The Architectural Optimization of Stretch-Formed Ceramic-Aluminum Microtruss Composites

by

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A thesis submitted in conformity with the requirements for the degree of Master’s of Applied Science
Department of Materials Science and Engineering
University of Toronto

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Abstract

Microtruss cellular materials have large internal surface areas and small cross-sectional strut dimensions, permitting surface modification to substantially enhance their mechanical performance. For instance, a ~400% increase in compressive strength with virtually no weight penalty can be induced by a hard anodized Al₂O₃ ceramic coating of only ~50 µm thickness. The present study seeks the optimal architecture of these composites by exploring three research challenges: architecture and degree of forming are interdependent due to stretch-forming, architecture and the material properties are interdependent due to work-hardening, and ceramic structural coatings add design complexity. Theoretical predictions and architectural optimizations demonstrated a potential weight reduction of ~3% to ~60% through the increase of internal truss angle for both annealed and work-hardened microtruss cores. While further validation is needed, experimental evidence in this study suggested the collapse in ceramic-aluminum microtruss composites could be considered as a mixture of composite strut global buckling and oxide local shell buckling mechanisms.
Acknowledgments

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<th>Description</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\sigma_{ys}$</td>
<td>Yield strength</td>
</tr>
<tr>
<td>$\sigma_{UTS}$</td>
<td>Ultimate tensile strength</td>
</tr>
<tr>
<td>$\sigma_{CR}$</td>
<td>Critical collapse strength or buckling strength of a column or strut</td>
</tr>
<tr>
<td>$\sigma_{MT}$</td>
<td>Theoretical compressive strength of a microtruss</td>
</tr>
<tr>
<td>$\sigma_{peak}$</td>
<td>Peak strength from a microtruss compressive stress-strain curve</td>
</tr>
<tr>
<td>$\sigma_{valley}$</td>
<td>Valley strength from a microtruss compressive stress-strain curve</td>
</tr>
<tr>
<td>$\sigma_{ref}$</td>
<td>Reference strength of a solid metal</td>
</tr>
<tr>
<td>$\rho_{MT}$</td>
<td>Density of a microtruss</td>
</tr>
<tr>
<td>$\rho_S$</td>
<td>Density of the sleeve</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>Density of the core</td>
</tr>
<tr>
<td>$\rho_{ref}$</td>
<td>Reference density of a solid metal</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$\varepsilon_{true}^{pl}$</td>
<td>True plastic strain</td>
</tr>
<tr>
<td>$K$</td>
<td>Hollomon work-hardening coefficient</td>
</tr>
<tr>
<td>$n$</td>
<td>Hollomon work-hardening exponent</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic modulus (also known as Young’s modulus)</td>
</tr>
<tr>
<td>$E_S$</td>
<td>Elastic modulus of the sleeve</td>
</tr>
<tr>
<td>$E_C$</td>
<td>Elastic modulus of the core</td>
</tr>
<tr>
<td>$E_T$</td>
<td>Tangent modulus</td>
</tr>
<tr>
<td>$E_{T,S}$</td>
<td>Tangent modulus of the sleeve</td>
</tr>
<tr>
<td>$E_{T,C}$</td>
<td>Tangent modulus of the core</td>
</tr>
<tr>
<td>$k$</td>
<td>Nodal rigidity constant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$L/r$</td>
<td>Slenderness ratio</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of gyration</td>
</tr>
<tr>
<td>$l$</td>
<td>Second moment of area (also known as moment of inertia)</td>
</tr>
<tr>
<td>$I_S$</td>
<td>Second moment of area of the sleeve</td>
</tr>
<tr>
<td>$I_C$</td>
<td>Second moment of area of the core</td>
</tr>
<tr>
<td>$A$</td>
<td>Cross-sectional area of a column</td>
</tr>
<tr>
<td>$A_S$</td>
<td>Cross-sectional area of the sleeve</td>
</tr>
<tr>
<td>$A_C$</td>
<td>Cross-sectional area of the core</td>
</tr>
<tr>
<td>$A_{Strut}$</td>
<td>Cross-sectional area of a strut</td>
</tr>
<tr>
<td>$A_{Cell}$</td>
<td>Projected area of a unit cell</td>
</tr>
<tr>
<td>$N_{strut}$</td>
<td>Number of struts per unit cell of the starting sheet</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of a column</td>
</tr>
<tr>
<td>$l$</td>
<td>Initial length of an internal strut before stretch-forming</td>
</tr>
<tr>
<td>$l_f$</td>
<td>Final formed length of an internal strut</td>
</tr>
<tr>
<td>$w$</td>
<td>Initial width of an internal strut before stretch-forming</td>
</tr>
<tr>
<td>$w_f$</td>
<td>Final formed width of an internal strut</td>
</tr>
<tr>
<td>$t$</td>
<td>Initial thickness of an internal strut before stretch-forming</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Final formed thickness of an internal strut</td>
</tr>
<tr>
<td>$s$</td>
<td>Anodic alumina coating thickness</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Microtruss internal truss angle</td>
</tr>
<tr>
<td>$\theta^\alpha_{max}$</td>
<td>Truss angle at which compressive strength is maximized</td>
</tr>
<tr>
<td>$\theta^F_{max}$</td>
<td>Truss angle at which $F$ is maximized ($\theta^F_{max} = \theta^\alpha_{max}$)</td>
</tr>
<tr>
<td>$\theta^M_{min}$</td>
<td>Optimal truss angle based on minimum mass approach</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>Coefficient representing starting metal sheet dimension</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Weight loss factor due to surface cleaning prior anodizing</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Anodic coating volume expansion factor</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>Coefficient representing normalized starting metal sheet dimension</td>
</tr>
<tr>
<td>$\bar{t}$</td>
<td>Non-dimensional starting strut thickness to length ratio</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>Non-dimensional coating thickness to starting strut thickness ratio</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>Non-dimensional weight</td>
</tr>
<tr>
<td>$\bar{F}$</td>
<td>Non-dimensional strength</td>
</tr>
<tr>
<td>$\bar{F}_{\text{elastic}}$</td>
<td>Non-dimensional strength based on elastic buckling assumption</td>
</tr>
<tr>
<td>$\bar{F}_{\text{plastic}}$</td>
<td>Non-dimensional strength based on plastic yielding assumption</td>
</tr>
<tr>
<td>$\bar{V}_s$</td>
<td>Volume fraction of the sleeve</td>
</tr>
<tr>
<td>$\bar{V}_c$</td>
<td>Volume fraction of the core</td>
</tr>
<tr>
<td>$\bar{V}_{\text{tot}}$</td>
<td>Total volume fraction of the metal solid in a microtruss</td>
</tr>
<tr>
<td>$W_{\text{global}}$</td>
<td>Relative weight of composite strut global buckling mechanism</td>
</tr>
<tr>
<td>$W_{\text{local}}$</td>
<td>Relative weight of oxide local shell buckling mechanism</td>
</tr>
<tr>
<td>$S_C$</td>
<td>Empirically fitted critical coating thickness</td>
</tr>
<tr>
<td>$\theta_C$</td>
<td>Empirically fitted critical truss angle</td>
</tr>
</tbody>
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1 Introduction

Cellular materials are hybrids of solid material and open space (air) [1]. Adjusting the proportion of open space to solid introduces a range of material property and density combinations. These cellular materials can, therefore, reach the low-density region in the material property space (see Figure 1.1), which would otherwise be unattainable by fully-dense materials [1, 2]. As a result, they are an attractive option for weight-limited load bearing [3-8] and impact energy absorption [9-12] applications in the automotive, marine, and aerospace industries. Current investigated examples such as: flooring materials, cargo ship structures, deployable energy absorbers, and jet blast deflectors are illustrated in Figure 1.2 [13-15]. Aside from mechanical applications, microtruss materials can also have multifunctional characteristics such as enhanced heat dissipation [16, 17] and pressure and fluid flow control [16].

![Figure 1.1 Material property chart of Young's modulus versus density](image)

*Figure 1.1 Material property chart of Young’s modulus versus density [2].*
Figure 1.2 Current research examples of cellular materials in automotive, marine, and aerospace industries: flooring materials (a), cargo ship structure (b), deployable energy absorber (c), and jet blast deflector (d); adapted from [13-15].
There are several ways to improve the mechanical performance of cellular materials. This study focuses on three strategies: the control of internal architecture, the choice of fabrication processes, and the enhancement of creating a composite.

First, several early studies focused on designing the internal architecture of these cellular structures. The two primary classes of these cellular structures are conventional stochastic metal foams and periodic microtruss cellular materials (see Figure 1.3) [3, 18]. The fundamental difference between metal foams and microtrusses lies in the internal connectivity (rather than uniformity) of their individual unit cells.

![Figure 1.3 An example of conventional open cell stochastic metal foams (a) and tetrahedral periodic microtruss cellular materials (b) [19, 20].](image)

To illustrate their difference, let us treat both foams and microtrusses as 3D space frames. According to Maxwell’s stability criterion, in order for a 3D pin-jointed frame to maintain a static position, each internal strut/beam ($b$) requires six equations of motion ($\Sigma F_{x,y,z} = 0; \Sigma M_{x,y,z} = 0$), while each connection/joint ($j$) can only provide three equations ($\Sigma F_{x,y,z} = 0$) [21]. Therefore, static equilibrium can only be achieved when the combination of struts and connections satisfies:

$$b \geq 3j - 6 \quad \text{or} \quad b - 3j + 6 \geq 0 \quad (1.1)$$
Figure 1.4 illustrates some examples of polyhedral unit cells with labels ‘yes’ or ‘no’ indicating whether they satisfy Maxwell’s stability criterion. The unit cells with freely rotatable (pin-joint) connections would collapse under an applied load if they do not satisfy this criterion. Stochastic metal foams have an internal strut connectivity (the average node-to-strut ratio) of 3~4 with unit cell structure similar to a 14-sided tetrakaidecahedron (unit cell number 8 in Figure 1.4) [18]. As a result, metal foams do not satisfy Maxwell’s criterion. Since the internal connection of metal foams cannot freely rotate, their internal ligaments would fail through bending deformation, and conventional metal foams are described as bending-dominated structures.

On the other hand, periodic microtrusses have an internal strut connectivity of 12 with a 1:1 ratio of tetrahedral and pyramidal unit cells (unit cell number 1 and 5 in Figure 1.4), which both satisfy Maxwell’s criterion [18]. As a result, externally applied loads in the microtruss are resolved through axial deformation and microtrusses are described as stretch-dominated structures. Due to this difference in failure mechanisms, microtrusses are more structurally stable than metal foams. At a relative density of 10%, microtruss materials can be three times as strong and ten times as stiff as conventional metal foams (see Figure 1.1, for more details, please refer to Refs.[1, 18]).
Second, the mechanical performance can also be increased by considering the metallurgical state of the material during fabrication. Microtruss materials can be fabricated using techniques such as investment casting [4, 5, 22-24], textile lay-up, rapid prototyping [6, 25, 26], and sheet-forming [27-31] (see reviews by Wadley [20] and Mines [32]). Among these, stretch-forming (a subset of sheet-forming) is a compelling approach since it relies only on simple sheet forming methods [27], and it provides additional work-hardening effects to the microtruss struts during fabrication.

Third, it is also possible to strengthen microtruss materials through surface modification because of their large internal surface areas combined with small strut cross-sections. For instance, electrodeposition of a ~50 µm thick coating of ultra-high strength nanocrystalline nickel was found to double and triple the compressive strength of low carbon steel and aluminum microtrusses, respectively [7, 8, 33]. Unfortunately, the relatively high density of the nanocrystalline coating means that a weight penalty is also added to the microtruss core. On the other hand, applying an anodic Al₂O₃ ceramic coating onto AA3003 aluminum microtrusses can provide a similar strengthening effect with virtually no additional weight penalty [34]. While it has been shown that ceramic structural coatings can provide promising strength enhancement, little is known about the failure mechanisms of these ceramic-aluminum composites and their optimization has yet to be studied.

Based on the aforementioned, there is a need to optimize the combined effects of microtruss architecture, work-hardening imported during stretch-forming and structural coating reinforcement. This research contributes to an enhanced understanding of stretch-formed microtrusses and structurally-coated microtruss composites. Architectural optimization permits the design of weight-efficient sandwich beams that have excellent strength and/or energy absorption performance, which would be invaluable in automotive, marine, and aerospace applications.

Hiu Ming (Bosco) Yu
In this document, Chapter 2 provides background information on microtrusses and microtruss composites, while Chapter 3 describes the experimental and analytical methods used in this study. Chapter 4 presents the results and discussion of architectural optimization. Chapter 5 describes potential future work. Chapter 6 concludes the findings of this study.

In optimizing ceramic-aluminum microtruss composites, three challenges will be faced. First, during stretch-forming, internal struts are subjected to tensile elongation and cross-sectional thinning, making architecture and degree of forming interdependent. Second, work-hardening from stretch-forming complicates the analysis since architecture and the material properties of the struts are also interdependent. Finally, ceramic structural coatings introduce another degree of freedom to this architectural design space. Due to the aforementioned challenges, Chapter 4 is divided into three main objectives: the optimization of stretch-formed annealed microtruss cores, stretch-formed work-hardened microtruss cores, and stretch-formed ceramic-aluminum microtruss composites.
2 Background Information

This chapter provides reviews literature concerning microtruss cores and microtruss composites. Section 2.1 compares the different fabrication techniques of microtruss cores. Section 2.2 reviews the research conducted to date in microtruss composites. Section 2.3 provides a review for anodizing, which is used in fabricating ceramic-aluminum microtruss composites. Section 2.4 presents other possible ceramic-based structural coating options.

2.1 Fabrication of Conventional Microtruss Cores

Conventional microtruss cores can be fabricated using techniques such as investment casting [4, 5, 22-24], textile lay-up, rapid prototyping [6, 25, 26], and stretch-forming [27-31] (see reviews by Wadley [20] and Mines [32]). The following subsections describe each of these fabrication processes in detail.
2.1.1 **Investment Casting**

Investment casting is a promising process used to fabricate complex structures such as microtruss cores [4, 5, 22-24]. In this process, ceramic shells are first coated on polymeric microtruss templates; molten metal is then run into the shell, burning off and displaces the polymer cores (see Figure 2.1a). Investment casting allows the formation of microtrusses without layer-by-layer joining methods, in order to construct almost perfect internal connections. Unfortunately, only highly castable alloys are suitable for this process [4, 5, 22-24]. Also, cast microtrusses tend to be relatively heavy, since low relative densities tend to result in the formation of casting defects (see Figure 2.1b) [4].

![Investment Casting Process](image1)

**Figure 2.1** Example of microtruss cores investment casting process (a) and the defects introduced when casting a low relative density structure (b); adapted from [4].

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2.1.2 **Textile Lay-Up**

In textile lay-up, metal wires or tubes are woven into uniform lattices structure (usually in square, diamond, or Kagome structures) followed by diffusion bonding/brazing (see Figure 2.2a) [35-38]. Lim and Kang showed the possibility of weaving corrugated wires into tetrahedral microtrusses (see Figure 2.2b) [39]. While textile lay-up is an efficient manufacturing process for the 2D lattices (square and diamond), it is much more complicated to massively produce 3D lattices (Kagome, and tetrahedral) using textile lay-up process without automation. Also, the rigidities of these textile lay-up structures could be weakened from the post-weaving brazing process.

![Figure 2.2 Example of textile lay-up of diamond structures (a) and tetrahedral microtrusses (b) [38, 39].](image-url)
2.1.3 Rapid Prototyping

Rapid prototyping, also known as 3D printing, permits the creation of complicated structures. In terms of microtrusses, this means that rapid prototyping allows for the formation of multiple architectures with flexible fabrication parameters. There are two variations of the rapid prototyping process used to fabricate microtruss cores: selective laser melting and self-propagating polymer waveguides (see Figure 2.3 for illustration). In the selective laser melting process, laser beams are used to melt metallic powders into metallic truss-like architectures [32]. In the self-propagating polymer waveguides processes, ultraviolet light beams are used to polymerize photosensitive monomers into truss-like templates [6]. These templates can then be metalized through electrodeposition [26] or electroless deposition [25]. The polymer cores can then be etched or melted to create hollow microtrusses. Due to the high resolution of rapid prototyping and the possibility of creating hollow microtrusses with wall thickness in sub-micron range, the relative density of these microtrusses can be as low as 0.01% [25]. Both selective laser melting and self-propagating polymer waveguides can fabricate multiple architectures efficiently. However, the multi-step process in the later-mentioned technique may increase the fabrication cost and limit the productivity.

Figure 2.3 Selective laser melting (a) and self-propagating polymer waveguide (b) made microtrusses [25, 40].

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2.1.4 **Sheet-Forming**

Simple sheet-forming processes can also be used to fabricate microtruss cores from perforated or expanded metal sheets [27-31]. Afterward, these corrugated layers can be joined by spot welding, diffusion bonding, or brazing methods. The most commonly used sheet-forming process involves folding metal sheets at the node regions using a bending brake (see Figure 2.4a). This folding method is the only known continuous manufacturing process of microtruss layers with high productivity. Stretch-forming, on the other hand, stretches the metal sheet in the out-of-plane direction at the node regions using alternating pins (see Figure 2.4b). During stretch-forming, each of the internal struts is deformed under tensile elongation and the struts are subjected to cross-sectional thinning. A range of microtruss architectures can be fabricated from a single starting perforated sheet and forming press, with the internal structure controlled by the degree of forming displacement. Additionally, plastic strain is introduced into the metal struts during tensile elongation, making the microtruss stronger due to intrinsic dislocation strengthening.

![Figure 2.4 Illustration of folding (a) and stretch-forming process (b); adapted from [20, 27].](image-url)
2.2 Microtruss Composites: Metal-Metal vs. Ceramic-Aluminum

The mechanical performance of microtruss cellular materials can be significantly enhanced through surface modification to create microtruss composites (also known as structurally coated microtrusses). Literature to date has showed two possible microtrusses composite types: metal-metal composites [7, 8, 33] and ceramic-aluminum composites [34]. In conventional surface treatment processes, protective coatings can be electrodeposited or anodized onto substrate surfaces to improve corrosion and/or wear resistance, while their strengthening effect is often ignored due to the small area fraction of the coatings. However, since microtruss cellular materials have large internal surface areas while the cross-sectional strut dimensions can be as small as hundreds of microns, only thin coatings of nanocrystalline metal were needed to significantly increase the compressive strength of low carbon steel and aluminum microtrusses, respectively [7, 8, 33]. These strength enhancements originate from the combined contribution of grain boundary strengthening within the coating and the large second moment of area of the coated materials. Figure 2.5a shows the cross-section of a nano-NiFe/low carbon steel microtruss composite [33]. Unfortunately, due to the relatively high density of the nanocrystalline coating, a weight penalty was also added as the microtruss core was coated. Meanwhile, light-weight Al₂O₃ ceramic coatings can be created on aluminum microtrusses through anodizing (see Figure 3.5b [34]). Compared to nanocrystalline coatings, these ceramic coatings provide a similar strengthening effect per coating thickness, yet the weight penalty is now minimized (see Figure 2.6) [34]. While these ceramic structural coatings have provided a promising strength enhancement to microtruss design, their failure mechanism has not been well studied. For instance, under compression, conventional nano-nickel-aluminum hybrids collapse through mid-strut ductile buckling with sleeve delaminations (see Figure 2.7a). In contrast, ceramic-aluminum hybrids collapse through local strut folding and oxide fractures near the hinge region (see Figure 2.7b). As a result, the present study further investigates this newly-reported failure mechanism and seeks for a properly designed architectural optimization for these ceramic-aluminum microtruss composites.
Figure 2.5 SEM image of the cross-section through nano-nickel/low carbon steel(a) and $A_2O_3$-Al(b) microtruss composites [33, 34].

Figure 2.6 Compressive peak strength as a function of coating thickness nano-nickel/low carbon steel and $A_2O_3$-Al microtruss composites (a) and strength-density property map (b) [34].

Figure 2.7 SEM images reviewing the global buckling of nano-nickel-Al microtruss composites (a) and the hinge failure of $A_2O_3$-Al microtruss composites (b) [33, 34].
2.3 Anodizing

Anodizing is a surface treatment process for stainless alloys such as titanium and aluminum. It is typically used to improve corrosion resistance, wear resistance, or paint adhesion [41]. Anodizing can be viewed as the reverse of electrodeposition, where the anode is the electrode coating site (see Figure 2.8 for schematic) [42]. The applied electric potential drives the aluminum surface to form a layer of protective oxide. The advantage of anodizing is that there is virtually no weight gain, as the coating material has low density.

![Figure 2.8 Schematics of anodizing setup (adapted from [42]) and ion transport during anodizing.](image)

In aluminum anodizing, there are two anodic reactions occurring on the Al$_2$O$_3$-Al interface ($2\text{Al} + 3\text{O}^{2-} \rightarrow \text{Al}_2\text{O}_3 + 6\text{e}^-$) and on the Al$_2$O$_3$-electrolyte interface ($2\text{Al}^{3+} + 3\text{H}_2\text{O} \rightarrow \text{Al}_2\text{O}_3 + 6\text{H}^+$), while a hydrogen evolution occurs at the cathode ($6\text{H}^+ + 6\text{e}^- \rightarrow 3\text{H}_2$) [43]. Figure 2.8 illustrates the ion transport of these two anodic reactions. The reaction at the Al$_2$O$_3$-Al interphase dominates since there is a greater supply of aluminum atoms at the substrate than in the electrolyte [41]. It is therefore generally assumed that the anodic oxide grows inward [43]. However, the atomic volume of Al$_2$O$_3$ is higher than the Al substrate and these oxide anodic coatings are highly porous. As a result, these coatings expand during anodizing (for more details, see Ref. [44]). The next two subsections review the formation theories of these porous oxides and the details of a specific type of anodizing (hard anodizing) for creating thick ceramic coatings.

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2.3.1 **Nano-porous Anodic Oxide**

Anodic oxides are highly porous, with channels arranged in a 2D hexagonal honeycomb pattern (see Figure 2.9a for schematic). Anodic coating thickness can range from 25 to 200 µm, while the pore diameter and the wall size range from 20 to 500 nm depending on the anodizing conditions. There are various theories about the formation of these nano-honeycomb structures; two of the most commonly mentioned theories are presented here.

An early theory postulated that the formation of the anodic nano-channel is due to local burning of the oxide [43]. At the beginning of anodizing, a layer of fully dense alumina builds up for a few nanometers, known as the barrier layer. This barrier layer may contain surface imperfections or different regions on the substrate surface may have localized higher electric fields, which in turn creates local heat zones [41]. As a result, the acidic electrolyte dissolves the oxide at these regions at a higher rate, resulting in porous nano-honeycomb structures. This theory also predicts that as the oxide builds up, the electro-conductivity of the substrate would decrease and reduce the growth rate of the oxide. A maximum thickness is achieved when the growth rate of the oxide is equal to its dissociation rate [43]. However, this theory does not explain the pore dimension dependency on anodizing potential [45].

![Figure 2.9 Schematic diagram of a nano-honeycomb (a) and the ion transport of a recently proposed oxide formation mechanism (b) [45, 46].](image-url)
Recently, a new explanation has been proposed by Runge and Pomis [45, 46]. In the initial stages of anodizing, multiple circular anodic oxide flakes grow on the substrate surface, and eventually approach each other. The repulsive force around these flakes is large enough to constrain the newly-formed oxides to grow in the out-of-plane direction, creating nano-channels. These nano-channels assist the ion transport within the oxide and they are never sealed during anodizing. Since the anodizing potential (voltage) controls the amount of ion transport, it therefore also controls the pore dimension (see Figure 2.9b). Runge and Pomis also revealed that the thickness of the barrier layers was in the same order magnitude as the channel wall thickness, which further supports the likelihood of this formation mechanism [45, 46]. Regardless of the formation mechanism, the pore dimension would alter elastic constants such as Elastic modulus (E) and Poisson’s ratio (ν). Thus, nano-indentation experiments are required to determine the mechanical strength of these porous oxides.

2.3.2 Hard Anodizing

The four common types of anodizing are: sulfuric acid based, chromic acid based, oxalic acid based, and hard anodizing [41]. Sulfuric acid based anodizing is often used for improving wear resistance, whereas chromic acid and oxalic based anodizing are used to create corrosion-resistant coatings. The first three anodizing processes usually produce oxides up to 65 µm thick. Hard anodizing is a special type of sulfuric acid based anodizing that operates at a lower temperature. The decrease in temperature reduces the corrosiveness of the acid bath and the oxide dissociation rate. As result, hard anodizing can produce an anodic aluminum oxide film that is thicker by an order of magnitude (up to 700 µm) [47]. Table 2.1 shows several common hard anodizing processes [41, 47-49]. Both thick film and Lasser’s hard anodizing can produce oxide thicknesses above 500 µm, making them the most attractive. However, thick film anodizing requires an industrial chiller to maintain a low operating temperature (3–5°C) and is difficult to control in a laboratory environment. Meanwhile, Lasser’s anodizing requires an applied voltage of up to 500 V, which is above the capacity of the power supply in the Hybrid Materials Design Group (BK Precision DC Power Supply 1630). Alumilite 225/226 anodizing requires relatively low applied voltage and easily-controlled temperature ranges. In this study, a hard anodizing process similar to alumilite 225/226 (with a higher current density to increase oxide growth rate) was used.

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Table 2.1 Common Hard Anodizing Processes for Aluminum Alloys [41, 47-49].

<table>
<thead>
<tr>
<th>Data Process</th>
<th>Solution (wt %)</th>
<th>Current Density (A/dm²)</th>
<th>Voltage (V)</th>
<th>Temperature (°C)</th>
<th>Thickness (µm)</th>
<th>Hardness [HV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Hard Anodizing</td>
<td>7% H₂SO₄</td>
<td>2-5</td>
<td>23 - 120</td>
<td>-5 to +5</td>
<td>0 - 250</td>
<td>N/A</td>
</tr>
<tr>
<td>Thick Films</td>
<td>3-5% (COOH)₂</td>
<td>1-2</td>
<td>40 - 60</td>
<td>3 - 5</td>
<td>0 - 625</td>
<td>N/A</td>
</tr>
<tr>
<td>Alumilite 225/226</td>
<td>12% H₂SO₄, 1% (COOH)₂</td>
<td>2.8</td>
<td>10 - 75</td>
<td>10</td>
<td>25-50</td>
<td>N/A</td>
</tr>
<tr>
<td>Martin Hard Coating</td>
<td>15% H₂SO₄</td>
<td>2.7</td>
<td>20 - 75</td>
<td>-4 to 0</td>
<td>50</td>
<td>N/A</td>
</tr>
<tr>
<td>Lasser</td>
<td>0.75% (COOH)₂</td>
<td>Voltage Controlled</td>
<td>50 - 500</td>
<td>1 - 7</td>
<td>700</td>
<td>N/A</td>
</tr>
<tr>
<td>Black Anodic Coating</td>
<td>50 g/l H₂SO₄, 0.1 g/l CaF₂, 0.5 g/l H₂SO₄, 1 g/l Cr₂(SO₄)₃</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>25-50</td>
<td>1400</td>
</tr>
<tr>
<td>Dowty Equipment Ltd.</td>
<td>10-30% H₂SO₄</td>
<td>0.25-1</td>
<td>15-90</td>
<td>-6 to +20</td>
<td>75-125</td>
<td>500-600</td>
</tr>
<tr>
<td>Dilute Sulfuric Acid</td>
<td>0.5-2.5% H₂SO₄</td>
<td>N/A</td>
<td>20 - 80</td>
<td>-5 to +5</td>
<td>150-200</td>
<td>450-520</td>
</tr>
<tr>
<td>Csokan and Hollo</td>
<td>0.1M H₂SO₄</td>
<td>N/A</td>
<td>45 - 60</td>
<td>-1 to +1</td>
<td>100-250</td>
<td>600-620</td>
</tr>
</tbody>
</table>
2.4 Other Ceramic-Based Structural Coatings

Anodic aluminum oxide (AAO) has a nano-honeycomb structure. The formation of the nano-honeycomb can be related to the equilibrium between oxide growth and dissolution [41] or to the expansion of oxide volume and the diffusion of oxygen anions [45, 46]. The porosity of nano-honeycombs may weaken the potential strength of the AAO. Therefore, three ceramic-based structural coatings are presented as possible future research directions. First, Plasma Electrolytic Oxidation (PEO) on the aluminum surface can create a denser coating [50]. The PEO coating can be up to three times as strong as the AAO (1600 HV vs. 537HV) [50]. PEO coating also tends to grow outward [51] rather than inward, making it even more structurally efficient than the AAO as it increases the second moment of area (I). Second, alumina-polymer composite coatings can be created by adding conductive monomers (amino-benzene) into the anodizing bath. Electroactive polymerization occurs during anodizing, causing the polymers and alumina to be co-deposited on the aluminum [52]. These coatings have a higher wear resistance [52], which may also correspond to an increase fracture strength. Lastly, the nano-honeycomb structure of the AAO can be used as a template to deposit carbon nano-tubes (CNT) or carbon nano-fibres (CNF) through carburizing to create hybrid coatings. Both the hardness and the elastic modulus increases two-fold after hybridization [53]. Figure 3.5 illustrates the aforesaid ceramic-based coatings.

![Figure 2.10 SEM image of PEO coating (a), TEM image of polymer-AAO hybrid coating (b), and SEM image of the CNF-AAO hybrid coating; adapted from [45, 53, 54].](image-url)
3 Methods

This chapter summarizes the experimental and analytical methods used in this study.

3.1 Experimental Methods

Stretch-formed ceramic-aluminum microtruss composites can be fabricated by a two-step process. In this study, square perforated aluminum AA3003 sheets were first stretch-formed into 3-D corrugated arrays. These aluminum microtruss cores were then surface-treated by hard anodizing to create a thin layer of ceramic structural coating, yielding microtruss composites. The following subsections describe the stretch-forming and hard anodizing processes.

3.1.1 Stretch-Forming of Aluminum Microtruss Cores

Microtruss cores with a large range of internal architecture variations can be created through the simple process of stretch-forming. In addition, work-hardening is introduced to the microtruss cores, which is not possible in other fabrication processes. In this study, aluminum AA3003 H14 perforated sheets (sheet thickness of $t = 0.74 \pm 0.01$ mm) were purchased from McNichols Perforated Products (Atlanta, GA). The perforation pattern had square holes (of size $l = 5.10$ mm) with an initial strut thickness (of $w = 1.25 \pm 0.01$ mm). The aluminum sheets were first annealed at 600 °C for 60 minutes, and then water quenched. This was followed by stretching the sheets under alternating pins in the out-of-plane direction to create pyramidal microtruss cores, after Ref. [27, 55] (see Figure 3.1).

Multiple forming/annealing cycles (with intermediate annealing treatments of 600 °C for 60 minutes) were then used to achieve a range of internal truss angles ($\theta = 21^\circ$ to $45^\circ$).

Figure 3.1 Schematic diagrams of a pyramidal unit prior (a) and during (b) stretch-forming [7, 55].
After stretch-forming, these microtruss cores can be tested as fabricated (work-hardened) or after another post-fabrication annealing treatment of 600 °C for 60 minutes to remove any fabrication-induced work-hardening (annealed). Both sample types were tested in uniaxial compression at a cross-head displacement rate of 1 mm/min using confinement plates that restricted the nodal displacement (see Figure 3.2a for schematic). Experimental compressive properties were obtained by correcting machine compliance (see Appendix A), a method similar to the one mentioned in Ref. [56]. Subsize coupons of annealed AA3003 (at 600 °C for 60 minutes) were tested in uniaxial tension (after Ref. [55]). Strains were measured from cross-head displacement with the correction of machine compliance, after Refs. [57, 58] (see Appendix B). The annealed AA3003 has an ultimate tensile strength of $\sigma_{UTS} = 110$ MPa, a yield strength of $\sigma_{YS} = 47$ MPa, a modulus of elasticity of $E = 69$ GPa, a Hollomon work-hardening coefficient of $K = 207$ MPa, and a Hollomon work-hardening exponent of $n = 0.275$. The tested material properties are in close agreement with the properties of the AA3003-O aluminum alloy reported in the Atlas of Stress-strain Curves [59]. Figure 3.2b shows the experimental tensile curve of the annealed aluminum. Microhardness testing showed that the annealed aluminum cores have an average hardness of $36 \pm 2$ HV (0.98 N applied load and 10 s dwell time).

![Figure 3.2](image)

Figure 3.2 Schematic diagrams of microtruss compression testing (a) and experimental tensile curve of annealed AA3003 aluminum (b).

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3.1.2 Structural Coating of Anodic Aluminum Oxide (AAO)

The stretch-formed AA3003 microtrusses were subjected to a surface cleaning treatment followed by hard anodizing to form a 0-50 µm thick anodic aluminum oxide on the surface of the microtruss cores (process conditions are summarized in Table 3.1). Samples were rinsed with de-ionized water between each step and after hard anodizing. The anodic oxide thickness was measured using optical micrographs of anodized internal struts at four different anodizing times: 15, 30, 45, and 60 minutes (see Figure 3.3 for an example). Figure 3.4 shows the measured oxide thickness as a function of time. Linear fitting indicates that the deposition rate is ~1.02 µm/min. As a result, the calculated coating thicknesses of the 5- and 10-minute anodized samples are 5 and 10 µm, while the measured coatings of the 15- and 45-minute anodized samples are 18 and 48 µm. Another set of starting facesheets was anodized under similar conditions (with a current density of 2.8 A/dm²) to show that the volume expansion factor (γ) of the oxide is ~1.6 (see analysis in Appendix C). Nanoindentation results showed that the anodic oxide has an elastic modulus of $E = 95.5 \pm 1.9$ GPa (see Appendix D for more details). Microhardness testing showed that the anodic coating was $537 \pm 38$ HV (using 0.98 N applied load and 10 s dwell time). The stretch-formed ceramic-aluminum microtruss composites were tested in uniaxial compression at a cross-head displacement rate of 1 mm/min using confinement plates that restricted the nodal displacement.

<table>
<thead>
<tr>
<th>Process Name</th>
<th>Solution</th>
<th>Time</th>
<th>Temperature</th>
<th>Current Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degreasing</td>
<td>5g/l Na₂CO₃, 5g/l Na₃PO₄</td>
<td>10 min</td>
<td>87 ± 5ºC</td>
<td>-----</td>
</tr>
<tr>
<td>Etching</td>
<td>5wt% NaOH</td>
<td>2 min</td>
<td>57 ± 2ºC</td>
<td>-----</td>
</tr>
<tr>
<td>De-smutting</td>
<td>20vol% HNO₃</td>
<td>1 min</td>
<td>33 ± 2ºC</td>
<td>-----</td>
</tr>
<tr>
<td>Hard anodizing</td>
<td>12wt% H₂SO₄, 1wt% (COOH)₂</td>
<td>0, 5, 10, 15, or 45 min</td>
<td>10 ± 1ºC</td>
<td>4 A/dm²</td>
</tr>
</tbody>
</table>
Figure 3.3 Strut cross-sections after 60-minutes of anodizing at 4A/dm$^2$.

Figure 3.4 Anodic oxide thickness measurements (at 4A/dm$^2$) as a function of anodizing time.
3.2 Analytical Methods

This section presents the collapse mechanism of conventional microtrusses under uniaxial compression and the concept of architectural optimization.

3.2.1 Collapse Mechanisms: Inelastic Buckling

Conventional metal microtruss cores (with internal strut connectivity of ~ 12) collapse under stretch-dominated failure mechanisms [3, 18]. To illustrate this failure mechanism, Figure 3.5 shows the collapse in compression of a pyramidal microtruss unit cell and a single internal strut in a Finite Element (FE) simulation using ABAQUS. A pure elastic stress-strain behaviour (E = 210 GPa) was assigned to both the pyramidal unit cell (on the left) and the single strut (on the right). In the case of the pyramidal unit cell, the outer edges of the four nodes in this unit cell were restricted from horizontal displacement in order to imitate the symmetry in an infinite lattice. The bottom face of the lowest strut in the unit cell was restricted from vertical displacement. The top face of the highest strut was prescribed with a uniform downward displacement up to half the height of the unit cell to simulate a uniaxial compression load. In the case of the single internal strut, the bottom face was restrict from vertical displacement and the top face was prescribed with a downward displacement up to half the height of the strut.

Figure 3.5 Illustration of the compression buckling failure mechanism in a unit cell of a pyramidal core.

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Under compression, the horizontal face struts in the pyramidal unit cell are not deformed. Since microtrusses are stretch-dominated materials, the applied loads transfers axially to the strut and the inclined core struts are deflected by buckling. The right side of the diagram shows the configuration of an internal strut after buckling instability: one side of the strut is in compression (indicated as red regions) while the other side is in tension (indicated as blue regions); illustrating the post-buckling stress bifurcation. In the case of a slender column/internal strut, the critical buckling strength ($\sigma_{CR}$) at the onset of instability can be calculated by the Euler buckling relation:

$$\sigma_{CR} = \frac{k^2 \pi^2 E}{(L/r)^2} = \frac{k^2 \pi^2 EI}{AL^2} \quad (3.1)$$

where $k$ is the rigidity constant of the nodes ($k = 1$ for idealized pin joints and $k = 2$ for idealized rigid joints [60, 61]), $E$ is the elastic modulus, $I$ is the second moment of area, $A$ is the cross sectional area, $L$ is the length of the struts, $r$ is the radius of gyration ($L/r$ is known as the slenderness ratio).
When the strut length is decreased to intermediate slenderness ratio, as is typical of microtruss materials, inelastic buckling becomes the governing failure mechanism. The inelastic critical buckling strength ($\sigma_{CR}$) can be estimated by replacing the elastic modulus ($E$) with the tangent modulus ($E_T$), in what is commonly known as the Shanley relation [60]:

$$\sigma_{CR} = \frac{k^2\pi^2E_T}{(L/r)^2} = \frac{k^2\pi^2E_T I}{AL^2} \quad (3.2)$$

where $E_T$ is the tangent modulus (the slope of the stress-strain curve, i.e. $E_T = \partial \sigma / \partial \varepsilon$). Shanley proposed that the critical strength can be calculated by plotting the intersection of the stress-strain curve with Equation (3.2) [60]. Figure 3.6 shows a working example of calculating the inelastic critical buckling strength of an annealed AA3003 column with $k = 1$ and $L/r = 100$. In this case, the critical strength is $\sim 20$ MPa.

A more convenient expression is to give the tangent modulus in terms of strength, where the stress-strain curve is approximated by a power-law Hollomon relation (i.e. $\sigma = K\varepsilon_{pl}^n$):

$$E_T = \left(\frac{\partial \varepsilon}{\partial \sigma}\right)^{-1} = \left(E^{-1} + n^{-1}K\frac{1-n}{n}\sigma\frac{1-n}{n}\right)^{-1} \quad (3.3)$$

where $K$ is the Hollomon work-hardening coefficient and $n$ is the Hollomon work-hardening exponent. Substituting Equation (3.2) into (3.3) expresses the critical strength as a single relation:

$$\frac{\sigma_{CR}}{E} + \frac{\sigma_{CR}^{1/n}}{n \cdot K^{1/n} (L/r)^2} - \frac{k^2\pi^2}{(L/r)^2} = 0 \quad (3.4).$$

Equation (3.4) allows the calculation of critical buckling strength through numerical iteration rather than the graphical method shown in Figure 3.6.

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3.2.2 Architectural Optimization

Stretch-forming allows multiple architectures to be fabricated from a given perforated sheet, defined by the starting strut thickness \( t \), width \( w \), and length \( l \). However, predicting the strength of stretch-formed microtrusses becomes more complex as the material properties change with internal truss angle \( \theta \) via the accumulation of plastic strain. Furthermore, anodizing introduces an extra degree of design freedom through the effect of coating thickness \( s \). Figure 3.7a shows a schematic diagram of the possible architectural parameters considered in this study and raises a critical design question: which architecture is best?

Here we illustrate a systematic method to determine the optimal architecture. This study’s concept of mass minimization is adapted from Ref. [62-66], where the optimal architecture is the structure that carries a required load at the lightest possible weight. Therefore, we define two parameters: the non-dimensional strength \( \bar{F} \) and the non-dimensional weight \( \bar{M} \):

\[
\bar{F} = \frac{\sigma_{MT}}{\sigma_{ref}} \quad \text{and} \quad \bar{M} = \frac{\rho_{MT}}{\rho_{ref}} \quad (3.5)
\]

where \( \sigma_{MT} \) is the strength of the microtruss, \( \sigma_{ref} \) is the reference strength, \( \rho_{MT} \) is the density of the microtruss, and \( \rho_{ref} \) is the reference density. Here, \( \sigma_{ref} \) and \( \rho_{ref} \) are defined to be the ultimate tensile strength and the density of the bulk aluminum AA3003 (\( \sigma_{ref} = 110 \) MPa and \( \rho_{ref} = 2.7 \) Mg/m\(^3\)).
To illustrate the architectural optimization analysis, Figure 3.7b shows an example of a hypothetical design space generated from two different architectural parameters, which could be any of the parameters listed in Figure 3.7a. Multiple properties such as non-dimensional weight ($\bar{M}$) can be created through different combinations of architectural parameters. For instance, $\bar{M}$ can range from 5% to 10%, shown as the grey contour lines in Figure 3.3b. Meanwhile, multiple architectures can produce different $\bar{F}$; a constant contour line of $\bar{F} = 10\%$ is shown in black. As we follow this constant contour of $\bar{F}$, multiple values of $\bar{M}$ are encountered. Only when the contour of $\bar{F}$ is tangential to $\bar{M}$ is minimum mass achieved. In this hypothetical example, the optimal architecture occurs at $\bar{M} = 5\%$. Since the gradient vectors of $\bar{F}$ and $\bar{M}$ are both orthogonal to their tangent vectors, the optimum occurs when the gradient vector of $\bar{F}$ is parallel to that of $\bar{M}$:

$$\nabla \bar{F} = \lambda \nabla \bar{M} \quad (3.6)$$

where $\lambda$ is the Lagrange multiplier.

Figure 3.7 Schematic diagrams of a microtruss architectural parameters (a) and a graphical example of optimization analysis (b).
The above optimization can also be determined mathematically by using the Lagrange multiplier in a constrained optimization [67]. Here we define a point, \( P(x, y) \), as a location within the architectural space. The optimization objective is to minimize \( \bar{M} \), which is subjected to the constraint of a constant level of \( \bar{F} = c \); i.e.:

\[
\min \{ \bar{M}(x, y) : \bar{F}(x, y) = c \} \quad (3.7).
\]

Note that the constant \( \bar{F}(x, y) = c \) is essentially a contour line (or tangent line) and it can be represented by its location \( (P) \) and its tangent unit vector \( (\hat{u}) \). Extrema occur when the rate of change of \( \bar{M} \) along this tangent vector is zero:

\[
\nabla \bar{M} \cdot \hat{u} = 0 \quad (3.8).
\]

Equation (3.8) also states that such a condition is only satisfied when the gradient vector of \( \bar{M} \) is orthogonal to the tangent vector \( (\hat{u}) \) of \( \bar{F} = c \). Meanwhile, the gradient vector of \( \bar{F} \) is essentially the normal vector of the surface and is by definition orthogonal to its tangent vector:

\[
\nabla \bar{F} \cdot \hat{u} = 0 \quad (3.9).
\]

Since the tangent vector is not a zero vector, Equations (3.8) and (3.9) state that the gradient vectors of \( \bar{F} \) and \( \bar{M} \) are parallel, a special case of the Karush-Kuhn-Tucker condition [68, 69]:

\[
\nabla \bar{F} = \lambda \nabla \bar{M} \quad (3.6)
\]

where \( \lambda \) is the Lagrange multiplier. In both the graphical and the analytical interpretation, we come to the same conclusion. Equation (3.6) will therefore be the governing relation for architectural optimization. A rigorous proof of the Karush-Kuhn-Tucker condition is omitted here; for more details, please refer to Ref. [67, 70, 71]. Although this issue does not apply to the system in this study, it is worth noting that Equation (3.6) does not always generate real solutions; a re-check by plotting the objective and constraint functions after optimization is advised [70].

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4 Results and Discussion

This study seeks to determine the optimal architecture(s) of stretch-formed ceramic-aluminum microtruss composites. The following subsections divide this objective into three parts: optimization of annealed microtruss cores, work-hardened microtruss cores, and structurally coated microtruss composites. Within each subsection the same order of discussion will be followed: strength prediction, experimental verification, and architectural optimization.
4.1 Annealed Microtruss Cores

4.1.1 Annealed Microtruss Cores: Strength Prediction

The architecture of stretch-formed microtrusses is specified by the geometry of the starting perforated sheet and the final forming angle after fabrication (Figure 4.1). During fabrication, the internal struts are elongated and their cross-sectional areas are reduced. The final strut length \( l_f \) increases from the initial value \( l \):

\[
l_f = l/\cos \theta = l \sec \theta \quad (4.1)
\]

where \( \theta \) is the internal truss angle. By assuming a constant strut volume and a uniform reduction in cross-sectional dimension, the strut width and thickness can be related to the initial width \( w \) and initial thickness \( t \) as:

\[
w_f = w \sqrt{\cos \theta} \quad (4.2\ a) \quad ; \quad t_f = t \sqrt{\cos \theta} \quad (4.2\ b).
\]

Figure 4.1 Schematic of a starting perforated AA3003 sheet (a) and the final architecture after stretch-forming (b); adapted from [72].
The compressive strength of the stretch-formed microtrusses depends on the internal truss angle. Since internal load resolution within the microtruss core is more efficient at higher truss angles, the compressive strength can be expressed as:

$$\sigma_{MT} = N_{strut} \cdot \sigma_{CR} \left( \frac{A_{strut}}{A_{cell}} \right) \sin \theta = 2 \cdot \sigma_{CR} \cdot \frac{w \cdot t}{(l + w)^2} \cos \theta \cdot \sin \theta \quad (4.3)$$

where $\sigma_{MT}$ is the compressive strength of the stretch-formed microtruss, $N_{strut}$ is the number of struts per unit cell, $\sigma_{CR}$ is the collapse strength of a single strut, $A_{strut}$ is the cross-sectional area of the strut, and $A_{cell}$ is the projected area of the unit cell.

However, as the truss angle increases, the strut slenderness ratio $(l/r)$, where $r$ is the radius of gyration) increases due to strut thinning and elongation, making the internal struts more susceptible to buckling failure:

$$\frac{L}{r} = \frac{\sqrt{12} \cdot l_f}{t_f} = \sqrt{12} \cdot \left( \frac{l}{t} \right) \cdot \sec^{3/2} \theta \quad (4.4).$$

Equations (4.3) and (4.4) suggest that the compressive strength can be maximized at one particular angle that balances the counteracting effects of load resolution and strut thinning. The compressive strength of the microtruss $(\sigma_{MT})$ can also be predicted by substituting Equations (4.3) and (4.4) into the inelastic buckling Equation (3.4):

$$\frac{\sigma_{MT}}{E \cdot \alpha \cdot \beta \cdot \cos \theta \cdot \sin \theta} + \frac{\sigma_{MT}^{1/n}}{n \cdot K^{1/n} \cdot \alpha \cdot \beta^{1/n} \cdot \cos^{(3+1)/n} \cdot \sin^{1/n} \theta} = 1 \quad (4.5)$$

where $\alpha$ and $\beta$ are two coefficients based on the starting metal sheet dimensions:

$$\alpha = \frac{k^2 \cdot \pi^2 \cdot t^2}{12 \cdot l^2} \quad (4.6 \ a) \quad \beta = \frac{2 \cdot w \cdot t}{(l + w)^2} \quad (4.6 \ b).$$

The compression strengths of the annealed aluminum microtruss cores can be calculated by Equation (4.5), using their initial strut dimensions \((w = 1.25 \pm 0.01 \text{ mm}, t = 0.74 \pm 0.01 \text{ mm}, \text{ and } l = 5.1 \text{ mm})\) and their annealed-state tensile properties \((E = 69 \text{ GPa}, K = 207 \text{ MPa}, \text{ and } n = 0.275)\).
4.1.2  Annealed Microtruss Cores: Experimental Verification

The theoretical calculated compressive strength of the stretch-formed microtrusses is then compared to the annealed AA3003 stretch-formed microtruss cores. Fourteen microtrusses were stretch-formed to a range of internal truss angles from 21º to 45º. They were then annealed to remove work-hardening in order to isolate the architectural effect before compression testing. Figure 4.2a shows the typical compressive stress-stain curve of these annealed microtrusses. The same type of inelastic buckling mechanism was seen for all angles. There was an elastic region (of slope $E$) before an initial peak stress ($\sigma_{\text{peak}}$), which was followed by a drop to a valley stress ($\sigma_{\text{valley}}$) and final densification. The experimental peak strengths ($\sigma_{\text{peak}}$) were then compared to the theoretical compressive strengths ($\sigma_{MT}$) at $k = 1$, $k = 1.43$, and $k = 2$, in Figure 4.2b (other compressive properties are documented in Appendix E). Both the theoretical calculation and the experimental results indicate that the strength first increases due to efficient load resolution, but eventually decreases due to strut thinning. The maximum compression strength for a given starting sheet geometry occurs at $\theta_{\text{max}} \sim 36^\circ$ in this study.

Figure 4.2 Compressive stress-strain curves of annealed microtruss cores at low and high angles (a), and compressive peak strength comparison between theoretical calculation and experiments (b).
It is worth noting that there is a fundamental difference between the critical bifurcation stresses predicted in Equation (4.5) and the peak strength measured experimentally. At the onset of buckling instability, the stress eventually bifurcates into compressive and tensile regions at either side of the column. This leads to local work-hardening near the deflection area, which may result in a higher buckling strength than the analytical prediction. A previous Finite Element (FE) simulation showed that the annealed AA3003 microtruss has a nodal rigidity of \( k = 1.43 \) at \( \theta = 35^\circ \), and its inelastic buckling strength is \(~15\%\) higher than its bifurcation stress [8]. On the other hand, the strut non-uniformity due to multiple forming cycles and/or imperfections may have a knock-down effect on their load carrying capabilities compared to that of an idealized strut. While they are not significant, the fluctuations in experimental data may reflect the effect of strut imperfections and/or the difference between bifurcation stress and buckling strength. In general, the experimental peak strengths (\( \sigma_{peak} \)) lie between the upper bound (\( k = 2 \)) and the lower bound (\( k = 1 \)) of the theoretical calculation, and therefore validate the strength prediction model.
4.1.3 **Annealed Microtruss Cores: Architectural Optimization**

As a conservative approach, an idealized pin-joint connections \((k = 1)\) are assumed in this analysis. The internal struts are also assumed to have square cross-sections (i.e. \(w = t\)) in order to maximize the strength-to-weight ratio for a single column. This allows us to generalize the analysis to consider all possible sheet geometries by creating the non-dimensional architectural parameter:

\[
\bar{t} = \frac{t}{l} = \frac{w}{l} \quad (4.7)
\]

where \(\bar{t}\) represents the original sheet dimension (starting strut cross-section to length ratio).

The non-dimensional strength (\(\bar{F}\)) in Equation (3.5) can be expressed as:

\[
\frac{\bar{F} \cdot \sigma_{ref}}{E \cdot \bar{\alpha} \cdot \bar{\beta} \cdot \cos^4\theta \cdot \sin\theta} + \frac{(\bar{F} \cdot \sigma_{ref})^{1/n}}{n \cdot k^{1/n} \cdot \bar{\alpha} \cdot \bar{\beta}^{3/n} \cdot \cos^{(3+1)/n}\theta \cdot \sin^{3/n}\theta} = 1 \quad (4.8)
\]

where \(\alpha\) and \(\beta\) are coefficients based on the starting metal sheet dimension:

\[
\bar{\alpha} = \frac{k^2 \cdot \pi^2}{12 \bar{t}^2} \quad (4.9a) \quad \bar{\beta} = \frac{2 \cdot \bar{t}^2}{(1 + \bar{t})^2} \quad (4.9b).
\]

The non-dimensional weight (\(\bar{M}\)) in Equation (3.5) can likewise be expressed as:

\[
\bar{M} = \frac{\rho_c}{\rho_{ref}} \cdot \frac{2V_{strut} + 1V_{node}}{A_{cell} \cdot H_{truss}} = \frac{\rho_c}{\rho_{ref}} \cdot \frac{2\bar{t}^2 + \bar{t}^3}{(1 + \bar{t})^2 \cdot (\tan\theta + \bar{t})} \quad (4.10)
\]

where \(\rho_c\) is the density of AA3003 core (\(\rho_{core} = 2.7 \text{ Mg/m}^3\)), \(V_{strut}\) is the strut volume, \(V_{node}\) is the node volume, \(A_{cell}\) is the projected area of the unit cell, and \(H_{truss}\) is the microtruss height.

Figure 4.3 shows an example of the surface plots of \(\bar{F}\) (a) and \(\bar{M}\) (b) of annealed AA3003 microtruss cores at different combinations of \(\theta\) and \(\bar{t}\). In general, \(\bar{M}\) increases as \(\bar{t}\) increases, and decreases as \(\theta\) increases. While \(\bar{F}\) also increases as \(\bar{t}\) increases, it passes through a local maximum with respect to \(\theta\) because of the counteracting effects of load resolution and strut thinning.

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One might want to maximize $F$ at a given $\bar{t}$ by selecting the appropriate truss angle. This angle ($\theta_{\text{max}}^F$) can be determined by setting $\partial F / \partial \theta = 0$ in Equation (4.8):

$$
\frac{F \cdot \sigma_{ref}(\cos^2 \theta - 4\sin^2 \theta)}{E \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \cos^5 \theta \cdot \sin^2 \theta} + \frac{(F \cdot \sigma_{ref})^{1/n} [\cos^2 \theta - (3n + 1)\sin^2 \theta]}{n^2 \cdot K^{1/n} \cdot \dot{\alpha} \cdot \dot{\beta}^{1/n} \cdot \cos^{(4+1)/n} \theta \cdot \sin^{(1+1)/n} \theta} = 0 \quad (4.11).
$$

While this expression is somewhat complicated, it can be satisfied in the unique condition when $\cos^2 \theta - 4\sin^2 \theta$ is negative and $[\cos^2 \theta - (3n + 1)\sin^2 \theta]$ is positive (and vice versa). This allows us to express the boundary of $\theta_{\text{max}}^F$ as:

$$
4\sin^2 \theta_{\text{max}}^F > \cos^2 \theta_{\text{max}}^F > (3n + 1)\sin^2 \theta_{\text{max}}^F \quad (4.12 \ a)
$$

or

$$
\frac{1}{\sqrt{4}} > \tan \theta_{\text{max}}^F > \frac{1}{\sqrt{3n + 1}} \quad (4.12 \ b).
$$

Note that the Hollomon work-hardening exponent ($n$) is between 1 (perfectly elastic) and 0 (perfectly elastic). Therefore, $\theta_{\text{max}}^F$ is between 26.6° and 45° for any given material. Combining Equations (4.8) and (4.11) permits an accurate prediction of $\theta_{\text{max}}^F$ for a given material and starting sheet dimension (more details are given in Appendix F).
However, maximizing $F$ at a given $\tilde{t}$ is not the only concern when designing microtrusses for weight-limited-applications. The optimal architecture ($\theta_{min}^M$) is the structure that carries a required load with the lightest possible weight, and can be determined when the Karush-Kuhn-Tucker condition [68, 69] mentioned in Section 3.2.2 is satisfied. For the sake of brevity, the calculated gradient vectors are summarized in Appendix G. Figure 4.4 shows the optimal trajectory $\theta_{min}^M$ of the annealed AA3003 microtruss cores, superimposing $\tilde{F}$, $\tilde{M}$, and $\theta_{max}^F$ over the $\theta - \tilde{t}$ design space. Note that $\theta_{max}^F$ ranges from 26.6º to 36º, while $\theta_{min}^M$ ranges from 60º to 64º. This means that it is more beneficial to stretch-form microtrusses to a higher truss angle in order to reduce density when weight is a design factor.

To illustrate this benefit, we compare the compressive performance of three architecture examples. Point A represents the $\theta_{max}^F$ of the annealed AA3003 in this study. For this specific $\tilde{t}$, the maximum strength is at $\tilde{F} = 0.0054$. Increasing the truss angle from point A ($\theta = 36^\circ$, $\tilde{t} = 0.145$, $\tilde{F} = 0.0054$, $\tilde{M} = 0.0392$) to point B ($\theta = 63^\circ$, $\tilde{t} = 0.145$, $\tilde{F} = 0.0028$, $\tilde{M} = 0.017$) decreases the strength, but reduces the density at a significantly greater rate, leading to a 20% increase in specific strength. Moving from point A to point C ($\theta = 61^\circ$, $\tilde{t} = 0.2$, $\tilde{F} = 0.0054$, $\tilde{M} = 0.0269$) allows the microtruss to carry the same amount of load but with 30% less mass (or a 46% increase in specific strength). Note that while point C has a higher strength-to-weight ratio than that of point B, they are both optimal. The choice between them primarily depends on the required load in the specific application. For instance, if a $\tilde{F} = 0.0028$ is required, the microtruss architectures at point B and point C both satisfy this constraint; however, point B offers a structure that is almost 40% lighter than that of point C, meaning that point C is over-designed. On the other hand, if a $\tilde{F} = 0.0054$ is required, point B does not satisfy the requirement and the architecture at point C should be used.
The trajectory of \( \theta_{min}^{G} \) in Figure 4.4 therefore represents the architectures that have the highest possible strength-to-weight ratio. The properties associated with these architectures are recalculated and compared to the experimental results of this study (see Figure 4.5). As the forming angle increases from 21° to 45°, the non-dimensional weight (or relative density) of the microtrusses in this study progressively decreases from \(~10\%\) to \(~4.5\%\). However, for a given load, the optimized microtruss cores are consistently lighter than all the experimental results. For instance, the lightest microtruss core fabricated in this study was at a truss angle of 45° with a \( \tilde{M} \) of 4.5% and a load carrying capability \( \tilde{F} \) of \(~1\%\) (or \( \sigma_{peak} \) of 1.05 MPa). At this given load carrying capability, the optimal architecture would have an \( \tilde{M} \) of 0.044, which represents a 3% weight reduction compared to the experimental results. This slight difference implies that the high-angles annealed microtruss cores tested in this study are already approaching the optimum. Nonetheless, it is always beneficial to further reduce mass by achieving the theoretical optimal architecture. This can be done by increasing \( \tilde{t} \) from 0.15 to 0.26 and \( \theta \) from 45° to 62°. In other words, microtrusses with even higher truss angles than those tested in this study are more desirable.
Figure 4.4 Contours of $\bar{F}$ and $\bar{M}$ superimposed with $\theta_{\text{max}}^\bar{F}$ and $\theta_{\text{min}}^\bar{M}$ of annealed AA3003 microtruss cores.

Figure 4.5 Optimal annealed properties compared to experimental results.
4.2 Work-Hardened Microtruss Cores

4.2.1 Work-Hardened Microtruss Cores: Strength Prediction

Section 4.1 showed that the strength of a microtruss can be predicted by solving Equation (4.5). For the convenience of this discussion, Equation (4.5) is presented again below:

\[
\frac{\sigma_{MT}}{E \cdot \alpha \cdot \beta \cdot \cos^4 \theta \cdot \sin \theta} + \frac{\sigma_{MT}^{1/n}}{n \cdot K^{1/n} \cdot \alpha \cdot \beta^{1/n} \cdot \cos^{(3+\frac{1}{n})} \theta \cdot \sin^{1/n} \theta} = 1 \quad (4.5)
\]

In the case of an annealed microtruss, Hollomon parameters (\(K\) and \(n\)) remain constant. However, for work-hardened microtrusses, the mechanical properties of the struts vary continuously with the increasing internal truss angle (or the degree of forming). This adds another level of complexity to the strength prediction. As a result, it is necessary to develop a relationship between forming truss angle and microtruss mechanical property.

For this reason, a method was created to theoretically calculate the evolution of the Hollomon parameters in terms of the plastic strain accumulated during fabrication and then to relate this plastic strain to the truss angle. This method relies on the assumption that, after unloading, a tensile work-hardened strut would follow Hooke’s law upon reloading up until the previously loaded stress (the new yield strength). Figure 4.6 compares the idealized and the experimentally-obtained tensile unload-reload engineering stress-strain curve of an annealed AA3003 subsize coupon. At the micro-scale, the experimental unload-reload curve follows a hysteresis loop pattern and catches up with the calculated curve with a \(\varepsilon \sim 0.001\) delay (see Figure 4.7b); nevertheless, both curves are essentially identical at the macro-scale (see Figure 4.7a). The above engineering load-reload curve (Figure 4.6a) can be converted into five true stress-strain curves with different yield strengths by shifting the entire stress-strain curve with a constant plastic strain in order to intercept at the origin. These five tensile curves represent the stress-strain behaviour after work-hardening and can be described by their own Hollomon relations.
The same procedure was carried out on an idealized 100-step unload-reload engineering stress-strain curve to create 100 true tensile curves for Hollomon fitting. Figure 4.7b shows the evolution of Hollomon parameters $K$ and $n$ in terms of the engineering yield strength. Both $K$ and $n$ decrease as the yield strength increases (Appendix H provides a more detailed explanation of this evolution).

The evolution of the Hollomon parameters is a function of the yield strength and plastic strain, while the total plastic true strain can be related to the truss angle by modifying Equation (4.1):

$$
\varepsilon_{pt}^{true} = \ln(\sec \theta) \quad (4.13).
$$

The above theoretical model allows us to calculate the inelastic buckling strength of the work-hardened microtruss as a function of truss angle, by substituting the Hollomon parameters at different angles into Equation (4.5).

Figure 4.6 Calculated and experimentally-obtained engineering tensile load-reload curves (a), and a magnified comparison (b).
4.2.2 Work-Hardened Microtruss Cores: Experimental Verification

In this section, the predicted compressive strengths of the work-hardened microtrusses are compared to the experimental peak strengths. Fourteen microtrusses with internal truss angle ranging from 21º to 45º were stretch-formed by multiple forming/annealing cycles with 80% deformation before failure at each cycle. Figure 4.8a compares the compressive stress-strain curve of the work-hardened microtrusses with annealed microtrusses at $\theta = 30^\circ$. A ~30% increase in strength is seen due to work-hardening. However, in order to stretch-form a microtruss beyond $\theta = 30^\circ$, an intermediate annealing step is required and each annealing step would reduce the strength. Figure 4.8b shows the comparison between experimental peak strength ($\sigma_{\text{peak}}$) and theoretical compressive strength ($\sigma_{\text{MT}}$) at $k = 1$, 1.43, and 2 (other compressive properties are documented in Appendix E).
The experimental results tend to be closer to the lower bound. This could be due to factors unaccounted for by the model. First, the effects of the difference in critical bifurcation stress and inelastic buckling strength and the possible strut non-uniformity mentioned previously also apply here. Second, the theoretical model created here predicts the compression buckling response using the materials’ tensile behaviour. As each of the individual struts was strengthened by tensile work-hardening, the Bauchinger effect may weaken the compression response through the following mechanism. During tensile prestrain, dislocations pile up along grain boundaries, introducing back stresses. This may induce preferred backward movements of dislocations during stress reversal (compression in our case), lowering the yield strength and introducing a permanent softening effect. Furthermore, the Al₆Mn intermetallic particles in aluminum AA3003 introduce heterogeneity, allowing preferred dislocation movement and amplifying the severity of the Bauchinger effect. Previous studies showed that strain reversal can reduce the effective yield strength by: ~23-47% in stainless steel, ~35-50% in electrolytic copper, and ~12-50% in dispersion-hardened aluminum alloys [73-79]. The Bauchinger effect may also affect the plastic regime in the stress-strain curve; pronounced roundings in the reverse loading curves [79-82] may increase the Hollomon parameter $n$. Finally, the non-uniformly distributed deformation during stretch-forming may result in softer regions right above or below the forming pin. Despite the Bauchinger uncertainty, the theoretical predictions are in close agreement with the experimental results.
4.2.3 **Work-Hardened Microtruss Cores: Architectural Optimization**

The non-dimensional strength ($\bar{F}$) of work-hardened microtrusses can also be calculated by solving Equation (4.8). For the convenience of this discussion, Equation (4.8) is presented again here:

$$
\frac{\bar{F} \cdot \sigma_{ref}}{E \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \cos^4 \theta \cdot \sin \theta} + \frac{\left(\bar{F} \cdot \sigma_{ref}\right)^{1/n}}{n \cdot K^{1/n} \cdot \dot{\alpha} \cdot \dot{\beta}^{1/n} \cdot \cos^{(3+\frac{1}{n})} \theta \cdot \sin^{3/n} \theta} = 1 \quad (4.8)
$$

However, the Hollomon parameters $K$ and $n$ are now functions of angle. Note that high-angle microtrusses are fabricated through multiple forming/annealing cycles, and the degree of work-hardening in each forming cycle can vary. As a result, there are virtually infinite possible manufacturing routines. For illustration purposes, Figure 4.9a shows the calculated $\bar{F}$ assuming that each strut is deformed to 80% of its maximum plastic strain before necking under each forming cycle (and $k = 1$). Figure 4.9b shows $\bar{M}$ of the work-hardened microtrusses.

![Figure 4.9](image)

**Figure 4.9** Calculated $\bar{F}$ (a) and $\bar{M}$ (b) of the work-hardened microtrusses as a function of $\theta$ and $\bar{\epsilon}$.
It is worth noting that work-hardening as an intrinsic strengthening effect that only introduces dislocation line defects at the microscopic level and does not affect the material density at the macroscopic level. As a result, the surface of $\bar{M}$ of the work-hardened microtrusses is identical to that of the annealed microtrusses. However, looking at $\bar{F}$ of the work-hardened microtrusses, not only is it always at a value higher than that of the annealed microtrusses, but it is also a piecewise surface due to the multiple forming/annealing cycles. Each of these discontinuities of the piecewise surface in Figure 4.9a) represents the softening from an annealing cycle. This piecewise surface is indifferentiable, which complicates the optimization process. Also, the optimal architectures are now dependent on the infinite possible manufacturing routes.

Therefore, we simplify the analysis by ignoring the manufacturing routes and only consider the optimal architectures at a given final work-hardened state (defined as the percentage of the true plastic strain deformation during forming versus at the necking point). Figure 4.10 shows the optimal trajectory ($\theta_{min}^{\bar{M}}$) of the microtruss cores starting from the annealed to the fully work-hardened case (using information from Figure 4.7b and satisfying the Karush-Kuhn-Tucker condition). The result of Figure 4.10 is summarized in Figure 4.11a. The optimal trajectory ($\theta_{min}^{\bar{M}}$) range increases from $60^\circ$-$64^\circ$ in the annealed state to $73$-$83^\circ$ in the fully work-hardened state. This means that the optimal architecture and material properties are interdependent. A change in $\theta_{max}^{\bar{F}}$ is also seen as the microtrusses are work-hardened.
The optimal compression properties of these work-hardened microtrusses can be determined by calculating their associated \( \bar{F} \) and \( \bar{M} \) (see Figure 4.11b). The optimal properties at 80\% work-hardening are compared to the experimental work-hardened microtrusses (see Figure 4.12). As the forming angle increases from 21\(^{\circ}\) to 45\(^{\circ}\), the work-hardened microtrusses tested in this study decrease in non-dimensional weight from ~10\% to ~4.5\%, while the non-dimensional strength fluctuates from one forming cycle to another. For a given load, the optimized microtruss cores are significantly lighter than all the experimental results. For instance, the lightest microtruss core fabricated in this study was at a truss angle of 45\(^{\circ}\) with a \( \bar{M} \) of ~4.5\% and a load carrying capability \( \bar{F} \) of ~1\%.

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At this given $F$, the optimal architecture would have 60% less mass. This can be achieved by increasing $\bar{t}$ from 0.15 to 0.2 and $\theta$ from 45° to 75°. High truss angles are strongly desirable for microtrusses in the work-hardened state.

Figure 4.11 Optimal trajectory $\theta_{\text{min}}$ (a) and optimal properties (b) at different work-hardened states.

Figure 4.12 Optimal annealed properties at 80% work-hardened state compared to experimental results.
4.3 Ceramic-Aluminum Microtruss Composites

4.3.1 Ceramic-Aluminum Microtruss Composites: Strength Predictions

The growth of $\text{Al}_2\text{O}_3$ ceramic structural coating through hard anodizing introduces a new degree of freedom to architectural design: coating thickness ($s$). Fundamentally, this single parameter represents the compound effect of the material properties and the distance from strut neutral axis of the ceramic coating. Both the architectural relation in Equation (4.2) and the strength prediction relation in Equation (4.5) for uncoated microtruss cores are no longer valid here.

In terms of architecture, anodic oxides grow inward and expand outward during anodizing. Therefore, the final strut dimensions $w_f$ and $t_f$ in Equation (4.2) are now modified to:

$$w_f = \delta \cdot w \sqrt{\cos \theta} - \frac{2s}{\gamma} \quad (4.14 \ a)$$

$$t_f = \delta \cdot t \sqrt{\cos \theta} - \frac{2s}{\gamma} \quad (4.14 \ b)$$

where $s$ is the anodic coating thickness, $\delta$ is the weight loss factor due to surface cleaning prior to anodizing, and $\gamma$ is the volume expansion factor of the anodic coating. A 2.5% weight loss was observed in Ref. [34] during the cleaning process used in this study; thus, $\delta$ is set as $\sqrt{0.975}$ to account for the cross-sectional thinning during cleaning. The volume expansion factor is taken to be $\gamma = 1.6$, based on the findings of Ref. [44] and the coating thickness measurements at a similar anodizing condition (see Appendix C).

In terms of compression performance, two types of failure mechanisms have been reported for ceramic-aluminum composites based on microscopy results (illustrated in Figure 4.13 for discussion purpose) [34]. At a thin coating thickness (0 to ~20 µm), conventional mid-strut buckling failure was seen. At a higher coating thickness, the failure mechanism changes to involve oxide fracture near the hinge region of the struts. This newly reported failure mechanism has not yet to be fully understood. To understand this mechanism, anodized microtrusses ($\theta = 43^\circ$ and $s = 48$ µm) were partially

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compressed to different strain levels. The axial cross-sections of their internal struts were observed under a scanning electron microscope (SEM). Backscatter scanning electron microscopy (Figure 4.14) shows the evolution of the cross-sections of the anodized microtrusses at different levels of compressive strain: $\varepsilon = 0$ (a), $\varepsilon = 0.24$ (b), $\varepsilon = 0.44$ (c), and $\varepsilon = 0.53$ (d).

Figure 4.13 SEM micrographs showing the overall strut failure mode for an unanodized aluminum core (a) and a ceramic-Al composite at $s = 38.5 \mu m$ (b) loaded to after peak stress [34].

Figure 4.14 Backscatter scanning electron micrographs of the anodized microtrusses at compressive strains of $\varepsilon = 0$ (a), $\varepsilon = 0.24$ (b), $\varepsilon = 0.44$ (c), and $\varepsilon = 0.53$ (d) [72].
Before compression, the hinge region between the node and the strut was seen to have a smaller cross-sectional thickness due to pin-indentation from stretch-forming (Figure 4.14a). As the microtruss is compressed towards (Figure 4.14b) and past its peak strength (Figure 4.14c and 4.14d), a series of oxide fracture sites are seen near this hinge area. These fracture sites continue from the edge of the node and expand along the strut length, while the remaining strut stays at an angle of ~40º (near the starting angle of θ = 43º) throughout the collapse sequence. More detail of the oxide failure can be observed at a higher magnification (see Figure 4.15). The manganese-rich Al₆Mn intermetallics [83] in the AA3003 are shown as elongated white particles. Before compression, these intermetallics are aligned parallel to the forming direction of the strut. As the strut collapses, the anodic coatings penetrate into the core, forcing the aluminum to undergo extensive plastic deformation and causing the second phase particles to follow a wave-like flow pattern (traced by the white arrows). The oxide penetrations also seem to follow a series of folding and fracturing events.

Figure 4.15 Backscatter scanning electron micrographs of the uncrushed (a) and partially crushed (b) ceramic-aluminum microtruss composites [72].
Another set of partially collapsed samples was characterized SEM, in order to examine the brittle fracture or the oxide. The fracture surface of the anodic oxide in nanoscale (Figure 4.16) shows a sequence of straw-like nano-structures, indicating that micro-cracks propagate by cleaving along the cell boundaries of the nano-honeycomb structure, which is consistent with the findings of Ref. [84-86].

![Figure 4.16](image)

Figure 4.16 High magnification micrograph of the fracture surface of the anodic oxide (a). Micro-cracks in normal anodic alumina propagate by cutting across the nano-pores (mode A), while cracks in hard anodic alumina propagate by cleaving along the cell boundaries (mode B) (b); adapted from [86].

Note that the fracture of brittle ceramics requires microscopic tensile stress. In the case of compression, these tensile stresses can only originate from strut/oxide deflection. Figure 4.15 shows that these deflections can be due to the folding of the oxide during compression. FE simulation of these ceramic-aluminum composite struts suggested that the ceramic sleeve deformed through local folding near the hinge region, resulting in a localized tensile stress within the oxide that can lead to brittle fracture [34]. Note that this local folding resembles the conventional tube wrinkling in bending [87-91] or local shell buckling in compression [92-96] (since microtrusses are stretch-dominated structures, the latter seems more probable). As a result, two analytical strength predictions based on composite strut global buckling (at low coating thickness) and oxide local shell buckling (at high coating thickness) are derived here to explain the collapse mechanisms of these ceramic-aluminum microtruss composites.
4.3.1.1 Composite Strut Global Buckling

Under uniaxial compression, conventional metal-metal microtruss composites collapse through inelastic buckling [7, 8, 33]. The collapse strength of a single strut is governed by the modified Shanley inelastic buckling relation (after Refs. [60, 93]):

\[
\sigma_{CR} = \frac{k^2 \pi^2 (E_{T,S} I_S + E_{T,C} I_C)}{(A_S + A_C) l^2} \tag{4.15}
\]

and

\[
\sigma_{CR} = \frac{\sigma_S A_S + \sigma_C A_C}{A_S + A_C} \tag{4.16}
\]

where the subscripts \( S \) and \( C \) represent the sleeve and the core, respectively. Equation (4.15) describes the resistance of the composite strut against deflection and a given amount of total stress. Note that both the tangent modulus of the sleeve \( (E_{T,S}) \) of the metal sleeve and the tangent modulus of the core \( (E_{T,C}) \) of the metal core continuously change with stress of the sleeve \( (\sigma_S) \) and stress of the core \( (\sigma_C) \). Equation (4.16) provides the isostrain relationship between the \( \sigma_S \) and \( \sigma_C \). The critical bifurcation stress is determined when these two equations are satisfied. The strengthening effect from creating a composite is due to the superior strength of the structural coating (represented by \( E_{T,S} \)) and the increase in the second moment of area \( (I_S) \), which occurs because the coatings are positioned further away from the neutral bending axis. In the present case, the increase in second moment of area is due to volume expansion as the aluminum core is progressively oxidized.

This composite strut buckling collapse mechanism is adapted to predict the collapse strength of ceramic-aluminum microtruss composites. Brittle ceramics show little plastic deformation before final fracture; it is assumed that the anodic \( \text{Al}_2\text{O}_3 \) coating in this study elastically deforms until it reaches buckling instability. Thus, the tangent modulus of the sleeve \( (E_{T,S}) \) in Equation (4.15) remains constant at its elastic modulus \( (E_S) \):

\[
\sigma_{CR} = \frac{k^2 \pi^2 (E_S I_S + E_{T,C} I_C)}{(A_S + A_C) l^2} \tag{4.17}
\]

Equation (4.17) is essentially stating that the aluminum core would collapse through inelastic buckling, whereas the ceramic sleeve would collapse through elastic buckling.
The strength of these microtruss composites can only be determined when the elastic modulus of these anodic coatings is known. However, the nano-porosity and amorphous microstructure of these anodic coatings reduce their rigidity from that of their fully dense-crystalline form. Nanoindentation tests were performed in collaboration with the University of Barcelona; the elastic modulus of the coating along the axial direction of the strut was determined to be $E_S = 95.5 \pm 1.9$ GPa (see Appendix D). The stress-strain relationship of the annealed AA3003 can be represented by its elastic modulus and Hollomon parameters ($E_c = 69$ GPa, $K_c = 207$ MPa, and $n_c = 0.275$). The collapse strength of the microtruss ($\sigma_{MT}^{global}$) can be calculated in one single equation by combining Equations (4.14), (4.16), (4.17), and the isostrain condition:

$$\sigma_c = \left\{ \left( \frac{l_c}{2 \cdot l^2 \cdot \pi^2 \cdot \sin \theta \cdot \cos^2 \theta} \cdot \frac{1}{E_c} \cdot n_c \cdot K_c^{1/n_c} \right) \right\} \frac{n_c}{1-n_c} \ (4.18\ a);$$

$$\sigma_s = \frac{\sigma_{MT}^{global} \cdot (l + w)^2}{2 \cdot \sin \theta} - A_c \cdot \sigma_c \ (4.18\ b);$$

$$\frac{\sigma_c}{E_c} + \left( \frac{\sigma_c}{K_c} \right)^{1/n_c} - \frac{\sigma_s}{E_S} = 0 \ (4.18\ c).$$
4.3.1.2 Oxide Local Shell Buckling

We assume that the ceramic coating acts as a hollow tube and collapses through local shell buckling. The instability occurs when the coating have reached the critical local shell buckling strength ($\sigma_S$). Note that the local shell buckling strength of a rectangular cross-sectioned tube with a thin shell wall can be approximated by the buckling of a thin plate [92]. Furthermore, the expansion of the oxide during anodizing leads to oxide discontinuities at the corners of the strut cross-sections. As a result, the oxide strength can be estimated by:

$$\sigma_S = \frac{\pi^2 \cdot E_S}{3 \cdot (1 - \nu^2)} \cdot \left( \frac{s}{w_f} \right)^2 \quad (4.19 \ a)$$

where $\nu$ is the Poisson’s ratio of the anodic $\text{Al}_2\text{O}_3$. Note that the nano-porosity of anodic oxide can affect the elastic constant $\nu$. A detailed analysis in Ref. [97] showed that the Poisson’s ratio ranges from $\nu = 0.23$ to 0.21 for porosity between 5 and 10%; this study assumes $\nu \sim 0.22$. The aluminum core is assumed to deform uniaxially until the strain at which instability occurs. With these assumptions, the strength of the AA3003 core ($\sigma_C$), the critical buckling strength of the composite strut ($\sigma_{CR}$), and the collapse strength of the microtruss ($\sigma_{MT}^{local}$) can be calculated by:

$$\frac{\sigma_C}{E_C} + \left( \frac{\sigma_C}{K_C} \right)^{1/n_C} - \frac{\sigma_S}{E_S} = 0 \quad (4.19 \ b);$$

$$\sigma_{CR} = \frac{\sigma_S A_S + \sigma_C A_C}{A_S + A_C} = \frac{\sigma_{MT}^{local} \cdot (l + w)^2}{2 \cdot \sin \theta} \quad (4.19 \ c).$$
4.3.2 Ceramic-Aluminum Microtruss Composites: Experimental Verification

To evaluate the aforementioned strength predictions, microtruss cores with a range of internal truss angles from 21° to 43° were stretch-formed through multiple forming/annealing cycles, followed by post-fabrication annealing to remove any residual work-hardening. These annealed trusses were then cleaned and hard anodized with various deposition times of 0, 5, 10, 15, and 45 minutes (associated with coating thicknesses of $s = 0, 5, 10, 18, \text{and } 48 \, \mu m$). Figure 4.17 shows the typical compressive stress-stain curves of these anodized microtrusses at $\theta = 43^\circ$. The peak strength progressively increases as coating thickness increases. Notably, a ~400% increase in strength is seen from only a ~50 µm thin ceramic reinforcement on aluminum cores that have a cross-sectional thickness of ~650 µm.

![Figure 4.17 Compressive stress-strain curves of anodized microtrusses (0, 5, 10, 18, and 48 µm) at $\theta = 43^\circ$.](image)

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Figure 4.18 compares the compressive tangent modulus (slope of the stress-strain curves) of these microtrusses. At a low strain level, all microtrusses elastically deform with nearly constant compressive modulus. After a strain of about $\varepsilon = 0.05$, the samples begin to deform plastically and the tangent modulus decreases, eventually dropping below zero. For the uncoated samples, the modulus drops gradually to zero; the first $x$-intercept is the strain when the peak strength is recorded ($\varepsilon_{\text{peak}}$) while the second $x$-intercept is the strain when the valley strength is recorded ($\varepsilon_{\text{valley}}$). Similar behaviour is seen for the 5 µm thick sample. However, a series of sharp fluctuations start to appear for samples at 10 µm or higher. The severity and frequency of these fluctuations also increase as coating thickness increases. At 10 µm, no sharp fluctuation is seen before $\varepsilon_{\text{peak}}$. At 18 µm, sharp fluctuations appear before $\varepsilon_{\text{peak}}$, but do not cross the $x$-intercept until $\varepsilon_{\text{peak}}$. At 48 µm, the modulus passes through large fluctuations as it progressively decreases; the first $x$-intercept is lower than the value of the $\varepsilon_{\text{peak}}$ while the last $x$-intercept is higher than the value of the $\varepsilon_{\text{valley}}$. These modulus fluctuations indicate strength disruptions at the microscopic level. They can either indicate oxide brittle fractures (or coating delaminations), which are consistent with oxide local shell buckling.
Figure 4.18 Compressive modulus curves of anodized microtrusses (0, 5, 10, 18, and 48 µm) at θ = 43°. Blue dots and red dots indicate the recorded peak strains and valley strains, respectively.
Table 4.1 summarizes and Figure 4.19 plots the experimental compressive peak strengths ($\sigma_{\text{peak}}$) of 35 different microtruss composites with different combinations of truss angles and coating thicknesses.

Table 4.1 Experimental Compressive Peak Strength of Ceramic-Aluminum Microtruss Composites.

<table>
<thead>
<tr>
<th>Coating thickness, $s$ [µm]</th>
<th>Peak strength, $\sigma_{\text{peak}}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21°</td>
</tr>
<tr>
<td>0</td>
<td>0.72</td>
</tr>
<tr>
<td>5</td>
<td>1.03</td>
</tr>
<tr>
<td>10</td>
<td>1.33</td>
</tr>
<tr>
<td>18</td>
<td>1.63</td>
</tr>
<tr>
<td>48</td>
<td>2.37</td>
</tr>
</tbody>
</table>

Figure 4.19 Compressive peak strength comparisons between experimental results and theoretical calculations based on global buckling and local shell buckling assumptions.
The experimental peak strengths ($\sigma_{\text{peak}}$) were also compared to the theoretical compressive strengths based on the assumption of composite strut global buckling ($\sigma_{\text{MT}}^{\text{global}}$) at $k = 1.43$ and oxide local shell buckling ($\sigma_{\text{MT}}^{\text{local}}$). For the global buckling assumption, good agreement is seen in the uncoated case, but the theoretical model overestimates the strength as coating thickness increases. As for the local shell buckling assumption, good agreement is seen in the 48 µm coating case, but the theoretical model underestimates the strength as coating thickness decreases. The aforesaid scenarios are superimposed in Figure 4.19. Figure 4.20 illustrates this overestimation of global buckling and underestimation of local shell buckling model by plotting the experimental strengths as functions of coating thickness ($s$). Within the architectural range of this study, the experimental results are always between the global buckling and local shell buckling predictions. The failure mechanism appears to transition from global buckling to local shell buckling, which agrees with the earlier microscopy results in Ref. [34].

Figure 4.20 Compressive peak strength comparisons between experimental results and theoretical calculations as functions of coating thickness. White dots represent experimental results. Blue and red region represents the upper bound and lower bound prediction by assuming global buckling local shell buckling, respectively.
It is worth noting that the transition from global buckling to local shell buckling had been reported in conventional tube compression tests [98-101]. However, Figure 4.20 implies the causes of the transition in conventional tubes versus the studied composite strut could be fundamentally different. In conventional tube compression, global buckling and local shell buckling are competing failure mechanisms, in which the dominate mechanism leads to the lowest possible collapse load, suppressing the characteristics of the other mechanism. In the present case, both the ceramic coating and the aluminum core interact at the material interface simultaneously contributing two distinct failure characteristics: core global buckling and sleeve shell buckling, and the actual failure strength lies between the boundaries of these two extreme cases. As a first attempt to quantify the contributions of these two failure mechanisms, we define the following transition function:

\[ \sigma_{\text{peak}} = W_{\text{global}} \cdot \sigma_{\text{MT}}^{\text{global}} + W_{\text{local}} \cdot \sigma_{\text{MT}}^{\text{local}} \quad (4.20) \]

where \( \sigma_{\text{peak}} \) is the experimentally obtained peak strength; \( W_{\text{global}} \) and \( W_{\text{local}} \) represent the relative weighting of the two failure mechanisms (where \( W_{\text{global}} + W_{\text{local}} = 1 \)). \( W_{\text{global}} \) and \( W_{\text{local}} \) can be determined graphically by looking at the differences between \( \sigma_{\text{peak}} \), \( \sigma_{\text{MT}}^{\text{global}} \), and \( \sigma_{\text{MT}}^{\text{local}} \), i.e.:

\[ W_{\text{global}} = \frac{\sigma_{\text{peak}} - \sigma_{\text{local}}}{\sigma_{\text{MT}}^{\text{global}} - \sigma_{\text{MT}}^{\text{local}}} \quad (4.21 \ a) ; \]

\[ W_{\text{local}} = \frac{\sigma_{\text{MT}}^{\text{global}} - \sigma_{\text{peak}}}{\sigma_{\text{MT}}^{\text{global}} - \sigma_{\text{MT}}^{\text{local}}} \quad (4.21 \ b) . \]
Figure 4.21 plots the $W_{\text{global}}$ and $W_{\text{local}}$ as functions of truss angle ($\theta$) and coating thickness ($s$). Note that each of the nodes on the surfaces represents an experimental data point. With increasing coating thickness, $W_{\text{local}}$ increases and $W_{\text{global}}$ decreases, consistent with the experimental observation in Ref. [34]. As the truss angle increases an opposite trend is seen, although it is not as significant, i.e. $W_{\text{local}}$ decrease and $W_{\text{global}}$ increases. This occurs because the aluminum core is subjected to strut thinning from stretch-forming fabrication. At a higher angle, for a given coating thickness, the coating is closer to the neutral bending axis; therefore, its collapse is similar to a column rather than a hollow tube. Finally, it is worth noting that the points at which $W_{\text{global}} = W_{\text{local}}$ indicate the transition from composite global buckling to oxide local shell buckling. Figure 4.21 shows such a transition occurs at coating thickness of about 10 ~ 20 µm, which agrees with the pervious microscopy results in Ref. [34].
The dependence of $W_{\text{local}}$ caused on the coating thickness can be represented by a simple power-law relation (i.e. $W_{\text{local}} = A \cdot s^B$). The decrease in $W_{\text{local}}$ caused by an increase in angle can also be represented by setting the power-law exponent as a function of angle. As a result, the following two-parameter empirical relation of $W_{\text{global}}$ and $W_{\text{local}}$ can be developed:

\[
W_{\text{local}} = \left(\frac{s}{S_C}\right)^{\sin^2\left(\frac{\pi}{2} \frac{\theta}{\theta_C}\right)} \quad (4.22 \ a);
\]

\[
W_{\text{global}} = 1 - \left(\frac{s}{S_C}\right)^{\sin^2\left(\frac{\pi}{2} \frac{\theta}{\theta_C}\right)} \quad (4.22 \ b)
\]

where $S_C$ is a critical coating thickness (in units of [$\mu$m]) and $\theta_C$ is a critical truss angle (in units of [°]). Empirical fitting showed that $S_C = 44.1 \pm 1.7$ $\mu$m, $\theta_C = 76.3 \pm 2.0^\circ$, with a coefficient of determination of $R^2 = 0.981$. Note that these fitting parameters can be used to describe the transition of failure mechanism. $S_C$ represents the coating thickness at which the transition from global buckling to local shell buckling is complete (i.e. $W_{\text{local}} = 1$), beyond which, collapse is only due to oxide local shell buckling. $\theta_C$ represents the truss angle at which the exponent is equal to one and the transition between global and local buckling progresses at a linear rate (i.e. $W_{\text{local}} = \frac{s}{S_C}$). At any angle lower than 76°, the failure mechanism transition occurs before a coating thickness of 22 $\mu$m ($S_c/2$) is reached. $S_C$ and $\theta_C$ also set the upper bound of the architectural space for which this empirical relation is valid. The compressive strength can then be expressed by combination Equations (4.20) and (4.22). Although this empirical method provides a functional relationship for the transition between the failure mechanisms, predicting beyond this study’s architectural range is not yet possible.
4.3.3 Ceramic-Aluminum Microtruss Composites: Architectural Optimization

While it is not yet possible to fully predict the compressive performance in order to optimize the architecture of these ceramic-aluminum microtruss composites, this section offers a general framework for optimizing these types of microtruss composites. Previously, we established that the optimal architecture is determined when the gradient vector of the non-dimensional strength ($\bar{F}$) is parallel to that of the non-dimensional weight ($\bar{M}$). Anodizing introduces an additional architectural parameter: coating thickness ($s$). To generalize the analysis to all architectural ranges, a non-dimensional parameter ($\bar{s}$) is defined as $\bar{s} = \frac{s}{t} = \frac{s}{w}$ to represent coating thickness. After anodizing, the overall density of the composite is influenced by the bulk density of the anodic coating. The non-dimensional weight $\bar{M}$ can be expressed as:

$$\bar{M} = \frac{\rho_{MT}}{\rho_{ref}} = \frac{V_s \rho_s + V_c \rho_c}{\rho_{ref}} = \frac{(V_{tot} - V_c) \rho_s + V_c \rho_c}{\rho_{ref}} \quad (4.21 \ a)$$

where $\rho_s$ is the density of the sleeve (varies depending on the porosity), $\rho_c$ is the density of the core (2.7 Mg/m$^3$ for aluminum), $V_s$ is the volume fraction of the sleeve, $V_c$ is the volume fraction of the core, and $V_{tot}$ is the total volume fraction of the solid. $V_{tot}$ and $V_c$ can then be expressed as:

$$\bar{V}_{tot} = \frac{2(\sec \theta - 2 \bar{s} \bar{\ell}) \left[ \bar{\ell} \left[ \delta \cos^{1/2} \theta + 2 \bar{s} \left( 1 - \frac{1}{\gamma} \right) \right]^2 + \left[ \bar{\ell} \left( \delta \cos^{1/2} \theta + 2 \bar{s} \left( 1 - \frac{1}{\gamma} \right) \right) \right]^3 \right]}{(1 + \bar{\ell})^2 \left[ \tan \theta + \bar{\ell} + 2 \bar{s} \bar{\ell} \right]} \quad (4.22 \ a);$$

$$\bar{V}_c = \frac{2 \left[ \bar{\ell} \left( \delta \cos^{1/2} \theta + 2 \bar{s} / \gamma \right) \right]^2 + \left[ \bar{\ell} \left( \delta \cos^{1/2} \theta + 2 \bar{s} / \gamma \right) \right]^3 \right]}{(1 + \bar{\ell})^2 \left[ \tan \theta + \bar{\ell} + 2 \bar{s} \bar{\ell} \right]} \quad (4.22 \ b).$$

The non-dimensional strength ($\bar{F}$), however, can only be determined when the failure mechanism of these microtruss composites is fully understood. The next chapter proposes some potential future work to achieve the more accurate prediction of $\bar{F}$.

Hiu Ming (Bosco) Yu
5 Future Work

The architectural optimization of ceramic-aluminum microtruss composites is only possible when their failure mechanism is fully understood. The pervious chapter developed an empirical relationship for expressing the transition between composite strut buckling and oxide local shell buckling. Further analysis is needed to confirm and explain this failure mechanism transition function as well as to extend it beyond the range of architectures considered. This chapter proposes some essential research topics for future investigation.

5.1 The Consideration of Finite Element Analysis (FE)

In general, the modulus and second moment of area of the oxide sleeve has a tendency to prevent the aluminum strut from global buckling, while at the same time, the good adhesion between oxide and core delays local shell buckling. As a result, failure of the composite strut involves a complicated interaction between aluminum core and the anodic oxide, requiring the assistance of Finite Element Analysis (FE). In the current study, the oxide fractured and penetrated into the aluminum core soon after local shell buckling occurred. FE simulation results showed that the local shell buckling of the oxide can result in localized tensile stress followed by oxide brittle fracture [34]. Fracture occurs when the principal tensile stress exceeds its tensile fracture strength ($\sigma_f$), which can be approximated by $\sigma_f = \frac{H \cdot \nu}{c_v}$, after Ref. [102] where $H$ is the hardness, $\nu$ is the Poisson’s ratio, and $c_v$ is a geometrical factor between 2 to 4. Nanoindentation results showed that the anodic oxide in this study had a hardness of $5.6 \pm 0.1$ GPa. Conventional hard anodized coating has a porosity of $5$-$15\%$ [41, 103]; thus, the Poisson’s ratio of the anodic alumina should range from $\nu = 0.21$-$0.23$ [97, 104, 105]. Based on this approximation, the tensile fracture strength of the anodic oxide is expected to be in the range of $\sigma_f = 310$-$640$ MPa. FE analysis also allows the consideration of other microscopic load disruptions such as coating delamination. In the case of nano-Ni/Al, coating delamination was seen near the hinge region [8], similar to the coating local shell buckling mechanism described in this study. Whether the coating fractures or delaminates after local shell buckling may depend on the ductility of the oxide and adhesive strength at the interface which can be modeled in FE simulation.
5.2 Optimization of Other Properties

During this investigation of these ceramic-aluminum microtruss composites, it was found that adding a ceramic coating can substantially increase not only the strength, but also the elastic modulus and the energy absorption. Tables 5.1 and 5.2 summarize the experimentally obtained compressive moduli and energy absorption parameter $J_{densification}$ values (defined as the area under the stress strain curve from half of the peak strength to twice of the peak strength: $J_{densification} = \int_{\varepsilon(\frac{2}{2}\sigma_{peak})}^{\varepsilon(2\sigma_{peak})} \partial \varepsilon \sigma \varepsilon$). A coating only ~50 µm thick induces more than a ten-fold increase in both compressive modulus (from 3 to 36 MPa) and energy absorption (from 280 to 2840 KJ/m$^3$). In the case of energy absorption, it is suspected that the increase is due to oxide folding and fracturing and the extensive plastic deformation of the core during compression (mentioned in Section 4.3). In addition, the remaining length of the strut is kept rigid during failure (rather than buckling in the middle) allowing the stress level to be maintained at an elevated value for a prolonged period of deformation. As a result, there remains a research opportunity to optimize the energy absorption characteristics of these microtruss composites.
Table 5.1 Experimental Compressive Modulus of Ceramic-Aluminum Microtruss Composites.

<table>
<thead>
<tr>
<th>Coating thickness, s [µm]</th>
<th>Compressive modulus, $E$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.17  5.08  7.36  12.38  13.78  14.39  16.58</td>
</tr>
<tr>
<td>5</td>
<td>4.14  6.05  9.33  14.18  15.61  19.39  22.69</td>
</tr>
<tr>
<td>10</td>
<td>6.08  8.09  12.5  17.18  19.16  24.03  23.98</td>
</tr>
<tr>
<td>18</td>
<td>6.51  9.36  11.54  19.72  24.02  24.14  28.01</td>
</tr>
<tr>
<td>48</td>
<td>9.76  14.36  20.5  26.34  31.98  32.38  35.52</td>
</tr>
</tbody>
</table>

Table 5.2 Experimental Energy Absorption of Ceramic-Aluminum Microtruss Composites.

<table>
<thead>
<tr>
<th>Coating thickness, s [µm]</th>
<th>$J_{densification}$ [KJ/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>270  390  580  680  540  510  490</td>
</tr>
<tr>
<td>5</td>
<td>320  530  680  1030  880  940  910</td>
</tr>
<tr>
<td>10</td>
<td>430  670  900  1270  1510  1370  1240</td>
</tr>
<tr>
<td>18</td>
<td>510  840  1000  1540  1900  1770  1810</td>
</tr>
<tr>
<td>48</td>
<td>760  1210  1700  2050  2590  2600  2840</td>
</tr>
</tbody>
</table>
6 Conclusion

The experimental results of the present study demonstrated that stretch-forming introduces intrinsic work-hardening effects and can strengthen AA3003 microtruss cores by ~30%, while a ~50 µm thick anodic ceramic coating can provide a ~400% improvement in strength (with virtually no weight penalty in both cases). The mechanical performance of the annealed microtruss cores, work hardened microtruss cores, and ceramic-aluminum microtruss composites can be further enhanced through architectural optimization. For instance, the compressive strength of the AA3003 stretch-formed microtruss cores examined in the present study is maximized at an internal truss angle ($\theta_{\text{max}}^F$) of ~36º, due to the counteracting effects of load resolution and strut thinning. Analytical derivations showed that these effects are always balanced at $\theta_{\text{max}}^F$ between 26.6º and 45º for any given material property and starting sheet geometry. However, when minimum weight is a design objective, the optimal architecture should be at the internal truss angle ($\theta_{\text{min}}^M$) giving the lightest possible weight for a given load carrying ability. For instance, the $\theta_{\text{min}}^M$ range increases from 60º-62º in the annealed state to 73º-83º in the 80% work-hardened state. In both cases, $\theta_{\text{min}}^M$ is 15º-56.4º higher than $\theta_{\text{max}}^F$. Figure 6.1 compares the experimental compressive performance of the annealed, work-hardened, and $s = 48$ µm coated microtrusses with the optimal performances of the annealed and 80% work-hardened states. In the case of annealed microtruss cores, optimization has the ability to save weight by 3%. Work-hardened microtruss cores tested in this study are in general 20-30% stronger than those that are annealed. Optimization can further improve them by saving up to 60% of their weight. While it is not yet possible to fully predict the compressive performance in order to optimize the architecture of these ceramic-aluminum microtruss composites, their experimental results are almost an order of magnitude stronger than both the annealed and work-hardened microtrusses cores. Therefore, there will be substantial benefit from ultimately determining the lightest possible ceramic-aluminum microtruss composites for a given load.

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Meanwhile, the theoretical predictions of ceramic-aluminum microtruss composites provide some insight into the transition in failure mechanism from global buckling to oxide local shell buckling. While the empirical formula is able, this formula was able to express the behaviour over the range of truss angles and coating thicknesses studied, its broader applicability remains to be seen. Consequently, substantial additional research, such as detailed microscopy and Finite Element Analysis, should be used to develop a more complete instability criterion. Fabricating these ceramic-aluminum microtruss composites at their optimal architectures will result in significant mechanical property improvements, rendering these composites attractive for light-weight, high-performance applications in the automotive, marine, and aerospace industries.

Figure 6.1 Compressive performance of the annealed, work-hardened, and anodized microtrusses with 48 µm thick coating experimental results in this study with the previously optimized annealed and 80% work-hardened microtrusses.
References


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Appendix A: Machine Compliance Correction in Compression Testing

All compression tests were performed in the SHIMADZU AG-I 50kN universal testing machine at a cross-head displacement rate of 1 mm/min. While the reaction force of the tested samples can be accurately measured using the load cell, the load frame itself undergoes elastic deformation, known as machine compliance, making cross-dead displacement calculation inaccurate. The specimen heights of the microtrusses in this study were in the range of ~3 to 5 mm, making the attachment of extensometers or strain gauges difficult. Ref. [56] summaries an alternative method to obtain specimen displacement by correcting machine compliance. At a given load, the total recorded cross-head displacement ($\delta_T$) can be expressed as [56]:

$$\delta_T = \delta_S + \delta_M \quad (A.1)$$

where $\delta_S$ is the sample displacement and $\delta_M$ is the displacement due to the machine compliance. Figure A.1a shows the compressive force-displacement curve of the SHIMADZU AG-I 50kN machine itself, also known as the machine compliance curve. This curve was fit to a bi-linear response with the following equations:

$$F = 36.243\delta_M \quad if \quad F \leq 4.058 \text{ kN} \quad (A.2 \ a);$$

$$F = 63.179 - 3.0163\delta_M \quad if \quad F > 4.058 \text{ kN} \quad (A.2 \ b)$$

where $F$ is the force (in units of kN), and $\delta_S$ is in units of [mm]. Figure A.1b shows an example of subtracting the machine displacement from the recorded compressive force-displacement of a tested sample used in this study ($\theta = 43^\circ$, $s = 48 \mu$m). At a load of $F = 3.5 \text{ kN}$, $\delta_R = 1.17 \text{ mm}$, while $\delta_C = 0.097 \text{ mm}$; thus, $\delta_S = 1.073 \text{ mm}$ and the machine compliance contributes ~8% of the total displacement in this example.
Table A.1 compares the compressive properties of this microtruss before and after machine compliance correction. Note that the strength parameters $\sigma_{\text{peak}}$ and $\sigma_{\text{valley}}$ remain unchanged before and after the correction, since the machine compliance only introduces extra displacement but does not affect the reaction force reading. As the focus of this study is compressive strength optimization, machine compliance does not affect the result of this study. However, changes are seen in parameters related to displacement. The uncorrected compressive modulus underestimates the actual compressive modulus, while the uncorrected densification energy overestimates the actual densification energy.

Table A.1 Compressive Properties of Sample ($\theta = 43^{\circ}$, $s = 48$ $\mu$m) Before and After Compliance Correction.

<table>
<thead>
<tr>
<th></th>
<th>Peak strength, $\sigma_{\text{peak}}$ [MPa]</th>
<th>Peak strain, $\varepsilon_{\text{peak}}$ [mm/mm]</th>
<th>Valley stress, $\sigma_{\text{valley}}$ [MPa]</th>
<th>Valley strain, $\varepsilon_{\text{valley}}$ [mm/mm]</th>
<th>Compressive modulus, $E$ [MPa]</th>
<th>Energy Absorption, $J_{\text{densification}}$ [KJ/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before correction</td>
<td>4.24</td>
<td>0.24</td>
<td>3.19</td>
<td>0.62</td>
<td>30.39</td>
<td>2960</td>
</tr>
<tr>
<td>After correction</td>
<td>4.24</td>
<td>0.22</td>
<td>3.19</td>
<td>0.61</td>
<td>35.52</td>
<td>2840</td>
</tr>
<tr>
<td>Percent difference</td>
<td>0 %</td>
<td>8.3 %</td>
<td>0 %</td>
<td>1.6 %</td>
<td>16.9 %</td>
<td>4.1 %</td>
</tr>
</tbody>
</table>
Appendix B: Machine Compliance Correction in Tensile Testing

A method of measuring strain values from non-standard tensile testing compose by correcting the machine compliance was adopted from Refs. [57, 58]. Compliance is defined as the inverse of the slope of the force-displacement curve (i.e. $C = \delta/F$). Equation (A.1) can be modified to [58]:

$$C_T = C_S + C_M \quad (B.1)$$

where $C_T$ is the total compliance, $C_S$ is the compliance of the sample, and $C_M$ is the displacement of the machine. Within the specified load range (i.e. 0-50 kN for SHIMADZU AG-I 50kN), the machine components elastically deforms. Therefore, the compliance of the machine can be calculated by [58]:

$$C_M = \frac{L}{A} \cdot \left( \frac{1}{E_T} - \frac{1}{E_S} \right) \quad (B.2)$$

where $E_T$ is the apparent modulus before compliance correction and $E_S$ is the elastic modulus of the specimen. $L$ and $A$ are the specimen gauge length and cross-sectional area. Unfortunately, the machine compliance in compression and tension is different since the components in the tensile testing grip may also elastically deform in tensile testing. Therefore, the compliance curve of the machine cannot be determined by the method in Appendix A. A specimen with known elastic modulus (i.e. $E_S = 69$ GPa for AA3003 aluminum alloys) can be used to determine the machine compliance for one specific specimen size. The stress and strain of the specimen can then be measured after calibration and compliance correction, i.e.:

$$\sigma_S = \frac{F}{A} \quad \text{and} \quad \varepsilon_S = \frac{\delta_T - F \cdot C_M}{L} \quad (B.3)$$

The tested material properties of the annealed AA3003 (at 600°C for 60 minutes) subsize coupons were close to those of the AA3003-O in the Atlas of Stress-strain Curve [59]. However, a more accurate technique is to measure the strain at the gauge area directly using three-dimensional digital image correlation technique [106, 107].

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Appendix C: Volume Expansion Factor of the Anodic Oxide

To calculate the volume expansion factor, four aluminum AA3003 sheet were cleaned and then anodized at 15, 30, 45, and 60 minutes using a similar deposition condition in this study (2.8 A/dm$^2$ as opposed to 4 A/dm$^2$). Figure C.1 shows the cross-sectional SEM and the optical micrograph on the uncoated (left) and coated (right) side of the same facesheets.

Figure C.1 Anodic oxide thickness micrograph of 15, 30, 45, and 60 minutes anodized coupons. Left: uncoated surface; right: coated surface.

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Note that the anodic oxide grows inward by consuming the aluminum core. Figure C.2 shows the measured oxide thickness and metal consumption as a function of time. The ratio between the oxide thickness and metal consumption is the volume expansion factor, and it was measured to be $\gamma \sim 1.6$, similar to the findings of Ref. [44]. The oxide deposition rate was 0.85 $\mu$m/min and the metal consumption rate was 0.52 $\mu$m/min.

Figure C.2 Anodic oxide thickness and metal consumption measurements as functions of anodizing times.
Appendix D: Elastic Modulus of the Anodic Oxide

Anodic aluminum oxide coatings from this study were analyzed at the University of Barcelona by nanoindentation to determine the elastic modulus. Nanoindentations were performed on 16 locations of the anodic oxide with 3 measurements at each location (see Figure D.1a). Figure D.1b shows the load-unload force-displacement curves of the alumina coating, Al₂O₃-Al interface, and the Al substrate. Figure D.1c shows the elastic modulus and hardness distribution. The alumina oxide has an elastic modulus of $95.5 \pm 1.9$ GPa, and a hardness of $5.6 \pm 0.1$ GPa, while the aluminum substrate has an elastic modulus of $78.8 \pm 5.0$ GPa, and a hardness of $0.8 \pm 0.0$ GPa.

Figure D.1 Nanoindentation experiment: indentation locations (a), force-displacement curves (b), and modulus and hardness distributions (c).
Appendix E: Summary of Tested Compressive Properties

This study focused on the compressive strength and the optimization of ceramic-aluminum microtruss composites. However, the mechanics of these ceramic-aluminum hybrids is a relatively new topic. There remain research opportunities on other compressive properties. For future researchers’ benefit, all the compressive properties tested in this study are documented here.

AA3003 Anneal Microtruss Cores:

Table E.1 Experimental Compressive Properties of AA3003 Annealed Microtruss Cores.

<table>
<thead>
<tr>
<th>Truss angle, θ [°]</th>
<th>Peak strength, σ_{peak} [MPa]</th>
<th>Peak strain, ε_{peak} [mm/mm]</th>
<th>Valley stress, σ_{valley} [MPa]</th>
<th>Valley strain, ε_{valley} [mm/mm]</th>
<th>Compressive modulus, E [MPa]</th>
<th>Energy Absorption, J_{densification} [KJ/m^3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21°</td>
<td>0.75</td>
<td>0.37</td>
<td>0.7</td>
<td>0.5</td>
<td>2.82</td>
<td>260</td>
</tr>
<tr>
<td>23°</td>
<td>0.83</td>
<td>0.35</td>
<td>0.72</td>
<td>0.56</td>
<td>3.5</td>
<td>340</td>
</tr>
<tr>
<td>25°</td>
<td>0.93</td>
<td>0.33</td>
<td>0.77</td>
<td>0.58</td>
<td>4.2</td>
<td>410</td>
</tr>
<tr>
<td>28°</td>
<td>1.05</td>
<td>0.28</td>
<td>0.85</td>
<td>0.54</td>
<td>5.86</td>
<td>540</td>
</tr>
<tr>
<td>30°</td>
<td>1.13</td>
<td>0.25</td>
<td>0.94</td>
<td>0.49</td>
<td>7.37</td>
<td>640</td>
</tr>
<tr>
<td>33°</td>
<td>1.18</td>
<td>0.19</td>
<td>0.99</td>
<td>20.37</td>
<td>10.71</td>
<td>900</td>
</tr>
<tr>
<td>36°</td>
<td>1.24</td>
<td>0.17</td>
<td>1</td>
<td>0.33</td>
<td>12.1</td>
<td>840</td>
</tr>
<tr>
<td>38°</td>
<td>1.16</td>
<td>0.16</td>
<td>0.81</td>
<td>0.39</td>
<td>11.74</td>
<td>620</td>
</tr>
<tr>
<td>40°</td>
<td>1.16</td>
<td>0.13</td>
<td>0.76</td>
<td>0.35</td>
<td>14.8</td>
<td>590</td>
</tr>
<tr>
<td>41°</td>
<td>1.15</td>
<td>0.12</td>
<td>0.83</td>
<td>0.31</td>
<td>16.45</td>
<td>620</td>
</tr>
<tr>
<td>42°</td>
<td>1.2</td>
<td>0.11</td>
<td>0.73</td>
<td>0.32</td>
<td>18.24</td>
<td>610</td>
</tr>
<tr>
<td>43°</td>
<td>1.16</td>
<td>0.11</td>
<td>0.69</td>
<td>0.33</td>
<td>17.27</td>
<td>580</td>
</tr>
<tr>
<td>44°</td>
<td>1.13</td>
<td>0.09</td>
<td>0.74</td>
<td>0.28</td>
<td>18.54</td>
<td>600</td>
</tr>
<tr>
<td>45°</td>
<td>1.05</td>
<td>0.09</td>
<td>0.59</td>
<td>0.31</td>
<td>16.82</td>
<td>480</td>
</tr>
</tbody>
</table>

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# AA3003 Work-hardened Microtruss Cores:

Table E.2 Experimental Compressive Properties of AA3003 Work-hardened Microtruss Cores.

<table>
<thead>
<tr>
<th>Truss angle, $\theta$ [$^\circ$]</th>
<th>Peak strength, $\sigma_{\text{peak}}$ [MPa]</th>
<th>Peak strain, $\varepsilon_{\text{peak}}$ [mm/mm]</th>
<th>Valley stress, $\sigma_{\text{valley}}$ [MPa]</th>
<th>Valley strain, $\varepsilon_{\text{valley}}$ [mm/mm]</th>
<th>Compressive modulus, $E$ [MPa]</th>
<th>Energy Absorption, $J_{\text{densification}}$ [KJ/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21°</td>
<td>1.06</td>
<td>0.3</td>
<td>0.85</td>
<td>0.51</td>
<td>4.22</td>
<td>380</td>
</tr>
<tr>
<td>23°</td>
<td>1.17</td>
<td>0.27</td>
<td>0.86</td>
<td>0.53</td>
<td>5.66</td>
<td>470</td>
</tr>
<tr>
<td>25°</td>
<td>1.32</td>
<td>0.23</td>
<td>0.87</td>
<td>0.55</td>
<td>7.86</td>
<td>580</td>
</tr>
<tr>
<td>28°</td>
<td>1.52</td>
<td>0.22</td>
<td>1.12</td>
<td>0.48</td>
<td>10.03</td>
<td>780</td>
</tr>
<tr>
<td>30°</td>
<td>1.66</td>
<td>0.18</td>
<td>1.22</td>
<td>0.44</td>
<td>13.55</td>
<td>930</td>
</tr>
<tr>
<td>33°</td>
<td>1.33</td>
<td>0.16</td>
<td>1.06</td>
<td>0.39</td>
<td>12.38</td>
<td>960</td>
</tr>
<tr>
<td>36°</td>
<td>1.5</td>
<td>0.14</td>
<td>1.07</td>
<td>0.32</td>
<td>16.38</td>
<td>1210</td>
</tr>
<tr>
<td>38°</td>
<td>1.64</td>
<td>0.12</td>
<td>1.07</td>
<td>0.3</td>
<td>20.58</td>
<td>1180</td>
</tr>
<tr>
<td>40°</td>
<td>1.21</td>
<td>0.12</td>
<td>0.91</td>
<td>0.29</td>
<td>15.43</td>
<td>740</td>
</tr>
<tr>
<td>41°</td>
<td>1.32</td>
<td>0.11</td>
<td>0.87</td>
<td>0.29</td>
<td>19.43</td>
<td>110</td>
</tr>
<tr>
<td>42°</td>
<td>1.43</td>
<td>0.1</td>
<td>0.76</td>
<td>0.33</td>
<td>22.15</td>
<td>830</td>
</tr>
<tr>
<td>43°</td>
<td>1.15</td>
<td>0.1</td>
<td>0.82</td>
<td>0.24</td>
<td>17.11</td>
<td>670</td>
</tr>
<tr>
<td>44°</td>
<td>1.23</td>
<td>0.09</td>
<td>0.75</td>
<td>0.27</td>
<td>20.68</td>
<td>650</td>
</tr>
<tr>
<td>45°</td>
<td>1.34</td>
<td>0.08</td>
<td>0.71</td>
<td>0.27</td>
<td>23.77</td>
<td>760</td>
</tr>
</tbody>
</table>
## Stretch–formed Ceramic-Aluminum Microtruss Composites:

Table E.3 Experimental Compressive Properties of Ceramic-Aluminum Microtruss Composites.

<table>
<thead>
<tr>
<th>Coating thickness, ( s ) [µm]</th>
<th>Peak strength, ( \sigma_{\text{peak}} ) [MPa]</th>
<th>Truss angle, ( \theta ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 µm</td>
<td>0.72   0.89  1.07  1.07  1.01  1.07  1.06</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>5 µm</td>
<td>1.03   1.25  1.37  1.52  1.63  1.51  1.66</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>10 µm</td>
<td>1.33   1.64  1.82  2.13  2.26  2.13  2.16</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>18 µm</td>
<td>1.63   2.01  2.25  2.74  2.85  2.87  2.90</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>48 µm</td>
<td>2.37   2.92  3.59  4.05  4.31  4.22  4.24</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coating thickness, ( s ) [µm]</th>
<th>Peak strain, ( \varepsilon_{\text{peak}} ) [mm/mm]</th>
<th>Truss angle, ( \theta ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 µm</td>
<td>0.36   0.29  0.23  0.15  0.13  0.12  0.1</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>5 µm</td>
<td>0.34   0.29  0.21  0.16  0.12  0.11  0.11</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>10 µm</td>
<td>0.31   0.28  0.21  0.18  0.14  0.16  0.13</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>18 µm</td>
<td>0.34   0.28  0.27  0.22  0.18  0.16  0.17</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>48 µm</td>
<td>0.28   0.24  0.25  0.27  0.27  0.23  0.22</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coating thickness, ( s ) [µm]</th>
<th>Valley stress, ( \sigma_{\text{valley}} ) [MPa]</th>
<th>Truss angle, ( \theta ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 µm</td>
<td>0.67   0.66  0.81  0.76  0.68  0.68  0.58</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>5 µm</td>
<td>0.99   0.96  1.07  1.05  0.82  0.77  0.73</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>10 µm</td>
<td>1.23   1.2   1.42  1.56  1.33  1.15  1.18</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>18 µm</td>
<td>1.46   1.53  1.37  2.05  2.23  1.57  1.97</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
<tr>
<td>48 µm</td>
<td>2.1    2.16  2.35  2.23  3.29  3.12  3.19</td>
<td>21°   25°  30°  35°  39°  41°  43°</td>
</tr>
</tbody>
</table>
Table E.4 Experimental Compressive Properties of Ceramic-Aluminum Microtruss Composites (continuous).

<table>
<thead>
<tr>
<th>Coating thickness, ( s ) [µm]</th>
<th>Valley strain, ( \varepsilon_{\text{valley}} ) [mm/mm]</th>
<th>Truss angle, ( \theta ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 µm</td>
<td>0.44 0.57 0.46 0.39 0.35 0.29 0.33</td>
<td>21° 25° 30° 35° 39° 41° 43°</td>
</tr>
<tr>
<td>5 µm</td>
<td>0.42 0.55 0.41 0.34 0.31 0.33 0.33</td>
<td></td>
</tr>
<tr>
<td>10 µm</td>
<td>0.45 0.56 0.46 0.38 0.32 0.29 0.28</td>
<td></td>
</tr>
<tr>
<td>18 µm</td>
<td>0.46 0.55 0.52 0.49 0.38 0.34 0.34</td>
<td></td>
</tr>
<tr>
<td>48 µm</td>
<td>0.43 0.55 0.64 0.7 0.63 0.61 0.61</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coating thickness, ( s ) [µm]</th>
<th>Compressive modulus, ( E ) [MPa]</th>
<th>Truss angle, ( \theta ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 µm</td>
<td>3.17 5.08 7.36 12.38 13.78 14.39 16.58</td>
<td>21° 25° 30° 35° 39° 41° 43°</td>
</tr>
<tr>
<td>5 µm</td>
<td>4.14 6.05 9.33 14.18 15.61 19.39 22.69</td>
<td></td>
</tr>
<tr>
<td>10 µm</td>
<td>6.08 8.09 12.5 17.18 19.16 24.03 23.98</td>
<td></td>
</tr>
<tr>
<td>18 µm</td>
<td>6.51 9.36 11.54 19.72 24.02 24.14 28.01</td>
<td></td>
</tr>
<tr>
<td>48 µm</td>
<td>9.76 14.36 20.5 26.34 31.98 32.38 35.52</td>
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</table>

<table>
<thead>
<tr>
<th>Coating thickness, ( s ) [µm]</th>
<th>Energy Absorption, ( J_{\text{densification}} ) [KJ/m³]</th>
<th>Truss angle, ( \theta ) [°]</th>
</tr>
</thead>
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<tr>
<td>0 µm</td>
<td>270 390 580 680 540 510 490</td>
<td>21° 25° 30° 35° 39° 41° 43°</td>
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<tr>
<td>5 µm</td>
<td>320 530 680 1030 880 940 910</td>
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</tr>
<tr>
<td>10 µm</td>
<td>430 670 900 1270 1510 1370 1240</td>
<td></td>
</tr>
<tr>
<td>18 µm</td>
<td>510 840 1000 1540 1900 1770 1810</td>
<td></td>
</tr>
<tr>
<td>48 µm</td>
<td>760 1210 1700 2050 2590 2600 2840</td>
<td></td>
</tr>
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</table>
Appendix F: Truss Angle with Maximum Compressive Strength for a Given Starting Sheet Geometry

Section 4.1.3 shows that the angle ($\theta_{\text{max}}^F$) which maximizes the non-dimensional strength ($F$) at a given $\bar{t}$ can be determined by setting $\partial F / \partial \theta = 0$. Here the author details the proof to include non-square strut cross-sections by determining the angle ($\theta_{\text{max}}^s$) which maximizes the compressive strength ($\sigma_{MT}$) for any starting sheet geometry ($w$, $t$, and $l$).

Elastic-Plastic Boundary:

The strength ($\sigma_{MT}$) of a stretch-formed microtruss is related to its architecture:

$$\sigma_{MT} = N_{\text{strut}} \cdot \sigma_{CR} \cdot \left( \frac{\Lambda_{\text{strut}}}{\Lambda_{\text{cell}}} \right) \sin \theta = 2 \cdot \sigma_{CR} \cdot \frac{w \cdot t}{(l + w)^2} \cos \theta \cdot \sin \theta \quad (G1)$$

where $\sigma_{CR}$ is the critical collapse strength of a single strut. Under elastic buckling, the critical collapse strength follows the Euler buckling relation [61], where $L/r$ is the strut slenderness ratio and can be expressed as:

$$\frac{L}{r} = \frac{\sqrt{12} \cdot l_f}{t_f} = \sqrt{12} \cdot \left( \frac{l}{t} \right) \cdot \sec^{3/2} \theta \quad (G2).$$

Therefore, the Euler buckling relation can be rewritten as:

$$\sigma_{CR} = \frac{k^2 \pi^2}{12} \cdot \left( \frac{t}{l} \right)^2 \cdot E \cdot \cos^3 \theta \quad (G3).$$

Equations (G2) and (G3) can be combined to calculate the elastic buckling strength of a microtruss ($\sigma_{MT}$):

$$\sigma_{MT} = 2 \cdot \frac{k^2 \pi^2}{12} \cdot \left( \frac{t}{l} \right)^2 \cdot E \cdot \frac{w \cdot t}{(l + w)^2} \cdot \cos^4 \theta \cdot \sin \theta \quad (G4).$$
Under plastic deformation, the critical collapse strength is estimated by the material’s ultimate tensile strength ($\sigma_{UTS}$), allowing $\sigma_{MT}$ to be expressed as:

$$\sigma_{MT} = 2 \cdot \sigma_{UTS} \cdot \frac{w \cdot t}{(l + w)^2} \cdot \cos \theta \cdot \sin \theta \quad (G5)$$

The angle at which the compressive strength is maximized ($\theta_{\sigma max}^\theta$) for a given starting sheet geometry can be calculated by the condition $\partial \sigma_{MT} / \partial \theta = 0$. Using Equation (G4), $\theta_{\sigma max}^\theta$ for a failure mechanism of elastic buckling is:

$$\cos^2 \theta - 4 \sin^2 \theta = 0 \quad ; \quad \tan \theta = \frac{1}{2} \quad ; \quad \theta_{\sigma max}^\theta = 26.6^\circ$$

According the Equation (A5), $\theta_{\sigma max}^\theta$ for plastic deformation is:

$$\cos^2 \theta - \sin^2 \theta = 0 \quad ; \quad \tan \theta = 1 \quad ; \quad \theta_{\sigma max}^\theta = 45^\circ$$

Note that the plastic deformation approach assumes that the material has a perfectly plastic stress-strain curve with a tangent modulus of zero. Metals have stress-strain behaviour with a tangent modulus lower than the elastic modulus but still above zero. The elastic buckling and plastic deformation approaches provide the upper bound and lower bound of $\theta_{\sigma max}^\theta$. Consequently, for any given starting metal, $\theta_{\sigma max}^\theta$ is always between $26.6^\circ$ and $45^\circ$.

**Inelastic Buckling:**

Under inelastic buckling, $\theta_{\sigma max}^\theta$ can be calculated using Equation (4.5) and setting its derivative $\partial \sigma_{MT} / \partial \theta = 0$:

$$\frac{\sigma_{MT}}{E \cdot \alpha \cdot \beta \cdot \cos^4 \theta \cdot \sin \theta} + \frac{\sigma_{MT}^{1/n}}{n \cdot K^{1/n} \cdot \alpha \cdot \beta^{1/n} \cdot \cos^{(3+\frac{1}{n})} \theta \cdot \sin^{1/n} \theta} = 1 \quad (G6)$$

while the maximum strength is achieved when the following expression is satisfied:

$$\frac{\sigma_{MT}(\cos^2 \theta - 4 \sin^2 \theta)}{E \cdot \alpha \cdot \beta \cdot \cos^5 \theta \cdot \sin^2 \theta} + \frac{\sigma_{MT}^{1/n}[\cos^2 \theta - (3n + 1) \sin^2 \theta]}{n^2 \cdot K^{1/n} \cdot \alpha \cdot \beta^{1/n} \cdot \cos^{(4+\frac{1}{n})} \theta \cdot \sin^{(1+\frac{1}{n})} \theta} = 0 \quad (G7)$$
While the latter expression is somewhat complicated, it can be satisfied in the unique condition when \((\cos^2\theta - 4\sin^2\theta)\) is negative and \([\cos^2\theta - (3n + 1)\sin^2\theta]\) is positive (and vice versa). This allows us to express the boundary of \(\theta_{max}^\sigma\) as:

\[
4\sin^2\theta_{max}^\sigma > \cos^2\theta_{max}^\sigma > (3n + 1)\sin^2\theta_{max}^\sigma \quad (G8\ a);
\]

or

\[
\frac{1}{\sqrt{4}} > \tan\theta_{max}^\sigma > \frac{1}{\sqrt{3n+1}} \quad (G8\ b).
\]

The following shows how to determine the angle that satisfies the aforesaid expression \(\theta_{max}^\sigma\):

\[
A = \frac{1}{E \cdot \alpha \cdot \beta \cdot \cos^4\theta \cdot \sin\theta}; \quad B = \frac{1}{n \cdot K^{1/n} \cdot \alpha \cdot \beta^{1/n} \cdot \cos\left(\frac{3+1}{n}\right) \theta \cdot \sin^{1/n}\theta}
\]

\[
C = \frac{(\cos^2\theta - 4\sin^2\theta)}{E \cdot \alpha \cdot \beta \cdot \cos^5\theta \cdot \sin^2\theta}; \quad D = \frac{[\cos^2\theta - (3n + 1)\sin^2\theta]}{n^2 \cdot K^{1/n} \cdot \alpha \cdot \beta^{1/n} \cdot \cos\left(\frac{4+1}{n}\right) \theta \cdot \sin^{(1+1)/n}\theta}
\]

\[
A\sigma_{MT} + B\sigma_{MT}^{1/n} = 1; \quad C\sigma_{MT} + D\sigma_{MT}^{1/n} = 0
\]

By solving Equation (G9), it is found that \(\sigma_{MT} = \left(\frac{D}{AD - BC}\right)\) and \(\sigma_{MT}^{1/n} = \left(\frac{C}{BC - AD}\right)\). Since \((\sigma_{MT}^{1/n})^n - \sigma_{MT} = 0\), an expression without \(\sigma_{MT}\) is derived to determine the \(\theta_{max}^\sigma\), i.e.:

\[
\left(\frac{C}{BC-AD}\right)^n - \left(\frac{D}{AD - BC}\right) = 0, \quad \text{with a boundary condition of } \tan^{-1}\frac{1}{\sqrt{4}} < \theta_{max}^\sigma < \tan^{-1}\frac{1}{\sqrt{(3n+1)}}.
\]

Note that this boundary converges with the boundary in the elastic buckling and plastic yielding approach when \(n = 0\) and 1. Therefore, for any starting sheets geometry with any metal system, \(\theta_{max}^\sigma\) can only be between 26.6º and 45º.
Appendix G: Gradient Vector of Non-Dimensional Strength and Non-Dimensional Weight

Only in this section, $\bar{F}$, $\bar{M}$, and $\bar{t}$ are referred to as $F$, $M$, and $t$ for convenience. Mathematical operations were performed using Maple, and the results are summarized as follows:

$$\frac{\partial \bar{M}}{\partial \theta} = -\frac{\rho_c(2t^2 + t^3)(1 + \tan^2 \theta)}{\rho_{ref}(1 + t)^2(\tan \theta + t)^2}$$

$$\frac{\partial \bar{M}}{\partial \bar{t}} = \frac{\rho_c(4t + 3t^2)}{\rho_{ref}(1 + t)^2(\tan \theta + t)} - \frac{2\rho_c(2t^2 + t^3)}{\rho_{ref}(1 + t)^3(\tan \theta + t)} - \frac{\rho_c(2t^2 + t^3)}{\rho_{ref}(1 + t)^2(\tan \theta + t)^2}$$

$$\frac{\partial \bar{F}}{\partial \theta} =$$

$$\frac{\partial \bar{F}}{\partial \bar{t}} =$$

Hiu Ming (Bosco) Yu
Appendix H: The Evolution of Holloman Parameters under Work-Hardening

In section 4.2.1, multiple true stress-strain curves were estimated from a single engineering stress-strain curve in order to determine the evolution of the Hollomon parameters $K$ and $n$. Here, a more in-depth analysis is shown to explain the behaviour of the decreasing $K$ and $n$ as the yield strength ($\sigma_{ys}$) increases. Figure I.1a shows the multi-tensile curve once again. Figure I.1b shows the same plot but in logarithmic scale.

![Multi-tensile curves created from a single stress-strain for Hollomon fitting (a) and multi-tensile curves in logarithmic scale (b).](image)

Using a logarithmic scale, the stress-strain curves can be approximated with bi-linear behaviour: an elastic region and a plastic region with the yield strength as the transitional point. Note the Hollomon power law ($\sigma = Ke^n$) can also be written as:

$$\ln \sigma = \ln K + n \ln \varepsilon \quad (I1)$$

where $\ln(K)$ is the $x$-intercept in Figure I.2b, and $n$ is the slope of the line. In the figure, we can see that as work-hardening progresses from curve 1 to 5, the yield strength increases. The Hollomon work-hardening exponent $n$ decreases mainly due to a decrease in the tangent modulus. The Hollomon work-hardening coefficient $K$ also decreases as shown from the $x$-intercepts. This is due to the increase in yield strength ($\sigma_{ys}$).
To illustrate this behaviour further, an example is picked by assuming that Hollomon fitting starts at the yield strength, where the yield strength is also defined as the proportional limit such that the Hollomon relation \( \sigma = K\varepsilon^n \) and Hooke’s Law \( \sigma = E\varepsilon \) would join at the proportional limit:

\[
\sigma_{ys} = K \left( \frac{\sigma_{ys}}{E} \right)^n \quad (12\ a);
\]

\[
\text{or } \ln K = n \cdot \ln E + (1 - n) \cdot \ln \sigma_{ys} \quad (12\ b)
\]

Note that the Hollomon work-hardening exponent \( n \) can only fall between 0 (perfect plasticity) and 1 (perfect elasticity). According to Equation (12), at \( n = 1 \):

\[ K = E = 69,000 \text{ MPa.} \]

At \( n = 0.5 \):

\[ K = e^{\ln \sigma_{ys} \cdot E} = \sqrt{\sigma_{ys} \cdot E} = \sqrt{47 \times 69,000} \text{ to } \sqrt{110 \times 69,000} = 1,800 \text{ to } 2,755 \text{ MPa} \]

At \( n = 0 \):

\[ K = \sigma_{ys} = \sigma_{UTS} = 110 \text{ MPa.} \]

In fact, the mathematical boundaries of the Hollomon work-hardening coefficient \( K \) are the ultimate tensile strength (110 MPa) and the elastic modulus (69,000 MPa). As \( n \) decreases and the yield strength increases, the Hollomon work-hardening coefficient \( K \) decreases from a value higher than \( \sigma_{UTS} \), to \( \sigma_{UTS} \) when the material is fully hardened.