajectories of each respective loading case, it was confirmed
that only the designs existing on the minimum mass trajectory were the lightest structures for a given
design load.

\subsection{Stiffness Constraint Validation}

Throughout the following experiments, a stiffness limit was applied to each loading model; the goal
being to avoid testing beams that deflected beyond the limits of the test frame. Of the 84 design points
tested, 4 beam designs exceeded $\delta = 3\%$: TRA1, TRA2, TRC1, and TEC1. From all tests, the primary
reasoning for poor deflection prediction was not based on the derived stiffness expressions, but instead
due anomalies that occurred in stubby and compliant beam tests. Though stiffness predictions for simply
supported beams became unreliable due to the beam sliding, the proposed stiffness constraint was proven
to be a valid testing method for the following study, considering stiffness prediction error for most beams
was within $\sim 15\%$. 
Chapter 5

Conclusion and Future Recommendations

Utilizing previous works in sandwich beam theory, strength and stiffness models for two new load cases were investigated: four point bending and uniformly distributed loading. To validate the new models, an extensive experiment phase was conducted, involving the design, fabrication and testing of 84 different test points, spanning across three different loading cases, with beams comprised of GFRP face sheets and two different core materials. To simulate uniformly distributed loading, a whippletree loading fixture was also introduced.

After comparing analytical predictions to experiment results, it was found that stiffness and strength models for both four point bending and uniformly distributed loading cases predicted experiment results within 15% error, despite anomalies that occurred in certain beam geometry cases. For short beams, shear force transmitted by the applied load was lost, resulting in side support indentation. As the accompanying deflection did not represent either bending or shear effects, stiffness prediction was unreliable. When compliant beams were tested, beam sliding was evident (attributed to the beams being simply supported).

During testing, stiffness and strength expressions were simplified, assuming the face sheet thickness of all conducted tests was small relative to the core, in addition to, assuming the cores were compliant. However, as beams comprised thicker face sheets or rigid cores were tested, results were under-predicted and the full terms were required.

By introducing a stiffness constraint to analytical models, eligible beam designs that were deemed overly compliant were removed from the considered design space. As design space regions were omitted, most beams predicted to fail by microbuckling were removed as well. During experiments, the same stiffness constraint was used to omit beam designs that were projected to deflect beyond the capabilities of the testing area. Of the 84 design points that were built around the implemented constraint, only 3 exceeded the set deflection ratio limit. The observed discrepancies were from overly compliant and short beams. Although the simply supported beams were prone to sliding, the implemented stiffness limit was proven to be a valid testing method, given the accuracy of the stiffness predictions.
For uniformly distributed load experiments, a whippletree loading testing fixture was constructed to simulate pressure loading. Though experiments were shown to correlate well with stiffness predictions, noticeable error margins were observed during failure analysis of GFRP/ROHACELL 51-IG beams. As ROHACELL 51-IG core shear occurred within a localized region (as opposed to EPS where shear yielding occurs over the entire beam length), the subtle differences in shear force distribution were more apparent. Though the following predictions were not compared directly to ideal pressure loading tests, results from the GFRP/EPS tests suggest that similar error margins would have been observed had an actual pressure load been used.

While deriving analytical models for all loading cases, buckling-induced adhesion failure was never considered. However, during four point bending tests, a single design point produced multiple occurrences of buckling-induced adhesion failure. Although beams were constructed with an strong adhesive, the underlying cause for the observed adhesion failure was accounted to flaws in fabrication, whereby air trapped in between the face sheet and core during curing lead to the development of bond gaps. Therefore, to prevent adhesion failure from occurring, beams are recommended to be fabricated with an adhesive of considerable strength compared to the face and core components, in addition to, employing fabrication techniques that prevent bond gaps (ie. vacuum bagging).

As stiffness and strength expressions for uniformly distributed loading have been introduced, further research is recommended for composite sandwich research, including beams comprised metal face sheets and metal foam, truss or honeycomb cores. To perform uniform loading experiments, several modifications are recommended for the whippletree loading fixture design, including a higher number of point load rollers and making the unit fully adjustable for different beam lengths. A final consideration for future research would be to investigate continuous (or fixed-end) composite sandwiches composed of fibre-reinforced polymer faces and polymer foam cores. By determining stiffness and strength expressions for continuous beams, composite sandwich panels supported by wireframe structures may be properly analyzed.

In conclusion, by providing new analytical models for stiffness and strength prediction, more complex loading cases may be considered, allowing for failure analysis and mass reduction methods to further improve structural performance.


